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UMI
R-CARD FENCE EDGE TREATMENT FOR COMPACT RANG REFLECTORS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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ABSTRACT

Popular edge treatments that are used for compact range reflectors are usually expensive. A new cost-effective reflector edge treatment is presented in this dissertation. The basic concept of the new treatment is using resistive sheets (R-cards) as a fence in front of the reflector edge to reduce the edge diffracted fields in the test zone area. This fence can be a new source of stray signal unless designed properly. Therefore, the resistance of this fence varies depending on the distance from the reflector edge. The proposed design approach starts from the 2D case where the fence geometry parameters can be found for optimum performance. The resistance of the R-cards is synthesized according to an optimum tapered aperture distribution. The complexity of the 3D problem is resolved by decomposing the problem in simpler 2D cases. Rectangular and circular rim reflectors will be targeted as examples in that they represent common rim shapes but different design approaches. The success of the R-card fence design will be demonstrated by simulated results and will be experimentally verified as well.
To my family
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iii</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>iv</td>
</tr>
<tr>
<td>Vita</td>
<td>v</td>
</tr>
<tr>
<td>List of Tables</td>
<td>ix</td>
</tr>
<tr>
<td>List of Figures</td>
<td>x</td>
</tr>
<tr>
<td>Chapters:</td>
<td></td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Organization of this Dissertation</td>
<td>4</td>
</tr>
<tr>
<td>2. R-cards and Analytic Techniques</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Introduction to R-cards</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Reflection and Transmission Coefficients of the R-card</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1 Parallel (TE) polarization</td>
<td>6</td>
</tr>
<tr>
<td>2.2.2 Perpendicular (TM) polarization</td>
<td>8</td>
</tr>
<tr>
<td>2.3 Iterative Physical Optics (IPO)</td>
<td>14</td>
</tr>
<tr>
<td>2.3.1 2D case</td>
<td>15</td>
</tr>
<tr>
<td>2.3.2 3D case</td>
<td>16</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>3.</td>
<td>R-card Design for the 2D Problem</td>
</tr>
<tr>
<td>3.1</td>
<td>Ray tracing</td>
</tr>
<tr>
<td>3.2</td>
<td>Aperture Study</td>
</tr>
<tr>
<td>3.3</td>
<td>Design Example</td>
</tr>
<tr>
<td>3.3.1</td>
<td>One R-card/edge at a time</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Down Range Performance</td>
</tr>
<tr>
<td>4.</td>
<td>R-CARD DESIGN FOR A 3D REFLECTOR WITH A RECTANGULAR RIM</td>
</tr>
<tr>
<td>4.1</td>
<td>R-card Design for a 3D Center-Fed Reflector</td>
</tr>
<tr>
<td>4.1.1</td>
<td>2D and 3D Reflector Strips</td>
</tr>
<tr>
<td>4.1.2</td>
<td>The Whole 3D Reflector</td>
</tr>
<tr>
<td>4.1.3</td>
<td>A Drawback</td>
</tr>
<tr>
<td>4.2</td>
<td>R-card Design for a 3D Offset Reflector</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Reflector Edge Diffraction</td>
</tr>
<tr>
<td>4.2.2</td>
<td>3D Reflector Strips</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Assembly of the R-card Segments</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Synthesis of the R-card Resistance</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Final Results</td>
</tr>
<tr>
<td>4.3</td>
<td>Summary</td>
</tr>
<tr>
<td>5.</td>
<td>R-CARD FENCE DESIGN FOR A CIRCULAR RIM COMPACT RANGE REFLECTOR</td>
</tr>
<tr>
<td>5.1</td>
<td>R-card Resistance Discretization and 2D Design</td>
</tr>
<tr>
<td>5.1.1</td>
<td>R-card Resistance Discretization</td>
</tr>
<tr>
<td>5.1.2</td>
<td>2D Offset Problem</td>
</tr>
<tr>
<td>5.1.3</td>
<td>2D Center-fed Problem</td>
</tr>
<tr>
<td>5.2</td>
<td>3D Geometry of the R-cards</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Shaped R-cards</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Flat R-cards</td>
</tr>
<tr>
<td>5.3</td>
<td>R-card Resistance Discretization</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Straight stair steps</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Quasi-circular stair steps</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Diffraction and caustic</td>
</tr>
<tr>
<td>5.4</td>
<td>Summary</td>
</tr>
</tbody>
</table>
6. MEASURED RESULTS ........................................................ 142
   6.1 Measurement Set-up ....................................................... 143
      6.1.1 Place ..................................................................... 144
      6.1.2 Materials and tools .................................................. 144
      6.1.3 Time ...................................................................... 145
      6.1.4 Construction ........................................................... 145
   6.2 Measured Data ............................................................... 146
   6.3 Summary ...................................................................... 153

7. Conclusion ....................................................................... 162

Appendices:
A. R-card Resistance Synthesis ................................................ 166
   A.1 Upper R-card .............................................................. 166
   A.2 Lower R-card .............................................................. 169
B. R-Card Segment Arrangement in Front of the Reflector Edge .... 172
C. R-Card Patching ............................................................... 176
D. Locating a Point with Respect to a Polygon ......................... 180
E. A Ring Encloses a Line ...................................................... 182
F. PATCHING A REFLECTOR OF CIRCULAR APERTURE .......... 185

Bibliography ....................................................................... 189
LIST OF TABLES

Table | Page
--- | ---
4.1 Upper R-card geometry parameters. Distances are in meters | 40
4.2 R-card geometric parameters for a 3D center-fed reflector. Dimensions are in meters | 57
4.3 R-card parameters for the vertical reflector strips. Dimensions are in meters | 75
4.4 R-card parameters for the horizontal reflector strip. Dimensions are in meters | 77
4.5 Coordinates of the R-card segments. Dimensions are in meters | 81
4.6 Aperture parameters for $D = 1.8$ reflector. Dimensions are in meters | 88
5.1 Values of the resistive sheets in the R-card stack placed in order | 107
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>TL model for R-card sheet in case of TE polarization.</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>TL model for R-card sheet in case of TM polarization.</td>
<td>9</td>
</tr>
<tr>
<td>2.3</td>
<td>R-card reflection coefficient versus angle of incidence for different values of resistance (TE).</td>
<td>12</td>
</tr>
<tr>
<td>2.4</td>
<td>R-card transmission coefficient versus angle of incidence for different values of resistance (TE).</td>
<td>12</td>
</tr>
<tr>
<td>2.5</td>
<td>R-card reflection coefficient versus angle of incidence for different values of resistance (TM).</td>
<td>13</td>
</tr>
<tr>
<td>2.6</td>
<td>R-card transmission coefficient versus angle of incidence for different values of resistance (TM).</td>
<td>13</td>
</tr>
<tr>
<td>2.7</td>
<td>Mutual current formulation for IPO-TE where $J_1$ generates current $J_2$ at point 2.</td>
<td>14</td>
</tr>
<tr>
<td>2.8</td>
<td>Mutual current formulation for IPO-TM where $J_1$ generates current $J_2$ at point 2.</td>
<td>16</td>
</tr>
<tr>
<td>2.9</td>
<td>Calculation of the PO equivalent current $\vec{J}$ on an R-card at the point of incidence P. The incident fields have to be analyzed into two components, parallel and perpendicular to the plane of incidence.</td>
<td>20</td>
</tr>
<tr>
<td>3.1</td>
<td>The R-card geometry controls the direction of the reflected rays.</td>
<td>25</td>
</tr>
<tr>
<td>3.2</td>
<td>The effect of the R-card geometry on directing the reflector scattered rays.</td>
<td>26</td>
</tr>
</tbody>
</table>
3.3 The effect of the R-card geometry on steering the triple-reflected rays. 27
3.4 Prototype GO tapered aperture distribution. .............................. 28
3.5 Different edge taper profiles. .................................................... 29
3.6 Diffracted fields in the test zone for different GO profiles with a 10 dB edge taper. ................................................................. 30
3.7 R-card resistance synthesized based on two transmission coefficients. 34
3.8 Geometry of the design example: an offset reflector with two well designed R-cards as reflector edge treatment. ............................. 35
3.9 Test zone fields due to each isolated R-card alone at 3, 6 and 12 GHz for the TM case. ................................................................. 36
3.10 Magnitude and phase of the test zone fields at 3, 6 and 12 GHz for the TM case. ................................................................. 37
3.11 Magnitude and phase of the down range test zone fields at 3, 6 and 12 GHz for the TM case. ................................................................. 38
4.1 Geometry of the designed R-cards for the 2D reflector. ................. 44
4.2 Geometry of the R-cards for 3D reflector strip. ............................ 44
4.3 A side-view for the R-cards and the 3D reflector strip which is similar to the 2D geometry shown in Figure 4.1. ........................... 45
4.4 The resistance distribution of the R-cards. .................................. 45
4.5 Test zone fields at 3 GHz for the 2D configuration (second iteration). 46
4.6 Test zone fields at 3 GHz for the 3D configuration (second iteration). 46
4.7 Test zone fields at 4 GHz for the 2D configuration (second iteration). 47
4.8 Test zone fields at 4 GHz for the 3D configuration (second iteration). 47
4.9 Test zone fields at 3 GHz after the first IPO iteration for the 2D configuration. ........................................ 48
4.10 Test zone fields at 3 GHz after the first IPO iteration for the 3D configuration. ........................................... 48
4.11 Test zone fields at 4 GHz after the first IPO iteration for the 2D configuration. ........................................... 49
4.12 Test zone fields at 4 GHz after the first IPO iteration for the 3D configuration. ........................................... 49
4.13 Test zone IPO field components produced by the 2D reflector at 3 GHz. ................................................. 50
4.14 Test zone IPO field components produced by the 3D reflector at 3 GHz. ................................................. 50
4.15 Test zone IPO field components produced by the 2D lower R-card at 3 GHz. ........................................... 51
4.16 Test zone IPO field components produced by the 3D lower R-card at 3 GHz. ........................................... 51
4.17 Test zone IPO field components produced by the 2D reflector at 4 GHz. ................................................. 52
4.18 Test zone IPO field components produced by the 3D reflector at 3 GHz. ................................................. 52
4.19 Test zone IPO field components produced by the 2D lower R-card at 4 GHz. ........................................... 53
4.20 Test zone IPO field components produced by the 3D lower R-card at 4 GHz. ........................................... 53
4.21 Trapezoidal R-card geometry. .................................................................................................................. 57
4.22 Coordinate geometry to place an R-card in its proper position. Both translation and rotation are performed. ........................................................................................................ 58
4.23 Geometry of a 3D center-fed reflector with four trapezoidal R-cards. ......................................................... 59
4.24 Pre-assumed GO aperture distribution with a 20 dB Kaiser-Bessel taper. .................................................. 64
4.25 Backward ray tracing to synthesize the R-card resistance point by point. 64
4.26 Resistance distribution of the trapezoidal R-cards. 65
4.27 Test zone fields at 3 GHz in a vertical central cut after the first IPO iteration for the 3D configuration. 66
4.28 Test zone fields at 3 GHz in a vertical central cut for the 3D configuration (second iteration). 66
4.29 Test zone fields at 4 GHz in a vertical central cut after the first IPO iteration for the 3D configuration. 67
4.30 Test zone fields at 4 GHz in a vertical central cut for the 3D configuration (second iteration). 67
4.31 Test zone IPO field components produced by the 3D reflector at 3 GHz. 68
4.32 Test zone IPO field components produced by the 3D lower R-card at 3 GHz. 68
4.33 Test zone IPO field components produced by the 3D reflector at 4 GHz. 69
4.34 Test zone IPO field components produced by the 3D lower R-card at 4 GHz. 69
4.35 Test zone fields at 4 GHz for the 2D configuration in case of TE polarization. 70
4.36 Test zone fields at 4 GHz in a vertical central cut for the 3D reflector fed by a vertically polarized feed. 70
4.37 Cones of diffracted rays for a 3D offset reflector with a square rim shape. 73
4.38 Contamination of the test zone field by diffracting rays emanating from the reflector edges. 74
4.39 Geometry of a vertical strip of an offset reflector with two R-cards. 77
4.40 Test zone fields at 3 GHz in a vertical central cut for the vertical reflector strip. 78
5.2 Stair step discretization for the upper R-card resistance distribution treating the 2D offset reflector. .......................... 109

5.3 Stair step discretization for the lower R-card resistance distribution treating the 2D offset reflector. .......................... 109

5.4 Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D offset reflector shown in Figure 5.1 at 3, 6 and 12 GHz for the TM polarization. Note that the stair step approximation of the R-card resistance has an insignificant effect on the performance. 110

5.5 Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D offset reflector shown in Figure 5.1 at 3, 6 and 12 GHz for the TE polarization. Note that the stair step approximation of the R-card resistance has an insignificant effect on the performance. 111

5.6 Geometry of the 2D center-fed reflector with R-card fence. ............... 113

5.7 Stair step discretization for the upper R-card resistance distribution treating the center-fed reflector. .......................... 114

5.8 Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D center-fed reflector shown in Figure 5.6 at 3, 6 and 12 GHz for the TM polarization. Both continuous and stair stepped R-card resistance are used. .......................... 115

5.9 Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D center-fed reflector shown in Figure 5.6 at 3, 6 and 12 GHz for the TE polarization. Both continuous and stair stepped R-card resistance are used. .......................... 116

5.10 A sketch showing the idea of shaping the R-cards in front of the reflector rim. The R-cards uniformly twist between the two given lines, $A_1B_1$ and $A_2B_2$, along a quarter of the circular rim after being transformed to a straight line. .......................... 117

5.11 Shaping the R-cards all around the reflector edge to follow its circular shape. The shape starts from the 2D design for the vertical and horizontal cuts. .......................... 119
5.12 Eight shaped R-cards following the reflector rim while satisfying the 2D design in the vertical and horizontal cuts. ...................................... 121

5.13 Geometry of the four R-cards obtained from the 2D to cover the basic vertical and horizontal cuts. ......................................................... 122

5.14 Sixteen flat triangular R-cards following the reflector rim while satisfying the 2D design in the vertical and horizontal cuts. ................. 123

5.15 The idea of Method B to obtain the geometry of the flat R-cards on the diagonals. ................................................................. 124

5.16 Eight flat R-cards following the reflector rim while satisfying the 2D design in the vertical and horizontal cuts. ......................... 127

5.17 Geometry of the straight stair stepped R-card resistance. ................. 128

5.18 Straight stair stepped R-card resistance distribution. ................. 130

5.19 Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is vertically polarized and $f = 3$ GHz. ............................................................ 131

5.20 Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is horizontally polarized and $f = 3$ GHz. ............................................................ 131

5.21 Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is vertically polarized and $f = 4$ GHz. ............................................................ 132

5.22 Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is horizontally polarized and $f = 4$ GHz. ............................................................ 132

5.23 Geometry of the quasi-circular stair stepped R-card resistance. .... 133

5.24 Quasi-circular stair stepped R-card resistance distribution. ........ 133
5.25 Magnitude of cross-range test zone fields in vertical and horizontal
cuts for the R-card treated, reflector shown in Figure 5.23. The feed
is vertically polarized and $f = 3$ GHz. ............................................................ 134

5.26 Magnitude of cross-range test zone fields in vertical and horizontal
cuts for the R-card treated, reflector shown in Figure 5.23. The feed
is horizontally polarized and $f = 3$ GHz. ........................................................ 135

5.27 Magnitude of cross-range test zone fields in vertical and horizontal
cuts for the R-card treated, reflector shown in Figure 5.23. The feed
is vertically polarized and $f = 4$ GHz. ................................................................. 135

5.28 Magnitude of cross-range test zone fields in vertical and horizontal
cuts for the R-card treated, reflector shown in Figure 5.23. The feed
is horizontally polarized and $f = 4$ GHz. ........................................................ 136

5.29 Rays diffracting from the circular reflector rim. ................................. 138

5.30 The intersection of the diffraction cones with a vertical plane in the
test zone. Only rays emanating from a few points on the right half of
the reflector edge are shown. .......................................................... 139

5.31 PO diffracted field calculated at 4 GHz in the central vertical cut. Note
that the diffracted field has a peak at the caustic location. .............. 140

6.1 One inch thick Styrofoam pieces are cut into the right shape and cov­
ered by paper with R-card contours plotted on. The reflector is waiting
behind after being tilted at the desired angle. ........................................ 146

6.2 The Styrofoam pieces are raised off the floor, assembled in the designed
fence geometry, and supported by wooden sticks of different heights.
Small cubes of Styrofoam are taped to the floor and used as bases for
the wooden sticks. .......................................................... 147

6.3 The higher side of the fence where a frame of 2-by-4 wood is built to
support the fence segments using Styrofoam cantilevers. This support
is to replace a large part of the wooden sticks support to allow sliding
the reflector under the fence umbrella. The opening in the frame is
made wider than the reflector. A flexible arrangement for the feed is
built as shown to provide different degrees of freedom to move the feed
to the focus and rotate it for different polarizations. .................. 148

xvii
6.4 The lower side of the fence shows the wooden frame supporting the fence segments with well fit Styrofoam cantilevers. The frame is made adjustable for fine tuning the height and orientation of each segment.

6.5 Preparing the R-card sheets by cutting the right values on the designated shape according to the quasi-circular contours. The R-card rolls also appear in the photo.

6.6 The tailor adds the final R-card layers from the specially-ordered R-card roll of 3000 Ω resistance value. This layer provides a well finished surface for the fence.

6.7 The R-cards are placed on the Styrofoam. The reflector is slid under the fence umbrella. The feed is focused. The probe arrangements are made where the positioner sits on a pair of 8-feet fiberglass ladders to scan the line of symmetry cut. The HP8510 network analyzer and the PC appear at the back of the photo.

6.8 Probed test zone fields at 3 GHz for V-pol.

6.9 Probed test zone fields at 3 GHz for H-pol.

6.10 Probed test zone fields at 4 GHz for V-pol.

6.11 Probed test zone fields at 4 GHz for H-pol.

6.12 Probed test zone fields at 5 GHz for V-pol.

6.13 Probed test zone fields at 5 GHz for H-pol.

6.14 Probed test zone fields at 6 GHz for V-pol.

6.15 Probed test zone fields at 6 GHz for H-pol.

6.16 Probed test zone fields at 7 GHz for V-pol.

6.17 Probed test zone fields at 7 GHz for H-pol.

6.18 Probed test zone fields at 8 GHz for V-pol.
6.19 Probed test zone fields at 8 GHz for H-pol. ................................................ 159
6.20 Probed test zone fields at 8 GHz for V-pol. ................................................ 160
6.21 Probed test zone fields at 9 GHz for H-pol. ................................................ 160
6.22 Probed test zone fields at 9 GHz for V-pol. ................................................ 161
6.23 Probed test zone fields at 10 GHz for H-pol. ................................................ 161
A.1 Upper R-card resistance evaluation using backward ray tracing. ............... 167
A.2 Forward ray tracing to evaluate the lower R-card resistance for an offset reflector. ................................................ 168
B.1 The R-card segment geometry arrangement in front of the reflector edge. 173
C.1 An R-card of four sided polygon shape is defined by its vertices $C_1, C_2, C_3, C_4$. Patching the R-card takes place resulting in a rectangular $M \times N$ matrix of patches. ................................................ 177
C.2 A quad with an interior angle greater than 180° is properly patched. .......................... 178
C.3 Two different schemes for patching triangular R-cards using additional corner to simulate a quadrilateral. ................................................ 179
D.1 Determination whether a point $P$ lies inside or outside N sided polygon. 181
E.1 Determination whether a ring, $C'_1, C'_2, \ldots, C'_N$ encloses a line or not defined by two points A and B. The ring is generally non planar. ...................... 182
E.2 Conversion of the 3D problem into a 2D one by projecting the whole geometry onto a plane perpendicular to $\mathbb{F}$. The line segment projection is a point $P$, the ring projection is a polygon and the free vectors $\mathcal{F}$'s remain the same. ................................................ 184
F.1 The central region of the circular grid. ................................................ 187
F.2 Two different types of rings used for gridding a circle. ................................ 187
F.3 A complete uniform gridding for a circle using triangular patches. ............... 188
1.1 INTRODUCTION

Antenna and RCS measurements require illuminating the object under test by a uniform plane wave. The primary methods to produce the plane wave are based on simply locating the test object at a large enough distance from an electromagnetic wave source to allow the spherical wave front to closely approximate a plane wave. The compact range is an alternative indoor measurement facility which produces such a uniform plane wave in a much shorter distance [1]. Offset parabolic reflectors have been widely used in many of the compact ranges to convert a spherical wavefront to a planar one. Unfortunately, stray signals diffracting from the reflector edges are a major source of error in such measurements [2]. The error comes from the imperfection of the plane wave in the target zone which appears as ripple distorting the uniformity of the test zone fields.

Different edge terminations have been used to reduce the edge diffracted stray signals in the test zone fields. Blended rolled edges and serrated edges are the most popular treatments used today to terminate compact range reflectors. By terminating
the concave reflector surface with a blended convex surface extension, the reflected rays are directed away from the test zone [3]-[4]. If the edge shaping is done slowly with a proper radius of curvature variation, creeping waves attach at the grazing boundary and propagate behind the reflector in a slow and smooth transition resulting in a great improvement in the uniformity of the target zone fields. In addition, as shown in [5], one can use a concave contour junction to further reduce the diffracted fields coming from the junction discontinuity between the blended rolled edge and the paraboloidal reflector.

By serrating the edge, a gradual transition is produced between the perfect conductor (PEC) reflector and free space. In terms of diffraction theory, serrating the reflector straight edge replaces the diffracting points on the edge and their diffracting cones that contaminate the target zone by new ones which spread the diffracting rays away from the test zone. This can only be accomplished by properly designing the serrations by properly choosing their number and orientation. The edges can either be straight as in [6] or shaped as in [7]. Nevertheless, the tips and valleys of the serration remain diffracting everywhere and contaminate the test zone with small amplitude corner diffraction terms.

Although the blended rolled edge has proven its superiority in reducing the variations in the test zone fields as discussed in [6], the serrated edge treatment is still used. This is mainly because serrating the reflector edge had been known for many years before the blended rolled edge treatment was discovered; even so; the serrated
edge treatment may still be preferred based on cost.

With the revolution in wireless communications and its conjunction with satellite technology, the need of more reliable antenna measurements has been increasing day after day. However, some of the compact ranges are still suffering from error problems in their measurements mainly because of the poor performance of their old edge treatment. These ranges are looking for an economical solution to correct and improve their performance. Serrated edge treatment is usually used in these ranges. One possible successful solution can be replacing the serrations with rolled edges as done in the ESL-OSU compact range about 15 years ago [3]. Unfortunately, both the cost and the anechoic chamber size limitations restrict this approach.

A new reflector edge treatment using resistive sheets, called R-cards, is proposed herein. This treatment can be a very suitable solution for old compact ranges that are seeking improvement as introduced in [8]. This new treatment is also expected to encourage and attract the attention of new compact range designers because of the following advantages:

- Significant ripple amplitude reduction.
- Good low frequency performance.
- Excellent down range performance.
- Simplicity.
- Low cost.
- Ability to improve existing compact range reflectors.
• Validity over a wide frequency bandwidth.
• Does not require a complex feed.
• No extra room space is needed.

1.2 Organization of this Dissertation

The rest of this Dissertation is organized as follows: Chapter 2 introduces the R-cards from the macroscopic point of view. A brief review of the numerical techniques used to analyze the reflector/R-card problem is also described in the same chapter. Chapter 3 presents the design approach for the 2D problem. An understanding for the different factors affecting the performance will be provided. Chapter 4 discusses the rectangular rim reflector problem showing how one can overcome the complexity added to the design problem. Analogy between the 2D and 3D cases will be given in Chapter 4 as well.

In Chapter 5, the R-card fence design for a circular rim reflector will be presented. Some practical issues will be considered in the design. Results will demonstrate the validity of the design approach. In Chapter 6, the measurements set-up and the steps to construct the R-card fence are explained. Measured results are to be shown. Chapter 7 concludes the research topic.

Appendix A explains the process of synthesizing the R-card resistance. Appendix B shows how the R-card segments are arranged in front of the reflector edge. Appendix C gives computational details of modeling and patching the R-cards. Appendices D and E describe some ray tracing tricks that are used many times during the course of an R-card design. Appendix F provides the mesh used for gridding the surface of the circular rim reflector.
CHAPTER 2

R-CARDS AND ANALYTIC TECHNIQUES

2.1 Introduction to R-cards

R-cards are resistive sheets that have some interesting electromagnetic applications especially in stealth technology. In most cases, they are attached to a metallic structure and work on decreasing the discontinuity between the very low resistance of the structure and that of free space. This metallic structure can be a radiator, horn antenna for example [9], or a scatterer such as a sub-reflector in a Gregorian system used in a dual chamber compact range [10]. For the problem at hand, the R-cards are not connected to the metallic structure but are simply mounted at a distance where they interact with the rest of the structure. They mainly control the direction of the diffracted rays and attenuate their amplitude.

The ability of the R-card to block the EM waves or let them go through is expressed in terms of the reflection and transmission coefficients, respectively. Three factors control the value of these coefficients at a certain local point on the R-card. These factors are the polarization of the incident wave, the angle of incidence, and the resistance value of the R-card at this local point.
2.2 Reflection and Transmission Coefficients of the R-card

Formulation of both the reflection and transmission coefficients of R-card for both TE and TM polarizations are presented in this section. The dependence of these coefficients on different factors are discussed. The Physical Optics (PO) technique is then reviewed and developed to a general iterative physical optics (IPO) used to analyze the reflector/R-card system. In this analysis, $\vec{E}_i^r$, $\vec{E}_r^r$, and $\vec{E}_r^t$ are defined as the incident, reflected, and transmitted electric fields; whereas $\vec{H}_i^r$, $\vec{H}_r^r$, and $\vec{H}_r^t$ define the corresponding magnetic fields. The surface unit normal vector at the point of incidence is given by $\hat{n}$. The equivalent surface current associated with the R-card at the point of incidence is defined by $\vec{J}_i^l$ for parallel (TE) polarization and $\vec{J}_i^p$ for perpendicular (TM) polarization. Although the reflection and transmission coefficients can be explicitly derived by applying different boundary conditions at the R-card surface [11], an alternative approach using transmission line (TL) model is used here as an efficient method to develop the same expressions in a few straightforward steps.

2.2.1 Parallel (TE) polarization

For TE polarized plane wave case, the reflection and transmission coefficients are defined as

$$\rho_\parallel \equiv \frac{\vec{H}_r^r}{\vec{H}_i^r}, \text{ and}$$

$$\tau_\parallel \equiv \frac{\vec{H}_r^t}{\vec{H}_i^t}. \tag{2.1}$$

$$\rho_\parallel \equiv \frac{\vec{H}_r^t}{\vec{H}_i^t}. \tag{2.2}$$
The well known TL model formulae for the reflection and transmission coefficients are

\[
\rho_{||} = \frac{Z_1 - Z_2}{Z_2 + Z_1}, \quad \text{and} \quad (2.3)
\]

\[
\tau_{||} = \frac{2Z_2}{Z_2 + Z_1}. \quad (2.4)
\]

From Figure 2.1, \(Z_1\) and \(Z_2\) are given by

\[
Z_1 = Z_{||}, \quad \text{and} \quad (2.5)
\]

\[
Z_2 = \frac{RZ_{||}}{R + Z_{||}}. \quad (2.6)
\]

Substituting \(Z_1\) and \(Z_2\) in Equations 2.3 and 2.4, one finds that

\[
\rho_{||} = \frac{Z_{||}}{Z_{||} + 2R}, \quad \text{and} \quad (2.7)
\]

\[
\tau_{||} = \frac{2R}{Z_{||} + 2R}. \quad (2.8)
\]

For oblique incidence and TE polarization, \(Z_{||}\) is given by

\[
Z_{||} = Z_o \cos \theta. \quad (2.9)
\]
where \( Z_0 \) is the free space impedance. Then, the reflection and transmission coefficients for the TE case become

\[
\rho_{\parallel} = \frac{\cos \theta}{\cos \theta + 2R/Z_0}, \quad \text{and} \quad (2.10) \\
\tau_{\parallel} = \frac{2R/Z_0}{\cos \theta + 2R/Z_0}. \quad (2.11)
\]

where

\( \theta \) is the angle of incidence (\( \leq 90^\circ \))
\( R \) is the value of R-card resistance in ohms/square, and
\( Z_0 \) is the air intrinsic impedance.

The surface current on the R-card sheet can be determined from the boundary conditions and is given by

\[
\vec{J}_{\parallel}^s = 2\rho_{\parallel} \hat{n} \times \vec{H}^i. \quad (2.12)
\]

For simplicity, the units of the R-card resistance, "ohms per square", will be reduced to "ohms" only for the rest of the dissertation.

### 2.2.2 Perpendicular (TM) polarization

The definition of the reflection and transmission coefficients in the case of TM polarization is

\[
\rho_{\perp} \equiv \frac{\vec{E}_r}{\vec{E}_i}, \quad \text{and} \quad (2.13) \\
\tau_{\perp} \equiv \frac{\vec{E}_t}{\vec{E}_i}. \quad (2.14)
\]

The TL model formulae for the reflection and transmission coefficients are
The sign change in the reflection coefficient between the TE and TM polarizations appears because of the difference in the coefficient definition which is based on the electric fields in the TM case and on the magnetic fields in the TE case.

From Figure 2.2, $Z_1$ and $Z_2$ are given by

$$Z_1 = Z_\perp, \quad \text{and}$$

$$Z_2 = \frac{RZ_\perp}{R + Z_\perp}.$$  \hspace{1cm} (2.17)

Substituting $Z_1$ and $Z_2$ into Equations 2.15 and 2.16, one obtains that

$$\rho_\perp = \frac{Z_\perp}{Z_\perp + 2R}, \quad \text{and}$$

$$\tau_\perp = \frac{2R}{Z_\perp + 2R}. \hspace{1cm} (2.19)$$

$$\rho_\perp = \frac{Z_\perp}{Z_\perp + 2R}, \quad \text{and}$$

$$\tau_\perp = \frac{2R}{Z_\perp + 2R}. \hspace{1cm} (2.20)$$
For oblique incidence, $Z_\perp$ is given by

$$Z_\perp = Z_o / \cos \theta. \quad (2.21)$$

Then, the reflection and transmission coefficients for the TM case become

$$\rho_\perp = \frac{-1}{1 + 2R \cos \theta / Z_o}, \quad \text{and} \quad (2.22)$$

$$\tau_\perp = \frac{2R \cos \theta / Z_o}{1 + 2R \cos \theta / Z_o}. \quad (2.23)$$

The surface current on the R-card sheet can be determined from the boundary conditions and is given by

$$\vec{J}_\perp = -2 \rho_\perp \vec{n} \times \vec{H}. \quad (2.24)$$

Plots for TE reflection and transmission coefficients versus the angle of incidence and for different practical values of the resistance $R$ are shown in Figure 2.3 and Figure 2.4, respectively, and the corresponding ones for TM mode are in Figure 2.5 and Figure 2.6. Some remarks on the behavior of these coefficients with the 3 different factors (polarization, $\theta$ and $R$) are

- Polarization dependence vanishes in case of normal incidence ($\theta = 0$) and can be neglected for small $\theta$. Up to $10^\circ$, one can consider the approximation $\rho_\parallel \approx |\rho_\perp|$ (compare Figure 2.3 with Figure 2.5) and $\tau_\parallel \approx \tau_\perp$ (compare Figure 2.4 with Figure 2.6). This approximation is obtained from the approximation $\cos \theta \approx 1$ for $\theta \leq 10^\circ$ which makes $Z_\parallel \approx Z_\perp$. This can be useful when polarization independent R-card design is needed.
• The rate of change of all coefficients increases with $\theta$ for fixed $R$. Plots are almost stable and flat for small $\theta$. They change more rapidly for larger $\theta$ ending up with a value of one or zero when $\theta$ reaches $90^\circ$.

• In general, for fixed $\theta$, coefficients are more sensitive to small values of $R$ than large ones. This can be easily explained by the knowledge of the combination of two parallel resistances being more sensitive to the smaller resistance. This behavior is stronger for small $\theta$. Practically, all curves of the same family for Figures 2.3 to 2.6, become very close to each other for $R > 1150 \, \Omega$. 
Figure 2.3: R-card reflection coefficient versus angle of incidence for different values of resistance (TE).

Figure 2.4: R-card transmission coefficient versus angle of incidence for different values of resistance (TE).
Figure 2.5: R-card reflection coefficient versus angle of incidence for different values of resistance (TM).

Figure 2.6: R-card transmission coefficient versus angle of incidence for different values of resistance (TM).
2.3 Iterative Physical Optics (IPO)

Physical Optics is a high frequency technique that is based on GO current evaluation instead of solving integral equations. In this problem, one needs to include the mutual interaction between the reflector and R-cards in the surface currents before integrating these currents to obtain the probed field in the quiet zone. This can be done by introducing the Iterative Physical Optics (IPO) which is a repetition of applying PO between two or more objects in a logical order. The basic formulations for calculating a current $\vec{J}_2$ at point 2 due to a surface current distribution $\vec{J}_1$ on another object can be derived in terms of the local normal vector $\hat{n}_2$ to the surface and the reflection coefficient of the surface at that point.
2.3.1 2D case

IPO for TE polarization

The magnetic vector potential \(d\vec{A}_1\) at point 2 generated by \(J_1 dl_1\) in Figure 2.7 is given by

\[
d\vec{A}_1 = -\frac{j}{4} \vec{J}_1 H_o^{(2)}(kr_{21}) dl_1. \tag{2.25}
\]

Using \(\vec{H} = \nabla \times \vec{A}\), the magnetic field at point 2 can be written as

\[
d\vec{H}_1 = -\frac{jk}{4}(\hat{r}_{21} \times \vec{J}_1)H_1^{(2)}(kr_{21}) dl_1. \tag{2.26}
\]

From Equation 2.12, and integrating over the whole current distribution \(\vec{J}_1\), the resulting current \(\vec{J}_2\) is

\[
\vec{J}_2 = \frac{jk\rho}{2} \int (\hat{n}_2 \times \hat{r}_{21} \times \vec{J}_1) H_1^{(2)}(kr) dl_1. \tag{2.27}
\]

IPO for TM polarization

The electric and magnetic fields generated by a filament current \(J_1 dl_1\) as shown in Figure 2.8 are given by

\[
d\vec{E}_1 = -\frac{kZ_o}{4} \vec{J}_1 H_o^{(2)}(kr_{21}) dl_1, \quad \text{and} \quad \tag{2.28}
d\vec{H}_1 = -\frac{jk}{4} H_1^{(2)}(kr_{21})(\hat{r}_{21} \times \vec{J}_1) dl_1. \tag{2.29}
\]

Substituting into Equation 2.24, the current filament \(\vec{J}_2\) at point 2 is given by

\[
d\vec{J}_2 = \frac{jk\rho}{2} H_1^{(2)}(kr_{21})(\hat{n}_2 \times \hat{r}_{21} \times \vec{J}_1) dl_1. \tag{2.30}
\]

with

\[
\hat{n} \times \hat{r}_{21} \times \vec{J}_1 = (\hat{n} \cdot \vec{J}_1)\hat{r}_{21} - (\hat{n}_2 \cdot \hat{r}_{21}) \vec{J}_1 \tag{2.31}
\]
Figure 2.8: Mutual current formulation for IPO-TM where \( \vec{J}_1 \) generates current \( \vec{J}_2 \) at point 2.

and \( \hat{n}_2 \cdot \vec{J}_1 = 0 \) for TM, the current at point 2 becomes

\[
d\vec{J}_2 = -\frac{jk\rho}{2}(\hat{n}_2 \cdot \hat{r}_{21})H_1^{(2)}(kr_{21})\vec{J}_1 dl_1.
\]

(2.32)

Integrating \( \vec{J}_1 \) over the whole surface leads to the final current \( \vec{J}_2 \) at point 2 as

\[
\vec{J}_2 = -\frac{jk\rho}{2} \int (\hat{n}_2 \cdot \hat{r}_{21})H_1^{(2)}(kr_{21})\vec{J}_1 dl_1.
\]

(2.33)

The above mutual currents for both polarizations, given in Equations 2.27 and 2.33, are very important in evaluating the iterative currents between the reflector and the R-cards. The reflection coefficients appearing in these expressions are always at point 2 and have special values of \( \rho_{\parallel} = 1 \) and \( \rho_{\perp} = -1 \) in case of perfect electric conductor (PEC) where \( R = 0 \).

2.3.2 3D case

Expressions of both reflection and transmission coefficients for both TE and TM polarization were obtained by transmission line model in Section 2.2. Moment Method (MM) is being used as an accurate analysis tool for the 2D problem [12]. However,
MM is very inefficient for electrically large geometries particularly 3D geometries. Thus, the iterative physical optics (IPO) was developed and used as an alternative efficient analysis tool to replace MM and still keeps sufficient accuracy. The 2D development is done for both TE and TM polarizations separately in the previous section. Combining both polarizations can determine the PO currents for the 3D problem, especially on the R-cards.

**PO Current on The R-card**

Knowing the incident field $\vec{E}^i$ and the reflection coefficients $\rho_\parallel, \rho_\perp$ of the R-card at the point of incidence, the physical optics surface current $\vec{J}$ can be obtained. The electric and magnetic fields need to be defined in terms of the plane of incidence.

The coordinate system $\hat{s}_i, \hat{t}_\parallel, \hat{t}_\perp$ can be formed if both the incident unit vector $\hat{s}_i$ and the R-card normal unit vector $\hat{n}$ are known. This may be done as follows:

$$\hat{t}_\perp = \frac{\hat{s}_i \times \hat{n}}{|\hat{s}_i \times \hat{n}|} \quad \text{and} \quad \hat{t}_\parallel = \hat{t}_\perp \times \hat{s}_i.$$  

(2.34)  

(2.35)

The incident electric field shown in Figure 2.9 can be expressed as

$$\vec{E}^i = \vec{E}_\parallel^i + \vec{E}_\perp^i$$  

(2.36)

where

$$\vec{E}_\parallel^i = (\vec{E}^i \cdot \hat{t}_\parallel) \hat{t}_\parallel \quad \text{and}$$  

(2.37)

$$\vec{E}_\perp^i = (\vec{E}^i \cdot \hat{t}_\perp) \hat{t}_\perp$$  

(2.38)

The incident magnetic field can be also expressed as

$$\vec{H}^i = \vec{H}_\parallel^i + \vec{H}_\perp^i$$  

(2.39)
where

\[ \vec{H}_{\|}^i = \frac{1}{Z_0} \hat{s}_i \times \vec{E}_{\|}^i \quad \text{and} \]
\[ \vec{H}_{\perp}^i = \frac{1}{Z_0} \hat{s}_i \times \vec{E}_{\perp}^i. \quad (2.40) \]

Replacing \( \vec{E}_{\|}^i \) and \( \vec{E}_{\perp}^i \) using Equations 2.37 and 2.38 with

\[ \vec{E}_{\|}^i = Z_o \vec{H}^i \times \hat{s}_i, \quad (2.42) \]
\[ \hat{s}_i \times \hat{t}_{\|} = \hat{t}_{\perp}, \quad \text{and} \]
\[ \hat{s}_i \times \hat{t}_{\perp} = -\hat{t}_{\|}, \quad (2.44) \]

one can get

\[ \vec{H}_{\|}^i = (\vec{H}^i \cdot \hat{t}_{\|}) \hat{t}_{\perp} \quad \text{and} \]
\[ \vec{H}_{\perp}^i = -(\vec{H}^i \cdot \hat{t}_{\perp}) \hat{t}_{\|}. \quad (2.46) \]

This leads to the components of the PO equivalent current in terms of the R-card reflection coefficients \( \rho_{\|} \) and \( \rho_{\perp} \) at point P such that

\[ \vec{J}_{\|} = 2\rho_{\|} \hat{n} \times \vec{H}_{\|}^i \quad \text{and} \]
\[ \vec{J}_{\perp} = -2\rho_{\perp} \hat{n} \times \vec{H}_{\perp}^i, \quad (2.48) \]

where

\[ \rho_{\|} = \frac{\cos \theta_i}{\cos \theta_i + 2R/Z_0}, \quad \text{and} \]
\[ \rho_{\perp} = \frac{-1}{1 + 2R \cos \theta_i/Z_0}. \quad (2.50) \]

Finally the total PO current is given by the sum of its two components

\[ \vec{J} = \vec{J}_{\|} + \vec{J}_{\perp}. \quad (2.51) \]
3D-IPO

For a perfect electric conductor (PEC), $\rho_\parallel = -\rho_\perp = 1$, and the PO current on the reflector is simply given by

$$\vec{J}_\perp = 2 \hat{n} \times \vec{H}^i. \quad (2.52)$$

The iteration of the PO currents between the R-cards and the reflector is performed exactly in a similar way as done in the 2D problem. It starts from the integrating the R-card currents produced by the feed (the zero current $\vec{J}_0^c$) to get the current component $\vec{J}_0^c$ on the reflector through the incident magnetic field $\vec{H}^0(\vec{J}_0^e)$. Then the reflector current $\vec{J}_0^c + \vec{J}_0^e$ is integrated to obtain the current $\vec{J}_0^c$ on the R-card. Then the R-card current becomes $\vec{J}_0^c = \vec{J}_0^e + \vec{J}_0^c$. By this, one iteration cycle ($It=1$) is completed and can be repeated as many as wanted. Fortunately, iterating more than twice adds very negligible information to the results and thus the efficiency is maintained. This fact is concluded from the 2D case and is also verified in the 3D case. The different components of the iterative currents are usually stored in compact binary files after multiplied by the PO patches.
Figure 2.9: Calculation of the PO equivalent current $\vec{J}$ on an R-card at the point of incidence $P$. The incident fields have to be analyzed into two components, parallel and perpendicular to the plane of incidence.
CHAPTER 3

R-CARD DESIGN FOR THE 2D PROBLEM

One of the important skills, that a talented engineer should have when facing a very complicated design problem, is to break the complexity into pieces. The ideas can then be tested quicker if they are going to work or not and a decision, to insistly pursue in one direction or flexibly switch to another, can be made while saving effort and time. Solving the 2D problem is one way to avoid complications associated with real life 3D engineering problems. It enables one to understand the major factors that should be considered in the design geometry and the choice of the analysis tool as well. In this chapter, the R-card design for the 2D problem is presented step by step to better understand the problem and show promising results for the R-card design that encouraged continued work and persistence in this direction.

3.1 Ray tracing

Placing the R-cards in front of the reflector introduces additional transmission, reflection and diffraction mechanisms that illuminate the inside of the anechoic chamber. While a few of these new mechanisms work in favor of improving the test zone fields, many of them have a negative effect on the performance and need to be controlled. The new mechanisms are produced from the interaction between the R-cards
with both the feed and the reflector. Other interactions such as the ones between the R-cards and the chamber walls are negligible assuming that the absorber on the walls is working properly. A long list of the undesired mechanisms can be considered for higher level R-card designs but for now, let’s focus on the following:

1. the reflection from the R-card due to direct feed illumination (see Figure 3.1),
2. reflector diffraction, R-card reflection, and reflector reflection (see Figure 3.2),
3. the triple reflection between the reflector and the R-card (see Figure 3.3).

Studying these interactions helps to minimize their negative impact on the test zone fields. To sense the effect of the R-card reflectivity, one can perform a ray tracing study. The first mechanism in the above list is shown in Figure 3.1. The choice of the R-card orientation can be very harmful to the test zone fields due to the specular reflection of the feed illumination. The R-cards should be properly oriented to deflect these rays away and prevent them from propagating even close to the test zone.

The interaction of the R-cards with the reflector is more complicated than with the feed, as represented by the second and third items in the above list. Because of the structure’s nature and the closeness of the R-cards to the reflector, multiple ray reflections between the R-card and the reflector play an important role in the R-card design. The reflector scattered rays going through double reflection before they go potentially into the test zone are shown in Figure 3.2.
The third and remaining strong new mechanism in the list is represented by the triple reflection starting and ending at the reflector as shown in Figure 3.3. These rays contaminate the test zone for improper R-card geometries as shown in Figure 3.3-(a) and preventing this from happening is not impossible if well placed R-cards are chosen as shown in Figure 3.3-(b).

3.2 Aperture Study

The R-card resistance distribution should be tapered to a high resistance at the R-card ends which are close to the reflector center, otherwise strong diffracted field will cause a significant degradation of the desired plane wave in the test zone. In the early stage of the work on this problem, the R-card resistance distribution was assumed to be double sided exponential defined by few optimized parameters [13]. The results were accepted at that time however it was noticed that the slope of the exponential resistance profile was more steeper than required which led to large slope diffraction terms in the test zone fields. A better solution can be obtained by finding the criteria to obtain a general R-card resistance. This criteria is given by the GO aperture field distribution which achieves the best performance in the test zone fields.

To properly design the R-card taper, a study of the tapered GO aperture field, such as the one shown in Figure 3.4, had to be performed. Different taper windows, similar to those given in Figure 3.5, are superimposed to the reflector aperture GO field, $f_{go}(y)$, from both ends to define the new tapered aperture field, $f_{ap}(y)$, as

$$f_{ap}(y) = f_{go}(y) \times \text{taper}(y).$$ (3.1)
The taper \( y \) function is given by

\[
taper(y) = \begin{cases} 
1 - \left[ \frac{1 - \text{win}(y)}{1 - \epsilon} \right] \cdot (1 - \gamma) & , y < y_1 \\
1 & , y_1 < y < y_2 \\
1 - \left[ \frac{1 - \text{win}(y)}{1 - \epsilon} \right] \cdot (1 - \gamma_u) & , y > y_2
\end{cases}
\] (3.2)

where

\[
\gamma_u = 10^{-bu/20} \quad \text{and} \quad \gamma_l = 10^{-bl/20}.
\] (3.3) (3.4)

Note that

\( y_1, y_2 \) are the ends of the uniform field portion

\( bl, bu \) are the lower and upper taper levels in dB

\( \text{win}(t) \) is a typical edge taper over the range \( t = 0 \rightarrow 1 \) where \( \text{win}(0) = 1 \) and \( \text{win}(1) = \epsilon \).

The parameters \( y \) and \( t \) have a linear relationship that satisfies \( t = 0 \) at \( y = y_1, y_2 \) and \( t = 1 \) at the reflector edges.
Figure 3.1: The R-card geometry controls the direction of the reflected rays.
Figure 3.2: The effect of the R-card geometry on directing the reflector scattered rays.

(a) Undesired R-card geometry.

(b) Desired R-card geometry.
Figure 3.3: The effect of the R-card geometry on steering the triple-reflected rays.
Figure 3.4: Prototype GO tapered aperture distribution.
The resulting aperture field is shown in Figure 3.4 where it coincides with the GO field in the middle and tapers towards the ends. An isotropic source with no taper, which is the worst case, is assumed to illuminate the reflector. The GO taper can actually be simulated by controlling the feed illumination during this aperture study to avoid other mechanisms generated by the R-cards. Details of this study can be found in [13] and only the final conclusion will be stated here.

It was found that, among the different edge taper profiles under investigation, the Kaiser-Bessel distribution gives the minimum diffraction level similar to that shown in Figure 3.6. The level of the field taper should be at least 20 dB at the reflector edge to achieve a valuable improvement in the test zone plane wave. In addition, the uniform portion of the GO aperture should be equal to or slightly larger than the
Figure 3.6: Diffracted fields in the test zone for different GO profiles with a 10 dB edge taper.
test zone. As expected, the size of the test zone is limited to the size of this uniform portion which is restricted by the lowest operating frequency.

The resistance of the R-cards is synthesized point by point to achieve the chosen Kaiser-Bessel GO aperture distribution with 20 dB taper. GO rays hit the R-card once, twice, or even more, before going to the test zone. As shown in Figure 3.7, direct rays from the feed to the reflector edge region hit the R-card before reaching the reflector and hit the R-card again at a different point after reflecting from the reflector before going to the aperture. The attenuation occurring in these two hits is expressed in terms of the transmission coefficients $T_1$ and $T_2$ at each point, respectively, as shown in Figure 3.7. The product of both transmission coefficients for every ray should satisfy

$$T_1 \times T_2 = \frac{f_{ap}}{f_{go}}$$

(3.5)

where $T_1$ and $T_2$ are functions of the local R-card resistance at each intersection point. A progressive algorithm [13] eases solving the nonlinear equation, (3.5), to obtain the upper and lower R-card resistance distributions. Backward ray tracing and forward ray tracing, explained in some detail in Appendix A, are necessary for the R-card resistance synthesis process. A 2D offset example will show the significant impact of this R-card design approach in the test zone fields.

### 3.3 Design Example

A 1.8 m wide, offset reflector with a 1.2 m focal length is used here as a design example. The frequency range of interest is 3-12 GHz and the expected size of the test zone is 0.9 m which is half of the reflector size. Note that the reflector size is only
18 wavelengths at 3 GHz. The test zone is located at a distance twice the focal length from the reflector vertex. The geometry of the reflector and the designed R-cards are shown in Figure 3.8. Each R-card is described by 1) the coordinates of its inner edge \((x_{1,2}, y_{1,2})\), 2) the tilt, \(\alpha_{1,2}\), relative to reflector axis, 3) the length, \(L_{1,2}\), and 4) the resistance distribution. The subscripts, 1 and 2, are for the upper and lower R-cards, respectively. The resistance distribution is truncated to 3000 Ω for practical reasons.\(^1\) Using this geometry, amplitude variations of less than 0.5 dB and phase variations within \(\pm 5\) degrees have been achieved in the test zone fields for both polarizations. The IPO results for the TM polarization at 3, 6, and 12 GHz will be discussed.

### 3.3.1 One R-card/edge at a time

Since the direct interaction between the R-cards is usually small and can be safely ignored, it is much easier to design the R-cards one at a time. This requires one to set the diffraction from one of the reflector edges to zero. This kind of separation is very easy if Geometrical Theory of Diffraction (GTD) [14] is used, where the diffraction coefficient can be set to zero when desired. To obtain sufficient accuracy with GTD, a more complicated GTD version, that includes higher order terms, must be used to include the slope and the R-card diffraction effect at low frequencies. For both MM and IPO analyses, this can be approximately done by extending the reflector with a tapered R-card following the parabolic contour [15]. This simulates a semi-infinite reflector where one of the edges disappears. Because the diffraction is maximum along the reflection shadow boundary (RSB), the upper reflector edge causes most significant ripple at the top of the test zone and so does the lower reflector edge at the bottom of the test zone. This is shown in Figure 3.9 before adding the R-cards.\(^1\) R-cards of high resistance values are difficult to obtain.

\(^1\)R-cards of high resistance values are difficult to obtain.
One can also notice that the lower edge diffracts more than the upper edge because the lower edge is closer to the feed and hit by stronger incident field. Optimizing the geometry of the upper R-card, and then the lower one, leads to the results also shown in Figure 3.9.

The amplitude and phase of the total cross range test zone fields for the reflector with both R-cards are shown in Figure 3.10. The significant ripple reduction over the frequency band is very obvious and it has been achieved because of the above understanding of the different factors.

It is important to mention that the optimization can be done for different frequencies that cover the whole frequency band of interest. Even though the R-card design was obtained at the lowest operating frequency, it will most probably be appropriate for higher frequencies but the other way around does not usually work. The optimum R-card parameters should always be within the constraints illustrated by ray tracing so that wide band performance can be obtained.

### 3.3.2 Down Range Performance

So far, it has been shown that the ripple level can be successfully reduced in the cross range of the test zone using the above R-card design. The down range performance is checked here as well for the same R-card design. Figure 3.11 shows the down range ripple behavior at the center of an extended test zone, $y = 0$, starting from $x = F = 1.2$ meters to $x = 5$ meters. Since the R-cards change the direction of the RSB away from the test zone, one should expect the reduced ripple level as shown from the results. Thus, these results suggest that it may be useful to place the
Figure 3.7: R-card resistance synthesized based on two transmission coefficients.

test zone further from the reflector with fences than normally done using conventional reflector approaches.
Figure 3.8: Geometry of the design example: an offset reflector with two well designed R-cards as reflector edge treatment.
Figure 3.9: Test zone fields due to each isolated R-card alone at 3, 6 and 12 GHz for the TM case.
Figure 3.10: Magnitude and phase of the test zone fields at 3, 6 and 12 GHz for the TM case.
Figure 3.11: Magnitude and phase of the down range test zone fields at 3, 6 and 12 GHz for the TM case.
CHAPTER 4

R-CARD DESIGN FOR A 3D REFLECTOR WITH A RECTANGULAR RIM

4.1 R-card Design for a 3D Center-Fed Reflector

Although compact range reflectors are always made offset to avoid feed blockage, it is easier to show the procedure for designing the R-cards for a center-fed reflector. The first step in attacking the complicated 3D reflector problem is to solve the corresponding 2D problem(s). The design of the R-card for a 2D reflector is much faster and easier to obtain. This was demonstrated by ray tracing and treating one edge at a time as discussed in Chapter 3 and in [13].

For the sake of a 2D to 3D correspondence, some constraints may apply in the process of finding the optimum 2D R-card as will be seen later. From the 2D R-card design, one can take the second step by applying the R-card design on a narrow reflector strip. Keeping the 3D reflector strip narrow allows one to limit the effect of the second curvature of the 3D paraboloid on the interaction between the reflector and the R-cards. Thus, one can guarantee good similarity between the 3D double-curved reflector and the 2D single curved cylindrical reflector. After making sure that the
R-cards work for reflector strips, the third and final step is to extend the R-cards along the complete circumference for the 3D reflector.

### 4.1.1 2D and 3D Reflector Strips

**Similarity in Geometry and Results**

For a symmetrical, 1.8 meters wide, 2D reflector, two R-cards are placed and, oriented, and their resistance distribution synthesized as shown in Figure 4.1.

Because of the symmetry, the parameters for the upper R-card alone are listed in Table 4.1.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$y_1$</th>
<th>$\alpha_1$</th>
<th>$L_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.54</td>
<td>0.50</td>
<td>115°</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 4.1: Upper R-card geometry parameters. Distances are in meters.

With the same parameters, R-cards are assembled in front of the 3D reflector strip as shown in Figure 4.2. While the width of the reflector strip is 0.4 meters, it is 0.6 meters for the R-cards. The side-view of the 3D configuration along the center of the strip shown in Figure 4.3 confirms the 3D to 2D geometry correspondence. The R-cards' resistance distribution, which is shown in Figure 4.4, is also exactly the same as the 2D case along the central cut.

After forcing the 2D and 3D strip geometries to look alike, similarity in the test zone probed fields should be expected. For verification and comparison purposes, different results for both cases are shown together. Only the TM polarization is
considered for the 2D case and the equivalent H-polarization is considered for the 3D vertical strip. The test zone fields at 3 GHz and 4 GHz, for both cases, are shown in Figures 4.5-4.8. These results are the second iteration output of the IPO code. Generally, agreement between the two cases can be seen from the behavior of the plots before and after placing the R-cards in front of the reflector. There are a few minor differences between the two cases which can be easily explained. The difference between the 2D wave spread factor, $1/\sqrt{\rho}$, and the 3D one, $1/r$, makes the GO fields flatter with less taper in the 2D case. Further, while the 2D feed is a line source which has a perfect cylindrical pattern illuminating the reflector, a Huygens source is used as the feed for the 3D reflector. The Huygens source has a $\cos^2 \theta/2$ pattern which adds a slight taper to the GO fields. These two differences result in a lower level of illumination of the 3D reflector edges and consequently less edge diffraction. Thus, slightly smaller ripple amplitude can be observed in the reflector fields for the 3D case.

Another difference between the 2D and 3D cases comes from the side edges of the 3D reflector strip. These two untreated edges also diffract into the test zone. Fortunately, their contribution is almost uniform along the vertical central cut where the probed fields are calculated. Their effect on the field ripple is negligible but they create a level shift between the PO and GO fields as shown in the 3D results. Obviously, this difference in level changes when the width of the reflector strip changes.

The above minor differences between the 2D and the 3D cases do not change the fact that the 3D R-card design, derived from the 2D case, succeeds in reducing the fluctuations in the test zone fields. On the other hand, 3D-IPO also converges very fast such that it is only necessary to consider the second iteration output for
the test zone fields. Actually, the first iteration fields, shown in Figures 4.9-4.12 for both cases are very close to the second iteration ones. This only happens when the R-cards are designed to properly interact with the reflector and the feed. Having the second iteration results very similar to the first ones indicates that the test zone ripple reduction is not coming from a phase cancelation mechanism. This guarantees that the design of the R-card is valid for a wide band of frequencies.

**IPO Terms**

One of the advantages of using IPO in the analysis is the ability to study the contribution of different currents on the total test zone fields. This can be useful in validating the R-card design and verifying the equivalence of the 2D and 3D cases.

The separate IPO field components of the reflector are shown in Figures 4.13 and 4.14 at 3 GHz for the 2D case and 3D case, respectively. Because of the symmetry, only the lower R-card field components are shown in Figures 4.15 and 4.16 at the same frequency, and also for both 2D and 3D cases. The zero iteration field is the one produced by the currents before any interaction between the reflector and the R-cards. Those currents are due to the direct feed illumination. The first iteration current on the reflector is produced by the zero order current on the R-cards. The first iteration current on the R-card is produced by the sum of the zero and first currents on the reflector. Then the second iteration current on the reflector is produced by the first current on the R-card and the second iteration current on the R-card is produced by the second current on the reflector.

From the obtained results, it is noted that the field term contributed from the second iteration is very weak. It is more than 40 dB down for the reflector and about
50 dB down for the R-card. This assures fast convergence of the IPO and validates the approximation made by neglecting higher iteration terms.

The reflector zero field is collimated and hence is the strongest in the test zone. It contains the undesired edge diffracted terms appearing as ripple. The R-card zero field is very low as long as the proper R-card orientation is made with respect to the feed. The field contribution of the first iteration is an effective component that reduces the ripple amplitude in the test zone. This is very clear at the lower edge of the test zone where the R-cards' first iteration component is maximum and is about 16 dB below the zero reflector field. This component represents the blockage of the lower R-card to the aperture fields, mainly the reflector lower edge diffracted fields, at the lower part of the test zone. The upper R-card behaves exactly the same way in terms of the upper part. The first term reflector field also contains the incident field attenuation caused by the R-cards to reduce the reflector edge illumination. Similar comments apply to the field components at 4 GHz shown in Figures 4.17-4.20. In the following section, the whole 3D reflector will be treated.
Figure 4.1: Geometry of the designed R-cards for the 2D reflector.

Figure 4.2: Geometry of the R-cards for 3D reflector strip.
Figure 4.3: A side-view for the R-cards and the 3D reflector strip which is similar to the 2D geometry shown in Figure 4.1.

Figure 4.4: The resistance distribution of the R-cards.
Figure 4.5: Test zone fields at 3 GHz for the 2D configuration (second iteration).

Figure 4.6: Test zone fields at 3 GHz for the 3D configuration (second iteration).
Figure 4.7: Test zone fields at 4 GHz for the 2D configuration (second iteration).

Figure 4.8: Test zone fields at 4 GHz for the 3D configuration (second iteration).
Figure 4.9: Test zone fields at 3 GHz after the first IPO iteration for the 2D configuration.

Figure 4.10: Test zone fields at 3 GHz after the first IPO iteration for the 3D configuration.
Figure 4.11: Test zone fields at 4 GHz after the first IPO iteration for the 2D configuration.

Figure 4.12: Test zone fields at 4 GHz after the first IPO iteration for the 3D configuration.
Figure 4.13: Test zone IPO field components produced by the 2D reflector at 3 GHz.

Figure 4.14: Test zone IPO field components produced by the 3D reflector at 3 GHz.
Figure 4.15: Test zone IPO field components produced by the 2D lower R-card at 3 GHz.

Figure 4.16: Test zone IPO field components produced by the 3D lower R-card at 3 GHz.
Figure 4.17: Test zone IPO field components produced by the 2D reflector at 4 GHz.

Figure 4.18: Test zone IPO field components produced by the 3D reflector at 3 GHz.
Figure 4.19: Test zone IPO field components produced by the 2D lower R-card at 4 GHz.

Figure 4.20: Test zone IPO field components produced by the 3D lower R-card at 4 GHz.
4.1.2 The Whole 3D Reflector
R-card Geometry

Equilateral trapezoidal R-cards are assumed as shown in Figure 4.21. The trapezoid is defined by 3 parameters; the top base $B_t$, the mean base $B_m$, and the height $L$. The R-card is initially located in the $x'-y'$ plane where the origin is at the center of the trapezoid. Patching takes place in the $x'-y'$ plane where both rectangular and triangular patches can be used. The centroid location and the area of each patch are calculated and stored in a rectangular matrix with a key for their indices to be used in the IPO integration. The trapezoid bottom base can be determined from

$$B_b = B_t + 2rL$$  \hspace{1cm} (4.1)

where

$$r = \frac{B_m - B_t}{L}.$$  \hspace{1cm} (4.2)

A rectangular R-card is a special case of the above trapezoid when $B_t = B_m$ is detected by the computer code. Then the value of $r$ vanishes, $B_b$ becomes equal to $B_m$ and $B_t$, and the triangular patches disappear and only the rectangular ones remain. After patching the R-card in the $x'-y'$ plane, it needs to be located and oriented properly in front of the reflector edge. Three angles of rotation, $\alpha_x$, $\alpha_y$, and $\alpha_z$, express the R-card rotation around the $x'$, $y'$, and $z'$ axes, respectively. Then translation $\vec{r}_0$ of three components, $(x_0, y_0, z_0)$, by a vector locates the rotated R-card into the correct place in front of the reflector edge. All patches undergo the same rotation and translation processes which can be expressed by the following vector equation:

$$\vec{r} = M \times \vec{r}' + \vec{r}_0$$  \hspace{1cm} (4.3)
where \( M \) is a \( 3 \times 3 \) rotation matrix whose elements are given by

\[
\begin{align*}
m_{11} &= \cos \alpha_y \cos \alpha_z \\
m_{12} &= -\cos \alpha_x \sin \alpha_z + \sin \alpha_x \sin \alpha_y \cos \alpha_z \\
m_{13} &= \sin \alpha_x \sin \alpha_z + \cos \alpha_x \sin \alpha_y \cos \alpha_z \\
m_{21} &= \cos \alpha_y \sin \alpha_z \\
m_{22} &= \cos \alpha_x \cos \alpha_z + \sin \alpha_x \sin \alpha_y \sin \alpha_z \\
m_{23} &= -\sin \alpha_x \cos \alpha_z + \cos \alpha_x \sin \alpha_y \sin \alpha_z \\
m_{31} &= -\sin \alpha_y \\
m_{32} &= \sin \alpha_x \cos \alpha_y \text{, and} \\
m_{33} &= \cos \alpha_x \cos \alpha_y
\end{align*}
\]

Note that the rotation is CCW for all axes and that the rotation comes before translation. The rotation angles of the R-cards can be calculated if the coordinates of any two corners, let's say \( C_1 \) and \( C_2 \), are given before and after rotation as shown in Figure 4.22. Assume they are \((x'_1, y'_1, 0)\) and \((x_1, y_1, z_1)\) for \( C_1 \) and \((x'_2, y'_2, 0)\) and \((x_2, y_2, z_2)\) for \( C_2 \), before and after rotation, respectively. The translation vector \( \vec{r}_o = x_o \hat{x} + y_o \hat{y} + z_o \hat{z} \) should also be known. Then the following six elements from the rotation matrix \( M \) can be determined from

\[
\begin{align*}
m_{11} &= \frac{(x_1 - x_o)y'_2 - (x_2 - x_o)y'_1}{x'_1y'_2 - x'_2y'_1} \\
m_{12} &= \frac{(x_1 - x_o)x'_2 - (x_2 - x_o)x'_1}{x'_1y'_2 - x'_2y'_1} \\
m_{21} &= \frac{(y_1 - y_o)y'_2 - (y_2 - y_o)y'_1}{x'_1y'_2 - x'_2y'_1} \\
m_{22} &= \frac{(y_1 - y_o)x'_2 - (y_2 - y_o)x'_1}{x'_2y'_1 - x'_1y'_2}
\end{align*}
\]

55
\[ m_{31} = \frac{(z_1 - z_0)y'_2 - (z_2 - z_0)y'_1}{x'_1y'_2 - x'_2y'_1} \] (4.17)

and

\[ m_{32} = \frac{(z_1 - z_0)x'_2 - (z_2 - z_0)x'_1}{x'_2y'_1 - x'_1y'_2}. \] (4.18)

The rotation angles are then calculated using

\[
\alpha_z = \tan^{-1}(m_{21}, m_{11})
\]

(4.19)

\[
\alpha_y = \tan^{-1}\left(-m_{31}, \frac{m_{21}}{\sin \alpha_z}\right) \quad \alpha_z \neq 0^\circ \text{ or } \alpha_y = \tan^{-1}\left(-m_{31}, \frac{m_{31}}{\cos \alpha_z}\right) \quad \alpha_z \neq 90^\circ
\]

(4.20)

(4.21)

and

\[
\alpha_x = \tan^{-1}(m_{32}, m_{22}m_{11} - m_{12}m_{21})
\]

(4.22)

where \( \tan^{-1}(y, x) = \tan^{-1}(y/x) \) and provides the angle in the right quarter depending on the signs of \( x \) and \( y \).

Thus, four R-card fences can be mounted in front of a center-fed reflector of a square aperture as shown in Figure 4.23. Table 4.2 describes the different R-card geometry parameters.
Table 4.2: R-card geometric parameters for a 3D center-fed reflector. Dimensions are in meters.

<table>
<thead>
<tr>
<th>R-card</th>
<th>$x_o$</th>
<th>$y_o$</th>
<th>$z_o$</th>
<th>$\alpha_x$</th>
<th>$\alpha_y$</th>
<th>$\alpha_z$</th>
<th>$B_m$</th>
<th>$L$</th>
<th>$B_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower</td>
<td>0</td>
<td>-0.7719</td>
<td>0.4132</td>
<td>25°</td>
<td>0°</td>
<td>0°</td>
<td>1.5438</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Left</td>
<td>-0.7719</td>
<td>0</td>
<td>0.4132</td>
<td>25°</td>
<td>0°</td>
<td>-90°</td>
<td>1.5438</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Upper</td>
<td>0</td>
<td>0.7719</td>
<td>0.4132</td>
<td>25°</td>
<td>0°</td>
<td>180°</td>
<td>1.5438</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Right</td>
<td>0.7719</td>
<td>0</td>
<td>0.4132</td>
<td>25°</td>
<td>0°</td>
<td>90°</td>
<td>1.5438</td>
<td>0.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 4.21: Trapezoidal R-card geometry.
Figure 4.22: Coordinate geometry to place an R-card in its proper position. Both translation and rotation are performed.
Figure 4.23: Geometry of a 3D center-fed reflector with four trapezoidal R-cards.
3D Aperture Distribution

Evaluation of the R-card resistance is based on the GO aperture distribution with a 20 dB Kaiser-Bessel taper as done in the 2D case. For the center-fed reflector, the aperture distribution is symmetrical and can be expressed by

\[ f_{ap}(x, y) = f_{go}(x, y) \times \text{taper}(x, y) \quad \text{.} \quad (4.23) \]

The \( \text{taper}(x, y) \) function is defined over the square aperture by

\[
\text{taper}(x, y) = \begin{cases} 
1 & t < Q', \quad \text{and} \\
1 - \left[ \frac{1 - \text{win}(t)}{1 - \gamma} \right] & \text{elsewhere}
\end{cases}
\quad (4.24)
\]

where

\[
t = \max(|x|, |y|)
\]

\[
\gamma = 10^{-b/20}
\]

\[
Q' = \text{the GO zone size (equal to or slightly bigger than the test zone)}
\]

\[
b = \text{the edge taper value in dB (usually 20 dB), and}
\]

\[
\text{win}(t) = \text{the Kaiser-Bessel taper window over the range } t = 0 \rightarrow 1
\]

where \( \text{win}(0) = 1 \) and \( \text{win}(1) = \varepsilon \).

A 20 dB tapered aperture with \( Q' = 0.5 \) is shown in Figure 4.24.

A recursive algorithm, based on backward ray tracing, is used to evaluate the R-cards' resistance distribution. Figure 4.25 shows a backward ray tracing for a point on the R-card in front of the upper reflector edge. The basic idea of this algorithm is to follow the intersection points of the rays with the R-card until one reaches a point with known resistance. In general, this point is in the air outside the R-card and therefore its resistance is infinity and the transmission coefficient is unity. The advantage of this algorithm is that there is no need to 1) sort the R-card patches, 2) start the resistance evaluation from a certain region on the R-card, 3) develop a search routine, or 4) create an interpolation process as done in the 2D case. The resultant R-card resistance distribution is shown in Figure 4.26.
The calculated fields, after first and second iterations, along the central vertical cut in the test zone for the above defined center-fed configuration at 3 and 4 GHz, are shown in Figures 4.27-4.30. Note the similarity of these results with the previous 3D-strip results. One can also note that at 3 GHz the second iteration output is a little bit different from the first iteration fields. This happens because the double curvature of the reflector, plus the larger size of the flat R-cards is now trapping more collimated energy which bounces back off the reflector to appear in the second iteration. Fortunately, this difference is small and it can hardly be observed at 4 GHz. Note the difference in the relative levels of the PO and GO fields after widening the reflector and in the presence of the 2 sided R-cards.

IPO terms can still be checked and compared with the 3D-strip case. Figures 4.31-4.34 show the reflector and the lower R-card IPO terms at 3 and 4 GHz. The major difference between these results and the 3D-strip results appears in the second iteration term of the reflector fields. This term is relatively higher than the 3D-strip case. This confirms the above remark that the curvature of the paraboloid and the four R-cards all around the reflector edge have a new interaction effect of trapping some more energy that shows up in the second iteration. This was not the case for the vertical 3D-strip case where the strip was narrow enough to hide the effect of the horizontal curvature of the reflector and there were no side R-cards that might share the upper and lower R-cards in blocking some of the collimated or bounced energy off the reflector. In spite of this only difference, good agreement is observed between the complete reflector and the 3D-strip reflector case.
4.1.3 A Drawback

One important issue in the 3D R-card design, based on the 2D one, is the spacing between the reflector edges and the R-cards. Since the 3D reflector is double-curved, any of the four edges is a parabolic contour which cannot keep a constant distance away from a flat R-card. It can be easily seen in Figure 4.23 where the distance between any reflector edge and its corresponding R-card is maximum in the middle and minimum at the reflector corners. While designing the 2D R-cards for this case, the R-cards were kept far enough from the reflector edge such that they did not intersect with the reflector surface. This is not the case for the 3D reflector. This represents a constraint on the R-cards' design since it is always preferable to place the R-cards as close as possible to the reflector edge to block/attenuate the diffracted fields before they spread out. The central region of the reflector edges is the major source of the stray rays going into the test zone as will be seen in the next section. The negative effect of this undesired large distance between the reflector edges and the R-cards shows up for the other polarization. For TE polarization in the 2D case, the test zone fields are shown in Figure 4.35. In the case of the 3D reflector, the test zone fields produced by a vertically polarized feed in a vertical central cut are shown in Figure 4.36. One should notice the reduction in the ripple level still exists but is not of the same amount as for the other polarization. The reason that the effect of the large distance between the reflector and the R-card does not appear in the TM polarization is because the R-card attenuation for large angles of incidence for this polarization is greater and the reflector edge diffraction itself is smaller. Placing the R-cards closer to the reflector edge helps to increase the angle of incidence on the R-cards for both the incident and diffracted waves. Recall that the R-card transmission
is polarization independent for the case of normal or near incidence. This drawback will be reduced for the offset reflector in Section 4.2, where a new solution will be offered.
Figure 4.24: Pre-assumed GO aperture distribution with a 20 dB Kaiser-Bessel taper.

Figure 4.25: Backward ray tracing to synthesize the R-card resistance point by point.
Figure 4.26: Resistance distribution of the trapezoidal R-cards.
Figure 4.27: Test zone fields at 3 GHz in a vertical central cut after the first IPO iteration for the 3D configuration.

Figure 4.28: Test zone fields at 3 GHz in a vertical central cut for the 3D configuration (second iteration).
Figure 4.29: Test zone fields at 4 GHz in a vertical central cut after the first IPO iteration for the 3D configuration.

Figure 4.30: Test zone fields at 4 GHz in a vertical central cut for the 3D configuration (second iteration).
Figure 4.31: Test zone IPO field components produced by the 3D reflector at 3 GHz.

Figure 4.32: Test zone IPO field components produced by the 3D lower R-card at 3 GHz.
Figure 4.33: Test zone IPO field components produced by the 3D reflector at 4 GHz.

Figure 4.34: Test zone IPO field components produced by the 3D lower R-card at 4 GHz.
Figure 4.35: Test zone fields at 4 GHz for the 2D configuration in case of TE polarization.

Figure 4.36: Test zone fields at 4 GHz in a vertical central cut for the 3D reflector fed by a vertically polarized feed.
4.2 R-card Design for a 3D Offset Reflector

An offset reflector is typical for compact range applications to avoid feed blockage problems. The R-card fence treatment should be applied to the 3D offset reflector. In the previous section, three major steps were used to illustrate the R-card edge treatment for a 3D center-fed reflector. These steps can also be applied to the offset case as studied here. Starting from the 2D case and going through the 3D reflector strips, a 3D offset reflector can have complete R-card fences all around the edges. The central regions of the reflector edges will require most of the attention and care in the design since they contribute the most to the undesired diffracted fields in the test zone. The R-cards will be carefully segmented to preserve the required small distance between the reflector edges and the R-cards.

4.2.1 Reflector Edge Diffraction

From the geometrical theory of diffraction, each incident ray that hits the reflector edge produces a cone of diffracted rays as shown in Figure 4.37(a). Only some of these diffracted rays corrupt the fields in the test zone. Those cones originating from the central regions of the reflector edges spread their diffracted rays into the test zone as shown from the front and side views in Figure 4.37(b) and (c). The cones close to the edge corners spread the diffracted rays away from the test zone. This can be illustrated from an observation point located in the test zone fields. Figure 4.38 shows the intersection lines of the diffracted cones from different reflector edges with a plane perpendicular to the reflector axis and located in test zone. It is verified that the central regions of the reflector's four edges are the main source of stray fields in the test zone and, consequently, more attention should be paid to these edge sections.
Fortunately, as shown in Section 4.1, the 2D R-card design is applied to the 3D reflector strips instead of the entire reflector. Since the treated edges of these strips model the diffraction from these important regions, the required attention is automatically taken care of as will be seen in the next section.
Figure 4.37: Cones of diffracted rays for a 3D offset reflector with a square rim shape.
Figure 4.38: Contamination of the test zone field by diffracting rays emanating from the reflector edges.
4.2.2 3D Reflector Strips

For the 3D offset reflector, a vertical strip corresponds to the 2D offset case while a horizontal strip corresponds to a 2D center-fed case. This means that two 2D R-card designs are needed; one for an offset reflector and another for a center-fed reflector.

**Vertical Strip**

As done before, from the 2D offset reflector case the R-card geometry can be determined to achieve a low ripple amplitude in the test zone. As a result, two R-cards, whose parameters are listed in Table 4.3, are placed in front of a vertical strip,

<table>
<thead>
<tr>
<th>R-card</th>
<th>x₀</th>
<th>y₀</th>
<th>z₀</th>
<th>αₓ</th>
<th>αᵧ</th>
<th>αz</th>
<th>Bₘ</th>
<th>L</th>
<th>Bₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower</td>
<td>0</td>
<td>0.0827</td>
<td>0.1418</td>
<td>27°</td>
<td>0°</td>
<td>0°</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Upper</td>
<td>0</td>
<td>1.5962</td>
<td>0.7566</td>
<td>-10°</td>
<td>0°</td>
<td>180°</td>
<td>0.5</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: R-card parameters for the vertical reflector strips. Dimensions are in meters.

as shown in Figure 4.39. The obtained test zone fields with reduced ripple amplitude are shown in Figure 4.40. Note that the feed is vertically polarized in this case.

The lower reflector edge diffracts more than the upper edge since the incident field is stronger (the 1/r range effect). The design of the lower R-card is more critical in this case and is more difficult to achieve because the R-card blocks/attenuates both the incident and the reflected fields in almost equal amounts. Note that the central point of the lower edge is on the paraboloid axis and the incident and reflected rays around this region are very close to being in opposite directions, which results in a similar angle of incidence on both sides of the lower R-card.

75
One important issue that should be noticed here is that the R-cards are made very close to the reflector edge, as the design demands. This means that extending the width of the reflector strip won’t be accompanied by extending the R-cards in the same plane, as will be seen later.

**Horizontal Strip**

In the case of the vertical strips, the incident, reflected and diffracted fields, the two diffracting points, and the probing cut, all lie in the same plane, the only plane of symmetry of the offset reflector. This makes the 2D-to-3D analogy of the R-card design very direct. Unfortunately, this is not the case when it comes to the horizontal reflector strip where the incident and reflected rays are not in the same plane. This affects the choice of orientation of the R-cards. The orientation of the R-cards that is brought from the 2D geometry can be applied either relative to the feed (incident rays) or to the test zone (reflected rays). It is found that having the R-card orientation applied relative to the reflected field is more rewarding in terms of reducing the amplitude of the ripple in the test zone fields. This comes from the fact that the R-cards usually have a greater impact on attenuating the diffracted fields with the reflected fields than on attenuating the incident fields that hit the reflector edge. The resulting strip test zone fields are shown in Figure 4.42.

The geometry of a horizontal strip with the two designed R-cards is shown in Figure 4.41. The R-cards are symmetrical and the parameters for one of the R-cards are listed in Table 4.4.
Table 4.4: R-card parameters for the horizontal reflector strip. Dimensions are in meters.

<table>
<thead>
<tr>
<th>R-card</th>
<th>$x_o$</th>
<th>$y_o$</th>
<th>$z_o$</th>
<th>$\alpha_x$</th>
<th>$\alpha_y$</th>
<th>$\alpha_z$</th>
<th>$B_m$</th>
<th>$L$</th>
<th>$B_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right</td>
<td>0.7719</td>
<td>0.9</td>
<td>0.4819</td>
<td>-26.4741°</td>
<td>18.5557°</td>
<td>-99.0055°</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Figure 4.39: Geometry of a vertical strip of an offset reflector with two R-cards.
Figure 4.40: Test zone fields at 3 GHz in a vertical central cut for the vertical reflector strip.

(a) 3D view

(b) Top view

Figure 4.41: Geometry of a horizontal strip of an offset reflector with two R-cards.
Figure 4.42: Test zone fields at 3 GHz in a horizontal central cut for the horizontal reflector strip.
4.2.3 Assembly of the R-card Segments

It is now time to extend the reflector strips and the R-cards to the whole and complete geometry. Since the R-cards are required to be close to the reflector edge, more than one R-card segment can be placed in front of each edge. The orientation of these segments should match both the curvature of the reflector edge and the design requirement for performance. The way that the R-cards are placed following the reflector edge curvature is described in Appendix A. The number of the R-card segments should be a minimum to avoid any new diffraction mechanisms coming from the wedges formed by any two adjacent segments. Three R-cards per edge is a good choice. The segments will look parallel to the reflector edge as shown in Figure 4.43. The distance between the reflector edge and the R-cards’ segments is different from one edge to another.

After having the R-card segments match the reflector edges, the orientation borrowed from the strip design should take place. The three R-card segments, that belong to the same reflector edge, rotate together around one axis that passes through the center of the middle segment. The axis of rotation is horizontal for the upper and lower R-card segments and it is vertical for the side ones. For the side R-cards, while the horizontal axis of rotation completely lies in the plane of the central segment, a tilted axis of rotation, that also lies in the plane of the central segments, does not provide as well as the vertical axis. Because of the choice of the vertical axis of rotation for the side R-cards, an undesired tilt, with respect to plane of symmetry, can be easily observed in the front view. This can lead to narrowing the lower part of the utilized test zone and affecting the uniformity of the required fields there. This can be adjusted in the process of the R-card trimming. Figure 4.44 shows the R-card
segments after their appropriate rotation.

To complete the assembly of the R-cards segments, one should force the R-card segments at the reflector corners to intersect in an acceptable manner. This can be done as shown in Figure 4.45.

Ten, out of the 12 segments, have been cut and trimmed. Only the two central segments of the upper and lower groups remain the same. The eight segments at the reflector corners now form a continuous surface. The central segment of each group of the side segments is forced to have a vertical inner edge to solve the tilt problem, mentioned above, so that the size of the test zone is preserved. Figure 4.46 shows the arrangement for the final R-card segments and their coordinates which are listed in Table 4.5.

<table>
<thead>
<tr>
<th>Edge</th>
<th>R-card</th>
<th>Corners (x, y, z)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>#1</td>
</tr>
<tr>
<td>E=1</td>
<td>1</td>
<td>0.300,-0.185,0.006</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.300,-0.185,0.006</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.936,-0.240,0.156</td>
</tr>
<tr>
<td>E=2</td>
<td>4</td>
<td>-0.936,-0.240,0.156</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.999,0.619,0.260</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>-1.088,1.181,0.451</td>
</tr>
<tr>
<td>E=3</td>
<td>7</td>
<td>-0.617,1.464,0.860</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>-0.300,1.350,0.800</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.300,1.350,0.800</td>
</tr>
<tr>
<td>E=4</td>
<td>10</td>
<td>0.500,1.181,0.725</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>0.500,0.619,0.493</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.676,0.307,0.362</td>
</tr>
</tbody>
</table>

Table 4.5: Coordinates of the R-card segments. Dimensions are in meters.

The new R-card segments, except for the unchanged ones, are no longer trapezoidal in shape. A new discretization for a general quadrilateral shape is now needed.
for the PO analysis. A general discretization for a quadrilateral is described in Appendix B.
Figure 4.43: Mounting the R-card segments in front of the reflector edges.
Figure 4.44: Proper orientation to the R-cards in front of each edge of the offset reflector.
Figure 4.45: Cutting and trimming the R-cards to form a continuous surface.
Figure 4.46: Arrangement of the R-card segments and their coordinates.
4.2.4 Synthesis of the R-card Resistance

Aperture Distribution

Since for an offset reflector, the aperture has only one axis of symmetry, the simple expression for the aperture distribution that was used in the case of the center-fed reflector in Section 4.1 is no longer valid. A more general aperture, based on Figure 4.47 is given by

\[ f_{ap}(x, y) = f_{go}(x, y) \times taper(x, y). \]  \hfill (4.25)

The taper\((x, y)\) function is defined over the rectangular aperture by

\[ taper(x, y) = \begin{cases} 1 & x_1 < x < x_2, \ y_1 < y < y_2, \ \text{and} \\ 1 - \left[ \frac{1 - \text{win}(t)}{1 - \epsilon} \right] \cdot (1 - \gamma) & \text{elsewhere} \end{cases} \]  \hfill (4.26)
where $\epsilon$ and $\gamma$ are the same as defined before and $t$ depends on location $(x, y)$ and is given by

$$ t = \left| \frac{\eta - \eta_t}{\eta_E - \eta_t} \right|. \quad (4.27) $$

For Region 1: $\eta = y$,  
$\eta_t = y_1$,  
$\eta_E = y_{off} - D/2$

Region 2: $\eta = x$,  
$\eta_t = x_1$,  
$\eta_E = -D/2$

Region 3: $\eta = y$,  
$\eta_t = y_2$,  
$\eta_E = y_{off} + D/2$

Region 4: $\eta = x$,  
$\eta_t = x_2$,  
$\eta_E = D/2$

Note that $x_2 = -x_1$ and the aperture can be a square ($D \times D$), as in our example, but the central region of uniform field, where $(taper(x, y) = 1)$, is generally a rectangle. This is due to offset of the reflector. For the parameters listed in Table 4.6, the aperture distribution is shown in Figure 4.48.

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$\gamma$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>1.4</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 4.6: Aperture parameters for $D = 1.8$ reflector. Dimensions are in meters.
Figure 4.48: The Kaiser-Bessel aperture distribution for the 3D offset reflector case.
Figure 4.49: R-card resistance should be determined along the radial directions which represent the projection of the traced ray between the aperture and the feed.

**Calculation of the R-card Resistance**

With the aid of ray tracing, the resistance is calculated point by point on different R-card segments to achieve the pre-assumed aperture distribution.

The calculation process of the 2D resistance distribution is better described as if it is the calculation of a 1D resistance distribution along a radial line as shown in Figure 4.49. These radials (\( \phi = \text{constants} \)) represent the projection of the traced rays between the aperture and the feed. While over each radial direction the resistance distribution at different points is dependent, although, it is independent of the resistance over other radials. The conversion between the cylindrical coordinates, \( \rho, \phi \) and the rectangular coordinates \( x, y \) needs to be used in both directions. This allows one to formulate an equation for the normalized resistance, \( \tilde{R}(\rho) = R(\rho)/Z_0 \) at \( \phi = \)
constant, as discussed below and as shown in Figure 4.50. This is not exactly the same as the 2D problem solved before because of the following two major differences:

1. the normal vector \( \hat{n} \) of the R-card may not be unique because of different segments, and

2. depending on \( \phi \), there may or may not be a gap or portion that should have infinite resistance.

Since \( \phi = \text{constant} \), \( \rho \) is the only independent variable over each radial. From Figure 4.50, the product of the two transmission coefficients \( T1 \) and \( T2 \) should satisfy

\[
T1(\rho_i) \times T2(\rho_r) = \frac{f_{sp}(\rho_r)}{f_{go}(\rho_r)} .
\]

(4.28)
Replacing $T_1$ and $T_2$ with their TM-polarization expressions leads to the following equation in the $\tilde{R}$:

$$\frac{2\tilde{R}(\rho_i) \cos \theta_i}{1 + 2\tilde{R}(\rho_i)} \cdot \frac{2\tilde{R}(\rho_r) \cos \theta_r}{1 + 2\tilde{R}(\rho_r)} = \frac{f_{ap}(\rho_r)}{f_{go}(\rho_r)}. \quad (4.29)$$

The angles, $\theta_i$ and $\theta_r$ are the angles of incidence of the incident and reflected rays, respectively, on the R-cards, as shown in Figure 4.50. The right hand side in Equation (4.29) is known from the aperture distribution. The left hand side relates the unknown $\tilde{R}$ at two different points, $\rho_i$ and $\rho_r$, which shows the dependency of the resistance along the radial. A simpler form of Equation (4.29) can be reached, after some arrangements, to obtain that

$$g_i g_r + g_i + g_r = h_{ir} \quad (4.30)$$

where

$$g_{ir} = \frac{1}{\tilde{R}(\rho_{ij})} \quad (4.31)$$

and

$$h_{ir} = 2 \left[ \frac{\cos \theta_i \cos \theta_r}{f_{ap}(\rho_r)/f_{go}(\rho_r)} - 1 \right]. \quad (4.32)$$

If $g$ is discretized into $N$ unknowns over the whole range of $\rho$, Equation (4.30) will produce a set of $N$ nonlinear equations in $N$ unknowns. The nonlinearity makes it difficult to solve instantaneously. Instead, Equation (4.30) can be solved recursively utilizing the fact that in most cases, the resistance is known over a portion of the range of $\rho$. Recall that the resistance is always infinity at the central region of the aperture where there is no R-cards. Thus, starting from any point on the R-card, one can trace the rays towards that region where the resistance is known and from
there, Equation (4.30) can be solved recursively until one returns to the starting point where the value of the resistance is required. The radials can be classified in terms of either:

- **Group A:** where \(|\pi/2 - \phi| < \pi/2 - \phi_{FB}\), or
- **Group B:** where \(|\pi/2 - \phi| \geq \pi/2 - \phi_{FB}\)

The recursive algorithm is divided into 3 major logical directions depending on the location of the starting point on the R-card where the resistance is to be found (see Figure 4.51). These directions are:

![Diagram of the R-card resistance synthesis algorithm](image-url)
1. Backward ray tracing mainly for the points on the upper and side R-cards. These points are located on the radials of group A.

2. Forward ray tracing mainly for the lower R-card points that are located on the radials of group A.

3. For group B radials, the rays are traced backward until the incident and reflected rays lie within a tube of a small radius $\delta$ around the reflector axis. Inside this tube, one can approximately consider that the incident and the reflected rays coincide on each other and hence $g_i = g_r$ in Equation (4.30) which becomes a simple second degree equation.

The angle $\phi_{FB}$ separates those radials that intersect with the inner sides$^2$ of the projected air opening in the R-card frame from those that do not intersect. In other words, the incident and reflected rays beyond $\phi_{FB}$ always intersect with the R-cards and do not have access to the air; thus, an alternative point, whose resistance is known, should start the recursive solution of Equation (4.30) as explained in the third item in the above list.

Calling the ray tracing backward or forward comes from the fact that the electromagnetic rays start from the feed and propagate to the aperture in the transmission mode. This defines the forward direction. The opposite direction, from the aperture to the feed, is called the backward direction.

The basic features of the backward ray tracing are described in the flowchart shown in Figure 4.52. The structure of the algorithm consists of two loops:

$^2$inner ring that consists of the inner sides of the R-card frame.
Figure 4.52: Backward ray tracing recursive algorithm for R-card resistance synthesis.
1. the ray tracing loop, which is shown in detail in the flowchart, determines, counts and stores the points necessary to solve Equation (4.30) recursively. The first point \( N = 1 \) is on the R-card where the resistance to be found and the last point should be in air. Information such as the dot products, \( \text{dot}^c = \hat{n} \cdot \hat{s}_r \), \( \text{dot}^i = \hat{n} \cdot \hat{s}_i \) and the taper, \( f_{ap}/f_{go} \), has to be stored during this loop since they are needed to calculate the resistance in the second loop. Detecting the last point in air is performed by checking if the incident ray is enclosed by the inner ring or not. Also detection if the intersection point is still on the same R-card or not is done as described in details in Appendices C and D.

2. a resistance calculation loop of size \( N \), as represented by the last block in the flowchart (exactly similar to the 2D case [13]), is easily performed using the collected information about the \( N \) points in the ray tracing loop.

The forward ray tracing algorithm is now easy to understand since its structure has similar elements as shown in Figure 4.53. The final block represents the second loop which can be reviewed from the 2D case.

To avoid extraneous complexity which adds little to the basic concept, it is recommended to avoid going into further detail of the above outlined algorithms. (Of course, one has to face these complicated details and some special cases while writing the computational code.) Switching to the final results at this stage should be sufficient for now. As an outcome of the resistance synthesis process in our offset reflector with the 20 dB Kaiser-Bessel aperture distribution, the resultant resistance distribution of different R-card segments is shown in Figure 4.54. A truncation of the high resistance values at 3000 \( \Omega \) is the case here as well. The interaction between different R-card segments and the effect of the discontinuity in the normal vector from one
Figure 4.53: Forward ray tracing recursive algorithm for R-card resistance synthesis.

Segment to another are clearly shown in the resistance distribution. By obtaining the R-card resistance, the design of the R-card is now complete and the probed fields in the test zone should be checked, as will be shown in the following section.
Figure 4.54: The resistance distribution for the different R-card segments.
4.2.5 Final Results

By performing the IPO integration on the reflector and the R-cards, the test zone fields are obtained in vertical and horizontal cuts through the center of the test zone. The 3D IPO is still not very efficient in terms of running time at high frequencies. For this reason, the fields will be calculated only at 3, 4, and 6 GHz for vertical polarization. The results are shown in order in Figures 4.55-4.57, respectively. The improvement in the fields with this R-card design is quite clear from the results. The agreement with the reflector strip result is also very noticeable.

4.3 Summary

The R-cards are designed for a 2D reflector that corresponds to the 3D geometry. Enough space between the R-cards and the reflector edge should be taken into consideration in the 2D R-card design. The 2D problem offers fast and accurate optimum R-card parameters. Taking these R-card parameters and applying them to the 3D reflector strip results in similar improved test zone fields. Different IPO terms show excellent agreement between the 2D and the 3D strip cases. Small differences, resulting from the feed pattern and the curvature of the reflector, hardly affect the agreement of the results in both cases. From the strips, the total 3D reflector is treated by a symmetrical frame of four R-cards. The R-cards are trapezoidal in shape and go through discretization, rotation and translation operations. The resistance distribution is synthesized by a recursive ray tracing algorithm to achieve a 20 dB tapered Kaiser-Bessel aperture distribution.

The complete design of the R-card fences for the offset reflector with a square aperture is accomplished and proven to achieve the ripple reduction goal in the test...
zone fields. The R-card design for the 2D reflector was initially obtained and then applied to the reflector strips. Then the arrangement for the R-card segments were forced to follow the curved reflector edge. Then, the R-card segments were connected in a proper way. One axis of symmetry was removed from the Kaiser-Bessel distribution expression to adapt for the special offset case. The resistance synthesis algorithm were explained in outline form and applied to obtain the R-card resistance distribution. Finally, the test zone fields were calculated to validate the design.
Figure 4.55: Test zone fields for the offset reflector at 3 GHz.
Figure 4.56: Test zone fields for the offset reflector at 4 GHz.
Figure 4.57: Test zone fields for the offset reflector at 6 GHz.
CHAPTER 5

R-CARD FENCE DESIGN FOR A CIRCULAR RIM COMPACT RANGE REFLECTOR

It is of great interest to generalize this R-card fence design for different reflector rim shapes. A circular rim reflector is popular and easy to obtain. Thus, design procedures for the R-card treatment for a reflector of a circular rim are discussed in this chapter.

The R-card design herein is more biased towards practical issues which make it easier to manufacture. For example the resistance of the R-card is difficult to realize point-by-point as done before. Instead, a stair step approximation is applied. This approximation is discussed in Section 5.1 for the 2D problems which will be the starting point of the solution of the 3D circular reflector. The steps to find the geometry of the R-card is introduced in Section 5.2. Section 5.3 presents two options of applying the discretized resistance distribution showing the results for each option. A comparison in the performance is also found in the same section with an explanation why they differ. This chapter concludes with a summary.
5.1 R-card Resistance Discretization and 2D Design

The speed and the efficiency of finding the R-card design using a 2D problem were utilized before starting the 3D problem. Both vertical and horizontal cuts were represented by two 2D problems; offset and center-fed reflector, respectively. The optimum parameters for the R-card geometry were obtained. The R-card resistance distribution was synthesized and approximated for practical implementation. The results show how much this approximation affects the performance.

5.1.1 R-card Resistance Discretization

While R-cards, of a continuous tapered resistance distribution, are usually very expensive to obtain, R-card sheets, of certain constant resistance values, are much less expensive. This encourages the designer to approximate the continuous R-card resistance by a stepped approximation and then evaluate its performance.

If two R-cards, of resistance values $R_1$ and $R_2$, are placed on top of each other, this results in an equivalent R-card whose new resistance value is the parallel combination or $R_1 R_2 / (R_1 + R_2)$. Thus, proper sizes of a number of different R-card sheets can be stacked to approximate any required resistance distribution. This will be demonstrated in the following examples which represent the 2D problems for the 3D case.

5.1.2 2D Offset Problem

The 2D offset, edge-treated, reflector system, shown in Figure 5.1, has two R-cards with tapered resistance values designed according to the procedures discussed in Chapter 2 and [13]. The approximation of this resistance can now take place as
shown in Figures 5.2 and 5.3 for the upper and lower R-cards, respectively. The stair steps are generated by finding points on the original distribution that match the available levels produced by different combinations of the R-cards. These points can be considered the matching points. The location of each stair step is midway between two successive matching points. There are 10 sheets for the upper R-card and 8 for the lower one. Their resistance values are listed in Table 5.1.2. These certain values are given and limited to the availability of the inexpensive R-card sheets. From the highest resistance, which is \(3000 \Omega\), one starts building the stack up by adding more sheets with proper resistance values at the proper locations. It is preferred to keep the height of the stair steps as small as possible. This is difficult to achieve at high resistance values where using two \(3000 \Omega\) sheets results in a 50\% reduction to 1500 \(\Omega\) level as shown in Figures 5.2 and 5.3. Fortunately, the taper rate is fast in the high resistance region. Adding a third \(3000 \Omega\) sheet takes the distribution to the 1000 \(\Omega\) level at the proper location. Although adding \(3000 \Omega\) sheets in shifted location guarantees the minimum heights of the stair steps, sheets of smaller resistance values are used to approximate the rest of the taper for two main reasons. The first reason is to prevent the number of sheets in the stack from being very large and the second reason is that sheets of smaller resistance values have better performance (tolerance) and are less expensive. Note that keeping the stair steps wider means a smaller number of sheets per the stack.

The validity of this approximation on the performance should now be checked out. The cross range test zone fields at a down range distance of \(x_{ap} = 100\) inches, before and after discretizing the R-card resistance, are shown in Figures 5.4 and 5.5 for TM and TE polarizations, respectively. Both the magnitude and phase of these fields are
calculated at 3, 6, and 12 GHz. It can be noticed that the difference between the results of the continuous resistance and the discrete one is small especially at 3 GHz. As can be expected, when the frequency increases, the approximation error slightly increases. At 12 GHz, the performance of the stacked R-card is still very good for both polarizations. It is also noticed that the amplitude of the test zone fields is less sensitive to the resistance approximation than the phase of the fields. In practice, at 12 GHz the phase will be also sensitive to other different factors (e.g, surface roughness) that are not modeled in this simulation. The results show amplitude variations of ±0.25 dB and phase variations of ±2° in most of the test zone area.

<table>
<thead>
<tr>
<th></th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
<th>R₄</th>
<th>R₅</th>
<th>R₆</th>
<th>R₇</th>
<th>R₈</th>
<th>R₉</th>
<th>R₁₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper R-card</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>1150</td>
<td>1150</td>
<td>375</td>
<td>247</td>
<td>131</td>
<td>55</td>
<td>36</td>
</tr>
<tr>
<td>Lower R-card</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
<td>1150</td>
<td>1150</td>
<td>375</td>
<td>247</td>
<td>131</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Values of the resistive sheets in the R-card stack placed in order.
Figure 5.1: Geometry of the 2D offset reflector with R-card fence.
Figure 5.2: Stair step discretization for the upper R-card resistance distribution treating the 2D offset reflector.

Figure 5.3: Stair step discretization for the lower R-card resistance distribution treating the 2D offset reflector.
Figure 5.4: Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D offset reflector shown in Figure 5.1 at 3, 6 and 12 GHz for the TM polarization. Note that the stair step approximation of the R-card resistance has an insignificant effect on the performance.
Figure 5.5: Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D offset reflector shown in Figure 5.1 at 3, 6 and 12 GHz for the TE polarization. Note that the stair step approximation of the R-card resistance has an insignificant effect on the performance.
5.1.3 2D Center-fed Problem

Another example that can verify the same results obtained with the stair stepped R-card resistance is the 2D center-fed reflector shown in Figure 5.6. The R-card geometry and the resistance distribution have been optimized for minimum variations in the test zone fields. The resistance distribution is approximated by stair steps as shown in Figure 5.7. A stack of 10 R-card sheets, exactly similar to the upper R-card in the offset case, is also used for this case. The matching point locations are determined again from the intersection between the continuous resistance distribution and the available discrete resistance levels. The test zone field results are plotted in Figures 5.8 and 5.9 for TM and TE polarizations, respectively and they actually show better agreement between the continuous and approximated resistance than the previous offset example especially at 6 and 12 GHz.

5.2 3D Geometry of the R-cards

5.2.1 Shaped R-cards

The 2D R-card design provides valuable information about the optimum R-card 3D geometry that is useful in the horizontal and vertical cuts ($\phi = 0, 180^\circ$ and $\phi = \pm \pi/2$). From this information, it is required to determine the geometry of the R-cards such that they properly cover the circular reflector rim. In this section the R-cards are defined by four points, not necessarily planar, since flat R-card segments will be considered later.

One possible way to force the R-cards to follow the shape of the reflector rim is the set-up shown in Figure 5.10. The geometric parameters$^3$ of the two lines, $A_1B_1$

$^3$position, orientation, and size
Figure 5.6: Geometry of the 2D center-fed reflector with R-card fence.
Figure 5.7: Stair step discretization for the upper R-card resistance distribution treating the center-fed reflector.

and $A_2B_2$, are borrowed from the 2D R-card designs presented in Section 5.1. These two lines are the boundaries of the shaped R-card that should cover the first quarter of the reflector rim defined from $\phi = 0$ to $\phi = 90^\circ$ and represented by the arc $P_1P_2$. To get the R-card geometry, $A_1B_1$ is moved to reach $A_2B_2$ in a linear transition with respect to $\phi$. This is best simulated by transforming the arc $P_1P_2$ to a straight line, as shown in the figure, then uniformly rotating and stretching the vectors, $\overrightarrow{P_1A_1}$ and $\overrightarrow{P_1B_1}$ at $\phi = 0$ until they reach $\overrightarrow{P_2A_2}$ and $\overrightarrow{P_2B_2}$ at $\phi = 90^\circ$. The tips of these vectors draw contours that represent the edges of the shaped R-cards when transformed back to the arc $P_1P_2$. For example if the shown quarter is to be covered by $N$ R-cards, then

$$\overrightarrow{A_k} = \overrightarrow{A_1} + (\overrightarrow{A_2} - \overrightarrow{A_1})(k - 1)/(N - 1)$$

and

$$A_{k_j} = A_{j_1} + (A_{j_2} - A_{j_1})(k_j - 1)/(N_1 - 1)$$

(5.1)
Figure 5.8: Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D center-fed reflector shown in Figure 5.6 at 3, 6 and 12 GHz for the TM polarization. Both continuous and stair stepped R-card resistance are used.
Figure 5.9: Magnitude and phase of the total cross-range test zone fields of the R-card treated, 2D center-fed reflector shown in Figure 5.6 at 3, 6 and 12 GHz for the TE polarization. Both continuous and stair stepped R-card resistance are used.
Figure 5.10: A sketch showing the idea of shaping the R-cards in front of the reflector rim. The R-cards uniformly twist between the two given lines, $A_1B_1$ and $A_2B_2$, along a quarter of the circular rim after being transformed to a straight line.
\[ \tilde{B}_k^t = \tilde{B}_1^t + (\tilde{B}_2^t - \tilde{B}_1^t)(k - 1)/(N - 1) \]  

(5.2)

where the transformed vectors \( \tilde{A}_{1,2}^t \) and \( \tilde{B}_{1,2}^t \) are given by

\[ \tilde{A}_{1,2}^t = \tilde{A}_{1,2} \quad \text{and} \]
\[ \tilde{B}_{1,2}^t = \tilde{B}_{1,2}. \]  

(5.3)

(5.4)

Transforming back to the reflector rim can be done using

\[ \tilde{A}_k = \tilde{A}_k + \tilde{P}_k \quad \text{and} \]
\[ \tilde{B}_k = \tilde{B}_k + \tilde{P}_k \]  

(5.5)

(5.6)

where

\[ \tilde{P}_k = \tilde{P}(\phi_k) \quad \text{and} \]
\[ \phi_k = \phi_1 + \frac{k - 1}{N - 1}(\phi_2 - \phi_1). \]  

(5.7)

(5.8)

Even though a linear transition is chosen to achieve the gradual twist in the shaped R-card, it is arbitrary and can be any other profile. Note that this method can be applied to any rim shape other than the circular shape.

The resultant geometry of the R-cards is shown in Figure 5.11 where the total number of R-card segments is 40 (N=10 per each quarter). Note that these R-card segments are not yet flat. Flattening the R-cards will be done next.

### 5.2.2 Flat R-cards

Dealing with a large number of R-card segments is very tedious. The number of the shaped R-cards can be reduced from 40 to only 8 without any change in the concept discussed in the previous section. This results in the fence geometry shown
Figure 5.11: Shaping the R-cards all around the reflector edge to follow its circular shape. The shape starts from the 2D design for the vertical and horizontal cuts.
in Figure 5.12. Since the segments are now larger in size, they are further away from being flat. A considerable amount of effort has been spent to find a suitable way to obtain flat R-cards all around the reflector edge. The effort was basically directed to connect the four R-card obtained from the 2D design and shown in Figure 5.13. It was easy to connect the upper R-card to the side ones but the real challenge came from connecting the side R-cards with the lower one since their orientations were not very compatible. One can summarize this experience in only two methods that have been found reasonable and can be applied to obtain the desired flat R-cards.

**Method A: Triangular R-cards**

One simple way to force these quadrilateral segments to be flat is to divide every four-sided segment into two triangles forming a new wedge along the diagonal. This will double the number of the segments from 8 to 16. Since each segment has two diagonals, there are two ways to obtain the two triangles. If one believes that it is better to avoid adding discontinuities close to the test zone at the basic central cuts, then the triangles should be obtained as shown in Figure 5.14.

**Method B: Tangential R-cards**

Another way to obtain flat R-cards from the eight shaped segments shown in Figure 5.12 is by forming the tangential planes at the connection lines. Four planes are already defined in the horizontal and vertical cuts (from the 2D solution) and the task is to find the ones on the diagonals. The line $A_kB_k$ is already known in this plane and can be described as a vector $\vec{w}$ as shown in Figure 5.15. Since the required plane is a connecting plane that should blend and give the required twist to two known planes in the principal cuts, one can find another vector $\vec{u}$ that works
Figure 5.12: Eight shaped R-cards following the reflector rim while satisfying the 2D design in the vertical and horizontal cuts.
Figure 5.13: Geometry of the four R-cards obtained from the 2D to cover the basic vertical and horizontal cuts.
Figure 5.14: Sixteen flat triangular R-cards following the reflector rim while satisfying the 2D design in the vertical and horizontal cuts.
Figure 5.15: The idea of Method B to obtain the geometry of the flat R-cards on the diagonals.
with \( \vec{w} \) to define the desired plane. The vector \( \vec{u} \) can be either one of the following blending vectors

\[
\vec{u}_A' = \vec{A}_{k+1} - \vec{A}_{k-1} \quad \text{and} \quad (5.9)
\]
\[
\vec{u}_B' = \vec{B}_{k+1} - \vec{B}_{k-1}. \quad (5.10)
\]

Since no preference can be decided, it may look fair to take the average sense. This is achieved through the normal vectors \( \vec{n}_1 \) and \( \vec{n}_2 \) defined by

\[
\vec{n}_1 = \vec{u}_A' \times \vec{w} \quad \text{and} \quad (5.11)
\]
\[
\vec{n}_2 = \vec{u}_B' \times \vec{w}. \quad (5.12)
\]

Normalizing the above normal vectors and taking the average results in the unit normal vector \( \hat{n} \) of the desired plane of the R-card, one finds that

\[
\hat{n} = \frac{\vec{n}_1 + \vec{n}_2}{|\vec{n}_1 + \vec{n}_2|}. \quad (5.13)
\]

After the R-card plane is defined, two vectors, \( \vec{u}_A \) and \( \vec{u}_B \) that represent the R-card edges which pass through the points \( A_k \) and \( B_k \) can be found from solving the following equations

\[
\hat{n} \cdot \vec{u}_A = 0 \quad \text{and} \quad (5.14)
\]
\[
\hat{n} \cdot \vec{u}_B = 0 \quad (5.15)
\]

in the z-component, \( \vec{u}_{A,B} \cdot \hat{z} \) which results in 125
Finally one obtains \( \vec{u}_A \) and \( \vec{u}_B \) from

\[
\vec{u}_{A,B} \cdot \hat{z} = \frac{n_x u_{xA,B}^t + n_y u_{yA,B}^t}{n_z}.
\] (5.16)

The above procedures can be described as if \( \vec{u}_A \) and \( \vec{u}_B \) rotate around \( AB \) such that the vectors, \( \vec{u}_A, \vec{u}_B, \) and \( \vec{w} \) become co-planar (\( \hat{n} \)).

The vectors, \( \vec{u}_{A,B} \), are known for the four R-cards on the basic cuts. Finding the intersection of these vectors with the adjacent planes result in defining the common edges of the R-cards. After direct trimming, the final obtained R-card geometry is as shown in Figure 5.16.

The advantage of method B over method A in flattening the R-card comes from the performance point of view since the R-cards on the basic cuts perfectly match the 2D design requirement. Meanwhile, although the triangular R-cards may look better, they create many new undesired wedges which may be located at the basic cuts resulting in the optimum orientation of the R-card being disturbed.

5.3 R-card Resistance Discretization

The R-card resistance is easier to build by stacking different constant resistance sheets to form stair steps. From the experience of the 2D problem, the results are very good with this approximation as was shown in Section 5.1. The task now is to apply this approximation to the 3D problem. The optimum levels of the stair steps are already known on the four R-cards on the vertical and horizontal cuts. Two
Figure 5.16: Eight flat R-cards following the reflector rim while satisfying the 2D design in the vertical and horizontal cuts.
approaches to cover the rest of the R-cards will be demonstrated in the following sections.

5.3.1 Straight stair steps

Geometry

The stair steps can still be kept straight on each R-card even though the reflector has a circular rim. This has again a more practical issue since straight levels are always easier to manufacture. Thus, the resistance distributions on the four R-cards are already known along the vertical and horizontal cuts as shown in Figure 5.17. The resistance of the intermediate/diagonal R-cards can be obtained by blending the resistance of those four R-cards in the main cuts.
Using similar stacks of resistive sheets in the 4 basic R-cards results in the same step levels on the diagonal R-cards. The advantage of having the same levels of the stair steps can be efficiently utilized by connecting similar levels together.

This can be done by connecting the edges of the resistive sheets as shown in Figure 5.17. This requires the determination of the points on each R-card side edges. These side edges are shared by every two adjacent R-cards. Thus, the resistance on the diagonal R-cards can be obtained by connecting these points on their side edges. It may be easier to find these points in the (x-y) plane after projecting the whole R-card frame. Recall that the aperture distribution can always be expressed in terms of the coordinates $x$ and $y$ only since the projection of the $z$ coordinates does not affect the resistance distribution. The resulting projection is shown in Figure 5.17.

For the basic four R-cards, the levels can be expressed in terms of one dimension only; $x$ for the horizontal cut R-cards and $y$ for the vertical cut R-cards. For example, for the side R-cards, the resistance at the $k^{th}$ step is given by

$$R = R_k \quad \text{for} \quad x_k < x < x_{k+1}. \quad (5.18)$$

Such a simple condition can not be applied to the diagonal R-cards since the steps are not uniform. Instead, for each given point $(x,y)$, a search algorithm has to find which step level this point belongs to. The resultant resistance distribution is plotted in Figure 5.18.

**Test zone fields**

As usual, to check on the R-card performance, the fields are calculated in the test zone before and after placing the R-card fence. Figures 5.19-5.20 show the magnitude of the test zone fields at 3 GHz in the central, vertical and horizontal cuts for vertically
Figure 5.18: Straight stair stepped R-card resistance distribution.

and horizontally polarized feeds, respectively. As can be seen, the ripple reduction is significant, from about 6 dB to less than 1.5 dB, and it is almost the same for both polarizations. A similar set of results at 4 GHz are shown in Figures 5.21-5.22. At this frequency, one starts to see the ripple amplitude increase in the vertical cut to exceed 8 dB in the absence of the R-cards. This is caused by the circular nature of the reflector rim as will be explained later. The R-cards, whose resistance values are discretized as straight stair steps, succeeds in reducing the ripple amplitude to less than 3 dB. The horizontal cut shows much better performance.

5.3.2 Quasi-circular stair steps

Geometry

The fact that the reflector rim is circular inspires another way of setting up the resistance distribution. It can be formed in a quasi-circular shape like that shown in Figure 5.24. It is of course more difficult to cut the R-cards with arc edges this
Figure 5.19: Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is vertically polarized and $f = 3 \text{ GHz}$.

Figure 5.20: Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is horizontally polarized and $f = 3 \text{ GHz}$. 

131
Figure 5.21: Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is vertically polarized and $f = 4 \text{ GHz}$.

Figure 5.22: Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.17. The feed is horizontally polarized and $f = 4 \text{ GHz}$.
Figure 5.23: Geometry of the quasi-circular stair stepped R-card resistance.

Figure 5.24: Quasi-circular stair stepped R-card resistance distribution.
way but hopefully it is worth doing it in terms of performance. The equation of the quasi-circular resistance contours (k^{th} step) in the range $\phi_1 < \phi \leq \phi_2$ is given by

$$\rho_k(\phi) = \rho_k(\phi_1) + \frac{\phi - \phi_1}{\phi_2 - \phi_1}[\rho_k(\phi_2) - \rho_k(\phi_1)]$$

(5.19)

where $\phi_{1,2}$ are the basic cuts at $\phi = 0, \pm 90^\circ$, and $180^\circ$.

Test zone fields

The test zone fields, calculated before and after the R-card treatment, are shown in Figures 5.25-5.28. Compared with the set of the results shown in the previous section, one can easily see that by using quasi-circular discretized resistance, the ripple amplitude is significantly reduced at 4 GHz. This means that such resistance distribution is more effective in attenuating and blocking the reflector edge diffracted fields from every possible point on the circular rim.
Figure 5.26: Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.23. The feed is horizontally polarized and $f = 3$ GHz.

Figure 5.27: Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.23. The feed is vertically polarized and $f = 4$ GHz.
Figure 5.28: Magnitude of cross-range test zone fields in vertical and horizontal cuts for the R-card treated, reflector shown in Figure 5.23. The feed is horizontally polarized and $f = 4 \text{ GHz}$. 
5.3.3 Diffraction and caustic

It is helpful to visualize the diffraction mechanisms resulting from the reflector rim under investigation to grasp the diffraction and caustic problems associated with this circular rim geometry. As a result, the diffraction cones from the circular rim of this reflector are shown in Figure 5.29.

Unlike rectangular rim reflectors, where only the central regions of the edges diffract to contaminate the test zone, the whole rim contributes in contaminating the test zone in case of the circular rim. To convince the reader of this fact, the trajectories of the diffraction cones intersecting a plane through the center of the test zone \( z = 100 \) inch) are shown in Figure 5.30. Note that only selected points on the right half of the reflector rim are considered. As can be seen, the test zone is contaminated by every single point on the reflector rim. This means that careful and uniform attention has to be made when designing the R-card fence all around the circular rim. A maximum of four points of diffraction normally contribute to each observation point except one caustic point \( (P) \) which has the contributions from an infinite number of diffraction rays emanating from the rim. Ray solutions fail at this point.\(^4\) The PO solution shows that the amplitude of the diffracted field is very large in the vicinity of the caustic and has a maximum value at the point itself as shown in Figure 5.31. Recall that such behavior did not appear in the case of the 3D reflector with a rectangular aperture. Furthermore, the ripple amplitude for a narrow reflector strip was of the same value as the whole rectangular reflector. In the circular rim

\(^4\)The caustic moves towards the center of the circle when the observation plane moves further from the reflector.
Figure 5.29: Rays diffracting from the circular reflector rim.
Figure 5.30: The intersection of the diffraction cones with a vertical plane in the test zone. Only rays emanating from a few points on the right half of the reflector edge are shown.
Figure 5.31: PO diffracted field calculated at 4 GHz in the central vertical cut. Note that the diffracted field has a peak at the caustic location.

case, the ripple amplitude increases with the width of the reflector strip because of the convex rim shape that directs the diffraction rays more to the test zone [5],[16].

5.4 Summary

The design procedures for an R-card fence as an edge treatment for circular rim reflector have been discussed in this chapter. The design starts from the 2D solution where the optimum geometries of the R-cards in the vertical and horizontal basic cuts are obtained. The continuous resistance distribution has been successfully replaced by a discretized distribution with insignificant impact on performance. The results for the 2D designs, offset and center-fed, have been shown to demonstrate these facts.

The 3D R-card geometry was then designed by following the circular reflector rim with shaped R-cards. Flattening the R-cards takes place by generating tangential
planes from the shaped R-card frame. Best performance, especially in the vertical cut, was obtained when quasi-circular stair steps resistance were used.

Tracing the diffraction rays emanating from the reflector edge showed that 1) unlike the rectangular rim, every point on the circular rim contributes in the test zone diffracted field and 2) rays intersect at one point, that lies on the line of symmetry, to form a caustic. The diffracted fields peak at this point. This makes the circular rim design more complex because all the rim points contribute to this undesired caustic diffraction level.

By understanding the diffraction mechanism for the circular rim reflector, the reason that the R-card fence should be properly designed to cover the whole edge has become clear and so does the use of the quasi-circular resistance distribution.
The design procedures for the R-card fence to work as an edge treatment for compact range reflectors have been introduced and discussed in the previous chapters. So far, only simulated results using computer codes have been shown to demonstrate the success of the design approach in improving the plane wave quality in the test zone. In this chapter, a brief description of the measurement set-up and procedures are provided. Finally, measured results are shown to verify the R-card fence concept.

It is necessary to mention that for the R-card design problem, the computer simulation has been an essential step before going through the experimental stage which is described in this chapter. The main reason for that is the large number of parameters that define the R-card fence leads to an infinite number of design possibilities which hardly reach an optimum design on trial and error basis. By simulation, simpler hypothetical problems, such as one edge at a time, 2D configurations, and reflector strips, were used to reduce the number of variables and lead to an optimum design step by step. Note also that physical intuition and sample cases resulted in better understanding of the impact of the R-card fence geometry. This kind of decomposition
of the problem into simpler ones could be only possible using computer simulation.

In some other problems, where an initial design can be guessed and the variables are few, empirical optimization can be feasible and much faster than the simulation which may or may not come later. The expense of the experimental optimization versus the time cost is still prohibited.

In our case, the simulated results were helpful to achieve the optimum design in the first place. It is much less expensive to change the geometry and the R-card dimensions in the computer simulation than actually wasting a huge amount of R-card material in trial prototypes which seemed to be very tedious if not impossible to work. So the appropriate approach is to invest time to generate the geometry parameters needed to build the R-card fence and then experimentally verify the design done here.

For a circular rim reflector, the design steps and the simulated results were discussed in Chapter 5. Based on that design, the set-up for the measurements will be described in Section 6.1. Collected data is shown in Section 6.2. Section 6.3 summarizes and concludes this chapter.

6.1 Measurement Set-up

In the following pages, a brief description of the measurement set-up is discussed. Good planning is the first step towards a successful and happy end for any measurement study.
6.1.1 Place

The first decision that had to be made whether to perform this measurement outdoor or indoor. Suggested outdoor areas such as the back yard of the ESL or on the roof of the ESL building would provide a low clutter environment. However; weather conditions from rain and wind could represent an obstacle while constructing the system and collecting the data. An indoor set-up was chosen to avoid such weather problems. In which case, time gating and post processing to the collected data solve the high clutter problems associated with indoor measurements. The dark room could not be reserved for the long and uncertain time needed for the construction of the fence and collecting data.

6.1.2 Materials and tools

A list of the essential materials and tools, needed to build the system, can be summarized in the following items:

1. Reflector (Diameter = 72 inches, F/D= 2/3)

2. R-cards rolls of constant values.

3. Styrofoam (1 and 2 inches thick)

4. Two 2-12 GHz standard AEL horns

5. Prober (linear motion)

6. PC and control software

7. Network Analyzer (HP8510)
8. RF cables

For the above list to be complete, many things should be added such as 2-by-4 wooden bars, wooden sticks, ladders, thread, nuts, bolts, tapes, driller, saw, keys, screw drivers, .... etc.

Fortunately, the ElectroScience Lab is well equipped with all the necessary tools and ready for such measurements. While the R-card rolls (3000 Ω and below) are available, ordering a roll of the 3000 Ω value R-card had to be done earlier. Shopping and testing R-card samples sent by different companies are typical procedures that one has to go through before placing an order.

6.1.3 Time

Since time is an important factor, some issues had to be taken care of such as:

1. ordering the required material early enough

2. reserving the measurement site

3. reserving the instruments and tools

Because tools and instruments are usually shared by many people, one should not exceed his time slot to allow others doing their own measurements on time. The whole process of constructing the fence and collecting data took approximately one month.

6.1.4 Construction

Different technical skills from being a carpenter to a tailor are required for building the designed R-card fence as described in Figures 6.1-6.7. Gravity was used to hold
the reflector and the R-card fence. This required the reflector to look up toward the ceiling. By doing this, the effect of the ground could also be avoided. The only drawback is that the prober had to be mounted close to the ceiling, such that the ceiling represented an obstacle that prevented a down range performance test. Two 8-feet fiberglass ladders were used to hold the prober.

6.2 Measured Data

In the following pages, measured test zone fields are plotted in Figures 6.8-6.23 from 2 to 10 GHz for both vertical and horizontal polarizations. In general, improved
Figure 6.2: The Styrofoam pieces are raised off the floor, assembled in the designed fence geometry, and supported by wooden sticks of different heights. Small cubes of Styrofoam are taped to the floor and used as bases for the wooden sticks.
Figure 6.3: The higher side of the fence where a frame of 2-by-4 wood is built to support the fence segments using Styrofoam cantilevers. This support is to replace a large part of the wooden sticks support to allow sliding the reflector under the fence umbrella. The opening in the frame is made wider than the reflector. A flexible arrangement for the feed is built as shown to provide different degrees of freedom to move the feed to the focus and rotate it for different polarizations.
Figure 6.4: The lower side of the fence shows the wooden frame supporting the fence segments with well fit Styrofoam cantilevers. The frame is made adjustable for fine tuning the height and orientation of each segment.
Figure 6.5: Preparing the R-card sheets by cutting the right values on the designated shape according to the quasi-circular contours. The R-card rolls also appear in the photo.
Figure 6.6: The *tailor* adds the final R-card layers from the specially-ordered R-card roll of 3000 $\Omega$ resistance value. This layer provides a well finished surface for the fence.
Figure 6.7: The R-cards are placed on the Styrofoam. The reflector is slid under the fence umbrella. The feed is focussed. The probe arrangements are made where the positioner sits on a pair of 8-feet fiberglass ladders to scan the line of symmetry cut. The HP8510 network analyzer and the PC appear at the back of the photo.
fields with smaller ripple amplitude appear in the test after adding the R-card fence. This is clearer at the lower frequencies of the band of interest at 3 and 4 GHz. Recall that the R-card design was based on its low frequency performance in the first place. The noise and clutter impact increases at higher frequencies. Reflector surface deformation also greatly impacts the data at higher frequencies.

6.3 Summary

The designed R-card fence has been built and assembled with a circular rim reflector and feed precisely enough and in a simple way to construct a prototype that can validate the design approach created by computer simulation before. The test zone fields have been probed along the line of symmetry before and after placing the R-card sheets. By time gating and processing the collected data, the clutter and noise are removed and only the desired fields are cleanly extracted. It has been experimentally proven that by using a well designed R-card fence, the reflector edge diffraction can be reduced to result in smaller variations in the test zone fields. This verifies the theoretical results developed during the course of this research.
Figure 6.8: Probed test zone fields at 3 GHz for V-pol.

Figure 6.9: Probed test zone fields at 3 GHz for H-pol.
Figure 6.10: Probed test zone fields at 4 GHz for V-pol.

Figure 6.11: Probed test zone fields at 4 GHz for H-pol.
Figure 6.12: Probed test zone fields at 5 GHz for V-pol.

Figure 6.13: Probed test zone fields at 5 GHz for H-pol.
Figure 6.14: Probed test zone fields at 6 GHz for V-pol.

Figure 6.15: Probed test zone fields at 6 GHz for H-pol.
Figure 6.16: Probed test zone fields at 7 GHz for V-pol.

Figure 6.17: Probed test zone fields at 7 GHz for H-pol.
Figure 6.18: Probed test zone fields at 8 GHz for V-pol.

Figure 6.19: Probed test zone fields at 8 GHz for H-pol.
Figure 6.20: Probed test zone fields at 8 GHz for V-pol.

Figure 6.21: Probed test zone fields at 9 GHz for H-pol.
Figure 6.22: Probed test zone fields at 9 GHz for V-pol.

Figure 6.23: Probed test zone fields at 10 GHz for H-pol.
CHAPTER 7

CONCLUSION

The R-card fence has been introduced and proven to be a successful edge treatment for compact range reflectors. The need of reducing the cost accompanied with popular edge treatments initially motivated the search of this new approach. The availability of the R-card sheets at low cost triggered the idea of using them as a fence to reduce the reflector edge diffraction. Fortunately, the idea has worked. The resistance of the R-card should be of low value near to the reflector edge to block the edge diffracted fields and taper to large values towards the end. If the R-card fence is properly designed, it will not create other significant error sources.

A simplified 2D version of the problem has been used to better understand the impact of the R-card fence in front of the reflector. New reflection and diffraction mechanisms have been visualized by ray tracing to examine the interaction between the R-card and the feed and the R-card and the reflector. Consequently, the initial proper R-card geometry has been recognized. The geometry, of each one of the two R-cards in the 2D case, has been optimized separately. The goal has always been to reduce the stray signal terms in the test zone fields.
The R-card resistance has been synthesized point-by-point according to a pre-defined aperture distribution following a Kaiser-Bessel taper. This kind of taper has been shown to produce optimum performance with minimum ripple amplitude in the test zone fields at the lowest operating frequency of interest. The results show a significant reduction in the variations of the plane wave in the test zone in both the cross range and down range directions. The achieved improvement has been stable over a wide band of frequencies with the low frequency-based design.

The R-card fence design procedure for a 3D reflector of a square aperture has been developed. The 2D-3D analogy has been utilized through designing the R-card fence for narrow reflector strips first. Very similar results have been shown for both the 2D and 3D strip cases. Four flat trapezoidal R-cards have been used to treat the four edges of a center-fed reflector. The obtained results have led to a conclusion that better performance can be achieved when using multi-segment R-cards per each reflector edge. Applying this approach to an offset reflector has verified that the R-cards have to be close enough to the reflector edge to block the diffracted rays before they spread out too far. Tracing the diffraction cones emanating from the reflector edge has specified the regions on the reflector edge that significantly contaminate the test zone. Consequently, the R-card design should be biased to cover these regions. Fortunately, this has been automatically included while using the reflector strips. These regions were found to be at the center of each reflector edge. A 3D ray tracing algorithm helped to synthesize the resistance of the R-card point-by-point based on the optimum aperture distribution. Using this approach for a practical example, it was found that the resultant fields in the test zone had smaller amplitude variations as expected.
To reduce the cost, commercially available circular rim reflectors have been used with the R-card fence. Although a circular rim reflector can be considered the worst choice as a compact range reflector, a successful R-card fence design has developed. One should note that the circular shape of the reflector rim directs the diffraction cones into the test zones which highly degrades the desired plane wave and results in large field variations. Thus, this circular rim design must take care of the whole reflector rim and still maintain the flatness of the R-cards was preserved. Both calculated and measured data have showed the success of this R-card design approach.

**Future Work**

A combination of an R-card fence and another types of reflector edge treatment can be the focus of future research. For example, one can attack the design of an R-card fence for serrated edge reflectors. In which case, the aperture taper due to the serration needs to be modeled. The R-card fence will only succeed if the aperture taper due to the combination of fence and serrations results in the desired optimum aperture taper.

A combination of rolled edge and R-card fence can also be thought of as tapering both the phase and the amplitude of the aperture fields. This might or might not result in better quality of plane wave in the test zone. As far as the author knows, this subject has not been investigated yet.

Shaping the R-card could also be another research target for future work. So far, the R-card designs have been based on flat, planar geometries.

A final possible application is to use the R-card in controlling the polarization. This comes from the dependence of the reflection and transmission coefficients of the R-card on polarization, angle of incidence and the ohmic resistance value. One
application is that the R-cards can act as polarization selective surfaces at small grazing angles.
R-card resistance is obtained by solving Equation 3.5 using the following progressive algorithms:

A.1 Upper R-card

For the upper R-card, backward ray tracing as shown in Figure A.1 is needed to realize the R-card resistance. The steps to obtain the resistance distribution for the upper R-card are as follows:

1. Divide the R-card into \( N_1 \) equi-spaced points each at \( y_k \) location given by

\[
y_k = y_1 + \frac{k - 1}{N_1 - 1} L_1 \sin \alpha_1, \quad k = 1, 2, \ldots, N_1. \tag{A.1}
\]

2. Resistance synthesis starts from the lowest point on the R-card and goes up. In other words, \( y \) changes in the range from \( y = y_1 \) to \( y_1 + L_1 \sin \alpha_1 \).

3. Trace the ray back horizontally to the reflector to get the intersection point \( (x_r, y_r) \) on the reflector.

4. Check if the feed ray \( (F, 0) \rightarrow (x_r, y_r) \) intersects with the R-card or not.
Figure A.1: Upper R-card resistance evaluation using backward ray tracing.

- If no, then set $T1 = 1$

- If yes, then calculate $T1$ at the intersection point $(x_t, y_t)$ using

$$T1 = \begin{cases} \frac{2R/Z_0}{\cos \theta_i + 2R/Z_0} & TE \\ \frac{2R \cos \theta_i / Z_0}{1 + 2R \cos \theta_i / Z_0} & TM \end{cases}$$

(A.2)

and

$$\cos \theta_i = |\hat{n} \cdot \hat{s}_i|$$

(A.3)

where

- $\hat{n}$ is the R-card normal unit vector
- $\hat{s}_i$ is the unit vector for the incident ray, and
- $R = R(y_t)$ R-card resistance at the point of the intersection interpolated from the previously evaluated resistances at lower coordinates.

Simple linear interpolation can be performed utilizing only two points below and above the point of the intersection $y_j < y_t < y_{j+1}$ using

$$j = \text{integer} \left\{ \frac{y_t - y_1}{L_1 \sin \alpha_1} (N_i - 1) \right\}$$

(A.4)
Reflector

\( R\text{-card} \)

\((x_r, y_r)\)

\( T2 \)

\( N2 \)

\( f_{ap}(y_r) \)

\( f_{go}(y_r) \)

\( (F, 0) \)

Figure A.2: Forward ray tracing to evaluate the lower R-card resistance for an offset reflector.

\[
R(y_i) = R(y_j) + \frac{y_k - y_j}{y_{j+1} - y_j}[R(y_{j+1}) - R(y_j)] \quad (A.5)
\]

5. Calculate \( T2 \) from

\[
T2 = \frac{f_{ap}(y_k)}{f_{go}(y_k) \cdot T1}. \quad (A.6)
\]

6. Obtain the normal incidence resistance \( R_{\text{norm}} \) at \( y_k \) from

\[
R_{\text{norm}}(y_k) = \frac{Z_0T2}{2(1 - T2)}. \quad (A.7)
\]

7. Determine the R-card resistance according to the polarization from

\[
R(y_k) = \begin{cases} 
R_{\text{norm}}(y_k) \cdot \cos \theta_r & TE \\
R_{\text{norm}}(y_k)/\cos \theta_r & TM 
\end{cases} \quad (A.8)
\]

and

\[
\cos \theta_r = |\hat{n} \cdot \hat{s}_r|. \quad (A.9)
\]
A.2 Lower R-card

In case of the lower R-card, a slightly different algorithm is applied. Forward ray tracing shown in Figure A.2 is needed for the lower R-card resistance evaluation this time and the procedures are as follows:

1. Divide the R-card into $N_2$ equi-spaced points at $y = y_k$'s in which

   \[ y_k = y_2 - \frac{k - 1}{N_2 - 1} L_2 |\sin \alpha_2|, \quad k = 1, 2, ..., N_2. \quad (A.10) \]

2. Resistance synthesis starts from the highest point on the R-card and goes down.

   In other words, $y$ changes in the range from $y = y_2$ to $y_2 - L_2 |\sin \alpha_2|$.

3. Trace the ray forward from the feed \( \rightarrow \) the R-card \( \rightarrow \) the intersection point \((x_r, y_r)\) on the reflector.

4. Check whether the horizontal ray, \((x_r, y_r) \rightarrow \) aperture, intersects with the R-card or not (i.e., if \( y_r < y_2 \) or not).

   - If no, then set \( T_2 = 1 \)
   - If yes, then check whether \(|y_r - y_k| > \Delta y\) where \( \Delta y = L_2/(N_2 - 1) \).

      - If \(|y_r - y_k| > \Delta y\) then

          (a) Interpolate to get \( R(y_r) \) from the resistance information for the previous points. This is done using

          \[
          j = \text{integer} \left\{ \frac{y_2 - y_r}{L_2 |\sin \alpha_2| (N_2 - 1)} \right\} \quad (A.11)\]

          \[
          R(y_r) = R(y_j) + \frac{y_r - y_j}{y_{j+1} - y_j} [R(y_{j+1}) - R(y_j)] \quad (A.12)\]

169
(b) Calculate $T_2$ at the intersection point using

$$T_2 = \begin{cases} \frac{2R(y_r)/Z_0}{\sin \alpha_2 + \frac{2R(y_r)/Z_0}{\sin \alpha_2}} & \text{TE} \\
\frac{1+2R\sin \alpha_2/Z_0}{\sin \alpha_2/Z_0} & \text{TM} \end{cases}$$

(A.13)

(c) Determine $T_1$ at $y_k$ from

$$T_1 = \frac{f_{ap}(y_r)}{f_{go}(y_r) \cdot T_2}.$$  

(A.14)

(d) Obtain $R_{\text{norm}}(y_k)$ from

$$R_{\text{norm}}(y_k) = \frac{Z_0T_1}{2(1-T_1)}.$$  

(A.15)

(e) Determine the R-card resistance according to the polarization from

$$R(y_k) = \begin{cases} R_{\text{norm}}(y_k) \cdot \cos \theta_i & \text{TE} \\
R_{\text{norm}}(y_k)/\cos \theta_i & \text{TM} \end{cases}$$

(A.16)

- If $|y_r - y_k| < \Delta y$ then $T_1$ and $T_2$ can not be separated and one can solve for $R(y_k)$ (or $r(y_k) = R(y_k)/Z_0$) directly from

$$r(y_k) = -B + \sqrt{B^2 - 4AC}.$$  

(A.17)

For the **TE case**

$$A = 1 - \zeta$$  

(A.18)

$$B = \zeta(a_1 + a_2), \text{ and}$$  

(A.19)

$$C = -\zeta a_1 a_2$$  

(A.20)

and for the **TM case**

$$A = b_1 b_2 (1/\zeta - 1)$$  

(A.21)

$$B = b_1 + b_2, \text{ and}$$  

(A.22)

$$C = -1$$  

(A.23)

170
where

\[ \zeta = \frac{f_{ap}}{f_{go}} \quad \text{(A.24)} \]

\[ a1 = \frac{1}{2} \hat{n} \cdot \hat{s}_r \quad \text{(A.25)} \]

\[ a2 = \frac{1}{2} \hat{n} \cdot \hat{s}_i \quad \text{(A.26)} \]

\[ b1 = 2 \hat{n} \cdot \hat{s}_r, \quad \text{and} \quad \text{(A.27)} \]

\[ b2 = 2 \hat{n} \cdot \hat{s}_i. \quad \text{(A.28)} \]
R-CARD SEGMENT ARRANGEMENT IN FRONT OF THE REFLECTOR EDGE

Assume \( K \) flat R-card segments are needed to be placed in front of a parabolic contour which represents the reflector edge as shown in Figure B.1. \( K \) is always an odd integer and each segment is defined by 3 parameters; e.g., the \( k^{th} \) segment is defined by the center \( c_k \), the tilt angle \( \gamma_k \), and the width \( W_k \). The segments are all arranged such that they are tangent to a shifted parabolic contour. The parabolic contour equation can be written as

\[
z = \frac{y^2}{4F} + C \tag{B.1}
\]

where \( F \) is the focal length and \( C \) is a constant. The tilt angle of the tangent to the parabola at any point \((y, z)\) is given by

\[
\gamma = \tan^{-1}(2F/y). \tag{B.2}
\]

In addition to the fact that the point \( c_k \) is at the center of the \( k^{th} \) R-card, it is also the tangential point which lies on the parabolic contour. Hence the tilt angle of the \( k^{th} \) segment can be given by

\[
\gamma_k = \tan^{-1}(2F/y_{c_k}). \tag{B.3}
\]
Figure B.1: The R-card segment geometry arrangement in front of the reflector edge.
If \( c_k, \gamma_k, \) and \( W_k \) are known, then the coordinates of the end points of the \( k^{th} \) segment are

\[
y_{p_1} = y_{c_k} + \frac{W_k}{2} \sin \gamma_k \tag{B.4}
\]
\[
z_{p_1} = z_{c_k} + \frac{W_k}{2} \cos \gamma_k \tag{B.5}
\]
\[
y_{p_2} = y_{c_k} - \frac{W_k}{2} \sin \gamma_k, \text{ and} \tag{B.6}
\]
\[
z_{p_2} = z_{c_k} - \frac{W_k}{2} \cos \gamma_k \tag{B.7}
\]

Considering the rectangular aperture reflector case, the coordinates of the central R-card where \( k = \text{mid} = \frac{K+1}{2} \) are given by

\[
y_{c_{\text{mid}}} = \begin{cases} y_{\text{off}} & \text{for side edges} \\ 0 & \text{for upper and lower edges} \end{cases}, \text{ and} \tag{B.8}
\]
\[
z_{c_{\text{mid}}} = \frac{y_{c_{\text{mid}}}}{4F} + C. \tag{B.9}
\]

The tilt angle \( \gamma_{\text{mid}} \) is calculated from Equation (B.3) and \( W_{\text{mid}} \) is initially chosen. Proceeding in two directions, the rest of the segments are determined. Let's determine the parameters \((c_{k+1}, \gamma_{k+1}, W_{k+1})\) of the \((k+1)^{th}\) segment noting that

\[
p_{1}^{k+1} = p_{2}^{k}. \tag{B.10}
\]

Then the central point \( c_{k+1} \) is determined from

\[
y_{c_{k+1}} = y_{p_{1}^{k+1}} + \sqrt{y_{p_{1}^{k+1}}^2 + 4F(C - z_{p_{1}^{k+1}}^2)} \text{ and} \tag{B.11}
\]
\[
z_{c_{k+1}} = \frac{y_{c_{k+1}}^2}{4F} + C. \tag{B.12}
\]

The tilt angle \( \gamma_{k+1} \) is then given by

\[
\gamma_{k+1} = \tan^{-1}(2F/y_{c_{k+1}}) \tag{B.13}
\]
and the width \( W_{k+1} \) can be finally obtained from

\[
W_{k+1} = \frac{2(y_{c_{k+1}} - y_{p_{c}^{++1}})}{\sin \gamma_{k+1}}.
\]  

(B.14)

The lower segments are determined using the same procedures, where \( atan(y, x) = \tan^{-1}(y/x) \), and provides the angle in the right quarter depending on the signs of \( x \) and \( y \).
APPENDIX C

R-CARD PATCHING

A general R-card shape is defined here and prepared for the PO integration. Every R-card is flat and is a four sided polygon (quadrilateral) in shape. The quadrilateral is defined by the coordinates of its four corners $C_1, C_2, C_3, C_4$. These corners have to be in a cyclic order. The R-card is divided into $M \times N$ small quadrilateral patches as shown in Figure C.1. The values of $M \times N$ are obtained from the longest adjacent sides and the required patch size. The centroid and the area of every patch are determined and stored. The flatness of the R-cards allows such quadrilateral patches to be used otherwise triangular patches have to be considered.

A different patching is needed if one of the interior angles of the quadrilateral is greater than 180°. Figure C.2 shows a patching scheme for such a case where the interior angle is greater than 180° at $C_3$. An automatic detection of the interior angles and the maximum values for $M$ and $N$ is included in the computer subroutine.

A polygon of more than four sides can be divided into quadrilaterals and hence patching takes place in the same manner. On the other hand, triangles can be treated as quadrilaterals by adding a virtual corner in the middle of a side (the longest is preferable) and re-ordering the corners. Since the angle at this additional corner will
be exactly equal to 180°, then two different patching schemes are possible as shown in Figure C.3.

Preserving the patch numbers in a rectangular matrix $M \times N$, which eases the storage process, is one of two differences between the R-card and reflector patching concepts. Since the reflector edge can be serrated, $N$ is usually a function of $i$. The second difference is the that triangular patches have to be used in the case of the reflector because of its non flat, paraboloid, surface. Triangular shapes guarantee both flat patches while having its vertices on a curved surface whose normal vector can be accurately approximated by the normal to the flat patch.

![Figure C.1: An R-card of four sided polygon shape is defined by its vertices $C_1, C_2, C_3, C_4$. Patching the R-card takes place resulting in a rectangular $M \times N$ matrix of patches.](image-url)
Figure C.2: A quad with an interior angle greater than 180° is properly patched.
Figure C.3: Two different schemes for patching triangular R-cards using additional corner to simulate a quadrilateral.
APPENDIX D

LOCATING A POINT WITH RESPECT TO A POLYGON

Let's assume that an $N$ sided planar plate is given by the coordinates of its $N$ corner points $C_1, C_2, ..., C_N$. These corners are in cyclic order. A point $P$ is also given in the same plane as shown in Figure D.1. It is required to determine whether the point $P$ lies inside or outside the plate.

Disrespecting the polygon shape, Cauchy criteria can generally be used for this purpose. Vectors $\vec{c}_1, \vec{c}_2, ..., \vec{c}_N$ connect the point $P$ with the polygon corners $C_1, C_2, ..., C_N$, respectively, and form the angles, $\theta_1, \theta_2, ..., \theta_N$ as shown in Figure D.1.

These angles can be determined precisely in the proper quadrant by using

\[
\sin \theta_j = \frac{\vec{c}_j \times \vec{c}_k \cdot \hat{n}}{||\vec{c}_j|| ||\vec{c}_k||} \quad \text{and} \quad (D.1)
\]

\[
\cos \theta_j = \frac{\vec{c}_j \cdot \vec{c}_k}{||\vec{c}_j|| ||\vec{c}_k||} \quad (D.2)
\]

taking into account that

\[
j = 1, 2, ..., N \quad \text{and} \quad (D.3)
\]

\[
k = \begin{cases} 
  j + 1 & j \leq N - 1 \\
  1 & j = N 
\end{cases} \quad (D.4)
\]

Note that $\hat{n}$ is the constant normal unit vector to the polygon plane.

180
Figure D.1: Determination whether a point $P$ lies inside or outside $N$ sided polygon.

Evaluating the sum $S$ of these angles by

$$S = \left| \sum_{j=1}^{N} \theta_j \right| \text{ radians} \quad (D.5)$$

The absolute value is considered since a total negative accumulation of the angles may result from the choice of the $\hat{n}$ direction or from the polygon corners rotation direction.

The Cauchy criteria can now be used to determine if $P$ is inside the polygon or outside by

$$S = \begin{cases} 2\pi & \text{if } P \text{ is inside} \\ 0 & \text{if } P \text{ is outside} \end{cases} \quad (D.6)$$

The algebraic mean of the above two distinctive values of $S$ can be used as the threshold upon which the decision of the point $P$ location can be made.
APPENDIX E

A RING ENCLOSES A LINE

A non planar ring is given in the space by the coordinates of its corners \( C'_1, C'_2, ..., C'_N \). Two points A and B, forming a line segment, are also given as shown in Figure E.1. It is required to determine if the ring encloses the line or not. Although the line is defined by a finite segment AB, it generally extends to infinity in both directions.

Figure E.1: Determination whether a ring, \( C'_1, C'_2, ..., C'_N \) encloses a line or not defined by two points A and B. The ring is generally non planar.
The easiest way to solve this 3D geometrical problem is to convert it into a 2D one. By projecting the whole geometry onto a plane perpendicular to the line AB, the problem becomes locating of a point with respect to a planar polygon. As shown in Figure E.2, the line is projected as a point P and the corners of the ring become the polygon corners $C_1, C_2, ..., C_N$. Again, the corners should be given in a cyclic order.

The decision whether P is inside or outside the polygon is exactly equivalent to the decision whether the ring encloses the line or not. As seen before, the sum of the angles $\theta$'s is the key. In order to get these angles, the vectors $\bar{c}$'s are to be determined. Let’s $\bar{F}$ be the vector connecting A and B and normal to the projection plane. The vectors, $\bar{a}_1, \bar{a}_2, ..., \bar{a}_N$, connect the point A to the ring corners $C'_1, C'_2, ..., C'_N$, respectively. As shown in Figure E.2, the free vectors $\bar{c}_j, j = 1, 2, ..., N$ can be determined from

$$\bar{c}_j = \bar{a}_j - \frac{\bar{a}_j \cdot \bar{F}}{|\bar{F}|^2} \bar{F} \quad \text{(E.1)}$$

The remaining procedures of evaluating $\theta$'s and the sum S follow exactly the same methods described in Appendix C.
Figure E.2: Conversion of the 3D problem into a 2D one by projecting the whole geometry onto a plane perpendicular to $F$. The line segment projection is a point $P$, the ring projection is a polygon and the free vectors $c$'s remain the same.
APPENDIX F

PATCHING A REFLECTOR OF CIRCULAR APERTURE

Small patches are needed to approximate the reflector curved surface for Physical Optics integration purpose. Triangular shape of the patches guarantees a unique normal vector for each patch. These triangles should have common vertices and sides and nearly equal areas. It is very difficult to accurately analyze reflectors with a circular rims using [17]. A circular aperture reflector can be patched as follows:

Given the circle diameter, \( D \), and the patch area, \( W^2 \).

1. Starting from the center, construct a square which is divided into 4 triangles as shown in Figure F.1. The number of nodes is \( N_1 = 4 \) (the square corners) and the radius, which is half of the square diagonal, \( r_1 = \frac{W}{\sqrt{2}} \).

2. The nodes will be doubled to \( N_2 = 8 \) to construct a ring of 12 triangles as shown in Figure F.1 where the outer rim radius is \( r_2 = W \sqrt{1 + \frac{6}{\pi}} \). In general, this ring can be defined as type A ring which is characterized by

\[
N_k = 2N_{k-1}
\]

\( F.1 \)

\(^5\)this comes from the square area \( 2r_1 \times 2r_1 = 4W^2 \).

\(^6\)It is preferred to use \( r_2 = W \sqrt{12/\pi} \) which does not obey the general \( k \) formulae.
where

\[ r_k = \sqrt{r_{k-1}^2 + \frac{3N_{k-1}W^2}{2\pi}} \]  \hspace{1cm} (F.2)
\[ NT_k = 3N_{k-1} \]  \hspace{1cm} (F.3)

**k** is the ring number

\[ N_{k-1}, N_k \] are the number of nodes on the inner and outer rims, respectively.

\[ NT_k \] is the number of the triangles contained in the ring.

3. Rings of a different type, called type B, can be also used. The \( k^{th} \) ring has

\[ N_k = N_{k-1} \]  \hspace{1cm} (F.4)
\[ r_k = \sqrt{r_{k-1}^2 + \frac{N_{k-1}W^2}{\pi}} \]  \hspace{1cm} (F.5)
\[ NT_k = 2N_{k-1} \]  \hspace{1cm} (F.6)

Both types of rings are shown in Figure F.2.

4. The choice between type A and type B rings is determined according to which is larger in value, \( T_A \) or \( T_B \). \( T_A \) and \( T_B \) are calculated from

\[ T_{A,B} = \left| \frac{r_k^{A,B} - r_{k-1}^{A,B}}{\sqrt{3}} - \frac{\pi r_k^{A,B}}{N_k^{A,B}} \right| \]  \hspace{1cm} (F.7)

While rings of type B are the more popular and common to use, rings of type A are used successively in the central region to rapidly improve the circular rim approximation starting from a square to a reasonable piecewise circular rim. A single type A ring is also used among many successive type B rings to preserve the circular rim approximation for larger radii by doubling the nodes. This is clearly shown in Figure F.3.
Figure F.1: The central region of the circular grid.

Figure F.2: Two different types of rings used for gridding a circle.
Figure F.3: A complete uniform gridding for a circle using triangular patches.
BIBLIOGRAPHY


