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UMI
EVALUATION OF REAL-TIME BUS ARRIVAL INFORMATION SYSTEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By
Sungjoon Lee, M.S.

*****

The Ohio State University
1999

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ABSTRACT

This study uses an expected utility approach to evaluate the performance of real-time bus arrival information systems (BAIS). Three BAIS alternatives are considered in terms of the real-time bus location data available to a "broadcaster" predicting bus arrival times for passengers at a bus stop: BAIS1 has no available real-time data; BAIS2 considers real-time data on the time since the most recent bus departure from the bus stop; BAIS3 considers real-time data on the headway at a point upstream from the bus stop in addition to the data available in BAIS2.

Bus and passenger arrivals at a stop are modeled as random variables, and passenger utility is modeled as a function of the actual waiting time and the predicted bus arrival time conveyed to the passenger upon arrival to the stop. This predicted time is assumed to be determined by the broadcaster who maximizes the passengers' expected utilities given the available real-time data. The expected utility across the passenger population is analytically formulated for each BAIS alternative. Since closed form solutions can be intractable, Monte Carlo simulation is used to evaluate the BAIS alternatives under different scenarios reflecting bus operating performance and passenger utility functions.
Simulation results show that the headway mean has an important effect on the expected utility of BAIS1 but little effect on the expected utilities of BAIS2 and BAIS3. The headway standard deviation, on the other hand, has little effect on the expected utility of BAIS1 but a large effect on the expected utility of BAIS2. The effect of the standard deviation on the expected utility of BAIS3 is tempered by the additional real-time data on the approaching bus. The various passenger utility functions examined have little effect on the expected utility patterns of any of the BAIS alternatives. A sensitivity analysis relating to the effect of the broadcaster’s knowledge shows great sensitivity to accuracy in the headway mean but little sensitivity to accuracy in headway standard deviation and utility function parameters. An application to three stops on a realistic bus system illustrates the reasonableness, value, and applicability of the developed methodology.
Dedicated to my mother,
my wife, Seonghee Choi,
and my sons, Sihoon and Donghoon
ACKNOWLEDGMENTS

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CHAPTER 1

INTRODUCTION

1.1 Motivation

The time that a bus passenger waits for his bus to arrive at the bus stop is one of the primary concerns of bus passengers and transit operators. This “waiting time” is a function of both when the bus arrives and when the passenger arrives. Since the operation of buses is affected by many factors, such as traffic condition, road condition, ridership, weather and driver characteristics, bus arrival times are inevitably variable. Moreover, the process by which bus passengers arrive at a bus stop is stochastic. Therefore, the passenger’s waiting time at a bus stop will be variable and difficult for the passenger to predict.

Because of this variability in bus arrival time, real-time information on when the next bus is expected to arrive could be useful for a bus passenger at the bus stop. Without such information, bus passengers might experience anxiety and have to pay constant attention to when the next bus will be arriving. On the other hand, bus arrival time information might allow bus passengers to engage in some diversionary activities (for
example, reading a newspaper comfortably, getting a cup of coffee) while waiting. Even if the waiting passenger does not engage in such activities, knowing when the bus will be arriving is likely to make the waiting time less unpleasant. It has been known empirically that passengers consider the time spent waiting for a bus more costly to them than the time incurred when riding a bus (Ben-Akiva and Lerman 1985). This higher level of discomfort is partly attributed to the uncertainty of the arrival time of the next bus. Real-time information on bus arrivals, therefore, has the potential to reduce the discomfort associated with waiting time.

A system providing bus arrival time information would be facilitated by an Automatic Vehicle Location (AVL) system. An AVL system is a technology that can track the vehicle's location in real-time. Real-time data on bus locations can then be used to forecast bus arrival times (Hounsell and McLeod, 1998; Gomez et al., 1998). The collected bus location data can also be archived and analyzed to determine historical bus operation characteristics that can in turn be used in conjunction with real-time data forecasting bus arrival times.

AVL technologies have been developing rapidly. Since 1968, many transit authorities have deployed and tested AVL systems on their bus fleets in attempts to improve service to the passengers. Various types of AVL technologies have been introduced, but these can be broadly classified into two major types: signpost and GPS technologies. We briefly describe these technologies next.
In the 1970s and 1980s, signposts were the more commonly used technology. A signpost is a vehicle detector installed along the roadside to collect bus location data. These "signposts" detect a signal emitted by buses upon their passing the signposts. Signpost systems are limited in their ability to track vehicles in operation because they can only collect bus location data at fixed pre-installed locations. If signposts are spaced too far apart, they cannot provide continuous bus location data in real-time. Another drawback of this technology also arises from their fixed location. If the bus route is changed, the signpost will have to be relocated to the new route.

In the 1990s, the Global Positioning System (GPS) was introduced as an alternate AVL technology for transit application. In this system, the GPS receiver's location is determined by decoding the signals from several satellites based on triangulation principles. When a GPS receiver is mounted on a bus, the location of the bus can be monitored by tracking the location of the GPS receiver. The bus can therefore be tracked whenever GPS signals can be received. This results in a more frequent bus location data and, hence, a finer spatial resolution than in the case of signposts. In addition, even if the bus route is altered, GPS can track the bus on the new route without any alteration of the AVL system. Because of these advantages and the declining cost of installing GPS systems, GPS has become the more attractive of the two systems. Despite the attractiveness of adopting GPS, it is important to point out that this technology has some limitations that can be relevant for some transit agencies under some conditions. Specifically, GPS receivers may not be able to receive the satellite signals at all times due to physical barriers such as tightly spaced high rise building. Nevertheless, even though
many bus agencies still use signposts, GPS is becoming the dominant AVL technology today (Okunieff, 1997).

With the development of AVL technologies, the interest in real-time Bus Arrival Information Systems (BAIS) and transit passenger information systems in general has emerged as an important aspect of Advanced Public Transportation Systems (APTS). Some transportation planners and researchers have studied transit passenger information system from various perspectives.

Balog and Smith (1992) designed a passenger information system at bus stops for demonstration on Route 18 of the London Transport system. This system enabled the implementation of a comprehensive monitoring program designed to measure the effect of information on passengers at bus stops (Atkins, 1994). Roadside signposts and current wheel-turn counts were used to monitor bus location, and an algorithm using real-time bus speeds and historical travel time data was used to predict bus arrival times to stops. The arrival time information was transmitted to the bus stops for display. Results on passenger perception and attitudes toward the information showed that passenger opinion was very positive as reflected in the considerable reduction in anxiety levels while waiting for the bus and the opportunity to make positive choices for diversionary activities on the basis of reliable information. The system was thought to be reliably and accurately operated, and it was considered satisfactory to the majority of passengers.

Sayers et al. (1994) developed an algorithm to improve the algorithm for bus travel time prediction used in the passenger information system on Route 18 of the London
Transport. Their prediction algorithm used a time series model based on the recently predicted travel time of the bus and weighted actual travel times of the three most recent buses. Best fit weights of variables using data over a specified time period were used.

Even though recent research has been directed towards bus passenger information systems, the benefit of such system under a wide variety of conditions is still not well understood. Hence, an evaluation framework for comparing various bus passenger information systems would be useful to bus system managers in deciding on where investments in information systems would be warranted.

A Bus Arrival Information System (BAIS) can be evaluated by its performance in providing reliable bus arrival time predictions. The performance of a BAIS alternative would vary according to the capability of the AVL system adopted for collecting real-time bus location data. A more capable AVL system may allow for forecasting bus arrival time more accurately, but it may cost more for installation and operation. The performance of a BAIS alternative may also vary according to the characteristics of bus operations and passengers' perception of the bus arrival time information. Hence, the choice of an appropriate BAIS may differ according to the characteristics of bus operations and passenger behavior. This study is concerned with the analysis of three BAIS alternatives under various bus operations and passenger behavior characteristics. More specifically, the focus is on the value of bus arrival information to passengers waiting to board the upcoming bus in terms of its reliability.
1.2 Evaluation of Bus Arrival Information System

While only a few researchers have addressed the evaluation of BAIS, many have studied the evaluation of Advanced Traveler Information System (ATIS) with concentration on auto travel and the value of information that assists travelers in making route and departure time choices. This section presents some ATIS evaluation studies and discusses the evaluation of BAIS alternatives addressed in this study.

An ATIS can be evaluated by conducting experiments on travelers' preferences under either hypothetical or actual conditions (Schofer, et al., 1993). A survey for evaluating an ATIS can be conducted by asking travelers how they would react to hypothetical information if it is provided to them (i.e., collecting stated preference data). An experiment can also be performed by observing travelers' behavior under specific information provided to them (i.e., collecting revealed preference data).

Surveys have been used by many researchers to evaluate ATIS which are not yet available to travelers. Mannering et al. (1995) surveyed travelers' preferences towards an in-vehicle traffic information system and used a logit model and regression analyses to quantify traveler ratings of the importance of in-vehicle system attributes. Khattak et al. (1996) explored how travelers may respond to information on unexpected congestion during the pre-trip stage. They investigated stated preferences on the route, departure time, and mode selection by conducting a survey. Polydoropoulou et al.(1996) studied travelers' responses for en-route switching decisions under various ATIS scenarios.
Frayer and Kroot (1996) studied consumer perceptions of the potential of intelligent transportation innovations using a focus group.

Survey methods have also been used to investigate user perceptions of ATIS in operation. Atkins (1994) investigated passenger perceptions and attitudes at bus stops for London Transport. Englisher (1996) investigated user perceptions of ATIS operated in the Boston metropolitan area in an effort to evaluate public acceptance and the utility of the information provided by this specific ATIS.

Results from such studies can be used to evaluate the benefits of traveler information under various scenarios in a simulation environment. Hickman and Wilson (1995) evaluated the effect of real-time transit information, including predicted vehicle travel times and arrival times to stops and stations, on path choices and passenger travel times to their destinations. Their methodology was applied to a Boston area corridor operated by the Massachusetts Bay Transportation Authority (MBTA). Their results, which were obtained through computer simulation, suggest that real-time information would yield only very modest improvements in passenger service measures such as origin-destination travel times and variability of trip time. Nevertheless, the value of improving qualitative performance measures, such as the quality of the waiting experience, can still be significant.

This study presents a simulation-based evaluation methodology that focuses on the value of bus arrival time information to passengers waiting to board an upcoming bus. The value is quantified in terms of the utility passengers derive from arrival time.
information given its reliability. Three BAIS alternatives are investigated under different bus operations parameters and different passenger preferences (i.e. utility functions). We assume that passenger utility is a function of the actual passenger waiting time and the bus arrival time predicted by the BAIS. Expected utility theory is then used to quantify the performance of the BAIS alternatives. An analytic formulation is developed, however, expected utilities are calculated through simulation for 3 BAIS alternatives using assumed passenger utility functions and probability distributions of bus and passenger arrivals at the bus stop.

1.3 Research Objectives

In this study, expected utility theory is used to evaluate BAIS alternatives. We consider three BAIS alternatives representing a spectrum of systems and calculate their expected utilities. We formulate the expected utility for each BAIS alternative in detail in Chapter 2. We mention here that the expected utility of a BAIS alternative is taken across the passenger population. This is equivalent to the expected utility of a random passenger. As mentioned earlier, the expected utility of a given BAIS alternative may vary according to bus transit operation characteristics and different types of passenger preferences. We investigate the expected utility changes in each BAIS alternative under various characteristics and preferences.

A bus transit operation is characterized by the following parameters: the headway mean, the headway standard deviation, and the correlation between the headway at the bus stop of interest and the headway at a point upstream of the bus stop. Headway at a
bus stop is the inter-arrival time of consecutive buses to the stop. Among other things, the headway is influenced by the number of buses in operation on the bus route at any given time. The headway can be reduced by increasing the number of buses in operation. For example, doubling the number of buses on a route would reduce the headway by half.

The headway standard deviation reflects the variability in consecutive bus arrivals at the bus stop. The standard deviation is influenced, among other things, by traffic conditions, road conditions, ridership levels, weather, and driver characteristics. For example, a bus is likely to arrive with more variability on a bus route having congested traffic and high ridership levels since the congestion will cause variability in travel velocities and high ridership will cause variability in dwell times due to high number of passengers boarding and alighting buses.

Correlation between the headway at the bus stop of interest and the headway at an upstream point from the bus stop captures the linear dependence between these two headways. Given a headway at an upstream point, the conditional headway standard deviation at the bus stop is small when there is a high correlation, while it is large when there is a low correlation. Chapter 3 discusses this correlation in more detail. Here we mention that the correlation is considered as a function of the distance between the bus stop and the upstream point.

Different passenger preferences are represented by different utility function specifications in this study. Specifically, we consider symmetric utility functions, asymmetric utility functions, and time dependent utility functions. A symmetric utility
function assumes that the passenger is equally sensitive to the accuracy of bus arrival time prediction regardless of whether the bus arrives earlier or later than the predicted arrival time. An asymmetric utility function assumes that the passenger is more sensitive to the accuracy of bus arrival time prediction depending on whether the bus arrives earlier or later than the predicted arrival time. A time dependent utility function assumes that the sensitivity of passenger utility to the accuracy of bus arrival time prediction depends on how long he actually waits.

The specific research questions addressed by this study then are:

- How do the expected utilities under the various BAIS alternatives change as the bus transit operations parameters change?
- How are the expected utilities under the various BAIS alternatives affected by the passenger utility function?
- As a subsequent result of the above questions, under what conditions do the BAIS alternatives appear most valuable?

To address these research questions, we develop a methodology for calculating the expected utility of a random passenger for various BAIS alternatives. We analytically formulate the expected utilities for the BAIS alternatives and then develop a simulation tool to calculate these expected utilities.

1.4 Dissertation Organization

This study consists of five chapters including this introductory chapter. Chapter 2 presents the methodology for evaluating the various BAIS alternatives. As mentioned
earlier, BAIS alternatives are evaluated by using an expected utility approach. Section 2.1 introduces the concept of the expected utility for BAIS evaluation. Section 2.2 discusses the BAIS alternatives considered in this study and the type of real-time data required by each BAIS alternative. Three BAIS alternatives are considered to represent a reasonable spectrum of technologies and design options. Section 2.3 develops formulations of the waiting time distribution as viewed by the bus operator (henceforth referred to as the bus arrival time broadcaster) under each BAIS alternative. Conditional and unconditional headway distribution functions are used to derive waiting time distributions. Section 2.4 develops the formulations that model the prediction of the bus arrival times by the broadcaster under each BAIS alternative. These predictions are based on the waiting time distributions derived in Section 2.3 and a specified passenger utility function. Section 2.5 discusses the quantification of the expected utility as the performance measure of each BAIS alternative.

The closed-formed solutions of the formulations of Chapter 2 are not tractable under a variety of assumptions and, therefore, we resort to computer simulation to calculate expected utilities. Chapter 3 develops the framework for the computer simulation. In the simulation, we consider a bus stop and generate bus headway and passenger realizations from given probability distribution functions. A Monte Carlo method is used to generate random headway and passenger arrivals. Section 3.1 presents an overview of the simulation. Section 3.2 discusses the headway and passenger generation. Section 3.2 also discusses the generation of headways at upstream points. These upstream headways represent data that may be available in a real-time bus location.
system. Correlation between headways at a bus stop and headways at an upstream point from the bus stop are explicitly considered. Section 3.3 discusses the computation of the expected utility from the simulation. Section 3.4 discusses an efficient way to determine the optimal bus arrival time the broadcaster provides to passengers at bus stops.

Chapter 4 presents expected utility results for each BAIS alternative and the effect of bus operations parameters and passenger utility functions on the expected utility. It also presents an empirical analysis using data obtained from a real bus system. Section 4.1 presents the effect of bus operations parameters. Expected utility results are presented as a function of headway mean, headway standard deviation and headway correlation. Section 4.2 presents the effects of the utility function. Three types of utility functions are used: symmetric utility functions, asymmetric utility functions and time dependent utility functions. Section 4.3 presents an empirical analysis for the data obtained from a real bus system. Data are collected at three bus stops on three different routes of The Ohio State University's Campus Area Bus System (CABS) in Columbus, Ohio. Headway distributions and correlation between headways at selected bus stops and upstream points are estimated and investigated, and expected utilities using these empirical data are computed.

Chapter 5 summarizes this research and discusses the implications of the results obtained. It also presents directions for future research.
CHAPTER 2

EVALUATION METHODOLOGY

2.1 Introduction

The objective of this research is to evaluate various Bus Arrival Information Systems (BAIS). This evaluation is performed by calculating the expected utility to a randomly arriving passenger at the bus stop for each BAIS. In this section, we present an overview of the concept of expected utility for BAIS evaluation.

Passengers may experience various actual waiting times for the next bus arrival because they arrive at different times within and across different headways. We consider a "broadcaster," who will inform passengers at a particular bus stop of the next bus arrival time by posting a "broadcast time". However, the next bus arrival time is uncertain to the broadcaster at the time that a particular passenger arrives at the bus stop. Hence, the broadcaster can predict the next bus arrival time by considering the probability distribution of the next bus arrival time based on the available historical and real-time information. In this study, historical data, such as unconditional and conditional bus headway distributions are assumed to be available to the broadcaster. Different types of
real-time data are available to the broadcaster depending on the characteristics of the various possible BAIS.

The broadcaster determines the broadcast time for a passenger based on her view of the time of arrival of the next bus reflecting the real-time data. At a given time, the distribution of the time until the arrival of the next bus is identical to the passenger waiting time distribution at this time. A distribution can be determined from the historical headway distribution. However, since the broadcaster can condition this passenger waiting time distribution on the real-time data available, the broadcaster's view of passenger waiting times is different depending on the real-time data available. Therefore, a passenger arriving at a given time in a system with one BAIS could receive a different broadcast time if he arrived at exactly same time but with a different BAIS in place.

The broadcaster is assumed to determine the broadcast time so as to maximize the expected utility of a passenger arriving at a specific time, where the expectation is taken over the distribution of the passenger's waiting times. That is, the broadcaster may change the broadcast time from one time to another so as to maximize the expected utility of a passenger who could be arriving at any instant. Therefore, passengers arriving at different times receive different broadcast times at their time of arrival, experience the different actual waiting times, and ultimately different utilities.

To evaluate a set of BAIS alternatives, we consider the entire set of experienced passenger utilities, i.e., the distribution of the individual passenger utilities. That is, we
assume that the overall performance of the system can be measured by the expected utility evaluated across the population of passengers. This expectation is equivalent to the expected utility of a random passenger in the system.

In this chapter, we formulate the methodology for calculating the expected utility for the three BAIS alternatives considered. In Section 2.2 we describe these three BAIS alternatives. In Section 2.3 we derive the mathematical expressions of the broadcaster’s view of the passenger waiting time distribution under each of the three BAIS alternatives. In Section 2.4 we derive the optimal broadcast time to be set by the broadcaster under each of the three BAIS alternatives. In Section 2.5 we derive the expressions for evaluating the overall performance of the three BAIS alternatives. Finally, we present the solution for the expected utility in the case of the exponential headway distribution.

We shall use the following notation:

\[ W \] = waiting time of a passenger for the next bus arrival,
\[ H \] = inter-arrival time of buses at a bus stop (headway),
\[ X \] = inter-arrival time of buses entered by a passenger at a bus stop (headway visited by a passenger),
\[ S \] = the location of the following bus at the time of passenger arrival (distance from bus stop),
\[ P \] = headway at the location of the following bus at the time of passenger arrival (upstream headway),
\[ R \] = time elapsed since the most recent bus departure at the time of passenger arrival,
\[ f_H \] = probability density function of \( H \),
\[ f_X \] = probability density function of \( X \),
\[ F_H \] = cumulative distribution function of \( H \),
\( f_w \) = probability density function of \( W \),
\( f_r \) = probability density function of \( R \),
\( f_{w,r} \) = joint probability density function of \( W \) and \( R \),
\( f_{w|R} \) = conditional probability density function of \( W \) given \( R \),
\( f_{p,s} \) = joint probability density function of \( P \) and \( S \),
\( f_{h|p,s} \) = conditional probability density function of \( H \) given \( P \) and \( S \),
\( F_{h|p,s} \) = conditional cumulative distribution function of \( H \) given \( P \) and \( S \), and
\( f_{w|r,p,s} \) = conditional probability density function of \( W \) given \( R \), \( P \) and \( S \),
\( f_{r,p,s} \) = joint probability density function of \( R \), \( P \) and \( S \),
\( f_{h,p,s} \) = joint probability density function of \( H \), \( P \) and \( S \).
\( f_{x,w} \) = joint probability density function of \( X \) and \( W \),
\( f_{r,x,p,s} \) = joint probability density function of \( R \), \( X \), \( P \) and \( S \),
\( f_{r,x|p,s} \) = conditional probability density function of \( R \) and \( X \) given \( P \) and \( S \),
\( f_{x,w|p,s} \) = conditional probability density function of \( X \) and \( W \) given \( P \) and \( S \),

Note that upper case variables are used to reflect random variable while their lower case counterparts used subsequently reflect realizations of random variables. All these variables are time-dependent, and the above notation would technically require an index for the time dimension. However, we will omit such an index for ease of presentation. Unless otherwise mentioned, the variables used relate to the time a passenger arrives at the considered bus stop.
2.2 Bus Arrival Information System Alternatives

We mentioned that the real-time data differ according to the type of BAIS and that the different real-time data affect the broadcaster’s view of passenger waiting times. In this section, we define the BAIS alternatives considered in this research and discuss how the real-time data affects the broadcaster’s view of the distribution of waiting time for each. While there can be many alternatives for BAIS, here we consider three to represent a reasonable spectrum of technology and design options. We illustrate the type of information that is useful in developing our methodology in Figure 2.1.

Two of the three BAIS alternatives considered reflect two kinds of real-time data. One is data on the time elapsed since the most recent bus departure from a considered bus stop at the time a passenger arrives at the bus stop, denoted \( r \) in Figure 2.1. The other is the data of bus headway taken at an upstream point of the considered bus stop, denoted by \( p \) in Figure 2.1, at the time a passenger arrives at the bus stop. We denoted the distance of the upstream point from the bus stop by \( s \) in Figure 2.1. The time of passenger arrival is shown by the arrow on the time axis in the figure. In this figure, \( h \) is the inter-arrival time (headway) between the lead bus and the following bus at the bus stop, \( w \) is the waiting time of the arriving passenger for the following bus arrival, \( t_t \) is the travel time of the lead bus for traveling the distance \( s \), and \( v \) is the average velocity of the following bus during traveling the distance \( s \).

We consider the following three BAIS alternatives in terms of the capability of providing real-time data to the broadcaster.
Base-Case: We denote this by BAIS1. BAIS1 represents a situation where no vehicle location system is installed. That is, no real-time data is available to the broadcaster to assist in determining the next bus arrival time (broadcast time). The broadcaster determines the distribution of passenger waiting times from the historical probability function of headways, $f_H$, only. Thus, the probability density function of the passenger waiting times as viewed by the broadcaster is only a function of the historical headway distribution, $f_H$, as follows:

$$f_w = g(f_H)$$
Previous Bus Departure Time: The second system considered is one which provides the real-time data on the time since the most recent bus departure \( r \) (a specific realization of the random variable \( R \), see Figure 2.1) from the considered bus stop. We denote this system by BAIS2. Knowing the time since the most recent bus departure allows the broadcaster to determine the waiting time distribution conditional on this time as follows:

\[
f_{w|R} = g(f_H, r)
\]  

(2.2)

The distribution of waiting time conditioned on the observed time since most recent bus departure \( r \) in general leads to a better estimate of the actual passenger waiting time than the unconditional waiting time distribution. However, in the case of random bus arrivals (i.e., bus arrivals following a Poisson process), the arrival time of the next bus cannot be estimated more accurately with this additional real-time data. We will discuss this in more detail in Section 2.6.

Upstream Headway: The third system considered is one which provides real-time data on the upstream headway, \( p \), and the location of the following bus at the moment of passenger arrival, \( s \), in addition to the time since the most recent bus departure, \( r \). We denote this case by BAIS3. The operator can now condition the passenger waiting time distribution on the time since the most recent bus departure \( r \), the upstream headway \( p \), and the location \( s \), of the following bus at the time of passenger arrival as follows:

\[
f_{w|R,P,S} = g(f_{H|P,S}, r, p, s)
\]  

(2.3)
In this case the broadcaster uses the headway distribution conditional on the upstream headway and the location of the following bus, instead of the unconditional headway distribution. This conditional headway distribution provides in general a better estimate of bus arrival time than the unconditional headway distribution.

The travel time of the lead bus from the location of the following bus at the time of passenger arrival to the bus stop, denoted by $t_r$ (see Figure 2.1), can also be conceivably used as real-time data for predicting the bus arrival time. More specifically, the passenger waiting time can be estimated by conditioning the travel time distribution of the following bus (which is same as the passenger waiting time distribution) on $t_r$. However, through close inspection it is evident that $t_r = p - r$ and, hence, in the context of BAIS3 the real-time data $t_r$ is redundant as it can be computed from the real-time data assumed available in BAIS3.

2.3 Waiting Time Distribution

In this section we derive the waiting time distribution for the three BAIS cases. When setting the broadcast time for a passenger, the broadcaster uses the waiting time probability density function associated with a passenger arriving at a specific time. We mentioned that the broadcaster will consider a different waiting time distribution according to the BAIS in place. Before we derive the waiting time distribution functions, we describe a basic assumption used in these derivations.

Passenger arrivals at a bus stop are assumed to follow a stochastic process. In general this process may be affected by the passengers’ knowledge of the bus schedule.
Passengers who know the bus schedule may attempt to arrive at the bus stop slightly before the published bus arrival time. However, when the bus headways are small, passenger arrivals will most likely not be influenced by the schedule, even if it is available. In this study, we assume that passengers arrive at bus stops independently of the bus schedule either because the bus headways are small enough or because passengers have no information about the bus schedule. These random arrivals, therefore, are assumed to follow a Poisson process with some mean arrival rate denoted by $\lambda$.

We now discuss the derivation of the waiting time distributions. In BAIS1, the broadcaster uses the unconditional historical probability distribution function to set the broadcast time, since no real-time data is available to her. In the case when passengers are assumed to arrive randomly to the bus stop, the probability density function of passenger waiting times until the arrival of the next bus is a function of the cumulative distribution of the bus headway, $F_H(w)$ and is given by (Larson and Odoni, 1981):

$$f_w(w) = \frac{1 - F_H(w)}{E[H]}$$

(2.4)

This expression takes into account the effect of random incidence, where it is more likely for a random passenger to arrive into a longer rather than shorter headway.

In BAIS2, the broadcaster uses a waiting time distribution conditional on the time $r$ since the departure of the previous bus from the bus stop considered. Note that $r$ corresponds to the time that has elapsed between the departure of the most recent bus and the passenger's arrival time. That is, it is the arrival time of the passenger, where time is
measured from the departure of the most recent bus. The waiting time distribution function conditioned on this time, $f_{w|R}$, can be derived from the historical headway distribution function, $f_H$, and $r$. To derive $f_{w|R}$, we derive the joint probability function $f_{w,R}$ and the marginal probability function $f_R$. The conditional probability function $f_{w|R}$ is then given by:

$$f_{w|R}(w|r) = \frac{f_{w,R}(w,r)}{f_R(r)}$$

(2.5)

To derive the marginal probability function $f_R$, note that headway $h$ is the sum of passenger waiting time $w$ and the time since most recent bus departure $r$, i.e. $h = w + r$. We are assuming that passenger arrivals follow a Poisson process. Therefore, the probability density function of $r$ is identical to that of $w$. More formally, the conditional probability distribution of $w$ given $X = h$, $f_{w|X}(w|h)$, follows a uniform distribution of value $1/h$ between 0 and $h$ since a passenger is equally likely to arrive anywhere in the given headway. For the same reason, the conditional probability distribution of $r$ given a headway, $f_{R|X}$, follows the same uniform distribution. Note that the distribution $f_w$ is determined from $f_{w|X}$ and $f_X$, and that the distribution $f_R$ is determined from $f_{R|X}$ and $f_X$. Since the conditional distributions $f_{w|X}$ and $f_{R|X}$ are identical, the probability distribution $f_w$ and $f_R$ are identical as well. Therefore, by replacing $w$ with $r$ in equation 2.4, the probability distribution function of $r$ is given by:
We now derive the joint probability $f_{w,R}$. Note that when the waiting time $w$ and the time since most recent bus departure $r$ are given, the headway $h$ is deterministic, since $h = w + r$. This indicates that the joint probability density function of any pair of variables among $w$, $r$ and $h$, determines the joint probability density function of any other pair of variables because given any two variables, the third variable is determined by $h = w + r$. Hence, the joint probability $f_{w,R}$ is given by:

$$f_{w,R}(w,r) = f_{X,W}(w+r,w)$$

(2.7)

The joint probability $f_{X,W}(h, w)$ is described by Larson and Odoni (1981) for the case when passengers are assumed to arrive randomly at the bus stop and is as follows:

$$f_{X,W}(h, w) = \frac{f_H(h)}{E[H]}, \quad 0 \leq w \leq h \leq \infty$$

(2.8)

Using equations 2.6, 2.7, and 2.8 in equation 2.5 the following proceeds:

$$f_{w|R}(w|r) = \frac{f_{w,R}(w,r)}{f_R(r)}$$

$$= \frac{f_H(w+r)}{1 - F_H(r)}$$

(2.9)

This equation indicates that the broadcaster's view of the waiting time distribution, conditional on the time since the most recent bus departure $r$, can be determined by shifting the headway distribution to the left by $r$, setting the value to zero for all $w < 0$, and
"scaling up" the portion of the headway distribution where \( h \) is greater than \( r \) so that the integration of the new headway distribution from \( w=0 \) to infinity is one.

In BAIS3, the broadcaster uses the waiting time distribution conditional on the time since the most recent bus departure, \( r \), upstream headway, \( p \), and the location of the following bus at the time of passenger arrival, \( s \). Given real-time data \( p \) and \( s \), the broadcaster can update the historical headway distribution by conditioning it on this data. Consequently, she can use the conditional headway distribution, \( f_{H|P,S} \), instead of the unconditional historical headway distribution, \( f_H \), to more accurately forecast the next bus arrival. More specifically, the waiting time probability density function conditional on \( r, p \) and \( s \), \( f_{w|r,P,S} \), can be derived from the headway probability density function conditional on \( p \) and \( s \), \( f_{H|P,S} \), and the time since most recent bus departure, \( r \). Through arguments similar to the those used when deriving equation 2.9, using \( f_{H|P,S} \) and \( F_{H|P,S} \) instead of \( f_H \) and \( F_H \) respectively, the conditional probability density function of the waiting time is given by:

\[
f_{w|r,P,S}(w|P,S) = \frac{f_{H|P,S}(w+r|P,S)}{1 - F_{H|P,S}(r|P,S)} \tag{2.10}
\]

Like equation 2.9, this equation indicates that the broadcaster's view of the waiting time distribution -- conditional on the time since the most recent bus departure, \( r \), upstream headway, \( p \), and the bus location, \( s \) -- can be determined by shifting the conditional headway distribution to the left by \( r \), setting the value to zero for all \( w<0 \), and "scaling
up" the portion of the conditional headway distribution where $h$ is greater than $r$ such that the new function integrates to one.

2.4 Broadcast Time

To set the broadcast time, we assume that a broadcaster considers the waiting time distribution associated with a passenger arriving at a specific time and a passenger utility function. In this section, we derive the broadcaster’s policy for setting the optimal broadcast time.

Upon arriving at a bus stop, passengers are assumed to be informed of the next bus arrival time (broadcast time) via a sign at the bus stop. We mentioned that the broadcast time can be changed as the real-time data are updated. Therefore, bus passengers may see several broadcast times while they are waiting for the next bus. In this study, we assume that the passenger utility is influenced only by the broadcast time, $b$, posted at the instant the passenger arrives and the time that the passenger actually waits after his arrival, $w$. We also assume that passengers do not abort their plan to take the bus even in the event of a long broadcast time.

Therefore, an individual passenger’s utility, $u(w,b)$, is assumed to be a function of waiting time and broadcast time. In this research, we consider that all passengers have the same utility function. The utility function is specified such that the passenger’s utility is largest when the bus arrives at the time indicated by the broadcast time and decreases as the difference between the actual waiting time $w$ and broadcast time $b$ increases. As is typical in utility theory (Edwards, 1992), we set the maximum and minimum values of
the utility function to 1 and 0, respectively. Figure 2.2 illustrates the general piecewise linear utility function used in this research and the following is its mathematical specification:

\[ u(w, b) = \begin{cases} 
0, & w < b - C_1 \\
1 - \frac{b - w}{C_1}, & b - C_1 \leq w < b \\
1 - \frac{w - b}{C_2}, & b \leq w < b + C_2 \\
0, & w \geq b + C_2
\end{cases} \]  

(2.11)

where:

- \( w \) = the actual waiting time
- \( b \) = the broadcast time
- \( C_1, C_2 \) = parameters that indicate the maximum allowable differences between broadcast time \( b \) and actual waiting time \( w \) beyond which the utility remains zero.

Figure 2.2 Illustration of utility function specified in equation 2.11.
We assume that the broadcaster sets the broadcast time to maximize the expected utility of a passenger arriving at a specific time, where the expectation is taken over her knowledge of the distribution of the waiting time $w$ at that arrival time.

BAIS1 provides no real-time bus location data. The optimal broadcast time for this case is the same for every passenger arriving at anytime, since the waiting time distribution is the same for every passenger as far as the broadcaster can know. There is no reason to provide passengers arriving at different times with different broadcast times for the next bus arrival, since no real-time bus location data are available. The broadcaster's view of the expected utility of a passenger for a given $b$, denoted by $EU_{BAIS1}(b)$, is determined by taking the expectation across all the possible outcomes of passenger utilities. This is achieved by integrating over the waiting time distribution $f_w$ as follows:

$$EU_{BAIS1}(b) = \int f_w(w)u(w,b)dw$$  \hfill (2.12)

Substituting equation 2.4 in equation 2.12 yields the following:

$$EU_{BAIS1}(b) = \int \frac{1 - F_H(w)}{E[H]} u(w,b)dw$$  \hfill (2.13)

The broadcaster determines the optimal broadcast time so as to maximize the expected utility. Therefore the optimal broadcast time $b^*$ is given by:
\[ b^* = \arg \max_b EU_{BAIS1}(b) \]
\[ = \arg \max_b \int \frac{1 - F_H(w)}{E[H]} u(w, b)dw \]  
(2.14)

BAIS2 provides real-time data on the time since the most recent bus departure, \( r \).

As was presented in Section 2.3, given the arrival time of an individual passenger, the original waiting time distribution based on the historical headway distribution is updated by conditioning on the real-time data \( r \) at that time. The optimal broadcast time in this case can, therefore, differ from passenger to passenger according to the real-time data available to the broadcaster at the time of passenger arrival. We denote the broadcast time for the passenger arriving at a particular time associated with real-time data \( r \) by \( b_r \). In the case of BAIS2, the expected utility of an individual passenger arriving at a particular time associated with \( r \), denoted by \( EU_{BAIS2}(b_r) \), is again determined by taking the expectation across all the possible outcomes of the passenger's utilities. This expectation is achieved by integrating over the conditional waiting time distribution \( f_{w|r} \), rather than the unconditional distribution used in the case of BAIS1, because the broadcaster under BAIS2 can refine this distribution by conditioning on real-time data \( r \). Therefore the expected utility is given by:

\[ EU_{BAIS2}(b_r) = \int f_{w|r}(w|r)u(w, b_r)dw \]  
(2.15)

Substituting equation 2.9 in equation 2.15 yields the following:
Therefore, the optimal broadcast time for the passenger arriving at a time associated with a particular time since most recent bus departure \( r \), \( b^*_{r} \), is given by:

\[
b^*_{r} = \arg \max_{b_r} EU_{BAIS2}(b_r) \]

\[
= \arg \max_{b_r} \int_{w} \frac{f_H(w+r)}{1-F_H(r)} u(w,b_r) dw
\]  

BAIS3 provides real-time data on the most recent bus departure \( r \), upstream headway \( p \), and the location of the following bus at the time of passenger arrival \( s \). Like the case of BAIS2, the optimal broadcast time in this case can differ from passenger to passenger, since the real-time data can vary with the time of passenger arrival. We denote the broadcast time for the passenger arriving at a particular time associated with time since most recent bus departure \( r \), upstream headway \( p \), and the location of the following bus \( s \) by \( b_{r,p,s} \). The expected utility for the passenger arriving at a time associated with \( r \), \( p \) and \( s \), denoted by \( EU_{BAIS3}(b_{r,p,s}) \), is given by taking the expectation across all the possible outcomes of passenger utilities, which is achieved by integrating over the conditional waiting time distribution \( f_{w|r,p,s} \). Again, this conditional distribution is used because it reflects the most updated distribution the broadcaster can arrive at based on the real-time data \( r \), \( p \), and \( s \). Therefore, the expected utility is given by:

\[
EU_{BAIS3}(b_{r,p,s}) = \int_{w} f_{w|r,p,s}(w|r,p,s)u(w,b_{r,p,s}) dw
\]
Substituting equation 2.10 in equation 2.18 yields the following:

\[ EU_{BAIS1}(b_{r,p,s}) = \int_{-\infty}^{\infty} \frac{f_{H|p,s}(w+r|p,s)}{1 - F_{H|p,s}(r|p,s)} u(w,b_{r,p,s}) dw \]  

(2.19)

Therefore, the optimal broadcast time for the passenger arriving at a time associated with a particular time since most recent bus departure \( r \), upstream headway \( p \), and the location of the following bus \( s \), \( b^*_{r,p,s} \), is given by:

\[ b^*_{r,p,s} = \arg \max_{b_{r,p,s}} EU_{BAIS1}(b_{r,p,s}) \]

\[ = \arg \max_{b_{r,p,s}} \int_{-\infty}^{\infty} \frac{f_{H|p,s}(w+r|p,s)}{1 - F_{H|p,s}(r|p,s)} u(w,b_{r,p,s}) dw \]  

(2.20)

### 2.5 Expected Utility for System Evaluation

We already mentioned that the overall system performance is evaluated by the expected utility of a randomly selected passenger. This expected utility is equivalent to the expectation of passenger utilities taken across the passenger population. In this section, we formulate this expected utility. In BAIS1, the expected utility of a specific passenger is the same for any passenger, since the waiting time distribution along with the optimal broadcast time is the same due to the lack of availability of real-time data. Therefore, the expected utility of a random passenger, denoted by \( EU_{BAIS1}^* \), is given by equation 2.13 with \( b \) set at its optimal value \( b^* \). This is given by:
In BAIS2, the expected utility for each passenger differs according to the real-time data on the time \( r \) that has elapsed since the most recent bus departure when the passenger arrives. Therefore, the expected utility of a random passenger in this case is determined by integrating the expected utility of the individual passenger as viewed by the broadcaster over the distribution of \( r \) as follows:

\[
EU_{BAIS2}^* = \int f_R(r)EU_{BAIS2}^*(b^*)dr
\]  

(2.22)

Substituting equations 2.6 and 2.16 in equation 2.22 yields the following:

\[
EU_{BAIS2}^* = \int \int \left( \frac{1-F_H(w)}{E[H]} \right) \left( \frac{f_H(w+r)}{1-F_H(r)} \right) u(w,b^*)dwdr
\]

(2.23)

In BAIS3, the expected utility for each passenger differs according to the real-time data on the time since most recent bus departure \( r \), upstream headway \( p \), and the location of the following bus \( s \) available when the passenger arrives. The expected utility of a random passenger in this case is determined by integrating the expected utility of the individual as viewed by the broadcaster over the joint distribution of the real-time data \( r \), \( p \), and \( s \) as follows:
The joint probability function of the real-time data $r$, $p$, and $s$ is derived by integrating the joint probability function of $r$, $h$, $p$, and $s$ with respect to $h$ as follows:

$$f_{r,p,s}(r,p,s) = \int_{h=r} f_{r,h,p,s}(r,h,p,s) \, dh$$

The variable $h$ has to be greater than or equal to $r$ because the time since the most recent bus departure cannot exceed the headway. Recall that the waiting time $w$ is known once the headway $h$ and the time since most recent bus departure $r$ are given, since $h = w + r$. Similar to what was seen in equation 2.7, this indicates that, given $p$ and $s$, the joint probability density function of any pair of variables among $w$, $r$ and $h$, determines the joint probability density function of any other pair of these variables, since given any two variables the other variable is deterministic by $h = w + r$. Therefore, the joint probability function of $r$, $h$, $p$, and $s$ is given by:

$$f_{r,h,p,s}(r,h,p,s) = f_{p,s}(p,s) f_{r,h|p,s}(r,h|p,s)$$

$$= f_{p,s}(p,s) f_{r|w|h,p,s}(r,w|h,p,s)$$

The joint probability density function of $h$ and $w$, $f_{x,w}$, is given by equation 2.8. We can modify this equation to determine an expression for $f_{x,w|h,p,s}$ by substituting $f_{h|h,p,s}$ for $f_{h}$ and $E[H|p,s]$ for $E[H]$ where $E[H|p,s]$ is the expected headway given...
upstream headway, $p$ and the location of following bus $s$. These substitutions result in the following:

$$f_{x,w|p,s}(h,w|p,s) = \frac{f_{H|p,s}(h|p,s)}{E[H|p,s]} \quad (2.27)$$

Substituting this expression in equation 2.26 results in the following:

$$f_{r,x,p,s}(r,h,p,s) = f_{p,s}(p,s) \frac{f_{H|p,s}(h|p,s)}{E(H|p,s)} \quad (2.28)$$

Substituting equation 2.28 in equation 2.25 and recognizing that $h>r$ results in the following:

$$f_{r,p,s}(r,p,s) = \int_{h=r}^{\infty} f_{p,s}(p,s) \frac{f_{H|p,s}(h|p,s)}{E(H|p,s)} dh$$

$$= f_{p,s}(p,s) \frac{1-F_{H|p,s}(r|p,s)}{E(H|p,s)} \quad (2.29)$$

Substituting equations 2.19 and 2.29 in equation 2.24 yields the following:

$$EU_{B33}^{\ast} = \iint_{s,p,r} f_{r,p,s}(r,p,s)EU_{B33}^{\ast}(b_{r,p,s}^{\ast}) dr dp ds$$

$$= \iiint_{s,p,r,w} \left( f_{p,s}(p,s) \frac{1-F_{H|p,s}(r|p,s)}{E[H|p,s]} \right) \left( \frac{f_{H|p,s}(w+r|p,s)}{1-F_{H|p,s}(r|p,s)} \right) \mu(w,b_{r,p,s}^{\ast}) dw dr dp ds$$

$$= \iiint_{s,p,r,w} \frac{f_{H|p,s}(w+r,p,s)}{E[H|p,s]} \mu(w,b_{r,p,s}^{\ast}) dw dr dp ds \quad (2.30)$$
CHAPTER 3

SIMULATION

3.1 Introduction

We developed an analytic formulation for calculating the expected utility of a random passenger for three BAIS alternatives. However, when solving for the expected utility using a variety of utility functions and bus headway distributions, closed form analytical solutions can become intractable. We will, therefore, use Monte Carlo simulation to solve the formulations presented in Section 2.5 under various scenarios.

In probabilistic problems, the Monte Carlo method is used to simulate the random process of interest in order to arrive at the desired solution numerically (Hammersley and Handscomb, 1964). In this research, Monte Carlo simulation is used to calculate the expected utility in the three BAIS alternatives.

To compute the expected utilities, bus headways are generated randomly from the assumed headway distribution function. Passenger arrivals are also generated from an assumed inter-arrival time distribution function. Based on these generated headways and passenger arrivals, the realization of waiting times of each passenger are determined. The
time until the next bus arrival provided to each arriving passenger is referred to as the broadcast time. This time is determined by considering the broadcaster’s view of passenger waiting times determined from historical knowledge of the bus headway distribution and the real-time data available under the various BAIS alternatives. We will discuss in more detail the broadcaster’s view of passenger waiting times subsequently. Here it suffices to mention that once the broadcast time for each passenger is determined under each BAIS alternative, the expected utility for a particular BAIS can be calculated by averaging out the passenger utilities across all the generated passengers.

To present the simulation method, we start by discussing the methodology for generating headways, passengers and upstream headways from the assumed distribution functions in Section 3.2. In Section 3.3, we will illustrate the expected utility calculation using an example for each of the three BAIS alternatives. In Section 3.4, we will discuss how the broadcast times are determined again for the three BAIS alternatives. Finally, In Section 3.5, we describe the computer implementation of this simulation and the corresponding expected utility computation process.

### 3.2 Simulation Methodology

In this section, we discuss how we generate headway, passenger and upstream headway samples from the assumed probability distributions. We discuss the generation of headway samples first. Then, we discuss the generation of passenger samples based on the generated headway samples. We then discuss the generation of upstream headway samples for each passenger.
3.2.1 Headway Generation

To generate realizations of the headways at the bus stop of interest, we model an observed headway as being the sum of the headway mean and a random term. Let the mean and the variance of the given headway distribution function be denoted $\bar{h}$ and $\sigma_h^2$, respectively. Then, a particular headway realization, denoted by $h_i$, is formulated as follows:

$$ h_i = \bar{h} + \epsilon_{H,i} $$ (3.1)

where:

$\epsilon_{H,i} =$ normally distributed random variables with 0 mean and a variance of $\sigma_h^2$.

Headway realizations are determined by generating random numbers $\epsilon_{H,i}$ from a normal distribution with mean 0 and variance $\sigma_h^2$ and adding these random numbers to the assumed mean, $\bar{h}$. The mean and variance of the generated headways are $\bar{h}$ and $\sigma_h^2$, respectively.

To illustrate the generation of headways, consider an example in which the headway distribution is normal with a mean and standard deviation of 10 minutes and 2 minutes, respectively. We generate the realizations of headway at the bus stop. Assume that we generate the random number $\epsilon_{H,i} = 1.27$ from the normal distribution function of 0 mean and a standard deviation of 2. Since the mean headway is assumed to be 10 minutes, based on equation 3.1 the generated headway observation is 11.27 (i.e. $10 + 1.27 = 11.27$). For the sake of developing an illustrative example that will be used
throughout this section, consider four additional headways generated in the same way.

The values of these data are shown in Table 3.1.

<table>
<thead>
<tr>
<th>$h_i$</th>
<th>$i$</th>
<th>$h_{i,d}$</th>
<th>$h_i$</th>
</tr>
</thead>
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<td>11.27</td>
</tr>
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<td>2</td>
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<td>3</td>
<td>2.21</td>
<td>12.21</td>
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</tr>
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<td>7.02</td>
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</tr>
<tr>
<td>5</td>
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<td>10.40</td>
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</tr>
</tbody>
</table>

Table 3.1 Example realizations of headway at a bus stop of interest

3.2.2 Passenger Generation

As already discussed, in this study, passenger arrivals are assumed to follow a Poisson process reflecting random arrivals. That is, the inter-arrival times between consecutively arriving passengers follow an exponential distribution given by the following:

$$f(t) = \lambda e^{-\lambda t}$$  \hspace{1cm} (3.2)

where:

t = inter-arrival time, and
\lambda = mean passenger arrival rate.

The expected value of exponentially distributed inter-arrival times with mean \lambda of arrival rate is $1/\lambda$ and, hence, the inter-arrival times on average decrease as the arrival...
rate increases. Naturally, we can generate more passenger realizations by increasing the arrival rate. However, the expected utility of a random passenger associated with any of the BAIS alternatives is independent of the passenger arrival rate, since passenger arrivals are totally random over time. In other words, the passenger waiting time distribution is independent of the arrival rate. This notion is clearly reflected in the analytic expected utility functions derived in Section 2.5. Nevertheless, the expected utility computed by averaging across the simulated passengers would be more accurate with a higher arrival rate. However, simulating with more passengers increases the computational effort. A trade-off between accuracy and computation time is struck in this study in an informal fashion.

To generate passenger arrivals, a number of inter-arrival times are sampled from the exponential distribution of equation 3.2. These, along with the generated bus arrival times, determine the realized passenger waiting times. Let \( t_j \), the \( j^{th} \) random number generated from equation 3.2, be the inter-arrival time observation between passengers \( j-1 \) and \( j \) for \( j \geq 2 \). For \( j = 1 \), \( t_1 \) represents the time between the first bus arrival and the subsequent first passenger arrival. This time also follows the same exponential distribution due to its memoriless property. Let \( h_i \), the \( i^{th} \) headway generated from equation 3.1, be the headway between the arrivals of buses \( i \) and \( i+1 \). The bus arrival times and passenger arrival times are depicted in Figure 4.1. In this figure time 0 is assumed to be the time of the initial bus arrival (bus 1). The waiting time \( w_{1} \), for the first passenger who arrives in headway 1 is given by the following:
\[ w_i = h_i - t_i \]  

(3.3)

Let the \( w_j \) be the waiting time for the \( j^{th} \) passenger who arrives in headway \( i \) (i.e., after bus \( i \) departs and before bus \( i+1 \) arrives). The arrival time of bus \( i+1 \) is given by the sum of headways 1 through \( i \). The \( j^{th} \) passenger arrival time is given by the sum of passenger inter-arrival times 1 through \( j \). Therefore, the waiting time \( w_j \) for the \( j^{th} \) passenger is given by:

\[ w_j = \sum_{m=1}^{i} h_m - \sum_{n=1}^{j} t_n \]  

(3.4)

Therefore, the time since most recent bus departure time for passenger \( j \), \( r_j \), is given by:

\[ r_j = h_i - w_j \]  

(3.5)
Variable:
- $h_i$: headway between $i^{th}$ and $(i+1)^{th}$ bus arrival
- $t_j$: inter-arrival time between $(j-1)^{th}$ and $j^{th}$ arriving passenger
- $w_j$: waiting time of $j^{th}$ arriving passenger
- $r_j$: time since most recent bus departure for $j^{th}$ arriving passenger

Legend:
- I: bus arrival
- X: passenger arrival

Figure 3.1  Passenger waiting times determined from generated headways and passengers

To illustrate the generation of passengers, an arrival rate of 0.3 passengers per minute is assumed. For example, consider the first passenger inter-arrival time generated from equation 3.2 to be 1.30. This fall within the first generated bus headway of 11.27 minutes. Based on equation 3.4, the waiting time of the first passenger is 9.97 minutes (i.e. $9.97 = 11.27 - 1.30$). Based on equation 3.5, the time since most recent bus departure for the first passenger is 1.30 minutes (i.e. $1.30 = 11.27 - 9.97$). Let the second generated inter-arrival time to be 1.98. This, too, falls within first headway since
1.30 + 1.98 < 11.27. Based on equation 3.4, the waiting time of the second passenger is 7.99 minutes (i.e. $7.99 = 11.27 - 1.30 - 1.98$). Based on equation 3.5, the time since most recent bus departure for the second passenger is 3.28 minutes (i.e. $3.28 = 11.27 - 7.99$).

The waiting times and times since most recent bus departure of the remaining passengers can likewise be determined from equations 3.4 and 3.5. Table 3.2 shows bus headways, passenger inter-arrival times, and resulting passenger waiting times and the times since most recent bus departure derived for five sets of headways used for illustrative purposes.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$h_i$</th>
<th>$\Sigma h_i$</th>
<th>$j$</th>
<th>$t_j$</th>
<th>$\Sigma t_j$</th>
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Table 3.2  Example generation of passenger arrivals
3.2.3 Upstream Headway Generation

Upstream headways are generated for use as real-time data in BAIS3. We defined upstream headway as the headway at the location of the following bus at the moment of passenger arrival, where the headway is taken between this “following” bus and the preceding bus. Hence, upstream headway has dependence with the headway at the stop of interest and the location of the following bus. In order to capture the dependence between headway at the stop of interest, the upstream headway of the same pair of buses at the location of the following bus when the passenger arrives, and this following bus location, we consider the correlation between headway and upstream headway as a function of the location of the following bus. Since the following bus location varies at the time of each passenger arrival, the correlation between headway and upstream headway varies for each passenger. Hence, the upstream headway needs to be generated for each passenger in a fashion that reflects the correlation between the headways and the upstream headways at the moment of the passenger arrival. We will discuss the dependence of the correlation between headway and upstream headway and the bus location and generation of the upstream headways subsequently. Here, we develop the general relationship between headway and upstream headway by using a correlation coefficient between headway and upstream headway. For this, we will use the following notation:

\[ \sigma_H^2 = \text{variance of headways } H \text{ at the stop of interest}, \]
\[ \sigma_P^2 = \text{variance of upstream headways } P \text{ at the location of the following bus when the passenger arrives}, \]
\[ \text{Cov}(H, P) = \text{covariance between headways } H \text{ at the stop of interest and upstream headways } P. \]
The strength of the linear dependence between the headway at a bus stop and the headway at a point upstream of the bus stop is characterized by the correlation coefficient which is given by the following:

\[ \rho = \frac{\text{Cov}(H, P)}{\sigma_H \sigma_P} \]  

(3.6)

The correlation coefficient \( \rho \) can be shown to fall between -1 and +1. A correlation of +1 indicates a perfect positive linear association between the headway and the upstream headway, while a correlation of -1 indicates a perfect negative linear association. If the relationship between the headway and the upstream headway is linearly independent, the correlation coefficient takes the value zero.

For a realistic bus operation, only a positive correlation between the headway and the upstream headway is expected. The correlation coefficient approaches +1 as the distance between the upstream point and the considered bus stop approaches 0, since the headways at two points would be identical if the distance between these points is 0 (i.e., they are same points). On the other hand, the correlation coefficient would be expected to approach 0 as the distance becomes sufficiently great. However, there is no operational reason to expect negative correlation between the headway and the upstream headway unless the bus operation is controlled with an explicit purpose of obtaining a negative correlation. This would imply that the headway is likely to be longer (shorter) than the headway mean at the stop of interest when the headway is smaller (longer) than the
headway mean upstream of the stop. This would be unusual in a normal bus operation situation. Therefore, we assume that the correlation coefficient is positive in this study.

We also assume that the parameters (e.g., mean and variance) of the headway and the upstream headway distributions are the same. With these assumptions, the correlation coefficient in Equation 3.6 becomes:

\[
\rho = \frac{\text{Cov}(H, P)}{\sigma_H^2}
\]  

(3.7)

with \( 0 \leq \rho \leq 1 \).

We use this equation to capture the relationship between the headway and the upstream headway. To develop the formulation, we use a first order autoregressive model, a common model used in time series analysis (e.g., Chatfield, 1975). Since we assume that the mean and variance of the bus stop headway and the upstream headway distributions are the same, we can use variables of the differences \((p - \bar{h})\) and \((h - \bar{h})\) in the first order autoregressive model which is given by the following:

\[
(p - \bar{h}) = \alpha(h - \bar{h}) + \varepsilon_p
\]  

(3.8)

where:

\( \alpha = \) the parameter that takes the value of the correlation coefficient \( \rho \), and

\( \varepsilon_p = \) a random variable whose mean is zero and variance is \((1 - \alpha^2)\sigma_H^2\).

Since there is no change in the means and the variances of the headway (henceforth unless otherwise mentioned the terms "headway" refers to the headway at the bus stop)
and upstream headway, the above autoregressive process is said to be stationary.

Equation 3.8 can be modified as follows:

\[ p = \bar{h} + \rho(h - \bar{h}) + \varepsilon_p \quad (3.9) \]

The mean of \( \varepsilon_p \) is assumed to be 0 without loss of generality. The variance of \( \varepsilon_p \), denoted by \( \sigma_p^2 \), is given by:

\[ \sigma_p^2 = (1 - \rho^2)\sigma_h^2 \quad (3.10) \]

In addition, given that marginal distribution of \( h \) is assumed to be normal, the marginal distribution of \( p \) and the conditional distribution of \( h \) given \( p \) are also normal.

Equations 3.9 and 3.10 provide the general relation between the upstream headway and the headway. Now, we discuss the dependence of the correlation coefficient and the following bus location at the moment of passenger arrival. In Section 2.1, we defined the location of the following bus at time of passenger arrival, denoted by \( s \), as the upstream distance between the bus stop of interest and the following bus location. We mentioned that the correlation coefficient between the headway and the upstream headway is considered to be a function of \( s \), \( \rho(s) \). In this study, we assume that the correlation coefficient is an exponential function of \( s \). Considering that the correlation coefficient is 1 when \( s \) equals 0, this exponential function is given by:

\[ \rho = e^{-as} \quad (3.11) \]

where:

\( a \) = parameter of the function.
Since we do not model the bus location, $s$, in this study, we use the waiting times computed from the generated headways and passengers to determine the correlation coefficient between the headway and the upstream headway for each passenger. The distance, $s$, is the product of the waiting time, $w$, and average bus velocity over the distance $s$, $v$ (see Figure 2.1):

$$ s = w \times v $$

(3.12)

The value of bus velocity, $v$, is uncertain at the time of passenger arrival, and therefore it is modeled as being the sum of the mean velocity $\bar{v}$ plus a random term $\xi$ as follows:

$$ v = \bar{v} + \xi $$

(3.13)

We assume that the random term $\xi$ is normally distributed. Substituting equation 3.13 in equation 3.12 yields the following:

$$ s = w(\bar{v} + \xi) = w\bar{v}(1 + \frac{\xi}{\bar{v}}) = w\bar{v}(1 + \varepsilon) $$

(3.14)

where:

$$ \varepsilon = \frac{\xi}{\bar{v}}. $$

The mean of the random variable $\varepsilon$ is assumed to be 0 without loss of generality. The standard deviation of this random variable is the same as the coefficient of variation of $v$, since the variance of $\xi$ is the same as the variance of $v$ from equation 3.13.

Substituting equation 3.14 in equation 3.11 yields the following:
\[ \rho = e^{-\alpha \bar{w}(1-\kappa)} \]

Equation 3.15 is an equation that allows us to determine the correlation coefficient between headway and upstream headway at the moment of each passenger arrival in the simulation by using the waiting time of the passenger. We will discuss in detail how to determine the correlation coefficient to generate upstream headway for each passenger subsequently.

We now discuss the generation of upstream headways given the generated headways and passenger waiting times. Figure 3.2 depicts the upstream headways, \( p_{j-2} \), \( p_{j-1} \) and \( p_j \), for the last three passengers (i.e. passengers \( j-2, j-1 \) and \( j \)) arriving during headway \( h_i \). The upstream headway for the last arriving passenger (passenger \( j \)), \( p_j \), is generated based on the headway \( h_i \). However, the upstream headway for the second to last passenger (passenger \( j-1 \)) is generated based on the generated \( p_j \) instead of \( h_i \) since the correlation coefficient between \( p_j \) and \( p_{j-1} \) has to be considered. Similarly, we generate upstream headways for each passenger (excluding the last passenger in the headway) based on the generated upstream headway of the next passenger to arrive. For example, \( p_{j-2} \) is generated based on the generated \( p_{j-1} \). We discuss the generation of \( p_j \) and \( p_{j-1} \) next.
Figure 3.2 Example of upstream headways for passengers arriving in headway $h_i$

To generate the upstream headway $p_j$, we use equation 3.9. The relationship between headway $i$, $h_i$, and the upstream headway for passenger $j$, $p_j$, is given by:

$$p_j = h + \rho_j (h_i - h) + \varepsilon_{p_j}$$  

(3.16)

where:

$\varepsilon_{p_j} =$ the random variable whose mean is 0 and variance is $(1 - \rho_j^2) \sigma_h^2$, and 

$\rho_j =$ the correlation coefficient between $h_i$ and $p_j$.

Based on equation 3.15, the correlation coefficient, $\rho_j$, is given by:

$$\rho_j = e^{-\sigma_w \tau (1 + \varepsilon_{p_j})}$$  

(3.17)
where:

\[ \mathcal{E}_{v,j} = \text{realization of } \mathcal{E}_v \text{ to determine } \rho_j. \]

To generate the upstream headway \( p_{j-1} \), we need to modify equation 3.16. Since the marginal distributions of \( p_j \) and \( p_{j-1} \) are assumed to be the same, the relationship between \( p_j \) and \( p_{j-1} \) is given by:

\[
p_{j-1} = \bar{h} + \rho_{j,j-1}(p_j - \bar{h}) + \varepsilon_{p_{j-1}} \tag{3.18}
\]

where:

\[ \varepsilon_{p_{j-1}} = \text{random variable whose mean is 0 and variance equals } (1 - \rho_{j,j-1}^2)\sigma_h^2, \text{ and} \]

\[ \rho_{j,j-1} = \text{the correlation coefficient between } p_j \text{ and } p_{j-1}. \]

During the time interval between the arrival of passengers \( j-1 \) and \( j \), the location of the following bus would have changed by \( s_{j-1} - s_j \) (see Figure 3.2). This distance, \( s_{j-1} - s_j \), can be estimated by the difference in the waiting times of these two passengers, \( w_{j-1} - w_j \). Using equation of 3.15, the correlation coefficient between \( p_j \) and \( p_{j-1} \), \( \rho_{j,j-1} \), is given by:

\[
\rho_{j,j-1} = e^{-\alpha (w_{j-1} - w_j) k(1 + \varepsilon_{v,j-1})} \tag{3.19}
\]

where:

\[ \mathcal{E}_{v,j-1} = \text{realization of } \mathcal{E}_v \text{ to determine } \rho_{j,j-1}. \]

The correlation coefficient between \( h_j \) and \( p_{j-1} \), denoted by \( \rho_{j-1} \), needs to be determined to predict the conditional headway distribution given upstream headway \( p_{j-1} \) for
forecasting optimal broadcast time when computing the expected utility. The correlation coefficient $\rho_{j-1}$ can be computed from $\rho_j$ and $\rho_{j-1}$ as follow:

$$
\rho_{j-1} = e^{-\alpha_j x_{j-1}}
$$

$$
= e^{-\alpha_j (x_j - x_{j-1})}
$$

$$
= e^{-\alpha_j} e^{-\alpha_j (s_j - s_{j-1})}
$$

$$
= \rho_j \rho_{j-1}
$$

(3.20)

To illustrate the generation of upstream headways, consider the following example that assumes specific values of $\alpha$, $\bar{v}$ and coefficient of variation of $v$. Let the parameter of the correlation function $\alpha$ and the coefficient of variation of the velocity distribution of $v$ be 0.0001 and 0.1, respectively. Let the mean bus velocity $\bar{v}$ be 600 meter per minute. We assumed that upstream headway distributions are the same as the headway distribution at the bus stop. As in the illustration of headway generation, the upstream headway is assumed to be normally distributed with 10 minute mean and 2 minute standard deviation.

We first determine the correlation coefficient and then generate upstream headways based on the determined correlation coefficient. Consider passenger 3 in Table 3.2, who arrives last in headway 1. From this table, headway 1 is $h_1 = 11.27$ minutes, and the waiting time of this passenger is 2.51 minutes. We then generate a random number for the realization of $\xi_{v,j}$ in equation 3.17 from the normal distribution with 0 mean and a standard deviation of 0.1 (since standard deviation of $\xi_{v,j}$ is the same of coefficient of
variation of \(v\)). Assume that the number is -0.032. Based on the equation 3.17 the correlation coefficient for this passenger is 
\[
\rho_3 = e^{-0.0001 \times (10 \times 60) \times 2.51 \times (1 - 0.032)} = 0.864.
\]
Based on equation 3.10, the standard deviation of the error term of equation 3.16 is 
\[
\sqrt{(1 - 0.864^2)} \times 2^2 = 1.01 \text{ minutes.}
\]
We then generate a random number from the normal distribution with 0 mean and a standard deviation of 1.01 (the value just calculated) for the upstream headway realization. Assume that this number is \(\varepsilon_{p,3} = 0.51\). The corresponding upstream headway observation for passenger 3 can be determined using equation 3.16 as 
\[
p_3 = \bar{h} + \rho_3 (h_1 - \bar{h}) + \varepsilon_{p,3} = 10 + 0.864 \times (11.27 - 10) + 0.51 = 11.61 \text{ minutes.}
\]

For another example, consider passenger 2 who arrives before passenger 3 (i.e. second last passenger in headway 1). The waiting time of this passenger is 7.99 minutes. Assume that we generate a random number of 0.031 for the realization of \(E_{v,j-1}\) in equation 3.19. Based on the equation 3.19 the correlation coefficient between upstream headways for passengers 2 and 3 is 
\[
\rho_{2,3} = e^{-0.0001 \times (10 \times 60) \times (7.99 - 2.51) \times (1 + 0.031)} = 0.712.
\]
Based on equation 3.10, the standard deviation of the error term of equation 3.18 is 
\[
\sqrt{(1 - 0.712^2)} \times 2^2 = 1.40 \text{ minutes.}
\]
Assume that we generate a random number \(\varepsilon_{p,2} = -1.29\) from the normal distribution with 0 mean and a standard deviation of 1.40 for the upstream headway realization. The corresponding upstream headway observation for passenger 2 can be determined using equation 3.18 as 
\[
p_2 = \bar{h} + \rho_{2,3} (p_3 - \bar{h}) + \varepsilon_{p,2} = 10 + 0.712 \times (11.61 - 10) - 1.29 = 9.86 \text{ minutes.}
\]
equation 3.20, the correlation coefficient between the headway at the bus stop of interest and the upstream headway for the passenger 3 is $\rho_2 = \rho_3, \rho_{2,3} = 0.864 \times 0.712 = 0.616$.

The values of the correlation coefficient and upstream headway for other passengers can be determined in a similar fashion. The generated correlation coefficients and upstream headways for other passengers are shown in Table 3.3.

<table>
<thead>
<tr>
<th>i</th>
<th>$h_i$</th>
<th>$j$</th>
<th>$w_j$</th>
<th>$\varepsilon_{\gamma,j}$</th>
<th>$\rho_{i,j-1}$</th>
<th>$\rho_j$</th>
<th>$\varepsilon_{\rho_j}$</th>
<th>$p_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.27</td>
<td>1</td>
<td>9.97</td>
<td>-0.154</td>
<td>0.904</td>
<td>0.557</td>
<td>0.70</td>
<td>10.57</td>
</tr>
<tr>
<td>1</td>
<td>11.27</td>
<td>2</td>
<td>7.99</td>
<td>0.031</td>
<td>0.712</td>
<td>0.616</td>
<td>-1.29</td>
<td>9.86</td>
</tr>
<tr>
<td>1</td>
<td>11.27</td>
<td>3</td>
<td>2.51</td>
<td>-0.032</td>
<td>0.864</td>
<td>0.51</td>
<td>11.61</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9.34</td>
<td>4</td>
<td>5.73</td>
<td>0.047</td>
<td>0.999</td>
<td>0.706</td>
<td>-0.09</td>
<td>9.83</td>
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<tr>
<td>2</td>
<td>9.34</td>
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<td>5.71</td>
<td>0.013</td>
<td>0.707</td>
<td>0.39</td>
<td>9.92</td>
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<tr>
<td>3</td>
<td>12.21</td>
<td>6</td>
<td>10.6</td>
<td>0.162</td>
<td>0.816</td>
<td>0.507</td>
<td>-0.73</td>
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</tr>
<tr>
<td>3</td>
<td>12.21</td>
<td>7</td>
<td>7.68</td>
<td>0.099</td>
<td>0.735</td>
<td>0.621</td>
<td>-1.59</td>
<td>11.13</td>
</tr>
<tr>
<td>3</td>
<td>12.21</td>
<td>8</td>
<td>3.01</td>
<td>0.022</td>
<td>0.950</td>
<td>0.846</td>
<td>0.89</td>
<td>13.70</td>
</tr>
<tr>
<td>3</td>
<td>12.21</td>
<td>9</td>
<td>2.17</td>
<td>-0.107</td>
<td>0.890</td>
<td>0.99</td>
<td>12.96</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7.02</td>
<td>10</td>
<td>6.75</td>
<td>0.011</td>
<td>0.956</td>
<td>0.672</td>
<td>0.08</td>
<td>8.34</td>
</tr>
<tr>
<td>4</td>
<td>7.02</td>
<td>11</td>
<td>6.00</td>
<td>-0.036</td>
<td>0.971</td>
<td>0.703</td>
<td>-0.88</td>
<td>8.18</td>
</tr>
<tr>
<td>4</td>
<td>7.02</td>
<td>12</td>
<td>5.49</td>
<td>0.096</td>
<td>0.876</td>
<td>0.724</td>
<td>0.55</td>
<td>9.03</td>
</tr>
<tr>
<td>4</td>
<td>7.02</td>
<td>13</td>
<td>3.47</td>
<td>-0.088</td>
<td>0.827</td>
<td>0.73</td>
<td>8.27</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10.4</td>
<td>14</td>
<td>9.79</td>
<td>-0.054</td>
<td>0.978</td>
<td>0.561</td>
<td>0.17</td>
<td>9.21</td>
</tr>
<tr>
<td>5</td>
<td>10.4</td>
<td>15</td>
<td>9.40</td>
<td>0.105</td>
<td>0.812</td>
<td>0.573</td>
<td>-1.38</td>
<td>9.02</td>
</tr>
<tr>
<td>5</td>
<td>10.4</td>
<td>16</td>
<td>6.26</td>
<td>-0.010</td>
<td>0.882</td>
<td>0.706</td>
<td>-0.78</td>
<td>10.49</td>
</tr>
<tr>
<td>5</td>
<td>10.4</td>
<td>17</td>
<td>4.14</td>
<td>-0.106</td>
<td>0.801</td>
<td>1.12</td>
<td>11.44</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3 Example realizations of upstream headways
3.3 Expected Utility Computation

We discussed the methods for generating headways, passenger arrivals and upstream headways from the assumed distribution functions, along with the corresponding computation of the waiting times. We now present the computation of the expected utilities from the simulation results. The expected utility is computed after determining the optimal broadcast time for each passenger. The broadcaster’s knowledge of the waiting time distribution is determined based on the real-time bus location data and the historical headway distribution. The generated headways can be used to approximate the historical headway distribution. The real-time data to the broadcaster has to also be consistent with the simulated bus headways, passenger arrivals and upstream headway.

Let the numbers of passenger and headway observations generated be $J$ and $I$, respectively. Recall that the waiting time for passenger $j$ is denoted $w_j$. Let $b_j$ denote the broadcast time provided to passenger $j$. The optimal broadcast time for passenger $j$ is denoted $b^*_j$. The $b^*_j$ is determined by the broadcaster based on the real-time data provided by either BAIS1, BAIS2, or BAIS3, along with the historical bus headway information. The utility of passenger $j$ is then given by $u(w_j, b^*_j)$. Since the probability of selecting any of $J$ passengers at random is $1/J$, the expected utility of a random passenger is given by the following:

$$EU = \frac{1}{J} \sum_{j=1}^{J} u(w_j, b^*_j)$$  (3.21)
Recall, BAIS1 is the case in which we assume that there is no real-time vehicle location data available to the broadcaster. In this case, the distribution based on the actual waiting times of all passengers can be used as the broadcaster's knowledge of the historical waiting time distribution. We discussed in section 2.4 that the optimal broadcast time is constant across all passengers in BAIS1. Let this constant broadcast time be denoted by \( b \). Based on the equations 2.14 and 3.21, and simulation results, the optimal broadcast \( b^* \) is then given by the following:

\[
b^* = \arg \max_b \frac{1}{J} \sum_{j=1}^{J} u(w_j, b)
\]  

(3.22)

To illustrate, consider the example data presented in Table 3.3 and a utility function given by the following:

\[
u(b, w) = \begin{cases} 
1 - \frac{|b - w|}{15}, & |b - w| < 15 \\
0, & \text{otherwise}
\end{cases}
\]  

(3.23)

The waiting times shown in Table 3.2 reflect the historical waiting time distribution. We assume that the broadcast time is determined to the nearest 0.01 minutes. Given \( b \), the expected utility of any passenger is calculated by \( EU = \frac{1}{J} \sum_{j=1}^{J} u(w_j, b) \). For example, for \( b=4 \), using the figures in Table 3.2 the expected utility is calculated as follows:

\[
EU(b = 4) = \frac{1}{17} \left( 1 - \frac{|4 - 9.97|}{15} \right) + \left( 1 - \frac{|4 - 7.99|}{15} \right) + \cdots + \left( 1 - \frac{|4 - 4.14|}{15} \right) = 0.810
\]
For \( b=5 \) the expected utility is calculated as follows:

\[
EU(b = 5) = \frac{1}{17} \left[ \left(1 - \frac{|5 - 9.97|}{15}\right) + \left(1 - \frac{|5 - 7.99|}{15}\right) + \ldots + \left(1 - \frac{|5 - 4.14|}{15}\right) \right] = 0.838
\]

In this example a value for \( b \) of 5 is preferred over a value of 4. The expected utility for every increment of 0.01 minutes for \( b \) in some specified range is computed and the time that maximizes the expected utility is determined. With this set of data, \( b^* = 6.03 \) minutes maximizes the expected utility and produces an expected utility value of 0.858. For this case, the broadcast time is set at 6.03 minutes for all passengers. Since broadcast time does not vary across passengers in the case of BAIS1, the expected utility of a random passenger is 0.858 as well.

As presented in this illustration, the optimal broadcast time can be determined by enumerating the expected utilities over an interval for \( b \) using some specified step size. We will discuss a more efficient way for determining the optimal broadcast time in Section 3.4.

BAIS2 is the case in which real-time data on the last bus departure time is available to the broadcaster. As discussed in Chapter 2, the broadcaster in this case has the ability to determine the optimal broadcast time for each passenger. Consider passenger \( j \). Based on equation 2.17, to determine the optimal broadcast time for passenger \( j \), the broadcaster determines the waiting time distribution conditional on the time since most recent bus departed from the bus stop for passenger \( j \), \( r_j \).
Recall that given the time since most recent bus departure for passenger \( j \), \( r_j \), the headway cannot be less than \( r_j \). Using \( k \) to index all the generated headway greater than \( r_j \), the waiting times conditioned on the time since most recent bus departure for passenger \( j \), \( w_{j,k} \), are given by the following:

\[
 w_{j,k} = h_k - r_j \tag{3.24}
\]

Let \( K_j \) denote the total number of headways greater than the time since most recent bus departure \( r_j \). The probability of randomly selecting such a headway is given by \( 1/K_j \).

The optimal broadcast time \( b^*_j \) for passenger \( j \) is given by the following:

\[
b^*_j = \arg \max_{b_j} \frac{1}{K_j} \sum_{k=1}^{K_j} u(w_{j,k}, b_j) \tag{3.25}
\]

To illustrate, consider finding the broadcast time for the first passenger generated in Table 3.2. The time since most recent bus departure for this passenger, \( r_1 \), is 1.30, whereas the headway distribution from Table 3.1 consists of the values 11.27, 9.34, 12.21, 7.02, and 10.40 each occurring with equal probability. The conditional waiting time distribution for this passenger can be determined from the value of \( w_{1,k} = h_k - 1.30 \), \( k=1,\ldots,5 \). For example, the value for \( w_{1,1} \) is given by 11.27-1.30 = 9.97 minutes. Four additional waiting times can be determined in the same way. Table 3.4 shows the conditional waiting times, given the time since most recent bus departure for passenger 1. Each of these times would be considered equally likely to occur with probability 1/5.
Using the values in Table 3.4, the optimal broadcast time for passenger 1 is determined in a similar way to that of BAIS1. For $b=7$, the expected utility for passenger 1 is given by the following:

$$EU(b = 7) = \frac{1}{5} \left[ \left( 1 - \frac{|7 - 9.97|}{15} \right) + \left( 1 - \frac{|7 - 8.04|}{15} \right) + \ldots + \left( 1 - \frac{|7 - 9.10|}{15} \right) \right] = 0.849$$

For $b=8$, the expected utility of passenger 1 is given by the following:

$$EU(b = 8) = \frac{1}{5} \left[ \left( 1 - \frac{|8 - 9.97|}{15} \right) + \left( 1 - \frac{|8 - 8.04|}{15} \right) + \ldots + \left( 1 - \frac{|8 - 9.10|}{15} \right) \right] = 0.889$$

In this example, a value for $b$ of 8 is preferred over a value of 7. By enumerating all the expected utilities using an increment of 0.01 minutes for $b$, we find that the expected utility for passenger 1 takes the maximum value of 0.905 for $b=9.10$. The optimal broadcast time for passenger 1 is therefore set at 9.10 minutes. We can determine the broadcast times for other passengers in a similar fashion. Table 3.5 shows the optimal broadcast times determined in this way for all passengers.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$r_i$</th>
<th>$k$</th>
<th>$h_{i,k}$</th>
<th>$w_{i,k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3</td>
<td>1</td>
<td>11.27</td>
<td>9.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>9.34</td>
<td>8.04</td>
</tr>
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<td></td>
<td></td>
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<td>10.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>7.02</td>
<td>5.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>10.40</td>
<td>9.10</td>
</tr>
</tbody>
</table>

Table 3.4 Waiting times in BAIS2 for passenger sample 1
Table 3.5 Broadcast times for an arriving passenger in the case of BAIS2

<table>
<thead>
<tr>
<th>headway(i)</th>
<th>passenger(j)</th>
<th>broadcast time ( b^*_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>9.10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7.09</td>
</tr>
<tr>
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<td>1.88</td>
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<td>6.27</td>
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<tr>
<td></td>
<td>17</td>
<td>4.16</td>
</tr>
</tbody>
</table>

The expected utility of a random passenger for BAIS2 is calculated according to equation 3.21 using the optimal broadcast times for each passenger shown in Table 3.5, and the actual waiting times for each passenger shown in Table 3.2. Thus, the expected utility of a random passenger for BAIS2 for this set of data is given by the following:

\[
EU_{BAIS2} = \frac{1}{17} \left( \left( 1 - \frac{9.10 - 9.97}{15} \right) + \left( 1 - \frac{7.09 - 7.99}{15} \right) + \ldots + \left( 1 - \frac{4.16 - 4.14}{15} \right) \right) = 0.907
\]
BAIS3 is the case in which the real-time data on the time since the most recent bus departure and the upstream headway are available to the broadcaster. Similar to the case of BAIS2, consider passenger \( j \). To determine the optimal broadcast time for passenger \( j \) based on equation 2.20, the broadcaster determines the waiting time distribution conditional on the time since most recent bus departure and upstream headway at the moment when passenger \( j \) arrives. Let real-time data on the upstream headway when passenger \( j \) arrives be denoted by \( p_r \). We generate the broadcaster’s view of conditional headways based on the given upstream headway \( p_r \) (observed by the broadcaster at the time of passenger arrival). Since the upstream headway \( p_r \) is generated based on a realization of the headway (see equation 3.16), the broadcaster’s view of the conditional headway distribution should be determined by Bayes’ Theorem as follows:

\[
 f_{H|P}(h|p_j) = \frac{f_{P|H}(p_j|h)f_{H}(h)}{f_{P}(p_j)} \quad (3.26)
\]

This may be difficult to implement, especially for real-time operations. We discuss this as a topic for future research in Chapter 5. Here, we assume that the broadcaster uses a heuristic model for the conditional headway distribution. Let \( \hat{h}_{j,i} \) denote the realization of the conditional headway at the bus stop of interest given upstream headway \( p_r \). We assume that the broadcaster uses the following heuristic:

\[
 \hat{h}_{j,i} = p_j + \epsilon_{p,j} \quad (3.27)
\]

where:
\( \varepsilon_{p,i} \) = realization of a normally distributed error term with 0 mean and standard deviation \( \sqrt{1 - \rho_i^2 \sigma_h} \) (i.e. realization of \( \varepsilon_p \), see equation 3.9),

\( \rho_i \) = correlation coefficient between \( p \) and \( h \) (see Table 3.3), and

\( \sigma_h \) = standard deviation of headway distribution.

This equation can be used to compute several realizations \( \hat{h}_{i,l} \) given \( p_i \) for \( l = 1, 2, \ldots, L \), where \( L \) is the total number of realizations of \( \hat{h}_{i,l} \). Once a \( \hat{h}_{i,l} \) is generated, the waiting time conditional on the time since most recent bus departure and the upstream headway is given by the following:

\[
w_{j,i} = \hat{h}_{j,i} - r_j
\]  

(3.28)

Recall that \( r_j \) is a real-time datum on the time since most recent bus departure for passenger \( j \). Similar to BAIS2, only conditional headways, \( \hat{h}_{j,l} \), greater than the time since most recent bus departure \( r_j \) are used to determine the conditional waiting times.

Let \( L_j \) denote the number of conditional headways greater than the time since the most recent bus departure \( r_j \). Since each realization is equally likely, the probability of each corresponding conditional waiting time for passenger \( j \) is given by \( 1/L_j \). Therefore, the optimal broadcast time \( b_j^* \) for passenger \( j \) is given by the following:

\[
b_j^* = \arg \max_{b_i} \frac{1}{L_j} \sum_{l=1}^{L_j} u(w_{j,i}, b_j)
\]  

(3.29)
To illustrate, consider finding the broadcast time for the first generated passenger of Table 3.2. The time since most recent bus departure $r_1$ and the upstream headway $p_1$ for this passenger are 1.30 and 10.57 respectively. The correlation coefficient $\rho_1$ is 0.557 (see Table 3.3). Recall that we assume the standard deviation of headway distribution is 2 minutes. Hence, the realization of random term $\varepsilon_p$, $\varepsilon_{p,d}$, of equation 3.27 is generated from the normal distribution with 0 mean and \( \sqrt{(1-0.557^2)2^2} = 1.66 \) standard deviation. In this example, five such conditional headways are generated. This happens to equal the number of generated headway $h$, but does not have to be. Assume that $\varepsilon_{p,d}$ are generated as 2.10, -0.86, 0.20, 0.94, and -1.65. Then we determine the conditional headway distribution from the random number generated. For instance, $\hat{h}_{i,1}$ is determined from equation 3.27 as follows:

$$\hat{h}_{i,1} = p_1 + \varepsilon_{p,i} = 10.57 + 2.10 = 12.67 \text{ minutes}$$

Subsequently, $w_{i,1}$ is determined as follows:

$$w_{i,1} = \hat{h}_{i,1} - r_1 = 12.67 - 1.3 = 11.37 \text{ minutes}$$

The other four waiting times for passenger 1 are determined in a similar fashion. Since all realization are equally likely, the probability of any of these five waiting times for passenger 1 is $1/5$. Table 3.6 shows the variables used to determine the waiting times for passenger 1 and the estimated waiting times.
Table 3.6  Waiting times in BAIS3 for passenger 1

For another example, consider passenger 4 of Table 3.2. The time since most recent bus departure $r_4$ and the upstream headway $p_4$ for this passenger are 3.61 and 9.83 respectively. The correlation coefficient $\rho_4$ is 0.706 (see Table 3.3). Assume that $\varepsilon_{p,t}$ are generated as -0.17, 1.44, -1.02, -1.60, and 0.52. Realization of $\hat{h}_{4,1}$ is determined from equation 3.27 as follow:

$$\hat{h}_{4,1} = p_4 + \varepsilon_{p,4} = 9.83 - 0.17 = 9.66$$

Subsequently, $w_{4,1}$ is determined as follow:

$$w_{4,1} = \hat{h}_{4,1} - r_4 = 9.66 - 3.61 = 6.05$$

The other four waiting times for this fourth passenger are determined in similar fashion. Table 3.7 presents the variables used to determine the waiting times for passenger 4 and the estimated waiting times.
Table 3.7 Waiting times in BAIS3 for passenger 4

<table>
<thead>
<tr>
<th>$j$</th>
<th>$r_i$</th>
<th>$p_i$</th>
<th>$l$</th>
<th>$\epsilon_{p,i}$</th>
<th>$w_{\mu}$</th>
<th>$1/L_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.61</td>
<td>9.84</td>
<td>1</td>
<td>-0.17</td>
<td>6.05</td>
<td>1/5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>1.44</td>
<td>7.66</td>
<td>1/5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>-1.02</td>
<td>5.20</td>
<td>1/5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>-1.60</td>
<td>4.62</td>
<td>1/5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>0.52</td>
<td>6.74</td>
<td>1/5</td>
</tr>
</tbody>
</table>

The distributions of waiting times for the other passengers are determined in similar fashion.

With the waiting time distribution and a specified utility function, the optimal broadcast time can be calculated as discussed in BAIS1 and BAIS2. Consider the first passenger, whose conditional waiting time distribution is shown in Table 3.6. Using the same utility function given by equation 3.23, for $b = 9$, the expected utility of passenger 1 is given by:

$$EU(b = 9) = \frac{1}{5} \left( \left( 1 - \frac{|9 - 11.37|}{15} \right) + \left( 1 - \frac{|9 - 8.41|}{15} \right) + \ldots + \left( 1 - \frac{|9 - 7.62|}{15} \right) \right) = 0.920$$

For $b = 8$, the expected utility of passenger 1 is given by:

$$EU(b = 8) = \frac{1}{5} \left( \left( 1 - \frac{|8 - 11.37|}{15} \right) + \left( 1 - \frac{|8 - 8.41|}{15} \right) + \ldots + \left( 1 - \frac{|8 - 7.62|}{15} \right) \right) = 0.895$$

Therefore, in this example a value for $b$ of 9 is preferred over a value of 8. By enumerating all the expected utility for using an increment of 0.01 minutes for $b$, we find
that the expected utility for passenger 1 is maximized when $b = 9.47$ minutes. The optimal broadcast time for passenger 1 is therefore set at 9.47 minutes.

For another example, consider passenger 4 whose conditional waiting time distribution is found in Table 3.7. For $b = 6$, the expected utility of passenger 4 is given by:

$$EU(b = 6) = \frac{1}{5} \left[ \left( 1 - \frac{|6 - 6.05|}{15} \right) + \left( 1 - \frac{|6 - 7.66|}{15} \right) + \ldots + \left( 1 - \frac{|6 - 6.74|}{15} \right) \right] = 0.938$$

For $b = 5$, the expected utility of passenger 4 is given by:

$$EU(b = 5) = \frac{1}{5} \left[ \left( 1 - \frac{|5 - 6.05|}{15} \right) + \left( 1 - \frac{|5 - 7.66|}{15} \right) + \ldots + \left( 1 - \frac{|5 - 6.74|}{15} \right) \right] = 0.920$$

Therefore a value for $b$ of 6 is preferred over a value of 5. By enumerating all the expected utility for using an increment of 0.01 minutes for $b$, we find that the expected utility for passenger 4 is maximized when $b = 6.05$. The optimal broadcast time for passenger 4 is therefore set at 6.05 minutes. The optimal broadcast times for other passengers can be found in a similar fashion.

Note that actual waiting times had been previously generated in the simulation for each passenger (see Table 3.2). When setting the optimal broadcast time $b_j^*$, the broadcaster is assumed not to know these actual waiting times. The optimal broadcast time is set by the simulated waiting time distribution conditioned on upstream headway and time since most recent bus departure. However, once the $b_j^*$ is set for a passenger, the actual waiting time data is available to determine that passenger utility $u(b_j^*, w_j)$.
For example, the passenger utility for passenger 1 and 4 are determined as:

\[
1 - \frac{|9.47 - 9.97|}{15} = 0.967 \quad \text{and} \quad 1 - \frac{|6.05 - 5.73|}{15} = 0.979,
\]

respectively. Table 3.8 shows the optimal broadcast time computed, the waiting time, and the resulting utility for each passenger in Table 3.3.

<table>
<thead>
<tr>
<th>headway ((i))</th>
<th>passenger ((j))</th>
<th>broadcast time ((b_j^*))</th>
<th>waiting time ((w_j))</th>
<th>passenger utility (u(b_j^*, w_j))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>9.47</td>
<td>9.97</td>
<td>0.967</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6.83</td>
<td>7.99</td>
<td>0.923</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.83</td>
<td>2.51</td>
<td>0.979</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6.05</td>
<td>5.73</td>
<td>0.979</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.52</td>
<td>5.71</td>
<td>0.946</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>8.69</td>
<td>10.6</td>
<td>0.872</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>6.37</td>
<td>7.68</td>
<td>0.913</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>4.13</td>
<td>3.01</td>
<td>0.925</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>2.79</td>
<td>2.17</td>
<td>0.958</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>8.81</td>
<td>6.75</td>
<td>0.862</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>7.90</td>
<td>6</td>
<td>0.873</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>7.97</td>
<td>5.49</td>
<td>0.835</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>5.22</td>
<td>3.47</td>
<td>0.884</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>9.15</td>
<td>9.79</td>
<td>0.957</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>8.64</td>
<td>9.4</td>
<td>0.949</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>6.41</td>
<td>6.26</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>5.09</td>
<td>4.14</td>
<td>0.936</td>
</tr>
</tbody>
</table>

Table 3.8  Broadcast times, waiting times and resulting passenger utilities in BSIS3

From the utility of each passenger, the expected utility of a random passenger for BAIS3 in this example can be calculated by:
Recall that the expected utilities for BAIS1 and BAIS2 are 0.858 and 0.907 respectively. As expected, when more real-time information used, the expected utility increases. This is because, as we discussed in Section 2.2, the broadcaster may predict the passenger waiting time better with more real-time information by conditioning the waiting time distribution on the real-time information. Hence, the expected utilities become progressively larger as the broadcaster moves from BAIS1 to BAIS2 to BAIS3.

### 3.4 Numerical Optimal Broadcast Time Solution

Recall that the expected utility of passenger \( j \), \( EU_j(b) \), is a function of the broadcast time \( b \), and that the broadcast time for passenger \( j \) is determined so as to maximize the expected utility of the passenger, that is, \( b_j^* = \arg \max_{b_j} EU_j(b) \). In the previous section, all values to the 0.01 minutes were enumerated in increments of 0.01 minutes. To be more efficient, we solve numerically for the optimal broadcast time based on the simulation results. A convergence parameter \( c_b \) is used to represent the maximum allowable error of the solution. The optimal broadcast time for a passenger is determined by searching within an interval between lower and upper bounds. We first discuss how to set these lower and upper bounds. Then we discuss the determination of an optimal broadcast time within this interval.

Since the optimal broadcast time can be any value within the waiting time distribution, the lower and upper bounds are determined so that this interval includes
most of the waiting time distribution. Recall that headway mean and standard deviation are denoted by $\bar{h}$ and $\sigma_h$, respectively. Since the headway distribution is assumed to be normal, we assume that the interval between $\bar{h} - 4\sigma_h$ and $\bar{h} + 4\sigma_h$ includes a sufficient portion of headway distribution for our purposes. Note that the waiting time distribution in BAIS1 is given by equation 2.4. Therefore, in BAIS1, the optimal broadcast time is sought out in the interval between 0 (lower bound) and $\bar{h} + 4\sigma_h$ (upper bound). For instance, in $\bar{h} = 10$ minute and $\sigma_h = 2$ minutes, the optimal broadcast time in BAIS1 is sought in the interval between 0 minute and $10 + 4 \times 2 = 18$ minutes.

In BAIS2, the waiting time distribution is determined by conditioning the headway distribution on real-time data $r$ for each passenger. The optimal broadcast time for a particular passenger in this case is assumed to be in the interval between $\bar{h} - 4\sigma_h - r$ (lower bound) and $\bar{h} + 4\sigma_h - r$ (upper bound). For instance, for the passenger having $r = 3$ minutes in a scenario where $\bar{h} = 10$ minutes and $\sigma_h = 2$ minutes, the bounds on the optimal broadcast time for this passenger in BAIS2 are computed as $10 - 4 \times 2 - 3 = -1$ minute and $10 + 4 \times 2 - 3 = 15$ minutes. The lower bound in this case is a negative value. Therefore, the optimal broadcast time is actually sought in the interval between 0 and 15 minutes. Note that the waiting time distribution in BAIS3 generally has smaller standard deviation than that in BAIS2. Hence, when seeking the optimal broadcast time for a passenger in BAIS3, we can conservatively use the same interval determined for the same passenger in BAIS2.
We now discuss how to search for the optimal broadcast time. As has been discussed previously, one approach is to conduct a “discrete enumeration” over the interval using a time step \( c_b \). For example, if we assume \( c_b \) is 1 minute and the broadcast time falls within the interval between 0 and 16 minutes, we would compute the expected utility for all integer values of \( b \) between 0 and 16, (i.e.,0, 1, 2, 3, ..., 16). Then, we would select the value of \( b \) associated with the maximum expected utility as the optimal broadcast time. Although this method is simple, it can be computationally intensive for large intervals or a small \( c_b \). A more computationally efficient method is presented next.

We can find the optimal broadcast time more efficiently using the Golden section method (Sheffi, 1985). The Golden section method is an interval reduction method which is useful in cases where the derivatives of the objective function are not easily evaluated. This method involves an iterative procedure to find the optimal point. This method works only when the objective function is concave or convex. We here numerically demonstrate that the expected utility as a function of broadcast time, \( EU_j(b) \), is concave for BAIS1 and BAIS2. Figure 3.3 shows the function, \( EU_j(b) \), is concave in \( b \) for various headway means and standard deviations and the times since most recent bus departure \( r \) (for BAIS2 only). The function \( EU_j(b) \) for BAIS3 can be assumed to be concave, since expected utility for BAIS3 is determined in a similar way to that of BAIS2 but using the reduced headway standard deviation due to conditioning of headway. The following paragraphs briefly review the Golden section method.
Figure 3.3  Demonstration of the concavity of the expected utility as a function of broadcast time, $EU_j(b)$
In the Golden section method, the original interval is reduced through the comparison of the utility at two points within the interval. This procedure is depicted in Figure 3.4. At the \( n \)th iteration, assume that the optimal point is within the interval defined by \( l^n \) (4 minutes in Figure 3.4) and \( u^n \) (16 minutes). Two interior points are assumed to be \( b^n_L \) (8.6 minute) and \( b^n_R \) (11.4 minutes). Since \( EU(t^n_L = 8.6) < EU(t^n_R = 11.4) \) and the \( EU_j(b) \) is concave, the optimal point must be to the right of \( b^n_L \) (8.6 minutes). Hence we discard the interval between \( l^n \) (4 minutes) and \( b^n_L \) (8.6 minute). The interval of the \((n+1)\)th iteration is now defined by \( l^{n+1} \) and \( u^{n+1} \) where \( l^{n+1} = b^n_L \) (8.6 minutes) and \( u^{n+1} = u^n \) (16 minutes). We continue the interval reduction with two new interior points \( b_L^{n+1} \) (11.4 minutes), and \( b_R^{n+1} \) (13.2 minutes). Such iterations continue until the length of the updated interval is less than or equal to twice that of the convergence parameter \( c \). The optimal solution is the average of the last two points.

The interior points at each iteration are set at a distance of 0.382 times the interval length from either end of the interval. This length is based on "Golden section" and, in general, leads to a quick reduction of the interval. For example, in the example depicted in Figure 3.4, at the \( n \)th iteration, \( l^n = 4 \) , \( u^n = 16 \) and the interval is 12 minutes (i.e. 16-4) long. Hence, the two interior points are set at 8.6 (\( = 4 + 0.382 \times 12 \)) and 11.4 (\( = 16 - 0.382 \times 12 \)). As mentioned above, since \( EU(t^n_L = 8.6) < EU(t^n_R = 11.4) \), the interval between \( l^n \) (4 minutes) and \( b^n_L \) (8.6 minute) is discarded. At the \((n+1)\)th iteration,
\( l^{n+1} = 8.6, u^{n+1} = 16 \) and the interval is 7.4 minutes (i.e. 16-8.6) long. The two interior points are set at 11.4 \((=8.6 + 0.382 \times 7.4)\) and 13.2 \((=16 - 0.382 \times 7.4)\).

![Figure 3.4](image)

<table>
<thead>
<tr>
<th>iteration</th>
<th>the interior points</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( l^n = 4 ) ( b_L^n = 8.6 ) ( b_R^n = 11.4 ) ( u^n = 16 )</td>
</tr>
<tr>
<td>( n+1 )</td>
<td>( l^{n+1} = 8.6 ) ( b_L^{n+1} = 11.4 ) ( b_R^{n+1} = 13.2 ) ( u^{n+1} = 16 )</td>
</tr>
<tr>
<td>( n+2 )</td>
<td>( l^{n+2} = 8.6 ) ( u^{n+2} = 13.2 )</td>
</tr>
</tbody>
</table>

Figure 3.4 Interval reduction by the Golden section method

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CHAPTER 4

RESULTS

We discussed the simulation of the expected utility calculation for the three BAIS alternatives in Chapter 3. In this chapter, we present and examine the simulation results. In section 4.1 we discuss the effect of the bus system parameters on the expected utility of the three BAIS alternatives considered. Then, in section 4.2, we discuss the effect of the passenger utility function on the expected utility.

4.1 Effect of Bus System Parameters

In this section, we investigate the effect of bus system parameters -- specifically the headway mean, headway standard deviation, and the correlation coefficient -- on the expected utility of the three BAIS alternatives. In order to do this, we assume a normal distribution function for the headway. We also assume that the decrease in passenger utility due to an early or a late bus arrival is symmetric and linear, and that passenger utility is independent of the actual passenger waiting time (We change some of these assumption in later sections). The utility function used in this section is as follows:
\[ u(w, b) = \begin{cases} 
0, & w < b - 10 \\
1 - \frac{|b - w|}{10}, & b - 10 \leq w < b + 10 \\
0, & b + 10 \leq w 
\end{cases} \] (4.1)

where, \( w \) is the actual waiting time and \( b \) is the broadcast time.

To simulate the expected utility, we set the number of simulated headways, passenger arrival rate, convergence criterion for solving broadcast time, and bus system parameter (i.e., headway mean, headway standard deviation) to the values found in Table 4.1. We also set the data used in the function of the correlation coefficient between headway and upstream headway (i.e., the values of \( a \) and \( \bar{v} \) of equation 3.15).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of headway samples</td>
<td>200</td>
</tr>
<tr>
<td>Passenger arrival rate (passengers per minute)</td>
<td>10</td>
</tr>
<tr>
<td>Convergence criterion for broadcast time (minutes)</td>
<td>0.1</td>
</tr>
<tr>
<td>Headway mean (minute)</td>
<td>10, 15, 20</td>
</tr>
<tr>
<td>Headway standard deviation (minute)</td>
<td>0, 1, 2, 3, and 4</td>
</tr>
<tr>
<td>Mean of bus velocity ( \bar{v} ) (m per sec)</td>
<td>10</td>
</tr>
<tr>
<td>Coefficient of variation of bus velocity parameter ( a )</td>
<td>0, 0.000025, 0.0001, 0.0004, 0.0016</td>
</tr>
</tbody>
</table>

Table 4.1 Data for simulation of the expected utility calculation.
4.1.1 Effect of Varying One Parameter

Headway mean

We begin our investigation by discussing the expected utility as a function of headway mean with fixed values of headway standard deviation and fixed function of the correlation coefficient. Figure 4.1 shows the expected utility for a 2-minute headway standard deviation and \( a = 0.0001 \) in the function of the correlation coefficient (see equation 3.15). From this figure, we observe that BAIS3 yields a higher expected utility than BAIS2, and BAIS2 yields a higher expected utility than BAIS1. This result is expected, since a broadcaster can estimate bus arrival times better with more real-time data. A better bus arrival time estimate results in a higher expected utility.

![Figure 4.1 Expected utility of a function of headway mean at 2-minutes headway standard deviation and \( a = 0.0001 \)]
We also observe that the expected utility decreases as the headway mean increases. The expected utility for BAIS1 begins to decrease markedly and eventually approaches 0 as the headway mean increases, while the expected utilities for BAIS2 and BAIS3 decrease very little and eventually stay at a certain non-zero constant value. This is mainly because the waiting time distribution of a passenger depends heavily on headway mean in BAIS1, but not as much in BAIS2 and BAIS3. The following paragraphs explain this in more detail. We first explain the pattern of the expected utilities in BAIS1 and BAIS2 by comparing them against each other. We then explain the pattern of the expected utility of BAIS3.

Note that the expected utility of a random passenger is determined by taking the integral of the expected utility of each passenger across the actual waiting time distribution. The expected utility of a particular passenger is determined by:

$$E[u(w,b)] = E \left[ 1 - \frac{|b - w|}{10} \right]$$

$$= 1 - \frac{E[|b - w|]}{10}$$  (4.2)

This means that the expected utility of a passenger is a function of the expected value of $|b - w|$ of the passenger. The expected value of $|b - w|$ of a passenger is obtained by integration of all possible $|b - w|$ over the broadcaster's view of the waiting time distribution under BAIS alternatives. Therefore, the expected value of $|b - w|$ is affected by the waiting time distribution and the optimal broadcast time. We will explain the pattern of the expected utility of a random passenger in BAIS1 and BAIS2 by
investigating the changes in the standard deviation of the broadcaster's view of the
waiting time distribution and the expected value of $|b - w|$ as a function of the mean
headway.

The broadcaster's view of the waiting time distribution is the same for all the
passengers in BAIS1, since no real-time data are available to the broadcaster. In the
broadcaster's view, a passenger arrival at a bus stop can be anywhere within a headway in
BAIS1. Hence, the waiting time distribution is affected largely by the headway mean.
Figure 4.2 illustrates the change in the waiting time distribution and the optimal broadcast
time in BAIS1 as the headway mean changes. Doubling the headway mean disperses the
waiting time distribution markedly. Assuming that optimal broadcast times are set at
roughly the same percentile of waiting time, the distribution of $|b - w|$ also disperse

![Figure 4.2 Waiting time distributions for 10- and 20-minute headway means with 2
minute standard deviation in BAIS1](image)

Figure 4.2 Waiting time distributions for 10- and 20-minute headway means with 2
minute standard deviation in BAIS1
markedly due to the doubling of headway mean. Hence, the expected value of \( lb - wl \) will increase markedly as the headway mean increases. We will illustrate this numerically subsequently.

On the other hand, in BAIS2, the broadcaster determines the waiting time distribution for each passenger by conditioning the headway distribution on the time since the most recent bus departure, \( r \). In this case, each passenger has a different waiting time distribution and broadcast time. To investigate the headway mean effect, we compare the expected value of \( lb - wl \) for passengers having the same percentile \( r \) for two different headway means. Figure 4.3 depicts the waiting time distributions in BAIS2 for a passenger having 50\(^{th} \)-percentile \( r \). We determined the values for 50\(^{th} \)-percentile \( r \) as 4.9 and 10.3 minutes for 10- and 20-minute headway means respectively. We explain this in more detail below in Figure 4.6. Figure 4.3 shows that the standard deviation of the waiting time distribution for this 50\(^{th} \)-percentile passenger does not change much when

![Figure 4.3 Waiting time distributions of the passenger having 50\(^{th} \)-percentile \( r \) for 10- and 20-minute headway means in BAIS2](image-url)
the headway mean is increased, but the mean simply shifts to the right. The optimal broadcast time for this passenger also shifts to the right to compensate for the shift to the right of headway mean. Hence, the \(|b - w|\) distribution for this passenger is changed little by the increase in headway mean, and the expected value of \(lb - wl\) for this passenger changes very little.

To illustrate the changes in standard deviation of the waiting time and expected values of \(|b - w|\) in BAIS1 and BAIS2, we simulate the waiting time distribution and the \(|b - w|\) distribution for 10- and 20-minute headway means with a 2-minute standard deviation. Figure 4.4 displays the simulated waiting time distribution for 10- and 20-minute headway mean in BAIS1. According to this graph, the distribution for a 20-minute headway mean is almost twice as wide as that for a 10-minute headway mean. The simulated standard deviations are 3.07 minutes and 5.87 minutes for 10- and 20-minute headway means respectively (a 2.80 minute increase). Consequently, the \(|b - w|\) distribution for the 20-minute headway mean becomes almost twice as wide as that for the 10-minute headway mean. Figure 4.5 displays the simulated \(|b - w|\) distribution for 10- and 20-minute headway means in BAIS1. The expected value of \(|b - w|\) are 2.57 minutes and 5.08 minutes, respectively, for the 10- and 20-minute headway mean (a 2.51 minutes increase).
Figure 4.4 Simulated waiting time distributions for 10- and 20-minute headway means in BAIS1 (2-minute headway standard deviation)
Figure 4.5  Simulated lb-wl distributions for 10- and 20-minute headway means in BAIS1 (2-minute headway standard deviation)
In BAIS2, we investigate the distributions of the waiting time and $lb - w_l$ for passengers having 50th-, 90th-, 95th-, and 99th-percentile $r$. Figure 4.6 displays the simulated cumulative distributions of $r$ for 10- and 20-minute headway mean. From this figure, we determine the 50th-, 90th-, 95th-, and 99th-percentile $r$ as 4.9, 9.4, 10.4 and 11.9 minutes, respectively, for 10-minute headway mean and 10.3, 18.0, 19.5, and 21.4 minutes, respectively, for a 20-minute headway mean. With these values, we simulate the standard deviation of waiting time distribution and the expected value of $lb - w_l$ for these passengers for 10- and 20-minute headway mean.

![Cumulative distribution of the time since most recent bus departure for the 10- and 20- minute headway mean](image)

Figure 4.6 Cumulative distribution of the time since most recent bus departure for the 10- and 20- minute headway mean
Figure 4.7 displays the waiting time distribution of the passenger having 50\textsuperscript{th}-percentile $r$ (i.e., $r$ = 4.9 and 10.3 minutes for 10- and 20-minute headway means respectively) in BAIS2. According to the graph, the distribution does not change much except for the shifted mean. The simulated standard deviations are 1.97, and 1.98 minutes for 10- and 20-minute headway means (a 0.01 minute increase). Figure 4.8 shows that the $|\bar{b} - w|$ distributions for the same passenger for 20-minute headway mean is almost the same as that for 10-minute headway mean, since the broadcast time for the 20-minute headway mean also shifts with the shifted waiting time distribution. The simulated expected value of $|\bar{b} - w|$ are 1.55 and 1.55 minutes for 10- and 20-minute headway means (no increase).

Figure 4.9 displays the waiting time distribution of the passenger having the 95\textsuperscript{th}-percentile $r$ (i.e., $r = 10.4$ and 19.5 minutes for 10- and 20-minute headway means respectively). According to the graph, the waiting time distribution does not change much either. The simulated standard deviations of waiting time distribution are 1.39 and 1.55 minutes for 10- and 20-minute headway means (a 0.23 minute increase). Figure 4.10 shows the distributions of $|\bar{b} - w|$ for the same passenger do not change much either by the headway mean. The expected value of $|\bar{b} - w|$ are 0.87 and 1.03 minutes for 10- and 20-minute headway means (a 0.16 minute increase).

We also simulated expected value of $|\bar{b} - w|$ for the passenger having 90\textsuperscript{th}- and 99\textsuperscript{th}-percentile $r$. Table 4.2 summarizes the changes in the standard deviations of the waiting time distributions and the expected values of $|\bar{b} - w|$ for the passengers having
50th-, 90th-, 95th- and 99th- percentile $r$ when the headway mean change. From this table, we see that the expected value of $|b - w|$ increases much more in BAIS1 than in BAIS2 as headway increases.

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<td>50 percentile</td>
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</tr>
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<td>90 percentile</td>
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<td></td>
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<td>0.17</td>
</tr>
<tr>
<td></td>
<td>99 percentile</td>
<td>0.94</td>
<td>0.99</td>
<td>0.05</td>
</tr>
<tr>
<td>lb-wl</td>
<td>BAIS1 2.57</td>
<td>5.08</td>
<td>2.51</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
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<td>90 percentile</td>
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</tr>
<tr>
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<td>95 percentile</td>
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<td>1.03</td>
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<td></td>
<td>99 percentile</td>
<td>0.73</td>
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<td>0.02</td>
</tr>
</tbody>
</table>

Table 4.2 Change of standard of deviation of waiting time distribution and mean of $|b-w|$ as the headway mean increases

This empirical illustration confirms the prior explanation of the expected utility patterns in BAIS1 and BAIS2. In summary, as headway mean increases, the waiting time distribution in BAIS1 disperses markedly. On the other hand, the waiting time distribution does not disperse as much in BAIS2. Hence, the expected value of $|b-w|$ in BAIS1 increases considerably as headway mean increases while it increases very little in BAIS2. This explains why the expected utility in BAIS1 decreases rapidly as headway mean increases while the expected utilities in BAIS2 decrease very little.
Figure 4.7 Simulated waiting time distributions for 10- and 20-minute headway means in BAIS2 for the passenger of 50th-percentile $r$ (2-minute headway standard deviation)
Figure 4.8 Simulated l-b-wl distributions for 10- and 20-minute headway means in BAIS2 for the passenger of 50th-percentile r (2-minute headway standard deviation)
Figure 4.9  Simulated waiting time distributions for 10- and 20-minute headway means in BAIS2 for the passenger of 95\textsuperscript{th}-percentile $r$ (2-minute headway standard deviation)
Figure 4.10 Simulated l_b-w_1 distributions for 10- and 20-minute headway means in BAIS2 for the passenger of 95th-percentile r (2-minute headway standard deviation)
We now discuss why the expected utility in BAIS1 converges to 0 as the headway mean increases while expected utility in BAIS2 converges to a certain non-zero value. In BAIS1, the waiting time distribution becomes flatter as headway mean increases. This means that the probability of a particular waiting time becomes smaller and gets close to 0 as the headway mean increases. Note that a passenger has positive utility only when she arrives within the time interval between \(b-10\) and \(b+10\) minutes, and she has 0 utility otherwise (see equation 4.1). As headway means increases, the proportion of passengers arriving between \(b-10\) and \(b+10\) minutes becomes smaller. As the headway mean becomes very large, this proportion approaches 0. This explains why the expected utility decreases and eventually gets close to 0 as headway mean increases. On the other hand, in BAIS2, the waiting time distribution of a particular passenger does not change much except for the shifted mean as the headway mean increases. Note that right tail of the waiting time distribution for a particular passenger in BAIS2 becomes less truncated due to the shifted mean as the headway mean increases. This means that the truncation of the waiting time distribution becomes negligible for most passengers when headway mean becomes very large. This explains why expected utility in BAIS2 stays at a constant value as headway mean increases.

Now, we discuss the pattern of expected utility in BAIS3. In BAIS3, a conditional headway distribution is used instead of an unconditional headway distribution for forecasting bus arrival time. Hence, the same explanation as BAIS2 can be given to the expected utility in BAIS3 decreasing very little as headway mean increases. However, from Figure 4.1, we observe that the expected utility in BAIS3 decreases faster.
than that in BAIS2. This is because the expected utility in BAIS3 is affected by the
correlation coefficient in addition to reason previously explained for BAIS2. The
following paragraphs explain this in more detail.

Recall that the variance of the conditional headway distribution in BAIS3,
denoted by $\sigma_c^2$ is given by (see equation 3.10):

$$\sigma_c^2 = (1 - \rho^2)\sigma^2_h$$

(4.3)

This means that the variance of the conditional headway distribution depends on the
correlation coefficient $\rho$ and the unconditional headway variance. Note that the
correlation coefficient $\rho$ is a function of the location of the following bus at the time of
passenger arrival $s$, i.e., $\rho = e^{-st}$ (see equation 3.11). As the headway mean increases, the
location of the following bus, $s$, for the passenger having same percentile waiting time
will increase, and consequently the correlation coefficient, $\rho$, observed for the passenger
will decrease. Therefore, the standard deviation of the conditional headway distribution
for the passenger will increase as headway mean increases. This explains why expected
utility in BAIS3 decreases faster than that in BAIS2.

The correlation coefficient for a certain passenger will eventually get close to 0 as
headway mean becomes very large, since the bus location, $s$, for the passenger becomes
very large as well. This indicates that standard deviation of the conditional headway
distribution for a certain passenger becomes close to the standard deviation of the
unconditional headway distribution as headway mean becomes very large. This explains
why the expected utility in BAIS3 will eventually approach the expected utility in BAIS2 as headway mean increases.

We now look at the values of BAIS2 and BAIS3. The values of BAIS2 and BAIS3 in this study, denoted by $V_{BAIS2}$ and $V_{BAIS3}$ respectively, are defined as follows:

$$V_{BAIS2} = \frac{EU_{BAIS2} - EU_{BAIS1}}{EU(\text{perfect}) - EU(\text{worst})}$$  (4.4a)

$$V_{BAIS3} = \frac{EU_{BAIS3} - EU_{BAIS1}}{EU(\text{perfect}) - EU(\text{worst})}$$  (4.4b)

where:

- $EU_{BAIS1}$ = the expected utility for BAIS1
- $EU_{BAIS2}$ = the expected utility for BAIS2
- $EU_{BAIS3}$ = the expected utility for BAIS3
- $EU(\text{perfect})$ = the expected utility with perfect broadcast time
- $EU(\text{worst})$ = the expected utility with worst broadcast time

$V_{BAIS2}$ and $V_{BAIS3}$ indicate the worth of BAIS2 and BAIS3 over BAIS1 in forecasting the bus arrival time respectively. Since we assume that maximum and minimum values of passenger utilities is 1 and 0, $EU(\text{perfect})$ and $EU(\text{worst})$ are 1 and 0 respectively. Hence, $V_{BAIS2}$ and $V_{BAIS3}$ can be given by as follow:

$$V_{BAIS2} = EU_{BAIS2} - EU_{BAIS1}$$  (4.5a)

$$V_{BAIS2} = EU_{BAIS3} - EU_{BAIS1}$$  (4.5b)
Figure 4.11 shows $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ as calculated by equations 4.5a and 4.5b. From Figure 4.11, we observe that $V[\text{BAIS3}]$ is greater than $V[\text{BAIS2}]$. We also observe that both $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ increase markedly at lower headway mean while their increases are less rapid for higher headway means. Moreover, the relative difference between $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ becomes smaller as headway mean increases.

These results imply that having real-time data $p$ in addition to the real-time data $r$ is valuable when $\alpha=0.0001$ in the function of the correlation coefficient (see equation 3.15). However, the value added by $p$ relative to the value added by $r$ decreases when the headway mean increases. They also imply that the real-time data $p$ and $r$ add values increase as the headway mean increases but do so at a decreasing rate.

![Figure 4.11 Value of BAIS as a function of headway mean at 2-minute headway standard deviation and $\alpha = 0.0001$](image)
Headway standard deviation:

We now investigate the expected utility as a function of headway standard deviation at fixed value of headway mean and fixed function of correlation coefficient function. Figure 4.12 displays the expected utilities at a 10-minute headway mean and $a=0.0001$ in the function of the correlation coefficient (see equation 3.15). From this graph, we observe that BAIS3 yields a higher expected utility than BAIS2 except at zero headway standard deviation, where they have equal expected utilities. We also observe that BAIS2 always yields a higher expected utility than BAIS1. Again, this is because the broadcaster can estimate the bus arrival time better when more real-time data is available.

![Figure 4.12](image-url)

Figure 4.12 Expected utility of a function of headway mean at 10-minutes headway mean and $a=0.0001$
We observe that the expected utilities are 1 at 0 headway standard deviation in BAIS2 and BAIS3. This is because, when the headway standard deviation is 0 (i.e., when the headway is deterministic), the broadcaster can forecast the exact bus arrival time based solely on the last bus departure time. That is, the broadcaster can forecast arrival time perfectly in both BAIS2 and BAIS3.

We observe that the expected utility generally decreases as the headway standard deviation increases. This is because an increase of headway standard deviation will increase the standard deviation of the waiting time distribution of a passenger in any BAIS alternative. The decrease in the expected utility is less in BAIS1 than in BAIS2. This is because the change in standard deviation of headway distribution does not heavily affect the waiting time distribution of a passenger in BAIS1, but it does affect the distribution heavily in BAIS2. The following paragraphs explain this in more detail. We first explain the pattern of the expected utilities in BAIS1 and BAIS2 by comparing them against each others. We then explain the pattern of the expected utility in BAIS3. As before, we will explain the pattern of the expected utility in BAIS1 and BAIS2 by investigating the changes in the waiting time distribution and the \( l_b-w_l \) distribution as the headway standard deviation changes and illustrating these changes with numerical examples.

We mentioned that the waiting time distribution is affected heavily by the headway mean in BAIS1. However, waiting time distribution in BAIS1 is not changed much by the change in headway standard deviation. Figure 4.13 shows the change of
Figure 4.13 Waiting time distributions for 2- and 4- minute headway standard deviations for 10 minute mean in BAIS1

waiting time distribution and the optimal broadcast time with the change in headway standard deviation from 2 minutes to 4 minutes for a 10 minute headway mean in BAIS1. As seen in the graph, the waiting time distribution does not disperse much with the increase in headway standard deviation. Since the waiting time distribution gets dispersed a little by the increase in headway standard deviation, the optimal broadcast time shifts to the right a little. This shift can compensate a little for the effect of the headway standard deviation in the $|b - w|$ distribution. Therefore, the $|b - w|$ distribution does not disperse much due to the increase in headway standard deviation.

On the other hand, in BAIS2, since waiting time distribution is determined for each passenger by conditioning the headway distribution on the time since the most recent bus departure, $r$, the waiting time distribution of a passenger gets dispersed as the
For the passenger having small $r$, we determine below (see Figure 4.18) the $10^{th}$-percentile $r$ as 1.1 and 1.3 minutes for 2- and 4-minutes headway standard deviation, respectively. Figure 4.14 depicts that the waiting time distribution of the $10^{th}$-percentile passenger $r$ is affected heavily by headway standard deviation in BAIS2. Doubling the headway standard deviation disperses the waiting time distribution markedly. The optimal broadcast time in this case shifts to the right little since the waiting time distribution for 4-minute headway standard deviation is truncated a little by the 0 value. Hence, the distribution of $|b - w|$ get dispersed much by an increase in headway standard deviation.

![Figure 4.14 Waiting time distributions of the passenger having 10th-percentile $r$ for 2- and 4-minute headway standard deviations with 2-minute mean in BAIS2.](image)
For the passenger having large \( r \), we determine the 95\(^{th}\)-percentile \( r \) below as 10.4 and 12.6 minutes for 2- and 4-minute headway standard deviations respectively. Figure 4.15 shows the waiting time distributions of the passenger having \( r=10.4 \) and 12.6 minutes in BAIS2 for 2- and 4-minute headway standard deviations respectively. According to this graph, the waiting time distribution of the passenger having 95\(^{th}\)-percentile \( r \) is affected heavily by headway standard deviation in BAIS2. The increase in headway standard deviation disperses the waiting time distribution much. The optimal broadcast time shifts to the right since the distribution get dispersed much to the right. Hence, the distribution of \( |b - w| \) gets dispersed much by the increase in headway standard deviation.

Figure 4.15 Waiting time distributions of the passenger having 95\(^{th}\)-percentile \( r \) for 2- and 4-minute headway standard deviations with 2-minute mean in BAIS2
To illustrate the changes in standard deviation of the waiting time distribution and expected value of $|b - w|$ in BAIS2 and BAIS3, we simulate the standard deviation of waiting time distribution and the expected value of $|b - w|$ for 2- and 4-minute headway standard deviation. Figure 4.16 displays the distributions of the waiting time for 2- and 4-minute headway standard deviations in BAIS1. The distribution does not get dispersed much by the increase in headway standard deviation. The standard deviations of the waiting time distribution are 3.07 and 3.70 minutes for 2- and 4-minute headway mean (a 0.63 minute increase). Accordingly, the $|b - w|$ distribution does not get dispersed much either. Figure 4.17 displays the $|b - w|$ distributions for 2- and 4-minute headway standard deviations in BAIS1. The expected value of $|b - w|$ are 2.57 and 3.04 minutes for 2- and 4-minute headway mean (a 0.47 minute increase).
Figure 4.16  Simulated waiting time distributions for 2- and 4-minute headway standard deviations in BAIS1 (10-minute headway mean)
Figure 4.17 Simulated lb-wl distributions for 2- and 4-minute headway standard deviations in BAIS1 (10-minute headway mean)
As before, in BAIS2, we simulate and analyze the waiting time and $|b - w|$ distributions for the passengers having 50th-, 90th-, 95th- and 99th-percentile $r$ for 2- and 4-minute headway standard deviations. Figure 4.18 displays the cumulative distributions of $r$ for 2- and 4-minute headway standard deviations. From this graph, we determine the 50th-, 90th-, 95th-, and 99th-percentile $r$ as 4.9, 9.4 and 10.4 and 11.9 minutes for the 2-minute headway standard deviation and 5.3, 10.7, 12.4 and 15.4 minutes for the 4-minute headway standard deviation. We use these values to simulate the standard deviation of waiting time and the expected value of $|b - w|$ for 2- and 4-minute headway standard deviations.

Figure 4.18 Cumulative distribution of the time since most recent bus departure $r$ for the 2- and 4- minute headway standard deviation
Figure 4.19 displays the waiting time distribution of the passenger having 50\textsuperscript{th}-percentile \( r \) for 2- and 4-minute headway standard deviation in BAIS2. From this graph, we see the waiting time distribution gets dispersed much by the increase in headway standard deviation. The standard deviations of waiting time distribution are 1.97, 3.20 minutes for the 2- and 4-minute headway standard deviations (a 1.23 minute increase). Accordingly, the \(|b-w|\) distribution gets dispersed much. Figure 4.20 displays the distributions of \(|b-w|\) of the passenger having 50\textsuperscript{th}-percentile \( r \) for 2- and 4-minute headway mean in BAIS2. The expected values of \(|b-w|\) are 1.55 and 2.56 minutes for the 2- and 4-minute headway standard deviations (a 1.01 minute increase).

Figure 4.21 displays the waiting time distribution for the passenger having 95\textsuperscript{th}-percentile \( r \) for the 2- and 4-minute headway means. The waiting time distribution for this passenger gets dispersed much too by the increase in headway standard deviation. The standard deviations of waiting time distributions are 1.13 and 2.00 minutes for the 2- and 4-minute headway standard deviation (a 0.87 minute increase). Figure 4.22 shows that the \(|b-w|\) distributions for the same passenger gets dispersed much as well. The expected value of \(|b-w|\) are 0.87 and 1.51 minutes for the 2- and 4-minute headway standard deviations (a 0.64 minute increase).

Similarly, we also investigate the waiting time distribution and \(|b-w|\) distribution of the passenger having 90\textsuperscript{th}- and 99\textsuperscript{th}-percentile \( r \). Table 4.3 summarizes the changes in standard deviation of the waiting time distribution and the expected value of \(|b-w|\) for the passengers having 50\textsuperscript{th}-, 90\textsuperscript{th}-, 95\textsuperscript{th}- and 99\textsuperscript{th}- percentile \( r \). This table shows that the
expected value of $|b-w|$ increases less in BAIS1 than in BAIS2 as headway standard deviation increases.

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Table 4.3 Change of standard of deviation of waiting time distribution and mean of lb-wl as the headway standard deviation increases

This empirical illustration confirms the prior explanation for the expected utility pattern in BAIS1 and BAIS2. In summary, as headway standard deviation increases, the waiting time distribution in BAIS1 does not get dispersed much. On the other hand, the waiting time distribution gets dispersed much in BAIS2. Hence, the expected value of $lb-wl$ in BAIS1 does not increase much as headway mean increases, while it increases markedly in BAIS2. This explains why the expected utility in BAIS2 decreases rapidly as headway standard deviation increases, while the expected utility in BAIS1 decreases much less.
Figure 4.19 Simulated waiting time distributions for 2- and 4-minute headway standard deviations in BAIS2 for the passenger of 50\textsuperscript{th}-percentile $r$ (10-minute headway mean)
Figure 4.20  Simulated |b-w| distributions for 2- and 4-minute headway standard deviations in BAIS2 for the passenger of 50th-percentile r (10-minute headway mean)
(a) for 2-minute headway standard deviation

(b) for 4-minute headway standard deviation

Figure 4.21 Simulated waiting time distributions for 2- and 4-minute headway standard deviations in BAIS2 for the passenger of 95th-percentile r (10-minute headway mean)
Figure 4.22 Simulated lb-wl distributions for 2- and 4-minute headway standard deviations in BAIS2 for the passenger of 95\textsuperscript{th} -percentile r (10-minute headway mean)
From Figure 4.12, we also observe that the decrease in the expected utility in BAIS3 is lower than in BAIS2. This is because the conditional headway distribution has a smaller variance than the unconditional headway distribution (see equation 4.3). We also observe that the expected utility line for BAIS3 starts approaching the expected utility line for BAIS2 as headway mean increases. This is because the conditional headway distribution is affected by not only unconditional headway distribution but also the correlation coefficient for a particular passenger which is affected by unconditional headway distribution as well. The next paragraph explains this in more detail.

As has been previously discussed in Chapter 3, the conditional headway standard deviation of a passenger, \( \sqrt{1 - \rho^2} \sigma_h \), is determined by the correlation coefficient and the unconditional headway standard deviation. An increase in the unconditional headway standard deviation increases the conditional headway standard deviation. At the same time, the correlation coefficient for the passenger, \( \rho \), also depends on the headway standard deviation, \( \sigma_h \). Recall that correlation coefficient, \( \rho \), is a function of the distance of the location of the following bus at the time of a passenger arrival, i.e., \( \rho = e^{-as} \) (see equation 3.11). Note that as the headway standard deviation increases, the distance, \( s \), for the passenger having a particular percentile waiting time increases, and consequently, correlation coefficient for the passenger, \( \rho \), decreases. Therefore, \( (1 - \rho^2) \) increases as headway standard deviation increases. As a result, the conditional headway standard deviation increases and approaches the unconditional headway standard deviation as
headway standard deviation increases. This is why the expected utility line for BAIS3 starts approaching the line for BAIS2 as headway standard deviation increases.

Figure 4.23 shows $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ as calculated by equations 4.5a and 4.5b. We observe that $V[\text{BAIS2}]$ is the same as $V[\text{BAIS3}]$ at zero headway standard deviation since the expected utilities for BAIS3 and BAIS2 are the same in the case of zero headway standard deviation. We also observe that the $V[\text{BAIS3}]$ is greater than $V[\text{BAIS2}]$ except at zero standard deviation, and that both $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ generally decrease as the headway standard deviation increases. $V[\text{BAIS2}]$ decreases more rapidly than $V[\text{BAIS3}]$.

These results imply that having additional real-time data of $p$ over the real-time data of $r$ is not worth when a bus system has 0 headway standard deviation. These results also imply that the value of the real-time data $r$ and $p$, decreases as the headway standard deviation of a bus system increases. However, the real-time data of $p$ over the real-time data of $r$ is worth when the headway standard deviation of a bus system is greater than 0, and becomes more worthy as the headway standard deviation of a bus system increases.
Figure 4.23 Value of BAIS as a function of headway standard deviation at 10-minute headway mean and 0.88 correlation coefficient
Correlation Coefficient

We now observe the effects of the correlation coefficient at fixed values of headway mean and headway standard deviation. Recall that the correlation coefficient, $\rho$, is modeled as being a function of the location of the following bus, $s$, at the time of passenger arrival i.e., $\rho = e^{-as}$ (see equation 3.11). To investigate the effect of correlation coefficient, we vary the parameter $a$ in the function for the correlation coefficient. To compare parameter $a$ to the corresponding correlation coefficient $\rho$, and the ratio of the conditional headway standard deviation and unconditional headway standard deviation, $\sigma_c/\sigma_h$, at a particular upstream point, we compute their values at a distance $s = 1000$ m upstream of the bus stop of interest. Note that $\sigma_c/\sigma_h$ is given by $\sqrt{1-\rho^2}$ (see equation 4.3). When $a=0$, the correlation coefficient is $e^{-0 \times 1000} = 1$, and $\sigma_c/\sigma_h$ is $\sqrt{1-1^2} = 0$. When $a=0.000025$, the correlation coefficient is $e^{-0.000025 \times 1000} = 0.975$, and $\sigma_c/\sigma_h$ is $\sqrt{1-0.975^2} = 0.221$. Table 4.4 shows various $a$ values and corresponding $\rho$ and $\sigma_c/\sigma_h$ at $s = 1000$ m.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\rho$</th>
<th>$\sigma_c/\sigma_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.000025</td>
<td>0.975</td>
<td>0.221</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.904</td>
<td>0.426</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.670</td>
<td>0.742</td>
</tr>
<tr>
<td>0.0016</td>
<td>0.202</td>
<td>0.979</td>
</tr>
<tr>
<td>0.0032</td>
<td>0.041</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Table 4.4 Parameter $a$ and corresponding $\rho$ and $\sigma_c/\sigma_h$ at $s = 1000$ m
As expected, as $a$ increases, the correlation coefficient (at $s=1000$ m) decreases and $\sigma_c / \sigma_H$ (at $s=1000$ m) increases. Next, we present the expected utility of BAIS3 as a function of parameter $a$. We also present the expected utility of BAIS3 as a function of $\rho$ and $\sigma_c / \sigma_H$ at $s=1000$m..

Figure 4.24 shows the expected utility at the 10-minute headway mean and 2-minute headway standard deviation. From this figure, we again observe that BAIS3 yields a higher expected utility than BAIS2, and that BAIS2 always yields a higher expected utility than BAIS1. Again, this is because the broadcaster can estimate the bus arrival time better with more real-time data.

![Figure 4.24](image.png)

Figure 4.24 Expected utility as a function of correlation function parameter $a$ at 10-minute headway mean and 2-minute headway standard deviation
As expected, the expected utilities for BAIS1 and BAIS2 remains constant, since the expected utilities for BAIS1 and BAIS2 are independent of the correlation coefficient. The real-time data on upstream headways are not used in BAIS1 and BAIS2.

We observe that expected utility in BAIS3 is 1 when $a=0$. This is because when $a=0$ the correlation coefficient between upstream and bus stop headways is always 1, and consequently the standard deviation of conditional headway distribution, 

$$\sigma_c = \sqrt{1 - \rho^2} \sigma_h$$ (see equation 4.3), is always 0. We also observe that the expected utility decreases as parameter $a$ increases and eventually approaches the expected utility of BIAS2. This is because as parameter $a$ gets sufficiently large, $\rho$ approaches zero and the standard deviation of conditional headway distribution $\sigma_c = \sqrt{1 - \rho^2} \sigma_h$ increases and approaches the standard deviation of the unconditioned headway $\sigma_h$.

We also observe that the decrease in the expected utility of BAIS3 becomes slower as $a$ value increases. This can be explained by the first and second derivatives of the conditional headway variance (see equation 4.3). The derivatives of the conditional headway variance with respect to parameter $a$ are given as follows:

$$\frac{d\sigma_c^2}{da} = \frac{d}{da}[(1 - \rho^2)\sigma_h^2] = \frac{d}{da}[(1 - e^{-2ar})\sigma_h^2] = 2se^{-2ar}\sigma_h^2 > 0$$ \hspace{1cm} (4.6a)

$$\frac{d^2\sigma_c^2}{da^2} = -4s^2e^{-2ar}\sigma_h^2 < 0$$ \hspace{1cm} (4.6b)

The positive sign of the first derivative indicates that as parameter $a$ increases the conditional variance $\sigma_c^2$ increases. Increases in $\sigma_c^2$ results in decreases in the expected
utility of BAIS3. Therefore, an increase in parameter $a$ decreases the expected utility. The negative sign of the second derivative indicates that the increase in the conditional variance $\sigma_c^2$ becomes less as $a$ increases. In other words, the effect of parameter $a$ on the conditional variance $\sigma_c^2$ decreases as $a$ gets larger. Hence, the size of the decrease in expected utility of BAIS3 brought about by an increase in parameter $a$ decreases as $a$ gets larger.

Figures 4.25 and 4.26 display the expected utility in BAIS3 as a function of correlation coefficient $\rho$ (at $s=1000$ m) and $\sigma_c/\sigma_{\mu}$ (at $s=1000$ m), respectively. From these figures, as previously discussed, the expected utilities for BAIS1 and BAIS2 remain constant, since the expected utilities for BAIS1 and BAIS2 are independent of the correlation coefficient.

From Figure 4.25, as expected, we observe that expected utility in BAIS3 gets closer to the expected utility of BIAS2 as $\rho$ get closer to zero, and it is 1 when $\rho=1$. We also observe that the expected utility increases as the correlation coefficient $\rho$ increases, and that the increase in expected utility becomes greater as the correlation coefficient increases. This is expected, since as the correlation coefficient $\rho$ increases the broadcaster can more accurately predict the arrival time when using real-time upstream headway data. This can also be explained by the first and second derivatives of the conditional headway variance with respect to parameter $\rho$. These derivatives are given as follows:
\[
\frac{d\sigma_C^2}{d\rho} = \frac{d}{d\rho} \left[ (1 - \rho^2)\sigma_H^2 \right] = -2\rho\sigma_H^2 < 0 \quad (4.7a)
\]

\[
\frac{d^2\sigma_C^2}{d\rho^2} = -2\sigma_H^2 > 0 \quad (4.7b)
\]

The negative sign of the first derivative indicates that as \( \rho \) increases the conditional headway variance \( \sigma_C^2 \) decreases. A decrease in \( \sigma_C^2 \) increases the expected utility of BAIS3. Therefore, an increase in \( \rho \) increases the expected utility. The negative sign of the second derivative indicates that the decrease in the conditional headway variance \( \sigma_C^2 \) becomes greater as \( \alpha \) increases. In other words, the effect of \( \rho \) on the conditional headway variance \( \sigma_C^2 \) increases as \( \rho \) gets larger. Hence, the size of the decrease in expected utility of BAIS3 brought about by an increase in \( \rho \) increases as \( \rho \) gets larger.

Figure 4.25  Expected utility as a function of correlation coefficient \( \rho \) (at \( s=1000 \) m) a 10-minute headway mean and 2-minutes headway standard deviation
From Figure 4.26, as expected, we observe that expected utility in BAIS3 gets closer to the expected utility of BIAS2 as $\sigma_c/\sigma_H$ get closer to 1, and it is 1 when $\sigma_c/\sigma_H = 0$. We also observe that the expected utility decreases as $\sigma_c/\sigma_H$ increases. This is expected since as $\sigma_c/\sigma_H$ increases the broadcaster can more accurately predict the arrival time when using the real-time upstream headway data.

Figure 4.26 Expected utility as a function of $\sigma_c/\sigma_H$ (at $s=1000$ m) at 10-minute headway mean and 2-minutes headway standard deviation
The additional values of BAIS2 and BAIS3 compared to BAIS1, V[BAIS2] and V[BAIS3] (see Equation 4.5a and 4.5b), as a function of parameter $a$ are displayed in Figure 4.27. From this figure, we observe that V[BAIS3] decreases and approaches V[BAIS2] as $a$ increases, while B[BAIS2] remains constant. These trends are easily understood by comparing the differences of the expected utilities in Figure 4.24.

Figure 4.27 Value of BAIS as a function of parameter $a$ at 10-minute headway mean and 2-minutes headway standard deviation
4.1.2 Effect of Two Parameters Varying Simultaneously

Headway mean and headway standard deviation

We now look at the expected utility as a function of headway standard deviation for various headway means. Figure 4.28 displays the expected utility for 3 cases of headway mean (i.e. 10, 15 and 20 minutes) at parameter $a=0.0001$ in the function of correlation coefficient. The bottom three curves represent the expected utilities of BAIS1, the middle three curves represent the expected utilities of BAIS2, and the top three (dotted) curves represent the expected utilities of BAIS3. This graph shows that the pattern of the expected utility seen in Figure 4.12 is repeated for each individual headway mean. The expected utilities decrease as the headway standard deviation increases, and the decrease in the expected utility is less in BAIS1 and BAIS3 than in BAIS2. The explanation for the expected utility patterns for 15- and 20-minute headway mean is the same as that explanation used for the 10-minute headway mean seen in Figure 4.12.

We also observe that the expected utility curves for a smaller headway mean lies far above the expected utility curve for a larger headway mean in BAIS1 while it lies only a little above in BAIS2 and BAIS3. Thus, at a given headway standard deviation, the effect of the headway mean on the expected utilities is much greater in BAIS1 than in BAIS2 and BAIS3. This effect was previously observed and explained in Figure 4.1 for a 2-minute headway standard deviation. Figure 4.28 confirms that the same pattern holds other standard deviations.
Figure 4.28 Expected utility as a function of headway standard deviation for various headway means at $a=0.0001$

Figure 4.29 displays the $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ calculated from equation 4.5a and 4.5b. The dotted lines are for BAIS3 and the solid lines are for BAIS2. Since the expected utility patterns are similar for each headway mean, the patterns of $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ observed in Figure 4.23 can again be observed for each headway mean. We also observe that $V[\text{BAIS2}]$ and $V[\text{BAIS3}]$ show higher values for the larger headway mean than for smaller headway means, a phenomenon that was also observed in Figure 4.11 and explained in the accompanying discussion.
Figure 4.29 Value of BAIS as a function of headway standard deviation for various headway mean at $a=0.0001$

Headway standard deviation and correlation coefficient

Figure 4.30 displays the expected utility as a function of headway standard deviation for the various values of the parameter $a$ in the function of the correlation coefficient at 10-minute headway mean. The dotted curves at the top are expected utilities for BAIS3 for the various $a$ values in the function of the correlation coefficient. The solid curves represent the expected utilities for BAIS2 and BAIS1. The expected utilities for BAIS1 and BAIS2 are not changed by the change of parameter $a$, since real-time data on upstream headways are not used in BAIS1 and BAIS2.
We observe again that the expected utility of BAIS3 remains at 1 when $\alpha = 0$ (i.e., $\rho = 1$). We also see that the expected utility of BAIS3 approaches the expected utility of BAIS2 when $\alpha$ is large (i.e. $\rho$ is close to 0), regardless of the value of $\sigma_H^2$. Again, this result is expected, since the variance of conditional headway, $(1 - \rho^3)\sigma_H^2$, is 0 when parameter $\alpha = 0$ and it approaches the unconditional headway variance $\sigma_H^2$ when $\alpha$ becomes very large. We also observe that the decrease in the expected utility in BAIS3 becomes faster as $\alpha$ increases. This is because the change in variance of the conditional headway distribution increases as $\alpha$ increases (i.e. $\rho$ decreases).
We now look at the expected utility as a function of parameter $a$. Figure 4.31 displays the expected utility as a function of parameter $a$ for various cases of headway standard deviation at the 10-minute headway mean. We again observe that, as parameter $a$ increases, the expected utility decreases at any headway standard deviation except 0. When $a=0$, the expected utility stays at 1 for any $\sigma_H$. We notice that each curve is approaching a constant utility. This constant utility is a function of $\sigma_H$ in BAIS2. Note that the additional information in BAIS3 simply reduces the conditional headway from $\sigma_H$ to $\sigma_C$ (i.e., $\sigma_C = \sqrt{1 - \rho^2 \sigma_H}$). Therefore, these curves approach larger expected utility for smaller $\sigma_H$.

![Graph showing expected utility as a function of parameter $a$ for various headway standard deviation at 10-minute headway mean.](image)

Figure 4.31 Expected utility of BAIS3 as a function of parameter $a$ for various headway standard deviation at 10-minute headway mean.
Figure 4.32 displays $V[\text{BAIS}_2]$ and $V[\text{BAIS}_3]$ as a function of the headway standard deviation for various values of parameter $a$ at 10-minute headway mean. We observe again that $V[\text{BAIS}_2]$ is not changed as parameter $a$ changes and always decreases as $\sigma_H$ increases. However, $V[\text{BAIS}_3]$ shows different patterns according to the parameter $a$. In the case of $a=0$ (i.e., $\rho=1$), $V[\text{BAIS}_3]$ increases as the headway standard deviation increases. This is because the expected utility for BAIS3 stays constant at 1 for $a=0$, and the expected utility for BAIS1 decreases with the headway standard deviation. For a large $a$ (i.e., $a=0.0004$) $V[\text{BAIS}_3]$ almost follows the pattern of $V[\text{BAIS}_2]$. At some values of $a$, it appears that there is a minimum to the $V[\text{BAIS}_3]$ curve (e.g., at $\sigma_H=1$ when $a=0.000025$).

![Figure 4.32 Value of BAIS as a function of headway standard deviation for various $a$ values at 10-minute headway mean](image-url)
4.2 Effect of Utility Function

In Section 4.1, we investigated the effect of the bus system parameters on the expected utility of the three BAIS alternatives. In this section, we will investigate the effect of the passenger utility function on the expected utility of the three BAIS alternatives. To do this, we parameterize $C$ in the utility function as $c_1 + \beta_1 w$ for $w < b$ and $c_2 + \beta_2 w$ for $w > b$ to capture various types of passenger preferences. The function used in this section, then, is as follows:

$$u(w,b) = \begin{cases} 
0, & w < \frac{b - c_1}{\beta_1} + 1 \\
1 - \frac{b - w}{c_1 + \beta_1 w}, & \frac{b - c_1}{\beta_1} + 1 \leq w < b \\
1 - \frac{w - b}{c_2 + \beta_2 w}, & b \leq w < \frac{b + c_2}{1 - \beta_2} \\
0, & \frac{b + c_2}{1 - \beta_2} \leq w
\end{cases}$$

(4.8)

We had set $c_1 = c_2 = 10$, and $\beta_1 = \beta_2 = 0$ to investigate the bus system parameters in Section 4.1. To investigate the effect of different utility functions, we now vary $c_1$, $c_2$, $\beta_1$, and $\beta_2$.

4.2.1 Symmetric Utility Function

In this section, we specify the utility function to allow for the changing sensitivity of passenger utility to the accuracy of broadcast time, i.e., $|b - w|$. In order to do this, we
will use a symmetric utility function. We set $c_1 = c_2 = c$ and $\beta_1 = \beta_2 = 0$. Thus, we specify the utility function as follows:

$$u(w, b) = \begin{cases} 
0, & w < b - c \\
1 - \frac{|b - w|}{c}, & b - c \leq w < b + c \\
0, & b + c \leq w
\end{cases}$$

(4.9)

We start by investigating the expected utility of BAIS1 as a function of headway mean for various $c$ values. Figure 4.43 displays the expected utilities in BAIS1 as a function of headway mean with 2-minute headway standard deviation for the various $c$ values. From this graph, we observe that the expected utility line for larger $c$ is located above that for smaller $c$. This means that, at a given headway mean, the expected utility becomes greater as $c$ increases. This can be explained by investigating the change in passenger utility due to the change in $c$ value, since expected utility is obtained by taking the integral of all possible passenger utilities over waiting time distribution. Note that the waiting time distribution is independent of the utility function (i.e. independent of $c$ value). This means that, at a given headway distribution, the waiting time distribution is not changed by the change in $c$ value. Hence, if the optimal broadcast time of the passenger is not changed by the increase in $c$ value, the expected utility of the passenger increases, since the term $\frac{|b - w|}{c}$ in the utility function decreases. However, the optimal broadcast time may change as $c$ value increases. This ability to change the optimal broadcast time will only increase the expected utility, since the broadcast time is
determined to maximize the expected utility of the passenger. Therefore, the expected utility with larger $c$ lies above that with smaller $c$.

![Figure 4.33](image)

Figure 4.33 Effect of symmetric utility function on the expected utility as a function of headway mean in BAIS1 (for 2-minute headway standard deviation)

We also observe that the patterns of decrease in the expected utility do not change much as $c$ increases. Thus, at any $c$ value, the expected utility decreases and eventually gets closer to 0 as headway mean increases. In Section 4.1.1, we mentioned the reason why the expected utility in BAIS1 decreases and gets closer to 0 at $c=10$ as headway mean increases. The same explanation can be given to the patterns of the expected utility for other $c$ values, since the waiting time distribution is independent of $c$ value.
We now look at the expected utility in BAIS2 and BAIS3 as a function of headway mean. Figures 4.34 and 4.35 display the expected utilities in BAIS2 and BAIS3 respectively for the various $c$ value with 2-minute headway standard deviation. From these graphs, we observe again that the expected utility line for the larger $c$ is located higher than that for the smaller $c$. As in BAIS1, the conditional waiting time distribution is independent of the utility function (i.e., $c$ value). Therefore, the same explanation as that given for BAIS1 can be given to these cases.

We also observe that the patterns of decrease in the expected utility do not change much as $c$ increases. Thus, at any value of $c$, the expected utilities in BAIS2 and BAIS3 decrease and eventually stay at a certain constant value as headway mean increases. In Section 4.1.1, we explained why the expected utilities in BAIS2 and BAIS3 decrease and stay at a certain value at $c=10$ as headway mean increases. Again, the same explanation can be given to the expected utility for other $c$ values, since the conditional waiting time distribution is independent of the utility function (i.e., $c$ value).
Figure 4.34 Effect of symmetric utility function on the expected utility as a function of headway mean in BAIS2 (for 2-minute headway standard deviation)

Figure 4.35 Effect of symmetric utility function on the expected utility as a function of headway mean in BAIS3 (for 2-minute headway standard deviation and $a=0.0001$)
We now look at the expected utility in BAIS1 as a function of headway standard deviation. Figure 4.36 displays the expected utilities in BAIS1 for the various $c$ values with 10-minute headway mean. From this graph, we observe again that the expected utility is greater for larger values of $c$ at any headway standard deviation. This was previously observed and explained.

![Graph showing expected utility in BAIS1 for various $c$ values](image)

**Figure 4.36** Effect of symmetric utility function on the expected utility as a function of headway standard deviation in BAIS1 (for 10-minute headway mean)

We also observe that the patterns of decrease in the expected utility do not change much as $c$ increases. Thus, at any $c$, the expected utility decreases as headway standard deviation increases. In Section 4.1.1, we explained why expected utility in BAIS1 decreases as the headway standard deviation increases for $c=10$. Again, this is because
standard deviation of waiting time distribution in BAIS1 increases as headway standard deviation increases. The same explanation can be given to the expected utility for larger $c$ value (i.e. $c = 20$), since the waiting time distribution for the 10-minute headway mean can be located within bounds of $b-20$ and $b+20$ minutes where passengers have non-zero utility. Here, we explain in more detail for the cases of smaller $c$ values in which some portion of waiting time distribution are out of the bounds of $b-c$ and $b+c$ minutes.

Figure 4.37 depicts that the waiting times distributions for 2- and 4-minute standard deviations and the bounds where passenger utility goes to 0 for $c=3$ based on the assumption of the broadcast time are the same. According to the graph, as the headway standard deviation increases, the waiting time distribution in BAIS1 gets dispersed, and therefore fewer passengers arrive within the bounds (i.e., between $b-3$ and $b+3$ minutes). This means more passengers have zero utility as headway standard deviation increases. The broadcast time may change a little due to the change in headway standard deviation. Nevertheless, the broadcast time would only be expected to change a little. For this reason, the expected utility decreases as standard deviation increases at $c=3$. The combination of this explanation and the previous explanation for $c=10$ in Section 4.1.1, can explain why expected utility decreases as headway standard deviation increases at any $c$ value.
We now look at the expected utility in BAIS2 and BAIS3 as a function of headway standard deviation. Figures 4.38 and 4.39 display the expected utilities in BAIS2 and BAIS3, respectively, for the various $c$ values with 10-minute headway mean. From these graphs, we observe again that the expected utility is greater for the larger value of $c$ in any headway standard deviation except 0 headway standard deviation. This was previously observed and explained.
Figure 4.38 Effect of symmetric utility function on the expected utility as a function of headway standard deviation in BAIS2 (for 10-minute headway mean)

Figure 4.39 Effect of symmetric utility function on the expected utility as a function of headway standard deviation in BAIS3 (for 10-minute headway mean and $a=0.0001$)
We also observe that the patterns of decrease in the expected utility do not change much as $c$ increases. Thus, at any $c$, the expected utility decreases as headway standard deviation increases. We explained why, in BAIS2 and BAIS3, expected utility decreases as headway standard deviation increases for $c=10$. Again, this is because the standard deviation of the conditional waiting time distribution increases as the headway standard deviation increases. The same explanation can be given to the expected utility for larger $c$ value, i.e. $c=20$.

As before, we can explain this in more detail for the cases of smaller $c$ values. We argue this for BAIS2 only, since the same argument can be given for BAIS3. Figure 4.40 show a conditional waiting time distributions for 2- and 4-minute headway standard deviation and bounds where passenger utility goes to 0 (i.e. $b-3$ and $b+3$ minutes) for $c=3$. Like BAIS1, as headway standard deviation increases, the conditional waiting time distribution gets more dispersed, and therefore fewer passengers arrive within the bounds. This means that more passengers have zero utility as the headway standard deviation increases. Therefore, the expected utility decreases as the standard deviation increases at $c=3$. The combination of this explanation and the previous explanation for $c=10$ in Section 4.1.1, can explain why expected utility decreases as the headway standard deviation increases at any $c$ value.
4.2.2 Asymmetric Utility Function

In this section, we specify the utility function so that it allows for different preferences depending on whether bus arrives early or late. In order to do this, we use an asymmetric utility function. This asymmetry is handled by setting $c_1 \neq c_2$. We keep $\beta_1 = \beta_2 = 0$ so that the expected utility function is specified as follows:
We consider two different asymmetric utility functions. One is the case where the passenger utility is more sensitive to the bus arriving earlier than the broadcast time \( (w < b) \) than to the bus arriving later than the broadcast time \( (w > b) \). The other is the case where the passenger utility is more sensitive to the bus arriving later than the broadcast time than to the bus arriving earlier than the broadcast time. We set \( C_1 = 3 \) and \( C_2 = 17 \) for the former case and \( C_1 = 17 \) and \( C_2 = 3 \) for latter case. The sum of \( C_1 \) and \( C_2 \) is maintained at a constant value (i.e., \( C_1 + C_2 = 20 \)) to eliminate the effect of length of range of nonzero utility (i.e., \( 2C = 20 \)) used in the previous section. The expected utility computed for each case is compared to the symmetric utility function (i.e., \( C_1 = C_2 = 10 \))

As in section 4.2.1, we start by investigating the expected utility of BAIS1 as a function of headway mean. Figure 4.4 displays the expected utilities in BAIS1 for asymmetric utility functions and the symmetric utility function used previously (i.e., \( C_1 = C_2 = 10 \)). We observe that the expected utility barely changes with the change in \( C_1 \) and \( C_2 \). However, the expected utility curve for the case of \( C_1 = 3 \) and \( C_2 = 17 \) lies slightly above that for the symmetric utility function (i.e., \( C_1 = 10 \) and \( C_2 = 10 \)), while the expected utility
curve for the case of $c_1=17$ and $c_2=3$ lies slightly below that of the symmetric case. The next paragraph explains this in more detail.

![Graph showing expected utility as a function of headway mean in BAIS1](image)

**Figure 4.41** Effect of asymmetric utility function on the expected utility as a function of headway mean in BAIS1 (for 2-minute headway standard deviation)

The optimal broadcast time $b^*$ for the case of $c_1=3$ and $c_2=17$ will be less than that for the symmetric case (i.e., $c_1=c_2=10$) since the broadcaster wants to make more passengers encounter waiting time greater than broadcast time ($w>b^*$) and fewer passengers encounter waiting time less than the broadcast time ($w<b^*$) because of the increased sensitivity to $w<b^*$. On the other hand, the optimal broadcast time for the case $c_1=17$ and $c_2=3$ will be larger than that for the symmetric case because of the increased sensitivity to $w>b^*$. These changes in broadcast times can conceivably compensate the changes in the expected utility due to the changes in $c_1$ and $c_2$. 

135
Figure 4.42 depicts the waiting time distribution in BAIS1 for a 2-minute headway standard deviation with 10-minute headway mean, as well as the optimal broadcast times determined from the simulation and utility functions for the above three cases. Note that the probability density of waiting time in BAIS1 decreases as waiting time increases. From the graph, we can see that the waiting time probability densities around the optimal broadcast time \( b^* \) are the highest for the case \( c_1=3 \) and \( c_2=17 \), second highest for the symmetric case, and lowest for the case \( c_1=17 \) and \( c_2=3 \). Hence, the expected utility -- which is determined by integrating the product of the utilities and probability density of obtained the utilities -- shall be the largest for the case \( c_1=3 \) and \( c_2=17 \), the second largest for the symmetric case, and the smallest for the case \( c_1=17 \) and \( c_2=3 \). Therefore, the expected utility curve for the case \( c_1=3 \) and \( c_2=17 \) would be expected to lie above the symmetric case curve, and the expected utility curve for the case \( c_1=17 \) and \( c_2=3 \) would be expected to lie below the symmetric case curve as seen in Figure 4.41. We also see in Figure 4.41 that the curves approach each other as headway mean increases. This is expected, since the slope of the waiting probability density curve decreases as the headway mean increases because the waiting time distribution gets more dispersed with increased headway mean. Thus, the differences between the waiting time probability densities around the optimal broadcast times for the three cases of utility functions become smaller as headway mean increases.
Figure 4.42 Illustration of a waiting time distribution in BAIS1 and three cases of utility functions with their optimal broadcast time.

Figures 4.43 and 4.44 show the expected utilities of BAIS2 and BAIS3 as a function of headway mean for 2-minute headway standard deviation and $a=0.0001$ (for BAIS3). Like BAIS1, we observe that the expected utility barely changes with the changes in $c_1$ and $c_2$. However, unlike BAIS1, the expected utility curve for the symmetric case ($c_1=c_2=10$) lies slightly above those for the cases of $c_1=3$ and $c_2=17$ and of $c_1=17$ and $c_2=3$. This pattern can be explained with similar arguments given for BAIS1. We explain this for BAIS2 only since the same argument can be given for BAIS3. Figure 4.45 depicts the waiting time distribution in BAIS2 for 2-minute headway standard deviation with 10-minute headway mean, as well as the optimal broadcast times determined from the simulation and utility functions for the above three cases. From the graph, we can see the probability density of waiting time around the optimal broadcast time is higher for the
Figure 4.43 Effect of asymmetric utility function on the expected utility as a function of headway mean in BAIS2 (for 2-minute headway standard deviation)

Figure 4.44 Effect of asymmetric utility function on the expected utility as a function of headway mean in BAIS3 (for 2-minute headway standard deviation and \( \alpha=0.0001 \))
symmetric case (i.e., $c_1 = c_2 = 10$) than for the asymmetric cases (i.e., $c_1 = 3$, $c_2 = 17$ and $c_1 = 17$, $c_2 = 3$). Hence, the expected utility shall be larger for the symmetric case than for the asymmetric cases. Therefore, the expected utility curve for the symmetric case would be expected to lie above the asymmetric case curves as seen in Figure 4.43.

Figure 4.46 displays the expected utility of BAIS1 as a function of headway standard deviation for a 10-minute headway mean. As in Figure 4.41, we observe that the expected utility is barely affected by the change in $c_1$ and $c_2$, but that the expected utility curve for the case $c_1 = 3$ and $c_2 = 17$ lies above that for the symmetric case, while the expected utility curve for the case $c_1 = 17$ and $c_2 = 3$ lies below that for the symmetric case. This pattern can be explained using the same type of arguments used for Figure 4.41. At zero standard deviation, the waiting time distribution in BAIS1 is uniformly distributed.
and therefore the probability densities of the waiting time around the optimal broadcasts time are the same for these three cases of utility function. Hence, the expected utilities for these three cases of utility functions will be the same.

Figures 4.47 and 4.48 show the expected utilities of BAIS2 and BAIS3 as a function of headway standard deviation for 10-minute headway mean. We again observe that the expected utility is barely affected by the change in $c_1$ and $c_2$, but the expected utility curve for the symmetric case (i.e., $c_1=c_2=10$) slightly lies above that for the asymmetric cases (i.e., $c_1=3$, $c_2=17$ and $c_1=17$, $c_2=3$). This pattern can be explained using the same type of arguments used in Figures 4.43 and 4.44. At zero standard deviation, the expected utility for BAIS2 and BAIS3 is 1 for any case of the utility function.
Figure 4.47 Effect of asymmetric utility function on the expected utility as a function of headway standard deviation in BAIS2 (for 10-minute headway mean)

Figure 4.48 Effect of asymmetric utility function on the expected utility as a function of headway standard deviation in BAIS3 (for 10-minute headway mean and $a=0.0001$)
4.2.3 Waiting Time Dependent Utility Function

In this section, we specify the utility function to capture passenger utility preferences that depend on the actual waiting time. The passenger’s feeling toward the same broadcast time accuracy might be different when she has a shorter waiting time than when she has a longer waiting time. The utility function in this section represents less sensitivity when there is a longer waiting time than when there is a shorter waiting time. That is, if the error in broadcast time is 1 minute, for example, this 1-minute error would seem less sensitive to the passenger if she waits 15 minutes than if she waits 5 minutes.

In order to do this, we set $\beta_1=\beta_2 \neq 0$ in equation 4.8. Specifically, we set $c_1=c_2=5$ and $\beta_1=\beta_2=\beta > 0$. Thus, we specify the utility functions as follows:

$$
u(w, b) = \begin{cases} 
0 & w < \frac{b-5}{1+\beta} \\
1-\frac{|b-w|}{5+\beta w} & \frac{b-5}{1+\beta} \leq w < \frac{b+5}{1-\beta} \\
0, & \frac{b+5}{1-\beta} \leq w 
\end{cases}
$$

In this equation, the upper bound, $\frac{b+5}{1-\beta}$, at which the passenger utility becomes 0, has a positive value for $0<\beta<1$, is infinite at $\beta=1$, and is negative for $\beta > 1$. Hence the equation is valid for only $0<\beta<1$. The positive value of parameter $\beta$ indicates that a passenger becomes less sensitive to the error of the broadcast time as the actual waiting time of the passenger increases. Specially, the value for $\beta > 1$ indicates that the increase
in the term of $5 + \beta w$ (i.e., decrease in sensitivity) is faster than the increase in the error of broadcast time (i.e., $|b - w|$) as waiting time increases for a given broadcast time. In other words, given a broadcast time, a passenger will have higher utility when he waits longer than when he waits shorter. The value for $\beta = 1$ indicates that the increase in the term of $5 + \beta w$ is equal to the increase in the error of broadcast time as waiting time increases for a given broadcast time. Hence, when $\beta \geq 1$, there is no upper limit at which the passenger utility becomes 0. In this case, the utility function can be given by:

$$u(w, b) = \begin{cases} 
0 & w < \frac{b - 5}{1 + \beta} \\
\frac{|b - w|}{5 + \beta w} & \frac{b - 5}{1 + \beta} \leq w
\end{cases} \quad (4.12)$$

We start by investigating the expected utility of BAIS1 as a function of headway mean for various time dependent utility functions. Figure 4.49 displays the expected utility at 2-minute headway standard deviation with four cases of the utility function. From this graph, we observe that the expected utility curves for larger $\beta$s lie above those for smaller $\beta$s. This means that, at a given headway mean, the expected utility increases as the value of $\beta$ increases. This can be explained by the same arguments used to explain the pattern of Figure 4.33. The waiting time distribution is independent of the utility function (i.e. independent of $\beta$ value). This means that waiting time distribution is not changed by the change in $\beta$ value. If the optimal broadcast time of a particular passenger is not changed by the increase in $\beta$ value, the expected utility of the passenger increases,
since the term $\frac{|b - w|}{5 + \beta w}$ in the utility function decreases. However, the optimal broadcast time can change as $\beta$ value increases. This change in optimal broadcast time will only increase the expected utility, since the broadcast time is determined to maximize the expected utility of the passenger. Therefore, the expected utility will increase for larger $\beta$ values for any headway mean.

For any $\beta$ value, the expected utility decreases as the headway mean increases, but does not necessarily approach zero. In Section 4.1.1, we mentioned why the expected utility in BAIS1 decreases and approaches zero as the headway mean increases for a symmetric utility function. The same explanation can be used to explain the expected utility patterns for $0 < \beta < 1$, since the passenger utility has positive values only within

![Figure 4.49 Effect of time dependent utility function on the expected utility as a function of headway mean in BAIS1 (for 2-minute headway standard deviation)](image-url)
limited bounds which are independent of the waiting time distribution (see equation 4.12). Thus, as headway mean increases, the probability of a particular waiting time decreases to 0, and consequently the expected utility gets close to zero.

When $\beta \geq 1$ there is no upper bound at which the passenger utility becomes 0. Therefore, from equation 4.12, each passenger utility can be determined by

$$u = 1 - \frac{|b - w|}{5 + \beta w} = 1 - \frac{|\frac{b}{w} - 1|}{\frac{5}{w} + \beta}$$

(4.13)

Note that as $\beta$ increases the optimal broadcast time decreases, since the passenger becomes more sensitive to error in broadcast time when the actual waiting time is shorter. Hence, as headway mean increases to infinity, the terms $b/w$ and $5/w$ in the above equation get closer to 0 since the waiting time, $w$, of a particular passenger increases to infinity. Therefore, as headway mean increases, the expected utility approaches the value $1 - \frac{1}{\beta}$. For example, when $\beta$ is 2, the expected utility of BAIS1 approaches to $1/2$.

We now look at the expected utility in BAIS2 and BAIS3 as a function of headway mean. Figures 4.50 and 4.51 display the expected utilities in BAIS2 and BAIS3, respectively, for the various $\beta$ values with a 2-minute headway standard deviation and at $\alpha=0.0001$ (for BAIS3). From these graphs, we observe again that the expected utility for a larger $\beta$ lies above that for a smaller $\beta$. This is can be explained by the same argument given for Figure 4.49.
Figure 4.50 Effect of time dependent utility function on the expected utility as a function of headway mean in BAIS2 (for 2-minute headway standard deviation)

Figure 4.51 Effect of time dependent utility function on the expected utility as a function of headway mean in BAIS3 (for 2-minute headway standard deviation and $a=0.0001$)
We also observe that, for any $\beta$ except $\beta=0$, the expected utility increases and eventually approaches 1 as the headway mean increases. For $\beta=0$, it decreases and eventually reaches a constant value. In Section 4.1.1, we discussed why the expected utilities of BAIS2 and BAIS3 decrease and approach constant values for the symmetric utility function (i.e., when $\beta=0$). To explain why the expected utility approaches 1 as the headway mean increases, recall that the waiting time distributions for BAIS2 and BAIS3 have increased mean but otherwise do not change much with the changes in the headway mean. As the headway mean increases, the actual waiting time, $w$, increases while $|b-w|$ does not change much, since the optimal broadcast time is also shifted to compensate for the shifted mean of the waiting time distribution. Therefore, when $\beta>0$, the term $\frac{|b-w|}{5+\beta w}$ in the utility function approaches zero as the headway mean increases. Since this term approach zero, the utility approaches 1 (see equation 4.11).

We now look at the expected utility in BAIS1 as a function of the headway standard deviation. Figure 4.52 displays the expected utilities in BAIS1 for the various $\beta$ values with a 10-minute headway mean. From this graph, we observe again that the expected utility curves for larger $\beta$s lie above those for smaller $\beta$s. This was previously observed and explained.

We also observe that the expected utility patterns in BAIS1 barely change with the change in the time dependent utility function (i.e., change in $\beta$ value). At any value of $c$, the expected utility decreases slightly as the headway standard deviation increases. In
Section 4.1.1, we discussed that the expected utility of BAIS1 decreases very little as the headway standard deviation increases, since the waiting time distribution in BAIS1 is not affected much by the headway standard deviation. This was explained when using the symmetric utility function (i.e., when $\beta=0$). Since the actual waiting time of a particular passenger is not affected by the value of $\beta$, the same arguments can be made for any value of $\beta$, and it is reasonable to see only slight decreases with increased headway standard deviation.

We now look at the expected utility in BAIS2 and BAIS3 as a function of headway standard deviation. Figures 4.53 and 4.54 display the expected utilities in

![Graph showing expected utility as a function of headway standard deviation.](image)

**Figure 4.52** Effect of time dependent utility function on the expected utility as a function of headway standard deviation in BAIS1 (for 10-minute headway mean)
Figure 4.53  Effect of time dependent utility function on the expected utility as a function of headway standard deviation in BAIS2 (for 10-minute headway mean)

Figure 4.54  Effect of time dependent utility function on the expected utility as a function of headway standard deviation in BAIS3 (for 10-minute headway mean and $\alpha=0.0001$)
BAIS2 and BAIS3, respectively, for the various $\beta$ values with a 10-minute headway mean. From these graphs, we observe again that the expected utility for larger value of $\beta$ lies above that for smaller value of $\beta$. This was previously observed and explained.

We also observe that the expected utility patterns in BAIS2 and BAIS3 do not seem to depend much on the $\beta$ value. At any $\beta$, the expected utility decreases as the headway standard deviation increases. We discussed that the expected utilities in BAIS2 and BAIS3 decrease markedly as the headway standard deviation increases, since the waiting time distributions in BAIS2 and BAIS3 are affected much by the headway standard deviation. This was explained when using symmetric utility function (i.e., when $\beta=0$). As in BAIS1, since actual waiting time is not affected by the value of $\beta$, the same arguments can be made for any value of $\beta$, and it is reasonable to see the same patterns of decreases with increased headway standard deviation.

4.3 Sensitivity Analysis for Broadcaster’s Knowledge

In the expected utility computation, we assumed that the broadcaster had perfect information on the historical bus operation data and the passenger utility function. This means that actual headway distribution would be identical to that considered by the broadcaster and that the utility function used to set the optimal broadcast times would be that which corresponds to passenger preferences. However, in reality, the broadcaster cannot have perfect knowledge of the actual headway distribution and of the utility function. Hence, the expected utilities simulated based on the perfect knowledge would
be different from the average of the actual passenger utilities taken across the entire passenger population.

The effect of the broadcaster's knowledge of the bus operation parameters and utility functions on the expected utility are investigated in a sensitivity analysis. The sensitivity analysis is performed by changing the broadcaster's knowledge of headway mean, headway standard deviation and parameters of utility functions in the simulation. The change in the expected utilities with the change in the broadcaster's knowledge of the bus operation parameters or utility function are investigated.

Specifically, in the sensitivity analysis, we assume bus operation parameters as $\bar{h} = 10$ minutes and $\sigma_h = 3$ minutes, and the utility function as given in the equation 4.10 with $c_1=10$ and $c_2=10$. When the broadcaster has perfect knowledge on these parameters, we calculate the expected utilities to be 0.706, 0.806 and 0.917 for BAIS1, BAIS2 and BAIS3 respectively. These expected utilities can also be approximated from Figure 4.12. We investigate the change of expected utility with respect to differences in the broadcaster's belief about the parameter values. For example, when the broadcaster believes that there is a 5-minute headway mean and has perfect knowledge of other parameters, the expected utilities are 0.669 (5.2 % decrease), 0.679 (16.6 % decrease) and 0.917 (no decrease) for BAIS1, BAIS2 and BAIS3 respectively. We found these values by using the "true" waiting times to determine the passengers' utilities when the broadcaster set the optimal broadcast times $b^*$ based on the erroneous belief that $\bar{h} = 5$. We varied the $\bar{h}$, $\sigma_h$, $c_1$, and $c_2$ individually by 50 % of the base values both by
increasing and decreasing the values and conducted similar analyses. Table 4.5 shows the results of this "one-at-a-time" sensitivity analyses.

<table>
<thead>
<tr>
<th>Broadcaster's belief</th>
<th>Expected utility</th>
<th>Change in expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BAIS1</td>
<td>BAIS2</td>
</tr>
<tr>
<td>with perfect knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>headway mean (min.)</td>
<td>5</td>
<td>0.669</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.656</td>
</tr>
<tr>
<td>std. dev. (min.)</td>
<td>1.5</td>
<td>0.705</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>0.706</td>
</tr>
<tr>
<td>c1</td>
<td>5</td>
<td>0.679</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.695</td>
</tr>
<tr>
<td>c2</td>
<td>5</td>
<td>0.690</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.696</td>
</tr>
</tbody>
</table>

Table 4.5 Sensitivity of expected utility to 50% changes in broadcaster brief of true value of headway mean, headway standard deviation and utility function parameters from base case (base case: $\bar{h} = 10$, $\sigma_h = 3$, $c_1 = 10$, $c_2 = 10$)

As seen in Table 4.5, expected utility is much more sensitive to the broadcaster's knowledge of headway mean than to her knowledge of other parameters in BAIS1 and BAIS2. This is fortunate, since it is probably easier to obtain accurate estimates of $\bar{h}$ than to obtain accurate estimates of the other parameters.

We also see that the decrease in expected utility brought about by erroneous headway mean beliefs is less in BAIS1 than in BAIS2. This is because broadcast times are the same to every passenger in BAIS1 due to the absence of real-time data. Thus, in BAIS1, the shifting the broadcast time (from the true optimal broadcast time) due to the wrong beliefs about the headway mean will actually increase utility for some passengers,
even though it will decrease utility for more passengers. On the other hand, since a broadcast time is set for each passenger in BAIS2, shifting the broadcast time due to the wrong beliefs about headway mean will decrease the utility of every passenger. We also see that erroneous headway mean beliefs do not change the expected utility of BAIS3. This is because our the heuristic forecasting model used by the broadcaster does not depend on the headway mean.

We also investigated the change in expected utility when varying all the previous parameters of the broadcaster’s belief simultaneously by 50% for the same base case. Table 4.6 shows the results. From this table we can see that error in the broadcaster’s beliefs about headway mean can intensify or alleviate the decrease in the expected utility brought about by error in the broadcaster’s beliefs about the utility function in BAIS1 and BAIS2. For example, when the broadcaster believes that $\bar{h} = 5$ minutes (i.e., smaller than the true $\bar{h} = 10$ minutes) and that the utility function parameters are $c_1 = 5$ and $c_2 = 15$, the expected utilities are 0.589 (16.6% decrease) and 0.579 (28.1% decrease) for BAIS1 and BAIS2, respectively. These decreases in the expected utility are greater than those due to the error in the broadcaster’s beliefs of the headway mean when other parameters are known perfectly (see Table 4.5). This is because the shift of the broadcast time due to the knowledge of smaller headway mean is intensified by the erroneous belief of the utility function parameters ($c_1 = 5$ and $c_2 = 15$) due to a greater sensitivity to early bus arrival. However, when the broadcaster believes that $\bar{h} = 15$ minutes (larger than the true $\bar{h} = 10$ minutes) and that utility function parameters are $c_1 = 5$ and $c_2 = 15$, the
expected utilities are 0.692 (2.0 % decrease) and 0.737 (8.6 % decrease) for BAIS1 and BAIS2, respectively. These decreases in the expected utility are smaller than those due to the error in the broadcaster’s belief of the headway mean when other parameters are known perfectly (see Table 4.5). This is because the shift of the broadcast time due to the knowledge of larger headway mean is alleviated by the erroneous belief of the utility function parameters \((c_1 = 5 \text{ and } c_2 = 15)\) due to smaller sensitivity to late bus arrival.

<table>
<thead>
<tr>
<th>Broadcaster’s belief</th>
<th>Expected utility</th>
<th>Change in expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>with perfect knowledge</td>
<td>0.706 0.806 0.917</td>
<td></td>
</tr>
<tr>
<td>mean = 5, std.dev = 1.5, (c_1 = 5 \text{, } c_2 = 15)</td>
<td>0.589 0.579 0.893</td>
<td>-16.6% -28.1% -2.7%</td>
</tr>
<tr>
<td>mean = 5, std.dev = 1.5, (c_1 = 15 \text{, } c_2 = 5)</td>
<td>0.697 0.670 0.916</td>
<td>-1.3% -16.8% -0.2%</td>
</tr>
<tr>
<td>mean = 15, std.dev = 4.5, (c_1 = 5 \text{, } c_2 = 15)</td>
<td>0.692 0.737 0.810</td>
<td>-2.0% -8.6% -11.7%</td>
</tr>
<tr>
<td>mean = 15, std.dev = 4.5, (c_1 = 15 \text{, } c_2 = 5)</td>
<td>0.436 0.320 0.824</td>
<td>-38.3% -60.3% -10.1%</td>
</tr>
</tbody>
</table>

Table 4.6  Sensitivity of expected utility to simultaneous 50 % changes in broadcaster belief of true value of headway mean, headway standard deviation and utility function parameters from base case (base case: \(\bar{h} = 10\), \(\sigma_h = 3\), \(c_1=10\), \(c_2=10\))

In BAIS3, since erroneous beliefs about the headway mean have no effect on the expected utility, as explained above, an erroneous increase in the broadcaster’s belief about the headway standard deviation intensifies much the decrease in the expected utility brought about by the erroneous broadcaster’s beliefs about the utility function parameters, while an erroneous decrease in the broadcaster’s beliefs about the headway standard
deviation intensifies the decrease in the expected utility brought about by error in the broadcaster's beliefs about the utility function parameters by only a small amount. For example, when the broadcaster believes that that $\sigma_H = 4.5$ minutes and that the utility function parameters are $c_1 = 5$ and $c_2 = 15$, the expected utility is 0.810 (11.7% decrease). However, when the broadcaster believes that $\sigma_H = 15$ minutes and that the utility function parameters are $c_1 = 5$ and $c_2 = 15$, the expected utility is 0.893 (2.7% decrease).

We also perform a similar sensitivity analysis for a base case with $\bar{h} = 20$ minutes and the other base case parameters set to the same values as those in the base case above (i.e., $\sigma_H = 3$, $c_1=10$, $c_2=10$). Again, we varied parameters by 50% one-at-a-time and simultaneously. Tables 4.7 and 4.8 show the results of this sensitivity analysis for the one-at-a-time and simultaneously. The results for this analysis are very similar to those for previous analysis.

<table>
<thead>
<tr>
<th>Broadcaster's belief</th>
<th>Expected utility</th>
<th>Change in expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BAIS1</td>
<td>BAIS2</td>
</tr>
<tr>
<td>with perfect knowledge</td>
<td>0.496</td>
<td>0.792</td>
</tr>
<tr>
<td>headway mean</td>
<td>0.431</td>
<td>0.335</td>
</tr>
<tr>
<td>(min.) 10</td>
<td>0.381</td>
<td>0.137</td>
</tr>
<tr>
<td>headway mean</td>
<td>0.496</td>
<td>0.789</td>
</tr>
<tr>
<td>(min.) 30</td>
<td>0.495</td>
<td>0.788</td>
</tr>
<tr>
<td>std. dev. 1.5</td>
<td>0.490</td>
<td>0.774</td>
</tr>
<tr>
<td>(min.) 4.5</td>
<td>0.481</td>
<td>0.783</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.478</td>
<td>0.770</td>
</tr>
<tr>
<td>5</td>
<td>0.490</td>
<td>0.785</td>
</tr>
<tr>
<td>15</td>
<td>0.481</td>
<td>0.783</td>
</tr>
</tbody>
</table>

Table 4.7 Sensitivity of expected utility to 50% changes in broadcaster belief of true value of headway mean, headway standard deviation and utility function parameters from base case (base case: $\bar{h} = 20$, $\sigma_H = 3$, $c_1=10$, $c_2=10$)
<table>
<thead>
<tr>
<th>Broadcaster's belief</th>
<th>Expected utility</th>
<th>Change in expected utility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BAIS1 BAIS2 BAIS3</td>
<td>BAIS1 BAIS2 BAIS3</td>
</tr>
<tr>
<td>with perfect knowledge</td>
<td>0.496 0.792 0.880</td>
<td></td>
</tr>
<tr>
<td>mean = 10, std.dev = 1.5, $c_1 = 5$, $c_2 = 15$</td>
<td>0.355 0.282 0.858</td>
<td>-28.5% -64.4% -2.6%</td>
</tr>
<tr>
<td>mean = 10, std.dev = 1.5, $c_1 = 15$, $c_2 = 5$</td>
<td>0.482 0.339 0.872</td>
<td>-2.8% -57.2% -0.9%</td>
</tr>
<tr>
<td>mean = 30, std.dev = 4.5, $c_1 = 5$, $c_2 = 15$</td>
<td>0.434 0.301 0.769</td>
<td>-12.6% -62.0% -12.6%</td>
</tr>
<tr>
<td>mean = 30, std.dev = 4.5, $c_1 = 15$, $c_2 = 5$</td>
<td>0.438 0.033 0.789</td>
<td>-11.8% -95.9% -10.3%</td>
</tr>
</tbody>
</table>

Table 4.8  Sensitivity of expected utility to simultaneous 50 % changes in broadcaster belief of true value of headway mean, headway standard deviation and utility function parameters from base case (base case: $\bar{h} = 20$, $\sigma_H = 3$, $c_1 = 10$, $c_2 = 10$)
4.4 Empirical Analysis of a Real Bus System

In this section, we use data taken from a real bus system to calculate passenger expected utilities for the three BAIS alternatives. We use bus operational data collected from The Ohio State University’s Campus Area Bus System (CABS) in Columbus, Ohio. In section 4.3.1, we examine the data taken from CABS, such as the headway distributions and the correlation coefficients of the headway and the upstream headway. In section 4.3.2, we examine the expected utilities calculated from the CABS’s data.

4.4.1 Historical Bus Operation Data

CABS provides transportation for The Ohio State University campus and the surrounding residential area. Figure 4.55 shows the CABS’s bus routes and schedules. The routes in this figure are about 2 to 7 miles long. Bus arrival schedules at bus stops are published in terms of headways. Published bus headways are between 6 and 30 minutes, depending on the route and time of day.
Figure 4.55 Ohio State University Campus Area Bus Service Route Map
(source: www.tg ohio-state.edu/cabspages.htm, visited on Jan. 20, 1999)
CABS recently installed a Global Positioning System (GPS) based vehicle location system for building a passenger information system. The system collects and archives a large amount of bus operation data. We analyze the archived GPS data to estimate the headways and correlation functions. The data collected by the system mainly pertain to the North Express, North and South Campus Loop and East Residential and Buckeye Village routes. We selected three bus routes having different scheduled headways to examine the bus data. Table 4.9 shows the bus operation data and bus stops for collecting data in these routes.

<table>
<thead>
<tr>
<th>Route</th>
<th>published headway (minutes)</th>
<th>runtime (minutes)</th>
<th>buses in operation</th>
<th>bus stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Express</td>
<td>6</td>
<td>24</td>
<td>4</td>
<td>North Dorm</td>
</tr>
<tr>
<td>Campus Loop South</td>
<td>10</td>
<td>30</td>
<td>3</td>
<td>Ohio Union</td>
</tr>
<tr>
<td>East Residential</td>
<td>15</td>
<td>30</td>
<td>2</td>
<td>North Dorm</td>
</tr>
</tbody>
</table>

Table 4.9 Selected bus routes for data collection

The archived bus operation data include information on bus location, bus velocity, time of observation, bus number, and route. From the bus location and the time of observation data, the times when the buses arrived at the selected bus stops were estimated. Headways at each selected bus stop were determined by the time differences in consecutive bus arrivals at the bus stop. The correlation function at each selected bus...
stop was also determined by the headways at the bus stop and upstream headways at several upstream points by increasing the distance from the bus stop. We used CABS's data collected between September 21 and October 31, 1998 to get headways and correlation coefficients at the selected bus stops.

**Headway:** We investigated headways at the three selected bus stops. Figure 4.56 displays the distributions (as frequency of headway) for the 6-minute published headway (at the North Dorm stop on the North Express route), the 10-minute published headway (at the Ohio Union stop on the Campus Loop South route), and the 15-minute published headway (at the North Dorm stop on the East Residential stop). This figure also shows the normal distributions with the empirically calculated mean and headway standard deviation in each case. Table 4.10 shows the various statistics for these distributions. The means of the headway observations are almost the same as the published headway. We also observe that the headway standard deviation increases as the headway mean increases. The headway standard deviations are 2.78 for the 6-minute headway, 4.19 for the 10-minute headway and 4.91 for the 15-minutes headway. This is intuitively expected, since a bus is likely to arrive off schedule less in a shorter headway because a bus must be operated within the bounds of the preceding and following buses. Thus, the preceding and following buses may keep a bus from deviating too far from the schedule of headway.
Figure 4.56 Headway distributions observed at selected bus stops
Table 4.10  Statistics for the headway distribution observed at selected bus stops

<table>
<thead>
<tr>
<th></th>
<th>Arps Hall</th>
<th>Ohio Union</th>
<th>North Dorm</th>
</tr>
</thead>
<tbody>
<tr>
<td>headway mean (minute)</td>
<td>6.01</td>
<td>10.34</td>
<td>15.08</td>
</tr>
<tr>
<td>headway standard deviation (minute)</td>
<td>2.78</td>
<td>4.19</td>
<td>4.91</td>
</tr>
<tr>
<td>skewness</td>
<td>1.03</td>
<td>0.72</td>
<td>0.13</td>
</tr>
</tbody>
</table>

We also observe that the frequency of headways smaller than the mean headway in the empirical distribution is smaller than the frequency in the corresponding normal distribution, while the frequency of headways larger than the mean headway is higher than in the corresponding normal distribution. That is, the observed headway distributions are skewed to left (i.e., positive skewness) as shown in Table 4.10. This is because the headways of early arriving buses are constrained by 0, but the headways of late arriving buses are not constrained. From Table 4.10, we observe that headway distribution is skewed more in a smaller headway mean than in a larger headway mean. This is because the headway of the following bus is more likely to be constrained by 0 in a smaller headway than in a larger headway because of the smaller time gap between the consecutive buses.

Figures 4.57 through 4.59 display the normal plots for these headway distributions. The vertical axis in the figure is a probability on a logarithmic scale. The horizontal axis represents the headway. The least square lines shown on the graphs represent the normal distributions in the cases of the same headway mean and headway
standard deviation. These normality test plots confirm the fact the headway distributions are skewed more severely as the headway mean decreases. That is, the empirical points are farther from the regression lines at the small headway (Arps Hall) stop than at the long headway (North Dorm) stop. Figure 4.57 shows the normality test plot for the 6-minute headway (at the Arps Hall bus stop). The empirical headway observations are located below the least square line for the smaller headways, then rise above the line for headways near the mean and fall below the line again for larger headways. This indicates that headways of early buses are constrained by 0; however, the headways of late buses are not constrained by any value. Therefore, the headway distribution is skewed in the positive direction. Figure 4.58 displays the normality plot for the 10-minute headway (at the Ohio Union bus stop). This plot is almost the same as that for the 6-minute headway. In this case, however, the plot of the headway observations deviates less from the least square line than that for the 6-minute headway reflecting the smaller value of skewness. Figure 4.59 displays the normal plot for the 15-minute headway (the North Dorm bus stop). In this case, the empirical headway observations follow the least square line very closely.
Figure 4.57 Normality plot for headways observed at the Arps Hall bus stop
Figure 4.58 Normality plot for headways observed at the Ohio Union bus stop
Figure 4.59 Normality plot for headways observed at the North Dorm bus stop
Correlation Coefficient: We now look at the observed the correlation coefficient function at the selected bus stops. We mention that the correlation coefficient is a function of the location of the following bus at the time of passenger arrival at a bus stop (which we have been denoting by a distance $s$ from bus stop). We calculate the correlation coefficient for several upstream points using equation 3.7, and plot the relationship between the correlation coefficient and distance $s$. Figures 4.60, 4.61 and 4.62 display these relationships for the 6-minute headway (at Arps Hall), 10-minute headway (at Ohio Union) and 15-minute headway (at North Dorm). The horizontal axis in the graph represents the distance from the bus stop to an upstream location. The vertical axis represents the correlation coefficient value between the headway at the bus stop and the headway at the corresponding upstream point. From these graphs, we observe that the correlation coefficient decreases as the distance increases. These results are reasonable, since more uncertainty would be expected to be involved in the prediction of bus arrival time as the distance of the bus increases. For example, the headway and upstream headway are correlated perfectly if the distance is 0 (because of certainty due to same points) and they become independent of each other if the distance become sufficiently large (because of very high uncertainty).

We also observe that the decrease in correlation coefficient does not follow the exponential function in which the rate of decrease is constant. This is because the traffic and road conditions are not homogeneous along the route. The way in which the correlation coefficient varies will depend on the traffic and road condition along the bus route. For example, in the road section having traffic lights and heavy vehicle and
pedestrian traffic, the correlation coefficient may be low since the bus travel times could be very different over the distance $s$, thereby changing the headway (see Figure 2.1). On the other hand, in road section having no traffic light and low vehicle and pedestrian traffic, the correlation coefficient may be high, since the bus travel times would be almost the same over the distance $s$, thereby not changing the headway much.

Figure 4.60  Observed correlation function at Arps Hall on North Express
Figure 4.61 Observed correlation function at Ohio Union on Campus Loop South

Figure 4.62 Observed correlation function at North Dorm on East Residential
4.4.2 Expected Utility

We calculate the empirical expected utilities for the three BAIS alternatives by using data obtained from CABS. We use the headway distribution and the correlation function observed at Ohio Union bus stop on Campus Loop South. The utility functions discussed in Section 4.1 (see equation 4.1) will be used for this analysis. We also simulate the corresponding hypothetical expected utilities from the developed model with the same headway means and standard deviation assuming a normal headway distribution and an exponential correlation coefficient function. The data for the hypothetical analysis is the same as that shown in Table 4.1 except for the bus velocity, which will be presented subsequently.

For empirical analysis of the expected utility, we generate the samples from the empirically observed function. Headway samples can be generated from the distribution shown in Figure 4.56b. Since we do not investigate the passenger arrival empirically, we assume passenger arrivals occur as a Poisson process in this analysis. Hence, passengers are generated using an exponential function for the inter-arrival time (equation 3.2). The upstream headway samples can be generated from the empirical correlation function. Figure 4.63 shows the empirical correlation function with respect to the distance of the following bus location. As seen in the figure, the function lies between two curves representing exponential functions with different parameter $a$. Figure 4.64 shows the observed relationship between the distance and the bus travel time (this is same as the passenger waiting time) from an upstream point to the Ohio Union bus stop.
Figure 4.63 Observed correlation function compared to two bounding exponential functions

Figure 4.64 Observed relationship between the distance and the travel time from an upstream point to the Ohio Union bus stop
From Figures 4.63 and 4.64, we can determine the empirical correlation function with respect to the waiting time, since waiting time is used for the generation of upstream headway samples in the empirical analysis. From Figure 4.64, we determine the average bus velocity as 3.9 meters per second. We use this velocity in the corresponding hypothetical analysis.

Figure 4.65 shows the empirical and hypothetical results of the expected utilities for BAIS1, BAIS2 and BAIS3. The three solid shapes represent the empirical results while the lines represent the corresponding hypothetical results. From this graph, we observe the empirical results seem to lie pretty close to the lines representing the hypothetical results. Thus, the developed model seems to produce a pretty good approximation of the results computed from the realistic bus operation data.

![Graph showing empirical and hypothetical results for BAIS1, BAIS2, and BAIS3.](image)

**Figure 4.65** Empirical results compared to the hypothetical results at the Ohio Union bus stop
However, we observe a pattern when comparing the two results. The empirical results of BAIS1 and BAIS2 seem to lie below the corresponding the curves for the hypothetical results. This is because the empirical headway distribution is skewed positively (see Figure 4.56b). We observed that the empirical headway distribution (at the Ohio Union bus stop) has a shorter left tail and longer right tail than the normal distribution. Due to the longer right tail, the waiting time distribution in BAIS1 gets more dispersed in the empirical headway distribution than in the corresponding normal headway distribution. This means that standard deviation of the waiting time distribution in BAIS1 is larger for the empirical headway distribution than for the corresponding normal distribution. Recall that a higher standard deviation in waiting time distribution leads to lower expected utility. This explains why the empirical expected utility in BAIS1 lies below the corresponding hypothetical expected utility line.

Recall that, in BAIS2, the waiting time distribution for a passenger is formed by a truncated headway distribution, since it is determined by conditioning the headway distribution on the time since most recent bus departure. A headway distribution with a long right tail leads to a more dispersed conditional waiting time distribution than the corresponding normal distribution when it is truncated. This means that the standard deviation of the waiting time distribution for the same percentile passenger is greater when using the empirical headway distribution than when using the corresponding normal distribution due to the longer right tail of empirical headway distribution. A higher standard deviation in the waiting time distribution leads lower expected utility. This
explains why the empirical result in BAIS2 lies below the corresponding hypothetical result line.

We now investigate the empirical correlation function to explain the results in BAIS3. In Figure 4.63, we observed that the empirical function of the correlation coefficient lies between curves representing hypothetical exponential functions for $a=0.00005$ and $a=0.0002$. Note that in the 10.34-minute headway mean, most passengers wait less than 10 minutes, which corresponds to 2000 m in distance for the following bus (see Figure 4.64). The empirical curve at distance less than 1000 m gets very close to the exponential function for $a=0.0002$. For longer distances the empirical function gets closer to the exponential function for $a=0.00005$. Hence, the empirical curve for the correlation coefficient at distances less than 2000 m can be considered to lie between these two hypothetical functions. Considering this, the empirical expected utility in BAIS3 would be expected to lie between the hypothetical expected utility lines for $a=0.00005$ and $a=0.0002$ in Figure 4.65.

The skewness of headway distribution can also affect the empirical results in BAIS3. Since the conditional headway distribution is used in BAIS3, a similar explanation used in BAIS2 can be given to BAIS3. There is another effect that causes the empirical expected utility to be smaller than the hypothetical expected utility. We mentioned that the waiting time distribution in BAIS1 (which is same as the actual waiting time distribution) gets dispersed due to the skewness of headway distribution. As the actual waiting time distribution gets dispersed, the waiting time of a particular
passenger increases and consequently the correlation coefficient used to generate the upstream headway for this passenger decreases. The decrease in the correlation coefficient leads to a decrease in the expected utility. This and the previous explanation explain why the empirical expected utility in BAIS3 would lie below the hypothetical result, and therefore lie below the $a = 0.0002$ curve.
CHAPTER 5

SUMMARY AND CONCLUSIONS

This chapter summarizes this research, discusses its limitations, and presents directions for future and continued research.

5.1 Research Objectives and Methods

The objective of this study was to develop a methodology for evaluating various Bus Arrival Information Systems (BAIS) in terms of their performance in providing bus arrival time predictions to passengers at bus stops. Furthermore, exploring the changes in the value of BAIS alternatives (reflecting different real-time vehicle data availability) as a result of changes in bus operations parameters (headway mean, headway variance and correlation between headways) and passenger preference (utility) functions was of interest.

We evaluated BAIS in terms of the expected utility provided to a random passengers at a bus stop. We formulated the analytic equations to determine the expected utility of a random passenger assuming three BAIS alternatives. We used probability density functions to model the stochasticity of bus operations and passenger arrivals. The
expected utility of a random passenger was determined by taking the expectation of the expected utility of the various passengers. The expected utility of a particular passenger is obtained by integrating the utility associated with the possible waiting times for this passenger given the broadcast time (i.e. bus arrival time) set by the broadcaster. The broadcast times were set to maximize a passenger’s expected utility when considered from the broadcaster’s point of view. To formulate the broadcaster’s view of passengers’ waiting time distributions, we assumed knowledge of historical headway density functions. In BAIS1, the optimal broadcast time is the same for every passenger, since the absence of real-time vehicle location data makes the unconditional waiting time distribution the same for every passenger. In BAIS2 and BAIS3, the optimal broadcast time differs from passenger to passenger according to the real-time data available to the broadcaster at the time of passenger arrival. Passengers were assumed to arrive according to a Poisson process. All passengers were also assumed to have identical utility functions that depend only on the difference between the actual waiting time and the broadcast time received upon arrival to the bus stop.

Since closed form solutions can be intractable for a wide variety of assumptions for our analytical formulation, we used Monte Carlo simulation to solve for the broadcast time and expected utilities. Bus and passenger arrival times at a bus stop were simulated by generating headway and passenger realizations from given probability density functions. Headway realizations were generated from normal distribution functions. Passenger realizations were generated assuming exponential passenger inter-arrival times (reflecting a Poisson passenger arrival process). To account for the effect of real-time bus
location data on refining the broadcaster’s arrival time predictions, we modeled the correlation between headways at a bus stop and headways at upstream points. Upstream headway realizations were generated for each passenger from an assumed normal density function for the headway at the stop and an exponential function modeling the correlation coefficient as a function of upstream distance from the stop. We simulated the expected utility for the BAIS alternatives under different scenarios reflecting different (1) bus operations parameters and (2) passenger utility functions. We also used the methodology developed to calculate the expected utility for BAIS alternatives under the operating conditions of the transit system serving the campus of The Ohio State University (Campus Area Bus System).

5.2 Findings and Discussion

We investigated the effect of bus system parameters on the expected utility. We observed that the BAIS alternative that assumed comprehensive real-time bus location data (BAIS3) exhibits a higher expected utility than the BAIS alternative that assumes that the broadcaster only knows the time elapsed since the most recent bus departure (BAIS2). Moreover, BAIS2 exhibits a higher expected utility than the BAIS that assumes only knowledge of the historical bus headway distribution (BAIS1). These results were expected, since a passenger should have higher expected utility in a BAIS alternative that uses more real-time data and therefore allows for better bus arrival time predictions. However, our results quantify the relative values.
We also observed that as the headway mean increases, the expected utility of BAIS1 decreases markedly and eventually approaches 0, while the expected utilities for BAIS2 and BAIS3 decrease very little and approach non-zero constant values. That is, changes in headway mean have a greater effect on the expected utility of BAIS1 than of BAIS2 or BAIS3. This implies that the relative values of BAIS2 and BAIS3 over BAIS1 increases as the bus system is operated under longer headways.

Moreover, as the headway standard deviation increases, the expected utility of BAIS2 decreases faster than that of BAIS1 and BAIS3. Nevertheless, the rate of decrease in BAIS3 depends on the correlation function. In any event, the relative value of BAIS2 over BAIS1 decreases as the bus system operation becomes more variable, while the relative value of BAIS3 over BAIS2 increases.

Finally, with regard to the effects of bus operations parameters, we observe that the expected utilities of BAIS1 and BAIS2 remain constant as the correlation between bus headways decreases. This result is expected due to the absence of real-time data at upstream points. On the other hand, the expected utility of BAIS3 decreases as the correlation decreases, since accuracy in predicting bus arrival times based on real-time data diminishes as the correlation decreases. Moreover, the expected utility of BAIS3 approaches that of BAIS2 as the headways become sufficiently uncorrelated. This implies that the relative value of BAIS2 over BAIS1 remains constant, while the relative value of BAIS3 over BAIS2 decreases as the bus operations reflects lower correlation between headways.
We also investigated the effect of passenger utility functions on the expected utility. The results discussed above assumed a symmetric utility function, i.e., one in which passengers have the same preference (disutility) for the error in broadcast time regardless of whether the bus arrives earlier or later than predicted. We varied these passenger preferences for accuracy of broadcast time. We observed that passengers with less sensitivity to errors exhibit a greater expected utilities than passengers with greater sensitivity. However, the patterns of how the expected utility vary with bus operations parameters do not change much with changes in the passenger sensitivity to errors.

We also investigated the effect of an asymmetric utility function, where the passenger preference (disutility) for the error in broadcast time is assumed to be different depending on whether the bus arrives earlier or later than predicted. We examined two cases of asymmetric utility function. One is the case where the passenger utility is more sensitive to the bus arriving earlier than the broadcast time than to the bus arriving later than the broadcast time. The other is the case where the passenger utility is more sensitive to the bus arriving later than the broadcast time than to the bus arriving earlier than the broadcast time. Both cases, however, reflect the same average sensitivity over the range on nonzero utilities. We observed that the expected utilities when considering asymmetric utility functions were almost the same as those when considering the symmetric utility functions under each BAIS alternative.

We also investigated the effect of utility functions that depend on the duration of the waiting time. With these functions, the effect of the same error in the broadcast time
is now different depending on the actual time a passenger waits. We used a waiting time dependent function representing higher sensitivity to errors occurring with shorter waiting times than to the same errors occurring with longer waiting times. We observed that the expected utility was greater under stronger dependence to waiting time duration. We also observed that most patterns of the expected utility with respect to bus operations parameters do not change much with this dependency. The exception is the situation where the expected utility was examined as a function of the headway mean. As the headway mean increases, the expected utilities of BAIS2 and BAIS3 increase slowly and approach 1 under waiting time dependent utility functions, while they decrease slowly and approach a constant value for the waiting time independent utility function. Nevertheless, the effect of headway mean on the expected utility of BAIS2 and BAIS3 is small when using either the waiting time dependent utility function or the waiting time independent utility function. Hence, the implications regarding the bus operations parameters barely change under the waiting time dependent utility functions.

These results indicates that the changes in the utility functions do not have much effect on the patterns of the expected utility as a function of bus operations parameters. In the above analyses, however, the utility function was assumed to be perfectly known to the broadcaster. That is, under each utility scenario the broadcaster was able to change the optimal broadcast time in order to compensate for changes in the utility function. Hence, if the broadcaster indeed has good knowledge of passenger utility functions the choice of the utility function is not of much concern in evaluating the BAIS alternatives considered. Of course, in reality the broadcaster may not have the luxury of knowing the
utilities functions of passengers. This issue was addressed in Section 4.3 with a specific symmetric utility function, and it is observed that expected utility is not affected much by the broadcaster's knowledge of the utility function.

For most of our results, we assumed that the broadcaster had perfect knowledge of the bus operations characteristics and passenger utility function. However, in reality she cannot have such perfect knowledge. This issue was addressed in Section 4.3 through the sensitivity analysis with limited specifications for bus operations parameters, utility function and broadcaster's knowledge. We found that expected utility exhibits great sensitivity to accuracy in the broadcaster's knowledge of the headway mean in BAIS1 and BAIS2 but little sensitivity to accuracy in the knowledge of headway standard deviation and utility function parameters for limited cases of bus systems and utility functions.

We investigated the expected utility of the three BAIS alternatives using data collected from a real bus system. Specifically, we applied the evaluation method developed to selected routes of The Ohio State University Campus's Area Bus System. We observed that the bus headway distributions were approximately normally distributed with means very close to published headway, but with a slightly positive skewness. We also estimated the correlation function using the field headway data. The empirical correlation functions, as expected, decrease with an increase in upstream distance. However, the rate of decrease changed according to the road and traffic condition along the route. We computed the expected utility using these empirical headway distributions and correlation functions using the simulation tool developed. We observed that the
empirical results were close to the results of the hypothetical results which assumed a
normal headway distribution and an exponential correlation function. Thus, the limited
field study was encouraging in that it indicated that the developed simulation model
provided a good approximation of realistic bus operations for the purposes of evaluating
bus arrival time information systems.

5.3 Limitations and Further Research

We calculated the expected utility for each BAIS based on a truncated normal bus
headway distribution. In the simulation, we generated the bus headway realization from
normal density functions, and discarded any negative headway realizations. Therefore,
the simulated headway distributions were not exactly normal due to this truncation. We
examined the number of the negative headways to investigate the effect of this
truncations. For example, in the case of a 10-minute headway mean and 4-minute
headway standard deviation, we discarded 2 negative headways during the generation of
200 headway samples. Table 5.1 shows the number of negative headways discarded
during the simulations when using various means and standard deviations.
As noticeable from this table, for larger headway means the simulated headways would reflect the mean and standard deviation of the original assumed distribution, since none or very few headway samples are discarded. However, for a small headway mean with a large standard deviation the generated headway means and standard deviations would be different than those assumed in the original distributions because of the truncation effect. Hence the expected utilities computed using the developed simulation tool may be inaccurate for a bus system with a small headway mean and large headway standard deviation. A more realistic approach might be to use the log-normal distribution to model bus headways under such conditions.

Table 5.2 shows the theoretically computed probability of obtaining negative headways in generation from the normal distribution. Multiplying by the 200 headways generated would yield the expected number of negative headways generated in 200 realizations. These numbers are reasonably close to those of Table 5.1. In simulations, we generally used headways greater than 10 minutes and headway standard deviations
less than 4 minutes. The probability of obtaining a negative headway is only 0.006 even in the case of 10-minute headway mean and 4-minute standard deviation.

<table>
<thead>
<tr>
<th>Headway standard deviation (min)</th>
<th>Headway mean (min)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
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<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.006</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.049</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.106</td>
<td>0.006</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.159</td>
<td>0.023</td>
<td>0.001</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2 Probability of getting negative headway in generation of headway

We also assumed that passenger arrivals follow a Poisson process. If passengers are to arrive according to a bus published schedule, this assumption is not likely to be valid. Such behavior would be more likely when the headway mean is large, since the long potential waiting time forces passengers to seek and use a bus schedule. It may be interesting to adapt the methodology to analyze cases where passengers arrive according to a schedule. This could be done by assuming and formulating a passenger arrival distribution based on a schedule. Since this passenger arrival distribution was used as an input, the other parts of the simulation would be identical.

We assumed that the broadcaster would use a heuristic prediction model to forecast bus arrival times in BAIS3. As discussed in Chapter 3, the proper conditional distribution of bus arrival times is given by Bayes' Theorem (see equation 3.26). Implementing this conditional headway distribution in a simulation study and in a manner
that could used in real-time would be a valuable future direction. Alternatively, the headway and the upstream headway can be modeled by a bivariate normal probability density function from which the conditional distributions of interest can be derived for the purpose of both generating upstream headways and to model the broadcaster’s determination of the headway distribution given the real-time data on upstream headway. Otherwise, better prediction heuristics could be devised and tested in future research.

As discussed in Section 5.2, more intensive and systematic sensitivity analyses of this kind would be a useful future direction.

We also assumed that only broadcast time at the time of a passenger arrival affects the utility of the passenger. However, the broadcast time would change with time, and therefore, the passenger may be affected by updated broadcast times as well. The revised broadcast times may be more accurate than the original ones because of the reduced uncertainty in bus arrival times as a bus approaches the stop. It is worthwhile to explore the effect of broadcast time updates on passenger utility and, consequently, to capture this effect in computing expected utilities. This updated broadcast time would have more significant effect in BAIS3 than in BAIS2, since the variance of the waiting time distribution of a passenger in BAIS3 decreases faster than in BAIS2 as the following bus approaches bus stop. In BAIS1, there is no updated broadcast time due to the absence of real-time data.

We also assumed that the bus location data for the broadcaster are accurate and updated continuously. Since bus location systems are prone to positioning error and
provide data periodically, rather than continuously, the bus location data inevitably involve some error. This error will have an effect on the broadcaster's predictions of bus arrival times. It may be pertinent to model such errors and capture their effect on the computation of optimal broadcast times and expected utilities.

We used The Ohio State University's Campus Area Bus system to analyze the expected utility empirically using field data. More empirical analyses for other bus systems would also be useful. Through empirical analysis, we would learn where most real bus systems fall within the spectrum of parameter values used in this research and see if real-time system would be truly benefit many transit systems.

This research focuses on the evaluation of various BAIS using an expected utility approach. How to integrate the expected utilities calculated with the approach developed here in an analytical methodology that also considers the cost of implementing and operating a BAIS is also a subject deserving future research.

In summary, there are many aspects to explore with future studies. Still, we feel that we have developed a framework and simulation programs to analyze the relative value of Bus Arrival Information Systems. We produced results that show that our framework and programs seem reasonable. These results also have helped provide insight on how the value of the various BAIS vary with operating parameters of the bus systems and of the passenger's preference (utility) functions. The quantification of value (utility) provided should also prove valuable in cost-effectiveness studies.
REFERENCE


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APPENDIX

COMPUTER PROGRAMS

program for calculating expected utility

Data file

Arrival rate  bus velocity (v)  coefficient of variation of v
1.0  10.0  0.1

Number of headway
200

Converge criterion
0.1

Number of data of a value
5
0.000001  0.000025  0.0001  0.0004  0.0016

Number of data of mean
3
10. 15.  20.

Number of data of standard deviation
5
4.  3.  2.  1.  0.

Parameters in utility function
1
C11  C12  C21  C22  wmax ------ parameters for utility function
3.  17.  3.  17.  20.
Program file

real w(500000),h(500000),p(500000)
real wdis(500000),expw(500000),hdwy(500000)
real euv(5,10,10,10),ccd(10),mean(10),var(10)
* ,bw2(500000),bw3(500000),cor(500000)

open(unit=10, file='data_sim',status='old')
read(10,*) ramda,v,cv
read(10,*) n_hw
read(10,*) bt_inc
read(10,*) ind_kc
read(10,*) (ccd(i),i=1,ind_kc)
read(10,*) ind_km
read(10,*) (mean(i),i=1,ind_km)
read(10,*) ind_k
read(10,*) (var(i),i=1,ind_k)
read(10,*) norun
read(10,*)

do ito=1,norun
read(10,*) cll,c12,c21,c22,wmax,ap1,ap2
close(10)
write(6,686) cll,c12,c21,c22, Wmax,ap1,ap2
686 format(/,' Cll=',f4.1,' C12=',f4.1*,' C21=',f4.1,' C22=',f4.1,' Wmax=',f4.1*,' ap1=',f4.1,' ap2=',f4.1)

do km=1,ind_km
hwmean=mean(km)
do k=1,ind_k
hwsd=var(k)
do kc=1,ind_kk
cc=ccd(kk)
ctsd=((2*(1-cc))**0.5)*hwsd
hwsd_d=((1-(cc)**2)**0.5)*hwsd
hwsd_s=hwsd
	nosam=0
ir=111
ir1=100
ir2=100+10
ir3=100+20
ir4=100+50
c------waiting time-----------------------------------------------
call waiting_time(w,nosam,hwmean,hwsd,ramda,n_hw,n_pa*
*,h,hdwy,p,ir,ir1,ir2,ir3,ir4,cv,t_headway
*,ave_headway,wdis,expw,ave_w,cc,kc,cor,v)
c--------------------------------------------
if (kc.eq.1)
call vlsl(bt_inc,hwmean,cl1,c21,c12,c22,nosam,w,t_headway,
p,b,eu,ave_w,wmax,apl,ap2,hwsd)

b_vlsl=b
euv(1,km,k,kc)=eu

ind=2
if(kc.eq.1)
call vlsl(bt_inc,cl1,c21,c12,c22,nosam,w,t_headway,
n_hw,h,p,eu_v2,hdwy,ind,hwsd,ctsd
* ,hwmean,ameanl,varil,bw2,wmax,apl,ap2,cor)
euv(2,km,k,kc)=eu_v2

call vlsl(bt_inc,cl1,c21,c12,c22,nosam,w,t_headway,
n_hw,wdis,expw,eu_v3,hdwy,ind,hwsd,ctsd
* ,hwmean,amean2,varil2,bw3,wmax,apl,ap2,cor)
euv(3,km,k,kc)=eu_v3

enddo

write(6,652) mean(km),nosam
652 format(2x,'mean=',f4.1,2x,'nosam=',i5)
endo

output-----------------------------------------------
do im=l,ind_km
write(6,654)(1000*cdd(k),k=l,7)
654 format(/,'mean',' SD ','v3=','v2=','v1')
do iv=l,ind_k
write(6,655)mean(im),var(iv),(euv(3,im,iv,ic),ic=l,7)
* ,euv(2,im,iv,1),euv(1,im,iv,1)
655 format(2x,f4.1,2x,f4.1,2x,9(f6.4,2x))
endo

do k=3,3
write(6,6540)(mean(i),i=l,3),(mean(i),i=l,3),(mean(i),i=l,3)
6540 format(/,'mean',' SD ','v3=','v2=','v1')
do iv=l,ind_k
write(6,6550)var(iv),(euv(3,im,iv,k),im=l,3)
* ,euv(2,im,iv,1),im=l,3),(euv(1,im,iv,1),im=l,3)
6550 format(2x,4x,2x,9(f6.4,2x))
endo
endo
endo
end
c Subroutine for BT and EU for VLI
subroutine vlsl(bt_inc, hwmean, c1l, c21, c12, c22, nosam, w, t_headway,
*  p, b, eu, ave_w, wmax, apl, ap2, hwsd)
c-----------------------------
real w(500000), p(500000)
b=0.
eu=0.
bm=0
bx=hwmean+4*hwsd
5
bc=(bx-bm)/2
do bt=bm,bx,bc
  euti=0
  do i=1,nosam
    if(bt.ge.w(i))c_maxl=cll+(c21-cll)*(w(i)/wmax)
    if(bt.lt.w(i))c_max2=cl2-i-(c22-cl2)  *  (w(i)/wmax)
    if(bt.ge.w(i))  util=(1.-(bt-w(i))/c_raxa)
    if(bt.lt.w(i)) util=(1.-(w(i)-bt)/c_max2)
    if(util.lt.0.)util=0.
    util=util/nosam
    euti=util+euti
  enddo
  eut=euti
  if(eut.gt.eu) b=bt
  if(eut.gt.eu) eu=eut
enddo
bm=b-bc/2
bx=b+bc/2
if(bc.gt.bt_inc) go to 5
return
end

c Subroutine for BT and EU for VLI
subroutine vls(bt_inc, cl1, c21, c12, c22, nosam, w, t_headway,
*  n_hw, x, y, eu_v, hdwy, ind, hwsd, ctsd
*  ,hwmean, anmean, vari, bw, wmax, apl, ap2, cor)
c-----------------------------
real wd(500000), x(500000), w(500000), y(500000), hdwy(500000)
*  ,bw(500000), cor(500000)
eu_v=0.
amean=0.
vari=0.
condhsd=1.
    id=1
    id2=1

do j=1,nosam
  if(ind.eq.3)condhsd=(1-(cor(j))**2.)**0.5
  eu=0.
  b=0.
  if(hwsd.eq.0.)b=w(j)
  if(hwsd.eq.0.)go to 8
if(ind.eq.2) then
  bm=hwmean-y(j)-4.*hwsd
  if(bm.lt.0.) bm=0.
  bx=hwmean-y(j)+4.*hwsd
else if(ind.eq.3) then
  bm=-1*y(j)-4.*condhsd*hwsd
  if(bm.lt.0.) bm=0.
  bx=-1*y(j)+4.*condhsd*hwsd
endif
7
  bc=(bx-bm)/2

c-for broadcaster headway ------
  if(ind.eq.3) then
    do i=1,n_hw
      x(i)=hwsd*gasdev(id)
    enddo
  else
    do i=1,n_hw
      x(i)=hwmean+hwsd*gasdev(id2)
    enddo
  endif
  c-set optimal broadcast time ----
  do bt=bm,bx,bc
    euti=0
    nu=0
    do i=1,n_hw
      wd(i)=condhsd*x(i)-y(j)
      if(wd(i).le.0) go to 2
      if(bt.ge.wd(i)) c_max1=c11+(c21-c11)*(wd(i)/wmax)
      if(bt.lt.wd(i)) c_max2=c12+(c22-c12)*(wd(i)/wmax)
      if(bt.ge.wd(i)) util=1.-((bt-wd(i))/c_max1)
      if(bt.lt.wd(i)) util=1.-((wd(i)-bt)/c_max2)
      if(util.lt.0) util=0.
      euti=util+euti
      nu=nu+1
    enddo
    eut=euti/nu
    if(eut.gt.eu) bt=bt
    if(eut.gt.eu) eu=eut
  enddo
  bm=b-bc/2
  bx=b+bc/2
  if(bc.gt.bt_inc) go to 7
  continue

8
  if(b.ge.w(j))c_max1=c11+(c21-c11)*w(j)/wmax
  if(b.lt.w(j))c_max2=c12+(c22-c12)*w(j)/wmax
  if(b.ge.w(j)) uty=1.-((b-w(j))/c_max1)
  if(b.lt.w(j)) uty=1.-((w(j)-b)/c_max2)
  if(uty.lt.0) uty=0.
  uty=uty/nosam
  eu_v=eu_v+uty
  bw(j)=(b-w(j))
  amean=amean+bw(j)
  enddo
195
amean=amean/nosam
do j=1,nosam
   vari=vari+((b-w(j))-amean)**2
endo
   vari=vari/nosam
return
end

c-----------------Subroutine of the waiting time distribution-----------------
* T _ H E A D W A Y , A V E _ H E A D W A Y , W D I S , E X P W ,
* A V E _ W , C C , K C , C O R , V )

INTEGER NOPH(200)
REAL PAS S, W (500000), H (500000), P (500000)
REAL W D I S (500000), EXPW(500000), HD W Y (500000)
* , R C C C ( 5 0 0 0 0 0 ) , R C C C 2 ( 5 0 0 0 0 0 ) , C O R ( 5 0 0 0 0 0 ) , R R L ( 5 0 0 0 0 0 )
T _ H E A D W A Y = 0 .
T _ W = 0
XP = 0.
nosam_p=0
hzero=0

do i hw=1,n_hw
   rcc=gasdev(idum)
   rl=hwsd*rcc
200 FORMAT(5,2x,6(f7.4,2x))
   headway=hwmwmean=rl
   wdis(ihw)=rl
   if(headway.le.0.) hzero=hzero+1
   if(headway.le.0.) go to 1
   h(ihw)=headway
endo

do ipa=1,100000
   c - passenger generation ------
   if(ipa.gt.1) then
      pass=expdev(idum4)/ramda
   else
      if(ihw.eq.1) pass=expdev(idum4)/ramda
   endif
   xp=xp+pass
   if(xp.le.headway) then
      nosam=nosam+1
      rrl(nosam)=rl
      p(nosam)=xp
      w(nosam)=headway-xp
      hdwy(nosam)=headway
      t_headway=t_headway+headway
      t_w=t_w+w(nosam)
endo
   rccc(nosam)=rcc

196
rccc2(nosam)=rcc2
else
    pass=xp-headway
dx=nosam-nosam_p
idx=idx+1
xp=0.
nosam_p=nosam
go to 31
endif
enddo
31 continue
enddo

c -upstream headway ----- 
do i=nosam,l,-l
14 rw=cv*gasdev(idum3)
rcc2=gasdev(idum2)
wind=0
if(hdwy(i).eq.hdwy(i+1)) wind=w(i+1)
if(i.eq.nosam) wind=0
if((l+rw).lt.0.) go to 14
corr=exp(-l*cc*v*60*(w(i)-wind)*(1+rw))
condhsd=1-(corr)**2)**0.5
hpp=hwmean+corr*rrl(i)
if(hdwy(i).eq.hdwy(i+1)) hpp=hwmean+corr*(hp-hwmean)
  r3=condhsd*rcc2
  hp=hpp+r3
cind=1.
if(hdwy(i).eq.hdwy(i+1)) cind=cor(i+1)
if(i.eq.nosam) cind=1.
cor(i)=corr*cind
exphw=hwmean+cor(i)*(hp-hwmean)
expw(i)=-1*(exphw-p(i))
endo
ave_w=t_w/nosam
2 return
end

c-----------------------------------------------
c Function for the exponential random number
function expdev(idum)
c-----------------------------------------------
integer idum
real expdev
1 x=ran(idum)
if(x.eq.0) goto 1
expdev=-log(x)
return
end
c Function for the normal random number

function gasdev(idum)
save iset,gset
data iset/0/
if(idum.lt.0) iset=0
if(idum.eq.0) then
   v1=2.*ran(idum)-1.
v2=2.*ran(idum)-1.
   rsq=v1**2+v2**2
   if(rsq.ge.1..or.rsq.eq.0.) goto 1
   fac=sqrt(-2.*log(rsq)/rsq)
gset=v1*fac
gasdev=v2*fac
   iset=1
else
   gasdev=gset
   iset=0
endif
return
end
## program for sensitivity analysis

### Data file

<table>
<thead>
<tr>
<th>Arrival rate</th>
<th>bus velocity($v$)</th>
<th>coefficient of variation of $v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Number of headway: 200

Converge criterion: 0.1

True $\alpha$ value: 1

True mean: 1

True standard deviation: 1

Braodcater's belief: $\alpha$ value, mean, standard deviation

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>10.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

True parameters in utility function:

<table>
<thead>
<tr>
<th>C11</th>
<th>C12</th>
<th>C21</th>
<th>C22</th>
<th>$v_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>20.0</td>
</tr>
</tbody>
</table>

$s_{C11}$, $s_{C12}$, $s_{C21}$, $s_{C22}$: Broadcaster's belief

<table>
<thead>
<tr>
<th>$s_{C11}$</th>
<th>$s_{C12}$</th>
<th>$s_{C21}$</th>
<th>$s_{C22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
</tbody>
</table>
Program file

real w(500000), h(500000), p(500000)
real wdis(500000), expw(500000), hdwy(500000)
real euv(5,10,10,10), ccd(10), mean(10), var(10)
* bw2(500000), bw3(500000), cor(500000), cor_s(500000)
real s_w(500000)
integer s_nosam

open(unit=10, file='data_sen-doc', status='old')
read(10,*)
read(10,*) ramda, v, cv
read(10,*)
read(10,*) n_hw
read(10,*)
read(10,*) bt_inc
read(10,*)
read(10,*) ind_kw
read(10,*) (ccd(i), i=1, ind_kw)
read(10,*)
read(10,*) ind_km
read(10,*) (mean(i), i=1, ind_km)
read(10,*)
read(10,*) ind_k
read(10,*) (var(i), i=1, ind_k)
read(10,*)
read(10,*) s_ccd, s_mean, s_var
read(10,*)
read(10,*) norun
read(10,*)

do ito=1, norun
read(10,*) c11, c12, c21, c22, wmax
read(10,*)
read(10,*) s_c11, s_c12, s_c21, s_c22
close(10)
write(6,686) c11, c12, c21, c22, wmax, apl, ap2
write(6,686) s_c11, s_c12, s_c21, s_c22


do km=1, ind_km
hwmean=mean(km)
do k=1, ind_k
hwsd=var(k)
do kc=1, ind_kw
cc=ccd(kc)
ctsd=((2*(1-cc))**0.5)*hwsd
hwsd_d=((1-(cc)**2)**0.5)*hwsd
hwsd_s=hwsd
hwmean_s=s_mean
hwsd_s=s_var
cc_s=s_ccd

200
nosam=0
ir=111
ir1=100
ir2=100+10
ir3=100+20
ir4=100+50

call waiting_time(w,nosam,hwmean,hwsd,ramda,n_hw,n_pa
* ,h,hw,y,p,ir,ir1,ir2,ir3,ir4,cv,t_headway
* ,ave_headway,wdis,expw,ave_w,cc,kc
* ,s_w,s_nosam,cor,v,cc_s,hwmean_s,cor_s,hwsd_s)
c--------BAIS1----------------------------------------------------------
if (kc.eq.1)
call vlsl(bt_inc,hwmean,cll,c21,c22,nosam,w,t_headway,
* ,p,b,eu,ave_w,wmax,ap1,ap2,hwsd
* ,s_w,s_nosam,s_cll,s_c21,s_c22)
b_vlsl=b
eu(1,km,k,kc)=eu
689 format(20x,'EU(VLS1)='f6.4)
c--------BAIS2----------------------------------------------------------
ind=2
if (kc.eq.1)
call vis(bt_inc,cll,c21,cl2,c22,nosam,w,t_headway,
* ,n_hw,h,p,eu_v2,hw,y,ind,hwsd,ctsd
* ,hwmean,amean1,vari1,bw2,wmax,ap1,ap2,cor
* ,cc_s,hwmean_s,hwsd_s,s_c11,s_c21,s_c12,s_c22,cc,cor_s)
eu(2,km,k,kc)=eu_v2
690 format(20x,'EU(VLS2)='f6.4)
c--------BAIS3----------------------------------------------------------
ind=3
call vis(bt_inc,cll,c21,cl2,c22,nosam,w,t_headway,
* ,n_hw,wsd,expw,eu_v3,hw,y,ind,hwsd,ctsd
* ,hwmean,amean2,vari2,bw3,wmax,ap1,ap2,cor
* ,cc_s,hwmean_s,hwsd_s,s_c11,s_c21,s_c12,s_c22,cc,cor_s)
eu(3,km,k,kc)=eu_v3
enddo
c--------output----------------------------------------------------------
doo=1,ind_km
write(6,654)(1000*ccd(k),k=1,7)
654 format(/,' mean',' SD ',2x,7('v3=','f4.1,2x','nosam='i5)
do iv=1,ind_k
write(6,655)(mean(im),var(iv),(eu(3,im,iv,ic),ic=1,7)
* ,eu(2,im,iv,1),eu(1,im,iv,1)
c Subroutine for BT and ED for VLll
subroutine vls1(bt_inc,hwmean,c11,c21,c12,c22,
    * nosam,w,t_headway,
    * p,b,eu,ave_w,wmax,apl,ap2,hwsd,
    * s_w,s_nosam,s_cl1,s_cl2,s_c21,s_c22)

    real w(500000),p(500000)
    real s_w(500000)
    integer s_nosam
    b=0.
    eu=0.
    bm=0
    bx=hwmean+4*hwsd
    bc=(bx-bm)/2
    do bt=bm,bx,bc
        euti=0
        c---modify sensitivity
        do i=1,nosam
            if(bt.ge.s_w(i))c_max1=s_cl1+(s_c21-s_cl1)*(s_w(i)/wmax)
            if(bt.lt.s_w(i))c_max2=s_cl2+(s_c22-s_cl2)*(s_w(i)/wmax)
            if(bt.ge.s_w(i)) util=(1.-bt)/c_max1
            if(bt.lt.s_w(i)) util=(1.-w(i)-bt)/c_max2
            if(util.lt.0.) util=0.
            util=util/nosam
            euti=util+euti
        enddo
        eut=euti
        if(eut.gt.eu) b=bt
        if(eut.gt.eu) eu=eut
    enddo
    bm=b-bc/2
    bx=b+bc/2
    if(bc.gt.bt_inc) go to 5
    euti=0.
    do i=1,nosam
        if(b.ge.w(i))c_max1=c11+(c21-c11)*(w(i)/wmax)
        if(b.lt.w(i))c_max2=c12+(c22-c12)*(w(i)/wmax)
        if(b.ge.w(i)) util=(1.-b)/c_max1
        if(b.lt.w(i)) util=(1.-w(i)-b)/c_max2
        if(util.lt.0.) util=0.
        util=util/nosam
        euti=util+euti
    enddo
    eu=euti
    return
end
c Subroutine for BT and EU for VLI
subroutine vis(bt_inc,c11,c12,c22,nosam,w,t_headway,
* n_hw,x,y,eu_v,hdwy,ind,hwsd,ctsd
* ,cc_s,hwmean_s,hwsd_s,s_cl1,s_cl2,s_c21,s_c22,cc,cor_s)

real w(500000),x(500000),w(500000),y(500000),hdwy(500000)
* eu_v=0.
* amean=0.
* vari=0.
condhsd=1.

id=1
id2=1

do j=1,nosam
if(ind.eq.3) condhsd=(1.-cor_s(j)**2.)**0.5

eu=0.
b=0.
if(hwsd.eq.0.)b=w(j)
if(hwsd.eq.0.)go to 8

if(ind.eq.2) then
  bm=hwmean-y(j)-4.*hwsd_s
  if(bm.lt.0.) bm=0.
  bx=hwmean-y(j)+4.*hwsd_s
else if(ind.eq.3) then
  bm=-1*y(j)-4.*condhsd*hwsd_s
  if(bm.lt.0.) bm=0.
  bx=-1*y(j)+4.*condhsd*hwsd_s
else
endif

bc=(bx-bm)/2
if(ind.eq.3)then
  do i=1,n_hw
    x(i)=hwsd_s*gasdev(id)
  enddo
else
  do i=1,n_hw
    x(i)=hwmean_s+hwsd_s*gasdev(id)
  enddo
endif

do bt=bm,bx,bc
  euti=0
  nu=0
  do i=1,n_hw
    wd(i)=condhsd*x(i)-y(j)
    if(wd(i).le.0) go to 2
    if(bt.ge.wd(i)) c_max1=s_cl1+(s_c21-s_cl1)*(wd(i)/wmax)
    if(bt.lt.wd(i)) c_max2=s_cl2+(s_c22-s_cl2)*(wd(i)/wmax)
    if(bt.ge.wd(i)) util=1.-((bt-wd(i))/c_max1)
    if(bt.lt.wd(i)) util=1.-((wd(i)-bt)/c_max2)
    if(util.lt.0.) util=0.
  euti=util+euti
  go to 7
  enddo

203
nu=nu+1
enddo

eut=euti/nu
if(eut.gt.eu) b=bt
if(eut.gt.eu) eu=eut
enddo

bm=b-bc/2
bx=b+bc/2
if(bc.gt.bt_inc) go to 7

continue

if(b.ge.w(j)) c_max1=c11+(c21-c11)*(w(j)/wmax)
if(b.lt.w(j)) c_max2=c12+(c22-c12)*(w(j)/wmax)
if(b.ge.w(j)) uty=1-((b-w(j))/c_max1)
if(b.lt.w(j)) uty=1-((w(j)-b)/c_max2)

if(uty.lt.0.) uty=0.
uty=uty/nosam
eu_v=eu_v+uty
bw(j)=(b-w(j))
amean=amean+bw(j)
enddo

amean=amean/nosam
do j=1,nosam
vari=vari+((b-w(j))-amean)**2
enddo

return
end

c---------Subroutine of the waiting time distribution------------
subroutine waiting_time(w,nosam,hwmean,hwsd,ramda,n_hw,*
  n_pa,h,hdwy,p,idum, iduml, idum2, idum3, idum4,cv,*
  t_headway,ave_headway,wdis,expw,ave_w,cc,kc,*
  s_w,s_nosam,cor,v,cc_s,hwmean_s,cor_s,hwsd_s)
  c-----------------------------------------------------------------
  integer no ph(200),s_nosam
  real pass,w(500000),h(500000),p(500000)
  real wdis(500000),expw(500000),hdwy(500000),s_w(500000)
  * ,rcc(500000),rcc2(500000),cor(500000),rrl(500000)
  * ,cor_s(500000)
  t_headway=0.
t_w=0
xp=0.
s_xp=0.
nosam_p=0
s_nosam=0
idum44=idum4

do ihw=1,n_hw
  rcc=gasdev(idum)
  rl=hwsd*rcc
 200 format(15,2x,6(f7.4,2x))
  headway=hwmean+rl
  wdis(ihw)=rl
  h(ihw)=headway
c --- broadcaster's belief headway distribution
   s_r1=rcc*hwsd_s
   s_headway=hwmean_s+s_r1
   do ipa=1,100000
     c ---- passenger sampling ----------------
     if(ipa.gt.l) then
       pass=expdev(idum4)/ramda
     else
       if(ihw.eq.1) pass=expdev(idum4)/ramda
     endif
     xp=xp+pass
     if(xp.le.headway) then
       nosam=nosam+1
       rrl(nosam)=rl
       p(nosam)=xp
       w(nosam)=headway-xp
       hdwy(nosam)=headway
       rccc(nosam)=rcc
       rccc2(nosam)=rcc2
     else
       pass=xp-headway
       idx=nosam-nosam_p
       idx=idx+l
       xp=0.
       nosam_p=nosam
       go to 31
     endif
     enddo
     31 continue
     do ipa=1,100000
       c ---- passenger sampling ----------------
       if(ipa.gt.l) then
         s_pass=expdev(idum44)/ramda
       else
         if(ihw.eq.1) s_pass=expdev(idum44)/ramda
       endif
       s_xp=s_xp+s_pass
       if(s_xp.le.s_headway) then
         s_nosam=s_nosam+1
         s_w(s_nosam)=s_headway-s_xp
       else
         s_pass=s_xp-s_headway
         idx_s=s_nosam-nosam_p_s
         idx_s=idx_s+l
         s_xp=0.
         nosam_p_s=s_nosam
         go to 32
       endif
     enddo
     32 continue
   enddo
c ---- sampling upstream headway ---------------------
do i=nosam,l,-1
   rw=cv*gasdev(idum3)
   rcc2=gasdev(idum2)
   wind=0
   if(hdwy(i).eq.hdwy(i+1)) wind=w(i+1)
   if(i.eq.nosam) wind=0
   if((1+rw).lt.0.) go to 14
   corr=exp(-1*cc*v*60*(w(i)-wind)*(1+rw))
   condh5=((1-(corr)**2)**0.5)
   hpp=hwmean+corr*rrl(i)
   if(hdwy(i).eq.hdwy(i+1)) hpp=hwmean-corr*(hp-hwmean)
   r3=condhsd*rcc2
   hp=hpp+r3
   cind=1.
   if(hdwy(i).eq.hdwy(i+1)) cind=cor(i+1)
   if(i.eq.nosam) cind=1.
   cor(i)=corr*cind
   cor_s(i)=exp(-l*v*60*w(i)*cc_s)
   exphw=hp
   expw(i)=-l*(exphw-p(i))
enddo
2 return
end

c --------------------------------------------------
c Function for the exponential random number
function expdev(idum)
integer idum
real expdev
   x=ran(idum)
1    if(x.eq.0) goto 1
1    expdev=-log(x)
1    return
end

c --------------------------------------------------
c Function for the normal random number
function gasdev(idum)
save iset,gset
data iset/0/
   if(idum.lt.0) iset=0
   if(idum.eq.0) then
      v1=2.*ran(idum)-1.
      v2=2.*ran(idum)-1.
      rsq=v1**2+v2**2
      if(rsq.ge.100..or.rsq.eq.0.) goto 1
      fac=sqrt(-2.*log(rsq)/rsq)
      gset=v1*fac
      gasdev=v2*fac
   endif
   iset=1
   else
      gasdev=gset
   iset=0
endif
return
end