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A STUDY OF THE INFLUENCE OF THE BAUSCHINGER EFFECT ON SPRINGBACK IN TWO-DIMENSIONAL SHEET METAL FORMING

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
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*****

The Ohio State University
1999

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Sheet metal forming is widely used in many industries, including the automobile industry, to produce net shape parts in large quantities. The processes involve product design, draw development (die-surface design), forming simulation (FEA), pattern cutting, soft-tool tryout, die design (solid die), and hard-tool tryout. Currently, these processes are very time-consuming and highly dependent on the experience of stamping engineer. For instance, development of dies for the new car panels generally takes approximately thirty-six months for completion. Of the processes just listed, draw-development and soft-tool tryout consume most of the time. To reduce both cost and lead time, the times required by these two processes must be decreased. FEA forming simulation is already widely utilized to check the formability during the draw-development process. In addition to formability concerns, springback is another significant problem that must be solved in order to form the exact shape of the product.

To predict the final shape of the deformed part after springback, the calculation of the internal stress distribution in the sheet metal must be accurate. Most researchers ignore the contribution of the Bauschinger effect when they determine the internal stress
distribution for the springback prediction. However, the Bauschinger effect exists when the element of sheet metal undergoes both loading and reverse loading processes, i.e. cyclical loading. The influence of the Bauschinger effect becomes more important when the deformation history of the sheet metal is more complicated. Most sheet metal elements undergo a complicated cyclical deformation history during the forming process. For an accurate prediction of springback, the Bauschinger effect must be considered to obtain an accurate internal stress distribution within the sheet metal after deformation.

Based on the foundation of the isotropic kinematic hardening model, the Mroz multiple surface model, plane strain assumptions, and experimental observation, a new incremental method and hardening model is proposed in this study. In comparison with methods based purely on isotropic hardening, kinematic hardening, and Mroz multiple surfaces models, this new methodology compares well with the experimental results for aluminum sheet metals undergoing multiple bending processes. As is well know, aluminum is one of the most difficult high strength sheet metals to simulate. This new hardening model is not only a generic method for springback prediction but also a hardening model for sheet metal forming process simulation.

An efficient and low cost multiple bending experiment has been designed and performed to investigate the influence of the Bauschinger effect on springback in sheet metal forming. Three different steel sheet metals (high strength, back hard, and AKDQ) and two types of aluminum (AA6111-T4 and AA6022-T4) were used as experimental
materials in this study. Based on the experimental results from this research, it can be seen that the influence of the Bauschinger effect on springback is more significant for aluminum than for steels. Therefore, both the deformation history and the Bauschinger effect must be considered to predict springback of aluminum stamping parts. Through this multiple bending experiment, the material parameters (called $CM$ values here) after reverse yield can be obtained. Then, these parameters can be applied to the new hardening model to get more accurate simulation results than can be obtained using the isotropic hardening model, kinematic hardening model, or Mroz multiple yield surfaces regardless of the tooling geometry and the clearance between die and punch.
DEDICATION

To My Wife, Jin-Meei, and Daughters, Isabella and Moriah
ACKNOWLEDGMENTS

I would like to thank my advisor, Professor Gary L. Kinzel, for his intellectual and moral supports, encouragement, valuable time, technical guidance, and sincere interest. Because of these, this research can be finished.

I also wish to thank Professor June K. Lee and Professor Rajiv Shivpuri as the dissertation committee and providing constructive comments and valuable suggestions.

I also would like to thank Professor Taylan Altan and Engineering Research Center for Net Shape Manufacturing for providing the experimental facilities and materials to make this research work successful. I also wish to thank Dr. Beender Yang for providing CMM facility to measure all experimental data in this research.

I also wish to thank Professor Robert Wagoner and CAMMAC since partial support of this research was from them.

I would like to thank my wife, Jin-Meei, for her continued moral support and encouragement, and for taking care of the whole family throughout my research work.
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<td>$E$</td>
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<tr>
<td>$E'$</td>
<td>Young’s modulus for plan strain</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Plastic modulus</td>
</tr>
<tr>
<td>$E_t$</td>
<td>The tangent modulus</td>
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<tr>
<td>$d\sigma_i$</td>
<td>Principal stress components increment ($i=1,2,3$)</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Proportional factor</td>
</tr>
</tbody>
</table>
\( \dot{\varepsilon}^p \)  An artificial total strain variable

\( \tilde{\varepsilon}^p \)  The total plastic strain of the reverse loading process

\( CM \)  A variable for the reverse loading process

\( \bar{\sigma}^{N+1} \)  The effective stress at step \( N+1 \)

\( \theta \)  The bending angle

\( \varepsilon_{\text{membrane}} \)  Membrane strain

\( \varepsilon_{\text{bending}} \)  Bending strain

\( r_{cm} \)  The radius of the middle layer

\( W \)  The width of the sheet metal

\( BM \)  The loading bending moment

\( f \)  Yield function

\( K \)  Strength coefficient in hardening laws

\( M \)  Shape factor in Hill's new yield function

\( n \)  Strain hardening exponent

\( \nu \)  Poisson's ratio

\( t \)  Sheet metal thickness

\( A_i \)  Stress area

\( r_{AA}^{'b} \)  The radius of layer AA' before springback

\( r_{AA}^{'a} \)  The radius of layer AA' after springback

\( d\varepsilon_{ii}^{e} \)  Elastic strain increment on principal direction \( i \)

\( d\varepsilon_{ii}^{p} \)  Plastic strain increment on principal direction \( i \)
$d\varepsilon_{ui}$  Total strain increment on principal direction $i$

$\varepsilon_{ui}^e$  Elastic strain on principal direction $i$

$\varepsilon_{ui}^p$  Plastic strain on principal direction $i$

$\varepsilon_{ui}$  Total strain on principal direction $i$

$n_{ij}$  The unit normal on direction $ij$

$l$  The length of sheet metal element
CHAPTER I

INTRODUCTION

1.1 General Remarks

Sheet metal forming is widely used in many industries, including the automobile industry, to produce net shape parts in large quantities. The process involves product design, draw development (die-surface design), forming simulation (FEA), pattern cutting, soft-tool tryout, die design (solid die), and hard-tool tryout. Currently, these processes are very time-consuming and highly dependent on the experience of the stamping engineer. For instance, development of dies for new car panels generally takes approximately thirty-six months for completion. Of the processes just listed, draw-development and soft-tool tryout consume most of the time. In order to reduce both cost and lead time, the times required by these two processes have to be decreased. FEA forming simulation is already widely and efficiently utilized to check the formability during the draw-development process. In addition to formability concerns, springback is another significant problem that must be solved in order to form the exact shape of the product. Problems dealing with springback are the major subject of this thesis.
Springback is a common phenomenon in sheet metal forming and can be affected by various parameters such as material properties, tooling geometry, friction between the sheet metal and the tooling, blank holder force, etc. As soon as the deformed part is removed from the die cavity, springback occurs, especially where bending, bending-unbending, and bending-unbending-reverse-bending are performed. Since springback causes the shape to change, this presents a major problem during the assembly process. In addition, this phenomenon complicates the die-design process.

Springback has to be compensated for by overcrowning, undercrowning, overbending, and underbending. In order to compensate for springback, quantitative prediction becomes very important during the draw development process. However, determining the compensation amount still depends largely on the designer’s experience; it is therefore a trial-and-error approach that can require significant amounts of tryout time, and also increases cost. This is the reason so many researchers have been trying to use either FEA or other analytic approaches to predict springback on the basis of the die surface geometry, boundary conditions, and the material properties of the sheet metal.

1.2 Motivation for the Current Work

To reduce the weight of cars, the automobile industry is moving towards high strength materials that have much lower weight-to-strength ratios than the traditional materials such as DQSK and AKDQ steels. Among these high strength materials, aluminum is a popular choice over steel due to its relatively light weight. However, predicting
springback for aluminum is more difficult than for other materials, and thus requires a longer process for die surface design (draw development process) than for steel.

It is normally assumed that the springback is caused by elastic unloading only. Therefore, given the unloading bending moment, the final shape of deformed part after springback can be computed directly. Furthermore, the accuracy of this unloading bending moment depends on the internal stress distribution within the sheet metal element. In other words, it is not possible to obtain an accurate springback prediction with a rough internal stress distribution alone.

Most of the sheet metal elements undergo complicated deformation histories during the forming process. This means that the Bauschinger effect exists within these elements since they may have one or more reverse yields. Therefore, the Bauschinger effect has to be considered for obtaining an accurate internal stress distribution within the sheet metal. Inclusion of the Bauschinger effect in modeling the internal stress calculation not only increases the accuracy of the springback prediction, especially for aluminum, but also has important implications on material savings.

A. Obtaining Great Accuracy in Springback Prediction:

The purpose of considering the Bauschinger effect is to obtain an accurate calculation of internal stress distribution. As is well known, the accuracy of internal stress directly affects the springback prediction. Furthermore, because Young's modulus of aluminum is only about one-third of that of steel, the result of an error in calculating internal stress
will greatly magnify the springback prediction error in aluminum as compared to steel.
Therefore, the Bauschinger effect must be considered, especially for aluminum, in order
to gain an accurate result on the springback prediction.

B. Material Savings:
If the designer can move the draw bead as close to the punch opening as possible and
reduce the size of the addenda area, material can be saved by reducing the sheet blank
size. Simultaneously, he also has to make sure the product can be formed. Once the
draw bead is close to the punch opening and the addenda area is small, some elements
that were outside of the draw bead will flow into the trim line and will be formed as a
"class 1", or a "class 2" surface, i.e. the flange. These elements that flow through the
draw bead undergo a complicated deformation. For instance, an element that flows
through a square bead has a deformation history composed of a sequence of stretching,
bending, reverse bending, bending, unbending, and stretching. Therefore, the
Bauschinger effect must be considered on this element in order to obtain an accurate
internal stress distribution. If the Bauschinger effect can be modeled accurately, the
designer will be able to design the die surface such that most of desired features can be
formed on the home position of the product on the draw die. Then flanging processes can
then be saved, and even the physical die number can be reduced as well. Accurate die
surface design that takes into consideration the Bauschinger effect will ultimately bring
about great saving by eliminating inaccuracies in initial die surface designs.
If the influence of the Bauschinger effect on springback can be accurately predicted, then material can be saved. This material saving would impact the cost of automobile production. For instance, carmakers produce millions of panels every year. The price of the aluminum is about three dollars per pound currently. If the automobile companies can reduce the blank size of each panel, then their cost would be reduced tremendously. Based on the above discussions, it can be concluded that the Bauschinger effect must be considered in order to predict springback accurately, especially for aluminum. However, most researchers ignore the influence of the Bauschinger effect when they determine the internal stress distribution for springback prediction. The new methodology developed in this dissertation will help to make these goals achievable for the sheet metal forming industry.

1.3 Research Objective and Scope of the Work

The main objective of this work is to develop a new hardening model that has the ability to model the Bauschinger effect well while the sheet metal has reverse yield. This model also can be utilized as a hardening model for large deformation. Furthermore, for isotropic hardening, kinematic hardening, the Mroz multiple yield surfaces model, and the new hardening model proposed here, a number of incremental equations have been derived to calculate the internal stress of the sheet metal element. These equations can be used for determining the internal stress distribution of the sheet metal, springback, and residual stress predictions after deformation under the plane strain situation.
The work comprises the following tasks:

1. Development of a new hardening model that can handle the Bauschinger effect well, especially for aluminum.
2. Development of the incremental equations for isotropic hardening model.
3. Development of the incremental equations for kinematic hardening model.
5. Development of the incremental equations for the new hardening model proposed in this dissertation.
6. Investigation of the influence of the Bauschinger effect on springback prediction through a multiple bending experiment.
7. Obtaining material parameters after reverse yield based on the multiple bending experiment (see item 5) and applying these parameters to the new hardening model (see item 4).
10. Integration in an analytical method of a procedure to predict the final geometry and residual stress after springback.

1.4 Dissertation Organization

This dissertation consists of seven chapters and three appendices. Chapter 1 provides a framework for dealing with the problem investigated in this study and outlines its objectives. Chapter 2 is composed of three subsections that review the literature on the
state of the art in hardening models, yield criteria, constitutive equations, the Bauschinger effect, and springback prediction. A new hardening model is proposed in Chapter 3. A number of incremental equations are then derived for the isotropic hardening model, kinematic hardening models, the Mroz multiple yield surfaces model, and the new hardening model proposed in Chapter 3. These equations can be used to determine the principal internal stresses of each fiber layer while it undergoes complicated deformations. Chapter 4 proposes a model for springback prediction by using the formulas derived in Chapter 3; this model was integrated in an analytical solution for springback prediction. A method for residual stress estimation was also derived in this chapter. Chapter 5 proposes and documents the results of an efficient and low cost multiple bending experiment. Through this experiment, the influence of the Bauschinger effect can be observed and the material parameters of the specimen after reverse yield can be determined and applied to the new hardening model proposed in Chapter 3. More experiments and simulation about aluminum sheet metal are completed in Chapter 6. Chapter 7 summarizes the research contributions of this dissertation to the state of the art and suggests future research.
Chapter 2 consists of three sections that review some of the literature that reflect the art of springback in the simulation of the sheet metal forming process. As is well known, constitutive laws, used to model material behavior, have the main influence on the sheet metal forming simulation results, so some hardening models, yield criterion, and constitutive equations are reviewed in Section 2.1. Section 2.2 reviews the physical phenomenon, experiments, and macroscopic models of the Bauschinger effect. Finally, Section 2.3 gives a review about the springback prediction methods including both analytical and FEM approaches.

2.1 Hardening Models and Constitutive Equations

2.1.1. Hardening Model

Isotropic hardening model, for example, the Von Mises yield criterion, is the most widely utilized model in sheet metal forming applications. The assumption of the isotropic hardening is that the yield surface expands uniformly in the stress space, and there is no translation once the material yields. The yield function for isotropic hardening model can
be written as Eq. (2.1.1) while the material is pressure-independent and has no Bauschinger effect.

\[ f(J^2) = k(\alpha) \] (2.1.1)

where the parameter \( \alpha \) can be defined by either the total equivalent plastic strain or total plastic work as shown on the Eq. (2.1.2).

\[ \alpha = \varepsilon^p = \int d\varepsilon^p \] or \[ \alpha = W^p = \int \sigma \, d\varepsilon^p \] (2.1.2)

However, the isotropic hardening model has no capability to model the behavior of the Bauschinger effect.

The kinematic hardening model has been proposed for modeling the Bauschinger effect and this and derivative models are reviewed as follows. Prager (1956) proposed a hypothesis of kinematic hardening. In his model, the initial yield surface translates in the stress space due to plastic deformation, but the size does not change at all. This kinematic hardening rule can be expressed as

\[ f(\sigma_y - \alpha_y) = k \] (2.1.3)

where \( \alpha_y \) called the back stress is the new yield surface center in the stress space, and \( k \) is a simple function of the initial yield surface.
In Prager's model, the back stress increment could be determined by

\[ d\alpha_\eta = Cde_\eta \]  

(2.1.4)

where \( C \) is a material constant.

Ziegler (1959) modified Prager's model and proposed another kinematic hardening rule where the yield surface translates as a rigid body motion in the direction of plastic strain increment. The back stress increment could be expressed as Eq. (2.1.5).

\[ d\alpha_\eta = d\mu(\sigma_\eta - \alpha_\eta) \]  

(2.1.5)

where \( d\mu \) is dependent on the history of plastic strain.

The above kinematic hardening models represent the stress-strain relation as a bilinear function, which in many cases is inadequate. As a result, Eisenberg and Phillips (1968) suggested a “nonlinear kinematic hardening concept” in an attempt to describe better the relation between stress and strain, a nonlinear function. Hodge (1957) proposed a combination of both kinematic and isotropic hardening rules which can be written as

\[ f(\sigma_\eta - \alpha_\eta) = k(k) \]  

(2.1.6)
He assumed the plastic strain increment can be linearly decomposed into two parts, one caused by kinematic hardening and the other by isotropic hardening. Thus, the plastic strain increment can be written as Eq. (2.1.7).

\[
d e_y^p = d e_y^k + d e_y^s = M d e_y^p + (1 - M) d e_y^p
\]  

(2.1.7)

where M is a value between one and zero.

When M is zero, above equation becomes pure kinematic hardening. On the other hand, it is pure isotropic hardening while M equals one. In addition to Ziegler and Hodge, Kadashevitch et al (1958) and Baltov et al (1965) also have done some modifications on Prager's kinematic hardening rule as well.

Moroz (1967, 1969) proposed the multiple yield surfaces model by introducing the concept of the "field of working-hardening modulii". This model has capability and ability to model the complex cyclical loading process; that is, the influence of the Bauschinger effect would be able to be modeled. The plastic modulus of this model is a piecewise linear function within these yield surfaces. Because this concept has been also utilized in this work, a more detail description and research results have been presented in the Chapter 3.

In addition, the two-surface model was proposed by Dafalias and Popov (1975) to model the behavior of the material undergone a complicated cyclic loading process. This model has only two yield surfaces, and the plastic modulus between these two yield surfaces is
in a continuous variation instead of a piece-wise function in the Mroz multiple surfaces model.

2.1.2 Yield Criteria

The most widely used yield criteria are listed and described as follows.

2.1.2.1. Tresca Criterion

Based on Coulomb's results on soil mechanics and his own experiment results on the metal extrusion, Tresca (1864) proposed a yield criterion, and this criterion can be expressed as

\[
\text{Max} \left[ \frac{1}{2} |\sigma_1 - \sigma_2|, \frac{1}{2} |\sigma_2 - \sigma_3|, \frac{1}{2} |\sigma_3 - \sigma_1| \right] = k
\] (2.1.8)

where \(k\) value can be determined by the unaxial test, and its value is equal to a half of the yield stress.

2.1.2.2. Von Mises Yield Criterion

Von Mises (1913) yield criterion, which can also be called a J2 criterion, can be written as Eqs. (2.9) or (2.10).

\[
\Phi = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2. \] (2.1.9)

\[
S_y^* = \frac{2}{3} \sigma_y^2 \] (2.1.10)

where \(S_y^*\) and \(\sigma_y\) are the deviatoric stress tensor and yield stress, respectively.
2.1.2.3. Hosford's Generalized Isotropic Yield Criterion

Hershey (1954) and Hosford (1972) proposed the generalized isotropic yield criterion shown as Eq. (2.1.11).

\[
\left[ \frac{(\sigma_1 - \sigma_2)^M + (\sigma_2 - \sigma_3)^M + (\sigma_1 - \sigma_3)^M}{2} \right]^{1/M} = Y
\]

(2.1.11)

where \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \) and \( 1 \leq M \leq \infty \) is proposed.

When \( M=1 \), the above equation becomes Tresca criterion, and becomes Von Mises criterion if \( M=2 \). Because it repeats its shape at lower values when \( M>2.767 \), the above equation becomes Tresca and Von Mises criteria when \( M= \infty \) and 4, respectively. For \( M = 6 \) and 8, the above function fits closely to the surfaces of BCC and FCC materials, respectively.

2.1.2.4. Hill's 48 Yield Criterion

Hill (1948) proposed an anisotropic yield criterion for orthotropic materials, and this function is shown below.

\[
\Phi = F(\sigma_{yy} - \sigma_{xx})^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H(\sigma_{xx} - \sigma_{yy})^2 + 2L\sigma_{yz}^2 + 2M\sigma_{xz}^2 + 2N\sigma_{xy}^2
\]

\[
= 2\overline{\sigma}^2
\]

(2.1.12)

where \( \sigma_0 \) is the stress expressed in the axes of orthotropic symmetry, and \( F, G, H, L, M, \) and \( N \) describe the plastic anisotropy.
For sheet metal forming process, plane stress assumption is adopted, and the above equation can be simplified by setting \( \sigma_x = \sigma_y = \sigma_{xy} = 0 \). The sheet metal is with planar anisotropy when it is anisotropic in its plane. However, if the plastic properties on any direction are the same, then it is called planar isotropy.

In the sheet metal forming simulation, it is assumed that only normal anisotropy exists with the sheet metal and plane stress assumption hold. The normal anisotropy is defined as the ratio of the width strain to the thickness strain. Based on this assumption, the above equation can be rewritten as Eq. (2.1.13).

\[
\Phi = \sigma_1^2 + \sigma_2^2 + \overline{R}(\sigma_1 - \sigma_2)^2 = (1 + \overline{R})\overline{\sigma}^2
\]  

(2.1.13)

The normal anisotropy can be calculated by Eq. (2.1.14).

\[
\overline{R} = \frac{\overline{R}_0 + 2*\overline{R}_{45} + \overline{R}_{90}}{4}
\]  

(2.1.14)

where \( \overline{R}_0, \overline{R}_{45}, \) and \( \overline{R}_{90} \) are the plastic anisotropy parameters of the 0, 45, and 90 degrees with respect to the cold rolling direction, respectively.

**2.1.2.5. Logan and Hosford**

Logan and Hosford (1980) proposed another yield criterion because Hill's 48 yield criterion is not a good description of the behavior of aluminum. Their function is shown in Eq. (2.1.15).
\[ \Phi = F(\sigma_y - \sigma_z)^m + G(\sigma_z - \sigma_x)^m + H(\sigma_x - \sigma_y)^m = 2\sigma^m \]  

(2.1.15)

It is a good approximation of polycrystal calculations for BCC and FCC metals while \( m \) is equal to 6 and 8, respectively.

2.1.2.6. Hill's 79 Yield Criterion:

Hill (1979) proposed another anisotropic yield criterion that is a generalization of the Logan and Hosford yield criterion.

\[ \Phi = F(\sigma_2 - \sigma_3)^M + G(\sigma_3 - \sigma_1)^M + H(\sigma_1 - \sigma_2)^M + A|2\sigma_1 - \sigma_2 - \sigma_3|^M + 
B|2\sigma_2 - \sigma_3 - \sigma_1|^M + C|2\sigma_3 - \sigma_1 - \sigma_2|^M = 2\bar{\sigma}^M \]  

(2.1.16)

Hill also proposed four truncated forms for in plane isotropy and plane stress. The fourth one is used extensively in the sheet metal forming analysis and shown below.

\[ \Phi = \frac{1}{2(1+R)}|\sigma_1 + \sigma_2|^M + \frac{(1+2R)}{2(1+R)}|\sigma_1 - \sigma_2|^M = \bar{\sigma}^M \]  

(2.1.17)

In addition to Hill's anisotropic yield criteria, some researchers also have proposed some other criteria that are listed below.

2.1.2.7. Barlat and Lian:

Equation (2.1.18) was proposed by Barlat and Lian (1989) and is expressed in the orthotropic symmetric axes.
\[ \Phi = a|K_1 + K_2|^m + a|K_1 - K_2|^m + (2 - a)|2K_2|^m = 2\bar{\sigma}^m \] (2.1.18)

where, \[ K_1 = \frac{\sigma_{xx} + h\sigma_{xy}}{2}, \quad K_2 = \sqrt{\left(\frac{\sigma_{xx} + h\sigma_{xy}}{2}\right)^2 + p^2\sigma_{xy}^2}, \] and where \( a, h, \) and \( p \) are anisotropic coefficients.

### 2.1.2.8. Others:

Barlat, Lege, and Brem (1991), Karafillis and Boyce (1993), and Barlat (1995) also proposed their anisotropic yield criteria. Equation (2.1.19) is the criterion of Barlat et al.

\[ \Phi = (3I_2)^{m-\frac{1}{2}} \left\{ [2 \cos\left(\frac{2\theta + \pi}{6}\right)]^m + [2 \cos\left(\frac{2\theta - 3\pi}{6}\right)]^m + [-2 \cos\left(\frac{2\theta + 5\pi}{6}\right)]^m \right\} = 2\bar{\sigma}^m (2.1.19) \]

### 2.1.3 Strain Hardening

Both uniaxial tensile test and bulge test are widely used to determine the relation of the measured effective strain and stress. However, Graf and Hosford (1993) proposed a plane-strain tension test method to obtain this relation. The strain hardening laws are obtained by fitting a curve to these experiment results. Some strain hardening rules are reviewed as follows.

Ludwik (1909) proposed an exponential equation shown below to fit the effective strain and stress relation.

\[ \bar{\sigma} = \sigma_y + K\bar{\varepsilon}^n \] (2.1.20)
Hollomon (1945) simplified the above equation as
\[ \sigma = K \varepsilon^n. \] (2.1.21)

Both Krupkowski (1945) and Swift (1952) added an initial strain to the Eq. (2.1.21) as shown below.

\[ \sigma = K(\varepsilon_o + \varepsilon)^n \] (2.1.22)

In addition, Eqs. (2.1.23) and (2.1.24) were proposed by Voce (1948) and Barlat as well as Jalinier (1985), respectively. The latter one (2.1.24) approximates well the dual phase steel.

\[ \sigma = A - B \exp(-C \varepsilon). \] (2.1.23)

and

\[ \sigma = \sigma_o + H \log(\varepsilon_o + \varepsilon). \] (2.1.24)

These four strain hardening rules, Eqs. (2.1.22)---(2.1.25) have been reviewed and compared by Rate and Welch (1983).
2.2 The Bauschinger Effect

2.2.1 General Remarks

The Bauschinger effect refers to the phenomenon that the yield stress of a metal object undergoing cyclical plastic deformation diminishes with the number of repetitions. Figure 2.1 illustrates such a drift of yield stress of a metal bar for the uniaxial stress case. The bar is under tension first until plastic deformation takes place. It is then subjected to compression beyond the reverse yield stress. As can be seen from Fig. 2.1, the yield stress in the compression direction is smaller than the original. This phenomenon was first reported by Bauschinger (1886), and is very important in engineering applications including sheet metal forming. Reviews of certain aspects of the Bauschinger effect have been made by Abelband Muir (1972), Sowerby and Uko (1979), and Bate and Wilson (1986).

Greater understanding of the Bauschinger effect has been obtained through some remarkable research on the part of other researchers. These contributions consist of both experiments and theoretical proposals, and the theoretical proposals can be divided into two main categories, macroscopic and microscopic methods. The continuum theory of plasticity is the backbone and foundation of the macroscopic method. This approach endeavors to describe the work hardening or strain hardening behavior of the materials, especially metals, which have undergone a complicated deformation history, including loading, unloading, and even reverse loading processes. The microscopic method focuses on the relation between the Bauschinger effect and the internal microstructure of the
material such as dislocations, matrix, particles, inclusions, etc.. This present section focuses only on the macroscopic approach.

![Diagram of the Bauschinger effect](image)

**Fig.2.1: The Phenomenon of the Bauschinger effect**

### 2.2.2 Some Definitions of the Bauschinger Effect

Moan, Sargent, and Embury (1973) defined a Bauschinger Effect Ratio, B.E.R., as

\[
B.E.R. = \frac{\sigma_B}{\sigma_F - \sigma_a}
\]

(2.2.1)

where \(\sigma_B\) and \(\sigma_F\) are the back stress and tensile flow stress respectively at strain \(\epsilon\), and \(\sigma_a\) is the initial yield stress of the virgin material.
This equation was also used by Looyd (1977) to reveal the relation of B.E.R and strain. Two different materials, Al-6% Ni and Al-7.6% Ca, were used for the test specimens. Ibrahim and Embury (1975) defined a so-called "Bauschinger Effect Parameter (BEP)" of which the definition is shown in Eq. (2.2.2). In their study, two different BCC materials, Armco iron and Zone refined niobium, were used as specimens for the experiment, and two plots, forward-plastic strain vs. BEP, were given.

\[
BE P = \frac{\sigma_F - \sigma_R}{\sigma_F - \sigma_o} = \frac{2\sigma_B}{\sigma_F - \sigma_o} \tag{2.2.2}
\]

where \( \sigma_F = \sigma_o + \sigma_{fpr} + \sigma_B, \) \( \sigma_R = \sigma_o + \sigma_{rpr} - \sigma_B, \) and \( \sigma_B \) and \( \sigma_{rpr} \) are the elastic back stresses which are determined by the sense of plastic pre-strain, and forward flow stress, respectively, and \( \sigma_o \), called basic flow stress, is estimated by extrapolating the part of the stress-strain curve back to the zero plastic strain.

Tan and Magnusson (1994) defined a Bauschinger effect factor as Eq. (2.2.3).

\[
B = \frac{|\sigma_f| - |\sigma_r|}{|\sigma_f|} \tag{2.2.3}
\]

where \( \sigma_f \) and \( \sigma_r \) are the forward stress at which the material starts to undergo unloading, and the reverse yield stress, respectively.

This Bauschinger effect factor was also used in their hardening model.
2.2.3 Experimental Measurement of the Bauschinger Effect

Regarding the measurement of the Bauschinger effect, some experimental approaches that have been published are reviewed as follows. The most common experimental methods for determining the Bauschinger effect are the uniaxial tension and compression, and cyclic torsion tests. For example, Rolfe, Haak, and Gross (1968) took specimens from bent plates with different orientations, including both transversal and longitudinal directions, then compressed the specimens. The Bauschinger effect was observed in the fact that the yield stress in the compression was lower than the yield stress in tension by as much as 30%. However, cyclical compression-tension test is not easy to utilize for determining the Bauschinger effect of the sheet metal because the sheet metal has a buckling problem while it is undergoing the compression test. Therefore, some special fixtures or equipment have to be devised. For example, Tan, Magnusson, and Persson, (1994) designed a fixture which can prevent buckling without introducing friction between the supports and the specimen and without introducing any restraint to lateral expansion of the sample as the axial load is applied. Tan et al. utilized this fixture first to apply the compressive load to specimens, and then applied the tensile load. The Bauschinger effect can be observed from their experimental data. Similarly, Kuwabara, Morita, Miyashita, and Takahashi (1995) also made a comb-shaped die for measuring in-plane compressive flow stress on the sheet metal. Then, they used this pre-compressed specimen for a tensile test (CT-test), and an in-plane compressive flow stress of uniaxially pre-stretched specimens (TC-test), and an uniaxial re-tensile flow stress of TC-test specimens (TCT-test). Alloy sheet A5182-O and AK-steel sheet were used as specimens in their experiments.
To avoid the buckling problem of the sheet metal, some pure bending and reverse bending experiments have been performed by, for example, Weinmann, Rosenberger, and Sanchez (1988), and Yoshida, Urabe, and Toropov (1998), and Jiang (1997). The Bauschinger effect phenomenon can be directly observed from their experimental data. Even if the relation of the stress and strain cannot be obtained directly, the Bauschinger effect can be observed from the bending moment and movement relation, as in, for example, Jiang's experiment (1997).

The shear test also has been used to test the Bauschinger effect of the sheet metal. For instance, Miyanchi, (1984, 1992) developed a test of planar simple shear to investigate the Bauschinger effect in both steel and aluminum sheets. This method is better than the torsion test because the torsion test cannot be applied to sheet metal. However, the cyclical torsion test is one of the most common approaches to determine the Bauschinger effect. For example, Takahashi, Shiono, Chida, and Endo (1984) used aluminum pipe as a specimen for the cyclical torsion test for obtaining the Bauschinger effect of the specimens, as did White et al. (1990). The torsion test approach for documenting the Bauschinger effect was also adopted by Rees (1981), and Takashi, and Shiono (1991), and Lindholm et al. (1980).

2.2.4 Some Macroscopic Models for the Bauschinger Effect

In the following sections, only macroscopic approaches are reviewed. The most commonly used models are as follows.
2.2.4.1 Mashing Model (elastic-plastic elements)

This model was proposed by Mashing (1923), and has been recently discussed by Asaro (1975). The mashing model comprises a number of elastic ideal plastic elements, and each element has a different yield stress. This model is the basis of many kinematic hardening models for representing the relation of the Bauschinger effect and the internal stress. A more detailed review of this model can be found in the study by Sowerby, and Uko (1979).

2.2.4.2 Linear and Nonlinear Kinematic Hardening Model

Prager (1956) proposed the hypothesis of the kinematic hardening model, which can also be used for modeling the Bauschinger effect and is the so-called “Ideal Bauschinger Effect” model. In this linear kinematic hardening model, the elastic limit on unloading and reverse loading is always double that of the initial yield stress. Ziegler (1959) modified Prager’s model. However, these two kinematic hardening models represent the stress-strain relation as a bilinear function, which in many cases is inadequate. In addition to Ziegler, Hodge (1957), Kadashevitch and Novozhilov (1958), and Baltov and Sawczuk (1965) also have made some modifications to Prager’s kinematic hardening rule.

To model the nonlinear part of the stress-strain curve, a nonlinear kinematic hardening model was first proposed by Armstrong and Frederick (1966). After Armstrong and Frederick, Chaboche (1977, 1986) made more refinements to the nonlinear kinematic hardening model. As mentioned before, the linear kinematic hardening model proposed
by Prager had no ability to capture the nonlinear elastic-plastic behavior during cyclical loading. Therefore, Armstrong and Frederick (1966), and Chaboche (1977) modified the back stress increment as

\[ d\alpha_y = C \cdot d\varepsilon_y^p - \gamma \cdot \alpha_y \cdot d\varepsilon_y^p \]  

(2.2.4)

where \( C \) and \( \gamma \) are positive material parameters, which can be obtained from both tensile or cyclic tests, and \( d\varepsilon_y^p \) is the increment of the equivalent plastic strain.

The second term on the right side of the Eq. (2.2.4) is called the "recall term". This term is very important for modeling the Bauschinger effect since it affects the plastic flow differently for tensile or compressive loading processes. The reason is that it depends on \( |d\varepsilon_y^p| \). This nonlinear hardening model is already implemented by a commercial program called PLIAG (1995, 1996), which is used for predicting the springback of the sheet metal formed part.

**2.2.4.3 Mixed Kinematic Hardening and Isotropic Hardening Model**

This concept was first proposed by Hodge (1957). He assumed that the plastic strain increment may be linearly decomposed into components that produce kinematic hardening, \( d\varepsilon_y^k \), and isotropic hardening, \( d\varepsilon_y^I \), as shown in Eq. (2.1.17).
The ratio of isotropic to kinematic hardening is defined by a mixed hardening parameter, $M$, which must be obtained from experimental observation. $M$ may be the value from 0 to 1. The relation between $\Delta e_{ij}^t$ and $\Delta e_{ij}^k$ is defined by Eq. (2.1.17)

Some researchers also considered that the yield surface undergoes translation, expansion, and distortion during the forming process. White, Bronkhorst, and Anand, (1990) proposed an improved isotropic-kinematic hardening model for the hardening behavior of metals. In their model, the isotropic hardening of the constitutive model can initially soften while the reverse loading is occurring.

2.2.4.4 Mroz Multiple Surfaces Model

This model was proposed by Mroz (1967, 1969) and has been applied by many researchers recently. Some examples are discussed as follows. Hunsaker, Vaughan, and Stricklin (1976) evaluated four hardening models, isotropic, Prager-Ziegler kinematic, the Mroz model, and mechanical sublayer model, on materials that have undergone a complex loading history, including reverse loading. By comparing the experimental data, they found that both the Mroz multiple yield surfaces model and mechanical sub-layer have a better fit with the experimental data than either the isotropic hardening model or the kinematic hardening model. Mroz's model also was used by Chu (1984, 1987) to derive an incremental constitutive equation for determining the formability of the three dimensional problem. Chu (1986) also utilized this incremental constitutive equation to analyze the stress and strain states and the energy dissipated for complicated loading history. This concept developed by Chu can also be used in both sheet metal forming and
structural failure analyses. Furthermore, Tang (1990, 1996) utilized Mroz's model and Chu's work (1984, 1987) in developing a constitutive equation by using Hill's 48 yield criterion for sheet metal forming application. As is widely known, if the internal stress of the deformed part can be precisely determined, then the springback prediction would be quite accurate. Because the deformation history is very complex and Tang's model can precisely calculate the internal stress distribution of the sheet metal after forming, this concept has already integrated into a 3D FEA code for predicting springback.

### 2.2.4.5 The Two Yield Surfaces Model

This model is used to determine the internal stress of the sheet metal undergoing a complex cyclic loading process. The Bauschinger effect exists in the cyclic loading process, so this model can be used to determine the influence of the Bauschinger effect.

As is well known, the two yield surfaces model was proposed by Krieg (1975) and by Dafalias and Popov (1976). This model is used to model the complicated cyclic loading process. With only two yield surfaces, the plastic modulus between these two yield surfaces is in a continuous variation, while the plastic modulus is a piecewise function in the Mroz multiple yield surfaces. The two yield surfaces model recently has been utilized by Takahashi and Ogata (1991) and Jiang (1997).
2.2.4.6 Other Models

In addition to the above models, some models were proposed by other researchers based on their observations of the material experiments and some of them are reviewed as follows.

Crafoord's Model:
The simplest model for considering the Bauschinger effect was proposed by Crafoord (1970). He assumed that the magnitude of the reverse yield stress is the same as the initial yield stress. This model has been used by Verguts and Soweby (1975) as one of their models for determining the internal bending moment of bent laminated sheet metals.

Tan's Model:
In 1994 Tan, Magnusson, and Persson (1994) provided a new hardening model incorporating the Bauschinger Effect Factor, $B$. For the plane stress case, their model can be expressed as

$$
s_1^2 + s_2^2 - s_1s_2 - s_iB(s_1 + s_2) = (1 - B)s_i^2
$$

(2.2.5)

where $s_i$ is the forward yield stress in tension and $B$ is called the Bauschinger Effect Factor which can be determined by experimental data and is defined as Eq. (2.2.6).
\[ B = \frac{|\sigma_f| - |\sigma_r|}{|\sigma_f|} \]  \hspace{1cm} (2.2.6)

where \( \sigma_f \) is the forward stress at which point the material starts to undergo unloading, and \( \sigma_r \) is the reverse yield stress.

When \( B = 0 \), Tan's hardening model is exactly the same as the Von Mises yield criterion.

Backlash Model:

Takashi and Shiono (1991) proposed a so-called backlash model for modeling the Bauschinger effect.

2.2.5 Physical Phenomenon of the Bauschinger Effect of the Sheet Metals

Based on several publications, some phenomena of the Bauschinger effect on the sheet metal are summarized below.

1. After reverse yielding, the reverse flow curve has a higher degree of roundness than the forward loading.

2. After reverse yielding, there is a transient region that disappears after the plastic strain is about 5\%–10\%. The reverse flow curve can be parallel or continue with the initial forward monotonic loading curve.

3. The strain aging raises the reverse yield stress and decreases the roundness of the reverse flow curve.
2.3 Springback Prediction

2.3.1 General Remarks

Once the formed part is removed from the tooling, the elastic recovery would occur, especially where bending and bending-unbending and even reverse bending were performed. This phenomenon is called springback. Because springback causes shape changes that can present major problems in the assembly process, this phenomenon complicates the draw development process. So, the springback has to be compensated by so called overcrowning and overbending techniques on the die surface that are based on experience and trial and error. This correction can be compensated either in the forming die or at the restrike operation. In order to compensate for this shape change amount during the draw development process, the springback quantitatively prediction becomes very important. Since springback depends on the material properties of the sheet metal, tooling geometry, and the friction, etc., it is very complex to predict. Therefore, this compensation is still heavily dependent on the designer’s experience, so it not only increases try-out time but also increases the cost.

In this section, two major categories about springback have been reviewed. One is to review what causes springback and how to reduce it, see subsection 2.3.2, and the other is to review the methodologies used to predict the amount of the springback. These methods can be classified into two main categories, analytic and FEA methods. Some papers are reviewed and discussed in the subsection 2.3.3.
2.3.2 Some Parameters Influencing Springback Result

2.3.2.1 Friction (Lubrication)

Any change in friction that increases the transverse stress gradient increases both springback and side-wall curl. Some experiments have been done by other researchers and revealed the relation of springback with friction.

Based on their experiment, Kato and Ueno (1968) stated that the springback increases when the low friction is on the die side of the drawn sheet metal, but springback decreases when the low friction is on the punch side. Kim and Thomson (1989) also had the same conclusion that reduction of friction on the punch side decreases the springback amount.

2.3.2.2 Material Properties

To reduce springback, Yoshida (1965) and Duncan and Bird (1978) proposed that the yield stress and the coefficient of the friction have to be as small as possible for producing more plastic deformation in the formed part. It can be concluded from their suggestions that springback and side wall curl increase with the yield strength. This kind of statement also can be found in the dissertation of Wang (1993), and in Wang, Kinzel, and Altan (1993). Levy (1984) has contributed an empirical formula to compute springback in terms of the yield strength for a variety of the die geometry. In 1993, Wang, Kinzel, and Altan (1993) also stated that the bending moment is greater for
materials with higher strength, strain hardening and normal anisotropy, which increase springback, under the same bending radius.

However, Lems, W. (1963) revealed that Young's modulus decreases with plastic strain. So the springback calculation becomes more complicated than a constant Young's modulus. Morestin and Boivin (1993) published a paper on how to utilize this concept to a springback calculation using the software, PLIAGE. In their experiments, they also found a recovery of the Young's modulus of plastified specimens after a few days but not for all steels tested. They stated that the Young's modulus would recover after several days. This phenomenon also has been observed in aluminum in the master thesis of Carden (1997). They called it time dependent springback.

Datsko and Hilsen (1984) concluded that springback decreases when the work hardening rate increases. In the same year, Davies (1984) also found that side-wall curl decreases as the strain hardening exponent increases.

2.3.2.3 The Bending Ratio R/t

Springback increases with the bending ratio, R/t where R and t are the bending radius and thickness of the sheet metal, respectively. Woo and Marshall (1959) derived some equations for springback calculation for both pure bending and bending with stretching cases. They also stated when the ratio of forming radius to metal thickness is larger, springback is greater and it can be reduced by applying a tension force at each end of the metal strip.
2.3.2.4 Tooling Clearance

Springback increases with the tool clearance ratio that is defined as the tooling gap/metal thickness regardless of the material. This phenomenon has been observed and found by Davies (1981, 1984), Liu (1982), Wang (1984), and Kim and Thomson (1989). Regarding the side wall curl, the same result as springback was observed by Umehara (1980), Hayashi (1984) and Kim and Thomson (1989); that is, side wall curl increases with the tooling clearance ratio. The reason springback increasing with tooling clearance is that the increase of the tool clearance ratio reduces the angle of the bending and the length of the contact with the punch, so the plastic deformation decreases. The same reason is also useful for side wall curl.

2.3.2.5 Bending with Tension

When tension is applied during, before, or after bending, the springback amount can be reduced by decreasing the stress gradient. The bending moment in the bent sheet would be smaller than the sheet metal undergoing bending without tension. The reason is that more plastic deformation happens during deformation. This phenomenon has been proved by many experiments and analysis papers and is discussed below.

Woo and Marshall (1959) derived some equations of springback calculation for both pure bending and bending with stretching cases. They also state when the ratio of forming radius to metal thickness is large, the springback is huge and springback can be reduced by applying a tension force at each end of the metal strip. Baba and Tozawa (1964)
finished some experiments of bending with tension. Based on their experiments, they found that springback decreases with an increase in the tensile force. UEDA and UENO (1981) published a paper with both experimental and analytical models and obtained good agreement between experiments and simulations. In their experiment, they used both T-M (a constant tension applied before and during bending) and T-M-T (a constant stress T1 applied before and during bending and a bigger stress T2 applied after bending). They concluded that springback decreases with increasing tension, and an additional tension (T2) is more effective than an initial tension (T1). Both Yu and Johnson (1982) and El-Domiaty as well as Shabaik (1984) declared and derived an equation and the influence of the tension force on the springback calculation. The difference between Yu et al (1982) and El-Domiaty et al (1984) is that Yu used an ideal plastic material model, but El-Domiaty used a work hardening material model.

Pourboughrat and Chandorkar (1992) also mentioned that parts formed by stretch forming have less springback than those formed by the draw-in operation. The reason is the tension force applied during the forming process. Kuwabara, Takahashi, Akiyama, and Miyashita (1995) did some experiments and analytic models for the springback predictions for the sheet metals undergoing (1) Stretching after bending (BS-process), (2) Bending with simultaneous stretching (SB-process), (3) SB-process followed by consecutive re-stretching (SBS-process), and (4) SB-process followed by unloading and successive re-stretching (SBUS-process).
2.3.3 The Methods for Springback prediction

The approaches utilized to solve the springback problem consist of two main categories: the analytic and finite element methods. A review of these two methods is described and discussed as follows.

2.3.3.1 Analytic Approach

2.3.3.1.1 Pure Bending Case

Using the elementary theory of bending, Garinder (1957) derived a generic and simplified mathematical analysis for the springback of metals with elastic-ideal plastic under pure bending. Based on his assumptions, the formula shown below can be obtained for springback calculation.

\[
\frac{R}{r} = 4\left(\frac{RS}{Et}\right)^3 - 3\left(\frac{RS}{Et}\right) + 1
\]  

(2.3.1)

where \( R \) and \( r \) are the radii of the middle layer of the sheet before and after springback, respectively, and \( S \) and \( t \) are the yield stress and thickness of sheet, respectively.

In the same paper, Crandall (1957) pointed out that the sheet metal is bent as a plate rather than a beam, so the stiffness of beam has to be replaced by the plate stiffness. Then, the above equation was modified as

\[
\frac{R}{r} = 4\left(\frac{RS(1 - \nu^2)}{Et}\right)^3 - 3\left(\frac{RS(1 - \nu^2)}{Et}\right) + 1
\]  

(2.3.2)
Queener and Angelis (1968) derived two formulas, one for springback and the other for residual stress, based on his assumptions. Their assumptions in this paper can be briefly summarized as follows: (1) pure bending, (2) plane strain, (3) isotropic and homogenous, (4) neutral axis always coinciding with middle surface, (5) neglecting shear stress and transverse stress, (6) small curvature bending, (7) neglecting thickness change, (8) the sheet metal obeying power work hardening law and Von Mise yield criterion. They obtained two equations shown below to calculate springback and residual stress, respectively.

\[
\frac{R_o}{R_f} = 1 - \frac{3K(1 - \nu^2)}{E(2 + n)(3/4)^{(1-n)/2}} \left( \frac{2R_o}{t} \right)^{1-n} + \left( \frac{2R_o}{t} \left( \frac{K}{E} \right)^{1/(1-n)} \right)^2 \left[ \frac{3(1 - \nu^2)^{(1-n)}}{(2 + n)(3/4)^{(1-n)/2}} \left( 1 - \nu + \nu^2 \right)^{(2-n)/2} - \frac{(1 - \nu^2)^3}{(1 - \nu + \nu^2)^{3/2}} \right] \quad (2.3.3)
\]

where \( R_o \) and \( R_f \) are the radii of the middle layer of the sheet before and after springback, respectively, and \( t \) is the thickness of sheet.

For \( 0 \leq |y| \leq R_o \frac{(K/E)^{1/(1-n)}}{(1 - \nu + \nu^2)^{1/2}} \):

\[
\sigma_R = \frac{E}{(1 - \nu^2)R_o} - \frac{3K(t/2)^{n-1}}{R_o^2(2 + n)(3/4)^{(1-n)/2} + \frac{24E \cdot R_o^2(K/E)^{3/(1-n)}}{t^3}} \times \left[ \frac{(1 - \nu^2)^{2-n}}{(2 + n)(3/4)^{(1-n)/2}} \left( 1 - \nu + \nu^2 \right)^{(2-n)/2} - \frac{(1 - \nu^2)^2}{3(1 - \nu + \nu^2)^{3/2}} \right] \quad (2.3.4.a)
\]
For $R_o \left( \frac{K}{E} \right)^{1/(1-n)} \frac{(1 - \nu^2)}{(1 - \nu + \nu^2)^{1/2}} \leq |y| \leq t/2$:

$$
\sigma_R = \frac{K}{(3/4)^{(1-n)/2} R_o} [\pm |y|^n - \frac{3y}{(2 + n)(t/2)^{1-n}}] + 24y \frac{E R_o^2}{t^3} \left( \frac{K}{E} \right)^{1/(1-n)} \times \\
\left[ \frac{(1 - \nu^2)^{2-n}}{(2 + n)(3/4)^{(1-n)/2} (1 - \nu + \nu^2)^{(2+n)/2}} - \frac{(1 - \nu^2)^2}{3(1 - \nu + \nu^2)^{3/2}} \right] 
$$

(2.3.4.b)

where $\sigma_R$ and $y$ are the residual circumferential stress and the distance from the center of sheet metal, respectively.

The positive and negative sign of the above equation corresponds to the sign of $y$. $y$ is defined as positive toward the outer edge of bent sheet metal. Otherwise, it is negative.

In the above equation, there is a jump discontinuity in the stress analysis at the elastic-plastic interface because of the difference in Poisson's ratio for elastic and plastic domains.

For most industrial applications, the ratio of the tooling radius with the metal sheet thickness is not large, $R_o / t < 30$, so Eqs. (2.3.3) and (2.3.4) can be simplified to Eqs. (2.3.5) and (2.3.6)

$$
\frac{R_o}{R_f} \approx 1 - \frac{3K(1 - \nu^2)}{E(2 + n)(3/4)^{(1-n)/2}} \left( \frac{2R_o}{t} \right)^{1-n} 
$$

(2.3.5)

$$
\sigma_R \approx \frac{K}{(3/4)^{(1-n)/2} R_o} [\pm |y|^n - \frac{3y}{(2 + n)(t/2)^{1-n}}] 
$$

(2.3.6)
Johnson and Yu (1981) showed the formulas of springback for the beam, plate, and circular plate undergoing work hardening, linear and nonlinear, material. They also simplified the problem by some assumptions that are the same as Garinder (1957) except their material is undergoing work hardening (at parts III and IV). However, in both parts I and II, the elastic perfect plastic material was adopted as well. To compare their results with other researchers, only the formulas of beam and plate with the linear work hardening material have been shown below.

For linear hardening material:

**Beam:**

\[
\frac{1}{R} = \frac{1}{R} - \frac{M}{EI} = \frac{2}{h} \frac{Y}{E} \left( \frac{1}{\gamma} - m \right)
\]

where \( \gamma = \frac{C}{(h/2)} \), \( m = \frac{M}{(1/6)yh^3} \), and \( h \), \( Y \), and \( \gamma \) are thickness of sheet, yield stress, and \( C \) and \( M \) are the elastic core height and moment caused by the internal stress.

**Plate:**

\[
\frac{1}{R_x} = \frac{1}{R_x} - \frac{M_x - \nu M_y}{EI} = \frac{2}{h} \frac{Y}{E} \left( \frac{1}{\gamma_x} - \frac{\nu}{\gamma_y} - m_x + \nu m_y \right)
\]

and

\[
\frac{1}{R_y} = \frac{1}{R_y} - \frac{M_y - \nu M_x}{EI} = \frac{2}{h} \frac{Y}{E} \left( \frac{1}{\gamma_y} - \frac{\nu}{\gamma_x} - m_y + \nu m_x \right)
\]

where \( M_a \) means \( M \) in the \( a \) direction and \( \nu \) is Poisson's ratio.
A series of discussions about the springback of rectangular plates under pure bending was made by Johnson and Yu (1981, 1982). But many limitations and simplifications have been made in their papers, so their applications in large metal forming processes is very limited (they assumed the deflection of the plate is smaller than its thickness).

In the springback analysis of Adams, Kasper, and Kurajian (1973), the volume change due to elastic strain has been included, which is widely ignored for the simplifying calculation, to calculate $\varepsilon_n$ in the plastic deformation. However, compared to the plane strain assumption, the error caused by volume change is very small. So it is reasonable to assume that volume is constant during the forming process.

Tan, Li, and Persson (1994) also derived two formulas shown in Eqs. (2.3.10) and (2.3.11) for springback and residual stress of plate sheet metal undergoing pure bending, respectively. They did an experiment by using X-ray diffraction with the layer-removal method to obtain the residual stress of the bending sheet. By comparing simulation results with experimental data, a sufficient agreement has been obtained for the small curvature pure bending process.

Their assumptions are listed as follows: (1) Small curvature bending, here $r/t>25$. (2) Neglecting strain in width direction, plane strain. (3) Neglecting normal stress, plane stress. (4) Cross plane remains plane during and after bending, neglecting shear stress. (5) The neutral axis always coincides with middle layer. (6) Isotropic hardening material, using Hollomon’s hardening law, and obeying Von Mises yielding criterion.
\[
\frac{1}{r'} = \frac{1}{r} - \frac{M}{rE' h^2}
\]  

(2.3.10.a)

with

\[
M = 2Er^2 \left( \frac{k}{E} \right)^{3/(1-n)} \left\{ \frac{(1-\nu^2)^2}{3(1-\nu + \nu^2)^{3/2}} - \frac{(2/\sqrt{3})^{n-1}}{n+2} \left( \frac{1-\nu^2}{\sqrt{1-\nu + \nu^2}} \right)^{n-2} \right\} + \\
\frac{t^n}{r} \frac{kt^2}{(n+2)(\sqrt{3})^{n-1}}
\]  

(2.3.10.b)

where \( r \) and \( r' \) are the radii of the neutral plane of the sheet strip before and after springback, respectively, and \( E' = \frac{E}{1-\nu^2} \) and \( h^2 = -rt + r^2 \ln \frac{r+t/2}{r-t/2} \), \( t \) is the thickness of the sheet, \( k \) is the strength coefficient, and \( n \) is the strain hardening exponent of Hollomon's law, respectively.

\[
\sigma_{\nu'}(y) = \sigma_x(y) + \sigma_{x'}(y)
\]  

(2.3.11.a)

with

\[
\sigma_{x'}(y) = -\frac{ym}{(r+y)h^2}
\]  

(2.3.11.b)

2.3.3.1.2 Pure Bending with Bauschinger Effect Consideration

The influence of the Bauschinger effect on springback has been considered by some other researchers with respect to the pure bending process. The cause for the Bauschinger effect is the fiber movement during the pure bending process. However, this influence of the fiber movement on the springback on pure bending is not very important since just a
few layers experience the influence of the Bauschinger effect. Some papers in which the Bauschinger effect is considered are reviewed below.

A theory concerning pure bending with plane strain assumption for rigid-perfect plastic material was proposed by Hill (1950). His formulas for the movement of fibers have been widely utilized even when the thickness reduction is considered zero in his model. Crafoord (1970) modeled the plane strain bending with rigid-work hardening materials and assumed that the flow stress models of fibers between the currently neutral and unstretched fibers are always equal to the original yield stress. Crafoord's model has been utilized by Verguts and Sowerby as one of their models for some engineering applications (1975).

Building on Crafoord's (1970) and Proksa's (1959) models, Dadras and Majlessi (1982) proposed two models of this region where the Bauschinger effect exists. Some discussions of their conclusions follow. Their models have a tangential stress discontinuity at the neutral layer because the rigid plastic material is assumed for their models. The neutral and unstretched layers move in the concave direction due to bending plastic deformation. The bending moment initially increases with deformation, but it gradually decreases when it reaches its maximum value. The cause of this phenomenon is the thickness reduction and the fact that the hardening rate declines at high strain.

To consider the Bauschinger effect on pure bending, Tan, Persson, and Magnusson (1995) proposed a model for sheet metals with rigid-plastic anisotroptic material. In their
paper, Hill’s quadratic yield criterion and the Ludwik hardening rule were used. The Bauschinger effect has only been considered between unstretched and neutral fibers by using the formula that was mentioned by Tan and Magnusson (1994) shown below.

\[ \sigma = \sigma_o - K|\varepsilon_p|^{n} \]  

(2.3.12)

where \( \sigma \) is the reverse yield stress, and \( \sigma_o \) as well as \( \varepsilon_p \) are the original yield stress and compressive pre strain, respectively.

They assumed that those fibers between neutral and unstretched layers have no reverse yielding and that reverse yield happens only at the unstretched fiber. This idea is totally different from Crafoord’s. Actually, the Bauschinger effect has to be considered from the original middle layer to the currently neutral fiber, but the influence of the Bauschinger effect between the original middle fiber to the current unstretched fiber is ignored in the Tan’s model. Finally, they pointed out that the model with the Bauschinger effect predicts a much greater reduction in the thickness than the model without the Bauschinger effect. They also stated that the effect of anisotropy on the material thinning of a pure bending condition is very small, but it has a relatively large contribution on the bending moment. They also constructed two models for pure bending. One is isotropic hardening with Voce’s law, the other uses kinematic with Ludwik’s law. They divided the sheet thickness into three zones: (1) the zone between the outer and strain free layers; (2) the zone between strain free and stress free (neutral) layers; and (3) the zone between stress free (neutral) and inner layers. They also obtained the formulas of stress and
bending moment of these three regions for the pure bending condition. The relations of
these layers for pure bending can be found as below.

\[ R_{outer} > R_{middle} > R_{strain\_free} > R_{stress\_free} > R_{inner} > 0 \]  

(2.3.13)

However, the above relations may be changed for the complicated draw forming process.
Based on their experimental and simulation results, some conclusions can be made as
below.

- Material thinning in pure bending is mainly caused by the Bauschinger effect and
  strain hardening.
- Anisotropy makes a very small contribution to thinning, but has a large effect on the
  bending moment.

Even though the above papers did not contribute any equation on the springback
calculation, they did offer some models for the internal stress calculation while the
material is undergoing pure bending. Once the internal stress can be obtained, the
removal bending moment can be calculated directly. Then the springback can be
determined.

2.3.3.1.3 Bending with Stretching

As mentioned before, the axial force has a great influence in springback. Johnson and Yu
(1982) showed its effect on springback for three kinds of deformed types: wholly elastic,
primary plastic, and secondary plastic of a beam with elastic-perfect plastic material.
Their results not only can be easily expressed in three elegant equations, but also can be drawn in the n-m plane, where n and m are the non-dimensional axial force and the bending moment, respectively. Based on the structure and approach of their paper, the results for these three regions of a beam with work hardening metals, $\bar{\sigma} = k\bar{e}^\alpha$, have been completed by El-Domiaty and Shabaik (1984) as well. El-Megharbel, El-Domiaty, and Shaker (1990) used the results of El-Domiaty and Shabaik (1984) to predict both springback and residual stress of the sheet metal after bending with stretching. Their results are shown below.

$$\frac{1}{R_f} - \frac{1}{R} = -\frac{M}{\partial M_e / \partial (1/R)} = -\frac{12M}{Eh^3} = -\frac{12M / M_e}{Eh^3 / M_e} = -\frac{12m}{Eh^3 / (Ybh^3 / 6)}$$ (2.3.14.a)

or

$$\frac{R_f}{R} = \frac{1}{1 - m\gamma}$$ (2.3.14.b)

where these notations here are the same as Eq. (2.3.7) and b is the width of sheet.

Each of these three regions has a different expression form for m, which is a function of $\delta$ and $\gamma$ (Eq. (2.3.7)). $\delta$ is a non-dimensional parameter and can be written as $\delta = d/(h/2)$, where d is the distance from the middle layer to the neutral axis. The above equation, shows that the axial force has a tremendous influence to decrease springback since d is primarily determined by the external axial force.

Kuwabara, Takahashi, Akiyama, and Miyashita Y. (1995) derived two springback formulas for stretch-bending and stretch-bending-restretching processes. In the stretch-
bending cases, the relation between the radii of curvature of the neutral plane of the sheet strip before and after springback can be expressed as

\[
\frac{1}{r_m} = \frac{1}{r_m} - \frac{12(1 - \nu^2)}{E(1 - \Delta \varepsilon_\theta)} \frac{1}{r_{TM}^3} \int_{R_0}^{R_0} \sigma_i'(r - r_m) \, dr
\]  

(2.3.15)

where \( r_m \) and \( r_m' \) are the radii of the neutral plane of the sheet strip before and after springback, respectively, and \( \Delta \varepsilon_\theta \) is the incremental compressive strain caused by stretch force unloading, and \( \sigma_i' \) is the stress expression form along the normal direction of the strip.

The value of \( t_{TM} \) can be calculated by

\[
t_{TM} = r_{ob} - R_d
\]  

(2.3.16)

where \( r_{ob} \) and \( R_d \) are the radii of curvature of the die and the convex outer surface of the sheet strip, respectively.

Similarly, the relation of \( r_m \) and \( r_m' \) of stretch-bending-re-stretching case can be written as

\[
\frac{1}{r_m'} = \frac{1}{r_m} - \frac{12(1 - \nu^2)}{E(1 - \Delta \varepsilon_\theta - \varepsilon_T)} \frac{1}{r_{TM}^3} \int_{R_d}^{R_0} \Sigma_i'(r - r_m) \, dr
\]  

(2.3.17)
where \( \varepsilon_{r2} \) is the tensile strain increment caused by the restretching force, and \( \Sigma' \) is the stress expression form along the normal direction of the strip, and \( \Sigma' \) has a different form in different regions.

The magnitude of Kuwabara's springback is defined as follows:

\[
\frac{\Delta R}{R_d} = \frac{R' - R_d}{R_d} \tag{2.3.18}
\]

where \( R' = r_m'' - t_{f,m} / 2 \).

The weak point of this model comes from neglecting the bending moment caused by the transverse residual stresses and from improperly assuming that \( \Delta \varepsilon_o \) is uniform across the sheet strip thickness so that some differences exist between the experimental and simulation results.

In their paper published in the 1995, the bending-unbending was not considered, so Kuwabara, Takahashi, Akiyama, and Ito (1996) introduced an analytic method to calculate the springback of the sheet subjected to bending-unbending under the tension forming process. In that paper, the contribution of the Bauschinger effect has been included. This approach is different from the paper published by Pourboghrat and Chu (1995) who used the kinematic hardening rule to model the Bauschinger effect. It can be concluded from their paper that the Bauschinger effect not only decreases the flow stress but also springback. Two removal moments (one caused by longitudinal stress and the
other caused by transverse stress) have influences on the springback in both the longitudinal plane and transverse plane. They divided the bending-unbending under tension process into five steps that are (1) initial stretching, (2) bending, (3) unbending, (4) unloading, and (5) springback processes.

After the springback process, the radii of curvatures of the central plane in the longitudinal and transverse directions can be written as follows.

\[ r_{\theta m} = \left( \frac{1}{r_{\theta ml}} - \frac{M_{\theta} - \nu \cdot M_{zl}}{D} \right)^{-1} \]  
\[ r_{z m} = \left( \frac{\nu \cdot M_{\theta} - M_{zl}}{D} \right)^{-1} \]

where \( D = \frac{E \cdot t^3}{12(1 - \nu^2)} \), \( t \) is the thickness after unloading process, and \( M_{\theta} \) and \( M_{zl} \) are the residual moments on the longitudinal and transverse directions, respectively, after the unloading process.

Chu (1986) used isotropic and anisotropic models to represent the material hardening properties. If the uniaxial stress-strain relation can be expressed as \( \sigma = f(\varepsilon) \), then the unloading behaviors of these two models are shown as Eqs. (2.3.20) and (2.3.21), and they have been used to predict the springback of the double-bend technique. This technique was developed by Davies and Liu (1984) and Liu (1984).
Isotropic:

\[
d\sigma = \begin{cases} 
  E \cdot d\varepsilon, & |\sigma| \leq \sigma_f \\
  f'(|\sigma|) \cdot d\varepsilon, & |\sigma| > \sigma_f 
\end{cases}
\]  

\[ (2.3.20) \]

anisotropic:

\[
d\sigma = \begin{cases} 
  f'(1/2|\Delta\varepsilon|) \cdot d\varepsilon, & |\sigma| \leq \sigma_f \\
  f'(|\sigma|) \cdot d\varepsilon, & |\sigma| > \sigma_f 
\end{cases}
\]  

\[ (2.3.21) \]

where \( \sigma_f \) is the previously obtained flow stress level.

While sheet metal undergoes only stretching and bending, the isotropic hardening model is the best choice for the material model. However, the isotropic model is insufficient for modeling the influence of the Bauschinger effect. Therefore, the anisotropic hardening model becomes the only model while the sheet is undergoing bending and unbending. In her paper, Chu utilized these two models to predict the springback of the double-bend technique. By comparing the results of the experimental data of Davies and Liu (1984) and Liu (1984), Chu found that the anisotropic model obtained a better result than the isotropic model. She also pointed out that the loading history is also a very important factor for predicting springback. Following the development of these two models, other researchers have also adopted one or/and the other of them as the material model such as Zhang and Lee (1995) and Kuwabara, Takahashi, Akiyama, and Ito (1996) and Pourboghrat and Chu (1995), Chu (1991), and Morestin, Boivin, and Silva (1996).

Chu (1991) derived a method, which describes the springback phenomenon in a series of proportional paths, to predict both simple springback and side-wall curl. This approach is
described in the following paragraphs. The isotropic-kinematic hardening rule and plane strain assumptions have been used. The relations between the true stress and the true strain of loading and reversed loading are different. They are as shown below.

For loading:

\[
|\sigma|/\sigma_y = \begin{cases} 
|\varepsilon|/\varepsilon_y, & |\varepsilon| < \varepsilon_y \\
(|\varepsilon|/\varepsilon_y)^n, & |\varepsilon| > \varepsilon_y 
\end{cases}
\]

(2.3.22)

For reversed loading (path initiated from a plastic state):

\[
|\Delta \sigma|/(2\sigma_y) = \begin{cases} 
\Delta |\varepsilon|/2\varepsilon_y, & \Delta |\varepsilon| < 2\varepsilon_y \\
(|\Delta |\varepsilon|/2\varepsilon_y)^n, & \Delta |\varepsilon| > 2\varepsilon_y 
\end{cases}
\]

(2.3.23)

where \( |\Delta \sigma| \) denotes the absolute value of the progression of a quantity from \( \sigma \).

For springback formula derivation, the stretch force and bending moment are very important because the stretch force (\( N \)) is used to check force equilibrium, and the bending moment is used to determine the removal bending moment. Chu normalized the regular stretch force and bending moment equations into the normalized forms. Both regular and normalized formulas are as shown below.

Regular:

\[ N = \int_0^t \sigma[k(z - z_n)] \cdot dz, \]

and

\[ M = \int_0^t \sigma[k(z - z_n)] \cdot (z - t/2) \cdot dz, \]

(2.3.24)
where \( k \) and \( z_n \) are the radius of curvature and the neutral axis position, respectively, and \( t \) is the thickness of the sheet metal.

Normalized:

\[
\overline{N} = \int_0^1 \sigma \left[ k \left( \bar{z} - \bar{z}_n \right) \right] \cdot d\bar{z},
\]

and

\[
\overline{M} = 6 \int_0^1 \sigma \left[ k \left( \bar{z} - \bar{z}_n \right) \right] \cdot (\bar{z} - 1/2) \cdot d\bar{z}
\]

In the above equations, the length is normalized by the sheet thickness, \( t \), and the stress as well as strain are normalized by \( \varepsilon_y \) and \( \sigma_y \). Curvature is normalized by \( (\varepsilon_y / t) \), and \( M \) and \( N \) are normalized by \( (\sigma_y \cdot t) \) and \( (\sigma_y \cdot t^2) / 6 \), respectively.

To determine the springback, Chu organized the springback phenomenon as the following series of proportional paths:

1. The sheet is formed from stress-free to \( (\bar{k}_o, \overline{N}_o) \) where \( \bar{k}_o = t / [\varepsilon_y (R_p + z_n)] \), in which \( R_p \) is the punch radius, and \( \overline{N}_o \) is the normalized in-plane force.

2. The stretching force is released from \( (\bar{k}_o, \overline{N}_o) \) to \( (\bar{k}_o, \overline{N} = 0) \).

3. After the tooling is removed, the bending moment within the sheet metal becomes zero as well \( (\overline{N} = 0, \overline{M} = 0) \).
Based on the above procedures and concepts, the formulas for $\bar{N}$ and $\bar{M}$ of each step are shown below.

For path 1:

$$\bar{N}_1(\bar{z}_{n1}) = \bar{N}_o = \int_0^{\bar{z}_{n1}} \left[-k_o(\bar{z} - \bar{z}_{n1})\right] d\bar{z} + \int_{\bar{z}_{n1}}^{\bar{z}_{n1}} k_o(\bar{z} - \bar{z}_{n1}) d\bar{z} + \int_{\bar{z}_{n1}}^{\bar{z}_{n1}} [k_o(\bar{z} - \bar{z}_{n1})] d\bar{z}.$$ 

For path 2:

$$\bar{N}_2(\bar{z}_{2}) = 0 = \int_0^{\bar{z}_{n2}} [\bar{z}_2 - k_o(\bar{z} - \bar{z}_{n1})] d\bar{z} + \int_{\bar{z}_{n1}}^{\bar{z}_{n1}} [k_o(\bar{z} - \bar{z}_{n1}) - \bar{z}_2] d\bar{z} + \int_{\bar{z}_{n1}}^{\bar{z}_{n1}} [\bar{z}_2(\bar{z} - \bar{z}_{n1})] d\bar{z}. $$

For path 3:

$$\bar{N}_3(\bar{z}_{n3}, \bar{k}_3) = 0 = \int_0^{\bar{z}_{n3}} k_3(\bar{z} - \bar{z}_{n3}) d\bar{z} - \int_{\bar{z}_{n3}}^{\bar{z}_{n3}} [2((\bar{z}_2 + k_3(\bar{z} - \bar{z}_{n3}))/2) - \bar{z}_2] d\bar{z}.$$ 

$$\bar{M}_3(\bar{z}_{n3}, \bar{k}_3) = 0 = \bar{M}_2 - \int_{\bar{z}_{n3}}^{\bar{z}_{n3}} 6k_3(\bar{z} - \bar{z}_{n3}) d\bar{z}$$ 

$$- \int_{\bar{z}_{n3}}^{\bar{z}_{n3}} [2((\bar{z}_2 + k_3(\bar{z} - \bar{z}_{n3}))/2) - \bar{z}_2] d\bar{z}. $$ (2.3.26)

The above equations have some limitations, which are $\bar{z}_{n1} > 1/k_o$, $\bar{z}_2 < 2$ and $\bar{z}_2 + k_3(1 - \bar{z}_{n3}) > 2$ . The reason is that this simplified approach assumes no plastic deformation happens during elastic unloading. The above non-linear equations for $\bar{z}_{n1}, \bar{z}_2, \bar{z}_{n3}$ and $\bar{k}_3$ can be solved through trial and error by using a computer program.
Once these variables are obtained, the simple springback can be calculated by using the following formula.

\[ \frac{R_f}{R_p} = \frac{k_o}{k_f} \]  

(2.3.27)

where \( k_j = (k_o - k_3) \), and \( R_p \) and \( R_f \) are the punch radius and final radius of curvature, respectively.

Side-wall curl also can be solved by a similar approach.

In Wang's dissertation (1993), he derived three formulas of springback for three different situations, and they are elementary bending theory, advanced bending theory, and bending under tension. Some elegant equations were obtained, and they can be used to solve the springback of the bending forming processes. His ideas and formulas are listed as follows.

1. For elementary bending theory (pure bending):

Here, the thinning has been ignored, and the neutral axis is always fixed in the middle of the sheet metal. Simple springback can be obtained by using the equation below.

\[ \frac{1}{R_n} - \frac{1}{R_n^'} = \frac{12(1 - \nu^2)}{wt^3E}(M_s + M_p) \]  

(2.3.28)

where \( R_n \) and \( R_n^' \) are the radii of the neutral axis before and after springback, and \( w \) is the width of the sheet metal, and \( M_s \) and \( M_p \) are the applied moment in the elastic and plastic deformation regions.
Both $M_e$ and $M_p$ can be determined by Eqs. (2.3.29) and (2.3.30) below.

$$M_e = \frac{2wR_n^2}{3} \frac{E}{1 - \nu^2} \varepsilon_{\varepsilon, o}^3$$  \hspace{1cm} (2.3.29)

where $\varepsilon_{\varepsilon, o} = \frac{F(1 - \nu^2)}{E} \sigma$

$$M_p = 2wkF^{n+1}R_n^2 \varepsilon_{\varepsilon, o}^2 e^{-2\varepsilon_{\varepsilon, o}^2/F} \sum_{j=0}^{\infty} \left( \frac{2^j - e^{\varepsilon_{\varepsilon, o}^2/F}}{j!(j+1+n)} \left[ (\ln(1 + \frac{t}{2R_n}) + \frac{\varepsilon_{\varepsilon, o}^2}{F})^{j+1+n} - \left( \frac{\varepsilon_{\varepsilon, o}^2}{F} \right)^{j+1+n} \right] \right)$$  \hspace{1cm} (2.3.30)

The value of $F$ depends on what kind of yield functions and situations are used.

2. For advanced bending theory (pure bending):

In this case, both thinning and movement of the neutral axis, caused by the normal stress, have been considered. The removal moment of this case can be expressed by the following equation, and Eq. (2.3.28) can be used again for springback prediction.

$$M_e + M_p = 1/2 \left[ P_i - \frac{Ck}{n+1} \left( \ln \frac{R_i}{R_i} \right)^{n+1} \right] (R_n^2 - R_i^2) + 1/2 \left[ P_o - \frac{Ck}{n+1} (\ln \frac{R_o}{R_n})^{n-1} \right] (R_o^2 - R_n^2)$$

$$+ CkR_n^2 \sum_{j=0}^{\infty} \frac{2^j}{(n+j+1)j!} \left[ (\ln \frac{R_n}{R_n})^{n+1-j} - \left( \ln \frac{R_i}{R_i} \right)^{n+1-j} \right]$$

$$+ CkR_i^2 \sum_{j=0}^{\infty} \frac{2^j}{(n+j+1)j!} \left[ (\ln \frac{R_i}{R_i})^{n-2-j} - \left( \ln \frac{R_o}{R_n} \right)^{n-2-j} \right]$$  \hspace{1cm} (2.3.31)

3. For bending under tension:
The removal moment can be expressed as below.

$$M = M_s + M_p = \frac{2}{3}E' R_n^2 \varepsilon_{e,0}^3 + kF^{-n+1} R_n^2 \varepsilon_e^2 \frac{e^{-\varepsilon_e - \varepsilon_0}}{F} \sum_{j=0}^{\infty} \frac{2j - e^{-\varepsilon_e - \varepsilon_0}}{(n + j + 1) j!} \left\{ [\left| \varepsilon_0 \right| + \frac{\varepsilon_0 - \varepsilon_{e,0}}{F}]^{n-1-j} \right\} + \left[ (\varepsilon_{\max} + \frac{\varepsilon_0 - \varepsilon_{e,0}}{F})^{n-1-j} - 2(\varepsilon_{e,0} + \frac{\varepsilon_0 - \varepsilon_{e,0}}{F})^{n-1-j} \right] \right\}$$

(2.3.32)

After the value of \(M\) is obtained, the springback amount can be found by using Eq. (2.3.28).

Zhang and Lee (1995, 1996) proposed a novel mathematical method to predict the springback of the 2-D plane strain bending forming problem. By comparing with Numisheet'93 experimental results and two step rail-shape sheet metal part (SAE paper No. 960596), their method obtained an excellent shape prediction. Their approach consists of the following steps:

1. The following equation will determine the longitudinal strain of the original middle layer: 
   \[ \varepsilon_{lmo} = \ln\left( \frac{\alpha R_{mo}}{L_o} \right) \], where \( \alpha \), \( R_{mo} \), and \( L_o \) are bending angle, the original radius of the middle layer and the original length of sheet of the original middle layer, respectively.

2. The longitudinal strain of each layer, \( \varepsilon_i \), is calculated by using Eq. (2.3.33). The corresponding stress of each layer, \( \sigma_i \), is calculated by using Eqs. (2.3.34) and (2.3.35).

\[
\varepsilon_i(\varepsilon_o, \varepsilon_{lmo}, R_i) = \varepsilon_{lmo} + \frac{1}{2} \ln\left( \frac{4[R_i^2 e^{2\varepsilon_{lmo}} + (z_0 + t_o / 2)(t_o + \sqrt{t_o^2 + 4R_i^2 e^{2\varepsilon_{lmo}}})]}{(t_o + \sqrt{t_o^2 + 4R_i^2 e^{2\varepsilon_{lmo}}})^2} \right) \]  

(2.3.33)
where $t_o$ and $R_i$ are the original thickness and the inner radius of the deformed sheet, i.e. the radius of tooling, respectively.

In the elastic region:

$$\sigma_1 = \frac{E}{1 - \nu^2} \varepsilon_1$$  \hspace{1cm} (2.3.34)

In the plastic region:

$$\sigma_1 = Kd_1 (\varepsilon_o + d_1|\varepsilon_1|)^n$$  \hspace{1cm} (2.3.35)

where $d_1 = \frac{1}{2} [2(1+r)]^{M-1} [(1+(1+2r)^{-1}(M-1)]^{M-1}, M$ and $M$ is the exponent in Hill's anisotropy yield function.

(3) The resulting force from the internal stress distribution can be calculated by using Eq. (2.3.36).

$$F = \int_{R_i}^{R_o} \sigma_1 (\varepsilon_1) dR$$  \hspace{1cm} (2.3.36)

where $R_o$ and $R_i$ are the external and internal radii of the deformed sheet, respectively.

(4) The resulting, $F$, and the transferred force through the cross section are then compared. If the difference is larger than a specified tolerance, then go back to step 1 and adjust the value of $\varepsilon_{imo}$ until the tolerance range is achieved.

(5) The bending loading moment on the middle of sheet is calculated by Eq. (2.3.37).
\[ M = \int_{R_1}^{R_2} \sigma_t(\varepsilon_t)[R - (R_o + R_t)/2]dR. \]  

(6) Using the formula below, compute the springback curvature caused by elastic unloading.

\[ \Delta K = \frac{M(1-\nu^2)}{EI}, \]  

where \( I \) is the moment of inertia after deformation (consider thickness thinning).

To analyze the springback of the forming part, they divided it into several regions with different features. For instance, the simulation cases of 1995 and 1996 have been divided into five and nine regions, respectively. Zhang and Lee used three methods to simulate the case of the Numisheet'93 conference; these methods are the new mathematical model, new mathematical model with kinematic hardening model, and the membrane theory plus bending strain. Comparison of these simulation results with the experimental data show that their new mathematical model has the best springback prediction among these methods.

Bordonaro and Lee (1996) used the method proposed by Zhang (1995) and the kinematic hardening models to redo the springback of the Numisheet'93 conference. They found the kinematic hardening model always underestimated the amount of springback. On the other hand, Zhang's method overestimated the springback amount.
Zhang and Hu (1997) published a paper more recently to describe Zhang's novel mathematical model again and simplified the equation of $\varepsilon_i$ as

$$
\varepsilon_i(z_o, \varepsilon_{1mo}, R_{mo}) = \varepsilon_{1mo} + \frac{1}{2} \ln(1 + \frac{2z_o}{R_{mo}} e^{-\varepsilon_{1mo}})
$$

(2.3.39)

which is simpler than the one used in 1995.

They also pointed out that the original middle surface, current middle surface, neutral surface, and the unstretched surface are not the same one even if most people assume they overlap during pure bending. Finally, they compared the new model's simulation results with Hill's pure bending model, membrane theory, and shell theory.

Most researchers assume that unloading is elastic and that no plastic deformation takes place during the springback process. Actually, the reverse yielding might happen especially when the ratio of the radius and the thickness is small. Then, the simulation results will be underestimated if the assumption is made that only elastic unloading occurs during springback. Both the relevant experiment and simulation have been done by Sanchez and Roberson (1996). Therefore, reverse yielding has to be included in the springback calculation. Because the thinning and bending moment are very important for the springback calculation, anisotropy and the Bauschinger effect have to be considered as well. However, most researchers consider only the Bauschinger effect while bending and unbending have taken place.
Zhang and Hu (1998) used Zhang's novel mathematical model to determine the stress and residual stress distributions in plane strain bending. The model proposed by them in 1997 has been utilized here to calculate the internal stress distribution before and after springback. Increment plastic theory is adopted and three different models, isotropic, kinematic, and directional hardening models, are used to determine the internal stress distribution when the sheet metal has undergone repeated bending, unbending, and reverse bending, that is, the Bauschinger effect has been considered. For the springback prediction and residual stress distribution, two methods have been compared. These two methods are pure elastic unloading and natural unloading. The concept of natural unloading, of which plastic deformation may might take place during the springback process, has been discussed by Sanchez, Roberson, and Gerdeen (1996).

2.3.3.1.4 Generic Deformation History

Morestin, Bovin, and Silva (1996) published an analytical method utilized in PLIAGE (old version, first version) for the springback prediction. In this paper, the Von Mises yield criterion and nonlinear kinematic hardening model proposed by Lemaitre and Chaboche have been used. The most important contribution of this method is that Young's modulus here is varied with plastic strain, and thus is not always constant. This phenomenon was discovered by Lems (1963). A piecewise linear function has been used by them to model the variation of the Young's modulus in PLIAGE. A brief description and discussion about this paper follows. Their assumptions are (1) plane strain in the y direction is made, \( d\varepsilon_y = 0 \), (2) plane stress in the x direction is made, \( d\sigma_y = 0 \), (3)
isotropic material is assumed, and (4) the neutral layer is fixed at the middle of the sheet metal.

From their assumptions, it can be deduced that each layer of fiber has a linear system with five parameters, \( d\sigma_z, d\sigma_y, d\alpha_x, d\alpha_y, d\alpha_z \) where \( d\alpha_i \) is the back stress increment (nonlinear kinematic hardening). These five equations are listed below.

\[
\begin{align*}
\frac{d\overline{\varepsilon}_y}{dt} &= \frac{3}{2} \cdot \frac{S_y \cdot d\sigma_y + S_z \cdot d\sigma_z}{\sigma_e \cdot [C \cdot \sigma_e - \gamma \cdot (S_x \cdot \alpha_x + S_y \cdot \alpha_y + S_z \cdot \alpha_z)]} + \frac{1}{E(x_f)} \cdot (d\sigma_y - \nu d\sigma_y) = 0 \quad (2.3.40) \\
\frac{d\overline{\varepsilon}_x}{dt} &= \frac{3}{2} \cdot \frac{S_y \cdot d\sigma_y + S_z \cdot d\sigma_z}{\sigma_e \cdot [C \cdot \sigma_e - \gamma \cdot (S_x \cdot \alpha_x + S_y \cdot \alpha_y + S_z \cdot \alpha_z)]} + \frac{1}{E(x_f)} \cdot (d\sigma_z - \nu d\sigma_y) \\
&= \frac{x_{\beta} \cdot d\rho}{\rho (\rho - x_{\beta})} + \frac{d\varepsilon_n^\sigma}{(1 + \varepsilon_n^\sigma)} \quad (2.3.41) \\
d\alpha_x &= \frac{3}{2} \cdot \frac{(C \cdot S_x - \frac{2}{3} \cdot \gamma \cdot \alpha_x \cdot \sigma_e) \cdot S_y \cdot d\sigma_y + S_z \cdot d\sigma_z}{\sigma_e \cdot [C \cdot \sigma_e - \gamma \cdot (S_x \cdot \alpha_x + S_y \cdot \alpha_y + S_z \cdot \alpha_z)]} \quad (2.3.42) \\
d\alpha_y &= \frac{3}{2} \cdot \frac{(C \cdot S_y - \frac{2}{3} \cdot \gamma \cdot \alpha_y \cdot \sigma_e) \cdot S_y \cdot d\sigma_y + S_z \cdot d\sigma_z}{\sigma_e \cdot [C \cdot \sigma_e - \gamma \cdot (S_x \cdot \alpha_x + S_y \cdot \alpha_y + S_z \cdot \alpha_z)]} \quad (2.3.43) \\
d\alpha_z &= \frac{3}{2} \cdot \frac{(C \cdot S_z - \frac{2}{3} \cdot \gamma \cdot \alpha_z \cdot \sigma_e) \cdot S_y \cdot d\sigma_y + S_z \cdot d\sigma_z}{\sigma_e \cdot [C \cdot \sigma_e - \gamma \cdot (S_x \cdot \alpha_x + S_y \cdot \alpha_y + S_z \cdot \alpha_z)]} \quad (2.3.44)
\end{align*}
\]

where both \( C \) and \( \gamma \) are the coefficients of the Lemaitre and Chaboche law, and \([S]\) as well as \( \sigma_e \) are the reduced deviatoric stress tensor and yield stress, respectively.

By solving this linear system layer by layer and increment by increment, the final stress state of the whole sheet metal can be computed after deformation. As the method used
by other researchers, the elastic recovery moment can be determined. Then the springback amount can be computed as well.

The new version of PLIAGE was published by Fenoglietto, Morestin, Boivin, and Deng (1995). The concept underlying this new version is based on the previous one. However, the orthotropic nature of the rolling sheet metal was not considered in the old version. Both the nonlinear kinematic hardening model proposed by Lemaitre and Chaboche and the orthotropic feature are utilized in the new version. One more improvement is that the Young’s modulus has been considered as a function varied with not only plastic strain but also depending on the rolling direction. Similarly, the Poisson ratio is also a function of plastic strain and direction. In order to calculate the elastic springback, the internal stress through the thickness of sheet metal has to be determined first, then the removal moment. Because kinematic hardening and orthotropic features are applied in this software, the derived procedure is different from others, e.g., the isotropic hardening model with constant Young’s modulus. But the basic idea is the same as others. The following description is the procedure and result of this new version of PLIAGE.

1. The deformed geometry is used to determine the total strain which can be written as

\[
d\bar{\varepsilon}_z = \frac{x_{\beta}}{\rho (\rho - x_{\beta})} \cdot d\rho + \frac{1}{1 + e^{\nu}_z} \cdot d\varepsilon^{\nu}_z \tag{2.3.45}
\]

where \( \bar{\varepsilon}_z \) is the total strain in the longitudinal direction (z direction), \( x_{\beta} \) is the distance from the neutral axis to fiber \( \beta \), \( \rho \) is the radius of neutral axis, and \( e^{\nu}_z \) is the tensile strain.
in z-direction, respectively. Here, the second increment $d\varepsilon_{zz}^\nu$ has to satisfy force equilibrium because of imposed tensile stress.

2. Compute the stress state in the thickness.

Here, Hill’s new yield function with kinematic hardening is used and rewritten as

$$f(\bar{\sigma}_y) = 1/2.F.(\bar{\sigma}_{yy} - \bar{\sigma}_{zz})^2 + 1/2.G.(\bar{\sigma}_{zz} - \bar{\sigma}_{xx})^2 + 1/2.H.(\bar{\sigma}_{xx} - \bar{\sigma}_{yy})^2 - 1/2 = 0$$  (2.3.46)

with $\bar{\sigma}_y = \sigma_y - \alpha_y$

and the modified incremental value of back stress is:

$$d\alpha_y = C_{zz} \cdot d\varepsilon_y^p - \gamma_z \cdot \alpha_y \cdot d\varepsilon_{eq:zz}^p, \quad (2.3.47)$$

where $\varepsilon_{eq:zz}^p$ is the equivalent plastic strain in the z direction.

3. Calculate the factor of proportionality $d\lambda$.

In this step, Drucker’s rule, consistency, and plastic energy equations are utilized so that $d\lambda$ can be computed by using the formula below.

$$d\lambda = \frac{\frac{\partial f}{\partial \alpha_{yy}} d\sigma_{yy} + \frac{\partial f}{\partial \alpha_{zz}} d\sigma_{zz}}{\sum_i \frac{\partial f}{\partial \alpha_i} [-C_z \cdot \frac{\partial f}{\partial \alpha_i} - \gamma_z \cdot \alpha_i \cdot \sqrt{F + G}]}$$  (2.3.48)

4. Determine the elastic and plastic strains.

After $d\lambda$ is obtained, the plastic strain can found by utilizing Drucker’s rule. The elastic strain also can be obtained through Hook’s law. Because of orthotropicity, the form of
expression of elastic strain is different from regular elasticity with constant Young's modulus and Poisson's ratio.

5. Find the total strain.

The total strain is the summation of elastic and plastic strains.

Because of the plane strain assumption where the total strain in the y direction is zero, the total strains on both yy and zz directions can be expressed as

\[
d \varepsilon_{yy} = [F \cdot (\overline{\sigma}_{yy} - \overline{\sigma}_z) + H \cdot (\overline{\sigma}_{yy} - \overline{\sigma}_{xx})] \cdot d \lambda + \frac{1}{E_{yy}(x_\rho)} \cdot d \sigma_{yy} - \frac{\nu_{yy}}{E_z(x_\rho)} \cdot d \sigma_z = 0, \quad (2.3.49)
\]

and

\[
d \varepsilon_z = [F \cdot (\overline{\sigma}_z - \overline{\sigma}_{yy}) + G \cdot (\overline{\sigma}_z - \overline{\sigma}_{xx})] \cdot d \lambda + \frac{\nu_{yz}}{E_{yy}(x_\rho)} \cdot d \sigma_{yy} - \frac{1}{E_z(x_\rho)} \cdot d \sigma_z
\]

\[
= \frac{x_\rho \cdot dp}{\rho \cdot (\rho - x_\rho)} + \frac{d \varepsilon^{\sigma}_z}{(1 + \varepsilon^{\sigma}_z)} \quad (2.3.50)
\]

Based on all the above equations, the incremental values of back stresses can be found by using the following equations.

\[
da_{xx} = d \lambda \cdot [C_x \cdot (G \cdot (\overline{\sigma}_{xx} - \overline{\sigma}_z) + H \cdot (\overline{\sigma}_{xx} - \overline{\sigma}_{yy})) - \gamma_{xx} \cdot \alpha_{xx} \cdot \sqrt{F + G}]
\]

\[
da_{yy} = d \lambda \cdot [C_y \cdot (F \cdot (\overline{\sigma}_{yy} - \overline{\sigma}_z) + H \cdot (\overline{\sigma}_{yy} - \overline{\sigma}_{xx})) - \gamma_{yy} \cdot \alpha_{yy} \cdot \sqrt{F + G}]
\]

\[
da_{zz} = d \lambda \cdot [C_z \cdot (F \cdot (\overline{\sigma}_z - \overline{\sigma}_{yy}) + G \cdot (\overline{\sigma}_z - \overline{\sigma}_{xx})) - \gamma_{zz} \cdot \alpha_{zz} \cdot \sqrt{F + G}]
\]
Therefore, a linear system is obtained with five parameters, $d\sigma_{zz}, d\sigma_{xy}, d\alpha_{xx}, d\alpha_{xy}, d\alpha_{zz}$.

It can be used to solve the stress of each fiber layer in the sheet metal, increment by increment. Then, the total removal moment can be determined when the stress status of the sheet is completely determined.

6. Compute the final geometry.

The springback of sheet metal can be found by the formulas shown below.

\[
\frac{1}{\rho_{\text{res}}} = \frac{1}{\rho} + \frac{1}{\rho_e} \quad \text{and} \quad \rho_e = \frac{\int_{-h/2}^{h/2} E_y(x_{\parallel}) \cdot b \cdot x_{\parallel}^2 \cdot dx_{\parallel}}{M_f \cdot (1 - \nu_{yz} \cdot \nu_{yx})}
\]

(2.3.54)

with

\[
M_f = \int_{-h/2}^{h/2} x_{\parallel} \cdot \sigma_{zz} \cdot b \cdot dx
\]

(2.3.55)

where $E_y(x_{\parallel})$ is the Young's modulus of layer $i$ in the $j$ direction, and $\nu_{yz}$ is the Poisson's ratio of layer $i$ in the $j$ direction.

2.3.3.2 FEM Approach

2.3.3.2.1 Shell Element

Oh and Kobayashi (1980) used the rigid-plastic material model for the loading phase simulation and the elastic-plastic material model for the springback phase to determine the springback prediction of the air bending process. The results obtained by using this approach are close to the results of a fully elastic-plastic simulation.
In a paper published by Tang (1987), the plane strain and axisymmetric shell bending element has been used to compute springback. Strictly speaking, another surface contact problem has to be solved in order to find the final shape of a product after the load is released and tooling has been removed. Since the above approach is very complicated, Tang proposed an approximate method to determine springback, and his approach is described as follows. Springback can be determined from the deformed part by releasing press and frictional forces or applying these forces with opposite signs to the deformed part.

Tang's method also has been used by Lee, Cho, Hambrecht, and Choudhry (1991) for springback simulation under the plane strain assumption. However, they used a rigid-plastic material model for the forming simulation and a perfectly elastic model for the springback simulation, instead of the elastic-plastic material model. Their simulation results were close to the results of Tang except the cases with a very low blank holder force. The reason is that the elastic strain is more important than in draw dominated problems.

Oñate and Agelet de Saracibar (1992) proposed another approach for predicting springback. They ignored the elastic effects during the loading process, but a flow formulation is utilized. For the springback process, all elements perform as fully elastic. Their method is better than the fully elastic-plastic procedure.
An efficient procedure for the sheet metal forming simulation has been discussed by Mercer, Nagtegaal, and Rebelo (1995). The forming simulation is finished by the explicit code and the springback simulation is completed by the implicit code. From the forming simulation to the springback simulation, the dynamic equilibrium has to be transformed into the static equilibrium. At the start of the springback simulation, they proposed to impose an artificial stress that is in dynamic equilibrium at the end of the forming simulation. The normal Newton-Raphson method has been used to do the iteration.

2.3.3.2.2 Membrane Element

Pourboghrat and Chandorkar of The Aluminum Company of America (1992) have developed a rigid-plastic, 2 node line element membrane code. This code can be used to predict the springback in the post-processing. Some simplifications and incorrect assumptions have been made in their code, and these are shown and described below.

1. Rigid-plastic material model is used for FEA code.
2. In the real world, membrane and bending strains are coupled together, but Pourboghrat and Chandorkar add a bending strain correction term to the membrane strain obtained from the FEM result.
3. The bending effect on the thinning has been neglected.
4. The internal stress change caused by bending-unbending has been ignored, but the bending-unbending process has caused tremendous internal stress during the forming process.
In addition to the above paper (1992), Pourboghrat and Chu (1995) of ALCOA presented an approach to calculate the springback due to bending and unbending. Their assumptions and derivations for the bending portion are based on the paper published in 1992, but the following assumptions about the unbending and stretching problem deserve to be mentioned. (1) Unbending occurs under the plane strain condition. (2) The plane section remains plane after unbending. (3) The centerline curvature of the sheet metal will be zero after bending and unbending. (4) Stretching of a bent element is caused by the following deformations: an elastic unbending, \( \Delta M_e \), a plastic unbending, \( \Delta M_p \), and a uniform stretching, \( \Delta e_t \). (5) Kinematic hardening governs the re-yielding in the reserve loading.

In this paper, the unloading centerline curvature of the element (after unbending and stretching), \( k_u \), can be expressed as

\[
k_u = -\frac{12(1-\nu^2)M_f}{Et_{UB}^3} \tag{2.3.56}
\]

\[
M_f = M^* - 2M_y - \frac{K'}{(k^*-\Delta k_e)^2}(\frac{(A-\Delta e_t)^n+2}{(A+\Delta e_t)^n+2}) + \Delta e_t \frac{(A-\Delta e_t)^n+1-(A+\Delta e_t)^n+1}{(n+1)} \tag{2.3.57}
\]

where \( M^* \) is the bending moment prior to unbending and stretching,

\[
A = (k^*-\Delta k_e)\frac{t_{UB}}{2} \quad \text{and} \quad M_y = \frac{2K'}{(2+n)}(\frac{t_B}{2})^{n+2}k^n \tag{2.3.58}
\]

where \( k_y = \frac{2(1-\nu^2)\sigma_y}{t_B E} \)

\[65\]
and \( \Delta e_t = 1 - \frac{t_{UB}}{t_g} \), and \( t_g \) as well as \( t_{UB} \) are the thickness before and after unbending-stretching, respectively.

Pourboghrat and Chu applied these formula to SHEET_S, a two node line element rigid-plastic implicit code created at OSU and compared the results with the experiments and simulation results of the ABAQUS. Based on the comparisons, it can be found that Pourboghrat's theory underestimated the springback and tension in the sheet metal.
CHAPTER 3

THE HARDENING MODELS FOR THE BAUSCHINGER EFFECT

3.1 Introduction

To predict the final shape of the deformed part after springback, the calculation of internal stress distribution of the sheet metal must be accurate. Actually, it is not possible to obtain a precise springback prediction with only a rough estimate of internal stress distribution within the sheet metal. Most researchers ignore the contribution of the Bauschinger effect when they determine the internal stress distribution for the springback prediction. However, the Bauschinger effect exists while the element of sheet metal undergoes both loading and reverse loading processes, i.e., cyclical loading. The influence of the Bauschinger effect becomes more important as the deformation history of the sheet metal is more complicated.

The isotropic hardening model makes no allowance for modeling the Bauschinger effect, and the kinematic hardening model cannot represent the material behavior well when the deformation history of sheet metal is complex. Therefore, Mroz's multiple yield surfaces, the field of working-hardening moduli, is also adopted here for modeling the
hardening behavior of the sheet metal with a complicated deformation history.

A more detailed derivation and comparison of the isotropic, kinematic hardening rules, and Mroz's multiple yield surfaces can be found in sections 3.3 and 3.4. In addition, based on the concept of the isotropic hardening model, the kinematic hardening model, the Mroz multiple yield surfaces, and experimental observation, a new hardening model is proposed. This new hardening model performs better than isotropic hardening, kinematic hardening, and the Mroz multiple yield surfaces models on the internal stress calculation. Some comparisons of internal stress calculation can be seen in this present chapter and Chapter 5.

3.2 Basic Assumptions

Nine basic assumptions are listed below. Based on these assumptions, the internal stress of each fiber layer within the sheet metal element can be determined. Then, the final shape of the deformed part, including the springback angle and side-wall curl, see Fig. 3.1, can be calculated.

1. The plane strain assumption is made for the principal direction 2: $d\varepsilon_2 = 0$.
2. The plane stress assumption is made for the principal direction 3: $d\sigma_3 = 0$.
3. The plane normal to the sheet surface remains plane during the deformation process.
4. The cold-rolled sheet metal is homogeneous with normal anisotropy.
5. The Bauschinger effect is considered.

6. The total tangential strain, on the principal direction 1, is the summation of the membrane and bending strains.

7. The middle of the sheet is considered to be bending strain free, but there may be a membrane strain.

8. The sheet metal conforms to the tooling surfaces in regions of contact.

9. Volume conservation is assumed, so the volume variation due to elastic deformation is neglected.

\[
\begin{align*}
\text{(a)} & \quad \text{Wall Opening} & \quad \text{(b)} & \quad \text{Wall Curl}
\end{align*}
\]

Fig. 3.1: (a) springback (b) side wall curl
3.3 Deformation History and Hardening Models

The deformation history is very important for determining the internal stress distribution within the sheet metal having deformation. For instance, two elements can have the same total strain but different stress values because their deformation histories are different in the strain space. Furthermore, the deformation history of a fiber layer can be any combination of the elastic loading, plastic loading, elastic unloading and elastic reverse loading, and reverse plastic deformations. To obtain the internal stresses of a fiber layer precisely, the Bauschinger effect must be considered if this fiber layer has reverse yielding. Also, a fiber layer might experience reverse yield more than once making this phenomenon even more important and complicated.

In the sheet metal forming industry, compression, one kind of reverse loading process, must always be eliminated to avoid wrinkles. So, both draw bar and draw bead are developed during the draw development process for eliminating this kind compression. For simulation purposes, a different kind of hardening model would obtain a different result on the internal stress value even though the same deformation history is applied. A more detailed discussion follows.

As mentioned before, the deformation history of the loading process can be divided into two steps. The first one is the elastic deformation and the other is the plastic deformation. Since the principal stress on the principal direction 3 is ignored and the plane strain assumption is taken to apply here, the stresses of the principal directions 1 and 2 during the elastic loading process can be expressed as Eq. (3.1).
For Von Mises yield criterion:

\[ |\varepsilon_{11}| < \sigma_y / \left[ E'(1 + \nu^2 - \nu)^{1/2} \right] \]

For Hill's 48 yield criterion:

\[ |\varepsilon_{11}| < \sigma_y / E' \left[ \left( 1 + \frac{\nu^2 - \frac{2R}{1+R}\nu}{1+R} \right)^{1/2} \right] \]

\[ \sigma_{11} = E' \times \varepsilon_{11} \]

\[ \sigma_{22} = \nu \times \sigma_{11} \] (3.1)

where \( E' = \frac{E}{1-\nu^2} \), \( \varepsilon_{11} \) is the total strain on the principal direction 1, and \( E \) and \( \nu \) are the Young's modulus and Poisson's ratio, respectively.

Regarding plastic deformation caused by the loading process, the stress increments have different computational methods according to the hardening models. These hardening models will be discussed in the sections 3.3.1, 3.3.2, 3.3.3, and 3.3.4. Likewise, the reverse loading process can be classified into two deformation types: elastic unloading and elastic reverse loading, and reverse plastic deformations. The loading process is used only for the first loading cycle for the virgin material. However, the concept of the reverse loading process is utilized repeatedly if the element undergoes cyclical loading.

For both elastic unloading and elastic reverse loading processes, the internal stresses of the sheet metal element on principal directions 1 and 2 can be calculated by Eq. (3.2).

\[ \sigma_{11} = \sigma_{11}^A + E' \times \varepsilon_{11}^{\text{reverse}} \]

\[ \sigma_{22} = \sigma_{22}^A + \nu \times E' \times \varepsilon_{11}^{\text{reverse}} \] (3.2)
where $\sigma_{11}'$ and $\sigma_{22}'$, referring to point A on Fig. 3.3, are the stresses of the sheet metal element which is the end of the loading process and $\varepsilon_{11}^{\text{reverse}}$ is the total reverse loading strain on the principal direction 1.

Four different hardening models will be discussed and their formulas of stress increments of plastic deformation will be found in the following subsections. More detailed derivative procedures are shown in Appendixes A, B, and C.

3.3.1 Isotropic Hardening Model

For the sake of simplification, the isotropic hardening model is widely applied in the sheet metal forming industry for forming simulations. The basic assumptions listed in section 3.2 are also made here except for the fifth one because this model has no capacity to model the Bauschinger effect. The theoretical justification underlying this hardening model can be found in the literature review section (Chapter 2). As mentioned before, most sheet metal elements have undergone a complicated deformation history, which includes loading, unloading, and reverse loading during the forming process. Due to the characteristic of isotropic hardening, the internal stress is always overestimated when the metal element is undergoing cyclical loading. Determination of the internal stresses for the loading, unloading, and reverse loading processes through the use of the isotropic hardening model is illustrated in the following paragraphs.

Loading Process:

When a sheet metal element undergoes large deformation, it is subjected to elastic deformation first, then plastic deformation. Equation (3.1) can be used for the element
undergoing elastic deformation. As soon as the material yields, i.e., the sheet metal element starts to show plastic deformation, the internal stress calculation is based on the hardening model. For isotropic hardening with Hill's 48 yield criterion, the stress increments on both principal directions 1 and 2 and the effective plastic strain increments can be determined by Eq. (3.3).

\[
\frac{d\sigma_{11}}{\text{denominator}} = \frac{E \times d\varepsilon_{11} \left\{ (1+R)^2 \times \left( E \times \sigma_{22}^2 + K \times \bar{\sigma}^2 \right) + E' \times R \times \sigma_{11} \left[ R \times \sigma_{11} - 2 \sigma_{22} (1+R) \right] \right\}}{\text{denominator}}
\]

\[
\frac{d\sigma_{22}}{\text{denominator}} = \frac{E \times d\varepsilon_{11} \left\{ (1+R) \times \left[ K \times \nu \times \bar{\sigma}^2 (1+R) + E' \times R \times \left( \sigma_{11} - \sigma_{22} \right)^2 \right] - E' \times \sigma_{11} \times \sigma_{22} \right\}}{\text{denominator}}
\]

\[
\frac{d\bar{\varepsilon}^p}{\text{denominator}} = \frac{E \times d\varepsilon_{11} \times \bar{\sigma} \left\{ R^2 \times (1-\nu) \times (\sigma_{11} - \sigma_{22}) + R \times \left[ \sigma_{11} \times (2-\nu) - \sigma_{22} \left( 1-2\nu \right) \right] + \sigma_{11} + \nu \times \sigma_{22} \right\}}{\text{denominator}}
\]

\[
\text{denominator} = (1+R)(1-\nu)\left[ K \bar{\sigma}^2 (1+R)(1+\nu) + 2E' \times R \left( \sigma_{11} - \sigma_{22} \right)^2 \right] + E \left( \sigma_{11}^2 + 2\nu \times \sigma_{11} \sigma_{22} + \sigma_{22}^2 \right).
\]

and $R$ is the normal anisotropy of the sheet metal, and $K$ is defined as

\[
K = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p}
\]

Both the denominator and $K$ are obtained from the data of the previous step, denotes step $n$ here, and applied to the current step, denotes step $n+1$. Regarding $K = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p}$, the
derivative procedure and its definition can be found in Appendix A. A more detailed derivation and explanation of Eqs. (3.3) and (3.4) can be found in Appendix A as well.

Once the stress increments are determined, the current step total stresses in both principal directions 1 and 2 can be computed by Eq. (3.5).

\[
\sigma_{11}^{n-1} = \sigma_{11}^{n} + d\sigma_{11}
\]

\[
\sigma_{22}^{n-1} = \sigma_{22}^{n} + d\sigma_{22}
\]

(3.5)

where \(n\) and \(n-1\) are the time steps, and \(d\sigma_{ij}\) is the stress increment between steps \(n\) and \(n+1\).

Similarly, the total effective plastic strain can be computed by Eq. (3.6).

\[
\bar{\varepsilon}^p = \bar{\varepsilon}^p + d\bar{\varepsilon}^p
\]

(3.6)

Unloading and Reverse Loading Processes:

For the elastic unloading and elastic reverse loading processes, Eq. (3.2) can be used to determine the internal stresses in both principal directions 1 and 2. Once the material has begun to experience reverse yield, Eqs. (3.3), (3.4), (3.5), and (3.6) can be utilized for determining the internal stress in both principal directions 1 and 2, as well as the effective strain. Figure 3.2 shows the stress-strain relation of the uniaxial stress case. By
observing this plot, the reverse yield stress and the $K = \frac{d\sigma}{d\varepsilon_p}$ of point B can be determined by

$$|\sigma_A - \sigma_B| = 2\sigma_A \tag{3.7.1}$$

$$\left| \frac{d\sigma}{d\varepsilon_p} \right|_B = \left| \frac{d\sigma}{d\varepsilon_p} \right|_A \tag{3.7.2}$$

Fig. 3.2: Plot of the relationship in the stress and strain in the uniaxial stress case
However, for the plane stress case the expanded yield surface must be utilized to determine the reverse yield stress, see Fig. 3.3. The reverse status is on the point B and its effective stress is equal to that of point A.

![Diagram](image)

Fig. 3.3: The method of finding the reversed yield stress for the isotropic hardening model

3.3.2 Kinematic Hardening Model

Due to some drawbacks in Prager's kinematic hardening model, Ziegler's kinematic hardening rule is utilized here. The same as with isotropic hardening, Eq. (3.1) can be used for elastic loading process, and Eq. (3.2) can be used for elastic unloading.
Regarding the plastic deformation process, all formulas concerning kinematic hardening are explained below, and a more detailed derivation can be seen in Appendices B and C.

**Loading Process:**

When the material starts to yield, the stress increments in both principal directions 1 and 2 can be computed by Eq. (3.8), and the total current stresses can be determined by Eq. (3.5) as well. To utilize Eq. (3.8) for determining the stress increments, both the plastic modulus, $E_p$, and proportional factor, $K_p$, have to be determined from the material experimental data, e.g., a tensile test. When Hill's 48 yield criterion is applied here, the proportional factor, $K_p$, and the plastic modulus, $E_p$, values of the yield surface can be estimated by Eqs. (3.9) and (3.10). A more detailed description and derivation of both the plastic modulus, $E_p$, and proportional factor, $K_p$, values can be found in Appendix B. Regarding Eq. (3.8), Appendix C shows all derivative procedures.

\[
d\sigma_{11} = \frac{E' \times d\varepsilon_{11} \left[ K_p \left( A^2 + B^2 + C^2 \right) + B^2 \times E' \right]}{K_p \left( 1 - \nu^2 \right) \left( A^2 + B^2 + C^2 \right) + E \left( A^2 + B^2 + 2\nu \times A \times B \right)}
\]

\[
d\sigma_{22} = \frac{E' \times d\varepsilon_{11} \left[ K_p \times \nu \left( A^2 + B^2 + C^2 \right) - A \times B \times E' \right]}{K_p \left( 1 - \nu^2 \right) \left( A^2 + B^2 + C^2 \right) + E \left( A^2 + B^2 + 2\nu \times A \times B \right)}
\]

where $A = \sigma_{11} - \alpha_{11} - \frac{R \sigma_{11} \left( \sigma_{22} - \alpha_{22} \right)}{1 + R} - \frac{R \sigma_{11} \left( \sigma_{11} - \alpha_{11} \right)}{1 + R}$, $B = \sigma_{22} - \alpha_{22} - \frac{R \sigma_{22} \left( \sigma_{11} - \alpha_{11} \right)}{1 + R}$, and

\[
C = -\frac{1}{1 + R} \left( \sigma_{11} - \alpha_{11} - \sigma_{22} - \alpha_{22} \right).
\]
The definitions of $K_p$ for Von Mises and Hill’s quadratic yield criteria are given below.

For the Von Mises yield criterion:

$$K_p = \frac{2}{3} E_p$$

For Hill’s 48 yield criterion:

$$K_p = \frac{(1+R)^2}{(1+R)^2 + 2R^2} E_p$$  \hspace{1cm} (3.9)

where $R$ is the normal anisotropy.

The definition of $E_p$ is

$$E_p = \frac{E \times E_i}{E - E_i}$$  \hspace{1cm} (3.10)

where $E$ is the Young’s modulus and $E_i$ is the tangent modulus, the slope of the stress-total strain at any given point. The physical meaning of $E_p$ is the same as $\bar{K} = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p}$ in Eq. (3.4).

Some researchers have argued that the ratio of principal stresses 1 and 2 is $\frac{R}{1+R}$ for Hill’s 48 yield criterion when the sheet metal has undergone plastic deformation. This argument is not correct since the basic plane strain assumption is contradicted. The reason can be found in Appendix A. The effective strain increment can be determined by Eq. (3.11).
Due to Ziegler's kinematic hardening rule utilized here, the back stress increments of the yield surface can be determined by Eq. (3.12) where \( d\mu \) can be estimated by Eq. (3.13). The derivations of Eqs. (3.8) and (3.13) can be found in Appendix C.

\[
d\alpha_y = d\mu (\sigma_y - \alpha_y)
\]

\[
d\mu = \frac{\left[\sigma_{11} - \alpha_{11} - \frac{R}{1+R} (\sigma_{22} - \alpha_{22})\right] d\sigma_{11} + \left[\sigma_{22} - \alpha_{22} - \frac{R}{1+R} (\sigma_{11} - \alpha_{11})\right] d\sigma_{22}}{\left[\sigma_{11} - \alpha_{11} - \frac{R}{1+R} (\sigma_{22} - \alpha_{22})\right] (\sigma_{11} - \alpha_{11}) + \left[\sigma_{22} - \alpha_{22} - \frac{R}{1+R} (\sigma_{11} - \alpha_{11})\right] (\sigma_{22} - \alpha_{22})}
\]

The new yield surface center can be calculated by Eq. (3.14).

\[
\sigma_{y^{*-1}} = \sigma_y^* + d\alpha_y
\]

**Unloading and Reverse Loading Processes:**

For elastic unloading and reversed elastic loading processes, Eq. (3.2) can be used to determine the internal stresses of both principal directions 1 and 2. Regarding plastic deformation, all equations used in the loading process for determining stress increments.
and back stress increments can be utilized here. Similarly, Eqs. (3.5) and (3.14) can be applied to compute total stresses and back stresses, respectively. It can be assumed that the Bauschinger effect is caused by the material memory, that is, the material still keeps its memory when it is just reloaded in the opposite direction. Because most experiments of materials have only information about the loading process, the $K_p$ value for materials in the loading process can be determined from the uniaxial tensile test. Due to the shortage of experimental data, the $K_p$ value for the reversed loading process cannot be obtained from the tensile test data. Therefore, an assumption has to be made in order to obtain the material constant, $K_p$, of the reverse loading process. Two different methods are described below for sheet metal, and these two methods use distinct artificial approaches to determine the value of $K_p$.

**Method 1, $K_{in1}$ (regular)**

During the deformation process, the total effective strain can be determined. Then, the $E_p$ can be computed from the plot of the effective stress vs. effective plastic strain, from the uniaxial tensile test. Once $E_p$ is available, $K_p$ can be calculated by Eq. (3.11).

**Method 2, $K_{in2}$ (an alternative method)**

When the material, e.g., sheet metal, just starts to experience elastic unloading and reverse yield, it still keeps the same $K_p$ value as it has at point A on Fig. 3.4, due to the material memory. An artificial coordinate, $\bar{\sigma} - \bar{\varepsilon}^p$, can be created as shown on Fig. 3.5,
where $\tilde{\varepsilon}^p$ represents the total plastic strain increased in the reverse loading process only.

A new artificial variable is defined as Eq. (3.17).

\[
\tilde{\varepsilon}^p = CM \ast | \tilde{\varepsilon}_A^p - \bar{\varepsilon}^p |
\]  

(3.17)

where $CM$ can be either a constant or a variable, and $\tilde{\varepsilon}_A^p$ is the total effective plastic strain at point A, just before elastic unloading.

Equation (3.17) is used to determine $\tilde{\varepsilon}^p$ and this value can be utilized to determine the $K_p$ value. Substitute $\tilde{\varepsilon}^p$ for $\bar{\varepsilon}^p$ as the effective plastic strain in Fig. 3.5, a plot of effective stress vs. effective plastic strain obtained from the uniaxial tensile test. Then, the magnitude of $K_p$ can be computed easily by Eq. (3.11). Analysis of Eq. (3.17) will demonstrate that the material has memory ability in this method, Kin2. Furthermore, when $\tilde{\varepsilon}^p$ is equal to $\bar{\varepsilon}^p$, the material becomes like a virgin material, that is, the material memory is recovered. $E_p$ can be determined by Eqs. (3.10), and (3.11) can be used to determine $K_p$ value when Hill’s 48 yield function is applied.
Fig. 3.4: Plot showing the approach to find the reversed yield stress for the kinematic hardening model
3.3.3 Mroz's Multiple Yield Surfaces Method

Mroz's theoretical concepts concerning multiple yield surfaces is that the yield surfaces are concentric at the origin of the stress space before the material undergoes plastic deformation. The yield surfaces become nonconcentric after plastic deformation. The advantage of the Mroz method is that the Bauschinger effect can be monitored even if the experimental data lacks information about reverse loading.

Once the material experiences plastic deformation, the first yield surface, the smallest one, starts to translate on the stress space. Then, the first yield surface would contact (tangentially without penetration) the second yield surface, and they would move together
when more plastic deformation occurs. Similarly, these two yield surfaces would move together to contact the third one when the material undergoes further deformation. A more detailed description about the movement of multiple yield surfaces can be found in the papers of Mroz (1967, 1969) in which the Von Mises yield criterion was used. Furthermore, Hill's 48 yield criterion and the anisotropic material are utilized in this dissertation.

For purposes of an earlier discussion, the deformation process has been divided into two categories, loading and reverse loading. The loading process is used only for the first loading cycle for the virgin material. However, the reverse loading process is utilized repeatedly if the element undergoes a cyclical loading process. This method is applied here for determining the internal stress distribution of the sheet metal when it has undergone a complicated deformation history. A more detailed description is given below.

**Loading Process:**

As shown in Fig. 3.6, all yield surfaces are concentric with respect to the origin of the stress space for the material that does not have any plastic deformation. During plastic deformation, the yield surfaces undergo no expansion, but translates as a rigid body on the stress space.

To consider the characteristics of the cold-rolled sheet metal anisotropy, Hill's 48 yield criterion is utilized here, and Ziegler's kinematic hardening rule is used to avoid the
drawbacks of Prager's kinematic hardening rule. Therefore, the equations derived for
kinematic hardening, in section 3.2, can be used here for EACH yield surface as well.
However, some changes must be made to obtain the benefit of Mroz multiple yield
surface approach, and those improvements can be found in the discussion that follows.

Fig. 3.6: All yield Surfaces before any plastic deformation

Once the material yields, the first yield surface, the smallest one, is actuated and
undergoes rigid body translation in the stress space. The $K_p$ value estimated by Eq.
(3.11) for the first yield surface is utilized, and Eq. (3.9) can be used to determine the
stress increments on the both principal directions 1 and 2. The movement of the smallest
yield surface currently actuated follows Ziegler's kinematic hardening rule, so Eqs.
(3.12), (3.13) and (3.14) can be used to determine the back stresses of this yield surface. Referring to Fig. 3.7, the first yield surface contacts, tangentially, the second yield surface, and then they both move together when the material undergoes more plastic deformation.

Because the second yield surface is then actuated, see Fig. 3.7, the $K_p$ value for the second yield surface is used to determine both the stress increments and total stresses. The total stresses and the stress increments of the current step are used to determine both the back stress increments and the back stresses of each yield surface. The stress increments of all the yield surfaces within and including the currently actuated one are the same and are used to determine the back stress increments of each yield surface.
Because the back stresses for these yield surfaces are different, their $d\mu$ values are distinct. Then, the back stress increments of one yield surface may be different from another. Likewise, Fig. 3.8 shows both first and second surfaces moving together and in a position about to contact the third one.

Fig. 3.8: Both the first and second surfaces are moving together

Furthermore, it can be concluded that when the stress status is between successive yield surfaces $N$ and $N-1$, the $K_p$ value of the yield surface $N$ must be used to determine both the stress increments and the total stresses. Simultaneously, the surfaces from $I$ to $N$ move together and their back stresses can be determined by Eqs. (3.12), (3.13), and (3.14). As soon as the yield surface $N$ contacts and remains tangent to surface $N+1$, the
surface $N+1$ is actuated. After the loading process is finished, all yield surfaces may be like those represented by Fig. 3.9. This figure can also be used to represent the starting point of the reverse loading process as well. To trace the complicated deformation history of the sheet metal element, the current total stresses and the back stresses of all yield surfaces must be stored.

![Diagram of stress-strain relationship](image)

**Fig. 3.9: All surfaces at the end of loading process**

**Unloading and Reverse Loading processes:**

Equation (3.2) can be used to determine the internal stresses while the sheet metal element undergoes elastic unloading and reverse elastic loading processes. Once the material starts to reverse yield, the first yield surface starts to move as shown in Fig. 3.10.
Then, the concept and the procedures of the reverse loading process are exactly the same as the loading process.

![Diagram](image)

*Fig. 3.10: The first yield surface translates in the reverse direction*

### 3.3.4 New Methodology

Before plastic deformation, all of the yield surfaces are concentric with the origin of the stress space (Fig. 3.6). Once the material yields, the first yield surface is actuated and starts to have rigid body translation and uniform expansion (see Fig. 3.11). As soon as the second surface is touched by the first surface (no penetration and keeping tangency), the second surface is actuated (see Fig. 3.12). These two surfaces move together and actuate the third one when the material undergoes more plastic deformation. The first
yield surface has only a rigid body translation while the second, currently actuated surface, has both a rigid body translation and uniform expansion. It can be concluded that the actuated yield surface both translates and expands during the plastic loading deformation process and all surfaces within the actuated surface have only rigid body translations. Figure 3.13 shows that the third surface is just actuated.

Fig. 3.11: The first surface undergoes both uniform expansion and translation
Fig. 3.12: The second surface is just actuated

Fig. 3.13: The third surface is just actuated
As shown on the Eq. (3.18), the total strain increment in principal direction 1 of each step, $d\varepsilon_1$, consists of a $d\varepsilon_1^i$, contributed to uniform expansion, and a $d\varepsilon_1^k$, contributed to rigid body translation.

$$d\varepsilon_{11} = d\varepsilon_{11}^i + d\varepsilon_{11}^k$$  \hspace{1cm} (3.18)

where $d\varepsilon_{11}^i = M d\varepsilon_{11}$, $d\varepsilon_{11}^k = (1 - M) d\varepsilon_{11}$, and $M$ is weight between 0 and 1.

The principal stress increments are shown on Eqs. (3.19) and (3.21). The incremental $d\varepsilon_{11}^i$ can be applied to Eqs. (3.19) and (3.20) to obtain the principal stress increments and effective plastic strain increment contributed by $d\varepsilon_{11}^i$. Similarly, Eqs. (3.21), (3.23),
and (3.24) are utilized to compute the principal stress increments contributed by \( d\varepsilon_{11}^k \), and Eq. (3.22) is used to determine the effective plastic strain increment contributed by \( d\varepsilon_{11}^k \). Furthermore, \( \overline{K} \) and \( K_p \) determined from the currently actuated surface are used for Eqs. (3.19) and (3.21).

\[
d\sigma'_{11} = \frac{E' \times d\varepsilon'_{11} \left( (1 + R)^2 \times (E' \times \sigma_{22}^2 + \overline{K} \times \sigma^2) + E' \times R \times \sigma_{11} \left[ R \times \sigma_{11} - 2\sigma_{22}(1 + R) \right] \right)}{\text{denominator}}
\]

\[
d\sigma'_{22} = \frac{E' \times d\varepsilon'_{11} \left( (1 + R) \times \left[ \overline{K} \times \nu \times \sigma^2 (1 + R) + E' \times R \times (\sigma_{11} - \sigma_{22})^2 \right] - E' \times \sigma_{11} \times \sigma_{22} \right)}{\text{denominator}}
\]

\[
d\overline{\varepsilon}^p = \frac{E' \times d\varepsilon'_{11} \times \sigma \left\{ (1 - \nu) \times (\sigma_{11} - \sigma_{22}) + R \times [\sigma_{11} (2 - \nu) - \sigma_{22} (1 - 2\nu)] + \sigma_{11} + \nu \times \sigma_{22} \right\}}{\text{denominator}}
\]

where

\[
\text{denominator} = (1 + R)(1 - \nu) \left[ \overline{K} \sigma^2 (1 + R)(1 + \nu) + 2E' \times R(\sigma_{11} - \sigma_{22})^2 \right] + E' \left( \sigma_{11}^2 + 2\nu \times \sigma_{11} \sigma_{22} + \sigma_{22}^2 \right).
\]

\( R \) is the normal anisotropy of the sheet metal, and \( \overline{K} \) is defined as

\[
\overline{K} = \frac{\sigma_{x+1} - \sigma_x}{\overline{\varepsilon}_{x+1} - \overline{\varepsilon}_x}
\]

(3.20)

where \( x \) is the currently actuated yield surface, \( \sigma_x \) is the effective stress of surface \( x \) on Fig. 3.6, and \( \overline{\varepsilon}^p \) is its corresponding effective plastic strain.
\[ d\sigma_{11}^k = \frac{E \times d\varepsilon_{11}^k \left[ K_p \left( A^2 + B^2 + C^2 \right) + B^2 \times E' \right]}{K_p \left( l - \nu^2 \right) \left( A^2 + B^2 + C^2 \right) + E' \left( A^2 + B^2 + 2\nu \times A \times B \right)} \]

\[ d\sigma_{22}^k = \frac{E \times d\varepsilon_{11}^k \left[ K_p \times \nu \left( A^2 + B^2 + C^2 \right) - A \times B \times E' \right]}{K_p \left( l - \nu^2 \right) \left( A^2 + B^2 + C^2 \right) + E' \left( A^2 + B^2 + 2\nu \times A \times B \right)} \]  

(3.21)

where 

\[ A = \sigma_{11} - \alpha_{11} - \frac{R}{1 + R} (\sigma_{22} - \alpha_{22}), \quad B = \sigma_{22} - \alpha_{22} - \frac{R}{1 + R} (\sigma_{11} - \alpha_{11}), \]

\[ C = -\frac{1}{1 + R} (\sigma_{11} - \alpha_{11} + \sigma_{22} - \alpha_{22}), \] and \( \alpha_{11} \) and \( \alpha_{22} \) are the back stresses of the currently actuated surface.

\[ d\bar{\varepsilon}^k = \sqrt{\frac{(1 + R)^2}{(1 + 2R)} \left( d\varepsilon_{11}^{pk} + d\varepsilon_{22}^{pk} + \frac{2R}{1 + R} \cdot d\varepsilon_{11}^{pk} \cdot d\varepsilon_{22}^{pk} \right)} \]  

(3.22)

where 

\[ d\varepsilon_{11}^{pk} = \frac{1}{K_p} \cdot \frac{A \cdot (A \cdot d\sigma_{11}^k + B \cdot d\sigma_{22}^k)}{(A^2 + B^2 + C^2)}, \quad d\varepsilon_{22}^{pk} = \frac{1}{K_p} \cdot \frac{B \cdot (A \cdot d\sigma_{11}^k + B \cdot d\sigma_{22}^k)}{(A^2 + B^2 + C^2)}, \] and

the definitions of \( A, B, \) and \( C \) can be seen in Eq. (3.21).

The definition of \( K_p \) for Hill's 48 quadratic yield criterion is

\[ K_p = \frac{(1 + R)^2}{(1 + R)^2 + 2R^2} E_p \]  

(3.23)

The plastic modulus, \( E_p \), can be determined based on a tensile test and is

\[ E_p = \frac{E \times E_t}{E - E_t} \]  

(3.24)

where \( E \) is the Young's modulus, and \( E_t \) is the tangent modulus, the slope of the stress-total strain curve between the currently actuated yield surface, say \( N \), and surface \( N + 1 \).
Once the principal stress increments are calculated, the current total principal stresses at step \( m+1 \) can be computed by Eq. (3.25). Equation (3.26) can be used to compute the total effective plastic strain.

\[
\sigma_{y}^{m+1} = \sigma_{y}^{m} + d\sigma_{y} \tag{3.25}
\]

where \( d\sigma_{y} = d\sigma_{y}^{i} + d\sigma_{y}^{k} \).

\[
\bar{\varepsilon}^{p} = \bar{\varepsilon}^{p} + d\bar{\varepsilon}^{p} \tag{3.26}
\]

where \( d\bar{\varepsilon}^{p} = d\bar{\varepsilon}^{p} + d\bar{\varepsilon}^{k} \).

\[d\alpha_{y} = d\mu(\sigma_{y} - \alpha_{y}) \tag{3.27}\]

\[d\mu = \frac{\left[\sigma_{11} - \alpha_{11} - \frac{R}{1+R}(\sigma_{22} - \alpha_{22})\right]d\sigma_{11}^k + \left[\sigma_{22} - \alpha_{22} - \frac{R}{1+R}(\sigma_{11} - \alpha_{11})\right]d\sigma_{22}^k}{\left[\sigma_{11} - \alpha_{11} - \frac{R}{1+R}(\sigma_{22} - \alpha_{22})\right](\sigma_{11} - \alpha_{11}) + \left[\sigma_{22} - \alpha_{22} - \frac{R}{1+R}(\sigma_{11} - \alpha_{11})\right](\sigma_{22} - \alpha_{22})} \tag{3.28}\]

\[d\mu = \frac{\left[\sigma_{11} - \alpha_{11} - \frac{R}{1+R}(\sigma_{22} - \alpha_{22})\right]d\sigma_{11} + \left[\sigma_{22} - \alpha_{22} - \frac{R}{1+R}(\sigma_{11} - \alpha_{11})\right]d\sigma_{22}}{\left[\sigma_{11} - \alpha_{11} - \frac{R}{1+R}(\sigma_{22} - \alpha_{22})\right](\sigma_{11} - \alpha_{11}) + \left[\sigma_{22} - \alpha_{22} - \frac{R}{1+R}(\sigma_{11} - \alpha_{11})\right](\sigma_{22} - \alpha_{22})} \tag{3.29}\]

The back stress increments for each yield surface can be determined by Eq. (3.27) where \( d\mu \) for the currently actuated surface can be estimated by Eq. (3.28), and Eq. (3.29) is for
all surfaces within the currently actuated one. The yield surface center at the current step \((m+1)\) can be calculated by using Eq. (3.30).

\[
\alpha_y^{m+1} = \alpha_y^m + d\alpha_y
\]

(3.30)

The total stresses and the stress increments for the current step are used to determine both the increment back stresses and the back stresses of each yield surface. The stress increments for all the yield surfaces within and including the currently actuated one are the same and are used to determine the back stress increments of each yield surface. Because the back stresses of these yield surfaces are different, their \(d\mu\) values are distinct. Then, the back stress increments of one yield surface may be different from another.

Furthermore, it can be concluded that when the stress status is between successive yield surfaces \(N\) and \(N-1\), the \(K_p\) value for surface \(N\) has to be used to determine the stress increments contributed by \(d\varepsilon_{11}^k\), and the \(\bar{K}\) for surface \(N\) is used to determine the stress increments contributed by \(d\varepsilon_{11}'\). Simultaneously, the surfaces from \(I\) to \(N\) move together, and their back stresses can be determined by Eqs. (3.27), (3.28), (3.29), and (3.30). Only the currently actuated surface, surface \(N\), has both rigid body translation and uniform expansion, and surfaces \(I\) to \(N-1\) have rigid body translation only. As soon as the yield surface of \(N\) contacts and remains tangent to surface \(N+1\), surface \(N+1\) is actuated. Then, the same approach can be used to determine the stress increments. After the loading process is finished, all yield surfaces would be like those represented by Fig. 96.
3.14. This figure also can be used to represent the starting point of the reverse loading process. To trace the complicated deformation history of the sheet metal element, the current total stresses and the back stresses of all yield surfaces must be stored.

![Diagram of yield surfaces](image)

**Fig. 3.15:** Yield surfaces in contact with the first surface undergoing expansion together

**Reverse Plastic Loading Deformation Processes:**

Once the material reverse yields in the reverse loading process, the first yield surface starts to undergo uniform expansion until it expands to a specific size. Simultaneously, other surfaces contacted with the first surface not only stay tangent with the first surface but also expand an amount equal to that of the first surface (Fig. 3.15). This new model is created on the basis of the isotropic hardening model, the kinematic hardening model, and the Mroz multiple yield surfaces model, and on experimental observations. There are
two assumptions that form the basis for the new method. One is that the surface cannot penetrate others, the Mroz multiple surface assumption. The other is from observations made by Tan, Magnusson, and Persson (1994). One of their experimental conclusions states that for all materials tested, the transient portions of the reverse flow curves terminate after a plastic strain of about 5%-10%, and the subsequent steady portions can be either a continuation of or PARALLEL to the initial flow curve. Equation (3.31) shows the maximum size that the first surface can expand in the reverse loading plastic deformation process.

\[
\bar{S}^{1}_{\text{new}} = \bar{S}^{1}_{\text{old}} + CM \times (\bar{S}^{2}_{\text{old}} - \bar{S}^{1}_{\text{old}}) \tag{3.31}
\]

where \(CM\) could be either a function or a constant value for each process and \(\bar{S}^{1}_{\text{new}}\) is the maximum size of the first yield surface that can be expanded during reverse loading process, \(\bar{S}^{1}_{\text{old}}\) and \(\bar{S}^{2}_{\text{old}}\) are the sizes of the first and second surfaces at the end of the loading process.

For Hill’s 48 yield criterion, the value of \(\bar{S}\) can be determined by

\[
\bar{S} = \sqrt{(\sigma_{11} - \alpha_{11})^2 + (\sigma_{22} - \alpha_{22})^2 - \frac{2r}{1+r}(\sigma_{11} - \alpha_{11})(\sigma_{22} - \alpha_{22})} \tag{3.32}
\]

The \(CM\) value for each process can be determined by a simple multiple bending experiment that is simple and inexpensive, and this \(CM\) value can be greater than one.
Fig. 3.16: The first surface showing movement in the reverse direction

Fig. 3.17: The first surface contacting the second surface in the reverse direction
During this uniform expansion process, it is assumed that the total strain increment is comprised by $de_{11}$ only so that $\mathcal{M}$ is zero in the Eq. (3.18). Then, $\bar{K}$ of the first yield surface and Eq. (3.19) are used to determine the principal stress increments and effective plastic strain increment. After they expand to the specific sizes, the first yield surface starts to have a rigid body translation in the reverse direction (see Fig. 3.17). During this rigid body translation, the total strain increment in Eq. (3.17) is comprised of $de_1$ only. The principal stress incremental can be determined by Eq. (3.21). $K_p$, and back stresses, $\alpha_{11}$ and $\alpha_{22}$, of the currently actuated surface (the smallest surface in Fig. 3.16 and second surface in Fig. 3.17 are used for Eq. (3.21). The activity of this new model is the same as that of the Mroz multiple surface method after the first yield surface finished the uniform expansion to the specific size.

On the other hand, if the first yield surface did not contact any surface during the loading process, the concepts of the loading process can be utilized for the reverse loading process as well until it contacts others.

3.4 A Comparison of These Methods in the Cyclical Loading Process

As is well known, aluminum is the sheet metal for which springback is most difficult to predict, and, at the same time, the most desirable material for the car maker in the future. The Bauschinger effect can be observed when the sheet metal undergoes a cyclical loading process. Therefore, the methods proposed in this chapter have been used to determine the internal stress variation of a fiber layer in the stress space. This layer is
undergone cyclical loading process. The material property of the AA6111-T4 can be seen on Table 3.1. A comparison of the internal stress variation of a fiber layer in the stress space of four different hardening models is discussed as follows.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus $E$</td>
<td>69 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio $v$</td>
<td>0.33</td>
</tr>
<tr>
<td>Thickness $t$</td>
<td>1.0mm</td>
</tr>
<tr>
<td>Strength Coefficient $K$</td>
<td>540.521 MPa</td>
</tr>
<tr>
<td>Strain Hardening Exponent $n$</td>
<td>0.25</td>
</tr>
<tr>
<td>Normal Anisotropy $R$</td>
<td>0.602</td>
</tr>
</tbody>
</table>

Table 3.1: Material properties of AA6111-T4

Figures 3.18, 3.19, 3.20, and 3.21 show the deformation history in the stress space for a fiber layer within the AA6111-T4 sheet metal while it is undergoing a cyclic loading process, for example, a multiple bending processes. The cyclic deformation first produces a positive strain of 0.1 in the principal direction 1. Next, the true strain is varied in the sequence of $-0.1$, 0.1, and $-0.1$. From Figs. 3.18 and 3.19, it can be seen that the isotropic hardening model overestimates the internal stress, while the kinematic hardening underestimates the internal stress. The Mroz multiple yield surfaces, in Fig. 3.20, gives better results than both isotropic hardening and kinematic hardening models,
but it can not represent the real material behavior. The reason is that the plastic shakedown occurs at the cycle three. Therefore, the new method is more flexible and can represent real material phenomena better than the others, as shown in Fig. 3.21. All parameters of this material, i.e., the $CM$ values in Eq. (3.31), used here for the new method can be found on the springback simulation section in Chapter 5.

Fig. 3.18: The simulation result of the isotropic hardening model
Fig. 3.19: The simulation result of the kinematic hardening model

Fig. 3.20: The simulation result of the Mroz multiple surfaces model
Fig. 3.21: The simulation result of the new model
CHAPTER 4

A MODEL FOR SPRINGBACK PREDICTION

4.1 Introduction

During the forming processes, portions of the deformed sheet undergo stretching and single- or multiple-bending processes, which cause springback and side-wall curl. A typical multi-bending process involves bending and then unbending, or bending, unbending, and then reverse bending. For these kinds of bending processes, the cyclic loading model has to be applied. For example, the models in the previous chapter used to obtain the internal stress throughout the metal thickness are critical to the accurate computation of the loaded bending moment, which in turns affects the accuracy of springback prediction. Just as the Bauschinger effect has decisive influence on internal stress calculation, its inclusion in the cyclic loading model can also lead to significant gain in accuracy.

Most of the published research neglects the contribution of the Bauschinger effect on the loaded bending-moment calculation. Therefore, the theoretical development of this study and simulation results are presented in the following sections.
4.2 Calculation of the Internal Stress of the Sheet Metal Element

In order to obtain an accurate final shape after springback, the internal stress distribution must be precise; this concern has been discussed in the literature review section concerning springback. As discussed before, most sheet metal elements undergo a complicated deformation process. The internal stress distribution within the sheet metal depends on the deformation history in strain space. Therefore, if the deformation history in the strain space can be well defined, then the internal stress distribution can be precisely determined by the tracing methods stated in Chapter 3. Since the method of obtaining the strain deformation history was not discussed in Chapter 3, that information will be given in the present section.

The total strain of each element is assumed to be the summation of both membrane and bending strains throughout the thickness. Therefore, compensation for the bending strain has to be computed and added to the membrane strain to gain the current total strain of the sheet metal element throughout its thickness. The membrane strain can also be determined by membrane FEA code. The method to estimate this compensation will be addressed in the next section. Equation (4.10) was developed as the means to determine its magnitude. It can also be assumed that each membrane element consists of many fiber layers, e.g., 50 layers, throughout the sheet metal thickness, and every fiber layer can be treated as an independent single unit which has only undergone cyclical compression and tension during the forming process. This single unit has no relation with the adjacent ones. A sheet metal element consists of many units (layers), and each unit (layer) has its own stress, tension or compression. Thus, the internal stress
distribution through the sheet metal element can be determined while the stress of every fiber layer is known. However, the force equilibrium throughout the thickness of the sheet metal is ignored to simplify the problem.

Figure 4.1 shows the profile of a simple die surface and assumes a sheet metal element has been drawn from zone 1 through both zones 2 and 3, then eventually to zone 4. This die is a simple example showing how to compute the total strain of this element during the forming process. A more detailed discussion revealing how to determine membrane and bending strains follows below.

As soon as the element is drawn into zone 2 area from zone 1, the bending strain, because of the bending occurring on the die shoulder, has to be added into the membrane strain to obtain the current total strain. Within zone 2, the membrane strain may also be increased, and the bending strain would be changed if thinning occurs in this area. Similarly, once the element is drawn into zone 3, the bending strain of this whole element would become zero due to the unbending process. Thus, the total strains for all layers within this sheet metal element would be the same as the membrane strain. Furthermore, while this element is drawn into zone 4, a kind of reverse bending happens, that is, the bending strain becomes a negative value in zone 4, in contrast to the positive values it had in zone 2. As demonstrated in Chapter 3, the Bauschinger effect exists in this sheet metal element when reverse plastic deformation occurred.
For instance, when a sheet metal element is drawn from zone 1 to zone 4, some layers of this element already underwent reverse yield two, three or even more times. So, it is obvious that the Bauschinger effect must be considered to obtain an accurate calculation of internal stress distribution within this element. The methods already proposed in Chapter 3 can be utilized here for determining the internal stress distribution by tracking their deformation histories on the strain space. Because the above simple die example is a generic surface, this concept can be applied to the punch surface as well.
4.3 The Related Formulas for Calculating Bending Strain

Figure 4.2 displays the geometry of the sheet metal before deformation, after being stretched, and after bending. At the end of the bending process, it is assumed that the inner radius of the sheet metal element is equal to the radius of the tooling. Likewise, the outer radius of the sheet metal element can be obtained by the principle of the volume conservation. As pointed out by Daras and Majlessi (1982), the pure bending also can cause metal thinning if the metal has undergone strain hardening. Simultaneously, they also showed that this thinning amount is extremely small. That is the reason the metal thinning is assumed to be caused by the membrane strain only, that is, the amount of thinning produced by the bending process is negligible: see the thickness of the sheet metal in Figs. 4.2 (b) and (c). Therefore, after stretching (see the difference between Figs. 4, the current thickness can be approximately determined by membrane analysis as in Eq. (4.1)).

\[ t = t_o \exp(-\varepsilon_m) , \]  \hspace{1cm} (4.1)

where \( t \) and \( t_o \) are the current and original thickness of the stretched sheet, and \( \varepsilon_m \) is the membrane strain along the longitudinal direction, principal direction 1.

After the bending process, see Figs. 4.2 (b) and (c), the radii of the external layer and of the current middle of the sheet metal layer, \( r_e \) and \( r_{cm} \), can be expressed by the equations shown below.

\[ r_e = r_i + t = r_i + t_o \exp(-\varepsilon_m) . \]  \hspace{1cm} (4.2)
\[ r_{cm} = r_i + \frac{t}{2} = r_i + \frac{t}{2} \exp(-\varepsilon_m). \] (4.3)

Because of the principal of volume conservation (whereby the volume change caused by the elastic deformation is negligible), the volume between fiber \( AA' \) and \( BB' \) in Fig. 4.2 (b) is the same before and after the bending process. Thus, \( r_{AA'} \) can be computed by Eq. (4.4).

\[ r_{AA'} = \sqrt{\frac{2* L * Z_{AA'}}{\theta} + r_i^2}, \] (4.4)

where \( \theta \) is the bending angle which can be obtained through the geometry of the bent sheet metal and, \( Z_{AA'} \) is a coordinate measured from the bottom of the sheet metal, see Fig. 4.3 (a). Because it is assumed that the current middle fiber layer is the bending strain free layer, Eq. (4.5) shows the formula for determining the bending angle.

\[ \theta = \frac{L}{(r_i + t / 2)}. \] (4.5)

where \( t \) and \( L \) are the thickness and the length of the sheet metal element after stretching.
(a) The geometry of a sheet metal element before deformation

(b) The geometry of a sheet metal element after stretching

(c) The geometry of a sheet metal element after stretching and

Fig. 4.2: The geometry of the sheet metal element during the forming process
Because of Eq. (4.5), Eq. (4.4) can be rewritten as Eq. (4.6).

\[ r_{AA'} = \sqrt{2 * Z_{AA'} * (r_i + t / 2) + r_i^{-2}}. \]

(4.6)

Because the thinning caused by the bending effect is negligible, after the sheet metal undergoes pure bending and pure unbending, the metal thickness recovers to the original thickness it had before the deformation process, see zone 3 of the Fig. 4.1. The same formulas used for the bending process can be applied again for the reverse bending process. Some modifications of the above equations are explained below.

Figures 4.3 (a) and (b) display the geometry of the sheet metal after the unbending and reverse bending processes, respectively. In order to calculate the bending strain after the reverse bending process, a new parameter, \( Z' \), measured from the top of the sheet metal element, see Fig. 4.3 (a), is defined as

\[ Z_{AA'}' = t - Z_{AA'} \]

(4.7)

Therefore, Eq. (4.6) can be rewritten as

\[ r_{AA'}' = \sqrt{2 * Z_{AA'}' * (r_i' + t / 2) + r_i'^{-2}} \]

(4.8)

where \( r_i' \) is the new tooling radius, see zone 4 in Fig. 4.1, at the contact area causing reverse bending on the sheet metal element.
As assumed before, the total strain of the sheet metal element is the summation of both bending and membrane strains shown by Eq. (4.9). As revealed in this equation, the total strain free layer can be determined, but the stress free layer has to be determined by tracing the deformation history proposed in Chapter 3 if the deformation history is complicated.

\[ \varepsilon = \varepsilon_{\text{membrane}} + \varepsilon_{\text{bending}} \]  

(4.9)

When the middle radius, \( r_{cm} \), the radius of the bending strain free layer, of the sheet metal is determined, the bending strain can be found by

\[ \varepsilon_{\text{bending}} = \ln \left( \frac{r}{r_{cm}} \right) \]  

(4.10)

where \( r \) is the radius of fiber layer of interest.
(a) The geometry of the sheet metal after unbending

(b) The geometry of the sheet metal after reverse bending

Fig. 4.3: The geometry of the sheet metal element after unbending and reverse bending
4.4 Determination of the Final Shape of the Deformed Part after Springback

Because it is assumed here that sheet metal always conforms to the tooling (membrane assumption), the curvatures of the sheet metal near the tool-sheet contact regions are ignored, that is, the curvatures of the sheet metal exist only where the tooling contact areas have curvatures— for example, zones 2 and 4 in Fig. 4.1. Furthermore, due to membrane assumption, the radius of the tooling has to be greater than or equal to the sheet metal thickness by a factor of five, especially for aluminum. Actually, in practical sheet metal forming practices, the ratio of radius and thickness is kept equal to or greater than five. Because of this practice, it can be assumed that the middle of the sheet metal has a shape similar to the tooling geometry but a different radius after the forming process, and the geometry of the inner sheet metal exactly conforms to the tooling surface. Therefore, the final shape prediction, after springback, can be represented by the final geometry of the current middle layer within the sheet metal.

Once the formed part is removed from the tooling, the elastic recovery occurs. This elastic recovery, springback, normally consists of both in-plane, shrinkage, and through the thickness, rotation. The influence of the shrinkage is not as important as the rotational on the formed part quality, and its amount is also very small in comparison with the rotation. Therefore, only the rotational part of the deformation has been considered here for the final shape prediction. After the internal stresses of all layers are computed, the next step is to determine the loading bending moment of each element.
4.4.1 Determination of the Loading Bending Moment

The loading bending moment of every sheet metal element is usually different and can be determined separately and independently. The reason is that each sheet metal element might have a different deformation history from the others. Similarly, the internal force of each element can be found while its internal stress distribution is obtained.

Because every fiber can be treated as an infinitesimally thin layer, the method proposed in Chapter 3 can be used to compute the principal internal stresses of each layer. For the sake of simplification, the stress distribution between two layers is approximately fitted by linear interpolation. Thus, the more layers throughout the thickness of the sheet metal for which internal stress distribution is calculated, the more accurate the stress distribution within the sheet metal will be. Springback is an elastic unloading process, so the loading bending moment must be calculated first for determining elastic unloading. The loading bending moment can be determined by

\[
M = W \int_{-z}^{z} \sigma_1 \cdot y \, dy, \quad (4.11)
\]

where \( \sigma_1 \) is the true stress in principal direction 1, \( W \) is the width of the sheet metal on principal direction 2, and \( y \) is measured from the middle of the sheet metal along direction 3, as shown in Fig. 4.4.

Because the shape of the stress distribution within two layers is either a triangle or a trapezoid, Eq. (4.11) can be simplified as Eq. (4.12).
\[ M = W \sum_{i=1}^{n} A_i \cdot y_i \]  

(4.12)

where \( A_i \) is the stress area between two fiber layers, and \( y_i \) is the distance from the centroid of area \( A_i \) to the middle fiber layer.

A simple case is discussed below to illustrate how to apply Eq. (4.12) for calculating the loading bending moment. Figure 4.5 displays the internal stress distribution within a sheet metal. The stress distribution between fiber layers \( AA' \) and \( BB' \) is a trapezoid, and the stress distribution between fiber layers \( CC' \) and \( DD' \) can be represented by two triangles as well.
Fig. 4.4: The principal directions of the sheet metal before deformation.

Fig. 4.5: An example showing how to determine the loading bending moment.

Because a trapezoid can be divided into two portions, a rectangle and a triangle, each component has to be calculated for computing the loading bending of this area by using
Eq. (4.12). Therefore, the loading bending moment caused by the stress distribution between layers $AA'$ and $BB'$ can be computed as is shown below.

\[
M_{AA'}^{BB'} = W \times \sum_{i=1}^{2} A_i \times y_i = W \times (A_1 \times y_1 + A_2 \times y_2) \\
= W \times \left\{ \sigma_{AA'} \times (y_B - y_A) \times \frac{(y_A + y_B)}{2} + [\sigma_{BB'} - \sigma_{AA'}] \times \frac{(y_B - y_A)}{2} \times [y_B - \frac{(y_B - y_A)}{3}] \right\}
\]

The first term of the above equation is for computing the rectangular area, and the other one is for the triangular area. This same computational method can be applied to all fiber layers throughout the thickness of sheet metal, then the total loading bending moment can be determined easily.

4.4.2 Determination the new curvature after rotational springback

If the loading bending moment is in the clockwise direction as shown in Fig. 4.6, then the removal bending moment would be in the counterclockwise direction. Figure 4.6(a) illustrates sheet metal that has undergone pure bending, and the direction of the loading bending moment that is clockwise. As seen in Fig. 4.6(b), which explains the strategy to determine rotational springback, one side of the sheet metal element can be assumed to be hinged, and the other side is in a counterclockwise rotation caused by the removal bending moment during the springback phenomenon. It is assumed that the magnitude of the removal bending moment is equal to the loading bending moment, but in the opposite
direction. On the other hand, the element would rotate in the clockwise direction while the loading bending moment is in the counterclockwise direction. As the example shown in Fig. 4.7, this sheet metal element has been deformed in a sequence of pure bending and reverse bending and its loading bending moment is in the counterclockwise direction.

To determine the radius of the middle of the sheet metal element after springback, the following equation can be used:

\[
\frac{1}{r_{cm}^{before}} - \frac{1}{r_{cm}^{after}} = \frac{M}{E*I} = \frac{12(1 - \nu^2) * M}{w * t^3 * E}
\]

(4.13)

where \( r_{cm}^{before} \) and \( r_{cm}^{after} \) are the radii of the middle of the sheet metal element before and after springback, respectively, and \( M \) is the loading bending moment estimated from Eq. (4.11).
Fig. 4.6: (a) A diagram of sheet metal that has undergone pure bending and the direction of the loading bending moment. (b) Illustration of the strategy to determine the rotational springback of (a).
Fig. 4.7: The internal stress distribution of a sheet metal element after pure bending and reverse bending processes.

4.4.3 The Residual Stress Distribution after Rotational Springback

Because it is assumed that springback is caused by elastic recovery only with no reverse plastic deformation, all fiber layers have no movement during the springback.
phenomenon. The reason is volume conservation and ignoring the volume change caused by the elastic deformation. The internal stresses of each fiber layer have been already computed before springback. The residual internal stress of the principal direction 1 of each fiber layer of the sheet metal after springback can be easily estimated by

$$\sigma_{1\text{ residual}} = \sigma_1 - E' \left[ \ln \frac{r_{AA'}^{\text{before}}}{r_{cm}^{\text{before}}} - \ln \frac{r_{AA'}^{\text{after}}}{r_{cm}^{\text{after}}} \right]$$  \hspace{1cm} (4.14)

where $r_{AA'}^{\text{before}}$ and $r_{AA'}^{\text{after}}$ are the radii of fiber layer $AA'$ (see Fig. 4.3 (C)), of the sheet metal before and after springback, respectively.

Once the residual stresses of all fiber layers are calculated, the residual stress distribution within the sheet metal throughout the thickness can be obtained easily by using linear interpolation.

For determining the final shape of the deformed part, one end of the part has to be hinged. Then compute the rotational elastic unloading, computational rotational by one element at a time, from the first element on the fixed side to the last element. For example, half of a symmetric part is used for the forming simulation, so the first node, at the center of the part, would be grounded, and hinged, to predict the final shape after springback. From the above discussion, the procedure to predict the final shape and residual stress distribution of one element of the deformed part after springback can be separated into the following steps.

1. Use the approaches proposed in Chapter 3 to calculate the internal stresses
of all layers within the element.

2. Use the concept proposed in this chapter to determine the internal stress distribution within the element.

3. Utilize Eq. (4.12) to determine the bending loading moment.

4. Use Eq. (4.13) to compute the final radius of the middle of the sheet metal element after springback.

5. Use Eq. (4.14) can to compute the residual stresses of all layers within the element.

6. Use liner interpolation to get the residual stress distribution.

4.5 Some Modifications Must be Made on the Section Form to Have Springback Capability

Because Section-Form is a kind of rigid plastic implicit code with a 2 noded line element, some modifications and assumptions must be made to achieve springback prediction. These suggestions are discussed and listed as follows.

1. Each membrane element consists of many fiber layers throughout the sheet metal thickness. The reason for having many fiber layers within the 2-node line element is to use the concept proposed in section 4.3 to determine the bending strain distribution throughout the sheet metal thickness.

2. Every fiber layer has undergone the uniaxial cyclical loading process, tension and compression loading, only during the forming process.

3. Every fiber layer is independent of other layers for the stress calculation.
4. If one node of the element is drawn into the new zone, for example from zone 1 into zone 2 in Fig. 4.2, then this whole element can be assumed to be in the new area, zone 2.

5. In order to trace the deformation history of each element, the strain change at each forming time step can be equally divided into many sub-steps, e.g., 100 steps.

6. The membrane strain of each element can be determined from the node coordinates of the membrane element.
CHAPTER 5

A MULTIPLE-BENDING EXPERIMENT AND ITS SIMULATION RESULTS

5.1 Introduction

During forming, most of the sheet metal elements undergo a complicated deformation process that may comprise a sequence of stretching, bending, unbending, and reverse bending processes. For example, when an element flows through a square or round bead, its deformation history consists of stretching and BRB (Bending, Reverse-Bending, Bending) processes. Furthermore, an additional sequence of unbending, bending, and unbending is involved when the sheet metal element flows through die shoulder into the cavity. For this kind of complicated deformation, the total strain method will not predict springback accurately because of the Bauschinger effect.

In order to investigate the influence of the Bauschinger effect on the springback phenomenon, a multiple-bending experiment has been designed and performed. The experimental results in this chapter show that two identical sheet metal specimens may have the same final total strains but distinct by different springback amounts. The reason
is that their deformation histories in the strain space are disparate. From these experimental results, it can be found that the influence of the Bauschinger effect on the springback depends to a great extent on not only the material type but also on the deformation history in the strain space.

The purpose of this work is to investigate the influence of the Bauschinger effect on springback in sheet forming. Then, an analytical model can be developed to include the Bauschinger effect in the springback prediction. If the sheet forming process can be accurately simulated, the designer can move the draw beads as close as possible to the punch opening, and the addenda area also can be decreased. Therefore, the blank sheet size can be reduced. In addition to the material savings, more accurate simulations can reduce soft tool tryout time.

5.2 Experiment Tooling, Materials, and Procedures

The tooling setup, specimen materials, and procedures of this efficient and low-cost multiple experiment follow.

Figure 5.1 is a drawing of the press and the tooling setup of this experiment, and WD-40 was used as the lubricant. Three different inserts were used, and their radii ($R$, in Fig. 5.1(b)) are 1/2, 3/8, and 3/16 inches. Aluminum AA6111-T4 (AL), high strength steel (HS), aluminum killed draw quality steel (AKDQ), and Bake Hard (BH) steel were studied. Their properties and thicknesses are summarized in Table 5.1 in which $K$ is the strength coefficient, $n$ is the hardening exponent, and $R$ is the anisotropy factor. During
the experiment, the gap, d, between the punch and the insert was fixed. As can be seen in Table 5.1, all materials have different thicknesses, so the clearances for these four materials are different. The dimensions of all of the specimens were 5"x1".

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<table>
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Table 5.1: Material properties of testing materials
Fig. 5.1: (a) Thirty-ton ERC linkage press and (b) The tooling setup
For each material type and insert, four different deformation processes were performed. To observe the repeatability of the test, three specimens for each case were tested. The experimental procedure is summarized in the following:

B: The specimen was fixed by the pad and then bent by the punch when it moved down as shown in Fig. 5.2.

BR: After the specimen was deformed in pure bending, it was turned over and bent in the reverse direction. The deformation sequence of the BR process is the combination of Figs. 5.2 and 5.3.

BRB: The specimen was then turned over again and bent.

BRBR: The specimen was again turned over and the bending process was repeated for a sequence of bending, reverse-bending, bending, and reverse-bending.
Fig. 5.2: The deformation sequence of bending process
Fig. 5.3: The deformation sequence of reverse bending process
5.3 Measurement Methods

For measuring the bending angles of a sheet metal before and after deformation, two approaches were utilized in this experiment.

5.3.1 Bending Angle Before Springback

It is not convenient to measure the bending angle during the deformation process; therefore, a simple numerical approach was used to determine the bending angle. A simple equation (Eq. (5.1)) based on a purely geometrical assumption and membrane assumption is used. Because the bending angle was always between 0 and 90 degrees for this experiment, the bending angle before springback can be computed from the geometry of the setup as

\[ L_2(1 - \cos \theta) - L_3 \times \cos \theta + (R_i + t/2) \times \theta \times \cos \theta - R_i \times \sin \theta = 0 \]  

(5.1)

where \( t \) is the sheet metal thickness, \( R_i \) is the tooling bending radius, \( \theta \) is the bending angle, and the definitions of \( L_2 \) and \( L_3 \) can be seen in Fig. 5.1(b).

5.3.2 Bending Angle After Springback

Figure 5.4 shows a cross section of an AA6111-T4 specimen after springback, with data points measured by a coordinate measuring machine (CMM). These points were measured around the middle of the specimen, and the distance between points is based on the profile curvatures. This means the distance between points would be greater if the
curvature is small. Based on the cross section of the specimen, the bending angle after springback can be computed by the following procedure:

1. Extract the inner portion of the cross section.
2. Use two linear equations to fit the two straight parts of the specimen.
3. Utilize these two equations to calculate the bending angle of this specimen after springback.

Fig. 5.4: A cross section measured by CMM of an AL_A specimen

5.4 Experimental Results

Based on the measurement methods used, the bending angles after springback of all specimens can be seen from the Tables 5.2 to 5.13. The abbreviations used in these tables are the following.
On the first row and column of these tables, e.g. AL_A of Table 5.2, the first prefix indicates the material type and the capital letter after the material type denotes the insert type. The insert types are listed as below.

_A: the tooling insert radius is 1/2 inch, and the gap between the die and punch is 1.55 mm.

_B: the tooling insert radius is 3/8 inch, and the gap between the die and punch is 1.35mm.

_C: the tooling insert radius is 3/16 inch, and the gap between the die and punch is 1.10mm.

Tables 5.3, 5.4, and 5.5 are the experimental results for AA6111-T4, and the results of the high strength steel can be seen in the Tables 5.6, 5.7, and 5.8. Tables 5.9 to 5.14 give the bending angles after springback for AKDQ and Bake Hard steels.
### Table 5.2: The experimental results for AL_A

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Table 5.11: The experimental results for BK_A
**Table 5.12: The experimental results for BK_B**

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<td>79.55°</td>
<td>78.98°</td>
<td>78.81°</td>
</tr>
<tr>
<td>Specimen 2</td>
<td>79.61°</td>
<td>79.24°</td>
<td>78.51°</td>
<td>79.16°</td>
</tr>
<tr>
<td>Specimen 3</td>
<td>79.60°</td>
<td>79.05°</td>
<td>78.50°</td>
<td>79.86°</td>
</tr>
<tr>
<td>Average</td>
<td>79.36°</td>
<td>79.28°</td>
<td>78.66°</td>
<td>79.28°</td>
</tr>
</tbody>
</table>

**Table 5.13: The experimental results for BK_C**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>B</th>
<th>BR</th>
<th>BRB</th>
<th>BRBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen 1</td>
<td>80.70°</td>
<td>80.30°</td>
<td>81.46°</td>
<td>81.04°</td>
</tr>
<tr>
<td>Specimen 2</td>
<td>80.59°</td>
<td>81.62°</td>
<td>80.95°</td>
<td>80.10°</td>
</tr>
<tr>
<td>Specimen 3</td>
<td>80.81°</td>
<td>81.56°</td>
<td>80.19°</td>
<td>81.14°</td>
</tr>
<tr>
<td>Average</td>
<td>80.70°</td>
<td>81.16°</td>
<td>80.87°</td>
<td>80.76°</td>
</tr>
</tbody>
</table>
5.5 The Multiple-Bending Experimental Results

Figures 5.5, 5.6, and 5.7 show all the experimental results for four materials for three different inserts. As discussed above, both B and BRB processes have the same final total strains throughout the thickness of the sheet metal. Similarly, both BR and BRBR have the same total strains on all specimens as well. However, their springback quantities are different because their deformation histories in the strain space are different.

From the experimental results, it is clear that the influence of the Bauschinger effect on all three steels is not very significant. For instance, for HS, AKDQ and Bake Hard steels, the springback variance between the B and BRB processes are insignificant; that is, the final total strain method can be utilized for these steels to determine the springback, and the error would be still acceptable. On the other hand, the influence of the Bauschinger effect on the springback prediction for aluminum is very significant, and its influence would be cumulative, e.g., as with AA6111-T4 here. This phenomenon can be seen in Table 5.2 and may be explained as follows.

A comparison of the results of B with BRB in Table 5.2 shows the difference is 2.66 degrees. Furthermore, the difference is 1.30 degrees for BR and BRBR. It is apparent from the significant difference between the measurement for B and BRB, for example, that the total strain method would not accurately predict the springback amounts for the aluminum sheet metal while it undergoes a cyclical deformation process. That is a reason that it is difficult to predict the springback in aluminum. It can be concluded that the
Bauschinger effect must be considered in the internal stress calculation when the sheet metal element, especially for aluminum, undergoes complicated cyclical deformation.

The experimental data also can be used to derive the material parameters for the sheet metal after reverse yielding (cyclical loading), and these parameters can be utilized for more precise springback predictions when the sheet metal undergoes complicated deformation processes.

Fig. 5.5: The experimental results of the 1/2-inch insert
Fig. 5.6: The experimental results of the 3/8-inch insert

Fig. 5.7: The experimental results of the 3/16-inch insert
5.6 Simulation Results

5.6.1 Springback Prediction

For this experiment, only aluminum AA6111-T4 has been used to verify the performance of the new hardening model. There are very important reasons to use aluminum as an example instead of steels. The first one is that springback is more difficult to predict with aluminum than with steels. The second reason is the influence of the Bauschinger effect is cumulative as aluminum sheet metal undergoes complicated deformation. However, only the experimental results of the 1/2-inch and 3/8-inch inserts are used here. The reason is the ration of R/t in the 3/16-inch insert is less than 5.0 and the equations here is for small curvatures only (see Chapter 3). Furthermore, this ration is always designed greater than 6.0. The experimental data for two different bending radii are given on the Fig. 5.8, called A-Die here, and Fig. 5.9, called B-Die here. The bending radii of A-die and B-Die are 3/8-inch and 1/2-inch, respectively. B, BR, BRB, and BRBR on Figs. 5.8, 5.9, 5.10, and 5.11 mean bending; bending, reverse bending; bending, reverse-bending, bending; and bending, reverse-bending, bending, reverse-bending. The bending and reverse bending angles of A and B dies are 86 and 85 degrees, respectively. These angles were determined by Eq. (5.1). The illustration of the bending and reverse bending angle, θ, can be seen on the Fig. 3.1(a).

Figure 5.9 shows the comparison of the experimental data and simulation results for the bending radius 1/2-inch insert. A similar comparison for the 3/8-inch insert radius can be seen on the Fig. 5.12. The data plotted on Figs. 5.9 and 5.12 can be seen on the Tables
5.14 and 5.15. Figs. 5.10 and 5.13 give the experimental results with the new method simulation result only. For these springback simulations, only one element is utilized to model this multiple bending process, so its computation time is very efficient. As concluded in the earlier discussion of the cyclical loading simulation, both isotropic hardening and kinematic hardening models are not able to model the influence of the Bauschinger effect even if the kinematic hardening model is the most popular method of handling the reverse yield problem. However, the results obtained by the Mroz method also cannot represent the real phenomenon of this material and forming processes. This new method shows the best results within these methods. A more detailed discussion about this simulation of the new method follows.

In order to simplify the model problem, only twenty yield surfaces were used to model AA6111-T4. The effective stress of the first one is set as yield stress and the twentieth is set as the effective stress of 0.5 true strain. It is assumed that 50 layers are uniformly distributed throughout the sheet metal thickness. The $M$ value for the loading process is set as 0.5 and the $CM$ values for BR, BRB, and BRBR are 1.0, 3.0, 0.0, respectively. These values are obtained from the experimental data using A-Die, and good fitting results are obtained, see Fig. 5.9. These $CM$ values can be used to represent the material properties of AA6111-T4 undergoing cyclical deformation process. After these values are obtained from the A-Die, they are also applied to the B-die, and the best springback prediction results are obtained among all methods, see Fig. 5.12.
Fig. 5.8: The experimental results using the A-Die

Fig. 5.9: The comparison of the experimental results with the simulation results of the four different methods for A-Die with material AA6111-T4
Table 5.14: Comparison of experimental data with four different methods for A-Die with AA6111-T4

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PROCESS TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
</tr>
<tr>
<td>Exp. Average</td>
<td>77.41°</td>
</tr>
<tr>
<td>Isotropic</td>
<td>77.66°</td>
</tr>
<tr>
<td>Kinematic</td>
<td>77.09°</td>
</tr>
<tr>
<td>Mroz</td>
<td>77.09°</td>
</tr>
<tr>
<td>New</td>
<td>77.42°</td>
</tr>
</tbody>
</table>
Fig. 5.10: Experimental data with the simulation results of the new method for A-Die with material AA6111-T4

Fig. 5.11: The experimental results using the B-Die
Fig. 5.12: The comparison of the experimental results with the simulation results of the four different methods for B-Die with material AA6111-T4

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PROCESS TYPE</th>
<th>B</th>
<th>BR</th>
<th>BRB</th>
<th>BRBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. Average</td>
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<td>73.03°</td>
<td>72.09°</td>
<td>71.74°</td>
<td></td>
</tr>
<tr>
<td>Isotropic</td>
<td>74.86°</td>
<td>71.94°</td>
<td>70.26°</td>
<td>69.02°</td>
<td></td>
</tr>
<tr>
<td>Kinematic</td>
<td>74.19°</td>
<td>81.43°</td>
<td>76.00°</td>
<td>82.07°</td>
<td></td>
</tr>
<tr>
<td>Mroz</td>
<td>74.19°</td>
<td>74.18°</td>
<td>74.22°</td>
<td>74.22°</td>
<td></td>
</tr>
<tr>
<td>New</td>
<td>74.59°</td>
<td>73.02°</td>
<td>71.95°</td>
<td>70.28°</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.15: Comparison of experimental data with four different methods for B-Die with AA6111-T4

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Fig. 5.13: Experimental data with the simulation results of the new method for B-Die with material AA6111-T4

5.6.2 Internal Stress Distribution and Residual Stress Estimation
The internal stress distributions of these four methods for both the A-Die and the B-Die are given on the Figs. 5.14-5.21. The method for computing the residual stress distribution, mentioned in Section 4.4.3, is utilized here for four different hardening models. The residual stress distribution after springback can be seen on the Figs. 5.22-5.29. By observing these figures, it can be concluded that this new model has the best internal stress distribution and residual stress analysis while isotropic hardening overestimated the material hardening result, and kinematic hardening model estimates the material as too soft during the multiple-bending process. This method is also better than the Mroz method due to its reality and flexibility.
Fig. 5.14: The internal stress distribution of the AA6111-T4 specimen under the B process using the A-Die

Fig. 5.15: The internal stress distribution of the AA6111-T4 specimen under the BR process using the A-Die
Fig. 5.16: The internal stress distribution of the AA6111-T4 specimen under the BRB process using the A-Die

Fig. 5.17: The internal stress distribution of the AA6111-T4 specimen under the BRBR process using the A-Die

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Fig. 5.18: The internal stress distribution of the AA6111-T4 specimen under the B process using the B-Die

Fig. 5.19: The internal stress distribution of the AA6111-T4 specimen under the BR process using the B-Die
Fig. 5.20: The internal stress distribution of the AA6111-T4 specimen under the BRB process using the B-Die

Fig. 5.21: The internal stress distribution of the AA6111-T4 specimen under the BRBR process using the B-Die
Fig. 5.22: The residual stress distribution of the AA6111-T4 specimen under the B process using the A-Die

Fig. 5.23: The residual stress distribution of the AA6111-T4 specimen under the BR process using the A-Die
Fig. 5.24: The residual stress distribution of the AA6111-T4 specimen under the BRB process using the A-Die

Fig. 5.25: The residual stress distribution of the AA6111-T4 specimen under the BRBR process using the A-Die
Fig. 5.26: The residual stress distribution of the AA6111-T4 specimen under the B process using the B-Die

Fig. 5.27: The residual stress distribution of the AA6111-T4 specimen under the BR process using the B-Die
Fig. 5.28: The residual stress distribution of the AA6111-T4 specimen under the BRB process using the B-Die

Fig. 5.29: The residual stress distribution of the AA6111-T4 specimen under the BRBR process using the B-Die

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5.7 Conclusion

Even though the experimental data and simulations in this dissertation are for multiple bending processes only, the concept of this new model can be applied in the real forming process while the deformation history of each element is known or well defined. The deformation history of each element can be determined through either the FEA code or designer expectation and/or experience. If FEA is used, this method is a sort of post processor. Otherwise, it becomes a purely analytic method for springback prediction. Furthermore, this new hardening model also can be used as a generic hardening model for metal forming. Based on this chapter, some conclusions are made below.

1. The deformation history of the sheet metal element in the strain space has to be considered for tracking its internal stress in the stress space.

2. A good hardening model for the cyclical loading process has to be derived, so the Bauschinger effect can be modeled well. This new method handles the Bauschinger effect well in order to predict springback.

3. An efficient, low-cost multiple-bending experiment has been designed to obtain the material parameters, $CM$ values here, of sheet metal after reverse yield. This parameter helps this new methodology to predict more accurate the springback result.
CHAPTER 6

SPRINGBACK PREDICTION IN ALUMINUM STAMPINGS

6.1 Introduction

AA6022-T4 and AA6111-T4 are the most widely utilized aluminum alloy sheet metals for outer panels of automotive vehicles manufactured in North America. As is well known, the quality requirements for the outer panels are more stringent than inner panels and Class 2 surfaces of outer panels. Therefore, these two materials were subjected to multiple bending experiments (see Chapter 5), to obtain the material parameters after reverse yield. The new hardening model, proposed in Chapter 3, was then utilized to simulate the springback phenomenon and to verify its performance in comparison with other hardening models. AA6111-T4 was utilized as the experimental material in Chapter 5. Both experimental data and simulation results can be found in Chapter 5. The present chapter documents one more experiment with the AA6111-T4 with a clearance different from that used in Chapter 5 in order to check the performance of the new hardening model proposed in Chapter 3. Furthermore, an experiment and simulation utilizing AA6022-T4 is documented in this chapter as well.
6.2 Experimental Procedure and Results of Multiple Bending Experiments With AA6022-T4 and 6111-T4

6.2.1 AA6022-T4

The configuration of the tooling is shown in Fig. 5.1(b). The 160-ton Minister Press (see Fig. 6.1) was used for this experiment. Figure 6.2 shows the experimental tooling setup. Three different inserts, the same as in Chapter 5, and three pads were used during the experiment. They are shown in Figs. 6.3 and 6.4, respectively.

<table>
<thead>
<tr>
<th></th>
<th>AA6022-T4 (0.92 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yield Strength</td>
</tr>
<tr>
<td>0 Degree</td>
<td>174.06</td>
</tr>
<tr>
<td>45 Degree</td>
<td>160.35</td>
</tr>
<tr>
<td>90 Degree</td>
<td>148.10</td>
</tr>
<tr>
<td>Average</td>
<td>160.72</td>
</tr>
</tbody>
</table>

Table 6.1: Material Properties of AA6022-T4

As mentioned in Chapter 5, four different deformation processes were applied to the AA6022-T4 to investigate the influence of the Bauschinger effect on the springback phenomenon. These processes are B, BR, BRB, and BRBR. To obtain a high degree of
accuracy in the experimental results, five specimens for each insert radius and for each radius were used. The clearance for this experiment for AA6022 was set close to 15% of the sheet metal thickness, that is, the gap $d$ in the Fig.5.1 (b) is 1.06 mm. The material properties of AA6022-T4 can be seen in Table 6.1. As in Chapter 5, CMM was utilized to measure the deformed specimens after springback. These measurements were taken one week after the experiment had been completed. The reason for the delay is that some researchers have reported the springback phenomenon of AA6022-T4 is dependent on time (see Carden's thesis [1997]). The experimental results with AA6022-T4 can be seen in Tables 6.1, 6.2, and 6.3 for 3 different insert radii, respectively.
Fig. 6.1: 160-ton Minister Press
Fig. 6.2: A picture of tooling as shown in Fig. 5.1 (b).

Fig. 6.3: A picture of three different radii inserts.
Fig. 6.4: A picture of three different pads for three inserts

<table>
<thead>
<tr>
<th>1/2 Inch Insert</th>
<th>Process Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA6022-T4</td>
<td>B</td>
</tr>
<tr>
<td>Specimen 1</td>
<td>76.56°</td>
</tr>
<tr>
<td>Specimen 2</td>
<td>76.21°</td>
</tr>
<tr>
<td>Specimen 3</td>
<td>76.62°</td>
</tr>
<tr>
<td>Specimen 4</td>
<td>76.13°</td>
</tr>
<tr>
<td>Specimen 5</td>
<td>76.49°</td>
</tr>
<tr>
<td>Average</td>
<td>76.40°</td>
</tr>
</tbody>
</table>

Table 6.2: The bending angle after springback: the experimental results of AA6022-T4 using 1/2-inch insert
### 3/8 Inch Insert Process Type

<table>
<thead>
<tr>
<th>AA6022-T4</th>
<th>B</th>
<th>BR</th>
<th>BRB</th>
<th>BRBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen 1</td>
<td>79.52°</td>
<td>77.71°</td>
<td>75.90°</td>
<td>75.43°</td>
</tr>
<tr>
<td>Specimen 2</td>
<td>79.69°</td>
<td>77.49°</td>
<td>76.50°</td>
<td>75.93°</td>
</tr>
<tr>
<td>Specimen 3</td>
<td>79.35°</td>
<td>77.87°</td>
<td>75.86°</td>
<td>75.81°</td>
</tr>
<tr>
<td>Specimen 4</td>
<td>79.35°</td>
<td>78.30°</td>
<td>76.73°</td>
<td>75.54°</td>
</tr>
<tr>
<td>Specimen 5</td>
<td>78.96°</td>
<td>78.01°</td>
<td>77.00°</td>
<td>75.74°</td>
</tr>
<tr>
<td>Average</td>
<td>79.37°</td>
<td>77.88°</td>
<td>76.40°</td>
<td>75.69°</td>
</tr>
</tbody>
</table>

Table 6.3: The bending angle after springback: the experimental results of AA6022-T4 using 3/8-inch insert

### 3/16 Inch Insert Process Type

<table>
<thead>
<tr>
<th>AA6022-T4</th>
<th>B</th>
<th>BR</th>
<th>BRB</th>
<th>BRBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen 1</td>
<td>81.93°</td>
<td>80.87°</td>
<td>80.16°</td>
<td>79.35°</td>
</tr>
<tr>
<td>Specimen 2</td>
<td>82.12°</td>
<td>81.32°</td>
<td>79.97°</td>
<td>79.33°</td>
</tr>
<tr>
<td>Specimen 3</td>
<td>82.48°</td>
<td>80.96°</td>
<td>80.25°</td>
<td>79.21°</td>
</tr>
<tr>
<td>Specimen 4</td>
<td>82.34°</td>
<td>81.04°</td>
<td>80.11°</td>
<td>79.28°</td>
</tr>
<tr>
<td>Specimen 5</td>
<td>81.92°</td>
<td>81.15°</td>
<td>80.04°</td>
<td>79.34°</td>
</tr>
<tr>
<td>Average</td>
<td>82.16°</td>
<td>81.07°</td>
<td>80.11°</td>
<td>79.30°</td>
</tr>
</tbody>
</table>

Table 6.4: The bending angle after springback: the experimental results of AA6022-T4 using 3/16-inch insert

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6.2.2 AA6111-T4

The material properties of aluminum AA6111-T4 can be seen in Table 5.1. Four different deformation processes have also been performed on AA6111-T4 with a 2.5mm clearance gap. The experimental results of this experiment are given in Tables 6.5, 6.6, and 6.7 for three different inserts, respectively.

<table>
<thead>
<tr>
<th>1/2 Inch Insert</th>
<th>Process Type</th>
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</thead>
<tbody>
<tr>
<td>AA6111-T4</td>
<td>B</td>
</tr>
<tr>
<td>Specimen 1</td>
<td>72.14°</td>
</tr>
<tr>
<td>Specimen 2</td>
<td>71.93°</td>
</tr>
<tr>
<td>Specimen 3</td>
<td>72.12°</td>
</tr>
<tr>
<td>Specimen 4</td>
<td>72.25°</td>
</tr>
<tr>
<td>Specimen 5</td>
<td>72.42°</td>
</tr>
<tr>
<td>Average</td>
<td>72.17°</td>
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</table>

Table 6.5: The bending angle after springback: the experimental results of AA6111-T4 using 1/2-inch insert
### 3/8 Inch Insert Process Type

<table>
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<th>Specimen</th>
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<th>BRB</th>
<th>BRBR</th>
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</thead>
<tbody>
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<td>71.86°</td>
<td>71.30°</td>
</tr>
<tr>
<td>2</td>
<td>75.34°</td>
<td>73.51°</td>
<td>72.23°</td>
<td>71.55°</td>
</tr>
<tr>
<td>3</td>
<td>75.63°</td>
<td>73.42°</td>
<td>71.76°</td>
<td>70.95°</td>
</tr>
<tr>
<td>4</td>
<td>74.93°</td>
<td>74.11°</td>
<td>71.30°</td>
<td>70.88°</td>
</tr>
<tr>
<td>5</td>
<td>73.82°</td>
<td>73.78°</td>
<td>72.14°</td>
<td>71.66°</td>
</tr>
<tr>
<td>Average</td>
<td>74.95°</td>
<td>73.46°</td>
<td>71.86°</td>
<td>71.27°</td>
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</table>

Table 6.6: The bending angle after springback: the experimental results of AA6111-T4 using 3/8-inch insert

### 3/16 Inch Insert Process Type

<table>
<thead>
<tr>
<th>Specimen</th>
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<th>BRB</th>
<th>BRBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76.65°</td>
<td>75.02°</td>
<td>74.74°</td>
<td>74.17°</td>
</tr>
<tr>
<td>2</td>
<td>76.26°</td>
<td>75.33°</td>
<td>74.82°</td>
<td>74.19°</td>
</tr>
<tr>
<td>3</td>
<td>76.35°</td>
<td>75.39°</td>
<td>74.41°</td>
<td>74.38°</td>
</tr>
<tr>
<td>4</td>
<td>76.36°</td>
<td>75.36°</td>
<td>74.93°</td>
<td>74.44°</td>
</tr>
<tr>
<td>5</td>
<td>76.17°</td>
<td>75.32°</td>
<td>74.88°</td>
<td>74.10°</td>
</tr>
<tr>
<td>Average</td>
<td>76.36°</td>
<td>75.28°</td>
<td>74.76°</td>
<td>74.26°</td>
</tr>
</tbody>
</table>

Table 6.7: The bending angle after springback: the experimental results of AA6111-T4 using 3/16-inch insert
6.3 Conclusions From The Experimental Results Using AA6022-T4 and AA6111-T4

The experimental results using AA6022-T4 with three different inserts (Fig. 6.3), with the same clearance, 115% t gap, can be seen on the Figs. 6.5, 6.6, and 6.7, respectively. The averages of the experimental results of these three inserts are shown in Fig. 6.8 in order to see the springback characteristics and repeatability of AA6022-T4. In these four figures, the definitions of B, BR, BRB, and BRBR are exactly the same as those recorded in Chapter 5. The actual bending angles after springback of three inserts can be seen on Tables 6.2, 6.3, and 6.4. By examining these figures, it can be seen that the performance of AA6022-T4 on the springback is the same as AA6111-T4, as shown in Chapters 5 and 6. The repeatability of this material on springback also can be confirmed by observing the similarity of the three curves in the Fig. 6.8.

A comparison of the results of B with BRB in Table 6.2 shows that the difference is 3.25 degrees. Furthermore, the difference is 2.55 degrees for BR and BRBR. It can be concluded from these different results that the total strain method would not accurately predict the springback amounts for aluminum when it undergoes a cyclic loading process. The same observations can be made from the data recorded from experiments using AA6022-T4 and AA6111-T4. Similarly to Figs. 6.5-6.8, a number of figures for AA6111-T4 with 2.50 mm gap are shown on Figs. 6.9-6.12. As noted above, most sheet metal elements undergo a complicated deformation history during the forming process. For that reason, it is difficult to predict the springback in aluminum. Because aluminum is widely used in the forming industry due to its high strength and low weight, it is
necessary to have a good methodology to accurately predict springback in order to save costs and tryout times. This observation reinforces the conclusion made early that the Bauschinger effect must be considered in the internal stress calculation when the sheet metal element undergoes complicated cyclical deformation, especially for aluminum.

Fig. 6.5: The experimental results using AA6022-T4 with 1/2-inch insert.
Fig. 6.6: The experimental results using AA6022-T4 with 3/8-inch insert.

Fig. 6.7: The experimental results using AA6022-T4 with 3/16-inch insert.
Fig. 6.8: The experimental results for all three inserts

![Graph showing bending angle after springback for different inserts and process types.]

Fig. 6.9: The experimental results using AA6111-T4 with 1/2-inch insert.

![Graph showing bending angle after springback for different cases and process types.]

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Fig. 6.10: The experimental results using AA6111-T4 with 3/8-inch insert.

Fig. 6.11: The experimental results using AA6111-T4 with 3/16-inch insert.
Fig. 6.12: The experimental results for all three inserts using AA6111-T4

6.4 Simulation Results

6.4.1 Springback Prediction with AA6022-T4

The springback simulation results with AA6022-T4 will be given in this section. Following the same procedure in Chapter 5, the material parameters (the \( CM \) values) after reverse yielding were obtained with the 1/2 inch insert, and then these parameters were applied to the results for the 3/8 inch and 3/16 inch inserts. The following discussion concerns the simulation procedure and results with these three inserts.

For these springback simulations, only one element is utilized to model this multiple bending process, so its computation time is very efficient. As concluded in the cyclical loading simulation, both isotropic hardening and kinematic hardening models are not able
to model the influence of the Bauschinger effect even though the kinematic hardening model is the most popular method to handle the reverse yielding problem. However, the results obtained by the Mroz method also cannot represent the real phenomenon of this material and forming processes. This new method shows the best results within these four hardening models referred to earlier. A more detailed discussion about this simulation of the new method follows.

The bending angles for 1/2-inch, 3/8-inch, and 3/16-inch inserts are 87, 88, and 87.5 degrees, respectively. These angles were obtained by using Eq. (5.1). In order to simplify the model problem, only twenty yield surfaces are used to model AA6022-T4. The effective stress of the first one is set as the yield stress and the twentieth is set as the effective stress of 0.5 true strain. It is assumed that 50 layers are uniformly distributed through the sheet metal thickness. The $M$ value for the loading process is set as 0.5 and the $CM$ values for BR, BRB, and BRBR are 1.2, 3.2, 0.0, respectively. These values are obtained from the 1/2-inch insert, and good fitting results are obtained (Fig. 6.13). The comparison of all experimental data and simulation results of the new methodology is shown on the Fig. 6.14. These $CM$ values can be used to represent the material property of AA6022-T4 undergoing the cyclical deformation process. After these values had been obtained from the 1/2-inch insert, they were also applied to 3/8-inch and 3/16-inch inserts, and the best springback prediction results were obtained among all four hardening models, see Figs. 6.15 and 6.17. The comparison of all experimental data and simulation results of 3/8-inch and 3/16-inch inserts can be seen on Figs. 6.16, and 6.18, respectively.
Tables 6.8, 6.9, and 6.10 give all simulation results by different methods for these three inserts.

Fig. 6.13: Comparison of experimental results with the results of the four different methods for AA6022-T4 with 1/2-inch insert
Fig. 6.14: Experimental data with the results of the new method for AA6022-T4 with 1/2-inch insert.

Fig. 6.15: Comparison of experimental results with the results of the four different methods for AA6022-T4 with 3/8-inch insert.
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Table 6.8: Comparison of experimental results with the results of the four different methods for AA6022-T4 with 1/2-inch insert
Fig. 6.16: Experimental data with the results of the new method for AA6022-T4 with 3/8-inch insert.

Table 6.9: Comparison of experimental results with the results of the four different methods for AA6022-T4 with 3/8-inch insert.

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Fig. 6.17: Comparison of experimental results with the results of the four different methods for AA6022-T4 with 3/16-inch insert

Fig. 6.18: Experimental data with the results of the new method for AA6022-T4 with 3/16-inch insert
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Table 6.10: The Comparison of Experiment data with 4 different methods for 3/16-inch insert with AA6022-T4

It can be seen from Figs. 6.17 and 6.18 that the simulation results of the new method for BRB and BRBR processes have large deviations from the experimental average. The reason for these deviations is that the equations derived are for small curvature only (see Chapter 3). The ratio of R/t in the 3/16-inch insert is close to 5.0 which is the boundary of small and big curvatures bending. A rule of thumb in the automobile industry for aluminum panel is to design the ratio between the fillet radius and sheet metal thickness to be 6.0 at least. Therefore this problem can be avoided. It also can be seen on these two figures, the simulation results of isotropic hardening are closed to the experimental data. The isotropic hardening model is impossible to gain the better results than the new model since it can not handle the Bauschinger effect. Here, isotropic hardening model is
just lucky to get the right answer because its equations have the same assumption as others here, that is, the assumption is the normal stress is ignored.

6.4.3 Internal Stress Distribution and Residual Stress Estimation on AA6022-T4

The internal stress distributions of these four methods for both the 1/2-inch insert and 3/8-inch insert are given in Figs. 6.19-6.26. The method discussed in Chapter 4 is utilized here to compute the residual stress distributions of four different approaches. The residual stress distributions after springback can be seen in the Figs. 6.27-6.34. Examination of these figures shows that this new model has the best internal stress distribution and residual stress analysis while isotropic hardening overestimates the material hardening result and kinematic hardening model calculates the material as too soft during the multiple bending process. As can be seen, this method is also better than the Mroz method because of its accuracy and flexibility.
Fig. 6.19: The internal stress distribution of AA6022-T4 Specimen undergoing the B process with a 1/2-inch insert

Fig. 6.20: The internal stress distribution of AA6022-T4 Specimen undergoing the BR process with a 1/2-inch insert
Fig. 6.21: The internal stress distribution of AA6022-T4 Specimen undergoing the BRB process with a 1/2-inch insert

Fig. 6.22: The internal stress distribution of AA6022-T4 Specimen undergoing the BRBR process with a 1/2-inch insert
Fig. 6.23: The internal stress distribution of AA6022-T4 Specimen undergoing the B process with a 3/8-inch insert

Fig. 6.24: The internal stress distribution of AA6022-T4 Specimen undergoing the BR process with a 3/8-inch insert
Fig. 6.25: The internal stress distribution of AA6022-T4 Specimen undergoing the BRB process with a 3/8-inch insert.

Fig. 6.26: The internal stress distribution of AA6022-T4 Specimen undergoing the BRBR process with a 3/8-inch insert.
Fig. 6.27: The residual stress distribution of AA6022-T4 specimen undergoing the B process with a 1/2-inch insert.

Fig. 6.28: The residual stress distribution of AA6022-T4 specimen undergoing the BR process with a 1/2-inch insert.
Fig. 6.29: The residual stress distribution of AA6022-T4 specimen undergoing the BRB process with a 1/2-inch insert.

Fig. 6.30: The residual stress distribution of AA6022-T4 specimen undergoing the BRBR process with a 1/2-inch insert.
Fig. 6.31: The residual stress distribution of AA6022-T4 specimen undergoing the B process with a 3/8-inch insert.

Fig. 6.32: The residual stress distribution of AA6022-T4 specimen undergoing the BR process with a 3/8-inch insert.
Fig. 6.33: The residual stress distribution of AA6022-T4 specimen undergoing the BRB process with a 3/8-inch insert

Fig. 6.34: The residual stress distribution of AA6022-T4 specimen undergoing the BRBR process with 3/8-inch insert
6.4.2 Springback Prediction on AA6111-T4

AA6111-T4 was used as one of the materials for the experiments documented in Chapter 5. The material parameters (the $CM$ values for the new hardening model) of AA6111-T4 were obtained as described in the simulation section of Chapter 5. To check the repeatability and the performance of this new model, the $CM$ values obtained in Chapter 5 were used as the parameters of AA6111-T4 for predicting springback of this multiple-bending experiment. For this experiment, the gap between the punch and the die (insert) was relatively large, i.e., 2.5 mm. The bending angles for 1/2- and 3/8- inch inserts are 82.3 and 83 degrees, respectively. These angles were obtained by using Eq. (5.1). For this experiment, the range of the specimens varies from 126.68 mm to 126.73 mm. So, the average length for this experiment was 126.70 mm. The input values of Eq. (5.1) for the insert with the 1/2-inch radius were $L_2 = 15.2$ mm, $L_3 = 23.25$ mm, $t = 1.0$ mm, and $R_i = 1/2$ inch. The input values for the insert with 3/8-inch radius were $L_2 = 12.025$ mm, $L_3 = 23.55$ mm, $t = 1.0$ mm, and $R_i = 3/8$ inch. Here, only the simulation results of the bending angles after springback were documented since the internal stress and residual stress distribution already have been discussed in Chapter 5.

Figure 6.35 illustrates a comparison of four hardening models with the average of the experimental data. It can be concluded again that the new hardening model has the best performance among these models. Figure 6.36 shows the comparison of the simulation result of the new methodology with the experimental data. The numerical values of Fig.
6.35 can be found on Table 6.11. The same conclusion can also be obtained by observing Figs. 6.37 and 6.38 that are the results for the 3/8-inch insert. By comparing Fig. 6.36 with Fig. 5.12, and Fig. 6.38 with Fig. 5.9, it can be seen that the repeatability of this new methodology is good regardless of the tooling geometry and clearance. However, it is also found from Figs. 6.36 and 5.12 that the simulation results of BRBR process for both clearances with 1/2-inch insert were overestimated. This phenomenon can be eliminated by using more precise equipment to obtain the $CM$ values and by using more specimens (only three specimens were used in Chapter 5 to gain the $CM$ values). As mentioned before, only one element was used for this simulation, so it is very efficient.

![Graph](image)

Fig. 6.35: Comparison of experimental results for four different methods using AA6111-T4 with 1/2-inch insert
Fig. 6.36: Experimental data with the results of the new method for AA6111-T4 with 1/2-inch radius insert

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Table 6.11: Comparison of experimental results for four different methods for AA6111-T4 with 1/2-inch radius insert
Fig. 6.37: Comparison of experimental results for four different methods using AA6111-T4 with 3/8-inch radius insert

Fig. 6.38: Experimental data with the results of the new method for AA6111-T4 with 3/8-inch radius insert
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Table 6.12: Comparison of experimental results for four different methods using AA6111-T4 with 3/8-inch radius insert
CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 Summary of the Work

Because springback causes changes in the shape of sheet metal panels, major problems in the assembly process can occur; this phenomenon thus complicates the draw development process. Therefore, springback has to be compensated for by overcrowning, undercrowning, overbending, and underbending techniques on the die surface; these techniques still depend on the engineer’s experience and the methods of trial and error. Since springback largely depends on the material properties of the sheet metal, tooling geometry, and the friction between the sheet metal and tooling, etc., it is very difficult to predict. Furthermore, most sheet metal elements undergo complicated deformation, like cyclical loading, during the forming process. The Bauschinger effect is also involved and complicates the internal stress calculation of the sheet metal elements. To predict springback precisely, the influence of the Bauschinger effect must be considered.

A number of incremental equations of isotropic hardening model, kinematic hardening model, and Mroz multiple yield surfaces model have been derived (see Chapter 3). However, because these three hardening models cannot handle the Bauschinger effect
well, a new hardening model has been developed. It is based on the concepts of the Mroz multiple yield surfaces model and the observations of experimental results made and documented by other studies (see Chapter 3). On the basis of the equations and hardening models discussed in Chapter 3, a model for springback prediction and residual stress estimation has been proposed and discussed in Chapter 4.

A multiple-bending experiment was designed and carried out (see Chapter 5). Through this experiment, the influence of the Bauschinger effect on the springback was observed. This experiment and its results were particularly important because it has been observed that the Bauschinger effect has more significant influence on the aluminum, AA 6111-T4, than on AKDQ, Bake Hard, and HS steels. Through this experiment, the material parameters after reverse yield were obtained; this experiment can be used in the future to obtain material parameters for any steel or aluminum sheet metal. The material parameters can be applied to the new hardening model proposed in Chapter 3 for obtaining more precise springback prediction, especially for aluminum stamping. Both AA6111-T4 and AA6022-T4 were used as the experimental materials for multiple bending experiment. Experimental and simulation results of these two materials are presented in Chapter 6.

7.2 Contributions to The Present State of Knowledge

An accurate internal stress distribution in sheet metal elements is not easy to obtain, especially when the Bauschinger effect exists within the deformed part. Therefore, a number of hardening models have been utilized and compared in the springback
prediction for the sheet metal forming process. Their implementation into an incremental analytic method provides an efficient and accurate prediction for springback during the draw development process. Some research contributions of this study on the state of art are:

1. A number of incremental equations have been obtained concerning the isotropic hardening model, the kinematic hardening model, the Mroz multiple yield surfaces model, and the new hardening model (see item 2). By using these incremental equations, the internal stresses of the principal directions, 1 and 2, can be computed when its deformation history in the strain space is known.

2. By utilizing the concepts of the isotropic hardening model, the kinematic hardening model, and the Mroz multiple yield surfaces model, and the observations from the experimental results made and documented by other studies, a new hardening model has been created. This hardening model allows modeling of the Bauschinger effect. In comparison with the isotropic hardening model, the kinematic hardening model, and the Mroz multiple yield surfaces model, this new model performs better than these models in internal stress calculation when the sheet metal undergoes complicated deformation processes.

3. The influence of the Bauschinger effect has been observed through a multiple-bending experiment. It was concluded that the influence of the Bauschinger effect on the aluminum sheet metals, AA6111-T4 and
AA6022-T4, is more significant than on steels, and its influence would be cumulative.

4. Through the low cost and efficient multiple-bending experiment, the material parameters of the sheet metal after reverse yielding can be obtained and these parameters can be applied to the new hardening model for accurate internal stress calculation.

5. An incremental method for springback prediction has been developed and this method produces good agreement with the experimental data of the multiple bending experiment.

6. An incremental method for residual stress estimation after springback has been developed.

7.3 Future Work

Based on the work of this dissertation, the following tasks are suggested for the future research.

1. Even though analytical solutions of the problems of springback prediction and residual stress estimation have been completed in this work, these concepts also can be used as a kind of post processor in an FEA code to determine the final geometry and residual stress of the deformed part after springback. For example, these concepts can be integrated into the Section_Form, a 2 nodes line element rigid plastic code.
2. A new hardening model has been documented in this dissertation. This hardening model has been used here to determine the internal stress of a fiber layer within the sheet metal when the deformation history is known. Furthermore, this hardening model also can be used as a hardening model for forming simulation.

3. The concepts proposed in this dissertation can be expanded to the three-dimensional sheet metal forming problem.

4. The phenomenon of springback is considered as elastic unloading only. Actually, the reverse yielding might occur during the springback process. Therefore, the equations derived for the springback calculation need to consider whether the sheet metal might have reverse yield during the springback process. Similarly, the reverse yield phenomenon needs to be considered in the residual stress estimation as well.

5. For springback prediction in this dissertation, only rotational springback is considered. In order to obtain greater accuracy in springback prediction, shrinking springback must be considered as well.

6. It is now possible to create a reference table, similar to a handbook, which will have the material parameters for most aluminum and steel sheet metals after reverse yield through the multiple bending experiment. These parameters can then be used for the new hardening model.
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This appendix derives the formula for isotropic hardening and explains how to obtain the required material property information from the tensile test.

As mentioned in the literature review, the isotropic hardening can be written as

$$F(\sigma, \alpha) = f(\sigma) - \bar{\sigma}^2(\bar{\varepsilon}^p) = 0$$  \hspace{1cm} (A.1)

where the parameter $\alpha$ can be defined by either the total equivalent plastic strain

$$\alpha = \varepsilon^p = \int |d\varepsilon^p|$$  \hspace{1cm} (A.2)

or the total plastic work as shown in Eq. (A.3).

$$\alpha = W^p = \int \sigma_y d\varepsilon^p_y$$  \hspace{1cm} (A.3)

Here, Eq. (A.2) is taken to be the definition of $\alpha$.

Based on Eq. (A.1), Hill's 48 yield function for plane stress with normal anisotropy can be written as
\[ F(\sigma, \bar{\varepsilon}^p) = \sigma_{11}^2 + \sigma_{22}^2 - \frac{2R}{1+R} \sigma_{11} \sigma_{22} - \bar{\sigma}^2(\bar{\varepsilon}^p) = 0 \]  

(A.4)

Because the equivalent stress can be written in terms of \( f(\sigma) \), see Eq. (A.1), the normality without any arbitrary constant can be written as

\[ d\varepsilon_j^p = \frac{\partial \bar{\sigma}}{\partial \sigma_j} d\bar{\varepsilon}^p \]  

(A.5)

So, the plastic strain increments in both principal directions 1 and 2 are

\[ d\varepsilon_{11}^p = \frac{\partial \bar{\sigma}}{\partial \sigma_{11}} d\bar{\varepsilon}^p = \frac{\left(\sigma_{11} - \frac{R}{1+R} \sigma_{22}\right)}{\bar{\sigma}} d\bar{\varepsilon}^p \]  

(A.6)

\[ d\varepsilon_{22}^p = \frac{\partial \bar{\sigma}}{\partial \sigma_{22}} d\bar{\varepsilon}^p = \frac{\left(\sigma_{22} - \frac{R}{1+R} \sigma_{11}\right)}{\bar{\sigma}} d\bar{\varepsilon}^p \]  

(A.7)

where \( \bar{\sigma} \) is effective stress and it can be calculated as

\[ \bar{\sigma} = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 - \frac{2R}{1+R} \sigma_{11} \sigma_{22}} \]  

(A.8)

Because of the plane stress assumption in the principal direction 3 and the plane strain assumption in the principal direction 2, the elastic strain increment of each step can be determined by

\[ d\varepsilon_{11}^e = \frac{1}{E'} (d\sigma_{11} - \nu \times d\sigma_{22}) \]  

(A.9)

\[ d\varepsilon_{22}^e = \frac{1}{E'} (d\sigma_{22} - \nu \times d\sigma_{11}) \]  

(A.10)

where \( E' = \frac{E}{1 - \nu^2} \), and \( \nu \) and \( E \) are Poisson’s ratio and Young’s modulus, respectively.
Because of the plane strain assumption in principal direction 2, the summation of the elastic and plastic strain increments in principal direction 2 has to be zero or Eq. (A.11).

\[ d\varepsilon_{22} = d\varepsilon_{22}^e + d\varepsilon_{22}^p = 0 \]  

(A.11)

Similarly, the increment strain in principal 1 direction is the summation of the elastic and plastic strains or Eq. (A.12).

\[ d\varepsilon_{11} = d\varepsilon_{11}^e + d\varepsilon_{11}^p \]  

(A.12)

The incremental strain on the principal 1 direction can be determined by the geometry change. Substitute Eqs. (A.6), (A.7), (A.9), and (A.10) into Eqs. (A.11) and (A.12) gives

\[ d\varepsilon_{11} = \frac{1}{E'} (d\sigma_{11} - \nu \times d\sigma_{22}) + \frac{\left(\sigma_{11} - \frac{R}{1+R} \sigma_{22}\right)}{\bar{\sigma}} d\bar{\varepsilon}^p \]  

(A.13)

\[ d\varepsilon_{22} = \frac{1}{E'} (d\sigma_{22} - \nu \times d\sigma_{11}) + \frac{\left(\sigma_{22} - \frac{R}{1+R} \sigma_{11}\right)}{\bar{\sigma}} d\bar{\varepsilon}^p = 0 \]  

(A.14)

Because the consistency must be satisfied, the next step is to use the consistency concept to obtain one more equation for determining the increment stresses and the increment of effective plastic strain.

Because of consistency and Eq.(A.1), Eq. (A.15) can be obtained

\[ dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial F}{\partial \varepsilon^p} d\varepsilon^p = 0 \]  

(A.15)

By using the chain rule, \( \frac{\partial F}{\partial \varepsilon^p} \) can be computed as
Because of Hill's 48 yield criterion and the normal stress ignored, the above equation becomes

\[ \frac{dF}{d\varepsilon^p} = \frac{\partial F}{\partial \sigma} \cdot \frac{d\sigma}{d\varepsilon^p} = 2\bar{\sigma} \frac{d\bar{\sigma}}{d\varepsilon^p} \]

Then,

\[ dF = \left(2\sigma_{11} - \frac{2R}{1 + R}\sigma_{22}\right)d\sigma_{11} + \left(2\sigma_{22} - \frac{2R}{1 + R}\sigma_{11}\right)d\sigma_{22} - 2\bar{\sigma} \frac{d\bar{\sigma}}{d\varepsilon^p} d\varepsilon^p = 0 \quad (A.16) \]

where \( \frac{d\bar{\sigma}}{d\varepsilon^p} \) can be determined from a tensile test, and the procedure is explained at the end of this Appendix. For a hardening material, \( \frac{d\bar{\sigma}}{d\varepsilon^p} \) is always greater than zero. However, it is zero for perfect plastic material and negative for softening material such as soil. The definition of \( \frac{d\bar{\sigma}}{d\varepsilon^p} \) is exactly the same as \( E_p \) of Eq. (B.10) of Appendix B.

To simplify the above equation, let \( \frac{d\bar{\sigma}}{d\varepsilon^p} = \bar{K} \). This is not a constant but a variable whose magnitude depends on the material properties and the location of the yield surface on the stress space.

Then, Eq. (A.17) becomes

\[ \left(\sigma_{11} - \frac{R}{1 + R}\sigma_{22}\right)d\sigma_{11} + \left(\sigma_{22} - \frac{R}{1 + R}\sigma_{11}\right)d\sigma_{22} - \bar{K} \times \bar{\sigma} \times d\varepsilon^p = 0 \quad (A.18) \]

Using the three Eqs. (A.13) and (A.14) and (A.18) to solve for \( d\sigma_{11} \), \( d\sigma_{22} \), and \( d\varepsilon^p \).
gives

\[ d\sigma_{11} = \frac{E \times d\varepsilon_{11} \left[ (1 + R)^2 \times (E \times \sigma_{22}^2 + \bar{K} \times \bar{\sigma}^2) + E \times R \times \sigma_{11} \left[ R \times \sigma_{11} - 2\sigma_{22} (1 + R) \right] \right]}{\text{denominator}} \]  

(A.19)

\[ d\sigma_{22} = \frac{E \times d\varepsilon_{11} \left[ (1 + R) \times \left[ \bar{K} \times \nu \times \bar{\sigma}^2 (1 + R) + E \times R \times (\sigma_{11} - \sigma_{22})^2 \right] - E \times \sigma_{11} \times \sigma_{22} \right]}{\text{denominator}} \]  

(A.20)

\[ \overline{d\varepsilon}_p = \frac{E \times d\varepsilon_{11} \times \bar{\sigma} \left[ R^2 (1 - \nu) \times (\sigma_{11} - \sigma_{22}) + R \times [\sigma_{11} (2 - \nu) - \sigma_{22} (1 - 2\nu)] + \sigma_{11} + \nu \times \sigma_{22} \right]}{\text{denominator}} \]  

(A.21)

where \( \text{denominator} \) is

\[ \text{denominator} = (1 + R)(1 - \nu) \left[ \bar{K} \bar{\sigma}^2 (1 + R) (1 + \nu) + 2E \times R (\sigma_{11} - \sigma_{22})^2 \right] + E \left( \sigma_{11}^2 + 2\nu \times \sigma_{11} \sigma_{22} + \sigma_{22}^2 \right), \]  

(A.22)

and \( \bar{K} \) is obtained from the previous step, say step \( n \), and applied to the current step, say step \( n+1 \).

The current step stresses in principal directions 1 and 2, and the total effective plastic strain can be computed by

\[ \sigma_{11}^{n+1} = \sigma_{11}^n + d\sigma_{11} \]  

(A.23)

\[ \sigma_{22}^{n+1} = \sigma_{22}^n + d\sigma_{22} \]  

(A.24)

where \( n \) and \( n+1 \) are the step numbers, and \( d\sigma_{ij} \) is the stress increment between steps \( n \) and \( n+1 \).
and \( n+1 \). Also
\[
\bar{\varepsilon}^p = \tilde{\varepsilon}^p + d\bar{\varepsilon}^p
\]

Because the effective plastic strain increment should not be less than zero, Eq. (A.15) can be used to check the solutions of all the above increments. Finally, an example will be utilized to show how to determine \( \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p} \), \( K \), from the tensile test, and a more detailed explanation and derivative procedure are shown as follows.

Normally, the stress strain relation of the tensile test in the plastic region can be expressed by fitting an equation. For example, Hollen’s hardening rule is used to represent the relation of stress and strain and shown below.
\[
\sigma = Ke^n
\]

where \( \varepsilon \) is the total strain in the tensile test, and \( K \) and \( n \) are the strength coefficient and the strain hardening exponent, respectively.

Because the information needed for Eqs. (A.13), (A.14), and (A.15) is \( \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p} \), the stress-strain plot of the tensile test has to be converted to stress-plastic strain plot for determining \( \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p} \). A brief derivative below shows how to get the relation of stress and plastic strain. For example, \( \sigma = Ke^n \) is used to be the filling equation for the relation of the stress and strain.
So,

\[ \sigma = K (\varepsilon^p + \varepsilon^s)^n = K \left( \frac{\sigma}{E} + \varepsilon^p \right)^n \]  \hspace{1cm} (A.27)

\[ \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} - \frac{\sigma}{E} = \varepsilon^p \]  \hspace{1cm} (A.28)

Then, \( \frac{d\sigma}{d\varepsilon^p} \) can be obtained by differentiating Eq. (A.27).

\[ d \left[ \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} - \frac{\sigma}{E} \right] \left/ d\varepsilon^p \right. = 1 \]  \hspace{1cm} (A.29)

\[ \frac{1}{n} \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} \cdot \frac{d\sigma}{d\varepsilon^p} - \frac{1}{E} \cdot \frac{d\sigma}{d\varepsilon^p} = 1 \]  \hspace{1cm} (A.30)

\[ \frac{d\sigma}{d\varepsilon^p} = \frac{1}{\left[ \frac{1}{n} \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} - \frac{1}{E} \right]} \]  \hspace{1cm} (A.31)

The above equation is for the uniaxial tensile test, so \( \frac{d\sigma}{d\varepsilon^p} \) can be written as

\[ \overline{K} = \frac{d\sigma}{d\varepsilon^p} = \frac{1}{\left[ \frac{1}{n} \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} - \frac{1}{E} \right]} \]  \hspace{1cm} (A.32)

This equation assumes that Young’s modulus is always constant regardless of the deformation history. No matter what kind of equation is used to describe the relation of stress and strain of the tensile test, this concept can still be utilized for determining \( \frac{d\sigma}{d\varepsilon^p} \).
APPENDIX B

FORMULA DERIVATION OF BOTH PLASTIC MODULUS, $E_p$, AND $K_p$ VALUES

Hill's 48 yield criterion can be written as

$$f = \sigma_{11}^2 + \sigma_{22}^2 - \frac{2R}{1 + R} \sigma_{11} \sigma_{22} + \frac{2}{1 + r} \left( \sigma_3^2 - \sigma_{11} \sigma_{33} - \sigma_{22} \sigma_{33} \right) - k^2 = 0$$  \hspace{1cm} (B.1)

Because normality has to be held for the plastic strain increment, the plastic strain increment can be expressed as

$$(n_y \sigma_{ij}) n_{kl} = K_p \varepsilon_{ij}^p$$  \hspace{1cm} (B.2)

where $K_p$ is a proportional factor, and $n_y$ is the unit normal projection in the $ij$ direction.

Also, $n_y$ can be written as

$$n_y = \frac{\partial f}{\partial \sigma_y} \frac{\partial f}{\partial \sigma_y}$$  \hspace{1cm} (B.3)

The tensile test is the most widely used experimental method to determine sheet metal properties, and the value of $K_p$ can be determined by tensile test data as explained in the
following procedures. Equation (B.1) is the yield function used for determining the $K_p$ value. To use Eq. (B.2) to estimate the $K_p$ value, the partial differential of each principal stress has to be found first as shown below.

\[
\frac{\partial \phi}{\partial \sigma_{11}} = 2\sigma_{11} - \frac{2R}{1+R}\sigma_{22} - \frac{2}{1+R}\sigma_{33} \quad (B.4)
\]

\[
\frac{\partial \phi}{\partial \sigma_{22}} = 2\sigma_{22} - \frac{2R}{1+R}\sigma_{11} - \frac{2}{1+R}\sigma_{33} \quad (B.5)
\]

\[
\frac{\partial \phi}{\partial \sigma_{33}} = \frac{2}{1+R}(2\sigma_{33} - \sigma_{11} - \sigma_{22}) \quad (B.6)
\]

Because the tensile test has stress in the principal 1 direction only, $\sigma_{22} = 0$ and $\sigma_{33} = 0$, and the above equations become

\[
\frac{\partial \phi}{\partial \sigma_{11}} = 2\sigma_{11} \quad (B.7)
\]

\[
\frac{\partial \phi}{\partial \sigma_{22}} = -\frac{2R}{1+R}(\sigma_{11}) \quad (B.8)
\]

\[
\frac{\partial \phi}{\partial \sigma_{33}} = -\frac{2R}{1+R}(\sigma_{11}) \quad (B.9)
\]

Only the relation of the increment plastic strain in direction 1 and the total stress in direction 1 can be observed from the tensile test data. Therefore, only the plastic strain increment in principal direction 1 can be calculated as shown below.

\[
d\epsilon_{11}^p = \frac{1}{K_p} \frac{\partial \phi}{\partial \sigma_{11}} \frac{\partial \phi}{\partial \sigma_{11}} d\sigma_{11} \quad (B.10)
\]
\[
d\varepsilon_{11}^p = \frac{1}{K_p} \frac{4\sigma_{11}^2}{4\sigma_{11}^2 + \frac{4R^2}{(1+R)^2} \sigma_{11}^2 + \frac{4R^2}{(1+R)^2} \sigma_{11}^2} \, d\sigma_{11}
\]  
(B.11)

\[
d\varepsilon_{11}^p = \frac{1}{K_p} \frac{(1+R)^2}{(1+R)^2 + 2R^2} \, d\sigma_{11}
\]  
(B.12)

Because \(d\varepsilon_{11}^p\) can also be written as

\[
d\varepsilon_{11}^p = \frac{1}{E_p} \, d\sigma_{11}
\]  
(B.13)

where \(E_p = \frac{E \times E_t}{E - E_t}\), \(E_t\) and is the tangent modulus, the slope of the stress - total strain at any given point.

\(K_p\) can be rewritten as

\[
K_p = \frac{(1+R)^2}{(1+R)^2 + 2R^2} E_p
\]  
(B.14)

By observing Eq. (B.14), it is obvious that the coefficient of the right hand side of the Eq. (B.14) is 2/3 when the material is isotropic, \(R = 1\). This concept also can be extended to Hill’s 79 yield function and others.
APPENDIX C

THE STRESS INCREMENT IN THE PLASTIC DEFORMATION REGION

As mentioned in Eq. (B.3) of Appendix B, the plastic strain increment can be expressed as

\[ d\varepsilon_{ij}^p = \frac{1}{K_p} \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl} \]  

(C.1)

where \( K_p \) can be determined by Eq. (B.14).

Because of the plane strain assumption in principal direction 2, the summation of the elastic and plastic strain increments in principal direction 2 has to be zero as shown by Eq. (C.3). Similarly, the incremental strain in principal direction 1 is the summation of elastic and plastic strains as shown on Eq. (C.2). However, the incremental strain in principal direction 1 can be determined by the geometry change. For example, if a sheet metal has been stretched, then the incremental strain in the 1 direction is the strain difference between steps 1 and 2.

\[ d\varepsilon_{11} = d\varepsilon_{11}^e + d\varepsilon_{11}^p \]  

(C.2)

\[ d\varepsilon_{22} = d\varepsilon_{22}^e + d\varepsilon_{22}^p = 0 \]  

(C.3)
where the superscripts e and p mean elastic and plastic, respectively.

Since Hooke’s law can still be used to determine elastic strain increment, the elastic part of Eqs. (C.2) and (C.3) can be rewritten as

\[ \begin{align*}
    d\varepsilon_{11}^e &= \frac{1}{E'}(d\sigma_{11} - \nu \times d\sigma_{22}) \\
    d\varepsilon_{22}^e &= \frac{1}{E'}(d\sigma_{22} - \nu \times d\sigma_{11})
\end{align*} \]  

(C.4)

and

(C.5)

where \( E' = \frac{E}{1 - \nu^2} \).

Since Ziegler’s kinematic hardening rule is utilized here, Hill’s 48 yield function shown in Eq. (B.1) has to be modified as

\[
\begin{align*}
    f &= (\sigma_{11} - \alpha_{11})^2 + (\sigma_{22} - \alpha_{22})^2 - \frac{2R}{1 + R} (\sigma_{11} - \alpha_{11})(\sigma_{22} - \alpha_{22}) + \\
    &\quad \frac{2}{1 + R} [(\sigma_{33} - \alpha_{33})^2 - (\sigma_{11} - \alpha_{11})(\sigma_{22} - \alpha_{22}) - (\sigma_{22} - \alpha_{22})(\sigma_{33} - \alpha_{33})] - k^2 = 0
\end{align*}
\]

(C.6)

By using both Eqs. (C.1) and (C.6), the plastic strain increments on both principal directions 1 and 2 can be written as

\[
\begin{align*}
    d\varepsilon_{11}^p &= \frac{1}{K_p} \left( \frac{\partial f}{\partial \sigma_{11}} \left( \frac{\partial f}{\partial \sigma_{11}} d\sigma_{11} + \frac{\partial f}{\partial \sigma_{22}} d\sigma_{22} + \frac{\partial f}{\partial \sigma_{33}} d\sigma_{33} \right) \right) \\
    &= \frac{1}{K_p} \left( \frac{\partial f}{\partial \sigma_{11}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{22}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{33}} \right)^2
\end{align*}
\]

(C.7)
\[ \frac{d\varepsilon_{22}^P}{K_p} = \frac{1}{2} \frac{\partial f}{\partial \sigma_{22}} \left( \frac{\partial f}{\partial \sigma_{11}} \frac{d\sigma_{11}}{d\sigma_{22}} + \frac{\partial f}{\partial \sigma_{22}} \frac{d\sigma_{22}}{d\sigma_{33}} \right) \]

\[ \left( \frac{\partial f}{\partial \sigma_{11}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{22}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{33}} \right)^2 \]  

(C.8)

Due to the plane stress assumption, \( d\sigma_{33} \) is equal to zero. Therefore, the above equations become

\[ d\varepsilon_{11}^P = \frac{1}{K_p} \frac{\partial f}{\partial \sigma_{11}} \left( \frac{\partial f}{\partial \sigma_{11}} \frac{d\sigma_{11}}{d\sigma_{22}} + \frac{\partial f}{\partial \sigma_{22}} \frac{d\sigma_{22}}{d\sigma_{33}} \right) \]

\[ \left( \frac{\partial f}{\partial \sigma_{11}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{22}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{33}} \right)^2 \]  

(C.9)

\[ d\varepsilon_{22}^P = \frac{1}{K_p} \frac{\partial f}{\partial \sigma_{22}} \left( \frac{\partial f}{\partial \sigma_{11}} \frac{d\sigma_{11}}{d\sigma_{22}} + \frac{\partial f}{\partial \sigma_{22}} \frac{d\sigma_{22}}{d\sigma_{33}} \right) \]

\[ \left( \frac{\partial f}{\partial \sigma_{11}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{22}} \right)^2 + \left( \frac{\partial f}{\partial \sigma_{33}} \right)^2 \]  

(C.10)

Equations (B.7), (B.8), and (B.9) can be rewritten as Eqs. (C.11), (C.12), and (C.13) while Eq. (C.6) is used as the yield function.

\[ \frac{\partial f}{\partial \sigma_{11}} = 2(\sigma_{11} - \alpha_{11}) - \frac{2R}{1 + R}(\sigma_{22} - \alpha_{22}) - \frac{2}{1 + R}(\sigma_{33} - \alpha_{33}) \]

(C.11)

\[ \frac{\partial f}{\partial \sigma_{22}} = 2(\sigma_{22} - \alpha_{22}) - \frac{2R}{1 + R}(\sigma_{11} - \alpha_{11}) - \frac{2}{1 + R}(\sigma_{33} - \alpha_{33}) \]

(C.12)

\[ \frac{\partial f}{\partial \sigma_{33}} = \frac{2}{1 + R} \left[ 2(\sigma_{33} - \alpha_{33}) - (\sigma_{11} - \alpha_{11}) - (\sigma_{22} - \alpha_{22}) \right] \]

(C.13)

By using Eqs. (C.11), (C.12), and (C.13) and setting \( \sigma_{33} \) and \( \alpha_{33} \) as zero, Eqs. (C.9) and (C.10) can be rewritten as
\[ d\varepsilon_{11}'' = \frac{1}{K_p} \left( \frac{2S_{11} - \frac{2R}{1+R} S_{22}}{2S_{11} - \frac{2R}{1+R} S_{22}} \right)^2 \left( \frac{2S_{11} - \frac{2R}{1+R} S_{22}}{2S_{11} - \frac{2R}{1+R} S_{22}} \right) d\sigma_{11} + \left( \frac{2S_{22} - \frac{2R}{1+R} S_{11}}{2S_{22} - \frac{2R}{1+R} S_{22}} \right) d\sigma_{22} \]

(C.14)

\[ d\varepsilon_{22}'' = \frac{1}{K_p} \left( \frac{2S_{22} - \frac{2R}{1+R} S_{11}}{2S_{11} - \frac{2R}{1+R} S_{22}} \right)^2 \left( \frac{2S_{11} - \frac{2R}{1+R} S_{22}}{2S_{11} - \frac{2R}{1+R} S_{22}} \right) d\sigma_{11} + \left( \frac{2S_{22} - \frac{2R}{1+R} S_{11}}{2S_{22} - \frac{2R}{1+R} S_{22}} \right) d\sigma_{22} \]

(C.15)

where \( S_y = \sigma_y - \alpha_y \) and \( \alpha_y \) is the back stress of the yield surface.

Because of Eqs. (C.4), (C.5), (C.14), and (C.15), Eqs. (C.2) and (C.3) can be rewritten as Eqs. (C.16) and (C.17).

\[ d\varepsilon_{11} = \frac{1}{E} (d\sigma_{11} - \nu \times d\sigma_{22}) + \]

\[ \frac{1}{K_p} \left( \frac{2S_{11} - \frac{2R}{1+R} S_{22}}{2S_{11} - \frac{2R}{1+R} S_{22}} \right)^2 \left( \frac{2S_{11} - \frac{2R}{1+R} S_{22}}{2S_{11} - \frac{2R}{1+R} S_{22}} \right) d\sigma_{11} + \left( \frac{2S_{22} - \frac{2R}{1+R} S_{11}}{2S_{22} - \frac{2R}{1+R} S_{22}} \right) d\sigma_{22} \]

(C.16)

\[ d\varepsilon_{22} = \frac{1}{E} (d\sigma_{22} - \nu \times d\sigma_{11}) + \]

\[ = 0 \]

(C.17)

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By solving the above two equations, the stress increments in both principal directions 1 and 2 can be obtained as follows.

\[
\begin{align*}
\dot{\sigma}_{11} &= \frac{E' \times d\varepsilon_{11} \left[K_p \left(A^2 + B^2 + C^2\right) + B^2 \times E'\right]}{K_p \left(1 - \nu^2\right)\left(A^2 + B^2 + C^2\right) + E' \left(A^2 + B^2 + 2\nu \times A \times B\right)} \\
\dot{\sigma}_{22} &= \frac{E' \times d\varepsilon_{11} \left[K_p \nu \left(A^2 + B^2 + C^2\right) - A \times B \times E'\right]}{K_p \left(1 - \nu^2\right)\left(A^2 + B^2 + C^2\right) + E' \left(A^2 + B^2 + 2\nu \times A \times B\right)}
\end{align*}
\]

where \( A = \sigma_{11} - \alpha_{11} - \frac{R}{1 + R} (\sigma_{22} - \alpha_{22}) \), \( B = \sigma_{22} - \alpha_{22} - \frac{R}{1 + R} (\sigma_{11} - \alpha_{11}) \), and \( C = \frac{-1}{1 + R} (\sigma_{11} - \alpha_{11} + \sigma_{22} - \alpha_{22}) \).

Finally, the procedure on how to determine \( d\mu \) is shown as follows. As seen in Eq. (C.6), the yield function for kinematic hardening can be expressed as

\[
F(\sigma, \alpha) = f(\sigma - \alpha) - K^2 = 0
\]

Due to consistency,

\[
\frac{\partial \tilde{F}}{\partial \sigma_y} d\sigma_y + \frac{\partial \tilde{F}}{\partial \alpha_y} d\alpha_y = 0
\]

Because of \( d\alpha_y = d\mu(\sigma_y - \alpha_y) \) and \( \frac{\partial \tilde{F}}{\partial \alpha_y} = -\frac{\partial \tilde{F}}{\partial \sigma_y} \), the above equation can be rewritten as

\[
\frac{\partial \tilde{F}}{\partial \sigma_y} d\sigma_y - d\mu \frac{\partial \tilde{F}}{\partial \sigma_y} (\sigma_y - \alpha_y) = 0
\]
so,

\[ d\mu = \frac{\partial f}{\partial \sigma_y} d\sigma_y \]

\[ = \frac{\partial f}{\partial \sigma_y} (\sigma_y - \alpha_y) \]  \hspace{1cm} (C.23)

Use Eq. (B.6), \( d\mu \) can be expressed as

\[
d\mu = \left[ \sigma_{11} - \alpha_{11} - \frac{R}{1 + R} (\sigma_{22} - \alpha_{22}) \right] d\sigma_{11} + \left[ \sigma_{22} - \alpha_{22} - \frac{R}{1 + R} (\sigma_{11} - \alpha_{11}) \right] d\sigma_{22}
\]

\[
= \left[ \sigma_{11} - \alpha_{11} - \frac{R}{1 + R} (\sigma_{22} - \alpha_{22}) (\sigma_{11} - \alpha_{11}) + \left[ \sigma_{22} - \alpha_{22} - \frac{R}{1 + R} (\sigma_{11} - \alpha_{11}) \right] (\sigma_{22} - \alpha_{22}) \right]
\]

\hspace{1cm} (C.24)