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WARPAGE PREDICTION AND ELIMINATION IN FILAMENT WOUND AND FIBER PLACED COMPOSITE SHELLS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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1999

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ABSTRACT

Filament winding and fiber placement are becoming common manufacturing processes throughout the aerospace industry for both launch vehicle and aircraft structures. The promise of significant weight reduction, compared to metallic structures, along with the advantages of automation, including reduced cost and increased quality control, make these attractive structural solutions. Recent demonstrations include: the Combined Experiments (CEP) sub-orbital demonstration flight, Boeing’s Sea Launch orbital vehicle, and the Beach Starship. These vehicles employed filament winding or fiber placement manufacturing processes and substantiated large performance gains.

However, as with many new technology development programs unforeseen complications often arise. In filament winding the tooling is generally in the form of a plug that has the composite fibers filament wound or fiber placed on the outer surface. To facilitate part-tooling separation the part is usually split or sectioned after cure and this is when problems often arise. Once the parts are released from the tooling they can demonstrate significant warpage. In many cases this warpage is large enough to make the part unusable. As the aerospace industry develops the need for larger composite structures with both the requirements of high performance and reduced manufacturing cost, understanding and controlling part warpage becomes an ever increasing necessity.
This research has identified the residual stresses responsible for the large distortions. It will be shown that the stresses develop during manufacturing and result from cure consolidation. The majority of cure consolidation in composites results from resin bleed-out and evacuation of entrapped air (voids). The magnitude is dependent on manufacturing parameters including cure pressure, winding tension, and material characteristics (i.e., pre-preg fiber volume fraction, resin viscosity, etc.).

In this dissertation a systematic procedure, or methodology, is developed to eliminate the processing induced warpage in filament wound and fiber placed composite parts. This is accomplished by first developing a through-thickness strain model based on fiber/resin cure consolidation and tooling thermal expansions. Both constant and variable consolidation models are derived and the lay-up or stacking sequence can be arbitrary (i.e., symmetric or unsymmetric). The strain model is then integrated with classical laminate theory and solutions for predicting and eliminating warpage obtained. The warpage elimination is accomplished by developing manufacturing tension control techniques that reduce and alter the residual stress profile to eliminate stress couples. The accuracy of the warpage prediction and elimination techniques are verified with experimental procedures. To facilitate this, cylindrical and flat panel test specimens were manufactured, the cure consolidation characterized, and the warpage measured. It was found that the predictions were accurate and the warpage could be reduced and eliminated in most cases. The ability to control warpage through manufacturing controls will benefit the aerospace industry by decreasing tooling and manufacturing development costs, and provide increased dimensional stability of filament wound and fiber placed structures.
To my wife Jean
ACKNOWLEDGMENTS

This work was sponsored by the U.S. Air Force through the Palace Knight program in conjunction with the Air Force Research Laboratory. I would like to thank all the people at the Air Force Research Laboratory for their support both financial and technical, and especially Dr. Steven Huybrechts and Lee Underwood, for making my research possible.

I would like to acknowledge my adviser, Dr. Herman Shen, for his guidance and wisdom, and for making my time at The Ohio State University both educational and enjoyable. I would also like to acknowledge the many other professors at The Ohio State University who gave me support and direction throughout my studies.

I am very grateful to my family: my father and mother, Stacie, and Jesse for all their support and encouragement. Finally, I am most grateful to my lovely wife Jean who gave so freely of her time, suffered long periods of separation, and supported me in so many ways it is not possible to list them here. My education would not have been possible without her.
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PUBLICATIONS

Dissertation Research Publications


Other Research Publications


FIELD OF STUDY

Major Field: Aeronautical and Astronautical Engineering
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NOMENCLATURE

\( u, v, w \)  
Displacement in the \( x, y, \) and \( z \) Directions, Respectively.

\( \varepsilon_{x,y} \)  
Strain in Global Coordinate System.

\( \varepsilon_{1,2} \)  
Strain in Fiber Coordinate System.

\( \varepsilon_{1o} \)  
Strain Resulting from Winding Tension.

\( \varepsilon_{1} \)  
Strain Resulting from Processing.

\( \varepsilon_{R} \)  
Strain Resulting From Tooling Thermal Expansion.

\( \sigma_{x,y} \)  
Stress in Global Coordinate System.

\( \sigma_{1,2} \)  
Stress in Fiber Coordinate System.

\( \Omega_{11,12} \)  
Stiffness Matrix Quantities.

\( \bar{\Omega}_{xx,xy} \)  
Transformed Stiffness Matrix Quantities.

\( [A] \)  
Extensional Stiffness Matrix.

\( [B] \)  
Bending-Stretching Coupling Matrix.

\( [D] \)  
Flexural Stiffness Matrix.

\( \kappa \)  
Curvature.

\( T \)  
Percent Consolidation.

\( \alpha_i, \alpha_f \)  
Coefficient of Thermal Expansion for Tooling and Fiber, Respectively.

\( \Delta t \)  
Temperature Change.

\( \theta \)  
Angle Between the Global and Fiber Coordinate System.

\( R \)  
Radius of Tooling.

\( r, z \)  
Distance from Tooling Face and Mid-Plane to Lamina, Respectively.

\( P^e \)  
Effective Cure Pressure, (ECP).

\( P^a \)  
Autoclave Pressure.

\( P^{strain} \)  
Strain Induced Cure Pressure.
CHAPTER 1

INTRODUCTION

1.1 Background

Filament winding and fiber placement are becoming common manufacturing processes throughout the aerospace industry for both launch vehicles and aircraft structures. The promise of significant weight reduction, compared to metallic structures, along with the advantages of automation, including reduced cost and increased quality control, make these attractive structural solutions. Some recent demonstrations of these benefits are: the Combined Experiments (CEP) sub-orbital demonstration flight, Boeing’s Sea Launch orbital launch vehicle, and the Beach Starship. These vehicles employed filament winding or fiber placement manufacturing processes and substantiated large performance gains. Future applications of composite material systems in the aerospace industry include: the X-33 Reusable Launch Vehicle, the Evolved Expendable Launch Vehicle (EELV), and Raytheon’s Premier I and Hawker Horizon business jets.

The CEP program best illustrates the potential benefits of automated manufacturing and weight reduction in composites. This program was comprised of two flights: the
first utilized an aluminum payload fairing and the second employed a composite filament wound fairing as shown in Figure 1. The dual flights provided a direct comparison of vehicle performance under real world conditions as opposed to a paper study. The composite payload fairing proved to be 60% lighter and reduced the manufacturing time by 88% [4]. This resulted in a significant decrease in cost and an increase in vehicle performance as shown in Figure 2.

![Figure 1: CEP Launch Vehicle and Payload Fairing](image)

Manufactured using Filament Winding.

However, as with many new technology development programs unforeseen complications often arise. In filament winding and fiber placement the tooling is generally in
the form of a plug that has the composite fibers filament wound or fiber placed on the outer surface as shown in Figure 3. To facilitate part-tooling separation the part is usually split, or sectioned, along the longitudinal axis after cure. This is the point where problems often arise. Once the parts are released from the tooling they demonstrate significant warpage. In the CEP program the warpage was severe enough to make the part unusable. Fortunately in this case the fairing could be separated from the tooling, and later from the vehicle, as one piece. If this had not been the case the insertion of composite technology and the associated performance increase would not have been possible. As the aerospace industry develops the need for larger composite structures with both the requirements of high performance and reduced manufacturing cost, under-
standing and controlling part warpage becomes an ever increasing necessity.

Figure 3: Closed-Section Filament Winding. Payload fairing for sub-orbital flight being manufactured at Air Force Research Laboratory.

1.2 Problem Statement

Warpage in composite parts is a serious problem across the manufacturing spectrum. This warpage results from residual stresses induced during manufacturing and can be described as an unwanted change in the radius of curvature, $R$. In many cases the warpage after manufacturing, often referred to as “spring-back”, “spring-out”, “spring-in”, or simply “spring” can be large enough to make the part unserviceable. This is especially true in relatively large thin composite shells that are separated or sectioned after cure. This is commonly found in filament winding and fiber placement pro-
cesses. These parts are considered “closed-sections” due to their geometry and because they can support fiber stress before cure (Figure 3). This is an important distinction between ‘open-sections’ where no fiber stress can be maintained before the matrix or resin has set. After the parts cure they are cut axially, or sectioned, to allow for separation from the tooling. At this point the sectioned edges are free and spring occurs.

In the past the process of tool iteration was commonly used to get a usable final part. In this process a tool with the desired geometry is created and a part produced. The spring-out or spring-in that develops is then measured. Based on this a new tool is produced and the whole process is iterated until a usable part is obtained. It is easily seen that this is prohibitively expensive on large scale structures. It also greatly restricts future design modifications since any change in geometry, material systems, fiber stacking sequence, or fiber orientation will start the iterative process anew.

To fully realize the potential of filament wound and fiber placed composites in the aerospace industry a better understanding of the underlying causes of warpage must be achieved. Utilizing this basic understanding, methods for reducing and eliminating the warpage must then be developed. In this dissertation we will formulate a methodology to accomplish this, but first a review of past research is in order.

1.3 Literature Search

Research into spring in composite materials has been ongoing virtually since the advent of modern composites with the majority being on open-section structures. Much work has been done in the areas of cure shrinkage [9 - 14], fiber/matrix thermal mis-
match [15 - 19], in-plane and out-of-plane hygrothermal loads [20 - 29], thick shell closed-section laminates [30 and 31], and tension-lag [32], which all lead to spring predictions of varying accuracy. These topics of research are discussed in the following sections.

The literature review will be broken into two categories: closed-section and open-section. The ability of a part to maintain stresses before cure, which is the definition used in this paper for closed-section parts, is of critical importance and determines how the residual stresses develop during manufacture. For this reason closed-section and open-section definitions will be used to categorize the literature review. Although the research accomplished for this dissertation is for closed-section parts, the concepts and theories developed for open-section parts lay the foundation for the research presented. Open-section parts will be covered first.

1.3.1. Open-Sections

Many of the early theories to predict spring attribute blame to the coefficient of thermal expansion (CTE) mismatch between the part and tooling. Major efforts were made to develop low or matched CTE tooling to eliminate the problem. This solution proved successful in many areas. However, a significant problem with spring continued to exist even with well matched part-tooling CTE’s. Because of this more basic approaches were investigated, such as hygrothermal swelling and chemical shrinkage, that are independent of the part-tooling CTE problem. An even more basic approach was proposed by Radford [9 and 10] when he investigated warpage of flat uni-direct-
Through-Thickness Fiber Volume Fraction Variations

In these flat plates the through-thickness hygrothermal swelling, chemical shrinkage, and complex geometry effects do not contribute. Yet the plates still exhibit significant spring. Therefore the distortion must be caused by material property variations in the through-thickness direction. Radford concluded from his studies that fiber volume fraction variations could be responsible for a major portion of the spring [9 and 10].

Most modern vacuum cured composites are top bled, meaning the excess resin is pulled out through a bleeder material from the surface opposite the tool face. This causes a higher fiber Volume Fraction \( V_f \) at the bled surface and a lower \( V_f \) at the tool face. \( V_f \) is simply the measurement of the percent fiber by volume in a fiber/matrix composite material. This parameter is extremely important in composite material since the fiber will dominate material properties such as stiffness and strength. For most fiber/matrix composites these properties may be in excess of 10 times that of the matrix.

To determine if the volume fractions could be experimentally observed an investigation of the \( V_f \) profile was performed by Radford and using a carbon fiber/epoxy prepreg system to accomplish this. The specimens were long thin uni-directional composite laminates (500 mm long by 25 mm wide and either 0.63 mm or 1.5 mm thick), and cured using an autoclave according to the prepreg manufacturer's recommendations. Long narrow specimens were used to eliminate the effects of curvature, \( k_2 \), in the off fiber direction. The \( V_f \) was determined using optical microscopy.
Radford found that there can be a significant variance in the $V_f$ of a laminate. The $V_f$ at the tooling surface was measured to be 41% while the $V_f$ at the vacuum bagged surface was found to be 58%. The $V_f$ away from the surface was more difficult to determine precisely because of data scatter. Utilizing an assumed spike profile volume fraction model, along with classical laminate plate theory, the curvature changes were predicted. The spike profile assumes the middle plies have a consistent $V_f$ and the outer plies account for the change.

The results of the classical laminate plate theory analysis showed that the predicted curvatures followed the same trends as that seen in the experiments. However, the theory showed significant error in predicting the magnitude of the spring. The difficulty in accurately matching the experimental results were two fold. First, the isothermal cure shrinkage in the resin, which is the basic physical phenomena responsible for the warpage or curvature change, depends on various cure parameters including: the final state of cure, dwell times in the cure cycle, and the maximum temperature used during cure. The percent shrinkage can range from 0.5 to 2.0% and depending on the value used in the analysis different degrees of accuracies are obtained. The second contributing factor was the volume fraction profile. The spike profile is simply an approximation and does not exactly represent the actual condition. These two factors reduce the accuracy of the model. However, the theory does give improved results when compared to previous work [11]. Radford also predicted that this variation in $V_f$ could account for the discrepancies in warping of male vs. female molded angled section that other authors had encountered. For the male mold the $V_f$ effect would be additive to the
Hygrothermal Swelling

As discussed above the hygrothermal swelling (or hygrothermal loading) also contributes to residual stresses. In papers by Kollar [20 and 22] formulas were derived to calculate the stresses, strains, and displacements caused by hygrothermal loading. The calculations were done for both plates and shells utilizing thin shell approximations along with laminate plate theory. Effective thermal expansion coefficients were also derived. These were used to account for the change in laminate thickness in a Finite Element Method (FEM) code. The through-thickness change, $\Delta h(z)$, is due to the hygrothermal loading and the Poisson’s effect from the in plane loading and is:

$$\Delta h(z) = \Delta h'(z) + \Delta h''(z)$$

(1)

where

$$\Delta h'(z) = \int_0^z CTE \, dz$$

$$\Delta h''(z) = \int_0^z \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} \begin{bmatrix} S_{perp} \\ \nu_{31} E_1 \\ \nu_{32} E_2 \end{bmatrix} \, dz$$

(2)

and $\alpha_z$ is the CTE in the out of plane direction.

For isotropic materials this out of plane expansion will not contribute to warpage but
for anisotropic materials it is important. The analysis was done for both constant and linearly varying thermal and moisture gradients. Sample problems were then investigated to determine the accuracy between the thin shell approximation (referred to as the Approximate solution) and a 3D elastic FEM code (which was treated as the Exact solution). A comparison was also made with a simplified solution that neglected the shell stretching terms which originate due to shell curvature. It was found that for linear temperature variations through the thickness the approximate and exact solutions agreed well when radius-to-thickness ratios were greater than 10, $R/h > 10$. With constant temperature variation profiles all three solutions agreed well. There was no comparison to test data in these papers.

**Chemical Shrinkage**

Two papers by Jain et al [13 and 14] extended the work of Kollar to include chemical shrinkage in shell structures. Chemical shrinkage is a complex function of the polymerization occurring during cure of the matrix and is difficult to characterize analytically. As a result the shrinkage was determined from experimental data. The through-thickness strain was specified by:

$$
\varepsilon^k_z(z) = \frac{\partial W}{\partial z} = \alpha^k_z \Delta T z + S^k_{\text{perp}} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} + \varepsilon^c_z(z)
$$  \hspace{1cm} (3)

As can be seen by comparing Equations (1 and 3) the addition to Kollar's theory is the chemical shrinkage term $\varepsilon^c_z(z)$. In their first paper the authors developed a theoretical model based on classical laminate plate theory and modified shell theory. Their
modification allowed for higher accuracy at lower radius-thickness ratios, \( R/h \). In applying conventional shell theory to a cylinder the strain in the direction of curvature can be represented by

\[
\varepsilon_y = \frac{1}{(1 + z/R)} (\varepsilon_y^0 + z\kappa_y)
\]

(4)

The standard practice for thin shells with \( R/h \geq 10 \) is to allow \( z/R \to 0 \). For their papers [13 and 14] Jain and Mai made a more elaborate substitution

\[
\frac{1}{(1 + z/R)} = 1 - \frac{z}{R} + \left(\frac{z}{R}\right)^2 - \left(\frac{z}{R}\right)^3
\]

(5)

This allowed for accurate results down to \( R/h \geq 2 \). This was necessary since these ratios are encountered in the angle sections they were investigating (Figure 4).

**Figure 4:** Open-Section Spring Investigation. Spring in angled sections deforms section from desired angle.
It was found that the predicted results agreed well with the predictions made by Kollar and other previously published results. It was also determined that the effects of the chemical shrinkage are important and must be included along with the hygrothermal swelling.

In the second paper by Jain et al [14] testing was done utilizing T300/934 carbon/epoxy unidirectional pre-preg tape, Ciba-Geigy BU230-E01 uniweave fabric and Ciba-Geigy RTM6 resin using resin transfer molding (RTM). The test specimens were simple angle sections manufactured using both female and male tooling (Figure 4). Different bend radii and bend angles were tested. It was found that, in general, the agreement between theoretical and experimental data was poor in thin laminates but became more accurate with thicker laminates, greater than 8 plies. Four reasons were given to explain the error: (1) difficulty in accurately measuring the spring, (2) material properties were determined from micromechanics and may be inaccurate, (3) resin shrinkage was not known and assumed to be 1.5%, and (4) consistencies of lay-up angle may not be good. Another problem discussed by the authors was bridging. This is the development of resin rich areas in the laminate during cure. This is similar to the effect found by Radford [9]. Depending on whether male or female tooling was used different levels of bridging were noted. It was also found that the spring-in did not depend on lay-up sequence or tool material (for symmetric lay-ups).

1.3.2. Closed-Sections

Closed-section parts differ from open section parts in that they are able to maintain
internal stresses before the matrix has cured. This can result in additional residual stresses being imparted to the final part along with those discussed above. Much of the research and theories developed for open-section parts have been applied to closed-sections with varying success. In this section the research accomplished specifically for closed-section parts is discussed.

**Thick Laminated Rings and Thermal Stresses**

One of the initial topics of research in closed-section structures was the investigation of the response of thick laminated composite rings to thermal stress by Ajit Roy [31]. In this research Roy looked at thick laminated rings. These rings had relatively low radius to wall-thickness ratios. In most cases the ratio was less then 10, which is the standard cutoff point for thin shell approximations. The interest in such thick-walled structures was for deep submersible applications.

According to Roy, processing of these thick structures is challenging and the residual stresses imparted can be significant. These stresses result from the anisotropy of the laminate, tooling CTE’s, and the large temperature differential between the cure and operational environments. Roy accomplished a ply-by-ply stress analysis for the multi-layered cylindrical sections, or rings (Figure 5). These rings are manufactured by filament winding a cylinder and cutting it into short sections. For the filament winding process each ply, or lamina, wound at an angle $\theta$ is accompanied by an additional ply with the opposite angle, $-\theta$. Roy treated each of these ply pairs as an orthotropic unit. Linear elastic material behavior was assumed and the rings were also assumed to be in plane stress in the $(r, \phi)$ or $(y, z)$ plane. The rings are symmetric so there will be no
Figure 5: Thick Laminate Cylindrical Sections (Rings). (a) Shows laminate with Tooling Mandrel. (b) Shows Loading applied by mandrel during cure. (c) Shows enlarged view of cut laminate with normal traction stresses, $q_{\mu}$, resulting from cure.

shear stress, $\tau_{r,\phi} = 0$. The constitutive relationships are:

$$\varepsilon^i_r - \alpha^i_r \Delta t = S^i_{rr} \left( \sigma^i_r - \frac{v^i_{r\phi}}{k_i^2} \sigma^i_\phi \right)$$

$$\varepsilon^i_\phi - \alpha^i_\phi \Delta t = S^i_{\phi\phi} \left( \sigma^i_\phi - \nu^i_{r\phi} \sigma^i_r \right)$$

where $\nu^i_{r\phi} = -S^i_{r\phi} / S^i_{\phi\phi}$, $k_i = \sqrt{S^i_{rr} / S^i_{\phi\phi}}$, $\Delta t = t_o - t_c$. For Roy's notation $t_o$
and \( t_o \) are the operating and cure temperatures respectively, \( S_{ij}(i, j = r, \phi) \) are the elements of the compliance matrix, and \( \alpha_i (i = r, \phi) \) are the coefficients of thermal expansion.

The equilibrium equation with circular symmetry is simply

\[
\frac{d\sigma_r^{(i)}}{dr} + \frac{\sigma_r^{(i)} - \sigma_\phi^{(i)}}{r} = 0
\]  

(7)

The kinematic equations can then be used along with Equations (6 and 7) to get the final form of the equilibrium equation

\[
\frac{d^2 u_r^{(i)}}{dr^2} + \frac{1}{r}\frac{du_r^{(i)}}{dr} - \frac{k_i^2}{r^2} = \frac{1}{r}[\alpha_r^{(i)} (1 - \nu_r^{(i)}) - \alpha_\phi^{(i)} (k_i^2 - \nu_r^{(i)})]\Delta t
\]  

(8)

Equation (8) is a linear second-order non-homogeneous equation and when solved gives a series of linear equations for the displacements, \( u_r^{(i)} (i = 1, 2, \ldots n) \), and stresses, \( \sigma_r^{(i)} (i = 1, 2, \ldots n) \). Utilizing the boundary conditions, the displacement and stress can be found and written in terms of the tractions \( q_i (i = 1, 2, \ldots n) \) (Figure 5).

Again, the rings being analyzed are manufactured using filament winding techniques and are considered thick shell laminates. The filaments are wound on the mandrel with varying levels of tension depending on manufacturing parameters including: geometry, material system, and winding angles. With the consolidation that occurs during cure the tension is almost entirely relieved in thick shell laminates. Because of this Roy ignored the winding tension effects. As will be shown later, for thin shells the winding tension effects cannot be ignored and become the dominate contributor to the
residual stresses. Roy also assumes that as the consolidation takes place during cure the resin viscosity of the matrix is very low and this allows the fibers to readjust and the mandrel to expand freely.

To compare the theoretical and experimental results the rings are cut and the opening or closing of the ring measured. When the ring is cut it is stress free along the cut surface. To determine the residual stresses a moment $M$, as shown in Figure 6, can be applied to force the ring back to its initial form. This moment, or stress couple, is the result of a hoop stress gradient through the thickness and can be expressed as

$$M = \sum_{i=1}^{n} \int_{r_i}^{r_{i+1}} \sigma_{\phi}^{(i)} r dr$$

(9)

The moment in Figure 6 can be positive or negative depending on whether the ring experiences spring-out or spring-in.

Figure 6: Stress Couple in Cut Ring. $M$ is moment required to either open or close ring to un-cut configuration.
It was found that the calculated results were within 15% of the experimental data. Part of this error is attributed to the unavailability of the material properties needed to get the compliance element $S_{rr}$. It was also found that for a constant wind angle, as $\theta$ was varied from $0 \rightarrow 90^\circ$ the spring-in increased from zero to a maximum at $\theta = 90^\circ$. To eliminate the spring in cylinders with varying wind angle it was found that the outer plies should be wound with a smaller angle. This is really only of academic interest since for the majority of structures the ply lay-up is determined by loading conditions.

The effects investigated by Roy account for the asymmetric properties of the thermal expansion in thick composite laminates. It will be shown later that for thin shells the winding tension/cure consolidation contributions to the stress couples will become dominate. However, this research is of interest since it was some of the first work done on closed-section structures. The methods developed for deriving and relating the spring measurements from experimental tests to the stress resultants and couples were used by Kollar [20 - 22] and are used throughout this dissertation.

*Tension-Lag*

Maji and Ganley [32] proposed that the spring in filament wound closed-sections was due to high tooling CTE and an associated tension-lag. In their experiments they used a relatively high CTE tooling material (Stainless Steel) along with a low CTE carbon fiber/epoxy composite material system. This particular material system uses a thermal autoclave cure cycle. The tooling was cylindrical and had multiple thin composite rings, or hoop sections, filament wound onto it for each test. The lay-ups all had 90
degree (hoop direction) ply orientation with various laminate thicknesses. Maji and Ganley proposed that as the part is cured the tooling mandrel will attempt to expand and place tensile stress on the composite ring. These tensile stresses are a result of the carbon fibers having a much lower CTE than the tooling mandrel. The stress will be in the form of a linear stress profile maximum at the tooling surface and decreasing as you move away from the surface. At a point part way through the laminate the stress caused by the mandrel expansion is no longer felt. They refer to this as a tension-lag effect since the tensile stress is not transmitted throughout the laminate. Once the epoxy has set the tension lag stress profile will be set in the ring and be independent of temperature changes. The stress profile that develops from the tension lag will induce a moment stress resultant on the laminate and can be calculated using Equation (9). This moment will result in spring once the rings are cut and free to deform (Figure 6). In this case the spring usually takes the form of spring-in.

Maji and Ganley also made direct measurements of the residual stress profile in uni-directional filament wound hoop sections utilizing strain gages. The gages were bonded to both the upper and lower surfaces of the rings. The rings were then slowly cut by taking shallow passes with a cooled diamond blade on a Computer Numerical Controlled (CNC) mill. The surfaces resulting from the cuts are stress free and were made near the strain gage locations. After each pass the strain was read. The strain reading is the result of the stress relief on the now free surface. Following this, a Finite Element model was made of the ring and the cuts. A unit load was then applied to the free surface of a the FEM model and interpolations made to match the FEM predicted
strains to that measured with the strain gages. In this manner the residual stresses could be inferred. An example of the measured residual stress is shown in Figure 7. As can be seen, the tensile stresses from the tooling expansion affects just over half the thickness of the ring. The smaller residual stresses towards the outer surface are attributed to error in the data. From this they assumed a strain profile in the through-thickness direction. This assumed profile accounts for the curvatures in the rings. This theory is discussed in more detail later in Chapter 2.

**Figure 7:** Measured Residual Strain in Composite Ring. Data from uni-direction carbon-Epoxy material system. Data courtesy Maji & Ganley.
1.4 Current Research Overview

The areas of research covered above are based on one or combinations of the following: (1) thermal mismatch between the matrix and fiber, (2) variations in the fiber volume fraction through the thickness, (3) through-thickness hygrothermal loading, (4) chemical shrinkage of the matrix, and (5) tooling CTE. In many respects they are related and are important due primarily to the anisotropy of a composite laminate. Another commonality of these, with the exception of (5), are that they analyze the residual stresses that develop after the matrix has set. This would appear to be the only way residual stresses could be imparted on a laminate. However, as will be shown later this is not the case.

Work being done at the Air Force Research Laboratory (AFRL) has resulted in spring measurements in excess of 10 times that predicted by these recent theories. It has been determined that the residual stresses responsible for these large distortions occur during manufacturing and are a result of the cure consolidation [33 - 35]. The majority of cure consolidation in composites results from resin bleed-out and evacuation of entrapped air (voids). The magnitude is dependent on manufacturing parameters including cure pressure, winding tension, and material characteristics (i.e., pre-preg fiber volume fraction, resin viscosity, etc.). The strain profile that develops is set once the resin cures and is therefore not a hygrothermal phenomena and is independent of cure temperature, or finished part operational environment.

All composite parts experience a degree of consolidation during cure. Autoclave curing, which is common in aerospace applications, exhibits a high degree of consoli-
This consolidation will tend to relieve any existing tension, or stress, in the fibers. The stress relief is a result of the changing arc length in the fibers as the part consolidates. Automation of composite part fabrication greatly exaggerates this problem by allowing the fiber to be placed on the tooling at elevated tensions leading to a large strain gradient through the laminate thickness. These are tensile strains since the fibers cannot exhibit compressive strains of any significant value. If during cure, the strain relief becomes compressive the individual fibers will simply buckle. This is the result of the matrix being uncured or in a liquid state at the time of consolidation. Without matrix support the buckling load of the individual fibers is essentially zero. An example of this is shown in Figure 8 where the outer plys have buckled during cure as is evident by the wrinkled surface appearance.

Figure 8: Consolidation Induced Fiber Buckling. Buckling of fibers experienced during thermal processing.
As previously stated the major differences between this and other theories is the time at which the strain gradient is introduced into the laminate and the part geometry being investigated. In the past residual stresses were attributed to through-thickness hygrothermal loads that develop while the part cools down from its cure temperature or in the final stages of polymerization. In both cases the matrix has already set. This difference results in changes to the way the strain gradients must be applied to classical laminate theory and the reference frames that must be used.

With respect to geometry, the effect we are investigating is only applicable to thin shell structures. As Roy found, the winding tension contribution to spring can be ignored in thicker laminates [30]. This is because even a relatively small percentage of consolidation will eliminate any pre-existing winding tension when small radius to thickness ratios exist, \( R/h < 10 \). However, with the large ratios found in aerospace structures the winding tension effect will become dominate.

The goal of this dissertation is to develop a manufacturing methodology for spring reduction and elimination in full size structures based on testing accomplished at the coupon level. To facilitate this the following will be accomplished: (1) gain a better understanding of the residual stresses within a filament wound cylinder, (2) develop a theoretical model that will predict the spring in laminates (both symmetric and asymmetric lay-ups), (3) characterize the cure consolidation and develop a consolidation prediction model based on empirical results, (4) integrate the theoretical and consolidation models to allow for spring prediction, and (5) develop a spring elimination manufacturing methodology. This will benefit industry in one of two ways. The first and
preferable outcome is, through manufacturing controls, total elimination of part deformation. However, in some cases this may not be possible and the spring must be accounted for during tooling design; this is a simple process once the spring can be predicted. In both instances the resulting final part will be dimensionally accurate and the process of tooling iteration will be eliminated.

1.5 Method of Solution

1. We will first develop a theoretical cure consolidation model to predict the strain gradient through the thickness of a laminate. The model will account for both symmetric and asymmetric lay-ups. The model will be developed in two forms: one form will assume constant consolidation, and the second will take into account variations in the consolidation through the thickness. This is covered in Chapter 2.

2. To determine the deformations caused by the residual stresses the strain model is integrated with modified classical laminate plate theory. Equivalent stress resultants and couples, which represent the residual stresses, are then calculated. These are applied, along with the appropriate boundary conditions and consolidation parameters, to a composite cylinder and the final deformations found. This topic is also covered in Chapter 2.

3. To allow for the application of the spring prediction model to commercial manufacturing, a consolidation prediction model is needed. This model will allow for the characterization of a material system at the coupon level. Once character-
ized the model can be used to predict consolidation for full scale structures. This will be an empirical based model and is covered in Chapter 3.

4. To verify the solutions and predictions, experiments are conducted to evaluate the accuracy of the theoretical and empirical models. This involved running tests to measure the amount of cure consolidation that occurs under different manufacturing conditions both for cylindrical sections and flat plate coupons. These experiments are covered in Chapter 4.

5. By integrating the prediction theory and consolidation characterizations models developed in Chapters (2, 3, & 4), spring predictions for general laminates are made and compared with the experimental data from Chapter 4. This topic is covered in Chapter 5.

6. In the last step a spring elimination manufacturing methodology is developed. This is accomplished by modifying the manufacturing process to input a counter residual stress profile to that developed during cure. This topic is covered in Chapter 6.

Throughout the analysis thin shell approximations will be used. The material is also assumed to behave in a linear elastic manner with small deformations.
CHAPTER 2

THEORETICAL WARPAGE FORMULATION

2.1 INTRODUCTION

In this chapter a theoretical formulation to predict warpage in filament wound cylindrical sections is developed. This formulation is based on the consolidation resulting from the cure process. We will first develop a theoretical cure consolidation model to predict the strain gradient through the thickness of a laminate. The model will account for both symmetric and asymmetric lay-ups. The model will be developed in two forms: one form will assume constant consolidation, and the second will take into account variations in consolidation through the thickness.

The next step will be to integrate the strain model with modified classical laminate plate theory. Equivalent stress resultants and couples are then calculated. These are applied, along with the appropriate boundary conditions and consolidation parameters, to a composite cylinder and the final deformations found. Classical laminate plate theory will be used throughout with minor modifications. These modifications will be noted as they occur.
2.2 Strain-Displacement Model (Constant Consolidation)

In this section equations representing the strain-displacement for a composite laminate ring will be developed based on a constant consolidation model. The resulting equations will predict the strain profile through the thickness of a filament wound composite cylindrical section.

2.2.1. Cure Consolidation

Consider a differential element, Figure 9, at distance \( R + r \) with a pre-existing winding tensile strain \( e^t_{10} \). For our convention letter subscripts \( x, y, z \) designate global coordinates and numerical subscripts \( 1, 2, 3 \) designate fiber coordinates [41]. Note, at this point we will ignore \( \Delta R \) but will account for it later in this chapter. The differential element is then displaced a distance \( \Delta r \) during cure. The arc length is \( ds = d\phi (R + r) \), where \( r = 0 \) at the tool face and \( r = r_o \) at the outer face of the part. \( r_o \) is specified before cure and part consolidation. In general \( \Delta r < 0 \) for a part that undergoes consolidation, or \( r_o = h \) for a part that was perfectly consolidated before cure, the ideal material. As the differential element moves \( \Delta r \) the arc length \( ds \) undergoes a deformation to a new arc length \( ds' \).

\[
ds' = d\phi (R + r + \Delta r) \tag{10}\\
\]

The change \( \Delta ds \) is then

\[
\Delta ds = ds' - ds = d\phi \Delta r \tag{11}\\
\]

If \( \Delta r < 0 \), which is generally the case, the deformation relieves any pre-existing ten-
The strain at any point through the thickness during the cure is then given by Equation (13). There exists only a fiber direction strain since at this point the matrix is a liquid and therefore cannot undergo a mechanical strain in the off-fiber direction. Note that this also means that there is no Poisson’s effect in the global sense.

\[ \varepsilon_1' = \varepsilon_{10}' + \frac{\Delta r}{R + r} \]  

\[ \varepsilon_1'(r) = \varepsilon_{10}' + \frac{\Delta r}{R + r} \]  

Figure 9: Shell Cross-Section. This figure shows an arbitrary differential element that is displaced during part consolidation.
Now a coordinate transformation to the midplane is made. This is done to allow a direct substitution into the equations derived later from classical laminate theory. The final cured thickness of the part is

\[ h = r_o + \Delta r_o \]  

(14)

where \( \Delta r_o \) is the displacement at \( r = r_o \).

The midplane can be defined as

\[ \text{midplane} = \frac{r_o + \Delta r_o}{2} \]  

(15)

and for laminate plate theory the distance from the midplane is

\[ z = r - \frac{r_o + \Delta r_o}{2} \]  

(16)

For a fully cured and consolidated part we will first assume the compaction is constant through the thickness. This is based on research accomplished by Springer and Loos [37 and 38]. In our case this is not an entirely accurate assumption and a correction will later be made to incorporate variable compaction. The research accomplished by Springer and Loos was for flat panels which have a constant cure pressure through the thickness. For a filament wound cylindrical section the cure pressure will not be constant and will result in some variation in consolidation. However, in many manufacturing processes the error is small and the constant consolidation assumption is acceptable. It also greatly simplifies the analysis accomplished in later chapters and allows for a closed form solution. The substitution is then
\[ \Delta r \equiv T r \quad (17) \]

Here \( T \) defines the amount of cure consolidation and is a dimensionless property measured for a particular manufacturing process. It can be simply thought of as the percent consolidation. The consolidation is dependent on the initial fiber tension, material system, and cure process or \( T = T(\text{tension}, \text{material}, \text{cure}) \). Making this substitution, Equation (16) becomes

\[ z = r - \frac{r_o + Tr_o}{2} \quad (18) \]

Solving Equation (18) for \( r \) and substituting into Equation (13) gives the strain as a function of the new variable, \( z \).

\[ \varepsilon_1(z) = \varepsilon_{1o} + \frac{T(z + r_o(1 + T)/2)}{R + z + r_o(1 + T)/2} \quad (19) \]

Equation (19) was developed for fibers wound or placed in the hoop direction \( \theta = 90^\circ \). For off-hoop fibers the wind angle must be taken into account. This is done by first looking at an individual fiber with an arbitrary wind angle \( \theta \), as shown in Figure 10. The fiber has an initial length \( L_f \) before cure and a length \( L_f + \Delta L_f \) after cure. As before we are defining the stretching of the fiber as positive consolidation. The fiber has a component of length in the hoop direction before and after cure of \( L_h \) and \( L_h + \Delta L_h \), respectively. The strain of the fiber is

\[ \varepsilon_1 = \frac{\Delta L_f}{L_f} \quad (20) \]
and we define the strain in the hoop direction as

\[ \varepsilon_h = \frac{\Delta L_h}{L_h} \]  

(21)

The desired form of the solution will be to represent the fiber strain, \( \varepsilon_1 \), in terms of the
hoop strain, \( \varepsilon_h \). To accomplish this we assume small displacements and use simple geometric relationships.

The length of the section of fiber before cure can be written as

\[
(L_f)^2 = (L_h)^2 + (L_a)^2
\]  
(22)

and after cure as

\[
(L_f + \Delta L_f)^2 = (L_h + \Delta L_h)^2 + (L_a)^2
\]  
(23)

We can also represent the length in terms of the wind angle as

\[
\frac{L_h}{L_f} = \sin \theta
\]  
(24)

Using Equations (22 and 23) to eliminate \( L_a \) gives

\[
2L_f \Delta L_f + (\Delta L_f)^2 = 2L_h \Delta L_h + (\Delta L_h)^2
\]  
(25)

For small displacements \((\Delta L)^2 \approx 0\) and Equation (25) simplifies to

\[
L_f \Delta L_f = L_h \Delta L_h
\]  
(26)

Solving Equation (24) for \( L_h \) and substituting into Equation (26) gives

\[
\Delta L_f = \Delta L_h \sin \theta
\]  
(27)

Substituting Equation (27) into Equation (20), followed by a substitution for \( L_f \) from Equation (24) gives:
\[
\varepsilon_1 = \frac{\Delta L_h \sin \theta}{L_f} = \frac{\Delta L_h \sin \theta}{L_h' \sin \theta} = \frac{\Delta L_h}{L_h} \sin^2 \theta = \varepsilon_h \sin^2 \theta
\] (28)

The effects of off-hoop direction angled plies can now be accounted for with a simple modification to Equation (19) which is

\[
\varepsilon'_1(z) = \varepsilon'_{1o} + \frac{T(z + r_o(1 + T/2))}{R + z + r_o(1 + T/2)} \sin^2 \theta
\] (29)

Equation (29) is the total strain in a fiber due to the pre-existing winding tension and the strain relief that occurs during cure. Note that this strain is set once the matrix has cured and is independent of temperature. A plot of the winding induced strain along with the strain gradient developed during processing is shown in Figures (11 and 12). It can be seen that as the consolidation factor \( T \) gets larger it passes a point at which the fiber strain would become compressive. However, as stated before this cannot happen since the buckling load of a fiber is essentially zero. The strain through the remainder of the thickness is then equal to zero. As will be discussed later, this attribute is of primary importance and greatly affects the spring predictions. In these figures the strain profile is shown as a continuous curve which will be the case for a uni-directional laminate. In more general laminate staking sequences this is not the case and the curve will be discontinuous. This is because a laminate is composed of distinct plies that will generally have different orientation and possibly different winding tensions. Laminate theory deals with this condition by solving for the stress resultants and couples by integrating over each individual ply and then summing the results.
Figure 11: Strain Profile Gradient Before Processing. Fiber strain at this point is due to winding tension and can vary depending on winding profile.

2.2.2. Tooling Expansion Effects

In a perfect world composite tooling would have zero Coefficient of Thermal Expansion (CTE) and be capable of handling an infinite number of processing cycles. This, of course, is not the case. The initial testing was to be done utilizing a zero CTE tool. However, this proved extremely difficult even in a research environment. Although the tooling used for the testing did have a low CTE it was sufficient to affect the data. This led to the addition of a tooling expansion term to the strain model developed above.

As before, consider a differential element a distance \( r \) from the tooling face, Figure 9. If the laminate did not experience consolidation, \( T = 0 \), Equation (29) will simply become
Figure 12: Strain Profile Gradient After Processing. The fiber is unable to support compression at this point during cure. Note that the curve will generally be discontinuous at each layer interface due to differences in winding tension and ply angle.

\[ \varepsilon_1^r(z) = \varepsilon_{1o} \]  

(30)

As the tooling undergoes thermal expansion during the cure cycle it will displace a distance \( \Delta R \). This expansion would cause an additional strain through the thickness equal to:

\[ \varepsilon_1^R(r) = \frac{\Delta R}{R + r} = \frac{R \Delta t \alpha_i}{R + r} \]  

(31)

where \( \Delta t \) is the temperature change, and \( \alpha_i \) is the CTE of the tooling. In the structures we are interested in \( R \gg r \). The strain is then:

\[ \varepsilon_1^R(r) = \varepsilon_{1o} = \Delta t \alpha_i \]  

(32)
and can be assumed constant through the laminate thickness. This strain will obviously be additive to the winding strain and give

\[ \varepsilon'_{1}(z) = \varepsilon'_{1o} + \Delta t \alpha \sin^{2} \theta \]  

(33)

where the right-hand-side of the equation is a new increased effective winding tension and, as before, the \( \sin^{2} \theta \) accounts for off axis plies. As can be seen in Equation (33) with no consolidation, \( T = 0 \), there is no strain gradient and therefore no spring. Note that this is assuming the curvatures resulting from mid-plane stretching, as discussed by Kollar[22], are being neglected. This is possible due to their small contribution as discussed earlier.

For practical applications consolidation does exist and \( T \neq 0 \). Superimposing the effect of consolidation on Equation (33) we get

\[ \varepsilon'_{1}(z) = \varepsilon'_{1o} + \left( \Delta t \alpha + \frac{T(z + r_0(1 + T)/2)}{R + z + r_0(1 + T)/2} \right) \sin^{2} \theta \]  

(34)

It can now be seen that the tooling expansion will not affect the slope of the strain gradient, but will augment the winding tension strain \( \varepsilon'_{1o} \) and effectively shift the zero strain point to the right. This will have the effect of increasing the spring in cases where the zero strain point is met, but will not affect the spring if the zero strain point was not crossed.

In practice, using high CTE tooling will change the slope of the strain gradient curve in some cases. This is because the tooling expansion increases the effective cure pressure and causes an increase of the consolidation parameter \( T \). As stated earlier \( T \) is a
function of cure pressure. How the effective cure pressure is increased is discussed in more detail in Chapter 3. Figure 13 shows some of the possible outcomes that tooling CTE can have on the strain profile.

![Strain Profile After Processing](image)

**Figure 13:** Effect of Tooling Expansion on Strain Profile. *Note that the curve will generally be discontinuous at each layer interface due to differences in winding tension and ply angle.*

The tension-lag and corresponding strain profile that Maji and Ganley [32] assumed is an empirical representation of the consolidation factor $T$ not being zero. If there was no consolidation during cure the tension-lag would be zero and the part stable. Maji and Ganley also found that spring depended on laminate thickness. This is the result of the zero strain point being crossed in the thicker laminates they examined. Once this point is crossed there is no residual stress in the outer plies. This effectively decreases the spring with respect to a thinner laminate. This assumes both laminates have the
same consolidation, which we will show to be an acceptable assumption in many cases. By comparing Figure 7 (the strain profile measured by Maji and Ganley) and Figure 12 (the profile obtained by assuming linear compaction) it can be seen that they show the same trend of linear decrease in strain through the laminate thickness. It can also be seen that the zero strain point predicted by the strain-displacement model is seen in the measurements taken by Maji and Ganley and for all practical purposes the strain through the remainder of the thickness is zero.

2.2.3. Fiber CTE Effects

The last effect that must be accounted for is the fiber CTE. For the carbon fiber specimens used for the testing in this dissertation the CTE is approximately zero so they will have no effect on the spring. However, for other higher CTE fibers the effect on the residual strain in the laminate will be similar to that of the tooling. The only difference is that the winding angle, $\theta$, will not affect the strain. The final equation for the strain profile is then Equation (35), where $\alpha_t$ and $\alpha_f$ are the CTE’s of the tooling and fiber respectively. Note that $\alpha_f$ is the CTE of the individual fiber and not the combined fiber-matrix CTE derived from micromechanics. Also, the denominator can be simplified using thin shell approximation, $R \gg z$, with little loss of accuracy to give

$$\varepsilon'(z) = \varepsilon'_1 - \Delta t \alpha_f + \left( \frac{\Delta t \alpha_t + T(z + r_o(1 + \gamma)/2)}{R} \right) \sin^2 \theta$$

(35)

2.3 STRAIN-DISPLACEMENT MODEL (VARIABLE CONSOLIDATION)

In the previous section the consolidation was assumed constant within the laminate.
As will be shown in later chapters in most cases this is a satisfactory assumption. However, there are cases when this approximation does break down. Specifically, when zero-spring manufacturing techniques are used the consolidation can vary significantly. The reason for this will be covered in depth in Chapter 6. In this section the changes required to incorporate variable consolidation into the strain-displacement model will be covered.

2.3.1. Cure Consolidation

Previously the consolidation was assumed constant and the displacement of a differential element was represented by Equation (17), repeated here for reference.

\[ \Delta r = T r \] (36)

We now investigate a more general case where the consolidation is a function of position, or \( T = T(r) \).

The displacement of a point at position \( r \) within the laminate, is

\[ \Delta r = \int_{0}^{r} T dr \] (37)

Since the individual lamina will have different consolidation we must examine the laminate in a ply-by-play manner. Summing over each layer, the displacement of the \( k^{th} \) lamina is
\[ \Delta r_k = \sum_{i=1}^{k} \int_{r_{i-1}}^{r_i} T_i dr \]  

(38)

where we assume the consolidation is piece wise constant over each lamina. Equation (38) can then be integrated to give the displacement of the \(k\)th lamina.

\[ \Delta r_k = \sum_{i=1}^{k} \left[ T_i (r_i - r_{i-1}) \right] \]  

(39)

Utilizing Equation (13) the strain within the \(k\)th lamina are

\[ \varepsilon'_{1,k} = \varepsilon'_{1,o} + \sum_{i=1}^{k} \left[ T_i (r_i - r_{i-1}) \right] / R \]  

(40)

The strain in the classical laminate reference frame is simply

\[ \varepsilon'_{1,k} = \varepsilon'_{1,o} + \sum_{i=1}^{k} \left[ T_i (z_i - z_{i-1}) \right] / R. \]  

(41)

The effects from the tooling and fiber CTE's can be accounted for in the same manner as before to give the final equation representing the strain within a laminate undergoing variable consolidation:

\[ \varepsilon'_{1,k} = \varepsilon'_{1,o} - \Delta t \alpha_f + \left( \Delta t \alpha_i + \sum_{i=1}^{k} \left[ T_i (z_i - z_{i-1}) \right] / R \right) \sin^2 \theta_k \]  

(42)

2.4 Modified Laminate Theory

Utilizing the strain model a modified form of the classical laminate theory can be developed which will end with the incorporation of Equation (35). Classical laminate
plate theory is used throughout this derivation unless otherwise noted.

The constitutive laws for a composites shells are [41]:

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} =
\begin{bmatrix}
A & B \\
B & D
\end{bmatrix}
\begin{bmatrix}
\varepsilon \\
\kappa
\end{bmatrix}
\]

(43)

where \([A]\) \([B]\) \& \([D]\) are the extensional, bending-stretching, and flexural stiffness matrices, respectively.

The stress resultants and couples can be represented by [41]:

\[
\begin{bmatrix}
N_T \\
M_T
\end{bmatrix} =
\int_{-h/2}^{h/2}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix}
dz
\]

(44)

where the \(T\) subscript indicates we are only interested in the contributions from cure consolidation.

Starting with the stresses in the fiber direction:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & 2Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 - \alpha_1 \Delta T - \beta_1 \Delta M - \varepsilon^e_1 \\
\varepsilon_2 - \alpha_2 \Delta T - \beta_2 \Delta M - \varepsilon^e_2 \\
\varepsilon_{12}
\end{bmatrix}
\]

(45)

where \(\varepsilon^e_1\) is the strain due to chemical shrinkage and plane stress is assumed, \(\sigma_{13} = \sigma_{23} = \sigma_3 = 0\). For this analysis the hygrothermal and chemical effects are neglected for simplicity. The stress is then:
\[ \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} \]  

(46)

Transforming to the global coordinate system gives:

\[ \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = [\mathcal{Q}] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \]  

(47)

where \([\mathcal{Q}] = [T]^{-1}[Q][T]\) are the commonly used stiffness matrix values in the global coordinate system and are referred to as the Q bars. The global strains are \(\{\varepsilon\}_{Global} = [T]^{-1}\{\varepsilon\}_{Fiber}\) and can be written as:

\[ \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 m^2 + \varepsilon_2 n^2 - 2mn\varepsilon_{12} \\ \varepsilon_1 n^2 + \varepsilon_2 m^2 + 2mn\varepsilon_{12} \\ \varepsilon_1 mn + \varepsilon_2 mn + (m^2 - n^2)\varepsilon_{12} \end{bmatrix} \]  

(48)

where \(x, y, z\) are the global indices, \(m = \cos(\theta)\), and \(n = \sin(\theta)\). We are using tensorial strain defined by

\[ \varepsilon_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i}) \]  

(49)

where \(i, j = x, y, z\) in a Cartesian coordinate system and a comma represents partial differentiation with respect to the second indice. For our convention the transformation matrix is [41]:
\[
[T] = \begin{bmatrix}
  n^2 & m^2 & 2mn \\
  m^2 & n^2 & -2mn \\
  mn & mn & (m^2 - n^2)
\end{bmatrix} \quad (50)
\]

and with tensorial strains \([T]^{-1}\) can be found simply by substituting \(\theta = -\theta\).

The global strains for a cylindrical shell can also be written as Equation (51) [13].

\[
\varepsilon_x = \frac{\varepsilon_{ox} - z \frac{\partial^2 w}{\partial x^2}}{1 + \frac{z}{R}}
\]

\[
\varepsilon_y = \frac{\varepsilon_{oy} + \left( \frac{1}{R} \frac{\partial v}{\partial y} - z \frac{\partial w}{\partial y^2} \right)}{1 + \frac{z}{R}}
\]

\[
\varepsilon_{xy} = \frac{\partial v}{\partial x} + z \left( \frac{1}{R} \frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial y \partial x} \right) + \frac{1}{1 + \frac{z}{R}} \left( \frac{\partial u}{\partial y} - z \frac{\partial^2 w}{\partial y^2} \right)
\]

Assuming \(\frac{z}{R} \ll 1\) and making the standard symbolic substitution for the curvature the strains become [39]:

\[
\begin{align*}
\varepsilon_x &= \varepsilon_{ox} - z \kappa_x \\
\varepsilon_y &= \varepsilon_{oy} - z \kappa_y \\
\varepsilon_{xy} &= \varepsilon_{oxy} - z \kappa_{xy}
\end{align*}
\]

(52)

where
\[ \kappa_x = \frac{\partial^2 w}{\partial x^2} \]
\[ \kappa_y = \frac{1}{R} \frac{\partial v}{\partial y} - z \frac{\partial^2 w}{\partial y^2} \]
\[ \kappa_{xy} = \frac{1}{R} \frac{\partial v}{\partial y} \frac{\partial w}{\partial x} - \frac{\partial w}{\partial y} \frac{\partial^2 w}{\partial y \partial x} \]

In this case the curvatures are equal to the differences between the inverse of the deformed and undeformed radii and are:

\[ \begin{bmatrix} \kappa_x \\ \kappa_y \end{bmatrix} = \begin{bmatrix} \frac{1}{R_x'} - \frac{1}{R_x} \\ \frac{1}{R_y'} - \frac{1}{R_y} \end{bmatrix} \]

where the primes are the deformed radii.

Substituting the above into Equation (44) and solving ply-by-ply according to laminate theory gives the combined internal force equations:

\[ \left[T\right] = \sum_{k=1}^{N} \begin{bmatrix} \frac{E_{x0}}{h_k} & \frac{E_{y0}}{h_{k-1}} & \frac{E_{xy0}}{h_{k-1}} \\ \frac{E_{y0}}{h_k} & \frac{E_{xy0}}{h_{k-1}} \end{bmatrix} \int_{h_{k-1}}^{h_k} dz + \left[\kappa\right] \int_{h_{k-1}}^{h_k} dz \]

\[ \left[M_T\right] = \sum_{k=1}^{N} \begin{bmatrix} \frac{E_{x0}}{h_k} & \frac{E_{y0}}{h_{k-1}} \end{bmatrix} z \int_{h_{k-1}}^{h_k} dz + \left[\kappa\right] \int_{h_{k-1}}^{h_k} z^2 dz \]

43
The first integral in the top term is similar to the \([A]\) matrix and the second integral is the \([B]\) matrix. The first integral is different from the standard \([A]\) matrix in that the strains are a function of \(z\). This is the result of the mid-plane strain being a function of \(z\), as was shown before. Similarly, in the bottom term the first integral is similar to the \([B]\) matrix and the second integral is the \([D]\) matrix. This is a deviation from the classical laminate plate theory. It appears to contradict the definition of mid-plane strain, but the \(z\) dependency is a result of the compaction which occurs during cure and is not a result of the change in curvature \(\kappa\). It will therefore be included in the mid-plane strain terms. During cure there is curvature due to the shell geometry, \(\kappa_g\), but no curvature due to the shell stresses, \(\kappa_{x, y, xy} = 0\). This is because the part is still being confined by the tooling. This \(z\) dependency, and the corresponding moment, results in the spring-in after part-tool separation. Making the substitution of \([B] & [D]\) gives:

\[
\begin{bmatrix} N \end{bmatrix} = \sum_{k=1}^{N} \left[ \tilde{\Omega} \right] \int_{h_{k-1}}^{h_k} \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \varepsilon_{xyo} \end{bmatrix}_k \, dz + [B]\{\kappa\} \\
\]

\[
\begin{bmatrix} M \end{bmatrix} = [D]\{\kappa\} + \sum_{k=1}^{N} \left[ \tilde{\Omega} \right] \int_{h_{k-1}}^{h_k} \begin{bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \varepsilon_{xyo} \end{bmatrix}_k \, zdz 
\]

where as stated before \(\varepsilon_{io} = \varepsilon_{io}(z)\) and must remain inside the integral.

Utilizing Equation (48) we can represent the midplane strain for each lamina as:
\[
\begin{bmatrix}
\varepsilon_{x_0} \\
\varepsilon_{y_0} \\
\varepsilon_{xy_0}
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_{1_0}m^2 + \varepsilon_{2_0}n^2 - 2mne_{1_2} \\
\varepsilon_{1_0}n^2 + \varepsilon_{2_0}m^2 + 2mne_{1_2} \\
\varepsilon_{1_0}mn + \varepsilon_{2_0}mn + (m^2 - n^2)e_{1_2}
\end{bmatrix}_k
\]

(57)

As stated before the effect we are investigating only occurs along the fiber direction so Equation (57) becomes:

\[
\begin{bmatrix}
\varepsilon_{x_0} \\
\varepsilon_{y_0} \\
\varepsilon_{xy_0}
\end{bmatrix}
= \begin{bmatrix}
m^2 - n^2 \\
n^2 - m^2 \\
(1 - \nu_{12})mn
\end{bmatrix}_k \varepsilon^i(z)
\]

(58)

Substituting Equations (29 and 58) into Equation (56) gives:

\[
\begin{bmatrix}
N_T
\end{bmatrix}
= \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \begin{bmatrix}
m^2 - n^2 \\
n^2 - m^2 \\
(1 - \nu_{12})mn
\end{bmatrix}_k \varepsilon^i(z)dz + \left[ B \right] \left\{ \kappa \right\}
\]

(59)

\[
\begin{bmatrix}
M_T
\end{bmatrix}
= \left[ D \right] \left\{ \kappa \right\} + \sum_{k=1}^{N} \int_{h_{k-1}}^{h_k} \begin{bmatrix}
m^2 - n^2 \\
n^2 - m^2 \\
(1 - \nu_{12})mn
\end{bmatrix}_k \varepsilon^i(z)zdz
\]

where \( \varepsilon^i(z) \) is defined in Equations (34 and 42).

For the specific cases we are interested in the part is still constrained by the tooling and the curvatures are zero. This gives the final form for determining the internal forces caused by cure consolidation:

45
\[
\begin{align*}
\begin{bmatrix} N_T \end{bmatrix} &= \sum_{k=1}^{N} \bar{Q}_k \int_{h_{k-1}}^{h_k} \left[ m^2 - \nu_{12} n^2 \right] e_1'(z) dz \\
\begin{bmatrix} M_T \end{bmatrix} &= \sum_{k=1}^{N} \bar{Q}_k \int_{h_{k-1}}^{h_k} \left[ m^2 - \nu_{12} n^2 \right] e_1'(z) zdz
\end{align*}
\] (60)

Applying the internal forces as stress resultants and stress couples to an unconstrained ring will produce the same deformations as when the parts are released from the tooling. The strains and curvature will be:

\[
\begin{bmatrix} \varepsilon_T \\ \kappa_T \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_T \\ M_T \end{bmatrix}
\] (61)

The general deformation of a composite cylinder with additional mechanical edge loads is:

\[
\begin{bmatrix} \varepsilon \\ \kappa \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{bmatrix} N_T + N \\ M_T + M \end{bmatrix}.
\] (62)

With the appropriate boundary conditions and consolidation parameters the spring can now be found utilizing Equations (60 & 62).

### 2.5 Composite Cylinder

Consider a cylinder as in Figure 9. For this case the curvatures and radii are
\(1/R_x = 0\) and \(R_y = R\) respectively. The displacements in the axial direction (\(x\)), the circumferential direction (\(\theta\)), and the radial direction (\(z\)) are \(u\), \(v\), and \(w\) respectively.

The strain displacement relationships in cylindrical coordinates are [39]:

\[
\begin{align*}
\varepsilon_x &= \frac{\partial u}{\partial x} \\
\varepsilon_\theta &= \frac{1}{R} \left( \frac{\partial v}{\partial \theta} - \frac{\partial w}{\partial \theta} \right) \\
\varepsilon_z &= \frac{\partial w}{\partial z} \\
\kappa_x &= -\frac{\partial^2 w}{\partial x^2} \\
\kappa_\theta &= \frac{1}{R^2} \left( \frac{\partial v}{\partial \theta} - \frac{\partial w}{\partial \theta} \right) \\
\kappa_z &= \frac{1}{R} \left( \frac{\partial v}{\partial x} - \frac{\partial w}{\partial x} \right)
\end{align*}
\]

\(\varepsilon_{xy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \) \hspace{1cm} (63)

The spring of the open-section cylinder is defined as [22]:

\[
S = \frac{\delta}{\gamma} = \frac{1}{\gamma} \int_0^\gamma \left( \frac{\partial v}{R \partial \theta} - \frac{\partial^2 w}{R \partial \theta^2} \right) d\theta = R \kappa_y
\]

where \(\gamma\) is the segment angle and \(\delta\) is the change in angle (Figure 14).

For the cylindrical sections tested in Chapter 4 the lay-up is symmetric and the \(B\) matrix is zero. The curvature resulting from cure consolidation is

\[
\kappa_y = \left[ D_{11} - D_{12} D_{22}^{-1} D_{23} \right] \sum_{k=1}^{N} \int_{l_{k-1}}^{l_k} \left[ \begin{array}{c} m^2 - n_{12}n^2 \\ n^2 - n_{12}m^2 \\ (1 - n_{12})mn \end{array} \right] \varepsilon_1(z) zdz
\]

For variable compaction Equation (65) takes the form

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\[ k_y = \left[ D_{12}^{-1} D_{22}^{-1} D_{23}^{-1} \right] \sum_{k=1}^{N} \left( \frac{1}{h_k} \int_{h_{k-1}}^{h_k} \frac{m^2 - \nu_{12} n^2}{n^2 - \nu_{12} m^2} \frac{\epsilon_1'(z) dz}{(1 - \nu_{12}) mn} \right) \]  

Equations (65 and 66) represent the final form and are used throughout the remainder of this dissertation for prediction and elimination of spring.

Undeformed

Deformed

Spring $S$ is Defined by $S = \delta / \gamma$

Figure 14: Graphical Spring Representation. Here \( \gamma \) is the section angle and \( \delta \) is the change in angle.
2.6 Conclusions

In this chapter a theoretical formulation was developed to predict spring in cylindrical sections based on cure consolidation. The formulation includes both constant and variable consolidation models. These models were integrated with modified classical laminate plate theory to derive equations predicting the residual stresses and associated stress resultants and couples. Utilizing these, additional equations were developed to predict the mid-plane strains and curvatures. With these equations the spring in a filament wound cylinder can be predicted if the consolidation is known. The next step is to characterize the consolidation to allow for implementation of this formulation in a manufacturing environment.
CHAPTER 3

CONSOLIDATION CHARACTERIZATION

3.1 INTRODUCTION

In this chapter the cure consolidation incurred during processing of filament wound and fiber placed closed-section composite shells is characterized. Cure consolidation is the major contributing factor of warpage in these structures and accurate characterization is crucial in predicting and controlling the deformation. In Chapter 2 models to predict the spring based on consolidation and winding tension were developed. For these models to be implemented successfully a thorough understanding of the scaling effects on consolidation must be acquired. This will enable characterization of materials at the coupon level. Our goal in this chapter is to gain a better understanding of the magnitude of cure consolidation and how manufacturing parameters and geometry variations contribute. This will allow for prediction of residual stresses within a laminate and lead to either elimination or reduction of these stresses and ultimately result in decreased spring and better material performance.

The majority of cure consolidation in composites results from resin bleed-out and evacuation of entrapped air (voids). The magnitude of the consolidation is dependent
on manufacturing parameters including cure pressure, winding tension, and material system characteristics (e.g., pre-preg fiber volume fraction, resin viscosity, etc.). In this chapter the cure pressure and winding tension contributions are investigated. Winding tension has a large effect on the final magnitude of consolidation and actually contributes in two ways: the first being the winding Lay-Down Efficiency (LDE) and the second being the Effective Cure Pressure (ECP). The winding Lay-Down Efficiency relates to how well the fibers are placed during manufacture. The more accurately the fibers are placed the less air is entrapped and the thinner the pre-cure thickness. This results in less consolidation during cure. The Effective Cure Pressure is the amount of pressure a laminate experiences during cure and is the summation of the autoclave and winding strain induced pressures. The ECP will help determine the post-cure thickness and likewise affect the consolidation.

The characterization will proceed in the following manner. First, a theoretical formulation is developed for the ECP based on coupon testing from Chapter 4. The next step is the investigation and integration of the LDE contributions. As with the ECP this is done by developing a formulation based on the physics of the problem along with data from Chapter 4. The final model will allow for prediction of consolidation in full-scale parts based on testing at the coupon level. This model is used extensively in Chapters (5 & 6) to predict and eliminate spring.

3.2 Effective Cure Pressure

Filament wound structures are normally of the closed-section type. Because of this
they are able to maintain tension in the fiber before the matrix has set. The tension is maintained by friction between lamina. This tension can come about in two ways: winding tension which may be constant or varied, and tension derived from tooling expansion which will occur if the part and tooling are not CTE matched. The tension, and associated hoop stress, will cause an increase in the consolidation. This increase can be dramatic when elevated winding tensions or high CTE tooling is employed. The fiber stress will act to increase the cure pressure, which is to say, it appears to the part as if a higher autoclave pressure is being used. The increase in cure pressure forces more resin from the laminate leading to higher volume fractions and greater consolidation. There is of course a maximum practical $V_f$ and it can range from 78% to 90% depending on packing structure [41]. This effect must be included to accurately characterize the consolidation. To accomplish this we will develop a model to account for the fiber strain and its effect on the cure consolidation. We will define the strain and autoclave pressure influences on consolidation as the Effective Cure Pressure (ECP).

3.2.1. Derivation

The ECP is essentially the summation of the pressures on a particular lamina. If we assume consolidation is a function of cure pressure each lamina will have a different consolidation factor, $T$, so we now have $T = T(P)$ which is to say $T = T(r)$ or $T = T(z)$ depending on reference frame. This means that the initial assumption in Chapter 2 of constant consolidation is no longer valid. However, it will be shown that for many circumstances it is an acceptable assumption.
To start our analysis we will assume that the ECP is dependent on the autoclave pressure and the pressure effects from the fiber strain (hoop stress), or

\[ P^e = P^a + P^{\text{strain}} \]  

(67)

We will ignore the autoclave pressure for now and concentrate on the strain-induced pressure. Assuming thin shell, the hoop stress in a cylinder is

\[ \sigma_h = \frac{P^{\text{strain}} (R + r)}{t} \]  

(68)

where, as before, \( R \) is the radius of the tooling, \( r \) is the distance from the tooling face to lamina, and \( t \) the lamina thickness. In our case we know the stress but not the pressure so we rearrange Equation (68) to get

\[ P^{\text{strain}} = \frac{\sigma_h t}{R + r} \]  

(69)

In a general laminate there exists \( N \) lamina and each will exert a pressure on the lamina below equal to Equation (69) (Figure 15). The total cure pressure on the lamina resulting from the fiber stress will be the summation of the pressures above, or

\[ P^{\text{strain}}_k = \sum_{i = k - 1}^{N} \frac{\sigma_{h,i} t}{R + r_i} \]  

(70)

As discussed previously, before the matrix has set there is no Poisson’s effect in the global sense and the stress-strain relationship for the hoop stress is simply

\[ \sigma_h = \varepsilon_1 E \]  

(71)
Section of Ring with N Layers

Each Individual Layer Contributes a Pressure $P_i$ to the Layers Below.

**Figure 15: Lamina with Fiber Strain Induced Cure Pressure.** Pressure derives from hoop stress.

Substituting into Equation (70) gives

$$P_{strain}^{strain} = \sum_{i = k+1}^{N} \frac{\varepsilon_{1,i}^t E_i t_i}{R + r_i}$$

(72)

where $\varepsilon_{1,i}^t$ is the fiber strain in the $i^{th}$ lamina resulting from the winding tension and tooling CTE. Equation (72) is for hoop wound rings. For off-axis laminates a correction must be made as in Section 2.2. Equation (73) is then the final expression for the cure pressure resulting from fiber strain.
\[ P_{\text{strain}}^k = \sum_{i=k+1}^{N} \frac{\varepsilon_{l_i} E_i t_i}{R + r_i} \sin^2 \theta_i \]  

(73)

The total effective cure pressure for the \( k^{th} \) lamina will then be

\[ P_k^e = P^a + \sum_{i=k+1}^{N} \frac{\varepsilon_{l_i} E_i t_i}{R + r_i} \sin^2 \theta_i \]  

(74)

where \( \varepsilon_{l_i} \) includes both the winding tension induced strain and the strain resulting from any tooling/part CTE mismatch. The ECP can now be calculated and used to predict the consolidation of a filament wound structure.

### 3.3 Empirical Consolidation Approximation

Based on the consolidation test results discussed in Chapter 4 an empirical approximation is presented here to predict the consolidation utilizing the effective cure pressure. The approximation is supported by comparison to data later in this chapter. We first assume the consolidation behaves like a second order polynomial function over the pressure range of interest

\[ T(P) = a + bP + cP^2. \]  

(75)

where \( a \) is the consolidation at zero pressure and will hereafter be referred to as \( T_0 \).

From intuition we can further assume there must be a maximum pressure \( (P_{\text{max}}) \) at which the laminate will have no more excess resin or entrapped air to bleed off and any additional pressure will result in no increase in consolidation. The laminate will become incompressible. The maximum consolidation corresponding to \( P_{\text{max}} \) is defined
as $T_{\text{max}}$. At this point it also appears logical to assume that for zero pressure the consolidation will be zero, but testing has shown that this is not the case and with no pressure there will be a relatively small consolidation equal to $T_o$. Based on these assumptions and observations we propose the consolidation curve will look as shown in Figure 16. Utilizing the boundary conditions:

\[
T(P_{\text{max}}) = T_{\text{max}}
\]

\[
\frac{\partial}{\partial P} T(P_{\text{max}}) = 0
\]

\[
T(0) = T_o
\]

the constant coefficient terms can be solved for and are:

\[
b = \frac{2(T_{\text{max}} - T_o)}{P_{\text{max}}}
\]

\[
c = \frac{T_o - T_{\text{max}}}{(P_{\text{max}})^2}
\]

where $T_o$, $T_{\text{max}}$, and $P_{\text{max}}$ must be determined from testing.

This leaves us with a simple equation representing the consolidation with constant coefficients that can be obtained from testing. This form significantly increases the difficulty of predicting the spring because the consolidation is no longer constant but depends on the pressure which in turn depends on the fiber strain which depends on the consolidation. This is a circular argument with a direct closed form solution not possible. We are now left with a problem that must be solved in an iterative fashion. This is
3.4 ECP EXPERIMENTAL RESULTS/DISCUSSION

In this section the results from the flat coupon tests are compared to the empirical formulation developed above. Specifically, we are verifying that consolidation is dependent on the pressure and can be modeled with a second order polynomial. Note, at this point we are not concerned with the source of the cure pressure (i.e. hoop stress or autoclave pressure) so flat coupons cured at various autoclave pressures can be used to simulate different ECP's. The initial step is to find the constant coefficient terms $T_0$, $b$, and $c$. The first method is to take measurements at zero pressure to directly deter-
mine $T_o$. Then data is taken at increasing autoclave pressures until $T_{max}$ and $P_{max}$ are found. All the coefficients can then be calculated using Equation (77).

However, the method above suffers from the requirement of taking data at zero pressure. The data at the lower pressures exhibit relatively more scatter. The second method is to take the first measurement at a relatively low pressure, 100 kPa for example. Then, as with the first method, data is taken at increasing pressures until $T_{max}$ and $P_{max}$ are found. A least square approximation using a second order polynomial can then be used to fit the data. This approximation gives the coefficients needed for Equation (75). Next, the accuracy of this approximation is discussed.

### 3.4.1. Empirical vs. Experimental Comparison

As can be seen in Figure 17, the match between the experimental and empirical consolidation predictions are very good. This verifies our assumption of cure pressure dependency and means that for a particular resin system the consolidation at any point within the laminate can be determined simply by knowing the ECP. It is interesting to note that for zero pressure there is still approximately 5% consolidation. A possible cause for this is the resin flow that occurs, regardless of pressure, when the resin is raised to the cure temperature. It is of academic interest only since in actual applications there will be a minimum of vacuum bag pressure applied to the part. A last note to this approximation is in regard to the range of use. If care is not taken to insure the ECP is in the proper range large errors can occur in the prediction. The computer programs discussed later do not allow the $P_e$ input to the consolidation equations to exceed $P_{max}$.
If $P^*$ exceeds $P_{\text{max}}$, then $P_{\text{max}}$ must be used for further consolidation calculations. This is a limitation of using a polynomial approximation. However, if care is taken the solution is easy to program.

![Graph showing empirical vs. experimental consolidation](image)

**Figure 17: Empirical vs. Experimental Consolidation.** Each experimental average data point represents the average of 4 coupons.

### 3.5 Displacement Model

Utilizing the empirical approximation and the Effective Cure Pressure models discussed above we will now develop a model that predicts the displacement of a lamina, or a point, within a filament wound laminate. In this development we are no longer assuming the consolidation is constant, but assume a dependency on the ECP as shown in Figure 17.

We now proceed in a similar fashion as in Chapter 2. The movement of a differential
element at any point within the laminate is

\[ \Delta r = \int_0^r T(P) \, dr \]  

(78)

Substituting the empirical representation for consolidation [Equation (75)] gives

\[ \Delta r = \int_0^r (T_0 + bP - cP^2) \, dr \]  

(79)

In this case we are looking at a ring with finite thickness so the cure pressure is actually the ECP and a function of \( r \):

\[ \Delta r = \int_0^r (T_0 + bP(r) - c(P(r)^2)^2) \, dr \]  

(80)

For a laminate the effective cure pressure is not continuous and \( \Delta r \) for the \( k \)th lamina must be found by summing over each layer below the one of interest. As in Chapter 2 we are assuming the pressure is piece wise constant for each lamina.

\[ \Delta r_k = \sum_{i=1}^{k} \int_{r_{i-1}}^{r_i} ((T_0 + bP_i - c(P_i^2)^2) \, dr \]  

(81)

This equation can be integrated to give the displacement of the \( k \)th lamina.

\[ \Delta r_k = \sum_{i=1}^{k} \left[ (T_0 + bP_i - c(P_i^2)^2)(r_i - r_{i-1}) \right] \]  

(82)

where \( P_i^e \) is defined by Equation (74).
Equation (82) represents the displacement of the \( k \)th lamina resulting from both the autoclave and strain induced pressure. As stated before this equation cannot be solved for directly since the strain \( \varepsilon_{i,j} \) is dependent on the displacement \( \Delta r_k \) through Equation (41) which is repeated here for reference.

\[
\varepsilon_{1,k}' = \varepsilon_{1,o}' - \Delta \alpha_t + \left( \Delta \alpha_t + \sum_{i=1}^{k} \left[ T_i(z_i-z_{i-1}) \right] / R \right) \sin^2 \theta_k \tag{83}
\]

where

\[
\sum_{i=1}^{k} T_i(z_i-z_{i-1}) = \Delta r_k \tag{84}
\]

The solution must be obtained by assuming an initial fiber strain and then iterating Equations (82 & 83) for each lamina until the solution converges. First, a reference frame change will be made to allow for integration with Classical Laminate Theory. Utilizing Equation (16) and thin shell approximations Equation (82) becomes

\[
\Delta r_k = \sum_{i=1}^{k} [(T_o + b P_i^e - c(P_i^e)^2)(z_i-z_{i-1})] \tag{85}
\]

where as before, \( z \) is measured from the laminate mid-plane.

### 3.6 Computer Solution

To solve for the consolidation a computer iterative program was developed and will be referred to as "Consolidation Gen". The solution utilizes Equations (83 and 85) in an iterative fashion to find the final consolidation. An iterative solution is required.
because of the circular dependence between the strains, displacements, and consolidation. The procedure implemented in the program is listed below.

1. Material system, manufacturing parameters, and tooling geometry input.

2. Effective Cure Pressure general equation defined as function of manufacturing parameters, \( P^e = P^e(P^a, e_{1,j}) \).

3. Ply interface height calculated, \( z_o, z_1 \ldots z_N \).

4. Initial fiber strain calculated.

5. Resulting consolidation calculated from Equation (75).

6. Lamina movement calculated from Equation (85).

7. Updated fiber strain calculated from Equation (83).

8. Steps 5-7 repeated until solution converges.

### 3.6.1. Solution Convergence

As can be seen from Figure 18 the solution for consolidation converges quickly. Within 3 iterative steps the solution has converged to within 0.05%. This is for a relatively thin laminate, but is representative of the test coupons utilized in this dissertation. In general the thicker the laminate the more steps required to find a solution, with 20 steps being sufficient for laminates up to 100 layers.

### 3.7 Effective Cure Pressure Applied to Cylindrical Sections

With the formulation and program developed above an investigation can be made
Consolidation vs. Iteration Step

![Chart showing consolidation vs. iteration step]

**Figure 18: Iterative Convergence Plot.** Graph represents top layer of twenty layer laminate wound with 45 N tension

into the effects, if any, the ECP has on the consolidation of a filament wound part. Note, we are ignoring the LDE contributions for now. Figure 19 shows the change in average consolidation for a representative filament wound cylinder utilizing 44 N winding tension, which equates to a 138 MPa winding stress. As expected the thicker laminates have higher consolidation. This is a result of higher ECP's. Also of interest is the convergence of all curves at the higher pressures. This is the result of the maximum consolidation pressure, $P_{\text{max}}$, being met by the autoclave pressure alone. If the autoclave pressure reaches $P_{\text{max}}$, the maximum consolidation will occur whether there is a hoop stress addition to the ECP or not. Because of this the additional complexity of taking into account the ECP in higher-pressure cure processes can be avoided. Above 500 kPa the error in neglecting the strain induced contribution to the ECP is less than 5%. How-

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ever, for lower-pressure cure processes, neglecting the hoop stress effects could result in errors greater than 30%.

![ECP vs. Autoclave Pressure](image)

**Figure 19: Effect of ECP on Consolidation.** Calculated for constant 44.5 N winding tension.

### 3.8 Lay-Down Efficiency

As discussed earlier the amount of air entrapped in a laminate during manufacture can be the dominate factor in consolidation. In general, the more tension used during winding the less the part will consolidate during cure. It is safe to assume that, if enough tension was used during winding, the strain induced pressure would force all entrapped air and excess resin out of the laminate. This would result in no consolidation during cure. This is, of course, not practical since there is a limit to the amount of tension a fiber can withstand during winding. Suffice to say it gives insight on why the
trend of decreasing consolidation with increasing winding tension exists. Another contributor is how accurately the fiber is placed. If there is an overlap or gap during winding this will entrap air and increase the consolidation. The winding accuracy is usually dependent on the capabilities of the winder and consistency of the material system being used. For our study we will group these effects under one parameter and define it as the Lay-Down Efficiency (LDE). Note, the winder accuracy and material system consistency are set and not controllable by the manufacturer. Because of this we will assume the LDE is a function of the winding tension only.

3.8.1. Empirical Lay-Down Efficiency Approximation

Unlike the empirical formulation of the ECP the LDE cannot be represented by a simple polynomial approximation. The consolidation curve must be broken into two segments with respect to winding stress: (1) an exponentially decreasing section and (2) a linearly decreasing section. These segments are shown in Figure 20.

The consolidation of the first segment is actually higher than what was found in the flat panel tests. As will be discussed in the Chapter 4 the flat panel specimens were laid up by hand. Some consolidation occurs during this lay-up and is not accounted for in the final measurements. This is why there is higher consolidation values for the lower winding stress rings than the $T_{max}$ found in the flat coupon tests. At these lower winding stresses a large amount of air is entrapped resulting in high consolidation values. This will be accounted for with an exponential term and give an additional constant coefficient.
Figure 20: Consolidation Segments. Each data point represents the average of three tests.

In the second segment the non-linear effect has disappeared and there is a linear relation between the winding stress and the consolidation. This can be modeled by a first order polynomial. Note, to account for scaling effects the tooling radius enters the equation in the denominator whenever the winding stress is used. The influence of the winding strain is actually dependent on its contribution to the hoop stress pressure so it must be inversely proportional to the radius as it was with the ECP. This will give an additional two constant LDE related coefficients for a total of three. The final equation is then
\[ T(P^e, \sigma_o, \theta) = \left\{ \frac{d - e \frac{\sigma_o}{R} \sin^2 \theta + \text{Exp}\left(\frac{(\sigma_o - f)t}{R}\right)}{T_0 + bP^e + c(P^e)^2} \right\} \]

where \( \sigma_o \) is the initial winding stress.

At this point the consolidation is both a function of the ECP, the initial winding stress, the tooling radius, and the fiber thickness. The last three terms can be grouped as the Lay-Down strain induced pressure, \( P_o^{\text{strain}} \), but Equation (86) is easier to work with as a function of the winding stress. This gives an addition of three constant coefficients terms, \( d, e, \) and \( f \) that must be determined from testing. Physical interpretations of the coefficients can be made: the coefficient \( d \) accounts for the accuracy of the winder and the consistency of the material system, \( e \) is the multiplier which scales the winding tension effect, and \( f \) accounts for the non-linear effects occurring at the low winding stresses. The coefficient \( f \) is also dependent on winder characteristics including roller set up, winding speed, and tensioner stability.

It must be noted that even though the ECP is also a function of the strain induced pressure, the two effects are separate and cannot be combined. As discussed before, the final hoop stress used to determine the ECP is not the initial winding stress, but the hoop stress remaining in the fiber after consolidation. The pressure, \( P_o^{\text{strain}} \), is the initial winding stress induced pressure and determines how tightly (efficiently) the fibers lay-down while winding.

The coefficients \( d \) and \( e \) can be found simply by analyzing data from the cylindrical
rings measured in Chapter 4. Care must be taken to insure the data points used are well into the linear range. Figure 21 shows the consolidation results for three different laminate thicknesses: 8, 10, and 12 plys. As can be clearly seen the higher the winding stress the lower the consolidation. The difference over the range investigated was as much as 300%, depending on laminate thickness. Again, this is because the higher the winding stress the thinner the initial lay-down thickness and the lower the consolidation. The data shown are for a 620 kPa autoclave cure. At this pressure the consolidation resulting from the ECP will be nearly $T_{max}$ (Figure 19). The only variation in cure will now be from lay-down thickness. We can then represent Equation (86) as

$$T(\sigma_o, \theta) = \left( d - e \frac{\sigma_o}{R} \sin^2 \theta \right) T_{max}$$

(87)

We are now left with the problem of pulling out the coefficients $d$ and $e$. Note, the exponential term contributions will have gone to zero at these winding stresses (we are looking at the linear data range, segment II). The procedure for accomplishing this is discussed in more detail in Section 3.9. Also of interest in Figure 21 is the trend of decreasing consolidation with increasing thickness. This can be attributed to measurement error. At the lower winding stresses there is considerable fiber buckling in the thicker laminates resulting in lower consolidation measurements. As the winding tension increases there is less buckling in the outer lamina and the data points begin to converge.
consolidation vs. winding stress

Figure 21: Lay-Down Efficiency Data. All data obtained using type II specimens.

3.9 **Consolidation Characterization Procedures/Recipe**

In this section a step by step procedure to find the required constant coefficients and the testing required is discussed. This is a recipe that utilizes the equations and techniques discussed previously in this chapter. The process is also shown in block diagram form in Figure 22.

1. Once the material system is chosen, the first step is to conduct flat panel coupon tests. Enough different autoclave pressures must be employed to insure an accurate determination of $T_o$, $T_{max}$, and $P_{max}$. A minimum of four points are required.

2. $T_o$, $T_{max}$, and $P_{max}$ are then used along with Equation (77), repeated here, to find $b$ and $c$. Alternatively, a second order polynomial curve fit can be used.
Figure 22: Consolidation Characterization Block Diagram. Block diagram represents process for characterizing consolidation.
\[ b = \frac{2(T_{\text{max}} - T_o)}{P_{\text{max}}} \] (88)

\[ c = \frac{T_o - T_{\text{max}}}{(P_{\text{max}})^2} \]

3. Next cylindrical ring tests are conducted at autoclave pressures equal to or greater than the \( P_{\text{max}} \) measured in step 2. These tests are conducted at winding stresses spanning the range of all possible stresses. This will insure the best curve fit for the specific manufacturing parameter space.

4. The ring consolidation data is graphed and a linear approximation made. Figure 23 shows a linear approximation using the standard Microsoft Excel linear approximation function. The linear equation is shown on the figure. Note that the data points used for the linear approximation of the winding stress must be in the linear range of the data, usually above 40 MPa winding stress.

5. Using the linear approximation generated above, and Equation (89), the coefficients \( d \) and \( e \) are determined. This is done by picking two stresses and then solving for the coefficients. For simplicity the test rings should be wound with \( \theta = 90^\circ \).

\[ T(\sigma_o) = \left( d - e\left(\frac{\sigma_o}{R}\right) \right) T_{\text{max}} \] (89)

6. The final coefficient to be determined is the low winding stress non-linear segment coefficient \( f \). The simplest method to accomplish this is a computer iter-
Consolidation vs. Stress

$$y = 0.0008e - 0.1812$$

Figure 23: Determination of LDE Coefficients. 8 ply laminate chosen for coefficient determination.

This is accomplished by sweeping $f$ in Equation (90) and finding the value corresponding to the minimum average error measured between the empirical and experimental data.

$$T(P^e, \sigma_o, \theta) = \left[ d - e^\frac{\sigma_o t}{R} \sin^2 \theta + \exp(\frac{\sigma_o - f}{R}) \right] \left[ T_o + b P^e + c (P^e)^2 \right] \tag{90}$$

7. At this point all the coefficients have been determined and the consolidation can now be predicted. Figure 24 shows the results utilizing the flat plate coupons and the ring data from Chapter 4. The approximation is good and captures both the linear and non-linear portions of the data. Figure 25 gives the same comparison for a lower pressure cure using the coefficients determined from the high pressure cure.
Figure 24: Empirical Consolidation vs. Experimental. Data represents 8, 10, and 12 Ply lamina cured at 620 kPa. Each point is the average of three tests.

Figure 25: Empirical Consolidation vs. Experimental. Data obtained from cure of 140 kPa.
8. The last step is to generate a composite graphical representation of the consolidation parameter space. Figure 26 is created by using Equation (86) and sweeping the values of the initial winding tension, and autoclave pressures. This accounts for the contributions of the Effective Cure Pressure and Lay-down Efficiency. Once this graph has been generated, the prediction of consolidation is simply a matter of entering the chart with the desired manufacturing parameters and pulling out the predicted consolidation. The curves are generated using the program "Consolidation Gen".

![Empirical Consolidation Graph](image)

**Figure 26: Empirical Consolidation Prediction.** Graph covers practical manufacturing envelope.

### 3.10 Consolidation Induced Fiber Buckling

In this section a brief investigation is made into the material performance degra-
tion resulting from consolidation. Depending on the amount of consolidation and laminate thickness the possibility exists for fibers to experience compressive loading during cure. As stated before this will lead to fiber buckling. The consolidation occurs before the resin has set and the buckling load of the individual fibers is essentially zero. This buckling will lead to the cured laminate having wavy fibers. This will result in decreased material strength, and to a lesser degree, decreased laminate stiffness. The magnitude of the performance decrease will not be discussed in this dissertation, but some guidelines are developed to reduce the possibility of fiber buckling.

As has been discussed in this chapter, various manufacturing parameters influence the amount of consolidation. The parameter that is of most interest for this analysis is the initial winding stress. The larger the winding stress the less likely a laminate is to experience fiber buckling. This is true for two reasons: first the higher the winding stress the lower the consolidation, and second the higher the winding stress the more strain release a fiber can tolerate before buckling. This is shown graphically in Figure 27.

This figure shows a comparison of the consolidation allowable before fiber buckling with the predicted consolidation. The predicted consolidation curve was taken from the 140 kPa curve graphed in Figure 26. The recommended design space is to the right of the intersection of the two curves. When the initial winding stress is maintained above this minimum point, the fibers will not experience compressive loading or the associated buckling. This minimum winding stress is defined as $\sigma_{o,\text{min}}$ and can be found by using the appropriate curve from Figure 26 along with the zero strain curve calculated.
using "Zero Strain Find". The program finds the consolidation when the strain of an individual lamina is zero. This is done using Equation (35).

![Diagram showing Allowable vs. Actual Consolidation]

Figure 27: Allowable Winding Stress Parameter Space. *Allowable space is to the right of the curve intersection.*

3.11 Conclusions

In this chapter the consolidation occurring in filament wound closed-section parts was characterized. The goal was to better understand the effects of: additional cure pressure resulting from the winding hoop stress, variation in accuracy of equipment and material systems, and the lay-down thickness. A series of empirical equations were developed to account for these effects. To accomplish this assumptions were made relating the consolidation to the cure pressure and the lay-down efficiency to winding tension. The accuracy of the assumptions and derived equations were then checked.
using coupon test data from Chapter 4. It was found that the ECP has significant impact on the consolidation and that an empirical approximation can predict these impacts. The effects of ECP on a representative filament wound cylinder were then investigated. It was found that for higher-pressure cure processes the error caused from ignoring the ECP would be small, less than 5%. However, for lower-pressure cures neglecting the hoop stress induced pressures could cause significant error, greater than 30%.

Next the Lay-Down Efficiency was investigated. It was found that the winding tension used on cylinders greatly affected the final consolidation. Unlike the ECP the winding tension and consolidation are inversely proportional and the change in consolidation for the range of tensions investigated was 300%.

Utilizing the equations developed for the ECP and LDE along with simple coupon testing, a recipe to characterize the consolidation of a filament wound cylindrical section was developed. This recipe concluded with the development of a graphical representations of the manufacturing parameter space and the resulting consolidation. This solves the problem of predicting and measuring the consolidation of full-scale parts by allowing the characterization of a material system at the coupon level.

Finally, a brief investigation was made into the problem of fiber buckling or fiber waviness. It was found that the buckling was the result of consolidation and that it could be controlled by properly characterizing the material system and maintaining minimum winding stresses.
CHAPTER 4

EXPERIMENTAL OVERVIEW

4.1 CYLINDRICAL SECTIONS (RINGS)

4.1.1. Introduction

To validate the theoretical solutions tests were conducted to measure the consolidation, $T$, and the spring-back, $S$, for specific manufacturing conditions. The consolidation and spring measurements were used in Chapter 3 and are used throughout the remainder of this dissertation. This chapter discusses the tests in detail.

The tests were conducted using both low CTE composite tooling and high CTE stainless steel tooling. Two types were used to insure that the theory was capturing the effects of the tooling CTE. Four categories of tests were conducted. In each category multiple series of test were run utilizing different winding tensions and laminate thicknesses. In each case the specimens were simple closed-section filament wound hoops (uni-directional rings with all plies running in the hoop or $y$ direction). The test categories are described below:

1. Composite tooling with specimens wound 5.1 cm wide.
2. Stainless steel tooling with specimens wound 5.1 cm wide.
3. Composite tooling with specimens wound 15.2 cm wide.

4. Stainless steel tooling with specimens wound 15.2 cm wide.

Specimens with different widths were used to determine if there were any edge effects contaminating the data. These categories were grouped into two specimen types: Type I = 5.1 cm wide and Type II = 15.2 cm wide. The narrow specimens are desired because they provide more data at a lower cost. This is discussed in more detail in Section 4.2.

4.1.2. Apparatus

This sub-section covers the equipment used for the consolidation and spring measurements. Also discussed is equipment calibration procedures and equipment operation.

The following is a list of the equipment with detailed discussion to follow.

*Winding Equipment and Tooling*

1. Entec 3-Axis Filament Winder (Figure 28).

2. Helman Engineering Inc. (HE) fiber tensioners with 0-70 N tension capability (Figure 30).

3. Chatillon DPP-10 Tension Gauge, 0-45 N (0-10 lbf).

4. Composite Low CTE Tooling Mandrel.

5. Stainless Steel High CTE Mandrel (Figure 32).
Measurement Equipment

6. Mitutoyo Depth Micrometer (Figure 34).

7. Two measurement support fixtures, one for each specimen type (Figure 34).

Vacuum Bagging Material & Processing Equipment

8. Airtech Wrighton 7000 Blue Bagging Film

9. Airtech Perforated Red Release Film.

10. Airtech Air Weave M7 White Breather Cloth.

11. Baron Autoclave (Figure 35).

Material System

12. The material system used for both the composite tooling and specimens was Fiberite IM7/977-2 Carbon Fiber Toughened Epoxy system. The properties are listed in Table 1.

3-Axis Entec Filament Winder

The Entec filament winder is a 3-axis computer numerically controlled winder. The winder can manufacture parts as large as 90 cm in diameter and 300 cm in length. The three axes are: (1) mandrel rotation, (2) vertical winding head translation, and (3) lateral winding head translation. Additional information on operation of the winder can be found in the operations manual [47]

Helman Engineering fiber tensioners

The tensioner system was manufactured by HE Inc. and can maintain from 0 to 70 N of tension. The system has 6 separate tensioners to allow for winding of 6 tows simulta-
neously. For consistency only one tensioner was used for the testing. Pictures of the control panel and tensioner system are shown in Figures (30 and 31). The tensioner was calibrated using the Chatillon DPP-10 Tension Gauge. The data and linear approximation are shown in Figure 29. The data are very consistent and fit the linear approximation with less then 1% variation.

The tensioners are designed to maintain a constant fiber tension regardless of material system type, ball size, or winding (pay-out) speed. The tensioner operates by utilizing a cantilevered arm load cell with feedback to control torque on tensioner motors. Additional information on detailed operation can be found in the HE operations manual [46].
Composite Low CTE Tooling Mandrel

The low CTE composite tooling mandrel was filament wound utilizing IM7/977-2 carbon/epoxy material system. This is the same material used for the test specimens. The lay-up of the tool is (0₂/±60/0₄/±60/0₄). This stacking sequence was chosen to maintain a similar CTE to the test specimens while maintaining the structural integrity required to withstand multiple winding and autoclave cure cycles. The calculated CTE of the mandrel is \(-0.06 \times 10^{-6}/°F\) which compares well with the value shown in Table 1 for individual fibers.

Stainless Steel High CTE Mandrel.

The stainless steel mandrel was used to insure the theory was capturing the effects of tooling thermal expansion. This mandrel is representative of the tooling commonly
used in industry for low to medium cost manufacturing processes. Where higher performance is required, lower CTE tooling is used. The low CTE tooling is usually more expensive and generally made of carbon fiber materials or low CTE metals such as Invar. The specific material is 304 Stainless with a CTE of approximately $12.6 \times 10^{-6}/°C$. The specimen thicknesses are an order of magnitude less than the mandrel thickness and the expansion of the mandrel can be considered unaffected by resistance of the test specimens.

**Depth-Micrometer and Test Fixtures.**

The depth micrometer was used along with the tooling fixture shown in Figure 34.
Figure 31: Tensioner Motors. The controller varies motor torque to maintain required tension depending on ball size.

The micrometer is a Mitutoyo Depth Micrometer and is accurate to 0.013 mm. The micrometer was used with two separate fixtures: one for the Type II specimens and one for the Type I specimens. The Type II fixture provided for measurements at three locations across the specimen width (Figure 33). The Type I fixture provided for measurement at the specimen center. A precision ground block was used along with the micrometer to keep the depth gage from indenting the specimen when pre-cure measurements were taken.
Figure 32: Stainless Steel Tooling Mandrel. The mandrel is covered with Airtech teflon release tape.

Test Fixture

The test fixtures are machined aluminum and were made specifically for the width of the specimen used. The fixture holds the depth micrometer at a consistent height above the mandrel and allows for measurement of the specimen thickness. The fixture shown in Figure 34 is the Type II (15.2 cm wide) fixture. The legs are machined to a point at the outer edges to prevent the fixture from tipping while sitting on the curvature of the mandrel.

Baron Autoclave

The autoclave used was manufactured by Baron Inc. It can operate at temperatures from 0 - 315 °C and pressures from 0 - 3100 kPa. The autoclave can be either computer
or manually controlled. The vacuum system on the autoclave has ten ports available but only one was used for each tooling mandrel. The control system can also record the actual cure cycle electronically or on hard copy. This feature was used to insure consistency between tests.

**Material System**

The material system used for the tests was Fiberite’s IM7/977-2 carbon/Epoxy material system. The fiber comes in the pre-preg form. Pre-preg indicates that the fiber has the resin pre-impregnated by the manufacturer. This provides precise control of the fiber volume fraction and results in higher and more consistent material properties. The pre-preg fiber is shipped refrigerated and must be maintained at approximately -18 °C.
Figure 34: Consolidation Measurement Fixture. The depth micrometer is used to measure pre and post-cure laminate thicknesses.

Figure 35: Baron Autoclave. The autoclave can be manual or computer controlled. All tests were conducted using computer control to insure consistency.
or $0 \, ^oF$. The fiber comes as one continuous fiber wrapped into spools of varying weight, as shown in Figure 36, and are referred to as "balls". The balls are numbered for tracking purposes, e.g., ball # 10, Lot 63182D. Within each lot the band-width (width of the tow) will change from ball to ball but the fiber volume fraction will remain constant. The recommended cure cycle is shown in Figure 37. It utilizes a high temperature high pressure cycle. The pressures used can be varied depending on application but the cure temperature is inflexible unless decreased material performance is acceptable.

<table>
<thead>
<tr>
<th>Ply Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus in Fiber Direction, $E_{11}$</td>
<td>172.4 GPa (25 Msi)</td>
</tr>
<tr>
<td>Young’s Modulus in Transverse Direction, $E_{22}$</td>
<td>10.1 GPa (1.5 Msi)</td>
</tr>
<tr>
<td>Shear Modulus, $G_{12}$</td>
<td>6.3 GPa (0.91 Msi)</td>
</tr>
<tr>
<td>Poisson’s Ratio, $\nu_{12}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion in Fiber Direction, $\alpha_1$</td>
<td>$-0.39 \times 10^{-6}/^\circ C (-0.22 \times 10^{-6}/^\circ F)$</td>
</tr>
<tr>
<td>Transverse Coefficient of Thermal Expansion, $\alpha_2$</td>
<td>$21.6 \times 10^{-6}/^\circ C (12.0 \times 10^{-6}/^\circ F)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiber Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus in Fiber Direction, $E_{11}$</td>
<td>262.0 GPa (38.0 Msi)</td>
</tr>
</tbody>
</table>

Table 1: IM7/977-2 Material Properties.

4.1.3. Manufacturing Procedures

The test specimens were manufactured on both the steel and composite tooling using the same manufacturing process. The process is specified below:
Figure 36: Fiber Ball. Fiber comes in pre-preg form with one continuous fiber wound to create a ball.

![Diagram of Fiber Ball with temperature and pressure parameters]

Figure 37: IM7/977-2 Carbon/Epoxy Recommended Cure Cycle. The material system is manufactured by Fibertite.

1. For the first test the tooling mandrels were covered with Airtech teflon release
tape and then coated with teflon 1700L McLube spray-on mold release. Prior to each additional test the mandrels were cleaned with Acetone and the mold release reapplied.

2. To start the winding process a tow was wound around the mandrel by hand and tacked to itself using a hot air gun. The number 1, or starting specimen, is to the far left in Figure 38. All specimens were filament wound with one tow (multiple tows can be wound simultaneously but one was used to obtain better consistency). Tension was maintained within $0.5 \ N$. Attempts were made to wind an entire specimen from one ball of fiber to eliminate any variation in tow thickness. This was not always possible but the balls were matched as closely as possible.

3. After all specimens were complete the thickness was measured using a depth micrometer and measurement fixture. The Type I specimens were measured at their center, with respect to width, at three circumferential locations ($90^\circ$, $180^\circ$, and $270^\circ$) around the mandrel with $0^\circ$ being the position they were cut. The Type II specimens were measured at three locations across their width at the same three circumferential positions as the Type I specimens. This gives 3 and 9 consolidation data points for each Type I and Type II specimen, respectively. Three measurements were taken across the width of the wider specimens to determine if the consolidation was constant across the width (no drop-off towards the edges) and to maximize the number of data points.

4. The mandrel was then vacuum bagged, as shown in Figure 39, and autoclave
cured according to the fiber manufacturer's recommendations.

5. Following cure the specimen thicknesses were again measured.

6. Once all the measurements were taken the specimens were cut along the zero degree line, removed from the mandrel, and the sprung radii measured using a dial micrometer.

4.2 Data Analysis

4.2.1. Introduction

To ensure the experimental procedures were resulting in credible data, two separate
specimen widths were tested. This section discusses the consistency and applicability of the specimen types. The primary question was, what minimum specimen width is required to ensure no edge effect contributions to the data? The narrower the specimen the less costly and time consuming it is to gather data; the Type I specimen provides three times the data for each test cycle. Approximately $0.5 \text{ cm}$ of washout exists on the edges of both specimen types (Figure 33). To determine the significance of the washout various comparisons are made between the data from these two types. The variations along the width of the Type II specimens are also investigated. The last factor explored was the time dependency of the winding strain. Since the resin is in a liquid state during winding (a highly viscous liquid state), the effect of consolidation and tension release before cure was a concern.

A specimen thickness of 8 plies was used for much of the analysis. Choosing the
quasi-optimum thickness is a trade-off between desiring maximum thickness to decrease the error resulting from instrument accuracy and the relative flatness of the specimen which is best for the thinner laminates. The flatness is controlled by fiber buckling which gives the surface a wavy appearance and makes post-cure measurements more difficult. For the composite tooling an 8 ply thickness resulted in little fiber buckling for a wide range of winding tensions but was still thick enough to keep the error contribution from instrument accuracy low.

4.2.2. Data Comparison Between Types

When comparing the data between specimen types, the two things of interest are consolidation and spring. Consolidation will be discussed first followed by spring comparisons.

Figure 40 shows a comparison of the consolidation vs. specimen type. As can be seen there is little difference between the Type I (5.1 cm wide) vs. the Type II (15.2 cm wide) specimens. All the data follows the same trend with an initial exponential decreasing segment followed by a linearly decreasing segment. The cause of this behavior was discussed in detail in Chapter 3. This indicates that the Type I specimens are sufficiently wide for use in consolidation characterization.

Figure 41 shows the spring vs. specimen type. In this case there is significant variation in the data. This indicates that the Type I specimens are not sufficiently wide for spring measurements. The question is, why would the Type I specimen work for consolidation and not for spring? The answer is actually quite simple.
Figure 40: Specimen Type Comparison. The Type I and Type II data points represent the average of 12 and 9 measurements, respectively.

For consolidation the measurements are point measurements taken at the center of the Type I specimen and at three points for the Type II. As can be seen in Figure 33 the measurement locations are well away from the edges and the wash-out is not likely to affect the consolidation data. On the other hand, the spring is obviously not a point measurement but an average of the residual stress couples across the width of a specimen. As might be expected, in the narrow specimen the washout becomes more significant. This indicates that for spring measurements the Type II specimens must be used.

4.2.3. Consolidation Measurements Across Specimen Width

The previous argument demonstrates that the consolidation measurements can be taken utilizing the Type I specimens. To further support this the variation in consolida-
Figure 41: *Spring vs. Specimen Type.* Each data point is the average of 9 separate tests.

tion across a specimen width is investigated. This can easily be accomplished since the
Type II specimens were measured at three locations with the outer measurement being
roughly the same distance from the edge as with the Type I specimens. Figure 42 shows
the results of the consolidation measurements across the thickness of a set of representa­
tive specimens wound with 40 N tension. An indication of edge effects in these tests
would be a local minimum or maximum value for consolidation at the center of the
specimen. As can be seen, all three curves have maximum and minimums at different
locations. These variations are attributed to data scatter. The same trend is seen with all
the Type II specimens with no significant change in consolidation over the width; the
average variation is less than 1% absolute consolidation. This supports the initial con­
clusion that the consolidation measurements can be made with the Type I specimens.
To summarize the testing analysis, the consolidation measurements can be made with the Type I specimens and were used for the consolidation characterization. However, for the spring measurements the wider Type II specimens must be used and are used in Chapters (5 & 6).

![Consolidation Variation vs. Width](image)

**Figure 42:** Consolidation Measurements Across Specimen Width. Each data point represents the average of 9 separate measurements.

### 4.2.4. Pre-Cure Time Dependence of Consolidation

Before cure the resin is in a high viscosity liquid state. There was a concern that the fibers would simply pull through the resin and release the tension before cure. This would not affect the final spring, but if this happened before the pre-cure measurements were taken the consolidation values would be lower. The theoretical models would still be applicable since they are only dependent on the magnitude of the consolidation, not
when it occurs. However, if the consolidation measurements are low, the spring predictions will be incorrect. They could be high or low depending on whether the zero strain point was crossed.

To investigate this a specimen one tow wide was wound and the thickness measured immediately. The thickness was then measured at various intervals. The surface appearance was also observed to see if there were any indications of resin propagating to the surface. If the fibers are pulling down through the resin the surface should become visibly resin rich. It was found that the thickness did not change by a measurable amount over the time frame of interest, 4 hours. There was also no change in surface appearance of the specimen. It is believed that the high viscosity and small size of the individual fibers prohibit any significant fiber movement. The tows are made of 15k bundles, which is to say there are 15,000 individual filaments making up one bundle. These act as a very fine screen that the resin cannot easily pass through. When the resin is raised to cure temperature the viscosity reduces to nearly that of water and this is when the consolidation occurs.

4.3 Flat Coupon Testing

This section covers the testing conducted to determine the constant ECP coefficients. To mimic the ECP occurring in filament winding, flat plate coupons were cured at different autoclave pressures. Since the ECP is simply the summation of autoclave and hoop stress induced pressures, a wide range of ECP's could be mimicked in this manner. Specifically, these tests determined $T_c$, $T_{mac}$ and $P_{mac}$. These are then used to
obtain the constant coefficient terms in Equation (75). Test were conducted at 1380, 965, 620, 345, and 172 kPa. The coupons were 7.62 x 10.16 cm. (3 x 4 in.) and made from Fiberite IM7/977-2 pre-preg carbon/epoxy resin system.

4.3.1. Apparatus

This sub-section covers the apparatus used in the flat coupon tests.

Winding Equipment and Tooling

1. 60 x 60 cm Precision Ground Steel Tooling Plate with 0.025 mm tolerance over the width of the plate.

2. Baron Autoclave. Same as discussed above.

3. Dial Micrometer. Same as discussed above.

Vacuum Bagging Material & Processing Equipment

4. Same Bagging Equipment and Procedures as discussed above.

Material System

5. The material system used was Fiberite IM7/977-2 Carbon Fiber Toughened Epoxy system. This material is in the tape form as opposed to the tow form used in filament winding. Otherwise the material has equivalent properties and handling procedures. The tape form comes as a roll 30.5 cm. (12 in.) in width. The rolls vary in length, but are on the order of 100+ meters. The tape is the same thickness as the tows used in filament winding. The tows are actually tape that has been slit into smaller widths so the consolidation properties should be identical.
4.3.2. Procedures

The procedures for this test are quite simple and are listed below.

1. The first step was to prepare the steel tooling plate. This was done by cleaning and then applying teflon sheeting to insure tool-coupon separation.

2. The next step was to prepare the coupon material. The carbon fiber tape was cut to coupon size, 7.62 x 10.16 cm. (3 x 4 in.). Each test had 8 coupons: four 10 plys thick and four 20 plys thick.

3. The individual plys were then stacked by hand to the required thickness (number
of plys) and placed on the tooling plate. A heat gun is used to tack the plys together and get them to lay flat. Steps 2 and 3 are shown in Figure 44.

4. The coupons were then vacuum bagged. Care was taken to insure the vacuum port was far enough from the coupons so as to not affect the consolidation.

5. The tooling plate was then placed in the autoclave and cured. Five different cure pressures were used, 1380, 965, 620, 345, and 172 kPa.

6. Once the cure was complete the coupons were: de-bagged, removed from the tooling plate, and the thickness measured.

The results for these tests are discussed in Chapter 3.
Figure 44: Flat Plate Coupon Test Preparation. The pre-preg tape was cut to size and stacked to desired thickness (ply number) utilizing a roller and heat gun.
CHAPTER 5

SPRING PREDICTION

5.1 INTRODUCTION

In this chapter a spring prediction methodology is developed and validated. This will be accomplished in four steps: (1) the trends in spring data are analyzed and compared on a point-by-point basis to the theoretical solutions developed in Chapter 2, (2) the solutions derived in Chapter 2 are integrated with the consolidation characterization results from Chapter 3, (3) the effects of cure pressure and winding tension on the spring and strain profiles are investigated, and finally (4) the integrated solution is compared to experimental data. Both low and high CTE tooling results are discussed. This will demonstrate that spring in filament wound parts can be accurately predicted and would enable a design engineer to modify the tooling and obtain a dimensionally accurate part.

5.2 SPRING DATA REVIEW

The first step in verifying the spring solutions is to look at the spring in a point-by-point fashion. This is done by looking at individual specimens, measuring the consolidation, inputting this value to the spring prediction formulation [Equation (66)],
and comparing it to experimental data. This would not be of great benefit in the manufacturing environment since the outcome of the testing could not be determined until full size tooling was available. However, it does provide a preliminary verification of the theory and leads to some interesting conclusions.

Figure 45 shows a comparison of the experimental vs. theoretical spring for a range of winding tensions. It is interesting to note that the data trends exhibit opposite slopes. Understanding why this occurs is critical to understanding the spring problem and will be discussed first. This is followed by discussion of the spring prediction accuracy.

![Spring vs. Tension](image)

**Figure 45:** *Spring vs. Tension*. The graph shows theoretical vs. experimental spring at various winding stress.

As discussed previously the spring is directly proportional to the consolidation. We also know that, in general, the consolidation decreases with increasing winding stress,
which is to say, consolidation is inversely proportional to winding stress. Knowing this we would expect the spring to decrease with higher winding tension. As can be seen in Figure 45 the high CTE tooling exhibits the expected trend, but the low CTE tooling exhibits the opposite trend. This is the result of the zero strain point being met at some point within the laminate; the outer lamina have experienced compressive loading and have buckled. As shown in Figure 12, at lower winding tensions and or higher cure consolidation values, the zero strain point will be crossed and the outer plies will not add to the residual stresses. The outer plies will, however, increase the value of the $D$ matrix (flexural stiffness matrix). This larger flexural stiffness and smaller stress couple will result in less spring. This can be seen for the composite tooling specimens where the spring actually increases with winding tension. Just the opposite of what was originally expected. The theory does capture this trend and predicts the magnitude of the spring quite accurately with less than 8% average error.

In the case of steel tooling the spring decreases with increasing tension, as would be expected. This differs from the composite tooling because of the high CTE of the steel. For the steel tooling the effective winding tension, which is simply the summation of the winding strain and the strain induced by the tooling CTE [Equation (33)], is relatively large and all lamina remain under tension during the cure process. We would get the same results from the composite tooling if higher winding tensions were used. This is evident in the data which shows a slope reversal at the higher winding stresses; the spring begins to decreases at approximately 90 MPa winding stress. The theoretical predictions again capture this trend and predict the magnitude of the total spring to
within 8% on average. The slope reversal in the composite tooling data is discussed in more detail later in this chapter.

5.3 Low CTE Tooling Spring Analysis

In this section the empirical consolidation model and theoretical spring formulations will be integrated to produce a general spring prediction theory. This generalized theory will allow for spring prediction in full scale tooling based on testing accomplished at the coupon level. This will eliminate the need for tooling iteration and reduce the cost of composite manufacturing. This is done utilizing the consolidation characterization from Chapters 3 and 4 along with the spring prediction equations in Chapter 2. These are integrated together in the program "Spring Predict". This program utilizes the Consolidation Gen. program along with Equation (65) or Equation (66) depending on the assumption of constant or variable consolidation. These equations are listed here for reference.

\[
\kappa_y = \left[ D_{12}^{-1} D_{22}^{-1} D_{23}^{-1} \right] \sum_{k=1}^{N} \left[ \delta_k \int_{h_{k-1}}^{h_k} \left[ \frac{m^2 - \nu_{12} n^2}{n^2 - \nu_{12} m^2} \right] e'_1(z) \, dz \right] \quad (91)
\]

\[
\kappa_y = \left[ D_{12}^{-1} D_{22}^{-1} D_{23}^{-1} \right] \sum_{k=1}^{N} \left[ \delta_k \int_{h_{k-1}}^{h_k} \left[ \frac{m^2 - \nu_{12} n^2}{n^2 - \nu_{12} m^2} \right] (e'_1, k) \, dz \right] \quad (92)
\]
The procedure used for determining the necessary coefficients is as stated in Chapter 3. This investigation is done primarily with 8 ply laminates. As discussed in Chapter 4, this is the quasi-optimum thickness with respect to data collection. In thought the integration is quite simple but in actuality the iterative bases of the Consolidation Gen program makes the integration somewhat complex. The Spring Prediction program utilizes the Consolidation Gen program on a discrete point-by-point basis and sweeps over a selected range to provide a graphical representation of the spring envelope. The programs are not discussed in detail but can be recreated utilizing the equations discussed previously. The results are shown in the following sub-sections.

5.3.1. Spring Prediction Analysis

Now that an integrated formulation to predict spring is available it is interesting to look at how varying parameters affect the spring. This is done by generating 3-Dimensional and 2-Dimensional graphic representations of the manufacturing parameter envelope. The 3-D graph is useful for visualization of the spring space, but is not conducive for pulling out specific data points. The 2-D representation can be used for this and enables a design engineer to easily read specific data points. This would be necessary in a design/manufacturing environment.

Figure 46 gives a 3-D representation of the spring envelope. The spring envelope covers all possible values bounded by practical manufacturing parameters. Figure 47 is a 2-D representation of the same envelope. Both figures provide a global picture of the manufacturing parameter space. Various points of interest can be readily seen in these
graphs. First, variations in autoclave pressure at lower pressures, for a particular winding stress, have significant effect on the spring. This results from the consolidation being dependent on the Effective Cure Pressure. At the lower cure pressures, variations in pressure have a large influence on the consolidation and cause significant change to the spring. However, as \( P^e \) approaches \( P_{max} \) the consolidation curves flatten out (Figure 19) and the spring becomes less dependent on pressure; the constant autoclave pressure lines began to converge. This supports our conclusions in Chapter 3 that the ECP influence can be neglected if higher pressure cures are being used.

**Spring Profile**

![Spring Profile](image)

*Figure 46: 3-D Spring Profile. Graph covers envelope of practical manufacturing parameters.*

Another interesting observation is in regards to the slope of the constant cure pressure curves. Specifically of interest is the slope of the curve after their peak value. At
the lower pressures the slope is flat, but at the higher pressures the curves exhibit a negative slope. This is due to the Effective Cure Pressure and the Lay-Down Efficiency counteracting each other. The ECP tends to increase the consolidation at higher winding stresses (the ECP is proportional to winding stress) while the LDE reduces the consolidation (the LDE is inversely proportional to winding stress). The opposing effects result in the consolidation for lower autoclave pressures being fairly constant across the winding spectrum. This can be seen in Figure 26 where the consolidation remains at approximately 5% regardless of the winding stress. At the higher pressure cures the hoop stress contribution to the ECP influenced consolidation are smaller and the curves have a pronounced negative slope. This is caused by the decreasing consolidation at the higher winding stresses; the LDE has become the dominant effect.

![Spring vs. Winding Stress](image)

**Figure 47: Spring vs. Winding Stress Envelope.** Graph used to determine specific values for spring.
5.3.2. Experimental Comparison/Discussion

The output from the program Spring Predict is compared with experimental data in Figure 48 for both variable and constant consolidation models. The results are good with both formulations capturing the slope reversal and predicting the absolute magnitude of the spring as well. The average error for the constant and variable consolidation models is less than 6% and either model could be used with little loss of accuracy. This error is small and can be attributed to the data scatter from the consolidation and spring measurements. It is interesting to note that both models have their maximum at approximately the same winding stress but the constant consolidation model has a lower maximum value. The larger spring results from the variable model having higher consolidation in the lower lamina and lower consolidation in the upper lamina (Figure 49). This decreases the strain more rapidly in the upper lamina thereby increasing the stress couple and spring.

Both models follow the slope reversal trend which indicates the formulation is correctly accounting for the effects of fiber buckling in the low CTE tooling. As the winding stress is increased, more and more of the layers remain in tension and contribute to the strain or stress profile. At a certain point all the layers remain in tension and as further winding stress is added the spring begins to decrease as the consolidation decreases. This can be easily seen in Figure 50 which shows the final strain profile for various winding stresses. At approximately 90 MPa winding stress all the layers remain in tension. This would correspond to the maximum spring (the worst winding tension with respect to spring). This also compares well to the maximum spring predicted in
Figure 47.

From this analysis it would seem logical to wind parts with the lowest possible winding stress thereby reducing the spring. However, as discussed in Chapter 3, if the zero strain point is met the fibers start to buckle (become wavy). This leads to significant reduction in material performance. Conversely, the more the winding tension is increase above 90 MPa the less spring. This would lead us to wind with very high tension, but this results in high residual stress which again decreases material performance. The solution that satisfies both low spring and good material performance is discussed in Chapter 6.

![Experimental vs. Predicted Spring](image)

Figure 48: Theoretical vs. Experimental Spring (Low CTE Tooling). The graph represents eight ply specimens. Each data point represents the average of three tests.
Figure 49: Variable Consolidation Profile. Graph compares variable and constant consolidating profiles.

Figure 50: Strain profile. The data shown was taken at 600 kPa autoclave pressure.
Appendix A contains additional plots of experimental vs. predicted spring for 10 and 12 ply thick laminates. The comparisons are good with errors less than 10%. It is interesting to note that for a given winding tension the spring decreases as the laminate thickness increases; the winding stress where the maximum spring occurs is shifted to the right. This can be seen more clearly in Figure 71 which shows the spring vs. winding tension for various laminates ranging from 2 to 20 plys thick. Again, this is the result of the outer lamina going into compression and not contributing to the residual stress couple. However, the additional lamina do contribute to the $D$ matrix which results in lower spring.

5.4 High CTE Tooling Spring Analysis

In this section the same analysis is followed for the high CTE tooling. The tooling represented here has a higher CTE than what would normally be used in an aerospace manufacturing environment. It does, however, give a good check of the theory at the outer bounds of practical tooling. The tests were done primarily with 8 and 12 ply laminates.

5.4.1. Spring Prediction Analysis

As before, 3-D and 2-D graphical representations of the spring envelope are generated using the full theoretical formulation. Again, some interesting observations can be made. In this case the spring is less sensitive to the cure pressure. This can be seen in Figures (51 and 52). There is little change in spring between constant pressure curves. From 0 - 900 kPa there is less than 15% change in the spring. This is due to the high
effective winding stress which makes the ECP almost entirely independent of cure pressure. This is to say, the strain induced pressure, $P_{\text{strain}}$, exceeds the pressure at which the laminate becomes incompressible, $P_{\text{max}}$, with little or no autoclave pressure.

\begin{center}
\textit{Spring Envelope (Steel Tooling)}
\end{center}

![Spring Envelope (Steel Tooling)](image)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{spring_envelope.eps}
\caption{3-D SpringProfile. Graph covers envelope of practical manufacturing parameters.}
\end{figure}

5.4.2. Experimental Comparison/Discussion

Figure 53 shows the theoretical vs. experimental spring predictions for the high CTE tooling. The predictions are good for both the constant and variable consolidation models. There is almost no difference in the spring predictions. In this case the slope of the spring prediction curves are negative for most of the winding stress range. This results from the zero strain point only being met at the lowest winding tensions. This can be seen in Figure 54 where any winding stress over 35 MPa insures tension
Spring vs. Winding Stress

Figure 52: Spring vs. Winding Stress Envelope. Graph used to determine specific values for spring.

throughout the laminate during cure. The maximum and average errors are 30% and 12% respectively. These results are not as good when compared to the low CTE tooling. There are two possible reasons for this and both result from the high effective winding stress. Again, this high stress comes from the summation of the winding and tooling expansion induced stresses. In some cases the stress is over 80% of the fiber failure stress.

First, the error may come from movement of the fibers in the lateral direction. This fiber movement takes the form of washout as shown in Figure 33. The washout in the high CTE tooling specimens was an order of magnitude greater than that of the low CTE tooling specimens. It is difficult to determine the significance of this contribution since it is hard to measure which fibers are moving. Without knowing this, the extent to
which lateral movement is affecting the strain profile and spring is hard to predict. For the lower CTE tooling the washout is small and the effects on the spring is negligible.

Another possible cause resulting from the high stress is fiber failure. In composites it is common to see individual fiber failure once 80% of the fiber strength is exceeded. This would probably have a much smaller effect on the spring than the washout, but it could still be significant. However, the results are good and considering the tooling is on the upper bounds of what is practical we considered these satisfactory.

**Figure 53: Theoretical vs. Experimental Spring (High CTE Tooling).** Over prediction results from the extremely high effective winding stress causing fiber movement in specimens.

### 5.5 Design Methodology Overview

In this section the procedures discussed in this chapter are condensed into a design methodology that can be used to obtain dimensionally accurate final parts. Once the
Summary of strain about the mid-plane results in spring.

spring in a part can be determined the process of tooling iteration can be eliminated. To obtain the correct tooling geometry it is simply a matter of: (1) following the procedures developed in Chapters 3 and 4 to determine the empirical consolidation coefficients, (2) integrating the consolidation model with the spring equations from Chapter 2, (3) predicting the part spring in the full scale tooling, and (4) adjusting the tooling geometry to accommodate the spring. This process is shown as a block diagram in Figure 55.

5.6 Conclusions

In this chapter it was shown that the theoretical solution developed in Chapter 2 could accurately predict the spring in filament wound cylindrical sections. More
DESIGN METHODOLOGY

Part Geometry & Material System

Characterize Consolidation
Coupon Testing

Spring Model
Constant or Variable Consolidation?

Integrate

Predict Spring

Modify Tooling
(Compensate for Spring)

Dimensionally Accurate Part

Figure 55: Design Methodology Block Diagram. Process for obtaining dimensionally accurate part.
importantly, it was shown that the prediction formulation integrated with the empirical consolidation models can predict spring based on coupon testing. This integrated model can also be used to determine the spring envelope and the effects of varying manufacturing parameters. These models and solutions allow a design engineer to adjust the manufacturing parameters to reduce the spring, or account for the spring in tool design. This will provide cost and time savings in the manufacturing process. In the following chapter the design methodology will be expanded to include spring elimination.
CHAPTER 6

SPRING ELIMINATION

6.1 INTRODUCTION

In the previous chapters a methodology was developed which allowed for the prediction of spring in filament wound cylinders. In this chapter the methodology will be extended to include the elimination/reduction of spring. An added advantage of the process will be the reduction and control of the residual stresses thereby enhancing material performance. This will be accomplished by modifying the manufacturing parameters. Specifically, the winding tension will be varied on a ply-by-ply basis to change the final cured strain profile. We will refer to this process as "Tension Control". The investigation will begin by deriving the required tension for spring elimination.

6.2 ZERO SPRING TENSION REQUIREMENTS

The process of spring elimination involves introducing an opposing moment into the laminate by changing the tension during winding, tension control. The required moment can be found by utilizing Equation (60), repeated here for reference.
This is the moment resulting from the residual stress and exists in the ring after cure. To eliminate the moment an opposing moment must be applied during manufacture.

A more visually intuitive way of stating the problem is to require the strain profiles shown in Figures (50 and 54) to be vertical, preferably at low strains. A vertical strain profile will eliminate the residual stress couples. This is done by varying the tension through the thickness. For uni-directional rings the process is simple and an optimum winding profile is easily obtained. For off-axis or asymmetric laminates the problem is more complex and an infinite number of solutions can be found. Both solutions are now discussed.

6.3 Low CTE Tooling Spring Elimination

For uni-directional hoops the calculations for required ply tension are straightforward and can be solved for directly if constant consolidation is assumed. The winding tension of the lamina above the base ply are simply adjusted to insure they have the same tension as the base ply after cure. The base ply is the #1 ply, or ply against the tooling face (Figure 15). As discussed before, this is more difficult if we assume variable consolidation based on the ECP and LDE. In Chapter 5, where constant tension was used throughout the winding process, the constant consolidation model had sufficient accuracy. As we will find later in this chapter, to eliminate the spring the winding
tension must vary significantly from ply-to-ply. This results in the lower lamina being wound with significantly less tension than normal. The consolidation of these low tension plys is more susceptible to ECP variations (Chapters 3 and 5) and in most cases this necessitates the use of the variable compaction model. The steps necessary to obtain the winding profile are discussed below.

1. As stated before the first step is to characterize the consolidation of the material system. This is done in the same manner as in Chapter 3.

2. The initial lamina stresses are defined by a winding gradient based stress starting with a base ply tension. The base tension choices are limited and usually equipment and tooling driven. Most tensioners have a minimum tension limit, and depending on tooling and ply orientation a minimum tension is required for correct fiber placement. The initial tension in the \( k \)th lamina is then defined by Equation (94) where \( G \) is the winding gradient, \( A \) is the cross-sectional area of the fiber, \( E_f \) is the modulus of the fiber, and \( \varepsilon_{base} \) is the strain in the base ply. The base strain is calculated from the base tension, \( b_n \), and should usually be chosen as the minimum value allowed by the equipment. This will result in the lowest average residual stress.

\[
\varepsilon_{o,k} = \varepsilon_{base} + G(k-1)/AE_{11} \tag{94}
\]

3. The post-cured strains are found by substituting Equation (94) into Equation (41). This equation requires an iterative solution because of the interdependence of the consolidation and strains. This was discussed previously.
\[ e_{1,k} = e_{base} + G(k-1)/AE_{11} + \sum_{i=1}^{k} \left[ T_k(z_i - z_{i-1})/R \right] \]

4. The gradient, \( G \), is then swept graphically up from zero until the strain is constant through the laminate. This results in zero spring. The choice of base tension and its effect on the required gradient will be discussed in more detail in the next section.

6.3.1. Spring Profile Analysis

Using this process a number of interesting graphical representations and general results can be obtained. Figure 56 shows the spring vs. tension gradient for a practical range of autoclave pressures, \( P^a \). Observing the constant autoclave pressure curves, the point where the curve crosses the horizontal axis corresponds to the zero spring gradient. It is interesting to note that as the pressure increases the zero spring crossover points shift to the right. This occurs because the cure pressure and consolidation are directly proportional. As the pressure increases the increase in consolidation results in a steeper strain profile and greater spring. To overcome this steeper strain profile a larger winding tension gradient is necessary. Additional plots are shown in Appendix B.1.

6.3.2. Strain Profile Analysis

Figure 57 shows the strain profile vs. the tension gradient. The data for this figure were obtained using \( P^a=600 \, kPa\). At a winding gradient of \( G=1.2 \) the strain profile is approximately vertical. This indicates that at this gradient, the spring should also be
approximately zero. This agrees with the graphical representation in Figure 56 which predicts the zero spring gradient for a 600 kPa cure to be approximately 1.2. The zero spring profile can also be observed by considering the first moment of the curves about the mid-plane. The resulting bending moment shown in Equation (93) is proportional to the first moment. It is easily seen that the first moment is zero for the $G=1.2$ curve. This is a comforting physical and visual interpretation of the spring problem. Additional plots are shown in Appendix B.3.

6.3.3. Base Tension Effects

The base tension can also have large effects on the spring and strain profiles. Figure 58 illustrates a comparison between constant pressure spring curves with different base
Strain Profile After Cure

Figure 57: Strain Profile. Data shown was obtained at 600 kPa and $b_t = 13 \text{ N.}$

tensions. The effect of increasing base tension is to shift the zero spring gradient to the left, a lower value. However, the higher base tension also increases the spring at the lower gradients and shifts the curve up. This occurs because the higher base tension results in more plys remaining in tension for a given winding stress. Since the winding gradient cannot begin to affect the spring until all plys remain in tension this allows the winding gradient to start reducing the spring earlier and results in the lower required zero spring gradient.

With these observations it would appear that a higher base tension is beneficial. In practice this is not the case since the absolute magnitude of the residual stress is higher. This would adversely impact material performance. This supports the initial statement that the base tension should be the lowest practical tension with respect to equipment
limitations.

**Base Tension Effects**

\[ P^* = 600 \text{ kPa} \]

![Graph showing Base Tension Effects](image)

*Figure 58: Base Tension Effects (Low CTE Tooling). Data shown was obtained at \( P^* = 600 \text{ kPa} \).*

### 6.3.4. Experimental Results/Discussion

In this section the experimental validations of the zero *spring* manufacturing methodology are discussed. Tests were conducted with both the composite and steel tooling. The steel tooling results will be discussed in a later section. It will be shown that with proper *tension control* the *spring* can be greatly reduced and eliminated in most cases.

Figure 59 shows a *spring* vs. tension gradient plot compared with experimental data. The data was taken at various values for \( G \) ranging from \( G = 0.7 - 1.8 \). The data and calculation were done with \( P^* = 600 \text{ kPa} \) and \( b_i = 10 \text{ N} \). The theoretical solution agrees well with the experimental, but underestimates the zero *spring* crossover point by approxi-
mately 14%. The theoretical solution also follows the trend of the experimental data well. As the gradient is increased the spring increases slightly and then rapidly drops off at approximately $G=1.0$. This is the gradient where all the plys remain in tension throughout the cure and the tension control begins to take effect. The tension gradient cannot begin to reduce the spring until the outer plys remain in tension. This can be seen by observing the strain profile shown in Figure 57 and visualizing its effect on the first moment. Additional plots are shown in Appendix B.

![Spring vs. Tension Gradient](image)

**Figure 59:** *Spring vs. Tension Gradient.* The results shown are for 600 kPa autoclave pressure.

Although the theory matches the experimental data well the reason for the theory consistently under predicting the zero strain crossover point is difficult to determine. The most likely rationale for this is the outer high tension plys pulling themselves down
through the lower tension plies. There has been no way found to verify this hypothesis. However, the results are very good and using the zero spring methodology will allow a design engineer to come within 15% of the correct gradient. Also, because of the consistency it should be possible to compensate for the error. This was demonstrated when tests were conducted at lower cure pressures.

Figure 60 shows a comparison of spring vs. tension gradient for a lower pressure cure, $P_a=140$ kPa. There are two experimental data points obtained for $G=1.2$ and 1.6. These points were chosen based on the results from the higher pressure cure. The theory predicted a gradient of slightly over $G=1.0$ would result in zero spring. Based on this a gradient of $G=1.2$, which is approximately 15% higher, was used in an attempt to get zero spring. To explore negative spring a gradient of $G=1.6$ was also used. As expected the results are the same as the higher pressure cure with the theory underestimating the strain gradient required to achieve zero spring. The difference between the theoretical and experimental is approximately 17%. This provides some confidence that the error is consistent given the same tooling geometry. Pictures of the uncontrolled (constant winding tension) and the two tension controlled specimens are shown in Figures (61 - 63). The spring in the uncontrolled, controlled, and over-controlled rings are, $S=0.203, -0.007, and -0.065$, respectively. For practical purposes the spring in the controlled specimen is zero. This demonstrates that the error can be compensated for and zero spring achieved.
Figure 60: Spring vs. Tension Gradient (Low Pressure). Results shown are for 140 kPa autoclave pressure.

Figure 61: Uncontrolled Specimen. Specimen wound with G=0.0. Represents spring with no tension control.
Figure 62: Tension Controlled Zero Spring Ring. Specimen wound with $G = 1.2$. Represents predicted zero spring tension control profile.

Figure 63: Tension Controlled Ring (Over Controlled). Specimen wound with $G = 1.6$. Represents higher than required tension control profile. Demonstrates tension control authority.
6.4 High CTE Tooling Spring Elimination

The analysis of the steel tooling is similar to that of the composite tooling. The only significant difference results from the higher effective winding tension. The effective winding tension was discussed in Chapter 2 and the governing equation is given by Equation (33). The following sub-sections discuss the result for the zero spring tension control techniques applied to the high CTE tooling.

6.4.1. Spring Profile Analysis

Figure 64 shows a comparison of the spring vs. tension gradient. Some important information is readily available from this plot. Specifically, the required gradient for zero spring is excessively large when compared to the low CTE tooling. For the low CTE tooling the necessary gradient was well within manufacturing limitations. With the high CTE tooling the required gradient of $G=1.9$ equates to a tension of $80 \, N$ in the outer ply; this is outside the range of many tensioners. Another concern would be fiber failure during winding. This occurs quite often when winding at elevated tensions. Because of this it can be concluded that spring cannot be totally eliminated in applications where high CTE tooling is employed. The results for specific tests are discussed in the following sections.

As with the composite tooling the cure pressure also has a significant effect on the spring. Increasing cure pressure shift the constant pressure curves up. This causes the required zero spring gradient to increase with pressure. This is expected and occurs because the consolidation is directly proportional to the cure pressure. The higher the
cure pressure the more consolidation and the more spring the tension control must overcome. This means the lowest practical cure pressure should be used to help eliminate the spring.

\[ \text{Spring vs. Tension Gradient} \]

\[
\begin{align*}
\text{Gradient (G)} & \quad \text{Spring (S)} \\
0.0 & \quad 0.70 \\
0.5 & \quad 0.60 \\
1.0 & \quad 0.50 \\
1.5 & \quad 0.40 \\
2.0 & \quad 0.30 \\
2.5 & \quad 0.20 \\
& \quad -0.10 \\
& \quad -0.20 \\
\end{align*}
\]

**Figure 64:** Spring vs. Gradient (High CTE Tooling). Data shown was obtained at 600 kPa.

6.4.2. Strain Profile Analysis

The strain vs. gradient profile for the high CTE tooling is shown in Figure 65. This figure was calculated using a 600 kPa cure. At a winding gradient of \( G = 1.9 \) the strain profile is approximately vertical. This indicates that at this gradient, the spring should be approximately zero. As before, this agrees with the graphical representation of the constant pressure spring curves shown in Figure 64. Another interesting difference with respect to the lower CTE tooling is that the zero strain point is not met at any
winding gradient. This is true for all cure pressures. This is why the spring vs. gradient profile starts with a negative slope with $G=0.0$. This is unlike the lower CTE tooling profile that begins with a positive slope. Appendix B.4 contains additional plots of strain profiles for various base tensions and cure pressures.

![Strain Profile After Cure](image)

**Figure 65**: Strain vs. Gradient (High CTE Tooling). Data shown was obtained at 600 kPa.

### 6.4.3. Base Winding Tension Effects

As with the low CTE tooling, an increase in base winding tension lowers the necessary gradient required to achieve zero spring. However, contrary to the low CTE tooling the increasing base tension does not have a corresponding increase in spring at the lower gradients. The spring is lower regardless of the gradient. This would again lead to an assumption that for spring elimination starting with a higher base winding tension
may be preferable. However, in practice this is not beneficial since it will also result in higher residual stress. The strain profile curve may be vertical, but it would be shifted to the right. These trends can be seen in more detail in Appendix B.2.

**Base Tension Effects**

\[ P^a = 600 \text{ kPa} \]

![Base Tension Effects](image)

**Figure 66:** Base Tension Effects (High CTE Tooling). Data shown was obtained at \( P^a = 600 \text{ kPa} \).

### 6.4.4. Experimental Results/Discussion.

Figure 67 shows the spring vs. tension gradient comparison between theoretical and experimental data taken at \( G = 1.1 \). The data and calculation were done with \( P^a = 600 \text{ kPa} \) and \( b_i = 12 \text{ N} \). This again shows that with tension control the spring can be greatly reduced. However, with the lower CTE tooling it is possible to eliminate the spring, where as with the higher CTE tooling the zero spring tension gradients are too severe. The fibers and or tensioners are generally not capable of withstanding or providing the
required tension magnitudes. From Figure 67 the required winding stress in the top ply would be approximately 200 MPa. This is below the ultimate stress of the fiber, but during the manufacturing process the fiber must be fed through multiple rollers, which usually results in fiber failure at stresses of this magnitude.

Another significant variation from the lower CTE is the error. In this case the theory appears to overestimate the experimental data. As with the spring predictions the error can be attributed to the large amount of washout that occurs with the higher CTE tooling. This results from the extremely high effective tensions which are near failure levels.

![Spring vs. Gradient](image)

Figure 67: Spring vs. Gradient (High CTE Tooling). Results shown are for 600 kPa autoclave pressure.
6.5 Considerations for Off-Axis and Asymmetric Laminates

For off-axis laminates the process is somewhat more complex. The procedure listed above will not suffice since the off-axis lamina will have a different consolidation based on the wind angle. In this case a computer routine must be employed that searches through individual ply tensions. There will be an infinite number of solutions that will give zero moment and therefore zero spring. Because of this, constraints must be placed to find a quasi-optimum solution. The driving constraint will usually be to find a solution that minimizes the average and mean residual stress, while not exceeding the manufacturing parameter envelope. Other than this the process is the same as with uni-directional rings.

6.6 Conclusions

In this Chapter the manufacturing methodology used to predict spring was expanded to include spring elimination. This was done by: (1) finding the tension requirements, (2) investigating the theoretical solutions for both high and low CTE tooling, and (3) validating the theoretical solutions by making comparisons to experimental data. With this expanded methodology we now have a design/manufacturing process that allows for spring reduction in all cases and elimination in most cases. A block diagram of this methodology is shown in Figure 68.

The first step was to investigate the tension requirements needed to eliminate spring. This was looked at in two ways. The first was to find the residual stress induced moment that develops during cure and apply a counter moment during winding by
varying the tension. The second, and more practical way, is to vary the winding tension to make the post cure tension in each lamina the same as that for the base lamina. This process is simple for uni-directional laminates, but becomes more complex for off-axis or asymmetric laminates. These have an infinite number of solutions and require a constraint based iterative process to find a quasi-optimum solution.

Because of the interdependency of the consolidation and strain a graphical method was developed for the actual tension control procedure. This was accomplished by specifying the tension profile based on a linear winding gradient and then sweeping the gradient to create a graphical representation of the spring envelope. Utilizing this graphical method the effects of base tension, cure pressure, and tooling CTE were investigated. It was found that the best solution to the spring problem started with choosing the lowest possible base tension and graphically finding the required zero spring winding gradient. This investigation showed that for low CTE tooling the gradient required for spring elimination was easily achieved. However, for higher CTE tooling spring elimination may not be possible because of the high tension required and the possibility of fiber failure.

The final step was to compare the theory to experimental data. It was determined that theory followed experimental data trend well and predicted the zero spring winding gradient to within 15% for the low CTE tooling. This error was also found to be consistent with respect to various cure pressures. Utilizing this consistency the error can be compensated for and the spring eliminated. This was shown in both high and low pressure cure cycles.
The investigation of the high CTE tooling confirmed that the spring could be reduced, but elimination would be difficult. Another noted difference between the low and high CTE tooling was that the theory overestimated the required gradient for zero spring. This is attributed to the extremely high effective winding tension associated with the steel tooling and the resulting washout.

This chapter clearly shows that the spring in filament wound cylinders can be eliminated or reduced by proper tension control techniques. An added benefit is the overall decrease in residual stresses which should result in increased material performance. This eliminates the need for tooling iteration and reduces the cost of manufacturing composite parts.
DESIGN METHODOLOGY

Part Geometry & Material System

Characterize Consolidation
Coupon Testing

Spring Model
Constant or Variable Consolidation?

Integrate

Predict Spring
Spring Predict

Modify Tooling
(Compensate for Spring)

Modify Manufacturing
(Spring Elimination)
Spring Elim.

Dimensionally Accurate Part

Figure 68: Design/Manufacturing Methodology Block Diagram. Block diagram shows steps to obtain dimensionally accurate Part.
CHAPTER 7

CONCLUSIONS

This research has developed a manufacturing methodology for spring prediction, reduction, and elimination in full size structures based on testing accomplished at the coupon level. This will benefit industry in one of two ways. The first and preferable outcome is, through manufacturing controls, total elimination of part deformation. However, in some cases this may not be possible and the spring must be accounted for during tooling design. In both instances the resulting final part will be dimensionally accurate and the process of tooling iteration will be eliminated.

In Chapter 2 a theoretical formulation was developed to predict spring in cylindrical sections based on cure consolidation. The formulation includes both constant and variable consolidation models. These models were integrated with modified Classical Laminate Plate Theory to derive equations predicting the residual stresses and associated stress resultants and couples. Utilizing the stress couples, additional equations were developed to predict the mid-plane strains and curvatures. With these equations the spring in a filament wound cylinder can be predicted once the consolidation is known.
In Chapter 3 the consolidation in filament wound closed-section parts was characterized for a common material system. A series of empirical equations were developed to account for the Effective Cure Pressure and Lay-Down Efficiency. To achieve this, assumptions were made relating the consolidation to the cure pressure and the lay-down thickness to winding tension. The accuracy of the assumptions and derived equations were then checked using coupon test data from Chapter 4. It was determined that the ECP has a significant effect on the consolidation and can be predicted through an empirical approximation as shown in Chapter 3. The effects of ECP on a representative filament wound cylinder were then explored. It was found that for higher-pressure cure processes the error caused from ignoring the ECP would be small, less than 5%. However, for lower-pressure cures neglecting the hoop stress induced pressures could cause significant error, greater than 30%.

Next the Lay-Down Efficiency was investigated. It was found that the winding tension used on cylinders greatly affected the final consolidation. The winding tension and consolidation are inversely proportional, and the change in consolidation for the range of tensions investigated was 300%.

Utilizing the equations developed for the ECP and LDE, along with simple coupon testing, a recipe to characterize the consolidation was developed. This recipe concluded with the development of graphical representations of the manufacturing parameter space and the resulting consolidation.

Chapter 5 demonstrated that the theoretical solution developed in Chapter 2 could accurately predict the spring. More importantly, it was shown that the theoretical solu-
tion integrated with the empirical consolidation model is capable of predicting spring based on coupon testing. This integrated model can also be used to determine the spring envelope and the effects of varying manufacturing parameters. These models and solutions enable a design engineer to adjust the manufacturing parameters to reduce the spring, or account for the spring in tool design.

In Chapter 6 the manufacturing methodology used to predict spring was expanded to include spring elimination. This expanded methodology provides a design/manufacturing process that allows for spring reduction in all cases and elimination in most cases. A block diagram of this methodology is shown in Figure 67.

The first step in eliminating spring was to investigate the tension requirements needed to eliminate spring. This was examined in two ways. The first was to find the residual stress induced moment that develops during cure and apply a counter moment during winding by varying the tension. The second, and more practical way, is to vary the winding tension to make the post cure tension in each lamina the same as that for the base lamina. This process is simple for uni-directional laminates, but becomes more complex for off-axis or asymmetric laminates. These have an infinite number of solutions and require a constraint based iterative process to find a quasi-optimum solution.

Because of the interdependency of the consolidation and strain a graphical method was developed for the actual tension control procedure. This was accomplished by specifying the tension profile based on a linear winding gradient and then sweeping the gradient to create a graphical representation of the spring envelope. Utilizing this graphical method the effects of base tension, cure pressure, and tooling CTE were
investigated. It was found that the best solution to the spring problem started with choosing the lowest possible base tension and graphically finding the required zero spring winding gradient. This investigation showed that for low CTE tooling the gradient required for spring elimination was easily achieved. However, for higher CTE tooling spring elimination may not be possible because of the high tension required and the possibility of fiber failure.

Finally, the integrated theory and experimental data were compared. It was determined that theory followed the experimental data trend well and predicted the zero spring winding gradient to within 15% for the lower CTE tooling. This error was also found to be consistent with respect to various cure pressures. Utilizing this consistency the error can be compensated for and the spring eliminated. This was shown in both high and low pressure cure cycles.

The investigation of the high CTE tooling confirmed that the spring could be reduced, but elimination would be difficult. Another noted difference was the theory overestimated the required gradient for zero spring. This is attributed to the extremely high effective winding tension associated with the steel tooling and the resulting washout.

This dissertation clearly shows that the spring in filament wound cylinders can be eliminated or reduced by proper tension control techniques. An added benefit is the overall decrease in residual stresses that should result in increased material performance. This eliminates the need for tooling iteration and will greatly reduce the cost of manufacturing composite structures. Currently AFRL has started applying this research
to various projects: on-orbit flywheel energy storage technology, the Minutaur Payload fairing, and cryogenic pressure vessels. Interesting to note is that the flywheel and pressure vessel application do not involve spring prediction or control, but involve predicting residual stress and the utilization of tension control to prevent fiber buckling; the fiber buckling has led to significant decreases in material performance in these areas.

As discussed in the introduction, research into part warpage in composites has been ongoing virtually since the advent of modern composite materials. The research presented here addresses and provides manufacturing techniques that should reduce or eliminate warpage in filament wound and fiber placed closed-section structures. However, much work still remains in this area including warpage control in more complex structures such as composite grid (Isogrid) and ChamberCore. These structures are now entering use in the aerospace industry and applying the methods developed here will be challenging.
BIBLIOGRAPHY


47. Entec 3-Axis Winder *Operations Manuel*.
APPENDIX A

SPRING PREDICTION PLOTS

![Experimental vs. Predicted Spring (10 Ply Laminate)]

Figure 69: Experimental vs. Predicted Spring (10 Ply Laminate).
Experimental vs. Predicted Spring
12 Ply Laminate

Figure 70: Experimental vs. Predicted Spring (12 Ply Laminate).

Spring vs Winding Stress

Figure 71: Spring vs. Winding Stress (Various Thicknesses).
APPENDIX B

ZERO SPRING PLOTS

B.1 SPRING VS. TENSION GRADIENT (LOW CTE TOOLING)

B. 1.1. VARIED BASE TENSION

Figure 72: Spring vs. Tension Gradient (Base Tension=4.5 N).
Figure 73: Spring vs. Tension Gradient (Base Tension=9.0 N).

Figure 74: Spring vs. Tension Gradient (Base Tension=13.5 N).
Figure 75: Spring vs. Tension Gradient (Base Tension=18.0 N).

Figure 76: Spring vs. Tension Gradient (Base Tension=22.5 N).
Spring vs. Gradient
Base Tension=27.0 N

Figure 77: Spring vs. Tension Gradient (Base Tension=27.0 N).
B.2 SPRING vs. TENSION GRADIENT (HIGH CTE TOOLING)

B. 2.1. VARIED BASE TENSION

**Figure 78:** Spring vs. Tension Gradient (Base Tension = 4.5 N).

**Figure 79:** Spring vs. Tension Gradient (Base Tension = 9.0 N).
Figure 80: Spring vs. Tension Gradient (Base Tension=13.5 N).

Figure 81: Spring vs. Tension Gradient (Base Tension=18.0 N).
Figure 82: Spring vs. Tension Gradient (Base Tension=22.5 N).

Figure 83: Spring vs. Tension Gradient (Base Tension=27.0 N).
B.3 STRAIN VS. TENSION GRADIENT (LOW CTE TOOLING)

B.3.1 VARIED CURE PRESSURE (CONSTANT BASE TENSION)

Figure 84: Strain vs. Tension Gradient ($P^a=0$ kPa).

Figure 85: Strain vs. Tension Gradient ($P^a=150$ kPa).
Figure 86: Strain vs. Tension Gradient ($P^a=300$ kPa).

Figure 87: Strain vs. Tension Gradient ($P^a=450$ kPa).
Figure 88: Strain vs. Tension Gradient ($P^*=600$ kPa).

Figure 89: Strain vs. Tension Gradient ($P^*=750$ kPa).
Figure 90: Strain vs. Tension Gradient (P^a=900 kPa).
B. 3.2. **Varied Base Tensions (Constant $P^2=600 \text{kPa}$)**

![Strain Profile](image)

**Figure 91:** Strain vs. Tension Gradient (Base Tension=4.5 N).

![Strain Profile](image)

**Figure 92:** Strain vs. Tension Gradient (Base Tension=9.0 N).
Figure 93: Strain vs. Tension Gradient (Base Tension=13.5 N).

Figure 94: Strain vs. Tension Gradient (Base Tension=18.0 N).
Figure 95: Strain vs. Tension Gradient (Base Tension=22.5 N).

Figure 96: Strain vs. Tension Gradient (Base Tension=27.0 N).
B.4 Strain vs. Tension Gradient (High CTE Tooling)

B. 4.1. Varied Cure Pressure (Constant Base Tension)

Figure 97: Strain vs. Tension Gradient ($P^\gamma=0$ kPa).

Figure 98: Strain vs. Tension Gradient ($P^\gamma=150$ kPa).
Figure 99: Strain vs. Tension Gradient ($P^* = 300$ kPa).

Figure 100: Strain vs. Tension Gradient ($P^* = 450$ kPa).
Figure 101: Strain vs. Tension Gradient ($P^a=600$ kPa).

Figure 102: Strain vs. Tension Gradient ($P^a=750$ kPa).
Figure 103: Strain vs. Tension Gradient ($P^a=900$ kPa).

B. 4.2. Varied Base Tensions (Constant $P^a=600$ kPa)

Figure 104: Strain vs. Tension Gradient (Base Tension=4.5 N).
Figure 105: Strain vs. Tension Gradient (Base Tension=9.0 N).

Figure 106: Strain vs. Tension Gradient (Base Tension=13.5 N).
Figure 107: Strain vs. Tension Gradient (Base Tension=18.0 N).

Figure 108: Strain vs. Tension Gradient (Base Tension=22.5 N).
Figure 109: Strain vs. Tension Gradient (Base Tension=27.0 N).