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EFFECTS OF MISALIGNMENT AND RESIDUAL STRESS ON
BUCKLING BEHAVIORS OF WELDED PLATE GIRDERS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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*****

The Ohio State University
1999

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ABSTRACT

In this study, an integrated process including the buckling analysis procedure, FEA models for misaligned joints, and residual stress characterization methods were used to investigate the misalignment and residual stress effects on the buckling behaviors of a welded girder panel.

Misalignment effects on 1-D compressed structures were studied first by misaligned columns subjected to an axial load. In order to stimulate the loading condition at the panels of a plate girder, simply supported misaligned plates subjected to uniform compression, in plane shear and in plane bending were modeled and studied by finite element analysis. Three more parameters were considered in the study: the magnitude of misalignment, the orientation of misalignment and the normalized slenderness ratio which is defined as the square root of yield stress divided by the square root of elastic bucking load. All of these analyses were conducted under two conditions: (1) with and (2) without considering the effects of residual stresses.

It was discovered that the normalized slenderness ratio, the magnitude of the misalignment, as well as the orientation of misalignment have a significant influence on the misalignment effects when residual stress effects are not considered. Structures with a lower normalized slenderness ratio (short columns or thick plates) are more sensitive to misalignment. Larger misalignment reduces more ultimate strength. When the applied force in the direction normal to the direction of misaligned joints, the ultimate strength of compressed structural members is more easily affected.

The inherent conductive shrinkage method was applied to characterize the residual stresses in the weld joint. A 1D conductive element, which has the equivalent weld size determined by the net heat input, is applied to simulate welds. The heat is
transferred to the base metal by the conductive elements. Based on the results from thermal analysis during the cooling period, the distribution of residual stress around the joint is determined.

Residual stress may either increase or decrease the buckling strength. When plates are subjected to in-plane shear, residual stress effects are dominant and the misalignment effect can be ignored. When plates are compressed uniformly, the misalignment effect on buckling strength is so important that the effects of residual stress can be ignored. For plates subjected to bending, residual stress improves the buckling strength if the normalized slenderness ratio of the plate is less than one and the buckling strength decreases when the ratio becomes larger than one.

Results from this study were applied to predict the buckling behaviors of a modified roll-on/roll-off ramp. The ramp failed during its proof testing because of shear buckling in the end panel of a girder when subjected to its test load. The FEA on the effects of joint misalignment, with or without considering residual stresses, was conducted using the numerical procedure proposed in the study. The results demonstrated that the failure of the ramp girder was due to inadequate design of the girder. Misalignment was found trivial due to shear dominant condition in the end panel of the web.
ACKNOWLEDGMENTS

I express my sincere appreciation to Dr. C. Tsai for his valuable guidance and insight throughout this research. My thanks also go to the other members of my advisory committee, Dr. Rokhlin and Dr. Kinzel, for their helpful suggestions and comments. I also thank Prof. Mansour at University of California, Berkley and Mr. Chi-Cheng Yang of George G. Sharp, Inc. for allowing me using the ramp data in my study. I thank my wife, Dr. Yong-Fang Kuo, for enduring with me during my scholastic endeavors. To my parents and parents in law, I am grateful for the encouragement you have always given me. Also, I would like to thank my friends Chin-Min Tso, Wentao Chen, Hsiu-Po Huang, Mark Moser, and Linda Matino for their extensive help. Finally, thanks go to my manager, Mark Zimmerman, and Director, Grant Bourgeois, at CASE-DWI for the flexibility at work and for allowing me to use the facility for this study.
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LIST OF SYMBOLS

A  Cross section of column; constant; observed point
a  Length of plate; constant
B  Constant; observed point; strain-displacement matrix
b  Width of plate; constant
C  Constant; observed point
D  Flexural rigidity; constant; observed point
E  Young's modulus
E_t  Tangent modulus
e  Eccentricity
F  Force
f  Frequency
f_{max}  Applicable maximum load and
f_y  Load where yielding happen
h  Height of column; thickness of plate
I  Moment of inertia
K  Element stiffness matrix
k  Geometric constant; eccentricity ratio
l  Length of column
N  Inplane forces
P  Applied force; critical load
P_n  Normalized stress
R  R square
r  Radius of gyration
S  Stress stiffness matrix
S  Parameter
\( t \)  Thickness of plate
U  Strain energy
u  Displacement
V  Displacement
\( V_u \)  Maximum shear strength
x  Rectangular coordinate
y  Deflection of column; rectangular coordinate
W  Work
w  Deflection of plate
\( \alpha \)  Coefficient of thermal expansion, constant; is ratio of ultimate stress divided by yield strength
\( \delta \)  Misalignment
\( \sigma_{re} \)  Residual stress
\( \sigma_y \)  Yield stress
\( \sigma_d \)  Dropping of elastic buckling stress
\( \sigma_{cr} \)  Critical buckling stress,
\( \sigma_{cr}^l \)  Reduced critical stress
\( \varepsilon \)  Strain
\( \gamma \)  Critical load of plate
\( \theta \)  Twist angle
\( \phi \)  Twist angle of tension field
\( \lambda' \)  Normalized slenderness ratio
\( \nu \)  Poisson’s ratio
\( \xi \)  Parameter
\( \tau \)  Shear critical stress
CHAPTER 1
INTRODUCTION

1.1 Background and Research Issues

A modified roll-on/roll-off ramp failed during its proof testing when subjected to its test load. Figure 1.1 shows that the ramp failed in the shear-buckling mode. Metallurgical evaluations in the fracture samples, which were cut from the failed ramp, found a serious misalignment condition between the girder web plates on top of and beneath the deck in the end panel near the upper support. Figure 1.2 shows the definition of misalignment that is the shift between the centerlines of joined components. A hypothesis relating girder failures to shear buckling, triggered by web misalignment, was proposed.

The assembling processes of build-up structures usually cause imperfection purposely or purposelessly. For example, the centerlines of two plates may not remain collinear after welding. Tolerance of misalignment between two joined plates has been defined by welding codes such as the AWS D1.1 [62]. However, the tolerance is determined based on the consideration of fatigue life instead of buckling strength.

Imperfections have been considered as a factor for reducing the buckling strength of welded structures. Yamba [1] defined imperfections as follows:

1. Geometrical imperfections
2. Residual stresses due to welding
3. Construction imperfections.

Geometrical imperfections are deviations from the planes of plates or from the straightness of columns. These imperfections will introduce internal eccentricity, which adds to the load eccentricity. Consequently, this results in a bending
Figure 1.1: The shear buckling at the top end panel of the failed ramp.

Figure 1.2: The definition of misalignment.
moment that may reduce the structure’s ultimate load-carrying capability in compression.

Residual stress induced by the welding process is another factor that may affect the buckling strength. Residual stress is induced by the non-linear plastic strain distribution that does not satisfy the compatibility condition. The incompatibility-induced internal stresses are commonly referred to as “residual stresses.” The residual stresses are in tension around the welds and are in compression away from the welds. The compressive residual stress may augment the bending moment caused by the externally applied compressive forces. This augmentative effect may reduce the buckling strength.

The fabrication of a built-up structure consists of many assembled parts. The misalignment condition is the most common phenomenon of construction imperfections in the fabrication of plate and column assemblies. Like geometrical imperfection, misalignment is a deviation from the plane of plates or from the colinearity of columns. However, the deviation is limited to near joints and not whole structures. The misalignment condition may act with the applied loads to reduce the ultimate load carry capability resulting from the increment of lateral displacements of compressed structural members.

Several analysis methods are commonly used to determine the buckling strength of structural members under compression. These methods include theoretical elastic buckling analysis, numerical simulation, and experiment.

Theoretical elastic bifurcation buckling analysis for compressed columns and plates predicts the buckling strength by solving the eigenvalue equation of the system. Eigenvalues are buckling loads and eigenvectors are buckling mode shapes. The lateral deflection is assumed to occur only when the applied load is greater than the critical buckling load, which is the smallest eigenvalue. A deficiency of this analysis is that it does not consider the behaviors of imperfections. Because of the deviations from the plane or straightness, the lateral deflection can be induced even before the load reaches the critical buckling load. This means that the bifurcation-type buckling will shift to the load and deflection-type buckling. This also implies that the
theoretical eigenvalue analysis becomes invalid when the imperfect condition is significant.

Besides the imperfection effects, the elastic buckling analysis does not consider the effects of yielding. For stub columns or stiffened plates, yielding may occur before buckling. It means that theoretical elastic buckling analysis will overestimate the strength. The welding-induced residual stresses are also from yielding during heating and cooling. Therefore, the elastic buckling analysis can not include the residual stress effect.

Furthermore, the plate girder of the failed ramp was subjected to the loads combination of bending, shear force, and normal compression. The geometry of the plate was in a trapezoidal shape. These two situations made it difficult to apply the formulas obtained from the theoretical derivation stated in the existing literature. The residual stresses induced by welding and by the misalignment condition between the upper and bottom web plates further complicated the problem.

Studies have been performed to cover the deficiencies of elastic buckling analyses by either numerical simulations or experiments. Numerical methods include iteration, the energy method, the finite difference method, and the finite element method. Buckling loads of imperfect structures can be obtained by simulating imperfections into the models. Yielding can also be considered by adding the yielding criteria and post yield behaviors of the compressed members into the model. Experimental methods are usually performed to predict the buckling behavior of structures that have similar shapes and properties. Imperfections are considered as parameters that may affect the buckling strength. Buckling loads of structures that have different degrees of imperfection were obtained by repeated tests. Based on the results from experiments, empirical formulas for explaining the effects of imperfection are derived. Table 1-1 summarizes some of the studies. A more detailed discussion is presented in Chapter 2.
<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Theoretical Derivation</th>
<th>Numerical Method</th>
<th>Experimental Method</th>
<th>Geometric Imperfection</th>
<th>Residual Stress</th>
<th>Misalignment</th>
<th>Buckling Criteria</th>
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* Not include buckling load obtained from eigenvalue, c: column, p: plate, s: shell, g: girder, b: beam

Table 1.1: Some studies in the buckling analyses.
Table 1.1 shows that most of the studies were focused on the effects of geometrical imperfection and residual stresses. The effects of misalignment and the interaction between residual stresses and misalignment have been rarely studied. Yamba [1] suggested accounting for the imperfection using the factor of safety. However, the procedure to determine the factor was not discussed in his dissertation. Ersvik [8] proposed adding a moment at the joint to account for the misalignment situation. However, only 1D column with small misalignment was presented. The magnitude of the moment is the amount of axial force multiplied by the misalignment. However, how to conduct analysis for misaligned jointed plates was not mentioned. Hu [18] proposed two types of misalignment modeling. First, the weld was treated as part of the tube, with shell elements as extensions of the tube wall. The thickness and material properties of these shell elements can be modeled differently from the tube according to the gap between cans and the properties of the weld material. However, the modeling of the welded narrow band may cause skewing in some of the elements along the weld that in turn will lead to poor convergence. Second, a series of rigid links along connected edges is applied to the model joint. The model is based on the assumption that welds are so strong that corresponding points at each side of the joint move at the rigid mode. This method has been proved by comparing the FEM results with the experiments of tube structures in his study. However, Hu's study was limited to tubular joint under axial compressive load, the validity of applying the model to plates which is subjected to either uniform compression, shear, or bending has not been investigated.

In addition to the inadequate knowledge of the misalignment effects at buckling strength, Table 1-1 also shows that there is just a little information on how to decide the buckling numerical from the load and displacement curves. It implies that numerical criteria to decide the buckling load for imperfect structures need to be developed before the investigation of misalignment effects.
1.2 Objective

The lack of knowledge of the misalignment and residual stress effects on buckling usually leads engineers to use larger safety factors in their design, which results in heavier structures and higher costs. To have a better and deeper understanding of the buckling behaviors of welded plate girders; this study is designed to investigate these effects of misalignment and residual stress on buckling strength. The buckling loads under different misalignment conditions were obtained by using the finite element analysis (FEA). A guide for the misalignment analysis of welded structures was proposed after summarizing the results of the investigation. In order to reach the objective, the following must be undertaken before investigating the misalignment and residual stress effects on buckling strength.

1. Develop and verify the numerical buckling analysis procedures and criteria.
2. Model misaligned joints.
3. Characterize residual stresses and distortion induced by welding.

Combining the conclusions from the above three studies, the relationships between misalignment and buckling strength were investigated under two conditions: (1) with and (2) without considering the effects of residual stresses. A summary from the investigation was presented for the columns and plates in the form of a normalized slenderness ratio ($\lambda'$) and a magnitude of misalignment. A real ramp example was used to test the summary. Figure 1.3 shows the flow chart of the study. The details of the study are stated in the following sections.

1.3 Method of Approach

1.3.1 Buckling Analysis

Because lateral displacement occurs before the load reaches its critical value for imperfect structures, observing the load and displacement curve is the way to decide buckling. To obtain buckling loads, a procedure for the load and displacement curve were developed. Three criteria were studied for determining buckling loads in the study. The most suitable criterion was obtained after comparing the predicted
Motivation
Failure of Ramp
- Load
- Geometry
- Residual Stress
- Misalignment

Failure of Ramp
- Load
- Geometry
- Residual Stress
- Misalignment

Literature Review
- Numerical
- Experimental
- Theoretical

Objective
Misalignment Effect on Buckling by FEA
- Load
- Geometry
- Residual Stress
- Misalignment

Develop Procedures for Load and Displacement Curve
- Load
- Material Properties
- Slenderness Ratio

Model Misalignment Joints
- Exact Joint Model (3D Solid With Fillet Joint)
- Simplified Joint Model (3D Solid Without Joint Model)
- Rigid Joint Model (2D Shell)

Plot Load and Displacement Curve

Propose Criterion

Obtain Buckling Load

Assess with Published Data
- Not Acceptable

Acceptable

Select Buckling Criteria

Study Numerical Models on Buckling Behaviors

Determine Approximate Misalignment Joint Model

Investigate Misalignment Effect
- Magnitude
- Load
- Effective Slenderness Ratio
- Residual Stress

Obtain Buckling Load under Residual Stress.

Assess with Published Data
- Not Acceptable

Acceptable

Apply Knowledge to Ramp Failure Analysis

Conclusions and Recommendations

Figure 1.3: Study flow chart.
results with existing values obtained from the literature review. The procedures for load and displacement curves and the three buckling criteria are stated as follows:

1.3.1.1 Numerical Procedure for the Load and Displacement Relationship

The curvature of a plate girder creates a vertical compression in the web due to a downward component of flange stress in a curved length of girder on the compression side and an upward component on the tension side [3,40]. Assuming the compression is uniform, the web plate can be considered either as a simply supported column subjected to an axial load or as a simply supported plate subjected to uniform compression on one side. When a simply supported plate girder is loaded in the plane of a web, shear force is significant near the supports. The part of the web between the two stiffeners can be considered a simply supported plate subjected to the action of uniform shear. The bending moment reaches its maximum value in the middle of the span where shear stresses are relatively small compared with the stress from bending at the section. The part of the web between two stiffeners near the middle span is in the condition of pure bending [3]. In order to obtain a rational basis for the buckling behavior of plate girder panels with a misaligned joint and residual stress, a comprehensive study should consist of the following four cases were simulated by a commercial finite element analysis code, ANSYS:

1. Columns under axial loads.
2. Plates under uniform axial loads.
3. Plates under uniform shear stress.
4. Plate subjected to pure bending.

Figure 1.4 shows above four loading conditions. The characteristic dimensions of columns and plates in the FEA models were derived from the normalized slenderness ratio (λ'). The definition of λ' for the columns is:

\[ \lambda' = \left( \frac{k \ell}{r} \right) \frac{\sigma_y}{\pi^2 E} \]  

(1-1)
where \( k \ell \) is the effective length of the column and \( k \) changes with the boundary conditions, \( r \) is the radius of gyration and is defined as \( r = \sqrt{\frac{I}{A}} \), \( I \) is the minimum moment of inertia of the cross section, \( A \) is the area of the cross section, \( E \) is Young’s modulus and \( \sigma_y \) is the yield stress.

The definition of \( \lambda' \) for plates is

\[
\lambda' = \frac{b}{t} \sqrt{\frac{12(1-\nu^2)\sigma_y}{k\pi^2E}}
\]

where \( b \) is the width, \( t \) is the thickness of the plate, \( k \) is the numerical factor, the magnitude of which depends on the aspect ratio and boundary conditions, and \( \nu \) is Poisson’s ratio.

The definition of a normalized slenderness ratio is the square root of yield stress divided by the square root of theoretical critical buckling stress. When \( \lambda' = 1 \), it
means that the critical buckling stress is equal to yield stress. When \( \lambda' > 1 \), it means that the buckling load is \( \frac{1}{\lambda'^2} \) times the yielding load and vice versa. For example, when \( \lambda' = 1.8 \), columns will buckle as the applied stress reaches \( \frac{1}{\lambda'^2} = \frac{1}{1.8^2} \approx 0.30 \) times the yield stress. The benefit of using the normalized slenderness ratio is that the results from one set of specified dimensions and material properties are generic, and the results are applicable to other circumstances. The parametric relationship of buckling loads obtained by the analysis of a set of \( \lambda' \) can be applied to other materials and dimensions. For example, columns with \( \ell = 1 \) and \( r = 0.1 \) and columns with \( \ell = 10 \) and \( r = 1 \) have the same critical buckling stress for the same material. Similarly, a simply supported plate with \( b = 1 \) and \( t = 0.1 \) and a plate with \( b = 10 \) and \( t = 1 \) also have the same nominal buckling stress when all the other variables in Eq. 1-2 are kept the same.

In order to investigate the buckling behaviors when the critical stress was higher or lower than the yield stress, six normalized slenderness ratios, which were 0.4, 0.7, 1, 1.3, 1.5, 1.8, were used in the modeling of FEA. The boundary conditions were simply supported. The length of the column was 10 meters. The radius of gyration changed with \( \lambda' \) and was determined by Eq. 1-1. The length and width of the plates were 1m and the thickness of the plate was determined by Eq. 1-2 and \( \lambda' \). Although the dimensions are given, the results from FEA are not limited to the specified dimension. It can be applied to different dimensions as long as \( \lambda' \) is the same. The assumptions for the FEA are presented below:

1. The material is homogeneous and isotropic steel. Bilinear isotropic hardening was used to simulate the stress-strain relationship after yielding. The slope of the strain hardening curve was assumed to be the tensile strength divided by the percent elongation of a 2-inch gage length. The effect of strain hardening is discussed in Chapter 3.

2. There are no geometrical imperfections in the original model. The plate was considered flat and the column, straight. The imperfection was modeled by applying a small concentrated load in the plates or columns.
After the dimensions, boundary conditions, and the above assumptions were determined, the load and displacement curve was recorded while the load was gradually increased until its peak, and then decreased. The analysis considered the elastic-plastic effect by applying the Von-Mises equivalent stress to determine yielding. The geometric non-linearity, such as large displacement, was also included. The analysis was repeated with the four cases and the six normalized slenderness ratios. A total of 24 load and displacement curves were studied to determine the numerical buckling criteria.

1.3.1.2 Buckling Criteria

Galambos [2] defined buckling load as the load at which a compressed element, member, or frame collapses in service or buckles during a loading test. According to the phenomenon of buckling, the buckling load can be the load that causes bifurcation buckling or the load with which the system reaches a maximum, or limit load without any previous bifurcation. By the above description and observing the load and displacement curves, the following three criteria were proposed for deciding on buckling loads numerically. The best criterion was suggested after comparing the buckling loads obtained form these three proposed criteria with published data. The definitions of these three criteria are shown in Figs.1.5 and 1.6. The criteria are:

1. Limit load,

2. The first load at which $\frac{du}{dP_n}$ is a local extreme value. $P_n$ is the normalized stress and $u$ is the lateral displacement. If there is no local extreme of $\frac{du}{dP_n}$, then the maximum load that has a convergent solution will be considered as the buckling load,

3. The first load where $\frac{d^2u}{dP_n^2}$ changes sign. If $\frac{d^2u}{dP_n^2}$ does not change sign, then the maximum load that has a convergent solution will be considered as the buckling load.
Figure 1.5 Definition of buckling criteria for structure which does not yield as load is applied.
A: limit load; B: local maximum of first derivative; C: sign change of second derivative
* Unit for derivatives: first derivative: $10^{-8}$ m second derivative: $10^{-13}$ m

Figure 1.6 Definition of buckling criteria for structure which yields as load is applied.
A,B,C: as defined in Fig. 2
* Unit for derivatives: first derivative: $10^{-2}$ m second derivative: 10 m
The limit load is the maximum load on the load displacement curve. The displacement is undefined at the curve when the load is greater than the limit load. It means that the catastrophic displacement may occur at the load. Therefore, the limit load was proposed to be the criterion for buckling.

The concept of the first criteria was from the phenomenon of buckling. As the applied load is larger than the critical buckling load, a small increment of loads will cause significant lateral displacement, which implies that \( \frac{du}{dP_n} \) will became large at the critical load. For a column analyzed elastically with a small deflection assumption, \( \frac{du}{dP_n} \) is infinite at the critical load. For structures considering the large deflection effects, the first derivative is finite, but still has a very large value. Therefore, it was proposed as a buckling criterion.

For structural members that have post-buckling strength, the curvature of the load and displacement curve changes after the applied load becomes larger than the theoretical critical load. After the applied load is larger than the limit load, displacement increases, while the load decreases. Both of these suggest that the second derivative of the load and displacement curves will change signs at a buckling load. So, the changed sign of the second derivative was proposed as a buckling criterion.

After the applied loads become larger than the buckling loads, the large deformation may cause difficulty in convergence. Although a new numerical technical, such as the arc search method, has been applied to solve the problems, sometimes, it just cannot get the converged solution. In this situation, displacement increases with the increment of applied loads in the finite element analysis until the solution diverges, which means that a local extreme for the first derivative and a changed sign of the second derivative could not be observed. Under this circumstance, the increments of the load should be as small as possible in order to make the maximum convergent load as close to the buckling load as possible. This is the reason why the maximum converged solution is considered as the buckling load when the local extreme for the first derivative or a change sign of the second derivative could not be observed from the load vs. displacement curve.
1.3.2 FEA Model for Misalignment

Misalignment is a common situation in the fabrication of welded structures. There are several methods that can be applied to simulate this situation. The most direct way to model a misaligned joint is to model it as a real joint. This means the joint is modeled with two fillet joints that connect two solid-element-type plates. Because the size of the welds is very small when it is compared with other dimensions, it needs fine mesh to model the welds and surrounding areas. In order to have a better aspect ratio, fine meshes are also needed. These mean that the number of elements and degrees of freedom will increase tremendously, which implies that more computation time is necessary to calculate the results. Because of this disadvantage, this method is applied only when the behaviors of the joints are of interest.

If the fillet joint is not simulated, the number of total elements may be reduced. However, the fine mesh near the joint is still needed to avoid poor aspect ratios. In addition to that, stress concentration and reduced stiffness near the joint may induce error in the finite element analysis.

Hu [18] proposed a simplified method by modeling the joint rigidly. Several equations are used to constrain nodes between different tubes. This method has been proved by comparing the FEM results with the experiments of tube structures.

In brief, there are at least the following three modeling methods to simulate misaligned joints. These models, which are derived from the above three modeling methods, are shown in Fig. 4.1 and are listed as follows:

1. Solid element with fillet at the joint,
2. Solid element without fillet at the joint,
3. Shell element with joints connected rigidly.

To select a model that can provide acceptable results with less number of elements for plate structures, these three models were compared with the buckling behavior under the same loading condition. A more detailed discussion follows in Chapter 4.
1.3.3 Residual Stress Characterization

A welding process analysis is a moving heat source problem including the following two steps: heating and cooling. Heat is input from nodes to nodes, which means a lot of load steps are required when a transient thermal FEA is performed. Applying the moving source method to calculate welding-induced residual stress may consume tremendous computational time, especially for structures that have many welds, due to the numerous load steps. The limit in applying the moving source method to real complex structures motivates the development of another skill.

Residual stresses are induced by the inherent strain which is the cumulative plastic strain induced by the nonlinear temperature distribution during welding. More detail description of inherent strain was stated in Chapter 4. If the inherent strain distribution can be simulated, it is not necessary to calculate the whole heating and cooling process in FEA, which will save a lot of computational time. Because at high temperature, the yield stress and Young’s modulus are very small, the residual stress is induced after the weld metal cools below 700° C [55]. Based on the phenomenon, the inherent strain method assumes that the welding-induced residual stresses and distortion can be characterized by the shrinkage at the weld and heat-affected zone during cooling.

The inherent strain is obtained by the following three steps:

1. Assign a high temperature to elements at welding zones as the initial condition.

2. Calculate the temperature in the structure member when the high temperature at the welding zone is gradually cooling to the room temperature.

3. Consider the temperature as a thermal load. An elastic-plastic FEA was performed to calculate residual stresses based on the results from thermal analysis.

Deciding the initial temperature and the size of welding zone is the first problem for applying the inherent strain method to predict residual stresses. How to simplify the modeling to get the balance between the time and accuracy is another issue. These topics are discussed at Chapter 4.
1.3.4 Misalignment Effects

Combining the study of buckling criteria, misaligned joint modeling, and residual stress simulation, the effects of the misalignment of a welded plate girder can be investigated.

Five misalignment conditions, which vary from 0.1 times thickness to 0.9 times thickness with a 0.2 times thickness increment, were used to calculate the buckling load of the four basic models shown in Fig. 1.4. To study the effects of normalized slenderness, four normalized slenderness ratios (0.4, 1, 1.3, 1.8) were applied to each case mentioned above. The buckling load and ultimate load were drawn with the change of misalignments and normalized slenderness ratios. A total of 80 data points were analyzed by regression.

The FEA was repeated by considering residual effects at plates that were subjected to in-plane uniform compression, shear, and bending. The buckling load and ultimate load curve was drawn with the change of misalignments and normalized slenderness ratios. A set of 60 new data points was analyzed by regression, also. The details are discussed in Chapter 5.

1.3.5 Verification of Misalignment Effects by Failure Analysis of a Plate Girder

The finite element model of the upper end section of the ramp was isolated from the failure ramp structure during the performance of buckling analysis. The appropriate boundary conditions were imposed along the cut boundaries to represent the various constraints from the neighboring structural members. The buckling criteria, the misaligned joint model and the residual stress simulation concluded from previous studies were applied to the panel model to investigate the effect of misalignment on buckling strength with and without the influence of residual stress.

The normalized slenderness ratio of the upper failed panel was estimated. The value was applied to the regression formula for predicting the panel’s buckling behavior under different misalignment and residual stress conditions. The predicted
trend was compared with the FEA results of the panel. A more detailed statement can be found in Chapter 6.

1.4 Anticipated Results

The following contributions are expected through this study:

1. Knowledge for the misalignment and residual stress effects on buckling strength.
2. FEA procedure for performing buckling analysis.
3. Criteria for numerical buckling analysis.
4. Recommendation for the misaligned joint model.
5. Modeling procedure(s) for residual stress characterization.
CHAPTER 2

LITERATURE REVIEW

As mentioned in Chapter 1, the following steps need to be conducted to investigate misalignment effects in a welded plate girder

1. Developing and verifying the numerical buckling analysis procedures and criteria.
3. Simulating residual stresses and distortion induced by welding.

These three topics are the focus for the literature review in this chapter. In addition, post-buckling strength and tension field theory are also reviewed to provide the knowledge of the ultimate strength for the compressed plate girder panel.

2.1 Buckling Analysis Procedures and Criteria

Knowledge of buckling behaviors is a key to developing analysis procedures and proposing the buckling criteria. Therefore, the behaviors of compressed members will be discussed first. The previous buckling analysis procedures and criteria will be stated later. The buckling loads decided by the proposed procedures and criteria need to be verified with real data. Data from experiments will also be reviewed in this section.

2.1.1 Buckling Behavior

According to the definition by Galambos [2], “to buckle” means to kink, wrinkle, bulge, or otherwise to lose its original shape because of elastic or inelastic
strain. The load at which a compressed element, member, or frame collapses in service or buckles in a loading test is the buckling load.

Problems in instability of compression members can be subdivided into two categories: those associated with the phenomenon called bifurcation of equilibrium, and those in which instability occurs when the system reaches a maximum, or limit, load without previous bifurcation. In the first case, a perfect member, when subjected to increasing load, initially deforms into one mode and then, at a load referred to as the critical load, the deformation suddenly changes into a different pattern. Axially compressed columns, plates, and cylindrical shells experience this type of instability. Critical loads are defined as buckling loads in these cases. By comparison, members of the latter category deform in a simple mode from the beginning of the loading until the maximum load is reached. Shallow arches and spherical caps subjected to uniform external pressure are examples of the second type of instability [2]. The maximum, or the limit, load is the buckling load in these cases. More discussion of these two types of buckling is offered below.

2.1.1.1 Bifurcation Buckling

“Bifurcation” is a term that relates to the load-deflection behavior of a perfectly straight and perfectly centered compression element at critical load. At critical load, a member can be in equilibrium in either a straight or slightly deflected configuration. A bifurcation results at a branch point in the plot of axial load versus lateral deflection, from which alternative load deflection plots are mathematically valid [2].

Bifurcation phenomena can also be explained by the potential energy of a rigid body system [3]. Consider, for instance, the three cases of equilibrium of a ball shown in Fig. 2.1. It can be concluded that the ball on the concave spherical surface (a) is in stable equilibrium, while the ball on the convex spherical surface (b) is in unstable equilibrium. The ball on the horizontal plane (c) is said to be in neutral equilibrium. Bifurcation happens in case (c), where the ball can roll in either direction and keep its equilibrium.
2.1.1.2 Limit-Load Buckling

Shallow arches and spherical caps subjected to uniform external pressure will have a load and displacement relationship as shown in Fig. 2.2. It can be observed that load and deformation increase simultaneously until a maximum or limit load is reached (point A at Curve 1). After the point beyond the threshold of the limit load, structures become unstable. When arches and spherical caps with a large rise-to-span ratio, they fail in an asymmetric mode as a results of bifurcation which is presented as curve 2. When the imperfection is large, the structures may fail even before the load reaches bifurcation-buckling load, which is shown by curve 3.

Usually, the bifurcation load is smaller than the limit load. For example, for a plate buckling elastically, the post-buckling strength will have the plate to resist more loads until the limit load is reached. In the case of severe imperfection, the situation is just the opposite. The limit load is lower than the bifurcation load and the limit load buckling will be observed. However, the threshold of bifurcation buckling when it becomes limit-load buckling has rarely been discussed.

2.1.1.3 Imperfection, Yielding and Post Buckling

According to the elastic buckling theory, a "perfect" compressed elements have three different types of post-buckling behavior which are typified by (1) the column, (2) the stiffened plates, and (3) the cylindrical shell [2]. Figure 2.3 illustrates
Figure 2.2: A typical load-deflection curve of the limit-load model: 1. limit-load curve, 2. bifurcation curve, 3. imperfection curve, point A: buckling load of load-deflection buckling (Galambos, 1988).

Figure 2.3: Load and deflection curve for elastic compressed elements (Galambos, 1988).
the post-buckling behaviors with light lines on the load-deflection curves beyond the critical load for each "perfect" element.

Imperfections will reduce the strength of compressed elements. The solid lines in Fig. 2.3 show the reduction of strength. Roorda [4] studied the post-buckling curves of initial imperfection systems (Fig. 2.4). It was concluded that small initial imperfections have only a negligible effect on the behavior of a system with stable post-buckling curves, but have a remarkable effect on the systems with unstable post-buckling curves.

![Figure 2.4: The effects of a small imperfection on the post buckling strength (Roorda, 1965).](image)

Yielding will further reduce the strength of a compressed element if the yielding load is smaller than the critical load. Figure 2.5 shows the effect of yielding on the columns [2]. The maximum load will approach the Euler load asymptotically even if an initial deflection, and/or an initial load eccentricity, is present as long as the material remains elastic (curve C in Fig. 2.5 (b)). If the critical load is greater than the yielding load, the post-buckling behavior is radically different from the elastic column: the bifurcation buckling will occur at the "tangent modulus "load (point D in Fig. 2.5 (c)), where the tangent modulus is the slope of the stress strain curve. Assuming the stiffness does not change by yielding, the load will asymptotically approach the "reduce modulus" load (E in Fig 2.5 (c)) [3]. In a real situation,
increasing the load beyond the tangent modulus load results in further yielding, which reduces the stiffness and causes the load deflection curve to achieve a peak (point F in Fig 2.5 (c)), beyond which it falls off. The imperfection will induce an extra moment that further reduces the strength of the columns. Curve G in Fig 2.5 (c) characterizes the performance of such a column.

![Graphs showing load vs. deflection](image)

Figure 2.5: The influence of yielding on the behaviors of columns (Galambos, 1988).

### 2.1.2 Buckling Analysis Procedures

Many studies have been carried out on obtaining the buckling loads by theoretical derivations or numerical methods. Most theoretical methods are limited to elastic buckling, which obtains the critical loads by solving the eigenvalue problem. Numerical methods can be applied to either bifurcation type or load and displacement type buckling analysis. The numerical procedures, which can apply in load-deflection buckling analysis, suggest performing a static analysis with a gradually increasing load. Following is a brief description of some typical methods.
2.1.2.1 Theoretical Derivation

Theoretical derivations have been created to calculate buckling loads for simple cases with specified boundary conditions such as “fixed” or “simply supported” [2,3,4,5]. The simple cases which can be analyzed theoretically are columns, beams, plates, or shells. These studies try to find the eigenvalues of the system. The smallest eigenvalue is the critical load which is the smallest load that will cause a lateral deflection of structures. Timoshenko [2] showed that the critical load of columns could be solved using the following equation:

\[ E I \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} = 0 \]  

(2-1)

where \( E \) is Young's modulus, \( I \) is the moment of inertia of the beam’s cross section, \( P \) is the axial load, \( y \) is the later deflection, and \( x \) is the coordinate of the column. The general solution for Eq. 2-1 is

\[ y = A \sin(kx) + B \cos(kx) + Cx + D \]  

(2-2)

where \( k = \left( \frac{P}{EI} \right)^{\frac{1}{2}} \).

Four boundary conditions are needed for Eq. 2-2 to obtain the solution. The buckling loads are the eigenvalues of these four equations.

Extending the beam buckling problem to a 2-D situation results in a plate buckling problem. The differential equation for this problem is:

\[ \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left( N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \]  

(2-3)

where \( w \) is the plate transverse displacement,

\[ D = \frac{E h^3}{12(1 - \nu^2)} \]

is the flexural rigidity of the plate, \( N_x, N_y \) and \( N_{xy} \) are the in-plane forces applied to the plate, \( h \) is the thickness, \( x \) and \( y \) are the coordinates of the plate, and \( \nu \) is Poisson’s ratio.

Equation 2-3 can be solved theoretically by assuming suitable admissible functions that satisfy the boundary conditions. For example, for a plate simply supported on its edges, the displacement \( w \) can be represented as:
\[ w = \sum_{m=1}^{a} \sum_{n=1}^{b} a_{mn} \sin\left(\frac{mn\pi x}{a}\right) \sin\left(\frac{mn\pi x}{b}\right) \]  

(2-4)

where \( a \) is the length and \( b \) is the width of the plate. The buckling loads are the eigenvalues that make the determinant of the set of equations equal to zero and \( a_{mn} \) nontrivial. For example, for a uniformly one-sided compressed plate, the critical stress, at \( n=1 \), can be represented as:

\[ \sigma_c = k \sigma_s \]

(2-5)

where \( k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \), \( \sigma_s = \frac{\pi^2 D}{hb^3} = \frac{\pi^2 E}{12(1-\nu^2)(\frac{h}{b})^3} \).

From Eq. 2-5 it can be observed that the critical buckling not only depends on the thickness of the plate, but also depends on the aspect ratio \( (a/b) \).

### 2.1.2.2 Numerical Method

Because the theoretical method can solve only special cases, numerical methods such as the iteration method, energy method, finite difference method, and finite element method have been applied to find an approximate solution for cases that are more complex.

Lee and Hauck [6] used the iteration method to find the buckling loads of columns under different end conditions. Nethercot [7] used the iteration method to calculate the hybrid steel I-Beam buckling loads. Ersvik [8] applied the iteration method to complex column assemblies. Although several applications have been made, the iteration method has a big disadvantage: it consumes too much computational time, especially when it is applied to analyzing complex structures.

Timoshenko [3] used the energy method to find the buckling loads of beams and plates. The energy method comes from the principle of virtual work \( \delta U = \delta W \). The method can be applied to structures that are more complex. For example, as the cross section changes in the axial direction or if coefficients of Eq. 2-1 are not constant, it will be very difficult to find the exact solution of Eq. 2-1, because the equation becomes nonlinear. In this situation, the energy method is applied to find the
approximated solution. The critical load can be represented by Timoshenko's formula [3] below:

$$P = \frac{\int_0^l El(y')^2 \, dx}{\int_0^l (y')^2 \, dx}$$  \hspace{1cm} (2-6)$$

for columns, and

$$\gamma = \frac{-D\iint \left\{ \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu)\left[ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} \, dx \, dy}{\iint \left[ N_x \left( \frac{\partial w}{\partial x} \right)^2 + N_y \left( \frac{\partial w}{\partial y} \right)^2 + 2N_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] \, dx \, dy}$$  \hspace{1cm} (2-7)$$

for plates. The energy method gives a very satisfying approximation to the true critical load, provided that the shape of the assumed curve is reasonably close to the exact curve.

The Rayleigh-Ritz [3] method assumed a closer shape function that can be obtained by supposing a number of assumed functions than by using a single assumed function. The displacements of column (y) and plates (w) in Eqs. 2-6 and 2-7 can be described by a set of admissible functions that are functions that satisfy boundary conditions. The minimum of P and γ can be calculated by finding the eigenvalues of the equations

$$\frac{\partial P}{\partial a_i} = 0 \text{ or } \frac{\partial \gamma}{\partial a_i} = 0$$  \hspace{1cm} (2-8)$$

The Galerkin method [9] is also based on series expansion. It is developed on the fact that some measure error in \( L(u) = 0 \) is minimized for any fixed value of N, if it satisfies simultaneously for \( m = 1, N \) the following conditions:

$$\int_V L(u) \phi_m \, dV = \int_V L(\sum_{n=1}^N a_n \phi_n) \phi_m \, dV = 0$$  \hspace{1cm} (2-9)$$

The Galerkin method is equivalent to the Rayleigh-Ritz method for the variations problem with a quadratic functional.

The finite difference method for a solution of differential equations constitutes another method for the reduction of the continuum to a system with a finite number of
degrees of freedom. Brush [9] applied the finite difference method to solve the buckling problems of columns, plates, and shells in his book. Kaladas [10] used the method to calculate the buckling load and natural frequencies of welded plates. Although the finite difference method may be a good method to perform buckling analyses, the available commercial codes are very few in structural analyses. Therefore, the finite difference method is not as popular as the finite element method in the field.

The finite element method (FEM) is a numerical method based on the principle of the energy method. Because the accuracy of the energy method depends on the admissible function, it is difficult to get an accurate result especially when the structures are complex and the admissible function is not clear. By dividing the structure member into several small elements, the FEM can overcome the deficiency of the energy method because the shape functions inside the small element are able to be defined clearly and reasonably. The theory of FEM has been stated in many books such as Burnett’s book [11] or Chandrupatla and Belegundu’s book [12]. The finite strip method [13, 14, 15] is used to reduce computational time and errors for thin walled structures in FEM buckling analyses. Some commercial codes such as ANSYS [16], ABAQUS [17] are used to perform finite element buckling analyses. Hu [18] used ANSYS and ABAQUS to obtain the buckling loads of welded offshore tube structures. Frank and Helwig [19] performed the buckling analysis of webs in asymmetric plate girders by ANSYS. Zarous and Redwood [20] used Nastran [21] to study the web buckling in thin webbed castrated beams.

Although there are different schemes and several commercial codes for applying FEM theory to buckling analysis, the basic FEM theory that derives its structure matrix by the assumption of virtual work is the same. The principle of virtual work assumes that a virtual change of internal strain energy must be equal to the change of external work due to applied loads, or:

\[ \delta U = \delta W \]  \hspace{1cm} (2-10)

where: \( U \) = strain energy

\( W \) = external work

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\[ \delta = \text{virtual operator.} \]

The general term of virtual strain energy is

\[ \delta U = \int_V \{\delta \varepsilon\}^t \{\sigma\} dV \quad (2-11) \]

where:
- \(\varepsilon\) = strain vector
- \(\sigma\) = stress vector
- \(V\) = volume of element

Neglect the thermal strain, and by the stress strain relationship:

\[ \{\sigma\} = [D]\{\varepsilon\} \quad (2-12) \]

where \([D]\) = stiffness matrix, then Eq. 2-11 can be represented as the following:

\[ \delta U = \int_V \{\delta \varepsilon\}^t [D] \{\varepsilon\} dV \quad (2-13) \]

The strain can be related to the nodal displacements by:

\[ \{\varepsilon\} = [B]\{u\} \quad (2-14) \]

where:
- \([B]\) = strain-displacement matrix, based on the element shape functions
- \(\{u\}\) = nodal displacement vector

Combine Eqs. 2-13 and 2-14 to give:

\[ \delta U = \{\delta u\}^t \int_V [B]^t [D][B]dV \{u\} = \delta \{u\}^t [K] \{u\} \quad (2-15) \]


The external work done by the forces applied on the node points can be represented as:

\[ \delta W = \delta \{u\}^t \{F\} \quad (2-16) \]

where \(\{F\}\) = nodal force applied to the element.

Combine Eqs. 2-15 and 2-16 and let \(\delta \{u\}\) not equal zero, the static analysis governing equation of FEM can be described as:

\[ [K]\{u\} = \{F\} \quad (2-17) \]

The stiffness of a structure changes with the stress state. The stiffening effect couples the in-plane and transverse displacements, and normally needs to be considered for thin structures with bending stiffness that is very small compared to axial stiffness, such as cables, thin beams, and shells. The effect of stress stiffening is
accounted for by generating and then using an additional stiffness matrix, hereinafter called the "stress stiffness matrix." The stress stiffness matrix is derived as the following [16]:

Considering the second order term of strain, strain can be represented as:

\[
\{\varepsilon\} = \{\varepsilon_i\} + \{\varepsilon_n\} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{1}{2} \left( \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j} \right)
\] (2-18)

The variation of strain is:

\[
\delta(\varepsilon) = \frac{1}{2} \left( \frac{\partial \delta u_i}{\partial x_j} + \frac{\partial \delta u_j}{\partial x_i} \right) + \left( \frac{\partial u_k}{\partial x_i} \frac{\partial \delta u_k}{\partial x_j} \right)
\] (2-19)

Apply Eq. 2-19 to 2-11 and neglect the second order nonlinear strain, it becomes

\[
\delta U = \int_V \{\delta \varepsilon\}^T \{\sigma\} dV
\]

\[
= \int_V \{\delta u\}^T \{B\}^T \{\sigma\} dV + \int_V \{\delta u\}^T \{G\} \{G\} \{u\} dV
\]

\[
= \{\delta u\}^T \int_V \{B\}^T \{D\} \{B\} dV \{u\} + \{\delta u\}^T \int_V \{G\} \{G\} \{u\} dV
\] (2-20)

Let \( S = \int G^T \sigma G dV \) (2-21)

\[
\begin{bmatrix}
\frac{\partial}{\partial x_1} \\
\frac{\partial}{\partial x_2} \\
\frac{\partial}{\partial x_3}
\end{bmatrix}
\]

where \( G = \begin{bmatrix}
\frac{\partial}{\partial x_1} \\
\frac{\partial}{\partial x_2} \\
\frac{\partial}{\partial x_3}
\end{bmatrix}
\)

The general formula of finite element analysis, Eq. 2-16, can be modified as:

\[
(K+S)[u] = [f]
\] (2-22)

Bifurcation buckling occurs as the second variation of potential energy vanishes, i.e.,

\[
(K+S)[\delta u] = 0
\] (2-23)

For a non-trial solution, the condition is

\[
\text{det}(K+S) = 0
\] (2-24)

Let \( s \) be the stiffening matrix as a unit force is applied, then Eq. 2-24 can be changed to:
\[ \det(K + \lambda s) = 0 \quad (2-25) \]

in which \( \lambda \) is the critical buckling load.

### 2.1.3 Buckling Criteria

The definition of elastic buckling is very clear. The buckling load is the eigenvalue of the equations that describe the system. The buckling mode is the eigenvector of these equations.

On the other hand, the definition of buckling of inelastic or imperfect structures is seldom discussed. The buckling load at this situation needs to be determined from the load and displacement curve. Galambos [2] defines the buckling load as the maximum load on the load-deflection curve, which corresponds to point A in Fig 2.2. This definition can be applied to imperfect structures. However, the definition will overestimate the strength for plates with small imperfections, because bifurcation buckling will occur before the maximum applied load can be reached. In other words, Galambos's definition does not consider the post-buckling strength.

Benson [27] defined buckling load for load-deflection buckling as the loads at which the two tangent lines on the load deflection curve intersect. However, the load-deflection curve is not linear; therefore, the chosen tangential points will change the buckling loads. This method cannot provide accurate buckling criteria.

Other researchers have suggested the use of the load at the intersection of the secant line with the test record. The secant line is drawn from the origin with a slope that is 95 percent of the recorded slope. This idea comes from fracture mechanics [38]. This assumption has the same problem as Benson's definition: since the curve is not linear, there is not a constant slope in the curve.

Fok [39] presented two numerical methods for estimating the buckling load from the experimental data of rectangular plates loading in the edges. The three-point technique is the use of three sets of such readings to form a system of nonlinear simulation equations, the solution of which yields the critical load, initial imperfection, and the constant governing the curvature of the load-deflection curves. The least squares technique employs the principle of least squares curve fitting to
these values. These methods are just focused on how to predict the load-displacement curve from the experiment but they do not propose criteria to calculate buckling from these curves. Fox’s study does not answer the question about how to determine the buckling load from the load-displacement curve.

2.1.4 Experiments

Experiments on columns, beams, and plates have been performed to verify the buckling theory or to derive empirical design formulas.

Van Kuren and Galambos [22] tested full-scale, as rolled, wide flange beam-columns to study the strength and deformation behaviors in the inelastic range. Hall [23] used the regression method to fit the 393 test data. The test data was fitted with the Euler hyperbola in the elastic range. The inelastic range of column strength was then represented by a sloping straight line from the Euler curve to a horizontal plateau in the lowest slenderness. More of Hall’s work will be represented in Chapter 3.

Usami and Fukumoto [24] tested 27 box columns with large width-thickness ratios. Based on their test results, the following empirical design formula was presented to predict the interaction buckling strength:

\[
\frac{f_{\text{max}}}{f_y} = \frac{C}{\lambda'}
\]  

(2-26)

where \(f_{\text{max}}\) is the applicable maximum load and \(f_y\) is the load where yielding happens. \(C\) is a constant related to the average residual stress:

\[
C = 0.85 - \frac{\sigma_{re}}{\sigma_y}
\]  

(2-27)

where \(\sigma_{re}\) is the residual stress, \(\lambda'\) is defined by Eq. 1-2.

Other compressed members were also tested besides beams, columns, and plates. Rao Adluri and Madugula [25] tested 26 hot-rolled steel angles under concentric axial compressive forces. Prion [26] studied the beam-column behavior of unstiffened fabricated steel tubes. Benson [27] performed a serious test on welded aluminum girders.
From comparing the test results and the results from the theoretical derivation or the numerical method, it is not unusual to see some deviations between the predicted values and experimental values. The deviations between theory and testing imply that something more is needed for consideration in the theoretical and numerical methods.

### 2.2 Misalignment

Misalignment is the centerline mismatch between two joined components. Because of the deviation from the straightness or plainness, an extra moment is induced as loads are applied. This moment may reduce the ultimate load carrying capability of compressed members.

As mentioned in Chapter, Yamba [1], Ersvik [8] and Hu [18] had worked on the issue of misalignment. Yamba's [1] study was not completed because how to determine the factor was not proposed in his dissertation. Ersvik's [8] work was focus on the modeling of misalignment of 1D beam-column structures. The effects of misalignment were only found in Hu's [18] dissertation. He found that the mismatch at the abutting ends is the principal parameter that initiates the local buckling and decreases the cross-sectional strength of the tube. The decrease in the maximum load was approximately proportional to the size of the mismatch. About a 3% decrease in ultimate load for every 0.5 mm increase was observed.

### 2.3 Residual Stress Simulation

Nethercot [7], Kitipornachi and Wong-Chung [34], and Weng [35] assumed a residual stress distribution pattern in their analyses to calculate the critical loads. The tensile yielded stress is assumed in the heat affection zone that is assumed to have the width of about 2 to 4 times the plate thickness. Constant compressive residual stresses are distributed in the rest of the parts.

Ueda and Yuan [36] predicted the residual stress in welded plates using inherent strains. The incompatible strain from nonlinear temperature distribution is regarded as the source of residual stress and is referred to as the inherent strain. They
found that the patterns of inherent strain vary little with changes in the welding conditions and the size of weld plates.

In butt joints, Yuan and Ueda [36] found the pattern of longitudinal inherent strains on the cross section at the plate center is the same and may be approximated by a trapezoid. The factor \( \frac{\alpha E T_{av}}{\sigma_{yb}} \) affects the magnitude and width of inherent strains. When \( \frac{\alpha E T_{av}}{\sigma_{yb}} \) is large, the width of the area where inherent strains exist increases while the magnitudes of inherent strains decrease. The symbols of the factor are defined as follows:

- \( \alpha \) is thermal expansion coefficient,
- \( E \) is Young’s modulus,
- \( \sigma_{yb} \) is the yield stress of the base metal,
- \( T_{av} \) is a parameter showing the intensity of heat input with respect to the plate size. The average temperature (\( T_{av} \)) is represented as:

\[
T_{av} = \frac{Q}{2\rho chB} \tag{2-28}
\]

Where

- \( Q \) is net heat input;
- \( \rho \) is density;
- \( h \) is thickness of plate;
- \( B \) is half width.

In T and I type joints, Yuan and Ueda [59] found that when \( T_{av} \) is less than 30°C, the inherent strain varies little against the width ratio of web and flange, \( B_w/B_F \). However, when \( T_{av} \) is large, the inherent strain distribution varies not only with the average temperature rise but also with the geometric ratio. That is, as the geometric ratio increases, the inherent strain arises in a smaller area for the same average temperature rise. This implies that \( T_{av} \) and \( B_w/B_F \) affect the bending deformation, which is proportional to the total strain in the longitudinal direction, and consequently affect the area of plastic strain.

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The residual stresses can be predicted using the following steps:

1. Find the inherent strain distribution obtained from the reference weld. The length and width of welds should be large enough so that only the longitudinal inherent strain exists in the middle part of the weld, and the transverse inherent strain remaining near the ends of the plate is not affected by the plate width.

2. Based on the reference inherent strain distribution, predict the inherent strain distribution for the specific welds whose geometry and heat input are different from the referenced strain.

3. Analyze residual stresses elastically, using the predicted inherent strain as the initial strain.

Shim, et al. [37] used the finite element method to calculate the residual stresses for thick plates. The plain strain assumption is used in the analysis to reduce the model from 3D to 2D. The heat input assumed a ramp function where the slope of the ramp was decided by experiment results. Two models were compared in the analysis: in the first model, the heat is applied to each pass and in the second model, each layer of the weld bead was considered as one lumped pass. After comparing the results with experimental data, the lump model was considered a reliable model with tolerable error and much less computational time. This method is suitable only for analyzing local residual stress around the weld of thick plates. For thin walled structures, such as plate girders, the structures are so complex that plain strain no longer exists. This method also consumes a lot of computing time if residual stress is calculated at each welding sequence. Shim’s method is not suitable in such a situation.

Hu [18] produced the residual stresses in his steel tube models by assuming a thermal expansion coefficient to elements adjacent to the welds, then decreasing the reference temperature to produce residual stresses. The amount of temperature variation and the width of thermal elements had to be adjusted to obtain the desired residual stress. He found that the residual stresses resulting from the welding process showed no effect on the conditions which caused the local buckling but had a certain effect on the maximum sectional strength and buckling mode shapes. The residual stresses may decrease the maximum cross section strength if the buckling occurs in the
elastic or elastic-plastic region, before entering the fully plastic zone. Because the method needs to adjust the temperature variation and width of thermal elements, it only can be applied when the residual stresses are known.

2.4 Effects of Geometrical Imperfection

Yamba defined geometrical imperfection as the deviations from the coplanar of the plates and from the straightness of the columns. He assumed that the imperfections introduce internal eccentricities that add to the load eccentricities. As a result, the bending deformation/load ratio is not linear; and consequently, the stress-deformation diagram has an increasing curvature. This relationship implies that the ultimate strength of a compression element is decided by the imperfection. He also found that the shape of the imperfection would affect the buckling strength. He indicated that the worst effects of these imperfections on the collapses of a plate are obtained when there are ripples in the plate in the direction of the stiffeners of a wave with a length similar to that of elastic critical buckling modes.

Ersvik [8] performed a series of non-linear analyses of multistory columns and found that the shape and magnitude of the initial deflection must be accounted for in making correct estimates of the possibility of failure. He also showed that the reduction in load-carrying capability decreases rapidly as the value of the slenderness ratio increases.

Bjorhovda [28] studied the effects of variations in the magnitude of the initial crookedness of columns. Figure (2-6) shows that the greater the deviation from the straightness, the more reductions will occur in the maximum carrying load. The results also show that the reduction of load-carrying capability decreases rapidly as the value of the slenderness ratio increases, which is consistent with Ersvik's conclusion.

For a column with pin ends, the Secant formula is recommended to offer another criterion for failure in imperfect conditions. Failure is predicted when the maximum compressive stress is larger than the yield stress. The maximum compressive stresses in the column can be represented as follows [29]:

36
For plate structures, no formula for governing the imperfections is proposed.

\[
\sigma_{\text{max}} = \frac{P}{A} \left[ 1 + \frac{cc}{r^2} \sec \left( \frac{L}{2r} \sqrt{\frac{P}{EA}} \right) \right]
\]

(2-29)

For plate structures, no formula for governing the imperfections is proposed.

Figure 2.6 Column curve bands for 112 columns, based on initial out-of-straightness of L/500, L/1000, and L/2000 (Bjorhovda, 1972).

2.5 Effects of Residual Stresses

For welded structures, the residual stresses are induced by a non-linear temperature distribution that cannot satisfy the compatibility equation [30]. To satisfy the compatibility equation, internal stresses and strains are induced. In the case of a plate welded along both longitudinal edges, as in the web of a welded I beam, the stress around the weld is in tension that is closed to the yield stress of the material. Large residual tension induces an almost uniform residual compression P_r over most of the width of the plate. When the plate buckles, the contribution of the residual tension to bending at a cross section is small, since the increments of deflection are
smaller at the edge of the plate than they are in the interior. The result of uniform residual compression $P_r$ is a bending moment, in addition to the moment from the externally applied compressive stress, $P$. The induced extra bending moment will reduce the critical stress [31].

Galambos [2] stated that columns made from flame-cut plates would have a greater strength than those made from a universal mill, because the universal mill causes compressive residual stress at the plate edges, whereas flame-cut plates have tensile residual stresses at the edges and compressive stress inside the plate. When plates are welded at the center, the compressive stress will reduce the tension induced from welding, so the flame-cut plates will provide better buckling strength.

Batterman and Johnston [32] found that because of nonlinearity, the separate effects of residual stress and initial curvature cannot be added to give a good approximation. They also found that residual stresses have little effect on the maximum strength of a very slender column, which has the strength approaching the Euler load.

Kaldas [10] used a finite difference technique to determine the in-plane residual stress field due to welding. The Rayleigh-Ritz method, with beam characteristic functions, is utilized for buckling analyses. He found that the residual stresses for multi-pass welds might be taken as the residual stress pattern caused by the pass with the largest actual energy input per weld length. The reason is that when a large weld is deposited on a smaller weld, the large weld completely ‘washes out’ the smaller weld. The welding sequence for welding around a plate has just a slight influence on the plateau. The welding induces residual stresses which will change the natural frequency and elastic critical load of plates. If the mode of free vibration is similar to the buckling mode, the following relationship between the natural frequency and buckling stress is proposed for plates under a unidirectional or hydrostatic loading condition.

$$\frac{\sigma_d}{\sigma_{cr}} + \left(\frac{f}{f_0}\right)^2 = 1$$  \hspace{1cm} (2-30)
where $\sigma_d$ is the dropping of elastic buckling stress, $\sigma_{cr}$ is the critical buckling stress, $f$ is the natural frequency after welding, and $f_0$ is the plate natural frequency before welding.

Dwight et al. [33] have studied the strength of simply supported square plates. They assumed the unloaded edges staying straight and derived the following empirical formula:

$$\sigma'_{cr} = \sigma_{cr} - \left[\left(\sigma_r + \sigma_y\right) \cos \frac{\pi \sigma_y}{2\left(\sigma_r + \sigma_y\right)} - \sigma_y\right]$$  \hspace{1cm} (2-31)

where $\sigma'_{cr}$ is the reduced critical stress, $\sigma_{cr}$ is the critical stress for unwelded plate of the same b/t, $\sigma_r$ is the residual stress and $\sigma_y$ is the yield stress of the material. For $\sigma_r \leq 0.2$ $\sigma_y$, Eq. 2-31 can be simplified to:

$$\sigma'_{cr} = \sigma_{cr} - \sigma_r$$  \hspace{1cm} (2-32)

Equation 2-32 is similar to Usami and Fukumoto’s [24] finding that nearly constant compressive stress can be measured in the center portion of each plate. The reduction in critical stress is close to the measured residual stress.

Benson [27] measured the residual stress in aluminum plate girders, using the sectioning method. The largest tensile stress was measured in the web of the most compact specimen and was about 77% of the yielding stress of the material in the heat-affected zone. The compressive stresses were smaller, but also of considerable size in base metal. However, the effect of residual stresses on the shear capability is uncertain.

Yamba [1] accounted for residual stresses in two ways: first, their magnitude can be directly deduced from the critical stress; second, they may be expressed in an equivalent geometrical imperfection, and the total resulting imperfection is used for calculation. He concluded that the influences of initial imperfections and residual welding stresses on the buckling behavior are evident. He also proposed that the residual stress would not have any effect when the width to thickness ratio (b/t) is less than 20.
2.6 Ultimate Load

As mentioned earlier, the post-buckling behaviors of stiffened perfect plates are stable. That means that the strength of stiffened plates may be much larger than the buckling load. For plate girders, the tension field action [2,3,31,40] made plate girders more resistant to loads after shear buckling. Therefore, it is very clear that a design based on buckling will be too conservative and the ultimate load must be considered in the designs of plate assemblies.

Although the post-buckling strength of the web plate is utilized, the criterion for permissible depth to thickness ratio and the ratio of the space between the stiffeners to the thickness of the web is still determined by the buckling consideration [2]. The buckling strength of webs considered in design is bending buckling, vertical buckling, and shearing buckling. AASHO [41] and AISC [42] have required h/t, a/h values to prevent bending, or vertical buckling. They also allow the shear buckling with the development of the tension field, where “h” is the height of the girder, “t” is the thickness of the web, and “a” is the space between the stiffeners.

For simply supported plates, the effect width method [1,2,3,31] can be used to derive the ultimate load if the critical stress is less than the yield stress. The stress will redistribute after buckling. The non-uniform post-buckling stress distribution can be replaced by the two rectangular stress blocks of intensity, $f_c$, and width $b_e/2$, where $b_e$ is the effect width. The effect width can be derived by assuming that it is the same width $b_e$ of a plate which buckles at the uniform stress of $f_c$, which is usually assumed to be the yield stress. By use of the effect width concept, some formulas for predicting ultimate load have been proposed. Kalyanaraman [43] proposed

$$\sigma_{av} = 1.19\sqrt{\sigma_{cr}}\sigma_y (1 - 0.30\sqrt{\frac{\sigma_{cr}}{\sigma_y}}) \quad (2-33)$$

AISI [44] specification for the ultimate load is:

$$\sigma_{av} = \sqrt{\sigma_{cr}\sigma_y} (1 - 0.22\sqrt{\frac{\sigma_{cr}}{\sigma_y}}) \quad (2-34)$$
When the web plate is subjected to bending, the effect width concept is also applied to replace the nonlinear compression distribution on a buckled web with linear distribution acting on the effective depth of the web. Basler and Thurlimann [45] assumed the depth to be 30t for a girder with h/t = 360. This enables the moment to be calculated if the effective depth is known.

Allen and Bulson [46] proposed that the maximum shear strength, for a plate with infinitely stiff edge members, can be estimated as:

\[ V_u = \tau_c bt + \frac{1}{2} \sigma_y bt \]  

(2-35)

where \( \sigma_y = \sqrt{\sigma_y^2 - 0.75 \tau_c^2} - 1.5 \tau_c \) provided that \( \tau_c << \sigma_y \).

For a web with infinite stiff flange, the total shear force is the sum of the critical stress and tension force component which acts in the vertical direction. For example:

\[ V_u = \tau_c bt + \frac{1}{2} \sigma_t bt \sin 2\phi \]  

(2-36)

This equation is equal to Eq. 2-35, as \( \phi = 45^\circ \) and the tension stress is equal to the yield stress.

Basler [47] considered flanges to be free and the tension field in the panel is reacted by the field in the adjacent panels. From the geometry of the panel, the effect width for the tensile field is:

\[ b_e = b \cos \phi - a \sin \phi \]  

(2-37)

The shear component from the tension field is:

\[ V_t = \sigma_t b_e t \sin \phi \]  

(2-38)

The ultimate stress for the partial tension field is

\[ V_u = \tau_c bt + \sigma_t (b \cos \phi - a \sin \phi) t \sin \phi \]  

(2-39)

The maximum of \( V_u \) is obtained by letting the first derivative of Eq. 2-39 be zero.

Many variations of the post-buckling tension field theory [48,49,50] have been developed since the Basler solution was published. The difference of these methods is
the decision of the effect width of the tension field and the consideration of the flange strength. A summary of these variations can be found in Galambos's book [2].

2.7 Summary

From the literature review, it is obvious that the eigenvalue buckling analysis cannot be applied to imperfect situations. The results from the experiments are different from the predicted ones because of imperfection.

Recent buckling analyses have taken the imperfections, such as residual stresses and geometrical imperfections, into consideration. However, the misalignment effects are seldom discussed, especially for the cases where misalignment is combined with residual stresses. Unfortunately, misalignment and residual stresses usually occur in a welded structure.

The finite element method has been applied to perform the buckling analysis for imperfect structures. However, the criteria for deriving the buckling loads from the load-deflection curve are seldom discussed. Furthermore, the modeling of the misalignment joint and the characterization of residual stresses for complex structures have rarely been studied. Therefore, in order to study misalignment and residual stress effects on welded structures, the following issues should be studied:

1. Buckling criteria for deciding buckling loads from load-displacement curves.
3. Simulation welding-induced residual stresses in complex structures.
CHAPTER 3

DECIDING ON THE BUCKLING CRITERIA

Critical loads of elastic buckling can be obtained from the eigenvalues of the equations that describe the system. However, for imperfect structures, the buckling load needs to be observed from the load-displacement curve of the system because imperfection may reduce the buckling strength. Therefore, criteria need to be established for deciding on the occurrence of load and displacement type buckling.

In this chapter, a simple example was used to illustrate why the eigenvalue buckling analysis cannot be applied to imperfect structures. The behavior of imperfect columns and plates under different loads was simulated by the FEM. Three criteria were applied to decide on the buckling loads from load-displacement curves. The appropriate criterion for different conditions was proposed after comparing the obtained buckling loads with the outcome from published documents.

3.1 The Insufficiency of Eigenvalue Buckling Analysis

The eigenvalue buckling analysis for columns and plates assumes that structural members do not deflect laterally until the load reaches its critical value. The critical buckling load of the system can be derived from the eigenvalue of the system, which is described in Chapter 2. However, when imperfections are considered, lateral displacement is incepted at the beginning of applying the load. Applying Eq. 2-24 to find the critical load of buckling may overestimate the buckling strength because the imperfection effects is not considered. Eq. 2-22 should be applied to calculate the load-displacement curve of the system when imperfection is concerned.
Figure 1.4 (a) shows a simply supported column, which is joined at the center of the column perfectly. From Eq. 2-1, the lateral deflection \( y \) of each segment can be described as:

\[
y_L = c_1 \cos \alpha x + c_2 \sin \alpha x
\]

\[
y_R = c_3 \cos \alpha x + c_4 \sin \alpha x
\]

(3-1)

where \( \alpha^2 = \frac{P}{EI} \), \( P \) is the axial load, \( E \) is Young’s Modulus, \( I \) is the moment of inertia, \( y_L \) is the left segment displacement, and \( y_R \) is the right segment displacement of the column. \( C \) refers to the coefficient.

Considering the boundary conditions: \( y=0 \) when at \( x=0 \) and \( x=\ell \), and at \( x=\ell/2 \), the displacement and rotation are continuous. The following four equations can be obtained.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \alpha \ell & \sin \alpha \ell & 0 \\
\cos \frac{\alpha \ell}{2} & \sin \frac{\alpha \ell}{2} & -\cos \frac{\alpha \ell}{2} & \sin \frac{\alpha \ell}{2} \\
-sin \frac{\alpha \ell}{2} & \cos \frac{\alpha \ell}{2} & -\sin \frac{\alpha \ell}{2} & \cos \frac{\alpha \ell}{2}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2 \\
c_3 \\
c_4
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(3-2)

Let the determinant of Eq. 3-2 be zero and get:

\[
\sin \alpha \ell =0
\]

(3-3)

Combining Eqs. 3-1 and 3-3, the first critical buckling load can be represented as

\[
P = \frac{EI\pi^2}{\ell^2}
\]

(3-4)

If these two segments are joined with misalignment, the column is no longer a perfect structure. A moment, as shown in Fig. 3.1, will be induced when the axial load is applied. This can be modeled as a perfect column subjected to an axial load at one end and a moment at the center of the column. Applying the same boundary conditions mentioned above for the perfect column, the load and deflection equations of the imperfect column can be represented as:
where $\delta$ is the distance between two central lines of the segments and is indicted in Fig. 3.2.

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & \cos\alpha & \sin\alpha \\
\cos\frac{\alpha\ell}{2} & \sin\frac{\alpha\ell}{2} & -\cos\frac{\alpha\ell}{2} & -\sin\frac{\alpha\ell}{2} \\
-\sin\frac{\alpha\ell}{2} & \cos\frac{\alpha\ell}{2} & \sin\frac{\alpha\ell}{2} & -\cos\frac{\alpha\ell}{2}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2 \\
c_3 \\
c_4
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
\delta \\
0
\end{bmatrix}
$$

(3-5)

Although Eq. 3-5 is similar to Eq. 3-2, the behaviors of columns under compression are different. Unlike eigenvalue buckling problems, lateral deflection of
the misaligned column is induced by the moment due to the misalignment before buckling takes place. It implies that the load-displacement curve can provide more information of buckling than the eigenvalue analysis does for imperfect structures.

Most of the commercial finite element analysis codes, such as ANSYS, NATRAN, and ABAQUS, perform linear buckling analyses by solving the eigenvalue equations. So, they are not capable of analyzing the effect of imperfections. Table 3.1 shows the eigenvalue buckling loads of a simply supported column with a normalized slender ratio of 4.6. Normalized slenderness is stated in Eq. 1-1 and is defined as the square root of the yield stress divided by the square root of buckling critical stress. The equivalent moment was added in the center of the column to simulate the misalignment joint [8]. Table 3.1 indicates that the critical load does not change no matter how the misalignment changes. Therefore, the linear eigenvalue buckling analysis cannot be applied to decide the buckling loads of imperfect structures. Buckling loads of imperfect structures should be determined from the load-displacement curve. It implies that a criterion to determine the inception of buckling from a load-displacement curve is necessary.

<table>
<thead>
<tr>
<th>Case</th>
<th>Eigenvalue Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misalignment 0.05 m</td>
<td>0.05σy</td>
</tr>
<tr>
<td>Misalignment 0.25 m</td>
<td>0.05σy</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison of FEM eigenvalue buckling analysis results. (Column with λ' = 4.6, theoretical critical stress 0.05σy)

### 3.2 The Behavior of Axial Loaded Columns

In order to have a criterion for determining the buckling loads, the behaviors of axial loaded columns which have normalized slenderness ratios 0.4, 0.7, 1.0, 1.3, 1.5 and 1.8, were observed first.
The normalized slenderness ratio for columns is defined by Eq. 1-1 as
\[ \lambda' = \left( \frac{k \ell}{r} \right) \sigma_y / \pi^2 E \]  
(1-1)

Because the normalized slenderness ratio is the square root of yield divided by the square root of theoretical critical buckling stress, a critical buckling stress is higher than the yield stress when \( \lambda' \) is less than one and vice versa. The range of the normalized slenderness ratio (0.4-1.8) represents columns which have an eigenvalue buckling stress varying from 6.25 times that of the yield stress to 0.3 times that of the yield stress. To avoid redundancy, only the results from the analyses of \( \lambda' = 0.4 \) and 1.8 are presented here. The compressive behaviors with \( \lambda' \) between 0.4 and 1.8 are varied between these two limits.

As in the procedure stated in Chapter 1, a simply supported column with the length of 10 m. was modeled to perform the finite element analysis. The mechanical properties of an AH-36 steel were applied to the model. The yield stress of AH-36 is 350 MPa (51 ksi) and the tensile strength is around 490-620 MPa (71-90 ksi) with a 22% elongation in a 2-inch gage [51,52]. These properties are listed in Table 3.2. Because the mechanical properties and dimensions of the column are normalized to the form of the normalized slenderness ratio, the results can be generalized to different properties and dimensions as long as the normalized slenderness ratio remains the same.

Elastic-plastic analyses were performed by a FEM commercial code: ANSYS. Bilinear isotropic hardening was used to simulate the stress-strain relationship after yielding. The slope of the strain hardening curve was assumed to be the tensile strength divided by the percent elongation of a 2-inch gage length. Because of large deflection after buckling, geometric non-linearity, such as large strain and large rotation, was included in the calculation. In addition, the stress-stiffening effect is considered in the analysis for structures modeling with beam or shell type elements. The Von-Mises equivalent stress was used to decide yielding. A small lateral perturbation, which causes a \( \ell/1,000 \) initial deflection by applying a small concentrated load in the middle of a perfect column, was applied in these analyses as...
an initial condition. The magnitude of the applied axial load was increased gradually until the reach of the buckling load, where the load may increase or decrease depending on the post-buckling strength.

<table>
<thead>
<tr>
<th>Temperature (°c)</th>
<th>70</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus (GPa)</td>
<td>200</td>
<td>195</td>
<td>185</td>
<td>163</td>
<td>135</td>
</tr>
<tr>
<td>Tangent Modulus (GPa)</td>
<td>1.31</td>
<td>1.98</td>
<td>1.33</td>
<td>0.43</td>
<td>0.11</td>
</tr>
<tr>
<td>Yield Stress (MPa)</td>
<td>350</td>
<td>306</td>
<td>263</td>
<td>131</td>
<td>43</td>
</tr>
<tr>
<td>Tensile Strength (MPa)</td>
<td>550</td>
<td>525</td>
<td>507</td>
<td>215</td>
<td>88</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.29</td>
<td>0.29</td>
<td>0.30</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>Thermal Expansion Coefficient (10⁻⁶/°c)</td>
<td>11.90</td>
<td>12.60</td>
<td>13.80</td>
<td>14.70</td>
<td>14.00</td>
</tr>
</tbody>
</table>

Table 3.2: The material properties of AH36 steel [51,52].

Figure 3.3 shows the load-displacement curve of a simply supported column with a normalized slenderness ratio of 1.8. The load-displacement curve for \( \lambda' = 0.4 \) is presented by Figure 3.6. Figures 3.4 and 3.7 present the first and second derivatives of each load-displacement curve. Figures 3.5 and 3.8 exhibit the stresses at the midsection of the column for these two normalized slenderness ratios.

The theoretical buckling stress determined by Euler's method is about 30% of the yield stress for columns with \( \lambda' = 1.8 \). Figure 3.3 indicates that the large lateral displacement does not occur until the load reaches its theoretical critical value (30% of the yield stress). The maximum of these two derivatives occurs at the maximum load that the columns can resist. From this, the following can be inferred:

1. When the applied axial load is below the theoretical critical value, the column is subjected mainly to the axial load. The stresses from bending are not important. The stresses at the cross section are small and almost uniform.
Figure 3.3: Displacement vs. normalized applied stress at the center of a column (slenderness = 1.8, axial load).

Figure 3.4: The first and second derivative vs. normalized applied stress at the center of a column (slenderness = 1.8, axial load).
Figure 3.5 Normalized stress vs. normalized applied stress at the center of a column (slenderness = 1.8, axial load).

Figure 3.6 Displacement vs. normalized applied stress at the center of a column (slenderness = 0.4, axial load).
Figure 3.7: The first and second derivative vs. normalized applied stress at the center of a column (slenderness = 0.4, axial load).

Figure 3.8: Normalized stress vs. normalized applied stress at the center of a column (slenderness = 0.4, axial load).
2. When the axial load is higher than the theoretical critical value, the column is subjected to both bending and axial stresses. The bending stress is due to the large lateral displacement from buckling. The lateral displacement is increasing so soon that the bending stress increases sharply.

From the above observation, the failure of the columns with $\lambda' = 1.8$ can be considered to include the following two steps:

1. Buckling initializes the large lateral displacement.
2. The large bending stress is induced from the large lateral displacement.

Failure is predicted to occur when there is stress on the outset fiber of the compressed side beyond the material's tensile stress.

Although the load-displacement curve in Fig. 3.6 is similar to that in Fig. 3.3, the explanation for the failure of the column with $\lambda' = 0.4$ is different. For $\lambda' = 1.8$, the large lateral displacement is due to buckling. Because of the large lateral displacement, the bending stress increases very fast, which results in the failure of the columns. For $\lambda' = 0.4$, the difference of stress between the top and bottom of the column is small. The cross section yields almost simultaneously. The lateral displacement becomes large after the column yields. This implies that the lateral deflection is due to yielding instead of buckling. Because of the yielding, the column can no longer resist any extra load.

From the failures of simply supported columns, the followings can be concluded:

1. For columns with a normalized slenderness ratio of 1.8, the large lateral displacement is due to buckling. After buckling, the stress will increase sharply until the column fails.

2. For columns with a normalized slenderness ratio of 0.4, the large displacement is due to yielding. Because of yielding, any extra lateral load will cause large displacement and then the column will fail.

3. For columns with a normalized slenderness ratio between 0.4 and 1.8, the column behavior is between the above two cases.
3.3 Simply Supported Square Plates under Different Loads

As stated in Chapter 1, to totally understand the buckling behavior of plate girders, the following loading conditions should be considered.

1. Simply supported plates uniformly compressed in one direction.
2. Simply supported plates compressed by shear force.
3. Simply supported plates compressed by bending.

As in the study on column behaviors, the normalized slenderness ratios were also chosen from 0.4 to 1.8, which represented the plates having an eigenvalue buckling stress varying from higher to lower than the yield stress of structural materials. The normalized slenderness ratio of the plates is defined by Eq. 1-2 in the following form.

\[
\lambda' = \frac{b}{t} \sqrt{\frac{12(1 - \nu^2)\sigma_y}{k\pi^2 E}}
\]

In these analyses, a 1mx1m simply supported plate was modeled for the finite element analysis. These three loading conditions are shown in Fig. 1.4. The AH-36 property and the same procedures stated in Chapter 1 and Section 3.2 were applied to these buckling analyses. Because the parameters are normalized, these results also can be generalized to other situations, as long as the normalized slenderness ratios are equal.

Figures 3.9, 3.12, and 3.15 exhibit the load-displacement curves of a plate with a normalized slenderness ratio of 1.8 subjected to uniform compression, pure shearing, and bending. It can be observed that the large lateral displacement does not occur until the load reaches its theoretical critical value (30% of the yield stress). Unlike the behavior of columns, plates can resist more loads after buckling, when subjected to compression or shear. This can be inferred from the load and displacement curve, where the displacement of the plate can increase only when more loads are added. This implies that a plate possesses post-buckling strength when it is subjected to uniform compression, shear or bending.
Figure 3.9: Displacement vs. normalized applied stress at the center of a plate (slenderness = 1.8, compression).

Figure 3.10: The first and second derivative vs. normalized applied stress at the center of a plate (slenderness = 1.8, compression).
Figure 3.11: Normalized stress vs. normalized applied stress at the center of a plate (slenderness = 1.8, compression).

Figure 3.12: Displacement vs. normalized applied stress at the center of a plate (slenderness = 1.8, shear).
Figure 3.13: The first and second derivative vs. normalized applied stress at the center of a plate (slenderness = 1.8, shear).

Figure 3.14: Normalized equivalent stress vs. normalized applied stress at the center of a plate (slenderness = 1.8, shear).
Figure 3.15: Displacement vs. normalized applied stress at the point D of a plate (slenderness = 1.8, bending).

Figure 3.16: The first and second derivative vs. normalized applied stress at the point D of a plate (slenderness = 1.8, bending).
Figure 3.17: Normalized equivalent stress vs. normalized applied stress at the point D of a plate (slenderness = 1.8, bending).

Figures 3.11, 3.14 and 3.17 show the stresses for plates under compression, shearing, or bending. It can be observed that:

1. When the load is below the theoretical critical value, the plate is subjected mainly to an in-plane compressive load. The stresses from bending are not important. The stresses at the cross section are small and almost identical.

2. When the applied load is higher than the theoretical critical value, the plate is subjected to both bending and compressive stress. The bending stress is due to the large lateral displacement from buckling.

The behaviors mentioned above are very similar to the reactions of axial compressed columns. Therefore, they should have similar failure mechanisms, i.e., the plate will buckle first. Buckling will induce large deflection that decreases the rigidity. Finally, the ultimate load is reached and the plate fails. The only difference is that the plates have post-buckling strength while the columns don't.
Figure 3.18: Displacement vs. normalized applied stress at the center of a plate slenderness = 0.4, compression.

Figure 3.19: The first and second derivative vs. normalized applied stress at the center of a plate (slenderness = 0.4, compression).
Figure 3.20: Normalized stress vs. normalized applied stress at the center of a plate (slenderness = 0.4, compression).

Figure 3.21: Displacement vs. normalized applied stress at the center of a plate (slenderness = 0.4, shear).
Figure 3.22: First and second derivative vs. normalized applied stress at the center of a plate (slenderness = 0.4, shear).

Figure 3.23: Normalized equivalent stress vs. normalized applied stress at the center of a plate (slenderness = 0.4, shear).
Figure 3.24: Displacement vs. normalized applied stress at the point D (slenderness = 0.4, bending).

Figure 3.25: The first and second derivative vs. normalized applied stress at the point D (slenderness = 0.4, bending).
Figures 3.18, 3.21, and 3.24 show the load and displacement relationship when the normalized slender ratio is equal to 0.4. The lateral displacements become large when the nominal applied stress is close to the yield stress. Figures 3.20, 3.23, and 3.26 display the stresses at the plate. The stresses are relatively uniform at the top, middle, and bottom of the plates, compared with the stresses shown in Figs. 3.11, 3.14, and 3.17 that have a $\lambda'$ of 1.8. As the discussion mentioned in the column's behavior, the lateral displacement is due to yielding because yielded structures can be deformed easily.

From these analyses, a similar conclusion for the failure of compressed column structures can be applied to the failure of compressed plate structures. Failure of the plates can be stated as follows.

1. For plates with a normalized slenderness ratio of 1.8, buckling induces lateral displacement. Stress will build up with the increment of displacement until failure.
2. For plates with a normalized slenderness ratio of 0.4, the large lateral displacement is due to yielding. Stress also increases with the increment of displacement until failure.

3.4 Buckling Criteria and Verifications

From the load-displacement curves of the axial compressed columns, it can be inferred that the applied load keeps increasing until the columns become unstable. The limit load, which is defined as the maximum load that a column can carry, can be considered as the buckling load in these cases. The first and second derivatives keep increasing when the load increases. No local extreme or change of the sign of the second derivative is observed. The maximums of these derivatives occur at the limited load. By defining the load where maximums of these derivatives occur as buckling load, these three criteria will get the same buckling loads in the column buckling analysis.

Figures 3.10, 3.13, and 3.16 display the first and the second derivatives of load-displacement curves of plates with a normalized slenderness of 1.8. The first local extreme of the first derivative occurs at the theoretical critical load. The second derivative changes its sign at this particular load. Therefore, by defining the load where the first derivative is a local extreme or the second derivative is changing signs, the critical buckling load can be found. Under this circumstance, plates buckle first and then there is failure after more loads are applied. Because of the post-buckling effect, it is obvious that the limit load criterion will overstate the buckling strength.

Figures 3.19, 3.22, and 3.25 present the first and second derivatives of the load-displacement curves of plate with a normalized slenderness ratio of 0.4. Because there is no post-buckling strength under this special circumstance, as stated in axial compressed columns and bent plates, the limit load criterion, the first derivative criterion, and the second derivative criterion will all get the same buckling load.

By combining the above observations, it can be concluded that the limit load can be applied to find the ultimate load and the derivative criteria (first and second)
can be applied to determine buckling for all the cases considered. Several comparisons are made to verify these conclusions in the following discussion.

3.4.1 Verification in Column Buckling

The buckling loads of columns from the three proposed criteria are compared with the results obtained from Euler’s solutions, which are stated in Chapter 2, and the results from Hall’s regression data.

Hall’s data set for regression analysis was derived from centrally loaded columns with 393 tests, which contain different degrees of imperfections in the data set. The data was divided into two categories: columns with \( l/r \) less than 110 and those with slenderness ratio 110 or greater. Hall derived a regression equation for materials with yield strength for 33 ksi -100 ksi (227.7 MPa-690 MPa). The regression equation is:

\[
\frac{F_{\text{ult}}}{F_y} = \begin{cases} 
1.3 - 0.57\lambda' & 0.35 \leq \lambda' \leq 1.53 \\
1.1 & \lambda' < 0.35 \\
\frac{\pi^2 E}{(kl/r)^2} & \lambda' > 1.53 
\end{cases}
\]  

(3-6)

where \( F_{\text{ult}} \) is the ultimate stress, \( F_y' \) is the nominal yield stress, \( \lambda' \) is the normalized slenderness described in Eq. 1.1.

Figure 3.27 shows the buckling load obtained from Euler’s formula, the proposed criteria, and the regression formula [23]. It can be observed that as \( \lambda' \) is greater than 1.53, both the regression and predicted results converge to Euler’s solutions. Because Hall’s formula has already included the effect of imperfection such as out of straightness and residual stress, it can be inferred that the strength of the column is decided by the normalized slender ratio for a column with a larger normalized slenderness ratio. Other factors, such as out of straightness, residual stresses and imperfections do not play an important role in deciding the ultimate strength of columns.
For short columns, the ultimate stress is greater than the yield stress because of the effect of strain hardening. Such values for higher critical stress can be observed experimentally when special precautions are taken against buckling at the yield point stress. Timoshenko [3] considered these higher ultimate stresses have no practical significance in the design of columns, because small imperfections will make the columns fail at the yield stress. Hall suggested that the ultimate stress for perfect structures should be between 1 and 1.1 times the yield stress.

Because a small out-of-straightness was imposed for inducing the lateral displacement, the ultimate loads of these imperfect columns from the FEM analysis are slightly less than the yield loads. These results match Timoshenko’s prediction.

The results from the proposed method and regression method have larger deviations as the normalized slenderness varies between 0.6 and 1.53. The results from FEA have a higher strength than the strength obtained from the regression formula.
This is reasonable, because the ultimate strengths in the FEA models only consider the effect of yielding. However, as mentioned before, columns in the intermediate range of slenderness have strengths not only depending on yield stress, but also on initial straightness, residual stresses, and other mechanical properties. Usually, out of straightness and compressive residual stress will reduce the strength of columns. Therefore, the regression method provides a lower value of strength than the proposed method does.

Figure 3.28 exhibits the comparison of strength of columns with initial out of straightness of $\ell/1000$. The upper and lower bounds are from Bjorhovde [28] who varied in the magnitude of the initial out of straightness of columns to study the changes in strength. There are 112 columns with a maximum initial out of straightness of $\ell/500$, $\ell/2000$, and $\ell/1000$ in the data set. Only the data with initial crookedness of $\ell/1000$ is used in the comparison. Figure 3.28 shows the predicted strength located between the upper and lower bands. This agreement with experimental results proves that the proposed FEA method can make a reasonable prediction for columns with crookedness.

The buckling strength under eccentric loading is also used to verify the proposed criteria. The eccentricity ratio is defined as:

$$k = \frac{ec}{r^2} \tag{3-7}$$

Change Eq.3-7 with the following set of relationships:

$$k = \frac{ec}{r^2} = \frac{eA}{r^2A} = \frac{eAP}{I/cP} = \frac{\frac{mc}{I}}{\frac{P}{a}} = \frac{\sigma_m}{\sigma_o} \tag{3-8}$$

where "c" is the height from the neutral axis to the edge of the cross section, "A" is the area, "r" is the radius of moment inertia, "I" is moment inertia and equals $Ar^2$, "P" is the axial load, and "e" is the eccentricity of loading, $\sigma_m$ is the stress from bending and $\sigma_o$ is the stress from the axial load. Equation 3.8 indicates that the eccentricity ratio...
Figure 3.28: Comparison of the strength of columns with an initial deflection of $\ell/1000$ (upper and lower bond from Bjorhovda, 1972).

Figure 3.29: Comparison of the strength of columns with a load eccentricity of 1 (experiment data from Ros and Brunner, 1965).
actually is the ratio of stress caused by bending and by axial compressing. Bleich and Ramsey [53] made a series of test data on the I-column with eccentricities \(k=0, 1, 3\). Figure 3.29 revealing the comparison of column strength as columns are subjected to eccentricity loads \(k=1\). The proposed method got critical strength that is slightly higher than the strength obtained from experiments because the effects of out of straightness and residual stresses were not considered in the analysis. However, this comparison also proved that the proposed method could provide reasonable and reliable results.

### 3.4.2 Verification in Plate Buckling

Because there is no experimental data available for simply supported plates, the buckling loads of plates from the three proposed criteria are compared only with the results obtained from the theoretical solutions illustrated in Chapter 2.

Figure 3.30 demonstrates the comparison of buckling load from the proposed procedures and criteria with Euler’s solution of simply supported plates under uniform compression. The results from the first and second derivatives are found the same under this condition. Therefore, the mark “derivative” in Fig. 3.30 means the results obtained by the derivative of the load displacement curve, regardless of which order is used. It can be observed that the limit load criteria will get a higher buckling load than those obtained by the derivative method and Euler’s when post-buckling strength is presented. When \(\lambda'\) is small, the loads obtained from the limit load method and the derivative method are higher than the yield load. The extra strength comes from the effect of strain hardening.

Figure 3.31 exhibits the comparison of simply supported plates under uniform shear. When \(\lambda'\) is large, the derivative method can predict buckling more precisely than the limit load method dose because of the post-buckling strength effect. Instead of at \(\lambda'\) equal to 1 in previous cases, the predicted buckling load is consistent with theoretical values when \(\lambda' \geq 1.3\). This is because the Von-Mises equivalent stress will
reach yield stress when applied nominal shear forces are equal to \(1/\sqrt{3}\sigma_y\), which is the critical stress for plates with a \(\lambda'\) of 1.3.

![Diagram showing the comparison of buckling loads of simply supported plates under uniform compression.](image)

Figure 3.30: Comparison of buckling loads of simply supported plates under uniform compression.

When \(\lambda'\) is smaller than 1.3, both criteria obtain failure loads higher than \(1/\sqrt{3}\sigma_y\) because of strain hardening. The ultimate strength cannot exceed tensile strength because plastic fracture occurs at the load. For structural steel, the ratio of tensile strength divided by yield strength is about 1.5 (the AH-36 steel has the tensile strength of about 490-620 MPa, which is about 1.4 to 1.7 times the yield stress). Based on the ratio, the maximum value of applied shear stress divided by yield stress should be 0.86 \((1.5/\sqrt{3})\). If the 1.1\(\sigma_y\) criterion proposed by Hall is chosen, the maximum of applied shear stress divided by yield stress should be less than 0.64. From the consideration of failure analysis, the 0.84 criterion is the ultimate shear load. For design, a conservative criterion 0.64 should be considered as the maximum load.
Figure 3.31: Comparison of buckling loads of simply supported plates subjected to pure shears (Hall's criteria from [23]).

Figure 3.32: Comparison of the buckling loads of simply supported plates subjected to bending.
Figure 3.32 reveals the comparison of simply supported plates under the action of bending. The post buckling strength is also observed when the normalized slenderness is large. Therefore, the derivative criterion can predict buckling strength and limit load criterion may overestimate the strength because of the post buckling strength. Because there is no globe yielding in the plate when the maximum applied stress is a little more than the yield stress, the ultimate strength of the plate is higher than the yield strength when $\lambda'$ is close to 1. However, as in the discussion above, the ratio of maximum stress divided by yield stress should be less than 1.5 to avoid plastic fracture.

### 3.5 Strain Hardening

As stated in Section 3.4, the ultimate stress may be higher than the yield stress because of the strain hardening effect. Hu [18] mentioned that larger strain hardening could improve the convergence in FEM analysis. However, the suitable value for the coefficient was not discussed in his study. Because the value will affect the ultimate strength of structures, the effect of strain hardening in FEM modeling needs to be studied.

Figure 3.33 shows the change of ultimate load of columns with a normalized slenderness ratio of 0.4 and 1.8 when the slopes of yielded stress strain curve change from zero to one. A slope of zero means perfect plastic condition and one means that there will be no yielding, which is the assumption of the elastic buckling analysis. Figure 3.33 indicates that when columns are buckling at the elastic zone ($\lambda' =1.8$), the strain hardening coefficient would not change the results because the stress is less than the yield stress. When columns fail in the plastic range, the ultimate strength will change linearly with the assumed slope of strain hardening. The lower bound of normalized ultimate stress should be 1, which is the stress of materials without the strain hardening effect. The ultimate load in the plastic range can be describes as:

$$P_{cr(\text{plastic})} = \frac{E L \pi^2}{\ell^2}$$

(3-9)
where $E_i$ is the slope of yielding stress strain curve. This result is similar to Shanley's prediction stated in Galambos's book [2]. Stowell [61] used the plasticity reduction factor ($\eta$) to predict the plastic buckling load under different boundary conditions. For the columns, Eq. 3-9 can be rearranged as:

$$P_{cr(\text{plastic})} = \eta P_{cr(\text{elastic})} \quad (3-10)$$

Where $\eta = \frac{E_i}{E}$, and $P_{cr(\text{elastic})}$ is the elastic critical load. Because plastic buckling occurs when the critical load is larger than the yielding load, Eq (3.10) can be rearranged to:

$$\eta P_{cr(\text{elastic})} \geq P_y \quad \Rightarrow \quad \eta \geq \frac{P_y}{P_{cr(\text{elastic})}} = \lambda'^2 \quad (3-11)$$

Equation (3-11) indicated that the plastic buckling could be observed only when $\eta \geq \lambda'^2$. For a column with a $\lambda'$ of 0.4, the minimum tangent modulus for equation (3-9) is 0.16. Figure 3.33 also suggests that the columns failed at the yielding load when the tangent modulus was less than 0.16 of Young's modulus.

For most structural steel, the tangent modulus is below 5% of Young's Modulus. The AH-36 material has tensile strength 490-620 MPa (71-90 ksi) with a 22% elongation in a 2-inch gage [51,52]. Therefore, the assumed tangent modulus in the FEM model is about 1.25% of Young's modulus. For slender columns, the tangent modulus does not affect the elastic buckling strength. For steel stub columns, most of the failure load will be close to the yield load because the tangent modulus of most structural steel is not large enough to induce plastic buckling. Therefore, this assumption can represent most of the steel structures.

The behavior of plates is similar to columns except that plates have post-buckling strength when $\lambda'$ is large. Different strain hardening assumptions may obtain different ultimate strengths. Figure 3.34 presents the load and displacement curves of simply supported plates with a $\lambda'$ of 1.8. Three properties simulated in the analysis are the real stress-strain curve of AH-36, 1.25%, and 50% of Young's modulus strain hardening slopes. The real stress-strain curve in this model includes four parts: the linear elastic region, perfect plasticity region, strain hardening region and necking
region. Real stress-strain curve modeling is seldom used because more iteration is
needed in deciding stress and strain, even though it can present the structure’s
behaviors more closely. Because of the large deflection after buckling, the bending
stress at the plate increases rapidly until yielding occurs. After yielding, the model
which has a tangent modulus of 50% of Young’s modulus is much stiffer than the
models that model tangent modules with a real stress-strain curve or 1.25% of
Young’s modulus.

![Graph showing change of buckling load at different Young's modulus](image)

Figure 3.33: Change of buckling load at different Young’s modulus for
columns with the normalized slenderness ratios of 1.8 and 0.4.

On the other hand, because of the perfect plastic region modeled in the stress-
strain curve, the post-buckling behaviors will be similar for models that consider a real
stress-strain curve or a small tangent modulus. Fig. 3.34 shows that buckling occurs in
the same load. These three curves are close to each other until the load where yielding
occurs. The model with 50% of Young’s modulus slope has the stiffest behavior.
Therefore, this model gets the largest ultimate load. On the other hand, the curves of

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models with a real stress-strain curve and 1.25% Young's modulus slope have almost the same ultimate load. It means the 1.25% assumption can be applied to model materials, such as steel, that have near perfect plastic behavior after yielding.

![Figure 3.34: Post buckling behavior of plate subjected to uniform compression with different strain hardening coefficient (slenderness =1.8)](image)


For plates under uniform compression, the plasticity reduction factor depends on the tangent modulus and the secant modulus [61]. The secant modulus ($E_s$) is defined as stress divided by strain. When stress is lower than yield stress, the secant modulus is equal to Young's modulus. As the stress is higher than yield stress, the secant modulus is between Young's modulus and the tangent modulus. For square plates under compression, the plasticity reduction factor can be represented as [61]:

$$
\eta = \frac{E_s}{E} \left( 0.114 + 0.886 \frac{E_t}{E_s} \right)
$$

(3-12)
Rearrange and simplify Eq. 3-12, the following can be derived:

\[ \eta = \frac{E_t}{E} + 0.114 \frac{\varepsilon_0}{\varepsilon} \]  \hspace{1cm} (3-13)

where \( \varepsilon \) is the strain and \( \varepsilon_0 \) is the strain at yielding. Usually, \( \varepsilon_0 \) is much less than \( \varepsilon \), so Eq. 3-13 can be simplified to

\[ \eta \approx \frac{E_t}{E}, \]  \hspace{1cm} (3-14)

which is the same as the column's plastic reduction factor. In other words, plastic buckling would not be observed when tangent modulus is small. Therefore, assigning 1.25% of Young's modulus as the tangent modulus of steel is a reasonable assumption.
CHAPTER 4

MISALIGNMENT MODELING AND RESIDUAL STRESS CHARACTERIZATION

To study the misalignment effects on welded structures by a finite element method, two modelings must be determined in advance. These two modelings are applied to the simulation of misalignment joint and residual stress. In this chapter, some selected previous works reviewed in Chapter 2 were fitted into finite element models. By comparing results from these works and finite element analysis, appropriate modeling methods were selected to be applied in the study of misalignment effect.

4.1 Misaligned Joint Model

Misalignment is a common situation in the fabrication of welded structures, especially when plates with different thickness are welded together. Misalignment may induce a bending moment that acts with external loads or residual stresses. The moment may increase the displacement and reduce the ultimate strength of the structure.

There are different methods to simulate misaligned joints. The most direct way to model a misalignment joint is to model it as a real joint. This means the joint is modeled as its real profile, which connects two plates that were simulated by a solid-type element. Theoretically, this model could provide the most real information around welds. However, this method is tedious and is very difficult to model when joint configuration is complicated. Moreover, because the size of welds is very small when compared with other dimensions, it requires relatively fine mesh to model the
welds and surrounding areas. Besides that, for most structural members, thickness is relatively small as it is compared with length or width. To have a better aspect ratio, fine meshes are also needed. These mean that the number of elements and degrees of freedom will increase tremendously when several welds need to be considered in the analysis, which implies more computation time is needed for the results. Because of this disadvantage, this method is applied only when the behaviors of joints are of interest. It is difficult to apply this method to large complex structures.

If the fillet joint is not simulated, the number of elements may be reduced. However, the fine mesh near the joint is still needed to avoid poor aspect ratios. In addition to that, stress concentration and the reduction stiffness near the joint may induce error of the finite element analysis.

Hu [18] proposed two methods to model misalignment tube joints. The first method considered the joint as the extension of the tube that is modeled by the shell element. The joint strength can be changed by changing the properties, such as thickness and Young's modulus, at the joint. The problem with this model is that the elements may have a very poor aspect ratio in the joint when the misalignment is small. The bad aspect ratio may induce a premature divergence that will cause the wrong interpretation of the buckling strength because the divergence may be due to a numerical reason but not the buckling. The second method is to model the joint rigidly. Several constrain equations are used between the nodes located on different tubes. This method has been proved by comparing the FEM results with the experiments of tube structures in his study.

To find a simplified model that can provide acceptable results with less number of elements for plate structures, the buckling behaviors of two simplified models, solid element without fillet at the joint and shell element with joints connected rigidly, were compared with real joint model under the same loading condition. Figure 4.1 shows these three models which are listed below.

1. Solid element with fillet at the joint,
2. Solid element without fillet at the joint,
3. Shell element with joints connected rigidly.
A simply supported plate with a normalized slenderness ratio ($\lambda'$) of 1.8 and a misalignment of one half plate thickness ($t$) is modeled with the above three methods. A pair of uniform compressions is applied in opposite sides. The stress and displacement at points, which are located in a quarter, middle, and three-quarters of the plate's length, are recorded. These observed points are presented in Fig. 4.2. The plates were modeled to have dimensions of 1m x 1m x 0.012m. The aspect ratios of shell element were 1 with a length of 5 cm in each side. Two layers of elements were assumed in the solid type model which simulated the plate with the element size of 5 cm x 5 cm x 0.6 cm, which means that the worst aspect ratio is 8.6 in the model. As the fillet weld was modeled, a set of wedge-type elements was applied to model the joint. The welding leg size was assumed to be 6 mm, which means finer elements with a width of around 6 mm are needed in the joint. Therefore, the solid element with fillet at the joint model has the largest number of elements.
Figure 4.3 shows the load and displacement curves under three models at point D of Fig. 4.2. It can be learned that the three curves are close to each other. Furthermore, by comparing the displacements at the same load, it can be inferred that the shell model is the most rigid model and the no-fillet weld model is the weakest one. This is because the rigid joint model has the smallest displacement and the no-fillet weld model has the largest displacement at the same load.

Stress near the joint is different because of the limit of modeling. For a shell with a rigid link model, the largest stress occurs at the outset fiber of the plate but it is very small in the no-fillet weld model, because it is free at the outermost fiber. The stress from the model with fillet welds is between these two conditions. In spite of the differences near the joint, stress away from the joint should be similar in these three models. Figure 4.4 exhibits the Von-Mises equivalent stresses at the middle layer of point A. The fillet weld model has the highest stress while the joint without the fillet weld model has the lowest stress. The stress in the rigid joints is between the fillet and no-fillet welding conditions. From the comparison, it can be concluded that these three models have similar conclusions on the buckling behavior.
Figure 4.3: Comparison of displacement among these three models.

Figure 4.4: Comparison of stresses at point A of the middle layer of the plate.
It is obvious that the aspect ratio will be poorer near the joint if the misalignment is either more or less than 0.5t in solid modeling. This will cause convergent difficulty in non-linear analysis. The rigid joint model is free from the bad aspect ratio in the thickness direction of the base metal. It also can provide approximate results with a lesser degree of freedom and less complexity in modeling, therefore, the rigid joint method is selected to model misalignment joints in the following study.

4.2 Residual Stress Characterization

When a metal is heated uniformly or linearly, it expands uniformly and no residual stresses are produced. On the other hand, if it is not heated linearly, thermal stresses and strains may develop in the metal. Most welding processes confined heat input to base metal in a small area, therefore, the temperature distribution in weldments is not linear. This implies that the residual stress will be induced. The fundamental relationships for a plane stress - residual stress field are stated as follows [30].

Without considering strain from creeping strain and due to phase transformations, the total strain can be represented as:

\[ \varepsilon_x = \varepsilon'_x + \varepsilon''_x \]
\[ \varepsilon_y = \varepsilon'_y + \varepsilon''_y \]
\[ \gamma_{xy} = \gamma'_{xy} + \gamma''_{xy} \]  

(4-1)

where

\[ \varepsilon_x, \varepsilon_y, \gamma_{xy} \] are components of the total strain

\[ \varepsilon'_x, \varepsilon'_y, \gamma'_{xy} \] are components of the elastic strain

\[ \varepsilon''_x, \varepsilon''_y, \gamma''_{xy} \] are components of the inelastic strain which includes thermal and plastic strain.

The total strain must satisfy the condition of compatibility:

\[ \left[ \frac{\partial^2 \varepsilon'_x}{\partial y^2} + \frac{\partial^2 \varepsilon'_y}{\partial x^2} - \frac{\partial^2 \gamma'_{xy}}{\partial x \partial y} \right] + \left[ \frac{\partial^2 \varepsilon''_x}{\partial y^2} + \frac{\partial^2 \varepsilon''_y}{\partial x^2} - \frac{\partial^2 \gamma''_{xy}}{\partial x \partial y} \right] = 0 \]  

(4-2)
From Hook’s Law, 
\[ \varepsilon'_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) \]
\[ \varepsilon'_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) \] 
\[ \gamma'_y = \frac{1}{G} \tau_{xy} \] 
and equilibrium conditions:
\[ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \]
\[ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \] 
(4-4)

, it can be known the first term of Eq. (4-2) is equal to zero.

Let the incompatibility \( R \) be:
\[ R = - \left[ \frac{\partial^2 \varepsilon''_x}{\partial y^2} + \frac{\partial^2 \varepsilon''_y}{\partial x^2} - \frac{\partial^2 \gamma''_{xy}}{\partial x \partial y} \right] \] 
(4-5)

If \( R \) is not equal to zero, it means the first term of Equation (4-2) is not equal to zero either. In order to satisfy the equilibrium condition, in addition to the elastic stress, residual stresses need to be considered in Equations (4-3) and (4-4). To speak in an easier way to judge whether residual stress is induced or not, it can be stated as follows:

Residual stress exists when the incompatibility \( R \) is not equal to zero.

**4.2.1 Welding-Induced Inherent Shrinkage Strain**

By considering incompatibility as the source of residual stress, Yuan and Ueda [36] defined the strains from incompatibility in welding as inherent strain.

Mathematically, the inherent strain can be described by the following formula [63]:

For two-dimensional plain stain condition, the stress can be presented as:
\[ \nabla^2 (\sigma_i + \sigma_j) = -\frac{E}{1-\nu} \nabla^2 (\alpha \theta) - [g(x, y) + \Delta g(x, y)] \]  
\[ (4-6) \]

where \( \alpha \) is the thermal expansion coefficient and \( \theta \) is the temperature which is governed by the conduction equation.

\[ \nabla \cdot \nabla (\lambda \theta) = \rho C_p \frac{\partial \theta}{\partial t} \]  
\[ (4-7) \]

\( g(x, y) \) is defined by the cumulative plastic strain from the beginning of the thermal cycle and is presented as:

\[ g(x, y) = \frac{E}{1-\nu^2} \left( \frac{\partial^2 \varepsilon_i^p}{\partial y^2} + \frac{\partial^2 \varepsilon_i^p}{\partial x^2} - \frac{\partial^2 \gamma_{xy}^p}{\partial x \partial y} \right) - \frac{\nu E}{1-\nu^2} \nabla^2 (\varepsilon_i^p + \varepsilon_y^p) \]  
\[ (4-8) \]

\( \Delta g(x, y) \) is determined by the change of plastic strain at each iteration. It is shown as:

\[ \Delta g(x, y) = \frac{E}{1-\nu^2} \left( \frac{\partial^2 \Delta \varepsilon_i^p}{\partial y^2} + \frac{\partial^2 \Delta \varepsilon_i^p}{\partial x^2} - \frac{\partial^2 \Delta \gamma_{xy}^p}{\partial x \partial y} \right) - \frac{\nu E}{1-\nu^2} \nabla^2 (\Delta \varepsilon_i^p + \Delta \varepsilon_y^p) \]  
\[ (4-9) \]

At the end of the thermal cycle, temperature of the plate goes back to the initial temperature. The stress in the plate is the residual stress. Equation (4-6) becomes

\[ \nabla^2 (\sigma_i^R + \sigma_j^R) + g^R(x, y) = -g'(x, y) \]  
\[ (4-10) \]

where the superscript \( R \) means stress and strain from the residual stress and the superscript \( I \) stands for the inherent strain. \( g'(x, y) \) is the same as the total cumulative plastic strain, \( g(x, y) \). The strain caused by the residual stress can be presented as:

\[ \varepsilon_i^D - \varepsilon_i' = \frac{1}{E} [\sigma_i^R - \nu(\sigma_y^R + \sigma_z^R)] + \varepsilon_i^{PR} \]

\[ \varepsilon_y^D - \varepsilon_y' = \frac{1}{E} [\sigma_y^R - \nu(\sigma_z^R + \sigma_i^R)] + \varepsilon_y^{PR} \]

\[ \varepsilon_z^D - (\varepsilon_z' + \varepsilon_y') = \frac{1}{E} [\sigma_z^R - \nu(\sigma_i^R + \sigma_y^R)] - \varepsilon_z^{PR} - \varepsilon_y^{PR} = \text{Cons tan} t \]

\[ \gamma_{xy}^D - \gamma_{xy}' = \frac{2(1+\nu)}{E} \tau_{xy}^R + \gamma_{xy}^{PR} \]  
\[ (4-11) \]
4.2.2 Residual Stress Characterization by the Inherent Shrinkage Method

Arc welding is performed by moving the electrode from the beginning to the end. Therefore, an arc welding thermal analysis can be considered as a moving heat source problem with two steps in the base metal: heating when the electrode is approaching and cooling when the electrode is away. However, to perform a thermal analysis that considers all of the moving activity in a real complex structure is very difficult because of the requirement of tremendous computational time. Because the application of the moving source method to real complex structures is limited, a new numerical skill to solve the issues needs to be developed.

Equation (4-10) shows that the residual stresses are determined by the inherent strain $g^f(x,y)$. If an equivalent inherent strain distribution for heating and cooling can be simulated, it is not necessary to calculate the thermal condition induced by the motion of electrode, which will save considerable computational time.

Figure 4.5 shows the peak temperature and Fig. 4.6 presents the longitudinal plastic strain under two welding conditions from a moving source finite element analysis [64]. The results show that the changes of peak temperature and plastic strain along the welding direction are virtually can be ignored if the deviations at both ends are not considered. Similar conclusions were found in Yuda's [36,55] studies. The pattern of longitudinal inherent does not change along the welding direction and it may be approximated by a trapezoid. The factor $\frac{e\xi T_{av}}{\eta_y}$ affects the magnitude and width of inherent strains (the symbols were defined in Chapter 2).

Because $T_{av}$ is defined as $\frac{Q}{\rho c h B}$, it implies that the amount of heat input is the key factor to decide the residual stress after the material properties, dimensions of weldments had been determined. At high temperatures, the yield stress and Young's modulus are very small, the magnitude of residual stress is not significant until the weld metal cools below $700^\circ$ C [55]. Based on this phenomenon, the inherent strain method predicted the welding-induced residual stresses and distortion by letting the
Figure 4.5: Peak temperature distribution obtained by moving source method under different heat input. (Tsai [64], 1999).
Figure 4.6: Longitudinal plastic strain distribution obtained by moving source method under different heat input, (Tsai [64], 1999).

(a) heat input: 603 cal/cm, travel speed: 7mm/s

(b) heat input: 905 cal/cm, travel speed: 7mm/s
temperature at weld and heat affected zone drop a certain amount to produce a
shrinkage that is equivalent to the total strain.

The inherent strain is obtained by the following three steps.

1. Prescribe a high temperature to elements at welding zones as the initial
condition.

2. Calculate the temperature in the structural members when the high
temperature at the welding zone is gradually cooled to room temperature. The thermal
properties, such as thermal conductivity, specific heat, convective heat transfer
coefficient, and density are considered in the calculation.

3. Consider the temperature distribution during cooling as a thermal load. An
elastic-plastic finite element analysis, which considers the material properties, such as
Young’s modulus and yield stress to be temperature dependent, is preformed to
calculate the stress from the beginning of cooling to the end. The stress at room
temperature is the residual stress.

Deciding the initial temperature and the size of welding zone is the first
problem for applying the inherent strain method to predict residual stresses. To have
an equal amount of heat input, the equivalent weld size is decided from the
equilibrium of heat input:

\[ Q = T_w \rho C_h W_{eq} \]

\[ \Rightarrow W_{eq} = \frac{Q}{T_w \rho C_h} \]  \hspace{1cm} (4-12)

where \(T_w\) is the temperature of weld at the beginning of cooling. The heat input
\(Q\) is equal to \(\eta VI/v\) where \(\eta\) is the arc efficient, \(V\) is the voltage, \(I\) is the current, and \(v\)
is the speed of the moving electrode. The values of arc efficiency for different welding
processes have been shown in several documents, such as in welding handbooks. If the
welding parameters are known, the net heat input will be known. Therefore, the
equivalent weld size can be determined.
4.2.3 Residual Stress Characterization by the Inherent Conductive Shrinkage Method

The inherent strain method described above has two major problems. The first problem is that the weld is so small when compared with the other dimensions of the structure that the relative fine mesh is needed to model the weld. The second problem is associated with assigning an initial temperature at the weld. This is because in the beginning, the high temperature is limited to the welding zone only, and a ramp type temperature distribution is assumed for modeling the initial temperature distribution. To have the ramp function temperature distribution, a set of very fine interface elements is needed between weld and the base metal. This will increase the number of elements and degrees of freedom when several welds are considered. The increment in degrees of freedom will cause a lot of computation time when analyzing a real complex structure.

If welds are modeled as a 1D element, the fine mesh at the weld is no longer necessary. A 1D conducting element, which has the same equivalent weld size is applied for substituting the 2D element that simulates welds. The heat is transferred to the base metal by the conductive elements which have the same conductivity as that of the base metal. The simplified inherent strain method is presented by Fig. 4.7. Because conductive elements were applied in the model, it was named "inherent conductive shrinkage method".

![Figure 4.7: The ideas of "inherent conductive shrinkage method"](image)
Because the area of the cross section is the same, the simplified model and inherent strain model contain the same energy at the weld. The mechanical strength of weld for the simplified model can be considered in FEA by letting that the node at the weld and the corresponding node at the base metal have the same longitudinal displacement.

The inherent shrinkage method and the inherent conductive shrinkage methods were compared with Yuan and Ueda's [59] results. Two 1000x150x6 (mm) mild steel plates were welded by a CO\textsubscript{2} gas shield arc welding with a current of 180 A, voltage of 20 V, and welding speed at 3 mm/s. The material properties of the welds, heat affected zone, and base metal are shown in Table 4-1. Yuan and Ueda assumed the arc efficiency to be 1, which makes the net heat input $Q$ be 1200 (J/mm) which induced a compressive residual stress at the magnitude of 104 MPa.

The size of equivalent welding zone and the peak temperature at weld in the beginning of cooling need to be determined to build the model. A parameter study was conducted to investigate the influence of initial peak temperature at weld under the same heat input. Initial temperatures varying from 800° C to 2400° C were assigned to the weld to find the change of residual stresses. The equivalent weld size is determined by Equation 4.12 with different initial peak temperature at weld. Figure 4.8 shows the change of the magnitude of residual stress with different initial temperatures. Because yielding occurs in the weld, the residual stress induced by the weld is proportion to the size of weld, which decrease linearly as the temperature increase. On the other hand, the residual stress induced by the base metal increase as the peak temperature increases because of higher heat input. The total residual stress varies between 95 MPa and 110 Mpa when the initial peak temperature is between 1200° C and 2000° C. It implies a reasonable initial peak temperature should lie in this range. Because the predicted residual stress is closest to the experiment one when the initial peak temperature is 1600° C, it was chosen as the initial peak temperature in the followed analysis. Figure 4.9 shows the residual stress distribution at a quarter of these butt jointed plates calculated by the “inherent conductive shrinkage method”.
Figure 4.8: The change of the magnitude of residual stress with different initial peak temperature.

Figure 4.9: The predicted residual stress distribution by the inherent conductive shrinkage method.
<table>
<thead>
<tr>
<th>Property</th>
<th>70</th>
<th>400</th>
<th>700</th>
<th>710</th>
<th>1600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>70</td>
<td>400</td>
<td>700</td>
<td>710</td>
<td>1600</td>
</tr>
<tr>
<td>Young's Modulus (GPa)</td>
<td>200</td>
<td>185</td>
<td>150</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Yield Stress (Base metal, MPa)</td>
<td>330</td>
<td>330</td>
<td>46</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Yield Stress (Weld, MPa)</td>
<td>480</td>
<td>480</td>
<td>46</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.29</td>
<td>0.30</td>
<td>0.32</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Thermal Expansion Coefficient (10^-6/°C)</td>
<td>11.90</td>
<td>13.80</td>
<td>14.90</td>
<td>14.00</td>
<td>10.00</td>
</tr>
<tr>
<td>Conductivity (W/m·°k)</td>
<td>65.20</td>
<td>50.70</td>
<td>32.90</td>
<td>28.90</td>
<td>10.00</td>
</tr>
<tr>
<td>Specific Heat (J/Kg·°k)</td>
<td>450</td>
<td>590</td>
<td>900</td>
<td>900*</td>
<td>900*</td>
</tr>
</tbody>
</table>

Table 4.1: The material properties applied at Yuan and Ueda's analysis[59].

Applying the same initial pear temperature (1600°C) and equivalent weld size to the inherent shrinkage method. It has a fine mesh in the equivalent welding zone for a better aspect ratio. A set of very narrow intermediate elements is located between the weld and the base metal to make the temperature close to ramp function.

Figure 4.10: The predicted residual stress distribution by the inherent shrinkage method.
Figure 4.10 shows the residual stress distribution. The comparison of the total degrees of freedom and the calculated residual stress with the initial temperature of 1600° C is shown in Table 4.2. This comparison indicates that the method for deciding initial temperature and equivalent weld size predicted the residual stress accurately. It also shows that the simplified method can obtain similar results with fewer degrees of freedom. Because the inherent conducted shrinkage method can obtain a satisfied result by fewer degree of freedoms, it was selected to calculate residual stress distribution in a plate in the following study.

<table>
<thead>
<tr>
<th>Method</th>
<th>Element Number</th>
<th>Degrees of Freedom</th>
<th>Residual Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inherent Shrinkage</td>
<td>912</td>
<td>5314</td>
<td>103</td>
</tr>
<tr>
<td>Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conductive Shrinkage</td>
<td>301</td>
<td>1953</td>
<td>102</td>
</tr>
<tr>
<td>Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yuan and Ueda's</td>
<td></td>
<td></td>
<td>104</td>
</tr>
</tbody>
</table>

Table 4.2: The comparison at number of element and degrees of freedom between inherent shrinkage method and conductive shrinkage method with Yuan and Ueda’s analysis [59].

The inherent conductive shrinkage method was also varied by comparing with the results of a welded H-Shape from Hall [23]. The shell type element is applied to simulate the behavior of flanges and web. The 1D bar element is applied in the inherent conductive strain method to simulate welds. The cross section of the H-Shape is shown in Fig. 4.10, which presents the flange plates of 9x3/4” and a web plate of 9x1/2”. An equivalent weld area is also shown in figure to present how the heat is input. The material of the shape is A7 which has a laboratory measured yield stress of 33.5 ksi (≈ 234.2 MPa) at room temperature. The temperature-dependent properties
listed in Table 4.3 are similar to AH36 except that A7 has a lower yield stress. The measured residual stress is 15 ksi (≈ 105 MPa) at the center of the web.

![Equivalent weld](image)

Flange Plate -9x3/4"
Web Plate -9x1/2"
Yield Stress-33.5 ksi
Weld Size - 6 mm

![Figure 4.11: The cross section of the H-shape column.](image)

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>70</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus (GPa)</td>
<td>200</td>
<td>195</td>
<td>185</td>
<td>163</td>
<td>135</td>
</tr>
<tr>
<td>Tangent Modulus (GPa)</td>
<td>1.31</td>
<td>1.98</td>
<td>1.33</td>
<td>0.43</td>
<td>0.11</td>
</tr>
<tr>
<td>Yield Stress (MPa)</td>
<td>231</td>
<td>202</td>
<td>115</td>
<td>87</td>
<td>29</td>
</tr>
<tr>
<td>Poison’s Ratio</td>
<td>0.29</td>
<td>0.29</td>
<td>0.30</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>Thermal Expansion Coefficient (10^-6/°C)</td>
<td>11.90</td>
<td>12.60</td>
<td>13.80</td>
<td>14.70</td>
<td>14.00</td>
</tr>
<tr>
<td>Conductivity (W/m·°K)</td>
<td>65.20</td>
<td>55.50</td>
<td>46.00</td>
<td>36.90</td>
<td>28.90</td>
</tr>
<tr>
<td>Specific Heat (J/Kg·°K)</td>
<td>450</td>
<td>500</td>
<td>590</td>
<td>730*</td>
<td>900</td>
</tr>
</tbody>
</table>

*: Maximum occurs at 700°C with the value of 2500

Table 4.3: The material properties of A7 steel [51.52].

It was assumed that a gas metal arc welding (GMAW) was used to weld these plates. For plates with a thickness ¾", the minimum leg size for the fillet weld is ¼" [58]. The welding parameters were as follows [60]: current: 400 A, voltage: 27 V, and arc speed 7mm/s. The arc efficiency for GMAW was about 75%, so the net heat input
Q was about 1200 J/mm. Because fillet welds were applied at each side of the web, the total net heat input was 2400 J/mm. Yuan and Ueda [59] assumed that the heat could be divided into two parts: $Q_f$ conducted to the flange and $Q_w$ to the web, which are proportional to the thickness respectively. That is,

$$Q_f = \frac{2Qh_f}{(2h_r+h_w)}$$
$$Q_w = \frac{Qh_w}{(2h_r+h_w)}$$  \hspace{1cm} (4-7)

From (4-7), the net heat input was 300 J/mm to the web and 900 J/mm to the flange. By Eq. (4-6), the equivalent weld size at web and flange were equal to:

$$B_w = \frac{600}{1600\rho C h_w} = 6.72 \text{ (mm)}$$
$$B_f = \frac{1800}{1600\rho C h_f} = 13.42 \text{ (mm)}$$

where $\rho C$ was 0.0044 J/mm$^\circ$C for steel.

![Figure 4.12: The comparison of residual stress from FEA (inherent conductive method) and from measuring at the web of H-shape column (Hall,[23],1981).](image-url)
Figure 4.13: The comparison of residual stress from FEA (inherent conductive method) and from measuring at the flange of H-shape column (Hall,[23],1981).

Figure 4.12 presents the stress distribution at the web and Figure 4.13 shows the stress at the flange in the middle of the H-shape column. The maximum compressive residual stress is about 95 MPa (about 40% of yield stress) at the web. The difference from these analyses and measured stress is within 5% of the yield stress. The predicted stresses at the edge of the flange have a larger difference with the measured value (7%). These two figures indicate that the proposed inherent conductive shrinkage method for residual stress can obtain a similar stress distribution pattern with small errors.

4.3 Buckling Analysis with Residual Stress

The effects of residual stress on buckling behavior were studied by combining the buckling analysis procedure, buckling criterion, and the residual stress simulation process together. These three topics mentioned above were stated and verified in Chapters 1, 3, and 4. The procedure and criteria for obtaining buckling loads with
residual stresses are the same as those that do not consider the effects of residual stress. The only difference is that residual stresses need to be simulated first.

The inherent conduct shrinkage methods mentioned in this Chapter were applied to simulate the residual stress in the study. After the residual stress is obtained, a load is applied at the desired positions. Because the Arc Search method was not applicable in ANSYS when the residual stress is considered, the gradually increasing load and Newton-Raphson method were applied for finding the load and displacement curve. The criteria proposed in Chapter 3 were applied to decide the buckling load.

4.3.1 Verification by H-Shape Column

The buckling strength and residual stress distributions of the welded H-Shape column from Hall [23] were used to verify the buckling analysis procedure. The residual stresses have been obtained and compared with experimented data in section 4.2. The comparisons showed that the calculated magnitudes of the residual stresses were close to the experiment data by the inherent conduct method.

The length of the column in the finite element analysis was 5.26 meters, which gave the column a normalized slenderness ratio of 1. The column was pined constrained at one end and constrained in the lateral direction at another end, where a gradually increasing axial load was applied. The constrained points and load-applied points are located at the center of the web. Rigid beams were modeled at the ends of flanges and web to keep the column “plane remains plane” in the analysis. The rigid beam model also can help convergence because the rigid beam model can avoid numerical stress concentration and local deformation at points where loads and constraints are applied.

As in the procedure stated above, a load vs. displacement curve were recorded for determining the buckling load. Table 4.4 demonstrates the comparison of buckling loads from the finite element analysis which simulates residual stresses by the inherent conductive shrinkage method. The calculated buckling loads had a difference of less than 5% of the experiment data. The sum of the buckling and residual stress was found to be close to the yield stress for columns with a normalized slenderness ratio of 1.
These results also matched Masubuchi’s prediction [56] that predicted the buckling load under the influence of welding-induced residual stress. These comparisons imply that the proposed FEA procedure and buckling determination criteria can predict the buckling load precisely.

4.3.2 Verification by Simply Supported Plates

As mentioned in Chapter 2, Dwight et al. [33] derived the following theoretical buckling strength formula for simply supported square plates:

\[
\sigma_{cr}^l = \sigma_{cr} - \left[ (\sigma_r + \sigma_y) \cos \frac{\pi \sigma_r}{2(\sigma_r + \sigma_y)} - \sigma_y \right] \quad (2-29)
\]

where \( \sigma_{cr}^l \) is the reduced critical stress
\( \sigma_{cr} \) is the critical stress for an unwelded plate of the same b/t.
\( \sigma_r \) is the residual stress and \( \sigma_y \) is the yield stress of the material.

When \( \sigma_r \) is less than or equal to 0.2 \( \sigma_y \), Eq. (2-29) can be simplified to:
\[
\sigma_{cr}^l = \sigma_{cr} - \sigma_r \quad (2-30)
\]

When \( \lambda' \) is less than 1, Equations (2-29) and (2-30) predicted a critical load that is greater than yield strength, which is physically impossible in most situations. For

<table>
<thead>
<tr>
<th>Method</th>
<th>FEA</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual Stress</td>
<td>95</td>
<td>105</td>
</tr>
<tr>
<td>(Web Center, MPa)</td>
<td>(41%)</td>
<td>(45%)</td>
</tr>
<tr>
<td>Residual Stress</td>
<td>94</td>
<td>110</td>
</tr>
<tr>
<td>(Flange Edge, MPa)</td>
<td>(40%)</td>
<td>(47%)</td>
</tr>
<tr>
<td>Buckling Load</td>
<td>150</td>
<td>149</td>
</tr>
<tr>
<td>(kN)</td>
<td>(56%)</td>
<td>(55%)</td>
</tr>
</tbody>
</table>

Key: ( ): Percentage of Yield Stress or Strength

Table 4.4: The comparison of the results from the finite element analysis and experiment

When \( \lambda' \) is less than 1, Equations (2-29) and (2-30) predicted a critical load that is greater than yield strength, which is physically impossible in most situations. For
thick plates, the out of plane displacement becomes less when a small disturbance is applied in the direction normal to the plane. The bending moment from residual stress will be smaller because of the reduction in displacement for thicker plates. Therefore, the welding-induced compressive residual stresses have little effect on thick plates because of the reduction in the bending moment. Yamba [2] concluded the studies on the residual stress and proposed that the residual stress will not have any effect when the width to thickness ratio is less than 20. A similar conclusion was also stated by Masubuch [56].

A series of FEM buckling analyses were performed to compare with the predictions stated above. The simply supported square plates in the FEM simulation were assumed to have the dimensions of 1m by 1m. The plate is subjected to uniform compression with the unloaded edges welded. Dwight's [33] assumption, which assumed unloaded edges to remain straight when the compression load was applied, was modeled at these analyses. The load and boundary condition are shown in Fig. 4.14.

![Figure 4.14: The loads and boundary conditions for verification of example 2.](image)

The properties of these square plates at the welding zone were assumed to be the properties of AH-36 steel which are listed in Table 3.2. It was assumed that a gas metal arc welding (GMAW) was used to weld these plates. For plates with a normalized slenderness ratio of 1.8, the thickness is about 12 mm (½") when both the length and the width are one meter. The welding parameters for the butt joint were as follows [60]: current: 400 A, voltage: 28 V, and arc speed 6 mm/s. The arc efficiency for GMAW is about 75% that makes the total heat input be 1400 J/mm. For plates
welded at the edge, half of the heat input predicted for the butt joint can be assumed. This assumption is based on that heat should be shared by the two plates that were welded together. The equivalent weld size under the assumption was about 8 mm. Figure 4.15 shows the residual stress in the direction parallel to the direction of compression. The tensile stress at the weld is slightly greater than the yield stress. The almost uniform compressive stress is distributed in the unaffected zone.

![Residual Stress Diagram](image)

**Figure 4.15**: Residual stress at the direction of uniform compression of a simply supported plate.

A parameter study was conducted by repeating the analyses with changing the thickness to vary the normalized slenderness ratio from 1.8 to 0.4. It is concluded that the residual stress obtained from the method is almost independent of the change of thickness. This results from the assumption of the shell type element, which has a uniform temperature in the thickness direction. The stress within the equivalent welding zone is the yield stress. The stresses outside the welding zone need to balance the shrinkage force from the equivalent welding zones. When the width of the equivalent welding zone keeps the same, the residual stress inside the plate should be the same. The calculated compressive residual stress at the plate is about 0.11 yield stress.
stress for plates with different thicknesses. According to Dwight’s formula, this implies that the difference of buckling strength between the results which considers the effects of residual stress and which does not consider the effects will be around 11% of the yield stress.

After the residual stresses were calculated, the compressive stress was applied at the sides where no welding was performed. The compression was gradually increased until the solution diverged. The load and displacement curves were recorded to decide the buckling load and ultimate load by the criteria proposed in Chapter 3.

<table>
<thead>
<tr>
<th>( \lambda' )</th>
<th>FEA (No Residual)</th>
<th>Dwight and Yamba (Eq.2-29)</th>
<th>Dwight and Yamba (Eq.2-30)</th>
<th>FEA (With Residual)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>0.7</td>
<td>1.00</td>
<td>Not defined</td>
<td>Not defined</td>
<td>1.00</td>
</tr>
<tr>
<td>1</td>
<td>0.99</td>
<td>0.90</td>
<td>0.89</td>
<td>0.80</td>
</tr>
<tr>
<td>1.3</td>
<td>0.59</td>
<td>0.50</td>
<td>0.48</td>
<td>0.43</td>
</tr>
<tr>
<td>1.5</td>
<td>0.44</td>
<td>0.35</td>
<td>0.33</td>
<td>0.30</td>
</tr>
<tr>
<td>1.8</td>
<td>0.30</td>
<td>0.21</td>
<td>0.20</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 4.5: Comparison of Dwight’s and Yamba’s results [1,33] with FEA results when residual stress effects are considered

Table 4.5 states the comparison of the buckling load decided by FEA, and by the combination of Dwight’s, and Yamba’s proposal. The results, which do not consider the residual stress effect, were assumed to be the base values. Dwight’s results were obtained by Equations (2.29) and (2.30) as \( \lambda' > 1 \). Yamba’s result is applied to a plate with \( b/t \) less than 20. For \( \lambda' > 1 \), the results from the FEA considering the effect of residual stresses are slightly less than Dwight’s formula. The difference decreases as \( \lambda' \) increases. It means that the proposed FEA procedure and buckling criteria can predict the buckling behavior closely when \( \lambda' \) is greater than 1. For \( \lambda' \) is less than 1, the FEA analysis shows the residual stress does not have a significant effect on the buckling strength as \( \lambda' \) is less than 0.7. The \( b/t \) value for plates
with a $\lambda'$ of 0.7 is about 31, which is larger than Yamba's criteria. It implies that the proposed buckling analysis method can predict the buckling behaviors with a larger b/t ratio range.

From the two verifications, it is concluded that the inherent conductive shrinkage method can be used to simulate the residual stress distribution. The magnitude of residual stresses at the welds is in tension and close to the yield stress. The stresses between the welds are in compression and are parallel to the direction of the welds. The proposed buckling analysis procedures and buckling criteria can predict the buckling strength when the effect of residual stresses is considered.
CHAPTER 5

MISALIGNMENT AND RESIDUAL STRESS EFFECTS

After determining the buckling criteria, misalignment modeling, and residual stress simulation, the effects of misalignment at the welded structure can be investigated. In order to develop a complete understanding of the misalignment effect, the misalignment effect was studied by two parameters: normalized slenderness ratio and the magnitude of misalignment. In the following study, two conditions are examined: (1) considering and (2) not considering the residual stress effects.

5.1 Misalignment Effects

In this section, the misalignment effects were investigated and stated case by case by four basic models shown in Figure 1.4. These four cases are (1) the simply supported column under the axial load, and (2) simply supported plates subjected to in-plane uniform compression, (3) shear, and (4) bending. Based on the results from Chapter 3, the limit load criterion was applied to decide the buckling of structures without post-buckling strength. The derivative criterion was applied when the post-buckling strength was presented. The shell type element with rigid link model, proposed and verified in Chapter 4, was chosen for misalignment joint modeling.

Five misalignment conditions, which vary from 0.1 thickness to 0.9 thickness with a 0.2 thickness increment, were used to calculate the buckling load of the four basic models shown in Fig. 1.4. To study the effects of normalized slenderness, four normalized slenderness ratios, 0.4, 1.0, 1.3, 1.8, were applied to each basic model mentioned above.
5.1.1 Simply Supported Column under the Axial Load

The buckling behaviors of axial loaded columns under misalignment conditions were studied by the finite element model and the loads and boundary conditions shown in Fig. 5.1. The finite element model is similar to the simply supported model analyzed in Chapter 3 with the exception of a misalignment joint at the center of the column. As mentioned previously, the normalized slenderness ratios for analysis were 0.4, 1.0, 1.3, 1.8, and the misalignments were varied from 0.1 time height (t) to 0.9 time height of the column with a 0.2 time height increment.

![Figure 5.1: Finite element model for misaligned column analysis](image)

The same FEA procedure performed in Chapter 3 was applied to the model. Figure 5.2 shows the load-displacement curves at misalignment 0.1 t and 0.9t for columns with a normalized slenderness ratio of 1.8. The load-displacement curves for columns of slenderness 0.4 under these two misalignment conditions are shown in Figure 5.3. From these two figures, it can be acknowledged that post-buckling strength is not important. Therefore, the limited load criterion can be applied to determine buckling.

Figure 5.4 shows the change of buckling load under a different magnitude of misalignment and normalized slenderness ratio. Because there is no post-buckling strength for the column, the buckling load is equal to the ultimate load. From the figure, the following several points can be observed.

1. For columns with larger normalized slenderness, misalignment has no significant effects on deciding the ultimate stress.
Figure 5.2: Displacement vs. normalized applied stress at the center of a column under two misalignment conditions (slenderness = 1.8)

Figure 5.3: Displacement vs. normalized applied stress at the center of a column under two misalignment conditions (slenderness = 0.4)
2. For columns with a normalized slenderness less than 1, misalignment can reduce the buckling strength significantly.

3. For columns with an intermediate slenderness, such as 1.3, the strength of the columns does not change as the misalignment is small and the strength decreases as the misalignment is large.

Figure 5.4: Effects of slenderness and misalignment at buckling strength of columns.

The reduction of ultimate strength is due to the bending stress induced from misalignment. Figure 5.5 exhibits the stress at the column with a $\lambda'$ of 1.8 and misalignment of 0.9t. Unlike the stresses shown in Fig. 3.5, which are uniform until the load reaches critical load, the stresses in Fig. 5.5 are different from the beginning of applying load. The difference is due to the bending stress from misalignments. The bending stress interacts with the axial load to increase the stress at the compressed side. It makes the columns yield at a lower load than the load obtained when no misalignment is modeled. This phenomenon can be further illustrated by Fig. 5.6, where the compressed side of a column, which has a 0.4 normalized slenderness ratio
Figure 5.5: Normalized stress vs. normalized applied stress at the center of a column (slenderness = 1.8, misalignment = 0.9t, axial load).

Figure 5.6: Normalized stress vs. normalized applied stress at the center of a column (slenderness = 0.4, misalignment = 0.9t, axial load).
and a 0.9t misalignment, yields before the failure of the column. Misalignment effects in short columns reduce buckling strength by early yielding. Because of the premature yielding, the stiffness of the column is reduced, resulting in a lower ultimate load.

For columns with a larger slenderness ratio, the buckling critical stress is lower than the yield stress. Buckling may occur before yielding. Therefore, the influence of misalignment is not so important. When the slenderness ratio is small, the buckling critical stress is higher or close to the yield stress. Yielding happens before buckling. Therefore, misalignment cannot be ignored for columns with a small normalized slenderness ratio.

5.1.2 Simply Supported Plate under Uniform Compression

A simply supported plate that is subjected to uniform compression was analyzed at different magnitudes of misalignment and normalized slenderness ratios. The load and boundary conditions are described in Figure 5.7. In the finite element model, length as well as width of the plate is one meter, while the thickness changes with the normalized slenderness ratio. The normalized slenderness ratios and the magnitudes of misalignments chosen for analysis were the same as the case mentioned in the column analyses.

![Figure 5.7: The load and boundary conditions of a simply supported plate subjected to compression in one direction.](image-url)
The load and displacement curves were calculated by gradually increasing the compression load at the edge, which is the same as the procedure stated in Chapter 3. After these curves are obtained, buckling loads are determined by the derivative criterion and the ultimate load are decided either by limit load criterion.

Figure 5.8 describes the changes of buckling loads with different misalignment magnitudes and with different normalized slenderness ratios. Buckling load decreases as misalignment increases when the normalized slenderness ratio is equal to 0.4 and 1. When the ratio is equal to 1.3, the buckling load remains unchanged when the misalignment is small. Nonetheless, buckling load decreases after misalignment is more than 0.5 time of thickness. When normalized slenderness ratio is equal to 1.8, an increase of buckling strength is observed. The increment of buckling load is the most significant difference in the results obtained from column analysis.

Figure 5.8: Effects of normalized slenderness ratio and misalignment buckling loads (compression).

Figure 5.9 presents the changes in ultimate loads. Because post-buckling strength is not important for plates with a normalized slenderness ratio less or close to
one, the buckling load is equal to the ultimate load in these cases. Therefore, a consistent result with buckling load can be accomplished on the ultimate load when the normalized slenderness ratio is less than 1.3. Although an increment in the buckling load is observed, the ultimate load still decreases when the misalignment increases for plates with a normalized slenderness ratio of 1.8. The trend of the ultimate load for plates under uniform compression is similar to the behaviors of columns under an axial force.

For plates with a $\lambda'$ of 1.8, the increasing of the buckling load is due to the change of the failure mode. Figure 5.10 shows the load displacement curves at the center point and point A (defined at Fig. 4.2) when the misalignment is 0.1t. The lateral deflections of point A and the center point are in opposite directions as the applied load is small. When the load is large enough to induce an elastic buckling, the

![Figure 5.9: Effects of normalized slenderness ratio and misalignment at ultimate load (compression).](image)

For plates with a $\lambda'$ of 1.8, the increasing of the buckling load is due to the change of the failure mode. Figure 5.10 shows the load displacement curves at the center point and point A (defined at Fig. 4.2) when the misalignment is 0.1t. The lateral deflections of point A and the center point are in opposite directions as the applied load is small. When the load is large enough to induce an elastic buckling, the...
Figure 5.10: Displacement vs. axial load at point A and center (perturbation, slenderness = 1.8, misalignment 0.1t, compression).

Figure 5.11: Displacement vs. axial load at point A and center (perturbation, slenderness = 1.8, misalignment 0.9t, compression).
displacements go to the same direction. The failure mode is similar to the first buckling mode of simply supported perfect plates. On the other hand, when the misalignment is large enough, the bending moment is so dominant that it forces the plates to bend in a mode that is similar to the second buckling mode. Figure 5.11 shows directions of the lateral deflection of point A and the center point always stays in opposite directions. Finally, the misaligned plate fails in a mode that is similar to the second buckling load. Because no elastic buckling occurs when the misalignment is large, the observed buckling load increases, instead of decreasing, when the misalignment increases.

However, bending from misalignment does reduce the ultimate load carrying capacity. Figure 5.12 demonstrates the above statement for a plate with a $\lambda'$ of 1.8 under two magnitudes of misalignment: 0.1t and 0.9t. When misalignment is 0.1t, bifurcation type buckling happens first. After buckling, the plate resists the extra load by post buckling strength, until its ultimate load is reached. When misalignment is equal to 0.9t, the lateral displacement keeps increasing with load increment until the ultimate load where the total carried load decreases while the displacement continues to increase. Because no elastic buckling occurs when the misalignment is equal to 0.9t, the ultimate load is considered to be the buckling load and that makes higher buckling load for plates with a larger misalignment. However, because misalignment induces large bending moment, a plate with a larger misalignment has a smaller ultimate load. This phenomenon can be observed in Fig. 5.12.

When will the buckling load increase with the magnitude of misalignment? Figure 5.13 can provide an answer. The maximum Von-Mises stress of the plate reaches yield stress under elastic critical buckling load when the misalignment is 0.9 times the plate thickness. However, a plate with a misalignment of 0.1 times the thickness does not yield at critical load. This indicates that when the maximum stress at the critical load is less than the yield stress, elastic buckling can happen. When the stress is higher than the yield stress, no elastic buckling is induced, and plates will fail at the ultimate load, which is considered to be the buckling load. Therefore, an increment of buckling load can be observed for plates with a significant post-buckling
Figure 5.12: Load and displacement curves for uniform compressed plates with different misalignment. Key: \( t \): thickness

Figure 5.13: Max. Von-Mises stresses for plate with a \( \lambda' \) of 1.8 under compression. Key: \( t \): thickness
strength, i.e., for plates that have a normalized slenderness ratio at least larger than one.

The first buckling mode may be caused by applying perturbation that is too large, which produces a deflection in the first mode. Figure 5.14 describes the load and displacement relationship for the 0.1t misaligned simply supported plate when initial perturbation is not applied. In the beginning, the plate deforms in the second mode. This can be recognized by the displacement at the center, which is close to zero and much less than the displacement at point A. When applied, the load reaches the critical load of the plate, the plate buckles, and all the points deflect in one direction, which means the plate buckles at the first buckling mode. Figure 5.14 indicates that the applied perturbation will is small enough that it does not affect the final buckling analysis results.

![Figure 5.14: Displacement vs. axial load at point A and center (no perturbation, slenderness = 1.8, misalignment 0.1t, compression).](image)

Because the change of the ultimate load is similar to the changes of the column, the same conclusion can be reached, i.e., the misalignment is important for plates with a lower normalized slenderness ratio. For plates with a greater normalized
slenderness ratio, misalignment is important only when the magnitude of the misalignment is large.

5.1.3 Simply Supported Plates under Shear

The behaviors of simply supported plates subjected to uniform shear stress were studied with the same misalignment magnitudes and normalized slenderness ratios ($\lambda'$) discussed above. Two misalignment conditions were studied. The first situation has misalignment in the middle section of plate. The second case has a diagonal misalignment joint. The results of the study are described as following.

5.1.3.1 Misalignment at the Middle Section

The load and boundary conditions are presented in Fig. 5.15. The finite element model for the analysis is the same as the model mentioned in Section 5.1.2 except for the applied force changing from uniform compression to uniform shear along the edges of the plate.

Figure 5.15: The load and boundary conditions of a simply supported plate subjected to shear in edges and misalignment at the middle section.

Load versus displacement curves under different misalignment conditions and normalized slenderness ratios were recorded first. As Section 5.1.2, buckling loads are
decided by the derivative criterion and the ultimate load is determined by the limit load criterion.

Figure 5.16 shows the change in shear buckling load when the misalignment and slenderness are changed. Buckling loads are reduced when the misalignment increases under the most normalized slenderness ratio. However, when the normalized slenderness ratio is 1.8, the buckling load remains unchanged as the misalignment is small; and it becomes larger when the misalignment is larger than half of the plate thickness. The shift in buckling mode is similar to the uniform compression case discussed in section 5.1.2. Therefore, the same reason can be applied to explain the phenomenon. In short, elastic buckling occurs when the combined stress from shear and misalignment at the theoretical critical buckling load is less than the yield stress. When combined stress is larger than the yield stress at the critical load, the plate deforms in the same shape until the ultimate load is reached if more loads are applied. For plates with a large normalized slenderness ratio, the ultimate load is larger than the buckling load. Therefore, an increment of the buckling load can be observed for plates with a large normalized slenderness ratio.

![Figure 5.16: Effects of slenderness and misalignment at buckling load for plates subjected to shear force with misalignment at middle section.](image-url)
Figure 5.17 reveals the changes of the ultimate load. It presents the same conclusion as what was found for plates under uniform compression. The ultimate load decreases when the misalignment increases, regardless of whether the buckling load increases or not. Although misalignment does reduce the ultimate load, the reduction of strength is not as notable as what columns and uniform compressed plates have. This may due to the orientation of misalignment is not normal to the direction of compressive force.

![Graph showing the effects of slenderness and misalignment at ultimate load for plates subjected to shear force with misalignment at middle section.](image)

**Figure 5.17:** Effects of slenderness and misalignment at ultimate load for plates subjected to shear force with misalignment at middle section.

Stress on the plate is another focus. As discussed in Chapter 3, the bilinear strain hardening assumption allows stress over tensile stress, which is physically impossible. To correct the assumption, tensile strength should be checked for plates with a small normalized slenderness ratio. The 1.5 times yield stress, which is recommended in Chapter 3, was applied to determine the occurrence of plastic fracture. It was found that the stress for plates with a normalized slenderness of 0.4 could resist stress higher than tensile strength under this model. The "rupture" in Figs.
5.16 and 5.17 refers to the load where the tensile strength is reached for the plates with a $\lambda'$ of 0.4. The rupture load almost decreases linearly in Figs 5.16 and 5.17 as the misalignment increases. This implies that the stress of the misaligned plates with a small normalized slenderness ratio is dominated by bending which is proportional to the misalignment.

5.1.3.2 Misalignment at the Diagonal Direction

To study the effect of misalignment orientation, the misalignment was assumed to occur in the diagonal direction which is normal to the direction of compression. Figure 5.18 shows the misalignment, load, and boundary conditions.

Figure 5.18: The load and boundary conditions of a simply supported plate subjected to shear in edges and misalignment at the diagonal direction.

Figure 5.19 displays the change of buckling and Figure 5.20 demonstrates the changes of ultimate load as the misalignment is in diagonal direction. It could be found that the shift of buckling load also occurs when the normalized slenderness is large. The ultimate load decreases as the misalignment increases. The reduction is much significant than the reduction found when the misalignment is at the middle section of a plate. This result implies that besides the normalized slenderness ratio and magnitude of misalignment, the direction of misalignment also affect the buckling and ultimate strength of plates under the action of shear.
Figure 5.19: Effects of slenderness and misalignment at buckling load for plates subjected to shear force with misalignment at diagonal direction.

Figure 5.20: Effects of slenderness and misalignment at ultimate load for plates subjected to shear force with misalignment at diagonal direction.
5.1.4 Simply Supported Plates under Bending

The buckling behavior of plates with misaligned joints under bending is studied by a finite element model. Two misalignment conditions: (1) the misaligned joint is located in the neutral axis of the plate, and (2) the misaligned joint is at the direction normal to the applied force were studied. The plate is modeled as simply supported along the edges. Moment, as modeling in chapter 3, is simulated by applying two opposite loads which gradually vary from positive to negative with zero at the neutral axis. The results of the two loading conditions are stated as following.

5.1.4.1 Misalignment at the Neutral Axis

![Diagram of Simply Supported Plate with Misalignment at Neutral Axis](image)

Figure 5.21: The load and boundary conditions of a simply supported plate subjected to bending with misalignment at the neutral axis.

Buckling loads and ultimate loads under different misalignment conditions and normalized slenderness ratios were determined by the load versus displacement curves and criteria proposed in Chapter 3. Figures 5.22 and 5.23 present the change of buckling load and ultimate load. These curves are almost parallel to each other, which means that for simply supported plates subjected to the action of bending, misalignment at neutral axis has no effect on the buckling behaviors. The is because the misalignment is located in the neutral axis, no bending is induced by the external applied force. Therefore, misalignment has no effect on buckling when the misaligned joint is as shown in Fig. 5.21.
Figure 5.22: Effects of slenderness and misalignment at buckling load for plates subjected to bending with misalignment at the neutral axis.

Figure 5.23: Effects of slenderness and misalignment at ultimate load for plates subjected to bending with misalignment at the neutral axis.
5.1.4.2 Misalignment at the Direction Normal to the Applied force

Figure 5.24: The load and boundary conditions of a simply supported plate subjected to bending with misalignment normal to the direction of applied force.

Another misalignment condition under bending was considered to evaluate the effect of misalignment orientation. The same procedure used for previous analyses was applied to find the buckling load and ultimate load. The buckling load was determined by the derivative criterion and ultimate load was predicted by the limit load criterion from the load and displacement curves.

Figure 5.25 shows the change of buckling load and Figure 5.26 demonstrates the change of ultimate load. The similar conclusion as the uniform compression case was found. Buckling load is found increased as the misalignment increases when the normalized slenderness is large. The ultimate load decreases as the misalignment increases. The smaller normalized slenderness ratio, the larger reduction can be observed. Comparing the results with the results predicted when misalignment is in neutral axis, it can be concluded that the direction of misalignment could also affect the buckling behaviors of plate under bending.
Figure 5.25: Effects of slenderness and misalignment at buckling load for plates subjected to bending with misalignment at the direction normal to the applied force.

Figure 5.26: Effects of slenderness and misalignment at ultimate load for plates subjected to bending with misalignment at the direction normal to the applied force.
5.2 Residual Stress effects

The situation described in Dwight's model was limited to a web welded to the flanges. It cannot represent the ordinary welding condition in the structures. For a plate girder, web plates are made around the edges of the plate to connect the flanges, decks, and stiffeners. Usually, an extra weld is needed to connect the plates between the stiffeners. The more appropriate welding situation for the plate panel should include an all around weld in the edges and a butt joint in the center. The welding configuration is demonstrated by Fig. 5.27, where the welding is made along the edges and the centerline of the plate.

![Welding Model Diagram](image)

Figure 5.27: The welding model for the web in a plate girder.

To compare the buckling behaviors with the previous analysis which did not consider the residual stress effect, once again, the plate was assumed to be 1m by 1m with thickness change with a normalized slenderness ratio. The inherent conductive method was employed to simulate residual stress for studying the buckling behaviors under the influence of residual stress. The same process to decide the equivalent weld size proposed in Section 4.3.2 was applied at this simulation. As in the analysis performed in Chapter 4, the welded edges of the plate were assumed to behave as a line. This means that there is no local deformation around the edges. The purpose for the assumption is to put the effect of stiffeners into consideration. When deformation occurs, the edges should deflect gradually because of the strength from the stiffeners.
in a real situation. Therefore, this is a reasonable assumption. Besides that, the line-at-edge assumption also can help convergence, because local deformation at the edges was avoided.

Figures 5.28 and 5.29 show the residual stress distribution in the direction normal and along the center weld line. The residual stress is in tension and over yield stress around the welding zone. The residual stress between welds is in compression and almost uniform. The magnitude of compressive residual stress is about 0.11 times the yield stress.

![Figure 5.28: Residual stress at the direction normal to the center weld.](image)

After calculation of the residual stresses, loads were applied to find the buckling strength under the influence of residual stress. Three loading conditions, uniform compression, uniform shear, and pure bending, were considered. The thickness of plate was changed to have the plate with a normalized slenderness ratio of 0.4, 1, 1.3, and 1.8 for above three loading conditions.
Figure 5.29: Residual stress at the direction along the center weld.

Figure 5.30 shows the difference of buckling and ultimate loads for plates subjected to uniform compression with and without the effects of residual stress. It can be read that the buckling loads when considering the residual stresses are larger than those obtained without considering the residual stress effects. This is a big difference from Dwight's proposal, which stated that the buckling loads of plates with residual stress can be predicted by subtracting the ratio of $\frac{\sigma_{\text{res}}}{\sigma_y}$ from the original buckling loads when $\lambda'$ is greater than 1. These results are also different from Yamba's [2] conclusion that proposed the residual stress having no effect when the width to thickness ratio is less than 20.

The only difference between the simulated welding situation and Dwight's model is the three welds in another direction. These three welds produce tension at the edges where loads are applied. The tension around the weld reduced the compressive force, so the buckling strength was improved. Therefore, the panel plates which are
welded to have tensile residual stress along the edges of compression have better buckling strength.

Figure 5.30: Comparison of buckling loads and ultimate loads for simply supported plates subjected to uniform compression. Key: *R: consider residual stresses.

For a simply supported plate subjected to shear, Figure 5.31 shows both the buckling load and ultimate strength decrease when the effects of residual stresses are considered. The reason for reduction in strength can be explained by the distribution of residual stresses. Figure 5.32 shows residual stress vectors. In the welding zone, the stress is in tension that acts along the welding direction. Around the corners where two weld lines are joined, the residual stresses are in compression to balance the tension forces in the weld. On the other hand, the applied shear force along the edges can be considered to be a concentrated force acting on the corners. One pair of the forces is
Figure 5.31: Comparison of buckling loads and ultimate loads for simply supported plates subjected to uniform shear.

Key: *R: consider residual stresses.

Figure 5.32: The magnitudes and directions of residual stress.
tension and another pair of the forces is compression. These forces are also shown in Fig. 5.32 with the arrow symbols to indicate the directions. It can be inferred that the compressive residual stress will interact with the external compression. The compressive residual stress will reduce the stabilized effects from the external tension. Because the compression effect increases and the tension effect decreases, the strength of the plates will be reduced.

Figure 5.33 presents the comparisons of buckling loads and ultimate loads between the results obtained with and without considering the effects of residual stresses for simply supported plates under bending. The buckling load and the ultimate load increase for plates with a normalized slenderness ratio ($\lambda'$) of 1.8 when the residual stresses are considered. For $\lambda' = 0.4$ and 1, the strength is a little smaller when the residual stresses are considered.

As mentioned before, the residual tension around the weld will provide the extra strength. For plates with $\lambda' = 1.8$, the critical stress at the top of the plates is less
than the yield stress. After the superposition of the stress, the tensile zone still exists as the externally applied stress reaches the critical stress. The tensile zone at the top of the plate will provide the extra stabilization effect, which gives the plates greater ultimate or buckling strength than the strength obtained without considering the residual stress. For plates with \( \lambda' = 1 \) or 0.4, the critical stresses at the top of the plates are greater than the yield stress. After the superposition of the stress, the tensile zones do not exist as the externally applied stresses reach the critical stress. This means that the stabilized effect from the tensile zones is no longer available. On the other hand, the residual compressive stresses will interact with the external compressive stresses to reduce the strength of plates. This is the reason why the ultimate or buckling strength will be smaller as the residual stresses are considered for plates when \( \lambda' \) is less than 1.

From the results, it can be understood that the effect of residual stress changes with the loading conditions. It is difficult to say whether residual stress will increase or decrease the strength. Generally, if the welding-induced tension can reduce the applied compressive force or provide extra constraints, such as the case of uniform compression and bending, the strength will improve. If the residual stress is in the same direction of applied force, the strength will be dramatically reduced.

### 5.3 Combined Misalignment and Residual Stress Effects

The same normalized slenderness ratios and magnitudes of misalignment of plates discussed in Section 5.1.2 were repeated to analyze the buckling behaviors under the combined effect of residual stress and misalignment.

The analysis procedures are the same as the procedure that performs buckling analysis for plates by considering residual stress. The magnitude of misalignment is also varied from 0.1 times the thickness to 0.9 times the thickness. The misaligned joint is modeled by a rigid link, which is stated in Chapter 4. The residual stress distribution was simulated by the inherent conductive shrinkage method.
5.3.1 Simply Supported Plates under Uniform Compression

Figures 5.34 and 5.35 illustrate the comparisons of buckling and ultimate loads between the results obtained from considering and not considering the effects of residual stresses when plates are subject to uniform compression.

When misalignment is zero, as mentioned before, the buckling loads and ultimate loads with considering the residual stresses are greater than those obtained without considering the residual stress effects. The results from considering and not considering the residual stress effects are getting closer to each other when the misalignments are increased at the joints. This implies that the misalignment effect dominates the strength. The ultimate strength under the influence of residual stress is slightly less than the strength obtained without considering residual stress.

Figure 5.34: The comparison of buckling load for plates under uniform compression.
Key: *R: consider residual stresses
The shift of buckling, mentioned in Section 5.1.1, is found to occur at lower loads than the load when the residual stress is not considered. The reason why the residual stress is so dominated can be explained as follows: the misalignment and the compressive residual stress will induce a bending moment to deform the plate in the mode similar to the second mode. Figure 5.36 illustrates the mechanism. Because of the extra moment, the strength is reduced and the shift of buckling load occurs at a lower load.

Figure 5.36: The moment induced by the compressive residual stress at a misaligned joint.
The following conclusion can be reached from the above observation. The effects of residual stress are not important when misalignment happens for plates under uniform compression, especially when the misalignment is large. This conclusion can also be understood by Figures 5.34 and 5.35, that the buckling loads are close to each other when the misalignment increases.

5.3.2 Simply Supported Plates under Shear

Figure 5.37 shows the comparisons of shear buckling loads. Fig. 5.38 presents the comparisons of ultimate loads. These results also include two situations: with considering and without considering the effects of residual stresses.

![Diagram showing comparisons of buckling load for plates under shear.](image)

Figure 5.37: The comparison of buckling load for plates under shear. Key: *R: consider residual stresses

In these two figures, both the buckling load and ultimate strength decrease when the effects of residual stresses are considered. As mentioned before, the reason for reduction in strength can be explained by the distribution of residual stresses shown in Figure 5.32. The residual stress is in the same direction of applied load that
dramatically reduces the strength of plates under the load of shear because the compression effect increases while the tension effect decreases.

![Graph showing the comparison of ultimate load for plates under shear](image)

**Figure 5.38**: The comparison of ultimate load for plates under shear.

Key: *R: consider residual stresses

The reduction of ultimate strength is more severe when \( \lambda' \) is small. The calculated ultimate load under the consideration of residual stress is close to the yielding load. Yielded structures cannot resist too much disturbance. The residual stress is a kind of disturbance that causes plates to fail without noticing any strain hardening effect, which is observed when residual stress is not modeled in the analysis under the shear load. Therefore, the maximum load carrying capacity for plates under shear and residual stress is the yielding load.

The ultimate strength of plates just slightly decreases as the misalignment increases when residual stress is considered. This means that the effects of misalignment can be ignored as the effects of residual stress are considered.
5.3.3 Simply Supported Plates under Bending

Figure 5.39 and 5.40 present the comparisons of buckling loads and ultimate loads between the results obtained with and without considering the effects of residual stresses. The strength varies slightly as the misalignments change. This illustrates that the misalignment at the neutral axis of plates is not important for plates under the action of bending. This is the same conclusion as what is found when the residual stress is not considered. Under a different magnitude of misalignment, the buckling load or ultimate load increases for plates with a normalized slenderness ratio \((\lambda')\) of 1.8 when the residual stresses are considered. For \(\lambda' = 0.4\) and 1, the strength is a little smaller when the residual stresses are considered. This change is due to the tensile residual stress at the weld discussed above.

![Diagram](image)

Figure 5.39: The comparison of buckling load for plates under bending. Key: *R: consider residual stresses
From the above statement, the following conclusions for plates subjected to bending can be made.

1. The residual stress will increase the strength as the normalized slenderness ratio is greater than 1 for simply supported plates subjected to bending.

2. The residual stress will slightly decrease the strength as the normalized slenderness is less than 1 for simply supported plates subjected to bending.

3. The effect of misalignment can be ignored whether or not the effect of residual stress is considered for simply supported plates subjected to bending.
In this chapter, a welded girder failure case is studied using the numerical procedures discussed in the previous chapters. This will demonstrate the applicability of this newly developed knowledge and numerical technique to a real-world industrial case.

The FSS T-AKR 287 class ramp is a portable ramp designed to provide the roll on/off function to two M1A1 tanks at the same time. During the proof test, the ramp capsized at the following test load:

- 32 long ton (LT) ramp self weight;
- 95 LT centered 5 ft from the upper end of the ramp and 8 ft off center;
74 LT centered 32 ft from the upper end of the ramp and 8 ft off center. This loading condition is shown in Figure 6.1.

Metallurgical evaluations on the fracture samples cut from the failed ramp found a series of misalignment conditions between the girder web plates on top of and beneath the deck in the end panel near the upper support. A hypothesis suggesting that the shear buckling be trigged by the misalignments instead of design insufficiency was proposed. Whether the shear buckling is due to the design insufficiency or due to the misalignment cannot be determined without further analysis. It became an argument between the designer and the manufacturer. However, the failed ramp will be a good example for applying the knowledge from previous findings to real failure analysis.

6.1 Finite Element Model

The finite element analyses include the following two parts:

1. Find the reaction forces at the supports.
2. Find the buckling and ultimate load of the critical panel.

Buckling was deemed to occur in the girder section if the reaction force at its neighboring support became greater than the buckling load determined from a buckling analysis.

The whole ramp model was used to decide the reaction force at the support when the ramp was subjected to the failure load during the test. Shell type elements were used for side girders and deck plates. Beam type elements were used for the cross beams and the longitudinal deck stringers. In the critical upper end panel of the outboard girder, refined meshes were used for more calculation accuracy (Fig. 6.2). The transitional displacement at the pin supports in the saddle was restrained in all directions (i.e., vertical, lateral, and longitudinal) for the inboard girder and in the vertical and longitudinal directions for the outboard girders. All the pins were free to rotate. The ramp was tilted by ten degrees from the surfaces. Because the detail of real stress distribution is not the focus for this step, the linear analysis was performed first to decide the reaction force at the supports. The reaction force at the supports does not change much if only locally yielding happens. Therefore, the linear analysis is a
Figure 6.2: The finite element model of ramp.
(GGS, MSC plan no. T-AKR 287-100-6632091)

Figure 6.3: Loads and boundary conditions for the panel buckling analysis.
reasonable approach with a lot of saving in CPU time. The reaction force at the support of failure side is about 760 kN.

Because of the complexity, it is very difficult to perform buckling analysis without making some simplifications and assumptions. The finite element model that represented the upper end section of the ramp was isolated from the ramp structure for the buckling analysis. The appropriate boundary conditions were imposed along the cut boundaries to represent the various constraints from the neighboring structural members. Figure 6.3 shows the boundary conditions of the isolated panel. The boundary of the panel at the first crossbeam from the upper end of the ramp was completely restrained from the translations and rotations. The transactional displacements along the boundary of the deck plates were restrained in the lateral direction but free of rotation. The lower vertical flange plate at the end of the girder was restrained from lateral translation due to the crossbeam support. The boundary conditions represent the stronger constraint conditions in the upper end panel of the outboard girder. The buckling strength determined from this isolated model will be the upper limit of buckling strength of the actual girder.

The buckling analysis proposed in the study was applied to find the buckling load of the panel. A continuous incremental load was applied at the support until the limit load was reached or the solution became divergent. The non-linear analyses, including the large strain, large rotation, and stress stiffening effects in the calculation were performed later. The arc search method was applied to allow the observation of the post-buckling behavior. A small perturbation in the style of the first modes is applied in the upper panel of the web. The mechanical properties of the structures are AH-36, which are listed in Table 3.2. A bi-linear stress strain relationship was assumed for the non-linear analysis. The yield strength of the steel used in the analysis is 350 MPa, the Young's Modulus is 200 GPa, and the Tangent Modulus is 1300 MPa. A refined element mesh was used in the buckled web plate to avoid buckling failure predictions from the numerical non-convergence.
Figure 6.4: Displacement vs. applied force at point A of the web. (misalignment = 0)

Figure 6.5: The first and second derivatives of the load and displacement curve (misalignment = 0; P: applied force).
Figure 6.6: The change of stress at the point A (misalignment = 0).

Figure 6.4 represents the load and displacement curve at the points where the large lateral displacement occurs. The buckling behavior is more similar to the load and deflection type buckling, because the lateral displacement is induced from the beginning of the load application. The buckling load is the limit load of the load and displacement curve that reaches its maximum at about 688 kN. This also means that the buckling load is the ultimate load. Figure 6.5 shows the first and second derivatives curves which reach the maximums at the limit load. This proves that the derivative method can predict the buckling load and limit load criterion can predicted the ultimate load. Figure 6.6 demonstrates the way the stresses increase dramatically after the limit load. This implies that the structures may become unstable after the buckling. Since the buckling load (688 kN) is smaller than the reaction force 760 (kN), it implies the shear buckling may be caused by the design insufficiency.

A logical scenario for the post-buckling behavior of the end panel may be realized by a series of chain effects. The upper panel was able to resist load with the
effect of tension field only after buckling. Because of the loss of rigidity at the upper panel, the entire reaction force was transferred into the lower panel and caused the rupture in the lower flanges at the cross section of sharper depth transition. At this instant, the whole panel began to lose its structural stability and became twisted. Finally, the ramp capsized. Figure 6.7 shows the magnitude and direction of principal stresses at the limit loads. The compressive stresses are acting in the diagonal direction of the web to cause the web to have a shear type buckling.

Figure 6.7 The magnitudes and directions of minimum principal stress.

Figure 6.8 indicates the Von-Mises equivalent stress at the panel after the load reaches the panel’s limit value. The stresses in the lower flanges at the cross section of sharper depth transition are close to the tensile strength, so the assumption of failure sequence is correct. Buckling at the upper panel caused the fracture of the lower flange and then the capsizing of the entire ramp. Figure 6.9 shows the high stress zone in the diagonal direction in the upper panel. The tension field action occurs in the upper panel. Figure 6.10 shows the out of plane displacement of the webs when a shear buckling occurs in the upper panel. Comparing Fig 6.10 with Fig. 1.1, the deformation
is very similar. It can be concluded that the FEM analyses can get very accurate results and that the prediction that "the failure may be caused by the design insufficiency" is highly possible.

The effects of the initial perturbation were also investigated. A second mode of perturbation was applied to the model with two small forces in opposite directions at the upper and lower panels of the web. Figure 6.11 shows the comparison of load and displacement at point A under first and second mode perturbation. Although the displacement is different, the ultimate load is just 3 kN different from the results.

![Figure 6.8: The distribution of maximum principal stress.](image)

obtained from the first buckling mode perturbation. Therefore, it would be safe to conclude that the small initial distribution would not change the buckling behavior of the ramp.
Figure 6.9: The Von-Mises equivalent stress and tension field at the web.

Figure 6.10: The out of plane displacement at the web.
Figure 6.11: The comparison of first mode and second mode perturbation.

6.2 Misalignment Effects

The rigid joint models mentioned in Chapter 4 along with the buckling analysis procedure and criteria proposed in Chapter 3 were used to study the buckling behavior under the influence of misalignments. Figure 6.12 shows the location and configuration of misalignment joints.

Two misalignment conditions, 3.5 mm and 8 mm, were investigated by FEM analysis under the same boundary conditions and the same procedures mentioned in Section 6.1. Figure 6.13 represents the buckling behavior of the panel with 3.5 mm and 8 mm misalignment. This indicates that they are all load and deflection type buckling. The buckling load is the maximum load achieved on these load-displacement curves. These loads of these panels are about 675-680 kN, which is close to the buckling load with no misalignment. These results show that the misalignment conditions do not play an important role in the buckling of the panel.
Figure 6.12: The finite element model for buckling analysis of misaligned end panels.

Figure 6.13: The comparison of load and displacement curve under
6.3. Residual Stress Effects

The residual stresses were simulated by the inherent conductive method mentioned in Chapter 4. The property changes of AH36 steel at different temperatures (Table 3.2) are assigned to the model. Like in previous analyses, the non-linear FEM analysis considers the large strain, large rotation, and stress stiffening effect in the calculation.

The thickness for the failed upper web and deck is 7 mm, and for the lower web, 15 mm, which requires a minimum 6 mm fillet leg size [58]. It is assumed that a gas metal arc welding process was used to join these plates. The welding parameters were as follows [60]: current: 430 (A), voltage: 30 V and arc speed: 12.7 mm/sec. The arc efficient for GMAW was about 75%, so the net heat input $Q$ was about 762 J/mm at each side of the deck. Considering the net heat for each plate is proportional to the thickness, the total heat for the upper web was as follows:

$$Q_{uw} = 2 \times Q_{h_{uw}} / (h_{uw} + h_{lw} + h_{d}) = 2 \times 762 \times 7 / (7 + 15 + 7) = 368 \text{ (J/mm)}$$

where “uw” represents the upper web, “lw” represents the lower web, “d” stands for deck, and “h” is the thickness. Similarly, the heat flowing to the deck was 368 (J/mm) and to the lower web was 788 (J/mm).

The 1600°C melting temperature was assumed as stated in Chapter 4. The equivalent weld size for the upper web and deck were:

$$B_{uw} = B_d = 368 / 1600 \rho C_{uw} = 7.46 \text{ (mm)}$$

For the lower web was:

$$B_{lw} = 788 / 1600 \rho C_{lw} = 7.46 \text{ (mm)}$$

where $\rho C$ was 0.0044 J/mm³ °C.

The thermal analysis was performed first, then the mechanical analysis was performed by considering the temperature as the thermal load. Figure 6.14 and 6.15 show the residual stress distribution in the longitudinal and vertical directions. These values suggest that the tension zone is around the welds and the residual stress inside the panel is in compression.
Figure 6.14: The longitudinal residual stress (misalignment = 8 mm).

Figure 6.15: The vertical residual stress (misalignment = 8 mm).
Figure 6.16 reveals the load and displacement curves for the panel with different misalignment conditions with the consideration of residual stress. The ultimate load is about 635 kN, which is slightly less than the buckling load when the residual stress effect is not considered. Figure 6.16 also shows that the misalignment effects are not important as the residual stress is considered.

6.4 Discussion

Because shear force is the most dominated factor in the end panel, the plate is assumed to be subjected to shear at the edges, which results in an equivalent compressive force in the longer diagonal direction.

From the study on Chapter 5, it can be conclude that misalignment effects depend on the normalized slenderness ratio, magnitude of misalignment and the orientation of misalignment. From Figure 6.7, it can be found that the force flows through from the bottom panel to the upper panel at an angle of about 45° relative to
the direction of misalignment. This is a condition similar to the case discussed in Section 5.1.3.1. Because of the orientation, just a little reduction in buckling and ultimate strength is expected as misalignment increases.

The residual stress caused compression in the diagonal direction of the upper panel. This direction is the same as the resultant compressive direction when the load is applied. Therefore, the buckling load under the influence of residual stress may reduce.

As the misalignment and residual stress considered together, residual stress effect is more important than misalignment. Therefore, misalignment does not change the buckling strength but a smaller buckling load is possible because of the residual stress.

Table 6.1 shows the ultimate loads of the ramp under different misalignment conditions. When the residual stress is not considered, the ultimate loads are around 675 to 688 (kN), with only a 2% difference. This means that the misalignment effects are not important. However, the residual stress reduces the ultimate strength. The results from the analysis matches the conclusion found in Chapter 5.

Because misalignment is not important, the misalignment model is not necessary in the FEA. Therefore, omitting the modeling of the misalignment joint will make the modeling of the ramp for FEA easier. However, because residual stress reduces the buckling strength, a FEA with the consideration of residual stress effects is necessary for shearing buckling analysis at the end panel.

<table>
<thead>
<tr>
<th>Misalignment (mm)</th>
<th>Without Residual Stress (kN)</th>
<th>With Residual Stress (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>688</td>
<td>635</td>
</tr>
<tr>
<td>3.5</td>
<td>673</td>
<td>635</td>
</tr>
<tr>
<td>8.0</td>
<td>675</td>
<td>635</td>
</tr>
</tbody>
</table>

Table 6.1: Comparison of failure loads under different misalignment.
CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations could be reached through the study.

7.1 Misalignment and Residual Stress Effects

The effect of misalignment is decided by the normalized slenderness ratio, along with the magnitude of misalignment, orientation, and residual stress.

Structures that have smaller normalized slenderness ratios are more sensitive to misalignment. For axial load columns, and simply supported plates subjected to uniform compression, the ultimate load reduces significantly at misalignment conditions when the normalized slenderness ratio is 0.4. As the slenderness ratio increases to 1.8, only a few reductions on ultimate load can be observed.

The magnitude of misalignment affects the buckling strength. Misalignment may increase the buckling strength by changing the failure modes for plates, which have higher normalized slenderness ratio, from the first buckling mode to the second mode when the magnitude of misalignment is large. Unlike the buckling loads, the ultimate loads keep reduction when the misalignment increases.

The reduction of ultimate strength also depends on the orientation of misalignment. When the misalignment in the direction normal to the external applied force, the misalignment effects are more significant than the situation that the misaligned joint is in the direction parallel to the applied force. The axial loaded columns and uniform compressed plates have the most significant misalignment.
effects on the ultimate loads. For plates under the action of shear with misalignment at the middle section and bending with misalignment in the neutral axis, the misalignment effects are not significant.

The effects of residual stress depend on the pattern of external loads. The residual stresses do not necessarily cause the reduction of buckling strength and ultimate strength. When residual stresses are considered, the strength will decrease for plates subjected to shear. On the other hand, the strength is increased when uniform compression is applied at a small misalignment situation. For plates under bending, the residual stress may increase or decrease the buckling strength, depending on the normalized slenderness ratio.

As the residual stress and misalignment are considered together, the misalignment effect is important when the plate is subjected to uniform compression. When shear or bending buckling is considered, instead of misalignment, the effects from the residual stresses are significant.

7.2 FEA Procedure for Buckling Analysis

A linear buckling analysis is always recommended to find the normalized slenderness ratio. The normalized slenderness ratio of any geometry can be obtained from a linear finite element buckling analysis. If the normalized slenderness is larger than 1 (or 1.3 for shear), the compressed structural member allows more tolerance to misalignment. Conducting a linear analysis rather than a non-linear one may reduce a lot the computation time.

When non-linear buckling analysis is conducted, the buckling load needs to be derived from the load and displacement curve. The applied load should increase gradually until its peak, and then decrease. The analysis should be elastic-plastic and considering the geometric non-linearity to account for the large deformation after buckling. The Von-Mises equivalent stress should be used to decide yielding in the calculation.

When the compressed members are subjected to uniform compression, thermal residual stress analysis may be omitted, because the effect of residual stress is
not so dominant in a misalignment situation. On the other hand, the misalignment joint
may not be modeled for shear and bending buckling analysis, in which case the
misalignment effect is not significant. With this knowledge, the buckling design and
analysis will be easier and will save more time.

7.3 The Numerical Buckling Criteria

For elastic buckling, the eigenvalue from linear analysis is the buckling load.
On the other hand, the numerical buckling criteria can be applied in the imperfect or
inelastic condition

The derivative method is recommended to determine the buckling load. The
limit load method is recommended for determining the ultimate load of plates. The
first derivative criterion and the second derivative criterion will get the same answer of
buckling loads in all cases. It does not matter which derivative is used to derive the
buckling loads.

7.4 Misalignment Model

Misaligned joints could be modeled by either solid, shell or beam type
elements. Theoretically, solid type element can provide the most precise information
around the joint at the cost of more complexity in modeling and much more time in
computing.

In this study, a simplified model that connects two misalignment plates with a
rigid link is recommended for the modeling. This method has been proved by
comparing the FEA results with the experiments of tube structures in Hu’s [18] study.
It also has been compared with the buckling behavior with a solid model with and
without the fillet joint in Chapter 4 of this dissertation. The comparison proved that the
rigid joint model can provide acceptable results with less number of elements for plate
structures.
7.5 Procedure for Residual Stress Characterization

To reduce the total degrees of freedom of the finite element model, the inherent conductive method, which employed a 1-D conductive type element to simulate welds, was proposed and verified in this study.

The analysis of this method consists of the following three steps:

1. Prescribe a high temperature to elements at welding zones as the initial condition. In the study, an initial temperature of 1600°C was recommended. The equivalent weld size was decided by the net heat input.

2. Calculate the temperature in the structural members when the high temperature at the welding zone is gradually cooled to room temperature. The thermal properties, such as thermal conductivity, specific heat, convective heat transfer coefficient, and density are considered in the calculation.

3. Consider the temperature distribution during cooling as a thermal load. An elastic-plastic finite element analysis, which considers the material properties, such as Young’s modulus and yield stress to be temperature dependent, is performed to calculate the stress from the beginning of cooling to the end. The stress at room temperature is the residual stress.

The residual stresses obtained from the “inherent conductive shrinkage” method were compared with Yuan and Ueda’s [59] results for a butt joint between two plates. The proposed method also compared with the results of a welded H-Shape from Hall [23]. The comparison shows that the proposed method can predict residual stress with satisfied accuracy by a much less degree of freedom.

7.6 Recommendations for Further Study

Although this study has focused on many issues, there still remain some limitations. The following recommendations are made for further research to make the study more solid and complete.
1. Design optimization: The key to avoid buckling is to constrain the compressed parts of the structures. For example, if a diagonal stiffener was added at the compressed direction of the plate girder's end panel, the shear buckling strength will improve a lot. However, adding more stiffeners will increase the weight and the cost of fabrication. Therefore, how to design a strong structure with less cost and weight will be an interested topic.

2. Fabrication optimization: The study shows that the residual tensile strength can help to stabilize structure. On the other hand, residual tensile stress is known can reduce the fatigue life and fracture resistance. How to have a optimized fabrication to account for all the factors is another important issue for the future study.
REFERENCES


64. Chen and Z. Feng, Modeling of Welding-Induced Plasticity for Distortion Prediction, presented at IIW-X/XV-RSDP Working Group Meeting, Paris, France, March 1999
IMAGE EVALUATION
TEST TARGET (QA-3)

1.0
1.1
1.25
1.4
1.6

1.0
1.1
1.25
1.4
1.6

150mm
6"

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