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NEW MODELING AND CONTROL DESIGN TECHNIQUES FOR AIRCRAFT STRUCTURAL DYNAMICS USING SMART MATERIALS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
The Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Seung-Keon Kwak, M.S.

*****

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ABSTRACT

In this research, new modeling and control techniques for aircraft structural systems are presented. The sensors and actuators used in this study are smart materials. The smart structural system, which is under external aerodynamic load, is modeled utilizing an integrated finite element method. The resulting model, presented in generalized coordinates, has a mass matrix, a non-symmetric aerodynamic damping matrix and a non-symmetric stiffness matrix (due to aerodynamic stiffness). Next the system is then transformed to a real but non-orthogonal modal coordinates, and a reduced order model is developed.

The control design consists of a flight mechanic control, a vibration control, and a linear matrix second order observer design. The flight mechanic control problem is to achieve a roll maneuver with a desired roll rate using a laminated flexible wing actuated by piezoelectric actuators. The equation of angular roll motion with nodal voltage actuation is derived from the finite element model. A new control design algorithm based on the ‘Reciprocal State Space’ framework is employed to achieve the desired roll rate. The deformed structure that obtains the specific roll rate is made up of new mass and stiffness matrices which are functions of the steady-state input voltage of the roll maneuver. For implementation, a laminated active twisting plate and a cantilevered beam
with piezoelectric actuators and sensor are chosen to achieve the active twisting motion and the active damping effect, respectively.

Another component of this study involves vibration damping of large scale aircraft components such as landing gear. The damping is achieved using the combination of active and passive controls. A steel tube, which is structurally equivalent to a Boeing 747 landing gear break rod, is selected as a test specimen. The expected goal is to dissipate the fundamental vibration mode of the tube. In order to accomplish this task, beam type dynamic absorber and a constrained layer damping method are used for passive vibration controls. In this case, both Matlab simulations and experimental results are provided for the dynamic absorber. A simulation result for the ‘Reciprocal State Space’ based optimal control scheme is also provided. Because of hardware limitations, real time experiments can not be performed for the optimal control problem. A fuzzy logic based controller employing acceleration measurement using piezoelectric actuators (PZT) is implemented in dSPACE for the active vibration control of the system. The integrated controller with passive and active components can absorb the fundamental mode of the system well according to both the simulation and experimental results.

A new matrix second order observer is employed to obtain more accurate estimate values for the second order system at hand in contrast to the traditional observer design based on the state space framework. In this portion of research, the issue of designing observers for linear matrix second order systems in the matrix second order framework before they are transformed to the state space framework is presented. Several important reasons are presented to justify the need for this approach, and the advantages of the second order observer are explained and contrasted with those of the typical first order
observer. Finally, the conditions of existence of the observer gains and the design methodologies are presented.
Dedicated to my parents

사랑하는 부모님께
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Major Field: Aeronautical and Astronautical Engineering

Studies in:

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Flight Mechanics and Control              M. Napolitano, Ph.D.
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CHAPTER 1

INTRODUCTION

In nature, a bird changes the shape of its wings in order to achieve proper flight performance. Humans have long pursued the dream of achieving bird-like flight performance. Efforts in this direction led to the development of rigid control surfaces such as the elevator, ailerons, rudder, and flaps for a conventional aircraft. However, these efforts are only adequate for limited flight envelopes and maneuvers. Many modern fighter aircrafts are required to possess rapid maneuverability at high speeds. If one attempts to accomplish this with traditional control surfaces, difficulties arise. For example, during a swift roll motion, the aircraft's wings are deformed significantly due to high dynamic pressure. This deformation causes the roll reversal phenomenon.

In recent years, a deformable structure design and control based on smart structure actuation and sensing have become an active topic of research in the aerospace community. A deformable wing structure may enable the aircraft to have faster and gentler maneuver as well as more efficient and safer performance (like that of a bird) by using the interaction among the elastic structures, smart materials, external aerodynamic pressure and the control system. Federal Research Laboratories such as Wright Laboratory, DARPA, and industrial companies such as Northrop Grumman are actively
engaged in the research and development of this challenging multidisciplinary area [34] [26].

Most of the previous modeling and control design research in this multidisciplinary area is confined to the integration of any two sub-disciplines such as structures and control, control and smart materials, and structures and smart materials. Only recently has the truly integrated problem of incorporating all four sub-disciplines, namely structures, aerodynamics, smart materials and control system design, been applied. This type of pure and straightforward integration of these four disciplines is realized to be a complicated task requiring expertise in all of these disciplines. In addition, for applications involving aircraft maneuvers, one should even add another sub-discipline, namely flight mechanics! One of the first attempts to incorporate structures, aerodynamics, control and flight mechanics (addressing specifically the roll maneuver) is the series of papers published by WL and Northrop Grumman researchers Khot, Eastep, Kari Appa and their colleagues [29, 30]. In this research, an attempt is made, perhaps for the first time, to achieve integration of all the above five disciplines! Efforts are undertaken to model and control the aeroelastic dynamics of a flexible wing structure embedded with piezoelectric material for actuation and sensing so that a roll maneuver with a desired roll rate is achieved by actively deforming the wing.

Typically, the motion of a piezoelectrically laminated flexible aircraft structures [46], can be expressed in the form of a linear ordinary second order differential equation in the ‘configuration or generalized’ coordinates. This system can then be put in the form of the well known ‘Matrix Second Order (MSO)’ system framework.
For flight mechanics problems, the static deformation of the wing is essential to obtain the desired performance of an aircraft. Thus, the deformed structural mass, damping, and stiffness matrices, which are functions of the steady-state voltage input, are also included in the control design model. The piezoelectric voltages serve as the control variables. To have a constant roll rate, a new framework called the ‘Reciprocal State Space’ framework is used for control design purposes. This new modeling and control design methodology is illustrated with the help of an example and its efficacy demonstrated clearly. First, the large model for the deformed structural dynamics in the generalized coordinates is transformed to a set of ‘non-orthogonal modal’ coordinates, and model reduction is carried out in these modal coordinates. Typically, this model is then converted to the ‘state space’ form.

In this study, the ‘reciprocal state space’ [42] representation is employed to implement acceleration feedback. A way to implement the control algorithm based on reciprocal state space is given in Chapter 3. Because of the hardware limitation, however, the continuous time experiment for the scheme is not available. Thus, a discrete time optimal control scheme for piezoelectric actuator and sensor laminated cantilevered beam is implemented to show how to reduce the vibration of wings.

Another component of this study lies in the vibration damping of landing gear component using piezoelectric actuators and intelligent fuzzy logic control. A beam type dynamic absorber is added to the structure to maximize the control effort. Next viscoelastic-constrained layer is added to enhance the damping effect.

The modal analysis of the steel tube and Boeing 747 landing gear break rod is done through finite element analysis using tetrahedral solid elements in ANSYS. In order
to reduce the model size and to achieve an optimal design, a simplified finite element model using beam elements is also built for both the tube and the dynamic absorber. It is observed that the simplified model matches well with the experimental results.

The active vibration control is implemented, counting on fuzzy logic in dSPACE platform due to easy implementation and robustness [37].

The resulting full state and state derivative feedback controllers are possible only when all the states and their derivative measurements are available. Practically, however, it is impossible to measure all the states and their derivatives. For example, the piezoelectric material laminated aircraft structure in aerodynamic field provides only displacement information through piezoelectric sensors. Thus, an observer is required to design a proper feedback controller. Traditionally, the observer is designed in the first order state-space framework. However, there are several noticeable problems in the first order observer designed by converting a second order system to a first order state space form. In this research, therefore, a new second order observer design is presented to obtain improved controller performance as well as to determine the minimum number and location of the sensors.
CHAPTER 2

INTEGRATED MODELING FOR FLEXIBLE AIRCRAFT STRUCTURES WITH PIEZOELECTRIC ACTUATORS

2.1 Finite Element Modeling for Piezo-laminated Flexible Aircraft Structures

The constitutive equations for a piezoelectric material are expressed as [43]

\[ T = \begin{bmatrix} c \end{bmatrix} S - \begin{bmatrix} e \end{bmatrix}^T E \]  \hspace{1cm} (2.1)

\[ D = \begin{bmatrix} e \end{bmatrix} S + \begin{bmatrix} e \end{bmatrix} E \]  \hspace{1cm} (2.2)

where

- \( T = [\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{yx} \quad \sigma_{xy}]^T \); stress vector
- \( S = [\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{zz} \quad \varepsilon_{yx} \quad \varepsilon_{xy}]^T \); strain vector
- \( E = [E_x \quad E_y \quad E_z]^T \); electrical field vector
- \( D = [D_x \quad D_y \quad D_z]^T \); electrical displacement vector

\[ \begin{bmatrix} c \end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{44}
\end{bmatrix} \]
In order to generate the finite element formulation of a flexible wing, an 8-node coupled brick element is employed as shown in Figure 2.1.

Figure 2.1: Coupled 8 Nodes 3-D Solid
The shape functions of the elements are expressed as

\[ N_i = \frac{1}{8} (1 + \xi \zeta_i)(1 + \eta \eta_i)(1 + \zeta \zeta_i) \]

Each node of the element has 4 degrees of freedom which are spatial displacements \((u, v, w)\) and voltage\((V)\). These displacements and voltage are coupled to each other according to the constitutive equations (Equations 2.1 and 2.2). The displacement fields with the shape functions in the finite element model are expressed as

\[ \mathbf{u} = [N_q] q \]

\[ \varphi = [N_\varphi] v \]

(2.3) \hspace{1cm} (2.4)

where \([N_q]\) and \([N_\varphi]\) are called the shape functions in the \(q\) and \(\varphi\) coordinates that are given by

\[
[N_q] = \begin{bmatrix}
N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots & N_8 & 0 & 0 \\
0 & N_1 & 0 & 0 & N_2 & 0 & \cdots & 0 & N_8 & 0 \\
0 & 0 & N_1 & 0 & 0 & N_2 & \cdots & 0 & 0 & N_8
\end{bmatrix}
\]

\[
[N_\varphi] = [N_1 \ N_2 \ \cdots \ N_8]
\]

\(\mathbf{u}\), \(q\) and \(v\) are the displacements, the generalized coordinates and voltage which are expressed as

\[ \mathbf{u} = [u \ v \ w]^T \]

\[ q = [u_1 \ v_1 \ w_1 \ u_2 \ v_2 \ w_2 \ \cdots \ u_8 \ v_8 \ w_8]^T \]

\[ v = [V_1 \ V_2 \ \cdots \ V_8]^T \]
Then, the strain and electric fields are written as

\[
S = [L_q][N_q]q = [B_q]q
\]  
(2.5)

\[
E = [L_\varphi][N_\varphi]v = [B_\varphi]v
\]  
(2.6)

where \([L_q]\) and \([L_\varphi]\) are mechanical and electrical operators. The matrices \([B_q]\) and \([B_\varphi]\) are expressed as

\[
[B_q] = \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & 0 & 0 & \frac{\partial N_2}{\partial \xi} & 0 & 0 & \ldots & \frac{\partial N_s}{\partial \xi} & 0 & 0 \\
0 & \frac{\partial N_1}{\partial \eta} & 0 & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \ldots & 0 & \frac{\partial N_s}{\partial \eta} & 0 \\
0 & 0 & \frac{\partial N_1}{\partial \zeta} & 0 & 0 & \frac{\partial N_2}{\partial \zeta} & \ldots & 0 & \frac{\partial N_s}{\partial \zeta} & \frac{\partial N_s}{\partial \zeta} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & \ldots & \frac{\partial N_s}{\partial \eta} & 0 & \frac{\partial N_s}{\partial \eta} \\
\frac{\partial N_1}{\partial \zeta} & \frac{\partial N_1}{\partial \zeta} & 0 & \frac{\partial N_2}{\partial \zeta} & 0 & \frac{\partial N_2}{\partial \zeta} & \ldots & \frac{\partial N_s}{\partial \zeta} & 0 & \frac{\partial N_s}{\partial \zeta} \\
\frac{\partial N_1}{\partial \xi} & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & \ldots & \frac{\partial N_s}{\partial \xi} & 0 & \frac{\partial N_s}{\partial \xi}
\end{bmatrix}
\]
2.2 Finite Element Modeling of Aerodynamic Pressure Distribution on the Aircraft Surfaces

The modeling of aerodynamic pressure distribution on the flexible wing structure is discussed will now be considered. According to piston theory [4, 11], the aerodynamic pressure on the surface of a wing, for a high Mach number ($M > 1.6$), is expressed as

$$
\Delta p = - \left[ \lambda \left( \frac{\partial w}{\partial x} \right) + g \left( \frac{\partial w}{\partial \tau} \right) \right]
$$

(2.7)

where $w$ is the nodal displacement in $z$ direction. The constant $\lambda$ and $g$ are expressed as following

$$
\lambda = \frac{2q}{\sqrt{M^2 - 1}}
$$

$$
g = \frac{\lambda}{U_a} \frac{M^2 - 2}{M^2 - 1}
$$

$$
q = \frac{1}{2} \rho_a U_a^2.
$$

$\rho_a$ and $U_a$ are air density and air velocity, respectively. Figure 2.2 shows a 32 degrees of freedom brick element with aerodynamic loads. According to Equation 2.7, these loads are changed by the vertical displacements of nodes. Here, let's assume that the loads change along the $x$ coordinate and the pressure difference between upper and lower surfaces of the wing act on the upper surface.
By substituting Equation 2.3 into Equation 2.7, we obtain

$$\Delta p = -\lambda [N_a]q_{,}\xi - g[N_a]_{,t}q$$  \hspace{1cm} (2.8)

where \([N_a]\) is the aerodynamic shape function which is given by

$$[N_a] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & N_{4,\xi=1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$
2.3 Equation of Motion via Hamilton’s Principle

The equation of motion of the wing can now be derived from Hamilton’s principle which is expressed as

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W) dt = 0$$

(2.9)

where $\delta T$, $\delta U$, and $\delta W$ are the first variation of kinetic energy, total potential energy and the work done by externally applied forces. The potential energy includes the mechanical strain energy, the electrical energy. Including all of the energy terms $\delta T$, $\delta U$, and $\delta W$ are expressed as [44, 2]

$$\delta T = \int_V \rho \dot{u} \delta \ddot{u} dV$$

(2.10)

$$\delta U = \int_{\Gamma} \left[ \delta \mathbf{S}^T [c] \mathbf{S} - \delta \mathbf{S}^T [\varepsilon] \mathbf{E} - \delta \mathbf{E}^T [\varepsilon] \mathbf{S} - \delta \mathbf{E}^T [\varepsilon] \mathbf{E} \right] dV$$

(2.11)

$$\delta W = \int_{S} \left[ \mathbf{t} \delta \mathbf{u} - Q \delta \varphi \right] dS$$

(2.12)

where $\rho$, $\mathbf{u}$, $\mathbf{t}$, and $Q$ are the mass density, displacement vector, surface traction and surface charge, respectively. By substituting Equation 2.10, Equation 2.11 and Equation 2.12 into Equation 2.9, one gets

$$\int \left[ \delta \mathbf{S}^T [c] \mathbf{S} - \delta \mathbf{S}^T [\varepsilon] \mathbf{E} - \delta \mathbf{E}^T [\varepsilon] \mathbf{S} - \delta \mathbf{E}^T [\varepsilon] \mathbf{E} + \rho \delta \mathbf{u} \dot{\mathbf{u}} \right] dV - \int_{S} \left[ \mathbf{t} \delta \mathbf{u} - Q \delta \varphi \right] dS = 0$$

(2.13)

By substituting Equation 2.3 through 2.6 into Equation 2.13, one can combine Hamilton’s Principle with the finite element method to obtain
Equation 2.14 can be reorganized as
\[
\frac{\partial}{\partial q^T} \{ [B_q]^T \eta \} dV q + \int [B_q]^T [e]^T [B_q] dV \eta + \int \rho \eta \eta [N_q] dS \eta + \int \alpha [N_q] dS = 0
\] 
(2.15)

From Equation 2.8, the aerodynamic loads are expressed in terms of the generalized coordinates of the element. Moreover, in Equation 2.15, the loads are grouped as the stiffness and damping coefficient matrices of the system. These coefficients are labeled as the aerodynamic stiffness and damping matrices. If Equation 2.15 is rewritten in a matrix form, the well known Matrix Second Order System [16] can be obtained as
\[
\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\eta} \\ \ddot{\eta} \end{bmatrix} + \begin{bmatrix} C_\lambda + K_\lambda \\ K_\psi \end{bmatrix} = 0
\] 
(2.16)

where
\[
M = \int_0^1 \int_0^1 \int_0^1 \rho \eta [N_q]^T [N_q] d\xi d\eta d\zeta
\]
\[
C_\lambda = \int_0^1 \int_0^1 \alpha [N_q]^T [N_q] d\xi d\eta d\zeta
\]
\[
K_\psi = \int_0^1 \int_0^1 \alpha [\lambda [N_q]^T [N_q] J_\psi d\xi d\eta d\zeta
\]
\[ K_{qp} = K_{qp}^T \]

\[ K_{qp} = \int_{-1}^{1} \int_{-1}^{1} [B_{\phi}]^T [\varepsilon][B_{\phi}] |J| d\xi d\eta d\zeta \]

and \( J \) and \( J_a \) are the Jacobian matrices.

\[
J = \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} & \frac{\partial N_5}{\partial \xi} & \frac{\partial N_6}{\partial \xi} & \frac{\partial N_7}{\partial \xi} & \frac{\partial N_8}{\partial \xi} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} & \frac{\partial N_5}{\partial \eta} & \frac{\partial N_6}{\partial \eta} & \frac{\partial N_7}{\partial \eta} & \frac{\partial N_8}{\partial \eta} \\
\frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \frac{\partial N_3}{\partial \zeta} & \frac{\partial N_4}{\partial \zeta} & \frac{\partial N_5}{\partial \zeta} & \frac{\partial N_6}{\partial \zeta} & \frac{\partial N_7}{\partial \zeta} & \frac{\partial N_8}{\partial \zeta}
\end{bmatrix}
\]

\[
J_a = \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} & \frac{\partial N_5}{\partial \xi} & \frac{\partial N_6}{\partial \xi} & \frac{\partial N_7}{\partial \xi} & \frac{\partial N_8}{\partial \xi} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} & \frac{\partial N_5}{\partial \eta} & \frac{\partial N_6}{\partial \eta} & \frac{\partial N_7}{\partial \eta} & \frac{\partial N_8}{\partial \eta} \\
\frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \frac{\partial N_3}{\partial \zeta} & \frac{\partial N_4}{\partial \zeta} & \frac{\partial N_5}{\partial \zeta} & \frac{\partial N_6}{\partial \zeta} & \frac{\partial N_7}{\partial \zeta} & \frac{\partial N_8}{\partial \zeta}
\end{bmatrix}
\begin{bmatrix}
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3 \\
x_4 & y_4 & z_4 \\
x_5 & y_5 & z_5 \\
x_6 & y_6 & z_6 \\
x_7 & y_7 & z_7 \\
x_8 & y_8 & z_8
\end{bmatrix}
\]

2.4 Model Reduction in Non-orthogonal Modal Coordinates

From Equation 2.16, the equation of motion with voltage actuation is written as

\[ M\ddot{q} + C_{\lambda}\dot{q} + (K_{q} + K_{\lambda})q = -K_{qp}v \]  

(2.17)

By pre-multiplying \( M^{-1} \) on both sides of Equation 2.17, one gets

\[ \ddot{q} + M^{-1}C_{\lambda}\dot{q} + M^{-1}(K_{q} + K_{\lambda})q = -M^{-1}K_{qp}v \]  

(2.18)
The modal equation of motion of the system can be obtained with a similarity transformation that diagonalizes the integrated stiffness matrix [38]. Let

\[ q = T \eta \]

where

\[ T = \begin{bmatrix} t_1 & t_2 & t_3 & \cdots & t_n \end{bmatrix} \]

\( t_i \) : eigenvector of the system

Then, Equation 2.18 can be written as

\[ \ddot{\eta} + C_\eta \dot{\eta} + \Lambda \eta = F_\eta \nu \tag{2.19} \]

where

\[ \eta = \begin{bmatrix} \eta_1 & \eta_2 & \eta_3 & \cdots & \eta_n \end{bmatrix}^T \]

\[ C_\eta = T^{-1} M^{-1} C \Lambda T \]

\[ \Lambda = T^{-1} M^{-1} (K_q + K_A) T = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n) \]

\[ F_\eta = -T^{-1} M^{-1} K_{q,\eta} \]

and \( \lambda_i \) are complex numbers. But the model in this system is too large for a meaningful control system design. For this reason, a transformation to the non-orthogonal (but real) modal coordinates, is carried out. The corresponding state space form of Equation 2.19 is written as

\[ \begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\Lambda & -C_\eta \end{bmatrix} \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix} + \begin{bmatrix} 0 \\ F_\eta \end{bmatrix} \nu \]

\[ \dot{x} = Ax + Bu \]

where \( x = [\eta \quad \dot{\eta}]^T \).
In this chapter, the roll maneuver with a constant roll rate and the resulting structural dynamic vibrations are analyzed. The structural dynamic deformations under the roll maneuver with a desired roll rate are modeled below. Figure 3.1 shows a plate wing with roll axis constraints.
3.1 Torsional Motion of Piezoelectric Continua

Since lift forces vary according to the angles of attack, $\alpha$, of a wing, which can be changed by the twisting motion of the wing, it is clear that one needs to impart twisting motion to the wing to generate the required moments for various flight maneuvers such as roll. Figure 3.2(a) and Figure 3.3 show the angle of attack and twisting motion of a wing, respectively. A pair of shear strains, $\gamma_{xy}$, in opposite directions are needed to produce the twisting moment of the wing as shown in Figure 3.2(b). The directional attachment technique of the piezoelectric actuators [36] is employed to achieve twisting motion of the wing.
Figure 3.2: Plate Wing, (a) Angle of Attack, (b) Shear Strains
Figure 3.3: Twisting Motion of a Wing
From Figure 3.4, the relation between the principal axes \((x', y', z')\) and reference axes \((x, y, z)\) are expressed as

\[
T' = [T_r] T
\]

\[
S' = [T_r] S
\]
where

\[
[T_r] = \begin{bmatrix}
 l_1^2 & m_1^2 & n_1^2 & 2m_1n_1 & 2n_1l_1 & 2n_1l_1 \\
 l_2^2 & m_2^2 & n_2^2 & 2m_2n_2 & 2n_2l_2 & 2n_2l_2 \\
 l_3^2 & m_3^2 & n_3^2 & 2m_3n_3 & 2n_3l_3 & 2n_3l_3 \\
 l_{21} & m_{21} & n_{21} & m_{21}n_{21} + m_{21}n_{21} & l_{21}n_{21} + l_{21}n_{21} & l_{21}m_{21} + l_{21}m_{21} \\
 l_{31} & m_{31} & n_{31} & m_{31}n_{31} + m_{31}n_{31} & l_{31}n_{31} + l_{31}n_{31} & l_{31}m_{31} + l_{31}m_{31} \\
 l_{31} & m_{31} & n_{31} & m_{31}n_{31} + m_{31}n_{31} & l_{31}n_{31} + l_{31}n_{31} & l_{31}m_{31} + l_{31}m_{31} \\
 2l_{12} & m_{12} & n_{12} & m_{12}n_{12} + m_{12}n_{12} & l_{12}n_{12} + l_{12}n_{12} & l_{12}m_{12} + l_{12}m_{12} \\
 2l_{13} & m_{13} & n_{13} & m_{13}n_{13} + m_{13}n_{13} & l_{13}n_{13} + l_{13}n_{13} & l_{13}m_{13} + l_{13}m_{13} \\
 2l_{23} & m_{23} & n_{23} & m_{23}n_{23} + m_{23}n_{23} & l_{23}n_{23} + l_{23}n_{23} & l_{23}m_{23} + l_{23}m_{23} \\
 2l_{23} & m_{23} & n_{23} & m_{23}n_{23} + m_{23}n_{23} & l_{23}n_{23} + l_{23}n_{23} & l_{23}m_{23} + l_{23}m_{23} \\
\end{bmatrix}
\]

\[
T = [\varepsilon]S - [\varepsilon]^T E
\]

where

\[
[T_r] = [T_r]^{-1}[\varepsilon][T_r]
\]

\[
[\varepsilon] = [\varepsilon]T
\]

Now, Equations 2.1 and 2.2 can be expressed as

\[
T = [\varepsilon]S - [\varepsilon]^T E
\]

\[
D = [\varepsilon]S + [\varepsilon]E
\]

where

\[
[l_1] = \cos \theta \\
[l_2] = -\sin \theta \\
[l_3] = 0
\]

\[
m_1 = \sin \theta \\
m_2 = \cos \theta \\
m_3 = 0
\]

\[
n_1 = 0 \\
n_2 = 0 \\
n_3 = 0
\]

It can be observed from Equation 3.1 and Equation 3.2, that \([\varepsilon]\) matrix, the modified piezoelectric constant, has a non-zero \(e_{36}\) entry. The torsional motion can be produced by this coefficient. As shown in Figure 3.4, two PVDF layers, bottom and upper layers, with
opposite skew angles  $\theta$ and $-\theta$ are required to generate the torque on the wing. The maximum shear strain, $\gamma_{xy}$, can be obtained when the skew angles are $\frac{\pi}{4}$.

3.2 Equation of Steady Roll for Deformable Wing

While the roll motion is expressed as an angular motion, in general, the finite element model is expressed in terms of generalized coordinates which are nodal displacements. In this research, therefore, the roll angular motion is obtained by modeling the deformation of the wing under the steady state roll rate which, in turn, is achieved by the steady state voltage actuation. Since the model of the wing is expressed with nodal displacements and voltages, the angular displacement cannot be directly modeled in generalized coordinates. The angular displacement of roll motion, shown in Figure 3.5, and its relationship to the input voltage can be obtained from the moment equilibrium equation of the wing. The roll equation of motion for the wing is expressed as

$$I \ddot{\phi} = M_R$$

where

$I = \frac{1}{3} ml^2$: Moment of inertia

$\phi$: Roll angle

$M_R$: Rolling moment due to lift

$m$: Mass of the wing

$l$: Length of the wing.
The rolling moment due to lift, $M_R$, is expressed as

$$M_R = \int_0^l L(y) \, dy$$

where $L(y)$ is lift force.

The lift, $L(y)$, is written as

$$L(y) = q c \zeta_t(y)$$

where

- $q = \frac{\rho U^2}{2}$: dynamic pressure
- $\rho$: air density
- $U$: air velocity
- $c$: airfoil span
- $\zeta_t$: lift coefficient

Figure 3.5: Roll Motion of a Wing
The lift coefficient, $C_L(y)$, can be expressed as a function of angle of attack ($\alpha$).

$$\frac{\partial C_L(y)}{\partial \alpha}[\alpha_0(y) + \alpha_e(y)]$$

$$= C_{L_0} + \frac{\partial C_L}{\partial \alpha} \alpha_e$$

(3.5)

where $\alpha_0(y)$ angles of attack for zero lift due to elastic twist and it is assumed that $\alpha_0(y)$ is zero. Note that, $\alpha_e(y)$ can be written as

$$\alpha_e(y) = \alpha_{pc}(y) - \frac{P_y}{U}$$

(3.6)

where $\alpha_{pc}(y)$ is the angle of attack produced by piezoelectric actuation and $\frac{P_y}{U}$ is the induced angle of attack due to roll rate ‘ $P$ ’. By substituting Equation 3.6 into Equation 3.5, the lift coefficient is written as

$$C_L(y) = C_{La} \left( \alpha_{pc}(y) - \frac{P_y}{U} \right)$$

(3.7)

where $C_{La} = \frac{\partial C_L(y)}{\partial \alpha}$. By introducing Equation 3.7 into Equation 3.4, the lift is expressed as

$$L(y) = C_{La} \rho c U^2 \left( \alpha_{pc}(y) - \frac{P_y}{U} \right)$$

Then, the rolling moment is obtained as

$$M_r = \frac{C_{La} \rho c U^2}{2} \int_0^\alpha \left[ \alpha_{pc}(y) - \frac{P_y}{U} \right] dy$$

(3.8)
By substitution Equation 3.8 into Equation 3.3 and using $P = \dot{\phi}$, the equation of motion for the roll can be expressed as

$$I \ddot{\phi} + \frac{C_{la} \rho c U l^2}{4} \dot{\phi} = \frac{C_{la} \rho c U^2}{2} \int_{0}^{l} \alpha_{pe} (y) dy$$

(3.9)

A finite element model analysis, as depicted in Figure 3.6, is required in order to find the relationship between the angle of attack, $\alpha_{pe}(y)$, and the nodal voltage input, $v$. For the purpose of modeling, it is assumed that chordwise segments of the wing remain rigid.

Figure 3.7 shows the cross sectional area and angle of attack, $\alpha$, of the wing.

![Figure 3.6: The Finite Element Model of the Wing Plate with Piezoelectric Lamina](image)
The angle $\alpha$ is the twisting angle due to the piezoelectric actuation in the element local coordinates. First, the angle of attack in the element coordinates, $\alpha_{pz}(\eta)$, has to be obtained through $\alpha_{pz}(y) \Rightarrow \alpha_{pz}(\eta)$. It is assumed that the angle $\alpha$ is constant through the chord line for the small twist angle. Since the angle of attack is measured from the centerline of the cross section of the wing, the angle $\alpha$ is expressed as

$$\alpha(y) = \frac{\partial w}{\partial x}_{z=0},$$

where $w$ is transverse displacement at nodes and $c_z$ is a $z$ coordinate value of center line of the wing. In the element local coordinates, the angle $\alpha$ is expressed as

$$\alpha_{pz}(\eta) = \frac{\partial w}{\partial \xi}_{\zeta=\xi_z},$$

where $c_\xi$ is the coordinate value for centerline of the cross sectional area of the wing in the local coordinate. Finally, the angle $\alpha$ is expressed in the generalized coordinates as

$$\alpha_{pz}(\eta) = L_{pw} N_q q = B_{pw}q$$

where
To build a full model, the angle of attack has to be integrated into the physical coordinates expressed as

\[
\int_{y_1}^{y_2} \alpha_\infty(y) \, dy
\]

where \( y_1 \) and \( y_2 \) are lower and upper bound for the element in the \( y \) coordinate as shown in Figure 3.8.

Figure 3.8: Finite Element Model for a Wing
Since the twisting angle is constant throughout the chord line, $\zeta$ can be selected as zero.

The modified shape function is expressed as

$$\Theta_i = \frac{1}{8}(1 + \eta \eta_i)(1 + c_\zeta \zeta_i)$$

The $y$ coordinate is written as

$$y = \sum_{i=1}^{8} \Theta_i y_i$$

From the chain rule

$$\frac{dy}{d\eta} = \sum_{i=1}^{8} \frac{d\Theta_i}{d\eta} y_i$$

The angle of attack for the $j^{th}$ structural element in the physical coordinate $y$ is expressed as

$$A_j = \int_{\eta_1}^{\eta_2} \alpha_{pc}(y) dy$$

$$= \int_{\eta_1}^{\eta_2} \alpha_{pc}(\eta) \sum_{i=1}^{8} \frac{d\Theta_i}{d\eta} y_i, d\eta$$

$$= \int_{\eta_1}^{\eta_2} B_{\alpha} \sum_{i=1}^{8} \frac{d\Theta_i}{d\eta} y_i, d\eta q_j$$

where

$q_j$: the generalized coordinates for $j^{th}$ element

$$B_\alpha = \begin{bmatrix} 0 & 0 & \int_{-1}^{1} \frac{\partial N_1}{\partial \xi} |_{\xi=\zeta} \ d\eta & 0 & \int_{-1}^{1} \frac{\partial N_2}{\partial \zeta} |_{\xi=\zeta} \ d\eta & \cdots & 0 & \int_{-1}^{1} \frac{\partial N_8}{\partial \zeta} |_{\xi=\zeta} \ d\eta \end{bmatrix}$$
Thus, the total integration of the angle of attack is written as

$$\int_0^l \alpha_{pc} (y) dy = \sum_{j=1}^{q} A_j$$

$$= B_{aq} q$$

where $B_{aq}$ is the $(1 \times n)$ matrix which contains $B_{aq} J_{\alpha_j}$ for the corresponding nodal displacement $q$. The nodal displacement vector $q$ can be expressed as the function of voltage input from the equation of motion of the cantilevered wing which is expressed as

$$M_q \ddot{q} + K_q q = -K_{q\theta} \nu$$

(3.10)

where $\nu$ is voltage input. Figure 3.9. shows the twisting motion of a cantilevered wing with voltage actuation.
Figure 3.9: A Twisting Motion of a Cantilevered Wing

Since the twisting angle is the static deflection of the wing, Equation 3.10 can be reduced to

\[ K_q q = -K_{qq} v_r \]

where \( v_r \) is voltage input for roll. Since the stiffness matrix of the cantilevered wing is not singular, the displacements are expressed as

\[ q = -K_q^{-1} K_{qq} v_r \] (3.11)

By substituting Equation 3.11 into Equation 3.9, the equation of motion for roll motion with voltage actuation is expressed as

\[
\dot{I} \ddot{\phi} + \frac{C_{l_0} \rho c U l^2}{4} \ddot{\phi} = -\frac{C_{l_0} \rho c U^2}{2} B_{ci} K_q^{-1} K_{qq} v_r
\] (3.12)
3.3 Full State Derivative Feedback Control Design for Roll Motion using Reciprocal State Space

State space representation is a useful tool to design controllers for linear systems, and many control design methods in the state space framework are available to achieve stabilization and regulation of the state variables. However, for the particular problem at hand, to achieve a desired constant roll rate, the state space based control design is cumbersome to use because the steady state constant roll rate implies infinite roll displacements as time passes. Therefore, the closed loop system is considered unstable.

To overcome this problem and still design a simple controller using available control system software for this desired roll rate achievement problem, a new framework called the 'Reciprocal State Space' framework is proposed [42].

The state space representation of Equation 3.12 is expressed as

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\phi}_r
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{3C_{\ell_m} \rho c U}{4m} \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\phi \\
\phi_r
\end{bmatrix} + \begin{bmatrix}
0 \\
-\frac{3C_{\ell_m} \rho c U^2}{2ml^2} [B_{a_1} \left[K_q \right]\left[K_{q_2}\right]^{1/4}
\end{bmatrix} v_r
\]

where \( x_r = [\phi \ \phi_r]^T \). The reciprocal state space representation of the system is expressed as

\[
x_r = G_r \dot{x}_r + H_r u_r
\]

\[
u_r = K_r \dot{x}_r
\]

where

\[
G_r = A_r^{-1}, \quad H_r = -A_r^{-1} B_r
\]
For regulation problem, coordinates transformation are expressed as

\[
\begin{align*}
\dot{\xi} &= \dot{x}_r + P \\
\xi &= x_r + E
\end{align*}
\]

where \( P = p_d \), \( E = p_d t \), and \( p_d \) is desired roll rates.

Then, Equation 3.13 is written as

\[
\begin{align*}
\dot{\xi} &= G_r \xi + H_r u_r \\
u_r &= K_r \xi
\end{align*}
\]

The performance index to be minimized is expressed as

\[
J = \int_0^T \left( \xi^T Q \xi + u^T R u \right) dt
\]

where \( Q \) and \( R \) are positive semi-definite matrix and positive definite matrix, respectively. According to references [31, 41], the LQR state feedback gain is written as

\[
K = R^{-1} H^T S
\]

The matrix \( S \) can be obtained from the associated Algebraic Matrix Riccati equation which is expressed as

\[
0 = S G + G^T S - S H R^{-1} H^T S + Q
\]

As an example, a PVDF plate wing with 2 layers and 16 elements, as shown in Figure 3.10, is selected for a control design whose objective is to achieve a desired constant roll rate.

Each layer has an opposite skew angle to generate torques of the wing. One half of the plate has eight 8-node brick elements with the total number of nodes being 30. In
other words, the system has 120 degrees of freedom, 90 for structural and 30 for electrical degrees of freedom. It is observed that the open loop system is unstable.

Although a pure structural system is neutrally stable, in general, the structure in the aerodynamic field is no longer stable due to the presence of the non-conservative aerodynamic field which bring forth some stiffness as well as damping. By applying the proposed control design technique, the closed loop system is not only stabilized but also a desired constant roll rate of 1.5 rad/sec is achieved. Figure 3.12 shows the roll rate responses for a selected set of weighting matrices. From this figure, it is clear that the desired roll velocity of 1.5 rad/sec is achieved by the controller. The corresponding roll angles are shown in Figure 3.11. The roll angle gradually increases as expected. Figure 3.13 shows the input voltage for the roll maneuver. The input voltage is used to build the deformed structure model which is explained in next section.

![Figure 3.10: Plate Wing (PVDF)](image-url)
Figure 3.11: Roll Angle Responses

Figure 3.12: Roll Rate Responses
3.4 Equation of Motion for Deformed Structure

The next task is to analyze the vibrational motion of the wing. A Considerable amount of research exists on the vibration suppression of flexible structures [21, 6, 14]. However, there are not many studies undertaken for modeling vibrations associated with flight maneuvers. In a roll maneuver with flexible wings, a desired angle of attack should be maintained to achieve a desired roll rate. In order to maintain the desired angle of attack, the deformation, twisting, of the wing is required. Therefore, a new equation of
motion for the deformed structure is necessary to dissipate the vibration of the wing as well as to obtain the desired roll motion.

The finite element model based on the deformed coordinates, $x_d$, $y_d$ and $z_d$ is formulated as shown in Figure 3.14.

![Figure 3.14: Deformed Coordinates](image)

The displacement fields are then written as

$$u_d = [N_q] q_d$$

$$\varphi_d = [N_\varphi] v_d$$

where

$$q = [u_{d_1}, v_{d_1}, w_{d_1}, u_{d_2}, v_{d_2}, w_{d_2}, ... , u_{d_k}, v_{d_k}, w_{d_k}]^T$$

$$\varphi = [V_{d_1}, V_{d_2}, ... , V_{d_k}]^T$$

The strain fields are expressed as
To obtain the equation of motion for the deformed structure, the following physical
coordinates are used,

\[ X_d = X + q_t \]  \hspace{1cm} (3.14)

where

\[
X_d = \begin{bmatrix} x_{d1} & y_{d1} & z_{d1} & x_{d2} & y_{d2} & z_{d2} & \cdots & x_{ds} & y_{ds} & z_{ds} \end{bmatrix}^T
\]

\[
X = \begin{bmatrix} x_1 & y_1 & z_1 & x_2 & y_2 & z_2 & \cdots & x_8 & y_8 & z_8 \end{bmatrix}^T
\]

\[
q_t = [q_{t1}, q_{t2}, \cdots, q_{ts}]
\]

From the original undeformed structure, the initial displacement, \( q_t \), can be obtained as

\[ q_t = -K_q^{-1}K_{qp}v_r \]  \hspace{1cm} (3.15)

where \( v_r \) is voltage input for the desired roll. By substituting Equation 3.15 into Equation
3.14, the deformed coordinates are expressed as

\[ X_d = X - K_q^{-1}K_{qp}v_r \]

Finally the equation of the motion for a deformed structure in the aerodynamic field is
then given by

\[
\begin{bmatrix} M_d & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{q}_d \\ \dot{v}_d \end{bmatrix} + \begin{bmatrix} C_{\alpha_d} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_d \\ \dot{v}_d \end{bmatrix} + \begin{bmatrix} K_d + K_{\alpha_d} \\ K_{\phi_{zd}} \\ K_{\phi_{pd}} \end{bmatrix} \begin{bmatrix} q_d \\ v_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

where

\[
M_d = \int_1^l \int_1^l \int_1^l \rho [N_q]_p [N_q]_d J_d d\xi d\eta d\zeta
\]

\[
C_{\alpha_d} = \int_1^l \int_1^l g [N_q]_p [N_q]_d J_{\alpha_d} d\xi d\eta
\]
and $J_d$ and $J_{au}$ are the Jacobian matrices.

$$J_d = \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} & \frac{\partial N_5}{\partial \xi} & \frac{\partial N_6}{\partial \xi} & \frac{\partial N_7}{\partial \xi} & \frac{\partial N_8}{\partial \xi} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} & \frac{\partial N_5}{\partial \eta} & \frac{\partial N_6}{\partial \eta} & \frac{\partial N_7}{\partial \eta} & \frac{\partial N_8}{\partial \eta} \\
\frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \xi} & \frac{\partial \xi}{\partial \xi} \\
\frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} & \frac{\partial N_5}{\partial \xi} & \frac{\partial N_6}{\partial \xi} & \frac{\partial N_7}{\partial \xi} & \frac{\partial N_8}{\partial \xi} \\
\frac{\partial \eta}{\partial \eta} & \frac{\partial \eta}{\partial \eta} & \frac{\partial \eta}{\partial \eta} & \frac{\partial \eta}{\partial \eta} & \frac{\partial \eta}{\partial \eta} & \frac{\partial \eta}{\partial \eta} & \frac{\partial \eta}{\partial \eta} & \frac{\partial \eta}{\partial \eta} \\
\frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} \\
\frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} & \frac{\partial \eta}{\partial \xi} \\
\frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta} & \frac{\partial \xi}{\partial \eta}
\end{bmatrix} \begin{bmatrix}
x_{d1} & y_{d1} & z_d \\
x_{d2} & y_{d2} & z_d \\
x_{d3} & y_{d3} & z_d \\
x_{d4} & y_{d4} & z_d \\
x_{d5} & y_{d5} & z_d \\
x_{d6} & y_{d6} & z_d \\
x_{d7} & y_{d7} & z_d \\
x_{d8} & y_{d8} & z_d
\end{bmatrix}$$

It is clear that the deformed structural mass, damping, and stiffness matrices are expressed as the functions of $v$, which is input voltage for the steady roll motion.

Figures 3.15 and 3.16 show the closed loop responses for the first and second modes of vibrations of the wing plate shown in Figure 3.10 which is deformed by the steady state
input voltage for the roll motion. Figure 3.17 shows the input voltage for the flexible modes.

Figure 3.15: Displacement Responses (First Flexible Mode)

Figure 3.16: Displacement Responses (Second Flexible Mode)
3.5 Experiments on Shape Control (Twisting) and Active Vibration Control

3.5.1 Twisting Plate

Different the case of the PVDF, the values of the piezoelectric constants $d_{31}$ and $d_{32}$ of the PZT are same. Thus, the twisting motion of the plate cannot be obtained by fully laminating the PZT actuator on the structure with skew angles. The alternate way to obtain the twisting motion of the plate is the directional attachment technique which is
suggested by Refs. [9, 8]. In this study, 6-PZT actuators are directionally (45° skew angle) attached for each side of a thin steel plate (12" × 3.5" × 0.08") as shown in Figure 3.18.

![Figure 3.18: Twisting Plate](image)

Figure 3.18: Twisting Plate
Figure 3.19 shows the twisting motion of the plate.

![Twisting Motion of the Plate](image)

3.5.2 Vibration Control for a Cantilevered Steel Beam

In this section, a new observer based control for vibration dissipation of a cantilevered beam with piezoelectric actuators and a sensor is presented. The ‘Optimal Control Theory’ is applied to dissipate the vibration in discrete-time. A first order observer is also designed to implement the full state feedback control. The finite element model procedure is used to build the model for the dynamics of the system. The model
consists of ‘11’ two-node bending elements. The model size is reduced to the $4 \times 4$ system in the modal coordinates to design ‘Linear Quadratic Regulator (LQR)’ control gains as well as to build state observer. Figure 3.20 shows the actual experimental setup of the beam.

Figure 3.20: Experimental Setup of the Cantilevered Beam (Ref. [37])
3.5.2.1 Modeling and Control Design

Figure 3.21 shows the finite element model of the beam and a 2-node bending element.

![Figure 3.21: (a) Finite Element Model of the Cantilevered Beam, (b) Beam Bending Element](image)

Figure 3.21: (a) Finite Element Model of the Cantilevered Beam, (b) Beam Bending Element
Actuator Design

The work done by electric field is expressed as

\[ \delta \mathbf{w} = \int_{V} \delta \mathbf{E} \cdot \mathbf{S} \, d\mathbf{V} \]
\[ \sigma = d_{31} E_f \]
\[ \varepsilon = h_z w(x,t) = h_u[N^s]q \]

where \( h_z \) is the distance from neutral axis to the actuator.

Since the electric field in \( z \) direction is expressed as

\[ E_f = \begin{bmatrix} N_{E_1} & N_{E_2} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \]

the actuator distribution matrix is written as

\[ K_{\sigma} = -\int_{0}^{l} E_{p} b_{31} d_{31} h_{u} [N^s]^T [N_E] \, dx \]

where \([N]\) and \([N_E]\) are matrices for structural and electrical shape functions, respectively. \( E_p \) is Young's modulus for the PZT.

Sensor Design

The electric field due to Mechanical Stress is

\[ E_m(x,t) = -g_{31} \sigma(x,t) = -g_{31} E_p \varepsilon(x,t) \]
\[ \varepsilon(x,t) = -h_z w(x,t) \]

where \( h_z \) is the distance from neutral axis to the sensor.

Thus, the sensor distribution matrix is expressed as

\[ K_{sen} = b_{2} g_{31} E_p h_u \int_{0}^{l} [N^s]^T \, dx \]
where \( b_s \) is the width of the sensor. As a result, the equation of motion of the beam is written as

\[
(M_s + M_p)\ddot{q} + (K_s + K_p)q = K_{sp}E_f
\]

and the measurement equation is expressed as

\[
E_m = K_{sen}q
\]

where

\[
M_s = \rho_s A_s \int_0^l [N]^T [N] dx
\]
\[
M_p = \rho_p A_p \int_0^l [N]^T [N] dx
\]
\[
K_s = E_s I_s \int_0^l [N^*]^T [N^*] dx
\]
\[
K_p = E_p I_p \int_0^l [N^*]^T [N^*] dx
\]

and the subscripts \( s \) and \( p \) are denoted as the structure and the piezoelectric materials, respectively. Since the system size is big (22×22) and first couples of modes are interested only, system size reduction is required. In order to reduce the system size, the modal coordinate transformation is applied. The modal coordinate transformation is expressed as

\[
q = T\eta
\]

where \( T \) is a eigenvector matrix and \( T^{-1}(M_s + M_p)T = I \). The equation of motion in the modal coordinates is written as

\[
\ddot{\eta} + \Lambda \eta = K_{sp}E_f
\]
\[
y = K_{sen}\eta
\]

where \( \Lambda \) is a diagonal frequency matrix. The state space representation of the reduced model is expressed as
\[
\begin{bmatrix}
\dot{\eta} \\
\dot{\bar{\eta}}
\end{bmatrix} = \begin{bmatrix}
0 & I \\
-\Lambda & 0
\end{bmatrix} \begin{bmatrix}
\eta \\
\bar{\eta}
\end{bmatrix} + \begin{bmatrix}
0 \\
K_{\eta \eta}
\end{bmatrix} E_f
\]
\[
y = \begin{bmatrix}
K_{\text{sens}} \\
0
\end{bmatrix} \begin{bmatrix}
\eta \\
\dot{\bar{\eta}}
\end{bmatrix}
\]

\[
\begin{align*}
x(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]

For the digital controller, the discretization of the continuous time system is required.

The discrete time system is written as
\[
\begin{align*}
x(k+1) &= A_d x(k) + B_d u(k) \\
y(k) &= C x(k)
\end{align*}
\]

where
\[
A_D = e^{A_T} \\
B_D = \int e^{A(t-\tau)} d\tau B
\]

Since only displacement from the PZT sensor can be measured, the estimator is necessary to implement the full state feedback control. A typical first order observer is expressed as
\[
\hat{x}(k+1) = A_D \hat{x}(k) + B_D u(k) + F(y(k) - \hat{y}(k)).
\]

Then, the augmented closed loop system is written as
\[
\begin{bmatrix}
x(k+1) \\
e(k+1)
\end{bmatrix} = \begin{bmatrix}
A_D - B_D G & B_D G \\
0 & A_D - FC
\end{bmatrix} \begin{bmatrix}
x(k) \\
e(k)
\end{bmatrix}
\]

where \( G \) and \( F \) are control gain and estimator gain matrices, respectively. The gains can be obtained from the ‘LQR’ design techniques which are explained in Section 3.3.
3.5.2.2 Experimental Result (dSPACE)

The dSPACE experimental diagram is shown in Figure 3.22 and the actual experimental result of the beam is shown in Figure 3.26. The weight matrices for the LQR design are

\[ Q_c = 1.0E3 \times I \]
\[ Q_e = 1.0E2 \times I \]
\[ R_c = I \]
\[ R_e = I \]

where the subscripts \( c \) and \( e \) are denoted as controller and estimator, respectively. From the Figure 3.23, the estimator based optimal controller can damp out the vibration of the beam well.
Figure 3.22: dSPACE Experiment
Figure 3.23: Time Responses
CHAPTER 4

ACTIVE AND PASSIVE VIBRATION CONTROL OF LANDING
GEAR COMPONENTS

4.1 Frequency Response Analysis for Passive Vibration Control Design through
Finite Element Model Approach

While closed-form solutions are desirable, they are not easily obtained for
distributed parameter systems [38]. Sometimes approximate solutions are more practical
and equally desirable for these systems. A discretization method like ‘finite element
analysis’ represents one way to obtain precise approximate solutions for distributed
parameter systems. In aerospace applications, generally, structural systems are too
complicated to have closed-form solutions. Thus, finite element analysis is one of the
methods used to analyze aircraft structures. Several studies [46, 16] have analyzed
flexible aircraft structures via finite element analysis. One focus of this study is to
dissipate bending vibrations of a Boeing 747 main landing gear break rod. A dynamic
absorber and constraint layer damping approaches are applied to isolate the vibrations.
Finite element analysis is employed to analyze the modal responses of the tube and to
design a dynamic absorber.
4.1.1 Finite Element Model (FEM) and Modal Analysis for a Steel Tube (Equivalent to a Boeing 747 Main Landing Gear Break Rod) using Beam Elements

It is true that the higher degree of freedom produces more accurate solutions in finite element model (FEM) analysis. However, the minimization of the degree of freedom without reducing the accuracy of the solution is also beneficial for computational issue. In order to achieve a relatively precise solution and to reduce the degrees of freedom for this bending vibration problem, beam elements, shown in Figure 4.2, are chosen to analyze the modal responses of the steel tube, as shown in Figure 4.1. Because of implementation issue and the cost of components, a steel tube was used to approximate the actual break rod, shown in Figure 4.3. A comparison of both the steel tube and the Boeing 747 brake rod is given later in the chapter. The overall finite element model of the tube is shown in Fig 4.4.
Figure 4.1: Steel Tube

Figure 4.2: 4 Degree of Freedom Beam Element
Figure 4.3: Boeing 747 Landing Gear Component

Figure 4.4: Finite Element Model of the Tube
The displacement field of the element is expressed as

\[
V(x, t) = N_1(x)v_1(t) + N_2(x)v_2(t) + N_3(x)v_3(t) + N_4(x)v_4(t)
\]

\[=[N]q\]  \hspace{1cm} (4.1)

where the vector of shape functions is given by, \([N]\), and the generalized coordinates \(q\) are expressed as

\[
q = [v_1 v_2 v_3 v_4]^T\]  \hspace{1cm} (4.2)

\[
[N] = [N_1 \ N_2 \ N_3 \ N_4]
\]

\[
N_1(x) = 1 - 3\left(\frac{x}{l}\right) + 2\left(\frac{x}{l}\right)^3
\]

\[
N_2(x) = \frac{x}{l}\left[\left(\frac{x}{l}\right) - 2\left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right]
\]

\[
N_3(x) = 3\left(\frac{x}{l}\right)^2 - 2\left(\frac{x}{l}\right)^3
\]

\[
N_4(x) = l\left[-\left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right]\]  \hspace{1cm} (4.3)

The strain energy of the system is expressed as

\[
U = \frac{1}{2} \int \sigma_x : \varepsilon_x \, dA = \frac{1}{2} \int E \varepsilon_x \cdot \varepsilon_x \, dA
\]

\[= \frac{1}{2} \int \varepsilon_x - \varepsilon_y \, dA\]  \hspace{1cm} (4.4)

where \(E\) is Young's Modulus. From the curvatures of the beam, the strain is written as

\[
-\frac{\varepsilon_x}{z} \equiv \frac{\partial^2 w}{\partial x^2} \quad \Rightarrow \quad \varepsilon_x = -z \frac{\partial^2 w}{\partial x^2}
\]  \hspace{1cm} (4.5)

Thus, the strain energy can be rewritten as

\[
U = \frac{E}{2} \int \left(z \frac{\partial^2 w}{\partial x^2}\right) \cdot \left(z \frac{\partial^2 w}{\partial x^2}\right) \, dA = \frac{E}{2} \int I(x) \left(\frac{\partial^2 w}{\partial x^2}\right)^2 \, dx
\]  \hspace{1cm} (4.6)
where $I(x)$ is the moment of inertia. Substituting Equation 4.1 into Equation 4.6 gives
the stiffness matrix for the element which is expressed as

$$[K] = E \int_0^l [N^*]^T [N^*] \, dx$$

(4.7)

where

$$[N^*] = \begin{bmatrix}
\frac{\partial^2 N_1}{\partial x^2} & \frac{\partial^2 N_2}{\partial x^2} & \frac{\partial^2 N_3}{\partial x^2} & \frac{\partial^2 N_4}{\partial x^2}
\end{bmatrix}$$

(4.8)

Substituting Equation 4.1 into Equation 2.10 gives the corresponding mass matrix which
is expressed as

$$[M] = \rho \int A(x) [N]^T [N] \, dx$$

(4.9)

where $\rho$ and $A$ are the mass density and cross-sectional area of the element,
respectively. The equation of motion of the tube is expressed as

$$[M][\ddot{q}] + [K][\dot{q}] = 0.$$  

(4.10)

The eigenvalue problem of Equation 4.10 is expressed as

$$[K][u] = \lambda[M][u]$$

(4.11)

where $\lambda = -s^2$ and $s$ is the frequency of the system. Table 4.1 shows the comparison of
the natural frequencies for the first two modes using the actual Boeing 747 break rod
(ANSYS), and the steel tube. A Matlab FEM program, a commercial package (ANSYS),
and experimental modal testing are used for validation. From the table, one can see that
the modal analysis methods are close to the actual experiments.
\begin{tabular}{|c|c|c|}
\hline
 & Weight (lb.) & Fist Mode (Hz) & Second Mode (Hz) \\
\hline
B 747 Break Rod & 14.2 & 478.58 & 1352 \\
\hline
Steel Tube & ANSYS (Solid) & 479.72 & 1248 \\
& Matlab (Beam) & 465.53 & 1283.3 \\
& Experiment & 464.3 & 1205.6 \\
\hline
\end{tabular}

Table 4.1: Modal Analysis

The corresponding mode shapes are given in Figure 4.5 through Figure 4.9. Figures 4.5 and 4.6 show the modal analysis of the Boeing 747 landing gear component with solid elements in ANSYS. The modal analysis of the tube with solid elements in ANSYS are given in Figures 4.7 and 4.8. The Matlab plot of the beam element analysis of the tube is shown in Figure 4.9. According to the table and figures, the results of finite element model analysis of the steel tube with beam elements are close to those of the landing gear components with solid elements.
Figure 4.5: First Mode of the Landing Gear Component (ANSYS)
Figure 4.6: Second Mode of the Landing Gear Component (ANSYS)
Figure 4.7: First Mode of the Steel Tube (ANSYS)
Figure 4.8: Second Mode of the Steel Tube (ANSYS)
4.1.2 FEM for Dynamic Absorber and Passive Vibration Control of the Integrated System

A number of studies have been completed for vibration control of beam type systems using dynamic absorbers [28, 40, 32, 1]. In Ref. [1], the optimal design techniques of the dynamic absorbing beam are explained. Multiple mode control with lumped mass dynamic absorbers is illustrated in Ref. [40]. In this study, the fundamental mode of the tube is damped using a dynamic absorber. Since the fundamental frequency of the tube is known as 464.3 Hz from both finite element analysis and experimental
results, the fundamental frequency of the dynamic absorber has to be close to that of the tube. A steel bar is chosen which is clamped at the center as a dynamic absorber. For the active control purpose, the minimum surface areas of the bar need to be greater than those of the actuators. The PZT actuator used in this study has the following dimensions $1.5" \times 2.72" \times 0.01"$. The dimension of the bar is thus chosen to be $1.75" \times 10.87" \times 0.5"$. Note the another constraint in the bar’s dimension is that the natural frequency must match the steel tube [39]. The fundamental frequency of the bar is 465 Hz. The finite element model and the actual experimental setup of the integrated system are shown in Figure 4.10 and Figure 4.11, respectively.

![Figure 4.10: Finite Element Model of the Integrated System](image)
Figure 4.11: Experimental Setup of the Tube with the Dynamic Absorber

Frequency responses from the Matlab simulations and the experimental results of the integrated system are given in Fig.4.12. From the figure, the experimental results are well matched to the simulations. The frequency for the fundamental mode of the integrated system is split into two modes at 378.12 Hz and 580 Hz, respectively. The model shape of the fundamental mode of the integrated system is shown in Figure 4.13. From the mode shape of the system, it is clearly seen that the absorber is 180° out of phase.
Figure 4.12: Frequency Responses of the Integrated System

Figure 4.13: Fundamental Mode Shape of the Integrated System
4.2 Constrained Layer Damping for the Dynamic Absorber

According to Ref. [39], the dynamic absorbers can be used as damping devices if the absorbers have damping. Since the damping ratio of the steel bar is very low, external damping effects need to be applied to the bar. The most common passive damping device for the beam is constrained layer damping [23, 5]. In this study, a three-layered sandwich bar as shown in Figure 4.14 is selected to maximize the damping effect. The actual thickness of each steel layer is a half of the thickness of original bar. The viscoelastic material called ‘DYAD 320’ from ‘SOUNDCOAT’ is selected as a constrained layer. Figure 4.15 shows the frequency responses for the constrained layer damping in comparison with the previous experimental results.

Figure 4.14: Constrained Layer
Figure 4.15: Frequency Responses with the Constrained Layer Damping
4.3 Active Vibration Control using PZT Actuators

Recently, active vibration control based on smart materials has increased in popularity. The reasons for the increase in popularity is twofold: First, these materials have the ability to be used as both sensors and actuators. Second, these materials are lightweight and can be easily attached to the surfaces of structures. Because of these reasons, smart structure based active control has been studied both theoretically [46, 16] and experimentally [37].

Figure 4.16: PZT Actuator Setup
4.3.1 Optimal Control Simulation based on Reciprocal State Space Framework

The finite element model procedure with piezoelectric actuator is expressed in Chapter 2. The equation of motion of the integrated system shown in Figure 4.16 is expressed as

\[(M_q + M_p)\ddot{q} + (K_q + K_p)q = -K_{qp}\nu\] (4.12)

where \(q\) and \(p\) are denoted as the structural layer and piezoelectric layer, respectively.

The state space representation of the system is expressed as

\[
\begin{bmatrix}
\dot{q} \\
\ddot{q}
\end{bmatrix} = 
\begin{bmatrix}
0 & I \\
-(M_q + M_p)^{-1}(K_q + K_p) & 0
\end{bmatrix}
\begin{bmatrix}
q \\
\dot{q}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
-(M_q + M_p)^{-1}K_{qp}
\end{bmatrix}\nu
\] (4.13)

or

\[x = Ax + Bu\] (4.14)

where \(x = [q \quad \dot{q}]^T\).

The reciprocal state space of the system is written as

\[x = Gx + Hu\] (4.15)

\[u = K\dot{x}\] (4.16)

where

\[G = A^{-1}, H = -A^{-1}B\]

For the LQR design technique, the performance index to be minimized is expressed as

\[J = \int_0^t [\dot{x}^TQ\dot{x} + u^TRu]dt\] (4.17)

where \(Q\) and \(R\) are positive semi-definite and positive definite matrices, respectively.

According to Ref [31, 41], the LQR state feedback gain is written as

\[K = R^{-1}H^TS\] (4.18)
The matrix $S$ can be obtained from the associated algebraic matrix Riccati Equation which is expressed as

$$0 = SG + G^T S - SHR^{-1}H^T S + Q$$  \hspace{1cm} (4.19)

Both the controller and estimator can be designed by applying LQR design techniques.

The controller and estimator of the system are given by the following

$$x = G\dot{x} + Hu$$
$$\dot{x} = G\dot{x} + Hu + F(y(t) - \hat{y}(t))$$  \hspace{1cm} (4.20)

$$y(t) = Cx, \quad u = -L\dot{x}$$
$$\hat{y}(t) = C\dot{x}$$

For implementation, the equations are reorganized as

$$\dot{x} = G^{-1}x + G^{-1}HL\dot{x} = Ax - BL\dot{x}$$
$$\dot{\hat{x}} = P^{-1}\dot{x} - P^{-1}FCK$$  \hspace{1cm} (4.21)

where $P = G - HL - FC$. The simulation diagram of the Equation is shown in Figure 4.17. According to the Figures 4.18 through 4.20, the reciprocal state space based controller with acceleration feedback can be used to dissipate the vibration of the system.
Figure 4.17: Simulation Diagram in Simulink
Figure 4.18: Time Responses for the Displacements
Figure 4.19: Time Responses for the Velocities
Figure 4.20: Time Responses for the Accelerations
4.3.2 Fuzzy Logic Control with Acceleration Measurement in dSPACE Platform

In this section, a fuzzy logic based active vibration controller is employed using the dSPACE platform to dissipate the vibration of the combined dynamic absorber and steel tube system. A detailed expression of the fuzzy logic controller is given in Ref. [37]. An accelerometer is attached to the one side of the dynamic absorber, and the measurement from the accelerometer is used as the input signal for the fuzzy controller. Note that zero acceleration means zero displacement in nonzero frequency modes. Thus, an acceleration measurement is sufficient to regulate the vibration.

For this specific problem, the rule base is divided to 11 different membership functions. The rule base of the fuzzy logic controller of the problem is shown in Table 4.2.

<table>
<thead>
<tr>
<th>Acceleration Measurement</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Output</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 4.2: Rule Base

Since the voltage range of the sensor is ±4V, the range of each interval in the rule base is 0.8V. The normalized membership functions of the controller for both input and output are shown in Figure 4.21.
Figure 4.21: Membership Functions

Figure 4.22 shows the frequency responses of the active control in comparison to the previous experimental results. Active vibration control using PZT actuators based on Fuzzy logic controller can reduce vibration magnitude by 50% of the constrained layered damping case.
Figure 4.22: Frequency Responses of the Active Control of the Tube
In this section, the issue of designing observers for linear matrix second order systems in the matrix second order framework before they transform to the state space framework is addressed. Several important reasons are presented to justify the need for this approach. In addition, the advantages of the second order observer are explained and contrasted with those of the typical first order observer. Taking various assumptions on available measurements into consideration, conditions of existence and design methodologies are presented to synthesize the observer gains.

Recently, stability and control issues for ‘Matrix Second Order (MSO)’ systems have attracted considerable attention especially for applications involving flexible aircraft structures and other aerospace applications as well.\cite{16, 42} For example, in the smart flexible aircraft structural control area, new smart deformable wings are proposed to enable the aircraft to have faster and gentler maneuvers as well as more efficient and safer performance (emulating a bird in flight) by using the interaction among the elastic structures, smart materials, and external aerodynamic loads. The dynamics of such multidisciplinary flexible aircraft structures can be expressed in the form of a finite dimensional multivariable MSO differential equation through the well known finite
The controllers and observers are designed, in general, after the MSO systems are converted to the standard first order state space framework.

However, recent researches [16, 42] recognize that there are several problems associated with the transformation of the MSO system to the state space framework. First, it is difficult to preserve the physical insight of the MSO system (with its mass, stiffness and damping coefficient matrices) when it is transformed to the first order state space form. Consider a typical MSO differential equation given by

$$ M\ddot{q} + C\dot{q} + Kq = Bf $$  \hspace{1cm} (5.1)

where $M_{nxn}$, $C_{nxn}$ and $K_{nxn}$ are mass, damping and stiffness matrices, respectively. $B_{nx1}$ is an actuator distribution matrix. $q_{nx1}$ and $f_{nx1}$ are generalized coordinate and external forcing vectors, respectively. The state-space representation for the above MSO differential equation is expressed as

$$ \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} f $$  \hspace{1cm} (5.2)

$$ \dot{x} = Ax + Bu $$

In the MSO system, as shown in Equation 5.1, valuable physical insight can be retained by analyzing the mass, damping, and stiffness coefficient matrices $M$, $C$ and $K$. The orthogonality of the system also can be determined by simply looking at the symmetric configuration of $M$ and $K$ matrices. However, this valuable physical insight is lost in the state space form because the mass, damping, and stiffness coefficient matrices are mixed into the system matrix $A$ as shown in Equation 5.2. Secondly, in the state space framework, the system size becomes large [16]. Thirdly and the most importantly, control designs in the state space framework do not exploit the availability of direct
acceleration measurements which are shown to be effective in feedback control to minimize control effort[42].

Moreover, in this chapter, another important problem is addressed regarding why the state space framework is inadequate particularly for observer design. Recently, it is pointed out explicitly in Ref. [7] that the correct estimate values for the second order system configuration coordinates may not be obtained through the first order observer design. The disadvantages of the observer design (for MSO systems) in the first order state space framework can be removed by designing the controllers and observers in the MSO framework before the controllers and observers transform to the state space framework as proposed in the literature [16, 42]. In this chapter, the need to design observers directly in MSO framework is clearly justified. In addition, new second observer design schemes are proposed by taking different scenarios of available measurements. Then the determination of these observer gains is shown as similar to the output feedback problem of the standard state space framework.

5.1 The Disadvantages of Observer Design in First Order State Space for MSO Systems

The first order state-space control design technique which was introduced in late 1950’s has been a very useful and important scheme to handle dynamic systems. The advantages of the state-space are followed. First, most of higher order dynamic systems can be described by a set of the first order differential equations. At second, it is easy to handle the multiple-input multiple-output (MIMO) system. Finally, a number of stability and control theories exist.
However, there are several disadvantages to handle the second order system in the first order state-space. In fact, not every higher order differential equation can be converted to the first order state space. Especially, the MSO differential equation cannot be transformed to the first order state space if the mass coefficient matrix is singular.

Another disadvantage is that it is impossible to apply the various combinations of measurements. For example, the acceleration measurement cannot be directly applied as a feedback signal in the first order state space based observer design. The most significant disadvantage among them is the mismatch of the estimated values for the velocity information, \( \hat{q} \) and \( \dot{q} \), in the first order observer design.[7] The typical first order observer is written as

\[
\begin{bmatrix}
\dot{\hat{q}} \\
\ddot{\hat{q}}
\end{bmatrix} = 
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
\hat{q} \\
\dot{\hat{q}}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
M^{-1}B
\end{bmatrix} u + 
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}[y - \hat{y}] 
\]

\( y = Nx \)

\( \hat{y} = N\hat{x} \)

where \( F_1 \) and \( F_2 \) are the observer gain matrices and \( N_{\text{bin}} \) is a measurement matrix.

Subtract Equation 5.3 from Equation 5.2, then

\[
\begin{bmatrix}
\dot{\hat{q}} - \dot{\hat{q}} \\
\ddot{\hat{q}} - \ddot{\hat{q}}
\end{bmatrix} = 
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
q - \hat{q} \\
\dot{q} - \dot{\hat{q}}
\end{bmatrix}
+ 
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}[N_1 \quad N_2]
\begin{bmatrix}
q - \hat{q} \\
\dot{q} - \dot{\hat{q}}
\end{bmatrix} 
\]

Let \( e_1 = q - \hat{q} \) and \( e_2 = \dot{q} - \dot{\hat{q}} \), then Equation 5.4 is written as

\[
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix} = 
\begin{bmatrix}
F_1N_1 & I + F_1N_2 \\
-M^{-1}K + F_2N_1 & -M^{-1}C + F_2N_2
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix} 
\]

(5.5)
From Equation 5.5, $\dot{e}_1 \neq e_2$ unless $F_1 = 0$, which is usually not zero in the first order observer. As a result, the unique values for $\hat{\dot{q}}$ and $\hat{\dot{q}}$ cannot be obtained through the first order observer. The differences between $\hat{\dot{q}}$ and $\hat{\dot{q}}$ are easily observed if Equation 5.3 is rewritten as

$$\dot{\hat{q}} = \dot{\hat{q}} + F_1 [y - \hat{y}]$$

### 5.2 New Approaches for Linear Matrix Second Order Observer

The disadvantages of the observer design (for MSO systems) in the first order state space framework can be removed by designing the controllers and observers in the MSO framework before they transform to the state space framework as proposed in the literature [16, 42]. Put it another way, this newly developed second order observer design scheme can achieve a unique $\hat{\dot{q}}$. The observer design scheme is written as

$$M\ddot{\hat{q}} + C\dot{\hat{q}} + K\hat{q} = B\dot{f} - L_d(y_d - \hat{y}_d) - L_v(y_v - \hat{y}_v) - L_a(y_a - \hat{y}_a)$$

(5.6)

where $L_d(n \times d)$, $L_v(n \times d)$, and $L_a(n \times d)$ are the observer gain matrices. $H_{d(l, \alpha)}$, $H_{v(l, \alpha)}$, and $H_{d(l, \alpha)}$ are the sensor distribution matrices. From Equation 5.6, a unique estimate value of the velocity information can be obtained through the second order observer. This can be more clearly demonstrated when the above observer scheme is written in the first order state-space form. If it is assumed that only displacement and velocity measurements are available, the Equation 5.6 can be rewritten as

$$M\ddot{\hat{q}} + C\dot{\hat{q}} + K\hat{q} = B\dot{f} - L_v(y_v - \hat{y}_v) - L_d(y_d - \hat{y}_d)$$

(5.7)
\[ y_v = H_v \dot{q}, \quad y_d = H_d \dot{q} \]

\[ \hat{y}_v = H_v \hat{\dot{q}}, \quad \hat{y}_d = H_d \dot{q} \]

The state-space form of the second order observer is expressed as

\[
\begin{bmatrix}
\dot{\hat{q}} \\
\dot{\hat{\dot{q}}}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
\hat{q} \\
\hat{\dot{q}}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
M^{-1}B
\end{bmatrix} f - \begin{bmatrix}
0 \\
M^{-1}L_d H_d \\
M^{-1}L_v H_v
\end{bmatrix}
\begin{bmatrix}
q - \hat{q} \\
\dot{q} - \hat{\dot{q}}
\end{bmatrix}
\]  

(5.8)

where

\[
\hat{x} = \begin{bmatrix} \hat{q} \\ \hat{\dot{q}} \end{bmatrix}^T
\]

\[
\hat{e} = \begin{bmatrix} q - \hat{q} \\ \dot{q} - \hat{\dot{q}} \end{bmatrix}^T
\]

From Equations 5.8 and 5.9, it is observed that the two partition matrices, \( G_{11} \) and \( G_{12} \), which are equivalent to \( F_1 \) in the first order observer in Equation 5.3, are inherently zero in the second order observer, while \( F_1 \) is usually not zero in the first order observer.

Because of these zero partition matrices, the partition matrices \( A_{11} \) and \( A_{12} \) of system matrix \( A \) in Equation 5.9 also remain as zero and identity matrices, respectively, in the closed loop error dynamics. In other words, the characteristics of the second order dynamics are preserved in the second order observer design. Moreover, since the first order form of the matrix second order observer, Equation 5.8, can preserve the advantages of the second order observer, the first order state space design schemes can still be applied to find the second order observer gains \( H_v \) and \( H_d \). Note that observer designs are always very much dependent on the assumptions made on the availability of measurements. It is important to realize that the observability criteria in the state space
framework with the assumed measurement equations may not guarantee the existence of the second order observer gains because of the special structure of the matrix second order observer as shown in Equation 5.8. This is the main reason and justification for the need to design observers directly in the MSO framework! In the next section, the design of observers in the matrix second order framework for each measurement case is discussed.

5.2.1 The Matrix Second Order Observer Design with the Displacement-Velocity Measurements

The matrix second order observer in case of displacement and velocity measurements is shown in Equation 5.7. By subtracting Equation 5.7 from the Equation 5.1 and letting \( e = q - \dot{q} \), the error dynamic of the second order observer is written as

\[
M\ddot{e} + C\dot{e} + Ke = L_v H_v \dot{q} + L_d H_d e
\]  
(5.10)

The state space form of Equation 5.10 is expressed as

\[
\begin{bmatrix}
\dot{e} \\
\ddot{e}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
e
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
M^{-1}L_d H_d & M^{-1}L_v H_v
\end{bmatrix}
\begin{bmatrix}
e \\
\dot{e}
\end{bmatrix}
\]  
(5.11)

The Equation 5.11 can be reorganized as

\[
\begin{bmatrix}
\dot{e} \\
\ddot{e}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
e
\end{bmatrix} +
\begin{bmatrix}
0 \\
M^{-1}
\end{bmatrix}
\begin{bmatrix}
L_d & L_v
\end{bmatrix}
\begin{bmatrix}
H_d & 0 \\
0 & H_v
\end{bmatrix}
\begin{bmatrix}
e \\
\dot{e}
\end{bmatrix}
\]  
(5.12)

\[
\ddot{e} = (A_t + F_t L_t H_t) \dot{e}
\]
It is interesting that Equation 5.12 is in the form of the closed loop dynamics of an output feedback control design. Consequently, these observer gains can be obtained through such output feedback controller design methods.

5.2.2 The Matrix Second Order Observer Design with Velocity-Acceleration Measurements

The matrix second order observer in case of the velocity-acceleration measurements is expressed as

\[ M \ddot{q} + C \dot{q} + Kq = Bf - L_{u}(y_{u} - \hat{y}_{u}) - L_{v}(y_{v} - \hat{y}_{v}) \]  \hspace{1cm} (5.13)

\[ y_{u} = H_{u} \dot{q}, \quad y_{v} = H_{v} \dot{q} \]
\[ \hat{y}_{u} = H_{u} \hat{q}, \quad \hat{y}_{v} = H_{v} \hat{q} \]

By subtracting Equation 5.13 from the Equation 5.1 and letting \( e = q - \hat{q} \), the error dynamic of the second order observer is written as

\[ M \ddot{e} + C \dot{e} + Ke = L_{u}H_{u} \dot{e} + L_{v}H_{v} \dot{e} \]  \hspace{1cm} (5.14)
Since the acceleration measurement cannot be directly used in the standard state space framework, the newly introduced ‘Reciprocal State Space’ [42] framework is applied to the system to find the observer gains. Equation 5.14 is rewritten as

\[ e = -K^{-1}M \ddot{e} - K^{-1}C \dot{e} + K^{-1}L_a H_a \ddot{e} + K^{-1}L_v H_v \dot{e} \]

or

\[
\begin{bmatrix}
\dot{e} \\
\ddot{e}
\end{bmatrix} = \begin{bmatrix}
-K^{-1}C & -K^{-1}M \\
I & 0
\end{bmatrix} \begin{bmatrix}
e \\
\dot{e}
\end{bmatrix} + \begin{bmatrix}
K^{-1}L_v H_v & K^{-1}L_a H_a
\end{bmatrix} \begin{bmatrix}
\dot{e} \\
\ddot{e}
\end{bmatrix}
\]

(5.15)

Equation 5.15 also can be reorganized in a form of the closed loop output feedback control as

\[
\begin{bmatrix}
\dot{e} \\
\ddot{e}
\end{bmatrix} = \left( A + F_2 L_2 H_2 \right) \bar{e}
\]

\[ A = \begin{bmatrix}
-K^{-1}C & -K^{-1}M \\
I & 0
\end{bmatrix} \quad (2n \times 2n) \]

\[ F_2 = \begin{bmatrix}
K^{-1}L_v & 0 \\
0 & H_v
\end{bmatrix} \quad (n \times l_2) \]

\[ L_2 = \begin{bmatrix}
H_v & 0 \\
0 & H_v
\end{bmatrix} \quad (l_2 \times 2n) \]

\[ \bar{e} = \begin{bmatrix}
e \\
\dot{e}
\end{bmatrix} \]

\[ l_1 = l_a + l_v \]

Since the stability criteria of the ‘Reciprocal State Space’ are same as those of the standard state space as proven in Ref. [42], the observer gain \( L_2 \) can be determined by applying the output feedback controller design schemes.
5.2.3 The Matrix Second Order Observer Design with Acceleration-Displacement Measurements

The matrix second order observer in case of the acceleration-displacement measurements is expressed as

\[ M \ddot{q} + C \dot{q} + Kq = Bf - L_a (y_a - \hat{y}_a) - L_d (y_d - \hat{y}_d) \]  \hspace{1cm} (5.17)

By subtracting Equation 5.17 from the Equation 5.1 and letting \( e = q - \hat{q} \), the error dynamics of the second order observer are written as

\[ M \ddot{e} + C \dot{e} + Ke = L_a H_a \dot{\hat{q}} + L_d H_d \dot{\hat{q}} \]  \hspace{1cm} (5.18)

The error dynamics of the system cannot be directly converted to either state space or reciprocal state space frameworks in the acceleration and displacement measurement case. Hence, a new approach is applied to obtain the observer gains, \( L_{a2} \) and \( L_{d2} \), as suggested in the Ref.[42]. As in Ref. [42], it is assumed that the sensor matrices, \( H_a \), \( H_v \) and \( H_d \) are identical. Let \( H_a \), \( H_v \) and \( H_d \) be \( H \). If the mass matrix, \( M \), is nonsingular, Equation 5.18 can be compared to Equation 5.10 to obtain the observer gains \( L_{a2} \) and \( L_{d2} \). Equations 5.10 and 5.18 can be rewritten as

\[ \ddot{e} + M^{-1} (C - L_v H) \dot{e} + M^{-1} (K - L_d H) e = 0 \]  \hspace{1cm} (5.19)

\[ \ddot{e} + (M - L_{a2} H)^{-1} C \dot{e} + (M - L_{a2} H)^{-1} (K - L_{d2} H) e = 0 \]  \hspace{1cm} (5.20)

By comparing Equation 5.19 with Equation 5.20, the observer gains \( L_{a2} \) and \( L_{d2} \) can be obtained as
\[ L_{a_2} = -M(C - L_v H)^{-1}L_v \] (5.21)
\[ L_{d_2} = L_d - (K - L_d H)(C - L_v H)^{-1}L_v \] (5.22)

Detailed steps to arrive to the above observer gains are given in the Ref.[42]. If the stiffness matrix, \( K \), is nonsingular, Equation 5.18 can be compared to Equation 5.14 to obtain the observer gains \( L_{a_2} \) and \( L_{d_2} \). Equations 5.14 and 5.18 can be rewritten as

\[
K^{-1}(M - L_a H)\ddot{e} + K^{-1}(C - L_v H)\dot{e} + e = 0
\]
\[
(K - L_{a_2} H)^{-1}(M - L_{a_2} H)\ddot{e} + (K - L_{d_2} H)^{-1}C\dot{e} + e = 0
\]

By applying the similar method to the \( M \) invertible case, the observer gains, \( L_{a_2} \) and \( L_{d_2} \), can be obtained as

\[ L_{a_2} = L_a - (M - L_a H)(K - L_v H)^{-1}L_v \] (5.23)
\[ L_{d_2} = -K(K - L_v H)^{-1}L_v \] (5.24)

Therefore, after obtaining the observer gains, \( L_v \) and \( L_a \) or \( L_d \), from either Equation 5.12 or Equation 5.16, the corresponding observer gains \( L_{a_2} \) and \( L_{d_2} \) can be obtained from Equations 5.21 and 5.22 or Equations 5.23 and 5.24.

5.2.4 Existence of Second Order Observer Gains is Similar to the State Space Output Feedback Control Problem

From the above discussion, it is clear that separate controllability and observability criteria are required to design observer based controllers for the MSO systems. While several controllability and observability criteria have been introduced for the MSO system [10, 25, 35, 20], the arguments made in the previous sections of this
research clearly demonstrated that these criteria are not useful especially for the observer
design. That most previous studies about the controllability and observability criteria of
the second order systems are based on their conversion to a first order form raises the
question followed. Can a second order controller and observer be designed if the system
is controllable and observable in the first order state-space framework? An answer is
‘Yes’ for the controller design but ‘No’ for the observer design. For example, the system,
\[
\begin{align*}
\ddot{q} + q &= u, \\
y &= q
\end{align*}
\]
cannot produce stable error dynamics in the second order observer although it is still
observable in the state space form [10, 25, 35, 20].

Therefore, in this section, new necessary and sufficient conditions for the
suggested second order observer design are introduced. As shown in Equations 5.12 and
5.16, the closed loop error dynamics are in the form of the closed loop output feedback
control. Thus, the existence of the observer gains is determined using the output
feedback controllability criteria. If the systems, Equations 5.12 and 5.16, are stabilizable,
the second order gain matrices can exist. As a result, the following conditions are
obtained.

**Theorem 1** When the mass matrix, $M$, is invertible, the necessary and sufficient
condition for the second order observer with velocity and displacement measurements is
that the triple $(A, H, F)$ is output feedback stabilizable.
Theorem 2 When the stiffness matrix, $K$, is invertible, the necessary and sufficient condition for the second order observer with velocity and acceleration measurements is that the triple $(A_2, H_2, F_2)$ is output feedback stabilizable.

The controllability and stabilizability of the output feedback have been important issues for the feedback control design scheme since the output feedback control is more practical than the full state feedback control. Several studies[19, 15, 45, 33] have done to determine the output feedback controllability and stabilizability. A typical closed loop system for the output feedback control is given as

$$A_c = (A + BK_C)$$

where,

$A : n \times n$

$B : n \times m$

$C : l \times n$

Theorem 4 If $A$ has distinct or repeated eigenvalues so that the eigenvalues of each Jordan block of the Jordan canonical form of $A$ are distinct, the stabilizability of the system by linear output feedback is feasible if and only if [19, 15, 45]

a) the $p$ fixed modes of the system are stable

b) $(A', B')$, which is the unstable subspace in the Jordan canonical form, is controllable

c) the rank of the output matrix $C$ satisfies: $l = n - p$

(The detail proof is provided in Ref.[19]).
Another necessary and sufficient condition for the output feedback stabilizability is given in Ref.[33].

**Theorem 5** A state space system is output feedback stabilizable if and only if

i) \((A, B)\) is stabilizable and \((A, C)\) is detectable,

ii) there exist real matrices \(Q\) and \(R\) such that

\[
QC + B^T S = R
\]

where \(S\) is the real symmetric nonnegative-definite solution of

\[
A^T S + SA - SBB^T S + C^T C + R^T R = 0
\]

The detail proof is given in Ref.[33]

5.3 Measurement Conditions and Perspectives in Matrix Second Order Framework

### 5.3.1 Conservative Non-Gyroscopic System

The conservative non-gyroscopic system is expressed as

\[
M\ddot{q} + Kq = Bf
\]  \hspace{1cm} (5.25)

where \(M_{\text{nxc}}\) and \(K_{\text{nxc}}\) are real symmetric matrices and \(M\) is positive definite.
5.3.1.1 Without Velocity Measurements

The matrix second order observers for the conservative non-gyroscopic system in case of **Displacement-Acceleration Measurement** are expressed as

\[ M\ddot{q} + K\dot{q} = Bf - L_\alpha (y_\alpha - \hat{y}_\alpha) - L_d (y_d - \hat{y}_d) \]  
\[ y_\alpha = H_\alpha \ddot{q}, \quad y_d = H_d \dot{q} \]
\[ \hat{y}_\alpha = H_\alpha \dot{\hat{q}}, \quad \hat{y}_d = H_d \ddot{\hat{q}} \]

and \( L_\alpha, L_d, H_\alpha, \) and \( H_d \) are \( n \times l_\alpha, n \times l_d, l_\alpha \times n, \) and \( l_d \times n \) matrices, respectively. By subtracting Equations 5.26 and 5.25, the error dynamics of the given observer are expressed as

\[ M\ddot{e} + Ke = L_\alpha H_\alpha \dot{e} + L_d H_d e \]

or

\[ M'\ddot{e} + K'e = 0 \]  
\[ M' = M - L_\alpha H_\alpha \]
\[ K' = K - L_d H_d \]

The eigenvalue problem of Equation 5.27 is expressed as

\[ \lambda u = \lambda u \]

where \( \lambda = -s^2 \). In order to have correct estimate values, the error dynamics of the system have to be asymptotically stable. Namely, every real part of the eigenvalues, \( s \), in Equation 5.28 has to be negative. Since \( \lambda \)s are categorized as real, pure imaginary, and complex numbers, however, all real parts of eigenvalues, \( s \), cannot be negative. Therefore, the error dynamics of Equation 5.27 cannot be asymptotically stable with any
combination of $L_d$ and $L^*$. As a result, velocity measurements are required to build a
second order observer for a conservative non-gyroscopic system.

**Measurement Condition 1** The velocity measurements are necessary to build the second order observer for the conservative non-gyroscopic system.

### 5.3.1.2 With Velocity Measurements

The second order observer for the conservative non-gyroscopic system with velocity measurements is expressed as

\[ M \ddot{\hat{q}} + K \dot{q} = B f - L_v (y_v - \dot{y}_v) \]  \hspace{1cm} (5.29)

\[ y_v = H_v \dot{q} \]

\[ \dot{y}_v = H_v \ddot{q} \]

where $L_v$ and $H_v$ are $n \times l_v$ and $l_v \times 1$ matrices, respectively. By subtracting Equation 5.29 from Equation 5.25, the error dynamics of the system are expressed as

\[ M \ddot{e} + K e = L_v H_v \dot{e} \]

or

\[ M \ddot{e} + C \dot{e} + K e = 0 \]  \hspace{1cm} (5.30)

where

\[ C = -L_v H_v \]  \hspace{1cm} (5.31)

From Equation 5.30, a necessary condition can be obtained. The state-space form of Equation 5.30 can be written as

\[ 92 \]
If $K$ is singular, $M^{-1}K$ is also singular. Thus, the system matrix of Equation 5.32 is singular with any $C$. As a result, the matrix $K$ has to be non-singular in order to have asymptotically stable error dynamics with velocity measurement.

**Design Observation 1** A necessary condition for the existence of the observer gain, $L_v$, is that stiffness matrix $K$ is non-singular.

According to the assumption of the conservative non-gyroscopic system, stiffness matrix $K$ can be symmetric semi-definite (Singular). Therefore, the following necessary condition is obtained.

**Measurement Condition 2** Displacement measurements are necessary to build the second order observer for the conservative non-gyroscopic systems if stiffness matrix $K$ is positive semi-definite.

Now, it is necessary to determine the conditions of measurement matrix $H_v$ in Equation 5.30 when a system has positive definite $K$. From Ref. [17], a matrix second order system is asymptotically stable if the matrices $M$, $K$, and $C$ are positive definite. If the $C$ is positive definite, thus, the error dynamics (Equation 5.30) are asymptotically stable. Let matrix $C_d$ be positive definite and dimension of $n \times n$. Then Equation 5.31 can be written as

$$C_d = L_v H_v$$  \hspace{1cm} (5.33)
Design Observation 2 A sufficient condition for the existence of the observer gain, $L_\nu$, is that $\text{rank}(H_\nu) \geq n$ for the conservative non-gyroscopic system with velocity measurements if stiffness matrix $K$ is positive definite.

5.3.2 Conservative Gyroscopic System

The matrix second order observer for the conservative gyroscopic system is expressed as

$$M\ddot{q} + G\dot{q} + Kq = Bf$$

where $M_{non}$ and $K_{non}$ are real symmetric matrices and $M$ is positive definite. $G_{non}$ is a skew symmetric matrix.

5.3.2.1 Displacement Measurement Case

The second order observer for the undamped gyroscopic system in case of ‘Displacement Measurement Only’ is expressed as

$$M\ddot{q} + G\dot{q} + K\dot{q} = Bf - L_d (\dot{y}_d - \dot{\hat{y}}_d)$$

$$y_d = H_d\dot{q}$$

$$\dot{\hat{y}}_d = H_d\dot{q}$$

The error dynamics of the system are expressed as

$$M\ddot{e} + G\dot{e} + Ke = L_d H_d e$$

or

$$M\ddot{e} + G\dot{e} + Ke = 0$$  \hspace{1cm} (5.34)
where

\[ K' = K - L_d H_d \]

The eigenvalue problem of Equation 5.34 is expressed as

\[ (s^2 M + sG + K') u = 0 \]  \hspace{1cm} (5.35)

Pre-multiply by \( u^H \) and let

\[ m = u^H M u, \quad i g = u^H G u, \quad k = u^H K' u \]

Equation 5.35 can be written as

\[ ms^2 + igs + k = 0, \quad m > 0, \quad g \text{ is real.} \]

Case 1) \( k \geq 0 \)

\[ s = \frac{- i (g \pm \sqrt{g^2 + 4mk'})}{2m} \]

Thus, \( s \) is always pure imaginary

Case 2) \( k < 0 \)

\[ s = \frac{- i g \pm \sqrt{-g^2 + 4mk'}}{2m}, \quad k' = -k \]

If \( 4mk' > g^2 \), \( s \) is a pair of complex numbers which has one positive real part and one negative real part. If \( 4mk' \leq 0 \), \( s \) is pure imaginary.

Case 3) \( k \) is a complex number. Let \( k = x + iy \). Then,

\[ s = \frac{- i g \pm \sqrt{g^2 + 4m(x + iy)}}{2m} \]

\[ = \frac{- i g \pm \sqrt{X + iY}}{2m} \]
where

\[ X = -g^2 - 4mx, \quad Y = -4my \]

Therefore, \( s \) is a pair of complex numbers which has one positive real part and one negative real part.

**Design Observation 3** The second order observer gain for the undamped gyroscopic system with only displacement measurement does not exist.

### 5.3.2.2 Acceleration Measurement Case

The second order observer for the undamped gyroscopic system in case of ‘Acceleration Measurement Only’ is expressed as

\[
\ddot{Mq} + \dot{G}q + Kq = Bf - L_u (y_u - \hat{y}_u)
\]

\[
y_u = H_a \ddot{q}
\]

\[
\hat{y}_u = H_a \dot{q}
\]

The error dynamics of the system are expressed as

\[
M\ddot{e} + \dot{G}e + Ke = L_u H_a \dot{e}
\]

or

\[
M'\ddot{e} + \dot{G}e + Ke = 0
\]

where

\[
M' = M - L_u H_a
\]  \hspace{1cm} (5.36)

The eigenvalue problem of Equation 5.36 is expressed as

\[
(s^2 M' + sG + K)u = 0
\]  \hspace{1cm} (5.37)
Premultiply by $u^H$ and let

$$m = u^H M' u, \quad ig = u^H Gu, \quad k = u^H Ku$$

Equation 5.37 can be written as

$$ms^2 + igs + k = 0, \quad k \geq 0, \quad g \text{ is real.}$$

Since $k \geq 0$, $s$ can be expressed as

$$s = \frac{-i\{g \pm \sqrt{g^2 + 4mk}\}}{2m}$$

Case 1) $m > 0$

$s$ is pure imaginary

Case 2) $m < 0$

$s$ is pure imaginary if $g^2 \geq 4mk$.

$s$ is a pair of complex numbers which has one positive real and one negative real if $g^2 < 4mk$.

Case 3) $m$ is complex conjugate

Let $m = x_1 \pm y_1 i$. Then,

$$s_{1,2} = \frac{-i\{g \pm \sqrt{g^2 + 4(x_1 + y_1 i)k}\}}{2(x_1 + y_1 i)}$$

$$= \frac{g \pm \sqrt{g^2 + 4x_1 k + 4y_1 ki}}{-2(y_1 - x_1 i)}$$

$$s_{3,4} = \frac{-i\{g \pm \sqrt{g^2 + 4(x_1 - y_1 i)k}\}}{2(x_1 - y_1 i)}$$

$$= \frac{g \pm \sqrt{g^2 + 4x_1 k - 4y_1 ki}}{-2(y_1 + x_1 i)}$$

(5.38) (5.39)
By applying Demoivre's Theorem, Equations 5.38 and 5.39 can be rewritten as

\[ s_{1,2} = \frac{g \pm (A + Bi)}{-2(y_1 - x_1i)} \]

\[ \frac{(gy_1 \pm Ay_1 \mp Bx_1) + (gx_1 \pm Ax_1 \pm By_1)i}{-2(x_1^2 + y_1^2)} \]

where

\[ A = \frac{1}{\sqrt{2}} \sqrt{g^4 + 8g^2x_1k + 16x_1^2k^2 + 16y_1^2k^2 + (g^2 + 4x_1k)} \]

\[ B = \frac{1}{\sqrt{2}} \sqrt{g^4 + 8g^2x_1k + 16x_1^2k^2 + 16y_1^2k^2 - (g^2 + 4x_1k)} \]

Equations 5.40 and 5.41 can be reorganized as

\[ s_{1,3} = \frac{(Bx_1 - Ay_1) - gy_1 \pm (gx_1 + Ax_1 + By_1)i}{2(x_1^2 + y_1^2)} \]

\[ s_{1,3} = \frac{-(Bx_1 - Ay_1) - gy_1 \pm (gx_1 - Ax_1 - By_1)i}{2(x_1^2 + y_1^2)} \]

By comparing the absolute values of \((Bx_1 - Ay_1)\) and \(gy_1\), the sign of \(\text{Re}(s)\) can be determined. From several steps of algebraic manipulations, the following result is obtained.

\[ (Bx_1 - Ay_1)^2 - g^2y_1^2 = B^2x_1^2 + B^2y_1^2 > 0 \]

Therefore, either \(s_{1,3}\) or \(s_{2,4}\) has a positive real part.
Design Observation 4 The second order observer gain for the undamped gyroscopic system with only acceleration measurement does not exist.

5.3.2.3 Velocity Measurement Case

The second order observer for the undamped gyroscopic system in case of 'Velocity Measurement Only' is expressed as

\[ M\ddot{q} + G\dot{q} + Kq = Bf - L_u(y_v - \dot{y}_v) \]

\[ y_v = H_v \dot{q} \]

\[ \dot{y}_v = H_v \dot{q} \]

The error dynamic of the system is expressed as

\[ M\ddot{e} + G\dot{e} + Ke = L_v H_v \dot{e} \]

or

\[ M\ddot{e} + (G + C)\dot{e} + Ke = 0 \]

\[ (s^2M + s(G + C) + K)e = 0 \]

(5.42)

where \( C = L_u H_v \). The eigenvalue problem of Equation 5.42 can be written as

\[ (s^2M + s(G + C) + K)e = 0 \]

Premultiply by \( u^H \) and let

\[ \dot{m} = u^H Ku, \quad \dot{ig} = u^H Gu, \quad \dot{k} = u^H Ku, \quad \dot{c} = u^H Cu \]

Equation 5.43 can be rewritten as

\[ ms^2 + (c + ig)s + k = 0, \quad m > 0, \quad k > 0 \]
where \( c \) and \( g \) are real. Then,

\[
s = \frac{-c - ig \pm \sqrt{c^2 + 2cgi - g^2 + 4mk}}{2m}
\]

\[
= \frac{-c - ig \pm \sqrt{X + iY}}{2m}
\]

(5.44)

where

\[
X = -c^2 - g^2 - 4mk, \quad Y = -2cg
\]

By applying Demoivre’s Theorem, Equation 5.44 can be expressed as

\[
s_{1,2} = \frac{-c - ig \pm \left( \frac{1}{\sqrt{2}} \sqrt{X^2 + Y^2 + X + \frac{i}{\sqrt{2}} \sqrt{X^2 + Y^2 - X}} \right)}{2m}, \quad \text{if } Y > 0
\]

\[
s_{1,2} = \frac{-c - ig \pm \left( \frac{1}{\sqrt{2}} \sqrt{X^2 + Y^2 + X - \frac{i}{\sqrt{2}} \sqrt{X^2 + Y^2 - X}} \right)}{2m}, \quad \text{if } Y < 0
\]

The real parts of the eigenvalues are

\[
\text{Re}(s) = \frac{-c \pm \frac{1}{\sqrt{2}} \sqrt{X^2 + Y^2 + X}}{2m}
\]

Since \( \sqrt{X^2 + Y^2 + X} > 0 \), if \( c \leq 0 \), one of \( \text{Re}(s) \) should be positive. If \( c > 0 \),

\[
\text{Re}(s) < 0 \quad \text{when} \quad \left| \frac{1}{\sqrt{2}} \sqrt{X^2 + Y^2 + X} \right| < |c|.
\]

In order to compare the two values,

\[
c^2 - \frac{1}{2} \left( \sqrt{X^2 + Y^2 + X} \right) = \frac{1}{2} \left( c^2 + g^2 + 4mk - \sqrt{X^2 + Y^2} \right)
\]

Since \( c^2 + g^2 + 4mk > 0 \) and \( \sqrt{X^2 + Y^2} > 0 \), if \( \left( c^2 + g^2 + 4mk \right) > \sqrt{X^2 + Y^2} \),

\( \text{Re}(s) < 0 \).
Thus, if \( c > 0 \), \( \text{Re}(s) \) is always negative when \( K \) is positive definite. Since the closed loop damping matrix \( C \) is expressed as
\[
C = L_n H_v,
\]
the following condition is obtained.

**Design Observation 5** A sufficient condition for the existence of the observer gain, \( L_v \), is that \( \text{rank}(H_v) \geq n \) for the conservative gyroscopic system with velocity measurements if stiffness matrix \( K \) is positive definite.

The stiffness matrix is not always positive definite. If the stiffness matrix is singular (Positive semi definite) one of \( \text{Re}(s) \) should be zero. Thus, \( K \) has to be positive definite. As a result, the following condition is also obtained.

**Measurement Condition 3** Displacement measurements are necessary to build the second order observer for conservative gyroscopic systems if stiffness matrix \( K \) is positive semi-definite.

### 5.3.3 Damped Non-Gyroscopic System

The matrix second order observer for the damped non-gyroscopic system is expressed as
\[
M\ddot{q} + C\dot{q} + Kq = Bf
\]
where $M_{xx}$, $C_{xx}$, and $K_{xx}$ are real symmetric matrix and $M$ is positive definite.

5.3.3.1 Without Velocity Measurement Case

The second order observer for the damped non-gyroscopic system without ‘Velocity Measurement’ is expressed as

$$M \ddot{q} + C \dot{q} + K \dot{q} = Bf - L_a (y_a - \dot{y}_a) - L_d (y_d - \dot{y}_d)$$

$$y_a = H_a \dot{q}, \quad y_d = H_d \dot{q}$$

$$\dot{y}_a = H_a \ddot{q}, \quad \dot{y}_d = H_d \ddot{q}$$

The error dynamic of the system is expressed as

$$M \ddot{\epsilon} + C \dot{\epsilon} + K \dot{\epsilon} = 0$$

or

$$M' \ddot{\epsilon} + C' \dot{\epsilon} + K' \dot{\epsilon} = 0 \quad (5.45)$$

The eigenvalue problem of Equation 5.45 is expressed as

$$(s^2 M' + sC' + K')u = 0$$

Pre-multiply by $u^H$ and let

$$m = u^H M' u, \quad c = u^H C u, \quad k = u^H K' u$$

Equation 5.46 can be written as

$$ms^2 + cs + k = 0, \quad c \geq 0$$

and
If \( c = 0 \), the system is neutrally stable or unstable. Thus, the following condition is obtained.

Equation 5.33 also can be expressed as

\[
H_v^T L_v^T = C_d^T
\]

Since \( C_d \) is positive definite, \( \text{rank}(C) \) is \( n \). Consequently, \( \text{rank}(H_v) \) has to be greater or equal to \( n \) in order for the solution \( L_v \) to exist.

**Design Observation 6** A necessary condition to build the matrix second order observer for the damped non-gyroscopic system without velocity measurement is that damping matrix \( C \) is non-singular.

### 5.3.3.2 With Velocity Measurement Case

The second order observer for the damped non-gyroscopic system with ‘Velocity Measurement’ is expressed as

\[
\dot{y}_v = H_v \dot{q}, \quad \ddot{y}_v = H_v \ddot{q}
\]

The error dynamics of the system are expressed as

\[
M \ddot{e} + C \dot{e} + K e = L_v H_v \dot{e}
\]
As shown in the previous section, the stiffness matrix $K$ has to be positive definite. If $C$ is positive definite, Equation 5.47 can be an asymptotically stable system when $M$ and $K$ are positive definite [16]. To have a positive definite $C$ matrix, the following theorem is useful.

**Theorem 5** Any indefinite symmetric matrix can be divided into a symmetric positive semi-definite matrix and a symmetric negative semi-definite matrix.

**Proof**

Let $A \in M_n$ and $A = A^T$. Then, $A$ can be diagonalized as

$$A = SAS^T$$

where $\Lambda$ and $S$ are diagonal matrix and orthogonal modal matrix, respectively. By separating positive entries and negative entries of the diagonal matrix $\Lambda$, the matrix $A$ can be written as

$$A = S\Lambda S^T = S(\Lambda_{pd} + \Lambda_{nd})S^T = S\Lambda_{pd}S^T + S\Lambda_{nd}S^T$$

where $\Lambda_{pd}$ and $\Lambda_{nd}$ are diagonal positive semi-definite and diagonal negative semi-definite matrices, respectively.

**Lemma 1** For any symmetric matrix, $A \in M_n$, $SAS^T$ is symmetric for all $S \in M_n$ [22].
Lemma 2 If two matrices are similar through similarity transformation, then they have the same eigenvalues.

Thus, the diagonal entries of $\Lambda_{p_{sd}}$ and $\Lambda_{n_{sd}}$ are the eigenvalues of $SA_{p_{sd}}S^T$ and $SA_{n_{sd}}S^T$, respectively.

Therefore, $SA_{p_{sd}}S^T$ and $SA_{n_{sd}}S^T$ are symmetric positive semi-definite and symmetric negative semi-definite.

By using the Theorem 5, Equation 5.47 can be written as

$$M\ddot{\mathbf{e}} + (C_{p_{sd}} + C_{n_{sd}})\dot{\mathbf{e}} + Ke = L\mathbf{e}$$

where $C_{p_{sd}}$ and $C_{n_{sd}}$ are positive semi-definite and negative semi-definite, respectively.

The ranks of each matrix are

$$\text{rank}(C) = n - r$$
$$\text{rank}(C_{p_{sd}}) = p$$
$$\text{rank}(C_{n_{sd}}) = n - (p + r)$$

where $r$ is the number of zero eigenvalues of $C$. Let's suppose that there is a damping matrix $C$ which is

$$C = SAS^T$$
where

\[
\Lambda = \begin{bmatrix}
  p_1 & 0 & 0 & \cdots & 0 \\
  0 & np_1 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \ddots & \vdots \\
  0 & 0 & np_2 & 0 & \cdots \\
  \vdots & \vdots & \vdots & \ddots & \ddots \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  0 & \cdots & 0 & np_3 & 0 \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  0 & \cdots & \cdots & \cdots & \cdots & 0 \\
  0 & \cdots & \cdots & \cdots & \cdots & np_i \\
\end{bmatrix}
\]

where \( p_i \) and \( np_i \) are positive and non-positive number, respectively, and

\[ rank(\Lambda) = n - r. \]

Then,

\[
\Lambda_{psd} = \begin{bmatrix}
  p_1 & 0 & 0 & \cdots & 0 \\
  0 & 0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \ddots & \vdots \\
  0 & 0 & 0 & np_2 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \ddots \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  0 & \cdots & \cdots & \cdots & \cdots & 0 \\
  0 & \cdots & \cdots & \cdots & \cdots & np_i \\
\end{bmatrix}, \quad rank(\Lambda_{psd}) = p
\]

\[
\Lambda_{nsd} = \begin{bmatrix}
  0 & 0 & 0 & \cdots & 0 \\
  0 & np_1 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \ddots & \vdots \\
  0 & 0 & np_2 & 0 & \cdots \\
  \vdots & \vdots & \vdots & \ddots & \ddots \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
  0 & \cdots & \cdots & \cdots & \cdots & 0 \\
  0 & \cdots & \cdots & \cdots & \cdots & np_i \\
\end{bmatrix}, \quad rank(\Lambda_{nsd}) = n - (p + r)
\]
The error dynamics can be asymptotically stable if the feedback is expressed as

\[ L_v H_v = S \Lambda_f S^T \]

where

\[
\Lambda_f = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & p_j - np_1 & 0 & \cdots \\
& 0 & p_j - np_2 & 0 & \cdots \\
& & \ddots & \ddots & 0 \\
& & & 0 & 0 \\
0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0
\end{bmatrix}
\]

Thus the rank of the matrix \( \Lambda_f \) has to be \( n - p \).

Therefore, the following condition can be obtained.

**Design Observation 7** *For the damped non-gyroscope system with velocity measurement, in order to have asymptotically stable error dynamics,*

1. it is necessary that \( \text{rank}(H_v) \geq n - p \), \( p \) : number of positive eigenvalues
2. it is sufficient if \( \text{rank}(H_v) \geq n \).

The observer gain \( L_v \) exists if and only if there exists the matrix \( S \Lambda_f S^T \) which lies on the column space of \( H_v \).
In this research, new modeling and control design techniques for aircraft structural dynamics using smart materials are presented. An attempt is made, perhaps for the first time, to model the dynamics of a smart flexible aircraft structure involving the integration of five disciplines which are structures, aerodynamics, smart materials, control and flight mechanics! Modeling of the dynamics is achieved using the finite element method in the ‘generalized’ coordinates. Because of the coupling between these subsystems, the resulting ‘Matrix Second Order System’ of equations consists of a symmetric positive definite mass matrix but a non-symmetric and indefinite damping (which includes aerodynamic damping) and stiffness (which includes aerodynamic and piezoelectric material stiffness, in addition to the standard structural stiffness) matrices!

The control objectives are, on the one hand, to sustain a roll maneuver with a desired roll rate and, on the other, to suppress the flexible mode vibrations by actively deforming a wing.

To control the roll maneuver, the piezoelectric voltage serves as the control variables. The voltage dependent mass, stiffness, and damping matrices are also determined to maintain the desired roll rate during the roll maneuver. The roll control is
designed in the new framework called ‘Reciprocal State Space’ which allows easy implementation of acceleration feedback control.

However, the control objective was limited only to roll maneuver. Therefore, the rest of the dynamics of the aircraft such as yaw and pitch controls are expected to be studied with smart flexible aircraft structure in future. The wind tunnel tests also will be required to verify the modeling and control algorithms for the flight mechanics.

In terms of the vibration problem, the large model in the generalized coordinates is transformed to a set of ‘non-orthogonal modal’ coordinates, and model reduction is carried out in these modal coordinates. This model is then converted to the ‘state space’ form, and a controller is designed to absorb the vibration using the standard state space control theory.

Another component of this study involves vibration control of a large scaled aircraft structure which is equivalent to the Boeing 747 landing gear component. Finite element model approaches are employed to analyze the modal responses of the tube and the landing gear component as well as to design a dynamic absorber. A constrained layer is added to the absorber to maximize the damping effect. In active control, piezoelectric actuators (PZT) are used to add additional damping effect to the system.

Due to hardware limitations, however, real time experiments for the optimal control in the ‘Reciprocal State Space’ could not be carried out in this research. Since the simulation of the real time experiment for the optimal control has been successful in simulink, nevertheless, this attractive acceleration feedback control scheme is believed possible to implement in dSPACE if the hardware is available.
In this study, to achieve the real time control experiment, fuzzy logic controller is employed to control the vibration in dSPACE platform due to the easy implementation and robustness of the controller [37]. According to the experimental results, the passive and active control can dissipate the vibration of the tube satisfactorily.

Since it has not been worked in the study to optimize the Fuzzy logic control effort, constrained layer damping and dynamic absorber, more studies in future are encouraged to increase the active control effects and reduce the mass of the dynamic absorber.

Finally, the issues of designing observers for linear matrix second order systems directly in the matrix second order framework without transforming the systems to the state space framework are addressed. Several important reasons are presented to justify the need for this approach, and the advantages of the second order observer are explained and contrasted those of the typical first order observer. Taking various assumptions on available measurements into consideration, then, the conditions of existence and design methodologies are presented to synthesize the controller gains. This concept of the direct matrix second order framework design is new and attractive especially for observer based controllers.

In this research, however, the matrix second order systems are limited to the symmetric mass, stiffness, and damping matrices. Therefore, more expansion of the second order observer study is expected for the general matrix second order system which has non-symmetric matrices
BIBLIOGRAPHY


