PIaget in 3Space:
Using the Three Mountains Test
To Design a
Constructivist Learning Environment

Dissertation

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ABSTRACT

This study addresses two problems: 1) how Piagetian research concerning coordinating perspectives can be used to design a constructivist learning environment; and 2) how research findings can guide the redesign of such environments. Two qualitative studies treat the first question: a computer-based replication of the Three Mountains Test and an investigation of a virtual learning environment. Methodological reflections address the second question.

The introduction presents Piaget’s theories of conceptual space, mental operations and operational schemes and the Three Mountains Test as a means for studying the development of projective space. A review of literature suggests that researchers’ theoretical perspectives have diverted attention from mental operations and limited the utility of findings for instructional design purposes.

The qualitative studies were conducted at a parent-run school with a sample of 14 students aged 11-12. Grounded theory development guided the analysis of video-recorded data.
The Three Mountains replication yields three results: 1) a description of an operational scheme for constructing perspectives; 2) a criterion linked to performance in the virtual environment; 3) requirements for the redesign of the virtual environment. The scheme represents a significant advance beyond accounts of constructing perspectives using mental rotation.

The virtual environment is engaging in terms of: 1) Assimilative Play, 2) Task Engagement, and 3) Affective Engagement – especially for teams that perform well on the Three Mountains Test. Four subspaces present opportunities for new design elements which scaffold development of the operational scheme.

Three redesign directions, guided by the proposition that perceptual activity scaffolds the development of conceptual activity, are suggested: 1) create tasks in truncated three dimensions as opportunities to discover and practice the operational scheme; 2) use interiors of objects as variations of the Three Mountains Test; 3) use the external environment for fully three dimensional tasks.

This study uses qualitative studies to contextualize findings and connect them to design purposes. The utility of grounded theory as a method for re-contextualizing and transferring findings is demonstrated by revealing a synthesis of the mental operations into an operational scheme and suggesting avenues for embedding new design elements in a virtual environment.
This work is dedicated to my life-partner,

Tessa Logan
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PREFACE

One astute reader of this study observed that she had to read it twice — once to get the lay of the land and a second time to understand its significance. With this preface I intend to provide a road map in the hope that one reading will suffice for all but the most ardent scholar.

There are three phases to this study:

1. The discovery of the mental operations involved in the construction of perspectives as potential design elements.
2. The investigation of the a virtual environment as a potential design context.
3. A proposed synthesis of the design elements with the design context to form a virtual learning environment for practicing the mental operations involved in the construction of perspectives.
The Discovery of the Mental Operations Involved in the Construction of Perspectives

Fifty years ago Piaget and Inhelder (1956/1948) published a study, The Child's Conception of Space, in which they outlined the process by which projective space is constituted. In a chapter entitled The Coordination of Perspectives they described many of the mental operations involved in constructing perspectives and argued that projective space is constituted through the process of constructing and coordinating perspectives. However, they stop short of describing an actual sequence of mental operations for constructing perspectives. Using a computerized version of the test devised by Piaget -- The Three Mountains Test -- this study captures on videotape evidence of the mental operations involved in the construction of perspectives and takes the additional step of describing an actual sequence of mental operations through which some children use the alignment of objects to construct perspectives.

A Virtual Environment as a Potential Design Context

In the second phase of this study the same 14 children who performed the virtual Three Mountains Test explore a virtual space station. Again videotape is used to capture the kids' interaction with the virtual environment. The children really enjoy the space station. They find it to be interesting, engaging and stimulating. An analysis of the space station exploration
indicates several opportunities to design in ways to practice the mental operations involved in the construction of perspectives which were discovered in phase one.

Synthesis of the Design Elements with the Design Context to Form a Virtual Learning Environment

The final phase brings together the design opportunities with the design needs to suggest interesting and engaging ways that children can discover and practice the mental operations involved in the construction of perspectives. It has long been recognized that the coordination of perspectives is one of the core activities of spatial intelligence, but until now nobody has proposed a concrete program for developing this form of spatial intelligence. This design synthesis represents a substantial step in that direction. The creation of a redesigned space station that incorporates activities which support discovering and practicing the mental operations involved in the construction of perspectives will be the next step.

The Study as an Example of Research-for-Design

On another level the study is presented as an example of how to use grounded theory development as a process for placing existing research — such as findings involving the Three Mountains Test — into a new context that is
relevant for instructional design. The discovery of new results was an added bonus that flowed, in part, from the application of new computer and video technology to an existing research situation.
INTRODUCTION

The Problem of the Coordination of Perspectives

The importance of \textit{spatial intelligence}\footnote{Many terms have been used to discuss spatial phenomena, including spatial abilities, spatial cognition, spatial visualization, spatial orientation, spatial perception, spatial representation and spatial intelligence. I will use the term spatial intelligence, which was originated by Howard Gardner (1983), primarily because this terminology locates spatial phenomena within a comprehensive matrix of intelligences.} in the contemporary world is growing rapidly. More and more areas of human endeavor are being reconfigured as computer-based domains, which require sophisticated spatial intelligence; design, engineering, architecture, geography, scientific visualization, and navigation, are but a few of the areas with such requirements. Each of these disciplines exploits 3-D representation and concomitant spatial intelligence in very specific ways. Professionals in each of these fields and many others as well must now be trained to develop specific
spatial abilities to utilize the advanced technologies within their fields. The ubiquity of this phenomena across so many fields suggests that the same technology that is being used in very specific ways in diverse fields might be used in a general way to help encourage the development of general spatial intelligence in school students. The purpose of this study is to investigate the possibility of using new 3-D computer technology to enhance the development of spatial intelligence in children.

There is no consensus on definitions of spatial abilities or the of other terms in use (Caplan, MacPherson, and Tobin, 1985; Elliot, 1987); however, Howard Gardner's review of the field provides some sense of how the term, spatial intelligence, is typically used. Gardner (1983) argues that spatial intelligence involves a family of abilities applied across arenas of widely differing scale, including abilities to recognize, create and transform images or representations, both mental and graphic (p. 176). While this conception can provide a useful backdrop for the study of spatial intelligence, it is too diffuse to guide the design of instructional technology.

A survey of the study of spatial cognition ranging from geography to psychology indicates that the process of coordinating multiple perspectives is central to the exercise of spatial intelligence at all scales of spatial activity, extending from object recognition to the construction of mental maps of territories (Tarr, Williams, Hayward, Guithier, 1998; Taylor and Tversky 1992; Tversky 1993; Couclelis 1992; Siegel and White, 1975). The centrality of this process suggests that it might provide a useful focus for the design of instructional technology aimed at enhancing spatial intelligence. This study
focuses on understanding the process of coordinating multiple perspectives. The results of this study will provide guidance for designing instructional media (3-D computer software, in this case) aimed at developing spatial intelligence.

Theoretical Framework

The line of research begun by Piaget and Inhelder (1948/1956) in *The Child's Conception of Space* and carried on by numerous others represents the most thorough theoretical and empirical examination of coordinating multiple perspectives in Western literature. Piaget observed that most adults functioning in modern Western society, especially those involved with scientific and mathematical activities, intuitively understand spatial relations. As with all of his research, Piaget turned to the study of children to seek the source of this intuitive understanding. In this research, Piaget investigated the developmental process through which people acquire the ability to coordinate multiple perspectives. Building upon 300 years of Western philosophic analysis, as well as his own prodigious career of psychological research, Piaget reasoned that the coordination of perspectives occurs in *conceptual space* through the use of *mental operations*. Piaget's empirical investigation of the coordination of perspectives revolutionized the study of spatial abilities. To understand the meaning of this intellectual revolution, one must examine the notions of *conceptual space* and *mental operations*; and how *mental operations* combine to constitute *schemes*.
Conceptual versus Perceptual Space

Conceptual space is distinguished from perceptual space by the fact that in conceptual space we deal with symbols or representations of objects, rather than with present objects as we do in perceptual space. The two-fold distinction between perception of present objects and conception which deals with symbols of absent objects has troubled philosophers and scientists throughout the modern era.

The notion that we have a separate conceptual or psychological space in which we construct and manipulate models of spatial relations is a thoroughly modern idea. Descartes's distinction between "extended matter" (res extensa) and "thinking matter" (res cognitans) provided the foundation within Western philosophy for the conception of a distinct conceptual space (Eliot, 1987). Modern Western philosophy began with Descartes's invention of the "mind" as a substance separate from physical reality. Descartes argued that while he must doubt his senses, he could trust his mind as the seat of rational thought. As Howard Gardner (1985) expressed it: "Descartes determined that the mind, an active reasoning entity, was the ultimate arbiter of truth." (p. 52). With Descartes's distinction came the first outline of a conceptual space apart from physical space.

When Descartes's rationalist position was answered by the British empiricists, who preferred to trust their senses as the source of dependable

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2 Especially, John Locke and David Hume
knowledge, they accepted unquestioningly Descartes's notion of a distinct conceptual space. In the ensuing debate the source of knowledge was contested, but not the view of mind as the seat of knowledge. As the polemic exchange continued, Locke contributed the notion of mental processes and the modern conception of mind started to take shape. In true empiricist fashion, Locke proposed a theory of spatial perception that was derived from sensory input—specifically sight and touch. Eliot (1987) summarized Locke's view:

The idea of space, according to Locke (1690, II, IV), is a simple one arrived at by sight and touch. Space differs from the body in that it is without solidity (solidity, not—as in Descartes—extension, for Locke is the primary quality of the body), without resistance against moving bodies, continuous, and immovable (Locke, 1690, II, XIII, 11-14). Strictly speaking, spatial perception is not directed at space itself, but at the spatiality of bodies and at spatial relations. (p. 28)

Locke, being grounded in the sensory side of the controversy emphasized spatial perception over spatial conception. However, he also, directed attention to spatial relations and the spatiality of bodies, rather than abstract space. Bishop Berkeley elaborated Locke's theory of spatial perception and further explored the interaction of sight and touch. In doing so, Berkeley laid the foundations for modern theories of perception (Eliot, 1987).

In the eighteenth century, Kant (1781/1958) sought to synthesize the rationalist position of Descartes with the conceptions of the British empiricist, in his Critique of Pure Reason. In the Kantian view, sense data are made sensible through the operation of innate schema and rules which govern our thought.
processes. With this intellectual move, Kant took a decisive step in providing the ground for the modern conception of spatial cognition. One of the principal fruits of the empiricist-rationalist debate and Kant’s attempted reconciliation has been the development of what may be termed the representation hypothesis. Simply stated it asserts that cognition consists of establishing internal representations of external objects and events and thinking consists of manipulating these internal representations (Winograd and Flores, 1986).

Richard Rorty (1979) summarizes this philosophic theme as follows:

[Philosophy] understands the foundations of knowledge and it finds these foundations in a study of man-as-knower, of the “mental processes” or the “activities of representation” which make knowledge possible. To know is to represent accurately what is outside the mind; so to understand the possibility and nature of knowledge is to understand the way in which the mind is able to construct such representations. Philosophy’s central concern is to be a general theory of representation. (pp. 3-4)

While the Cartesian distinction has proved very useful for the evolution of a notion of conceptual space and for the development of the representational hypothesis, Descartes’s writing has also given rise to the faulty assumption of mind/body dualism that has characterized Western philosophy since Descartes. Winograd and Flores (1986) characterize this assumption in the following manner:

3 A frequent corollary of the conception is that truth is based on the correspondence between internal representations and external objects and events; however, I believe that Piaget, von Glasersfeld and others have demonstrated that a form of the representational hypothesis is also consistent with a constructivist theory of knowing. See von Glasersfeld (1995).
This understanding, which goes hand in hand with what we have called the 'rationalistic orientation', includes a kind of mind-body dualism that accepts the existence of two separate domains of phenomena, the objective world of physical reality, and the subjective mental world of an individual's thoughts and feelings.
(p. 30 [emphasis in original])

The legacy of this over-drawn distinction between physical and mental realities, between mind and body, has served both to advance and to hamper the development of adequate theories of spatial perception and spatial cognition. As result of the mind-body distinction, most contemporary theories still suffer from a bias against multimodal and embodied conceptions\(^4\) of spatial perception and cognition (Varela, F., Thompson, E., and Rosch, E., 1991; Millar, 1994).

While the first chapter of modern Western philosophy yielded many useful ideas — conceptual space, mental processes, thought as schemata, and the role of experience— it remained for Piaget and other developmentalists to provide a more adequate account of how schemata (and schemes) arise from experience. The bridge between the early work and that of Piaget was provided primarily by Ernest Cassirer, a German neo-Kantian writing in the early part of the 20th century. In his monumental four volume work, the Philosophy of Symbolic Forms, Cassirer (1923/1955, 1924/1956, 1929/1957) sought to establish the fundamental role of the development of symbol systems in human affairs (Olson, 1983; Eliot, 1987). Writing in his first work in

\(^4\) For several reasons, which will be explained in the methodology chapter, this study focuses on the normally-dominant visual component of spatial perception. However, Piaget's theory is quite consistent with, and to some degree, entails a multimodal approach.
the English language, An Essay on Man, Cassirer (1944) expressed this theme:

Man has, as it were, discovered a new method of adapting himself to his environment. Between the receptor system and the effector system, which are found in all animal species, we find in man a third link which we may describe as the symbolic system. This new acquisition transforms the whole of human life. As compared with the other animals man lives not merely in a broader reality; he lives, so to speak in a new dimension of reality. (pp. 42-43 [emphasis in original])

Cassirer propounded this thesis at a time when behaviorism was dominating American psychology. In doing so, he profoundly influenced European developmental psychologists, such as Werner and Piaget, especially in their conception of the development of spatial intelligence (Eliot, 1987). Cassirer argued that only humans are capable of developing a symbolic or conceptual understanding of spatial relations. He contended that our spatial knowledge develops in terms of three different realms of experience: action space, perceptual space and conceptual space. Two constant themes in his work are the developmental character of our spatial knowledge and the fundamental distinction between our concrete acquaintance with space and our abstract knowledge of space. On this last topic Cassirer (1944) said:

Acquaintance means only presentation; knowledge includes and presupposes representation. The representation of an object is quite a different act from the mere handling of the object. The latter demands nothing but a definite series of actions, of bodily movements coordinated with each other or following each other. . . . But the representation of space and space relations means much more. . . . We must have a general conception of the object,
and regard it from different angles in order to find its relation to other objects. We must locate it and determine its position in a general system. (pp. 67-68 [emphasis added])

Clearly, Cassirer's philosophy fore-shadowed the path that Piaget's work would take and served to further clarify the distinction between conceptual space on one hand, and perceptual spaces on the other.

**Understanding Operations**

If conceptual space is the *where* that we create as a distinctly human arena for the exercise of our spatial intelligence, then mental operations are the *how* through which we constitute conceptual space as an arena for human activity. In Piaget's theory, mental operations are the basic internal actions through which we manipulate and transform symbols or representations in conceptual space. Piaget (1977) defined mental operations as "actions which are internalizable, reversible and coordinated into systems characterized by laws which apply to the system as a whole" (p. 456). Unpacking this terse definition requires a detailed examination of several of Piaget's works ranging over diverse topics. However, the search will be worth the effort, for, in my view, the concept of mental operations is the notion that is most central to Piaget's conception of intelligence in general, and spatial intelligence in particular.

Piaget's program of genetic epistemology is fundamentally concerned with the psychological development of operations, the coordination of sets of
operations into structured wholes, and the transformations of such structures that characterize the movement from one stage of intellectual development to another. In his 1957, volume, *Logic and Psychology*, Piaget (1977) delineated his conception of operations:

Psychologically, operations are actions which are internalizable, reversible and coordinated into systems characterized by laws which apply to the system as a whole. They are actions, since they are carried out on objects before being performed on symbols. They are internalizable, since they can also be carried out in thought without losing their original character of actions. They are reversible as against simple actions which are irreversible. In this way, the operation of combining can be inverted immediately into the operation of dissociating ... Finally, since operations do not exist in isolation, they are connected in the form of *structured wholes*. (p. 456 [emphasis in original])

In Piaget's view operations are patterned activities or procedures which originate as sensorimotor or perceptual activities and gradually are internalized. In Gilbert Ryle's terms, operations involve *knowing how* rather than *knowing that*. They are procedural knowledge. After they are internalized, the patterns of action are gradually separated from concrete objects and applied to symbols. Ultimately, the abstracted pattern of action constitutes the operation. Piaget equates internalization with *representation formation*, which occurs in the preoperational stage of development. During this period, typically between ages 2 and 7, the growth of the *symbolic function* – language, symbolic play, and deferred imitation – supports the development of the capacity for representation formation. Piaget (1977) summarized this
process thus: "As a result of the symbolic function, 'representation formation', that is to say, the internalization of actions into thoughts, becomes possible" (p. 457).

In the years since Piaget wrote, others have suggested that conceptual operations may derive from sensorimotor activity. Experientialists, such as Lakoff and Johnson, argue that abstract conceptual structures and rational thought emerge from the bodily or sensorimotor level. Lakoff (1988) expressed this assumption in the following:

Meaningful conceptual structures arise from two sources: (1) from the structured nature of bodily and social experience and (2) from our innate capacity to imaginatively project from certain well-structured aspects of bodily and interactional experience to abstract conceptual structures. Rational thought is the application of very general cognitive processes—focusing, scanning, superimposition, figure-ground reversal, etc.—to such structures. (p.189)

Likewise, proponents of embodied cognition such as Varela, Thompson, and Rosch (1991), and Maturana and Varela (1987) argue that cognitive structures emerge from recurrent sensorimotor patterns. However, only Piaget detailed and substantiated empirically a connection between mental operations and actions, namely, that operations are actions that are first carried out on objects, but later are performed internally on symbols.

A second qualification that Piaget places on operations that differentiates them from other actions is that they are reversible. This notion is often reflected in the use of the phrase reversible operations. When speaking of sensorimotor or perceptual activities, Piaget emphasizes the notion that
reversible actions are ones in which a child can return to the starting point of the action. Piaget retains this action-oriented conception of reversibility through his discussion of the stage of concrete operations. In this stage, actions begin with concrete objects and end with reference to the same concrete objects. The objects may be subject to a sequence of operations between the beginning and end states, but the whole process is grounded in the concrete objects to which it refers. For Piaget the appearance of logical thinking is the hallmark of the formal operational stage of development. Piaget (1977) summed up the central role of reversibility in the emergence of formal logical operations from concrete operations in the following paragraph:

The various types of thought activity which arise during the preceding period [preoperational thought] finally attain a state of "mobile" equilibrium, that is to say, they acquire the character of reversibility (of being able to return to their original state or starting point). In this way, logical operations result from the coordination of the actions of combining, dissociating, ordering, and the setting up of correspondences, which then acquire the form of reversible systems. (p. 458 [emphases added])

Later in his 1969 book, The Psychology of the Child, Piaget (1969/1977) further delineated his concept of reversibility at the level of formal operations by arguing that there are the following two fundamental forms of reversibility:

The first of these forms of reversibility is inversion or negation; its characteristic is that the inverse operation combined with the corresponding direct operation cancel the whole thing out: \( +A -A = 0 \).
The second form of reversibility is *reciprocity* or symmetry. Its characteristic is that the original operation combined with its reciprocal operation results in an equivalency. If, for example the original operation consists of introducing a difference between A and B in the form of A < B, and the reciprocal operation consists in canceling this difference or expressing it the other way around, you end up with the equivalence A = A ...(pp. 399-400 [emphasis in original])

Reversibility, both in its concrete manifestation as being able to return to the original state or starting point and its formal expression either as *inversion* or as *reciprocity*, is the basis for repeated or recurrent patterns of action. The very possibility of stable operations and sequences of operations rests on reversibility as a primitive property. As a basis for recurrent patterns of action, reversibility provides a common ground with other theorists of biological epistemology such as Maturana.  

Piaget’s third requirement for mental operations is that they do not exist in isolation, [but rather] they are connected in the form of structured wholes. In Piaget’s terms, the interconnected set or ensemble of operations is a structure of human intelligence. Structures cohere because they have an internal logic that obtains among the ensemble of operations. Gruber and Voneche (1977), editors of *The Essential Piaget*, captured this sense of structure in the following introductory remarks:

Piaget’s structures are not things or beliefs, but coherent sets of mental operations which can be applied to things or beliefs or to anything else in the individual’s psychological space. For

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5For example, see Maturana and Varela (1987): *The Tree of Knowledge: The Biological Roots of Human Understanding.*
example, the belief in the conservation of matter when shapes are
deformed is not, in this sense, a "structure." Rather, the set of
operations by which this belief is arrived at is a structure. Piaget
does not claim that 8-year-old children all over the world
spontaneously discover the conservation of matter; rather, they
develop a set of operations that permits them to make this
discovery when presented with a problem that can be solved if
they do so. What matters is not a particular set of beliefs but a
general set of operations. (p. xxxiii)

What are Schemes?

In addition to the notions of conceptual space and mental operations, a
third term which must be thoroughly explored to understand Piaget's study of
spatial intelligence is *schemes*. While Piaget transformed the concept from a
static form to a dynamic program for mental activity, Kant deserves credit for
devising and explicating the initial conception. In *The Mind's New Science*,
Howard Gardner (1985) summarized Kant's *invention of schemata* in the
following manner:

Kant...devised yet another level of analysis — that of *schema*, or
*schemata* — interposed between the raw sensory information and
the abstract a priori categories...In devising this explanatory
apparatus, Kant sought to determine the circumstances under
which the categories can find concrete employment. A schema
serves as a mediating representation which is intellectual in one
sense, sensible in another. Thus, a schema is directly activated in
terms of sensory experience and yet can be plausibly thought to
provide an interpretation of that experience. As a cognitive
scientist might put it today, Kant had entered the world of "mental representation". (p. 58 [emphasis in original])
From Cassirer, Piaget received the neo-Kantian baton and undertook the task of empirically investigating how schemata develop through experience. As Margaret Boden (1980) noted:

He [Piaget] is firmly situated in a continuing philosophic tradition, within which his closest intellectual companion is Kant. But much as Kant was speaking to Hume, Piaget is speaking to the nativist Kant, urging a concentration on developmental issues which in Kantian epistemology were ignored. (p. 158)

In Piaget's hands schemata moved from static forms of knowledge, in Kantian terms, to whole series of dynamic procedures and transformations through which knowledge is constructed. In his later writings Piaget used the terms *schemata* and *schemes* to signify these two different notions (Boden, 1979). This distinction has often been lost in translation, which has resulted in frequent misunderstanding of Piaget, especially by English-speaking people (von Glasersfeld, 1995). Indeed, in the preface to the 1969 English translation of *The Mechanisms of Perception*, Piaget (1969) cautioned:

Experience has shown that the translation is always less well understood than the original and that it sometimes gives rise to misinterpretations which then persist. In addition, some distinctions, which appear to the author to be clear in his own tongue, are difficult to render in the second language: for example, the distinction between the terms 'scheme' and 'schema'. In our usage, these terms correspond to quite distinct realities, the one operative (a scheme of action in the sense of instrument of generalization) and the other figurative (a figural or topographical schema). (p. ix)
Recalling the meaning of the English term, *scheme*, in the sense of a *systematic plan of action* is helpful in understanding Piaget's distinction. However, Piaget's notion of scheme is somewhat broader, for it includes, indeed, is typified by unconscious or tacit patterns of action that have developed through recurrent activities. As is so often the case with Piaget, his conception of scheme is grounded in his understanding of sensorimotor activity. In *The Mechanisms of Perception*, Piaget (1961/1969) writes:

> All repeatable sensory-motor activities give rise to schematisation in the sense that whenever actions are repeated, a generalization occurs on the basis of common structures or schemes to which new situations are assimilated in so far as they are equivalent to those which give rise to the schemes. (p. 189)

One cannot over-estimate the importance of this distinction (and the parallel distinction between *operative* and *figurative* knowledge) in understanding Piaget's contribution to our understanding of the development of all intelligences, and spatial intelligence in particular. Boden (1980) acknowledged the import of these distinctions in recognizing Piaget's innovation over prior psychologists (as well as philosophers) who have used schemata as an explanatory concept:

> Th[e] distinction between figurative and operative knowledge is variously expressed by Piaget as corresponding to the passive and active aspects of knowledge: . . . to the data or content known and procedures or transformations effected on it. Mental imagery in

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6 In this regard, it is analogous to the notion of *structural coupling* developed by Maturana.

7 For Piaget, *sensorimotor* means "activities [which] entail the intervention of motor activity over and above the organization of sense data" (Piaget 1969, p. 189).
whether involved in perception, memory or imagination — is predominantly figurative insofar as one considers its content only; but changing or using the image involves the operative aspects of the mind. (p. 68)

In summary, schemes are recurrent sequences of mental operations: they are the active, creative series of procedures and transformations of the mind. They are the programs through which we construct and process our experience. Along with his explication of mental operations, Piaget’s other primary innovation was to conceptualize and investigate empirically the development of schemes (von Glasersfeld, 1995).

The Development of Projective Space

Piaget’s research revealed that children acquire the ability to coordinate perspectives in the normal course of development as they consolidate their command of concrete operations and begin to move into formal operations in their early teenage years. In this period of development, a basic level of this capacity becomes a spontaneous or unconscious activity characterizing ordinary interaction with spatial relations. Prior to this point, children’s understanding of spatial relations is piecemeal, being concerned only with the parts within a single object or with the relations between pairs of objects, but not with the relations among all objects. This piecemeal understanding is termed topological, which signifies, among other things, that the space constituted through this understanding lacks a comprehensive framework that
embraces all objects. The development of the coordination of perspectives provides the child with a comprehensive framework that can encompass all objects and relations. This conception of a unified space is referred to as projective space because the mental operations involved in projecting shapes from one plane to another are integral to the coordination of perspectives.

Perhaps the clearest way to convey what is meant by unified space is to examine how the use of perspective since the Renaissance has provided a strong sense of unified space in representational art.® Prior to the Renaissance, artists did not portray groups of figures and their environs in perspective. Rather, they were represented as isolated figures with a diminished, stylized background. During the Renaissance the rise of naturalistic drawing which sought to represent bodies and emotions realistically, and the invention of representation in perspective revolutionized art (Ivins, 1946; Field, 1997). The discovery of methods for constructing representations in perspective allowed artists to create paintings with a striking degree of realism --- they are represented as the eye would see them, at least from the appropriate viewpoint. With the ubiquitous use of perspective drawing in recent centuries and the invention of photography, which (subject to the characteristics of the lens) captures scenes in perspective, modern viewers have come to expect perspective in rendering and are startled when it is absent.

® In an interesting parallel, Piaget has demonstrated that representational space is a developmental intermediary between perceptual and conceptual spaces.
The invention of methods for constructing representations in perspective is generally credited to two 15th century Italians, Brunelleschi and Alberti (Field, 1997). Brunelleschi, the engineer, first demonstrated the new method for drawing in perspective by actually doing it. Alberti, the humanist, popularized the method by describing it in a small volume, Della pittura, which he dedicated to Brunelleschi. William Ivins (1946) captured the significance of this event in the following bold statement:

The history of art during the five hundred years that have elapsed since Alberti wrote has been little more than the story of the slow diffusion of his ideas through Europe. The other thing in Alberti's book that marked the coming of a new attitude far removed from that of the Greeks was his description of the earliest known geometrical scheme for depicting objects in a unified space, or in other words what we today call perspective. (pp. 67-68 [emphasis added])

What Ivins suggests in this remarkable quote is that a perspective represents a unified space, albeit a static one in that it is associated with a particular point of view. However, through the process of coordination of perspectives, one can construct a system of viewpoints and corresponding perspectives, and thereby constitute a unified conceptual space, called projective space.

Piaget's work demonstrates that the development of topological space precedes the development of projective space. In the introductory summary to The Child's Conception of Space Piaget (1948 / 1956) described this developmental sequence thus:

[T]hese investigations will show that prior to organizing a projective and euclidean space, the child starts by building up and
using certain primitive relationships such as proximity and separation, order and enclosure. Such relationships correspond to those termed “topological” by geometricians, and similarly [are] regarded by them as elementary from the standpoint of the theoretical construction of space. (p. xii)

In Piaget’s conception, the dual developments of projective and Euclidian space are the pivotal achievements in acquiring spatial intelligence. They represent the final consolidation of concrete operations and form the intuitive basis for the subsequent development of formal spatial operations, such as geometry. In Piaget’s view, projective space is more fundamental than Euclidian space to the functioning and extension of spatial intelligence; Euclidian operations only refine projective space by adding a metric for measurement. The notion that projective space, in some sense, subsumes Euclidian space reflects a mathematician’s view of the relation of projective geometry to Euclidean geometry. In discussing the evolution of projective geometry, William Ivins says:

In 1859 Cayley, who had been thinking very hard about anharmonic ratios and the line at infinity, perceived that perspective geometry instead of being a minor province in the empire of Euclid was logically a much vaster empire of which Euclidean geometry was a subordinate division. (p. 102-103)

However, for the study of spatial intelligence, it is the psychological, rather than the logical, primacy of projective space that is of greatest consequence. Projective space is psychologically primary, that is, it forms the
basis for our intuitive sense of unified space, for the following three reasons:

1. It is intuitive because it conforms to our visual experience—we see in perspective (at least after about age two).
2. It represents a conception of unified space because the coordination of perspectives systematically relates positions to perspectives, and changes in position to changes in perspective. It is a dynamic system in that it addresses both states and transformations of those states. As such, it is an intuitive model for understanding transformations.
3. It forms the basis for our intuitive sense of unified space because the mental operations that constitute projective space become tacit when they are mastered.

Building upon the Western philosophic tradition, Piaget distinguished between conceptual and perceptual space. In Piaget's account, conceptual space has its own development trajectory apart from the development of our ability to perceive spatial relations, which he refers to as perceptual space. Indeed, Piaget and Inhelder (1948/1956) open The Child's Conception of Space by stating:

The chief obstacle to any developmental study of the psychology of space derives from the circumstance that the evolution of spatial relations proceeds at two different levels. It is a process that takes place at the perceptual level and at the level of thought or imagination. (p. 3)

Long after the ability to perceive projective and Euclidian relations, is developed in the first two years of life, conceptual space begins to arise at about age 7 or 8.

In the interim, the acquisition of language, drawing, and other forms of iconic and symbolic representation provide the basis for an intermediate arena, representational space, which is gradually internalized to form a basis for
conceptual space. Conceptual space emerges from representational space, first as internalized imitation (mental images) which are tied to perception, and later as mental operations which are freed from current perception. Activity in representational space is characterized by frequent "perceptual activity" commencing with changes of centration or decentration [focus] and consisting of comparisons, transpositions, anticipations, and the like" (p. 16). Perceptual activity plays the same role in the genesis of conceptual space as sensorimotor activity does in the nascence of perceptual space. Perceptual activity is gradually internalized as reversible operations which begin and end with reference to concrete objects. These concrete operations, along with other similarly concrete operations such as seriation, are generalized and abstracted to furnish the basis for formal operations. Finally, in early adolescence, projective and Euclidean space typically appear as forms of conceptual space. Piaget and Inhelder summarize this evolution in the following manner:

Thus, thought has the task of reproducing at its own level (of representation as distinct from direct perception) everything that perception has so far achieved within the limited field of direct contact with the object. (p. 13)

Piaget and Inhelder's research reveals the same pattern in each of these parallel but temporally separated developmental sequences: mastery of topological relationships provides the ground for the subsequent development of projective and Euclidean relationships. Topological relations, such as proximity and separation, order and enclosure, are characterized by Piaget as
intuitive and psychologically primitive relations. They are piecemeal relations which deal with only one or two objects at a time. Whereas topological relations concern simple one-to-one correspondences, projective and Euclidean spaces deal with much more complex situations. The primary reason for this greater complexity is that projective spaces conserve straight line and certain proportions called cross-ratios and Euclidean spaces conserve angles, curves, and distances during transformations, while in general topological spaces do not. Conservation of these relational properties requires a general system of organization — a projective system. Piaget's research demonstrates that the development of projective and Euclidean spaces results from the operational coordination of spatial relations within a general system. As operational coordination expands from the relations among multiple actual perspectives to the relations among all possible perspectives, the operational level moves from concrete toward formal. However, in Piagetian theory, formal operations are exclusive to the logical operation of formal systems such as geometry. Consequently, the coordination of perspectives remains at the level of concrete operations.

In his study of the development of projective space, Piaget undertook to describe and understand the operations through which a general projective system is constructed in conceptual space. He understood that his description of the operations and stages were inferential constructions of an outside observer. However, through the use of a group of ingenious tests, Piaget and Inhelder systematically observed behavior which illuminated the conceptual operations
involved in constructing projective straight lines, discriminating between viewpoints of subject and object, spontaneously rendering in perspective, and, finally, coordinating multiple perspectives. In Piaget's approach, the basic achievement of projective space rests on a systematic coordination of perspectives.

Figure 1.1: Piaget and Inhelder's Three Mountains Test

To study the development of the coordination of perspectives, Piaget and Inhelder devised the *Three Mountains Test*, which was used to pose spatial problems to 100 children 4-12 years old. Since it was based on a tabletop display (Figure 1.1), the test dealt with the before-behind and right-left dimensions of conceptual space, while disregarding the up-down dimension. The spatial problems required that the children conceptually construct perspectives as they would appear from viewpoints other than their own. Based on the results of this test, Piaget and Inhelder (1948/1956) proposed a four-stage developmental process through which the children achieved operational coordination of perspectives. These four stages (using Piaget and Inhelder's numeration) are the following:

**IIA.** The child confined to reproducing his own point of view.

**IIB.** Transitional reactions. Attempts to distinguish between different viewpoints.

**IIIA.** Genuine but incomplete relativity. (Discovery that the left-right and before-behind relations vary according to the position of the observer.)

**IIIB.** Complete relativity of perspectives. (Existence of an operational scheme for coordinating perspectives.) (pp. 209-246.)

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10 In this study I use the terms viewpoint or point of view (POV) to designate the position and orientation of an actual or potential viewer; and, perspective to indicate what one would see from that POV, especially the spatial relations.
In the first two stages, the child is dominated by the perception, unable to conceptually construct other perspectives. In the third stage, independent operations relating to either the left-right or the before-behind dimensions allow the child to begin to construct other perspectives. Finally, in the fourth stage, the simultaneous coordination of both the left-right and the before-behind dimensions, as well as several other operations involving projection in depth, conservation of straight lines in transformation, and occlusion of distance sections by nearer sections, combine to afford systematic coordination of perspectives. Piaget and Inhelder (1948/1956) describe this accomplishment in the following manner:

In sum, at substage IIIb the operations required to co-ordinate perspectives are complete, and in the following quite independent forms. First, to each position of the observer there corresponds a particular set of left-right, before-behind relations between the objects constituting the group of mountains. These are governed by the projections and section appropriate to the visual plane of the observer. . . .

Second, between each perspective viewpoint valid for a given position of the observer and each of the others, there is also a correspondence expressed by specific changes of left-right, before-behind relations, and consequently by changes of the appropriate projections and sections. It is this correspondence between all possible points of view which constitutes coordination of perspectives. That is to say, it provides the essential unity and homogeneity of projective space, complete at this level though only at a rudimentary form. (pp. 241-2)

In summary, Piaget and Inhelder's research demonstrated that by the final stage of the development of projective space children have acquired a
two-part operational scheme. One part of this scheme concerns correspondences between perspectives and POVs; the other, deals with correspondences between changes in POV and changes in perspectives. This operational scheme, the coordination of perspectives scheme, is composed of a set of concrete operations. The set of operations fits Piaget's criteria for operations, namely, actions which are internalizable, reversible and coordinated into systems characterized by laws which apply to the system as a whole. Piaget and Inhelder used the Three Mountains Test to study the development of this set of operations in children as they achieved operational coordination of perspectives.

Outline of the Study

This study addresses two inter-related problems: 1.) on a methodological level it addresses the question of how research findings can be used to guide the design of constructivist instructional media; 2.) on a substantive level it addresses the question of how the body of Piagetian research concerning the coordination of perspectives can be used to design constructivist instructional media. The primary focus of the study is on the substantive question. The study's approach to the substantive question is an instance of the methodological question. Hence, reflections on the substantive study address the methodological question. In this regard, the reflections will constitute a case

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The argument developed in this study centers on the need to re-contextualize research findings when they are to be used to guide design in a new medium.
study addressing a meta-methodological question, that is, one dealing a type of methodology.\textsuperscript{12} The aim of the case study is a proof of concept; that is, a demonstration of the viability of the approach illustrated in this study.

Chapter one introduced the general substantive topic for the study -- spatial intelligence -- and laid out a theoretical framework for the substantive study. The theory section concentrated on Piaget's notions of conceptual space, mental operations and operational schemes. The Three Mountains Test was presented as the canonical means for studying the development of projective space by children. Chapter two is an examination of the use of the Three Mountains Test in the research literature, focusing on how researchers' theoretical perspectives and research interests have shaped their findings in ways which often limit their utility for instructional design purposes. Chapter three presents a rationale for conducting an inter-related pair of studies -- a computer-based replication of the Three Mountains Test and a qualitative study of a virtual learning environment. It also describes the research situation and the methodology used in the two studies. Chapter four reports the analysis of the computer-based replication of the Three Mountains Test. Chapter five is a report of the learning environment study, as well as background data gathered through a short questionnaire. Chapter six concludes with substantive conclusions and design criteria, future research and design directions and finally, methodological reflections.

\textsuperscript{12} The rationale for this levels distinction lies within Russell's Theory of Types, which concerns the relations between different logical levels. Since the Theory of Types is relevant to several aspects of this study, I will discuss the theory in Appendix A.
CHAPTER 2

LITERATURE REVIEW

The Three Mountains Test in the Literature

In the four decades since *The Child's Conception of Space* first appeared in English, scores of studies which use the Three Mountains Test as a point of departure have been published. Depending on one's philosophic commitments, one can view these studies as "efforts by others to replicate or to extend it [the Three Mountains experiment], using a composite of Piaget's methods" (Elliot, 1987, p. 107) or as derivative studies that have investigated interesting issues raised by Piaget's work, while turning away from the central question addressed by Piaget. As I discussed in Chapter One, Piaget and Inhelder used the Three Mountains Test to study the development of a set of mental operations in children as they achieved operational coordination of perspectives. Subsequent researchers have, for the most part, left the investigation of this set of operations - this scheme for coordinating perspectives - as Piaget and Inhelder left it at the conclusion of their study. Instead of further
clarifying the scheme for coordinating perspectives, they have pursued other related topics under the rubric of perspective-taking, while paying homage to Piaget as their precursor.

In my review of studies that derive from the Three Mountains Test, I find that interpretations of Piaget's work and subsequent derivative investigations were shaped by three of the most powerful currents in twentieth century Western thought: the linguistic philosophy of Wittgenstein, cognitivism, as derived from rational empiricism, and psychoanalytic theory.

Linguistic Philosophy

Starting with Wittgenstein's conception of language games, many social scientists have turned increasing attention to the role of rules, rituals, frames, and scripts in social interaction (Mischel, 1974). Within this general movement, psychologists have focused on the social aspects of the coordination of perspectives. In doing so, they have recast the question of coordinating perspectives from one of individual cognition, which addresses mental operations, to one of social cognition, which addresses how children learn "to make inferences about another person's visual experiences" (Flavell, 1974). In this move, the research situation of the Three Mountains Test, which involves either constructing the perspective of a hypothetical person -- the little man -- or that of the researcher, has been elevated to the status of a paradigmatic situation.

John Flavell, then a professor of psychology at the Institute of Child Development, University of Minnesota, and a primary interpreter of Piaget for American audiences, has been a leader in this reconceptualization. Flavell
retained Piaget's interest in the mental operations involved in constructing and coordinating perspectives, but de-emphasized its importance. Building on three bodies of research – Piaget's study of the coordination of perspectives, a major replication by Laurendeau and Pinard, and his own research using a modified version of the Three Mountains Test – Flavell (Flavell, 1974; Flavell, Omanson, and Latham, 1978) developed an interpretation that focuses on the perception of the researcher or other observer and the ability of children to make inferences about others' perceptions. In an influential 1974 article, entitled "The Development of Inferences about Others", Flavell presented a four stage model of perspective-taking, which he characterized as "a developmental analysis of percept inference" (p. 66). As this phrase suggests, the model deals with inferences about others' perception, not with the construction of perspectives:

Level 0. "knowledge that others have visual experiences."

Level 1. "the ability to represent the fact that others do see objects."

Level 2. "the ability to represent the added fact that others see these objects in various perspectives, that is, see various qualitatively-describable views of them"

Level 3. "the ability to represent with fair quantitative precision another's retinal, painter's eye image of a visual display, complete with attempts to reproduce projective rather than real sizes"\(^\text{13}\)

(Flavell, 1974, p. 111)

\(^{13}\)This characterization is reminiscent of Gibson's theory of direct perception, which was quite influential at that time, but which opposes Piaget's theory of perception.
While trying to maintain a parallel with Piaget's model, Flavell clearly moved away from Piaget's concern with the mental operations through which projective space is constituted to his own interest in social perception, which he accorded a more advanced developmental role. While describing his stage model, he asserted: "The higher the level, in other words, the more clearly and unambiguously one is dealing with inferences about percepts rather than with objects (Flavell, 1974, p. 100 [emphasis in the original])." The implication — that as one moves to higher developmental levels, one deals more with percepts rather than with objects — marks Flavell's split from Piaget. Whereas Piaget's line of investigation carried him from consideration of how projective space is concretely constituted to the formal operations of geometry, Flavell's line of investigation carried him from knowledge about objects to inferences about percepts. Whereas Piaget consistently pursued the procedural knowledge inherent in mental operations, Flavell turned to declarative knowledge in the form of rules governing inferences.

Flavell formalized this split with Piaget's approach in a subsequent article entitled "Solving Spatial Perspective-taking Problems by Rule Versus Computation: A Developmental Study" (Flavell, Omanson, and Latham, 1978). The authors began this article with the assertion that "A distinction can be made between rules and computation in the area of spatial perspective-taking and its development" (p. 462). Before turning to an elaboration of the rule development model which is described above, Flavell briefly considered the computation half of the distinction he has proposed. He defined computation as "the actual cognitive processes subjects use to estimate ('compute') how an object display appears to another person who sees it from a different position."
Having defined computation in a way that is consistent with a focus on social inferences, Flavell dispensed with the problem of computation by referring to several processes suggested by others. These processes included mentally rotating the display or the subject mentally rotating themselves into the other’s position. These suggestions, which will be considered later, refer to Shepard and Metzler’s (1971) work on the mental rotation of single objects, rather than to Piaget’s work on the construction of projective space through the coordination of perspectives.

Having dealt with computation, Flavell returned to considering rules for solving perspective-taking problems. In his view, “Rules refer to general relationships among observer positions and observer visual experiences, relationships that are essentially invariant across displays (Flavell, Omanson, and Latham, 1978, p. 462). In a series of articles published over a decade, Flavell and his associates described a set of such rules for inferences. The rules are considered to represent two developmental levels. Newcombe (1989) summarized these two rule sets:

At Level 1, children know the that and the what of other people’s visual experience, but not the nature of that experience in detail (Masangkay, et al., 1974). At Level 2, children have acquired rules such as the following: one observer has only one view of an object or array (Salatas & Flavell, 1976), observers in different positions have different views (Flavell, Flavell, Green & Wilcox, 1981; Flavell, Omanson, and Latham, 1978; Salatas & Flavell, 1976), observers in the same position have the same view (Flavell, et al., 1981; Flavell, Omanson, and Latham, 1978). . . .( pp. 208-209 [emphasis in original])
Newcombe lists seven such Level 2 rules that Flavell and his associates documented. All seem to be sensible rules of perception; however, as Newcombe (1989) noted: “The developmental relationship of acquisition of Level 2 rules to acquisition of the ability to compute is not clear” (p. 209 [emphasis added]).

As the articles quoted above indicate, at about this time Flavell and other researchers in the field began writing about perspective-taking tasks, rather than using Piaget’s notion of the coordination of perspectives. This change in terminology, which was grounded in Flavell’s prior work on role-taking (Nigl, 1981), signaled a broadening and diffusing of the field of study. Investigations ranged from Flavell’s own work on social inferences to a task in which “the child and/or the experimenter wore colored glasses and the child was asked to describe how a white card appeared to each of them” (Liben, 1978) were labeled perspective-taking tasks. While all these articles dealing with perspective-taking paid homage to Piaget, they strayed far afield from his concern with documenting the mental operations involved in constituting projective space.

Another group of researchers who pursued a rule-based approach to perspective-taking is represented by Fishbein and his associates. Judging by the multiple cross-references to each other’s work Fishbein and Flavell influenced each other extensively. In attempting to show the relevance of social as well as cognitive factors in the coordination of perspectives, Fishbein and his associates postulated stages of social development that function in addition to the

14 However, as I argued in Chapter One the more significant issue which Piaget addressed is the relationship between perception and conception, or the ability to compute alternative perspectives.
cognitive stages of the coordination of perspectives studied by Piaget (Nigl, 1981). The social stages were called egocentric, non-egocentric, and emphatic.

Elliot (1987) summarized Fishbein’s thesis in the following manner:

Rather than describe a progression of levels of knowledge, from awareness of the existence of objects to an awareness of computational inferences, Fishbein and his associates suggested that the same phenomena could be explained in terms of rules. Specifically, they suggested three rules: 1) young children’s rule is basically egocentric or ‘you see what I see’, 2) somewhat older children use the non-egocentric rule of ‘if you aren’t in my place, you don’t see what I see’; and 3) still older children’s rule use is basically emphatic or ‘if I were in your place, I would see what you see.’ (Elliot, 1987, p. 109. [emphasis in original])

Fishbein and associates argued that the social factors operate orthogonally to (independently from) the conceptual factors in determining performance in coordination of perspectives tasks (Nigl and Fishbein, 1974).

In a 1974 paper entitled “Perception and Conception in the Coordination of Perspectives”, Nigl and Fishbein set out to clarify the relative contribution of perceptual and conceptual factors to the coordination of perspectives, emphasizing these factors, while minimizing the effect of social ones. In the first two of a series of three experiments, they found, much as Piaget and Inhelder did, that conception predominated when children attempted to construct alternative perspectives and that a marked improvement in performance occurred between ages 9 and 11. Based on these results, the authors proposed a heuristic model to describe the cognitive factors involved:

Three types of cognitive processes — extraction, matching and inhibition — seem to be involved in children’s performance on the
nonfrontal positions, but only extraction and matching on the frontal position. The initial process is one of extracting sufficient information from the three-dimensional array to build up a schema or image of how the objects are spatially arranged. . . . Second, the children must extract sufficient information from each of the choice stimuli (photographs) to build up an image of how the represented objects are spatially arranged. . . . Third, they must compare these two types of images. . . . Finally, for the nonfrontal perspectives, the children must inhibit making a match based on their own view of the display in order to avoid making an egocentric error. (pp. 863-864)

This model, since it is based on generic cognitive processes such as extracting and comparing, would be of little use in guiding design work. In fact, it seems a step back from the suggestive descriptions of mental operations provided by Piaget and Inhelder. To conclude their paper, Nigl and Fishbein offered a "more detailed discussion of the imagery used in these tasks" (p. 865). The ensuing discussion relies on the stated, but unsubstantiated, assumption that the cognitive processes involved are based on children's ability to build and manipulate mental images. In this move the authors called upon many of assumptions of cognitivism, the prevailing American psychological orientation of the later half of the twentieth century (Gardner, 1985).

Cognitivism

Cognitivism revolves around two central notions: symbolic representation and computation. As discussed in Chapter One, theories of symbolic representation have been developing in Western philosophy since the Enlightenment. Frequently, contemporary cognitivists harken back to Descartes in presenting their conception (Fodor, 1975). Computation is
understood as the manipulation of symbolic representations – either propositions or images. Cognitivism builds upon the rationalist assumption of representations and complements it with a notion of computation developed throughout the nineteenth and early twentieth centuries within the field of symbolic logic. Varela, Thompson, and Rosch, (1991) summarized this view as follows:

The central intuition behind cognitivism is that intelligence—human intelligence included—so resembles computation in its essential characteristics that cognition can actually be defined as computations of symbolic representations. (p. 40)

The reasoning is both circular and compelling (perhaps because of its circularity). For example, one of its principal spokesmen, Jerry Fodor, argued that if mental processes are computational, there must be representations upon which computations can be performed (Gardner, 1985). The reasoning is also compelling because of the obvious and pervasive power of computers. Moreover, it is persuasive because we all have experience with thinking, and sometimes our thinking involves manipulation of representations, either symbolic or pictorial.

Cognitivists hypothesize an “irreducible” (Haugland, 1981) level of cognition, called the “level of representation” (Gardner, 1985):

Cognitive science is predicated on the belief that it is legitimate—in fact, necessary—to posit a separate level of analysis which can be called the “level of representation.” When working at this level of analysis, a scientist traffics in such representational entities as symbols, rules, images—the stuff of representation which is found between input and output—and, in addition, explores the ways in which these representational entities are joined, transformed, or contrasted with one another. This level is necessary to explain the variety of human behavior, action and thought. (p. 38)
Within the ranks of cognitivists, there is an ongoing controversy over whether representations involve propositions or images or both. The work of Shepard and Metzler (1971) on the mental rotation of images has been a major impetus to proponents of image representation.

In explaining the cognitive processes involved in their model, Nigl and Fishbein (1974) cited the work of Shepard and Metzler on mental rotation and suggested that "Such a strategy could conceivably be used on the nonfrontal task, in coordination of perspectives experiments (p. 866, [emphasis added])."

The authors continued by elaborating how this notion might be applied to the coordination of perspectives task: "The child could 'image' the array and then track its constantly changing appearance as they imagined it moving to the spatial position specified by the experimenter's position" (p. 866). However, they go on to note that research by Huttenlocher and Presson (1973) suggested that this strategy cannot be effectively implemented by either children or adults.

The alternative strategy proposed by Huttenlocher and Presson (1973) also entails mental rotation, albeit in a two-step process. Nigl and Fishbein's (1974) summary of this process is as follows:

The first step involves assessing the relation between the array and the experimenter who occupies a specific viewing position. The second step seems to entail rotation of this array-experimenter unit until the experimenter is imagined to occupy the subject's own position. (p. 866)

These two rotational strategies — one described by Huttenlocher and Presson and the other by Shepard and Metzler — are, of course, the two referred to by Flavell earlier. A major review article, "The Development of Spatial
Perspective-Taking”, published fifteen years later (Newcombe 1989), indicated that research on the relevance of mental rotation strategies to the coordination of perspectives has not advanced in the interim. Writing in 1985, Weatherford suggested that mental rotation and perspective-taking involve different mental operations:

Very few model space studies of mental rotation have been conducted to date, and those that have been carried out have typically been designed to compare this spatial ability with that of perspective-taking ability. Like the latter, mental rotation seems to be an age-related ability. However, it seems to be easier than perspective taking and has produced different error patterns, suggesting that perhaps different mental operations or strategies underlie these two distinct skills (Huttenlocher and Presson, 1973).

(p. 49)

Nora Newcombe’s own research, as well as that of her frequent collaborators, J. Huttenlocher and L. Liben15, reflects a deep commitment to the tenets of cognitivism, while eschewing the social orientation of Flavell and Fishbein. Writing from a cognitivist theoretical perspective, they construed Piaget and Inhelder in information-processing and representational terms. For example, Newcombe (1989) wrote: “One hypothesis — that of Piaget and Inhelder, as well as others — is that the ability to compute others’ views depends on the nature of children’s systems for encoding the location of objects in space” (p. 210). In another article, Newcombe and Huttenlocher (1992) asserted: “In particular, he [Piaget] held that children could not succeed at perspective-taking problems because they coded spatial location in a fashion

15 For example, see Newcombe and Huttenlocher (1992) or Liben, Patterson and Newcombe (1981).
fundamentally different from that of adults" (p. 635). The language of spatial-coding supports the notion that the objects in a perspective-taking task form an array, which can be mentally rotated. This interpretation contradicts Piaget's notion -- which was discussed in Chapter One -- that a unified space, within which coding could occur, is constituted through mental operations.

Having transformed the question of how perspectives are constructed into a question of how locations are coded, Newcombe and Huttenlocher's research has been directed at discerning the role of landmarks and frames of reference in perspective-taking tasks. In a series of papers, they proposed a developmental model of location coding that they contrast with Piaget's theory concerning the progression from topological space to projective space (Newcombe, 1989, Sec. III, pp. 212-217). Newcombe summarized this developmental model as the following:

In summary, coding of single targets may develop from association with single landmarks, to proximity to single landmarks, to distance from frameworks of landmarks with the third stage possibly including a sequence from local to overall frameworks. Internal coding develops more slowly, and is often not used even by adults and even in situations where it would be helpful. (p. 217)

As one might surmise from this summary, the perspective-taking research conducted by Newcombe and her associates has utilized research situations in which external landmarks -- desks, cabinets, windows -- have figured prominently. Within this conceptual framework, Newcombe and Huttenlocher have addressed the question of "'internal coding of an array of targets' -- that is, remembering the relationships among the targets so that the whole is seen as forming a specified shape. . . . Such a coding might make spatial
transformation tasks easier, in a mental rotation process (e.g. Shepard and Metzler, 1971) (Newcombe, 1989, p. 217 [emphasis added]).” Citing Huttenlocher and Presson (1979) and Presson (1982), Newcombe concluded that neither children nor adults use such encoding, but she offered no alternative explanation of how the construction of alternative perspectives is accomplished.

Psychoanalytic Theory

Psychoanalytic theory is a third intellectual tradition that has shaped research about the coordination of perspective. The influence of this stream of thought has flowed primarily through the concept of egocentrism, which Piaget borrowed from Jung. According to Morss (1987), the term entered Piaget’s writing in the early 1920’s with the publication of The Language and Thought of the Child (1923/1960) and Judgment and Reasoning in the Child (1924/1928). Morss (1987) noted that, “In terms of theory, egocentrism was defined by Piaget in terms of the dichotomous model of the 1920’s lecture: as an ‘intermediate form’ between autism and intelligence (Piaget, 1923/1960, p. 45) (p. 266 [emphasis in original]).” Later, Morss explained, “Up to the early 1930s, egocentrism for Piaget remained a cognitive distortion, based on the slow dissolution of infantile autism (p. 270).”

Piaget’s notion of egocentrism began to change when he started working with spatial concepts. He began to differentiate the social concept of point of view from the spatial concept of constructing a perspective from a viewpoint. However, since he continued to use the term in a descriptive sense,
the seeds of confusion remained in his writing. Newcombe (1989) described this transition in Piaget’s usage: “Egocentrism was regarded as an explanatory construct in Piaget’s early writing (e.g. Piaget 1923/1926), but by the time he worked on space, he had changed his position. Egocentrism had become a descriptive attribute, a characteristic of the early stages of several developmental sequences.” (p. 206)

While it is quite possible to read The Child’s Concept of Space (Piaget and Inhelder, 1948/1956) and to understand Piaget’s limited descriptive use of the term as the inability\textsuperscript{16} to construct perspectives, many did not. In an article entitled “The Construction of Perspectives: Piaget’s Alternative to Spatial Egocentrism”, written 30 years after the translation of The Child’s Concept of Space into English, John Morss (1987) presented a construction of perspectives interpretation as an alternative to the current orthodoxy based on egocentrism. The legacy of “the language of egocentrism” (Coie, Costanzo, and Farnhill, 1972) has persisted, even when some thought it had been laid to rest. Eliot (1987) described this situation:

In the 1940s, Piaget referred to egocentrism as the inability to take a different viewpoint from one’s own. Wallon (1945) strongly criticized this construct, and because it was not theoretically necessary, Piaget ceased employing it by the late 1940s. As Montangero [1985] notes, researchers who employed or criticized the construct of egocentrism in the late 1960s probably did not realize that, from the point of view of Piagetian theory, they were dealing with a “theoretical ghost.” (p. 101)

This theoretical ghost was alive well into the 1980s (and is, perhaps, still flourishing today). For example, a 1981 volume entitled The Development of

\textsuperscript{16} The term inability is used here to mean the absence of the set of necessary mental operations
According to Piaget, one of the most important changes that occurs during cognitive development is the decline of egocentrism. Operationally (in terms of performance on spatial tasks) egocentricity can be defined as a child’s unawareness that perspectives other than his own, exist (Piaget and Inhelder, 1956)(p. 17)

Similarly, while speaking of Piaget and Inhelder’s study (1948/1956) in a 1983 article, Rosser wrote ,“The failure of these young children was attributed to the pervasive egocentrism inherent in pre-operational thinking”(p. 661). Even Laurendeau and Pinard’s classic study (1970), The Development of the Concept of Space in the Child, which closely follows Piaget’s research design, succumbed to the use of egocentrism as an explanatory concept. For example, they wrote: “This analysis shows in fact that if the child prefers one of the partially identical pictures, it is because his efforts at decentration are still shackled by very strong egocentric resistances” [p. 378. emphasis added].

Perhaps one reason why the ghost of egocentrism has persisted so tenaciously in the study of spatial perspective-taking is that most neo-Piagetian theory has been concerned with the validity of the stage theory and the problem of decalages (uneven development across areas) (Feldman, 1994; Gruber and Voneche, 1995). The concept of egocentrism has been entangled in broader theoretical questions that have little to do with spatial perspective-

17 Laurendeau and Pinard’s study is describe extensively in Chapter Three.
taking. Newcombe (1989) noted this appropriation of spatial studies for other theoretical purposes: “The perspective-taking problem has been most popularly believed to index a global characteristic called ‘egocentrism’, said to be exhibited in many other cognitive tasks, as well as in social understanding” (p. 204). The net result of the persistence of the concept of egocentrism has been that there have been far more studies of the existence, prevalence and timing of egocentric errors than there have been of the mental operations involved in coordinating perspectives.

In closing this survey of research on the coordination of perspectives, I join John Morss (1987) in his conclusion to The Construction of Perspectives: Piaget’s Alternative to Spatial Egocentrism:

If Piaget’s alternative theory were to be assimilated into contemporary discussion, it might be possible to finally move beyond the concept of spatial egocentrism. This misleading formulation could perhaps be forgiven and forgotten. Instead, attention could be directed wholeheartedly at the positive capabilities of the young child... (p. 277)

In a similar, though less dismissive sense, I would argue that concern with inferences about percepts and coding of locations has diverted attention from the essential Piagetian question of how alternative perspectives are constructed. While recognizing the value in these other research interests, it is time to turn attention back to understanding the mental operations involved in the coordination of perspectives.
CHAPTER 3

METHODOLOGY

Understanding

In *positivist* philosophy of science, methodology generally refers to procedures aimed at removing or controlling the *situatedness* of research findings. The intent is to create *positive knowledge* that is free of its social, cultural, historical, and theoretical context. The litmus test of such research resides in the findings themselves: are they valid, reliable, and generalizable? (Lincoln and Guba, 1985). In contrast, the methodological intent here (and, I believe, in all good qualitative research) concerns procedures which assure that the situatedness of the research is sufficiently communicated. The intent is to represent knowledge-in-context that will engender a shared understanding with the reader. The litmus test of this mode of research lies in the understanding: is it sufficient to allow the reader to understand and act in other related contexts?\(^{18}\) This statement, of course, calls for some explanation of

\(^{18}\) The other contexts may be educational, research or, as in this case, design arenas.
what constitutes *sufficient understanding* and *related contexts*. The position proposed in this study is that, for instructional designers at least, adequate understanding can be generated through a small-scale replication of the relevant research and the relation of contexts can be established through a qualitative study of the prospective context.¹⁹ The chapters that follow document my quest to develop a sufficient understanding of the coordination of perspectives as revealed through Piaget's Three Mountains Test and to connect that understanding to the design of a computer-generated virtual environment which I hope will eventually serve as a constructivist learning environment.

**In Search of a Method**

This study began with the realization that new 3-D computer technologies, which are creating increased demand for spatial intelligence, might also be used to enhance the development of spatial intelligence. Based on this insight, I began to develop a prototype of a *constructivist learning environment* (Dede, 1995). I created the prototype using an architectural design package called *Virtus WalkThrough Pro*, which renders 3-D computer-generated representations in real time. The virtual environment that I created is a multi-level space station. Figure 3.1 shows a view of the outside of the space station as it would appear from a virtual distance of 100 yards away.²⁰ Since the

¹⁹ Ultimately, the tests of the adequacy of understanding and the strength of relationship between contexts are *pragmatic* questions to be answered by the success of projects.

²⁰ The space station was rendered in color on the computer; however, microfilm publishing of dissertations can accommodate only grayscale images. Consequently, all images have been
VirtusWalkThrough Pro program renders representations in real time, the virtual environment is one that participants could freely explore — both inside and outside.

Figure 3.1: Virtual Space Station from 100 Yards

In designing the space station I incorporated features that called for the exercise of spatial intelligence. Research on factors which contribute to the development of spatial intelligence is scarce, but there is some evidence that certain activities, such as building models or playing with Legos, are correlated with better scores on tests of spatial intelligence (Deno, 1995, Baenninger and Newcombe, 1989). The design of the space station was guided by general

reproduced in grayscale.
notions found in the literature, such as the importance of frames of reference (Tversky, 1993, Millar, 1994), the role of bodily orientation in establishing frames of reference (Lakoff, 1987, Johnson, 1987, Lakoff and Johnson, 1980), the importance of scale (Montello, 1993, Couclelis, 1992), and the process of coordinating multiple views or perspectives (Piaget and Inhelder, 1948/1956). However, I found little in the literature to indicate which specific conceptual skills constitute spatial intelligence. I reasoned that such specific skills might suggest design elements that could be incorporated into the design of the space station.

My exploration of the literature on spatial intelligence led me to conclude that two higher-order or composite mental skills, mental rotation and coordination of perspectives, are at the core of spatial intelligence (Elliot, 1987, Gardner, 1985, Lohman, 1979). Each of these skills has been investigated for decades using paradigmatic tests – the Mental Rotations Test (MRT), to examine mental rotation and the Three Mountains Test, to study the coordination of perspectives. One research review (Corballis, 1982) argued that mental rotations and the MRT constitute a research paradigm within psychology. Another review (Elliot, 1987) noted that the Three Mountains study has been replicated over a hundred times.

I first attempted to incorporate elements of the Mental Rotations Test in the space station design. However, I found that the objects used in the test are treated as unitary objects: the images are mentally rotated, just as one would rotate the physical object (Shepard and Metzler, 1971; Vandenberg and Kuse, 1978). Figure 3.2 shows an example from the MRT (Vandenberg, 1971; Vandenberg and Kuse, 1978). Two of the four images are to be matched with a
target image (not shown). The two incorrect images are rotated versions of the mirror image of the target; hence, the left or right-handedness of their features are reversed from those of the target. David Olson (Olson and Bialystok, 1983) demonstrated that people could be trained to perform mental rotations by showing them a film of the rotation process, but this kind of instruction seems like simply training to the test. I could see no way to draw out facets of the rotation process that would suggest design elements for the space station. Consequently, I turned my attention to research using the Three Mountains Test.

![Image of Mental Rotation Test Diagram](image-url)

Figure 3.2: The Mental Rotation Test

When I looked more thoroughly at work done with the Three Mountains Test, I found that Piaget and Inhelder (1948/1956) describe the mental operation required for the coordination of perspectives in a manner that
approaches the level of specificity necessary for design purposes. Based on their research using the Three Mountains Test, they describe a four-stage process through which children achieve coordination of perspectives.

In the first two stages, the child is dominated by perception: he or she is unable to construct other perspectives conceptually. In the third stage, independent operations relating to either the left-right or the before-behind dimension allow the child to begin to construct other perspectives. Finally, in the fourth stage, the simultaneous coordination of both the left-right and the before-behind dimensions, as well as several other operations involving projection in depth, conservation of straight lines in transformation, and occlusion of distance sections by nearer sections, combine to afford systematic coordination of perspectives.

Although I studied Piaget and Inhelder's work at length, I never found a clear description of the basic mental operations used in the coordination of perspectives. Consequently, I turned to the literature on the Three Mountains Test. There I found that even the best research focused on the developmental stages and the types of errors common at each level, rather than on the mental operations (Laurendeau and Pinard, 1970; Coie, Costanzo, and Famhill, 1973). This search led me to the conclusion that I must do two studies: one to identify the mental operations and a second to find ways to incorporate the mental operations in the design of the space station. Since Piaget's work with the Three Mountains Test had provided me with the most promising approach to mental operations, I chose to make the first study a small scale replication of the Three Mountains Test aimed specifically at revealing the basic mental operations. To insure that the results would be compatible with the design
environment, namely a computer-generated 3-D environment, I designed the replication study using that same medium. Figures 3.3 depicts the computer-based version of the Three Mountains Test that I developed to replace the tradition tabletop clay models used by Piaget and the many others who replicated the Three Mountains study.\textsuperscript{21} (See Figure 1.1 in Chapter One for a representation of Piaget's tabletop model.) The computer model was created using \textit{Virtus Walkthrough Pro}, the same software package used to produce the space station.

![Figure 3.3: The Three Mountains Test on a Computer](image)

\textsuperscript{21} The computer-based model was rendered in color. The darkest mountain was blue, the medium-shaded mountain was red, and the lightest was yellow. The plain on which the mountains rest was green and the background was light blue.
I reasoned that investigating the mental operations in a computerized version of the Three Mountains Test would provide one half of the connection between mental operations and the potential constructivist learning environment of the space station. The other half of the linkage would come from a qualitative study of the space station as a potential constructivist learning environment in which specific design elements might be embedded. These two studies — linked by a common representational medium, linked by a common research site and situation, linked by a focus on mental operations and linked by a common qualitative methodology — would form a prototype of a research-for-design methodology. The intermediate criterion for this methodology would be the goodness of fit between the design elements and the learning environment. At this stage of development of the learning environment, the two studies should provide a basis for fitting the design elements into the overall environmental design. Ultimately, further research in the form of field testing the next generation of the space station, which would include the new design elements, would provide the basis for judging the educative value and effectiveness of the design. However, that step is beyond the scope of this study. In conclusion, the prototype methodology demonstrated in this study offers an answer to the first of my two research problems: namely, how research findings can be used to guide the design of constructivist instructional media. With this introduction in mind, I will now

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22 The notion of research-for-design that I am proposing involves (re)contextualizing research to link design criteria with a body of substantive knowledge. In this respect it is distinct from formative evaluation which is performed within the design process.

23 This is but one of several answers to this question. I do not mean to imply that everyone must only use their own research for design guidance, but rather that research findings must be re-contextualized in the new medium for which one is designing. Research-for-Design is
turn to a description of the research design for the two qualitative studies — the Three Mountains Test replication and an investigation of the Space Station Exploration.

The Site and Sample

The site for this study is Prairie View Community School (PVCS) in Rural, Ohio. PVCS is a parent-governed cooperative school which serves about 70 students in K through 6th grades. It provides a developmentally-appropriate curriculum based on cooperative project-based learning. While it serves a wide economic spectrum, the educational background of member families is uniformly high, with over 40% of the families including at least one person with a PhD. I secured entry to PVCS as a research site through my long association with the school, which includes being a member parent for six years, serving on the Board of Directors for four years, and volunteering in the classroom frequently over the past six years. Based on this established relationship, the director of the school provided permission to conduct the study at PVCS and arranged for one of the school’s Macintosh computers to be placed in a private room to be used as the research site.

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24 Prairie View Community School is a fictional name assigned to the school.
25 The educational level of the parents is indicative of the likelihood that the children have had the advantage of playing with the kinds of toys and games that have been shown to be correlated with higher scores on tests of spatial ability.
The computer was placed on a table with about 18 inches between the edge of the table and the computer screen. Two chairs for the participants were placed in front of the table facing the computer. During the Three Mountains part of the data collection, 8 1/2"X 11" laminated screen shots were placed on the table in front of the participants. During the Space Station Exploration part, the keyboard and mouse pad were placed in front of the computer. An omnidirectional audio microphone was placed to the left of the computer on the tabletop. A video camcorder was placed either to the left of the participants for the Three Mountains Test or directly behind the participants for the Space Station Exploration.

The participants for this study were 14 children drawn from the 1997-98 Intermediate class (grades 5 & 6) at PVCS. The study included 14 of the 17 children in the class. Eight were 11 years old and six were 12 years old. Piaget's work, as well as Laurendeau and Pinard's, suggests that the zone for proximal development (ZPD) for the transition to perspective space typically occurs at age 11 -12. Laurendeau and Pinard's sample included 50 children at each age level from six to twelve. Their developmental scale divided the third stage, Piaget's IIIA, into 2 substages, which were labeled 2A & 2B, because their scale started with stage 0. Based on Laurendeau and Pinard's sample the probability of 11 year-olds being at stage two is 56%; for stage three, it is 22%. The probability of 12 year-olds being at stage two is 62%; for stage three, it is 28%. Since the goal of this study was to study intensively the transition from stage two to stage three, the sampling was purposeful (Patton, 1990) and aimed at adequately displaying the transition dynamics. If the population I was sampling from had the same characteristics as Laurendeau and Pinard's
population, then I would have expected a sample of eight 11 year-olds and six
12 year-olds to include three or four children who are at stage three and eight
children who are at various points in stage two. Based on the educational level
of the children's parents, I expected the PVCS sample to display a
developmental distribution equal to or higher than the distribution found in
Laurendeau and Pinard's sample. Consequently, I expected that the sample of
14 children drawn from the 1997-98 Intermediate class (grades 5 & 6) at PVCS
to illuminate adequately the mental operations characteristic of the transition
from Piagetian stage two to stage three in the coordination of perspectives.

The Research Situation

The data were collected in a single two hour session, which was broken
into three parts: part I of the Three Mountains Test, done individually, the Space
Station Exploration, done in pairs, and part II of the Three Mountains Test, also
done in pairs. The children selected their own partners for the paired activities.
Each pair arrived and left at the same time. In addition to the activities
described below in the section on the Three Mountains Test and the Space
Station Exploration, the children completed a brief questionnaire.26 The
purpose of the questionnaire was to ascertain background information
regarding the child's use of computers and other media, their patterns of
engagement in other activities that research (Deno, 1995) has shown to be
correlated with spatial intelligence, and the educational and vocational
background of their family.

26 See Appendix B for a copy of the questionnaire.
The questionnaire and the interaction with the two computer programs - the Three Mountains Test and the Space Station Exploration — formed a single integrated session. I began the session by giving both of the participants a general introduction. Following this, I asked one of the children to step outside the room and fill out the questionnaire while the other did part I of the Three Mountains Test. I then reversed the situation, with the second child completing the questionnaire while the first did the test. I then brought them together to explore the space station as a team and to do part II of the Three Mountains Test. The session concluded with a short de-briefing session.

Data were collected by both audio-taping and video-taping the children as they interacted with the computer programs. This procedure allowed me to capture the action on the computer screen and the comments of the child on the same tape. Video recording was done from behind or from the side, in part to assure that individual children would not be easily identifiable. Furthermore, in keeping with confidentiality requirements of the university, the tapes will remain with me, as the researcher and, (with the exception of my advisor) will not be available for others to view. However, I may wish to use clips from the tapes as part of conference presentations. If I use clips, they will be edited to insure that no children are identifiable.

The data I collected using a video camera was undoubtedly shaped by the rapport I established and maintained with the children. I have known all of

27 See Appendix C for the script of the general introduction.
28 In Laurendeau and Pinard’s study as well as my own, part II of the Three Mountains Test proved to add very little to the first part; consequently, my focus is primarily on the first. In general when I write about the Three Mountains test, without qualification, I mean the first part.
the children for six years or longer. They know me primarily as the father of
one of their classmates and as a parent who has helped supervise field trips.
However, I took several steps to shape the affective and task definitions of the
research situation. My letter inviting their participation in the study was
addressed to the child with an enclosed letter for their parents. The
invitation\textsuperscript{30}, promised a fun experience with a serious purpose and either two
movie passes or a gift certificate to a bookstore as a token of appreciation.
Following the initial invitation, I contacted the children directly about
scheduling a time and matching them with partners. In nearly all cases I was
able to pair children with one of their top three choices for partners and to
schedule a mutually acceptable time for the research session. I began the
sessions by picking up the pair of children and driving them to the school. This
gave me an opportunity to chat with the kids and answer any questions they
had about the research. In the research sessions I used the prepared materials -
both print and HyperStudio media -- as vehicles for conveying the task-
related definition of the situation, while I assumed an informal attitude. I tried
to be informative by answering any questions and explaining the relevance of
the Three Mountains Test in terms of its contribution to the design of the space
station. I also sought to support a fun atmosphere by joining in the laughter
and joking that surrounded the children's attempts to master navigating in 3-D,
which often included running into virtual walls (signaled by computer-
generated sounds). In administering the Three Mountains Test I followed a
rigid protocol, diverging from the script only to answer questions or to assure

\textsuperscript{30}See Appendix D for the text of the invitation. The accompanying letter to the parents,
which is not reproduced here, conveyed the same information as the children's letter.
the children that they were doing fine. In conducting the Space Station Exploration, however, I had no fixed protocol and adapted my procedure to the spontaneous activity of the children. For example, when someone found a way to get outside the space station, into outer space, I followed their lead and allowed them to explore that part of the environment too. I also incorporated such serendipitous inventions in my emergent protocol by intimating the possibility of outer space exploration to subsequent participants.

Virtual Piaget: the Three Mountains Test on a Computer

In the decades since Piaget and Inhelder's original study, there have been hundreds of efforts to replicate or extend it, often with modifications of the testing situation, the response options, and/or the scoring methods (Eliot, 1987). Laurendeau and Pinard's 1970 study, The Development of the Concept of Space in the Child, represents one of the most thorough efforts at replicating and verifying Piaget and Inhelder's study. Laurendeau and Pinard examined the responses of 500 children aged 4 1/2 to 12 to a modified version of the Three Mountains Test (Figure 3.4).^{31}

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^{31} Piaget and Inhelder's original study included 100 children aged 6 to 12.
Figure 3.4: Modified Version of the Three Mountains Test

Note. From The Development of the Concept of Space in the Child. (Figure 17, p.314) by M.Laurendeau and A. Pinard. New York: International Universities Press. Copyright 1970 International Universities Press. Reprinted with permission.
Laurendeau and Pinard described their test situation in the following manner:

The experimental apparatus [Figure 3.4] consists of a plain green cardboard square (52 cm. on a side), on which three colored paper cones representing a group of mountains . . . are placed. The square base is made of four smaller pieces of equal size (26 cm. on a side) bound together by strips of canvas, and the cross formed by these two lines can serve as a reference point for the subject wishing to estimate the degree of overlap of the mountains. The three paper cones are different in color and size and are placed at fixed positions on the board. The largest, a red cone, has a 20 cm. base diameter and is 11.5 cm. high; the middle-sized cone is blue and has a 14 cm. base diameter and a height of 7.5 cm.; finally, the smallest cone, colored yellow, has a base diameter of 9 cm. and is 5 cm. high. (Laurendeau and Pinard, 1970. p. 313.)

To test the participant's construction of alternative perspectives, Laurendeau and Pinard set the experimental apparatus (Figure 3.4) on a table in front of the participant, placed a small toy man (3 cm. high) at one of the alternative positions (e.g. position F in problem # 1, see Figure 3.4), and asked the participant to choose which of five pictures the little man might have taken from that point of view. The five alternative perspectives were represented by line drawings shown in Figure 3.5.

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32 In this study I use the terms viewpoint or point of view (POV) to designate the position and orientation of an actual or potential viewer and the term perspective to indicate what one would see from that POV, especially the spatial relations.

33 The apparatus was placed in the orientation shown in Figure 3.5, with the subject facing position A and the researcher standing on the opposite side of the table.
Figure 3.5: Iconic Representations of Alternative Perspectives

Note. From The Development of the Concept of Space in the Child. (Figure 18, p.315) by M. Laurendeau and A. Pinard. New York: International Universities Press. Copyright 1970 International Universities Press. Reprinted with permission.
Laurendeau and Pinard describe these drawings as follows:

The material also includes [see Figure 3.5] a set of nine sketched scenes (18 x 14 cm. each) which reproduce nine different perspectives in miniature; two of these, however (pictures H and I), are in fact impossible, given the actual positions of the mountains. (Laurendeau and Pinard, 1970. p. 313.)

The five alternatives were laid out on the table in front of the participant in a fixed order. For example, in problem # 1 the choices (laid out from right to left) were G, A, D, F, H. The participant was asked to choose the picture that corresponds to the picture the little man would have taken if he had a camera. Once the participant had chosen, they were asked to justify the choice and to say why each of the alternatives was incorrect. The justifications for choosing and rejecting pictures were recorded as the data for that participant.

In the introduction to the test, the researcher modeled a correct response using language that highlighted the left-right and before-behind dimensions of the problem. In the introductory example the little man is placed right in front of the participant, and the participant is given two choices — the one representing the POV from which the participant is perceiving the scene (A), and one representing the POV from the corner (D). In modeling the correct way to respond, the researcher was instructed to say the following:

You see, the man is placed right in front of you. So he sees the same thing that you do: he sees the red mountain in the back, and the blue one to the left here, and then the yellow one to the right, here. So you see, he sees this picture [designate card A]. On this picture, the red one is in the back, then the blue is to the left, and the yellow one is to the right, in front of the red one. On the other picture [show card D] the red one is in front of the yellow
one, and the blue one is on the wrong side. To see it like this, he would have to go somewhere else, do you understand? (p. 453 [emphasis in the original]

With this more iconic set of representations and more rigid protocol, Laurendeau and Pinard replicated the major aspects of Piaget and Inhelder's study using a much larger sample. Laurendeau and Pinard focused on the test, per se, and the protocols generated by the test, whereas Piaget and Inhelder used the Three Mountains Test to investigate the underlying mental operations. However, Laurendeau and Pinard's meticulous analysis of the 500 test protocols allowed them to construct a four-stage model that bears striking similarity to Piaget's model and therefore confirms the original study.

For several reasons — including the clarity of their stage model; the detailed description of their testing, scoring and interpreting methods; and their iconic representations — I used the Laurendeau and Pinard study as the model for my small scale replication of the Three Mountains Test. Laurendeau and Pinard's version of this test, including their data collection protocol, was used, with the following modification: the tabletop model was replaced by a 3-D computer representation of the three mountains, and their 2-dimensional iconic representations of alternative perspectives were replaced by screen shots of the 3-D computer representation.

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34 Compare this tabletop model to Piaget and Inhelder's, which is shown in Figure 1.1, of Chapter One.
Figure 3.6: The Three Mountains Test on a Computer

Figure 3.7: Screen Shots of Alternative Perspectives
Figures 3.6 and 3.7 depict the situation as it was presented to the participants in Problem #1: the large image on the top represents the computer screen showing the three mountains and the little man placed in the middle of the right edge facing the mountains. The computer-generated figure of a little man replaces a small doll used in Laurendeau and Pinard's research to indicate the point of view (POV) from which the participant is to construct a perspective. The miniature pictures, labeled H - G, represent the five 8 1/2” X 11” screen shots that were laid out in front of the participants. The designations H - G correspond to positions on the perimeter of the three mountains plain, and the order of placement of H - G follows the protocol established by Laurendeau and Pinard in their study.

In creating my computer model I retained the proportionality of Laurendeau and Pinard's research apparatus. I also fixed the participant's POV in relation to the model. Figure 3.8 depicts this situation. The circular object on the right of the figure represents the location of the participant's view in relation to the computer model.

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35 Part 1 of the Three Mountains Test is composed of three problem situations, which will be described in greater detail in a later section of this chapter.

36 Figure 3.6 and 3.7 are presented as a unit to duplicate the situation encountered by the participants: the top figure represents the image that appeared on the computer screen, while the miniatures in Figure 3.7 represent the images that were laid before them. This practice of presenting the two figures as a unit is continued throughout this report.

37 This configuration is equivalent to placing Laurendeau and Pinard's 20" x 20" model on a countertop 2 1/2 feet in front of the participant and 8" below their line of sight.
This construction fixes the POV of the participant at a height appropriate for a typical 11 - 12 year-old in relation to the plane of the mountains.\textsuperscript{38} Fixing the POV of the participants eliminates an unacknowledged source of variation in the Laurendeau and Pinard's setup: namely, the difference in POV between a short six-year-old and a tall twelve-year-old. This arrangement also addresses a major methodological concern of Laurendeau and Pinard – the fear that subjects could lean over to look from other positions, especially the front left corner (position B in Figure 3.4). The concern that participants could turn the conceptual task of constructing alternative perspectives into a perceptual task led them to consider perspective B (when the participant is at A) as an egocentric response. As the analysis in Chapter Four will show, this assumption has very significant consequences. However, the medium used in

\textsuperscript{38} In keeping with the fiction that the cones are mountains I refer to the base as the plain of the mountains. It is, of course, also true that the base is a planar surface in the geometric sense of planes.
this study eliminates that concern, because the POV is fixed. Regardless of how
the participants may turn or lean, they will see only the predetermined
perspective.

The screen shots, which replaced Laurendeau and Pinard’s line drawings
of alternative perspectives, were obtained by locating the virtual camera in

*Virtus* on top of the little figure’s head. Since *Virtus* renders everything in
perspective, the screen shots more accurately represented the relative size of
the mountains when seen from different perspectives. This heightened the
salience of the before-behind dimension, especially in the screen shots (Figure
3.7), which are dramatically different from the line drawings used by
Laurendeau and Pinard (Figure 3.5). A second variation resulted from the fact
that the computer image, like the screen shots, was clearly a representation.
While the tabletop model is certainly also a representation, it has a physical
presence. In contrast, with the computer model it was clear that *all is*
*representation*. One consequence of the representational character of the
computer model is that the children felt free to indicate positions on the
computer screen, as well as on the screen shots. This difference in the research
situations generated a category of non-verbal interaction with the model that
was absent in Laurendeau and Pinard’s version (or if present, is not reported.)

Using this general setup, participants were presented with three
problems. Figure 3.9 shows the relevant positions for the three problems and
the alternative perspectives that were offered. The model depicted in Figure
3.9 corresponds to Laurendeau and Pinard’s setup, which is shown in Figure
3.4. For problems #1 and #2 the participants were shown the view of the
computer model shown in Figure 3.9 (but without the labels). For problem #1,
the little man was placed at position F; for problem #2, he was placed at position C. The layout for problem #1 is shown in Figures 3.6 and 3.7 of this chapter. Similar images of the layout for problem #2 are shown in Figures 4.3 and 4.4 of Chapter Four.

Figure 3.9: The Computer Model with Points Labeled

Prior to solving problem #3 the participant's POV is changed. In Laurendeau and Pinard's study the participant physically moves to the opposite side of the model, standing close to position E (see Figure 3.9). In my procedure, the participant is presented with a different view of the computer model, shown in Figure 3.10, which represents the view of the model facing position E. The small figure is then placed in the computer model at the upper
right-hand corner of Figure 3.10 (position B). The complete layout for problem 
#3, including screen shots of alternative perspectives, is shown in Figures 4.5 
and 4.6 of Chapter Four.

Figure 3.10: The Computer Model Viewed from the Opposite Side

Using the computer model and screen shots, I duplicated, as closely as 
possible, Laurendeau and Pinard's procedure. In the introduction to the test I 
modeled a correct response using language that highlighted the left-right and 
before-behind dimensions of the problem, just as Laurendeau and Pinard had 
done. For each of the three problems, the appropriate view of the model was 
displayed and five 8 1/2" X 11" screen shots were laid out in front of the 
participants. For example in problem #1, the layout of which is shown in
Figures 6 and 7, the screen shots, labeled H - G, correspond to positions on the perimeter of the three mountains plain. After placing the screen shots before the participant, I asked the participant to choose the screen shot that corresponds to the picture the little man would have taken if he had a camera. Once the participant had chosen, I asked them to justify their choice and to say why each of the alternatives was incorrect. The justifications for choosing and rejecting pictures was recorded as the data for that participant.

Part II of the Three Mountains Test

Part II of the Three Mountains Test was not conducted immediately after part I: the Space Station Exploration, which lasted about an hour, intervened. However, I will describe the procedure for part II at this time since its procedure utilizes the same setup as part I.

In part II of the Three Mountains Test the participants are presented with the same view of the computer model as was used in problems #1 and #2 of part I. This is the view shown in Figure 3.9, but without the labels. The problem presented in part II consists of identifying the POVs from which five perspective screen shots were taken. The five screen shots are shown in Figure 3.11. The screen shots were presented one at a time, starting with the shot on the right. Participants were asked to point to the location on the computer model from which the shot was taken. Since Figure 3.9 does have the positions labeled, one can easily see the correspondence between the positions and the screen shots, except for screen shot I. This picture (I) represents an impossible shot that was constructed using the mirror image of a shot taken from the front
left corner of Figure 3.10. Since it is a mirror image, the left-right relations of the two end mountains are reversed while the middle mountain remains in place.

Figure 3.11: POVs to be Identified in Part II

In my procedure, participants were working in pairs when they addressed this task. The rationale for this decision is discussed later in conjunction with the Space Station Exploration, which was also conducted in pairs.

**Space Station Exploration**

The procedure used for the Space Station Exploration was a continuation of the procedure used in the Three Mountains study. After participants had individually completed part I of the Three Mountains Test, I brought the pair together and introduced them to the space station. Piagetian research typically concentrates on the individual in an effort to identify the core operations that characterize human intelligence. This is certainly the case in Piaget's study of
spatial intelligence. Indeed, Piaget is often criticized for neglecting the social dimension of development. Von Glasersfeld, however, argues that this criticism is unwarranted, noting "Piaget’s repeated observation that the most frequent occasions for accommodation are provided by interactions with others." (von Glasersfeld, 1995, p. 67). Be this as it may, Piaget’s research methodology is definitely focused on the individual. In my replication of the Three Mountains Test, evidence about the character of mental operations was elicited by asking the individual participants to verbally justify each of their decisions. However, in the exploration of the space station, a more social research context was employed. The participants were asked to explore the virtual space station in pairs for two reasons: one theoretical, one methodological. Following Vygotsky (1978), Mead (1964) and Bruner (1986), I believe that social construction of mental operations is at least as significant as individual construction through reflective abstraction. Observing the conversation of a pair working cooperatively offers a natural window on the mental operations and their social construction.

The introduction to the space station consisted of the following:

1.) A verbal introduction to the task (the participants were also given a printed copy of the introduction to help them remember the specific information they were asked to gather),

2.) A HyperStudio multimedia program,

3.) A printed Navigation Guide that the pair could refer to as they explored.

The Space Station Introduction, reproduced in Appendix E, presented the
notion of taking turns in performing the navigational responsibilities, oriented the participants to the nature of the HyperStudio presentation, and defined the ostensible task for exploring the space station. The task was introduced in the following manner:

After you have explored the Space Station for a while, I would like for you to collect some information from inside the Space Station:

1. Find the sum of the numbers on objects in red rooms.
2. Find the sum of the numbers on yellow objects.

The HyperStudio introduction to the space station begins with orienting information, starting with an image of the space station as it would appear from 100 yards away (Figure 3.12)
Welcome to the launch pad for Space Station TETRA

Figure 3.12: Opening Card of HyperStudio Introduction

The next half dozen cards describe various aspects of the space station. For example, the following card (Figure 3.13) provides information about the scale and construction of the space station.
The Space Station is made of geometric rooms and tubular halls. The halls are made from tubes which are 10 ft. wide and 50 ft. long.

The rooms are geometric shapes: tetrahedrons, cubes, and octahedrons.

Figure 3.13: Scale and Construction Details

The button that reads Enter here for a little test ride activates a Quicktime movie that moves the participant's POV down one of the hallways, simulating movement within the space station.

Another card (Figure 3.14) alerts participants to the notion that the windows might prove useful for navigating in the virtual environment of the space station.
Here is a view from one of the many windows in the space station. A look outside is often helpful for getting oriented in a 3-dimensional maze like the space station.

Figure 3.14: View from a Space Station Window

Yet another card (Figure 3.15) gives an example of the shapes-with-numbers that are the objects of the search task.
After being oriented to the space station, the participants were introduced to the method of controlling movement in the space station using combinations of keys and mouse movements. Various combinations support moving forward and backward, pivoting right and left, tilting one’s view up and down, sliding right and left, and sliding up and down. The navigational method was demonstrated in two different ways, with half of the teams of participants receiving instruction in each of the two modes. Half of the teams
used a HyperStudio program simulating the effects of the mouse-and-key combinations using a series of short Quicktime movies. The teams watched as I demonstrated and explained the navigation method. Following demonstration of the navigational method, the teams was given a printed navigational guide to use as a reminder. After the introduction, the POV of the participants was shifted to inside the space station. With one person controlling the mouse and the other controlling the keyboard, they found their POV situated in a hallway of the space station. Figure 3.16 depicts the initial view of the participants inside the virtual space station.

Figure 3.16: In a Hallway of the Space Station
Figure 3.16 represents the view seen from the middle of a 50' long by 10' diameter purple hallway, looking through a blue cubicle room and on into another purple hall. The upper panels of the hallway are transparent; thus, stars are visible in both of the windows. The space station is located in a cluster of stars, which are hollow Platonic solids -- cubes, tetrahedrons and octahedrons. The virtual dimensions of the stars are of the same scale as the rooms in the space station. On several occasions participants got outside the space station and wondered if they could get into the stars — a few did!

The space station is a large, somewhat complex structure composed of three rings of rooms and hallways. Each ring is about 300 feet in circumference. There are 16 rooms and 17 hallways connecting them. The rooms are hollow Platonic solids -- cubes, tetrahedrons, and octahedrons. Their dimensions are such that they would fit within a cube that is 12 feet on a side. The halls are 10 feet in diameter and about 50 feet long, sometimes with a right angle turn in the middle. The middle level is connected to each of the other levels by single vertical tubes, which the participant must slide up or down to change levels. In addition there are half a dozen diagonal tubes that do not connect levels, but rather lead to dead-ends. I described the diagonal tubes, which are quite difficult to navigate, as traps. However, several of the participants chose to explore them.

Once inside the virtual space station, the participants explored the environment, gathering information as they went. In contrast to the Three Mountains Test situation, the virtual space station is a laissez faire environment which places no explicit demands on the participants. It is possible to navigate

39 In discussing the space station I treat virtual dimensions as if they were real without continually noting that they are virtual dimensions.
the environment informed only by perception; however, one of the design hypotheses is that the coordination of perspectives to construct a projective space would be helpful. The request for specific information encouraged the participants to explore systematically, but they also wandered freely. As the participants explored the space station, I videotaped the computer screen which was displaying their POV within (or sometimes outside) the space station. I also recorded their conversation as they collaborated in the navigation and exploration.

Methods of Data Analysis

*Grounded theory development* provided the methodological foundation and guided the data analysis for both of the qualitative studies. Although there are no canons for *research-for-design*, grounded theory development is admirably suited to the task. Grounded theory is constructed *inductively* through qualitative research (Patton, 1990, Strauss and Corbin, 1990). As Patton (1990) notes, "The strategy of inductive designs is to allow the important analysis dimensions to emerge from patterns found in the cases under study without presupposing in advance what the important dimensions will be" (p. 44).

In its use of inductive procedures, grounded theory development stands in contrast to the use of a priori theory, which requires that we assimilate the new data, new phenomena into a pre-existing theory. In Piagetian terms, grounded theory development is the complementary accommodatory process. Good science, good intellectual development, I presume, uses the two in tandem. Hence, I began my study with an exposition of several key concepts.
drawn from Piagetian theory. These concepts provided a theoretical sensitivity to my data analysis -- they attuned me to look for certain kind of patterns, without constraining me to look only for those patterns. Strauss and Corbin describe the role of theoretical sensitivity in grounded theory development in the following manner:

Theoretical sensitivity refers to the attribute of having insight, the ability to give meaning to data, the capacity to understand, and the capability to separate the pertinent from that which isn’t. All this is done in conceptual rather than concrete terms. (Strauss and Corbin, 1990, p. 42)

Grounded theory is particularly pertinent to research-for-design because of the of its attention to contextual factors and their bearing on transferability. Lincoln and Guba (1985) explained this concern:

Grounded theory, that is, theory that follows from data rather than preceding them (as in conventional inquiry) is a necessary consequence of the naturalistic paradigm that posits multiple realities and makes transferability dependent on local contextual factors. (pp. 204-5 [emphasis added])

As I noted earlier, a central question for this study is how findings can be transferred from one medium in which the research was conducted to another medium in which designs will incorporate those findings.

Strauss and Corbin (1990) argue that “well-constructed grounded theory will meet [the following] four central criteria”:

1. Fit
2. Understanding
3. Generality

4. Control (p. 22)

Close correspondence or fit with the substantive phenomena follows from the systematic application of inductive procedures. I have already noted the importance of understanding for the transfer of findings from research to design contexts. The criterion of generality merits comment. In regard to generality the authors state:

If the data upon which it is based are comprehensive and the interpretations conceptual and broad, then the theory should be abstract enough and include sufficient variation to make it applicable to a variety of contexts related to that phenomenon. (p. 23)

This quotation points to the role of grounded theory as a vehicle for bridging different, but related contexts. It is not claimed that the theory is context-free, but rather that it can comprehend a variety of contexts through concrete links to the various contexts. This, of course, is exactly what is required to re-contextualize findings in design projects.

The criterion of control also calls for some explanation. Control has long been considered a hallmark of positivist science. In the tripartite division of research paradigms initiated by Habermas (1968/1971), prediction and control are typically presented as the dominant interest in positivist methodology. Clearly, grounded theory development, both in its conception and use, represents an interpretivist methodology. Hence, the notion of control must be related to that context. Strauss and Corbin (1990) elaborate their sense of
Finally, the theory should provide control with regard to action toward the phenomenon. This is because the hypotheses proposing relationships among concepts — which later may be used to guide action — are systematically derived from actual data related to that (and only that) phenomenon. Furthermore, the conditions to which it applies should be clearly spelled out; therefore, the conditions should apply specifically to a given situation. (p. 23)

Thus, in their use of the term, the control afforded by grounded theory is a constraint on the users of the theory, not on the subjects of the research — it is the basis for action that is informed by the theory, but specifically limited to the phenomenon addressed in the theory. This notion of informed but limited action aligns well with my interest in using grounded theory to guide the design of instructional media.

As a procedure, grounded theory development is simply the systematic process of moving from description to descriptive categories and from descriptive categories to analytic categories and analytic statements. Detailed descriptions provide a basis for constructing descriptive coding categories, which summarize the cases in the categories. Gathering descriptive categories into analytic categories and linking those with relational statements form the framework of the grounded theory. More detailed accounts of the mechanisms that operationalize the analytic propositions complete the theory.

Grounded theory development procedures guided the data analysis in both of the qualitative studies I conducted. In the Three Mountains study there was significant a priori influence from Piagetian notions from the beginning.
However, when I turned my attention to the data transcripts, I tried to set aside Piaget's conceptual framework and take a fresh look at the data. Working with the raw data, I developed a set of nine descriptive coding categories. In the process of moving to analytic categories I realized there was a striking correspondence between Piaget's theoretical description of projective operations and the descriptive categories I had generated. At this point my grounded theory began to interact directly with the a priori theory. The internal logic of Piaget's set of operations was that they are hierarchical. This notion was based only on a general structuralist argument relating these operations to other sets of operations in mathematics, not directly on Piaget's empirical work with the Three Mountains Test. In my analysis I was able to demonstrate not only a congruence with Piaget's hierarchy, but also how the operations collaborate as a coherent set to enable the coordination of perspectives.

Validity

In qualitative research, concerns with validity are often replaced by efforts to establish trustworthiness. (Lincoln and Guba, 1985) Trustworthiness is engendered in the communication between the researcher/writer and the reader. As I argued in the beginning of this chapter:

The intent [here] is to represent knowledge-in-context that will engender a shared understanding with the reader. The litmus test of this mode of research lies in the understanding: is it sufficient to allow the reader to understand and act in other related contexts.

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In the design and execution of this study I took several steps to ensure the trustworthiness of my research. The first was to use two devices — both video and audio recording — to capture the raw data and insure an adequate basis for accurate transcription. The second step was to view and listen to the recordings repeatedly to gain something akin to triangulation. Lincoln and Guba (1985) point out that triangulation can be achieved through “the use of multiple and different sources, methods, investigators, and theories” (p. 305). By using multiple passes at multiple recordings I tried to create an accurate transcript — one that is superior to what an individual recording notes during an experiment might acquire. The third measure I incorporated was to use images and text to recreate the research situation as closely as possible. Then, having recreated the context I displayed a substantial portion of the data to enable readers to more easily imagine the interaction of the children with the computer programs. Finally, I depicted my inductive reasoning process as I used the grounded theory development process to move from descriptive categories to analytic conclusions. Ultimately, readers’ judgments of whether they would have drawn similar inferences from the data will determine the trustworthiness of this study.

**Ethics**

The question of ethics is always a relevant question in research involving human participants. In my study I presented the purposes and procedure in a straightforward manner. There was no deception or omission
of relevant information. I facilitated participation in the research sessions by pairing the children with their friends and by providing transportation. Judging by both the explicit feedback given in the de-briefing session and the affect captured on the video, the sessions were quite enjoyable for the participants. All of the kids expressed interest in participating in a follow-up session involving the space station. Finally, as a quid pro quo for participation I gave each child either two movie tickets or a gift certificate to a bookstore — resulting in an implicit wage of $4.00 per hour for participation!

Timeline

This study was conducted over the past two and a half years. Development of the space station using Virtus WalkThrough Pro was begun in the summer of 1996. A prototype of the space station was tested informally late in 1996. The HyperStudio introduction to the space station was developed and tested in the winter of 1997. A literature review and the search for a method, which was recounted in the opening of this chapter, occupied the remainder of 1997 and reached into early 1998. During the winter of 1998 several prototypes of the mental rotation test were developed using HyperStudio and Virtus and then discarded. The computer-based version of the Three Mountains Test was developed using Virtus during the spring of 1998. The research protocol was developed in late spring and the data were collected in early summer. The later half of 1998 was devoted to analysis of the data and production of this document.
CHAPTER 4

ANALYSIS OF THE THREE MOUNTAINS TEST DATA

Data Analysis of Three Mountains Test

The analysis of the Three Mountains Test data was conducted in iterations, as is typical of grounded theory development. Since the data were recorded on video tape, I was able to return repeatedly to the raw data to ask increasingly refined questions. After reviewing the video tapes to get a gestalt impression of each session, I used only the audio tapes to transcribe the verbal data. The data consists of my queries and the participants' verbal indications of which screen shot they thought corresponded to the POV of the little figure that I had placed in the computer model.
Figures 4.1 and 4.2 depict the situation as it was presented to the participants in Problem #1: the large image on the top represents the computer
screen showing the three mountains and the little man place in the middle of the right edge facing the mountains. The miniature figures, labeled H - G represent the five 8 1/2” X 11” screen shots that were laid out in front of the participants. The designations H - G correspond to positions on the perimeter of the three mountains plain and the order of placement of H - G follows the protocol established by Laurendeau and Pinard in their study. Since the options were laid out in front of the participants, answers were typically expressed as this one or that one. To capture the pointing that accompanied these expressions I reviewed the video tapes and added notations to the transcripts to reflect which screen shot participants were indicating. At the same time, I recorded the gestures participants used as non-verbal expressions their responses or reasons. Often I found participants saying this side or that side and indicating with their left or right hands. I also observed that participant frequently pointed to positions on the screen or on the screen shots to specify locations or points of view. This process of adding indications and gestures to the verbal transcript produced an augmented transcript that more closely reflects the participants’ reasoning and their expression of that reasoning as it is captured on the video tape.

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40 Participants generally referred to the figure as the little man. To keep my narrative consistent with the transcripts, I will follow that convention.

41 See Chapter 3 for a fuller description of the research situation.

42 L & P scored only the verbal justifications of their subjects, and insisted that subjects explicitly mention both left-right and before-behind dimensions to demonstrate coordination of perspective. In my scoring I counted nonverbal indications of these dimensions as well as their verbal expression.

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Developing Coding Categories

With the video record close at hand for instant reviews, I then worked through the transcripts developing and assigning coding categories that described the justifications offered by the participants for their choice of which screen shots they thought were correct and incorrect from the POV of the little figure. As I worked through the transcripts, I constructed a set of nine rationales or categories of reasoning being used. These nine categories are as followings:

**LR (Left-Right).** This is the basic projective relation of one object being to the left or right of another object from a particular POV; or of an object being on the left or right side of the field of vision from a particular POV.

**FB (Front/Back).** Along with LR, this is a basic projective relation which denotes one object being in front of or in back of another object from a particular POV. It does not signify that one object is directly behind another, but only that it is farther from the observer in the same general direction, from a particular POV.

**Per (Perspective).** The appearance of the size and shape of objects changes with changes in point of view. Comments which refer to these changes relative to a particular POV are coded Per.

**AIn (Alignment).** Comments that one object is directly behind another from a particular POV or that two objects and a POV lie in a straight line are coded AIn.

**Sec (Section).** Comments that from a particular POV one object partially blocks or occludes one’s view of another object are coded Sec. Along with alignment, to which it is closely related, section is subject to fine degrees of articulation: one can specify, not only that one object partially occludes another, but also which side is occluded and how much remains visible.
FR (Frame of Reference). The vertical and horizontal lines on the surface underlying the mountains (referred to as the plain in the text) offers a possible frame of reference for identifying various POV. Comments which refer to these lines are coded FR.

Rev (Reversed). For some screen shots, especially those that are mirror images of actual screen shots, the most economical way of expressing the relation of one screen shot to the correct POV is to say that the object relations are reversed or switched. Comments which refer to one shot being the reverse of another or that the end objects in a series are switched, while the middle one remains in place are coded Rev.

Imp (Impossible). Several of the screen shots represent impossible POV—they are constructed by taking the mirror image of actual screen shots. Explicit recognition of the impossibility of the shots is coded Imp.

POV (Point of View). A common response to the question why a screen shot is wrong for a particular POV is to indicate to which location or POV that shot corresponds. Part II of L & P's version of The Three Mountains Test exploits this mode of response: the test involves presenting a single picture and asking which POV that picture represents.

Using this set of coding categories I scored the protocols and summarized the justifications offered by the participants. I found that six participants solved all three of the spatial problems with ease; two solved all three problems, but with some difficulty; and six participants erred in solving one or more of the problems. This distribution tends to confirm that this sample captured a cross-section of participants in the process of consolidating their capacity to coordinate perspectives.43

43 This distribution is also reasonably comparable to L & P's sample. They did not report the exact proportion who answered all three correctly; however they did report the proportion that they rated at each level of development. In their scheme, level three is composed of participants who got all three correct and gave adequate justifications. Using their proportions one would expect my sample to include three or four level 3 participants. Using their proportions one would expect my sample to include eight level 2 participants. The level 2
Two Global Strategies

The justifications and comments of the participants, both those who solved all of the problems and those who erred, displayed several salient patterns. In solving the first problem, which nearly everyone answered correctly (13 out of 14), participants typically chose the correct perspective and described it using the same terms I had modeled in the warm-up example. In that example, I said something like the following, which was taken from an actual transcript:

OK, and you can see that the red mountain is in the back, and the blue mountain’s on the left, and the yellow mountain’s on the right. And in this one [indicating the incorrect one] the red mountain’s on the left, the blue mountain is on the right, so that’s all wrong. And the yellow mountain is in the back, so it would be this one[indicating the correct one].

Following my modeling, on the first problem almost everyone (11 out of 14) described their chosen perspective in Left-Right (LR) and Before-Behind (FB) terms. On subsequent problems only about half of the participants followed the modeling with the rest of them using some other combination of categories to establish the rationale for their choices. Once they had explained their choice of the correct perspective, the participants employed one of following two global strategies to justify their rejection of the remaining four perspectives:

Participants would also include some number who got all three correct, but gave inadequate justifications according to L & P’s protocol. Since L & P did not report the number who got all three problems correct, I could not statistically test the equivalence of the two sample distributions.
1. **POV Strategy.** In the POV Strategy the participant justifies rejection of a perspective by indicating which alternative POV corresponds to that perspective. For example, given picture D (see Figure 4.2), the participant might correctly say that it is what you would see if you stood in the back left corner, or they might point to that corner on the computer screen. This strategy was a frequent one for dealing with the *perceived* perspective (picture A), but it was also used extensively for *alternate constructed* perspectives by several participants. While nine participants used the strategy at least once, three used it almost exclusively in dealing with the 12 perspective that they were rejecting. One of those three employed the strategy 11 times, while the other two each used it 9 times. It is noteworthy that all three of those who used the POV strategy got all three of the problems correct. Hence, POV appears to be not only a global strategy, but also a relatively highly developed one that relies on *routinely coordinating POVs and perspectives.* However, it is also worth noting that all of those using this strategy mis-identified several POVs. In fact, one of the participants who was among the pair that got all three right, but only with difficulty, misjudged 3 of the 9 POVs when using this strategy. Thus, while the POV strategy appears to be an *advanced* strategy, used by the more competent in the coordination of perspectives, it can also be a source of errors. In a couple of instances participants recognized that impossible shots were, in fact, impossible. This rare occurrence may well be the most advanced version of the POV strategy.
2. Perspective Reference Strategy. In this strategy the participants used the constructed perspective, which corresponds to the given POV, as the basis for critiquing and rejecting other perspectives. The critiques take the form of either describing in detail how the picture would have to be changed to match the reference perspective; or pointing to positions that objects would have to assume to match the reference; or by indicating how the picture contradicts the reference perspective. Differential use of these three modes of critique -- by detail, by position, and by contradiction -- seems to reflect differences in style of expression, rather than differences in underlying mental operations. However, the whole question of to what degree these expressions reflect mental operations merits further consideration.

Here is an example from Keith's response to problem # 2 in which he uses reference by detail first and then complements that with an implicit reference by contradiction:

Because in his view the blue mountain would be on his right,..., and closer to him. And the red mountain would be farther away. And this view shows that the red mountain is closer to him than the blue one.

Unlike, the POV strategy, which operates only as a global strategy, the Reference strategy, in both the detail and contradiction modes, involves a second level of analysis that involves the operations denoted by most of the other coding categories. Left-right, before-behind dimensions, alignment,

44 To assure confidentiality all names in this study are pseudonyms.
section, perspective and reversal operations all figure prominently in both the process of establishing a reference perspective corresponding to the given POV and in critiquing alternative perspectives. As coding categories, these terms signify the operations being employed by participants constructing reference perspectives and using them to critique other perspectives. The role of operations is apparent in an unremarkable way when participants are systematically constructing correct perspectives and consistently using them as a reference. However, as we shall see, the role of operations is more striking when participants are struggling and making mistakes. This is what one would expect, for the mental operations involved in coordinating perspectives normally operate as tacit process knowledge. Once they are mastered at around age 12, specific operations disappear from view, both for the participant and for an outside observer. It is only during this brief transition period when they are being mastered that the mental operations involved in coordinating perspectives are observable. Even then, they are apparent only when participants err and thereby reveal the operations that they are struggling to learn.

Upon reflection it is easy to see why people would use these two strategies. The structure of the situation lends itself to the development of these two strategies. Recall that, in Piaget's terms, the coordination of perspectives involves establishing correspondences between 1.) POVs and perspectives, and

---

45 See pp. 91-92 for definitions of the coding categories.
46 This is similar to the phenomenon of breakdown that Heidegger and Winograd and Flores have commented upon: we only notice tacit aspects of activity e.g. technology, when it fails to work.
2.) changes in POV and changes in perspectives. Basically there are two terms in these relations: POVs and perspectives. Each of the strategies takes one of those terms as the basis for critiquing alternative perspectives.

Piaget and Inhelder did not discuss the two global strategies -- perhaps because they dealt with the full developmental range from barest beginnings around age 6 to consolidation around age 12, whereas I have focused only on the final transition to consolidation. The POV strategy appears only among the most sophisticated in my sample, and may not have been so obvious in Piaget's sample. However, there is a striking correspondence between the set of detailed operations that I have uncovered and the set of elementary projective concepts described by Piaget and Inhelder (1948/1956) in their General Conclusions chapter of The Child's Conception of Space. In a section entitled Sub-logical operations constituting projective relationships, they delineate eight elementary projective operations, of which they say the following:

...projective operations perform a vital role in bringing about global co-ordination of space, a role which must now be summarized from the standpoint of operational structure. Projective concepts take account, not only of internal topological relationships, but also of the shapes of figures, their relative positions and apparent distances, though always in relation to a specific point of view. (p. 467)

The projective operations (Piaget and Inhelder, 1948/1956, pp. 469-72) and their corresponding coding categories are shown in Table 4.1 below:
<table>
<thead>
<tr>
<th>Piaget's Projective Operations</th>
<th>Coding Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition &amp; Subtraction of Projective Elements</td>
<td><strong>Sec</strong> (Section). Comments that from a particular <strong>POV</strong> one object partially blocks or occludes one's view of another object are coded Sec.</td>
</tr>
<tr>
<td>Rectilinear Order</td>
<td><strong>Aln</strong> (Alignment). Comments that one object is directly behind another from a particular <strong>POV</strong> or that two objects and a <strong>POV</strong> lie in a straight line are coded Aln.</td>
</tr>
<tr>
<td>Complementary Perspective Relations</td>
<td><strong>Rev</strong> (Reversed). Comments which refer to one shot being the <em>reverse</em> of another or that the end objects in a series are <em>switched</em>, while the middle one remains in place are coded Rev.</td>
</tr>
<tr>
<td>Symmetrical Interval Relations</td>
<td><em>(Conserves middle term)</em>[^47]</td>
</tr>
<tr>
<td>One-One Multiplication of Elements</td>
<td><strong>LR</strong> (Left-Right). This is the basic projective relation of one object being to the left or right of another object from a particular <strong>POV</strong>. <strong>FB</strong> (Front/Back). This is a basic projective relation which denotes one object being in front of or in back of another object from a particular <strong>POV</strong>.</td>
</tr>
<tr>
<td>One-One Multiplication of Relations</td>
<td><strong>Per</strong> (Perspective). The appearance of the size and shape of objects changes with changes in point of view.</td>
</tr>
</tbody>
</table>

Table 4.1: Match between Coding Categories and Projective Operations

[^47]: This operation maintains the middle term through transformations such as reversal. In my protocols it was denoted by the term *switched*, which was included in the reversal category.
Piaget and Inhelder argue that these operations build upon each other in hierarchical fashion; that is, that Alignment depends on Section, Reversal depends on Alignment, and Section, and so on. Hence, skill in higher operations requires command of more elementary operations, and coordinating perspectives demands mastery of the set of operations. Furthermore, the operations function as a set and the set includes reversible operations. In short, the set fits Piaget's criteria for operations, namely, they are "actions which are internalizable, reversible and coordinated into systems characterized by laws which apply to the system as a whole." (Piaget and Inhelder, 1948/1956)

In the preceding and succeeding sections of this chapter Piaget and Inhelder delineate eight corresponding operations for topological and Euclidean spaces, respectively. These, as well as the projective operations, are presented as part of a general structuralist argument relating sub-logical operations to more familiar logical operations such as addition and subtraction. Piaget and Inhelder do not take the additional step of explicitly relating the projective operations to their empirical observations, as I have done.

Piaget and Inhelder (1948/1956) contend that our idea of space is constituted through projective operations. Regarding the functioning of projective operations they propound the following three points:

1. Projective operations develop out of more basic topological operations.

2. As a set, projective operations have an internal logic, the logic of concrete operations.

3. Projective operations are of a different logical type than the formal operations of geometry. They characterize this logical type as sub-logical. (pp. 449-50)
By far the most significant notion here is that projective operations are of a different *logical type*—it is central to their claim that our idea of space is constituted through projective operations. Piaget and Inhelder justify the notion that projective operations form a distinct logical level and their characterization as sub-logical by a reference to Russell's *Theory of Types.*\(^{48}\) The fundamental idea here is that through the use of projective operations we create the conceptual space within which formal logical operations then function. The process of creating the conceptual space (and objects within that space) is, in some sense, prior to and of a different logical order than operations which act on collections of objects (or symbols of those objects) within conceptual space. Another way of saying this is that one must have an intuition of unified space, which is supplied through projective operations, before one can do geometry, which involves formal logical operations. Hence, projective operations are a different logical type in just the way portrayed by Whitehead and Russell.

With both the two global strategies and the correspondence between my coding categories and Piaget's hierarchy of projective operations in mind, I would now like to turn to the dominant pattern that emerged in my analysis of the efforts of participants who erred in solving the spatial problems presented by the Three Mountains Test.

\(^{48}\) See Appendix A: Theory of Types.
Difficulties Arising from Alignment and Section Problems

Difficulties in constructing correct alignments and sections contributed to most of the errors observed in the Three Mountains Test. To help visualize the situation recall the following definitions of these coding categories:

**Aln (Alignment).** Comments that one object is directly behind another from a particular POV or that two objects and a POV lie in a straight line are coded Aln.

**Sec (Section).** Comments that from a particular POV one object partially blocks or occludes one's view of another object are coded Sec. Along with alignment, to which it is closely related, section is subject to fine degrees of articulation: one can specify, not only that one object partially occludes another, but also which side is occluded and how much remains visible.

A pattern of confusion and errors arose for participants who had difficulty constructing correct alignment and section relations among two of the mountains and a POV. Of the nine errors made, three were committed by one participant whose only response was to attribute their perceived perspective to every alternative POV. In the literature this is usually referred to as the egocentric response. All of the remaining six errors were alignment and section errors produced by participants who had not yet mastered the operations necessary to construct alternative POVs consistently.
Three of the errors and one illuminating near-error were made on problem two. The situation in problem #2 is similar to problem #1, which was
described in the beginning of this chapter. The participant is facing the computer screen with Figure 4.3. displayed on the screen. The display is exactly the same as the previous problem, except the little figure has been moved to the middle of the left side, facing the mountains. Screen shots, just like the miniature screen shots shown in Figure 4.4 (miniatures E - B), of the alternative perspectives are laid out on the desk in front of the participant. The designations E - B refer to positions around the perimeter of the plain in Figure 4.3 and hence, refer to alternative POVs. Likewise, miniature figures [ E ] - [ B ] are the alternative perspectives corresponding to the POVs. In the selected transcripts that are used to illustrate points the following typographical conventions have been used: the words of the researcher and the participants are shown in ordinary type. Researcher's comments and descriptions of gestures made by the participant are enclosed in parentheses and shown in bold type, e.g. (Points to the right side of the screen). Indications of screen shots, which have letters assigned to them, are enclosed in brackets and designated by the letter of screen shot in bold type, e.g. [ C ]. Also, the mountains are often referred to by only their color, e.g. red to indicate the red mountain.

Bob is a typical example of a participant who is in the process of mastering alignment, but just could not construct it correctly in Problem #2. While he constructed correct perspectives on both of the other problems, he was unable get the correct alignment on #2, and consequently chose perspective [ B ] rather than [ C ]. Bob's difficulty is illustrated in the following transcript:
**Bob: Problem #2**

Bob: This one. [B] Because it's in front (Points to picture, at blue) and these (Red and yellow) are in the back. That's just how he would see it. (Points to screen)

Me: Would you run through that again?

Bob: This one. [B] or wait a minute... this one [C] because this is (Points to screen) sticking out here. Yes, this one. (Back to B) Because the red one is partially behind here and yellow one is off to the side (Points to picture) and a blue one is in front of both them.

Me: So that's the correct one? [B]

Bob: Yes.

....

Me: Okay, and why is this one incorrect? [C] (The correct one)

Bob: Because it (Points to picture, at blue) should be on the left so he can see the yellow one, because the yellow one is off to the side of the blue one. (Points to screen)

....

In this example Bob vacillates between [C], the correct perspective, and the neighboring corner shot, [B]. He settles on [B] because he thinks yellow should be visible. Later in justifying his rejection of [C], he predicates his left-right (LR) construction of blue’s position on his prior commitment to yellow remaining visible: Because it (Points to picture, at blue) should be on the left so he can see the yellow one.

Ron, like Bob, fixes on an alignment that keeps yellow in the picture. Although he expresses some doubt at the end, Ron does not construct an
alignment that explicitly incorporates both yellow and blue with a POV.

**Ron:** Problem #2

Ron: that one. [ B ]

Me: okay, why do you say it is that one?

Ron: well, *(Points to the picture)*, the blue one is in front and the orange (red) one and the yellow one are in the same places that they would be if he was standing there.

....

Me: and why is this one incorrect? [ C ]

Ron: well, the yellow one would still be in the picture.

....

Me: okay, and you said this was the correct one [ B ].

Ron: I think so, I’m not sure???(Expresses some doubt)

Contrast Ron and Bob with Laura who had no difficulty constructing an alignment and section description that explicitly incorporated all three objects with a POV. Here is her response to perspectives [ C ] and [ B ], the two which Ron and Bob confused:

**Laura:** Problem #2

Laura: I think he would take this picture. [ C ]

Me: okay, and why you say that one?

Laura: well, he is looking pretty much almost all at the blue one, but he can see the red one because it is more to the side, but the
yellow one is right behind it so he can't see the yellow, so there would be no yellow.

....

Me: and why is it not this one? [ B ]

Laura: because the blue one, again, would be to the right (Gestures to the right), and the yellow would not be there. The red, ...(Shakes head, no) the blue would be closer again.

As this case illustrates, Laura was able to construct a complex alignment that started with him (the little man in the picture), then moves to the blue mountain and finally on to the yellow. In this regard Laura says, "he is looking pretty much almost all at the blue one, but he can see the red one because it is more to the side, ...so he can't see the yellow...." Similarly, Keith constructed a correct three term alignment saying, "Because from his view, the blue mountain would be blocking out the yellow one, and the red mountain would on, to the left of the blue one". In both of these cases the participants provided a complex statement that included a three term alignment description in a single expression. In its general form a complex alignment expression describes a line with the little figure at one end, the blue mountain in the middle and the red mountain at the other end. In contrast to this complex expression, Ron and Bob offered a set of simpler expressions that relate pairs of objects — figure to mountain or mountain to mountain.

Chris was another person who found it difficult to distinguish between perspectives [ B ] and [ C ].
Chris: Problem #2

Chris: Hmm... ... I know it not either of these. [E, F]. I guess this one, [A], but I'm not quite sure.

Me: OK, why do you think it's this one? [A]

....

Chris: I don't really have any reasons besides that it seems like that would be the view, but I'm not sure. I think it's this one [A], but it could be this one [C] or this one [B], but I think it's this one. [A]

Me: You think this [A] is the best one? Can you explain why that fits?

Chris: Or maybe this one [C]... No this one! [C] Wait, I don't know.

Me: OK, you think this is the best one now. [C]

Chris: I don't know!

Me: Well, explain why it would not be this one. [E]

Chris: It would not be this one because, wait, because the blue one has to be in the front, because he's closer to the blue. (She taps C)

Me: OK, why would it be this one, then?[C]

Chris: Because the blue one is in the front and the red behind it and I'm thinking now maybe the blue one would cover up the yellow... maybe, I'm not sure. (She has it correct, but just is not sure. The question is whether the blue would occlude the yellow.)

....

OK, and how about this one? this one right here? [B]
Why might it be this one? Wait! Because, Wait, this one [B] or this one [C], Why might it be this one? Because, it just looks kind of like that one: this one [B] or this one [C]. I don’t know which one it is (exasperated). I don’t know.

In her nearly-successful attempt, Chris constructs pairs of terms, blue-red and blue-yellow. She says, "the blue one is in the front and the red behind it...maybe the blue one would cover up the yellow." Like Ron and Bob, Chris never consolidates her construction of the alignment into a three term expression.

Finally, there is the case of Amy who had difficulties with alignment in all three problems. She manages to solve two of the three problems. In the following transcript she discovers a correct alignment almost by chance:

Amy: Problem #2

Amy: this one. [A]

Me: why you say this one?

Amy: because, it is kind of hard to tell, but the yellow one is over to the right and then the red one is right next to it and a blue one is in the middle a bit over. (But the blue is not in the middle in the picture she chose, which is [A]. She is having trouble constructing the alignment and section. However, she describes the correct perspective more closely than the picture represents!)

....

Me: okay, why is it not this one? [C] (This is the correct one.)

Amy: because, pretty much the same, but yes, the same, wait! Okay, because the yellow one is not really in the picture either and I think the yellow one would be.

Me: okay and this is the one you said it would be. [A]
Okay and why would it be this one?

Amy: (sigh), because the yellow one is over to the right and the red one is next to it. And then the blue one is almost even with the yellow one. (**She points to the picture and traces an imaginary line with her finger, an alignment of the blue and the yellow.**) Wait! Yes, this one is tricky, but the yellow...actually it is not this one [A] now that I think about it.

Me: okay, do you want to look at them all again and think about which one you think it is? It is definitely one of them. I can guarantee that.

Amy: yes. I think it is actually this one. [C] (**Right this time**)  

Me: OK. Can you explained me why you think it is this one? [C]

Amy: because the blue one, I think, would be covering the yellow one up.

Me: okay.

Amy: and the blue one's over to left farther than the red one. And the red one is more to the right. (**This is confusing. Does she mean in the ego view, where the blue is on the left, but that is irrelevant, or in the perspective C, where it is highly relevant, but she is saying them backwards? The blue is more to the right which is why it occludes the yellow.**)

....

Me: okay, why would not be this one? [B]

Amy: (sigh) because the blue is to the left instead of on the right.

In this interesting protocol Amy almost stumbles upon the correct answer by tracing the alignment on the computer screen. In this way, she physically
constructs a complex expression of alignment which allows her to conclude that yellow would be occluded by blue.

This contrast between the complex expressions of alignment of Laura, Keith and Amy, who solved the problem and the simpler, piecemeal expressions of Ron, Bob and Chris, who did not solve the problem, suggests that a systematic examination of this pattern in the rest of the sample is warranted. Indeed, in the whole sample everyone who solved the problem described the alignment in a complex expression, which linked the little man, the blue and the yellow mountains. The most typical form of this expression was, like Laura and Keith, to say that from the little man's POV the yellow is behind the blue, and therefore, he can't see the yellow. By including the conclusion that therefore, he can't see the yellow this form of the expression strongly implies that all three objects are in alignment. Other examples of this form include Sam who said, "[if I were standing like this] the yellow one would be behind the blue one so that I could not see it"; and Tom who said, "the yellow is right behind the blue one so that he wouldn't be able to see it." Two participants, both of whom got all three problems correct and who seem to be among the most competent in this task, used a weaker form of this expression that lacked the conclusion. One was, Bill who said, "I would imagine seeing this blue Mountain mainly in front ... The yellow Mountain would be somewhere over here. (Points to picture, indicating behind the blue.)" The other form of this expression was to note that from his POV blue and yellow are in the same line, and therefore, he would not be able to see yellow. This form was used by two people: Julie who said it in the strong form: "... it looks as though these (Yellow and blue) are in almost in the exact same line .... so he probably wouldn't be able to

49 Sam's mode of expression was to place himself in the little man's position.
see this one..., the yellow one.... the blue is in the way”; and Sarah, who used the weaker form saying, “they (Blue and yellow) are about in the same line, that way from his perspective. ....”

In conclusion, it is quite striking that all the participants who got Problem #2 correct used a complex expression of alignment, while all of those who got the problem wrong had difficulties with alignment and used a simple form of expression to describe the situation. For now, this suggests an interesting hypothesis to be tested in our analysis of the responses to problem # 3, which also presented alignment problems for several participants. Moreover, this observation, if it is substantiated in our examination of problem # 3, suggests some important instructional design considerations. It also raises once again the question of the relation between the coordination of projective mental operations and linguistic expressions used to describe the operations. This analysis implies either that expressions reflect operations and their coordination, or that expression both reflect and may serve as a linguistic tool for coordinating projective mental operations. Either interpretation will have significant implications for instructional design.

50 There is no reason to believe this difference simply reflects different speech patterns or different levels of linguistic ability. While I did not collect data that explicitly addresses this question, six years of interaction with the participants, including a prior qualitative study conducted in their class, leads me to believe that the group using simpler expressions is as competent in linguistic matters as the rest of the class.
Alignment Difficulties with Problem #3

Figure 4.5: Situation for Problem #3

Figure 4.6: Alternative Perspectives for Problem #3
The remaining three errors were made on problem #3. The situation in problem #3 is somewhat different from problems #1 and #2. Prior to the presentation of problem #3, the participant's POV is moved to the opposite side of the 3-D display. As a result the participant is facing the computer screen with Figure 5. displayed on the screen. The display represents the opposite side of the mountains from the previous problem. The little figure has been moved to the upper right corner, as is shown in Figure 4.5. Screen shots, just like the miniature screen shots shown in figures A through B, of the alternative perspectives are laid out on the desk in front of the participant. (The designations A - B refer to positions around the perimeter of the plain in Figure 4.5. and hence, refer to alternative POVs. Likewise, miniature figures [ A ] - [ B ] show the alternative perspectives corresponding to the POVs.)

The responses to this problem followed the same general pattern as in problem #2: those who solved the problem used a complex expression to describe the alignment, while those who answered incorrectly had difficulties constructing a correct alignment of the figure, then the blue mountain, and finally, the red mountain. Bill, who gave a good example of a correct response, said: "if I say I'm the little man (Points to screen), I would see this blue one and this red one and then to the side of the red one would be the yellow mountain". All told, nine of the ten correct answers incorporated similar complex expressions of alignment.

Ron, as an example of someone who had problems, seemed unaware of the alignment question and focused only on blue being on the left side and yellow being on the right. As a result he had trouble distinguishing between perspectives [ A ] and [ B ]. As the following transcript shows he vacillated
without a clear means of deciding:

Ron: Problem #3

Ron: okay, so which one would be? Right from that corner? That one. [B]

Ron: well, the blue one would be on the left side, that (Points to the yellow on the picture) would be on the right side

....
Me: and why is this one wrong? [A]

Ron: that one might be it, I don't know, I think that one is it [A], I didn't look at them very carefully.

Me: okay, why you say this one is it now? [A]

Ron: well, because it is the right way. That one actually from, it would be from from there, (Points to the picture, AF corner) it would be from, no.
Me: take your time.

Ron: that one is wrong.

Me: take your time, we have plenty of time.

Ron: no, I think this one is wrong. [A] I'm not sure.

Me: okay, you're going back and forth between this one [B] and this one. [A]

Ron: I think it is that one. [B]

Me: okay you are sticking with this one. [B]
Okay, what is the problem with this one? [A]
Ron: I don't know. It still looks good and... 
This one could just be closer up. And that's why there's less of a picture.

Me: yes? Okay, so do you want to leave it that it could be either one of these two?

Ron: okay.

Amy, in contrast, was aware of the alignment question and came close to constructing it correctly, but failed to fine tune her alignment enough to recognize the occlusion of red by blue. Like Ron, she confused perspective [ A ] with [ B ], because "I think the blue one is covering too much of the red one."

Amy: Problem #3

Me: This one, this one, or this one?

Amy: Okay, I think it would be this one. [ A ]

Me: Okay, why do you say it would be that one?

Amy: Because the blue one would be the closest to you. It would be the first one. The red one would be behind it ( But in the one she chose, A, red is farther back, but not behind blue. More section than alignment. ) and the yellow would be sort of, because the yellow it is behind the red one here ( From her ego perspective ), so it would be to the right. ( In the little man's perspective )

Me: Okay, why would it not be this one? [ B ] ( The correct one )

Amy: (sigh) because I think the blue one is covering too much of the red one and I think it would cover up less of the red one. ( Clearly, again, a problem of section )
Julie, likewise, did not realize the importance of constructing an alignment; instead she mentally fixated on the left-right dimension, insisting that "the red would be to the left of the blue." This narrow focus led her to choose [ D ] in spite of its conflicts with the alignment of red and blue with the figure, as well as, the placement of yellow.

**Julie: Problem 3**

Julie: I think the blue would be the first thing he saw and then the red would be to the left of the blue behind it and then he wouldn't be able to see the yellow or it would be right next to the red. But none of these seem to match what I'm thinking. See, in almost all of these, except for this [ E ], the red is to the right of the blue. That's why I think it would be this [ D ].

Alright I'm going to take out this [ A ] because I'm almost positive the yellow would be behind the red. If its next to it, you wouldn't see yellow in front of it. Well, it would either be back here (On A, she points behind red) or next to it. It wouldn't be in front. Maybe I don't want to do that. I'll leave it in..... [ A ] is like a backward view of [ D ]... um......I can't tell if he'd be seeing the red more, whether its directly behind the blue or more to the left or more the right. From here it looks like he would be seeing it more to the left, but it seems like more than half is over to the other side. I'm going to, I think, take out this one [ A ]. I still think the yellow would be farther behind the red or next to it, and that he would be seeing more of the blue than just the end. OK... Alright this is where I'm confused, whether it's the left [ D ] or the right [ B ]. OK, I guess I'm going to say this one [ D ]. Take out this [ B ] because I think that well.........Ya, I am going to take out this [ B ] because I think the yellow is farther back and the red would be on the other side. He would see more of the other side.
Me: OK.

Julie: Now I should sat why its this one. Well the blue is, well the yellow is behind the red, and red is on the right of the blue like it is in here and, no the left of the blue, sorry. And the blue is not directly in front of it, but a little off to the right. None of these exactly match what I though it would look like, but this is probably the closest.

Conceptualizing Alignment and Section Operations

All of the alignment problems revolve around distinguishing perspectives [A], [B] and [C], which correspond to POVs lying in the front [A], the front left corner [B] and the left side [C]. To help conceptualize the operational task of constructing a perspective view from C while positioned at A, I have used Figure 4.1, labeled the relevant points, as well as two points (X & Y) intermediate between A & B and B & C (See Figure 4.7). I have also added miniature screen shots of the perspectives taken from POVs A - C with the direction of focus being the center of the plain. These miniatures mimic the perspectives one would experience walking from point A to point C along the edge of the plain with a focus on the center of the plain. They show the changing alignment of mountains one would perceive if one traversed this path. Contrary to convention, but in keeping with Laurendeau and Pinard's protocol, the positions designated A - C are to be read from right to left.

In the following analysis I have retained the convention of indicating perspectives (as represented in screen shots) with bracketed bold letters. I use bold unbracketed letters to represent the point of view from which the corresponding screen shots were taken.

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As one proceeded from A to X moving along the perimeter, the red mountain would begin to disappear behind the blue. An imaginary line drawn from the
left-most point of the base of the red mountain to the right-most point of the base of the blue mountain and continued on until it intersected the front edge somewhere between A and X. This alignment line (aa* in Figure 4.9.), since it lies on the plane of the bases of the mountains, would indicate the critical point at which the red mountain would begin to disappear behind the blue. Similar imaginary lines (bb* and cc*, in Figure 4.9.) could be constructed to indicate the critical points between B and Y where the red mountain would be completely occluded by the blue, and the critical point between Y and C where the yellow mountain would vanish behind the blue. Focusing consciously or unconsciously on such imaginary alignment lines would facilitate constructing the perspective from C or Y or B. For some it was that easy: for example, Keith who said, "Because from his view (C), the blue mountain would be blocking out the yellow one, and the red mountain would be ... to the left of the blue one."

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52 A physical prop for scaffolding this process might involve providing students with a flexible transparent ruler that they could place on the computer screen to help them construct imaginary alignment lines.
In a similar manner, constructing imaginary lines $aa^*$ and $cc^*$ (Figure 4.10) would have helped Amy and Ron distinguish between perspectives $A$ and $B$. Beyond line $aa^*$ (while moving from $A$ to $B$) red begins to be occluded by blue. Likewise, beyond $cc^*$ red begins to reappear on the left side of blue. Judging by the construction of these two lines one can see that from POV $B$, red would be substantially obscured by blue, but, would not be visible on the left side of blue. On this basis $\{B\}$ is clearly distinguishable from $\{A\}$ on one side and $\{C\}$ on the other side.
Figure 4.10: Imaginary Lines from the Opposite POV

Figure 4.11: Alternative Perspectives for Points A - C
As these two examples demonstrate, the construction of lines of alignment (and implied sections) directly supports the construction of alternative perspectives. Our knowledge of how reversal works can add structural detail to our construction; however, alignment and section are the basic operations. Left-right and before-behind, as projective dimensions, give us tools to reason and communicate about the construction, but they come into play only after basic alignment constructions have been made. The left-right dimension, which is based on the bilateral symmetry of our bodies, depends upon a prior alignment of the center line of our bodies with some external reference point. The before-behind dimension also depends on prior assessment of alignment — before-behind loses its utility to the degree that the object of attention diverges from a prior alignment which established the left-right dimension. Somewhere around 45 degrees divergence from the reference alignment, before-behind starts to become beside — to the left or right. Hence, the coordination of left-right and before-behind dimensions is consequent upon the construction of appropriate alignment and sections.

An Operational Scheme

The operational scheme I infer from these observations and this analysis is as follows:

1. Focus on appropriate details in the situation.\textsuperscript{53} For example, in Figure 4.10, the near, right side bases of the red and blue mountains are

\textsuperscript{53} As with all operational knowledge, experience and, perhaps, instruction guide us in determining what are the appropriate details to support alignment and section constructions.
appropriate points of detail, much more so than the tops of the red and blue mountains. This operation of focusing involves the control of perceptual activity by conceptual operations as was discussed in Chapter One.

2. Construct imaginary lines linking the details with points close to the desired POV. In Figure 4.10, these temporary lines of alignment would be lines $aa^*$, $bb^*$ and $cc^*$. The imaginary line $bb^*$ which connects the far side base points of red and blue would be the most difficult to imagine. Constructing that line requires knowing that in perspective the circular bases of the mountains become flattened ellipses. Recalling and applying this content knowledge is essential for constructing hidden lines such as $bb^*$.

3. Coordinate the temporary lines of alignment and draw out implications. For example, in Figure 4.10, for POVs along the right edge, lines $bb^*$ and $cc^*$ delimit the small region within which red would be directly behind blue. Beyond $bb^*$ (closer to $B$) a section of red would be visible on the right of blue. Beyond $cc^*$ (closer to $C$) a section of red would be visible on the left of blue.

4. Use the temporary alignment lines to construct the central line of alignment linking the POV with the objects. In Figure 4.10, this would entail a process of successive approximations that would project the central line roughly parallel to $cc^*$ and place the red mountain almost
directly behind the blue one, but slightly to the right since B lies beyond bb*. This would imply a central line that projects from B through the right side of blue to the center of red.

5. **Use the central alignment line as a frame of reference to assign left-right and before-behind relations.** This operation allows both internal checking and communication about the correspondence of a perspective with a POV.

6. **Draw out the implications of the alignment.** In Figure 4.10, this would mean realizing that the yellow is to the right of the central line and recalling that the placement of lines aa*, bb* and cc* imply that red is behind blue with some showing on the right.

The scheme for coordinating a single POV with a perspective comprises these six steps, some of which represent multiple operations. Coordinating multiple perspectives, of course, involves rapidly applying this alignment scheme to multiple situations. The alignment scheme described here is an inferential model of the program of mental operations necessary for coordinating perspectives. It is based solely on the the three mountains test. To the degree that the three mountains test situation is prototypical of projective space, the alignment model is a prototype of the construction process for projective space. I assume this alignment model is but one among several schemes for the coordination of perspectives. However, it is one that lends itself to providing guidance for the design of constructivist learning environments.
Data Analysis of Part II of the Three Mountains Test

Part II of the Three Mountains Test was conducted at the end of the research session, right after the Space Station Exploration. The same teams (see Chapter Five for vignettes of the teams) that explored the space station were presented with five perspectives (Figure 4.12).

Figure 4.12: Screen Shots of Perspectives for Part II

For each of the perspectives the team was asked to point to the position or POV on the screen image that corresponds to perspective: the question I asked was “Where would the little man need to stand to take this picture?” Figure 4.13 depicts the screen image presented to the teams, except the positions were not labeled. Notice that perspective [I] represents an impossible POV — the screen shot is of the mirror image of perspective as it would be seen from the upper right-hand corner in Figure 4.13. (the unlabeled EF corner)
Laurendeau and Pinard found Part II to be considerably easier than Part I of the Three Mountains Test. After comparing the two parts of the test they decided to use only the first part in generating a developmental scale. Part II was used only as a secondary check on their analysis of Part I (Laurendeau and Pinard, 1970, pp. 321-327). I had intended to use Part II in a similar secondary sense; however, my analysis of Part I cast the results of Part II in an interesting light.

Among the seven teams in this study, all teams got the first four items correct. This is not surprising — they were collaborating in teams and Laurendeau and Pinard's subjects had found the first four problems to be
relatively easy. In their study the 11- and 12-year-olds, working individually, were correct at least 60% of the time on items 1 - 4. However, in both studies, problem five (the impossible POV) had a different outcome. In Laurendeau and Pinard’s study, only 20% of the 11- and 12-year-olds, correctly recognized that the image was impossible. In this study two of the seven teams recognized that the screen shot was impossible. Laurendeau and Pinard attributed this massive failure to the specious nature of the problem and an over-reliance on dominant features. (1970, p. 324). I have a slightly different interpretation.

Part II explicitly calls for use of the POV strategy discussed earlier in this chapter. At that point I describe the POV strategy as a relatively highly developed one that relies on routinely coordinating POVs and perspectives. The data in this study suggest that all teams initially used the POV strategy, relying primarily on the prominent position and proximity of the red mountain. Members of two teams, then, went on to check their answers using operations of the alignment scheme. For example, in one team, Ron first sensed a visual contradiction saying “But the yellow one would have to be over there [on the other side of the red].” and Tom added the conclusion saying, “It would be that corner [EF] if these were reversed”. In making this judgment, Tom computed an alignment, recognized the implied left-right relations, and realized that the yellow and blue mountains should have been reversed. The team composed of Sue and Laura, likewise, realized that the image represented an impossible POV. In both cases, the members mutually scaffolded each other constructions to quickly arrive at a correct solution, suggesting that with feedback to prompt checking of responses, impossible shots could be used to encourage practice of the mental operations involved in the construction of perspectives.

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CHAPTER 5

ANALYSIS OF THE SPACE STATION DATA

Introduction

The analysis of the space station exploration was conducted in several stages. The first step was to convert the visually and auditorially-rich video tapes of kids careening around a virtual space station into transcripts that imposed a space and time framework onto their activity. To do this, I used the three levels of the space station and the color of rooms within each level to describe location and the timer on the VCR to assign a time coordinate. Within this space-time framework, I recorded the action and speech of the participants.

Below is a typical snippet from one of the transcripts:

<table>
<thead>
<tr>
<th>Location</th>
<th>Time</th>
<th>Person</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purple</td>
<td>2:40</td>
<td>Sue</td>
<td>What's that down there?</td>
</tr>
<tr>
<td></td>
<td>3:00</td>
<td></td>
<td>(They slide down the tube.)</td>
</tr>
<tr>
<td>Lower</td>
<td>3:20</td>
<td>Laura</td>
<td>Whoa! (on reaching the bottom)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Giggles)</td>
</tr>
</tbody>
</table>
BlueStart 3:30
4:30 Laura  Whoa! What was that? (Sees start cube.)
Laura  Go in here or in here.
Sue  Wherever you wish to go.
5:15 Sue  You are good at controlling the mouse.
Laura  Pretty!
5:40 Sue  It's a red room.
Laura  Are there any objects in it?
Red 5:45 Sue  Reminds me of Myst.
(They look out the long hall window.)

Initially, I produced complete transcripts; however, as I developed a set of categories, I became more selective, ignoring mutual direction-giving and other incidental conversation. Even with this selectivity, some transcripts still required 15 pages. I used these transcripts to create a set of descriptive categories or themes. However, since some of the most interesting patterns of activity involved extended sequences of moves throughout the space station, I created a means of summarizing the whole session into a single table. To complete this table I assigned a sequential number to each location that the team visited and then entered the series appropriate to each location in the table. For example, if the team began on the middle level in the blue room (as most did) and then circled through the four rooms on that level twice, the middle line of their table would appear as in Figure 5.1

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Sessions in the space station last from 28 to 52 minutes with the average being 40 minutes.
This example indicates that the team visited the blue room (middle level) first and fifth in their journey; the green room second and sixth, etc. Both the middle and the lower levels are continuous loops. The top level, however, is discontinuous -- there is no connecting hallway between the terminal rooms. In addition to the colored rooms in the space station the Space - Time record includes other locations, such as diagonal traps, inside computers, inside stars, or just in outer space, where some participants spent significant amounts of time.

Vignettes of the Seven Teams

In the following section I introduce the seven teams of participants by presenting the Space - Time Record of their sojourn in the space station. This introduction also describes the participants in terms of their performance on the Three Mountains Test and their responses to the brief questionnaire, which was
described in Chapter Three. The data from the test and the questionnaire are offered only to convey a more vivid sense of the participants. Given the small sample size and the absence of striking patterns, I see no significance beyond their descriptive value. I also use the vignettes of the teams to introduce themes or categories of activity that are generally applicable, but are well-illustrated by a particular team. In a subsequent section I will consider the significance of these themes.

I discuss the results of the Three Mountains Test in terms of two categories: those who got all the problems correct and those who missed one or more problems. As the transcripts in Chapter Four indicated, the participants who missed some of the problems are very actively struggling with the mental operations required — they are learners or apprentices of the process. Among those who got all the problems correct, even the most sophisticated, made careless errors in their justifications or in Part II of the test. In keeping with the apprentice terminology, I refer to this other category as journeykids. Among the fourteen participants there were six apprentices and eight journeykids.

There are two categories of data from the questionnaire that I will use: 1. whether the children describe themselves as frequent computer users, and 2. the total number of spatial-related activities reported by the children in part II of the questionnaire. Six of the fourteen participants answered yes to the question: "Do you use it (your home computer) frequently?". Part II of the questionnaire was composed of fourteen activities that prior research has shown are correlated with higher test scores on some measures of spatial ability. Total positive responses to these queries ranged from three to twelve,
with a median of nine. In the discussion I will refer to whether the participant was below, at, or above the median on spatial-related activities.

**Team One: Chris and Becky**

Chris and Becky are both young (eleven-year-olds) spatial apprentices. Both scored below the median on the spatial-related activities. Neither described herself as being a frequent user of their home computer. In fact, Becky is probably the closest to a non-user in the sample, being the only one who did not say she liked to use computers.

Chris and Becky were conservative explorers. They spent the majority of their time on the middle level, where they began. They made over three complete circuits of that level and only one each of the top and lower levels. They were very task-oriented, taking care to note the size and shape of objects and directing their exploration toward gathering the requisite information. They made very good use of the windows, frequently using them to orient themselves and plan next steps. Chris, in particular, became more adventurous as the session proceed. Toward the end she began to move faster, experimenting with the virtual weightlessness by flipping and spinning. — I refer to this type of activity as movement play. She also started making play sounds to mimic the sounds of motion and collision — “Enroom!” , “Boosh!” , “Shuuu!” , “Boom!” . Both Chris and Becky were quite impressed by the model of the space station that is located on the upper level exclaiming, “Oh, my gosh, is that the space station?” “Oh, cool! This is neat!”. However, they did not use the model to orient themselves or plan further explorations. The use of
windows for orientation and planning, the mimicking of sounds — sound play — and movement play are three important themes that arose first in this session.

Team Two: Sue and Laura

Sue and Laura are two bold young adventurers. Sue responded above the median in terms of spatial-related activities. She is a frequent PC user — she commented that the space station reminded her of Myst. Laura scored right on the median in spatial-related activities. Both proved to be quite proficient journeykids in the Three Mountains Test.

In the space station, this team spread their time fairly equally among the several levels. However, they did not limit themselves to just the space station proper. They explored one of the diagonal traps, got themselves into and out of a computer and mastered the art of slipping through the walls of the space station, an activity not planned during the design of the space station. This bit of serendipity opened a whole new arena for exploration and, eventually, for my designs. The addition of this realm for exploration presented exciting possibilities as Laura evidenced when she yelled, on her third or fourth trip outside the space station, “We’re in outer spaaaace!” Clearly, for Sue and Laura exploring the space station from the outside was just as interesting and, perhaps, more than exploring it from the inside.

55 At first I thought they might have discovered a seam where the walls were not completely attached. However, after several more passages through the walls, I realized that the computer they were working on was running at a processor speed at least five times the speed of the machine on which I developed the space station. My hypothesis is that the higher speed of the processor sometimes allows the movement routine to exceed the capacity of the collision detection routine; hence when moving at a high speed one can slip through the walls.
A second theme, in addition to exploration, that this pair initiated was imaginative play. At one point Sue, without explanation, imported a dramatic reference: she began calling Laura first "Sargent Wilson", then "Sargent Lawson", asking her to "Bring me up, Sargent Wilson". It was as if they were characters in a play navigating a space vehicle. Another innovation introduced by this team was to use the model of the space station to map out their strategy for further exploration. They combined this mapping activity with use of the windows, as Chris and Becky had done, and with exploration outside to get a quite full picture of the whole space station. In the process, they totally lost track of their task and concluded without reporting the information I had requested.56

Team Three: Bill and Amy

The team composed of Bill and Amy presents an interesting situation. Bill is probably the most sophisticated computer user and certainly was one the most skilled performers on the Three Mountains Test. Amy, on the other hand, is a young learner on the spatial task and a relative novice computer user - at least for gaming. While Amy indicated that she is a frequent computer user, she acknowledged during the session that she does not play computer games. Bill, in contrast, expressed interest in the overall design project, inquiring if I intended "to sell this as a computer game after it has been fully modified?", suggesting that I should have an outside gateway and asking "How do you get the graphics to work?" 57 He also demonstrated great facility in navigating

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56 I am sure they would have reconstructed it if I had asked at the end of the exploration, but I did not.
around the space station. In fact, when I returned to the room to begin the space station exploration session, I found that he had come in, figured out the controls, and begun cruising rapidly around the lower level. He continued to *cruise* while Amy got oriented, which accounts for the disproportionate activity on the lower level. Later, he skillfully *tacked up and down* a diagonal tube, which for most people is a frustrating trap. Interestingly, both Bill and Amy responded below the median in terms of the spatial activities, which, of course, is testament to the fact that the relation between those activities and spatial ability is correlational, not causal.

Under Bill's leadership, the team quickly exhausted the space station, exploring all areas and gathering the required information. When they inadvertently slipped through a wall into space, Bill's response was "I thought I would take advantage of the situation and map the space station." Later, when they came across the holographic model of the space station on the upper level, Bill actually took time to do a sketch of the space station.

This team was so efficient that they completed gathering the information in only 28 minutes, whereas other groups reluctantly quit after 45 or 50 minutes, and then only because we were constrained to wrap up at a certain time. Even so, this team gathered the requested information and covered more territory than all but one other team—almost eight circuits of levels of the space station.

This team, far more than any other team, had an internal conflict between one member who was oriented toward the task and the other who

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57 From a prior study with many of these same kids, I know that Bill has done programming and a bit of hacking.
wished to explore. The task, of course, was only given as a pretext to encourage exploration. Regardless of the task, I am convinced, Bill would have continued much longer and engaged in much greater challenges if the opportunity had been there. However, to Amy’s credit, this was one of the few teams that actually reported the requested information at the end of the session.

**Team Four: Phil and Keith**

Phil and Keith are the quintessential explorers — the boy counterparts to the team of Sue and Laura. They are both journeykids in terms of the Three Mountains test and at or above the median in the spatial-related activities. Unlike the girl explorers, these two are among the older group (sixth-graders). Neither described themselves as frequent computer users; however, they were both very engaged with exploring the space station.

In 40 minutes they went everywhere and did (nearly) everything, including exploring outside the space station three times, probing a diagonal trap, getting stuck for five minutes inside a computer and ending stuck inside a star. This team travelled extensively: in addition to their many side excursions they circuited levels of the space station nearly nine times. I think they could have gone on much longer — mainly outside — if we hadn't run out of time.

Phil and Keith expanded on the sound play motif. In addition to sound mimicry — "zoom", "errroom!", "bang!", "boom!" — they added imaginative sounds, especially saying "ow!" every time they ran into a wall or object. At one point, Phil discovered that he could keep a beat by repeatedly hitting the
wall. Another time, Keith addressed a computer: "Hello, computer. What do you say? Do you say 'Hey'?") (This interaction suggest some interesting design possibilities.) They also proposed exiting a computer through its CDROM drive. I categorized this notion, as well as several other implicit and explicit suggestions put forth by the participants, under the theme suggestions.

The imaginative play of this team is also suggestive. They evolved the idea that by pressing the right key combination, they could "elevator up" or "elevator down". They also alluded imaginatively to other contexts -- sometimes private jokes and sometimes more public references, as in: "Houston we have a problem. We are stuck in a computer!" This, of course, is another example of imaginative play that was recognized in Sue and Laura's transcript.

This team, like several others, gathered lots of information about colors of rooms and objects from which they could have gleaned the requested information, but the task was eclipsed by the excitement of exploring.

Team Five: Julie and Sarah

Julie and Sarah presented an odd mixture. Both are young girls, but there the similarity ends. Julie scored as an apprentice on the Three Mountains Test, was below the median in terms of spatial activities and called herself a frequent computer user. Sarah, in contrast, performed as a journey kid on the Three Mountains Test, was above the median on spatial activities and is not a frequent computer user.
Both inside and outside the testing situation Sarah has impressed me as being an extremely kinesthetic person. In the space station simulation she frequently shifted, tilted or rotated her head or body, rather than using the mouse to control her point of view. She does not seem to be someone who relates easily to computers. On the other hand, her accomplished performance on the Three Mountains Test and her frequent participation in spatial-related activities, seem to indicate that her kinesthetic intelligence is working well for her and helping her master the mental operations involved in the coordination of perspectives. She presents an interesting contrast to Bill, who was mentioned earlier — he scored low on the spatial-related activities, but performed very well on the Three Mountains Test.

Julie and Sarah’s time in the space station was a combination of fun and frustration. Of their 45 minutes, they spent nearly a third trying to get out of a trap or trying to get back into the space station. Unlike some other teams, their excursions into these realms were frustrating diversions rather than adventures. At one point Julie plaintively asked, “Are we the most confused of everybody? Probably!” With these several distractions, they did not cover all of the space station — they missed half of the top level and part of the bottom level. All told, they circled levels of the space station only about three and one half times, which is a little more than half the average. However, in the end, Julie’s parting words were, “That was neat-o, keen-o!”

While they had fun, this team also found the experience often frustrating. Julie turned this frustration into a constructive suggestion: “If someone spends more than eight minutes on this [a diagonal trap], you should have a button on the screen you can click and it will show you the way out.”
Team Six: Ron and Tom

Ron and Tom are two older boys who are both frequent PC users and who both register high on the spatial activities scale. Tom did well on the Three Mountains Test; however, Ron is a bit of an anomaly. Like Bill, he is an experienced computer gamer; for example, he tacked up and down diagonal tubes with ease. But, in spite of his age, PC use and spatial-related activities, he still was only an apprentice in the Three Mountains Test. This could be testing error, but his expressions fit the pattern for the others who were having troubles integrating the mental operations into a functional scheme for coordinating perspectives. I surmise that he is someone who could benefit quickly from some scaffolded practice in the necessary operations.

In many respects Ron and Tom's exploration of the space station was similar to that of Phil and Keith, but scaled back. They toured levels of the space station a little over 5 time compared to nearly nine times for Phil and Keith, and they had only one brief sojourn outside the space station. They engaged in imaginative play. Ron's classic expression was “I don't think we are in Kansas anymore!”

Ron is apparently an accomplished computer game player as evidenced by his ability to tack up and down diagonal tubes. Tom, on the other hand, is a novice who enjoyed the lack of gravity, doing somersaults and spins. They both engaged in frequent sound play (“Ow”, “Swoosh!”, “Vroom”). They got excited by their accomplishments and were frustrated that there were not more levels to explore. However, they did get outside: “Wow, we’re in outer space!” – just as time ran out. Ron’s final comment was, “That was cool!”
Team Seven: Bob and Sam

Bob and Sam are intrepid explorers much like the teams #2 and #4 or, at least a close second. Neither are frequent computer users. Bob, the older of the two, was above the median in spatial activities, but scored only as an apprentice on the Three Mountains Test. Sam, on the other hand, was at a median level in spatial activities and scored as a journeykid on the Three Mountains Test.

This team covered all levels, but spent more than half their time on the middle level, making 5 complete or partial circuits of it. They also spent a significant amount of time outside, including an eight minute final romp which ended with them trying to crash into a star when time ran out. In fifteen minutes they changed from wanting to avoid the outside: “Let’s not go outside anymore.” to seeking outside adventures: “I think we should go outside the space station.”

They were probably the most exuberant team, saying, for example, “Wow, this is cool!” and “Wow, this is fun!” several times. Like all the other teams, they engaged in a lot of sound play, adding a whistling sound to the collective repertoire as they descended a tube.

Their imaginative play was also noteworthy. At one point Sam said auspiciously, “We are space peoples.” Another time Bob, while proposing that they go into a star said he wanted “to go where only one other person has gone” -- an obvious allusion to Star Trek.
Constructing the Virtual Environment

The space station simulation is simply patterns of pixels on a screen. These patterns represent shapes – simple Platonic solids and regular tubes – that I have joined together and called a space station. The underlying software, *Virtus WalkThrough Pro*, allows the user to control their point of view (POV) with the mouse and keyboard. The software very efficiently creates the illusion of an environment by rendering in realtime sights and sounds appropriate to the user’s POV. If the presentation is effective, the user *assimilates* the representations into their repertoire of operational schemes; and, in doing so constructs the virtual environment. Through the active construction processes of the user, the patterns of pixels (and sounds) become a virtual environment.

The data that I recorded and which I have introduced in the above vignettes are reflections of those active construction processes. The themes that I described are manifestations of sensorimotor, perceptual, representational and conceptual schemes employed by the participants as they enacted the space station and interacted with that construction. Examining patterns in those themes will provide a basis for judging the potential effectiveness of the space station as a virtual learning environment. They will also furnish grounds for an abstract model of the space station in terms of which I can consider design revisions, including the inclusion of design elements aimed specifically at supporting the invention and practice of the mental operations involved in the coordination of perspectives. In discussing the themes and patterns I will necessarily use language that tends to reify the representations that compose the space station. Likewise, I will write about the participants as if they were
separate from the constructed environment, when in fact what I observed was the participants interacting with the environment as they constructed it. Much of what I observed reflected prior experience with computer games — or the lack of such experience. Some people manipulated the computer controls with great ease indicating considerable prior experience; others operated the controls with difficulty. However, several teams showed marked improvement in handling the controls during the single 45 minute session, suggesting that lack of experience with computer games would not be an enduring barrier.

Patterns of Activity, Patterns of Engagement

The interior of the space station is a very sparse environment. There are about a half dozen geometric objects with numbers inscribed on their surfaces scattered throughout the rooms. There are two computers with information pertaining to a complex puzzle displayed on their screens. The participants in this study were informed about the big puzzle to which the information of the computer screen refers, but they were not trying to solve that puzzle in this exercise. Their task was simply to collect some of the information on the geometric shapes. The other elements of the space station interior are a scale model of the space station located in one of the terminal rooms on the top level, spacy music that played only on the bottom level and a rainbow carpet that sometimes appeared on the lower level.

The seven teams in this study explored this environment, nominally in search of some simple information. On average they traversed the top level a little over once each. They circled the bottom level an average of twice. Finally,
since the middle level communicates between the other two and because they typically began there, they averaged just over three circuits of that level. Six of the seven teams ventured outside the space station; one went out five times, but the average was twice. All but one of the teams explored the diagonal traps, some with considerable success. Two teams went into the computers. Finally, three teams tried to enter stars — one team was successful.

Assimilative Play

The teams explored widely and came to terms with the space station — assimilated it to their operational schemes — through a variety of play mechanisms. These were illustrated earlier in the vignettes, but I will summarize them now. All seven of the teams engaged in sound play. Their sound play assumed two variations: 1. background sounds such as “Zoom!” “Vroom!” and “Bang!” which served to fill in sounds implied by the action, but missing from the soundtrack, 2. immersive sounds, most notably, “Ow!” or “Ouch!”, when colliding with walls or objects. Four of the seven teams engaged in immersive sound play. One participant, Phil, even, in mock seriousness, said, “I hurt myself” (8:35).

A second form of assimilative play was imaginative play, which was practiced by six of the seven teams. In this form of play outside references were imported to the situation, which I believe served to contextualize and make familiar the strange environment. Sometimes it took the form of allusions which acknowledged the strangeness as when Ron said “I don’t think we are in Kansas anymore” (13:40). Other times it served to connect to other exploratory
adventures, for example when several teams used the phrase “Forward ho!”.
A third variant is illustrated by Keith’s use of the phrase to *elevator up* to
assimilate the unusual experience of sliding up a long tube into the familiar
experience of riding an elevator.

A third form of play utilized by nearly all the teams was experimenting
with movement in a virtual weightless environment. People spun around, did
back flips and somersaults and other movements which are possible only in a
weightless state. I believe this form of play helped acclimate the participants,
particularly those who are novices at computer gaming, to the virtual
environment.

A fourth form of play exercised by only two teams is what I call *joint
control play*. Navigational control is achieved through a combination of
keyboard controls and mouse movements. The research situation was set up so
that one member of the team controlled the mouse while the other controlled
the keyboard. The results of the mouse movements are contingent upon which
keys are being pressed. Normally, the POV of the team moves forward when
the cursor is place above the middle of the screen. However, if the team
member controlling the keyboard suddenly presses the *option* key, the POV
will move directly upward instead. In two teams, Sue and Laura and Phil and
Keith, several times the keyboard controller, without consultation, pressed the
*option* key and the team’s POV suddenly shot upwards. In both cases, it was
obviously done repeatedly as a non-verbal joke.
Task Engagement

The ostensible task for the teams was to find several pieces of information. All the teams attended to this task and, with only one exception, returned something from which the correct answers could be gleaned. However, the larger assignment was exploration. The Introduction to the Space Station made this quite clear when it said:

Here is a computer Space Station for you to explore. You can explore it together and talk about how to get around... After you have explored the Space Station for a while, I would like for you to collect some information from inside the Space Station. ... (Chapter Three, Appendix X)

Both in pursuing the information-gathering and in exploring all the teams followed my suggestion and made at least occasional use of the many windows to orient themselves and plan their activities.

Moving beyond the information-gather task, five of the seven teams explored beyond the space station. They went outside, into diagonal traps, into computers and even into stars. To aid in this larger exploration, three of the teams used the scale model of the space that is located in one of the top level terminal rooms as 3-dimensional map. Using it they mapped out where they had been and where they wanted to go. This use of the Holographic Model for mapping distinguished these three teams as more advance adventurers.

A third task-oriented theme that emerged was either implicitly or explicitly offering suggestions for the improvement of the space station. In one way or another, all the teams, except one, made suggestions that could be used
to improve the space station. They range from allowing escape from the computer through the CDROM drive to providing a doorway (Sue) or a gateway (Bill) to get back into the space station. All these suggestions reflect an engagement with the design of the space station which could facilitate information-gathering, exploration and other tasks.

Affective Engagement

The display and expression of emotions and feelings are what we normally use to judge engagement with an activity. The participants revealed their feelings about the space station exploration in several ways. All of the teams expressed interest, excitement and delight with some aspects of the experience. Often this excitement was conveyed through the tone and manner of their expression, which is vividly recorded on the video tape. However, only a shadow of the expression remains on written transcripts. I used exclamation points to record such expressions. Although such an assessment is quite subjective, I found that I made such notations on a average of about half a dozen per transcript, with one transcript receiving twelve. Suffice it to say, the participants were excited during the space station exploration.

In addition to expressions of excitement, several participants expressed confusion and/or frustration. The two boys, (Bill and Ron) whom I identified as probably the most experience computer gamers, both conveyed frustration at there not being more levels or places to go in the space station. Julie and Sarah, too, said they were frustrated, but more with the confusing environment and their inability to navigate easily. They were also the team that said "Are we the
most confused of everybody? Probably!” At one point Sue and Laura said they were confused; however, their confusion was momentary, since they proved to be among the most adept explorers.

Another indicator of affective reactions was the general emotional tenor of the sessions. With only one exception the sessions were filled with frequent laughter and giggles, as if the participants were doing something enjoyable together. The one exception was team three in which the tension over information-gathering versus exploration was already noted. That session was certainly congenial, but not as jovial as the others.

Finally, the spontaneous exclamations of the kids were direct indications of how they felt about the experience of exploring the space station. In every session one or often both of the participants said either some part of the experience or the whole experience was cool, neat, fun awesome or some similar expression of positive feeling.

Conclusions

Several conclusions that can guide design and suggest directions for further research seem warranted by the foregoing analysis.

1. The space station environment is a viable arena for the exercise of spatial intelligence. While the space station exploration was enjoyable and engaging for virtually everyone, it was more so for those with well-developed operational schemes for the coordination of perspectives. A review of the vignettes reveals that there were only two teams in which both of the participants were journeykids. Significantly, these were also the teams that I
identified in the vignettes as being the most adventuresome and the teams which appeared to be the most fully engaged in the themes analyzed. In particular, team two (Sue and Laura) and team four (Phil and Keith) were two of the three teams that used the model space station for 3-D mapping; they were the only teams that engaged in joint control play; and finally, team four was one of the three teams that engaged in immersive sound play. Moreover, these two teams ventured outside as much as the other five teams combined. They were the only two teams to explore the interior of computers and were two of the three teams that tried to enter stars. In so far as the Three Mountains Test is an indicator of spatial intelligence, the space station is an arena for the exercise of spatial intelligence, at least for these four young people.

2. The space station environment is a viable arena for the exercise of spatial intelligence for kids having the spectrum of spatial abilities represented by this group of 14 children. It remains an open question whether the engagement of the other teams, each of which includes at least one apprentice, was, proportional to their respective development of operational schemes for spatial intelligence. However, there is certainly reason to suppose that is the case — both theoretically and empirically. In terms of the data, other teams engaged with the space station environment similarly to the two advanced teams, but to lesser degrees: other teams explored outside the space station; other teams noticed the 3-D model; other teams engaged in immersive sound play, etc. However, it remains an open question whether a year later, or after an hour's more experience with the space station, the other teams would have engaged just as the advanced teams did.

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58 It was actually only Bill who individually constructed a map of the space station. Interestingly, he is also the most sophisticated performer on the Three Mountains Test.
3. The space station environment includes several spaces and places that offer various design possibilities. The space station is not, as I originally thought, a complex unified structure. As the participants in the study demonstrated, it is composed of a macro environment, the field of stars, a meso environment, the space station proper, and various potential micro environments, the interior of computers, for example. The different environments have different characteristics, some of which may be very significant for design purposes. For example, the middle level of the space station gets lots of traffic, suggesting the possibility of locating a sequence of progressively more difficult activities here.

4. The design of the space station could easily incorporate more levels, more micro environments, more furniture and objects. Micro environments could be developed in their own right -- the interior of computers could contain a panoply of components within which the participants might be challenged to navigate. Likewise, the field of stars could have an inherent structure or structures which require identification and navigation. However, the various environments require easy access. Gateways, doorways and passages between environments require careful planning and design: anything that can be gotten into must have away out!

Conclusions and lessons learned from considering the space station in light of this body of data quickly glide into design revisions. I will conclude this chapter on the analysis of the Space Station Exploration here and leave the outline of design revisions for the final chapter.
CHAPTER 6

CONCLUSIONS

Introduction

This study addresses two problems: 1) how the body of Piagetian research concerning the coordination of perspectives can be used to design a constructivist learning environment; and 2) more generally, how research findings can be used to guide the design of constructivist instructional media. The first question -- the question of Piaget in 3Space -- is approached through two qualitative studies: a computer-based replication of the Three Mountains Test and a qualitative study of a prototype virtual learning environment. In this chapter I will review the results of these two inquiries and consider how they can be joined in a design process; that is, how the understanding gained through the Three Mountains study can be used to guide the redesign of the space station. The outcome of this process will be a set of design directions for the next iteration of the space station design. Next, I will outline ideas for future research that flow from this
study. Finally, I will address the second question with methodological reflections on the process I have used to contextualize research findings for design purposes.

Results of the Three Mountains Test Study

The Three Mountains study yielded three primary results. The first result is a description of an operational scheme for the construction of alternative perspectives. The scheme — the alignment scheme — incorporates the set of projective operations described by Piaget and Inhelder; however, it goes beyond Piaget and Inhelder's findings by proposing how the construction process may actually be accomplished. This finding represents a significant advance beyond previous attempts to account for the construction of alternative perspectives using notions of mental rotation.

The second outcome of the Three Mountains study is the discovery of a means for relating the two studies. Analysis of the data from the Space Station Exploration study, indicates that there is a strong correspondence between performance on the Three Mountains Test and performance in the space station. Consequently, in so far as the Three Mountains Test reflects some aspect of spatial intelligence, it is an aspect that is common to performance in these two contexts (at least for this group of children).

The third product of the Three Mountains study is the specification of a detailed set of requirements for the design of learning environments aimed at scaffolding the development of ability to construct alternative perspectives. Following Vygotsky (1978), scaffolding is typically interpreted to involve cues, modelling,
These requirements include opportunities to discover and practice the six operations delineated in Table 6.1, both individually and in combinations. Piaget indicates and my own research demonstrates that these operations function as a hierarchy; thus, practicing them in combinations seems sensible. In addition to these basic operations, the scheme entails perceptual and representational activities involving recognizing appropriate details and constructing imaginary alignment lines. Discovering and practicing these operations, as well as the whole alignment scheme, complete the set of design requirements.

or suggested strategies. In this study, scaffolding is extended to encompass the notion of embedded opportunities to discover and practice a set of operations.
<table>
<thead>
<tr>
<th>Piaget's Projective Operations</th>
<th>Coding Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition &amp; Subtraction of Projective Elements</td>
<td><strong>Sec</strong> (Section). Comments that from a particular POV one object partially blocks or occludes one's view of another object are coded Sec.</td>
</tr>
<tr>
<td>Rectilinear Order</td>
<td><strong>Aln</strong> (Alignment). Comments that one object is directly behind another from a particular POV or that two objects and a POV lie in a straight line are coded Aln.</td>
</tr>
<tr>
<td>Complementary Perspective Relations</td>
<td><strong>Rev</strong> (Reversed). Comments which refer to one shot being the reverse of another or that the end objects in a series are switched, while the middle one remains in place are coded Rev.</td>
</tr>
<tr>
<td>Symmetrical Interval Relations</td>
<td>(Conserves middle term)</td>
</tr>
<tr>
<td>One-One Multiplication of Elements</td>
<td><strong>LR</strong> (Left-Right). This is the basic projective relation of one object being to the left or right of another object from a particular POV. <strong>FB</strong> (Front/Back). This is a basic projective relation which denotes one object being in front of or in back of another object from a particular POV.</td>
</tr>
<tr>
<td>One-One Multiplication of Relations</td>
<td><strong>Per</strong> (Perspective). The appearance of the size and shape of objects changes with changes in point of view.</td>
</tr>
</tbody>
</table>

Table 6.1: Coding Categories and Piaget's Projective Operations
Results of the Space Station Exploration Study

The first product of this study is a global assessment of the space station environment, primarily in terms of engagement. Three categories of engagement describe the data: 1) Assimilative Play, 2) Task Engagement, and 3) Affective Engagement. In terms of all three categories, the space station environment is an engaging one for the participants. This is the case for all seven teams, but, more so for the two teams in which both members performed well on the Three Mountains Test.

The second result, which follows from the first, is a confirmation that the space station is an appropriate environment for dealing with spatial intelligence, especially those aspects related to the coordination of perspectives. The intuition that an environment like the space station might support the development of spatial intelligence gave impetus to the space station project and guided its initial design. This intuition was substantiated by both the global assessment and the evidence of a relation between performance on the Three Mountains Test and performance in the Space Station Exploration. At least for this limited sample, the space station appears to be a fitting environment for exercising and developing spatial intelligence.

The final fruit of the analysis of the Space Station Exploration is the identification of a group of opportunities for redesign of the space station environment. Four subspaces of the total environment appear to be promising locations for embedding new design elements. They are all present in embryonic form; hence, they will be relatively easy to develop. Moreover, they may be prototypical of categories that can be expanded.
The first subspace is the field of stars. It is a fully three-dimensional array that could be structured and configured in many ways. Currently, it consists of three types of Platonic solids -- tetrahedron, octahedron and cubes. Other satellites -- moons, artificial satellites, smaller space stations -- could be added. Subsets of the array could be structured into constellations that are recognizable only from specific viewpoints. The second subspace is the space station itself especially the mid level, where high traffic patterns suggest it as an arena for recurrent puzzles that are presented with increasingly greater demands. A third category of subspaces comprises the interiors of objects. In the current version, this category is best represented by the interiors of computers; however, the interiors of stars, satellites and machines, as well as computers, offer many opportunities for embedding microworlds that incorporate design elements. The final category consists of the interfaces between other subspaces. The Space Station Exploration study documents many instances where participants were eager to move from one subspace to another. This interest in passage could be channeled into many conceptual spatial challenges.

Design Directions

Combining the results of these two studies to create a set of design directions requires a theory to relate systematically the design requirements flowing from the first study to the design opportunities presented in the second. This theory concerns how perceptual and conceptual activities relate to each other. Piaget struggled with this topic throughout much of his career, culminating in the volume The Mechanisms of Perception (1961/1969). In this
book and in other writings reaching back at least to *The Concept of Space in the Child* (1948/1956) Piaget distinguished between figurative structures and operative structures; that is, between the content and the process levels of cognition. At the same time he argued that *perceptual activity*, as process, derives from sensorimotor activity and continues to develop in interaction with conceptual activity. The proposition that I would like to put forth is that *within this interaction between perceptual activity and conceptual activity, we use perceptual activity to scaffold the development of conceptual activity, at least in the realm of the construction of alternative perspectives*. By this proposition I mean that we use perceptual activity to model, cue, check, and support our construction of alternative perspectives when we use the alignment model. This may seem like an almost self-evident notion; however, in the literature on perspective-taking, perceptual activity has been seen as evidence of *egocentrism* rather than as a support for the development of conceptual skills. For example, this interpretation led Laurendeau and Pinard to classify a set of critical responses as egocentric, which profoundly affected their interpretation.60

Given the notion that, at least within the purview of this study, perceptual activity can be used to scaffold the development of conceptual activity, several design directions follow easily from the results of the two studies. The first direction would be to devise a series of tasks in *truncated three dimensions*.61 In this series, the alternative perspectives to be

60 See Chapter Two, pages 41 - 44, for a discussion of the concept of *egocentric* responses and Chapter Four, page 100, for a discussion of the critical responses.
61 By truncated I mean that, like in the Three Mountains Test, one of the dimensions is only partially represented. In the case of the Three Mountains Test, the truncated dimension is the vertical dimension.
constructed would move progressively from zero to 90 degrees deviance from
the participant's POV, much as the alternative POV was moved in the analysis
of alignments in Chapter Four. Recognizing significant details and constructing
temporary lines of alignment would be the primary objectives of these
activities. Feedback or the opportunity to retry would be essential. The main
structure of the space station, especially interfaces between levels or areas and
between the interior and outer space might be locations for such tasks. Traffic
patterns could be monitored and the task varied with each successive
encounter. The goal of these tasks would be to provide opportunities for the
participants to invent/discover and practice the alignment scheme and the
operations it comprises.

A second design direction would be to use the interiors of objects, like
computers, to reproduce variants on the Three Mountains Test. In a computer
the components could be used to fashion a truncated 3-D environment similar
to the Three Mountains Test. Following Keith's suggestion, which was
recounted in Chapter Four, exit from the computer could be through the
CDROM or some other port. Participants could have the choice of searching
through the maze of components to find the way out or of using information
to quickly identify the escape route by constructing an alternative perspective.
A diagram "stenciled" on the inside of the computer could depict the
perspective to be constructed.

A third design direction would be to take advantage of the fact that the
field of stars (and other satellites) occupies a fully three dimensional space.
Tasks such as finding a particular star using only an picture of the field of stars
taken from a different POV could truly challenge participants.
Ideas for Future Research

This has been a very fruitful study — many ideas for future directions have occurred to me. I will mention just four of them that I will probably pursue. The first research direction is, obviously, to do a qualitative study of the space station, revised as discussed above. If I am able to implement even a fraction of the innovations implied by the design directions, I will have rich new environment worthy of a detailed study. The second research direction is to conduct a larger scale study of the computer-based version of the Three Mountains Test with a sample that has a wider age range and greater diversity in backgrounds. My objectives in such an inquiry would be to determine whether the alignment model applies to the experience of other children and to see if other schemes emerge from a larger pool of experience.

In addition to these direct extensions of the two constituent studies of this inquiry, two more radical spin-offs appeal to me. The first is to extend the Three Mountains Test into a fully three dimensional framework. Using something like the field of stars in the space station environment, one could devise a 3-D test of the coordination of perspectives. The Platonic solids in the field could play the same role as the mountains in the traditional test. A boundary -- perhaps a semi-transparent cube -- would be required to delimit the field and allow the designation of alternative POVs. Provision of a 3-D set of axes would complete the parallel with the traditional Three Mountains Test. To maintain continuity with prior research using the Three Mountains Test, all other aspects of the research methodology would remain the same as in Chapters Three and Four of this study.
The second fundamental extension of this research would require the collaboration of a mathematician. While conducting this study, I have observed that several authors have suggested that important formal mathematical concepts have been developed in conjunction with notions of perspective and projective space. As I noted in Chapter One, Piaget argues that projective space forms the basis for our intuitive sense of a unified space, within which the formal operations of geometry can be performed. In a volume entitled *The Invention of Infinity*, J. V. Field (1997) notes that both the intuitive sense and the formal mathematical sense of infinity were affected by the constructions of perspective by Renaissance artists. Field (1997) argues that Desargues, the originator of projective geometry, was "the first mathematician to get the idea of infinity properly under control" (p. 196). It may be that with Desargues's work on projective geometry, the formal operations of geometry were separated from the concrete operations that constitute projective space and the intuitively-sensible distant point of perspective construction was distinguished from the mathematical notion of infinity. However, it is intriguing to observe that Desargues's Theorem, the fundamental basis for projective geometry, rests on formally establishing the alignment of three points, just as the alignment model for the construction of alternative perspective proposed in this study rests on intuitively establishing the alignment of three points. Moreover, formal mathematical concepts, such as critical points, figure in the alignment.

62 The formal statement of Desargues's Theorem is "If two triangles are in perspective from a point, the three intersections of their corresponding sides (extended) lie on one line" (Oglivy, 1969, p. 89). Oglivy does not define what is meant by "in perspective from a point"; however, the meaning is evident in his discussion of the figure used to illustrate the proof. The point referred to is the center of projection. In reference to this figure, Oglivy writes: "In Fig. 63, triangle $\triangle ABC$ of Plane $\Sigma$ (sigma) is projected to triangle $\triangle A'B'C'$ of plane $\Sigma'$, $O$ being the center of projection." (Oglivy, 1969, p. 86)
scheme. In addition, there are other possible connections to be investigated – for example, Brook Taylor, of Taylor series fame, wrote a volume entitled *Linear Perspective* (1715). I am not a mathematician; consequently, I cannot judge the relevance of these observations. However, the notion that an intuitive basis for formal mathematical concepts could be developed using a constructivist learning environment similar to the space station is a very attractive idea – one that merits further investigation!

**Limits of the Study**

The limitations of this study are complements of its strengths. The study focused on one method of re-contextualizing research findings for design purposes: it did not compare this method with alternatives. The study examined closely the experience of a small group (14 children) from an atypical school: it did not utilize a large and representative sample; hence generalization is not warranted. The Three Mountains study focused on 11 and 12-year-olds to investigate a crucial transition period: it did not examine other age groups, such as 6 to 10-year-olds, for whom equally interesting developmental transitions may be occurring. The Three Mountains study focused on illuminating the *mental operations* through which young people construct and coordinate alternative perspectives: it did not inquire into why the participants were at a particular developmental stage – either in terms of antecedent conditions or possible gender-related differences. With the exception of translating the test into a computer context, the Three Mountains study adhered closely to the research protocol established by Piaget and Laurendeau.
and Pinard: it did not explore the implications of translating the test into a full 3-dimensional representation. The methodology exploited the advantages in capturing the nuances of human behavior and communication afforded by the use of video technology: it did not develop the theoretical rationale for such use, nor compare it with other methods. The Space Station Exploration employed and emergent design and procedure, supporting discovery and serendipity: its conclusions were, appropriately, more descriptive and evaluative than the Three Mountains Test analysis. The final limitation is simply a limitation, with no complementary strength. It is that the study, while extensive, was limited to one cycle of the overall development process: it does not include the next phase of field testing a new generation of the space station that incorporates design elements based on the Three Mountains test.

**Methodological Observations**

The global methodology of this study was to use small scale qualitative studies to contextualize research findings and connect them to a prospective design context. A central question for this study has been how findings can be transferred from one medium in which the research was conducted to another medium in which designs will incorporate those findings. Grounded theory development was used, in part, because of its emphasis on transferability of findings. The developers of this method point to the role of grounded theory as a vehicle for bridging different, but related contexts. They do not claim that the theory is context-free, but rather that it can comprehend a variety of contexts through concrete links to the various contexts. This, of course, is
exactly what is required to re-contextualize findings in design projects.

The utility of grounded theory development as a method for re-contextualizing and transferring findings from a research to a design context far exceeded my expectations. The qualitative study of the Three Mountains Test, not only gave greater substance to Piaget and Inhelder's findings, but also revealed a synthesis of the mental operations into an operational scheme for constructing alternative perspectives. The qualitative study of the Space Station Exploration, not only yielded a clear description of interaction with the environment, but also, suggested avenues for embedding the required design elements in the space station design. The combination of these two studies proved to be a useful, albeit time-consuming, method for transferring research findings to a design context.

The success of this method suggests two further steps. The first step would be to systematically compare this research-for-design method with the more conventional approach of contextualizing research findings through a literature review. A second direction to pursue would be to investigate the possibility of using different media — perhaps multimedia published on CDROMs — to include more of the research context in published reports. This procedure would allow instructional designers and other users of the research to re-contextualize more easily the findings for their own purposes.

Denouement

This study has been a journey — it began with some interesting design concepts and intimations of questions worthy of pursuit. It has surprised and
delighted me, unfolding in ways I could not anticipate. Now it ends with interesting design directions, insights that merit sharing and more questions deserving pursuit. It has been a journey worth the price of passage.


APPENDIX A

THE THEORY OF TYPES

Gregory Bateson is one of the few social scientists to extract Whitehead and Russell's theory of types from *Principia Mathematica* and apply it to social phenomena -- his comments will be most helpful in understanding what Piaget and Inhelder mean by the assertion that projective operations are of a different logical type. In an essay entitled, *The Logical Categories of Learning and Communication*[^3], Bateson said:

> ...the theory asserts that no class can, in a formal logical or mathematical discourse, be a member of itself; that a class of classes cannot be one of the classes which are its members; that a name is not the thing named. . . . . it is not at all unusual for theorists of behavior science to commit errors which are precisely analogous to the error of classifying the name with the thing named -- or eating the menu instead of the dinner -- an error of logical typing. (p. 280)

One of the most common errors of logical typing involves mixing categories of different levels of abstraction. Bateson explains the logical foundations of this sort of error:

...a class cannot be one of those items which are correctly classified as its nonmembers... we shall commit an error in formal discourse if we count the class of chairs among the items within the class nonchairs. Inasmuch as no class can be a member of itself, the class of nonchairs clearly cannot be a nonchair. ... (a) that the class of chairs is of the same level of abstraction (i.e. the same logical type) as the class of nonchairs; and further, (b) that is the class of chairs is not a chair, then, correspondingly, the class of nonchairs is not a nonchair.64

This logic may be applied to the notion of egocentric responses. If egocentric responses are the absence of projective operations, then they are a class of non-projective operations, and hence they are of a different logical type than projective operations.

Drawing upon Russell's theory of types, Piaget and Inhelder argue that concrete projective operations are of a different logical type than the formal operations of geometry: -- in their terms, they are the sub-logical operations through which we constitute projective space.

64 ibid, p. 280.
APPENDIX B

QUESTIONNAIRE

Circle One (circle none if you don’t know or it does not apply to you)

Yes  No  Do you have a computer at home?
Yes  No  Do you use it frequently?
Yes  No  Do you like to use computers?
Yes  No  Does your mother use a computer at work?
Yes  No  Is your mother a teacher or professor?
Yes  No  Does your father use a computer at work?
Yes  No  Is your father a teacher or professor?
Yes  No  Do you have a TV at home?
Yes  No  Do you watch TV frequently?
Yes  No  Do you watch mostly public TV?

For the next questions, answer for the time since kindergarten.

Yes  No  Have you played with Legos?
Yes  No  Have you played with other building blocks?
Yes  No  Have you repaired toys or bicycles?
Yes  No  Helped repair automobiles?
Yes  No  Repaired equipment?
Yes  No  Built race car sets?
Yes  No  Used power tools?
Yes  No  Built train sets?
Yes  No  Made clothes from patterns?
Yes  No  Played with building set?
Yes  No  Built scenery for plays?
Yes  No  Built models?
Yes  No  Traced routes on a map?
Yes  No  Navigated in a car?
I would like your help with two computer programs that deal with your perception of space in computers. The second program is a virtual space station that I have been developing. I think you will enjoy exploring it. The first program is a test of how you perceive space. The results of the test will help me improve the design of the space station.

The first program is based on a test of spatial perception that has been used for over fifty years. Traditionally, it has been done using a tabletop model of three mountains on a plain—it's called the Three Mountains Test. I want to see how it works on a computer. I would like you to do the first part individually.

After you have each done the first part of the Three Mountains Test, I will bring you together, and you can explore the space station as a team. After you have explored the space station, for about an hour, I would like you to finish the second part of the Three Mountains test, but as a pair.
I would like your help with some research I am doing. I am testing some computer programs to see how young people deal with space on computers. The results of this research will help me get my degree from Ohio State University.

During July I will be collecting the data for my research. I hope to include all of the 1997-98 PVCS Intermediate class in my study. Naturally, travel and vacations may exclude some, but I anticipate that most of the class will be able to join in. I am writing to invite you to participate, and to request your parents’ permission for you to be included in my study.

Participation in the study will require a single 2-hour session, which will be conducted at PVCS. During the session, you will be asked to test 2 computer programs. The programs run on ordinary Macintosh computers like the ones regularly used at PVCS. One of the programs is a little test of how you perceive space. The other program lets you explore a virtual space station. Neither program requires any special equipment, so it should provide a familiar, safe and, hopefully, enjoyable experience.

I will collect data by video-taping you as you interact with the computer programs. This will allow me to capture the action on the computer screen and your comments on the same tape. Since the recording will be from behind, you will not be easily identifiable. Furthermore, the tapes will remain with me and, will not be available for others to view. However, I may wish to use clips from the tapes (in which individual children are not identifiable) as part of a
conference presentation. I will also want to interview you about your
computer experience and other activities. These brief interviews will be tape-
recorded. I am also interested in cooperative learning, so I will ask you to
interact with the programs both individually and in pairs. To arrange pairings, I
am asking everyone to suggest 3 other classmates you would like to be
grouped with.

I will provide transportation to and from the sessions. To help your busy
parents with their transportation schedules I will be happy to pick you up either
at your home or at another location, and return you either to your home or to
another location. To show my appreciation for participating in my study I want
to give you your choice of either 2 tickets to the _____ theater or a gift
certificate to the _____ Book Center, as well as my deep gratitude.

If you are willing and your parents are willing to have you participate in
this study, I will need the following:

1. A permission form signed by both parent and child.
2. A schedule of workable dates and times in July.
3. A list of 3 classmates (from last year) for potential groupings.

If you have any questions, please call me at 123-4567. Enclosed is a letter
for your parents--please pass it on to them. I will call you about specific
arrangements when I receive your reply.
SPACE STATION INTRODUCTION

Here is a computer Space Station for you to explore. You can explore it together and talk about how to get around. You will need to decide who will guide the mouse and who will help. You can change roles whenever you want.

The Space Station begins with a little program to show you how use the mouse and keyboard to get around in the Space Station. There are several pictures of the Space Station in the introductory program. Look at them closely -- they will help you find your way around the Space Station. I have printed out a navigation guide that summarizes the instructions in the introductory program.

After you have explored the Space Station for a while, I would like for you to collect some information from inside the Space Station:

1. Find the sum of the numbers on objects in red rooms.

2. Find the sum of the numbers on yellow objects.