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Principles for the design and development of large travel Magnetic Suspension Actuators

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

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ABSTRACT

Electromagnetic systems are at the forefront of available technologies that possess the potential to deliver advanced actuation systems. Their contactless nature is recognized to be a key asset that eliminates friction, and helps realize accurate positioning in multiple degrees of freedom at high bandwidths. These characteristics have been realized in the current generation of positioners, but the developments suffer from small workspace volume which limits their widespread applicability. To address these shortcomings, this dissertation focuses on the design and development of a Magnetic Suspension Actuator (MSA) with large travel in multiple degrees of freedom.

Critical to the development of such an actuation system, is the formulation of the principles for the design of two basic components: (1) The electromagnetic actuation strategy that supports the characteristics demanded of the actuation system, and (2) the control algorithm that regulates the performance of the MSA to the desired specifications. In the design of the actuation scheme, a methodology is first developed to synthesize optimal actuation strategies comprising of only DC electromagnets. In order to reduce the size and the power requirements of the electromagnets in these actuation schemes, permanent magnets are combined with DC electromagnets to generate Permanent-Electro-Magnet (PEM) combinations. Optimal design of the PEM combinations that maximizes the force characteristics in relationship to the
actuation requirements, while maintaining flotor manipulability is addressed. The PEM combinations are then incorporated into the design of the horizontal and vertical actuation schemes that result in the desired actuation properties.

Formulation of the principles for the design of the compensation scheme, first involves the development of a robust nonlinear control algorithm, for realizing large and accurate travel in the MSA. Geometric feedback linearization technique using the nonlinear model of the electromagnetic force is employed to formulate the controller. To provide robustness to uncertainty/disturbance, an algorithm based on discrete time-delay control is formulated. The performance of the robust nonlinear control algorithm is experimentally investigated on a single degree of freedom MSA.

Tracking accuracy in the robust nonlinear control strategy degrades with increasing frequency and amplitude of the desired trajectory, due to the presence of ferromagnetic hysteresis in the electromagnetic actuators. The Preisach independent domain model is utilized to capture all of the essential characteristics of the hysteresis nonlinearity in electromagnetic actuators. The Preisach model is inverted and incorporated into the robust nonlinear control algorithm to compensate for hysteretic effects. The performance of this approach is verified experimentally.
In praise of Lord Sri Krishna
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I am grateful to my advisor Dr. C. H. Menq for his constant guidance and support during the entire course of my doctoral studies. It has been an honor and a privilege to have the opportunity to work with him. I am also indebted to Dr. K. Srinivasan and Dr. V. I. Utkin not only for their comments and suggestions on this dissertation, but also for having helped in many ways during my graduate studies at OSU. My thanks to the students at the Coordinate Metrology and Measurement Laboratory for providing a friendly and scholarly environment to pursue this research. Experiments performed in this dissertation would not have been possible without the help of Joseph West, Jerry Kingzett, and Keith Rogers, Laboratory Supervisors in the Department of Mechanical Engineering. Finally, I express my deepest appreciation for my parents for their sacrifices in helping me realize my objectives.
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CHAPTER 1

INTRODUCTION

Precision engineering is at the forefront of technologies resulting in technically advanced products that deliver superior performance with longer life. Many of these products incorporate components manufactured to increased tolerances, with improved quality control and greater miniaturization. This necessitates manufacturing processes and machines, operating in regimes of micro or even nano technologies. Fabrication processes such as photolithography demand positioning tolerances in the range of 1-10 nanometers, while micro machining of silicon wafers requires accuracies of $0.5 - 5 \mu m$ [27]. In achieving these submicron accuracies, the fabrication technique relies on extremely high fidelity in transfer of the tool motion profiles onto workpiece features with the desired surface qualities. Researchers have reported that tool motion accuracy, tool travel, tool stiffness and bandwidth are the critical factors in the success of precision fabrication [42] and inspection processes [4].

Actuation mechanisms in precision fabrication processes are designed to provide the necessary tool stiffness, and the nanometric accuracies that enable stable machining at high bandwidths [14, 53]. Recent developments in piezo-electric Fast Tool Servos (FTS) [33] and magneto-stiction actuators [13] are being utilized to generate accurate geometries with fine tolerances in asymmetric turning operations [15, 49]
with variations less than 40 nanometers at good repeatability. However, the size of features that may be produced by these actuation mechanisms is restricted to about 50-200 micron, which hinders widespread commercial applications, such as turning of ferro-metallic cams. To overcome these large travel restrictions, some of the machines [12] are designed with bearings that couple several actuators. The bearings constrain the motion to the desired degrees of freedom while the actuators regulate the tool position within these degrees of freedom. The precision attainable via this technique is limited due to the inaccuracies of the bearing elements. Gas and fluid bearings used in diamond turning machines eliminate surface contact but are difficult to model and control at high bandwidths due to their nonlinear dynamics [21].

Precision inspection applications such as electrical and mechanical probing of semiconductor wafers demand high positioning tolerances over large areas (100 nanometers over an 8 inch wafer [4]), and multiple degrees of freedom actuation with high resolution and bandwidth. High positioning resolution over a large area is usually obtained by cascading coarse-fine manipulators. In this configuration, the coarse manipulator provides the large travel in multiple degrees of freedom, while the fine manipulator positions with high accuracy around the nominal operating point of the coarse manipulator. This scheme usually involves complex mechanical couplings which are inherently difficult to model and control due to the presence of nonlinearities such as clearances and friction. Therefore even though this configuration is able to realize high positioning resolutions over large areas in multiple degrees of freedom, it is restricted by its small bandwidth at which it inspects the wafers.
The development of magnetic suspension actuators can potentially solve these problems and make significant contributions in the areas of high precision fabrication. Since magnetic suspension eliminates contact and friction, the problems of roughness and expensive tolerancing in mechanical bearings are eliminated. Actuation schemes based on magnetic suspensions utilize the same set of electromagnets for realizing large travel in multiple degrees of freedom actuation, as well as suspension. This eliminates the alignment problems encountered with mechanical bearings, and the bandwidth limitations associated with coarse-fine manipulators. In addition, the contactless nature of magnetic suspension provides several key advantages such as reduced mechanical maintenance, unlimited life, and no lubrication. Since the Magnetic Suspension Actuator (MSA) is actively controlled in all its degrees of freedom, several of its properties are functions of the control algorithm that can be manipulated in real time. This enables instantaneous actuator balancing, vibration compensation, adjustable stiffness and damping, and decoupled motion along the coordinate degrees of freedom. Using advanced compensation strategies, robustness to plant parameter variations and external disturbances can also be incorporated. For these reasons, magnetic suspension actuators are an attractive solution to the problem of precision motion control in multiple degrees of freedom with large travel, high resolution and bandwidth. The following section details the important issues in the development of these systems that are being addressed.
1.1 Statement of the problem

Magnetic suspension has been extensively exploited in the development of contactless bearings and magnetically levitated vehicles (MAGLEV) for ground transportation systems. Both of these systems are narrow gap devices, in which the suspended object (flotor) moves close to the electromagnet pole-faces. Since the strength of the magnetic field decreases rapidly with increasing distance, narrow gap operation enables high force densities. Magnetic suspension actuators based on the same principle have been investigated in literature, where the entire travel of the actuator is confined to a limited region around a nominal operating point in the magnetic field. Magnetic suspension actuators based on this principle are characterized by either multiple DOF actuation with small workspace volume [26] or large travel in one/two degrees of freedom with low dynamic stiffness [79]. Electromagnet actuation schemes in these systems are designed to provide large flux densities in regions close to the electromagnet pole-faces that automatically precludes large travel. The control algorithm utilized to regulate the performance of these actuators is based on an approximate linear model of the actual nonlinear field distribution at the nominal operating point. The linear algorithms perform satisfactorily in small regions around the nominal operating point, but the performance degrades with increasing travel. The development of a large travel, multiple degrees of freedom MSA warrants further investigations in issues relating to the design of electromagnet configurations and the synthesis of appropriate control algorithms. This research aims at developing a set of principles for the design and development of large travel, magnetic suspension actuators. The basic issues and the solution approaches relating to such an investigation are outlined in the following sections.
1.1.1 Principles for the development of the electro-magnetic actuation scheme

In the design of the MSA, several schemes exist for generation of magnetic fields in space and consequent suspension of the floating body. Utilization of actively controlled Direct Current (DC) electromagnets, or of Lorentz force based electrodynamic actuators are two popular methods for generating magnetic suspensions. In comparison to DC electromagnets, electrodynamic actuation provides higher force densities due to advances in permanent magnet technologies, and the higher current sourcing capabilities of the AC power supplies. However, linear motors can be designed to provide large travel in only one/two degrees of freedom, when the flotor is constrained to move at constant air-gaps. If large travel along all the three translational degrees of freedom is required, then actively controlled DC electromagnets are the only feasible generators of magnetic field. Therefore, the dissertation first focuses on the development of a rational design principle for synthesizing electromagnet arrangements in a MSA.

The current generation of magnetic suspension actuators employ a general design principle in which electromagnets are arranged in push-pull configurations. Since an electromagnet can only attract a ferro-magnetic body, the push-pull configuration of two electromagnets is equivalent to a bi-directional actuator. Therefore, magnetic suspension actuators with \( n \) DOF usually have \( 2n \) or more electromagnets located in pairs on opposite sides of the flotor. However, systematic analysis of the electromagnetic force model suggests that the number of electromagnets required to suspend a \( n \) DOF actuator may be reduced, by a more astute location and orientation of the electromagnets. This reduction in the number of electromagnets not only leads
to a lighter and a more compact mechanism, but also reduces the number of power supplies that lowers the cost and simplifies the control algorithm. Consequently, the development of a rational approach for synthesizing minimum electromagnet actuation schemes for the Magnetic Suspension Actuator, that are optimal with respect to its large travel requirements, is first addressed.

The electromagnet actuation scheme designed above, can be further enhanced by incorporating permanent magnets with DC electromagnets to give Permanent-Electro-Magnet (PEM) combinations. These PEM combinations provide some critical advantages in the design of the actuation scheme for magnetic suspension actuators. The permanent magnet in the PEM combinations provide a strong DC component of the magnetic flux which is utilized in supporting the weight of the flotor. Also, the permanent magnets provide significantly higher forces so that it is easier to obtain large travel from the actuation scheme in all of the translational degrees of freedom. The combination of these two factors reduces the size of electromagnets in the actuation scheme, thereby making it more compact and requiring less electric power, in comparison to the one comprising of only DC electromagnets.

However, actuation schemes with PEM combinations need to be carefully designed since maximization of force characteristics often leads to loss of flotor manipulability. At small distances, the DC electromagnet interaction is particularly strong and overwhelms the forces from the permanent magnets, leading to a loss of manipulability. At large airgaps, forces from the permanent magnets dominate the interaction. Therefore, synthesis of actuation strategies comprising of PEM combinations that not only ensure the actuation characteristics but also the manipulability of the flotor in
the entire operational space of the MSA, is an important design issue that is addressed in this research.

1.1.2 Principles for the development of the control algorithm

As detailed in the previous section, much of the current state of the art in magnetic suspension actuators is based on the narrow gap principle of operation in which the floating body moves close to the electromagnet pole-face. The control problem then is to provide tracking within the confines of a limited region around a nominal operating point in the magnetic field. Usually linear control strategies are employed, that are based on an approximate linear force model of the actual nonlinear force distribution, at the nominal operating point. The tracking performance of the linear control strategies continuously degrades with increasing deviations from the operating point. until the system destabilizes for a set of fixed controller gains. However in the case of large travel, excursions from the operating point are typical. Therefore, linear control strategies cannot be employed. Further, the electromagnetic force model is typically uncertain, which may further change during operation. The MSA may also experience unmeasurable external disturbance forces in applications such as precision machining. Consequently, the performance requirements of the MSA dictate that its controllers incorporate the nonlinear model of electromagnetic interaction, and must be robust to modeling variations and external disturbances while providing precision tracking at high bandwidths.

Development of the robust nonlinear control algorithm is approached using the geometric feedback linearization technique [60, 56]. Geometric feedback linearization utilizes the model of the electromagnetic force interaction to cancel the nonlinearities.
and reduce the plant to a linear controllable form. Cancellation of the nonlinearities renders the system susceptible to modeling uncertainties, parameter variations and external disturbances. An approach to robust feedback linearization is investigated by estimating the effective uncertainty as a function of the system states and tracking error, through the concept of discrete time delay control [50]. This compensation algorithm is implemented on a single degree of freedom, magnetic suspension system and large travel, accurate tracking and enhanced dynamic stiffness in face of significant modeling uncertainties and external forces are demonstrated.

Tracking accuracy in electromagnetic actuation system degrades with increasing frequency and amplitude of the desired trajectory, due to the presence of ferromagnetic hysteresis. Ferromagnetic hysteresis is a complex nonlinearity with memory that may result in multiple outputs for a given input, depending on its time history. It manifests strongly in a large travel MSA, where electromagnets are driven with current signals that undergo full scale excursions, driving the core well into ferromagnetic saturation. Unmodeled hysteresis leads to inaccuracy in trajectory tracking at high frequencies with large amplitudes, and the distortion in performance is significant in context of high accuracy tracking applications [67]. To overcome this limitation, it is important to develop hysteresis models that capture all of the essential characteristics relevant to electromagnetic actuators, and can also be incorporated in the control algorithm formulated above. Development of these models, their experimental verification, model inversion, and consequent hysteresis compensation are the final steps in the development of the complete compensation strategy for the MSA.
1.2 Scope and significance of the research

The research focuses on the formulation of general principles for the design and development of a large travel, multiple degrees of freedom MSA. To this end, it addresses specific issues in the development of electromagnetic actuation schemes, and the synthesis of robust, nonlinear, discrete time controllers, as outlined in the previous section. The formulation of these principles is an important step in the eventual development of the MSA with two key properties: Large travel in multiple degrees of freedom, and enhanced stiffness. The combination of these properties will not only increase the workspace volume of the actuator that may be accessed with high resolution but will also facilitate the utilization of the MSA in situations where it is dynamically coupled to the environment, and experiences large interaction forces. It may therefore be used for advanced precision fabrication applications such as micro machining, and photolithography.

In addition to enabling a generation of magnetic suspension actuators with enhanced capabilities, investigations of the issues in the design of electromagnetic actuation schemes and control system contribute to the current set of results in the following specific aspects:

- **Synthesis of optimal electromagnet arrangements:** In the large travel MSA, the electromagnetic force changes in magnitude as well as direction with variations in the airgap. In designing a minimum electromagnet configuration for the MSA, it is necessary to account for these nonlinear variations, and ensure the manipulability of the flotor at each point in the travel range of the MSA. This nonlinear variation of the forces distinguishes the problem of
electromagnet location in a large travel MSA, from similar problems in the areas of robotic grasping where unidirectional actuation forces are also encountered. The linear dependence between the components of the finger forces when friction is modeled in grasping problems has resulted in several synthesis results, none of which are applicable to the problem of nonlinear variation of forces in the MSA. Chapter (3) presents tests for synthesis of electromagnet locations that guarantee flotor manipulability at each point within its three DOF translational space, in face of the nonlinear variations.

- **Synthesis of PEM combinations for large travel:** A systematic procedure for designing PEM combinations, given a desired travel range is formulated. Unique to this design procedure is its ability to maximize the available force from the PEM combination, while preserving flotor manipulability in the complete travel range. When incorporated into the actuation scheme for the large travel MSA, PEM combinations help in reducing the size of the electromagnets and the associated power requirements.

- **Compensation of unmatched uncertainty:** When the uncertainty in the state-space representation of a physical system occurs at the same level of differentiation as the control input, it is termed as *matched* uncertainty. Robust feedback linearization techniques such as sliding mode control, Lyapunov function based compensation, and adaptive estimation are employed in compensation of matched uncertainty. The matching conditions are typically restrictive for arbitrary physical systems, particularly in the case of magnetic suspension when its voltage-current characteristics are modeled, and the nature of the uncertainty
is unmatched. Past research in compensation of unmatched uncertainties imposes restrictions on the uncertainty formulation, that are not satisfied by the suspension equations. Chapter (5) removes these restrictions by formulating an approach that regulates the system output by estimating the uncertainty as a function of the system states and the tracking error through discrete time-delay control, for systems that are state feedback linearizable.

- **Compensation of hysteresis in electromagnetic actuators:** Even though ferromagnetic hysteresis has been studied extensively in literature, its model based compensation in electromagnetic actuators is not addressed. The compensation methodology is restricted by the either the number of inputs, or by the nature of the force trajectories. To overcome these problems. Chapter (6) first utilizes the Preisach independent domain model to capture all of the relevant hysteresis characteristics in the electromagnetic actuator. It then incorporates the Preisach model of hysteresis in the control algorithm that regulates the electromagnetic actuator to hysteresis free operation.

### 1.3 Dissertation overview

The rest of the dissertation is organized as follows. A review of the current state of the art in magnetic suspension actuators is provided in Chapter (2). It traces the sequence of developments in magnetic suspensions, magnetic bearings, and magnetic actuators. A brief review of the basic principles of development of these systems is presented, and their influence on the development of magnetic suspension actuators is documented. In the latter half of the chapter, a survey of the magnetic actuation schemes and control algorithms in magnetic suspension actuators is presented. This
survey summarizes some of the inherent limitations in the developments, and provides a context for the work that is presented in the following chapters.

Chapter (3) formulates the principles for the development of the actuation scheme consisting only of DC electromagnets. The solution approach first develops a systematic procedure for modeling the electromagnet interactions between the flotor and the electromagnets. The model is then utilized in synthesizing optimal actuation strategies comprising of 7, 8, and 9 electromagnets, that realize the desired travel range and ensure the complete manipulability of the flotor in the six degree of freedom space.

In order to reduce the size of DC electromagnets, and the associated current requirements, design of the actuation scheme with Permanent-Electro-Magnet (PEM) combinations is addressed in Chapter (4). The design process first optimizes the electromagnets in relationship to the actuation requirements. The optimized electromagnets are utilized in synthesizing PEM combinations that ensure flotor manipulability. The PEM combinations are then incorporated into the design of the horizontal and vertical actuation schemes that result in the desired actuation characteristics.

Chapter (5) formulates the principles for the development of the control algorithm. It presents a robust nonlinear compensation algorithm for realizing large travel in magnetic suspension systems suffering from parameter variations, and external disturbance forces. A geometric feedback linearization technique that utilizes the nonlinear model of the electromagnetic force is employed to obtain large travel. Robustness to uncertainty/disturbance in the feedback linearized system is achieved through a discrete time delay control based compensation algorithm. The performance of this
algorithm is experimentally verified on a single degree of freedom magnetic suspension system.

Tracking accuracy in the above robust nonlinear control strategy degrades with increasing frequency and amplitude of the desired trajectory, due to the presence of ferromagnetic hysteresis in electromagnetic actuators. An approach using the Preisach independent domain model is used to capture the essential characteristics of the hysteresis nonlinearity in Chapter (6). The model is inverted and incorporated in the control strategy that regulates the electromagnetic actuator and compensates for the hysteretic effects. Performance of this approach is experimentally verified on an electromagnetic actuator.

Finally, Chapter (7) summarizes the results and the contributions of this dissertation. Some topics of future research that may enhance the performance capabilities of the MSA are discussed.
This research focuses on the design and development of magnetic suspension actuators with large and accurate travel in multiple degrees of freedom. Magnetic suspension systems have been extensively studied in literature, resulting in numerous applications, ranging from levitated transportation trains to magnetically suspended bearings. A brief review of the basic principles of development of these systems is presented in the first section of this chapter. The constant influence of these basic principles in the development of magnetic suspension actuators is then documented by surveying major advances in the area. Finally, the advantages and the limitations of these efforts in relationship to our research are examined in context of the design of the electromagnetic actuation scheme, and the design of the control architecture. Such a literature review will provide the context and the motivation for developments detailed in the successive chapters.

2.1 Background in magnetic suspensions

Several methods exist for supporting moving or rotating masses that utilize the electromagnetic interaction between permanent magnets, actively controlled electromagnets, coils magnetized by time varying currents and superconducting magnets.
Schemes utilizing forces of attraction are called suspension techniques and those using forces of repulsion are termed as levitation techniques. Detailed descriptions and comparisons for each of these techniques is found in [29, 9]. The forces in an attractive scheme tend to increase with decreasing coil separation, and vice-versa for repulsive schemes. Since the goal is flotation at a constant height, attractive schemes are said to be inherently unstable while the repulsive schemes are stable. By varying the excitation current appropriately, the attractive scheme can be stabilized.

Suspension of ferro-magnetic bodies using actively controlled DC electromagnets is the most popular method of obtaining magnetic suspensions. Ferro-magnetic bodies develop strong intrinsic magnetic fields when placed in an external field generated by an electromagnet. This intrinsic field aids external magnetization which attracts the floating body to the electromagnet. By actively modulating the strength of this external field, the floating body can be stabilized. Such systems have been extensively employed in the development of contactless magnetic bearings [45, 51, 48], and magnetically suspended ground transportation systems [55]. A comprehensive bibliography of this area is presented in [73]. Both these systems are essentially narrow gap electromagnetic devices, in which the floating body moves close to the electromagnet pole-face. This narrow gap operation characteristic has two important consequences: Firstly, the electromagnets in these actuators are designed and configured to provide high flux densities in regions close to the electromagnet pole-faces. Secondly, the control algorithm in these narrow gap devices is based on an approximate linear model of the actual nonlinear field distribution at the nominal operating point. The linear algorithms perform satisfactorily in small regions around the operating point, and
provide accurate stabilization and maximum stiffness in a single position and orientation. This is in sharp contrast to the objectives of the MSA where large travel is one of the desired features. This key difference in the performance objective makes the design and development process of the MSA, quite distinct from that of magnetic bearings and ground transportation systems.

As opposed to magnetic bearings and ground transportation systems, magnetic suspension actuators providing linear motion have been investigated by only a few researchers. A survey of the key developments leading to the latest generation of magnetic suspension actuators is presented in the following section.

2.2 Prior developments in magnetic suspension actuators

DC electromagnets have been the most popular method for generating magnetic fields in magnetic suspension actuators. A magnetic levitation servo for flexible assembly [72] using DC electromagnets was developed to provide 5 DOF motion with micron resolution. The servo was primarily utilized in flexible assembly as a programmable Remote Center Compliance device. The maximum travel of the floating body in translation and rotation were ±.6 mm and ±.7 degrees, respectively. Maximum stiffness of 136 Newton/mm and a positioning accuracy of 3 microns were obtained when actuated in only one coordinate direction. The device could also be used as a force and torque sensor with a resolution of .04 Newton and .001 Newton-meter, respectively. The control architecture was based on the linearized model of the actual nonlinear plant at the operating point. The performance of the system was sensitive to the modeling accuracy of the electromagnet characteristics.
Another actuator for use as an analytical semiconductor probing tool was developed to provide positioning along two axes [5]. The actuation scheme consisted of six pairs of permanent magnets and current carrying coils, that were used to generate the magnetic field. Using a control law based on the linearized model of the plant dynamics, a positioning repeatability of 0.2 microns was obtained for travel lengths of about 1 mm. The authors report that levitation stability was not robust, and disturbances that might occur in real probing situations caused the system to fail.

In a series of developments, Trumper [67, 26] has developed an atomic-scale precision motion control stage. It is a magnetically suspended, six degrees of freedom, precision motion control actuator with sub-nanometer positioning stability inside a 100 micron cube of travel. It utilizes 12 electromagnets and 6 capacitance probes to provide control forces and position feedback. The flotor is floated in oil to achieve neutral buoyancy which offsets its weight, and to provide high frequency disturbance rejection. The actuator is designed for use as a sample positioning stage for scanning tunneling microscopy. Experimental results show peak to peak noise of .04 nanometers on a command step input of approximately .1 nanometers.

In a similar development, Arling and Kohler [1] developed a 6 DOF positioner for silicon wafers for use in integrated circuit production. It consists of a stage that is supported by 16 electromagnets arranged in 8 pairs that are located symmetrically on its periphery. Due to the unipolar characteristics of the electromagnet actuators, the pairings provide a push-pull effect. The actuator is instrumented with capacitance sensors to measure the relative position of the flotor with respect to the stator, and a laser interferometer to measure the global coordinates since the 6 DOF positioner is mounted on a coarse positioning stage for larger travel capabilities. The stage
possesses a total travel of 300 microns in each of the 3 coordinate translational directions, with minimal rotational capabilities. A gain scheduling controller achieved a 20 nanometer peak to peak positioning stability in response to step input command of 50 microns with a broad band noise disturbance of amplitude 308 nanometers applied at the base of the actuator.

Magnetic fields can also be generated through electrodynamic actuation, a concept that was extensively used in the development of magnetically levitated ground transportation systems. This approach utilizes the force experienced by a current carrying conductor in a static magnetic field (Lorentz force) to modulate the position of the floating body. In most configurations, the floating body is either instrumented with the permanent magnets or with a current carrying coil. The current is actively controlled in time, and the motion of the flotor regulated. Such a scheme has been implemented in the development of a magnetic bearing stage for photolithography [70, 68, 80, 69]. The stage is suspended in 5 degrees of freedom by DC electromagnets, and driven in the sixth DOF by a linear motor. In this configuration, an array of Halbach permanent magnets is attached to the flotor, while an array of current carrying coils is attached to the stator. The stator windings consist of two phases, such that by independently modulating the amplitude of each of the currents, the required axial and normal components of forces are controlled. The stage possesses a linear travel of 50 mm and a positioning stability of 3 nanometers peak to peak in the axial direction. It may be observed that the flotor in this configuration is constrained to move at a constant air-gap to realize large travel, which is possible only in one or two translational DOF. However, electrodynamic actuation schemes provide higher force densities due to advancements in permanent magnet technology
that generates stronger magnetic fields. These advancements can be incorporated in the development of combinations of permanent and DC electromagnets that are detailed in Chapter (4).

The recent development of a surface motor driven precision positioning system [65, 64] is based on actuation through linear motors detailed above. In this development, an actuator levitated by three air bearing pads is driven by three linear motors in two translational and 1 rotational degrees of freedom. The guidance and positioning of the stage's planar motion is achieved via the simultaneous operation of three identical PID servo control systems, each employing a laser interferometer feedback loop. Experimental results indicate that a positioning resolution of 10-20 nanometers and a yaw angle accuracy of about ±0.08 micro-radians are achieved for an $x - y$ travel range of 10 mm. This actuator finds application for X-ray photolithography in semiconductor fabrication.

These investigations clearly demonstrate the viability of utilizing magnetic suspensions for ultra fine motion control in mechanical assemblies, even with Angstrom accuracies. However, all the efforts have focused on the development of accurate, fine positioning devices. Even though some of the actuators possess large travel in one/two translational directions, none of the reports address realization of large travel in all of the three translational directions. The reasons for these limitations lie in the design of their actuation strategies and controller architectures that are detailed in the following sections.
2.3 Electromagnetic actuation schemes

As detailed in the previous section, electromagnetic actuation schemes have been designed using combinations of linear motors, actively controlled DC electromagnets and permanent magnets. In comparison to DC electromagnets, linear motors provide higher force densities due to advances in permanent magnet technologies, and the higher current sourcing capabilities of the AC power supplies. However, linear motors can be designed to provide large travel in only one or two degrees of freedom, where the flotor is constrained to move at constant air-gaps. Extension to three or more translational DOF leads to loss of performance efficiencies, thereby necessitating the design of electromagnetic actuation schemes with combination of permanent and DC electromagnets.

Using the design philosophy of magnetic bearings, DC electromagnets have been employed in push-pull configurations in recent generation of magnetic suspension actuators, since an electromagnet can actuate a magnetic material only in the attractive direction. Therefore actuators with $n$ degrees of freedom have $2n$ or more electromagnets located in pairs on opposite sides of the flotor. Intrinsically, this configuration assumes an electromagnetic interaction force that is primarily directed along the normal joining the electromagnet pole-face to ferromagnetic targets on the flotor. In actual practice though, the electromagnetic interaction consists of three components, the largest one being directed normal to the electromagnet pole-face. The other two components are relatively smaller and lie in the plane of the electromagnet pole-face. These in-plane components introduce a coupling between the orthogonal coordinate directions that can be used not only to actuate the flotor, but also to manipulate it at significantly larger air-gaps, by optimizing the resulting magnetic field distribution.
Some researchers recognized the limitations of linear motors in multiple degrees of freedom actuation [67, 46] and attempted to alleviate the restrictions imposed by DC electromagnets by incorporating permanent magnets in the actuation scheme. The permanent magnets in these combinations provide high flux density which is essential for exerting large forces, while the electromagnets are used to modulate the effective net force. Since combinations of permanent magnets and DC electromagnets can be designed to provide bi-directional actuation, it also simplifies the design of the actuation scheme and the control algorithm. However at small air-gaps, the permanent magnet interaction in these bi-directional actuators is particularly strong and overwhelms the forces from the electromagnets, thus leading to the loss of flotor stability. Trumper [67] reported that the flotor usually crashed into the actuator at small airgaps. Such problems have hindered advanced applications of permanent magnet combinations. and the development of the general principles for the design and placement of these actuators for the large travel MSA is addressed in Chapter (4) of this dissertation.

2.4 Control algorithms

The realization of large travel, accurate tracking, high bandwidth and dynamic stiffness is not only a function of the electromagnetic actuation scheme, but also of the control algorithm designed to regulate the performance of the system. The realization of desirable actuator characteristics, have been examined extensively in the context of magnetic suspension bearings [54]. As pointed out earlier, magnetic suspension bearings are narrow gap devices that seek to maintain their nominal operating position, inspite of external disturbances. Linear controllers based on the linearized model of
the electromagnetic force distribution around the operating point, are utilized to regulate performance. Higher stiffnesses in these controllers is obtained through either off-line design methods such as $H_\infty$ compensation [17, 44], or through online estimation and compensation such as Time Delay control [82]. Since the recent generation of MSA have been built using electromagnets in push-pull configurations, similar to those employed in magnetic bearings, researchers have pursued the same principles in design of control systems for magnetic suspension actuators. In these systems however, changing positions of the flotor correspond to changing models of the electromagnetic force distribution. The linear controller performs satisfactorily in small regions around the nominal operating point, but the performance degrades with increasing travel. The requirement of large travel therefore necessitates the utilization of nonlinear control algorithms.

Nonlinear control algorithms utilize the nonlinear model of the electromagnetic field distribution, describing significantly large regions around the electromagnet. Geometric feedback linearization techniques were investigated [71] for a one degree of freedom magnetic suspension system. It was experimentally demonstrated that the controller yielded transient responses largely independent of the operating point air gap, when distances were increased four times in comparison to linear controllers. Even though the controller performs satisfactorily in achieving the travel range, it is sensitive to deviations from the nominal model. Implementation of feedback linearization technique requires extremely accurate system models, otherwise controller performance degrades in the presence of uncertainties. Since the dynamics of external disturbances are inherently unknown, dynamic stiffness is an issue with these controllers. Reports addressing utilization of robust feedback linearization schemes for a
large travel and a high stiffness MSA are not found in literature, and are addressed in this dissertation.

2.5 Conclusions

A review of the current state of the art in magnetic suspension actuators confirms the viability of using magnetic suspensions for ultra fine motion control applications. All of the previous investigations have focused on the development of accurate, fine positioning devices, and some of these actuators possess large travel in one or two translational degrees of freedom. However, reports addressing the realization of large travel in all three translational directions with high bandwidth are not found in literature. A review of the actuation strategies, controller architectures and sensing methodologies, reveals their inherent limitations with respect to the large travel and high stiffness requirements, demanded of the MSA. The following chapters detail approaches in overcoming these limitations, and enabling the development of a large travel Magnetic Suspension Actuator.
CHAPTER 3

DESIGN OF ACTUATION SCHEMES WITH DC ELECTROMAGNETS

This chapter addresses the problem of electromagnet location in a large travel, multiple degrees of freedom Magnetic Suspension Actuator (MSA). It presents a systematic procedure for developing a model of the electromagnetic interactions between the flotor and the electromagnets, through finite element calibration. The resulting model provides an accurate description of the magnetic field at large air-gaps, and is scalable with number of electromagnets. This model is utilized for synthesizing an actuation strategy that minimizes the number of electromagnets, while ensuring the complete manipulability of the flotor. Since electromagnet forces are unidirectional in nature, a feasible actuation strategy ensures that the current vector commanded to the electromagnets is always positive, for all possible locations of the flotor. A design procedure is formulated through which optimal configurations consisting of 7, 8, and 9 electromagnets are synthesized that realize the desired travel range and ensure the complete manipulability of the flotor in the six degree of freedom space.
3.1 Introduction

As opposed to narrow gap magnetic suspension actuators, the requirement of large travel in multiple DOF necessitates the development of actuation strategies that are able to provide optimal flux densities in all regions of the MSA, not just close to the electromagnet polefaces. These optimal flux densities can be obtained by either using Lorentz force based electrodynamic actuators [24], actively controlled DC electromagnets [1] or a combination of the two [68]. Even though electrodynamic actuation provides higher force densities, it can only be designed to provide large travel in one/two DOF, where the flotor is constrained to move at constant airgaps [65, 64]. Consequently, design of six DOF magnetic suspension actuators is usually pursued with actuation schemes comprising of Direct Current (DC) electromagnets. However, a DC electromagnet can only attract a magnetic body, thereby providing uni-directional actuation. The usual approach then is to arrange these electromagnets in a push-pull configuration so that the entire actuation scheme consists of bi-directional actuators [80]. This arrangement necessitates using 2n or more electromagnets to suspend and stabilize a flotor in n DOF. However, systematic analysis of the electromagnetic force model suggests that the number of electromagnets required to suspend a n DOF actuator may be reduced by a more astute location and orientation of the electromagnets. This reduction in the number of electromagnets, not only leads to a lighter and a more compact mechanism, but also reduces the number of power supplies that lowers the cost and simplifies the control algorithm. Consequently, the research develops a rational approach for synthesizing minimum electromagnet actuation schemes for magnetic suspension actuators that are optimal with respect to its large travel requirements.
Actuation strategies with minimum number of electromagnets are synthesized by modeling and analyzing all the components of the interaction force between the electromagnet and the ferromagnetic flotor. This interaction force consists of three components, the largest one directed normal to the electromagnet poleface. The other two components are relatively smaller and lie in the plane of the electromagnet poleface. These inplane components introduce a coupling between the orthogonal coordinate directions, and can in some situations be used to actuate the flotor. Iwaki [28] investigated these couplings, and using results from fixturing theory [35], developed a rational approach for suspending a floating body (flotor) in $n$ DOF with less than $2n$ electromagnets. However, this algorithm assumed a linear scaling of the electromagnet forces with both the airgap and the current variables, a situation that exists only in a small region around the nominal operating point. In a large travel MSA on the other hand, the variation is nonlinear with respect to both the current and the airgap variables. Minimum electromagnet configurations for the large travel requirement must ensure that the uni-directional electromagnet forces balance the external wrench on the flotor at each point in its operational workspace. This manipulability condition in context of unidirectional actuation forces is different from the ones encountered in robotic manipulability analysis [81]. Ensuring this manipulability condition in face of the nonlinear variations of the actuating forces also distinguishes this particular problem from other synthesis cases in grasping literature [52], where unidirectional actuation forces are also encountered.

The solution approach to the above problem first develops a systematic procedure for modeling the electromagnet interactions between the flotor and the multiple electromagnets surrounding it, by using computational approaches in Section (3.2). The
resulting model is accurate at large airgaps and scalable with the number of electromagnets. This model is utilized in synthesizing a minimum electromagnet actuation strategy that ensures the manipulability condition. This is accomplished in a two step process. In the first step, the electromagnet arrangement is tested for flotor manipulability at its nominal location. The electromagnet orientations in the resulting configuration are then selected for optimal conditioning of the current vector. In the second step, the electromagnet arrangement obtained previously is then examined for its manipulability characteristics in the entire travel range of the MSA. This is accomplished by employing a test based on results in polynomial positivity and Sturm's Theorem. Using this procedure, optimal configurations of 7, 8, and 9 electromagnets are synthesized in the last section that realize the desired travel range and ensure the complete manipulability of the flotor in the six degree of freedom space.

3.2 Development of the electromagnetic force model for the MSA

The Magnetic Suspension Actuator consists of multiple electromagnets located around the flotor that are utilized to suspend and modulate its position. One such arrangement of electromagnets along with the sensing system is depicted in Figure (3.1) in which the flotor is suspended using nine electromagnets. The electromagnetic interaction depends on the geometrical and material properties of the electromagnets, the material properties of the flotor, its position and orientation relative to the electromagnets, and the current being supplied to the electromagnets. The model for the electromagnetic interaction can be developed using three approaches: Lumped parameter magnetic circuit analysis, computational methods, and experimental calibration.
Figure 3.1: Arrangement of the electromagnets and the sensing system in the MSA.

Figure 3.2: Schematic of an electromagnet interacting with an inclined plate.
The magnetic circuit analysis works best for systems with small airgaps with negligible leakage flux, and thus cannot be applied in the case of the large travel MSA. The computational approach primarily employs the finite element method for the solution of electromagnetic equations. This method provides flexibility in describing interactions between electromagnets and flotor of various geometries at large airgaps. For synthesis of electromagnet locations, the computational model is employed. Once an optimal electromagnet actuation strategy is designed, it is experimentally calibrated for its electromagnetic model that is suitable for high precision control of the MSA. Consequently, such a combination of computational method and experimental calibration is utilized to derive the electromagnetic force model for the MSA.

In deriving the electromagnetic force model for the MSA, it is observed that the size of the flotor is significantly larger in comparison to the polefaces of the electromagnets. Consequently, the interactions between the electromagnets in the MSA are minimal and can be ignored [67]. The force model for the complete MSA can then be derived by superimposing the models of the interactions between the flotor and each of the electromagnets. This interaction between a single electromagnet and the flotor in turn, can be effectively described by analyzing the magnetic forces between the electromagnet under consideration, and the face of the flotor closest to it. This situation corresponds to that of a metallic plate immersed in a magnetic field due to an electromagnet. Using a computational methods, a model of the forces on a metallic plate due to an electromagnet, as a function of its relative position and orientation, are derived in the following section. The results are then superimposed to derive the force model for the complete MSA.
3.2.1 Calculation of magnetic forces on an inclined plate

Consider the case of a square plate of thickness $t$ and length $l$, placed at a distance $h_i$ above electromagnet $i$, as shown in Figure (3.2). To obtain an expression for the forces that are exerted on the plate due to the electromagnet $i$, a reference coordinate system $[\hat{x}_o, \hat{y}_o, \hat{z}_o]$, and a pair of local coordinate systems $[\hat{x}_i, \hat{y}_i, \hat{z}_i]$ and $[\hat{u}_i, \hat{v}_i, \hat{w}_i]$ are established. In these coordinate systems, $\hat{z}_i$ is normal to the $i^{th}$ plate, while $\hat{w}_i$ lies along the axis of electromagnet $i$. $\hat{v}_i$ is selected to be orthogonal to both $\hat{w}_i$ and $\hat{z}_i$, as described by the following equation:

$$\hat{v}_i = \frac{\hat{z}_i \times \hat{w}_i}{|\hat{z}_i \times \hat{w}_i|}$$

(3.1)

and if $\hat{z}_i \times \hat{w}_i = 0$, then $\hat{v}_i$ can be an arbitrary vector in the plane of the electromagnet poleface. The relative orientation between the electromagnet and the plate is characterized by two angles. The first of these is the plate inclination angle $\alpha_i$, between the plate normal $\hat{z}_i$ and the electromagnet axis $\hat{w}_i$. The second is the inplane rotation angle $\beta_i$ between $\hat{y}_i$ and $\hat{v}_i$. Using this nomenclature, transformation within these coordinate systems can be expressed as:

$$[\hat{u}_i, \hat{v}_i, \hat{w}_i] = [\hat{x}_i, \hat{y}_i, \hat{z}_i] R_{\hat{u}_i}^{\hat{x}_i}$$

(3.2)

where $R_{\hat{u}_i}^{\hat{x}_i}$ is the associated rotation matrix and is given as:

$$R_{\hat{u}_i}^{\hat{x}_i} = \begin{bmatrix}
\cos \beta_i \cos \alpha_i & -\sin \beta_i & \cos \beta_i \sin \alpha_i \\
\sin \beta_i \cos \alpha_i & \cos \beta_i & \sin \beta_i \sin \alpha_i \\
-\sin \alpha_i & 0 & \cos \alpha_i
\end{bmatrix}$$

(3.3)

Similarly, the transformation from the local coordinate system $[\hat{x}_i, \hat{y}_i, \hat{z}_i]$ to the reference coordinate system is expressed as:

$$[\hat{x}_i, \hat{y}_i, \hat{z}_i] = [\hat{x}_o, \hat{y}_o, \hat{z}_o] R_i^0,$$

(3.4)

30
<table>
<thead>
<tr>
<th>Physical characteristic</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the core</td>
<td>.0685 m</td>
</tr>
<tr>
<td>Inner radius of coil</td>
<td>.0359 m</td>
</tr>
<tr>
<td>Outer radius of coil</td>
<td>.0703 m</td>
</tr>
<tr>
<td>Number of turns</td>
<td>4000</td>
</tr>
<tr>
<td>Demagnetization constant</td>
<td>.44</td>
</tr>
<tr>
<td>Coil resistance</td>
<td>65 ohms</td>
</tr>
<tr>
<td>Electromagnet inductance</td>
<td>25 mH</td>
</tr>
</tbody>
</table>

Table 3.1: A listing of the physical parameters of the electromagnet.

Figure 3.3: Schematic of an arrangement with seven electromagnets.
where $R_{f}$ is the corresponding rotation matrix. Using these transformations, forces derived in the electromagnet coordinate $[\hat{u}, \hat{v}, \hat{w}]$ are expressed in the reference coordinate system $[\hat{x}, \hat{y}, \hat{z}]$ through the following equations:

$$
\begin{bmatrix}
\hat{f}_x \\
\hat{f}_y \\
\hat{f}_z
\end{bmatrix} = R_{\hat{u} \hat{z}}^x 
\begin{bmatrix}
\hat{f}_x \\
\hat{f}_y \\
\hat{f}_z
\end{bmatrix}
$$

Similarly, torques derived in the electromagnet coordinate system are transformed to the reference coordinate system as follows:

$$
\begin{align*}
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix} &= R_{\hat{u} \hat{z}}^x 
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix} \\
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix} &= R_{\hat{u} \hat{z}}^x 
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix} + [\rho_i^o] R_{\hat{u} \hat{z}}^{o} 
\begin{bmatrix}
\hat{f}_x \\
\hat{f}_y \\
\hat{f}_z
\end{bmatrix}.
\end{align*}
$$

where $\rho_i^o = [\rho_{i_x}^o, \rho_{i_y}^o, \rho_{i_z}^o]^T$ is the position vector from reference coordinate system to the local coordinate system, and $[\rho_i^o]$ is defined as follows:

$$
[\rho_i^o] = 
\begin{bmatrix}
0 & -\rho_{i_z}^o & \rho_{i_y}^o \\
\rho_{i_z}^o & 0 & -\rho_{i_x}^o \\
-\rho_{i_y}^o & \rho_{i_x}^o & 0
\end{bmatrix}.
$$

To determine electromagnet forces $[f_x, f_y, f_z]^T$ and torques $[\tau_x, \tau_y, \tau_z]^T$ as a function of the relative distance $h_i$, orientation angles $\alpha_i, \beta_i$, and electromagnet current $I_i$, three dimensional electromagnetic finite element simulations are performed using the software Maxwell. When the plate is parallel to the electromagnet poleface, it experiences a normal force $f_{\hat{w}}$, with the other five components $f_{\hat{x}}, f_{\hat{y}}, \tau_{\hat{x}}, \tau_{\hat{y}}, \tau_{\hat{z}}$, being identically zero. The variation of $f_{\hat{w}}$ with the half length of the plate $l/2$ is graphed in Figure (3.4), for an electromagnet with the dimensions listed in Table
(3.1), with a current of 3 amperes and placed 2 mm below the plate. Initially, \( f_{\dot{w}} \) rises with increasing lengths of the plate but saturates when the plate half length exceeds the radius of the electromagnet coil. It is observed from Figure (3.4) that a square plate of length 40 cm (half length = 20 cm) may be considered to be effectively infinite in comparison to the electromagnet of Table (3.1). In such a situation, forces \([f_{u}, f_{\dot{u}}, f_{\dot{w}}]\)^T and torques \([\tau_{\dot{u}}, \tau_{\dot{u}}, \tau_{\dot{w}}]\)^T are independent of the inplane rotation angle \( \beta_i \). Consequently, electromagnet forces and torques need to be calibrated only with respect to one orientation angle \( \alpha_i \), along with the relative distance \( h_i \) and electromagnet current \( I_i \).

Due to the symmetry in the interaction between the electromagnet and the infinite plate, force component \( f_{\dot{u}} \) and torque components \( \tau_{\dot{u}}, \tau_{\dot{w}} \) are identically zero for all values of \( h_i \) and \( \alpha_i \). Further, as the electromagnet forces are linear with respect to the square of the current \( I_i^2 \) \[46\], the remaining components of the electromagnetic force and torque are expressed as:

\[
\begin{align*}
    f_{\dot{u}} &= -pf_{u}(h_i, \alpha_i)I_i^2 \\
    f_{\dot{w}} &= pf_{\dot{w}}(h_i, \alpha_i)I_i^2 \\
    \tau_{\dot{u}} &= pr_{\tau}(h_i, \alpha_i)I_i^2
\end{align*}
\]

(3.8)

where \( pf_{u}(h_i, \alpha_i), pf_{\dot{w}}(h_i, \alpha_i), pr_{\tau}(h_i, \alpha_i) \) are polynomials in \( h_i, \alpha_i \) determined through finite element simulations. Figures (3.5) and (3.6) plot the variation of \( f_{\dot{u}} \) and \( f_{\dot{w}} \), respectively, with \( h_i \) and \( \alpha_i \), for \( I_i = 3 \) amperes. Since the relationship is nonlinear, \( p_{f_{\dot{u}}}, p_{f_{\dot{w}}} \) are approximated by surface fitting second order polynomials in \( h_i \) and \( \alpha_i \). It is observed that the magnitude of both \( f_{\dot{u}} \) and \( f_{\dot{w}} \) increases with increasing inclination. Also, \( f_{\dot{u}} \) is zero when the plate is normal to the electromagnet. Similarly.
Figure (3.7) plots the variation of $\tau_{i}$, with $h_i$, $\alpha_i$. Again, $p_{\tau_i}$ is obtained by fitting a second order polynomial in $h_i$ and $\alpha_i$.

The electromagnet forces and torques determined above can be transformed to the reference coordinate system using Equations (3.5) and (3.6). These results are utilized in the following section to determine the force model for the complete MSA.

### 3.2.2 Derivation of the force model for the complete MSA

Consider a Magnetic Suspension Actuator consisting of a ferro-metallic flotor suspended by seven electromagnets, as shown in Figure (3.3). Since each side of flotor is much larger in comparison to the corresponding electromagnet poleface, the interaction between the electromagnets is minimal. The net force on the flotor is derived by superimposing the individual forces between the electromagnets and the corresponding faces on the flotor. Superimposing the expressions for the electromagnetic forces between a single electromagnet and a metallic plate that were derived in the previous section, the net force and torque on the flotor is expressed as follows:

$$\begin{bmatrix} f_{x_o} \\ f_{y_o} \\ f_{z_o} \end{bmatrix} = \sum_{i=1}^{n} R_i^o \begin{bmatrix} f_{x_i} \\ f_{y_i} \\ f_{z_i} \end{bmatrix}$$

$$\begin{bmatrix} \tau_{x_o} \\ \tau_{y_o} \\ \tau_{z_o} \end{bmatrix} = \sum_{i=1}^{n} R_i^o \begin{bmatrix} \tau_{x_i} \\ \tau_{y_i} \\ \tau_{z_i} \end{bmatrix} + \sum_{i=1}^{n} [\rho_i^o] R_i^o \begin{bmatrix} f_{x_i} \\ f_{y_i} \\ f_{z_i} \end{bmatrix}.$$  (3.9)

where $n$ is the total number of electromagnets in the MSA. Using relations (3.5, 3.6) and (3.8), the above equations can be expressed as:

$$\begin{bmatrix} f_{x_o} \\ f_{y_o} \\ f_{z_o} \end{bmatrix} = \sum_{i=1}^{n} R_i^o R_i^{\tilde{x}_i} \begin{bmatrix} p_{f_{x_i}} (h_i, \alpha_i) \\ 0 \\ p_{f_{z_i}} (h_i, \alpha_i) \end{bmatrix} I_i^2$$

$$\begin{bmatrix} \tau_{x_o} \\ \tau_{y_o} \\ \tau_{z_o} \end{bmatrix} = \sum_{i=1}^{n} R_i^o R_i^{\tilde{x}_i} \begin{bmatrix} 0 \\ p_{\tau_{x_i}} (h_i, \alpha_i) \\ 0 \end{bmatrix} I_i^2 + \sum_{i=1}^{n} [\rho_i^o] R_i^o R_i^{\tilde{x}_i} \begin{bmatrix} p_{f_{x_i}} (h_i, \alpha_i) \\ 0 \\ p_{f_{z_i}} (h_i, \alpha_i) \end{bmatrix}.$$  (3.10)
Figure 3.4: Variation of $f_{\dot{\omega}}$ with plate half length.

Figure 3.5: Variation of $f_{\dot{\omega}}$ with airgap $h_i$ and inclination $\alpha_i$.  

35
Figure 3.6: Variation of $f_{\alpha}$ with airgap $h$ and inclination $\alpha$.

Figure 3.7: Variation of $r_{\alpha}$ with airgap $h$ and inclination $\alpha$. 
which relate the net force and torque to the current $I_i$ and the geometrical parameters $h_i, \alpha_i, \beta_i$ of each of the electromagnets. Using compact notation, the above equation can be recast as:

$$\begin{pmatrix} F_o \\ \tau_o \end{pmatrix} = G(h_1, \cdots, h_n, \alpha_1, \cdots, \alpha_n, \beta_1, \cdots, \beta_n)I$$

(3.11)

where $G$ is the electromagnet location matrix, and $I = (I_1^2, I_2^2, \ldots, I_n^2)^T$. and $F_o, \tau_o$ are the force and torque vectors in the reference coordinate system. The model is linear with respect to the square of the electromagnet currents, and nonlinear with respect to the airgap variables and inclination angles. This force model is next analyzed for synthesis of optimal electromagnet locations.

### 3.3 Synthesis of optimal electromagnet locations

The synthesis problem in the design of Magnetic Suspension Actuators is the rational selection and location of electromagnets in space to ensure the manipulability of the flotor. Electromagnetic actuation is unidirectional, since ferromagnetic bodies are always attracted to DC electromagnets. The force experienced by an inclined plate in an electromagnetic field is proportional to $I_i^2$ at a constant airgap (Equation (3.8)), verifying its unidirectional characteristic. It is therefore necessary to systematically locate these electromagnets so that the unidirectional electromagnetic forces may actuate the flotor in all the degrees of freedom. This requires that the six components of the net force on the flotor (Equation (3.10)) may be modulated independently of each other, by systematically varying the currents in the electromagnets. One commonly used arrangement is to locate the electromagnets in a push-pull configuration where pairs of electromagnets are located on opposite sides of the cube. This configuration
ensures the manipulability of the flotor in the three DOF space, consisting of translations along the three coordinate DOF. For a MSA with the design constraints of Table (3.2), this arrangement is impractical since twelve electromagnets are required to ensure manipulability in a six DOF space. Real time control and coordination of such a large number of electromagnets is a difficult problem, and it is desirable to utilize a configuration that optimizes on the number and location of the electromagnets.

The push-pull configuration primarily utilizes the component of the interaction force that is normal to the electromagnet poleface, in actuating the flotor in that direction. The inplane force components that are relatively smaller in magnitude, are ignored and are treated as disturbance forces. However, the displacement of the flotor due to the inplane components introduces a coupling between the orthogonal coordinate directions, that can be utilized to actuate the flotor without arranging the electromagnets in a push-pull configuration. Referring to Figure (3.3), flotor displacement in the positive $\hat{x}_o$ direction may be induced by utilizing the inplane $f_{z_3}$ component of Electromagnet (3). Thus, if the electromagnets are suitably designed and located, the flotor may be manipulated in all the six DOF by using less than twelve electromagnets. Since the electromagnet location matrix in Equation (3.10) is nonlinear with respect to the airgap, any given arrangement of electromagnets must ensure the manipulability of the flotor in the entire operational space of the MSA.

In the next section, the manipulability condition is first investigated at the nominal location of the flotor. Any electromagnet arrangement that satisfies the manipulability condition at the nominal flotor location is then optimized for the conditioning of the current vector by modulating the electromagnet inclinations. The following section then examines the electromagnet location matrix for flotor manipulability in
Table 3.2: Design constraints for the MSA.

<table>
<thead>
<tr>
<th>Property</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of degrees of freedom</td>
<td>6</td>
</tr>
<tr>
<td>Travel range in translational DOF</td>
<td>±3 mm</td>
</tr>
<tr>
<td>Number of electromagnets</td>
<td>6-12</td>
</tr>
<tr>
<td>Inclination range of the electromagnets</td>
<td>±12.7 degrees</td>
</tr>
</tbody>
</table>

the entire operational space of the MSA, using results on polynomial positivity and Sturm’s Theorem.

3.3.1 Manipulability condition at the nominal position of the flotor

Using Equation (3.11), the force model of the MSA at the nominal location of the flotor can be expressed as:

\[
\begin{pmatrix}
F_o \\
\tau_o
\end{pmatrix} = G^0(h_1^0, \ldots, h_n^0, \alpha_1, \ldots, \alpha_n)I
\]

(3.12)

where \( h_i^0 \) are the nominal airgaps between the electromagnets and the flotor at its nominal location. Manipulability of the flotor at the nominal location requires that each component of the net force may be modulated independently. This implies that there exists a positive \( I \) for any given \( (F_o, \tau_o)^T \) in \( \mathbb{R}^6 \). \( I \) is said to be positive when each of its component is positive and greater than zero. Ability to command \( (F_o, \tau_o)^T \) in \( \mathbb{R}^6 \) requires that \( G^0 \) be of rank 6. In this situation, the solution to Equation (3.11) at the nominal location with \( n > 6 \), can be expressed as:

\[
I^0 = I_p^0 + \lambda I_n^0
\]

(3.13)
where $\mathbf{t}_p^0$ is the particular solution to Equation (3.11) usually found through the pseudo-inverse approach, and is not necessarily positive. $\mathbf{t}_n^0$ is the null space solution, so that $\mathbf{G}^0 \mathbf{t}_n^0 = \mathbf{0}$. If the electromagnets are arranged so that the resulting null space vector is positive, then $\lambda$ can always be appropriately selected to make $\mathbf{t}^0$ positive for any value of $\mathbf{t}_p^0$. Physically, $\mathbf{t}_n^0$ corresponds to the components of the forces exerted by the electromagnets that cancel each other, and produce a zero net force on the flotor. A geometrical interpretation of these null space forces is provided in the following section.

**Geometrical interpretation of the manipulability condition**

The electromagnet location matrix $\mathbf{G}^0$ may be partitioned into two sub-matrices: $\mathbf{G}_p^0$ and $\mathbf{G}_a^0$, whose dimensions are $(6 \times 6)$ and $(6 \times n - 6)$ respectively. $\mathbf{G}_p^0$ is selected so that all of its columns are linearly independent, and is of rank 6. Then it is possible to express the null space equation $\mathbf{G}^0 \mathbf{t}_n^0 = \mathbf{0}$ as:

$$
\begin{bmatrix}
\mathbf{G}_p^0 & \mathbf{G}_a^0
\end{bmatrix}
\begin{bmatrix}
\mathbf{t}_p^0 \\
\mathbf{t}_n^0
\end{bmatrix}
= \mathbf{0}
$$

$$
\mathbf{G}_p^0 \mathbf{t}_p^0 = -\mathbf{G}_a^0 \mathbf{t}_n^0
$$

$$
\mathbf{t}_p^0 = -\mathbf{G}_a^{0-1} \mathbf{G}_a^0 \mathbf{t}_n^0
$$

(3.14)

A geometrical interpretation of the above equation is shown in Figures (3.8) and (3.9) for a three DOF flotor actuated by four electromagnets. The schematic of this configuration is shown in Figure (3.8) in which the flotor translates along the three coordinate directions. Electromagnets (1), (2), and (3) constitute the matrix $\mathbf{G}_p^0$ since the force vectors are linearly independent. Figure (3.9) depicts a force cone $\mathbf{OP}_1 \mathbf{OP}_2 \mathbf{OP}_3$ that characterizes the linear combination $\mathbf{G}_p^0 [I_1^2 I_2^2 I_3^2]^T$. Vertices $\mathbf{OP}_j$ represent the force vectors corresponding to electromagnet with the current $I_j^2 = 1$. All other
possible scalings of this force vector corresponding to different values of $I_j^2$ lie along $OP_j$. The sides of the force cone bound all the possible linear combinations of the force vectors, taken two at a time. Thus, the vectors resulting from the linear combinations of $OP_1$ and $OP_2$ lie along the face $OP_1P_2$ and are bounded by vertices $OP_1$ and $OP_2$. Similarly, all linear combinations of the three force vectors constituting $G_p^0$ lie inside the force cone $OP_1P_2P_3$, and are bounded by the three sides $OP_1P_2, OP_2P_3$ and $OP_3P_1$. Conversely, any point inside the force cone $OP_1P_2P_3$ is positively spanned by the three primary force vectors constituting the matrix $G_p^0$. Therefore if $-G_p^0I_{na}$ can be made to lie in this force cone, then Equation (3.14) is satisfied, and a positive null space is obtained. $G_p^0$ for this example is the force vector corresponding to Electromagnet (4) in Figure (3.8), which is located opposite Electromagnet (3). Since its force vector is a function of variables $h_4$, and $\alpha_4$, it is possible to modulate the electromagnet airgap and its inclination to achieve the desired orientation of the vector inside the force cone. With the addition of the appropriate null space component $G_a^0$, the force cone of Figure (3.9) changes to a sphere and spans $\mathbb{R}^3$. Thus the electromagnet location matrix $G^0$ is able to position the net force vector at any arbitrary point in the $\mathbb{R}^3$ space. Consequently, if $G_p^{0^{-1}}G_a^0$ is positive in Equation (3.14), then $-G_a^0I_{na}$ lies inside the force cone and the null space is guaranteed to be positive at the nominal location of the flotor. Location and orientation of this vector in the force cone and the associated conditioning of the null space is discussed in the following section.
Figure 3.8: Schematic of an arrangement with four electromagnets.

Figure 3.9: The force cone corresponding to the four electromagnet arrangement.
3.3.2 Optimal conditioning of the null space current vector

Using Equations (3.14), the null space of the electromagnet location matrix is expressed as:

\[
\begin{pmatrix}
I_{np} \\
I_{na}
\end{pmatrix} = \begin{pmatrix}
-G_p^{-1} G_d I_{na} \\
I_{na}
\end{pmatrix}
\]  \hspace{1cm} (3.15)

where \( I_n \) is \((n \times 1)\) vector, and \( n \) is the number of electromagnets utilized in the arrangement. Null space condition is the ratio of the largest component of \( |I_n| \) to its smallest component and is mathematically expressed as:

\[
\mathcal{N} = \frac{\max |I_n|}{\min |I_n|}
\]  \hspace{1cm} (3.16)

\( \mathcal{N} \) varies with flotor movement in the operational space of the MSA. It is desirable that at each point in the MSA workspace, \( \mathcal{N} \) is close to unity. This enables an equitable loading of power supplies, which reduces their size and improves performance. To this effect, \( \mathcal{N} \) is first optimized at the nominal location of the flotor with the inclinations of the electromagnets as the variables. This ensures that the \(-G_d^0 I_{na}\) lies towards the center of the force cone. This increases the ability of the electromagnet configuration in accommodating larger variations in the flotor positions. This critical property is utilized in synthesizing electromagnet configurations that provide positive null space components as the flotor moves in its operational space. Therefore, the steps in realizing a properly conditioned null space at the nominal location of the flotor are summarized below:

1. The primary electromagnets should be located so that \( G_p^0 \) is of full rank.

2. The primary electromagnets should be configured so that \(-G_d^0 I_{na}\) lies inside the force cone.
3. Finally, the orientations of both the primary and additional electromagnets should be optimized so that the null space condition ratio $\mathcal{N}$ is as close to 1 as possible.

The application of these steps to the synthesis of electromagnet locations for a 3 DOF model of the MSA is presented in the following section.

**Synthesis example: 3 DOF model with 4 electromagnets**

Consider the 3 DOF model of the MSA with four electromagnets depicted in Figure (3.8). All of the electromagnets are identical and their parameters are listed in Table (3.1). At the nominal location of the flotor, the airgap between each of the electromagnets and the corresponding face of the flotor is 16 mm. In a conventional
synthesis strategy, each of the electromagnets is positioned parallel to the corresponding flotor face. Since the inclination angles are zero, the force vectors exerted by the electromagnets on the flotor lie along the coordinate axes of the force cone. Consequently, the force vector $OP_1$ of Electromagnet (1) is oriented along the $\hat{y}_o$ axis, while $OP_2$ and $OP_3$ lie along the $\hat{x}_o$ and the $\hat{z}_o$ axis respectively. Thus the resultant of the forces from Electromagnets (1), (2) and (3) spans $\mathbb{R}^3$. To command forces in $\mathbb{R}^3$, the only option available in this conventional synthesis strategy is to locate 3 additional electromagnets with force vectors $OP_4$, $OP_5$ and $OP_6$ along the $-\hat{z}_o$, $-\hat{x}_o$ and $-\hat{y}_o$ axis, respectively. This push-pull arrangement of electromagnets consequently requires 6 electromagnets to manipulate a 3 DOF actuator.

As opposed to the conventional synthesis strategy, it is possible to incline the electromagnets to the flotor. This introduces inplane components in the forces exerted by the electromagnets on the flotor. The force cone corresponding to this situation is depicted in Figure (3.10). Electromagnets (1), (2) and (3) are each parallel to the corresponding face of the flotor. Consequently, the force vector of Electromagnet (1) lies along $OP_1$ that is coincident with the $\hat{y}_o$ axis. Similarly, the force vectors of Electromagnets (2) and (3) lie along $OP_2$ and $OP_3$, respectively. The resultant of the forces from the three electromagnets lies in the semi-infinite volume $OP_1P_2P_3$. In this example, $G^0_p$ is numerically expressed as:

$$G^0_p = \begin{bmatrix} 0 & 213 & 0 \\ 213 & 0 & 0 \\ 0 & 0 & 213 \end{bmatrix}$$

(3.17)

The fourth electromagnet constitutes $G^0_a$. The synthesis problem reduces to locating Electromagnet (4) so that $-G^0_aI_n$ lies in the force cone $OP_1P_2P_3$. Clearly, as there is only one additional electromagnet, it may be placed opposite to any of the three
primary electromagnets, on the other side of the flotor. As depicted in Figure (3.8), Electromagnet (4) is placed opposite Electromagnet (3), at a nominal airgap of 16 mm. When the electromagnet is inclined at angles of \( \alpha_4 = 4.24 \) and \( \beta_4 = -44.96 \) degrees, \(-G^0_4\) lies along \( OP_4^0 \) outside the force cone of Figure (3.10). At this point, the null space is not positive. When the electromagnet inclination is changed to \( \alpha_4 = 4.24 \) and \( \beta_4 = -135.03 \) degrees, \(-G^0_4\) lies along \( OP_4^1 \) in the force cone. For this orientation and location of Electromagnet (4), the null space condition ratio \( \mathcal{N} \) equals 19.15, and the null space is given as \( \mathbf{P}_n^0 = \begin{bmatrix} 0.0471 & 0.0471 & 0.9058 & 0.9023 \end{bmatrix}^T \). With increasing inclination of Electromagnet (4), the inplane components strengthen, resulting in null space condition ratio \( \mathcal{N} \) of 8.9884, at inclination angles of \( \alpha_4 = 8.47 \) and \( \beta_4 = -135.15 \) degrees, represented by \( OP_4^2 \). Similarly, the null space condition ratio improves to 6.1635 for inclination angles of \( \alpha_4 = 12.70 \) and \( \beta_4 = -135.35 \) degrees, at the point \( P_4^3 \) in the force cone. Therefore, as the force vector \(-G^0_4\) moves towards the center of the force cone, the null space condition ratio decreases. This indicates an increased ability of the electromagnet configuration in accommodating larger variations in flotor positions.

By selecting the orientation of both the primary and the additional electromagnets appropriately, it is possible to optimize the null space condition ratio \( \mathcal{N} \). For the configuration presented above with \( \alpha_4 = 12.70 \) and \( \beta_4 = -135.35 \) degrees, \( \mathcal{N} \) at the extremal location of the flotor degrades to 10.82. In contrast, with optimized inclinations of \( \alpha_1 = 9.49, \alpha_2 = 9.50, \alpha_3 = 12.70, \alpha_4 = 12.70 \) degrees, and \( \beta_1 = 161.57, \beta_2 = -71.02, \beta_3 = -135.35, \beta_4 = -135.35 \) degrees, the null space condition ratio at the nominal location of the flotor is found to be 2.32. As the flotor reaches the extremal position in the operational workspace, \( \mathcal{N} \) maximizes to 4.33. Consequently.
<table>
<thead>
<tr>
<th>Electromagnet</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.24°</td>
<td>135.04°</td>
</tr>
<tr>
<td>2</td>
<td>4.24°</td>
<td>-135.04°</td>
</tr>
<tr>
<td>3</td>
<td>0.00°</td>
<td>0.00°</td>
</tr>
<tr>
<td>4</td>
<td>0.00°</td>
<td>0.00°</td>
</tr>
<tr>
<td>5</td>
<td>0.00°</td>
<td>0.00°</td>
</tr>
<tr>
<td>6</td>
<td>4.24°</td>
<td>-44.96°</td>
</tr>
<tr>
<td>7</td>
<td>4.24°</td>
<td>-44.96°</td>
</tr>
</tbody>
</table>

Table 3.3: Parameter listing for the seven electromagnet configuration with positive null space.

<table>
<thead>
<tr>
<th>Electromagnet</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
</tr>
</thead>
<tbody>
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<td>1.41°</td>
<td>135.00°</td>
</tr>
<tr>
<td>2</td>
<td>0.00°</td>
<td>0.00°</td>
</tr>
<tr>
<td>3</td>
<td>0.71°</td>
<td>-135.00°</td>
</tr>
<tr>
<td>4</td>
<td>0.71°</td>
<td>45.00°</td>
</tr>
<tr>
<td>5</td>
<td>0.00°</td>
<td>0.00°</td>
</tr>
<tr>
<td>6</td>
<td>0.25°</td>
<td>-90.00°</td>
</tr>
<tr>
<td>7</td>
<td>0.53°</td>
<td>-90.00°</td>
</tr>
</tbody>
</table>

Table 3.4: Parameter listing for the seven electromagnet configuration with non positive null space.
a 40% improvement in the null space condition ratio is obtained through optimization of the electromagnet inclination angles.

This optimized electromagnet configuration needs to be examined for null space positivity in the entire operational space of the MSA. This is accomplished by using results in polynomial positivity as detailed in the following section.

### 3.3.3 Test for null space positivity in the operational space of the MSA

Once the electromagnets are arranged to provide a positive null space at the nominal locations of the flotor, the electromagnet location matrix $G$ of Equation (3.11) is a function only of the airgap $h$, since the inclination variables $\alpha$ remain essentially constant as the travel range of the flotor in the rotational DOF is limited to ±0.5 degrees (see Table (3.2)). The dimension of the null space associated with this electromagnet location matrix $G$ equals $n - 6$, where $n$ is the total number of electromagnets. Denoting $G^j$ to be the electromagnet location matrix of dimension $(6 \times 7)$ corresponding to the $j^{th}$ basis vector of the null space current $I_n^j$ that is to be computed, then the null space current satisfies $G^j I_n^j = 0$ and is written as:

$$
G^j = \begin{bmatrix}
G^j(1,\cdot) & G^j(2,\cdot) & \cdots & G^j(6,\cdot)
\end{bmatrix}^T
$$

$$
I_n^j = G^j(1,\cdot) \otimes G^j(2,\cdot) \otimes \cdots \otimes G^j(6,\cdot)
$$

in which $I_n^j$ is expressed in terms of a six dimensional cross product. Each component of the null space basis vectors is a function of the airgap variable $h = [h_{x_0}, h_{y_0}, h_{z_0}]^T$.

---

1 The travel range in the rotational DOF is constrained by the available sensing instrumentation. Laser interferometric sensing is employed in translational DOF for large airgaps with nanometric accuracies which restricts the deviations in the orientations of the flotor. The analysis is general and the presentation in this section can be extended to include variations in the flotor orientation.
where $h_{x_0}, h_{y_0}, h_{z_0}$ are the flotor displacements from the nominal position in the reference coordinate system. Therefore the positivity of all the components of the null space current basis vectors $I_n^i$ needs to be examined for the range of variations in $h_{x_0}, h_{y_0}, h_{z_0}$ given in Table (3.2). The following section presents the analysis for examining positivity of each component of the null space current basis vectors at each point in the operational space of the MSA.

**Test for null space positivity in 3 translational DOF**

As the flotor translates in the 3 translational degrees of freedom, the $k^{th}$ component of the null space current basis vector $I_n^i$ (Equation (3.18)) is expressed as:

$$P(h_{x_o}, h_{y_o}, h_{z_o}) = \Sigma_{i=0}^n a_i(h_{y_o}, h_{z_o})h_{x_o}^i$$ (3.19)

where $P(h_{x_o}, h_{y_o}, h_{z_o})$ is a real polynomial. The objective is to examine the positivity of the above polynomial in the interval $D^3 \subset \mathbb{R}^3$ for all $h_i \in \left[ h_i^l, h_i^u \right]$ with the three values of $i$ belonging to the set $[x_o, y_o, z_o]$. Naturally, $D^3$ corresponds to the travel range in the translational DOF given in Table (3.2). If the function in Equation (3.19) is positive at a point in $D^3$ and possesses no real roots in the given interval, then it is also positive over the entire interval of parameter variations. This is established using the results on polynomial positivity by Walach and Zeheb [77], which states that $P(h_{x_o}, h_{y_o}, h_{z_o})$ is positive for all $h \in D^3$ if and only if:

- All the 2 dimensional polynomials arrived at by substituting the end points of the interval of $h_i(i = x_o, y_o, z_o)$ in $P(h_{x_o}, h_{y_o}, h_{z_o})$ one at a time are positive in $D^2$.

- The following set of 3 equations in 3 variables

$$P(h_{x_o}, h_{y_o}, h_{z_o}) = 0$$

49
Figure 3.11: Graphs (a)-(g) depict the positivity of the polynomials obtained by substituting the interval endpoints for the first component of the null space current in the seven electromagnet case.
\[ \frac{\partial P_i}{\partial h_i} = 0 \quad \forall i \in \{\hat{x}_o, \hat{y}_o\} \] (3.20)

has no solution in \( D^3 \).

Checking the positivity of the two dimensional polynomials derived by substituting the interval endpoints requires the recursive utilization of the above procedure. The second condition given by Equation (3.20) is established systematically using the theory of resultants given by Bickart and Jury [3]. This test is utilized to test the null space positivity of a seven electromagnet configuration as outlined below.

For the electromagnet arrangement depicted in Figure (3.3), and the electromagnet inclination parameters listed in Table (3.3), the first component of the null space current is expressed as:

\[ P_1(h_{\hat{x}_o}, h_{\hat{y}_o}, h_{\hat{z}_o}) = a_2(h_{\hat{y}_o}, h_{\hat{z}_o})h_{\hat{x}_o}^2 + a_1(h_{\hat{y}_o}, h_{\hat{z}_o})h_{\hat{x}_o} + a_0(h_{\hat{y}_o}, h_{\hat{z}_o}) \] (3.21)

where the polynomial coefficients \( a_i \) for \( i \in [0, 1, 2] \) are given as follows:

\[
a_i(h_{\hat{y}_o}, h_{\hat{z}_o}) = \sum_{j=0}^{3} b_{i,j}(h_{\hat{z}_o})h_{\hat{y}_o}^j \]

\[
b_{i,j}(h_{\hat{z}_o}) = \sum_{k=0}^{3} c_{i,j,k}h_{\hat{z}_o}^k
\] (3.22)

for all possible values of \( i, j \). It is required to verify that \( P_1(h_{\hat{x}_o}, h_{\hat{y}_o}, h_{\hat{z}_o}) > 0 \) for \( h_{\hat{x}_o, \hat{y}_o, \hat{z}_o} \in [a = 12 \text{ mm}, b = 20 \text{ mm}] \). Implementing each of the steps in the test listed above:

- By substituting the end points of the interval in each of the coordinate directions, it needs to be verified that the following polynomials is positive in \( D^2 \):

\[
P_1(h_{\hat{x}_o}, h_{\hat{y}_o}, h_{\hat{z}_o}^a) > 0
\]

\[
P_1(h_{\hat{x}_o}, h_{\hat{y}_o}, h_{\hat{z}_o}^b) > 0
\]
Figure 3.12: Graphs (a)-(g) plot the null space current components versus variations in $h_{\bar{z}_o}$, $h_{\bar{y}_o}$ along the coordinate axes. Each graph depicts the magnitude of the current component at $h_{\bar{z}_o}$ values of .012, .016 and .020 meter.
Selecting the first of these polynomials $P_1(h_{x_0}, h_{y_0}, h_{z_0})$ and applying the test recursively, the following set of polynomials are generated by substituting the end points of the intervals along $h_{x_0}$ and $h_{y_0}$:

\[
P_1(h_{x_0}^a, h_{y_0}^a, h_{z_0}^a) = 0.86h_{x_0}^2 - 0.01169h_{x_0} + 0.000143284
\]
\[
P_1(h_{x_0}^b, h_{y_0}^b, h_{z_0}^b) = 0.807h_{x_0}^2 - 0.0109h_{x_0} + 0.000134
\]
\[
P_1(h_{x_0}^a, h_{y_0}^b, h_{z_0}^a) = 0.392h_{y_0}^4 - 0.021321h_{y_0}^3 + 0.000459h_{y_0}^2 - 0.00000491h_{y_0} + 0.0000000342
\]
\[
P_1(h_{x_0}^b, h_{y_0}^b, h_{z_0}^a) = 0.78h_{y_0}^4 - 0.0426h_{y_0}^3 + 0.000918h_{y_0}^2 - 0.0000098h_{y_0} + 0.00000068(3.24)
\]

Using Sturm’s theorem or numerically, it is easily verified that each of these polynomials has no real roots. Further, since the constant term is positive, these polynomials is positive in $D^2$. Further, using Maple it is verified that the two polynomials $P_1(h_{x_0}, h_{y_0}, h_{z_0})$ and $\partial P_1(h_{x_0}, h_{y_0}, h_{z_0})/\partial h_{x_0}$ have no roots, thus verifying the condition given by Equation (3.20). Consequently, $P_1(h_{x_0}, h_{y_0}, h_{z_0})$ is positive in $D^2$, which is verified in Figure (3.11). Graph(a).

Similarly, the rest of the polynomials in Equation (3.23) are checked for positivity in $D^2$. This fact is verified in Graphs (b)-(f) in Figure (3.11).

The final step in establishing positivity of the first component of null current $P_1(h_{x_0}, h_{y_0}, h_{z_0})$ is to verify the condition given by Equation (3.20). To this end, it suffices to show that $\partial P_1/\partial h_{x_0} = a_1(h_{y_0}, h_{z_0})h_{x_0} + a_0(h_{y_0}, h_{z_0})$ has no positive
roots. Using the test again to establish this fact, the six polynomials obtained by substituting the end points of the interval $D^3$ are all positive. Further, by forming the resultant between $\frac{\partial P_1}{\partial h_{\xi_0}}$ and $\frac{\partial^2 P_1}{\partial h_{\xi_0}^2} = a_1((h_{\gamma_0}, h_{\zeta_0})$, it is found that these equations have no common zeros, thus satisfying the second condition implied by Equation (3.20). Consequently, $\frac{\partial P_1}{\partial h_{\xi_0}}$ has no zeros which implies that the first component of null current is positive.

The above test predicts that the first component of the null space current is positive in the entire operational space of the MSA. This is verified in Figure (3.12) which graphs each component of the null space current versus variations in $h_{\xi_0}$, $h_{\gamma_0}$ and $h_{\zeta_0}$. The seventh component of the null space current is unity. It is observed that the null space current is positive for the selected values of $h$ and the current variation is significantly nonlinear. Simply computing the null space current at various flotor locations may yield unreliable results, as demonstrated by the following example.

For the seven electromagnet case depicted in Figure (3.3), and the corresponding parameters listed in Table (3.4), the null space current vector at the nominal location of the flotor is $(0.0113, 3.7887, 0.3734, 0.0081, 3.7828, 0.1824, 0.1901)$. A plot of the variation of the null space current components with motion along $h_{\xi_0}$ in Figures (3.13) and (3.14) reveals that the null space is positive at the nominal location, as well as the end points of the operational space. However for $h_{\xi_0} \in [12.5 \text{ mm}, 15.0 \text{ mm}]$, the first and the fourth components of the null space current are negative. By simply computing the null space current at a finite number of locations in the operational space $D^3$, it is possible to miss this non-positive variation of the null space current. Consequently, a test for polynomial positivity with analytically guaranteed results is necessary which is employed in this section.
Figure 3.13: Graphs (a)-(g) plot the first six null space current components versus variations in $h_2$, for the seven electromagnet case.

Figure 3.14: Variation of the seventh null space component with $h_2$, for the seven electromagnet configuration.
<table>
<thead>
<tr>
<th>Electromagnet</th>
<th>$\alpha_i$</th>
<th>$\beta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$9.00^\circ$</td>
<td>$180.00^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>$9.00^\circ$</td>
<td>$180.00^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>$9.00^\circ$</td>
<td>$180.00^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>$6.40^\circ$</td>
<td>$-173.52^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>$6.40^\circ$</td>
<td>$173.52^\circ$</td>
</tr>
<tr>
<td>6</td>
<td>$4.90^\circ$</td>
<td>$0.00^\circ$</td>
</tr>
<tr>
<td>7</td>
<td>$4.90^\circ$</td>
<td>$0.00^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Travel in x direction</th>
<th>±3 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travel in y direction</td>
<td>±3 mm</td>
</tr>
<tr>
<td>Travel in z direction</td>
<td>±3 mm</td>
</tr>
</tbody>
</table>

$N_{\text{max}}$ | 3.27
$N_{\text{min}}$ | 1.51

Table 3.5: Parameter and property listing for the seven electromagnet configuration.

### 3.4 Electromagnet arrangements for complete manipulability of the 6 DOF flotor

By using the test for null space measure at the nominal location of the flotor and the test for null space positivity, three optimized configurations of 7, 8 and 9 electromagnets are developed that ensure the complete manipulability of the flotor. These arrangements are compared in the following sections.

#### 3.4.1 Seven electromagnet configuration

Figure (3.3) shows the seven electromagnet configuration designed for complete manipulability of the flotor in 6 DOF. The optimized electromagnet parameters obtained by following the procedure outlined in the preceding sections are listed in Table (3.5). It is desirable that the electromagnets be placed close to the flotor faces...
to minimize the airgap, and increase the effective force. Thus each of the electromagnets are placed at the limit of the desired travel range from the nominal position of the flotor. It is also observed that all the electromagnets are inclined to the normal. With increasing inclination, the inplane component of the forces on the flotor increases, thereby improving the conditioning of the associated null space current vector. Table (3.5) also lists the travel range to be ±3 mm in each of the translational directions, which corresponds to the design requirements. The conditioning of the null space current vector equals 1.51 at the nominal position of the flotor and maximizes to 3.27 at the endpoint of the interval. The ability to improve null space conditioning is restricted due to the availability of only one additional electromagnet to help modulate forces in $\mathbb{R}^6$. These restrictions on improved conditioning of the null space vector at the interval endpoint restricts applications of this configuration to situations requiring small to medium travel, or when power supplies delivering the required current are available. The improvement in the conditioning ratio with the utilization of one and two additional electromagnets is investigated in the following sections.

3.4.2 Eight electromagnet configuration

The eight electromagnet configuration is depicted in Figure (3.15), with the optimized electromagnet parameters listed in Table (3.6). In comparison to the seven electromagnet configuration of Table (3.5), improvements of 15% and 26% in $N_{\max}$ and $N_{\min}$ respectively, are observed. With the addition of another electromagnet in the remaining direction, it is possible to further improve the null space conditioning. This is investigated in the nine electromagnet configuration.
Figure 3.15: Schematic of configuration with eight electromagnets.

<table>
<thead>
<tr>
<th>Electromagnet</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.70°</td>
<td>135.35°</td>
</tr>
<tr>
<td>2</td>
<td>12.70°</td>
<td>-135.35°</td>
</tr>
<tr>
<td>3</td>
<td>12.70°</td>
<td>-44.64°</td>
</tr>
<tr>
<td>4</td>
<td>3.26°</td>
<td>120.83°</td>
</tr>
<tr>
<td>5</td>
<td>3.26°</td>
<td>-59.17°</td>
</tr>
<tr>
<td>6</td>
<td>3.85°</td>
<td>83.67°</td>
</tr>
<tr>
<td>7</td>
<td>3.85°</td>
<td>83.67°</td>
</tr>
<tr>
<td>8</td>
<td>5.20°</td>
<td>-78.33°</td>
</tr>
</tbody>
</table>

|                  |                  |
| Travel in \( x \) direction | ±3 mm            |
| Travel in \( y \) direction | ±3 mm            |
| Travel in \( z \) direction | ±3 mm            |

|                  |                  |
| \( N \) max      | 2.80             |
| \( N \) min      | 1.11             |

Table 3.6: Parameter and property listing for the eight electromagnet configuration.
Figure 3.16: Schematic of the configuration with nine electromagnets.

<table>
<thead>
<tr>
<th>Electromagnet</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.65°</td>
<td>-44.84°</td>
</tr>
<tr>
<td>2</td>
<td>12.65°</td>
<td>135.16°</td>
</tr>
<tr>
<td>3</td>
<td>12.65°</td>
<td>44.84°</td>
</tr>
<tr>
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<td>11.36°</td>
<td>128.00°</td>
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<tr>
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<td>11.36°</td>
<td>-52.00°</td>
</tr>
<tr>
<td>6</td>
<td>11.36°</td>
<td>52.00°</td>
</tr>
<tr>
<td>7</td>
<td>8.12°</td>
<td>28.75°</td>
</tr>
<tr>
<td>8</td>
<td>7.09°</td>
<td>-4.10°</td>
</tr>
<tr>
<td>9</td>
<td>9.62°</td>
<td>159.41°</td>
</tr>
</tbody>
</table>

Table 3.7: Parameter and property listing for the nine electromagnet configuration.
3.4.3 Nine electromagnet configuration

The nine electromagnet configuration is balanced with respect to the number of electromagnets in each axis of translation, as depicted in Figure (3.16). The minimum value of $N$ equals 1.07 which increases to a maximum of 2.9 at the extremities of the travel range, as listed in Table (3.7). This improvement in the null space conditioning is important when severe constraints are placed on the maximum current capabilities of the power supplies. Therefore, even though it is possible to completely manipulate a six degrees of freedom flotor with only seven electromagnets, power supply characteristics may dictate that a nine electromagnet configuration is more suitable for practical implementation.

3.5 Conclusions

This chapter addresses the problem of electromagnet location in a large travel, multiple degrees of freedom, Magnetic Suspension Actuator. In the first part of the chapter, a mathematical model of the electromagnetic interaction between the flotor and the electromagnets around it, is derived using the finite element approach. This approach ensures that the model is accurate for the entire range of travel of the MSA, especially at large airgaps between the flotor and the electromagnets. The resulting force model for the MSA is linear with the square of electromagnetic current, and nonlinear in the airgap variables. This model is then utilized for synthesizing an actuation strategy that ensures the manipulability of the flotor while minimizing the number of electromagnets.

The number of electromagnets are minimized by employing configurations that utilize all the components of the interaction force between the electromagnet and the
flotor. However, due to the unidirectional nature of electromagnetic forces, it is necessary that the current vector commanded to the electromagnets be positive. This is accomplished by ensuring the positivity of the null space associated with the electromagnet location matrix in a two step process. In the first step, the manipulability condition is first formulated at the nominal location of the flotor. Any electromagnet arrangement that satisfies this condition at the nominal location is then optimized by modulating the electromagnet orientations for its null space current conditioning. In the second step, this electromagnet arrangement is examined for positivity of the null space current at all possible flotor locations in the operational space of the MSA. Since the null space current is represented by multivariable polynomials, a test based on polynomial positivity and Sturm's Theorem is employed.

Using the procedure outlined above, optimized electromagnet arrangements are synthesized that ensure the complete manipulability of the flotor in the six DOF space. Even though the arrangement of seven electromagnets realizes the desired travel along the three coordinate axes, null space conditioning cannot be improved significantly which may limit its application to situations requiring small to medium range travel. Utilization of eight electromagnets results in a significant improvement over the seven electromagnet case. The best results are obtained with a configuration of nine electromagnets with a minimum condition ratio of 1.07 which increases to 2.69 at the interval endpoints. This improved conditioning may be important in situations when the maximum current available from the power supplies is constrained. Therefore, the electromagnet modeling and location approach suggested in this chapter leads to a more practical, and efficient actuation strategy for the MSA.
CHAPTER 4

DESIGN OF THE ELECTROMAGNETIC ACTUATION SCHEME WITH PEM COMBINATIONS

This chapter addresses the design of the electromagnetic actuation scheme for the large travel Magnetic Suspension Actuator, using Permanent-Electro-Magnet (PEM) combinations that generate higher forces at larger airgaps. In contrast to the developments of the previous chapter, where the design of the actuation scheme was addressed by optimally locating DC electromagnets, PEM combinations help reduce the size and current requirements of the electromagnets. However, utilization of permanent magnets leads to issues in modeling and manipulability of PEM combinations. A methodology for addressing these issues and designing the actuation scheme for the MSA is presented in this chapter.

4.1 Introduction

In the previous chapter, design of the electromagnetic actuation strategy was addressed using optimally located DC electromagnets. Since no force is generated when the electromagnet is unenergized, such an actuation system requires current not only to control the motion of the flotor but also to support its weight. Additionally, in
order to meet the large travel requirement desired of the Magnetic Suspension Actuator, high flux densities at large airgaps are desired to ensure flotor manipulability. The combination of these two factors results in large electromagnets, with high current requirements, consequently leading to electromagnet heating, cost escalation, and problems in the design of high performance power supplies.

In order to reduce the size of the electromagnets and the associated current requirements, permanent magnets can be combined with DC electromagnets to generate Permanent-Electro-Magnet combinations that give critical advantages in the design of the actuation scheme for the Magnetic Suspension Actuator. The permanent magnets in the PEM combinations are used to provide a strong DC component of the magnetic flux which is utilized in supporting the weight of the flotor. Also, the permanent magnets provide significant forces so that it is easier to obtain large travel from the actuation scheme in all of the translational degrees of freedom. The DC electromagnets in the PEM combinations are used to actively control the flotor by modulating the current in the coil of the electromagnets. Therefore the actuation scheme becomes compact and requires less electric power in comparison to the one comprising of only DC electromagnets.

However, actuation schemes with PEM combinations need to be carefully designed since maximization of force characteristics often leads to loss of manipulability. At small distances, the DC electromagnet-target interaction is particularly strong and overwhelms the forces from the permanent magnets, leading to a loss of manipulability. At large airgaps, forces from the permanent magnets dominate the interaction. Therefore, synthesis of actuation strategies comprising of these PEM combinations that not only ensure the desired force characteristics but also the manipulability of
the flotor in the entire operational space of the MSA, is an important design issue that is addressed in this chapter.

The large travel requirement of the actuation scheme necessitates modeling of the magnetic field distribution at large distances from the electromagnet poleface. This description of the electromagnetic field is nonlinear, with respect to the spatial variables. The computational approach that was adopted in the previous chapter for modeling magnetic forces due to DC electromagnets at large airgaps, can also be utilized for modeling PEM interactions. However, this approach is inadvisable due to the computational expense associated with finite element characterization of the complete actuation scheme. As an alternative, a systematic procedure for assembling the complete model of the actuation scheme from individual models of the PEM components is investigated. The electromagnetic interactions within each of these PEM components can be conveniently characterized through finite element simulations. Based on this characterization, an aggregate mathematical model for the complete actuation scheme is developed. The model is scalable with the number of PEM combinations, and provides a reasonably accurate description of the magnetic field at large airgaps.

Using the above methodology, this chapter develops an electromagnetic actuation strategy that supports the large travel requirements. This encompasses the design and size optimization of the electromagnets, which are then utilized in synthesizing PEM combinations. The design process for the PEM combinations selects permanent magnets that satisfy the large travel objectives of the actuation strategy, and ensure complete flotor manipulability in the operational space. The PEM combinations are
4.2 Permanent-Electro-Magnet (PEM) combinations

To obtain high forces at large airgaps, PEM combinations are utilized. The advent of high performance rare earth magnets with large coercive force provides the impetus to use PEM combinations in the design of the actuation scheme. One such arrangement of permanent magnets in conjunction with an electromagnet is shown in Figure (4.1). The E-core electromagnet interacts with a ferromagnetic target on the flotor. This interactions produces an attractive force. However, two pairs of permanent magnets are arranged to provide a repulsive force between the flotor and the stator. Thus this combinations of permanent magnets with an electromagnet provides bi-directional actuation. In addition, permanent magnets can also be utilized next incorporated into the design of the horizontal and vertical actuation schemes that give the desired actuation characteristics.
to passively support the weight of the flotor. The permanent magnets in this arrangement provide the necessary force to suspend the system, while the electromagnets are used for stabilization and control. Thus the PEM combinations not only decrease the size of the electromagnets, but also reduce the current and slew rate requirements demanded of the power supplies.

The main challenge in designing actuation schemes with PEM combinations is to prevent manipulability problems that are frequently encountered. When the airgap is small, the forces due to the electromagnets strongly dominate those of the permanent magnets, leading to a loss of manipulability over the net interaction force. At large airgaps on the other hand, permanent magnet forces are significantly larger than electromagnet forces, leading again to a loss of manipulability. The approach utilized in this research is to incorporate the manipulability constraint in the design process, so that both the electromagnets and permanent magnets are sized suitably. This methodology is detailed in Section (4.4).

The procedure for the design of the actuation scheme comprises of four steps: Design of the electromagnets, design of the PEM combinations, design of the horizontal actuation scheme, and finally the design of the vertical actuation scheme. The following section, details the issues and the procedures in the synthesis of DC electromagnets.

4.3 Design of DC electromagnets

Design of a DC electromagnet involves the selection of the geometry, the size, the material, and the winding parameters for the electromagnet. Both cylindrical
(used in previous chapter) [67] as well as E shaped cores [26] have been used in previous developments of magnetically suspended actuators. In comparison to cylindrical electromagnets, E-core provide higher forces due to reduced flux fringing and leakage. Since most of the flux is confined within the ferromagnetic laminates in E-core electromagnets, lumped parameter models can be developed, on the basis of which design and optimization of electromagnet dimensions can be pursued. Finally, since E-core electromagnets are extensively utilized in transformer industry, they are easily available in standard dimensions. Cylindrical electromagnets on the other hand, need to be custom fabricated. Due to these three reasons, E-core geometry is selected for the electromagnet to be utilized in the design of PEM combinations.

The approach to determining the optimal size of the electromagnet involves first the development of the lumped parameter model of the electromagnet. The lumped parameter model is compared with the finite element model, to understand its limitations. The lumped parameter model is then used to optimize the electromagnet dimensions given constraints on the current and voltage characteristics of the power supplies. These constraints reflect both the static as well as the dynamic performance requirements expected from the electromagnetic actuator. The force versus current versus airgap characteristics for the optimal electromagnet are then obtained through three dimensional finite element simulations. This process of designing the E-core electromagnet is described in the following sections, starting first with the derivation of the lumped parameter model, and its comparison to finite element models.
4.3.1 Development of the E-core electromagnet model

An E-core electromagnet interacting with a target is depicted in Figure (4.2). The electromagnet is energized by current $i$ flowing in a $N$ turn winding around its central leg. The magnetic circuit corresponding to the electromagnetic interactions is depicted in Figure (4.3), where $\mathcal{R}_1$ is the reluctance associated with the paths 1234 and 1674, and $\mathcal{R}_2$ with path 14. The assumptions associated with this modeling procedure are detailed in [34]. Expressing the reluctance in terms of the geometrical parameters of the electromagnet:

$$\mathcal{R}_1 = \left( \frac{l_c}{2\mu_0\mu_t A_t} + \frac{t_e + 0.5l_c}{\mu_0\mu_e A_{eel}} \right) + \frac{h}{\mu_0 A_{gel}} = k_1 + k_2 h$$  \hspace{1cm} (4.1)

$$\mathcal{R}_2 = \frac{t_e}{\mu_0\mu_e A_{cel}} + \frac{h}{\mu_0 A_{gel}} = k_3 + k_4 h$$  \hspace{1cm} (4.2)

where $l_c$ and $t_e$ denote the length and the height of the electromagnet, respectively. $h$ is the airgap between the electromagnet and the target, while $A_{cel}$ and $A_{eel}$ are the cross-sectional areas associated with end legs and the central leg of the electromagnet, respectively. Similarly, cross-sectional areas are denoted by $A_{gel}$ and $A_{gel}$ in the case of airgaps. and by $A_t$ for the target. $\mu_o, \mu_e$ and $\mu_t$ are permeabilities of air, electromagnet and target materials. With these definitions, the force between the electromagnet and target is expressed as:

$$F = \frac{(0.5k_2 + k_4)}{(0.5k_1 + k_3 + (0.5k_2 + k_4) h)^2} \frac{N^2i^2}{2}$$  \hspace{1cm} (4.3)

where the force acts in the downward direction along the $-z_o$ axis, towards the electromagnet. Recognizing that both $\mu_t$ and $\mu_e$ are much greater than 1, the above formula can be approximated as:

$$F = \frac{0.5N^2i^2}{(0.5k_2 + k_4) h^2}$$  \hspace{1cm} (4.4)
Figure 4.2: A schematic of the E-core electromagnet interacting with the target.

Figure 4.3: The magnetic circuit corresponding to the E-core electromagnet interacting with a target.
The force predicted by the above equation is compared with the results of the finite element simulations in Figure (4.4), for an E-core electromagnet energized with a 100 turn winding carrying a current of 9.25 ampere. The parameters of the E-core electromagnet used in the simulations are listed in Table (4.1). The three dimensional finite element model is simulated using the software Maxwell-3D. Since the predictions of the lumped parameter model approximately follow the finite element simulations in Figure (4.4), it can be utilized in optimizing the electromagnet parameters. Force versus current versus airgap characteristics of the optimal electromagnet can be determined more precisely by either experimental calibration or finite element simulations.

4.3.2 Optimization of the E-core electromagnet parameters

The first step in optimizing the electromagnet parameters is to relate all of its dimensions in terms of the electromagnet height $t_e$ and the electromagnet length $l_e$. Expressing the thickness of the target as $t_t = f_t t_e$, the width of the target as $w_t = f_w t_e$, the width of the electromagnet as $w_e = f_w t_e$, length of the end legs of the electromagnet as $l_{el} = f_{el} l_e$, and the length of the central leg of the electromagnet as $l_{cl} = f_{cl} l_e$, the expression for force in Equation (4.4) can be rewritten as:

$$ F = \frac{0.5 \mu_0 f_{cl} f_w f_{el} t_e l_e N_i^2 i^2}{(0.5 f_{cl} + f_{el}) h^2} \quad (4.5) $$

The above equation can then be analyzed to optimize the electromagnet parameters in terms of its basic dimensions $t_e$ and $l_e$. To determine the optimal relationship between the lengths of the end legs of the electromagnet to its central leg, the force of the electromagnet is computed for various values of $f_{el}$, using the above equation. For the electromagnet of dimensions listed in Table (4.1), Figure (4.5(a)) plots the variation
Figure 4.4: A comparison of the force versus airgap characteristics predicted by the lumped parameter model with FEM simulations at $i = 9.25$ ampere.

<table>
<thead>
<tr>
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<th>Value</th>
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</thead>
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<td>Length of electromagnet $l_e$</td>
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</tr>
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<td>Width of electromagnet $w_e$</td>
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<tr>
<td>Height of electromagnet $t_e$</td>
<td>36.00 mm</td>
</tr>
<tr>
<td>Length of end leg $l_{el}$</td>
<td>10.00 mm</td>
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<tr>
<td>Length of central leg $l_{cd}$</td>
<td>21.00 mm</td>
</tr>
<tr>
<td>Height of central leg $t_{cd}$</td>
<td>30.00 mm</td>
</tr>
<tr>
<td>Number of winding turns $N$</td>
<td>100</td>
</tr>
<tr>
<td>Diameter of winding wire $d_w$</td>
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<tr>
<td>Specific resistance of wire $R_w$</td>
<td>.05296 Ohm/meter</td>
</tr>
<tr>
<td>Material of the electromagnet</td>
<td>Steel</td>
</tr>
</tbody>
</table>

Table 4.1: Physical characteristics of the preliminary electromagnet used in simulations.
in electromagnet force as $f_{cl}$ is changed from 0.05 to 0.5. It is observed that the maximum force is obtained when $f_{cl} = 0.2778$ which gives an optimal $f_{cl} = 0.5544$. Next, we observe that Equation (4.5) is actually independent of $t_e$, since $w_e = f_w t_e$, and both $f_{cl}$, $f_{el}$ are independent of $t_e$. This fact is verified in Figure (4.5(b)) numerically. Increasing width of the electromagnet increases the length of the winding for fixed number of turns, and consequently increases the voltage requirement of the power supply. By similar reasoning, it is observed that the electromagnetic force $F$ increases linearly with electromagnetic width $w_e$, as numerically verified in Figure (4.5(c)). Finally, Equation (4.5) is also independent of $l_e$ as $l_{cl} = f_{cl} l_e$ whose effect has already been studied in Figure (4.5(a)). Consequently, analysis of Equation (4.5) leads to optimal values for $f_{el}$ and $f_{cl}$, along with the conclusion that an optimal value of $w_e$ needs to be calculated, by incorporating constraints on the available voltage and current available through the power supply.

The $N$ turn winding of the electromagnet resides on its central leg, whose length is $l_{cl}$ and width $w_e$ as shown in Figure (4.2). The length of wire required for an $N$ turn winding around this leg is expressed as:

$$L_w = 2 \left( \frac{N_l l_{cl}}{d_w} + Q_l \right) (l_{cl} + w_e) + 2 t_{cl} (N_l - 1) N_l + 4 Q_l N_l d_w$$

(4.6)

where $L_w$ is the total length of the wire, $N_l$ is the number of layers of winding on the central leg of the electromagnet and is computed through the relationship $N = N_l l_{cl}/d_w + Q_l$. $d_w$ is the diameter of the winding wire and $t_{cl}$ is the height of the central leg as depicted in Figure (4.2). If $R_w$ denotes the resistance per unit length of the wire, then the total resistance of the winding is $L_w R_w$. Using a 22 gage wire with a resistance per unit length value of $R_w = 0.05296$ ohms/meter, and constraining the total resistance $R_w = 6.25$ Ohms, Equation (4.5) is optimized for the
Figure 4.5: Optimization of electromagnet force with respect to its various parameters: (a) Variation with $f_{et}$, (b) variation with $t_e$, and (c) variation with $w_e$. Graph (d) plots the force versus airgap characteristics of the optimal electromagnet from the lumped parameter model and the FEM simulations at $i = 1.0$ ampere.
The electromagnet force $F$ with number of winding turns $N$, width of the electromagnet $w_e$, and the length of the end leg $l_{el}$ as the variables. The constraint optimization procedure yields an optimal electromagnet of $N = 700$ turns, with $w_e = 47.30$ mm. and $l_{el} = 36.0$ mm, whose force versus airgap characteristic for $i = 1.0$ ampere is graphed in Figure (4.5(d)). For this optimized electromagnet, a maximum force of 280.06 Newton is obtained at $i = 1.0$ ampere. Comparison with finite element simulation results for its force versus airgap characteristic reveals close correspondence with the predictions of the lumped parameter model of Equation (4.4), in the case of the optimal electromagnet.

The final step in the complete optimization of the E-core electromagnet is to incorporate the constraints of dynamic performance in the optimization procedure. The inductance of the electromagnet depicted in Figure (4.2) corresponding to the force expressed in Equation (4.5), as be written as:

$$L_m = \frac{N^2}{(0.5k_2 + k_4) h}$$  \hspace{1cm} (4.7)

where $L_m$ is the effective inductance of the electromagnet and target pair. The inductance depends on the number of winding turns $N$ and the airgap $h$ between the electromagnet and the target, in addition to the geometrical parameters. The variation of inductance $L_m$ with airgap $h$ is depicted in Figure (4.6) for the electromagnet with parameters listed in Table (4.1). It is observed that inductance increases with decreasing airgap $h$, between the electromagnet and the target. Therefore, the value of inductance at an airgap of 1 mm, representing the minimum working airgap for the electromagnet actuator is used in computation for the required voltage and denoted as $L_{m_{\text{max}}}$. 

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Figure 4.6: Variation of electromagnet inductance $L_m$ with airgap $h$.

Maximum voltage is required in dynamic operation when either the current is ramped from 0 to $i_{\text{max}}$ or ramped down from $i_{\text{max}}$ to 0, in one sampling instant $\Delta t$. The voltage required in this operation is given by the following equation:

$$V_d = \frac{R_w e^{-R_w/L_{\text{max}} \Delta t} i_{\text{max}}}{1 - e^{-R_w/L_{\text{max}} \Delta t}}$$ (4.8)

where $V_d$ is the voltage required for the dynamic operation described above. Qualitatively, it is observed in the above equation that a decreasing $N$ reduces $L_m$, which in turn reduces $V_d$. However, a decreasing $N$ requires a corresponding increase in $i_{\text{max}}$ to keep the magnetization $Ni_{\text{max}}$ constant. This increase in current leads to higher heating, which may impose constraints on continuous operation of the electromagnet.

For the case of the optimal electromagnet, computation using Equation (4.8) at an airgap $h = 1.0$ mm, with constant magnetization $Ni_{\text{max}} = 700$ ampere-turn, yields a value of $V_d = 160$ volts for $i_{\text{max}} = 14$ amperes, and $V_d = 1500$ volts for $i_{\text{max}} = 1.55$.
amperes. Clearly, both these values are infeasible with respect to precision power supplies available commercially. Therefore, Equations (4.7) and (4.8) are incorporated as constraints in the optimization procedure that maximizes electromagnet force $F$ in Equation (4.4), with maximum values of voltage at $V_d = 70$ volts, along with a maximum $R_w = 6.25$ Ohms. The procedure yields an optimal electromagnet with $N = 71$ turns, and $i_{\text{max}} = 10.0$ amperes. Comparison of the force versus airgap characteristics obtained through lumped parameter model and finite element simulations are graphed in Figure (4.7), at $i_{\text{max}} = 10.0$ ampere. The curves from the two modeling procedures are approximately coincident, verifying the fact that the lumped parameter model is able to accurately predict forces for the optimal electromagnet. The corresponding variation of $V_d$ with airgap $h$ at $i_{\text{max}} = 10.0$ ampere is shown in Figure (4.8). Maximum $V_d = 50$ volts at an airgap $h = 1.0$ mm is required for dynamic operation of the optimal electromagnet.

4.3.3 Selection of laminates for the electromagnet-target pair

E-core electromagnets such as the one depicted in Figure (4.2) are either constructed out of laminates or powdered iron cores to minimize ferromagnetic hysteresis. Each grain particle in powdered iron cores is coated with plastic to insulate it from adjacent particles, to maximize resistance to circulating eddy currents. However, this property is obtained at the expense of relatively poorer magnetic properties in comparison to ferromagnetic laminates, consequently reducing the force between the electromagnet and the target. Consequently, E-core constructed out of ferromagnetic laminates are usually employed in various transformer applications and some of the previous developments in magnetically suspended actuators [78]. The final step in
Figure 4.7: Force versus airgap characteristics with $i_{\text{max}} = 10.0$ ampere for the optimal electromagnet incorporating the constraint on maximum available voltage.

Figure 4.8: Variation of maximum voltage $V_d$ with airgap $h$ for dynamic operation of the electromagnet with $i_{\text{max}} = 10.0$ ampere.
the design of the E-cores electromagnets is selection of the commercially available laminates that most closely provide the force-current-airgap characteristics obtained with the optimal electromagnet, designed in the previous section.

Commercial E-core electromagnets are available in either silicon steel or nickel/steel alloys. Silicon steel laminates provide higher forces than nickel/steel alloys for the same input current, but display hysteresis characteristics. Since compensation of hysteresis is addressed in the development of the control architecture in this research, silicon steel laminates were selected to exploit their capabilities to carry high flux densities. Commercial laminates under the trade name of EI-62 (Sub-Tronics Inc., Stacy, MN) with a 5/8 inch center leg were used in the construction of the E-core electromagnet. The thickness of each of the laminations is .014 inches which are stacked together to obtain an electromagnet with \( w = 1.0 \) inch. The dimensions of the electromagnet and the winding parameters of the E-core electromagnet manufactured using these laminates are listed in Table (4.2). Comparison of the force versus airgap characteristics for current values of 10.5, 8.5, 6.25 and 4.20 amperes in the manufactured electromagnet, and for 10, 8, 6 and 4 amperes in the optimal electromagnet are shown in Figure (4.9(a)-(d)). Due to the small differences in the size of the two electromagnets, slightly higher current values are required in the manufactured electromagnet to obtain the desired force versus airgap characteristics.

The final component in the design of the electromagnetic actuator is the design of the ferromagnetic target. The ferromagnetic target is also constructed out of laminations made of EI-62 silicon steel of thickness .014 inches. However, the target is oversized in comparison to E-core electromagnet so that the forces do not change as the target moves in the \( x_o \) and \( y_o \) directions of the reference coordinate system.
Figure 4.9: Comparison of the force versus airgap characteristics at current of 10.5 and 10.0 ampere in (a), 8.5 and 8.0 in (b), 6.25 and 6.0 in (c), and 4.2 and 4.0 in (d) for the manufactured and designed electromagnets, respectively.
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<tr>
<td>Specific resistance of wire $R_w$</td>
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</tr>
<tr>
<td>Thickness of the laminates</td>
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</tr>
<tr>
<td>Material of the laminates</td>
<td>Silicon steel (EI-62)</td>
</tr>
</tbody>
</table>

Table 4.2: Physical characteristics of the optimal electromagnet.

Figure (4.2)) due to three dimensional translation of the flotor. For this reason, the target is oversized and has dimensions of $l_t = 2.25$ inches, and $w_t = 1.30$ inches. The height of the target $t_t = 0.375$ inches. The laminations were stacked together and dipped in electrical grade varnish to provide rigidity and strength to the structure.

The E-core electromagnetic actuator designed above, is only one of the two components that constitute PEM combinations. The electromagnetic actuator is combined with permanent magnets, arranged in repulsive configurations to derived the complete combination. The issues and the approaches in sizing and selection of the permanent magnets in relationship to the actuation characteristics desired of the PEM combinations are detailed in the following section.
4.4 Design of PEM combinations

There are essentially two issues in the design of the PEM combinations. The first of these issues relates to the modeling of the interactions between the permanent magnets. As described in Section (4.2), pairs of permanent magnets in PEM combinations are usually arranged in a repulsive configuration. In such a configuration, the majority of the flux lines from each of the permanent magnets is forced to leak into the airgap, and travel to the opposite pole. Consequently, description of interaction forces in repulsive configurations of permanent magnets is not possible using simple equations of the lumped parameter approach. Therefore, description through field equations becomes necessary [46], which may be solved using various computational methods. In this research, finite element method is employed for determining the force versus airgap characteristics for the permanent magnet pairs that are employed in the actuation strategy. Simulations are performed for pairs of permanent magnets arranged in repulsive configurations, to determine interaction forces as a function of the permanent magnet length and diameter. Suitable dimensions are then selected, in conjunction with the characteristics of the designed E-core electromagnet, that satisfy the objectives of the performance scheme, and the manipulability conditions associated with the PEM combination that are described next.

The net force from the PEM combination consists of two components: The attractive force being provided by the electromagnet-target pair, and the repulsive force from the permanent magnets. At each point within the range of variation of the airgap, it is important to ensure that the desired net force can be commanded by appropriately modulating the current input to the electromagnet. This requires
Figure 4.10: Force versus current characteristics of the PEM combination for the vertical actuation scheme at various airgaps. (a) $h = 1$ mm, (b) $h = 3$ mm, (c) $h = 5$ mm, (d) $h = 7$ mm, and (e) $h = 9$ mm. Graph (f) depicts the force versus airgap characteristic at $i = 0$ ampere.
the appropriate sizing of the permanent magnets in relationship to the force-current-airgap characteristics of the electromagnet, so that as the current is varied from zero to its maximum value in the electromagnet, the net force from the PEM combination changes from repulsive to attractive force. This is referred to as the manipulability condition. Consider the optimal electromagnet with parameters listed in Table (4.2) in combination with a pair of NdFe35 permanent magnets, each of length 21 mm. and of radii 3 and 7 mm. The force versus current versus airgap characteristics of this PEM combination are depicted in Figure (4.10). It plots the force versus current characteristics of the PEM combination as the airgap is changed from 1 to 9 mm. in steps of 2 mm. At an airgap of 1 mm, as the current is varied from 0 to 1400 ampere-turns, the net force changes from 13.04 to -329.65 Newton. At an airgap of 9 mm. the corresponding values for the same range of variation of current are 2.76 and -0.18 Newton. Consequently, as the airgap is varied, net force lies in range given by \{F_{\text{max}}(h), F_{\text{min}}(h)\}, where \( F_{\text{min}} \) and \( F_{\text{max}} \) are necessarily negative and positive, respectively, as the current to the electromagnet is varied. This condition ensures manipulability of the PEM combination. \( F_{\text{min}} \) and \( F_{\text{max}} \) are functions of airgap \( h \) and their values depends on the characteristics of both the electromagnet and the permanent magnets.

The six degree of freedom motion of the flotor is depicted in Figure (4.11). Translation along the \( x \) and the \( y \) axis, and rotation around the \( z \) axis are termed as the degrees of freedom in the horizontal plane. Similarly, translation along the \( z \) axis, with rotations around the \( x \) and the \( y \) axis are termed as vertical degrees of freedom. The design of the complete actuation scheme consists of designing the actuation schemes for the horizontal and the vertical degrees of freedom. Due to constraints of
Figure 4.11: The six degrees of freedom associated with the flotor.

laser interferometric sensing, angular rotations are actively minimized by the control scheme. If the variation of the forces in the vertical (horizontal) actuation scheme can be minimized as the flotor translates in the horizontal (vertical) plane, then the design process can be decoupled. Realization of this condition through appropriate design of the PEM combinations, for both the horizontal and vertical actuation schemes is detailed in the following sections.

4.5 Design of the vertical actuation scheme

The first step in the design of the vertical actuation scheme is to select the size and number of permanent magnet pairs, which placed in combination with the E-core optimal electromagnet (with parameters listed in Table (4.2)) ensure the manipulability condition for the PEM combination. For this purpose, series of FEM simulations are performed for pairs of cylindrical permanent magnets of varying diameters and
lengths, placed in repulsive configurations, in an airgap range of 1-9 mm. The resulting interaction force is then expressed as a function of the permanent magnet radius, length and airgap. In the finite element simulations, Neodymium-Iron (NdFe) is selected to be the material for the permanent magnets due to its high energy product. This calibration of the permanent magnets in combination with the lumped parameter model of the optimal E-core electromagnet, is then optimized. Since the permanent magnets support the flotor weight, the objective of the optimization procedure is to maximize the repulsive force of the permanent magnets, while satisfying the manipulability condition. Results of this optimization procedure are discussed below.

The above design procedure results in permanent magnets of radii 3 and 7 mm, with lengths of 21 mm. The permanent magnet with the larger radii is to be mounted on the flotor, while the smaller magnet is placed on the stator. The permanent magnet placed on the stator is smaller in diameter, so that the force of the PEM combination along the \( z_o \) axis, does not vary significantly with motion along the \( x_o \) and the \( y_o \) axes. Such a design ensures that the desired decoupling between the horizontal and vertical actuation schemes is maximized. Similarly, the target corresponding to the E-core electromagnet is also oversized, as described in Section (4.3.3) to maximize the decoupling. The force versus airgap characteristics of the permanent magnets designed above are graphed in Figure (4.12). A maximum repulsive force of 13.04 Newton at an airgap of 1 mm, and minimum force of 2.76 Newton at airgap of 9 mm is obtained. Dimensions and properties of commercial permanent magnets which best approximate the designed permanent magnets are listed in Table (4.3).
<table>
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</tr>
<tr>
<td>Length of PM on flotor</td>
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</tr>
<tr>
<td>Material of permanent magnet</td>
<td>Neodymium-Iron</td>
</tr>
<tr>
<td>Maximum energy product</td>
<td>35.0 MG.Oe</td>
</tr>
<tr>
<td>Residual flux density</td>
<td>12.0 KG</td>
</tr>
<tr>
<td>Coercive force</td>
<td>11.0 KOe</td>
</tr>
</tbody>
</table>

Table 4.3: Physical characteristics of the permanent magnets in the PEM combination designed for the vertical actuation scheme.

Figure 4.12: Force versus airgap characteristic of the permanent magnet pair designed for the vertical PEM combination in comparison to the NdFe and SaCo permanent magnets available commercially.
of the designed permanent magnets, as observed from Figure (4.12). Finally, the complete force versus airgap versus current characteristics of the PEM combination are graphed in Figure (4.10). It is observed that at the maximum airgap of 9 mm, the minimum force is -0.18 Newton at 1400 ampere-turn, which indicates that the designed permanent magnets are indeed the largest possible for the optimal E-core electromagnet. The complete force characteristics of the PEM combination for the vertical actuation scheme are listed in Table (4.4).

Associated with the vertical actuation scheme are three degrees of freedom as depicted in Figure (4.11): Rotation around the $x_o$ and the $y_o$ axes, and translation around the $z_o$ axis. Consequently, a minimum of three PEM combinations are required to modulate these degrees of freedom. Since the PEM combination provides bi-directional actuation, all of the actuators are located on the bottom surface of the flotor. This leaves the top face of the flotor free for mounting any payload or instrumentation. The three PEM combinations are placed in a triangular arrangement at

Table 4.4: Actuation characteristics of the PEM combinations designed for the vertical actuation scheme.

<table>
<thead>
<tr>
<th>Actuation characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum airgap</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>Maximum airgap</td>
<td>9.0 mm</td>
</tr>
<tr>
<td>Maximum operating current</td>
<td>1400 ampere-turn</td>
</tr>
<tr>
<td>Minimum force at min. airgap</td>
<td>-316.61 Newton</td>
</tr>
<tr>
<td>Maximum force at min. airgap</td>
<td>13.04 Newton</td>
</tr>
<tr>
<td>Minimum force at max. airgap</td>
<td>-0.15 Newton</td>
</tr>
<tr>
<td>Maximum force at min. airgap</td>
<td>2.76 Newton</td>
</tr>
</tbody>
</table>

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Figure 4.13: Configuration of the vertical actuation scheme.
the bottom surface of the flotor, as shown in Figure (4.13). In the figure, the flotor (pink box) is embedded with three targets (shown in cyan) and three permanent magnets (also shown in cyan). The corresponding E-core electromagnets (shown in green) and the other half of the permanent magnet pairs (shown in red) mount on the stator, not shown in the figure. The PEM combinations are located so that the turning moments are maximized, while the interference between the magnetic fields of the electromagnets and the permanent magnets is minimized. Consequently, the electromagnets are located at the vertices of an equilateral triangle, while each of the three permanent magnets sets is located at the mid-point of the three triangle edges.

4.6 Design of the horizontal actuation scheme

The horizontal actuation scheme primarily modulates the three degrees of freedom associated with the flotor as depicted in Figure (4.11): Translation along the \(x_o\) and the \(y_o\) axes, and rotation around the \(z_o\) axis. Unlike the vertical actuation scheme, the PEM combinations in this case can be placed along any or all of the four sides in the horizontal plane of the flotor, without any restrictions. Consequently, the characteristics desired of the PEM combinations in the horizontal actuation scheme are different from the PEM combinations designed for the vertical actuation scheme. To maximize the forces along the \(x_o\) and the \(y_o\) axes, PEM combinations in the horizontal actuation scheme are arranged in a push-pull configuration as depicted in Figure (4.14). When the flotor translates in the \(-y_o\) direction to its maximum displacement of \(-y_{\text{max}}\) from the nominal position, permanent magnets in PEM combination \(P_{y_1}\) and the electromagnet in PEM combination \(P_{y_2}\) provide a net force in the \(+y_o\) direction to modulate the position of the flotor. Similarly, as the flotor moves by a distance \(y_{\text{max}}\)
in the $+y_o$ direction, the electromagnet in PEM combination $P_{y_2}$ and the permanent magnets in PEM combination $P_{y_1}$ provide the necessary force to manipulate the flo-tor within its travel range. Consequently, the arrangement of PEM combinations $P_{y_1}$ and $P_{y_2}$ in a push-pull configuration has the potential to provide larger forces in the horizontal plane. Design of PEM combinations in push-pull configurations that max-imize the force characteristics of the horizontal actuation scheme, while maintaining its manipulability is described in the following paragraphs.

The design of the PEM combination consists of two components: The electromagnets and the permanent magnets. As the range of variation of the airgap $h$ in Figure (4.2) remains 1-9 mm, the optimal electromagnet with parameters listed in Table (4.2) is again utilized for sizing the permanent magnets, and maximizing the force characteristics of the horizontal PEM combinations. To obtain a model of the forces between pairs of permanent magnets arranged in a repulsive configuration, FEM simulations are performed for permanent magnets of varying diameters and lengths, placed in an airgap range of 1-9 mm. The resulting interaction force is expressed as a function of the permanent magnet radius, length and airgap. This model of the permanent magnets in combination with the force-current-airgap characteristics of the optimal electromagnet is then incorporated into an optimization scheme that generates PEM combinations for push-pull configurations of Figure (4.14). At each point within the travel range of $\{-y_{\text{max}}, y_{\text{max}}\}$, the net force due to the two PEM combinations varies between $\{F_{y_0}^{\text{min}}(y), F_{y_0}^{\text{max}}(y)\}$ as the current is changed from 0 to $i_{\text{max}}$ in each of the two electromagnets. The objective of the optimization procedure then is to maximize the minimum of $\{|F_{y_0}^{\text{min}}(y)|, |F_{y_0}^{\text{max}}(y)|\}$ for $y \in \{-y_{\text{max}}, y_{\text{max}}\}$, while subjected to the
Figure 4.14: Schematic of two PEM combinations arranged in a push-pull configuration for a single axis in the horizontal actuation scheme.

Figure 4.15: Force versus airgap characteristic of the permanent magnet pair designed for the horizontal PEM combination in comparison to the NdFe permanent magnets available commercially.
constraint of satisfying the manipulability condition by maintaining $F_{y_0}^{\min}(y) < 0$ and $F_{y_0}^{\max}(y) > 0$ for all $y$.

The above design procedure results in a PEM combination of configuration shown in Figure (4.1), and arranged in a push-pull configuration of Figure (4.14). The permanent magnets are of radii 4.30 and 8.30 mm and length 19.0 mm. As in the case of the vertical actuation scheme, the larger permanent magnet is placed on the flotor to minimize variation of permanent magnet forces, as the flotor moves in the $z_o$ direction. The force versus airgap characteristics of one of the permanent magnet pairs is graphed in Figure (4.15). A maximum repulsive force of 29.7094 Newton at an airgap of 1 mm, and minimum force of 9.0537 mm at an airgap of 9 mm is obtained. Corresponding dimensions and properties of commercially available permanent magnets selected for the horizontal PEM combinations are listed in Table (4.5), and the force versus airgap characteristic graphed in Figure (4.15). Maximum and minimum force characteristics of the complete push-pull pair comprising of the permanent magnets, and the optimal electromagnets are graphed in Figure (4.16). The two curves in the graph correspond to the maximum and minimum net force that is obtained as the flotor translates along the $y_o$ direction in a travel range of $\{y_{\min} = -4.0, y_{\max} = 4.0\}$ mm. At $y_{\min} = -4.0$ mm, the maximum and minimum values of the net force are 30.70 Newton and -210.14 Newton, respectively. The magnitude of the minimum force is much larger since the electromagnet of PEM combination $P_{y2}$, easily dominates the forces from the permanent magnets and the electromagnet of PEM combination $P_{y1}$. Corresponding situation is also observed at $y_{\max} = 4.0$ mm. Finally, at each point of the interval $\{y_{\min}, y_{\max}\}$, it is observed
Table 4.5: Physical characteristics of the permanent magnets in the PEM combination designed for the horizontal actuation scheme.

<table>
<thead>
<tr>
<th>Physical characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of PM on stator</td>
<td>9.39 mm</td>
</tr>
<tr>
<td>Diameter of PM on flotor</td>
<td>19.05 mm</td>
</tr>
<tr>
<td>Length of PM on stator</td>
<td>19.05 mm</td>
</tr>
<tr>
<td>Length of PM on flotor</td>
<td>19.05 mm</td>
</tr>
<tr>
<td>Material of permanent magnet</td>
<td>Neodymium-Iron</td>
</tr>
<tr>
<td>Maximum energy product</td>
<td>35.0 MG.Oe</td>
</tr>
<tr>
<td>Residual flux density</td>
<td>12.0 KG</td>
</tr>
<tr>
<td>Coercive force</td>
<td>11.0 KOe</td>
</tr>
</tbody>
</table>

Figure 4.16: Force characteristic of the PEM combination pair designed for the horizontal actuation scheme.
that the corresponding range of forces that may be commanded lie in the range \( \{-|F^\text{min}_{y_0}(y)|, |F^\text{max}_{y_0}(y)|\} \), indicating that the manipulability condition is satisfied.

The push-pull configuration of Figure (4.14) is modified to include three PEM combinations along the \( y_o \) axis, as depicted in Figure (4.17). PEM combinations \( P_{y2} \) and \( P_{y3} \) are placed off axis \( y_o \), so that corresponding forces \( F_{y2} \) and \( F_{y3} \) can be utilized to modulate the rotation \( \theta_z \) around the \( z_o \) axis. To preserve the force characteristics of Figure (4.16), the lengths of the permanent magnets pairs in PEM combinations \( P_{y2} \) and \( P_{y3} \) are shortened to 12.70 mm. and the maximum current in the electromagnets of PEM combinations \( P_{y2} \) and \( P_{y3} \) reduced to 6 amperes. The force-airgap-current characteristics for the push-pull configuration with three PEM combinations, as arranged in Figure (4.17) are depicted in Figure (4.18). The maximum and minimum net force curves approximately follow those depicted in Figure (4.16), and consequently maintain the manipulability condition in the entire travel range.

To maintain symmetry in the horizontal actuation scheme, and to maximize the torque \( T_z \) around the \( z_o \) axis in Figure (4.17), the push-pull configuration designed for the \( y_o \) axis is duplicated along the \( x_o \) direction also. A solid model of the complete horizontal actuation scheme is depicted in Figure (4.19). The blue box is the stator while the flotor (pink box) is surrounded by six sets of PEM combinations, three along each of the horizontal axis. In each of the PEM combinations, the electromagnet and a set of permanent magnets are mounted to the stator, while the target and the other set of permanent magnets are embedded in the flotor. Even though three PEM combinations can actuate the flotor in the horizontal degrees of freedom, additional PEM combinations provide higher forces along each axis, and optimize the current.
Figure 4.17: Schematic of three PEM combinations along a single axis of the horizontal actuation scheme.

Figure 4.18: Force characteristic of the three PEM combinations designed for a single axis of the horizontal actuation scheme.
Figure 4.19: Configuration of the horizontal actuation scheme.
### Table 4.6: Actuation characteristics along a single axis of the horizontal actuation scheme.

<table>
<thead>
<tr>
<th>Actuation characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum airgap</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>Maximum airgap</td>
<td>9.0 mm</td>
</tr>
<tr>
<td>Maximum operating current</td>
<td>1400 ampere-turn</td>
</tr>
<tr>
<td>Minimum force at min. airgap</td>
<td>-199.10 Newton</td>
</tr>
<tr>
<td>Maximum force at min. airgap</td>
<td>23.40 Newton</td>
</tr>
<tr>
<td>Minimum force at max. airgap</td>
<td>-34.50 Newton</td>
</tr>
<tr>
<td>Maximum force at max. airgap</td>
<td>205.81 Newton</td>
</tr>
</tbody>
</table>

requirements of the electromagnet at large airgap. As per the design, the horizontal actuation scheme provides translation of 8 mm along each axis, with minimal and maximal airgaps of 1 and 9 mm, respectively. The actuation characteristics of the horizontal actuation scheme are summarized in Table (4.6).

### 4.7 Conclusions

An actuation scheme comprising of Permanent-Electro-Magnet combinations that enables large travel in all of the three translational degrees of freedom, is designed for the MSA. Combination of permanent magnets with electromagnets helps generate higher forces at larger airgaps, which is critical in limiting both the size of the electromagnets, and the current requirement from the power supplies. To overcome manipulability problems traditionally associated with the design of PEM combinations, a synthesis methodology is developed. In this methodology, design and size optimization of electromagnets is first performed in response to the desired travel range, while incorporating the power constraints imposed by the power supplies. Thereafter, the
design process selects the permanent magnets which in combination with the optimal electromagnets, maximize the output force while ensuring flotor manipulability in the desired travel range.

The design of the complete actuation scheme using PEM combinations is performed in a two step process. The vertical actuation scheme is designed first using PEM combinations consisting of a single electromagnet and a pair of permanent magnets. Three PEM combinations are located only on the bottom side of the flotor, so that the top surface is free to mount payload or instrumentation. The PEM combinations for the horizontal actuation scheme comprise of one electromagnets and two pairs of permanent magnets. Six PEM combinations are utilized, three along each of the two orthogonal coordinate axes in the horizontal plane. The three PEM combinations along a axis are arranged in a push-pull configuration to maximize the force and torque characteristics, while maintaining manipulability in the desired travel range. Maximum electromagnet current and voltage values of 10 amperes and 50 volts respectively, are required to modulate the position of the flotor within its complete operational space. Due to the power constraints incorporated in the design process for the electromagnets, these current and voltage values can be commanded with commercially available power supplies. Consequently, the design methodology presented in this chapter results in a feasible electromagnetic actuation scheme for the large travel MSA.
CHAPTER 5

DESIGN OF A ROBUST NONLINEAR COMPENSATION SCHEME

The chapter presents a robust nonlinear compensation algorithm for realizing large travel in magnetic suspension systems suffering from parameter variations and external disturbance forces. A geometric feedback linearization technique that utilizes the complete nonlinear description of the electromagnetic field distribution is employed to obtain large travel. Robustness to uncertainties in the feedback linearized system is achieved through the development of a discrete time delay control based compensation algorithm. In comparison to previous developments, the new scheme removes the constraints of triangularity conditions in compensation of unmatched uncertainties. The performance of this algorithm is experimentally verified on a single degree of freedom magnetic suspension system for situations requiring stabilization, trajectory tracking and disturbance rejection.

5.1 Introduction

The earliest developments in magnetic suspension systems can be traced to contactless magnetic bearings [51] and levitated trains [55], examples of narrow gap devices in which the control problem was one of regulation to a desired setpoint.
Current technology in MSA is based on the same narrow gap principle, except that the control problem is to provide tracking of the actuator endpoint within the confines of a limited region around a nominal operating point in the magnetic field. End effector tracking is important because the MSA is utilized for applications in mechanical inspection and fabrication. These devices employ linear control strategies, that are based on an approximate linear force model of the actual nonlinear force distribution, at the nominal operating point [45]. The tracking performance of the linear control strategies continuously degrades with increasing deviations from the nominal operating point [36], until the system destabilizes for a set of fixed controller gains. To realize larger travel, gain scheduling approaches were utilized in which the nonlinear model was successively linearized at various operating points, with appropriate controller gains for each of these locations [32]. This corresponded to a linear controller with changing gains, at discrete intervals. Even though gain scheduling controllers ensured larger travel, strict requirements on tracking accuracy resulted in fine discretizations of the operating range with large look tables of controller gains. The size of these multidimensional lookup tables that could be implemented in real time control, ultimately restricted the travel range in MSA [1]. As an alternative to gain scheduling, geometric feedback linearization techniques [60, 56] utilizing the complete nonlinear description of the electromagnetic field distribution were implemented by Trumper [71]. The controller yielded transient responses that were largely independent of the operating point air gap, and significantly larger travel was realized in comparison to linear controllers.

Geometric feedback linearization however, relies on an accurate plant model so that the nonlinearities are canceled and the plant reduced to a linear controllable
Cancellation of the nonlinearities renders the system susceptible to modeling uncertainties, and parameter variations. Since magnetic systems suffer from parameter variations [23] in the form of resistance changes, drift in magnetization curve constants and variations in disturbance forces, it becomes necessary to investigate robust geometric feedback linearization algorithms that compensate for these effects. In this chapter, we focus on the development of one such robust feedback linearization algorithm for magnetic suspension actuators that guarantee performance despite parameter perturbations.

Magnetic suspension systems in general suffer from two principal components of uncertainty: The first of these are the parameter variations, where the electromagnetic coil characteristics change due to resistance heating, and the coefficients of the B-H curve drift with temperature. The second important source of uncertainty is the external interaction forces on the suspended body which are typically unknown. Robustness to parameter variations relaxes the accuracy constraints on the force-current-airgap characterization of the electromagnets, while resistance to external forces determines the dynamic stiffness of the suspension. If the dynamical equations for the voltage-current characteristics of the power supply are not modeled in the suspension equations, then the parameter variations and the external disturbance force terms appear in the same level of differentiation as the system input, which happens to be the coil current in this case. Uncertainties that occur in the same order of differentiation as the control inputs are termed as matched uncertainties. Nonlinear systems with matched uncertainties have been addressed extensively in literature, and several robust feedback linearization techniques such as sliding mode control [75], Lyapunov functions [59, 58], and adaptive estimation [38, 63] have been utilized for
effective compensation. The matching conditions are typically restrictive for arbitrary nonlinear systems, and if the voltage-current dynamics of the power supply are modeled in the suspension equations, then the uncertainty terms appear in levels of differentiation other than that of the voltage input to the system, and are therefore termed as unmatched uncertainties. Modeling of the voltage-current characteristics and consequent compensation of the resulting unmatched uncertainties is essential to an efficient and cost-effective design of the magnetic suspension system. By ignoring the dynamics of the voltage-current characteristics, the controller may demand arbitrarily high current slewing rates which in turn lead to an oversized power supply system. Therefore, compensation of unmatched uncertainties in nonlinear control system design is an important step in context of magnetic suspension systems.

Reports addressing compensation of nonlinear systems with unmatched uncertainties are few in literature, with techniques based on Lyapunov functions [16, 10] and variable structure algorithms [57] being investigated for such systems. Variable structure as well as the Lyapunov function based controllers utilize switching control inputs based on a priori bounds of the uncertainty to steer the system states to the desired trajectories in the presence of parameter variations and external disturbances. To compensate for unmatched uncertainties, both Slotine and Hedrick [57], and Freeman and Kokotovic [16] utilize the techniques of integrator backstepping [39], described in Section (5.3) of this chapter. However, the uncertainties are constrained by the triangularity conditions [39], which limits the applicability to general nonlinear systems. Recent investigations of Chen [7] have reduced the constraints of the triangularity conditions by the so-called extended matching conditions, which are applicable to a larger class of structural systems. However, the uncertainty in
the magnetic suspension equations do not subscribe to either the triangularity or the extended matching conditions.

As an alternative to sliding mode and Lyapunov controllers, this chapter proposes an approach in estimating the uncertainty as a function of the system states and the tracking error through the concept of discrete time delay control [50]. The objective is to determine the uncertainty content at each of the differentiation levels by utilizing the information present in the plant outputs and inputs from the previous sampling instants. If the maximum frequency contents of the uncertainty are significantly slower than the sampling rate, then the total uncertainty in the plant is estimated and fed-back to achieve robust compensation. Such an approach improves upon two of the major limitations encountered in the previous investigations, namely the constraints of triangularity conditions and no parameter variations in the input channel. With these improvements, it becomes possible to utilize the algorithm for precision motion control of magnetic suspension systems, and demonstrate large travel, accurate tracking and enhanced dynamic stiffness in face of significant modeling uncertainties and external disturbing forces.

The chapter is organized as follows: The following section presents the problem formulation and the design objective, while Section (5.3) details the control algorithm. A magnetic suspension system is modeled and analyzed in Section (5.4), while experimental results are presented in Section (5.5). Finally, conclusions are drawn in Section (5.6).
5.2 Problem formulation

Consider a perturbed single input nonlinear systems of the form\textsuperscript{2}:

\[
\dot{x} = \hat{f}(x) + \hat{f}(x, \theta(t)) + [\hat{g}(x) + \hat{g}(x, \theta(t))]u.
\]  

(5.1)

where \( \hat{f}(x), \hat{g}(x) \) are assumed to be \( C^\infty \) vector fields in \( \mathbb{R}^n \). with \( \hat{f}(0) = 0, \hat{g}(x) \neq 0, \forall x \in \mathbb{R}^n \) and \( u \in \mathbb{R} \). \( \theta(t) \) is a vector of unknown time varying parameters or disturbances which takes values in \( \mathbb{R}^p \). The uncertainty terms, \( \hat{f}(\cdot), \hat{g}(\cdot) \) are smooth vector fields in the neighborhood of the origin \( x = 0 \) with \( \hat{f}(x = 0, \cdot) \) not necessarily zero at this point. It is assumed that the nominal model without the uncertainty terms

\[
\dot{x} = \hat{f}(x) + \hat{g}(x)u,
\]  

(5.2)

is locally feedback linearizable around the origin \([60, 56]\) with the transformation, \( z = T(x) : U \rightarrow \mathbb{R}^n \). The system above has relative degree \( n \). Differentiating this transformation componentwise with respect to time.

\[
\begin{align*}
\dot{z}_1 &= \frac{\partial T_1(x)}{\partial x} (\hat{f}(x) + \hat{g}(x)u) = T_2 = z_2 \\
\dot{z}_2 &= \frac{\partial T_2(x)}{\partial x} (\hat{f}(x) + \hat{g}(x)u) = T_3 = z_3 \\
&\vdots \\
\dot{z}_n &= \frac{\partial T_n(x)}{\partial x} (\hat{f}(x) + \hat{g}(x)u) = \frac{1}{\beta(x)}(u - \alpha(x)) = v
\end{align*}
\]  

(5.3)

the nonlinear system transforms to a linear companion form with a new input \( v \). The control input \( v \) is related to the actual control \( u \) through a mapping of nonlinear scalar functions \( \alpha(x), \beta(x) \) as given by Equation (5.3). The transformation \( z = T(x) \)

\textsuperscript{2}In Sections 5.2 and 5.3, symbols with and without indices refer to vectors and scalar components, respectively.
is invertible throughout the domain, except for some finite number of singularity points. With this representation, it is possible to synthesize controllers based on linear design principles to realize performance specifications such as stabilization and trajectory tracking.

However, due to the \textit{a priori} knowledge of only the nominal model, implementation of the transformation \( z = T(x) \) on the uncertain plant Equations (5.1), results in imperfect feedback linearization. Differentiating the transformation \( z = T(x) \) componentwise once again, and using the relations developed in Equation (5.3),

\[
\begin{align*}
\dot{z}_1 &= T_2 + \frac{\partial T_1}{\partial x} (\dot{f}(x, \theta(t)) + \dot{g}(x, \theta(t))u) = z_2 + \eta_1(z, v, \theta(t)) \\
\dot{z}_2 &= T_3 + \frac{\partial T_2}{\partial x} (\dot{f}(x, \theta(t)) + \dot{g}(x, \theta(t))u) = z_3 + \eta_2(z, v, \theta(t)) \\
& \vdots \\
\dot{z}_n &= v + \frac{\partial T_n}{\partial x} (\dot{f}(x, \theta(t)) + \dot{g}(x, \theta(t))u) = v + \eta_n(z, v, \theta(t)) 
\end{align*}
\]  

we observe that the imperfectly feedback linearized system of Equations (5.4) is still nonlinear. The uncertainty terms \( \eta_i(t) \) are nonlinear functions of the states \( z(t) \), the control input \( v(t) \) and the uncertain parameters, \( \theta(t) \). Furthermore, the uncertain terms \( \eta_i(t) \) occur at levels of differentiation, different from that of the control input \( v \). Since the uncertainty does not necessarily lie in the range space of the input channel, it is termed as \textit{unmatched} uncertainty. Since the system is of relative degree \( n \), derivatives of the control input do not appear in the unmatched uncertainty terms. Consequently, it is possible to determine the effective uncertainty at the last level of differentiation. Further, if \( \eta_2 \) depends only on \( (z_1, \theta(t)) \), \( \eta_3 \) depends only on
\((z_1, z_2, \theta(t))\) and so on, then the uncertainty is said to satisfy the **triangularity conditions** [39]. It is clear that the uncertainty in this formulation does not subscribe to these restrictive conditions.

Since the magnetic suspension systems is controlled digitally, the continuous time model of Equations (5.4) needs to be expressed in discrete time. By selecting an appropriate sampling interval that maintains feedback linearizability in the discrete time domain [62], Equations (5.4) can be discretized using a difference scheme of the form

\[
\hat{z}_i(t) |_{t = k\Delta t} = \frac{z_i(k + 1) - z_i(k)}{\Delta t}
\]  

(5.5)

to

\[
z_1(k + 1) = z_2(k)\Delta t + \eta_1(k)\Delta t + z_1(k)
\]

\[
z_2(k + 1) = z_3(k)\Delta t + \eta_2(k)\Delta t + z_2(k)
\]

\[
\vdots
\]

\[
z_n(k + 1) = v(k)\Delta t + \eta_n(k)\Delta t + z_n(k)
\]

(5.6)

with a corresponding nonlinear feedback of the form \(u(k) = \alpha(x(k)) + \beta(x(k))v(k)\).

For an \(n\) dimensional single input system expressed as chain of perturbed integrators, the design objective is formulated as follows.

**Design objective:** The goal is to regulate the state \(z_1\), regarded as the system output using a static state feedback control law \(v(k) = v(x(k))\). For any given \(\delta > 0\), it is desired that the trajectories of Equation (5.6) enter a residual set \(\Omega \subset \mathbb{R}^n\) after a finite time, and then stay there for all time thereafter i.e. \(z \in \Omega \Rightarrow |z_1| \leq \delta\) for all time \(t > t_0\).
Although the design objective is stated in terms of the $\delta$-regulation of the output $z_1$, the same formulation includes the $\delta$-tracking of bounded reference signals having bounded derivatives, as is demonstrated in Marino and Tomei [39]. Additionally, the formulation does not constrain the uncertainties to either subscribe to the triangularity conditions or be independent of the input channels. Our solution procedure to the control problem is detailed in the following section.

5.3 Control algorithm

In this section, we use the earlier developments in back stepping design in combination with discrete time time delay control to construct a feedback law that regulates the system of Equation (5.6) to the nominal model approximately. The control law is then modified to achieve rigorous $\delta$-regulation of the output.

5.3.1 Approximate model regulation

**Step 1:** Selecting the first level of differentiation from Equation (5.6) and substituting, $\ddot{z}_1(k) = z_1(k), \ddot{z}_2(k) = z_2(k) - q_1(k)$

$$
\ddot{z}_1(k+1) = \ddot{z}_2(k)\Delta t + \ddot{z}_1(k) + \eta_1(k)\Delta t + q_1(k)\Delta t \tag{5.7}
$$

where $q_1(k)$ is synthetic input to compensate for the uncertainty. The uncertainty $\eta_1(k)$ is unknown at time instant $k$, and the synthetic input uses the value of the uncertainty at the previous sampling instant

$$
q_1(k) = -(\ddot{z}_1(k) - \ddot{z}_2(k-1)\Delta t - \ddot{z}_1(k-1) - q_1(k-1)\Delta t)/(\Delta t) \tag{5.8}
$$
so that \( q_1(k) = -\eta_1(k - 1) \), to regulate the first level of differentiation. The first level of differentiation with the above equation is expressed as

\[
\ddot{z}_1(k + 1) = \ddot{z}_2(k)\Delta t + \dot{z}_1(k) + (\eta_1(k) - \eta_1(k - 1))\Delta t. \tag{5.9}
\]

If the dynamics of the uncertainty are slow in comparison to the the sampling rate, then \( \eta_1(k - 1) \approx \eta_1(k) \) and Equation (5.9) may be written out as

\[
\ddot{z}_1(k + 1) = \ddot{z}_2(k)\Delta t + \dot{z}_1(k) + O(\Delta t). \tag{5.10}
\]

Thus \( O(\Delta t) \) regulation may be achieved for the first level of differentiation, provided the sampling rate is significantly higher than the maximum frequency components of the uncertainty.

**Step 2:** Selecting the second level of differentiation from Equation (5.6), and substituting \( \dot{z}_2(k) = z_3(k) - q_2(k) \) along with the definition for \( \ddot{z}_2(k) \), we obtain

\[
\ddot{z}_2(k + 1) = \ddot{z}_3(k)\Delta t + \dot{z}_2(k) + \eta_2(k)\Delta t + q_1(k) - q_1(k + 1) + q_2(k)\Delta t. \tag{5.11}
\]

Due to compensation of unmatched uncertainty in the first level of differentiation, the uncertainty terms in the second level consist of two components, \( \eta_2(k) \) and the derivative of \( \eta_1(k) \). Since it was assumed that the uncertainty terms \( \ddot{f}(\cdot), \ddot{g}(\cdot) \) are smooth vector fields in Section (5.2), the control effort \( q_1(k + 1) \) can be approximated by Taylor’s series expansion as follows:

\[
q_1(k + 1) = 2q_1(k) - q_1(k - 1) + O(\Delta t)^2. \tag{5.12}
\]

The second level of differentiation can then be expressed as

\[
\ddot{z}_2(k + 1) = \ddot{z}_3(k)\Delta t + \dot{z}_2(k) - (q_1(k) - q_1(k - 1)) + \eta_2(k)\Delta t + q_2(k)\Delta t + O(\Delta t)^2. \tag{5.13}
\]
Similar to Step (1), the synthetic input $q_2(k)$ is derived as

$$q_2(k) = -\eta_2(k - 1) + \frac{q_1(k) - q_1(k - 1)}{\Delta t}. \quad (5.14)$$

Substituting the above expression in the previous equation, one obtains

$$\ddot{z}_2(k + 1) = \ddot{z}_3(k)\Delta t + \ddot{z}_2(k) + (\eta_2(k) - \eta_2(k - 1))\Delta t + O(\Delta t)^2. \quad (5.15)$$

This equation can also be expressed as

$$\ddot{z}_2(k + 1) = \ddot{z}_3(k)\Delta t + \ddot{z}_2(k) + O(\Delta t) \quad (5.16)$$

if $\eta_2(k - 1) \approx \eta_2(k)$ at elevated sampling rates.

**Step 3:** Implementing the above procedure for all the intermediate levels of differentiation in Equation (5.6), and substituting $\bar{v}(k) = v(k) - q_n(k)$ in the last level, one derives

$$\ddot{z}_n(k + 1) = \bar{v}(k)\Delta t + \ddot{z}_n(k) + (\eta_n(k) - \eta_n(k - 1))\Delta t + O(\Delta t)^2. \quad (5.17)$$

using $q_n(k) = -\eta_n(k - 1) + (q_{n-1}(k) - q_{n-1}(k - 1))/\Delta t$. The entire system is therefore regulated to

$$\ddot{z}_1(k + 1) = \ddot{z}_2(k)\Delta t + \ddot{z}_1(k) + O(\Delta t)$$
$$\ddot{z}_2(k + 1) = \ddot{z}_3(k)\Delta t + \ddot{z}_2(k) + O(\Delta t)$$
$$\vdots$$
$$\ddot{z}_n(k + 1) = \bar{v}(k)\Delta t + \ddot{z}_n(k) + O(\Delta t) \quad (5.18)$$

under the control

$$v(k) = \bar{v}(k) + q_n(k)$$
\[ q_n(k) = -\eta_n(k-1) + \frac{q_{n-1}(k) - q_{n-1}(k-1)}{\Delta t} \]

\[ q_1(k) = -\eta_1(k-1) \]

\[ \bar{v}(k) = \mu(z_1(k), \ldots, z_n(k), q_1(k), \ldots, q_n(k)) \quad (5.19) \]

where \( \bar{v}(k) \) is pole placement feedback controller that realizes the desired transient response. From this control algorithm, the following remarks may be made:

1. The perturbed plant of Equation (5.6) is regulated to the approximate nominal model of Equation (5.18), by estimating the uncertainty components from the previous sampling instants. Therefore, it is desirable that \( \Delta t \) be sufficiently small, and ideally \( \Delta t \to 0 \). However, the selection of an arbitrarily small sampling interval is constrained by the noise sensitivity considerations and the computational resources [50].

2. Due to the presence of \( O(\Delta t) \) in Equation (5.18), \( \delta \)-regulation of the output cannot be guaranteed. Therefore, the control law is modified to achieve the desired design objective as detailed in the following section.

5.3.2 \( \delta \)-regulation of the output

Representing the desired dynamics as

\[ w_1(k + 1) = w_2(k)\Delta t + w_1(k) \]

\[ w_2(k + 1) = w_3(k)\Delta t + w_2(k) \]

\[ \vdots \]

\[ w_n(k + 1) = \bar{v}(k)\Delta t + w_n(k) \quad (5.20) \]
and defining $e_n(k) = w_n(k) - \bar{z}_n(k)$, an additional compensation component is added to the control effort at each level of differentiation to provide $\delta$-regulation of the output. The various steps in the algorithm are outlined below:

**Step 1:** Selecting the first level of differentiation as per the first of step of the previous section, and using $q_1(k) = -\eta_1(k-1)\Delta t + e_1(k)$, we obtain

\[
\bar{z}_1(k+1) = \bar{z}_2(k)\Delta t + \bar{z}_1(k) + (\eta_1(k) - \eta_1(k-1))\Delta t + e_1(k).
\]  

(5.21)

Subtracting the above equation from the first level of differentiation in Equation (5.20), one derives

\[
e_1(k+1) = e_2(k)\Delta t + O(\Delta t).
\]

(5.22)

**Step 2:** Similarly, following the second step of the previous section, and defining $q_2(k) = -\eta_2(k-1) + (q_1(k) - q_1(k-1))\Delta t + e_2(k)$, the second level of differentiation is expressed as

\[
\bar{z}_2(k+1) = \bar{z}_3(k)\Delta t + \bar{z}_2(k) + (\eta_2(k) - \eta_2(k-1))\Delta t + e_2(k) + O(\Delta t)^2
\]

(5.23)

which is subtracted from the second equation of Equation Set (5.20) to give

\[
e_2(k+1) = e_3(k)\Delta t + O(\Delta t).
\]

(5.24)

**Step 3:** Repeating the above procedure for all of the intermediate levels of differentiation in Equation (5.6), and substituting $\bar{v}(k) = v(k) - q_n(k)$ in the last level, one derives

\[
\bar{z}_n(k+1) = \bar{v}(k)\Delta t + \bar{z}_n(k) + (\eta_n(k) - \eta_n(k-1))\Delta t + e_n(k) + O(\Delta t)^2.
\]

(5.25)

using $q_n(k) = -\eta_n(k-1) + (q_{n-1}(k) - q_{n-1}(k-1))\Delta t + e_n(k)$. Subtracting the above equation from the last one in Equation Set (5.20), we obtain

\[
e_n(k+1) = O(\Delta t)
\]

(5.26)
since the nominal control effort $\bar{u}(k)$ is the same for both the desired and actual dynamics.

From Equations (5.22), (5.24) and (5.26), the complete error dynamics for the system are written out as

$$e_1(k+1) = e_2(k)\Delta t + O(\Delta t)$$  \hspace{1cm} (5.27)
$$e_2(k+1) = e_3(k)\Delta t + O(\Delta t)$$  \hspace{1cm} (5.28)
$$\vdots$$
$$e_{n-1}(k+1) = e_n(k)\Delta t + O(\Delta t)$$  \hspace{1cm} (5.29)
$$e_n(k+1) = O(\Delta t).$$  \hspace{1cm} (5.30)

The above equation implies that $e_n(k + 1)$ is regulated to $O(\Delta t)$. Since $e_n(k)\Delta t$ is $O(\Delta t)^2$, it can be concluded from Equation (5.29) that $e_{n-1}(k + 1)$ is also regulated to $O(\Delta t)$. Using these relationships recursively, it is inferred from Equation (5.28) that $e_2(k + 1)$ is regulated to $O(\Delta t)$. This fact in conjunction with Equation (5.27) implies that $e_1(k + 1)$ is also of $O(\Delta t)$.

Since $e_1(k) = w_1(k) - \bar{z}_1(k)$, it is concluded that $z_1(k)$ converges to $w_1(k)$ with an accuracy of $O(\Delta t) = \delta$, since $\bar{z}_1(k) = z_1(k)$. Therefore, by sampling sufficiently fast enough, $\delta$-regulation or tracking can be achieved.

### 5.4 Magnetic suspension system: Modeling and system analysis

The main focus of this implementation is to demonstrate nonlinear control of a magnetic suspension system with an aim to realizing large travel. Since geometric feedback linearization techniques are susceptible to uncertainties, a robust nonlinear compensation algorithm is employed that compensates for both the parameter
variations in the plant model, and the external disturbance forces that act on the system. We demonstrate that robustness to parameter variations relaxes the accuracy constraints on the force-current-airgap characterization of the electromagnet, while robustness to external disturbance forces enhances the stiffness of the system. In the following paragraphs, the various components of a magnetic suspension system are modeled and analyzed, to help understand and interpret the experimental results.

### 5.4.1 Electromagnet subsystem

For the magnetic suspension system shown in Figure (5.1), the equations for force and coil voltage obtained by the lumped parameter model of the system [67] are generally expressed as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{b}{m} \left( \frac{i}{a + x_1} \right)^2 + g - \frac{f_d}{m} \\
\dot{i} &= \frac{ix_2}{a + x_1} - \frac{R(a + x_1)i}{2b} + \frac{(a + x_1)u}{2b},
\end{align*}
\]  

(5.31)
where \( u \) is the input coil voltage, \( f_d \) is the external force, \( i \) is the coil current, \( g \) is the gravitational constant, \( m \) is the mass of the ball, \( a, b \) are magnetic constants for the setup and \( R \) is the coil resistance. \( x_1, x_2 \) represent the ball position and velocity respectively. The system satisfies the conditions for state feedback linearization if \( a \ll x_1 \), and the following coordinate transformation

\[
\begin{align*}
  z_1 &= x_1 \\
  z_2 &= x_2 \\
  z_3 &= -\frac{b}{m} \left( \frac{i}{a + x_1} \right)^2 + g \\
  v &= \frac{i}{m(a + x_1)}(Ri - u),
\end{align*}
\]

linearizes the system to a companion form of Equation (5.3). In the ideal interaction of an electromagnet with a point mass, \( a = 0 \), and the electromagnetic force is infinite at \( x = 0 \). However in experimental situations, the electromagnetic force approaches a constant value as the ball touches the electromagnet poleface and is accounted by \( a \). \( a \) is found to be much less than the limiting value of \( x_1 \) which is the radius of the suspended ball. To characterize the force relationship in the physical setup, experiments are performed. In these experiments, a ball of known weight is placed under the electromagnet that is energized by current in steady state conditions. The current in the electromagnet is gradually increased till the ball lifts off. The lift off current for various ball positions constitutes an approximate current-airgap curve for the electromagnet, for a fixed value of the magnetic force. This procedure is then repeated for various balls of varying weights to obtain a complete force-current-airgap characterization of the electromagnet that is expressed as

\[
F = \frac{.1617i^2}{(3155.3x_1^2 - 31.64x_1 + .945)^2}.
\]
where \( x_1 \) is position in meters and \( i \) is current in amperes. The current versus airgap relationship is quadratic in the experimental model of Equation (5.33), while it is linear in the lumped parameter model of Equation (5.31), for constant values of force \( F \). A linear approximation to the quadratic relationship of the experimental model can be extracted in which \( a, b \) equal \( 0.0244 \) and \( 1.66 \times 10^{-4} \), respectively for \( x_1 \) in the range of \( 0.15 - 0.33 \) inches. Instead of approximating this relationship, the robust control algorithm is designed using the relatively more accurate experimental force-current-airgap characterization (Equation (5.33)), by a derivation process that is outlined for the lumped parameter model.

The nature of the experimental model is consistent with those reported in literature [37] but suffers from calibration errors. To determine the actual model, the system is identified from the input-output data obtained under closed loop control. For a sine wave trajectory of frequency 1 Hertz and an amplitude of 0.10 inches, the identified force-current-airgap relationship for the electromagnet is expressed as

\[
F = \frac{0.1617i^2}{(1872.13x_1^2 + 48.74x_1 + 4844)^2}.
\]  
(5.34)

A comparison between the experimental and the identified models in Figure (5.2) depicts significant discrepancy between the two curves. The approximate model was obtained under steady state conditions, in contrast to the input-output model that is identified under dynamic loading. Suspension parameters such as inductance that are typically assumed to be constant in steady state analysis, are actually functions of the ball position under dynamic conditions. These assumptions introduce discrepancies between the two characterizations of the electromagnet as is observable in Figure (5.2). A complete nonlinear, dynamic characterization of the electromagnet is a challenging and arduous process [20], and we demonstrate that a robust nonlinear
controller based on the experimental model based on static experimental calibration, provides satisfactory performance operating under significant model uncertainties.

5.4.2 Position sensor

The position sensor is an infra red emitter detector pair, that is integrated with the magnetic suspension system as shown in the schematic of Figure (5.1). It works by illuminating the suspended ball with an infra red beam of light that is reflected back to the detector. The voltage output of the detector is proportional to the intensity of the reflected light which is a measure of the ball position. The nonlinear relationship between the sensor output (voltage) to the ball position is depicted in Figure (5.3). The sensor has a operational range of .05 — .50 inches. The deviations between the sensor calibration and the actual displacement are of the order of .001 inches. This constraint sets the limit to the positioning accuracy that may be obtained in closed loop control. However, the sensor resolution is approximately .00025 — .00033 inches, which indicates the precision attainable through the digital controller. To eliminate noise, the sensor data is filtered with a first order Butterworth filter with a cut-off frequency of 5 Hz. This limits the resulting bandwidth of the closed loop system to 4.3 Hz.

5.4.3 Real time control system

The control algorithm of Section (5.3) is implemented on 486 based PC compatible micro-computer with a clock of 33 MHz. The A/D, D/A cards have 12-bit resolution, and the control algorithm operates at a sampling rate of 1000 Hz. The mass of the ball is about 16.5 grams, and it requires a current of approximately 1.0 ampere to stabilize this ball, at a distance of approximately .28 inches from the electromagnet.
Figure 5.2: A comparison between the experimental and the identified model for a ball of weight 16.5 grams. A sine wave of frequency 1 Hz and amplitude .10 inches is used as the system input in obtaining the identified model.

Figure 5.3: Experimental calibration of the position sensor.
Table 5.1: A listing of the physical characteristics of the magnetic suspension system

<table>
<thead>
<tr>
<th>Physical characteristic</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum travel</td>
<td>.33 inches</td>
</tr>
<tr>
<td>Electromagnet resistance</td>
<td>65 ohms</td>
</tr>
<tr>
<td>Accuracy of optical sensor</td>
<td>.001 inches</td>
</tr>
<tr>
<td>Resolution of optical sensor</td>
<td>.00025 - .00033 inches</td>
</tr>
<tr>
<td>Cut-off frequency of sensor filter</td>
<td>5 Hertz</td>
</tr>
<tr>
<td>Weight of the suspended ball</td>
<td>16.5 grams</td>
</tr>
<tr>
<td>Radius of the suspended ball</td>
<td>.3125 inches</td>
</tr>
<tr>
<td>Sampling rate of control algorithm</td>
<td>1000 Hertz</td>
</tr>
</tbody>
</table>

5.5 Experimental results in robust nonlinear compensation

The magnetic suspension system is designed to investigate the characteristics of the control algorithm, and the results in stabilization, large travel, trajectory tracking, and stiffness need to be interpreted in context of its physical characteristics. For example, the maximum possible travel available in this setup is .33 inches (8.382 mm), so the performance of the nonlinear controller is investigated in this range. Secondly, the force-current-airgap model is approximate in nature as described in Section (5.4), and therefore all results are obtained under significant parameter variations. Finally, the optical sensor provides an accuracy of about .001 inches (25 micron) at a cut-off frequency of 5 Hz, which constrains the maximum accuracies and the largest bandwidths realizable from the system. The simulation and experimental results are discussed together under the following three headings.
5.5.1 Travel and stabilization

Figures (5.4)-(5.7) compare and contrast the performance of three different types of controllers for ball stabilization at three different heights from the electromagnet pole. The stabilization performance of the linear PD controller is depicted in Figure (5.4) for step inputs corresponding to .28, .25 and .22 inches. The gains of the linear controller are designed using a nominal model obtained by linearization of the magnetic suspension equations at the operating point of .28 inches and .9 amperes. It is observed that for step inputs of .28 and .25 inches, the linear controller provides satisfactory stabilization performance, whereas the response degrades for a step input of .22 inches. Due to the nonlinear nature of the system, the uncertainty on the nominal linear model increases with larger ball displacements from the nominal operating point of .28 inches. At a displacement of .22 inches, the linear controller is unable to compensate for the uncertainty and marginal stabilization is obtained. Thus a fixed gains linear controller cannot be utilized for realizing large travel in a magnetic suspension system.

Figure (5.5) depicts the performance of the feedback linearization controller in combination with a linear PD controller based on the approximate, experimental model of Section (5.4), for the same set of input trajectories. The nonlinear controller stabilizes the plant for commanded step inputs of .28, .25 and .22 inches. It is observed that with decreasing distance to the electromagnet pole, the stabilization accuracy degrades. This is due to the increasing deviation between the approximate experimental and actual plant models, as shown in Figure (5.2). Therefore, even though the feedback linearization controller based on the approximate model
Figure 5.4: Stabilization performance of the linear PD controller.

Figure 5.5: Stabilization performance of the feedback linearization controller.
Figure 5.6: Stabilization performance of the robust feedback linearization controller.

Figure 5.7: Stabilization performance of the robust feedback linearization controller for larger travel.
provides stabilization over a larger range in comparison to the linear controller, the stabilization accuracy is unsatisfactory for precision motion control.

Figure (5.6) demonstrates the performance of the robust feedback linearization controller in combination with a linear PD controller for step inputs of .28, .25 and .22 inches. The ball stabilizes to an accuracy of ±0.0005 inches for all the three inputs, which corresponds to the accuracy of the position sensor. For the same set of gains, robust stabilization performance to step inputs of .14, .16 and .18 inches is shown in Figure (5.7). These results verify the robustness of the controller to parameter variations, and satisfactory stabilization accuracy for large travel.

The current curves for the stabilization performance of the robust feedback linearization controller are depicted in Figure (5.8) for step inputs of .28 and .25 inches and in Figure (5.9) for the step input of .22 inches. After the initial transient, the control current settles down to a constant steady state value which decreases as the ball stabilizes closer to the electromagnet. However, it is observed from the graphs that this decrease is not proportional to the difference in the stabilization heights, due to the nonlinear nature of both the controller and the plant. The current in each of these graphs remains under 2 amperes, which is the maximum output available from the power supply.

### 5.5.2 Trajectory tracking performance

The trajectory tracking performance of both the feedback linearization, and the robust feedback linearization controllers is depicted in Figures (5.10)-(5.12). Figure (5.10) shows the trajectory tracking results of a feedback linearization controller when the input is a 1 Hz sine wave of amplitude .04 inches. It is apparent that the feedback
Figure 5.8: Current curves for the stabilization performance of the robust feedback linearization controller corresponding to step inputs of .28 and .25 inches.

Figure 5.9: Current curve for the stabilization performance of the robust feedback linearization controller corresponding to step input of .22 inches.
Figure 5.10: Tracking performance of the feedback linearization controller.

Figure 5.11: Tracking performance of the robust feedback linearization controller.
Figure 5.12: Tracking errors with the robust feedback linearization controller.

Figure 5.13: Tracking performance of the robust feedback linearization controller for larger travel.
linearization controller is unable to compensate for the differences in the nominal and the actual nonlinear models. Figure (5.11) depicts the performance of the robust feedback linearization controller to the same input. The simulation results are output trajectories of the nominal model in response to the the selected input. Therefore, the objective of the robust feedback linearization controller is to regulate the plant output to the simulation results. The corresponding trajectory tracking error curves are presented in Figure (5.12). It is observed that the tracking errors are within ±0.0015 inches of the trajectory, which is of the same order of magnitude as the accuracy of the position sensor. In fact, the error curve has two distinct components: 1 Hertz low frequency component, and the superimposed high frequency oscillations. The 1 Hz low frequency error component is systematic and occurs due to the finite roll-off characteristics of the noise filter. This error component is about 2% of the trajectory amplitude, which corresponds to the magnitude error at 1 Hertz, of the frequency response curve of the nominally feedback linearized closed loop system. Therefore, this component can be compensated with the design of an appropriate feedforward controller, which will also make the tracking precision closer to the actual resolution of the sensor.

Figure (5.13) shows the performance of the controller in tracking a 1 Hz sine wave of amplitude .08 inches. The gains of the controller are identical to those utilized in Figures (5.11) and (5.12). The initial transient response degrades due to the slow dynamics of the 5 Hz Butterworth filter incorporated in the feedback loop to eliminate sensor noise. Figures (5.14)-(5.15) depict the current curves corresponding to Figures (5.10), (5.11) and (5.13). The current curve of Figure (5.15) depicts the modulation of the control effort by the robust feedback linearization controller in achieving perfect
Figure 5.14: Current curve for the tracking performance of the feedback linearization controller.

Figure 5.15: Current curve for the tracking performance of the robust feedback linearization controller.
tracking as opposed to the feedback linearization controller (Figure (5.14)). Figure (5.16) shows a larger range in the variation of the current curve (.74-1.18 amperes) in response to a .08 inch amplitude sine wave in comparison to the corresponding variations (.78-.94 amperes) for a sine wave of amplitude .04 inches, depicted in Figure (5.15). In either case, Figures (5.11)-(5.13) verify the consistent performance of the algorithm in situations of large travel that demands nonlinear compensation with model regulation.

5.5.3 Dynamic stiffness

The evaluation of the dynamic stiffness of the magnetic suspension system requires the application of an external disturbance force on the ball and the observation of the corresponding deviations from its nominal trajectory. This necessitates the development of a special fixture that can impress a known external force on the ball as
it transverses its trajectory. Due to the unavailability of such a device, it is decided to change the value of the gravitational constant $g$, in the nominal model. Since $g = 9.81 \text{ m/sec/sec}$ in the physical plant, the objective of the controller is to regulate the uncertain plant to the nominal model. Thus changing the value of $g$ in the nominal model is equivalent to exerting an external force on the physical system. Figures (5.17) and (5.18) depict the performance of the robust feedback linearization control when the nominal value of $g = 8.0 \text{ m/sec/sec}$ which is equivalent to 3.0 grams of force on the ball in the downward direction. The tracking errors are confined to within .003 inches of the nominal trajectory, and to within .0005 inches of the tracking errors in Figure (5.12). This indicates an effective stiffness of 236.22 grams/mm for the suspension. It is concluded from the tracking errors that the controller performs adequately during model regulation and rejects external disturbing forces.

5.6 Conclusions

The chapter addresses precision motion control of a magnetic suspension system for realizing large travel. We argue that the requirement of large travel in magnetic suspension systems necessitates the development of nonlinear control algorithms based on geometric feedback linearization techniques that utilize the complete description of the electromagnetic field distribution. Geometric feedback linearization techniques are sensitive to model uncertainties, and have therefore not been widely employed for control of these systems. Magnetic suspension systems in particular suffer from two principal components of uncertainty: Parameters variations in the components of the electromagnetic circuit, and the external interaction forces on the suspended body. Robustness to parameter variations relaxes the accuracy constraints
Figure 5.17: Compensation of external disturbance force by the robust feedback linearization algorithm.

Figure 5.18: The tracking error curve during compensation of external disturbance forces by the robust feedback linearization algorithm.
on the force-current-airgap characterization of the electromagnet, while resistance to external forces enhances the dynamic stiffness of the suspension. Effective compensation of these uncertainties through nonlinear control system design is vital in development of large travel suspension systems.

An algorithm for compensation of unmatched uncertainties in feedback linearized systems based on discrete time delay control is proposed. If the maximum frequency contents of the uncertainty are significantly slower than the sampling rate, then the algorithm achieves robust compensation. The performance of the robust nonlinear control algorithm is experimentally investigated with respect to stabilization, trajectory tracking and disturbance compensation. In each of these experiments, the controller is designed using the approximate nonlinear model of the system which is significantly different from the actual plant model. For a fixed set of gains, the robust nonlinear controller stabilizes the systems for a large range of ball positions with an accuracy of ±.0005 inches. The realizable accuracy in this implementation is limited by the sensor characteristics. Higher stabilization accuracies are possible with laser interferometric sensors. For the same set of inputs, the linear controller destabilizes while the nominal feedback linearization controller suffers from poor stabilization accuracy. In trajectory tracking performance evaluation, the robust nonlinear controller provides tracking accuracies of ±.0015 inches which are of the same order of magnitude as the accuracy of the position sensor. Additionally, when the suspended ball is impressed with an external disturbance force, the controller provides adequate model regulation and rejection of disturbance forces with a tracking accuracy of .003 inches. Thus the experimental results demonstrate the consistent performance of
the algorithm in realizing large travel inspite of parameter variations and external
disturbances.
CHAPTER 6

HYSTERESIS COMPENSATION IN ELECTROMAGNETIC ACTUATORS

The key features of the Magnetic Suspension Actuator, such as its large travel, high tracking accuracy, and large bandwidth are critically dependent on its control algorithm. Robust nonlinear controllers for magnetic suspension systems were developed in the previous chapter, and their performance in realizing large travel in face of uncertainty/disturbance was experimentally verified. However, realization of accurate trajectory tracking capabilities at high frequencies in a large travel range is hindered by the presence of ferromagnetic hysteresis in electromagnetic actuators. Modeling and compensation of hysteresis is a key issue in the development of the complete control algorithm for the MSA that achieves the performance objectives. To this end, an approach using the Preisach independent domain model is used to capture the essential characteristics of the hysteresis nonlinearity. This model is then inverted and incorporated in the control strategy that regulates the electromagnetic actuator and compensates for the hysteretic effects. Development and experimental verification of this approach is detailed in this chapter.
6.1 Introduction

Ferromagnetic hysteresis is a complex nonlinearity with memory that may result in multiple outputs for a given input, depending on its time history [41]. It manifests strongly in a large travel MSA where electromagnets are driven with current signals that undergo full scale excursions, driving the core well into ferromagnetic saturation. Unmodeled hysteresis leads to inaccuracy in trajectory tracking at high frequencies with large amplitudes, and the distortion in performance is significant in context of nanometric tracking accuracy applications, as investigated in Section (6.2) of this chapter [67, 2, 61]. To overcome this limitation, it is important to develop hysteresis models that not only capture all of the essential characteristics relevant to electromagnetic actuators, but are also suitable for control system design and real-time implementation. Development of these models, their experimental verification, model inversion, and consequent hysteresis compensation are addressed in this chapter.

Hysteresis in soft ferromagnetic materials has been studied extensively, and two distinct types of models have been proposed to capture the observed hysteretic characteristics. The first group of models is derived from the underlying physics of hysteresis and combined with empirical factors to describe the observed characteristics [30, 31]. However, these models have limited applicability, as the physical basis of some of the hysteresis characteristics is not completely understood [8]. Further considerable effort is required in identifying and tuning the model parameters to accurately describe the hysteresis nonlinearity. The second group of models are phenomenological in nature and are essentially mathematical constructs to describe the observed phenomenon [66, 25]. Among these, the Preisach independent domain model has found
widespread acceptance for modeling hysteresis in soft ferromagnetic materials [41]. Among its appealing features are its ability to model complex hysteresis loops, a well defined identification algorithm, and a convenient numerical simulation form. Recent extensions to the Preisach model that include dependence on output rate of change [11, 40], description of accommodation phenomena [41], and formulations for vector hysteresis [76] have increased its range of applicability to ferromagnetic systems. Consequently, Preisach model based compensation of hysteresis nonlinearity is pursued in this research.

Model based compensation of hysteresis in electromagnetic actuators has been addressed in only a few reports. Usually a simplified model of hysteresis employing a linear parameterization of the major and minor loops is developed [22, 61]. At the expense of modeling accuracy, linear parameterization aids development of robust compensation strategies such as adaptive control and sliding mode estimation. However, effectiveness of sliding mode and adaptive control strategies in compensating complex electromagnetic hysteresis loops is still an issue, since none of these reports address experimental implementation. As an alternative, compensation of hysteresis effects in piezo-electric and smart material actuation systems using Preisach model based control architectures is being attempted. A simplified learning scheme driven by the differences between the output of the Preisach model and the hysteretic plant in conjunction with a PID controller achieves the desired compensation [18]. However, the learning is restricted to specific trajectories, and needs to be re-tuned for different inputs. A passivity based controller using only the direct Preisach model has also been suggested, but the formulation is specific to velocity control [19]. To overcome these limitations, a control scheme based on Preisach model inversion for hysteresis
compensation in electromagnetic actuators is presented in this chapter. Since the Preisach model captures all the relevant hysteresis characteristics, this compensation method is not restricted by the number of inputs, or the nature of their trajectories. Further this compensation algorithm can be incorporated in a robust control architecture comprising of uncertainty/disturbance observers in the feedback loop (Section (6.2)), to preserve performance in face of model parameter variations and external disturbances. Consequently, high accuracy regulation of the electromagnetic actuator to hysteresis free operation is addressed by the compensation strategy.

With the above objective, Section (6.2) first investigates the effects of ferromagnetic hysteresis on MSA performance, following which Section (6.3) develops the Preisach model for variations in all of the three relevant input variables for the electromagnetic actuator: Current, airgap and orientation. Issues in model identification and numerical simulation are discussed next, following which the effectiveness of the Preisach model in predicting hysteresis is investigated experimentally. The inverse Preisach models are formulated and incorporated in an open-loop control scheme for hysteresis compensation in Section (6.5). The performance of this scheme is experimentally verified for cases of variation in only the input current, variation in both current and airgap, and for the general case of variation in current, airgap and orientation. In the final sections of the chapter, dynamic performance is also investigated and conclusions are drawn.

6.2 Effect of ferromagnetic hysteresis on MSA performance

The effect of ferromagnetic hysteresis on tracking performance is investigated on a single degree of freedom Magnetic Suspension Actuator, similar to the setup used
in the previous chapter. As depicted in Figure (6.1), the system consists of an E-core electromagnet interacting with a ferromagnetic target. The electromagnetic interaction force between the electromagnet and the target is modulated by controlling the current $i(t)$, in response to deviations in the airgap $h(t)$ between the electromagnet and the target. Geometrical and physical specifications for the E-core electromagnet and the target are detailed in Section (6.4). The objective of the control algorithm is to regulate the position of the target $h(t)$ with high accuracy during fast trajectory tracking applications, in the presence of ferromagnetic hysteresis.

A schematic of the control architecture used to regulate the position of the target is depicted in Figure (6.2). It generalizes the robust feedback linearization algorithm developed in the previous chapter, for achieving nanometric tracking accuracies, at high bandwidths, in presence of higher frequency uncertainty/disturbances [6]. Since geometric feedback linearization controller is synthesized using a first order approximation to the Zero Order Hold (ZOH) in the control loop in Chapter (5), small deviations are observed in the system response when implemented on a continuous time plant. These small deviations are significant in applications demanding high precision tracking performance. Consequently, an additional feedback linearization compensator is incorporated that calculates a compensating component based on the time average of the error due to the approximate representation of the ZOH. The second component in the controller architecture is the uncertainty/disturbance observer [74, 6], that provides robustness to modeling uncertainty and unmeasured external disturbances. Finally, feedforward controllers are designed [43] to improve the bandwidth of the regulated linear system comprising of the nonlinear plant, nonlinear feedback linearization and uncertainty/disturbance observers. As discussed in
Figure 6.1: A single degree of freedom Magnetic Suspension Actuator setup.

Figure 6.2: Schematic of the control architecture used for high performance control.
the following paragraph, this controller architecture provides high accuracy trajectory tracking capabilities during large amplitude excursions of the target.

To evaluate the performance of the above control architecture, a force model of the E-core electromagnet interacting with the target is developed through experimental calibration. The force-current-airgap characterization of the model is depicted in Figure (6.3). It is observed that the model is nonlinear with respect to both electromagnetic current $i(t)$ and the airgap $h(t)$. For the purpose of tracking a sine wave of frequency 20 Hz and amplitude 0.5 mm, the nonlinear model is inverted in the feedback linearization module, and a third order preview filter [43] is employed as a feedforward controller to extend the bandwidth and the precision of the nominal closed loop system. A tracking error of approximately 6.8 nm is observed in Figure (6.4). The corresponding current profile is depicted in Figure (6.5), and it ranges between 3.22-5.95 amperes. Consequently, it can be concluded that the controller architecture is able to satisfactorily regulate the system performance with nanometric accuracy, for large variations in airgap and input current.

The next step is to investigate the performance of the above control architecture on electromagnetic actuators with hysteresis. For this purpose, Preisach independent domain model of hysteresis in developed. Model development and identification are detailed in Section (6.3) of this chapter. Experimental results reported in Section (6.4), convincingly demonstrate its ability to model hysteresis in electromagnetic actuators made of soft ferromagnetic material. Force-current-airgap characteristics of this model recorded experimentally are graphed in Figure (6.6). It is observed that each of the force-current curves at a constant airgap is a loop. When current is monotonically increased, force increases along the lower curve. On the other hand.
Figure 6.3: Experimental force-current-airgap characteristics of the E-core electromagnet.

Figure 6.4: Performance of the control architecture in trajectory tracking a 20 Hz sine wave with the non-hysteretic plant.
Figure 6.5: Current commanded to the electromagnet in trajectory tracking a 20 Hz sine wave with the non-hysteretic plant.

Figure 6.6: Experimental force-current-airgap characterization of the E-core electromagnet for both increasing and decreasing current variations.
when current is decreased from its maximum value, force decreases along the upper curve. It is also observed that the force-current-airgap characterization of Figure (6.3), were actually the lower segments of the hysteresis loops displayed in Figure (6.6). Performance of the control architecture that inverts the nonlinear model of Figure (6.3) in the feedback linearization module, on a Magnetic Suspension Actuator plant with hysteresis is presented next.

The degradation in trajectory tracking performance when the control architecture is implemented on a Magnetic Suspension Actuator with hysteresis, is shown in Figure (6.7). It is observed that the maximum error is 415 nanometers, 61 times larger than the 6.8 nanometers that is observed in the case of the non-hysteretic plant. Clearly, this degradation in performance is unacceptable for applications requiring high precision trajectory tracking. The trajectory tracking error is due to the discrepancy between the current profiles commanded by the controller depicted in Figure (6.5) for the non-hysteretic plant, and in Figure (6.8) for the hysteretic plant. It is observed that the disturbance observer is unable to compensate for hysteresis and restore the current profile, consequently leading to trajectory tracking error. To restore tracking performance, Preisach model of hysteresis is inverted in the feedback linearization module of the control architecture. Preisach model inversion is detailed in Section (6.5) of this chapter. The corresponding tracking performance is depicted in Figure (6.9). It is observed that the tracking performance is very similar to that of Figure (6.4), which was obtained with the non-hysteretic plant. Closer inspection reveals that the maximum difference between the two trajectories lies within ±0.05 nanometers.
Figure 6.7: Controller performance in trajectory tracking a 20 Hz sine wave with the hysteretic plant.

Figure 6.8: Current commanded to the electromagnet in trajectory tracking a 20 Hz sine wave with a hysteretic plant.
Figure 6.9: Performance of the controller with the inverse Preisach model in trajectory tracking a 20 Hz sine wave on the hysteretic plant.

It is concluded from the above set of simulations, that ferromagnetic hysteresis results in severe performance degradation in context of high accuracy tracking applications for a large travel MSA. Robust control strategies employing non-hysteretic models of ferromagnetic interaction are unable to provide adequate compensation. To achieve the performance accuracies demanded of the system, it becomes necessary to employ hysteresis models that provide adequate compensation, and regulate the electromagnetic actuator to hysteresis free operation. Modeling, identification and compensation of hysteresis in electromagnetic actuators made of soft ferromagnetic material is detailed in the following sections of this chapter.
6.3 Preisach modeling of hysteresis

Hysteresis is encountered in a large range of applications that usually involve magnetic, ferro-electric, mechanical or optical systems. However in each of these applications, hysteresis is characterized as a multi-branch nonlinearity for which branch-to-branch transitions of the output occur after input extrema [41]. In most situations and for ferromagnetic hysteresis in particular, the branches of the hysteresis nonlinearity are determined by the past extremum values of the input, while the speed of input variations between the extremum points has no influence on branching. This is illustrated in Figure (6.10) where two different inputs \( u_1(t) \) and \( u_2(t) \) with the same extremum values but different time traces otherwise, result in the same hysteresis curves. Hysteresis nonlinearities are also characterized by the presence of loops as shown in Figure (6.10(c)). However, looping is a particular case of branching when the input varies cyclically between the same two extremum values. So a hysteresis model that successfully captures the branching phenomenon will also adequately describe the looping properties. Finally, hysteresis nonlinearities usually display non-local memory. At any given reachable point in the hysteresis diagram, there exist an infinite number of paths representing future outputs of the hysteretic system. The path selected depends on the past time history of the input. Clearly hysteresis is a complex phenomenon, and its model must describe the characteristics of branching and non-local memory. To this effect, the development of the Preisach model of hysteresis is detailed in the following paragraphs.

The mathematical description of the Preisach model consists of an infinite set of simple hysteresis operators \( \gamma_{\alpha \beta} \), that are depicted in Figure (6.11). Each of these operators is represented as a rectangular loop on the input-output graph. In the figure.
Figure 6.10: Independence of hysteresis nonlinearity with respect to speed of input variations. (a) Time trace of trajectory $u_1(t)$. (b) Time trace of trajectory $u_2(t)$. (c) Hysteresis curve for both the input trajectories with the same extrema.

$\alpha$ and $\beta$ are the values of the input $u(t)$, at which the output $\gamma_{\alpha\beta}u(t)$ switches between unity and zero. As the input $u(t)$ is monotonically increased from zero, the ascending branch $abcde$ is followed. However, when the input is monotonically decreased from $+\infty$ to 0, the output follows along the descending branch edfba. Associated with each of these operators $\gamma_{\alpha\beta}$ is an arbitrary weight function $\mu(\alpha, \beta)$, termed as the Preisach function. With these definitions, the Preisach model is expressed as:

$$f(t) = \Gamma u(t) = \int \int \mu(\alpha, \beta)\gamma_{\alpha\beta}u(t)d\alpha d\beta$$  \hspace{1cm} (6.1)

for all $\alpha \geq \beta$. $\Gamma$ denotes the Preisach hysteresis operator as defined by the above equation. In this model, the same input $u(t)$ is applied to all the hysteresis operators. The output of these operators is multiplied by the corresponding Preisach functions $\mu(\alpha, \beta)$, and then summed continuously over all possible values of $\alpha$ and $\beta$. As the output of each of the hysteresis operators depends only on the current value of the input and its direction of change, the operator by itself models only hysteresis with local memory. However, the super-imposition of these hysteresis operators provides a
Figure 6.11: The hysteresis operator.

Figure 6.12: The triangle $T$ in the $\alpha - \beta$ plane.
model with non-local memory. The mechanism of storing this time history information in the Preisach model is revealed through its geometric interpretation.

### 6.3.1 Geometrical interpretation of the Preisach model

The geometrical interpretation of the Preisach model is facilitated by representing Equation (6.1) graphically in a $\alpha - \beta$ plane, as shown in Figure (6.12). As indicated earlier, associated with each hysteresis operator $\hat{\gamma}_{\alpha \beta}$ is a unique Preisach function $\mu(\alpha, \beta)$. Consequently, there exists a one-to-one correspondence between operators $\hat{\gamma}_{\alpha \beta}$ and the feasible points $(\alpha, \beta)$ on this plane. At each point, the corresponding hysteresis operator has switching values given by the coordinates of the points itself.

The feasible region in the plane is the triangle $T$ depicted in Figure (6.12), since the physics of the problem dictates that $\alpha \geq \beta$. The hypotenuse of this triangle is the line $\alpha = \beta$, with the coordinates of the vertex being $(\alpha_0, \beta_0)$, representing the maximal value of the input that is being modeled. Therefore, the Preisach function $\mu(\alpha, \beta)$ equals zero outside triangle $T$.

To understand the mechanisms of the Preisach model, consider the case of hysteresis formation in electromagnetic actuators when the driving input is current $i(t)$, and the resulting output is force $f(t)$. When the current to the electromagnet is zero, the output of all hysteresis operators is zero. As the input is monotonically increased to a value $\alpha_1$ at time $t_1$ as shown in Figure (6.13(a)), all hysteresis operators with $\alpha$ switching values less than $\alpha_1$ are turned on, and their output equals $+1$. This leads to a sub-division of the triangle $T$ into two parts: $S^+(t)$ consisting of point $(\alpha, \beta)$ where the output of the hysteresis operators $\hat{\gamma}_{\alpha \beta}i(t) = +1$, and $S^0(t)$ where the output is zero. The demarcation between the two sets is a horizontal line given
Figure 6.13: Mechanism of storage of input history in the Preisach model. Graphs (a)-(c) depict the time trajectory of current input to the electromagnet. Graphs (d)-(f) show the corresponding sub-divisions of the triangle $T$, and the formation of the interface $L(t)$. 
by the equation $\alpha = \alpha_1 = i(t_1)$, and shown in Figure (6.13(d)). As the current is monotonically increased from 0 to $\alpha_1$, this horizontal line moves vertically up, and its motion is terminated when the input reaches its maximum value.

Current is monotonically decreased from $\alpha_1$ to a value $\beta_1$ at time $t_2$. At this current value, all hysteresis operators in the set $S^+(t)$ with $\beta$ switching values greater than $\beta_1$ are turned off, with their output at zero. Thus the original sub-division of triangle $T$ in Figure (6.13(d)) is modified, with the appearance of a vertical link in addition to the original horizontal link. As depicted in Figure (6.13(e)), the vertical link moves from right to left as the current is monotonically decreased from $\alpha_1$ to $\beta_1$. At the minimum value of current, the equation of the link is given by $\beta = \beta_1 = i(t_2)$.

The boundary between the regions $S^+(t)$ and $S^0(t)$ is denoted by $L(t)$, with the coordinates of its vertex being $(\alpha_1, \beta_1)$. In the third step, current is again increased to a value $\alpha_2$ at time $t_3$, which is less than $\alpha_1$. This increase in the input results in the formation of a new horizontal link in $L(t)$ at $\alpha = \alpha_2 = i(t_3)$. Next as current is decreased again to $\beta_2$ greater than $\beta_1$, a new vertical link appears on the interface function $L(t)$. The appearance of the final two links in $L(t)$ is illustrated in Figure (6.13(f)). Consequently at any given instant of time, the triangle $T$ is sub-divided into two regions $S^+(t)$ and $S^0(t)$, depending on the set of hysteresis operators that are turned on and turned off, respectively. The interface $L(t)$ between these two regions possesses vertex coordinates that correspond to the local maxima and minima of the current input at previous instants of time. The final link of $L(t)$ is attached to the triangle hypotenuse $\alpha = \beta$, and is horizontal or vertical depending on whether current was increased or decreased in the previous step.
Figure 6.14: Triangle $T$ for the experimental identification of the Preisach model.

Figure 6.15: Triangle $T$ for numerical implementation of Preisach model when input is decreasing.
With this geometrical interpretation, Equation (6.1) can be expressed as:

$$f(t) = \Gamma u(t) = \int \int \mu(\alpha, \beta) d\alpha d\beta \quad \forall (\alpha, \beta) \in S^+(t) \quad (6.2)$$

as \(\tilde{\gamma}_{\alpha\beta}u(t) = 0, \forall (\alpha, \beta) \in S^0(t)\). Therefore, the output of the Preisach model depends on the sub-division of the triangle \(T\) into the two sets \(S^+(t)\) and \(S^0(t)\). This sub-division is determined by the interface \(L(t)\) which depends on the past extremum values of the input. Consequently, the Preisach model is able to predict hysteresis phenomenon with non-local memory.

The next important step in characterizing the hysteresis nonlinearity with Preisach models is to determine the number of required hysteresis operators \(\tilde{\gamma}_{\alpha\beta}\) and their associated Preisach functions \(\mu(\alpha, \beta)\). A systematic identification procedure and a discrete numerical implementation form for Equation (6.2) were developed by Mayergoyz [41], that are rederived for the case of force versus current hysteresis, in the following section.

### 6.3.2 Identification of the Preisach model

To determine \(\mu(\alpha, \beta)\) in the Preisach model, current is monotonically increased from zero to some value \(\alpha_1\), and then decreased to a value \(\beta_1\) that is greater than zero. At this instant of time, the sub-division of triangle \(T\) into the two sets \(S^+(t)\) and \(S^0(t)\) is shown in Figure (6.14). If \(f_{\alpha_1}\) denotes the output of the hysteresis nonlinearity at current value of \(\alpha_1\), and \(f_{\alpha_1\beta_1}\) denotes the output after current has been decreased to \(\beta_1\) from its maximum value of \(\alpha_1\), then a new function \(F_{\alpha_1,\beta_1}\) is defined as follows:

$$F_{\alpha_1,\beta_1} = f_{\alpha_1} - f_{\alpha_1\beta_1} \quad (6.3)$$

In Figure (6.14), we observe that the shaded area \(T(\alpha_1, \beta_1)\) is subtracted from the set \(S^+(t)\) when the current is decreased from \(\alpha_1\) to \(\beta_1\). Consequently, the integral over

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the area \( T(\alpha_1, \beta_1) \) equals the difference in hysteresis outputs at current values of \( \alpha_1 \) and \( \beta_1 \). This relationship is mathematically expressed as:

\[
F_{\alpha_1, \beta_1} = \int_{\beta_1}^{\alpha_1} \int_{\beta}^{\alpha} \mu(\alpha, \beta) d\alpha d\beta \quad (6.4)
\]

Differentiating this expression first with respect to \( \beta_1 \) and then with respect to \( \alpha_1 \) in combination with Equation (6.3) gives rise to the relationship:

\[
\mu(\alpha_1, \beta_1) = -\frac{\partial^2 F_{\alpha_1, \beta_1}}{\partial \alpha_1 \partial \beta_1} \quad (6.5)
\]

Consequently \( \mu(\alpha_1, \beta_1) \) is a function of \( F_{\alpha_1, \beta_1} \) that can be determined experimentally by first increasing the input current to \( \alpha_1 \) and then decreasing it to \( \beta_1 \). Theoretically by determining \( f_{\alpha_1, \beta_1} \) for each point in the triangle \( T \), \( \mu(\alpha_1, \beta_1) \) can be found for the complete Preisach model given by Equation (6.1).

The Preisach model is numerically implemented by employing experimentally determined functions \( f_{\alpha_1, \beta_1} \), that are used for finding the Preisach functions \( \mu(\alpha, \beta) \). The objective is to determine an explicit formula for the output of the hysteresis nonlinearity \( f(t) \) in terms of the experimental data \( f_{\alpha_1, \beta_1} \), and the time history of the current input given by \( L(t) \) in Figure (6.15). In the figure, the coordinates of the vertices of \( L(t) \) which are alternating series of dominant input extrema, are represented by coordinates \( (\alpha = M_k, \beta = m_k) \). The positive set \( S^+(t) \) can then be subdivided into \( n \) trapezoids \( Q_k \), so that:

\[
f(t) = \sum_{k=1}^{n(t)} \int \int \mu(\alpha, \beta) d\alpha d\beta \quad \forall (\alpha, \beta) \in Q_k(t) \quad (6.6)
\]

Since the number \( n \) of the trapezoids and their shapes may change with time, consequently both \( n \) and \( Q_k \) are functions of time in the above equation. Through Equation
Figure 6.16: Triangle $T$ for numerical implementation of Preisach model when input is increasing.

Figure 6.17: Electromagnetic actuator used for experimental verification of Preisach model.
(6.4), it is known that:

\[
\int \int \mu(\alpha, \beta) \, d\alpha \, d\beta = F(M_k, m_{k-1}) \quad \forall (\alpha, \beta) \in T(M_k, m_{k-1}) \tag{6.7}
\]

\[
\int \int \mu(\alpha, \beta) \, d\alpha \, d\beta = F(M_k, m_k) \quad \forall (\alpha, \beta) \in T(M_k, m_k) \tag{6.8}
\]

Since geometrically each trapezoid \(Q_k\) can be expressed as a difference of two triangles \(T(M_k, m_{k-1})\) and \(T(M_k, m_k)\), one derives:

\[
\int \int \mu(\alpha, \beta) \, d\alpha \, d\beta = F(M_k, m_{k-1}) - F(M_k, m_k) \quad \forall (\alpha, \beta) \in Q_k(t) \tag{6.9}
\]

Summing over the entire area \(S^+(t)\) of triangle \(T\) and using Equation (6.6), the output \(f(t)\) of the hysteresis nonlinearity is written out as:

\[
f(t) = \sum_{k=1}^{n(t)} [F(M_k, m_{k-1}) - F(M_k, m_k)] \tag{6.10}
\]

if the input is decreasing in which case the last link is vertical as shown in Figure (6.15). As observed in this figure, \(m_n\) equals \(i(t)\), so that the above equation is also expressed as:

\[
f(t) = \sum_{k=1}^{n(t)-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] + F(M_n, m_{n-1}) - F(M_n, i(t)) \tag{6.11}
\]

where the summation is performed over \((n-1)\) trapezoids at time instant \(t\). Since \(L(t)\) is due to the past history of the input \(i(t)\), the first term in the above equation \(\sum_{k=1}^{n(t)-1} [F(M_k, m_{k-1}) - F(M_k, m_k)]\) is already available in computer implementation. Consequently, at time instant \(t\), the alternating series of dominant input extrema \((M_k, m_k)\), constituting the coordinates of the interface \(L(t)\) are updated, and the output of the hysteresis nonlinearity calculated using the above equation.

For the case when the input is increasing and the last link is horizontal, the output of the hysteresis nonlinearity is similarly derived, and expressed as:

\[
f(t) = \sum_{k=1}^{n(t)-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] + F(i(t), m_{n-1}) \tag{6.12}
\]

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with the corresponding triangle depicted in Figure (6.16). Consequently, the Preisach model as expressed by Equations (6.10) and (6.12) relates the output \( f(t) \) to experimentally measured data by eliminating the need for differentiation as required in Equation (6.5), or double integration of Equation (6.1).

Numerical implementation of the above form of the Preisach model requires the experimental determination of \( F_{\alpha_1,\beta_1} \) at a finite number of points within the triangle \( T \). For this purpose, a square mesh covering the triangle \( T \) is created, and the corresponding values of \( f_{\alpha_1,\beta_1} \) are experimentally determined. The parameters of the Preisach model \( \{ F_{\alpha_1,\beta_1} \} \) are calculated using Equation (6.3). The alternating series of dominant extrema \( (M_k, m_k) \) are calculated from the time history of the input current to the electromagnet, to determine the vertices of the interface \( L(t) \). This series is continuously updated at each instant of time. Finally, the output \( f(t) \) of the hysteresis nonlinearity is calculated using the Preisach model parameters \( \{ F_{\alpha_1,\beta_1} \} \), in Equations (6.10) and (6.12). In the experiments reported in this chapter, \( \alpha_o = \beta_o = 6.5 \) amperes.

A grid of points with \( \Delta\alpha = \Delta\beta = 0.5 \) amperes in the complete range of variation of \( \alpha \) and \( \beta \) on the triangle \( T \) is created. \( f_{\alpha_1,\beta_1} \) are experimentally determined for these grid points, and the Preisach model parameters are calculated. If the value of the input current does not lie at the grid points of the triangle \( T \), interpolation is employed to determine the corresponding \( F_{\alpha_1,\beta_1} \).

### 6.4 Experimental verification

To experimentally verify the performance of Preisach models in predicting hysteresis nonlinearities in electromagnetic actuators, forces from an E-core electromagnet interacting with an iron target as shown in Figure (6.17) are measured. The E-core
electromagnet is made of silicon steel with a commercial trade name of EI-62. Both the electromagnet as well as the target are made of laminations that are .014 inches thick. The dimensions of the E-core electromagnet and the target are shown in Figure (6.17). The electromagnet is energized using a Kepco ATE 150-7M linear power supply in a current range of 0 to 7 amperes. The forces are measured using a ATI Gamma series six axis force/torque sensor, mounted on the ferromagnetic target. The sensing system has a force resolution of .003 Newton along the $x_o$ and $y_o$ directions, and .006 Newton along the $z_o$ direction. The torque sensing resolution is .00018 Newton-meter in all the three directions.

To minimize flexibility and ensure alignment, the test apparatus consisting of the E-core electromagnet, the iron target and the force sensor is mounted between the cross-slides of the Instron testing machine. The airgap between the electromagnet and the target is varied by moving the Instron cross-slides. This motion has a resolution of .005 inches. The relative orientation between the electromagnet and the target is modulated using a goniometer with an angular positioning resolution of .005 arc-min. Both the force sensor and the power supply are controlled through a Pentium PC instrumented with data acquisition and control cards.

The forces between the electromagnet and the target depend on the current input, the airgap separation and their relative orientation. Since the target is considerably larger than the electromagnet poleface as shown in Figure (6.17), variation of forces with translation in the $x_o$ and $y_o$ directions, and rotation around the $z_o$ axis is negligible. As the target is effectively infinite in comparison to the electromagnet, it is sufficient to calibrate electromagnet forces with respect to rotation around a single axis. This calibration procedure is clearly detailed in Chapter (3) for the case of a
Figure 6.18: Rising triangular current trajectory input to electromagnetic actuator for experimental verification of Preisach model.

Figure 6.19: Comparison of force profiles obtained through experiments and Preisach model prediction for a rising triangular current trajectory.
Figure 6.20: Comparison of hysteresis predicted by the Preisach model with experimental data for the last loop of the rising triangular trajectory.

Figure 6.21: Comparison of hysteresis predicted by the Preisach model with experimental data for the minor loop of the rising triangular trajectory.
cylindrical electromagnet interacting with a metallic plate. In the experimental setup of Figure (6.17), the relative orientation is varied by rotating the electromagnet $\theta_y$ degrees about the $y$ axis of the $\{x, y, z\}$ coordinate system, established at the surface of the electromagnet. The $y$ axis is always parallel to and $h(t)$ mm below, the $y_o$ axis of the reference coordinate system $\{x_o, y_o, z_o\}$ fixed to the target. Consequently, hysteresis in this electromagnetic actuator is a function of three input variables: Current $i(t)$, airgap $h(t)$, and the rotation $\theta_y(t)$. The measured output in all the experiments in the normal component of the force $f_{z_o}(t)$ between the electromagnet and the target, and concisely denoted as $f(t)$. Performance of the Preisach model in predicting hysteresis variations with respect to these variables is presented in the following sections.

6.4.1 Variation of force with input current

The performance of the Preisach model in predicting hysteresis nonlinearity for slow variations in input current is investigated for three different types of trajectories. In the first case, a rising triangular current profile depicted in Figure (6.18) is employed. The experimentally measured output values agree well with the Preisach model predictions are shown in Figure (6.19). The difference in electromagnetic force for rising and falling values of current, is plotted in Figures (6.20) and (6.21). During the time span of $t = 100$ to $t = 150$ seconds, Preisach model predictions compare well with the experimentally observed values in Figure (6.20). Similar correspondence is observed for the time span of $t = 60$ to $t = 100$ seconds in Figure (6.21). The small discrepancy at low values of current is due to the limited number of calibration points available in the corresponding region of the Preisach triangle. This error can be eliminated by increasing the number of calibration points. The rising triangular
Figure 6.22: Comparison of force profiles obtained through experiments and Preisach model prediction for the decaying sinusoid current trajectory.

Figure 6.23: A general current profile input to the actuator for Preisach model verification.
trajectory leads to a simple interface $L(t)$ in the triangle $T$. Since each successive peak is higher than the previous one, it wipes out the previous dominant maxima. Therefore, at the last peak corresponding to time $t = 125$ seconds, the interface $L(t)$ is a horizontal line at $\alpha = i(t) = 5$ amperes.

The second trajectory investigated is a decaying sine wave, the force profile of which is shown in Figure (6.22). Since each successive peak and valley is less than the previous one, this kind of trajectory results in a series of dominant extrema corresponding to each of the maxima and minima in the time trajectory. Consequently, the interface $L(t)$ in the triangle $T$ is a staircase with as many steps as the extrema in the trajectory. It is observed from Figure (6.22) that the Preisach model accurately predicts the force output of the hysteresis nonlinearity in comparison to the experimentally measured force profile. The maximum error between the experimental and
the model predicted hysteresis values is about .15 Newton, occurring at a hysteresis peak of 1.01 Newton and corresponding current value of 3.75 amperes.

The final trajectory being investigated in Figure (6.23) consists of four components: A rising ramp from 0-12.5 seconds, a decaying sinusoid with initial amplitude of 2.5 Newton from 12.5-50 seconds, a rising sinusoid from 50-100 seconds, and finally a decaying sinusoid with a much smaller initial amplitude of 1.0 Newton. The purpose of this current trajectory is to simulate general profiles that may be commanded to the electromagnetic actuator in closed loop control, and to investigate the performance of the Preisach model in these situations. Corresponding force profiles obtained experimentally and calculated through the Preisach model are graphed in Figure (6.24), which verifies the consistent performance of the Preisach model in predicting the hysteresis nonlinearity for variations in input current.
Figure 6.26: Performance of the two input Preisach model in predicting hysteresis output at airgap $h = 1.66$ mm.

Figure 6.27: Performance of the two input Preisach model in predicting hysteresis output at airgap $h = 2.93$ mm.
6.4.2 Variation of force with current and airgap

The force output of the electromagnetic actuator in Figure (6.17) depends not only on current, but also on the airgap \( h(t) \) between the electromagnet and the target. In the general case, the hysteresis nonlinearity may manifest not only for cyclical variations in current, but also for cyclical variations in the airgap \( h(t) \). Hysteresis effects in the electromagnetic actuators with respect to airgap at a constant current excitation of 6.5 amperes are investigated in Figure (6.25). As the airgap is changed from 1.27 mm to 4.06 mm in steps of .254 mm, the force versus airgap plots are almost coincident, when the airgap is first increased and then decreased. Consequently, the hysteretic effect of airgap variation on force is negligible, and the following simplified two input Preisach model is utilized [41]:

\[
f(t) = \int \int \mu(\alpha, \beta, h(t)) \, d\alpha d\beta \quad \forall \alpha \geq \beta \tag{6.13}
\]

This two input model recognizes the dependence of the actuator force on airgap by incorporating the dependence of \( \mu \) on not only \( \alpha \) and \( \beta \) but also the airgap variable \( h(t) \). To determine the parameters of the Preisach model in this case, nine equally spaced calibration points in an airgap interval of 1.27 mm to 3.302 mm are selected. At each point on this interval, a set of \( F_{\alpha_1, \beta_1} \) values as per Equation (6.3) for the complete grid of triangle \( T \) is determined. The gridding of triangle \( T \) is identical to the one used in the case of force variation with current, as reported in Section (6.3.2). Finally for a given value of \( h(t) \), corresponding values of \( F_{\alpha_1, \beta_1} \) are determined using second order polynomial interpolation.

The performance of this two input Preisach model in modeling hysteresis is investigated at airgaps of 1.66 mm and 2.93 mm in Figures (6.26) and (6.27), respectively.
A triangular current trajectory of peak value 6.5 amperes is employed in each case. It is observed that the Preisach model is able to accurately predict the force trajectory for each of the airgaps, with the same set of calibration data that is acquired using the procedure outlined above.

6.4.3 Variation of force with current, airgap and orientation

The force of the electromagnetic actuator shown in Figure (6.17), is also a function of the relative orientation between the electromagnet and the target, in addition to its dependence on the airgap and current variables. To model hysteresis with respect to all of these variables, the two input Preisach model of the previous section is extended as follows:

\[ f(t) = \int \int \mu(\alpha, \beta, h(t), \theta_y(t)) \, d\alpha d\beta \quad \forall \alpha \geq \beta \quad (6.14) \]

where \( \theta_y(t) \) is the rotation of the electromagnet around the \( y \) axis in Figure (6.17). The Preisach functions \( \mu \) in the three input Preisach model are functions of both the airgap \( h(t) \) and the orientation \( \theta_y(t) \). Hysteresis is predicted in a variation range of \( 1.3462 - 1.8542 \) mm for the airgap variable \( h(t) \), and from \(-3\) to \(+3\) degrees for the orientation variable \( \theta_y(t) \). Parameters of the three input Preisach model are obtained at calibration points of \( \theta_y = 0, 1.0, 2.0 \) and \( 3.0 \) degrees at three airgap values of \( h = 1.3462, 1.6002 \) and \( 1.8542 \) mm. Since the variation of the output force is symmetrical with respect to changes in \( \theta_y \), this model calibration predicts hysteresis output for the complete range of variation of \( \theta_y \). At any given value of \( h(t) \) and \( \theta_y(t) \), the corresponding Preisach model parameters are calculated using second order polynomial interpolation.
Figure 6.28: Performance the Preisach model in predicting hysteresis output at airgap $h = 1.47$ mm and orientation $\theta_y = 2.5^\circ$.

Figure 6.29: Performance the Preisach model in predicting hysteresis output at airgap $h = 1.70$ mm and orientation $\theta_y = 0.6^\circ$. 

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The performance of the three input Preisach model is investigated at \( h = 1.47 \) mm and \( \theta_y = 2.5 \) degree in Figure (6.28), and at \( h = 1.70 \) mm and \( \theta_y = 0.6 \) degree in Figure (6.29). Since both these points are distinct from the calibration points, Preisach model parameters are obtained by interpolation. Comparison of experimental data with Preisach model prediction in Figures (6.28) and (6.29) indicate accurate prediction of force trajectories. Consequently, the three input Preisach model is able to predict force trajectories for a range of variations in current, airgap and orientation values.

6.5 Inversion of the Preisach model of hysteresis

As described in the previous section, hysteresis in electromagnetic actuator is predicted well by Preisach models with respect to variations in input current, airgap and relative orientation. This presents an opportunity to design controllers that correct these hysteretic effects, and improve accuracy of the commanded force. Essential to the synthesis of such controller architectures is the development of the inverse Preisach model. Formulation of the inverse Preisach model for a single input is formulated first in the following paragraphs, following which it is extended to two and three inputs.

Given the Preisach model parameters \( \{F_{\alpha,\beta}\} \) and the associated interface \( L(t) \) for the triangle \( T \), the inverse Preisach model determines the current \( i(t + \Delta t) \) that will result in a desired force \( f_d(t + \Delta t) \) at the next instant, for fixed values of airgap \( h \) and orientation \( \theta_y \). In the formulation of the inverse Preisach model, two distinct cases corresponding to decreasing \( f_d(t + \Delta t) < f_d(t) \), and increasing \( f_d(t + \Delta t) > f_d(t) \) need to be considered. When \( f_d(t + \Delta t) = f_d(t) \), then \( i(t + \Delta t) = i(t) \).
6.5.1 Case of decreasing desired force

Consider a Preisach model with parameters \( \{ F_{\alpha_i, \beta_i} \} \) and an interface \( L(t) \) shown in Figure (6.30). The vertices of this interface are characterized by an alternating series of dominant maxima \( \{ M_k \} \) and dominant minima \( \{ m_k \} \). The current value of the input is \( i(t) \) corresponding to a desired force \( f_d(t) \). As the force desired in the next step \( f_d(t + \Delta t) \) is less than \( f_d(t) \), the final link of interface \( L(t + \Delta t) \) at instant \( t + \Delta t \) must be vertical at some \( \beta = i(t + \Delta t) < i(t) \) so that \( S^+(t + \Delta t) < S^+(t) \). This situation is shown in Figure (6.31). The process of inversion involves determining this value of \( \beta \) at instant \( t + \Delta t \).

The basic issue in the inversion process is to isolate the values of \( \alpha \) and \( j \) at which a current input of \( i(t + \Delta t) \) to the hysteresis nonlinearity results in a force output of \( f_d(t + \Delta t) \). To this end, we denote the region under the interface \( L(t) \) till some vertex \( (M_l, m_l) \) as \( S^+_l(t) \) and the force corresponding to it as \( f_{S^+_l}(t) \). Regions \( S^+_l(t) \) and \( S^+_{l+1}(t) \) corresponding to vertices \( (M_l, m_l) \) and \( (M_{l+1}, m_{l+1}) \) respectively, are shown in Figure (6.30). The first step in Preisach model inversion is to select an index \( l \in k = 1, \ldots, n \) so that \( f_{S^+_l}(t) < f_d(t + \Delta t) < f_{S^+_{l+1}}(t) \). It is possible that either \( f_{S^+_l}(t) \) or \( f_{S^+_{l+1}}(t) \) is equal to \( f_d(t + \Delta t) \). This condition expresses the fact that the inverse corresponding to \( f_d(t + \Delta t) \) lies on the line \( \alpha = M_{l+1} \) which is one component of the required solution. The required value of \( \beta = i(t + \Delta t) \) is obtained by recognizing in Figure (6.31) that:

\[
f_d(t + \Delta t) = f_{S^+_l}(t) + f_{\Delta S^+_l}(t) \tag{6.15}
\]

where \( f_{\Delta S^+_l}(t) \) is the force corresponding to the area \( \Delta S^+_l(t) = S^+(t + \Delta t) - S^+_l(t) \). \( F_{\alpha_i, \beta_i} \) in the region \( m_l < \beta = i(t + \Delta t) < m_{l+1} \) at \( \alpha = M_{l+1} \) is found by interpolation and denoted as \( G(M_{l+1}, \beta) \), a polynomial function in \( \beta \). Expressing the equation in
Figure 6.30: Triangle $T$ at instant $t$ corresponding to the case of decreasing desired force.

Figure 6.31: Triangle $T$ at instant $t + \Delta t$ corresponding to the case of decreasing desired force.
terms of the Preisach model parameters with the help of Equation (6.12), one obtains:
\[ f_d(t + \Delta t) = \sum_{k=1}^{l} [F(M_k, m_{k-1}) - F(M_k, m_k)] + F(M_{l+1}, m_l) - G(M_{l+1}, \beta) \] (6.16)

Inverting the above, the solution to the inverse problem is derived to be:
\[ i(t + \Delta t) = \beta = G^{-1} \left( \sum_{k=1}^{l} [F(M_k, m_{k-1}) - F(M_k, m_k)] + F(M_{l+1}, m_l) - f_d(t + \Delta t) \right) \] (6.17)
evaluated at \( \alpha = M_{l+1} \) when \( f_d(t + \Delta t) < f_d(t) \).

### 6.5.2 Case of increasing desired force

Similar to the previous development, consider the situation depicted in Figures (6.32) and (6.33). The current value of the input is \( i(t) \) corresponding to \( f_d(t) \). Since \( f_d(t + \Delta t) \) is greater than \( f_d(t) \), the last link of interface \( L(t + \Delta t) \) is horizontal at \( \alpha = i(t + \Delta t) > i(t) \), so that \( S^+(t + \Delta t) > S^+(t) \). Regions \( S_i^+(t) \) and \( S_{i+1}^+(t) \) corresponding to vertices \( (M_i, m_i) \) and \( (M_{i+1}, m_{i+1}) \) respectively, are shown in Figure (6.32). Selecting the index \( l \in k = 1, \ldots, n \) so that \( f^{S_{l+1}^+(t)} < f_d(t + \Delta t) < f^{S_l^+(t)} \), the inverse is guaranteed to lie on the line \( \beta = m_l \). As in the previous case, the corresponding value of \( \alpha = i(t + \Delta t) \) is obtained with the following set of equations:
\[ f_d(t + \Delta t) = f^{S_{l+1}^+(t)} + f^{\Delta S_{l+1}^+(t)} \] (6.18)

the geometry of area \( S_{l+1}^+(t) \) is shown in Figure (6.32) and \( \Delta S_{l+1}^+(t) = S^+(t + \Delta t) - S_{l+1}^+(t) \) as per Figure (6.33). \( F_{\alpha, \beta} \) in the region \( M_{i+1} < \alpha = i(t + \Delta t) < M_l \) at \( \beta = m_i \) is found by interpolation and denoted as \( G(\alpha, m_i) \), a polynomial function in \( \alpha \). Recasting the above equation in Preisach model parameters by substituting Equation (6.12), one derives:
\[ f_d(t + \Delta t) = \sum_{k=1}^{l-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] + F(M_{l+1}, m_l) + G(\alpha, m_i) \] (6.19)
Figure 6.32: Triangle \( T \) at instant \( t \) corresponding to the case of increasing desired force.

Figure 6.33: Triangle \( T \) at instant \( t + \Delta t \) corresponding to the case of increasing desired force.
Inverting the above, the solution to the inverse problem is derived to be:

\[ i(t+\Delta t) = \alpha = G^{-1}\left(f_d(t + \Delta t) - \sum_{k=1}^{l-1} [F(M_k, m_{k-1}) - F(M_k, m_k)] - F(M_{l+1}, m_l)\right) \]

(6.20)

evaluated at \( \beta = m_l \) when \( f_d(t + \Delta t) > f_d(t) \).

The inverse solution procedure given by Equations (6.17) and (6.20) essentially consists of three components: Updating of the array of dominant input extrema \( \{M_k, m_k\} \) at each instant of time, calculation of the output force corresponding to the area under the interface \( L(t) \), and the determination of the index \( l \) to find the value of \( \alpha \) or \( \beta \) at which the inverse solution may lie. The computationally intensive second component of determining the force corresponding to the area under \( L(t) \) is performed recursively at each instant \( t \), and is consequently implementable in real-time computer control.

Inversion of the two input and the three input Preisach models follows the above procedure closely. Along with the calibration data and the input history \( i(t) \), the inverse three input Preisach model determines the current \( i(t + \Delta t) \) that will result in a desired force \( f_d(t + \Delta t) \) at the next instant for specific values of airgap \( h(t) \) and relative orientation \( \theta_y(t) \). The problem can be solved using the methodology outlined above, once the Preisach model parameters \( \{F_{\alpha_1, \theta_1}\} \) at \( h(t) \) and \( \theta_y(t) \) are established. This is accomplished through interpolation using calibration data points. In the examples that are presented in the next section, second order polynomial interpolation is employed.
Figure 6.34: Open-loop compensation of hysteresis in electromagnetic actuators.

Figure 6.35: Force versus current graphs at $h = 1.29$ mm for the lumped parameter, finite element and experimental models.
6.6 Hysteresis compensation through Preisach model inversion

Based on the results presented in Sections (6.4.1) to (6.4.3), it is observed that the Preisach models capture the hysteresis characteristics of electromagnetic actuators accurately and comprehensively. Consequently, it is possible to pursue model based compensation of hysteresis nonlinearity in these actuators using an open-loop compensation strategy depicted in Figure (6.34). The force profile desired from the electromagnetic actuator is input to the inverse Preisach model developed in Section (6.5) and described by Equations (6.17) and (6.20). The inverse model predicts the current trajectory that would achieve the desired force profile, and compensate for hysteresis. This current trajectory is then input to the electromagnetic actuator on the experimental test-bench described in Section (6.4), and the resulting forces recorded. The deviation between the desired forces and those recorded experimentally, is then a measure of the success of this compensation strategy.

The performance of the compensation strategy is experimentally investigated for four different cases: Force variation with current only, variation in current and airgap, variation in current, airgap and orientation, and finally, dynamic performance. Results for each of these cases are presented in the following sections.

6.6.1 Compensation for variation in input current

The performance of the compensation scheme is first experimentally investigated for variations of force with input current at fixed values of airgap and relative orientation. In the experiments, the electromagnetic force model is obtained through three different methodologies: Lumped parameter analysis, finite element modeling,
and experimental calibration. The standard lumped parameter force model for an E-core electromagnet is employed [34], with the dimensions given in Figure (6.17). The finite element model for the electromagnetic actuator is obtained by performing three dimensional simulations using Maxwell-3D. Lastly, experimental calibration is employed to determine the Preisach model parameters \{F_{a1}, a3\} in the range of variation of current. The calibration curve for each of these approaches is depicted in Figure (6.35) at an airgap \( h = 1.29 \text{ mm} \) and orientation \( \theta_y = 0 \text{ degree} \). It is observed that the finite element calibration approximately follows the experimental graph, while significant deviation is observed between lumped parameter and experimental calibrations. This difference is due to the large value of the airgap, which causes significant flux fringing and leakage and is not modeled in the lumped parameter approach. An inverse model for each of force models is developed, and the performance of the open-loop compensation scheme of Figure (6.34) is experimentally investigated.

The desired force profile \( f_d \) in these experiments consists of three components as depicted in Figure (6.36): A rising ramp from 0-25 seconds, a decaying sinusoid in 25-45 second range, followed by a rising sinusoid. A comparison of the experimental profiles obtained with the inverse of each of the three force models is depicted in Figure (6.36). Significant error is observed in the case of the compensation scheme with the inverse lumped parameter model. The compensation strategy in this case is unable to accurately compensate for either the actuator force-current nonlinearity or hysteresis effects. With the inverse finite element model, the maximum compensation error is reduced to about 3.2 Newton, down from 5.75 Newton in the lumped parameter case. Even though the inverse finite element model is able to compensate...
Figure 6.36: Performance of the compensation scheme with the inverse lumped parameter, finite element, and Preisach models.

Figure 6.37: The error curve obtained with the inverse Preisach model at $h = 1.29$ mm.
in large part for the force-current nonlinearity, the error component due to the hysteresis nonlinearity distorts the response. In comparison, the inverse Preisach model provides accurate compensation for both the force-current as well as hysteresis nonlinearities as observed in Figure (6.36). The experimental force trajectory for this case completely overlaps the desired force profile, and the two curves are coincident. The corresponding error curve between these two trajectories is depicted in Figure (6.37), and is bounded in a region of 0.25 Newton. Consequently, the open-loop compensation scheme based on the inverse Preisach model provides accurate and precise regulation of the electromagnetic actuator in face of both hysteretic and force-current nonlinearities.

6.6.2 Compensation for variation in current and airgap

The performance and the calibration of the hysteresis model with respect to variations in the input current and airgap at a fixed orientation, were described in Section (6.4.2). On the basis of these results, performance of the open-loop hysteresis compensation scheme at airgaps of $h = 1.66$ mm and $h = 2.93$ mm is shown in Figures (6.38) and (6.39), respectively at a fixed orientation of $\theta_y = 0.0$ degree. The desired force trajectory is similar to the one employed for the previous case, comprising of a sequence of rising ramp, decaying sinusoid followed by a rising sinusoid. In both the cases, good agreement between the desired and experimentally obtained force profiles is observed. At $h = 1.66$ mm, maximum error between the desired and experimental values is about 0.10 Newton at the trajectory maxima of 15.5 Newton. The corresponding value at $h = 2.99$ mm is about 0.06 Newton, occurring at the
Figure 6.38: Performance of the compensation scheme with the two input inverse Preisach model at $h = 1.66$ mm.

Figure 6.39: Performance of the compensation scheme with the two input inverse Preisach model at $h = 2.93$ mm.
trajectory peaks. Consequently, the two input Preisach model is able to compensate for hysteresis with respect to both current and airgap.

6.6.3 Compensation for variation in current, airgap and orientation

Figures (6.40) and (6.41) depict the performance of the three input inverse Preisach model in compensating for hysteresis at airgap and orientations of $h = 1.47$ mm, $\theta_y = 2.5$ degree, and $h = 1.70$ mm, $\theta_y = 0.6$ degree, respectively. The calibration process for the Preisach model is described in Section (6.4.3). The desired force trajectory is these figures is general in nature, and consists of a sequence of increasing and decreasing ramps of varying slopes and magnitudes. At both these locations, the airgap and orientation of the target do not coincide with the calibration points, so interpolation is employed. Examination of Figures (6.40) and (6.41) reveals that the three input Preisach model is able to regulate the electromagnetic force to the desired trajectory.

6.6.4 Dynamic performance

Electromagnetic hysteresis in general does not change with the frequency of excitation, and the associated hysteretic core losses are proportional to this frequency [47]. However in implementation, this relationship is invariably corrupted by the presence of circulating eddy currents in the cores that induce frequency dependence in the observed electromagnetic force. To minimize these effects, both the E-core and the electromagnet are constructed from powdered metal, Anchor Steel TC-80 material (Hoeganaes Corporation, Riverton, NJ), whose each particle is insulated from the others by a thin plastic coating. The performance of the inverse Preisach model
Figure 6.40: Performance of the compensation scheme with the three input inverse Preisach model at \( h = 1.47 \) mm and \( \theta_y = 2.5 \) degree.

Figure 6.41: Performance of the compensation scheme with the three input inverse Preisach model at \( h = 1.70 \) mm and \( \theta_y = 0.6 \) degree.
Figure 6.42: Performance of the compensation scheme when the frequency of the desired force trajectory is swept from 10-50 Hertz.

Figure 6.43: The error curve between desired and experimental force profiles when the frequency of the input trajectory is swept from 10-50 Hertz.
in compensating for hysteresis during fast variations of the input trajectory is then investigated on this electromagnetic actuator.

Figures (6.42) and (6.43) depict the performance of the open-loop control scheme in compensating for hysteresis as the frequency of the input trajectory is swept from 10-50 Hertz in a time span of 0.3 seconds. Figure (6.42) compares the time profile of the desired force trajectory with that of the experimental data recorded at 10 KHz sampling rate. In plotting this graph, the desired force profile is previewed by 3 steps to compensate for the .3 msec phase delay in the six axis force sensing instrumentation, which has a natural frequency of 7000 Hz in the vertical directions when unloaded. Hysteresis curves recorded by a single axis force transducer of natural frequency 70 KHz, at input excitation frequencies of 25 and 90 Hertz are shown in Figure (6.44). The absence of significant deviation between the loops demonstrates that observed hysteresis is indeed independent of excitation frequency in a range of 25-90 Hertz.

The error curve when the frequency of the desired input trajectory is swept from 10-100 Hertz in a time span of 0.4 seconds is plotted in Figure (6.45). A linear dependence between error and frequency is observed, corresponding to the increasing phase of the six axis force measuring system with increasing frequency. Consequently, it is concluded that the developed Preisach models satisfactorily regulate actuator performance in face of hysteretic losses up to frequencies of 100 Hertz. Investigation of performance at higher frequencies demands larger sampling rates that are not possible with the existing experimental setup.
Figure 6.44: A comparison of the hysteresis loops obtained with input excitation of frequency 25 and 90 Hertz.

Figure 6.45: The error curve between desired and experimental force profiles when the frequency of the input trajectory is swept from 10-100 Hertz.
6.7 Conclusions

This chapter addresses a key issue in the development of the control architecture for the high performance Magnetic Suspension Actuator, by focusing on hysteresis modeling and compensation in electromagnetic actuators made of soft ferromagnetic materials. Even though the observed hysteresis values are small, simulations results indicate significant degradation in performance while tracking fast trajectories with large amplitudes, in context of nanometric accuracies that are demanded of the system. Performance is restored only when the controller incorporates a comprehensive ferromagnetic hysteresis model, and is able to regulate the electromagnetic actuators to hysteresis free operation. To this end, the Preisach independent domain model is utilized to capture all of the essential characteristics of the hysteresis nonlinearity in electromagnetic actuators. Experimental results convincingly demonstrate its ability to accurately model the output of the hysteresis nonlinearity with respect to variations in input current, airgap and orientation.

To enable compensation of hysteresis in electromagnetic actuators, development of inverse Preisach models is addressed. The formulation of the inverse Preisach model is suitable for real-time computer implementation as it employs recursive relationships that are calculated once and stored. The performance of an open-loop compensation strategy employing these inverse Preisach models is experimentally investigated. Results demonstrate the consistent performance of the control strategy based on Preisach model inversion, in hysteresis compensation for variations in input current, airgap and orientation. Finally, effective compensation of hysteresis for variations upto 100 Hz of the desired force trajectory is also obtained. It may be possible to compensate for faster variations in the desired force profile, but could not
be experimentally verified due to limitations imposed by the maximum sampling and control rates available in the instrumentation.

Development of the Preisach models of ferromagnetic hysteresis completes the formulation of the high performance control architecture depicted in Figure (6.2), for the Magnetic Suspension Actuator. The feedback linearization module implements the inverse Preisach model to compensate both for ferromagnetic hysteresis and the force-current-airgap nonlinearity of the electromagnetic actuator, thus enabling large travel. In conjunction with the uncertainty/disturbance observer and feedforward controller components, large travel at high frequencies with nanometric accuracies is obtained, thereby satisfying the high performance requirements of the MSA.
CHAPTER 7

CONCLUSIONS

7.1 Summary

The main focus of this dissertation was to enable the development of large travel Magnetic Suspension Actuators, for realizing accurate travel at high bandwidths in multiple degrees of freedom. Critical to the development of such an actuation system, was the formulation of the principles for the design of two basic components: (1) The electromagnetic actuation strategy that supports the key characteristics demanded of the actuation system, and the (2) control algorithm that regulates the performance of the MSA to the desired specifications. Due to the stringent performance specifications demanded of the MSA, significant issues were encountered in each of the two tasks. The issues, the approaches that were formulated and results obtained therein, are summarized below.

Development of the electromagnetic actuation scheme using DC electromagnets is first addressed. A systematic procedure for developing a model of the the electromagnetic interactions between the flotor and the electromagnets, through finite element calibration is formulated. The resulting model provides an accurate description of the magnetic field at large airgaps, and is scalable with the number of electromagnets. This model is utilized for synthesizing an actuation strategy that minimizes the
number of electromagnets, while ensuring the complete manipulability of the flotor. The resulting actuation strategy is optimized by modulating the inclination angles of the electromagnets, so that the magnetic forces are maximized. Using this procedure, optimal configurations consisting of 7, 8, and 9 electromagnets are synthesized that realize the desired travel range and ensure flotor manipulability in the six degree of freedom space.

In order to reduce the size and the power requirements of the electromagnets in the actuation schemes designed above, permanent magnets are combined with DC electromagnets to generate Permanent-Electro-Magnet (PEM) combinations. The permanent magnets in the PEM combinations provide a strong DC component of the magnetic flux which is utilized in supporting the weight of the flotor. The permanent magnets also provide significantly higher forces so that it is easier to obtain large travel from the actuation scheme in all of the translational degrees of freedom. However when PEM combinations are designed to maximize the force characteristics, manipulability problems are frequently encountered. Optimal design of the PEM combinations that maximizes the force characteristics in relationship to the actuation requirements, while maintaining manipulability is addressed. The optimal PEM combinations are then incorporated into the design of the horizontal and vertical actuation schemes that result in the desired actuation characteristics.

Formulation of the principles for the design of the compensation system, first involves the development of a robust nonlinear control algorithm, for realizing large and accurate travel in the MSA. Geometric feedback linearization techniques using the the nonlinear model of the electromagnetic force are employed to formulate the controller. However such techniques are sensitive to model uncertainties and external disturbance
forces that may act on the MSA. To provide robustness to uncertainty/disturbance, an algorithm based on discrete time-delay control is formulated. If the maximum frequency contents of the uncertainty are significantly slower than the sampling rate, then the algorithm achieves robust compensation. The performance of the robust nonlinear control algorithm is experimentally investigated with respect to stabilization, trajectory tracking and disturbance compensation in a single degree of freedom MSA.

Tracking accuracy in the robust nonlinear control strategy degrades with increasing frequency and amplitude of the desired trajectory, due to the presence of ferromagnetic hysteresis in the electromagnetic actuators. Even though the experimentally observed hysteresis values are small, simulations results indicate significant degradation in performance while tracking fast trajectories with large amplitudes, in context of nanometric accuracies that are demanded of the system. Performance is restored only when the controller incorporates a comprehensive ferromagnetic hysteresis model, and is able to regulate the electromagnetic actuators to hysteresis free operation. Hence the Preisach independent domain model is utilized to capture all of the essential characteristics of the hysteresis nonlinearity in electromagnetic actuators. The Preisach model is inverted and incorporated into the robust nonlinear control algorithm to regulate the electromagnetic actuator, and compensate for hysteretic effects. Experimental results demonstrate the consistent performance of this approach in compensating both for the current-force-airgap nonlinearity, and ferromagnetic hysteresis.
7.2 Contributions

This dissertation lays the foundations for the development of next generation of Magnetic Suspension Actuators with characteristics that are critical to success of the precision engineering industry. The property of large and accurate travel in multiple degrees of freedom, will enable applications such as photolithography and scanning tunneling microscopy to be executed with higher precision at faster bandwidths. The high stiffness characteristics of the MSA in combination with its large travel property are vital to precision metal cutting, non-circular turning, and micro-machining where high tracking accuracy at high bandwidths in face of external disturbance forces is required. Therefore the key features of the MSA are essential for improving the range of products that may be produced by ultra precision fabrication, and consequently improving productivity and part quality.

In addition to enabling a generation of magnetic suspension actuators with enhanced capabilities, investigations of the issues in the design of electromagnetic actuation scheme and control algorithm has resulted in technical contributions in the four specific areas:

1. A methodology for synthesizing optimum electromagnet actuation strategies for large travel, multiple DOF, Magnetic Suspension Actuator is formulated. The actuation strategy minimizes the number of electromagnets while maintaining flotor manipulability in the complete operational workspace of the MSA.

2. A systematic procedure for designing Permanent-Electro-Magnet (PEM) combinations is devised, for a given travel range. The design procedure maximizes
the available force from the PEM combination, while preserving flotor manip­
ulability. Incorporation of the PEM combinations into the actuation scheme.
decreases the size of the electromagnets and the associated power requirements.

3. A robust nonlinear control algorithm based on geometric feedback linearization
and discrete time-delay control is formulated, for compensation of unmatched
uncertainties occurring in magnetic suspension actuators. In comparison to pre­
vious approaches, this algorithm does not restrict the uncertainty formulation.
but requires the sampling rates to be significantly faster than the maximum
frequency contents of the uncertainty.

4. A strategy for model based compensation of ferromagnetic hysteresis in electro­
magnetic actuators is developed. The essential characteristics of the hystere­
sis nonlinearity are captured through the Preisach independent domain model.
which is then incorporated into the robust nonlinear control architecture.

7.3 Future research

Once the configuration of the electromagnetic actuation scheme has been finalized
based on the principles formulated in Chapter (3) for the case of DC electromagnets.
and in Chapter (4) for PEM combinations, a force distribution algorithm needs to
be synthesized. As detailed in Chapter (3), there are many possible solutions to the
force distribution problem given by Equation (3.11). The particular solution may
be derived in a number of ways, including the pseudo-inverse approach suggested in
Chapter (3). The homogeneous solution involves a free parameter λ, that must be
scaled to make the total solution positive. An approach for the appropriate selection
of this free parameter in relationship to the particular solution, while minimizing the

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current and slew rate demanded of the power supplies, needs to investigated for the
development of the complete control architecture.

The realization of the desired characteristics in the Magnetic Suspension Actuator
also depends on the properties of the sensing and the electronic systems. The sensing
system must consist of non-contacting sensors that transduce flotor displacements in
six degrees of freedom. For this purpose, laser interferometric sensors are usually
employed. Laser interferometric sensors provide resolution upto 1.2 nanometers with
a sensing range of 40 meters, and a response bandwidth of upto 10 KHz. However,
current laser interferometer setups are limited in their angular range to a total dis­
placement of less than 1 degree when measuring large linear travel. This characteristic
limits the maximum angular travel realizable from the MSA. Consequently, the design
and development of a six degree of freedom laser interferometric sensing system that
maximizes the angular range is an important issue in the development of the large
travel MSA.

The electronic systems include the real time buses, the power supplies, the proces­
sors and the interface boards. The real time control architecture has to be designed to
implement elevated sampling rates of 1000-2000 Hertz within a system that generates
control and sampling actions at accurate intervals of time, and minimizes variations
in the sampling periods. These requirements will necessitate the use of high speed
digital signal processors, with dedicated algorithms to perform specific tasks. The
variability in the control and sampling actions generated with such a hardware needs
to be experimentally determined, and its effect on system performance analyzed.

The above discussion points to the major issues that need to be studied in context
of large travel Magnetic Suspension Actuators. By formulating the principles for the
design and development of MSA, this dissertation establishes a foundation for the successful prototyping of such systems.


