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MODELING OF GROUND GRID AND METALLIC STRUCTURE CURRENTS 
AND ANALYSIS OF THEIR EFFECT ON THE MAGNETIC FIELD 
IN HIGH VOLTAGE AC SUBSTATIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for 
the Degree Doctor of Philosophy in the 
Graduate School of The Ohio State University

by

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*****

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ABSTRACT

There has been a strong interest in power frequency magnetic field generated by high voltage facilities due to the public concern of their effect on the human health. Currents are the source of magnetic field. Therefore, a complete knowledge of currents in high voltage ac substations is necessary to determine the magnetic field distribution in and near the substations. The most difficult currents to determine are those in substation ground grids and metallic structures. A comprehensive review of previous research on modeling and calculation of the current distribution is presented first. A mathematical model for accurately computing the ground grid and metallic structure currents is then developed. In the model, the ground grid is divided by finite segments and accurate equations for the ground grid segment leakage current distribution are used. The developed model considers not only the current leakage to the soil but also resistive and inductive effects, and can be applied to any configuration of ground grids and metallic structures. Finally, the accuracy of the model is demonstrated by comparison between the results of calculations, and field measurements. Comparison is also made between the calculated and measured magnetic field flux density. In addition, all current sources in the substations and their contributions to the magnetic fields are discussed.
The developed mathematical model is essential for accurately modeling of substation current distribution, for investigation of magnetic field distribution and magnetic field management in power substations, and for determination of ground grid resistance, potential distribution and ground grid design.
Dedicated to my family
ACKNOWLEDGMENTS

I would like to express my profound gratitude to my advisor, Professor Donald G. Kasten for his invaluable guidance, encouragement and support which make this dissertation possible, and for all his time and efforts spent in every phase of this study.

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My special thanks go to my husband, Xinda Li, for his understanding, support and assistance. Additional thanks to my son, David Li, for his smiles and being my joy and pride. My sincere thanks to my mother and father for their continuous encouragement and help.
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<tr>
<td>B</td>
<td>magnetic flux density</td>
<td>Wb m$^{-2}$ or T</td>
</tr>
<tr>
<td></td>
<td>(1T=1Wb m$^{-2}$ )</td>
<td></td>
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<tr>
<td>Bj</td>
<td>magnetic flux density produced by jth segment</td>
<td>Wb m$^{-2}$ or T</td>
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<tr>
<td>d</td>
<td>distance between two segments</td>
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<td>V m$^{-1}$</td>
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<td>I_i</td>
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<tr>
<td>$l$</td>
<td>conductor length $\text{m}$</td>
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<td>$l_{rod}$</td>
<td>length of ground rod $\text{m}$</td>
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<td>$L$</td>
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<td>$n$</td>
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<td>$O_j$</td>
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</tr>
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<td>$\alpha$</td>
<td>$= \tan^{-1}(k_s)$</td>
<td>rad</td>
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<tr>
<td>$\delta_j$</td>
<td>current density distribution factor</td>
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<tr>
<td>$\theta$</td>
<td>angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\mu$</td>
<td>permeability of medium</td>
<td>H m⁻¹</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>permeability of free space ($= 4 \pi \times 10^{-7}$)</td>
<td>H m⁻¹</td>
</tr>
<tr>
<td>$\rho$</td>
<td>resistivity</td>
<td>Ω m</td>
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\( \sigma \) conductivity \( \Omega^{-1} \text{ m}^{-1} \)

\( \phi_i' \) potential induced on segment \( j \) due to unit current flowing from segment \( k \) into earth \( \text{V} \)

\( \phi \) magnetic flux (linkage) \( \text{Wb} \)

\( \phi_{ik} \) magnetic flux through \( i \)th segment due to current of \( k \)th segment \( \text{Wb} \)

\( \omega \) angular frequency \( \text{rad/sec} \)

*Note: Where there is an arrow "→" above the symbol such as \( \vec{B} \), it represents a vector symbol with direction as shown in the dissertation; where there is "^" above the symbol such as \( \hat{l} \), it represents a complex vector.*
CHAPTER 1
INTRODUCTION

1.1 Importance of the Dissertation and Statement of the Problem

There has been a strong interest across the United States and worldwide, in recent years, in power frequency (50 or 60 Hz) magnetic fields due to the public concern that exposure to magnetic fields might have a harmful effect on human health [1-3]. A great deal of investigation has been conducted since early 1979 when Edward Leeper and Nancy Wertheimer [4] first suggested close correlation between power frequency magnetic fields and certain forms of leukemia and brain cancer. The United States Energy Policy Act of 1992 underscored the national importance of the issue of electromagnetic field exposure and the U.S. Department of Energy established an EMF RAPID Program in May 1994 [5,6] as a reaction to the enactment of the Energy Act. Many researchers worldwide have focused on magnetic fields created by power lines and high voltage substation equipment, because they produce continuous and strong magnetic fields. The studies have attempted to quantify the magnetic field effect, but no one has presented a clear answer to the problem of the impact of magnetic fields on the human body.

Besides the public health concern described above, the increasing use of electronic equipment requires an accurate prediction of electromagnetic fields generated by power
system components, since they can be exposed to the electromagnetic fields. Furthermore, knowledge of the magnetic field distribution is important and necessary for ground grid and substation design.

Therefore, a complete understanding and an accurate determination of the magnetic field near electric power transmission and distribution lines and high voltage ac substations is very important. Many papers [7-16] are concerned with magnetic fields generated by power lines. Only a few studies were conducted in the area of magnetic field environment in and near substations [17-28]. EPRI founded several research projects at The Ohio State University for determining the electromagnetic field distribution in and around high voltage ac substations [20-21, 25-27]. Principal investigators were Professors Kasten and Sebo. It is a complicated task to accurately predict magnetic fields in substations due to many factors influencing the magnetic fields. Currents are the source of magnetic fields; the magnetic field strength is critically dependent on the magnitude and phase angle of these currents. As long as all the possible source currents are accurately determined, the magnetic field strength can be calculated using the models in papers [26,27]. Therefore, an essential step in defining a strategy to manage magnetic fields is to accurately predict the various field sources. In general, it is very difficult to predict all the sources of the magnetic fields in substations because of its complex configuration. In this dissertation, various source currents are discussed. Then, a literature review related to prediction of those source currents and considerations in the magnetic field study is presented. A mathematical model to accurately determine the ground grid and metallic structure current distribution in substations is developed. The model is then implemented
in a computer program to calculate current distribution of any configuration of the ground grid and metallic structures. The calculation results are compared with field measurements and with available results from other research efforts. The accuracy of the model is therefore demonstrated.

1.2 Identification of Magnetic Field Sources in AC Substations

A substation is a complex concentration of magnetic field sources. The main equipment in high voltage ac substations are transformers, switchgear, reactors, buswork and the incoming and outgoing power lines. Although the buswork and power lines are the dominant current carrying components, other source currents may provide a significant contribution to the magnetic field, magnitude and phase angle. In this section, the other factors which cannot be neglected when performing magnetic field strength calculations will be discussed.

- Neutral wire currents

Adding to the complexity of the substation field sources is the fact that the feeder loads are not fully balanced. When the system is unbalanced, significant return currents may flow in neutral lines. Currents in the neutral wires cause deviation of the magnetic flux density magnitude compared with that under the assumption of balanced conditions [7,10,18,20,25].

As power line and bus currents can be measured, the neutral line currents can also be obtained by measurements. Using these measured currents as input to a computer
program, the magnetic field strength can easily be calculated based on known theoretical formulae.

- **Ground grid currents**

  As mentioned above, if the currents are unbalanced due to unbalanced loads on the power lines, especially for the outgoing lines which are closer to the loads, there are return currents flowing in both neutral wires and into the earth.

  In and around substations, some of the return currents flow through buried conductors. The value and direction of the ground conductor currents are closely related to the location of the buried conductors, transformers, power lines, buswork and other current carrying equipment. The ground conductor currents contribute to the magnetic field [24,25]. Because the ground conductors are buried in soil, it is difficult to conduct current measurements on the ground grid.

- **Metallic structure currents**

  There is inductive coupling between various current carrying components in substations such as metallic structures and ground grid conductors. The eddy currents in metallic structures and other induced currents in substations cause an associated magnetic field which must be considered where considering fields close to the structures.

  As discussed above, the most difficult currents to determine are those in substation ground grids and metallic structures, since it is difficult to measure currents in the buried ground grid and in metallic structures.
1.3 Current Considerations for Unbalanced Power Systems

As discussed in section 1.2, when a power system is unbalanced, significant return currents may flow in the ground grid and metallic structures. The following will discuss the possible system conditions and various system configurations under which the ground grid and metallic structure currents could be significant.

- Balanced and symmetrical system

For a balanced and symmetrical three-phase power system, the 60Hz currents and voltage are equal in magnitude and 120 degrees apart in phase. Therefore, there is no current flowing in the neutral conductor, ground grid or metallic structures under balanced conditions for a three phase power system. However, the power system is usually unbalanced. System unbalance is caused by unbalanced phase impedances or unbalanced loads, or the combination of both. In general, it is the load unbalance which is the major cause of system unbalance.

- Unbalanced load

Three phase power systems are designed to deliver large quantities of power to a great number of diverse loads of single phase and three phase. In order to maintain a balanced system, single phase loads are distributed as equal as possible among the three phases. In lower voltage (LV) power distribution systems, the individual loads are changing with time and therefore frequently unbalanced. In addition, unbalanced load conditions can arise on a medium voltage (MV) system, where single phase distribution
transformers and spur-line supplies are tapped off the three phase network and these loads are not balanced across the three phases.

- **Power system configurations under unbalanced conditions**

  As discussed above, the power system will in general be unbalanced both in amplitude and phase angle due to unbalanced loads on the line, especially on distribution lines. The unbalanced return current will flow through either the neutral wire or the earth if a ground current return path is provided or both.

  For delta-wye connected transformers, the secondary usually has a grounded neutral. Therefore, there is a path for ground current when the system is unbalanced. System unbalance will also result in ground currents for a wye-wye transformer connection with both primary and secondary neutrals grounded. However, there will be no current return path and therefore no ground current when either primary or secondary neutral is not grounded. For a delta-delta connection of a transformer, there is no inherent means for providing a neutral ground.

  Generally, a distribution transformer has delta-wye configuration with grounded neutral, and significant currents may flow in the ground grid due to unbalanced loads. Where the grounded neutral wire is attached to the metallic structure in a substation, the unbalanced currents will flow into the structure and the current in the metallic structure has to be considered.
1.4 Objectives and Contributions of the Dissertation

As discussed above, the most difficult currents to determine in the power substations are those in substation grounding grids and in metallic support structures. The objectives of this research are to develop a mathematical model so that the ground grid and metallic structure currents can be accurately calculated and magnetic fields generated by those currents can then be determined.

The major contributions of this dissertation are:

1. An accurate mathematical model is developed which can calculate current distribution both in the ground grid and in metallic support structures.
2. The mutual inductance between conductor segments is included for all possible positions, configurations and directions in the proposed model.
3. Voltage drop in ground grid conductors due to resistance, self and mutual inductance is considered in leakage current calculations; leakage current from ground rod can be calculated.
4. Both leakage current and longitudinal current within the ground grid are considered in the proposed model.
5. Validity of the model is demonstrated by comparison with field measurements.
1.5 Outline of the Dissertation

The dissertation is composed of 7 chapters and 2 appendices. Following the Introduction, Chapter 2 gives a comprehensive review of related literature, including substation magnetic field calculations generated by different current sources and substation ground grid modeling methods for current distribution. Chapters 3 and 4 describe the modeling techniques for ground grid and steel structure currents in the power substations, considering leakage, self and mutual effects on the current distribution. In Chapter 5, the developed mathematical model is applied to the calculation of current distribution in a practical substation for which the measurement data is available. Comparison between the calculated and measured results will demonstrate the accuracy of the modeling techniques. Chapter 6 shows a comparison between the calculated and measured magnetic field flux density and describes all current sources in the substation and their effects on the magnetic fields. Chapter 7 summarizes the conclusions and future work.

Appendices A and B list the calculated results and the developed computer programs, respectively.
A great deal of research [6-16] has been conducted on the magnetic fields generated by power lines since the early 1970’s. In the past ten years, a few studies [17-28] have been reported on magnetic fields produced by ac substations. Prof. Sebo et al developed scale models to investigate the electromagnetic field distribution in ac substations [19-21]. Papers studied substation magnetic fields by measurements [22-25]. This dissertation emphasis is on the calculation of power frequency magnetic fields and the field sources from high voltage ac substations.

2.1 Three - Dimensional Magnetic Field Calculations in Substations

Prof. Kasten et al [26,27] first developed a computer program to predict the magnetic field distribution of high voltage ac substations by using the Biot-Savart law as described below. It is assumed that all currents in a substation are confined to wires and conductors, and the earth is nonmagnetic. Then, by using the Biot-Savart law the magnetic flux density can be expressed as (refer to Fig. 2.1)
where \( \mu_0 \) is the permeability of free space \( (\mu_0 = 4\pi \times 10^{-7} \text{ H} \cdot \text{m}^{-1}) \), \( \vec{r}_i \) is a vector from the source to the field point, \( \hat{I} \) is the path current (complex quantity), and \( \vec{dl} \) is a differential element at \( l \) in the direction of the current reference.

![Diagram](image)

**Figure 2.1: Description of variables for the Biot-Savart law**

A closed form solution of the Biot-Savart law (Eq. 2.1) has been developed as shown in Eq. 2.2 for straight line segments (see Fig. 2.2) [26]. The resulting magnetic flux density can be calculated by approximating the current path as straight lines and setting the current constant in each segment.
\[ \hat{\mathbf{B}}_j = \mu_0 \sum_{i=1}^{N} \frac{\hat{I}_i}{4\pi|\vec{r}_{ij}|} \left( \sin \theta_{i1} + \sin \theta_{i2} \right) \hat{a}_i \times \hat{a}_y \]

(2.2)

where \( N \) is the number of segments, \( \hat{I}_i \) is the current in segment \( i \), \( \vec{r}_{ij} \) is the vector from the center of segment \( i \) to \( j \), \( \hat{a}_i \) is a unit vector in the direction of segment \( i \), and \( \hat{a}_y \) is a unit vector in the direction of vector \( \vec{r}_y \).

Fig. 2.2 Discrete approximation of the Biot-Savart law

Based on the above theory, a computer program has been developed. The validity of the program was demonstrated by comparison with the scale model and field test results of the magnetic fields. The program has been applied to predict the magnetic field in and
around a few practical substations such as the Bixby 345kV switchyard and the Madison 69/13kV substation. All currents used in the magnetic field calculation were obtained by measurements.

Hayashi et al [28] also used Biot-Savart's law to develop their magnetic field calculation program. The calculation of 60Hz magnetic field was conducted in the 187kV switchyard of a 187/66kV substation.

2.2 Substation Magnetic Field Generated by Power Lines and Buses

Most studies [18-23] on the magnetic field in substations involve only power lines and buses and their source currents. For example, the authors of paper [28] have made the following assumptions for the calculation of substation magnetic fields:

1. The three phase currents are completely balanced so that there are no neutral wire currents;
2. Currents in the ground grid conductors are neglected;
3. Field distortion due to steel structure is ignored;
4. Induced currents on the counterpoise wires and ground wires are ignored.

Based on the above assumptions, the magnetic field is only influenced by currents and the configuration of transmission lines and buses. Therefore, they only discussed the following obvious parameters which influence the value of magnetic field: current distribution on the transmission lines and buses; bus phasing arrangement and the height and spacing of the transmission lines and buses. The ground grid and metallic structure current contribution to the magnetic field is neglected.
Olsen and Lyon [29] found an equivalent source to replace the actual sources by processing magnetic flux density measurement data. This method used the maximum likelihood estimation technique to calculate the coefficients of multiple spherical and finite length cylindrical sources. The resulting equivalent source then was used to calculate the magnetic field strength. This paper only discussed the current sources from buses and transformers. Other sources, such as ground grid current and induced currents in metallic structures, were not discussed in this paper.

2.3 Substation Magnetic Field Generated by Other Sources

Except the most obvious parameters of power lines and buses as mentioned above which significantly influence the magnetic field, many other factors such as current unbalance (phase and amplitude), ground currents and metallic structure currents may have a significant contribution to the magnetic field. However, there are only a few publications with discussions of factors that may contribute to the total magnetic field.

Recently, Hayashi et al [30] investigated the effects of eddy currents in conductive objects by numerically estimating the magnetic flux density in the vicinity of the objects using the $A^* - \Omega$ method, the modified vector potential method. The $A^* - \Omega$ method employs a modified vector potential $A^*$ in the conducting regions and a scalar potential $\Omega$ in the non-conducting regions. The conductive objects employed are of three configurations: square plate, cylindrical bar and H-shaped bar modeling metal materials used in ac substations. The three shaped conductive objects are exposed to a uniform magnetic field of power frequency. The calculated magnetic flux density in the vicinity of
the objects shows that the original uniform field is influenced by the conductive objects. Actually, the magnetic field in substations is highly nonuniform, which makes it difficult to predict the influence on the magnetic field by induced eddy currents in metallic structures. The paper did not calculate the magnetic flux density in practical substations with metallic structures present.

Kasten and Sebo first made measurements of metallic structure currents in a specific substation by measuring the currents in the interconnected conductors between the structure and the ground grid and quantitatively discussed the magnetic field distortion due to metallic structure currents [25,31]. It is very difficult to measure structure currents and no other report has been found for conducting the structure current measurements. In addition, there is no publication concerning metallic structure current calculations. Therefore, it is necessary to develop a model to calculate the metallic structure current in substations.

Kasten and Sebo et al [25,31] discuss the influence of other current carrying objects such as neutral lines, metal structures and ground conductors on the magnitude of the magnetic field by conducting measurements in a 69kV/13kV substation. Daily and Dawalibi [24] discuss magnetic fields generated by a typical distribution substation, considering currents in the grounding systems, distribution feeder neutrals, overhead ground wires and induced currents in equipment structures and ground grid loops.

Only a few papers include the modeling and calculation of substation ground grid currents for magnetic field studies [32, 38-40]. However, there are a few studies which have been conducted on the ground grid current distribution for different purposes. The
following literature review will discuss all the modeling techniques for substation ground grid current distribution.

2.4 Modeling of Ground Grid Currents in Substations

2.4.1 Dawalibi’s Method for Calculating Single Ground Conductor Current Distribution

Dawalibi [32] first calculated currents in a buried short conductor in his study on magnetic field generated by overhead and buried short conductors. Fig.2.3 shows a buried conductor with a length of $l$. It is assumed that $I_l$ and $I_r$ (longitudinal currents at the extremities of the conductor) and the total leakage current, $S$, are known. Then, the longitudinal current at any point $u$ along the conductor can be calculated as follows:

$$I(u) = a \cdot u + b$$

(2.3)

where $a = S/l$ and $b = I_l$ for $0 < u < l/2$; $a = -S/l$ and $b = I_r + S$ for $l/2 < u < l$.
Eq. 2.3 is derived by assuming that the current is leaked uniformly from the conductor into the soil.

2.4.2 Earth Leakage Current Calculations

The purpose of ground grid leakage current calculation for all of the following reviewed papers is to restrict the touch voltage and step voltage within safe limits, and to keep ground resistance or ground potential rise (GPR) small [42]. Ground resistance is calculated by dividing ground potential rise by the injected current to the ground grid. GPR is defined as the maximum voltage that a station grounding grid may attain relative to a distant grounding point to be at the potential of remote earth. Touch voltage is defined as the potential difference between the GPR and the earth surface potential at the point where a person is standing, while at the same time having their hands in contact with a grounded structure. Step voltage is the difference in earth surface potential experienced by a person bridging a distance of 1 meter with their feet without contacting any other grounded object [42]. Knowledge of the leakage current is essential to obtain the GPR and ground potential difference.

The earth leakage current of a ground conductor is non-uniform. Dawalibi and Mukhedkar [33] presented a method to calculate the leakage current distribution considering non-uniform leakage currents. In their paper, a ground grid was first divided into n segments with the same length, du. Then the current in segment j is

\[ i_j = \delta_j i \, du \quad i = I/L \] (2.4)
where $I_j$ is the total current leaked to earth from segment $j$; $\delta_j$ is the current density distribution factor which takes into account the non-uniform current density along segment $j$; $I$ is the total injected current into the ground grid; $L$ is the total length of ground grid, and $n$ is the number of segments. It is obvious that

$$I = I_1 + I_2 + \ldots + I_j + \ldots + I_n$$

$$n = L / d_u$$

(2.5)

Dawalibi and Mukhedkar derived the following equation by solving Eqs. 2.4 and 2.5:

$$\sum_{j=1}^{n} \delta_j = n$$

(2.6)

The following formula was proposed to determine $\delta_j$:

$$\delta_j = \alpha \frac{d_{qj}}{d_{fj}}$$

(2.7)

where $\alpha$ is a constant chosen to satisfy Eq. 2.6; $d_{qj}$ is the distance between segment $j$ and the center of the ground grid to take into account the effect of ground grid configuration; $d_{fj}$ is the distance between segment $j$ and the fault location to take into account the effect of fault proximity. It can be seen that $\alpha$ is an estimated value and therefore $\delta_j$ is not an accurate factor when taking into account non-uniform current distribution.

Later, Dawalibi and Mukhedkar [34, 35] used another method to calculate the non-uniform current density distribution factor $\delta_j$ by solving the following simultaneous equations:
\[
\delta_1 V_{11} + \delta_2 V_{21} + \ldots + \delta_i V_{1i} + \ldots + \delta_n V_{ni} = V
\]
\[
\delta_1 V_{12} + \delta_2 V_{22} + \ldots + \delta_i V_{12} + \ldots + \delta_n V_{n2} = V
\]
\[
\ldots
\]
\[
\delta_1 V_{ij} + \delta_2 V_{2j} + \ldots + \delta_i V_{ij} + \ldots + \delta_n V_{nj} = V
\]
\[
\ldots
\]
\[
\delta_1 V_{1n} + \delta_2 V_{2n} + \ldots + \delta_i V_{in} + \ldots + \delta_n V_{nn} = V
\]

where \( V \) is the ground grid potential rise; \( V_{ij} \) is the potential induced by segment \( i \) on segment \( j \). Eq. 2.8 was derived by assuming that the ground grid elements are at the same potential.

Heppe [36] further deduced a set of equations to calculate the induced potential, \( V_{ij} \), for parallel, perpendicular and angled line segments as shown in Fig. 2.4. As segment \( i \) is close to segment \( j \), the line source equations will lead to errors. For this reason, Kouteynikoff [37] considered the \( i \)th segment of cylindrical shape as segment \( i \) is close to segment \( j \) and induced potential on segment \( i \) due to segment \( j \), \( \phi_i' \), was calculated as follows:

\[
\phi_i'(r,z) = \frac{I \rho}{\pi^2 ab} \int_0^{l_j} \frac{K_0(\lambda r)}{K_1(\lambda a)} \cos \lambda z \sin \frac{\lambda b d\lambda}{2 \lambda^2}
\]  

(2.9)

where \( a \) is the radius of the grid element, \( b \) is the length of \( j \)th segment \( (l_j) \), \( r \) and \( z \) are coordinates at the induced point on segment \( i \), \( K_0 \) and \( K_1 \) are the second kind of modified Bessel functions of the zero and first order, respectively.

The induced potential, \( V_{ij} \), is the mean value of \( \phi_i' \) when the point moves along segment \( i \):
\[ V_y = \frac{1}{l_i} \int \phi' ds \]  

(2.10)

where \( l_i \) is the length of the ith element and \( s \) is measured along the ith segment.

**Figure 2.4: Angled segments**

### 2.4.3 Dawalibi's Method to Calculate Longitudinal Currents of a Ground Grid

Modeling of ground grid leakage currents as discussed above is based on the assumption that all points on the ground grid surface are at the same potential. In the study on magnetic field generated by overhead and buried short conductors, Dawalibi [38] further pointed out that there is a voltage drop in a ground grid conductor when it is
subject to longitudinal currents. Therefore, it is necessary to consider the self-impedance of ground grid conductors and the mutual effects between conductors. In [38], the self-impedance of a conductor is expressed as the sum of internal and external impedances. The internal impedance was calculated using the well established equation for a cylindrical conductor. The external impedance is expressed as the mutual impedance between an infinitesimally thin filament at the center of the conductor and an insulated filament at its surface.

As shown in Fig. 2.5, the mutual impedance between two conductors insulated from each other of finite lengths with angle $\beta$ is given by the following equations [38]:

$$Z_m = Q_m + j\omega L_m \tag{2.11}$$

$$L_m = \cos \beta \int_{c}^{d} \int_{A}^{B} P(R) ds dt \tag{2.12}$$

$$Q_m = Q(AC) - Q(AD) + Q(BD) - Q(BC) \tag{2.13}$$

$$P(R) = \frac{j\omega \mu}{2\pi} \cdot \frac{1 - (1 + \gamma R)e^{-\gamma R}}{\gamma^2 R^2} \tag{2.14}$$

$$Q(R) = \frac{1}{2\pi(\sigma + j\omega\varepsilon)} \cdot \frac{e^{-\gamma R}}{R} \tag{2.15}$$

where $R$ is the radial distance between element ds and dt, and $\sigma$, $\mu$, $\varepsilon$, $\gamma$ are the conductivity, permeability, permittivity and propagation constant of soil respectively.
Taking into account the conductor self and mutual effects, Dawalibi [38] used electromagnetic field theory and a hybrid approach to solve a set of simultaneous linear equations to obtain the ground grid segment longitudinal currents. The hybrid method is based on both electromagnetic field theory and electric circuit concepts.

(1) Field Theory Approach [38]

Electromagnetic theory shows that

$$E = - (\nabla V + j \omega A)$$

where $V$ is the electric potential, $A$ is the magnetic potential and $E$ is the electric field strength.
By solving the above fundamental electromagnetic equation, Dawalibi [38] derived the following equation:

\[ Z_k I_k - Y_k^{-1} \frac{d^2 I_k}{du^2} + j \omega \sum_{i=1}^{n} A_{ik} + \sum_{i=1}^{n} \frac{\partial V_{ik}}{\partial u} = 0 \]  

(2.17)

The longitudinal current, \( I_k \), at any point, \( u \), along segment \( k \) is given by

\[ I_k(u) = a_k u + b_k \]  

(2.18)

where \( Z_k \) is the unit length internal impedance of the ground conductor; \( Y_k \) is the unit length admittance of the ground conductor; \( V_{ik} \) is the potential contributions of segment \( i \) on segment \( k \). Similarly, \( A_{ik} \) is the magnetic potential contribution of segment \( i \) on segment \( k \).

Assuming that the grid conductors are divided into \( n \) segments, Eqs. 2.17 and 2.18 generate \( 2n \) unknowns represented by the coefficients \( a_k \) and \( b_k \), and therefore introduce \( 2n \times 2n \) simultaneous linear equations to solve for \( a_k \) and \( b_k \). It can be seen that the above approach involves complicated analytical expressions and introduces computation difficulties. Dawalibi [38] then uses the following hybrid approach to simplify the above model.

(2) Hybrid Approach [38]

The first assumption for the hybrid approach is: \( Y_k = \infty \) \((Y_k^{-1} = 0)\) for uncoated conductors, resulting that the second term of Eq. 2.17 tends to approach zero. The following equation (Eq. 2.19) is obtained by integrating Eq. 2.17 from left to right along the portion of the segment as shown in Fig. 2.6:
The other assumption for the hybrid approach is that the longitudinal current in one half of segment $i$ is equal to the current at the closest conductor node. Based on this assumption, the longitudinal current can be taken out of the integral sign of Eq. 2.19 and the following equation is obtained:

$$\int_s^f Z_k I_k + j\omega \sum_i \int_s^f A_{ik} + \sum_i [V_{ik}(f) - V_{ik}(s)] = 0$$ (2.19)

where $Z_{ik} = j\omega \int_s^f A_{ik} / I_i$, $U_k = \sum_i [V_{ik}(f) - V_{ik}(s)]$ and 

Figure 2.6: Simplified representation of a conductor segment [38]
the subscript \( p \) indicates that the current applies to either left or right portion of the segment.

At zero frequency, \( \omega = 0 \) \( (Z_a = j \omega \int \frac{A}{I} = 0) \), Eq. 2.20 becomes:

\[
Z_k \frac{I_k}{2} + U_k = 0
\]  

(2.21)

Eq. 2.21 can be reduced to \( n \times n \) simultaneous equations by solving \( n \) unknown segment currents.

Due to the assumptions, the accuracy of the above model depends on frequency, ground grid configuration and the number of ground grid segments.

### 2.4.4 Network Approach for Current Distribution by Mader and Zaffanella

Mader and Zaffanella [39] presented an electrical network model and formulas to calculate resistance and self-inductance of conductors and pipes and mutual inductance between closely spaced conductors in a residential distribution system. The resistance of the \( k \)th segment of a ground grid, \( R_k \), was calculated as:

\[
R_k = \frac{\rho \cdot l_k}{A}
\]  

(2.22)

where \( \rho \) is the specific resistivity of the material in ohm-meters, \( l_k \) is the length of the \( k \)th segment in meters, and \( A \) is the effective cross sectional area in \( m^2 \).

The self-inductance, \( L_k \), of a ground grid pipe segment \( k \) and the mutual inductance, \( M_{ik} \), for a pair of side-by-side parallel pipe segments, \( i \) and \( k \), of the same length, \( l \), were calculated using equations from Grover [43]:

24
\[ L_k = 2 \times 10^{-7} \left[ \ln \left( \frac{2l}{r_1} \right) - 1 + \ln(\zeta) \right] \]  
\hspace{1cm} (2.23)

\[ \ln(\zeta) = 0.25 \left( 1 - \left( \frac{r_1}{r_2} \right)^{1.57} \right) \]  
\hspace{1cm} (2.24)

\[ M_{\text{MK}} = 2 \times 10^{-7} \left[ \ln \left( \frac{l}{d} + \sqrt{1 + \frac{l^2}{d^2}} \right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right] \]  
\hspace{1cm} (2.25)

where \( r_1 \) and \( r_2 \) are outer and inner radii of a ground pipe respectively; \( d \) is the distance between the \( i \)th and \( k \)th ground pipe segments.

Then the coefficient of mutual inductance \( K \) was obtained by

\[ K = \frac{M}{\sqrt{L_i L_k}} \]  
\hspace{1cm} (2.26)

where \( L_i \) and \( L_k \) are the self inductances of the two parallel segment conductors. By inputting the value of \( K \) into commercial software (PSPICE, MicroSim Corporation, Irvine, California), the ground current distribution can be calculated.

There is inductive coupling between each conductor and every other conductor. In this paper [39], the equation of mutual inductance is limited to a pair of side-by-side parallel conductors of the same length. The leakage currents are neglected. In addition, the model can only be applied to distribution wires and water pipes.
2.4.5 Selby and Dawalibi's Approach for Current Distribution at High and Low Frequencies

Selby and Dawalibi [40] discussed an alternative method to determine the current distribution in networks of overhead and buried conductors. Their approach for the high frequency range (30-300 MHz) is based on antenna theory and least square point matching technique. The approach for the low frequency range employed a power minimization algorithm. The induced currents are determined by explicitly imposing Faraday's law. This method imposed some constraints such as constant current flows along conductor segments, Kirchhoff's law for current flows at nodes, and minimal Joule heat loss. The method was not applied to the solution of the current distribution for any configuration of ground grid conductors. This paper discussed only the leakage current distribution of a simple ground grid configuration.

2.5 Summary and Conclusions

Most studies have not included the magnetic field generated by ground grid and metallic structures. A few researchers have discussed the contribution of magnetic field from ground grid and metallic structures by measurement. None of the publications has discussed and calculated metallic structure currents in substations.

Most methods for modeling and calculation of ground grid current distributions have the following limitations:
(1) A ground grid is divided by \( n \) segments so that leakage current along each segment is uniform. When the segment is not short enough or the segment is at a ground grid intersection, its leakage current is non-uniform.

(2) It is assumed that the potential at the ground grid is the same at all points on the surface of the grid. However, there are voltage drops in ground grid conductors. In a few papers, resistance, self impedance and mutual impedance of each ground grid segment are included in current calculations. But their methods are limited to the linear filaments.

(3) Ground rod leakage currents are neglected.

(4) The network approach did not include leakage currents in the modeling method.

In addition, previous leakage current models are only used for substation ground grid design and are not applied for magnetic field studies.

Therefore, a more elaborate and accurate model needs to be developed for calculating the current distribution both in the ground grid of any configurations and in the metallic support structures. Leakage currents from ground rods shall also be included. Both leakage currents and network currents within the ground grid shall be considered.
CHAPTER 3

MODELING OF GROUND GRID

LEAKAGE CURRENT DISTRIBUTION

A substation ground grid is composed of horizontal conductors and vertical ground rods buried in the earth. Fig. 3.1 shows an example of a typical ground grid. The

![Diagram of a ground grid with labels for parallel with x axis, parallel with y axis, and ground rod.]

Figure 3.1: An example of ground grid
purpose of a substation grounding grid is to keep the step and touch voltages within the safety tolerance limits for personnel and to keep the ground resistance small. At normal operation conditions, there may be multi-currents injected in the ground grid. The injected currents will flow through the ground grid and part of the current will leak into the soil. The following mathematical model is developed to accurately determine the ground grid leakage current distribution.

In this chapter, an accurate modeling method, Green function method, is used for the calculation of mutual resistance. Boundary element technique is applied to the calculation of ground grid leakage currents. The expression of potential, \( \phi \), induced by a cylindrical segment with uniform current density is derived.

### 3.1 Theory of Electromagnetics

Assuming a current \( I \) is entering a ground grid embedded in uniform soil with soil resistivity \( \rho \), by theory of electromagnetics [44], the potential, \( U \), at any point, \( P(x,y,z) \), can be represented by Laplace’s equation as follows:

\[
\nabla \frac{1}{\rho} \nabla U = 0
\]

(3.1)

The electric field strength is

\[
\vec{E} = -\nabla U = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} - \frac{\partial U}{\partial z} \hat{k}
\]

(3.2)

The current density is

\[
\vec{J} = \sigma \vec{E}
\]

(3.3)

where \( \sigma \) is the conductivity of the conducting medium.

By solving the above equations using Green’s function theorem [49], we obtain
\begin{equation}
U(P) = \iint_{s} G(P,Q)J(Q)ds
\end{equation} \tag{3.4}

where \(G(P,Q)\) is Green's function presenting the potential induced at point \(P\) by a unit current flowing into a surface point \(Q\) on a ground grid conductor, which is related to the geometry of the grid conductor.

The total leakage current is
\begin{equation}
I = \iint_{s} J(Q)ds
\end{equation} \tag{3.5}

### 3.2 Boundary Element Method [37,45,46]

By breaking the ground grid into \(n\) linear segments, the leakage current is given by
\begin{equation}
I = \sum_{j=1}^{n} I_{j}
\end{equation} \tag{3.6}

and the potential at point \(P\) is
\begin{equation}
U(P) = \sum_{j=1}^{n} G(P,O_{j})I_{j}
\end{equation} \tag{3.7}

where \(O_{j}\) is the center of segment \(j\), \(I_{j}\) is the current flowing from the \(j\)th segment into the soil and \(G(P,O_{j})\) is the potential at point \(P\) induced by unit current flowing through the \(j\)th segment.

For a point \(P\) located on the surface of segment \(i\), \(G(i,j)\) represents the mutual resistance between segments \(i\) and \(j\). As a result we have
\begin{equation}
U(i) = \sum_{j=1}^{n} R_{ij} \cdot I_{j} \quad \text{or} \quad \sum_{j=1}^{n} R_{ij} \cdot I_{j} - U(i) = 0 \quad (i=1,2...n)
\end{equation} \tag{3.8}
U(i) is the potential rise of segment i. Assume the potential at the reference point is $U_0$. Then the nodal voltage, $V_i$, can be calculated as described in Chapter 4. Therefore,

$$U(i) = U_0 + \frac{(V_i + V_{i+1})}{2}$$

Thus, Eq. 3.8 becomes

$$\sum_{j=1}^{n} R_{ij} \cdot I_j - U_0 = \frac{(V_i + V_{i+1})}{2} = V(i)$$

(3.9)

Total leakage current, $I_{\text{leak}}$, is the sum of the known injected current, $I_{ij}$, to the grid:

$$I = \sum_{ij} I_{ij} = I_{\text{leak}}$$

(3.10)

Then, Eq. 3.6 becomes:

$$\sum_{j} I_j = \sum_{ij} I_{ij} = I_{\text{leak}}$$

(3.11)

Eq. 3.9 and 3.11 can be expressed as the following simultaneous equations:

$$\begin{bmatrix}
R_{11} & R_{12} & R_{1n} & -1 \\
R_{21} & R_{22} & R_{2n} & -1 \\
\vdots & \vdots & \vdots & \vdots \\
R_{n1} & \cdots & R_{nn} & -1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix}
= 
\begin{bmatrix}
V(1) \\
V(2) \\
\vdots \\
V(n)
\end{bmatrix}
- 
\begin{bmatrix}
U_0 \\
I_{\text{leak}}
\end{bmatrix}$$

(3.12)

As long as $R_{ij}$ and $V(i)$ are known, the segment leakage current, $I_j$, and the ground grid reference point potential rise, $U_0$, can be calculated by solving Eq. (3.12).
3.3 Calculation of $R_j$ and $G(P,O_j)$, Green Function Method and Point Source Method

As described above, $G(P,O_j)$ is the potential induced at point $P$ when a unit current is injected into the earth by a single segment $j$. Where the distance from point $P$ to the $j$th segment is much longer than the length, $l_j$, of that segment, the segment can be considered as a point source. Then, we have

$$G(P,O_j) = \rho \cdot \left( \frac{1}{r} + \frac{1}{r'} \right) \sqrt{\frac{4\pi}{r}}$$ (3.13)

where $r$ is the distance between center point, $O_i$, of the $i$th segment and point $P$; $r'$ is the distance between the image point of $O_j$ and point $P$ ($r, r' \gg l_j$).

When $P$ is close to the $j$th segment, especially where $P$ is on the surface of the $j$th segment, the point source equation will lead to large errors. In this case, the Green function method as described below is used to calculate the mutual resistance, where, the $j$th segment is considered as a cylindrical shape. Solving Laplace’s equation (3.1) by separation of the variables, the following accurate expression is derived (the derivation will be presented in section 3.5):

$$\phi'_k(r,z) = \frac{\rho}{\pi^2 ab} \int_0^\infty \frac{K_0(\lambda r) \cos \lambda z \sin \frac{\lambda b}{2 \lambda^2}}{K_1(\lambda a)} \frac{d\lambda}{\lambda^2}$$ (3.14)

where $\phi'$ is the potential rise, $a$ is the radius of the grid element, $b$ is the length of the element, $r$ and $z$ are coordinates (see Fig. 3.2), $K_0$ and $K_1$ are the modified Bessel functions of the second kind of zero and first order respectively. The axis for the segment
is as shown in Fig. 3.2. The z axis passes through the center of the cylindrical segment and the r axis is radial to the segment. Point P lies on a plane formed by the axis r and z.

Assume the ground grid is buried in soil at depth h. By the method of images, an image source is used to meet boundary conditions at the surface of the earth, then we have

\[ G(P, O_j) = \phi_i^j(r, z) + \phi_i^j(r', z') \]  

where \( r' \) and \( z' \) is the new coordinates of the image source segment as shown in Fig. 3.3.

As indicated in Fig. 3.3, we have

\[ r' = \sqrt{r^2 + (2 \cdot h)^2} \quad z' = z \]  

The mutual resistance between \( i \) and \( j \) is the mean value of \( G(P, O_j) \) when P moves along \( i \):

\[ R_v = \frac{1}{l_i} \int G(P_{ij}, O_j) \, ds \]  

where, \( l_i \) is the length of the ith element.
Figure 3.3: Image of segment, j.

Figure 3.4: Mutual resistance approximation
The integral of Eq. 3.17 is very complicated and will require considerable computer time to accurately calculate $R_{ij}$. Therefore, the following approximate average method is used. Related to three points of the $i$th segment, $P_1$, $P_2$ and $P_3$ (two end points and the center point of the $i$th segment) as shown in Fig. 3.4, calculate $G(P_1, O_j)$, $G(P_2, O_j)$ and $G(P_3, O_j)$ using Eq. 3.14 and 3.15. The approximate value of $R_{ij}$ is taken as the average of those three point values:

$$R_{ij} = \frac{G(P_1, O_j) + G(P_2, O_j) + G(P_3, O_j)}{3} \quad (3.18)$$

Calculation [37] shows that the above approximate method is accurate. More points can be chosen on segment $i$ if the accuracy needs to be improved.

### 3.4 Expressions of $K_0$ and $K_1$ [42]

In Eq. 3.14, the modified Bessel’s functions of the second kind of zero and first order, $K_0$ and $K_1$, were required. They are expressed as follows:

$$K_0(x) = -\left[\ln(x/2) + \nu\right] \cdot I_0(x) + \sum_{k=1}^{\infty} \frac{1}{k^2} \left(1 + \frac{1}{2} + \ldots + \frac{1}{k}\right) \left(\frac{x}{2}\right)^{2k}$$

$$K_1(x) = \ln\left(\frac{x}{2} + \nu\right) \cdot I_1(x) - \frac{1}{2} \sum_{k=0}^{\infty} \frac{1}{k! (1+k)!} \left(\frac{x}{2}\right)^{2k+1} \left(\sum_{m=1}^{k} \frac{1}{m} \sum_{\substack{m \mid l \cdot k}} \frac{1}{l} \right) + \frac{1}{x}$$

where $\nu = 0.577216$;

$I_0(x)$ and $I_1(x)$ are the modified Bessel’s functions of the first kind of zero and first order with following expressions:
\begin{align*}
I_0(x) &= \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left( \frac{x}{2} \right)^{2k} ; \\
I_1(x) &= \sum_{k=0}^{\infty} \frac{1}{k!(k+1)!} \left( \frac{x}{2} \right)^{2k+1} 
\end{align*}

(3.21)

Fig. 3.5 shows the Bessel function variation with variable \( x \).

![Bessel function graph](image)

Figure 3.5: Modified Bessel function [47]

3.5 Derivation of the Equation for \( \psi \)

As described in section 3.2, when segment \( j \) is close to segment \( k \) considering the segment as a point source will lead to errors for mutual resistance calculation. In this case, the segment is considered as a cylindrical shape as shown in Fig. 3.2 and the accurate calculation for potential on segment \( j \) is expressed in Eq. 3.14. Derivation of Eq. 3.14 is presented in the following sections.
3.5.1 Problem Analysis

It is assumed that a ground grid is embedded in a homogeneous resistive medium which extends to infinity in all directions. Since the ground grid is divided into small segments, the following simplifications can be made (refer to Fig. 3.2):

(1) Current density, \( J \), is constant on the surface of each segment;

(2) Both ends of the segment are extended to infinity and the current density is zero on the extended surface;

(3) Presence of other segments of the ground grid is neglected.

Due to the above simplifications, the problem has perfect circular symmetry, thus allowing the use of cylindrical coordinates (Fig. 3.6).

Figure 3.6: Cylindrical coordinates
3.5.2 Theory of Electromagnetics

Assuming a current I is entering a grounding grid embedded in homogeneous medium with soil resistivity \( \rho \), the potential, \( \phi \), at any point, \( P(r, \theta, z) \), can be represented as Laplace's equation in cylindrical coordinates (see Fig. 3.6) as follows:

\[
\nabla^2 \phi(r, \theta, z) = \frac{1}{r} \frac{\partial}{\partial \theta} \left( r \frac{\partial \phi}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial z^2} = 0
\]  

(3.22)

It is obvious that \( \frac{\partial^2 \phi}{\partial \theta^2} = 0 \) because of circular symmetry. Then, Eq. (3.22) becomes:

\[
\nabla^2 \phi(r, \theta, z) = \frac{1}{r} \frac{\partial}{\partial \theta} \left( r \frac{\partial \phi}{\partial \theta} \right) + \frac{\partial^2 \phi}{\partial z^2} = 0
\]  

(3.23)

Further simplification of Eq. 3.23, we have

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]  

(3.24)

The electric field strength is \( \vec{E} = -\nabla \phi \)  

(3.25)

The current density is \( \vec{J} = \frac{\vec{E}}{\rho} = -\frac{\nabla \phi}{\rho} \)  

(3.26)

Therefore, based on the assumptions in section 3.5.1, we have

\[
J = \frac{1}{\rho} \left( \frac{\partial \phi(r, z)}{\partial r} \right)_{\rho a} = \begin{cases} 
-\sigma & |z| < b / 2 \\
0 & |z| > b / 2 
\end{cases}
\]  

(3.27)

Thus, we have the following boundary conditions:

\[
\left( \frac{\partial \phi(r, z)}{\partial r} \right)_{\rho a} = \begin{cases} 
-\rho \cdot \sigma & |z| < b / 2 \\
0 & |z| > b / 2 
\end{cases}
\]  

(3.28)
\( \phi(r,z) = \phi(r,-z) \) \hspace{1cm} (3.29)

\( \phi(r,z) \) bounded \hspace{1cm} (3.30)

The potential \( \phi(r,z) \) must satisfy Laplace’s Eq. 3.24 and boundary conditions Eq. 3.28-3.30, which define an external Neumann problem or the second boundary problem for an infinite cylinder.

3.5.3 Solution for \( \phi(r,z) \) Using the Method of Separation of Variables \([48]\)

By the method of separation of variables, we assume a solution of \( \phi(r,z) \) in the following form:

\[ \phi(r,z) = T(r) \cdot Q(z) \] \hspace{1cm} (3.31)

where, \( T(r) \) and \( Q(z) \) are, respectively, functions of \( r \) and \( z \) alone. Substituting Eq. 3.31 into Eq. 3.24, we obtain

\[ \frac{1}{T} \frac{\partial^2 T}{\partial r^2} + \frac{1}{r \cdot T} \frac{\partial T}{\partial r} = -\frac{1}{Q} \frac{\partial^2 Q}{\partial z^2} \] \hspace{1cm} (3.32)

The left side of Eq. 3.32 is a function of \( r \) only, and the right side of Eq. 3.32 depends only upon \( z \), so that both sides of Eq. 3.32 must be equal to a constant as shown by the following equations

\[ \frac{T''}{T} + \frac{T'}{r \cdot T} = C \] \hspace{1cm} (3.33)

\[ \frac{Q''}{Q} = -C \] \hspace{1cm} (3.34)

where \( C \) is a separation constant to be determined by boundary conditions. We investigate three possible cases for \( C \): \( C < 0, C = 0, C > 0 \).
Case 1: $C < 0$. The general solution of Eq. 3.34 in this case is of the form

$$Q(z) = C_1 \cdot e^{\sqrt{-C}z} + C_2 \cdot e^{-\sqrt{-C}z}$$

(3.35)

where $C_1$ and $C_2$ are arbitrary constants. Eq. 3.35 tends to have $Q(z) \mid_{z \to \infty} \to \infty$, which is contradictory with boundary condition given by Eq. 3.30, that is $\phi(r, z)$ bounded. The solution does not exist.

Case 2: $C = 0$. Here, the general solution of Eq. 3.34 is

$$Q(z) = C_1 z + C_2$$

(3.36)

Eq. 3.36 tends to have $Q(z) \mid_{z \to \infty} \to 0$, which is contradictory with boundary condition Eq. 3.35. The solution does not exist.

Case 3: $C > 0$. Let $C = \lambda^2$, then Eq. 3.34 becomes

$$Q'' + \lambda^2 Q = 0$$

(3.37)

In this case, assume the solution has the following form:

$$Q(z) = C_1 \cos(\lambda z) + C_2 \sin(\lambda z)$$

(3.38)

From the boundary condition given by Eq. 3.29, $\phi(r, z) = \phi(r, -z)$, we have $C_2 = 0$.

Thus, $Q(x) = C_1 \cos(\lambda x)$

(3.39)

Substituting $C = \lambda^2$ into Eq. 3.33, we obtain

$$r^2 T''(r) + r T'(r) - \lambda^2 r^2 T(r) = 0$$

(3.40)

Let $x = \lambda r$, and

$$y(x) = T(r) = T(x/\lambda)$$

(3.41)

Differentiation both sides of Eq. 3.41 with respect to $x$ yields
\[ \lambda y'(x) = T'(r) \quad \text{and} \quad \lambda^2 y''(x) = T(x/\lambda) \] (3.42)

As a result, Eq. 3.40 becomes

\[ x^2 y''(x) + x y'(x) - x^2 y(x) = 0 \] (3.43)

Comparing Eq. 3.43 with the standard form of Bessel Equation of order \( n \):

\[ x^2 y''(x) + x y'(x) - (x^2 + n^2) y(x) = 0 \] (3.44)
we can see that Eq. 3.43 is Bessel Equation of order zero. The general solution of Bessel equation of order zero is

\[ y(x) = C_3 I_0(x) + C_4 K_0(x) \] (3.45)
where \( I_0 \) and \( K_0 \) are, respectively, the modified Bessel functions of the first and second kind of order zero:

\[ I_0 = \sum_{k=0}^{\infty} \left( \frac{x}{2} \right)^{2k} \frac{\Gamma(k+1)}{(k!)^2} = 1 + \frac{1}{1!} \left( \frac{x}{2} \right)^2 + \frac{1}{2!} \left( \frac{x}{2} \right)^4 + \frac{1}{3!} \left( \frac{x}{2} \right)^6 + \cdots \] (3.46)

\[ K_0(x) = -I_0(x) \left[ \ln(x/2) + \nu \right] + \sum_{k=1}^{\infty} \frac{\left( \frac{x}{2} \right)^{2k}}{k!^2} \sum_{m=0}^{\infty} \frac{1}{m!} \left( 1 + \frac{1}{2} + \cdots + \frac{1}{k} \right) \left( \frac{1}{2} \right)^{2k} \] (3.47)

As \( r \to \infty \) and \( x \to \infty \), we have \( I_0(x) = I_0(\lambda r) \to 0 \). Since \( \phi(r, z) \) is bounded (boundary condition Eq. 3.30), we must have

\[ C_3 = 0. \]

Thus, Eq. 3.45 becomes

\[ y(x) = C_4 K_0(x) = C_4 K_0(\lambda r) = T_\lambda(r) \] (3.48)

Substituting Eq. 3.39 and Eq. 3.48 into Eq. 3.31, we obtain
\[
\phi_\lambda(r, z) = T_\lambda(r) \cdot Q_\lambda(z) = C_\lambda K_\nu(\lambda r) \cdot C_\lambda \cos(\lambda z)
\]
\[
= C_\lambda K_\nu(\lambda r) \cdot \cos(\lambda z)
\] (3.49)

where \( C_\lambda \) = constant to be determined by boundary condition (Eq. 3.28).

The above solution, Eq. 3.49, satisfies the Laplace equation, Eq. 3.24, and boundary conditions, Eq. 3.29-3.30. Since \( \lambda \neq 0 \) and \( \lambda > 0 \), considering all eigenvalues of \( \lambda \), we obtain the following solution:

\[
\phi(r, z) = \int_0^\infty C_\lambda K_\nu(\lambda r) \cdot \cos(\lambda z) d\lambda
\] (3.50)

Differentiating both sides of Eq. 3.50:

\[
\left( \frac{\partial \phi(r, z)}{\partial r} \right)_{r=a} = \left[ \int_0^\infty C_\lambda \frac{\partial K_\nu(\lambda r)}{\partial r} \cos(\lambda z) d\lambda \right]_{r=a}
\] (3.51)

\[
\frac{d}{dx} K_\nu(r, z) = -K_\nu(\lambda x)
\] (3.52)

By boundary condition, Eq. 3.28, we obtain

\[
\left( \frac{\partial \phi(r, z)}{\partial r} \right)_{r=a} = -\int_0^\infty C_\lambda \cdot \lambda K_\nu(\lambda a) \cdot \cos(\lambda z) d\lambda = \begin{cases} -\rho \sigma & |z| < b / 2 \\ 0 & |z| > b / 2 \end{cases}
\] (3.53)

Applying the following equation

\[
\int_0^\infty \frac{\cos(\lambda z) \cdot \sin(\lambda \cdot b / 2)}{\lambda} d\lambda = \begin{cases} \pi / 2 & |z| < b / 2 \\ 0 & |z| > b / 2 \end{cases}
\] (3.54)

to Eq. 3.53 we can have
\[
\int_{0}^{\infty} \frac{\cos(\lambda z) \cdot \sin(\lambda \cdot b / 2)}{\lambda} \cdot \frac{C_{2} \cdot \lambda^{2} K_{1}(\lambda a)}{\sin(\lambda \cdot b / 2)} d\lambda
\]

\[
= \begin{cases}
\frac{\pi C_{2} \cdot \lambda^{2} K_{1}(\lambda a)}{2 \sin(\lambda \cdot b / 2)} & |z| < b / 2 \\
0 & |z| > b / 2
\end{cases} 
\]

Eq. 3.55 holds as

\[
C_{2} = \frac{\sin(\lambda \cdot b / 2)}{\lambda^{2} K_{1}(\lambda a)} \cdot \frac{2}{\pi} \cdot \rho \cdot \sigma 
\]

It is assumed that a unit current flows into the conductor segment (see Fig. 3.2) with uniform current density on the segment surface:

\[
\sigma = \frac{1}{2\pi a \cdot b} 
\]

Substituting Eq. 3.56-3.57 into Eq. 3.50, we obtain

\[
\phi^{*}(r, z) = \int_{0}^{\infty} \frac{\sin(\lambda b / 2)}{\lambda^{2} K_{1}(\lambda a)} \cdot \frac{1}{2\pi a b} K_{0}(\lambda \cdot r) \cos(\lambda \cdot z) d\lambda 
\]

\[
= \frac{\rho}{\pi^{2} a b} \int_{0}^{\infty} K_{0}(\lambda r) \cos \lambda z \sin \frac{\lambda b}{2} d\lambda 
\]

where \( K_{0} \) and \( K_{1} \) are the modified Bessel functions of the second kind of zero and first order, respectively. Eq. 3.58 represents the relationship as previously expressed in Eq. 3.14.
3.6 Computation Methods

A ground grid is typically composed of horizontally buried conductors either in parallel with a chosen x, y axis or not parallel with either axis, and vertical ground rods as illustrated in Fig. 3.1. There may be a number of currents injected into the grid.

Based on the mathematical model as discussed in this chapter, a computer program is developed to calculate ground grid leakage currents. In the computation, the ground grid is first divided by n segments; self and mutual resistance matrix is then created; finally, ground grid segment leakage currents are calculated by solving simultaneous equation given by Eq. 3.12.

3.6.1 Divide Ground Grid into n Segments

It is assumed that a ground grid is divided into \( n_1 \) segments in parallel with the x axis, \( n_2 \) segments parallel with the y axis, \( n_3 \) segments in other directions and \( n_4 \) ground rod segments. Then, the total number of segments in the ground grid is \( n = n_1 + n_2 + n_3 + n_4 \). From the known coordinates of each ground grid conductor end points, the coordinates of each segment center point \((x_i, y_i)\) can be determined. The coordinates for segments that are parallel with either the x axis or y axis can be easily determined. For the segments not parallel with either the x axis or y axis, the slope \(k_s\) of the segment can be determined from the known end point coordinates, and thus the center point of each segment can be obtained as follows:

\[
\alpha = \tan^{-1}(k_s) \quad x_i = l_i \cos(\alpha) \quad y_i = l_i \sin(\alpha) \quad (3.59)
\]

where \(l_i\) is the length of segment, \(i\).
3.6.2. **Self and Mutual Resistance Matrix Calculation**

Calculations show that the point source method of section 3.3 is accurate for mutual resistance when the distance between the two conductor segments is five times greater than the length of conductor segments (see Table 3.1). Table 3.1 shows a comparison of calculated results using the point source method and the Green function method. It can be seen that the error is under 1% for the point source method when the distance between the two conductor segments is five times greater than the length of conductor segments. A ground grid can be divided into many segments with different length. When \( r > 5l_i \), the point source method is applied as described in the following:

(1) between horizontally buried conductor segments with buried depth, \( h \), in the earth:

\[
R_y = \rho \cdot \left( \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} + \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (2 \cdot h)^2}} \right)
\]  

(3.60)

<table>
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<th>distance (d) /length (L)</th>
<th>( R_{ik} ) (point method)</th>
<th>( R_{ik} ) (Green's function)</th>
<th>Error (%)</th>
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<td>1.7119</td>
<td>1.6985</td>
<td>0.789</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison between the point source method and the Green function method for computation of ground grid segment mutual resistance
(2) between horizontally buried conductor segment and ground rod having length, \( l_{rod} \):

\[
R_y = \rho \cdot \left( \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (l_{rod}/2)^2}} + \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (2 \cdot h + l_{rod})^2}} \right)
\]

(3.61)

(3) between ground rods:

\[
R_y = \rho \cdot \left( \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} + \frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (2 \cdot h + l_{rod})^2}} \right)
\]

(3.62)

In Eqs. 3.60 - 3.62, the direction of conductor segments have no effect on the results as the conductor segments are considered as point sources with coordinates at center point.

When the distance of between two conductor segments is less than five times of the length of the conductor segment, we apply Eq. 3.14-3.18 to calculate self and mutual resistance. In Eq. 3.14, the new coordinates of \( r, z \) and \( r' \) and \( z' \) are created as follows:

1) between segments, \( i \) and \( j \), in parallel with \( x \) axis; between segment, \( i \), in parallel with \( x \) axis and segment, \( j \), in parallel with \( y \) axis; between segment, \( i \), in parallel with \( x \) axis and non parallel segment, \( j \); between ground rods, \( i \) and \( j \):

\[
r = \text{abs} (y_i - y_j) \quad z = z' = \text{abs} (x_i - x_j)
\]

(3.63)

\[
r' = \sqrt{(y_i - y_j)^2 + (2 \cdot h)^2}
\]

(3.64)

2) between segment, \( i \), in parallel with \( x \) axis and ground rod segment, \( j \):

\[
z = z' = \text{abs} (x_i - x_j) \quad r = \sqrt{(y_i - y_j)^2 + (l_{rod}/2)^2}
\]

(3.65)

\[
r' = \sqrt{(y_i - y_j)^2 + (2 \cdot h + l_{rod})^2}
\]

(3.66)
(3) between segments, i and j, in parallel with y axis; between segment, i, in parallel with y axis and non-parallel segment, j:

\[ r = \text{abs} \left( x_i - x_j \right) \quad z = z' = \text{abs} \left( y_i - y_j \right) \]  
\[ r' = \sqrt{(x_i - x_j)^2 + (2 \cdot h)^2} \]  
\[ z = z' = \text{abs} \left( y_i - y_j \right) \quad r = \sqrt{(x_i - x_j)^2 + \left( l_{\text{rod}} / 2 \right)^2} \]  
\[ r' = \sqrt{(x_i - x_j)^2 + (2 \cdot h + l_{\text{rod}})^2} \]  

(4) between segment, i, in parallel with y axis and ground rod segment, j:

(5) between segments, i and j, of non-parallel segments (see Fig. 3.7):

Figure 3.7: Nonparallel segment
\[ d = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]  \hspace{1cm} (3.71)

\[ \alpha_1 = \tan^{-1} \frac{\text{abs}(y_i - y_j)}{\text{abs}(x_i - x_j)} \]
\[ \alpha_2 = \pi / 2 - \tan^{-1} k_y - \alpha_1 \]  \hspace{1cm} (3.72)

\[ z = z' = d \sin \alpha_2 \]
\[ r = d \cos \alpha_2 \]  \hspace{1cm} (3.73)

\[ r' = \sqrt{(d \cdot \cos \alpha_2)^2 + (2 \cdot h)^2} \]  \hspace{1cm} (3.74)

(6) between non-parallel segment, \( i \), and ground rod segment, \( j \):

\[ z = z' = d \sin \alpha_2 \]
\[ r' = \sqrt{(d \cdot \cos \alpha_2)^2 + (l_{rod}/2)^2} \]  \hspace{1cm} (3.75)

\[ r' = \sqrt{(d \cdot \cos \alpha_2)^2 + (2 \cdot h + l_{rod})^2} \]  \hspace{1cm} (3.76)

(7) Self resistance for all segments:

Similarly, self-resistance can be calculated by choosing segment, \( j \), on the surface of conductor segment, \( i \). Therefore, we have

\[ r = a \ \text{(radii of grid conductor)} \]

As a result, we have the following self and mutual resistance matrix, \( R_{\text{self}} \) (\( n = n_1 + n_2 + n_3 + n_4 \)), \( (R_{ij} = R_{ji}) \):

\[
R_{n \times n} = \begin{bmatrix}
R_{1_{n_1 \times n_1}} & R_{2_{n_1 \times n_2}} & R_{3_{n_1 \times n_3}} & R_{I_{n_1 \times n_4}} \\
R_{2_{n_2 \times n_1}}^T & R_{3_{n_2 \times n_2}} & R_{4_{n_2 \times n_3}} & R_{5_{n_2 \times n_4}} \\
R_{3_{n_3 \times n_1}}^T & R_{4_{n_3 \times n_2}}^T & R_{5_{n_3 \times n_3}} & R_{6_{n_3 \times n_4}} \\
R_{4_{n_4 \times n_1}}^T & R_{5_{n_4 \times n_2}}^T & R_{6_{n_4 \times n_3}} & R_{7_{n_4 \times n_4}}
\end{bmatrix}_{n \times n} \hspace{1cm} (3.77)
\]
where

- \( R_1 \) is \( n_1 \times n_1 \) order matrix of mutual resistance between segments in parallel with \( x \) axis;
- \( R_2 \) is \( n_1 \times n_2 \) order matrix of mutual resistance between segment, \( i \), in parallel with \( x \) axis and segment, \( j \), in parallel with \( y \) axis;
- \( R_3 \) is \( n_1 \times n_3 \) order matrix of mutual resistance between segment, \( i \), in parallel with \( x \) axis and non-parallel conductor segment, \( j \);
- \( R_4 \) is \( n_2 \times n_2 \) order matrix of mutual resistance between segments in parallel with \( y \) axis;
- \( R_5 \) is \( n_2 \times n_3 \) order matrix of mutual resistance between segment, \( i \), in parallel with \( y \) axis and non-parallel conductor segment, \( j \);
- \( R_6 \) is \( n_3 \times n_3 \) order matrix of mutual resistance between non-parallel conductor segments;
- \( R_7 \) is \( n_1 \times n_4 \) order matrix of mutual resistance between segment, \( i \), in parallel with \( x \) axis and ground rod segment, \( j \);
- \( R_8 \) is \( n_2 \times n_4 \) order matrix of mutual resistance between segment, \( i \), in parallel with \( y \) axis and ground rod segment, \( j \);
- \( R_9 \) is \( n_3 \times n_4 \) order matrix of mutual resistance between non-parallel segment, \( i \), and ground rod segment, \( j \);
- \( R_{10} \) is \( n_4 \times n_4 \) order matrix of mutual resistance between ground rod segments. There is no magnetic coupling between perpendicular conductors.
3.6.3 Computation Algorithm

A computer program is developed based on the above computation method. The following summarizes the algorithm.

(1) Input conductor radii, \( a \), soil resistivity, \( \rho \), ground grid buried depth, \( h \), injected current, \( I \), coordinates of horizontally buried conductor and ground rod end points, \( X \) and \( Y \), and length of ground rod, \( l_{rod} \).

(2) Divide ground grid conductors in parallel with \( x \) axis by \( n_1 \) segments, ground grid conductors in parallel with \( y \) axis by \( n_2 \) segments, non-parallel ground grid conductors by \( n_3 \) segments and ground rod by \( n_4 \) segments; produce coordinates of ground grid segment central point, \( x_j \) and \( y_j \).

(3) Use the point source method (Eqs. 3.60-3.62) to calculate mutual resistance, \( R_{ij} \), if the distance between segments is greater than five times of the length of segment. Otherwise, use the Green function method (Eqs. 3.63-3.70 and Eqs. 3.14-3.18) to calculate mutual resistance, \( R_{ij} \).

(4) Create mutual resistance matrix, \( R \) (Eq. 3.77).

(5) Use the calculated results of nodal voltage for each segment potential rise from network calculation to be discussed in the next chapter.

(6) Calculate segment leakage currents by method discussed in section 3.2 (Eq. 3.12).
3.7 Summary

An accurate modeling method, Green function method, is used for the calculation of mutual resistance. Calculation shows that point source method for mutual resistance calculation is adequate when the distance between two conductor segments is five times greater than the length of that conductor segment. Boundary element technique is applied to the calculation of ground grid leakage currents for any ground grid configurations. The expression of potential, $\phi$, induced by a cylindrical segment with uniform current density is derived. A few computation strategies are used on the development of the computer program. The developed computer program can be applied to accurately calculate leakage current distribution of any shaped ground grid in power substations.
Chapter 3 discussed the ground grid leakage current calculation. Besides the leakage current, there are currents flowing through the ground grid conductors which results in a voltage drop due to conductor resistance and inductance. In this chapter, electrical network theory will be used to calculate the current distribution along the ground grid and metallic structure network. Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) describe the current distribution. A specific ground grid network is shown in Fig. 4.1 and a specific metallic structure network is shown in Fig. 4.2 for which field measurement results are available (see Chapter 5 of this dissertation). The following mathematical model can be used to calculate the current distribution for any three dimensional ground grid and metallic structure.
Figure 4.1: Ground grid network of Madison 69/13kV substation

Figure 4.2: Metallic structure network of Madison 69/13kV substation
4.1 Modeling of Ground Grid and Metallic Structure Currents

The ground grid and metallic structures will be modeled as a lumped resistance and inductance network. Assume that the ground grid and the metallic structure network are divided into branch segments. Each segment is modeled as a lumped resistance and self-inductance. In addition, there are mutual inductances between branch segments.

4.1.1 Resistance, $R_k$, of the kth Branch Segment of the Ground Grid and Metallic Structures

The following well known equation is used to calculate the kth branch segment resistance, $R_k$, in ohms:

$$R_k = \frac{\rho \cdot l_k}{A}$$

(4.1)

where $\rho$ is the specific resistivity of the material in ohm-meters, $l_k$ is the length of the kth branch segment in meters, and $A$ is the effective cross sectional area in m$^2$. Ground grid conductors are usually made of copper with a typical resistivity of $1.8 \times 10^{-8}$ ohm-meter; metallic structure is usually made of steel with a typical resistivity of $2 \times 10^{-7}$ ohm-meter.
4.1.2 Self Inductance, \( L_k \), of the \( k \)th Branch Segment

The self inductance is defined as follows:

\[
L = \frac{\phi}{I} = \frac{1}{I} \oint_{s} \mathbf{B} \cdot \hat{n} ds
\]  

(4.2)

where \( \phi \) is the flux on surface \( s \), which is generated by a current carrying loop, \( I \). \( \mathbf{B} \) and \( \phi \) are both directly proportional to current and thus \( L \) is independent of the magnitude of current.

Self-inductance of the ground grid conductor segment \( k \) is calculated using Eq. 4.3 from [43]:

\[
L_k = 2 \times 10^{-7} I_k \left[ \ln \left( \frac{2I_k}{a} \right) - 1 + 0.25 \beta \right]
\]  

(4.3)

where the factor \( \beta \) is taken as 1 for nonmagnetic materials; \( a \) is the radius of ground grid conductors in meters.

Self-inductance for the metallic structure is calculated by finding the equivalent radius from the effective cross sectional area and substituting it in Eq. 4.3.

4.1.3 Mutual Inductance, \( M_{ik} \), Between \( i \)th and \( k \)th Branch Segments

The mutual inductance is defined as follows:

\[
M_{ik} = \frac{\phi_{ik}}{I_k} = \frac{1}{I_k} \oint_{s_i} \mathbf{B}_k \cdot \hat{n} ds
\]  

(4.4)
where $\phi_{ik}$ is the flux through segment $i$ due to current $I_k$ on segment $k$. In the absence of magnetic materials, mutual and self-inductances are independent of the value of the currents and depend only on the geometry of the system.

Mutual inductance is calculated by using equations from Grover [38] as described in the following.

1) Mutual inductance for a pair of side-by-side parallel grid conductors of the same length $l_k$:

$$M_{ik} = 2 \times 10^{-7} l_k \left[ \ln \left( \frac{l_k}{d} + \sqrt{1 + \frac{d^2}{l_k^2}} \right) - \sqrt{1 + \frac{d^2}{l_k^2} + \frac{d}{l_k}} \right]$$  \hspace{1cm} (4.5)

where $d$ is the distance between the $i$th and $k$th conductors (see Fig. 4.3 (a)).

2) Mutual inductance for any parallel grid conductors with length $l_i$ and $l_k$ (see Fig. 4.3 (b)):

$$M_{ik} = 10^{-7} \left[ \frac{\alpha \sinh^{-1} \frac{\alpha}{d} - \beta \sinh^{-1} \frac{\beta}{d} - \gamma \sinh^{-1} \frac{\gamma}{d} + \delta \sinh^{-1} \frac{\delta}{d}}{d} \right]$$

$$- \sqrt{\alpha^2 + d^2 + \sqrt{\beta^2 + d^2 + \sqrt{\gamma^2 + d^2} - \sqrt{\delta^2 + d^2}}}$$  \hspace{1cm} (4.6)

where, $\alpha = l_i + l_k + \delta$, $\beta = l_i + \delta$, $\gamma = l_k + \delta$.

3) Mutual inductance for conductors, $l_i$ and $l_k$, meeting at a point (Fig. 4.3 (c)):

$$M_{ik} = 2 \times 10^{-7} \cos \theta \left[ l_i \cdot \tanh^{-1} \frac{l_k}{l_i + S} - l_k \cdot \tanh^{-1} \frac{l_i}{l_k + S} \right]$$  \hspace{1cm} (4.7)

where $S = l_i \sqrt{1 + \frac{l_k}{l_i^2} - 2 \cdot \frac{l_k}{l_i} \cdot \cos \theta}$  \hspace{1cm} (4.8)
Figure 4.3: Mutual inductance between conductors
(4) As shown in Fig. 4.3 (d), mutual inductance between two conductor segments of lengths \( l_i \) and \( l_k \) having an angle \( \theta \) in the same plane, not meeting:

\[
\frac{M_{lk}}{2 \cos \theta} = 10^{-7} \left[ (\mu + l_i) \tanh^{-1} \frac{l_k}{S_1 + S_2} + (\nu + l_k) \tanh^{-1} \frac{l_i}{S_1 + S_4} - \mu \tanh^{-1} \frac{l_k}{S_2 + S_4} - \nu \tanh^{-1} \frac{l_i}{S_2 + S_3} \right]
\]

(4.9)

where

\[
2 \cos \theta = \frac{S_4^2 - S_3^2 + S_2^2 - S_1^2}{l_i \cdot l_k}
\]

(4.10)

\[
\mu = \frac{2l_i \left( S_2^2 - S_3^2 - l_i^2 \right) + \left( S_4^2 - S_3^2 + S_2^2 - S_1^2 \right) \cdot \left( S_4^2 - S_3^2 - l_k^2 \right)}{4 \cdot l_i^2 \cdot l_k^2 - \left( S_4^2 - S_3^2 + S_2^2 - S_1^2 \right)^2}
\]

(4.11)

\[
\nu = \frac{2l_k \left( S_4^2 - S_3^2 - l_k^2 \right) + \left( S_4^2 - S_3^2 + S_2^2 - S_1^2 \right) \cdot \left( S_2^2 - S_3^2 - l_i^2 \right)}{4 \cdot l_i^2 \cdot l_k^2 - \left( S_4^2 - S_3^2 + S_2^2 - S_1^2 \right)^2}
\]

(4.12)

Note: As \( S_3 = 0 \), the general expression as shown for Fig. 4.3 (d) will become the specific as shown for Fig. 4.3 (c). In this case, calculations show that using Eqs. 4.9 - 4.12 for \( M_{lk} \) leads to the same results as using Eqs. 4.7 - 4.8.

(5) As shown in Fig. 4.3 (e), mutual inductance between two conductor segments of lengths \( l_i \) and \( l_k \) having an angle \( \theta \) in any desired positions:

\[
\frac{M_{lk}}{2 \cos \theta} = 10^{-7} \left[ (\mu + l_i) \tanh^{-1} \frac{l_k}{S_1 + S_2} + (\nu + l_k) \tanh^{-1} \frac{l_i}{S_1 + S_4} - \mu \tanh^{-1} \frac{l_k}{S_2 + S_4} - \nu \tanh^{-1} \frac{l_i}{S_2 + S_3} \right] \frac{\omega \cdot \phi}{\sin \theta}
\]

(4.13)
where \( 2 \cos \theta = \frac{S_1^2 - S_2^2 + \sum_{i=1}^{l-1} S_i^2}{l_i \cdot l_k} \)  

\( \delta^2 = S_3^2 - \mu^2 - \nu^2 + 2\mu\nu \cos \theta \)  

\[
\mu = \frac{2l_k^2(S_2^2 - S_3^2 - l_i^2) + (S_4^2 - S_3^2 + S_2^2 - S_1^2)(S_4^2 - S_3^2 - l_k^2)}{4 \cdot l_i^2 \cdot l_k^2 - (S_4^2 - S_3^2 + S_2^2 - S_1^2)^2} \cdot l_i
\]  

\[
\nu = \frac{2l_i^2(S_4^2 - S_3^2 - l_i^2) + (S_4^2 - S_3^2 + S_2^2 - S_1^2)(S_2^2 - S_3^2 - l_k^2)}{4 \cdot l_i^2 \cdot l_k^2 - (S_4^2 - S_3^2 + S_2^2 - S_1^2)^2} \cdot l_k
\]  

\[
\omega = \tan^{-1}\left[ \frac{\delta^2 \cos \theta + (\mu + l_i)(\nu + l_k) \sin^2 \theta}{\delta S_1 \sin \theta} \right] - \tan^{-1}\left[ \frac{\delta^2 \cos \theta + (\mu + l_i) \cdot \nu \sin^2 \theta}{\delta S_2 \sin \theta} \right] + \\
+ \tan^{-1}\left[ \frac{\delta^2 \cos \theta + \mu \cdot \nu \sin^2 \theta}{\delta S_3 \sin \theta} \right] - \tan^{-1}\left[ \frac{\delta^2 \cos \theta + \mu \cdot (\nu + l_k) \sin^2 \theta}{\delta S_4 \sin \theta} \right]
\]  

As described above, the position of conductor segments will have great influence on the mutual inductance calculation. Since the ground grid and metallic structure have complicated configurations, the mutual inductance calculation will be discussed in detail in section 4.3.

4.2 Network Analysis [50,51]

4.2.1 Branch Impedance Matrix

Assume the ground grid and metallic structure network have b branches and n+1 nodes. The branch impedance matrix, \( Z(\omega) \), is:
\[ Z(j\omega) = \begin{bmatrix}
R_1 + j\omega L_1 & j\omega M_{12} & \cdots & j\omega M_{1b} \\
j\omega M_{21} & R_2 + j\omega L_2 & \cdots & j\omega M_{2b} \\
\vdots & \vdots & \ddots & \vdots \\
j\omega M_{b1} & j\omega M_{b2} & \cdots & R_b + j\omega L_b
\end{bmatrix} \]  

(4.19)

The branch admittance matrix is:

\[ Y(j\omega) = Z(j\omega)^{-1} = \begin{bmatrix}
R_1 + j\omega L_1 & M_{12} & \cdots & j\omega M_{1b} \\
j\omega M_{21} & R_2 + j\omega L_2 & \cdots & j\omega M_{2b} \\
\vdots & \vdots & \ddots & \vdots \\
j\omega M_{b1} & M_{b2} & \cdots & R_b + j\omega L_b
\end{bmatrix}^{-1} \]  

(4.20)

4.2.2 Nodal analysis

Incidence matrix, \( A \), is used to relate branches and nodes.

The incidence matrix \( A \) is a rectangular matrix of order \( n \times b \), whose \( ij \)th element is defined by the following:

- \( a_{ij} = +1 \) if branch \( j \) is directed away from node \( i \)
- \( a_{ij} = -1 \) if branch \( j \) is directed toward node \( i \)
- \( a_{ij} = 0 \) if branch \( j \) is not incident with node \( i \)

Based on KCL, the following equations have been derived.

The nodal admittance matrix \( [Y_N]_{n \times n} \) is given by

\[ [Y_N]_{n \times n} = AYA^T \]  

(4.21)

Then we have

\[ Y_N V_N = I_{\mathcal{N}} \]  

(4.22)

where \( V_N \) is the \( n \times 1 \) column vector of node voltages in an \( n+1 \) node network (one node is chosen as a reference point), \( I_{\mathcal{N}} \) is the \( n \times 1 \) node current vector and the \( k \)th row element of \( I_{\mathcal{N}} \) represents the source current injected into node \( k \).
The solution of Eq. 4.22 is:

$$V_N = Y_N^{-1} I_{jN}$$  \hspace{1cm} (4.23)

Then, we have:

Branch voltage: $V_b = A^T V_N$  \hspace{1cm} (4.24)

Branch current: $I_b = YY_b$  \hspace{1cm} (4.25)

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_b \\
\end{bmatrix} =
\begin{bmatrix}
Y_{11} & Y_{12} & \ldots & Y_{1b} \\
Y_{21} & Y_{22} & \ldots & Y_{2b} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{b1} & Y_{b2} & \ldots & Y_{bb} \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_b \\
\end{bmatrix}
\]

(4.26)

### 4.3 Computation Method

#### 4.3.1 Coordinate Setup

It is assumed that a ground grid has $n_1$ branches in parallel with $x$ axis, $n_2$ branches in parallel with $y$ axis, $n_3$ branches in other directions and $n_5$ metallic structure branches. Then, the total branches of the ground grid and metallic structure network is $b$ ($b = n_1 + n_2 + n_3 + n_5$). $n_4$ is the number of ground rod segments which were considered in the leakage current calculation of Chapter 3. There are no network currents flowing in ground rods. Therefore, ground rod branches are not included in network calculation. As described in Chapter 3, the coordinates of each branch conductor center point $(x_i, y_i)$ is automatically created in the computer program.
4.3.2 Mutual Inductance Calculation

Direction and position of the conductor branches will influence the mutual inductance as described in section 4.1.3. Therefore, it is a complicated task to calculate the mutual inductance considering all different situations of any configuration of ground grid and structure networks. The following cases are considered in the mutual inductance calculations.

Case 1: Parallel conductor segments

As shown in Fig. 4.4 (a), the distance $d$ and $\delta$ is positive when they are substituted into Eqs. 4.5 and 4.6. Therefore, we use absolute value for the following calculations for $d$ and $\delta$.

(1) Between ground grid segment $i$ and $k$ in parallel with $x$ axis

\[
\begin{align*}
    d &= |(y_i - y_k)| \\
    \delta &= [(x_i - x_k)] - l_i / 2 - l_k / 2
\end{align*}
\]

(2) Between ground grid segment $i$ and metallic structure segment $k$ in parallel with $x$ axis

As shown in Fig. 4.3 (b), we have

\[
\begin{align*}
    d &= \sqrt{(y_i - y_k)^2 + (h + h_s)^2} \\
    \delta &= [(x_i - x_k)] - l_i / 2 - l_k / 2
\end{align*}
\]

where $h$ is the buried depth of ground grid and $h_s$ is the height of steel structure segment.
Figure 4.4: Mutual inductance between parallel conductors

(3) Between ground grid segment i and k in parallel with y axis

Similar to case 1 (1), we have

$$d = \left| (x_i - x_k) \right|$$  \hspace{1cm} (4.31)

$$\delta = \left| (y_i - y_k) \right| - l_i / 2 - l_k / 2$$  \hspace{1cm} (4.32)

(4) Between ground grid segment i and metallic structure segment k in parallel with y axis
Similar to case 1 (2), we have

\[ d = \sqrt{(x_i - x_k)^2 + (2h + h_y)^2} \]  
\( (4.33) \)

\[ \delta = |(y_i - y_k)| - l_i / 2 - l_k / 2 \]  
\( (4.34) \)

Substitution of \( d \) and \( \delta \) into Eq. 4.5 for a pair of side-by-side parallel segments and into Eq. 4.6 for other parallel segments will give the mutual inductance, \( M_{ik} \).

Similarly, the mutual inductance, \( M_{ik} \) can be obtained for parallel conductor segments in direction \( z \) by employing new coordinates.

From Eqs. 4.7 - 4.18 and Fig. 4.3, it can be seen that the main parameters for calculating mutual resistance between nonparallel conductor segments are the segment distance, \( S_1, S_2, S_3 \), and \( S_4 \). As long as \( S_1, S_2, S_3 \), and \( S_4 \) are determined, the other parameters and mutual resistance can be calculated using Eqs. 4.10 - 4.12 and Eqs. 4.14 - 4.18. The following cases will be discussed to calculate \( S_1, S_2, S_3 \), and \( S_4 \).

Case 2: Between segment, \( i \), in parallel with \( x \) axis and nonparallel segment, \( k \), with slope of \( k_x > 0 \)

(1) As shown in Fig. 4.5 (a), we have

\[ S_1 = \sqrt{\left(x_k + \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2}\right)^2 + \left(y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i\right)^2} \]  
\( (4.35) \)

\[ S_2 = \sqrt{\left(x_k + \frac{l_k}{2} \cdot \cos \alpha - x_i - \frac{l_i}{2}\right)^2 + \left(y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i\right)^2} \]  
\( (4.36) \)

\[ S_3 = \sqrt{\left(x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i - \frac{l_i}{2}\right)^2 + \left(y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i\right)^2} \]  
\( (4.37) \)
\[ S_4 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2} \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]  

(2) As shown in Fig. 4.5(b), we have

\[ S_1 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i - \frac{l_i}{2} \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]

\[ S_2 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha - x_i - \frac{l_i}{2} \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]

\[ S_3 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2} \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]

\[ S_4 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2} \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]

(3) As shown in Fig. 4.5(c), we have

\[ S_1 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2} \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]

\[ S_2 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i - \frac{l_i}{2} \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]

\[ S_3 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2} \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]

\[ S_4 = \sqrt{\left( x_k + l_k \cdot \cos \alpha - x_i - \frac{l_i}{2} \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]
Figure 4.5: Mutual inductance between conductor in parallel with x axis and nonparallel conductor with slope $k_\alpha > 0$
Case 3 Between segment, i, in parallel with x axis and nonparallel segment, k, with slope of $k_* < 0$

(1) As shown in Fig. 4.6 (a), we have:

$$S_1 = \sqrt{\left(x + \frac{l_k}{2} \cos \alpha - x_i - \frac{l_i}{2}\right)^2 + \left(y + \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

$$S_3 = \sqrt{\left(x - \frac{l_k}{2} \cos \alpha - x_i + \frac{l_i}{2}\right)^2 + \left(y + \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

$$S_2 = \sqrt{\left(x - \frac{l_k}{2} \cos \alpha - x_i - \frac{l_i}{2}\right)^2 + \left(y + \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

$$S_4 = \sqrt{\left(x + \frac{l_k}{2} \cos \alpha - x_i + \frac{l_i}{2}\right)^2 + \left(y - \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

(4.47)  (4.48)  (4.49)  (4.50)

(2) As shown in Fig. 4.6 (b), we have:

$$S_1 = \sqrt{\left(x - \frac{l_k}{2} \cos \alpha - x_i - \frac{l_i}{2}\right)^2 + \left(y + \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

$$S_2 = \sqrt{\left(x - \frac{l_k}{2} \cos \alpha - x_i + \frac{l_i}{2}\right)^2 + \left(y + \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

$$S_3 = \sqrt{\left(x + \frac{l_k}{2} \cos \alpha - x_i + \frac{l_i}{2}\right)^2 + \left(y - \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

$$S_4 = \sqrt{\left(x + \frac{l_k}{2} \cos \alpha - x_i - \frac{l_i}{2}\right)^2 + \left(y - \frac{l_k}{2} \sin \alpha - y_i\right)^2}$$

(4.51)  (4.52)  (4.53)  (4.54)
Figure 4.6: Mutual inductance between conductor in parallel with x axis and nonparallel conductor with slope $k_s < 0$
(3) As shown in Fig. 4.6 (c), we have:

\[ S_1 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2} \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]  

(4.55)

\[ S_2 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha - x_i - \frac{l_i}{2} \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]  

(4.56)

\[ S_3 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i - \frac{l_i}{2} \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]  

(4.57)

\[ S_4 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha - x_i + \frac{l_i}{2} \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha - y_i \right)^2} \]  

(4.58)

Similarly, \( S_1, S_2, S_3 \) and \( S_4 \) can be obtained for all the cases between segment, \( i \), in parallel with \( y \) axis and nonparallel segment, \( k \), with slope, \( k_\alpha > 0 \) and \( k_\alpha < 0 \).

Case 4: Nonparallel segments, \( i \) and \( k \)

(1) As shown in Fig. 4.7 (a), we have

\[ S_1 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

(4.59)

\[ S_2 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

(4.60)

\[ S_3 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

(4.61)

\[ S_4 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

(4.62)
(2) As shown in Fig. 4.7 (b), we have:

\[ S_1 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

\[ S_2 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

\[ S_3 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

\[ S_4 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]

(3) As shown in Fig. 4.7 (c), we have:

\[ S_1 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

\[ S_2 = \sqrt{\left( x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

\[ S_3 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]  

\[ S_4 = \sqrt{\left( x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left( y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \]
Figure 4.7: Mutual inductance between nonparallel conductors
As shown in Fig. 4.7 (d), we have:

\[ S_1 = \sqrt{\left(x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left(y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \quad (4.71) \]

\[ S_2 = \sqrt{\left(x_k - \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left(y_k + \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \quad (4.72) \]

\[ S_3 = \sqrt{\left(x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i + \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left(y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i + \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \quad (4.73) \]

\[ S_4 = \sqrt{\left(x_k + \frac{l_k}{2} \cdot \cos \alpha_k - x_i - \frac{l_i}{2} \cdot \cos \alpha_i \right)^2 + \left(y_k - \frac{l_k}{2} \cdot \sin \alpha_k - y_i - \frac{l_i}{2} \cdot \sin \alpha_i \right)^2} \quad (4.74) \]

By definition of mutual inductance, we have \( M_{ij} = M_{ji} \) and \( M_{ij} = 0 \) if two conductors are perpendicular. In mutual inductance calculation, we consider the following cases: between parallel conductor segments; between parallel and non-parallel conductor segment; between ground grid conductor and metallic structure segments including parallel and non-parallel conductor segments in different orientations.

As a result, we have the following self and mutual inductance matrix, \( M_{ab} \) (\( b = n_1 + n_2 + n_3 + n_4 \)):

\[
M = \begin{bmatrix}
M_{x,n_1 \times n_1} & 0_{n_1 \times n_2} & \cdots & 0_{n_1 \times n_3} & Ms_{1,n_1 \times n_4} \\
0_{n_2 \times n_1} & M_{x,n_2 \times n_2} & \cdots & 0_{n_2 \times n_3} & Ms_{2,n_2 \times n_4} \\
M_{x,n_3 \times n_1, \text{par.}} & M_{y,n_3 \times n_2, \text{par.}} & \cdots & M_{x,n_3 \times n_3, \text{par.}} & Ms_{3,n_3 \times n_4} \\
Ms_{1, n_4 \times n_1} & Ms_{2,n_4 \times n_2} & \cdots & Ms_{3,n_4 \times n_3} & Ms_{4,n_4 \times n_4}
\end{bmatrix}_{b \times b} \quad (4.75)
\]

where
- $M_{x,x}$ is $n_1 \times n_1$ order of mutual inductance matrix between branch segments in parallel with $x$ axis; since branch segment, $i$, in parallel with $x$ axis and branch segment, $j$, in parallel with $y$ axis are perpendicular, there is no inductive coupling and mutual inductance is zero;

- $M_{x,ap,per}$ is $n_1 \times n_3$ order of mutual inductance matrix between branch segment, $i$, in parallel with $x$ axis and non-parallel conductor branch segment, $j$;

- $M_{y,y}$ is $n_2 \times n_2$ order of mutual inductance matrix between segments in parallel with $y$ axis;

- $M_{y,ap,per}$ is $n_2 \times n_3$ order of mutual inductance matrix between segment, $i$, in parallel with $y$ axis and non-parallel conductor segment, $j$;

- $M_{ap,per}$ is $n_3 \times n_3$ order of mutual inductance matrix between non-parallel conductor segments;

- $M_{s,1}$ is $n_1 \times n_5$ order of mutual resistance matrix between segment, $i$, in parallel with $x$ axis and metallic structure branch segment, $j$;

- $M_{s,2}$ is $n_2 \times n_5$ order of mutual inductance matrix between branch segment, $i$, in parallel with $y$ axis and metallic structure branch segment, $j$;

- $M_{s,3}$ is $n_3 \times n_5$ order of mutual inductance matrix between non-parallel branch segment, $i$, and metallic structure branch segment, $j$;

- $M_{s,4}$ is $n_5 \times n_5$ order of mutual inductance matrix between metallic structures. Therefore, the mutual inductance between conductor segments with different directions and configurations are included in the above mutual inductance matrix.
4.3.3 Computation Algorithm

The following is a step by step description of the algorithm based on the above computation method.

(1) Input grid conductor radius, \( a \), metallic structure equivalent radius, \( r_e \), copper ground grid resistivity, \( \rho_c \), steel structure resistivity, \( \rho_s \), ground grid buried depth, \( h \), injected current, \( I_j \), and the associate coordinates, coordinates of horizontally buried conductor and metallic structure end points, \( X \) and \( Y \), and vertical length of steel structure, \( h_s \).

(2) Divide ground grid conductors in parallel with x axis into \( n_1 \) branch segments, ground grid conductors in parallel with y axis into \( n_2 \) branch segments, non-parallel ground grid conductors into \( n_3 \) branch segments and steel structures into \( n_s \) segments; produce coordinates of branch segment central point, \( x_i \) and \( y_i \). The divided segments may have various different lengths. It is assumed that leakage current is uniformly distributed within a ground grid segment.

(3) Create incidence matrix, \( A \).

(4) Create mutual resistance matrix, \( M \) (Eq. 4.75).

(5) Calculate self-resistance and self-inductance of branch conductor segments (Eqs. 4.1 and 4.3).

(6) Calculate network nodal voltages, \( V_N \), by solving Eqs. 4.21 and 4.23.

(7) Calculate branch currents, \( I_b \), by solving Eqs. 4.24 and 4.26.
4.4 Summary

A calculation method has been proposed and developed for calculating the three-dimensional current distribution of a ground grid and metallic structure network. In the calculation, the ground grid and metallic structure is divided into n branches. Each branch segment resistance and self inductance are included in the study. Mutual inductances between branch segments of all possible orientations are also included. The segment potential rise calculated in this chapter is used in Eq. 3.12 of Chapter 3 for leakage current calculation. Total branch segment currents are the sum of leakage and network currents. Based on the proposed mathematical model, a computer program is developed and listed in Appendix B.
CHAPTER 5

CALCULATION RESULTS AND VERIFICATION OF
MATHEMATICAL MODEL

The mathematical model proposed in Chapters 3 and 4 can be used to determine both ground grid and metallic structure currents. Comparisons are made between the calculated results and the field measurements. The good agreement between the simulated and measured results demonstrates the applicability of the modeling technique. The developed model is also applied to other ground grids and the calculated data is then compared with other calculated/measured results which are available in literature [34,37].

5.1 Madison Substation [32]

Fig. 5.1 shows the 69/13.8kV Madison substation. The Madison substation is owned and operated by Columbus Southern Power. It has two outgoing 13.8kV lines and one 69kV line feeding the station. There is a 13.8kV recloser for each of the two lines and one for the low side of the transformer, and a voltage regulator on the 13.8kV bus. The transformer is rated at 2MVA and is normally operated at a voltage ratio of 67/13.7kV. The three phase transformer is of Δ-Y configuration with secondary neutral grounded. Fig.
5.2 shows the Madison substation ground grid and Fig. 5.3 shows the metallic structure on the 13.8kV side of the Madison substation. These figures are similar to Fig. 4.1 and 4.2 which were used to illustrate the development of the network model. Since the transformer primary is connected in delta, there will be no current flow in the overhead ground wire that connects to the metallic structures. However, there can be induced currents in metallic structure, though for this substation it is negligible.

The currents indicated in Figs. 5.2 and 5.3 show the segment currents of the ground grid and metallic structure network for which measurement data is available for comparison purposes. All currents were measured using clip-on 1000/1 amp current...
Figure 5.2: Madison substation ground grid with measured currents identified

Figure 5.3: Madison substation 13.8kV side metallic structure with measured currents identified
transformers. The current transformer secondaries were connected to a central switching panel to expedite the measurement process. An HP 3577A network analyzer was used to measure the magnitude and phase angle of each current relative to a fixed reference. The total time required for measuring all currents was approximately 10 minutes. Over the entire period of the field measurements, the magnitudes and angles of the currents did not vary significantly.

5.2 Field Measurement Results of Madison Substation

Measured currents of the 67/13.8kV Madison substation network shown in Figs. 5.2 and Fig. 5.3 are listed in Tables 5.1 - 5.3. Table 5.1 shows the measured injected currents to the Madison substation network. The north line neutral and east line neutral are terminated at the same point on the steel structure, therefore the sum of both line neutral currents is the neutral current, \( I_{\text{neutral}} \), injected to the steel structure. Since the

<table>
<thead>
<tr>
<th>Injected currents</th>
<th>transformer current ( I_{t_1} )</th>
<th>transformer current ( I_{t_2} )</th>
<th>regulator current ( I_{r_1} )</th>
<th>regulator current ( I_{r_2} )</th>
<th>neutral wire current ( I_{\text{neutral}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>magnitude (amps)</td>
<td>26.07</td>
<td>12.84</td>
<td>1.17</td>
<td>1.65</td>
<td>36.877</td>
</tr>
<tr>
<td>angle (degrees)</td>
<td>-135.4</td>
<td>-120.10</td>
<td>-127.4</td>
<td>-113.75</td>
<td>75.33</td>
</tr>
</tbody>
</table>

Table 5.1: Injected currents to the network
<table>
<thead>
<tr>
<th>current</th>
<th>magnitude (amps)</th>
<th>angle (degrees)</th>
<th>current</th>
<th>magnitude (amps)</th>
<th>angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>12.1</td>
<td>40.33</td>
<td>$I_4$</td>
<td>10.62</td>
<td>46.26</td>
</tr>
<tr>
<td>$I_2$</td>
<td>7.61</td>
<td>13.43</td>
<td>$I_5$</td>
<td>3.95</td>
<td>50.83</td>
</tr>
<tr>
<td>$I_3$</td>
<td>5.31</td>
<td>76.35</td>
<td>$I_6$</td>
<td>2.77</td>
<td>67.37</td>
</tr>
</tbody>
</table>

Table 5.2: Metallic structure currents shown in Fig. 5.3

<table>
<thead>
<tr>
<th>segments parallel with x axis</th>
<th>magnitude (amps)</th>
<th>angle (degrees)</th>
<th>segments parallel with x axis</th>
<th>magnitude (amps)</th>
<th>angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>2.45</td>
<td>178.11</td>
<td>$I_7$</td>
<td>4.15</td>
<td>80.08</td>
</tr>
<tr>
<td>$I_2$</td>
<td>2.75</td>
<td>53.07</td>
<td>$I_8$</td>
<td>8.71</td>
<td>-118</td>
</tr>
<tr>
<td>$I_3$</td>
<td>2.41</td>
<td>177.27</td>
<td>$I_9$</td>
<td>17.45</td>
<td>57.11</td>
</tr>
<tr>
<td>$I_4$</td>
<td>20.4</td>
<td>-141</td>
<td>$I_{10}$</td>
<td>7.3</td>
<td>73</td>
</tr>
<tr>
<td>$I_5$</td>
<td>8.37</td>
<td>36.4</td>
<td>$I_{11}$</td>
<td>7.4</td>
<td>61.4</td>
</tr>
<tr>
<td>$I_6$</td>
<td>9.9</td>
<td>-144.3</td>
<td>$I_{12}$</td>
<td>3.5</td>
<td>74</td>
</tr>
</tbody>
</table>

Table 5.3: Ground grid currents shown in Fig. 5.2
transformer secondary neutral is grounded, there are transformer currents, $I_{j1}$ and $I_{j2}$, injected to the ground grid. Regulator currents, $I_{j3}$ and $I_{j4}$, injected to the ground grid are also shown in Table 5.1. As the transformer tank and regulator tank are grounded at two locations as indicated in Fig. 5.2, each has two currents injected to the ground. Tables 5.2 and 5.3 list the measured steel structure and ground grid currents identified in Figs. 5.2 and 5.3, respectively.

5.3 Calculation of Ground Grid and Metallic Structure Current Distribution of Madison Substation

The computer program developed in Chapters 3 and 4 is used to calculate the Madison substation ground grid and metallic structure current distribution (see sections 3.6.3 and 4.3.3).

5.3.1 Input the known data

(1) Input the coordinates of the ground grid and the metallic structure network shown in Fig. 5.4 (a) and (b).

(2) Input the injected currents as given in Table 5.1.

(3) Choose soil resistivity as 100 $\Omega\cdot$m for moist soil [42].

(4) Other known data for this network: length of ground rod is 10 feet; height of metallic structure is 18 feet and the depth of ground grid is 1.5 feet.
5.3.2 Calculation results and discussions

Fig. 5.4 shows the segment numbering for the ground grid and steel structures of Madison Substation. Note that the numbering of ground grid segments shown in Fig. 5.4 is for computer calculation. Any comparisons between calculated and measured results will use the segment numbers identified for measurement purposes as shown in Fig. 5.2. In Fig. 5.4, \( n_1 \) and \( n_2 \) represent the number of ground grid segments parallel with \( x \) axis and \( y \) axis respectively; \( n_3 \) represents the number of non-parallel ground grid segments; \( n_4 \) and \( n_5 \) represent the number of ground rod and steel structure segments respectively. The length of conductor segments need not to be the same. In this calculation, the Madison ground grid conductors parallel with the \( x \) axis are divided into 24 segments \( (n_1 = 24) \); the ground grid conductors parallel with the \( y \) axis are divided into 18 segments \( (n_4 = 18) \); non-parallel conductors are divided into 2 segments \( (n_3 = 2) \); there are 9 ground rods \( (n_4 = 9) \) and 13 metallic structure branches \( (n_5 = 13) \). The ground grid conductor segment length is varying from 4.5 feet to 10 feet.

Table 5.4 is a list of calculated resistances and self-inductances of the ground grid segments for network current calculation purposes; Table 5.5 is a partial list of calculated mutual inductance of the ground grid; Table 5.6 shows self and mutual resistance for leakage current calculation purposes; Table 5.7 is a partial list of calculated currents for the ground grid for which the measured data is available (Fig. 5.2). More complete list of calculated results is given in Appendix A. As shown in Table 5.7, the network currents are
Figure 5.4: Ground grid and steel structure divided by $n$ segments ($n=n_1+n_2+n_3+n_4+n_5$)
calculated considering (1) grid conductor resistance $R$ only, (2) grid conductor resistance $R$ and self inductance $L$, (3) and grid conductor resistance $R$, self inductance $L$ and mutual inductance $M$. The currents calculated as shown in Table 5.7 are at power frequency (60 Hz). As can be seen from Table 5.7, when self inductance and mutual inductance are included in the calculation, the results show little change. Therefore, ground grid conductor segment, self and mutual inductance can be neglected at power frequency. However, both self and mutual inductance shown in Tables 5.4 and 5.5 will have significant contribution on the current distribution at high frequency since self and mutual impedance ($j\omega L$ and $j\omega M$) increase proportionally with frequency ($\omega$). From calculated results shown in Table 5.7, it also can be seen that the leakage currents can be neglected at normal operation conditions of substation. When the injected currents are very large, especially at fault conditions, the leakage currents are not negligible.

The Madison substation ground potential rise (GPR), $U_0$, at reference point is 51.3 V.
### Table 5.4: Resistance and self-inductance of ground grid segments as identified in Fig. 5.4a (partial list)

<table>
<thead>
<tr>
<th>Segment, k, parallel w/ x axis</th>
<th>Resistance, $R_k$ $10^3 \times$</th>
<th>Self-inductance, $L_k$ $10^6 \times$</th>
<th>Segment, $n_1 + k$, parallel w/ y axis</th>
<th>Resistance, $R_{n_1+k}$ $10^3 \times$</th>
<th>Self-inductance, $L_{n_1+k}$ $10^5 \times$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2584</td>
<td>0.2518</td>
<td>k = 1</td>
<td>0.1604</td>
<td>0.1432</td>
</tr>
<tr>
<td>11</td>
<td>0.3208</td>
<td>0.3245</td>
<td>k = 8</td>
<td>0.2138</td>
<td>0.2015</td>
</tr>
<tr>
<td>12</td>
<td>0.2138</td>
<td>0.2015</td>
<td>k = 9</td>
<td>0.2049</td>
<td>0.1916</td>
</tr>
<tr>
<td>16</td>
<td>0.2183</td>
<td>0.2085</td>
<td>k = 13</td>
<td>0.1871</td>
<td>0.172</td>
</tr>
<tr>
<td>17</td>
<td>0.2183</td>
<td>0.2085</td>
<td>k = 14</td>
<td>0.1871</td>
<td>0.172</td>
</tr>
<tr>
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### Table 5.5: Mutual inductance of ground grid segments as identified in Fig. 5.4a (partial list)

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<th>Segment, k, parallel w/ x axis</th>
<th>Mutual inductance, $M_{1,k}$ $10^3 \times$</th>
<th>Segment, k, parallel w/ y axis</th>
<th>Mutual inductance, $M_{1+k, n_1+k}$ $10^6 \times$</th>
<th>Segment, k, parallel w/ x axis &amp; non-parallel segment</th>
<th>Mutual inductance, $M_{k,n_1+n_2+k}$ $10^4 \times$</th>
<th>Segment, k, parallel w/ y axis &amp; non-parallel segment</th>
<th>Mutual inductance, $M_{k,n_1+n_2+2}$ $10^5 \times$</th>
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<td>k</td>
<td>10^3 x</td>
<td>k</td>
<td>10^4 x</td>
<td>k</td>
<td>10^5 x</td>
</tr>
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85
<table>
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<tr>
<th>Segment number, k, parallel w/ x axis</th>
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<th>Between non-parallel segment and parallel w/ x axis $R_{k,n1+n2+1}$</th>
<th>Segment number, k, parallel w/ y axis</th>
<th>Between segments parallel w/ x axis and y axis, $R_{1,n1+k}$</th>
<th>Between non-parallel segment and parallel w/ y axis $R_{k,n1+n2+1}$</th>
<th>Ground rod segment number</th>
<th>Between ground rod segment and parallel w/ x axis $R_{n1+n1+n2+n3+k}$</th>
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Table 5.6: Self and mutual resistance for leakage current calculation, the ground grid segment number as identified in Fig. 5.4a (partial list)
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<th>$I_m$ (Amp) (R only)</th>
<th>$I_m$ (Amp) (R and L)</th>
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<th>Angle (R only)</th>
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<th>Angle (R,L,M)</th>
<th>leakage (Amp)</th>
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<td>97.98</td>
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Table 5.7: Calculated current of ground grid segments
(refer to Fig. 5.2 for segment number)
5.4 Comparison Between Calculated Results and Measurements

Figs. 5.5-5.8 show the comparison between the measured and calculated currents of Madison substation ground grid (see Fig. 5.2 for segment number sequence). Fig. 5.5 shows the comparison of current amplitude and phase angle of ground grid conductor segments between measured data and calculated results considering conductor segment resistance only. It can be seen that the calculated current amplitude agrees well with the measured data. Generally, the calculated phase angle agrees well with the measured data. However, there are angle deviations for those currents in segments, 1, 2, 3 and 13, 15, 16, 17, 18 and 19. It can be seen from Fig. 5.5 (a), the currents in those segments are very small (under 3 amps) compared to the other segment currents and the error is negligible. Fig. 5.6 shows the comparison of current amplitude and phase angle between measured data and calculated results considering conductor segment resistance and self-inductance. Fig. 5.7 shows the comparison of current amplitude and phase angle between measured data and calculated results considering conductor segment resistance, self-inductance and mutual inductance. Fig. 5.8 shows the comparison of current amplitude and phase angle between measured data and calculated results including leakage currents. It can be seen that there is little improvement on the calculations when the segment self inductance, mutual inductance and leakage currents are included in the calculation. As discussed in section 5.3, conductor inductance and leakage current are not significant for the ground
Figure 5.5: Ground grid network current distribution (R only)
Figure 5.6: Ground grid network current distribution (R and L)
Figure 5.7: Ground grid network current distribution (R, L and M)
Figure 5.8: Ground grid current distribution (R, L, M and leakage)
grid current distribution at power frequency and normal operation conditions. Comparisons in Figs. 5.5-5.8 show that the simulated and measured results agree well. In the comparisons, the agreement of current amplitude is within 15% between calculated and measured. The deviation in phase angle of small currents is due to some uncertainty of system operation at time of measurement. In addition, ground rod leakage currents is also calculated. The ground rod leakage currents are very small (under 0.1 Amp) and can be neglected.

Figs. 5.9 (a) - (f) show a comparison of metallic structure currents between measured and calculated data considering conductor resistance, self-inductance and mutual inductance. The difference between measured and simulated current magnitude considering conductor resistance only is within 23%. However, the difference is reduced to within 15% when self inductance is included in the calculation. Therefore, inductance should be considered for calculating structure current distribution in substations.

It can be seen from the above comparison, the computed results agree reasonably well with the measurements, which demonstrates the relatively high degree of accuracy of the proposed modeling technique.
Figure 5.9: Metallic structure current distribution of Fig. 5.3
5.5 **Comparison With Other Results Available in Literature**

In this section, the proposed model is compared with other applications found in literature [34, 37, 40]. Fig. 5.10 shows leakage current distribution along side conductors of a 20 meter square ground grid (see Fig. 5.11). The ground grid conductor radius is 0.16 meters and the buried depth is 1 meter. The injected current to the ground grid is 1500 amps. The solid line of Fig. 5.10 shows the calculated current distribution. The symbol “*” of Fig. 5.10 indicates the measured data obtained from the literature [40]. In the leakage current calculation, the voltage drop across the ground grid conductors is included. A good agreement between the measured and calculated is obtained.

![Comparison with the measured data in [40]](image)

**Figure 5.10:** Comparison with the measured data in [40]
Table 5.8 shows the ground grid resistance obtained by Dawalibi and Mukhedkar's method [34], Koutenikoff's method [37] and the proposed method (see section 3.6.3). The listed ground grid configuration and parameters of Table 5.8 are identical to the ones used in [34, 37]. Ground grid resistance is equal to the ratio of ground potential rise (GPR) and injected current to the grid. Ground potential rise and leakage currents are calculated by solving Eq. 3.12 of Chapter 3. The comparison of the ground grid resistance will also demonstrate the application of the proposed modeling technique. Good agreement with the other methods can be seen from Table 5.8.
Table 5.8: Comparison with other methods

Errors exist in both simulation and measurement. However, from the above comparisons, it is concluded that the proposed method for current distribution of ground grid and steel structure is sufficiently accurate in any applications.
5.6 Summary

In this chapter, the calculated results for current distribution by the proposed methods in Chapters 3 and 4 are compared with the measured data and other previous calculation methods available. Good agreement of the results has been found. In the calculation, conductor segment resistance $R$, self inductance $L$ and mutual inductance $M$ are considered. It is found that the conductor resistance plays a dominant role for the current distribution at power frequency.
Magnetic flux density measurements were conducted at the 69/13 kV Madison substation as shown in Fig. 5.1. This chapter presents a comparison between the measured and calculated results of magnetic fields.

6.1 Station Description

The field measurements for actual field data regarding magnetic flux density distribution and substation currents were obtained in the Madison substation [31]. Fig. 6.1 shows an overall layout of Madison substation. Fig. 6.2 gives details related to the layout of the substation equipment and buswork, all of which are within the inner fenced area. Note that the X-Y axis and origin are also indicated in Figure 6.2. Field measurements and calculated data will be referenced to this coordinate system. The outer fenced area is approximately 400 ft x 400 ft and the inner fenced area is 140 ft east to west and 100 ft and 120 ft north to south along the east side and west side, respectively.
Figure 6.1: Layout of Madison substation

Figure 6.2: Layout of Madison Substation within inner fence showing buswork and equipment
6.2 Comparison of Measured and Calculated Magnetic Flux Density Values

The effect of various current sources within the substation on the resulting magnetic flux density distribution was determined using program MAGFLD developed by The Ohio-State University [26,27]. To determine this effect, the current in portions of the station were considered separately, without any other currents included, e.g., the current in the ground grid only on the contribution of the magnetic flux density with all other currents at zero. The cases were as follows:

- **3-phase** only 3-phase measured currents in 13 kV and 69 kV buses and lines
- **neutral** only the neutral current in the two 13 kV lines
- **Grd. grid** only the current in the various elements of the ground grid including the transformer and voltage regulator grounding
- **steel Str.** only the current in the steel structure associated with the 13 kV yard
- **total** includes the currents from all four of the above cases.

There were many magnetic flux density measurement traverses taken. Fig. 6.3 shows several of these traverses on the layout of the station as given in Fig. 6.1. There are five traverses identified outside the inner fence and two inside the inner fence.

Figs. 6.4 - 6.5 show a comparison between the measured and calculated quantities of $B_{res} \quad (= [B_x^2 + B_y^2 + B_z^2]^{1/2})$ for a traverse at $Y = 51.3$ feet and at $Y = 79.3$ feet, respectively. Figs. 6.6 - 6.10 show the results for the 5 traverses on Figure 6.3 that are
outside the inner fence, \( Y = -125.5 \), \( Y = -4.5 \) feet, \( Y = 125.5 \) feet, \( X = -1.5 \) feet and \( X = 144.5 \) feet, respectively. For the above comparisons shown in Figs. 6.4 - 6.10, several cases are included in the calculation: (1) magnetic flux density induced by 3-phase currents only; (2) magnetic flux density induced by neutral current only; (3) magnetic flux density induced by ground grid only; (4) magnetic flux density induced by steel structure only; (5) total magnetic flux density induced by all of the above currents. The agreement between the measured and calculated magnetic flux density including all the above source current contributions are generally quite good. The difference in Fig. 6.4 between measured and calculated results may be due to the currents in the transformer tank wall.

![Diagram showing specific measurement traverses](image)

Figure 6.3: Layout of station showing specific measurement traverses
Figure 6.4: $B_{\text{res}}$ for traverse at $Y = 51.3$ ft

Figure 6.5: $B_{\text{res}}$ for traverse at $Y = 79.3$ ft
Figure 6.6: $B_{\text{res}}$ for traverse at $Y = -125.5$ ft

Figure 6.7: $B_{\text{res}}$ for traverse at $Y = -4.5$ ft

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Figure 6.8: $B_{res}$ for traverse at $Y = 125.5$ ft

Figure 6.9: $B_{res}$ for traverse at $X = -1.5$ ft
6.3 Analysis of Various Current Sources on the Contribution to Magnetic Fields

Figures 6.4 to 6.10 show that three phase currents only are not adequate and the resulting magnetic flux density does not always agree with the measured results. However, as neutral line currents, ground grid and steel structure currents are included, the resulting total magnetic flux density agrees well with the measured results. Therefore, all four major groups of current carrying elements in the substation shall be included: three-phase buswork and three phase overhead line conductors; 13kV overhead line neutral conductors; ground grid; and steel structures.
6.3.1 Magnetic Fields Generated From Three Phase and Neutral Conductors

Figs. 6.4 - 6.10 show that three phase buswork and three phase overhead line conductors are the major contributors to the total magnetic field. Overhead neutral conductors provide a return path for three phase unbalanced currents. When the loads are unbalanced, the neutral currents can provide a significant contribution to the total magnetic field. From Figs. 6.4 and 6.9, where the profile is under the north 13 kV line, note that the contribution from the neutral is significant (10 mG). As the direction of neutral current is opposite to the three phase unbalanced currents, it cancels partially the magnetic flux density from the three phase line currents. Whereas, the east 13kV line does not have as large a component of neutral current, and therefore, the effect of the neutral is less (Figs. 6.5 and 6.8). The profiles shown in Fig. 6.7 and 6.10 are on the sides of the station away from the 13kV lines and do not show as significant contribution from the neutral conductors. However, for both of these, as the profile moves closer to the 13kV lines, an increase in the effect is noticeable. The profiles shown in Fig. 6.6 are far away from the station and the total magnetic flux density is under 1 mG.

The 69kV incoming line has no neutral current flow because the transformer is delta connected on the 69kV side.
6.3.2 Magnetic Fields Generated From Ground Grid and Steel Structures

As can be seen from Figs. 5.2 and 5.3, the 13 kV neutrals are connected to the steel structure which is connected to the ground grid which is connected to the transformer secondary neutral grounding bushing. Therefore, the ground grid and steel structures are tied essentially together and provide a path for any return current in the 13 kV line neutrals to the transformer grounded neutral.

Fig. 6.11-6.13 [25] are contour plots of magnetic flux density for an area extending 20 feet outside the inner fenced area of the substation. Fig. 6.11 includes the effects of only the ground grid currents and Fig. 6.12 includes the effects of only the steel structure currents. For comparison purposes, Fig. 6.13 shows the resulting contour plots for all station current carrying components included. Note that in each of the figures an outline of the ground grid configuration and the steel structure configuration as seen from a top view are shown; connections (nodes) between segments are identified with small circles. From Fig. 6.11 and 6.12 it is obvious that the effects of the ground grid and steel structure are significant over and near the segments. Within a radius of 25 ft the level is 5 mG or greater and in the area above the ground grid the field is 10 mG or greater. Therefore, the levels of the field produced by ground grid and steel structure current components in their vicinity are not negligible.
Figure 6.11: $B_{\text{res}}$ (mG) for Madison substation, ground grid only

Figure 6.12: $B_{\text{res}}$ (mG) for Madison substation, steel structure only
6.4 Discussion of the Influence of Ground Grid Parameters on Magnetic Field

In most existing substations, the exact coordinates of the ground grid may not be known. This section is to investigate how much the inaccuracy of ground grid coordinates will influence the calculated ground grid current distribution and the generated magnetic field flux density. Fig. 6.14 shows a 40m x 30m ground grid with different conductor spacing. In the calculations, the ground grid conductors are divided into 31 segments and the injected current to the ground grid is assumed to be 1000 amp. Table 6.1 shows the calculated ground grid current distribution. Table 6.2 shows the calculated magnetic flux density generated by the currents at traverse A from P1 to P2.
Figure 6.14: 40m x 30m substation ground grid with different grid conductor spacing
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<th>Fig. 6.1 (a) spacing 10,10,10</th>
<th>Fig. 6.1 (b) spacing 5,15,10</th>
<th>relative diff. between (a) and (b) %</th>
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<th>relative diff. between (a) and (c) %</th>
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**Table 6.1:** Conductor currents of ground grid shown in Fig. 6.14
Table 6.2: Magnetic flux density (mG) from current sources of Table 6.1, spacing between points = 5 m

From Table 6.1, it can be seen that the relative difference of currents between Fig. 6.14 (a) and (d) are within 10%, where the ground grid conductor spacing is within 1 meter deviation. Table 6.2 shows that magnetic flux density at traverse A is within 10% when the ground grid conductor spacing is within 1 meter deviation (see the value of relative difference between Fig. 6.14 (a) and (d) of Table 6.2). As the distance between the current source and the magnetic observation point increases, the relative magnetic flux density difference decreases due to the spacing deviation of a ground grid.

6.5 Summary

This chapter presents a comparison between measured and calculated magnetic flux density using MAGFLD developed by The Ohio State University [26,27]. Good agreement of the results is obtained when all the possible current sources are included in
the magnetic flux density calculation. The effect of all the current sources in a substation is discussed. The currents in three phase buswork, three phase overhead line conductors and unbalanced neutral conductor tend to dominate the total magnetic flux density. In the vicinity of the ground grid and metallic structures, contribution to the magnetic flux density from the currents in the ground grid and metallic structure can reach above 10 mG, which is 30% of the total magnetic flux density, and can not be neglected. Therefore, the proposed model in Chapters 3 and 4 for calculating ground grid and metallic structure current distribution is essential to determine magnetic field distribution in substations. It is found that deviation of ground grid configuration within one meter has little effect on the total current and magnetic field flux density distribution.
Chapter 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

The statement of the problem of this research, a description of all the current sources contributing to magnetic fields in ac substation, and the objectives, contributions and outline of the dissertation are described in Chapter 1.

In this dissertation, previous methods for ground grid and metallic structure current distribution are reviewed and limitations inherent in them are outlined (Chapter 2). Based on the boundary element method and theory of electromagnetics and network modeling technique, an accurate mathematical model for calculating the magnetic source current distribution of a ground grid and metallic structure is proposed in Chapters 3 and 4. Chapter 5 verifies the accuracy of the proposed model by comparison of the calculated results with the field measured data from a practical substation and with the available data in literature [34, 37, 40]; the various parameters of the ground grid and metallic structure on the effect of the calculation results are discussed. In Chapter 6, the effects of all current sources in a substation on the magnetic flux density are determined and discussed; agreement between the measured and the calculated magnetic flux density
is found when all the current sources are included in the magnetic field flux density calculation.

The following conclusions can be drawn from this research:

(1) The proposed model includes both leakage current and network currents within the ground grid, and is accurate for calculating current distribution in the ground grid.

(2) Modeling of currents in metallic support structure is developed.

(3) A computer program is developed based on the established mathematical model to determine current distribution of any shaped ground grid and metallic structure configuration.

(4) The actual field measurement data is used for comparison purposes, including all source current data, magnitudes and angles, and the resulting magnetic flux density data. Most research efforts discussed in literature have not had such data available. This is another unique feature of the effort presented in this dissertation.

(5) Good agreement between the simulated and field measured results demonstrates the accuracy of the modeling technique. Good agreement is also found between this proposed and the other data available in literature.

(6) When the contributions on the total magnetic flux density from the ground grid and metallic structure currents are not included, the error can reach above 30% between the calculated and measured magnetic flux density. When all the source currents including ground grid and steel structure currents are considered in the magnetic flux density
calculation, the error between the calculated and measured is within 10%. Therefore, effects of the ground grid and metallic structure currents, in addition to the three-phase line and bus currents, balanced or unbalanced, on the magnetic flux density in and nearby the substation can not be neglected.

(7) Ground grid and steel structure self inductance L and mutual inductance M are considered in the proposed model.

(8) Within one meter of deviation on the ground grid conductor coordinate data, the resulting current and magnetic flux density distribution are within 10% error and therefore it is allowed to have conductor coordinate deviation.

(9) The proposed model is essential to determine ground grid and metallic structure current distribution, to investigate substation magnetic field distribution and to design a substation ground grid.

7.2 Recommendations for Future Work

This dissertation offers the following suggestions for future work:

(1) Some loads on the distribution lines are nonlinear and generate harmonic currents. The harmonic currents need to be included in the calculation of current distribution; and the effects of the harmonic currents on the magnetic fields should be investigated.

(2) Currents in metallic tanks such as transformers, reactors and regulators are not included in this dissertation. There may be currents in the transformer tank wall, which are
not included in this effort. Modeling of those currents and investigation of their effect on the magnetic fields are recommended.

(3) There may be induced currents in a substation metallic fence, which needs to be included in the study.

(4) Eddy currents in different shapes of metallic structures are very complicated when harmonic currents are considered. Modeling of those currents needs extensive work.
APPENDIX A

PARTIAL LIST OF CALCULATION RESULTS

Tables A.1-A.5 list the calculated results for the Madison Substation ground grid and steel structure network. The divided conductor segment sequence numbers are shown in Fig. 5.4.

<table>
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<th>Segment, k, parallel w/ x axis</th>
<th>Resistance ( R_k (\Omega) ) ( 10^{-3} \times )</th>
<th>Self Inductance ( L_k (H) ) ( 10^{-3} \times )</th>
<th>Segment, ( n_1 + k ), parallel w/ y axis</th>
<th>Resistance ( R_{n_1+k} (\Omega) ) ( 10^{-3} \times )</th>
<th>Self Inductance ( L_{n_1+k} (H) ) ( 10^{-3} \times )</th>
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Table A.1: Resistances and self inductances of ground grid segments of Fig. 5.4 (a)
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<td></td>
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<td>22</td>
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</table>

Table A.2: Partial list of mutual inductances between ground grid segments of Fig. 5.4 (a)
<table>
<thead>
<tr>
<th>Segment number, (k), parallel w/ x axis</th>
<th>Between segments parallel w/ x axis, (R_k (i = 1) , (\Omega))</th>
<th>Between nonparallel segments and parallel w/ x axis, (R_{ki+n1+n2+1} , (\Omega))</th>
<th>Segment number, (k), parallel w/ y axis</th>
<th>Between segments parallel w/ y axis, (R_{1n1+k} , (\Omega))</th>
<th>Between nonparallel segment and parallel w/ y axis, (R_{kn1+n2+1} , (\Omega))</th>
<th>Ground rod segment number (k)</th>
<th>Between ground rod segments &amp; parallel w/ x axis, (R_{1n1+n2+n3+k} , (\Omega))</th>
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<tbody>
<tr>
<td>1</td>
<td>38.9563</td>
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<td>2</td>
<td>1.7501</td>
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<td>1.8324</td>
<td>Between nonparallel segments</td>
<td>Ground rod segment #</td>
<td>Between ground rod (R_{n1+n2+n3+1, n1+n2+n3+k} , (\Omega))</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.5494</td>
<td>(R_{n1+n2+1, n1+n2+2} , =2.1673)</td>
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<td></td>
</tr>
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<td>Between ground rod</td>
<td>(R_{n1+n2+n3+1, n1+n2+n3+k} , (\Omega))</td>
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Table A.3: Partial list of mutual resistances, \(R_k\), between ground grid segments of Fig. 5.4 (a) for leakage current calculation purposes

121
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<thead>
<tr>
<th>segment number</th>
<th>( I_m ) measured</th>
<th>Angle measured</th>
<th>( I_m ) calculated</th>
<th>Error (%)</th>
<th>Angle calculated</th>
<th>Error (%)</th>
</tr>
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<tbody>
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<td>2.28</td>
<td>7.73</td>
<td>133.42</td>
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<td>61.4</td>
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<td>74</td>
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Average error: 8.42% for amplitude and 21.44% for phase angle.

Table A.4 Comparison between measured and calculated ground grid current
(refer to Fig. 5.2 for segment number)

<table>
<thead>
<tr>
<th>segment number</th>
<th>( I_m ) measu.</th>
<th>angle measu.</th>
<th>( I_m ) calcul. (R only)</th>
<th>err (%)</th>
<th>angle calcul. (R only)</th>
<th>error (%)</th>
<th>( I_m ) calcul. (R.L&amp;M)</th>
<th>err (%)</th>
<th>angle calcul. (R.L&amp;M)</th>
<th>error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.1</td>
<td>40.33</td>
<td>12.20</td>
<td>0.81</td>
<td>54.41</td>
<td>25.88</td>
<td>11.91</td>
<td>1.57</td>
<td>53.71</td>
<td>24.92</td>
</tr>
<tr>
<td>2</td>
<td>7.96</td>
<td>13.45</td>
<td>8.41</td>
<td>0.46</td>
<td>46.67</td>
<td>71.18</td>
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<td>4.69</td>
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<td>10.59</td>
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</table>

Table A.5 Comparison between measured and calculated steel structure current
(refer to Fig. 5.3 for segment number)
APPENDIX B

LIST OF PROGRAMS

Matlab programs for the calculation of ground grid and metallic structure current distribution for AC substations are listed as follows.

1. Main Calculation Sequence

Calcul.m

inputcal % input data
measure % measured data for comparison
data % parameter calculation
coord % coordinate setup
netwkcal % A matrix
mutual1; mutual2; mutual3; mutual4; mutual5; mik;
matrixc % ground grid network current
strumut % structure current
R1ij; R2ij; R3ij; R4ij; R5ij; R6ij; R7ij; R8ij; R9ij; R10ij; Rij;
un % segment potential
leakcal % leakage current

% ground grid currents: network current + leakage current
netwk_lk % considering R, L and mutual inductance, M and leakage.
end

2. Input Known Data

inputcal.m

clear all
global ra ps dp;
% Input coordinates of ground grid conductors
% input coordinates of parallel conductors in one direction:
% LX  x1  y1  x2  y2
XY1=[0  0  29  0
     0  27 29  27
     33  3  82  3
     33 24  82 24];
% (2) input coordinates of parallel conductors in another direction:
% LY
XY2=[0  0  0  27
     44  3 44  24
     65  3 65  24
     82  3 82  24];
% (3) input coordinates of nonparallel conductors
% LU
XY3=[29  0  33  3
     29  27 33  24];

% (4) Input coordinates of ground rod
XYROD=[0  0; 29  0; 53  3; 82  3;
       44 13; 0 27; 29 27; 53 24; 82 24];

% (5) length of ground rod, Lrod in feet:
Lrod=10;
% (6) vertical length of metallic structure, h, in feet
h=18;
% metallic structure equivalent cross section, re^2*pi, square meters
% (7) re in meters
re=0.03;

% (8) Input coordinates of metallic structure two end points
% x1  y1  z1  x2  y2  z2
XYs=[53  7  0  53  7  h
       65  7  0  65  7  h
       78  7  0  78  7  h
       53 20  0  53 20  h
       65 20  0  65 20  h
       78 20  0  78 20  h
       53  h  65  h
       65  h  78  h
       53  h  65  h
       65  h  78  h
       53  h  65  h
       53  h  65  h
       53  h  65  h
       78  h  78  h];
% (9) Input coordinates of injected currents
%        
x1  y1  z1
XYinj=[44  7  0
      59  3  0
      33  24  0
      59  24  0
      79  7 h];

% (10) input the injected currents:
Ij=[26.7*exp(-j*135.4*pi/180)
   1.17*exp(-j*127.4*pi/180)
   12.84*exp(-j*120.1*pi/180)
   1.65*exp(-j*113.75*pi/180)
   36.877*exp(j*75.33*pi/180)];

% (11) input other data:
% Ground grid conductor radius: ra in meter
% buried depth of ground grid: dp in feet
% copper ground grid resistivity: p Ohm*meter
% soil resistivity: ps Ohm*meter
% steel structure resistivity: pl
ra=0.007;    dp=1.5;
p=18.e-9;    ps=100;    pl=2.e-7;
end
% end of input data

3. Input Measured Data for Comparison

measure.m
% input measured metallic structure currents:
Is=[12.1*exp(j*40.33*pi/180)
   7.61*exp(j*13.45*pi/180)
   5.31*exp(j*76.35*pi/180)
   10.62*exp(j*46.26*pi/180)
   3.95*exp(j*50.83*pi/180)
   2.77*exp(j*67.37*pi/180)];

% input measured ground grid currents
%        
Im    Angle
Imea=[2.45  178.11;  2.75  53.07;  2.41  177.27;
      20.4  -141;    8.37  36.4;    9.9  -144.3;
      4.15  80.08;   8.71  -118;   17.45  57.11;
      7.3    73;    7.4    61.4;    3.5    74;
      2.75  53.07;   18.8  -136;   9.34  55.65
      0.12  -139.28;  0.75  97.19;   2.41  -2.74;
      2.79  128.32];
4. Divide Ground Grid and Steel Structure Network by a Segments

data.m

% number of parallel conductors: m1 and m2;
% number of nonparallel conductors: m3
global ra ps dp;
    m1=size(XY1,1); m2=size(XY2,1); m3=size(XY3,1);

% length of conductors parallel with x axis: LX
% length of conductor segments parallel with x axis: L1
% # of segments for one conductor: nx; ny; nz
    mm1=0;
    for i=1:m1-1
        if XY1(i,1)==XY1(1,1)
            mm1=mm1+1;
        else
            end
    end
    nx1=4; nx2=8;
    for i=1:mm1
        LX(i)=abs(XY1(i,3)-XY1(i,1));
        for ii=1:nx1
            L1((i-1)*nx1+ii)=LX(i)/nx1;
        end
    end
    for i=mm1+1:m1;
        LX(i)=abs(XY1(i,3)-XY1(i,1));
        for ii=1:nx2
            L1(mm1*nx1+(i-mm1-1)*nx2+ii)=LX(i)/nx2;
        end
    end
% # of conductor segments parallel with x axis: n1
    n1=mm1*nx1+(m1-mm1)*nx2;
% length of conductors in y direction: LY
% length of conductor segments parallel with y axis: L2
    mm2=0;
    for i=1:m2-1
        if XY2(i,2)==XY2(1,2)
            mm2=mm2+1;
        else
            end
    end
end

ny1=6; ny2=4;
for i=1:mm2
    LY(i)=abs(XY2(i,4)-XY2(i,2));
    for il=1:ny1
        L2((i-1)*ny1+il)=LY(i)/ny1;
    end
end

for i=mm2+1:m2
    LY(i)=abs(XY2(i,4)-XY2(i,2));
    for il=1:ny2
        L2(mm2*ny1+(i-mm2-1)*ny2+il)=LY(i)/ny2;
    end
end

% # of conductor segments in parallel with y axis: n2
n2=mm2*ny1+(m2-mm2)*ny2;

% length of nonparallel conductors: LU
% length of conductor segments: L3
    LU=sqrt((XY3(i,4)-XY3(i,2))^2+(XY3(i,3)-XY3(i,1))^2);
    for i=1:m3
        L3(i)=sqrt((XY3(i,4)-XY3(i,2))^2+(XY3(i,3)-XY3(i,1))^2);
    end

% # of nonparallel conductor segments : n3
n3=m3;

% # of ground rod, n4.
    n4=size(XYROD,1);

% # of structure branches, n5
    n5=size(XYs,1);

% total number of segments:
n=n1+n2+n3+n4+n5

% total number of injected current:
nj=size(Ij,1);

% total leakage current to soil:
    Isleak=sum(Ij);

% measured metallic structure currents: Is
    Isum=sum(Is);

% # of structures connected to ground grid
    nn1=size(Is,1);

% segment length due to current injection point, ground
% rod and conductor intersection, which makes the change of
% segment length setup
    L1(mm1*nx1+2)=4.875; L1(mm1*nx1+1*nx2+2)=4.875;
    L1(mm1*nx1+3)=9; L1(mm1*nx1+1*nx2+3)=9;
    L1(mm1*nx1+4)=6; L1(mm1*nx1+1*nx2+4)=6;
\[
\begin{align*}
L_1(m_1*n_1+5) &= 6; & L_1(m_1*n_1+1*n_2+5) &= 6; \\
L_1(m_1*n_1+6) &= 4.75; & L_1(m_1*n_1+1*n_2+6) &= 4.75; \\
L_2(1*n_1+1) &= 4; & L_2(1*n_1+2) &= 6; & L_2(1*n_1+3) &= 5.75;
\end{align*}
\]

% length of metallic structure branches, L4
for i=1:n1
    L4(i) = h;
end
for i=n1+1:n1+(n1-2)
    L4(i) = abs(XYs(i,1)-XYs(i,4));
end
for i=n1+(n1-2)+1:n5
    L4(i) = abs(XYs(i,2)-XYs(i,5));
end

5. Calculate Segment Coordinates

coord.m
% coordinate setup
% First step: considering the structure currents as the injected currents
% to the ground, and calculate the ground grid network currents.

global c;
% Divide ground grid to n segments:
\[
n = n_1+n_2+n_3;
\]
% 1 foot = 0.3048m; 1m = 3.28 feet; let
\[
c = 0.3048;
\]
% derivation of coordinate of ground grid segments; considering
% middle point of ground grid segment.
% coordinates of conductor segments in parallel with x axis: X1 and Y1
% coordinates of conductor segments in parallel with x axis: X2 and Y2
% coordinates of nonparallel conductor segments: X3 and Y3
% X1, Y1
for i=1:mm1
    X1((i-1)*n1+1)=L1((i-1)*n1+1)/2;
    Y1((i-1)*n1+1)=XY1(i,2);
    for il=2:nx1
        ii=(i-1)*nx1+il;
        X1(ii)=X1(ii-1)+L1(ii-1)/2+L1(ii)/2;
        Y1((i-1)*nx1+il)=XY1(i,2);
    end
end

for i=mm1+1:mm1

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\[ \begin{align*}
&\text{for } i = 2: nx2 \\
&\quad X(i+1) = X(i) + L1(i)/2 + L1(i+1)/2; \\
&\quad Y(i+1) = Y(i) + Y2(i+1)/2; \\
&\text{end} \\
\end{align*} \]

\[ \begin{align*}
&\text{for } i = 1: mn2 \\
&\quad X2((i-1)*ny1+1) = X2(i+1); \\
&\quad Y2((i-1)*ny1+1) = Y2(i+1); \\
&\quad \text{for } i1 = 2: ny1 \\
&\quad \quad ii = (i-1)*ny1+1; \\
&\quad \quad Y2(ii) = Y2(ii-1) + L2(ii-1)/2 + L2(ii)/2; \\
&\quad \quad X2((i-1)*ny1+1) = X2(i+1); \\
&\quad \text{end} \\
&\text{end} \\
\end{align*} \]

\[ \begin{align*}
&\text{for } i = mm2+1:m2 \\
&\quad ii = mm2*ny1 + i-mm2-1)*ny2; \\
&\quad Y2(ii+1) = Y2(i+1); \\
&\quad X2(ii+1) = X2(i+1); \\
&\quad \text{for } i1 = 2: ny2 \\
&\quad \quad Y2(ii+i1) = Y2(ii+i1-1) + L2(ii+i1-1)/2 + L2(ii+i1)/2; \\
&\quad \quad X2(ii+i1) = X2(i+1); \\
&\text{end} \\
\end{align*} \]

\[ \begin{align*}
&\% \text{ slope of nonparallel conductor segments: } Ks \\
\end{align*} \]

\[ \begin{align*}
&\text{for } k = 1:n3 \\
&\quad Ks(k) = (XY3(k,4) - XY3(k,2)) / (XY3(k,3) - XY3(k,1)); \\
&\quad Angle(k) = \text{atan}(Ks(k)); \\
&\quad X3(k) = XY3(k,1) + L3(k) \cdot \cos(Angle(k)) / 2; \\
&\quad Y3(k) = XY3(k,2) + L3(k) \cdot \sin(Angle(k)) / 2; \\
&\text{end} \\
\end{align*} \]

6. Calculate Network Incidence Matrix

\texttt{netwkcalc.m} \\
% incidence matrix: A \\
% number of nodes: \\
\texttt{nn = (nx1+1)*m1-1;} \\
% notes on conductors in x direction \\
\texttt{for } i = 1:n1
A(i,i)=1; A(i+1,i)=-1;
end
for i=1:m
    A(nx1*i,nx1*i+1)=0;
end
for k=1:n2-ny1
    A(k,k)=1; A(k+nx1,k)=-1;
end
for i=1:ny1
    A(n1+i,nx1*i)=-1;
end
for k=1:ny1
    A(n1+k,ny1*(m2-1)+k)=1;
end
for k=1:ny1-1
    A(n1+k+1,k)-1;
end

7. Calculate Mutual Inductance

mutual1.m
% between parallel conductor segments of unequal/equal length
% in parallel with x axis
for k=1:nx1
    for k1=nx1+1:n1
        d=abs(Y1(k1)-Y1(k));
        d1=sqrt((Y1(k1)-Y1(k))^2+(X1(k1)-X1(k))^2);
        dt=abs(X1(k1)-X1(k))-L1(k1)/2-L1(k)/2;
        if d1==d
            af=L1(k)+L1(k1)+dt; bt=L1(k)+dt; gm=L1(k1)+dt;
            M1(k,k1)=1. e-7*(af*asinh(af/d)-bt*asinh(bt/d)-gm*asinh(gm/d)+dt*asinh(dt/d));
            sq1=sqrt(af^2+d^2); sq2=sqrt(bt^2+d^2);
            sq3=sqrt(gm^2+d^2); sq4=sqrt(dt^2+d^2);
            M1(k,k1)=M1(k,k1)-1. e-7*(sq1-sq2-sq3+sq4);
        end
    end
end
for k=nx1+1:nx1*mm1
    for k1=nx1*mm1+1:n1
        d=abs(Y1(k1)-Y1(k));
        d1=sqrt((Y1(k1)-Y1(k))^2+(X1(k1)-X1(k))^2);
        dt=abs(X1(k1)-X1(k))-L1(k1)/2-L1(k)/2;
        if d1==d
            af=L1(k)+L1(k1)+dt; bt=L1(k)+dt; gm=L1(k1)+dt;
            M1(k,k1)=M1(k,k1)-1. e-7*(sq1-sq2-sq3+sq4);
        end
    end
end

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M1(k,k1)=1.e-7*(af*asinh(af/d)-bt*asinh(bt/d)-gm*asinh(gm/d)+dt*asinh(dt/d));
    sq1=sqrt(af^2+d'^2); sq2=sqrt(bt^2+d^2);
    sq3=sqrt(gm^2+d^2); sq4=sqrt(dt^2+d'^2);
    M1(k,k1)=M1(k,k1)-1.e-7*(sq1-sq2-sq3+sq4);
end
end
end

for k=nx1*mm1+1:nx1*mm1+nx2
    for k1=nx1*mm1+nx2+1:n1
        d=abs(Y1(k1)-Y1(k));
        d1=sqrt((Y1(k1)-Y1(k))^2+(X1(k1)-X1(k))^2);
        dt=abs(X1(k1)-X1(k))-L1(k)/2-L1(k)/2;
        if d1=d
            af=L1(k)+L1(k)+dt; bt=L1(k)+dt; gm=L1(k)+dt;
            M1(k,k1)=1.e-7*(af*asinh(af/d)-bt*asinh(bt/d)-gm*asinh(gm/d)+dt*asinh(dt/d));
            sq1=sqrt(af^2+d'^2); sq2=sqrt(bt^2+d^2);
            sq3=sqrt(gm^2+d^2); sq4=sqrt(dt^2+d'^2);
            M1(k,k1)=M1(k,k1)-1.e-7*(sq1-sq2-sq3+sq4);
        end
    end
end

% between side by side parallel conductor segments of the same length
% in parallel with x axis
    k=nx1+1;
    d=Y1(k)-Y1(1);
    sq=sqrt(1+d^2/L1(k)^2);
    M1(1,k)=2.e-7*L1(k)*(log(L1(k)/d+sq)-sq+d/L1(k));
    for i=2:nx1
        M1(i,nx1+i)=M1(1,nx1+1);
    end
for i=1:nx2
    k=nx1*mm1+i;
    k1=nx1*mm1+nx2+i;
    d=Y1(k1)-Y1(k);
    sq=sqrt(1+d^2/L1(k)^2);
    M1(k,k1)=2.e-7*L1(k)*(log(L1(k)/d+sq)-sq+d/L1(k));
end
M1(i,k)=M1(k,i)
for i=1:n1-1
    for k=i+1:n1
        M1(k,i)=M1(i,k);
    end
end

end
mutual2.m

% between parallel conductor segments of unequal/equal length
% in parallel with y axis
for k=1:ny1
  for k1=ny1+1:ny1*mm2+ny2
    d=abs(X2(k1)-X2(k));
    dt=abs(Y2(k1)-Y2(k))-L2(k1)/2-L2(k)/2;
    af=L2(k)+L2(k1)+dt; bt=L2(k)+dt; gm=L2(k1)+dt;
    % i=n1+k; i1=n1+k1;
    M2(k,k1)=1.e-7*(af*asinh(af/d)-bt*asinh(bt/d)-gm*asinh(gm/d)+dt*asinh(dt/d));
    sq1=sqrt(af^2+d^2); sq2=sqrt(bt^2+d^2);
    sq3=sqrt(gm^2+d^2); sq4=sqrt(dt^2+d^2);
    M2(k,k1)=M2(k,k1)-1.e-7*(sq1-sq2-sq3+sq4);
  end
end
for k = l:nyl
  for kl=nyl+lm2+ny2+l:n2
    d=abs(X2(kl)-X2(k));
    dt=abs(Y2(kl)-Y2(k))-L2(kl)/2-L2(k)/2;
    af=L2(k)+L2(kl)+dt; bt=L2(k)+dt; gm=L2(kl)+dt;
    % i=n1+k; i1=n1+k1;
    M2(k,k1)=1.e-7*(af*asinh(af/d)-bt*asinh(bt/d)-gm*asinh(gm/d)+dt*asinh(dt/d));
    sq1=sqrt(af^2+d^2); sq2=sqrt(bt^2+d^2);
    sq3=sqrt(gm^2+d^2); sq4=sqrt(dt^2+d^2);
    M2(k,k1)=M2(k,k1)-1.e-7*(sq1-sq2-sq3+sq4);
  end
end
for k=ny1*mm2+1:nyl*mm2+ny2
  for k1=ny1*mm2+ny2+1:n2
    d=abs(X2(k1)-X2(k));
    dt=abs(Y2(k1)-Y2(k))-L2(k1)/2-L2(k)/2;
    af=L2(k)+L2(k1)+dt; bt=L2(k)+dt; gm=L2(k1)+dt;
    % i=n1+k; i1=n1+k1;
    M2(k,k1)=1.e-7*(af*asinh(af/d)-bt*asinh(bt/d)-gm*asinh(gm/d)+dt*asinh(dt/d));
    sq1=sqrt(af^2+d^2); sq2=sqrt(bt^2+d^2);
    sq3=sqrt(gm^2+d^2); sq4=sqrt(dt^2+d^2);
    M2(k,k1)=M2(k,k1)-1.e-7*(sq1-sq2-sq3+sq4);
  end
end

% between side by side parallel conductor segments of the same length
% in parallel with y axis
for i=1:ny2
    k=ny1*mm2+ny2+i;
    k1=ny1*mm2+ny2*2+i;
    d=X2(k1)-X2(k);
    sq=sqrt(1+d^2/L2(k)^2);
    M2(k,k1)=2.*e-7*L2(k)*(log(L2(k)/d)+sq)-sq+d/L2(k);
end
for k=ny1*mm2+ny2+1:ny1*mm2+ny2*2
    for k1=ny1*mm2+ny2*2+1:n2
        d=abs(X2(k1)-X2(k));
        d1=sqrt((X2(k1)-X2(k))^2+(X2(k1)-X2(k))^2);
        dt=abs(Y2(k1)-Y2(k))-L2(k1)/2-L2(k)/2;
        if d1==d
            af=L2(k)+L2(k1)+dt; bt=L2(k)+dt; gm=L2(k1)+dt;
            % i=n1+k; il=n1+k1;
            M2(k,k1)=1.*e-7*(af*asinh(af/d)-bt*asinh(bt/d)-gm*asinh(gm/d)+dt*asinh(dt/d));
            sq1=sqrt(af^2+d^2); sq2=sqrt(bt^2+d^2);
            sq3=sqrt(gm^2+d^2); sq4=sqrt(dt^2+d^2);
            M2(k,k1)=M2(k,k1)-1.*e-7*(sq1-sq2-sq3+sq4);
        end
    end
end
% M2(i,k)=M2(k,i)
for i=1:n2-1
    for k=i+1:n2
        M2(k,i)=M2(i,k);
    end
end

mutual3.m
% between nonparallel conductor segments of unequal/equal length
% in parallel with x axis
% case 1
% for k=1:m3
%     if k==m3
%         for i=nx1+1:nx1*mm1
%             D3=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)-L1(i)/2)^2;
%             D3=sqrt(D3);
%             D1=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)+L1(i)/2)^2;
%             D1=sqrt(D1);
%             D4=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)+L1(i)/2)^2;
%             D4=sqrt(D4);
%             D2=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)-L1(i)/2)^2;
%             D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L1(i)*L3(k));
u=(2*L1(i)^2*(D2^2-D3^2-L1(k)^2)+af*(D4^2-D3^2-L1(i)^2))*L3(k);
u=u/(4*L3(k)^2*L1(i)^2-af^2);
v=(2*L3(k)^2*(D4^2-D3^2-L1(k)^2)+af*(D2^2-D3^2-L1(i)^2))*L1(i);
v=v/(4*L3(k)^2*L1(i)^2-af^2);
M3(i,k)=1.e-7*cos2*((u+L3(k))*atan(L1(i)/(D1+D2))+(v+L1(i))*atan(L3(k)/(D1+D4)));
M3(i,k)=M3(i,k)-1.e-7*cos2*(u*atan(L1(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));
end
else
for i=1:nx1
D3=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)-L1(i)/2)^2;
D3=sqrt(D3);
D1=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)+L1(i)/2)^2;
D1=sqrt(D1);
D4=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)+L1(i)/2)^2;
D4=sqrt(D4);
D2=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)-L1(i)/2)^2;
D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L1(i)*L3(k));
u=(2*L1(i)^2*(D2^2-D3^2-L1(k)^2)+af*(D4^2-D3^2-L1(i)^2))*L3(k);
u=u/(4*L3(k)^2*L1(i)^2-af^2);
v=(2*L3(k)^2*(D4^2-D3^2-L1(k)^2)+af*(D2^2-D3^2-L1(i)^2))*L1(i);
v=v/(4*L3(k)^2*L1(i)^2-af^2);
M3(i,k)=1.e-7*cos2*((u+L3(k))*atan(L1(i)/(D1+D2))+(v+L1(i))*atan(L3(k)/(D1+D4)));
M3(i,k)=M3(i,k)-1.e-7*cos2*(u*atan(L1(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));
end
end

% case 2
for k=1:m3
if k==m3
for i=1:nx1
D3=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)-L1(i)/2)^2;
D3=sqrt(D3);
D1=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)+L1(i)/2)^2;
D1=sqrt(D1);
D4=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)+L1(i)/2)^2;
D4=sqrt(D4);
D2=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)-L1(i)/2)^2;
D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
end
end
end

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\[ u = \frac{u}{(4L^3(L)^2L^1(i)^2 - \alpha f^2)}; \]
\[ v = \frac{v}{(2L^3(L)^2(D^4^2-D^3^2-L^1(i)^2)+\alpha f(D^2^2-D^3^2-L^3(k)^2))}; \]
\[ M_{3(i,k)} = 1.0 \cdot \cos^2\left((u+L^3(k)) \cdot \arctan(L^1(i)/(D^1+D^2))+(v+L^1(i)) \cdot \arctan(L^3(k)/(D^1+D^4))\right); \]
\[ M_{3(i,k)} = M_{3(i,k)} - 1.0 \cdot \cos^2\left(u \cdot \arctan(L^1(i)/(D^3+D^4)) + v \cdot \arctan(L^3(k)/(D^2+D^3))\right); \]

else

for i = nx1*mm1+nx2+1:nx1*mm1+nx2+4

\[ D^3 = (XY^3(k,4) - Y^1(i))^2 + (XY^3(k,3) - X^1(i) - L^1(i)/2)^2; \]
\[ D^3 = \sqrt{D^3}; \]
\[ D^1 = (XY^3(k,2) - Y^1(i))^2 + (XY^3(k,1) - X^1(i) + L^1(i)/2)^2; \]
\[ D^1 = \sqrt{D^1}; \]
\[ D^4 = (XY^3(k,4) - Y^1(i))^2 + (XY^3(k,3) - X^1(i) + L^1(i)/2)^2; \]
\[ D^4 = \sqrt{D^4}; \]
\[ D^2 = (XY^3(k,2) - Y^1(i))^2 + (XY^3(k,1) - X^1(i) - L^1(i)/2)^2; \]
\[ D^2 = \sqrt{D^2}; \]
\[ \alpha f = D^4^2 - D^3^2 + D^2^2 - D^1^2; \]
\[ \cos^2 = \alpha f \cdot L^1(i) \cdot L^3(k); \]
\[ u = \frac{(2L^3(L)^2(D^2^2-D^3^2-L^3(k)^2)+\alpha f(D^4^2-D^3^2-L^3(k)^2)) \cdot L^1(i)}{4L^3(k)^2}; \]
\[ v = \frac{(2L^3(L)^2(D^4^2-D^3^2-L^1(i)^2)+\alpha f(D^2^2-D^3^2-L^3(k)^2)) \cdot L^1(i)}{4L^3(k)^2}; \]
\[ M_{3(i,k)} = 1.0 \cdot \cos^2\left((u+L^3(k)) \cdot \arctan(L^1(i)/(D^1+D^2))+(v+L^1(i)) \cdot \arctan(L^3(k)/(D^1+D^4))\right); \]
\[ M_{3(i,k)} = M_{3(i,k)} - 1.0 \cdot \cos^2\left(u \cdot \arctan(L^1(i)/(D^3+D^4)) + v \cdot \arctan(L^3(k)/(D^2+D^3))\right); \]
end
end

% case 3

for k = 1:m3

if k = m3

for i = nx1*mm1+nx2+1:nx1*mm1+nx2*(m1-mm1)

\[ D^3 = (XY^3(k,4) - Y^1(i))^2 + (XY^3(k,3) - X^1(i) + L^1(i)/2)^2; \]
\[ D^3 = \sqrt{D^3}; \]
\[ D^1 = (XY^3(k,2) - Y^1(i))^2 + (XY^3(k,1) - X^1(i) - L^1(i)/2)^2; \]
\[ D^1 = \sqrt{D^1}; \]
\[ D^4 = (XY^3(k,4) - Y^1(i))^2 + (XY^3(k,3) - X^1(i) + L^1(i)/2)^2; \]
\[ D^4 = \sqrt{D^4}; \]
\[ D^2 = (XY^3(k,2) - Y^1(i))^2 + (XY^3(k,1) - X^1(i) + L^1(i)/2)^2; \]
\[ D^2 = \sqrt{D^2}; \]
\[ \alpha f = D^4^2 - D^3^2 + D^2^2 - D^1^2; \]
\[ \cos^2 = \alpha f \cdot L^1(i) \cdot L^3(k); \]
\[ u = \frac{(2L^1(i)^2(D^2^2-D^3^2-L^3(k)^2)+\alpha f(D^4^2-D^3^2-L^1(i)^2)) \cdot L^3(k)}{4L^3(k)^2}; \]
\[ v = \frac{(2L^3(L)^2(D^4^2-D^3^2-L^1(i)^2)+\alpha f(D^2^2-D^3^2-L^3(k)^2)) \cdot L^1(i)}{4L^3(k)^2}; \]

end
end

end
end
v=(2*L3(k)^2*(D4^2-D3^2-L1(i)^2)+af*(D2^2-D3^2-L3(k)^2))*L1(i);
v=v/(4*L3(k)^2*L1(i)^2-af^2);
M3(i,k)=1.e-7*cos2*((u+L3(k))*atan(L1(i)/(D1+D2))+(v+L1(i))*atan(L3(k)/(D1+D4)));
M3(i,k)=M3(i,k)-1.e-7*cos2*(u*atan(L1(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));
end
else
for i=nx1*mm1+1:nx1*mm1+nx2
D3=(XY3(k,4)-Y1(i)))*2+(XY3(k,3)-X1(i)+L1(i)/2)*2;
D3=sqrt(D3);
D1=(XY3(k,2)-Y1(i)))*2+(XY3(k,1)-X1(i)-L1(i)/2)*2;
D1=sqrt(D1);
D4=(XY3(k,4)-Y1(i)))*2+(XY3(k,3)-X1(i)-L1(i)/2)*2;
D4=sqrt(D4);
D2=(XY3(k,2)-Y1(i)))*2+(XY3(k,1)-X1(i)+L1(i)/2)*2;
D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L1(i)*L3(k));
end
for k=1:m3
if k==m3
for i=nx1*mm1+5:nx1*mm1+nx2
D3=(XY3(k,4)-Y1(i)))*2+(XY3(k,3)-X1(i)+L1(i)/2)*2;
D3=sqrt(D3);
D1=(XY3(k,2)-Y1(i)))*2+(XY3(k,1)-X1(i)-L1(i)/2)*2;
D1=sqrt(D1);
D4=(XY3(k,4)-Y1(i)))*2+(XY3(k,3)-X1(i)-L1(i)/2)*2;
D4=sqrt(D4);
D2=(XY3(k,2)-Y1(i)))*2+(XY3(k,1)-X1(i)+L1(i)/2)*2;
D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L1(i)*L3(k));
end
end
end
M3(i,k)=M3(i,k)-1.e-7*cos2*(u*atan(L1(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));   
end
 else
   for i=nx1*mm1+nx2+5:nx1*mm1+nx2*(m1-mm1)
      D3=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)+L1(i)/2)^2;
      D3=sqrt(D3);
      D1=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)-L1(i)/2)^2;
      D1=sqrt(D1);
      D4=(XY3(k,4)-Y1(i))^2+(XY3(k,3)-X1(i)-L1(i)/2)^2;
      D4=sqrt(D4);
      D2=(XY3(k,2)-Y1(i))^2+(XY3(k,1)-X1(i)+L1(i)/2)^2;
      D2=sqrt(D2);
      af=D4^2-D3^2+D2^2-D1^2;
      cos2=af/(L1(i)*L3(k));
      u=(2*L1(i)^2*(D2^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-L1(i)^2))*L3(k);
      u=u/(4*L3(k)^2*L1(i)^2-af^2);
      v=(2*L3(k)^2*(D4^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L1(i);
      v=v/(4*L3(k)^2*L2(i)^2-af^2);
   end
end
end

mutual4.m
% between nonparallel conductor segments of unequal/equal length
% in parallel with y axis
% case 1

k=m3;
for i=1:ny1*mm2
   D3=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)-L2(i)/2)^2;
   D3=sqrt(D3);
   D1=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
   D1=sqrt(D1);
   D4=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)+L2(i)/2)^2;
   D4=sqrt(D4);
   D2=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)-L2(i)/2)^2;
   D2=sqrt(D2);
   af=D4^2-D3^2+D2^2-D1^2;
   cos2=af/(L2(i)*L3(k));
   u=(2*L2(i)^2*(D2^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-L1(i)^2))*L3(k);
   u=u/(4*L3(k)^2*L2(i)^2-af^2);
   v=(2*L3(k)^2*(D4^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L2(i);
   v=v/(4*L3(k)^2*L2(i)^2-af^2);
end
\[ M_{4(i,k)} = 1.07 \times \cos^2 ((u + L_3(k)) \times \text{atan}(L_2(i)/(D_1 + D_2)) + (v + L_2(i)) \times \text{atan}(L_3(k)/(D_1 + D_4))); \]
\[ M_{4(i,k)} = M_{4(i,k)} - 1.07 \times \cos^2 ((u \times \text{atan}(L_2(i)/(D_3 + D_4)) + v \times \text{atan}(L_3(k)/(D_2 + D_3))); \]

\% case 2

\begin{verbatim}
  for k=1:m3
    if k==m3
      for i=ny1*mm2+1:ny1*mm2+ny2-1
        D3=(X3(k,3)-X2(i))^2+(X3(k,4)-Y2(i)-L2(i)/2)^2;
        D3=sqrt(D3);
        D1=(X3(k,1)-X2(i))^2+(X3(k,2)-Y2(i)+L2(i)/2)^2;
        D1=sqrt(D1);
        D4=(X3(k,3)-X2(i))^2+(X3(k,4)-Y2(i)+L2(i)/2)^2;
        D4=sqrt(D4);
        D2=(X3(k,1)-X2(i))^2+(X3(k,2)-Y2(i)-L2(i)/2)^2;
        D2=sqrt(D2);
        af=D4^2-D3^2+D2^2-D1^2;
        cos2=af/L2(i)*L3(k));
        u=(2*L2(i)*D3^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-D2(i)^2))*L3(k);
        u=u/(4*L3(k)*L2(i)^2-af^2);
        v=(2*L3(k)*D3^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L2(i);
        v=v/(4*L3(k)*L2(i)^2-af^2);
        M4(i,k)=1.07 \times \cos^2((u+L3(k)) \times \text{atan}(L2(i)/(D1+D2))+(v+L2(i)) \times \text{atan}(L3(k)/(D1+D4)));
        M4(i,k)=M4(i,k)-1.07 \times \cos^2((u \times \text{atan}(L2(i)/(D3+D4)) + v \times \text{atan}(L3(k)/(D2+D3)));
      end
    else
      for i=ny1*mm2+ny2+1:n2
        D3=(X3(k,3)-X2(i))^2+(X3(k,4)-Y2(i)-L2(i)/2)^2;
        D3=sqrt(D3);
        D1=(X3(k,1)-X2(i))^2+(X3(k,2)-Y2(i)+L2(i)/2)^2;
        D1=sqrt(D1);
        D4=(X3(k,3)-X2(i))^2+(X3(k,4)-Y2(i)+L2(i)/2)^2;
        D4=sqrt(D4);
        D2=(X3(k,1)-X2(i))^2+(X3(k,2)-Y2(i)-L2(i)/2)^2;
        D2=sqrt(D2);
        af=D4^2-D3^2+D2^2-D1^2;
        cos2=af/L2(i)*L3(k);
        u=(2*L2(i)*D3^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-D2(i)^2))*L3(k);
        u=u/(4*L3(k)*L2(i)^2-af^2);
        v=(2*L3(k)*D3^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L2(i);
        v=v/(4*L3(k)*L2(i)^2-af^2);
        M4(i,k)=1.07 \times \cos^2((u+L3(k)) \times \text{atan}(L2(i)/(D1+D2))+(v+L2(i)) \times \text{atan}(L3(k)/(D1+D4)));
        M4(i,k)=M4(i,k)-1.07 \times \cos^2((u \times \text{atan}(L2(i)/(D3+D4)) + v \times \text{atan}(L3(k)/(D2+D3)));
      end
    end
\end{verbatim}
i=ny1+1;
D3=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)-L2(i)/2)^2;
D3=sqrt(D3);
D1=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)+L2(i)/2)^2;
D1=sqrt(D1);
D4=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
D4=sqrt(D4);
D2=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)-L2(i)/2)^2;
D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L2(i)*L3(k));

u=(2*L2(i)^2*(D2^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-L2(i)^2))*L3(k);
u=u/(4*L3(k)^2*L2(i)-af^2);
v=(2*L3(k)^2*(D4^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L2(i);
v=v/(4*L3(k)^2*L2(i)-af^2);

M4(i,k)=1.e-7*cos2*((u+L3(k))*atan(L2(i)/(D1+D2))+v*atan(L3(k)/(D1+D4)));
M4(i,k)=M4(i,k)-1.e-7*cos2*(u*atan(L2(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));
end

% case 3
for k=1:m3
if k==m3
for i=ny1*mm2+ny2+1:n2
D3=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
D3=sqrt(D3);
D1=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)-L2(i)/2)^2;
D1=sqrt(D1);
D4=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
D4=sqrt(D4);
D2=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)+L2(i)/2)^2;
D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L2(i)*L3(k));

u=(2*L2(i)^2*(D2^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-L2(i)^2))*L3(k);
u=u/(4*L3(k)^2*L2(i)-af^2);
v=(2*L3(k)^2*(D4^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L2(i);
v=v/(4*L3(k)^2*L2(i)-af^2);

M4(i,k)=1.e-7*cos2*((u+L3(k))*atan(L2(i)/(D1+D2))+v*atan(L3(k)/(D1+D4)));
M4(i,k)=M4(i,k)-1.e-7*cos2*(u*atan(L2(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));
end

end

i=ny1*mm2+ny2;
D3=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
D3=sqrt(D3);
D1=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)-L2(i)/2)^2;
D1=sqrt(D1);
D4=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)-L2(i)/2)^2;
D4=sqrt(D4);
D2=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)+L2(i)/2)^2;
D2=sqrt(D2);
af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L2(i)^*L3(k));
u=(2*L2(i)^2*(D2^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-L2(i)^2))*L3(k);
u=u/(4*L3(k)^*L2(i)^2+af*2);
v=(2*L3(k)^2*(D4^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L2(i);
v=v/(4*L3(k)^*L2(i)^2-af*2);
M4(i,k)=1.0e-7*cos2*((u+L3(k))*atan(L2(i)/(D1+D2))+(v+L2(i))*atan(L3(k)/(D1+D4)));
M4(i,k)=M4(i,k)-1.0e-7*cos2*(u*atan(L2(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));
else
  for i=ny1*mm2+2:ny1*mm2+ny2
    D3=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
    D3=sqrt(D3);
    D1=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)-L2(i)/2)^2;
    D1=sqrt(D1);
    D4=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)-L2(i)/2)^2;
    D4=sqrt(D4);
    D2=(XY3(k,1)-X2(i))^2+(XY3(k,2)-Y2(i)+L2(i)/2)^2;
    D2=sqrt(D2);
    af=D4^2-D3^2+D2^2-D1^2;
cos2=af/(L2(i)^*L3(k));
u=(2*L2(i)^2*(D2^2-D3^2-L3(k)^2)+af*(D4^2-D3^2-L2(i)^2))*L3(k);
u=u/(4*L3(k)^*L2(i)^2+af*2);
v=(2*L3(k)^2*(D4^2-D3^2-L2(k)^2)+af*(D2^2-D3^2-L3(k)^2))*L2(i);
v=v/(4*L3(k)^*L2(i)^2-af*2);
M4(i,k)=1.0e-7*cos2*((u+L3(k))*atan(L2(i)/(D1+D2))+(v+L2(i))*atan(L3(k)/(D1+D4)));
M4(i,k)=M4(i,k)-1.0e-7*cos2*(u*atan(L2(i)/(D3+D4))+v*atan(L3(k)/(D2+D3)));
end
end

% case 4
k=1:m3-1
for i=1:ny1*mm2
  D3=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
  D3=sqrt(D3);
  D1=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)-L2(i)/2)^2;
  D1=sqrt(D1);
  D4=(XY3(k,3)-X2(i))^2+(XY3(k,4)-Y2(i)+L2(i)/2)^2;
\[ D_4 = \sqrt{D_4}; \]
\[ D_2 = (X_{Y3}(k,1) - X_{2}(i))^2 + (X_{Y3}(k,2) - Y_{2}(i) - L_2(i)/2)^2; \]
\[ D_2 = \sqrt{D_2}; \]
\[ af = D_4^2 - D_3^2 + D_2^2 - D_1^2; \]
\[ \cos^2 = af/(L_2(i)*L_3(k)); \]
\[ u = (2*L_2(i)^2*D_2^2 - D_3^2 - L_3(k)^2 + af*(D_4^2 - D_3^2 - L_2(i)^2))*L_3(k); \]
\[ v = (2*L_3(k)^2*D_4^2 - D_3^2 - L_2(k)^2 + af*(D_2^2 - D_3^2 - L_3(k)^2))*L_2(i); \]
\[ \text{mutual5.m} \]
\[ \text{end} \]
\text{end}

\text{mutual5.m}
\text{\% between nonparallel segments}
\text{for } k = 2 : m_3
\text{for } k_1 = 1 : m_3 - 1
\[ D_3 = (X_{Y3}(k,3) - X_{Y3}(k_1,3))^2 + (X_{Y3}(k,4) - X_{Y3}(k_1,4))^2; \]
\[ D_3 = \sqrt{D_3}; \]
\[ D_1 = (X_{Y3}(k,1) - X_{Y3}(k_1,1))^2 + (X_{Y3}(k,2) - X_{Y3}(k_1,2))^2; \]
\[ D_1 = \sqrt{D_1}; \]
\[ D_4 = (X_{Y3}(k,3) - X_{Y3}(k_1,1))^2 + (X_{Y3}(k,4) - X_{Y3}(k_1,2))^2; \]
\[ D_4 = \sqrt{D_4}; \]
\[ D_2 = (X_{Y3}(k,1) - X_{Y3}(k_1,3))^2 + (X_{Y3}(k,2) - X_{Y3}(k_1,4))^2; \]
\[ D_2 = \sqrt{D_2}; \]
\[ af = D_4^2 - D_3^2 + D_2^2 - D_1^2; \]
\[ \cos^2 = af/(L_3(k_1)*L_3(k)); \]
\[ u = (2*L_3(k_1)^2*D_2^2 - D_3^2 - L_3(k_1)^2 + af*(D_4^2 - D_3^2 - L_3(k_1)^2))*L_3(k); \]
\[ v = (2*L_3(k_1)^2*D_4^2 - D_3^2 - L_3(k_1)^2 + af*(D_2^2 - D_3^2 - L_3(k_1)^2))*L_3(k); \]
\[ \text{M}_5(k_1,k) = 1.\times 10^{-7} \cos^2 ((u + L_3(k_1))*\text{atan}(L_2(i)/(D_1 + D_2)) + (v + L_3(k_1))*\text{atan}(L_3(k)/(D_1 + D_4))); \]
\[ \text{M}_5(k_1,k) = \text{M}_5(k_1,k) - 1.\times 10^{-7} \cos^2 (u*\text{atan}(L_2(i)/(D_3 + D_4)) + v*\text{atan}(L_3(k)/(D_2 + D_3))); \]
\text{end}
\text{end}

% M_1(i,k) = M_1(k,i)
\text{for } i = 1 : n_3 - 1
\text{for } k = i + 1 : n_3
\text{M}_5(k,i) = \text{M}_5(i,k); \]
\text{end}
\text{end}

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% combination of all mutual inductance: M1, M2, M3, M4 and M5
M0(n1,n2)=0;
M=c*[M1 M0 M3
     M0' M2 M4
     M3' M4' M5];
end

10. Calculate Ground Grid and Steel Structure Network Currents

matrixc.m
% considering ground grid resistance, self and mutual inductance
% resistance: Rk; self inductance of ground grid segments: Lk
% Mutual inductance of ground grid segments: M
% Ground grid segment radius: ra in meter
% copper ground grid resistivity: p Ohm*meter
ra=0.007; p=18.e-9;
% self inductance: Lk
for k=1:n1
    Rk(k)=p*c*L1(k)/pi/ra^2;
    Lk(k)=2.e-7*c*L1(k)*(log(2*c*L1(k)/ra)-1+0.25*1);
end
for k=1:n2
    Rk(n1+k)=p*c*L2(k)/pi/ra^2;
    Lk(n1+k)=2.e-7*c*L2(k)*(log(2*c*L2(k)/ra)-1+0.25*1);
end
for k=1:n3
    Rk(n1+n2+k)=p*c*L3(k)/pi/ra^2;
    Lk(n1+n2+k)=2.e-7*c*L3(k)*(log(2*c*L3(k)/ra)-1+0.25*1);
end
% ground grid network has b branches (segments) and n+1 nodes, then
b=n1+n2+n3
% branch impedance matrix: Z
w=2*pi*60;
for i=1:b-1
    for k=i+1:b
        Z(i,k)=j*w*M(i,k);
    end
end
% Z(i,k)=Z(k,i)
for i=1:b-1
for k=i+1:b
    Z(k,i)=Z(i,k);
end
end
for k=1:b
    Z(k,k)=Rk(k)+j*w*Lk(k);
end
% branch admittance matrix:
Y=inv(Z);
% nodal admittance matrix: Yn
Yn=AA*Y;
Yn=Yn*AA';
% injected current: Ijn
% consider structure currents as the known injected current
% # of notes: nn
    nn=mm1*(nx1+1)+(m1-mm1)*(nx2+1)+mm2*(ny1-1)+(m2-mm2)*(ny2-1)-1
for i=nn
    IN(i)=0;
end
Ijn(30-1)=real(Ij(1)); Ijn(13-1)=real(Ij(2));
Ijn(17-1)=real(Ij(3)); Ijn(21-1)=real(Ij(4));
Ijn(12-1)=real(Is(1)); Ijn(14-1)=real(Is(2));
Ijn(40-1)=real(Is(3)); Ijn(20-1)=real(Is(4));
Ijn(22-1)=real(Is(5)); Ijn(41-1)=real(Is(6));
Ijn(nn)=0;
Ijn(30-1)=imag(Ij(1)); Ijn(13-1)=imag(Ij(2));
Ijn(17-1)=imag(Ij(3)); Ijn(21-1)=imag(Ij(4));
Ijn(12-1)=imag(Is(1)); Ijn(14-1)=imag(Is(2));
Ijn(40-1)=imag(Is(3)); Ijn(20-1)=imag(Is(4));
Ijn(22-1)=imag(Is(5)); Ijn(41-1)=imag(Is(6));
Ijn(nn)=0;
% nodal voltage: Vn
Vnn=inv(Yn);
Vn=Vnn*Ijn'; Vn=Vnn*Ijn';
% branch voltage: Vb; branch current: Ib
Vb=AA*Vn; Vb1=AA'*Vn1;
Ib=Y*Vb; Ib1=Y*Vb1;
Ib=Ib+Ib1*j;
Im=abs(Ib); Iang=angle(Ib)*180/pi;
Im=[Im   Iang];
end
% coordinate calculation results with the measured
Ic=[Ib(4); Ib(8); Ib(9); Ib(11); -Ib(12); Ib(13); -Ib(14);
    Ib(18); -Ib(19); -Ib(20); -Ib(21); -Ib(23); Ib(28);
-Ib(31); Ib(33); Ib(41); Ib(43); Ib(44)];
Ical=abs(Ic); Iangle=angle(Ic)*180/pi;

% total # of measured values: num
n=1; ncal=[Ical Iangle];

% Compare the calculated results with the measured values
num=size(lmea);
plot(1:1:num, Iangle,'+', 1:1:num, Ang,'*')

% title('Ground grid current phase angle')
xlabel('Grid conductor segment number')
ylabel('Current angle (degree)')
text(8,180,'* measured')
text(8,160,'+ calculated-considering R, L & M')

% title('Ground grid current distribution')
xlabel('Ground grid conductor segment number')
ylabel('Current amplitude (Amp)')
text(8,240,'* measured')
text(8,230,'+ calculated-considering R, L & M')
axis([0 20 0 26]);

9. Calculate Steel Structure Current Distribution

strures.m
% considering network resistance, self inductance and mutual inductance
% segment resistance: Rk; self inductance; mutual inductance, M
for k=1:n1
    Rk(k)=p*c*L1(k)/pi/ra^2;
    Lk(k)=2.e-7*c*L1(k)*(log(2*c*L1(k)/ra)-1+0.25*1);
end
for k=1:n2
    Rk(n1+k)=p*c*L2(k)/pi/ra^2;
    Lk(n1+k)=2.e-7*c*L2(k)*(log(2*c*L2(k)/ra)-1+0.25*1);
end
for k=1:n3
    Rk(n1+n2+k)=p*c*L3(k)/pi/ra^2;
    Lk(n1+n2+k)=2.e-7*c*L3(k)*(log(2*c*L3(k)/ra)-1+0.25*1);
end
for k=1:n5
% Rk(n1+n2+n3+k)=p1*L4(k)/sl/sl/c;
    Rk(n1+n2+n3+k)=p1*c*L4(k)/pi/re^2;
    Lk(n1+n2+n3+k)=2.e-7*c*L4(k)*(log(2*c*L4(k)/re)-1+0.25*1);
end
% ground grid network has b branches (=# of segments), then
b=n1+n2+n3+n5;

% branch impedance matrix: Z
w=2*pi*60;
for i=1:b-1
    for k=i+1:b
        Z(i,k)=j*w*M(i,k);
    end
end

% Z(i,k)=Z(k,i)
for i=1:b-1
    for k=i+1:b
        Z(k,i)=Z(k,i);
    end
end
for k=1:b
    Z(k,k)=Rk(k)+j*w*Lk(k);
end

% branch admittance matrix:
Y=inv(Z);

% nodal admittance matrix: Yn
Yn=AA*Y;
Yn=Yn*AA';

% injected current: Ijn
%** consider structure currents as the known injected current**
% # of nodes: nn
nn=size(AA, 1);
Ijn(30-1)=real(Ij(1)); Ijn(13-1)=real(Ij(2));
Ijn(17-1)=real(Ij(3)); Ijn(21-1)=real(Ij(4));
Ijn(30-1)=imag(Ij(1)); Ijn(13-1)=imag(Ij(2));
Ijn(17-1)=imag(Ij(3)); Ijn(21-1)=imag(Ij(4));
Ijn(nn)=real(Ij(5)-Isleak); Ijn(nn)=imag(Ij(5)-Isleak);

% nodal voltage: Vn
Vnn=inv(Yn);
Vn=Vnn*Ijn'; Vn1=Vnn*Ijn1';

% branch voltage: Vb; branch current: Ib
Vb=AA*Vn; Vb1=AA'*Vn1;
Ib=Y*Vb; Ib1=Y*Vb1;
Ib=Ib+Ib1*j;
Im=abs(Ib); Iang=angle(Ib)*180/pi;
IIm=[Im Iang]
end

% coordinate calculation results with the measured
for i=1:nn1
Ic(i)=Ib(n-1+i);
end
Ical=abs(Ic)'; IangleF=angle(Ic)*180/pi;

% total # of measured values: num
Ical=[Ical Iangle];

% Compare the calculated results with the measured values
Imea=[abs(Im)];
Ang=[angle(Im)*180/pi];
num=size(Imea);
plot(1:num, Imea,'+', 1:num, Ang,'*')

% title('Current Phase Angle')
xlabel('Structure segment number')
ylabel('Current angle (degree)')

% axis([0.5 6.5 0 80]);
plot(1:num, Ical,'+', 1:num, Imea,'*')

% title('Current Distribution')
xlabel('Structure segment number')
ylabel('Current amplitude (Amp)')

% axis([0.5 6.5 1.5 14]);

end

10. Calculate Mutual Resistance

Rlij.m

global bb c r ra z ps;
% parallel w/ x axis
for i=1:n1
    for k=i:n1
        d=sqrt((X1(i)-X1(k))^2+(Y1(i)-Y1(k))^2)*c;
        if d>4*L1(i)*c
            % point source approximation
            d1=sqrt((X1(i)-X1(k))^2+(Y1(i)-Y1(k))^2+(2*dp)^2)*c;
            R1(i,k)=ps*(1/d+1/d1)/4/pi;
        else
            % Green's function approach
            % New coordinates (r,z)
            % at middle point of segment, k
            z=abs(X1(k)-X1(i))*c;
            if Y1(k)-Y1(i)==0
                r=ra;
        end
    end
end
else
    r=abs(Y1(k)-Y1(i))*c;
end
bb=L1(i)*c;
g=quad('mbessel',0.00001,2);

% considering image of the segment, k
r=c*sqrt((Y1(k)-Y1(i))^2+(2*dp)^2);
gg=quad('mbessel',0.00001,2);
g=abs(g+gg);

% at end point of segment, k
z=abs(X1(k)-L1(k)/2-X1(i))*c;
if Y1(k)-Y1(i)==0
    r=ra;
else
    r=abs(Y1(k)-Y1(i))*c;
end
g1=quad('mbessel',0.00001,2);

% considering image of the segment, k
r=c*sqrt((Y1(k)-Y1(i))^2+(2*dp)^2);
gg1=quad('mbessel',0.00001,2);
g1=abs(g1+gg1);

% at other end point of segment
z=abs(X1(k)+L1(k)/2-X1(i))*c;
if Y1(k)-Y1(i)==0
    r=ra;
else
    r=abs(Y1(k)-Y1(i))*c;
end
g2=quad('mbessel',0.00001,2);

% considering image of the segment
r=c*sqrt((Y1(k)-Y1(i))^2+(2*dp)^2);
gg2=quad('mbessel',0.0001,1);
g2=abs(g2+gg2);

% mean value along segment, k
R1(i,k)=(g1+g1+g2)/3;
end
end

R2ij.m

global bb c r ra z ps;
% between segments in parallel w/ x axis and y axis
for i=6:n1
    for k=1:n2
\[ d = \sqrt{(X_1(i) - X_2(k))^2 + (Y_1(i) - Y_2(k))^2} \times c; \]

\[ \text{if } d > 4 \times L_1(i) \times c \]

\% point source approximation

\[ d_1 = \sqrt{(X_1(i) - X_2(k))^2 + (Y_1(i) - Y_2(k))^2 + (2 \times d_2)^2} \times c; \]

\[ R_2(i,k) = \frac{ps \times (1/d_1 + 1/d_1)/4/\pi}{4}; \]

\% Green's function approach

\% New coordinates \((r,z)\)

\% at middle point of segment, \(i\)

\[ z = \text{abs}(X_1(i) - X_2(k)) \times c; \]

\[ r = \text{abs}(Y_1(i) - Y_2(k)) \times c; \]

\[ bb = L_1(i) \times c; \]

\[ g = \text{quad('mbessel',0.00001,2)}; \]

\% considering image of the segment, \(k\)

\[ r = c \times \sqrt{(Y_1(i) - Y_2(k))^2 + (2 \times d_2)^2}; \]

\[ gg = \text{quad('mbessel',0.00001,2)}; \]

\[ g = \text{abs}(g + gg); \]

\% at end point of segment, \(i\)

\[ z = \text{abs}(X_1(i) - L_1(i)/2 - X_2(k)) \times c; \]

\[ r = \text{abs}(Y_1(i) - Y_2(k)) \times c; \]

\[ g_1 = \text{quad('mbessel',0.00001,2)}; \]

\% considering image of the segment, \(k\)

\[ r = c \times \sqrt{(Y_1(i) - Y_2(k))^2 + (2 \times d_2)^2}; \]

\[ gg_1 = \text{quad('mbessel',0.00001,2)}; \]

\[ g_1 = \text{abs}(g_1 + gg_1); \]

\% at other end point of segment

\[ z = \text{abs}(X_1(i) + L_1(i)/2 - X_2(k)) \times c; \]

\[ r = \text{abs}(Y_1(i) - Y_2(k)) \times c; \]

\[ g_2 = \text{quad('mbessel',0.00001,2)}; \]

\% considering image of the segment

\[ r = c \times \sqrt{(Y_1(i) - Y_2(k))^2 + (2 \times d_2)^2}; \]

\[ gg_2 = \text{quad('mbessel',0.0001,1)}; \]

\[ g_2 = \text{abs}(g_2 + gg_2); \]

\% mean value along segment, \(k\)

\[ R_2(i,k) = (g + g_1 + g_2)/3; \]

end

end

R3ij.m

\% Between segment in parallel w/ x axis and nonparallel segment

for \(i = 1:n1\)

for \(k = 1:n3\)
\[ d = \sqrt{(X_1(i) - X_3(k))^2 + (Y_1(i) - Y_3(k))^2} \times c; \]

\[
\text{if} \ d > 4 \times L_1(i) \times c
\]

% point source approximation

\[
d_1 = \sqrt{(X_1(i) - X_3(k))^2 + (Y_1(i) - Y_3(k))^2 + (2 \times d)^2} \times c;
\]

\[
R_3(i,k) = ps \times (1/d + 1/d_1)/4/\pi;
\]

else

% Green's function approach

% New coordinates (r, z)
% at middle point of segment, i
\[
z = \text{abs}(X_1(i) - X_3(k)) \times c; \quad r = \text{abs}(Y_1(i) - Y_3(k)) \times c;
\]

% considering image of the segment, k
\[
r = c \times \sqrt{(Y_1(i) - Y_3(k))^2 + (2 \times d)^2};
\]

\[
gg = \text{quad('mbessel',0.00001,2)';}
\]

\[
g = \text{abs}(g + gg);
\]

% at end point of segment, i
\[
z = \text{abs}(X_1(i) - L_1(i)/2 - X_3(k)) \times c;
\]

% considering image of the segment, k
\[
r = c \times \sqrt{(Y_1(i) - Y_3(k))^2 + (2 \times d)^2};
\]

\[
gg1 = \text{quad('mbessel',0.00001,2)';}
\]

\[
g1 = \text{abs}(g1 + gg1);
\]

% at other end point of segment
\[
z = \text{abs}(X_1(i) + L_1(i)/2 - X_3(k)) \times c;
\]

% considering image of the segment
\[
r = c \times \sqrt{(Y_1(i) - Y_3(k))^2 + (2 \times d)^2};
\]

\[
gg2 = \text{quad('mbessel',0.00001,1)';}
\]

\[
g2 = \text{abs}(g2 + gg2);
\]

% mean value along segment, k
\[
R_3(i,k) = (g + g1 + g2)/3;
\]

end

end

R4ij.m

global bb c r ra z ps;

% between segments which are in parallel w/ y axis
for i=1:n2
  for k=i:n2
\[ d = \sqrt{(Y2(i) - Y2(k))^2 + (X2(i) - X2(k))^2} \cdot c; \]
if \( d > 4 \cdot L2(i) \cdot c \)

\% point source approximation
\[ d1 = \sqrt{(Y2(i) - Y2(k))^2 + (X2(i) - X2(k))^2 + (2 \cdot dp)^2} \cdot c; \]
\[ R4(i,k) = ps \cdot (1/d + 1/d1)/4/pi; \]
else

\% Green's function approach
\% New coordinates \((r,z)\)
\% at middle point of segment, \(k\)
\[ z = \text{abs}(Y2(k) - Y2(i)) \cdot c; \]
if \( X2(k) - X2(i) \neq 0 \)
\[ r = ra; \]
else
\[ r = \text{abs}(X2(k) - X2(i)) \cdot c; \]
end
\[ bb = L2(i) \cdot c; \]
\[ g = \text{quad('mbessel',0.00001,2)}; \]

\% considering image of the segment, \(k\)
\[ r = c \cdot \sqrt{(X2(k) - X2(i))^2 + (2 \cdot dp)^2}; \]
\[ gg = \text{quad('mbessel',0.00001,2)}; \]
\[ g = \text{abs}(g + gg); \]

\% at end point of segment, \(k\)
\[ z = \text{abs}(Y2(k) - L2(k)/2 - Y2(i)) \cdot c; \]
if \( X2(k) - X2(i) \neq 0 \)
\[ r = ra; \]
else
\[ r = \text{abs}(X2(k) - X2(i)) \cdot c; \]
end
\[ gl = \text{quad('mbessel',0.00001,2)}; \]

\% considering image of the segment, \(k\)
\[ r = c \cdot \sqrt{(X2(k) - X2(i))^2 + (2 \cdot dp)^2}; \]
\[ gg1 = \text{quad('mbessel',0.00001,2)}; \]
\[ g1 = \text{abs}(g1 + gg1); \]

\% at other end point of segment
\[ z = \text{abs}(Y2(k) + L2(k)/2 - Y2(i)) \cdot c; \]
if \( X2(k) - X2(i) \neq 0 \)
\[ r = ra; \]
else
\[ r = \text{abs}(X2(k) - X2(i)) \cdot c; \]
end
\[ g2 = \text{quad('mbessel',0.00001,2)}; \]

\% considering image of the segment
\[ r = c \cdot \sqrt{(X2(k) - X2(i))^2 + (2 \cdot dp)^2}; \]
gg2=quad('mbessel',0.0001,1);
g2=abs(g2+gg2);

% mean value along segment, k
R4(i,k)=(g+g1+g2)/3;
end
end
end

R5ij.m

global bb c r ra z ps;
% between segment in parallel w/ y axis and nonparallel segment
for i=1:n2
    for k=1:n3
        d=sqrt((Y2Ci-Y3(k))^2+(X2(i)-X3(k))^2)*c;
        if d>4*L2(i)*c
            d1=sqrt((Y2(i)-Y3(k))^2+(X2(i)-X3(k))^2+(2*dp)^2)*c;
            R5(i,k)=ps*(1/d+1/d1)/4/pi;
        else
            % Green's function approach
            % New coordinates (r,z)
            V o
            % at middle point of segment, i
            z=abs(Y2(i)-Y3(k))*c; r=abs(X2(i)-X3(k))*c;
            bb=L2(i)*c;
            g=quad('mbessel',0.00001,2);
            % considering image of the segment, k
            r=c*sqrt((X2(i)-X3(k))^2+(2*dp)^2);
            gg=quad('mbessel',0.00001,2);
            g=abs(g+gg);
            % at end point of segment, i
            z=abs(Y2(i)-L2(i)/2-Y3(k))*c;
            r=abs(X2(i)-X3(k))*c;
            g1=quad('mbessel',0.00001,2);
            % considering image of the segment, k
            r=c*sqrt((X2(i)-X3(k))^2+(2*dp)^2);
            gg1=quad('mbessel',0.00001,2);
            g1=abs(g1+gg1);
            % at other end point of segment
            z=abs(Y2(i)+L2(i)/2-Y3(k))*c;
            r=abs(X2(i)-X3(k))*c;
            g2=quad('mbessel',0.00001,2);
            % considering image of the segment
            r=c*sqrt((X2(i)-X3(k))^2+(2*dp)^2);
\begin{verbatim}
gg2=quad('mbessel',0.0001,1);
g2=abs(g2+gg2);

% mean value along segment, k
R5(i,k)=(g+g1+g2)/3;
end
end

R6ij.m

global bb c r ra z ps;
% between nonparallel segments
for i=1:n3
  for k=i:n3
    d=sqrt((X3(i)-X3(k))^2+(Y3(i)-Y3(k))^2)*c;
    if d>4*L3(i)*c
      % point source approximation
      d1=sqrt((X3(i)-X3(k))^2+(Y3(i)-Y3(k))^2+(2*dpr2)^2)*c;
      R6(i,k)=ps*(1/d+1/d1)^4/pi;
    else
      % Green's function approach
      % New coordinates (r,z)
      if d == 0
        r=ra; z=0;
      else
        af=atan(abs(Y3(i)-Y3(k))/abs(X3(i)-X3(k)));
        z=d*sin(af); r=d*cos(af);
      end
      bb=L3(i)*c;
      g=quad('mbessel',0.00001,2);
      % considering image of the segment, i
      r=sqrt(r^2+(2*dp*c)^2);
      gg=quad('mbessel',0.00001,2);
      g=abs(g+gg);
    end
  d=sqrt((XY3(i,1)-X3(k))^2+(XY3(i,2)-Y3(k))^2)*c;
    af=atan(abs(XY3(i,2)-Y3(k))/abs(XY3(i,1)-X3(k)));
    z=d*sin(af); r=d*cos(af);
    bb=L3(i)*c;
    g1=quad('mbessel',0.00001,2);
    % considering image of the segment, i
    r=sqrt(r^2+(2*dp*c)^2);
    gg1=quad('mbessel',0.00001,2);
\end{verbatim}
gl=abs(gl+gg1);

% at other end point of segment

d=sqrt((XY3(i,3)-X3(k))^2+(XY3(i,4)-Y3(k))^2)*c;

af=atan(abs(XY3(i,4)-Y3(k))/abs(XY3(i,3)-X3(k)));

z=d*sin(af); r=d*cos(af);

bb=L3(i)^c;

g2=quad('mbessel',0.00001,2);

% considering image of the segment, i

r=sqrt(r^2+(2*dp*c)^2);

gg2=quad('mbessel',0.0001,1);

g2=abs(g2+gg2);

% mean value along segment, k

R6(i,k)=(g+g1+g2)/3;

end

end

R7ia.m

global bb c r ra z ps;

% between segment in parallel w/ x axis and ground rod

for i=23:n1
    for k=1:n4
        d=sqrt((X1(i)-XYROD(k,1))^2+(Y1(i)-XYROD(k,2))^2+(Lrod/2)^2)*c;

        if d>4*H(i)*c

        d1=sqrt((X10)-XYROD(k,l))2+(Y1(i)-XYROD(k,2))2+(Lrod/2+2*dp)^2)*c;

        R7(i,k)=ps*(1/d+1/d1)/4/pi;

        else

        % Green's function approach

        % New coordinates (r,z)

        z=abs(X1(i)-XYROD(k,1))*c;

        r=c*sqrt((Y1(i)-XYROD(k,2))2+(Lrod/2)^2);

        bb=L1(i)^c;

        g=quad('mbessel',0.00001,2);

        % considering image of the segment, k

        r=c*sqrt((Y1(i)-XYROD(k,2))2+(Lrod/2+2*dp)^2);

        gg=quad('mbessel',0.00001,2);

        g=abs(g+gg);

        % at end point of segment, i

        z=abs(X1(i)-L1(i)/2-XYROD(k,1))*c;

        r=c*sqrt((Y1(i)-XYROD(k,2))2+(Lrod/2)^2);

        gl=quad('mbessel',0.00001,2);
% considering image of the segment, k
   \( g_{gl} = \text{quad('mbessel', 0.00001, 2)}; \)
   \( g_1 = \text{abs}(g_1 + g_{gl}); \)
   \( r = c \times \sqrt{(Y_1(i) - XYROD(k,2))^2 + (L_{rod}/2 + 2 \times dp)^2}; \)
% at other end point of segment
   \( z = \text{abs}(X_1(i) + L_1(i)/2 - XYROD(k,1)) \times c; \)
   \( r = c \times \sqrt{(Y_1(i) - XYROD(k,2))^2 + (L_{rod}/2)^2}; \)
   \( g_2 = \text{quad('mbessel', 0.0001, 1)}; \)
   \( g_2 = \text{abs}(g_2 + g_{gg}); \)
% considering image of the segment
   \( r = c \times \sqrt{(Y_1(i) - XYROD(k,2))^2 + (L_{rod}/2 + 2 \times dp)^2}; \)
   \( g_{gg} = \text{quad('mbessel', 0.00001, 2)}; \)
% mean value along segment, i
   \( R_7(i,k) = (g + g_1 + g_2) / 3; \)
end
end

RSij.m

global bb c r ra z ps;
% between segment in parallel w/ y axis and ground rod
   for i=1:n2
      for k=1:n4
         \( d = \sqrt{(X_2(i) - XYROD(k,1))^2 + (Y_2(i) - XYROD(k,2))^2 + (L_{rod}/2)^2 \times c}; \)
         if \( d > 4 \times L_2(i) \times c \)
         \( d_1 = \sqrt{(X_2(i) - XYROD(k,1))^2 + (Y_2(i) - XYROD(k,2))^2 + (L_{rod} + 2 \times dp)^2 \times c}; \)
         \( R_{8i}(i,k) = ps \times (1/d + 1/d_1) / 4/\pi; \)
         else
         \% Green's function approach
         \% New coordinates (r,z)
% at middle point of segment, i
            \( z = \text{abs}(Y_2(i) - XYROD(k,2)) \times c; \)
            \( r = c \times \sqrt{(X_2(i) - XYROD(k,1))^2 + (L_{rod}/2)^2}; \)
            \( bb = L_2(i) \times c; \)
            \( g = \text{quad('mbessel', 0.00001, 2)}; \)
% considering image of the segment, k
            \( r = c \times \sqrt{(X_2(i) - XYROD(k,1))^2 + (L_{rod}/2 + 2 \times dp)^2}; \)
            \( g_{gg} = \text{quad('mbessel', 0.00001, 2)}; \)
            \( g = \text{abs}(g + g_{gg}); \)
% at end point of segment, i
            \( z = \text{abs}(Y_2(i) - L_2(i)/2 - XYROD(k,2)) \times c; \)
            \( r = c \times \sqrt{(X_2(i) - XYROD(k,1))^2 + (L_{rod}/2)^2}; \)
global bb c r ra z ps;
% between ground rods
for i=1:n4
  for k=i+1:n4
    d=sqrt((XYROD(i,1)-XYROD(k,1))^2+(XYROD(i,2)-XYROD(k,2))^2)*c;
    dist(i,k)=d/Lrod/c;
  end
end
% point source approximation
  d1=sqrt((XYROD(i,1)-XYROD(k,1))^2+(XYROD(i,2)-XYROD(k,2))^2)+(Lrod/2+2*dp)^2)*c;
  R10(i,k)=ps*(1/d1+1/d)/4/pi;
end
% Green's function approach
% at middle point of segment, i
  r=ra; z=0;
  bb=Lrod*c;
  g=quad('mbessel',0.00001,2);
% considering image of the segment, i
  r=sqrt(ra^2+(Lrod/2+2*dp)^2)*c;
  gg=quad('mbessel',0.00001,2);
  g=abs(g+gg);
% at end point of segment, i
  r=ra; z=c*Lrod/2;
\[
g_1 = \text{quad('mbessel',0.00001,2)}; \\
\text{considering image of the segment, i}
\]
\[
r = \sqrt{r_i^2 + (L_{rod}/2+2*dp)^2}; \\
g_{gl} = \text{quad('mbessel',0.00001,2)}; \\
g_1 = \text{abs(g1+gg1)}; \\
\text{mean value along segment, i}
\]
\[
g = (g+g1)/2; \\
\text{for } k=1:n4 \\
R10(k,k) = g; \\
\text{end}
\]

**Rij.m**

\[
\% \text{ mutual resistance: combination of all cases: R1, R2, ..., R10} \\
\% R1(i,k) = R1(k,i) \\
\% \text{for } i=1:n1 \\
\% \text{for } k=i+1:n1 \\
R1(k,i) = R1(i,k); \\
\% \text{end} \\
\% \text{end}
\]

\[
\% R4(i,k) = R4(k,i) \\
\% \text{for } i=1:n2 \\
\% \text{for } k=i+1:n2 \\
R4(k,i) = R4(i,k); \\
\% \text{end} \\
\% \text{end}
\]

\[
\% R6(i,k) = R6(k,i) \\
\% \text{for } i=1:n3 \\
\% \text{for } k=i+1:n3 \\
R6(k,i) = R6(i,k); \\
\% \text{end} \\
\% \text{end}
\]

\[
\% R10(i,k) = R10(k,i) \\
\% \text{for } i=1:n4 \\
\% \text{for } k=i+1:n4 \\
R10(k,i) = R10(i,k); \\
\% \text{end} \\
\% \text{end}
\]

\[
R = [R1 \ R2 \ R3 \ R7 \ R2' \ R4 \ R5 \ R8 \ R3' \ R5' \ R6 \ R9 \ R7' \ R8' \ R9' \ R10];
\]

end
11. Calculate Segment Potential

un.m
% Calculate voltage between segment and reference point, Un
% Potential on segment: U0+Un
% segment in parallel with x axis:
    Un(1)=Vn(1)/2;
    for i=2:n1-1
        Un(i)=(Vn(i)+Vn(i-1))/2;
    end
    Un(n1)=(Vn(n1)+Vn(n1-1))/2;
    Un(mm1*nx1+nx2)=(Vn(n1)+Vn(mm1*nx1+nx2-1))/2;
    for i=1:mm1
        Un(nx1*i)=(Vn(n1+n2-mm1*2+i-1)+Vn(nx1*i-1))/2;
    end

% segment in parallel with y axis:
    Un(n1+1)=Vn(n1)/2;
    for i=n1+2:n1+ny1*mm2-1
        Un(i)=(Un(i-1)+Un(i-2))/2;
    end
    for k=1:m2-mm2
        for i=n1+ny1*mm2+ny2*(k-1)+1:n1+ny1*mm2+ny2*k
            Un(i)=(Un(i-k-1)+Un(i-k-2))/2;
        end
    end
    Un(n1+ny1)=(Vn(nx1)+Vn(n1+ny1-2))/2;
    Un(n1+ny1+1)=(Vn(n1+ny1-1)+Vn(nx1*mm1+2))/2;
    Un(n1+ny1+ny2+1)=(Vn(n1+ny1+ny2-2)+Vn(nx1*mm1+5))/2;
    Un(n1+ny1+ny2*2+1)=(Vn(n1+ny1+ny2*2-3)+Vn(n1+n2-2))/2;
    Un(n1+ny1+ny2+2)=(Vn(nx1*mm1+nx2+2)+Vn(n1+ny1+ny2-3))/2;
    Un(n1+ny1+ny2*2)=(Vn(nx1*mm1+nx2+5)+Vn(n1+ny1+ny2*2-4))/2;
    Un(n1+ny1+ny2*3)=(Vn(n1+n2-1)+Vn(n1+n2-5))/2;

% nonparallel segments
    for i=1:n3
        Un(n1+n2+i)=(Vn(nx1*mm1+nx2*(i-1))+Vn(n1+n2+i-5))/2;
    end

% ground rods
    Un(n1+n2+n3+1)=0; Un(n1+n2+n3+2)=Vn(n1+n2-4);
    Un(n1+n2+n3+3)=Vn(nx1*mm1+3); Un(n1+n2+n3+4)=Vn(n1+n2-2);
    Un(n1+n2+n3+6)=Vn(nx1); Un(n1+n2+n3+7)=Vn(n1+n2-3);
    Un(n1+n2+n3+8)=Vn(nx1*mm1+nx2+3); Un(n1+n2+n3+n4)=Vn(n1+n2-1);
Un(n1+n2+n3+n4)=Vn(n1+ny1);
end

12. Calculate Ground Grid Leakage Current

% ground grid leakage current calculation, including ground rod
% # of segment: nn;
nn=n+n4;
% It is assumed that the potential at the reference point: U0
% # of unknown to be solved: nn+1: segment leakage currents (nn) + U0 (1)
for i=1:nn
    ONE(i)=1;
end
C=[R -ONE'
    ONE 0];
% voltage between segment and reference point, Un
% Potential on segment: U0+Un
CC=inv(C); Un=[Un Isleak];
Ileak=CC*Un';
Imleak=abs(Ileak); Ileakang=angle(Ileak)*180/pi;
end

13. Calculate Current Distribution with All Parameters Included

% considering ground grid resistance and self and mutual inductance
% In addition, leakage currents to soil are considered
% branch voltage: Vb; branch current: Ib
%     Im=abs(Ib); Iang=angle(Ib)*180/pi;
%     If=[Im Iang]
%     Imleak=abs(Ileak); Ileakang=angle(Ileak)*180/pi;
% coordinate calculation results with the measured
Ic=[Ib(4)+Ileak(4); Ib(8)-Ileak(8); Ib(9)+Ileak(9); Ib(11)+Ileak(11);
    -Ib(12)-Ileak(12)-Ileak(n+3); Ib(13)+Ileak(13); -Ib(14)-Ileak(14);
    Ib(18)+Ileak(18); -Ib(19)-Ileak(19); -Ib(20)-Ileak(20)-Ileak(n+8);
    -Ib(21)-Ileak(21); -Ib(23)-Ileak(23); Ib(28)+Ileak(28);
    -Ib(31)-Ileak(31); -Ib(33)-Ileak(33)-Ileak(n+5); Ib(35)+Ileak(35);
    Ib(41)+Ileak(41); -Ib(43)-Ileak(43); Ib(44)+Ileak(44)];
Ical=abs(Ic); Iangle=angle(Ic)*180/pi;

% total # of measured values: num
Ilcal=[Ical Iangle];

% Compare the calculated results with the measured values
159
num = size(Imea);
plot(1:num, Iangle,'+', 1:num, Ang,'*')

title('Ground Grid Current Phase Angle')
xlabel('Ground grid conductor segment number')
ylabel('Current angle (degree)')
text(6,185,'* measured')
text(6,170,'+ calculated-Network(R & L & M)+Leakage')

plot(1:num, Ical,'+', 1:num, Imea,'*')

title('Ground Grid Current Distribution')
xlabel('Ground grid conductor segment number')
ylabel('Current amplitude (Amp)')
text(4,24,'* measured')
text(4,23,'+ calculated-Network(R & L & M)+Leakage')

axis([0 20 0 26]);
BIBLIOGRAPHY


