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AN EXPERIMENTAL STUDY OF MIXING AND NOISE IN A SUPERSONIC RECTANGULAR JET WITH MODIFIED TRAILING EDGES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

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* * * * *

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ABSTRACT

A rectangular nozzle with design Mach number 2 and aspect ratio 3 is used to investigate the effects of various geometric modifications to the nozzle trailing edge on gas mixing and jet noise. Flow visualizations are carried out by the laser sheet illumination technique, while noise is measured by two microphones. Surface flow visualizations by the kerosene-lampblack technique and wall pressure measurements are performed to investigate the major sources of streamwise vorticity. Variation of thrust is measured by an axial thrust measuring system. In the ideally expanded case, no measurable mixing enhancement or noise reduction was observed. On the other hand, significant mixing enhancement was achieved for all modifications in the underexpanded cases. The near field mixing performance of modified nozzles, which generate kidney type pairs of streamwise vortices, were found to be better, while at farther downstream locations, nozzles which generate mushroom type pairs of streamwise vortices performed better. When the modification was used on both sides of the longer nozzle dimension, mixing enhancement was additive of that by each modification. When the jet was overexpanded, relative mixing enhancement was reduced with downstream locations because of the much enhanced mixing of the baseline nozzle due to the flapping motion. Only nozzles which generate mushroom type pair vortices showed better mixing at farther downstream locations. The mixing enhancement
for nozzles with double sided modifications was almost in the same level of that for nozzles with single sided modifications. The measured wall pressure and surface flow data suggests that the major source of streamwise vorticity is most likely the spanwise pressure gradient on the modification.

The reductions of the overall sound pressure level for the modified nozzles were greater for the overexpanded cases than for the underexpanded cases. These reductions are probably related to the three dimensional distortions of shock cell structure which could be inferred from the cross sectional deformations and more directly from surface flow images and pressure data. The fundamental screech mode on the minor axis for the baseline nozzle was antisymmetric at $M_j=1.75$. The modes for the modified nozzles were changed to oblique modes due to the modifications. Regardless of the modifications, the modes were antisymmetric both on the minor and major axes when jets were slightly underexpanded. The fundamental screech frequency measured agreed well with the theoretical value. No measurable thrust loss or gain was observed for nozzles with single or double sided trailing edge modifications. Therefore, the use of trailing edge modifications is a very promising technique, which can significantly enhance mixing and reduce shock noise with minimal thrust loss, especially in the non-ideally expanded cases.
Dedicated to my wife, children, and our families
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CHAPTER 1

INTRODUCTION

When one designs a machine or a system, many factors are considered such as performance, the effects on environment, safety, and so on. The priority of these factors varies from application to application. For example, in a supersonic jet as a propulsion device, a quieter jet has been demanded for commercial aircraft such as the Concorde. In this case, the supersonic jet noise may be one of the major issues in the design of the jet. On the other hand, the jet noise may not be a major concern for a military aircraft. What has been required in a military aircraft is the ability to be invisible or stealthy to radar and infrared detection systems. The Stealth NightHawk Fighter is the only aircraft in which the low-observable stealth technology has been exploited. To be invisible to an infrared detection system, the exhausted jet fluid of an aircraft should be mixed rapidly with the ambient air, which will result in a low detectable jet infrared signature. Fortunately, the enhancement of mixing of a jet also results in the reduction of jet noise, in general.

One of the puzzling things in supersonic mixing layers is that the mixing or spreading rate, dictated by spanwise or ring-type vortical structures, is significantly reduced as the compressibility increases. Many mixing enhancement techniques have been explored to
overcome the reduced mixing capability in highly compressible flows. One proposed way to enhance mixing is to generate streamwise vortices which are not significantly affected by the compressibility. The use of simple tabs at the jet exit has been proven to be effective not only in enhancing mixing but also in reducing jet noise by generating streamwise vortices. Although the use of tabs is very effective in mixing enhancement and noise reduction, it results in the loss of thrust due to the blockage effects of the tabs. Thus, an alternative technique of using trailing edge modifications, is suggested to enhance mixing and to reduce noise with minimal thrust loss. Since the trailing edge modifications are simple cut-outs on the splitter plate, they do not protrude into the jet stream and in turn no significant thrust loss is expected.

The purposes of the study are to enhance mixing and to reduce jet noise with minimal thrust loss by generating strong streamwise vortices through trailing edge modifications in a rectangular jet. As will be discussed later, the reason behind using rectangular nozzles is that they have superior mixing and noise characteristics when compared with circular jets. To investigate the role of streamwise vortices and mixing enhancement, laser based flow visualization techniques are used. The effects of trailing edge modifications on near- and far-field noise are measured by two microphones. Detailed discussions on the source of streamwise vortices and thrust results are also be presented.
CHAPTER 2

BACKGROUND

2.1 Introduction

Much work has been done on mixing enhancements and noise reduction in subsonic and supersonic jets to meet the noise regulations for commercial airplanes and/or to reduce the jet infrared signature for military airplanes. Noise and mixing are closely related. The enhanced mixing, in general, results in reduced jet noise and the jet noise affects mixing characteristics. As shown in Fig. 2.1, the jet mixing region consists of near- and far-field regions. The nearfield region strongly depends on the nozzle geometry, while the normalized velocity profile in the farfield region collapses into a single profile regardless of the nozzle geometry. Thus, the farfield region is called a self-similar region. In contrast to an incompressible jet, supersonic jets can have a nozzle exit pressure higher (underexpansion) or lower (overexpansion) than the ambient pressure. In these non-ideally expanded cases, there are shock waves in the jet plume as sketched in Fig. 2.2 (a) and (c). The first shock wave is attached to the nozzle for the overexpanded case, while it is off the nozzle exit in the underexpanded case since expansion waves are generated at the nozzle lip.
As a measure of the compressibility in supersonic flows, the convective Mach number, based on the assumption that there are stagnation points in mixing layers formed isentropically, was introduced by Bogdanoff [1983] and Papamoschou and Roshko [1986]. The convective Mach numbers for the high speed stream \( M_{c1} \) and the low speed stream \( M_{c2} \) are defined as \( M_{c1} = (U_1 - U_c)/a_1 \) and \( M_{c2} = (U_c - U_2)/a_2 \), respectively. Where \( U_c \) is the convection velocity of structures, \( U_1 \) and \( U_2 \) are the velocity of the high speed and low speed streams, and \( a_1 \) and \( a_2 \) are the speed of sound for the high speed and low speed streams, respectively. When the specific heat ratios for the two streams are equal, \( \gamma_1 = \gamma_2 \), the convective Mach number \( M_c \) is reduced to: \( M_c = M_{c1} = M_{c2} = (U_1 - U_2)/(a_1 + a_2) = \Delta U/(a_1 + a_2) \), where \( \Delta U = U_1 - U_2 \). Although this convective Mach number is controversial as a measure of compressibility, it is a quite valid parameter for representing the overall compressibility of a supersonic flow, especially when \( M_c \) is less than 0.6. As the convective Mach number increases, the spreading rate of a mixing layer significantly decreases when compared with that for incompressible mixing layers. The instability analysis shows that this reduction in mixing rates is due to the much decreased amplification rates of the instability waves as the convective Mach number increases [Sandham and Reynolds 1991]. It has been found that large scale structures are two dimensional in subsonic and low convective Mach number supersonic flows, while they are three dimensional or oblique in higher Mach number supersonic flows [Papamoschou 1989, Sandham and Reynolds 1991, Elliott et al. 1992a, Clemens and Mungal 1992, Krothapalli and Strykowski 1996]. The dynamics of these large scale structures is a major parameter for controlling both mixing and noise in a jet.

Typical mixing enhancement techniques explored so far in supersonic flows are the
use of mechanical tabs [Ahuja and Brown 1989, Samimy et al. 1991], swirl generating vanes [Naughton and Settles 1992], and the modifications of nozzle trailing edges [Wlezien and Kibens 1988, Martens et al. 1996b]. The use of tabs appears to be an effective way to enhance mixing and to reduce noise in both subsonic and supersonic flows with some thrust penalty. Trailing edge modified nozzles were investigated to determine whether they could excite oblique large scale structures and so increase mixing rates in supersonic flows [Martens et al. 1996b]. Unfortunately, they did not excite large scale structures as effectively as expected. As will be discussed later, the streamwise vortices generated by tabs or by other means are not significantly affected by the compressibility contrary to the spanwise or ring vortices, so the generation of streamwise vortices is a powerful way to enhance mixing in a supersonic jet.

A major jet noise source in subsonic flows is turbulent mixing noise related to large and small eddies. When a supersonic jet is operated at an off-design condition, the jet generates shock associated noise, such as screech tones and broadband shock associated noise, due to the interaction of large scale structures with these shock cell structures in the jet plume. In addition, the jet can generate eddy Mach wave radiation when turbulent structures travel downstream supersonically, relative to the ambient sound speed. The frequency and directivity of the supersonic jet noise are quite well predicted theoretically for simple nozzles, while the amplitude is not [Tam 1995].

The purpose of the study is to investigate mixing enhancement and noise reduction through trailing edge modifications with minimal thrust loss. The effects of nozzle exit geometry and the fully expanded jet Mach number are to be explored in the study. The ratio
of jet to ambient temperature along with these two parameters are the major factors which significantly affect mixing and noise in a jet. The ultimate purposes of the trailing edge modifications are to control or excite the large scale structures in a jet plume for the ideally expanded case, and to generate streamwise vortices so as to enhance mixing and reduce noise for the non-ideally expanded cases. The importance of these major parameters and large scale structures on mixing and noise will be discussed in the following sections. In addition, the supersonic jet noise sources will be discussed.

2.2 Major Parameters Governing Mixing and Noise Generation in a Supersonic Jet

As mentioned above, the major parameters which significantly affect mixing and noise characteristics in supersonic jets are the fully expanded jet Mach number, the ratio of jet to the ambient temperature and the jet exit geometry. The effect of each parameter is now discussed in detail.

2.2.1 The effects of the fully expanded jet Mach number on mixing and noise

For a given nozzle, mixing and noise generation strongly depend on the fully expanded jet Mach number, which is calculated from the ratio of jet stagnation to the ambient pressure. As the fully expanded jet Mach number increased, the spreading rate of a jet varied drastically for the convergent nozzles [Gutmark et al. 1990] and for the converging-diverging rectangular nozzles [Martens et al. 1996b]. The flapping and
symmetric motions of a jet plume also depend on the fully expanded jet Mach number [Gutmark et al. 1990, Raman 1996]. Circular jets experience mode switching, from axisymmetric to antisymmetric/flapping motion, as the fully expanded jet Mach number increases [Gutmark et al. 1990]. Many noise measurement results show the significant variations of the amplitude of screech tones with the fully expanded jet Mach number [Gutmark et al. 1990, Krothapalli and Strykowski 1996, Raman 1996]. The linear stability analysis of Krothapalli and Strykowski [1996] for a convergent circular jet reveals that as the fully expanded jet Mach number increases, the large scale structures are three dimensional or oblique and the amplification rates of the most unstable waves decrease. This analysis is similar to the Sandham and Reynolds’ [1991] finding in a plane mixing layer. As a matter of fact, the fully expanded jet Mach number is the most significant parameter in mixing and noise for a jet.

2.2.2 The effects of the ratio of jet to ambient temperature on mixing and noise

The effects of the ratio of jet to ambient temperature on mixing are not well documented. However, a few studies of these effects on noise were done by Rosfjord and Toms [1975], Tanna et al. [1975], Tam et al. [1991], and Krothapalli and Strykowski [1996]. According to Rosfjord and Toms [1975], the screech frequency continuously decreases with the nozzle pressure ratio or the jet Mach number in a high temperature jet. In a hot and moderate Mach number jet, the jet generates intense Mach wave radiation, because the instability waves or large scale turbulent structures generally propagate at
supersonic speeds relative to the ambient speed of sound [Tam et al. 1991]. Krothapalli and Strykowski's [1996] analysis shows that as the ratio of jet to ambient temperature increases, the screech tone amplitude decreases due to the relatively much enhanced three-dimensional instability waves.

2.2.3 The effects of jet exit geometry on mixing and noise

Many studies on non-axisymmetric nozzles both in subsonic and supersonic flows have been done partially due to their benefits in mixing and noise over axisymmetric jets. Non-axisymmetric jets, especially elliptic and rectangular jets, have much enhanced mixing rates and reduced noise levels compared with circular jets. Since the spreading rate of the minor axis side of an elliptic or a rectangular jet, in general, is higher than that of the major axis side, the jet plume experiences axis switching in subsonic flows [Ho and Gutmark 1987, Hussain and Husain 1989, Grinstein 1993, 1995] and in supersonic flows [Schadow et al. 1989]. However, no axis switching was found in the experiments of Gutmark et al. [1991], and Martens et al. [1996b]. The spreading rate at the minor axis plane was higher than at the major axis plane in Martens et al.'s experiments, while the spreading rates were almost the same at both the minor and major axis planes in Gutmark et al.'s cases. It is believed that enhanced mixing in a non-axisymmetric jet is partially attributed to the axis switching. The possible causes for the axis switching in a non-axisymmetric jet are nonuniform azimuthal jet exit conditions, self induction [Gutmark et al. 1991, Grinstein 1993, 1995], and
streamwise vortices [Reeder, 1994]. The effects of trailing edge modifications will be discussed later.

2.3 Large Scale Spanwise (or Ring) and Streamwise Structures

It is generally agreed that large scale structures play an important role in the mixing and noise generation of jets. Brown and Roshko [1974] were first to show that spanwise organized, large scale structures exist in plane mixing layers and that they are responsible for the entrainment and mixing. They are also an important source of jet noise production [Tam 1975, Tam and Morris 1980, Seiner 1984, Seiner et al. 1993]. In incompressible and low convective Mach number compressible mixing layers, large scale structures are two-dimensional “roller” structures [Clemens and Mungal, 1990]. These structures are generated due to the Kelvin-Helmholtz instability [Bernal and Roshko 1986, Tam 1989, 1991]. As the convective Mach number increases, the obliqueness of a large scale structure increases [Papamoschou 1989, Sandham and Reynolds 1991, Elliott et al. 1992a, Clemens and Mungal 1992, Krothapalli and Strykowski 1996]. Furthermore, these structures are not well organized and are three dimensional at higher convective Mach numbers [Clemens and Mungal, 1992]. Sandham and Reynolds suggested that the critical convective Mach number for the emergence of oblique waves is about 0.6. They also presented a formula predicting the oblique angle of a structure at a Mach number greater than 0.6: $M_c \cos \beta = 0.6$, where $\beta$ is the angle between the instability wave direction and flow direction.

In contrast to spanwise or ring-like vortical structures, the streamwise vortices are not
much affected by the compressible effects [Elliott et al., 1992b], as mentioned above. This was shown numerically and experimentally. As a matter of fact, many mixing enhancement techniques have been attempted to generate streamwise vortices with or without being aware of this fact. Although the amplification rate of spanwise instability waves decreases with the increasing convective Mach number, the effects of the compressibility on the evolution of streamwise vorticity fields in a free shear layer are minimal, as mentioned above. Thus, generating streamwise vortices in supersonic jets would be an effective way of enhancing mixing in a shear layer.

The existence of the streamwise vortices in an incompressible plane mixing layer was proposed by Bernal and Roshko [1986]. Zapryagaev and Solotchin [1988] found streamwise vortices in an underexpanded circular jet and claimed that these vortices were generated from the Taylor-Goertler instability. Novopashin and Perepelkin [1989] studied the conditions which caused “petal structures”, streamwise aligned wave like structures, in sonic orifices. They showed that the Reynolds number and the ratio of orifice edge roughness to the jet diameter are the decisive parameters in the generation of the petal structures. In addition, they suggested that the stability loss was probably related to the flow peculiarity at the jet boundary as a positive radial pressure gradient in the mixing layer. The origin of streamwise vortices in a circular water jet was studied experimentally by Liepman and Gharib [1992]. They concluded that streamwise vortices were generated from the braid region of the large scale structures in the jet. Crown-shaped circular nozzles were used by Longmire et al. [1992] to enhance mixing in an incompressible flow by generating streamwise vortices. Grinstein [1993] found hair pin like structures around the corner regions of subsonic
rectangular jets through numerical flow visualizations. These structures are counter rotating vortex pairs. After roller type vortex rings breakdown, these pipe shaped streamwise vortices fill the jet plume. According to Gutmark et al. [1995], underexpanded jets have naturally occurring streamwise vortices, which are located between the intercepting shock and the jet boundary. These vortices are located within the first shock cell for underexpanded circular jets and are not seen in all cases [Krothapalli et al. 1991]. Arnette et al. [1993] showed the existence of spatially stationary streamwise vortices in underexpanded convergent and converging-diverging nozzles by visualizations and measurements of pressure fields along the azimuthal direction. They argued that these streamwise vortices resulted from the Taylor-Goertler instability.

Much enhanced mixing via trailing edge modifications was reported in underexpanded rectangular jets by Martens et al. [1996b]. They showed that the enhancements in mixing were related to the generation of streamwise vortices by the modified trailing edges for underexpanded cases. Zaman et al. [1991] and Samimy et al. [1993] used tabs to increase mixing and to reduce noise. Their results show that the tabs generated streamwise vortices and these vortices increased mixing and reduced noise both in the subsonic and supersonic circular jet flows, supporting the belief that streamwise vortices are not so affected by the compressibility as claimed by Elliott et al. [1992b]. These ample experimental evidences show that streamwise vortices exist in an underexpanded jet and that these vortices play an important role in mixing and entrainment. The scale of streamwise vortices generated by tabs or trailing edge modifications is much larger than that occurring naturally in a nozzle with simple exit geometry.
2.4 Mixing Enhancement Techniques in a Jet

Mixing enhancement techniques which have been attempted so far are categorized as active and passive excitations. The active excitement means the use of external energy in a form of mechanical, electrical, or acoustic energy to excite instability waves in mixing layers. On the other hand, the passive excitement implies that the excitement of instability waves is attempted without using external energy such as placing a cavity around the jet plume, using tabs, or trailing edge modifications.

Here are a few examples for active excitation techniques. Lepicovsky et al. [1986] studied the effects of acoustic excitations on the mixing of a jet. A faster center line velocity decay was measured in subsonic and supersonic jets with acoustic excitations and the most effective excitations were achieved at Strouhal numbers, $St_0 = \frac{fD}{U}$, between 0.35 and 0.5 at all operating conditions examined. In the definition of Strouhal number, $f$ is the excitation frequency and $D$ and $U$ are the jet exit diameter and velocity, respectively. A wedge shaped mechanical actuator driven by piezoelectric wafers was adopted to excite a jet from a converging nozzle over a Mach number range of 0.7 to 1.2 in Kibens and Glezer's [1992] experiments. They found that the excitation was effective for high aspect ratio jets and that a flapping mode was excited and the jet orientation could be altered depending on the excitation frequency. Martens et al. [1996a] used the glow discharge excitations to excite either two-dimensional or oblique instability waves in low Reynolds number plane mixing layers of convective Mach number 0.51 and 0.64. The increases of mixing rates were measured only in the higher convective Mach number layer.
Several passive excitation techniques will be cited in this section. The effects of mechanical tabs on mixing and screech tones in heated and unheated, subsonic and supersonic jets were studied by Ahuja and Brown [1989]. Their results show that the tabs suppress screech tones and cause center line velocity to decay faster, which signifies enhanced mixing. However, the effects of tabs on thrust and the reason for this enhanced mixing were left unanswered. Zaman et al. [1991], Samimy et al. [1993], and Reeder and Samimy [1996] furthered these studies by using advanced visualization techniques and by measuring the flow field. Their studies include the effects of the number of tabs, tab geometries and tab locations on mixing and far field noise. They showed that the streamwise vortices generated by the tabs were responsible for the enhanced mixing, the disappearance of screech noise, and the reduction in broadband shock associated noise. As mentioned above, their results support the Elliott et al.'s [1992b] claim that the effects of compressibility on streamwise vortices are minimal. Open rectangular and semi-circular cavities, placed adjacent to a fully expanded Mach number 2.0 jet, were used to excite instability waves [Yu et al. 1994, Yu et al. 1995]. Well organized large scale structures, which are hard to be seen in such a high convective Mach number flow, were found in the experiments. Mie scattering visualizations show drastically enhanced growth rates in the excited side of the jet. Martens et al. [1996b] attempted to excite or amplify oblique instability waves in Mach number 2.0 rectangular jets via trailing edge modifications. These modifications of trailing edges produced enhanced mixing rates for underexpanded cases. The effects of these trailing edge modifications on mixing, noise, and thrust are the major
concerns of this study. Thus, the spanwise oblique large scale structures and trailing edge modifications are to be discussed in the following section in detail.

2.5 Oblique Large Scale Structures and Trailing Edge Modifications

There are ample experimental and theoretical evidences that large scale structures are oblique relative to the streamwise direction for a convective Mach number greater than 0.6. As will be shown, this fact has been substantiated both experimentally and theoretically.

Papamoschou [1989] showed that large scale structures in a plane mixing layer are oblique and three-dimensional at a high convective Mach number. Based on this finding and the fact that the growth rate increases with the decrease of the convective mach number, three trailing edge devices were attempted to enhance mixing without success. The three devices are a vortex generator, slanted trip wires, and a saw-tooth extension. By using the linear stability theory, Sandham and Reynolds [1991] revealed that the amplification rates of instability waves decreased as the convective Mach number increased and that the most amplified waves are oblique at a Mach number greater than 0.6. The effects of the convective Mach number on large scale structures were tested in plane mixing layers for three convective Mach numbers, 0.28, 0.62 and 0.79, by Clemens and Mungal [1992]. At a higher convective Mach number, large scale structures were three-dimensional and no organized two-dimensional structures were found in their experiments.

Several trailing edge modification experiments are presented and cited again here in detail, simply because their effects are the major concern of this study. In the above
mentioned Longmire et al.'s [1992] experiments, the crown-shaped attachments to circular 
jets generated counter rotating streamwise vortices and in turn these vortices increased the 
mixing in incompressible circular jets. Raman [1996b] studied the effects of the 
nonuniformities in the shock cell structures for rectangular jets having slanted cut outs. The 
beveled nozzles produced spanwise oblique shock cells and in turn radiated weaker and 
unsteady screech. In the experiments of Martens et al. [1996b] mentioned in the previous 
sections, the saw tooth-like trailing modifications were not found effective in exciting or 
amplifying oblique instability waves for perfectly expanded jets, whose modification angles 
were based on the Sandham and Reynolds' [1991] theoretical relationship. However, when 
jets were underexpanded, the jets with modified edges resulted in enhanced mixing rates 
compared with the rate of a simple rectangular jet. Their experiments do not include the 
effects of the trailing edge modifications on noise generations and thrust variations, and the 
modification angle is limited to a single value. Thus, the objectives of this study are to 
optimize the modification which is best for mixing enhancements and/or noise reductions 
depending upon applications, and to measure the thrust in order to compare mixing and noise 
performance of each nozzle based on the thrust loss.

2.6 Noise Generation Mechanism in a Supersonic Jet

Large scale structures play an important role in noise generation in supersonic jets 
as well as in mixing. Supersonic jets could have four kinds of noise components such as fine 
scale mixing noise, broadband shock associated noise, screech tones, and eddy Mach wave
radiation when conditions for them are fulfilled. The fine scale mixing is generated from very small scale turbulence, while the others are related to large scale structures in mixing layers. Each noise component has unique spectral characteristics and directivity. A major part of screech tones and broadband shock associated noise propagates upstream, while that of the eddy Mach wave propagates downstream. It is believed that the directivity and spectral characteristics of each noise component are quite well known and that they are theoretically predictable with the exception of the sound pressure level. When shock cells exist in a jet plume, either screech or broadband shock associated noise is the dominant noise. Both noise components are generated from the interaction of large scale structures with shock cells in a jet plume. The screech tones come from strong interactions and the broadband shock associated noise comes from weak interactions [Powell 1953b, Tam and Tanna 1982]. The wavelength of the broadband shock associated noise is shorter than that of the eddy Mach wave radiation.

Since the fine scale mixing noise is negligible when compared with screech tones, broadband shock associated noise, and eddy Mach wave radiation, these three sources will be discussed further in following subsections. Also, the fine scale mixing noise is much less understood in comparison with the other three.

2.6.1 Broadband Shock Associated Noise

Broadband shock associated noise is generated from the weak interaction of large scale structures with quasi-periodic shock cells in a jet plume, while the screech is generated
from the strong interaction between them [Tam, 1995]. The dominant part of this noise has upstream directivity. Harper-Bourne and Fisher [1973] modeled this noise component successfully. According to them, turbulent structures whose coherent length scales persisted over several shock cells would effectively radiate this type of noise due to positive reinforcements. A semi-empirical stochastic model theory, which uses three flow variables of the mean flow (the large scale turbulent structure, the shock cell, and the disturbance arising from the interaction between the large scale structures and shock cells), was developed by Tam [1987, 1991, 1992]. The calculated results for noise spectra at various angles relative to jet axis show good agreement with the experimental results of Norum and Seiner [1982]. Kim et al.’s [1993] simulation results for the broadband shock associated noise agree well with Yamamoto et al.’s [1984] data.

2.6.2 Screech Tone

When a supersonic jet is imperfectly expanded, it could generate high pitch narrowband noise called screech. The screech was first studied by Powell [1953b]. According to him, screech tones are generated by sustained feedback loops, which consist of the excitation of the intrinsic instability waves in the initial jet shear layer, the interaction of these instability waves with shock cell structures and in turn radiation of acoustic noise, and feedback of acoustic waves to the initial shear layers. The major parameters governing screech characteristics are the jet Mach number, jet temperature, and nozzle exit dimension.

To generate screech tones, quasi-periodic shock cell structures are necessary. These
shock cell structures were predicted by Prandtl [1904] and later by Pack [1950] by using the vortex sheet model, which assumes mixing layers are very thin. Tam et al. [1985] calculated shock cell spacing and pressure distributions by adopting the multiple-scale expansion. Their results agree well with the experimental results of Norum and Seiner [1982]. An explicit formula predicting the screech frequency of a circular jet was derived by Tam et al. [1985] as:

\[ \frac{f_j D_j}{U_j} = 0.67 \left( M_j^2 - 1 \right)^{-1/2} \left[ 1 + 0.7M_j \left( 1 + \frac{\gamma - 1}{2} M_j^2 \right)^{-1/2} \left( \frac{T_o}{T_x} \right)^{1/2} \right] \]  

(2.1)

where \( f_j \) is the fundamental screech frequency, and \( T_o \) and \( T_x \) are the total temperature of the jet and the ambient temperature, respectively, and \( U_j \) is the fully expanded jet exit velocity. \( D_j \) is calculated from the exit diameter \( D \), the design Mach number \( M_d \), and the jet Mach number \( M_j \) as:

\[ \frac{D_j}{D} = \left[ \frac{1 + (\gamma - 1)M_j^2 / 2}{1 + (\gamma - 1)M_d^2 / 2} \right]^{(\gamma + 1)/(\gamma - 1)} \left( \frac{M_d}{M_j} \right)^{1/2} \]  

(2.2)

Tam [1988] derived explicit formulas for shock cell spacing in elliptic and rectangular jets and a formula predicting the screech frequency in a rectangular jet, which agree well with experimental data. The shock cell spacing, \( L_s \), is predicted by the following formula:

\[ L_s \approx 2h \left( M_j - 1 \right)^{1/2} \left[ 1 + \frac{1}{\pi} \frac{h}{b} \right]^{1/2} \]  

(2.3)

and the screech frequency formula is given as:
\[ \frac{fh}{u_j} = \frac{\frac{u_c}{u_j}}{2 (1 + \frac{u_c}{a_*}) (M_j^2 - 1)^{\frac{1}{2}} \left[ \left( \frac{h}{b} \right)^2 + 1 \right]^{\frac{1}{2}}} b + h \left[ M_d \left( \frac{1 + \frac{\gamma - 1}{2} M_j^2}{1 + \frac{\gamma - 1}{2} M_d^2} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} - 1 \right] + 1 }^{-1} \]  

(2.4)

where \( b \) and \( h \) are the major and minor axis length respectively, \( u_c \) is the convection velocity of instability waves, \( u_j \) is the jet exit velocity, and \( f \) is the screech frequency. The above formula slightly overestimates the screech frequency compared with Krothapalli et al.'s [1986] and Powell's [1953a] data, when the empirical relationship of Harper-Bourne and Fisher's [1973], \( u_c = 0.7 u_j \), is used. Even for beveled rectangular jets, Tam et al. [1996] successfully derived a screech frequency formula.

As the fully expanded jet Mach number increases, the screech frequency decreases due to the increase of shock cell spacing. Furthermore, the amplitude of screech tones decreases with the increase of jet Mach number after reaching a peak at a moderate Mach number. The increased jet temperature, in general, results in the decrease of screech amplitudes. Krothapalli and Strykowski [1996] argued that the decrease of the screech amplitude for Mach numbers greater than a certain value in an underexpanded jet was related to the lack of two-dimensional coherent structures. Three possible causes of the cessation of screech in underexpanded jets were discussed by Raman [1996a]: The frequency mismatch between screech frequency and the band of most amplified instability waves, the weakening of screech producing shock due to the Mach disk, and reduction of acoustic feedback and receptivity at high levels of underexpansions. Through detailed reasoning, he found that the cessation of screech at a high Mach number was due to the blockage effects.
of the barrel shock and in turn poor excitability of instability waves at the jet exit. The importance of quasi-periodic shock cells was studied by Raman [1996b]. As described before, generally weaker and unsteady screech was observed for rectangular jets with slanted exits which produced oblique shock cells in the jet plume.

2.6.3 Eddy Mach Wave Radiation

By using the large scale instability theory, the eddy Mach wave radiation is well predicted. When instability waves propagate downstream at a supersonic speed relative to the ambient sound speed, they effectively produce intense noise. This dominant noise propagates downstream. This radiation will be the dominant noise in perfectly expanded jets where there are no shock cells in a jet plume. In laminar compressible flows, Lees and Lin [1946] tried to estimate this kind of noise by using a stability analysis. The importance of large scale structures is considered in their analysis. The noise radiated at a narrow angle relative to the jet axis was well predicted in compressible shear layers by using a multiple scale asymptotic expansion [Tam and Morris, 1980]. Tam and Burton [1984] found that noise emissions will occur for any disturbance which has a supersonic speed. It was revealed by Tam et al. [1991] that the highest sound pressure level of the far field noise radiates at a direction and frequency closely matching the Mach wave radiation and frequency of the most amplified instability wave of the jet. According to Ffowcs-Williams [1963], the eddy Mach radiation is very efficient, so it can convert between 0.1 and 1 % of the total mechanical energy of a jet to noise.

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Papamoschou and Debiasi [1998] were able to eliminate this type of noise by using a coaxial jet. Since the Mach wave radiation is generated by large scale structures which move supersonically with respect to the surrounding fluid, the elimination of the Mach wave radiation can be achieved by making the convection velocity of large scale structures subsonic. By adjusting the flow condition of the outer jet, the convection velocity of large scale structures, both in the mixing layers between the core and outer jet and between the outer jet and the ambient air, became subsonic relative to the surrounding fluid.
Fig. 2.1 Sketch of a supersonic jet.
Fig. 2.2 Sketch of three different flow regimes of a supersonic nozzle.

(a) Underexpanded jet

(b) Ideally expanded case

(c) Overexpanded case
3.1 Introduction

The effects of trailing edge modifications on mixing and noise in a supersonic rectangular jet were investigated by simple visualization techniques and by using two microphones. Mixing area and the information on large scale structures in the jet plume were obtained from the cross sectional images at several downstream locations. To investigate screech mode variation with the modification, nearfield noise was acquired by two microphones. The Overall Sound Pressure Level (OSPL) variation with the modifications at farfield was also measured by the two microphones. To figure out the source of streamwise vorticity, the three dimensional pressure distribution on the modification surface was obtained by spanwise and streamwise arrays of pressure taps. The surface flow on the modification was visualized by a surface flow visualization technique before measuring the pressure distribution with pressure taps.
3.2 Air supply system

All the experiments were conducted at the Aeronautical and Astronautical Research Laboratory at The Ohio State University. The compressed air from two four-stage compressors is stored in two cylindrical tanks with a total capacity of 42.5 m$^3$ at 16.5 MPa pressure. The air is filtered and dried before being stored in the tanks. Thus, the jet air is drier than the ambient air. The stored air is then throttled down to the required stagnation pressure using control valves. As shown in Fig. 3.1, the air is introduced to the stagnation chamber, with 19 cm diameter and 120 cm length, radially by four 1.9 cm i.d. (inside diameter) flexible hoses in order to remove any axial force caused by the inlet flow momentum. For the larger nozzles, which will be described later, the air enters the stagnation chamber axially by a 3.8 cm i.d. flexible hose not only because the mass flow rate is too high but because thrust measurements are not performed for the larger nozzle. After being passed through a perforated plate placed in the middle of the chamber, the air is exhausted into the ambient air through an aspect ratio 3 rectangular nozzle. The nozzle is attached to a faceplate by an extension to provide enough room for optical access. For the larger nozzle, the extension is not necessary since it has enough room for optical access.

3.3 Nozzle Types

Two kinds of design Mach number 2 aspect ratio 3 rectangular nozzles are used in the present study. The dimensions of the larger nozzle are 1 inch by 3 inch (2.54 cm high
and 7.52 cm wide), while they are 3/8 inch by 1 1/8 inch (0.95 cm high and 2.86 cm wide) for the smaller nozzle. The nozzle exit equivalent diameters, \( D_{eq} = \left( \frac{4A_{ex}}{\pi} \right)^{1/2} \), are 1.95 inches (49.6 mm) and 0.733 inches (18.6 mm) for the larger and smaller baseline nozzles, respectively.

### 3.3.1 Larger nozzles with single-sided trailing edge modification

The schematic of the baseline nozzle and the ten nozzles with single trailing edge modification used in the experiments are shown in Fig. 3.2. The larger baseline nozzle is a half-nozzle with a 1 mm thick splitter plate on one side and a 44 mm thick lip on the other side (Fig. 3.2 (a)). The thickness for the other two sides is 19 mm. The jet air reaches the design Mach number of the larger nozzle 6 mm upstream of the trailing edge modifications. Thus, the development of the flow within the nozzle is not affected by the presence of the trailing edge modification. The modifications are simple serrations on the splitter plate with an angle of 35° to 90° relative to the spanwise direction. The larger nozzle with single trailing edge modification was used to find out what kind of serration or cut-out is good for mixing enhancement and noise reduction. It is expected that the investigation of flow structures for a nozzle with single modification is less complex since there would be no interaction between mixing layers of the jet in the near field.

One of the purposes of the trailing edge modification is to excite oblique large scale structures in the mixing layer for the ideally expanded case. From the Sandham and Reynolds' [1991] theoretical relationship between the convection Mach number (\( M_c \)) and the
angle of large scale structures ($\beta$), i.e., $M_\infty \sin \beta = 0.6$, the angle $\beta$ relative to the streamwise direction is 45° since the convective Mach number is 0.85 in the ideally expanded case. Two other angles, 35° and 55°, were also examined.

### 3.3.2 Smaller nozzles with double-sided trailing edge modification

The baseline nozzle for double-sided trailing edge modification experiments is a full nozzle, which is smaller than the one used in the single-sided modification experiments and has the capability of trailing edge modifications on one or two sides. The exit area of the new nozzle has been reduced so that hot jet experiments can be conducted as the heater capacity is not sufficient for the larger nozzle. The schematic of the baseline nozzle and the trailing edge modified nozzles and modification types are shown in Fig. 3.3. The nozzle name represents the types of modifications on both sides. For example, nozzle N65 has modifications M6 and M5. Based on the previous single-sided modification results [Samimy et al. 1997a, b], four types of modifications M3, M4, M5, and M6 were selected because of their comparable mixing and noise performance. By combining any two types of modifications, six double trailing edge modified nozzles, nozzle N33, N43, N44, N55, N65, and N66 as depicted in Fig. 3.3, were selected as good candidates for enhancing mixing.

### 3.4 Flow visualization techniques

Flow field visualizations are carried out to gather information on large scale structures and mixing rate or level variations with the trailing edge modification. Most of
the visualizations of instantaneous jet cross sectional images are conducted by a simple visualization technique, which is called the laser sheet illumination technique [Arnette et al. 1993].

3.4.1 Basic Mie/Rayleigh scattering visualization technique

As shown in Fig. 3.4, the instantaneous jet cross section is illuminated by a sheet of the laser light from a Spectra Physics Model GCR-4 pulsed Nd:YAG laser. The laser light with 532nm wavelength from the frequency doubled Nd:YAG laser is scattered by condensed water particles in the mixing layer, which are formed when the cold and dry jet air is mixed with the humid and warm ambient air. Thus, only mixing layers are visualized by the technique. The particle size is on the order of 50nm, so the particles can follow the jet flow faithfully [Samimy and Lele 1991]. The scattering by the particles is in the intermediate range between Mie and Rayleigh scattering, so it is appropriate to call it Mie/Rayleigh scattering. The laser is capable of producing high energy pulse up to 700 mJ/pulse when the frequency is doubled. Since the pulse duration is only 9 ns, the flow is effectively frozen and instantaneous images can be acquired. The jet fluid convects only 5 μm for the pulse duration at $M_j=2.5$ where the jet exit velocity is approximately 560 m/s.

A sheet of the laser light is formed by cylindrical and spherical lenses. The instantaneous jet cross sectional images are taken by a 14-bit Princeton Instruments intensified CCD camera. The camera is controlled by a Princeton Instruments PG-200 programmable pulse generator which interfaces the camera controller with the laser. The
images are recorded on the hard disk of a 486 personal computer. From the 50 instantaneous images, mixing areas are calculated.

The visualizations of the jet cross section are performed at two downstream locations of 2 and 8\(D_{eq}\) for the larger nozzle as listed in Table 3.1. For the smaller nozzle with double-sided trailing edge modification, the visualizations are conducted at four downstream locations, 1, 2, 4, and 8\(D_{eq}\), to get enough information on mixing area variations and large scale structure development and evolution with downstream locations. All the experiments conducted with the smaller nozzle are listed in Table 3.2. Reynolds numbers, \(Re = \frac{\rho_{exit} U_{exit} D_{eq}}{\mu_{exit}}\) based on the nozzle equivalent diameter \(D_{eq}\) are listed at the bottom of each table.

3.4.2 Image processing

The instantaneous cross sectional images are contaminated by scattering from dust particles in the flow and the nozzle surface which scatters stray laser light. The background scattering in the images can be removed by subtracting the background image (flow-off image) from the signal images (flow-on images). The background images are taken just after the signal images are recorded with the same conditions without the jet flow. However, the dust images cannot be removed by this technique.

One option is to use a temperature-broadened iodine filter (interested readers are directed to Elliott et al. [1994]) in front of the ICCD camera to filter out both the dust and background images. The idea of the temperature broadened iodine filter is to eliminate
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<td>5.35x10^6</td>
<td>7.22x10^6</td>
<td>1.19x10^7</td>
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</tbody>
</table>

N_FR: Farfield noise measurements
N_NR.FR: Nearfield and Farfield noise measurements
V_2_8: Flow visualizations at 2 and 8D_{eq}
V_2: Flow visualizations at 2D_{eq}
SFV_TAP: Surface flow visualization and three dimensional pressure field measurement on the nozzle inside wall
\* Re=\rho_{exit}U_{exit}D_{eq}/\mu_{exit}

Table 3.1 Experiments conducted for larger nozzles with single trailing edge modification.
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<td>$2.48 \times 10^6$</td>
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</table>

$^*$ Thrust measurement
$^+$ Flow visualizations at 1, 2, 4, and 8 $D_{eq}$

$Re = \rho_{cut} U_{cut} D_{eq} / \mu_{exit}$, is $8.79 \times 10^5$ at $M_j = 1.5$

Table 3.2 Experiments conducted for smaller nozzles with double-sided trailing edge modification.
background images from stationary surfaces and slow moving dust particles by setting the laser frequency at the absorption well of the filter. Since the scattering from fast moving particles experiences Doppler shift, only the image from these particles pass through the filter. However, the iodine filter cannot remove the dust images in the mixing layer, since the dust particles move almost at the same speed of the jet flow.

Thus, a digital filtering technique is used in the present experiments to remove the dust image both from the ambient and jet air. The characteristics of the dust images in an instantaneous image are that they have relatively greater intensity gradients, defined as the ratio of the intensity difference to the pixel distance, and that they are isolated and smaller sized than the signal image. Based on the characteristics of the dust images, a program is coded to remove the dust images. After processing each instantaneous image, the image is further processed by using the image processing toolbox of Matlab. By these processes, dust free instantaneous images are obtained.

3.5 Surface flow visualizations and wall pressure measurements on a modification

The three dimensional pressure distribution on a modification is expected when the jet is non-ideally expanded as depicted in Fig. 3.5. The spanwise pressure gradient is thought to be the major source of streamwise vorticity. Since vortex dynamics governs the jet development, the information on the pressure distribution on the modification wall is highly desired. Before measuring the wall pressure distribution on the modification, the surface flow visualization is required to obtain the range of the downstream influence zone. The
surface flow visualization is performed by the kerosene-lampblack surface-streak trace method well described by Settles and Teng [1983].

### 3.5.1 Kerosene-lampblack surface-streak trace method

This surface flow visualization technique is developed by Settles [1975] to be used for blow down facilities. Contrary to the conventional surface flow visualization technique which uses oil-pigment mixture, the surface streak trace by kerosene-lampblack mixtures is not affected by the tunnel shutdown since the kerosene evaporates completely before the airflow is stopped [Settles and Teng, 1983]. The surface flow image marked by the remaining pigment is taken by ordinary transparent adhesive tape. Their microscopic observation reveals that the deposited lampblack particles are typically in the range of a few μm or less. Thus, the flow disturbance by the deposited pigment particles is expected to be negligible. According to Settles and Teng [1983], the spatial resolution of the technique is on the order of 0.25 mm.

In the present experiments, the mixture of black powder paint and kerosene is used to visualize the surface flow field on a modification. Before each test, the mixture is applied 12 mm upstream of the cut-out. Kerosene is used as a carrier fluid of black paint powder during test and it evaporates completely before the jet air is shut down. Thus, the surface flow pattern is not affected by the shutdown. Just after shutdown, the modification is taken apart and the surface flow pattern traced by the paint powder is recorded on 3M 4.9 cm wide
adhesive transparent tape. The image is imprinted on white paper by pressing the tape on it. Even after being photocopied, the surface flow image is quite good.

3.5.2 Pressure measurements by arrays of pressure taps

The expected three dimensional wall pressure distribution on the modification is measured by placing spanwise and streamwise arrays of pressure taps as shown in Fig. 3.6. The static pressure is converted into a digital value by a pressure transducer of manually switchable 12-channel Scanivalve. The digital value is displayed by a Newport Model 2001-4 Read-out. By scanning the mean static pressure along spanwise and streamwise directions, the three dimensional pressure distribution is obtained. Since the read-out just shows numeric values proportional to the static pressure, the read-out and pressure transducer are calibrated by using a pressure gauge.

3.6 Noise measurements

Farfield noise is measured to investigate the OSPL reduction by the single-sided modifications. On the other hand, the screech mode variation with the trailing edge modification is investigated by measuring nearfield noise. Both nearfield and farfield noise is measured by two B & K Model 4135, 6.35 mm diameter condenser microphones. The microphones are omnidirectional within 3, 6, and 12 dB up to 12.5, 20, and 31.5 kHz, respectively. However, in the present noise measurements reported herein, the directivity is
within 1 and 2.5 dB up to 20 and 31.5 kHz, respectively, since the microphone axis angle relative to the jet noise sources is less than 40°. Each microphone is attached to a Bruel and Kjaer Model 2633A preamplifier. The signals from both microphones were amplified by a Bruel and Kjaer Model 5935 dual-microphone amplifier. According to the manufacturer’s specifications, the output of the amplifier is linear when the root mean square input value is less than 5 V. The acoustic signals were acquired by a Datel PC414-A2 12-bit Analog-to-Digital (A/D) board, with a sampling rate of 100 kHz per channel.

The acoustic data were saved on the hard disk of a 486 personal computer. One hundred blocks of 4096 samples were acquired with a corresponding total sampling time of 4.096 seconds. The sampling and storing of data are performed by the accompanying MS-DOS based Datel software PC414-SET. By using a Bruel and Kjaer Type 4231 Sound Level Calibrator, which is an electronically driven constant sound level generator at 1 kHz, each microphone was calibrated after the acquisition of each data set. The OSPL, averaged spectra, and phase angles are calculated by using the signal processing toolbox of Matlab.

3.6.1 Farfield noise

For the measurement of farfield noise, both microphones are placed 30 nozzle equivalent diameters, \( D_{eq} = (4A/\pi)^{1/3} = 4.96 \) cm, away from the jet axis on the jet exit plane: One on the major axis and the other on the minor axis of the nozzle. A much wider area of nearby surface than in the nearfield noise measurements was covered by the acoustic form to minimize the effects of acoustic reflections on the OSPL. The acoustic foam is made of
open-cell polyurethane which effectively absorbs acoustic waves in the range of 1000-25,000 Hz [Raman, 1996]. The background noise (without the jet flow) is also acquired to eliminate its effects on the farfield noise level.

### 3.6.2 Nearfield noise

To determine the changes in the screech mode due to the modification of the nozzle trailing edge, nearfield noise measurements were performed. As shown in Fig. 3.7, two microphones were placed either on the minor axis (3.6D₁ apart) or on the major axis (3.6D₂ apart). In each case, both microphones were placed 3.6D₆ off from the jet axis. The angle of microphones with the jet axis was 150° in both cases. According to Krothapalli and Strykowski [1996], the principal direction of the fundamental screech is approximately 150° from the jet axis. For acoustic measurements, the face of the stagnation chamber and all the nearby surfaces were covered with acoustic foam to reduce acoustic reflections.

### 3.7 Thrust measuring system

Thrust variations with trailing edge modifications are measured by a thrust measuring system, which was built for axial thrust measurements as shown in Fig. 3.1. To minimize the inlet flow effects on thrust, the compressed air is introduced into the stagnation chamber, radially, and perpendicular to the jet axis. The hoses stay flexible even after they are pressurized up to the maximum stagnation pressure (at Mᵥ=2.5), so the effects of inlet flow
on thrust are expected to be negligible. The stagnation chamber, two chamber supports, and two 5.08 cm diameter shafts are assembled together and so are free to move axially in four guiding and supporting Thomson linear bearings. The vertical cantilever transmits thrust generated by the jet into the load cell installed on the table. The load cell is a Sensotec Model 13 100 lb (445 N) capacity and the thrust values are displayed by a Sensotec Model GM read-out. The load cell and read-out are calibrated by the shunt calibration function built into the read-out. To prevent the stagnation chamber from slamming into the load cell at start up and to minimize frictional effects on thrust, a pre-load of approximately 60 N is used.

Thrust is measured at four fully expanded jet Mach numbers of 1.75, 2.0, 2.2, and 2.5 for the baseline nozzle and nozzle NB6 (=N6), NB4 (=N4), N55, N65, and N66. The letter B in the name of a nozzle represents the baseline plate, which does not have cut-outs.
Fig. 3.1 Schematic of the thrust measuring system. The nozzle, nozzle extension, two stagnation supports, and vertical cantilever are assembled together and move together.
(a) Side and end views of the baseline nozzle

(b) Baseline trailing edge (MB) and modifications M1-M10 for the big nozzle

(c) Nozzle NB  (d) Nozzle N1  (e) Nozzle N2  (f) Nozzle N3

(g) Nozzle B4  (h) Nozzle N5  (i) Nozzle N6  (j) Nozzle N7

(k) Nozzle B8  (l) Nozzle N9  (m) Nozzle N10

Fig. 3.2 Schematic of the larger nozzle configurations (Thick lines represent thick lips in (b) - (l)).
Fig. 3.3 Schematic of the small nozzle with double sided trailing edge modifications.
Fig. 3.4 Schematic of the flow visualization system.
Fig. 3.5 Anticipated three dimensional pressure distributions, spanwise pressure gradients on the inside wall of a nozzle for the underexpanded case, and the induced streamwise vortices.
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<th>z-dir.(P)</th>
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1) All units are in mm

Fig. 3.6 The location of pressure taps on modifications M5 (top) and M6 (bottom).
Fig. 3.7 Microphone positions for far and near field noise measurements.
CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

Extensive experiments were performed to investigate not only the effects of trailing edge modifications on the mixing and noise of the jet, but also the source of streamwise vorticity generated by the trailing edges. These experiments are flow visualization, noise measurement, and wall pressure measurement on the modifications. From the instantaneous jet cross sectional images, the mixing level and mixing rate of a jet are calculated. The mixing performance of nozzles with single-sided or double-sided trailing edge modifications was compared by using the calculated mixing level and rate. The jet development and evolution with the downstream location, which are governed by the dynamics of large scale vortical structures such as streamwise vortices, were also investigated from the instantaneous and average cross sectional images.

The screech mode variation with the trailing edge modification was investigated by measuring nearfield noise. Since large scale structures are responsible for shock associated noise, the screech mode is expected to change with the trailing edge modification. Farfield
noise was also measured to obtain quantitative OSPL variations due to the trailing edge modifications. Although thrust loss or gain was expected to be negligible, the thrust variation due to trailing edge modifications was measured by the thrust measuring system.

A detailed discussion on the source of the large scale structures is presented first because the vortex dynamics governs jet development and mixing.

4.2 Sources of Streamwise and Spanwise Vorticity

Since vortex dynamics governs jet development and mixing, some discussion of the topic as a follow up of Samimy et al.'s work [1997b] and Kim et al.'s [1998] is warranted. Three major sources of streamwise vorticity in the supersonic jet with modified trailing edges are believed to be: 1) the vorticity convected downstream in the boundary layer approaching the modified region, 2) the anticipated spanwise pressure gradient on the jet inside surface as shown schematically in Fig. 3.5, and 3) the potential baroclinic torque due to misalignment of pressure and density gradients.

4.2.1 Source 1: Vorticity convected downstream in the boundary layer approaching cut-outs

Source 1, the vorticity convected downstream in the boundary layer, experiences vortex stretching and reorientation in passing through the modified flow regions. As was believed to be the case in the jet with tabs [Zaman et al. 1994], this source is much weaker
than the vorticity generated by a spanwise pressure gradient in the vicinity of the modified trailing edges. If source 1 is a strong source of streamwise vorticity, one would observe similar streamwise structures in the ideally expanded case as in the non-ideally expanded cases since source 1 is present in all flow regimes of the present experiments. As will be discussed later, no noticeable streamwise structures were observed in the ideally expanded case. For this reason, source 1 is negligible compared with source 2, which is the spanwise pressure gradient on the modification.

4.2.2 Source 2: spanwise pressure gradient

The second streamwise vorticity source, source 2, is the spanwise pressure gradient on the modified trailing edge as schematically shown in Fig. 3.5. This source is similar to the streamwise vorticity due to the spanwise pressure gradient on a tab [Samimy et al. 1997b], which is called “source 2” by Zaman et al. [1994]. Samimy et al. conjectured that a sheet of vorticity is shed along cut-out edges by the spanwise pressure gradient and that the streamwise vorticity generated is the “skew-induced” type [Bradshaw, 1987], for nozzles with oblique cut-out edges such as nozzle N33 and N44.

It was Lighthill who first introduced the vorticity production term by a wall pressure gradient. He called the term vorticity flux density and it is written as \(-\nu \left( \frac{\partial \Omega_x}{\partial y} \right)\), where \(\nu\) is viscosity of the fluid, \(\Omega_x\) is x-direction vorticity, and y is normal to the wall. For incompressible boundary flows, span- and stream-wise vorticity fluxes are described as [Lighthill 1963, Reynolds and Carr 1985, and Honkan and Andreopoulos 1997].
\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial z} \right)_w = \left( \nu \frac{\partial \Omega_x}{\partial y} \right)_w. \tag{4.1}
\]

\[
\left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right)_w = \left( \nu \frac{\partial \Omega_z}{\partial y} \right)_w. \tag{4.2}
\]

where \(x, y, \) and \(z\) are for streamwise, normal, and spanwise directions, respectively, as shown in Fig. 3.5, and the subscript \(w\) represents values on the wall. Basically, equations 4.1 and 4.2 are derived from the momentum equation and the non-slip condition on the wall. A similar equation is obtained for compressible boundary layer flows. The vorticity flux by the spanwise pressure gradient is reduced from:

\[
\left( \frac{\partial p}{\partial z} \right)_w = \frac{\partial}{\partial z} \left( \mu \Omega_x \right) = \left( \Omega_x \frac{\partial \mu}{\partial y} \right)_w + \left( \mu \frac{\partial \Omega_z}{\partial y} \right)_w. \tag{4.3}
\]

Since the viscosity of the air is dependent on temperature only, i.e. \(\mu = \mu(T)\), equation 4.3 is reduced to the same equation as equation 4.1 when the wall is adiabatic. In the present experiments, artificial heating or cooling of the nozzle wall was not performed. In addition, the viscosity of the air is a weak function of temperature as seen in Maxwell’s viscosity-temperature relationship as:

\[
\frac{\mu}{\mu_o} = \left( \frac{T}{T_o} \right)^{0.666}, \tag{4.4}
\]

where \(\mu_o\) is the reference viscosity of air at the reference temperature \(T_o\). Thus, equation 4.1 and 4.2 are still valid even for compressible flows without significant artificial heating or cooling as in the case of the present experiments, where the wall can be considered as
roughly adiabatic for this reason. The physical meaning of equations 4.1 and 4.2 is that vorticity is generated on the wall by a pressure gradient and deposited into the jet stream [Lighthill 1963, Honkan and Andreopolos 1997].

Span- and stream-wise pressure gradients are expected in both the over- and under-expanded cases in the present experiments due to downstream influences through the subsonic region of the boundary layer and due to the spanwise propagation of disturbances caused by cut-outs. The extent of the downstream influence zone in the underexpanded cases is expected to be on the same scale as the boundary layer thickness, which is on the order of a millimeter and a fraction of millimeter for the larger and smaller nozzles, respectively. On the other hand, the range of the disturbance propagation due to cut-outs is dependent on the design Mach number and the type of trailing edge modification in the underexpanded case. Since the design Mach number of the larger nozzle is 2.0, the forward expansion wave propagates at approximately 30° relative to the streamwise direction. When the jet is overexpanded, the extent of downstream influence varies with the jet exit pressure as will be further discussed.

4.2.2.1 Surface flow visualizations

Before measuring the expected three dimensional pressure distribution on the inside surface of a modification, the surface flow pattern was obtained by using a mixture of kerosene and black paint powder. The acquired surface streak patterns for the baseline nozzle and modifications M3, M4, M5, and M6 are shown in Figs. 4.1-4.5. The mixture is
applied about 12 mm upstream of each cut-out just before each test. To avoid or to minimize start-up and shut-down effects on the streak pattern, the pressure controlling valve was opened within 0.2 sec, stayed at the desired pressure 5 sec, and shut down within less than 0.2 sec as shown in Fig. 4.6. Since kerosene is volatile, it is completely evaporated before the jet is shut down. Thus, a streak pattern, which is free from shut-down effects, can be acquired as mentioned above. The flow disturbance by the deposited paint particles is negligible because the remaining paint powder on the modification surface is in the range of a few μm or less [Settles and Teng 1983].

As shown in Figs. 4.2 and 4.3, the downstream influence zone for modifications M3 and M4 is about 2 mm from cut-out edges when the jet was underexpanded. For the baseline splitter plate, the influence zone was not visualized as shown in Fig. 4.1 because the flow direction was not changed. However, it is expected that the zone may be on the same order as that for modifications M3 and M4. The zone is very wide for modifications M5 and M6 as respectively shown in Fig. 4.4 and 4.5 because the downstream of the forward expansion line, starting from cut-out corners and about 30° off the jet axis, is affected by the cut-out. Thus, the spanwise pressure gradient for modifications M5 and M6 is smaller than that for modifications M3 and M4 as will be shown in the next subsection. As shown in Fig. 4.7, three dimensional boundary layers are expected to be developed in the downstream influence zone for modifications M5 and M6 for the underexpanded case.

In contrast to the case for underexpansion, the downstream influence zone is strongly dependent on the jet exit pressure or the fully expanded jet Mach number. For the fully expanded jet Mach number 1.75, the separation line marked on the modification is about 45°
relative to the jet axis for all tested modifications except for the baseline splitter plate. By the three dimensional separation line on the modification, a three dimensional separation shock is expected to be generated at the upper edge of the boundary layer. As the fully expanded jet Mach number or the jet exit pressure decreases, the angle of the separation line increases and eventually becomes 90° relative to the jet axis at $M_j = 1.5$ as depicted in Fig. 4.7. The spanwise separation line on a modification is marked at $M_j = 1.5$ for all modifications tested. Thus, the spanwise pressure gradient is expected to be small at $M_j$ of 1.5 or less.

4.2.2.2 Measured wall pressure distribution

The expected three dimensional pressure distribution, as depicted schematically in Fig. 3.5, was measured by span- and stream-wise arrays of pressure tabs based on the surface flow images for modifications M5 and M6. The location of each tap on the modification is tabulated in Fig. 3.6. All surface flow visualizations and wall pressure measurements were carried out for the larger nozzle to take advantage of the thicker boundary layer, which makes the surface flow visualization and pressure measurements easier than that for the smaller nozzle.

The measured spanwise pressure distributions for modifications M5 and M6 at five flow regimes, $M_j = 1.5, 1.75, 2.0, 2.2,$ and $2.5$, is shown in Figs. 4.8 and 4.9. The measured exit pressure was normalized with the ambient pressure of 14.4 psi (99.3 kPa). In addition, the spanwise distance $z$ is normalized by the cut-out width $W_{co}$, which is equal to 1/3 of the
larger nozzle width. The measured streamwise pressure distributions along S- and D-array (in Fig. 3.6) for the modifications are shown in Fig. 4.10-4.13. In these figures, the streamwise distance x from the jet exit was normalized by cut-out length \( L_\text{co} \) (38.1 mm), which is equal to \( 3/2 \) of \( W_\text{co} \).

**Ideally Expanded Case**

As expected, no significant span- or stream-wise pressure gradients were measured in the ideally expanded case as shown in Figs. 4.8-4.13. This negligible streamwise vorticity source resulted in almost the same jet cross section regardless of the type of modifications as will be shown in the flow visualization sections.

**Underexpanded Cases**

Not surprisingly, the normalized pressure distribution for underexpanded cases is similar for different \( M_j \) of 2.2 and 2.5. This is due to the fact that the expansion fan initiated from the cut-out corners does not depend on \( M_j \) but does depend on design Mach number as mentioned above. By the spanwise pressure gradient, a pair of counter rotating streamwise vortices is formed with the same direction as a log rolling down the “pressure hill” [Lighthill 1963, Zaman et al. 1994, Honkan and Andreopolos 1997] as shown in Fig. 4.14. Figures 4.14 and 4.15 are built from the surface flow visualization and pressure measurement data. For modifications M3 and M4, the pressure gradient is deduced from the surface flow
visualization data. The downstream influence zone through the subsonic boundary layer, which is estimated by the pressure gradient shown in Figs. 4.12 and 4.13, is in the same range of 2mm that is measured from surface flow images.

The measured spanwise pressure gradients, \( \frac{\partial p}{\partial z} \), for both modifications M5 and M6 are strong enough to generate streamwise vorticity on the wall by equation 4.1. However, equations 4.1 and 4.2 do not show any direct connection between vorticity itself and the magnitude of the pressure gradient, but show a circulation flow rate per unit length into the jet flow on the wall by the pressure gradient, when the unit of any side of equation 4.1 is considered.

**Overexpanded Cases**

In contrast to the underexpanded cases, the pressure distribution on a modification varies with the fully expanded jet Mach number \( M_j \) in the overexpanded cases, since the separation location and accompanying separation shock location significantly depend on \( M_j \). The dependence of separation location on \( M_j \) is depicted in Fig. 4.7 in the overexpanded cases. At \( M_j = 1.75 \) for modification M5, the pressure hill is similar to that for underexpanded cases at cross section B-B in Fig. 4.15. However, the pressure gradient may have an opposite direction at a further upstream location A-A. The three dimensional pressure gradient shown in Fig. 4.15 is adverse to the jet flow and so results in three dimensional separation on the wall, which was marked on the surface flow images. The sense of vorticity of the separation bubble is the opposite direction of that depicted in the figure because of the separation.
Thus, the interpretation of the spanwise pressure gradient as a streamwise vorticity source is not straightforward.

At \( M_j = 1.5 \) the spanwise pressure gradient was very small due to a two-dimensional separation shock generated well upstream of the cut-out as can be seen in Figs. 4.1-4.3. By the two-dimensional separation shock at \( M_j = 1.5 \) which eliminates spanwise pressure gradients, the jet cross section at \( x/D_{eq} = 2 \) is similar regardless of the type of cut-out as will be shown in Figs. 4.18-4.23.

### 4.2.3 Source 3: baroclinic torque

The third source of streamwise vorticity is the baroclinic torque. The baroclinic torque is a phenomenon that is generated in compressible flows in general, and is present when the pressure and density gradients are not parallel. In the vorticity transport equation for the mean flow shown in equation 4.5 [Morkovin, 1992], the second term on the right hand side represents the baroclinic torque.

\[
U \cdot \nabla \left( \frac{\Omega}{\rho} \right) = \frac{\Omega}{\rho} \cdot \nabla U - \frac{1}{\rho} \nabla \left( \frac{1}{\rho} \nabla p \right) \times \nabla p + \text{viscous term} + \text{Reynolds stress term} \quad (4.5)
\]

The underscores represent vectors and \( U, \Omega, \rho, \) and \( p \) are velocity, vorticity, density, and pressure, respectively. If one assumes a simple two-dimensional expansion of a bounded flow, as shown in Fig. 4.16, the baroclinic torque that would generate streamwise vorticity is zero. The misalignment of the pressure and density gradients in a non-zero pressure
gradient boundary layer is due to the density gradient generated by the temperature variation across the boundary layer. Thus, the baroclinic torque is negligible for adiabatic incompressible and low Mach number subsonic boundary layer flows, where the temperature variations across the boundary layer are negligible. On the other hand in very high speed flows, this could be a strong source of vorticity [Marble et al. 1990, Waitz et al. 1992].

As shown in Fig. 4.16, the only non-zero baroclinic component, which is normal to both pressure and density gradients, is the spanwise component. As a result, a vortex, which is rolling down the pressure hill, may experience negative spanwise baroclinic torque. When a vortex rolls down a three-dimensional pressure hill in a three-dimensional boundary layer, the vorticity component, which is aligned to the isobar of the pressure hill, undergoes negative baroclinic torque as in two-dimensional boundary layers. So far, the discussion is based on qualitative analysis, and so it is hard to estimate the relative contribution of baroclinic torque to the resultant vorticity. This is simply because the baroclinic torque and pressure gradient terms are not in the same equation.

In the experiment with a converging tabbed nozzle, Zaman et al. [1994] concluded that baroclinic torque must not be significant simply because a similar overall effect of the tab was observed in supersonic and subsonic flow conditions. Since the Mach number at the pressure hill, where a vortex is generated, was around 1 and so the density gradient across the boundary layer was small, the baroclinic torque is expected to be negligible in their case. However in the present case, the Mach number in the area where a vortex is rolled up is around 2, and the density gradient is greater than that of their case. In fact, the sense of the observed streamwise vortices always match with the sign of a vortex which would be
generated by spanwise pressure gradient, although the vortex experienced negative torque as discussed. By this reason, baroclinic torque seems to be negligible in the present case also. In the Mach number 2 tunnel flow, Donohue et al. [1992] found that the streamwise vortex generated by a ramp (in the author’s opinion the source of the streamwise vorticity was a spanwise pressure gradient on the ramp) is stronger than the one generated by baroclinic torque. In addition, the effect of streamwise vortices generated by pure baroclinic torque on the mean flow was negligible. This supports that the vorticity generated by baroclinic torque might be negligible compared with that caused by the spanwise pressure gradient in the present case.

4.2.4 The effects of expansion/compression on vorticity

Another issue to be addressed is whether the vortex stretching or compression is important in supersonic jets in non-ideally expanded conditions where the vortices generated by the cut-outs go through significant expansion in the underexpanded flow regimes and compression in the overexpanded flow regimes. Two approaches were tried: one is the use of the vorticity transport equation 4.5 in quasi-two-dimensional boundary layer flow, and the other is the use of the conservation of angular momentum of a vortex as the same analysis, Tennekes and Lumley [1992] used in incompressible flows. The first approach is more complicated than the latter, but it can calculate the effect of baroclinic torque. On the other hand, the latter analysis cannot estimate the effect of baroclinic torque, because it only uses upstream and downstream conditions.
4.2.4.1 The use of the vorticity transport equation

To investigate this issue, the vorticity transport equation 4.5 is considered again. According to Bradshaw [1987], the viscous term is much smaller than the first term on the right hand side, representing vortex stretching and reorientation. By neglecting the viscous term, equation 4.5 can be re-written for the streamwise component as:

\[
V_s \frac{\partial (\Omega_s / \rho)}{\partial s} + V_b \frac{\partial (\Omega_b / \rho)}{\partial b} + V_n \frac{\partial (\Omega_n / \rho)}{\partial n} = \frac{\Omega_s}{\rho} \frac{\partial V_s}{\partial s} + \frac{\Omega_b}{\rho} \frac{\partial V_b}{\partial b} + \frac{\Omega_n}{\rho} \frac{\partial V_n}{\partial n} + BC_s. \tag{4.6}
\]

where \(BC_s\) is the baroclinic component for the \(s\)-direction and it is zero as shown in Fig. 4.16. In the equation, the first term on the right hand side represents vortex stretching, while the second and third terms represent vortex tilting by normal and spanwise gradients of streamwise velocity, respectively. The vortex tilting terms mean that streamwise vorticity can be induced by rotating or tilting a normal or spanwise vortex into the streamwise direction.

To focus on streamwise vorticity change through expansion, a streamwise vortex tube in a quasi-two-dimensional flow, a flow similar to that shown in Fig. 4.16, is considered. Because \(V_b = 0, \partial (\Omega_b / \rho) / \partial b = 0, BC_s = 0, V_s >> V_n\) and \(\partial (\Omega_s / \rho) / \partial s >> \partial (\Omega_s / \rho) / \partial n\), equation 4.6 is reduced to:

\[
V_s \frac{\partial (\Omega_s / \rho)}{\partial s} = \frac{\Omega_s}{\rho} \frac{\partial V_s}{\partial s}, \tag{4.7}
\]
Thus, the strength of streamwise vortices decreases through expansion for the underexpanded jets due to a faster decrease of density in comparison with the increase in velocity. For the expansion from \( M_j = 2.0 \) to \( M_j = 2.5 \), the density decreases approximately 45%, and the velocity increases 15%. On the other hand, vortex strength increases through compression owing to the compression of vortex diameter, which is similar to vortex stretching in incompressible flows. For the compression from \( M_j = 2.0 \) to \( M_j = 1.75 \), the density increases approximately 31%, and the velocity decreases 8%.

Similarly, the spanwise (b-direction) and normal (n-direction) component variations of vorticity through expansion/compression can be obtained. Let's consider a spanwise vortex, then \( \Omega_s = \Omega_n = 0 \). In addition, \( V_s >> V_n \) and \( \partial(\Omega_b/\rho)/\partial z >> \partial(\Omega_b/\rho)/\partial n \). Thus, equation 4.5 is reduced for a spanwise vortex,

\[
\nabla_s \frac{\partial (\Omega_b / \rho)}{\partial s} = BC_s, \tag{4.9}
\]

\[
\nabla_s \frac{\partial (\Omega_b / \rho)}{\partial s} = \frac{1}{\rho^2} \left( \frac{\partial \rho}{\partial n} \frac{\partial \rho}{\partial s} - \frac{\partial \rho}{\partial s} \frac{\partial \rho}{\partial n} \right),
\]

\[
\rho = \rho(s,n), \quad p = p(s,n), \quad \nabla_s = \nabla_s(s,n),
\]

\[
\frac{\partial (\Omega_b / \rho)}{\partial s} = \frac{1}{\rho^2 \nabla_s} \left( \frac{\partial \rho}{\partial n} \frac{\partial \rho}{\partial s} - \frac{\partial \rho}{\partial s} \frac{\partial \rho}{\partial n} \right),
\]

integrate over \( s \).
\[ \Omega_n = \rho \int \frac{1}{\rho} \left( \frac{\partial p}{\partial n} \frac{\partial p}{\partial s} - \frac{\partial p}{\partial s} \frac{\partial p}{\partial n} \right) ds + (\text{constant}) \rho. \] (4.10)

For a vortex normal to the wall, \( \Omega_s = \Omega_n = 0 \). Since \( V_y = 0, \) \( BC_n = 0, \) \( V_y \gg V_n \) and \( \partial(\Omega_n)/\partial s \gg \partial(\Omega_n/\rho)/\partial n, \) equation 4.5 is rewritten as for the normal vortex,

\[ V_s \frac{\partial(\Omega_n/\rho)}{\partial s} = \frac{\Omega_n}{\rho} \frac{\partial V_n}{\partial n} + BC_n = 0, \] (4.11)

integrate over \( s. \)

\[ \Omega_n \Omega_n = \rho \frac{\rho_1}{\rho_2} \Omega_n \int_{\rho_1 s}^{\rho_2 s} \frac{1}{\rho} \frac{\partial^2 \rho}{\partial n^2} ds. \] (4.12)

When \( \partial V_y/\partial n = 0, \) equation 4.12 is reduced to:

\[ \Omega_n \Omega_n = \frac{\rho_2}{\rho_1} \Omega_n \Omega_n. \] (4.13)

Contrary to a streamwise vortex, a simple relationship like equation 4.8 was not obtained for spanwise and normal vortices. Thus, it is not easy to estimate the change of spanwise and normal vorticity through expansion/compression without solving them numerically.
4.2.4.2 The use of the conservation of angular momentum of a vortex

A more simple analysis to figure out the effect of expansion/compression was adopted for quasi-two-dimensional flow by using the conservation of angular momentum. The disadvantage of this approach is that it cannot estimate the effects of baroclinic torque. However, this approach provides much clear insight of the compression or expansion effects on vorticity, simply because it does not use unclear assumptions and is not complicated. With this relatively simple analysis, the vorticity and turbulence level variations through an expansion or compression can also be explained.

Vorticity variation

For a streamwise vortex filament in one dimensional supersonic flow, as shown in Fig. 4.17, the vortex radius $R_2$ and the streamwise vorticity $\Omega_2$ after expansion can be calculated by using the angular momentum conservation law.

$$I_1 \Omega_1^{(1)} = I_2 \Omega_2^{(2)}.$$  (4.14)

Since $I = \frac{1}{2} mR^2$,

$$R_1^2 \Omega_1^{(1)} = R_2^2 \Omega_2^{(2)},$$

where $I$ is the moment of inertia for the vortex filament with radius $R$, numeric subscripts and numeric superscripts inside parentheses represent upstream and downstream locations.
Let $k_A = \sqrt{A_2/A_1} = \sqrt{\frac{M_1}{M_2} \left( \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}}$, then

\[
\frac{R_2}{R_1} = k_A,
\]

and

\[
\Omega_{s}^{(2)} = \Omega_{s}^{(1)} / k_A^2 = A_1 \Omega_{s}^{(1)} / A_2.
\] (4.15)

where $A$ and $M$ respectively represent the cross sectional area of the vortex filament and Mach number of the flow and $\gamma$ is the specific heat ratio of the jet fluid. The relationship between $k_A$ and Mach number is obtained from the isentropic calculation. By using the mass flow rate conservation law, $(\rho AV)_1 = (\rho AV)_2$, equation 4.15 becomes the same equation as equation 4.8, which was reduced from the vorticity transport equation. In fact, equations 4.8 and 4.15 indicate that the circulation of a vortex is conserved when there are no vorticity sources. Contrary to the incompressible counterpart, the vortex stretching by expansion in a supersonic flow results in the decrease of vorticity as proven by equation 4.8 and 4.15.

For a spanwise vortex, the length of an eddy is elongated by $k_A$ through expansion. In other word, $k_A = L_2/L_1$ for a spanwise vortex filament. From the mass conservation law, the vorticity variation by the expansion is written as:

\[
\rho_1 \pi R_1^2 L_1 = \rho_2 \pi R_2^2 L_2,
\]
\[
\left( \frac{R_2}{R_1} \right)^2 = \frac{\rho_1}{\rho_2} \frac{L_1}{L_2} = \frac{\rho_1}{\rho_2} \frac{1}{k_A},
\]

\[
= \left( 1 + \frac{\gamma - 1}{2} \frac{M_2^2}{M_1^2} \right)^{-\frac{1}{\gamma - 1}} \left( \frac{M_2}{M_1} \right)^{\frac{1}{2}} \left( 1 + \frac{\gamma - 1}{2} \frac{M_2^2}{M_1^2} \right)^{\frac{1}{2}} = \left( \frac{M_2}{M_1} \right)^{\frac{1}{2}}.
\]

\[
= \left( 1 + \frac{\gamma - 1}{2} \frac{M_2^2}{M_1^2} \right)^{-\frac{1}{4}} \left( \frac{M_2}{M_1} \right)^{\frac{1}{2}} \left( 1 + \frac{\gamma - 1}{2} \frac{M_2^2}{M_1^2} \right)^{\frac{1}{2}} = \left( \frac{U_2}{U_1} \right)^{\frac{1}{2}} \left( \frac{\rho_1}{\rho_2} \right)^{\frac{1}{2}},
\]

\[
\frac{R_2}{R_1} = \left( \frac{U_2}{U_1} \right)^{\frac{1}{4}} \left( \frac{\rho_1}{\rho_2} \right)^{\frac{1}{4}} = k_{sp},
\]

(4.16)

where \( \rho \) is the fluid density, \( L \) is the length of the vortex filament, \( U \) is streamwise mean velocity, and \( \rho_1/\rho_2 \) is from the isentropic relation. Thus, the vorticity after expansion for the spanwise vortex \( \Omega_{sp}^{(2)} \) is:

\[
\Omega_{sp}^{(2)} = \left( \frac{R_1}{R_2} \right)^2 \Omega_{sp}^{(1)} = \Omega_{sp}^{(1)} / k_{sp}^2 = \sqrt{\frac{U_1}{U_2}} \frac{\rho_2}{\rho_1} \Omega_{sp}^{(1)} = \sqrt{\frac{\rho_1}{\rho_2} \frac{U_1}{U_2} \frac{\rho_2}{\rho_1}} \Omega_{sp}^{(1)}.
\]

(4.17)
Equations 4.8 and 4.18 show how the density change through expansion/compression plays an important role in the vorticity transport. To estimate the density effects on vorticity quantitatively, the relative vorticity after expansion/compression over the downstream influence zone for the present experiments is listed in Table 4.1. In the calculation, the upstream condition of the downstream influence zone is always fixed at $M_d=2.0$, while the downstream Mach number is the same as the fully expanded Mach number $M_j$. For both streamwise and spanwise vorticity, the effects of density, $\rho_2/\rho_1$, surpass those of vortex stretching, $U_2/U_1$ or longitudinal elongation, $k_A$. Thus, vorticity decreases through expansion, while it increases through compression.

\[ \Omega_{sp}^{(2)} = \sqrt{\frac{A_2}{A_1}} \frac{\rho_2}{\rho_1} \Omega_{sp}^{(1)} = k_A \frac{\rho_2}{\rho_1} \Omega_{sp}^{(1)}. \]  

(4.18)

Table 4.1 Streamwise and spanwise vorticity variations through expansion or compression.

<table>
<thead>
<tr>
<th>$M_j$</th>
<th>Streamwise vorticity, by Eq. (4.8)</th>
<th>Spanwise vorticity, by Eq. (4.18)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_2/U_1$</td>
<td>$\rho_2/\rho_1$</td>
</tr>
<tr>
<td>1.5</td>
<td>0.8356</td>
<td>1.7168</td>
</tr>
<tr>
<td>1.75</td>
<td>0.9243</td>
<td>1.3165</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2.2</td>
<td>1.0520</td>
<td>0.8001</td>
</tr>
<tr>
<td>2.5</td>
<td>1.1180</td>
<td>0.5724</td>
</tr>
</tbody>
</table>
**Turbulence intensity variation**

It is generally said that turbulence consists of the superposition of eddies of ever-smaller sizes [Hinze 1975], or that turbulence is characterized by high levels of fluctuating vorticity [Tennekes and Lumley 1972, Kundu 1990]. Thus, vortex dynamics plays an essential role in the description of turbulent flows [Tennekes and Lumley 1972]. These provide sufficient grounds for using equations 4.8 and 4.15 to calculate turbulence variations through expansion or compression.

Let's calculate turbulence level variations through expansion or compression. Let $\sigma_j$ be the rms value of the j-component of velocity fluctuation. Spanwise vortices generates streamwise velocity fluctuation, while streamwise and one spanwise vortices generate the other spanwise velocity fluctuation. Thus, streamwise and spanwise rms values are written as:

$$\sigma_u^{(1)} = \sqrt{\frac{(\omega_1 R_1)_{sp}^2 + (\omega_1 R_1)_{sp}^2}{2}},$$

$$\sigma_v^{(1)} = \sigma_w^{(1)} = \sqrt{\frac{(\omega_1 R_1)_{sp}^2 + (\omega_1 R_1)_{st}^2}{2}},$$

where $\sigma_u$ is streamwise rms value of velocity fluctuation. $\sigma_v$ and $\sigma_w$ are spanwise rms values of velocity fluctuation, and the superscript inside () and numeric subscript represent the value at a certain location (1 for upstream and 2 for downstream).
If one can assume \((\Omega_1)_{sp} = (\Omega_1)_{st} = \Omega_1\) and \((R_1)_{sp} = (R_1)_{st} = R_1\),

then

\[
\sigma_u^{(1)} = \sigma_v^{(1)} = \sigma_w^{(1)} = \sigma_1 R_1.
\]

Turbulence after expansion or compression \((\sigma_j)^{(2)}\) is:

\[
(\sigma_u)^{(2)} = \sqrt{\frac{(\Omega_2 R_2)_{sp}^2 + (\Omega_2 R_2)_{sp}^2}{2}} = \sqrt{\frac{(\Omega_1 R_1 \frac{1}{k_{sp}})_{sp}^2 + (\Omega_1 R_1 \frac{1}{k_{sp}})_{sp}^2}{2}} = \frac{\Omega_1 R_1}{k_{sp}} = \frac{(\sigma_u)^{(1)}}{k_{sp}},
\]

(4.19)

\[
(\sigma_v)^{(2)} = \sqrt{\frac{(\Omega_2 R_2)_{sp}^2 + (\Omega_2 R_2)_{st}^2}{2}} = \sqrt{\frac{(\Omega_1 R_1 \frac{1}{k_{sp}} k_A R_1)_{sp}^2 + (\Omega_1 R_1 \frac{1}{k_A} k_A R_1)_{sp}^2}{2}} = \Omega_1 R_1 \sqrt{\frac{1}{2} \left( \frac{1}{k_{sp}^2} + \frac{1}{k_A^2} \right)} = (\sigma_v)^{(1)} \sqrt{\frac{1}{2} \left( \frac{1}{k_{sp}^2} + \frac{1}{k_A^2} \right)}.
\]

(4.20)

From equations 4.19 and 4.20, it is clear that expansion or favorable pressure gradient results in the decrease of turbulence intensity, while compression increases it. This trend agrees well with experimental results in a supersonic wind tunnel for expansion [Arnette 1995, Arnette et al. 1998], compression [Fernando and Smith 1990], and for both expansion and compression [Wier et al. 1998]. Quantitative comparison, listed in Table 4.2, shows
reasonably good agreement between predicted and measured turbulence intensity ratios. The measured data for expansion is taken at 1.5 times the incoming boundary layer thickness downstream in the middle of boundary layer. In the same way, the streamwise turbulence intensity data for compression is taken in the middle of the boundary layer. Surprisingly, the agreement between the predicted and measured data is quite good in both boundary layers which experience expansion and compression. Much better agreement is seen when the expansion is gradual. This may support the feasibility of the simple analysis. The analysis of turbulence level variations through compression or expansion given above is valid on the assumption that turbulence of the flow is dominated by the velocity fluctuation due to eddies.

<table>
<thead>
<tr>
<th></th>
<th>$\rho_z/\rho_t$</th>
<th>$k_A$</th>
<th>$k_p$</th>
<th>$\sigma_u^{(2)}/\sigma_u^{(1)}$</th>
<th>$\sigma_v^{(2)}/\sigma_v^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Predicted</td>
<td>Measured</td>
</tr>
<tr>
<td>7° Exp.</td>
<td>0.663</td>
<td>1.203</td>
<td>1.119</td>
<td>0.8934</td>
<td>0.87$^+$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.89$^+$</td>
</tr>
<tr>
<td>14° Exp.</td>
<td>0.419</td>
<td>1.487</td>
<td>1.266</td>
<td>0.7896</td>
<td>0.83$^+$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.796$^*$</td>
</tr>
<tr>
<td>8° Comp.</td>
<td>1.494</td>
<td>0.842</td>
<td>0.730</td>
<td>1.371</td>
<td>1.369</td>
</tr>
<tr>
<td></td>
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$^+$: Data from Arnette [1995], where $^+$ for centered expansion and $^*$ for gradual expansion
$^\#$: Data from Fernando and Smith [1990]

Table 4.2 The decrease of turbulence intensity through expansion in a supersonic boundary layer.
4.3 Flow Visualizations for Larger Nozzles with Single-sided Trailing Edge Modifications

The primary objectives of flow visualizations for larger nozzles with single sided modifications are to determine the optimum trailing edge modification and to visualize large scale structures, which play major roles both in mixing and noise generation. The interpretation of visualization image of large scale structures by the trailing edge modifications is expected to be much easier with single-sided modification, because the interaction of mixing layers is minimized. Based on the information collected in the single modification experiments with a half nozzle, double sided trailing edge modifications will be determined to produce maximum mixing enhancement. The mixing performance of each nozzle was evaluated in terms of mixing level and shape factor, which represent mixing potential [Glawe et al. 1995].

The idea of using trailing edge modifications for mixing enhancement is to excite or amplify oblique large scale structures in the ideally expanded case and to generate streamwise vortices for non-ideally expanded cases. Based on Sandham and Reynolds’ theoretical relationship, the cut-out angle was calculated to be 45° relative to the spanwise direction. In Martens et al.’s work [1996b], it was not clear whether the ineffectiveness of the 45° cut-out in exciting expected oblique large scale structures in the ideally expanded case was due to the mismatch of the flow structure angle with the cut-out angle. Thus, two additional modification angles, 35 and 55° relative to the spanwise direction, were also used with nozzles 3 and 4, since both nozzles showed better mixing and noise reductions in
comparision with other nozzles. Two additional 90°, relative to the spanwise direction, modifications were also investigated since it was expected that stronger streamwise vortices could be generated by these modification. The other parameters, such as cut-out depth and width, might also affect mixing performance. However, the cut-out depth is fixed at half of the nozzle span, 38.1 mm for the larger nozzles and 14.3 mm for the smaller nozzles.

Visualizations were carried out for five flow regimes at two downstream locations as shown in Table 3.1.

4.3.1 Jet cross sectional deformation

Since the laser pulse duration is just 9 ns and the exposure time is dictated by it, acquiring instantaneous images even for high speed supersonic flows is possible. The fluid displacement within the laser pulse duration is just 5 μm at the jet maximum velocity of 560 m/s. Figures 4.18-4.20 show instantaneous, average, and rms images for various nozzles at x/D_{eq} = 2 at various operating conditions. Figs. 4.19 and 4.22 show the average of 50 instantaneous images shown in Figs. 4.18 and 4.21, respectively. The bright areas represent mixing regions which were marked by condensed water particles. Since there is no water condensations outside the mixing region, the jet potential core and the ambient air, the outside of the mixing region is dark. These particles are formed when the moisture in the relatively warm and humid ambient air is cooled down and condensed by the cold and dry jet air through mixing. The size of the these particles is on the order of 50 nm and so it is small enough to follow the jet flow faithfully [Samimy and Lele 1991]. Those images shown
in Figs. 4.18 and 4.21 are typical for 50 images at each flow regime. Jet cross sections were deformed severely depending on the operating conditions and trailing edge modifications.

The rms images shown in Figs. 4.20, 4.23, and 4.26 represent intensity fluctuations in the jet mixing layer. Since there are negligible flow fluctuations in the jet potential core and in the ambient air, the rms values are low (looks dark in the images). Although the rms values are scalar fluctuations, a higher rms value indicates higher mixing activity to some degree.

The major parameters affecting the jet cross sectional deformation are the fully expanded jet Mach number, the type of modification, the dynamics of large scale structures such as streamwise vortices, and the cross stream flow. Since the dynamics of large scale structures and the cross stream flow strongly depend on the fully expanded jet Mach number, the jet cross sectional deformation will be discussed separately for different flow regimes.

For single sided trailing edge modified nozzles, cross sectional visualizations were performed at only two downstream locations $x/D_{eq}=2$ and 8, which represent relative near field and farther downstream locations, respectively. The purposes of the visualizations for the larger nozzle with single sided trailing edge modifications are to find out the best modification at all flow regimes and to check if any cut-out is good for exciting oblique large scale structures in the ideally expanded case as mentioned earlier.

### 4.3.1.1 Ideally expanded case

One of the objectives of the present experiments is to excite oblique large scale structures which are expected to have oblique 45° angle relative to the jet axis. As
mentioned earlier, the cut-out 45° angle is based on Sandham and Reynolds' [1992] theoretical value for the convection Mach number 0.83 at $M_j=2.0$. Martens et al. [1996b] used four different modifications M1, M2, M3, and M4, as shown in Fig. 3.2, with the cut-out angle of 45° to excite or amplify the oblique large scale structures. Although their results show no significant excitation, it was not clear whether the ineffectiveness of exciting by cut-out edges was due to the mismatch of the angle between the cut-out and oblique large scale structures as mentioned above. Thus, two additional cut-out angles were used on modifications M3 and M4 to check if the ineffectiveness was related to the cut-out angle. As shown in Fig. 3.2, the additional modifications are M7, M8, M9, and M10.

For none of the different cut-out angles, the jet cross sectional images at $x/D_{eq}=2$ show significant excitation or amplification of the mixing layer at the ideally expanded case $M_j=2.0$, as shown in Figs. 4.18-4.23. This result is consistent with Papamoschou’s [1989] experimental results in a plane mixing layer. In his case, he used a saw-tooth extension at the end of splitter plate, which resulted in negligible mixing enhancements, to excite large scale structures. In the figures, the slightly thicker mixing layers around the cut-out is most likely due to earlier development of mixing layers compared with unmodified edges.

4.3.1.2 Underexpanded cases

The distortions of jet cross sections is increased as the degree of underexpansion is increased as can be seen in Figs. 4.18 to 4.26. The outward cross streamwise flow and the strength of the streamwise vortices become stronger as the degree of underexpansion
increases due to the increased pressure difference between the jet plume and the ambient air at the nozzle exit. These two parameters seem to determine the shape of the cross section of a jet. However, their relative contribution on the deformation of the jet cross section varies with downstream locations. At the upstream location \( x/D_{eq} = 2 \), both cross stream flows and streamwise vortices probably play important roles in the jet cross sectional deformation. On the other hand, at the downstream location \( x/D_{eq} = 8 \), the dynamics of streamwise vortices probably is dominant in the deformation of the jet cross section.

The modifications M3 through M10 can be categorized as two groups: group A and group B. Group A nozzles, N3, N5, N9, and N10 nozzles, have a kidney type pair of streamwise vortices which entrain the ambient air into the jet, while group B nozzles have a mushroom type pair of counter rotating streamwise vortices which eject the jet air into the ambient air in the underexpanded cases, as depicted in Fig. 4.27. Because of the difference in the dynamics of vortices, mixing and jet development characteristics with downstream locations are different for each group.

Images at \( x/D_{eq} = 2 \)

As depicted in Fig. 4.28, the jet flow is redirected by expansion fan or waves upon approaching the cut-out edges. The forward Mach wave line only depends on the design Mach number, while the rearward Mach wave line depends on the fully expanded jet Mach number \( M_j \). The expansion fan is three dimensional for nozzles with trailing edge modifications and so the flow around the modification is expected to be three dimensional.
As discussed in section 4.2.2.1, three dimensional surface flow patterns were measured by the surface flow visualization technique. Schematic of cross stream velocity distribution at the jet exit plane is depicted in Fig. 4.29, which is deduced from the measured surface flow and pressure field on the modification and the nozzle block. In addition to the cross stream flow, a pair of streamwise vortices generated by the spanwise pressure gradient on the modification, as discussed before, would play an important role in the jet cross sectional deformation.

At the near field location, the jet cross section of the flow is spatially stationary, thus the instantaneous jet cross section images are similar to the matching average image. Even from the rms image, it is possible to guess the type of modification. The average and rms images for nozzle N5 and N6 clearly show the presence of a pair of counter rotating streamwise vortices. However, for other nozzles with different modifications, the presence of streamwise vortices is only inferred from the surface flow images and cross sectional images. The strength of streamwise vortices for modifications M5 and M6 appeared to be stronger than that for other modifications. This is partially because of the orientation factor of cut-out edges, defined as the sine of the cut-out angle as shown in Fig. 3.2. For nozzles with modifications M5 or M6, the orientation factor is 1 while for others it is less than 1. The orientation factor 1 means that the shed vortex along the cut-out edge is aligned to the streamwise direction, thus it only has a streamwise vorticity component. On the other hand, the orientation factor 0 means that the shed vortex does not have a streamwise vorticity component. Thus, the streamwise vortices for nozzles with modifications M5 or M6 are expected to be stronger because the orientation factors for these modifications are equal to 1.
At this downstream location, the jet cross sectional deformation could be explained to some degree with the cross stream velocity distribution as depicted in Fig. 4.29. For nozzles N5 and N6, further distortions of mixing layers by the strong streamwise vortices are observed in average and rms images as shown in Figs. 4.19-4.20 and 4.22-4.23. However, the streamwise vortices present in nozzles N3 and N4 cannot be visualized with the present technique, but contribute to the jet cross sectional distortion. A two dimensional vortex generates an azimuthal velocity field, which not only induces mutually induced velocity $V_i$ for a pair of vortices as shown in Fig. 4.27 and 4.29, but also causes nearby mixing layers to be deformed as observed in the cross sectional images for nozzles N5 and N6. As will be discussed later, the vortex induced deformation of mixing layers is much clearer at $x/D_{eq}=1$.

Group A nozzles, N3, N5, N9, and N10, generate a kidney type pair of vortices, which entrain the ambient air into the jet. While group B nozzles, N4, N6, N7, and N8, generate a mushroom type pair of vortices, which eject the jet fluid into the ambient air. The kidney type pair of vortices for group A nozzles induces mutual induction velocity $V_i$ toward the jet axis, which suppresses the lateral expansion of the jet. As the downstream distance increases, it is expected that two vortices in a pair will interact. As a result, the entraining capability will be reduced after the interaction. Thus, it seems desirable to make the distance of the two vortices larger to enhance further downstream mixing performance for group A nozzles. The interaction of a vortex pair with downstream locations will be discussed in the section of flow visualizations for the smaller nozzles with double sided trailing edge modifications, since visualizations were carried out at only two locations for single side
modified nozzles.

On the other hand, for group B nozzles, the mushroom type pair of vortices causes the vortex pair to move away from the jet axis as shown in Fig. 4.27 (b), where $V_i$ is the mutually induced velocity by the pair of vortices. Because of the added velocity $V_i$ on the cross stream velocity $V_c$, the jet is expanded laterally more than in group A nozzles. According to the potential theory, the mutual induction velocity ($V_i$) is proportional to the vortex strength ($\Gamma$) and inversely proportional to the vortex center-to-center distance ($r_o$):

$$V_i \propto \frac{\Gamma}{r_o}. $$

Since the mixing performance is proportional to the $\Gamma/r_o$ [Zaman 1996], a narrower cut-out, which will have a smaller $r_o$, is expected to perform better for group B nozzles. The pair of mushroom type streamwise vortices acts as a fluid pump, which helps the jet air ejected much faster.

Vertical elongations, the jet expansion along the major axis, of the jet cross section at $x/D_{eq} = 2$ can be explained in terms of the jet exit pressure and corner vortices. In an incompressible square jet, it was found that a pair of counter rotating streamwise corner vortices are responsible for the axis switching [Quinn 1992]. According to Zaman [1994], an axis switching caused by a pair of counter rotating vortices in an underexpanded sonic rectangular jet. Thus, it is believed that a pair of streamwise corner vortices are partially responsible for the elongation. The other possible reason for the elongation is the jet exit pressure. Fig. 4.30 shows streamwise pressure distributions on the inside wall of the shorter dimension. Comparing the exit pressure with the degree of vertical elongation of each nozzle, there seems a direct relationship between them. Thus, the exit pressure also appears to be partially responsible for the vertical elongation of the jet cross section.
Images at x/D_{eq}=8

Since nozzles N7-N10 did not show significant mixing enhancement in comparison with nozzles N3 or N4 at x/D_{eq}=2, flow visualizations for these nozzles at x/D_{eq}=8 were not performed. Figures 4.24-4.26 show the cross-sectional images at x/D_{eq}=8. The interaction between counter rotating pair of vortices is observed for group A nozzles, nozzle N3 and N5. As will be further discussed in a later section, the interaction seems to be responsible for decreased mixing performance at further downstream locations for group A nozzles.

For group B nozzles, nozzle N4 and N6, each vortex causes the mate vortex to move away from the jet axis as depicted in Fig. 4.27 and 4.29. As a result, the destructive interaction, which was observed for group A nozzles, between two pair vortices was not observed. These mushroom type pair vortices acted as a fluid pump and resulted in a further laterally expanded jet. As long as the strength of the pair of vortices remains sufficiently high, the pumping action by the pair of vortices may enhance mixing performance significantly even at further downstream locations.

4.3.1.3 Overexpanded cases

Contrary to the underexpanded cases, the overall flow pattern on the modification strongly depends on the fully expanded jet Mach number M_j as discussed in section 4.2.2.1 when the jet is overexpanded. The flow pattern variation with M_j is due to the change of the separation line within the nozzle depending on the jet exit pressure and accompanying
separation shock waves as depicted in Fig. 4.7. As in the underexpanded cases, the cross stream velocity distribution caused by the separation shock and streamwise vortices appeared to be responsible for the jet cross sectional deformation. A schematic of the cross stream velocity distributions at the jet exit plane is shown in Fig. 4.31, which were deduced from the surface flow visualization images and the measured pressure distribution on the modification. As mentioned earlier, the role of streamwise vortices in the overexpanded cases is not straightforward due to the flow separation within the nozzle.

**Moderately overexpanded case, \( M_j = 1.5 \)**

Two dimensional oblique separation shock was generated when the jet was moderately overexpanded as depicted in Fig. 4.31 (a). Because of the two dimensional shock, almost similar jet cross section images were observed regardless of the type of modifications at \( x/D_{eq} = 2 \) and 8. Slight indentations into the jet axis around cut-outs at \( x/D_{eq} = 2 \) are most likely due to slightly lower jet exit pressure as measured and shown in Figs. 4.8-4.11. Since the spanwise pressure gradient on the modification, which appeared to be a major source of streamwise vorticity, is not significant, spanwise structures may be dominant structures at the upstream location.

At the downstream location \( x/D_{eq} = 8 \), spanwise structures appear to be broken. As will be shown in section 4.5.2.3, spanwise structures observed up to \( x/D_{eq} = 4 \) appeared to be broken down at \( x/D_{eq} = 8 \) for some double sided trailing edge modified nozzles (see Figs. 4.10-4.16). Thus, random streamwise structures induced by vortex tilting may be responsible
for the jet cross sectional deformation. Considering the very weak streamwise vorticity sources, i.e., very low spanwise pressure gradient, the contribution of streamwise vortices on the jet cross sectional deformation may be negligible. The shape of the jet cross section at \(x/D_{eq}=2\) appeared to be preserved up to \(x/D_{eq}=8\). This fact provides reasonable evidence for the conjecture. Thus, the cross stream flow caused by two dimensional separation shock waves appeared to be responsible for the jet cross sectional deformation at this Mach number.

Jet vectoring, the deflection of the jet plume direction, was observed at this Mach number as shown in Figs. 4.25-4.28. In the figures, the shift of jet center to the left of the images is due to the jet vectoring. The jet vectoring for the baseline nozzle is more significant than any other nozzles. After being deflected to the left, when it is viewed in front of the jet, by the separation shock, the jet is redirected to the right slightly by the reflected separation shock as shown in Fig. 4.31 (a). The angle of jet vectoring appeared to be dependent on the location of the separation shock. Considering \(y\)-directional momentum balance by cross stream flows, the jet deflection angle will be maximum when the separation shock generated on the splitter plate just passes over the right edge of the nozzle exit.

**Slightly overexpanded case, \(M_j=1.75\)**

As in the underexpanded cases, the cross stream velocity distribution and streamwise vortices appeared to play major roles at the upstream location \(x/D_{eq}=2\). The qualitative cross stream velocity distribution at the jet exit plane, which is shown in Fig. 4.31, was deduced
from the surface flow visualization images (Figs. 4.1-4.5) and the measured pressure data (Figs 4.8-4.13). By the three dimensional separation and reflected shock waves, very complex three dimensional cross stream flow seems to be developed at the jet exit plane. As will be discussed in the noise section, the jet shows flapping motion at this Mach number.

The cross sectional deformation of a jet at $x/D_{eq}=2$ is reasonably well explained by using the shock induced cross stream flow distribution and momentum balance of the cross flow across the cross section. The cross section of nozzle N5, especially the left side of the mixing layer, looks strange when one compares the cross stream velocity distribution for the nozzle in Fig. 4.31 (e) and the cross sectional image in Figs. 4.18-4.20. This bulged shape of the left mixing layer is most likely due to much greater $y$-directional flow momentum caused by the two conical shock waves than negative $y$-directional momentum generated by the shock initiated from the left edge of the jet.

The role of streamwise vortices on the jet cross sectional deformation is questionable at this Mach number, not only because no vortex-like structures are observed in the images but also because the adverse pressure gradient, as shown in Figs. 4.8-4.11 and Fig. 4.15, appeared to result in flow separation as in the surface flow images of Figs. 4.1-4.5. In the separation for a two dimensional flow, the separation bubble does not convect with the flow but stays at the separation region with very unsteady characteristics [Smith 1982]. In the present case, a three dimensional separation was observed. The sense of the vorticity for the separation bubble is the opposite direction of that shown in Fig. 4.15. The average and instantaneous cross sectional images do not show any vivid evidence of the presence of strong enough streamwise vortices in the jet plume. If there are such vortices, the
deformation of the jet cross section may be more curved shaped as the curved cloud around the eye of a hurricane.

The flow is more complex for nozzle N5 due to a much more complicated spanwise and streamwise pressure gradient as shown in Fig. 4.15 (c). For this nozzle, the sign of the spanwise pressure gradient is changed with downstream location. For the complex flow features, including three dimensional shock waves and separation, the role of streamwise vortices on the jet cross sectional deformation is not straight forward. More experimental work is required to figure out the sense and strength of streamwise vortices in this flow regime. However, the contribution of streamwise vortices, if any, is not expected to be as significant as that of cross stream flow when considering the shape of the jet cross section.

At the farther downstream location $x/D_{eq}=8$, very random streamwise structures appeared to be present as shown in the instantaneous image Fig. 4.24. Because of the random and weak characteristics of the streamwise vortices, further severe jet cross sectional deformation was not observed. As a result, the shape of the jet cross section of a nozzle at the upstream location appeared to be preserved up to the farther downstream location.

Significant mixing enhancement was observed for the baseline nozzle NB at $x/D_{eq}=8$. This enhancement is due to the flapping motion of the jet. For the other nozzles, the jet center is shifted to the left due to the jet vectoring. Considering the jet center shift distance from the nozzle axis, the jet vectoring angle at $M_j=1.75$ is greater than that at $M_j=1.5$. As depicted in Fig. 4.31, $y$-directional momentum generated by separation shocks from the right side is not so much canceled out by either reflected shock or oblique shock waves generated from the right side of the nozzle (viewed in front of the nozzle). As mentioned before, the
deflection of the jet would decrease as the separation shock moves upstream, since negative y-directional momentum, by the reflected shock, increases with the shock movement.

4.3.2 Overall Mixing

To assess the overall mixing performance of the nozzles with various trailing edge modifications, one must determine the extent of the mixing region between the jet and the ambient air. For the reason discussed next, the calculations must be done on instantaneous images. Then, the average overall mixing must be calculated from the instantaneous mixing. The average images for $M_j = 1.75$ for the baseline nozzle shown in Figs. 4.19 and 4.22 indicate a thicker mixing layer in comparison with that of the other nozzles. However, the jet for this case was screeching and flapping as will be discussed in the noise section. The instantaneous images for this case clearly indicated the effects of this flapping motion which causes an alternating preferential entrainment and mixing on one side or the other of the mixing layer. Because of this flapping motion, the average jet mixing layer thickness may appear to have increased when in actuality the thickened mixing layer was due to the flapping motion. Thus, the more accurate calculation of the mixing is to determine the instantaneous mixing first, then average the instantaneous values to determine the average mixing.

Another issue which should be discussed is the repeatability of the mixing region for different conditions such as the ambient temperature and humidity. As discussed above, the mixing layer visualization was performed by illuminating condensed water particles, which were formed when the relatively cold and dry jet air was mixed with the warm and humid
ambient air. It is believed that the extent of the mixing obtained from the visualization would, to some degree, depend on the ambient temperature and humidity levels. The experiments were conducted in a laboratory environment with no control on these two parameters over a several month period. Thus, a proper normalizing process for the mixing area is required to remove or minimize the weather effects. Unfortunately, this issue was raised after all visualizations for nozzles with single sided trailing edge modification were carried out. To minimize the weather effects, different data processing was used for different downstream locations. For the nozzles with double sided trailing edge modifications, a reference mixing area was acquired to calibrate the change in weather conditions. The reference mixing area is taken with the baseline nozzle just prior to or just after each test, and so the weather condition effects on mixing area can be calibrated.

In calculating the mixing areas shown in Fig. 4.32 for \( x/D_{eq}=2 \), the mixing area, for the ideally expanded flow regime, was divided into two parts along the major axis; one half, the left side of the major axis, was affected and the other half was unaffected by the trailing edge modifications. The doubled mixing area of the unaffected half was used to normalize all the mixing areas for that nozzle operated in various flow regimes. The reason for this normalization was to remove or minimize the weather effects.

The procedure outlined above, could not be used at a further downstream location, \( x/D_{eq}=8 \), where the cut-outs affect the entire cross-section of the jet as seen in Figs. 4.24-4.26. Since the results presented in Fig. 4.33 showed that the cut-outs have minor effects in the ideally expanded case, it was decided to normalize the mixing region for each nozzle in different flow regimes with the mixing region for the same nozzle in the ideally expanded

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flow regime. The results for the baseline nozzle and nozzles N3-6 are shown in Fig. 4.33. This normalization might affect the results by 5 to 10%, but it should not change the overall trend in a major way.

**Ideally expanded case, $M_j=2.0$**

Figure 4.32 shows the overall mixing area for each nozzle at five different flow conditions in the jet near field region ($x/D_{eq}=2$). As one can observe visually from the images, the effects of various cut-outs in the ideally expanded flow regime are relatively small in comparison with those in the non-ideally expanded flow regimes. According to Martens et al. [1996b], the mixing enhancement at $M_j=2.0$ is due to the earlier development of the mixing layer around the cut-out. Considering flow visualization and far field noise data, which will be presented later, the cut-out or the trailing edge modification seems to be not effective in the excitation or amplification of oblique large scale structures in the mixing layer in this flow regime.

**Underexpanded cases, $M_j=2.2$ and 2.5**

When the jet is underexpanded, significant mixing enhancement up to 38 and 54% over the baseline nozzle can be seen in Figs. 4.32 and 4.33 at $x/D_{eq}=2$ and 8, respectively. The mixing performance of nozzles in group A (nozzles N3, N5, N9, and N10) at the near field region, in general, is superior than that of nozzles in group B (nozzles N4, N6, N7, and
N8) in the underexpanded flow regime, while the group B nozzles have superior performance in the overexpanded flow regime. As discussed in subsection 4.3.1, group A nozzles generate a kidney type streamwise pair of streamwise vortices, while group B nozzles generate a mushroom type streamwise pair of vortices. Thus, the kidney type streamwise pair of streamwise vortices, generated by group A nozzle, is more beneficial in the near field mixing. Nozzles N5 and N6 outperform the other nozzles in group A and B, respectively. These nozzles have the strongest streamwise pair of vortices, partially because of their greater streamwise orientation factor as discussed above.

For the further downstream location at x/D_{eq}=8, the mixing performance of group B nozzles is much better than that of group A nozzles at M_j=2.5. As in the near field region, nozzle N5 and N6 showed best mixing performance in each group. The mixing enhancement of nozzle N5 and N6 over the baseline nozzle NB is respectively as much as 22 and 54%. However, the mixing levels for groups A and B are in the same range when the jet is slightly underexpanded.

The strength of the dominant streamwise vorticity source, which is the spanwise pressure gradient on a modification, increases with the degree of underexpansion as shown in Figs. 4.15 and 4.16. For a given geometry, the increased streamwise vorticity source would result in a stronger streamwise pair of vortices and in turn would cause the pair vortices to interact with each other faster for group A nozzles. As will be discussed further in the later subsection on flow visualizations for the smaller nozzle, the mixing rate is reduced after the vortex pair interaction between upper and lower vortices in Fig. 4.27 is started. The entrainment capability of the kidney type pair vortices for group A nozzles may
begin to decrease, as the lower edge of the upper vortex interacts with the upper edge of the lower vortex. On the other hand, for group B nozzles, the stronger mushroom type pair vortices act as a stronger fluid pump and so enhance mixing performance much more. Most probably, the different vortex dynamics between group A and B nozzles resulted in “mixing performance switch” with downstream locations. Because of the mixing performance switch with downstream locations, nozzle N5 is best in mixing at the near field downstream location, while it is nozzle N6 that shows best mixing at the farther downstream locations.

**Overexpanded cases, $M_j=1.5$ and 1.75**

Group B nozzles showed better mixing performance than group A nozzles both at the near field and further downstream locations as shown in Figs. 4.32 and 4.33 in the overexpanded cases. At the near field location, the mixing enhancement of nozzle N6 over the baseline nozzle is 32 and 59% at $M_j=1.5$ and 1.75, respectively. The amount of mixing enhancement for nozzle N6 is reduced to 7 ($M_j=1.5$) and 43% ($M_j=1.75$) at $x/D_{eq}=8$ location. Nozzle N3 shows mixing level reduction, rather than enhancement, as much as 24 and 16% respectively at $M_j=1.5$ and 1.75 at $x/D_{eq}=8$ location.

As will be discussed in the noise result section, the baseline nozzle shows strong flapping motion, which results in enhanced mixing and increased shock associated noise. Thus, the suppression of the flapping motion would result in decreased mixing to some degree, even though it reduces shock noise.

For group A nozzles, the cross stream flow is developed in such a way that the jet is
shrunk toward the jet center as depicted in Fig. 4.31. On the other hand, for group B nozzles, the center region of the jet air is ejected into the ambient air by the cross stream. In addition, the jet cross section is vertically elongated by the cross flow developed by the separation shock wave as depicted in the figure and as can be observed in Fig. 4.25. The elongation of the jet along the major axis and the ejection of the jet along the minor axis, by cross flow and/or relatively weak streamwise pair vortices, may be responsible for the better mixing performance of group B nozzles.

4.4 Noise Measurements

All the noise measurements were carried out for nozzles with single sided trailing edge modifications. Far field noise was measured to investigate Overall Sound Pressure Level (OSPL) changes with trailing edge modifications and with the fully expanded jet Mach number, while near field noise was acquired to measure screech modes related to flow instability modes. Microphone positions are shown in Fig. 3.7 for far- and near-field noise measurements. The noise measurements were not performed in an anechoic chamber, thus the measured OSPL is used for approximate comparison of noise reduction performance by trailing edge modifications.

4.4.1 Far Field Noise Results

The farfield noise was measured to investigate the effects of the trailing edge modifications on OSPL. Two microphones were placed at 30 D_{eq}, the jet exit equivalent
diameter, away from the jet center on the jet exit plane: One on the major axis and the other on the minor axis as depicted in Fig. 3.7 (a). The OSPL difference between two microphones was less than 0.6 dB, which is the repeatability of measurements for the given conditions. The spectra from two microphones were almost the same. Thus, only the results for the minor axis are presented and discussed from this point on.

The farfield spectra for seven nozzles, the baseline nozzle and nozzles N1-6, are shown in Figs 4.34-4.38 at $M_j=1.5, 1.75, 2.0, 2.2,$ and $2.5,$ respectively. As mentioned earlier, the spectra are the average of 100 spectra, whose sampling size is 4096. Since the sampling rate was 100 kHz, the bandwidth or frequency resolution is 24.4 Hz. The characteristics of the spectra were changed depending on the type of modifications. The effects of trailing edge modifications on the spectra are more significant at the overexpanded operating conditions than at the perfectly- and under-expanded conditions.

**Ideally expanded case**

As shown in Fig. 4.36, the changes in the spectra at the perfectly expanded condition are negligible. The maximum sound pressure level at 1000-3000 Hz is almost the same regardless of the type of modifications. This maximum noise band is related to mixing noise. This is consistent with flow visualization data, which does not show significant changes in flow fields. The OSPL variations with different nozzles, as shown in Fig. 4.39, are within the repeatability of the OSPL for a given nozzle and given operating conditions. At this Mach number, the mixing noise is the dominant noise source because there are no shock
waves in the jet plume, and because the convective velocity is not supersonic relative to the ambient sound speed to generate Mach wave radiation.

As was shown in the previous section, the flow visualizations did not show any significant changes at this ideally expanded condition. From the flow visualization and far field noise results, a conclusion can be drawn: the modifications are not effective in the excitations/amplifications of oblique large scale structures, which are expected to have a 45° angle inclination with respect to the jet axis, in the ideally expanded case as originally conjectured.

**Underexpanded cases**

Surprisingly, the variations in the spectra and OSPL for the underexpanded conditions did not match the major distortions in the cross sectional at these conditions. As in the spectra for baseline nozzles, there are no noticeable screech tones in the underexpanded case. If the contribution of the screech noise to OSPLs at $M_j=2.2$ is as large as at $M_j=1.75$, the OSPL reduction at $M_j=2.2$ would also be comparable with that at $M_j=1.75$. However, the contribution of the screech noise is relatively small in the underexpanded case because there is no strong screech tones. Thus, the OSPLs were not affected much even though the distortions of the jet cross section were significant.

The measured frequency at the peak sound pressure level of Broadband Shock Associated Noise (BSAN) for the baseline nozzle agrees well with the predicted value by the Tam's formula [1988] as shown in Table 4.3. The peak frequency varied respectively
between 20 and 40% with the type of modifications at $M_j = 2.2$ and 2.5. Since the frequency of shock noise is inversely proportional to the shock cell spacing, the variation of the peak BSAN frequency is most likely related to the shock cell structure variation due to the trailing edge modification, which can be inferred from the surface flow images in Figs. 4.1-4.5 and the wall pressure distribution in Figs. 4.8-4.13.

<table>
<thead>
<tr>
<th>Predicted</th>
<th>BSAN, Hz</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NB</td>
<td>N1</td>
</tr>
<tr>
<td>$M_j = 2.2$</td>
<td>2710</td>
<td>2840</td>
</tr>
<tr>
<td>$M_j = 2.5$</td>
<td>1980</td>
<td>2180</td>
</tr>
</tbody>
</table>

Table 4.3 Measured peak frequencies of BSAN for different nozzles.

Fig. 4.40 shows the variation of spectra with the jet Mach number, $M_j$, for the baseline nozzle. As the jet Mach number increases, the center frequency of BSAN shifts to lower values due to the increased shock cell lengths as was theoretically predicted by equation 2.3 and shown in Fig. 4.41.

Only nozzle N3 and N5 reduced OSPL by 1.0 and 1.5 dB at $M_j = 1.5$ and 1.0 and 1.1 dB at $M_j = 2.5$, respectively, as shown in Fig. 4.39. Except for these nozzles, the changes in OSPL for other nozzles are negligible considering the repeatability of OSPL, 0.6 dB. As a matter of fact, the maximum amplitudes of BSAN for modified nozzles decreased a little, but the increases of these bandwidths negated the reductions at the underexpanded

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conditions. The reduction of the peak sound pressure level of the BSAN is in the range of
2 to 4 dB at $M_j = 2.2$ and 3 to 4 dB at $M_j = 2.5$, respectively, as shown in Figs. 4.37 and 4.38. In both underexpanded cases, the spectra at a frequency greater than 10 kHz is almost identical regardless of the type of modifications. The reduction of OSPL of nozzle N6 at $M_j = 2.5$ is comparable with that for nozzle N3 or N5, while it is negligible at $M_j = 2.2$. Thus, group A nozzles, nozzles N3 and N5, seem to be better in noise reduction in the underexpanded cases.

**Overexpanded cases**

As could be seen in Figs. 4.34 and 4.35, significant OSPL reduction is expected not only due to reduced screech tone amplitude but also due to reduced BSAN level. By the trailing edge modification, the maximum sound pressure level and peak frequency of BSAN varied significantly at $M_j = 1.5$. The peak sound pressure level reduction is as much as 6 to 8 dB for nozzles N2-6, while it is only 2 dB for nozzle N1 as shown in Fig. 4.34. The reduction of BSAN for nozzle N6 is so significant that it is hard to recognize the presence of BSAN. For all modifications, the spectral content for mixing noise, at the frequency range of 1-3 kHz, does not appear to change at all. In fact, the peak sound pressure level of BSAN for nozzles N2-6 is reduced to that for the mixing noise, thus both BSAN and mixing noise are comparable noise sources at this Mach number of $M_j = 1.5$.

The shock noise is much reduced when the jet is slightly overexpanded $M_j = 1.75$ as shown in Fig. 4.35. The spectra for nozzles N1 and N2 are roughly identical to that for the
baseline nozzle, and in turn no significant noise reduction is expected for these nozzles. On the other hand, the reduction of screech tone and BSAN level for nozzles N3-6 are significant. For group A nozzles, nozzles N3 and N5, the spectrum is free from any screech tones. Surprisingly, no shock noise, BSAN and screech tone, is observed for nozzle N5. This may result from the earlier destruction of shock cell structures, which is an essential part of shock noise. The earlier destruction can be inferred from the surface flow images as shown in Fig. 4.4 and the measured pressure distribution on the modification. For group B nozzles, nozzles N4 and N6, there are still screech tones even though their levels are much reduced. The variation of the frequency of shock noise will be discussed in the near field noise section.

The decrease in the maximum amplitudes of Broadband Shock Associated Noise (BSAN) resulted in significant OSPL reductions in the overexpanded conditions as shown in Fig. 4.39. As expected from the spectra, the OSPL reduction for nozzles N1 and N2 is negligible. In general, the OSPL reduction for group A nozzles is greater than that for group B nozzles. The OSPL reduction for nozzle N5 is best and it is as much as 3 and 4 dB at $M_j=1.5$ and $M_j=1.75$, respectively. Considering that 3 dB reduction means the decrease of total acoustic energy to half of the original value, the reduction by nozzle N5 is significant. Although the reduction level for group B nozzles is slightly less than that for group A nozzles, the noise reduction performance for both group nozzles is comparable, especially at $M_j=1.5$. 

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4.4.2 Near Field Noise Results

Nearfield noise was measured at 3.5D_{eq} off the jet center line, as shown in Fig. 3.7 (b), to investigate screech modes, which are closely related to flow field modes as mentioned by Raman and Rice [1994]. In this section, the mode variations with the trailing edge modifications at M_{j} = 1.75 and 2.2 will be discussed. Since jets were screeching only at these Mach numbers as shown in Figs. 4.34-4.38, the near field noise measurements were performed at these conditions only. The connection between flow field and noise will be presented in the following section. Near field noise was performed only for the baseline nozzles, thick and thin lip nozzles, and nozzles N1-4 because the major interest of the study has been focused on mixing.

Spectra and ternary phase angles for each nozzle are shown in Figs. 4.42-4.45. As in the spectrum for far field noise, the spectrum is the average of 100 spectra of 4096 samples. Thus, the frequency resolution is 24.4 Hz because the sampling rate was 100 kHz. Figures 4.42 and 4.43 are for the minor axis plane, while Figs. 4.44 and 4.45 are for the major axis plane. For a clear representation of modes, the phase was displayed in a ternary form: The lower, higher, and middle values represent symmetric, antisymmetric, and oblique modes, respectively. Screech modes are considered as symmetric or antisymmetric when phase angles are within 0±40° or 180±40°, respectively, while modes are oblique if the phase angles are outside of these regions.
4.4.2.1 On the minor axis

On the minor axis the fundamental screech modes for all nozzles with modified trailing edges are oblique at $M_j = 1.75$, while at $M_j = 2.2$ they are still antisymmetric modes regardless of the modifications as respectively shown in Figs. 4.42 and 4.43. This is puzzling when the cross sectional images shown earlier are recalled. For both Mach numbers, the degrees of cross sectional deformations was almost the same. Thus, the mode change by trailing edge modifications was also expected at $M_j = 2.2$. At this point, it is hard to figure out what caused the different mode variations at different Mach numbers. Martens et al.'s [1996b] visualizations at the perfectly- and under-expanded cases do not show any significant differences between the thick and thin lip baseline nozzles, which is consistent with almost the same noise spectra for both nozzles as shown in Fig. 4.43. However, at $M_j = 1.75$ the screech frequencies are quite different between the two nozzles. This is probably associated with different geometric conditions at the jet exit and the receptivity of the shear layer to these conditions. The fundamental screech frequency shifts ranged from -293 to 928 Hz at $M_j = 1.75$ as shown in Table 4.4. At $M_j = 2.2$, the screech frequencies only for nozzle N2 and N3 are changed. The rearranged shock cell structures, which are due to the cut-outs and can be inferred from the surface flow images and the wall pressure distributions on a modification shown before, are probably responsible for these frequency shifts.
Table 4.4 Screech frequencies, phase angles, and coherence in the minor axis plane.

The second and third harmonics are clearly seen at $M_j=1.75$ as shown in Fig. 4.42. The screech modes are alternating between symmetric and antisymmetric modes for the baseline nozzles: The first (fundamental) and third harmonics are antisymmetric and the second and fourth ones are symmetric as in Raman and Rice's [1994] experiments. When
the jet was underexpanded, the screech modes were not changed with the trailing edge modifications as depicted in Fig. 4.43. The second and higher order harmonics were not observed at $M_j = 2.2$.

Two competing screech tones were detected for the baseline nozzles at $M_j = 2.2$ on the minor axis. Since the parameters, which determine the screech frequency, are the shock cell length and the convective velocity of large scale structures, a jet can have two competing modes if there are two slightly different shock cell lengths or convection velocities. This conjecture also needs more investigation. To find out whether the competing screech tones are steady and/or coherent within the time resolution, an amplitude versus time-frequency plot is shown in Fig. 4.46 for the thin-lip baseline nozzle at $M_j = 2.2$. In the figure, eight sample blocks of 1024 samples were averaged. Since the sampling rate was 100 kHz, the time and frequency resolutions are 0.08 sec and 97.7 Hz, respectively. Both competing tones are quite steady, but it is hard to tell whether they are concurrent or mutually exclusive due to poor time resolution of the analysis.

4.4.2.2 On the major axis

As shown in Figs. 4.44 and 4.45, the amplitude of screech-like tones at $M_j = 2.2$ is relatively low, and so it might be controversial to define them as screech tones. Thus, it needs to be discussed whether the screech-like spikes are screech tones or components of broadband shock associated noise. The bandwidth of the screech-like spikes is smaller than the one-third bandwidth around the peak frequency of them. According to Tam's [1988]
theory, the center frequency of broadband shock associated noise is much higher than the screech frequency. In addition, the coherence values are greater than that for the peak of broadband shock associated noise. Judging from these facts, it is quite reasonable to regard the screech-like spikes as screech tones. The easiest way to determine whether a spike is a screech tone is to acquire spectra at two or more different angles and to compare the spike frequencies. If the frequency changes with the measuring angles, the spike is not a screech tone since the screech frequency is independent of measuring direction, while the frequency of BSAN experiences Doppler shifts.

The screech modes, on the major axis, seem to experience a mode switching from the symmetric to antisymmetric mode with the jet Mach number as could be seen in Figs. 4.44 and 4.45. The fundamental screech modes for all nozzles except the thick-lip baseline nozzle and nozzle N2 appear to be symmetric at the lower Mach number ($M_j = 1.75$), while at the higher Mach number ($M_j = 2.2$) those for all nozzles appear to be spanwise antisymmetric. This suggests that a jet can have spanwise antisymmetric motions at a higher jet Mach number. Raman and Rice [1994] showed that a rectangular jet, similar to the baseline nozzles, had spanwise antisymmetric modes at a certain range of Mach numbers. Since only two Mach numbers were used in these experiments, it is not known whether the spanwise antisymmetric modes are also present in the moderately underexpanded case, $M_j = 2.5$. When jets are slightly underexpanded, the fundamental screech modes do not seem to be affected by the modifications on the major axis as on the minor axis. The oblique modes at $M_j = 1.75$ are observed only for nozzle N2 and the thick-lip baseline nozzle. Since nozzle N2 is not symmetric along the major axis as shown in Fig. 3.2, the oblique mode for this nozzle was
expected, while the reason for oblique behavior of the thick-lip baseline nozzle is not known.

The screech frequency, phase angle, and coherence are summarized in Table 4.5. One interesting thing is that the coherence of the second harmonic tone is higher than that of the fundamental tone. As depicted in Fig. 4.47, the amplitude of the fundamental tone is modulating more than that of the second harmonic tone. This probably results in lower coherence of the fundamental screech tone.

<table>
<thead>
<tr>
<th>Nozzle</th>
<th>Screech frequency, Hz</th>
<th>Phase, degree</th>
<th>Coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>first</td>
<td>second</td>
<td>first</td>
</tr>
<tr>
<td>Baseline, thick lip</td>
<td>5470</td>
<td>-20</td>
<td>0.85</td>
</tr>
<tr>
<td>Baseline, thin lip</td>
<td>2637</td>
<td>5298</td>
<td>-17.4</td>
</tr>
<tr>
<td>Nozzle N1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Nozzle N2</td>
<td>2783</td>
<td>-131.2</td>
<td>0.95</td>
</tr>
<tr>
<td>Nozzle N3</td>
<td>3002</td>
<td>11</td>
<td>0.74</td>
</tr>
<tr>
<td>Nozzle N4</td>
<td>3316</td>
<td>-0.8</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 4.5 Screech frequencies, phase angles, and coherence in the major axis plane at $M_j=1.75$.

4.4.2.3 Mixing noise

The humps in the low frequency regions are probably from the turbulent mixing noise. The center frequency is 1000 Hz at $M_j=1.75$ as shown in Fig. 4.44. The amplitudes and frequencies remain nearly unchanged regardless of the trailing edge modifications at a given Mach number, while the screech frequencies and the center frequencies of broadband
shock associated noise are varying with the modifications. This suggests that the humps in the lower frequency regions are not related to shock cell structures but related to turbulent mixing.

4.4.2.4 Strouhal number variations for the fundamental screech tone

The measured fundamental screech frequencies agreed well with the Tam's [1988] theoretical values (equation 2.4) as shown in Fig. 4.48, where the solid and dashed lines represent the Strouhal number variations of the fundamental screech tone and the broadband shock associated noise for the baseline nozzles, respectively. In calculating Strouhal number $S_{th}$, the fundamental screech frequency ($f_{sc}$), the fully expanded jet exit velocity ($U_j$) and the nozzle exit height ($h$) were used: $S_{th} = f_{sc}h/U_j$. When a jet is screeching, the Strouhal number of the fundamental screech tone corresponds to the jet preferred Strouhal number forming a standing wave sustained by the downstream traveling hydrodynamic wave and upstream traveling acoustic wave. If these two frequencies do not match, the amplitude of screech will be decreased or eliminated due to this frequency mismatch.

The Strouhal numbers for nozzle 3 and 4 are well off from the Tam's theoretical values [1988] suggesting the shock cell lengths are changed significantly for these nozzles. Although shock cell structures were not measured, significant deformation of shock cell structures due to trailing edge modifications can be inferred from surface flow visualization images (Figs. 4.1-4.5) and the measured pressure field on the modification (Figs. 4.8-4.13). The shifts of Strouhal number from the theoretical values are partially attributed to the
rearrangement of shock cell structures and/or to the changes of the convection velocity of large scale structures due to modifications. Nozzle N1, with an excellent agreement between theoretical and experimental results for the fundamental screech, shows the least changes in both mixing and noise.

4.4.2.5 Steadiness of screech tones

To investigate whether screech tones at $M_j=1.75$ are steady, time-frequency plots are shown in Figs. 4.49-4.53 respectively for the thin lip baseline nozzle and nozzles N1-4. The time and frequency resolutions are respectively 0.04 sec and 97.7 Hz as in Fig. 4.47. The fundamental screech tone for the thin lip baseline nozzle and nozzles N1-2 is very steady, while the second and third harmonics are relatively unsteady. For nozzles N3 and N4, the fundamental screech tone is not so steady as that for the baseline nozzle.

4.4.3 The Connection between Screech and Flow Field Modes

The fundamental screech mode has a corresponding flow instability mode since a screech tone is a stationary wave generated by a downstream propagating flow instability wave and an upstream propagating acoustic wave. Raman and Rice [1994] showed that these two modes, screech mode and flow instability mode, are the same by comparing the mode from microphones and that from hot film probes. The effects of antisymmetric motions for the thin-lip baseline nozzle is shown in Fig. 4.54. The instantaneous images are not
sequentially taken, thus, not correlated in time. They are typical images showing "left-thick", "right-thick", and "both-thin" mixing layers within the 50 instantaneous images of one set. There were no "both-thick" mixing layers. The number of images for each case is almost the same. These suggest that the thick mixing layers are mutually exclusive on the minor axis plane, and that the jet is in antisymmetric motions as measured with two microphones. The antisymmetric motion is usually called flapping motion.

4.4.4 Concluding remarks for the experiments with the larger nozzle

For the half nozzles with single sided trailing edge modifications, flow visualizations at two downstream locations were performed to investigate mixing enhancement through trailing edge modifications. In addition, near- and far-field noise measurements were carried out to compare noise reduction performance of the nozzles with single sided trailing edge modifications. A detailed discussion on the source of streamwise vorticity has been provided also. The spanwise pressure gradient, which is believed to be the major streamwise vorticity source, was measured by an array of pressure taps on the modification.

As far as mixing is concerned, it was found that group A nozzles (nozzles N3, N5, N9, and N10) outperform at the near field downstream location while group B nozzles (nozzles N4, N6, N7, and N8) outperform at the further downstream location of $x/D_{eq}=8$ when the jet is underexpanded. This mixing performance switch with downstream location is believed to be due to different vortex dynamics of kidney type (for group A nozzles) and mushroom type (for group B nozzles) pair vortices. In the overexpanded cases, Group B
nozzles performed better in mixing at locations. Nozzle N5 and N6 were found best in overall mixing performance respectively in group A and B nozzles.

The OSPL reduction was greater for group A nozzles than that for group B nozzles. As in the mixing performance, nozzle N5 and N6 performed best in OSPL reduction in each corresponding nozzle group. In general, the OSPL reduction by the trailing edge modification was greater in the overexpanded cases, where the baseline nozzle produced strong screech tones and BSAN. Considering the severe cross sectional distortion in the underexpanded cases, the reduction of OSPL is surprisingly small. Overall, nozzle N5 was found to be the best in OSPL reduction. Most likely, the stronger streamwise vortices generated by the spanwise wall pressure gradient on the modification are responsible for the better mixing of these nozzles.

In the ideally expanded case, no noticeable variations both in mixing and OSPL were observed for nozzles with single sided trailing edge modifications. To check if the ineffectiveness of the modifications with 45° cut-outs is due to the mismatch of the cut-out angle with the angle of large scale structures, two additional cut-out angles 35 and 55° were also used to make nozzles N7-10. As was shown, these additional nozzles did not show any notable mixing enhancement. Thus, the excitation or amplification of large scale structures by cut-outs/trailing edge modifications seems to be unsuccessful.

The flow visualization data from Martens et al. [1996b] and noise data for the present study show that both the mixing enhancement and noise reduction of nozzles N1 and N2 is negligible. For nozzles N7-N10, their mixing performance is in the same range of that for nozzle N3 or N4. Thus, modifications M1, M2, and M7-10 will not be used further in the
experiments for the smaller nozzles with double sided trailing edge modifications. Although nozzles N5 and N6 were best in each nozzle group, modifications M3 and M4 will be used because their performance is comparable with that for modifications M5 and M6.

4.5 Flow Visualizations for Smaller Nozzles with Double-sided Trailing Edge Modifications

The baseline nozzle used so far is a half nozzle: One side is flat (splitter plate) and the other is contoured to get $M_d=2.0$. As mentioned earlier, only the splitter plate was modified to investigate mixing enhancements and noise reduction caused by modifications on the splitter plate. Thus, the flow field deformations were restricted to only one side. The half-nozzle with one splitter plate was used to isolate jet deformations to only one side, so that it would be much easier to investigate the effects of the modifications on mixing and noise by comparing the mixing regions of the modified side with those of the unmodified side. In addition, the nozzle would provide less possibility of mixing layer interactions at least in the jet near field, because only the mixing layer on the splitter plate side experiences severe distortions. This also makes the investigations easier.

From the results for the half-nozzle with one splitter plate, the general idea of how the modifications enhanced mixing and reduced noise was acquired. Based on the findings from the experiments for the larger nozzle with single sided trailing edge modifications, double trailing edge modifications were designed. The modification of trailing edges would be different for different applications even though the nozzle which has better mixing
characteristics appeared to reduce noise as well. Unfortunately, the noise issue has been dropped out since the single training edge modification experiments were done. Thus, how to maximize mixing with minimal thrust loss is the primary object of the experiment for the smaller nozzle.

4.5.1 Selection of trailing edge modifications for maximum mixing

In the work with single trailing edge modified nozzles, noise measurements and flow visualizations at \( x/D_{eq} = 2 \) and 8 showed that 1) the modifications did not significantly enhance mixing in the ideally expanded case, 2) the modifications, however, did significantly enhance mixing and reduce noise when the jet operated in non-ideally expanded flow regimes, and 3) the mixing enhancement and noise reduction are closely related to the streamwise vorticity generated by the modifications. In the experiments for the smaller nozzles, modifications on both the trailing edges of the nozzle were used to investigate the interaction between the structures generated by the two trailing edges, and the effects of the modifications on mixing enhancement. Also, the visualizations were carried out at two more \( x/D_{eq} \) locations to improve the evaluation of the effects of trailing edge modifications. The \( x/D_{eq} \) locations for the smaller nozzle with double sided trailing edge modifications were 1, 2, 4, and 8. In addition, the thrust measuring system, discussed earlier, was designed and used to investigate the effects of trailing edge modifications on the thrust force generated by the jet. The experiments performed with the smaller nozzles are listed in Table 3.2.

Based on the results obtained with the single-trailing edge modified nozzle
experiments discussed in section 4.3 and the results of Martens et al. [1996b], four of the modifications that produced better mixing and noise performance were used to form six nozzles with the trailing edges modified on both sides: four of them were symmetric and two others asymmetric as shown in Fig. 3.3. The double-sided modifications were combinations of modifications type M3, M4, M5, and M6 (Fig. 3.3 (b)). To obtain reference data, a baseline nozzle was also used.

In the visualizations of the larger nozzles with single sided trailing edge modifications, \( M_j = 1.75 \) and 2.5 appeared to be a good representative for fully expanded jet Mach numbers respectively for overexpansion and underexpansion. Thus, the flow regimes explored for the smaller nozzles were limited to \( M_j = 1.75, 2.0, \) and 2.5 to more focus on the jet and vortex evolution with downstream locations.

### 4.5.2 Cross sectional deformation

Figures 4.55-4.62 show instantaneous and average images of the cross section of the jet at \( x/D_{eq} = 1, 2, 4, \) and 8 for the baseline nozzle and six modified nozzles at the nominal design Mach of 2.0, at the overexpanded case of \( M_j = 1.75, \) and at the underexpanded case of \( M_j = 2.5.\) In the figures, only mixing regions are visualized by water particles, which are formed when the cold and dry jet air mixes with the humid and warm ambient air entrained into the jet plume. It should be mentioned that the degree of expansion for the underexpanded case with the jet exit pressure to ambient pressure ratio \( (P_e/P_a) \) of approximately 2.2 is much stronger than the degree of compression for the overexpanded
case with the jet exit pressure to ambient pressure ratio \((P_e/P_a)\) of approximately 0.7.

The major concerns of the section are vortex-vortex interaction, the interaction between mixing layers, and vortex and cross sectional development with downstream locations, which were not discussed in the similar section for the larger nozzles with single sided trailing edge modifications. In that section, these kinds of discussions were not made simply because the visualizations were limited to only two downstream locations.

### 4.5.2.1 Ideally expanded case, \(M_j=2.0\)

As was observed in the earlier work for the ideally expanded flow regime [Martens et al. 1996b, Samimy et al. 1997a, b] and as presented in section 4.3, the cut-outs or trailing edge modifications do not seem to affect the jet cross section in any significant fashion. This is partially due to the lack of the main streamwise vorticity source of surface pressure gradients in the zone of downstream influence, indicated schematically in Figs. 4.14 and 4.15. The lack of cross stream flow is also partially responsible for this.

### 4.5.2.2 Underexpanded case, \(M_j=2.5\)

For the underexpanded case, the streamwise structures due to the M5 and M6 type cut-outs are very clearly shown at \(x/D_{eq}=1\) and 2 as can be seen in Figs. 4.55-4.58. In Figs. 4.55 and 4.56, hurricane-like vortical structures are observed in the mixing layer around modification M5 at \(M_j=2.5\). These structures signify the existence of streamwise vortices.
which are generated by the spanwise pressure gradient on the modification as shown in Figs. 4.8-4.9 and 4.14. The streamwise structures due to the M3 and M4 type cut-outs, however, can only be inferred from the jet cross section deformation at all $x/D_{eq}$ locations. These results are again consistent with Samimy et al.’s results [1997b]. They conjectured that the streamwise vorticity shed due to the surface pressure gradients in the zone of downstream influence by the cut-outs are aligned in the streamwise direction for M5 and M6 type cut-outs, while for M3 and M4 type cut-outs, the shed vorticity, originally aligned with the cut-outs, must go through a reorientation process by the term $\partial V_y/\partial b$ in equation 4.6 ($\partial U/\partial z$ with the coordinate system in Fig. 3.5) to form streamwise vorticity. Thus, the streamwise vortices seem to be stronger and better defined for the former cases.

The vortex-vortex interaction of a pair of vortices can be observed in the mixing layer around modifications M3 and M5, which generate a kidney type pair of vortices. When Figs. 4.55 and 4.57 or Figs. 4.56 and 4.58 are compared, the interaction between the vortex mates begins between $x/D_{eq}=1$ and 2 for modification M5. For modification M3, the interaction takes place farther downstream location in between $x/D_{eq}=4$ and 8 as observed in Figs. 4.58 and 4.60. The downstream location of the interaction may depend on the distance between two vortices and the strength of the vortices. Modification M5 has a stronger streamwise pair of vortices and a shorter distance between them than the pair of vortices generated by modification M3. As a result, the interaction between mate vortices for modification M5 took place at a location further upstream than for modification M3. As will be discussed in the overall mixing section, this unfavorable interaction results in the deterioration of the entrainment capability of the kidney type pair of vortices. Thus, it is desirable to increase
the distance between vortices in a pair so as to enhance the far field mixing capability.

On the other hand, the mutual interaction for the mushroom type pair of vortices, generated by modification M4 or M6, benefits mixing by causing the pair of vortices to move faster away from the jet center as shown in Fig. 4.27. As discussed in section 4.3, the mushroom type pair of vortices act as a fluid pump and in turn result in enhanced mixing. No unfavorable interactions between vortex mates for modification M4 or M6 were observed at any downstream locations as shown in Figs. 4.55-4.62.

As the jet cross section is expanding from the original cross section in the underexpanded case, the pair of streamwise structures imparted by each modified trailing edge seemed to develop almost independent of each other even beyond $x/D_{eq}$ of 4 in the experiments with smaller nozzles, although a slight interaction between left and right mixing layers was expected and observed in the experiment with the larger nozzles. For example, the left side of the jet cross sections for nozzles N65 and N66 at $x/D_{eq} = 4$ (Fig. 4.59 or 4.60), due to the trailing edge M6, are very similar, even though the other sides of the cut-outs have different trailing edges M5 and M6, respectively. The implication is that for this flow regime, at downstream locations up to $4D_{eq}$ where interaction begins, each cut-out acts almost independently, and the effect should be almost additive.

4.5.2.3 Overexpanded case, $M_j=1.75$

For the overexpanded case, even though the pressure ratio is small ($P_c/P_a \approx 0.7$), the compression in the jet cross section causes the mixing layers from two sides to interact right
away. For example, this interaction is obvious for N43, N44, N65, and N66, even at $x/D_{eq} = 1$ as observed in Figs. 4.55 and 4.56. The interaction between left and right mixing layers is much stronger for nozzles N44 and N66, which in turn causes the potential core to decay faster than for the other nozzles as observed in Fig. 4.60, where the potential core, the dark region inside the mixing region, is not seen for the two nozzles at $x/D_{eq} = 4$. The cross sectional deformation for symmetric nozzles, the baseline nozzle, nozzles N33, N44, N55, and N66, is and symmetric as observed in Figs. 4.55-4.62. When the modification was used only on one side for the larger nozzle, no visible interference was observed at $x/D_{eq} = 2$ as shown in Figs. 4.18-4.23.

The jet cross sectional deformation in the overexpanded case seems to be dictated by the cross stream velocity distribution at the jet exit, jet flapping motions, and relatively weak streamwise vortices. Most likely, the cross stream velocity distribution shown in Fig. 4.63 is responsible for the jet initial cross sectional deformation at $x/D_{eq} = 1$ and 2. At $x/D_{eq} = 2$ and 4, the flapping motion of the jet, presented in the following section, is partially responsible for the jet cross sectional deformation. On the other hand, streamwise structures may play a dominant role in the cross sectional deformation at farther downstream locations of $x/D_{eq} = 4$ and 8. The flapping motion of the jet may be responsible for the development of the jet mixing layer for flapping nozzles, all nozzles except nozzles N33 and N43, at $x/D_{eq} = 2$ and 4.

Nozzle N43 shows the most significant cross sectional deformation at up to $x/D_{eq} = 4$. Most probably, the asymmetric cross stream flow at the jet exit is responsible for the relatively severe deformation at upstream locations of $x/D_{eq} = 1$ and 2. A similar deformation
was expected for nozzle N65 since this nozzle is also asymmetric about the major axis. Contrary to the expectation, the mixing layer of nozzle N65 is quite symmetric about the major axis. The cross stream flow in nozzle N43 is developed almost independently up to the jet exit because the cut-outs M3 and M4 are well secluded from each other in the spanwise direction. On the other hand, the cross flow from one modification in nozzle N65 may be affected by that from the other modification well inside the nozzle, and in turn the cross streamwise momentum by each modification is balanced out at upstream locations, which would result in a symmetric mixing layer as shown in the images. This conjecture is based on the surface visualization and wall pressure data for the larger nozzle as discussed in section 4.2.

The sense of streamwise vortices by the spanwise pressure gradient in the overexpanded case is not as straightforward as in the underexpanded case discussed above. Since the cross stream flow is directed up to the pressure hill, shown in Figs. 4.8-4.9, and 4.15, the flow undergoes separation, which was marked on surface flow images shown in Figs. 4.2-4.5. When the separation takes place, the sense of the separation vortex is the opposite direction of the one shown in Fig. 4.15 as mentioned earlier. In case of two dimensional flow, the circulation of the separation bubble is not convected by the mean flow [Smith 1982]. More experimental work is required to define the role of streamwise vortices in the overexpanded case due to complex three dimensional flow fields and shock waves, which involve separation as in the experiments for the larger nozzle. However, the cross sectional deformation at the upstream locations of \( x/D_{eq}=1 \) and 2 is quite reasonably explained by using the cross stream velocity distribution at the jet exit, which is shown in
Surprisingly, no jet vectoring was observed for the smaller nozzles with double sided trailing edge modifications, while all larger nozzles except the larger baseline nozzle showed jet vectoring at this Mach number. As shown in Fig. 4.63, the y-directional momentum by each side of cross flow may offset each other for the smaller nozzles, while y-directional momentum from the modification exceeds that from unmodified side (left) for the larger nozzles.

The effects of modifications on jet flapping motion

A statistical based analysis was adopted to investigate the flapping motion of the jet at the overexpanded case $M_j=1.75$. When a jet is flapping, the relatively thick mixing layers are mutually exclusive as shown in Fig. 4.54. According to Messersmith and Dutton [1996] and Gruber and Nejad [1997], the size of flow structure and orientation can be drawn from the two dimensional spatial correlation field. Thus, the effects of trailing edge modifications on the flapping motion can be evaluated from the spatial correlation.

The formulae for the two dimensional spatial correlation are written as:

$$I_n'(j, k) = I_n(j, k) - \bar{I}(j, k), \quad (4.21)$$

$$I_{mn}(j, k) = \left[ \frac{1}{N} \sum_{n=1}^{N} I'(j, k)^2 \right]^{1/2}, \quad (4.22)$$

and

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\[
R(j, k) = \frac{1}{N} \sum_{n=1}^{N} \frac{I_n'(j, k) I_n'(j_{ref}, k_{ref})}{I_{rms}(j, k) I_{rms}(j_{ref}, k_{ref})},
\]

where \(I(j,k)\) and \(I_n'(j,k)\) are respectively the instantaneous intensity and intensity fluctuation at pixel location \((j,k)\) in \(n\)-th instantaneous image of total \(N\) images, \(I(j,k)\), with - over, is the average intensity at pixel \((j,k)\), \(I_{rms}(j,k)\) is the rms intensity at \((j,k)\), and \(R(j,k)\) is the correlation coefficient at pixel \((j,k)\). The numerator in equation 4.23 represents the ensemble average of \(N\) instantaneous covariance at a pixel \((j,k)\). Equation 4.23 represents how much the intensity fluctuation at pixel \((j,k)\) is correlated with that at the reference pixel \((j_{ref},k_{ref})\) and so its value is 1 when the pixel is equal to the reference pixel \((j_{ref},k_{ref})\). The correlation coefficient can have from -1 to 1 [Bendat and Piersol 1986], with a value of 1 for perfect correlation and a value of 0 for no correlation.

In the present study, the reference position was selected at a position where the rms intensity was the local maximum of the mixing region for each data set. The calculated two dimensional correlation at \(M_j=1.75\) at four downstream locations are shown in Figs. 4.64-4.70 respectively for the baseline nozzle and nozzles N33, N43, N44, N55, N65, and N66. No noticeable spatial correlation was observed in other flow regimes. In the figures, thick and thin lines are respectively for positive and negative correlation coefficients, and the rectangular box represent the outer boundary of an image.

A mutually exclusive behavior of left and right mixing layers was observed at \(x/D_{eq}=2\) and 4 for the baseline nozzle as shown in Fig. 4.64. Most likely, the mutually exclusive behavior of the mixing layers are related with the jet flapping motion. Only for
nozzles N33 and N43, no notable flapping motion was observed as shown in Figs. 4.65 and 4.66. The extent or degree of flapping of the jet appeared to be much reduced by double sided trailing edge modifications for other nozzles in general. In addition, the modification seemed to cause the flapping to take place earlier and to be decayed faster. Although noise was not measured for the smaller nozzle, nozzles N33 and N43 are expected to result in significant OSPL reduction since the jet is not significantly flapping, which is closely related with the jet screech.

4.5.3 Overall mixing

Mixing areas were calculated from 50 instantaneous images in the same way as described by Samimy et al.[1997b], and in section 4.3.2. In an instantaneous image, a pixel with a greater intensity than a threshold value counted as a part of the mixing region. After calculating the mixing area for each instantaneous image, ensemble averages of the 50 images were obtained. In the previous experiments for the larger nozzle, it was determined that to obtain an accurate measure of the mixing region, a reference mixing region is required to take into account the change in the room temperature and humidity during experimentation, as these parameters affect the condensation process in the jet mixing region. The reference mixing area for each nozzle was acquired at $M_j = 2.0$ using the baseline nozzle after flow visualizations for the nozzle operated at the three flow regimes were carried out. All the mixing areas were normalized by the associated reference value. Thus, mixing areas are expected to be repeatable for different weather conditions by using this normalization.
technique.

Figure 4.71 shows the mixing areas normalized by the appropriate reference mixing area for the baseline and all the modified nozzles, at three different flow regimes, and at all four \(x/D_{eq}\) locations. The results at \(x/D_{eq} = 1\) are less accurate than the other locations as the reflection from the nozzle, due to the close proximity of the laser sheet to the nozzle at this location, made the background subtraction and the calculation of threshold intensity of the images much more difficult for this location [Samimy et al. 1997b]. As one could also infer directly from the images shown in Figs. 4.55-4.62, the mixing enhancement due to trailing edges is minimum at \(M_j = 2.0\), relatively small at \(M_j = 1.75\) as the jet is flapping in this flow regime for the baseline case and the mixing is naturally substantially increased, and quite substantial at \(M_j = 2.5\).

**Underexpanded case, \(M_j = 2.5\)**

Figure 4.72 shows the evolution of mixing area with \(x/D_{eq}\) for both over- and underexpanded flow regimes for all the nozzles. One could get both mixing level and mixing rate, or entrainment rate, from this figure. Note that the results are normalized, at each \(x/D_{eq}\), with the mixing area for the baseline nozzle at \(M_j = 2.0\) at that location. Thus, the mixing area for the underexpanded jet results shown in Fig. 4.72 (b), is substantially larger than that of the reference case at all locations. The mixing level (or area) for nozzles N44 and N66 are much higher than the baseline case in every streamwise location, and also show higher growth rate. On each modified side for these two nozzles, there is a pair of mushroom type
streamwise vortices that eject the jet fluid into the ambient by the fluid pump action of the pair vortices. This process seems to continue all the way to $x/D_{eq} = 8$, and the vortices seem to be still quite strong at this location. The mixing level for the modifications in nozzle N66 is better than that of N44 because the streamwise vortices for N66 are stronger due to their initial alignment with the streamwise direction or larger orientation factor of cut-out edge, as discussed earlier.

The initial mixing levels for nozzles N55 and N33 are much higher than the baseline case, especially for N55, but the mixing rate is similar to the baseline case in the downstream locations. Nozzle N33 has the largest growth rate until $x/D_{eq} = 4$. Each one of the modifications for these nozzles generate a pair of streamwise vortices that entrain ambient air into the mixing region. Similar to the difference between the nozzles N66 and N44, the modifications for the nozzle N55 generate stronger vortices than nozzle N33. However, as shown in Figs. 4.55-4.62, the pair of streamwise vortices generated by each cut-out in N55 are much closer to each other than those in N33, and the interaction of the pair of vortices begins much earlier than for nozzle N33 as discussed in section 4.5.2. This unfavorable interaction between kidney type vortices for nozzles N33 and N55 deteriorates mixing performance, and results in the decrease of mixing rate as shown in Fig. 4.72. While the pair of vortices in N33 does not interact until $x/D_{eq} = 4$, and thus keeps entraining a large amount of ambient air into the jet, the pair in N55 starts to interact beyond $x/D_{eq} = 1$, and thus limits their entrainment capabilities. In the figure, the decrease of mixing level for nozzle N55 began at an earlier downstream location than for nozzle N33 due to the earlier streamwise vortex interaction. As mentioned in section 4.5.2, it is desirable to make vortex pair distance
larger for the kidney type pair of vortices to enhance farther downstream mixing performance.

Any nozzles with modification M4 and/or M6 (Nozzle N43, N44, N65, and N66), which generates mushroom type pair vortices, showed the increase of relative mixing level and rate as observed in Fig. 4.72 (b). As mentioned above in section 4.5.2, the mushroom type pair vortices act as a fluid pump and do not result in unfavorable interaction as for nozzles with only modifications M3 or M5. According to [Zaman, 1996], the entrainment is proportional to $\Gamma/r_o$ for the mushroom type pair vortices, where $\Gamma$ is the circulation and $r_o$ is the distance between vortices. For the nozzle N66, the pair of vortices has stronger vortices than for nozzle N44 due to a greater vortex orientation factor, and roughly the same distance as can be inferred from the instantaneous images. Thus, the mixing level and rate of nozzle N66 are greater than those of nozzle N44 at a downstream location as observed in Fig. 4.72 (b). The nozzles N43 and N65, have characteristics in between N44 and N66, and N33 and N55, as expected.

**Overexpanded case, $M_j=1.75$**

The mixing enhancement due to trailing edge modification in the overexpanded flow regime is smaller in comparison with that in the underexpanded flow regime. In fact, by $x/D_{eq} = 8$, the baseline case has higher mixing level than some of the modified nozzles. There are three main reasons for this: 1) the jet cross section is contracting initially due to the cross stream flow as shown in Fig. 4.63, 2) the jet for the baseline case is flapping which
provides significant enhancement for the baseline case, and 3) no strong vortical activities seen to be present for this flow regime. In contrast to the underexpanded cases where the interference of mixing layers are not visually observed and so each mixing layer develops almost independently all the way down to $x/D_{eq}=8$, the mixing layers interact for the overexpanded case immediately due to the reduced cross section of the jet caused by compression waves.

When the mixing levels for the larger and smaller nozzles, respectively shown in Figs. 4.32-4.33 and Fig. 4.71, are compared, the use of double sided trailing edge modifications did not significantly increase the mixing level, but rather remained about the same. This may be due to inward directing cross flows caused by compression, which restrict outward development of the jet. Although, the mixing level for nozzles with modifications is higher than that for the baseline nozzle at $x/D_{eq}=1$, 2, and 4, only nozzle N43, N44, and N66 show better mixing than the baseline nozzle at $x/D_{eq}=8$ due to much increased mixing of the baseline nozzle caused by the flapping motion. As was observed in the experiments for the larger nozzles, the mixing performance of nozzles with modification M4 or M6 appeared to be better in general. It is nozzle N43 which had a severely distorted jet cross section and showed increased mixing rate over $x/D_{eq}=4$. Again, nozzle N43 has characteristics roughly between N33 and N44, while nozzle N65 has characteristics roughly between N55 and N66.
4.6 Thrust variations with the modification

Thrust generated by the jet is measured at four flow regimes, $M_j=1.75, 2.0, 2.2, \text{ and } 2.5$, by the thrust measuring system shown schematically in Fig. 3.1, and discussed earlier in the experimental facility and techniques section. To get the repeatability of the system, five separate measurements were carried out at the same Mach number over two days for the baseline nozzle. The repeatability ranges are $\pm 1.15, \pm 0.58, \pm 0.50, \text{ and } \pm 0.30\%$ of the measured average thrust at each Mach number for $M_j=1.75, 2.0, 2.2, \text{ and } 2.5$, respectively. The repeatability improves at higher thrust levels, as it seems there is a fixed inaccuracy associated with the measurements. In a given supersonic nozzle operated in supersonic flow regimes, theoretical thrust ($T_{th}$) for an ideal gas is a function of the stagnation ($p_o$) and ambient pressure ($p_a$) as shown in equation 4.21:

$$T_{th} = m V_e + A_e (p_e - p_a)$$

$$= \gamma M_d p_o A^* \sqrt{\left(\frac{2}{\gamma + 1}\right)^{(\gamma-1)/(\gamma+1)}} \left[1 + \frac{\gamma - 1}{2} M_d^2 \right] +$$

$$\frac{A^*}{M_d} \left[ \frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M_d^2 \right)^{(\gamma+1)/2(\gamma-1)} \right]^{-\gamma} \left( p_o \left(1 + \frac{\gamma - 1}{2} M_d^2 \right)^{\gamma-1} - p_a \right),$$

$$T_{th} = f(p_o, p_a, \gamma, A^*, M_d). \quad (4.21)$$

where $m$ is mass flow rate, $V_e$ is jet exit velocity, $\gamma$ is the specific heat ratio of the jet fluid,
$A_e$ is the nozzle exit area, $A^*$ is the nozzle throat area, and $M_d$ is the design Mach number of the nozzle. The stagnation pressure was controlled within ±0.5 psi because thrust is a strong function of the stagnation pressure, and the ambient pressure was recorded after each experiment to correct for its effect on the thrust. As shown in the equation, theoretical thrust for the ideal gas is independent of the stagnation temperature.

Although vortex generating tabs are an effective way of enhancing mixing not only for the underexpanded cases but also for the ideally expanded case, they result in thrust loss [Samimy et al. 1993, Zaman et al. 1994]. However, it was conjectured by Samimy et al. [1997 a, b] that the thrust loss associated with trailing edge modifications would be very small or non-existent, since the modified trailing edges did not protrude into the flow. The measured and theoretical thrust results are depicted in Fig. 4.73 for six nozzles at four flow regimes. The agreement between the experimental and theoretical thrust results is excellent. The inviscid theoretical calculation slightly overpredicts the thrust values due to neglecting the viscous effects. The variations of the measured thrust values for all modified nozzles tested, in a given operating condition, are within the repeatability range of the baseline nozzle, indicating no thrust loss or gain due to the trailing edge modifications. Thus, the technique based on trailing edge modifications is a promising technique to provide enhanced mixing and reduced noise without thrust penalty.
Fig. 4.1 Surface flow streak footprints on the inside wall of the baseline splitter plate. The flow direction is from bottom to top of the page.
Fig. 4.2 Surface flow streak footprints on the inside wall of modification M3. The flow direction is from bottom to top of the page.
Fig. 4.3 Surface flow streak footprints on the inside wall of modification M4. The flow direction is from bottom to top of the page.
Fig. 4.4 Surface flow streak footprint on the inside surface of modification M5. The flow direction is from bottom to top of the page.
Fig. 4.5 Surface flow streak footprint on the inside surface of modification M6. The flow direction is from bottom to top of the page.
Fig. 4.6 Stagnation pressure cycle for the optimum surface flow visualization.
Fig. 4.7 The zone of the downstream influence and resultant surface flow patterns on modification M6 for underexpanded (top) and overexpanded (bottom) case.
Fig. 4.8 Spanwise pressure distributions for M5 at five flow regimes.
Fig. 4.9 Spanwise pressure distributions for M6 at five flow regimes.
Fig. 4.10 Streamwise pressure distributions for M5 at five flow regimes.
Fig. 4.11 Streamwise pressure distributions for M6 at five flow regimes.
Fig. 4.12 Streamwise pressure distributions just upstream of the cut-out of M5 at five flow regimes.
Fig. 4.13 Streamwise pressure distributions just upstream of the cut-out of M6 at five flow regimes.
Fig. 4.14 Three dimensional pressure distributions and spanwise pressure gradients on the inside wall of a nozzle for the underexpanded case, and the induced streamwise vortices.
Fig. 4.15 Three dimensional pressure distributions, spanwise pressure gradients on the inside wall of a nozzle for the overexpanded case, and the induced streamwise vortices. The sense of vorticity for the separation bubble is opposite sign to that shown in the figure.
Baroclinic Component (BC):

\[
BC_s = \frac{1}{\rho^2} \left[ \frac{\partial p}{\partial b} \frac{\partial p}{\partial n} - \frac{\partial p}{\partial n} \frac{\partial p}{\partial b} \right] = 0
\]

\[
BC_b = \frac{1}{\rho^2} \left[ \frac{\partial p}{\partial n} \frac{\partial p}{\partial s} - \frac{\partial p}{\partial s} \frac{\partial p}{\partial n} \right] < 0
\]

\[
BC_n = \frac{1}{\rho^2} \left[ \frac{\partial p}{\partial s} \frac{\partial p}{\partial b} - \frac{\partial p}{\partial b} \frac{\partial p}{\partial s} \right] = 0
\]

Fig. 4.16 Vorticity transport in two dimensional compressible boundary layers.
Fig. 4.17 Vortex stretching and bulging through expansion. In contrast to the incompressible vortex stretching, the vortex element experiences not only stretching but also bulging.
Fig. 4.18 Instantaneous cross sectional images at $x/D_{eq}=2$. The physical image size is $4.6 D_{eq}$ wide and $2.8 D_{eq}$ high (230 mm x 140mm).
Fig. 4.19 Average cross sectional images at $x/D_{eq}=2$. 

(a) Baseline nozzle  (b) Nozzle N3  (c) Nozzle N5  (d) Nozzle N9  (e) Nozzle N10
Fig. 4.20 RMS cross sectional images at $x/D_{eq}=2$. 

(a) Baseline nozzle  (b) Nozzle N3  (c) Nozzle N5  (d) Nozzle N9  (e) Nozzle N10
Fig. 4.21 Instantaneous cross sectional images at $x/D_{eq} = 2$. The physical image size is 4.6 $D_{eq}$ wide and 2.8 $D_{eq}$ high (230 mm x 140mm).
Fig. 4.22 Average cross sectional images at $x/D_\text{eq}=2$. 
Fig. 4.23 RMS cross sectional images at $x/D_e = 2$. 

(a) Baseline nozzle  (b) Nozzle N4  (c) Nozzle N6  (d) Nozzle N7  (e) Nozzle N8
Fig. 4.24 Instantaneous cross sectional images at $x/D_{eq}=8$. The physical image size is 7.3 $D_{eq}$ wide and 4.5 $D_{eq}$ high (362 mm x 223 mm).
Fig. 4.25 Average cross sectional images at $x/D_{eq}=8$. 
Fig. 4.26 RMS cross sectional images at x/D_{eq}=8.
Fig. 4.27 Two different groups of modifications: one, group A (a), has a pair of vortices which entrains the ambient air and the other, group B (b), has a counter rotating pair vortices which ejects the jet air into the ambient air in the underexpanded case. The sense of vorticity for the overexpanded case is not clear.
Fig. 4.28 Shock and expansion waves generated from the beginning corner of the cut-out at $M_j = 1.75$ and 2.5, respectively.
Fig. 4.29 Schematic of deduced cross stream velocity distributions at the nozzle exit and the signs of pairs of streamwise vortices in the underexpanded case. The straight arrow indicates the direction of mutually induced velocity on the mating vortex.
Fig. 4.30 Streamwise pressure distributions on a side wall at $M_j=2.5$. 
Fig. 4.31 Schematic of cross stream velocity distributions and shock lines, which was deduced from surface flow and pressure data, at the nozzle exit; (a) at $M_j=1.5$, (b)-(f) at $M_j=1.75$. 
Fig. 4.32 Variations of mixing region with the fully expanded jet Mach number at x/D_q=2.
Fig. 4.33 Variations of mixing region with the fully expanded jet Mach number at $x/D_{eq}=8$. 
Fig. 4.34 Variations of farfield spectra with trailing edge modifications at $M_f=1.5$. 
Fig. 4.35 Variations of farfield spectra with trailing edge modifications at $M_j=1.75$. 
Fig. 4.36 Variations of farfield spectra with trailing edge modifications at $M_f=2.0$. 
Fig. 4.37 Variations of farfield spectra with trailing edge modifications at $M_f=2.2$. 
Fig. 4.38 Variations of farfield spectra with trailing edge modifications at $M_j=2.5$. 
Fig. 4.39 Variations of OSPL with trailing edge modifications at the farfield.
Fig. 4.40 Variations of farfield spectra with the fully expanded jet Mach number for the thin lip baseline nozzle.
Fig. 4.41 The increase of shock cell spacing with the jet Mach number.
Fig. 4.42 Near field spectra and ternary displays of phase for six nozzles at $M_f=1.75$ on the minor axis. The higher, lower and middle dashed line values respectively represent antisymmetric, symmetric, and oblique modes.
Fig. 4.43 Near field spectra and ternary displays of phase for six nozzles at $M_j=2.2$ on the minor axis.
Fig. 4.44 Near field spectra and ternary displays of phase for six nozzles at $M_j=1.75$ on the major axis.
Fig. 4.45 Near field spectra and ternary displays of phase for six nozzles at $M_j=2.2$ on the major axis.
Fig. 4.46 Time-frequency plot for the thin-lip baseline nozzle at $M_j=2.2$ on the minor axis. Time and frequency resolutions are respectively 0.08 sec and 48.8 Hz.
Fig. 4.47 Time-frequency plot for the thin-lip baseline nozzle at $M_f=1.75$ on the major axis. Time resolution and frequency bandwidth are respectively 0.041 sec and 97.7 Hz.
Fig. 4.48 Variations of Strouhal number for the fundamental screech tone with the fully expanded jet Mach number.
Fig. 4.49 Sound pressure level versus time-frequency plot for the thin lip baseline nozzle $M_f=1.75$ on the minor axis.
Fig. 4.50 Sound pressure level versus time-frequency plot for nozzle N1 $M=1.75$ on the minor axis.
Fig. 4.51 Sound pressure level versus time-frequency plot for nozzle N2 $M_i=1.75$ on the minor axis.
Fig. 4.52 Sound pressure level versus time-frequency plot for nozzle N3 $M_\infty=1.75$ on the minor axis.
Fig. 4.53 Sound pressure level versus time-frequency plot for nozzle N4 $M_r=1.75$ on the minor axis.
Fig. 4.54 Instantaneous images, showing flapping motion of the jet plume, for thin baseline nozzle at $M_j=1.75$. Images are not connected in time.
Fig. 4.55 Instantaneous cross sectional images at $x/D_{eq}=1$. The physical image size is $6.8D_{eq}$ wide and $4.1D_{eq}$ high (126.7 mm x 75.7 mm).
Fig. 4.56 Average cross sectional images at $x/D_{eq} = 1$. 
Fig. 4.57 Instantaneous cross sectional images at $x/D_{eq}=2$. The physical image size is 6.5 $D_{eq}$ wide and 3.9 $D_{eq}$ high (121.5 mm x 71.8 mm).
Fig. 4.58 Average cross sectional images at $x/D_{eq} = 2$. 

$M_j = 1.75$  $M_j = 2.0$  $M_j = 2.50$
Fig. 4.59 Instantaneous cross sectional images at $x/D_{eq}=4$. The physical image size is $6.4D_{eq}$ wide and $3.8D_{eq}$ high (120.0 mm x 70.8 mm).
Fig. 4.60  Average cross sectional images at $x/D_{eq}=4$. 
Fig. 4.61 Instantaneous cross sectional images at $x/D_{eq}=8$. The physical image size is $9.5D_{eq}$ wide and $5.6D_{eq}$ high (176.8 mm x 103.8 mm).
Fig. 4.62 Average cross sectional images at $x/D_e=8$. 
Fig. 4.63 Inferred cross stream velocity distributions at the nozzle exit and the signs of pairs of streamwise vortices and direction of their induced velocity in the underexpanded case. The lines inside the nozzle represent shock and expansion waves: Thick lines for shock waves and thin lines for expansion waves.
Fig. 4.64 Spatial correlation with downstream location for the smaller baseline nozzle NBB (Thick and thin contours are respectively for positive and negative values).
Fig. 4.65 Spatial correlation with downstream location for nozzle N33. (Thick and thin contours are respectively for positive and negative values)
Fig. 4.66 Spatial correlation with downstream location for nozzle N43. (Thick and thin contours are respectively for positive and negative values)
Fig. 4.67 Spatial correlation with downstream location for nozzle N44. (Thick and thin contours are respectively for positive and negative values)
Fig. 4.68 Spatial correlation with downstream location for nozzle N55. (Thick and thin contours are respectively for positive and negative values)
Fig. 4.69 Spatial correlation with downstream location for nozzle N65. (Thick and thin contours are respectively for positive and negative values)
Fig. 4.70 Spatial correlation with downstream location for nozzle N66. (Thick and thin contours are respectively for positive and negative values)
Fig. 4.71 Normalized mixing area at four downstream locations.
Fig. 4.72 Evolution of the normalized mixing area with downstream locations.
Fig. 4.73 Variations of thrust with the fully expanded jet Mach number.
CHAPTER 5

CONCLUSIONS

The effects of trailing edge modifications on mixing and noise in aspect ratio 3 rectangular jets were investigated at $M_j=1.5, 1.75, 2.0, 2.2,$ and $2.5$ for the larger nozzles with single sided trailing edge modifications. Based on the mixing and noise results for the larger nozzles, four types of modifications were selected to form six smaller nozzles with double sided trailing edge modifications so as to maximize the mixing enhancement. The four modifications, $M_3, M_4, M_5,$ and $M_6,$ were found to have comparable mixing and noise characteristics in the experiments with the larger nozzles, which have just one modification on the right side.

To verify whether the ineffectiveness of $45^\circ$ cut-outs in the ideally expanded case is due to the mismatch of the angle between large scale structures and the cut-out edge, two additional modification angles $35^\circ$ and $55^\circ,$ relative to the spanwise direction, were investigated. It was found that the trailing edge modification was not an effective way to excite or amplify oblique large scale structures in the ideally expanded case. In the perfectly expanded case, $M_j=2.0,$ no significant changes in either mixing or noise were measured. This is consistent with the Martens et al.'s [1996b] experiments, where the $45^\circ$ cut-outs did
not effectively excite oblique structures at this condition.

In the underexpanded cases, group A nozzles, which generate a kidney type pair of vortices as in nozzles N3 and N5, performed better in mixing at the near field downstream location than group B nozzles, which generate a mushroom type pair of vortices as in nozzles N4 and N6. While at a farther downstream location, the mixing performance of group B nozzles was better than that of group A nozzles. This mixing performance switch for different type of modification with downstream locations was found to be due to the different vortical dynamics of kidney and mushroom type streamwise pair of vortices. The kidney type streamwise pair of vortices entrained the ambient air into the jet and experienced unfavorable interaction between vortex mates at some downstream location. As a result, the capability of mixing at a farther downstream location is much reduced by the interaction. On the other hand, the mushroom type streamwise pair of vortices acted as a fluid pump, which ejected the jet fluid into the ambient air. Thus, relative mixing performance with downstream locations increased for group B nozzles, while it decreased for group A nozzles. The measured spanwise pressure gradient on the modification is believed to be the major source of the streamwise vorticity.

When the jet is overexpanded, the identification and role of streamwise vortices were not as clear due to the complex three dimensional features of the flow, which include three dimensional shock waves and separation. Inferring from surface flow visualizations and wall pressure measurement results, the cross stream flow due to shock waves appeared to be responsible for the jet cross sectional deformation at the jet near field downstream location. The separation lines marked on the surface flow images depended strongly on the fully
expanded jet Mach number $M_j$. At $M_j=1.5$, the jet cross section was almost the same regardless of the type of modification, due to the two dimensional separation shock wave generated well upstream of cut-outs. The mixing performance of group B nozzles was better than that of group A nozzles at both $x/D_{eq}=2$ and 8. As in the tab case, the amount of mixing enhancement in the overexpanded cases was relatively small compared with that in the underexpanded cases.

The reductions of OSPL for modified nozzles were greater for the overexpanded cases than that for the underexpanded cases. These reductions are most probably related to the three dimensional distortions of shock cell structure which could be inferred from the cross sectional deformations and more directly from the surface flow images and wall pressure data. These three dimensional distortions of shock cell structures provide unfavorable conditions for screeching and so lead to reduced shock associated noise: Screech tones and broadband shock associated noise. Group A nozzles were better in noise reduction than group B nozzles in general. The OSLP reduction for nozzle N5 is as much as 4 dB at $M_j=1.75$. As can be inferred from the surface flow images on the modification, the shock cell structures were almost broken down well inside the nozzle. This may result in the almost shock noise free spectrum for nozzle N5.

The fundamental screech modes on the minor axis for the baseline nozzles were antisymmetric at $M_j=1.75$. The modes for the modified nozzles were changed to oblique modes due to the modifications. However, on the major axis the modes for all nozzles except nozzle 2 and the thick-lip baseline nozzle were symmetric. Regardless of modifications, the modes were antisymmetric both on the minor and major axes when the
jets were slightly underexpanded. On the major axis, the thin-lip baseline nozzle appears to experience mode switching from symmetric to antisymmetric. The turbulent mixing noise was not changed substantially by trailing edge modifications. The fundamental screech frequency was well predicted by Tam's [1988] formula.

In the experiment for the larger nozzles with single sided modifications, it was found that modifications M3, M4, M5, and M6 are comparable in mixing enhancement and noise reduction. From these four modifications, six smaller nozzles, which have modifications on both of the long dimensional sides, were configured to investigate the interaction between vortex mates and mixing layers and the evolution of streamwise vortices with downstream locations. Flow regimes were limited to $M_j = 1.75, 2.0, \text{ and } 2.5$, since they are representative jet Mach numbers for overexpansion, ideal expansion, and underexpansion, respectively. Cross sectional visualizations were carried out at four downstream locations, $x/D_{eq} = 1, 2, 4,$ and $8$, to acquire the jet development or evolution with downstream locations. With these detailed visualizations, additional information about mixing rate, vortex-vortex interaction, and the interaction between mixing layers was acquired.

Although nozzles N33 and N55, which generate kidney type pair vortices at both sides, outperformed in mixing at near field locations, their mixing rates were much reduced after the vortex-vortex interaction in the pair vortices began in the underexpanded case. For these kinds of modifications, it seems to be much better to make the vortex center distance of the pair vortices longer in order to delay the interaction and so enhance downstream mixing performance. The interaction between left and right mixing layers were not observed up to $x/D_{eq} = 4$, and so both side mixing layers developed almost independently in the
underexpanded case. Thus, mixing enhancement for nozzles with double sided trailing edge modifications was additive. The nozzles with modifications generating a mushroom type streamwise pair of vortices showed the increased mixing rate with downstream locations. The pumping action of the mushroom type pair of vortices seems to be responsible for the increase in mixing rate.

In the overexpanded case, no significant increase of mixing by the use of double sided trailing edge modifications was observed. The mixing enhancement with double sided trailing edge modifications was almost in the same range of that with single sided trailing edge modifications. This may be related to the nature of the cross stream flow at the exit, which directs into the nozzle center and so restricts the development of the mixing layers.

No significant gain or loss of thrust was observed for the smaller nozzles with single and double sided trailing edge modifications. Therefore, the use of trailing edge modifications, as a passive excitement technique, is a promising technique since it can enhance mixing and reduce noise with minimal thrust loss.
BIOGRAPHY


