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MEASUREMENT OF THE $^{112}\text{Sn}(p,\gamma)^{113}\text{Sb}$, $^{119}\text{Sn}(p,\gamma)^{120}\text{Sb}$, AND $^{96}\text{Zr}(p,\gamma)^{97}\text{Nb}$ CROSS SECTIONS AND ASTROPHYSICAL S-FACTORS AT LOW ENERGIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Frank R. Chloupek, B.S.

* * * * *

The Ohio State University

1998

Dissertation Committee:

Dr. Richard N. Boyd, Advisor
Dr. Evan Sugarbaker
Dr. Terry Walker
Dr. Richard Kass
Dr. Glyn Meyrick

Approved by

Advisor
Department of Physics
ABSTRACT

One of the least well understood processes of nucleosynthesis is the p-process. The best site or sites for this process are still undetermined and many likely contributing cross sections have not been measured directly. This work presents experimental results for three previously unmeasured (p,γ) reactions in an attempt to extend experimental knowledge of these reactions to a higher mass section of the nuclear chart.

Measurements were made of the (p,γ) reaction cross sections for three target nuclei: $^{96}$Zr, $^{112}$Sn, and $^{118}$Sn at the 11 MV FN Tandem Van de Graaff Accelerator at the University Notre Dame in South Bend, Indiana. Incident proton energies ranged from 2.8 MeV to 8.5 MeV. The astrophysical S-factors are also calculated.

The cross section measurements and S-factors are compared to Hauser-Feshbach theoretical calculations using the NON-SMOKER simulation to judge the applicability of the Hauser-Feshbach formalism to these reactions.
To my family and friends,
for all their love and support.
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VITA

June 25, 1968 ................................. Born - South Holland, IL

1990 ................................. B.S. Physics and Mathematics
Michigan State University
East Lansing, MI

1990-1991 ........................................ University Fellow
Department of Physics
The Ohio State University
Columbus, OH

1992-1993, 1994-present .................. Graduate Teaching and
Research Assistant
Department of Physics
The Ohio State University
Columbus, OH

Publications

Study of the Beta-delayed Neutron Emission of $^{19}$C., A. Ozawa, G. Raimann,
R.N. Boyd, F.R. Chloupek, M. Fujimaki, K. Kimura, H. Kitagawa, T. Kobayashi,
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Boyd, M. Hellstrom, D.J. Morrissey, M.J. Balbes, F.R. Chloupek, M. Fauerbach, C.A.
Mitchell, R. Pfaff, C.F. Powell, G. Raimann, B.M. Sherrill, M. Steiner, J. Vandegriff,
Fields of Study

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CHAPTER 1

INTRODUCTION

Space is big. Really big. You just won't believe how mind bogglingly big it is. I mean you may think it's a long way down the road to the chemist, but that's just peanuts to space.

- The introduction from The Hitchhiker's Guide to the Galaxy by Douglas Adams

1.1 Prologue

This thesis discusses the measurement of \((p,\gamma)\) reaction cross sections for three target nuclei: \(^{96}\text{Zr}\), \(^{112}\text{Sn}\), and \(^{119}\text{Sn}\) with incident proton energies ranging from 2.8 MeV to 8.5 MeV. The astrophysical S-factors are also calculated. The cross section measurements were made at the 11 MV FN Tandem Van de Graaff Accelerator at Notre Dame University in South Bend, Indiana. Two experimental runs were made during April/May 1997 and during September 1997.

The cross section measurements were compared to Hauser-Feshbach calculations from the NON-SMOKER program described in [Rauscher, et al. 1997].
1.2 Stellar Synthesis

In the standard Big Bang model, processes of nucleosynthesis occurring during the first few minutes after the universe's creation result in a universe of mostly $^1$H and $^4$He, with small amounts of $^2$H, $^3$He, $^6$Li, and $^7$Li. The lack of stable nuclei at nuclear masses $A=5$ and $A=8$, coupled with the lack of free neutrons, prevent the standard scenario from producing appreciable quantities of heavier elements, although some inhomogeneous Big Bang scenarios could produce detectable amounts of heavier nuclei.

The majority of the elements are synthesized in stellar environments. A cloud of gas composed mainly of hydrogen and helium collapses under its own gravitational attraction, raising the temperature and density in the interior to a level such that nuclear fusion begins to occur. These nuclear reactions release energy to temporarily halt the gravitational collapse. The stages of burning that a star undergoes depend upon the mass of the star and the composition of the material the cloud is formed of. First generation stars (Pop II) contain near primordial material. Later generation stars (Pop I) also contain heavier elements synthesized in earlier generations of stars. The accepted belief is that some of the elements synthesized by stars are ejected back into the interstellar medium through supernovae explosions.

Stellar synthesis typically produces only nuclei with atomic masses up to iron and nickel, since, with the binding energy per nucleon maximized, these are the most stable elements. To synthesize elements heavier than iron, other processes, such as the r-process, s-process, and p-process are needed. To simulate these processes, one needs
to keep track of reactions involving hundreds of different nuclei. These scenarios can be very sensitive to the parameters of temperature and density. It is also important to try to identify possible astrophysical sites where such temperatures and densities can be achieved for each of the processes.

1.3 Experiment

To measure the cross section, an isotopically enriched target with atomic number A and proton number Z was irradiated by a proton beam for several minutes. After the irradiation the target was then placed near a detector to observe the decays of the A+1,Z+1 nucleus. The detectors were at a different site from the irradiation to protect them from the radiation coming from the proton beam and short lived radiation products while enabling us to place the detector very near the target, subtending a larger solid angle to achieve a higher efficiency. We used two different detector types: for the $^{113}$Sb and $^{120}$Sb decays we used two bismuth germinate oxide (BGO) detectors, while for the $^{97}$Nb decays we used a hyperpure germanium (HPGe) detector. Through measuring the number of decays in a given time interval, and with a knowledge of the detector's photopeak efficiency, the total number of nuclei created and the reaction cross section were determined.

The ideal target nucleus will have a ($p,\gamma$) reaction product with a half life of around 5-60 minutes. This is long enough to enable the target to be placed near the detectors and have readings taken, but short enough that the target can be subjected to multiple irradiations in an experimental run. Additionally, there should be a
detectable γ-ray emission and contamination from other reactions (notably (p,n)) should be minimal in the area of the γ-ray peak.

1.4 Motivation

P-process calculations typically require the computation of extensive numbers of reactions. Many of the cross sections of these reactions have not been experimentally measured, leaving these calculations to rely on theoretical estimates using the Hauser-Feshbach formalism developed by [Hauser and Feshbach 1952]. Few cross sections that have been experimentally determined: [Sauter and Käppeler 1997] has recently presented (p,γ) reaction data for 92Mo, 94Mo, 95Mo, and 98Mo; [Laird, et al. 1987] measured 90Zr(p,γ)91Nb; [Fülöp, et al. 1995] determined 70Ge(α,γ)74Se; and [Somorjai, et al. 1996] measured 144Sn(α,γ)148Gd. Additionally, several (n,γ) reactions on stable nuclei are known and tabulated in [Bao and Käppeler 1987].

The reactions that we selected were not necessarily those critical to the p-process, but those where the activation technique could be applied. As such, we are testing the applicability of the NON-SMOKER Hauser-Feshbach code rather than measuring specific key p-process reactions.

Chapter 2 introduces relevant aspects of nuclear astrophysics, compound nucleus reactions, the Hauser-Feshbach formalism, and beta-decay. Chapter 3 discusses stellar synthesis along with the p-process and possible p-process sites. Chapter 4 describes the experimental apparatus used to make the measurements along with the calibrations of same. Chapter 5 presents the results of the experiments along with the
calculation of the cross sections and S-factors. comparing the results to the Hauser-Feshbach code NON-SMOKER. Chapter 6 closes with the conclusions from the experiment.
CHAPTER 2

THEORY

*The concepts initially formed by abstraction from particular situations or experimental complexes acquire a life of their own.*

- Werner Heisenberg

Before we begin a discussion of stellar nucleosynthesis in general, or the p-process in particular, we must introduce a few concepts of nuclear astrophysics. This chapter outlines some needed nuclear physics concepts and discusses the formalism used to model reactions involving medium or heavy mass nuclei.

2.1 Q-value

Nuclear reactions either produce or consume energy. If there is a nuclear reaction with \( n \) incident particles, each with mass \( M_i \), and \( m \) outgoing particles, each with mass
\( M'_i \), the Q-value is the difference in the masses between the incoming and outgoing particles and is given by equation 2.1:

\[
Q = (\sum_{i=1}^{n} M_i - \sum_{j=1}^{m} M'_j)c^2
\]  

(2.1)

In most processes of nucleosynthesis, the masses in equation 2.1 should be nuclear masses since we are dealing with fully ionized nuclei. Positive Q-values mean that the reaction is exothermic and produces energy. The energy released is either carried off by the outgoing particles as kinetic energy or emitted as electromagnetic radiation (\( \gamma \)-rays). Negative Q-values mean the reaction is endothermic and consumes energy. Endothermic reactions require that the kinetic energies of the particles be sufficient to allow the reaction to proceed. This energy is called the threshold energy.

### 2.2 Cross Section

Along with the amount of energy liberated or consumed, we also wish to know the probability of a particular reaction taking place. This value is called the cross section, \( \sigma \), which is measured in units of length squared. Cross sections are measured in units called barns (\( \text{b} \)), with \( 1\text{b}=10^{-24}\text{cm}^2 \). Smaller cross sections are sometimes measured in millibarns \( (10^{-3}\text{b}) \), microbarns \( (10^{-6}\text{b}) \), or nanobarns \( (10^{-9}\text{b}) \). Reactions mediated by the strong force typically have the largest cross sections, while those mediated by the weak force normally have the smallest cross sections, with electromagnetic
reactions generally in between. Three sample reactions illustrating this relationship are shown below in Table 2.1 from [Rolfs and Rodney 1988].

<table>
<thead>
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<th>Reaction type</th>
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<th>Value (barns)</th>
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<tr>
<td>$^{15}\text{N}(p,\alpha)^{12}\text{C}$</td>
<td>Strong</td>
<td>2.0 MeV</td>
<td>$0.5$</td>
</tr>
<tr>
<td>$^3\text{He}(\alpha,\gamma)^7\text{Be}$</td>
<td>Electromagnetic</td>
<td>2.0 MeV</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>$p(p,e^+\nu)d$</td>
<td>Weak</td>
<td>2.0 MeV</td>
<td>$10^{-20}$</td>
</tr>
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</table>

Table 2.1: Examples of cross section values.

### 2.3 Reaction rates

In a situation with $N_i$ incident particles and $N_t$ target particles, with a relative velocity $v$ between them, the reaction rate $r$ is given by $N_i$ multiplied by the flux of incident particles $N_t v$, multiplied by the individual cross section $\sigma(v)$ for the two nuclei to produce the reaction of interest at that velocity.

$$r = N_i N_t v \sigma(v)$$  \hspace{1cm} (2.2)

Another type of reaction that is important is that of a nuclear decay. If a particle $X$ is created by the decay of a particle $Y$, the rate of this process is related to the mean lifetime $\tau_y$ of the particle $Y$.

$$r_X = N_Y / \tau_y$$  \hspace{1cm} (2.3)
In a stellar environment, we do not have particles with a single fixed velocity. There are a range of values depending on the temperature $T$, represented by a probability distribution $\phi(v)$ such that $\int_0^\infty \phi(v)dv = 1$. Thus the cross section we need is not a fixed cross section, but an average over the velocity distribution. This then is called the reaction rate per particle pair, and is given by:

$$< \sigma v > = \int_0^\infty \phi(v)v\sigma(v)dv$$  \hspace{1cm} (2.4)

Applying equation 2.4 to equation 2.2 results in:

$$r_{12} = \frac{N_1N_2}{1 + \delta_{12}} < \sigma v >$$ \hspace{1cm} (2.5)

where the Kronecker delta $\delta_{12}$ is included to prevent double counting for reactions between identical particles.

For a nondegenerate stellar gas with nuclei at nonrelativistic velocities, the velocities are given by a Maxwell-Boltzmann distribution, equation 2.6, where $m$ is the particle mass and $k$ is Boltzmann's constant 8.617 eV/K.

$$\phi(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}}$$  \hspace{1cm} (2.6)

To determine $< \sigma v >$ in a stellar environment requires integrations over the velocity distributions of both nuclei, $v_1$ and $v_2$. Then $< \sigma v >$ is given by equation 2.7, where $v$ is the relative velocity between the two particles.

$$< \sigma v > = \int_0^\infty \int_0^\infty \phi(v_1)\phi(v_2)v\sigma(v)dv_1dv_2$$  \hspace{1cm} (2.7)
To solve equation 2.7, we will first transform this to the center of mass system, introducing the center of mass velocity \( V = v_1 + v_2 \), the reduced mass \( \mu = \frac{m_1 m_2}{m_1 + m_2} \), the total mass \( M = m_1 + m_2 \), and the center of mass energy \( E = \frac{1}{2} \mu v^2 \). We can transform the variables and then integrate over \( V \).

\[
\phi(V) = 4\pi V^2 \left( \frac{M}{2\pi kT} \right)^{3/2} e^{-\frac{MV^2}{2kT}}
\]

\[
\phi(v) = 4\pi v^2 \left( \frac{\mu}{2\pi kT} \right)^{3/2} e^{-\frac{\mu v^2}{2kT}}
\]

\[
<\sigma v> = \int_0^\infty \int_0^\infty \phi(V) \phi(v) v \sigma(v) dV dv
\]

\[
<\sigma v> = 4\pi \left( \frac{\mu}{2\pi kT} \right)^{3/2} \int_0^\infty v^3 \sigma(v) e^{-\frac{\mu v^2}{2kT}} dv
\]

\[
<\sigma v> = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E e^{-\frac{E}{kT}} dE
\]

2.4 Rate Equations

Often, several different reactions are simultaneously occurring to produce and destroy particular nuclei. The rate of change of a nuclear species \( N_x \) is related to its production rate \( r^{\text{prod}} \) and its destruction rate \( r^{\text{dest}} \). Using equations 2.2, 2.3, and 2.5, we can express the rates as the sum of all the possible production less all the possible destruction reactions. Here we have \( n \) possible production reactions, \( m \) possible decays, \( o \) possible destruction reactions, and the decay of the X particle itself. If the particle is stable then the term \( N_x/\tau_x \) is not present.
Equation 2.15 is often not as complex as it seems, since there are generally only a few contributing production and destruction reactions in a given environment. Many processes of nucleosynthesis involve networks of reactions involving several nuclei which are simulated through solving coupled equations of the type 2.15.

2.5 S-factor and Gamow Peak

It is sometimes convenient to express the cross section in a form that separates out the non-nuclear effects. For energies much less than the Coulomb barrier, we have a Coulomb portion of the cross section \( \propto e^{-2\eta} \), where \( \eta = \frac{Z_i Z_f e^2}{2\pi \hbar v} \) is the Sommerfield parameter. Analogous to the classical cross section expression in section 2.2, there is also a term relating to the square of de Broglie wavelength \( \propto 1/E \). Removing these two factors from the cross section, all the purely nuclear effects are subsumed in a factor \( S(E) \):

\[
S(E) = \frac{\sigma}{e^{2\eta/\sqrt{E}/E}} \tag{2.16}
\]
The reason this formulation is useful is that for nonresonant reactions, \( S(E) \) is often a smoothly varying function of energy, and an experimental value of \( S(E) \) can be more easily extrapolated to low energies. This is useful because many astrophysical nuclear reactions occur in environments with low center of mass energies. Since the cross sections for two individual particles to react are generally small at these energies, the energies of interest are lower than those which can be experimentally measured. One generally measures the reaction at higher energies and then is forced to extrapolate to the region of interest.

If we substitute equation 2.16 into equation 2.12 and assume that \( S(E) = S(E_0) \) then we can obtain:

\[
<\sigma v> = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \sigma(E_0) \int_0^\infty e^{-\frac{E}{4T} - \frac{b}{E^{1/3}}} dE
\]

where:

\[
b = (2\mu)^{1/2} \pi e^2 Z_1 Z_2 / \hbar
\]

The integral in equation 2.17 is similar to a Gaussian integration. Its peak will give us the most probable energy for a reaction to occur, the effective mean energy for a reaction at a temperature \( T \). This is called the Gamow peak and occurs at:

\[
E_0 = \left( \frac{b k T}{2} \right)^{2/3}
\]
2.6 Compound Nuclear Reactions

When a particle of low (a few MeV) energy impacts upon a medium or heavy nucleus the number of different reaction pathways that can occur can be quite large and exhibit many narrow, closely spaced resonances. For example, an incident particle can strike one of the nucleons in the target, transferring some of its energy. Both the incident particle and the struck nucleon can undergo many further collisions within the nucleus, resulting in the incident particle energy possibly being shared between many different states in the target nucleus. In this section we will briefly discuss the mechanisms relevant to these reactions. For further discussions on this topic, the reader is directed to [Vogt 1968], [Marmier and Sheldon 1970], [Satchler 1990], [Feshbach 1992], or [Fröbrich and Lipperheide 1996].

It has been shown that for the energies and nuclei studied in this work, it is reasonable to assume that we have no direct reaction component, i.e., no particles quickly escape the nucleus and that the target nucleus plus the incident proton form a bound state termed a compound nucleus. Such reactions are characterized by the many narrow states due to the large number of possible collisions that can occur, hence a large number of states in the compound nucleus contribute to the reaction. Assuming the compound nucleus exists for a long time ($\approx 10^{-16} - 10^{-18}$ sec) compared to the time it takes for a particle to cross a nucleus ($\approx 10^{-22}$ sec), we may treat the available states as a statistical sum of all the possible compound nucleus levels. The compound nucleus that is formed can then decay. We assume that the decay probabilities are independent of the way the compound nucleus is formed, i.e., the
compound nucleus does not "remember" how it was formed, other than to conserve particle number, energy, spin, and parity. This is called the Bohr Assumption and is supported in the experimental nature of these reactions, such as the generally isotropic angular distribution of the reaction products. A violation of this isotropy would indicate a direct component of the reaction. The \((p,\gamma)\) reactions studied in this work can be assumed to proceed through a compound nucleus as shown pictorially in figure 2.1.

![Diagram of compound nucleus formation](image)

**Figure 2.1:** Pictorial representation of compound nucleus formation during a \((p,\gamma)\) reaction

The cross section of the reaction depends upon the energy levels of the intermediate nucleus. The nucleus can decay by light particle evaporation or by fission. If one is looking for a specific exit reaction, the cross section to a specific final state depends upon how many competing exit channels are open to the reaction. Therefore, if one looks for a particular exit channel, the cross section is given by equation 2.20, where \(\sigma_a^C\) is the cross section to form the compound state, \(G_m\) is the branching ratio to the state of interest, and \(\sum_{i=1}^{3} G_i\) is the sum over all possible decay states. The
The subscript $a$ represents the initial state of the system, called the incoming channel. The subscript $b$ represents the final state of the system, called the outgoing channel. $J$ is the total spin.

$$\sigma_{a\rightarrow b}^J = \sigma_a \frac{G_m}{\sum_{i=1}^{a} G_i}$$  \hspace{1cm} (2.20)$$

One interesting aspect of 2.20 is that the decay channels are competing with one another, so that the opening of a new decay channel will decrease the cross section to the other exit channels.

### 2.6.1 Hauser-Feshbach Formalism

The theoretical framework used in calculating $\sigma_{a\rightarrow b}^J$ is the Hauser-Feshbach formalism, initially put forth by [Hauser and Feshbach 1952] to model the inelastic scattering of neutrons. Its basic principles have been extended to the reaction of incident protons, $\alpha$-particles, and other particles such as deuterons with medium to heavy nuclei. It is not appropriate to the interactions of these particles with light nuclei since these reactions typically only exhibit a few broadly spaced resonances and are more appropriately treated as proceeding directly through the few available states.

To express equation 2.20 in terms of the Hauser-Feshbach formalism, we first need to introduce the reciprocity theorem. This states that the cross section for a nuclear reaction $a \rightarrow b$ is related to the inverse reaction $b \rightarrow a$ by equation 2.21, where $k_i$ is the wave number for the appropriate particles.
Additionally, we need to define the partial fusion cross section \( \sigma^J_F(a) \), which is the cross section for the initial state \( a \) to form the compound nucleus, regardless of the exit channel of the compound nucleus. Thus, the partial fusion cross section is the sum over all possible exit channels as given by equation 2.22.

\[
\sigma^J_F(a) = \sum_b \sigma^J_{a \rightarrow b}
\]  

(2.22)

Inverting this gives an expression for the cross section in terms of the partial fusion cross section and the branching ratio \( G \).

\[
\sigma^J_{a \rightarrow b} = \sigma^J_F(a) G_b
\]  

(2.23)

Switching the indices in equation 2.23 gives:

\[
\sigma^J_{b \rightarrow a} = \sigma^J_F(b) G_a
\]  

(2.24)

If we substitute equations 2.23 and 2.24 into equation 2.23, we obtain the following relationship:

\[
\frac{k_a^2 \sigma^J_F(a)}{G_a} = \frac{k_b^2 \sigma^J_F(a)}{G_b}
\]  

(2.25)

The left hand side depends only upon the channel \( a \) and the right hand side depends only upon the channel \( b \). This means that they must both be equal to a common
value, $X$, which is channel independent. Thus, the probability of a compound nucleus to decay to a specific exit channel is:

$$G_i = \frac{k_i^2 \sigma_F^i(i)}{X} \quad (2.26)$$

Since the compound nucleus must decay, the sum over all the exit channels is normalized to 1.

$$\sum_{i=1}^{n} G_i = 1 \quad (2.27)$$

By summing both sides of equation 2.26 over all the decay channels $i$ available to the compound nucleus we can get an expression for $X$.

$$X = \sum_{i=1}^{n} k_i^2 \sigma_F^i(i) \quad (2.28)$$

This implies that we can express the cross section from an initial state $a$ to a final state $b$ ($\sigma_{a \rightarrow b}^J$) solely in terms of the partial fusion cross sections ($\sigma_F(c)$).

$$\sigma_{a \rightarrow b}^J = k_b^2 \frac{\sigma_F^b(a)\sigma_F^b(b)}{\sum_c k_c^2 \sigma_F^c(c)} \quad (2.29)$$

### 2.6.2 Black Nucleus Model

To evaluate equation 2.29 we need to introduce characteristics of nuclear interaction models. We will examine a class of models called "optical models." In these models, the incoming particles are represented by plane waves.
One example is the black nuclear model where a strong interaction immediately forms a compound nucleus for all incoming waves incident on the nucleus that penetrate into the nucleus. In this case, the cross section to form a compound nucleus is:

$$\sigma^C_a = \frac{\pi}{k_a^2} (2J + 1) T_a^J$$  \hspace{1cm} (2.30)

where $T_a^J$ is the probability of an incident particle being absorbed by the nucleus.

If we have partial waves with $l > 0$, or if we have charged incident particles, the centrifugal and Coulomb barriers can result in $T_a^J < 1$.

If we examine the case of S-wave neutrons, with the assumption that there are only incoming waves inside the nuclear surface, with a complex wave number $k = k_0 + ib$ inside the nucleus, then from [Satchler 1990], we can define a reflection coefficient:

$$R = \left| \frac{k - k_0}{k + k_0} \right|^2$$  \hspace{1cm} (2.31)

and a transmission coefficient:

$$T = 1 - R = \frac{4k k_0}{|k + k_0|^2}$$  \hspace{1cm} (2.32)

where $k$ is the incoming wave number. If we are in the low energy neutron limit of $k_0 \gg k$, $T \approx 4k k_0/(k_0^2 + b^2)$. Thus the cross section for the formation of the compound nucleus becomes:

$$\sigma^J_{a \rightarrow b} = \frac{4\pi k_0}{k_a (k_0^2 + b^2)}$$  \hspace{1cm} (2.33)
However, in more general terms, we can express the compound cross section in terms of the transmission coefficients. Substituting equation 2.30 in equation 2.29 results in the Hauser-Feshbach formula:

\[
\sigma_{a\rightarrow b}^J = \frac{\pi}{k_a^2} (2J + 1) \frac{T_a^J T_b^J}{\sum_c T_c^J}
\]  

(2.34)

Here \(k_a^2\) is the incident particle's momentum squared and \(J\) is the total angular momentum of the system. The \(T_i^J\)'s are the transmission coefficients and their value depends upon the specific nuclear model used to calculate the shape of the interaction potential.

### 2.6.3 Cloudy Crystal Ball Model

The black nucleus model is not an accurate model for describing the partial fusion cross sections since its main assumption, that every incident particle that penetrates the surface is absorbed, is an oversimplification. An improvement on this is the "cloudy crystal ball" model where the incident particle exists for a short time inside the nucleus as an independent particle. The interaction, at least for neutrons incident upon nuclei, is represented in terms of a complex potential \(U = V + iW\), where the \(W\) term is responsible for absorption.

A simple square well potential, \(U = -V - iW(V > W)\), can be chosen, where the \(W\) is responsible for absorption. However, a potential that could better describe the interaction between incident charged particles and nuclei is expressed as the sum
of three parts: a central potential, a Coulomb potential, and a potential for the spin-orbit interaction.

\[ U_{\text{toy}} = U_{\text{central}} + U_{\text{Coulomb}} + U_{\text{SO}} \]  

(2.35)

The central potential has real and imaginary parts given by:

\[ U_{\text{central}} = -V_c f(r, R_{w c}, a_{w c}) - iW_c f(r, R_{w c}, a_{w c}) + i4aW_D \frac{d}{dr} f(r, R_{w c}, a_{w c}) \]  

(2.36)

where we have introduced the Woods-Saxon form factor as a refinement of the simple square well potential.

\[ f(r, R, a) = \frac{1}{1 + e^{\frac{r-R}{a}}} \]  

(2.37)

\( R \) and \( a \) as constants describing the average width and curvature of the potential.

The final term in equation 2.36 is a surface term, since absorption of an incident particle is more likely to occur at the nuclear surface. Taking the derivative of \( f \) causes such a term to be significant only where \( f \) is changing, i.e., at the nuclear surface.

The Coulomb term is the Coulomb potential between a homogeneous charged sphere of charge \( Z e \) with radius \( R_c \) and a particle of charge \( z e \),

\[ U_{\text{Coulomb}} = \frac{Zze^2}{2R_c} \left( 3 - \frac{r^2}{R_c^2} \right) \theta(r - R_c) + \frac{Zze^2}{r} \left( 1 - \theta(r - R_c) \right) \]  

(2.38)

where the theta function \( \theta(x) = 0 \) for \( x < 0 \) and \( = 1 \) for \( x > 0 \).
Finally, there is a spin orbit term:

\[ U_{SO} = -V_{SO}(\sigma \cdot \mathbf{l}) \left( \frac{\hbar}{m_c c} \right)^2 \frac{1}{r} \frac{d}{dr} f(r, R_{so}, a_{so}) \]  

(2.39)

This leaves a total of 12 different parameters (not including \( R_c \), since scattering results do not typically depend strongly on this parameter [Wong 1990], which is usually set to \( R_c = 1.2A^{1/3}\text{fm.} \)) The optical model parameters may be found through fitting to experimental scattering data as in [Perey 1963] and [Becchetti and Greenlees 1969].

An alternative way to derive the optical model potentials is to realize that the potential for nucleon-nucleus scattering is the result of an average interaction between an incident nucleon and all the constituents of the target nucleus. Thus, by obtaining a good expression of the nucleon density, the optical model parameters can be derived. Such a method was used by [Jeukenne, et al. 1977] and is applied in the NON-SMOKER code.

### 2.6.4 Transmission Coefficients

In this context we calculate the \( S \) (scattering) and \( T \) (transmission) matrices. The reaction cross section is the sum over all possible reaction cross sections excluding elastic scattering:

\[ \sigma_{reac}^J = \frac{\pi}{k^2} (2J + 1) \sum_{s, s', l', l} |U_{a's'\ell', a}^J|^2 \]  

(2.40)
where the sum is over all possible quantum numbers. Following [Vogt 1968] we note
that the matrix $U$ is both unitary and symmetric

$$\sum_{a's'l'} (U_{i}^{a's'l',asl}) (U_{j}^{a's'l',asl})^* = 1$$  \hspace{1cm} (2.41)

Thus it follows that we can use 2.41 to eliminate the sum over the primed terms in
2.40:

$$\sigma_{\text{reac}}^J = \frac{\pi}{k_a^2} (2J + 1) \sum_{sl} (1 - |U_{i}^{j,asl,asl}|^2)$$  \hspace{1cm} (2.42)

with the elastic cross section as:

$$\sigma_{\text{elas}}^J = \frac{\pi}{k_a^2} (2J + 1) \sum_{sl} |1 - U_{i}^{j,asl,asl}|^2$$  \hspace{1cm} (2.43)

To find the transmission coefficients, we take the energy averages of each side.

$$\overline{\sigma}_{\text{reac}}^J = \frac{\pi}{k_a^2} (2J + 1) \sum_{sl} (1 - \overline{|U_{i}^{j,asl,asl}|^2})$$  \hspace{1cm} (2.44)

$$\overline{\sigma}_{\text{elas}}^J = \frac{\pi}{k_a^2} (2J + 1) \sum_{sl} (|1 - \overline{U_{i}^{j,asl,asl}}|^2)$$  \hspace{1cm} (2.45)

$$\overline{\sigma}_{\text{elas}}^J = \frac{\pi}{k_a^2} (2J + 1) \sum_{sl} (|1 - \overline{U_{i}^{j,asl,asl}}|^2 + |\overline{U_{i}^{j,asl,asl}}|^2 - |\overline{U_{i}^{j,asl,asl}}|^4)$$  \hspace{1cm} (2.46)

Following [Vogt 1968], the last two terms are the part of the elastic scattering cross
section that proceeds through the compound nucleus. This gives an energy average
absorption cross section of

$$\overline{\sigma}_{\text{abs}}^J = \frac{\pi}{k_a^2} (2J + 1) \sum_{sl} (1 - |\overline{U_{i}^{j,asl,asl}}|^2)$$  \hspace{1cm} (2.47)
thus identifying this with equation 2.30 we can conclude that the transmission coefficients are given by:

\[ T_{asl.asl}^J = 1 - |\overline{U}_{asl.asl}^J|^2 \]  

(2.48)

For the elastic channels we can identify the transmission coefficients with the phase shifts \( \delta \) of the optical models.

\[ T_{opt} = 1 - |e^{2i\delta}|^2 \]  

(2.49)

### 2.6.5 Level Densities - Evaporation Model

An inherent assumption in the Hauser-Feshbach formalism is that the density of states in the compound nucleus is high, allowing us to take an energy average over the many narrow resonances. To do this we need an expression for the level densities in the compound nucleus. The application of the Fermi level density model provides a framework to achieve this. In this model, the neutrons and the protons are treated as a degenerate Fermi gas. We can consider the nucleus as a gas of \( n \) particles at some temperature \( T \). With this model we can relate the excitation energies to the temperature and Boltzmann's constant \( k = 0.861 \times 10^{-10} \text{ MeV K} \):

\[ E^* = kT \]  

(2.50)
We can use a thermodynamic expansion given in [Marmier and Sheldon 1970] relating the excitation energy to the energy at zero temperature $E_0$.

$$E(T) = E(0) \left( 1 + \frac{3\pi^2 n^2 k^2 T^2}{20 E(0)^2} + \cdots \right)$$

(2.51)

The excitation energy can then be defined as:

$$E^* = E(T) - E(0) = \frac{3\pi^2 n^2}{20 e_0^2} (kT)^2$$

(2.52)

Now, through introducing some formulae from [Marmier and Sheldon 1970], we can rewrite the Boltzmann equation as:

$$S = k \left[ \ln(\rho(E^*)) - \ln(\rho(0)) \right]$$

(2.53)

where the entropy is related to the temperature via:

$$S = \int_0^T \frac{dE^*}{T}$$

(2.54)

$dE^*$ is a function of temperature, to get its explicit dependence we use equation 2.52 and take the derivative of each side.

$$dE^* = \frac{6\pi^2 n^2}{20 e_0^2} k^2 T$$

(2.55)

Substituting equation 2.55 in equation 2.54 allows us to compute the integral giving the entropy.
where the constant $a$ is defined as:

$$a = \frac{3\pi^2 n^2}{20e_0^2}$$  

(2.60)

Substituting equation 2.59 in equation 2.53 results in the Bethe level density:

$$\rho(E^*) = \rho(0)e^{2\sqrt{aE}}$$  

(2.61)

The Fermi gas model was refined by [Gilbert and Cameron 1965] to a more complex form, termed the “back shifted Fermi Model.” where has a small shift $\delta$ in the energy levels and also explicitly includes a spin dependent component.

$$\rho(U, J) = \frac{1}{2} F(U, J) \rho(U)$$  

(2.62)

where $U = E - \delta$ and the spin and space terms get separated as below. The reader is directed to [Gilbert and Cameron 1965] for a derivation.

$$\rho(U) = \frac{1}{\sqrt{2\sigma 12a^{1/4}}} \frac{e^{2\sqrt{aU}}}{E^{5/4}}$$  

(2.63)
\[ F(U, J) = \frac{2J + 1}{2\sigma} e^{-\frac{U(J+1)}{2\sigma^2}} \]  
\[ \sigma^2 = \frac{2mAR^2}{r\hbar^2}(U/a)^{1/2} \]

However, at low energies this formula diverges and is found experimentally to be well represented instead by equation 2.66.

\[ \rho(U) = \frac{e^{U/T}}{T} \]

To fully describe \( \rho(U) \) the low and high energy expressions are tangentially fitted together as in [Gilbert and Cameron 1965]. This can then be used to determine the density of states available for the compound nucleus.

Using the optical model transmission coefficients, and the density of states available by the back shifted Fermi gas, the cross section can be calculated. In this experiment, we compared our data to the NON-SMOKER calculations described in [Rauscher, et al. 1997], where the cross section is found by computing the sum of the transmission coefficients for the lowest energies up to the highest experimentally known state. For higher states the sum is replaced by an integral over the density of states. The transmission coefficients are obtained by solving the Schrödinger equation and the integral over the density of states is computed by numerical methods.

Given the moderately good agreement at these energies of the Hauser-Feshbach formalism to observed reactions, it is usually this formalism, and the various related nuclear models that are used in calculating the medium to heavy nuclear reactions that are involved in simulating the r, s and p processes, described in chapter 3.
2.7 Beta Decays

For more detailed discussions of beta decay the reader is directed to any general nuclear text, including [Siegbahn 1955], [Segre 1964], [DeShalit and Feshbach 1974], and [Krane 1988]. The term “beta decay” here really encompasses 3 related processes: electron emission, positron emission, and electron capture.

Electron emission: \[ ^{Z}X \rightarrow ^{Z+1}X' + e^- + \bar{\nu}_e \]  

Positron emission: \[ ^{Z}X \rightarrow ^{Z-1}X' + e^+ + \nu_e \]  

Electron capture: \[ ^{Z}X + e^- \rightarrow ^{Z-1}X' + \nu_e \]

Here, we will use the term β-ray to represent either an emitted positron or an emitted electron. An electron neutrino or antineutrino is also emitted in a beta decay process. Due to the conservation of momentum and energy between the three final products and the initial system, the kinetic energy is carried off mainly between the electron neutrino and the β-ray. The recoil nucleus' kinetic energy can generally be ignored, as is shown by the following argument.

The recoil energy of the resulting nucleus is given by the following:

\[ E_{\text{nuc}} = \frac{p_{\text{nuc}}^2}{2m_{\text{nuc}}} \]  

Due to the conservation of momentum between the three final products, we can replace the nuclear momentum with the neutrino and electron momenta.
To estimate the maximum size of $E_{\text{nuc}}$, we examine the limit where the electron energy is large. Then equation 2.71 can be expressed as the following:

$$E_{\text{nuc}} = \frac{(p_e + p_{\nu})^2}{2M_{\text{nuc}}}$$  \hspace{1cm} (2.71)

Typical electron kinetic energies are around $1 \text{ MeV}/c^2$. Typical nuclear masses involved in $\beta$-decay are around 100 amu. This gives a result of $E_{\text{nuc}} \approx 13 \text{ eV}$. This can be safely ignored in most calculations including the ones following in this section. However, the recoil kinetic energy is, in principle, observable and in some cases must be measured if one is studying certain features of $\beta$-decays such as the angular correlation between $\beta$-rays and neutrinos.

The emission of the neutrino means that there are three reaction products. Hence, the $\beta$-ray does not have a fixed energy, but rather a distribution of energies with an endpoint equal to the reaction's Q-value (see section 2.1) minus the mass of the electron neutrino. It was this shape of the $\beta$-ray energy spectrum that led Wolfgang Pauli to postulate the existence of the neutrino in the 1930s.

Since the electron neutrino mass is zero in the standard model of particle physics, and is measured to be experimentally at most a few eV (for electron neutrinos), the endpoint is effectively the Q-value of the reaction. The Q-values for the types of
reactions are given by the following, where the $M$'s are nuclear masses, $m_e$ is the electron (or positron) mass, and $B_e$ is the binding energy of the electron captured.

$$Q_{\beta^+} = M_z - M_{z+1} + m_e$$
$$Q_{\beta^-} = M_z - M_{z-1} + m_e$$
$$Q_{EC} = M_z - M_{z-1} - B_e + m_e$$

The theory of beta decay was developed by Fermi, and based on Fermi's Golden Rule, equation 2.77, a general result for any transition. For a derivation, the reader is directed to Appendix A in [Segre 1964].

$$\lambda = \frac{2\pi}{\hbar} |H_{fi}|^2 \rho(E_f)$$

with

$$H_{fi} = \int \psi_i^* H \psi_f dv$$

where $\lambda$ is the decay rate, $\rho(E_f)$ is the density of final states and the matrix element $H_{fi}$ is given by equation 2.78, where $H$ is the interaction Hamiltonian, $\psi_i$ is the initial wave function, and $\psi_f$ is the final wave function.

We are interested in the differential form 2.79 which gives the rate for a decay to a state with a specific electron and neutrino energy.

$$d\lambda = \frac{2\pi}{\hbar} |H_{fi}|^2 \frac{dN}{dE}$$
2.7.1 Density of States

To evaluate equation 2.79, we need an expression for the density of states $dN/dE$. While $\beta$-decay is a 3 body decay with energy and momentum conserved, we have seen that the recoil of energy of the nucleus is small, and in our case can be ignored. Thus the total energy released $E$ is given by equation 2.80.

$$E = E_\nu + E_e = cp_\nu + (m^2c^4 + c^2p_e^2)^{1/2} \quad (2.80)$$

Now we express the density of states for the electron and neutrino as an expression in phase space over a normalization volume $V$:

$$\frac{dN_e}{dp_e} = \frac{4\pi V}{(2\pi\hbar)^3} p_e^2 \quad (2.81)$$

$$\frac{dN_\nu}{dp_\nu} = \frac{4\pi V}{(2\pi\hbar)^3} p_\nu^2 \quad (2.82)$$

$$dN_\nu dN_e = \frac{16\pi^2 V^2}{(2\pi\hbar)^6} p_\nu^6 dp_\nu p_e^6 dp_e \delta(E - E_e - E_\nu) \quad (2.83)$$

where the $\delta$ function is included to conserve energy.

This gives us an expression for the density of states for a particular neutrino and electron momentum. However the neutrino and electron momenta are related through equation 2.80.

$$E - E_e = E_\nu = cp_\nu \quad (2.84)$$
Integrating over the neutrino momentum results in equation 2.85:

\[ dN_e \frac{dN_\nu}{dE} = \frac{16\pi^2V^2}{(2\pi \hbar)^6c^3}(E - m^2c^4 + c^2p^2_e)p^2_e dp_e \]  

(2.85)

Thus, applying equation 2.85 we can rewrite equation 2.79 as follows:

\[ d\lambda = \frac{2\pi}{\hbar}|H_{fi}|^2(4\pi)^2 \frac{V^2(E - m^2c^4 + c^2p^2_e)^{1/2}}{(2\pi \hbar)^6c^3} dp_e \]  

(2.86)

### 2.7.2 Matrix Elements

To proceed further we return to the matrix elements from equation 2.78. Since a 3-decay creates a free antineutrino and free electron, we can decompose the final wave \( \psi_f \) into a nuclear part \( \psi'_i \) and parts for the electron \( \phi_e \) and antineutrino \( \phi_\nu \). The initial wave function is purely nuclear. Dropping the prime on \( \psi'_i \) the matrix element can now be expressed as equation 2.87.

\[ H_{fi} = \int [\psi_f^* \phi_e^* \phi_\nu^*] H \psi_i dv \]  

(2.87)

The function \( \phi_\nu^* \) represents an antineutrino going backward in time. By the assumption of Time Reversal Symmetry, this time-reversed antineutrino wave function is equivalent to that of a neutrino progressing forward in time. Thus \( \phi_\nu^* = \phi_\nu \). Applying this result to equation 2.87 results in equation 2.88.

\[ H_{fi} = \int [\psi_f^* \phi_e^*] H \psi_i \phi_\nu dv \]  

(2.88)
To simplify equation 2.88 we treat the electron and neutrino as free particles, expressing them as plane waves over a normalization volume $V$.

$$\phi_e(r) = \frac{1}{\sqrt{V}} e^{i\mathbf{p} \cdot \mathbf{r}/\hbar}$$  \hspace{1cm} (2.89)
$$\phi_\nu(r) = \frac{1}{\sqrt{V}} e^{i\mathbf{q} \cdot \mathbf{r}/\hbar}$$  \hspace{1cm} (2.90)

Over nuclear distances ($\approx 10^{-15}$m), for energies typical for beta decay reactions (a few MeV), we expand the exponential.

$$e^{i\mathbf{p} \cdot \mathbf{r}/\hbar} = 1 + \frac{i\mathbf{p} \cdot \mathbf{r}}{\hbar} + \cdots$$  \hspace{1cm} (2.91)
$$e^{i\mathbf{q} \cdot \mathbf{r}/\hbar} = 1 + \frac{i\mathbf{q} \cdot \mathbf{r}}{\hbar} + \cdots$$  \hspace{1cm} (2.92)

The factor $\frac{p_F}{\hbar} \approx .01 - .1$. Thus the series quickly converges and we can keep only the leading term. An exception to this is when the matrix element vanishes for the first term, and then the other terms are the only contribution to the decay. These so-called "forbidden" decays are briefly described in section 2.7.4.

It is convenient to express the matrix elements of the Hamiltonian $H_{fi}$ in terms of a constant $g$ describing the strength of the interaction and a term $M_{fi}$ which acts as an operator between the nuclear states.

$$H_{fi} = g M_{fi} = g \int \psi_f^* M \psi_i d\nu + \cdots$$  \hspace{1cm} (2.93)
Here we drop the higher order terms which depend on the electron and neutrino momenta. There is also a Coulomb interaction between electron and the residual nucleus, which we will ignore temporarily, treating it in section 2.7.3.

To evaluate the matrix elements, let us first consider the possible interactions that can occur. Since $\beta^-$-decay converts a neutron to a proton, while $\beta^+$-decay and electron capture convert a proton to a neutron, the form of interaction can be considered in terms of the isospin raising and lowering operators, $\tau^{(\pm)}$. Then, as outlined in [DeShalit and Feshbach 1974] and [Wong 1990], according to the V-A theory, there can be a "Fermi" polar vector component $C_F\tau^{(\pm)}$ and a "Gamow-Teller" axial vector component $C_{GT}\tau^{(\pm)}\sigma$:

$$M_{fi}^{spin} = C_F M_F + C_{GT} M_{GT} \tag{2.94}$$

where the Fermi portion is:

$$|M_F|^2 = \frac{1}{2J+1} \sum_{f,i} |< f | \sum_k \tau^{(\pm)}(k)|i>|^2 \tag{2.95}$$

and the Gamow-Teller portion is:

$$|M_{GT}|^2 = \frac{1}{2J+1} \sum_{f,i} \sum_j |< f | \sum_k \tau^{(\pm)}(k)\sigma_j(k)|i>|^2 \tag{2.96}$$

The approximations made are valid only if the matrix elements $M_{fi}$ do not vanish at $r=0$. For this to happen the emitted $\beta$-ray and neutrino can not carry any angular momentum $j$. Then, the only angular momentum change in the decay can then come from the intrinsic spins of the neutrino and electron, which have $s = 1/2$. In this case
the spins may be parallel (S=1) and the decay is called a Gamow-Teller decay or they may be anti-parallel (S=0) and the decay is called a Fermi decay. Allowed 3-decays may be pure Fermi, pure Gamow-Teller, or a mixture of both. A 3-decay is allowed if the nuclear spin change $\Delta J = 0$ or 1 and parity is conserved.

### 2.7.3 Total Decay Rate

To derive the total decay rate for all possible electron momenta, we will integrate equation 2.88 over all possible momenta of the electrons. The range of integration is from 0 to $p_{\text{max}} = \frac{1}{c}(W^2 - m^2c^4)^{1/2}$.

$$d\lambda = g^2 \frac{|M_{fi}|^2}{2\pi c^3 \hbar^2} \int_0^{p_{\text{max}}} \left(W - (m^2c^4 + c^2p^2_c)^{1/2}\right)^{1/2} p_c^2 dp_c$$  \tag{2.97}

However, one thing we have ignored is the Coulomb interaction between the electron and the residual nucleus. This can be accounted for classically by a factor $F(Z, E_e) = 2\pi \zeta(1 - e^{-2\zeta})^{1/2}$, where $\zeta = Ze^2/\hbar v_e$. $Z$ is the nuclear charge, and $v_e$ is the velocity of the electron. If the nucleus is light or the electron energy is high, $F(Z, E_e) \approx 1$. Relativistically $F(Z, E_e)$ is more complex, and is given in [DeShalit and Feshbach 1974] by equation 2.98 where $R$ is the nuclear radius and $s^2 = 1 - (\frac{Ze}{137})^2$.

$$F(Z, E_e) = 2(2kR)^{2(s+1)} \frac{1 + s}{s^2 + \zeta} \left| \frac{e^{\pi\zeta/2}\Gamma(s + 1 + i\zeta)}{\Gamma(2s + 1)} \right|^2$$  \tag{2.98}

Carrying out the integration of equation 2.97 in the $F(Z, E_e) = 1$ limit results in:
\[
\int_0^{p_{\text{max}}} \left( E - (m^2c^4 + c^2p_e^2)^{1/2} \right)^{\frac{1}{2}} p_e^2 dp_e = -\frac{p_{\text{max}}}{4mc} - \frac{p_{\text{max}}^3}{12m^3c^3} + \frac{p_{\text{max}}^3}{30m^5c^5} \frac{(1+p_{\text{max}}^2)^{1/2}}{4} \log \left( \frac{p_{\text{max}}}{mc} + (1 + \frac{p_{\text{max}}}{mc})^{1/2} \right) \quad (2.99)
\]

In a general case we would also have to include the factor \( F(Z, E_0) \) in this integral. The result of this integration with \( F(Z, E_0) \) included is represented by a factor \( f(Z, E_0) \), which now includes this Coulomb dependence.

\[
\lambda = \frac{g^2|M_{fi}|^2m^5c^4}{2\pi c^3h^3} f(Z, E_0) \quad (2.100)
\]

Expressing equation 2.100 in terms of the halflife for a particular nucleus gives:

\[
t_{1/2} = \frac{\ln 2}{\lambda} = \ln 2 \frac{2\pi c^3h^7}{g^2|M_{fi}|^2m^5c^4f(Z, E_0)} \quad (2.101)
\]

or, equivalently:

\[
f(Z, E_0)t_{1/2} = \frac{\ln 2}{\lambda} = \ln 2 \frac{2\pi c^3h^7}{g^2|M_{fi}|^2m^5c^4} \quad (2.102)
\]

It is often these "\( f t_{1/2} \) values" which are quoted as an example of the strength of a particular \( \beta \)-decay. The greater the \( f t_{1/2} \) value, the longer lived the parent nucleus is and the weaker the \( \beta \)-decay.

### 2.7.4 Forbidden Decays

Decays which do not fulfill the conditions associated with the Fermi or Gamow-Teller decays are the so-called "forbidden" decays. These decays have longer half lives.
than "allowed" decays. However, allowed decays are occasionally not permitted. An example of this is in the $^{113}\text{Sn}$ decay where the initial and final states have different parity. In this case both $|M_F|^2$ and $|M_{GT}|^2$ vanish. These $\beta$-decays can only occur when an electron or neutrino is emitted with non-zero angular momentum. These decays generally have larger $f_{t_{1/2}}$ values, since there is less phase space open to the decay. To compute these decays, one would have to evaluate 2.88 while keeping higher order terms in the exponential expansion of the neutrino and electron wave functions.

A $\beta$-decay is called first forbidden if the nuclear spin change $\Delta J = 0$ or 1 or 2 and parity changes. With increasing angular momentum change, the degree of forbiddenness increases. In general, a $\beta$-decay is n-th order forbidden by the selection rules $\Delta J = n, n + 1$ and $\Delta \pi = (-1)^n$. As can be seen in table 2.2, the decays we are considering in this work are all allowed decays, with one exception being a small background reaction from the decay of $^{113}\text{Sn}$, which is first forbidden. The fact that this decay is first forbidden leads it to have a much longer halftime than the other decays mentioned.

<table>
<thead>
<tr>
<th>Parent</th>
<th>Spin(par)</th>
<th>Daughter</th>
<th>Spin(par)</th>
<th>Decay Type</th>
<th>$t_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{97}\text{Nb}$</td>
<td>$9/2^+$</td>
<td>$^{97}\text{Mo (656 keV)}$</td>
<td>$7/2^+$</td>
<td>$\beta^-$ Allowed</td>
<td>72 min</td>
</tr>
<tr>
<td>$^{113}\text{Sb}$</td>
<td>$5/2^+$</td>
<td>$^{113}\text{Sn (498 keV)}$</td>
<td>$3/2^+,5/2^+$</td>
<td>$\beta^+$ Allowed</td>
<td>6.7 min</td>
</tr>
<tr>
<td>$^{113}\text{Sn}$</td>
<td>$1/2^+$</td>
<td>$^{113}\text{In (391 keV)}$</td>
<td>$1/2^-$</td>
<td>EC First Forbidden</td>
<td>115 days</td>
</tr>
<tr>
<td>$^{120}\text{Sb}$</td>
<td>$1^+$</td>
<td>$^{120}\text{Sn (ground)}$</td>
<td>$0^+$</td>
<td>$\beta^+$ Allowed</td>
<td>15.9 min</td>
</tr>
</tbody>
</table>

Table 2.2: Beta decay reactions in this work. The state of the daughter nucleus is the state that has the highest branching ratio, other states in the nucleus may contribute to a lesser degree.
2.7.5 β – γ Correlations

As can be seen in table 2.2, β-decay transitions will not always proceed directly to the ground state of a daughter nucleus, but to one or more excited states. These excited states then γ-decay to the ground state. These decays happen very quickly, and it has been experimentally observed that allowed γ-decays are not correlated with the β-decay. However, if the β-decay is forbidden, the γ-decay will sometimes exhibit a correlation. The β-decays we are dealing with in this experiment are all allowed decays however, and have been assumed to have isotropic correlations.
The motivation behind the study of (p,γ) reactions on intermediate mass nuclei is to develop empirical data to use in improving models of element synthesis. These reactions are important in several theoretical models of nucleosynthesis processes called "p-processes". To understand why these models have been developed, we must first examine the element synthesis process that occur in stars and other related processes. It is within this context that the need for a "p-process" is evident.

This chapter will give a general overview of the element synthesis that occurs in a stellar environment. It will describe the normal stellar burning stages, along with the synthesis of elements greater than iron in the "r-process" and "s-process." Finally, it
will undertake a more detailed examination of the p-process, giving a description of some of the different p-process models and the possible astrophysical sites where they may occur.

3.1 Element Synthesis

The question of how the elements in the universe are synthesized is one of the fundamental problems of nuclear astrophysics. The first attempt at a comprehensive explanation was put forward in 1957 by Burbidge, Burbidge, Fowler, and Hoyle in [Burbidge, et al. 1957]. Many general overviews exist, among the most recent being by [Wallerstein, et al. 1997].

Within this framework the universe is believed to begin in a hot Big Bang. The universe expands and cools, passing from a quark-gluon plasma phase into hadrons. When the universe is a few minutes old, the temperatures and densities are such that the weak reactions can no longer keep equilibrium between the protons and neutrons. As the universe cools further, the protons and neutrons undergo a series of nuclear reactions termed Big Bang Nucleosynthesis (BBN). This process is responsible for producing $^4\text{He}$, along with small amounts of $^2\text{H}$, $^3\text{He}$, $^6\text{Li}$, and $^7\text{Li}$. It is blocked from producing significant amounts of other nuclei by the lack of stable nuclei with nuclear masses $A=5$ or mass $A=8$, although some of the Inhomogeneous Big Bang models can produce small amounts of higher mass elements. Synthesis of significant amounts of heavier elements requires further processing in stars, supernovae, or other astrophysical sites.
3.2 Stellar Burning

Stellar nucleosynthesis is a complex process and will only be briefly summarized in the sections to follow. For expanded discussions of the topics and ideas presented here, the reader is directed to [Wallerstein, et al. 1997] and [Rolfs and Rodney 1988].

3.2.1 Hydrogen Burning

The gas in the interstellar medium is composed of hydrogen, helium, and a small amount of heavier elements ("metals"). First generation (Pop II) stars have negligible amounts of metals (around .01 - .02%), because they are formed from near primordial materials. Second or later generation stars such as the sun (Pop I) have increased metallicity (around 1 - 2%) since they are formed after first generation stars have ejected material back into the interstellar medium through events such as supernova explosions.

A star begins with a cloud of gas collapsing under its own gravity. As the gas cloud collapses, the virial theorem states that the gravitational potential energy lost through the star’s collapse is converted in equal amounts to thermal energy and radiation. The temperature and density of the gas cloud continue to increase during the collapse as energy is simultaneously radiated away. If the total mass of the gas cloud is greater than around \(0.1M_\odot\), where \(M_\odot\) is equal to the mass of the sun, the center of the cloud will eventually reach a temperature around \(10^6\) Kelvin. At this temperature, the
energy of the protons in the core is great enough that thermonuclear fusion reactions occur. The energy released by these reactions balances the energy radiated from the star, temporarily halting the gravitational collapse and reestablishing hydrostatic equilibrium.

**Proton-Proton Chains**

Since the most abundant elements present are hydrogen and helium, and since they have the lowest Coulomb barriers, any nuclear process will first occur involving these elements. One might initially think that hydrogen burning could proceed via the electromagnetic reaction $p(p,\gamma)^2\text{He}$, but $^2\text{He}$ is unbound and reverts quickly back to two protons. One might wonder if the unstable $^2\text{He}$ could exist a short time as a resonance and capture a proton via $^2\text{He}(p,\gamma)^3\text{Li}$, but $^3\text{Li}$ is also unbound. Taking advantage of the $^4\text{He}$ present does not allow for the electromagnetic reaction $^4\text{He}(p,\gamma)^5\text{Li}$. since $^5\text{Li}$ has a very short half-life ($t_{1/2} \approx 10^{-21}$ sec), quickly decaying back to $^4\text{He} + p$.

Since the strong and electromagnetic reaction pathways are blocked, hydrogen burning proceeds via the weak reaction $p(p,e^+\nu)d$. The Q-value of this reaction is 1.44 MeV. Of this, 1.02 MeV goes into creating the electron-positron pair and the remaining 0.42 MeV is distributed between the outgoing particles as kinetic energy. The kinetic energies of the positron and deuteron are rapidly reabsorbed by the star as these interact with other particles in the stellar interior. The cross section for a neutrino interaction is mediated by the weak force, and the neutrino almost always escapes from the star, carrying off an average 0.26 MeV of energy. Since the $p(p,e^+\nu)d$
reaction is governed by the weak force, it proceeds slowly in the stellar interior. As given in [Rolfs and Rodney 1988] the average lifetime of a proton in the sun is about $10^{10}$ years.

Once deuterium has been synthesized, further reactions can occur. Referring to equation 2.5, we can see that the strong $d + d$ reactions will be suppressed by the very low abundance of deuterium. Since protons are very abundant, the most likely reaction pathway is $d(p,\gamma)^{3}\text{He}$, with a $Q$-value of 5.494 MeV. This is an electromagnetic reaction, and proceeds quicker than the weak $p(e^+\nu)d$, meaning that what little free deuterium is created rapidly burns to $^{3}\text{He}$. While the lifetime of a proton in the sun's interior is $10^{10}$ years, the lifetime of a deuteron is 1.6 seconds ([Rolfs and Rodney 1988]). The final step in this process involves $^{3}\text{He}(^{3}\text{He},2p)^{4}\text{He}$, with a $Q$-value of 12.860 MeV. The reaction $^{3}\text{He}(^{3}\text{He},\gamma)^{6}\text{Be}$ does occur, but has a smaller cross section by about 4 orders of magnitude and is not a significant reaction pathway. Reactions involving $d + ^{3}\text{He}$ will be suppressed by the low deuterium abundance in the star. This reaction chain is called the p-p-I process, and results in four protons producing one $\alpha$-particle, two positrons, and two neutrinos.

A star can also use existing $^{4}\text{He}$ as a catalyst in hydrogen burning. Occasionally, $^{3}\text{He}$ will capture an $\alpha$-particle to make $^{7}\text{Be}$, with a $Q$-value of 1.587 MeV. The $^{7}\text{Be}$ can then decay by capturing one of its electrons to $^{7}\text{Li}$. This capture can occur directly to the ground state (89.6%), producing a 862 keV neutrino, or to the first excited state (10.4%), producing a 384 keV neutrino and a subsequent 478 keV $\gamma$-ray. As in the p-p-I process, the neutrinos escape the star, taking their energy with them. While the
electron capture of $^7$Be has a mean decay time of 79.6 days on earth. It is suppressed in stars because most of the $^7$Be is in a partially or fully ionized state, depending on the stellar temperature. Free electrons must be captured for $^7$Be to decay. In the sun, this lengthens the mean lifetime of $^7$Be to around 140 days. For further details on this, see [Rolfs and Rodney 1988]. $^7$Li then captures a proton and breaks up into two $\alpha$-particles, with a Q-value of 17.347 MeV. Notice that while there are two $\alpha$-particles at the end of this reaction sequence, called the p-p-II chain, an existing $\alpha$-particle was used as a catalyst, resulting in the net production of one $\alpha$-particle, two positrons, and three neutrinos, while destroying four protons.

Occasionally, $^7$Be will capture a proton ($Q = 0.14$ MeV) before it can electron capture, producing $^8$B. In the solar environment, this is a rare process, accounting for approximately only .2% of the total p-p process reactions. $^8$B is radioactive with a lifetime of 1.1 seconds, decaying to an excited state of $^8$Be at 2.94 MeV. $^8$Be*. This has a Q-value of 15.04 MeV and produces a very energetic neutrino with an average energy of 7.30 MeV. This $^8$Be* rapidly ($\tau_{1/2} = 4 \times 10^{-22}$ sec) breaks up into two $\alpha$-particles, with a Q-value of 2.94 MeV. As in the p-p-II chain, this series of reactions, termed the p-p-III chain, used an existing $\alpha$-particle is used as a catalyst, resulting in the net production of one $\alpha$-particle, two positrons, and three neutrinos and the destruction of four protons. A summary of the three p-p chains is presented in table 3.1.
Table 3.1: p-p chain reactions and energy released.

<table>
<thead>
<tr>
<th>Chain</th>
<th>Solar Branching %</th>
<th>Reactions</th>
<th>Q-Value (MeV)</th>
<th>Energy Loss %</th>
</tr>
</thead>
</table>
| p-p-I | 86                | p(p,e+ν)d  \\
d(ν,γ)³He  \\
³He(³He,2p)⁴He | 26.20        | 2.0           |
| p-p-II| 14                | p(p,e+ν)d  \\
d(ν,γ)³He  \\
³He(α,γ)⁷Be  \\
⁷Be(e-,ν)⁷Li  \\
⁷Li(ν,α)α   | 25.66        | 4.0           |
| p-p-III| .02              | p(p,e+ν)d  \\
d(ν,γ)³He  \\
³He(α,γ)⁷Be  \\
⁷Be(p,γ)⁸B  \\
⁸B(e+ν)⁸Be^+  \\
⁸Be^+(α)α | 19.17        | 28.3          |

CNO cycles

In second generation or later (Pop I) stars there are some metals produced by first generation stars. If the star is massive enough (around 1.4\(M_\odot\)) a high enough temperature will be reached in the core to allow for additional paths of hydrogen burning called the CNO cycles. At temperatures above 20 × 10⁶K with 1% of the C, N, or O nuclei present. the CNO cycle becomes more efficient at energy generation than the p-p chains.

Since carbon is the most abundant of these nuclei, the process begins with proton capture on carbon, initiating the following series of reactions:

\[
^{12}\text{C}(p,γ)^{13}\text{N}(e^+ν)^{13}\text{C} (p,γ)^{14}\text{N}(p,γ)^{15}\text{O}(e^+ν)^{15}\text{N} (p,α)^{12}\text{C}
\]
The net result of this is that 4 protons are converted into an alpha particle, 2 positrons and 2 neutrinos, with a Q value of 26.73 MeV. The neutrinos produced have low energy, leaving most of the energy reabsorbed by the star.

It is possible to "break out" of the CNO cycle via $^{15}\text{N}(p,\gamma)^{16}\text{O}$. However, this reaction normally gets returned to the main CNO cycle by $^{16}\text{N}(p,\gamma)^{17}\text{F} (e^+\nu)^{17}\text{O}(p,\alpha)^{14}\text{N}$. These two cycles together are called the CNO bi-cycle, and shown in figure 3.1

![The CNO bi-cycle](image)

Figure 3.1: The reactions involved in the CNO bi-cycle.

There are additional break-out branches from the CNO bi-cycle into other cycles. These branches produce small amounts of heavier nuclei but release little energy compared to the main CNO bi-cycle. At higher temperatures, the nuclei in the CNO
bi-cycle such as $^{13}\text{N}$ do not undergo beta decay but instead proton capture via reactions such as $^{13}\text{N}(p,\gamma)^{14}\text{O}$. beginning the hot CNO (HCNO) cycle. As temperatures increase, other reactions, such as the NeNa and MgAl cycles, occur. For a further discussion the reader is referred to [Rolfs and Rodney 1988].

3.2.2 Helium Burning

Ultimately, the core of a star becomes depleted of hydrogen, leaving mostly helium along with a smattering of heavier elements. With the loss of thermal pressure from the hydrogen burning reactions the core again begins to contract due to gravity. Hydrogen burning still continues in a shell around the core, which expands due to the inner part of the shell being heated by the helium core, and hence the star becomes a red giant. When the inner core of the star reaches a temperature around $100 - 200 \times 10^6\text{K}$, and a density of $10^2 - 10^5 \text{g/cm}^3$, helium burning can occur.

There is a barrier to helium burning in that $^8\text{Be}$ is unstable, decaying to two $\alpha$ particles. However, it does have a long enough lifetime ($\tau_{1/2} \approx 10^{-16}\text{sec}$) to enable some $^8\text{Be}$ to exist in equilibrium with the $^4\text{He}$ nuclei. Occasionally, an additional $\alpha$ particle is captured by the $^8\text{Be}$ nucleus in the “triple-\(\alpha\) process” to create $^{12}\text{C}$. While such a reaction would normally be unlikely, it is greatly enhanced by an s-wave resonance near the $^8\text{Be} + \alpha$ threshold. Occasionally, this excited state will decay back to the $^{12}\text{C}$ ground state, releasing 7.65 MeV of energy. Since the 7.65 MeV state is 0+ and the ground state is also 0+, the decay typically proceeds through a 2+
excited state at 4.43 MeV, with a very small branching directly to the ground state through pair emission.

$^{12}$C can react further with an alpha particle through $^{12}$C($\alpha,\gamma$)$^{16}$O. The reaction proceeds through two subthreshold resonances in $^{16}$O along with a direct component. Further alpha capture through $^{16}$O($\alpha,\gamma$)$^{20}$Ne is suppressed by the reaction's higher Coulomb barrier, low Q-value, and lack of resonances, effectively stopping the helium burning process with $^{16}$O. For further details see [Rolfs and Rodney 1988] or Hale in [Wallerstein, et al. 1997].

3.2.3 Carbon-Oxygen-Neon Burning

Eventually, helium becomes exhausted in the stellar core, initiating further gravitational contraction. If the star has low mass, the contraction will be halted by electron degeneracy pressure and it will become a carbon-oxygen white dwarf. If the star is larger than 8-10 $M_\odot$, the gravitational collapse is too strong and the carbon-oxygen core will contract further. Carbon will begin to burn at temperatures around $8 \times 10^9$K and densities of $3 \times 10^8$ g/cm$^3$. Carbon will burn before oxygen, since the $^{12}$C + $^{12}$C reaction has the lowest Coulomb barrier of the possible reactions. Three main strong carbon burning reactions can occur: $^{12}$C($^{12}$C,$\alpha$)$^{20}$Ne ($Q = 4.62$ MeV), $^{12}$C($^{12}$C,$p$)$^{23}$Na ($Q = 2.24$ MeV), and $^{12}$C($^{12}$C,$n$)$^{23}$Mg ($Q = -2.60$ MeV). $^{12}$C + $^{16}$O will also occur but at a much lower rate due to the higher Coulomb barrier.
Eventually this carbon fuel is exhausted and the core contracts further, increasing its density and temperature. When the temperature has risen to about $1 \times 10^9$ K, photodissociation will begin to occur on $^{20}\text{Ne}$, splitting off an $\alpha$-particle via $^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$. This liberated $\alpha$-particle then reacts with another $^{20}\text{Ne}$ nucleus via $^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$, resulting in an oxygen-manganese core. The temperature of the core increases further until oxygen burning occurs at $2 \times 10^9$ K, which produces mainly silicon and sulfur along with several other nuclei. For further details see [Rolfs and Rodney 1988] or Barnes in [Wallerstein, et al. 1997]. At the end of this phase a $^{28}\text{Si}$ core remains.

### 3.2.4 Silicon Burning

The reaction $^{28}\text{Si} + ^{28}\text{Si}$ does not occur because of its high Coulomb barrier. Instead, as the temperature rises any remaining nuclei lighter than $^{28}\text{Si}$ get photodissociated, with the free protons, neutrons and $\alpha$-particles combining with the lighter nuclei to produce more silicon in the core. Then, at a temperature of about $3.5 \times 10^9$ K, $^{28}\text{Si}$ itself begins to be dissociated by ($\gamma,\alpha$) reactions. The free $\alpha$-particles are then captured by other $^{28}\text{Si}$ nuclei in a series of ($\alpha,\gamma$) reactions proceeding to nickel:

$$^{28}\text{Si}(\alpha,\gamma)^{32}\text{S}(\alpha,\gamma)^{36}\text{Ar}(\alpha,\gamma)^{40}\text{Ca}(\alpha,\gamma)^{44}\text{Ti}(\alpha,\gamma)^{48}\text{Cr}(\alpha,\gamma)^{52}\text{Fe}(\alpha,\gamma)^{56}\text{Ni}$$

This is an oversimplification, as other reactions involving protons, neutrons, $\alpha$-particles, and photons also are involved, as the entire core undergoes a network of energy releasing reactions. For further details see [Rolfs and Rodney 1988] or Barnes in [Wallerstein, et al. 1997].
3.2.5 Supernova Explosion

After the $^{28}\text{Si}$ in the core is exhausted, the star is left with an iron-nickel core. The outer layers of the star are still undergoing the earlier stages of stellar burning, giving the star a characteristic “onion” structure as shown in figure 3.2. With the end of silicon burning, the core resumes its collapse. The details of this collapse are complex depending upon the stellar parameters and are beyond the scope of this work to discuss here. A general overview is contained in [Rolfs and Rodney 1988].

![Onion structure of a pre-supernova star with main burning reactions indicated](image)

Figure 3.2: “Onion” structure of a pre-supernova star with main burning reactions indicated. (not to scale)

In brief, further element synthesis does not occur because the binding energy per nucleon is maximized: that is, further alpha or proton capture to halt the collapse does not occur because these reactions have a negative Q-value. The lack of radiative pressure results in a loss of hydrostatic equilibrium in the core and it collapses, with
the core temperature continuing to rise. Eventually, high energy photons dissociate all elements back to protons and neutrons. The core collapse ends in a neutron star or black hole. The mechanics of this collapse may cause a shock wave that heats up the outer regions of the core creating new "explosive burning" processes of nucleosynthesis in the outer stellar shells.

3.3 R-Process

It is in this collapse that many believe some of the elements heavier than iron are synthesised. One suggestion, put forward initially in [Burbidge, et al. 1957] was that of a large flux of neutrons reacting on "seed nuclei" (primarily $^{56}$Fe) in a series of rapid neutron capture reactions termed the r-process. General overviews of the r-process occurs in [Rolfs and Rodney 1988], [Meyer 1994], and Hoffman and Timmes in [Wallerstein, et al. 1997]. In this environment, the density of neutrons is very high, so the time scale for a neutron capture on most nuclei is much less than the time scale for a $\beta$-decay. The reaction pathway of this process therefore occurs far from the region of the chart of the nuclides where stable nuclei exist. However, at a certain point, either a nucleus is formed that has a short half-life and $\beta$-decays before a neutron capture can occur, or, alternately, a nucleus is formed in which the last neutron is so weakly bound that it is quickly photodissociated by thermal photons in the high temperature environment in which the r-process occurs. Eventually, these nuclei will $\beta$-decay. The $\beta$-decay product then can undergo further neutron captures. Since, in this environment, $\beta$-decays are slow compared to neutron captures, material builds up
at these waiting points. The longer the $\beta$-decay half-life, the more material builds up. When the neutron captures cease, the nuclei produced $\beta$-decay back to stable nuclei. If the system is reaches equilibrium between the ($n,\gamma$) and ($\gamma,n$) reactions during the r-processing, the ratios of these waiting point nuclei will be determined only by the $\beta$-decay lifetimes, and not by the strengths of the neutron capture or photodissociation reactions. The r-process will eventually terminate when the product fissions into lighter nuclei. This endpoint is not clearly known, but is believed to be somewhere near $A=270$. This means that the r-process can produce very heavy nuclei, such as $^{238}\text{U}$, $^{235}\text{U}$, and $^{232}\text{Th}$, that cannot be produced in the s-process, described in section 3.4.

The r-process likely does not occur in any stable environment because of the high temperatures and neutron densities required. One site that has been considered for the r-process is a supernova where the core collapses to a neutron star. This site is attractive because the material first undergoes photodissociation to neutrons and $\alpha$-particles. This process is more fully described in section 3.9 as it has been suggested as a mechanism for the production of some of the p-process nuclei. In this model, $\alpha$-particles and free neutrons undergo a series of reactions that result in a distribution of seed nuclei around $Z \approx 36-40$, $A \approx 100$. These serve as the seeds for neutron capture reactions.

Other sites have been considered, such as inhomegenous primordial nucleosynthesis, jets in stellar cores, binary interactions, explosive burning in helium or carbon supernovae shells, neutron star accretion disks, novae, etcetera.
3.4 S-Process

The r-process can only produce some of the nuclei heavier than iron. A second neutron capture process must exist and is termed the s-process. In this process the neutron capture rates are slower, but occur over a longer period of time. The s-process therefore proceeds through nuclei that are stable or long-lived, as shown in figure 3.3. If the lifetime is shorter than the time for neutron capture, the nucleus will eventually β-decay. The decay product may then capture another neutron. Also, to reproduce the observed s-process abundances, it likely does not occur in a single location, but in several different sites. A general overview of the s-process can be found in [Rolfs and Rodney 1988], [Meyer 1994], and Smith in [Wallerstein, et al. 1997]. The s-process proceeds to $^{209}$Bi, which is stable, while $^{210}$Bi decays via α-particle emission.

Since the s-process is a slowly occurring process, it must take place in a stable astrophysical environment(s). One site may originate with helium burning in the cores of massive stars where the reaction chain $^{14}$N(α,γ)$^{18}$F(β$^+$)$^{18}$O(α,γ)$^{22}$Ne(α,n)$^{25}$Mg can produce free neutrons to be used in an s-process, a mechanism which could allow the material produced to be ejected into the interstellar medium through strong winds from this star.

A second site may occur at the helium burning shell in asymptotic giant branch stars. These stars have convective helium and hydrogen shells, with a largely inert carbon-oxygen core. $^{13}$C is created in the hydrogen burning shell by $^{12}$C(p,γ)$^{13}$N(β$^+$)$^{13}$C. Mixing of this material into the helium burning region results in the reaction $^{13}$C(α,n)$^{16}$O,
Figure 3.3: Part of the s-process pathway, with selected s, r, and p process nuclei indicated. The r-process nuclei are likely produced by rapid neutron capture processes that β-decay back to stable nuclei. (from [Meyer 1994])
liberating neutrons. These neutrons can then be captured by other nuclei in an s-process.

3.5 P-Process Neccessity

Certain nuclei (called the “p-process” nuclei) on the proton-rich side of stability cannot be produced through the r-process or s-process. They are “blocked” by a stable nucleus on the beta decay path back from the r or s process nuclei path. However, these nuclei exist and can be observed on earth. Clearly, one or more astrophysical scenarios are needed to explain the production of the p-process nuclei in their observed abundances. Peaks in the distribution of the nuclei occur at the neutron closed shells \( N = 50, 82 \) and the closed proton shell \( Z = 50 \).

<table>
<thead>
<tr>
<th>Table 3.2: The p-process nuclei</th>
</tr>
</thead>
<tbody>
<tr>
<td>The p-process nuclei</td>
</tr>
<tr>
<td>( ^{74}\text{Se} )</td>
</tr>
<tr>
<td>( ^{106}\text{Cd} )</td>
</tr>
<tr>
<td>( ^{132}\text{Ba} )</td>
</tr>
<tr>
<td>( ^{168}\text{Yb} )</td>
</tr>
</tbody>
</table>

We need a model to synthesize the p-process nuclei (listed in table 3.2) in the abundances that are found in the solar system. There are 34 nuclei which are either partially or totally formed through the p-process. The task of devising a process to correctly reproduce 34 abundances is daunting. General discussions of the p-process are in [Meyer 1994] and Boyd in [Wallerstein, et al. 1997].
3.6 Proton Capture Sites

The obvious first process to consider is proton capture on a group of seed nuclei, analogous to either the r or s-processes (see sections 3.3 and 3.4). The first suggested site for the p-process was the hydrogen rich outer envelope of a massive star heated up during its supernova explosion. Details of such a site are in [Burbidge. et al. 1957], [Cameron 1957], and [Ito 1961]. The assumption is that the p-nuclei are produced by \((p,\gamma)\) reactions on a set of "seed" material which has already undergone r and s processing. The temperature required for this processing is high \((1 - 3 \times 10^9 \text{ K})\), with a range of temperatures needed to produce both the lower mass and higher mass p-nuclei. The greater the mass of the nucleus undergoing proton capture, the greater the temperature needed to penetrate the Coulomb barrier. However, the probability of photodisintegration by thermal photons through a \((\gamma,n)\) reaction is more likely than proton capture at temperatures around \(2.5 \times 10^9 \text{ K}\) for the heavier nuclei. Thus the higher mass nuclei cannot be produced through a pure proton capture process.

Since pure proton capture cannot produce all the p-nuclei, it suggests an alternative reaction path of photodissociation on a group of seed nuclei. Initially, the nuclei photodissociate by \((\gamma,n)\) reactions. When the products become proton rich, the \((\gamma,p)\) and \((\gamma,\alpha)\) reactions become stronger. If the temperature cools rapidly and does not reach equilibrium, the burning will be incomplete, leaving proton rich nuclei present.
In this scenario the \((p,\gamma)\) reactions produce the light p-nuclei and \((\gamma,n)\) reactions produce the heavier p-nuclei. [Audouze and Truran 1975] extended this to a reaction network of \((p,\gamma), (n,\gamma), (\alpha,\gamma), (\alpha,p), (\alpha,n),\) and \((p,n)\) reactions. They assumed temperatures of \(1 - 2 \times 10^9\)K, densities around \(10^4\) g/cm\(^3\), about 10 seconds of processing, and an initial distribution of matter equalling solar abundances of hydrogen, helium, and r and s process elements. With a lower temperature, the \((p,\gamma)\) and \((\gamma,n)\) reactions will not produce appreciable amounts of the p-process nuclei. While this model, within these parameters, is able to produce most of the p-nuclei, the light p-nuclei are underproduced. However [Woosley and Howard 1978] showed that such a site was not possible due to the parameters needed. Specifically, to achieve the required temperature of \(2 \times 10^9\)K for the 10 seconds needed to produce the p-process nuclei would require densities in excess of \(10^3\)g/cm\(^3\) even with the seed abundances proposed by [Audouze and Truran 1975] increased by a factor of 100. \(10^3\)g/cm\(^3\) is the approximate density in the helium-burning shells of stars. This seems to exclude the hydrogen-envelope as a site for producing an appreciable amount of the p-process nuclei.

### 3.7 The \(\gamma\)-process

Another possible synthesis mechanism for the p-process nuclei is the \(\gamma\)-process. It is postulated to occur in a high temperature \((2 - 3 \times 10^9)\)K environment, which provides a source of high energy thermal photons to cause photodissociation reactions. A series of \((\gamma,n), (\gamma,p),\) and \((\gamma,\alpha)\) reactions then begins on heavy elements produced
by the r and s process. Initially, \((\gamma,n)\) reactions predominate, since neutron emission does not have a Coulomb barrier. As the nuclei get more proton rich, the \((\gamma,n)\) reactions either reach a nucleus that positron-decays or a nucleus where \((\gamma,p)\) or \((\gamma,\alpha)\) reactions become energetically favored. If the nucleus has an even proton number, \((\gamma,\alpha)\) will likely compete with \((\gamma,n)\). while if it has an odd proton number, \((\gamma,p)\) may contribute. The reaction path here is critical, since this determines what nuclei are left to eventually positron-decay back to stability. Also [Woosley and Howard 1978]
assumed that the inverse reactions created by the liberated particles or any other sources were negligible. While this model succeeds in reproducing the heavier p-nuclei fairly well when one takes an average over a temperature range, it underproduces the light p-nuclei.

It has been suggested by [Arnould 1976] that the p-process nuclei may originate from the oxygen-neon shell in stars. [Arnould 1976] presented a model where an oxygen burning shell of a star could produce elements of the p-process nuclei if there were r and s process seeds at around solar abundances. The model primarily produces the heavier p-process nuclei, while underproducing the light ones. The model has a problem, however, in transporting the fragile p-nuclei from the shell where they were produced, out into the cosmos. [Woosley and Howard 1978] modified this model, choosing instead as a site a supernova explosion. This provided a transport on the p-process nuclei to the interstellar medium.

[Prantzos, et al. 1990] attempted to produce p-process nuclei through a model of the SN 1987A explosion. They were able to reproduce the p-process abundances
within a factor of 3 of solar abundances for about half of the p-process nuclei. The seed nuclei for the process came from an s-process originating during the star's helium burning phase. [Rayet, et al. 1990] modelled the process as a series of different temperature shells. [Rayet, et al. 1995] extended this modelling of Type II supernovae to stars in a range of 13-25 solar masses. These stars would have temperatures ranging from $1.8 - 3.3 \times 10^9$K in the oxygen-neon shell. The reaction network included not only the various $(\gamma,n)$, $(\gamma,p)$, $(\gamma,\alpha)$ reactions and their inverses, but also the carbon and oxygen burning reactions described in section 3.2.3. This complete network involves over 1000 nuclei and 10000 reactions, using experimental data when available (for neutron captures and some stable nuclei below iron) and theoretical Hauser-Feshbach formalism for those reactions that have not been measured. The seeds for the p-process are s-process nuclei produced from helium burning. In these models, however, the light p-nuclei are still generally underproduced.

3.8 RP-Process

[Wallace and Woosley 1981] outlined the rp-process, which is characterized by a series of rapid proton captures. If the temperature and/or density of a stellar region undergoing the HCNO cycle increases, eventually there is a "breakout" reaction of $^{15}\text{O}(\alpha,p)^{19}\text{Ne}$. Normally, the $^{19}\text{Ne}$ will positron-decay to $^{19}\text{F}$ and be recaptured by the HCNO cycle. At a certain temperature and density, the probability for proton capture on $^{19}\text{Ne}$ will become significant.
At this point, the nucleus does not return to the HCNO cycle but undergoes a series of rapid proton captures analogous to the r-process. The proton captures continue until one of two things happen: either the resulting nucleus formed by a \((p,\gamma)\) reaction would be unbound, which means the nucleus must wait to positron decay, or the nucleus is bound but has a very short half-life and positron decays before it can undergo a proton capture. This occurs for some of the heavier nuclei, which means that the rp-process may not extend to the proton drip line.

### 3.8.1 Low Temperature Scenarios

One site that has been considered for the rp-process is a situation at a "low" \((T \approx 0.75 \times 10^9 \text{K})\) temperatures with a 2\% concentration of metals as \(^{14}\text{O}\) and \(^{15}\text{O}\). [Wallace and Woosley 1981] showed that such a site leads to a series of rapid proton captures that mainly produce \(^{56}\text{Ni}\). The termination of the process at \(^{56}\text{Ni}\) is seen by examining the characteristics of this nucleus. It is a doubly magic nucleus, and has a half-life on earth of 6.1 days, decaying purely by electron capture. The half-life will be longer in an ionized environment since free electrons will have to be captured rather than bound ones. Destruction of \(^{56}\text{Ni}\) is difficult as a high Coulomb barrier impedes an \((\alpha,p)\) reaction while \(^{56}\text{Ni}(p,\gamma)^{57}\text{Cu}\) is impeded by photodissociation back to \(^{56}\text{Ni}\). After approximately 40 seconds of rp-processing, the \(^{14}\text{O}\) and \(^{15}\text{O}\) are destroyed. If the processing times are lengthened to greater than 200 sec, elements heavier than \(^{56}\text{Ni}\) will begin to be produced through \(^{56}\text{Ni}(p,\gamma)^{57}\text{Cu}\).
3.8.2 Dwarfs/Novae

Another possibility has been suggested by [Howard, et. al 1991], where the $\gamma$-process occurs during the explosion of a carbon-oxygen white dwarf. The advantage of this site is that several flashes of helium burning can serve as the source of the $s$-process nuclei which act as the seeds for the $p$-process. This can allow for concentrations of $s$-process nuclei in very high abundances. Near the surface of the carbon-oxygen core there is a region between $2.4 - 3.2 \times 10^9$K where approximately 1 second of burning can make the $p$-nuclei. The carbon burning results in liberating protons which can be captured by the seed nuclei through a series of $(p,\gamma)$ reactions to produce the light $p$-nuclei. One example is

$$^{86}\text{Kr}(p,\gamma)^{87}\text{Rb}(p,\gamma)^{88}\text{Sr}(p,\gamma)^{89}\text{Y}(p,\gamma)^{90}\text{Zr}(p,\gamma)^{91}\text{Nb}(p,\gamma)^{92}\text{Mo}$$

which produces $^{90}$Zr and some $^{92}$Mo.

Additionally, photodissociation by $(\gamma,n)$ contributes via reactions such as

$$^{98}\text{Mo}(\gamma,n)^{97}\text{Mo}(\gamma,n)^{96}\text{Mo}(\gamma,n)^{95}\text{Mo}(\gamma,n)^{94}\text{Mo}(\gamma,n)^{93}\text{Mo}(\gamma,n)^{92}\text{Mo}$$

which synthesizes $^{92}$Mo and $^{94}$Mo. Other nuclei such as $^{78}$Kr, $^{96}$Ru, and $^{106}$Cd are also produced through proton capture.

The final abundance of the $p$-process isotopes produced depends upon the actual temperature dependence of the areas along with the precise nuclear reactions and other parameters. For these parameter ranges chosen however, this site underproduces the heavier $p$-nuclei relative to the lighter nuclei.
3.8.3 Thorne-Zytkow Objects

Another possible site for the rp-process concerns Thorne-Zytkow objects, stars with degenerate neutron star cores described in [Thorne and Zytkow 1977]. It has been shown by [Biehle 1991] that such stars with a total mass of over $12M_\odot$ could be stable through an rp-process of burning. In this situation $^{12}$C, $^{14}$N, and $^{16}$O act as seeds for a series of proton captures.

<table>
<thead>
<tr>
<th>Seed Nucleus</th>
<th>Product</th>
<th>Daughter</th>
<th>Mean Lifetime (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C</td>
<td>$^{18}$Ne</td>
<td>$^{18}$F</td>
<td>2.41</td>
</tr>
<tr>
<td>$^{18}$F</td>
<td>$^{22}$Mg</td>
<td>$^{22}$Na</td>
<td>5.57</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>$^{26}$Si</td>
<td>$^{26}$Al</td>
<td>3.17</td>
</tr>
<tr>
<td>$^{26}$Al</td>
<td>$^{30}$S</td>
<td>$^{30}$P</td>
<td>1.70</td>
</tr>
<tr>
<td>$^{30}$P</td>
<td>$^{68}$Se</td>
<td>$^{68}$As</td>
<td>140</td>
</tr>
<tr>
<td>$^{68}$As</td>
<td>$^{72}$Kr</td>
<td>$^{72}$Br</td>
<td>25.1</td>
</tr>
<tr>
<td>$^{72}$Br</td>
<td>$^{77}$Sr</td>
<td>$^{77}$Rb</td>
<td>13</td>
</tr>
<tr>
<td>$^{77}$Rb</td>
<td>$^{81}$Zr</td>
<td>$^{81}$Y</td>
<td>900</td>
</tr>
<tr>
<td>$^{81}$Y</td>
<td>$^{82}$Zr</td>
<td>$^{82}$Y</td>
<td>800</td>
</tr>
<tr>
<td>$^{82}$Y</td>
<td>$^{86}$Mo</td>
<td>$^{86}$Nb</td>
<td>30</td>
</tr>
<tr>
<td>$^{86}$Nb</td>
<td>$^{91}$Ru</td>
<td>$^{91}$Tc</td>
<td>30</td>
</tr>
<tr>
<td>$^{91}$Tc</td>
<td>$^{96}$Pd</td>
<td>$^{96}$Rh, $^{96}$Ru</td>
<td>end</td>
</tr>
</tbody>
</table>

Table 3.3: rp-process with $^{12}$C seed and waiting points.

Thorne-Zytkow objects have a hydrogen burning envelope with a degenerate neutron core. This core heats the inner layers of the envelope to around $10^9$K. Such a star is highly convective, with mixing of the layers. The very hot inner envelope of the star acts as a site for the rp-burning reactions summarized in table 3.3. Eventually, the $(p,\gamma)$ reactions are halted because the resulting nucleus is unbound, and the nucleus waits to positron decay. Nuclei which decay quickly (on the order of a few seconds)
remain for further processing. Longer lived nuclei such as $^{68}$Se ($\tau_{1/2} = 140$ sec), $^{81}$Zr ($\tau_{1/2} = 900$ sec), and $^{82}$Zr ($\tau_{1/2} = 800$ sec) tend to be swept out by the convective currents into the cooler regions of the envelope, stopping most of the rp-process at $^{68}$Se. However, the convection does carry some daughters back into the burning region where they can undergo further processing, becoming, in effect, a series of rp-processes. This synthesizes appreciable amounts of the lighter p-process nuclei.

Thorne-Zytkow objects are still speculative. No accepted observations of one have been made. One could search for lines of molecules of rp-process hydrides, such as $^{84}$SrH, in the spectra of giant stars, but such abundances have not yet been seen.

3.9 The $\alpha$-Process

Another possibility is the $\alpha$-process suggested by [Woosley and Hoffman 1992]. This is also a possible site for the r-process (see section 3.3). Here, material undergoes photodissociation to $\alpha$-partices and neutrons. The material then expands and cools so rapidly that equilibrium is never reached, leaving element abundences mostly consisting of nuclei between krypton and zirconium around mass 100.

[Woosley, et al. 1994] and [Fuller and Meyer 1995] considered a site of a supernova collapse with a neutron star core. In this scenario, a neutrino wind occurs for a few (1-10) seconds following the creation of a neutron star after a supernova collapse. As this expands outward, the neutron to proton ratio is determined by the neutrino and antineutrino interactions on free protons and neutrons: $n(\nu_e,e^-)p$, $p(\bar{\nu}_e,e^+)n$. 

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In the supernova models of [Qian, et al. 1993] the electron antineutrinos have higher energy (16MeV vs 11MeV) giving an electron fraction \( Y_e = \frac{p}{n+p} = \frac{1}{1+\bar{\nu}_e/\nu_e} \) = 0.38 - 0.47. Once this region starts to expand and cool, the protons and neutrons rapidly combine to form \( \alpha \)-particles and neutrons, most free protons are absorbed in the \( \alpha \)-particles. Neutrino capture on \( \alpha \)-particles or heavier nuclei rapidly become minimal as the weak reactions freeze out. A network of \((\alpha,\gamma)\), \((n,\gamma)\), \((p,\gamma)\), \((p,n)\), \((\alpha,p)\), \((\alpha,n)\) reactions and their inverses ensues, ultimately producing many of the r-process nuclei. These models, however, are not successful at producing appreciable amounts of the p-process nuclei.

[Hoffman, et al. 1996] speculated that the electron fraction \( Y_e = \frac{p}{n+p} \) could be higher because of threshold effects of electron antineutrino capture by protons. If \( Y_e \) were raised to the range of .484 - .488, amounts of the light p-nuclei are created within the first few seconds of expansion through charged particle reactions. There is a threshold for the reaction of electron antineutrino capture on protons, which will increase the proton richness of the matter outflow. This change in \( Y_e \) will lead to production of the light p-nuclei \(^{74}\text{Se}, \, ^{78}\text{Kr}, \, ^{84}\text{Sr}, \, \) and \(^{92}\text{Mo}\). It is a very fine change, however, and these models depend very critically on the \( Y_e \) parameter. Also, they do not seem to be able to synthesize some of the other p-nuclei, such as \(^{94}\text{Mo}, \, ^{96}\text{Ru}, \) or \(^{98}\text{Ru}, \) in their observed abundances.
3.10 The $\nu$-process

Certain p-process nuclei, such as $^{138}$La and $^{180}$Ta, are not produced well by any of the mechanisms heretofore discussed. A possible method to synthesize these nuclei is a $\nu$-process where a large flux of neutrinos from a type II supernova explosion pass through the outer shells causing ($\nu$,p), ($\nu$,n), or ($\nu$,e) reactions. [Woosley, et al. 1990] has shown that such a process could produce solar abundances of $^{138}$La and $^{180}$Ta through ($\nu$,n) reactions off of the $^{139}$La and $^{181}$Ta present in the neon shell. There is also a contribution from $^{138}$Ba($\nu$,e)$^{138}$La. These nuclei are very difficult to produce in other scenarios, so a $\nu$-process would work well with one or more of the other scenarios to produce the p-process nuclei.
CHAPTER 4

EXPERIMENTAL PROCEDURE

Don't do it the way I did it.
Do it the way I meant it.

- Otto Preminger, movie director

Our data were collected at the 11MV FN Tandem Van de Graaff Accelerator at Notre Dame University in South Bend, Indiana. Two runs were carried out—the first in April/May 1997 and the second in September 1997. The first experiment measured the $^{112}$Sn$(p,\gamma)^{113}$Sb reaction over a range of incident proton energies from 3.5 to 8.5 MeV. The second experiment improved the experimental technique with this target, enabling us to reduce the minimum incident proton energy to 2.8 MeV. We also measured the $^{119}$Sn$(p,\gamma)^{120}$Sb reaction with incident proton energies of 2.8 to 6.5 MeV and the $^{96}$Zr$(p,\gamma)^{97}$Nb reaction with incident proton energies from 3.5 to 6.0 MeV. This chapter provides a discussion of the apparatus and methods used to obtain the experimental data.
4.1 Accelerator Facility

The Notre Dame accelerator facility is shown in figure 4.1 [NDwebsite]. The targets were irradiated in the general purpose area (indicated in figure 4.1 by X). After the irradiation was completed, the targets were taken to an area where they were placed between two bismuth germinate oxide (BGO) detectors (indicated by *) or next to one high-purity germanium (HPGe) detector (indicated by **). The detectors were at a different location than the irradiation area since we wished to protect the detectors from radiation by the proton beam or by short lived products from the target irradiation. Also by having a separate detector area, we were able to place the detectors closer to the target and subtend a larger solid angle. A Van de Graaff accelerator was an appropriate choice for this experiment since we were interested in having a beam with a well defined energy, enough beam time to do repeated irradiations, but were less concerned with having a very intense or high energy beam.

4.1.1 Ion Source

The Notre Dame facility uses a SNICS (Source of Negative Ions by Cesium Sputtering) source. The source is briefly described at [NECwebsite] and in [Rolfs and Rodney 1988]. A cesium oven produces a vapor which condenses on a hot tungsten surface. The cesium that condenses becomes positively charged. Cesium is used because its low ionization energy ($W_i = 3.89$ eV) makes it extremely susceptible to ionization when condensing on a material like tungsten having a high
work function ($W_a = 4.53 \text{ eV}$) and is very accepting of electrons. This is called the Langmuir effect, and the ratio of positive ions $N_p$ to neutral ions $N_0$ is given in [Rolfs and Rodney 1988] as:

$$\frac{N_p}{N_0} = e^{-\frac{W_a - W_F}{kT}}$$  \hspace{1cm} (4.1)

This results in a factor of 355 for a surface heated to a temperature of 1000 C. The ionized cesium is extracted and is attracted toward a cathode. The atoms strike the cathode, causing it to sputter negative ions. These negative ions then enter the beam pipe. The low energy beam then is focused by an Einzel lens and a set of low energy steering quadrupole magnets as shown in figure 4.2.
Figure 4.2: Schematic of the FN Tandem Van de Graaff accelerator beamline. (diagram provided by Notre Dame University)
4.1.2 Tandems

After traversing the low energy section of the beam line, the negative ion beam enters the FN Tandem Van de Graaff as shown in figure 4.3. A general overview of the operation of Van de Graaff accelerators is contained in [Livingston and Blewett 1962] and [Bygrave, et al. 1970]. Briefly, electric charge is sprayed onto a belt which is carried to and collected by a high voltage terminal. To prevent corona discharges and sparks, the high voltage terminal is surrounded by a pressurized gas of nitrogen and carbon dioxide, along with sulfur-hexafluoride, which acts as an insulator. This enables the terminal to reach voltages in the MV range. The terminal reaches a voltage of \( V = \frac{Q}{C} \), where \( C \) is the terminal capacitance (150 \( \times 10^{-12} \) farads for this machine). To reach higher machine voltages, more charge is used. Charge must be continuously placed on the belt not only to replace the normal current leakage from the terminal, but also to replace the beam, which represents another portion of the leakage current.

The negative ion beam enters and is accelerated toward the terminal through a tube comprised of equipotential planes. The beam then passes through a carbon stripper foil which removes some of the electrons, creating a positive ion beam. The positive beam then is further accelerated through a second tube to a high velocity away from the terminal. In Van de Graaff accelerators, the beam is very stable. Measured terminal ripples were on the order of a few keV for our experiment, out of a beam energy of several MeV.
Figure 4.3: Schematic of the 11MV FN Tandem Van de Graaff at the University of Notre Dame. (diagram provided by Notre Dame University)
4.1.3 High Energy Beam Line

The positive ion beam created in the FN Tandem is steered through the beam line by a series of steering dipole and focusing quadrupole magnets as shown in figure 4.2. The main analyzing magnet is used to select both the type of beam and the beam energy. A switching magnet directs the beam down the appropriate beam line. The high energy steerers are used to make fine adjustments and thus tune the beam.

4.1.4 Target Area

The beam passed through a 6 mm collimator in the R2D2 chamber and then a 4 mm collimator in the target chamber before it impacted the target as shown in figure 4.4. The collimators were there to prevent the beam from hitting the target frame and beam walls during tuning. Irradiation of the aluminum target frame can create long lived isotopes such as $^{22}$Na, which would provide an additional source of background radiation.

The target was mounted in a target holder placed upon an electrically grounded movable metal rod. This provided the ability to have the beam spot strike different places on the target. The first irradiation of the $^{112}$Sn target, which was at a current of a few microamps, accidentally melted a small hole in the target. The surface area of the target was large enough that we were able to avoid the hole for subsequent
runs. We reduced the current to under 200 nanoamps to avoid melting the target further.

The beam exited the small target chamber down a length of beam pipe until it came to a Faraday cup, which also doubled as a beam stop. The Faraday cup provided a current reading. The Faraday cup was insulated and had a suppressor ring -300 Volts in front to repel electrons which might be knocked off the Faraday Cup by the incident proton beam. If the Faraday cup was not suppressed, these electrons could travel down the beam line and strike the Faraday cup, which would mean that the current reading from the cup would be smaller than the beam current striking the target.

Figure 4.4: Schematic layout of target chamber (not to scale).
4.1.5 Optical Alignment

The beam spot was approximately 2-3 mm in diameter when it struck the target, as shown by the diameter of the hole burned in the $^{112}$Sn target. It was important to align the beamline and collimators properly, which was done using a target blank and an optical telescope at the end of the beamline. The exact distance from the top of the target rod to the center of the blank was noted. This enabled us to move the target spot to different areas of the target in the case of carbon buildup (see section 5.3.2) or if we damaged a portion of the target as noted above. Additionally, the Faraday cup at the end of the beam line was heavy and weighed down the beam line. Thus, care had to be taken to ensure that adequate support was under the beam pipe after the target chamber to prevent it from sagging.

4.1.6 Beam Tuning

Once we had optically aligned the beamline, we had to align the beam. We inserted a blank target (which was essentially a large hole) in the chamber and brought the terminal up to the required voltage. Then we injected beam into the beam line seeking to maximize the amount of current reaching the Faraday cup while minimizing the current on the collimators. Initially we had a significant amount of beam striking a section of the beam pipe upstream from the R2D2 chamber. This triggered $\gamma$-ray detectors in the room. The section of beam being irradiated was a “nose” used to reduce the beam pipe’s diameter. This unwanted irradiation occurred because the
magnets' location had been selected to produce a beam focus in the center of the large R2D2 chamber. Our target was about another 3 meters downstream, with the Faraday cup being approximately an additional 1.5 meters further. To address this we moved the final steering magnet closer to the R2D2 chamber along with moving the "nose" further downstream. This enabled us to reduce the amount of beam striking the beam pipe to a negligible level. Even with this change however, the last set of quadrupoles were required to be slightly defocused to maximize the Faraday cup current and achieve a reasonable beam focus. This seemed to have the effect of making the beam very sensitive to the magnet settings and exacerbated the problem of beam scattering off the target detailed in section 4.1.7. Despite these changes, the beam quality was good, with the measured current from the collimators being only a few nanoamps at most.

4.1.7 Beam Current

The beam impacting the target typically had a current in the range of fifty to two hundred nanoamps. We attempted to measure the beam current on the target by using a Brookhaven Current Integrator (BCI) reading obtained from a Faraday cup at the end of the beamline. The BCI outputted a pulse each time a certain amount of charge was collected. The charge scale was selectable by the user, but we typically used a $10^{-7}$ scale, which resulted in 100 pulses for each $10^{-7}$ Coulombs of charge collected. The BCI was outputted to a scalar and readings were taken from the scalar at regular intervals. For the $^{112}$Sn target (producing $^{113}$Sb with $t_{1/2} = 6.7$ minutes),
we took readings at 3 minute intervals. For the $^{119}$Sn target (producing $^{120}$Sb with $t_{1/2} = 15.9$ minutes) and $^{96}$Zr target (producing $^{97}$Nb with $t_{1/2} = 72$ minutes), we used 5 minute intervals. Dividing the scalar readings by the time interval gave an average current reading for that interval. This procedure of multiple readings was done to determine the degree of variation in the beam current during the irradiation. Our readings showed that the beam current was stable to within 3% for the intervals used.

We had intended this measurement to be the current measurement as well. However we discovered during the first experiment that a significant fraction of the beam was scattering off the target and striking the beam pipe before reaching the Faraday cup. The amount of beam scatter depended on the energy of the proton beam and varied from 20% of the beam being scattered up to 65% for the lowest incident proton energies. This amount of beam scatter also depended strongly upon the specific beam tune, perhaps because we were far from the "optimized" center of the R2D2 chamber.

We were able to obtain an accurate current reading through a measurement of the current before and after the target was irradiated. The current reading after the target was irradiated was used as the current measurement. Measurements taken during the run showed that the current was very stable during the run, with the average current varying by less than 3% for the 3 or 5 minute integration intervals we used.

During the second experiment we moved the Faraday cup closer to the target. However, because of the need to isolate the Faraday cup, we were limited in how
close it could be located to the target chamber. Therefore, we were not able to reduce appreciably the fraction of the beam striking the beam pipe prior to the Faraday cup. Hence, we used the same procedure to determine the beam current – taking a reading of the BCI scalar output with the target removed after each data run. We continued to record the BCI reading during the target run in case the beam current became unstable. As before, the average current variations were less than 3% for the 3 or 5 minute integration intervals we used.

4.2 Targets

The targets used in this experiment and some of their relevant properties are listed in table 4.1. The $^{112}$Sn and $^{119}$Sn targets were borrowed from the Indiana University Cyclotron Facility (IUCF). The $^{12}$C target was borrowed from Notre Dame University.
to estimate the contribution to background from carbon buildup (see section 5.3.2). Enriched $^{96}$Zr foil was purchased from the Trace Sciences Corporation, cut in two pieces and mounted on target holders. This produced two identical $^{96}$Zr targets.

The experimental data were obtained from the $^{112}$Sn, $^{119}$Sn, and $^{96}$Zr targets. We chose these targets because they had $(p,\gamma)$ reaction products with half-lives that were from a few minutes to a little over an hour, enabling us to detect the product but yet have multiple irradiations of the target. They also had to have a $\gamma$-ray or positron emission that could be detected by the BGOs (greater than around 250 keV) and were chosen so background $(p,n)$ reactions would not swamp the $(p,\gamma)$ decays.

4.3 $\gamma$-ray interactions in detectors

Interactions of $\gamma$-rays with matter are described in [Siegbahn 1955] and [Segre 1964]. There are three possible interactions that the incident $\gamma$-rays can have with the detector. The most important for our purposes is the photoelectric effect. In this interaction the photon is absorbed by an atom in the material, resulting in the emission of an electron from one of the outer shells. Due to the conservation of energy, this electron carries the energy of the incident photon less its binding energy. In the case of a scintillation detector as described by [Bicron Brochure], this electron is reabsorbed in the material and its energy, along with the residual binding energy, is reabsorbed in the scintillation material becoming part of the photopeak. The photopeak energy then, is equal to the energy of the incident photon.
Another possible γ-ray interaction is Compton scattering, where an incident photon scatters off an electron, losing some of its energy in the process. The scattered photon may further interact in the detector, eventually being absorbed through the photoelectric effect. This adds to the photopeak. Conversely the Compton scattered photon may escape the detector, resulting in a signal in the detector proportional to the energy deposited.

For photons with energies above 1022 keV, it is possible for pair production to occur, where the photon radiates energy through production of an electron-positron pair. The positron annihilates with an electron, giving rise to an additional two 511 keV γ-rays. These γ-rays can either be photoelectrically absorbed or Compton scattered. However, pair production does not become significant until the γ-ray energy reaches a few MeV. Very high energy γ-rays can produce an electromagnetic shower of several electron-positron pairs. None of the reaction products we were interested in had significant γ-ray emissions at those energies.

### 4.4 BGO Detectors

Two bismuth germinate oxide (BGO) scintillation detectors were used in both experiments. Bismuth germinate oxide, or Bi$_4$Ge$_3$O$_{12}$, is well described in [Knoll 1979] and [Bicron Brochure]. Its high density (7.13 g/cm$^3$) and the presence of bismuth (Z=83) make BGO an efficient γ-ray absorber compared to other scintillators such as thallium-activated sodium iodide (NaI(Tl)), since the photoelectric cross section and
hence the photopeak efficiency is approximately proportional to $Z^4$. These characteristics made BGO an appropriate material to choose as a detector for this experiment.

The BGO collects the energy deposited in the crystal by the incident photon and reemits it through scintillation light. The BGO scintillation light is not reabsorbed by the crystal since the energy of the light photons is less than that needed to activate the crystal. One area where a BGO crystal is inferior to other organic scintillators such as NaI(Tl) is in its low scintillation efficiency. Scintillation efficiency is defined as the percentage of deposited particle energy that is reemitted as scintillation light. From [Knoll 1979], a NaI(Tl) crystal has a scintillation efficiency of about 13%. while a BGO has an efficiency of only about 1%. This means that a 1 MeV photon absorbed in the BGO will be converted to approximately 10 keV in scintillation photons. Since each photon has an energy of around 3 eV, this would result in approximately $3.3 \times 10^3$ photons produced.

To collect and amplify this small light signal, the BGO crystal is attached to a photomultiplier (PM) tube. The basic structure of a photomultiplier tube is shown in 4.5 and is well described in [Knoll 1979].

Photons scatter electrons from the charged surface of a semi-transparent photocathode. These electrons are attracted to the first dynode set at some positive voltage. Upon striking the dynode, the incident electron scatters several other electrons, which are in turn attracted to the second dynode. The number of electrons emitted per incident electron varies from material to material, but is typically around 10. A series of these dynodes, each connected to a consecutively larger voltage, is present in the
Figure 4.5: Photomultiplier tube structure. (from [Knoll 1979])
PM tube, thus the signal gets amplified multiple times. The photomultiplier tubes in our BGO detector outputted signals of about 3 millivolts when exposed to the 662 keV $\gamma$-rays from a $^{137}$Cs source.

4.4.1 Physical Setup

The target was placed between the two detectors for the first experiment as shown in figure 4.6 and for the second experiment as shown in figure 4.7.

![Diagram of the BGO detector set-up for the first experiment.](image)

Figure 4.6: Diagram of the BGO detector set-up for the first experiment. (not to scale)

For the first experiment, a spot was marked between the two detectors for consistency in the geometric set up. For the second experiment, we improved this by placing a lead brick with a groove cut in it between the detectors. Instead of trying
to place the target holder on a spot, it was placed in a groove just wide enough to accept the base of the target holder, reducing the positioning uncertainty. The BGOs were placed inside stainless steel tubes and then surrounded by 4 inches of lead on all sides. This arrangement was chosen on the basis of tests at The Ohio State University to minimize the lowest background contribution (see section 5.3.1). Around the lead shielding six plastic scintillators were used in an anti-coincidence arrangement to veto cosmic ray background events. This gave approximately a 10% further reduction in background over the lead and stainless steel shielding alone. As seen in figures 5.4, 5.5, and 5.8, the products of the $^{112}\text{Sn}(p,\gamma)$ and $^{119}\text{Sn}(p,\gamma)$ reactions, $^{113}\text{Sb}$ and $^{120}\text{Sb}$, both have positron emission as part of their decay scheme. While it is difficult to
directly detect the positron (see section 5.2.1) one can detect the 511 keV $\gamma$-rays produced by the eventual annihilation of the positrons with electrons. To facilitate this, .375 inch aluminum plates were placed between the detector and the target to absorb the positrons. After the first experiment, we decided that the minimal extra shielding provided by the second outer ring was not worth the difficulty of making the detecting geometry asymmetric. Also for the second experiment we moved the two BGO detectors closer together to increase the solid angle they subtended. This enabled us to obtain measurements for lower incident proton energies.

4.4.2 BGO Efficiency

A critical parameter in the experiment is the efficiency of the BGO detectors. To measure this, we conducted tests at The Ohio State University with the standard sources of $^{137}\text{Cs}$, $^{22}\text{Na}$, and $^{60}\text{Co}$. The sources were placed 60 cm from the face of the detector. Knowing the intensity of the sources in $\mu$Ci ($1 \mu$Ci = $3.7 \times 10^4$ decays/second) and the solid angle that the detector face subtends, one can calculate the photopeak efficiency of the BGO detector for $\gamma$-rays passing through it. Here $\text{Int}$ is the intensity, $t_0$ is the time when the source's intensity was last measured. $\text{Dec}_{\exp}$ is the number of decay particles that are incident on the detector surface. $br_{\gamma}$ is the branching ratio for the particular $\gamma$-ray of interest and $\Omega$ is the solid angle the detector subtends.

$$\text{Int}(t) = \text{Int}(t_0) \times e^{-\ln2 \times (t-t_0) / t_{1/2}} \quad (4.2)$$
\[ Dec_{\text{total}}/\text{sec} = \text{Intensity} \times br_\gamma \times 3.7 \times 10^4 \text{decays/\mu Ci} \quad (4.3) \]

\[ Dec_{\text{exp}}/\text{sec} = \frac{Dec_{\text{total}}/\text{sec}}{4\pi} \Omega \quad (4.4) \]

\[ \text{Eff} = \frac{Dec_{\text{exp}}}{Dec_{\text{meas}}} \quad (4.5) \]

By placing the source a large distance from the detectors, we were able to assume that the \( \gamma \)-rays were normally incident upon the detector face and passed through the full length of the detector. When we used the \(^{60}\text{Co}\) source we noted that the two photopeaks overlapped because of the resolution constraints of the BGOs. Thus we used three points in the energy calibration, two \( \gamma \)-rays from \(^{22}\text{Na}\) and one \( \gamma \)-ray from \(^{137}\text{Cs}\). The results are summarized in table 4.2.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Energy</th>
<th>Counts</th>
<th>Total ( \gamma )-rays Incident</th>
<th>Efficiency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{22}\text{Na})</td>
<td>511</td>
<td>34161</td>
<td>37942</td>
<td>90</td>
</tr>
<tr>
<td>(^{137}\text{Cs})</td>
<td>661.7</td>
<td>45737</td>
<td>586350</td>
<td>78</td>
</tr>
<tr>
<td>(^{22}\text{Na})</td>
<td>1274.5</td>
<td>12680</td>
<td>20801</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 4.2: Photopeak efficiency measurements for bismuth germinate oxide (BGO) detector.

However, in the experiment we have instances where the \( \gamma \)-rays are not normally incident on the detector face. To account for this we assumed that the probability of a \( \gamma \)-ray being absorbed in a section of detector varied exponentially with detector thickness. From our measurements we can calculate \( r_{\text{half}} \), the length of the BGO crystal which gives us a 50% chance of absorbing a \( \gamma \)-ray with a particular energy. This is given by the BGO length (\( BGO\text{L} \)) and the BGO efficiency (\( BGO\text{EFF} \)) measured above.
\[ r_{\text{half}} = -\ln 2 \times BGOL/\ln(1 - BGOEFF) \] (4.6)

Thus the chance that a \( \gamma \)-ray with a path length \( D \) is photoelectrically absorbed is:

\[ \text{abs} = 1 - e^{-\frac{\ln 2 \times D}{r_{\text{half}}}} \] (4.7)

where \( BGOEFF \), and hence \( r_{\text{half}} \), are functions of the \( \gamma \)-ray energy.

### 4.4.3 Aluminum Plates

The 511 keV \( \gamma \)-rays come from electron-positron annihilation (see section 5.2.1) which does not occur at the source. To stop the positrons before the detector we used .375 inch aluminum plates. These plates also had the effect of absorbing or scattering a percentage of the \( \gamma \)-rays. To measure the effect of the \( \gamma \)-ray absorption/scattering in the aluminum, we used the standard sources of \(^{22}\text{Na}, \, ^{137}\text{Cs}, \) and \(^{60}\text{Co}\) to measure the amount of absorption and scattering by different thicknesses of aluminum.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Energy</th>
<th>None</th>
<th>.25 in</th>
<th>.375 in</th>
<th>.625 in</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{22}\text{Na})</td>
<td>511</td>
<td>101463</td>
<td>92524</td>
<td>87688</td>
<td>77813</td>
<td>6816</td>
</tr>
<tr>
<td>(^{137}\text{Cs})</td>
<td>661.7</td>
<td>345749</td>
<td>316301</td>
<td>301630</td>
<td>272310</td>
<td>9223</td>
</tr>
<tr>
<td>(^{60}\text{Co})</td>
<td>1173</td>
<td>112429</td>
<td>104563</td>
<td>100421</td>
<td>92381</td>
<td>2043</td>
</tr>
<tr>
<td>(^{22}\text{Na})</td>
<td>1274.5</td>
<td>35826</td>
<td>33451</td>
<td>32195</td>
<td>29602</td>
<td>2516</td>
</tr>
<tr>
<td>(^{60}\text{Co})</td>
<td>1332</td>
<td>104536</td>
<td>97810</td>
<td>94526</td>
<td>87095</td>
<td>3063</td>
</tr>
</tbody>
</table>

Table 4.3: Absorption and scattering for standard \( \gamma \)-ray sources by several different thicknesses of aluminum.
Using the results in table 4.3 we fitted the absorption fractional loss ($frac_{loss}$) to the aluminum thickness ($x$) as a straight line fit for each energy.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Energy (keV)</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}$Na</td>
<td>511</td>
<td>$frac_{loss} = .400 \times -2.58 \times 10^{-3}$</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>661.7</td>
<td>$frac_{loss} = .349 \times x + 1.00 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1173</td>
<td>$frac_{loss} = .291 \times x - 5.09 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>1274.5</td>
<td>$frac_{loss} = .299 \times x - 1.66 \times 10^{-3}$</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1332</td>
<td>$frac_{loss} = .274 \times x - 1.54 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 4.4: Aluminum absorption fits for standard sources.

Now, if we now look at the overall energy dependence of these fits, we get an overall fit of $frac_{loss} = -0.0132E + 0.453x$, where $E$ is the $\gamma$-ray energy.

For the absorption of the positrons, we referred to a graph of penetration depth from [Marion 1968] shown in figure 4.8. From this we determined the penetration depth of the positrons into the aluminum. This also provided a check that there was sufficient aluminum to stop the most energetic positrons. A second check of this would be to look for a “tail” on the 511 keV $\gamma$-ray peak consisting of positrons that reach the detector and decay inside it, giving up their last bit of energy. One of the 511 keV $\gamma$-rays could be captured in the detector and add to the positron energy, causing a “tail” on the $\gamma$-ray peak. We saw no evidence of this tail in the data.

There is a geometrical effect that is present in the experiment as well. In the actual experiment, the $\gamma$-rays are not perpendicular to the detector or to the aluminum, some strike at more oblique angles. Additionally, the 511 keV $\gamma$-rays that are emitted through the electron-positron annihilation may either pass through a fraction of one
Figure 4.8: Penetration of electrons into aluminum. (Taken from [Marion 1968])
aluminum absorber, or through more than one, depending on where the positron is stopped. To take all of these effects into account we used a Monte Carlo simulation.

4.4.4 Monte Carlo Simulation

To calculate the full efficiency (detector plus geometric) of the BGOs for a specific $\gamma$-ray energy, a Monte Carlo simulation of the experiment was developed to calculate these efficiencies [Murphy and Chloupek 1997]. The target was treated as a point source set a distance from the detectors corresponding to the experimental set-ups shown in figures 4.6 and 4.7. Then the efficiency for each relevant $\gamma$-ray was calculated by simulating the emission of around $10^8$ $\gamma$-rays at the energy of interest, using a spherically isotropic random distribution of angles. For those $\gamma$-rays that intersected the detector, there was a possibility of absorption/scattering through the aluminum, and then a possibility of detection in the BGO as measured above. To figure out the possibility of detection, we used the measured value from equation 4.6 and assumed that the absorption probability varied exponentially with distance as given by equation 4.7. This gave us an overall efficiency for the gamma peaks.

The situation for the 511 keV $\gamma$-rays is slightly more complex. We first assumed an isotropic emission of around $10^8$ positrons at random angles, with energies randomly distributed according to the expected positron spectrum. The positron then traveled until it struck either the aluminum absorber or the lead shielding. It penetrated an average distance into the aluminum given by [Marion 1968] and annihilated with an electron, creating two 511 keV $\gamma$-rays emerging back to back at another randomly
selected angle. The γ-rays emerge back to back because of conservation of momentum. Then we calculated the probability of each photon being detected, giving us an efficiency for 511 keV γ-ray singles. The Monte Carlo program also tracked both γ-rays, allowing for the computation of efficiency for 511 keV γ-ray coincidences in both detectors.

<table>
<thead>
<tr>
<th>γ Energy (keV)</th>
<th>Expt 1 E1 Eff.%</th>
<th>Expt 1 E2 Eff.%</th>
<th>Expt 2 Eff.%</th>
</tr>
</thead>
<tbody>
<tr>
<td>334</td>
<td>5.20</td>
<td>4.63</td>
<td>9.74</td>
</tr>
<tr>
<td>498</td>
<td>4.69</td>
<td>4.17</td>
<td>8.68</td>
</tr>
<tr>
<td>511s (113Sb)</td>
<td>7.10</td>
<td>6.55</td>
<td>13.45</td>
</tr>
<tr>
<td>511s (120Sb)</td>
<td></td>
<td>13.42</td>
<td></td>
</tr>
<tr>
<td>935</td>
<td>3.82</td>
<td>3.42</td>
<td>6.97</td>
</tr>
<tr>
<td>940</td>
<td>3.82</td>
<td>3.42</td>
<td>6.96</td>
</tr>
<tr>
<td>1013</td>
<td>3.69</td>
<td>3.31</td>
<td>6.73</td>
</tr>
<tr>
<td>1018</td>
<td>3.69</td>
<td>3.31</td>
<td>6.71</td>
</tr>
<tr>
<td>511c (113Sb)</td>
<td>0.645</td>
<td>0.645</td>
<td>2.28</td>
</tr>
<tr>
<td>511c (120Sb)</td>
<td></td>
<td></td>
<td>2.30</td>
</tr>
</tbody>
</table>

Table 4.5: Photopeak efficiency measurements for the bismuth germinate oxide (BGO) detectors for selected γ-ray energies with geometric effects included. The 511s refers to 511 keV γ-ray singles, and 511c refers to 511 keV γ-ray coincidences in both detectors.

4.4.5 Electronics and Data Acquisition

For the first experiment, two signals were taken from the preamplifier base of each BGO – an anode signal and a cathode signal. The cathode signal was the signal read in by the data acquisition system. The anode signal was used to set up the anti-coincidence. In the second experiment, we had difficulty using the anode signal due
to its small amplitude and instead split the cathode signal. One effect of this was that the overall event signals were smaller in the second experiment even with maximum amplification, so that we were only using a portion of the available ADC channels. There were still an adequate number of ADC channels to resolve the individual decay branches. Each plastic scintillator also had a signal taken from its base. The signals were processed as indicated in diagram 4.9.

The raw cathode signals were negative and had amplitude of around a few millivolts (mV). The cathode signals were amplified using a gain of around 1000 and a shaping time of 1 μsec. Their polarity was also reversed to make them positive. The signals were then outputted into the system as ADC channels 1 and 2.

Both the BGO anode and plastic scintillator signals were shaped by timing filter amplifiers (TFAs) to give a signal with a fast rise time to use for the timing signals. The amplification gain used differed for the different plastics and anode signals due to different raw signals coming from the detectors, but ranged from gains of 1 to 20. Rise times of the output signals were in the range of a few nanoseconds, with fall times slightly longer. The signals were negative. The fast TFA signals were outputted to constant fraction discriminators (CFDs) which converted the signals into logic pulses. The logic pulses originating from the 6 plastic scintillators were summed by a Fan In Fan Out unit, creating one pulse so that a veto signal could be sent to the master strobe whenever the plastic signal occurred in coincidence with the summed BGO anode signals.

90
The output of the BGO anode signal from the CFD was inputted into a time-to-amplitude converter (TAC). One of the signals was fed in directly as the START signal. The second BGO signal was delayed about 5 microseconds and used as the STOP signal. The TAC converts the relative timing between the two signals into a positive proportionate electronic signal with an amplitude of a few volts. This signal was inputted as channel 0 on the ADC. In the case where there was a signal on E1 and not on E2, or vice versa, the TAC would fail to convert, giving no signal.

We also fed a clock signal into channel 3 of the ADC. Thus for each event recorded, we had 4 parameters: TAC signal, E1 signal, E2 signal, and clock. We recorded an event whenever the master strobe “event in” signal fired, where the overall trigger condition being that either of the BGO detectors having a signal, but no signal in any of the plastic scintillators.

4.4.6 Dead Time

The data acquisition system takes a finite time to process events. If an event occurs while the system is busy it will not be processed. To correct for this, we needed to know the dead time of the apparatus, i.e., how often the data acquisition system was busy and would not accept a signal. We determined this as shown in figure 4.10. The master strobe event in goes into a KS 2197 K-BUS. When the system is busy, a “busy out” signal is activated. The coincidence between that signal and a 100Hz oscillator is read into a quad scalar. Once every 5 minutes, as determined by an independent
Figure 4.9: Electronics diagram for experiment 1.
1000 KHz oscillator, both these numbers along with a clock reading were written to a file. The overall live time fraction was determined by equation 4.8.

\[ \text{frac}_{\text{live}} = 1 - \frac{t_{\text{busy}}}{t_{\text{tot}}} \]  

(4.8)

where \( t_{\text{busy}} \) is the time the apparatus was busy and \( t_{\text{tot}} \) is the total interval time.

---

**Figure 4.10:** Electronics diagram of timing information.
4.5 Hyperpure Germanium Detectors

We also made use of a hyperpure germanium (HPGe) detector. We took advantage of the resolution to identify the gamma spectra more accurately as described in section 5.2. Additionally, we used these detectors to measure the cross section of the (p,\gamma) reaction on \(^{96}\text{Zr}\). The \(^{96}\text{Zr}(p,n)\) reaction contributes to the overall \(\gamma\)-ray yield. The numerous \(\gamma\)-ray peaks make it impossible to resolve the \(^{97}\text{Nb}\) peak with the BGO detectors.

In a BGO, or any other scintillation counter, the energy resolution is somewhat poor [Knoll 1979]. The process of converting the radiation energy to a light pulse involves a number of steps and the pulse created consists of only a few thousand electrons, leading to large statistical fluctuations. As described in [Knoll 1979], solid state detectors, such as HPGe, use particle-hole pairs to convey the signal. The germanium must be refined to a very high purity to allow the charge carriers (which are either residual impurities or crystal defects) to penetrate the material. The detector crystal must be continuously cooled with liquid nitrogen to keep the leakage currents low. An advantage of HPGe detectors over doped germanium detectors, such as germanium doped with lithium, is that the HPGe may be stored at room temperature between uses.
4.5.1 HPGe Efficiency

The efficiency measurement of the HPGe detector was simpler than for the BGOs. Since the only relevant emission for $^{97}$Nb was a single $\gamma$-ray (no positron emission), we only were concerned with the efficiency for a single energy (658 keV) emitted at the point that the target was located at. To measure the efficiency, we took the standard sources of $^{137}$Cs, $^{22}$Na, and $^{60}$Co, and placed them at the same location the irradiated $^{96}$Zr target occupied.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Energy (keV)</th>
<th>Counts</th>
<th>Total $\gamma$ Incident</th>
<th>Efficiency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}$Na</td>
<td>511</td>
<td>494743</td>
<td>13976200</td>
<td>3.54</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>511</td>
<td>841720</td>
<td>31290000</td>
<td>2.69</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>661.7</td>
<td>693931</td>
<td>36328600</td>
<td>1.91</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>661.7</td>
<td>1390613</td>
<td>70770000</td>
<td>1.96</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1173.2</td>
<td>311743</td>
<td>21163067</td>
<td>1.47</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1173.2</td>
<td>567220</td>
<td>47380000</td>
<td>1.20</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>1274.5</td>
<td>129686</td>
<td>10672000</td>
<td>1.22</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>1274.5</td>
<td>223306</td>
<td>17400000</td>
<td>1.28</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1332.5</td>
<td>285690</td>
<td>21167533</td>
<td>1.35</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1332.5</td>
<td>517201</td>
<td>47390000</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 4.6: Efficiency measurements for hyperpure germanium detector.

As we see from the data in table 4.6, the efficiency measurements have significant errors in them, possibly due to the dead time corrections, since $frac{live}{active}$ given by equation 4.8 was between 0.1 and 0.5. However, because of the proximity of the 658 keV $^{97}$Nb $\gamma$-ray to the 661.7 keV $^{137}$Cs $\gamma$-ray, we simply used the efficiency measured from this source as the efficiency for the 658 keV $\gamma$-ray.
4.5.2 Electronics and Data Acquisition

One signal was outputted by the hyperpure germanium detector. The signal was divided with one half being amplified and then fed into the system as shown in figure 4.11. The other half was sent into a TFA, and then into a CFD to create a logic signal, which was then used to generate the “event in” signal.

![Electronics Diagram](image)

Figure 4.11: Electronics diagram for the hyperpure germanium arrangement.

4.6 Computer System

The signals were processed by a CAMAC data acquisition system. The CAMAC system was limited to processing around 6,000 signals per second, which led to significant dead times even at fairly low event levels.

The CAMAC signals were processed by the HHRIF data acquisition system, developed by the Holifield Radioactive Ion Beam Facility in Oak Ridge, TN. This data
acquisition system is described further in [Meïßner 1995]. The output of this was four data points for each event recorded: a TAC value, an E1 value, an E2 value, and a clock value. A program was written to convert these into ZEBRA n-tuples which we then analyzed using the CERN Physics Analysis Workstation (PAW) program, described in [CERN 1995].
CHAPTER 5

ANALYSIS AND RESULTS

All you have to do [to protect yourself from radiation] is go down to the bottom of your swimming pool and hold your breath.

- David Miller.
Department of Energy spokesperson

After the ZEBRA N-tuples have been written, the event by event data must be analyzed to extract the cross sections and astrophysical S-factors. This section discusses the procedure used to calculate the cross sections from the experimental data, discusses the sources of error and background, presents the results, and compares them to the NON-SMOKER theoretical predictions.
5.1 Binning and Cuts

The data were processed off-line using computer facilities at The Ohio State University. The event-by-event N-tuples were converted to histograms using the CERN Physics Analysis Workstation (PAW) software package [CERN 1995], dividing each data set into intervals coinciding with the dead time intervals described in section 4.4.6. For the $^{112}$Sn and $^{119}$Sn targets, four histograms were produced for each interval: a spectrum of single events in detector E1, a spectrum of single events in detector E2, a spectrum of events in detector E1 that had a positive TAC signal and coincided with events in the 498-511 keV peak in detector E2, and a spectrum of events in detector E2 that had a positive TAC signal and coincided with events in the 498-511 keV peak in detector E1. For the $^{96}$Zr target, only one spectrum was produced—that of events in the HPGe detector.

5.2 Spectrum Analysis

In order to analyze the emissions of the activated targets in detail, each of the targets was placed in front of a HPGe detector to identify the gamma peaks. The detector was calibrated using the standard sources of $^{137}$Cs, $^{22}$Na, and $^{60}$Co.

This gave us straight line fits (using the Physica fitting package described in [Chuma 1994]) of

$$E = [(1.6112 \pm .0014) \times Ch - (25.6 \pm .9)] MeV \quad (5.1)$$
Table 5.1: Calibration results from the hyperpure germanium detector.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Gamma Energy (keV)</th>
<th>Channel (Expt 1)</th>
<th>Channel (Expt 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}\text{Na}$</td>
<td>511.0</td>
<td>333.5</td>
<td>461</td>
</tr>
<tr>
<td>$^{137}\text{Cs}$</td>
<td>661.6</td>
<td>426</td>
<td>590.5</td>
</tr>
<tr>
<td>$^{60}\text{Co}$</td>
<td>1173.2</td>
<td>744</td>
<td>1031.5</td>
</tr>
<tr>
<td>$^{22}\text{Na}$</td>
<td>1274.5</td>
<td>807</td>
<td>1118.5</td>
</tr>
<tr>
<td>$^{60}\text{Co}$</td>
<td>1332.5</td>
<td>843</td>
<td>1169</td>
</tr>
</tbody>
</table>

\[ E = [(1.1605 \pm 0.0004) \times Ch - (23.8 \pm 4)] \text{MeV} \]  

for the second experiment. The fits had chi-squared per degree of freedom of 1.27 and 0.23 respectively.

### 5.2.1 Positron Emission

One might wonder why we only detected $\gamma$-rays. Two of the nuclei we were observing, $^{113}\text{Sb}$ and $^{120}\text{Sb}$, have positron emission as part of their decay scheme. The detector efficiency for the positrons should be higher than that for the $\gamma$-rays and make the positrons easier to detect. Additionally, unlike the $\gamma$-rays, the positrons are not emitted with a definite energy. They have a continuous spectrum of energies with a definite endpoint. This makes their identification more difficult, especially in a complex spectrum where several positrons, electrons, and photons are emitted.
Another difficulty is that the positron emitted eventually collides with an electron and annihilates, creating two 511 keV $\gamma$-rays. If the positron annihilation would occur in a BGO detector, one or both of the $\gamma$-rays may also be absorbed in the detector. This would result in not one positron spectrum, but three—the original spectrum, plus additional spectra of positron + a 511 keV $\gamma$-ray and positron + two 511 keV $\gamma$-rays. making the actual determination of the number of positrons emitted by the source very difficult. There is also an aluminum cover on the BGO which will absorb some fraction of the positrons, making it difficult to measure the detector efficiency.

Thus, we wished not only to ignore the positrons, but to stop them completely before they reach the BGOs, and detect instead the 511 keV $\gamma$-rays created from the electron-positron annihilation. To do this we used aluminum plates to stop the positrons as described in section 4.4.3.

5.2.2 $^{112}$Sn spectrum

For the first experiment we obtained a spectrum from the HPGe detector from the irradiated $^{112}$Sn target. These produced the spectra shown in figures 5.1, 5.2, and 5.3.

From these plots we were able to identify a number of $\gamma$-ray peaks from the $^{112}$Sn irradiation as shown in table 5.2. Note that the peak at 1009 keV is actually a 511 keV $\gamma$-ray resulting from a positron emission in coincidence with a subsequent 498 keV $\gamma$-ray from the decay of an excited state of $^{113}$Sn. Since the excited state decays very quickly, the two $\gamma$-rays add in the detector as a coincidence. As we see from the
Figure 5.1: Spectrum of the irradiated $^{112}$Sn target with from the HPGe detector.

decay scheme in figures 5.4 and 5.5, from [Table of Isotopes, 8th ed.] we expect this coincidence.

One might question whether this is a true coincidence or pile-up from two separate decays of $^{113}$Sb. We believe this to be a coincidence with negligible pile-up contribution because expected pile-up peaks at 996 ($498+498$) keV and 1022 ($511+511$) keV are not evident. These should appear if there is any noticeable pile-up contribution to the 1009 keV peak. However, there are no true coincidences in one detector between
two 498 keV $\gamma$-rays or two 511 keV $\gamma$-rays. Since, as shown in figure 5.3, these peaks are not evident, we find that all coincidences are real and that pile-up is negligible. The HPGe detector, while it had good resolution of the $\gamma$-ray peaks, had a much lower efficiency than the BGO detectors. In the experiments, the data from the $^{112}\text{Sn}$ and $^{118}\text{Sn}$ targets used the BGO detectors to take advantage of their efficiency.

There is an additional contribution to the irradiated $^{112}\text{Sn}$ spectrum from $^{113}\text{Sn}$. The $^{113}\text{Sb}$ produced by the $(p,\gamma)$ reaction decays to $^{113}\text{Sn}$ either by positron emission
Figure 5.3: Greatly expanded spectrum of irradiated $^{112}$Sn target from the HPGe detector.

or electron capture as shown in figure 5.6. $^{113}$Sn is itself radioactive with a halflife of 115 days, decaying to $^{113}$In with $\gamma$-ray emissions as shown in table 5.4 [NNDC website].
Table 5.2: Identifiable peaks from the HPGe detector for the irradiated $^{112}$Sn target. Notice the absence of peaks at 996 keV and 1022 keV, indicating that the 1009 keV peak is formed by a true coincidence of a 498 keV $\gamma$-ray with a 511 keV $\gamma$-ray and not pile-up.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Energy</th>
<th>Nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>221</td>
<td>334</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>258</td>
<td>392</td>
<td>$^{113}$Sn</td>
</tr>
<tr>
<td>325</td>
<td>498</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>333</td>
<td>511</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>392</td>
<td>603</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>499</td>
<td>779</td>
<td>unknown</td>
</tr>
<tr>
<td>597</td>
<td>935</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>600</td>
<td>940</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>642</td>
<td>1009</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>645</td>
<td>1013</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>648</td>
<td>1018</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>728</td>
<td>1147</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>923</td>
<td>1459</td>
<td>$^{113}$Sb</td>
</tr>
<tr>
<td>983</td>
<td>1557</td>
<td>$^{113}$Sb</td>
</tr>
</tbody>
</table>

5.2.3 $^{119}$Sn spectrum

A similar procedure was used to calibrate the energies of the $^{119}$Sn target peaks. Using a spectrum from a HPGe detector shown in figure 5.7 and equation 5.2 we were able to identify the following $\gamma$-rays as shown in table 5.5.

The complete $^{119}$Sn decay scheme is shown in figure 5.6 [NNDCwebsite].
5.2.4 $^{96}$Zr spectrum

The irradiated $^{96}$Zr spectrum, shown in figure 5.9 was more complex than the $^{112}$Sn or $^{118}$Sn spectra. Partly this was due to contamination from other zirconium isotopes in the target as shown in table 4.1, but mostly this was due to the $(p,n)$ reaction on $^{96}$Nb as can be seen in table 5.7.

The complete $^{97}$Nb decay scheme is shown in figure 5.10 [Table of Isotopes. 8th ed.] and the relevant $\gamma$-ray emissions are given in table 5.8 [NNDCwebsite].

We had attempted in the first experiment to measure the $^{96}$Zr$(p,\gamma)^{97}$Nb reaction using the BGOs, but were unable to separate the 658 keV $\gamma$-ray from $^{97}$Nb from
Figure 5.5: Decay scheme of $^{113}\text{Sb}$ – part 2.

Figure 5.6: Decay scheme of $^{113}\text{Sn}$. 

107
<table>
<thead>
<tr>
<th>γ Energy (keV)</th>
<th>γ Uncertainty (keV)</th>
<th>Branching Ratio</th>
<th>BR Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>332.41</td>
<td>0.05</td>
<td>14.8</td>
<td>0.8</td>
</tr>
<tr>
<td>497.96</td>
<td>0.09</td>
<td>80.0</td>
<td>2.0</td>
</tr>
<tr>
<td>511.137</td>
<td>137.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>708.40</td>
<td>0.10</td>
<td>0.40</td>
<td>0.03</td>
</tr>
<tr>
<td>935.77</td>
<td>0.06</td>
<td>1.71</td>
<td>0.10</td>
</tr>
<tr>
<td>940.63</td>
<td>0.06</td>
<td>2.62</td>
<td>0.15</td>
</tr>
<tr>
<td>1013.28</td>
<td>0.06</td>
<td>0.91</td>
<td>0.06</td>
</tr>
<tr>
<td>1018.12</td>
<td>0.06</td>
<td>0.48</td>
<td>0.03</td>
</tr>
<tr>
<td>1146.6</td>
<td>0.4</td>
<td>0.45</td>
<td>0.04</td>
</tr>
<tr>
<td>1234.2</td>
<td>0.3</td>
<td>0.45</td>
<td>0.06</td>
</tr>
<tr>
<td>1556.30</td>
<td>0.20</td>
<td>1.05</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 5.3: $^{113}$Sb decay γ-rays—Branching ratios greater than .4

<table>
<thead>
<tr>
<th>γ Energy (keV)</th>
<th>γ Uncertainty (keV)</th>
<th>Branching Ratio</th>
<th>BR Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>255.06</td>
<td>0.05</td>
<td>1.82</td>
<td>0.09</td>
</tr>
<tr>
<td>391.688</td>
<td>0.015</td>
<td>64</td>
<td>2.0</td>
</tr>
<tr>
<td>638.03</td>
<td>0.08</td>
<td>0.00954</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: $^{113}$Sn decay γ-rays.

the other decay products (both β-rays and γ-rays) of $^{96}$Nb and other contaminants. Thus, we used the HPGe detector in order to resolve this peak.

### 5.3 Background

One of the challenges in any experiment is to be able to separate the detector signal from the unwanted background. To do this, one tries to reduce the background to
as low a level as practicable. There were several different contributions to the overall background in these experiments.

### 5.3.1 Natural Background

We sought to reduce the natural background as much as possible. We tested the BGO detectors at OSU to determine ways of reducing the ambient background. We tested various combinations of stainless steel, lead, and aluminum. Ultimately, we
Figure 5.8: Decay scheme of $^{120}$Sb.
Table 5.5: Visible peaks with an irradiated $^{119}$Sn target using a HPGe detector.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\gamma$ Energy (keV)</th>
<th>Nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>155</td>
<td>158</td>
<td>$^{117}$Sb</td>
</tr>
<tr>
<td>461</td>
<td>511</td>
<td>$^{120}$Sb</td>
</tr>
<tr>
<td>507</td>
<td>563</td>
<td>$^{122}$Sb</td>
</tr>
<tr>
<td>523</td>
<td>583</td>
<td>unknown</td>
</tr>
<tr>
<td>546</td>
<td>610</td>
<td>unknown</td>
</tr>
<tr>
<td>806</td>
<td>912</td>
<td>unknown</td>
</tr>
<tr>
<td>824</td>
<td>930</td>
<td>$^{116}$Sb</td>
</tr>
<tr>
<td>1135</td>
<td>1293</td>
<td>$^{116}$Sb</td>
</tr>
<tr>
<td>1280</td>
<td>1460</td>
<td>$^{40}$K</td>
</tr>
</tbody>
</table>

Table 5.6: $^{120}$Sb decay gammas.

<table>
<thead>
<tr>
<th>$\gamma$ Energy (keV)</th>
<th>$\gamma$ Uncertainty (keV)</th>
<th>Branching Ratio</th>
<th>BR Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>511.5</td>
<td>0.3</td>
<td>82.03</td>
<td></td>
</tr>
<tr>
<td>703.8</td>
<td>0.3</td>
<td>0.149</td>
<td>0.013</td>
</tr>
<tr>
<td>988.6</td>
<td>0.7</td>
<td>0.063</td>
<td>0.006</td>
</tr>
<tr>
<td>1171.2</td>
<td>0.3</td>
<td>1.69</td>
<td>0.090</td>
</tr>
</tbody>
</table>

used a stainless steel cylinder of 0.75 inch thickness around each BGO. For one BGO we also used a thinner cylinder of 0.375 inch thickness for the first experiment. We dispensed with this thinner cylinder in the second experiment after we concluded that making the experimental geometry symmetric was more important than the marginal background reduction conveyed. Then we surrounded the stainless steel and detectors with approximately 4 inches of lead bricks, taking care to stagger the bricks to avoid holes in the lead shielding.
<table>
<thead>
<tr>
<th>Channel</th>
<th>$\gamma$ Energy (keV)</th>
<th>Nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>209</td>
<td>216</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>322</td>
<td>350</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>341</td>
<td>372</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>395</td>
<td>435</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>417</td>
<td>460</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>435</td>
<td>480</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>461</td>
<td>511</td>
<td>cosmic ray background</td>
</tr>
<tr>
<td>511</td>
<td>568</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>530</td>
<td>592</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>588</td>
<td>657</td>
<td>$^{97}\text{Nb}$</td>
</tr>
<tr>
<td>641</td>
<td>719</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>691</td>
<td>778</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>719</td>
<td>810</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>753</td>
<td>849</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>826</td>
<td>934</td>
<td>$^{93}\text{Nb}$</td>
</tr>
<tr>
<td>961</td>
<td>1091</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>989</td>
<td>1128</td>
<td>$^{98}\text{Nb}$</td>
</tr>
<tr>
<td>1054</td>
<td>1200</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>1087</td>
<td>1238</td>
<td>unknown</td>
</tr>
<tr>
<td>1181</td>
<td>1347</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>1243</td>
<td>1441</td>
<td>$^{96}\text{Nb}$</td>
</tr>
<tr>
<td>1262</td>
<td>1460</td>
<td>$^{40}\text{K}$</td>
</tr>
<tr>
<td>1311</td>
<td>1498</td>
<td>$^{96}\text{Nb}$</td>
</tr>
</tbody>
</table>

Table 5.7: Identifiable peaks from HPGe detector for the irradiated $^{96}\text{Zr}$ target.
This configuration of lead and stainless steel reduced the background by about a factor of 10 from that of an unshielded BGO. To further reduce the background we also used 6 plastic scintillators in coincidence with the BGO signal as a veto to eliminate some of the 511 keV γ-ray cosmic ray background. The major remaining sources of background were $^{40}$K, producing a 1438 keV γ-ray; and other γ-rays and β-rays produced by the decay of naturally occurring radioactive uranium and thorium isotopes. These isotopes were present in our lead and steel shielding and even in the

Figure 5.9: Spectrum of irradiated $^{96}$Zr target from the HPGe detector.
<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Uncertainty (keV)</th>
<th>BR</th>
<th>BR Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>178.0</td>
<td>0.3</td>
<td>0.049</td>
<td>0.010</td>
</tr>
<tr>
<td>238.4</td>
<td>0.3</td>
<td>0.049</td>
<td>0.010</td>
</tr>
<tr>
<td>549.25</td>
<td>0.2</td>
<td>0.049</td>
<td>0.010</td>
</tr>
<tr>
<td>657.94</td>
<td>0.09</td>
<td>0.049</td>
<td>0.009</td>
</tr>
<tr>
<td>719.53</td>
<td>0.19</td>
<td>0.090</td>
<td>0.009</td>
</tr>
<tr>
<td>857.46</td>
<td>0.21</td>
<td>0.045</td>
<td>0.007</td>
</tr>
<tr>
<td>909.55</td>
<td>0.14</td>
<td>0.040</td>
<td>0.007</td>
</tr>
<tr>
<td>1024.4</td>
<td>0.3</td>
<td>1.09</td>
<td>0.07</td>
</tr>
<tr>
<td>1117.02</td>
<td>0.18</td>
<td>0.085</td>
<td>0.008</td>
</tr>
<tr>
<td>1148.6</td>
<td>0.3</td>
<td>0.049</td>
<td>0.010</td>
</tr>
<tr>
<td>1268.62</td>
<td>0.10</td>
<td>0.147</td>
<td>0.020</td>
</tr>
<tr>
<td>1515.66</td>
<td>0.19</td>
<td>0.122</td>
<td>0.013</td>
</tr>
<tr>
<td>1629.09</td>
<td>0.22</td>
<td>0.025</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Table 5.8: $^{97}$Nb decay $\gamma$-rays.

Figure 5.10: Decay scheme of $^{97}$Nb.
detector itself. The use of additional lead shielding could not reduce this background since the lead itself has some radioactive contamination. The steel was placed inside the lead since its content of radioactive impurities is lower than the lead.

An example of the natural background shown by the BGO detector is shown in figure 5.11. Given the small count rate compared to other background sources, such as Compton scattered photons, we felt justified in ignoring the structure of the background and simply treating it, along with the Compton scattering and beta emission of all of the irradiation products as discussed in section 5.3.3, as part of an overall exponential background.

Figure 5.11: Background spectrum from the BGO detector for a five minute interval.
5.3.2 Carbon and Oxygen Buildup

When a target in a vacuum system is subject to radiation, carbon and oxygen may build up on the target. Carbon can result from oil and grease from the target pumps contaminating the vacuum in the target chamber. This carbon can accumulate on the heated target surface. Oxygen can result from the oxidization of the target while in the atmosphere. This buildup can then react with the incident beam. Two reactions that may be of possible concern are $^{12}$C($p, \gamma$)$^{13}$N and $^{16}$O($p, \gamma$)$^{17}$F.

$^{17}$F decays via positron emission. This could cause difficulties as we are trying to detect 511 keV $\gamma$-rays emitted from other positron emitting products. However, $^{17}$F only has a 64.5 second halflife, thus any that would be produced during target irradiation should decay away quickly. In the unlikely event that $^{17}$F should be produced in large quantities, the short half life should make its existence very evident.

$^{13}$N also decays by positron emission with a halflife of 10 minutes. While it might be possible to separate a 10 minute halflife from the 6.7 minute halflife of $^{113}$Sb and the 15.9 minute halflife of $^{120}$Sb, it could be difficult if the $^{13}$N contribution was large. Complicating the matter is that the amount of $^{12}$C accumulating on the target varies from run to run, and increases throughout the experiment.

To test whether or not the buildup of $^{12}$C was a legitimate concern, we irradiated a $^{12}$C target (see table 4.1) with a thickness of 100 $\mu g/cm^2$, much thicker than the few $\mu g/cm^2$ of carbon buildup that we were expecting. The spectrum produced is shown in figure 5.12. The spectrum has been fitted to a Gaussian for the 511 keV $\gamma$-ray plus
Figure 5.12: Spectrum from a $^{12}$C target irradiated with 8.50 MeV protons—five minute decay interval. The fit is between channels 200 to 1000 and the peak around channel 1350 is from naturally occurring $^{40}$K.

an exponential for the residual background. Given that this target is 20-100 times thicker than the thin carbon buildup layer we might expect, and that the 8.5 MeV incident protons are the most energetic incident protons used in the experimental runs, we can assume that a maximum of only about 10 counts over a five minute interval in a 511 keV $\gamma$-ray peak would be from carbon buildup, a number that we can safely ignore.
5.3.3 Compton Background

A third source of background is the multiple Compton scatters of incident $\gamma$-rays as discussed in section 4.3. The Compton scatters of $\gamma$-rays of several different energies combine to form an overall background. Since the largest source of these Compton scattered photons is the target itself, this background will largely scale with the overall count rate. Thus we represented this background as an exponential with variable amplitude and curvature.

5.3.4 $^{119}$Sn Target Impurities

A final contribution of background radiation results from impurities in the target. For the $^{96}$Zr, we used the hyperpure germanium detectors and were able to resolve the peaks sufficiently. The $^{112}$Sn target was very pure, with only a contamination of 1.0% $^{114}$Sn. This impurity did not contribute appreciably to the radiation yield. As one can see in the target table 4.1, the $^{119}$Sn target also contained a number of different isotopes, including $^{120}$Sn. This $^{120}$Sn can react by a (p,n) reaction to form $^{120}$Sb. We must estimate the amount of $^{120}$Sb created by this reaction and subtract it from the total calculated yield of $^{120}$Sb. This cross section was measured by [Johnson, et al. 1977], and we used their measured values of the cross section, interpolating when needed, to get the cross section values for our energies. The number of nuclei created is given by inverting equations 5.27 and 5.20, derived later in section 5.6.
This amount is then subtracted from the measured amount of $^{120}\text{Sb}$ to get the contribution from the $(p,\gamma)$ reaction. The amounts of nuclei created by other possible contamination reactions were estimated the same way, using the [Johnson, et al. 1977] data if available, and the Hauser-Feshbach NON-SMOKER code output of [Rauscher, et al. 1997] for those unmeasured reactions. The amount of $^{116}\text{Sb}$ ($t_{1/2} = 15\text{ min}$) made was calculated < 0.1% of the $^{120}\text{Sb}$ and was ignored. $^{117}\text{Sb}$ ($t_{1/2} = 2.8\text{ hours}$) is made, but it has a low probability for positron emission (1.7%) and should not contribute appreciable to the 511 keV $\gamma$-ray radiation from the target. The $^{122}\text{Sb}$ 558 keV $\gamma$-ray intensity is similarly reduced by that isotope’s relatively long ($t_{1/2} = 2.7\text{ days}$) half-life.

5.4 Fitting of Spectra

5.4.1 $^{112}\text{Sn}$ Singles Fitting

After the gamma spectrum peaks had been identified, the histograms were fitted using the routines in PAW. The $^{112}\text{Sn}$ singles spectra were fit with Gaussians for the 498-511 keV peak, the 935-1018 peak, and the 334 keV peak (with the area fixed in order that it had the correct area in comparison to the 498-511 keV peak). Additionally, we had to account for the 392 keV $\gamma$-ray from the $^{113}\text{Sn}$. 
\(^{113}\)Sn has a half life of 115 days, which means effectively that any activity created from the decay of \(^{113}\)Sb is present for the duration of the experiment. The magnitude of the 392 keV \(\gamma\)-ray peak will only increase as more \(^{113}\)Sn is produced as a result of the irradiation of the \(^{112}\)Sn target. The 392 keV peak becomes prominent during the later time intervals of a run when most of the \(^{113}\)Sb has decayed away, and is also significant during the runs at low energy.

To account for this peak, we took the late time intervals of runs where the 392 keV peak was very prominent, and fitted the histograms using a Gaussian centered at the 392 keV peak, a fit for the \(^{113}\)Sb that was also present, and an exponential to represent the residual background. This allowed us to calculate the parameters of the 392 keV peak. This result was then incorporated into the fitting routines of the earlier time intervals as a fixed Gaussian for instances where the 392 keV peak was not as prominent. We felt justified in using this method because the parameters of the 392 keV peak varied only slightly throughout the experiment, with there often being no discernible difference in the peak's parameters from one run to the next.

One of the spectra from detector E1, along with the individual peak fits and overall composite fit, is shown in figure 5.13. The fits for E1 were started at channel 260 and continued up to channel 1100. We did this to avoid fitting to the extreme low energy background, which appears to be cut off because of a threshold effect, or to fit to the small higher energy peaks.

One of the spectra from detector E2, along with the individual peak fits and overall composite fit, is shown figure 5.14. The fits for E2 were started at channel
Figure 5.13: Singles spectrum from detector E1—7 MeV protons—5 to 10 minutes, first experiment, May 4, 1997. The components of the fit and the total fit are also plotted.
Figure 5.14: Singles spectrum from detector E2—7 MeV protons—5 to 10 Minutes, first experiment, May 4, 1997. The components of the fit and the total fit are also plotted.

...and continued up to channel 1150. We did this to avoid fitting to the extreme low energy background, or to fit to the small higher energy peaks.

For the second experiment the fits followed the same procedure. Because the raw signals were smaller as described in 4.4.5, the actual fit parameters had to be altered.
5.4.2 $^{112}\text{Sn}$ Coincidence Fitting

The $^{112}\text{Sn}$ coincident spectra were determined by an event having a TAC signal (a sample TAC spectrum is shown in figure 5.15) and an event in the 498/511 peak in the other BGO detector. These spectra were fitted in a manner similarly to the singles spectra, with one difference being that due to the coincident situation, the small area of the peak for the 935-1018 keV $\gamma$-rays was also constrained to have the correct area as compared to the 498-511 keV peak. A sample spectrum, including the peak and overall fit, is shown in figure 5.16.

![TAC Spectrum from irradiated $^{112}\text{Sn}$ (750 keV protons)](image)

Figure 5.15: Sample TAC spectrum.
Figure 5.16: Coincident spectrum from detector E1—7 MeV protons—5 to 10 Minutes. first experiment, May 4, 1997. The components of the fit and the total fit are also plotted.
5.4.3 $^{119}\text{Sn Fitting}$

The fitting of the $^{119}\text{Sn}$ spectra proceeded similarly to the $^{112}\text{Sn}$ spectra except the only significant peak was the 511 keV peak. The spectra were fitted to a Gaussian for the 511 keV $\gamma$-ray, plus an exponential for the background. This was the fit used for the both the single and the coincident spectra. Examples of the single and coincident fits are shown in figures 5.17 and 5.18.

![Graph showing $^{119}\text{Sn}$ spectra fitting](image)

Figure 5.17: Singles spectrum from detector E1—5.5 MeV protons—0 to 5 minutes, second experiment, September 19, 1997. The components of the fit and the total fit are also plotted.
Figure 5.18: Coincident spectrum from detector E1—5.5 MeV protons—0 to 5 minutes, second experiment. September 19, 1997. The components of the fit and the total fit are also plotted.

5.4.4 $^{96}$Zr Fitting

Fitting was simpler in the case of the $^{96}$Zr target irradiation because the HPGe detector was used, and since, as shown in table 5.8, there was no positron emission and only one prominent $\gamma$-ray emission to be concerned with. An expanded view of the spectrum is shown in figure 5.19. The $\gamma$-ray peak did sit atop a background
produced by various $\beta$-decays of target products and Compton scatterings of $\gamma$-rays. To determine the true number of counts in the peak, we took the number of counts in the five channel peak and subtracted the background from it, which we assumed to be the average of the 5 channels immediately before and the 5 channels immediately after the peak, with a separation of one channel between the intervals.

Figure 5.19: Spectrum from irradiated $^{96}$Zr, second experiment, September 20, 1997.
5.5 Yield calculation of (p,γ) decays.

Once the peak values have been extracted using PAW, the next step is to calculate the total number of decays from the relevant irradiation product in that time interval.

As shown in sections 4.4.2 and 4.5.1, an overall efficiency for the detectors was calculated. This was used with the branching ratios for the decays to calculate the fraction of total decays, $frac_{peak}$, that the number of counts in each peak represents.

5.5.1 $^{113}$Sb Produced

There are three main peaks in the irradiated $^{112}$Sn target spectrum to fit. a 498-511 keV singles peak, a 935-1018 keV singles peak, and a 511 keV coincident peak.

Table 4.5 shows the results of the calculations of the efficiencies for the detector for each individual gamma peak ($eff_{peak}$). To calculate the overall fraction we also need to know the branching ratios ($br_{peak}$) given in table 5.4.

The 498-511 keV peak has contributions from a single 498 keV or a single 511 keV γ-ray hitting the detector. However, as the decay schemes in figures 5.4 and 5.5 show, we may have instances where the 511 keV γ-ray will be in coincidence with another γ-ray or the background in the same detector, adding their energies together, correspondingly reducing the counts in the peak. This correction accounts for a reduction in the peak fraction by about 10%. This is the largest of all the corrections that are made to the peak fractions. These events have to be subtracted from the
total single events. We can see in figure 5.3 that we do observe such coincidences. Thus the total 498-511 fraction is given by equation 5.4.

\[
frac{498-511}{} = \text{eff}_{498\text{br}_{498}} + \text{eff}_{511\text{br}_{511}} - \text{eff}_x\text{br}_{x}\text{eff}_{511\text{br}_{511}} \tag{5.4}
\]

The calculation for the 935-1018 peak is similar. We have contributions from 935 keV, 940 keV, 1013 keV, and 1018 keV \(\gamma\)-rays, along with a contribution from a 498 keV \(\gamma\)-ray that adds in coincidence with a 511 keV \(\gamma\)-ray. Then we must subtract events in which one of the single \(\gamma\)-rays will be in coincidence with a 511 keV \(\gamma\)-ray. Note from the decay scheme of \(^{113}\text{Sb}\) in figure 5.6 that the 498 keV \(\gamma\)-ray will not be in coincidence with anything except perhaps a 511 keV \(\gamma\)-ray.

\[
frac{935-1018}{} = \text{eff}_{935\text{br}_{935}} + \text{eff}_{940\text{br}_{940}} + \text{eff}_{1013\text{br}_{1013}} + \text{eff}_{1018\text{br}_{1018}} + \text{eff}_{498\text{br}_{498}}\text{eff}_{511\text{br}_{511}} - \text{eff}_{511\text{br}_{511}} + (\text{eff}_{935\text{br}_{935}} + \text{eff}_{940\text{br}_{940}} + \text{eff}_{1013\text{br}_{1013}} + \text{eff}_{1018\text{br}_{1018}})(5.5)
\]

The final fraction to calculate is the 511 coincidence peak. Since the 511 keV \(\gamma\)-rays are emitted in electron-positron annihilation, that contribution is not simply calculated by \(\text{eff}_{511\text{br}_{511}}\text{eff}_{511\text{br}_{511}}\). As described in section 4.4.4, the calculation of the 511 keV coincidences was done using a Monte Carlo code, as were the 511 keV singles. Now, we also have the possibility of a 511 keV \(\gamma\)-ray and a 498 keV \(\gamma\)-ray being in coincidence. Finally we have to subtract those instances where a 511 keV \(\gamma\)-ray coincidence occurs along with a non 498 keV \(\gamma\)-ray hitting the detector. Note

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that, due to the geometry of the first experiment as shown in figure 4.6 the efficiencies for the first and second detector are not equal.

\[
\frac{\text{frac}_{511c}}{511c} = \text{eff}_{511c} \text{br}_{511} + \text{eff}_{498} \text{br}_{498} \text{eff}_{511} \text{br}_{511}
\]

\[
+ \text{eff}_{498} \text{br}_{498} \text{eff}_{511} \text{br}_{511} - \text{eff}_{511c} \text{br}_{511}(\text{eff}_x + \text{eff}_2) \text{br}_x
\]  

(5.6)

5.5.2 \(^{120}\text{Sb Produced}\)

Only the 511 keV \(\gamma\)-ray is prominent in the irradiated \(^{119}\text{Sn}\) target. A Gaussian was used to fit this peak for the single and the coincident spectra. An exponential was used to describe of the residual background.

The 511 keV peak has contributions from a single 511 keV \(\gamma\)-ray hitting the detector. However, we may have instances where the 511 keV \(\gamma\)-ray will be in coincidence with another \(\gamma\)-ray in the same detector and add, correspondingly reducing the counts in the peak. These events have to be subtracted from the single events. Thus the total 511 keV fraction is given by equation 5.7.

\[
\text{frac}_{511} = \text{eff}_{511c} \text{br}_{511} - \text{eff}_x \text{br}_x \text{eff}_{511} \text{br}_{511}
\]  

(5.7)

Additionally we must calculate the 511 coincidence peak fraction. Since the 511 keV \(\gamma\)-rays are emitted in positron-electron annihilation, that contribution is not simply calculated by \(\text{eff}_{511c} \text{br}_{511} \text{eff}_{511} \text{br}_{511}\). The calculation of the 511 keV coincidences was done using the Monte Carlo simulation, as were the 511 keV singles. Now, we
also have to subtract the possibility of a 511 keV γ-ray coincidence occurring along with another γ-ray hitting the detector. This gives us a coincidence fraction given by equation 5.8.

\[
frac{c511}{c} = \text{eff}_{511c} br_{511} - 2 \text{eff}_{511c} br_{511} \text{eff}_{br} 
\]

(5.8)

### 5.5.3 ⁹⁷Nb Produced

The irradiated ⁹⁶Zr spectrum shown in figure 5.19 is trivial to analyze as there is only one γ-ray peak at 658 keV and no significant coincidences. Thus the total 658 keV fraction is given by equation 5.9.

\[
frac{c658}{c} = \text{eff}_{658} br_{658}
\]

(5.9)

### 5.5.4 Total Decays

Once \(frac_{\text{peak}}\) from the relevant equations 5.4 - 5.9 is determined, the next step is to calculate the total number of decays \(N_{\text{dec}}\) that occurred in the interval. To do this we take the number of counts calculated in the peak \(N_{\text{peak}}\) and divide by the fraction of total decays that peak represents, also taking into account the detector livetime, given by equation 4.8. The total number of nuclei that decayed during the time interval then is given by equation 5.10.
\[ N_{dec} = \frac{N_{peak}}{\text{frac}_{peak} \times \text{frac}_{live}} \]  

(5.10)

5.5.5 Nuclei Created

From the total number of decays in an interval \( t_1 \) to \( t_2 \), we can compute the number of total nuclei that were created during the irradiation. At time \( t = 0 \), we have \( N_0 \) nuclei present, of which \( N_{cre} \) were created during the irradiation with \( N_{left} \) leftover from past irradiations. Here \( \lambda = \ln(2)/t_{1/2} \), where \( t_{1/2} \) is the half life of the nucleus in question. In the case of calculating the \(^{120}\text{Sb}\) produced, we must also subtract from \( N_{cre} \) the number of nuclei created in the \(^{120}\text{Sn}(p,n)^{120}\text{Sb}\) reaction as detailed in 5.3.4.

\[ N(t_1) = N_0 e^{-\lambda t_1} \]  

(5.11)

\[ N(t_2) = N_0 e^{-\lambda t_2} \]  

(5.12)

\[ N_{dec} = N(t_1) - N(t_2) = N_0(e^{-\lambda t_1} - e^{-\lambda t_2}) \]  

(5.13)

\[ N_{dec} = N_0 e^{-\lambda t_1}(1 - e^{-\lambda(t_2-t_1)}) \]  

(5.14)

\[ N_{cre} = \frac{N_{dec} \times e^{\lambda t_1}}{(1 - e^{-\lambda(t_2-t_1)})} - N_{left} \]  

(5.15)

5.6 Cross Section

After equation 5.15 the rate of production can be calculated. Due to the thickness of the target and the strength of the reaction, we can assume that the target is not
"used up". The reaction rate is relatively slow compared to the number of nuclei in the target. Thus, we produce nuclei at a constant rate \( r \). However, the nuclei that we are producing are radioactive and decay away at a rate \( \lambda N \) where \( N \) is the number of nuclei present. If we irradiate for a period of time \( t_{irr} \) we can calculate this rate.

\[
\frac{dN}{dt} = r - \lambda N \quad (5.16)
\]

\[
\int_0^{N_{cre}} \frac{dN}{r - \lambda N} = \int_0^{t_{irr}} dt \quad (5.17)
\]

\[
(ln(r - \lambda N_{cre}) - ln r)/\lambda = t_{irr} \quad (5.18)
\]

\[
1 - \lambda N_{cre}/r = e^{-\lambda t_{irr}} \quad (5.19)
\]

\[
r = \frac{\lambda N_{cre}}{1 - e^{-\lambda t_{irr}}} \quad (5.20)
\]

Now, from the rate \( r \) we can calculate the cross section \( \sigma \). The rate is related to the flux of the incident protons \( (\Phi_p) \) times the number of target particles per unit area \( (N_t) \), and the cross section \( \sigma \).

\[
r = \Phi_p N_t \sigma \quad (5.21)
\]

\[
\sigma = r/\Phi_p N_t \quad (5.22)
\]

\[
\Phi_p = I/q_p \quad (5.23)
\]

\[
N_t = m_t \times purity/m_{atom} \times Area \quad (5.24)
\]

\[
m_t = \rho_t \times Area \times d \quad (5.25)
\]

\[
\sigma = \frac{r}{\frac{I}{q_p \times d \times purity \over m_{atom}}} \quad (5.26)
\]
\[ \sigma = \frac{r q_{p} m_{atom}}{I d \rho \times purity} \] (5.27)

5.7 Uncertainties

Of great importance are the uncertainties associated with the cross section measurement. There are a number of different contributions to the final uncertainty in our measurements.

5.7.1 Statistical Errors/Dead Time

One class of uncertainties concerns each individual measurement and can be added in quadrature for the entire run. There is a statistical error in each peak due to the number of counts in the peak. If we consider the distribution to be identical to a Poisson distribution, this can be estimated as a percentage error of \( \sqrt{N}/N \), where \( N \) is the number of counts in a peak. This ranged from less than 1% to over 10%.

We also had an uncertainty that resulted from the dead time measurements. For the first experiment, the live time fractions were measured to \( \pm 0.005 \). For the second experiment they were measured to \( \pm 0.002 \). The dead time errors were significant in the first experiment for those readings with very low \( (f r a c_{live} < .2 \) in some cases) live time fractions. An example is where we have a live time fraction of .20, the \( \pm 0.005 \) uncertainty in the measurement results in an overall uncertainty of around 10%.
5.7.2 Run Systematic Uncertainties

There is a second class of uncertainties that affected all cross section measurements made throughout an energy run. These included an uncertainty due to the current measurement (estimated at 3%) and numerous errors associated with the placement of the target.

To calculate the contribution from each of these placement uncertainties, we estimated 1 sigma standard deviations for each relevant parameter and ran the same Monte Carlo code that we used for the detector efficiencies at ± 1 sigma, and the average value to get a percentage error. The results are shown in tables 5.9 and 5.10. The placement errors for the zirconium are not calculated in this way but instead we assumed a ±5% overall error that gets folded in with the uncertainty in the HPGe efficiency calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Detector</th>
<th>498 %</th>
<th>511s %</th>
<th>940 %</th>
<th>511c %</th>
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<td>.1 inches</td>
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Table 5.9: Sources of uncertainty for $^{112}$Sn target in experiment 1. RALU: Distance between the target and aluminum absorbers; TGTH: Target height; TGTX: Target offset off line between detectors; TGTZ: Target offset on line between detectors.
Then we took the errors and added them in quadrature to get a value of the uncertainty for that run. For the second experiment it was slightly simpler since, due to the symmetric geometry, the geometric E1 uncertainties equal the E2 uncertainties. Also, certain aspects of the detector set-up were altered, such as the position of the aluminum, so they could no longer vary from run to run.

<table>
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<tr>
<th>Parameter</th>
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<th>511s %</th>
<th>940 %</th>
<th>511c %</th>
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Table 5.10: Sources of uncertainty for tin targets in experiment 2. TGTH: Target height. TGTX: Target offset off line between detectors, TGTZ: Target offset on line between detectors.

5.7.3 Experiment Systematic Uncertainties

Finally, there are those uncertainties that were systematic throughout the experiment. These had to do with the geometric set up of the experiment, uncertainties in the branching ratios for the decays, uncertainties in the absorption of $\gamma$-rays in the aluminum and steel, and uncertainties in the $\gamma$-ray detection efficiency. The uncertainties in the isotope concentration were negligible in comparison.

Then we took the uncertainties in tables 5.11 and 5.12, and added them in quadrature to get a overall value of the uncertainty in the experimental results.
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<th>Parameter</th>
<th>Value</th>
<th>Detector</th>
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<th>511s %</th>
<th>940 %</th>
<th>511c %</th>
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<td>E2</td>
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<td>.5</td>
<td>.2</td>
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Table 5.11: Systematic sources of uncertainty for $^{112}$Sn target in Experiment 1. BGOD: BGO Diameter; BGOL: BGO Length; RSEP: Separation between detector and target; RALU: Distance between the target and aluminum absorbers; TGTH: Target height; DT1H: Detector 1 height; DT2H: Detector 2 height; TGTX: Target offset to side; DT1X: Detector 1 offset to side; DT2X: Detector 2 offset to side; BPEN: Beta penetration depth; PBDM: Lead “box” dimensions; ALCO: Aluminum absorption efficiency; BGOC: BGO efficiency; TGTZ: Target offset on line between detectors.
Table 5.12: Systematic sources of uncertainty for tin targets in experiment 2. While the positrons emitted have slightly different energies, their errors are approximately equal. BGOD: BGO Diameter; BGOL: BGO Length; RSEP: Separation between detector and target; RALU: Distance between the target and aluminum absorbers; TGTH: Target height; DT1H: Detector 1 height; DT2H: Detector 2 height; TGTX: Target offset to side; DT1X: Detector 1 offset to side; DT2X: Detector 2 offset to side; BPEN: Beta penetration depth; PBDM: Lead "box" dimensions; ALCO: Aluminum absorption efficiency; BGOC: BGO efficiency; TGTZ: Target offset on line between detectors.

<table>
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<th>Parameter</th>
<th>Value</th>
<th>498 Unc.%</th>
<th>511s Unc.%</th>
<th>940 Unc.%</th>
<th>511c Unc.%</th>
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</thead>
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<td>.14</td>
<td>.4</td>
</tr>
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<td>.5</td>
<td>1</td>
<td>.5</td>
</tr>
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<td>3</td>
<td>2.5</td>
<td>.7</td>
</tr>
<tr>
<td>RALU</td>
<td>.1 inches</td>
<td>.7</td>
<td>.7</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>TGTH</td>
<td>.1 cm</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.4</td>
</tr>
<tr>
<td>DT1H</td>
<td>.1 cm</td>
<td>.2</td>
<td>.2</td>
<td>.2</td>
<td>.4</td>
</tr>
<tr>
<td>DT2H</td>
<td>.1 cm</td>
<td>.2</td>
<td>.2</td>
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<td>.4</td>
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<td>.14</td>
<td>.1</td>
<td>.35</td>
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<td>.08</td>
<td>.1</td>
<td>.1</td>
<td>.35</td>
</tr>
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<td>.17</td>
<td>.7</td>
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<td></td>
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<tr>
<td>PBDM</td>
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<td>.1</td>
<td></td>
<td>.3</td>
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<tr>
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<td>1.8</td>
<td>.3</td>
<td>3</td>
</tr>
<tr>
<td>BGOC</td>
<td>fit</td>
<td>3.5</td>
<td>3.5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>TGTZ</td>
<td>.1 cm</td>
<td>3.2</td>
<td>3</td>
<td>3</td>
<td>.02</td>
</tr>
<tr>
<td>TGTZ(E1+E2)</td>
<td>.1 cm</td>
<td>.04</td>
<td>.04</td>
<td>.2</td>
<td>.02</td>
</tr>
</tbody>
</table>
5.8 Overall Cross Section

Now that we have calculated the inferred cross sections for each time interval and one peak, we must put them all together to get an overall value for the $^{112}\text{Sn}(p,\gamma)^{113}\text{Sb}$, $^{119}\text{Sn}(p,\gamma)^{120}\text{Sb}$, and $^{96}\text{Zr}(p,\gamma)^{97}\text{Nb}$ reactions.

5.8.1 $^{112}\text{Sn} (p,\gamma)^{113}\text{Sb}$ and $^{119}\text{Sn}(p,\gamma)^{120}\text{Sb}$

The division into time intervals gave us a number of different spectra for each energy. A few points with extremely large dead times similarly had large error bars. When averaging values together we used a weighted mean.

The agreements between time intervals and between singles and coincidences support the fitting routines and Monte Carlo calculations used. However, when the count rate decreases, the signal to noise ratio decreases and the background contribution becomes significant and its structure more defined. Fitting an exponential to it becomes a poorer approximation. Plus, reactions on some very small contaminants in the target can give rise to longer halflife products, producing extra 511 keV $\gamma$-rays. While these are not significant for the early time divisions, (see section 5.3.4) they may contribute when one is several halflives after the target irradiation has ceased. Also, the BGO resolution spreads out the $\gamma$-ray peaks over dozens of channels, so when dealing with counts of a few hundred in a peak over a five minute interval, one would not expect a good fit. When averaging together cross section values, we took
weighted means of those at the beginning of the individual energy runs (usually the first 30-45 minutes).

We deduced 3 different measurements of the cross section from each detector for each energy for the $^{112}\text{Sn} \ (p,\gamma)^{113}\text{Sb}$ reaction: a measurement from the 498-511 peak, a measurement from the 935-1018 peak, and measurement for the 511 coincidence peak. For the $^{119}\text{Sn} \ (p,\gamma)^{120}\text{Sb}$ reaction we obtained 2 different measurements of the cross section from each detector for each energy: a measurement of from the 511 singles peak and a measurement from the 511 coincidence peak. The coincident measurement cross section, however, was about 20% higher than the singles data, even after all the corrections detailed in section 5.3.4 had been made. We postulate that one of the contamination reactions, perhaps the $^{116}\text{Sn} \ (p,\gamma)^{117}\text{Sb}$ reaction may be stronger than the NON-SMOKER calculations imply. This could cause not only a contamination in the 511 keV $\gamma$-ray peak (since $^{117}\text{Sb}$ is a positron emitter) but also coincidences between these 511 keV $\gamma$-rays and other $\gamma$-rays. In this work we have presented the singles data, with the understanding that this data may also have contamination from another reaction and should be considered only an upper bound.

As we saw in section 5.7, one of the major sources of error is a small displacement along the axis between the detectors, but that this error cancels to first order if we take $E_1+ E_2$. To arrive at an overall average value we took error weighted means of $E_1+ E_2$ 498-511 singles, $E_1+ E_2$ 935-1018, and $E_1+ E_2$ 511 coincidences. Only in the case of the lowest energy points did the use of weighted means have a discernible effect,
this being due to the large statistical errors of some of the small E1+E2 935-1018 and coincident peaks.

5.8.2 \(^{96}\text{Zr}(p, \gamma)^{97}\text{Nb}\)

Since the runs were broken up into different intervals, we averaged those individual cross sections together to get an overall value for the cross section. As there is only one gamma peak and one detector in this experiment, that was all that needed to be done. The long half life of the \(^{97}\text{Nb}\) meant that there were significant counts in the 658 keV peak even at the end of the intervals for our counting run, this also means, however, that \(N_{\text{eff}}\) in equation 5.15 was a nontrivial correction to calculating the number of nuclei created during an irradiation.

5.9 Average Energies

In calculating the cross sections, we are interested not in the incident proton energy persay, but in an average center of mass energy. The center of mass energy is given by equation 5.28:

\[
E_{\text{cm}} = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v^2
\]  

(5.28)

Since the incident proton energy is \(\frac{1}{2} m_1 v^2\), the center of mass energy differs from the incident proton energy by a multiplicative factor of \(m_t/(m_t + m_p)\), where \(m_t\) is the mass of the target atom.
More difficult to determine is the average energy the reaction occurs at. The amount of energy lost by the incident proton was calculated by the program BABEL [Bowsher 1982], the output of which is shown in table 5.13.

<table>
<thead>
<tr>
<th>Incident Energy</th>
<th>$^{96}$Zr Loss (keV)</th>
<th>$^{112}$Sn Loss (keV)</th>
<th>$^{119}$Sn Loss (keV)</th>
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<td>157.8</td>
<td>142.8</td>
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<td>74.4</td>
<td>67.4</td>
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Table 5.13: Energy loss of protons in target materials.

However, the cross section varies with energy, so we need to average it in with equation 5.27. The mean energy $\bar{E}$ of the reaction is given by

$$\bar{E} = \frac{\int_{E_0}^{E_1} E \sigma(E) dE}{\int_{E_0}^{E_1} \sigma(E) dE}$$

(5.29)
It behooves us to express the cross section in terms of the S-factor as given in equation 2.16. We then assume that the S-factor $S(E)$ is roughly constant over the range of energies and cancels from the upper and lower integrals.

$$\bar{E} = \frac{\int_{E_0}^{E_1} \frac{e^{2\pi nZ/\sqrt{E}}}{\sqrt{E}} dE}{\int_{E_0}^{E_1} \frac{1}{E} e^{2\pi nZ/\sqrt{E}} dE}$$  \hspace{1cm} (5.30)

It is this value of the energy that we use to calculate the cross section.

<table>
<thead>
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<th>$^{119}$Sn Energy (keV)</th>
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Table 5.14: Mean energy of protons for cross sections in target materials.
5.10 $^{112}\text{Sn} (p,\gamma)^{113}\text{Sb}$ Cross Section

The $^{112}\text{Sn}(p,\gamma)^{113}\text{Sb}$ reaction traces out the following graph as shown in figure 5.20. Notice that this cross section drops off after 8.0 MeV, due to the opening of the (p.n) channel at 7.8 MeV. The NON-SMOKER code output is shown as a curve, while our data are shown as individual points.

![Graph](image)

Figure 5.20: Deduced cross section for $^{112}\text{Sn}$. The NON-SMOKER output is shown as a curve, while our data are shown as individual points with error bars.
5.11 $^{119}\text{Sn} \,(p,\gamma)^{120}\text{Sb} \text{ Cross Section}$

The $^{119}\text{Sn}(p,\gamma)^{120}\text{Sb}$ reaction traces out the following graph as shown in figure 5.21. The NON-SMOKER code output is shown as a curve, while our data are shown as individual points with error bars. Note, as described in 5.8.1, these should be considered as upper bounds due to the possibility of contamination.

![Graph showing deduced upper bound cross section for $^{119}\text{Sn}$. The NON-SMOKER output is shown as a curve, while our data are shown as individual points with error bars.](image)

Figure 5.21: Deduced upper bound cross section for $^{119}\text{Sn}$. The NON-SMOKER output is shown as a curve, while our data are shown as individual points with error bars.
5.12 \(^{96}\text{Zr} (p,\gamma)^{97}\text{Nb} \) Cross Section

The \(^{96}\text{Zr}(p,\gamma)^{97}\text{Nb} \) reaction traces out the following graph as shown in figure 5.22. The NON-SMOKER code output is shown as a curve, while our data are shown as individual points with error bars.

![Graph of \(^{96}\text{Zr}(p,\gamma)^{97}\text{Nb} \) Cross Section](image)

Figure 5.22: Deduced cross section for \(^{96}\text{Zr} \). The NON-SMOKER output is shown as a curve, while our data are shown as individual points with error bars.
5.13 S-factor and Gamow Window

Of interest to astrophysics is the astrophysical S-factor, which is the cross section with the Coulomb energy and DeBroglie wavelength dependent factors removed. This was described in section 2.5 and is given by equation 2.16.

\[ S(E) = \frac{\sigma}{e^{-2\pi\hbar/\sqrt{E/E}}} \quad (5.31) \]

Applying equation 5.31 to our results, we obtain the results shown in figures 5.23, 5.24, and 5.25. The NON-SMOKER code output are shown as curves, while our data are shown as individual points with error bars.

We also may wonder what the Gamow peak is for the relevant reactions. This was described in section 2.5 and is calculated for several temperatures of interest in table 5.15.

<table>
<thead>
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<th>Reaction</th>
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<th>( T = 1.0 \times 10^9 )</th>
<th>( T = 2.0 \times 10^9 )</th>
<th>( T = 3.0 \times 10^9 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.76</td>
<td>1.20</td>
<td>1.91</td>
<td>2.50</td>
</tr>
<tr>
<td>(^{112}\text{Sn}(p,\gamma)^{113}\text{Sb})</td>
<td>0.85</td>
<td>1.35</td>
<td>2.14</td>
<td>2.81</td>
</tr>
<tr>
<td>(^{119}\text{Sn}(p,\gamma)^{120}\text{Sb})</td>
<td>0.85</td>
<td>1.35</td>
<td>2.14</td>
<td>2.81</td>
</tr>
</tbody>
</table>

Table 5.15: Gamow peak value for selected temperatures – temperatures are in Kelvin, energies are in MeV.
Figure 5.23: Deduced S-factor for $^{112}\text{Sn}(p,\gamma)^{113}\text{Sb}$. The NON-SMOKER output is shown as a curve, while our data are shown as individual points with error bars.
Figure 5.24: Deduced upper bound S-factor for $^{119}\text{Sn} (p,\gamma)^{120}\text{Sb}$. The NON-SMOKER output is shown as a curve, while our data are shown as individual points with error bars.
Figure 5.25: Deduced S-factor for $^{96}\text{Zr}(p,\gamma)^{97}\text{Nb}$. The NON-SMOKER output is shown as a curve, while our data are shown as individual points with error bars.
CHAPTER 6

CONCLUSIONS

*It should have a big exciting finish -
like an earthquake, a catechism of nature.*

- Harry Rapf, former MGM executive in the 1930s and ’40s

The activation technique of measuring (p,γ) cross sections can be called a success. Three different cross sections have been measured by this technique. The addition of this (p,γ) cross section data to that of [Laird, et al. 1987] and [Sauter and Käppeler 1997] extends the experimental knowledge of these cross sections into the mass ranges around A ≈ 100 – 120. In contrast to [Sauter and Käppeler 1997], who found (p,γ) experimental cross sections on molybdenum isotopes approximately an order of magnitude higher than expected, we found that the (p,γ) reactions on ⁹⁶Zr and ¹¹²Sn were generally in agreement with the NON-SMOker calculation method described by [Rauscher, et al. 1997]. It is noted however, that [Sauter and Käppeler 1997] used the older SMOker code of [Cowan, et al. 1991] which may lead to somewhat different theoretical results.
There is evidence in our data that the cross section for $^{112}\text{Sn}(p,\gamma)^{113}\text{Sb}$ is higher at lower incident proton energies, and lower in the region where the (p,n) reaction channel opens. There is also an indication that the $^{119}\text{Sn}(p,\gamma)^{120}\text{Sb}$ reaction is stronger than the NON-SMOKER calculations indicate, however, there is a possibility of contamination, so this claim must be made with caution. An experimental measurement of $^{116}\text{Sn}(p,\gamma)^{117}\text{Sb}$ would serve to clarify this result better.

As our results are fairly consistent with the NON-SMOKER calculations, one would not expect a refinement of this code based upon this experimental data to have a great effect. Neither would one anticipate changes in the results of the p-process codes dependent upon these reactions. The reaction closest to the proton drip line, $^{112}\text{Sn}(p,\gamma)^{113}\text{Sb}$, is in reasonable agreement with the theoretical calculation although there is some evidence for a higher S-factor at astrophysically significant energies.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Reaction</th>
<th>Product</th>
<th>Product Halflife</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{98}\text{Ru}$</td>
<td>(p,γ)</td>
<td>$^{97}\text{Rh}$</td>
<td>31 min</td>
</tr>
<tr>
<td>$^{102}\text{Pd}$</td>
<td>(p,γ)</td>
<td>$^{103}\text{Ag}$</td>
<td>1.1 hours</td>
</tr>
<tr>
<td>$^{106}\text{Cd}$</td>
<td>(p,γ)</td>
<td>$^{107}\text{In}$</td>
<td>33 min</td>
</tr>
<tr>
<td>$^{106}\text{Cd}$</td>
<td>(α,γ)</td>
<td>$^{110}\text{Sn}$</td>
<td>4 hours</td>
</tr>
<tr>
<td>$^{112}\text{Sn}$</td>
<td>(α,γ)</td>
<td>$^{116}\text{Te}$</td>
<td>2.5 hours</td>
</tr>
<tr>
<td>$^{116}\text{Sn}$</td>
<td>(p,γ)</td>
<td>$^{117}\text{Sb}$</td>
<td>2.8 hours</td>
</tr>
<tr>
<td>$^{135}\text{Ba}$</td>
<td>(p,γ)</td>
<td>$^{136}\text{La}$</td>
<td>9.9 min</td>
</tr>
<tr>
<td>$^{163}\text{Dy}$</td>
<td>(p,γ)</td>
<td>$^{164}\text{Ho}$</td>
<td>29 min</td>
</tr>
</tbody>
</table>

Table 6.1: Possible targets for future studies using the experimental technique described in this work.
The activation technique used here should be applicable to studying \((p, \gamma)\) reactions involving other nuclei, including those of higher mass. One difficulty in measuring data on the proton rich side however, is the availability of sufficiently isotopically pure targets. This is complicated by the fact that many of the stable nuclei of interest have low abundance, and are often p-process nuclei themselves. As was seen in the case of the \(^{119}\text{Sn}\) target, even small impurities can cause difficulty in interpreting the experimental results. Setting aside that criterion for the moment, the technique used in this experiment should be applicable to the targets in table 6.1. This would enable the study of \((p, \gamma)\) reactions over a wider parameter space, even extending up to \(^{163}\text{Dy}\). and provide a better test of the Hauser-Feshbach formalism. Conceivably, this technique could be extended to the study of \((\alpha, \gamma)\) reactions as well, although the high energy needed for the reaction to overcome the Coulomb barrier could result in reactions such as \((\alpha, 2n)\), \((\alpha, p)\), and \((\alpha, d)\) contributing to the decay of the compound nucleus, making the identification of the \((\alpha, \gamma)\) reaction product more difficult.


