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A STUDY OF THE POTENTIAL BENEFITS
OF USING INTERACTIVE SOFTWARE IN HIGH SCHOOL GEOMETRY
SOLELY FOR WHOLE CLASS DEMONSTRATION AND DISCUSSION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

by

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* * * * *

The Ohio State University
1998

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ABSTRACT

This study investigated whether students in geometry classes using one TI-92 calculator with Cabri Geometry software for whole class demonstration and discussion could overcome obstacles to the use of diagrams, become proficient in conjecturing and proof, and whether benefits differ by gender. Subjects were 55 students taught by the researcher during the 1996-97 year.

Individual student use of calculators for one week followed by extensive use of the TI-92 for whole-class demonstration and discussion helped students realize the value of seeing a number of examples illustrating a concept, understand how to generate a conjecture, and realize different diagrams can yield the same conclusion. Writing paragraphs explaining use of the TI-92 to construct geometric figures helped students overcome difficulties with diagrams. At the end of the study, many students could apply theorems proven for one diagram to similar diagrams and modify diagrams to develop conjectures or proof. Although conjectures were often based on appearance, a number of students produced viable conjectures particularly when asked to conjecture about specific parts of a diagram. Some students could indicate how a proof would proceed, and a few provided good justification for conjectures.
Results of the Fennema-Sherman Mathematics as a Male Domain Scale indicated that females viewed mathematics as less of a male domain than did males at the beginning and end of the study (p<.05). Males scored significantly higher on the Attitude Toward Success in Mathematics Scale at the end of the study than at the beginning (p<.05).

On comparisons of abilities to apply geometric concepts to diagrams, males scored significantly higher on worksheets completed later in the study period than on those completed earlier and significantly higher than females on those completed later in the study (p<.05). On worksheet problems dealing with developing conjectures, females scored significantly higher on worksheets completed toward the end of the study than on those completed earlier (p<.05).
To Lori
ACKNOWLEDGMENTS

I wish to thank my adviser, Dr. Sigrid Wagner, for her advice, encouragement, and patience during my six-plus years as a doctoral student in Mathematics Education at the Ohio State University. I also want to thank Dr. Doug Owens for always being available to assist, advise, and encourage and Dr. Bob Brown for taking time from a busy schedule as OSU Mathematics Department Chairman to participate on my dissertation committee. I consider each of these to be the highest examples of university teaching. Thanks also to Michelle Reed for getting my draft dissertation to the graduate school on time.

I want to thank Mr. John Jenkins, Principal of Alamogordo High School; Ms. Karen Couch, Assistant Superintendent for Human Resources; and Ms. Clarissa Johnson, Assistant Superintendent for Pupil Services of the Alamogordo, New Mexico School District for allowing me to conduct research in their district. Thanks to the students in my geometry classes for perservering through a year in which the teacher was also learning about geometry, students, teaching, and using technology to teach geometry.

Thanks to my wife, Lori, whose frequent gentle--and occasionally not-so-gentle--push encouraged me to start this program and enabled me to reach this point, and my daughter, Meghann, for being patient when I wasn't very patient in providing assistance on mathematics homework assignments.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Statement of the Problem</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Creating Mathematics</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Use of Diagrams</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Interactive Geometry Software</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Gender and Interactive Software</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Research Questions</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Theoretical Rationale</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Constructivism</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Constructivism and Classroom Discourse</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Social Constructivism</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>Learning about Mathematics</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Vygotsky</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Van Hiele</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Significance</td>
<td>28</td>
</tr>
<tr>
<td>2.</td>
<td>Review of the Literature</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Conjecturing</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Diagrams</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>Interactive Geometry Software, Conjecturing, and Diagrams</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>Classroom Discourse</td>
<td>49</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fennema-Sherman Mathematics Attitude Scales</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>Data Types and Collection Methods</td>
<td>69</td>
</tr>
<tr>
<td>3</td>
<td>Fennema-Sherman Mathematics Attitude Scales Used in the Present Study</td>
<td>87</td>
</tr>
<tr>
<td>4</td>
<td>Fennema-Sherman Scale Reliabilities as Reported by Fennema-Sherman (1976) and as Determined in the Present Study</td>
<td>91</td>
</tr>
<tr>
<td>5</td>
<td>Means for First and Third Administrations of the Fennema-Sherman Scales</td>
<td>93</td>
</tr>
<tr>
<td>6</td>
<td>Attitude Survey Statements Developed by the Researcher</td>
<td>97</td>
</tr>
<tr>
<td>7</td>
<td>Researcher-Developed Items as Originally Grouped by the Researcher</td>
<td>99</td>
</tr>
<tr>
<td>8</td>
<td>Opinions as to the Topics Addressed by Researcher-Developed Items</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>Item Discrimination Indices Correlating Scores on Researcher-Developed Items with Total Group Scores for the First Administration of the Attitude Survey</td>
<td>103</td>
</tr>
<tr>
<td>10</td>
<td>Mean Scores by Group for the First and Third Administrations of the Researcher-Developed Items</td>
<td>105</td>
</tr>
<tr>
<td>11</td>
<td>Mean Scores for Selected Researcher-Developed Items for the First and Third Administration of the Attitude Survey Instrument</td>
<td>106</td>
</tr>
<tr>
<td>12</td>
<td>Collection Methods and Periods for Data Used in the Study</td>
<td>112</td>
</tr>
<tr>
<td>13</td>
<td>Problems from the Preliminary and Pythagorean Theorem Worksheets Used in the Analyses of Students’ Abilities to Apply Definitions, Postulates, or Theorems to Diagrams</td>
<td>115</td>
</tr>
</tbody>
</table>
25 Numbers and Percentages of Students Answering Selected Preliminary Worksheet Problems Correctly and Numbers and Percentages of Students Basing Answers on the Appearance of the Diagram ......................................................... 142

26 Preliminary, Similar Triangle, and Circle Worksheet Problems Used to Investigate Students' Abilities to Develop Conjectures .......................................................... 149

27 Rubric for Scoring Conjectures Developed by Students .......................................................... 152

28 Means as Percents, Standard Deviations, and p-Values for Comparisons of Mean Scores on Conjecturing Problems from Preliminary, Similar Triangle, and Circle Worksheets ........................................................... 154

29 Problems 4, 5, and 6 from the Homework Assignment Which Were Used in the Analysis of Students' Abilities to Prove Conjectures .................................................. 162

30 Rubric for Scoring the Justification Given by Students to Problems on the Homework Assignment and Circle Worksheet ........................................................................ 163

31 Means as Percents, Standard Deviations, and p-Values for Comparisons of Mean Scores from the Proof Problems on the Homework Assignment and Circle Worksheets .................................................................................. 164

32 Analysis of Homework Problems Showing Numbers of Female and Male Students Who Were Able to Provide Adequate Proof for Problems 4, 5, and 6 .......................................................... 165

33 Proof of Similar Triangle Worksheet Problem 1: If two sides of a triangle are cut by a line parallel to the third side, then two similar triangles are formed ........................................................................................................................................ 167

34 Methods of Proof Used by Students on Similar Triangle Worksheet Problems 2, 3, and 4 .................................................................................................................. 168

35 Summary of Significant Statistical Findings Related to Research Questions 1 and 2 .......................................................................................................................... 187
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Triangle-Angle Bisector Theorem (A line bisecting one angle of a triangle divides the opposite side into segments proportional to the other two sides of the triangle) Applied to Angle ABC</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>An Illustration of the Effect of Moving Point B on Segments AD and DC When the Triangle-Angle Bisector Theorem is Applied to Angle ABC</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>A &quot;Trianquad&quot;—a Triangle and a Quadrilateral with a Common Vertex</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>Some Students See Segment AD as an Altitude of Triangle ABC, Some as a Side of Triangles ADB and ADC but Many Cannot See Both</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>The Nine Dot Problem: Connect All Nine Dots without Raising the Pencil</td>
<td>39</td>
</tr>
<tr>
<td>6</td>
<td>An Impossible Triangle—the Sum of the Lengths of Two Sides is Less than the Length of the Third Side</td>
<td>42</td>
</tr>
<tr>
<td>7</td>
<td>Handout Given to Students to Illustrate TI-92 Screens and Menus</td>
<td>70</td>
</tr>
<tr>
<td>8</td>
<td>Activities Used in Class to Introduce Students to the TI-92</td>
<td>72</td>
</tr>
<tr>
<td>9</td>
<td>A Portion of the Worksheet Dealing with Medians of a Triangle</td>
<td>73</td>
</tr>
<tr>
<td>10</td>
<td>Group Assignment Involving Use of the TI-92</td>
<td>75</td>
</tr>
<tr>
<td>11</td>
<td>Group Assignment Evaluation Form</td>
<td>76</td>
</tr>
<tr>
<td>12</td>
<td>Tangents to a Circle from a Point are Congruent (PX  ≡ PY)</td>
<td>126</td>
</tr>
<tr>
<td>13</td>
<td>Circle Worksheet Part I, Problem 3a: Given: Two tangent circles; $\overline{EF}$ is a common external tangent; $\overline{GH}$ is the common internal tangent. Discover and prove something interesting about point G</td>
<td>127</td>
</tr>
</tbody>
</table>
Definition: An Inscribed Angle is an angle whose vertex is on a circle and whose sides are chords of the circle. (\( \angle ABC \) is an inscribed angle.)

Theorem: The measure of an inscribed angle is equal to one-half the measure of its intercepted arc. (\( m\angle ABC = \frac{1}{2} \text{arc AC} \).)

Circle Worksheet Part II, Problem 3: Make and prove a conjecture regarding \( m\angle RST \). NOTE: \( \overline{RT} \) is a diameter of circle Q.

The “Easiest” Solutions to Preliminary Worksheet Problems 20 (Draw an angle complementary to angle 1) and 21 (Draw an angle supplementary to angle 1).

A Second Solution Method for Preliminary Worksheet Problems 20 (Draw an angle complementary to angle 1) and 21 (Draw an angle supplementary to angle 1).

A Third Solution Method for Preliminary Worksheet Problems 20 (Draw an angle complementary to angle 1) and 21 (Draw an angle supplementary to angle 1).

Solutions to Similar Triangle Worksheet Problem 5: Show how you could find triangles similar to those shown by the solid lines.

Circle Worksheet Part I, Problem 2: Line segments PA, PB, and RS are tangents to the circle. Explain why \( PR + RS + SP = PA + PB \).

Construction of a Rhombus.

One Method of Constructing a Kite.

Constructing a Kite Using a Rectangle.

Circle Worksheet, Problem 3, Part II: Make a conjecture regarding the relationship between \( \angle PNM \) and \( \angle PQM \).

An Example of Rhombus Construction.

Points of Intersection of Medians, Altitudes, and Perpendicular Bisectors.

Another Example of Intersection Points.

Triangle Proportionality Theorem.
| A.4 | Triangle Proportionality Theorem - Another Example | 218 |
| A.5 | Three Parallel Lines and Two Transversals | 219 |
| A.6 | Line DF Moved | 219 |
| A.7 | Orientation of Line DF Changed | 220 |
| A.8 | Triangle-Angle Bisector Theorem | 220 |
| A.9 | Triangle-Angle Bisector Theorem with Point B Moved | 221 |
| A.10 | Triangle-Angle Bisector Theorem with Point C Moved | 222 |
| A.11 | Circles with Perpendiculars | 223 |
| A.12 | Equal Ratios | 224 |
| A.13 | Changing Ratios | 224 |
| A.14 | Right Triangle | 225 |
| A.15 | Right Triangle ABC | 227 |
| A.16 | The Pythagorean Theorem Illustrated | 227 |
| A.17 | Another Illustration of \(c^2 = a^2 + b^2\) | 228 |
| A.18 | Euclid's Proof | 229 |
| A.19 | Converse of Pythagorean Theorem | 230 |
| A.20 | \(\angle I > 90^\circ\) | 231 |
| A.21 | \(\angle I < 90^\circ\) | 231 |
| A.22 | 45°-45°-90° Triangle | 232 |
| A.23 | Data For 45°-45°-90° Triangle | 232 |
| A.24 | Data For 30°-60°-90° Triangle | 233 |
| A.25 | 47°-43°-90° Triangle | 234 |
CHAPTER 1

STATEMENT OF THE PROBLEM

The purpose of this study was to determine whether high school geometry students benefit when interactive geometry software is used solely for whole-class demonstration and discussion. Previous research has shown that when interactive software is used with individual students or pairs of students, it can aid students in overcoming obstacles commonly associated with the use of diagrams and can help students to become conjecturers—to become creators of mathematics rather than merely digestors of someone else's mathematics.

Many researchers have investigated the potentially divergent impact of technology on the learning of males and females. Due to gender-based differences in attitudes toward technology and its use, it is possible that any benefits which accrue to students due to the use of interactive geometry software might differ between female and male students. This study investigated students’ abilities to use diagrams and to develop conjectures in a class where interactive software was used only for whole-class demonstration and discussion and also looked at whether gender differences exist in these abilities.
Creating Mathematics

The fact that the primary methods of teaching mathematics that have long been used in the United States as well as in other countries are not adequate to prepare students for life in a modern technological society has been recognized by mathematics teachers, teacher educators, and professional organizations. In *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (MSEB, 1989), the National Research Council states, "Evidence from many sources shows that the least effective mode of mathematics learning is the one that prevails in most of America's classrooms: lecturing and listening" (p. 57). The NRC goes on to state that while this type of teaching might enable students to retain material long enough to do well on a standardized test it does not promote long-term learning, higher-order thinking skills, or problem-solving abilities. In *Professional Standards for Teaching Mathematics*, the National Council of Teachers of Mathematics (NCTM, 1991) recognizes the inadequacy of present mathematics teaching methods and states the need for five major shifts in the teaching of mathematics. Among these are a need to move away from the ideas that the teacher is the only source of mathematical knowledge in the classroom and that all mathematics problems have one right answer that can be found in a few minutes by applying a set of memorized procedures.

As an alternative to the way mathematics has traditionally been taught, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) state that students must be given the opportunity to "conjecture, test, and build arguments about a conjecture's validity" (p. 5). Secondary students should be given:
Experiences designed to foster continued intellectual curiosity and increasing independence [that] should encourage students to become self-directed learners who routinely engage in constructing, symbolizing, applying, and generalizing mathematical ideas. (p. 128)

The NRC (MSEB, 1989) states that a recognition of the inadequacy of present methods of teaching mathematics must result in shifts away from “an authoritarian model based on ‘transmission of knowledge’ to a student-centered practice featuring ‘stimulation of learning’” (p. 81) and away from “preoccupation with inculcating routine skills to developing broad-based mathematical power” (p. 82).

Schwartz (1993) says that we should require students to create in any course we require them to take. He says that we would not think of having students take English without having them create English in the form of compositions or poetry yet we have long taught students only the mathematics that has been created by other people and have not given students the opportunity to create mathematics. He goes on to say, “I assert that the essence of mathematical creativity lies in the making and exploring of mathematical conjectures” (p. 8). He acknowledges that part of the reason why we have not required students to create in mathematics classes is that teaching mathematics via conjecturing is difficult without the proper tools to support it. Within the past few years, tools that enable students to conjecture and thus to create and experience mathematics as a working mathematician have become available in the form of interactive geometry software such as the Geometric Supposer, the Geometer’s Sketchpad, Cabri Geometry, and others.
Use of Diagrams

Obstacles to the use of diagrams in geometry relate to ways in which students view diagrams and are able to manipulate them and, according to Yerushalmi (1993), fall into three categories:

(1) In high school geometry, a single, specific diagram is often used and is intended to be a model for a class of objects, but whatever specific diagram is used has characteristics of its own that do not necessarily apply to the class of objects it is intended to model. One difficulty students have is determining which characteristics apply to the specific diagram and which apply to the class of objects.

(2) In many geometry textbooks, diagrams are commonly presented in a "standard" position. For example, isosceles triangles may always be pictured with the base at the bottom. If students only see diagrams in this "standard" position, they may not be able to recognize the same figure when shown in a "non-standard" position.

(3) Students may not be able to see a diagram in different ways. They may concentrate on a certain part or parts of the diagram and not on the whole or they may concentrate on the whole and ignore specific parts. This could cause them to miss certain key features of the diagram. Also they may see the diagram as a completed figure and feel that they are not allowed to modify or add to it. Modifying diagrams, particularly adding auxiliary lines, can be a key step in developing conjectures or to developing arguments to support conjectures.
Interactive Geometry Software

Interactive geometry software is a fairly recent development. Several different programs exist such as The Geometric Supposers, The Geometer’s Sketchpad, Cabri Geometry, and others. Specific characteristics vary depending on the particular piece of software, but in general, this type of software enables students to make accurate geometric constructions of the type normally done with a straight edge and compass very quickly and easily. For example, Cabri Geometry on the TI-92 calculator has eight pull-down menus containing tools that enable the user to construct angles, lines, line segments, circles, triangles, and other polygons. To construct a triangle, the user selects the triangle tool, designates three points that are to be the vertices, and the calculator draws the triangle. The software also enables the user to construct parallel lines, perpendicular lines, midpoints, perpendicular bisectors, and angle bisectors. The user can easily construct the medians of a triangle by using the midpoint tool to find the midpoint of each side of the triangle and then using the line segment tool to construct the segment from each vertex to the midpoint of the opposite side.

In addition to compass-and-straight-edge constructions, the user can measure distance between two points, length of a line segment, perimeter or area of a polygon, or slope of a line by selecting the appropriate tool and then pointing at the figure that is to be measured. A calculate tool enables the user to compute ratios of line segments, differences between distances, or other measures that may be of interest in verifying a conjecture. Cabri geometry has a check tool that enables the user to determine whether
specified points are collinear and whether specified lines are perpendicular or parallel. The software also contains tools that enable the user to rotate, dilate, and reflect figures.

A feature that Cabri geometry has but that some other software does not is the ability to "grab" a point, move it around, and see what changes take place. For example, a user could construct a triangle and its medians and see that the medians intersect in a point. He or she could then grab a vertex of the triangle and move it to different positions. As the sides of the triangle are stretched or shrunk by moving the vertex, the line segments that were constructed to be medians change so that they are medians of the new triangles that result from moving the vertex. This tool gives the user the ability to construct numerous examples (of triangles in this case) very easily and see what happens to various parts of the diagram and generalize relationships.

Gender and Interactive Software

Several researchers including Brunner (1992) and Nelson and Watson (1991) indicate that there is a gender difference in the ways technology is viewed and used among elementary and secondary students and that this difference grows as students grow older. Sex differences in the attitudes of high school students towards mathematics have been documented in a number of studies (Collis, 1987) with females having more negative attitudes than males. In regard to geometry, Clements and Battista (1992) reviewed research that indicates on the whole that male students outperform female students in geometry. A reason often given for this is that males possess greater spatial visualization abilities as evidenced by consistently higher scores on spatial visualization tests. Since mathematics achievement correlates positively with spatial visualization, it follows that
males would perform better in geometry. With attitude and spatial visualization factors in mind, it is possible that any benefits to students achieved by using interactive software to teach geometry might not be the same for female students as for male students.

Research Questions

This study addresses two primary research questions associated with the ability to use diagrams and the ability to conjecture and prove conjectures. Within each of these two primary areas, the question of differences based on gender is also addressed. The research questions are:

A. Are high school students in a geometry class taught using interactive software only in a whole-class setting able to overcome the obstacles stated by Yerushalmy (1993) to the effective use of diagrams? Are there gender-based differences in students’ abilities to overcome such obstacles when the software is used in this manner?

B. Are high school students in a geometry class taught using interactive software only in a whole-class setting able to form conjectures and produce convincing arguments supporting their conjectures? Are there gender-based differences in students’ abilities to develop and prove conjectures when the software is used in this manner?

Theoretical Rationale

This section discusses the theoretical basis of the position that interactive software used in a whole-class setting in a high school geometry class can benefit students by helping them overcome obstacles associated with diagrams and by enabling them to create mathematics through conjecturing and defending conjectures.
Constructivism

As a theory of learning, constructivism contends that knowledge is constructed by the individual learner based on perceptions, experiences, and previous knowledge. Knowledge construction is a continuing process. As the learner is confronted with experiences that differ from the expected or that present information in conflict with previous constructions, a state of cognitive conflict or disequilibrium is encountered (Simon, 1995). The processes of assimilation, “the filtering or modification of the input” (Piaget & Inhelder, 1969, p. 6) to fit already existing schemes, or accommodation, “the modification of internal schemes to fit reality” (Piaget & Inhelder, p. 6), must then be employed to resolve the conflict and reestablish a state of equilibrium. This done, the process can start over again if new, conflicting information is encountered. Through this cycle, knowledge constructions are continually made and updated.

As indicated earlier, Schwartz (1993) contends that we have an obligation to provide all students the opportunity to create mathematics and that the essence of creating mathematics is “in the making and exploring of mathematical conjectures” (p. 8). Both NCTM (1989) and NRC (MSEB, 1989) state that mathematics classes should be taught in a way that enables students to develop mathematical power, a term that NCTM states, “denotes an individual’s abilities to explore, conjecture, and reason logically” (p. 5). NRC states, “Students construct meaning as they learn mathematics. ... It is students’ acts of construction and invention that build their mathematical power” (p. 59). After describing constructivism, NCTM says, “This constructive, active view of the learning process must
be reflected in the way much of mathematics is taught” (p. 10). Thus, constructivism is a fundamental theoretical basis for reform in the teaching of mathematics.

The process of teaching through conjecturing is a constructivist process in which students use their knowledge and experiences to form conjectures and then test those conjectures and either retain or modify them in light of newly gained knowledge. They experience conflict when a conjecture they believe to be true turns out to be false and, through assimilation or accommodation, must modify their knowledge based on the new information. Yerushalmy and Chazan (1993) describe lab sessions during which students in their studies worked with Geometric Supposer software as follows:

In a lab session, students are typically given a problem and asked to (a) record numerical and visual information gleaned from using the Supposer, (b) formulate conjectures about these data, (c) offer arguments or proofs to support the conjectures, and (d) note any other relevant thoughts, questions, or concerns. (p. 29)

Because interactive geometry software was created to support conjecturing (Battista & Clements, 1995; Schwartz, 1993), constructivism also provides much of the theoretical basis for the present research on using such software to teach high school geometry in a whole-class situation.

One goal of a constructivist teacher is to present situations to students that give rise to cognitive conflict. Wood (1993b) states, “Teachers should provide students with instructional activities that will give rise to problematic situations” (p. 16). In describing his thoughts and actions in teaching a class of preservice elementary teachers the concept of area, Simon (1995) states, “I tried in different ways to promote disequilibrium so the students would reconsider the issue” (p. 129). Interactive geometry software aids in
accomplishing this function. The Geometric Supposer makes it possible for students to easily create and then investigate a large number of accurate geometric figures—many more than could be considered in a traditional geometry class (Yerushalmy & Chazan, 1993). This property of the software gives students the freedom to actively explore their own areas of interest giving them the opportunity to experience cognitive conflict as they explore. This conflict must be resolved, and in the process, knowledge is constructed. Schwartz (1993) states that tools such as the Geometric Supposer enable users to “probe their own understanding of a domain as well as devise new relationships among the objects of the domain” (p. 9). In the present study, because only one copy of the software was available to the class, the teacher was usually the primary operator of the software but actively sought and took directions from students on directions to pursue in exploring geometric figures. Thus students were able to explore to a degree and the teacher was able to provide situations which produced cognitive conflict.

For example, in one lesson the Triangle-Angle Bisector Theorem which states that a line bisecting one angle of a triangle divides the opposite side into segments proportional to the other two sides of the triangle was discussed. Using the TI-92, the teacher constructed a triangle and a line bisecting one of its angles and then used the measurement tool on the calculator to measure line segments and the calculate tool to calculate the ratios of the lengths of segments thus verifying the theorem for this specific example as shown in Figure 1.

The teacher asked students to predict what would happen if point B were moved? There were several conjectures. Many thought the lengths of segments AD and DC would
Figure 1. Triangle-Angle Bisector Theorem (A line bisecting one angle of a triangle divides the opposite side into segments proportional to the other two sides of the triangle) Applied to Angle ABC

remain constant. Using the "grab" tool on the calculator, the teacher moved point B and students saw that, contrary to their predictions, the lengths of segments AD and DC changed. The teacher asked the class to think about why those lengths change when point B is moved and the reason they change was discussed by the class. Because BD is the bisector of angle ABC, if B is moved and changes the size of angle ABC, then, since BD is the bisector of angle ABC it must also move. This changes the lengths of segments AD and DC. As stated in the theorem, the ratios remain equal. This is illustrated in Figure 2.

Both Schwartz (1993) and Yerushalmy (1993) discuss conjecturing, induction, and generalization and the close ties among these processes. Schwartz states, "The mental acts of thinking inductively and generalizing are at the heart of what mathematics students ought to learn to do" (p. 9). Yerushalmy states:

Generalizations are a particular kind of conjecture, created by reasoning from the specific to the general. ... In induction the generalization is created from the examination of instances or examples. As an instance or a set of instances is examined, certain properties are identified. The given example is then taken as a member of a larger set and its properties are imputed to the larger set. Thus, in induction, a generalization is induced from examples. (p. 58)
Generalizing is then a form of conjecturing and as such is also a constructivist process. Software such as that used in the present study seeks to develop students' generalizing skills by presenting them with large quantities of relatively easily obtained visual information with which to work.

Figure 2. An Illustration of the Effect of Moving Point B on Segments AD and DC When the Triangle-Angle Bisector Theorem is Applied to Angle ABC

Several researchers (Confrey, 1990; Noddings, 1990) have distinguished between weak and strong acts of construction of mathematical knowledge. Even in rote learning situations, students are constructing knowledge but most will be performing weak acts of construction characterized by memorization, imitation of examples, and isolation from the body of knowledge. The desire for the mathematics classroom is that students perform strong acts of construction. Confrey states that the fundamental quality of a strong or powerful construction is that it is believed by the student. Several authors (Battista & Clements, 1995; Lampert, 1993; Wilson, 1993) indicate that many students in geometry class see no reason for proof. Lampert states that often students believe if a teacher gives them a statement to prove or if a statement comes from the textbook, it must be true so
why bother proving it. For some students, the fact that something looks to be true in a
diagram or several similar diagrams is sufficient to prove it. Battista and Clements discuss
studies done using the Geometric Supposer and conclude that students using the software
"believed that theorems generated with Supposer software needed to be proved before
they could be accepted as true" (p. 51). Thus, the Supposer enabled the student to make a
strong construction of the need for proof. Schwartz (1993) states that one of the benefits
of using the Supposer is, “Students coming to understand deeply and for themselves the
need for formal proof in mathematics” (p. 14). In the study described, Cabri Geometry
software was used to explore geometric diagrams to see if using the software in a whole-
class situation would also produce student recognition and understanding of the need for
proof.

Confrey (1990) lists several additional characteristics of strong constructions.
Among them are “A potential to act as a tool for further considerations; ...A guide for
future actions; and ... An ability to be justified and defended” (p. 112). Interactive
geometry software when used to support conjecturing as in the present study aids the
development of each of these characteristics in students. The diagrams that the software
enables a student to make are themselves knowledge constructions that guide the student
in developing conjectures (knowledge constructions). The student then develops an
argument to justify or defend the conjecture.

Constructivism and Classroom Discourse

In studies using the Geometric Supposer software (Yerushalmi & Houde, 1986),
students have used the software either individually or in pairs during lab sessions that

13
normally take place two days each week. An important activity that occurs on some of
the days when lab sessions are not held is whole class discussions of the results of lab
sessions. Thus, even in classes in which students use the software individually or in pairs,
the importance of student-student and student-teacher interaction in a whole class session
is recognized and incorporated into the teaching method. In the present study because the
software was used only for whole class demonstration and discussion, some of the benefits
which could have been achieved by students using it individually or in pairs may have been
lost but additional benefits may have been achieved through the additional time spent in
whole class demonstration and discussion. Thus, classroom discourse is an essential
element in the present study.

Classroom discourse that enables effective student-student and student-teacher
interaction is a key ingredient in the constructivist theory of learning. Confrey (1990)
states that, “Constructivism commits one to teaching students how to create more
powerful constructions” (p. 112). She describes such instruction as “inherently
interactive” (p. 112). In describing teaching models that have proven successful in
enabling students to perform strong acts of construction, Noddings (1990) says, “All of
these methods share a common characteristic: They are all highly interactive” (p. 17).
One reason such instruction is highly interactive is that discourse between teacher and
student and among students causes students to be confronted with the ideas of others that
they then must compare with their own ideas. If the new ideas are in conflict with their
own, the learner experiences cognitive conflict that must be resolved (Wood, Cobb, &
Yackel, 1993). Through this process learning takes place. Wood (1993a) states:
Social interaction could give rise to cognitive conflict for the individuals. The situations that children find problematic take a variety of forms and can include resolving obstacles or contradictions that arise when they attempt to make sense of a situation in terms of their current concepts or procedures, accounting for a surprising outcome, verbalizing their mathematical thinking, resolving conflicting points of view, developing a framework that accommodates alternative solution methods, and formulating an explanation to clarify another child’s solution attempt. (p. 11)

Studies with the Geometric Supposer (Gordon, 1993; Lampert, 1993; Yerushalmy, 1993; Yerushalmy & Chazan, 1993) in which students use the software individually or in pairs in a lab setting several days each week and then meet as a whole class to discuss the results of their lab work verify that use of the software causes students to be confronted with the thinking of other students and with the need to justify, modify, or defend their own thinking in terms of that of other students. Lampert’s study of teachers using the Supposer shows that interaction among students and between students and teacher as they discuss their findings make it necessary for the teacher to be willing to modify lessons to go where the discussion leads instead of sticking strictly to set plans.

Another reason classroom discourse is essential to a constructivist perspective of teaching is that students are viewed as active constructors of knowledge. To perform strong acts of construction, students must be actively involved in the learning process. In describing radical constructivism, Bauersfeld (1992) states, “Every cognitive construction is not passively received but (a) is a person’s individual construction, and (b) results from adapting to constraints and acting upon challenges” (p. 21). This implies that, “The student will need ample opportunities to develop, present, and discuss her/his personal constructions and ideas, and to check their convergence” (p. 21) and, “the permanent
support of an attitude of curiosity, of inquisitiveness, of searching for a pattern and regularities, of expecting to find surprising issues, appear to be markedly helpful” (p. 23).

In describing his thoughts in regard to teaching area concepts, Simon (1995) states, “I considered, however, that for a student to follow an explanation might not require the same level of understanding as would be needed to generate an explanation” (p. 127). That is, the activity on the part of the student required to explain the concept would generate a deeper understanding than merely listening to someone else explain it.

An essential technique in using interactive software is having students explain their thinking--what they did, what they discovered, what they think it means. This can be done in classes not using the software but giving students the ability to easily explore geometry seems to encourage them to explore, to take pride in what they discover, and to want to explain it to others (Lampert, 1993; Yerushalmy, 1993; Yerushalmy & Chazan, 1993).

Gordon (1993) states:

They [students] learn to value their own ideas and those of others. This leads to a growing ability to communicate about mathematics and to integrate individual and group learning. Above all, they assume an increasing amount of responsibility for the making and transmitting of knowledge in the classroom. (p. 233)

Quoting one student from his Supposer class in which no textbook was used, Healy (1993) states:

The thing I hate most about the Build-a-Book class is that I’m always tired. The reason is I stay up late trying to figure out how I can make the other people in class understand how I think and, especially, how I can change their minds to agree with me. (p. 93)

Foletta (1994) found that use of the Geometer's Sketchpad enabled lower achieving students to participate more in classroom discourse.
The implications of a constructivist perspective for teaching mathematics are many and classroom discourse is essential. Noddings (1990) states:

The emphasis on construction forces us to probe deeply into students' activity. How firm a grasp do they have on the material? What can they do with it? What misconceptions do they entertain? Even if they are producing wrong answers, are they constructing in a way that is mathematically recognizable? These are among the questions we need to ask in order to teach effectively. (p. 14)

Simon (1995) describes such teaching in the following manner:

The only thing that is predictable in teaching is that classroom activities will not go as predicted. Although the teacher creates an initial goal and plan for instruction, it generally must be modified many times (perhaps continually) during the study of a particular conceptual area. As students begin to engage in the planned activities, the teacher communicates with and observes the students, which leads the teacher to new understandings of the students' conceptions. The learning environment evolves as a result of interaction among the teacher and students as they engage in the mathematical content. (p. 133)

Simon (1993) states that the teacher is no longer the dispenser of information but rather the architect of learning opportunities that include whole-class discussions. Steffe and D'Ambrosio (1995) state, "One reason for stressing interactive mathematical communication as a principal currency of a mathematics teacher is that social interaction underlies all teaching actions" (p. 156). Woods (1993a) states, "The teacher's role was to listen to her students as they expressed their thinking about the activities and to build from these thoughts by asking questions and making suggestions to create opportunities for their learning" (p. 9), and "collaborative settings created situations in which the teacher could learn about individual students' mathematical thinking as she listened, observed, and asked questions while interacting with the groups" (p. 12). Wood, Cobb and Yackel (1993) state:
The challenge for teachers is to find ways to facilitate and build on their student’s ideas to encourage the construction of increasingly powerful conceptual operations. This requires listening to children’s explanations and developing an understanding of the underlying conceptual operations that underscore children’s thinking. (p. 67)

Confrey (1990), Bauersfeld (1992), Steffe and D’Ambrosio (1995), and Wood (1993b) all discuss the need for the teacher to view children’s constructions of mathematics, which may seem irrational to the teacher, as rational expressions of their understanding. Confrey indicates that the differences in the ways children view mathematical ideas “are not simply reducible to missing pieces or absent techniques or methods; children’s ideas also possess a different form of argument, are built from different materials, and are based on different experiences” (p. 109). Children’s ideas may make sense only within their limited framework or may be genuine alternatives. Through interaction with the child, the teacher needs to probe the child’s understanding to determine the current status of constructions and then decide how instruction can best proceed. Instead of trying to convince the child that he or she is wrong based on the teacher’s mathematical expectation and continuing to question the child until he or she gives the “correct” answer, Bauersfeld suggests that a “wrong” answer can be used as a starting point for instruction. Even though the answer may not be what was expected, there are likely parts of the child’s construction of the concept that are on the right track. Through discourse with the child, the teacher can identify those constructions and use them as a positive starting point for further instruction. He states:

Adapting the actual tasks towards a possible extension of what the student can already do more or less successfully (Vygotsky’s “zone of proximal development”) will surely function less frustratingly and more encouragingly than the usual forcing into contradictions and the continued “squeezing” questioning. (p. 26)
An important product of teacher-student interaction discussed by several researchers (Confrey, 1990; Simon, 1995; Steffe & D’Ambrosio, 1995) is the teacher’s construction of a mental model of the child’s mathematical knowledge. Confrey describes it as a case study of each student’s understanding. She states:

Through their interactions with students regarding their knowledge of subject matter, teachers construct a tentative path upon which students may move to construct a mathematical idea more consonant with accepted mathematical knowledge. Teachers, however, must be prepared for the likelihood that students’ constructions will not coincide with their own, and encourage the students’ expression of their beliefs so that teachers come to understand student beliefs. (p. 112)

She describes how the teacher involved in her study of a summer classroom at Mount Holyoke College would analyze the difficulties he thought each of his students would have with a particular topic and then, based on his knowledge of that student, plan “a tentative solution path” (p. 120).

Simon (1995) describes this path as a hypothetical learning trajectory—hypothetical because, just as student’s knowledge of mathematics is constructed, so too is the teacher’s understanding of that knowledge. This trajectory is composed of a goal for learning, the activities designed to achieve that goal, and “a prediction of how the students’ thinking and understanding will evolve in the context of the learning activities” (p. 136). As instruction proceeds, both the student’s knowledge of mathematics and the teacher’s understanding of that knowledge are continually updated necessitating changes in the hypothetical learning trajectory. Teacher-student interaction is again essential to this process. “Students’ thinking and understanding is taken seriously and given a central
place in the design and implementation of instruction. ...Understanding students’ thinking is a continual process of data collection and hypothesis generation” (p. 141).

In their model, Steffe and D'Ambrosio (1995) refer to the hypothetical learning trajectory as the zone of potential construction which is:

Determined by the teacher as she interprets the schemes and operations available to the student and anticipates the student’s actions when solving different tasks in the context of interactive mathematical communication. The anticipation is based on the teacher’s knowledge of other students’ ways of operating, on the teacher’s knowledge of the particular mathematics of that student, and on the results of the teacher’s interactions with the student. (p. 154)

Research using interactive geometry software, in most cases the Geometric Supposer, indicates that teaching changes dramatically when these tools are used. It is no longer possible to let the textbook dictate the next day’s lesson. Lampert (1993) states, “the Supposer takes the ordering of the learning agenda away from the teacher” (p. 165).

In her study, teachers using the Supposer commented that one of their most difficult tasks was in understanding the thinking of students and following the students’ leads through the subject. They stated it was difficult keeping track of where each individual student was in terms of the subject matter. Clements and Battista (1992) report that in classrooms using various computer environments which would include interactive geometry software, it is more difficult for students to hide what they do not know. This indicates that teachers using such tools should find it somewhat easier to at least know what students do not know. Gordon (1993) states that teachers using the Supposer “display an uncommon eagerness to discern the varying learning styles of their students and to find ways to modify their teaching to address these varying styles” (p. 234), “realize that the scope and
sequence of the traditional geometry course is not sacred” (p. 234), and “learned to let go, to give up their role as the only authority in the classroom” (p. 234).

Schoenfeld (1986) lists four techniques that teachers can use in a classroom to focus student’s attention on the thought processes they might use in solving problems. Two of these would be especially applicable when using interactive software for whole class demonstration and discussion. In the first, the teacher serves as a role model and instead of presenting only the polished solution to a problem as is usually done in a lecture class, models the processes that yield the solution—the false starts, recoveries, mistakes, insights, and how they can be used to solve the problem. In the second, the teacher serves more as a scribe or orchestrator rather than leader, lets students take the lead in discussing or presenting a topic, but steps in periodically to comment on the direction the class is taking.

Social Constructivism

According to Simon (1995), social constructivism combines psychological and sociological aspects of the classroom. The psychological perspective refers primarily to the individual—the individual’s knowledge of and about mathematics, the individual’s knowledge of the understandings of mathematics that other students have, and the individual’s knowledge of the ways mathematics classes function. The sociological perspective refers primarily to the group of learners and focuses on two factors: taken-as-shared mathematical knowledge and social norms. Each individual continually updates his or her knowledge constructions as new experiences take place or new information is encountered. Since knowledge construction is based on individual interpretations and
understandings, it is impossible for one individual to have specific knowledge of the understandings of other individuals although, through interaction, a group can develop a sense of understandings that they share in common. Taken-as-shared knowledge refers to such sensed common understandings while social norms refer to the practices established and accepted by students and teacher as to what constitutes effective and expected participation in an environment in which learning mathematics is valued (Cobb, Yackel, & Wood, 1993; Simon, 1995).

Classroom discourse allows negotiation of meaning to take place and taken-as-shared knowledge to be developed. Wood, Cobb, and Yackel (1993) describe the process as follows:

The tensions that exist between the meanings held by the teacher or by other students create the necessity to take another’s perspective into account and to engage in negotiation so that meanings can be established that are then taken-as-shared. The manner in which the teacher and students mutually constitute the basis for mathematical activity in the discussion creates the possibility for establishing genuine communication discourse and creates opportunities for students to learn. (p. 59-60)

Not only does it create opportunities for students to learn—it also allows students to experience the way mathematics is actually developed. Bauersfeld (1992) says:

Only the active participation in the related language game and the permanent negotiation of meaning can lead to the constitution or accomplishment of taken-as-shared meanings for actions, symbols, and objects. Regulations and norms will emerge from the permanent interaction between teacher and students, as well as among the students themselves, rather than from an explicit talking about something without living it. (p. 21)

Lampert (1993) indicates that use of the Supposer makes students more independent learners and that information such as definitions, terminology, and labeling are often
discovered or invented by the students instead of being "doled out to students according to a careful agenda" (p. 170). Gordon (1993) states that teachers using the Supposer learned to value the ideas of students and to give up their role as the only source of authority on the subject matter and that students using the Supposer "become geometers—nimble ones for the secondary school setting" (p. 231). Yerushalmy and Houde (1986) indicate that the Supposer "can encourage students to behave like geometers" (p. 418).

The importance of social norms in quality classroom discourse has been emphasized by a number of researchers (Bauersfeld, 1992; Confrey, 1990; Simon, 1995; Wood, 1993b; Wood, Cobb, & Yackel, 1993). Social norms inform students as to what is expected of them both in regard to explaining their own mathematical thinking and in regard to valuing the ideas of others. Indeed, without social norms, effective discourse could not take place. Both aspects are important and are primarily the responsibility of the teacher to establish.

Students must know that in the mathematics classroom, they are expected to explain their mathematical reasoning processes—there is no other option. In their study of second-grade students, Wood, Cobb, and Yackel (1993) indicate that, by her actions, the teacher made it clear to students that they had an obligation to provide justification for their thinking. Simon (1995) felt that his students' "competence in providing justification would grow as they engaged in discussions in which the demand for justification was consistently present" (p. 125). Student participation and justification of reasoning are key aspects to a classroom using interactive geometry software to support a conjecturing approach to teaching.
If students are to put their thought processes on display not only for the teacher but for other students to see, then an attitude of respect for the ideas of others is an absolute necessity (Wood, 1993b). In their study of second-grade children, Wood, Cobb, and Yackel (1993) state, "It was necessary for children to be afforded opportunities to discuss their solution methods in a setting that encouraged them to explain and justify their reasoning and thinking" (p. 55). In discussing this same idea, Bauersfeld (1992) says:

Many mathematics teachers are quite rigid in their verbal aspirations and their related evaluations of students' utterances. But they are quite permissive in the social organisation of their class. Under the integrating perspective the opposite way round appears to be a more promising one: to accept and encourage students' mathematical utterances within very wide limits with respect to how it is said, as long as a serious background (reason, argument, etc.) can be identified. But to be rigid about keeping the social regulations, namely, insisting on listening to others' inventions and explanations, keeping turn-taking order, taking seriously the others' serious contributions, etc. (p. 24)

The establishment of social norms that require respect for the ideas of others is an absolute necessity for successful learning in a classroom using interactive software to teach geometry using a whole-class demonstration and discussion format. Students' discussing the thought processes that lead them to particular conjectures is an essential element in students' encountering cognitive conflict that enables them to make more powerful constructions of mathematical knowledge. Lampert (1988) describes making a conjecture as:

Taking a risk; it requires the admission that one's assumptions are open to revision, that one's insights may have been limited, that one's conclusions may have been inappropriate. While possibly garnering recognition for inventiveness, letting other interested persons in on one's conjectures increases personal vulnerability. Courage and modesty are appropriate to participation in mathematical activity because truth remains tentative, even as the proof of a conjecture evolves. (p. 3).
**Learning About Mathematics**

Classroom discourse is important not only to students' learning of mathematics but also to their learning about mathematics. A major criticism of traditional methods of teaching mathematics is that these methods do not give students a true picture of what mathematics is or of how mathematicians work (CUPM, 1971; Davis, 1986; NCTM, 1989). The manner in which mathematics has been presented in most textbooks and in most mathematics classes totally ignores the interactive nature of the development of mathematics. Many students view mathematics as a set of rules to be memorized (Schoenfeld, 1989), feel all problems can be solved in a few minutes (Schoenfeld, 1988b), and seldom see a requirement for any novel or innovative work (Doyle, 1988). Silver, Kilpatrick, and Schlesinger (1990) indicate that the traditional way mathematics has been taught causes students to think that mathematics "emerged fully formed from Euclid's brow and has remained fixed ever since" (p. 8). In describing mathematics—the mathematics that students seldom see—Schoenfeld (1988a) states:

Indeed, "figuring it out" is what mathematics is all about. ....Mathematicians spend most of their time making sense of things. Doing mathematics is sense-making and becoming a mathematician includes developing (or internalizing) the mathematician's aesthetic, a predilection to analyze and understand, perceive structure and structural relationships, to see how things fit together. (p. 87).

Schwartz (1993) feels that appropriate software such as the Geometric Supposer can help students "figure it out." Yerushalmy and Houde (1986) state that because of the wealth of numerical and visual information it makes available and because conjectures can be tested easily, the Supposer makes it possible for students to behave like working mathematicians.
Vygotsky

Vygotsky (1978) defines the zone of proximal development as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (p. 86). He indicates learning takes place only within the zone of proximal development that “awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers” (p. 90). A primary reason for the development of interactive geometry software is to enable students to test conjectures easily and quickly. In so doing, it fills the role of a more capable peer by enabling the student to do something he or she would not otherwise be able to do.

Yerushalmy (1993) states, “The Supposer was designed to focus and direct users’ attention by providing certain options and not others and by making some things easy to do while making others difficult or impossible” (pp. 61-62). Schwartz (1993) indicates that the rationale for putting some construction tools into Supposer software was to make it easier for the user to explore geometry so that the user would explore geometry. All of Euclidean geometry could be derived from tools that only make lines and arcs but Schwartz says:

If a user of the software must make an elaborate series of straight line and circular arc constructions in order to draw a median or an angle bisector, that user is unlikely to explore playfully the properties of geometric constructions that are rife with medians and angle bisectors. (p. 11)
A simple example of Cabri Geometry software acting as a more capable peer lies in the measuring of an angle. To measure an angle correctly using Cabri Geometry, the student must designate three points making up the angle in the proper order, that is, with the vertex point as the center point. If the student does not do this, the software will not measure the angle the student wants to measure. The student must learn to designate the angle according to established conventions in order to use the software. Supposersoftware functions as a more capable peer by quickly and accurately drawing a number of diagrams that the student can then investigate to see what they have in common. It would be very tedious and sometimes impossible for the student to make such drawings without the software--many would simply not do it. The software also enables the student to quickly check whether two lines are parallel, whether two lines are perpendicular, whether three points are collinear, and to accurately measure such things as the lengths of line segments or the areas of figures. These tools are also things that the student alone either could not do accurately or that would be very difficult and time consuming to do.

Van Hiele

Dina van Hiele-Geldorf and Pierre Marie van Hiele proposed a model of geometric thinking composed of five levels through which students pass in developing geometric understanding. The van Hieles felt that teachers should give students a wide variety of learning experiences in geometry to help them progress between levels. They proposed a five-step process of instruction designed to assist a student in progressing from one level to the next. This instructional model emphasizes the importance of both teacher-student and student-student interaction in learning geometry. They felt that higher levels were not
achieved through teacher lecturing but through appropriate exercises appropriately sequenced (Clements & Battista, 1992). They felt questioning of students to be a crucial part of instruction. Students should know “how” and “why” and should be asked to come up with alternative explanations or proofs at the appropriate levels (Crowley, 1987).

According to Hoffer (1981), at Van Hiele Level 1, a student can recognize a shape as a whole but not be aware of properties of that shape; at Level 2, the student can recognize and analyze properties of figures; and at Level 3, the student can understand interrelationships among figures. As mentioned earlier, one obstacle discussed by Yerushalmy and Chazan (1993) to the use of diagrams in developing conjectures is the difficulty students have in changing from a focus on the whole diagram to a focus on particular parts of the diagram or from a focus on one part of the diagram to a focus on another part. The difficulty in changing focus when using diagrams could mean that the student is at one Van Hiele level and needs to progress to the next higher level before he or she is capable of changing focus. Yerushalmy and Chazan state that one reason for development of the Geometric Supposer was to help students overcome this obstacle by giving them a number of diagrams to consider as opposed to the one diagram they might see in a textbook. This would aid students in progressing through the Van Hiele levels.

**Significance**

The present study yields information on whether the use of interactive geometry software for whole-class demonstration and discussion helps students to overcome common obstacles associated with the use of diagrams. It provides data on methods students used to overcome those obstacles, on problems they encountered in attempting to
do so, on effective ways to use the software in the classroom, and on teacher workload inherent in using the software in this manner on a daily basis.

The study also provides data on whether use of the software solely in a whole-class situation helps students to become more adept at developing and proving conjectures. Gordon (1993) summarized research relating to students using the Geometric Supposer individually or in pairs in the classroom by saying that students were curious, desired to explore the subject, were willing to take risks and make mistakes, and seemed “to value new ideas and to recognize that there are occasions when they themselves are the originators of such ideas” (p. 231). Students develop intellectual independence and learn to value their own ideas and the ideas of others. These characteristics are necessary for students who are to be creators of mathematics. If use of the software in a whole-class setting also helps to develop these same characteristics in students, it will aid them in becoming conjecturers.

In previous studies which identified benefits associated with the use of interactive geometry software, students had access to the software individually or in pairs. Due to cost, many schools are not able to afford the numbers of computers or calculators necessary to give students this kind of access and therefore do not use the software. Many of these schools could afford one computer or calculator with interactive software for each geometry class. If similar benefits can be achieved through use of interactive geometry software in whole-class demonstration and discussion, then these schools could be motivated to invest in the software and many more students will be able to benefit from the technology.
This study contributes to the body of knowledge on gender differences in the learning of mathematics and the use of technology. To date, there have been few studies that investigated the relationship of gender to the use of interactive geometry software.

Using interactive software to teach high school geometry could improve students' attitudes toward mathematics and teachers' attitudes towards teaching geometry. In studies of interactive technologies in general, Adkins (1993) found that students liked learning from such courseware, a feeling echoed by McGivney (1990) in regard to a mathematics course developed to satisfy a university-wide mathematics requirement. Teachers who pioneered use of the Geometric Supposer in high school geometry indicate that student attitudes toward mathematics improve when they are able to take a more active role in the classroom (Gordon, 1993; Healy, 1993; Houde, 1993). This study investigates student attitudes toward learning geometry before and after the use of interactive software.

Lampert (1993) states that teachers using the Geometric Supposer felt that the additional time and effort required to use it was worthwhile and that they would use it again and encourage others to do so. In discussing research findings in regard to teachers using the Geometric Supposer, Gordon (1993) says teachers, “started to do mathematics instead of simply teaching the subject” (p. 233), and displayed “a growing willingness to explore new technologies and their potential for enhancing the teaching and learning of mathematics” (p. 233). Because the researcher was also the teacher of the class and was the only teacher involved, this study is limited in the amount of information it provides on changes in teacher attitudes due to use of the software. It does include reactions of the
researcher to teaching geometry using interactive software for the first time and comparisons of using the software to using the textbook.
This chapter is a review of literature pertinent to various aspects of the present study. It begins with a review of research on the use of conjecturing and diagrams in mathematics classes and then discusses the results of research which investigated the use of interactive geometry software on students' abilities to conjecture and to use diagrams. Other related topics such as classroom discourse, the reform movement in mathematics education, and gender issues are then discussed.

Conjecturing

The ability of students to generate reasonable conjectures is seen as a major element in the development of true mathematical ability. The National Council of Teachers of Mathematics indicates that an essential element of a student's development of mathematical power is the ability to conjecture (NCTM, 1989). Schwartz (1993) says that the essence of creating mathematics is "in the making and exploring of mathematical conjectures" (p. 8). Yerushalmy (1993) states, "The ability to extend rules using correct generalization techniques is a major characteristic of mathematical competence" (p. 61).
Yerushalmy (1993) describes conjecturing as a method people use to reach conclusions in which statements thought to be true but not known to be true are made. Conjectures can arise from a number of sources including induction, generalization, explanation, deduction, belief, experience, and proof. The types of conjecturing of interest in the present study arise from the processes of generalization and induction both of which involve reasoning from the specific to the general. In induction, properties identified in an example or set of examples are conjectured to be true for a larger set of objects.

Yerushalmy describes generalization as, “a process acted out on a statement,” in which, “a more specific statement becomes a more general statement” (p. 58). Generalization involves extending either a proven statement or a conjecture to include a larger set of objects than previously included.

Using Lakatos’ (1976) description of a mathematics classroom, Chazan (1995) suggests that, “rather than the examination of a collection of examples leading to a conjecture, conjectures lead to the examination of examples,” and “stronger conjectures are built out of the corroboration and refutation of prior conjectures” (pp. 15-16). Thus, conjectures can come from the examination of examples but also conjectures can lead to the examination of examples. The idea is that in order to determine what data to look at when investigating specific examples, the investigator must have had some conjecture in mind.

Yerushalmy (1993) reviews research into the difficulties that students have in making generalizations. One difficulty is overgeneralization in which students either observe no differences in the structure of new examples from the structure of examples to
which a rule is known to apply and erroneously extend a rule to cover the new examples or try to apply rules formed for special cases to all cases (Matz, 1982). Research by Nickerson and others (1985) indicates that in making conjectures about the relationships between two factors, students often focus only on situations in which both factors are valid and fail to consider other possibilities such as situations in which one or both factors may not apply. In so doing, they miss many possibilities for conjectures.

Chazan (1995) observed and studied high school classes in which teachers were attempting to use a conjecturing approach to teaching geometry. He describes the difficulties that students have in generating conjectures even when the patterns in the data they are exploring are very apparent to the teacher. Feeling somewhat frustrated by the students’ inability to generate conjectures, teachers sometimes attempt to direct the attention of students to certain data by providing a chart indicating what measurements students should make and record as they examine diagrams. Chazan states that student difficulties in developing conjectures are even greater in such instances.

Proof and disproof are essential elements of conjecturing. One study by Yerushalmy (1993) used a pretest and a posttest to assess students’ conjecturing abilities and another study used a test to assess students’ abilities to prove true statements. At the beginning of the school year, students showed the common difficulties with the use of diagrams and very marginal abilities to produce conjectures or generalizations. Many seemed to think that three examples were sufficient to confirm a conjecture and, similarly, that three counterexamples were required to disprove one. This is in line with the statement of Clements and Battista (1992) that in proof students often do not distinguish
between measurement evidence and deductive proof—many think measuring can prove a statement for all members of an infinite class of objects and that a deductive proof applies only to one specific diagram. Often students see no reason for proving something that is either in the textbook or which the teacher has already called a theorem (Battista & Clements, 1995; Lampert, 1993; Wilson, 1993). Bishop (1986) states that traditional methods of teaching geometry have led to proof meaning “learning someone’s proof of someone else’s theorem” (p. 153).

Clements and Battista (1992) indicate that many secondary school students are at Van Hiele level two or below and, because the ability to construct proofs occurs at level three or beyond, proof is beyond the reach of many students at the beginning of a year of high school geometry. This feeling is echoed by Hoffer (1981) who along with Clements and Battista suggests changes in teaching methods and classroom activities to raise the Van Hiele levels of students prior to introducing formal proof. Mikusa (1995) looked into methods used by elementary and secondary school students to justify their thinking and found that less than one percent used formal proof. Many high school students used what he termed Intuitive Affirmation in which the student makes a statement thought to be self-evident and therefore makes no attempt to validate that statement.

The testing of conjectures necessitates looking for nonexamples which prove the conjecture false. Hershkovitz, Bruckheimer, and Vinner (1987) indicate that students bring images of various geometric concepts based on their experiences and beliefs to the geometry classroom and these concept images can have a negative impact on their ability or desire to look for nonexamples. Yerushalmy (1993) states that previous studies on
generalization in the social sciences and in mathematics education show that, “common biases to plausible generalizations include an insufficient quantity of samples and a tendency to choose examples that confirm an hypothesis” (p. 82).

**Diagrams**

Although meant to be an aid to students, the diagrams that are used to illustrate concepts such as theorems or definitions in geometry can cause considerable difficulties for students just beginning their study of mathematics. Yerushalmy (1993) groups these obstacles to the use of diagrams into the three categories discussed in the following paragraphs.

**Determining essential characteristics of a diagram.** The difficulties that students have with diagrams arise largely from the fact that most theorems or definitions in geometry apply to a broad class or family of objects not just to one object but when we try to use a diagram to illustrate a theorem, we must do so with a specific diagram. That diagram has characteristics of its own which may or may not be common to the family of objects to which the theorem or definition applies. Discerning which characteristics are common to the family of objects to which the theorem applies and which apply only to the specific diagram causes great difficulties for many students. Schwartz (1993) describes this dilemma in this manner:

One can construct a regular 3-gon, a regular 17-gon, a regular (any particular number)-gon but one cannot construct a regular N-gon. Similarly, one can construct any particular triangle, but one cannot construct a triangle that is any triangle. If one constructs a diagram of a triangle, then aside from the size of the triangle, there is only one such triangle. (This is in sharp contrast to the situation in algebra where a notation system allows one to write F(x) = mx + b to denote any linear function.) The geometry one wishes to learn, teach, and make does not
deal with the properties of particular shapes but rather with the properties of classes of shapes. (p. 10)

The work of Hershkovitz, Bruckheimer, and Vinner (1987) with elementary students as well as preservice and in-service elementary teachers indicates that all groups have difficulties in distinguishing the essential characteristics of a diagram (those characteristics which must be present for the diagram to be an illustration of a particular concept) from nonessential characteristics (those which only particular examples have). Through a series of diagrams illustrating examples and nonexamples of the concept, they led students and teachers to a definition of what they called a “trianquad,” a triangle and quadrilateral with a common vertex. Although teachers did better than students in defining the trianquad based on the diagrams given, members of both groups stated unnecessary or incorrect attributes in their definitions.

Recognizing figures. Students often have difficulties identifying geometric figures when the figures are shown in positions different from those in which students are used to seeing them. This problem has been exacerbated by textbooks which commonly show figures in only one position such as right triangles only with the right angle at the bottom. Hershkovitz, Bruckheimer, and Vinner (1987) asked elementary school students and teachers to identify right triangles shown in different positions and found that both groups could identify right triangles drawn with the base at the bottom, had more difficulty identifying right triangles when the right angle was rotated through 45 degrees, and had most difficulty when the right angle was shown at the top of the figure. They also asked the students and teachers in their study to draw the altitudes of several triangles and the
diagonals of several quadrilaterals shown in diagrams. They found that teachers did little
better than students in these activities with many members of both groups failing to draw
the altitudes of an oblique triangle outside of the triangle or the diagonals of a concave
quadrilateral outside the quadrilateral. They conclude that the concept images held by
teachers and students of what an altitude or diagonal should look like precluded them
from applying the definition correctly.

Changing views of a diagram. This difficulty relates to students' abilities to view
diagrams in different ways, to change from a view of the total diagram to a concentration
on particular parts or vice versa, or their concept of a diagram as an unchangeable entity
which thus prohibits them from adding to it. This difficulty is also illustrated in the work
of Hershkovitz, Bruckheimer, and Vinner (1987) on trianquads in which both teachers and
students had difficulty recognizing that the definition of a trianquad applied in diagrams
such as that shown in Figure 3.

![Figure 3. A "Trianquad"—a Triangle and a Quadrilateral with a Common Vertex](image-url)
Another example given by Yerushalmy (1993) is shown in Figure 4 in which students either see segment AD as an altitude of triangle ABC and fail to see it as a side of triangles ABD and ADC or see it as a side of those triangles and fail to see it as an altitude of the larger triangle.

![Figure 4](image-url)  
**Figure 4.** Some Students See Segment AD as an Altitude of Triangle ABC, Some as a side of Triangles ADB and ADC but Many Cannot See Both

Bishop (1986) points out that this "geometrical rigidity" is exploited in problems such as counting the number of squares on a checkerboard or connecting the nine dots in Figure 5 without lifting the pencil. Geometric rigidity may also help explain students'...
reluctance to add segments or other geometric figures to the diagrams which illustrate a theorem, definition or problem. They feel that, for some reason, this is beyond the rules of the game and they are not allowed to do so. The inability to distinguish parts from the whole could relate to Van Hiele levels. A student must be at level two in order to do this but research cited by Clements and Battista (1992) indicates that many high school students are below level two even after completing a course in geometry. Work must be done to raise the Van Hiele levels of students in order to enable them to overcome this obstacle.

**Why students may not draw---or use---diagrams.** Simon (1986) reports on a study which investigated why students did not draw diagrams to help solve problems in which it seemed that the use of a diagram would be helpful. While this is somewhat different from the previous discussion of the difficulties students have in using diagrams they are given in a textbook or by the teacher, it seems that the reasons that students do not draw diagrams could also help explain the difficulties they have in using diagrams they are given. Simon's study identified five characteristics of students which seem to impact whether or not they choose to draw a diagram.

First, it is impossible to draw a diagram which helps to solve a problem without an understanding of certain concepts related to the problem. Similarly, if a student does not have a grasp of concepts related to a given diagram, then he or she will not be able to use the diagram to solve a problem, complete a proof, or develop a conjecture.

Before a student will draw a diagram to help solve a problem, the student must feel capable of drawing and manipulating diagrams, must have experience which indicates that
diagrams can be helpful in problem solving, and must think to draw the diagram. This would also apply to diagrams students are given. While these diagrams are already drawn, the student must feel capable of manipulating the given diagram and feel that he or she is allowed to do so. Through experience, the student must see the value in using given diagrams and must think to use them.

According to Simon (1986), how a student views mathematics and himself or herself as a mathematics student also impacts willingness to draw a diagram. If the student sees mathematics as a set of disconnected rules which cannot be understood, feels that there is always only one correct way to do a mathematics problem, and sees themselves as incapable of understanding and doing mathematics, then he or she will not be likely to attempt drawing a diagram to investigate ways of approaching a problem. It seems apparent that students holding these views would also be less likely to productively use the diagrams they are given. If a student feels they have no control over the mathematics they are doing, then they would not feel it permissible to manipulate a given diagram. They might feel that there is only one “correct” way to view a given diagram and refrain from trying to see it in other ways which could lead to a solution of the problem.

Lastly, before a student will draw a diagram as an aid in solving a problem, the student must want to solve the problem correctly. While we often take it for granted that students want to solve problems correctly, Simon (1986) points out that this may not always be the case. For the student experiencing mathematics anxiety, the more immediate goal may be to just get the problem over with regardless of whether it is done
correctly or not. Taking the time to draw—or to use—a diagram only increases the time it takes to do a problem for such a student thus increasing the time that they must deal with the anxiety.

**Misleading diagrams.** Sometimes, instead of helping a student to solve a problem, an inappropriate or incorrect diagram can lead a student to an incorrect solution. For example, is it possible to draw a triangle with sides of the lengths 5 inches, 6 inches, and 12 inches? The answer is no because the sum of the lengths of any two sides of a triangle must be greater than the length of the third side, but such a triangle can be drawn and labeled on the chalk board as in Figure 6. When this is done, many students will conclude that it is possible to draw such a triangle because it was just done.

![Figure 6. An Impossible Triangle—the Sum of the Lengths of Two Sides is Less Than the Length of the Third Side](image)

In the preface to *Geometry Turned On*, King and Schattschneider (1997) give a similar example in which an incorrectly drawn diagram leads to a seemingly valid proof that all triangles are isosceles. If the student believes the diagram, then he or she will most likely believe the incorrect result.
Interactive Geometry Software, Conjecturing, and Diagrams

Although there are several interactive geometry software products available, most of the research done to this point has used the Geometric Supposer and much of that research is described in various chapters of *The Geometric Supposer: What is it a Case of?* including Lampert (1993), Yerushalmy (1993), Yerushalmy and Chazan (1993), and others. The findings of these studies and others indicate that students in classes using the Supposer learn as much geometry content as students in more traditional classes (Bobango, 1987; Gordon, 1993). This should help alleviate some of the concerns of teachers and students about whether students can learn what they need to learn in a class where the syllabus is determined by student explorations (Lampert, 1993). On the other hand, some of this concern may be unjustified because other studies indicate that the majority of students do not learn geometry when it is taught using traditional methods at least if proof writing ability is used to determine whether or not students know geometry. Hirschhorn and Thompson (1996) report on a study by Senk (1985) that found that only 30 percent of geometry students master proof in a year-long course. Battista and Clements (1995) report that “formal deduction among students who have studied secondary school geometry is nearly absent” (p. 48). McCoy (1990) compared two similar geometry classes, both with experienced, well-qualified teachers, both using the same text, but one using the Supposer during class time approximately once every two weeks and the other not using the Supposer. She found that students in the Supposer class scored significantly higher than the non-Supposer class on a posttest provided by the textbook publisher. She also found that Supposer students scored significantly higher on
problems requiring higher order thinking skills and on applications problems while performance on tasks requiring lower order thinking was about the same for the two groups.

Obstacles to the use of diagrams. Yerushalmy and Chazan (1993) presents a summary of research into whether the Geometric Supposer could aid students in overcoming the previously mentioned obstacles to the use of diagrams. Studies reported were conducted in high school geometry classes in the U. S. (primarily the Boston area) and in Israel. Results indicate that use of the Supposer does help students overcome those obstacles. A 1984-85 study, in which two classes used the Supposer and one comparison class did not, showed that few of the students in any of the classes used diagrams on a pretest, 80% of the non-Supposer group did not use diagrams on the posttest, but many Supposer students did use diagrams on the posttest. A study of two eighth-grade classes in Israel showed Supposer students had fewer difficulties with diagrams in non-standard position than did non-Supposer students. Other indications of Supposer students overcoming obstacles to the use of diagrams are that they “worked with many diagrams, saw them as instances of a single class, and were able to abstract the features that characterize the class, leaving behind the particular, non-characteristic aspects of the individual diagram” (p. 38); “imputed movement to diagrams, treated individual diagrams as snapshots of a process occurring to one underlying configuration, and thought of those snapshots as a class of figures with common characteristics” (p. 38); and “when working without the computer on a single member of a class, were even able to imagine other
members of that class and build arguments on this basis” (p. 38). Summarizing their research, Yerushalmy and Chazan state:

When given problems to analyze, the students in this study are willing and able to focus on different parts of diagrams, look for nonstandard diagrams, and add auxiliary lines to diagrams in order to derive conjectures. They are also adept at creating and examining sequences of related diagrams. Their use of these abilities led to many interesting, novel conjectures. Finally, the skills in working with diagrams are also helpful to these students when proving their conjectures. They are able to abstract the variant and invariant aspects of the set of objects in a series of diagrams and to use auxiliary lines and their ability to focus on parts of diagrams to derive proofs. (p. 54)

In the previously mentioned problem of proving that all triangles are isosceles, King and Schattschneider (1997) emphasize the necessity of accurate diagrams and point out that interactive geometry software makes the erroneous diagrams which lead to such conclusions highly unlikely if not impossible.

Generalization abilities. Yerushalmy (1993) uses the same studies as Yerushalmy and Chazan (1993) but focuses on use of the Supposer to aid students’ generalization abilities. In the studies reported by Yerushalmy, as the year progressed, Supposer students were more able to look at a given statement in a variety of ways and produce more original conjectures and were better able to consider a given diagram as one of a series of diagrams and thus to produce generalizations. They were able to look at diagrams from different viewpoints and produce conjectures based on these different views of the diagrams. Results of the argument test showed the Supposer students more capable of producing convincing proofs. This is in contrast to findings of Bobango (1987) who studied 72 high school geometry students in both regular and honors classes and found no significant difference in proof writing ability between those using the Supposer and those
not using it. Some of the difference could be explained by the fact that Bobango's findings were based on a 20-day treatment whereas studies by Yerushalmy and others generally covered a longer period of time. Yerushalmy states that Supposer students appreciate the ease with which they can generate examples with the software and that they learn to use extreme cases and nonexamples to support their conjectures. Battista and Clements discuss studies using the Geometric Supposer and conclude that the software helped students understand the need for proof, a feeling echoed by Schwartz (1993).

Lester (1996) studied 47 female high school geometry students divided into a control group which used ruler, protractor, and compass for geometric constructions and an experimental group which used the Geometer's Sketchpad software. She found that use of the software to create geometric diagrams and use of its dynamic capabilities to modify and manipulate those diagrams increased students' learning and understanding of geometric concepts. Elchuck (1992) studied the impact that the dynamic capabilities of the software had on students' conjecture-making abilities. He randomly divided 157 ninth-grade students into one group which used the full capabilities of either the Geometric Supposer or Geometer's Sketchpad software and another group which had access to all of its capabilities except the ability to modify figures by dragging selected parts. He found no difference in the conjecture-making abilities of the two groups of students.

Chazan (1995) indicates that the type of problems used with interactive software determines whether the software is an aid or hindrance to students' conjecturing abilities. If a teacher uses what he terms "discovery" problems or "induction by enumeration" in
which the teacher leads students to “discover” what the teacher has in mind, then use of
the software can enforce the notions that many students have that the teacher is the sole
possessor of mathematical knowledge in the classroom, that mathematics problems have
only one correct solution, and that mathematics is not a subject to be actively explored.
On the other hand, if a teacher uses “induction by intuition” or “guided inquiry” problems
which may suggest avenues of exploration but assume that “students must take
responsibility to decide which question to explore and which measurements to choose to
test the hypotheses they have developed based on their intuitions (from prior experience
and knowledge developed in the class)” (p. 23), then interactive software can aid students
in becoming conjecturers, help them to develop mathematical power, and enhance their
image of themselves as students of mathematics. He concludes by saying:

If such problems (discovery problems) degenerate into an activity where students
are guessing what is on the teacher’s mind, the teacher is still the ultimate authority
and arbiter of correctness of students’ ideas; the teacher’s ideas are the most
important ideas in the classroom. By way of contrast, the guided inquiry type of
problem supports a teaching style which alters the roles of students and teachers.
It emphasizes the importance of students’ ideas, allows students some choice, and
changes the teacher’s role from herder to that of facilitator, coach and community
builder. (pp. 26-27)

Changing roles of teachers and students. Lampert (1993) reports on the views of
teachers from a diverse group of schools in the Boston area who were using the Supposer
to teach high school geometry during the 1986-87 school year. Teachers were
interviewed and were also observed as they taught using the Geometric Supposer and as
they participated in monthly meetings. In most schools, the Supposer was used in lab
situations in which students worked individually or in pairs several days each week.

Students met as a whole class on non-lab days. Lampert says:

In the classrooms I studied, the software, the computers, and the commitment to spend regular class sessions doing lab problems functioned together to redefine both the nature of geometric knowledge and the teachers’ and students assumptions about their respective roles in the acquisition of that knowledge. The Supposer became a new authority on what it meant to do geometry and thus on what teachers needed to teach and what students needed to know and know how to do. The technology was in no sense a replacement for either the teacher or the textbook however. It was an additional resource in the classroom that put both of those more familiar elements in a different perspective. (p. 152)

In most classes, instead of following set lesson plans or following the textbook, students’ experiments in using the Supposer determined the direction the course would take. This caused several problems identified by Lampert (1993). Both teachers and students were uncertain as to whether students would learn what they were “supposed to” in such an environment, teachers had difficulty figuring out how to guide student learning, teachers had to relinquish the idea that the study of Euclidean geometry can proceed in only one path—that dictated by the textbook—and both students and teachers had to give up the notion that all math problems have only one correct answer. Another significant change results from the fact that when the direction of the course is determined by student exploration, the teacher will not always be prepared to answer every question or work every problem that arises. Teachers have to be willing to say, “I don’t know,” or, “I’ll figure it out and show you tomorrow.” This challenges the idea that the teacher is the holder of all mathematical knowledge in the classroom and is responsible for passing it down to students. In many cases, students are able to figure it out and also to help other students to do so. Comments from students in Healy’s (1993) classes bear this out. One
student stated, "Our class used to believe Mr. Healy was the basis of knowledge for
gometry, and we used to bug him for answers. No matter what he says now, we don’t
feel any obligation to agree with him" (p. 100). Another said, "You actually have to use a
part of your brain you don’t use very often in school. The thinking part. ...We’re all
given a chance to speak our thoughts and opinions. I love it. I loved being forced to
think."
(p. 103). Lampert states, "All of these structural innovations in the way geometry
is communicated to learners results in a substantial increase in students’ capacities to learn
mathematics independently from teachers" (p. 169).

Classroom Discourse

The importance of classroom discourse to a constructivist theory of learning and in
reforming the teaching of mathematics was discussed earlier. This section reviews some
of the literature pertaining to classroom discourse.

Classroom discourse refers to communications between teacher and students or
among students in a classroom situation. It is the method by which the majority of
instruction is imparted, that students use to demonstrate learning, and is part of the
individual identity of teachers and students (Cazden, 1986). Therefore, classroom
discourse is an important part of the educational experience. It can range from lecture in
which the teacher does all the talking and students are only passive participants to
discussion in which students are active participants doing most of the talking with the
teacher being an informed listener who may occasionally help guide the discussion and
who evaluates the depth of student understanding. Edwards and Mercer (1987) and
Cazden categorize much of classroom discourse as falling into an initiation-response-
evaluation structure in which, for example, the teacher asks a question, the student provides an answer, and the teacher evaluates the answer. These researchers provide detailed discussions of the factors that operate in the classroom in forming and guiding discourse including explicit and implicit rules, verbal and nonverbal cues, misunderstandings, and the potential impact of different cultural backgrounds on participation in discourse. Students learn the "rules" of classroom discourse and respond accordingly.

In a study of interaction in algebra classes, Whalen (1994) found students to be bound by the initiation-response-evaluation pattern learned through years of experience in the mathematics classroom culture. She found that interaction increased significantly when the lesson was not from the textbook or from worksheets. In such cases, students often felt the teacher was unprepared to teach and that this gave them license to be more independent, to take more control of the classroom situation, and to break out of the normal patterns of classroom communication. Healy (1993) comments that in his "no book" class, he did not ask questions or participate in class activities for one to three months--sufficient time to allow students to build confidence in themselves so that their confidence would not be shaken when he asked a question. This led some students to feel that he did not know the subject matter and was not capable of teaching them but it also led to the students assuming responsibility for their own learning and that of others.

Nonverbal communications from students make up some of the cues that teachers use in making critical decisions in teaching and therefore are an important part of classroom discourse. In a review of research on nonverbal communications, Miller (1986)
states, "many more feelings and intentions are sent and received nonverbally than verbally" (p. 6) and therefore, teachers need to be better receivers and transmitters of nonverbal communications. Neill (1991) discusses nonverbal communications in depth and states that, in many cases, the message transmitted nonverbally is more truthful than that given verbally. Among the reasons students use nonverbal communications are a lack of opportunity to communicate verbally, feelings of insecurity or inequality, and the need to transmit messages that they cannot express verbally for some reason. Being able to receive and accurately interpret nonverbal communications greatly enhances discourse and quality of learning. Neill indicates, for example, that students often express lack of understanding through facial expressions and that students learn to use nonverbal communications to avoid verbal interaction with the teacher.

Green and Weade (1987) discuss the importance and complexity of interaction in lesson formulation, delivery, and reception. "Lessons are constructed and negotiated during interactions between teacher and students" (p. 6), and consequently, participants must "monitor continually what is occurring, how it is unfolding, and who is participating or required to participate" (pp. 6-7). They emphasize that the participant must be able to simultaneously attend to what is being learned, to how it is being approached, and to the social structure of the classroom—what are the rules governing who can speak and what constitutes an appropriate response. These rules can vary several times during the course of one lesson and students must be able to recognize when changes occur and adjust their behavior accordingly. Participants must also attend to the "historical" aspects of the lesson—what happened earlier in the class period influences how they are to respond to
situations that occur later in the period. Green and Weade provide the following description of the overall complexities of the classroom interaction process in the course of progressing through a lesson:

The teacher must decide who can talk when, where, about what, and for what purpose; weigh the effect of student participation on the forward flow of the lesson, provide feedback to students, meet individual student needs and maintain group and lesson direction. Students are coparticipants in the construction of classroom lessons. As such, they must also monitor the teacher expectations as signaled during delivery; determine when, how, and whether to participate; monitor the academic, social, and activity demands; construct, interpret, and reconstruct text (e.g., read, write); observe teacher responses to others as well as to self to determine expectations for what to know and what to do....At any given point in the lesson under construction, in order to interpret what is meant, participants must process both the context of the message and its delivery. (p. 9)

While discourse does not necessarily determine quality of learning (Noddings, 1990), the consensus of opinion seems to be that a style that involves a higher degree of student involvement leads to higher quality of learning for most students (Cobb, Yackel, & Wood, 1993; Edwards & Mercer, 1987; NCTM, 1991).

Lampert (1988) states that in the culture of the mathematics classroom, the teacher and textbook are the sources of knowledge, mathematics is not a subject for exploration, and knowing mathematics means getting right answers. Reform efforts in mathematics education seek to shift this definition of what it means to know mathematics, and, in order to make this shift, “teachers and students need to do different sorts of activities together with different kinds of roles and responsibilities” (p. 9). In the class that is the focus of the study, she was responsible for picking out problems that were not solved by merely applying some known algorithm to get the correct answer but that gave opportunity for discussion through multiple possibilities for solution. Students were responsible for
discussion—for explaining their insights and thinking. She states, "the solution is more than the answer, just as the problem is more than the question. Generating a strategy and arguing for its legitimacy indicates what the student knows about mathematics" (pp. 11-12). Thus, effective discourse between teacher and students and among students is essential for students to know what it really means to do mathematics.

The Reform Movement

The recommendations of several professional organizations involved in reform movements reflect general agreement with the findings of the research cited above in regard to the importance of classroom discourse and the type of discourse essential to a quality learning environment. Adopting a constructivist viewpoint, the National Council of Teachers of Mathematics Professional Standards for Teaching Mathematics (NCTM, 1991) emphasizes classroom discourse as an essential element in an environment that promotes student construction of mathematical knowledge and also emphasizes the essential role the teacher has in establishing this type of discourse. Three of the standards specifically address the essential nature of discourse in the mathematics classroom and the critical role teachers play in establishing such discourse:

1. Standard Two states that to effectively orchestrate classroom discourse, the teacher must be able to decide "when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with difficulty" (p. 35). The teacher should be able to monitor the participation of each student in classroom discussion and to decide "when and how to encourage each student to participate" (p. 35).
2. Standard Three addresses the students' role in discourse and states that teachers should:

Promote classroom discourse in which students listen to, respond to, and question the teacher and one another; ...initiate problems and questions; make conjectures and present solutions; ...try to convince themselves and one another of the validity of particular representations, solutions, conjectures and answers. (p. 45)

3. Standard Four discusses enhancing classroom discourse and indicates that the goal should be to “establish a discourse that is focused on exploring mathematical ideas, not just on reporting correct answers” (p. 52). This standard goes on to state that teachers must encourage students to use a variety of tools in expressing mathematical ideas and should accept various means for communicating about mathematics.

The classroom in which this type of discourse takes place is different from the traditional classroom in which the teacher does most of the talking and students are passive. The *Professional Standards for Teaching Mathematics* (NCTM, 1991) emphasizes that the type of classroom discourse just described cannot take place in a lecture format or when students are asked only lower level questions. NCTM states, “In order for teachers to maximize communications with and among students, they should minimize the amount of time they themselves dominate classroom discussions” (p. 96).

NCTM (1991) also addresses the teachers' responsibility for establishing a classroom environment in which strong acts of knowledge construction by each student can take place. A key to this is establishing in the classroom a sense of genuine respect for the ideas of others and of valuing reason and sense-making. Specifically, the *Professional Standards for Teaching Mathematics* states that the teacher should establish an
environment that fosters the learning of mathematics by "respecting and valuing students' ideas, ways of thinking, and mathematical dispositions; and by consistently expecting and encouraging students to take intellectual risks by raising questions and formulating conjectures" (p. 57). NCTM states:

In order to facilitate learning by all students, teachers must also be perceptive and skillful in analyzing the culture of the classroom, looking out for patterns of inequality, dominance, and low expectations that are primary causes of nonparticipation by many students. Engaging every student in the discourse of the class requires considerable skill as well as an appreciation of, and respect for, students' diversity. (p. 34)

Similar views are taken by other organizations. The National Research Council (MSEB, 1989) states:

Teachers need to listen as much as they need to speak. They need to resist the temptation to control classroom ideas so that students can gain a sense of ownership over what they are learning. Doing this requires genuine give-and-take in the mathematics classroom both among students and between students and teachers. The best way to develop logical thinking is to encourage open discussion and honest criticism of ideas. (pp. 59-60)

The Carnegie Forum (1986) also states the need for students to be active learners and says that schools must change their focus from "passive acquisition of facts and routines to the active application of ideas to problems" (p. 8). This requires teachers who can support that type of learning, who can "help students see patterns of meaning where others see only confusion, [and]...foster genuine creativity in students" (p. 8). In their overall recommendations on the reform of teacher education, the Holmes Group (1986) states that professional, competent teachers "know that teaching and learning are interactive" (p. 29) and, therefore, such teachers avoid lectures that dispense fact after fact and force students to be only passive observers. These teachers have the ability to "interpret the
misunderstandings students bring to and develop during lessons, they identify students’
misconceptions, and question their surface responses that mask true learning” (p. 29).
After summarizing their recommendations of mathematical course content for future
mathematics teachers, the Committee on the Undergraduate Program in Mathematics
(CUPM, 1971) of the Mathematical Association of America goes on to state:

Communication is essential in mathematics, and prospective teachers must pay
special attention to all of the ways mathematics is most effectively communicated. They
should be led to regard mathematics as a creative activity—something which
one does rather than merely something which one learns. The active participation
of the student in the process of discovering and communicating mathematical ideas
is crucial for his real understanding. Courses should be taught in ways that foster
active student involvement in the development and presentation of mathematical
ideas. (p. 20)

Chazan (1995) suggests that the use of interactive software in high school
geometry can run counter to the reform efforts described depending on the type of
problems teachers use with the software. If teachers use problems which direct students
toward specific conjectures, they may be reinforcing the ideas that the teacher is the sole
possessor of mathematical knowledge in the classroom, that math problems have only one
correct solution, and that exploration in mathematics is limited to discovering that
solution. On the other hand, problems in which students must decide what data to collect
and what questions to explore can help to overcome these notions and thus strengthen the
reform effort.

Gender and Technology

Brunner (1992) states, “There is a growing body of research documenting a gender
difference in the way we use technology” (p. 6). She reports on a study in which men and
women viewed as technology experts were asked to fantasize about future developments in their areas of expertise. While there was some overlap in responses by the sexes, most followed gender lines. She states,

The women in this study saw their technological instruments as people connectors, as communication and collaboration devices. The men tended to see them as extensions of their power over the physical universe. On the whole, the women’s technological fantasies were sensual, were embedded in human relationships, served to integrate their public and private lives, and were relatively modest in aspiration. The men’s far more epic technological fantasies were about absolute power, tremendous speed, and unlimited knowledge. (p. 7)

Nelson and Watson (1991) reviewed literature on computer use and attitudes of school children toward technology. They state that no differences between the sexes in computer use and attitudes are apparent in preschool and early elementary grades. Differences begin to appear by the third or fourth grade with girls seemingly less interested in computer use until, by the teen years, a number of studies report that girls dislike computers while boys enjoy them and thus become more skillful in using them. In the 1970s and early 1980s, gender differences in computer use and skills were primarily a component of math anxiety since computers and computer programming were closely associated with mathematics classes. This has changed as computer use has expanded to other subject areas, but gender differences in attitudes toward and use of computers are still apparent. One factor contributing to this difference is the differing ways girls and boys are socialized and the differing opportunities they have for computer use. Parental attitudes are a large factor. Boys receive more encouragement and are given more opportunities for computer use at home and more opportunity to attend computer camps and classes. Tittle (1986) states that even where total enrollments in computer-related
courses may not vary widely, the types of courses do vary with a very large percentage of
girls being enrolled in word-processing courses and a relatively small percentage in
programming courses. Turkle (1984) as cited by Nelson and Watson feels that a reason
for these differences lies in the fact that girls are often taught negotiation, compromise,
and give and take whereas boys are taught decisiveness and the imposition of will. Tittle
reports evidence that the objectives of socialization for boys are often self-reliance and
independence which is not often the case for girls.

Another factor in gender differences cited by Nelson and Watson (1991) is
software selection. Research studies have documented male dominance in characters and
storyline in children’s textbooks and more recently in educational software programming.
In addition, such resources often show men in a wider variety of professions and playing a
wider variety of roles. Tittle (1986) states that “gender related differences in interests
need to be considered in software development in order to affect attitudes” (p. 1164).

Collis (1987) studied the relationship between sex and attitudes towards
computers and attitudes towards mathematics of secondary school students and found the
association between attitudes towards mathematics and attitudes towards computers to be
significant. This confirms the results of earlier studies and suggests that negative attitudes
towards mathematics may help generate negative attitudes towards computers. One
reason for this is that the introduction of computers into the curriculum has often been
closely associated with mathematics. Other researchers suggest that the use of computers
may help improve the attitudes of females towards mathematics but Collis’ study did not
confirm this.
Senk and Usiskin (1983), as reported by Clements and Battista (1992), found that females and males possessed equal proof writing skills and were at essentially the same Van Hiele levels at the beginning of the year but that males outperformed females on both beginning and end-of-year content tests and that males were at higher Van Hiele levels at the end of the year. They feel this could be due to the differences in educational experiences available to the two groups. Others feel that girls may do as well as boys on proof writing because girls tend to follow the rules more carefully and thus do better on tasks that are accompanied by a formal set of rules. Others indicate that girls prefer verbal modes of thought while boys prefer nonverbal modes.

Some remedies to this situation have been tried with some success. Clements and Battista (1992) report that in a study by Flores (1990) use of the University of Chicago School Mathematics Project geometry materials appeared to reduce or even reverse this gap. They conclude by stating:

Thus, there is evidence suggesting that males and females, or at least subgroups thereof, may differ in the processes they use to solve mathematics, particularly geometry problems. There seem to be differences along the spatial dimension. However, these differences have not yet been adequately investigated. (p. 456)

They caution that research on gender differences is a complex undertaking and that lack of gender difference on a particular task is not an indication that males and females are using the same methods to approach problems.

**Significance of the Present Study**

Other studies (Yerushalmy, 1993; Yerushalmy & Chazan, 1993) have investigated the potential benefits to students of use of interactive geometry software in teaching high
school geometry. The present study differs in two aspects. In other studies, the software was used on a computer workstation while the present study used Cabri Geometry on the TI-92 calculator. This has several potential benefits. First, the calculator is much less expensive than the computer workstation and this difference in cost could make the technology available to many more students. Second, the calculator is much more mobile than the computer workstation and therefore access to the software is not limited to times that the class can obtain access to a computer laboratory. Secondary schools normally have limited computer facilities and teachers must compete for their use. Use of the TI-92 makes it possible to utilize the interactive software in a regular classroom setting without having to take the class to a computer laboratory.

In previous studies, the software has been used either by individual students or pairs of students. In the present study, one copy of the software was used for whole-class demonstration and discussion. If some of the benefits achieved through use with individual students or pairs of students can also be achieved when the software is used in a whole-class setting, the cost to schools of purchasing the software will be further reduced and it can be made available to many more students.

Other studies which have investigated benefits to students from the use of interactive geometry software have not looked at whether benefits differ by gender as the present study does.

Previous studies (Adkins, 1993; Gordon, 1993; Healy, 1993; Houde, 1993; McGivney, 1990) have indicated that student attitudes improve when interactive software is used in the classroom but these conclusions have been based primarily on teacher
observations and student comments. Attitude measurement instruments were not used. The present study uses a such an instrument to compare attitudes by gender at both the beginning and end of time that interactive software was used in the classroom and to compare attitudes at the beginning of that time period with attitudes at the end to see if there were any significant changes in attitudes which might be attributed to use of the software.

Most other studies in which interactive software has been used in the classroom have been more limited in scope either taking place over a short period of time or using the software in a computer laboratory situation only a few times a week. In the present study, students observed the software being used much more frequently and throughout an entire semester. The increased frequency of use over a longer time period could change the impact that the software has on students.
CHAPTER 3

METHODS AND PROCEDURES

This study investigated whether students in geometry classes using one TI-92 calculator with Cabri geometry software solely for whole class demonstration and discussion could overcome obstacles to the use of diagrams, become more proficient in conjecturing and proof, and whether any benefits achieved might differ by gender. The study was conducted from August 1996 to May 1997 and included both quantitative and qualitative portions. The quantitative portion investigated student attitudes towards factors related to the learning of geometry. The qualitative portion included worksheets, interviews, and student presentations and sought to determine students' abilities to use diagrams at various times during the study and whether there were changes in these abilities as the study progressed.

Site

The study was conducted in two high school geometry classes taught by the researcher at Alamogordo High School, Alamogordo, New Mexico during the 1996-97 school year. Alamogordo is a city in southeastern New Mexico with a population of approximately 30,000. Alamogordo High School is the only high school in the city and
has an enrollment of approximately 2200 students. In addition to permanent residents of Alamogordo and the surrounding area, the high school draws a number of students from families assigned to nearby Holloman Air Force Base.

**Subjects**

The subjects were the students enrolled in two geometry classes taught by the researcher during the 1996-97 school year. Students were assigned to the two classes using the school's normal procedure for assigning students to classes and no effort was made to get any particular students or groups of students into those classes. The numbers of students in each class varied throughout the year. There were a total of 57 students assigned to the two classes at the beginning of the school year. Of those, 44 students completed the school year in the same class. Other students came into the two classes during the school year so that at the end of the year, there were 49 students in the two classes.

**Instrumentation**

Several methods were used to assess the impact of the use of the interactive geometry software. Among these were an attitude survey; homework assignments, quizzes, tests, and worksheets completed by students; interviews with selected students; and student group presentations of theorems related to areas of polygons.

**Attitude surveys.** A Likert-type attitude survey instrument was developed to assess the attitudes of students towards various factors associated with the learning and doing of mathematics. Of the 66 questions on the instrument, 42 were taken from the Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976) and the
remaining 24 were developed by the researcher to obtain information about areas unique to the present study.

The Fennema-Shernan Mathematics Attitudes Scales (Fennema & Sherman, 1976) are made up of nine subscales which were designed to assess certain factors which might impact a student's decision whether or not to study mathematics as well as the student's ability to learn mathematics. Factors addressed in the scales could impact females and males in differing ways, and so the scales can be used to assess whether there are differences in the attitudes of female and male students. Table 1 lists the titles and purposes of each of the nine subscales. The abbreviations shown in the table for each subscale will be used in various places throughout this document in referring to questions taken from the subscales.

Each subscale consists of 12 statements to which the subject responds in one of five ways: strongly agree, agree, undecided, disagree, or strongly disagree. Of the 12 statements in each subscale, six are positively worded, that is, worded so that a response of agree or strongly agree indicates a positive attitude in regard to that factor and six are negatively worded, that is, worded so that such a response indicates a negative attitude in regard to that factor. To limit the length of the attitude survey instrument used in the present study to a reasonable level, six statements from each of seven of the subscales (Confidence in Learning Mathematics, Attitude Toward Success in Mathematics, Teacher, Mathematics as a Male Domain, Usefulness of Mathematics, Mathematics Anxiety, and Effectance Motivation in Mathematics) were selected for inclusion. Statements selected were those that appeared to the researcher to most closely parallel the purpose of the
## Table 1

**Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976)**

<table>
<thead>
<tr>
<th>Title and Abbreviation</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence in Learning Mathematics Scale (C)</td>
<td>Assess student confidence in their ability to learn mathematics and to do well in mathematics classes.</td>
</tr>
<tr>
<td>Mother Scale (M)</td>
<td>Assess student perceptions of their mother’s attitudes toward their ability to learn mathematics</td>
</tr>
<tr>
<td>Father Scale (F)</td>
<td>Assess student perceptions of their father’s attitudes toward their ability to learn mathematics</td>
</tr>
<tr>
<td>Attitude Toward Success in Mathematics Scale (AS)</td>
<td>Assess student perceptions of the consequences of success or failure in mathematics</td>
</tr>
<tr>
<td>Teacher Scale (T)</td>
<td>Assess student perceptions of their teacher’s attitudes toward their ability to learn mathematics</td>
</tr>
<tr>
<td>Mathematics as a Male Domain (MD)</td>
<td>Assess student perceptions of mathematics as a domain more appropriate to males than females</td>
</tr>
<tr>
<td>Usefulness of Mathematics Scale (U)</td>
<td>Assess student perceptions of the usefulness of mathematics both in their current school situation and in their future plans</td>
</tr>
<tr>
<td>Mathematics Anxiety Scale (A)</td>
<td>Assess student feelings of anxiety or dread associated with being confronted with mathematical tasks</td>
</tr>
<tr>
<td>Effectance Motivation in Mathematics Scale (E)</td>
<td>Assess student perceptions of their desire to be involved in mathematical activities</td>
</tr>
</tbody>
</table>
present study. No statements from the Mother or Father scales were used. This decision was made primarily to limit the length of the survey instrument and because these scales seemed to have less applicability to the present study than the others. Of the six statements selected from each subscale, three were positively worded and three were negatively worded. Selected statements are shown in Chapter 4.

An additional 24 statements were developed by the researcher and included in the attitude survey instrument. These statements were necessary to assess student attitudes toward items unique to the present study which are not addressed in the Fennema-Sherman scales such as attitudes towards what constitutes proof in mathematics and attitudes towards the use and manipulation of diagrams which are included with problems in geometry. These statements are shown in Chapter 4. Statements from the Fennema-Sherman scales along with statements developed by the researcher were randomly mixed to produce the attitude assessment instrument which is shown in Appendix B.

**Preliminary worksheet, homework, and quiz.** Students were given a worksheet, homework assignment, and a quiz, each of which required the interpretation of diagrams and the making of conjectures. These were given to assess the initial position of students in regard to use of diagrams and conjecturing. The worksheet is shown in Appendix C.

**Worksheets.** A primary element of the study was three worksheets which were developed by the researcher and completed by all students in both geometry classes. Worksheets, which are shown in Appendix C, dealt with three specific topics: similar triangles, the Pythagorean Theorem, and circles.
Interviews. Following completion of the worksheets on similar triangles and circles, 16 students were interviewed to discuss in more detail their responses to specific questions on the worksheets and their rationale for those responses. This was done to enable the researcher to gain further insight into how students used the diagrams; whether they were able to overcome the obstacles commonly associated with diagrams and, if so, how they overcame each obstacle; how they developed conjectures based on the diagrams; how they developed arguments to support their conjectures; and their views of the strengths of those arguments. Interview questions were based on individual student responses to items on each worksheet so there were no specific interview protocols. Criteria for selecting students for interviews are explained in detail in the procedures section of this chapter and in Chapter 5.

Student presentations. Students were divided into groups of four or five and each group was assigned a theorem related to the area of a specific polygon to present to the rest of the class. Group presentations were set up by the teacher so that all students in the group had to do some part of the presentation but the students decided which student would do which part of the presentation. Assignments of students to particular groups were determined in all cases by the teacher.

Other data collection. In addition to those discussed above, several other devices were used to collect data related to the purpose of the study. These included a teacher journal and student explanations of how to construct various geometric figures using the tools available on the TI-92.
Procedures

Table 2 summarizes data collection procedures which are described in more detail in the paragraphs which follow. With the exception of interviews which were conducted with 16 selected students as described earlier and the journal completed by the teacher/researcher, subjects of data collection types were all students in both geometry classes taught by the researcher.

Assessment of initial position. The worksheet, homework assignment, and quiz used to assess the initial position of students in regard to use of diagrams and conjecturing were completed by all students in both classes in October 1996, prior to use of the TI-92 in either class. Students completed the worksheet and quiz working alone in class. The homework assignment was given to students one day and turned in on the next day. Each of these instruments required students to interpret diagrams and to make conjectures. They were evaluated by the teacher/researcher and each counted as a grade in the class. Examples of problems and results are reported in Chapter 5.

Introduction to the TI-92. To acquaint students more thoroughly with the tools which are available on the TI-92 geometry package so that they would have a better understanding of what was happening later when the calculator was used in front of the whole class, arrangements were made with Texas Instruments Incorporated to borrow 30 TI-92s for one week toward the end of January 1997. During that week, students used the calculators individually in class everyday. Students were given a handout (Figure 7) showing the menus available on the TI-92 geometry package. They had this handout available for use throughout the remainder of the year.
Table 2

*Data Types and Collection Methods*

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Collection Method</th>
<th>Collection Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude Survey</td>
<td>66-question Likert-type survey instrument</td>
<td>October 18 and 29, 1996 May 22, 1997</td>
</tr>
<tr>
<td>Preliminary Worksheet, Homework, and Quiz</td>
<td>Student papers completed at home or in class</td>
<td>October 1996</td>
</tr>
<tr>
<td>Student Journals on reactions to using the TI-92</td>
<td>Written journals kept by students daily for one week</td>
<td>January 1997</td>
</tr>
<tr>
<td>Worksheets on Similar Triangles, the Pythagorean Theorem, and Circles</td>
<td>Written problems completed by students in class</td>
<td>February to May 1997</td>
</tr>
<tr>
<td>Interviews</td>
<td>Audio taped and transcribed</td>
<td>February to May 1997</td>
</tr>
<tr>
<td>Group Presentations</td>
<td>Video taped</td>
<td>April 1997</td>
</tr>
<tr>
<td>Lessons using the TI-92 with Cabri Geometry</td>
<td>Some video taped</td>
<td>January to May 1997</td>
</tr>
<tr>
<td>Teacher Journal</td>
<td>Typed entries completed periodically by the teacher/researcher</td>
<td>August 1996 to May 1997</td>
</tr>
<tr>
<td>Student papers on constructions</td>
<td>Written tests, quizzes, and homework completed by students</td>
<td>January to May 1997</td>
</tr>
</tbody>
</table>
Figure 7. Handout Given to Students to Illustrate TI-92 Screens and Menus
During the first several days of the week that the calculators were available, the teacher had students follow him through some examples to illustrate use of the various tools available on the calculator (Figure 8). During the last few minutes of class each day, students worked individually or in groups on simple assignments taken from *92 Geometric Explorations on the TI-92* (Keyton, 1996). These included activities such as constructing an angle and its bisector, then constructing perpendicular segments from a point on the bisector to the sides of the angle, measuring the lengths of these perpendicular segments and looking for a relationship. Students then moved the point around on the bisector and were able to observe that the lengths of the perpendicular segments change but remain equal to each other. This illustrates that any point on the bisector of an angle is equidistant from the sides of the angle.

During the last three days of the week, students worked individually or in small groups with the teacher’s assistance to complete a worksheet developed by the teacher. The worksheet was designed to give students experience using a variety of the tools available on the TI-92. Separate sections had students construct the medians, angle bisectors, perpendicular bisectors of sides, and altitudes of a triangle which they had constructed on the TI-92 and then explore relationships among these segments. The portion of the worksheet dealing with medians is shown in Figure 9. In addition to other assignments, students kept a daily journal of their reactions to the use of the calculator.

**Whole-class activities.** Beginning in January, classes were taught using the TI-92 equipped with an overhead projection device whenever it was feasible to do so. This was a key element of the study which sought to determine whether use of the TI-92 in such a
TI-92 INTRODUCTION

Explore the various menus on the TI-92 and get practice drawing and working with objects.

    b. Construct a segment which intersects the line.
    c. Label the end points of the segment.
    d. Construct the intersection point of the line segment and line.
    e. Construct and label a point on the line.

    b. Draw a triangle.
    c. Draw a pentagon.
    d. Draw a regular octagon.

F4:  a. Construct a line segment, then construct a line perpendicular to it.
    b. Construct two parallel lines.
    c. Construct a triangle and construct the midpoints of all sides.
    d. Construct a line segment, then construct its perpendicular bisector.
    e. Construct an angle by constructing two line segments with a common endpoint. Bisect the angle.

    b. Construct a quadrilateral and measure the lengths of its sides.
       1. Use Calculate to add the lengths of the sides.
       2. Use Distance and Length to find its perimeter
       3. Use Area to find its area.
       4. Use check property to check whether sides are perpendicular or parallel.
    c. Try to construct three points in a straight line, then use check property to see if they are collinear.

F7:  a. Construct a line segment.
    b. Construct the midpoint of the line segment.
    c. Use Label to label the three line segments.
    d. Use Hide/Show to hide the segment.

F8:  a. Show how to clear the screen

Figure 8. Activities Used in Class to Introduce Students to the TI-92
I. Construct a triangle ABC. (F3 3: Triangle)

Create the midpoints of each side of the triangle. (F4 3: Midpoint)
Label the midpt of AB as point X, the midpt of BC as point Y, and the midpt of CA as point Z.

Construct segments AX, BY, and CZ. (F2 5: Segment)

What are these segments called?

What do you observe about the three segments?

Grab one of the vertices of the triangle and move it around.

Does the relationship appear to be true for other triangles?

Make a conjecture regarding the medians of a triangle.

Label the intersection point as point O. (F2 3: Intersection Point and F7 4: Label)

Measure distances AO, OY, BO, OZ, YO, and OA. (F6 1: Distance and length)

Find a relationship among these measurements.

Grab a vertex of the triangle and move it around.

Does this relationship continue to hold true?

Make another conjecture regarding the medians of a triangle.

Figure 9. A Portion of the Worksheet Dealing with Medians of a Triangle
way would impact students’ attitudes toward geometry and students’ abilities to use

diagrams. School policies made it necessary to follow the sequence of topics in the
textbook fairly closely. The TI-92 was used to introduce theorems and definitions, to
explore concepts, to pose and explore problems, and to answer student questions. Sample
lesson outlines and illustrations showing how the calculator was used in the classes are
given in Appendix A. These are taken from video recordings of actual class sessions
which took place during the course of the study and are only a sample of lessons in which
the TI-92 was used.

Student group presentations of theorems related to the area of polygons took place
during April. Rather than the teacher presenting the lessons using the TI-92 and soliciting
participation of the students as had been done with most past lessons, students were
responsible for preparing and presenting the lessons. Part of the presentation included use
of the TI-92. Groups were set up so that in one of the classes, they consisted of a mix of
males and females while in the other class, they were composed of either all males or all
females. This was done in an effort to gather data on how students would distribute the
tasks to be completed, particularly the one related to use of the calculator, and what
differences there might be in mixed groups as opposed to same sex groups. Figure 10
shows the group assignments as given to students. Group presentations were video taped
and were evaluated by the teacher using the form in Figure 11. All students were given a
copy of the teacher’s evaluation and comments on their group’s presentation.

Attitude survey. The attitude survey instrument described earlier was administered
to all students in both geometry classes three times during the school year. The first two
GEOMETRY ASSIGNMENT - Group Presentations

During the next few weeks, we will be studying the area of polygons, circles, and sectors of circles (Chapter 11 in the book). As part of this study, I will divide the class into groups of four or five students and each group will be responsible for presenting one theorem related to the area of a polygon to the rest of the class. Groups and the specific theorem assigned to each group are shown below.

I will give you two class days, Wednesday and Thursday, April 9th and 10th, to work with your group and prepare your presentation. Parts of the presentation should include:

A. A statement of the theorem with an explanation of its meaning and definitions of any terms needed for the class to understand it.
B. An explanation of the proof of the theorem. (Proofs are outlined in the textbook.)
C. A demonstration of the theorem using the TI-92. A member of the group can do this demonstration or the group can tell me what needs to be done and I will operate the calculator.
D. Two example problems which illustrate use of the theorem. These can be taken from the book or other sources.

Each student in the group must present part of the presentation to the class. Your group can determine how you want to do this. In addition to the class presentation, each member of the group must hand in a written summary of the group’s presentation. This should include all of the items listed above. You will be graded both on your presentation to the class and on the written summary. This grade will count as a test grade. Student presentations will be video recorded.

You will be tested on the material presented by other students so you should be taking notes and asking questions during these presentations. When students are making their presentations to the class, I expect others to be listening and taking notes. If I have to ask you to be quiet or pay attention during someone else’s presentation, I will deduct points from your grade.

Figure 10. Group Assignment Involving Use of the TI-92
**Evaluation:** The following criteria will be used to evaluate your presentation:

<p>| | | | | | | | | | | |</p>
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<tbody>
<tr>
<td>1. Did the presentation cover all the required items?</td>
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<td>10</td>
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<td>2. Were unfamiliar terms explained so that the rest of the class could understand them?</td>
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<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>3. Was the theorem presented in a way which enabled the class to understand its meaning?</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>4. Was the proof of the theorem presented in a way which enabled most students to understand it?</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>6</td>
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<td>10</td>
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<tr>
<td>5. TI-92 Demonstration:</td>
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<tr>
<td>- Enhance understanding of the theorem?</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
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<td>10</td>
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<tr>
<td>- Make use of the calculator's capabilities?</td>
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<td>6. Examples:</td>
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</tr>
<tr>
<td>- Presented in a logical, understandable manner?</td>
<td>1</td>
<td>2</td>
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<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>- Relate to the theorem presented?</td>
<td></td>
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<tr>
<td>- Varied to present different aspects of the theorem?</td>
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<tr>
<td>7. Presenters:</td>
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</tr>
<tr>
<td>- Knowledgeable of the subject and well-prepared?</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>- Speak clearly, loudly, use good grammar?</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>- Answer questions?</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>- Hold the interest of other students?</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Figure 11. Group Assignment Evaluation Form*
administrations took place fairly close together (October 18, 1996 and October 29, 1996) so that the test-retest method of estimating the reliability of the instrument could be used. Although Fennema and Sherman (1976) report high reliabilities associated with their attitude scales, it was necessary to recheck reliabilities because only a portion of each of the seven subscales was being used and it was also necessary to check the reliability of the items developed by the researcher.

Various authors discuss the amount of time which should be allowed to elapse between administration of an instrument if the test-retest method of estimating reliability is to be used. Hennerson, Morris, and Fitz-Gibbon (1987) suggest that a one-month interval is a good rule-of-thumb to use to avoid actual changes in attitude of subjects while also minimizing the possibility of subjects remembering how they responded the first time they completed the scale. Mueller (1986) suggests an interval of several weeks. In this study, the instruments were given fairly close together (11 days). This was done for several reasons. First, even though the TI-92 was not being used, students were getting further and further into geometry on a daily basis and it seemed appropriate to give the retest before too much additional subject matter had been covered. Second, because there were 66 items on the attitude scale, it was felt that students would not recall specifically how they responded on very many of the items. Last, administering the attitude instrument for the second time on October 29th fit well into the schedule of activities for both classes.

Results for the Fennema-Sherman scales were subjected to statistical tests. Two sample t-tests were used to compare results of the first administration of the instrument to determine if there were significant differences in the attitudes of males and females at the
beginning of the study. The instrument was given a third time to all students in both
classes again on May 22, 1997. Paired t-tests were used to compare results of this third
administration with those of the first administration to see if there had been any significant
changes in the attitudes of female or male students during the course of the study. Two
sample t-tests were again used to compare results of the third administration of the survey
to determine if there were significant differences in the attitudes of males and females at
the end of the study. Results of all administrations are reported in Chapter 4.

The researcher-developed items were grouped into five categories: Problem,
Discourse, Proof, Diagram, and Mathematics. Statistical analyses similar to that
conducted for the Fennema-Sherman (1976) scales were conducted on the grouped items.

**Worksheets.** The three worksheets on similar triangles, circles, and the
Pythagorean Theorem were composed of problems that focused on students’ abilities to
overcome the three obstacles to the use of diagrams which were discussed in Chapter 1
and on their abilities to develop conjectures related to specific diagrams or problems.
These topics were first covered with students using the TI-92 in a whole class setting and
then students completed homework assignments, quizzes, and tests on each topic.
Students completed two of the worksheets (similar triangles and circles) immediately after
that topic had been studied in class. The similar triangle worksheet was completed on
February 13th and the circle worksheet on April 7th. The third worksheet on the
Pythagorean Theorem was completed on May 5th several months after the textbook
section on that topic had been covered in class but students had been assigned a number of
additional problems requiring use of the theorem in the interim. All worksheets were
completed by students working individually during class time without using notes or the textbook and were given to the teacher prior to the end of that class period. Although worksheets were graded rather leniently, they were graded and counted as one homework assignment toward the student's grade. This was done in an attempt to motivate students to put forth their best effort in completing the worksheets. Results are discussed in Chapter 5.

**Interviews.** The 16 students interviewed following completion of the similar triangle and circle worksheets were selected using specific criteria to give a wide range of interests and abilities. Eight students, four female and four male, were selected from each of the two classes. By the time interviews began, the first semester had already been completed and the teacher was familiar with the work habits and abilities of the students. Of the eight students selected from each class, two (one female and one male) were judged by the teacher to be very capable geometry students and two (one female and one male) were students who had done poorly during the first semester of the course. The other four students from each class (two females and two males) were students whose performance during the first semester put them somewhere in the middle of the class. As an incentive to participate in the interviews, students were given a "homework pass" for each interview they participated in. The pass could be used by the student to improve a previous homework grade or to exempt them from doing a subsequent assignment.

Prior to the interviews, the instructor reviewed the worksheets completed by each of these students and, based on that review, determined questions appropriate for that particular student. Although in general, the same worksheet problems were discussed
with all of the interviewed students, specific questions asked of each student were based
primarily on how that student responded to the particular questions on the worksheet and
so questions varied somewhat from student to student. The goal was to complete all
interviews within one week after students had completed the worksheet so that it would
be fairly fresh in their minds. In some cases, scheduling difficulties made it impossible to
conduct all interviews within this time frame. Detailed analysis of the worksheets and
interviews is given in Chapter 5.

Other data collection. In addition to those discussed above, several other devices
were used to collect data related to the purpose of the study. The teacher kept a journal
which was updated periodically as he discovered things pertinent to the results of the
study. As experience was gained in using the interactive geometry software in front of the
class, it seemed that a valuable exercise for students to complete related to having them
explain how they would construct particular geometric figures using the tools available on
the TI-92. Such questions were given as parts of homework assignments, tests, and as
part of the essay portion of the semester examination. Results of all these are discussed in
detail in Chapter 5.

Limitations

One limitation of the study lies in the fact that the researcher was also the teacher
of the two geometry classes used in the study. This meant that subjects in the study knew
that the teacher/researcher would also be determining their course grades for geometry.
This factor could have influenced their responses to some data collection items during the
course of the study. This is particularly crucial on items such as the attitude surveys and
interviews where students might have responded more positively or negatively than they would have otherwise, depending on their attitude toward the teacher. On items such as homework, quizzes, and worksheets which were part of the class grade, this factor could have had a positive impact in that it could have motivated them to put forth a better effort than they might otherwise have. The nature of the study and the extent of data collection efforts were two factors which made it necessary to do the study in the classes taught by the researcher. On the positive side, this enabled the researcher to know the students better than he would have had he been in the class for only a short period of time or only occasionally. A more in-depth knowledge of individual student abilities and motivation could enable the researcher to more effectively select worksheet and interview questions, for example.

In regard to the attitude survey, students were asked to put their names on the papers each time they completed the survey. This was done so that the researcher could match responses from particular students in analyzing the data over the three administrations of the instrument. In all cases, it was emphasized to students that their honest opinions were desired, that their responses would in no way impact their grades in the class, and that the teacher/researcher would not look at the survey instruments until after final grades in the class had been determined. Another method of coding the attitude survey instruments could have been developed but it would have been more cumbersome and probably would have involved use of a third party to administer the instruments. The researcher felt that students would give accurate responses and that putting their names on
the papers might, in some cases, motivate them to give more accurate responses than they
would have had they responded anonymously.

Another limitation is that subjects for the study were not selected randomly. At
the time the study commenced, students had already been assigned to geometry classes for
the school year and it was not possible to reassign them. It was felt that allowing the
guidance office to assign students to the study classes using their normal procedures
would be adequate.

Another limitation lies in the fact that the study was conducted in one high school
in a relatively small southeastern New Mexico city. Particular characteristics associated
with that city and school could mean that results of the study are not generalizable to
other populations. The same would be true of any particular city or school in which the
study might have been conducted.

The number of items on the attitude assessment instrument could possibly be seen
as a limitation. The length of the scale (66 items) is longer than recommended by some
researchers (Mueller, 1986) and this could have caused some subjects to become bored
with it and just circle responses instead of reading each statement and providing accurate
responses. By the time students completed the instrument for the first time, school had
been in session for almost two months and the teacher had built a rapport with students
which should have helped to minimize this possibility. At each administration of the
instrument, the researcher explained its purpose and how it would be used in the study and
students were asked to provide their most accurate responses. The idea behind this was to
help students feel more a part of the study effort.
On the other hand, results of the attitude survey are reported by scale for the questions from the Fennema-Sherman (1976) instrument. The fact that only half of the questions from each scale were used could limit the reliability of results. Appropriate statistical techniques were used to estimate reliabilities and are reported in Chapter 4.

Some of the researcher-developed items on the attitude assessment scale proved to be unreliable and so were discarded after the instrument had been administered. Other items dealt with relatively independent topics and so again there were a small number of items on which to gage the possible change in attitudes of each student in regard to a particular topic. Still it appears that some useful data can be gleaned from these questions.

Due to scheduling conflicts, it was impossible to interview all 16 students immediately after completion of the worksheets. When longer periods of time elapsed between completion of the worksheet and interview, the researcher gave the student a copy of the worksheet to review prior to the interview. The fact that other factors could have influenced responses to interview questions is not seen as a major problem because the object was to determine changes in students’ abilities to use diagrams or develop conjectures during the course of the entire study.

Chapter 5 describes how students’ scores for the various types of problems on the four worksheets were computed. It then describes statistical analyses which were conducted to compare those scores. Because different types of problems were used on the four worksheets, it should be remembered that significant statistical findings could be the result of other factors such as the differences in the problems on the worksheets rather than a result of use of the TI-92 with Cabri Geometry.
As discussed in chapter three, a Likert-type attitude survey instrument was developed for use in the study. Of the 66 statements on the instrument, 42 were taken from the Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976) and the remaining 24 were developed by the researcher. The first objective of the survey instrument was to see if there would be any changes in the attitudes of students in regard to various factors associated with the learning and doing of mathematics over the course of a year in which the TI-92 with Cabri Geometry was used fairly extensively for whole class demonstration and discussion. The second was to identify any differences in the attitudes of female and male students at the beginning of the school year and at the end of the year. This chapter explains how items on the instrument were scored, how Fennema-Sherman items were selected, and how researcher-developed items were developed and selected. It then discusses validity and reliability issues in regard to both the Fennema-Sherman scales and the researcher-developed items. Finally, it presents statistical analyses.
of students’ responses to the instrument. The instrument, as given to students, is shown at Appendix B.

**Scoring of the Attitude Survey Instrument**

Students responded to each question by circling one of five choices (strongly agree, agree, undecided, disagree, and strongly disagree) which were listed just below each question. To reduce the chances of a response set (simply agreeing or disagreeing to all statements) on the part of a respondent, some statements were worded so that agreeing with them will indicate a positive attitude toward that factor while others were worded so that agreement will indicate a negative attitude. It is desirable to have a scoring system in which a higher score indicates a more positive attitude and a lower score a more negative attitude regardless of whether the statement itself is positive or negative. The coding of a statement as positive or negative is a key factor in this scoring system. For statements coded as positive, a “strongly agree” response received a score of five, an “agree” received a score of four, and so on, with a “strongly disagree” response receiving a score of one. For statements coded as negative, the scoring was reversed. The first step in scoring the survey instruments was to determine a score for each response made by a student using the above procedure. Scores for each response were first entered into a grid and then into a computer spreadsheet which tallied total scores for each student as well as the mean score across all respondents for each statement, the standard deviation for each statement, and the number of responses by category. For each of the three administrations of the survey, a separate spreadsheet was used to tally responses by females and males and by Fennema-Sherman (1976) items and researcher-developed items.
Fennema-Sherman Items

Table 3 shows the statements from the Fennema-Sherman (1976) scales which were used in the instrument and, for comparison purposes, also shows Fennema-Sherman items not used.

Validity

To establish the validity of their scales, Fennema and Sherman used a process for establishing content validity. They state:

To establish content validity, each scale dimension was defined; each author independently wrote items representing the dimension and judged the validity of the other author's items. Items which were agreed upon as measuring an aspect of the dimension were selected with attention being given to covering the range of the dimension. (p. 5-6)

In the original scales, many of the items which are positively worded correspond to very similar negatively worded items. For example, in the Attitude toward Success in Mathematics Scale, item four states, "It would be really great to win a prize in mathematics" (p.24), while item seven states, "Winning a prize in mathematics would make me feel unpleasantly conspicuous" (p. 24). For the present study, item four was used while item seven was not. To maintain validity, this procedure was used where possible in selecting items for the attitude survey instrument.

Reliability

Reliability of the Fennema-Sherman (1976) items was established as part of the original research in which those items were developed. Henerson (1987) indicates that use of a portion of the items from a published scale does not present critical reliability problems as long as positive evidence of internal consistency is obtained through a
Table 3

Fennema-Sherman Mathematics Attitude Scales Used in the Present Study (Fennema & Sherman, 1976, pp. 21, 24 – 29)

<table>
<thead>
<tr>
<th>FS#</th>
<th>Scale and Statement</th>
<th>#</th>
<th>P/N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Mathematics Anxiety Scale (A)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Math doesn’t scare me at all.</td>
<td>57</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>It wouldn’t bother me at all to take more math courses.</td>
<td>65</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>I haven’t usually worried about being able to solve math problems</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>I almost never have gotten shook up during a math test</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>I usually have been at ease during math tests.</td>
<td>64</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>I usually have been at ease in math classes.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>Mathematics usually makes me feel uncomfortable and nervous.</td>
<td>58</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>Mathematics makes me feel uncomfortable, restless, irritable, and impatient.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>I get a sinking feeling when I think of trying hard math problems.</td>
<td>12</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>My mind goes blank and I am unable to think clearly when working mathematics.</td>
<td>6</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>A math test would scare me.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>Mathematics makes me feel uneasy and confused</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td><strong>Attitude toward Success in Mathematics (AS)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>It would make me happy to be recognized as an excellent student in math.</td>
<td>24</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>I’d be proud to be the outstanding student in math.</td>
<td>38</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>I’d be happy to get top grades in mathematics.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>It would be really great to win a prize in mathematics.</td>
<td>2</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>Being first in a mathematics competition would make me pleased.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>Being regarded as smart in math would be regarded as a great thing</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>Winning a prize in mathematics would make me feel unpleasantly conspicuous.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>People would think I was some kind of a grind if I got A’s in math.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>If I had good grades in math, I would try to hide it.</td>
<td>41</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>If I got the highest grade in math I’d prefer that no one knew.</td>
<td>46</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>It would make people like me less if I were a really good math student.</td>
<td>30</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>I don’t like people to think I’m smart in math.</td>
<td>NI</td>
<td>N</td>
</tr>
</tbody>
</table>

Note. FS# indicates the number of that item in the applicable Fennema-Sherman Scale. P/N indicates whether an agree response indicates a positive or negative attitude. NI indicates that item was not included in the present study.
### Table 3—continued

**Fennema-Sherman Mathematics Attitude Scales Used in the Present Study (Fennema & Sherman, 1976, pp. 21, 24 - 29)**

<table>
<thead>
<tr>
<th>FS#</th>
<th>Scale and Statement</th>
<th>#</th>
<th>P/N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Confidence in Learning Mathematics (C)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Generally I have felt secure about attempting mathematics.</td>
<td>35</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>I am sure I could do advanced work in mathematics.</td>
<td>13</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>I am sure that I can learn mathematics.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>I think I could handle more difficult mathematics.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>I can get good grades in mathematics.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>I have a lot of self-confidence when it comes to math.</td>
<td>63</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>I'm no good in math.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>I don't think I could do advanced mathematics.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>I'm not the type to do well in mathematics</td>
<td>54</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>For some reason even though I study, math seems unusually hard for me.</td>
<td>33</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>Most subjects I can handle Okay, but I have a knack for flubbing up math.</td>
<td>50</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>Math has been my worst subject.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td><strong>Effectance Motivation in Mathematics (E)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>I like math puzzles.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>Mathematics is enjoyable and stimulating to me.</td>
<td>7</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>When a math problem arises that I can’t immediately solve, I stick with it until I have the solution.</td>
<td>34</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>Once I start trying to work on a math puzzle, I find it hard to stop.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>When a question is left unanswered in math class, I continue to think about it afterward.</td>
<td>20</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>I am challenged by math problems I can’t understand immediately.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>Figuring out mathematical problems does not appeal to me.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>The challenge of math problems does not appeal to me.</td>
<td>61</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>Math puzzles are boring.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>I don’t see how some people can spend so much time on math and seem to enjoy it.</td>
<td>21</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>I would rather have someone give me the solution to a difficult math problem than to have to work it out for myself.</td>
<td>14</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>I do as little work in math as possible.</td>
<td>NI</td>
<td>N</td>
</tr>
</tbody>
</table>

*Note. FS# indicates the number of that item in the applicable Fennema-Sherman Scale. P/N indicates whether an agree response indicates a positive or negative attitude. NI indicates that item was not included in the present study.*
### Table 3—continued

*Fennema-Sherman Mathematics Attitude Scales Used in the Present Study (Fennema & Sherman, 1976, pp. 21, 24 - 29)*

<table>
<thead>
<tr>
<th>FS#</th>
<th>Scale and Statement</th>
<th>#</th>
<th>P/N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Mathematics as a Male Domain (MD)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Females are as good as males in geometry.</td>
<td>9</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>Studying mathematics is just as appropriate for women as for men.</td>
<td>62</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>I would trust a woman just as much as I would trust a man to figure out important calculations.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>Girls can do just as well as boys in mathematics.</td>
<td>49</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>Males are not naturally better than females in mathematics.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>Women certainly are logical enough to do well in mathematics.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>It's hard to believe a female could be a genius in mathematics.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>When a woman has to solve a math problem, it is feminine to ask a man for help.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>I would have more faith in the answer for a math problem solved by a man than a woman.</td>
<td>19</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>Girls who enjoy studying math are a bit peculiar.</td>
<td>53</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>Mathematics is for men; arithmetic is for women.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>I'd expect a woman mathematician to be a masculine type of person.</td>
<td>37</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td><strong>Teacher (T)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>My teachers have encouraged me to study more mathematics.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>My teachers think I'm the kind of person who could do well in math.</td>
<td>23</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>Math teachers have made me feel I have the ability to go on in math.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>My math teachers would encourage me to take all the math I can.</td>
<td>8</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>My math teachers have been interested in my progress in math.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>I would talk to my math teachers about a career which uses math.</td>
<td>59</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>When it comes to anything serious I have felt ignored when talking to math teachers.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>I have found it hard to win the respect of math teachers.</td>
<td>42</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>My teachers think advanced math is a waste of time for me.</td>
<td>18</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>Getting a mathematics teacher to take me seriously has usually been a problem.</td>
<td>3</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>My teachers would think I wasn't serious if I told them I was interested in a career in science or mathematics.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>I have had a hard time getting teachers to talk seriously with me about mathematics.</td>
<td>NI</td>
<td>N</td>
</tr>
</tbody>
</table>

*Note.* FS# indicates the number of that item in the applicable Fennema-Sherman Scale. P/N indicates whether an agree response indicates a positive or negative attitude. NI indicates that item was not included in the present study.
Table 3—continued

Fennema-Sherman Mathematics Attitude Scales Used in the Present Study (Fennema & Sherman, 1976, pp. 21, 24 - 29)

<table>
<thead>
<tr>
<th>FS#</th>
<th>Scale and Statement</th>
<th>#</th>
<th>P/N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Usefulness of Mathematics (U)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>I'll need mathematics for my future work.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>I study mathematics because I know how useful it will be.</td>
<td>26</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>Knowing mathematics will help me earn a living.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>Mathematics is a worthwhile and necessary subject.</td>
<td>NI</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>I'll need a firm mastery of mathematics for my future work.</td>
<td>11</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>I will use mathematics in many ways as an adult.</td>
<td>15</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>Mathematics is of no relevance to my life.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>8</td>
<td>Mathematics will not be important to me in my life’s work.</td>
<td>22</td>
<td>N</td>
</tr>
<tr>
<td>9</td>
<td>I see mathematics as a subject I will rarely use in my daily life as an adult.</td>
<td>NI</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>Taking mathematics is a waste of time.</td>
<td>52</td>
<td>N</td>
</tr>
<tr>
<td>11</td>
<td>In terms of my adult life it is not important for me to do well in mathematics in high school.</td>
<td>28</td>
<td>N</td>
</tr>
<tr>
<td>12</td>
<td>I expect to have little use for mathematics when I get out of school.</td>
<td>NI</td>
<td>N</td>
</tr>
</tbody>
</table>

Note. FS# indicates the number of that item in the applicable Fennema-Sherman Scale. P/N indicates whether an agree response indicates a positive or negative attitude. NI indicates that item was not included in the present study.
split-half reliability coefficient or other methods. In the present study, a respectable split-half reliability coefficient of 0.78 was obtained using data from the first administration of the instrument.

The test-retest method was used to compare results of the first administration of the instrument on October 18, 1996 and with the second on October 29, 1996 to estimate its overall reliability. Scores of the same 34 students, 19 females and 15 males, were used. Results of the two administrations were correlated using the Pearson Product-Moment Correlation Coefficient. Table 4 gives the reliabilities for each of the Fennema-Sherman scales as reported in Fennema-Sherman (1976) using the split halves method and as determined in the present study using the test-retest method.

Table 4

Fennema-Sherman Scale Reliabilities as Reported by Fennema-Sherman (1976) and as Determined in the Present Study

<table>
<thead>
<tr>
<th>Scale</th>
<th>Fennema-Sherman Split Half Reliabilities</th>
<th>Present Study Test-Retest Reliabilities*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Anxiety (A)</td>
<td>.89</td>
<td>.81</td>
</tr>
<tr>
<td>Attitude toward Success in Mathematics (AS)</td>
<td>.87</td>
<td>.56</td>
</tr>
<tr>
<td>Confidence in Learning Mathematics (C)</td>
<td>.93</td>
<td>.82</td>
</tr>
<tr>
<td>Effectance Motivation in Mathematics (E)</td>
<td>.87</td>
<td>.86</td>
</tr>
<tr>
<td>Mathematics as a Male Domain (MD)</td>
<td>.87</td>
<td>.95</td>
</tr>
<tr>
<td>Teacher (T)</td>
<td>.88</td>
<td>.79</td>
</tr>
<tr>
<td>Usefulness of Mathematics (U)</td>
<td>.88</td>
<td>.78</td>
</tr>
</tbody>
</table>

*n = 34.
Mueller (1986) indicates that a well-constructed scale could have a reliability of .80 or higher and Henerson (1987) states that reliabilities of .70 or above are considered respectable. As shown in Table 4, all of the present study test-retest reliabilities are close to or greater than .80 except the reliability of the Attitude toward Success in Mathematics scale in the present study which had a reliability of .56. On this scale, there were two students whose scores on the first administration were considerably different than on the second which took place 11 days later. One scored a total of 22 the first time and 29 the second. The other scored 30 the first time and 21 the second. Elimination of both scores for these two students would bring the overall test-retest reliability of this scale up to approximately .73.

Statistical Analyses of Student Responses

An individual student's total score on each scale gives an indication of that student's attitudes. Because there are six questions per scale, a total score between 18 and 30 would indicate a favorable attitude while a score between 6 and 18 would indicate a negative attitude. For the Mathematics Anxiety scale, "favorable" means less anxiety toward mathematics and for the Mathematics as a Male Domain scale, "favorable" means that mathematics is not viewed as a predominantly male domain. Table 5 shows means for total scores on the Fennema-Sherman (1976) scales for female and male students for the first and third administrations of the survey instrument.

For females, all means except on the U scale increased from the first to the third administrations although most only slightly. Scores for A, C, and E scales were slightly
Table 5

Mean Scores for First and Third Administrations of the Fennema-Sherman Scales

<table>
<thead>
<tr>
<th>Fennema-Sherman scale</th>
<th>Females</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st</td>
<td>3rd</td>
</tr>
<tr>
<td>Mathematics Anxiety (A)</td>
<td>16.74</td>
<td>17.53</td>
</tr>
<tr>
<td>Attitude toward Success in Mathematics (AS)</td>
<td>24.05</td>
<td>25.05</td>
</tr>
<tr>
<td>Confidence in Learning Mathematics (C)</td>
<td>16.89</td>
<td>17.16</td>
</tr>
<tr>
<td>Effectance Motivation in Mathematics (E)</td>
<td>17.42</td>
<td>17.84</td>
</tr>
<tr>
<td>Mathematics as a Male Domain (MD)</td>
<td>27.42</td>
<td>28.05</td>
</tr>
<tr>
<td>Teacher (T)</td>
<td>20.79</td>
<td>20.84</td>
</tr>
<tr>
<td>Usefulness of Mathematics (U)</td>
<td>21.00</td>
<td>20.84</td>
</tr>
</tbody>
</table>

\*n = 19. \*\*n = 14.

below 18, the neutral point, while scores for all other scales were above 20 for both administrations. For males, there were increases on the A and AS scales but decreases on all others. All totals were above the neutral point for the first administration. The score for the E scale dropped to slightly below that point on the third administration. The most positive attitudes were shown on the AS scale for both female and male students and on the MD scale for female students. The high means on the AS scale indicates that students have a favorable view toward success in mathematics. Several of the statements in this scale address whether or not students would want others to know that they are good mathematics students. The high scores on this scale indicate they either do not view it as detrimental to themselves if others know of their success in mathematics or, if it is detrimental, they are willing to accept the consequences. The high mean for females on the Mathematics as a Male Domain scale indicates that the girls in the two classes state that they do not view mathematics as a male domain.
Items taken from the Fennema-Sherman scales were used to determine whether there was a significant difference in the attitudes of females as compared to males at the beginning of the school year and again at the end of the school year and also to determine if there was any significant change in the attitudes of males or females over the course of the school year. Two-sample t-tests were used to compare the scores of females to those of males at the beginning and again at the end of the study. According to Hays (1994), use of a t-test to compare means of two samples involves the assumptions that both populations are normally distributed and have equal variance. Hays goes on to state that for samples of moderate size or larger, departures from normality have little impact on the conclusions. Because of the sample sizes and number of questions in the present study, normality was not considered an issue. In cases where doubts existed, nonparametric tests were used to verify results. Prior to testing each hypothesis, tests for homogeneity of variance were conducted. These tests indicated a significant difference at the .10 significance level between the variances for males and females on the start date for the Attitude toward Success in Mathematics, Mathematics as a Male Domain, and Teacher Scales, and on the end date for the Anxiety, Mathematics as a Male Domain, and Teacher Scales. In these cases, a two sample t-test which accounts for unequal variance was used.

On the start date of the study, the only variable showing a statistically significant difference at the .05 level was Mathematics as a Male Domain which had a p-value of .004. Therefore the null hypothesis of no difference in means between males and females on this variable at the beginning of the study is rejected and the alternate hypothesis of unequal means is accepted. A higher score on this scale indicates that an individual does
not view mathematics as a male domain. Because the mean score for females (27.42) is higher than that for males (22.00), this can be interpreted as indicating that, at the beginning of the study, the females in the study viewed mathematics as less of a male domain than did the males in the study.

Comparisons of means at the end of the study also resulted in the only statistically significant difference at the .05 level being for Mathematics as a Male Domain with a p-value of .01. Therefore, the null hypothesis of no difference in means between males and females on this variable at the end of the study is rejected and the alternate hypothesis of unequal means is accepted. Once again, the mean for females (28.05) was higher than that for males (21.00) indicating that the females in the study view mathematics as less of a male domain than do the males in the study.

Paired t-tests were conducted using a .05 level of significance to determine whether there were any statistically significant changes in the views of males or females on any of the variables from start date to end date. Results showed no significant differences in the views of females. For males, the only significant difference was on the variable Attitude toward Success in Mathematics which had a p-value of .009. The higher mean on the third administration of the survey (25.14) as compared to the first administration (23.14) indicates a more positive attitude on this variable from beginning to end of the study.

Keppel (1991) discusses the cumulative effect of type I error when doing a number of statistical tests. He says, "When we conduct a number of comparisons on a set of means, there is an FW (familywise) error rate with which we must contend and this error
rate increases directly with the number of comparisons we test” (p. 165). In this study, a number of hypotheses were tested with three shown to be significant. It must be kept in mind when interpreting the results that there is the possibility that those results were due to this cumulative error rate rather than to a true statistical difference.

**Researcher-Developed Items**

The researcher-developed statements, shown in Table 6, address other areas of concern in mathematics education. Investigation of student responses led to the elimination of items 55, 66, and 27. The intent of statement number 55 was to solicit student opinions about the course. The idea was that, if they did not like the course, they would recommend it to their enemies. Even though comments were not requested, several students responded strongly agree and wrote comments stating that all students need to take geometry and so they would recommend the course to their friends as well as their enemies.

The intent of item 66 was to see if a semester of exposure to interactive geometry software in a whole-class setting in which they were asked to make conjectures about what would be true in certain situations and then prove those conjectures would change student opinions of whether they could create mathematics themselves or whether they were relegated to being only users of mathematics created by others. Review of the responses indicated that there was some confusion about what it means to “create” mathematics. Does creating mathematics mean that one must develop a theorem that has not been seen before or does solving a problem constitute creating mathematics? Some students apparently felt the latter was true.
### Table 6

**Attitude Survey Statements Developed by the Researcher**

<table>
<thead>
<tr>
<th>#</th>
<th>Statement</th>
<th>P/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Math is an obstacle standing in the way of graduating from high school.</td>
<td>N</td>
</tr>
<tr>
<td>4</td>
<td>There is only one way to solve a math problem and the teacher should just tell us how to do it.</td>
<td>N</td>
</tr>
<tr>
<td>5</td>
<td>A theorem is true only for the diagram that is shown with it.</td>
<td>N</td>
</tr>
<tr>
<td>10</td>
<td>It's okay for me to add lines to the diagrams given in problems.</td>
<td>P</td>
</tr>
<tr>
<td>16</td>
<td>Mathematics can be beautiful.</td>
<td>P</td>
</tr>
<tr>
<td>17</td>
<td>Math problems have one right answer.</td>
<td>N</td>
</tr>
<tr>
<td>25</td>
<td>Math class is more interesting when we discuss what we are learning.</td>
<td>P</td>
</tr>
<tr>
<td>27</td>
<td>It is unfair for a teacher to give a problem I haven't seen before on a test.</td>
<td>N</td>
</tr>
<tr>
<td>29</td>
<td>A right angle can be drawn in a lot of different positions.</td>
<td>P</td>
</tr>
<tr>
<td>31</td>
<td>If I can't solve a math problem in 5 minutes, there is no sense in working on it any longer.</td>
<td>N</td>
</tr>
<tr>
<td>32</td>
<td>I can work out a proof by thinking logically.</td>
<td>P</td>
</tr>
<tr>
<td>36</td>
<td>You can prove a theorem in geometry by measuring angles with a protractor.</td>
<td>N</td>
</tr>
<tr>
<td>39</td>
<td>Even if I make a mistake, I learn from working problems on the board in class.</td>
<td>P</td>
</tr>
<tr>
<td>40</td>
<td>Mathematicians are creative people.</td>
<td>P</td>
</tr>
<tr>
<td>43</td>
<td>To solve a math problem you have to use a rule you memorized earlier.</td>
<td>N</td>
</tr>
<tr>
<td>44</td>
<td>In math class a student should only have to sit and listen.</td>
<td>N</td>
</tr>
<tr>
<td>45</td>
<td>A triangle should always be drawn with the base at the bottom.</td>
<td>N</td>
</tr>
<tr>
<td>47</td>
<td>Working in groups with other students helps me to understand math.</td>
<td>P</td>
</tr>
<tr>
<td>48</td>
<td>I learn more in math class when I can ask questions of the teacher and other students.</td>
<td>P</td>
</tr>
<tr>
<td>51</td>
<td>The teacher should tell us how to do the problems and not ask a bunch of questions.</td>
<td>N</td>
</tr>
<tr>
<td>55</td>
<td>I would recommend this course to one of my enemies.</td>
<td>N</td>
</tr>
<tr>
<td>56</td>
<td>I enjoy the challenge of answering questions in math class.</td>
<td>P</td>
</tr>
<tr>
<td>60</td>
<td>Doing proofs in geometry teaches you how to think logically.</td>
<td>P</td>
</tr>
<tr>
<td>66</td>
<td>I can create mathematics.</td>
<td>P</td>
</tr>
</tbody>
</table>

*Note.* # indicates the number of the statement on the attitude survey instrument. P/N indicates whether an agree response indicates a positive or negative attitude.
The intent of statement number 27 was not to indicate that the only exercises which should be on tests were specific ones which students had worked previously but exercises essentially like those which students had done in class or on homework. Some students interpreted this to mean only specific problems they had seen before.

Validity

One method discussed by Henerson (1987) to establish construct validity is to give the instrument to a group of experts in the field without indicating its purpose and ask them to state their conclusions about what the instrument appears to be measuring. To address this aspect of validity, the researcher-developed items were grouped by the researcher into five categories as shown in Table 7. These grouped items, without category headings, were then given to an expert group consisting of eight persons who had recently completed or were nearing completion of doctoral studies in mathematics education at the Ohio State University. These individuals were asked to indicate the topic they thought each group of statements addressed and their answers, which are shown in Table 8, were compared with topics the researcher intended the statements to address.

Although category titles are stated somewhat differently, there is general agreement between the researcher and expert panel members that the statements in groups III, IV, and V address student attitudes about proof, diagrams, and mathematics respectively. The researcher intended the statements in group II to address the topic of discourse in the mathematics classroom. As shown in Table 8, none of the experts used the word "discourse" in describing the topic addressed by those statements but their responses indicate the same general idea. The statements in group I were intended by the
Table 7

Researcher-Developed Items as Originally Grouped by the Researcher

<table>
<thead>
<tr>
<th>#</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
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<td>31</td>
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<td>43</td>
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</tr>
<tr>
<td>4</td>
<td>There is only one way to solve a math problem and the teacher should just tell us how to do it.</td>
</tr>
<tr>
<td>25</td>
<td>Math class is more interesting when we discuss what we are learning.</td>
</tr>
<tr>
<td>39</td>
<td>Even if I make a mistake, I learn from working problems on the board in class.</td>
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<td>44</td>
<td>In math class a student should only have to sit and listen.</td>
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<td>48</td>
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</tr>
<tr>
<td>51</td>
<td>The teacher should tell us how to do the problems and not ask a bunch of questions.</td>
</tr>
<tr>
<td>56</td>
<td>I enjoy the challenge of answering questions in math class.</td>
</tr>
<tr>
<td>32</td>
<td>I can work out a proof by thinking logically.</td>
</tr>
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<td>36</td>
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<td>45</td>
<td>A triangle should always be drawn with the base at the bottom.</td>
</tr>
<tr>
<td>1</td>
<td>Math is an obstacle standing in the way of graduating from high school.</td>
</tr>
<tr>
<td>16</td>
<td>Mathematics can be beautiful.</td>
</tr>
<tr>
<td>40</td>
<td>Mathematicians are creative people.</td>
</tr>
</tbody>
</table>
Table 8

*Opinions as to the Topics Addressed by Researcher-Developed Statements*

<table>
<thead>
<tr>
<th>Respondent</th>
<th>Opinions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group I</strong></td>
<td></td>
</tr>
<tr>
<td>Researcher</td>
<td>Student attitudes about the nature of mathematics problems</td>
</tr>
<tr>
<td>Expert 1</td>
<td>Skills—following rules—not necessarily understanding the concepts</td>
</tr>
<tr>
<td>Expert 2</td>
<td>What it means to “do” mathematics</td>
</tr>
<tr>
<td>Expert 3</td>
<td>Nature and value of problem solving</td>
</tr>
<tr>
<td>Expert 4</td>
<td>Assessment—answers on a test</td>
</tr>
<tr>
<td>Expert 5</td>
<td>Assessment in the mathematics class</td>
</tr>
<tr>
<td>Expert 6</td>
<td>The nature of mathematics</td>
</tr>
<tr>
<td>Expert 7</td>
<td>Beliefs about mathematical problem solving</td>
</tr>
<tr>
<td>Expert 8</td>
<td>The nature of solving mathematics problems</td>
</tr>
<tr>
<td><strong>Group II</strong></td>
<td></td>
</tr>
<tr>
<td>Researcher</td>
<td>Student attitudes about discourse in the mathematics classroom</td>
</tr>
<tr>
<td>Expert 1</td>
<td>Comprehension and learning for understanding (cooperative learning)</td>
</tr>
<tr>
<td>Expert 2</td>
<td>What it means (takes) to learn mathematics</td>
</tr>
<tr>
<td>Expert 3</td>
<td>Role of teacher and role of student</td>
</tr>
<tr>
<td>Expert 4</td>
<td>Learning—lecture vs. doing—how do students learn mathematics</td>
</tr>
<tr>
<td>Expert 5</td>
<td>Pedagogy in the math class</td>
</tr>
<tr>
<td>Expert 6</td>
<td>How students learn mathematics</td>
</tr>
<tr>
<td>Expert 7</td>
<td>Beliefs about learning mathematics</td>
</tr>
<tr>
<td>Expert 8</td>
<td>How we learn mathematics</td>
</tr>
<tr>
<td><strong>Group III</strong></td>
<td></td>
</tr>
<tr>
<td>Researcher</td>
<td>Student attitudes about proof in geometry</td>
</tr>
<tr>
<td>Expert 1</td>
<td>Proofs and logical thinking—comprehension</td>
</tr>
<tr>
<td>Expert 2</td>
<td>The role of logical thinking in/and geometry</td>
</tr>
<tr>
<td>Expert 3</td>
<td>Nature of a proof in mathematics</td>
</tr>
<tr>
<td>Expert 4</td>
<td>Mathematical reasoning—What is necessary for mathematical proof?</td>
</tr>
<tr>
<td>Expert 5</td>
<td>Geometry proofs</td>
</tr>
<tr>
<td>Expert 6</td>
<td>What is taken as sufficient for proof in mathematics</td>
</tr>
<tr>
<td>Expert 7</td>
<td>Beliefs about doing proofs</td>
</tr>
<tr>
<td>Expert 8</td>
<td>The nature and purpose of proof</td>
</tr>
</tbody>
</table>
Table 8—continued

Opinions as to the Topics Addressed by Researcher-Developed Statements

<table>
<thead>
<tr>
<th>Respondent</th>
<th>Opinions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Group IV</strong></td>
</tr>
<tr>
<td><strong>Researcher</strong></td>
<td>Student attitudes about the use of diagrams in geometry</td>
</tr>
<tr>
<td>Expert 1</td>
<td>Freedom to change things—or add</td>
</tr>
<tr>
<td>Expert 2</td>
<td>The role of visual representation</td>
</tr>
<tr>
<td>Expert 3</td>
<td>Meaning of diagrams as thinking tools</td>
</tr>
<tr>
<td>Expert 4</td>
<td>Generalization or constructions</td>
</tr>
<tr>
<td>Expert 5</td>
<td>Making generalizations</td>
</tr>
<tr>
<td>Expert 6</td>
<td>How diagrams influence/help/hinder/prove theorems in geometry</td>
</tr>
<tr>
<td>Expert 7</td>
<td>Beliefs about geometric diagrams</td>
</tr>
<tr>
<td>Expert 8</td>
<td>Visualization, orientation and purpose of diagrams</td>
</tr>
<tr>
<td></td>
<td><strong>Group V</strong></td>
</tr>
<tr>
<td><strong>Researcher</strong></td>
<td>Student attitudes about mathematics in general.</td>
</tr>
<tr>
<td>Expert 1</td>
<td>Attitude toward mathematics</td>
</tr>
<tr>
<td>Expert 2</td>
<td>No response</td>
</tr>
<tr>
<td>Expert 3</td>
<td>Math as obstacle or as opportunity</td>
</tr>
<tr>
<td>Expert 4</td>
<td>What is mathematics (purpose or content)?</td>
</tr>
<tr>
<td>Expert 5</td>
<td>Attitudes about mathematics</td>
</tr>
<tr>
<td>Expert 6</td>
<td>About the beauty/creativity involved in mathematics</td>
</tr>
<tr>
<td>Expert 7</td>
<td>Beliefs about the nature of mathematics</td>
</tr>
<tr>
<td>Expert 8</td>
<td>Personal beliefs about and relationship with mathematics</td>
</tr>
</tbody>
</table>

101
researcher to address student attitudes towards problems they are expected to solve in mathematics classes. There was less agreement among the experts on this group than on the others. Four indicated that the statements addressed attitudes about problem solving while two felt they related to assessment or to answering test questions and two indicated that the statements related to the nature of mathematics in general. After review of the expert responses and another review of the statements, items 4, 39, and 51 were moved to group I. All contain the words "problem" or "problems" and it was felt they fit better in the group intended to address student attitudes toward problems.

Henerson (1987) states that a criterion-group study in which a group thought to possess an abundance of the construct in question scores high while a group thought deficient in the construct scores low builds a good case for validity. The expert group was also asked to respond to each individual item with strongly agree, agree, undecided, disagree, or strongly disagree. Results obtained from them were later compared with results obtained from the first administration of the instrument to the students in the two geometry classes. For the 21 retained items, 105 is the highest possible score indicating a very positive attitude, 63 is the neutral score, and 21 would be the lowest possible score indicating a very negative attitude. Average total score for the expert panel was 96.1, a highly positive score, while students averaged a total score of 72.8. While this score is somewhat above neutral, it is still substantially below the score of the expert panel.

Reliability

After grouping of the items as described above, item discrimination indices were computed by correlating individual item scores with total group scores on the first
administration of the attitude survey using the Pearson Product-Moment Correlation Coefficient. As shown in Table 9, after elimination of item 47 which correlated .20 with the total for the discourse group, all other items correlated between .46 and .76 with totals for their respective groups. These 20 items were retained for further analyses. To address the overall reliability of the researcher-developed items, the Pearson Product-Moment Correlation Coefficient was computed using total scores on those items of the same 34 students from the first and second administrations of the instrument. A correlation of 0.743 was obtained indicating respectable overall reliability.

Table 9

Item Discrimination Indices Correlating Scores on Researcher-Developed Items with Total Group Scores for the First Administration of the Attitude Survey

<table>
<thead>
<tr>
<th>Group</th>
<th>Statement Numbers and Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: Problem</td>
<td></td>
</tr>
<tr>
<td>Statement Number</td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
</tr>
<tr>
<td>II: Discourse</td>
<td></td>
</tr>
<tr>
<td>Statement Number</td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
</tr>
<tr>
<td>III: Proof</td>
<td></td>
</tr>
<tr>
<td>Statement Number</td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
</tr>
<tr>
<td>IV: Diagram</td>
<td></td>
</tr>
<tr>
<td>Statement Number</td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
</tr>
<tr>
<td>V: Mathematics</td>
<td></td>
</tr>
<tr>
<td>Statement Number</td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td></td>
</tr>
</tbody>
</table>

Note. n = 34.
**Statistical Analysis of Responses**

An individual student's total score on each scale gives an indication of that student's attitudes. Table 10 shows means for total scores on each of the five groups of researcher-developed items for female and male students for the first and third administrations of the survey instrument. For females, no means decreased from the first to the third administrations although one stayed the same and others increased only slightly. Mean scores for females on all groups were above the neutral points shown in Table 10 with means for the proof and mathematics categories being only slightly above neutral. For males, the mean score for the problem group declined slightly from the first to third administration while that for the mathematics group stayed the same and all others increased slightly. All means were above the neutral points shown in Table 10 for both administrations although some only slightly. This indicates no areas of overall poor attitudes among the female or male students at the beginning or at the end of the year but also no areas of strongly positive attitudes.

Statistical analysis of grouped researcher-developed items was conducted to determine whether there was a significant difference in the attitudes of females as compared to males at the beginning of the school year and again at the end of the school year and also to determine if there was any significant change in the attitudes of males or females over the course of the school year on any of the five groups of statements. Two-sample t-tests were used to compare the scores of females to those of males at the beginning and again at the end of the study and paired t-tests were used to compare the
Table 10

*Mean Scores by Group for the First and Third Administrations of the Researcher-Developed Items*

<table>
<thead>
<tr>
<th>Group</th>
<th># Items</th>
<th>Possible Scores</th>
<th>Female Mean</th>
<th>Male Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1st</td>
<td>3rd</td>
</tr>
<tr>
<td>Problem</td>
<td>6</td>
<td>6 30 18</td>
<td>19.74</td>
<td>20.68</td>
</tr>
<tr>
<td>Discourse</td>
<td>4</td>
<td>4 20 12</td>
<td>14.74</td>
<td>15.00</td>
</tr>
<tr>
<td>Proof</td>
<td>3</td>
<td>3 15 9</td>
<td>10.11</td>
<td>10.89</td>
</tr>
<tr>
<td>Diagram</td>
<td>4</td>
<td>4 20 12</td>
<td>15.16</td>
<td>15.47</td>
</tr>
<tr>
<td>Math</td>
<td>3</td>
<td>3 15 9</td>
<td>9.79</td>
<td>9.79</td>
</tr>
</tbody>
</table>

*n = 19.  b*n = 14.

scores of each gender at the beginning of the study with scores for that gender at the end of the study. Because of the sample sizes and number of questions in the present study, normality was not considered an issue. In cases where doubts existed, nonparametric tests were used to verify results. For the comparisons of males with females, prior to testing each hypothesis, tests for homogeneity of variance were conducted. These tests indicated a significant difference at the .10 level between the variances for males and females on the end date for the Mathematics Scale. In this case, a two sample *t*-test which accounts for unequal variance was used.

The *t*-tests showed no significant differences at the .05 level between the attitudes of males and females either at the start date or at the end date of the study and no significant change in attitudes for either gender from start to end of the study on any of the groups of items.
Because individual items do address areas of concern to mathematics educators and to this study, the results for some individual items will be discussed. Table 11 lists the items and gives mean scores for males and females at the beginning and end of the study.

Table 11

<table>
<thead>
<tr>
<th>Statement</th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st</td>
<td>3rd</td>
</tr>
<tr>
<td>1. Math is an obstacle standing in the way of graduating from high school.</td>
<td>3.68</td>
<td>3.58</td>
</tr>
<tr>
<td>16. Mathematics can be beautiful.</td>
<td>2.37</td>
<td>2.58</td>
</tr>
<tr>
<td>25. Math class is more interesting when we discuss what we are learning.</td>
<td>3.63</td>
<td>3.68</td>
</tr>
<tr>
<td>29. A right angle can be drawn in a lot of different positions.</td>
<td>4.05</td>
<td>3.89</td>
</tr>
<tr>
<td>31. If I can’t solve a math problem in five minutes, there is no sense in working on it any longer.</td>
<td>3.95</td>
<td>3.79</td>
</tr>
<tr>
<td>43. To solve a math problem, you have to use a rule you memorized earlier.</td>
<td>2.53</td>
<td>2.47</td>
</tr>
<tr>
<td>44. In math class, a student should only have to sit and listen.</td>
<td>3.89</td>
<td>4.05</td>
</tr>
<tr>
<td>45. A triangle should always be drawn with the base at the bottom.</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>48. I learn more in math class when I ask questions of the teacher and other students.</td>
<td>4.21</td>
<td>4.16</td>
</tr>
</tbody>
</table>

Note. For each item, the highest possible score is 5, the lowest possible score is 1, and the neutral score is 3. a n = 17. b n = 14.

The only scores showing a negative attitude are on items 16 and 43. For item 16, this means that many students do not associate beauty with mathematics. Item 43 could identify part of the reason for that. Many students feel that memorization has much to do with success in mathematics. Means for these questions changed little from the beginning.
to the end of the study perhaps indicating that use of the calculator did not impact these views.

The only differences in means greater than 0.5 were for items 29 and 31. For item 29, the mean scores for males and females were about the same at the beginning of the study; however, at the end of the study, the mean score for males was 0.54 higher than that for females. The score for males increased during the year while that for females declined slightly. Again, one of the goals of the use of interactive software in the classroom is to help students realize that geometric figures can be drawn in many different positions so this increase to a score above 4.0 for males is encouraging and could be a result of repeated use of the software in the classroom. It is disappointing that there is no corresponding increase for females.

For item 31, the mean score for males decreased by 0.57 from the beginning to the end of the study. Because there was no similar decline for females, this resulted in a difference in mean scores of males and females of 0.72 at the end of the study with females having the higher score. This change for males is difficult to explain. One of the goals of use of interactive software in the classroom is to encourage students to explore mathematics. To explore, they must be willing to spend time on problems so this result certainly runs contrary to the goal.

Items with mean scores greater than or equal to 4.0, which indicates a highly positive attitude, are item 25 for males at the end of the study, item 29 for females at the beginning and for males both at the beginning and end of the study, item 44 for females at the end of the study, item 45 for females at the beginning and end of the study and for
males at the end of the study, and item 48 for both males and females at both the beginning and end of the study. Item 29 has already been discussed. Item 45 relates to it. The high positive score on this item indicates that many students do not agree with the statement and realize that triangles can be drawn in many different positions. Items 25, 44, and 48 all deal with classroom discourse. High mean scores on these items indicate that students prefer to be active in math class rather than merely listening to the teacher. One goal of the use of interactive geometry software on the TI-92 is to draw students into discussion and involvement in the class. Because means on these questions were relatively high at both the beginning and end of the study with no significant changes, no change in attitude can be attributed to calculator use.

Discussion

Results of this study indicate that few significant changes took place in the attitudes of students during the course of a year in which the TI-92 with Cabri Geometry software was used extensively for whole-class demonstration and discussion. The only significant change from the beginning to the end of the study was for males on the variable Attitude toward Success in Mathematics. Statements on this scale relate to whether or not a student would want to be recognized by his or her peers as a good mathematics student. The change over the year for males could relate to a change in their attitudes caused at least in part by exposure to use of the calculator. That is, repeated exposure to what could be done with the calculator caused them to feel it would be a positive factor to be known by other students as someone capable of doing such
things on a sophisticated calculator. There is always the possibility that other factors may have brought about this change in attitude.

The only significant differences found in the study between male and female students was on the Mathematics as a Male Domain scale. Results showed both at the beginning and end of the study, that females viewed mathematics as less of a male domain than did males. Mean scores on this scale were very high for females. The mean at the beginning was 27.42 and at the end 28.05 compared to the highest possible score of 30.00. Dispersion was relatively low with standard deviations of 1.66 at the beginning and 2.01 at the end. This means that both at the beginning and end of the study, scores for females were grouped closely together at the high end of the scale. Mean scores for males both at the beginning and end of the study were 22.00 and 21.00 respectively. These are substantially above the neutral score of 18.00 indicating that, overall, the males also do not view mathematics as a male domain. Dispersion was much greater for males with standard deviations of 5.55 at the beginning of the study and 8.43 at the end. This would indicate that there are some who hold strong views that mathematics is a male domain.

The lowest possible score is 6. On the first administration of the survey, there were males with scores of 6 and 13, and on the last, there were scores of 6, 7, 9, and 10. Low scores for females were 23 at the beginning of the study and 22 at the end. Both of these are considerably above the neutral point of 18.

Because significant differences between the scores of females and males on the Mathematics as a Male Domain existed both at the beginning and at the end of the study, there was no change which could be attributed to calculator use in the classes. Mueller
(1986) points out that a difficulty with attitude surveys is that people often do not act in accordance with their expressed views. The fact that the females in the classes state that they do not view mathematics as a male domain does not mean that they will not act as if it is a male domain.
CHAPTER 5

ANALYSIS OF DATA RELATED TO DIAGRAMS, CONJECTURE, AND PROOF

The two primary research questions addressed by this study are:

A. Are high school students in a geometry class taught using interactive software only in a whole-class setting able to overcome the obstacles stated by Yerushafany (1993) to the effective use of diagrams? Are there gender-based differences in students’ abilities to overcome such obstacles when the software is used in this manner?

B. Are high school students in a geometry class taught using interactive software only in a whole-class setting able to form conjectures and produce convincing arguments supporting their conjectures? Are there gender-based differences in students’ abilities to develop and prove conjectures when the software is used in this manner?

This chapter first discusses the analysis of data collected during the study using the methods shown in Table 12 as they pertain to each research question and then discusses other related findings. All worksheets, the homework assignment, and the quiz are shown in Appendix C. The similar triangle, and circle worksheets were completed individually by students in class immediately after sections in the textbook dealing with those topics
had been covered using the TI-92 with Cabri Geometry for whole-class demonstration and
discussion whenever possible and after the students had completed several homework
assignments and taken a chapter test on the topic. The worksheets on similar triangles and
circles were followed by interviews with eight female and eight male students to further
examine their answers to the worksheet questions. The Pythagorean Theorem worksheet
was completed several months after the textbook sections on the Pythagorean Theorem
had been covered but, in the interim, students had worked with other right triangle
applications. No student interviews were conducted following completion of this
worksheet.

In the analyses of the data, a number of statistical tests were used. The caution
stated in Chapter 4 regarding the cumulative effect of type I error when doing a number of
statistical tests also applies to these analyses. It must be kept in mind when interpreting
the results that there is the possibility that some significant results could be due to this

<table>
<thead>
<tr>
<th>Collection Method</th>
<th>Collection Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary Worksheet</td>
<td>October 1996</td>
</tr>
<tr>
<td>Preliminary Homework</td>
<td>October 1996</td>
</tr>
<tr>
<td>Preliminary Quiz</td>
<td>October 1996</td>
</tr>
<tr>
<td>Worksheet and interviews on Similar Triangles</td>
<td>February 1997</td>
</tr>
<tr>
<td>Worksheet and interviews on Circles</td>
<td>April 1997</td>
</tr>
<tr>
<td>Worksheet on the Pythagorean Theorem</td>
<td>May 1997</td>
</tr>
<tr>
<td>Student Group Presentations</td>
<td>April 1997</td>
</tr>
<tr>
<td>Student Journals on reactions to using the TI-92</td>
<td>January 1997</td>
</tr>
<tr>
<td>Teacher Journal</td>
<td>September 1996 - May 1997</td>
</tr>
<tr>
<td>Student papers on constructions</td>
<td>January - May 1997</td>
</tr>
</tbody>
</table>
cumulative error rate rather than to a true statistical difference. According to Hays (1994), use of a t-test to compare means of two samples involves the assumptions that both populations are normally distributed and have equal variance. Hays goes on to state that for samples of moderate size or larger, departures from normality have little impact on the conclusions. For many of the samples used in these analyses, the Shapiro-Wilk W Test indicated that the data were not normally distributed. In all cases, either the Wilcoxon Signed Rank Test or the Wilcoxon Rank Sum Test, as appropriate, were also used and results compared with those of the t-tests. In all but two cases which will be discussed later, results were the same. Prior to testing each hypothesis, tests for homogeneity of variance were conducted.

Research Question One: Diagrams

The first question addresses the abilities of high school students in a geometry class taught using interactive software only in a whole-class setting to overcome obstacles to the effective use of diagrams and whether such use of the technology might impact the genders in different ways. The obstacles as stated by Yerushalmy (1993) are:

--Students often have difficulty determining which characteristics apply to the specific diagram used to illustrate a geometric concept such as a theorem and which characteristics apply to the class of objects the diagram represents, that is, the set of objects to which the geometric concept applies.

--Students sometimes become accustomed to seeing diagrams in "standard" positions, may not be able to recognize the same figure when shown in a different position, and therefore may not be able to apply definitions and theorems to that figure.

--Many students are not able to view geometric figures in a variety of ways and many feel that they cannot modify diagrams shown in textbooks or given by teachers. These adversely impact students' abilities to examine relationships among parts of a diagram and therefore their ability to make conjectures.
To address these obstacles in the present study, questions on the preliminary worksheet, the preliminary quiz, the similar triangle worksheet, and Pythagorean Theorem worksheet were separated into three categories which will be discussed separately:

1. Applying definitions, postulates, or theorems to diagrams;
2. Adding lines to diagrams;
3. Identifying figures in nonstandard positions.

**Applying Definitions, Postulates, or Theorems to Diagrams**

Selected problems from the preliminary worksheet completed in October, the similar triangle worksheet completed in February, and the Pythagorean Theorem Worksheet completed in May were used to evaluate students on this category of questions. Table 13 shows the problems from each worksheet which were used in this comparison. These problems relate to difficulties students have in determining which characteristics apply to a specific diagram used to illustrate a geometric concept and which apply to the class of objects the diagram represents and to difficulties students have in focusing their attention on specific parts of a diagram. In each problem, a definition, postulate, or theorem is first stated and illustrated with a diagram. Students are then asked to apply the definition, postulate, or theorem to other diagrams. On the similar triangle worksheet, Problem 1 was used as the illustration. It was discussed and proven in class before students began work on other problems, all of which are applications of the theorem proven in Problem 1.
Table 13

Problems from Preliminary and Pythagorean Theorem Worksheets Used in the Analyses of Students’ Abilities to Apply Definitions, Postulates, or Theorems to Diagrams

### Preliminary Worksheet

1. **Theorem:** Vertical angles are Congruent.  
   **Illustration:** In the diagram, angle 1 and angle 2 are congruent.

In each of the following diagrams, tell which angles are congruent.

<table>
<thead>
<tr>
<th>A.</th>
<th>B.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram A" /></td>
<td><img src="image2.png" alt="Diagram B" /></td>
</tr>
</tbody>
</table>

22. **Theorem:** If two lines are perpendicular, then they form congruent adjacent angles.

**Illustration:**  
Given: $AC \perp DB$ then $\angle ABD \cong \angle DBC$

In each of the following diagrams, tell which angles if any are congruent.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Diagram A" /></td>
<td><img src="image4.png" alt="Diagram B" /></td>
<td><img src="image5.png" alt="Diagram C" /></td>
</tr>
</tbody>
</table>

Given: $XZ \perp WY$  
Given: $CF \perp AE$  
Given: $XZ \equiv WY$

23. **Theorem:** If two lines form congruent adjacent angles, then they are perpendicular.

**Illustration:**  
Given: $\angle 1 \equiv \angle 2$, then $m \perp n$

In each of the following diagrams, tell which lines are perpendicular.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image6.png" alt="Diagram A" /></td>
<td><img src="image7.png" alt="Diagram B" /></td>
<td><img src="image8.png" alt="Diagram C" /></td>
</tr>
</tbody>
</table>

Given: $\triangle BCF \equiv \triangle DCF$  
Given: $\angle 1 \equiv \angle 2$, $\angle 3 \equiv \angle 4$
Problems from Preliminary and Pythagorean Theorem Worksheets Used in the Analyses of Students' Abilities to Apply Definitions, Postulates, or Theorems to Diagrams

25. Theorem: If two angles are supplements of the same angle, then they are congruent.
Illustration:

Given: \( \angle 1 \) and \( \angle 2 \) are supplementary
\( \angle 1 \) and \( \angle 3 \) are supplementary
Then \( \angle 2 \equiv \angle 3 \).

In the following diagrams, tell which angles are congruent.
A.

Given: \( \angle 1 \) and \( \angle 2 \) are supplementary
\( \angle 2 \) and \( \angle 3 \) are supplementary

B.

Given: \( \angle 4 \) and \( \angle 6 \) are supplementary
\( \angle 5 \) and \( \angle 6 \) are supplementary

C.

Given: \( \angle 1 \) and \( \angle 2 \) are supplementary
\( \angle 2 \) and \( \angle 3 \) are supplementary

26. Theorem: If two angles are complements of the same angle, then the angles are congruent.
Illustration:

Given: \( \angle 1 \) and \( \angle 2 \) are complementary
\( \angle 1 \) and \( \angle 3 \) are complementary
Then \( \angle 2 \equiv \angle 3 \).

In each of the following diagrams, tell which angles are congruent.
A.

Given: \( \angle 4 \) and \( \angle 5 \) are complementary
\( \angle 5 \) and \( \angle 6 \) are complementary

B.

Given: \( \angle 2 \) and \( \angle 3 \) are complementary
\( \angle 2 \) and \( \angle 6 \) are complementary
Problems from Preliminary and Pythagorean Theorem Worksheets Used in the Analyses of Students' Abilities to Apply Definitions, Postulates, or Theorems to Diagrams

Similar Triangle Worksheet Problems

1. Given: DE \parallel AC

Prove: \triangle ABC \sim \triangle DBC

9. In the figure below, name all the similar triangles you can.

10. In the figure below, is \triangle NOP \sim \triangle MOQ? Explain.

19. Name all pairs of similar triangles in the figure below.

20. Figure OPQRST is a regular hexagon. Tell which triangles are similar and support your conclusions.

Pythagorean Theorem Worksheet Problems

Pythagorean Theorem: In a right triangle, the square of the hypotenuse equals the sum of the squares of the two sides.

Example for Problems 1 and 2: In triangle ABC, c^2 = a^2 + b^2.
Problems from Preliminary and Pythagorean Theorem Worksheets Used in the Analyses of Students' Abilities to Apply Definitions, Postulates, or Theorems to Diagrams

1. Find the value of $c$ in the following:

![Diagram A]

2. Find the value of $x$ in the following:

![Diagram B]

Examples for problems 7 through 11:

- In any $\triangle ABC$, if $c^2 = a^2 + b^2$, then $m\angle C = 90^\circ$.
- In any $\triangle ABC$, if $c^2 < a^2 + b^2$, then $m\angle C < 90^\circ$.
- In any $\triangle ABC$, if $c^2 > a^2 + b^2$, then $m\angle C > 90^\circ$.

Referring to the diagram below, is triangle $ABC$ a right, acute, or obtuse triangle?

7. 

![Diagram C]

8. 

![Diagram D]

9. In triangle $RST$, which angle do you think would be largest?

![Diagram E]

10. In triangle $ABC$, if $m\angle A = 67^\circ$ and $m\angle B = 84^\circ$, which side would be shortest? Longest?

11. In triangles $ABC$ and $DEF$, if $m\angle A < m\angle D$, what would be the relationship between $BC$ and $EF$?

![Diagram F]
Statistical analysis. Problems were graded and each student was given a score for the problems on each worksheet reflecting the percent of problems answered correctly. Worksheets completed by the same 21 girls and 13 boys were used for each comparison. Paired $t$-tests and the Wilcoxon Signed Rank Test at the .05 level were used to compare overall mean scores for the 34 students on each of the three worksheets to determine whether there were statistically significant changes in mean scores from one worksheet to the next. The same tests at the .05 level were also used to compare mean scores of females on each worksheet and to compare mean scores of males on each worksheet to determine if there were statistically significant changes in performance by gender from one worksheet to the next. Two sample $t$-tests and the Wilcoxon Rank Sum Test at the .05 level were used to compare mean scores of females with mean scores of males on each worksheet to determine if there were statistically significant differences in performance by gender. Where two-sample $t$-tests were used, tests for homogeneity of variance showed no significant differences at the .10 level between the variances for males and females in the scores on any of the three worksheets. Table 14 shows means and standard deviations for all groups on the three worksheets and $p$-values for all comparisons of means.

Overall, the only significant difference was in the mean score on the similar triangle worksheet as compared to the Pythagorean Theorem worksheet. Because mean scores were higher on the Pythagorean Theorem worksheet, this indicates that scores were significantly better on that worksheet than on the similar triangle worksheet.

There were no significant differences in the mean scores of females on any of the worksheets. For males, results of both the $t$-test and Wilcoxon Signed Rank Test show a
significant difference in mean scores on the similar triangle worksheet as compared to the
Pythagorean Theorem worksheet. Results of the Wilcoxon Test also shows a significant
difference at the .05 level in mean scores on the preliminary worksheet as compared to the
Pythagorean Theorem worksheet. Sample data on both of these worksheets appears not
to be normally distributed so results of the Wilcoxon Test were used. Because the mean
score on the Pythagorean Theorem worksheet was higher than those on either the similar
triangle or preliminary worksheets, this indicates that the performance of males in the
study was significantly better on the problems from the Pythagorean Theorem worksheet
than on the problems from the other two worksheets.

Table 14

Means as Percents, Standard Deviations, and p-Values for Comparisons of Mean Scores
on Problems from the Preliminary, Similar Triangle, and Circle Worksheets Used in the
Analysis of Students' Abilities to Apply Definitions, Postulates, or Theorems to Diagrams

<table>
<thead>
<tr>
<th>Worksheet</th>
<th>M</th>
<th>SD</th>
<th>Preliminary - Similar A t-test</th>
<th>Wilcoxon d p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students a</td>
<td>72.15</td>
<td>19.77</td>
<td>.089</td>
<td>.092</td>
</tr>
<tr>
<td>Males b</td>
<td>73.38</td>
<td>23.32</td>
<td>.391</td>
<td>.300</td>
</tr>
<tr>
<td>Females c</td>
<td>71.38</td>
<td>17.80</td>
<td>.151</td>
<td>.164</td>
</tr>
<tr>
<td>Similar Triangle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students a</td>
<td>64.71</td>
<td>22.29</td>
<td>.017</td>
<td>.027</td>
</tr>
<tr>
<td>Males b</td>
<td>67.31</td>
<td>21.37</td>
<td>.006</td>
<td>.009</td>
</tr>
<tr>
<td>Females c</td>
<td>63.10</td>
<td>23.21</td>
<td>.251</td>
<td>.287</td>
</tr>
<tr>
<td>Pythagorean Thm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students a</td>
<td>76.18</td>
<td>18.57</td>
<td>.329</td>
<td>.158</td>
</tr>
<tr>
<td>Males b</td>
<td>84.77</td>
<td>13.69</td>
<td>.075</td>
<td>.044</td>
</tr>
<tr>
<td>Females c</td>
<td>70.86</td>
<td>19.46</td>
<td>.923</td>
<td>.768</td>
</tr>
<tr>
<td>Male b - Female c</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary</td>
<td>.779</td>
<td>.519</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Similar Triangle</td>
<td>.600</td>
<td>.788</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>.031</td>
<td>.031</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*n = 35.  b n = 13.  c n = 22.  d Wilcoxon Signed Rank Test.  e Wilcoxon Rank Sum Test.
The only statistically significant difference at the .05 level between the performance of females and males was on the Pythagorean Theorem worksheet on which the mean score for females was 70.86% and for males was 84.77%. This indicates that males performed significantly better on the problems from this worksheet than did females.

**Quiz analysis.** The quiz, which consisted of the four problems shown in Table 15 also dealt with the application of basic geometric concepts to diagrams. Its goal was to ascertain whether students who could identify the types of angles in a simpler diagram could also do so in a more complicated one. The quiz was designed so that questions in problems 1 and 2 correspond to similar questions in problems 3 and 4. This correspondence and the topic of each question are shown in Table 16. The diagrams for problems 1 and 2 are fairly simple and answering these problems correctly requires fairly straightforward applications of the definitions involved. The diagrams for problems 3 and 4 are more complex and answering these questions requires students to concentrate on certain aspects of the diagram and to exclude others.

Each student was given two grades for the quiz, one reflecting the percent of simple problems answered correctly, and the other reflecting the percent of more complex problems answered correctly. *T*-tests and appropriate Wilcoxon Tests at the .05 level of significance were used to compare mean scores to determine whether there were significant differences in the overall mean scores on the two groups of problems, by gender on the two groups, and whether there were differences between male and female students on either the simple or more complex problems. Means and standard deviations

121
Table 15

*Quiz Problems 1 through 4*

**Quiz Question 1:** In the diagram above, name a pair of corresponding, alternate-interior, and same-side interior angles.

**Quiz Question 2:** In the diagram above,

a. An interior $\angle$ of $\triangle ABC$ is: 

b. An interior $\angle$ of $\triangle ABC$ is: 

c. $m\angle 1 + m\angle 2 + m\angle 3 =$ 

d. $m\angle 4 = m\angle 4$

**Quiz Question 3:** In the diagram above,

a. An interior $\angle$ of $\triangle XYZ$ is: 

b. An exterior $\angle$ of $\triangle XYZ$ is: 

c. $m\angle 2 + m\angle 5 + m\angle 12 =$ 

d. $m\angle 6 + m\angle 11 = m\angle 9$

**Quiz Question 4:** In the diagram above,

a. $m\angle 2 + m\angle 5 + m\angle 12 =$ 

b. $m\angle 12 + m\angle 9 = m\angle 15 + m\angle 9$ 
and $m\angle 12 + m\angle 9 = m\angle 15 + m\angle 14$

c. $m\angle 3 = m\angle 11 + m\angle 14$

d. Line $n$ is a transversal which crosses lines $l$ and $m$. Name a pair of alternate interior angles.

e. Line $p$ is a transversal which crosses lines $l$ and $m$. Name a pair of corresponding angles.
Table 16

**Topics and Related Quiz Problems**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Related Problems</th>
<th>Simple Problem</th>
<th>Complex Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding $\angle$</td>
<td>1a</td>
<td></td>
<td>4e</td>
</tr>
<tr>
<td>Alternate Interior $\angle$</td>
<td>1b</td>
<td></td>
<td>4d</td>
</tr>
<tr>
<td>Interior Angles of a $\Delta$</td>
<td>2a</td>
<td>3a</td>
<td></td>
</tr>
<tr>
<td>Exterior Angles of a $\Delta$</td>
<td>2b</td>
<td>3b</td>
<td></td>
</tr>
<tr>
<td>Exterior Angles of a $\Delta$</td>
<td>2d</td>
<td>3d</td>
<td></td>
</tr>
<tr>
<td>Exterior Angles of a $\Delta$</td>
<td>2d</td>
<td>3d</td>
<td>4b</td>
</tr>
<tr>
<td>Exterior Angles of a $\Delta$</td>
<td>2d</td>
<td>4c</td>
<td></td>
</tr>
<tr>
<td>Sum of Angles of a $\Delta$</td>
<td>2c</td>
<td>3c</td>
<td></td>
</tr>
<tr>
<td>Sum of Angles of a $\Delta$</td>
<td>2c</td>
<td></td>
<td>4a</td>
</tr>
</tbody>
</table>

for simple and complex problems by groups are shown in Table 17 as are p-values for the comparisons. Results showed no significant differences in the performance of females as compared to males on either the simple or more complex problems. There were significant differences in the mean scores for simple problems as compared to complex problems for all students, for male students, and for female students. In all cases, mean scores on the simpler problems were significantly higher than scores on the more complex problems indicating that students had more difficulties with the questions relating to the more complex diagrams. This also indicates that a significant number of students who demonstrated basic understanding of the concepts by answering the simple problems correctly were unable to apply the concepts to the more complex diagrams.

The last column of Table 18 shows the numbers of students who answered the question for the easier diagram correctly but answered the question for the more difficult
Table 17

Means as Percents, Standard Deviations, and p-Values for Comparisons of Mean Scores on Simple and Complex Quiz Problems

<table>
<thead>
<tr>
<th>Category</th>
<th>M</th>
<th>SD</th>
<th>p-Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students (n = 49)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simple</td>
<td>83.10</td>
<td>24.19</td>
<td>All Students Simple - Complex</td>
</tr>
<tr>
<td>Complex</td>
<td>63.67</td>
<td>23.43</td>
<td>t-test Wilcoxon*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&lt;.001 &lt;.001</td>
</tr>
<tr>
<td>Female (n = 29)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simple</td>
<td>80.72</td>
<td>23.06</td>
<td>Females Simple - Complex</td>
</tr>
<tr>
<td>Complex</td>
<td>63.38</td>
<td>22.35</td>
<td>t-test Wilcoxon*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&lt;.001 &lt;.001</td>
</tr>
<tr>
<td>Male (n = 20)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simple</td>
<td>86.55</td>
<td>25.95</td>
<td>Males Simple - Complex</td>
</tr>
<tr>
<td>Complex</td>
<td>64.10</td>
<td>25.51</td>
<td>t-test Wilcoxon*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&lt;.001 &lt;.001</td>
</tr>
<tr>
<td>Male-Female</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simple</td>
<td>.413</td>
<td>.222</td>
<td>Wilcoxon*</td>
</tr>
<tr>
<td>Complex</td>
<td>.917</td>
<td>.732</td>
<td></td>
</tr>
</tbody>
</table>

*Wilcoxon Signed Rank Test.  bWilcoxon Rank Sum Test.

diagram incorrectly. These students seem to understand the concept but may have difficulty focusing on certain parts of a more difficult diagram to apply it. In this regard, students had most difficulty with questions dealing with corresponding and alternate interior angles of parallel lines and with exterior angles of a triangle. The addition of line \( n \) and the additional angles created at the common intersection of lines \( l \), \( n \), and \( p \) seems to have caused the most difficulty for students on Problem 4e. In Problem 4d, line \( p \) which creates angles 10 and 11 seems to have caused students the most difficulty in identifying alternate interior angles.

Twenty-seven girls and seventeen boys missed question 4.b. A substantial number of girls (18) and boys (12) answered the corresponding question about the simpler diagram correctly. For 4.b. most students correctly answered angles 11 and 14, the two
Table 18

*Quiz Analysis Showing Numbers of Students Who Answered Simple Questions Correctly but Answered More Complex Questions on the Same Topic Incorrectly*

<table>
<thead>
<tr>
<th>Topic and Related Problems</th>
<th>Gender</th>
<th>Simple Correct</th>
<th>Complex Correct</th>
<th>Simple Correct/Complex Incorrect&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Corresponding ( \angle )s</strong>&lt;sup&gt;a&lt;/sup&gt; (1a/4e)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>20 (69%)</td>
<td>14 (48%)</td>
<td>8 (28%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>18 (90%)</td>
<td>13 (65%)</td>
<td>5 (25%)</td>
</tr>
<tr>
<td><strong>Alternate Interior ( \angle )s</strong>&lt;sup&gt;c&lt;/sup&gt; (1b/4d)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>24 (83%)</td>
<td>18 (62%)</td>
<td>5 (17%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>18 (90%)</td>
<td>9 (45%)</td>
<td>9 (45%)</td>
</tr>
<tr>
<td><strong>Interior Angles of a ( \Delta )</strong> (2a/3a)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>28 (97%)</td>
<td>27 (93%)</td>
<td>1 (3%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>18 (90%)</td>
<td>19 (95%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td><strong>Exterior Angles of a ( \Delta )</strong> (2b/3b)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>27 (93%)</td>
<td>26 (90%)</td>
<td>2 (7%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>18 (90%)</td>
<td>17 (85%)</td>
<td>1 (5%)</td>
</tr>
<tr>
<td><strong>Exterior Angles of a ( \Delta )</strong> (2d/3d)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>18 (62%)</td>
<td>18 (62%)</td>
<td>2 (7%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>12 (60%)</td>
<td>11 (55%)</td>
<td>1 (5%)</td>
</tr>
<tr>
<td><strong>Exterior Angles of a ( \Delta )</strong> (2d/4b)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>18 (62%)</td>
<td>2 (7%)</td>
<td>16 (55%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>12 (60%)</td>
<td>3 (15%)</td>
<td>9 (45%)</td>
</tr>
<tr>
<td><strong>Exterior Angles of a ( \Delta )</strong> (2d/4c)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>18 (62%)</td>
<td>12 (41%)</td>
<td>8 (26%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>12 (60%)</td>
<td>8 (40%)</td>
<td>4 (20%)</td>
</tr>
<tr>
<td><strong>Sum of Angles of a ( \Delta )</strong> (2c/3c)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>23 (79%)</td>
<td>23 (79%)</td>
<td>1 (3%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>19 (95%)</td>
<td>17 (85%)</td>
<td>2 (10%)</td>
</tr>
<tr>
<td><strong>Sum of Angles of a ( \Delta )</strong> (2c/4a)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Girls</td>
<td>23 (79%)</td>
<td>24 (83%)</td>
<td>1 (3%)</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>19 (95%)</td>
<td>18 (90%)</td>
<td>1 (5%)</td>
</tr>
</tbody>
</table>

<sup>a</sup>\( n = 29 \) girls and 20 boys.

<sup>b</sup> Indicates questions on the quiz which address the same topic with the first being a more simple problem than the second.

<sup>c</sup>Number of students answering simple problem correctly, complex problem incorrectly.
vertical angles. Angles 9 and 12 together form an exterior angle of the triangle and the sums of their measures would be equal to the sum of the measures of the two nonadjacent interior angles of the triangle, angles 4 and 7. Few students gave this correct answer. A substantial number of both girls and boys answered 2.d. correctly but missed 4.c. indicating that they may not have seen the triangle in the diagram for question 4.

Worksheet on circles. Although the worksheet on circles dealt primarily with conjecturing and proof, there were indications that more complicated diagrams caused difficulties for some students. In Part I of the worksheet, the corollary, "Tangents to a circle from a point are congruent," was stated and illustrated by the diagram shown in Figure 12. Problem 3a of Part I is shown in Figure 13.

The diagram shown in Figure 14 was used to illustrate the definition of an inscribed angle and the theorem which states that the measure of an inscribed angle is equal to one-half the measure of its intercepted arc. Problem 3 of Part II then asked students to make and prove a conjecture regarding the measure of angle RST in Figure 15, given that segment RT is a diameter of the circle.

\[ \text{Figure 12. Tangents to a Circle from a Point are Congruent (PX \cong PY)} \]
Figure 13. Circle Worksheet Part I, Problem 3a: Given: Two tangent circles; $EF$ is a common external tangent; $GH$ is the common internal tangent. Discover and prove something interesting about point $G$.

Figure 14. Definition: An Inscribed Angle is an angle whose vertex is on a circle and whose sides are chords of the circle. ($\angle ABC$ is an inscribed angle.) Theorem: The measure of an inscribed angle is equal to one-half the measure of its intercepted arc.

\[ m\angle ABC = \frac{1}{2} \text{arc AC}. \]

Figure 15. Circle Worksheet Part II, Problem 3: Make and prove a conjecture regarding $m\angle RST$. NOTE: $RT$ is a diameter of circle $Q$. 

127
The following excerpts are from an interview with one of the girls.

Researcher: (Part I) Number 3a, you said, "Point G forms a right triangle." What do you mean, "So EG equals GF?"
Student: No it doesn't form....
R: No, not necessarily. Do you see how you could apply the theorem to point G?
S: Um, hmm. EG and GH are congruent and GF and GH are congruent.

R: In this one (Part II, Problem 3), it says make a conjecture regarding the measure of angle RST. You said, "The measure of angle R is equal to arc ST. RQ is congruent to QT." Okay, it says make a conjecture about the measure of this angle.
S: What could I say about that?
R: Yeah.
S: It's half the measurement of arc RT.
R: Which is what? What is the measurement of this arc?
S: 180
R: So what would this angle be?
S: 90.
R: Right.
S: Oh!

In the above examples, part of the problem could have been with the difficulty of the diagram because, when the researcher isolated the student's attention to specific parts of the diagram, she was then able to apply the theorem correctly. The following student also had difficulty with Problem 3 of Part I and was able to apply the theorem correctly with some help. She indicates that the difficulty of the diagram caused problems.

Researcher: On this one, you said G is the midpoint of EF because triangle HEF has a median which is GH.
Student: I didn't really understand that one. I'm not sure.
R: Do you see how you could apply the theorem to number three?
S: EH and EF would be congruent.
R: EH and EF?
S: Yeah, because of the tangent.
R: But see this isn’t really a point of tangency because if I extended this (EH), it would go through the circle.
S: Oh, okay. I know, it's GH and GF.
R: Yeah, GH and GF would be congruent.
S: And GH and GE.
R: And then these two would be congruent and so EG and GF would be congruent. Why do you think you didn’t see that before?
S: I don’t know. It’s just confusing.
R: Why?
S: Because there’s a lot of lines and stuff.
R: So if you just crossed out one circle and looked at the other one?
S: It would be easier.

The following student indicates as did several others that the two circles shown together on the diagram of Part I, Problem 3 were confusing to him.

Researcher: This one you said, “Point G is equally distant from the circle.” I don’t know what that means. What do you mean by equally distant from the circle?
Student: I guess it was supposed to be the distance from the midpoint.
R: From the center of the circle to G?
S: Yeah. From the center of the circle to GF, they were all the same distance so I thought that GF and GE would be equal and G to H would be equal to either one.
R: Okay, so did you see how you could apply this theorem to this?
S: No, I don’t see any way to apply the theorem.
R: If you block out one of the circles.
S: Oh, right there!
R: Then if you block out the other circle, the same thing applies to the other circle.
S: I see it now.
R: So is the diagram too complicated?
S: There are too many lines so it just threw me off of the theorem.

Adding Lines to Diagrams

The problems shown in Table 19 from the preliminary, similar triangle, and Pythagorean Theorem worksheets were meant to address difficulties students might have in adding to given diagrams or drawing diagrams of their own. While some of the problems could be solved without adding to or drawing diagrams, for most students the search for a solution would be simplified by adding to the given diagram or by drawing an appropriate diagram of their own.
Table 19

Problems from the Preliminary, Similar Triangle, and Pythagorean Theorem Worksheets Which Either Required Students to Add to Diagrams or Would Be Simplified by the Use of Diagrams

Preliminary Worksheet

20. Draw an angle complementary to angle 1.


Similar Triangle Worksheet

Based on number 1 above (see Table 13), show how you could find triangles similar to those shown in numbers 5 and 6.

5. \[ \triangle XYZ \]

6. \[ \triangle ABC \]

11. Construct a triangle similar to \( \triangle ABC \) with base on line \( m \).

12. Given that the lines below are parallel, show how you could construct two similar triangles.

15. Quadrilateral \( WXYZ \) is a trapezoid. Show two similar triangles and support your conclusions.

m
Problems from the Preliminary, Similar Triangle, and Pythagorean Theorem Worksheets Which Either Required Students to Add to Diagrams or Would Be Simplified by the Use of Diagrams

Pythagorean Theorem Worksheet Problems
3. Find the length of a diagonal in the rectangle below.

4. A 10 foot ladder leans against a wall so that the base of the ladder is 4 feet from the wall. Draw a diagram and then determine how high up on the wall the ladder will reach. (NO DIAGRAM WAS GIVEN FOR THIS PROBLEM.)

5. On the axes below, find the distance from the point (-2, -3) to (4, 5).

6. An airplane sits on a runway and is 300 feet from the base of a 1200 foot tower. The plane then taxis down the runway at the rate of 200 feet per minute for two minutes. How far is the plane from the base of the tower? How far is the plane from the top of the tower?
Statistical analysis. The problems were first scored using the rubric shown in Table 20. As with the previous category of problems, each student was given an overall score for the problems on each worksheet. Paired t-tests and the Wilcoxon Signed Rank Test at the .05 level were first used to compare overall mean scores for the 34 students on Table 20.

**Rubric for Scoring Worksheet Problems Dealing with Adding Lines to Diagrams**

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student did not add to given diagram or draw a separate diagram</td>
</tr>
<tr>
<td>1</td>
<td>Student added to the given diagram incorrectly or drew a separate diagram incorrectly</td>
</tr>
<tr>
<td>2</td>
<td>Student drew a correct diagram but not on the given diagram</td>
</tr>
<tr>
<td>3</td>
<td>Student correctly added to the given diagram</td>
</tr>
</tbody>
</table>

each of the three worksheets to determine whether there were statistically significant changes in mean scores from one worksheet to the next. The same tests were also used to compare mean scores of females on each worksheet and to compare mean scores of males on each worksheet to determine if there were statistically significant changes in performance by gender from one worksheet to the next. Two sample t-tests and the Wilcoxon Rank Sum Test at the .05 level were used to compare mean scores of females with mean scores of males on each worksheet to determine if there were statistically significant differences in performance by gender. Worksheets completed by the same 21 girls and 13 boys were used for each comparison. Where two-sample t-tests were used, tests for homogeneity of variance were first conducted. These tests showed a significant
difference at the .10 level between the variances for males and females on the Pythagorean Theorem worksheet (p = .063). In this case, a two sample t-test which accounts for unequal variance was used. Table 21 shows means and standard deviations for all groups on the three worksheets and p-values for all comparisons of means.

T-tests showed a significant difference in the means of the similar triangle worksheet and the Pythagorean Theorem worksheet for students overall but the Wilcoxon Signed Ranks Test did not show this difference. Because the data appear not to be normal based on the Shapiro-Wilk W Test and normal probability plots, results of the Wilcoxon Test were used. This means no significant differences overall on mean scores. This analysis also showed no significant differences in the mean scores of females or of males from one worksheet to the next and no significant differences in the mean scores of females as compared to males on any worksheet.

Discussion of student responses. Three solutions given by students for preliminary worksheet problems 20 and 21 (see Table 19) are shown by the dotted lines in figures 16, 17, and 18. That shown in Figure 16 was most frequently given (12 girls and 9 boys) and is perhaps the easiest—if students are willing to add lines to given diagrams. The solutions shown in Figures 17 and 18 seem to indicate an unwillingness to add lines to the given diagram. Those who used the method of Figure 17 (5 girls and 2 boys) drew a separate diagram while those who used the method of Figure 18 (3 girls and 2 boys) seemed to realize that they could solve the problem by adding lines but shied away from actually adding lines to the existing diagram. Instead they drew separate figures very close to the existing diagram.
Table 21

Means as Percents, Standard Deviations, and p-Values for Comparisons of Mean Scores on Problems from the Preliminary, Similar Triangle, and Pythagorean Theorem Worksheets Used in the Analysis of Students’ Responses to Problems Requiring Them to Add to Existing Diagrams or to Draw Diagrams

<table>
<thead>
<tr>
<th>Worksheet</th>
<th>M</th>
<th>SD</th>
<th>p-Values</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Preliminary-Similar Δ</td>
<td>t-tests</td>
<td>Wilcoxon&lt;sup&gt;d&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>80.88</td>
<td>26.71</td>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>.064</td>
<td>.086</td>
<td></td>
</tr>
<tr>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
<td>87.23</td>
<td>21.67</td>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
<td>.109</td>
<td>.147</td>
<td></td>
</tr>
<tr>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
<td>76.95</td>
<td>29.20</td>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
<td>.280</td>
<td>.327</td>
<td></td>
</tr>
<tr>
<td>Similar Triangle</td>
<td></td>
<td></td>
<td>Similar Δ - Pyth. Thm</td>
<td>t-tests</td>
<td>Wilcoxon&lt;sup&gt;d&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>69.53</td>
<td>23.43</td>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>.047</td>
<td>.148</td>
<td></td>
</tr>
<tr>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
<td>71.92</td>
<td>27.14</td>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
<td>.314</td>
<td>.485</td>
<td></td>
</tr>
<tr>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
<td>68.05</td>
<td>21.40</td>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
<td>.082</td>
<td>.191</td>
<td></td>
</tr>
<tr>
<td>Pythagorean Thm</td>
<td></td>
<td></td>
<td>Prelim - Pyth. Thm</td>
<td>t-tests</td>
<td>Wilcoxon&lt;sup&gt;d&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>78.56</td>
<td>15.09</td>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>.649</td>
<td>.421</td>
<td></td>
</tr>
<tr>
<td>Male&lt;sup&gt;b&lt;/sup&gt;</td>
<td>80.69</td>
<td>10.31</td>
<td>Male&lt;sup&gt;b&lt;/sup&gt;</td>
<td>.272</td>
<td>.139</td>
<td></td>
</tr>
<tr>
<td>Female&lt;sup&gt;c&lt;/sup&gt;</td>
<td>77.24</td>
<td>17.53</td>
<td>Female&lt;sup&gt;c&lt;/sup&gt;</td>
<td>.970</td>
<td>.943</td>
<td></td>
</tr>
<tr>
<td>Males&lt;sup&gt;b&lt;/sup&gt; - Female&lt;sup&gt;c&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td>t-tests</td>
<td>Wilcoxon&lt;sup&gt;e&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>Preliminary</td>
<td>.282</td>
<td>.348</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Similar Triangle</td>
<td>.646</td>
<td>.530</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>.475</td>
<td>.635</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>n = 34.  <sup>b</sup>n = 13.  <sup>c</sup>n = 21.  <sup>d</sup>Wilcoxon Signed Rank Test.  <sup>e</sup>Wilcoxon Rank Sum Test.

As discussed earlier, Problem 1 of the similar triangle worksheet (see Table 13) had been worked in class as an illustration before students completed the worksheet. To respond to Problems 5, 6, 11, 12, and 15 (see Table 19), students could apply the theorem of Problem 1 and either add line segments or other geometric figures to the diagrams provided or produce separate diagrams. For example, the most common solutions to Problem 5 are shown in Figure 19 in which the dotted line segments indicate those added by students.
Figure 16: The “Easiest” Solutions to Preliminary Worksheet Problems 20 (Draw an angle complementary to angle 1) and 21 (Draw an angle supplementary to angle 1)

Figure 17: A Second Solution Method for Preliminary Worksheet Problems 20 (Draw an angle complementary to angle 1) and 21 (Draw an angle supplementary to angle 1)

Figure 18: A Third Solution Method for Preliminary Worksheet Problems 20 (Draw an angle complementary to angle 1) and 21 (Draw an angle supplementary to angle 1)
Figure 19. Solutions to Similar Triangle Worksheet Problem 5: Show how you could find triangles similar to those shown by the solid lines.

The following exchange took place in an interview with a girl who did add lines to the diagrams of all problems but indicates some hesitancy to do so.

Researcher: Do you have any problem with adding extra lines to a diagram?
Student: No, not really. But it did take awhile to figure out what to do on that first one, 5 and 6.
R: Why do you think that was the case?
S: I don’t know. Just because I didn’t know if we were supposed to add lines or not but I figured out we were.

Instead of adding to the diagrams of Problems 5 and 6, several students gave valid written explanations of how to draw the similar triangles by constructing lines parallel to one side of the triangle. These students did add to the diagrams of the other problems. One girl said the reason was that it was easier to explain in words what to do on problems 5 and 6 but difficult to do so on the others. A boy said he wrote out the explanations to five and six but did the drawings on the others because he needed to finish the paper. Another boy added lines to the diagrams of problems 11 and 12 but did not on the other three. When asked why, he explained that it was easy for him to visualize the figure in
problems 5, 6, and 15 without doing the drawing but on problems 11 and 12 he could not visualize what the diagram looked like without doing the actual drawing.

Although the circle worksheet dealt primarily with conjecturing and proof, whether or not they could add to diagrams caused difficulty for some students on that worksheet also. In attempting to solve Problem 2 of Part I, one girl added a line segment joining points A and B in Figure 20 but then questioned if she was allowed to do so as indicated in the following interview excerpt.

Researcher: This one, number 2, you said, “PA and PB are congruent so PS and PR are congruent.” Yeah, PA and PB, that’s just an application of the theorem. Why would PS and PR be congruent?
Student: Because they’re, you know...they’re not tangents, right?
R: No. Do you see how you could apply the theorem?
S: Yes, because two lines are parallel lines so the corresponding angles....
R: Okay, so you are saying if you drew AB?
S: Yeah. Or you can’t do that, right?
R: Why not?
S: You can? I think so.

![Figure 20](image)

*Figure 20.* Circle Worksheet Part I, Problem 2: Line segments PA, PB, and RS are tangents to the circle. Explain why PR + RS + SP = PA + PB.
Identifying Figures in Nonstandard Positions

Preliminary worksheet problems 5, 6, 7, 9, and 10 shown in Table 22 relate to the difficulty that students might have in identifying figures in “nonstandard” positions or in identifying figures which are part of more complicated diagrams. Because this was the only worksheet containing problems solely related to identifying figures in nonstandard position, no statistical analysis of these problems was conducted. Difficulties that students have with other types of problems which are addressed elsewhere stem in part from difficulty in recognizing figures in nonstandard positions. Table 23 shows the number of girls and boys who answered each question correctly. Many who missed the questions listed some but not all of the correct parts.

In these problems, students had little difficulty in identifying right angles, parallel lines, and the square but had more difficulty with perpendicular lines. A sizable number of students (14 girls and 6 boys) missed Problem 10 but most who missed it (13 girls and all 6 boys) identified the figure as either a right angle or a right triangle. In the diagram for Problem 10, angle NOR appears to be a right angle and it seems they were basing their answer on what appeared to be true from the diagram rather than information they were given as true.

Answers based on the appearance of the diagram. In reviewing students’ work on some problems on the preliminary worksheet, it became apparent that they were often basing their answers on what seemed to be true based on the appearance of the diagram used to illustrate the concept rather than on the stated facts. In so doing, they were attributing characteristics of the specific diagram to the family of figures it was meant to
5. In which of the following diagrams are there lines which appear to be perpendicular?

A. \[ \]  B. \[ \]  C. \[ \]  D. \[ \]  E. \[ \]  F. \[ \]

6. In which of the following diagrams do there appear to be right angles?

A. \[ \]  B. \[ \]  C. \[ \]  D. \[ \]  E. \[ \]

7. In which of the following diagrams do there appear to be parallel lines?

A. \[ \]  B. \[ \]  C. \[ \]  D. \[ \]  E. \[ \]

9. Figure ACEG appears to be a __________.

10. Figure NOR is a ________________.
Table 23

Numbers of Girls and Boys Answering Problems 5, 6, 7, 9, and 10 of the Preliminary Worksheet Correctly

<table>
<thead>
<tr>
<th>Problem</th>
<th>Concept</th>
<th>Number Answering Correctly</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Girls&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>5</td>
<td>Perpendicular Lines</td>
<td>5 (21%)</td>
</tr>
<tr>
<td>6</td>
<td>Right Angles</td>
<td>19 (79%)</td>
</tr>
<tr>
<td>7</td>
<td>Parallel Lines</td>
<td>20 (83%)</td>
</tr>
<tr>
<td>8</td>
<td>Square</td>
<td>16 (67%)</td>
</tr>
<tr>
<td>10</td>
<td>Triangle or Angle</td>
<td>10 (42%)</td>
</tr>
</tbody>
</table>

<sup>a</sup>n = 24.  <sup>b</sup>n = 15.

represent. This was true for many of the problems from the preliminary worksheet shown in Table 13 and also for some other problems from that worksheet which are shown in Table 24. Table 25 lists the problems for which students seemed to base their conclusions on the appearance of the diagram and indicates by gender the number of students answering each problem correctly and the number who gave incorrect answers which seem to have been based on the appearance of the diagram.

In the diagram illustrating the theorem of Problem 1 (see Table 13), the intersecting line segments which form the vertical angles were drawn so that they appear to be perpendicular with one being horizontal and one vertical. This was done to see if students would assume that a characteristic of the diagram which accompanied the theorem (perpendicular lines) also applied to the class of figures the diagram was intended to represent (any pair of intersecting lines). This did not appear to be the case.
In the diagram below, M is the midpoint of segment KT. Name all congruent segments.

In the figure below, segment BE bisects angle ABC. Name all congruent angles.

In the figure below, D is the midpoint of line segment BE. Which line segments are congruent?

In the following diagrams, which angles are complementary?

In Problem 11, the only information students were given was that point M is the midpoint of segment KT. This would mean that segments KM and MT are congruent yet many students stated that other segments such as HM and ML or HJ and TK were congruent. This was apparently based on the appearance of the diagram. In Problem 12, the only angles known to be congruent based on the given information are angles ABE and EBC, yet some students stated that other pairs of angles such as 1 and 2, 3 and 4, or 1
Table 25

Numbers and Percentages of Students Answering Selected Preliminary Worksheet Problems Correctly and Numbers and Percentages of Students Basing Answers on the Appearance of the Diagram.

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Girls (n = 24)</th>
<th></th>
<th></th>
<th></th>
<th>Boys (n = 15)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct Answer</td>
<td>Answer based on appearance</td>
<td>Correct Answer</td>
<td>Answer based on appearance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B</td>
<td>24 (100%)</td>
<td>2 (8%)</td>
<td>15 (100%)</td>
<td>0 (0%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>14 (58%)</td>
<td>12 (50%)</td>
<td>12 (80%)</td>
<td>3 (20%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>14 (58%)</td>
<td>9 (38%)</td>
<td>13 (87%)</td>
<td>3 (20%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>12 (50%)</td>
<td>7 (29%)</td>
<td>12 (80%)</td>
<td>3 (20%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19B</td>
<td>11 (46%)</td>
<td>0 (0%)</td>
<td>12 (80%)</td>
<td>1 (7%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19C</td>
<td>12 (50%)</td>
<td>2 (8%)</td>
<td>10 (67%)</td>
<td>3 (20%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22A</td>
<td>22 (92%)</td>
<td>0 (0%)</td>
<td>15 (100%)</td>
<td>0 (0%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22B</td>
<td>21 (88%)</td>
<td>9 (38%)</td>
<td>10 (67%)</td>
<td>6 (40%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22C</td>
<td>5 (21%)</td>
<td>4 (17%)</td>
<td>6 (40%)</td>
<td>0 (0%)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>23A</td>
<td>20 (83%)</td>
<td>0 (0%)</td>
<td>12 (80%)</td>
<td>0 (0%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23B</td>
<td>14 (58%)</td>
<td>8 (33%)</td>
<td>11 (73%)</td>
<td>1 (7%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23C</td>
<td>21 (88%)</td>
<td>0 (0%)</td>
<td>10 (67%)</td>
<td>0 (0%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25A</td>
<td>9 (38%)</td>
<td>11 (46%)</td>
<td>11 (73%)</td>
<td>2 (13%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25B</td>
<td>10 (42%)</td>
<td>7 (29%)</td>
<td>8 (53%)</td>
<td>3 (20%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26A</td>
<td>15 (62%)</td>
<td>3 (12%)</td>
<td>11 (73%)</td>
<td>1 (7%)</td>
<td></td>
<td></td>
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<tr>
<td>26B</td>
<td>12 (50%)</td>
<td>3 (12%)</td>
<td>7 (47%)</td>
<td>0 (0%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. In some cases, students gave the correct answer along with incorrect answers which seem to be based on the appearance of the diagram. These were scored as a correct answer and also an answer based on appearance. *Indicates that the student’s answer seemed to be based on the appearance of the diagram used to illustrate the concept.*
and 4 are congruent. Problem 13 states that D is the midpoint of BE, and the only segments known to be congruent would therefore be segments BD and DE which do not look congruent in the diagram. All of the segments appear to be the same length in the diagram and a number of students stated that other segments were congruent. There were some interesting answers to this question. In addition to stating that segments BD and DE are congruent, one student said that segments BC and CD are congruent, apparently basing the first answer on the given information but the second on the appearance of the diagram. Another student wrote, "Not True," beside the statement in the problem which said that point D is the midpoint of segment BE.

In the diagram illustrating the theorem of Problem 22 (see Table 13), the perpendicular lines are purposely drawn so that one is horizontal and the other vertical. Students had little difficulty with part A which was a direct application of the theorem in a situation only slightly different from the given illustration but had much more difficulty with part B which included the perpendicular lines as part of another more complicated figure. The most common errors appear to result from assuming that things are true because they appear to be true in the diagram without having any information to back up those assumptions. Ten students apparently assumed that other lines in the diagram are perpendicular because they appear to be. Five stated that all eight angles in the diagram which look like right angles are congruent.

Students had little difficulty with parts A and C of Problem 23 (see Table 13) but in part B, 9 apparently assumed that other lines were perpendicular because they appear to be so in the diagram. The most common incorrect answer was that lines CG and AE are
perpendicular. The fact that 32 of the 39 students answered part A correctly does not necessarily mean that they understood the concept. The diagram for that problem shows only two line segments so stating that those two segments are perpendicular would be the only possible answer.

In Problem 25 (see Table 13), the illustration accompanying the theorem uses angles which appear to be right angles. Students are then asked to apply the theorem first to a diagram very similar to the illustration and then to two different diagrams. Again, many students seemed to base their answers on the appearance of the diagram rather than on the given information. The reason that many students (32) answered part B correctly may be in part because there are only two angles in the diagram that appear to be congruent. In Problem 26 (see Table 13), several students who gave the correct answers added additional incorrect information which seemed to be based solely on the appearance of the diagram.

Assuming that things are true based on the diagrams used to illustrate the theorems caused difficulties with some problems on the other worksheets as well. On the similar triangle worksheet, the diagrams for problems 10 and 19 (see Table 13) are very similar except that Problem 19 has two additional line segments. Most students answered Problem 10 correctly but then many either missed Problem 19 entirely or stated that there are similar triangles in the diagram which, in fact, are not known to be similar. The diagram for Problem 9 (see Table 13) caused no major problems but the very similar diagram in Problem 19 did cause problems for many students. As in other problems already discussed, the main difficulty seems to be that students assume things to be true
from a diagram which are not known to be true. In this case they seem to be assuming that segments BD and AE and segments AC and FD in Problem 19 are parallel. Most did not make this assumption in Problem 10, the less complicated diagram, but did so in Problem 19.

On the Pythagorean Theorem worksheet (see Table 13), problems 7 and 8 ask students to tell whether the triangles in the diagram are acute, right, or obtuse. For five girls and one boy who missed Problem 8, there may have been a misunderstanding of the diagram. All computed $8^2 + 18^2$, compared that sum with $15^2$, and then stated that the triangle is acute because $c^2 < a^2 + b^2$. They apparently used the sides in the same order that they were used in the diagrams illustrating the theorems with no regard to the orientation of the triangle. Students did substantially better on Problem 7 than on Problem 8 even though the two problems are similar. This is probably because, in Problem 7, the sum of the squares of the two smaller sides is equal to the square of the longer side and so no interpretation had to be made as to whether the angle would be acute or obtuse. In Problem 8, these figures are not equal and students had to interpret the meaning of the inequality in terms of the diagrams.

On Problem 9, three girls made the correct computation but then said that $\angle T$ would be largest because the sum of the squares of ST and RT would be the greatest of the sums of the squares of any two sides. These students seem to not be applying the information given in the diagram correctly. One other girl did the computation correctly but then apparently did not know what to do with it and said that the triangle is equilateral because it looks like it is equilateral.
The most interesting outcome of Problem 11 is that 8 girls and 3 boys stated that the 2 sides would be congruent with many of those giving the side-angle-side congruency theorem as justification. In the diagram which accompanies the problem, the two triangles look congruent. These students apparently placed the looks of the diagram ahead of what they were told in the problem.

**Student Group Presentations**

The student group presentations on the areas of polygons which took place in April also relate to the question of gender differences in abilities to use diagrams (see Figures 10 and 11). Group composition was determined by the teacher/researcher so that each group had four or five students of varying ability levels and so that, in the third period class, groups consisted of a mix of boys and girls while in the fourth period class, groups consisted of either all boys or all girls. A key element of the assignment was demonstration of the theorems related to the areas of polygons using the TI-92. For this part of the presentation, students were to construct the polygon, measure appropriate line segments, and demonstrate the area formula. Because students had little actual experience using the TI-92, they had been told that they could either do the calculator demonstration themselves or they could give instructions to the teacher who would do the actual operation of the calculator.

Of the six mixed-gender groups in the Period 3 class, boys did five of the calculator presentations with two operating the calculator themselves and three giving instructions to the teacher. The one girl from this class who did the calculator
presentation gave instructions to the teacher. In the Period 4 class, the two boys and four girls doing this part of the presentation all chose to operate the calculator themselves.

Of the eight students who chose to operate the calculator themselves, two of the four boys and all of the four girls gave remarkable demonstrations of use of the TI-92 considering their very limited experience in actually operating the calculator. They not only constructed the polygons but also used the measurement, calculate, and drag tools to demonstrate the formulas. It is significant that four of five girls doing this part of the presentation chose to operate the calculator themselves and all were extremely proficient in use of the calculator and showed no hesitancy in demonstrating their abilities to use it in front of the class. Unfortunately, time did not allow another group project in which the make up of the groups could be reversed to see if the girls who did such a good job would also do the calculator portion of the presentation when part of a mixed-gender group.

Research Question Two: Conjecturing and Proof

The second research question asks: Are high school students in a geometry class taught using interactive software only in a whole-class setting able to form conjectures and produce convincing arguments supporting their conjectures? Are there gender-based differences in students' abilities to develop and prove conjectures when the software is used in this manner? This question is tied closely to the first in that overcoming the difficulties associated with the use of diagrams is essential to conjecturing ability.

To address these questions in the present study, selected questions on the preliminary worksheet, the similar triangle worksheet, the homework assignment, and the circle worksheet were separated into two categories which will be discussed separately:
1. Developing viable conjectures;

2. Providing justification for conjectures.

Developing Viable Conjectures

Problems on the preliminary, similar triangle, and circle worksheets shown in Table 26 dealt with students’ abilities to develop conjectures. Conjectures made by students were first scored using a rubric similar to that used by Yerushalmy, Chazan, and Gordon (1987). The rubric and examples are shown in Table 27. It is sometimes difficult to score conjectures made by students because it is often not clear exactly what the student was looking at or thinking about when he or she developed the conjecture. Chazan and Houde (1989) indicate that conjectures should not be scored as right or wrong because their validity lies in the thought processes of the conjecturer. In scoring these problems, the student was given the benefit of the doubt. For example, in some cases a valid conjecture may have been made by a student based solely on the appearance of a particular diagram with no thought to how the given information and previously studied concepts apply to the diagram. In these cases, students were given credit for valid conjectures.

It should also be pointed out that most of the problems students were given to conjecture about in the present study likely fit the category of problems which Yerushalmy, Chazan, and Gordon (1987) term directly related to the school curriculum and which have, to the more experienced eye, obvious answers. They are not the type of problems which give students the opportunity to investigate extensively and to develop truly original conjectures regarding relationships that might exist which had not or would
**Table 26**

*Preliminary, Similar Triangle, and Circle Worksheet Problems Used to Investigate Students' Abilities to Develop Conjectures.*

---

**Preliminary Worksheet**

15. Given: $m\angle AOC = m\angle BOD$, what conjectures can you make?  
16. Given $RS = PS$ and $ST = SQ$, what conjectures can you make?

---

18. Given angles 4 and 6 are supplementary, what conjectures can you make?

---

27. Given $\angle 2 \equiv \angle 3$, what conjectures can you make?  
32. Given $\angle 7 \equiv \angle 8$, what conjectures can you make?

---

**Similar Triangle Worksheet**

State any conjectures you can make about the triangles shown in numbers 2 through 4 and give arguments to support your conjectures.

2.  
16. Given $\angle 1 \equiv \angle 2$ and $\angle 3 \equiv \angle 4$, state any conjectures you think are true and support your conjectures.
Circle Worksheet - Part I Problems

1. Quadrilateral ABCD is circumscribed about a circle. Discover and prove a relationship between \( AB + DC \) and \( AD + BC \).

2. \( \text{Diagram of a quadrilateral with a circle inscribed} \)

3. Given: Two tangent circles; \( EF \) is a common external tangent; \( GH \) is the common internal tangent. Discover and prove something interesting about point G.
Table 26—continued

Preliminary, Similar Triangle, and Circle Worksheet Problems Used to Investigate Students' Abilities to Develop Conjectures.

Circle Worksheet - Part II Problems

2. Make a conjecture regarding the relationship between $\angle PNM$ and $\angle PQM$ in the diagram below. Prove your conjecture.

3. Make and prove a conjecture regarding $m\angle RST$ below. NOTE: $\overline{RT}$ is a diameter of circle Q.

4. Make and prove a conjecture regarding opposite angles of Quadrilateral ABCD which is inscribed in circle O.
Table 27

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
<th>Problem: Given RS = PS and ST = SQ, what conjectures can you make?</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No conjecture or conjecture repeats given information.</td>
<td>RS = PS and ST = SQ</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Conjecture restates given information in another form or states the obvious.</td>
<td>RS ≡ PS and ST ≡ SQ or ∠s PSR and TSQ are vertical angles.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Conjecture seems to be based on the appearance of the diagram and does not apply to other diagrams.</td>
<td>RS ≡ ST</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Conjecture seems to be a generalization beyond merely the given information or the obvious but does not go as far as #4 below.</td>
<td>PS + ST = RS + SQ</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Conjecture uses the given information along with previously studied theorems to reach a conclusion about the diagram.</td>
<td>RT = PQ</td>
<td></td>
</tr>
</tbody>
</table>

not be covered in class. The reason for using the simpler type of problem in the present study was primarily to give more students the opportunity to be successful at developing some type of conjecture. Of the students in their study, Yerushalmy, Chazan, and Gordon (1987) state,

For almost all the students, conjectures presented real problems. Knowing what to conjecture about, discerning patterns and relationships, and generating conjectures were all hard work. For some students, conjecture-making was the province of "smart people." (p. 48)

The same was certainly true of students in the present study. Assignments in which students were asked to make more original conjectures had little success. Few students
were successful at them and a high percentage would not make a serious attempt to develop conjectures. For many students, conjecturing problems seem to fall into the same category as proofs in geometry or word problems in algebra—they will not even attempt to do them and are willing to suffer the consequences in terms of their grades rather than exert the mental effort required to successfully complete the problem. This includes some otherwise very successful students and relates to students’ overall attitudes toward geometry which will be discussed later. Due to the lack of success with students attempting to conjecture about more general problems, the teacher decided to use conjecture problems in which students were asked to make conjectures about the relationships of certain parts of a diagram instead of about the diagram overall. These “directed” conjectures helped students to focus their attention on particular parts of the diagram when looking for relationships.

**Statistical analysis.** Scores reflecting the percentage of possible points earned for the problems on each worksheet were computed for each student. Paired *t*-tests and Wilcoxon Signed Rank Tests at the .05 level were first used to compare overall mean scores for the 34 students on each of the three worksheets and then to compare mean scores of females on each worksheet and to compare mean scores of males on each worksheet. Two sample *t*-tests and the Wilcoxon Rank Sum Test at the .05 level were used to compare mean scores of females with mean scores of males on each worksheet to determine if there were statistically significant differences in performance by gender. Where two-sample *t*-tests were used, tests for homogeneity of variance showed no significant differences at the .10 level between the variances for males and females in the
scores on any of the three worksheets. Table 28 shows means and standard deviations for all groups on the three worksheets and p-values for all comparisons of means. None of the comparisons of male and female mean scores were significant at the .05 level for any worksheet. Comparisons of mean scores for all students and by gender showed the mean scores on the similar triangle worksheet to be significantly higher than the mean scores of

Table 28

Means as Percents, Standard Deviations, and p-Values for Comparisons of Mean Scores on Conjecturing Problems from Preliminary, Similar Triangle, and Circle Worksheets

<table>
<thead>
<tr>
<th>Worksheet</th>
<th>M</th>
<th>SD</th>
<th>p-Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Prelim - Similar A</td>
</tr>
<tr>
<td>Preliminary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>63.29</td>
<td>24.52</td>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
<td>66.92</td>
<td>27.12</td>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
<td>61.14</td>
<td>23.24</td>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>Similar Triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>84.94</td>
<td>26.37</td>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
<td>83.62</td>
<td>30.89</td>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
<td>85.73</td>
<td>24.06</td>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>Circle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
<td>68.71</td>
<td>28.06</td>
<td>All Students&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Male&lt;sup&gt;b&lt;/sup&gt;</td>
<td>62.31</td>
<td>33.27</td>
<td>Males&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Female&lt;sup&gt;c&lt;/sup&gt;</td>
<td>72.50</td>
<td>24.53</td>
<td>Females&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Male<sup>b</sup> - Female<sup>c</sup> | t-tests | Wilcoxon<sup>e</sup>
| Preliminary        | .508    | .334                   |
| Similar Triangle   | .823    | .782                   |
| Circle             | .306    | .410                   |

<sup>a</sup>n = 35  <sup>b</sup>n = 13  <sup>c</sup>n = 22.  <sup>d</sup>Wilcoxon Signed Rank Test.  <sup>e</sup>Wilcoxon Rank Sum Test.
preliminary worksheet. The reason for the higher scores on the similar triangle worksheet is likely due to the fact that all problems which students were asked to conjecture about were direct applications of the theorem of Problem 1 on that worksheet (see Table 13). Even though students were not told that the other problems on the worksheet were applications of Problem 1, if they recognized that relationship as many did, then they were able to successfully complete the other problems. The comparisons of the preliminary worksheet and circle worksheet are probably more valid. Even though the problems on the circle worksheet related to theorems stated in earlier problems on the worksheet, they were not direct applications of those theorems in the same way they had been applied in the earlier problems. In these comparisons, the only significant results were for females whose mean score on the circle worksheet (72.50) was significantly higher than their mean score on the preliminary worksheet (61.14). This indicates that they performed significantly better on the circle worksheet which they completed later in the year after being exposed to use of the calculator throughout the study period. There was no corresponding increase of males in the comparisons of these two worksheets.

Discussion of student responses. When the preliminary worksheet was given to students in October, the class had discussed the meaning of a conjecture and how one might go about developing a conjecture and the teacher had demonstrated the process in class, but students had little experience actually making conjectures on their own. As would be expected, students did not perform well on conjecturing problems at this point in the year. In many cases, when students stated a viable conjecture for the problem, they also stated conjectures which were obviously not true. An example is Problem 27 (see
Table 26) in which one student stated that \( m\angle 1 = m\angle 4 \) which is correct but then also stated that \( m\angle 1 = m\angle 2 \).

In many cases students merely repeated the given information as a conjecture or restated it in another way. For example, in Problem 18 (see Table 26), it is given that two angles are supplementary. A number of students "conjectured" that the sum of these angles would be 180 degrees. Many of the conjectures listed by students were direct applications of definitions, postulates, or theorems that had been previously covered fairly extensively in class. For example in Problem 16 (see Table 26), many students stated only that \( \angle RSP \equiv \angle QST \) or \( \angle PST \equiv \angle RST \), a direct application of the vertical angle theorem. While these statements are true, they are trivial conjectures at best. Students most likely listed these trivial conjectures either because the problem instructed them to make a conjecture and those were the only things they could think of or because those were the obvious true statements and, once they came up with one conjecture, they did not carry their thought process any further to come up with others. The goal of conjecturing is to bring students to a point where they can make viable conjectures which are neither obviously true nor obviously false but which stand a good chance of being true and require some further investigation.

In the interviews following the completion of the similar triangle worksheet, students were asked to explain the meaning of conjecture and to tell how they would develop a conjecture. Although some students stated that they did not know what a conjecture is, the answers indicate that many do understand the meaning of the term. One girl stated that a conjecture is, "A statement that you've stated that's not proven yet,"

156
while a boy said, "It’s a thing you say that you think is true but don’t know it’s true," and another girl said, "It’s a conclusion about the information that you are given." When asked how they would come up with a conjecture, the answer of one student is fairly typical:

You just look at the picture that you’re given and, based on theorems and stuff that you know, you make a relationship. It’s just a thing that you think is true that you have to prove.

The responses of other students provide more insight into their thinking. The following student felt that a conjecture applies to a specific diagram and is stated using the letters from the diagram while a theorem is stated in general terms and applies to a family of figures.

Researcher: What is a conjecture?
Student: It’s kinda like a theorem. It kinda proves a theorem but it’s different from a theorem. It proves a theorem in a way like for a specific triangle or a specific....
R: So you’re saying a conjecture would apply to a specific diagram?
S: Yes. It’s like a proof kinda but in words. That’s what I think.
R: So you’re saying a conjecture would apply to a specific diagram. What would a theorem apply to?
S: To like all diagrams I guess. Kind of like all triangles or all right triangles or something like that.
R: So how would you come up with a conjecture?
S: A conjecture would be like specific type letters and stuff from a specific diagram.
R: Then a theorem would apply to what?
S: Figures like that one.

The following student did not seem to understand the difference between a conjecture and theorem at the beginning of the discussion but then figured it out during the interview.

Researcher: First of all, what is a conjecture?
Student: Like something in geometry that’s true. Its been proven.
R: What’s a theorem?
S: A theorem has been proven also.
R: What's the difference between a theorem and a conjecture?
S: A theorem is like a long kinda thing.
R: The reason I asked you is I proved this in class (Problem 1) and then I said (on problems 2 through 4) state any conjectures.
S: It's like a thing you think is true and then you have to prove it.
R: A conjecture is?
S: If you prove it true, then it could be a theorem.
R: So a theorem is something that's already been proven then.
S: Yes.

One girl stated her conjectures for each of problems 2 through 4 as, “That any triangle with parallel lines will make two similar triangles,” and another girl stated her conjectures as, “If a triangle is inside another triangle and is sharing an angle and is parallel then the triangles are similar.” These are restatements of the theorem of Problem 1 but the students did not seem to realize that.

Conjectures based on appearance. It is clear from the interviews that some students based their conjectures primarily on the appearance of the diagram. When asked how she knew that the triangles she drew in Problem 11 of the similar triangle worksheet (Table 19) were similar, one girl said that she just drew them so that they looked like they were similar. When asked how he came up with triangles ABC, DBE, and AFG being similar in Problem 9 (Figure 13), one boy said, “They just look similar.” In his answer to number 11 (Table 19), he stated that angle ABC in the given diagram looks like a right angle and he drew another triangle to the right of that diagram with a right angle in the corresponding position.

The same is true of the problems on the circle worksheet. For Problem 3 of part I (Table 26), when asked how she came up with the conjecture that G is the midpoint of segment EF, one girl replied, “It looks like it.” She gave the same reason when asked how
she came up with the conjecture that $\angle RST$ is a right angle in Problem 3 of part II (Table 26). Another girl had difficulty applying the theorem of part two because of the looks of the diagram. She stated, "I don't see how it could be half of that because like the way I see it is like ABC is that whole thing and how could this whole thing be half of this right here and it looks like it's the same size."

**Impact of hands-on use of the TI-92.** For one week in January, the students had individual access to TI-92s through the Texas Instruments Corporation calculator loan program. One of the activities they did during that week was to construct triangles and then construct the medians, angle bisectors, perpendicular bisectors of the sides, and altitudes and see what observations they could make. The activity was designed to get them to use a lot of the different tools on the 92; familiarize them with medians, altitudes, angle bisectors, and perpendicular bisectors; and acquaint them with some theorems they would study later in the year. In addition to the expected conjectures, the following were stated:

1. Two altitudes of an obtuse triangle lie outside the triangle and one inside.

2. Altitudes form 90° angles with the opposite side.

3. Medians bisect the opposite sides.

These points had already been studied in class and may seem obvious but they emphasize that doing the actual constructions on the TI-92 helps students to understand what some of the definitions mean.

Other conjectures were:

4. The altitudes of an obtuse triangle form obtuse angles.
5. Medians are not the same length except in an equilateral triangle.

6. The medians can be used to find the center of an equilateral triangle. (What is the center of a triangle? Each of the four points mentioned earlier is a center of a triangle. In an equilateral triangle, the four points would all be the same, so it would be an appropriate definition of a center of a triangle.)

From their comments, hands-on experience with the calculator seems to have helped some students to grasp some important basic ideas. Several commented on how the calculator could be used to easily generate a number of examples which could then be used to help develop a conjecture. The teacher continued to emphasize this point throughout the year when using the calculator in front of the class. He used it repeatedly to explain the differences between inductive and deductive reasoning and to emphasize that numerous examples do not prove a conjecture whereas one counterexample disproves one. This may have helped some students to understand and to see the need for proof.

It appears, again from their comments, that use of the calculators helped students understand the idea of conjecturing and, although some state that developing conjectures is still difficult for them, a number seemed to feel that they had a better idea of how to generate a viable conjecture after using the calculators for a week. Several commented that use of the calculators helped them to understand the theorems more quickly.

Perhaps the most important comments related to use of the calculators are those which show that some students realized that a number of different people were arriving at the same conclusion through different diagrams and those which relate the satisfaction that students derived from being able to do something with the calculator.
Providing Justification for Conjectures (Proof)

On October 11, 1996, students were given the last three problems of the worksheet shown in Appendix C as a homework assignment (see Table 29). The first three similar problems had been completed earlier and discussed in class. The following analysis of students' abilities to provide justification for their conjectures will compare Problems 4, 5, and 6 from that homework assignment with the problems from the circle worksheet shown in Table 26. Conjecture problems on the preliminary worksheet did not ask students to provide justification for their conjectures and conjecture problems from the similar triangle worksheet could all be proven by direct application of the first problem on that worksheet and so do not provide a valid indication of students' abilities to justify their conjectures.

Statistical analysis. Each student was given an overall score for the problems on the homework assignment and the circle worksheet using the rubric shown in Table 30. Work completed by the same 22 girls and 9 boys were used for each comparison. Paired t-tests and the Wilcoxon Signed Rank Test at the .05 level were first used to compare overall mean scores for the 31 students on the two assignments to determine whether there were statistically significant changes in mean scores from one worksheet to the next. The same tests at the .05 level were also used to compare mean scores of females on each assignment and to compare mean scores of males on each assignment to determine if there were statistically significant changes in performance by gender from one worksheet to the next. Two sample t-tests and the Wilcoxon Rank Sum Test at the .05
Table 29

*Problems 4, 5, and 6 from the Homework Assignment Which Were Used in the Analysis of Students' Abilities to Prove Conjectures*

| Diagram |

4. $\angle 6$ would be supplementary to which of $\angle 13$, $\angle 14$, $\angle 15$, or $\angle 16$? Outline a proof of your conjecture.

5. $\angle 6$ would be congruent to which of $\angle 13$, $\angle 14$, $\angle 15$, or $\angle 16$? Outline a proof of your conjecture.

6. Make a conjecture about the relationship between $\angle 9$ and $\angle 14$. Outline a proof of your conjecture.

Level were used to compare mean scores of females with mean scores of males on each assignment to determine if there were statistically significant differences in performance by gender. Where two-sample t-tests were used, tests for homogeneity of variance were first conducted. These tests showed no significant differences at the .10 level between the variances for males or for females on either the homework assignment or the circle worksheet.
Table 30

Rubric for Scoring the Justification given by Students to Problems on the Homework Assignment and Circle Worksheet.

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student made no attempt to justify the stated conjectures or merely restated the given information.</td>
</tr>
<tr>
<td>1</td>
<td>Student stated some valid facts related to the problem other than the given information but did not indicate that they knew how to justify the conjecture.</td>
</tr>
<tr>
<td>2</td>
<td>Student provided an indication that he or she realized the basic idea of how to justify the conjecture but made no attempt to adequately explain the reasoning.</td>
</tr>
<tr>
<td>3</td>
<td>Student showed that he or she knew the basic idea of how to justify the conjecture and made progress in putting together a logical argument but the argument was not complete.</td>
</tr>
<tr>
<td>4</td>
<td>Student provided sound and complete justification for his or her conjecture in some form.</td>
</tr>
</tbody>
</table>

Note. Students were told they could provide justification in whatever form they preferred and that justification did not have to be in the form of two column proofs.

Table 31 shows the means and standard deviations by group for each of the two assignments as well as the p-values for comparisons of mean scores of the homework assignment with the circle worksheet.

The only statistically significant differences in mean scores at the .05 level were in the comparisons of the homework assignment and circle worksheet for all 31 students and for female students. In both cases mean scores on the homework assignment given early in the year were significantly higher than mean scores on the worksheet given late in the year. On the surface, this would seem to indicate that these students’ proof-writing abilities declined over the year but there may be other factors which influenced this...
outcome. First, because the homework assignment was not given in class, students may have worked together on the assignment and individual scores may not truly reflect individual abilities. On the other hand, mean scores for all groups on both the homework assignment and the circle worksheet are very low indicating very marginal proof-writing ability for many students both at the beginning and at the end of the year. Mean scores are low because a number of students received a score of 0 according to the rubric in Table 30 for several of the problems. There are two possible reasons for this. As indicated earlier, many students seemed to have the attitude that they could not do proofs. This attitude continued for some throughout the year to the extent that when they saw that a problem asked for a proof or for justification of their conjectures, they would not make any attempt to do it. A second factor is attitude toward geometry overall. A number of students see
no reason for studying geometry, put no effort into it, and are willing to accept the consequences of this attitude.

Discussion of student responses. In spite of the rather dismal overall performance on proof problems, some students did make good progress in their abilities to justify conjectures and there are some interesting outcomes when individual student responses to the problems are analyzed.

Table 32 shows the numbers of male and female students who produced adequate proofs of their conjectures for problems 4, 5, and 6 on the homework assignment. In this case, adequate proof is defined to be a score of 3 or 4 according to the rubric in Table 30. As mentioned previously, because these problems were given as part of a homework assignment, it is likely that some students received help from others so the table may overstate the numbers of students who were capable of providing adequate proofs.

Table 32

Analysis of Homework Problems Showing Numbers of Female and Male Students Who Were Able to Provide Adequate Proof for Problems 4, 5, and 6

<table>
<thead>
<tr>
<th>Problem</th>
<th>Adequate Proof</th>
<th>Girls</th>
<th>Boys</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. ∠6 would be supplementary to which of ∠13, ∠14, ∠15, or ∠16? Outline a proof of your conjecture.</td>
<td>7 (32%)</td>
<td>3 (33%)</td>
<td></td>
</tr>
<tr>
<td>5. ∠6 would be congruent to which of ∠13, ∠14, ∠15, or ∠16? Outline a proof of your conjecture.</td>
<td>17 (77%)</td>
<td>3 (33%)</td>
<td></td>
</tr>
<tr>
<td>6. Make a conjecture about the relationship between ∠9 and ∠14. Outline a proof of your conjecture.</td>
<td>6 (27%)</td>
<td>1 (11%)</td>
<td></td>
</tr>
</tbody>
</table>

Note. Adequate Proof is defined to be a score of 3 or 4 according to the rubric of Table 30. *n = 22.  †n = 9.
Most students who presented a valid argument supporting their conjectures did so in the traditional two-column format even though they were told that they could use other formats. This format is emphasized by the textbook and, up to that point, had been the format used most in class. Some students merely repeated the given information as a proof while others seemed to write whatever definitions, postulates, or theorems they could remember regardless of whether they related to the problem or not. These responses could indicate a lack of understanding of the meaning of proof. Some seemed to know why their conjectures were true but were unable to present a complete argument. For example, on Problem 4, one girl who conjectured correctly that angle $6$ is supplementary to angles 14 and 15, wrote, "$6$ is the same as 13 and added to $6 = 180." $\

She may have meant that the measures of angles $6$ and 13 are the same and when the measures of angles 14 or 15 are added to that of angle $6$ the result is 180. For Problem 5, another girl wrote, "Angle $6$ is corresponding to angle 12; angle 12 is corresponding to angle 16; angle $6$ is congruent to angle 16." It seems reasonable in this case to assume she knew the applicable definitions and theorems but did not quite know how to put them on paper. For Problem 6, a girl wrote, "Take out $\angle 10$ and $\angle 13$ then $\angle 9$ and $\angle 14$ are supp." She seemed to realize that since lines $l$ and $m$ are parallel, you could just "slide" line $l$ over so that it is on top of line $m$ and then angles 9 and 14 together would form a straight angle and be supplementary. Some students gave proofs which indicated a solid understanding of how to use the applicable postulates, definitions, and theorems to support their conclusions.
Applying a proof to other diagrams. Although the proof problems from the similar triangle worksheet were not scored using the rubric of Table 30, they do provide some insights into students' thinking and understanding of proof. Problem 1 (see Table 13) of the similar triangle worksheet had been proven in class as shown in Table 33. Problems 2, 3, and 4 (see Table 26) asked students to state and prove conjectures regarding triangles much like those in the diagram for Problem 1 but in different positions. All could be proven by direct application of the theorem of Problem 1.

Table 33

Proof of Similar Triangle Worksheet Problem 1: If two sides of a triangle are cut by a line parallel to the third side, then two similar triangles are formed.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $DE \parallel AC$</td>
<td>Given</td>
</tr>
<tr>
<td>2. $\angle BAC \equiv \angle BDE$</td>
<td>If two parallel lines are cut by a transversal, then corresponding angles are congruent.</td>
</tr>
<tr>
<td>$\angle BDE \equiv \angle BCA$</td>
<td></td>
</tr>
<tr>
<td>3. $\triangle ABC \sim \triangle DBE$</td>
<td>If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. (Angle-Angle Postulate)</td>
</tr>
</tbody>
</table>

Table 34 shows results for these problems and also shows the numbers of students who used Problem 1 (Use #1) as the justification for their conjectures as well as the number who basically reproduced the above proof and used the angle-angle postulate (Use AA) as justification.

As can be seen from the Table 34, most students stated a valid conjecture for all three problems and also gave some indication of how to proceed with a proof of their
Table 34

*Methods of Proof Used by Students on Similar Triangle Worksheet Problems 2, 3, and 4*

<table>
<thead>
<tr>
<th>Problem</th>
<th>Stated Conjecture</th>
<th>Gave Supporting Argument</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Girls&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Boys&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>Use #1</td>
<td>Use AA</td>
</tr>
<tr>
<td>2</td>
<td>22 (96%)</td>
<td>15 (94%)</td>
</tr>
<tr>
<td>3</td>
<td>22 (96%)</td>
<td>14 (88%)</td>
</tr>
<tr>
<td>4</td>
<td>22 (96%)</td>
<td>13 (81%)</td>
</tr>
</tbody>
</table>

<sup>a</sup>n = 23.  <sup>b</sup>n = 16.

conjectures. Most conjectures involved stating which triangles would be similar although a few students stated that certain sides of the triangles would be proportional. The theorem in Problem 1 together with the fact that it had been covered in class just prior to the time that students completed the worksheet certainly gave the students some direction as to what conjecture to make. The results probably would not have been as good had the students been given only problems 2, 3, and 4 sometime later and asked to make any conjectures they felt were true. Nevertheless, it is encouraging that so many students seemed to understand the meaning of Problem 1 and could apply it to the other problems.

Most students did not use the theorem of Problem 1 as the basis for their proofs of problems 2, 3, and 4 but instead gave some indication that the method of proof would be by using the Angle-Angle (AA) Postulate. For some, this meant restating the steps of the proof for Problem 1 with the letters changed appropriately. Others stated something like the following for number two, "\( \angle A = \angle B \) and \( \angle D = \angle E \) because of corresponding angles so therefore it will make \( \triangle BCD \) similar to \( \triangle ACE \) because of the AA Postulate."
Others merely stated, "AA Postulate," or "Corresponding Angles." In some cases, the latter two responses probably do not indicate that the student understood how to prove the conjecture but only that he or she felt the proof would most likely follow the same pattern as the proof of Problem 1.

Those who rewrote the proof of Problem 1 with appropriate changes as their justification for problems 2 through 4 gave several reasons for doing so. One said that since he had to show his work, he thought more was required than merely saying that the others were true by the theorem in Problem 1. Others did not realize that, once proven, Problem 1 could be taken as a theorem and used in subsequent problems. When it was pointed out to one boy during the interview that he could use the theorem of Problem 1, he asked, "Oh, you don't have to go through the whole thing again?" A girl said she restated the proofs so that she would remember how to do the problem if she looked at it again a few weeks later.

The results of these problems indicate that most students did not have difficulty applying the theorem of Problem 1 to the different diagrams in problems 2, 3, and 4. They did not seem to believe that, since the diagram in Problem 1 appears to be an equilateral triangle, the theorem applies only to equilateral triangles. For Problem 2, one girl stated, "\( \triangle AEC \) is similar to \( \triangle BCD \) because of what was proven above. This triangle is just upside-down."

For most students, the right triangle in the diagram for Problem 4 did not cause any difficulties in applying the theorem of Problem 1. In the following interview excerpt,
the student seemed somewhat unsure but then gave a good reason for why the right
triangle makes no difference.

Researcher: The fact that this one is a right triangle, number 4, doesn't make any
difference?
Student: (Some hesitancy) No, because the Triangle Proportionality Theorem didn't say-
it said for all triangles.

Another student gave a very similar response:

Researcher: The example we used was not a right triangle but number 4 and number 6
were right triangles. Does that cause you any difficulty in applying this to right triangles?
Student: No, because the thing said that a triangle that has parallel sides, that it would be
similar.

The diagram for Problem 4 did cause difficulties for several students indicating that
they were either unsure as to whether they could apply the theorem of Problem 1 to it
directly or that they did not believe that they could. One of the boys stated the similar
triangle conjectures for problems 2 and 3 correctly and stated that both were true due to
the proof from Problem 1. He stated the similar triangle conjecture correctly for Problem
4 but gave inadequate justification for his answer. In discussing these problems in the
interview, the following exchange took place.

Researcher: So how did you come up with the conjectures on these?
Student: I don't remember.
R: Look at number one. Can you tell me how you came up with it?
S: This proof applies to these two because this one is almost exactly the same as that one.
That's why I said that. (The student is saying that the proof for Problem 1 applies to
problems 2 and 3 and that is why he wrote "Proof from #1" as his supporting argument
for these conjectures.) .... And this one (referring to Problem 4) had a right angle so it
couldn't be exactly the same as these two because neither one of these had right angles.
R: Okay, so when you look at this diagram (for Problem 1), you say it's not the same as
this one, the one with the right angle?
S: Yeah, they're not the same.
R: Because this one has a right angle?
S: Yeah, that’s why.

Problem 15 of the similar triangle worksheet (Table 19) asked students to show two similar triangles and support their conclusions. Of the 23 girls and 16 boys completing the worksheet, only 10 girls and 8 boys gave justification for their answers. One boy extended the two sides of the trapezoid until they met at a point and then wrote, “My conclusion is based on #1’s proof.” In the interview he said that after extending the two sides, it would be exactly the same as number one. Others gave similar explanations.

Problem 16 of the similar triangle worksheet is shown in Table 26. The idea behind the problem was that students would see that because $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, they could either apply the angle-angle postulate directly or see that the two line segments are parallel because corresponding angles are congruent and then apply the theorem of Problem 1 to arrive at a conjecture that two triangles are similar. Most students (16 of 23 girls and 15 of 16 boys) gave the similar triangle conjecture. The diagram on the worksheet did not include the labels of the points as shown in Table 26. After labeling those points as shown, a significant number (13 girls and 8 boys) also stated that line segments $AE$ and $BD$ are parallel and two girls added that corresponding parts of the triangles would be proportional. One student’s conjecture read, “If two sides of a triangle are cut by a line and corresponding angles are congruent then the line and the base of the triangle are parallel and the two triangles are similar.” In stating her conjecture this way, she combined the two conjectures about similar triangles and parallel lines and in a way which applied not just to the specific diagram given in the problem.
Most students correctly stated either the angle-angle postulate (9 girls and 12 boys) or the theorem from Problem 1 (2 girls and 2 boys) as justification for their conjecture that the two triangles are similar and most who conjectured that the two line segments are parallel gave the fact that corresponding angles are congruent as justification. This is not sufficient to conclude that the students understand proof but does show that they were able, in this situation, to state reasons for the truth of their conjectures. One girl stated a correct proportion conjecture and as justification stated, "If a line parallel to one side of a triangle intersects the other two sides, then it divides them proportionally." This is true, but it is not clear if she understood why the two line segments are parallel or if she was assuming they are because they look that way in the diagram.

Most students had difficulty with Problem 1 of Part I of the circle worksheet (see Table 26). One boy applied the theorem to each vertex of the quadrilateral and made congruency marks on the diagram to show which segments would be congruent. He then saw that both of the sums $AB + DC$ and $AD + BC$ had one of each type congruency mark and so reasoned that the sums must be equal. In an attempt to prove that the sums are equal, a girl labeled the points of tangency and realized that the theorem would apply to each separate point. She then assigned numbers to represent the segment lengths, assigning the same numbers to congruent segments. By adding the numbers assigned to the segments making up $AB + DC$ and the segments making up $AD + BC$, she found that the two sums were equal and so reasoned that they would always be equal. She apparently thought that verifying a relationship by using a specific example such as she...
used when she applied the numbers to the diagram was sufficient to prove the two sums
equal. She used a similar process on Problem 3 of part I (see Table 26).

Several students tried to apply the theorem of part I directly to the sides of the
quadrilateral to show that the two sums are equal as shown in the following excerpt from
an interview.

Researcher: You said, "AD is congruent to DC due to the theorem and AB is congruent
to BC also because of the theorem." Is that what the theorem says?
Student: No, it's basically saying that B to this point would be congruent to B to this
point.
R: To the point of tangency?
S: Yes. I guess I just applied this theorem wrong.

In many of the instances where students provided a correct conjecture but without
justification, they were able to produce the justification during the interview. The
following excerpt came from the interview with one of the girls and shows that she tried
initially to apply the theorem directly to prove sides of the quadrilateral congruent but then
realized how to apply it correctly.

Researcher: We had this theorem that said tangents to a circle from a point are congruent.
Then I asked you basically to apply that theorem in three different situations. On this one,
you said AB and DC are tangent to circle O but they are not congruent because they do
not meet at a point. Well, that's true, but if you look at the way you marked this, what
would be congruent?
Student: Like AD and AB, they would be congruent because they meet at point A.
R: Is that what the theorem says?
S: (No Response)
R: If you look at the way you marked these, see here's three (marks) and here's three
(marks), what did you mean by that? And what did you mean by these marks? And then
you labeled the points where they are tangent as X, N, Y, and F.
S: Yeah, they were congruent.
R: So this part....
S: ...and that part are congruent.
R: Okay, so you applied the theorem correctly. So you recognized that you could apply the theorem to those parts.
S: Yeah.

In Problem 3 of Part I of the circle worksheet (see Table 26), the majority of students stated the conjectures that point G is the midpoint of segment EF or that segments EG and GF are congruent but many may have based their conjectures primarily on the looks of the diagram. Numbers providing sufficient proof for this problem were low as with Problem 1. The most common error in justification was the statement that segments EG and GF are congruent because tangents to a circle from a point are congruent. These students did not seem to realize that the corollary applies only to tangents to the same circle and that it could not be applied to tangents to two different circles. Again, the more complicated diagram seems to have caused difficulty for some students.

In Problem 2 of Part II of the circle worksheet (see Table 26), most students who stated the correct conjecture but gave an invalid proof based their proof on lines NP and QM being parallel apparently basing that assumption on the appearance of the diagram. In Problem 3 of Part II of the circle worksheet, few students were able to provide sufficient justification to support their conjectures that angle RST is a right angle. For some, the conjecture may have again been based primarily on the appearance of the diagram. A much lower percentage of students gave a valid conjecture for Problem 4 perhaps because it is difficult to find a conjecture based solely on the appearance of the diagram.
Some students were able to provide good justification for their conjectures. When asked to justify his conjecture that the angles would be supplementary in Problem 4, one boy explained very succinctly that the sum of the arcs intercepted by the angles would be 360° because, “this arc right here plus this arc would equals the whole circle,” so the sum of the angles would be 180° making the angles supplementary.

Other Findings

During the course of the study, several findings not directly related to the three primary research questions became apparent to the researcher. These will be discussed in this section.

Student Group Presentations

As previously mentioned, during the month of April, students in both classes gave group presentations on theorems related to the areas of polygons (See Figure 10). As part of the presentation, students were to use the TI-92 to construct the polygons, measure appropriate line segments, and demonstrate the area formulas. Because students had little hands-on experience with the calculator, they were told that they could either operate the calculator themselves or give instructions to the teacher who would operate it.

Six of the eight students who chose to operate the calculator themselves gave remarkable demonstrations of use of the TI-92 considering their very limited experience in actually operating the calculator. They not only constructed the polygons but also used the measurement, calculate, and drag tools to demonstrate the formulas. As an example, the rhombus is one of the more difficult figures to construct accurately. One boy who was demonstrating the formula for the area of a rhombus on the calculator, used several
theorems previously covered by the class in his construction (see Figure 21). He first
drew a line segment AB to be one of the diagonals of the rhombus and then constructed
the line which is the perpendicular bisector of that segment because he knew the diagonals
of a rhombus are perpendicular bisectors of each other. He put a point C on the
perpendicular bisector and then drew line segments AC and BC to be two sides of the
rhombus. He knew these segments would be congruent because of the theorem which
says that any point on the perpendicular bisector of a segment is equidistant from the
endpoints of the segment. He then constructed one line through point B parallel to side
AC and another line through point A parallel to side BC. The point D where these two
lines intersect is the fourth vertex of the rhombus. After constructing the rhombus in this
way, he measured the lengths of the two diagonals using the measurement tool and then,
using the calculate tool, calculated the formula \( A = \frac{1}{2}d_1d_2 \) to get the area of the
rhombus. This was a student who normally showed very little interest in geometry and did
very little work.

Figure 21. Construction of a Rhombus
Other students who demonstrated formulas for the areas of other polygons did an equally thorough job. While these students probably chose to do the calculator demonstration because they felt confident in doing it, it demonstrates that they had learned a great deal about use of the TI-92 and were able to apply some of the theorems they had learned to construct the required figures. It also demonstrates for these students a good ability to work with diagrams.

**Student Explanations of Constructions**

In preparing for class, the teacher/researcher found that he often had to spend a good deal of time figuring out how to construct different figures on the TI-92 so that he could demonstrate a theorem to the class or conduct some other class activity. He realized that this would also be a good activity for students to help them overcome some of the obstacles associated with the use of diagrams because it forces them to think more deeply about the properties of the geometric figures they are studying. Normally when students draw a rectangle, for example, they hurriedly sketch a four-sided figure that may look something like a rectangle. Even if they use a straight edge and construct the figure neatly, they probably give little thought to the properties of the figure they are constructing. Requiring them to explain how they would construct a rectangle using the tools available on the TI-92 forces them to explain that both pairs of opposite sides must be drawn so that they are parallel and that consecutive sides must be perpendicular. There are several ways to construct a rectangle and other figures using the tools available on the TI-92 and this can lead to good discussion of various theorems. This activity also gives students practice explaining their thinking so that others can understand. The explanations
often give the teacher insight into student thinking and misperceptions which can then be discussed and corrected.

During the course of the study, students were given several assignments in which they had to explain how to construct various geometric figures. The following discussion provides examples of some student construction methods and benefits of using construction problems such as these.

**Homework assignment.** One set of construction problems was given to students on March 20, 1997 as a homework assignment. Students had time to work together in class on it and most did so. The first problem stated: Given a circle, tell how to circumscribe a pentagon about it. The most common solution was to draw a circle, draw five radii of the circle, and then draw lines perpendicular to each of the five radii at the points where the radii intersect the circle. Some students merely said to draw lines at the endpoints of each of the five radii. Using the TI-92 in class, it was shown that lines which look like they touch the circle in only one point, i.e. which look like tangent lines, usually touch in more than one point and usually do not form right angles with a radius. This can be used to emphasize the point that each side of a circumscribed polygon is tangent to the circle and that a tangent line is perpendicular to the radius drawn to the point of tangency.

Some students said that the five points had to be equidistant from each other so it was pointed out that the instructions did not say that the pentagon had to be regular. Students who said that the five points should be equidistant from each other did not include instructions for how to find those five points. This gave the opportunity to discuss how that might be done.
Some students gave instructions which work but which include unnecessary steps. One boy said to first draw a circle with two perpendicular diameters and then draw a radius of the circle which makes a 45 degree angle with one of the diameters. (He did not tell how to do that.) He then said to draw tangents to the endpoint of each radii using the perpendicular lines tool. One girl said, “Draw two segments, one on the top and one on the bottom, perpendicular to the circle (and) then draw two other lines on the other sides parallel to the circle.” Instructions such as this give a lot of insight into what the student understands and does not understand. The teacher can ask the student what she means by a segment perpendicular to a circle—perhaps she means perpendicular to a diameter at its endpoints. One boy who knew how to do the construction wrote, “Draw 5 different radii spread around the circle and make 5 perpendicular lines.” This can be used to show him that, even though he knows what he wants to do, his instructions are not thorough enough to tell someone else how to do it. Some students confused the terms “circumscribed” and “inscribed” and told how to construct an inscribed circle. This activity let the teacher know of this confusion so that it could be corrected.

The second problem stated: Given a right triangle, tell how to inscribe it in a circle. The class had previously discussed the theorem which states that the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices. Most students applied this theorem and said to first find the midpoint of the hypotenuse and then draw a circle with that point as center and the distance from that point to one of the vertices as radius. Because of the theorem, the circle would go through all three vertices of the triangle. Application of that theorem to explain how to do this construction requires
students to put together several pieces of information which might seem rather straightforward to a more experienced observer but which often are not so obvious to high school geometry students. They must understand that all points on a circle are equidistant from the center. They must realize that the midpoint of the hypotenuse being equidistant from all three vertices means that the distance from that point to any vertex is the same as the distance from that point to any other vertex. Finally, they must combine these pieces of information to realize that, if they use the midpoint of the hypotenuse as the center of a circle and the distance from that point to any vertex as radius, the circle will have to go through the other two vertices. The ability to combine information in this way to reach a conclusion is crucial to doing not only geometry but mathematics in general.

Some students said to find the midpoint of the hypotenuse and use it as the center of the circle but then said to “expand the circle until each vertex touches the circle.” This would indicate that they realized that the midpoint of the hypotenuse would be the center of the circle but did not realize that all they had to do was use the distance from that point to any vertex as radius and the circle would have to go through the other vertices. Several students said to find the midpoint of the hypotenuse and then to “draw a circle using the midpoint as the centerpoint.” They did not tell what to use as radius. This points out that constructions such as these can be good exercises in getting students to thoroughly explain how to do something.

In answering this problem, some students said to first draw a circle, then construct two perpendicular lines which intersect the circle so that the right angle is on the circle. Then draw the segment which is the hypotenuse. The is not a correct solution because the
statement of the problem started with, "Given a right triangle." This can be used to emphasize to students that starting with a specific circle and inscribing a right triangle in it is not the same as starting with a specific right triangle and circumscribing a circle about it. Other students also started with the circle but then said to construct two perpendicular radii and then draw the hypotenuse. They apparently did not understand the meaning of an inscribed polygon. Another girl said to draw a circle, put three points on it, connect two of the points, and then connect those to the third point so that they form a 90 degree angle. She did not explain how to insure that they would form a 90 degree angle.

**Semester examination.** As part of the semester exam, students were given two construction problems similar to the ones above. One asked students to explain how to construct a kite using the tools available on the TI-92. The most common and perhaps easiest explanation of how to construct a kite used the theorem that one diagonal of a kite is the perpendicular bisector of the other. One girl explained the construction as follows:

> Using the tools available on the TI-92, a kite can be drawn by first drawing a segment vertically (it doesn't have to be vertical) across the screen. Then, construct the perpendicular bisector of the segment. Pick two points on the perpendicular bisector (on opposite sides of the segment) and connect the endpoints of the segment and two chosen points on the perpendicular bisector to form a kite.

Many other students used this basic method although some were not as thorough in their explanations. Others basically did the same construction with one major flaw. They started with the segment but then constructed a perpendicular line to that segment instead of the perpendicular bisector. The quadrilateral obtained by connecting the points would not necessarily be a kite. This is an easily corrected error which gives the opportunity to
explain the difference between perpendicular lines and the perpendicular bisector of a line segment and why, in this case, one must use the perpendicular bisector.

Several students used a circle in their construction of a kite. The circle insures that one pair of consecutive sides are congruent. One student’s explanation was as follows:

1. Make a line.
2. Make a line perpendicular to that line.
3. Label the intersection point.
4. Make a circle with the intersection point the center.
5. Make a segment joining the perpendicular line to the original line but on the circle.
6. Make another segment on the bottom joining the perpendicular line to the original, but on the circle to the segment point.
7. Follow instructions five and six for other side, but make segments extend past the circle on the original line.

This construction is shown in Figure 22.

![Figure 22. One Method of Constructing a Kite](image)

Two other methods were used which, if thoroughly explained, would be valid.

One involved placing two isosceles triangles base to base. Students who used this method normally did not explain how to construct the isosceles triangles or how to put their bases together. This could easily be done using two intersecting circles with different radii. The
other method involved first constructing a rectangle, which was explained in a previous problem, then constructing the perpendicular bisector of one side of the rectangle. Another line through the rectangle perpendicular to the other two sides (but not the perpendicular bisector) is then drawn. Connecting the points where the two additional lines intersect the rectangle results in a kite. This method is shown in Figure 23. Overall, there were a number of valid methods of constructing a kite. While some are more laborious than others, they show good understanding of some geometric concepts and the ability to apply those concepts together to achieve the desired result.

![Figure 23. Constructing a Kite Using a Rectangle](image)

Construction problems such as those discussed above show whether or not students really understand the characteristics of various geometric figures and provide the teacher with a good opportunity to see where the student might have misunderstandings so that those misunderstandings can be cleared up before they cause further difficulties including difficulties with the use of diagrams. These construction problems can be fairly simple or very complicated and thus provide the opportunity for students to work with a
variety of types of diagrams. Continued exposure to more complex diagrams which they
determine how to construct could help students overcome several of the difficulties
associated with the use of diagrams. In particular, it should help students overcome
hesitancy to add to diagrams and also help them to learn to look at the whole diagram as
well as its various parts.

Because many of the construction problems can be done in more than one way,
they provide good opportunity for students to apply a variety of previously learned
definitions and theorems, lead to some lively class discussion, and provide a challenge for
students to find a more simple construction method than other students or the teacher.
They also give students practice in writing instructions so that others can understand and
follow those instructions to construct the appropriate figure.

Student Attitudes Toward geometry

Many students do not consider the study of geometry important and take it only
because it is required. Many seem to dislike geometry intensely and are very vocal about
that. This includes some who dislike math in general but also some who claim that they
like other math courses they have been in, primarily algebra I. Some claim that they see a
reason for taking algebra—that they see some application to their future needs—but they
see no reason for taking geometry. Some students claim that this is the reason that they
put forth so little effort in geometry. They only want to get through the course so that
they can move on to other courses with more real-world applications. Many students
seem content to get by with C’s and D’s in geometry when they are capable of getting A’s
and B’s. Most have given little thought to what they might learn from the study of
geometry and do not seem to realize that failure to exert effort in geometry could jeopardize their future plans.

Problems with the Textbook

The textbook book can be a hindrance in helping students to become conjecturers particularly when the sequence of material as presented in the text must be followed fairly closely. On one worksheet, students were asked to draw parallel lines with several transversals, measure corresponding, alternate interior, and same-side interior angles and then to conjecture about the relationships among these angles. Rather than trying to do any serious thinking about what the relationships could be, many students just looked ahead in the book to see what the theorems were and then wrote those as their conjectures. This continued throughout the year and, as a result, some students got little experience with actually developing a conjecture on their own and so never learned what it means to develop a conjecture. Even in a situation where the sequence of material in the textbook must be followed fairly closely, it is necessary to spend adequate time on topics not covered in the text to help students become conjecturers. In the present study, the teacher/researcher presented a series of lessons on kites, a topic not covered in the textbook the class was using.

Use of Loaned Calculators

Use of the loaned calculators for one week early in the study period was a very worthwhile and necessary part of the study because it gave the students personal experience in using the calculator which they could draw upon to help understand what
was happening when the teacher used the calculator in front of the whole class to explain or demonstrate a theorem or concept later in the year.

This hands-on experience with the calculators helped some students grasp important ideas. Some realized the value of seeing a number of examples illustrating a theorem or concept. Use of the calculators helped students understand the idea of conjecturing and a number seemed to feel that they had a better idea of how to generate a viable conjecture after using the calculators for a week. Use of the calculators helped some students realize that different diagrams could be used to reach the same conclusion. Even though the week in which students had hands-on experience with the calculator occurred early in the study period, some students developed more interesting and more complex conjectures during that week than later in the study when they did not have personal access to the calculator.

Summary of Findings Reported in Chapter Five

Table 35 summarizes the significant statistical findings reported in Chapter 5. The preliminary worksheet, similar triangle worksheet, homework assignment, and quiz were all completed early in the study period, whereas the Pythagorean Theorem and circle worksheets were completed late in the study period. In the comparisons of student abilities to apply definitions, postulates, or theorems to diagrams, the higher mean scores on the Pythagorean Theorem worksheet as compared to the similar triangle worksheet for all students indicates an increased ability to use diagrams. This could indicate that use of the TI-92 with Cabri Geometry in a whole-class setting had a positive impact on students' abilities to overcome obstacles associated with the use of diagrams. Other factors such as
### Table 35

**Summary of Significant Statistical Findings Related to Research Questions 1 and 2**

<table>
<thead>
<tr>
<th>Category</th>
<th>Comparison</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Applying definitions, postulates, or theorems to diagrams</strong></td>
<td></td>
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<tr>
<td></td>
<td><strong>Similar ∆ - Pythagorean Theorem Worksheets for all students</strong>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Scores were significantly higher on the Pythagorean Theorem Worksheet than on the Similar ∆ Worksheet.</td>
</tr>
<tr>
<td></td>
<td><strong>Similar Triangle - Pythagorean Theorem Worksheets for males</strong></td>
<td>Scores were significantly higher on the Pythagorean Theorem Worksheet than on the similar triangle Worksheet.</td>
</tr>
<tr>
<td></td>
<td><strong>Preliminary - Pythagorean Theorem Worksheets for Males</strong></td>
<td>Scores were significantly higher on the Pythagorean Theorem Worksheet than on the preliminary worksheet.</td>
</tr>
<tr>
<td></td>
<td><strong>Males - Females on the Pythagorean Theorem Worksheet</strong></td>
<td>Males scored significantly higher than females on the Pythagorean Theorem Worksheet.</td>
</tr>
<tr>
<td></td>
<td><strong>Quiz/simple problems - Quiz/complex problems for all students</strong>, males, and females</td>
<td>All groups scored significantly higher on the simple problems than on the more complex problems.</td>
</tr>
<tr>
<td></td>
<td><strong>Developing viable conjectures</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Similar ∆ - Preliminary Worksheets for all students</strong>, males, and females</td>
<td>Scores were significantly higher on the Similar ∆ Worksheet than on the Preliminary Worksheet for all groups.</td>
</tr>
<tr>
<td></td>
<td><strong>Similar ∆ - Circle Worksheets for all students</strong>, males, and females</td>
<td>Scores were significantly higher on the Similar ∆ Worksheet than on the Circle Worksheet for all groups.</td>
</tr>
<tr>
<td></td>
<td><strong>Preliminary - Circle Worksheet for females</strong></td>
<td>Scores were significantly higher on the Circle Worksheet than on the Preliminary Worksheet.</td>
</tr>
<tr>
<td></td>
<td><strong>Providing justification for conjectures (Proof)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Homework Assignment - Circle Worksheet for all students</strong> and for females</td>
<td>Scores were significantly higher on the Homework Assignment than on the Circle Worksheet.</td>
</tr>
</tbody>
</table>

<sup>a</sup>α = .05. <sup>b</sup>The term “all students” refers to comparisons in which male and female students were grouped together as opposed to comparisons of males to males or females to females.

187
differences in the specific problems on the worksheets and exposure to other teaching methods during the course of the study could also have had a positive influence on students' abilities to use diagrams. Because males also scored significantly higher on the Pythagorean Theorem worksheet than on either the similar triangle worksheet or the preliminary worksheet and significantly higher than females on the Pythagorean Theorem worksheet, use of the TI-92 in this manner could have had a larger positive impact on males than on females.

In this same category of problems, all comparisons of the simpler quiz problems with the more complex problems indicated that mean scores were significantly higher for the simpler problems than for the more complex ones. Perhaps this is to be expected, but the important point is that some students who demonstrated understanding of the concepts by applying them correctly to the simpler diagrams had difficulty applying the same concepts to the more complicated diagrams. These students apparently had difficulties in interpreting and using the diagrams properly.

In the groups of problems dealing with developing conjectures, there were mixed results. Comparisons for all students, for males, and for females showed higher mean scores on the similar triangle worksheet than on the preliminary worksheet and on the similar triangle worksheet than on the circle worksheet. As previously discussed, this may be more a factor of the type of problem on each worksheet than an indication that conjecturing ability decreased as the study progressed. The problems on the similar triangle worksheet for which students were asked to make conjectures required the application of a specific theorem which had been covered in class just prior to completion.
of the worksheet. This certainly could have been a factor in students performing well on that worksheet. Comparisons of conjecturing problems from the preliminary and circle worksheets is likely a more valid indication of increased conjecturing ability. In this case females scored significantly higher on the circle worksheet than on the preliminary worksheet. There was not a corresponding significant increase for males. This could indicate that calculator use had a more positive impact on females in this category of problems than on males.

As mentioned previously, for proof problems, the significantly higher scores on the homework assignment as compared to the circle worksheet may have occurred at least in part because students collaborated with each other on the homework problems whereas the worksheet problems were completed by individual students during class.

The results discussed above along with the results of the qualitative portion of the study are summarized below.

- At the beginning of the study, students exhibited the difficulties with diagrams discussed by Yerushalmy (1993). They had considerably more difficulty answering questions related to more complicated diagrams even when they had previously demonstrated an understanding of the concept in a question related to a simpler diagram.

- As the year progressed, a number of students were able to overcome some of the difficulties with diagrams including difficulties relating a specific diagram to a family of figures.

- Most students could apply a theorem proven for one diagram to other simple diagrams but, as the diagrams became more complicated, many had difficulty focusing their attention on specific parts of the diagram and were not able to apply the theorem correctly. This continued for some throughout the year. Late in the year, a number of students were able to use diagrams related to the Pythagorean Theorem correctly, interpret word problems, draw accurate diagrams, and develop correct solutions.
Some students had no hesitancy in adding lines or other figures to the diagrams given in the text or by the teacher but others students continued to be uncertain throughout the year as to whether it was permissible for them to do so.

Even with little hands-on experience with the TI-92, some students became remarkably proficient in its use and were able to use it to construct and manipulate complicated diagrams. Some students who show little interest in geometry otherwise, show a good deal of interest and ability when using the calculator.

Asking students to write paragraphs explaining how they would use the tools available on the TI-92 to construct various geometric figures forces students to think more deeply about the characteristics of geometric figures, forces them to apply theorems and definitions they have learned, exposes misconceptions they have, and helps them overcome obstacles to the use of diagrams.

At the beginning of the year, students had difficulties in producing viable conjectures. This was particularly true when they were given only a diagram and asked to make conjectures about it. This continued for some students throughout the year.

Conjecturing ability improved for some students as the year progressed. Students were better able to produce viable conjectures when asked to conjecture about the relationship between specific parts of a diagram. The difficulty of the diagram seems to hinder many in applying known theorems or corollaries to a diagram to develop a conjecture.

Conjectures are often based on the appearance of the diagram. This was true at the beginning of the school year and continued for a number of students throughout the year. This could indicate a problem with separating properties of the specific diagram with those of the family of diagrams it is intended to represent.

At the beginning of the year, most students had difficulty with proof. Some were able to prove fairly simple statements but many seemed not to comprehend what is meant by proving a statement in mathematics. This continued through the year for many students but, toward the end of the year, a number were able to at least indicate an approach to a proof and several showed good understanding of proof for students at their stage of mathematical maturity.

With some individual attention, such as in an interview, some students were able to figure out things on their own that they previously had not been able to do or not taken time to do. This applies to the use of diagrams and also to conjecturing and proof.
- Students seem to understand that the TI-92 quickly produces a number of examples that can be used to test relationships. Even at the end of the year, some students believe that when a relationship holds for many examples demonstrated on the calculator, then the conjecture is proven.

- Negative student attitudes toward geometry adversely affect their performance in class. A number of students, including some who do very well in other classes, are vocal in their dislike of geometry, state that they see no reason for taking it, and put forth little effort to do well in the class. They seem willing to accept the consequences of a low grade in geometry rather than put forth the mental effort necessary to develop viable conjectures or proofs.

- The textbook can be a hindrance in students becoming conjecturers. To help students become conjecturers, some work from sources other than the text is necessary so that they cannot rely on the book for their conjectures instead of thinking on their own.

- Use of loaned calculators for one week early in the study was essential to giving students a basis for understanding and following what the teacher was doing when he used the calculator throughout the remainder of the study for whole class demonstration and discussion.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

This chapter provides a brief overview of study methodology, relates the findings of the quantitative and qualitative portions of the study to each of the research questions, and provides conclusions and recommendations.

Methodology

The study was conducted in two geometry classes taught by the researcher during the 1996-97 school year in a large high school in the southwestern United States. Students had personal access to TI-92 calculators with Cabri Geometry for one week early in the study. After that, one TI-92 was used extensively in whole class demonstration and discussion to introduce and investigate topics in geometry. Attitude surveys were conducted at the beginning and again at the end of the study period to determine whether there were changes in students' attitudes toward a number of factors related to mathematics as a result of use of the calculator and to determine if there were gender-based differences in attitudes. Instruments designed to assess students' abilities to use diagrams and to develop and prove conjectures were administered several times during the course of the study to examine the impact that frequent use of the TI-92 with Cabri
Geometry in a whole-class setting might have on student progress in these areas. Results were also analyzed to determine if there were gender-based differences in student progress which might be attributed to calculator use. Results of analyses of the attitude survey are reported in Chapter 4. Results of analyses of students’ abilities to overcome obstacles associated with the use of diagrams are reported in Chapter 5. (See Tables 14, 17, 21, 28, 31, and 35.)

Research Question One

Are high school students in a geometry class taught using interactive software only in a whole-class setting able to overcome the obstacles stated by Yerushalmy (1993) to the effective use of diagrams? Are there gender-based differences in students’ abilities to overcome such obstacles when the software is used in this manner?

These obstacles relate to abilities to distinguish between characteristics of a specific diagram used to illustrate a concept and characteristics of the class of figures represented by the specific diagram, abilities to identify geometric figures in a variety of positions, abilities to view geometric diagrams in a variety of ways, and reluctance to add to given diagrams. Students’ responses to the preliminary instruments showed that many students exhibited some difficulties in all of these areas. Many incorrectly attempted to apply characteristics of diagrams used to illustrate theorems to the class of objects represented by the diagrams. Many who could recognize and apply theorems to a figure shown in one position had difficulties recognizing the same figure when shown in different positions. A number who could recognize and apply theorems to a figure shown by itself or in a relatively simple diagram had difficulties recognizing the figure and therefore
difficulties in applying theorems when the same figure was part of a more complicated diagram. Some students either felt that they were not allowed to modify diagrams accompanying a problem or showed hesitancy to do so.

As the study progressed, these obstacles continued to be a problem for some students but others made progress in overcoming them. On the similar triangle worksheet completed in February many students still exhibited these difficulties. Most could apply a theorem proven for one diagram to other diagrams but, as the diagrams became more complicated, many had difficulty in focusing their attention on specific parts of the diagram. Most students added lines to some diagrams but showed reluctance to do so on others and a few stated they did not know if they were allowed to do so. On the Pythagorean Theorem Worksheet, given in May, most students were able to use diagrams correctly on relatively simple problems but still had difficulties on more complicated diagrams. Most added line segments where needed to complete problems and a substantial number of students seemed able to interpret word problems correctly, draw accurate diagrams, and determine correct solutions. As with other worksheets, students had more difficulty with the more complicated diagrams.

On all of the worksheets, a number of students seemed to base their conjectures and conclusions primarily on the appearance of the given diagrams and often placed what appeared to be true based on the looks of the diagram ahead of any deductive reasoning and, at times, even ahead of what they were given to be true in the particular problem. This indicates a continuing difficulty with distinguishing the characteristics of the diagram accompanying a problem with the class of figures that diagram represents. Hands-on use
of the calculators helped some students to realize that there are essential characteristics of a diagram to which a theorem or problem applies but many different diagrams can have those essential characteristics.

A number of students exhibited good proficiency toward the end of the study period in constructing and manipulating diagrams as shown by their ability to construct fairly complicated figures using the TI-92 calculator even though they had very limited hands-on experience with it. The use of construction problems which require students to tell how to construct certain geometric figures using the tools available on the calculator also helped them to overcome these obstacles by requiring them to think more deeply about specific characteristics of those figures.

On problems comparing abilities to apply definitions, postulates, or theorems to diagrams, students scored significantly higher on the Pythagorean Theorem worksheet completed late in the study period than on the similar triangle worksheet which was completed earlier in the study period. This could indicate an increased ability to use diagrams and that use of the TI-92 with Cabri Geometry in a whole-class setting had a positive impact on students’ abilities to overcome obstacles associated with the use of diagrams. Males also scored significantly higher on the Pythagorean Theorem worksheet than on either the preliminary worksheet which was completed early in the study period or the similar triangle worksheet and significantly higher than females on the Pythagorean Theorem worksheet. This could indicate that use of the TI-92 for whole-class demonstration and discussion had a larger positive impact on males than on females in overcoming the obstacles associated with the use of diagrams. Other factors such as
differences in the specific problems on the worksheets and exposure to other teaching methods during the course of the study could also have had a positive influence on students' abilities to use diagrams.

On the attitude survey, mean scores for males on all four items related to the use of diagrams increased from the beginning to the end of the year indicating a better understanding of the proper use of diagrams. For females, scores increased for two items but decreased for the other two.

Research Question Two

Are high school students in a geometry class taught using interactive software only in a whole-class setting able to form conjectures and produce convincing arguments supporting their conjectures? Are there gender-based differences in students' abilities to develop and prove conjectures when the software is used in this manner?

On the preliminary instruments, students exhibited difficulties in developing viable conjectures particularly when given a diagram and asked to list all conjectures which they think might be true. They did better when asked to conjecture about relationships among particular parts of a given diagram. Early in the year, a few students were able to write simple proofs but the vast majority showed a lack of understanding of the concept.

Hands-on experience with the calculator for one week helped some students to gain a better understanding of conjecturing. During that week, many students developed more interesting conjectures which required deeper thought than they did later in the year without the calculators when many stated only obvious truths which were the direct result of definitions or theorems.
On the similar triangle worksheet completed in February, many students were able to make conjectures when asked to conjecture about the relationship among specified parts of a diagram, many were able to give some indication of the basis of a proof, but few were able to develop solid proofs. The difficulty of the diagram seemed to hinder many in applying known theorems or corollaries to a diagram to develop a conjecture.

On the circle worksheet given in April, more students seemed to understand the value of having a number of examples which can be used to test relationships. Many were able to develop conjectures when asked to conjecture about relationships among specific parts of a diagram. Although some were able to give a basis for proof of their conjectures, many still showed little understanding of what it means to prove a statement in mathematics. This is in agreement with statements by Battista and Clements (1995) and Hirschhorn and Thompson (1996). A number of students still think that showing that a relationship holds for a number of examples constitutes proof, an attitude which continued for some students to the end of the year. A few students showed good understanding of proof for high school geometry students and were able to produce good proofs of their conjectures. This is in agreement with the findings of Gordon (1993) and Yerushalmy and Houde (1986) that some students using interactive software become good geometers.

Many students base their conjectures on the looks of the diagram and many were content to give obvious conjectures which followed directly from definitions or theorems. This could be the result of a lack of real understanding of what it means to conjecture and shows the need to explore this topic more deeply. By requiring students to think more
deeply about relationships among parts of a geometric figure, the construction problems
can help students become better conjecturers.

On worksheet problems dealing with developing conjectures, females scored
significantly higher on the circle worksheet than on the preliminary worksheet. There was
not a corresponding significant increase for males. This could indicate that calculator use
had a more positive impact on females in this category of problems than on males.

On proof problems, students as a whole and female students as a group scored
significantly higher on the homework assignment given early in the study period than on
the circle worksheet. This may have occurred at least in part because students
collaborated with each other on the homework problems and may not be an indication that
proof-writing abilities decreased over the course of the study. In general, proof-writing
abilities were very low both at the beginning and the end of the study. Part of the reason
for this may be due to students’ attitudes toward proof problems.

Statements on the attitude survey which pertain to conjecturing and proof show
mixed results. Statements such as, “Math problems have one right answer;” or, “If I can’t
solve a math problem in 5 minutes, there is no sense in working on it any longer;” and “To
solve a math problem you have to use a rule you memorized earlier,” generally run
contrary to the nature of conjecturing and proof. On these statements, mean scores
decreased from the beginning to the end of the year in all but one instance. This indicates
a less positive attitude toward conjecturing and proof. On the other hand, mean scores
increased somewhat on the statement, “There is only one way to solve a math problem and
the teacher should just tell us how to do it.” This could indicate an understanding that
different students can come up with different conjectures and different methods of proof for the same problem. Mean scores increased for both males and females on the statement, “I can work out a proof by thinking logically,” which could indicate improved confidence in their abilities to produce proofs. The increase for males was considerably greater than that for females.

Other Results of the Attitude Survey Instrument Related to Gender Differences

On the Fennema-Sherman (1976) portion of the attitude survey instrument, the only significant difference found between male and female students was on the Mathematics as a Male Domain scale. Results showed that both at the beginning and end of the study, females viewed mathematics as less of a male domain than did males. Scores for females were grouped closely together at the high end of the scale both at the beginning and at the end of the study indicating strong expressed views that mathematics is not a male domain. Mean scores for males were well above the neutral point indicating that males also did not view mathematics as a primarily male domain. Because significant differences between the scores of females and males existed both at the beginning and at the end of the study, there was no change which could be attributed to calculator use in the classes. Other parts of the study indicate that the females may not always act in accordance with these strongly expressed views. This is a difficulty in attitude surveys discussed by Mueller (1986). For example, in the group projects, the fact that in five of six mixed gender groups, males did the calculator portions of the presentations may indicate that the females allowed the males to do what they felt was a “male activity.” On the attitude survey, males scored considerably higher than females at the end of the year.
on the statement, "I enjoy the challenge of answering questions in math class," which could indicate that females are less comfortable with doing something traditionally considered to be more of a male role.

The only significant change from the beginning to the end of the study was for males on the variable "Attitude toward Success in Mathematics." This would indicate an improved attitude on the part of males toward being recognized by peers as good mathematics students and could be the result of use of the calculator. There was no corresponding significant change for females on this scale.

For the researcher-developed items on the attitude survey instrument, there were considerable differences in the mean responses of males and females on some items. On the statement, "If I can't solve a math problem in 5 minutes, there is no sense in working on it any longer," mean score for females was considerably higher than that for males at the end of the study. On items related to classroom discourse such as, "Math class is more interesting when we discuss what we are learning," or "In math class a student should only have to sit and listen," and "Working in groups with other students helps me to understand math," mean scores for males were considerably higher than scores for females at the end of the year indicating that males more than females prefer discussion and working with others in math class.

Conclusions

Based on the above discussion, the results of the study indicate that use of the TI-92 with Cabri Geometry software for whole class demonstration and discussion throughout the second semester of the school year had mixed results in helping students to
overcome difficulties with diagrams and in helping students to become conjecturers and to be able to provide justification for their conjectures.

Statistical analysis of worksheet results indicates that use of the software in this manner may have had a positive impact on students' overcoming the obstacles to the use of diagrams identified by Yerushalmy (1993) with a larger impact on males than on females and may have had a positive impact on students' willingness to modify the diagrams they are given. In regard to conjecturing, results indicate that calculator use may have had a more positive impact on females than on males.

Some students continued to exhibit difficulties with the interpretation and use of diagrams throughout the semester and some continued to have difficulty in producing conjectures and valid proofs of their conjectures at the end of the study. On the other hand, a number of students seem to have overcome some of the difficulties associated with use of diagrams and to have developed an understanding of what it means to conjecture and what it means to prove a statement in mathematics. While these improvements may not have occurred solely due to use of the calculator and interactive software in the classes, it appears that some of the improvement was due to students' continued exposure to use of the calculator to explore geometric concepts. These findings agree with those of Yerushalmy (1993) and Yerushalmy and Chazan (1993) in studies where Geometric Supposer software was used in situations where students had access to the software either individually or in pairs. The inability of students to develop comprehensive proofs after a year of high school geometry is well-documented (Battista and Clements, 1995; Hirschhorn & Thompson, 1996).
There were some differences in the results for female and male students as discussed above. Use of the calculator may have had an impact on some of these differences. The improved attitude of males toward recognition as good mathematics students could be the result of viewing the ability to use a complex calculator as a positive factor in the eyes of their peers. Because the ability to use such a tool has been viewed as more of a male than a female role, females may not view the ability to use the calculator as a positive factor. This is in agreement with research by Brunner (1992) and Nelson and Watson (1991) which found gender differences in the way the use of technology is viewed. Use of interactive software to construct geometric figures on the TI-92 requires visualization skills. If males possess greater visualization skills than do females as indicated in research reviewed by Clements and Battista (1992), then greater improvement in the ability to use diagrams correctly for males than for females could be the result of the males’ greater visualization abilities. This apparently does not apply to all students. In the present study, the four girls who did the calculator portions of the group projects in the all-female groups were judged by the researcher to be among the top six students in the two classes in ability to use the calculator and two of the girls were judged to be the best in the two classes.

The Most Important Results

In the opinion of the researcher, the two most important results of this study are:

(a) identification of the overwhelming extent to which students base their problem solutions on the appearance of a given diagram, and
(b) the demonstrated value of problems in which students must explain how to construct geometric figures using tools available on the TI-92.

**Solutions based on appearance.** In all parts of the study, whether dealing with students’ abilities to interpret and use diagrams or their abilities to produce conjectures and arguments supporting those conjectures, results showed that a substantial number of students placed the appearance of the diagram above everything else—above deductive reasoning and even above given information—in developing a problem solution. This tendency seems to have decreased for some students as the year progressed, but instruments showed even at the end of the study that a number of students based their conclusions primarily on the appearance of the given diagram.

One consequence of this is the placing of unnecessary restrictions on conjectures or theorems. This relates to the first of Yerushalmy’s (1993) stated obstacles to the use of diagrams. For example, problem 3 of Part II of the circle worksheet given in April, asked students to conjecture about the relationship between angles PNM and PQM in the diagram shown in Figure 24 and then to justify their conjectures. Most conjectured that the two angles are congruent but many assumed that line segments NP and QM are parallel because they appear to be in the diagram. When used in conjunction with the theorem that the measure of an inscribed angle is equal to one-half its intercepted arc, the diagram in Figure 24 illustrates that two inscribed angles which intercept the same arc are congruent. The assumption that the two segments are parallel unnecessarily limits the theorem to a special case.
The consequences of the tendency to base conclusions on the appearance of a diagram goes beyond limiting the scope of the theorem. For example, after assuming that the two segments in Figure 24 are parallel, many students then tried to use the facts they had learned about various types of angles associated with parallel lines to justify their conjecture that the two angles are congruent. A number even erroneously stated that the angles are congruent because they are alternate interior angles or corresponding angles of parallel lines cut by a transversal. By leading their search for justification into unproductive areas, the assumption that the segments are parallel prevented them from pursuing a more productive course in spite of the fact that the definition of an inscribed angle and the theorem stating that the measure of an inscribed angle is equal to one-half the measure of its intercepted arc were given on the same worksheet just above this problem. Their conclusion based on the appearance of the diagram prevented their finding valid justification for their conjecture—even though the basis for valid justification was right in front of them.

Figure 24. Circle Worksheet, Problem 3, Part II: Make a conjecture regarding the relationship between $\angle PNM$ and $\angle PQM$. 

![Diagram of a circle with segments NPM and QPM]
Because so much of the students' work is based on the appearance of the diagrams they are given, students must be given problems in which they cannot draw conclusions from appearance. An extreme example discussed earlier is problem 13 of the preliminary worksheet which is repeated below.

13. In the figure below, D is the midpoint of line segment BE. Which line segments are congruent?

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A B C D E F G H
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In this problem, D does not look like the midpoint of line segment BE. Such problems may be necessary if students are to learn to base their conclusions on logic rather than appearance. Less radical examples such as problem 4 of Part II of the circle worksheet (see Table 25) could also be used.

In journal and interview comments, several students stated that the week of hands-on experience using the calculator and the almost daily use of the calculator in a whole-class situation by the teacher to illustrate theorems and investigate problems helped them to realize that the calculator quickly and easily produces a number of examples of geometric figures to which conjectures apply. When using the calculators individually, they realized that different students were reaching the same conclusion through different diagrams. Seeing a number of examples of diagrams to which a concept applied helped some to begin to distinguish between the essential and nonessential characteristics of a diagram.

The value of construction problems. Requiring students to explain in detail how to construct both simple and complex geometric figures using the tools available on the TI-
92 can have a number of important benefits including helping students overcome some of the obstacles associated with the use of diagrams. One week of hands-on experience with the calculator early in the study period along with continual use of the TI-92 by the teacher to introduce concepts and theorems is essential to this type of activity.

As indicated by Simon (1995), requiring a student to explain how to construct a geometric figure requires the student to have a deeper level of understanding than is necessary to merely listen to or even to follow someone else's explanation. In explaining how to construct a rectangle, for example, students must explain that opposite sides have to be constructed parallel to each other and adjacent sides perpendicular to each other. In developing such explanations on their own, students encounter areas where their knowledge is lacking, that is, they encounter cognitive conflict, and with it the opportunity to construct new knowledge.

Students in the classes studied in the present research often showed pride in being able to explain to other students how to accurately construct geometric figures and thus learned to value their own thinking and that of others. These are benefits achieved through use of interactive software by individual students or pairs of students identified in research by Gordon (1993), Lampert (1993), Yerushalmy (1993), and Yerushalmy and Chazan (1993) which can also accrue to students in classes where the software is used solely in a whole-class setting. Listening to other students explain their construction methods causes students to be confronted with the thinking of others and with the need to justify, defend, or modify their own thinking. Most geometric figures can be constructed in a variety of ways using the tools available on the TI-92 and the different methods of
construction can lead to valuable discussion in class of which is the "best" way and to competition among students to develop the most efficient method. These were illustrated in the two geometry classes in the study when students produced a number of ways of constructing a kite and showed pride in describing their methods to other students. On another occasion, after one student had explained how to construct a rhombus as described in chapter 5 (see Figure 21), another student who normally showed little interest in the class said that he had an easier method of making the construction and proceeded to describe his method to the class. He said to first construct a circle O (see Figure 25) and then draw two radii OA and OB of the circle. Because these segments are radii of the same circle, he knew they would give two adjacent congruent sides of the rhombus. He then said to draw a line parallel to segment OA through point B and another line parallel to segment OB through point A. The intersection point C of these two parallel lines is the fourth vertex of the rhombus.

Depending on how thoroughly individual students explain their thinking, construction problems can give the teacher an opportunity to look more deeply into a student's thinking, identify areas where the student's knowledge is lacking, and then

![An Example of Rhombus Construction](image.png)

*Figure 25. An Example of Rhombus Construction*
modify teaching activities to create opportunities for the student to encounter cognitive conflict and modify their thinking. These are essential activities for a constructivist teacher identified by Noddings (1990), Simon (1995), Wood (1993a), and others. As discussed by Confrey (1990), Simon, and Steffe and D’Ambrosio (1995) such activities give a teacher the opportunity to construct a mental model of the student’s mathematical knowledge at that point. As expressed by Confrey, Bauersfeld (1992), Steffe and D’Ambrosio, and Wood (1993b), teachers in these situations need to view student’s sometimes incorrect explanations of these geometric constructions as rational explanations of their thinking at that point and work from there to modify the student’s thinking.

Going through the process of describing the constructions in detail helps students to understand characteristics of figures they are studying. In one problem, students were asked to describe how to construct a right triangle and a circle circumscribed about the triangle. After constructing the triangle, most used the theorem that the midpoint of the hypotenuse is equidistant from the three vertices to construct the circle. To apply that theorem to explain how to do the construction, students must understand that all points on a circle are equidistant from the center, that the midpoint of the hypotenuse being equidistant from all three vertices means that the distance from that point to any vertex is the same as the distance from that point to any other vertex, and, they must combine these pieces of information to realize that, if they use the midpoint of the hypotenuse as the center of a circle and the distance from that point to any vertex as radius, the circle will have to go through the other two vertices. The ability to combine information in this way to reach a conclusion is crucial to doing not only geometry but mathematics in general.
Implications for Teaching

Two of the most important implications for teaching are discussed above. Those are the tendency of students to base their conclusions on the appearance of given diagrams and the value of activities which require students to explain how to construct various geometric figures using the tools available on the TI-92.

Teachers should be aware that use of the TI-92 with Cabri Geometry Software solely for whole-class demonstration and discussion may have a positive impact on students’ overcoming the obstacles to the use of diagrams identified by Yerushalmy (1993) with a larger impact on males than on females. Using the TI-92 to construct a number of figures that satisfy stipulated conditions but look different can help students identify essential and nonessential characteristics of a diagram. In regard to developing conjectures, such use may have a more positive impact on females than on males.

In a number of cases, students were able to figure out things based on focused questions from the researcher during the interviews that they claimed not to know prior to the interview. In some cases, the questions helped students to focus their attention on parts of a diagram and may have helped them to overcome one of the obstacles associated with diagrams thus enabling them to complete the problem or develop a conjecture.

Borrowing calculators through the Texas Instruments Corporation calculator loan program so that every student has access to one for at least one week early in the study was a crucial part of the study which gave students the background necessary to understand what was happening in class the remainder of the year. This was also crucial to the value of the construction problems as described earlier.
Many students have difficulty with conjecturing particularly when asked to list all statements they think may be true about an entire diagram. When asked to develop conjectures, some students merely restate the given information in their own words or state the obvious results of definitions. For example, given that line segments AB and CD are congruent, some students will conjecture that the lengths of the two segments are the same. Beginning with activities which enable students to make such seemingly trivial conjectures, moving on to activities in which they are asked to conjecture about specific parts of a diagram, and then using activities which ask students to make conjectures about diagrams in general may help them overcome obstacles with diagrams and become better conjecturers.

The ease with which a great number of diagrams illustrating a particular conjecture can be generated using interactive software may tend to reinforce in students the idea that many examples constitute proof. This is contrary to findings in research using the Geometric Supposer as reported by Clements and Battista (1992). When using the calculator to generate examples, the teacher needs to continually emphasize that many examples do not prove a conjecture to be true while one counterexample disproves a conjecture. Examples in which several diagrams supporting a conjecture are first constructed followed by one diagram disproving the conjecture could help change students' belief in measurement as proof.

The textbook can be a detriment to getting students to think deeply and come up with viable conjectures. The teacher needs to develop some activities outside the textbook so that students cannot rely so heavily on it. In a school where the sequence of
material as presented in the textbook must be followed, there will be some constraints on optimum use of interactive software as described by Lampert (1993).

Recommendations

The following recommendations for future research are based on experience gained in the present study. It seems worthwhile to develop a valid and reliable survey instrument which could be used to measure changes in student attitudes toward those items which the researcher-developed items in this study tried to approach. That is, does use of the TI-92 and interactive geometry software improve student attitudes toward and understanding of the use of diagrams, conjecturing, and proof? Does it impact student attitudes toward their abilities to produce conjectures and proofs? Does it impact widely held views about mathematics in general such as those expressed in the statements about mathematics problems taking no more than five minutes, mathematics being mostly memorization, and mathematics problems having only one right answer? Does use of the calculator impact females and males differently in these areas? This would help to clarify some questions raised in the present study such as whether or not use of interactive geometry software is more beneficial to males in overcoming difficulties associated with the use of diagrams than to females.

The present study investigated whether use of one TI-92 with Cabri Geometry software used solely for whole class demonstration and discussion helped students overcome difficulties with diagrams, produce conjectures and proofs, and whether there were differences in these for males and females. If such benefits can be achieved through the use of only one calculator in a classroom, then, due to the reduced cost of purchasing
only one calculator, these benefits can be made available to many more students. The present study did not compare a classroom in which the TI-92 was not used to one in which it was used. Such a comparison would help to clarify whether any benefits achieved were due to use of the interactive software or were the result of other factors.

Research similar to that described in this study should be done in geometry classes in which TI-92s are available for students to use on a daily basis either individually or in pairs. Some of the results of the current study indicate that positive results could be greater if students had calculators to use on a daily basis and this should be investigated. This would also help to clarify whether results obtained in studies by researchers such as Yerushalmy and Chazan (1993) which used Geometric Supposer software on computers can also be obtained through use of the TI-92 with Cabri Geometry software.

In the group projects conducted toward the end of the study period, mixed-gender groups were used in one class and same-gender groups were used in the other. In five of the six mixed-gender groups, males did the calculator portions of the group presentations. In the four all-female groups in the other class, the girls who did the calculator portions of the presentation were extremely proficient in use of the calculator and showed no hesitancy in demonstrating their abilities to use it in front of the class. Time did not allow another group project in which the make up of the groups could be reversed to see if the girls who did such a good job would also do the calculator portion of the presentation when part of a mixed-gender group. Further research into this would be interesting.

Use of interactive geometry software such as Cabri Geometry on the TI-92 requires visualization skills. If males possess greater visualization skills than do females,
extensive use of interactive software could benefit males more than females. The present study indicated the possibility of such differences. More research is needed into the potentially different impact of the use of interactive geometry software on male and female students to determine for example whether some of the findings of Doenges (1996) in regard to the use of graphing calculators would also apply to the use of interactive geometry software.

Many students do not know why they are taking geometry, take it only because it is a required course, and have a negative attitude toward the study of geometry which persists throughout the year. While these attitudes may be true of other required courses including other required mathematics courses, the observations of the researcher in studying these two classes throughout one school year indicate that these negative attitudes seem to be stronger in the case of geometry. For many students, this means that they put little effort into the study of geometry and therefore get little out of a course which could help them to develop a greater understanding of mathematics in particular and of logical reasoning in general. Measures should be taken to inform students of the benefits and value of learning geometry and its applicability to modern times. This could be done by emphasizing the relationship of geometry to other mathematics courses which students take but also by emphasizing the relationship between logical thinking in geometry and the logical thinking necessary to solve problems which students face in life on a day to day basis. Activities used by Fawcett (1995) in his geometry classes of the 1930s where, for example, he educated students to critically analyze any argument using logical reasoning, would seem to be beneficial today.
Appendix A

Sample Lessons Using the TI-92
This appendix gives examples of lessons that were taught by the researcher using one TI-92 with Cabri Geometry and the overhead projection unit. During the course of the second semester of the school year, several class sessions in which the calculator was used were video recorded. The following is taken from those video recordings. It is not intended to be a transcription of everything that happened in the class but is meant to demonstrate how the TI-92 was used. These represent only a small portion of the classes in which the TI-92 was used. Illustrations are taken directly from the TI-92 screen.

February 6, 1997

This lesson took place the week after we had the loaner calculators from Texas Instruments. It is a review of some items that students had been asked to do on worksheets while using the calculators on their own. One of the activities students did was to draw a triangle and construct the altitudes, medians, and perpendicular bisectors of sides. They saw that the three altitudes intersect in a common point as do the medians and perpendicular bisectors of sides. By moving one or more of the vertices around, they were able to see that this holds for a number of examples of triangles. The teacher asked them to look at those three intersection points and see if they could determine a relationship. The first thing discussed in this lesson was that idea. To speed things up during class, the teacher had previously constructed a triangle with altitudes, medians, and perpendicular bisectors of sides. The point of intersection of the altitudes was labeled A,
of the medians M, and of the perpendicular bisectors B. This is shown in figure A.1 in
which the line segments have been hidden to reduce the complexity of the diagram. The
relationship is that the three segments are collinear. This was verified in class using the
check property feature of the TI-92.

Figure A.1: Points of Intersection of Medians, Altitudes, and Perpendicular Bisectors

This was also checked by constructing a line through B and A, moving one of the vertices
of the triangle around, and showing that point M always stays on that line. Figure A.2
illustrates this for another triangle.

Figure A.2: Another Example of Intersection Points
The remainder of the lesson covered theorems dealing with ratios and proportions. Students had examined all but one of these relationships using the borrowed calculators on their own so this was a review. First, triangle ABC was constructed and then line segment DE parallel to side AC was constructed. Students were asked to state a relationship between triangle ABC and DBE. (They are similar triangles.) Students were then asked to tell what ratio would be equal to \( \frac{DB}{DA} \). By the Triangle Proportionality Theorem, \( \frac{DB}{DA} = \frac{BE}{EC} \). Those segments were measured and ratios computed to show that the proportion is true for this example. This is illustrated in figure A.3.

![Figure A.3: Triangle Proportionality Theorem](image)

Using this figure students were asked to tell what would happen to the lengths of the segments and to the ratios if segment DE were moved to another position in the triangle but still parallel to AC. They correctly reasoned that the lengths of all segments and the ratios would change but, according to the theorem, the ratios would remain equal. This is illustrated in figure A.4.
Figure A.4: Triangle Proportionality Theorem - Another Example

Students were then asked what would happen if point C was moved. Would the lengths of segments DB and DA change? How about BE and EC? Would the ratios change? There were several conjectures about what might happen. Point C was then moved to show that DB and DA stay the same while BE and EC change. Some students were surprised to see that the ratios stay the same. No student agreed to explain why this happens so the teacher explained that according to the theorem, it is always true that

\[
\frac{DB}{DA} = \frac{BE}{EC}
\]

so if segments DB and DA don't change, then their ratio can't change and therefore the ratio of BE and EC must also remain constant.

Next, three parallel lines cut by two transversals were drawn and labeled as shown in figure A.5. The calculator was used to demonstrate that \( \frac{AB}{BC} = \frac{DE}{EF} \). Students were then asked what would happen to the lengths of these segments and ratios if line BE were moved up and down. They correctly reasoned that the segment lengths and ratios would change but that the ratios would always be equal.
Students were then asked what would happen if line DF were moved. The realized that if it moved only to the right or left but maintained the same orientation in relation to the parallel lines, that nothing would change. (See figure A.6)

Students were then asked what would happen if line DF was moved so that its orientation with respect to the parallel lines changed. There were several conjectures as to what might happen. The calculator was used to demonstrate that the lengths of segments DE and EF would change but the ratio of DE to EF would remain constant. As in the
previous theorem, this is because AB and AC don’t change when line DF is moved and because \( \frac{AB}{BC} = \frac{DE}{EF} \), the ratio of DE to EF must remain the same. (See figure A.7)

![Figure A.7: Orientation of Line DF Changed](image)

One other topic which students had not previously seen was covered in this lesson. That is the Triangle-Angle Bisector Theorem which states that a line bisecting one angle of a triangle divides the opposite side into segments proportional to the other two sides of the triangle. The diagram was drawn, segments measured, and the theorem verified as shown in figure A.8.

![Figure A.8: Triangle-Angle Bisector Theorem](image)
Students were asked what would happen if point B were moved? Again there were several conjectures. Many thought that the lengths of segments AD and DC would remain constant. Using the calculator, students saw that these lengths change. The reason why they change was discussed. Because BD is the bisector of angle ABC, if B is moved and changes the size of angle ABC, then, since BD is the bisector of angle ABC it must also move. This changes the lengths of segments AD and DC. As stated in the theorem, the ratios remain equal. This is illustrated in figure A.9.

Figure A.9: Triangle-Angle Bisector Theorem with Point B Moved

One student then asked what would happen if point C moved so the teacher explored this with the class. In this case the length of segment AB remains constant but the lengths of all other segments change. Therefore the ratios change but are always equal to each other. This is shown in figure A.10 which should be compared with figure A.9.
February 14, 1997

The purpose of the lesson was to introduce students to geometric means. To do this, Activity 28, Geometric Means, from *92 Geometric Explorations on the TI-92* (Keyton, 1996) was copied and given to the students. Prior to copying, labels on the diagrams in the original version of this activity were changed to correspond to labels in an identical diagram in the textbook. Students were asked to follow along as the teacher went over the worksheet on the TI-92 and to copy on their worksheets what was being done on the overhead unit. This was done in an effort to keep students from merely sitting and watching what the teacher was doing and to keep them actively involved in what was going on in class.

In going through this activity, a line segment AB is created first, then the midpoint of the line segment M is created using the midpoint tool. Next, a circle with its center at point M and radius equal to MA is constructed. Then a point N is created anywhere on segment AB and a perpendicular line to segment AB through point N is drawn. The intersection point of the perpendicular line and the circle is labeled point C. Finally, the
measurement tool is used to measure and record the lengths of segments AN, CN, and NB. When all this is done, a figure such as the following results.

![Figure A.11: Circle with perpendiculars](image)

The instructions for the activity next ask students to discover a relationship among the three numbers representing the lengths of AN, NB, and CN. In this study, students did not have access to calculators. Had they, over time they might have become adept at finding relationships. In this case the teacher led them through the remainder of the activity calculating the ratios \( \frac{AN}{CN} \) and \( \frac{CN}{NB} \). Students saw that these ratios were equal so that the proportion \( \frac{AN}{CN} = \frac{CN}{NB} \) is true. The definition of the geometric mean was given and students saw that CN is the geometric mean between AN and NB.
Figure A.12: Equal Ratios

The teacher then “grabbed” point N and moved the perpendicular line along line segment AB. As this was done, students could see that the lengths of each of the segments AN, NB, and CN changed and that the value of the ratios \( \frac{AN}{CN} \) and \( \frac{CN}{NB} \) changed but that the proportion \( \frac{AN}{CN} = \frac{CN}{NB} \) held true for each example created by moving point N.

Figure A.13: Changing Ratios

The teacher then hid the circle, the perpendicular line, and point M in the diagram and drew line segments AC, CB, and NB. This left only right triangle ABC with its
altitude CN. He mentioned a theorem not yet covered in this class which states that an angle inscribed in a semicircle is a right angle. Therefore, triangle ABC is a right triangle. He showed that this was true in this diagram by measuring angle ACB.

![Diagram of Right Triangle](image)

Figure A.14: Right Triangle

The teacher then referred students to the diagram in the text which is similar to the one shown above. The diagram in the text accompanies a theorem which states that when the altitude is drawn to the hypotenuse of a right triangle, three similar triangles are formed. This theorem and similar triangles were then used as another way of illustrating that CN is the geometric mean between AN and NB, that is, because corresponding parts of similar triangles are proportional, then \( \frac{AN}{CN} = \frac{CN}{NB} \). This illustrated the first corollary to the theorem which was given in the text which states that the altitude to the hypotenuse of a right triangle is the geometric mean between the segments of the hypotenuse. Finally, a second corollary, also the result of the above similar triangles, was introduced which states
that the leg of a right triangle is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to it.

A somewhat different approach to introducing the topic was used in the second geometry class. Instead of going through all of the steps on the worksheet, the teacher started with the right triangle which had already been measured and labeled. This was done to speed things up and to allow less time in which students were only watching what the teacher was doing and were not actively involved in the lesson. Although labeling and measuring things on the TI-92 does not take a great deal of time, it can seem like a lot of time when the teacher is doing it in front of the class and students are only watching. Often, when parts of a diagram are labeled and measured, the calculator places the labels in places which cover up other key parts of the diagram and they must be moved elsewhere. Again this takes time and is time in which students may not be actively involved in the lesson. In the second class, the right triangle ABC was shown to the students (see figure A.14) and then altitude CN was moved along segment AB creating other right triangles and showing that the proportion \( \frac{AN}{CN} = \frac{CN}{NB} \) is true for all examples shown. The other theorems and corollaries were introduced as in the previous class.

February 18, 1997

This lesson used the TI-92 and Cabri Geometry to introduce the Pythagorean Theorem. First a right triangle was drawn. Students had previously been ask to tell how they could construct a right triangle using the tools available on the TI-92 and several methods of doing this were discussed. In this case, a line segment AC is first drawn and
then a perpendicular line to AC through point C is constructed. A point B is placed somewhere on the perpendicular line, the perpendicular line is hidden, and segments BC and AC are constructed. This gives a diagram such as that shown in figure A.15.

![Figure A.15: Right Triangle ABC](image)

Students were aware that the Pythagorean Theorem is often stated $c^2 = a^2 + b^2$. The diagram was modified to show the origin of this statement of the theorem. Sides a, b, and c were measured and the quantities $c^2$ and $a^2 + b^2$ were calculated so that students could see that those quantities are equal in this example. This is shown in figure A.16.

![Figure A.16: The Pythagorean Theorem Illustrated](image)
The teacher then "grabbed" point B and moved it around so that a number of examples of right triangles were formed. When B is moved, the lengths of sides a, b, and c change as do the values of $c^2$ and $a^2 + b^2$ but, in all cases, $c^2 = a^2 + b^2$. This is illustrated in figure A.17.

![Figure A.17: Another Illustration of $c^2 = a^2 + b^2$](image)

Next, a small right triangle ABC was constructed and, using a macro on the TI-92, squares on the sides and the hypotenuse of triangle ABC were constructed. Using the area tool, the areas of the squares were determined and the areas of the two squares on the legs were added together to show that their sum was the same as the square of the hypotenuse. Again, a point of the triangle was moved around to show this relationship is true in many examples. This illustrates Euclid's proof of the Pythagorean Theorem (See figure A.18).
Examples illustrating the Pythagorean Theorem were then done on the chalk board. It is cumbersome to do them on the TI-92 because it is time consuming to draw line segments with specified lengths.

February 20, 1997

This lesson dealt with the converse of the Pythagorean Theorem. Students were first asked to state the converse: If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle. This was illustrated on the TI-92 using the last part of Activity 47 in 92 Geometric Explorations on the TI-92 (Keyton, 1996). In this activity, a triangle LID is constructed. For this lesson, the teacher initially constructed the triangle so that angle I was a right angle. Students were not aware of this. The sides and hypotenuse of the triangle were measured and the square of the hypotenuse was compared with the sum of the squares of the sides. These two quantities were equal. Angle I was then measured to show that it was a right angle. Figure A.19 illustrates this.
Students had been asked to think about what the relationship among the sides and hypotenuse would be if the measure of angle I were greater than 90 degrees or less than 90 degrees. This question was discussed in class. The teacher asked what would happen in the diagram above if the lengths of sides LI and ID stayed constant but the measure of angle I became greater than 90 degrees. (For the purposes of illustration, it seemed important to simplify things by keeping the lengths of LI and ID constant so that only the measure of angle I changed. This would enable students to concentrate on what happened to the length of the hypotenuse and to the comparison of the sum of the squares of the legs to the square of the third side. To construct a triangle in which two sides would stay the same length when a vertex was moved, the teacher used the compass tool on the TI-92). Using the diagram on the TI-92, students saw that in this case, the square of the hypotenuse became greater than the sum of the squares of the legs (See figure A.20).
Students were also asked what would happen if the measure of angle I were less than 90 degrees. This outcome is illustrated in figure A.21.

These two diagrams were then related to the corresponding theorems in the textbook.

February 24, 1997

This lesson deals with special right triangles. Although it is not specifically an activity in 92 Geometric Activities for the TI-92 (Keyton, 1996), it closely follows activities in that book. First the teacher demonstrated how to use the TI-92 to draw a 45°-45°-90° triangle. This is done by first constructing a line segment AX, then using the
numerical edit and rotate tools on the TI-92 to rotate that segment 45° and then rotate the resulting segment again through 45°. Points were labeled and sides were measured to give a diagram such as the following (figure A.22).

![Figure A.22: 45°-45°-90° Triangle](image)

The teacher then split the screen on the TI-92 and used the Collect Data tool to put the measurements of the sides of the triangle into a table. He then grabbed one of the vertices of the triangle and moved it around so that different 45°-45°-90° triangles were shown. Periodically, more data was collected so that finally, the table contained data on 5 such triangles. This is illustrated in figure A.23.

![Figure A.23: Data for 45°-45°-90° Triangles](image)
The same procedure was followed for $30^\circ$-$60^\circ$-$90^\circ$ triangles giving the data shown in figure A.24.

![Figure A.24: Data for 30°-60°-90° Triangle](image)

These sets of data gave students five examples of each type of triangle to work with. For both sets of data students were asked to find a relationship among the numbers. For the $45^\circ$-$45^\circ$-$90^\circ$ triangles, their first reaction was that two of the numbers were equal and their second was that the square of the larger equals the sum of the squares of the two smaller numbers. The teacher asked them to find other relationships. The idea is that by examining ratios of the sets of numbers, students could see that these ratios are always constant so that in a $45^\circ$-$45^\circ$-$90^\circ$ triangle the ratio is $1:1: \sqrt{2}$ and in a $30^\circ$-$60^\circ$-$90^\circ$ triangle, the ratio is $1: \sqrt{3} :2$.

March 4, 1997

The class had been studying the tangent function. This lesson was meant to illustrate to students where the numbers they get on their calculator when they use the
tangent function or when they look up the tangent function in a table come from. The teacher first told students that they were going to construct a right triangle and ask students to give him the values they wanted to use for the two acute angles of the triangle. The first class used 47 and 43 degrees. The second class used 50 and 40 degrees. To construct a right triangle with acute angles of 47 and 43 degrees, a procedure like that used for constructing the 45°-45°-90° and 30°-60°-90° triangles was used. The lengths of sides of the triangle were measured. Students were then asked how they would compute the tangent of the 47 degree angle. They said by dividing the opposite side by the adjacent side. This was done on the calculator with the results shown in figure A.25.

![Figure A.25: 47°-43°-90° Triangle](image)

The teacher then switched to the home screen of the TI-92 and used the tangent function of the calculator to compute the tangent of 47 degrees. This came out to be 1.07237. He had students do the same on their calculators. He then went back to the geometry screen and asked students what would happen if he moved a vertex of the triangle around. After a little discussion, he explained that the angle measures would stay the same, 47 and 43
degrees, because they were constructed to be those sizes. Students said that the lengths of
sides would change but the ratio of the sides, the tangent, would stay the same. This
proved to be true when a vertex was actually moved on the calculator. The teacher
emphasized that this illustrates the point that the tangent function depends on the measure
of the angle not on the size of the triangle. The same was then done for the 43 degree
angle in the first class and for the 50 and 40 degree angles in the second class.

March 19, 1997

In this lesson, the TI-92 was used in the discussion of three problems from the
previous day's homework assignment. The first problem states that a triangle with legs of
lengths six and eight is inscribed in a circle and asks students to find the radius of the circle
(Jurgensen, Brown & Jurgensen, 1994). The teacher asked students to tell how to
construct a right triangle on the TI-92. One said to draw a line and another line
perpendicular to it. The teacher drew a line segment with a line perpendicular to it at one
end point and then put another point on the line. Connecting the three points gives the
right triangle. The teacher asked, "What do you know about the midpoint of the
hypotenuse of a right triangle?" A student responded that it is equidistant from the three
vertices. The teacher used the midpoint tool to find the midpoint of the hypotenuse and
then constructed the circumcircle using the midpoint of the hypotenuse as the center and
the distance to one of the vertices as radius. He then asked students how they would find
the radius of the circle and one explained how to use the Pythagorean Theorem to find the
hypotenuse of the triangle which is the diameter of the circle and then divide that by two
to get the radius. (See figure A. 26)
The second problem stated that 2 concentric circles have radii of 15 and 7 centimeters. It went on to say that a diameter AB of the larger circle intersects the smaller circle at points C and D. It then asks students to give two possible values for the length of AC (Jurgensen, Brown & Jurgensen, 1994). The teacher explained how to construct two concentric circles using the numerical edit tool on the TI-92 and circles with radii of 15 and 7 millimeters were constructed. (Using centimeters would have made the diagram too big for the calculator screen.) Points A, B, C, and D were labeled as in the problem statement (see figure A. 27) and possible values for AC were discussed. If points are labeled as in figure A.27, the value would be 22 millimeters as was verified using the calculator. If the labels of points C and D are reversed, the length of AC would be 8 centimeters. The class discussed how to get these answers without using the calculator and measuring.
The third problem states that two congruent circles Q and R intersect at points C and D and asks students to determine what type of figure QDRC would be and why segments CD and QR would be perpendicular bisectors of each other. It then states asks students to find the length of segment CD if QC = 17 and QR = 30 (Jurgensen, Brown & Jurgensen, 1994). The teacher used a radius of seven millimeters and drew the two concentric circles again using the measurement transfer tool (see figure A.28).

Students said that the figure would be a rhombus. When asked why, one said because the sides are all radii of congruent circles. As to why CD and QR are perpendicular bisectors, students explained that the diagonals of a rhombus are perpendicular bisectors of each
A student then explained how to find the length of CD using the facts that the diagonals are perpendicular along with the Pythagorean Theorem.

March 21, 1997

For the previous day's homework assignment, the teacher told students to write out explanations of how they would construct certain figures using the tools available on the TI-92. Those procedures were discussed in class on this day. The first problem was to tell how to circumscribe a polygon about a given circle. One student said to first draw five radii of the circle. Another then said to connect the end points of the radii. The teacher explained that this would give an inscribed pentagon rather than a circumscribed one. The first student then said to draw perpendiculars to the five radii at their endpoints and the resulting figure would be a pentagon (see figure A.29).

![Figure A.29: A Circumscribed Pentagon](image)

The teacher asked why you couldn't just start with the circle and draw tangent lines without drawing the radii. A student explained that you couldn't be sure that the lines touched the circle in only one point. The teacher demonstrated this on the TI-92.
emphasizing that the only way to be sure the lines touch the circle in only one point, that is, that they are tangents to the circle, is to draw the radii and the perpendicular lines.

The second problem was to circumscribe a circle about a given right triangle. A student explained how to do it using the fact that the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices. See figure A.26.

The third problem was to draw two internally tangent circles. After one false start, a student said to draw a line segment, find the midpoint of the segment, and then construct a perpendicular to the segment at one end point. She said to use the midpoint of the segment as the center of one circle and the distance from the midpoint to one end as the radius. That resulted in one circle. She got confused on how to draw the other. The teacher told her not to give up because she was on the right track. He then showed how to construct the second circle using the length of the segment as its radius. See figure A.30.

Figure A.30: Internally Tangent Circles
The fourth problem involved constructing two externally tangent circles. One student said to first draw a line segment then draw two circles, one on either side, that touch the segment in the same point. The teacher drew a segment and one circle and explained that if the segment was tangent to the circle, it would have to be perpendicular to the radius drawn to the point of tangency. He then drew the radius and measured the angle which turned out to be about 87 degrees. See figure A.31.

Figure A.31: A False Start

The teacher asked how you could be sure the angle was 90 degrees. One student said to draw a perpendicular line to the segment at its midpoint. The teacher explained that it wouldn’t have to be at the midpoint but that the perpendicular line was necessary. The student said to use the endpoints of the segment as the centers of the circles with the radii being the distances from the endpoints to the point of intersection of the segment and the perpendicular line. This is illustrated in figure A.32.
The fifth problem asked students how to draw a line tangent to two given nonintersecting circles. The teacher admitted that in spite of many attempts, he still hadn’t figured out how to do this one. Several students proposed methods. One involved constructing the segment connecting the centers of the two circles and then constructing perpendiculars to that segment at the center of each circle. Students then said to make a rectangle out of the figure. The teacher asked where the fourth side should go. One student said at the endpoint of the radius of the larger circle. This was done and it was obvious that that segment was not tangent to the smaller circle (see figure A.33).

Figure A.32: Externally Tangent Circles

Figure A.33: An Attempt to Construct an External Tangent
The teacher explained that it would be much simpler to start with the line and then construct two circles which were tangent to it and showed how this could be done. Another student said to first construct the two circles, construct a line through the center of one and then a line parallel to that through the center of the other. His idea was that the line connecting the two points where the parallel lines intersected the circles would be tangent to both circles (see figure A.34).

Figure A.34: Another Attempt to Construct an External Tangent

The teacher explained that the line most likely would not be tangent to both of the two circles. That student asked to borrow the calculator for the remainder of the class period to work on the problem. The problem was never solved but did result in some good discussion.

May 8, 1997

The topic of this lesson is kites, a topic mentioned only once in the textbook used by the class and then only in regard to one problem. Many students, when asked to make conjectures about a topic in the text, would just look at the theorems or corollaries in the
next few sections in the text to come up with their conjectures. The teacher wanted to spend approximately two weeks on a topic that was not covered in the text to see if students could develop conjectures when they didn’t have the text to rely on. Kites were chosen because kites are quadrilaterals and the students had already studied other types of quadrilaterals. The day prior to this lesson, the teacher had given the definition of a kite: A kite is a quadrilateral with two distinct pairs of consecutive equal sides. The class had discussed the definition and looked at examples of kites. For homework, students were to tell how they could construct a kite using the tools available on the TI-92 and then develop as many conjectures as they could in regard to kites.

Three students, one girl and two boys, gave their methods of constructing kites. One of the boys came up to the front of the class and operated the calculator himself showing the class how he constructed a kite. The other two students told the teacher what steps to take in constructing the kite but the teacher operated the calculator. The following are the instructions for constructing a kite and figures showing each.

The girl volunteered to go first and gave the teacher these steps in constructing a kite.

1. First, make a circle with center at point A.
2. Make a chord BD of the circle.
3. Draw segments AB and AD so that you have an isosceles triangle ABD.
4. Find the midpoint of segment BD and label it point X.
5. Draw a ray from A through X.
6. Put a point C on the ray somewhere to the right of point X.
7. Draw segments BC and DC.

Figure ABCD will be a kite.

![Diagram of a kite with labels A, B, C, D, and X]

Figure A35: First Method of Constructing a Kite

It should be pointed out that the teacher asked students to label their kites as shown in this figure to make it easier to talk about various parts of the kite in class.

The boy who did the demonstration himself next gave his method for constructing a kite.

1. Draw a line segment BD.
2. Draw the perpendicular bisector of BD.
3. Put two points A and C on the perpendicular bisector, one on either side of BD.
4. Draw segments AB, BC, CD, and DA.

Figure ABCD is a kite. When the teacher asked how he knew that the figure had two pairs of consecutive congruent sides, the boy responded that any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

That theorem had been studied by the class earlier in the year.
Several students commented that this was a much easier method than the first one. The teacher told them that both methods were valid and the assignment had been to come up with a way of doing it.

Prior to the beginning of class, one of the boys had shown the teacher his method of constructing a kite and asked the teacher if it was valid. It is a rather unique method so the teacher asked him to share it with the class.

1. Draw a rectangle.

2. Draw a square at one end of the rectangle with the length of a side of the square being equal to one of the sides of the rectangle.

3. Draw the diagonals of the square and rectangle.

When points are labeled as shown in figure A.37, quadrilateral ABCD is a kite.
The next portion of the lesson involved having students come up with as many conjectures regarding kites as they could. Students gave their conjectures while the teacher wrote them on the board. The following are the conjectures. Labels refer to the “standard” method of labeling a kite as discussed earlier.

1. \( m \angle BXC = 90^\circ \) (The same student said that the measures of all four angles with vertex at point X would be 90 and this was equivalent to saying that segments AC and BD, the diagonals of the kite, are perpendicular.)

2. \( \triangle ABC \cong \triangle ADC \)

3. \( \overline{AC} \) bisects \( \overline{BD} \)

4. \( \angle ABC \cong \angle ADC \)

5. \( \overline{AX} \cong \overline{XB} \)

6. \( \overline{CX} \) bisects \( \angle BCD \) and \( \overline{AX} \) bisects \( \angle BAD \)

7. \( \triangle AXY \cong \triangle AXD \)

Each of these conjectures (except those dealing with congruent triangles) was examined on the TI-92 to see if they seemed to be true for a number of examples. This
was done by constructing the figure, making appropriate measurements, dragging points, and seeing if the relationship held. All conjectures with the exception of $AX \equiv XB$ seemed to be true. The fact that this one was shown not to be true by using the TI-92 gave the teacher an opportunity to show how a nonexample can disprove a conjecture and also to emphasize that not all conjectures that we think may be true actually turn out to be true. The assignment for the next day was to prove four of the conjectures (numbers 1, 2, 3, and 6 above.)
Appendix B

Attitude Survey
ATTITUDE SURVEY

On the following pages is a series of statements. There are no correct answers for these statements. They have been set up in a way which permits you to indicate the extent to which you agree or disagree with the ideas expressed. Suppose the statement is:

Example 1. I like mathematics.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

As you read the statement, you will know whether you agree or disagree. If you strongly agree, circle “strongly agree” on the line below the question. If you agree but with reservations, that is, you do not fully agree, circle “agree” on the line below the question. If you disagree with the idea, indicate the extent to which you disagree by circling “disagree” or “strongly disagree” on the line below the question. If you neither agree or disagree, that is, if you are uncertain, circle “undecided” on the line below the question. Also, if you cannot answer a question, circle “undecided” on the line below the question. Now mark Example 1. Do the same for Example 2 below.

Example 2. Math is very interesting to me.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

Do not spend much time with any statement, but be sure to answer every statement. Work fast but carefully.

There are no “right” or “wrong” answers. The only correct responses are those that are true for you. Whenever possible, let the things that have happened to you help you make a choice.

THE PURPOSE OF THIS INVENTORY IS TO GIVE ME AN INDICATION OF THE ATTITUDES OF STUDENTS TOWARDS VARIOUS IDEAS RELATED TO LEARNING MATHEMATICS. YOUR RESPONSES WILL HAVE NO IMPACT ON YOUR GRADE IN THE COURSE EITHER POSITIVELY OR NEGATIVELY. NO ONE OTHER THAN MYSELF WILL KNOW WHAT YOUR RESPONSES ARE.

1. Math is an obstacle standing in the way of graduating from high school.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

2. It would be really great to win a prize in mathematics.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

3. Getting a mathematics teacher to take me seriously has usually been a problem.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

4. There is only one way to solve a math problem and the teacher should just tell us how to do it.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

5. A theorem is true only for the diagram that is shown with it.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

6. My mind goes blank and I am unable to think clearly when working mathematics.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

7. Mathematics is enjoyable and stimulating to me.

Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree
8. My math teachers would encourage me to take all the math I can.  
   | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
9. Females are as good as males in geometry.  
   | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
10. It's okay for me to add lines to the diagrams given in problems.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
11. I'll need a firm mastery of mathematics for my future work.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
12. I get a sinking feeling when I think of trying hard math problems.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
13. I am sure I could do advanced work in mathematics.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
14. I would rather have someone give me the solution to a difficult math problem than have to work it out for myself.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
15. I will use mathematics in many ways as an adult.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
16. Mathematics can be beautiful.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
17. Math problems have one right answer.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
18. My teachers think advanced math is a waste of time for me.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
19. I would have more faith in the answer for a math problem solved by a man than a woman.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
20. When a question is left unanswered in math class, I continue to think about it afterward.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
21. I don't see how some people can spend so much time on math and seem to enjoy it.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
22. Mathematics will not be important to me in my life's work.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
23. My teachers think I'm the kind of person who could do well in mathematics.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
24. It would make me happy to be recognized as an excellent student in math.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
25. Math class is more interesting when we discuss what we are learning.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
26. I study mathematics because I know how useful it will be.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
27. It is unfair for a teacher to give a problem I haven't seen before on a test.  
    | Strongly Agree | Agree | Undecided | Disagree | Strongly Disagree |
28. In terms of my adult life it is not important for me to do well in mathematics in high school.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

29. A right angle can be drawn in a lot of different positions.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

30. It would make people like me less if I were a really good math student.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

31. If I can't solve a math problem in 5 minutes, there is no sense in working on it any longer.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

32. I can work out a proof by thinking logically.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

33. For some reason even though I study, math seems unusually hard for me.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

34. When a math problem arises that I can't immediately solve, I stick with it until I have the solution.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

35. Generally I have felt secure about attempting mathematics.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

36. You can prove a theorem in geometry by measuring angles with a protractor.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

37. I would expect a woman mathematician to be a masculine type of person.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

38. I'd be proud to be the outstanding student in math.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

39. Even if I make a mistake, I learn from working problems on the board in class.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

40. Mathematicians are creative people.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

41. If I had good grades in math, I would try to hide it.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

42. I have found it hard to win the respect of math teachers.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

43. To solve a math problem you have to use a rule you memorized earlier.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

44. In math class a student should only have to sit and listen.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

45. A triangle should always be drawn with the base at the bottom.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

46. If I got the highest grade in math I'd prefer that no one knew.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree

47. Working in groups with other students helps me to understand math.
   Strongly Agree  Agree  Undecided  Disagree  Strongly Disagree
<p>| | | | | | | |</p>
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>48. I learn more in math class when I can ask questions of the teacher and other students.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>49. Girls can do just as well as boys in mathematics.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>50. Most subjects I can handle okay, but I have a knack for flubbing up math.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>51. The teacher should tell us how to do the problems and not ask a bunch of questions.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>52. Taking mathematics is a waste of time.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>53. Girls who enjoy studying math are a bit peculiar.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>54. I'm not the type to do well in mathematics</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>55. I would recommend this course to one of my enemies.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>56. I enjoy the challenge of answering questions in math class.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>57. Math doesn't scare me at all.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>58. Mathematics usually makes me feel uncomfortable and nervous.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>59. I would talk to my math teachers about a career which uses math.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>60. Doing proofs in geometry teaches you how to think logically.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>61. The challenge of math problems does not appeal to me.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>62. Studying mathematics is just as appropriate for women as for men.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>63. I have a lot of self-confidence when it comes to math.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>64. I usually have been at ease during math tests.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>65. It wouldn't bother me at all to take more math courses.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
<tr>
<td>66. I can create mathematics.</td>
<td>Strongly Agree</td>
<td>Agree</td>
<td>Undecided</td>
<td>Disagree</td>
<td>Strongly Disagree</td>
<td></td>
</tr>
</tbody>
</table>

252
Appendix C

Worksheets and Assignments
Name_________________________

Answer all questions working by yourself. This assignment is worth 20 points toward your homework grade.

1. Theorem: Vertical angles are congruent.
Illustration: In the diagram, angle 1 and angle 2 are congruent.

In each of the following diagrams, tell which angles are congruent.

A. 

B. 

2. Complete the statement for each diagram.

A. 

B. 

\[ m \angle ABD = m \angle \_ + m \angle \_ \]

\[ m \angle MNO = m \angle \_ + m \angle \_ \]

3. Complete the statement for each diagram.

A. 

B. 

\[ XZ = \_ + \_ \]

\[ AE = \_ + \_ \]

4. 

\[ m \angle TZW = \_ ? \]

Figure C.1: Preliminary Worksheet
5. In which of the following diagrams are there lines which appear to be perpendicular?

A.  
B.  
C.  
D.  
E.  
F.  

6. In which of the following diagrams do there appear to be right angles?

A.  
B.  
C.  
D.  
E.  

7. In which of the following diagrams do there appear to be parallel lines?

A.  
B.  
C.  
D.  

8. In the following diagram, which triangles appear to be congruent to triangle ABC?

A.  
B.  
C.  
D.  
E.  

9. Figure ACEG appears to be a ________.
10. Figure NOR is a  

11. In the diagram below, \( M \) is the midpoint of \( \overline{KT} \). Name all congruent segments.

12. In the figure below, \( BE \) bisects angle \( ABC \). Name all congruent angles.

13. In the figure below, \( D \) is the midpoint of line segment \( BE \). Which line segments are congruent? 

14. Fill in the blank.

\[ m \angle 1 + m \angle 2 = m \angle \] 

15. Given: \( m \angle AOC = m \angle BOD \). What conjectures can you make?

16. Given \( RS = PS \) and \( ST = SQ \). What conjectures can you make?
17. Given: $\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
What conjectures can you make?

18. Given angles $4$ and $6$ are supplementary. What conjectures can you make?

19. In the following diagrams, which angles are complementary?

20. Draw an angle complementary to angle $1$.


22. Theorem: If two lines are perpendicular, then they form congruent adjacent angles.

Illustration:

In each of the following diagrams, tell which angles if any are congruent.

A. 

B. 

C. 

Given: $\overline{XZ} \perp \overline{WY}$

Given: $\overline{CF} \perp \overline{AE}$

Given: $\overline{XZ} \cong \overline{WY}$

257
23. Theorem: If two lines form congruent adjacent angles, then they are perpendicular.

Illustration:

![Diagram showing two lines forming congruent adjacent angles.](image)

Given: $\angle 1 \cong \angle 2$

Then $\perp$

In each of the following diagrams, tell which lines are perpendicular.

A. 

B. 

C. 

24. Find the measures of all angles in the diagram below.

![Diagram with angles labeled 1, 2, and 3.](image)

25. Theorem: If two angles are supplements of the same angle, then they are congruent.

Illustration:

![Diagram showing two angles adjacent to a line labeled 1, 2, and 3.](image)

Given: $\angle 1$ and $\angle 2$ supplementary, $\angle 2$ and $\angle 3$ supplementary

Then $\angle 2 \cong \angle 3$

In the following diagrams, tell which angles are congruent.

A. 

B. 

C. 

Answer: 

Answer: 

Answer: 

258
26. **Theorem:** If two angles are complements of the same angle, then the angles are congruent.

Illustration:

\[ \angle 1 \quad \angle 2 \quad \angle 3 \]

In each of the following diagrams, tell which angles are congruent.

A.

\[ \angle 4 \quad \angle 5 \quad \angle 6 \]

Answer: __________

\[ \angle 5 \quad \angle 6 \quad \angle 3 \]

Answer: __________

\[ \angle 2 \quad \angle 3 \]

Answer: __________

27. **Given:** \( \angle 2 \cong \angle 3 \)

What conjectures can you make?

28. **Given:** Angle 1 and Angle 3 are supplementary. Name another angle supplementary to angle 1.

29. If angle QRS is a right angle, name two complementary angles.

30. (a) Name a supplement of angle AOE.

(b) Name a supplement of angle COD.

(c) Name a complement of angle EOD.

31. **Given:** \( \angle 1 \cong \angle 2 \)

What conjectures can you make?

32. **Given:** \( \angle 7 \cong \angle 8 \)

What conjectures can you make?
1. Make a conjecture about the relationship between $\angle 4$ and $\angle 5$.

Outline a proof for your conjecture.

2. Make a conjecture about the relationship between $\angle 5$ and $\angle 12$.

Outline a proof for your conjecture.

3. $\angle 4$ would be congruent to which of $\angle 9$, $\angle 10$, $\angle 11$, or $\angle 12$?

Outline a proof of your conjecture.

Figure C.2: Preliminary Homework Assignment
4. $\angle 6$ would be supplementary to which of $\angle 13, \angle 14, \angle 15$, or $\angle 16$?

Outline a proof of your conjecture.

5. $\angle 6$ would be congruent to which of $\angle 13, \angle 14, \angle 15$, or $\angle 16$?

Outline a proof of your conjecture.

6. Make a conjecture about the relationship between $\angle 9$ and $\angle 14$.

Outline a proof of your conjecture.
GEOMETRY QUIZ

1. In the diagram below, name a pair of
   a. Corresponding angles __________
   b. Alternate interior angles __________
   c. Same-side interior angles __________

2. Answer the following for the diagram below:
   a. An interior angle of triangle ABC is ∠_____.
   b. An exterior angle of triangle ABC is ∠_____.
   c. m∠1 + m∠2 + m∠3 = ________
   d. m∠_____ + m∠_____ = m∠4

3. Answer the following questions for the diagram below.
   a. An interior angle of triangle XYZ is ∠_____.
   b. An exterior angle of triangle XYZ is ∠_____.
   c. m∠2 + m∠5 + m∠12 = ________
   d. m∠_____ + m∠_____ = m∠9

4. Answer the following questions for the diagram below.
   a. m∠4 + m∠7 + m∠10 = ________.
   b. m∠12 + m∠9 = m∠____ + m∠____ and
      m∠12 + m∠9 = m∠____ + m∠____
   c. m∠3 = + m∠____ + m∠____
   d. Line n is a transversal which crosses lines l and m.
      Name a pair of alternate interior angles.
   e. Line p is a transversal which crosses lines l and m.
      Name a pair of corresponding angles.

Figure C.3: Preliminary Quiz

262
SIMILAR TRIANGLES

1. Given: DE \parallel AC

Prove: \triangle ABC \cong \triangle DBE

State any conjectures you can make about the triangles shown in numbers 2 through 4 and give arguments to support your conjectures.

2.

3.

4.

Based on number 1 above, show how you could find triangles similar to those shown in numbers 5 and 6.

5.

6.

Figure C.4: Worksheet on Similar Triangles
7. Label each of the following as right, acute, or obtuse triangles.

8. Label each of the following as equilateral, isosceles, or scalene triangles.

9. In the figure below, name all the similar triangles you can.

10. In the figure below, is $\triangle NOP \sim \triangle MOQ$? Explain.

11. Construct a triangle similar to $\triangle ABC$ with base on line $m$.

12. Given that the lines below are parallel, show how you could draw two similar triangles.
13. Quadrilateral $ABCD$ is a parallelogram. Find the values of $x$ and $y$.

14. Quadrilateral $PQRS$ is a parallelogram. Find the values of $x$ and $y$.

15. Quadrilateral $WXYZ$ is a trapezoid. Show two similar triangles and support your conclusions.

16. Given: $\angle 1 \equiv \angle 2$

   $\angle 3 \equiv \angle 4$

   State any conjectures you think are true and support your conjectures.
17. You are standing at point A which is 15 feet from the river. Explain how you could use similar triangles to find the width of the river at point B. (Assume the banks of the river are parallel.)

18. You are exactly 6 feet tall. Explain how you could use similar triangles to estimate the height of the flagpole.

19. Name all pairs of similar triangles in the figure below.

20. Figure OPQRST is a regular hexagon. Tell which triangles are similar and support your conclusions.
Part I: Tangents

Corollary to Theorem 9-1: Tangents to a circle from a point are congruent.
Example: PX is tangent to circle O at point X, PY is tangent to circle O at point Y.
Then PX = PY.

1. Quadrilateral ABCD is circumscribed about a circle. Discover and prove a relationship between AB + DC and AD + BC.

2. PA, PB, and RS are tangents. Explain why PR + RS + SP = PA + PB.

3. Given: Two tangent circles; EF is a common external tangent; GH is the common internal tangent.
   a. Discover and prove something interesting about point G.
   b. Discover and prove something interesting about \( \angle \text{EHF} \)

Figure C.5: Worksheet on Circles
Part II: Inscribed angles

Definition: An inscribed angle is an angle whose vertex is on a circle and whose sides are chords of the circle. In the figure to the right, ∠ABC is an inscribed angle.

1. Which of the following angles would be inscribed angles? Explain.

Theorem 9-7: The measure of an inscribed angle is equal to half the measure of its intercepted arc.

Example: \( \angle ABC = \frac{1}{2} \angle AC \)

2. Make a conjecture regarding the relationship between \( \angle PNM \) and \( \angle PQM \) in the diagram below. Prove your conjecture.

3. Make and prove a conjecture regarding \( \angle RST \) below. NOTE: \( RT \) is a diameter of circle Q.

4. Make and prove a conjecture regarding opposite angles of Quadrilateral ABCD which is inscribed in circle O.
GEOMETRY WORKSHEET - Pythagorean Theorem

Pythagorean Theorem: In a right triangle, the square of the hypotenuse equals the sum of the squares of the two sides.

Example: In Triangle ABC, \( c^2 = a^2 + b^2 \)

Show your work on all problems.
1. Find the value of \( c \) in the following:
   ![Triangle](image)
   \( A \)
   \( B \)
   \( C \)
   \( 3 \)
   \( 4 \)

2. Find the value of \( x \) in the following:
   ![Triangle](image)
   \( 10 \)
   \( 24 \)

3. Find the length of a diagonal of the rectangle below.
   ![Rectangle](image)

4. A 10 foot ladder leans against a wall so that the base of the ladder is 4 feet from the wall. Draw a diagram and then determine how high up on the wall will the ladder reach?

5. On the axes below, find the distance from the point (-2, -3) to (4, 5).
   ![Axes](image)

6. An airplane sits on a runway and is 300 feet from the base of a 1200 foot tower. The plane then taxis down the runway at the rate of 200 feet per minute for two minutes. How far is the plane from the base of the tower? How far is the plane from the top of the tower?

Figure C.6: Worksheet on the Pythagorean Theorem
In any triangle ABC,
if \( c^2 = a^2 + b^2 \),
then \( \angle C = 90^\circ \)

In any triangle ABC,
if \( c^2 < a^2 + b^2 \),
then \( \angle C < 90^\circ \)

In any triangle ABC,
if \( c^2 > a^2 + b^2 \),
then \( \angle C > 90^\circ \)

7. Referring to the diagram below, is triangle ABC a right, acute, or obtuse triangle? Explain.

8. Referring to the diagram below, is triangle ABC a right, acute, or obtuse triangle? Explain your reasoning.

9. In triangle RST below, which angle do you think would be largest? Explain your reasoning.

10. In triangle ABC, if \( \angle A = 67^\circ \) and \( \angle B = 84^\circ \), which side would be shortest? Longest? Explain your reasoning.

11. In triangles ABC and DEF below, if \( \angle A < \angle D \), what would be the relationship between BC and EF? Explain your reasoning.

12. In triangles QRS and TUV below, if RS > UV, what would be the relationship between \( \angle Q \) and \( \angle T \)? Explain your reasoning.
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