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TIME-DEPENDENT PATTERN FORMATION IN FLUID DYNAMICAL SYSTEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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ABSTRACT

Time-dependent states have been studied in three fluid dynamical systems. The first state is called “spatio-temporal intermittency” and is studied in the Taylor-Dean system, which concerns the flow found in a fluid driven by rotating concentric cylinders with a partially filled gap. The second investigation considers the flow found in two immiscible fluid layers driven by a temperature gradient imposed perpendicular to the interface, the so-called “two-layer Rayleigh-Bénard system”. The third time-dependent state is found in the “Weissenberg effect,” which is the climbing up a rotating rod by a non-Newtonian fluid.

Spatio-temporal intermittency is a flow state with both laminar and turbulent regions co-existing in time and space. This state, as found in the Taylor-Dean system, is analyzed in the framework of second-order phase transitions using statistical methods.

The two-layer Rayleigh-Bénard system is explored using silicone oil and a perfluorinated hydrocarbon as the working fluids. The apparatus has an annular geometry and uses horizontal shadowgraphs to probe the flow. A phase diagram has been produced showing the onset temperature difference vs. $l_1$, the ratio of the depth of the bottom layer to the overall cell depth. A time-dependent state, which involves
changes in the alignment of the convection rolls in the layers. is found at a temperature difference slightly larger than that required for convection. This time-dependent state has been characterized by finding the wavelengths and periods of the pattern as well as the velocities of the rolls.

Lastly, the Weissenberg effect has been shown to exhibit the "Ruelle-Takens" route to chaos. The sequence of instabilities from one frequency to two frequencies and finally three frequencies has been documented at various temperatures. It is found that the frequencies of the primary and secondary instabilities are constant fractions of the rod rotation rate. It is also found that these fractions are independent of temperature.
To Carol and Merle.

my parents.
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CHAPTER 1

Introduction

Nature has surrounded us with many different types of fluid dynamical systems. Each system has an impact on life in some direct or indirect manner. For example, flow in the mantle pushes around the plates of the crust and rivers carry ships to the ocean. All of these fluid systems are driven by some source of energy. Gravity pulls water down a hill and the core heats the mantle. Each system deals with this energy in a different way, typically organizing itself into some type of flow pattern. Some of these patterns are benign while others can be very malignant. i.e., the atmosphere organizing itself into pretty cloud formations and gentle spring breezes, or into deadly tornadoes and hurricanes. Oceans can nonchalantly lap against the beach or form waves that toss ships. A better understanding of how fluid flows organize themselves into these flow patterns in response to external forces should be useful in predicting the more violent and dangerous types of these patterns and in understanding fluid flow. (For a review of pattern formation, see Cross and Hohenberg[8].)

Since it is extremely difficult to deal with the large number of variables in real-life systems (i.e., hurricanes depend on ocean currents, wind speed, wind shear, heat exchange between the ocean and atmosphere, moisture content of the air, Coriolis
forces and more), well-controlled experiments are needed to shed some light on the more complex natural phenomena. Typically these systems take advantage of some aspect of nature (e.g., symmetry) to simplify the problem. Two of the first and most studied experiments are the Taylor-Couette and the Rayleigh-Bénard systems. For a review of both systems see Koschmieder[3] or Busse[9], for just the latter see Normand et al. [10]. The Taylor-Couette system consists of two concentric cylinders with a small gap between them. The gap is filled with some type of fluid that is driven into motion by rotating one or both of the cylinders, either in the same direction or the opposite direction. Varying the rotation rates and relative direction of the cylinders leads to many different types of patterns [11]. The flow can be as simple as a featureless azimuthal rotation at slow speeds (called Couette flow), or as complex as spiral (or barber pole) turbulence. Many of these flow states are understood, but the origins of some are still a mystery.

The Rayleigh-Bénard system consists of a layer of fluid bounded above and below by plates which are either heat-conducting or insulating. A temperature difference, $\Delta T$, is maintained between them, typically with the hotter on the bottom. When $\Delta T$ is small, heat is conducted from the hot plate to the cold plate and the fluid is motionless. When $\Delta T$ exceeds some critical value, $\Delta T_c$, the fluid starts to convect. This flow organizes itself into many different patterns depending on how far $\Delta T$ is above critical, the size of the apparatus and the nature of the fluid[12]. As in the Taylor-Couette system, many aspects of Rayleigh-Bénard convection are understood while some questions remain.
One of the more interesting aspects of both the Taylor-Couette and Rayleigh-Bénard systems is that both show time-dependent flow states at appropriate values of the control parameter ($\Delta T$ in the Rayleigh-Bénard system, rotation rates of the cylinders in the Taylor-Couette system). Since most natural phenomena exhibit time dependence, it is important to understand time dependence in these laboratory apparatus. Unfortunately, these time-dependent states occur at control parameters above the onset to the first pattern. It would be useful to study systems that exhibit time dependent states at or near the onset to pattern formation. This work will describe experiments with three such systems, namely the Taylor-Dean system, the two-layer Rayleigh-Bénard system and the Weissenberg effect.

The Taylor-Dean system has been shown to exhibit time dependent states at onset with the outer cylinder rotating and the inner cylinder stationary by Mutabazi and Andereck[13]. One of these states is called “spatiotemporal intermittency” (STI) and is the focus of this work. STI is characterized by the co-existence of laminar and turbulent regions in the flow. Chapter 2 will discuss STI and experiments which characterize it in the Taylor-Dean system.

The two-layer Rayleigh-Bénard system has also been shown to exhibit time dependent states by Colovas[14] and Andereck. et al. [15] in a rectangular geometry. Busse and Sommerman[16] have found time dependence in an annular geometry using thermistors to probe the flow. Chapter 3 will discuss the two-layer Rayleigh-Bénard system and describe the time dependent states found in an annular geometry using an optical technique to probe the flow.
Yet another system that exhibits time-dependent states is the climbing up a rotating rod by a non-Newtonian fluid. This phenomenon was first characterized by Weissenberg[17] and has been given the name "Weissenberg Effect". Experiments characterizing this phenomenon will be discussed in Chapter 4.
CHAPTER 2

Spatiotemporal Intermittency in the Taylor-Dean System

2.1 Introduction

Turbulence has perplexed scientists and engineers for a long time. It can play a part in everything from how well a clothes washer works to the safety of passengers on an airplane. Great strides have been made during the last few years towards understanding turbulence [18, 19, 20], but there is still room for progress. Both the phenomenon and the underlying causes of turbulence are difficult to understand. One potentially useful approach to understanding turbulence is to understand the transition from organized flow to turbulent flows. One of the routes to turbulence has been given the name "spatiotemporal intermittency" or STI. STI generically is characterized by the co-existence of laminar (organized) and turbulent flows side-by-side in the same system.

Interest in STI was piqued by the work of Pomeau [21]. Pomeau conjectured that a similarity existed between STI and directed percolation and that STI may show some features of second-order phase transitions. STI has been found in many
diverse experimental systems including: boundary layer flow\cite{22}: plane Poiseuille flow\cite{23, 24}: the Taylor-Couette system\cite{11, 25, 26}: pipe flow\cite{27}: Rayleigh-Bénard convection\cite{28, 29, 30, 31}: the printer's instability\cite{32, 33, 34}: the Faraday system\cite{35}: and electromagnetically forced linear vortices\cite{36}. Work on STI can also be found in numerical systems such as: coupled map lattices\cite{37, 38, 39}: the damped Kuramoto-Sivashinsky equation\cite{40}: the complex Ginzburg-Landau equation \cite{41}: and probabilistic cellular automata \cite{42}. Many of these works show Pomeau's conjecture has some validity. The works also show the characteristic features of STI are slightly different from system to system and therefore, STI is not universal in character. This lack of universality has been attributed to either the different nature of the laminar or background states or to the existence of spurious long wavelength correlations in the system\cite{43}.

This chapter will focus on a study of STI in the Taylor-Dean system as reported by Degen, Mutabazi, and Andereck\cite{44}. Various techniques will be used to characterize the STI. Like previous work, it will also show STI is not universal in character but it does have features of second-order phase transitions.

2.1.1 The Taylor-Dean System

As mentioned in Chapter 1, the Taylor-Dean system is a variant of the classic Taylor-Couette system. The Taylor-Dean system used in this experiment has been described in \cite{1}. It consists of two horizontal, concentric cylinders with a small gap between them. The inner cylinder is made from black Delrin plastic with radius $a = 4.49\, cm$. 
The outer cylinder is made from Duran glass and has radius \( b = 5.08 \text{ cm} \). The gap is labeled \( d = b - a = 0.59 \text{ cm} \) and the radius ratio is defined as \( \eta = a/b = 0.883 \). The cylinders can rotate independently, but for this work the inner is stationary and the outer rotates with speed \( \Omega \). The cylinder is driven into rotation by a CompuMotor\textsuperscript{TM} stepper motor Model AX33-93. The motor is controlled with a CompuMotor AX Series controller interfaced to a PDP-11 computer. The gap between the cylinders is partially filled with fluid, leaving an air space at the top. See Figure 2.1 for a cross-sectional view. The filling level is chosen to be \( n = \Theta_f/2\pi = 0.75 \) (where \( \Theta_f \) is the angular measure of the azimuthal extent of the fluid), although the particular value does not have a significant influence on the phenomena reported here[1]. The end conditions of the system are fixed by a set of rings that are attached to either the inner cylinder or to the inner surface of the outer cylinder at a distance \( L = 53.40 \text{ cm} \) apart. This gives an aspect ratio \( \Gamma = L/d = 90 \), large enough to realize a one-dimensional extended system. The set of rings that attaches to the outer cylinder are made from Teflon, while the second set that attaches to the inner cylinder are made from Plexiglas.

The fluids used in this study, either water or a 28\% by mass mixture of glycerol and water, are Newtonian and are characterized by their kinematic viscosity \( \nu \) and density \( \rho \) (see Table 2.1). The equations governing the flow in the Taylor-Dean system are the continuity equation

\[
\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} = 0 \tag{2.1}
\]
Figure 2.1: Geometry of the Taylor-Dean apparatus[1].
<table>
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<th>CGS Units</th>
<th>Water</th>
<th>28% Glycerol in Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\nu) kinematic viscosity</td>
<td>(cm^2 s^{-1})</td>
<td>(0.98 \cdot 10^{-2})</td>
</tr>
<tr>
<td>(\rho) density</td>
<td>(gm cm^{-3})</td>
<td>(0.998)</td>
</tr>
</tbody>
</table>

Table 2.1: Properties of the fluids used in the Taylor-Dean system [6].

and the cylindrical form of the Navier-Stokes equation[45]

\[
\frac{\partial v_r}{\partial t} + (\mathbf{v} \cdot \nabla)v_r = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \nabla^2 v_r - \frac{2}{r^2} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} \right) \tag{2.2}
\]

\[
\frac{\partial v_\phi}{\partial t} + (\mathbf{v} \cdot \nabla)v_\phi + \frac{\nu v_r}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \phi} + \nu \left( \nabla^2 v_\phi + \frac{2}{r^2} \frac{\partial v_\phi}{\partial \phi} - \frac{v_\phi}{r^2} \right) \tag{2.3}
\]

\[
\frac{\partial v_z}{\partial t} + (\mathbf{v} \cdot \nabla)v_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 v_z \tag{2.4}
\]

where

\[
(\mathbf{v} \cdot \nabla)f = v_r \frac{\partial f}{\partial r} + \frac{\nu v_r}{r} \frac{\partial f}{\partial \phi} + v_z \frac{\partial f}{\partial z} \tag{2.5}
\]

and

\[
\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}. \tag{2.6}
\]

Following the work of Normand et al. [46], the velocity is scaled by \(\Omega b\). distance by \(d\). pressure by \(\rho \nu \Omega b / d\) and time by \(d^2 / \nu\).

The dimensionless boundary conditions on the bulk flow at the cylinder walls are

(inner cylinder stationary)

\(v_\phi(a) = 0\) and \(v_\phi(b) = \Omega.\) \(\tag{2.7}\)

At the free surfaces, the boundary conditions are

\[\frac{\partial v_r}{\partial \phi} = 0\) and \(v_\phi = 0.\] \(\tag{2.8}\)
Analysis of the bulk flow by Mutabazi et al. [2] has shown the velocity profile found in Figure 2.2, where \( x \) is a dimensionless radial variable from the inner cylinder to the outer cylinder. The flow at the free surfaces takes the form of a recirculation roll as found by Normand et al. [46].

Following the paper by Mutabazi et al. [47], the control parameter \( \Omega \) is rescaled to be the Taylor Number:

\[
Ta = \frac{\Omega bd}{\nu} \left( \frac{d}{a} \right)^{1/2}.
\]  

(2.9)

All of the results are reported as functions of \( Ta \).

Visualization of the flow pattern is achieved by adding 1\% (by volume) Kalliroscope\textsuperscript{TM} AQ 1000[48] to the working fluid. The Kalliroscope flakes align with velocity gradients in the flow. In a typical roll pattern, flow along the cylinder axially or azimuthally appears bright due to reflections from the flakes. Flow in the radial direction appears dark because the flakes are seen edge-on and have little surface area to reflect the light. Most of the light travels past the flakes to the black inner cylinder. In the data presented below, dark (low intensity) regions correspond to (roughly) radial flow, while light (high intensity) regions are axial or azimuthal flow.

### 2.2 Experimental procedure

The experiments begin with mixing the solution of water, glycerol and Kalliroscope (or just water and Kalliroscope). The solution (the procedure is the same for both) is allowed to mix in the system at high \( Ta \) for at least 12 hrs prior to data capture.
Figure 2.2: Base flow velocity profile far from the free surfaces: $V(x) = x(3x - 2)$. The flow has a nodal surface at $x_0 = 2/3$ and it is potentially centrifugally unstable in the zone $1/3 < x < 2/3$ and stable elsewhere[2].
This is to insure complete mixing and to remove any spurious temperature variations. The accuracy of the solution is determined by systematically increasing the control parameter $\Omega$ from laminar flow, until the flow state called “stationary Dean rolls” is found, see Figure 2.3. Dean rolls are the first instability found in “Dean flow”. Dean flow is flow around a curved surface driven by a pressure difference. This state occurs at $Ta = 115[1]$, see Figure 2.4. Taking this value of $Ta$ and the experimentally determined onset value of $\Omega$ allows a calculation of $\nu$ using Equation 2.9. Once the value of $\nu$ is known, the solution has been characterized and further exploration can proceed.

The investigation of STI begins with setting the control parameter. $Ta \sim 150$. This puts the system in the stationary Dean rolls (see Figure 2.3) just below the onset of oscillatory Dean rolls, see Figure 2.4. After an hour wait, $Ta$ is increased by roughly 1.5% or less. (Typical values for the increase in $Ta$ are $\Delta Ta \approx 2.6$.) A period of 20 min is allowed to pass for the system to reach a steady state before a data file is captured. The data consists of the intensity of light reflected off of the Kalliroscope present in the flow. High intensity areas show flow perpendicular to the line of sight, while dark areas represent radial flow parallel to the line of sight. The light is collected with a 1024 pixel linear CCD array focused 1 cm below the free surface on the front face. (The front face is defined as the face that has the outer cylinder traveling upward.) The data is captured from the middle 30 cm of the system along the axis. The CCD array was interfaced to a PDP-11 Computer through a CAMAC system. 5000 lines of data are collected at intervals of $\Delta t = 0.14$ sec between the lines. which
Figure 2.3: Photograph of Dean rolls. \( Ta = 124 \), showing middle 30 cm of system.
Figure 2.4: Diagram of the different states observed in the Taylor-Dean system when the inner cylinder is fixed. Photographs of some of these states can be found in Figures 2.3, 2.5, 2.6 and 2.7.
Figure 2.5: Photograph of the Taylor-Dean system. $Ta = 244$, showing middle 30 cm of system. Some turbulence is present on the right and left edges.
Figure 2.6: Photograph of the Taylor-Dean system. $Ta = 265$, showing middle 30 cm of system. Both turbulent and laminar regions can be seen.
Figure 2.7: Photograph of the Taylor-Dean system. $Ta = 283$, showing middle 30 cm of system. Nearly all of the flow is turbulent.
gives a total time of approximately 12 min for a complete data file. Finally, \( Ta \) is increased by \( \Delta Ta \) again and the procedure is repeated. A data set typically consists of about 50 data files, which covers a range of \( Ta \approx 150 \rightarrow 300 \).

The data files are transferred to the Physics Department Vax cluster using a bipedal data transfer algorithm. Once on the Vax, the analysis begins by creating a space-time diagram. Space-time diagrams are produced by plotting the data as intensity versus axial-position at the given time interval, see Figures 2.8, 2.9, 2.10 and 2.11. These plots can then be analyzed to determine the flow state, and the wavelengths and velocity of the pattern, in addition to being the starting point for many other analysis procedures.

### 2.2.1 Binarization of the space-time diagrams

To efficiently extract a useful picture of the behavior in the intermittent state, it is necessary to first identify the turbulent and laminar domains. A binary representation is the simplest approach and the one usually taken. Binarization of the space-time diagrams makes use of the fast time variation within the turbulent bursts compared with laminar regions. It also emphasizes the interest is in the burst itself, not the structure of the turbulence. Taking advantage of the latter, the data is first reduced by taking an average of 10 space points. This reduces the data set from 1024 pixels in space to 102. The next step is to calculate the second difference in time of the intensity pattern \( I''(t) = I(t+2) - 2I(t+1) + I(t) \). To determine if a pixel is turbulent, the second difference \( I'' \) is compared to the space average of \( I'' \). If the
Figure 2.8: Space-time diagram of the traveling roll state in the Taylor-Dean system at $Ta = 100$. The horizontal axis is the spatial dimension $x$ scaled by the gap size $d$. The vertical axis is the time $t$ scaled by the viscous diffusion time $d^2/\nu$. 
Figure 2.9: Space-time diagram of the mixed state of traveling and stationary rolls for $Ta = 111$. 
Figure 2.10: Space-time diagram of the stationary roll pattern at $Ta = 115$. 

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Figure 2.11: Space-time diagram for the oscillatory roll pattern at $Ta = 174$. 
Figure 2.12: Space-time diagram of pattern at $Ta = 235$, slightly above onset to STI.
Figure 2.13: Space-time diagram of pattern at $Ta = 240$, showing both turbulent and laminar domains.
Figure 2.14: Space-time diagram of pattern at $Ta = 254$. Turbulence almost completely fills the system.
difference. $I'' - < I'' >$, is greater than an arbitrary cutoff value. the pixel is said to be turbulent. otherwise it is labeled laminar. This produces a binary space-time diagram of laminar and turbulent regions.

Since the binarization procedure is not perfect, a data massaging routine is then applied. The routine makes use of the observation that a turbulent burst is completely turbulent within its boundaries in space and time. i.e., there should not be laminar pixels inside a turbulent burst. Also, the turbulent bursts are of a finite size and spurious turbulent pixels must be removed. To implement this removal process, the routine’s first step is to check if each turbulent pixel has two or more turbulent neighbors. If it does, it remains turbulent. If it does not, it is switched to laminar. The boundary pixel ($i = 1$) for each line is used to determine the number of neighbors for the second ($i = 2$) pixel and then ignored. The pixel at the end of the line is handled similarly. This results in a loss of $\sim 5$ pixels around the edges of the data file after the completion of the procedure. In a second pass over the data, if any laminar pixel has three or more turbulent neighbors, it is switched to turbulent. A third pass is similar to the first, except that a turbulent pixel must now have three or more neighbors to remain turbulent. During the next pass over the data, if the $i^{th}$ pixel is turbulent and the $i + 5$ (in space) pixel is turbulent, all pixels in between are also considered turbulent. The fifth pass involves the same procedure, except it checks the fifth pixel away in time and the last pass checks the fifth pixel in space again. This routine is designed to remove spurious turbulent pixels and to fill in gaps
in the turbulent domains that are not real. A version of this program can be found in Appendix A.

Variations of this routine have been used with different nearest neighbor requirements, and different massaging lengths. A comparison with the raw data shows the procedure is insensitive to the parameter variations when applied with a corresponding slight adjustment of the cutoff value. In addition, the procedure as described above was applied with numerous cutoff values. The binarized output then is compared with the raw data to ensure its accuracy. Examples of binarized space-time diagrams can be found in Figures 2.15, 2.16 and 2.17 where dark is turbulent and light is laminar. The binarized space-time diagram serves as the starting point to calculate numerous statistical properties of STI.

2.2.2 Spatial average power spectrum of the space-time diagrams

To further characterize the space-time diagrams, a spatial average power spectrum is calculated. The first step in the process is to subtract out the average of each line of data. Each shifted line of data is then run individually through the power spectrum routine found in Numerical Recipes[49]. The power spectra from each line in the file are then averaged to produce the required spatial average power spectrum, hence referred to as the power spectrum. Spatial average power spectra can be analyzed to determine the wavelength(s) of the flow pattern. As will be described below, they
Figure 2.15: Binarized version of Figure 2.12. The dark regions indicate the turbulent flow. Some pixels around the edges of the plot are lost due to the binarization procedure.
Figure 2.16: Binarized version of Figure 2.13.
Figure 2.17: Binarized version of Figure 2.14. The system is almost completely turbulent.
will also be used to calculate the background noise fraction $f_N$. Some examples of power spectra can be found in Figures 2.18, 2.19 and 2.20.

2.3 Experimental results

2.3.1 Sequence of pattern states

The sequence of the pattern states observed in the system is given in Figure 2.4. The first roll pattern occurs in the form of traveling inclined rolls at approximately $Ta = 97$. These rolls appear near the recirculation zone and their axial extent grows with the control parameter until they fill the whole system (Figure 2.8). The transition does not show any hysteresis when ramping up and down within our resolution of 1%. The drift velocity $v_d = 27$ and the wavelength of the pattern is $\lambda = 1.170[47]$.

For $Ta \in [100.115]$, the pattern is a mixed state of traveling rolls superposed on a stationary roll pattern with different wavelengths $\lambda_1$ and $\lambda_2$ (Figure 2.9).

For $Ta = 115$, the mixed state becomes stationary. This transition is subcritical and highly hysteretic. It is characterized by the existence of a front separating the disappearing chaotic state and the emerging almost stationary roll pattern (Figure 2.3). The space-time plot exhibits an asymmetric roll pattern where large and small rolls alternate in a chaotic way, giving rise to mean wavelengths in a ratio $2/3$ (Figure 2.10)[13].

The stationary roll pattern becomes unstable to monoperiodic oscillations for $Ta \in [150.203]$ (Figure 2.11), followed by biperiodic oscillations for $Ta \in [203.225]$. 

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Figure 2.18: Power spectrum at $Ta = 235$. Most of the power is in a few dominate frequencies indicating a small background noise fraction $f_N$. 

32
Figure 2.19: Power spectrum at $Ta = 240$. The background is increasing indicating a growing $f_N$. 
Figure 2.20: Power spectrum at $Ta = 254$. Most of the power is found in the background, indicating a large $f_x$. 
The biperiodic oscillations are characterized by two different frequencies. The first oscillation frequency corresponds to a wavy-vortex-like instability while the second frequency corresponds to a modulational-type instability. It has been observed in particular that the oscillations occur between two neighboring rolls inside the large wavelength and have properties similar to a collective mode with fixed nodes.

Upon a further increase of $T_a$, the oscillating pattern exhibits a continuous nucleation of spatiotemporal defects (collisions at some space positions and roll generation at others) which reduce the correlation length of the pattern and induce a chaotic behavior. These spatiotemporal defects generate turbulent bursts in our system which last a short time before disappearing from the flow pattern. See Figures 2.6 and 2.12. For larger values of $T_a$, the spatiotemporal intermittency is sustained and the typical size of laminar domains decreases. Examples of this can be found in Figures 2.7, 2.13 and 2.14.

The flow state beyond STI is very turbulent and has not been characterized by the available experimental tools. It should be mentioned that the flow exhibits some underlying spatially coherent rolls with a mean wavelength comparable to the large wavelength of the stationary pattern.

2.3.2 Spatiotemporal intermittency

Spatiotemporal intermittency is characterized by the coexistence of a spatially ordered pattern (considered laminar flow) with turbulent patches or bursts which appear
irregularly in time and in different positions. This can be seen in the raw space-time diagrams and their binarized versions. Figures 2.12 & 2.15, 2.13 & 2.16, and 2.14 & 2.17. By using an analogy with thermodynamics, STI can be considered as a two-phase state with one stable and the other metastable. The first turbulent bursts result from the Eckhaus-type instability when rolls collide or are generated. This process forces the pattern to adjust its wavelength and to nucleate localized perturbations which will be driven away in the azimuthal direction. These bursts occur spontaneously, do not propagate into the neighboring laminar state, and decay rapidly. For higher $Ta$, the turbulent bursts grow in time and size by propagating into neighboring laminar domains. The turbulence spreads in this regime: the size of laminar domains diminishes as the control parameter increases, giving rise to an almost completely turbulent state. Therefore we distinguish two dynamic types of turbulent bursts: localized turbulent patches resulting from the nucleation of defects and active turbulent bursts which tend to destroy laminar domains.

Behavior of the turbulent fraction

Using the binarization procedure on the space-time diagrams of the flow pattern as described above, we have represented in black the turbulent domains and in white the laminar ones (see Figures 2.15, 2.16 and 2.17). This representation allows us to easily measure the time-averaged spatial turbulent fraction (referred to simply as the turbulent fraction, $f$) as well as the temporal turbulent fraction (called also the intermittency factor, $\gamma$). $f$ is defined as the ratio of the total area of turbulent
patches to the total area of the space-time diagram. $\gamma$ is the ratio of the mean duration of turbulent patches to the total measurement time at a particular point in space. Operationally, $f$ is calculated by simply counting the number of turbulent pixels in the binarized space-time diagrams and dividing by the total number of pixels. $\gamma$ is calculated by counting the number of turbulent pixels along one line in time of the space-time diagram and dividing by the length of that line.

The turbulent fraction plays the role of the order parameter in this problem and varies with the control parameter $Ta$ (Figures 2.21 and 2.22). The behavior of the turbulent fraction depends on the boundary conditions imposed on the flow: whether the end rings are attached to the inner cylinder or rotate with the outer cylinder affects the details of the transition process. Since the behavior of $f$ and $\gamma$ are similar, the following will concentrate only on the time-averaged spatial turbulent fraction, $f$.

When the end rings are attached to the fixed inner cylinder, the transition to spatiotemporal intermittency occurs via a perfect bifurcation with a definite onset. The spatial turbulent fraction exhibits a net threshold at $Ta_0 = 228 \pm 1$ indicating the transition to STI (Figure 2.21). Near the onset of STI, the turbulent fraction grows as a power law with $Ta$ (Equation 2.10).

$$f(Ta) = f_0 \left(\frac{Ta - Ta_0}{Ta_0}\right)^\beta$$

(2.10)

The exponent is measured as $\beta = 1.30 \pm 0.26$. The intermittency factor also shows a net transition with a similar behavior (Figure 2.22).

When the end rings are attached to the rotating outer cylinder, the rings induce Ekman cells which create turbulence near the ends for $Ta \geq 190$. This end region
Figure 2.21: Time averaged spatial turbulent fraction as a function of the control parameter: the crests (+) correspond to the case when the end rings are attached to the inner cylinder, the squares (□) correspond to the case when the end rings rotate with the outer cylinder. In the latter case, the imperfect nature of the bifurcation comes from the Ekman cells, which induce a localized turbulence to the ends before the onset of spatiotemporal intermittency in the bulk flow.
Figure 2.22: Intermittency factor measured at different positions along cylinder axis:
(+) at $x = 8.7$. (□) at $x = 16.8$. (*) at $x = 29.1$ and (◊) at $x = 38.3$. 
turbulence forces the oscillatory Dean rolls and induces localized turbulent bursts. In this case, the transition to STI is slightly modified and becomes imperfect. Therefore, for $Ta < Ta_0$, turbulent bursts already occur in the flow, while at $Ta = Ta_0$, the turbulent fraction has grown to approximately 10% (Figure 2.21). We have measured the different parameters for this regime and have found that other than the imperfect nature of the transition and the slope of the curves $f(Ta)$, the other characteristics were less modified by the end turbulence. Therefore, STI is a robust property of the flow and the imperfection has a real impact only on details of the transition.

**Behavior of the noise fraction**

Analysis of the spatial average power spectrum described above shows a systematic increase in the broadband background noise as $Ta$ is increased. In order to characterize this more fully, the noise fraction $f_N$ is calculated. $f_N$ is defined as the ratio of the area in the background to the total area of the spatial power spectrum. The noise fraction behaves essentially the same as the turbulent fraction but has the added benefit of not depending on an arbitrary cutoff, see Figure 2.23. The similarity between the noise and turbulent fractions is further proof of the robust character of STI and validates the procedure used to calculate the turbulent fraction.
Figure 2.23: Noise background fraction ($f_N$) as an alternative measure of the turbulent fraction ($f$).
Distribution of the laminar and turbulent domains

One of the ways to characterize STI used previously by Chaté and Manneville[40, 38] and others[30, 31, 29, 26] is to look at the distribution of laminar and turbulent domains as a function of their size, both in length and in time. These histograms give a quantitative picture of the global average spatial (or temporal) structure of the flow[38]. For example, a histogram exhibiting an exponential dependence shows the existence of a length scale present in the flow. A histogram with an algebraic dependence shows the lack of a finite characteristic scale (or alternatively that all scales are present in the flow). A crossover from one type of dependence to the other indicates a change in the nature of the flow. For example, Willaime, Cardoso and Tabeling[36] report a crossover from algebraic to exponential dependence in their study of forced electromagnetic vortices. They explain this crossover by differences in the nature of the turbulence. The algebraic regime is characterized by isolated patches of turbulence while the exponential regime is characterized by connected patches of turbulence. (Connected patches first appear in the system as distinct, separated turbulent areas. They subsequently expand and connect into large patches later in time. The isolated patches appear and disappear without connecting to their neighbors.)

The procedure to find these distributions starts with the binarized space-time diagram and simply measures the size of each turbulent (or laminar) domain. Once the length of a particular domain is known, the appropriate counter in the histogram is increased and another domain can be analyzed. Example histograms can be found
in Figures 2.24 and 2.25. The program implementing this procedure can be found in Appendix A. Once the distribution is found, its shape is fit with the appropriate functional form using the commercially available program Origin™ by MicroCal.

For values of $Ta$ close to the onset of STI, the distributions of the laminar domains have algebraic dependence, as

$$N(l) \propto l^{-\mu}$$

(also called a power law) over a factor of five range in domain size (Figure 2.24). somewhat smaller than found in other experiments on STI [29]. We have found that, for 3 separate values of the cutoff, the exponent $\mu = 1.7 \pm 0.1$ is constant for $Ta \in [228, 234]$. It should be noted that domains smaller than $\ln(x) = 1$ represent laminar structures of the order of the roll size and smaller. As previously discussed, the internal structure of the turbulent burst is outside the scope of this work and therefore the smallest domains will not be included in the fits. Furthermore, the longer domains (close to the size of the system) in the histograms of the algebraic regime exhibit some exponential falloff reminiscent of the crossover regime found in Rayleigh-Bénard convection [29].

As mentioned previously, the existence of the algebraic regime shows the laminar domains appearing in the system at all length scales. i.e., there is no preferred laminar domain size. This is due to the nature of the turbulent domains. The turbulence is appearing spontaneously throughout the flow and quickly disappears. Whenever a turbulent burst occurs, a long laminar domain is split into two shorter ones[38]. Because there are no preferred locations for the bursting to occur, the laminar domains
Figure 2.24: Histogram of the laminar domains in the algebraic regime ($Ta = 233$). Dotted line is fit with Equation 2.11 to region bounded by vertical lines.
Figure 2.25: Histogram of the laminar domains in the exponential regime ($Ta = 237$). Dotted line is fit with Equation 2.12 from vertical line to end of data.
are split in a random fashion with no dominate characteristic lengths. Since the
turbulent fraction \( f \) (Figure 2.21) is small in this regime, the turbulent bursts appear
infrequently and (probably) one at a time. The laminar domains are bounded on one
domains are bounded on one end by turbulence and on the other by the system boundary and hence, all lengths up
to the total system length appear in the flow. This also implies that an infinite system
would still have infinite laminar domains even with a small amount of turbulence
present in the flow. This becomes easier to understand with the identification that an
algebraic dependence is mathematically similar to an exponential dependence with an
infinite characteristic length[50]. Therefore, the algebraic dependence will break down
when something appears in the flow to constrain the laminar domains other than the
ends of the system. The only candidate to supply this constraint is another turbulent
burst. When two turbulent bursts appear in the system at the same time, the laminar
domain between them will be constrained and have a definite characteristic size. As
\( Ta \) is increased, \( f \) becomes larger and the probability for finding two (or more)
turbulent domains in the system at the same time increases. When this probability
becomes large enough, the histograms no longer show an algebraic dependence.

In place of the algebraic decay, the distribution of the laminar domains with their
size follows an exponential law for \( Ta \in [237, 270] \).

\[ N(l) \propto e^{-ml}. \]  

(2.12)

also called exponential decay. The quantity \( 1/m = l_c \) is the characteristic width of
the laminar domains. It decreases with \( Ta \) with power law dependence as

\[ l_c(Ta) = l_0(Ta - Ta_c)^{-\omega}. \]  

(2.13)
where $\alpha = 0.64$ (Figure 2.26), meaning that the scales of the laminar domains are becoming small rapidly as $Ta$ is increased in the exponential regime. The value $Ta_c = 237$ was chosen to be the threshold of the second transition in STI: below this value, the turbulent domains are embedded in a laminar background and above it, the laminar zones are embedded in a turbulent background. The distribution of the turbulent domains exhibits only an exponential regime following Equation 2.12 where now $1/m = l_i$ is the characteristic width of the turbulent bursts. Their characteristic width $l_i$ increases with $Ta$ as in Figure 2.27. As mentioned above, the turbulent domains enter the system with a characteristic size and therefore never show a power law region.

The temporal histograms show similar behavior. In particular, the algebraic regime and exponential regimes are seen: the power law gives an exponent $\mu_t = 1.7$ for $Ta \in [228, 237]$ and the characteristic time (lifetime) of the laminar domains decreases with the control parameter after the law in Equation 2.13 where $\alpha_t = 0.73$. The lifetime of the turbulent domains grows with $Ta$ (Figure 2.28).

Correlation function and length

To complete the statistical analysis of STI in the Taylor-Dean system, we have computed the correlation function and correlation length of the binarized space-time plots. The (spatial) correlation function is defined as

$$C(x) = \frac{<I(z+x,t)I(z,t)>}{<I(z,t)^2>} \quad (2.14)$$
Figure 2.26: Inverse of the characteristic width of the laminar domains in the exponential regime. Solid line is fit with Equation 2.13 to region bounded by vertical lines.
Figure 2.27: Characteristic width of the turbulent domains in the spatiotemporal regime (with rotating end rings).
Figure 2.28: Lifetime of laminar domains as a function of the control parameter.
where $l(z,t) = 0$ or 1. Below the transition we assign 1 to the turbulent pixels and 0 to the laminar pixels. We reverse the assignments above the transition. This switch is necessitated by the fact that the correlation length of the most prominent component of the flow is effectively infinite (or the system size). Below the transition the laminar state dominates, while above the transition the turbulent state dominates. It is most useful to calculate the correlation length for the secondary state in a background of the dominate state. This method of calculating the correlation function is borrowed from the study of spin lattices [51, 52], where only the fluctuations on top of the background are important. From the resulting correlation function (Figure 2.29) we have calculated the correlation length $\xi$ by fitting it with Equation 2.12 where now $1/m = \xi$. This is done separately for both laminar and turbulent domains. The obtained correlation lengths diverge near $T_{ac} = 237$ (Figure 2.30) showing that there is a separation between two distinct regimes. For $Ta < T_{ac}$ the correlation length of the turbulent domains grows and becomes infinite at $Ta = 237$, while for $Ta > 237$, the correlation length of the laminar regions decreases. Therefore, the correlation length divergence can be considered as the hallmark of the critical point corresponding to the transition to the exponential regime. The behavior of the correlation length near $T_{ac}$ can be represented as follows

$$
\xi \sim \begin{cases} 
(Ta - T_{ac})^{-\nu} & \text{for } Ta > T_{ac} \\
(T_{ac} - Ta)^{-\nu'} & \text{for } Ta < T_{ac} 
\end{cases}
$$

(2.15)

The best fit gives the following values of the critical exponents for the correlation length: $\nu \approx 0.53, \nu' \approx 1.20$. Therefore, for $Ta < T_{ac}$, the turbulent domains are the fluctuations developing in a laminar background. while for $Ta > T_{ac}$, the laminar
zones are the fluctuations dying in the turbulent background. The difference between \( \nu \) and \( \nu' \) might be attributed to the difference in the nature of the turbulent bursts below and above \( Ta_c \). The correlation length measured that way is approximately the same as that obtained from the raw space-time diagrams.

2.4 Discussion

Spatiotemporal intermittency in the Taylor-Dean system exhibits two regimes with two threshold values. The first corresponds to an algebraic regime with a threshold \( Ta_a = 228 \), and the second regime is exponential with a threshold at \( Ta_c = 237 \). For \( Ta < 237 \), the turbulent bursts (metastable phase) nucleate spontaneously and are localized in space and time. The characteristic width of the laminar domains is infinite, i.e., approximately the same as the length of the whole system. For \( Ta > 237 \), the turbulent bursts (stable phase) form a connected pattern with an embedding of laminar zones (metastable phase) whose characteristic width decreases significantly with \( Ta \) to a few wavelengths of the initial pattern. The correlation length of the laminar and turbulent domains diverge near the value \( Ta_c = 237 \), which is considered the critical point corresponding to the onset of the exponential regime.

In connection with Pomeau’s conjecture, the value \( Ta_c = 237 \) corresponds to a critical point for a second order phase transition. However, the critical exponents measured in our experiment (\( J, \alpha, \nu, \nu' \)) are different from those predicted by theories of critical phenomena (Landau mean field theory [52], directed percolation theory [53],...). STI observed in other one dimensional extended systems exhibits two distinct
Figure 2.29: Correlation function of the binarized space-time plot ($Ta = 233$): measured points (+) are shown, along with a solid line representing the exponential fit following Equation 2.12. Horizontal axis ($x$) is spatial dimension scaled by $d$ the gap size.
Figure 2.30: The correlation length $\xi$ divergence near the critical point. Solid lines are fits with Equation 2.15.
regimes (algebraic and exponential) but the exponents characterizing each regime and the turbulent fraction differ from one experiment to another. In Table 2.2 we give the values of different exponents from various experiments. The differences originate apparently in the behavior of the laminar background flow pattern on which STI sets in. In some cases there are localized bursts, as in Rayleigh-Bénard convection [29], or in the Taylor-Dean system. In the printer's instability there are dilation waves which may explain the difference in the coefficients. Therefore it is unlikely that all reported experiments belong to the same class of STI universality even though they exhibit the main features of Pomeau's conjecture. It is worthwhile to notice that the different numerical simulations on partial differential equations [40, 54], on coupled map lattices [37] or on cellular automata [42] have also shown spatiotemporal intermittency with different exponents.

2.5 Conclusion

Spatiotemporal intermittency in the Taylor-Dean system has been characterized using numerous methods from experimental data. The first method calculated the turbulent fraction $f$ from binarized versions of space-time diagrams. With stationary end boundary conditions, $f$ grows from zero at $Ta_s = 228$. With rotating end conditions, Ekman cells induce turbulence in the flow below $Ta_s$ with $f$ reaching ~10% at $Ta_s$. Above $Ta_s$, $f$ grows as in the stationary case, albeit with a slightly different functional form. The data also show two different dynamical regimes. The first exhibits algebraic decay in the spatial histograms of the laminar domains with an onset at
Table 2.2: Experimental values of different exponents measured in some of the systems exhibiting STI.

<table>
<thead>
<tr>
<th>Experimental System</th>
<th>$\beta$</th>
<th>$\mu_s$</th>
<th>$\mu_t$</th>
<th>$\alpha_s$</th>
<th>$\alpha_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convection in annulus [29]</td>
<td>1.9 ± 0.1</td>
<td>1.9</td>
<td></td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Convection in channel [54]</td>
<td>0.3 ± 0.05</td>
<td>1.6 ± 0.2</td>
<td>2.0 ± 0.2</td>
<td>0.50 ± 0.05</td>
<td>0.50 ± 0.05</td>
</tr>
<tr>
<td>Convection in annulus [54]</td>
<td>1.7 ± 0.1</td>
<td>2.0 ± 0.1</td>
<td></td>
<td>$\sim$ 0.50</td>
<td>$\sim$ 0.50</td>
</tr>
<tr>
<td>Roll coating system [33]</td>
<td>0.45 ± 0.05</td>
<td>0.63 ± 0.02</td>
<td>0.61 ± 0.02</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Taylor-Dean system</td>
<td>1.33 ± 0.26</td>
<td>1.67 ± 0.14</td>
<td>1.74 ± 0.16</td>
<td>$\sim$ 0.64</td>
<td>$\sim$ 0.73</td>
</tr>
</tbody>
</table>

$Ta_c$. This algebraic dependence shows the laminar domains appearing in the system at all length scales. At $Ta_c = 237$ and above, the laminar domains follow an exponential decay, evidence of a characteristic length scale. The turbulent domains exhibit exponential decay for all control parameters. The temporal histograms exhibit similar dependences as their spatial counterparts.

A third method used to characterize the STI is the binary correlation function. Below some critical $Ta$, the turbulent domains exist in a background of laminar flow. Therefore, the correlation function is calculated with turbulent pixels set to 1 and laminar pixels set to 0. Above the critical $Ta$, the turbulent domains dominate and act as a background for the laminar domains. This necessitates a reversal of the
pixels such that laminar is set to 1 and turbulent to 0. Calculating the correlation length from the binary correlation function in this manner shows a divergence at $Ta_c = 237$. The same critical $Ta_c$ as found with the algebraic/exponential crossover in the histograms. This procedure of reversing the pixel designation above and below the threshold is borrowed from the study of spin lattices[52] and second-order phase transitions. This is the first attempt at applying this procedure to STI.

This divergence of the correlation length confirms Pomeau's conjecture[21] that STI has characteristics of second order phase transitions in equilibrium systems with $Ta_c$ as the transition point. However, the measured exponents in the Taylor-Dean system differ from those obtained in other experiments (see Table 2.2). This implies that the detailed behavior of spatiotemporal intermittency is not universal in character, although its qualitative nature is similar for the several systems investigated thus far.

Several factors differentiate STI found in the Taylor-Dean system from STI in other systems and may be partly responsible for the lack of universal character. First, the turbulence inside a turbulent burst has more fine-scale structure than the bursts associated with other systems. For example, STI in Rayleigh-Bénard convection[30] is primarily large fluctuations in the wavelength of the roll pattern. Second, the turbulence appears on top of, and destroys the background Taylor vortex state. The turbulence found in the Taylor-Dean system is closer to hydrodynamic turbulence than that found in many of the other systems. Third, the Taylor-Dean system has a mean flow which carries the turbulence azimuthally away from the free surface. And
finally, it has been shown that the end conditions affect the transition to STI. The exact path through which these factors change the nature of STI remains for further study.
CHAPTER 3

The Two-layer Rayleigh-Bénard System

3.1 Introduction

Fluids flow under the influence of many different driving forces. Chapters 2 and 4 are concerned with flows driven by rotating cylinders. This chapter will concentrate on a flow driven by temperature differences. This flow is generically called “thermal convection” and can be found in many places around us. For example, the sun heats the ground, which in turn heats the air. This hot air wants to rise and the cold air at higher altitude wants to fall. How these two tendencies (along with numerous other factors) organize gives rise to the weather. The first scientific observations on thermal convection are thought to have been by Count Rumford in 1870[55]. Credit for the first controlled experiments on convection goes to Bénard[56]. The first attempt at a theory to explain Bénard’s observations was made by Lord Rayleigh[57].

Since Bénard’s seminal work, many refinements have been made to the experiment that now bears the name. “The Rayleigh-Bénard System.” For reviews of convection see Normand et al. [10]. Cross and Hohenberg[8], or Koschmieder[3] and references therein.
3.2 The Single Layer Rayleigh-Bénard System

The classic single layer Rayleigh-Bénard system is shown schematically in Figure 3.1. As described in Chapter 1, it consists of an infinite horizontal layer of fluid of height $d$, bounded above and below by planar surfaces. The bottom surface is maintained at temperature $T$, while the top is at a (typically) lower temperature $T - \Delta T$.

The Rayleigh-Bénard system requires a number of equations to describe the motion of the fluid. The most important are the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (3.1)$$

the Navier-Stokes equation

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \rho \mathbf{g} \hat{z} + \mu \nabla^2 \mathbf{v}. \quad (3.2)$$

and the thermal equation

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T = \kappa \nabla^2 T \quad (3.3)$$

where $\mathbf{v}$ is the velocity vector, $\rho$ the density, $p$ the pressure, $\mathbf{g}$ the acceleration due to gravity, $\hat{z}$ the vertical direction, $\mu$ the dynamic viscosity, and $\kappa = k/\rho C_p$ the thermal diffusivity with $k$ as the thermal conductivity and $C_p$ the specific heat. The remaining equations describe the variation of each fluid property with temperature.

It turns out that most of the property variations are extremely small and can be safely ignored in many laboratory situations. Possible exceptions include working with high applied temperature differences (e.g., the viscosity of most silicone oils
Figure 3.1: Schematic of the single layer Rayleigh-Bénard system.
changes by 10% with a 6°C temperature change) or fluids with unusual properties such as the density inversion found in water at 4°C. It was independently shown by Oberbeck[58] and Boussinesq[59] that only the density variation needs to be included in the equations under the conditions found in most Rayleigh-Bénard experiments, as

$$\rho = \rho_o (1 + \alpha \Delta T) \quad (3.4)$$

where $\rho_o$ is the density at $T$, and $\alpha$ is the coefficient of thermal expansion. It was further shown that the only term in the equation where this variation is relevant is in the gravitational force term. This method of ignoring fluid property variations has been given the name “Boussinesq approximation” and greatly simplifies an otherwise intractable problem. Discussion of the problem when variations in the other fluid properties (so-called non-Boussinesq flows) can be found in [3], but will not be touched upon here.

Further simplification of the equations is achieved by non-dimensionalizing the variables with the appropriate scales. Following Drazin and Reid[60], distances are scaled with $d$, time with $d^2/\kappa$, velocity with $\kappa/d$, temperature with $\Delta T$ and pressure with $\rho_o \kappa^2/d^2$.

Yet another simplification can be used if we only consider the case of infinitesimal motion in the fluid. As mentioned in Chapter 1, the first state at small $\Delta T$ is conduction. In the conduction state, the fluid is motionless and only conducts heat from the hotter surface to the cooler. At a certain critical value of the temperature difference $\Delta T_c$, the fluid becomes unstable to motion. However, near $\Delta T_c$, the fluid
motion is still going to be very slow, and the temperature field is going to be similar to that found in the conduction state. Again following Drazin and Reid[60], let us define a new set of variables such that $v$ is the velocity and

$$T(x, y, z) = T_o(z) + \theta(x, y, z)$$

(3.5)

is the temperature field. where $\theta$ is a small perturbation on top of the conduction state temperature $T_o(z)$. Since both $v$ and $\theta$ are considered infinitesimal, any term in the equations that is proportional to $v^2$, $\theta^2$, or $v\theta$ will be eliminated, thereby linearizing the equations.

Using the Boussinesq approximation, non-dimensionalizing, and linearizing the equations for Rayleigh-Bénard convection leads to the following: the continuity equation (Equation 3.1) becomes

$$\nabla \cdot v = 0.$$  

(3.6)

the Navier-Stokes equation (Equation 3.2) becomes

$$\frac{\partial v}{\partial t} = -\nabla p + Ra Pr \theta \hat{z} + Pr \nabla^2 v.$$  

(3.7)

and the thermal condition (Equation 3.3) is now

$$\frac{\partial \theta}{\partial t} - u = Pr \nabla^2 \theta,$$  

(3.8)

where $u$ is defined as the $\hat{z}$ component of the velocity. The two new dimensionless numbers are the Rayleigh number

$$Ra = \frac{g \alpha d^3 \Delta T}{\kappa \nu},$$

(3.9)

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and the Prandtl number

\[ Pr = \frac{\nu}{\kappa}. \] (3.10)

Ra is a non-dimensionalized temperature difference and is the relevant control parameter for the Rayleigh-Bénard system. Pr is a measure of the relative importance of the two dissipative processes in the problem. A high Pr indicates that viscosity is responsible for most of the dissipation by limiting any local velocity variation. A low Pr indicates thermal diffusion is controlling the process by dissipating any temperature imbalances which are necessary for the buoyancy effects.

The boundary conditions on the flow can take one of two forms at the bounding surfaces. The first is called the "free boundary" and requires that

\[ w = \frac{\partial w}{\partial z} = \theta = 0. \] (3.11)

This situation describes the fluid being bounded by vacuum, but is reasonably approximated when the fluid is bounded by a gas. The second type of boundary condition is the "rigid boundary"

\[ w = \frac{\partial^2 w}{\partial z^2} = \theta = 0. \] (3.12)

A metal plate is a typical rigid boundary condition. These boundary conditions combine to form three possibilities for a single fluid layer: rigid-rigid, rigid-free and free-free.

The analysis of Equations 3.6, 3.7 and 3.8 along with appropriate boundary conditions (either Equation 3.11 or 3.12) can be found in numerous texts, including Drazin and Reid[60] and Koschmieder[3]. The main result is \( Ra_c \), the critical value of the
Rayleigh number found in Table 3.1. A second result is the onset wavelength of the pattern $\lambda_c$.

At values $Ra > Ra_c$, the flow takes the form of rolls, shown schematically in Figure 3.2. It should be noted that the analysis of the linear equations is only applicable for values of $Ra$ very near $Ra_c$. Furthermore, the linear equations do not specify the orientation nor the exact nature of the roll structure. An analysis of the "weakly non-linear" or the "fully non-linear" equations is required to determine the specifics of the flow. An extensive theoretical study of convection has been made by Busse and coworkers [61, 62, 63, 64, 65, 66, 4]. A summary of Busse's findings is known as the "Busse balloon" (Figure 3.3). The Busse balloon is a three-dimensional ($Ra, Pr, \alpha$) ($\alpha = 2\pi/\lambda$ is the wavenumber of the pattern) phase diagram which defines the stability boundaries between the possible states in Rayleigh-Bénard convection. Many of the features of the Busse balloon have been confirmed by experiment.

Too many experiments have been performed on the Rayleigh-Bénard system to list them all here. However, the review articles by Normand, Pomeau & Velarde[10] and more recently by Cross & Hohenberg[8], and the book by Koschmieder[3] have extensive bibliographies. Some data is common throughout most of the experiments.
Figure 3.2: Schematic of Rayleigh-Bénard convection rolls, where $\lambda$ is the wavelength of the pattern. from [3].
Figure 3.3: The Busse Balloon from [4]. Details explaining the notation can be found in [4].
and worth summarizing here. For example, the critical values for $Ra_c$ and $\lambda_c$ for the rigid-rigid and rigid-free boundary conditions (given in Table 3.1) have been confirmed over and over again. Also, the onset state is almost always time-independent stationary rolls (exceptions occur at low $Pr$ and with non-Boussinesq flows\[8\]). In typical experiments, time-dependence only appears at a minimum $Ra > 2Ra_c$ and for experiments with high $Pr$, time-dependence can require $Ra > 100Ra_c$\[30\] (or more) before appearing.

### 3.3 The Two-layer Rayleigh-Bénard System

As mentioned previously, the single layer Rayleigh-Bénard system has been studied extensively. However, the simple act of adding a second layer of immiscible fluid to the system opens up a vast parameter space that is yet to be thoroughly explored. Unlike the single-layer system described above, the flow in a two-layer system not only depends on $Ra$ and $Pr$, but also depends on the heights of the fluid layers and all of the fluid parameters (thermal conductivity, thermal expansion, viscosity, ...). Additionally, new driving mechanisms associated with the interface (surface tension and/or deformation) could play a roll in the dynamics. These new factors combine to produce new flow states which cannot be found in the single layer system. Many of these new states are described in the text by Renardy and Joseph\[67\]. One of the more interesting predictions is the possibility of time-dependent states at the onset to convection. This is an exciting prospect due to the applicability of the weakly nonlinear equations near the onset to convection, which may make the theoretical problem...
of studying time-dependent patterns slightly more tractable. This is in contrast to the one-layer Rayleigh-Bénard system in which time-dependent states only appear well above onset, where only the fully non-linear equations apply.

Much of the interest in the two-layer Rayleigh-Bénard system has been generated by two practical problems, namely mantle convection and encapsulated crystal growth. Geophysicists typically try to model the Earth's mantle in one of three ways: single-layer, two-layer, and two-layer with flushing events. The first model is virtually identical to the single-layer Rayleigh-Bénard system. The second model is similar to the system studied here, while the third is a hybrid of the first two models. Much theoretical work has been done on the two-layer model of mantle convection [68, 69, 70, 71, 72], however experimental data is quite limited. The second system, encapsulated crystal growth[73], concerns the solidification of a crystal which is bounded above and below by fluids. It is necessary to understand the flows associated with such a system in order to ensure the desired crystal characteristics. Experimental studies with the two-layer Rayleigh-Bénard system will provide useful information for dealing with both of these practical problems.

The two-layer Rayleigh-Bénard system is shown schematically in Figure 3.4. By convention, the bottom layer is labeled 1 while the top is 2. All quantities specific to each layer will have the appropriate subscript. i.e., the bottom layer density will be \( \rho_1 \), the top layer density will be \( \rho_2 \), etc. Two new quantities come into the problem to describe the relative height of the layers. The "depth fractions" are defined as

\[
l_1 = \frac{d_1}{d} \quad \text{and} \quad l_2 = \frac{d_2}{d} = 1 - l_1
\]

(3.13)
representing the height of each layer relative to the overall height[74]. An alternative form is the "depth ratio" given as

$$a = \frac{d_2}{d_1} = \frac{l_2}{l_1}$$  \hspace{1cm} (3.14)

being the ratio of the height of the top layer to that of the bottom[5]. The data presented in this work will use $l_1$ as the preferred variable.

Since the fluids are immiscible, mixing across the interface is negligible. Therefore, each layer of fluid follows its own set of equations and the interface is handled through the appropriate boundary conditions. The nondimensionalization scheme used by Renardy and Renardy[74] for the two-layer system is the same as that given above for the single layer system using the bottom fluid parameters. The only exception is that the temperature is not only rescaled but also shifted by $T_o$. Using this nondimensionalization scheme and the Boussinesq approximation (no linearization), the equations for the bottom layer are the continuity equation

$$\nabla \cdot \mathbf{v} = 0.$$  \hspace{1cm} (3.15)

the Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + Ra Pr \theta \mathbf{\hat{z}} + Pr \nabla^2 \mathbf{v}$$  \hspace{1cm} (3.16)

and the thermal equation

$$\frac{\partial \theta}{\partial t} + (\mathbf{v} \cdot \nabla) \theta = \nabla^2 \theta$$  \hspace{1cm} (3.17)

where both $Ra$ and $Pr$ are defined as above (Equations 3.9 and 3.10) using the bottom fluid parameters. The top layer of fluid follows Equation 3.15.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -r \nabla p + \frac{Ra Pr}{3} \theta \mathbf{\hat{z}} + \frac{r}{m} Pr \nabla^2 \mathbf{v}.$$  \hspace{1cm} (3.18)
Figure 3.4: Schematic of the two-layer Rayleigh-Bénard system.
and

\[ \frac{\partial \theta}{\partial t} + (\mathbf{v} \cdot \nabla) \theta = \frac{1}{\gamma} \nabla^2 \theta \quad (3.19) \]

where the new variables are the following ratios of the fluid parameters

\[ m = \frac{\mu_1}{\mu_2}, \ r = \frac{\rho_1}{\rho_2}, \ \gamma = \frac{\kappa_1}{\kappa_2}, \ \zeta = \frac{k_1}{k_2} \text{ and } \beta = \frac{\alpha_1}{\alpha_2}. \quad (3.20) \]

In addition to the equations of motion, the boundary conditions need to be specified. At the bottom plate, the conditions are

\[ \mathbf{v} = 0, \ \theta = 1. \quad (3.21) \]

and at the top plate

\[ \mathbf{v} = 0, \ \theta = 0. \quad (3.22) \]

The interface between the two layers is more complicated. If it is assumed that the interface stays flat. (which is true for the experiments reported here) then the boundary conditions at the interface simplify to

\[ u_1 = u_2, \ v_1 = v_2, \ w_1 = w_2 = 0 \text{ and } \theta_1 = \theta_2. \quad (3.23) \]

where \( \mathbf{v} = (u, v, w) \). The first solution of Equations 3.15. 3.16. 3.17. 3.18 and 3.19 with these boundary conditions (Equations 3.21. 3.22 and 3.23) describes the conduction state[74] and is simply

\[ \mathbf{v} = 0 \quad (3.24) \]

\[ \theta_1 = 1 - A_1 z. \quad (3.25) \]

\[ \theta_2 = A_2 (1 - z) \quad (3.26) \]
where $\theta_1$ is the temperature in the bottom fluid ($0 \leq z \leq l_1$). $\theta_2$ is for $l_1 \leq z \leq l_2$ with

$$A_1 = \frac{1}{l_1 + \zeta l_2}, \quad A_2 = \zeta A_1.$$  \hfill (3.27)

It should be noted that these equations leave out the terms containing surface tension contributions. Surface tension can enter the problem in either of two ways: curvature of the interface and Marangoni effects. As stated above, the interface between the fluids used here does not show signs of curvature. Marangoni effects are gradients in the surface tension at the interface caused by temperature variations. This would enter the problem as a stress at the interface. Fortunately, Marangoni effects only play a part in the dynamics when the thickness of the layers is small (for experiments where Marangoni effects dominate, see Schatz et al. [75] or Koschmieder[3]). Given the size of the system used for this work and following Rasenat et al. [76], Marangoni effects will also be ignored for this work.

Given the conduction state in Equations 3.24, 3.25 and 3.26, a linearization can be performed around it. If $\tilde{\theta}$, $\tilde{v}$, and $\tilde{p}$ are the perturbations to the temperature, velocity, and pressure respectively, the linearized equations for the bottom layer become

$$\nabla \cdot \tilde{v} = 0. \quad (3.28)$$

$$\frac{\partial \tilde{v}}{\partial t} = -\nabla \tilde{p} + Ra Pr \tilde{\theta} \tilde{z} + Pr \nabla^2 \tilde{v}. \quad (3.29)$$

$$\frac{\partial \tilde{\theta}}{\partial t} - A_1 \tilde{w} = \nabla^2 \tilde{\theta} \quad (3.30)$$

and for the top layer

$$\nabla \cdot \tilde{v} = 0. \quad (3.31)$$
\[
\frac{\partial \tilde{\nu}}{\partial t} + = -r \nabla \tilde{p} + \frac{Ra Pr}{\beta} \frac{\partial \tilde{\theta}}{\partial z} + \frac{r}{m} Pr \nabla^2 \tilde{\nu}. \tag{3.32}
\]

\[
\frac{\partial \tilde{\theta}}{\partial t} - A_2 \tilde{\psi} = \frac{1}{\gamma} \nabla^2 \tilde{\theta}. \tag{3.33}
\]

Analysis of the linear equations by Colinet and Legros\cite{5} for a specific value of the fluids parameters yields a bifurcation diagram as shown in Figure 3.5. Their work shows the possibility of three states depending on \(a\), the depth ratio (Equation 3.14) of the two fluids (or alternatively \(l_1\) the depth fraction).

Colinet and Legros found that above \(\Delta T_c\) (or \(Ra_c\)), each layer forms its own set of convection rolls similar to those found in the single layer Rayleigh-Bénard system (see Figure 3.2). The different flow states at onset in the two-layer Rayleigh-Bénard system concern the alignment of the two sets of rolls. Two of the simplest possible flow states are stationary and are shown in Figure 3.6. The first is called “mechanical coupling” (MC) (or alternatively viscous coupling). MC is the state where hot upflow in one layer is above (or below) cold downflow in the other layer. The fluid flows in the same direction above and below the interface, \(i.e.,\) the rolls rotate in a gear-like fashion. The second stationary state is called “thermal coupling” (TC). TC is characterized by the hot upflow in one layer being above (or below) the hot upflow in the other layer. Likewise, the cold downflows are also aligned. This causes the rolls to turn in opposite directions above and below the interface, thereby setting up a region of high shear. The third basic state found in the two-layer system is time-dependent. As first shown by Gershuni and Zhukovitskii\cite{77}. As further described by Colinet and Legros\cite{5}, it takes one of two forms depending on the exact nature of the experimental configuration. In confined systems, the time-dependent state is an
Figure 3.5: Critical bifurcation diagram from [5] for a particular set of fluids. The thick lines are stability boundaries with TC for $a > 1.067$, MC for $a < 0.92$ and oscillatory in between. The inset graph is $\omega$ vs. $a$ with $\omega$ as the frequency of the oscillatory instability. Note, the horizontal axis is $a$, the depth ratio.
oscillation between MC and TC, while in extended systems it manifests itself as a traveling wave state that is neither exactly TC nor MC.

It turns out that the generic characteristics of Figure 3.5 are robust: TC is present at high values of $a$ (or low values of $l_1$), MC is at low values of $a$ and between the MC and TC is a region of time-dependence. The exact details of the size of the time-dependent regime and its location are dependent on the particular values of the fluid parameters[77, 5, 76, 78, 79]. For some sets of fluids, the time-dependent region is so small as to probably be inaccessible to experiments or possibly non-existent at the onset to convection.

Further analysis of the two layer problem looking at the weakly non-linear and fully non-linear equations has been carried out by many researchers[5, 80, 81, 67, 82, 83, 78, 74]. They have shown that, in addition to the states described above (TC and MC), a system with an extended geometry (both horizontal dimensions are large compared to the height) can have a variety of 3-dimensional patterns. An extensive listing of these can be found in the text by Renardy and Joseph[67]. Other authors have investigated the effects of surface tension and/or deformable interfaces[84, 72, 69, 70, 71, 68]. The interesting idealized case of a 1:2 ratio in the pattern wavelengths has been studied by Proctor and Jones[85].

### 3.3.1 Previous experiments

Experimental work on the two-layer Rayleigh-Bénard system has been carried out by a number of researchers. One of the first investigations appears to have been by Zeren
Figure 3.6: Cartoon of the stationary states found in the two-layer Rayleigh-Bénard system. (a) thermal coupling. (b) mechanical coupling.
and Reynolds\[84\]. They report confirmation of the theoretical prediction for $Ra_e$ when heating from below, but heating from above did not show any of the predicted instabilities. This work predates the theoretical prediction of time-dependent onset. Later work by Rasenat et al. [76] appears to have worked around the edges of the time-dependent region. They investigated the flows at values of $l_1$ just below and just above the predicted oscillatory region. Experiments by Prakash and Koster\[86\] have concentrated on the mechanisms associated with the two distinct stationary coupling modes (TC and MC) and how they vary with fluid properties. Prakash and Koster do not report a time-dependent state. Cardin et al. [87, 88, 89] have reported a time-dependent state. However, their state does not involve coupling oscillations and has characteristics far from those predicted by linear theory. For example, the observed period is 20 hrs compared to predictions which are on the order of 1 hr. Secondly, the reported onset to the time-dependent state is at $Ra = 9.7Ra_e$, well above the predicted onset. The first observation of the predicted state is from Andereck et al. [15, 90] and Colovas\[14\]. Recently, Busse and Sommerman\[16\] have reported observation of a time-dependent state in an annular system using thermistors to probe the flow.

### 3.4 Apparatus

The apparatus used for these experiments (shown in Figures 3.7 and 3.8) borrows many ideas from Colovas\[14\]. His apparatus had a rectangular configuration with height and width the same as that used here. The present geometry is annular.
however, chosen to better mimic the infinite systems used in theory. Unlike the rectangular system, the lack of azimuthal boundary conditions removes any restrictions on the motion of the rolls and their size is only limited by the requirement of an integral number of rolls. The mean radius is chosen large enough to minimize curvature effects and the gap width is selected so that the flow is quasi-one-dimensional.

The first parts of the apparatus to be discussed are the upper and lower plates. Both plates are made from aluminum, due to its high thermal conductivity. The top plate is annular with outer diameter 161 mm, inner diameter 59.1 mm, and height 44 mm. The top of the top plate has two grooves machined into it which act as water channels. A thin stainless steel sheet covers the water channels and is sealed with o-rings. Temperature controlled water (described below) enters the outer ring through the SS plate at one point along the circumference from above. The water is extracted at a point diametrically opposite to the inlet. The inlet and outlet for the inner water channel are opposite to that for the outer. This is to insure that any rise in the temperature of the water as it flows through the cell is offset by water flowing in the opposite direction in the other ring. Eight small holes are drilled in the side of the plate 8 mm above the bottom surface. The depth of the holes is such that they end at the mean radius of the plate. These holes are to allow thermistors to be imbedded in the plate to monitor the temperature. A single hole is drilled diagonally into the side of the plate to allow access to the working fluids. The last features of the top plate are two o-ring grooves machined into the bottom surface. These accept
Figure 3.7: Cross sectional view of the annular two-layer Rayleigh-Bénard system.
Figure 3.5: Photograph of the system.
o-rings which complete the seal between the plate and the rings which make up the cell holding the working fluids.

The bottom plate is constructed similarly to the top. It has identical o-ring grooves machined into its top for the same purpose. Likewise it has eight holes in the side for thermistors. The outer and inner radii are the same as the top plate. However, the height of the bottom plate is 25.4 mm and the bottom surface is machined smooth. The temperature of the bottom plate is maintained via a Minco Products, Inc. film heater. The heater is model HR5553R88.8L12A. It has an outer diameter of 159 mm, inner diameter of 51 mm, and a resistance of 88.8 Ω. Control of the heater will be described below. The heater is compressed against the bottom plate with a 1 cm thick plexiglas plate that is machined to match the outer and inner radii of the plate.

The working fluids are contained between two rings. The first rings that were used were made from glass. They were purchased from the OSU Glass Blowing shop and were cut from off-the-shelf tubing. It quickly became evident that the radii around these rings varied too much and hindered the visualization technique. Therefore, an alternative was sought. Specially made glass rings were prohibitively expensive. It was decided to try plexiglas and eventually the following rings were produced.

The outer ring has outer diameter 139.5 mm and inner diameter 126.6 mm. The inner ring has outer diameter 88.8 mm and inner diameter 76.1 mm. Both have a height \( d = 12.2 \text{ mm} \). The gap is 18.9 mm wide and the aspect ratio \( \Gamma = \frac{2\pi r_m}{d} = 27.7 \), where \( r_m = 53.9 \text{ mm} \) is the mean radius. Construction of the rings proceeded using the following procedure:
1. Begin with 50 mm thick plexiglas sheet. Machine into a cylinder with outer radius slightly (~ 3 mm) larger (smaller for the inner radius) than the final dimensions.

2. Anneal the plexiglas according to manufacturer's instructions.

3. Return cylinder to lathe, machine until just slightly over desired dimensions.


5. Cut off top of cylinder. The polishing process usually rounds this edge, which must be removed.

6. Cut off ring to desired height.

The original 50 mm plexiglas yields two rings of the appropriate final height.

The plexiglas rings are sandwiched between the aluminum plates. O-rings provide the seal between the plexiglas and the aluminum. The o-rings are constructed from 3 mm neoprene o-ring cord from the Apple Rubber Company, Inc. It should be noted that the neoprene o-rings are softer than standard BUNA o-rings. This requires that the o-ring grooves be shallower than the typical grooves. Specifically, the grooves used in the aluminum plates are 1.9 mm deep. The ends of the o-ring cord are connected using Superglue™. It should also be noted that some working fluids can leach monomer out of the o-rings. To minimize the effects of this on the experiment, the cord is soaked in silicone oil for at least 2 weeks prior to sizing for the final o-rings.

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Significant shrinkage of the o-ring takes place during the soaking. For more discussion on this see Colovas[14].

3.4.1 Visualization

Visualization of the flow pattern is achieved using the shadowgraph technique. For a discussion of the shadowgraph technique see the text by Goldstein[91] or Jenkins[92].

The shadowgraph technique takes advantage of the coupling between the temperature of the fluid and the index of refraction $n$. For most fluids (including those used in this experiment, as determined from data sheets supplied by the manufacturers), $n$ decreases with increasing temperature. To implement the shadowgraph technique, light must pass through the fluid. The light is bent away from regions of hot fluid ($i.e.$, hot upflow) and into regions of cold fluid ($i.e.$, cold downflow). A convection pattern monitored with the shadowgraph shows bright regions and dark regions corresponding to cold and hot regions of the flow respectively. A schematic of the shadowgraph technique is shown in Figure 3.9. It should be noted that the shadowgraph technique is local perpendicular to the direction of light propagation and integral along it.

The typical implementation of the shadowgraph technique for single layer Rayleigh-Bénard convection is to view from above. Since shadowgraphs are integral along the direction of propagation, it would be difficult to separate the two layers using this approach. To alleviate this, it was decided to view the flow from the side. In an annular geometry, this requires viewing in a radial direction.
Figure 3.9: Schematic of the shadowgraph technique.
The portion of the apparatus used to observe the flow follows from [93, 94] and is shown in cross section in Figure 3.7. The light source is a SuperBright LED from LiteOn Optoelectronics. A lens is molded onto the front of the LED to concentrate the light. This must be removed by cleaving it with a razor blade prior to use in this application. (Light polishing of the surface will remove any defects after the cleaving.)

The LED is positioned at the focal point of a 50 mm diffraction limited achromatic lens, which produces collimated light. This collimated light travels down the axis of the system where it reflects off of a 45° conical mirror, giving radially outward propagating light. Next, the light traverses the plexiglas rings and the convecting fluids. Then the light is reflected back radially inward by a cylindrical mirror. The light makes a second pass through the fluid before striking the conical mirror again to exit the system along the axis. A portion of the light is directed into a Pulnix Model TMC-7CN CCD camera with a beam splitter. The CCD camera is connected to a computer that captures images and data.

The beam splitter and lens were purchased from commercial sources. The 45° conical mirror and the cylindrical mirror were made in the Physics Department Shop from 7075 aluminum. (7075 aluminum is harder than other alloys and therefore polishes to a smoother surface.) The procedure to produce these mirrors is identical to the procedure used to polish the plexiglas rings, including the appropriate annealing process. Great care must be taken to insure the accuracy of the angle on the conical mirror, and the consistency of the radius on the cylindrical mirror.
Using this method to probe the system produces images such as that in Figure 3.10. Moving radially outward, the first series of alternating bright and dark lines represents the roll structure in the top layer. The next fuzzy ring is the interface. The outer series of bright and dark lines is the rolls in the bottom layer.

The procedure to align all of the various optical components has varied in detail, but has normally been as follows:

1. Position the LED at the focal point of the lens.
2. Level the conical mirror using a laser. The beam is directed axially downward (verified with a level) onto the mirror and then radially outward. The beam should be horizontal (also verified with a level) at different positions around the mirror.
3. Align the axis of the conical mirror with the axis of the system.
4. Position the beam splitter in the light path, at a 45° angle to the plane of the system.
5. Mount the CCD camera to capture the light from the beam splitter, making sure the light travels along the axis of the camera.
6. Position the cylindrical mirror at the appropriate height, leveling it, and aligning its axis with the axis of the system.
7. Adjust the focus and depth of field on the camera lens.
8. Adjust the brightness of the LED.
Figure 3.10: Sample image of the two-layer Rayleigh-Bénard system in the time-dependent regime. The image has been divided by a background image to remove any optical defects. Note, the image shows TC at the bottom right and MC at the top. Since this is an image of the time-dependent regime, a subsequent image will show different couplings at different locations.
The final refinement to the shadowgraph typically requires a fine-tuning of the positions of all of the components.

3.4.2 Temperature control

As mentioned above, the crucial control variable for Rayleigh-Bénard convection is the temperature difference $\Delta T$ between the plates. The temperature of the top plate is controlled with constant temperature water flowing through the water channels described above. The temperature of the water is controlled with a Brinkmann Instruments Lauda RM6 recirculating temperature bath. Due to the design of the RM6, short time transients are sometimes introduced into the water temperature. To smooth out these transients, a low-pass temperature filter is installed along the water line between the RM6 and the top plate. This low-pass filter is a large container filled with wire mesh. As the water enters the filter, it mixes with the water already present to equilibrate the temperatures. The wire mesh not only aids the mixing, it also serves as a heat conductor to dissipate any thermal gradients that arise. The set point of the RM6 is controlled with a 486/DX33 computer using a CyberResearch, Inc. DAC (digital-to-analog) 6 channel output card model CYDDA06. Unfortunately, the RM6 is not factory configured for external control. The set point inside the RM6 is controlled via a potentiometer and a reference voltage. External control is achieved by removing the sweep lead from the potentiometer and applying a voltage to it relative to one end of the potentiometer. It should be emphasized that the low side of the potentiometer is much higher ($\sim 30\,\text{volts}$) than normal earth ground. Since the
DAC's ground level is set by the PC motherboard and will not float. The difference in the ground levels of the DAC and RM6 make direct connection impossible. Either the DAC tries to pull the RM6 down, or the RM6 pulls the DAC up. This causes a current surge that could destroy either or both components. (Unfortunately this has been confirmed and the repair bills will prove it.) To alleviate this problem, it is necessary to install an optical isolator between the two instruments. The optical isolator was designed by the Department of Physics Electronics Group and carries their designation D693A-1. The main working component is a Burr-Brown ISO-121-G optical isolator chip. The optical isolator maintains the output voltage difference equal to the input voltage difference. However, the low side voltage may drift to any value and so alleviates the discrepancy between the two instruments.

Controlling the set point of the RM6 with the DAC is simple once the technical problems have been overcome. The set point varies linearly with the applied voltage difference. This however can cause a minor problem. If the voltage difference goes to zero, the set point of the RM6 falls to $-10^\circ C$. Since the working fluid is water, loss of the voltage difference would cause the system to freeze and quite possibly damage the RM6. To counter this possibility, a relay is installed along the power line into the RM6. The control circuit for the relay monitors the output of the DAC and shuts down the RM6 should the voltage difference drop too low.
In addition to setting the temperature of the top plate, the temperature must also be monitored. The temperature of each plate is probed using a series of Thermometrics P65DB103J thermistors placed in the holes described above. The temperature of the thermistors is proportional to their resistance as

\[ T(R) = A + B \ln \left( \frac{R}{R_0} \right) + C \ln^2 \left( \frac{R}{R_0} \right) + D \ln^3 \left( \frac{R}{R_0} \right) \]  

(3.34)

where \( A, B, C, D \) and \( R_0 \) are constants. A calibration has been performed to determine the values of these constants for each individual thermistor. The temperature standard that was used is a calibrated Thermometrics CSP60BA252M Serial Number 392-947 thermistor, traceable to known standards. The resistance of the thermistors is measured with a Keithley Instruments 2000/2000SCAN DMM. The SCAN feature allows the DMM to probe up to 10 thermistors using a series of internal relays. The DMM is interfaced to the same PC as that used to control the RM6 via the RS-232 serial communications port. This allows storage of a temperature time series for later analysis.

As described above, a heater has been attached to the bottom plate to control its temperature. The voltage (or current) applied to the heater is supplied by a Kepco ABC 30-0.3M linearly programmable power supply. The output of the power supply is controlled via the same DAC used to control the RM6. The output of the DAC is controlled by the PC using a Proportional-Integral-Differential (PID) control scheme. The PID scheme uses the known temperature of the bottom plate and the desired set point to calculate an error \( E = T_{set} - T_b \). The error is used in

\[ V = K \left[ (1 + I_t)E - E_1 + D_t (E - 2E_1 + E_2) \right] + V_i \]  

(3.35)
to calculate $V$, the voltage output of DAC to the power supply[95]. Here $K$ is the proportional constant, $I_t$ is the integration time, $D_t$ is the differentiation time, and the subscripts refer to the appropriate values measured at previous time steps. The operational values of $K = 400$, $I_t = 150$, and $D_t = 40$ were determined by trial and error during the initial setup stage of the project to minimize the variation of the temperature of the bottom plate over time. The size of the time step is also a variable that can change the accuracy of the PID scheme. The response of the system used here is slow and hence the time step is 30 sec for this experiment. The program implementing the temperature control can be found in Appendix B.

Using this PID scheme to control the bottom plate and the water to control the top plate have yielded temperature accuracies of $\delta T_t = \pm 0.005^\circ C$ and $\delta T_s = \pm 0.003^\circ C$. These give $\delta \Delta T = \pm 0.007^\circ C$ for the temperature difference. The horizontal temperature variation is less than $\pm 0.005^\circ C$. The mean temperature of the system is kept at $21^\circ C$.

### 3.5 Fluid selection

The most important parts of the experiment are the fluids. Great care must be exercised in the selection process. Several factors are taken into consideration, including immiscibility, optical properties, meniscus effects, and the onset Rayleigh number (see Equation 3.9). The immiscibility criterion is necessitated by the use of two layers of fluid with a distinct interface. While no two fluids are ever 100% immiscible, some combinations can be close.
The requirements on the optical properties follow from the use of the shadowgraph technique. First, the fluids must be clear to allow the light to propagate through them. Second, the variation of $n$ with temperature must be large enough to allow the shadowgraph method to work.

As shown in [75, 76], meniscus effects can have a large impact on the flows. Ideally, the contact line between the two fluids would be perpendicular to the side wall and straight. If the contact line is not straight or the meniscus too large, the variations in the heights of the two layers can change the flow states. This must be minimized by the fluid selection as well as by the proper choice of the sidewall material.

The last criterion rests on the search for the oscillatory onset and that the fluids follow the Boussinesq approximation near onset. This latter requirements dictates that the critical temperature difference $\Delta T_c$ have a reasonable value. As shown in Colinet and Legros[5], one criterion for the oscillatory onset is that the critical Rayleigh numbers $Ra_c$ in each layer be approximately equal. Since the temperature of the fluids at onset is nearly the same as in the conduction state, the so-called "conduction assumption" can be used to calculate the interface temperature between the two fluids. This method assumes that each fluid is merely conducting heat from the bottom plate to the top and the interface temperature is determined by Newton's law of thermal conduction. Use of this assumption gives an interface temperature of

$$T_{int} = \frac{d_2 k_1 (T_o - \Delta T) + d_1 k_2 T_o}{d_2 k_1 + d_1 k_2}.$$  (3.36)
This definition for the interface temperature can be used to slightly modify Equation 3.9 to define Rayleigh Numbers for each layer as

\[
Ra_1 = \frac{g \alpha_1 (T_o - T_{int}) d_1^2}{\kappa_1 \nu_1} \quad \text{and} \quad Ra_2 = \frac{g \alpha_2 (T_{int} - (T_o - \Delta T)) d_2^2}{\kappa_2 \nu_2}
\]  

(3.37)

Setting \(Ra_1 = Ra_2\) and folding in Equation 3.36, allows a calculation of the relative depths of the two layers at which the onsets coincide. If the appropriate value of \(Ra_c\) is taken from Table 3.1, an estimate of \(\Delta T_c\) can also be made.

Satisfying all of these criteria can be difficult with readily available fluids. A long search finally led to the combination of Rhône Poulenc's Rhodorsil™ 47v10 silicone oil over 3M Corporation's Fluorinert™ FC70. FC70 is a perfluorinated hydrocarbon that is essentially chemically inert. Both of these fluids are clear and their physical properties appear in Table 3.2. They produce a reasonably small meniscus and a straight contact line with the plexiglas side walls. Calculation of the onset Rayleigh Number gives \(\Delta T_c \sim 0.76^\circ C\) and \(l_1 = 0.43\), both reasonable values. An added bonus is the relative harmlessness of both fluids. Of course, these fluids are not 100% immiscible. The 47v10 dissolves a small amount of the FC70. This does not appear to have an effect on the experiments reported here after saturation has occurred. To this end, a small amount of FC70 is added to the 47v10 and the mixture is agitated for some time (usually on the order of weeks) before it is put into the apparatus. Once the 47v10 is saturated with FC70, the characteristics of the flow states are stable for a period of at least months. More discussion on the effects of this mixing can be found in [14, 76].
<table>
<thead>
<tr>
<th>Property</th>
<th>CGS Units</th>
<th>47v10</th>
<th>FC70</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) density</td>
<td>g cm(^{-3} )</td>
<td>0.927</td>
<td>1.94</td>
</tr>
<tr>
<td>( \nu ) kinematic viscosity</td>
<td>cm(^2) s(^{-1} )</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>( \kappa ) thermal diffusivity</td>
<td>cm(^2) s(^{-1} )</td>
<td>8.601 \times 10(^{-4} )</td>
<td>3.446 \times 10(^{-4} )</td>
</tr>
<tr>
<td>( \alpha ) thermal expansion</td>
<td>K(^{-1} )</td>
<td>1.08 \times 10(^{-3} )</td>
<td>1.0 \times 10(^{-3} )</td>
</tr>
<tr>
<td>( k ) thermal conductivity</td>
<td>g cm (^{-3} ) K(^{-1} )</td>
<td>1.3 \times 10(^4 )</td>
<td>0.70 \times 10(^4 )</td>
</tr>
<tr>
<td>( C_p ) specific heat</td>
<td>cm(^2) s(^{-2} ) K(^{-1} )</td>
<td>1.63 \times 10(^7 )</td>
<td>1.047 \times 10(^7 )</td>
</tr>
</tbody>
</table>

Table 3.2: Physical properties of the 47v10 and FC70 fluids. Data taken from specification sheets provided by the manufacturers.

### 3.6 Experimental procedure

The experimental procedure begins by leveling the surface of the bottom plate using an Applied Geomechanics 900 Series tiltmeter, which is accurate to \( \pm 0.01^\circ \). Next, the working surfaces of the plates and the plexiglas rings are cleaned with an Alconox\textsuperscript{TM} /water solution and then rinsed with distilled water. A second rinse in isopropanol removes any remaining oil. A final cleaning is performed using a lens cleaning solution.

Assembly of the cell is done with great care to ensure the plexiglas rings are concentric to each other and the plates. Finally, the cell is placed in the support system and the temperature control is initiated.

Once the temperature of the plates has stabilized, the following procedure is used:

1. Fill the system with FC70 to the desired height.

2. Allow the temperature of the FC70 to equilibrate to the cell and confirm the height measurement.

3. Add saturated 47v10 to fill the rest of the cell.
4. Set $\Delta T \sim 3^\circ C$ for at least 12 hrs so that any final mixing can take place.

5. Return to $\Delta T \sim 0.3^\circ C$ so that the system can return to the conduction state.

6. Perform a final measurement of the heights of the layers.

7. Position the optics.

8. Begin data taking routine.

It should be noted that the height measurements are taken with a Wild floor cathetometer accurate to $\pm 0.02 mm$.

### 3.6.1 Data acquisition procedure

As mentioned above, the data are captured by a Pulnix TM-7CN CCD camera. The camera is interfaced to a 386 computer via an Imaging Technology Inc. FG-100-AT Image Processor board which captures, displays, and stores the images. The data procedure starts with capturing an image of the conduction state as a background. Next, $\Delta T$ is increased by approximately 0.1$^\circ C$. The system is left to equilibrate at the new $\Delta T$ for at least 6 hrs. After the equilibration period, an image is captured and compared to the background to detect the presence of a convective roll structure. If no rolls exist, $\Delta T$ is increased again and the procedure repeated. If a roll structure is found, a time series is captured.
A time series of whole images would quickly exhaust the available hard disk space. Instead, a line of data is captured through the middle of each layer, shown schematically in Figure 3.11. Since the image is in the form of concentric circles, the lines of data are also circles of pixels. A typical data file consists of 520 or 1030 circles captured every 30 sec. Each circle is straightened and plotted as intensity vs. position to create a space-time diagram similar to those described in Chapter 2. Typically, it is necessary to remove a time average background at each space point. This removes any stationary optical defects and leaves any time dependent features. For example see 3.12, 3.13 and 3.14. Space-time diagrams can be analyzed to determine if roll motion is present, the wavelength of the pattern, and the period of the motion. It should be noted that the space dimension of the space-time diagrams will be in terms of the angle $\theta$ around the cell, in units of degrees ($^\circ$). A conversion from the angle to a length can be made by using the mean circumference of the system $C_m = 2\pi r_m = 338.7 \text{ mm}$ as $x = \theta C_m / 360$.

### 3.7 Experimental results

The primary result of this work is a phase diagram showing the onset $\Delta T$ vs. $l_1$ (Figure 3.15). Time-dependent convection is found in the range $0.357 \leq l_1 \leq 0.382$ at a $\Delta T$ roughly $0.1^\circ C$ above the onset to stationary convection. Unfortunately, the desired result of time-dependence at the onset to convection has not been found with this set of fluids. However, a recent theoretical analysis by Renardy[78] has shown that the time-dependent state is not expected for this set of fluids. Renardy predicts
Figure 3.11: Schematic showing circles of data captured for time series files.
Figure 3.12: Background subtracted space-time diagram showing the roll motion at $\Delta T = 1.103^\circ C$, $l_1 = 0.375$. (a) is upper layer, (b) is lower layer. Vertical axis is time (min). Horizontal axis is $\theta(\circ)$ around cell.
Figure 3.13: Background subtracted space-time diagram showing roll motion at $\Delta T = 1.189^\circ C$, $l_1 = 0.375$. (a) is upper layer. (b) is lower layer.
Figure 3.14: Background subtracted space-time diagram showing roll motion at \( \Delta \theta = 1.567^\circ C \), \( \ell_1 = 0.375 \). (a) is upper layer. (b) is lower layer.
a time-dependent state in the range $0.415 \leq l_1 \leq 0.417$ which is different than the experimental range. Likewise, the predicted $\Delta T = 0.64$ is below the experimental value $\Delta T = 0.9$ for $l_1 = 0.40$, the only depth fraction with both a theoretical prediction and experimental verification.

The region of time dependent behavior and the onset $\Delta T$ to stationary convection are identical to the findings of Colovas[14] who performed experiments using the same fluid combination in a system with rectangular geometry. However, $\Delta T$ necessary for the onset to time dependence is less in the annular system than that found in the rectangular system. This could possibly be attributed to confinement effects in the rectangular system due to the presence of endwalls.

The stationary convection at $l_1 \leq 0.357$ shows strong evidence of thermal coupling while states at $l_1 \geq 0.382$ show stationary mechanical coupling. This orientation, with TC at low $l_1$, MC at high $l_1$, and the time-dependence in between, confirms the general prediction of Colinet and Legros[5] as shown in Figure 3.5. It should be noted that these states have been explored to at least $\Delta T = 2^\circ C$ (twice the onset to convection) and sometimes to $\Delta T = 3^\circ C$. The time-independent states are stationary throughout the range of $\Delta T$.

### 3.7.1 The time-dependent state

The analysis of the time-dependent state begins with a fast-Fourier transform (FFT) (similar to that described in Chapter 2) in both the spatial and temporal directions.
Figure 3.15: Onset temperature difference to stationary thermal coupling (●), stationary mechanical coupling (○) and time-dependent (□) convection. Solid line serves only as a guide to the eye.
The spatial FFT yields the wavelength of the pattern while the temporal FFT yields the period.

The wavelength (λ) of the pattern is shown in Figure 3.16 as a function of ΔT. It is evident that the wavelength is roughly constant over the range of ΔT with a value of λ = 15.6° = 14.7 mm. This can be expressed in terms of Renardy’s dimensionless wavenumber α = 2πd/λ ≈ 5.2. This falls in Renardy’s predicted range α = 4.5 – 5.5[78] for this fluid combination. Colovas’[14] rectangular system exhibits α ≈ 7, higher than the value found here. This difference in the wavenumbers might be attributed to the end conditions of the systems. Colovas’ system is constrained by end walls while the annulus does not have end walls to constrain the roll pattern.

The period (τ) of the time-dependent state is shown in Figure 3.17. This figure shows an increase from τ ~ 50 min at low ΔT to τ ~ 80 min at higher ΔT. Renardy[78] predicts an onset period of τ = 40 min which is somewhat smaller than the measured value. The prediction does not extend above onset. Colovas[14] finds a period of τ = 47 min which is roughly constant over the range of ΔT. The onset values are virtually identical. The difference between the values at high ΔT will be discussed below.

Further analysis of the time-dependent state focuses on the positions of the roll boundaries and their motion. A program was written to find the downflow boundary, which appears in the space-time diagram as a high value. The program finds the average of pixels i to i + 5 and compares it to the averages of pixels i - 6 to i - 1 and i + 6 to i + 11. If the middle average is larger than the two neighbor averages
Figure 3.16: The wavelength ($\lambda$) vs. $\Delta T$ for $l_1 = 0.375$. (□) is lower layer. (●) is upper layer. Error bars determined by the full width at half maximum of the power spectra used to determine the wavelength.
Figure 3.17: The period ($\tau$) vs. $\Delta T$ for $l_1 = 0.375$. (□) is lower layer. (●) is upper layer.
pixel $i$ is labeled as a upflow roll boundary. This procedure is used to identify the upflow roll boundaries throughout the space-time diagram (see Figures 3.18, 3.19 and 3.20). Two things can be observed in these figures. First are coupling oscillations. Where the different symbols fall on top of one another, the upflow roll boundaries in each layer are aligned. This indicates a thermal coupling state. In other parts of the figure, the different symbols do not overlap, indicating mechanical coupling.

The second observation concerns the movement of the roll boundaries. At $\Delta T$ near onset, the roll motion is relatively smooth and linear (Figures 3.18 and 3.19). At higher $\Delta T$, the roll motion becomes less smooth (Figure 3.20). In other words, the velocity is relatively constant over long periods at $\Delta T$ near onset while the velocity changes with time at higher $\Delta T$. Closer examination of Figure 3.20 shows that the velocity is slower when the roll boundaries coincide and faster when the boundaries are separated. This fast/slow or stick/slip motion is qualitatively similar to the "ratcheting motion" described by Gorman et al. [96] in cellular flames. The difference in the velocities is summarized in Figure 3.21, which shows a representative roll boundary from two different values of $\Delta T$ or in Figure 3.22 which shows the velocities vs. time for the same roll boundaries. The stick/slip motion exhibits a slow velocity $v_s \approx 0.12 \text{ mm/min}$ and a fast velocity $v_f \approx 0.63 \text{ mm/min}$. The velocity of a roll boundary is found by fitting a line to pixels $i$ through $i + 30$ using a Numerical Recipes[49] routine. The slope of the line is taken as the velocity of pixel $i + 15$. This procedure is repeated over the complete range in $i$. 
Figure 3.18: Downflow roll boundaries of the state found in Figure 3.12. (×) is upper layer. (□) is lower layer.
Figure 3.19: Downflow roll boundaries of the state found in Figure 3.13. (×) is upper layer. (□) is lower layer.
Figure 3.20: Downflow roll boundaries of the state found in Figure 3.14. (×) is upper layer. (□) is lower layer.
Figure 3.21: One representative roll boundary taken from Figure 3.19 (o) showing smooth motion at $\Delta T = 1.189^\circ C$ and Figure 3.20 (x) showing rougher motion at $\Delta T = 1.567^\circ C$. 
Figure 3.22: Velocity vs. time for the roll boundaries shown in Figure 3.21. (o) at $\Delta T = 1.189^\circ C$. (*) at $\Delta T = 1.567^\circ C$. 
It can be conjectured that this stick/slip motion plays a roll in the increase of \( \tau \) as \( \Delta T \) increases. The slow motion in part of the cycle increases \( \tau \) as seen in Figure 3.17.

### 3.8 Conclusion

Experiments on the two-layer Rayleigh-Bénard system using Rhône Poulenc’s Rhodorsil 47v10 silicone oil over 3M’s Fluorinert FC70 have been described. The predicted time-dependent state for this set of fluids\[78\] has been observed using an optical technique (shadowgraphs) in an extended annular system for the first time. While this state is not coincident with the onset to convection (for this set of fluids), it is significantly closer to onset than that found in the single layer system. The onset to time-dependence in the single layer system is at least \( \Delta T = 2\Delta T_c \) and typically on the order of \( \Delta T = 100\Delta T_c \). The time-dependent state found here is only 10% above the onset to a stationary flow pattern. The general predictions of Colinet and Legros\[5\] of thermal coupling at low \( l_1 \), mechanical coupling at high \( l_1 \), and time-dependence in between have been confirmed. Renardy’s\[78\] predictions for the period and wavenumber for the time-dependent state in this set of fluids have also been confirmed.

Colovas’\[14\] experiments with the same fluids in a rectangular geometry show a phase diagram that has stationary onsets virtually identical to that found here. Likewise, the region of time-dependent convection is the same. However, time-dependence is found \( \sim 0.1^\circ C \) above the stationary state in the annulus and \( \sim 0.2^\circ C \) above in
the rectangle. The periods of the time-dependent states are the same near the onset to time-dependence. However, the period in Colovas' work is constant as $\Delta T$ is increased while the period increases with $\Delta T$ in this work. Finally, the wavenumbers of the patterns are different with $\alpha \approx 7$ in the rectangle and $\alpha \approx 5.2$ in the annulus.

The rolls undergo relatively smooth motion at $\Delta T$ near the onset to time-dependence. However, at $\Delta T$ high above onset, the rolls show non-uniform motion. This fast/slow or stick/slip motion is similar to the "ratcheting motion" found in cellular flames.

This experiment lays the groundwork for future investigations of the two-layer Rayleigh-Bénard system in an annular geometry. The next step with this set of fluids is to find the mechanism associated with the transition from smooth motion to the stick/slip motion. Also, the states found at the extremes of $l_1$ remain to be documented. The next phase of experimentation on this system should probably focus on the fluids (47v2 silicone oil over water) which exhibit time-dependence coincident with the onset to convection as found in the rectangular system by Colovas[14]. Additionally, the states involving deformation of the interface are yet to be explored in any geometry. Next generations of the apparatus need to expand beyond the quasi-one-dimensional nature of the annulus and rectangle. A plethora of new states have been predicted for experiments in extended geometries[67].
CHAPTER 4

Time-dependence in the Weissenberg effect

4.1 Introduction

Chapters 2 and 3 concern the flow of Newtonian fluids. This chapter is concerned with characterizing one of the many phenomena found in non-Newtonian fluids, namely the Weissenberg effect. This effect was first systematically studied by Weissenberg[17] and hence named after him.

Before describing the Weissenberg effect itself, a discussion of Newtonian vs. non-Newtonian fluids is in order. The term “Newtonian fluid” is used to describe a fluid that has a viscosity (dynamic $\mu$ or kinematic $\nu$) which is independent of the motion of the fluid and is named after Newton who first proposed this model[97]. The model enters into the Navier-Stokes equation as

$$\nu \nabla^2 \mathbf{v}$$

(4.1)

which can be seen in the last term on the right side of Equations 2.2, 2.3 or 3.2. Water, silicone oil (at low shear) and mineral oil are typical Newtonian fluids.
The term "non-Newtonian fluid" describes any liquid which doesn't fit the definition of a Newtonian fluid. For example, mayonnaise, ketchup, paint and Silly Putty, are common substances that are non-Newtonian fluids. Some non-Newtonian fluids have varying viscosities. The viscosity of these fluids changes depending on what is being done to them. For example, mayonnaise is perfectly happy to sit in a lump when left alone. However, as soon as it is pushed with a knife across the bread, it easily spreads. (This is an example of a thixotropic or shear-thinning fluid.) Other non-Newtonian fluids have "elastic" properties. For example, when Silly Putty is stretched and released, it will slowly pull itself back into a blob. A third type of non-Newtonian fluid is called "visco-elastic", which combine the varying viscosities with elastic effects. The outcome of all of this is that Equation 4.1 is no longer sufficient to describe the viscosity contribution to the Navier-Stokes equation.

Many researchers have spent lifetimes trying to devise terms to replace Equation 4.1. Numerous models can be found in rheology texts such as Macosko or Joseph. Two of these models are

\[ \nu \nabla^2 \mathbf{v} + \nu_1 (\nabla^2 \mathbf{v})^2 \]  

and

\[ \nu_2 (\nabla^2 \mathbf{v})^a \]  

where \( a \) is a power dependent on the specific fluid under consideration. The first is called a "second-order" model while the second is a "power law".

While some of the models fit some of the data quite well, no particular model stands out as correct under all circumstances. Typically, workers in this field choose
their favorite model and adjust the parameters until it fits the data. This is hardly satisfactory. Some of the models might even violate fundamental laws of physics. For example, the second term in the second-order model (Equation 4.2) has been shown to drop out of the equations after consideration of symmetry and the laws of thermodynamics[100, 101], leaving only the Newtonian term, which is insufficient to describe the phenomena. It has been suggested by Brand[100] that a detailed examination of the Weissenberg effect may aid in showing where these second-order models break down and possibly inspire a better model of some non-Newtonian fluids.

4.1.1 The Weissenberg effect

When a spinning rod is immersed in a pool of non-Newtonian fluid, sometimes the fluid (depending on its exact nature) will climb up the rod. Side views are shown in Figures 4.1, 4.2, 4.3 and 4.4 while top views appear in Figures 4.5 and 4.6. This is called the Weissenberg effect (or simply rod-climbing). It is a manifestation of the elastic properties of the fluid. Typically, a long chain polymer is dissolved in a viscous solvent to produce the most pronounced climbs. Roughly speaking, longer polymer chains produce stronger elastic forces, which are related to the height of the climbing fluid.

The Weissenberg effect is the result of “normal stresses”. When typical fluids undergo shear, viscosity produces a stress in the direction of the shear. (The term “shear” is used in this context to describe the result of pushing part of a fluid past another part. The “direction” of the shear is taken to be the direction of the relative
Figure 4.1: Side view of the Weissenberg effect in STP\textsuperscript{TM} at $\omega = 0.0 \ rev/sec$ showing the meniscus. Note, rod diameter is 1.27 cm.
Figure 4.2: Side view of the Weissenberg effect in STP\textsuperscript{TM} at $\omega = 1.0 \, \text{rev/sec}$ showing the concave shape of the climbing fluid. Note, rod diameter is 1.27 cm.
Figure 4.3: Side view of the Weissenberg effect in STP™ at $\omega = 2.0 \text{ rev/sec}$ showing the convex shape of the climbing fluid. Note. rod diameter is 1.27 cm.
Figure 4.4: Side view of the Weissenberg effect in STP\textsuperscript{TM} at $\omega = 2.3 \text{ rev/sec}$ showing the broken axial symmetry found in the time-dependent regime. Note. rod diameter is $1.27 \text{ cm}$.
Figure 4.5: Top view of the Weissenberg effect in STP™ at $\omega = 2.0 \, \text{rev/sec}$ showing the axisymmetric climbing fluid. The circle in the center of the picture is the top of the rod with diameter 1.27 cm.
Figure 4.6: Sequential top views of the Weissenberg effect in STP™ at \( \omega = 2.3 \, \text{rev/sec} \) showing the broken axial symmetry, captured every 0.3 sec. The circle at the centers of the pictures is the top of the rod with diameter 1.27 cm. The larger circles are concentric with the rod to serve as a reference. (a) shows the climbing fluid extending toward the bottom left, (b) toward the bottom right, (c) toward the top right and (d) toward the top left.
motion.) Normal stresses due to elastic properties in the fluid are stresses perpendicular to the direction of shear. For example, imagine two parallel plates moving past each other. If the gap between them is filled with a Newtonian fluid, each plate experiences a drag force due to viscosity that will oppose the motion. If however, the gap is filled with an elastic fluid, the plates will not only experience the viscous drag, but also a force perpendicular to the motion that will (typically) drive the plates apart. This force perpendicular to the shear is due to the normal stresses.

In the Weissenberg effect, the spinning rod produces shear in the azimuthal direction around the rod. The normal stresses, which are perpendicular to the azimuthal direction, push fluid radially inward. The fluid near the rod has no place to go except up. Eventually, gravitational and surface tension[102] forces on the climbing fluid balance the upward force and the result is a stable blob of fluid above the background pool.

Another way to think about the Weissenberg effect is to imagine that the elastic polymers are essentially rubber bands. As the rod rotates, these rubber bands are organized into a quasi-concentric pattern and stretched. They respond to the stretching by squeezing in, which produces the “normal stress” described above.

A mathematical analysis of the Weissenberg effect and a series of experiments have been performed by Joseph and numerous colleagues[102, 103, 104, 105, 106, 107, 108, 109]. A nice summary can be found in Joseph’s book[99]. Studies of the bulk flow in the fluid have been performed by Eitelberg[110, 111]. All of these authors appear to have concentrated on the primary state and only briefly mention the existence of
time-dependent states at higher rod rotation rates. Characterization of some of the time-dependent states is the focus of this work.

4.2 Apparatus

The apparatus used in these experiments is shown in Figures 4.7 and 4.8. Initial design, construction, and debugging of the apparatus was performed by Kahle[112]. The design of the apparatus took into account the desire to observe the climbing fluid from all angles. The container is made from a plexiglas cylinder bonded to a plexiglas plate which serves as the bottom. The rod enters the container from below, leaving the top open for visualization. A precision drill chuck holds the rod in place and is supported by bearings. This method of supporting the rod was chosen to facilitate easy changes of the rod. An aluminum disk holds an oil seal which is fitted over the rod and is sealed flush with the bottom of the container. It should be noted that early experiments showed a localized heating due to friction between the rod and seal. A 3.2 mm plexiglas sheet serves as a false bottom and isolates the fluid from the aluminum disk. Subsequent temperature measurements have shown this to virtually eliminate the problem[112]. A flexible coupler is used to connect the drill chuck to a Compumotor™ stepper motor model A/AX83-93 which drives the rod into rotation at a rate $\omega$. The motor is driven by a Compumotor™ AX Series controller. The rods are made out of drill rod, which has a precisely ground radius. The data that appear in this work were obtained with a 1.27 cm diameter rod.
Figure 4.7: Schematic of the Weissenberg apparatus.
Figure 4.8: Photograph of the Weissenberg apparatus.
Visualization of the climbing fluid is done from above with a Loral Fairchild 3456 pixel linear array CCD camera. The CCD camera is interfaced to a computer through a MuTech MV-1000/1100 interface board. Illumination of the climbing fluid is with a standard 50W halogen spotlight. The spotlight is turned on and off via a relay switch controlled through the computer's parallel port. This allows a minimum amount of heat from the bulb to be introduced into the system. Additionally, three sheets of heat (infrared radiation) absorbing glass are positioned along the light path to further reduce the heating of the fluid.

The apparatus is contained inside a plexiglas box for temperature control. Constant temperature water is circulated through a radiator. A fan circulates the air inside the box past the radiator. A thermistor is immersed into the fluid to measure the temperature. The resistance of the thermistor is monitored by a Hewlett-Packard 3478A DMM which is interfaced to the computer via the GPIB interface bus. It should be noted that all heat sources including the driving motor, the fan motor, and the light bulb are outside the containment box. The temperature controlled water is provided by a Brinkmann Instruments Lauda RM6 recirculating temperature bath.

4.2.1 The fluid

The selection of the fluid was driven by two considerations. First, the fluid should be readily available. Second, the desire to recreate previous experiments to verify the apparatus. To this end, we follow the lead of Joseph et al. [103, 106, 105] in choosing First Brand Corporation's STP™ Oil Treatment. STP™ is sold as a viscosity index
improver for automobile engines. While the exact content of STP™ is a trade secret, the main ingredients are listed in Table 4.1. Characterization of STP™ in the form of the "constitutive constants of second order fluids" can be found in [105, 113] and references therein.

Since the exact content of STP™ is unknown, some attempts have been made to mix a solution with known ingredients. The first mixture consisted of ~ 1% by weight of a high molecular weight (750,000-1,000,000) polyisobutylene (PIB) (Exxon’s Vistanex™ L-80) in mineral oil. A second solution included ~ 1% by weight of a medium molecular weight (51,000-56,000) PIB (Exxon’s Vistanex™ LM-MH) and mineral oil. These solutions were dissolved at room temperature for about six months.

### Table 4.1: Ingredients of Oil treatment. Data taken from a material safety data sheet provided by the manufacturer[7].

<table>
<thead>
<tr>
<th>Material</th>
<th>Content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemically neutralized heavy naphthenic petroleum distillates</td>
<td>15 - 40</td>
</tr>
<tr>
<td>Solvent-dewaxed heavy paraffinic petroleum distillates</td>
<td>60 - 80</td>
</tr>
<tr>
<td>Zinc dialkyldithiophosphate</td>
<td>3 - 11</td>
</tr>
<tr>
<td>Calcium sulfonate</td>
<td>1 - 4</td>
</tr>
<tr>
<td>Amine grafted ethylene/propylene copolymer</td>
<td>3 - 11</td>
</tr>
</tbody>
</table>
with occasional stirring. Tests with the Weissenberg apparatus showed minimal climbing in either solution. Consequently, a third solution was mixed with ~4% by weight high molecular weight PIB in mineral oil. To speed up the mixing process, this solution was subjected to heating (the temperature should never exceed 300°C or the polymer will break down) and continuous stirring. This procedure required about three months for complete mixing. Initial tests show this solution will indeed climb a rotating rod. Unfortunately, time constraints have precluded further investigation with this fluid.

While one of the solutions described above does exhibit the Weissenberg effect, the long mixing time makes it difficult to use. Alternatives based on the “Boger Fluids”[114] have also been produced. The Boger fluids have been extensively tested by the rheological community and their properties are readily available. Two solutions have been prepared. The first consists of 1.7% PIB, 63% kerosene and 36% polybutene (by weight), while the second contains 3.2% PIB, 60% kerosene and 37% polybutene. The PIB is the same Exxon Vistanex™ L-80 used above. The research grade kerosene is from Fisher Scientific and the polybutene is British Petroleum’s Hyvis™-3. The PIB and kerosene are combined first and mixed for 1-1.5 weeks. After the PIB has dissolved in the kerosene, the polybutene is added. The complete solution is then mixed for an additional 2-3 weeks. This gives a total mixing time of roughly 5 weeks, much less than the 3 months needed by the PIB/mineral oil solutions. Tests show no Weissenberg effect in the first (1.7% PIB) solution, while the second fluid climbs...
slightly less than the STP™. Again, only initial testing has been performed with these fluids.

4.3 Experimental procedure

The initial step of the procedure is the assembly of the apparatus. Great care is exercised to ensure that the rod is perfectly centered and perpendicular to the container. Unwanted wobble of the rod must be eliminated. Next, the container is filled with STP to the desired height. For this work, the height of the STP™ is 2.7 cm. Since the filling process invariably introduces air bubbles into the fluid, it is left to sit for an extended time (~ 24 hrs) so the bubbles migrate to the surface. During this waiting period, the camera and light source can be put into position. Once the bubbles are gone and everything is satisfactory, the plexiglas box can be sealed. Next, the fan and water are turned on for a period of at least 18 hr for the temperature of the system to equilibrate.

Once the initial equilibration period has passed, a data series can be initiated. A typical data series begins at a value of $\omega$ below the onset to the time-dependent state. Once the desired value of $\omega$ is set, the system is maintained at this value for ~ 1 hr, following which a data file is collected. Data files typically consist of 32768 lines taken at time intervals of $\delta t = 0.00922$ sec between the lines. Each line is usually 400 pixels wide. After the data set is captured, $\omega$ is increased by $\delta \omega \approx 0.01 \, rev/sec$ or less, and the equilibrium period initiated. As mentioned above, the light is on only during the
data capture portion of the routine. The data series typically ends when the climbing liquid has started to exhibit chaotic time-dependence.

4.3.1 Data Analysis

Since the main goal of this work is to characterize the time-dependence of the climbing fluid, it is necessary to determine the position of the edge of the climbing fluid as a function of time $p(t)$. Once $p(t)$ is known, standard techniques such as power spectra analysis and the construction of return maps can be used to characterize the phenomena.

Determining the edge of the climbing fluid depends primarily on the nature of the original data file. The method chosen here is to illuminate the climbing fluid from an angle such that only light from the edge of the climbing fluid enters the camera, see Figure 4.9. Therefore, the raw data set shows a line of high intensity indicating the edge of the climbing fluid in a dark background, see Figures 4.10, 4.11, 4.12 and 4.13.

Once the data sets have been captured in this form, the next task is to determine the position of the edge of the climbing fluid. This was originally done by sequentially comparing each pixel to an arbitrary cutoff. The first pixel whose intensity was larger than the cutoff was labeled as the edge and the examination of the next line began. Unfortunately, random variations somewhere in the system made this less than perfect. Possible sources for these variations include: changes in the reflectivity of the fluid (dust on the surface or an air bubble), electronic noise, inconsistencies in the
Figure 4.9: Diagram showing the lighting scheme.
Figure 4.10: Grayscale image of a raw data file. \( \omega = 1.769 \text{ rev/sec} \) at \( T = 16.6^\circ C \).
Figure 4.11: Grayscale image of a raw data file. $\omega = 1.899 \text{ rev/sec}$ at $T = 16.6^\circ C$. 
Figure 4.12: Grayscale image of a raw data file. \( \omega = 2.015 \text{ rev/sec} \) at \( T = 16.6^\circ C \).
Figure 4.13: Grayscale image of a raw data file. $\omega = 2.510 \text{ rev/sec}$ at $T = 16.6^\circ C$. 
lighting (60 Hz AC power?), analog-to-digital noise, and many others. To eliminate these effects, an averaging of the pixels is performed prior to the comparison. Through trial and error, a procedure of averaging three spatial pixels and four sequential lines was finally settled upon. The program implementing this procedure can be found in Appendix C. This averaging scheme reduces the file from 32768 lines to 8192 positions of the edge of the climbing fluid \( p(t) \). Examples of the output from this program can be seen in Figures 4.14, 4.15, 4.16 and 4.17.

Once \( p(t) \) has been found, a fast-Fourier transform technique is used to determine the power spectrum. The power spectrum routine is from Numerical Recipes[49]. Example power spectra can be seen in Figures 4.18, 4.19 and 4.20. The power spectra are then analyzed to determine the dominate frequencies and associated power present in the flow.

In addition to power spectra, the edge of the climbing fluid data is analyzed using return maps. Return maps are produced by mapping time series data onto a coordinate system of \((x, y, z)\) as \((p(t), p(t + \delta t), p(t + 2\delta t))\) (or higher dimensions if necessary), where \(\delta t\) is called the time delay (more on \(\delta t\) below). As \(t\) is incremented, the points plot out a trajectory in the space. If \(p(t)\) is constant \((i.e., \text{the climbing fluid doesn't move})\), the map will be a single point \((i.e., \text{all of the data will fall on top of each other})\). If \(p(t)\) is singly periodic, the trajectory will be a circle, if doubly periodic a torus. Finally, if \(p(t)\) is random, the trajectory will eventually completely fill the space. For a discussion on return maps, see the books by Abarbanal[115], Holden[116] or Bergé. Pomeau & Vidal[117].
Figure 4.14: Position of the edge of the climbing fluid in Figure 4.10.
Figure 4.15: Position of the edge of the climbing fluid in Figure 4.11.
Figure 4.16: Position of the edge of the climbing fluid in Figure 4.12.
Figure 4.17: Position of the edge of the climbing fluid in Figure 4.13.
Figure 4.1S: Power spectrum of Figure 4.14.
Figure 4.19: Power spectrum of Figure 4.15.
Figure 4.20: Power spectrum of Figure 4.16.
The characteristics of the delay map depend on the particular value of the time delay $\delta t$. Following Abarbanel[115], the time delay is chosen as the $\delta t$ at the first minimum of the “average mutual information.” The average mutual information is found with

$$I(\delta t) = \sum_{p(t),p(t+\delta t)} P(p(t),p(t+\delta t)) \log_2 \left[ \frac{P(p(t),p(t+\delta t))}{P(p(t))P(p(t+\delta t))} \right]$$

(4.4)

where $P(p(t))$ is the probability density function and $P(p(t),p(t+\delta t))$ is the joint probability density function[115]. An example average mutual information function can be seen in Figure 4.21.

Return maps can be quite complicated even with the best $\delta t$. A Poincaré section is used to reduce the level of complication of a high dimension map. To produce a Poincaré section, a plane is cut through the map and a mark is noted wherever the trajectory passes through the plane from a particular side. If the trajectory is a circle, the Poincaré section will show a single point (see Figure 4.22). If the trajectory is a toroid, then the marks on the Poincaré section will form a simple closed loop. As the trajectory gets more complicated, so too will the Poincaré section. For completely random data, the Poincaré section will be completely filled. For a discussion of Poincaré sections see Bergé et al. [117].

Operationally, the return maps are calculated from $p(t)$ using a Fortran program that can be found in Appendix C. The first step in this process is to subtract out the average of $p(t).$ yielding $p(t) - < p >$. This step merely shifts the $p(t)$ from oscillating around some unknown value to oscillating around 0. Since the data is oscillating around zero, the Poincaré section is chosen to be the $z = 0$ plane. This
Figure 4.21: Average mutual information function for $\omega = 2.495 \text{ rev/sec}, T = 18.8^\circ C$
Figure 4.22: Sample Poincaré section when the trajectory is a circle.
translates into marking the Poincaré section when the trajectory crosses the $z = 0$ plane from below. The next step in the process is to determine when the trajectory has crossed the plane \(i.e.,\) find when \(z(i) < 0\) and \(z(i + 1) > 0\). Once this is known a \textit{Numerical Recipes}[49] routine is used to interpolate the point \((x, y, 0)\) from the trajectory. Five points along the trajectory are fit with a 4\(^{th}\) order polynomial for this task. Once the point is found, this process is repeated for other points along the trajectory.

### 4.4 Experimental results

The acquisition procedure described above yields a series of data files at increasing values of the rod rotation rate \(\omega\). Analysis of these files follows the above procedure, first finding the position of the climbing fluid \(p(t)\), second finding the power spectrum, and finally the Poincaré sections. \(p(t)\) and the power spectra are analyzed in conjunction to determine the onset of the primary (called the wobble) and secondary (the modulation) instability frequencies. These onset values are labeled \(\omega_p\) and \(\omega_s\) respectively. In addition, the power in each frequency can also be found.

Analysis of \(p(t)\) and the power spectra for \(\omega < \omega_p\) shows an extremely small time dependence with the frequency of the instability \(f = \omega\) (see Figures 4.14 and 4.18). This shows that the rod is not perfectly aligned, but is very close. The plot of \(p(t)\) at \(\omega \cong \omega_p\) shows the onset to the wobble but with a decrease in the amplitude as in Figure 4.23. The corresponding power spectrum shows the emergence of the primary instability frequency, \(f_p\). Investigation of this decaying amplitude leads to
an analysis of the temperature of the fluid during the data file as plotted in Figure 4.24. The temperature of the fluid rises \( \sim 0.03 \, ^\circ C \) during the data capture routine. This temperature rise seems to be the only source for the decrease in the amplitude of the wobble. The next data file in the series shows a strong wobble with a consistent amplitude during the complete file.

The temperature dependence of \( \omega_p \) can be clearly shown by capturing a series of data sets at different temperatures. Analysis of a series of data sets shows the temperature dependence of \( \omega_p \) as plotted in Figure 4.25. Higher STP\textsuperscript{TM} temperatures require higher \( \omega_p \) at the onset to the primary instability. This seems to confirm the idea of the decaying amplitude at \( \omega \equiv \omega_p \) being due to a slight increase in the temperature.

Above the onset of the primary frequency but below the onset of the secondary \( (\omega_p < \omega < \omega_s) \), the wobble is clearly evident (see Figure 4.15). Analysis of the power spectra (Figure 4.19) shows the amplitude of the primary frequency increases from zero with increasing \( \omega \) as in Figure 4.26. Attempts to fit the amplitude of the primary instability of a number of data sets with

\[
A \propto (\omega - \omega_p)\alpha
\]

shows a range of \( \alpha \sim 0.1 \rightarrow 0.4 \). with a typical fit shown in Figure 4.26.

At \( \omega = \omega_s \), a second frequency \( f_s \) enters into the system as a modulation on top of the primary (see Figures 4.16 and 4.20). A plot of \( \omega_p \) and \( \omega_s \) vs. temperature appears as a phase diagram for the system in Figure 4.27. \( f_s \) is incommensurate with \( f_p \). Many other peaks also appear in the power spectrum at \( \omega_s \). Two of the largest
Figure 4.23: Position of the climbing fluid just at the onset to the primary instability $\omega = 1.839 \, rev/sec$ showing a decrease in the amplitude.
Figure 4.24: Temperature of the fluid during the capture of the data file shown in Figure 4.23.
Figure 4.25: Critical rotation rate for the onset to the primary frequency: $\omega_p$ vs. temperature.
Figure 4.26: The amplitude of $f_p$ vs. $\omega$ at $T = 16.6^\circ C$. The solid line is a fit with Equation 4.5 with $\alpha = 0.22$. 
are the sum peak at \( f_p + f_s \) and the difference peak at \( f_p - f_s \). A power spectrum with all of the peaks identified can be seen in Figure 4.28. A plot of \( f_p \) and \( f_s \) vs. \( \omega \) for one data set is shown in Figure 4.29. This plot shows a linear increase in both \( f_p \) and \( f_s \) with \( \omega \). Perhaps a more revealing graph can be found in Figure 4.30 which shows \( f_p/\omega \) and \( f_s/\omega \) vs. \( \omega \). This shows \( f_p/\omega = 0.42 \) and \( f_s/\omega = 0.061 \) independent of \( \omega \).

Combining the plots of \( f_p/\omega \) and \( f_s/\omega \) vs. \( \omega \) from different data sets yields the plot in Figure 4.31. As can be seen, the ratios \( f_p/\omega \) and \( f_s/\omega \) are nearly identical regardless of the particular data set or the temperature of the STP™. Data sets taken with decreasing \( \omega \) show similar results. No hysteresis has been observed within our resolution.

In addition to the frequencies, the amplitude of the secondary instability has also been analyzed. Figure 4.32 shows this amplitude vs. \( \omega \). Like the primary, the secondary grows in amplitude from zero with increasing \( \omega \). However, a comparison of the amplitude of the primary and secondary instabilities shows the latter is always significantly smaller than the former (Figure 4.33). Attempts to fit Equation 4.5 to the power in the secondary show a range of \( \alpha = 0.2 \rightarrow 0.4 \) for various data sets, with a typical fit in Figure 4.32.

There is some evidence of a third frequency entering into the motion at rod rotation rates above the onset to the secondary instability \((\omega > \omega_s)\) (see Figure 4.34). However, the peak on the power spectrum is short lived and quickly splits into two new peaks (at higher \( \omega \)). This is possibly due to the appearance of a fourth and fifth frequencies
Figure 4.27: Phase diagram for the Weissenberg effect with rod diameter 1.27 cm and STP depth 2.7 cm showing the onset rotation rate for the primary (□) and secondary (*) instabilities vs. temperature.
Figure 4.28: Power spectrum at $\omega = 2.049\ rev/sec$ at $T = 16.6^\circ C$ with all of the peaks identified.
Figure 4.29: The primary $f_p$ and secondary $f_s$ frequencies vs. rod rotation rate $\omega$ at $T = 16.6^\circ C$. The error bars are on the order of the size of the symbol.
Figure 4.30: The primary and secondary frequencies plotted as $f_p/\omega$ and $f_s/\omega$ vs. $\omega$. Same data as shown in Figure 4.29.
Figure 4.31: The ratio of the primary and secondary frequencies to $\omega$ ($f_p/\omega$ and $f_s/\omega$) vs. $\omega$ from different data sets. $\ast$ at $T = 15.4^\circ C$. $\bigtriangleup$ at $T = 16.6^\circ C$. $\diamondsuit$ at $T = 16.6^\circ C$. $\circ$ at $T = 16.6^\circ C$. $\square$ at $T = 18.8^\circ C$. Sets at the same temperature were taken at different times to show reproducibility.
Figure 4.32: Amplitude of the secondary instability at $T = 16.6^\circ C$. Solid line is a fit with Equation 4.5 where $\alpha = 0.21$. 
Figure 4.33: Amplitude of both the primary (□) and secondary (★) instabilities at $T = 15.4^\circ C$. 
or a mis-identification of the third frequency. More detailed study is needed to identify this elusive third frequency.

Upon further increases in $\omega$, the time-dependence of $p(t)$ begins to have a chaotic character (see Figures 4.13 and 4.17). The power spectra begin to show significant increases of the power in the harmonics relative to the power in the primary frequencies. Additionally, a broad-band background begins to appear (Figure 4.35). More discussion of the chaotic nature of the flow will appear in the next section.

### 4.4.1 Poincaré sections

The process of finding Poincaré sections begins with a calculation of the "average mutual information" function (Equation 4.4). The first minimum of the average mutual information is used as the time delay for calculating the Poincaré section as described above.

The Poincaré sections for data files with $\omega < \omega_p$ are similar to that in Figure 4.36. Since the value of $p(t) - < p >$ is essentially zero, the Poincaré section is a small dot at $(0,0)$. At $\omega_p < \omega < \omega_3$, the points on the Poincaré section move away from $(0,0)$ and the spot grows as in Figure 4.37. The ideal Poincaré section for this single frequency time series would be a single point located at some value $(x, y)$. Since there is some noise in the experimental $p(t)$, the single point is smeared out into a finite sized spot.

At $\omega = \omega_3$, the Poincaré section expands and begins to show a more complex structure (Figure 4.38). Increasing $\omega$ above $\omega_3$ (but below the more chaotic region)
Figure 4.34: Evidence of a third frequency entering the motion identified with the arrow. Data taken from $\omega = 2.059 \text{ rev/sec}$ at $T = 16.6^\circ C$. 
Figure 4.35: Power spectrum of Figure 4.17, showing broad-band background.
Figure 4.36: Poincaré section for $\omega = 1.899 \text{ rev/sec}$ at $T = 16.6^\circ C$. $p(t)$ for this file is similar to that found in Figure 4.14.
Figure 4.37: Poincaré section for $\omega = 2.029\, \text{rev/sec}$ at $T = 16.6\, ^{\circ}\text{C}$, above the onset of the wobble but below the modulation.
shows Poincaré sections as in Figure 4.39. This shows a further expansion and a
definite structure of the points. The Poincaré section continues to grow at still higher
\( \omega \) (Figure 4.40). Some structure is still evident indicating the flow is not random, but
rather chaotic in nature.

4.5 Conclusion

The Weissenberg Effect (rod-climbing) has been studied in First Brand Corporation’s
STP™ oil additive. The transitions from a steady state to a time dependent state
with one frequency, two frequencies and more have been identified and characterized
as a function of rod rotation rate \( \omega \) and at various temperatures of the fluid for the first
time. The onset rotation rate to the primary frequency \( \omega_p \) increases with increasing
temperature. The value of the ratio of the primary frequency to the rotation rate
\( f_p/\omega = 0.42 \) is independent of the temperature and \( \omega \). The amplitude of the primary
frequency increases with \( \omega \).

A second frequency \( (f_s) \) enters into the motion at a rod rotation rate of \( \omega_s > \omega_p \).
Like the primary, the ratio of the frequency of the secondary instability to \( \omega \) is also
independent of temperature and \( \omega \) with a value of \( f_s/\omega = 0.062 \). There is some
evidence suggesting the existence of a third frequency. However, further work needs
to be done to conclusively identify it.

At still higher values of \( \omega \), broadband background begins to appear in the power
spectra. This broadband background signals the onset of chaotic time dependence
in the flow. This sequence of states, from one frequency to two frequencies to three
Figure 4.38: Poincaré section for $\omega = \omega_3 = 2.059 \text{ rev/sec}$ at $T = 16.6^\circ C$. 
Figure 4.39: Poincaré section for $\omega = 2.089 \text{ rev/sec}$ at $T = 16.6^\circ C$. 
Figure 4.40: Poincaré section of the more chaotic region for $\omega = 2.329 \text{ rev/sec}$ at $T = 16.6^\circ C$. 
frequencies and then chaos, is reminiscent of the “Ruelle-Takens” scenario. (For a dis-
cussion of the Ruelle-Takens route to chaos, see Bergé et al. [117].) This similarity to
the Ruelle-Takens scenario suggests that the Weissenberg effect is a low-dimensional
system with few active degrees of freedom. Further refinement of the apparatus is
necessary for direct measurement of the dimension of the system to confirm this
conjecture.

In addition to the frequency, Poincaré sections have been used to characterize the
flow states. The Poincaré sections support the conclusions based on the power spectra
analysis of the appearance of chaotic time dependence at $\omega \gg \omega_s$.

The characterization of the transition to chaos presented here will serve as a
foundation for future work with this system. Some of the many open questions with
the Weissenberg effect in STP$^\text{TM}$ include:

1. Are the ratios $f_p/\omega$ and $f_s/\omega$ constant for a larger range of temperatures?

2. Do $f_p/\omega$ and $f_s/\omega$ change with the size of the rod?

3. Are there limits to the rod size which causes a fluid climb?

4. How quickly does the fluid respond to rapid changes in the rod rotation rate?
   \emph{i.e.}, How fast does the fluid reach full height with a sudden increase in $\omega$ or fall
to zero height with a sudden stop in $\omega$?

5. What is the mechanism behind the “flower instability” observed by Beavers and
   Joseph[104] with a time-dependent $\omega$?
Additionally, these same questions need to be answered for other fluids which exhibit the Weissenberg effect. As mentioned above, STP™ has not been fully characterized and this limits attempts at modeling the system. Repeating this investigation with a fluid that has been fully characterized (or characterizing the STP™) is a required step toward achieving Brand's goal[100] of a better model for visco-elastic fluids.
Appendix A

Analysis program used for the characterization of STI in the Taylor-Dean system.

The Fortran analysis program implementing the binarization procedure, calculation of the histograms and binary correlation function described in Chapter 2. The curve fitting has been done using MicroCal’s Origin™.

c
program name: hist.for found on the Physics Dept.

character*30 file2,file3,file4,file5
character*32 file6
character*31 file7
character*5 fil
character*21 dir2,dir3,dir4,dir5
character*23 dir6
character*22 dir7
character*9 file1
integer n1(102),n2(102),data(6000,102),xd1(6000,102)
integer ibuf(102),n3(5000),n4(5000)
INTEGER in1,in2,nframe
real perc,c(50),avsq,sum(50),r(102)
common/datac/counter

c enter the arbitrary cutoff
write(6,*),'enter diff'
read(5,*),diff
write(6,*),'this program does the turbulence for frames'
write(6,*),'1 through x; please enter x'
read(5,*),nframe
nn=nframe-5
write(6,*),'enter n1 output directory'
read(5,'(A)')dir2
write(6,*)'enter n2 output directory'
read(5,'(A)')dir3
write(6,*)'enter time laminar domain directory'
read(5,'(A)')dir5
write(6,*)'enter time turb domain directory'
read(5,'(A)')dir4
write(6,*)'enter binary corr data directory w/o average'
read(5,'(A)')dir6
write(6,*)'enter binary corr data directory w/
  switched points'
read(5,'(A)')dir7
WRITE(6,*)'Input filename w/o extensions'
read(5,'(A)')fil
write(file1,fil)
1111 WRITE(6,*) fil
format(a5,'.cnv')
write(file2,fil)
format(a21,a5,'.dat')
write(file3,fil)
format(a21,a5,'.dat')
write(file4,fil)
format(a21,a5,'.dat')
write(file5,fil)
format(a21,a5,'.dat')
write(file6,fil)
format(a21,a5,'.dat')
write(file7,fil)
format(a22,a5,'.dat')
C input raw data file
counter=0
open(2,file=file1,status='old', access='direct',
  & format='unformatted',recl=102)
read(2,rec=1) numpts,number,dt,fi,fo,ft,ri,ro,ibk
do in=1,102
  n1(in)=0
  n2(in)=0
endo
do in=1,nframe
  read(2,rec=in+1)ibuf
  do j=1,102
    data(in,j)=ibuf(j)
  endo
endo
close(2)

c run binarization procedure in function at bottom
call datacrunch(data,xd1,1,nframe,diff)
c turbulent fraction from binarization procedure
perc=counter/((float(nframe)-7.)*96.)*100.
write(6,*)'% turb is',perc

c find histogram of spatial size of turbulent domains
do in1=4,nframe-5
  ic=0
  do in2=4,99
    if (xd1(in1,in2).eq.1) then
      ic=ic+1
    else
      if (ic.ne.0) n1(ic)=n1(ic)+1
      ic=0
    endif
  enddo
  if (ic.ne.0) n1(ic)=n1(ic)+1
enddo
open(1,file=file2, status='new')
do i=1,102
  if (n1(i).gt.0) write(1,*)i,n1(i)
enddo
write(1,*)10000,10000
write(1,*)counter,perc
write(1,*)diff
close(1)

c find histogram of spatial size of laminar domains
do in1=4,nframe-5
  ic=0
  do in2=4,99
    if (xd1(in1,in2).eq.0) then
      ic=ic+1
    else
      if (ic.ne.0) n2(ic)=n2(ic)+1
      ic=0
    endif
  enddo
  if (ic.ne.0) n2(ic)=n2(ic)+1
enddo
open(1,file=file3, status='new')
do i=1,102
  if (n2(i).gt.0) write(1,*)i,n2(i)
enddo
write(1,*)10000,10000
write(1,*)counter,perc
write(1,*)diff
close(1)
c find histogram of temporal size of turbulent domains
   do in=1,5000
      n4(in)=0
      n3(in)=0
   enddo
   do in2=4,99
      ic=0
      do in1=4,nn
         if (xd1(in1,in2).eq.1) then
            ic=ic+1
         else
            if (ic.ne.0) n3(ic)=n3(ic)+1
            ic=0
         endif
      enddo
      if (ic.ne.0) n3(ic)=n3(ic)+1
   enddo
   open (1, file=file4, status='new')
   do i=1,nframe
      if (n3(i).gt.0) write(1,*),i,n3(i)
   enddo
   write(1,*),10000,10000
   write(1,*),counter,perc
   write(1,*),diff
   close(1)

c find histogram of temporal size of laminar domains
   do in2=4,99
      ic=0
      do in1=4,nn
         if (xd1(in1,in2).eq.0) then
            ic=ic+1
         else
            if (ic.ne.0) n4(ic)=n4(ic)+1
            ic=0
         endif
      enddo
      if (ic.ne.0) n4(ic)=n4(ic)+1
   enddo
   open(1,file=file5,status='new')
   do i=1,nframe
      if (n4(i).gt.0) write(1,*),i,n4(i)
   enddo
   write(1,*),10000,10000
   write(1,*),counter,perc
   write(1,*),diff
   close(1)
c find binary correlation laminar = 0, turbulent = 1
  do k=1,45
    c(k)=0.0
    sum(k)=0.0
  enddo
  do i=4,nn
    avsq=0.0
    do j=1,3
      r(j)=0.0
      r(j+99)=0.0
    enddo
    do j=4,99
      r(j)=float(xd1(i,j))
      avsq=avsq+r(j)*r(j)
    enddo
    if (avsq.eq.0) goto 121
    avsq=avsq/96.
    do j=1,45
      do k=4,100-j
        c(j)=c(j)+r(k)*r(j+k-1)/avsq
        sum(j)=sum(j)+1.0
      enddo
    enddo
  121 enddo
  open(1,file=file6,status='new')
  do i=1,45
    if (sum(i).eq.0) then
      write(1,*),i,i-1,0.0
    else
      write(1,*),i,i-1,c(i)/sum(i)
    endif
  enddo
  close(1)

  c find binary correlation laminar = 1, turbulent = 0
  do k=1,45
    c(k)=0.0
    sum(k)=0.0
  enddo
  do i=4,nn
    avsq=0.0
    do j=1,3
      r(j)=0.0
      r(j+99)=0.0
    enddo
    do j=4,99
      r(j)=float(xd1(i,j))-1.0
    enddo
    open(1,file=file6,status='new')
  enddo

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avsq=avsq+r(j)*r(j)
enddo
if (avsq.eq.0) goto 131
avsq=avsq/96.
do j=1,45
do k=4,100-j
   c(j)=c(j)+r(k)*r(j+k-1)/avsq
   sum(j)=sum(j)+1.0
enddo
enddo

131 enddo

open1, file=file7, status='new')
do i=1,45
   if (sum(i).eq.0.0) then
      write(1,*),i,i-1,0.0
   else
      write(1,*),i,i-1,c(i)/sum(i)
   endif
enddo
close(1)
write6,*),'counter=',counter
write6,*),'1: to do another graph, 2: to stop'
read5,*), int2
if (int2.eq.1) goto 1111
END

c implement binarization scheme
Subroutine datacrunch(data,xd1,bframe,nframe,diff)
real avd(102)
integer xd1(6000,102),data(6000,102)
integer iymin,iymax,bframe,nframe
common/datac/counter
iymin=1
iymax=102
c take first t derivative
do inl=bframe,nframe-1
do in2=iymin,iymax
   xd1(in1,in2)=data(in1,in2)-data(in1+1,in2)
enddo
dndo
c take second time der
do in1=bframe,nframe
do in2=iymin,iymax
   data(in1,in2)=0
enddo
dndo
do in1=bframe,nframe-2
    do in2=iymin,iymax
        data(in1,in2)=xd1(in1,in2)-xd1(in1+1,in2)
    enddo
endo

C take average
    do in2=iymin,iymax
        avd(in2)=0.0
        do in1=bframe,nframe-2
            avd(in2)=avd(in2)+data(in1,in2)
        enddo
        avd(in2)=avd(in2)/(nframe-bframe-1)
    enddo

c compare to arbitrary cutoff
    do in1=bframe,nframe
        do in2=iymin,iymax
            xd1(in1,in2)=0
        enddo
    enddo
    do in1=bframe,nframe-2
        do in2=iymin,iymax
            if (abs(data(in1,in2)-avd(in2)).gt.diff) then
                xd1(in1,in2)=1
            endif
        enddo
    enddo

c smooth data
    do in1=bframe,nframe
        do in2=iymin,iymax
            data(in1,in2)=0
        enddo
    enddo
    do in1=bframe+1,nframe-3
        do in2=iymin+1,iymax-1
            if (xd1(in1,in2).eq.1) then
                ii=xd1(in1-1,in2-1)+xd1(in1-1,in2)+xd1(in1-1,in2+1)
                ii=ii+xd1(in1,in2-1)+xd1(in1,in2+1)+xd1(in1+1,in2-1)
                ii=ii+xd1(in1+1,in2)+xd1(in1+1,in2+1)
                if (ii.gt.1) data(in1,in2)=1
            endif
        enddo
    enddo

c fill data
    do in1=bframe,nframe
        do in2=iymin,iymax
            data(in1,in2)=1
        enddo
\begin{verbatim}
xdl(in1,in2)=0 enddo enddo

do i=bframe+2,nframe-4
do in2=iymin+2,iymax-2
  if (data(i,in2).eq.1) then
    xdl(i,in2)=1
  else
    jj=data(i-1,in2-1)+data(i-1,in2)+data(i-1,in2+1)
    jj=jj+data(i,in2-1)+data(i,in2+1)+data(i+1,in2-1)
    jj=jj+data(i+1,in2)+data(i+1,in2+1)
    if (jj.gt.2) xdl(i,in2)=1 endif
  enddo
enddo

c smooth data
do in1=bframe,nframe
do in2=iymin,iymax
  data(in1,in2)=0 enddo enddo

do in1=bframe+3,nframe-5
do in2=iymin+3,iymax-3
  if (xdl(in1,in2).eq.1) then
    ii=xdl(in1-1,in2-1)+xdl(in1-1,in2)+xdl(in1-1,in2+1)
    ii=ii+xdl(in1,in2-1)+xdl(in1,in2+1)+xdl(in1+1,in2-1)
    ii=ii+xdl(in1+1,in2)+xdl(in1+1,in2+1)
    if (ii.gt.2) data(in1,in2)=1 endif
  enddo enddo

c fill x lines
do in1=bframe,nframe
do in2=iymin,iymax
  xdl(in1,in2)=0 enddo enddo

do in1=bframe+3,nframe-5
do in2=iymax-2,iymax+4
  data(in1,in2)=0 enddo enddo

do in1=bframe+3,nframe-5
do in2=iymin+3,iymax-3
\end{verbatim}
if (data(in1,in2).eq.1) then
  xdl(in1,in2)=1
if (data(in1,in2+5).eq.1) then
  do in=in2+1,in2+5
    xdl(in1,in)=1
  enddo
endif
endif
enddo
c fill t line
  do inl=bframe,nframe
    do in2=iymin,iymax
      data(inl,in2)=0
    enddo
  enddo
do in1=nframe-4,nframe+2
  do in2=iymin+2,iymax-2
    xdl(in1,in2)=0
  enddo
endo
do in2=iymin+3,iymax-3
  do in1=bframe+3,nframe-5
    if (xdl(in1,in2).eq.1) then
      data(in1,in2)=1
    enddo
  enddo
do in1=bframe+3,nframe-5
  do in2=iymin+2,iymax+4
    data(in1,in2)=0
  enddo
endo
c fill x line data
  do in1=bframe,nframe
    do in2=iymin,iymax
      xdl(in1,in2)=0
    enddo
  enddo
do in1=bframe+3,nframe-5
  do in2=iymin+3,iymax-3
    if (data(in1,in2).eq.1) then
      xdl(in1,in2)=1
    enddo
  enddo
endo
if (data(in1, in2+5).eq.1) then
  do in=in2+1, in2+5
    xdl(in1, in)=1
  enddo
endif
endif
enddo
enddo

count area
  do in1=bframe+3, nframe-5
    do in2=iymin+3, iymax-3
      if (xd1(in1, in2).eq.1) counter=counter+1
    enddo
  enddo
return
end
# Appendix B

Rayleigh-Bénard system

Programs used with the two-layer
```c
#include <stdlib.h>
#include "seqdlgl.h"
#include "xqwl_dlg.h"
#include "pid2_dlg.h"

int i, mResult, m, k, nReset;
int pidChan, pidDel, TopChan, NumChan;
int idComDev, nFileHandle, BotDac=0, TopDac=5;
char cTemp[20], FileName[0] = "pidxq*. dat", cTBuf[200];
int IncDel, Series=0, SeriesCount;
float TempInc, rTemp;
float Error, oError, oError2, pidRes, AveT=21.0, TDiff;
float pidout, pidold, pidSet, pidTi, pidTd, Time, TIncrement
float BotSet, TopSet, TTemp;
float Pa[], Pb[], Pc[], Pd[], PR[], Temp, z, Resist[];
unsigned ipidout;
BOOL file;
static char SampleClass[32] = "SampleClass";
OFSTRUCT ofline1;

// function prototypes
static BOOL FirstInstance( HANDLE );
static BOOL AnyInstanceC HANDLE, int, LPSTR );
LONG FAR PASCAL WindowProc(HWND, UINT, UINT, LONG);
void SizeWindow(HWND, short, short);
BOOL GetFileName(int, HWND, char *);
void keithley(HWND);

/*
 * WinMain - initialization, message loop
 */
int PASCAL WinMain( HANDLE this_inst, HANDLE prev_inst,
LPSTR cmdline, int cmdshow )
{
    MSG msg;
#endif __WIND0WS.386__
    sprintf( SampleClass,"SampleClass%d", this_inst );
    prev_inst = 0;
#endif
    if( !prev_inst ) {
        if( !FirstInstance( this_inst ) ) return( FALSE );
    }
}
if(!AnyInstance(this_inst, cmdshow, cmdline)) return( FALSE);

while( GetMessage( &msg, NULL, 0 , 0 ) ) {
    TranslateMessage( &msg );
    DispatchMessage( &msg );
}
return( msg.wParam );

} /* WinMain */

/*
 * FirstInstance - register window class for the application, 
 * and do any other application initialization
 */
static BOOL FirstInstance( HANDLE this_inst )
{
    WNDCLASS wc;
    BOOL rc;

    /*
     * set up and register window class
     */
    wc.style = CS_HREDRAW | CS_VREDRAW;
    wc.lpfnWndProc = (LPVOID) WindowProc;
    wc.cbClsExtra = 0 ;
    wc.cbWndExtra = sizeof( DWORD );
    wc.hInstance = this_inst;
    wc.hIcon = LoadIcon( this_inst, "Camlcon" );
    wc.hCursor = LoadCursor( NULL, IDC_ARROW );
    wc.hbrBackground = GetStockObject( WHITE_BRUSH );
    wc.lpszMenuName = "SampleMenu";
    wc.lpszClassName = SampleClass;
    rc = RegisterClass( &wc );
    return( rc );
} /* FirstInstance */

/*
 * AnyInstance - do work required for every instance 
 * of the application:
 * create the window, initialize data
 */
static BOOL AnyInstance(HANDLE this_inst, int cmdshow, LPSTR cmdline)
{
    HWND hwnd;

/* 
 * create main window 
 */
hwnd = CreateWindow( 
    SampleClass, // class 
    "Proportional-Integral-Differential", // caption 
    WS_OVERLAPPEDWINDOW, // style 
    0, // init. x pos 
    0, // init. y pos 
    700, // init. x size 
    80, // init. y size 
    NULL, // parent window 
    NULL, // menu handle 
    this_inst, // program handle 
    NULL // create parms 
);
if( !hwnd ) return( FALSE );

/*-------------------------------------------------------------*/
-windowProc - handle messages for the main application------------------------------------------*/
LONG FAR PASCAL WindowProc( HWND hwnd, UINT msg, 
    UINT wparam, LONG lparam )
{
HANDLE hInstance; //handle variable for dialog box
FARPROC lpfnDlgProc;
BOOL nDresult;
char cTbuf[100], inBuf[200];
int nReply;
static int nTimer=0;
int ComSiz, ComRes, err, ComErr;
DCB FAR mdcb;
COMSTAT FAR* lpStat;
switch( msg )
{
{  
case WM_COMMAND:  
    switch( LOWORD( wparam ) )  
    {  
    case IDM_SERIES:  
        if(Series==1)  
        {  
            nReply=MessageBox(hwnd,"Turn off  
            series temperature increase?",  
            "Inquiry",MB_YESNO);  
            if(nReply==IDYES)  
                Series=0;  
        }  
        else  
        {  
            hInstance=GetWindowWord(hwnd,GWW_HINSTANCE);  
            lpfnDlgProc= MakeProcInstance(DialogProc2,hInstance);  
            nDresult=DialogBoxParam(hInstance,"DIALOG_5",  
                hwnd,lpfnDlgProc,NULL);  
            FreeProcInstance(lpfnDlgProc);  
            if(nDresult==TRUE)  
            {  
                sprintf(cTbuf,"Time delay in minutes %d\n  
                Increment = %f",IncDel,TempInc);  
                nReply=MessageBox(hwnd,cTbuf,"Correct?",  
                MB_YESNO);  
                if(nReply==IDYES)  
                    Series=1;  
                    IncDel*=2;  
                    SeriesCount=0;  
            }  
        }  
        break;  
    case IDM_SETUP: //get sequence parameters  
        if(nTimer==0){  
            /*calibration data for Thermometrics thermistors 1-9 using  
            calibrated reference */  
            Pa[1]=25.25254;  
            Pc[1]=-4.26214;  
            Pd[1]=5.60894;  
            PR[1]=10240.04814;  
            Pa[2]=25.0733;  
            188
Pc[2]=-0.85727;
Pd[2]=2.03617;
PR[2]=10210.17522;
Pc[3]=1.41617;
Pd[3]=-0.0185;
PR[3]=10059.56224;
Pc[4]=1.71546;
Pd[4]=-0.25961;
PR[4]=10256.36034;
Pc[5]=1.61209;
Pd[5]=-0.17458;
PR[5]=9911.73672;
Pc[6]=0.74271;
Pd[6]=0.535;
PR[6]=10047.69187;
Pc[7]=1.57738;
Pd[7]=-0.17158;
PR[7]=10022.18841;
Pa[8]=25.0.16688;
Pb[8]=-22.98961;
Pc[8]=1.60257;
Pd[8]=-0.16258;
PR[8]=10278.74796;
Pa[9]=24.73839;
Pc[9]=1.51027;
Pd[9]=-0.08448;
PR[9]=9916.20792;
Pa[10]=25.;
Pb[10]=-20.;
Pc[10]=2.;
Pd[10]=1.;
PR[10]=10000.;
// RS-232 communications
idComDev=OpenComm("COM2",1024,128);
if (idComDev <0) {
    MessageBox(hwnd,"error in OpenComm", "Info",MB_OK);
    ComErr=GetCommError(idComDev,lpStat);
    wsprintf(cTbuf,"GetComError num %d",ComErr);
    MessageBox(hwnd,cTbuf,"Info",MB_OK);
    return 0;
}
err=GetCommState(idComDev,&mdcb);
if (err<0){
    wsprintf(cTbuf,"getcommstate = %d",err);
    MessageBox(hwnd,cTbuf,"Info",MB_OK);
}
err=BuildCommDCB("COM2:9600,n,8,1",&mdcb);
if(err<0){
    MessageBox(hwnd,"error in BuildComm", "Info",MB_OK);
    return 0;
}
err=SetCommState(&mdcb);
if(err<0){
    MessageBox(hwnd,"error in SetCommState", "Info",MB_OK);
    return 0;
}
// flush transmission buffer
ComRes=FlushComm(idComDev,0);
if(ComRes<0){
    MessageBox(hwnd,"Problem flushing trans buffer", "Info",MB_OK);
}
hInstance=GetWindowWord(hwnd,GWW_HINSTANCE);
lpfnDlgProc= MakeProcInstance(DialogProc, hInstance);
nDresult=DialogBoxParam(hInstance,"DIALOG_1", hwnd,lpfnDlgProc,NULL);
FreeProcInstance(lpfnDlgProc);
if(nDresult==TRUE)
{
    NumChan=9;
    keithley(hwnd);
}
nReset=0;
TDiff=pidSet;
BotSet=AveT+0.5*TDiff;
TopSet=BotSet-TDiff;
sprintf(cTbuf,"Top T %8.5f, Bot T %8.5f",
TopSet,BotSet);
MessageBox(hwnd,cTbuf,"Set Temperatures",MB_OK);

/* subtract 0.35 from top set point to correct for difference between
panel read out and actual temperature */

//calculate the set point for the DAC to control the RM6
TopSet=TopSet-0.35;
 pidout=2463.54+71.657*TopSet;
 ipidout=pidout;
 mResult=cbAOut(0,TopDac,0,ipidout);
 if(mResult!=0){
   wsprintf(cTbuf,"Error %d",mResult);
   MessageBox(hwnd,cTbuf,"ERROR",MB_OK);
 }
 pidold=0;
 pidTi=pidDel/pidTi;
 pidTd=pidTd/pidDel;
 oError=0;
 oError2=0;
 Time=0.0;
 TIncrement=pidDel;
 TIncrement=TIncrement/60.;
}
} //nDresult
} //nTimer
break;
case IDM_STATUS:
 sprintf(cTbuf,"Top T %8.5f, Bot T %8.5f, file is %d",
 ,TTemp,Temp,file);
 MessageBox(hwnd,cTbuf,"Temperatures",MB_OK);
break;
case IDM_FILE:
 if(file==TRUE)
{
   wsprintf(cTbuf,"Stop recording data and close
 file?");
   nDresult=MessageBox(hwnd,cTbuf,"Query",
   ,MB_OKCANCEL);
   if(nDresult==TRUE){
_lclose(nFileHandle);

file=FALSE;

}
}
else
{
  wsprintf(cTbuf,"Start recording data?");
  nDresult=MessageBox(hwnd,cTbuf,"Query",
                    MB_OKCANCEL);
  if(nDresult==TRUE)
  {
    nFileHandle=OpenFile(Filename,&ofline1,
                         OF_CREATE);
    if (nFileHandle==0)
      MessageBox(hwnd,"File Error!",
                 "ERROR",MB_OK);
    file=TRUE;
  }
}
break;
case IDM_GO:
  if(nTimer==0){
    nTimer = SetTimer(hwnd,1,pidDel*1000,NULL);
    if(nTimer==0)
      MessageBox(hwnd,"No timers left", "Message", MB_OK);
  }
break;
case IDM_CHANGE:
  hInstance=GetWindowWord(hwnd,GWW_HINSTANCE);
  lpfnDlgProc= MakeProcInstance(DialogProc2,hInstance);
  nDresult=DialogBoxParam(hInstance,"DIAL0G_2", hwnd, 
                           lpfnDlgProc,NULL);
  FreeProcInstance(lpfnDlgProc);
  if(nDresult==TRUE)
  {
    TDiff=pidRes;
    BotSet=AveT+0.5*TDiff;
    TopSet=BotSet-TDiff;
    sprintf(cTbuf,"Top T %8.5f, Bot T %8.5f",TopSet,BotSet);
    MessageBox(hwnd,cTbuf,"Set Temperatures",MB_OK);
    /* subtract 0.35 from top set point to correct for difference between
panel read out and actual temperature */

    TopSet=TopSet-0.35;
    pidout=2463.54+71.657*TopSet;
    ipidout=pidout;
    mResult=cbAQut(0,TopDac,0,ipidout);
    if(mResult!=0){
        wsprintf(cTbuf,"Error %d",mResult);
        MessageBox(hwnd,cTbuf,"ERROR",MB_OK);
    }
} break;
case IDM_STOP:
    KillTimer(hwnd,1);
    nTimer=0;
    break;
case IDM_EXIT:
    if(!nTimer){
        if(file==TRUE)
            _lclose(nFileHandle);
        ComRes=CloseComm(idComDev);
        if(ComRes<0){
            MessageBox(hwnd,"error closing com port","Info",MB_OK);
        }
        nDresult=MessageBox(hwnd,"Zero Bot DAC?\n Top to 3950","Query",MB_OKCANCEL);
        if(nDresult==IDOK){
            mResult=cbAQut(0,BotDac,0,0);
            if(mResult!=0){
                wsprintf(cTbuf,"Error %d",mResult);
                MessageBox(hwnd,cTbuf,"ERROR",MB_OK);
            }
            mResult=cbAQut(0,TopDac,0,3950);
            if(mResult!=0){
                wsprintf(cTbuf,"Error %d",mResult);
                MessageBox(hwnd,cTbuf,"ERROR",MB_OK);
            }
            PostQuitMessage( 0 );
        }
    } break;
} break;

193
case WM_TIMER:
    SeriesCount++;
    if(nReset!=0)
    {
        wsprintf(cTBuf,":TRAC:FEED:CONT NEXT\r") ;
        ComSiz=1strlen(cTBuf);
        ComRes=WriteComm(idComDev,cTBuf,ComSiz);
        if(ComRes!=ComSiz){
            MessageBox(hwnd,"error in FEED","Info",MB_OK);
        }
    }
    wsprintf(cTBuf,":INIT\r") ;
    ComSiz=1strlen(cTBuf);
    ComRes=WriteComm(idComDev,cTBuf,ComSiz);
    if(ComRes!=ComSiz){
        MessageBox(hwnd,"error in INIT","Info",MB_OK);
    }
    nReset=1;
pwait(NumChan);
    wsprintf(cTBuf,":TRAC:DATA?\r") ;
    ComSiz=1strlen(cTBuf);
    ComRes=WriteComm(idComDev,cTBuf,ComSiz);
    if(ComRes!=ComSiz){
        MessageBox(hwnd,"error in TRAC:DATA","Info",MB_OK);
    }
pwait(2);
    wsprintf(inBuf,"\r") ;
    ComRes=ReadComm(idComDev,inBuf,NumChan*16);
    for(k=1;k<=9;k++)
    {
        wsprintf(cTemp,"\r") ;
        for(m=0;m<=14;m++)
            cTemp[m]=inBuf[m+(k-1)*16];
        Resist[k]=atof(cTemp);
    }
    // start PID calculations as found in X.Q.Wang's thesis pg 44
    // pidTi=pidDel/pidTi, pidTd=pidTd/pidDel
    if((Resist[pidChan]>5000)&&(Resist[pidChan]<15000))
    {
        z=log(Resist[pidChan]/PR[pidChan]);
        Temp=Pa[pidChan]+Pb[pidChan]*z+Pc[pidChan]*z*z
            +Pd[pidChan]*z*z*z;
        z=log(Resist[TopChan]/PR[TopChan]);
Temp = Pa[TopChan]+Pb[TopChan]*z+Pc[TopChan]*z*z
   +Pd[TopChan]*z*z*z;
Error = BotSet - Temp;
// check if Temp is out of bounds
if((Temp>40.0) || (Temp<10.0))
   Error = 0.0;
pidout = pidK*((1+pidTi)*Error - oError
   +pidTd*(Error - 2*oError + oError2)) + pidold;
oError2 = oError2;
oError = Error;
if(pidout > 3050.)
   pidout = 3050.;
if(pidout < 0.0)
   pidout = 0.;
/* set pidold = pidout after limit check to avoid spurious changes
   to pid system when it thinks it is driving the D/A beyond limits*/
   pidold = pidout;
   ipidout = pidout;
   mResult = cbAOut(0, BotDac, 0, ipidout);
   if (mResult != 0){
      wsprintf(cTbuf, "Error %d", mResult);
      MessageBox(hwnd, cTbuf, "ERROR", MB_OK);
   }
else
{
   // reset RS-232 communications
   MessageBeep(0);
   // close port
   ComRes = CloseComm(idComDev);
   if(ComRes < 0){
      MessageBox(hwnd, "error closing com port", "Info", MB_OK);
   }
pwait(4);
   // open port
   idComDev = OpenComm("COM2", 1024, 128);
   if (idComDev < 0) {
      MessageBox(hwnd, "error in OpenComm", "Info", MB_OK);
      ComErr = GetCommError(idComDev, lpStat);
      wsprintf(cTbuf, "GetCommError num %d", ComErr);
      MessageBox(hwnd, cTbuf, "Info", MB_OK);
      return 0;
err=GetCommState(idComDev,&mdcb);
if (err<0){
    wsprintf(cTbuf,"getcommstate = %d",err);
    MessageBox(hwnd,cTbuf,"Info",MB_OK);
}

err=BuildCommDCB("COM2:9600,n,8,1",&mdcb);
if(err<0){
    MessageBox(hwnd,"error in BuildComm", "Info",MB_OK);
    return 0;
}

err=SetCommState(&mdcb);
if(err<0){
    MessageBox(hwnd,"error in SetCommState", "Info",MB_OK);
    return 0;
}

// flush transmission buffer
ComRes=FlushComm(idComDev,0);
if(ComRes<0){
    MessageBox(hwnd,"Problem flushing trans buffer", "Info",MB_OK);
}

// reset keithley
keithley(hwnd);
nReset=0;

Time=Time+TIncrement;
if(file==TRUE){
    sprintf(cTbuf, " %f",Time);
    _write(nFileHandle,cTbuf,lstrlen(cTbuf));
    for(k=1;k<=9;k++)
    {
        z=log(Resist[k]/PR[k]);
        rTemp=Pa[k]+Pb[k]*z+Pc[k]*z*z+Pd[k]*z*z*z;
        sprintf(cTbuf," %8.5f",rTemp);
        _write(nFileHandle,cTbuf,lstrlen(cTbuf));
    }
    sprintf(cTbuf, " %d\r\n",ipidout);
    _write(nFileHandle,cTbuf,lstrlen(cTbuf));
}

/* Do stuff necessary for series control*/
if(Series==1)
    if(SeriesCount==IncDel)
    {
        SeriesCount=0;
        TDiff+=TempInc;
        BotSet=AveT+0.5*TDiff;
        TopSet=BotSet-TDiff;
        TopSet=TopSet-0.35;
        pidout=2463.54+71.657*TopSet;
        ipidout=pidout;
        mResult=cbAQut(0,TopDac,0,ipidout);
        if(mResult!=0)
        {
            wsprintf(cTbuf,"Error %d",mResult);
            MessageBox(hwnd,cTbuf,"ERROR", MB_OK);
        }
    }
break;
    case WM_CLOSE:
        PostMessage(hwnd,WM_COMMAND,IDM_EXIT,0);
        break;
    default:
        return(  DefWindowProc(  hwnd, msg, wparam, lparam));
    }
return(  0L );
} /* WindowProc */

/******************************************************/
/* Function: SizeWindow */
/* Purpose: Size application window to fit */
/* with the loaded image size. */
/* Parameter: hwnd -- window handle to be set */
/* width -- window width (client area) */
/* height -- window height (client area) */
/* Returns: NONE */
/******************************************************/

void SizeWindow(HWND hwnd, short width, short height)
{
    RECT wRect;
    short xf, yf, yc, ym;
    xf = GetSystemMetrics(SM_CXFRAME);
    yf = GetSystemMetrics(SM_CYFRAME);
yc = GetSystemMetrics(SM_CYCAPTION);
ym = GetSystemMetrics(SM_CYMENUS); GetWindowRect(hwnd, &wRect);
SetWindowPos(hwnd, 0,
    wRect.left,
    wRect.top,
    width + 2*xf,
    height + 2*yf + ym + yc,
    SWP_NOZORDER);

} /* end of SizeWindow */

/* process dialog boxes it input parameters */

BOOL FAR PASCAL DialogProc(HWND hDlg, WORD wMessage,
    WORD wParam,LONG lParam)
{
    HWND hwndMyEdit;
    BOOL FAR* lpfFrameErr;
    BOOL FAR* lpfWaitErr;
    switch(wMessage)
    {
        case WM_INI T DIALOG:
            return(TRUE);
        case WM_COMMAND:
            switch (wParam)
            {
            case PID_OK:
                EndDialog(hDlg,TRUE);
                return(TRUE);
            case PID_CANCEL:
                EndDialog(hDlg,FALSE);
                return (TRUE);
            case PID_BOT:
                hwndMyEdit = GetDlgItem(hDlg, PID_BOT);
                SendMessage(hwndMyEdit, WM_GETTEXT,
                    sizeof(cTemp), (LPARAM) ((LPSTR) cTemp));
                pidChan=atoi(cTemp);
                return (TRUE);
            case PID_TOP:
                hwndMyEdit = GetDlgItem(hDlg, PID_TOP);
                SendMessage(hwndMyEdit, WM_GETTEXT,
                    sizeof(cTemp), (LPARAM) ((LPSTR) cTemp));
                }
TopChan=atoi(cTemp);
return (TRUE);
case PID_SAMPRT:
    hwndMyEdit = GetDlgItem(hDlg, PID_SAMPRT);
    SendMessage(hwndMyEdit, WM_GETTEXT,
        sizeof(cTemp),(LPARAM) ((LPSTR) cTemp));
    pidDel=atoi(cTemp);
    return (TRUE);
case PID_SETPT:
    hwndMyEdit = GetDlgItem(hDlg, PID_SETPT);
    SendMessage(hwndMyEdit, WM_GETTEXT,
        sizeof(cTemp),(LPARAM) ((LPSTR) cTemp));
    pidSet=atof(cTemp);
    return (TRUE);
case PID_DERTIM:
    hwndMyEdit = GetDlgItem(hDlg, PID_DERTIM);
    SendMessage(hwndMyEdit, WM_GETTEXT,
        sizeof(cTemp),(LPARAM) ((LPSTR) cTemp));
    pidTd=atof(cTemp);
    return (TRUE);
case PID_INTTIM:
    hwndMyEdit = GetDlgItem(hDlg, PID_INTTIM);
    SendMessage(hwndMyEdit, WM_GETTEXT,
        sizeof(cTemp),(LPARAM) ((LPSTR) cTemp));
    pidTi=atof(cTemp);
    return (TRUE);
case PID_K:
    hwndMyEdit = GetDlgItem(hDlg, PID_K);
    SendMessage(hwndMyEdit, WM_GETTEXT,
        sizeof(cTemp),(LPARAM) ((LPSTR) cTemp));
    pidK=atof(cTemp);
    return (TRUE);
    }
break;
return(FALSE);
*/

BOOL FAR PASCAL DialogProc2(HWND hDlg, WORD wMessage,
    WORD wParam,LONG lParam)
{
    HWND hwndMyEdit;
BOOL FAR* lpfFrameErr;
BOOL FAR* lpfWaitErr;
switch(wMessage)
{
    case WM_INITDIALOG:
        return(TRUE);
    case WM_COMMAND:
        switch (wParam)
        {
            case PID_OK2:
                EndDialog(hDlg,TRUE);
                return(TRUE);
            case PID_CANCEL2:
                EndDialog(hDlg,FALSE);
                return (TRUE);
            case PID_RESET:
                hwndMyEdit = GetDlgItem(hDlg, PID_RESET);
                SendMessage(hwndMyEdit, WM_GETTEXT,
                strlen(cTemp),(LPARAM) ((LPSTR) cTemp));
                pidRes=atof(cTemp);
                return (TRUE);
            case PID_INCDEL:
                hwndMyEdit=GetDlgItem(hDlg,PID_INCDEL);
                SendMessage(hwndMyEdit, WM_GETTEXT,
                strlen(cTemp),(LPARAM) ((LPSTR) cTemp));
                IncDel=atoi(cTemp);
                return(TRUE);
            case PID_TEMPINC:
                hwndMyEdit = GetDlgItem(hDlg, PID_TEMPINC);
                SendMessage(hwndMyEdit, WM_GETTEXT,
                strlen(cTemp),(LPARAM) ((LPSTR) cTemp));
                TempInc=atof(cTemp);
                return(TRUE);
        }
        break;
}
return(FALSE);

/* send commands to the Keithley 2000/2000SCAN to initiate */
/* resistance measurements in 'shot' mode */

void keithley(HWND hwnd)
{

}
int ComSiz, ComRes;
char cTBuf[100];

wsprintf(cTBuf, "\r*RST\r");
ComSiz = lstrlen(cTBuf);
ComRes = WriteComm(idComDev, &cTBuf, ComSiz);
if (ComRes != ComSiz) {
    wsprintf(cTBuf, "Error in *RST num %d", ComRes);
    MessageBox(hwnd, cTBuf, "Info", MB_OK);
}

wsprintf(cTBuf, ":STAT: PRES:*CLS\r");
ComSiz = lstrlen(cTBuf);
ComRes = WriteComm(idComDev, &cTBuf, ComSiz);
if (ComRes != ComSiz) {
    wsprintf(cTBuf, "Error in stat:presnum %d", ComRes);
    MessageBox(hwnd, cTBuf, "Info", MB_OK);
}

wsprintf(cTBuf, ":SENS:FUNC 'RES'\r");
ComSiz = lstrlen(cTBuf);
ComRes = WriteComm(idComDev, &cTBuf, ComSiz);
if (ComRes != ComSiz) {
    wsprintf(cTBuf, "Error in :conf:res num %d", ComRes);
    MessageBox(hwnd, cTBuf, "Info", MB_OK);
}

wsprintf(cTBuf, ":STAT: MEAS: ENAB 512\r");
ComSiz = lstrlen(cTBuf);
ComRes = WriteComm(idComDev, &cTBuf, ComSiz);
if (ComRes != ComSiz) {
    wsprintf(cTBuf, "Error in STAT:MEAS num %d", ComRes);
    MessageBox(hwnd, cTBuf, "Info", MB_OK);
}

// wait 4 seconds for DMM to process commands
pwait(4);
if (NumChan == 10)
    wsprintf(cTBuf, ":SAMP:COUN 10\r");
else
    wsprintf(cTBuf, ":SAMP:COUN %ld\r", NumChan);
ComSiz = lstrlen(cTBuf);
ComRes = WriteComm(idComDev, &cTBuf, ComSiz);
if (ComRes != ComSiz) {
    wsprintf(cTBuf, "Error in :SAMP:COUN num %d", ComRes);
MessageBox(hwnd,cTbuf,"Info",MB_OK);
}
wsprintf(cTBuf,":\TRIG:COUN 1\r")
ComSiz=1strlen(cTBuf);
ComRes=WriteComm(idComDev,&cTBuf,ComSiz);
if(ComRes!=ComSiz){
wsprintf(cTbuf,"Error in :TRIG:COUN num %d",
ComRes);
MessageBox(hwnd,cTbuf,"Info",MB_OK);
}
if(NumChan==10)
wsprintf(cTBuf,":\TRAC:POIN 10\r")
else
wsprintf(cTBuf,":\TRAC:POIN %ld\r",NumChan);
ComSiz=1strlen(cTBuf);
ComRes=WriteComm(idComDev,&cTBuf,ComSiz);
if(ComRes!=ComSiz){
wsprintf(cTbuf,"Error in :TRAC:POIN num %d",
ComRes);
MessageBox(hwnd,cTbuf,"Info",MB_OK);
}
wsprintf(cTBuf,":\TRAC:FEED:SENS1\r")
ComSiz=1strlen(cTBuf);
ComRes=WriteComm(idComDev,&cTBuf,ComSiz);
if(ComRes!=ComSiz){
wsprintf(cTbuf,"Error TRAC:FEED SENS %d",
ComRes);
MessageBox(hwnd,cTbuf,"Info",MB_OK);
}
wsprintf(cTBuf,":\TRAC:FEED:CONT NEXT\r")
ComSiz=1strlen(cTBuf);
ComRes=WriteComm(idComDev,&cTBuf,ComSiz);
if(ComRes!=ComSiz){
wsprintf(cTbuf,"Error TRAC:FEED:CONT %d",
ComRes);
MessageBox(hwnd,cTbuf,"Info",MB_OK);
}
if(TopChan==10)
wsprintf(cTBuf,":\ROUT:SCAN (@%ld:10)\r",
pidChan);
else
wsprintf(cTBuf,":\ROUT:SCAN (@1:9)\r")
ComSiz=1strlen(cTBuf);
ComRes=WriteComm(idComDev,&cTBuf,ComSiz);
202
if (ComRes != ComSiz) {
    MessageBox(hWnd, "error in roust: sca", "Info", MB_OK);
}

wsprintf(cTBuf, "ROUT:SCAN:LSEL INT\r\n");
ComSiz = lstrlen(cTBuf);
ComRes = WriteComm(idComDev, &cTBuf, ComSiz);
if (ComRes != ComSiz) {
    MessageBox(hWnd, "error in LSEL INT", "Info", MB_OK);
}

/*------------ end of pidxqw.c ---------------*/
Appendix C

Programs used with the study of the Weissenberg effect

The first program is used to find the edge of the climb. It is written in C under the VMS system. The second calculates the Poincaré sections using Fortran under the VMS system. The plotting routine is DI-3000. Programs implementing Numerical Recipes[49] FFT routines are also used. Commercial programs used include Spyglass™ and MicroCal's Origin™.

/* edge finding routine */
/* program edge2.c found on the Physics Dept. */
/* Alpha cluster

#include <stdio.h>
#include <stdlib.h>

FILE *infile,*outfile;
char inpfile[50],outpfile[50];
unsigned char outnum;
int cutoff,endframe,numPix,pix[3200],edge[16410];
int i,j,ave,k,pixval;

int main (void)
{
    printf("input file:\n");
    gets(inpfile);
    printf("output file:\n");
    gets(outpfile);
    printf("enter number of lines\n");
    scanf("%d",&endframe);
    printf("enter number of pixels\n");
scanf("%d",&numPix);
printf("enter cutoff value\n");
scanf("%d",&cutoff);
if ((infile=fopen(inpfile,"r")) == NULL)
{
    printf("Can’t open input file\n");
    exit(1);
}
if ((outfile=fopen(outpfile,"w")) == NULL)
{
    printf("Can’t open output file\n");
    exit(1);
}

/* find the edge of the blob by adding the intensity of a pixel at four sequential times, then computing the average of three neighbors in space and finally comparing with an arbitrary cutoff */
cutoff*=3;
for(j=0;j<endframe;j+=4)
{
    for(i=0;i<numPix;i++)
    {
        outnum=getc(infile);
        pix[i]=outnum;
    }
    outnum=getc(infile);
    for(i=0;i<numPix;i++)
    {
        outnum=getc(infile);
        pixval=outnum;
        pix[i]+=pixval;
    }
    outnum=getc(infile);
    for(i=0;i<numPix;i++)
    {
        outnum=getc(infile);
        pixval=outnum;
        pix[i]+=pixval;
    }
    outnum=getc(infile);
    for(i=0;i<numPix;i++)
    {
        outnum=getc(infile);
        pixval=outnum;
        pix[i]+=pixval;
    }
    outnum=getc(infile);
    for(i=0;i<numPix;i++)
    {
outnum=getc(infile);
pixval=outnum;
pix[i]+=pixval;
pix[i]/=4;
}
outnum=getc(infile);
for(i=numPix-8;i>=0;i-=1)
{
    ave=pix[i];
    for(k=1;k<=2;k++)
        ave+=pix[i+k];
    if(ave>cutoff)
    {
        edge[j]=i;
        i=-i;
    }
}
fclose(infile);
for(j=0; j<endframe; j+=4)
    fprintf(outfile, "%d %d
", j+1, edge[j]);
}

Program poin.for to find Poincare sections.
found on the Physics Dept. Alpha cluster

PROGRAM poincare
real z,xa(4),ya(4),point1(3000),point2(3000)
real x(8200), ave, plane
INTEGER i,j,delay,end,dumm
CHARACTER*40 infle,outfle
REAL XV(4), YV(4), ZV(4)
character*30 xlab,ylab,title
INTEGER bframe,in1,in2,name
COMMON/XY1bl/xmin,ymin,xmax,ymax,iyax
vx1=-0.80
vx2=0.80
vy1=-0.60
vy2=0.60
ndvc=3
ifont = 7

c DI-3000 initiation commands
CALL JBEGIN
CALL JDINIT (NDVC)
CALL JDEVON (NDVC)
CALL JVSPAC (-1.,1.,-0.75,0.75)
CALL JWINDO (-100.,100.,-75.,75.)
CALL JDFONT (ifont)
CALL JDPINT (0)

write(ylab,*)'y'
write(xlab,*)'x'

100 write(6,*) 'File to read:'
read(5, '(A)') infle
write(6,*) 'What is the delay?'
read(5,*) delay
write(title,1001)delay
1001 format('poin.for ',i4)
OPEN(unit=l,file=infle,status='old')
end=8000
DO i=1,end
   read(i,*) t,dumm
   x(i)=float(dumm)
enddo
close(1)
c subtract average from data
ave=x(1)
do i=2,end
   ave=ave+x(i)
endo
d=ave/float(end)
ave=int(d)
do i=1,end
   x(i)=x(i)-ave
endo
c plane set exactly to 0 gives fitting routine problems
plane=0.00001
k=0
DO 20 i=4+delay+delay,end
   IF(x(i).le.plane .and. x(i+1).gt.plane) THEN
   c if this is true, then the plane is pierced
   c use the NumRec polint routine to interpolate, do
   c x and y separately. fitting a 4th order poly to 5 points.
   k=k+1
20
do m=1,2
    ya(m)=x(i+m-2-d delaying)
    ya(m+2)=x(i+m-d delaying)
    xa(m)=x(i+m-2)
    xa(m+2)=x(i+m)
endo
call polint(xa,ya,4,plane,y,dy)
c check to see if polint returns a valid point
if(y.eq.1000) then
    k=k-1
    goto 20
else
    point1(k)=y
endif
do m=1,2
    ya(m)=x(i+m-2-d delaying)
    ya(m+2)=x(i+m-d delaying)
    xa(m)=x(i+m-2)
    xa(m+2)=x(i+m)
endo
call polint(xa,ya,4,plane,y,dy)
if(y.eq.1000) then
    k=k-1
    goto 20
else
    point2(k)=y
endif
endif
20 CONTINUE
c find min and max of data for plots
xmin=-150.
xmax=150.
ymin=xmin
ymax=xmax
name=name+1
call jframe
CALL JVPORT (vx1,vx2,vy1,vy2)
CALL JWINDO (0.,1.,0.,1.)
call jwclip(.false.)
CALL JROOPEN (name)
CALL JIWIND (XV,YV,ZV)
CALL JPOLGN (XV,YV,4)
CALL JSIZE(0.015,0.03)
CALL JJJUST(1,2)
call jbase(0.,1.,0.)
call jmove(-0.1,0.4)
call jhstrg(ylab)
call jbase(1.,0.,0.)
call jmove(0.4,-0.09)
call jhstrg(xlab)
call jsize(0.022,0.045)
CALL JMOVE(0.05,1.1)
call jhstrg(infile)
call jmove(0.45,1.1)
CALL JHSTRG(title)
call jsize(0.015,0.03)
call xlabel
   CALL JRCLOS (name)
CALL JWINDO (xmin,xmax,ymin,ymax)
CALL JWCLIP (.TRUE.)
CALL JOPEN
CALL J1styl(0)
call jcmark(2)

DO i=1,k
    call jmark(point1(i),point2(i))
enddo

CALL JCLOSE
call jvisbl(name,0)
call jpurge(name)
write(6,*) '1: to do another graph, 2: to stop'
read(5,*) int2
if (int2.eq.1) goto 100
CALL JDEVOF (NDVC)
CALL JDEND (NDVC)
CALL JEND
END
BIBLIOGRAPHY


[56] H. Benard, Revue Générale des Sciences Pures et Appliquées 11, 1261 (1900).
[57] L. Rayleigh, Phil. Mag. 32, 528 (1916).


