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A Multiple Reflection Cell
Photonic True-Time Delay Device
for Phased Array Antennas

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

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* * * * *

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This paper describes an optical unit designed to provide true-time delays (TTDs) for phased array antennas. The TTD device is a multiple reflection optical mirror configuration with a spatial light modulator for switching. This device can provide: a) ultra-wide bandwidth, b) compact size (folded paths with repetitive refocusing to effectively remove beam spread), c) parallel processing of hundreds of independent array element time delays for transmit and receive modes, d) design flexibility for time delays of picoseconds to microseconds, and e) radar system configuration flexibility. Proof-of-concept hardware demonstrations were built to verify time delays and measure system performance factors. In addition, theoretical analysis of the device is presented along with implications for real-world implementation and integration into a multi-element array in transmit and receive mode.
This is dedicated to all the magical people that have made my life worth living.
ACKNOWLEDGMENTS

I would like to thank my two advisers, Dr. Stuart A. Collins, Jr. and Dr. Betty Lise Anderson. Without their patience, guidance, and support, this work would not have been possible. I would also like to thank my husband who helped me survive graduate school. Finally, I would like to thank my graduate school friends. They helped keep my soul alive through this arduous process.
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CHAPTER 1

INTRODUCTION

The design and laboratory demonstration of an optical true-time delay (TTD) device for phased array microwave antennas are presented in this dissertation. The individual elements of a microwave phased array antenna each need signals with individual time delays. The methods used presently include lengths of coax cable, which are bulky and expensive, or computer controlled phase shifts which limit bandwidth. This work describes a multiple reflection optical mirror configuration with a spatial light modulator for switching. This design uses a compact optical arrangement and produces TTD with ultra-wide bandwidth for a wide range of simultaneous radar frequency components.

Phased array antennas have the potential for a wide variety of applications from surveillance, tracking, astronomy, and geodesy to wireless and satellite communication. The beams emitted from phased array antennas, that consist of a number of independent antenna elements, can be electronically scanned in direction by reprogramming the phase with which the signals of the individual elements are combined.

The performance of phased arrays so far, however, has been limited because phase-shifting electronics are intrinsically narrow-band. By replacing phase shifts with true-time delays, signals from different elements can be correlated independently of
frequency. Electronically implementing the true-time delay is generally impractical because of the many long lengths of stripline, waveguides, or coaxial cable needed, which are expensive, bulky, and temperature sensitive. Because long paths are comparatively easy to obtain optically, photonic systems promise a means of obtaining the beam agility of array systems combined with wide bandwidth [1] to [5].
1.1 Description of Phased Arrays

A phased array antenna consists of an array of individual antennas (called elements) emitting or receiving coherently. The radar beams emitted or received from each element add together in amplitude and phase to form one composite signal, Figure 1.1. In transmitting, for example, the composite beamfront direction is controlled not by mechanically rotating a single element (e.g. dish) antenna, but rather by changing the emission timing for each element relative to the others in the array. For example, if all the elements start to emit at the same time, the beamfront is parallel to the array plane (propagating in the array broadside direction), as in Figure 1.1a. If a true-time delay is applied linearly across the array so that each element emits at $m\Delta t$, where $m$ is the element index, and $\Delta t$ is the incremental time delay, the beamfront is formed at some angle, $\theta$, the steering angle from broadside. By geometry, the angle formed is $\theta = \arcsin\left(\frac{L/c}{d/c}\right) = \arcsin\left(\frac{\Delta t}{c}\right)$, where $\Delta t$ is the incremental time delay, and from Figure 1.1, $d$ is inter-element spacing, $L$ is the wave propagation distance between adjacent elements to obtain the desired steering angle, $\theta$, and $c$ is the speed of light. This dissertation is concerned with a simple compact broadband photonic scheme for providing the requisite time delays.

1.2 A Novel Photonic True-Time Delay Device

1.2.1 Single Cell Time Delay

We describe here a true-time delay device that falls into the free-space category, but uses a multiple-pass optical cell [6],[7] with refocusing mirrors that has the advantage of avoiding beam spreading problems. The optical true-time delay approach presented here has its roots in a three-mirror optical cell that was originally designed
by John White for spectroscopic measurements requiring very long optical paths through a gaseous sample [8]. The original White cell has been in common use in the spectroscopic community since its inception. White later made modifications for special applications [9]. The original White cell has also been used for time delay, either as a tapped delay line [10] or for pulse train generation [11]. Other uses indirectly related to time delay include acoustic pulsing [12], and cineholography [13]. The White cell has even been adapted for optical computing [14].

The true-time delay beam steering photonic device proposed uses two cells to provide multiple reflections in a mirror cavity with continuous refocusing, while a spatial light modulator selects between different optical paths on each reflection [15]. The basic framework of the design studied here is the White cell, a confocal arrangement of three spherical mirrors (A, B and C), all of the same radius of curvature, R, as shown in Figure 1.2. The basis of the time delay is the multiple bounces back and forth through the cell that create a long optical path; we have developed a way to make the path lengths controllable.

The three cell mirrors are arranged with one mirror (A) opposite the other two (B and C) and separated by a distance equal to their common radius of curvature. As a consequence of paraxial ray optics and the fact that the radius of curvature is twice the focal length of a spherical mirror, Mirror A is conjugate to itself through either Mirrors B or C. Thus an object on the surface of Mirror A will be imaged back onto Mirror A, by either Mirror B or C. Also, Mirrors B and C are image conjugates of each other through Mirror A.

This confocal arrangement allows an optical beam input near the edge of Mirror A (focused as an image onto an input turning mirror) to travel continuously back
A, B, and C: spherical mirrors
CC(B) and CC(C): center of curvature locations for B and C

Figure 1.2: The basic White cell configuration, adapted from [1].
and forth (B-A-C-A-B, etc.) and be re-imaged on Mirror A after every round trip through the cell (see Figure 1.2). The beam will be refocused each trip through the cell to a diffraction limited spot, thus effectively eliminating propagation beam spread and allowing for this compact system design.

After N reflections, the beam exits the cell by reflecting off an output turning mirror beyond the edge of Mirror A. The number of reflections is determined by the input beam location distance from the physical center of A, the mirror diameters, and the separation of the centers of curvature of B and C, CC(B) and CC(C) (see Figure 1.2). Once the cell is designed, the total number of reflections, N, can easily be changed by changing the separations of the centers of curvature, CC(B) and CC(C).

The key to the design presented here is the introduction of controllable switching by replacing the spherical mirror A with an optical switching device. One such optical switch might be a nematic liquid crystal spatial light modulator (SLM) and polarizing beam splitter (PBS) (see Figure 1.3). A lens must be combined with the SLM (which only has a flat internal mirror) to replace the focusing properties of the spherical mirror A. The focal length of this lens can be adjusted to allow for inclusion of a PBS. Controlling the SLM state at a point can then cause deflection of the beam by switching the polarization by 90° (at any reflection) whereupon the polarizing beam splitter reflects the beam out of the cell.

The simplest way to obtain variable time delay is to use one switching cell and switch the beam out of the cell after a certain number of reflections. Controlling the number of trips through the cell allows for changing the optical path length and therefore time delay (see Figure 1.3). The incremental time delay is the travel time associated with one round trip. The number of reflections before switching equals
Figure 1.3: The basic single cell configuration, adapted from [1].
the time delay value, thus we call this type of delay "additive". The total number of reflections on the SLM before the beam walks off the edge, $N$, would determine the total number of different time delays possible. The maximum delay possible would be the time for the maximum number of trips through the cell, or approximately $N \times \frac{2R}{c}$ (neglecting the beamsplitter refractive index).

1.2.2 Compound Cell Time Delay

Building on the simple single cell concept, a more advanced compound cell system has also been devised with the advantages of simpler output optics and a greater number of time delays (see Figure 1.4). Here two White cells of different optical lengths are constructed perpendicularly, sharing a common polarizing beam splitter (PBS) and spatial light modulator (SLM). The different path lengths are introduced by changing positions (and also therefore the focal lengths) of the spherical mirrors in
the cells, or, by changing the refractive index of the material in one of the cells. The PBS would be used to direct polarization-switched reflections from the SLM between the two reflection cells. The beam would make all \( N \) (\( N \) is now fixed) reflections on the SLM and then exit the system. The path delay would be determined by how many times the beam was switched to each of the different path lengths available between all the different spherical mirrors. For this compound cell, the time delay is then proportional to the *difference* between available path lengths, thus being called "differential" delay. For the compound cell, as in the single cell, one set of \( N \) image reflections (small spots) on the SLM corresponds to delay control for one antenna element.

Next, we introduce the concept of adding parallel columns of reflected spots, allowing full use of the spatial light modulator's two-dimensional pixel array for parallel element control (see figure 1.5). We will show that we can input multiple beams by inputting each beam at different distances to the left and right of the optic plane (see Figure 1.5). Thus, we use off-axis reflections in the cell to make highly parallel processing possible.

The final parallel processing system concept therefore includes parallel light beams focused to a row of spots at the input turning mirror of the cell (see Figure 1.5). This results in the desired parallel columns (of spot images) in the \( y \)-direction; two columns for each incoming beam, left and right of the optical plane. After the appropriate delay, the beams exit the cell and proceed to a photodetector, each possessing a unique time delay.

The result is a set of optical pulses or signals produced, one for each antenna element. Each optical signal is then connected to a microwave signal and fed to the
Spatial Light Modulator (SLM) Front View:
Spot Pattern for 7 Input Beams

Note: Square SLM is used here.
Gray arrows indicate the rows of reflection spots (in x-direction)
for one input beam (A).

Figure 1.5: Parallel processing of multiple beam off-plane reflections on the SLM, adapted from [1].
associated antenna element. Each optical signal has its own independently controlled delay. These concepts will be further explained and extended in Chapter 2.

Approaches to optical true-time delay tend to fall into two categories: those using fibers [16] to [26] and those using long free-space paths [27] to [30]. Some fiber approaches use multiple optical switches or broadcast the light over all possible paths at once. Wavelength division multiplexing schemes have recently been developed using fiber Bragg gratings [19, 24]. Free-space systems have also used multiple optical switches for switching the beams between sequential optical paths. These optical switches are usually liquid crystal-based. All of these will be discussed in more detail in Section 1.4.

The present approach differs from previous free-space approaches in that it uses only one optical switch or spatial light modulator (SLM), and one beam splitter, rather than one or more SLM for each stage. The device controls a separate light beam for each antenna element, and modifies the length of the path that each beam travels through the cavity. Many beams circulate through the cell simultaneously, and the length of the path each beam travels is independently controlled. The device is fully reciprocal in that it can be used for either transmitting or receiving.

1.3 Further Background on Phased Arrays

1.3.1 System-Level Description

A simplified system diagram is shown in Figure 1.6. A low power microwave source generates the microwave signal that is fed into the beam steering system. The radar signals then are fed to the transmit/receive (T/R) modules along with amplitude control signals. The T/R modules are made of high power microwave amplifiers for
transmitting (or low noise amplifiers for receiving), switches for changing between transmit and receive modes, and the actual individual antenna elements that make up the array. Amplitude control lines control the gains of the amplifiers that operate on the signal sent from the beam steering unit. The result is a steered and shaped composite radar beam.

The true-time delay required can be converted to a phase shift between elements, hence the name phased array. This is done using the relation \( \Delta \phi = \omega \Delta t \), where \( \Delta \phi \) is the phase difference between elements, \( \omega \) is the angular frequency of the radar carrier, and \( \Delta t = \frac{\Delta \tau}{c} \) is the true-time delay difference between elements. Note that a particular phase shift is equivalent to the time delay required only at the calculated frequency. In other words, the phase shift required for beam steering is a function of frequency.
1.3.2 Advantages and Disadvantages

Phased array antennas have significant advantages over other antennas [31]. No mechanical moving parts are required for antenna steering, producing a more robust system with much lower maintenance costs. Since individual element control is inherent in a phased array, an array can be designed to allow for conformal antenna shapes. The amplitude control permits changing the beam shape when desired, which is impossible with single element antennas. This same element control is also very advantageous for receive-signal real-time and off-line signal processing. The mechanical, electromagnetic and signal processing advantages of phased arrays have resulted in the use of this type of antenna in some of the most physically demanding and technically advanced missions.

The drawbacks are that phased arrays are expensive, complex, and in some cases heavy and bulky, in large part due to the electronic beam steering system. With the important advantages mentioned previously, it is no wonder that phased arrays are the target of electrical engineering research to reduce cost, size and complexity in subsystems such as the beam steering unit.

1.3.3 Current Beam Steering Methods in Phased Arrays

Two main electronic methods are used for beam steering: phase shifting and true-time delay. Both of these electronic methods, however, have serious drawbacks.

Electronic phase shifters (e.g. diode and ferro-electric) are used at each element to provide modulo-$2\pi$ phase shifting. Each phase shifter is tuned to the nominal center frequency of the radar signal, and provides the same phase shift to all frequencies
passed. In the frequency domain interpretation, the phase shifter’s transfer function has constant phase response [21].

As discussed previously, the phase shift required for a certain beam direction is only equivalent to the time delay at one frequency, $\Delta \phi = w_o \Delta t = w_o \frac{d\sin \theta}{c}$. This is a major drawback to phase-shift steering devices which can only supply the same phase shift to all frequencies. With a constant phase shift (produced by a phase shifter), frequencies other than $w_o$ will be steered in different directions. This directional spreading of a multi-frequency (wideband) beam is termed beam squint (Figure 1.7).
For narrowband continuous wave (CW) signals, the phase shifting method works well. For wideband pulsed signals, phase shifting produces unacceptable beam squint at bandwidths greater than about 5% of design center frequency [21]. This undesirable consequence means phase shifters cannot be used for high bandwidth missions such as mobile communications, spread spectrum or high resolution millimeter wave radars [21].

Unlike phase shifting, however, TTD does provide steering over a broad bandwidth. Again, the Fourier transform of a time delay provides a frequency domain interpretation to compare to phase shifting: the time delay transfer function has a ramped phase function so that the phase shift is linearly proportional to the frequency of the signal [5]. Thus, in a broadband pulsed signal, each frequency component is phase shifted appropriately to steer each component beamfront in the same direction therefore there is no beam squint, even for ultra-wide bandwidths. In a narrowband CW signal, the beamfront is steered but there is no advantage over using phase shifters. For pulsed signals, however, true-time delay has the extremely important advantage over phase shifters of steering ultra-wide bandwidth signals with no beam squint.

The true-time delay method produces signal time delays by physically changing the propagation time for each element’s signal. The number of different time delays required for the system is dependent on the element spacing, the maximum desired steering angle, and the steering resolution. Electronic true-time delay steering of a phased array requires switching the signal for each element between transmission lines (dispersionless coaxial cables) of different lengths, resulting in a large, heavy, expensive, and complex cable and switching system to steer the array.
Figure 1.8: Transmit (a) and receive (b) configurations for an optical true-time delay device.

Despite the bandwidth advantages, electronic true-time delay is rarely implemented due to the hardware bulk, weight, expense, power loss, RF interference, crosstalk and system complexity incurred when using lengths of coax to produce the time delays [31],[32]. Electronic true-time delay is only used in a few of the most critical, high-priced ground based systems. Most phased arrays today use electronic phase shifters. Beam squint, however, limits phase shifter arrays to narrowband pulsed signals, preventing full realization of pulsed system bandwidth. It is clear that a better method of true-time delay is necessary for performance improvement in phased arrays [33].
1.3.4 System Integration

Figure 1.8 shows in more detail how a photonic true-time delay device would fit into a phased array antenna system. We see two configurations, one for transmitting and one for receiving. Our optical TTD device is shown in the center of each part. There is one optical beam through the device for each antenna element; each optical beam is independently delayed.

In the transmit mode, shown in Figure 1.8a, a separate polarized and RF-modulated light beam is introduced into the cell for each of N antenna elements. The N beams can be generated by N modulated light sources, or by using a 1xN splitter as shown in the Figure and a single RF-modulated light source. The light source(s) can be of any single wavelength, and need not necessarily be lasers since the true-time delay device will work for incoherent as well as coherent light.

The N identical RF modulated light beams are introduced into the TTD device and are given the N individual time delays associated with the desired steering angle. Each independently time-delayed beam is detected by a separate photodetector which drives a transmit module (amplifier and antenna array element). The amplitude of the beam is individually adjusted for beam shaping by electronic amplification.

Figure 1.8b shows the receive case, where N array element signals modulate N independent light beams. The beams are input into the TTD device, and are individually delayed to form the desired receive pattern.
1.4 Review of Previous TTD Photonic Work

1.4.1 Photonic Phase Shifters

Photonics have been applied to both phase shifting and true-time delay beam steering schemes. The photonic phase shifting schemes, however, are as inherently narrowband as their electronic counterparts, [34] to [45]. Various complex photonic methods have been used: acousto-optical shifting [40], optical heterodyning [44], Fourier spatial processing [36], etc. Due to the bandwidth limitations, phase shifters are not the optimal solution for beam steering and, therefore, will not be discussed.

1.4.2 Photonic True-Time Delay Devices

Fiber Optics Methods

Fiber delay line systems have received the most consideration of all the photonic TTD methods under development to date since they evolved from work on electromagnetic interference-free (EMI-free) delivery of the control signals to the transmit/receive (T/R) modules [46]. Fiber TTD systems have even been used in proof-of-concept demonstration antenna arrays [17, 16, 47, 48, 49].

Fiber optic TTD systems work by using complex switching systems to move the optical signal power between various lengths of fibers that form the different optical paths for the time delay. Without this switching, a fiber for each different time delay would be required for each element, amounting to huge fiber networks. The key to each different system design is therefore the switching method or architecture used.

Some novel fiber switching schemes have been proposed. One method used a rotating optical commutator to switch the optical beam between fibers of different
lengths [23]. Another method used a recirculating delay system to repetitively circu-
late light through a loop of fiber by using a single-pole single-throw fiber optic switch [50]. Another fiber system employed a matrix of opto-electronic switches to direct the output from an array of fiber delay lines to the T/R modules [51]. One fiber TTD system used an architecture based on frequency division multiplexing so that a single fiber amplifier could carry all signals simultaneously to the array elements.

In some antenna demonstrations, optically implemented systems have been hy-
bridized with electronic microstrip delay lines for the shortest time delays because of accuracy, switch cost constraints and the difficulty in using fiber for picosecond delay times [17, 50]. The minimum time delay is limited physically by how small a piece of fiber can be manipulated and accurately sized [50].

**Opto-electronic Integrated Circuit Methods**

Opto-electronic integrated circuits (OEICs) are beginning to be investigated for broadband radar time delay units. The production of integrated optical delay lines (waveguides) has been described [52, 20]. OEICs are semiconductor photonic devices (such as waveguides and diode lasers) built on a single substrate so that standard chip integration methods can be used. OEICs have greater optical path length accuracy than physical fiber cuts, and greater compactness and physical robustness than fibers [52]. However, OEICs still have insertion, switching and waveguide losses and are very limited in the possible lengths of time delay by their physical size.

**Free Space Methods**

Free space methods have been avoided in the past due to mechanical positioner cost and complexity [46]. Free space methods have the potential, however, to have
much lower insertion and switching losses than fiber systems. These methods also have the potential to perform spatial optical signal processing, along with time delays [32].

In free space, an optical beam is selectively switched between unguided paths of different optical lengths. Switching is typically done by changing beam polarization (for example, with a spatial light modulator (SLM)) and then passing the beam through a polarizing beam splitter (PBS) that directs different polarizations into different paths. The path differences can be created via media of different refractive indices or by different physical distances.

In one particular scheme, polarization switching into different birefringent materials (index switching) provides different time delays, Figure 1.9 [3]. The system uses polarized light, sequential SLMs and various length birefringent crystals. The spatial
light modulators rotate the polarization plane to be along either the fast or slow axis of the birefringent materials. A major drawback to this and most other free space methods is diffractive spreading of each beam as it propagates, causing inter-element-signal crosstalk as the beams overlap, reduced output power and requiring a larger physical system size due to a larger beam spot.

Physical path delays can also be created by cascading bulk optical branching elements (such as prisms and mirrors), Figure 1.10. The laser beam is spatially filtered with the iris and expanded with lenses (not shown) to pass through all 25 channels of the SLM. In each channel, the polarization can be independently rotated to either pass through both PBSs providing a short delay or to be reflected by the PBSs providing a long delay. Cascaded delay stages separated by optical switches made of SLM polarization rotators and PBSs formed one example of this type of
system. Each stage is a factor of two longer than the preceding one, thus each stage provides one bit in the total (e.g. $2^3$ or $2^4$) time delay. [4, 53, 54, 55, 29, 56, 25]. This type of system requires large physical distances and large numbers of components for multiple bits of time delay.

One unique free-space TTD device used "rubber" mirrors or deformable mirror devices (DMDs) to create the time delays in the frequency domain [5]. DMDs have many small reflecting elements that can be individually translated and rotated distances on the order of several wavelengths of light. In essence, this system works as a frequency domain filter. Optical frequency components of a modulated light beam are spatially separated via a diffraction grating, Figure 1.11, and projected onto the aperture of the DMD. Translation and tilt of the DMD elements is used to create a linear phase ramp across the aperture (and thus across the frequency band). In

Figure 1.11: Deformable mirror device (DMD) TTD concept, adapted from [5].
the frequency domain, time delay is a linear ramp phase shift function. This system has several drawbacks: large system size, optical components with moving parts, and true-time delay bandwidth limited to the size and movement range of the DMD.

In comparison to the devices just described, the TTD device of this work has improved on the drawbacks of these other systems. This TTD device has no mechanically moving parts. It has no beam spread due to its repeated refocussing. This device uses only one SLM instead of multiple ones and is more compact than linearly concatenated free-space designs. Finally, our device has no inherent bandwidth limit and is only limited in bandwidth by the optical modulators and detectors available.

1.5 Research Problem Statement

The purpose of this work is to demonstrate essential features of operation of the novel photonic TTD system just described and to investigate performance parameters necessary for implementation of a device suitable for a phased array antenna. This device would replace the electronic beam steering unit in Figure 1.6.

1.6 Contents of the Dissertation

The following chapters contain information on the details of the design, demonstration and integration issues. In Chapter 2, we describe the design and explore related optical theory on imaging within the cell and fields within the cell affecting crosstalk. Chapter 3 contains the detailed descriptions of all the laboratory demonstration experiments and the data obtained. Two implementations of the device are used to demonstrate all aspects of time delay control. In addition, performance parameters are investigated in depth. The performance parameters are round trip
power loss and two kinds of crosstalk, polarization-related and pixel-related. Chapter 4 contains further analysis of the demonstration results and their relation to device implementation and system integration. Finally, Chapter 5 provides a summary of the dissertation results and suggestions for further work.
CHAPTER 2

DESIGN DESCRIPTION AND RELATED THEORY

Having reviewed the background of the problem of producing true-time delay (TTD) and introduced a new solution in Chapter 1, we now proceed with a more detailed explanation of the compound cell and examination of the theoretical background of the compound-cell design. This will provide background to better understand the experimental work in Chapter 3. Here we introduce the operation of several incremental modifications to the basic White cell that, in total, result in the final compound-cell design. With this groundwork, we discuss the choice of optical lengths and distances to meet any time delay design criteria and develop a spot size model. Overall, this chapter provides the methodology needed to understand and design a compound-cell device for any phased array antenna.

The chapter is divided into two major sections. In the first, Section 2.1, we start with an introduction, then describe the operation of a White cell, a single cell design, to be called type I, a type II compound-cell design, and finally a type III compound-cell design. The ranges of time delays possible with each design are discussed next. The section concludes with a formal derivation of the imaging conditions within the device and relates these to the desired scale of the time delay. In the second major
section, Section 2.2, we derive expressions for fields within the device that will affect the performance capability and limitations of the device.

2.1 Theory of Operation

2.1.1 Introduction

The True-Time Delay (TTD) device described here has, as its basis of operation, many independent beams that make a controlled number of passes back and forth through an optical cavity. The device is based on the White Cell [8], an optical cavity consisting of three spherical mirrors, which recirculates a beam many times through the cell and refocuses the beam on each pass. The true-time delay device described here operates similarly in that it recirculates many beams simultaneously, but has specific extensions to achieve the desired purpose. First, the time each beam spends bouncing back and forth in the cell is independently controlled using one spatial light modulator. Second, we use two cells of different optical path lengths, but that have one mirror in common, that mirror being the surface of the spatial light modulator. This configuration increases the range of possible transit times.

We begin by reviewing the operation of the original White Cell. Then we discuss the first modifications for the generation of variable true-time delays, including the incorporation of the spatial light modulator into a type I device. Next we describe the type II compound cell approach. The final design, type III, is described. Finally, an expression for the range of time delay produced by each type device is derived and the expressions for the corresponding focal lengths and distances are derived.
2.1.2 The Basic White Cell

The original White Cell is shown in Figure 2.1. It consists of three identical spherical mirrors, all of the same radius of curvature. Mirror A is separated from Mirrors B and C by a distance equal to their radii of curvature. The center of curvature of Mirror A lies on the optical axis, and falls between Mirror B and C. The alignment of Mirrors B and C is such that their centers of curvature are located on Mirror A, each a distance $\delta$ above and below the optical axis, respectively. Light from Mirror B is imaged by Mirror A onto Mirror C and vice versa. Also, light from Mirror A is imaged by Mirror B back onto Mirror A or by Mirror C back onto Mirror A. All imaging conditions have unity magnification. This assures that all the spots on Mirror A are imaged to the same size.
Figure 2.2: Operation of the White Cell: the beam is introduced into the cell via an input turning mirror, expands to fill Mirror B, and is imaged to a point again on Mirror A. Drawing not to scale.

In operation, light is injected into the White Cell, either alongside Mirror A from the bottom right or, as shown in Figure 2.2, from a spot imaged onto a small, flat mirror called the input turning mirror. The input light in the beam is conditioned to expand onto Mirror B. Mirror B refocuses the beam to another spot image on Mirror A. Transverse object and image distances are all equal because all imaging conditions in the White cell have unity magnitude. Thus if the beam enters the cell a distance $d_1$ from Mirror B's center of curvature, it will be focussed at a point the same distance $d_1$ on the opposite side of the center of curvature. From there, the beam is reflected by Mirror A and expands again onto Mirror C, as shown in Figure 2.3. Since the point being imaged by Mirror C is located a distance $d_2$ from the center of curvature
Figure 2.3: The beam expands again, this time filling mirror C, and is focused to a new point on Mirror A. Drawing not to scale.

of Mirror C, Mirror C refocuses the light beam to a new spot $d_2$ away from Mirror C's center of curvature on the opposite side.

This process continues as shown in Figure 2.4, which is drawn looking at Mirror A from the right in the previous figure. The centers of curvature of Mirrors B and C are shown, and the spots, alternating from top to bottom of Mirror A, are numbered in the sequence in which they appear. The optical axis is at the center of the figure, coinciding with spot number 8.

The spots eventually miss Mirror A and the beam exits the cell, bypassing Mirror A and being turned out by an output turning mirror as shown in the figure. The number of total bounces is determined by the spacing $2\delta$ between the centers of curvature of mirrors B and C, and by the overall size $D$ of Mirror A.
Figure 2.4: Sequence of spots for an input beam in line with centers of curvature of Mirrors B and C. Optical axis intersects mirror at center, coincident with spot 8. Drawing not to scale.
Now consider expressions for the spot pattern in terms of x-axis locations. First we present a generic expression, written in terms of \( x_B \), the location of the center of curvature of spherical mirror B. The center of curvature of mirror C is at \( x_B + 2\delta \). Let the input spot location be \( x = 0 \). Then the general expressions for the x-axis locations of the spots are written in terms of \( x_B \) and center of curvature separation, \( 2\delta \). They are

\[
\begin{align*}
  x_{even} &= m_{even}2\delta \quad \text{for even spot numbers, } m_{even} = 0, 2, 4, \ldots \\
  x_{odd} &= 2x_B - 2\delta(m_{odd} - 1) \quad \text{for odd spot numbers, } m_{odd} = 1, 3, 5, \ldots
\end{align*}
\]

In Figure 2.4, we have used a particular expression for \( x_B \):

\[
x_B = (n + \frac{1}{2})2\delta
\]

where \( n \) is always an integer and related to the total number of spots on the SLM. Then the resulting spot locations are

\[
\begin{align*}
  x_{even} &= m_{even}2\delta \\
  x_{odd} &= (2n + 2 - m_{odd})2\delta
\end{align*}
\]

The spots can never have the same x-axis location since \( 2n + 2 - m_{odd} \neq m_{even} \) for any integer value of \( n \). In fact, the spots are always separated by an integer multiple of \( 2\delta \), \( x_{even} - x_{odd} = 2\delta(m_{even} + m_{odd} - 2n - 2) \). The smallest x-axis separation between spots is \( 2\delta \) when \( m_{even} + m_{odd} - 2n - 2 = 1 \). For this pattern, the total number of spots, \( q \), on the SLM is then \( q = 2n + 1 \), and we see \( n = 7 \) in figure 2.4. Note that for this choice of \( x_B \) the number of spots on Mirror A (the SLM) is always odd.
2.1.3 The Type I Single Cell

We now describe the first set of modifications to the White Cell to adapt it to variable true-time delay applications. The modifications are shown in Figure 2.5. The first modification is to change Mirror A from a curved mirror to a flat one perpendicular to the optic axis and add a lens of focal length \( f \), where \( f \) is equal to the radius of curvature of the original spherical mirror. In this case, the lens-mirror combination is optically equivalent to the mirror it replaces. Next, we replace the flat mirror (Mirror A) with a reflective Spatial Light Modulator (SLM). The SLM, which can be either electronically or optically addressed, is configured to rotate the direction of polarization of the reflected beam by 90 degrees at any particular pixel that is selected. Finally, we add a polarizing beamsplitter, adjusting the distances to Mirrors B and C to maintain imaging of Mirror B onto Mirror C and vice versa.

Figure 2.5: The Type I cell. One beam is shown making two full transits of the cell, or twice the minimum path. Drawing not to scale.
We require the input light to be polarized in the plane of the paper in Figure 2.5. The beamsplitter is taken to reflect light polarized in the plane perpendicular to the paper but transmits light polarized parallel to the plane of the paper. In this way, light reflecting from a selected pixel is turned out of the cell. We refer to a cell with these modifications as Type I.

We will be interested in the time delays experienced by the light pulses propagating through the cell. We see that the incremental time delays, \( \Delta t \), are in units of the transit time \( T_{A(B,C)} \) from Mirror A (now the SLM) to either Mirror B or C and back, where

\[
\Delta t = T_{A(B,C)} = \frac{2[R + (n - 1)S]}{c} \text{ secs}
\]

where \( R \) is the distance between Mirror A and either Mirror B or C, \( n \) is the refractive index of the beamsplitter cube, \( S \) is the length of the cube, and \( c \) is the velocity of light in free space.

The type I cell can be used directly for time delay. With no SLM pixels set for polarization switching, the beam will bounce through the cell the same number of times and exit the cell. This would constitute the longest possible path for the beam and longest time delay. If a certain pixel is set to switch polarization, the beam is then turned out of the cell by the PBS after the corresponding number of bounces to reach that pixel. Thus, the choice of pixel controls the number of bounces and hence the propagation time delay. The delay is proportional to the number of bounces.

One drawback to the Type I device is that the output is unwieldy to detect because the beam leaves the time-delay cell from a different point and at a different angle depending on the particular delay chosen. An imaging lens could be used to
direct the output beam to a linear detector. Such detectors are slow due to their large area and are not suitable for microwave frequency detection.

**Cell Spot Patterns**

In Figure 2.4, we showed the set of spot images from the original White cell on the spatial light modulator. There, the input spot was introduced on the same line as the centers of curvature of the two spherical mirrors. There are many more sets of spot patterns with various advantages in a time-delay device. Here we discuss them.

In general, there are two ways to generate different spot patterns. The first method is to change the position of the input beam on the input turning mirror. This means an input spot is no longer at \( y = 0 \) but has some y-axis displacement, call it \( y = y_o \). This affects the y-location of all the resultant spots in the pattern. Each different value of \( y_o \) leads to a similar spot pattern, but in a unique location. The second method to change the spot pattern is to change the location of the centers of curvature of the spherical mirrors, e.g. CC(B) and CC(C). This displacement changes the relative x-axis location of the spots and provides many more similar patterns with unique locations. Combining these two methods can provide different spot patterns with advantages for time delay devices. We now provide further details on the two methods and resulting advantages.

**Multiple Beams from Displacing the Input Spot**

We now discuss the first method of changing the spot pattern, y-displacement of the input beam and see that multiple beams in the time delay device result. These patterns, adapted for a time-delay device by Collins [15], are similar to the absorption cell work of Bernstein and Herzberg [57].

34
In Figure 2.4, the input spot is on the same line as the centers of curvature of Mirrors B and C. In contrast, Figure 2.6a shows Mirror A the same as Figure 2.4 but it shows spots which are off the optical axis by an amount $y_o$, as is spot 0. Mirror A is now shown in Figures 2.6a as a square section of a spherical mirror, rather than round, as in Figure 2.4. The centers of curvature of Mirrors B and C, however, still have the same x-axis positions used in Section 2.1.2. Also, in Figure 2.6a, input and output turning mirrors are shown and have been extended the length of Mirror A. The circles represent spots of light imaged onto the SLM or turning mirror.

Spherical mirror imaging about the center of curvature still holds so that an image is as far from the center of curvature as is the object but on the other side of the center of curvature. The result is that now consecutive image spots alternate top and bottom as in Figure 2.4, but also left and right as shown. The complete spot pattern is seen in Figure 2.6b. The complete pattern has two columns of spots for each input spot. Even spots have displacement of $+y_o$ and odd spots have displacement of $-y_o$.

The analytical expressions for the centers of curvature and the spot locations with y-location values added to the expressions from Section 2.1.2 are

Centers of Curvature

$$x_B = (n + \frac{1}{2})2\delta, \quad y_B = 0 \quad n \text{ an integer} \quad (2.7)$$

$$x_C = x_B + 2\delta, \quad y_C = 0 \quad (2.8)$$

Spot Pattern

$$x_{ev} = m_{ev}2\delta, \quad y_{ev} = +y_o \quad (2.9)$$

$$x_{odd} = (2n + 2 - m_{odd})2\delta, \quad y_{odd} = -y_o \quad (2.10)$$

35
Figure 2.6: (a) Construction used to predict location of next spot. The mirror has been made a square section of a spherical mirror. (b) Completed spot pattern. Drawing not to scale.
It can be seen that neighboring spots in one column are separated by a distance of $4\delta$. Comparing the spot locations between the two columns, we see that the two columns of spots for a given beam do not line up in rows, but actually alternate on every other row. For this reason, we call this spot pattern the "alternating" spot pattern. In addition, there is an odd number of controllable spots for this beam on the SLM.

All input beams come into the cell on the bottom and exit the cell on the top. The output spots for one turning mirror location are in the same column on the output turning mirror above the input spot.

We now discuss several items shown in Figure 2.7. Figure 2.7 shows the same view of the Mirror A as Figure 2.6. Different input spots are numbered along the bottom. The letters B and C in Figure 2.7 represent the centers of curvature of Mirrors B and C. The numbers on the right side indicate the bounce number as the light bounces off Mirrors B and C and back. We label the input turning mirror bounce as spot 0 and then the output turning mirror becomes $m = 10$, with spots 1 through 9 on the SLM.

In Figure 2.6, the input spot on the turning mirror was at $+y_o$. A pattern similar to that in Figure 2.6, but with a different value of $y_o$, is shown in gray for input spot 5 for reference.

Now consider an input spot with $y$-location of $-y_o$. The resulting spot pattern for the $-y_o$ case is illustrated in Figure 2.7 for input spot 7, shown by the blackened spots.
Figure 2.7: An array of input beams, with the sequences for input beams 5 and 7 highlighted in gray and black, respectively. Drawing not to scale.
We note that the pattern of the black spots in Figure 2.7 is the mirror image of the pattern in gray spots. The combination of the two patterns nicely fills in all the spaces in the columns of beams 5 and 7. The spot locations for the \(-y_o\) input location are

\[
\begin{align*}
    x_{ev} &= m_{ev}2\delta, \quad y_{ev} = -y_o \\
    x_{odd} &= (2n + 2 - m_{odd})2\delta, \quad y_{odd} = +y_o
\end{align*}
\]  

(2.11) \hspace{2cm} (2.12)

Now, let us introduce a full row of input spots as shown on the extended input turning mirror in Figure 2.7. The numbers below the input turning mirror in the figure designate the input spots. There are an odd number, \(N = 11\), of input spots in the figure. The input spots have \(y\)-axis input locations of \(y_o = \pm 2n_i\delta\), for \(n_i = 1 \ldots \frac{N-1}{2}\) where \(n_i\) is the input beam increment. Input spot 6 initiates a sequence of image spots that all lie on one column through the optic axis and the two centers of curvature. This one column has the original White cell spot pattern illustrated in Figure 2.4. All the other input spots, however, result in alternating spot patterns. Input spots to the right of input spot 6, have similar patterns to that shown for input spot 7, but at greater unique \(y\)-axis locations. Input spots to the left of input spot 6 have alternating spot patterns similar to that shown in gray for spot 5, a mirror image pattern of those on the right side. Thus, the columns of spots to the left and right of the center column interleave so that all spots land on independent pixel locations on Mirror A, i.e. the spatial light modulator.

We now see that it is possible to accommodate many independent beams by using a row of off-axis inputs, as shown in Figure 2.7. In addition, high density is possible
with intra-row spot interleaving, produced by introducing input beams on both sides of the optical axis on the input turning mirror. The result is a matrix of spots on both sides of the optic axis, all with uniquely controllable pixel locations.

We can now start to make the connection with phased array microwave antennas. Phased antenna arrays require many independently variable time delays, so we introduce multiple light beams into the TTD device. We associate one light beam with each antenna element. The TTD device then simultaneously generates independent delays for each antenna element. This means we now have a compact, highly parallel time delay device.

Improvements from an Alternative Centers of Curvature Locations

We now discuss the second method of changing the spot pattern, new x-location of the centers of curvature. The resulting pattern, adapted for a time delay device by the author of this work, is similar to the work of Schulz-DuBois [10]. We will find this second pattern provides significant advantages in input and output and allows an additional input multiplexing scheme for hundreds of input beams [58] that is not possible with the first or alternating spot pattern without altering the shape of the SLM. This input multiplexing scheme leads to a way of effectively using a larger number of image elements on the SLM, which will be described in the next section.

The new spot pattern is shown in Figure 2.8. Again, the letters B and C in Figure 2.8 represent the centers of curvature of Mirrors B and C. The centers of curvature of Mirrors B and C are slightly closer to the bottom turning mirror. The pattern of bounces for a beam 5 is shown with darkened spots. The resulting spot pattern is now aligned horizontally with two spots per row and vertically with two columns. For this reason we call this pattern the “opposing” spot pattern.
Figure 2.8: The opposing spot pattern of time delay spot images with input and output on the same side of the SLM for each input beam. Drawing not to scale.
This pattern has an even number of controllable bounces on the SLM as shown in Figure 2.8 and can result in one more controllable spot on the SLM than the previous arrangement. In addition, both the input and output are on the bottom of the device, which allows for re-use of the input optics. Note this means that the so-called input turning mirror now is divided so as to have half input spots (left) and half output spots (right). Similarly for the turning mirror on the top of the SLM. Top and bottom, and left and right input and output can be reversed as needed, also adding to the versatility of this setup.

We can now examine the expressions for the locations of the individual spots. Again, the input spot location is $x = 0$, the center of curvature of spherical mirror B is labelled $x_B$, and the center of curvature of mirror C is $x_B + 2\delta$. Recall, the x-axis locations of the spots for reflections about the centers of curvature were generically given as

$$x_{ev} = m_{ev}2\delta, \quad y_{ev} = +y_o \quad \text{for even spot numbers, } m_{ev} = 0, 2, 4\ldots \quad (2.13)$$

$$x_{odd} = 2x_B - 2\delta(m_{odd} - 1), \quad y_{odd} = -y_o \quad (2.14)$$

for odd spot numbers, $m_{odd} = 1, 3, 5\ldots$

The difference in this spot pattern is that the location of the center of curvature of Mirror B is now different from that in Equation 2.3.
It is an integer multiple of the separation distance between the centers of curvature. 

\[ x_B = 2n\delta, \quad \text{where } n \text{ is an integer and again related to the total number of spots.} \]

In Figure 2.8, \( n=5 \) and for the new spot pattern, there are \( 2n \) spots on the SLM. The resulting spot locations are now

\[
\begin{align*}
\text{Centers of Curvature} \\
x_B &= 2n\delta, \quad y_B = 0 \\
x_C &= x_B + 2\delta, \quad y_C = 0
\end{align*}
\]

\[
\begin{align*}
\text{Spot Locations} \\
x_{ev} &= m_{ev}2\delta, \quad y_{ev} = +y_o \\
x_{odd} &= (2n + 1 - m_{odd})2\delta, \quad y_{odd} = -y_o
\end{align*}
\]

The condition \( m_{ev} = (2n + 1 - m_{odd}) \) can be true with various combinations of \( m_{ev} \) and \( m_{odd} \) for any value of \( n \), which results in \( x_{ev} = x_{odd} \). This means there will be multiple pairs of spots that have the same x-axis location and line up in the same row, as seen in the figure. Neighboring spots in one column are still separated by \( 4\delta \).

The same number of input beams as the previous spot pattern (excluding the center row) can be used by placing half the input spots on the bottom turning mirror and half on the top turning mirror as shown in Figure 2.8. Interleaving is now by rows, instead of between interlaced alternating spots as in the previous configuration.

The ability to input and output on the same side of the SLM with the opposing spot pattern has distinct advantages in the laboratory demonstration. Two aspects make testing and aligning the design much easier. Both of these changes can be done without realigning the output optics. The first aspect is that the number of spots can be changed easily by moving \( x_C \), which allows us to study power loss at the output.
The second aspect is that the spot separation can be changed independently of the number of spots on the LV by moving the center of curvature of both mirrors. The spot spacing can be changed easily by moving $x_B$ and $x_C$ closer, which works well for testing pixel interference. Further, the opposing spot pattern allows re-use of input optics for output purposes.

**Highly Parallel Input Method**

We next consider the total number of useful bounces the spot patterns produce on the SLM. Here, we assume the SLM has a square array of image elements. Previously, in Figures 2.2 and 2.2, we have shown a square spot matrix implying that the total number of bounces is equal to the number of input beams. Actually, for either pattern, we are not limited to a square spot matrix but can have a rectangular spot matrix. The only limit on the input row is the size of the SLM. The number of bounces, however, is limited by power loss, to be discussed in Chapter 3. This means that, in practice, for a large number of input beams, the number of bounces will be less than the number of inputs and only a small fraction of the pixels on the SLM are being used.

The opposing spot time delay configuration makes possible a slightly different input pattern that allows a greater use of the SLM area. We find that many more unused SLM image elements can be exploited by inputting a column of input spots for each previous input spot as shown in Figure 2.9. The different symbols correspond to different beams so the spot pattern can be traced. Each of these beams strikes the SLM at a unique set of pixels and can be independently controlled over the full range of time delays. For example consider a 256 x 256 pixel SLM. For 16 bounces on the SLM (which provides 81 increments of time delay) there can be $256/8 = 32$ beams
Figure 2.9: The opposing spot pattern with a matrix of input beams on the turning mirror uses a much larger area of the SLM. Drawing not to scale.
introduced in each column, and with $N = 128$ input and 128 output columns, for a total of $128 \times 33 = 4.096$ input beams, corresponding to 4.086 antenna elements. If the number of bounces on the SLM is increased to 20 (121 increments of time delay), there are 25 useable spots in a column and the total number of independent antenna elements supported is $128 \times 25 = 3200$.

### 2.1.4 The Type II Compound Cell

This section continues with further improvements to the time delay device. Another true-time delay photonic device can be implemented by next adding a second pair of identical spherical mirrors (call them E and F) as shown in Figure 2.10. We will refer to this configuration as Type II. Mirrors E and F have the same focal lengths, but these are different from those of Mirrors B and C.
Now we have dual cells, joined at the beamsplitter. A single lens next to the SLM cannot satisfy the focusing conditions for both the cell containing (B, C) and the cell of (E, F). This problem is solved by using two different lenses of focal lengths $f_I$ and $f_{II}$, which are now placed on the output sides of the polarizing beamsplitter as shown in Figure 2.10. At this point, we have dual Type I cells. Each of these cells can thus be considered a unit cell of a compound cell device. The images of the centers of curvature of the pairs of corresponding mirrors, (B, E) and (C, F), imaged through their respective cell lenses, are coincident so that the spots form the exact same pattern when passing through either unit cell. Formulas for the focal lengths of the compound cell lenses and mirrors will derived later in section 2.1.7.

In the compound cell configuration of Figure 2.10, there are different transit times in the two cells because of their different path lengths. In this configuration, instead of ejecting a given beam after a predetermined number of transits, each beam makes the same total number of transits through the device and exits by striking the output turning mirror. Now, however, the spatial light modulator is used to control how many of those transits are made to Mirrors B or C and how many are made to Mirrors E or F. The minimum possible delay corresponds to a path having all the bounces through the shorter cell and the maximum delay results from making all passes through the longer cell. The smallest delay increment, $\Delta_{II}$, is the difference between the two-way path lengths through the shorter and longer cells:

$$\Delta_{II} = T_{A(E,F)} - T_{A(B,C)}$$  \hspace{1cm} (2.19)

where the $T$'s are found using Equation 2.6. The number of possible delay increments is equal to the number of bounces in the cell. As in the type I device, maximum delay is directly proportional to the total number of bounces on the SLM.
The lengths of the delay increments are determined by the optical distance to the mirror pairs from the SLM. For this reason, the device is suitable for quite small delays as well as large ones. To get very long differential delays, on the order of nanoseconds, one would choose one cell of the device to have a very long path and the other cell to have as short a path as possible. A very long path could be folded around the outside of the smaller path in the interest of compactness. For smaller delays, the two cells would be designed to be close in dimensions, or to use materials of different refractive indices. Delay increments smaller than 10 ps should be possible.

A major advantage of the type II cell is a more advantageous output arrangement than with the type I cell. In contrast to the type I device, the compound cell configuration has the advantage of producing one fixed-direction output beam for each input beam, regardless of the time delay selected. Every light beam makes the same number of trips through the device even though the particular path varies with the delay chosen. Every beam makes a predetermined number of spots on the spatial light modulator, and leaves the cell from a point on the output mirror determined only by that beam's entrance point on the input turning mirror. The result is that the optics required to couple the outputs to the antenna elements, or corresponding optical fiber in the case of remoting, is fixed and identical regardless of the delay increment selected.

2.1.5 The Type III Compound Cell

We describe here our final modification: we remove the restriction that the two spherical focusing mirrors in each unit cell be identical. Figure 2.11 shows this final compound cell configuration for the case where Mirrors E and F have different focal
Figure 2.11: The Type III final compound cell TTD device shown here has a considerably greater range of time delays than the Type II device. Drawing not to scale.
lengths. Mirrors B and C could also be made different from each other, but are shown the same in the figure without loss of generality. Again lens $f_{II}$ in Unit Cell II is chosen to image Mirror E onto Mirror F. Although the lenses are different for the two unit cells, the imaging is still 1:1 for any of the paths and the centers of curvature are coincident. As in the type II device, each light beam hits the SLM in the same pattern of spots regardless of which path that beam follows in the compound cell. Each beam exits the cell at a fixed location independent of delay, thus maintaining the advantageous output of the type II device.

The added advantage of the type III device, over the type II device, is the production of more many delay increments from the same number of bounces. This will be shown quantitatively in the next section. The compound cell device produces time delay constituted by the difference between a fixed number of pathlengths, thus leading to the term "differential" delay. The type III device does this also, but has available more different sized pathlengths from which to choose, extending the range of time delays for the same number of bounces. Thus, the time delay from a type III device is dependent on both the total number of controllable bounces on the SLM and the number of different pathlengths possible. In the next section, we present a quantitative discussion of time delays with the devices.

2.1.6 Time Delay with the True-Time Delay Devices

We now derive expressions for the time delays available from the both the type I single cell and the types II and III compound cell devices.
Type I Time Delays

In a Type I device, additive delays are produced. To produce the time delay, a beam is turned out of the cell after a certain number of transits. This produces an additive delay with the amount of time delay proportional to the number of bounces on the SLM before the beam polarization is switched. The total time delay is given by

\[ T_{\text{out}} = m_{\text{select}}T_{A(B,C)} = m_{\text{select}}\frac{2[R + (n - 1)S]}{c} \text{ secs} \quad (2.20) \]

where \( m_{\text{select}} \) is the number of bounces on the SLM before the beam polarization is switched, \( R \) is the distance between Mirror A and either Mirror B or C, \( n \) is the refractive index of the beamsplitter cube, \( S \) is the length of the cube, and \( c \) is the velocity of light in free space. For the type I device, the total number of bounces possible on the SLM directly translates into the total number of increments of time delay available.

Type II Time Delay

In a Type II device, a differential delay is produced. There is only one size of pathlength difference between the two cells, so there is only one time delay increment size. The total time delay is given by

\[ T_{\text{out}} = m_{\text{select}}\Delta_{II} = m_{\text{select}}T_{A(E,F)} - T_{A(B,C)} \text{ secs} \quad (2.21) \]

where \( m_{\text{select}} \) is now the number of bounces in the longer cell, and \( T_{A(\text{mirror,mirror})} \) is the round-trip transit time through the appropriate cell. For the type II device, the total number of bounces possible on the SLM directly translates into the total number of increments of time delay available.
Type III Compound Cell Time Delay

Time delay in the type III compound cell is dependent on the total number of bounces on the SLM (as in type I and II), and also on the number of different paths available. To determine the total number of attainable time delays in a compound cell device, we must begin by examining the possible paths of the beams.

From the input turning mirror, the geometry is such that all beams must go to Mirror B. From B the beams return to the SLM, completing the first transit, at which point their individual polarizations are either rotated or not. If the polarization of a particular beam is not rotated on the first bounce, it proceeds to Mirror C. If the polarization is rotated, the beam goes to Mirror F. After the second transit, each beam can go either to B or to E, based again on whether the corresponding pixel on the SLM is activated.

We wish to distinguish the mirrors for later quantitative analysis. We start by labelling the mirrors as odd or even, depending on the number of the bounce from the SLM or the input turning mirror. Thus Mirrors B and E are even since the beam can visit these mirrors only after an even numbered bounce on the SLM (or input turning mirror), and similarly C and F are called odd. Light leaving an odd mirror can only be directed to an even mirror, and vice versa. A transition diagram is shown in Figure 2.12. In this figure, the allowed path switches between the spherical mirrors of the two unit cells are indicated with arrows. The spherical mirrors are indicated by circled letters, B, C, E, and F, in their respective locations within the compound cell diagram.

We next quantitatively distinguish the bounces to the mirrors. Let $m$ be the number of times each independent beam travels from the spatial light modulator (or...
input turning mirror) to any one of Mirrors B, C, E or F and back to the SLM. For example, in Figure 2.8, \( m \) equals eleven. We label the image spots on the SLM to correspond to the number of trips, such that the input spot on the input turning mirror is bounce 0 and the output spot on the output turning mirror is bounce 11. With the opposing spot pattern, the symmetry is such that \( m \) will always be an odd number; thus there will always be \( \frac{m-1}{2} \) bounces off the odd mirrors and \( \frac{m-1}{2} + 1 \) bounces off the even mirrors.

We can now write the expressions for the transit time through the device for any path. Let \( L_B, L_C, L_E, \) and \( L_F \) be twice the optical distance from the spatial light modulator to the corresponding mirror, as shown in Figure 2.11. Also let \( i \) be the number of bounces off (even) Mirror E, and \( j \) be the number of bounces off (odd) Mirror F. The values of \( i \) and \( j \) for any particular light beam are governed by the

---

Figure 2.12: A transition diagram for the bounces between even spherical mirrors (B and E) and odd spherical mirrors (C and F). Drawing not to scale.
particular pixels being activated on the SLM. Then there will be \( \frac{m-1}{2} + 1 - i \) bounces off the other even mirror, B, and \( \frac{m-1}{2} - j \) bounces off the other odd mirror, C.

The total transit time is then given by

\[
T = \frac{1}{c} [L_Ei + L_B\left(\frac{m-1}{2} + 1 - i\right) + L_Fj + L_C\left(\frac{m-1}{2} - j\right)]
\]

(2.22)

\[
= \frac{1}{c} [L_B + \frac{m-1}{2}(L_B + L_C) + (L_E - L_B)i + (L_F - L_C)j]
\]

(2.23)

Because the total number of round trips, \( m \), is fixed, Equation 2.23 shows that the total transit time has a constant portion

\[
T_o = \frac{1}{c} [L_B + \frac{m-1}{2}(L_B + L_C)]
\]

(2.24)

and a variable portion

\[
T_v = \frac{1}{c} [(L_E - L_B)i + (L_F - L_C)j]
\]

(2.25)

The latter part, the controllable delay time, \( T_v \), depends only on the two path differences \( (L_E - L_B) \) and \( (L_F - L_C) \) and the selection of \( i \) and \( j \). In the type II device, where \( L_E = L_F \), \( L_C = L_B \), and \( (L_E - L_B) = (L_F - L_C) \), there is only one path difference or increment size available. Thus we see that having different spherical mirror distances, we end up with more path differences or delay increments available with the same number of bounces.

Next, we wish to optimize the choice of \( L_E, L_F, L_B \) and \( L_C \) to get the maximum time delay possible in one complete sequence of delays. We seek to optimize Equation 2.25 for a given number of transits \( m \). Suppose we want the minimum delay increment to be \( \Delta \) seconds. Let us assume that the distances to Mirrors E and F are longer than those to Mirrors B and C. We also assume that \( (L_F - L_C) > (L_E - L_B) \).
First, we examine the allowed range of time delays determined by the counters $i$ and $j$. For the odd mirrors, either Mirror C or Mirror F can be selected each time, so that counter $j$ can range from 0 to $\frac{m-1}{2}$. For the even mirrors, the beam must first go to Mirror B; Mirror E cannot be selected until after the beam has struck the SLM at least once. Mirror E can thus be selected a maximum of $\frac{m-1}{2} + 1 - 1$ times, so the counter $i$ can range from 0 to $\frac{m-1}{2}$ also.

The minimum possible total delay will be obtained if all bounces are between Mirror B and C, that is, $i = j = 0$. All paths will have at least this minimum delay $T_o$ in common. The next possible shortest delay would be obtained if the beam were diverted one time to the even mirror having the shorter additional distance one time, that is, it goes to E instead of B on only one of the even passes, or $i = 1$ and $j = 0$. This means for this case, the beam still visits Mirror C on every odd pass and never goes to Mirror F. For a given desired delay increment $\Delta$, we choose the distances to Mirrors E and B such that

$$\frac{1}{c}(L_E - L_B) = \Delta \quad (2.26)$$

To produce delays of $\Delta, 2\Delta, 3\Delta, \ldots, \frac{m-1}{2}\Delta$, etc., the beam is deflected to Mirror E the corresponding number of times. This is accomplished by incrementing $i$, the number of bounces to Mirror E. The maximum value $i$ can attain is $\frac{m-1}{2}$, giving the maximum delay of $\frac{m-1}{2}\Delta$ by going to Mirror E rather than Mirror B for all but the first bounce.

To continue the sequence, then, one would like to increment $j$, the number of visits to the longer distance odd mirror, and reset $i$ to zero.
Recall our assumption that $L_F > L_C$. Thus one chooses the focal lengths of Mirrors F and C such that
\[
\frac{1}{c}(L_F - L_C) = \left(\frac{m-1}{2}\right)\Delta + \Delta \quad (2.27)
\]
\[
= \frac{m+1}{2}\Delta 
\quad (2.28)
\]

In effect we are counting in a base $\frac{m+1}{2}$ system, where $i$ is the digit in the ones place, and $j$ the digit in the $\frac{m+1}{2}$'s place, the next significant digit.

The maximum possible delay results when the beam passes from Mirrors E to F and back as often as possible, so that there are $j = \frac{m-1}{2}$ trips to Mirror F and $i = \frac{m-1}{2}$ trips to Mirror E. The maximum time delay is therefore
\[
T_{\text{max}} - T_o = \frac{m-1}{2}\Delta + \left(\frac{m-1}{2}\right)\left(\frac{m+1}{2}\right)\Delta \quad (2.29)
\]
\[
= \Delta\left[\frac{m^2 + 2m - 3}{4}\right] \quad (2.30)
\]

We see that the maximum delay goes quadratically as the number of bounces. For 17 bounces, 81 different individual delays, from $(0, \cdots, 80)\Delta$, are possible. This is a great advantage over the other two types of cells in which the number of time delays are proportional only to $m$, not as in the type III.

Note that there are no particular constraints on the actual choices of lengths $L_B, L_C, L_E,$ and $L_F$, only on the differences $L_E - L_B$ and $L_F - L_C$. Therefore, to keep the overall device size as small as possible, it is advantageous to make $L_B$ and $L_C$ as small as possible, to keep $L_E$ and $L_F$ small. As long as the difference $L_E - L_C$ is the appropriate length with respect to $L_F - L_C$, then the lengths can be chosen at will.

The lengths of the delay increments are determined by the optical distance to the mirror pairs from the SLM. To get very long differential delays, on the order of
nanoseconds, one would choose one cell of the device to have a very long path and the other cell to have as short a path as possible. As mentioned earlier, a very long path could be folded around the outside of the smaller path for compactness. For smaller delays, the two cells would be designed to be close in dimensions, or to use materials of different refractive indices. Delay increments smaller than 10 ps should be possible.

We have now examined the theory of operation of the TTD devices. We have also looked at parallel processing, spot patterns and the resultant amount of time delay available with each design. For phased array radars requiring large numbers of time delays, the best solution is the type III compound-cell device. A drawback to the types I and II delay devices is that the time delay is directly proportional to the number of bounces in the cell. In the type III compound-cell, the time delays are proportional to the square of the number of bounces. In addition, there is only one output beam for each channel, and the direction of each output beam is independent of the time delay, so the output optics are simple. Now we turn to the paraxial optical imaging equations for the type III true-time-delay device.

2.1.7 Imaging Equations and Relation to Time Delay

Here we derive expressions for the focal lengths of the cell lenses and spherical mirrors and tell how these are related to the desired time delay increment size, $\Delta$, and then number of bounces, $m$, for the optimal type III device. We do this in three steps: 1) we choose mirror distances $L_B$ and $L_C$ for convenience, 2) choose mirror distances $L_E$ and $L_F$ to give desired time delays, and 3) solve for focal lengths of cell lenses and spherical mirrors from the imaging equations. In doing so, we show that
there is diffraction limited focusing throughout any and all paths taken through the
device by reimaging each pass at the SLM plane.

From the previous section, we first chose the distances from the SLM to Mirrors
C, D, E, and F given the minimum desired time delay increment $\Delta$ and number of
bounces, $m$. As shown in Figure 2.13, we define the optical distances from the SLM
to Mirrors B, C, E, and F and back as $L_B, L_C, L_E$ and $L_F$. Without loss of generality,
we choose $L_B$ for convenience and set $L_C = L_B$, also for convenience. Then, letting
c be the speed of light in a vacuum, the required optical distances, from Equations
2.26 and 2.28, are

$$L_E = c\Delta + L_B$$  \hspace{1cm} (2.31)

$$L_F = \left(\frac{m+1}{2}\right)c\Delta + L_B$$  \hspace{1cm} (2.32)

Once these desired optical distances, $L_B, L_C, L_E$ and $L_F$, are established, the focal
lengths of the Mirrors B, C, E, and F and of Lenses I and II can then be obtained
using the unit cell imaging conditions.

We consider Unit Cell II with the uneven mirror distances first. We call the
distances from Cell Lens II to spherical mirrors E and F, $d_E$ and $d_F$. We define $\Sigma d_d$
to be the physical composite distance from the SLM plane to the cell lens, and choose
it to be the same for both cells (see Figure 2.13). The composite distance includes
glass layers, air gaps and the PBS cube. Thus we can relate $d_E$ to $L_E$ and $d_F$ to $L_F$
using

$$d_E = \frac{L_E}{2} - \Sigma(n_d d_d)$$  \hspace{1cm} (2.33)

$$d_F = \frac{L_F}{2} - \Sigma(n_d d_d)$$  \hspace{1cm} (2.34)
Figure 2.13: The compound cell device with distances labelled for both unit cells. Drawing not to scale.
where $\Sigma n_d d_d$ is the composite optical distance for composite distance $\Sigma d_d$ and $n_d$ is the composite refractive indices. After choosing a suitable value for $\Sigma d_d$ and solving for the distances $d_E$, $d_F$, etc., we proceed to find the focal lengths for the Cell Lens II, $f_{II}$, and spherical mirrors, $f_E$ and $f_F$ from the imaging conditions.

To derive the imaging expressions, we first describe the optical configuration for a unit cell with different spherical mirrors, as shown in Figure 2.14. Here the optics are diagrammed for clarity as consecutively linear optical systems with spherical mirrors replaced by the equivalent lenses. This nicely represents the actual reflective cell design, described in detail in Section 2.1.5. There are three configurations shown representing the three imaging conditions: a) SLM onto SLM through Mirror E. b) SLM onto SLM through Mirror F, and c) Mirror E onto Mirror F.

Imaging condition a) is shown in Figure 2.14a. Moving left to right, we see the SLM plane, followed by the beamsplitter and the cell lens. After being refracted by the cell lens of focal length $f_l$, the beam goes to Mirror E, represented by a lens of focal length $f_E$, and then back through the cell lens and onto the SLM plane. Figure 2.14b is the same as (a), except that $f_l, D_E, f_E$ now go to $f_{II}, d_f, f_F$. In Figure 2.14c, left to right, we see Mirror E, followed by the cell lens. $f_{II}$, the SLM plane, the cell lens again, and back to Mirror F. Between all optics there are labelled physical distances.

We now summarize the geometrical optics matrix methods used here. The lateral displacement, $x_i$, and angle, $\theta_i$, of a ray with the axis at any plane, "i", in the optical system can then be determined by multiplying an input plane ray in column vector form, $\begin{bmatrix} x_o \\ n_o \theta_o \end{bmatrix}$, by the intervening matrices for each translation and refraction as they occur. We represent translation matrices $\begin{bmatrix} 1 & d_x \\ 0 & 1 \end{bmatrix}$ with $T_i$ and refractive
Figure 2.14: The optical diagram for the three imaging conditions for matrix calculation of the type III compound cell true-time delay device. Drawing not to scale.
matrices \[ \begin{bmatrix} 1 & 0 \\ -I_i & 1 \end{bmatrix} \] with \( R_i \). Thus to find, for example, the location and angle of a ray starting from the SLM plane and ending at the spherical mirror E, we use the following equation:

\[
\begin{bmatrix}
    x_E \\
    \frac{n_E \theta_E}{c_E}
\end{bmatrix}
= \begin{bmatrix}
    1 & \frac{n}{c} \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    1 & 0 \\
    -I & 1
\end{bmatrix}
\begin{bmatrix}
    1 & \Sigma \left( \frac{n}{n-1} \right) \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    x_{SLM} \\
    \frac{n_{SLM} \theta_{SLM}}{c_{SLM}}
\end{bmatrix}
\]

(2.35)

\[
= T_E * R_{II} * T_{\Sigma_E} * \begin{bmatrix}
    x_{SLM} \\
    \frac{n_{SLM} \theta_{SLM}}{c_{SLM}}
\end{bmatrix}
\]

(2.36)

\[
= M_E * \begin{bmatrix}
    x_{SLM} \\
    \frac{n_{SLM} \theta_{SLM}}{c_{SLM}}
\end{bmatrix}
\]

(2.37)

\[
= \begin{bmatrix}
    A_E & B_E \\
    C_E & D_E
\end{bmatrix}
* \begin{bmatrix}
    x_{SLM} \\
    \frac{n_{SLM} \theta_{SLM}}{c_{SLM}}
\end{bmatrix}
\]

(2.38)

where \( \begin{bmatrix}
    x_{SLM} \\
    \frac{n_{SLM} \theta_{SLM}}{c_{SLM}}
\end{bmatrix} \) is ray vector at the input SLM plane and \( \begin{bmatrix}
    x_E \\
    \frac{n_E \theta_E}{c_E}
\end{bmatrix} \) is the output ray vector at the plane of Mirror E. \( \Sigma \left( \frac{n}{n-1} \right) \) results from the product of several translation matrices for material of different refractive index. For this example, the final system matrix from the input to the output plane is \( M_E \). For this example, there is not imaging between the input and output planes. If the input and output planes are to be image conjugates of each other, it is well known that the B matrix element of the final system matrix must then be equal to zero.
Using this convention, we form the optical system matrices relating the input and output planes for the configurations in Figure 2.14:

\[
M_{SLME} = T_{\Sigma_n} \cdot R_{f_{II}} \cdot T_{d_E} \cdot R_{f_{II}} \cdot T_{d_E} \cdot R_{f_{II}} \cdot T_{\Sigma_n}
\]  
(SLM to SLM imaging through Mirror E) (2.39)

\[
M_{SLMF} = T_{\Sigma_n} \cdot R_{f_{II}} \cdot T_{d_F} \cdot R_{f_{II}} \cdot T_{d_F} \cdot R_{f_{II}} \cdot T_{\Sigma_n}
\]  
(SLM to SLM imaging through Mirror F) (2.40)

\[
M_{EF} = T_{d_F} \cdot R_{f_{II}} \cdot T_{\Sigma_n} \cdot T_{\Sigma_n} \cdot R_{f_{II}} \cdot T_{d_E}
\]  
(Mirror E to Mirror F) (2.41)

We use the ray matrix equations to obtain the system matrices \(M_{SLME}, M_{SLMF}\) and \(M_{EF}\). Then, for each case, we set the B matrix elements equal to zero, to ensure imaging in each case. When the B matrix element is zero, it means all rays emerging from one point in the input plane will arrive at one image point in the output plane.

Requiring that the SLM be imaged onto itself through mirrors E and F respectively, the first two configurations in the figure, gives

\[
\left[2 \left(1 - \frac{(\Sigma d_e)}{f_{II}}\right) - \frac{d_E}{f_E} + \frac{(\Sigma d_e)}{f_E} \left(\frac{d_E}{f_{II}} - 1\right)\right] = 0 \quad (2.42)
\]

\[
\left[2 \left(1 - \frac{(\Sigma d_e)}{f_{II}}\right) - \frac{d_F}{f_F} + \frac{(\Sigma d_e)}{f_F} \left(\frac{d_F}{f_{II}} - 1\right)\right] = 0 \quad (2.43)
\]

Requiring that mirror E be imaged onto mirror F, the last configuration in the figure, we have

\[
d_E \left(1 - \frac{d_F}{f_{II}}\right) + d_F \left(1 - \frac{d_E}{f_{II}}\right) + 2(\Sigma d_e) \left(1 - \frac{d_F}{f_{II}}\right) \left(1 - \frac{d_E}{f_{II}}\right) = 0. \quad (2.44)
\]
With the three resulting equations, we solve for the three unknowns: \( f_E \), \( f_F \) and \( f_{II} \). Equation 2.44 reduces to

\[
\begin{align*}
 f_{II}^2 \left\{ d_E + d_F + 2\left( \frac{d_d}{n_d} \right) \right\} - 2f_{II} \left\{ \left( \frac{d_d}{n_d} \right) d_E + \left( \frac{d_d}{n_d} \right) d_F + d_E d_F \right\} + 2\left( \frac{d_d}{n_d} \right) d_E d_F = 0 \\
(2.45)
\end{align*}
\]

We solve this quadratic for \( f_{II} \):

\[
 f_{II} = \frac{d_E d_F + \left( \frac{d_d}{n_d} \right) \left( d_E + d_F \right)}{d_E + d_F + 2\left( \frac{d_d}{n_d} \right)} \pm \frac{\sqrt{\left( d_E^2 d_F^2 + (d_E - d_F)^2 \left( \frac{d_d}{n_d} \right)^2 \right)}}{d_E + d_F + 2\left( \frac{d_d}{n_d} \right)} \\
(2.46)
\]

Now, we can use this result with Equations 2.42 and 2.43 to solve for the focal lengths \( f_E \) and \( f_F \):

\[
\begin{align*}
 f_E &= \frac{f_{II} \left( \frac{d_d}{n_d} + d_F \right) - d_F \left( \frac{d_d}{n_d} \right)}{2(f_{II} - \frac{d_d}{n_d})} \\
 f_F &= \frac{f_{II} \left( \frac{d_d}{n_d} + d_E \right) - d_E \left( \frac{d_d}{n_d} \right)}{2(f_{II} - \frac{d_d}{n_d})} \\
(2.47) \quad (2.48)
\end{align*}
\]

For the solution for Unit Cell I, we replace \( L_E \) with \( L_B \), \( d_E \) with \( d_B \), \( L_F \) with \( L_C \), and \( d_F \) with \( d_C \). Then we set \( d_B = d_C \) and \( f_B = f_C \). The unit cell solution with equal focal length spherical mirrors reduces to the case of a unit cell with the distance from the cell lens to both spherical mirrors equal to the cell lens focal length. The solution is then of a much simpler form:

\[
\begin{align*}
 f_I &= \frac{\left( d_B^2 + 2d_B \left( \frac{d_d}{n_d} \right) \right) \pm d_B^2}{2(d_B + \frac{d_d}{n_d})} \\
 &= d_B, \quad \frac{d_B \left( \frac{d_d}{n_d} \right)}{d_B + \frac{d_d}{n_d}} \\
 f_B = f_C &= \frac{f_I \left( \frac{d_d}{n_d} + d_B \right) - d_B \left( \frac{d_d}{n_d} \right)}{\frac{\left( \frac{d_d}{n_d} \right) f_I - \left( \frac{d_d}{n_d} \right)^2}{\frac{\left( \frac{d_d}{n_d} \right) f_I - \left( \frac{d_d}{n_d} \right)^2}} \\
(2.49) \quad (2.50) \quad (2.51)
\end{align*}
\]

For the first \( f_I \) solution, the distance from the spherical mirrors to the cell lens is equal to the focal length of the cell lens which produces plane waves on the SLM. For
the second \( f_t \) solution, the spherical mirror is imaged onto the SLM and then again onto the other spherical mirror. This design uses the first solution for \( f_t = d_B \).

Now, we have shown that with this design, all the imaging conditions are maintained in each unit cell such that for all paths possible there is unity magnification from the SLM surface onto itself. All output beams exit the cell in one SLM column in a fixed direction independent of the time delay path traveled through the cell.

### 2.2 Optical Fields in the Cell

We now turn our attention to an examination of optical beam shapes and sizes in the TTD device. The removal of beam propagation divergence by refocussing is a prime advantage of this design and in this section, we examine the fields propagating in the cell. We first present a field model based on diffraction theory and then use it to predict beam size and shape as the beam propagates through the cell. We then examine necessary mirror curvature and gain an indication of focussing robustness.

First we describe the optical configuration used, as shown in Figure 2.15. Here the optics are diagrammed as a consecutive linear system for clarity, but still represent the actual reflective cell design, described in detail in Section 2.1.5. We see an input beam from the input aperture mask on the left. The input aperture mask, to be further described in Chapter 3, is a mask with circular apertures to create small beams from a much larger beam spot. The input mask is followed by an input imaging lens used to image the aperture onto the input turning mirror plane (marked in the diagram). After the input turning mirror, the beam is inside the unit cell and first encounters a polarizing beamsplitter (PBS) followed by the cell lens. After passing through the cell lens the first time, the beam reaches spherical mirror B. From spherical mirror
Figure 2.15: The optical diagram for matrix calculation and diffraction theory analysis of the compound true-time delay device. The input aperture mask on the left has round holes. Drawing not to scale.
B (SMB), the beam is reflected back through the cell lens, then the PBS, and then onto the SLM.

After reaching the SLM the first time, the beam is alternately reflected back and forth through two paths to the two spherical mirrors. Each of the two paths consists of translation through the front glass plate of the SLM, translation to and through the PBS, translation to the cell lens, refraction by the cell lens, translation to the spherical mirror, refraction by the spherical mirror, translation to and refraction by the cell lens, translation back to and through the PBS, translation back to the SLM and through the glass cover to the flat reflecting surface of the SLM.

Table 2.1 contains example component values which will be used for diffractive size comparisons. These values are very similar to those configured in the laboratory, with typical distances and focal length values of the input optics chosen to provide a 2X demagnification from the input aperture mask to the turning mirror. For this theoretical example, the input turning mirror is placed in the same plane as the SLM.

Again, we represent all optical paths with paraxial geometrical optics matrix methods. Thus to find the location and angle of a ray starting from the input aperture and ending at the input turning mirror, we use the following equation:

\[
\begin{bmatrix}
x_{tm} \\
\theta_{tm}
\end{bmatrix} = \begin{bmatrix} 1 & \frac{d_{a2}}{1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{-f_i} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{d_{a3}}{1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_o \\ \theta_o \end{bmatrix}
\]

\( (2.52) \)

\[
= T_{a2} \cdot R_{f_i} \cdot T_{a3} \cdot \begin{bmatrix} x_o \\ \theta_o \end{bmatrix}
\]

\( (2.53) \)

\[
= M_{tm} \cdot \begin{bmatrix} x_o \\ \theta_o \end{bmatrix}
\]

\( (2.54) \)

\[
= \begin{bmatrix} A_{tm} & B_{tm} \\ C_{tm} & D_{tm} \end{bmatrix} \cdot \begin{bmatrix} x_o \\ \theta_o \end{bmatrix}
\]

\( (2.55) \)
### Matrix Distance Values

<table>
<thead>
<tr>
<th>Distance Description</th>
<th>Distance Label and Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>input aperture mask to input imaging lens</td>
<td>$a_1 = 300 \text{ mm}$</td>
</tr>
<tr>
<td>input imaging lens to input turning mirror</td>
<td>$a_2 = 150 \text{ mm}$</td>
</tr>
<tr>
<td>input turning mirror to PBS cube</td>
<td>$a_3 = 40 \text{ mm}$</td>
</tr>
<tr>
<td>PBS cube width</td>
<td>$d_2 = 38 \text{ mm}$</td>
</tr>
<tr>
<td>PBS cube to cell lens</td>
<td>$d_3 = 15 \text{ mm}$</td>
</tr>
<tr>
<td>cell lens to spherical mirror B</td>
<td>$d_B = 400 \text{ mm}$</td>
</tr>
<tr>
<td>cell lens to spherical mirror C</td>
<td>$d_C = 400 \text{ mm}$</td>
</tr>
<tr>
<td>SLM (51 mm diam.) to PBS cube</td>
<td>$d_1 = 35 \text{ mm}$</td>
</tr>
<tr>
<td>SLM front glass plate $\approx \frac{1}{4}$ inch thick</td>
<td>$d_0 = 7 \text{ mm}$</td>
</tr>
</tbody>
</table>

### Component Characteristics

<table>
<thead>
<tr>
<th>Component</th>
<th>Characteristic Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>input imaging lens (38 mm diam.)</td>
<td>$f_i = 1004 \text{ mm}$</td>
</tr>
<tr>
<td>cell lens (51 mm diam.)</td>
<td>$f_I = 400 \text{ mm}$</td>
</tr>
<tr>
<td>SMB spherical mirror B (51 mm diam.)</td>
<td>$f_B = 250 \text{ mm}$</td>
</tr>
<tr>
<td>SMC spherical mirror C (51 mm diam.)</td>
<td>$f_C = 250 \text{ mm}$</td>
</tr>
</tbody>
</table>

Table 2.1: Example values for a compound cell TTD device for diffractive size comparisons.
where \( \begin{bmatrix} x_o \\ \theta_o \end{bmatrix} \) represents the input plane ray and \( \begin{bmatrix} x_{lm} \\ \theta_{lm} \end{bmatrix} \) represents the output ray at the turning mirror plane. Similarly, we form other optical system matrices relating other paths in the device:

\[
M_{lm} = T_{a2} \ast R_{f1} \ast T_{a3} \quad \text{(input aperture mask to input turning mirror) (2.56)}
\]

\[
M_{cl} = T_{d3} \ast T_{d2} \ast T_{a3} \quad \text{(input turning mirror to cell lens) (2.57)}
\]

\[
M_B = T_{db} \ast R_{f2} \ast M_{cl} \quad \text{(input turning mirror to spherical mirror B) (2.58)}
\]

\[
M_{SLM1} = T_{do} \ast T_{d1} \ast T_{d2} \ast T_{ds} \ast R_{f2} \ast T_{db} \ast R_{fB} \ast M_B
\]

\[
\text{(input turning mirror to first bounce on SLM) (2.59)}
\]

\[
M_{unit} = T_{do} \ast T_{d1} \ast T_{d2} \ast T_{ds} \ast R_{f1} \ast T_{db} \ast R_{fB} \ast T_{d3} \ast T_{d2} \ast T_{d1} \ast T_{do}
\]

\[
\text{(round trip through cell from SLM to SLM) (2.60)}
\]

### 2.2.1 Formula for Fields in the Cell

Now, we wish to find the field patterns for the beams anywhere in the device, after the beams emanate from the round holes in the input aperture mask. The mask holes can be off center by \( r_2 \). We would like to relate the geometrical optics matrix parameters to diffraction patterns for laser light. We do this by using the Lens Matrix Diffraction Integral (LMDI) [59]:

\[
E_i(r_i) = \frac{jk e^{-jkL_0}}{2\pi B} \int dr_0 E_0(r_0) e^{\frac{jk}{\lambda} |r_0|^2} e^{\frac{-jk}{\lambda} (|r_0|^2 - 2r_0 \cdot r_i + |r_i|^2)} \tag{2.61}
\]

In Equation 2.61, the matrix parameters, A, B, and D, come from the system matrix calculated from the input to output planes. The terms \( E_0(r_0) \) and \( E_1(r_i) \) are the input and output plane electromagnetic field amplitudes, respectively, in terms of the input and output location vectors, \( r_0 \) and \( r_i \), and with the scalar approximation with wave number \( k = \frac{2\pi}{\lambda} \) for monochromatic light. The term \( e^{\frac{-jk}{\lambda} |r_0|^2} \) adds a generic
input spherical wave phase term and wave radius of curvature at the input plane. The phase propagation term outside the integrand accounts for propagation from the input plane an optical distance $L_0$ along the axis to the output plane.

We start by making one substitution and then perform several simplification steps. To account for an off-axis input aperture, we transform to transverse coordinates, $r_0'$, centered on the aperture, $r_0 = r_0' + \tilde{r}_2$. Leading phase terms are dropped, since they will not affect the calculation of the diffraction intensity pattern. Integrand coefficient terms are grouped according to powers of $r_0'$. These actions result in Equation 2.62:

$$E_1(r_1) \approx \frac{k}{2\pi B} \int dr_0' E_0(r_0') e^{-\frac{ik}{2} |\tilde{r}_1|^2 \left(\frac{1}{k} + \frac{i}{4}\right)} e^{-\frac{ik}{2} (\tilde{r}_0' r_1 + Ar_1 - r_1)}$$  \hspace{1cm} (2.62)

Now we can use a change in coordinates to center the output field, by substituting for the phase term exponent:

$$\tilde{r}_1' = \tilde{r}_1 - \tilde{r}_2 (A + \frac{B}{R})$$  \hspace{1cm} (2.63)

Substituting Equation 2.63, we obtain Equation 2.64:

$$E_1(r_1) \approx \frac{k}{2\pi B} \int dr_0' E_0(r_0') e^{-\frac{ik}{2} |\tilde{r}_1'|^2 \left(\frac{1}{k} + \frac{i}{4}\right)} e^{-\frac{ik}{2} (\tilde{r}_0' \tilde{r}_1')}$$  \hspace{1cm} (2.64)

We now change to cylindrical coordinates to match the symmetry of the circular aperture in the input mask, which has diameter “a”. For this we use $r_0' = (\rho_0, \phi_0)$ and $r_1' = (\rho_1, \phi_1)$. Equation 2.65 results:

$$E_1(\rho_1, \phi_1) = \frac{k}{2\pi B} \int \int \rho_0 d\rho_0 d\phi_0 E_0(\rho_0, \phi_0) e^{-\frac{ik}{2} \rho_0^2 \left(\frac{1}{k} + \frac{i}{4}\right)} e^{-\frac{ik}{2} \rho_0 \rho_1 \cos(\phi_0 - \phi_1)}$$  \hspace{1cm} (2.65)
The situation is invariant under coordinate rotation, so for convenience we choose \( \phi_1 = 0 \). With unity field amplitude over the aperture limits, the integral becomes separable:

\[
E_1(\rho_1, \phi_1) = \frac{k}{2\pi B} \int_0^{\frac{\pi}{2}} \int_0^B \rho_0 \rho e^{-\frac{ik}{2} b_0^2(k + \frac{\phi_1}{2})} \int_0^{2\pi} d\phi_0 e^{i k \rho_0 \rho_1 \cos(\phi_0)}
\]

(2.66)

\[
= \frac{k}{B} \int_0^{\frac{\pi}{2}} \int_0^B \rho_0 \rho e^{-\frac{ik}{2} b_0^2(k + \frac{\phi_1}{2})} J_0 \left( \frac{k \rho_0 \rho_1}{B} \right)
\]

where \( J_0 \) is the Bessel function of order zero. Finally, we use two substitutions to transform to normalized radial variables, \( v_0' \), on the input plane and \( v_1' \) on the output plane:

\[
v_0' = \rho_0 \sqrt{\frac{2}{\lambda B}} \sqrt{\frac{B}{R} + A}
\]

(2.67)

\[
v_1' = \frac{\rho_1}{\sqrt{\frac{B}{R} + A}} \sqrt{\frac{2}{\lambda B}}
\]

(2.68)

to get a form similar to the Fresnel diffraction integral:

\[
E_1 \left( v_1' \sqrt{\frac{\lambda B}{2} \left( \frac{B}{R} + A \right)} \right) \approx \frac{\pi}{B \sqrt{R} + A} \int_0^{\frac{\pi}{2}} e^{-\frac{2\pi v_0'}{R} + A} v_0' dv_0' e^{-\frac{j \pi v_0'^2}{2}} J_0 (\pi v_0' v_1')
\]

(2.69)

We see the integrand limit now defines a "Fresnel number" parameter, \( N \), where

\[
N = \sqrt{\frac{\alpha^2}{2\lambda B} \left( \frac{B}{R} + A \right)}
\]

(2.70)

This concludes the development of the model for fields in the cell. We now apply this model in the next section.

### 2.2.2 Application of the Formula to the Compound Cell Fields

Now we use Equation 2.69 to examine various aspects of the compound cell TTD device. First we note that the off-axis lateral shift, \( r_2 \), does not affect the shape of the diffraction pattern, which is still a convolution with a Bessel function. and is
determined only by the upper limit. The shape is determined uniquely by the Fresnel number, \( N \). The lateral shift affects only the location and size of the pattern. The lateral shift affects this through \( v_1' \) which is a function of \( r_2^2 \). The curvature, \( \frac{1}{R} \), of the input fields at the aperture also affects the location and size of the pattern. From here on, we assume a plane wave input field at the input aperture mask with \( R = \infty \), which removes all the \( \frac{R}{R} \) terms.

If the input and output planes examined are image conjugates, element \( B \) of the optical system matrix, \( M \), will be equal to zero. In that case, since we have assumed infinite apertures in the optical system, convolution with the Bessel function becomes convolution with a delta function [60]. The delta function sifts out a magnified version of the input, in other words, perfect imaging.

We now start from the input aperture, and calculate the field widths at selected optical components. We use these results to examine relative effects of aperture sizes within the optical system and thereby determine the validity of the infinite aperture assumption in this case.

**Fields at the Input Lens**

The first aperture is the input imaging lens. Using only the translation matrix for distance \( a_1 \), from the input aperture mask to the input imaging lens, the matrix elements are used in the Lens Matrix Diffraction Integral to derive and plot the diffraction pattern. The associated lens matrix is \( \begin{bmatrix} 1 & 300 \\ 0 & 1 \end{bmatrix} \). The pattern at the input imaging lens, shown in Figure 2.16, has a Fresnel number of \( N=0.7 \) and total width including first sidelobes of 1.6 mm. The 38 mm lens can be easily approximated as an infinite diameter aperture.
Figure 2.16: Radial plot of the field intensity pattern at the input lens plane.
Fields at the Turning Mirror

With an effectively infinite aperture, there is close to perfect imaging at the turning mirror. This means we can approximate the fields at the turning mirror as an image of the circular aperture of the input mask with an image diameter of 200 μm. The turning mirror, at 5 mm width, can thus still be approximated as infinitely dimensioned.

Fields at the Cell Lens

We now start looking at the fields inside the cell itself, continuing with the fields at the input turning mirror. We find the optical system matrix from the aperture plane through the turning mirror plane, through the PBS cube, and up to the cell lens plane for use in the LMDI formula to be \([ -1.3 \quad -160.0067 ] \quad \begin{array} { r } { -0.01 } \\ { -2 } \end{array} \). The diffraction pattern at the cell lens has a Fresnel number of \( N=1.1 \) and total width including first sidelobe of about 0.9 mm, as shown in Figure 2.17. Therefore, the 51 mm diameter cell lens can be considered infinite in extent in comparison.

Fields at the Spherical Mirror

The final relative aperture sizes that must be examined are those of spherical mirrors B and C. We continue the path, passing through the cell lens and on to the spherical mirror. The associated matrix is \([ -4 \quad -800 ] \quad \begin{array} { r } { -0.0067 } \\ { -1.6 } \end{array} \). The resulting pattern has a Fresnel number of \( N=0.9 \) and a total width of 4.4 mm including the first sidelobes, shown in Figure 2.18. The use of a 51 mm diameter mirror meets the infinite aperture approximation. All beams coincide at the spherical mirrors, so the spots can be centered on the mirror. Since the beams coincide, the spherical
Figure 2.17: Radial plot of the field intensity pattern at the cell lens plane.
Figure 2.18: Radial plot of the field intensity pattern at the spherical mirror plane.
mirror size could be reduced and still meet the infinite aperture requirement within
tolerance. Size reduction is desirable to reduce size and cost of the apparatus.

**Fields at the SLM**

Finally we consider the fields at the SLM, which is a conjugate plane to the input
turning mirror and the input aperture mask. Given that there are no intervening
limiting apertures, we should expect an image diameter of 200 μm at the 51 mm
diameter SLM face. With this relative size ratio, we expect no aperture effects there
for repetitive trips through the cell.

We have now shown that apertures within an example device can be approximated
as infinite. Thus, the Lens Matrix Diffraction Integral is a reasonable model for
diffractive propagation in the TTD device.

**Edge Effects**

Despite the infinite aperture approximation in relation to the field sizes, edge
effects will occur even if they are very small. At each optical component in the cell,
size reduction optimization requires the beam to pass near the edge of an optical
component.

Minimizing the distance between image spots on the SLM requires the beam spot
on the turning mirror be moved as close to the mirror edge as possible. The PBS
cube and the cell lens would be sized to just permit the beams with greatest angle to
pass through. Minimization of the spherical mirrors requires the diameter to contain
the field energy in the pattern necessary to maintain power loss criteria. In each of
these cases, the possibility of diffractive effects from edge interaction exists despite
the size differential between the component size and the beam.
The entrance pupil of the TTD device is spherical mirror B. The subtended angle of this 2-inch circular aperture imaged through the cell lens is approximately \( \varphi = \frac{51 \text{ mm}}{400 \text{ mm}} = \frac{1}{8} \) radians. The associated point response function for this aperture has width of \( \frac{2.44\lambda}{\varphi} = 19.5\lambda \approx 10\mu m \). Further minimization of the spherical mirrors will increase imaged spot diffraction.

**Depth of Focus**

We here examine longitudinal focussing robustness in terms of the field results. The actual requirement for optimum performance with a pixellated device is optical power well contained on the pixel footprint, not a geometrical image. Power containment is necessary to avoid crosstalk caused by energy leakage out of the pixel and even into other pixels. In our device, we are achieving this power containment by imaging in a White cell base.

The depth of focus gives an indication of where the diffraction pattern is bounded by the geometric shadow of the aperture. That is, where the beam power is contained in a well-contained region. Within this region, the energy would still be contained on the pixel if the SLM plane was slightly shifted in location by defocussing.

We use the LMDI to calculate the fields pattern at several distances from perfect imaging on the SLM to determine an approximate value for the depth of focus. In Figure 2.19, we see four patterns at 1, 5, 10 and 15 mm after the first bounce on the SLM. From these example patterns, we see the pattern is starting to spread wider than the image width of 200 \( \mu m \) at about 10 mm from one side of the SLM, in Figure 2.19c. The fields are symmetric about the SLM focal plane, thus the full depth of focus is twice the one-sided depth or 20 mm. In a repetitively focussed cell, with 20 bounces, a 20 mm depth of focus would allow approximately 1 mm longitudinal
Figure 2.19: Radial plot of the field intensity pattern at small distances after first bounce on SLM to observe depth of focus.
defocussing on each bounce, while still maintaining the concentration of energy in the area of the pixel. This provides longitudinal focussing robustness.

**Spherical Mirror Curvature**

We now use the field results above to examine the beam size at the spherical mirror in light of another question. Is the illuminated area on the spherical mirror small enough that the mirror can be approximated by a plane mirror?

The substitution of a plane mirror would require that the difference in phase from the center to the edge of the field pattern on the spherical mirror be less that some acceptable fraction of a wavelength, e.g. $\frac{1}{10}$ wave. Recall from the preceding examples that the spot halfwidth at the spherical mirrors was predicted to be 2.2 mm including the first sidelobes. We calculate the phase difference between the edge of the pattern and the center of the pattern using the exponential phase term from the field equation:

$$phase = 2 \left( \frac{kx^2}{2R} \right)$$

$$= 2 \left( \frac{(2\pi)x^2}{2\lambda R} \right)$$

$$= \frac{(2)(2\pi)2.2 \ mm^2}{2(0.514 \times 10^{-3} \ mm)(500 \ mm)}$$

$$= 18.8(2\pi) \ radians$$

Here we estimate the radius of curvature of the field, $R$, as 500 mm. The difference $\approx 19\lambda$ at 514 nm. Therefore, plane mirrors cannot be substituted for the spherical mirrors despite the relatively small spot size. A spot width of 4 $\mu$m would be required to meet the $\frac{1}{10}\lambda$ phase variation requirement.
2.3 Chapter Summary

In this chapter, we provide a detailed examination of the theoretical background of the true-time delay device design. With this groundwork, we discuss the choice of optical lengths and distances to meet any time delay design criteria, and develop a model of the fields in the cell. The chapter was divided into two major sections. In the first, Section 2.1, we developed the design, discuss the time delay available, and then derived the imaging equations that provide the solution for the lens and mirror focal lengths. In the second major section, Section 2.2, we derived expressions for and examined the fields within the device. Overall, this chapter provided the methodology needed to design a true-time delay device for any phased array antenna.
3.1 Introduction

In this chapter, the main thrust of the dissertation work is presented. The design was conceived and validated on paper, yet laboratory demonstration is essential for verification of the theory and to discover potential performance or construction issues not obvious from the theory. Thus the laboratory experiments were proposed to address two main purposes: 1) to provide proof that the design will produce suitable time delays with potential for use in phased array antennas and 2) to examine performance parameters that affect design decisions for radar system integration.

For the first purpose, two experimental configurations are used. This includes an initial time delay demonstration of optical switching in a type I multiple reflection cell and a time delay demonstration of one unit cell of the type III compound-cell device. For the type III device demonstration, only one unit cell is available in our laboratory, however, only one unit cell is required for proof of time delay with the compound-cell device. This proof also includes input, output and detection methods that could be used in a future radar beam-steering prototype. At least 9 increments
of $\approx 7.3$ nanosecond time delay are demonstrated on three separate channels using short rise-time laser pulses.

For the second purpose, examining performance parameters, the work is done on either various combinations of the separate components themselves or on the unit cell of the type III compound cell design. Here, the single type III unit cell is a natural starting place for the research of this device since not all of the performance parameters can be obtained from the output of the complete compound-cell device.

The key performance aspects are in the areas of power loss and crosstalk. These areas are important for operational modeling of the full compound-cell device. The key parameters measured, which affect quality and quantity of time-delay with the compound-cell device, are round trip power loss, individual component loss analysis, polarization-related crosstalk and pixel-related crosstalk.

In this chapter, we describe experimental configurations and present measurement results. In Section 3.2, we describe the time delay demonstration with the type I device, Section 3.2.1, and with the type III unit cell of the compound-cell device, Section 3.2.2. In total, the data from these two experiments clearly show that the compound-cell design produces time delay suitable for a phased array antenna. Sections 3.3 through 3.4 contain data on the key performance parameters. These are round trip power loss, 3.3.1, component losses, 3.3.2, polarization-related crosstalk, 3.4.1, and pixel-related crosstalk, 3.4.2, respectively.
3.2 Time Delay

In this section, time delay will be demonstrated using two experimental approaches that together show all aspects of how time delay is generated by a type III compound-cell TTD device. With these demonstrations, we show that the compound-cell design can produce time delay suitable for a phased array antenna. The two approaches to time delay were produced by two different demonstration devices, a type I device and the unit cell of the type III compound-cell device. First, the type I device demonstration was used to show that the optical switching premise of the cell design did work. Next, one unit cell of the type III compound-cell device was constructed to obtain proof of time delay with the type III approach and performance information for the compound-cell design.

In Section 3.2.1, we demonstrate time delay produced with the type I device. This shows the ability to switch beams with the spatial light modulator in the cell. In Section 3.2.2, we present the demonstration of a unit cell of the type III compound-cell device.

3.2.1 Type I Device Demonstration of Time Delay

In this section we describe the time delay demonstration with the type I device. This demonstration shows the operation of the optical switch and path delay inherent both in this type I device and in the type III compound-cell TTD device.

Figure 3.1 shows the type I device. Here we see a Pockel cell optical gating device in the lower right corner for creating sharp rise time pulses from a continuous wave laser beam. The input laser beam is indicated in the lower right corner, coming from the 514 nm Argon source described in Appendix B. The semitransparent mirror next
Figure 3.1: The type I time delay demonstration device consisted of a single White Cell with an optical switch. An optical control system selects the time delay. The output is detected when the beam exits the cell. Drawing not to scale.
to the Pockel cell splits off energy to the trigger detector, the output of which is then routed to the digital sampling oscilloscope (DSO) at the top of the figure. In the center of the figure is the main part of the unit cell. At the far left are a control beam and scanning slit to apply light to the write-side of the liquid crystal light valve (LCLV), next to the slit. The light valve and its operation are described in detail in Appendix A. On the read-side (right) of the light valve is a 546 nm polarizing beam splitter (PBS) and above it an auxiliary mirror and input turning mirror. To the right of the beamsplitter is the cell lens, and at the far right are two identical dielectric spherical mirrors. Finally, at the top of the figure is the output section with an output lens and signal detector, also feeding the oscilloscope.

NOTE: In Chapter 2, the spot diagram, Figure 2.4, was shown as a sideview corresponding to the plan view of the cell shown in Figure 2.3. In this chapter, we are discussing the laboratory apparatus. For this case, one would have to be above the optics table to see the plan view of the device. The natural viewing position for the spot pattern on the LCLV is at ninety degrees to the plan view. Thus our discussion of spot pattern directions will hereafter differ by ninety degrees from Chapter 2 descriptions. Specifically, the two columns of spots resulting from each input beam are now two rows. The beams bounce from left to right when facing the LCLV, not top to bottom. The input turning mirror is at the top left of the LCLV and the output turning mirror is at the bottom left. A column of spots is input onto the turning mirror. A 3-D view is shown as if from a position at the side of the optics table in Figure 3.1.

The auxiliary mirror is necessary since the turning mirror cannot be in the same plane as the light valve reflecting surface (inside a housing), the turning mirror and
auxiliary mirror constitute an image plane conjugate to the light valve to keep the imaging conditions correct within the cell. With this configuration, only every other bounce is controllable by polarization switching, i.e. when the bounce occurs on the light valve.

A focused vertical slit of light is horizontally translated to align with the corresponding bounce position to serve as write beam. Since there is only one row of bounce positions in this demonstration, a vertical slit is sufficient rather than a spot. Use of a vertical slit eliminates the need for vertical alignment of the control light.

In this concept demonstration, each time-delay output appeared at a different image location, requiring movement of the detector. The output lens is adjusted to focus onto the detector the particular beam direction exiting the cell from the light valve plane. The detector and lens must be moved to intercept the beam at a different angle from each different time delay position.

In operation, the horizontally polarized input beam is chopped into 250 nsec pulses when passed through the Pockel cell. Part of the beam is used to trigger the DSO. The main part of the beam then reflects off the turning mirror (bounce 0), passes through the cell lens and strikes spherical mirror B. The beam then reflects back through the cell lens, passing through the PBS and this time striking the light valve (bounce 1). The region of the light valve at bounce 1 is set to keep the polarization horizontal so the beam passes back through the PBS and the cell lens and then strikes spherical mirror C. Spherical mirror C then reflects the beam back through the cell to the auxiliary mirror (bounce 2), which is in the same plane as the input turning mirror. At bounce 2, the beam polarization does not change when reflected off the auxiliary mirror and the beam returns to spherical mirror B, then back through the
Figure 3.2: The data for several different time-delays are shown in Figure 3.2. Several time delay cell output traces are overlaid on channel one. The trigger trace is shown on channel four.

cell to bounce 3 on the light valve. The bounce 3 position on the light valve is set to change the polarization by the control light coming through the slit on the write-side of the light valve. After the polarization is changed the beam is then ejected from the cell towards the output lens.

The data for several different time-delays are shown in Figure 3.2. Here we see plots of intensity vs. time. Four traces of time delay cell output are superimposed by the scope memory on channel one (top). A single trigger trace is shown on channel four (bottom). Inverting amplifiers were used so the pulses goes negative. All we see here is the leading edge of the 250 nsec-long inverted pulses.
The type I demonstration provides direct evidence of time delay and optical switching capabilities. Figure 3.2 shows time delay increments of 0.7.2.14.0. and 21.3 nsec. The average time delay is 7.1 nsec, which agrees with the measured cell size delay of 6.8 nsec within the estimated measurement error of $\pm 1.2$ nsec. These time delays clearly demonstrate the optical switching capability produced by the spatial light modulator in a basic multiple reflection cell, which is the basis for the compound-cell TTD device.

3.2.2 Type III Unit Cell Demonstration of Time Delay

In this section, we use one half of a type III compound cell to demonstrate time delay. For this demonstration, the apparatus is first described. Next, data are presented. This includes demonstration of the time delay produced with the unit cell of the type III compound cell apparatus using short rise-time laser pulses. At least sixteen increments of time delay are available on three independent laser beam inputs. This time delay output translates into producing 4 bits of radar time delay resolution on three independent channels that could be used for steering a phased array antenna in transmit mode. The type III unit cell apparatus is also used for power loss and crosstalk measurements, which are described later in Sections 3.3 and 3.4.

Unit Cell Laboratory Apparatus

Here we overview the type III demonstration apparatus used for the time delay demonstration. Complete information on the total configuration and alignment are given in Appendix B. This detailed appendix is provided for the type III apparatus because it represents a more optimal time delay approach than the type I and is used for the majority of measurements in this dissertation. The single type III unit cell is
Figure 3.3: The type III demonstration device consists of one of the two unit cells making up the type III compound cell, along with input and output optics. Drawing not to scale.

A natural starting place for the research of this device since not all of the performance parameters can be obtained from the output of the complete compound-cell device.

The apparatus is shown in Figure 3.3. There we see in the center the optical switch made up of the liquid crystal light valve (LCLV) and 532 nm polarizing beam splitter (PBS) at left center. At right of center are the spherical mirrors and separating them from the PBS is cell lens I. This uses the opposing spot configuration so that the input and output turning mirrors are one above the other next to the LCLV. Only the top
input mirror is seen. At the bottom of the figure, there is an input field lens to the left of the input aperture mask. This causes the three beams to be superimposed on the spherical mirrors. The input folding mirror and the input/output (IO) imaging lens, which images the aperture mask onto the turning mirror, complete the input optics portion. The output optics re-uses the IO imaging lens and adds two additional folding mirrors to re-direct the output beams onto a fast-response detector. The input light passes the first folding mirror while light from the output turning mirror is reflected off it.

The trigger detector (not shown) is a Thorlabs Det 020 and the layout is such that it requires coaxial cable routing to the DSO several times the length of that to the signal detector. The time delay signal detector is a Thorlabs Det 010/FDS010.

The two laser sources, not shown, used in the type III time delay demonstration are a pulsed Q-switched frequency-doubled Nd:YAG laser, and bore-sighted with it a 3 mW continuous wave (CW) argon ion laser. Both laser beams are configured for polarization parallel to the table (which will hereafter be referred to as horizontally polarized). The CW argon beam is used for co-alignment of the Nd:YAG. The Q-switched Nd:YAG laser produces a nanosecond rise-time pulse and is used for time delay measurements. The optics for conditioning each of these laser beams are not shown here, but are fully described in Appendix B.

Input laser light from the pulsed Nd:YAG source module enters the apparatus along the bottom of Figure 3.3. The input field lens conditions the light to coincide the separate beams into one spot on the spherical mirror. The light then illuminates the input aperture mask with three circular holes. All three beams from the input aperture mask could be individually time delayed if multiple detectors were available.
Only one fast detector was available for the purposes of this time delay measurement. Therefore, the input spot mask was blocked to pass only one input beam. The single beam exits the aperture mask and propagates beneath the edge of the output folding mirror adjacent to the input mask. The input folding mirror and input imaging lens then focus the incoming beam onto a plane near the input turning mirror and conjugate to the LCLV. The input turning mirror directs the beam into the single unit cell. The beam undergoes multiple reflections and is continually refocussed on the LC plane. The LCLV voltage is set to allow the polarization to stay horizontal, so all the beam energy stays within this unit cell (unit cell I). The number of reflections is controlled by the alignment of the spherical mirrors by setting the number of bounces. The bounces can be adjusted by changing the locations of the centers of curvature of the two spherical mirrors. After the desired number of bounces is made, the beam bounces onto the output turning mirror instead of the LCLV and is directed out of the cell. The output optics direct the beam reflecting off of the output turning mirror and exiting the cell onto the fast-response detector.

**Time Delay Demonstration with the type III Unit Cell**

In this section, the time delay measurements are described and data are presented. We demonstrate time delay with this one unit cell of the type III compound-cell device by changing the number of bounces in the cell before the beam exits. This shows the robustness of the multiple reflection unit cell alignment and the operation of the input and output optics, independent of the time delay bounce number or direction.

The number of bounces in the unit cell is changed by adjusting the position of the center of curvature of spherical mirror C as described under the opposing spot configuration in Chapter 2. The bounces alternate from top to bottom row, but
output can only be obtained from bounces on the bottom row reaching the output turning mirror. If the spot on the input turning mirror is defined to be the zeroth bounce as in Chapter 2, then possible output bounces are the odd bounce numbers, 1, 3, 5, etc. Therefore, time-delay output increments in this configuration increase in multiples of two round-trip times through the cell. For this particular cell, the time delay for two round trips is nominally 7 nsec. All bounce numbers are verified visually.

Figure 3.4 shows traces of DSO scope data of output intensity vs. time. There are two traces in each part: signal on the top and trigger reference on the bottom. Figure 3.4a shows the first possible cell output, where bounce 0 was on the input turning mirror and bounce 1 went to the output turning mirror then out of the cell. The signal pulse on channel 1 is slightly ahead of the trigger pulse on channel 2 due to the difference in coaxial cable length from the two detectors to the digital sampling oscilloscope (DSO). However, on a DSO, the trigger does not need to occur before the sampled events. The gain of the scope in these traces is scaled to fit the two traces on the same axes and to more clearly show the time delay present.

Figure 3.4b shows the time-delay by outputting bounce 3. By moving the centers of curvature of spherical mirrors B and C, bounces 1 and 2 are positioned above each other on the far side of the light valve in the same column, which results in bounce 3 hitting the output turning mirror directly beneath bounce 0. Thus, between bounce 1 output and bounce 3 output, there are two round-trips through the cell, or \approx 7 nsec of delay. The measured delay is 7.34 nsec, measured from half-max of the trigger reference pulse to the half-max of the delayed pulse.
Figure 3.4: a) The minimum or reference time delay through the cell; b) the time delay is 6.5 nsec from bounce 3 to bounce 1; c) the time delay for bounce 5 to bounce 1 is 14.9 nsec; d) the time delay for bounce 11 to bounce 1 is 36.5 nsec; e) the time delay is 67.7 for bounce 19 to bounce 1.
Figure 3.4c shows the time-delay for outputting bounce 5. Once bounce 1 is desirably positioned on the LCLV, only the center of curvature of spherical mirror C needs to be adjusted to create more bounces and align them in columns. Thus bounce 4 is above bounce 1 on the right side of the LCLV; bounce 2 is above bounce 3 in the center of the pattern; and bounce 5 is below bounce 0 and exits on the output turning mirror. The measured delay for bounce 5 with four round trips from the reference pulse is 14.5 nsec.

Figure 3.4d shows the time-delay after outputting bounce 11. Here, the signal trace contains two pulses, the larger one time delayed and the other smaller one coincident with the trigger pulse. As shown in Figure 3.3, the output detector in the actual apparatus faces the input optics module. Therefore, the “leakage” pulse appearing coincident with the trigger pulse is probably due to light pollution on the detector as the pulsed laser fires. This noise is only seen here and in the next plot because the gain has been increased to account for signal loss after so many bounces in the cell. This type of crosstalk would not be present in a working system due to baffling, etc. The signal width varies also. At this high gain setting on the scope, the background noise level is higher and the DSO’s repetitive sampling scheme results in the ragged composite signal seen here. The time delay for bounce 11 with nine round trips from the reference pulse is 36.5 nsec.

Figure 3.4e shows the time-delay after outputting bounce 19. After 19 bounces, the energy coming through the cell is apparently comparable in strength to the scattered light hitting the detector, as the two pulses appear somewhat similar in magnitude. The time delay for bounce 19 with nine round trips from the reference pulse is 65.7 nsec.
To summarize the time delay measurements: a) in Figure 3.4a, we see the minimum or reference time delay through the cell; b) in Figure 3.4b, the time delay is 7.34 nsec from bounce 3 to the first bounce, which represents a time difference for two round trips through the cell and is the minimum detectable increment with this demonstration configuration c) in Figure 3.4c, the time delay to reference for bounce 5 is 14.5 nsec or a multiple of two increments of 7.25 nsec average length d) in Figure 3.4d, the time delay to reference for bounce 11 is 36.5 nsec or a multiple of five increments of 7.3 nsec average length e) in Figure 3.4e, the time delay is 65.7 nsec, corresponding to bounce 19, 18 round trips or nine increments of average 7.3 nsec. This gives an overall average increment time of 7.3 nsec from the four example time delay measurements. These figures clearly show the time delay inherent in the device. Switching pathlengths produces time delay and this is the basic principle of the design.

These time delay measurements with the type III apparatus show the robustness of the multiple reflection unit cell alignment and the operation of the input and output optics, independent of the time delay bounce number or direction.

Finally, we demonstrate the capability of providing independent output control. With our one control spot, we can individually activate any one spot in the matrix of spots on the LCLV surface. As an illustration, two cases of spots being activated and documented in photographs. The configuration for photography is shown in Figure 3.5.

The photography configuration is similar to that for time delay except that the image spots on the LCLV read-side are imaged out the reflected side of the cube onto a ground glass screen which is photographed. Because the PBS is designed for the
Figure 3.5: The configuration for photographing activated LCLV pixels demonstrating independent control. Drawing not to scale.
532 nm wavelength, light at 514 nm partially “leaks” through on each bounce so that multiple spots on each row can be visualized for demonstration purposes. We see in the center of Figure 3.6 the optical switch made up of the liquid crystal light valve (LCLV) and 532 nm polarizing beam splitter (PBS) at left center. At right of center are the spherical mirrors and separating them from the PBS is cell lens I. Only the top input mirror is seen. At the bottom of the figure, there is an input field lens to the left of the input aperture mask. The input folding mirror and the input/output (IO) imaging lens complete the input optics portion. A lens that images the LCLV surface is seen near the top of the figure, on the reflecting side of the PBS cube. A nearby flat mirror directs the outgoing light onto a ground glass viewing screen. The camera is mounted on the opposite side of the screen.

Input laser light from the Argon source module enters the apparatus along the bottom of Figure 3.5. The laser light illuminates the input aperture mask with three circular holes. Passing throught the input field lens conditions the light to coincide the separate beams into one spot on the spherical mirror. The three beams exit the aperture mask and propagate beneath the edge of the output folding mirror adjacent to the input mask. The input folding mirror and input imaging lens then focus the incoming beams onto a plane near the input turning mirror and conjugate to the LCLV. The input turning mirror directs the beams into the single unit cell. The beams undergo multiple reflections and are continually refocussed on the LC plane. The LCLV voltage is set to allow Nd:YAG polarization to stay horizontal, so only part of the Argon power stays within the unit cell on each bounce. The rest of the power is reflected out by the PBS cube so that a dim image of the spots on the LCLV for the three input beams is focussed onto the ground glass viewing screen. The number
of reflections is controlled by the alignment of the spherical mirrors. The control spot on the write-side of the LCLV is aligned with one particular read-side spot to activate that spot and switch most of the Argon power out of the cell for that one spot. This activated spot appears much brighter on the ground glass screen, clearly indicating which spot is being activated at the control position. Two different spots are activated and photographed to show independent control on the matrix of spots generated on the LCLV by the three input beams.

In Figure 3.6a, the middle row has spot 3 activated so that it appears bright with most of the beam energy being turned out through the PBS. In Figure 3.4b, spot 2 is activated. For the compound cell, the bright activated spot would represent the point at which the beam would be turned into the other unit cell. This simple sequential demonstration with the optically driven LCLV demonstrates that independent control of the complete spot pattern in the compound cell TTD design is possible.

In this time delay section, we have demonstrated all aspects of the time delay capabilities of the full type III compound-cell TTD design. We first showed the polarization switching capabilities with the type I apparatus. Polarization switching is needed to switch the beam path between the two unit cells of the type III compound-cell device. We next showed the robustness of the type III design, where all beams from the cell exit in the same location and direction independent of the associated time delay or number of bounces. Lastly, independent switching control for the matrix of spots was shown. The combination of these three separate demonstrations indicates that the type III compound-cell TTD design is sound.

As a sidelight to the time delay, we also demonstrated compound-cell input/output and control apparatus that can next be used in a radar beam steering demonstration.
Figure 3.6: To demonstrate independent spot control qualitatively, two different spots were activated sequentially. In (a), top photo, bounce 5 on row 2 is activated. In (b), bottom photo, bounce 3 on row 2 is activated. Photos not to scale.
in the future. With these demonstrations, we have shown that the compound-cell device can produce time delay suitable for a phased array antenna.

In the remaining sections, we present data and discuss key parameters measured for performance modeling of the full type III compound-cell device. These are round trip power loss, component power loss, polarization-related crosstalk and pixel-related crosstalk. In the next section, we start with beam power loss in traversing the unit cell.

3.3 Power Loss Budget

The power loss in the system will determine the quantity of time delays possible with this system. The useable power level of the signal has a finite lower limit because of detector sensitivity and ability to demodulate the signal back into microwave frequencies. This lower limit combined with the power loss per cell trip will determine the maximum possible number of bounces off the spatial light modulator. The number of trips then determines the maximum time delay possible, as outlined in Chapter 2. Minimizing power loss of the cell will optimize the design.

In this section, we first examine round trip power loss at the output of the type III unit cell in Section 3.3.1. Next, in Section 3.3.2, individual component losses are measured. Finally, in Section 3.3.3, we compare all the power loss factors concurrently.

3.3.1 Round Trip Power Loss

To examine signal power loss, the same type III unit cell configuration was used as for the previous section on time delay, as shown in Figure 3.7. In the figure, we see in the input/output (I/O) optics, and the unit cell composed of the liquid crystal light valve, PBS cube, cell lens and spherical mirrors.
Figure 3.7: The configuration used to measure round trip power loss in the type III unit cell demonstration device. Drawing not to scale.
The output was monitored by a large, relatively slow detector instead of the smaller, faster detector used for pulsed laser time delay measurements. The one centimeter diameter detector allowed the capture of the total output energy with minimal alignment problems. The identical measurement was made for time delay, except that the detector was used to determine relative pulse power (peak height), normalized to laser source power (see Appendix B). Absolute power measurements are not required, since we are interested in power ratios (e.g. reflectance, etc.). However, for meaningful ratios, the laser power measured must fall within the detector's linear range. The detector and power reducing optical density (OD) filters were analyzed for linearity and power reduction, respectively, at the working wavelength of 532 nm. Measuring the detector linearity and calibrating the filters are described in detail in Appendix C.

Power data was taken from the odd outputs for 1 to 13 bounces on the LCLV, which was the maximum extent of the linear range obtainable with the combined detector and calibrated neutral density (ND) filters.

The power values are shown plotted as log power vs. round trip number in Figure 3.8. This plot shows the behavior is exponential such that $P_{out} = P_{in} \exp(n/\tau)$, where $n$ is the bounce number and $\tau = -0.35$. For a single round-trip, the transmission factor is 0.7, that is, the power loss is 30%. A single round trip is composed of transmission through the cell lens and PBS cube twice, reflection off of the LCLV and the dielectric spherical mirror once each. In the next few sections, the individual components are examined for reflectance or transmittance, as appropriate, with the intent to isolate the causes of this relatively high loss.
Figure 3.8: The round-trip power values were plotted as log power vs. round trip number. A reasonable fit to the curve is an exponential factor of 0.7 for transmission.
3.3.2 Component Losses

In this section, reflectance (‘R’) or transmittance (‘T’) are measured for the components of the unit cell: the cell lens, the spherical mirrors, the polarizing beam splitter cube and the liquid crystal light valve. Error determination for all measurements is discussed in Appendix C.

There are two contributions to power loss from the signal channel: non-interfering and interfering. Power loss out of the signal channel lowers the signal-to-noise-ratio (SNR). Component losses, defined as 1-R for mirrors and 1-T for lens, can be due to absorption and diffuse scattering, which are non-interfering losses. Interfering losses are non-absorptive losses which deflect the light to other paths. They are more detrimental to SNR because they contribute to the noise component (crosstalk) while also reducing the signal component. Interfering crosstalk-type losses will be examined in later sections.

Cell Lens Transmittance

Energy that is not transmitted through the cell lens constitutes loss due to this component. The cell lens is a broadband antireflection-coated, 51mm diameter, BK-7 bi-convex lens from Newport. Cell lens transmittance was measured near normal incidence using the configuration shown in Figure 3.9. Here we see that light from the Nd:YAG source is input through ND filters adjusted to keep the beam energy within the detector’s linear range. A small metallic folding mirror is used to turn the beam 90 degrees toward the lens so that the output detector does not look back towards the laser source. The detector is placed on the optic axis behind the lens. Power is
Figure 3.9: The configuration used to measure cell lens transmittance in the type III unit cell demonstration device. Drawing not to scale.
Figure 3.10: The configuration to measure reflectance of the spherical mirrors near normal incidence. Drawing not to scale.

measured with and without the lens in front of the detector, using peak pulse voltage values from the digital sampling oscilloscope (DSO) as described in Appendix C.

The cell lens transmittance measured was 0.994, indicating 0.006 loss factor. The coating is specified to be less than 1/2% reflective, and the measured value of transmittance agrees with this within the measurement error of 7%, as discussed in Appendix C.
Spherical Mirror Reflectance

Energy that is not specularly reflected from the spherical mirrors constitutes loss for this component. The two dielectric spherical mirrors used are glass blanks custom coated for 546 nm at normal incidence. The reflectance of the spherical mirrors was measured at near normal incidence, using the configuration shown in Figure 3.10. Here, we see that light from the Nd:YAG source is input through ND filters adjusted to keep the beam energy within the detector's linear range. A small metallic mirror is used to turn the beam toward the mirror under test. The detector is placed behind the small metallic folding mirror to measure the beam power as it is reflected back along side the folding mirror at near normal incidence. To obtain input beam power, the detector was also moved to measure the beam power in front of the spherical mirror.

The measured reflectance of dielectric spherical mirrors B and C was 0.987 and 0.985. We do not have manufacturer's specifications for these particular coatings. Comparable modern coatings (for example, Melles Griot) have an angular acceptance of ±15 degrees and ±50 nm spectral width for reflectance of 0.993. Measurements of both spherical mirrors indicate performance to modern standards within the measurement error of 7%, as described in Appendix C.

Polarizing Beam Splitter (PBS) Transmittance

In this section, we examine power losses in the PBS cube that are due to diffuse scattering, absorption and surface Fresnel reflections. Diffuse scattering, absorption, and surface reflections contribute to signal power loss, while incomplete polarization separation contributes to both signal power loss and increased crosstalk. Thus, the
PBS cube is a critical factor in the overall device signal-to-noise ratio (SNR). The cube was a CVI BK-7 polarizing beam splitter cube designed for the 532nm laser-line with broadband AR coatings.

To measure the PBS cube losses, the ratio of total output to total input is calculated. Input and output measurements were done using the configuration in Figure 3.11. There we see the Nd:YAG laser beam, horizontally polarized. The laser beam is aimed onto the input aperture with an adjustable iris sized to prevent overflow of the beam spot on the turning mirror. The beam then proceeds to the input turning mirror, which sends the beam through the PBS cube and onto the large-area detector, whose area is much larger than the beam. A portion of the beam is also reflected off the PBS. The detector is placed first on the transmitting and then on the reflecting side and the outputs are measured as shown in Figure 3.11 using the DSO as detailed in Appendix C. Finally, the cube is removed and the beam coming from the turning mirror is measured for the total input power value.

The total throughput measured from both sides of the cube for horizontal polarization input is ≈97%. Total throughput is composed of 96.4% on the transmitted side and 0.4% on the reflected side. The cube is specified by the manufacturer to have negligible absorption loss, thus assuming less than 0.5% loss on each coated surface reflection, we expect a total throughput of 99%. The measured value is greater than the specification, but the two are consistent within the estimated measurement error of 7%, as described in Appendix C.
Figure 3.11: To determine PBS cube losses, measurements were taken to allow calculation of the ratio of total output to total input. Drawing not to scale.
Light Valve Reflectance

The objective of this test was to determine possible sources of loss in the LCLV read-side. This was done by measuring reflectance of several reflections on the read-side at several angles and at two different wavelengths. The first set of measurements were done with Argon to gain an initial assessment of power loss in the light valve. The second set of measurements was configured to closely match the TTD device demonstration using the Nd:YAG laser. The main surfaces of concern for power loss are the front surface of the broadband AR-coated plate, the index-matching mineral oil layer between the front AR-coated plate and the glass LCLV cover, and the liquid crystal/dielectric mirror layer behind the liquid crystal layer. These layers were described in Appendix A.

Figure 3.12 shows the measurement apparatus for the Argon laser measurements. Further details on apparatus used is included in Table 3.1. The input Argon beam, horizontally polarized, is passed through a narrow slit to form a very small spot to aid in lateral separation of Fresnel reflections. The conditioned beam is then folded by Mirrors M1, M2 and M3, and passed through a lens and a scatter-blocking aperture before being reflected off the LCLV. The beam is at an angle to the surface to separate light reflected from various surfaces. Undesired reflections are blocked out so that only the desired reflection reached the detector. Total reflected power from all reflections is also measured to isolate light lost due to Fresnel reflections from loss due to scattering and absorption. The detector is then moved to intercept the beam before the LCLV to measure total input power for ratio calculation.
Folding Mirrors: M1, M2 and M3

Figure 3.12: The configuration used to measure reflectance of the liquid crystal light valve read-side with the Argon laser. Drawing not to scale.
Table 3.1: LCLV reflectivities measured with Argon, 514 nm.

The brightest Fresnel reflections were from the front coated glass plate and were very small, 0.01%, measured at 13 degrees from normal. The total reflectance from the entire light valve (including all reflected power) was 62% at 13 degrees.

Only total beam reflectance is measured next, neglecting Fresnel losses which are seen to be small, above. The second set of measurements is taken to more closely match the conditions of TTD cell operation. The pulsed Nd:YAG laser beam is used, bias voltage is applied to the LCLV as described in Appendix A and measurements are taken at smaller incident angles.

Figure 3.13 shows the measurement apparatus for the Nd:YAG laser measurements. Details on apparatus given in Table 3.1 still apply. The input Nd:YAG beam is horizontally polarized. The incoming beam is folded by Mirrors M1, M2, and M3,
Figure 3.13: The configuration used to measure reflectance of the liquid crystal light valve read-side with the Nd:YAG laser. Drawing not to scale.
Table 3.2: LCLV reflectances measured with Nd:YAG, 532 nm.

<table>
<thead>
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<th>Percent Reflectance</th>
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<td>Total Reflectance at 13 degrees</td>
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</tr>
<tr>
<td>Total Reflectance at 9 degrees</td>
<td>68</td>
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<td>Total Reflectance at 5 degrees</td>
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</tbody>
</table>

and passes through a scatter blocking iris before being reflected off the LCLV. The iris is \( \approx 3 - 4 \) mm in diameter. Different angles are selected by moving the position of Mirror M3 and redirecting M2. Total reflected power is measured by a detector in the beam reflecting off the LCLV. The detector is also moved to intercept the beam before the LCLV to measure total input power for ratio calculation.

The measured values are shown in Table 3.2. Measured total reflectance is 59% at 13 degrees, 68% at 9 degrees, and 74% at 5 degrees, within 7% error as discussed in Appendix C. Reducing the angle of incidence appears to reduce the loss. Thus, at small angles, there is <26% loss in power reflecting from the light valve due to absorption and diffuse scattering.

This high loss value prompted us to investigate another type of liquid crystal device to suggest what role the liquid crystals play in scattering and absorption. We measured transmittance of a twisted nematic LC clock display with the configuration shown in Figure 3.14. Beam power was measured with and without the LC display in front of the detector. Transmission loss was measured to be 9%, which could be mostly attributed to Fresnel reflections off the uncoated front and back surfaces.
Figure 3.14: The configuration used to measure reflectance of the twisted nematic liquid crystal clock display. Drawing not to scale.
This is a significantly lower value than the 26% loss for the reflective LCLV measured above.

To hypothesize the reason for the difference in loss between the two LC devices, one might first consider the differences between them. The LCLV is reflective, and has an embedded broadband dielectric reflecting layer. A poor reflecting surface could easily cause large losses compared to the transmissive display device which has no such layer. In addition, our LCLV has parallel nematic liquid crystal configuration and the display has a twisted nematic liquid crystal configuration. Further research into the actual loss mechanisms at work in the light valve was beyond the scope of this work, but would be important for future developmental work.

3.3.3 Comparison Discussion

At this point, we stop to consider the preceding measurements in light of round trip power loss. Exclusive of the light valve, the rest of the components, the cell lens, spherical mirrors and PBS cube, represent a round trip transmission of

\[
0.964^2(PBS \text{ twice}) \times 0.994^2(\text{cell lens twice}) \times 0.986(\text{spherical mirror}) = 0.905
\]

(3.1)

Inclusion of the light valve brings the round trip transmission to 0.90*0.74=0.66% or a round trip loss of 34%. This value derived from individual component measurements is consistent with the 30% loss derived from the cell output after multiple round trips, Section 3.3.1, within measurement error. Having investigated non-crosstalk power loss in the time delay signal, we now turn to examining power lost to other time delay channels within the system which creates crosstalk.
3.4 Crosstalk

Power diverted from one signal to any other results in even greater reduction in SNR than signal loss out of the system, since crosstalk-noise power is increased concurrently. There are two sources of crosstalk possible in the compound-cell TTD device. The first, polarization crosstalk, is investigated in this section. The second, pixel crosstalk, a liquid crystal effect, is examined in the next section.

3.4.1 Polarization Crosstalk

We define polarization crosstalk as unwanted changes in path direction caused by imperfect operation of components that deal with polarization. Unwanted power following an alternate path through the system can result in a false signal at incorrect times. This results in power at multiple time delays at a single channel output. The two main components involved in polarization crosstalk are the PBS cube and the LCLV.

PBS Cube Polarization Separation

In this section, we examine the ability of the cube to separate light of two polarizations. The cube and the LCLV comprise the optical switch needed to change time delay for one channel in the compound-cell TTD device. In our TTD device, one channel represents time delays on one input beam. Ideally, in the unit cell composed of spherical mirrors B and C, there is only light with p-polarization which is always transmitted by the cube. In reality, some p-polarization power exits the reflected side of the cube. Likewise, in the unit cell with mirrors E and F, ideally only light with s-polarization is reflected by the cube. If any power leaks into the wrong unit cell on
any channel, that leakage power will undergo incorrect time delay and appear at an undesirable time on that channel's output.

In this section, we measure the ability of the cube to maintain separation between the two polarization components on a single pass through the cube, using the Nd:YAG laser. We do this in three steps: a) measure the cube's angular response, b) set the cube at the optimal angle, and c) measure polarization separation ratios with the cube.

**Polarizing Beam Splitter Cube Angular Response**

Before polarization measurements, the PBS cube transmittance and reflectance performance is optimized for compound-cell time delay by angle-tuning the cube. The PBS cube used in the demonstration is AR-coated, 1.5 inches on a side and designed for the 532 nm laser line. The cube has the following nominal 90-degree manufacturer's specifications for transmittance and reflectance values for the s- and p-polarization components: $T_p=0.95$, $R_s=0.999$, $R_p=0.050$, $T_s=0.001$. These values show that polarization separation is poor for the p-component, that is, the cube still reflects some light when all light should be transmitted. For use in the compound-cell TTD device, we want to angle tune the cube, i.e. adjust the angle, to increase $T_p$ to keep more p-polarization power in the transmitting side. This would result in degrading the s-polarization separation. The goal was also to more evenly balance the power loss ($R_p$ or $T_s$) in the transmitting and reflecting sides, respectively. Equal power loss in both polarizations would make the SNR the same for all time-delay paths.

The PBS cube is angle tuned using the set-up shown in Figure 3.15. The cube, on its platform, is first mounted on a rotating post-holder-base, graduated in degrees.
Figure 3.15: Configuration for measuring PBS cube output over a range of angles to document angle tuning. Drawing not to scale.
Next, it is visually aligned to be parallel to the LCLV front face. The figure shows the horizontally p-polarized Nd:YAG input beam coming off the turning mirror and passing through the PBS cube. The detector is placed in the beam on the reflected side of the cube. Approximately 5% of the horizontally-polarized beam is reflected out the “s-side” of the cell and onto the detector. To map the angular behavior in this region, the reflected output is measured over a range of angles. The cube platform was rotated in 0.5 degree increments, and at each setting, the detector is repositioned as the cube is rotated and power is measured. The cube’s reflected output was measured to show the position of the minimum and the width of the angular behavior curve.

Figure 3.16 shows a plot of the cube data with degree markings centered on the minimum. The data in Figure 3.16 is plotted relative to the maximum power value measured. Here we see that the cube performance curve has uniform response in a region with angular width of only about one-half degree. The cube’s angular response affects the optimal angular width for ray paths within the cell.

To put the cube’s angular response width into perspective, on the current cell, the angular spread is ≈3 degrees, between the beams going from spherical mirror B to C (an inch apart) by reflecting off the center of the LCLV (approximately 500 mm away). The cell’s angular beam spread is therefore six times larger than the cube’s angular bandwidth. Thus, PBS cube performance in the TTD cell will vary depending on the particular beam path. A practical cell design would therefore require minimizing the angular beam spread in the cell. In addition, this stock cube was not optimized for this particular use. Custom beamsplitters could be designed that have wider angular bandwidth [61].

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Figure 3.16: Plot of cube tuning data with degree markings centered on the minimum.
**Setting Optimal Angle for PBS Cube**

Once the cube's angular behavior was measured, the cube was locked into place at the optimal position, which was that of minimum reflected output. To accomplish this, a horizontally polarized Nd:YAG beam is sent through the cube and the reflected output is viewed on a white screen. The cube is rotated until the reflected output was at a minimum. The cube's rotating base is locked into place with the locking set screw.

The optimization of cube performance preceded all other measurements. The cube remained fixed throughout the rest of the project. The cube was not moved again until the very last intra-cell measurement, which measured the cube losses due to scattering, absorption and surface reflections, described previously in Section 3.3.2. The final angle from the cube to the optical axis was documented by triangulation with the front surface Fresnel reflection using a portable HeNe laser. The cube face was turned at 88.4 degrees from the optical axis toward the spherical mirrors.

**Polarization Separation by the PBS Cube**

After angle tuning, the transmittance (T) and reflectance (R) values for the cube were determined. Figure 3.17 shows the measurement configuration using the Nd:YAG laser. Appendix B describes the Nd:YAG laser source configuration. The figure shows two quarter-wave plates and a Glan-Laser polarizer that are used to rotate and purify the beam for vertical polarization. The typical beam aperture iris size used is 3-4 mm. The polarized laser beam (p- or s-polarized) transits the PBS cube after leaving the input turning mirror. The detector is aligned alternately on either output of the cube to measure transmitted and reflected outputs in both polarizations.
Figure 3.17: Configuration for measuring transmittance and reflectance values for the polarizing beamsplitter cube. Drawing not to scale.
Given the transmitted and reflected output power for each polarization, the transmittance (T) and reflectance (R) were calculated by incorporating measured losses to allow comparison with manufacturer’s specifications. Thus, we use \( T_s + R_s = .97 \) and \( T_p + R_p = .97 \) which gives the following values for the cube: \( T_p = 0.967, R_s = .969, R_p = 0.003, T_s = 0.001 \). Estimated measurement error is \( \approx 7\% \), as described in Appendix C. The transmittance and reflectance values show that angle tuning has reduced the p-polarization transmittance loss and more evenly balanced the output values for each side of the cube, as desired.

These values can also be used to determine contrast ratio (CR) as follows: transmitted \( CR = \frac{T_p}{T_s} = 967 \), reflected \( CR = \frac{R_s}{R_p} = 323 \). Nominal contrast ratios (CRs) from the manufacturer’s specifications would be 950 and 20 for transmitted and reflected CRs, respectively, without angle tuning. The contrast ratios after angle tuning show that polarization splitting capabilities are still three times greater on the transmitted side, but greatly improved over the un-tuned PBS performance.

As mentioned previously, this stock cube was not optimal for this particular use, and custom designing of the thin-film beam splitting surface for even polarization splitting could be pursued in further development efforts. Nevertheless, even for this particular cube, these values translate into a maximum single pass crosstalk level of \( 20\log(CR) = -50dB\), which is quite acceptable.

**Polarization Separation by the LCLV with the PBS Cube**

In the previous section, we described the polarization splitting action of the PBS cube alone. In this section, we now describe the combined action of the PBS cube and the LCLV, which together control the actual switching operation in the compound-cell TTD device.
Figure 3.18: Configuration to measure the combined cube/LCLV transmittance and reflectance. Drawing not to scale.
Figure 3.18 shows the cube/LCLV transmittance and reflectance measurement configuration. Important components are the LCLV and spherical mirrors in the center left of the figure. The p-polarized light beam comes in through the cell input lens at center left, is reflected off the turning mirror, TM, and goes through the cube. The beam is reflected off spherical mirror B, goes back through the cube again and onto the LCLV. The light is then converted to s-polarization or remains p-polarized, depending on the bias voltage or control light intensity on the LCLV. The detector is positioned to measure power output consecutively for transmitted and reflected light, and for the cases when the LCLV is set to return s- and p-polarization respectively.

Having measured the transmitted and reflected output power for each polarization, the transmittance (T) and reflectance (R) values for the LCLV/cube combination are calculated by incorporating measured losses, as before. The following values were obtained for the cube: \( T_p = 0.958, R_s = 0.842, R_p = 0.012, T_s = 0.128 \). Estimated measurement error is \( \approx 7\% \), as described in Appendix C. These values were also used to determine the contrast ratio (CR) as before: transmitted \( CR = \frac{T_p}{R_s} = 7 \), reflected \( CR = \frac{R_s}{R_p} = 70 \). We see that the LCLV with the cube degrades the polarization splitting performance as compared to the cube alone. Single pass crosstalk based on the LCLV/cube performance is estimated to be \(-20\log(7)=-17\) dB, well down from the \(-50\) dB of the cube alone. This implies the LCLV is the source of the degradation due to incompletely switching light from one linear polarization to the other, instead returning elliptically polarized light.

The polarization separation results show that the current LCLV is not producing complete polarization switching. Liquid crystal orientation and voltage bias level are optimized as described in Appendix A. Further optimization has not been pursued.
since this light valve was chosen for availability not suitability. We attribute the observed performance to the light beams not transiting the liquid crystal layer in a direction normal to the substrate. Better performance should be possible with a modern twisted nematic light valve designed for greater contrast ratios over wider angles. Based on current technology contrast ratio values of 100, polarization-related single pass crosstalk values should be at \(-20 \log(\text{detected contrast ratio}) = -40 \text{ dB}\), at a minimum.

In this section, we have looked at the polarization separation of the PBS cube by itself as well as in combination with the LCLV. The results show that the current LCLV is the limiting factor.

### 3.4.2 Pixel-Related Crosstalk

This section will describe the second type of inter-beam interaction, that which occurs between pixels on the LCLV. Here, a pixel is taken to mean a focussed time-delay spot on the "read" side of the LCLV and its associated uniform liquid crystal response region. The LC response region results from light-activation by the controlling spot on the "write" side and provides a region where light passing through will have its polarization completely switched, i.e. horizontal is changed into vertical and vice versa. Since each time-delay spot must be switched independently, the energy in each spot must be kept separated or else incorrect switching of part of the signal energy will occur.

To assure that all the time delay power in one spot is completely within the full polarization-switching response region, the control spot width on the write-side is purposely designed to be larger than the input time-delay spot on the read side. This
is to allow for a region outside the time delay spot area in which the liquid crystal’s orientation can relax. In this LC relaxation zone, incomplete polarization switching will occur. Thus, a control spot and its resulting LC relaxation zone constitute a pixel for the optically controlled light valve used. Electronically controlled light valves are physically pixellated and therefore, have different LC response spatial functions.

Three measurements are taken with the demonstration device to observe the effects of spot size and separation on pixel-related crosstalk: a) the read spot size used is documented with a spatial profile, b) the liquid crystal response to the write spot is documented with a spatial profile, and c) the spatial interaction of the two spots is measured over a range of separation distances by profiling a convolution of the two spots on the read-side.

Read Spot Spatial Profile

The time delay beam spot size on the read-side is measured using the apparatus shown in Figure 3.19. There we see an Argon beam being directed into the center of the cell. Lens 11 images the LCLV liquid crystal layer onto a nominal 10 μm aperture in front of a photomultiplier tube (PMT). The PMT and attached pinhole are scanned across the magnified image of the LC layer spot. The resulting data is then calibrated to the size of the time delay spot on the LC layer with the measured de-magnification factor. This calibrated plot is shown in Figure 3.20. The spot width of the time delay spot at the liquid crystal plane was 136 μm at FWHM, 194 μm wide at the 10% level, and 227 μm wide at the background noise level.

The input aperture mask holes were nominally 400 μm in diameter and 1000 μm center in separation. The IO imaging lens is chosen and positioned to provide the desired demagnification (≈2X) from the input aperture mask down to the desired
Figure 3.19: Configuration to measure the time delay focussed spot size on the liquid crystal light valve (LCLV) read-side. Drawing not to scale.
Figure 3.20: Plot of the PMT scan of the time delay focussed spot size, scaled to the size on the LCLV plane.
input image size, 200 \( \mu m \), at the LC conjugate plane. The measured mask hole sizes were an average of 396 \( \mu m \) in diameter. The measured demagnification was 0.53, giving a predicted spot size on the LCLV of 210 \( \mu m \). The measured time delay spot size demagnified to the LC plane size was 136 \( \mu m \) at FWHM. This discrepancy is attributed to difficulty in measuring the input aperture mask size, error in locating the PMT in the output image plane, and overall measurement error.

One also notices the appearance of a diffraction pattern in the spot pattern. This is expected and can also be examined using the Lens Matrix Diffraction Integral (LMDI) equations from Section 2.2. Using measured distances and optical component values, the system matrix is calculated from the input aperture mask to the PMT measurement plane and the resultant pattern is shown in Figure 3.21, alongside the raw spatial data taken at the PMT plane. The pattern in the figure was a reasonable match with the raw data; only the outer sidelobes do not appear in the measured data. The calculated pattern has a Fresnel number of 2.2. Considering that many patterns are possible within the measurement error since the integral is very sensitive, the LMDI model for the fields in the cell agrees with measurements.

**Write Spot Response Spatial Profile**

This section describes the measurement of the size of the LCLV write-side response to the read-side control spot. A control spot is imaged onto the fiber optics of the write-side faceplate. The faceplate transfers the optical image to the photoconductive layer, as described in Appendix A. The liquid crystal layer responds to the voltage created by the photoconductive layer. This response region, in which the polarization switching varies from full at the center to partial at the tails, is visible on the “read” side of the LCLV through an analyzer. The entire affected LCLV area must be
Figure 3.21: Plot of (a) the calculated fields of the read-side spot imaged onto the PMT plane; (b) raw data from the PMT scan of the read-side spot, and (c) direct comparison of the two curves.
illuminated so that the total response can be mapped on the “read” side for a given optical input.

In Figure 3.22, we see the measurement configuration for the response region of the LCLV to the optical control spot. The LCLV is in the center. In this configuration, the entire read side of the LCLV was illuminated with the light from a Mercury arc lamp, through a sheet polarizer, color filter, and turning mirror.

On the write-side of the LCLV, light from a projector shines on a mask with an aperture in it. The mask position is controlled by an actuator driving a translation stage. The aperture is imaged onto the LCLV with a fast lens. The light intensity from the projector is controlled by a variac and set to obtain maximum contrast ratio in the read-side output plane.

The control mask hole was nominally designed to be 900 μm in diameter. The fast demagnifying lens provided a nominal demagnification of ≈3X from the control mask down to the desired read spot size, 300 μm, at the LCLV read side fiberoptic faceplate. After construction, the mask hole size was measured at 980 μm in diameter. The lens distances for the focussed control mask provided a measured demagnification value of .27. This leads to a predicted value of 265 μm diameter spot on the LCLV write side faceplate.

The LC plane was imaged using a lens at a high magnification of ≈10X onto the photomultiplier, which has a nominal 100 μm pinhole in front of it. The pinhole size was chosen to be able to get a response from the PMT with the low light levels from the arc lamp. The PMT is fixed at the location of the image plane.

The image of the read-side response region was moved across the photomultiplier by moving the write-side control spot using the control spot mask actuator. The
Figure 3.22: Configuration to measure the liquid crystal light valve response to the write-side optical control spot. Drawing not to scale.
Figure 3.23: Plot of the PMT scan to measure the liquid crystal light valve (LCLV) read-side response to the write-side control spot, scaled to the size at the LCLV plane.
resulting profile is shown in Figure 3.23, demagnified to the LC plane size for comparison to the time delay spot profile in Figure 3.20. We see in Figure 3.23 that the uniform response region is 235 \( \mu \text{m} \) measured between the 95% points, 331 \( \mu \text{m} \) measured between the 90% points and 541 \( \mu \text{m} \) at FWHM.

To assure that all the time delay power in one spot is completely within the full polarization-switching response region, the control spot width on the write-side is purposely designed to be larger than the input time-delay spot on the read side. This oversizing is to allow for a region for the liquid crystal’s orientation to relax by neighbor interaction. In this LC relaxation zone, incomplete polarization switching will occur. Previous work on these types of light valves showed a relaxation zone width of 25 \( \mu \text{m} \) for a write spot 175 \( \mu \text{m} \) in diameter [62]. Further investigations of point response functions for the particular light valve used were outside the scope of this work. Based on the previous work, we chose as a nominal design a 200 \( \mu \text{m} \) read spot and a 300 \( \mu \text{m} \) write spot, allowing for a 50 \( \mu \text{m} \) wide relaxation ring inside the write spot.

Referring again to Figure 3.23, we see the expected tails of partial response outside the uniform response region, due to interaction between neighboring liquid crystal molecules. These partial-response tail regions constitute a zone where uniform polarization control is not possible and no energy from the time delay spot should fall. In addition, energy from a time delay spot on an adjacent beam that falls into the partial-response zone would also have its polarization incorrectly rotated. These partial-response regions necessarily create a buffer between time-delay spots on the same and different channels, the width of which is dictated by the level of acceptable
interaction, here considered as crosstalk. Any time-delay spot energy in the partial-response zone will result in crosstalk between the desired time-delay and other time delays, on the same or other channels. The size of the necessary buffer zone thus determines minimum spot separation and therefore maximum packing density in the TTD device.

We have now discussed why adjacent time delay spots on the read-side should be separated to the level of crosstalk desired by the width of the LCLV response tail. For this particular light valve and the spot sizes selected, -19dB crosstalk level would require a separation of 165 \( \mu m \) between time-delay spots. This 19 dB crosstalk separation zone is outlined with dotted lines on the partial response tails of the profile in Figure 3.23.

Recall that all the power in one time delay spot should fall completely within the uniform polarization-switched response region. By comparing Figures 3.20 and 3.23, we see that the nominal time delay spot energy is contained within a total width of 227 \( \mu m \) and the uniformly LCLV response region is 235 \( \mu m \). Therefore, for this demonstration, all time delay energy falls within the uniform switching region as desired.

Another design goal, however, is to minimize device size. This would require identically matching the width of the uniform LCLV response with that of the total time-delay spot width. In this device, the LCLV response region was 235 \( \mu m \) when only 227 \( \mu m \) is needed. This discrepancy is partially attributed to variation in beam spot magnification, varying liquid crystal response and measurement error. Further efforts on spot size matching were not pursued at this time since our purpose was proof of concept. Response size matching is a matter of design, and will be dependent on
the input response of the particular optical switch used. Electronically controlled light valves are physically pixellated and therefore, have different LC response functions.

**Response Zone and Read Spot Spatial Interaction**

A demonstration was done to show the effect of the partial-response zone of one pixel on a neighboring one as a function of separation distance. There are three steps in the measurement. The first step is to align a detector to measure imaged output from the first read-side bounce (bounce 1) on the LCLV using the vertically polarized co-aligned Argon beam reflected by the PBS cube. The second step is to switch back to the horizontally polarized Nd:YAG beam so that the beam proceeds through the PBS cube instead of onto the detector. The last step is to scan the write-side control beam across the location of the bounce 1 pixel, producing some output on the detector. This shows pixel crosstalk as a neighboring pixel, set for polarization rotation, moves nearer to and across the bounce 1 pixel, set for no rotation.

In Figure 3.24, we see the measurement configuration for the third step, scanning the control beam across the bounce 1 read pixel. The LCLV is in the center. On the read side, the beam first enters the cell from the turning mirror on the left and proceeds through the PBS cube and cell lens to spherical mirror B. The beam is then reflected back through the cell until it reaches the LCLV at the bounce 1 position. In earlier measurement steps, this pixel is not illuminated on the write-side, so it does not rotate polarization. The beam would then continue back through the cell and be blocked at spherical mirror C to prevent further reflections in the cell.

On the write-side of the LCLV, light from a projector shines on the control spot mask with the aperture in it. The mask position is controlled by an actuator driving a translation stage. The aperture is imaged onto the LCLV with a fast lens. The light
Figure 3.24: Configuration to measure the read-side output from scanning the control spot on the write-side across the location of the bounce 1 spot on the read-side, demonstrating neighboring pixel interaction as a function of separation distance. Drawing not to scale.
intensity from the projector is controlled by a variac and set to obtain maximum polarization rotation in the read-side output plane. Here the control spot on the write-side has been moved to partially illuminate the bounce 1 pixel so that some of the beam is reflected by the PBS cube onto the detector.

On the LCLV read-side, we see that the bounce 1 pixel is imaged through the reflected side of the PBS cube using a lens at high magnification of \( \approx 10\times \) onto the large area detector. This was done in an earlier step of the measurement by using the co-aligned vertically polarized Argon beam for visual alignment. After alignment, the beam is switched to horizontally polarized Nd:YAG. A voltage bias appropriate for no polarization rotation is applied to the light valve. Output beam pulses are measured with the detector and digital sampling oscilloscope as described in Appendix C.

The control spot on the write-side is then swept across the read-side spot location with the actuated translation stage. The effect of the control spot proximity to the bounce 1 spot output through the PBS cube is recorded as a function of position in Figure 3.25. Essentially, this plot constitutes the central horizontal line contour through a two-dimensional convolution of the two spot patterns (the LCLV response to the control spot and the bounce 1 read-side spot). We see the FWHM of the pattern is 635 \( \mu \text{m} \). This is basically consistent with the width of a convolution of the response spot (FWHM 541 \( \mu \text{m} \)) and the read-spot (FWHM 136 \( \mu \text{m} \)). The expected convolution width is approximately equal to the sum of the widths of the two convolved patterns or 677 \( \mu \text{m} \).

In summary, we have demonstrated the LCLV response region shape that determines minimum size and maximum packing density. For size minimization, the goal is to identically match full-scale LCLV response width with the time-delay spot width.
Figure 3.25: Plot to demonstrate neighboring pixel interaction as a function of separation distance.
to the level of crosstalk acceptable. Response size matching is a design parameter
determined by the response size of the particular light valve used. Packing density
is determined by spot size and spot separation. Spot separation is also determined
by the width of the partial-polarization-switching tails on the LCLV response region.
Adjacent time delay spots on the read-side would have to be separated by the width
of the LCLV response tail to the level of crosstalk crosstalk desired. Thus, for this
particular light valve, -19dB crosstalk would require a separation of 165 μm between
time-delay spots.

3.5 Chapter Summary

In this chapter, the main thrust of the dissertation work is presented. The labora­tory experiments addressed two main purposes: 1) to provide proof that the design
will produce suitable time delays with potential for use in phased array antennas and
2) to examine performance parameters that affect design decisions for radar system
integration. We describe experimental configurations and present measurement re­sults. In Section 3.2, we describe the time delay demonstration with the type I device
and with the type III unit cell of the compound-cell device. Together, the data from
these two experiments clearly show that the compound-cell design produces time de­lay suitable for a phased array antenna. Sections 3.3 and 3.4 present data on the
key performance parameters. These are the power loss budget for round trip power
loss and component losses, and the two kinds of crosstalk, polarization-related and
pixel-related. The LCLV proved to be the limiting factor in both power loss and
crosstalk.
CHAPTER 4

PRACTICAL CONSIDERATIONS FOR APPLICATION TO PHASED ARRAYS

4.1 Introduction

In this chapter, having given analytical performance predictions and demonstrated proof of concept, we examine the implications of the demonstration results of Chapter 3 with respect to overall integration of the TTD device with phased array antenna systems. We will discuss design topics at several levels. First, at the system level, we discuss two topics: radar system interface and radar bandwidth supported. Then, at the device level, we discuss input/output methods, alignment sensitivities, and construction options. Finally, at the component level, we examine lens and cube power loss and crosstalk, and a detailed discussion on optimizing various aspects of the liquid crystal light valve.

4.2 Radar System Interface

Most photonic true-time delay systems are designed to be used as optical channels to carry amplitude-modulated light signals to and from the individual transmit/recieve (T/R) modules in the phased array antenna, while providing the proper time delay for transmitting or receiving [54, 55, 29].
In transmit mode, a TTD system generally sits between the low power microwave source and the transmitting amplifiers, allowing use of one source (called single spigot architecture) by providing power division as well as time delay control. This is shown in Figure 4.1a. This configuration has the advantage of requiring only one microwave source and allows remote location of the time delay device from the source or the antenna array itself, if advantageous to the installation.

An alternate transmit mode source configuration is branch-fed from several sources. In such a case, several time delay systems may be need to be co-located with each microwave source. Each microwave source drives multiple electro-optic modulators for multiple light sources or one modulator for a single light source that is then split...
into many input beams. The output light beams of the TTD device are then con­verted by appropriate transducers back into electronic signals to provide time delays suitable for a microwave amplifier.

In receive mode, shown in Figure 4.1, a time delay device would receive light signals from light sources modulated by the T/R modules acting as receivers. These modulated light signals are then time delayed as appropriate for receive beam steering. Here, a modulator is required for each antenna element.

A reciprocal transmit/receive antenna system would require a separate light source modulator for each antenna element. The light modulator would be driven by either the transmitting microwave signal in transmit mode or the received signal from the antenna element in receive mode. Depending on the position of the transmit/receive
switch controlled by the radar system. A possible configuration for a reciprocal system with a type III device is shown in Figure 4.2.

The laboratory device for this work is currently configured to accommodate a single source input supporting several T/R channels in the transmit mode. However, the laboratory device is versatile in that it could be adapted to any radar source or antenna remoting configuration, using additional light sources, modulators and detectors, as just described.

4.3 Radar Bandwidth Supported

True-time delay has its payoff in increasing the useable bandwidth of a phased array antenna system. Ultra-wide bandwidth (UWB) signals are usually produced with one of four methods: quasi-impulse, monocycle, step chirp and noise radar [63]. Impulse radars have pulse widths of 300 psec to several nanoseconds, generated by fast switching devices such as an arc discharge. Monocycle signals are generated by gating a continuous wave (CW) frequency source and can have pulsewidths of a nanosecond. Step chirped systems vary the transmitting frequency in discrete steps with a programmable CW sources and can be slowly pulsed to aid in receiver noise reduction [64]. An example step chirp system at OSU has a range of 2 to 18 GHz [64]. Noise radar signals are generated with bandlimited noise and are time delayed and cross-correlated when received to determine target range. The OSU noise radar has a frequency range of 50-600 MHz [65].

Most of the UWB frequencies just given are within the range of advanced photonic components [66, 58]. Modulation speed for a currently available bulk electro-optic amplitude modulator is 200 MHz. Modulated diode lasers have bandwidths in the
tens of GHz. Detector speed is higher, with a current high speed photodetector bandwidth of 60 GHz. However, these components are independent of the compound cell true-time delay device of this work. Our TTD device can support any optical bandwidth for which photonic modulators and detectors operate. This means that, at the current time, our device could be configured for all UWB radars except some of the fastest impulse radars. There are also some array element limitations.

4.4 Light Input/Output

The present type III laboratory configuration uses an input aperture mask to form several independent input laser beams from one Nd:YAG laser beam spot. The number of input beams produced by this method is limited by the spot size of the incident laser beam. Beam expanders could be used to expand the main beam spot size and increase the number of input beams produced. The power in each input beam is somewhat variable, however, due to the spatial profile of the main beam spot. Spatial filtering should be used for spatial consistency. This method is rather wasteful in terms of beam power, since, for example, the fill factor for our mask was 20%.

A microlens array could be used instead of the aperture mask would reduce the loss associated with creating the input beams. The microlens array would also replace the input imaging lens, but a large field lens on the main beam prior to the microlens array or a spherical input turning mirror would be required to maintain input beam coincidence on the spherical mirrors. In addition, for use in a reciprocal transmitting and receiving antenna system, this method would require rerouting the multiple input beams through separate bulk modulators. After modulation, the input beams would
need to be realigned by way of identical length optical paths into a small beam matrix on the input turning mirror. However, as was discussed in the previous section, bulk modulators are limited in bandwidth. An alternative method would be preferred for reciprocal systems.

A second method more suitable for higher bandwidth and reciprocal transmitting and receiving systems is the use of individual directly modulated laser diode sources for each input beam. Figure 4.2 provides an illustration of an example reciprocal system. The use of individual sources has the additional advantage of less input power loss. Each source would be equipped with a graded index (GRIN) lenslet for collimating. The sources could be mounted in a matrix configuration and demagnified onto the input turning mirror with a field lens and input imaging lens, as before.

For output, the exiting beams need to be separated and then individually aimed onto separate detectors or as a group onto a detector array, with the caveat of identical optical path lengths following reflection from the output turning mirror. The matrix of output beams on the output turning mirror could be imaged onto a matrix of GRIN lenses, each coupling to a multimode fiber. The fiber could be used to transport each output light beam to another location for detection if the device is employed in a remoted antenna system. The output fibers would then couple to a matrix of fast detectors, perhaps integrated in a small area chip configuration. For a 256X256 SLM with a 20 bounce spot pattern, there is an output matrix of 128X25 beams. Using a first approximation of 5 mm square area per detector, the output detector matrix would be 64X10 cm in area.
4.5 Alignment Issues

We now address sensitivity of the device to vibration and resultant misalignment. Theoretically, the device can function with either coherent or incoherent light, such as from an arc lamp. However, the use of lasers with large depth of focus is more advantageous since this permits some longitudinal defocussing without inducing large crosstalk or power loss. Lateral misalignments, however, could cause crosstalk by changing spot location on the SLM pixels and detectors. The small number of transits ($\approx 20$) make the TTD device less susceptible to alignment errors as compared to spectroscopic White cells which may use hundreds of bounces [9]. However, for nanosecond time scale delay increments with $q = 20$, the number of bounces on the SLM, the round trip difference (C-F) would be approximately ten feet. The distance to spherical mirror F would be approximately 5 feet. For a $10\mu$m pixel separation, angular control must be within 0.005 degrees. A solid construction is proposed in the next section that would reduce vibration and misalignment.

One concern for the White cell has historically been astigmatism [67, 68, 69]. This is of concern to spectroscopy when the number of cell transits can be in the hundreds and the pathlengths in tens of meters. Again, the true-time delay device presented here requires a comparatively small number of refocusings with relatively short pathlengths. In addition, we are using large depth of focus lasers instead of incoherent light as White did. Astigmatism was not expected to be a problem and indeed we have not observed any noticeable spot shape changes in our experiments.
4.6 Construction Options

The unit can be ruggedized by using a solid material instead of air as shown in Figure 4.3. This figure shows a possible solid block construction. The optical path lengths in the two cells are made different by varying the refractive index or the length of the material between the beamsplitter and the mirror pairs. The spherical mirror surfaces can be ground onto the ends of the glass or other optical material, and those surfaces can then be coated for enhanced reflectance. Some change in material will be necessary to produce the lenses at the outputs of the beamsplitter. This solid construction is appealing because it is rugged and could be made comparatively temperature insensitive.
4.7 PBS Cube Power Losses

In Chapter 3, we saw that power loss is component dependent and the two largest contributors in our apparatus are the light valve and the polarizing beamsplitter. Power loss in the light valve will be discussed shortly in Section 4.9. Here we discuss power loss from the cube.

Our PBS cube was purchased from stock and not optimized for this specific purpose. Cubes can be custom designed to meet certain specifications [61]. One specific improvement would be to reduce overall power loss in the cube to less than the 3% measured. This refers specifically to total losses to reflection and absorption for high peak power laser pulses.

4.8 PBS Cube and Lens Crosstalk

4.8.1 Cube and Lens Fresnel Reflection Crosstalk

Fresnel reflection crosstalk is defined to be light from Fresnel reflections that has incorrect time delay and that actually reaches the detector. Only light passing through the most limiting aperture prior to a detector will reach a detector's active area. The most limiting aperture for each detector is the small spot area on the output turning mirror which is imaged onto the detector's active area. Light from Fresnel reflections on one beam that enters the limiting aperture of another beam is considered to be intra-channel crosstalk noise. Light from Fresnel reflections on one beam that enters the limiting aperture for that same beam is then intra-channel crosstalk noise.

We first consider Fresnel reflections from the lens and cube surfaces within the cell. Light from curved surfaces within the cell, such as the cell lens, will undergo beam divergence. Only a tiny fraction of the rays from these diverging beams could
possibly hit the output turning mirror and pass through any one limiting aperture and actually reach a detector. Similarly, Fresnel reflections from the PBS cube faces, not being directed at the proper angles necessary for refocussing in the cell, will undergo beam divergence and could only have a tiny fraction of rays from each bounce possibly reach any of the detectors. Fresnel reflection crosstalk noise from these two components should therefore be negligible.

4.8.2 Cube Polarization Splitting Crosstalk

Single pass polarization-related crosstalk was shown to be limited by the LCLV and the PBS cube in Chapter 3. Polarization-related crosstalk from the LCLV will be discussed in Section 4.9. Here we consider polarization-related crosstalk from the cube. First, we define contrast ratio and discuss how it relates to crosstalk. Then we consider performance of our current PBS cube and a possible alternative that would provide improved performance. Finally, an analysis is presented on the cumulative effects of a given contrast ratio on detected polarization splitting crosstalk.

The contrast ratio (CR), defined as the ratio of polarization outputs at each of the two output ports of the PBS cube, is a parameter used to quantify the polarization separation capability and as such determines the polarization splitting crosstalk due to the cube. The PBS cube offers one area for improved polarization-related crosstalk performance. In Chapter 3, we measured contrast ratios of 967 and 323 for the transmitting and reflecting sides of our angle-tuned cube respectively. Optimal crosstalk values would result with maximal polarization splitting capability, or contrast ratio, that is the same in both directions, transmitted and reflected, so that receiver SNR is not time delay dependent.
An alternative to a custom glass PBS cube is a calcite polarizing beamsplitter such as a Glan-Thomson cube which can provide up to $10^5$ polarization separation [61]. In this type of cube, the angle between the propagation directions of the two polarized components is less than ninety degrees (e.g. 45 degrees), but this presents no problems with our TTD device [61]. The unit can be adjusted to accommodate a calcite PBS cube by moving the spherical mirrors of the second unit cell to align with the calcite cube output direction and the centers of curvature aligned with those of the first unit cell as before. One limitation to the calcite cubes is that the size of the natural crystal material limits apertures to less than 20 mm on a side. However, with our proposed SLMs pixel pitch of 60 $\mu$m (50 $\mu$m pixel with 10 $\mu$m gap width), this would still allow addressing of a 200X200 pixel SLM with a stock 12 mm calcite cube. Some of these numbers will be used in our analysis.

We now analyze the effects on crosstalk of imperfect polarization separation by the PBS cube. Polarization splitting crosstalk in the cube is defined as polarized light that is incorrectly separated by the cube into an incorrect time delay path and still reaches the output detector. This is energy that is not diverted to the proper unit cell and thus becomes crosstalk noise. Considering a first case for two bounces off the LCLV, we then extend to the analysis to three or more bounces.

In order to compute crosstalk and therefore dynamic range levels, we use a two part method: find the amplitude of the individual noise pulses created at the output of the cell and find their time delay to determine if any noise pulses add together to create higher amplitude noise. We will assume that we have an ideal LCLV that perfectly rotates the polarization when necessary, lossless lenses and spherical mirrors, and an imperfect cube. The cube incorrectly splits light of each polarization with the
fractions of $R_p, T_p, R_s$ and $T_s$ for the reflectance and transmittance values for p- and s-polarizations, respectively. For incoming p-polarized light, the cube transmits $T_p$ and reflects $R_p$. For incoming s-polarized light the cube transmits $T_s$ and reflects $R_s$. For polarizing beamsplitter cubes, $T_p \gg R_p$ and $R_s \gg T_s$. For our purposes, we assume a balanced cube with $T_p = R_s$ and $R_p = T_s$. For this example, we choose the desired time delay path as all p-polarized, such that the LCLV is in the all OFF-state. This path is given by the time delay path of BCB for this example with two bounces on the LCLV.

We follow the fractions produced on each pass through the cube on a branching tree diagram, shown in Figure 4.4a. The incoming light with p-polarization leaves the input turning mirror, and in passing through the cube is split by the cube into $T_p$ (desired signal) and $R_p$ (noise) fractions. These fractions are reflected by their respective spherical mirrors, B and E, and then return to the cube.

We then designate the fraction of power remaining by a string of $R_p$ and $T_p$ factors ordered so that the most recent operation is on the right. Each fraction is then partially split to form four new fractions, of varying powers, but all of horizontal polarization. The fraction labelled $R_p T_p$ in Figure 4.4a has been transmitted by the cube after reflection from spherical mirror E. It leaves the cell and only contributes to power loss not crosstalk. The portion labelled $R_p R_p$ has been reflected off spherical mirror E and then reflected by the cube back to the LCLV and thus stays in the cell. The branch labelled $T_p T_p$ has been reflected off of spherical mirror B and then transmitted by the cube to the LCLV. The fraction $T_p R_p$ is reflected out the side of the cube after reflecting off spherical mirror B and leaves the cell. Thus we see that
PBS Cube Assumptions (assuming perfect light valve):

\[ P \text{ in } T_p \quad T_p > R_p \]
\[ S \text{ in } R_s \quad R_s > T_s \]

(a)

\[ T_p = R_s \quad R_p = T_s \]

(b)

Figure 4.4: Configuration for analysis of polarization crosstalk due to PBS cube polarization separation. (a, top) Assumptions. (b, bottom) Noise fraction tree diagram.
when the last two terms in a power fraction are $T_p R_p$ or $R_p T_p$ the beam exits the unused side of the beamsplitter and leaves the cell, not contributing to crosstalk.

Continuing down through the tree, we see that on the third pass through the cube out to the spherical mirrors again, a leakage term to the third power in $R_p$ is produced. It and all the terms followed by the symbol X are dropped since they are higher order loss terms and are neglected because they are severa orders of magnitude smaller than lower order noise terms. On the fourth pass through the cube back to the LCLV for the second bounce, terms followed by the symbol O drop out because they exit the cell out the unused cube face. After the fifth pass to the spherical mirrors and the sixth pass out to the output turning mirror, the remaining terms all leave the cell by reflecting off the output turning mirror.

Terms with incorrect time delay terms constitute crosstalk noise. The desired time delay path desired is represented by the fraction $T_p^6$. Thus, we are concerned about all the other terms since those are fractions that had the wrong polarization at some point and therefore travelled the wrong path in the cell. Figure 4.4b shows all the other fractions remaining in the cell. The non-signal terms that remain are $R_p^2 T_p^4$, $T_p^2 R_p^2 T_p^2$ and $T_p^4 R_p^2$. We see that all the noise terms have the term $R_p^2$ in common. The term $R_p^2$ means the noise fraction went to either mirror E or mirror F. We note that terms with $R_p^2$ in them have time delay paths with only one trip to an incorrect spherical mirror. It is important to consider if these noise pulses have the same time delay.

To analyze the time delay paths of exiting noise fractions, we examine a case with $q = 4$ bounces on the light valve or five round trips through the cell, the last trip being to exit the cell. For the $q = 4$ case with the LCLV still in the all OFF state, the
delay path of BCBCB is the desired result. The accompanying noise terms can have only $R_p^2$, i.e. only one trip to either spherical mirror E or F in each time delay path. These are BCECB, ECBCB, BCBCB, BFBCB, and BCBFB. The first three terms have the same time delay, having only one trip to mirror E each. The last two terms also have the same time delay, having one trip to mirror F each. This means that for $q = 4$ bounces on the SLM, there are at most $\frac{q}{2} + 1$ terms that would add together with the same time delay. Therefore the worst case of additive polarization splitting noise has an amplitude of $(\frac{q}{2} + 1)T_p^{2q}R_p^2$ and the signal pulse has amplitude of $T_p^{2q+2}$.

Note that this worst case only occurs in relatively few time delay configurations. The detected output rf dynamic range level, $D_{\text{cube}}$, is the ratio of the output signal pulse power to the power of the largest noise fraction. This gives a measure of how well the signal pulse stands out against the crosstalk noise background. Note that here we have assumed high bandwidth pulses that are narrower than the minimum time delay increment such that the noise and signal pulses never overlap in the time domain.

Recalling the relationship $R_p = T_s$ so that we may substitute the contrast ratio term, $\frac{T_p}{R_s}$, the dynamic range for this scenario is then

$$D_{\text{cube}} = -20\log\left(\frac{T_p^{2q+2}}{(\frac{q}{2} + 1)T_p^{2q}R_p^2}\right) = -20\log\left(\frac{T_p^2}{(\frac{q}{2} + 1)R_p^2}\right) = -20\log\left(\frac{C R_p^2}{\frac{q}{2} + 1}\right)$$

By symmetry, we can see the vertically polarized case, where the LCLV switches polarization, is the same. In that case, the terms $T_p$ and $R_p$ are replaced with $R_s$ and $T_s$, respectively. The replaced terms are identical in numerical value with a balanced cube, thus the same crosstalk value results.
For a calcite contrast ratio of $10^5 : 1 \ (CR = 10^5)$ and $q = 20$ bounces on the LCLV, this gives an rf dynamic range of -179 dB (optical crosstalk of -89 dB). For a calcite cube with $q = 62$, the rf dynamic range is -170 dB rf. For our tuned glass PBS cube with contrast ratio of $CR = 323$, the rf dynamic range is -79 dB (optical crosstalk -29 dB) for $q = 20$ and still -70 dB rf for $q = 62$.

Recall from the section on multiple columns of input spots that for a fixed number of pixels on an SLM, the number of bounces on the SLM per input beam affects the total number of input beams possible. From the formula above for beamsplitter crosstalk, we see that a tradeoff must be made between the number of bounces leading to the number of time delay increments and the level of crosstalk. If the number of time delay increments is to be maximized at the cost of number of input beams controlled, then crosstalk will go up. However, for $q = 62$ bounces, the rf dynamic range is still -170 dB rf for a calcite cube and -70 dB rf for a glass cube. This shows that with even a modest glass PBS cube and maximal bounces, the limiting factor on polarization crosstalk is the LCLV not the cube. The LCLV is examined in detail in the next section.

4.9 Liquid Crystal Light Valve

From the demonstration measurements in Chapter 3, it is apparent that the liquid crystal light valve is the limiting factor in the device performance. This section is devoted to a discussion of those limitations and proposals for a custom-designed light valve that would improve on these limiting aspects. We discuss switching speed, power loss, spot density, pixel-related crosstalk and finally, the polarization-related crosstalks of Fresnel reflections and polarization switching.
4.9.1 Switching Speed

A target speed for changing the beam direction of phased array antennas has been stated as 1 \( \mu \text{sec} \) [70]. TTD devices also need to be computer controlled to allow integration with the beamsteering subsystem. In the subject TTD device, time delay switching speed is determined by the SLM switching speed and the speed of the SLM control device.

The switching speed of our LCLV is specified to be \( \leq 70 \text{ msec} \) [71]. For the demonstrations in this work with this optically driven light valve, realistic switching speeds are not possible nor are they required and the optical write light was controlled by manual switching of applied light and spot locations. Light control for our optical SLM could be computerized for driving an antenna system by the use of a cathode ray tube controlled by computer. Some efforts with new methods of driving nematic devices have reportedly produced switching speeds of 50 \( \mu \text{sec} \) [72]. However, it is expected that other light valve technologies such as electronically addressed ferroelectric liquid crystals may provide better solutions for TTD purposes, at least in terms of speed.

There are various SLM technologies available [73], with different switching speeds. Response times of 50 \( \mu \text{sec} \) have been measured for ferroelectric liquid crystals (FLC’s) [74, 75], and response times as fast as 13 \( \mu \text{sec} \) are predicted [76]. The FLC’s have the low loss associated with liquid crystals, and can be made in arrays as large as 256 x 256. Ultra-fast charge-coupled device (CCD) sensors have frame rates of 100 million frames/sec or 10 nsec per frame for a 256X188 pixel device [77]. Therefore, it seems possible that a ferroelectric LCLV optimized for use in this device could be produced with frame rates of 1 \( \mu \text{sec} \).
4.9.2 LCLV Power Loss

The liquid crystal light valve is the component with the largest power loss. The dielectric mirror is broadband and optimized for wavelengths longer than 514 nm [71]. In addition, a separate, randomly chosen uncoated twisted nematic display cell has measured loss of only 9%, which can mostly be attributed to Fresnel reflection losses. Therefore, the relatively high loss in our light valve is very likely due to a lossy broadband dielectric mirror. Optimization studies were not performed on the light valve used here since it was chosen due to availability, and not specifically designed for this wavelength.

Light valves have not been optimized to reduce reflective loss. Reflective loss is not typically a serious factor in single-reflection applications such as display for which SLMs are typically designed. For this reason, we propose a LCLV optimized for this TTD device at one wavelength for low loss upon reflection.

The proposed light valve would have a single wavelength multilayer dielectric, and thus be expected to have a reflectance of $\approx 99\%$, based on the little loss attributed to the liquid crystals themselves in our twisted nematic transmission measurement. This reflecting surface construction should be possible for either analog optically driven light valves or on-chip electronically driven light valves.

This new reflectance value for the light valve would reduce the overall cell loss to 11%, as compared to our current total measured value of 30%. Since we are able to detect signal pulses after 19 round trips fairly easily with the current cell, we assume we could also detect at least two more bounces for a total of 21 round trips with the losses we have presently. For 21 round trips through the cell with our light valve, the total optical signal power has decreased to $0.7^{21} = 5.6 \times 10^{-4}$ or -32.5 dB.
Given a light valve with an improved reflectance, the optical power would be down to $0.89^{21} = 8.6 \times 10^{-2}$ or only -10.6 dB, a significant improvement. Given that the cell is still functional after a 32.5 dB decrease in signal power, one could increase the number of time delay bounces. For -32 dB loss and transmission of 0.89, $m=63$ round trips can occur in the cell which would produce 1024 time delay increments.

### 4.9.3 Spot Density on the LCLV

Spot size and spot separation determine spot density on the spatial light modulator in the TTD device. In Chapter 3, we saw that pixel spatial response determines pixel-related crosstalk and therefore required spot separation on the SLM. Spot size on the SLM is limited by pixel size in both an optically-controlled and an electrically-controlled SLM.

Beam spread is another factor to consider regarding pixel size in our proposed TTD-optimized LCLV. In one commercial FLC LCLV, the pixel size is 14 \( \mu m \) square. Given \( \lambda \) is free space wavelength, \( R \) is the wavefront radius of curvature, and \( D \) is the aperture diameter, this size spot has an approximate beam spread of

\[
\frac{\lambda R}{D} \approx \frac{(0.532 \ \mu m)(500 \ mm)}{14 \ \mu m} \approx 19 \ mm
\]  

at the spherical mirrors, which is rather large. In addition, there must be room for a turning mirror along one side and close to the front of the SLM, without blocking the expanding beam coming from the SLM reflecting surface. If we assume 1 mm thickness for the front surface glass for our proposed on-chip LCLV and for our input turning mirror, then we must have a Rayleigh range greater than 2 mm. Therefore, we propose a 50 \( \mu m \) pixel size in our TTD LCLV. The Rayleigh range for a circular
spot of diameter 50 μm illuminated with plane waves is

\[
\frac{D^2}{2\lambda} = \frac{(50 \text{ μm}^2)}{2(0.532 \text{ μm})} = 2.3 \text{ mm}
\]  \hspace{1cm} (4.4)

This pixel size has a beam spread of only 5.3 mm at the spherical mirror plane, which is more advantageous for compactness.

In the optically-controlled LCLV with analog pixel size, however, other factors in the choice of spot size need to be considered. Because of multiple pass power loss in the cell, we want to use as much power in each input beam as possible, to give good electronic SNR. This results in high irradiance levels at the SLM surface. Thus physical damage to the SLM must be considered in the design.

The spot size is one factor in determining the irradiance level at the SLM surface. In the optically-controlled SLM where the reflecting medium is continuous, the pixel size has a lower limit but no upper limit. Thus, the spot size can be adjusted to maintain total power level but reduce irradiance at the SLM surface. The manufacturer's tested no-damage irradiance level for LCLV's was 10kW/cm² [78]. Our own tests indicated a no-damage irradiance level was approximately 200 kW/cm² at 10 Hz. This would mean, for example, that a 50 mJ 10 nsec Nd:YAG pulse would require 4.6 optical density filtering at the input aperture spot mask plane, when a 400 μm spot is demagnified to a spot size of 200 μm onto the SLM plane.

Irradiance levels at the SLM are also dependent on laser pulsewidth as well as spot size. Irradiance levels will therefore also be affected by the radar signal pulsewidth which will control the modulated light pulsewidth in the time delay input signals. Ultra-wide band impulse radar signals can have pulsewidths as short as 1 nsec and
even into the 10s of picoseconds [63]. For very short radar signal pulses, the correspon-
ding laser power will have to be very high to maintain total pulse energy. This
may require a trade-off of enlarging the spot on the SLM to avoid physical damage.

4.9.4 Pixel-Related Crosstalk

We have shown that for an optically-addressed LCLV, pixel-related crosstalk is
basically dependent on the image spot spatial profile and pixel response of the par-
ticular light valve. Pixel response in Hughes LCLVs has been previously studied [62].
Our concern for this research is to obtain read and write spot sizes that demonstrate
minimal pixel crosstalk. We achieve minimal pixel crosstalk by using a read spot max-
imum width of 227 \(\mu m\) and a write spot with flat response region of 331 \(\mu m\) across,
thus totally containing all the read spot power. For this particular light valve, the
resulting liquid crystal neighbor response from the write spot creates tails of 165 \(\mu m\)
(10% to 90%). Thus the addressable pixels must be separated by 165 \(\mu m\) to achieve rf
dynamic range values of -19 dB; greater separation will produce even better dynamic
range values. These spot sizes were chosen for convenience and control of fluence on
the light valve surface. Different read and write spot sizes would result in different
pixel separations.

Electronically addressed light valves may be physically pixellated [79]. Energy
falling on the interpixel regions will result in power loss but not interfere with the
neighboring pixel. Thus, electronically addressed SLMs may offer pixel crosstalk
advantages. There are tradeoffs, however, that must be examined, such as pixel size
and high power limit.
4.9.5 Polarization-Related Crosstalk

In this section, we examine two types of polarization crosstalk related to the LCLV. The first type of crosstalk is based on single bounce contrast ratio performance, designated polarization switching crosstalk. As a prelude to this crosstalk analysis, we discuss our light valve's poor contrast ratio performance. We then examine and eliminate one possible cause of that poor performance, that of off-normal incidence on the light valve. By eliminating this possibility, we conclude that it is our particular light valve that is not optimal for this use, but we find that there are others reportedly available that should give us better performance. Then, given an optimized contrast ratio, we analyze the resultant cumulative detected polarization switching crosstalk value. We derive a closed-form expression based on LCLV contrast ratio performance and predict a value based on best reported performance.

The second type of crosstalk, Fresnel reflection crosstalk, is not dependent of LCLV performance, but is a direct result of Fresnel reflections from the front surface of the LCLV. Power circulating in the cell from Fresnel reflections has not been or is incorrectly polarization switched by the LCLV. This type of crosstalk is dependent on the physics of available anti-reflection coatings. We again derive a closed-form expression for the cumulative detected Fresnel reflection crosstalk and predict worst-case values based on typical coating performance.

Light Valve Contrast Ratio

Single bounce polarization-related crosstalk was shown to be limited by our LCLV in Chapter 3. The LCLV contrast ratio (CR), defined as the ratio of OFF/ON output at the transmitted side of a perfect analyzer and ON/OFF output at the
reflected side of a perfect analyzer, is a parameter that affects polarization switching crosstalk from the cell. The higher the contrast ratio, the better the ratio of signal to noise pulse amplitude. In addition, balanced contrast ratios from both ports are desired so that more noise is not created in half the delay paths of the device. In Chapter 3, we measured, contrast ratios of 967 and 323 for the transmitted and reflected sides, respectively, of the PBS cube alone, when splitting highly polarized light. This cube is therefore not perfect, however, contrast ratio values dropped to 7 and 70, respectively, when the LCLV was combined with the cube, as occurs in actual polarization switching operation. This comparison shows the light valve is responsible for the overall low and unbalanced polarization switching performance in our device.

Our light valve configuration was optimized for best contrast ratio during the laboratory demonstration construction. This involved alignment of the liquid crystals axis and optimizing the voltage bias and control light power levels, as described in Appendix A. The liquid crystals must be aligned for the incoming polarization vector and the address scheme must uniformly rotate the liquid crystal molecules. We use an optically addressed light valve and thus polarization is most uniformly switched where the write light intensity is most uniform. Our control spot spatial profile was shown to be uniform within 5% in Chapter 3.

In addition, polarization switching requires optical control at only two operating points of a parallel nematic light valve, which is designed to provide grayscale instead of binary output. This means precise write-light intensity control is required. Demonstration voltage bias was maintained by an extremely stable Keithley signal generator. Write-side optical power was controlled by a variac and no appreciable
drift was observed. A binary device, such as a ferroelectric or twisted nematic liquid crystal light valve might provide more consistent control of contrast between the on/off polarization switching states. Since this light valve was used because of availability not suitability, no further configuration optimization studies were conducted. However, it is possible that an inherent property of the TTD cell design, that of off-normal-incidence, may have affected LCLV performance.

In this TTD application, the optimal SLM needs to have a high contrast ratio through either side of the analyzer in a cone of angles near the normal. We now wish to look at a first-order analysis of the effect of incidence angle on liquid crystal birefringence to determine impact on contrast ratio, which is the significant parameter in crosstalk prediction.

**Contrast Ratio for Off-Normal Incidence on Parallel Nematic LCLV**

We will use a first-order approximation for the effect of incoming beam angle on an ideal parallel nematic device and solve for the output through an analyzer. The ON state is shown in Figure 4.5. Here we see a cross-section view of the LCLV in the y-z plane. The incoming light is propagating in the y-z plane and polarized at 45 degrees into the plane of the paper, with in-phase x and y-z components. The liquid crystal cell is of thickness such that \( \frac{\pi}{2} \) relative phase delay between the two polarization components is achieved when normally reflected through the device and the crystals are not tipped (no voltage applied), the 'ON' state. With full voltage applied, the crystals are tipped 90 degrees and no polarization rotation occurs for normal reflection, the 'OFF' state. The liquid crystals have a typical ordinary refractive index value of \( n_o = 1.52 \) in the x-z plane for the short axis and a typical
extra-ordinary refractive index value of $n_e = 1.73$ in the y-z plane for the long axis of the liquid crystal molecules.

Based on the effective refractive index for tilted crystals given in Appendix A, we now proceed to derive the off-normal electric field polarization after passing through the liquid crystals. We then compute the output through an analyzer and examine the leakage percentage in the OFF state.

The incoming electric-field is

$\vec{E}_{\text{in}} = \left( \frac{\hat{x} + \hat{y}}{\sqrt{2}} \right) E_o e^{i\omega t}$ \hspace{1cm} (4.5)

For the incoming x-component of polarization, there is no change in refractive index with tip in the y-z plane, neither due to tip of the crystals or angle of the incoming light vector. The x-component refractive index is thus equal to $n_o$. The resultant
phase change due to the liquid crystals for the x-component is

\[ \phi_s = k \cos \theta_o \int_0^{2d} n_o dz \approx kn_o2d \cos \theta_o \]  

(4.6)

where \( \theta_o \) is the angle between the incoming x-component propagation direction in the liquid crystal material and the z-axis, which is normal to the long axis of the liquid crystals. For the y-component of polarization, the refractive index changes as a function of y-z tilt. The effective refractive index is a combination of fast and slow indices as a function of angle, called the effective refractive index:

\[ \frac{1}{n_{eff}^2} = \frac{\cos^2 \theta_e(z)}{n_e^2} + \frac{\sin^2 \theta_e(z)}{n_o^2} \]  

(4.7)

where \( \theta_e \) is defined here as the angle between the incoming y-component propagation direction in the liquid crystal material and the z-axis, which is normal to the long axis of the liquid crystals. Thus the y-z phase shift becomes

\[ \phi_p = k \int_0^{2d} \frac{dz \cos \theta_e}{\sqrt{\frac{\cos^2 \theta_e(z)}{n_e^2} + \frac{\sin^2 \theta_e(z)}{n_o^2}}} \approx \frac{k2d \cos \theta_e}{\sqrt{\frac{\cos^2 \theta_e}{n_e^2} + \frac{\sin^2 \theta_e}{n_o^2}}} \]  

(4.8)

The resultant output electric field with phase shifts is

\[ \vec{E}_{out} = \frac{E_o}{\sqrt{2}} \left( \hat{x}e^{-j\phi_s} + \hat{y}e^{-j\phi_p} \right) e^{jwt} \]  

(4.9)

with \( \phi_s \) and \( \phi_p \) given above.

The analyzer is set to pass polarization at 45 degrees out of the plane of the paper. The field expression for the light exiting the analyzer is then the dot product of the analyzer direction \((-\hat{x}, +\hat{y})\) and the output field:

\[ | \vec{E} | = \frac{E_o}{\sqrt{2}} \left( \hat{x}e^{-j\phi_s} + \hat{y}e^{-j\phi_p} \right) \cdot \left( \frac{-\hat{x} + \hat{y}}{\sqrt{2}} \right) \]  

(4.10)

\[ = \frac{E_o}{2} \left( -e^{-j\phi_s} + e^{-j\phi_p} \right) \]  

(4.11)
The irradiance of the analyzer output is then

\[ I = | \bar{S} | = \frac{| \bar{E} |^2}{2Z_o} \]  
\[ = \frac{E_o^2}{4Z_o} (1 - \cos(\phi_s - \phi_p)) \]  
\[ = \frac{E_o^2}{2Z_o} \sin^2 \left( \frac{\phi_s - \phi_p}{2} \right) \]

Using \( \phi_s \) and \( \phi_p \) from above then gives the final expression for output irradiance for the ON state:

\[ | \bar{S} | = \frac{E_o^2}{2Z_o} \sin^2 \left[ 2kd \left( n_o \cos \theta_o - \frac{\cos \theta_e}{\sqrt{\frac{\cos^2 \theta_e}{n_e^2} + \frac{\sin^2 \theta_e}{n_o^2}}} \right) \right] \]

We now solve for the liquid crystal cell thickness, \( d \), for \( \pi \) phase change for normally incident input when the crystals are vertical:

\[ \phi_p = k \int_0^{2d} n_p dz \approx kn_e 2d \]  
\[ \phi_s = k \int_0^{2d} n_s dz \approx kn_o 2d \]  
\[ \phi_p - \phi_s = \pi = k2d(n_e - n_o) \]  
\[ d = \frac{\pi}{2k(n_e - n_o)} \]

With this value of \( d \), we find ON-state irradiance proportional to

\[ S_1 = \sin^2 \left[ \left( \frac{\pi}{2(n_e - n_o)} \right) \left( n_o \cos \theta_o - \frac{\cos \theta_e}{\sqrt{\frac{\cos^2 \theta_e}{n_e^2} + \frac{\sin^2 \theta_e}{n_o^2}}} \right) \right] \]

Similarly, it can be shown that the field in the \( \frac{\pi}{\sqrt{2}} \) polarization direction (the leakage field) is given by

\[ C_1 = \cos^2 \left[ \left( \frac{\pi}{2(n_e - n_o)} \right) \left( n_o \cos \theta_o - \frac{\cos \theta_e}{\sqrt{\frac{\cos^2 \theta_e}{n_e^2} + \frac{\sin^2 \theta_e}{n_o^2}}} \right) \right] \]
We now examine the OFF-state configuration as shown in Figure 4.6. We see this configuration can be decomposed identically to the ON-state if the tip angle of the incoming field is now replaced by \( \theta'' = \frac{\pi}{2} - \theta \). Substituting this expression for our \( \theta \)s in the previous scenarios gives OFF-state irradiance output proportional to

\[
S_2 = \sin^2 \left( \frac{\pi}{2(n_e - n_o)} \right) \left( n_o \cos \theta_o - \frac{\cos \theta_e}{\sqrt{\sin^2 \theta_e n_2^2 + \cos^2 \theta_e n_2^2}} \right) \quad (4.22)
\]

Similarly, the output term in the aligned polarizer direction in the OFF state is

\[
C_2 = \cos^2 \left( \frac{\pi}{2(n_e - n_o)} \right) \left( n_o \cos \theta_o - \frac{\cos \theta_e}{\sqrt{\sin^2 \theta_e n_2^2 + \cos^2 \theta_e n_2^2}} \right) \quad (4.23)
\]

For our demonstration cell, we use a worst-case angle of

\[
\theta = \frac{6.35 \text{ cm}}{50 \text{ cm}} = 0.127 \text{ radians} \quad (4.24)
\]

Figure 4.6: Configuration for OFF-state analysis of off-axis input beams through parallel nematic liquid crystal light valve.
outside the light valve and inside $\theta_o = \frac{\alpha}{n_o}$ and $\theta_e = \frac{\alpha}{n_e}$. This angle is an estimate of the angle resulting from proceeding from one spherical mirror to the outside edge of the spot pattern on the SLM and then to the other spherical mirror. It is the largest deviation from normal within the cell, being larger than the estimated angle from one row on the SLM to one spherical mirror and back to the another row on the SLM, as occurs in the spot pattern. For our cell’s off-normal angle, the ON/OFF ratio through crossed polarizer ($\frac{\text{ON}}{\text{OFF}}$), equivalent to the reflected CR, is $\approx 4 \times 10^3$, and the OFF/ON ratio through aligned polarizers ($\frac{\text{OFF}}{\text{ON}}$) is $\approx 3 \times 10^4$, much better than that observed. We conclude that the off-normal angles present in our device had very little impact on the contrast ratio performance in our LCLV. However, off-normal angles in the device design should be kept to a minimum and performance at off-normal incidence should be optimized in any light valve considered for this application.

Single wavelength contrast ratios of 100:1 have been reported for ferroelectrics (FLCs) [79]. However, recent work with FLCs have shown that this reported contrast ratio only represents the value measured through crossed polarizers, which is typically all that is important for display purposes. Contrast ratios viewed through parallel polarizers are reportedly only 20:1 [80]. This is comparable to our transmitted-side cell configuration and for which we measured a contrast ratio of 7:1. Again, this low contrast ratio will produce less satisfactory crosstalk values if used in our device, since we require high contrast ratio through both ports of the analyzer (our PBS cube) so that polarization crosstalk noise is not created in either side of the cell.

High contrast ratio performance in nematic light valves has been reported. Contrast ratios for nematic light valves as high as 1000:1 have been reported in two-port
device work [72, 4]. We expect that an optimal light valve could be configured to provide this contrast ratio for our purpose as well.

**LCLV Polarization Switching Crosstalk Analysis**

Given that contrast ratio improvement appears possible with use of a specialized LCLV, we now proceed to analyze the cumulative effects of a given single bounce contrast ratio on detected polarization switching crosstalk. Polarization switching crosstalk is defined as light from incorrect polarization switching by the LCLV that has incorrect time delay and reaches the detector. Once we determine the sources and amplitudes of the crosstalk, we use the highest amplitude crosstalk pulse to calculate worst case dynamic range. Here dynamic range refers to the ratio of peak power of the signal to the peak power of the single largest crosstalk component. This gives a measure of how well the signal pulse stands out against the crosstalk noise background.

In order to compute crosstalk, we use a two part method: find the amplitude of the individual noise pulses created at the output of the cell and find their time delay to determine whether the noise pulses add together at any of the time delays. First, we examine the case of incoming horizontal polarization with a desired path of all p-polarization for four bounces on the LCLV; the LCLV is in the all OFF state. Having examined the crosstalk associated with the cube, it is a reasonable assumption to use a cube that acts as a perfect polarization splitter. We also assume lossless lens and spherical mirrors. We will assume that the LCLV incorrectly switches a fraction of the light. For incoming p-polarized light, the light valve returns $X(p)$ and $Y(s)$ fractions, where $X(p) > Y(s)$. For incoming s-polarized light, the light valve returns $W(s)$ and $Z(p)$ fractions, where $W(s) > Z(p)$. Finally, we note that $X=W$ and $Y=Z$. 

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We now follow the fractions produced on each pass through the light valve on a branching tree diagram, shown in Figure 4.7a. The incoming p-polarization goes through the cube untouched, reflects off spherical mirror B, passes back through the cube and reaches the LCLV. The LCLV then splits the p-polarized light into X and Y fractions, where the X fraction is the desired signal and the Y fraction is the noise. These fractions are directed appropriately for their polarization by the cube to spherical mirrors C and F. The X fraction is reflected by spherical mirror B back to the LCLV where it is split again into XX and XY, where the most recent operation is on the right. Here, XX represents the desired signal and XY is a new noise term. The Y fraction is reflected by spherical mirror F back to the LCLV where it is split into YZ and YW, which both represent noise. Note that the rightmost variable in every term also tells which polarization that fraction has at the time. For example, YZ is p-polarized, while YW is s-polarized. The current state of polarization allows one to follow the path of each noise pulse through the cell as it is split by the PBS cube. The four fractions, XX, XY, YZ and YW, then pass through the cell and are reflected back to the LCLV where each fraction is then partially rotated to form two new fractions, of varying powers and polarizations. The fractioning continues at every bounce on the LCLV. After an even number of bounces on the LCLV, four in this case, the beam exits the cell by reflection off the output turning mirror, on bounce 5.

The desired time delay path is represented by the fraction $X^4$ in Figure 4.7b. To compare magnitudes, we condense each fraction into a term containing only Xs and Ys, using the relationship $X=W$ and $Y=Z$, shown at the output of the tree in Figure 4.7b. We are interested in only the largest magnitude noise pulses, those of only single order in Y, since the $Y^2$ and higher terms will be much smaller. There are four
LCLV Operation (assuming perfect cube):

(a) \[ \begin{align*}
P & \in - X(p) \\
Y & \in - Y(s) \\
S & \in - W(s)
\end{align*} \]

\[ X > Y \]

\[ X = W \]

\[ Y = Z \]

\[ W > Z \]

(b) \[ \begin{align*}
XY \\
YW \\
YZ \\
XX \\
XYZ \\
XYW \\
XYZ \\
XZY \\
XYZX \\
XYZY \\
XYZZ
\end{align*} \]

Figure 4.7: Configuration for analysis of polarization switching crosstalk with a liquid crystal light valve. (a, top) Assumptions. (b, middle) Switched component tree diagram.
such terms: XXXY, XXXW, XYWW, and YWWW, each with magnitude of $X^3Y$.

Do these pulses have the same time delay, thus reducing the dynamic range?

To compute the time delay paths of all the noise pulses, we first translate all the fractions into spherical mirror bounces, using the polarization splitting properties of the cube and the polarization at each pass through the cell. The translated paths are shown in the parallel tree in Figure 4.8. Since the desired time delay path was BCBCB, all others are incorrect. Each incorrect trip to either E or F corresponds to an incorrect fraction created above, which are the s-polarized fractions, $Y(s)$ and $W(s)$. We are most concerned with the time delay of the highest magnitude fractions from above. These four fractions correspond to time delays of BCBCE, BCBFE, BCEFE, and BFEFE. Each of these has a unique time delay, thus we can generalize that the largest amplitude noise pulses do not have identical time delays. Therefore the strongest noise pulse has amplitude of $X^{q-1}Y$ and the signal pulse has amplitude of $X^q$ where $q$ is the (even) number of bounces on the LCLV for the opposing spot configuration). Using the relationship $Y = Z$ to allow us to substitute with the contrast ratio, $\frac{X}{Z}$, the resulting output rf dynamic range is

$$D_{\text{switch}} = -20\log\left(\frac{X^q}{X^{q-1}Y}\right) = 20\log\left(\frac{Y}{Y}\right)$$

$$= -20\log(CR) \quad (4.26)$$

For a single bounce contrast ratio of 1000:1 (CR=1000) as is expected with an optimized light valve, the rf dynamic range is -60 dB (optical crosstalk of -30 dB), independent of the number of bounces.
Figure 4.8: Time delay path tree diagram for analysis of polarization switching crosstalk with a liquid crystal light valve.
LCLV Fresnel Reflection Crosstalk Analysis

Having concluded our discussion of light valve contrast ratio and resultant polarization switching crosstalk, we now turn to another phenomenon in the LCLV that creates another type of polarization-related crosstalk, Fresnel reflection.

We begin by reviewing Fresnel reflection crosstalk, defined to be light from Fresnel reflections that have incorrect time delay and that actually reach the detector. Only light passing through the most limiting aperture prior to a detector will reach a detector's active area. The most limiting aperture for each detector is the small spot area on the output turning mirror which is imaged onto the detector's active area. Any light of incorrect time delay that is coincident with the cell bounce paths will reach the detector and cause crosstalk. This is the case with light valve Fresnel reflections.

We begin by examining desired operation of a LCLV. A typical LCLV consists of a front layer of glass covering a layer of liquid crystals with are adjacent to a reflecting mirror surface. At each pixel, the liquid crystal birefringence is such as to switch polarization or not. In designed cell operation, a main signal beam, representing one time delay channel, passes in through the front surface of glass, passes through the liquid crystals, reflects off the embedded mirror, passes back through the liquid crystals and finally back out through the front plate of glass.

In dealing with Fresnel reflections off the LCLV, we first consider what takes place on a single bounce and then combine the effects of many bounces. Finally, then, we consider these results in an example to generalize whether any of the noise effects are additive.
For our single bounce analysis, we assume nominal values for multilayer anti-reflection coated glass of 99.5% transmitted and 0.005% reflected power. We also assume perfect reflection at the back of the liquid crystal cell and ignore power loss other than Fresnel reflections. Finally, we assume the next interface encountered, that of the liquid crystal to glass plate interface, is relatively index-matched and negligible Fresnel reflections are created here.

First we examine a single bounce off the LCLV. For each main beam passing through the LCLV, there are two Fresnel reflections possible. The first Fresnel reflection is reflected back off the front surface of the glass plate as the main beam passes through and has an irradiance of \(0.005I_{in}\), where \(I_{in}\) is the input irradiance. The main beam is then reflected and passes back out through the liquid crystal layer and the front glass plate. When the main beam reaches the front of the glass plate, a second Fresnel reflection is created that goes back through the plate, liquid crystal layer, and is reflected back out again. This second Fresnel reflection, upon exiting the LCLV, has an irradiance of \(0.005(0.995^2)I_{in} \approx 0.005I_{in}\).

Because the glass layer is relatively thin compared to the other dimensions in the cell and the reflection angles are so small, there is no appreciable sidestep or time delay for these two reflections. Therefore, they pass through the TTD cell as would the main beam. The light from these two Fresnel reflections appears at the same set of spots on the LCLV as the main beam and eventually reaches the output turning mirror and is imaged onto the detector active area along with the main beam, thus forming a source of noise if they have an incorrect time delay. In that they reach the detector for the main beam that created them, these noise pulses are intra-channel.
We must consider the effects of polarization on the time delay path. In other words, do these Fresnel reflections have different time delays than the main beam, making them now crosstalk noise pulses? If light from either Fresnel reflection leaving the LCLV has the same polarization as the main beam leaving the LCLV, then it is indistinguishable from the main beam and is not noise. If light from either Fresnel reflections leaving the LCLV has a different polarization than the main beam, it will be reflected by the PBS cube into a different path through the TTD device and appear at an different and incorrect time delay.

If this happens, then this is noise and we must consider two factors to compute the crosstalk: the individual amplitude of the cumulative noise pulses created and whether the noise pulses superimpose at any of the time delays. It is possible that reflections at different bounces can combine irradiances and this further decreases the signal-to-noise pulse amplitude ratio. We use this ratio to determine crosstalk or dynamic range level.

We look at an OFF pixel set for no polarization switching of the light. In this case, both Fresnel reflections are the same final polarization as the main beam and they are not noise. Now look at a pixel is turned ON to switch polarization. The main beam’s output polarization is switched from its incoming polarization. The first Fresnel reflection is not switched, never passing through the liquid crystal layer. The second Fresnel component goes through two round trips through the liquid crystal cell and is re-switched back to the original incoming polarization, thus ending up in the same state as the first Fresnel reflection. The outgoing polarization state of both Fresnel reflections is opposite that of the outgoing main beam. At this point, they become noise signals and will now travel an incorrect path through the cell.
Thus, we see that Fresnel reflections that are noise only occur on passes through the LCLV when the pixel is turned ON to switch the polarization. In addition, the two Fresnel reflection components created in one LCLV reflection, have essentially the same power, polarization, direction and time delay and therefore add together. Thus each ON pixel bounce produces a Fresnel noise signal of $2 \times 0.5\% = 1\%$ relative power.

What about multiple reflections as the noise pulses pass through the cell? These “primary” Fresnel reflections will create “secondary” Fresnel reflections when passing back through the LCLV, but these will be of higher order, having amplitude $0.005^2$. We neglect them since we seek the highest amplitude noise to compute worst case dynamic range. Thus, we are only concerned with primary Fresnel reflections which have deviated from the desired cell path only once on initial creation at a single ON pixel reflection. These two primary Fresnel reflections then undergo the same relative power losses passing through the rest of the cell, such that when they exit the cell, their amplitude ratio to the main beam, $R_{\text{amp}}$, is the same for crosstalk purposes, still that of $R_{\text{amp}} = \frac{99}{1}$.

Now, we examine the resultant time delay after multiple bounces to see if noise signals add in irradiance at the detector at one incorrect time delay to create a higher combined noise level. We use a graphical approach. Consider the illustration shown in Figure 4.9 which symbolically shows the bounces taken by the main beam and the Fresnel reflections created at several bounces. We see two horizontal sections in the figure. The top section is a row of spots labeled q-4 to q. To the right of the spots, there is a rectangle representing the output turning mirror. The section at the bottom is a set of lines and labels representing trips to the spherical mirrors,
which are labelled for each path. Each path has a different pattern representing the respective reflection component.

Consider the simplest non-trivial case that results in more than one Fresnel noise pulse being generated to gain an indication of whether the bounces will add together. This case is for a $2\Delta$ time delay, which is produced by switching twice to spherical mirror E instead of spherical mirror B. The desired switch to mirror E instead of B could occur at any even numbered bounce, after returning from spherical mirror C. Here we choose the switches to occur as late in the bounce pattern as possible to reduce the number of switches required and noise pulses generated. The last reflection out of the cell always occurs from an even mirror, either B or E. Therefore, having the last pixel switch the beam to mirror E removes the necessity to switch one more
time back to mirror C and then reflect to mirror B to exit the cell. So for $2\Delta$ time delay, the last three spots are turned ON, and are shown as darkened circles.

We now examine the reflections off these last three pixels and onto the output turning mirror. The main beam for this particular case is shown as a solid line. It started at bounce 1 (not shown) and bounced back and forth between spherical mirror B, the LCLV and spherical mirror C, only the last few of which are shown. The first \( q - 3 \) bonces are thus assumed to have the path BCBC...BC. At bounce \( q - 2 \), the main beam has its polarization switched and proceeds to mirror E as shown. The main beam path switches back to C on bounce \( q - 1 \), then to E again on bounce \( q \). The beam then exits the cell via the output turning mirror. Thus the main beam path, shown as a solid line, has a path of BCBC..CECE.

Since the polarization of the main beam is not rotated until bounce \( q - 2 \), all primary Fresnel reflections created by these prior bounces have the same polarization state as the main signal and cannot be distinguished from it. Therefore we start with a main beam of 100% power relative to bounce \( q - 2 \).

Next we label the primary Fresnel reflections generated at each activated pixel and indicate which spherical mirror they will then proceed to on that and all subsequent bounces. The \( (q - 2) \)th Fresnel reflection is not polarization switched on bounce \( q - 2 \), but is switched on bounces \( q - 1 \) and \( q \) and is shown with a dotted line. Therefore its path is C-B-F-B. The \( (q - 1) \)th Fresnel reflection is created on bounce \( q - 1 \) (squiggly line) and then is switched on bounce \( q \), for a path of C-E-F-B. The Fresnel created on the \( q \)th bounce (dashed line) has a path of C-E-C-B.

Figure 4.9b shows the expected time delay trace for this particular example. The desired time delay signal had the path of BCBC...BCECE, which is the minimum
time delay of all trips between mirrors B and C, and two minimum increments of $\Delta = \frac{L_F-L_C}{c}$. The Fresnel noise pulse from the bounce $q-2$ with path $...BCEFB$ has amplitude $0.005I_{in}$ and is located at time delay $+10\Delta$, where we have chosen $L_F-L_C = 10(L_E-L_B)$ for illustration purposes. The Fresnel noise pulse from bounce $q-1$ with path $...BCEFB$, is shown at time delay $+11\Delta$, with amplitude $0.005I_{in}$. The Fresnel noise pulse from bounce $q$ with path $...BCECB$ and is shown at time delay $+1\Delta$, with amplitude $0.005I_{in}$.

From this example, we can generalize that all primary Fresnel reflection pulses exiting the cell have unique time delays and do not add together at the output. The detected relative amplitude ratio, $R_{amp} = \frac{29}{1}$ gives a detected Fresnel reflection dynamic range, $D_{Fresnel}$, of

$$D_{Fresnel} = -20\log(R_{amp}) = -40 \text{ dB rf (20 dB optical)} \quad (4.28)$$

We now summarize our analysis of Fresnel-related polarization crosstalk. Primary Fresnel reflections off the LCLV are the worst case of Fresnel noise. This type of crosstalk is inter-channel since it makes noise on the same channel that created it. Secondary Fresnel reflections off the LCLV are several orders of magnitude smaller than primary ones and are neglected. Each primary Fresnel noise pulse appears at a different time delay, either before or after the desired signal time delay pulse and does not add in amplitude with other noise pulses. Therefore detected Fresnel reflection crosstalk is not dependent on the number of bounces. Fresnel reflections represent a fundamental physical limit on TTD cell crosstalk since this is energy that can never be polarization-controlled.
4.10 Quasi-Continuous Wave System Performance

In previous sections we have worked with very short pulse signals. This may not always be the case. Here we consider very long signals, i.e. microwave signals with pulse widths longer that the minimum time delay increment or continuous wave (CW) systems such that overlap of the time delayed signal with the noise fractions in the time domain is possible. The signal-to-noise ratio (SNR) is now used as a measure of system preformance. The SNR here means the ratio of signal power to the total power in all the noise fractions.

The optical noise fractions are small amplitude duplicates of the optical signal, which is amplitude modulated by a microwave source. The noise fractions, having an incorrect time delay, add coherently to the main signal as they are modulo-2π phase shifted versions of the main signal. The main signal cannot be disstiquished from the noise fractions as was done in the previous noise calculations, in which we looked at dynamic range for very narrow pulses that did not overlap. Thus we predict the power in the signal versus the power in the noise fractions as a measure of system performance in the quasi-CW case.

Previously, the signal and resultant noise fraction amplitudes were derived for several cases, but we only compared the power of the signal to the power of the single noise fraction with the largest amplitude. Now, we wish to add the powers of all the noise fractions. Noise terms with the same time delay add coherently to each other. If there are \( u \) noise terms, where \( u \) is an integer, with the same time delay and amplitude of \( A \), then the combined amplitude is \( uA \), which results in a voltage signal with magnitude \( uA \) at the optical detector output. The associated microwave noise power is then the voltage squared, \((uA)^2\). Noise fractions with different time
delays are treated as randomly phased signals that add incoherently, since the time delay in the noise fractions will vary greatly depending on the time delay selected in the device, the spot activation pattern implemented, etc. For \( v \) noise fractions with different time delays, each has a power of \( A^2 \) and the total noise power is then \( vA^2 \).

With these caveats in mind, we start by re-examining the noise fractions from the PBS cube polarization splitting crosstalk. For worst case, all the noise fractions are split between two time delays, thus we have two fractions with amplitudes \( (\frac{q}{2} + 1)T_p^{2q-2}R_p^2 \) and \( (\frac{q}{2})T_p^{2q-2}R_p^2 \), and a signal amplitude of \( T_p^{2q} \). The SNR is then

\[
\text{SNR}_{\text{cube}} = -10 \log \left( \frac{(T_p^{2q})^2}{\left( (\frac{q}{2} + 1)^2 + (\frac{q}{2})^2 \right) (T_p^{2q-2}R_p^2)^2} \right)
\]

For \( q = 62 \) and \( CR = 10^5 \), we get \( \text{SNR}_{\text{cube}} = -167 \) dB rf.

For LCLV polarization switching noise, the total number of terms produced in this scenario is proportional to a binomial expansion of the number of bounces, as in \((X + Y)^q\). One of the expanded terms represents the signal term amplitude, \( X^q \). The rest of the terms in \( Y \) represent noise fractions. For \( q = 62 \) bounces, this is a large number of terms with increasing powers of the leakage fraction \( Y \), e.g. \( Y^2, Y^3 \), etc. Here we ignore noise leakage terms with exponents higher than first order. The first order terms typically have different time delays and we treat them independently. Thus we have \( q \) first order noise terms of amplitude \( X^{q-1}Y \). We get for SNR

\[
\text{SNR}_{\text{switch}} = -10 \log \left( \frac{(X^q)^2}{q(X^{q-1}Y)^2} \right)
\]

\[
\text{SNR}_{\text{switch}} = -10 \log \left( \frac{CR^2}{q} \right)
\]

For \( q = 62 \) and \( CR = 10^3 \), we get \( \text{SNR} < -40 \) dB rf.
For LCLV Fresnel reflection noise fractions, there are \( q \) fractions, all with various time delays which we treat independently and a relative amplitude of 0.01. The relative signal amplitude is 0.99. Thus we get for SNR

\[
SNR_{Fresnel} = -10 \log \left( \frac{0.99^2}{q(0.01)^2} \right). \tag{4.31}
\]

For \( q = 62 \), we get \( SNR = -22 \text{ dB rf} \). We see that for quasi-CW systems, Fresnel reflection noise is the major noise source in the system. This noise can be controlled with quality anti-reflection coatings. For this calculation we assumed a nominal value of 0.5%, but we believe this value could be improved with state-of-the-art coating technology.

### 4.11 Summary

In this chapter, we have discussed the implications of the demonstration results on integration of the true-time delay device with a phased array antenna system for beamsteering. We described the system level issues of possible radar system interfaces and that all bandwidths can be supported except for extreme impulse radars. At the device level, we examined input/output, alignment issues, and construction options. At the component level, we discussed power losses and crosstalk for the PBS cube and the cell lens. Finally, we examined limitations of our LCLV and made proposals for an optimized custom device in regards to switching speed, power loss, spot density and crosstalk.
CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary

The purpose of this work was to demonstrate essential features of operation of a novel photonic true-time delay device and to investigate performance parameters necessary for implementation of the device in a phased array antenna. The individual elements of a microwave phased array antenna need individual time delays. The methods used presently include lengths of coax cable, which are bulky and expensive, or phase shifts, which limit bandwidth. This work described a multiple reflection optical mirror configuration with a spatial light modulator for switching.

Among the advantages of this approach is the physical size of the TTD device, which is determined by the number of input beams, the delay increment, and the SLM resolution. The constant refocusing in the cells means that diffraction over long optical paths does not imply an increase in physical size of the unit. The unit can be made very compact even for long delays. The longest path(s) can be folded around the outside of the other mirrors. Also, the unit can be made rugged by using solid blocks of materials. Another advantage is flexibility in light source requirements, since there are no restrictions on source wavelength, coherence or modulated bandwidth.
Finally, the device is highly parallel in nature and allows production of independent time delay for thousands of antenna elements with one spatial light modulator. The design is fully reciprocal in that it can be used equally well in receive or transmit mode. The design presented here can provide photonic true-time delay for the high performance wideband phased array antennas of the future.

In Chapter 1, the problem was stated and the TTD device introduced as a solution. Additional background on phased array antennas was given, as well as descriptions of previous photonic work in this area.

In Chapter 2, we provided a detailed examination of the theoretical background of the true-time delay device design. With this groundwork, we discussed the choice of optical lengths and distances to meet any time delay design criteria, and developed a model of the fields in the cell. The chapter was divided into two major sections. In the first major section, we developed the design, discussed the time delay available, and then derived the imaging equations that provide the solution for the lens and mirror focal lengths. In the second major section, we derived expressions for and examined the fields within the device. Overall, this chapter provided the methodology needed to design a true-time delay device for any phased array antenna.

In Chapter 3, the main thrust of the dissertation work was presented. The laboratory experiments addressed two main purposes: 1) to provide proof that the design will produce suitable time delays with potential for use in phased array antennas and 2) to examine performance parameters that affect design decisions for radar system integration. We described experimental configurations and presented measurement results. We described the time delay demonstration with the type I device and with the type III unit cell of the compound-cell device. Together, the data from these
two experiments clearly showed that the compound-cell design produces time delay suitable for a phased array antenna. The last two sections presented data on the key performance parameters. These are the power loss budget for round trip power loss and component losses, and the two kinds of crosstalk, polarization-related and pixel-related.

In Chapter 4, we have discussed the implications of the demonstration results on integration of the true-time delay device with a phased array antenna system for beamsteering. We described possible radar system interfaces and bandwidths supported. We examined limitations of our LCLV and made proposals for an optimized custom device in regard to power loss, crosstalk, spot density, and switching speed. For the entire device, we examined alignment issues, and construction options. Finally we discussed power losses and crosstalk due to the PBS cube.

This document also included three appendices. These covered the liquid crystal light valve, the type III compound cell laboratory configuration, and the signal processing methods used for measurements.

5.2 Conclusions

Time Delay

• The design procedure was shown analytically.

  – Time delays were proportional to $m^2$, where $m$ is the number of round trips through the device.
  
  – Formulas for focal lengths and distances were derived based on time delays desired.
- Formulas for fields in the cell were derived and compared well with measured irradiance patterns.

- Aspects of time delay control with the compound cell TTD device were demonstrated.
  
  - Demonstrated the control of the polarization switch made up of the SLM and PBS cube.
  
  - Demonstrated variable time delay paths in the cell with a minimal increment detectable of two round trips, or $\Delta t \approx 7$ nsec.
  
  - Two spot patterns were presented that provide configuration flexibility for various TTD situations.

**Power Loss**

- Power loss from the spherical mirrors and lenses are not limiting factors in this device.

- The PBS cube had measured loss of 3%, but should not be a limiting factor when optimized.

- Power loss by the LCLV is the most limiting. Our LCLV had a measured loss of 26% per reflection. Optimized devices are expected to reduce this loss to that equivalent to the spherical mirror, cube and lens.

- Expected Best Case: Assuming optimized components, 63 round trips and 1024 time delay increments should be possible.
Crosstalk

- Crosstalk due to the spherical mirrors and lenses are not limiting factors.

- Crosstalk induced by the PBS cube is not a limiting factor. Polarization separation dynamic range in the narrow pulse case, \( D_{\text{cube}} \), is a function of contrast ratio squared \( (CR)^2 \), where contrast ratio is defined as the ratio of ON output to OFF output measured at one exit port of the PBS cube, and \( q \), the number of bounces on the light valve in the opposite spot configuration. The cumulative detected polarization separation dynamic range will be \( D_{\text{cube}} = -70 \text{ dB rf} \) with \( CR=323 \) and \( q=62 \).

- Crosstalk produced by the LCLV comes from several factors, one of which is limiting. These values refer to the narrow pulse case.
  - The small angles of the reflections in the cell are expected to have negligible effect in producing crosstalk from the LCLV.
  - Polarization switching dynamic range due to the light valve, \( D_{\text{switch}} \), is a function of contrast ratio. The cumulative detected polarization switching dynamic range will be \( D_{\text{switch}} = -60 \text{ dB rf} \) for \( CR = 1000 \) for the best light valve performance reported.
  - Fresnel reflection crosstalk produced by the LCLV is expected to be the largest noise contribution and a function of the ratio of Fresnel transmission fraction, \( T_F \), and Fresnel reflection fraction, \( R_F \). The cumulative detected LCLV Fresnel reflections dynamic range will be \( D_{\text{Fresnel}} = -40 \text{ dB rf} \) with \( \frac{T_F}{2R_F} = \frac{0.99}{2 \times 0.05} \), a fundamental limit based on a nominal anti-reflection coating reflectance of 0.5% and is independent of the number of bounces.
• Expected Best Case: For the $m = 63$ ($q = 62$) case of 63 round trips through the opposite spot configuration, the best possible custom-made components for crosstalk purposes suggests an overall dynamic range of $\approx -40 \text{ dB } rf$ from the detector for the narrow pulse case, which is quite acceptable.

5.3 Other Applications of the Device

Another radar application not related to antenna steering is in the field of noise radar [81]. Noise radar consists of broadband noise emitted towards and reflected from a target. The returned noise signal is then cross-correlated at different delay times with a reference of the original signal. The time delay thus controls the target range available with the noise radar. Here, the time delay application is relevant to processing the signal rather than steering the antenna. Currently, pre-determined fixed coaxial cables are used which are cumbersome and inflexible. The TTD device presented here represents a variable delay method to allow greater and more flexible range gating with a noise radar system [81].

Besides the applicability to phased array radar time delay, this research has many other non-radar potential applications. New spectrometers are being proposed even today based on the White cell, for such applications as atmospheric transmissometers [82]. Programmable length spectrometers may present a possible use of this TTD device. Laser acoustic Doppler imaging systems for biomedical and industrial uses may also be potential application areas for time delays [83, 11, 12]. Time lapse holography requires coherent time delayed pulses of laser light which could easily be produced with this system [13]. Another possible use is as a tapped delay line in optical signal processing or in digital beamforming [38]. A programmable TTD device
could also be used to compensate for multipath interference in wireless systems [58] or for polarization component dispersion in communication fibers [84].

5.4 Future Work

Further work on the type III device should concentrate on loss reduction and further demonstration of radar integration.

The spatial light modulator, currently an optically-controlled nematic liquid crystal light valve, has been shown to be the cause of the majority of loss. Acquisition of a lower loss SLM should be pursued. In addition to lower loss, interior input/output turning mirrors in the plane of the SLM reflecting surface should be incorporated into the SLM design.

A further advancement would be the construction of a type III complete compound cell with both unit cells. The time delay size should be determined by the wavelength and resolution requirements of the proposed antenna beamsteering prototype. A complete type III compound cell will allow actual demonstration of time delay beamsteering and will require programmable control. Programmable control can be produced with the current SLM by incorporating a computer-controlled CRT imaged onto the SLM write-side. Alternatively, an electronically-controlled ferroelectric prototype from a manufacturer such as Displaytech could be substituted for the current SLM.

Once a compound cell with computer control is available, integration into a small phased array antenna for beamsteering demonstrations should be pursued. This would require radar signal-to-photon conversion and vice versa. Assuming amplitude modulation of the light beam by the radar signal, an electro-optical modulator
of appropriate bandwidth and a commensurate detector would be required to allow integration with the radar unit and antenna.
APPENDIX A

HUGHES LIQUID CRYSTAL LIGHT VALVE

The basis of the time delay devices described herein is a spatial light modulator, which for this work is a Hughes liquid crystal light valve (LCLV). It is described in this appendix. We also detail the procedure used to configure the light valve for the TTD demonstration and measurements.

A.1 The Hughes Liquid Crystal Light Valve

A diagram of the LCLV cross-section is shown in Figure A.1. There we see that the LCLV consists of many layers sandwiched together. The main layers are the photoconductor layer left of center which controls the input information (write-side) and the liquid crystal layer right of center which controls the output information (read-side). For this work, we used the LCLV as a birefringence mirror.

We now describe all the layers of the LCLV, moving from left to right [7]. First on the left is a fiber optic faceplate that transports light from the write-side face to a transparent Indium Tin Oxide (ITO) conductive layer [7]. Once light passes through the transparent conductor, it strikes the light sensitive CdS semiconductor layer. Next to the CdS layer is a light blocking CdTe layer to separate write-light from reaching the output and to prevent read-side light from reaching the CdS [85, 86]. To
the right of the light blocking layer is a dielectric mirror. This broadband mirror is optimized for wavelengths greater than 514 nm [71]. On the right side of the dielectric mirror is the liquid crystal (LC) layer, held between two transparent walls that make up the “cell” containing the liquid crystals. These outer walls have striated surfaces to control the direction of the liquid crystal molecules. The molecules in our LCLV are in parallel with no voltage applied. The LC layer is approximately 10 μm thick [71]. To the right of the LC cell is a second ITO transparent conductor, and then an uncoated glass plate comprising the read-side face of the LCLV. In addition, our light valve has an anti-reflective coated glass plate attached to the front glass plate with index-matching fluid.

In operation, voltage is applied to the ITO layers. A fraction of that appears across the LCs. The amount across the LC layer depends on applied voltage and
light incident on the photoconductor on the write or input side. When no voltage is applied across the LC layer, the molecules are arranged in parallel from one side of the layer to the other. As voltage across the LC layer is applied, the LC molecules rotate about the fast axis due to an induced dipole moment on the molecules. One set is shown rotated in Figure A.1. The tip of the molecules changes the refractive index behavior of the liquid crystals. The main action involved in the read or output side of light valve is in the birefringence of the liquid crystals [6]. The liquid crystals are oblong birefringent molecules, with two different refractive indices for light polarized along the long and short axes of the molecules.

For light polarized along the narrow direction of the molecules, called the fast axis, the refractive index, \( n_o \), stays constant independent of tip. For light polarized along the extra-ordinary or slow axis, the refractive index, \( n_e \), is dependent on tip angle, \( \Theta \), as follows [7]:

\[
\text{A.1} \quad n_{\text{eff}} = \frac{n_en_o}{\sqrt{n_e^2 \sin^2 \Theta - n_o^2 \cos^2 \Theta}}
\]

where \( n_e \) is the extra-ordinary refractive index, \( n_o \) is the ordinary index, and \( \Theta \) is the tip angle, measured between the molecule axis and the face of the cell. In practice, light is polarized at 45° to the fast and slow axis with equal field components along both axes. The light propagates through the LC cell, reflects off the mirror and comes back through the cell. This results in a phase delay, \( \Phi \), between the two polarization components [7]:

\[
\text{A.2} \quad \Phi = \frac{2\pi (2d)}{\lambda} (n_{\text{eff}} - n_o)
\]

where \( d \) is the LC layer thickness and \( \lambda \) is the free space wavelength of the read-light.
The basic LCLV operation is shown in Figure A.2 [6]. Write-light for one pixel comes from the left and is transported to the photoconductive layer by the fiber optic faceplate. Once light strikes the CdS photoconductive layer, the charge transport is altered and the voltage across the liquid crystal layer is changed for that pixel. Read-light enters from the right, linearly polarized at 45° to the fast and slow birefringent axes. The read-light passes through the molecules, reflects off the dielectric mirror and passes back through the LC layer. The phase difference, $\Phi$, between the polarization components along the fast and slow axis in general results in elliptically polarized light. The output light is passed through an analyzer perpendicular to the incoming read-light polarization direction. The read light intensity through the analyzer then exhibits the typical sine-squared phase difference behavior of birefringent materials. A typical LCLV oscillatory output is shown in Figure A.3.
Figure A.3: Characteristic curve of the read-side output through an analyzer for a liquid crystal light valve (LCLV). Adapted from [7].
For time delay use, only the left-most read-out minimum and maximum intensity states are required, corresponding to phase shifts of modulo-$2\pi$ and $-\pi$, respectively between the polarization components. A voltage of $\approx 4.5$ VAC and frequency 2 kHz is applied across the transparent conductors to set the operating point of the LCLV at the minimum read-light output through the analyzer (point A on Figure A.3). Then write-light of appropriate intensity is applied to produce the $\pi$ phase change necessary for maximum read-light output. This occurs at point B in Figure A.3. Application of light thus controls time delay path switching at certain pixels. All pixels can be switched at once by applying $\approx 6$ VAC to the conductors in lieu of control light. This can be convenient for test purposes.

A.2 Power Test with the Light Valve

In the laboratory demonstrations described, maximal laser power was needed to allow the detection of multiple bounces. We addressed this issue prior to the demonstration by conducting qualitative destructive testing with a similar LCLV by subjecting it to pulsed laser power higher than manufacturer’s specifications. Consultation with the manufacturer suggested that the CdS photocathode layer would break down before the LC layer. A comparable LCLV was subjected to step increases of pulsed laser power, with 1, 2, 10 and 600 pulses at each step. After each exposure, the LCLV response was visually evaluated under polarized mercury light through an analyzer. There was no clear evidence (dark image spots, physically charred areas) of LCLV damage up to fluence levels of approximately 30 mJ per pulse and 200 kW/cm$^2$ irradiance on the light valve surface. Therefore, it seems that high power exposure should have had minimal impact; however, hidden damage may have existed.
A.3 Light Valve Alignment

The un-biased liquid crystals (LCs) must be optimally aligned with the incoming polarization vector. The un-biased crystals, in this case, are parallel to the face of the light valve (LCLV), but should be rotated around the optical axis to 45-degrees to the incoming polarization vector. In this state, a local maximum is produced at the analyzer (PBS cube) output. We monitored alignment progress by observing the maximum through the analyzer with a detector and oscilloscope. During unbiased alignment, we discovered that aligning the crystal axis by manually rotating the LCLV housing produced an unstable response from this LCLV. We deduced that the LCLV responded to objects at greater than ambient temperatures that were close to the fiber optic faceplate (e.g. a human hand and a coffee cup filled with hot liquid). Optical fibers common in the LCLV time frame transmitted energy in the near infra-red [87], thus this response was feasible. Therefore, to eliminate the response instability, a handle was attached to the LCLV housing so that manual rotation of the housing could be done without exposing the fiber-optic write-side faceplate to any non-equilibrium temperature objects. This meticulous liquid crystal alignment process should have optimized the procedural accuracy as well as possible.

To further document the behavior of this particular LCLV in the operating region for time delay, output through an analyzer (cube's reflected side) was recorded over the operating range of voltage input, at approximately every 50 mV as shown in Figure A.4. Here we see the usual oscillatory behavior of the LCLV response as the crystal tip increases with voltage input. For compound-cell operation, a voltage bias is applied to place the LCLV output at the first minimum, point A, which we call the unactivated or OFF state since there is nominally no polarization switching at
Figure A.4: To document the behavior of the particular LCLV in the operating region for time delay, output through an analyzer (PBS cube's reflected side) was recorded over the operating range of voltage input, at approximately every 50 mV.

this operating point. The activated or ON state then refers to the application of additional voltage (via photoconductive input) that places the LCLV operating point at the first maximum, point B, where nominally there is complete linear polarization switching. Figure A.4 shows that detector output at the first maximum is lower than the threshold detector output at low voltages. This means this maximum is sub-optimal. LCLV output at threshold voltage is dependent on the thickness of the
LCLV, and can range from no polarization switching (no analyzer output) to full linear polarization switching (full analyzer output). However, above threshold, the first maximum in the LCLV operation curve should represent the output for full linear polarization switching, and therefore should *always* be greater than or equal to the low voltage output. This curve thus indicates that this particular light valve is only producing elliptical polarization, not full linear polarization switching as desired. The final result in the time delay cell is power leakage into the incorrect time delay path.
This appendix provides detailed information on the total apparatus of the type III compound cell demonstration apparatus and on alignment of the unit cell of the device. This apparatus is used for most of the measurements in the dissertation. To organize the discussion of the research apparatus of the type III demonstration configuration, the apparatus is grouped into four modules as shown in Figure B.1: laser sources, unit cell, input/output and control. First, we give an overview of the apparatus operation, then proceed to describe the configuration of the four modules in detail. Finally, we discuss detection calibration procedures for the power detectors and neutral density filters.

The time delay apparatus is shown in Figure B.2. There we see the optical switch made up of the liquid crystal light valve (LCLV) and 532 nm polarizing beam splitter (PBS) at left center. At right center are the spherical mirrors and separating them from the PBS is cell lens $f_l$. The input and output turning mirrors are one above the other next to the LCLV so that only the top input mirror is seen. At the bottom of the figure, there is an input field lens to the left of the input aperture mask. The input/output (IO) folding mirror and the IO imaging lens complete the IO module.
Figure B.1: A system diagram of the four modules of the type III demonstration apparatus.

The output optics re-uses the IO imaging lens and adds two additional folding mirrors to re-direct the output beams onto a detector. The trigger detector is not shown. The layout is such that the trigger detector requires coaxial cable routing to the digital sampling oscilloscope (DSO) several times the length of that to the signal detector.

Input laser light from the pulsed Nd:YAG source module enters the apparatus along the bottom of Figure B.2. The input field lens conditions the light for proper focussing inside the cell. The light then illuminates the input aperture mask with three circular holes. All three beams from the input aperture mask could be individually time delayed if multiple detectors were available. Only one fast detector was available for the purposes of this time delay measurement, therefore, the input spot mask was blocked to pass only one input beam. The single beam exits the aperture mask and propagates beneath the edge of the output folding mirror adjacent to the
Figure B.2: A top view of three of the type III configuration modules: the input/output module, the unit cell module, and the control module. The laser sources module is not shown. Drawing not to scale.
input mask. The input folding mirror and input imaging lens then focus the incoming beam onto the LCLV conjugate plane near the input turning mirror. The input turning mirror directs the beam into the unit cell. The beam undergoes multiple reflections and is continually refocused on the LC plane. The optical switch is set to allow the polarization to stay horizontal, so all the beam energy stays within this unit cell (unit cell I). The number of reflections is controlled by the alignment of the spherical mirrors. The number of bounces can be adjusted by changing the locations of the centers of curvature of the two spherical mirrors. After the desired number of bounces is made, the beam bounces onto the output turning mirror instead of the LCLV and is directed out of the cell. The output optics direct the exiting beam onto the detector plane.

B.1 Laser Sources Module

Two laser sources are used in the laboratory demonstration, a pulsed Q-switched frequency-doubled 532 nm Nd:YAG laser, and bore-sighted with it a 3 mW continuous wave (CW) 514 nm argon ion laser. Both laser beams are configured with their polarizations parallel to the table (which will herein be referred to as horizontally polarized). The CW argon beam is co-aligned with the Nd:YAG. The argon is used first for alignment since the CW beam is easier to see. The Q-switched Nd:YAG laser produces a five nanosecond rise-time pulse. Both lasers are used for various power measurements. The optics for conditioning each of these laser beams are discussed in the next two sections, B.1.1 and B.1.2.
B.1.1 Argon Laser Source Configuration

The argon ion laser used is an American Lasers Company model 60C/60B, serial number 60-12/83-068DH. This laser is equipped with a Littrow prism for wavelength selection; the laser is tuned to the 514 nm wavelength. The output beam is vertically polarized to the plane of the table.

Figure B.3 shows the Argon laser source configuration. Horizontal polarization is required, so the vertically polarized beam is passed through two plastic Bausch and Lomb quarter-wave plates (QWPs). To purify the polarization further, the beam is passed through a Glan-Thomson polarizing (GTP) cube adjusted to pass polarization parallel to the table. The GTP is mounted in a rotatable mount with 360 degree markings, with direction indicators for parallel (horizontal) and perpendicular (vertical). The beam is then folded by metallic mirror, M1, toward two adjustable iris
apertures (A1 and A2), used for co-aligning the Nd:YAG laser. After passing through
the two irises, the Argon beam is turned toward the demonstration cell by metallic
folding mirror M2, and exits the source module.

Co-alignment Procedure

The Argon beam is first aligned as desired with the two metallic folding mirrors M1
and M2. Then the two irises are stopped down and centered laterally and vertically
on the Argon beam, by eye. Once centered, they are never moved again, unless
realignment becomes necessary. These irises are then opened for full power operation
or stopped down during Nd:YAG co-alignment.

Variations

Certain procedures required differently polarized light. To obtain vertically polar­
ized light, the QWP's are removed and the GTP is rotated to the vertical direction
(perpendicular to the plane of the table). To obtain circularly polarized light, only
one QWP is used and the GTP is removed. To produce 45-degree linearly polarized
light, the QWP’s are removed and the GTP is set to 45-degrees to the table. The use
of circularly polarized light or 45-degree linearly polarized light is sometimes helpful
for cell or detector alignment with the PBS cube. These two types of polarization
provide output in both transmitted and reflected directions through the PBS.

B.1.2 Nd:YAG Source Configuration

The Nd:YAG laser used is a Continuum Q-switched model 660B-10, serial number
197, with frequency-doubling crystal and a wavelength filter. The laser produces 10
nanosecond pulses at a wavelength of 532 nm, horizontally polarized and nominally
60 mJ pulses at rep rates of either 2, 5 or 10 Hz.
Figure B.4: The Nd:YAG source module configuration. (a, top) Optics inside the safety enclosure. (b, bottom) Related exterior optics for laser safety enclosure system. Drawings not to scale.
Figure B.4 shows the Nd:YAG source module configuration. A cardboard box safety enclosure covers two prisms, a glass plate, a high power detector and two beam stops, shown in Figure B.4b. When the beam first exits the laser cavity, it is reflected off the front surfaces of two uncoated glass prisms, one above the other. This serves two purposes. The first is to adjust the beam height to that of the Argon laser by adjusting the height of the stacked pair of prisms. Second, the reflections provide a factor of 100 power reduction (optical density or OD of 2). The beam exits the box through a baffle, used to reduce light scatter out of the enclosure.

Back inside the safety enclosure, the main portion of the Nd:YAG beam is dumped to a beam block, but a portion is used for power monitoring. After refracting through the first prism, the dumped beam is passed through a quarter inch thick glass plate. The Fresnel reflections from the front and back surfaces of this glass plate are aimed down at a Newport 818T-30 high power thermopile detector, serial number 2156, lying face up on the table. The data is displayed on a Newport 1815-C Optical Power Meter, serial number 1609. The main beam passing through the glass plate then hits the beam stop inside the safety enclosure. The secondary beam transmitted through the bottom prism also hits a beam block on the table surface.

Once the experimental beam exits the safety enclosure, a small amount of the beam is deviated for a trigger signal, shown in Figure B.4b. This is accomplished by passing the beam through a glass microscope slide cover slip. The cover slip's Fresnel reflections are aimed at a photodetector behind a set of neutral density filters. The output at this detector feeds one channel of the oscilloscope and is used as the scope trigger and reference beam. The amount of attenuation and the detectors change depending on laser power output and experimental configuration. The trigger detector
used for time delay measurements is a Thorlabs FDS200. The trigger detector used for power measurements is the Newport 818SL, SN 4866. Since it is possible to change alignment when exchanging neutral density filters, the trigger location is moved for some measurements where alignment was critical. The trigger detector is moved closer to the cell apparatus by intercepting the beam after mirror M4. Reducing the beam propagation distance after passing through the neutral density filters reduces the size of the alignment error.

After passing through the cover slip, the experimental beam is folded by two metallic mirrors, M1 and M2, used for two-axis alignment onto the remove-and-replace metallic mirror, M3. Mirror M3 directs the beam through the two co-alignment irises. Once adequate alignment is achieved through the stopped-down irises, alignment is complete and the irises are re-opened for the actual experiment. After passing through the two irises, the experimental beam is turned into the cell by one last metallic folding mirror, M4, and exits the source module.

**B.2 Unit Cell Module**

The unit cell module consists of one side of the type III compound-cell device including an optical switch. Figure B.2 shows a top view of the unit cell module, which is composed of the spatial light modulator (SLM), the turning mirrors, the polarizing beam splitter, cell lens I and two spherical mirrors. The optical switch is used to change the polarization of the beams to switch the light from one cell to the other in the compound-cell device.

The SLM is a Hughes liquid crystal light valve (LCLV) that was available in our laboratory. This particular LCLV is parallel nematic, and optically controlled.
The LCLV is further described in Appendix A. It has an image-transfer fiber-optic faceplate on its write-side and an uncoated glass faceplate on its read-side. To reduce the Fresnel-reflection loss on the read-side, an anti-reflection (AR) coated cover glass was added on the LCLV read-side faceplate, mounted with a layer of index-matching mineral oil between the coated cover glass and the uncoated faceplate. The entire light valve is mounted in an aluminum mounting frame fixed to the table. The LCLV mount allows rotation about vertical and horizontal axes perpendicular to the optical axis of the cell, to aid in aligning the LCLV perpendicularly to the axis. The LCLV can also be rotated about the optical axis and locked into place to aid in alignment of the untipped liquid crystal axes to the incoming polarization vector.

The turning mirrors are mounted in front of the LCLV front surface. The turning mirror mounting bracket is bolted to the front of the LCLV mount. Figure B.5 shows these two small turning mirrors have vertical beveled edges to allow multiply reflecting beams in the cell to pass close to the beveled edge for close spot spacing on the LCLV. The input turning mirror is above the output turning mirror, relative to the optics table. The two mirrors are each glued to brass-rod mounts that can be rotated about vertical and horizontal axes and translates horizontally, relative to the optical axis of the cell as shown in Figure B.5.

The particular LCLV used had concentric rings of varying polarization response at any particular voltage input, probably due to variation in the thickness of the liquid crystal layer. There did exist an inner circle of uniform spatial response of about 14 mm in diameter, more than enough for our needs. The input and output turning mirrors were positioned on the left side of the uniform response area (looking at the
Figure B.5: An example turning mirror. The input turning mirror is on top and the output turning mirror is inverted underneath it. The arrows show directions of motion available on both turning mirrors. Drawing not to scale.
LCLV read side), so that the maximum uniform area could be used for switching time delay beam spots.

Located next to the LCLV/turning mirror complex is the polarizing beam splitter (PBS) cube, designed for optimal polarization separation at the 532 nm Nd:YAG laser line. This cube is AR-coated, 1.5 inches on a side and center-mounted on a cube platform. The platform post holder is mounted on a graduated rotating base. The platform is positioned as closely as possible to the LCLV while still allowing for angle-tuning rotation. The cube angle was adjusted to maximize throughput of p-polarization to the compound-cell's transmitted side (straight through the cube). Angle-tuning is discussed further in the main text.

The cell lens is mounted next to the PBS cube. It is broadband AR-coated and has a focal length of 400 mm. The lens is mounted on a kinematic baseplate for easy removal and replacement. The cell lens is removed to allow a large beam from a mercury arc lamp, discussed next, to be directed through the PBS cube onto the LCLV face for focussing and spot pattern alignment.

A mercury arc lamp, shown separately in Figure B.6, is used to flood the LCLV read-side for focussing and aligning the control spot. The arc lamp is mounted outside the cell and the beam is directed onto the LCLV with a plane folding mirror temporarily mounted inside the cell, in front of the spherical mirrors. The arc lamp output is collimated, horizontally polarized, and line filtered for the 546 nm mercury line, chosen for its proximity to the 532 nm Nd:YAG line. This incoherent beam is chosen because it has a short depth-of-focus for precision in focussing. The collimating lenses are chosen to give a beam large enough to fully cover the LCLV read-side.
Figure B.6: A beam from a mercury arc lamp is directed into the cell to illuminate the entire face of the liquid crystal light valve for focusing and alignment purposes. Drawing not to scale.
This makes it easier to locate the control spot position for aligning the control spot with the read-side time delay spot pattern.

B.2.1 Choosing Focal Lengths and Distances

Focussing Light in the Unit Cell

There are two main processes involved in focussing light within the unit cell. The first is to externally focus input light onto a plane conjugate to the LC plane. This ensures that the light enters the cell correctly conditioned to allow multiple reflection paths and continual reimaging. The second process is to focus light internally with the cell optics in terms of the LCLV, cell lens and spherical mirror distances to obtain the desired images.

As a prelude to external focussing, the liquid crystal (LC) plane, that is the dielectric reflective surface at the back of the thin liquid crystal layer within the LCLV, is first imaged through the PBS. An LC imaging lens is used to magnify the image from the LC plane through the reflected side of the PBS cube onto a viewing screen, at about 10X magnification. This provides a means of viewing both the external and internal focussing processes. To properly set the image distances of this LC imaging lens, a transparency with sharp-edged patterns is attached directly to the LCLV write-side fiber-optic faceplate. By attaching the transparency directly to the faceplate, the need for focussing onto the write-side is eliminated which removes an additional source of focussing error. The LCLV is then activated and the mercury lamp beam used to read out the transparency image on the LC plane read-side. Once imaging is achieved, the LC imaging lens and the LCLV are fixed in position. This assures that one always views the LC plane, or, as in external focussing, any planes conjugate to the LC plane.

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Ideally, the input turning mirror is located in the same plane as the LC plane, and is therefore conjugate to it. In that case, the input beam could be directly focussed onto the turning mirror/LC plane. In our case, the LC plane is behind a half-inch of glass in the LCLV housing. Therefore, the input and output turning mirrors cannot physically be placed in the same plane as the LC plane. The turning mirrors are located as close as possible to the front of the LCLV housing, but the LC conjugate plane lays a short distance in front of the turning mirror.

As part of the external focussing process, a movable "object", is required to locate the LC conjugate plane on which to focus the input beam. For this, we use the Argon laser beam to illuminate a small aperture covered with a diffuser, Figure B.7, to make the light emanating from the object diverge over a large range of angles. This decreases the depth of focus giving focussing precision. The polarization is set

Figure B.7: A photo of the focussing object; a small aperture covered with diffusing material.
vertically so that the light is reflected by the beamsplitter. The object is mounted to allow translation for focussing. The beam is aimed through the focussing object onto the input turning mirror. The input turning mirror aims the beam through the PBS cube and onto the magnified focussing screen. The location of the focussing object is then adjusted until its image on the LCLV viewing screen is sharp, indicating that it is then in a position conjugate to the LC plane.

Next, we turn to the adjustment of the internal cell optics for focussing light within the cell. This involves setting the cell optics distances for imaging. Two optical distances are important internally within the cell: the spherical-mirror-to-cell-lens distance and the cell-lens-to-LCLV distance. By building only one side of the compound-cell device, the spherical mirror focal lengths are less constrained. The only constraints are that the round trip time between spherical mirrors be greater than the oscilloscope temporal resolution and that the configuration fit on the table.

We chose to build the “B/C” side of the type III compound cell, as shown in Figure 2.11 in Chapter 2. Two identical multilayer dielectric spherical mirrors coated for 546 nm, of 250 mm focal length, were available. Since the spherical mirror focal lengths are equal, the spherical-mirror-to-cell-lens separation is then equal to the focal length of the cell lens. The cell lens focal length of 400 mm was chosen from available stock sizes since it allowed acceptable space for the PBS cube. The spherical-mirror-to-cell-lens distance is then fixed at the manufacturer's stated focal length distance by physically measuring the 400 mm and mounting the spherical mirrors and cell lens onto a small breadboard, as shown in Figure B.8. This breadboard is 12X18 inches, three-quarter inch thick, and sits on top of the optics table.
Figure B.8: Illustration of the moveable breadboard in the unit cell demonstration configuration. The cell lens and two spherical mirrors are mounted on the breadboard for easy focussing. Drawing not to scale.
Since the cell-lens-to-spherical-mirror distance is now already in place, the cell-lens-to-LCLV distance remains to be adjusted within the cell. Moving the LCLV and input/output optics would present tedious input beam realignment requirements with each adjustment. Therefore, it is most advantageous to move the cell-lens-and-spherical-mirror combination, already fixed in relation to each other, by translating the breadboard with guides bolted along its sides. The breadboard is initially positioned at roughly the correct distance from the LCLV, based on the lens calculations in Chapter 2. During final focussing, the breadboard slides the cell lens toward/away from the LCLV while keeping the cell lens and spherical mirrors fixed in their relative locations.

With the cell approximately adjusted, we next adjust the cell optics to obtain two rows of spot images in the cell for fine focussing. At this point, the focussing object with the diffuser is still in position at the LC plane conjugate position and illuminated with the Argon laser. The input turning mirror is then aimed so that the input beam hits spherical mirror B, passes through the lens and is reflected off the LCLV to be returned to spherical mirror C. The spherical mirror centers of curvature are adjusted until several multiple images of the focussing object appear on the LCLV.

After obtaining multiple images, the final adjustment of the LCLV-cell-lens distance can be accomplished to finely focus light inside the cell. When in focus, all the images are sharp, of the same size, and in straight rows. To accomplish this last step, the breadboard, with cell lens and spherical mirrors mounted, is translated until the images on the LCLV were all the same size. The rows can be adjusted to be parallel to the table by adjusting the centers of curvature of the spherical mirrors.
**Focussing Phenomenon**

As part of the cell focussing process, an interesting phenomenon was observed. As the lens/spherical mirror board moved toward the LCLV, the rows of spots on the top and bottom of the LCLV would converge. If the board moved away, the rows diverged. The explanation possibly lies with the focussing properties of the cell. The lens-spherical mirror-lens combination can be considered a "compound lens" with principal planes. When the board moves toward the LCLV, the round trip distance from the LCLV back to itself is reduced. Taking the turning mirror as the fixed object plane, then the object-compound lens distance was reduced as the board moved toward the LCLV. As the object-compound lens distance decreases, the compound lens-image distance would increase. But the LCLV (the desired image plane) to compound lens distance was also reduced by moving the board closer, therefore the image plane is no longer on the LCLV. If one considered the cell on an unfolded optic axis diagram, then the new image plane would now be located behind the LCLV plane. Due to the LCLV reflecting surface, the actual image plane is actually folded back and physically lies in front of the LCLV inside the cell. The image is only the first image plane, due to the multiple refocussing in the cell. This first image plane becomes the second object plane. Now the second object-compound lens distance is even smaller. The second image plane would then be even farther in front of the LCLV and closer to the cell lens. The multiple image planes get continually farther from the LCLV plane with each bounce. This shows up as a decrease in size of each set of spot images on the top and bottom, respectively. The top and bottom rows appear to converge independently since they are represent rows of separate images.
Conversely, the diverging rows can be explained as follows. When the lens-spherical mirror compound lens board is moved back or away from the input object on the turning mirror, the LCLV to LCLV distance increases. The first image plane then moves forward along the optic axis an amount \( \Delta \), while the LCLV plane moves back an identical amount, so the separation becomes \( 2\Delta \). This first image becomes the second object and the resulting second image plane is now \( 4\Delta \) from the LCLV. This means the rays have diverged even further from the second image plane when they reach the LCLV plane. With each bounce, the separation gets bigger and the light fields on the LCLV plane diverge further on the top and bottom, respectively. Photos of this phenomenon are provided in Figure B.9. Awareness of the converging and diverging row pattern is useful when focussing the cell.

This completes the switching module and cell focus descriptions. The next subsection describes the input/output module.

B.3 Input/Output Module

The input/output (IO) module performs the two separate functions of input and output. The IO module is shown in Figure B.2 and in Figure B.10 in detailed side view. The input turning mirror is on top of the output turning mirror so only one is visible in the figure. All mirrors used in the IO module are metallic front surface mirrors. Input and output functions are described separately here.

Referring to Figure B.10, the portion of the IO module used for input is made up of an input field lens with an input aperture mask next to it, an IO folding mirror, an IO imaging lens and the input turning mirror. For input, the IO module creates and focusses input beams onto the LC conjugate plane in front of the input
Figure B.9: As part of the cell focussing process, the rows of spots on the top and bottom of the LCLV would converge or diverge.
Figure B.10: Side view of the input/output module. Drawing not to scale.
turning mirror. There are two purposes for the input optics: to create multiple sharply focussed images on the LC conjugate plane, and to cause the input beams to converge at the same spot on the back spherical mirrors. This latter requirement reduces the angular spread of beams in the cell and reduces the physical size necessary for the spherical mirrors.

To form the multiple beams, the input beam from the laser source is passed through a multi-aperture input aperture mask. Small, sharp-edged spots are desired for refocussing onto the read-side of the LCLV. Small spots allow for higher packing density on the LCLV, and sharp-edges are required for separation of the independent “pixels” or spots. The input aperture mask has 3 circular holes, nominally 396 \( \mu \text{m} \) in diameter and 1000 \( \mu \text{m} \) center separation. The beams exiting the mask are appropriately conditioned as described next.

The IO imaging lens is chosen and positioned to provide the desired demagnification (2X) from the input aperture mask down to the desired input image size, (221 \( \mu \text{m} \)), at the LC conjugate plane. Once the focal length of this lens is chosen, the resulting conjugate plane for the spherical mirrors is then approximately located by calculation and measurement.

The input field lens immediately next to the input aperture mask is chosen and positioned to make coincident the beams from the mask at the spherical mirror plane. Once the location of the spherical mirror conjugate plane in front of the cell is found, the field lens focal length is determined as the distance from the input aperture mask to the spherical mirror conjugate plane. With this optical arrangement, the beams coming out of the input aperture mask will all coincide at the spherical mirror conjugate plane, and therefore also at both spherical mirrors inside the cell. The field
lens is placed in its approximate position and is fine-adjusted to obtain coincidence of the beams at spherical mirror B on a viewing card.

Referring to Figure B.10, the portion of the IO module used for output is made up of an output turning mirror, the IO imaging lens, the IO folding mirror re-used, and two additional output folding mirrors that conveniently direct the exiting beams to a detector. For output, the IO module conditions the beams exiting the cell for detection and conversion to electronic time delay signals for the antenna elements.

The output beams exit the cell by way of the bottom output turning mirror after the prescribed number of bounces. The output beams are refracted by the IO imaging lens so that they pass above the incoming beams. Near and above the input aperture mask, two flat folding mirrors are used to redirect the output beams to the detector. After passing through the IO imaging lens, the three beams were separated and diverging enough to be individually detected.

B.4 Control Module

The control module provides the write light for the optical control of the LCLV. Figure B.2 shows the control module, consisting of the control light source (a slide projector), a control spot mask with translatable mounting, and a control imaging lens for demagnifying the mask and imaging onto the LCLV.

An optical "pixel" was created by illuminating the mask which contained a single circular hole. A beam from the slide-projector was used for illumination. A frosted glass slide was loaded in the projector holder to eliminate the annoying image of the projector bulb filament.
The mask was mounted on a three-axis translation stage with two left-right micrometers to allow a wider range of adjustment. Manual adjustment was available on the vertical control spot position, and the horizontal control spot position could be adjusted with an electro-mechanical actuator and motion controller.

The mask hole was 980 \( \mu m \) in diameter. The demagnifying lens reduced the mask spot to a diameter of 265 \( \mu m \) at the LCLV write-side fiber-optic faceplate. This size was designed to be approximately 300 \( \mu m \) to provide full LCLV response over the region of the 200 \( \mu m \) time delay spot on the read-side, based on a partial response zone of 50 \( \mu m \) around the diameter of the spot. This was done to control crosstalk, as discussed in the main text Section 3.4. The control spot was manually aligned on one row of spots on the read-side. Then the motion controller could be used to sweep the control spot along the row, activating the spots in the row one at a time.

### B.5 Detection Calibrations

Linear detection is required for voltage measurements used to calculate relative power ratios. For this type of measurement, we are comparing amplitudes of voltage pulses in relative ratios such as reflection and transmission. Absolute power values and laser pulse shapes are not needed, only linear response to power input, and calibrated values for the neutral density filters that are used. Uncalibrated detectors larger than the detected beams are used to reduce alignment and light collection errors. Determination of the linear range of these detectors and calibration of neutral density filters are described in this section.

The linear range of the detectors, seen in Figure B.11, is found by recording pulse amplitudes with one detector, over a wide power range in 2X (0.3 ND) increments
Newport 818–SL Detector Linearity

Figure B.11: Plot of the power output of the large area detector used to determine the range of linearity.
and selecting the desired limits on the measured plot. We start by measuring the pulse peak voltage with and without filtration of one 0.3 ND filter. Then various filters and a Jodon variable beam splitter wheel are used in place of the one filter to adjust beam power back to the previous value. The 0.3 ND filter is then placed in the beam again and another reading is taken. These steps are repeated to give power readings with the detector at every 0.3 ND. The range of the tested detector is taken to be 215 mV to 8 mV, but most signal outputs were adjusted to be well between these voltage extremes.

Neutral density filters are used to adjust the signal beam power to stay within the linear range of each detector. Uncalibrated attenuation filters are used on the reference detector, but are fixed in value throughout a set of measurements and do not affect the ratios. Calibrated filters must be used with the signal detector so that attenuator adjustments between measurements can be accounted for in power ratios. These calculations are further described in Appendix C.

Several neutral density filters are calibrated for experimental use. Only ND filters with relatively small attenuation (two 1.0 ND filters, a 0.3 filter and a 0.4 ND filter) can be calibrated since detector output has to stay in the linear range with and without the filter in the beam. Nominal filters are first placed in the beam in front of the detector to adjust power to the upper end of detector's linear range. Then the filter under test is placed in front of the detector and power is remeasured. A laser beam power reference measurement is made and used to normalize the ratio calculation for each filter. The values measured for the four ND filters are given in Table B.1.

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<table>
<thead>
<tr>
<th>Nominal Filter Value</th>
<th>Measured Value at 532 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 ND #1</td>
<td>11.4 or 1.06 ND</td>
</tr>
<tr>
<td>1.0 ND #2</td>
<td>10.7 or 1.03 ND</td>
</tr>
<tr>
<td>0.4 ND</td>
<td>2.64 or 0.421 ND</td>
</tr>
<tr>
<td>0.3 ND</td>
<td>2.10 or 0.323 ND</td>
</tr>
</tbody>
</table>

Table B.1: Measured values of ND filters used as the calibrated filters for power measurements.
In this work, two types of data are processed. The two types of data are time measurements and peak voltages. Two different digital sampling oscilloscopes (DSOs) are used. This appendix provides details on the DSOs, how measurements are taken with them and how the data is processed. The two DSOs used are a Hewlett-Packard HP54501A and a Gould 4072. Features and specifications of the two DSOs will be described first. We then describe the use of the scopes in taking the measurements and data processing methods.

C.1 Basic Oscilloscope Specifications

C.1.1 The HP54501A

The HP54501A is a basic digital storage oscilloscope with two full amplitude channels and a sampling rate of 10 Msamples/sec [88]. The scope has an instantaneous bandwidth of 1 MHz and a repetitive signal bandwidth of 100 MHz. The repetitive bandwidth is a result of the oscilloscope's ability to sample repeated acquisitions of a repetitive signal at random time locations to fill in the trace. The minimum timebase is 2 nsec/div and timebase accuracy is .005%. Timebase resolution is 100
psec. Maximum vertical gain is 5 mV/div and gain accuracy is ±1.5% of full range. Vertical quantization resolution is 0.4% of full range, but 0.8% for 5mV/div [88].

An averaging mode is available. A repetitively sampled signal is composed of samples taken at random time intervals to cover a wider region of the signal. Thus, to average at a certain time, samples that fall into a small time range or “bucket” are averaged together [89]. “...adjacent timebuckets are not averaged together. The algorithm, therefore, does not affect the signal itself, but rather filters out rapid deviations of the signal from its steady-state condition.”[89] The algorithm used for continuous averaging [89] is

\[
\text{Ave}_N = \frac{\sum_{i=1}^{N} V_i}{N} = \frac{\sum_{i=1}^{N-1} V_i + V_N}{N} = \text{Ave}_{N-1} + \frac{V_N - \text{Ave}_{N-1}}{N}
\]

(C.1)

(C.2)

where the \( V_i \)'s are the voltages appearing in one time bucket. One way to think of the averaging algorithm is that it computes the most current average displayed by weighting the most recent data as shown in the second term of the expression and then adding the result to the last average. Noise from the lasers, detectors, coax cable connectors and oscilloscope, etc. combines to produce a fluctuating signal. Averaging mode is used to reduce the fluctuations in the signal.

C.1.2 The Gould 4072

The Gould 4072 is a full-featured DSO with two full amplitude channels and a sampling rate of 400 Msamples/sec [90]. The instantaneous bandwidth is 100 MHz and the repetitive bandwidth is 5 GHz. Gould calls this mode ETS or equivalent time sampling. Maximum vertical gain is 2mV/div and gain accuracy is ±3% of full scale. Vertical resolution is 8 bits or 0.4% of full scale. Minimum time base is 20 nsec/div.
Figure C.1: Time delay data taken with the type I device using the HP 54501A oscilloscope.

and timebase accuracy is ±3%. Time base resolution is 200 psec. Averaging mode is available [90].

C.2 Time Data

The first type of measurement taken is time data. There were two different experiments which used different DSOs. The objective is to observe relative time delay between two pulse leading edges. Exact pulse shape is not critical, only linear system response. Both scopes, however, are operated in repetitive sampling mode to monitor laser pulses as well as possible. In addition, both scopes are operated in averaging mode.
For the type I time delay measurements, signal and trigger traces are captured by
the HP DSO at each time delay position (Figure C.1). Electro-optically gated 514
nm pulses (risetime ≤ 1 nsec) are used. A fast detector (risetime ≤ 1 nsec) is used for
the trigger signal. A beam splitter sends a portion of the main beam to the trigger
detector prior to entering the time delay cell. An avalanche-photodiode detector with
risetime of ≤200 psec is used at the output of the time delay cell. Both detectors are
amplified by 20 dB amplifiers with dc-150 MHz bandwidth.

The trigger point is used as a reference for all the time delayed signal traces.
Oscilloscope memory is used to store, recall and overlay several time delayed traces,
based on the trigger as reference. Traces were averages of 8 acquisitions. Time delay
is determined by measuring distance on the display plots between time delayed signals
on Channel 1.

To estimate time measurement error, the horizontal spread over several acquisi­
tions is determined. The time spread of five traces, each composed of an average of
8 acquisitions, is taken as a measure of uncertainty for any horizontal measurement.
The spread was ±1.2 nsec. For time difference calculations, the uncertainty is the
root of the sum of the squares of uncertainty for each time measurement, so that time
delay uncertainty is estimated to be

$$\delta_{delay} = \sqrt{1.2^2 + 1.2^2} = 1.7 \text{ nsec}$$

(C.3)

For the type III unit cell time delay measurements, signal and trigger traces are
captured by the Gould DSO at each time delay position (Figure C.2), usually averaged
over 4 or 8 acquisitions. Q-switched 532 nm Nd:YAG pulses (risetime 5 nsec) are used.
Similar fast detectors (risetime ≤1 nsec) are used for signal and trigger channels. No
amplification is used. Neutral density (ND) filters are used to cut down the power
Figure C.2: Time delay taken with the type III unit cell device and the Gould oscilloscope.
of the Nd:YAG pulses. Trigger beam and time delay beam attenuation is different, since the time delay beam suffers more loss propagating through the cell. Filters were changed to keep detector output below 300 mV.

The raw voltage traces from the signal and trigger detectors are downloaded to a computer. Time delay is then determined using an algorithm that calculates the time difference between the half maxs of the trigger pulse and the time delay pulse in each case. Using the raw data reduces the error due to trigger pulse jitter.

At greater time delay values, the delayed signal is noisier and there is more uncertainty in the time estimation. To estimate a bound on uncertainty, we used bounce 19 as the worst case and determined an estimate of horizontal uncertainty, based on signal variation and scope error. We measured the vertical spread at pulse peak for bounce 19 to be 20 mV (scale not shown on the plot) and the slope at halfmax to be 1.2X10^8 V/sec, which gives a horizontal uncertainty of ± 0.08 nsec. We include the 3% error in the DSO time base as multiplicative uncertainty and the resolution error as additive.

The resolution uncertainty is estimated from the standard deviation of a uniformly distributed random variable with standard deviation \( \sigma_{res} \), where

\[
\delta_{res} = \sigma_{res} = \frac{\text{range}}{\sqrt{12}}
\]

\[
= \frac{200 \ \text{psec}}{\sqrt{12}}
\]

\[
= .058 \ \text{nsec}
\]

\[ \text{(C.4)} \]

\[ \text{(C.5)} \]
Using the derivative method of uncertainty estimation, we first get an estimate on uncertainty for a single time trace:

\[
\left( \frac{\delta_{\text{time}1}}{\text{time}} \right)^2 = \left( \frac{\delta_{\text{trace}}}{\text{time}} \right)^2 + \left( \frac{\delta_{\text{accur}}}{\text{time}} \right)^2
\]

\[
\left( \frac{\delta_{\text{time}1}}{\text{time}} \right)^2 = \left( \frac{\delta_{\text{trace}}}{\text{time}} \right)^2 + \left( \frac{\text{time base accur}}{\text{time}} \right)^2
\]

\[
\left( \frac{\delta_{\text{time}1}}{7.3} \right)^2 = \left( \frac{0.08}{7.3} \right)^2 + \left( \frac{0.03(20)}{7.3} \right)^2
\]

\[
\delta_{\text{time}1} = 0.6 \text{ nsec}
\]

Then we add the resolution error as the root of the sum of the squares:

\[
\delta_{\text{time}} = \sqrt{\delta_{\text{res}}^2 + \delta_{\text{time}1}^2}
\]

\[
\delta_{\text{time}} = \sqrt{0.058^2 + 0.6^2}
\]

\[
\delta_{\text{time}} = 0.6 \text{ nsec}
\]

With this value for single measurement time uncertainty, we estimate the uncertainty for time difference calculations as root of the sum of the squares of the single measurement uncertainty:

\[
\delta_{\text{delay}} = \sqrt{\delta_{\text{time}(a)}^2 + \delta_{\text{time}(b)}^2}
\]

\[
= \sqrt{0.6^2 + 0.6^2}
\]

\[
= 0.9 \text{ nsec}
\]

This results in a time delay uncertainty for this method of ± 0.9 nsec.

C.3 Voltage Data

The second type of measurement is voltage measurements used to calculate relative optical power ratios. Absolute power values and laser pulse shapes are not needed,
only linear response to power input. Therefore, uncalibrated large detectors are used to reduce alignment and light collection errors. Only the Gould DSO is used to take these voltage measurements and two identical detectors larger than the associated beams (risetime $\leq 2$ $\mu$sec) are used without amplification for the reference and experimental channels. For this type of measurement, we are comparing amplitudes of detector output pulses in relative ratios such as reflection and transmission. The linear range of the detectors was determined as described in Appendix B and taken to be 215 mV to 8 mV, but most signal outputs were adjusted to be well between these voltage extremes.

Changes in the Nd:YAG laser source output power between measurements will distort the power ratio calculations. To reduce this effect, a fixed detector larger than the beam receiving a fraction of the laser beam power prior to entering the apparatus is used as a reference value with which to normalize each measured voltage value. Each measured signal voltage is divided by the concurrent laser power reference reading.

Neutral density filters are used to adjust the cell output beam power to stay within the linear range of each detector. Uncalibrated attenuation filters are used on the reference detector, but are fixed in value throughout a set of measurements and do not affect the ratios. Calibrated filters must be used with the signal detector so that attenuator adjustments between measurements can be accounted for in power ratios. Several neutral density filters are calibrated for experimental use as described in Appendix B.
The voltage measurements are done using the DSO's internal datum and cursor lines. The datum is set to the pre-pulse signal level and the scope math processor is used to find the maximum value in the pulse trace.

We define the following values: \( I \), the input signal peak pulse voltage; \( I_R \), the laser reference peak pulse voltage for the input measurement; \( O \), the output signal peak pulse voltage; \( O_R \), the laser reference peak pulse voltage for the output measurement. Thus each ratio calculation is a product as in the following example:

\[
Reflection = R = \frac{O}{O_R} \frac{I_R}{I}
\]  

(C.9)

We now consider uncertainty for voltage power ratio calculations. The error in the voltage measurement is the vertical accuracy plus the additive error of the AD resolution [88]. Using the derivative method of uncertainty estimation, then for a product function such as the ratio above, uncorrelated uncertainties add as follows:

\[
\frac{\delta_R^2}{R} = \frac{\delta_O^2}{O} + \frac{\delta_I^2}{I} + \frac{\delta_{I_R}^2}{I_R} + \frac{\delta_{O_R}^2}{O_R}
\]  

(C.10)

Given the Gould's accuracy specification of 3\% full scale represents 3\( \sigma \), then 1\% full scale is \( \sigma \) [91]. We assume an average of \( \frac{5}{8} \) full-scale for a typical 100 mV pulse, on a scale of 20 mV/div with eight divisions. The voltage accuracy uncertainty is then \( \delta_{accu} = \frac{8}{5} \sigma = 1.6\% \). The quantization uncertainty is estimated from the standard deviation of a uniformly distributed random variable with standard deviation \( \sigma_{quant} \).
For a voltage measurement, we have additive error so we find uncertainty as the root of the sum of the squares of the uncertainties of DSO accuracy and quantization. Typical uncertainty for voltage measurements is then

\[ \delta_{voltage} \approx \sqrt{\delta_{accur}^2 + \delta_{quant}^2} \]  

\[ \approx \sqrt{(0.016(100))^2 + 0.6^2} \text{ mV} \]  

\[ \approx 1.7 \text{ mV out of 100 mV} \]

For the uncertainty in each reference term, we use only the quantization error. Finally, using all these uncertainties, we get a total uncertainty in a typical ratio calculation with Equation C.10:

\[ \frac{\delta_R}{R} = \sqrt{\frac{\delta_O^2}{O} + \frac{\delta_I^2}{I} + \frac{\delta_{IR}^2}{IR} + \frac{\delta_{OR}^2}{OR}} \]  

\[ = \sqrt{1.7\%^2 + 1.7\%^2 + 1.7\%^2 + 1.7\%^2} \]  

\[ \approx 3\% \]

Thus we have an estimated uncertainty of ±3% for each power ratio calculation, e.g. reflection, transmission, polarization ratio, etc.
BIBLIOGRAPHY


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