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Design and Development of Enhanced Ground Penetrating Radar Systems for the Detection and Classification of Unexploded Ordnances and Land Mines

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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1997

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ABSTRACT

Ground penetrating radar (GPR) has wide applications. This dissertation discusses two important applications which have been gaining a lot of attention recently worldwide. They are unexploded ordnance (UXO) and land mine detection and classification. Although there exist many UXO and land mine detection systems, most of them face high false alarm rates due to the lack of proper target classification techniques. In this document, the complex natural resonance (CNR) signature is used for both UXO and land mine classification. The theoretical CNR's of real world UXO's are calculated numerically using a body of revolution moment method code. The precessed results from actual field data also demonstrate the feasibility of such classification technique.

Two GPR systems dedicated to the detection and classification of UXO and land mine, respectively, is presented. Actual field data collected from these systems are analyzed. A new type of GPR antenna design – Horn-Fed Bowtie (HFB), is found in an effort of overcoming problems commonly faced by many other GPR antennas. This new design leads to many improvements such as stability, sensitivity, gain,..., etc. as shown in the measured data.

Two new data processing techniques – the late-time spectrum (LTS) and surface clutter reduction (SCR) concepts are also developed to improve the UXO and land mine detection and classification capability. The former improve the signal-to-clutter
ratio by removing the surface scattering from the data. The latter calculates the resonant spectrum efficiently using the late-time impulse response. Both two techniques can also be applied to many other GPR applications.

In order to understand more about the dielectric object resonant mechanisms related to land mine application, the CNR behavior of a lossless dielectric sphere immersed in a lossless medium is investigated analytically and numerically. The CNR's of dielectric spheres are obtained from the complex poles of the well known Mie solution. This study shows that both internal and external resonances exist. However, only the internal ones are practical significance because of their higher Q values. It is also found that one may be able to enhance the internal resonances by increasing the dielectric contrast between the target and its ambient medium.
To my family
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CHAPTER 1

INTRODUCTION

Electromagnetic ground penetrating radar (GPR) has a wide variety of applications in geology, archeology, civil engineering and military clearance. The performance of a GPR system depends on the application goal, the target types, the environment, the measurement technique, data processing and final interpretation. A GPR system that works well for all applications does not exist. An optimum GPR system can only be obtained via a careful customized design. Although there are a variety of GPR systems, this dissertation focuses only on near-surface systems as illustrated in Figure 1.1, where the sensor is located close the air-ground interface. Each system can be divided into a radar, sensor and data processing units. Note that these three units may not be physically separated. In order to better understand GPR system issues, a quick overview is given below.

1.1 Overview about GPR System – Practical Considerations

When designing or choosing a GPR system, one must consider both theoretical and practical aspects. Some of the fundamental ones are listed below:

- target type
Ground Penetration Radar (GPR) 

Radio Frequency Interference (RFI) 

\[ \varepsilon_r = 1 \quad \sigma = 0 \]

Rough Surface

Figure 1.1: Configuration of a near-surface GPR system.
• detection
• classification
• medium properties
• depth
• data collection
• processing requirement
• radar unit physical constraints
• environment conditions
• operator training
• budget.

Every item listed above is closely related to the design and selection of the radar approach, radar operation mode, antenna, measurement technique, processing technique, ...., etc. These will all be discussed further in the following sections.

Target Type

Information about the target to be detected is very important for selecting a proper sensor type, measurement approach and processing method. Four major issues are discussed below:

• Material - The material contrast between the target and its environment plays an important role in choosing the sensor type. For example, a plastic mine
with dielectric constant of 3 buried in dry sand with dielectric constant of 4, will be difficult to detect for a radar system, better known as GPR. It is even invisible for a magnetometer system. In this case, an acoustic radar might be more sensitive if one can couple the energy into the medium.

- **Dimensions** – In many applications, the sensitivity and frequency range are related to the target dimensions and depth. For example, a larger conductive target has lower resonant frequencies than a smaller one. In some applications such as UXO detection, the target depth is also related to the target dimension.

- **Shapes** – The shape feature of a target is helpful for detection and identification in image processing. For GPR, an optimum radiated polarization is related to the target shape and orientation. For example, most UXO’s have elongated bodies which scatter strongly linearly polarized signals, which is very different from underground layered structures.

- **Hazardous or Non-Hazardous** – In designing and applying a GPR system for UXO, mine or radiative waste, safety is an important issue.

**Detection and Classification**

1. **existence** – The goal of the measurement is to detect the presence of the target. High signal-to-noise and signal to clutter ratios are the major goal in system design.

2. **location** – A radar which is aimed at not only detecting but also classifying a target requires both high sensitivity and high resolution. A radar's resolution can be enhanced via a high gain sensor arrangement (array) or multiple look
data (synthetic aperture array, SAR). Using a sensor array has the advantage of performing real time detection. SAR does not need a large sensor array and, thus, is more compact and less expensive.

3. **classification** – This type of application relies on good sensitivity, fine resolution and smart processing.

**Medium Properties**

1. **surface roughness** – A GPR system located close to the ground surface is subjected to performance changes at different heights. A low directivity system located away from the surface will suffer from a poorer signal-to-clutter ratio (SCR) due to surface roughness scattering. A high directivity system away from the surface may pick up big surface reflections when a terrain spot is normal to the direction of the incoming beam.

2. **stability** – The medium properties often change with environment such as weather and seasons. This fact can affect the performance of a radar which is optimized under certain assumed constant conditions.

3. **homogeneity** – If a target is located within an inhomogeneous medium with respect to the radar source, any inhomogeneity will scatter and become a source of clutter. Detection of the target becomes more difficult in a low signal to clutter ratio environment. Two common types of inhomogeneities are the discrete layered structures and the continuous property variation as a function of depth.
4. isotropy – When the medium is anisotropic; i.e., its properties are a function of propagation direction, multiple propagation modes may occur. Sea ice structures, represents an anisotropic medium for electromagnetic wave propagation due to the crystal structures. This can cause some location ambiguities. For electromagnetic waves, the electric field polarization may also change as it propagates along an anisotropic medium. Almost all current GPR systems are designed for isotropic medium applications.

5. soil loss – When energy propagates through a medium, the power density per unit area decreases because of beam spreading and medium loss. If the medium has finite conductivity, the electromagnetic energy is lost to heat while it propagates. The medium loss also affects the maximum detection depth.

Depth

Depth specification is an important radar parameter. The common rule is, of course, the deeper the better. However, one must be more practical in specifying the maximum detection depth.

1. fixed depth or variable depth – Most of the current GPR systems are used for detecting targets at variable depths. The range of depth is directly related to the propagation time range. This should be considered in setting the radar's pulse repetition frequency (PRF). The slower the PRF the longer it takes to collect the data. If a frequency scan radar is used, the frequency step should be small enough to cover the desired time range. The total frequency bandwidth also determines the range resolution.
For some particular applications, only targets located at a certain depth are wanted. For these applications, a special system design could be used to enhance the performance. For example, a focused beam could be used to focus the energy to increase the SNR, SCR and detection depth. Hardware time domain gating can also improve spectral domain SNR and SCR.

2. maximum depth – The maximum depth is determined by the propagation loss, system sensitivity, noise level, clutter level, interference level, system output power and processing gain.

Data Collection and Processing Requirements

1. data collection speed – The data collection speed is determined by the radar sampling rate, PRF, position updating speed, data transfer rate, data writing rate and audio/video output speed (if any). UXO detection is an example where faster data collection speed is desired because of the large survey area.

2. data storage and transportation – A potential problem using a fast data collection rate is the storage of huge amounts of data. This needs to be considered in order to have an un-interrupted measurement. An auxiliary storage device could be used when the primary storage device is full. For real time applications, only a small amount of data needs to be temporarily stored for audio/video output or processing. In this case, dynamic RAM is ideal because of its fast data access speed.

3. data processing speed – Data processing usually becomes the most time consuming part of the whole system if it is done in software. It is also critical for
real time applications. Parallel processing is one way of reducing the processing time.

Physical Constraints

1. **size and weight limitations** – A GPR with high resolution or deep penetration normally has large dimensions due to the array arrangement and low frequency requirement. An oversized or overweight GPR system may be useless for some applications where terrain is an important factor.

2. **sensor location** – Most GPR systems must be located as close to the target as possible, which can reduce the propagation loss and increase the energy coupled to the target. Subsurface GPR such as a bore-hole GPR which requires inserting the antenna into the ground is a destructive type. Most applications require a non-destructive measurement where the sensors are close but not on the ground surface. There are three major limitations for such systems. First, surface roughness can make the radar hard to move at high speed with its sensor very close to the surface. Second, the variation of height between the sensor and the surface may change the sensor’s radiated energy which is very hard to calibrate out. Third, some applications such as a radar for detecting anti-personnel mines, require a minimum separation between the surface and any part of the radar. Notice that if a radar is optimized with its sensor on or very close to the surface, its performance may become very poor when the sensor is moved away from the surface.
Environment Conditions

The environment in which the GPR is to be used should also be considered during the design phase. For example, in a city area, the vibrations due to the traffic, various heat sources, lots of iron material, the available measurement space and radio frequency interference (RFI) can create extra limitations to GPR system performance.

Operator Training

To optimize the performance of a GPR system requires knowledge of the radar limitations, various operational methods and operator's experience. The complexity and time involved in training a common user have always been important to a consumer. Therefore a GPR system should be automated as much as possible. Among them, data storage, data calibration and data processing are major items.

The author has been involved in a variety of GPR applications such as underground pipe detection, unexploded ordnance (UXO) and land mines detection. Through the course of study, a good antenna, a good data processing and a good operator have always been the keys to successful GPR operation. A good operator does not need to understand the details about the antenna design or the data processing algorithm but he (or she) needs to know their limitations and optimum operational modes. Another very important thing often neglected is the combination of antenna design and data processing. A new processing technique may lead to a new antenna design and visa versa. This is often neglected due to the lack of good communication between the data processing people and antenna engineers or because of over estimating the power of signal processing. Being open minded also plays an important role. For the same target detection problem, there may be many different types
of sensors attacking different physical phenomena associated with the target and its environment. Each has its advantages and limitations. By a proper combination of different types of sensors, the final result may far exceed what each individual system can achieve. Fortunately, this has become a common understanding in UXO and land mine detection. Again, a "new sensor" which is the combination of different sensors has opened up a whole new window for the signal processing community. On the other hand, the maturity of neural network and fuzzy logic in signal processing also make data fusion among different sensors practical. However, what is the best way to do the data fusion? There may be no answer to this question. To the author, feature fusion seems to be the most efficient way. Features obtained from an individual type of sensor reveal one physical property of the target and its environment. By including different physical properties, one can improve the target detection and classification. It is this belief that keeps the development of individual type of sensor going. The study to be discussed in this document was motivated by the same belief.

UXO and land mine clearance have gained a lot of attention after the "cold war". This is based on both military safety and humanitarian considerations. The problem of UXO clearance arises because of the large amount of UXO's buried under the ground at military munition proving grounds. These uncleared sites become hazardous for any land development. The research for UXO clearance has been progressing for decades with limited success. A common problem encountered in UXO and land mine detection is the high false alarm rate. A unique feature provided by a GPR system is its potential to perform target classification, which may greatly reduce the false alarm rate. Chan et. al. in an earlier study successfully utilized complex natural resonance (CNR) information to identify anti-tank mines [10]. Recently, it
also has been shown that a CNR technique can also be applied for UXO classification [12]. The use of the same technique to identify the anti-personnel mines is also under investigation.

A UXO detection and classification system called the Subsurface Ordnance Clearance System (SOCS) developed by the ElectroScience Laboratory, Battelle and Naval EOD Technical Division (NAVEODTECHDIV) is presented in Chapter 2. The classification technique is based on the extracted complex natural resonances (CNR’s) using the total least square (TLS) Prony’s method. Theoretical results for full-scale UXO’s are calculated using a body of revolution moment method (BORMM) code. A practical UXO classification yields an estimate of the UXO length from its CNR information. Successful UXO classification examples from actual measured data are presented. Although successful UXO detection and classification has been obtained from the data collected using existing GPR antennais, there are some common disadvantages. These are low directivity, ground coupling and antenna ringing. Its improvement leads to a better GPR antenna design to be discussed in next chapter.

Chapter 3 presents a novel GPR antenna design called the Horn-Fed Bowtie (HFB) antenna. This new design features higher gain, less ring, improved stability and sensitivity, reduced weight and size, wider frequency bandwidth and reduced ground interface reflection when compared with common GPR antennas. The comparison of measured results clearly showed that the new antenna is superior to the existing SOCS antenna.

The ESL land mine detection and classification GPR system, or the ESL Land Mine Radar for short, is presented in Chapter 4. This system features a focused beam to avoid the energy spreading for stand-off measurements. The OSU-ESL compact
range radar [9] now operated in a frequency band from 1 to 6 GHz is used as the source. CNR of land mines are obtained from the backscattered data for classification purposes. The first prototype antenna illuminated the ground at the Brewster angle to reduce the air-interface reflection. However, the elongated illumination area was found to suffer from severe clutter that overlapped in time with the target return. The second prototype which illuminates the ground at normal incident was found to have a big advantage of being able to separating the target from surface clutter despite of its higher clutter level.

Land mine detection and classification is much more complicated than the UXO problem. Most land mines are much smaller than UXO's and are buried at shallow depths. Their non-metallic bodies make them difficult to detect because of the poor electrical contrast with the ambient medium. This also results in highly damped natural resonances, which are more difficult to obtain from processing. However, as will be shown in Chapter 5, with proper processing, one can still obtain the resonance spectrum and use it for classification. Measured data for a particular AP mine (Lory) buried at different depths is used to demonstrate the feasibility of land mine detection and classification.

In Chapter 5, the Surface Clutter Reduction (SCR) and the Late-Time Spectrum (LTS) data processing are introduced. The former helps reduce surface clutter by synthesizing the clutter and then removing it from the data. The latter obtains the natural resonant frequencies by applying the Fourier transformation to the late-time signal. The SCR processing improves the target detection capability. The LTS processing provides a efficient way of getting the target's resonant signatures. Both processing technique have been combined to the data collected from the ESL Land
Mine Radar. The results has shown great improvement in land mine detection and classification performance.

For highly conductive GPR targets, such as UXO’s, the CNR classification technique has shown reasonable success. However, for dielectric GPR target, such as plastic land mines, the problem becomes much more challenging due to the much more complicated resonance phenomena. In order to understand the resonance behavior associated with a dielectric target immersed in a dielectric medium, a dielectric sphere target is chosen due to its simple geometry and well known scattered fields. In Chapter 6, the complex resonance poles associated a dielectric sphere are closely investigated. The dielectric constant of the sphere may be less or greater than that of the ambient medium. Each leads to quite different resonance behavior. The resonances of special cases such as perfect electrical conducting (PEC) sphere, perfect magnetic conducting (PMC) sphere, PEC cavity and PMC cavity are also examined. It will be shown that the dielectric sphere resonances are closely related to the resonances of those special cases. Both internal and external resonances were found and some of them may be strongly coupled. It will also be shown that the resonance spectrum and the damping behavior can be explained using simple physical interpretations.

Finally, the study is concluded in the last chapter where possible future efforts are addressed.
CHAPTER 2

THE DETECTION AND CLASSIFICATION OF BURIED UNEXPLODED ORDNANCE (UXO)

Buried UXO clearance has gained increased attention worldwide. Although most UXO's are constructed of steel and might be detected using metal detectors or magnetometers, large amounts of shrapnel scattered around the contaminated area will result in high false alarm rates. The need for a reliable UXO classification is beyond question. In order to penetrate deeper into a lossy ground, a radar must operate at a relatively low frequency band (< 300 MHz). This prevents the image techniques from being a good UXO classification tool due to its low resolution. Although an electromagnetic induction sensor shows promise in detecting UXO’s, its target classification capability is still under investigation [4, 7].

Straight wires produce a very strong resonance at specific frequencies when the wire length is a multiple of a half wavelength, particularly in terms of late time scattering characteristics. Consequently, Hayes [27] used the CNR’s of such wires to measure the earth’s electrical parameters. In his paper, the CNR’s are to be used to measure the length of the wire (or UXO) and this becomes a UXO signature. The polarization response of the UXO also contributes to its identification. It has been shown in the literature that the late time target response from a transient radar
can be expanded using a singularity expansion method (SEM), [5], [32], [36], [39], etc.. Each non-zero singularity corresponds to a CNR. Many other CNR signature applications can also be found in the literature. Tesche used the SEM to find the CNR poles for a thin wire scattering in free space [50]. Michalski and Pearson applied the SEM to characterize a loop antenna immersed in a lossless and homogeneous medium [35]. Chan, et al [10], [11] utilized the CNR signatures to discriminate a plastic anti-tank mine and some simple-geometry metallic targets buried at shallow depths for different media. It is observed that these resonances are more highly damped and thus discrimination becomes more difficult for plastic targets. In their study, an empirical formula was also proposed to predict the CNR change when a target is moved from free-space into a different non-ferrite simple lossy medium with known electrical parameters. That formula agrees with the exact relation derived later by Baum [6].

Two well known advantages of using the CNR signatures for target classification are their target orientation independence and the small number of the required characterizing parameters. Although the CNR signatures (resonant frequency and damping rate) are orientation independent, the strength of each CNR is affected by the target orientation as well as the excitation source. Vitebskiy and Carin [52] also found that when a conducting wire is buried close the air-ground surface, the wire-interface interaction can change the CNR signature. Most UXO's have a simple geometry and the scattered fields can be easily calculated using any numerical modeling technique, such as BORMM, finite element method (FEM) or finite difference time domain (FDTD) method. Therefore, their free space CNR signatures can be easily obtained using any CNR extraction method.
Conventional Prony method for CNR extraction is known to perform poorly in the presence of noise. An improved CNR extraction technique, the total least square TLS-Prony method, proposed by Yu [43] will be used instead for CNR extraction in this paper. The TLS-Prony method greatly reduces the variance of calculated CNR for data contaminated by random noise. Another complex pole extraction technique, the matrix pencil method [44], also gives better results than the conventional Prony method for noise corrupted data. The Cramer-Rao bound of a CNR in noise has also been derived by Steedly and Moses [47]. Although each CNR contains both resonant frequencies and damping factors, our study of UXO's buried in a lossy medium indicated that the damping factor is very sensitive to noise or clutter and is not reliable for practical UXO classification purposes. The UXO signature used herein is the imaginary part of the complex frequency extracted from the late time scattered field.

Finally, it is observed that there is likely to be a substantial amount of shrapnel present. This may add additional resonances but will not modify the UXO resonance as long as it is not too close to the UXO. A researcher was once asked if a shell that simply split down the middle on explosion could be discarded. The obvious answer was "you better dig it up". The conclusion of course is that if the shrapnel retains its original shape and length/diameter ratio, it must be treated as a live UXO.

2.1 Theoretical Background

2.1.1 Late Time Impulse and CNR Signature

As shown by Morgan [36], the impulse response from a perfectly electrical conducting (PEC) target can be expressed as
\[ \tilde{H}(\bar{r},t) = u(t-t_0) \sum C_n(\bar{r})\tilde{h}_n(\bar{r})e^{st} + \tilde{H}^s(\bar{r},t) + \tilde{H}^{PO}(\bar{r},t), \]  

(2.1)

where \( t_0 = |\bar{r}|/c \), \( \tilde{h}_n(\bar{r})e^{st} \) are the natural modes which satisfy the source-free wave equation, \( \tilde{H}^s(\bar{r},t) \) is the startup scattering of the natural modes, \( \tilde{H}^{PO}(\bar{r},t) \) is the physical optics scattered field. Both \( \tilde{H}^s(\bar{r},t) \) and \( \tilde{H}^{PO}(\bar{r},t) \) constitute the early-time response and contain only singularities of the excitation source. These natural modes start to reverberate across the target after the transient incident field passes by the target. These modes stabilize after the multiply reflected currents form these damped oscillations. For complex targets or targets containing highly-damped CNR's, it is difficult to separate the early-time and late-time responses. In this case, the CNR extraction process is then forced to fit the early-time, non-resonant signals into the SEM which results in erroneous CNR's. Note that Equation 2.1 does not consider the presence of noise and radio frequency interference (RFI). Fortunately, the UXO response is not highly damped. Both noise and RFI will appear in the early-time region as well as the late-time region. Since noise is a non-resonant signal it causes the same problem as previous early-time signals. RFI introduces extra non-damped CNR's, which could be separated if they are previously identified. Any clutter appearing in the late-time region will also cause error. If the the excitation source contains poles, such as antenna ringing and cable reflections, it also adds additional non-target CNR's to the final spectrum. Proper calibration or filtering is usually used to remove them. Noise, RFI, clutter and antenna ringing are four major error sources in applying the CNR classification technique.

To show how the CNR signatures are related to the geometric signatures of a test target, the backscattered fields from PEC wires are studied using the following
Figures. Figure 2.1 shows the scattered field results for an obliquely incident plane wave illuminating a 3-foot long conducting wires with different L/D ratios. As the wire becomes thicker, the resonance frequency decreases and the peak becomes broader (larger damping factor). The change of the target's shape changes the induced current distribution and results in different CNR's. Figure 2.2 plots the oblique incident radar cross sections (RCS) for straight wires with three different lengths. The length-to-diameter (L/D) ratio is fixed at 500 for this case. The global magnitude variation is due to the early-time scattering and the local resonance peaks are due to the late-time resonances. The location and width of each peak are determined by the resonant frequency and damping factor of each CNR. Harmonic CNR's are also observable in each case. Distinctive peak distributions clearly indicate that they are different targets. If the incidence angle changes, the shape and location of each peak will remain the same but the magnitude will change. The corresponding transient responses are also shown in Figures 2.3–2.5. The transient response of the 3-foot wire obviously contains a lower frequency components than the 1-foot or 2-foot one and its resonances also lasts longer in time. The object now is to extract the resonant frequencies and the damping factors of all modes contained in these transient responses.

2.1.2 Time Domain CNR Extraction

Consider a time domain waveform, \( f(t) \), which can be expressed as

\[
f(t - t_0) = \sum_{m=1}^{N} c_m e^{s_m(t-t_0)} u(t-t_0)
\]  

where \( s_m = \alpha_m + j2\pi f_m \) are poles in the complex frequency plane, \( \alpha_m, f_m \) and \( c_m \) are, respectively, the damping factors, resonant frequencies and complex amplitudes associated with the \( m \)th CNR mode. Unlike the \( s'_m \), the \( c'_m \) are functions of both target
Figure 2.1: Free space RCS of a 3-foot PEC wire with different diameters.

Figure 2.2: Free space RCS of PEC wires with different lengths (L/D=500).
Figure 2.3: Transient response of a 1-foot long wire.

Figure 2.4: Transient response of a 2-foot long wire.
Figure 2.5: Transient response of a 3-foot long wire.

orientation and excitation source. note that $u(t)$ is a unit Heaviside function, and $t_0$ marks the beginning of late time response. From now on, $t_0$ is assumed to be zero for convenience without losing in generality. Since $f(t)$ is real, all $s_m$ can be paired into complex conjugated pairs. Discretizing Equation 2.2, one obtains that

$$f((n-1)\Delta t) = \sum_{m=1}^{N} c_m e^{s_m(n-1)\Delta t}, \quad n = 1, \ldots, L,$$

(2.3)

where $\Delta t$ is the sampling period which should satisfy Nyquist sampling criteria to avoid higher mode aliasing. There are a total of $N$ CNR's and $L = 2N$ is the number of data points. The above equation will be rewritten in the following simpler form for discussion convenience:

$$f_n \equiv f((n-1)\Delta t) = \sum_{m=1}^{N} c_m z_m^{n-1}, \quad n = 1, \ldots, L,$$

(2.4)

where $z_m = e^{s_m\Delta t}$. The above complicated non-linear curve fitting problem was solved by Prony [42] by realizing that $f_n$ also satisfies the following autoregressive difference
equation
\[ f_n = - \sum_{m=1}^{N} a_m f_{n-m} \quad \text{for} \quad N + 1 \leq n \leq 2N. \]  
(2.5)

The coefficients \( a_m \) can be solved using \( 2N \) data points, \( f_1, f_2, \ldots, f_{2N} \), and the following matrix equation,
\[
\begin{bmatrix}
  f_N & f_{N-1} & \cdots & f_1 \\
  f_{N+1} & f_N & \cdots & f_2 \\
  \vdots & \vdots & \ddots & \vdots \\
  f_{2N-1} & f_{2N-2} & \cdots & f_N
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2 \\
  \vdots \\
  a_N
\end{bmatrix}
= -
\begin{bmatrix}
  f_{N+1} \\
  f_{N+2} \\
  \vdots \\
  f_{2N}
\end{bmatrix}. 
\]  
(2.6)

Applying complex Z-transform to the above difference equation, it is easy to show that the coefficients \( a_m \) are also the coefficients of the following \( N \)th order characteristic polynomial
\[ 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N} = 0. \]  
(2.7)

The roots for the above equation then give the poles \( z_m = e^{\omega_m \Delta t}, m = 1, 2, \ldots, N \).

Using Equation 2.4, the residues, \( c_m \) can then be found by solving the equation,
\[
\begin{bmatrix}
  1 & 1 & \cdots & 1 \\
  z_1 & z_2 & \cdots & z_N \\
  \vdots & \vdots & \ddots & \vdots \\
  z_1^{N-1} & z_2^{N-1} & \cdots & z_N^{N-1}
\end{bmatrix}
\begin{bmatrix}
  c_1 \\
  c_2 \\
  \vdots \\
  c_N
\end{bmatrix}
= 
\begin{bmatrix}
  f_1 \\
  f_2 \\
  \vdots \\
  f_N
\end{bmatrix}. 
\]  
(2.8)

The above three-step procedure (Equation 2.6–2.8) for extracting poles, \( z_m \), and the residues, \( c_m \) is called the Prony method. If \( L > 2N \), Equation 2.5 is simply a linear forward prediction for \( N + 1 \leq n \leq L \). In this case, coefficients \( a_m \) can be solved in a least square error sense. This modified Prony method is known as the extended Prony method or LS-Prony method [28], [34]. The least square error solution can be obtained by solving \( (F^H F) a = F^H f \). Note that LS-Prony method is equivalent to the covariance method of linear prediction as shown by Marple [33] and Kay [29]. LS-Prony has been shown to gives better results than the original
Prony method in the presence of noise. Tufts and Kumaresan further reduced the pole variance by using the principle eigenvalue method to solve a matrix of lower rank, \( M (M < L) \) in a least square error sense. The noise effect is reduced by using the singular value decomposition (SVD) method to separate the noise effect associated with the insignificant eigenvalues from the pole calculation. The TLS-Prony method proposed by Yu [43] further improves the Kumaresan-Prony method by treating the data matrix \( F \) and vector \( f \) simultaneously in the least square error process. A rigorous statistical analysis of TLS-Prony method was done by Steedly et al. [48]. When the signal contains known component and noise, the pole variance can be further reduced by using the linearly constrained total least squares LCTLS-Prony method [17]. Throughout the rest of this dissertation, TLS-Prony method will be used for CNR extraction.

2.2 CNR’s From UXO Models Obtained From Computed Scattered Field

In this section, the CNR’s of four different UXO in free space will be calculated via computational modeling. All UXO’s are assumed to be perfectly electrical conducting (PEC). First, the free-space back scattered fields from each UXO is calculated using BOR MM algorithm. For example, one can refer to the geometry shown in Figure 2.6, where a full-scaled computational model of an UXO #2083 (see inserted photo in Figure 9). The transient response can be obtained by Fourier transforming the calculated frequency domain data and is shown in Figure 2.7. The TLS-Prony method is then applied to the late-time response to extract CNR’s which are shown in Figure 2.8 along with the CNR’s of three other UXO’s. Time – frequency behavior of CNR signatures can also be obtained by applying the TLS Prony’s method to each
data subset within a moving window. Figures 2.9–2.12 plot the processed discrete CNR as a function of time for four selected UXO's. Each has different weight, shape and length. In each plot's header, "W" is the window width, "Nor()" is the 0 dB normalization factor and "(L, NS)" are the parameters for the TLS-Prony method. Different gray scale levels represent different CNR magnitude. The impulse response is also given at the bottom of each figure. Quite different CNR signatures are observed in the late-time region for different UXO's. As the UXO's length increases, the resonant frequency of the fundamental mode decreases. Harmonic resonances are also visible if they are strong enough. For UXO #2083, the odd modes are not visible due to weak excitation. The gray scale shows that the magnitude of each CNR decays exponentially as time increases. This damping factor behavior is implicit in each CNR plotted as a function of time. Two artificial resonances are introduced at 50MHz and close to 950MHz because of the discontinuities of spectrum. It is also found that, in all cases, the half wavelength resonant lengths corresponding to the lowest CNR modes for UXO #2083, #2082, #2077 and #2075 are 30.0, 54.5, 13.3 and 8.6 inches, respectively. The actual lengths are found to be 25.4, 39.0, 11.3 and 7.5 inches, respectively. It is interesting to notice that a greater difference is observed for the UXO #2082 which has a curved body. The resonant length is 140% of its actual length (front tip - to - back tip). This is because of the longer curved path along the UXO body. As one might expect, the calculated poles become unstable in the early-time region where the non-resonant specular reflections and diffractions dominate.

Once the free space CNR's are obtained, one can predict the corresponding CNR's when the same UXO is buried in a lossy simple medium using the following Baum's
transformation:

\[ S = -\frac{\sigma}{2\varepsilon_0\varepsilon_r} + \left[\frac{\sigma}{2\varepsilon_0\varepsilon_r}\right]^2 + \left(\frac{S^{(0)}}{\varepsilon_r}\right)^2 \right]^{1/2}. \]

Note that \( S^{(0)} = \alpha_0 + j\omega_0 \) is a free space CNR pole and \( S = \alpha + j\omega \) is the corresponding pole in a medium of permittivity, \( \varepsilon_r \), and conductivity, \( \sigma \).

### 2.3 Measurement Result from Subsurface Ordnance Clearance System (SOCS)

An actual UXO classification measurement example is given in this section. The data was collected using the Subsurface Ordnance Clearance System (SOCS) owned by the Naval Explosive Ordnance Disposal Technology Division (NAVEODTECH-DIV), as shown in Figure 2.13. The autonomous vehicle houses the main controller, the global positioning system (GPS) and the power generator and is designed by
Figure 2.7: Calculated transient response of UXO #2083.

Figure 2.8: CNR poles extracted from numerical UXO models.
Figure 2.9: CNR versus time (UXO#2083).
Figure 2.10: CNR versus time (UXO#2082).
Figure 2.11: CNR versus time (UXO#2075).
Figure 2.12: CNR versus time (UXO #2077).
Figure 2.13: Picture of SOCS during an autonomous survey.

the Robotic division of the Wright Laboratory at the Tyndall AFB. Both magnetic and GPR sensors are located at the rear of SOCS. The GPR sends out a 250-volt, 3 nano second impulse through a cross-bowtie antenna (see figure 2.14) constructed and tested by the author at the ElectroScience laboratory [40]. This cross-polarization arrangement is designed to minimize the direct antenna coupling as well as to avoid picking up the reflections from the ground surface and any layered structure in the ground. The antenna is operated in frequency from 20 to 200 MHz. Although the antenna itself can be used to collect both co-polarized and cross-polarized data. The current SOCS radar only collects the cross-polarized data. SOCS is design to survey a field autonomously using pre-planned paths at a speed of approximately 1.5 acres/hour[8]. The data to be shown here was measured at the UXO test site located at the Wright Laboratory, Tyndall AFB. This is a pile of 200x200x6 cubic feet
Figure 2.14: Existing SOCS cross-bowtie dipole antenna.
of white beach sand sitting on top of an existing beach. Three calibration thin metallic pipes and various UXO were buried using different depths up to 6 feet at various orientations. The relative permittivity and conductivity of the sand at the time of measurement were 5 and 0.004 mho/m, respectively. The collected data was first processed by Battelle using 3D synthetic aperture array (SAR) method to obtain a 3D image [25]. The energy at all depths up to 6 feet was then added to form an 2D energy gray scale map, as shown in Figure 2.15, where known UXO locations are marked by small circles. Each UXO's orientation is also shown by a line with the length of the line being four times the actual UXO length for visual reasons. Good detection is clearly indicated by the high correlation in the overlay. Some targets located the lower right corner were not detected simply because the system didn't go over them. Some detected anomalies such as the lower left corner were later found to be unknown targets such as metallic box, metallic bar and can, etc. Note that the survey directions were taken to be east-west and south-north, any UXO oriented 45 degree with respect to the survey direction would have weak cross-polarization responses because of the special antenna arrangement. Figure 2.16 plots an ensemble of the measured data as the radar moved across three known buried metallic pipes whose layout is also shown in Figure 2.17. The detection of these pipes are clearly indicated by three groups of arcs. It is noticed that each group shows different oscillation periods in the late-time portion of the signal. A window is next applied to locate a single target (the right most one in this case) to be classified. This target selection is done automatically by thresholding the three dimensional imaged data. The TLS-Prony method is then used to calculate CNR's for each waveform of the windowed data. The final discrete spectrum is plotted in Figure 2.18 where the vertical scale is
Figure 2.15: Overlay of measured energy gray scale map and the known UXO locations marked by “o”. The projected UXO orientation is indicated by a line and the length of the line is four times of the actual UXO length.
the frequency and the horizontal scale is the position. A distinctive line spectrum is found at a frequency near 75 MHz which corresponds to a three foot resonant length in free space. Table 2.3 compares the estimated lengths for some UXO's found in this test site with their actual lengths. It is noticed that BDU33 has a similar curved body and tail fins as the UXO #2082 discussed in the previous section. As already found in that section, the estimated length was 140% of the actual length. The estimate length is also 140% of the actual length for the BDU33 case. All other
CALIBRATION CYLINDERS BURIED
AT TYNDALL AFB TEST SITE

Figure 2.17: Layout of calibration conducting pipes.

Figure 2.18: Processed resonance line spectrum for 3-foot long pipe buried in sand.
UXO’s listed in the table show good agreement between the estimated lengths and their actual ones.

<table>
<thead>
<tr>
<th>Target Type</th>
<th>Depth (ft)</th>
<th>Dominant CNR</th>
<th>Predicted L(in)</th>
<th>Actual L(in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1' cylinder</td>
<td>1.038</td>
<td>$-0.169 + 1.325i$</td>
<td>12.53</td>
<td>12</td>
</tr>
<tr>
<td>2' cylinder</td>
<td>3.664</td>
<td>$-0.107 + 0.701i$</td>
<td>23.66</td>
<td>24</td>
</tr>
<tr>
<td>3' cylinder</td>
<td>5.896</td>
<td>$-0.062 + 0.478i$</td>
<td>36.75</td>
<td>36</td>
</tr>
<tr>
<td>UXO 2000lb</td>
<td>5.129</td>
<td>$-0.062 + 0.178i$</td>
<td>91.39</td>
<td>90</td>
</tr>
<tr>
<td>UXO 8” shell</td>
<td>2.068</td>
<td>$-0.120 + 0.498i$</td>
<td>33.27</td>
<td>30</td>
</tr>
<tr>
<td>UXO 81mm</td>
<td>2.458</td>
<td>$-0.062 + 0.601i$</td>
<td>20.46</td>
<td>18</td>
</tr>
<tr>
<td>UXO BDU33</td>
<td>4.739</td>
<td>$-0.060 + 0.479i$</td>
<td>34.59</td>
<td>25</td>
</tr>
<tr>
<td>UXO BDU33</td>
<td>4.361</td>
<td>$-0.062 + 0.478i$</td>
<td>34.66</td>
<td>25</td>
</tr>
<tr>
<td>USSR mortar</td>
<td>2.894</td>
<td>$-0.100 + 0.642i$</td>
<td>25.71</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Table 2.1: Comparison of CNR estimated lengths and actual lengths of various targets.

2.4 Conclusion

The application of using the CNR signatures to classify buried UXO’s has been presented in this chapter. The BORMM code has been used to obtain the theoretical CNR’s for the UXO’s. Baum’s transformation has been used to transform a CNR obtained when the target is buried in a lossy medium to a CNR when the target is in free space. Actual measured data for anonymous UXO’s buried in sand was processed to obtain a length estimate of each detected UXO based on extracted CNR signatures. These estimated lengths correlated very well with the actual length and verify the feasibility of using this technique for UXO classification. While most of the UXO’s in this sand media were detected and identified, the remaining UXO’s would have been detected if the antenna had been rotated by 45 degrees. This, of course, is
in itself a signature. Although it was not included in this paper, reasonable success of UXO classification for more lossy soils ($\varepsilon_r = 25, \sigma = 0.07 \text{mho/m}$) was also achieved. Improving the identification capability in more lossy soils is an ongoing study. For lossy and inhomogeneous medium such as wet clay result in poorer signal-to-clutter and signal-to-noise ratios which degrade the CNR calculation. In many practical cases, clutter appearing near the target has been found to cause major errors in extracted CNR's.

Although the current cross-bowtie antenna works reasonably well, several problems have been found during field measurements. First, the antenna weights 200 pounds, which makes it difficult to maneuver. Second, the low gain property of a dipole antenna makes it less efficient and more susceptible to above-ground RFI interferences, even though the antenna is operated close to the ground in order to couple more energy into the ground. In addition, its characteristics becomes very sensitive to the ground condition and make it unstable. In next chapter, a new type of GPR antenna – the Horn-Fed Bowtie (HFB), will be introduced to overcome the above three difficulties.
CHAPTER 3

A NEW GROUND-BASED GPR ANTENNA DESIGN

In the previous chapter, it was pointed out that the current SOCS antenna has faced three disadvantages - heavy weight, low gain and instability. In fact, these are also major disadvantages for most other near-surface GPR antennas. A new type of antenna design was motivated by the intention to overcome the disadvantages associated with the existing SOCS antenna as well as most GPR antennas. This new design combines the advantages of a TEM horn, flat bowtie dipole and dielectric loading to provide more stable impedance matching, higher directivity and greater size reduction. This new concept will be discussed. Measured data from this new antenna will be compared with the data from the previously constructed cross-bowtie dipole antenna [40]. This data will show that this new antenna provides significant improvements.

3.1 Common Problems of GPR Antennas

The four major problems associated with most GPR antennas are: (a) ground coupling (b) ground interface reflection (c) antenna ringing (d) low directivity and RFI shielding. These are illustrated in Figure 3.1.
In order to couple the energy into the ground more efficiently, most GPR antennas are located very close to the ground or even contact the ground. The higher ground permittivity with respect to air tends to pull more energy into the ground. This is similar to a dielectric loaded capacitor in the static case. Since the antenna feed position is so close to the ground, it is also influenced by the ground properties. If one assumes a smooth surface and known homogeneous permittivity, one can carefully match the antenna impedance specifically tuned for such cases. But once the surface is rough or the ground becomes more inhomogeneous, like most real field environment, the antenna's impedance becomes unstable and causes a lot of system problems.

Some antennas are operated at a certain distance away from the surface in order to be less affected by the ground and also to have better mobility. In these cases, the air-ground interface then creates a reflection whose amplitude depends on the relative permittivity contrast between the ground-air interface. This undesired surface reflection not only reduces the antenna efficiency but also obscures the detection of shallow targets. Furthermore, if antenna ringing occurs between the surface reflection and feed, deep target detection will become very difficult.

Antenna ringing (or resonance) occurs when the transmitted signal current bounces back and forth on the antenna. Take a flat bowtie dipole as an example, the antenna is excited at the feed point via a cable. Current then flows along the conducting arms toward the ends and decays as it propagates due to radiation. The rest of the current flows back toward the feed point where partial reflection may occur if the antenna's surge impedance is not perfectly matched to the cable. The feed point reflected currents then flow toward the ends and the whole process repeats again. In the time domain, an impulsive input will be mutually reflected and creates radiated fields of a
damped oscillation nature. From the reciprocal property, an incoming field will also excite similar antenna resonances at the receiving port. Thus, the presence of antenna ringing greatly reduces the time domain resolution and becomes an undesired clutter term for target detection. Further, this behavior occurs after any reflection. Thus, shallow target can produce a signal that obscures the scattering from a deeper target. Lossy material such as resistive cards or absorber can be used to absorb the bouncing currents at the price of decreased efficiency. A better but more difficult approach is to match the antenna's surge impedance to the input cable over the operational band. Then the current flowing back to the feed point will be more completely absorbed. As mentioned earlier, when the feed point is close to the ground, the surge impedance becomes a function of the ground properties which makes it difficult to maintain a good match. In this case, the use of absorbing materials becomes a better choice regardless of its low efficiency.

A bowtie dipole antenna in free space radiates symmetrically with respect to its dipole axis. When the dipole is placed close to the air-ground interface, more energy tends to radiate into the ground. In addition to that, surface waves are also launched at both ends. However, there is still a significant amount of energy radiating into the upper space. This means that the antenna will be sensitive to any close by objects as well as RFI. Although this can be improved by using proper shielding which adds both weight and size to the antenna.

3.2 New Horn-Fed Bowtie (HFB) Antenna Design for Ground-based GPR Applications

Figures 3.2, 3.3 and 3.4 illustrate the geometry of this new antenna. The launcher section is basically a TEM horn filled with a lossless dielectric material. The TEM
Figure 3.1: Antenna interaction with ground interface.
horn is chosen because of its wideband characteristic and that its surge impedance can be easily adjusted by changing the plate angle, horn angle and dielectric loading. The dielectric loading is used to reduce the air-ground reflection by choosing its permittivity close to that of the ground, which is illustrated in Figure 3.5. Another important advantage of the dielectric loading is to increase the effective dimensions of the horn such that a reasonable directivity can be obtained at low frequencies. The ground coupling section is formed by a flat bowtie dipole whose arms are terminated by a tapered resistive R-card to prevent antenna ringing as will be shown latter. The launcher and coupling sections are connected through the transition section which is formed by smoothly bending the walls of the TEM horn into arms of the flat bowtie dipole as shown in Figure 3.2.

The new HFB antenna has the following features:

- **Improved Surge Impedance Stability** :

  Since the feed point is located at the top of the TEM horn and is away from the ground, it is quite independent of the ground properties. It is determined only
Figure 3.3: Top view of horn-fed bowtie dipole.

Figure 3.4: Side view of horn-fed bowtie dipole.
Figure 3.5: (a) Strong air-ground interface reflection when the TEM horn is not filled. (b) Reduced interface surface reflection when the TEM horn is filled with dielectric material which has a permittivity close to that of ground.

by the dielectric constant of the cavity material and the horn geometry. Once matched, it becomes independent of the ground condition and the reflections from the feed point are minimized for all ground conditions.

- **Reduced Ringing**:
  In addition to the tapered R-card termination, the better, more stable surge impedance matching also helps reduce the antenna ring down.

- **reduced interface reflection**:
  A unique feature of the HFB antenna is the use of dielectric loading inside the TEM horn section. This not only improves the low frequency directivity but also reduces the surface reflection by choosing a dielectric constant close to that of the test ground since the reflection for normal plane wave incidence is
approximately given by

\[ R = \frac{\sqrt{\varepsilon_{\text{filling}}} - \sqrt{\varepsilon_{\text{ground}}}}{\sqrt{\varepsilon_{\text{filling}}} + \sqrt{\varepsilon_{\text{ground}}}}. \]  \hspace{1cm} (3.1)

- **Improved RF Shielding:**

  The combination of a directive horn and dielectric loading causes most of the electromagnetic fields to be confined within the horn section resulting in very little leakage. This results in better RFI immunity, which means that no shielding is required. More design details will be discussed in the following sections.

### 3.2.1 Lossless Dielectric Loading

In order to be able to match the permittivity of the ground it is important to find a lossless material that is variable, low cost and reasonable weight. It was found that one could mix distilled water and isopropyl alcohol for this application. The permittivity values of distilled water and isopropyl alcohol are 80 and 18, respectively. By mixing them with different volume ratio, it was found that the mixture permittivity ranges from 18 to 80, which is shown in Figure 3.6. The horizontal scale indicates the (isopropyl alcohol/water) volume ratio. The vertical scale corresponds to the measured dielectric constant at 40 MHz. However, it was found that a liquid material has several practical disadvantages. For example, its volume changes with temperature and it may also change into gas or solid states. A better low loss material is a solid material formed by mixing Barium Titanate (BaTiO\textsubscript{3}) powder and Epoxy. This also provides a wide range of permittivity. Figure 3.7 shows the measured permittivity of BaTiO\textsubscript{3}/Epoxy mixture with different weight ratios.
Figure 3.6: Variation of permittivity for water/alcohol mixture.

Figure 3.7: Variation of permittivity for BaTiO$_3$/epoxy mixture.
3.2.2 Surge Impedance of Dielectric Loaded TEM Horns

A TEM horn formed by two triangular conducting plates, as shown Figure 3.8, has been widely used due to its simplicity and broadband characteristics [53]. A setup for measuring the surge impedance of a TEM horn with dielectric loading is illustrated in Figure 3.9. A monopole of finite length conducting triangular plate is mounted on a finite conducting ground plane. A thin plexiglass cavity filled with the /alcohol/water mixture was inserted in between the conducting plate and the ground plane. An HP8753C network analyzer was then used to measure the reflection coefficient at the feed point. The end effects from the plate and the ground plane were removed using time domain gating. Figure 3.10 shows the measured surge impedance of the horn with a 60 degree plate angle and various horn angles without any dielectric filling. This result agrees with calculated data predicted by Maloney et. al. [31]. Figure 3.11 plots the measured surge impedance of a TEM horn (horn angle = plate angle = 20 degrees) with different dielectric loading using the liquid mixture described earlier. An empirical formula, Equation 3.2, was found to fit the measured data reasonably well for several cases of different plate angles and horn angles. It is given by

\[ R(\varepsilon) = R(\infty) + R(1) \frac{1 + A}{1 + \varepsilon^2} \]  

where \( R(1) \) and \( R(\infty) \) are the resistance values for \( \varepsilon_r = 1 \) and \( \varepsilon_r = \infty \), respectively.

3.2.3 The Effect of Air Gap

In practice, the antenna may not be contacted to the ground but, instead, is raised above the ground, which leaves an air gap. It is important to know the relationship between the plane wave reflection coefficient and the gap height. Figure 3.12 plots the reflection amplitudes versus gap heights in terms of the free space wavelength.
Figure 3.8: Finite length TEM horn.

Figure 3.9: Setup for measuring the surge impedance for a TEM horn.
Figure 3.10: Surge impedance for a 60-degree plate angle TEM horn with different flare angles.
Surge Impedance of A Loaded TEM Horn

Figure 3.11: Surge impedance of a 20 degree TEM horn loaded with different dielectric constant material.

The permittivity of the top and bottom medium are assumed to be the same here and are 4, 9, 16 and 25, respectively. Only a height less than a tenth of wavelength is considered here. As expected, when the height is zero the reflection vanishes and it increases as the height increases. It is also observed that the reflection increases as the ground permittivity increases. This result suggests that the gap needs to be smaller for higher permittivity environments to avoid significant reflections. When the permittivity of the top and bottom medium are not the same, the corresponding reflection versus gap height plot is shown in Figure 3.13. In this case, the minimum reflection coefficient is obtained when the height is zero but it is not a zero level. Note that the reflection is determined by Equation 3.1.
Figure 3.12: The reflection amplitude for different air gap heights ($\varepsilon_1 = \varepsilon_2$ case).
Figure 3.13: The reflection amplitude for different air gap height ($\varepsilon_1 \neq \varepsilon_2$ case).
3.2.4 Prototype HFB Antenna Measurements

Two prototype antennas with the above new features have been built and tested as shown in Figures 3.14 and 3.15. Figure 3.16 shows the cavity made to conform the shape of antenna shown in Figure 3.14. This cavity is then inserted into the horn section to obtain dielectric loading. A liquid water/alcohol mixture with a permittivity of 25 was used as the dielectric filling. The measured data collected at the ElectroScience Laboratory (ESL) slant pipe site (see Figure 3.23) shows that both prototype antenna have at least 20 dB more sensitivity than the original SOCS antennas. In this measurement, a long conducting wire was first inserted into of the buried PVC pipe (see Figure 3.23). The wire was then removed without moving the antenna. The relative permittivity of the ground was about 25 at the time of measurement. The scattered fields from the wire alone was then obtained by subtracting the measured data with and without the wire. Figure 3.17 summarizes the comparison between the current SOCS antenna and the two new prototype antennas. It was also found that the new antenna has less antenna ringing due to its better impedance matching. It should be noted that a tapered resistive card was not used for either of the prototype antennas.

3.3 New HFB Antenna Measurements

Recently, a refined HFB antenna was constructed for field measurements. Its geometry is illustrated in Figure 3.18. A solid BaTiO₃ mixture with a relative permittivity of 9 was then used to fill the TEM horn section. Two detachable tapered R-card extensions are also included. The antenna is mounted on a sled which is used to tow the antenna along the ground surface. A photo of the new antenna can be
Figure 3.14: First (large) prototype of new horn-fed bowtie-dipole antenna built at ESL.
Figure 3.15: Second (small) prototype of new horn-fed bowtie-dipole antenna built at ESL.
Figure 3.16: Removable cavity used for large prototype new antenna.
<table>
<thead>
<tr>
<th></th>
<th>CURRENT SOCS ANTENNA</th>
<th>LARGE PROTOTYPE NEW ANTENNA</th>
<th>SMALL PROTOTYPE NEW ANTENNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>WEIGHT</td>
<td>285 lb.</td>
<td>18 lb. (unloaded)</td>
<td>12 lb. (unloaded)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>69 lb. (loaded)</td>
<td>25 lb. (loaded)</td>
</tr>
<tr>
<td>HEIGHT</td>
<td>18 in.</td>
<td>57 in.</td>
<td>22 in.</td>
</tr>
<tr>
<td>LENGTH</td>
<td>5 ft.</td>
<td>6 ft.</td>
<td>5.3 ft.</td>
</tr>
<tr>
<td>SENSITIVITY</td>
<td>0 dB (co-pol.)</td>
<td>+10 dB (unloaded)</td>
<td>+13 dB (unloaded)</td>
</tr>
<tr>
<td></td>
<td>-10 dB (cross-pol.)</td>
<td>&gt;=15 dB (loaded)</td>
<td>&gt;=18 dB (loaded)</td>
</tr>
<tr>
<td>IMPEDANCE STABILITY</td>
<td>ground dependent</td>
<td>nearly constant</td>
<td>nearly constant</td>
</tr>
<tr>
<td>INTERFERENCE</td>
<td>need shield</td>
<td>no shield</td>
<td>no shield</td>
</tr>
<tr>
<td>BANDWIDTH</td>
<td>20 - 200 MHz</td>
<td>20 - 200 MHz</td>
<td>20 - 200 MHz</td>
</tr>
<tr>
<td>ANTENNA RINGING</td>
<td>-10dB / 10ns</td>
<td>-20dB / 10ns</td>
<td>-20dB / 10ns</td>
</tr>
</tbody>
</table>

Figure 3.17: Comparison between the current SOCS antenna and the new prototype antennas.
The impulse response associated with this new HFB antenna is shown in Figure 3.20, which was measured using the previously described slant pipe and long wire setup. The top and bottom figures correspond to the time and frequency domain responses, respectively. The responses from the existing SOCS antenna are also overlayed for comparison. As one can see, the new HFB antenna has better responses in terms of sensitivity, bandwidth and flatness. The flat frequency response indicates that the antenna does not ring. All these are desirable features for impulse radars and GPR applications.

As already mentioned, a unique feature of the new HFB antenna is the use of dielectric loading to reduce the surface reflection arising from the air-ground interface. This is demonstrated by the measurement result shown in Figure 3.21, where the reflected signal was recorded with the antenna sitting on the floor and raised by 18 inches. When the antenna is raised, the permittivity mismatch occurring at the antenna aperture creates a reflection as indicated by the dashed line. When the antenna is sitting on the floor, better permittivity continuity between the dielectric filling and the ground greatly reduces the surface reflection as indicated in these results.

As mentioned earlier, a resistive card (R-card) can be used to avoid the current being bounced back from the ends of the antenna. A R-card with exponential tapering from 3 Ω to 200 Ω was used to terminate the bowtie dipole. A reflection measurement with the antenna raised 18 inches above a concrete floor was done to compare the cases with and without R-card. These results are shown in Figure 3.22. It is observed that
Figure 3.18: Newly constructed HFB antenna with solid dielectric filling made of BatO$_3$/Epoxy mixture. A 0-180 hybrid is used as a balun to feed the antenna.
Figure 3.19: Photo of newly constructed HFB antenna.
Figure 3.20: Measured impulse response of the new HFB antenna using a long wire and slant PVC pipe setup.
Figure 3.21: Surface reflection variation caused by moving the antenna off the ground.
the use of R-cards indeed reduce the late time signal level, which is related to the antenna ringing. It should be noted that neither the resistance profile nor the R-card length is optimized for the current design. Additional reduction should be expected by using a longer R-card section.

The radiation pattern is another important characteristic for any antenna design. For GPR applications the radiation pattern is also determined by the ground properties and needs to be measured in the presence of the ground. To find out the radiation pattern for the new HFB antenna, the setup shown in Figure 3.23 was used. The ground medium is dry sand with a relative permittivity around 4. A long dipole antenna was inserted into a PVC pipe which was approximately 3 feet below the surface. The HFB antenna under test was then moved across the PVC pipe along a straight line on the surface with its polarization parallel to the dipole. At each position, the received signal by the HFB antenna was recorded as a function of frequency. By doing so, a two dimensional data set was obtained which contains the frequency versus position information. This gives the H-plane pattern of the HFB antenna. The frequency domain data can also be transformed into the time domain by applying a Fourier transformation. The final result is shown in Figure 3.24. A broad H-plane pattern is observed in this measurement, which is desirable since one can then have multiple looks at a target as the antenna passes over it.

To compare the sensitivity of the new HFB antenna with that of the existing SOCS antenna, a long conducting wire is used as the reference target as described earlier. The two antennas were used to measure the wire response. The measurement was done in a dry sand environment. The results are shown in Figure 3.25. As one can clearly see, the new HFB antenna received a signal more than 15 dB higher
Figure 3.22: Reflection measurements of the new HFB antenna with and without the extended tapered R-card section. The antenna is raised 18” above a concrete floor.
Figure 3.23: Buried slant PVC pipe setup for static measurement (top) and pattern measurement (bottom).
Figure 3.24: One-way transmission pattern measured from the setup shown in previous figure.
than that obtained by the existing SOCS antenna did. It should be reminded that the SOCS antenna measures the cross-polarized response. This increase in this sensitivity is due to the better directivity provided by the TEM horn section as well as being co-polarized. Both antennas were also used simply as receivers to monitor RFI, the results showed essentially the same signal levels. This suggests that the new HFB antenna performs as well as the SOCS antenna in terms of RFI shielding although the latter uses R-card, absorber and conducting walls to provide shielding. This again is attributed to the better performance achieved using the new HFB antenna.

The next comparison between the new HFB and SOCS antennas is the clutter level. First, measured data from the SOCS antenna evaluated using the results shown in Figure 3.26 where the dashed line plots the wire minus no wire result of the slant pipe measurement and solid line plots the unsubtracted wire data. Notice that the raw data is dominated by the direct antenna coupling which is less than -30 dB because of the orthogonal polarity design between the transmitting and receiving dipole. The decaying tail is attributed to the antenna resonance which was found to be around 50MHz. Although the direct coupling is reasonably low and the ringing is dropping reasonably fast, it is still not low and fast enough such that at the time when the target response arrives (dashed line) the signal to antenna clutter ratio is basically unity. If the antenna term is stable enough during the actual measurement where the antenna is moved along the ground, most of the antenna term can be subtracted out using an ensemble average subtraction approach. As discussed early, it would require the antenna to be away from the surface for the results to be stable; however, this results in less energy being coupled into the ground and a reduction of sensitivity. The reason for the long tail is as follows. Let's assume that the antenna
Figure 3.25: Sensitivity comparison of new HFB antenna and the existing SOCS antenna in sand environment.
ring down is 100dB per cycle, which is very ideal. As stated earlier, the antenna’s
resonant frequency is about 50 MHz because of its length. The resonance cycle is
then 20 ns. Then the ring down is only 100/20=5 dB/ns. It is interesting to see
that although the antenna has large ring down per cycle by , perhaps, well matching
and proper resistively loading the antenna’s length results in effectively a slow ring
down. This seems to suggest that the antenna should be either rather short or rather
long. A very short antenna results in fast decaying per nanosecond provided the
attenuation/cycle can be maintained. A very long antenna avoids the second ringing
peak appearing within the time of interest. The above assumes that one can still
achieve 100 dB/cycle decaying. In practice, a very short antenna has low radiation
efficiency, resistive loading does not work well and a fairly good match may be -20
dB. This means that a very short antenna tends to have smaller ringing decaying per
cycle, which then results in slower decaying per nanosecond. How long is long enough?
It is determined by the radars dynamic range, P (dB) soil loss, A (dB/m) and soil
dielectric constant, $\varepsilon_r$. The maximum target range $R$ defined by the SNR=1 criteria
can be expressed as

$$R = \frac{P}{2A} \quad (m). \quad (3.3)$$

The target time range, $T$, is then given by

$$T = \frac{2R\sqrt{\varepsilon_r}}{c}. \quad (3.4)$$

Then the antenna’s half length, $L_H$, should be longer than $T \times c$ (meters), i.e.,

$$L_H > \frac{P}{A\sqrt{\varepsilon_r}}. \quad (3.5)$$

The above assumes that the wave propagating along the antenna arm has the velocity
of free space. For example, if a radar has 90 dB dynamic range, the soil loss is 50
dB/meter and the dielectric constant is 25, then $L_H$ must be longer than $\frac{90}{50} \times 5 = 7$ meters. The total length of the antenna will then be 14 meters! The smaller the soil loss or the higher the soil dielectric constant, the longer the antenna. However, a long antenna which satisfies the above criteria is not convenient for field tests and is impractical.

Figure 3.27 compares the raw data and the wire-no wire response collected using the new HFB antenna. The antenna clutter appears to be much higher than that of SOCS antenna due the feed mismatch reflection. At the arrival time of target response (dashed line) the signal to antenna clutter ratio is slightly less than that of SOCS antenna although the target response is much higher than what obtained using SOCS antenna as shown in Figure 3.25. The higher antenna clutter is caused by the presence of reflection at the antenna feed. This is normal for co-polarized reflection measurements. Recall that SOCS antenna performs the transmission measurement where the transmitting and the receiving antennas are separated. The orthogonal arrangement helps greatly reduce the coupling between the antennas. As will be shown shortly, although the new HFB antenna has a higher clutter level than SOCS antenna, it is very stationary and thus can be removed easily. This is because the feed point is designed to be away from ground influence. On the other hand, although the SOCS antenna has lower clutter level, it is not stationary due the the ground coupling. Therefore, the processed results from the new HFB antenna show less clutter.

The new HFB antenna has been actually mounted on the SOCS chasis to perform field test at Tyndall AFB UXO test facility. Two UXO's were buried in sand. The first UXO was buried close to the surface, and the second was buried at one foot depth. The antenna was moved across the two UXO along a straight line. The
Figure 3.26: Impulse response of wire (solid line) and wire-no wire (dashed line) using SOCS antenna.
Figure 3.27: Impulse response of wire (solid line) and wire-no wire (dashed line) using HFB antenna.
processed results are shown in Figure 3.28. The horizontal scale is proportional to
distance and the vertical scale is the signal time delay which is proportional to the
depth in downward direction. Two targets are clearly observed. The first shallow
target shows a much stronger response than the second one. The same measurement
was also performed using the existing SOCS cross-bowtie antenna for comparison.
The result is shown in Figure 3.29. One can also clearly see the two test targets. This
preliminary test shows comparable detection ability for both the new HFB antenna
and the old SOCS antenna; however, the latter weights almost four times heavier than
the former. Comparing Figure 3.28 with Figure 3.29, the former seems to have less
clutter. This can be seen better by examining the raw data as shown in Figures 3.31
and 3.30. In Figure 3.31, the first big peak corresponds to the SOCS antenna’s direct
coupling occurring at the feed points. As one can see, this coupling term is changing
significantly from location to location. On the other hand, the first negative peak
in Figure 3.30 corresponds to the feed reflection of the new antenna, which remains
fairly independent of locations. This stability improvement is expected in the HFB
design and is achieved by raising the feed point away from the ground interface (see
Figure 3.2).

3.4 Conclusions

A performance comparison between the new HFB antenna and the existing SOCS
antenna is summarized in Table 3.4. The surge impedance in the current HFB design
was not optimized, but even so, its reflection coefficient is about -15 dB. This can
be improved by changing either the horn angle or the plate angle. The relative
permittivity of the current dielectric filling was 9, which may not be the best choice for
GPR Data From Scan #1 to #99 (ps26203a.gpr)

Figure 3.28: Measured data using the new HFB antenna for two UXO's buried in the sand.
Figure 3.29: Measured data using the old SOCS antenna for two UXO's buried in the sand.
Figure 3.30: New HFN antenna raw data

Figure 3.31: Old SOCS antenna raw data.

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SOCS Antenna vs New HFB Antenna

<table>
<thead>
<tr>
<th></th>
<th>SOCS Antenna</th>
<th>New HFB Antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>200 pounds</td>
<td>50 pounds</td>
</tr>
<tr>
<td>size</td>
<td>60”×60”×18”</td>
<td>18”×71”×21”</td>
</tr>
<tr>
<td>Matching</td>
<td>-20dB (varying)</td>
<td>-15dB (fixed)</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>–</td>
<td>15–20 dB better</td>
</tr>
<tr>
<td>RFI Level</td>
<td>-60 dBm</td>
<td>-60 dBm</td>
</tr>
<tr>
<td>10dB Bandwidth</td>
<td>100 MHz</td>
<td>250MHz</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison between the current SOCS antenna and the new HFB antenna.

high dielectric ground such as wet clay. Changing the permittivity requires changing either the horn or the plate angle to maintain a good impedance match. It was found that the BaTiO$_3$/Epoxy mixture forms a strong solid filling which performs much better than a liquid mixture. It is well known that a cross-bowtie dipole antenna has advantages such as low antenna coupling, low ground layer background and full polarization capability over a single flat bowtie dipole. Therefore, an interesting extension of the current HFB antenna design is to design a cross-polarized version of the HFB antenna. This remains as a topic for future development.
CHAPTER 4

ESL LAND MINE DETECTION AND CLASSIFICATION SYSTEMS

In Chapter 2, a UXO detection and classification radar system was introduced. The UXO classification using a CNR technique was also discussed. For that application, highly conductive targets and their resonances were studied. In this chapter, a land mine detection and classification system developed by the ElectroScience Laboratory is presented. The classification of different land mines will be based on their resonant frequencies. The previous UXO research can be applied directly except that anti-personnel (AP) mines are usually shallower and their resonance frequencies are much higher. However, more and more land mines are made of plastic to avoid being detected by simple metal detectors. In addition, their resonance mechanisms are far more complicated than metallic targets. Unlike metallic UXO's whose resonances are external to the structure, both internal and external CNR's exist in plastic land mines. This will be studied in more details in Chapter 6. Earlier plastic anti-tank mine research was done by Chan et. al. [10]. They also demonstrated that the internal resonances are dominant for at least one dielectric mine in an earth environment. The dielectric property difference between the plastic mines and the surrounding medium makes them detectable using a ground penetration radar (GPR). The ElectroScience
Laboratory (ESL) and Battelle are currently developing a stand-off land mine radar (LMR) system for the Office of Special Technology (OST). The present approach was proposed by Dr. Leon Peters Jr. and will be discussed in this chapter. Measured data from the prototype system shows promising detection capability. Since there are many other non-metallic objects, such as rocks, distributed ground voids and roots, existing in the natural environment, GPR systems for this application are plagued false alarms. Another current research goal, therefore, is to apply the CNR technique to non-metallic target classification and thereby, reduce this false alarm rate problem.

4.1 The First Prototype of ESL Land Mine Radar (LMR) System

The ESL LMR system features compact range radar [9], ultra wideband (UWB) [30] feed antenna and a focused reflector concept as illustrated in Figure 4.1. The radar is operated at a 1-6 GHz frequency range. The reflector diameter is 28”. The focus spot size at 2 GHz is about 6” for the -6 dB contour and 10” for -10 dB contour. A major advantage of the LMR system is the reduced surface clutter because of reduced illumination area. A focused beam also prevents the energy from spreading out. The surface reflection is minimized by choosing the incident angle close to the Brewster angle. More detail description about the system can be found in [13] and [45].

A prototype system was mounted on an ATV to perform field tests, as shown in Figure 4.2. Figure 4.3 shows an example of measured data taken while moving the vehicle-mounted antenna focused reflector along the ground surface. There are three targets located at the 22, 70 and 140 inch positions. These targets are a 4-inch diameter, half inch thick rock, an SB33 mine and a Lory mine, respectively. All three targets are sitting on the ground surface which is covered by 4 inch tall grass. The
data was collected using a frequency sweep system over 1-6 GHz frequency band. The center ray of the focused beam is at about 45 degree incident angle with a two way focussed spot ranging from approximate 20" to 4" for 1 to 6GHz, respectively. It is seen that the surface grass creates significant scattering. One can also notice the vertical variation of the scattering band. This is due to the terrain altitude variation which changes the distance between the antenna and the illumination spot. The curve shown at the bottom of plot is the accumulated energy, $\sum |z[n]|^2$, for each scan. It is interesting to see the correlation between the high energy peaks and the true target locations except for the SB33 target whose signal is buried under grass scattering signals.
Figure 4.4 shows the results after applying the autocorrelation to the previous data shown in Figure 4.3. Since the terrain surface is usually not flat, the time delays from different ground spots are different. This can be corrected by applying autocorrelation to each scanned data. After processing, the maximum response is always located at the zero lag position. For a sampled data sequence, $x[n]$, the corresponding autocorrelation function, $r_{xx}$, is typically defined as

$$r_{xx}(k) = \frac{\sum_{n=0}^{N-1-k} x[n]x^*[n]}{\sum_{n=0}^{N-1} |x[n]|^2}.$$

(4.1)

It is observed that different delays shown in the previous figure have been corrected. Also, a lower energy floor is shown in the bottom curve due to the reduction of noise. Since surface clutter due to the grass, specular or small pebbles have shorter time responses, after this time delay adjustment, it is much easier to separate the surface clutter (top portion of the figure) and the late time signals due to the ringing of targets. This late time resonant behavior will be discussed more in Chapter 5.

Figure 4.5 replots the data shown in Figure 4.4 in log scale. The horizontal line shows the break point between early and late time signals. For now, this break point is chosen empirically such that most of the early time scattering band corresponds to surface clutter and specular reflections is removed. This choice will be automated by calculating the energy density as a function of time and applying a proper threshold. The data below the horizontal line is then processed to obtain the power spectral density which is simply the Fourier transform of its autocorrelation function, $r_{xx}(\tau)$ (see Equation 4.1) [46]. This result is shown in Figure 4.6. The dark regions correspond to the dominant resonant frequencies of the targets. It is observed that a rock with similar size as the mine, also shows resonance frequencies within the same band as
Figure 4.2: Prototype LMR system mounted on an ATV.
Figure 4.3: Time-domain waterfall plot of multiple scan data BEFORE applying autocorrelation processing.
Figure 4.4: Time-domain waterfall plot of multiple scan data AFTER applying autocorrelation processing.
Figure 4.5: Autocorrelated data in log scale for selecting the tail portion of the data.

mine. The middle target (SB33) is invisible in the final result because it’s dominant resonance frequency is high and it died out too fast. If one moves up the break point to include more early time data, it will show up along with the surface clutter.

Raw data of another example is shown in Figure 4.7, where the targets sat on an unpaved road surface which is full of small rocks. Mine SB33 and Lory were located at positions 25 and 90, respectively. A 2 inch long thin rock was also put between the mines. Compared to Figure 4.3, it is noticed that early clutter have reduced significantly since there was no grass in the current case. The corresponding autocorrelated data and the final PSD result are also shown in Figures 4.8 and 4.9,
Figure 4.6: The final power spectrum for the late time portion of signals.

respectively. These results clearly show the resonant structures for both the SB33 and Lory land mines. Each of them have distinctive resonant frequency combinations.

Although the previous data has shown good detection and classification for land mines sitting on the surface, several disadvantages were also discovered. First, the down range location of the focussed spot is very sensitive to the tilt angle. Second, the spot location is also sensitive to the terrain variation. Third, the clutter caused
Figure 4.7: Raw data of targets sat on an unpaved road bed.
Figure 4.8: Autocorrelated data of targets sat on an unpaved road bed.
Figure 4.9: Final LTS result for targets sat on an unpaved road bed.
by rough surface scattering extends in time domain due to different arrival time of the
different rays. All three disadvantages are the direct result of the large incident angle
of the original prototype. Although, the Brewster angle minimizes the reflection for
a plane interface, the surface roughness scattering causes a severe clutter problem.
This is especially true for buried mines whose weak returns are buried under surface
clutter and are difficult to separate. Furthermore, the contamination in the late-time
data makes it more difficult to extract the resonant signatures. This problem can be
improved by re-orienting the system and thus changing the incident angle, as will be
discussed in the next section.

4.2 The Improved Prototype Land Mine Radar (LMR) System

In previous section, it was found that the low-beam configuration results in a
time-extended surface clutter which obscure the desired signal from buried mines.
That configuration is also too sensitive to the tilt angle and terrain variation. After
re-orienting the reflector and the feed antenna, an improved high-beam configuration
was developed, as shown in Figure 4.10. The new incident angle is about 20 degrees
from the surface normal direction.

To test the new configuration, a plastic anti-personnel mine (Lory) which is shown
in Figure 4.11 was buried at various depths in the ESL dirt box. The whole anten-
na/radar unit then scans over an arc length of 3 feet on the ground and collects
a frequency data set at a 0.7 inch position increment. The OSU-ESL compact range
radar [9] was operated in the 1 to 6 GHz frequency band to collect the data. The data
was then processed to obtain both the envelope versus position and frequency versus
position plots. The soil characteristics for the test site are shown in Figure 4.12.

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Figure 4.10: High-beam reflector setup.
Figure 4.11: Lory mine.
Figure 4.12: Soil characteristics of the ESL dirt box.
It will be shown in Chapter 5 that the surface reflection can be reduced by synthesizing the reflected signal with a priori knowledge of the system response and the shape of the bandpass window applied before transforming the frequency domain data to the time domain. The following results are obtained after the surface clutter reduction process has been applied.

Figures 4.13 to 4.16 show the processed envelope ensemble results for Lory buried at 1", 2", 3" and 4", respectively. The ground surface is located near -6 ns position. All the 1", 2" and 3" deep mines are clearly detected. For the 4" depth, the mine response becomes very weak compared with its environment. The corresponding resonance spectrum obtained using the late-time spectrum (LTS) technique to be discussed in Chapter 5 are also shown in Figures 4.17-4.20. Note that distinctive resonant spectrum peaks are observed at the positions close to where the mine is located. The bright bands between 4GHz and 5GHz are related to the system response which can be calibrated out.

4.3 Conclusions

Two prototype land mine radar systems have been presented. Both systems launch a focussed beam to perform stand-off measurements. This focussed beam concept prevent the energy from spreading out much wider than the mine. Its area-reduced illumination also reduces the surface clutter level and thus improves the signal-to-clutter ratio. The first prototype system used the Brewster angle to minimize surface reflection. This system was found to be too sensitive to the reflector tilt angle and terrain variation. It also suffered from time-extended surface clutter. It has been shown to be able to detect and classify land mines sitting on the surface, where the
Figure 4.13: Envelope ensemble of Lory mine buried at 1" depth.

Figure 4.14: Envelope ensemble of Lory mine buried at 2" depth.
Figure 4.15: Envelope ensemble of Lory mine buried at 3" depth.

Figure 4.16: Envelope ensemble of Lory mine buried at 4" depth.
Figure 4.17: Processed PSD ensemble of Lory mine buried at 1" depth.

Figure 4.18: Processed PSD ensemble of Lory mine buried at 2" depth.
Figure 4.19: Processed PSD ensemble of Lory mine buried at 3" depth.

Figure 4.20: Processed PSD ensemble of Lory mine buried at 4" depth.
target's response dominates. Even so, buried land mines were found to be difficult to detect using this configuration. The second improved prototype system was obtained by re-orienting the reflector and feed antenna such that the focussed beam has a high incident angle, which is about 20 degree with respect to the surface normal direction. Measured and processed data for an anti-personnel mine buried in soil at various depths showed great detection and classification capability using this new system. Even though strong surface scattering exists for this incident angle, the results show that the clutter data collected by the improved prototype can be easily separated and processed out. Furthermore better resonant spectrum performance has been achieved because the late-time portion of data is not contaminated by surface clutter. The processing techniques for reducing the surface scattering and calculating the resonant spectrum will be discussed in the following chapter. Tests of a Such system mounted on human are currently under investigation (see Figure 4.21).
Figure 4.21: ESL LMR system mounted on human.
Two new processing techniques – the Late-Time Spectrum (LTS) technique and the Surface Clutter Reduction (SCR) technique are introduced. The LTS processing calculates the resonant spectrum by Fourier transforming the late-time portion of data, where the natural resonances dominate the response. The SCR processing technique reduces the early-time clutter by synthesizing the clutter and then removing it from the data. Both processing techniques were developed while processing the data collected using the new Land Mine Radar (LMR) system. The processed results have shown great detection and classification improvement.

5.1 Late-Time Spectrum (LTS) Processing Technique

In Chapter 2, the complex natural resonance (CNR) signature have been used for UXO classification with reasonable success for low loss medium like sand but with very limited success for high loss medium. In that application, the CNR’s were extracted using the TLS-Prony method [43]. The study showed that damping factors are too sensitive for any practical applications. Large damping factors also reduce usable data length and make it difficult to separate the specular scattering from the
resonant scattering. Although the TLS-Prony method provides a high resolution spectrum, the presence of non-resonant scattering causes errors in extracted CNR's using the TLS-Prony method. For external resonance targets, such as unexploded ordnance, the damping factors are related to target geometry and ambient medium loss. For internal resonance targets, such as plastic mines, damping factors are related to target shapes and the dielectric contrast between the targets and the surrounding medium. The less the contrast, the higher the damping factor and the shorter the usable data length. The dielectric constant of plastic land mines normally ranges from 2 to 6, which is close to that of dry sand ($\approx 4$) or dry clay ($\approx 12$ to 25). Therefore, the CNR's of plastic land mines are usually highly damped such that the results from the TLS-Prony method becomes highly unstable.

A new simple technique which uses the CNR concept and considers the practical limitations discussed above has been developed at the ElectroScience Laboratory (ESL) for preliminary target classification. This new technique examines the spectrum associated with the resonant portion of the target's impulse response and is called the Late-Time Spectrum Technique (LTS). Significant improvement has been obtained by separating the resonant data from non-resonant clutter response, such as surface roughness, grass and small rocks. Compared to the regular time-frequency processing, LTS method only processes the late time portion of the data once instead of multiple times for each time-shifted segment of the data. In addition, since only FFT is required, LTS can be performed extremely fast which makes real time land mine classification more feasible. Compared to the Prony type CNR extraction techniques, the LTS method gives resonant frequencies with lower resolution but improved stability. It is also much easier to implement an automatic LTS process scheme. One
can also use an autocorrelation processing approach to correct different time delays due to the terrain variation and also to improve the signal to noise ratio in the data.

To emphasize the importance of this early time and late time separation, the scattered fields of a plastic land mine (P4MK1) was measured in a compact range and is analyzed as shown in Figure 5.1 which plots the regular time-frequency distribution. The target was measured in free space over a 2 to 12 GHz frequency band. The incident angle is 70 degree away from the normal direction of mine’s front surface with the electric field parallel to the surface. One can clearly see that the early time \( t < 51 \text{ ns} \) region is dominated by the specular scattering including reflections and diffractions which have wideband response. After 52 ns, the resonance becomes apparent and shows one dominant resonance frequency around 4.5 GHz. Note that some high frequency resonances may exist. Since the damping factors for those higher modes are normally large, they tend to die out quickly and are difficult to find.

5.2 LTS Processing Flow Chart

Figure 5.2 shows the current processing flow chart. First, the frequency sweep data is collected. A built-in hardware time gate is used to isolate the undesired signals arising from any nearby scattering such as antenna mismatch or scattered fields from the reflector’s edges and operator. The data is then transformed into the time domain via typical inverse digital Fourier transform (IDFT). The undesired surface clutter is reduced by preprocessing the transform data which will be discussed in the next section. Here, let us focus on the LTS processing part.
Figure 5.1: Time-Frequency distribution for the P4MK1 mine measured in free space with a 70 degree angle of incident.
Figure 5.2: Late-time spectrum (LTS) processing flow chart.
5.3 Automatic Late Time Selection

In the previous study [13], an autocorrelation processing approach was used to realign the different time shift due to terrain variation. After realignment, the fixed beginning time point of the late time signal is specified based upon experience. A new technique has been introduced to select the beginning time point automatically without using the autocorrelation which is time consuming. This new technique will be demonstrated via the following example. Figure 5.3 plots an ensemble of measured envelope data for the Lory mine buried 2 inch deep. The SCR processed data was collected using the ESL LMR system (see Chapter 4). Comparing this data with the previous data collected using the 70 degree angle incidence, one finds that the buried mine shows up more clearly. Figure 5.4 plots the normalized backward cumulated amplitude curves for positions 15, 25 and 35. For each time waveform, let us calculate the cumulated sum of the signal amplitude backward within a prespecified time range. Then one can normalize this calculated result to unity. The onset point of the late time portion is determined by choosing a threshold ( normally 0.5 – 0.8 ). Figure 5.5 shows the onset of the late time signal for different positions if the threshold is chosen to be 0.5. As one can see, this new technique finds the proper onset point regardless of the terrain variation. After determining the range of the late-time signal, one simply calculates the corresponding frequency spectrum via the Fourier transformation.

5.4 Surface Clutter Reduction (SCR) Processing Technique

As discussed in the earlier section, one needs to select the late time portion of data to obtain the resonance spectrum associated with the target. This needs to be done
Figure 5.3: Ensemble plot of measured envelope data for the Lory mine buried 2 inch deep.

Figure 5.4: Backward cumulated amplitude curves for three different positions.
Figure 5.5: Automatic late-time onset selection using an amplitude cumulated sum technique.

in the time domain. Since the ESL LMR system collects the data in the frequency domain, one has to transform it into the time domain. The frequency truncation at both ends (1 GHz and 6 GHz) modify the true impulse response by convolving it with a \( \sin(x)/x \) type of artificial signal which will be called the "base waveform". This base waveform is introduced when transforming a finite bandwidth frequency response with a unit amplitude and zero phase signal. Therefore, a true impulse time response becomes a \( \sin(x)/x \) response which has many sidelobes on both sides of the position of the impulse. The presence of these sidelobes prevent one from detecting other weaker signals. It is well known that this kind of situation can be improved by applying a proper bandpass window (e.g. Kaiser window) prior to the frequency-to-time transformation. Normally, one would obtain reduced sidelobes. Figure 5.6
Figure 5.6: Frequency response weighted by different Kaiser windows.

plots the curves of different windowed frequency responses. The corresponding time responses are shown in Figure 5.7. If two signals are too close together or if one of them is very weak, it is still difficult to detect in their presence in the sidelobes. This is exactly the situation that occurs to the land mine measurement, where the mines are buried very close to the surface. When the ground is illuminated with a radar beam, the surface will reflect and scatter first and then the target signal will return shortly. When the dielectric contrast between the target and the soil is small, the target response is weak and may be buried under the sidelobes of the surface signal. This new surface reflection reduction processing has been developed to remove most of the undesired reflections arising at the ground surface.
The shape of the mainlobe and sidelobes are determined by the system frequency response and the bandpass window, which is fixed and known. The frequency response of the reflected signal from a flat surface is the same as the illumination source. Therefore, as long as one knows the peak amplitude of the reflected signal, one can synthesize its waveform and then subtract it from the total waveform. This is demonstrated by the results shown in Figure 5.8. The solid line plots the original time response from an actual radar data with the Lory mine buried 2 inch deep. The dash line plots the synthesized response associated with the surface reflection. Figure 5.9 plots the final response after the synthesized surface reflection has been subtracted. As one can easily see, this result provides much clearer responses that can be used to detect deep provided their scattered fields occur in the time slot for which the sin(x)/x signal has been subtracted. This clutter subtraction process can
be repeated until all the early time clutter terms have been removed. It should be noticed that any surface scattering mechanism which has a near-flat spectrum can also be removed by the SCR processing.

The following example demonstrates the associated enhancement in target detection capability. Figure 5.10 shows the measured result (envelope ensemble) for a Lory mine buried 3" deep. One can clearly see the strong surface reflection and the mine response underneath is only barely seen. The discontinuity at the surface reflection is caused by the trench dug when burying the mine. In this case, the signal(mine)-to-clutter(surface) is much less than one. Figure 5.11 plots the envelope after removing the dominant surface reflection term within the -5ns and -7ns time frame. An improved in signal-to-clutter ratio is clearly seen. Notice that there is still strong scattering existing between the -7 and -5 ns time slots. This is due to

Figure 5.8: The original radar data and the synthesized surface reflection.
the distributed illumination caused by oblique incidence. After 5 iterations of SCR processing, the results is shown in Figure 5.12. As one can easily see, the signal-to-clutter ratio has been improved and the buried mine is now clearly visible. When the target is too close to the surface, the processing will also try to remove the target’s specular signal as well, as shown in Figure 4.13. This, however, would not affect the late time response associated with the target resonances.

5.5 Conclusions

Two new processing techniques – the Late-Time Spectrum (LTS) technique and the Surface Clutter Reduction (SCR) technique were introduced. These techniques have been shown to greatly improve the detection and classification of plastic land mines. More examples can also be found in Chapter 4. The SCR processing improves
Figure 5.10: Envelope of the measured data with the Lory mine buried 3" deep before removing the surface reflection.

Figure 5.11: Envelope of the measured data with the Lory mine buried 3" deep after one iteration of SCR processing.
Figure 5.12: Envelope of the measured data with the Lory mine buried 3" deep after five iterations of SCR processing.

The target detection capability by removing surface clutter. The LTS processing provides an efficient way of obtaining the resonant frequencies which improves the target classification capability. Although both LTS and SCR techniques were developed for processing the data collected from ESL LMR system, their applications, however, are much more general than this.
CHAPTER 6

ELECTROMAGNETIC RESONANCES BY DIELECTRIC SPHERES

The CNR behavior associated with a dielectric spheres immersed in a dielectric medium will be investigated here. This study was motivated by the application of plastic land mine classification using the CNR signatures. The sphere geometry is chosen because of the availability of an exact solution. The results generated by this example can then be interpreted in terms of physical mechanisms to provide insight concerning resonances for more general shapes. To simplify the problem, both the sphere and its ambient medium are assumed to be homogeneous, isotropic and lossless. The scattered fields are obtained first by using the well known normal mode expansions and matching the internal and external fields at the boundary. However, as will be shown, the expression of normal mode expansion only describes the three dimensional standing waves, which does not provide much physical insight about individual wave mechanisms. A Poisson sum transformation is then applied to the normal mode expansion to separate different wave mechanism by deforming the integration path to include different poles in the integrand. These poles arise from the zeros of the denominators of the expansion coefficients and are related to the complex resonances. Throughout this chapter, an \(e^{i\omega t}\) time dependence is used.
The resonances for both dielectric spheres and dielectric bubbles are studied. The difference depends on whether the internal permittivity is higher or lower than that of the ambient medium. It will be shown that, the resonances can be separated into external and internal modes. The internal resonances are excited due to the internal bouncing waves which experience multiple reflections. The external modes are actually caused by the surface waves which creep along the sphere outer surface. The behavior of resonant poles for different internal and external permittivity will be investigated. The physical background of different resonances will also be explored by investigating some special cases (PEC or PMC) first. It will be shown that the resonances phenomena for a general dielectric sphere (or bubble) are highly related to these special cases.

Even a simple sphere can have very complicated scattering mechanisms. Rainbow and glory scattering, for example, have been studied for decades by numerous authors [41, 37]. Here, the focus is on resonance phenomena alone. Although Conwell et al [15] have studied the resonant spectra of dielectric spheres, they have not separated the surface wave poles from the internal resonant poles. They also dealt with only one case where the refractive index of the sphere is 1.4 and the ambient medium is free space. A more general situation will be discussed here. When the internal permittivity is greater than the external one, it is a typical dielectric sphere scattering problem. On the other hand, when the external permittivity is greater, it becomes a dielectric "bubble" scattering problem. Four special cases include perfect electric conducting (PEC) sphere, perfect magnetic conducting (PMC) sphere, PEC spherical cavity and PMC spherical cavity. Many authors have studied the surface waves propagating along the surfaces of PEC cylinders or spheres. These particular waves are related to
the zeros of \( H^{(2)}_n(k_d a), H^{(2)'}_n(k_d a) \) (cylindrical) or The resonances of a PEC cylindrical or spherical cavity can also be found in the literature. These resonance associated with internal bouncing waves are determined by zeros of \( J_n(k_d a), J'_n(k_d a) \) (cylindrical case) or \( J_n'(k_d a), J_n''(k_d a) \) (spherical case). Note that for each integer order, \( n \), there are multiple zeros which, in turn, give multiple solutions to \( k_d \) and \( k_x \). The numbers shown on the figures presented later refer to the order number, \( n \). The complex wave number, \( k_d \) and \( k_x \) can then be related to the physical resonant frequency and damping factor via

\[
k = \frac{\omega}{v},
\]

\[
= 2\pi f - j\alpha,
\]

where \( v \) is the phase velocity, \( \omega \) is the complex frequency in radian unit, \( f \) is the resonant frequency in Hertz and \( \alpha \) is the damping factor. Several authors have studied about the resonances of dielectric spheres and some general objects. Franz and Beckmann [21] studied the creeping waves for objects of finite conductivity. Barber et al [3] used the resonance to characterize axisymmetric dielectric objects. An excellent physical and mathematical discussion about the high-frequency scattering by a dielectric sphere is given by Nussenzveig [37, 38].

### 6.1 Scattered Fields by a Dielectric Sphere

Figure 6.1 represents a dielectric sphere immersed in a infinite dielectric medium. The relative permittivity and permeability for the internal and external region are \((\varepsilon_d, \mu_d)\) and \((\varepsilon_x, \mu_x)\), respectively. An \( x \)-polarized and \( z \)-traveling incident plane wave can then be expressed in spherical normal mode expansion as [26],
Figure 6.1: A plane wave incident upon a dielectric sphere.

\[ E_r^i = -j \frac{E_0 \cos \phi}{(k_ar)^2} \sum_{n=0}^{\infty} j^{-n}(2n+1)J_n(k_ar)P_n^1(\cos \theta), \]  \hspace{1cm} (6.2)

where \( P_n^1 \) is the associated Legendre function. Note that \( P_0^1(\cos \theta) = 0 \) and

\[ \hat{B}_n(z) = \sqrt{\frac{\pi z}{2}} B_{n+\frac{1}{2}}(z), \]  \hspace{1cm} (6.3)

with \( B \) representing the cylindrical Bessel functions, \( J, Y \), or the Hankel functions, \( H^{(1)}, H^{(2)} \).

It is sometimes more convenient to use the vector potentials, \( \vec{A} = \vec{r} A_r \) and \( \vec{F} = \vec{r} F_r \). \( \vec{F} \) is used to generate the H mode (TE to r) and \( \vec{A} \) determines E mode (TM to r). The electrical and magnetic fields can then be determined by

\[ \vec{E} = -\nabla \times (\vec{r} F_r) + \frac{1}{j \omega \varepsilon} \nabla \times \nabla (\vec{r} A_r) \]

and

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\[ \vec{H} = -\nabla \times (\hat{r} A_r) + \frac{1}{j \omega \varepsilon} \nabla \times \nabla (\hat{r} F_r). \] (6.4)

In spherical coordinate, one can rewrite the above equations explicitly as

\[
\begin{align*}
E_r &= \frac{1}{j \omega \varepsilon} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) A_r \\
E_\theta &= \frac{-1}{r \sin \theta} \frac{\partial F_r}{\partial \phi} + \frac{1}{j \omega \varepsilon r \partial \theta} \frac{\partial^2 A_r}{\partial \theta^2} \\
E_\phi &= \frac{1}{r} \frac{\partial F_r}{\partial \theta} + \frac{1}{j \omega \varepsilon r \partial \phi} \frac{\partial^2 A_r}{\partial \phi^2} \\
H_r &= \frac{1}{j \omega \mu} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) F_r \\
H_\theta &= \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{1}{j \omega \mu r \partial \phi} \frac{\partial^2 F_r}{\partial \phi^2} \\
H_\phi &= \frac{1}{r} \frac{\partial A_r}{\partial \phi} + \frac{1}{j \omega \mu r \sin \theta} \frac{\partial^2 F_r}{\partial \phi \partial \theta}
\end{align*}
\] (6.5)

and

\[
\begin{align*}
H_r &= \frac{1}{j \omega \mu} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) F_r \\
H_\theta &= \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{1}{j \omega \mu r \partial \phi} \frac{\partial^2 F_r}{\partial \phi^2} \\
H_\phi &= \frac{1}{r} \frac{\partial A_r}{\partial \phi} + \frac{1}{j \omega \mu r \sin \theta} \frac{\partial^2 F_r}{\partial \phi \partial \theta}
\end{align*}
\]

The incident plane wave can then be expressed in terms of magnetic and electrical vector potentials, \( \vec{A}^i = \hat{r} A_r^i \) and \( \vec{F}^i = \hat{r} F_r^i \) such that

\[
A_r^i = \frac{E_0 \cos \phi}{k_z Z_\varepsilon} \sum_{n=0}^{\infty} a_n \hat{n} n(k_z r) P_n^1(\cos \theta)
\]

and

\[
F_r^i = \frac{E_0 \sin \phi}{k_z} \sum_{n=0}^{\infty} a_n \hat{n} n(k_z r) P_n^1(\cos \theta)
\] (6.6)

where

\[
a_n = \frac{j^{-n}(2n + 1)}{n(n + 1)}.
\] (6.7)

The scattered potentials can then be expressed as
external fields \((r > a)\)

\[
A_r^{*+} = \frac{E_0 \cos \phi}{k_z Z_x} \sum_{n=0}^{\infty} b_n \hat{H}_n^{(2)}(k_z r) P_n^1(\cos \theta)
\]

and

\[
P_r^{*+} = \frac{E_0 \sin \phi}{k_z} \sum_{n=0}^{\infty} c_n \hat{H}_n^{(2)}(k_z r) P_n^1(\cos \theta).
\] (6.8)

internal fields \((r < a)\)

\[
A_r^{* -} = \frac{E_0 \cos \phi}{k_z Z_x} \sum_{n=0}^{\infty} d_n \hat{j}_n(k_d r) P_n^1(\cos \theta)
\]

and

\[
P_r^{* -} = \frac{E_0 \sin \phi}{k_z} \sum_{n=0}^{\infty} e_n \hat{j}_n(k_d r) P_n^1(\cos \theta).
\] (6.9)

To simplify the future expressions, we define \(\beta \equiv k_x a\) and \(\alpha \equiv k_d a\). The boundary condition are the continuity of \(E_\theta, E_\phi, H_\theta\) and \(H_\phi\) at \(r = a\), which require that

\[
b_n = -a_n \frac{Z_x \hat{j}_n(\beta) \hat{j}_n(\alpha) - Z_d \hat{j}_n(\beta) \hat{j}_n'(\alpha)}{Z_x \hat{H}_n^{(2)}(\beta) \hat{j}_n(\alpha) - Z_d \hat{H}_n^{(2)}(\beta) \hat{j}_n'(\alpha)}
\]

\[
c_n = -a_n \frac{Z_x \hat{j}_n(\beta) \hat{j}_n'(\alpha) - Z_d \hat{j}_n'(\beta) \hat{j}_n(\alpha)}{Z_x \hat{H}_n^{(2)}(\beta) \hat{j}_n(\alpha) - Z_d \hat{H}_n^{(2)}(\beta) \hat{j}_n'(\alpha)}
\] (6.10)

\[
d_n = -a_n \frac{j Z_x \hat{j}_n(\beta) \hat{j}_n(\alpha) - j Z_d \hat{H}_n^{(2)}(\beta) \hat{j}_n'(\alpha)}{Z_x \hat{H}_n^{(2)}(\beta) \hat{j}_n(\alpha) - Z_d \hat{H}_n^{(2)}(\beta) \hat{j}_n'(\alpha)}
\]

and

\[
e_n = -a_n \frac{-j Z_d}{Z_x \hat{H}_n^{(2)}(\beta) \hat{j}_n'(\alpha) - Z_d \hat{H}_n^{(2)}(\beta) \hat{j}_n'(\alpha)}.
\]

Expressions of \(b_n, c_n, d_n\) and \(e_n\) can be further combined into

\[
f_n(n + \frac{1}{2}, \beta) = -\frac{N e_i \hat{j}_n(\beta) \hat{j}_n'(\alpha) - \hat{j}_n'(\beta) \hat{j}_n(\alpha)}{N e_i \hat{H}_n^{(2)}(\beta) \hat{j}_n'(\alpha) - \hat{H}_n^{(2)}(\beta) \hat{j}_n(\alpha)}
\] (6.11)
and

\[ g_i(n + \frac{1}{2}, \beta) = \frac{-j}{Ne_i \hat{H}_n^{(3)}(\beta) \hat{J}_n^x(\alpha) - \hat{H}_n^{(3)y}(\beta) \hat{J}_n(\alpha)} \]  

(6.12)

where

\[ N = \frac{Z_e}{Z_d} \]

\( TE\) mode: \( e_1 = 1 \), and

\( TM\) mode: \( e_2 = \frac{1}{N^2} \).

### 6.1.1 Relative and Scaling Properties of Poles

Two important physical properties can be drawn from Equations 6.10.

**Relative Property:**

Let us consider the following scenarios:

Scenario 1: \( \epsilon_d = \tau \epsilon_0, \epsilon_z = \epsilon_0, \mu_d = \mu_z \)

Scenario 2: \( \epsilon_d = \epsilon_0, \epsilon_z = \frac{\epsilon_0}{\tau^2}, \mu_d = \mu_z \)

Scenario 3: \( \mu_d = \mu_0, \mu_z = \tau^2 \mu_0, \epsilon_d = \epsilon_z \)

Scenario 4: \( \mu_d = \frac{\mu_0}{\tau}, \mu_z = \mu_0, \epsilon_d = \epsilon_z \)

Scenario 5: \( Z_e = Z_d, \alpha/\beta = \tau \), i.e., \( \frac{\epsilon_4}{\mu_4} = \frac{\mu_4}{\epsilon_4} = \tau^2 \).

**Scenario (1)&(2)**

Since \( N = \frac{Z_e}{Z_d} = \frac{\sqrt{\mu_0/\epsilon_0}}{\sqrt{\mu_d/\epsilon_d}} = \tau \) and \( \frac{\alpha}{\beta} = \frac{\sqrt{\mu_0/\epsilon_0}}{\sqrt{\mu_0/\epsilon_0}} = N \), both cases have exactly the same set of expansion coefficients (Equation 6.10). Therefore, they both have the same set of poles. The characteristic equation is
\[ N e_1 \hat{H}_n^{(2)}(\beta) \hat{J}_n'(N\beta) - \hat{H}_n^{(2)'}(\beta) \hat{J}_n(N\beta) = 0. \]  

\text{Scenario (3) & (4)}

In this case, \( N = \frac{Z_{d}}{Z_{a}} = \frac{\sqrt{\mu_{d}/\varepsilon_{a}}}{\sqrt{\mu_{a}/\varepsilon_{a}}} = \tau \) and \( \frac{\alpha}{\beta} = \frac{\sqrt{\mu_{d}\varepsilon_{a}}}{\sqrt{\mu_{a}\varepsilon_{d}}} = 1/N \). Again these two scenarios share the same set of expansion coefficients (Equation 6.10) and poles. The characteristic equations are

\[ Ne_1 \hat{H}_n^{(2)}(\beta) \hat{J}_n'(\beta/N) - \hat{H}_n^{(2)'}(\beta) \hat{J}_n(\beta/N) = 0. \]  

Although the \( N \) values for Scenarios (1)–(4) are the same, (1) & (2) do not have the same set of poles as (3) & (4).

\text{Scenario (5)}

In this case, although the characteristic impedances of the internal and the external medium are matched, different wave velocities result in scattering and resonances. It is also interesting to notice that both TE and TM modes now share the same set of poles corresponding to the zeros of the characteristic equation such that

\[ \hat{H}_n^{(2)}(\beta) \hat{J}_n'(\alpha) - \hat{H}_n^{(2)'}(\beta) \hat{J}_n(\alpha) = 0. \]  

From the above discussions, one can conclude that the sphere scattering and resonance problem in a pure magnetic or electric environment are determined by the relative permeability or relative permittivity contrast between the sphere and its ambient medium. Their absolute values cause a scaling effect on the complex resonant frequencies, that will be discussed next.
6.1.2 Complex Frequency Scaling Property

The characteristic equation (the denominator in Equation 6.11) can be written as

\[ N \varepsilon_i \hat{H}_n^{(2)}(\omega, \sqrt{\mu_z \varepsilon_z a}) \hat{J}_n(\omega, \sqrt{\mu_d \varepsilon_d a}) - \hat{H}_n^{(2)}(\omega, \sqrt{\mu_z \varepsilon_z a}) \hat{J}_n(\omega, \sqrt{\mu_d \varepsilon_d a}) = 0. \]  

(6.16)

The poles in the complex \( \omega \)-plane can be obtained by finding the zeros of the above equation. Now, let's consider a new problem with

\[ \varepsilon_z = \tau \varepsilon_z, \]
\[ \mu_z = \tau \mu_z, \]
\[ \varepsilon_d = \tau \varepsilon_d, \text{ and} \]
\[ \mu_d = \tau \mu_d. \]

The new characteristic equation becomes

\[ N \varepsilon_i \hat{H}_n^{(2)}(\bar{\omega}, \sqrt{\mu_z \varepsilon_z a}) \hat{J}_n(\bar{\omega}, \sqrt{\mu_d \varepsilon_d a}) - \hat{H}_n^{(2)}(\bar{\omega}, \sqrt{\mu_z \varepsilon_z a}) \hat{J}_n(\bar{\omega}, \sqrt{\mu_d \varepsilon_d a}) = 0, \]  

(6.17)

where \( \bar{\omega} = \tau \omega \). Comparing Equations 6.17 and 6.16, it is easy to see the equivalence between them. One can also conclude that if \( \omega_s \) are the roots for Equation 6.16 then \( \omega_s/\tau \) corresponds to the roots of Equation 6.17. That is, the complex frequencies of the new problem are related to the complex frequencies by a simple scaling factor, \( \tau \).

6.1.3 Standing Wave Representation versus Propagating Wave Representation

In far field region, where \( r \to \infty \), one can then replace \( \hat{H}_n^{(2)}(kr) \) by its large argument approximation such that

\[ \hat{H}_n^{(2)} \to j^{n+1} e^{-jx} \text{ as } x \to \infty. \]  

(6.18)
Then Equation 6.8 becomes

$$F_{r+} = jW \sum_{n=0}^{\infty} \frac{2n + 1}{n(n + 1)} f_1(n + \frac{1}{2}, \beta) P_n^1(\cos \theta)$$  \hspace{1cm} (6.19)

where

$$W = \frac{E_0 \sin \phi}{k_0} e^{-jka}. \hspace{1cm} (6.20)$$

Recall the Poisson sum formula is given by

$$\sum_{l=0}^{\infty} \phi(l + \frac{1}{2}, x) = \sum_{m=-\infty}^{\infty} (-1)^m \int_{0}^{\infty} \phi(\lambda, x) e^{\frac{2\pi \lambda d}{\lambda}} d\lambda. \hspace{1cm} (6.21)$$

Applying the above formula to Equation 6.19, one obtains that

$$F_{r+} = jW \sum_{m=-\infty}^{\infty} (-1)^m \int_{0}^{\infty} \frac{2\lambda}{\lambda^2 - \frac{1}{4}} f_1(\lambda, \beta) P_{n-\frac{1}{2}}^1(\cos \theta) e^{2\pi \lambda x} d\lambda. \hspace{1cm} (6.22)$$

The above transformation from the normal mode expansion to the integration has physical significance. The normal mode expansion organizes the fields in standing wave patterns by adding bounced and diffracted waves whereas the Poisson sum gives the waves themselves. For example, a well known property of the Bessel function is

$$j_{n-\frac{1}{2}}(\pi) = \frac{\tilde{H}^{(1)}_{n-\frac{1}{2}}(x) + \tilde{H}^{(2)}_{n-\frac{1}{2}}(x)}{2}. \hspace{1cm} (6.23)$$

It is obvious that the standing radial wave function on the left-hand side are resulted from the superposition of the incoming and outgoing radial waves. Similarly, it can be shown that [38]

$$P_{n-\frac{1}{2}}(\cos \theta) = Q^{(1)}_{n-\frac{1}{2}}(\cos \theta) + Q^{(2)}_{n-\frac{1}{2}}(\cos \theta) \hspace{1cm} (6.24)$$

which describes the standing angular wave function as a superposition of two angular waves propagating at opposite directions.
The main contributions of the integration in Equation 6.22 are from the singularities such as the saddle point and complex pole in the integrand. The saddle point gives the stationary phase solution which is related to the reflected field. The complex poles are the zeros of the denominator of \( f_i(\lambda, \beta) \). That is

\[
N e_{ij} \tilde{H}^{(2)}_{\lambda - \frac{1}{2}}(\beta) \tilde{J}_{\lambda - \frac{1}{2}}(\alpha) - \tilde{H}^{(2)}_{\lambda - \frac{1}{2}}(\beta) \tilde{J}_{\lambda - \frac{1}{2}}(\alpha) = 0
\]  

(6.25)

where \( i = 1 \) for TE modes and \( i = 2 \) for TM modes. Recall that \( \beta = \omega \sqrt{\varepsilon_\perp \mu_\parallel} \) and \( \alpha = \omega \sqrt{\varepsilon_d \mu_d} = N \beta \). Therefore, the complex resonance frequency, \( \omega \), can then be obtained by solving for the zeros of Equation 6.25. By investigating the behavior of these zeros, one is able to achieve a better understanding about the physics of the dielectric sphere resonances. This is exactly what will be done in the following sections. Some special cases such as a PEC sphere, PEC cavity, PMC sphere and PMC cavity, will be investigated first, which are strongly related to a more general case which will be discussed later.

6.2 Special Cases – PEC and PMC Sphere or Cavities

Resonances of special cases like PEC (or PMC) sphere in a dielectric medium and a dielectric cavity with PEC (or PMC) walls can be immediately obtained from Equation 6.25 by choosing special values for \((\varepsilon_d, \mu_d)\) and \((\varepsilon_\perp, \mu_\parallel)\).

6.2.1 PEC Sphere \((\varepsilon_d \to \infty, \varepsilon_d \mu_d =\text{constant})\)

In this case, \( N \to \infty \) and the external characteristic equations from the denominator of Equation 6.11 are
Using the relation 6.3, the above equations are equivalent to

\[ H^{(2)}(\beta) = 0 \quad \text{(TEr)}, \quad \text{and} \]
\[ H^{(2)'\prime}(\beta) = 0 \quad \text{(TMr)}, \quad \text{(6.26)} \]

which are the same characteristic functions associated with a PEC sphere given in [26].

**Physical Interpretation**

If one solves Equation 6.27 for the complex \( \lambda \) with \( \beta \) fixed, one obtains multiple poles, \( \lambda_i \), known as "Regge poles" [16] in the complex \( \lambda \)-plane. Each pole, \( \lambda_i \) corresponds to the propagation constant of the \( i \)th creeping wave. The real and imaginary parts correspond to the phase and amplitude variation, respectively, along the direction of propagation. The sign of the imaginary parts are chosen to be positive \( (e^{j\omega t}) \) to avoid amplitude catastrophe. The relation between creeping waves and normal modes have been discussed by Dragonette and Flax [18] – "for the modal eigenfrequencies \( \omega_n \), one may state physically from the foregoing that the \( i \)th eigenfrequency of the \( n \)th mode is that frequency at which the speed \( c_i(\omega) \) of the \( i \)th creeping wave coincides with the wave speed \( c_n(\omega) \) of the \( n \)th modal vibration". In their cylindrical case, the speed of the \( i \)th creeping wave and the speed of the \( n \)th mode were defined as

\[ c_n(\omega) = \frac{an}{\omega} \quad \text{and} \quad \text{(6.28)} \]
\[ c_1(\omega) = \frac{a\omega}{\lambda_1}. \] (6.29)

Therefore the modal eigenfrequencies can be found by letting \( c_n = c_1 \).

For the spherical case, the above statement needs to be modified. First, let us consider surface waves which are launched from a point source located at the south pole of a sphere and propagating toward the north pole along the "great circle" paths. All the rays will meet at the north pole and continue propagating toward the south pole, where the next cycle starts. At the north pole caustic, each individual ray experiences a 180° phase advance. Therefore, the phase delay for each ray to finish a round trip is \( \Delta \phi = \text{Re}(\lambda) \times 2\pi - \pi \). The resonant condition then requires that \( \Delta \phi = n(2\pi) \), where \( n = 1, 2, \ldots \), or, \( \text{Re}(\lambda) = n + \frac{1}{2} \). This conclusion agrees with what given by Nussenzveig [24]. Therefore for the spherical case, one only has to redefine the equivalent mode velocity as \( c_n(\omega) = \frac{a\omega}{n + \frac{1}{2}} \) as opposed to that defined in Equation 6.28. Then, one can solve for the complex mode frequencies (or eigenfrequencies), \( \beta_{nl} \), from Equation 6.27 by letting \( \lambda = n + \frac{1}{2} \). Here, the corresponding poles (roots) are obtained graphically in the quantized complex \( k_a \) plane for \( |k_a| < 6 \), and are plotted in Figure 6.2. The discretization error is 0.06 for the real part and 0.01 for the imaginary part. Only the physical poles located at the forth quadrant with negative imaginary parts are selected. The poles associated with TE and TM modes are marked by "o" and "+", respectively. The number beside each curve indicates its model order, \( n \). Since there are multiple modes associated each \( n \) value. Each string of poles collects poles of the same mode number for different order number, \( n \). For example, the left most string collects the first TM modes for different \( n \) values, the second string from the left connects the first TE modes for different \( n \) values, the third string from the left connects the second TM modes for different \( n \) values.
values and so on. The real and imaginary parts of each pole are directly related to the resonant frequency and damping factor via Equation 6.27. Therefore, the poles located at far right of the Figure 6.2 are highly damped and of little practical use.

Numerous authors have studied these type of poles. Franz [20] called the surface waves associated with these poles "creeping waves" because the waves "creep" along the surface. The same wave mechanisms are called "Franz waves" in the acoustic scattering problem for a "rigid sphere" in fluids [23, 22]. It was found there that this type of wave propagates in the external medium and is mainly determined by the surface geometry instead of the interior material property. These surface waves are launched by grazing incident rays. This has been shown experimentally by Ashkin [2]. Fahlen and Bryant [19] also provided an astonishing visualization of surface waves on water droplets. Once the surface waves are launched, they propagate along the surface with attenuation due to the continuous reradiation in the tangent direction. The physical explanation for the existence of surface waves may be considered as a continuous process of ray diffraction occurring at the surface. The diffraction coefficient of a smooth transparent object has been derived by Chen [14].

**Large Argument Uniform Asymptotic Expansions of Zeros**

Streifer and Kodis [49] found the following asymptotic expansion for the poles of Equation 6.27:

\[ \lambda \sim \beta + e^{-j\pi/3} \xi / \gamma \quad (6.30) \]

where \( \gamma = (\frac{3}{\beta})^{1/3} \ll 1 \), and

\[ \xi = \omega - \delta \quad (6.31) \]
Figure 6.2: Normalized complex poles for a PEC sphere. TE and TM poles are marked by "o" and "+", respectively. The numbers indicate the order.
Note that $x_l$ is defined as the $l$th zero of Airy functions $Ai(-x)$ (TE) or $Ai'(-x)$ (TM), and

\[
\delta_l = -e^{-\frac{i\pi}{3}x_l^2}{20}\gamma^2 + e^{\frac{i\pi}{2}x_l^2}{1400}\gamma^4 - \left(\frac{281x_l^4}{4536000} - \frac{29x_l}{12600}\right)\gamma^6 \\
+ e^{\frac{-i\pi}{6}M \gamma} \left\{1 + e^{\frac{-i\pi}{3}x_l^2}{6}(1 + \frac{1}{M^2})\gamma^2 - e^{\frac{i\pi}{3}x_l^2}{20}\left(\frac{3}{2M^4} + \frac{13}{9M^2} - \frac{1}{18}\right)\gamma^4 \right\} - \frac{e^{i\pi/6}x_l}{36M} \left(\frac{1}{M^4} + \frac{2}{M^2} + 1\right)\gamma^5 + o(\gamma^7)
\]  

(6.32)

where $M = (N^2 - 1)^{1/2}$. For the PEC sphere, $M \to \infty$, only the first three terms in Equation 6.32 are significant. By letting $\lambda_l = n + \frac{1}{2}$ and keeping just the first term of $\delta_l$, one can obtain the asymptotic approximation of $\beta_l$ by solving Equation 6.30. The results are compared with the actual poles obtained from the prescribed graphical method in Figure 6.3 for TE poles and in Figure 6.4 for TM poles. One can clearly see that the asymptotic approximation of poles from Equation 6.30 agrees very well with true poles.

An important feature about this type of surface waves is that they are determined by the geometry and the material property of external medium. It will be shown that this is also true when the PEC sphere is replaced by a dielectric bubble with internal permittivity less than that of the ambient medium.

### 6.2.2 PMC Sphere $\mu_d \to \infty$, $\varepsilon_d \mu_d =$constant

In this case, $Z_d \to \infty$, $N \to 0$, and the characteristic equations from the denominator of Equation 6.11 are given by

\[
\hat{H}^{(2)}_{\lambda-rac{1}{2}}(\beta) = 0 \quad \text{(TEr)} \quad \text{and} \quad (6.33)
\]

\[
\hat{H}^{(2)}_{\lambda-rac{1}{2}}(\beta) = 0 \quad \text{(TMr)}
\]

(6.34)
Figure 6.3: Comparison of poles obtained directly from the characteristic function using the graphical method (marked by "x") and from the asymptotic approximation (marked by "o") of the zeros of $H_{n+\frac{1}{2}}^{(2)}(ka)$. 
Figure 6.4: Comparison of poles obtained directly from the characteristic function using the graphical method (marked by "x") and from the asymptotic approximation (marked by "o") of the zeros of $H_{n+rac{1}{2}}^{(2)}(ka)$. 
Compare these expressions with Equation 6.27 for the PEC sphere case, one can immediately recognize the same set of poles except that the TE poles for the previous case have now become TM poles for the PMC case and vice versa.

6.2.3 PEC Cavity $\epsilon_z \to \infty, \epsilon_z \mu_z = \text{constant}$

When $Z_e \to 0$, the internal characteristic equations are obtained from Equation 6.12 by letting $N$ vanish, i.e.,

\[
\begin{align*}
\hat{J}_{\lambda-\frac{1}{2}}(\alpha) &= 0 \quad \text{(TMr),} \\
\hat{J}_{\lambda-\frac{1}{2}}(\alpha) &= 0 \quad \text{(TEr).}
\end{align*}
\]

These are exactly the same characteristic equations for PEC spherical cavity given in [26] where tables of the zeros can also be found. Using the relation 6.3, the above equations are equivalent to

\[
\begin{align*}
J'_\lambda(\alpha) &= 0 \quad \text{(TMr),} \\
J_\lambda(\alpha) &= 0 \quad \text{(TEr).}
\end{align*}
\]

Recall that

\[
J_\lambda(z) = \frac{H^{(1)}_\lambda(z) + H^{(2)}_\lambda(z)}{2}
\]

The asymptotic expansion of $H^{(1)}_\lambda(z)$ for $z \gg \lambda$ is given by [1]

\[
H^{(1)}_\lambda(z) \sim \sqrt{\frac{2}{\pi^2 z \sin \gamma}} e^{-j\frac{\pi}{4}} e^{jz(sin\gamma - \gamma \cos \gamma)} \sum_{n=0}^{\infty} A_n \frac{\Gamma(n + \frac{1}{2}) e^{\frac{3\pi}{4}}}{\left(\frac{z \sin \gamma}{2}\right)^n},
\]

where $\Gamma$ is the Gamma function and $\cos \gamma = \lambda/z$. If one keeps only the $n = 0$ term and makes the substitutions, $z = k\delta r$, $A_0 = 1$ and $\Gamma(\frac{1}{2}) = \sqrt{\pi}$, one obtains that
Consider Figure 6.5, where an internal wave is launched at a surface point “A” and leaves the surface at an angle of $\gamma$. An arbitrary radial line intercepts the wave at “F” and $\angle OFC = \pi/2 - \zeta = \theta + (\pi/2 - \gamma)$. The total phase change along the radial direction from “A” to “C” can be expressed as

$$-\phi = -\int_{r}^{b} \vec{k}_d \cdot d\vec{R}. \quad (6.42)$$

Figure 6.6 shows an expanded view around point “F”, where $d\theta$ is a small angle increment and $dR$ is the corresponding radial distance change. Since $\theta$ is small, two radial line $\vec{OD}$ and $\vec{OE}$ are of equal length, $R$ and assumed to be parallel to each
other. The angle formed by $DF$ and $DE$ is denoted as $\zeta$. Then one obtains the following relations:

\[ \zeta = \gamma - \theta \quad (6.43) \]
\[ R = \frac{b}{\sin(\pi/2 - \zeta)} \quad (6.44) \]
\[ R = \frac{r \cos \gamma}{\cos \zeta} \]
\[ \bar{k} \cdot d\bar{r} = k_4 \sin \zeta dR \quad (6.45) \]
\[ dR = \bar{D}E \tan \zeta \quad (6.46) \]
\[ dR = (Rd\theta) \tan \zeta \quad \text{and} \]
\[ dR = \frac{r \cos \gamma}{\cos \zeta} \tan \zeta d\theta. \]

Using the above relation and Equation 6.42, one finds that
\[
\phi = k_d r \cos \gamma \int_0^\gamma \tan^2(\theta - \gamma) d\theta \\
\phi = -k_d r (\sin \gamma - \gamma \cos \gamma).
\] (6.47)

Combining Equations 6.42 and 6.47, the final radial direction phase factor is given by

\[
e^{i k_d r (\sin \gamma - \gamma \cos \gamma)}.
\] (6.48)

Comparing this expression with Equation 6.41, one can immediately see that both have the same radial direction phase factor. Therefore, \( H^{(1)}_\lambda(k_d r) \) is related to internal bounced waves which bounce away from the surface at an angles, \( \gamma \), defined by \( \cos \gamma = \lambda / (k_d a) \). Similarly, \( H^{(2)}_\lambda(k_d r) \) is related to the outgoing waves which propagate toward the surface and get reflected. The total field distribution in the radial direction become a standing wave formed by the inward and outward waves, i.e. \( J_\lambda(k_d r) \). It should be noted that there results agree with the predictions of Nussenzveig [38, 24] using the semiclassical scattering mechanics. In their study, the distance "b" refers to an "impact parameter" associated with the semiclassical collision problem. Notice that if \( \lambda \) is fixed and \( z \) increases then \( \gamma \) increases and \( b \) decreases. The suggests that the large argument approximation, \( z \gg \lambda \) should correspond to waves toward the sphere center. This condition is easily satisfied for lower order modes who have small \( \lambda \). Let the zeros for \( J_\lambda(z) \) and \( J_\lambda(z)' \) be denoted by \( j_{\lambda,s} \) and \( j_{\lambda,s}' \). Using the McMahon's expansion for \( s \gg \lambda \) [1] and the substitution \( \lambda = n + \frac{1}{2} \), one obtains

\[
j_{n+\frac{1}{2},s} \sim \pi \left(\frac{n}{2} + s\right), \quad \text{and} \quad (6.49)
\]
where \( s = 0, 1, 2, \ldots \). Then the resonance frequencies can be found from the relation

\[
\omega = k_d a, \text{ such that } z = k_d a \text{, such that }
\]

\[
\frac{\omega}{2a} \sim \frac{n - 1}{2} + s
\]

(6.52)

(6.53)

(6.54)

For a given \( n \), there are multiple resonance modes determined by mode number \( s \). The frequency separation, \( \Delta f = \frac{\omega}{2a} \), suggests that these resonances occur at frequencies such that there is an integer number of wavelengths in \( 2a \). Under such a condition, the resonances are related to waves that bounce back-and-forth between the inner surface and the sphere center. The center of the sphere acts like a PEC boundary and vanishing small electric field. This is expected since the symmetry would result in a total cancelation of electric fields at the center.

6.2.4 PMC Cavity \( \mu_z \rightarrow \infty, \varepsilon_z \mu_z = \text{constant} \)

In this case, \( Z_z \rightarrow \infty \), and the characteristic equations are

\[
\hat{J}_{\lambda-rac{1}{2}}(\alpha) = 0 \quad \text{(TMr)}, \quad \text{and} \quad J_{\lambda-rac{1}{2}}(\alpha) = 0 \quad \text{(TEr)}.
\]

(6.55)

(6.56)

These are very similar to Equation 6.35 except that the poles for TE and TM mode are interchanged. Since the physical interpretation of the resonances are similar to that for PEC cavity case, no further discussion is necessary.
6.3 Resonances of a Dielectric Bubble ($\epsilon_d < \epsilon_x$)

For a dielectric sphere where $N > 1$, one can find the complex poles in $(k a)$-plane from Equation 6.25. It is better to start with an example where $\epsilon_d = \epsilon_0, \epsilon_x = 6\epsilon_0$ and $\mu_d = \mu_x = \mu_0$. Again, the poles are obtained using the graphical method and the results are shown in Figure 6.7 where $TE$ and $TM$ modes are marked by "o" and "+", respectively. It should be noticed that this figure is currently in the $k a$ space; i.e., it is normalized by the external medium. One can also choose to normalize it by the internal medium, which is better for studying the internal resonances. Two different groups of poles can be clearly distinguished by their different separation between adjacent modes. The first group of poles which have smaller vertical separation and larger imaginary parts correspond to higher damped resonances. They are associated with surface waves creeping along the outer surface. Since these surface waves are propagating along the interface and on the high dielectric side, they are beyond the critical angle and thus, the sphere appears to be impenetrable. Therefore, these surface wave poles are expected to be similar to those for a PMC sphere. The second group of resonances have poles closer to the real axis and thus are less damped. These resonances are associated with internal bouncing modes similar to inside a PEC spherical cavity. Both types of resonances will be investigated next.

External Modes

Figure 6.8 compares the current poles with those from PMC sphere, where the TE and TM poles for the PMC sphere are marked by "x" and "x", respectively. It is observed that the TM poles for the current dielectric bubble (marked by "+") are located very close to the TM mode poles (marked by "x") for the PMC sphere. It
Figure 6.7: Poles in complex $k_xa$-plane for dielectric bubble.
is also noticed that the TE\(_n\) are located close to TM\(_{n+1}\) mode poles for the PMC sphere. This unique behavior can be understood by consider the extreme case when \(\varepsilon_x \to \infty\). Notice that \(\beta\) also approaches \(\infty\) since \(\mu_x\) is fixed. From Equation 6.25, one obtains the following new characteristic equations

\[
\begin{align*}
\hat{H}_n^{(2)}(\beta) &= 0 \quad \text{(TM), and} \\
\hat{H}_n^{(2)'}(\beta) &= 0 \quad \text{(TE).}
\end{align*}
\]  

(6.57)  

(6.58)

Using the relation \(\hat{B}_n'(x) = -\hat{B}_{n+1}(x) + \frac{n}{\beta} \hat{B}_n(x)\), the second equation can be changed into

\[
-\hat{H}_{n+1}^{(2)}(\beta) + \frac{n+1}{\beta} \hat{H}_n^{(2)}(\beta) = 0.
\]

\[
\rightarrow \hat{H}_{n+1}^{(2)}(\beta) = 0 \quad \text{since} \beta \to \infty.
\]

(6.59)

From the above, it is found that when \(\varepsilon_x \gg \varepsilon d\), TM\(_n\) mode surface wave poles are close to TM\(_n\) poles for PMC sphere and the TE\(_n\) mode surface wave poles are close to TM\(_{n+1}\) poles for PMC sphere. This agrees with what was observed earlier in Figure 6.8. Therefore, the resonances associated with the first group of poles with small vertical separation are excited by the the surface waves creeping along the outer surface similar to those for impenetrable sphere. Since they are propagating almost completely in the external medium, their resonance frequencies are determined only by geometry and the material property of the external medium. This is demonstrated by the results shown in Figure 6.9. Since the poles are normalized against external medium, the surface waves are not affected much by the material inside the sphere. Thus, the surface waves poles for both cases are close to each other. Notice that the internal pole location have changed substantially.
Figure 6.8: Comparison of poles from dielectric bubble and from PMC sphere. The poles are plotted in the $k_x \alpha$-plane.
Figure 6.9: Comparison of surface wave poles for dielectric bubbles with different internal permittivity. The poles are plotted in the $k_xa$-plane.
Internal Modes

In order to study the internal resonances, it is better to normalize the poles against the internal medium. This is done by plotting the poles in the complex $k_d$-plane, as shown in Figure 6.10. The differences between the Figures 6.10 and 6.7 is the different scaling. Figure 6.10 also uses smaller horizontal range in order to find more details about the internal resonances. There are four major features associated with the internal resonant poles:

(1) All the poles are located to the right of $Im(k_d a) \approx 4.3$ line, as shown by the dash line.

(2) Smaller imaginary parts compared to the surface wave poles suggest lower damped resonances and more practical use.

(3) TE modes have smaller imaginary parts than those of TM modes. In fact, all TE modes tends to have the same imaginary parts.

(4) Well pole location separation between the internal modes and external modes suggests weak coupling between the them.

(5) The frequency separation between two adjacent modes is determined by the internal wave velocity and independent of the ambient medium.

The following discussions will reveal more details about the internal modes.

The Frequency of Internal Resonances

Physically, a dielectric bubble and a PEC spherical cavity of the same size and same internal material are similar as far as the internal resonances are concerned.
Figure 6.10: Poles in the complex $k_d a$-plane for dielectric bubble.
<table>
<thead>
<tr>
<th>n</th>
<th>TE</th>
<th>TM</th>
<th>TE</th>
<th>TM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.52</td>
<td>2.81</td>
<td>4.49</td>
<td>2.74</td>
</tr>
<tr>
<td>2</td>
<td>5.80</td>
<td>4.00</td>
<td>5.76</td>
<td>3.87</td>
</tr>
<tr>
<td>3</td>
<td>7.02</td>
<td>5.12</td>
<td>6.99</td>
<td>4.97</td>
</tr>
<tr>
<td>4</td>
<td>8.20</td>
<td>6.28</td>
<td>8.18</td>
<td>6.06</td>
</tr>
<tr>
<td>5</td>
<td>9.38</td>
<td>7.38</td>
<td>9.36</td>
<td>7.14</td>
</tr>
</tbody>
</table>

However, the former experience damping due to the energy leaking out of the bubble whereas no damping is present for the latter. The resonant frequencies are determined by the coherent condition considering the phase variations arising from the wave propagation distance, the phase of the reflection coefficient of each bounce and the caustic position. Since all three factors are the same for a dielectric bubble and a PEC spherical cavity, one would expect them to have very similar resonant spectrum. To verify this, normalized resonant frequencies, $Re(k_n a)$, of the first few modes, ($n = 1, \ldots, 5, s = 1, 2; z_{n,s}$ denote the zeros), for the dielectric bubble and PEC cavity are compared in Tables 6.3 and 6.2. It should be noted that the poles for a PEC cavity are real numbers and are the zeros of $\hat{J}_n(z)$ and $\hat{J}_n'(z)$. Considering the numerical accuracy in our calculation, Table 6.3 and 6.2 verify the previous prediction, i.e., the internal resonant frequencies for a dielectric bubble are very close to those for a PEC spherical cavity.

**The Damping of Internal Resonances**

Figure 6.11 plots the TE and TM reflection coefficients for an plane wave which is obliquely incident upon a plane interface between two homogeneous media characterized by $\varepsilon_d = \varepsilon_0$ and $\varepsilon_s = 6\varepsilon_0$. The coefficients are plotted as a function of $\sin \theta_i$, where
<table>
<thead>
<tr>
<th>n</th>
<th>TE</th>
<th>TM</th>
<th>TE</th>
<th>TM</th>
</tr>
</thead>
<tbody>
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<td>7.73</td>
<td>6.12</td>
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<tr>
<td>2</td>
<td>9.13</td>
<td>7.51</td>
<td>9.10</td>
<td>7.44</td>
</tr>
<tr>
<td>3</td>
<td>10.48</td>
<td>8.77</td>
<td>10.42</td>
<td>8.72</td>
</tr>
<tr>
<td>4</td>
<td>11.68</td>
<td>9.98</td>
<td>11.71</td>
<td>9.97</td>
</tr>
<tr>
<td>5</td>
<td>12.99</td>
<td>11.24</td>
<td>12.97</td>
<td>11.19</td>
</tr>
</tbody>
</table>

Table 6.1: Comparing the normalized resonant frequencies $Re(k da)$ of the lower modes ($n = 1, \ldots, 5, p = 2$) for dielectric bubble with those for PEC spherical cavity.

Table 6.2: Comparing the normalized resonant frequencies $Re(k da)$ of the lower modes ($n = 1, \ldots, 5, p = 1$) for dielectric bubble with those for PEC spherical cavity.

$\theta_i$ is the incident angle with respect to the surface normal direction. It is observed that the TE mode reflection amplitude is always greater than $\approx 0.42$, which is the reflection amplitude for normal incident case; i.e., $\frac{2\pi-\theta}{2\pi+\theta}$. For TM mode, the reflection amplitude is less than 0.42 for $\theta_i$ less than the Brewster angle. At the Brewster angle, there is no reflection. As $\theta_i$ continue to increase above the Brewster angle, the reflection amplitude grows again with reversed phase. In Section 6.2.3, it was shown that the incident angle of a bouncing wave inside a PEC spherical cavity is $\frac{\pi}{2} - \gamma$, where $\cos \gamma = \frac{n+1/2}{k da}$. Therefore the incident angle is related to the mode number by

$$\sin \theta_i = \frac{n + 1/2}{\omega_{n,p} \sqrt{\varepsilon_d \mu_d a}}.$$  \hspace{1cm} (6.60)

Notice that $\omega_{n,p}$ is a complex number, $n$ is the order number and $p$ is mode number. For example $\omega_{1,1,2}$ corresponds to the first and second roots of the characteristic function for $n = 1$. For internal resonant poles, the real part of $\omega_{n,p} \sqrt{\varepsilon_d \mu_d a}$ is much
Table 6.3: The incident angles associated with the internal TM resonant modes shown in Figure 6.10 (dashed line).

<table>
<thead>
<tr>
<th>n</th>
<th>$\sin \theta_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.562</td>
<td>32.2</td>
</tr>
<tr>
<td>1</td>
<td>0.678</td>
<td>38.8</td>
</tr>
<tr>
<td>2</td>
<td>0.755</td>
<td>43.3</td>
</tr>
<tr>
<td>3</td>
<td>0.799</td>
<td>45.8</td>
</tr>
<tr>
<td>4</td>
<td>0.841</td>
<td>48.2</td>
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<tr>
<td>5</td>
<td>0.873</td>
<td>50.0</td>
</tr>
<tr>
<td>6</td>
<td>0.896</td>
<td>51.3</td>
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<tr>
<td>7</td>
<td>0.907</td>
<td>52.0</td>
</tr>
<tr>
<td>8</td>
<td>0.931</td>
<td>53.3</td>
</tr>
</tbody>
</table>

\[
\sin \theta_i \approx \frac{n + 1/2}{\frac{1}{2} \text{Re}(\omega_n \sqrt{\varepsilon_d \mu_d a})}.
\]  

(6.61)

In order to have a real angle, one must impose another condition such that

\[
n + \frac{1}{2} < \frac{1}{2} \text{Re}(\omega_n \sqrt{\varepsilon_d \mu_d a}).
\]  

(6.62)

From Equation 6.61, one can find the incident angle of a particular mode. For example, the incident angles associated with the TM modes linked by a dashed line in Figure 6.10 are listed the result in Table 6.3. For this group, as mode number increases the incident angle also increases, and the wave moves away from the axial direction. For other poles close to the \( \text{Im}(k_d a = 0.43) \) line, the incident angles are closer to the normal direction and the waves are closer to the axial direction.

Consider a situation like Figure 6.13 where a wave is reflected multiple times inside a dielectric bubble. Let us now define the following new quantities:

- \( N \): total number of bounces to close the loop (i.e. resonant condition)
- \( L \): total traveling distance for the loop after \( N \) bounces
- \( T \): total time required to close the loop
- \( v_d \): phase velocity inside the bubble as if it is infinite large
- \( R^{TE,TM} \): TE, and TM, planar reflection coefficients at each bounce,
Figure 6.11: Reflection amplitude of TE and TM plane waves incident upon a half space medium versus $\sin(\theta_i)$, where $\theta_i$ is the incident angle.

Figure 6.12: Transmission amplitude of TE and TM plane waves incident upon a half space medium versus $\sin(\theta_i)$, where $\theta_i$ is the incident angle.
Figure 6.13: An internal wave going through multiple internal reflections.

\[ \alpha^TE, \alpha^TM : \text{effective damping factors measured by T intervals.} \]

It is easy to find that

\[ L = N(2a \cos \theta_i) \]
\[ T = \frac{L}{v_d} \]
\[ \omega^{TE, TM} = 2\pi f^{TE, TM} - j \alpha^{TE, TM} \]
\[ e^{-\alpha^{TE, TM}T} \equiv |R^{TE, TM}|^N. \]

From the above expressions, it is found that

\[ \alpha^{TE, TM} = -\frac{v_d}{2a \cos \theta_i} \ln|R^{TE, TM}| \]
and the effective normalized damping factor is defined as

\[ \chi_e \equiv \text{Im}(k_d a) = \frac{\text{Im}(\omega)}{v_d} a = -\frac{1}{2 \cos \theta_i} \ln|R^{TE, TM}|. \]

Figure 6.14 plots \( \chi_e \) versus \( \sin \theta_i \) using Equation 6.69. It is interesting to notice that \( \chi_e \) for both TE and TM mode are always larger than 0.43, which is the number
obtained for the normal incident case. That is, the wave is bounced back and forth along the axial direction, similar to the well known "Fabry-Perot Resonator". It is also noticed that TE modes have much smaller values, i.e., smaller damping, than TM modes. For both modes, the effective normalized damping factors increase as \( \sin \theta \) increases. From the Equation 6.61, this suggests that the damping increases as the order number \( n \) increases. All of these predict features (1) and (3) correctly.

Therefore, the damping of the internal resonances is caused by an energy leakage process. The leakage happens whenever an internal waves is incident upon the internal-external interface and is partially transmitted. As shown earlier, the resonant frequencies of a dielectric bubble are similar to those of a PEC spherical cavity and are mainly determined by the bubble geometry; whereas, the damping factors are determined by the difference of electromagnetic properties between the internal and external material. The plane interface assumption used in the above discussion may not be entirely valid for lower order modes when the reflection and transmission coefficients become functions of the curvature; nevertheless, the physical interpretation about the damping factor is still valid.

In Figure 6.9, the poles for two dielectric bubbles with different internal permittivity are compared by plotting the poles in \( \text{Im}(k_x a), \text{Re}(k_x a) \) plane, which clearly shows that the surface poles are mainly independent of the internal material. For internal resonances, it is found that the internal resonances of a bubble are very independent of external medium. To demonstrate this the poles are plotted in a \( -\text{Im}(k_d a)\frac{2}{\text{Im}(R_0)}, \text{Re}(k_d a) \) plane, where \( R_0 \) is the normal incident reflection coefficient observed from the internal side. For example, from Figure 6.11, it is found that for a free space bubble immersed in a ambient medium with dielectric constant of 6,

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Figure 6.14: Effective damping factors, $\alpha^{TE, TM}$, versus $\sin \theta_i$.

$R_0 \equiv 0.42$. The imaginary axis is normalized by a multiplication factor $-\frac{2}{\ln|R_0|}$. This normalization tends to shift the imaginary parts of the near-axis resonances toward unity. Figure 6.15 compare the internal resonance poles for two bubbles with relative permittivity of 1 and 2, respectively, immersed in the same type of medium with relative permittivity of 6. The internal relative permittivity are 1 and 2, respectively. It is observed that locations of near-axis resonant poles whose imaginary are close to unity almost remain unchanged. Since the normalization is only a function of internal permittivity, this indicates that the near-axis resonances are only a function of internal permittivity and independent of external medium. It is also noticed that a group of non-axial resonance poles circles by dash line shows larger horizontal deviation for different internal permittivity. This is expected because that the actual reflection
Figure 6.15: Comparison of normalized internal resonance poles for dielectric bubbles with different internal permittivity.

coefficient is function of incident angle therefore $R_0$ will not be able to normalize the damping factors for off-axis resonances.
6.4 Resonances of Dielectric Spheres ($\varepsilon_d > \varepsilon_z$)

6.4.1 Internal Resonances

Resonant Frequencies

In Section 6.3, it was shown that the resonant frequencies inside a dielectric bubble are very close to those inside a PEC spherical cavity (see Tables 6.3 and 6.2) since both have similar geometrical properties except that the dielectric bubble has penetrable boundary which results in damped resonances. Similarly, one would expect the internal resonant frequencies of a dielectric sphere to be close to those of a PMC spherical cavity with the same dielectric filling. This can be verified by comparing the real parts of the poles associated with a dielectric sphere with those associated with a PMC cavity. Tables 6.4.1 and 6.4.1 compare the real part of the first and second pole, respectively, for $n = 1, 2, \ldots, 5$ of a dielectric sphere with relative permittivity equal to 3 with those of a PMC cavity. Tables 6.4.1 to 6.4.1 show similar comparison for other cases where the sphere's relative permittivity is equal to 6 and 9, respectively. From these comparisons, it is confirmed that the resonant frequencies of a dielectric sphere are close to those of PMC cavity filled with the same relative permittivity for the second or higher modes for each order, $n$.

Resonant Damping Factors

The damping factors associated with the resonances inside a dielectric sphere are studied in this section. In Section 6.3, it was found that the damping factors of the resonances inside a dielectric bubble are closely related to the plane interface reflection/transmission problem. Therefore let us start with the reflection/transmission problem shown in the insert of Figure 6.16, where a plane wave is obliquely incident...
### Table 6.4: Comparing the normalized resonant frequencies Re($k_d\alpha$) of the first pole, $z_{n,1}$, of order $n = 1, \ldots, 5$, for a dielectric sphere ($\varepsilon_r = 3$) with those for a PMC spherical cavity.

<table>
<thead>
<tr>
<th>$n$</th>
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<th>TE</th>
<th>TM</th>
<th>TE</th>
</tr>
</thead>
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<td>4.49</td>
<td>2.74</td>
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<td>5.78</td>
<td>4.12</td>
<td>5.76</td>
<td>3.87</td>
</tr>
<tr>
<td>3</td>
<td>7.14</td>
<td>5.38</td>
<td>6.99</td>
<td>4.97</td>
</tr>
<tr>
<td>4</td>
<td>8.77</td>
<td>6.52</td>
<td>8.18</td>
<td>6.06</td>
</tr>
<tr>
<td>5</td>
<td>10.52</td>
<td>7.72</td>
<td>9.36</td>
<td>7.14</td>
</tr>
</tbody>
</table>

### Table 6.5: Comparing the normalized resonant frequencies Re($k_d\alpha$) of the second pole, $z_{n,2}$, of order $n = 1, \ldots, 5$, for a dielectric sphere ($\varepsilon_r = 3$) with those for a PMC spherical cavity.

<table>
<thead>
<tr>
<th>$n$</th>
<th>TM</th>
<th>TE</th>
<th>TM</th>
<th>TE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.78</td>
<td>6.15</td>
<td>7.73</td>
<td>6.12</td>
</tr>
<tr>
<td>2</td>
<td>9.11</td>
<td>7.47</td>
<td>9.10</td>
<td>7.44</td>
</tr>
<tr>
<td>3</td>
<td>10.46</td>
<td>8.80</td>
<td>10.42</td>
<td>8.72</td>
</tr>
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<td>11.71</td>
<td>9.97</td>
</tr>
<tr>
<td>5</td>
<td>13.01</td>
<td>11.29</td>
<td>12.97</td>
<td>11.19</td>
</tr>
</tbody>
</table>

### Table 6.6: Comparing the normalized resonant frequencies Re($k_d\alpha$) of the first pole, $z_{n,1}$, of order $n = 1, \ldots, 5$, for a dielectric sphere ($\varepsilon_r = 6$) with those for a PMC spherical cavity.

<table>
<thead>
<tr>
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<th>TM</th>
<th>TE</th>
</tr>
</thead>
<tbody>
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<td>4.49</td>
<td>2.98</td>
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<td>2.74</td>
</tr>
<tr>
<td>2</td>
<td>5.14</td>
<td>4.33</td>
<td>5.76</td>
<td>3.87</td>
</tr>
<tr>
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<td>6.43</td>
<td>5.60</td>
<td>6.99</td>
<td>4.97</td>
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<td>7.69</td>
<td>6.83</td>
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<td>6.06</td>
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<td>5</td>
<td>9.02</td>
<td>8.06</td>
<td>9.36</td>
<td>7.14</td>
</tr>
</tbody>
</table>

Table 6.4: Comparing the normalized resonant frequencies $Re(k_d\alpha)$ of the first pole, $z_{n,1}$, of order $n = 1, \ldots, 5$, for a dielectric sphere ($\varepsilon_r = 3$) with those for a PMC spherical cavity.

Table 6.5: Comparing the normalized resonant frequencies $Re(k_d\alpha)$ of the second pole, $z_{n,2}$, of order $n = 1, \ldots, 5$, for a dielectric sphere ($\varepsilon_r = 3$) with those for a PMC spherical cavity.

Table 6.6: Comparing the normalized resonant frequencies $Re(k_d\alpha)$ of the first pole, $z_{n,1}$, of order $n = 1, \ldots, 5$, for a dielectric sphere ($\varepsilon_r = 6$) with those for a PMC spherical cavity.
<table>
<thead>
<tr>
<th>Dielectric Sphere ((\varepsilon_r = 6))</th>
<th>PMC Spherical Cavity</th>
</tr>
</thead>
<tbody>
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<td>7.78</td>
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<tr>
<td>2</td>
<td>9.14</td>
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<tr>
<td>3</td>
<td>10.37</td>
</tr>
<tr>
<td>4</td>
<td>11.32</td>
</tr>
<tr>
<td>5</td>
<td>12.25</td>
</tr>
</tbody>
</table>

Table 6.7: Comparing the normalized resonant frequencies \(Re(k_d\alpha)\) of the second pole, \(z_{n,2}\), of order \(n = 1, \ldots, 5\), for a dielectric sphere (\(\varepsilon_r = 6\)) with those for a PMC spherical cavity.

<table>
<thead>
<tr>
<th>Dielectric Sphere ((\varepsilon_r = 9))</th>
<th>PMC Spherical Cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>TM</td>
</tr>
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<td>1</td>
<td>4.37</td>
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<tr>
<td>2</td>
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<td>3</td>
<td>6.71</td>
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<tr>
<td>4</td>
<td>8.03</td>
</tr>
<tr>
<td>5</td>
<td>9.23</td>
</tr>
</tbody>
</table>

Table 6.8: Comparing the normalized resonant frequencies \(Re(k_d\alpha)\) of the first pole, \(z_{n,1}\), of order \(n = 1, \ldots, 5\), for a dielectric sphere (\(\varepsilon_r = 9\)) with those for a PMC spherical cavity.

<table>
<thead>
<tr>
<th>Dielectric Sphere ((\varepsilon_r = 9))</th>
<th>PMC Spherical Cavity</th>
</tr>
</thead>
<tbody>
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<td>2</td>
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<td>4</td>
<td>11.23</td>
</tr>
<tr>
<td>5</td>
<td>12.52</td>
</tr>
</tbody>
</table>

Table 6.9: Comparing the normalized resonant frequencies \(Re(k_d\alpha)\) of the second pole, \(z_{n,2}\), of order \(n = 1, \ldots, 5\), for a dielectric sphere (\(\varepsilon_r = 9\)) with those for a PMC spherical cavity.
upon a plane interface from a higher permittivity medium toward a lower permittivity medium. The TE and TM mode reflection and transmission coefficients can then be written as

\[ R_{TM} = \frac{Z_2 \cos \theta_t - Z_d \cos \theta_i}{Z_2 \cos \theta_t + Z_d \cos \theta_i} \]  
\[ R_{TE} = \frac{Z_2 \cos \theta_t - Z_d \cos \theta_i}{Z_2/\cos \theta_t + Z_d/\cos \theta_i} \]  
\[ T_{TM} = 1 + R_{TM} \]  
\[ T_{TE} = 1 + R_{TE} \]  

where \( Z_2 = \sqrt{\mu_2 / \varepsilon_2} \), \( Z_d = \sqrt{\mu_d / \varepsilon_d} \), \( \theta_i \) and \( \theta_t \) are incident angle and transmission angle, respectively. The phase matching condition along the interface requires that

\[ \theta_t = \sin^{-1} \frac{k_d}{k_2} \sin \theta_i, \quad \text{or} \quad \theta_t = \sin^{-1} \sqrt{\frac{\varepsilon_d}{\varepsilon_2}} \sin \theta_i. \]  

When \( \sin \theta_i > \sqrt{\frac{\varepsilon_2}{\varepsilon_d}} \), \( \theta_t \) becomes imaginary, total reflection results. As a result, the transmitted field propagates along the boundary and its amplitude decays exponentially in the direction away from the interface. The larger \( \theta_i \), the faster the decay.

Figure 6.16 and 6.17 plot the magnitude of reflection and transmission coefficients, respectively, as functions of \( \sin \theta_i \). The solid and dash line correspond to TE and TM modes, respectively. The relative permittivity is chosen to be 3 for the incident side and 1 for the transmitted side. As one can see, when \( \sin \theta_i \) is greater than \( 1/\sqrt{3} \approx 0.58 \), both \( R_{TE} \) and \( R_{TM} \) become complex numbers with unit amplitude. It is also noticed that a zero reflection angle exists for TM mode. This particular angle is usually referred to as the "Brewster angle" and is denoted by \( \theta_B \), where

\[ \sin \theta_B = \frac{1}{\sqrt{1+\varepsilon_d/\varepsilon_2}}. \]  

This happens when the propagation of the reflected waves coincide with the polarization of the transmitted wave. The corresponding normalized
Figure 6.16: Reflection coefficients for TE and TM modes incident upon a plane interface.
Figure 6.17: Transmission coefficients for TE and TM modes incident upon a plane interface.
effective damping factor, $\chi_e$, defined in Equation 6.69, for internal bounced waves is also plotted in Figure 6.18. It is noticed that $\chi_e$ for the TE modes are always smaller than $\chi_0(\approx 0.66)$ and decrease as $\sin \theta_i$ increases until $\theta_i$ reaches the critical angle, $\theta_c$. Beyond the critical angle $\chi_e^{TE}$ becomes zero. For the TM mode, $\chi_e$ first increases from $\chi_0$ as $\theta_i$ increases in the $\theta_i < \theta_B$ region. In $\theta_B < \theta_i < \theta_c$ region $\chi_e^{TM}$ decreases to zero and remains zero for $\theta_i > \theta_c$.

After reviewing the previous plane interface reflection/transmission problem, the attention now returns to the investigation of internal resonances of the previous dielectric sphere problem. Figure 6.19 shows the poles in complex $k_d a$-plane for a dielectric sphere ($\epsilon_d = \epsilon_0$) in free space. The plotting region is focused on resonance modes.

Figure 6.18: Normalized effective damping factor, $\chi_e$. 
The poles can be separated into two groups: TE modes (connected by solid lines) and TM modes (connected by dash lines). It should be noted that only the lowest nine poles for each string, i.e., \( n = 1, 2, \ldots, 9 \), are significant. It is noticed that all TE mode poles are located in the region where \( 0 < -\text{Im}(k_d a) < 0.66 \) and there imaginary parts decrease as the order, \( n \), increases. This agrees the behavior predicted in Figure 6.18. For TM modes, as the mode number increases, the poles first move away from \(-\text{Im}(k_d a) = 0.66\) line and then turn around such that they move toward the real axis. For example, The lowest string of poles turns around at \( n = 2 \) mode and the second string turns around at \( n = 5 \) mode. This particular example may not show this general behavior, it is more evident if one looks at the pole distribution for a dielectric sphere with the relative permittivity of 9 as shown in Figure 6.20. More turning back of TM mode may be observed. The lowest string turns around right at the first mode, the second string turns around at third mode, and so forth. First, it is intriguing to notice that this seems to agree with the behavior predicted by the normalized effective damping factor (see Figure 6.18) again. Further investigation about the turning point using the previous relation, \( \sin \theta_t \approx \frac{n+1/2}{\text{Re}(k_d a)} \), shows that the turning points are close to the Brewster angle. Take Figure 6.19 as an example, at the turning points, the ratio of \( \frac{n+1/2}{k_d a} \) correspond to \( \frac{2.5}{4} = 0.63 \) and \( \frac{5.5}{10.4} = 0.53 \), which are close where the Brewster angle is, i.e., \( \sin \theta_B \approx 0.5 \). More examples can be found in Figure 6.20, where the ratio of the turning points correspond to \( \frac{1.5}{4.3} = 0.35, \frac{3.5}{10.1} = 0.35, \frac{5.5}{16} = 0.34 \) and \( \frac{5.5}{20.8} = 0.31 \), which are also close to the Brewster angle (\( \sin \theta_B \approx 0.3162 \)). The deviation might be due to the fact that the plane interface approximation as well as the condition \( \text{Re}(k_d a) \gg \text{Im}(k_d a) \) become less valid for lower frequency modes.
Figure 6.19: Normalized poles in complex $k_d a$-plane for a dielectric sphere ($\varepsilon_d = 3\varepsilon_0$) in free space.
Figure 6.20: Normalized poles in complex $k_d a$-plane for a dielectric sphere ($\varepsilon_d = 9\varepsilon_0$) in free space.
6.4.2 External Resonances

It has been shown that the resonant frequencies and damping factors of external modes for a dielectric bubble are very similar to those for a impenetrable PMC sphere due to the similar surface wave mechanism caused by the continuous curvature diffraction. It was also shown that those external modes are insensitive to the internal medium. Similar properties will be shown to exist for the external resonances associated with a dielectric sphere except that the PMC sphere is replace by a PEC sphere and the damping factor of TE\(_r\) are more sensitive to the internal medium.

Figure 6.21 plots the poles for a dielectric sphere (\(\varepsilon_r = 9\)) in free space in complex \((k_x a)^2\)-plane, where the vertical and horizontal axes correspond to \(\text{Re}(k_x a)\) and \(\text{Im}(k_x a)\), respectively. The TE\(_r\) and TM\(_r\) mode poles are marked by "o" and "+", respectively. For comparison, the normalized TE\(_r\) and TM\(_r\) poles associated with a PEC sphere are also plotted and marked by "x" and "\(*\)", respectively. The following discussion focuses on the external resonant poles belonging to the strings marked by \(A, B, C, \ldots\), etc. and all the internal mode poles will be discussed later in other sections. As one can see, the external resonance pole locations for a high dielectric sphere are very close to those for a PEC sphere except for string \(A\). The normalized damping factors, \(\text{Im}(k_x a)\), seems to deviate more than the resonant frequencies. Larger difference appears at the first TM string, \(A\), which are located close to the internal resonance poles and thus are more strongly coupled to them. This difference is even more pronounced in the case of lower dielectric contrast as shown in Figure 6.22, where the poles for a sphere with a dielectric constant (\(\varepsilon_r = 6\)) are plotted. Again, all TM\(_r\) modes are still close to PEC poles except for those belonging to the first string, \(A\), which start to show more variation in pole location instead of being along
Figure 6.21: Normalized poles in complex $k_x a$-plane for a dielectric sphere ($\varepsilon_d = 9\varepsilon_0$) in free space.
a nice smooth curve as are the other surface wave poles. This is because the internal resonance poles move farther to the right due to higher damping. Being closer to the A-string poles, these internal resonance poles alter the A-string pole locations and make them deviate more from the PEC poles. It is also observed that all the external TE\(_r\) mode poles marked by "o" move farther away from PEC poles toward the higher damping direction (i.e., right of the figure). Nevertheless, all TE\(_r\) mode resonant frequencies remain close to those associated with a PEC sphere. In earlier discussions it was found that the resonant condition of an impenetrable PEC sphere are determined by the phase velocities of the surface waves creeping along the sphere's outer surface. Looking at similar resonant frequencies between the dielectric sphere and the PEC sphere, it is concluded that the external resonances of a dielectric sphere are cause by the surface waves surrounding the sphere's outer surface using almost the same phase velocities but larger attenuation along the propagating direction.

For impenetrable spheres, the attenuation is due to the continuous power lost by radiating waves away from the spherical surface along the tangent direction. For a dielectric sphere, in addition to the radiation tangent to the surface, part of energy is continuously shedding into the sphere at the critical angle direction. The existence of such wave mechanism was first suggested by van der Hulst [51] while studying the resonant peaks observed in the scattered fields from rain droplets. This unique process does not occur in the dielectric bubble case since no internal critical angle exists.

As the dielectric contrast decreases, the A-string pole locations become severely distorted by the internal resonance poles which move further away from the real axis as discussed before. This is shown in Figure 6.23. All other surface wave poles also
Figure 6.22: Normalized poles in complex $k_a$-plane for a dielectric sphere ($\varepsilon_d = 6\varepsilon_0$) in free space.
move farther away from the PEC poles to the higher damping direction, as shown in Figure 6.23. In a extreme case where \( \epsilon_d = \epsilon_\infty \), all poles move infinitely toward the right; i.e., \( \text{Im}(k_xa) \to \infty \). The result is that all resonances disappear, which agree with what is expected for a homogeneous medium.

### 6.5 Conclusions

The main contribution of this study is to provide a simple physical interpretation for predicting the resonance behavior of a general dielectric sphere. This is achieved by (1) interpreting the internal standing waves as multiple bounced waves which are
obliquely incident upon the sphere boundary (2) relating the resonances associated
with impenetrable cavities and spheres to those associated with a general dielectric
sphere (3) using the plane-interface reflection/transmission model to predict the be-
havior of internal resonance damping factor.

The complex resonances of a dielectric sphere (or bubble) were separated into ex-
ternal modes and internal modes. The resonance behavior of the external modes are
caused by the surface waves which creep along the sphere's outer surface and have
been shown to be very similar to impenetrable spheres. For a dielectric sphere, the
external modes are similar to those associated with a PEC sphere. On the other hand,
the external modes of a dielectric bubble are similar to those associated with a PMC
sphere. External modes were also found to have much larger damping factors than
external modes and thus, are less important in practice. The resonant frequencies of
the external modes are mainly independent of internal dielectric property. However,
their damping factors are determined by the internal/external dielectric contrast. The
damping factor deviation from that of an impenetrable sphere is normally insignifi-
cant except for a low-contrast dielectric sphere; i.e., the internal dielectric constant
is greater than, but close to, the external one. In this case, the surface waves keep
shedding energy into the sphere at the critical angle. The internal resonances associ-
ated with a dielectric sphere (or bubble) were shown to be related to internal bounced
waves which experience multiple reflections. The resonant frequencies were shown to
be close to those of an impenetrable spherical cavity filled with the same dielectric
material. More precisely, the internal resonant frequencies of a dielectric sphere and
bubble are close to those of a PMC and PEC spherical cavity, respectively. It was
found that the internal resonances associated with a dielectric bubble are mainly in
the axial direction. The damping of internal resonances are mainly determined by
the amount of energy transmitted out at each internal reflection. For either dielec-
tric sphere or bubble, it was found that the damping associated with the axial \( TE_r \)
modes are less than that associated with axial \( TM_r \) modes. For dielectric spheres,
it is interesting to find that the higher \( TM_r \) modes may have very small damping
factors when the internal incident angle is larger than the Brewster.
CHAPTER 7

CONCLUSIONS

7.1 Summary and Conclusions

The design and development of ground penetrating radar (GPR) systems for the
detection and classification of UXO and land mine were discussed in this dissertation.
The radar system is described to optimize the signal-to-noise and signal-to-clutter
ratios. This lead to the development of new antenna concepts. The classification in
all cases has been based on the targets' complex natural resonances (CNR).

In Chapter 2, the theoretical background about the CNR's associated with PEC
targets and the processing algorithm used to extract CNR information were presented.
It was also shown that CNR technique can be used to classify the UXO's by estimating
their lengths with good success provided the soil is not too lossy. The estimated UXO
lengths using their first resonance were found to be 10% to 20% longer than their
physical lengths which is related to their shape. Numerical modeling using a body
of revolution moment method code was used to calculate the free space scattered
fields for several different UXO's. These fields were then processed using the TLS-
Prony method to obtain the free space CNRs, which can also be used to obtain the
corresponding CNRs in different electrical medium using Baum's transformation [6].
These computations confirmed that the 10% to 20% additional length was because of
the smaller length-to-diameter ratios of the UXO's. This numerical simulation serves as a important tool to generate UXO CNR databases for future classification.

Field measured data was collected by the Subsurface Ordnance Clearance System (SOCS) with its GPR antenna designed by the author. The data were then processed to obtained 3D image for UXO detection and CNR extraction for UXO classification. The UXO length estimates from the CNR information showed good correlation with their physical dimensions. It was also found that the existing antenna can be further improved.

Motivated by trying to improve the size, weight, bandwidth, sensitivity and stability of the existing SOCS antenna, a novel antenna design concept, the dielectric-loaded horn-fed bowtie (HFB) dipole was investigated in Chapter 3. This new design concept was then carried further to build two prototype antennas and one operational antenna. After evaluating many dielectric filling options, it was determined that the original liquid mixture had to be replaced by a better solid (BaTiO3/Epoxy) from both construction and stability considerations. A better way of constructing the new antenna was also found in cooperation with the antenna fabricator. Various measurements were performed to test the new antenna and the results were compared with those obtained from the existing antenna. This work not only verified the original design concept but also illustrated that the new HFB antenna performs better than the existing antenna in many ways such as sensitivity, RF interference immunity, size and weight reduction and matching stability.

The ESL land mine radar (LMR) prototype system for the detection and classification of land mines, especially for plastic ones, was presented in Chapter 4. This
LMR system features stand-off measurement using a focussed beam such that a better signal-to-surface clutter ratio is achieved. It was found that the surface clutter plays an important role in land mine detection and classification because of this clutter term is so close to the target return in time. It was found that a low-angle incidence configuration suffers from the time-stretched surface clutter which makes it extremely difficult to separate out the target response for detection and classification. On the other hand, a high-angle incidence system receives a strong surface scattering, but it can be processed out to expose the target response. Improved detection and classification results were obtained from actual measured data for buried plastic mines. This study serves as a good example of a successful combination of radar system design and signal processing development. Clear detection of plastic land mines buried in a normal soil up to 3" was obtained which is outstanding. The CNR signature was also obtained for land mine classification. Compared to UXO classification, the CNR's of a plastic mine were found to be much complicated and more difficult to obtain. Special processing techniques were developed for this purpose.

Two special processing techniques, the Surface Clutter Reduction (SCR) and Late-Time Spectrum (LTS) techniques, were developed to detect and classify plastic land mines. These topics were discussed in Chapter 5. The former technique improves the signal-to-clutter ratio by removing the surface scattering from the data. The latter calculates the resonant spectrum efficiently using the late-time impulse response. Processed results for a particular type of land mine (LORY) buried in normal soil up to 3" deep clearly demonstrates the system improvements. Both techniques can also be applied to many other GPR applications.
Motivated by the observation of plastic land mine CNR's, a better understanding of the CNR behavior of dielectric targets embedded in a dielectric medium was needed. A dielectric sphere was chosen based on its well known scattered field solutions. The complex resonance pole behavior was investigated rigorously for both high and low dielectric spheres. Two type of resonances were found, internal and external resonances. Some external resonances can also couple to the internal resonances for the low dielectric contrast case, where the permittivity of the sphere is close to that of its ambient medium. It was also found that the external resonances are closely related to the resonances found in perfectly electrical conducting (PEC) and magnetic conducting (PMC) spheres. The internal resonances, on the other hand, are closely related to the resonances found in PEC and PMC spherical cavities. It was also shown that the internal resonances are excited by internal bouncing waves and the external resonances are excited by the surface waves which creep along the outer surface of the sphere. A simple plane interface reflection/transmission model was shown to accurately predict the behavior of damping factors, which play an important role in determining whether the corresponding poles are useful. Since a highly damped resonance is difficult to detect and thus is of little practical use. Through the understanding of the CNR behavior associated with a dielectric sphere, it is now easier to predict the CNR behavior associated more complex dielectric targets.

7.2 Potential Future Research

There is still much future work to be done to improve the classification performance using the CNR technique. Some of the important ones are listed below.

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CNR Extraction from Multiple Looking Data

The current algorithm extracts CNR information from a single waveform recorded at a single position. However, when the antenna is moved along the surface, it will receive additional responses from the same target for different look angles. Each response received at one antenna position correspond to one target response. Since the CNR's remain unchanged but the clutter is changed, more accurate and stable CNR information should be obtained if one combines multiple looking angle data properly. To do this, one has to modify the current TLS-Prony algorithm to include multiple waveforms.

Cross-polarization HFB antenna

It is well known that a cross-dipole arrangement has lower antenna and ground clutter compared to a single dipole in GPR applications. Therefore, the future improvement of the new Horn-Fed Bowtie (HFB) antenna design can be achieved by designing a new cross-polarization version. Any antenna that reduces the direct response from the antenna itself and surface clutter will generate improved results.

Possible Mobile Radar

Currently a modified radar operating from 1–6 GHz has been constructed. This radar is currently undergoing tests. While considerably smaller than the compact range radar, it still must be vehicular (or cart) mounted. It seems practical to reduce the radar size substantially, perhaps to the extent that it could be mounted in a backpack.
Soil Modification

From the dielectric sphere study, it was found the most external resonance modes are highly damped. Therefore, internal modes are more important in practice. It was also learned that the damping of the internal resonances are determined by the dielectric contrast between the target and its ambient medium. If one can modify the permittivity of the ambient medium, he may be able to enhance the internal resonances by decreasing their damping factors. This study is currently under investigation.

CNR Study for General-Shaped Dielectric Targets

Although, the dielectric sphere study has provided insight about the dielectric target resonances, most real world targets, however, have much more complex geometries and analytic solutions are often not available. Numerical modeling techniques such as the finite difference time domain (FDTD) technique or the finite element method (FEM) should be used as tools to extend the knowledge about more general dielectric target resonances.

Clutter Model

While this study has focussed attention on the particular target to be detected, little information is available on the clutter in such earth media. It is important to expand the understanding of the clutter characteristics since it could be used a priori to determine the success of a given GPR to detect a specific target in a clutter environment.
BIBLIOGRAPHY


