INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI
A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor MI 48106-1346 USA
313/761-4700 800/521-0600
BUSINESS CYCLE MEASUREMENT
USING A DYNAMIC FACTOR MODEL
WITH DURATION
DEPENDENT TRANSITIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Tracy L. Foertsch, M.A.

*****

The Ohio State University
1997

Dissertation Committee:
Professor Stephen G. Cecchetti, Adviser
Professor Pok-sang Lam
Professor Masao Ogaki

Approved by

[Signature]
Adviser
Department of Economics
ABSTRACT

The official NBER chronology of business cycle peaks and troughs is the basis both for studies on asymmetries in business cycle behavior and for the hypothesis that the business cycle has systematically changed in the period following W.W.II, with contractions decreasing and expansions increasing in length. The accuracy of such findings hinges upon the historical consistency of the NBER-dated cyclical peaks and troughs, with such consistency being defined in part by the uniform application of a single procedure for dating business cycle turning points. Our focus is the development and application of such a procedure and the subsequent remeasurement of U.S. business cycles.

In developing our dating procedure, we introduce time-varying transition probabilities into a multivariate algorithm for identifying turning points in the comovements of series. In doing so, we utilize a regime switching model in which the evolution of the transition probabilities is conditioned on an additional economic variable whose value is inferred from the number of periods the economy has been in the current phase. It is the introduction of such duration-dependent transition probabilities which enables us to detect asymmetries in the duration dependence of
expansions and contractions without reference to an existing business cycle chronology.

In applying our dating algorithm to U.S. business cycle indicators, the outcome is twofold. First, we obtain results characterizing the duration dependence of post-W.W.II business cycles, concluding that post-W.W.II contractions exhibit marked positive duration dependence while post-W.W.II expansions exhibit no duration dependence, positive or negative. More specifically, our dating algorithm predicts that while the postwar economy moves out of a recession after only 12 months it has better than a 95% probability of remaining in an expansion even after two years. Second, we derive a business cycle chronology extending from 1885 to 1996. While our postwar chronology approximates that of the NBER, our interwar chronology omits two contractions while our pre-W.W.I chronology omits four contractions identified by Burns and Mitchell. In both cases, estimates of the growth rate of a common factor component would seem to suggest that these omitted episodes are merely periods of decelerated growth and not periods of recession.
Dedicated to my parents
ACKNOWLEDGMENTS

I wish to thank Stephen Cecchetti, Pok-sang Lam, and Masao Ogaki for their helpful comments and intellectual support. Without their help, this thesis would not have been possible. I also wish to thank Masanori Hashimoto for his constant encouragement and enthusiastic support both during the completion of this thesis and my graduate career.

I am grateful to Chang-jin Kim for providing me with a copy of the computer program used in his paper and to Marcelle Chauvet for help in acquiring some of the data used here. I am also grateful to Thomas McCurdy for help with various computational problems encountered in the process of completing this thesis.

Finally, I wish to thank all of those who helped me with various computer problems, including Andrew Paine and Roy McCarthy.
VITA

November 21, 1969..............................Born - Marion, Ohio


1991 - present.......................................Graduate Teaching and Research Associate, The Ohio State University

FIELDS OF STUDY

Major Field: Economics
TABLE OF CONTENTS

Abstract ........................................................................................................................... ii
Dedication ...................................................................................................................... iv
Acknowledgments ........................................................................................................... v
Vita ........................................................................................................................................ vi
List of Tables .................................................................................................................. ix
List of Figures ................................................................................................................ xi

Chapters:
1. Introduction .............................................................................................................. 1
2. What is a Business Cycle? ...................................................................................... 10
3. The FTP and DDTP Dynamic Factor Models ....................................................... 23
   3.1 The FTP Model ...................................................................................... 28
   3.2 The DDTP Model ................................................................................... 32
4. Methodological Issues ............................................................................................. 37
   4.1 The DDTP Filtering Algorithm .............................................................. 38
   4.2 The DDTP Smoothing Algorithms ........................................................ 41
5. Application to Inter- and Postwar Coincident Indicators ...................................... 46
   5.1 Estimation of the FTP and DDTP Models ............................................ 50
   5.2 Identification of Inter- and Postwar Business Cycle Turning Points .......... 55
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Application to Prewar Coincident Indicators</td>
<td>60</td>
</tr>
<tr>
<td>6.1 Estimation of the FTP and DDTP Models</td>
<td>66</td>
</tr>
<tr>
<td>6.2 Identification of Prewar Business Cycle Turning Points</td>
<td>71</td>
</tr>
<tr>
<td>7. Concluding Remarks</td>
<td>83</td>
</tr>
<tr>
<td>List of References</td>
<td>90</td>
</tr>
<tr>
<td>Appendices</td>
<td></td>
</tr>
<tr>
<td>A. Proof of Identification of the AR(2) State Space Model</td>
<td>101</td>
</tr>
<tr>
<td>B. Filtering and Estimation of the FTP Model</td>
<td>108</td>
</tr>
<tr>
<td>B.1 State Space Representation</td>
<td>109</td>
</tr>
<tr>
<td>B.1.1 Motivation</td>
<td>110</td>
</tr>
<tr>
<td>B.1.2 The State Space Model</td>
<td>115</td>
</tr>
<tr>
<td>B.2 The Modified Kalman Filter</td>
<td>120</td>
</tr>
<tr>
<td>B.2.1 The Basic Filtering Algorithm</td>
<td>121</td>
</tr>
<tr>
<td>B.2.2 Estimation and Initialization</td>
<td>131</td>
</tr>
<tr>
<td>C. Filtering and Estimation of the DDTP Model</td>
<td>136</td>
</tr>
<tr>
<td>C.1 Computation of the Sparse Transition Matrix and Initialization of the Filter</td>
<td>137</td>
</tr>
<tr>
<td>C.1.1 Initialization of the Markovian Filter</td>
<td>137</td>
</tr>
<tr>
<td>C.1.2 Initialization of the Kalman Filter</td>
<td>140</td>
</tr>
<tr>
<td>C.2 Iterative Structure of the Kalman Filter</td>
<td>141</td>
</tr>
<tr>
<td>C.3 Iterative Structure of the Markovian Filter</td>
<td>143</td>
</tr>
<tr>
<td>D. Figures, Tables, and Illustrations</td>
<td>147</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fixed Transition Probability Model: Postwar Monthly</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td>Coincident Indicators, 1947.04 - 1996.09</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Duration-Dependent Transition Probability Model: Postwar</td>
<td>153</td>
</tr>
<tr>
<td></td>
<td>Monthly Coincident Indicators, 1947.04 - 1996.09</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Fixed Transition Probability Model: Interwar Monthly</td>
<td>159</td>
</tr>
<tr>
<td></td>
<td>Coincident Indicators, 1919.04 - 1939.03</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Duration-Dependent Transition Probability Model: Interwar</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>Monthly Coincident Indicators, 1919.04 - 1939.03</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Comparison of Inter- and Postwar Peaks and Troughs,</td>
<td>162</td>
</tr>
<tr>
<td></td>
<td>1919.04 - 1996.09</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Inter- and Postwar Peaks and Troughs, 1919.04 - 1996.09, from the DDTP</td>
<td>163</td>
</tr>
<tr>
<td></td>
<td>and FTP Dynamic Factor Models</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Ratio of Jerome’s Index of Factory Employment to Fabricant’s</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td>Index of Wage Earners in Manufacturing Industries in</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Census Years</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Comparison of the Growth Rates of Jerome’s Index of Factory</td>
<td>169</td>
</tr>
<tr>
<td></td>
<td>Employment and Fabricant’s Index of Wage Earners in</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Manufacturing Industries, Summary Statistics</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Comparison of Average Growth Rates and Standard Deviations,</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td>Prewar Monthly Coincident Indicators</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Fixed Transition Probability Model: Prewar Monthly</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>Coincident Indicators, 1885.04 - 1920.03</td>
<td></td>
</tr>
</tbody>
</table>
Duration-Dependent Transition Probability Model: Prewar Monthly Coincident Indicators, 1885.04 - 1920.03 ........................................ 176

Prewar Peaks and Troughs, 1885.04 - 1920.03, Identified via the Inferred Probabilities that $S_t = 0$ Conditional on $Y_T = \{y_1, \ldots, y_T\}$ ........................................ 180

Prewar Peaks and Troughs, 1885.04 - 1920.03, Identified via Estimates of the Growth Rate of the Common Factor, $\Delta f_{it}$ ............................................................. 181

Business Cycle Peaks and Troughs from the DDTP Model and Average Phase Durations ........................................ 184

Tests of the Null Hypothesis of No Postwar Duration Stabilization ........................................ 186


Fixed Transition Probability Model: Interwar Monthly Coincident Indicators, 1919.04 - 1948.03 ........................................ 189

Duration-Dependent Transition Probability Model: Interwar Monthly Coincident Indicators, 1919.04 - 1948.03 ........................................ 190
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Log Growth Rates of Industrial Production, 1987 = 100, 1919.02 - 1996.09</td>
<td>148</td>
</tr>
<tr>
<td>2</td>
<td>Log Growth Rates of Employees in Nonagricultural Establishments, 1929.02 - 1996.09</td>
<td>149</td>
</tr>
<tr>
<td>3</td>
<td>Log Growth Rates of Personal Income, Constant Dollars, 1929.02 - 1996.09</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on $Y_r = {y_1, \ldots, y_r}$, FTP Model of Monthly Coincident Indicators</td>
<td>151</td>
</tr>
<tr>
<td>5</td>
<td>Duration-Dependent Transition Probabilities, DDTP Model of Postwar Monthly Coincident Indicators</td>
<td>154</td>
</tr>
<tr>
<td>6</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on $Y_{r-1} = {y_1, \ldots, y_{r-1}}$, FTP Model of Postwar Monthly Coincident Indicators</td>
<td>155</td>
</tr>
<tr>
<td>7</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on $Y_r = {y_1, \ldots, y_r}$, FTP Model of Postwar Monthly Coincident Indicators</td>
<td>156</td>
</tr>
<tr>
<td>8</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on $Y_{r-1} = {y_1, \ldots, y_{r-1}}$, DDTP Model of Postwar Monthly Coincident Indicators</td>
<td>157</td>
</tr>
<tr>
<td>Page</td>
<td>Title</td>
<td>Reference</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>9</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on</td>
<td>$Y_t = {y_1, \ldots, y_T}$, DDTP Model of Postwar Monthly Coincident</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Indicators</td>
</tr>
<tr>
<td>10</td>
<td>Duration-Dependent Transition Probabilities, DDTP Model</td>
<td>of Interwar Monthly Coincident Indicators</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on</td>
<td>$Y_{t-1} = {y_1, \ldots, y_{t-1}}$, FTP Model of Interwar Monthly Coincident</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Indicators</td>
</tr>
<tr>
<td>12</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on</td>
<td>$Y_t = {y_1, \ldots, y_T}$, DDTP Model of Interwar Monthly Coincident</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Indicators</td>
</tr>
<tr>
<td>13</td>
<td>Estimates of the Growth of the Common Factor, $\Delta f_{nt}$, DDTP</td>
<td>Model of Interwar Coincident Indicators</td>
</tr>
<tr>
<td>14</td>
<td>Estimates of the Growth of the Common Factor, $\Delta f_{nt}$, DDTP</td>
<td>Model of Postwar Coincident Indicators</td>
</tr>
<tr>
<td>15</td>
<td>Fabricant's Annual Index of Wage Earners Employed in</td>
<td>Manufacturing vs. Jerome's Index of Factory Employment,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1889 - 1923 (Log Levels)</td>
</tr>
<tr>
<td>16</td>
<td>Log Growth Rates of the Jerome/BLS Index of Factory Employment,</td>
<td>1889.02 - 1920.03</td>
</tr>
<tr>
<td>17</td>
<td>Log Growth Rates of Bank Clearings Outside New York City,</td>
<td>1885.04 - 1920.03</td>
</tr>
<tr>
<td>18</td>
<td>Log Growth Rates of Total Imports, 1885.04 - 1920.03</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Log Growth Rates of the Miron-Romer Index of Industrial,</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Duration-Dependent Transition Probabilities, DDTP Model</td>
<td>of Prewar Monthly Coincident Indicators</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on $Y_{t-1} = {y_1, \ldots, y_{t-1}}$, FTP Model of Prewar Monthly Coincident Indicators</td>
<td>178</td>
</tr>
<tr>
<td>22</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on $Y_{t-1} = {y_1, \ldots, y_{t-1}}$, DDTP Model of Prewar Monthly Coincident Indicators</td>
<td>179</td>
</tr>
<tr>
<td>23</td>
<td>Inferred Probabilities that $S_t = 0$ Conditional on $Y_t = {y_1, \ldots, y_T}$, DDTP Model of Prewar Monthly Coincident Indicators</td>
<td>182</td>
</tr>
<tr>
<td>24</td>
<td>Estimates of the Growth of the Common Factor, $\Delta f_{it}$, DDTP Model of Prewar Coincident Indicators</td>
<td>183</td>
</tr>
<tr>
<td>25</td>
<td>Duration-Dependent Transition Probabilities, DDTP Model of Postwar Quarterly Coincident Indicators</td>
<td>191</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

Perhaps as an acknowledgment of the seven decades of work pursued by the National Bureau of Economic Research (NBER), the official NBER chronology of business cycle peaks and troughs has motivated a broad spectrum of research on both the nature and the evolution of business cycle phenomena. With respect to the former, the NBER chronology has been used not only to investigate asymmetries in business cycle behavior (Falk, 1986; Neftci, 1979; Wecker, 1979) but also to analyze the duration dependence associated with its individual expansionary and contractionary phases (Diebold and Rudebusch, 1990; Diebold, Rudebusch, and Sichel, 1993; Sichel, 1991). With respect to the latter, it has served as a basis for the hypothesis that the business cycle has systematically changed in the period following World War II (W.W.II), with this change first being optimistically summarized by Burns (1960, 1-2) as follows:

It is a fact of the highest importance, I think, that although our economy continues to be swayed by the business cycle, its impact on the lives and
fortunes of individuals has been substantially reduced in our generation. More than twenty-five years have elapsed since we last experienced a financial panic or a deep depression of production and employment. Over twenty years have elapsed since we last had a severe business recession. Between the end of the second world war and the present, we have experienced four recessions, but each was a relatively mild setback. Since 1937 we have had five recessions, the longest of which lasted only 13 months. There is no parallel for such a sequence of mild -- or such a sequence of brief -- contractions, at least during the past hundred years in our country....In considering problems of the future, we can proceed more surely by recognizing the changes in economic organization which already appear to have done much to blunt the impact of business cycles.

Burns' (1960) postwar "stabilization" hypothesis suggests two separate avenues of inquiry. The first focuses upon the possibility that the amplitudes of business cycle fluctuations have diminished during the postwar period, with amplitude being measured either by peak-to-trough declines in or by the standard deviations of the growth rate of output (Baily, 1978; DeLong and Summers, 1986); the second points to the possibility that the duration of the individual phases of the business cycle have stabilized between the pre- and post-W.W.II periods, with the duration of contractions -- or, the number of months elapsing between a business cycle peak and the subsequent business cycle trough -- becoming shorter and the duration of expansions -- or, the number of months elapsing between a business cycle trough and the subsequent business cycle peak -- becoming longer (Moore and Zarnowitz, 1986b).

While DeLong and Summers (1986) and Diebold and Rudebusch (1992), respectively, find empirical evidence in support of postwar volatility and duration stabilization, a recurring criticism of both hypotheses has been that any apparent shift
toward shorter, less volatile recessions is merely a “figment” of the data. Romer (1986a-c, 1989), for example, argues that if postwar series are constructed using components and methods approximating those of the prewar aggregates or if the prewar aggregates are reconstructed to reflect certain postwar relationships, the gulf between pre- and postwar volatility is diminished. In particular, she argues that many of the prewar aggregates from which conclusions regarding postwar volatility stabilization are drawn exhibit an “exaggerated cyclicality” because of the manner in which they were constructed. For instance, Frickey-like indexes of industrial production are based only on materials production rather than both materials and products production as is the case with the modern Federal Reserve Board index of industrial production. Because materials are more strongly procyclical, Romer argues that such aggregates are most likely more volatile than the underlying economy they are meant to proxy. In addition, Romer criticizes the Lebergott (1964) unemployment series not only because it assumes that the labor force does not vary with the business cycle but also because it assumes that employment in some sectors moves one-for-one with output in that sector. With respect to duration stabilization, Watson (1994) similarly points out that a dearth of prewar data required the early NBER researchers

1 Zarnowitz (1992) and Lebergott (1986) provide insightful criticisms of Romer’s reconstructed prewar aggregates. Balke and Gordon (1989) and Weir (1986), respectively, provide alternative estimates of prewar real GNP and unemployment that are consistent with Burns’ (1960) stabilization hypothesis.

2 Both the Lebergott (1964, Tables 9.2 and A-15) unemployment data and the Frickey (1947) index of industrial production are annual series. The Frickey series extends from 1860 to 1914; the Lebergott series extends from 1890 to 1930.
to date the prewar business cycle turning points using cyclically more sensitive series. Once this bias in the chronology is accounted for by dating postwar business cycles using comparable postwar data, duration stabilization in the postwar era vanishes, with expansions decreasing and contractions increasing in length.

The ultimate resolution of this volatility stabilization debate rests with the derivation of a reliable measure of the quantitative size of fluctuations in the prewar period. This may in turn only be obtained from an aggregate measure of the prewar U.S. economy that is directly comparable to a similar aggregate measure of the postwar U.S. economy. As pointed out by Diebold and Rudebusch (1992), the paucity of prewar source data makes the construction of such an incontrovertible prewar aggregate extremely difficult. The resolution of the duration stabilization debate, on the other hand, depends upon the derivation of a historically consistent chronology of business cycle turning points. Historical consistency in the case of a business cycle chronology is reflected not only in the use of data that is broadly comparable between the pre- and postwar eras but also in the uniform application of a single procedure for dating business cycle peaks and troughs. Our focus in these chapters is the development and application of such a procedure. As a starting point, we build upon the methodological contributions of Chauvet (1996) and Kim and Yoo (1995), who separately introduce multivariate algorithms for dating turning points in the comovements of series which are assumed to move contemporaneously with overall economic conditions. Given the dearth of prewar data, it is the application of such a
mechanical dating algorithm to the available pre- and post-W.W.II measures of aggregate economic activity which might ultimately yield an historically consistent chronology of business cycle turning points and, hence, an avenue for the ultimate resolution of the duration stabilization debate. Beyond this, our objective is twofold.

First, we turn our attention to those features of the cycle omitted particularly from Chauvet’s (1996) statistical model. Previous studies by Hamilton (1988, 1989, 1993), Lam (1990), Boldin (1994), and Filardo (1993, 1994), among others, suggest that those economic aggregates which are generally considered to mirror the overall state of the economy may be characterized as exhibiting asymmetries arising from four sources -- differences in the mean growth rates associated with the expansionary and contractionary states, differences in the transition probabilities between the same, differences in the unconditional expected duration of each phase, and differences in the duration dependence of each phase. Because Chauvet (1996) treats her transition probabilities as fixed, or because she assumes that the probability of moving from an expansion to an expansion or from a contraction to a contraction is constant, her statistical model does not capture this final feature of the business cycle -- particularly, the duration dependence in postwar contractions documented by Diebold and Rudebusch (1990), Sichel (1991), and Diebold, Rudebusch, and Sichel (1993), among others. We take account of this, or we take account of the possibility that the probability of remaining in a contractionary state is dependent upon the number of
months the economy has been in a contraction, by introducing time-varying transition probabilities (TVTP) into her dynamic factor model with regime switching.3

In doing so, we utilize a regime switching model described by Durland and McCurdy (1994). Durland and McCurdys' (1994) nonlinear model, like those in Diebold, Lee, and Weinbach (1994), Ghysels (1992), and Filardo (1993, 1994), conditions the evolution of the transition probabilities on an additional economic variable; however, unlike these other researchers who define this variable in terms of some observable economic fundamental or indicator, Durland and McCurdy (1994) take the novel approach of actually inferring its value from the number of periods the observable series has been in the current phase. By using Durland and McCurdys' methodology then, we are then making our transition probabilities not only time-dependent but also duration-dependent. It is the introduction of such duration-dependent transition probabilities (DDTP) which allows us to detect asymmetries in the duration dependence of expansions and contractions without reference to an existing business cycle chronology.

With respect to this second objective, both Kim and Yoo (1995) and Chauvet (1996) apply their dating algorithms only to the identification of postwar business cycle turning points. Although they generate business cycle chronologies that closely mimic the NBER chronology, it would be useful to apply their dynamic linear model with

3 Kim and Yoo (1995) do estimate a TVTP dynamic factor model; however, they condition the evolution of the transition probabilities on the Composite Index of Eleven Leading Economic Indicators.
regime switching (both with and without DDTP) to the dating of business cycles over a subsample of the pre-World War I (W.W.I) period considered by Burns and Mitchell (1947), in our case 1885 - 1920. Such a project is motivated by the following excerpt taken from *Measuring Business Cycles* (1947, 95):

This is not to say that the reference dates must remain in their present state of rough approximation. Most of them were originally fixed in something of a hurry, revisions have been confined mainly to large and conspicuous errors, and no revision has been made for several years. Surely, the time is ripe for a thorough review that would take account of extensive new statistical materials, and of the knowledge gained about business cycles and the mechanics of setting reference dates since the present chronology was worked out.

Romer (1994) has documented the nature of some of these “rough approximations” and the inconsistencies to which they have led in the pre- and interwar business cycle chronologies. For instance, modern NBER practices dictate that reference dates be identified from data that has been seasonally adjusted but not detrended. Romer (1994) is able to demonstrate, however, that the pre-1927 turning points are at least partially derived from the analysis of two detrended series, the A.T.&T. Business Index and the Snyder Clearings Index of Business. The use of such series potentially imparts a growth cycle bias on the pre-1927 turning points, resulting in a chronology that systematically identifies phases of below average growth as periods of decline of cyclical proportions (Zarnowitz, 1992). In addition to this, the NBER currently sets reference cycle dates using broad-based measures of real
economic activity such as industrial production and nonagricultural employment. The A.T.&T. Index differs markedly from these modern economic aggregates in that it contains nominal series. As a result of a decidedly downward trend in prices over the thirty years following the Civil War, it is likely that cyclical movements show up more strongly in these nominal aggregates than in their real equivalents (Zarnowitz, 1992). Romer (1994) argues that the likely result of the use of nominal as opposed to real series over this interval is a prewar NBER chronology that systematically dates turning points earlier than the postwar chronology.

Romer (1994) corrects these, among other, inconsistencies in the pre- and interwar chronologies by deriving, from the postwar chronology, a simple set of ex-ante rules for dating business cycle turning points. She then applies these to the prewar Miron-Romer industrial production series (1884.02 - 1940.12). The resulting prewar business cycle chronology does differ from that given by Burns and Mitchell in *Measuring Business Cycles* (1947); in particular, it challenges the notion that postwar contractions are shorter and postwar expansions longer than their prewar equivalents. However, Romer’s (1994) use of only industrial production is inconsistent not only with the repeated references in *Measuring Business Cycles* (1947) to the analysis of the specific cycles of hundreds of different series but also with the modern NBER practice of not identifying business cycle turning points from the specific cycle of a single aggregate (Hall, 1981). We deal with that inconsistency represented by Romer’s univariate dating algorithm by applying to groups of pre- and interwar series a
multivariate dating algorithm. For the fixed transition probability (FTP) and the DDTP dynamic factor models, the outcome of doing so is not only a series of prewar estimates of the comovements among these measures of real economic activity but also a series of prewar probabilities of being in a recession, both of which allow us to date prewar business cycles mechanically.

The remainder of this work proceeds as follows: Chapter 2 provides additional motivation for our study by briefly describing the nature of the business cycle in the context of our statistical model. Chapter 3 reviews the FTP dynamic factor model and briefly develops the DDTP dynamic factor model. With respect to the latter of these objectives, emphasis is placed upon demonstrating how DDTP may be feasibly introduced into Chauvet (1996) and Kim and Yoos' (1995) statistical framework. The details of the modified filtering algorithms used to infer both the probability of being in an unobserved state and the mean of the unobserved common factor are left to Appendixes B and C. Chapter 4 discusses those methodological issues relating to the implementation of the nonlinear filter we use in estimating these models as well as to the computation of the smoothing formulas we use in deriving business cycle chronologies. Chapter 5 applies the FTP and DDTP versions of the model discussed in Chapters 3 and 4 to business cycle indicators data covering the inter- and postwar periods in the United States (U.S.); Chapter 6 does the same for business cycle indicators data covering the pre-W.W.I period in the U.S. Chapter 7 offers some concluding remarks.
CHAPTER 2

WHAT IS A BUSINESS CYCLE?

In their landmark work, Burns and Mitchell (1947, 3) define business cycles in the following manner:

Business cycles are a type of fluctuation found in the aggregate economy of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic;....

This early definition of the business cycle has remained relatively unchanged in the fifty years since the publication of Burns and Mitchells' *Measuring Business Cycles* (1947), with Moore proposing only the following modification to the official NBER definition of a recession (Hall, 1981):

A recession is a recurring period of decline in total output, income, employment, and trade, usually lasting six months to a year and marked by widespread contractions in many sectors of the economy.
As is evident from both this original definition as well as from Moore's modification thereof, "a basic view of the business cycle that underlies the Bureau's measurements..." is that "business cycles are due to the predominance of agreement in timing among specific cycles" (Zarnowitz, 1963, 179-180), implying that the dating of a cyclical peak or trough is not the outcome of the analysis of a single aggregate but rather the result of the analysis of the comovements among multiple aggregates, with each aggregate representing a different facet of economic activity. This view is reflected in the following description of the NBER dating methodology (Burns and Mitchell, 1947, 76-77):

Our first step toward identifying business cycles was to identify the turns of general business activity indicated by [Thorp's (1926)] annals. Next, the evidence of the annals was checked against indexes of business conditions and other series of broad coverage. In most cases, these varied records pointed clearly to some one year as the time when a cyclical turn occurred. When there was conflict of evidence, additional statistical series were examined and historical accounts of business conditions consulted, until we felt it safe to write down an interval within which a cyclical turn in general business probably occurred. We then proceeded to refine the approximate dates by arraying the cyclical turns in the more important monthly or quarterly series we had for the time and country.

Those cyclical turns ultimately identified from these arrays of the cyclical turns of the specific series comprise the peaks and troughs of the "reference cycle." This reference cycle in turn functions as a proxy for an unobtainable ideal -- a single, precise indicator of the condition of the aggregate economy.
There are those who contend that such a proxy for overall economic activity is readily obtainable in the form of some comprehensive measure of output such as real GNP or industrial production and that the use of either one of these in identifying business cycle peaks and troughs is preferable to the NBER's purely subjective methodology of setting reference dates from the comovements of series. For instance, Cloos (1963) argues that the concept of general business is more closely related to a comprehensive measure of output like industrial production and that a chronology derived from the identification of its peaks and troughs would, as such, correspond much better to most economists and non-economists understanding of the business cycle. In addition, he argues that a chronology derived from the identification of the peaks and troughs of a single aggregate would remove much of the "fuzziness" surrounding the NBER-dated cyclical peaks and troughs.\(^4\)

However tempting it may be to equate business cycle turning points with the turning points of a single aggregate, reasons remain for not doing so. To begin, the Federal Reserve Board index of industrial production does not begin in its present monthly form as a weighted average of the indexes of both "materials" and "products" production until 1919; prior to this no similar comprehensive index of production is available. For the W.W.I and pre-W.W.I years (1869-1921), real GNP is available only

\(^{4}\) The extent of this fuzziness is gauged by the multitude of competing chronologies, the majority of which identify the same general periods of expansion and contraction as the NBER but differ as to the placement of a cyclical peak or trough in a given month (Persons, 1931; Ayres, 1939; Fels, 1959; Cloos, 1963).
on an annual basis (Romer, 1989; Balke and Gordon, 1989). Real quarterly GNP data is available after 1921; however, Moore (1961, 43-44 and 133) stresses that a dearth of quarterly statistics prior to 1929 makes the estimates of many of the constituent components of national output crude approximations at best. In fact, real GNP does not begin in its present quarterly form until two years after the end of W.W.II, in 1947. With respect to the years preceding the 1920s then, we are left without the modern indicators of aggregate output. Unfortunately, suitable replacements are unlikely to be found among the available comprehensive indexes and indicators of pre-W.W.I production.

For instance, one of three indicators of production and/or general business conditions discussed by Mitchell (1927, 189-360) is Frederick Macaulay's pig iron production series. This particular series is characterized by long, continuous coverage of its one component, with monthly data extending back to 1877 and through the 1940s. However, in determining the timing of pre-W.W.I business cycle turning points, a series based upon more than simply the production of a single commodity is desirable. An alternative to pig-iron production suggested by Mitchell (1927) is the trend-adjusted A.T.&T. Business Index. Unfortunately, the A.T.&T. Index is no more an ideal proxy for production in the aggregate economy than Macaulay's pig iron series (Mitchell, 1927, 294-295). From its inception in 1877 to 1884, the A.T.&T. index is

---

5 Balke and Gordon (1986) construct estimates of real quarterly GNP for the prewar period, beginning in 1875.1, from two sources. These are Person's (1931) index of industrial production and trade and the Federal Reserve Board index of (total) industrial production.
based solely upon pig iron production. The addition of blast furnace capacity and bank clearings outside New York City in 1885 and wholesale prices in 1892 broadens its coverage; however, these last two aggregates are at best only indirect indicators of production, with the former of these -- bank clearings -- representing the dollar totals of checks and drafts drawn on individual banks and credited to the accounts of other banks through city clearing houses and with the latter -- wholesale prices -- measuring the average change in the prices of nonagricultural commodities sold in primary markets.\(^6\) The fact thus remains that by modern standards the A.T.&T. index -- at least for the pre-W.W.I years -- is not a comprehensive indicator of output. The same may be said with respect to a third alternative offered by Mitchell (1927, 297-299) -- Persons' Index of Production and Trade. In this case, the index from its beginning in 1877 to 1902 is based solely upon pig iron production and bank clearings in seven cities; in both 1903 and 1915 its composition changes, with Persons adding merchandise imports, gross railroad earnings, and industrial employment in 1903 and removing all dollar valued series and adding cotton consumption and steel ingots production after 1915.\(^7\) As this listing of contributing series indicates, the only direct measure of output included in the pre-W.W.I portion of index is pig iron production.

In addition, Romer (1994, 323) argues that these three rather abrupt changes in

\(^6\) Additional changes to the A.T.&T. Index were made in 1903, 1909, 1913, 1919, and 1921; in 1922, all nominal series were dropped from the index (Mitchell, 1927, 294-295).

\(^7\) The post-1915 series is comprised of freight car loadings, pig-iron production, steel ingot production (added in 1919), raw cotton consumption, and industrial employment (Mitchell, 1927, 299).
coverage "may mean that apparent changes in behavior are due to changes in the data rather than to genuine changes in economic forces."

Similar arguments may be made with respect to the one pre-W.W.I production index included in Moore (1961, 39 and 130-131). The Babson Index of the Physical Volume of Business Activity, which Moore (1961) begins in 1889 although an extension is available back to 1871 (U.S. Department of Commerce, 1949, 322-323), measures movements in the physical volume of commercial and industrial production, including agricultural marketing and processing. Unlike the two comprehensive indicators described above (the A.T.&T. and Persons' index), the Babson index includes no nominal components, with the index being constructed by Babson as a base-year weighted aggregate of seasonally adjusted constant dollar magnitudes and physical volumes. Also, unlike the two previously discussed indicators, Babson's index is based upon a broader concept of production, with the 1889 components from which the index is constructed including coke and pig iron production along with cotton takings, bituminous/anthracite coal mining, and crude petroleum production. Miron and Romer (1990) identify two flaws in Babson. First, in addition to the 1889 production components listed above, the Babson index includes the gross value of imports and exports, making this not only an index of production but also of trade. Second, the Babson index is available only in a seasonally adjusted form, an attribute Miron and Romer (1990, 322) consider a particularly serious drawback given that
"early methods of seasonal adjustment involved complicated procedures that typically did more than remove regular seasonal fluctuations from the data."

Of course, one who is inclined to argue that business cycles are best measured via the ups and downs of a single aggregate can use the above critique of the available comprehensive indicators of production as a starting point for the construction of a more ideal proxy of pre-W.W.I output. This is exactly what Miron and Romer (1990) attempt with their monthly index of industrial production. Extending from 1884.02 to 1940.12, their series is constructed by combining, via historical value-added weights, various monthly indexes of the physical output of thirteen manufacturing and mineral products. Miron and Romer (1990) apply three criteria in the selection of these thirteen series. First, to avoid shifts in the coverage of their aggregate index, Miron and Romer include only reliable and consistent monthly indexes that date back to 1884. Second, to avoid overstating the cyclical behavior of the series over the deflationary periods following the Civil War and extending throughout the pre-W.W.I period, Miron and Romer exclude all nominally valued series. Finally, because their interest is in the construction of an index of production, Miron and Romer (1990) exclude all indirect proxies of output such as bank clearings and prices and include the physical

---

8 These thirteen series incorporate the following (Miron and Romer, 1990, 333-335): (1) metals (pig iron capacity and tin imports); (2) food (sugar meltings at four ports, cattle receipts in Chicago, hog receipts in Chicago, Minneapolis flour shipments, and coffee imports); (3) minerals (anthracite coal shipments, Connelsville coke shipments, and crude petroleum products); (4) apparel products (wool receipts in Boston, raw silk imports); (5) rubber products (crude rubber imports).
volumes of imports only if they may be used as proxies for the output of some domestically produced good.⁹

A comparison of these three criteria with the descriptions of the existing pre-W.W.I aggregates given above indicates that the Miron-Romer (1990) index is an improvement over the available alternatives. For instance, all four comprehensive measures of output, with the exception of pig iron production, include some components which are only indirect indicators of business activity. Two series -- Persons' Index of Production and Trade and the A.T.&T. Business Index -- include not only nominal components but also sometimes dramatic shifts in the types of series covered. Finally, all four series -- bar none -- encompass a narrow range of industrial output, with all but the Babson Index incorporating only pig iron production. This said, even Miron and Romer (1990) admit that their index is imperfect. In particular, an emphasis upon the use of long, consistent series as well as a paucity of prewar data combine to produce an index of the physical volume of production that is comprised exclusively of primary commodities. Because such primary commodities, or "materials," are typically more volatile than finished goods, or "products," it is likely that Miron and Romers' index of industrial production is more "cyclically sensitive" than either an alternative prewar index that would encompass both "materials" and

⁹ For example, crude rubber imports are included in the aggregate Miron-Romer index as a proxy for the output of rubber goods; coffee imports are included as a proxy for the production of processed coffee (Miron and Romer, 1990, 333-335).
“products” production (i.e., a prewar equivalent of the modern Federal Reserve Board Index of Industrial Production) or the actual economy which it is meant to proxy.  

As the above discussion of the available pre-W.W.I indicators demonstrates, a fundamental lack of prewar data impedes the construction of a single comprehensive measure of pre-W.W.I output. In measuring prewar business cycles, this reality leaves us with two options. We can date business cycles by equating reference cycle turning points with the peaks and troughs of one of these imperfect indicators, as Romer (1994) does; or we can take advantage of the available historical indicators of aggregate and sometimes disaggregate business activity, dating reference cycle turning points from clusters of specific cycle turning points, as Burns and Mitchell (1947) suggest. Adopting the latter of these approaches is advantageous in two respects. First, by identifying reference cycle turning points from groups of indicators for which the composition may change as a result of data availability but for which the defined area of coverage of the data does not, we can construct a reference cycle which is the best possible approximation (given the available data) of the underlying economy and, hence, of a single, comprehensive indicator of aggregate economic activity. In other words, we can compensate for the paucity of prewar data by applying to our dating algorithm varying direct and indirect measures of economic activity (e.g., income,  

---

10 “Volatility” here refers to the standard deviation of the growth rates of the series; increased “cyclical sensitivity or volatility” refers to the increased incidence of business cycle contractions. Watson (1994) provides a more detailed discussion of this possible volatility bias in the prewar data and its consequences with respect to the average durations of the phases and the duration stabilization debate.
trade, manufacturing activity, employment). Such an approach would be particularly necessary in the years preceding the 1870s. For here it is the case not only that all the comprehensive indexes reviewed by Mitchell (1927) and Moore (1961) are unavailable but also that only 19 monthly or quarterly series are available (Burns and Mitchell, 1947, 81-82). Three of these are disaggregate measures of production (hog receipts in Chicago, cattle receipts in Chicago, and boot and shoe shipments from Boston) which are of little value in setting reference cycle peaks and troughs; eight are financial variables; and eight are price series. Because all are indirect and/or disaggregate indicators and, hence, unsuitable proxies for aggregate production, one would seem to have little choice but to use a multivariate methodology that captures comovements among these series in establishing a chronology of business cycle peaks and troughs extending back to 1860.

Second, both Watson (1994) and Romer (1994) argue that shifts in phase durations between the pre- and post-W.W.II periods are the result of inconsistencies in the manner in which NBER researchers identified pre- and post-W.W.II business cycle turning points. Romer (1994) attempts to correct these methodological inconsistencies in a univariate context, applying dating rules inferred from the postwar NBER chronology to a prewar index of industrial production. We address this same issue in a multivariate context, applying a multivariate dating algorithm to groups of pre- and postwar series representing direct and indirect measures of various facets of economic activity. Such an approach in turn allows us to address one frequent criticism of the
NBER's subjective methodology -- namely, the sometimes considerable "fuzziness" surrounding its placement of cyclical peaks and troughs -- by allowing us to take a purely mechanical approach to dating business cycle turning points. In particular, we use both estimates of an unobserved component summarizing comovements among the series and inferred probabilities that the aggregate economy is in a recessionary phase to derive a reference cycle chronology.

The above discussion has attempted to justify the use of a multivariate approach to dating business cycle turning points in the pre-W.W.I era. Reasons of consistency alone would seem to justify the subsequent extension of our multivariate algorithm into the inter- and postwar period -- if only to obtain a postwar chronology whose construction approximates that of the prewar chronology. However, another reason remains for taking a multivariate approach to the dating of inter- and postwar business cycles. The years following W.W.I witnessed a veritable explosion in the number and quality of indicators of aggregate economic activity (Burns and Mitchell, 1947, 81-82). In 1920, for instance, 665 monthly and quarterly series were available; by 1930, this number had jumped to 710, with 154 of these relating to production alone. In 1914, the only comprehensive measure of employment available was a Bureau of Labor Statistics index of factory employment; no aggregate measures of production, income, or trade and sales existed. By 1919, however, the modern Federal Reserve Board index of industrial production had come into existence. By 1929, there also existed aggregate measures of employment in nonagricultural establishments and of personal
income; by the beginning of the postwar period, there also existed a comprehensive measure of manufacturing and trade sales in the U.S. (Moore, 1961). The selection of just one of these aggregates to serve as a proxy for overall economic activity would seem to produce a "fuzziness" of its own, with different series reaching their peak and trough levels at different times. Friedman (1961, 453-454) points out that this alone argues for the indispensability of a reference cycle chronology based upon the comovements of several series. In particular, he writes

The availability of the...reference cycle dates gives a general-purpose timing scale that obviates the necessity of choosing any single series as an index of that elusive concept "general business." The reference chronology can be used to explore the timing relation between another series and general business....This is a fairly crude technique...But it is one of the few techniques currently available...; it not only saves much labor but, more important, permits comparable observations over a much longer period.

In these chapters, we take a mechanical approach to the multivariate identification of business cycle turning points. We should point out, however, that mechanical algorithms for identifying the peaks and troughs of a single aggregate do exist. Two prominent examples of these are the Bry-Boschan (1971) procedure and Romer's (1994) loss rules for identifying specific cycles. The Bry-Boschan (1971) algorithm is a rather complicated set of ad hoc filters and rules for isolating peaks and troughs in an individual economic time series. Romer's (1994) algorithm is a relatively simple set of three rules which are based upon the cumulative loss of log output. The rules themselves are identified by matching the postwar NBER chronology with
postwar industrial production data and by then identifying the losses in the log of industrial production consistent with (1) the identification of a peak-to-peak whole cycle, (2) a distinction being drawn between local and maximum peaks and troughs, and (3) the dating of a business cycle peak or trough after the actual peak or trough in the series (i.e., the identification of a business cycle plateau).

Romer's (1994) algorithm is perhaps less-applicable than that of Bry-Boschan (1971) primarily because it is derived via an analysis of only postwar industrial production data. However, both algorithms represent *ex-ante* rules for identifying cyclical peaks and troughs, with the Bry-Boschan (1971) procedure isolating local minima and maxima in a series subject to constraints on both the length and amplitude of expansions and contractions.

The multivariate algorithm we derive here differs from the Bry-Boschan (1971) procedure and Romer's loss rules by identifying and dating turning points in the comovements among series not on the basis of a set of rules regarding amplitude or length derived from an existing chronology but on the basis of an inferred probability that the economy is in a recessionary or contractionary state. The following two chapters provide a more technical discussion of the statistical model and algorithm we apply in the multivariate dating of pre- and post-W.W.II business cycle turning points. Following the description of the basic setup of our model in Chapter 3, the reader may proceed to the results presented in Chapters 5 and 6 without any loss of continuity.
CHAPTER 3

THE FTP AND DDTP DYNAMIC FACTOR MODELS

Using a single-index model of macroeconomic variables that are hypothesized to move contemporaneously with overall economic conditions, Stock and Watson (1989, 1991) develop an experimental index of coincident economic indicators. Their approach to cataloging the broad-based swings in economic activity defined as the reference cycle is unique in the sense that it endows upon this intuitive concept a precise, mathematical definition. In particular, in the Stock and Watson model, the reference cycle is nothing more than a unobserved cyclical component that is assumed to describe all underlying business cycle trends. Although this so-called common factor is unobserved, by assuming that it affects several macroeconomic series in the same manner, Stock and Watson (1989, 1991) are able to use a Kalman filter to estimate its expected conditional value from observations on the coincident series. This expected conditional value in turn gives them a composite index of the coincident indicators, or a summary of cyclical path followed by the reference cycle.
In a review of the modern study of business cycles, Diebold and Rudebusch (1996) point out that while the Stock and Watson (1989, 1991) model is capable of capturing comovements among economic variables it remains silent with respect to a second defining characteristic of the business cycle, namely its nonlinearity arising from recurrent, discrete switches between alternative states of the world. Hamilton's (1989) Markov model addresses this in that it explicitly captures regime switches at the turning points between periods of high (positive) growth, labeled expansions, and periods of low (negative) growth, labeled recessions. This in turn enables one to use the data to make dating decisions, with the incidence of a peak or trough being determined not subjectively but from the conditional probability that the inferred state in a given period is expansionary or contractionary. Unlike the Stock and Watson (1989, 1991) model, the Hamilton (1989) model also allows one to identify certain asymmetries between the phases, with such asymmetries being represented by differences in the estimated mean growth rates and in the transition probabilities associated with the expansionary and contractionary states.

The Hamilton (1989) model is not without its flaws, however. To begin, it is a univariate filtering process, implying that any business cycle chronology derived from it is based not upon the turning points of a reference cycle but rather upon the turning points of the specific cycle of a selected economic aggregate. This characteristic need not be problematic. Using quarterly real GNP extending from 1952.1 to 1984.4, Hamilton (1989) reproduces the official NBER chronology; however, applying the
same FTP Markov model to monthly industrial production data, Filardo (1994) finds
only a weak correlation between the growth phases identified by the estimated
probabilities and the expansions and contractions identified by the NBER.\textsuperscript{11} This
highlights a second, more significant, problem -- the failure of the Hamilton model
(1989), when applied to series whose frequency is monthly, to account for several
historical business cycle episodes. Although one can argue that the NBER chronology
is itself only the best judgment of a committee of experts and, therefore, open to
reevaluation, it must also be pointed out that the power of the Hamilton model to
detect business cycle turning points arises from its ability detect outliers in growth
rates. As such, the noise inherent in monthly data has at minimum the potential to
mask what might otherwise be identified by the univariate filter as a peak or trough,
making the Hamilton (1989) model a potentially poor tool for the mechanical dating of
cyclical turning points.

Diebold and Rudebusch (1996) suggest an alternative approach to the
mechanical dating of business cycles. Building upon the work of both Hamilton (1989)
and Stock and Watson (1989, 1991), they propose a multivariate dynamic factor model
with regime switching which, because it incorporates the single-index framework
developed in Stock and Watson, captures comovements among business cycle

\textsuperscript{11} Boldin (1994) finds a much stronger correlation when he applies a Markov model with FTP to
monthly unemployment rates, as does Filardo (1994) when he applies a Markov model with TVTP to
monthly industrial production data. Lam (1995) finds only a weak correlation between the identified
growth phases and the NBER chronology when he applies Hamilton's (1989) model to quarterly real
GNP extending from 1952.2 - 1993.3.
indicators and which, because it allows the mean growth rate of the unobserved common factor to vary with the state of the world, produces the conditional probabilities needed to derive a business cycle chronology. Until recently, estimation of their proposed framework was not feasible because it necessitated both estimation of the conditional mean of an unobserved common factor and inference on a latent Markov variable. Separate studies by Chib (1993), Shephard (1994), and Carter and Kohn (1994) as well as by Kim (1994) have provided suitable estimation methods, with Shephard (1994) and Carter and Kohn (1994) developing efficient multi-move Gibbs sampling techniques and with Kim (1994) providing an easily implementable approximate-maximum likelihood based approach. With respect to the former of these, Shephard (1994) and Carter and Kohn (1994) suggest iterative simulation methods that can be used to produce Bayesian estimates of the unobserved regime indicator and the unobserved common growth factor; because theirs is an inference free methodology, an exact likelihood function is obtained. With respect to the latter, Kim combines Hamilton's (1989) Markovian filter with a discrete version of the Kalman filter to produce an algorithm for solving a dynamic factor model with regime switching.\footnote{Kim's (1994) work complements that of Highfield (1990) and Shumway and Stoffer (1991), who separately introduce algorithms for solving generalized state space models characterized by switches.} because Kim's methodology requires that the posterior value of the state vector, which is a mixture of normal distributions for $t > 2$, $t = 1, \ldots, T$, be approximated as a normal
distribution at each iteration, it does not produce the exact log-likelihood function but rather an approximation thereof.

Kim and Yoo (1995) and Chauvet (1996) take up estimation of that multivariate dynamic factor model suggested by Diebold and Rudebusch (1996) using the approximate-maximum likelihood methods developed in Kim (1994); Kim and Nelson (1995) do the same using the more computationally time-consuming simulation techniques given in Shephard (1994) and Carter and Kohn (1994). One very clear advantage of the Bayesian inference pursued by Kim and Nelson (1995) are the sequences of regime indicators, \( S_\tau = [S_1, S_2, \ldots, S_\tau] \), generated by each run of the Gibbs sample. These allow Kim and Nelson (1995) to realize the pairs, \( [S_\tau = j, S_{\tau-1} = i] \), \( i, j = 0,1 \), from which they calculate not only the frequency of transitions at \( \tau \) but also the duration of recessions or recoveries up to \( \tau - 1 \). From the latter of these quantities, they conduct multivariate tests of postwar duration dependence.

Kim and Nelson’s (1995) tests are derived without explicitly incorporating into that framework suggested by Diebold and Rudebusch (1996) time- or duration-dependent transition probabilities. Our contribution here is to incorporate these into a dynamic factor model with regime switching, in the process constructing a filtering

algorithm which may be applied to generalized state space models with duration-dependent state transitions.

Section (3.1): The FTP Model

Like Kim and Yoo (1995) and Chauvet (1996), we consider the following modification of the Stock and Watson (1989, 1991) model:

\[ \Delta Y_i = \gamma_i(L) \Delta f_i + \Delta v_i \quad i = 1, \ldots, n \quad (1) \]

\[ \Delta f_i = \mu_0 + \mu_i S_i + \phi(L) \Delta f_{i-1} + \eta_i \quad S_i = 0, 1 \quad (2) \]

\[ \Delta v_i = D_i(L) \Delta v_{i-1} + \varepsilon_i, \quad i = 1, \ldots, n \quad (3) \]

for which

\[ \eta_i \sim \text{iid} \ N(0, \sigma_{\eta}^2), \]

\[ \varepsilon_i \sim \text{i.i.d.} \ N(0, \sigma_{\varepsilon}^2), \]

\[ D_i(L) = \text{diag}(d_1(L), \ldots, d_n(L)), \]

and

\[ p_{ji} = \Pr[S_i = j | S_{i-1} = i], \quad \sum_{j=0}^i p_{ji} = 1 \quad \forall \ i \text{ for } M \text{ states}. \]

The first two of these restrictions imply that \( \Delta f_i \), the growth rate of the unobserved common factor, and \( \Delta v_i \), the growth rate of the \( i \)th idiosyncratic component, are mutually and serially uncorrelated at all leads and lags; the second and third restrictions
imply that $\Delta v_{i,j}$ is uncorrelated with all other idiosyncratic components, $\Delta v_{i,j}$, $i,j = 1, \ldots, n$, $i \neq j$, at all leads and lags. These first three assumptions, combined with the normalization $\sigma^2 = 1$, jointly ensure identification of the state space model derived from equations (1) - (3).\textsuperscript{14} The final restriction given above implies that a two-state, first-order Markov process governs all switches between the recessionary, $S_i = 0$, and expansionary, $S_i = 1$, states. Those nonlinearities arising from these switches are reflected in the mean growth rates, with $\mu(S_i = 0)/(1 - \phi(L)) = \mu_0/(1 - \phi(L))$ defining state 0 as a decelerated (or negative) growth state and $\mu(S_i = 1)/(1 - \phi(L)) = (\mu_0 + \mu_1)/(1 - \phi(L))$ defining state 1 as an accelerated (or positive) growth state.

Returning to that model presented in equations (1) - (3), $Y_a$ in (1) represents the log of $i$th endogenous observable variable hypothesized to move contemporaneously with overall economic conditions; $\phi(L)$, $\gamma(L)$, and $D(L)$ in (1) - (3) represent, respectively, a scalar, a vector, and a matrix lag polynomial, with $L$ denoting the lag operator and with $\Delta = 1 - L$. The vector lag polynomial, $\gamma_i(L)$, gauges the responsiveness of the $i$th coincident indicator to movements in the underlying business cycle component. Setting $L = 0$ restricts the $i$th series to move only contemporaneously with the business cycle; setting $L = q$, $q > 0$, allows the $i$th

\textsuperscript{14} Setting $\sigma^2 = 1$ identifies the scale of $\Delta f_i$. Setting a single $\gamma_i = 1$, $i = 1, \ldots, n$, achieves the same objective.
series to lag the business cycle by possibly $q$ periods. In Stock and Watson (1989), $L = 0$ in the case of the coincident indicators, industrial production, personal income less transfer payments, and manufacturing and trade sales; $L = 4$ in the case of the lagging indicator, employee hours in nonagricultural establishments.

Finally, equations (2) and (3) jointly indicate that all cyclical dynamics in this modified Stock and Watson (1989, 1991) model are generated by one of two sources, idiosyncratic shocks entering via fluctuations in $\varepsilon_t$ and common shocks entering via fluctuations in $\eta_t$. Equations (1) - (3), in combination with the first three restrictions ensure that all comovements exhibited by the $n$ series arise from a single source, fluctuations in $\eta_t$.

The state space model into which we cast (1) - (3) is not unique. By first filtering $\Delta y_t$, $\gamma_t\Delta f_t$, and $\Delta \nu_t$ in (1) by $D(L)$ and by then assuming that (2) follows an AR(2) process, (3) follows an AR(1) process, and that all $n = 4$ indicators are coincident, we obtain the following state space model for which the dimension of the state vector, $\alpha_t$, is independent of the number of idiosyncratic components, $\Delta \nu_t$, $i = 1, \ldots, n$:

$$
\begin{bmatrix}
\Delta y_{1t} \\
\Delta y_{2t} \\
\Delta y_{3t} \\
\Delta y_{4t}
\end{bmatrix}
= 
\begin{bmatrix}
d_1 & d_3 & 0 & 0 \\
d_2 & d_4 & 0 & 0 \\
d_3 & d_5 & 0 & 0 \\
d_4 & d_6 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\gamma_1 & -d_1\gamma_1 & 0 & 0 \\
0 & \gamma_2 & -d_2\gamma_2 & 0 \\
0 & 0 & \gamma_3 & -d_3\gamma_3 \\
0 & 0 & 0 & \gamma_4 -d_4\gamma_4
\end{bmatrix}
\begin{bmatrix}
\Delta y_{t-1} \\
\Delta y_{t-2} \\
\Delta y_{t-3} \\
\Delta y_{t-4}
\end{bmatrix}
+ 
\begin{bmatrix}
\Delta f_t \\
\Delta f_{t-1} \\
\Delta f_{t-2} \\
\Delta f_{t-3}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t} \\
\varepsilon_{3t} \\
\varepsilon_{4t}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t} \\
\varepsilon_{3t} \\
\varepsilon_{4t}
\end{bmatrix}
$$

(4)
with (4) being summarized by

\[ \Delta Y_t = \beta + Z\alpha_t + \xi_t, \quad \xi_t \sim N(0, H) \]  

(4')

and with (5) being summarized by

\[ \alpha_t = \mu(S_t) + T\alpha_{t-1} + Ru_t, \quad u_t \sim N(0, Q). \]

(5')

Equation (4') is the measurement equation; equation (5') is the transition equation. The term \( f_{t-1} \) is included as the final element of \( \alpha_t \) to permit estimation of the composite index in its levels form via the identity, \( f_{t-1} = \Delta f_{t-1} + f_{t-2} \).

---

15 Because not all series considered here begin in the first month included in the interval of estimation, (4') is multiplied an \( n \times n \) matrix \( W_t \), whose rows are a subset of the \( n \times n \) identity matrix. In the case that the time \( t \) observation corresponding to the third coincident indicator is missing, we substitute the following into (4') to reduce the dimension of \( \Delta Y_t, \beta, Z, \) and \( H \) by one:

\[ \Delta Y'_t = W_t\Delta Y_t, \quad \beta' = W_t\beta, \quad Z' = W_tZ, \quad \text{and} \quad H' = W_tHW_t', \]

where

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

In the case that the time \( t \) observations corresponding to both the third and fourth coincident indicators are missing, we substitute the same into (4'), with

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

so that the dimension of \( \Delta Y_t, \beta, Z, \) and \( H \) is reduced by two.

16 Estimates of the level of the index may be extracted by first setting \( f_{t-1} = 100 \cdot \ln(\hat{f}_{t-1}) \), where \( \hat{f}_{t-1} \) is the desired factor level, and by then exponentiating \( f_{t-1} \) and indexing \( \hat{f}_{t-1} \) to equal 100 in a selected month and year.
Section (3.2): The DDTP Model

The DDTP dynamic factor model is subject to the same structure and restrictions as the FTP model given by equations (1) - (3); however, in the case of DDTP, the two-state, first-order Markov transition probability matrix represented by

$$ p_{ji} = \Pr[S_i = j | S_{t-1} = i], \sum_{j=0}^{1} p_{ji} = 1 \forall i $$

is replaced by

$$ p_{j(d)} = \Pr[S_i = j | S_{t-1} = i, D_{t-1} = d], \sum_{d=0}^{1} \sum_{j=0}^{1} p_{j(d)} = 1 \forall i, $$

where $D_{t-1}$ is an integer-valued, random variable summarizing the number of periods the economy has been in the current state as of time $t-1$. Because the conditional transition probabilities summarized by $p_{j(d)}$ are dependent upon more than simply the first lag of $S_t$, it is necessary for us to parameterize them using a functional form that both maps $D_{t-1}$ into the unit interval and nests a first-order Markov process characterized by fixed transition probabilities. Following Filardo (1993, 1994) and Durland and McCurdy (1994), we select the logistic functional form, obtaining the following specification of $p_{j(d)}$ in the two state case:\footnote{If $b_0 = 0$, no duration dependence is associated with recessions; if $b_1 = 0$, no duration dependence is associated with expansions. If $b_0 = b_1 = 0$, (6) is consistent with a FTP first-order Markov process.}
The variable \( \tau \) in equation (6) warrants further explanation. In Durland and McCurdy (1994), whose univariate filtering algorithm we incorporate into that multivariate filtering algorithm introduced in Kim (1994), Hamilton’s (1989) first-order Markov specification is extended to a \( \tau \)-order system, \( 1 \leq \tau < N \), where \( N \) is the length of the time series being modeled. So that the focus of the analysis is shifted from the implementation of a univariate filtering algorithm characterized by a higher order Markov process to the implementation of a univariate filtering algorithm characterized by duration dependence, the effects of the \( \tau \)-lags of the Markov states are collapsed into \( d \) by replacing \( \Pr(S_t = i|S_{t-\tau} = i, \ldots, S_{t-\tau} = i) \) with \( \Pr(S_t = i|S_{t-\tau} = i, D_{t-\tau} = \tau) \). The practical result is the imposition of restrictions on a
(2τ × 2τ) transition matrix such that the only nonzero elements are those associated with first-order transitions between \( S_t \) and \( S_{t-1} \).

Durland and McCurdys' (1994) treatment of TVTP differs from that of other researchers. In particular, the value of duration variable, \( D_{t-1} \), upon which Durland and McCurdy (1994) condition their time-varying transition probabilities is inferred from the observable series, \( Y_t = \{y_1, \ldots, y_t\} \). All other researchers who have introduced TVTP into a Markov regime-switching model have conditioned their time-varying probabilities on an additional observable economic variable, e.g., seasonal indicators in the case of Ghysels (1992) and the Composite Index of Eleven Leading Economic Indicators in the case of Filardo (1994). This alternative approach is much easier to implement in both the univariate and multivariate cases in that at each pass through the filter one need only compute a \((M' × 1)\) vector of inferred prediction and filtering probabilities, where \( r \) is the number of latent Markov variables carried at each iteration. With Durland and McCurdy (1994), it is necessary to compute a \((M' × \tau)\) matrix of such probabilities at each iteration so that there exists a vector of conditional probabilities corresponding to each possible value of \( d \). To make their filter computationally feasible, Durland and McCurdy (1994) restrict \( d ≤ \tau \), with the optimal value of \( \tau \), in essence the upper limit on the memory of the Markov process, being chosen via a grid search for which the maximized value of the log-likelihood is the criteria.
While being computationally demanding, Durland and McCurdys' (1994) approach to introducing TVTP into a Markov model is advantageous in two respects. First, it does not require that we condition our transition probabilities on a business cycle predictor whose turning points precede those of the computed reference cycle. Because we incorporate into our analysis the inter- and prewar periods, this is particularly important given the dearth of pre-W.W.II analogues to those leading indicators utilized in Filardo (1994). Second, it allows us to incorporate explicitly into that dynamic factor model given by equations (1) - (3) not only asymmetries in the duration dependence of the two phases but also a parsimoniously summarized higher-order Markov process, effectively increasing the amount of information to the filter without concomitantly increasing the number of transition parameters.

We incorporate these higher-order processes into our log-likelihood by introducing the inferred duration variable, $D_{t-1}$, into the conditional density of $y_t$. For the DDTP dynamic factor model then, the conditional likelihood, denoted $f^*$, is given by

$$f^*(y_t | Y_{t-1}) = \sum_{s_{t-1} = 0}^{1} \sum_{s_{t-1} = 0}^{d} \hat{f}(y_t | S_t = s_t, S_{t-1} = s_{t-1}, \ldots, S_{t-r} = s_{t-r}, Y_{t-1}, D_{t-1} = d)$$

$$\times \Pr(S_t = s_t, Y_{t-1}, D_{t-1} = d) \times P(S_{t-1} = s_{t-1}, \ldots, S_{t-r} = s_{t-r}, D_{t-1} = d | Y_{t-1})$$

---

18 Those business cycle predictors considered by Filardo (1994) include the Composite Index of Eleven Leading Economic Indicators (1948.01 - 1992.08), the Stock and Watson (1989) Experimental Index of Leading Indicators (1960.01 - 1992.08), Standard and Poor's Composite Stock Index (1948.01 - 1992.08), and the Federal Funds Rate (1953.05 - 1992.08).
with \( \hat{f}(y_t | s_t = j, s_{t-1} = i, Y_{t-1}, D_{t-1} = d) = (2\pi)^{-N/2} |F_t^{-j(d)}|^{-1/2} \exp\left(-\frac{1}{2} \eta_{t-1}^{\mu(d)} F_t^{j(d)} \eta_{t-1}^\mu(d) \right) \)

and with the log-likelihood function in turn being given by \( L(\theta) = \sum_{t=1}^{T} \ln \left[ f^*(y_t | Y_{t-1}, \theta) \right] \).

The vector \( \theta \) consists of the system parameters given in (4') and (5') and the transition probability parameters given in (6) and (8).

One notable difference between our nonlinear filter and that of Durland and McCurdy (1994) is conditionality of \( \hat{f}(y_t | \cdot) \) on \( D_{t-1} \). The conditional density derived via our nonlinear filter is made a function of the duration variable \( D_{t-1} \) as a result of the “collapsing” process we employ in reducing the optimal number of estimates of the posterior distribution of \( \alpha_t \). This collapsing process is discussed in greater detail in the next chapter. All other details of our filtering algorithm are left to Appendix C.
CHAPTER 4

METHODOLOGICAL ISSUES

Two methodological issues warrant discussion in this chapter. First, we provide a skeletal outline of the DDTP filter used in the estimation of our nonlinear dynamic factor model. In this context, we review a "collapsing" process by which the computational burden brought on by the introduction of multiple states of the world into a fixed coefficient state space model is reduced. Second, we derive the duration-dependent smoothing algorithms with which we infer the state of the economy and compute estimates of the unobserved state vector at $t$ conditional upon $Y_t$. In Chapter 5, we use the smoothed probabilities generated by the former of these algorithms to date interwar and post-W.W.II business cycle turning points; in Chapter 6, we use the smoothed estimates of the unobserved common factor generated by the latter to date pre-W.W.I business cycle turning points. Throughout this chapter, as well as in all that follow, we make the simplifying assumption of carrying only two latent Markov variables at anyone iteration — $S_t$ and its first lag, $S_{t-1}$. 
Section (4.1): The DDTP Filtering Algorithm

The objective of Kim's (1994) nonlinear filter for estimating a generalized state space model characterized by FTP is the formation at each date $t$ of a forecast of the unobserved state vector, $\alpha_t$, conditional not simply upon $Y_{t-1}$ but also upon the unobserved latent variables, $S_t = j$ and $S_{t-1} = i$. Adopting Kim's (1994) notation, this forecast is denoted

$$a^u_{t|t-1} = E(\alpha_t|Y_{t-1}, S_t = j, S_{t-1} = i),$$

where $a^u_{t|t-1}$ represents one of a set of $M^2$, $M = 2$, different forecasts, with one such forecast corresponding to each possible combination of $j$ and $i$. Associated with these are $M^2$ different mean squared error matrices, represented here by

$$p^u_{t|t-1} = E\left( (\alpha_t - a^u_{t|t-1})(\alpha_t - a^u_{t|t-1})' | Y_{t-1}, S_t = j, S_{t-1} = i \right).$$

The nonlinear filter used here in estimating a dynamic factor model into which duration-dependent transitions have been introduced is derived via some minor modifications to these equations. In particular, the conditional forecast of the unobserved state vector $\alpha_t$ is now represented by

$$a^{u(d)}_{t|t-1} = E(\alpha_t|Y_{t-1}, S_t = j, S_{t-1} = i, D_{t-1} = d),$$

while its associated matrix of mean squared errors is denoted by

$$p^{u(d)}_{t|t-1} = E\left( (\alpha_t - a^{u(d)}_{t|t-1})(\alpha_t - a^{u(d)}_{t|t-1})' | Y_{t-1}, S_t = j, S_{t-1} = i, D_{t-1} = d \right).$$
In contrast to the FTP case given above, $a_{t-1}^{(d)}$ is now the forecast of $\alpha_t$ conditional not only upon the economy attaining state $j$ at $t$ and state $i$ at $t-1$ but also upon the economy having attained state $i$ for $d$ consecutive periods at $t-1$. The recursive algorithm by which $a_{t-1}^{(d)}$ and $p_{t-1}^{(d)}$ are computed is written as follows:

\begin{align}
\alpha_{t-1}^{(d)} &= T\alpha_{t-1|t-1}^{(d)}, \\
p_{t-1}^{(d)} &= Tp_{t-1|t-1}^{(d)}T' + RQR', \\
\eta_{t-1}^{(d)} &= y_t - Z\alpha_{t-1}^{(d)}, \\
F_t^{(d)} &= Zp_{t-1}^{(d)}Z' + H, \\
K_t^{(d)} &= p_{t-1}^{(d)}Z\left(F_t^{(d)}\right)^{-1}, \\
a_{t}^{(d)} &= \eta_{t}^{(d)} + K_t^{(d)}\eta_{t-1}^{(d)}, \\
p_{t}^{(d)} &= (I - K_t^{(d)}T)p_{t-1}^{(d)},
\end{align}

where $a_{t-1}^{(d)}$ and $p_{t-1}^{(d)}$ are the first two moments of the posterior distribution of $\alpha_{t-1}$ conditional upon information up to $t-1$ given $S_{t-1} = i$ and $D_{t-1} = d$; $a_{t-1}^{(d)}$ and $p_{t-1}^{(d)}$ are the first two moments of the prior distribution of $\alpha_t$ conditional upon information up to $t-1$ given $S_t = j$, $S_{t-1} = i$, and $D_{t-1} = d$; and $K_t^{(d)}$ is the Kalman gain. $\eta_{t-1}^{(d)}$ and $F_t^{(d)}$ denote, respectively, the conditional forecast error and variance of $y_t$. In
(9.2) and (9.4), respectively, \( Q = \text{var}(u_t) = \sigma^2_\eta \) while
\[
H = \text{var}(\xi_t) = \text{diag}(\sigma^2_{\xi_1}, \ldots, \sigma^2_{\xi_n}).
\]

An inspection of equations (9.1) - (9.7) reveals that while (9.6) - (9.7) produce estimates of the first two moments of the posterior distribution that are jointly conditional upon \( D_{t-2} = d-1, S_{t-1} = i, \) and \( S_{t-2} = h \) equations (9.1) - (9.2) take as inputs only those estimates that are conditional upon \( D_{t-1} = d \) and \( S_{t-1} = i \) alone. To obtain \( a^\text{th}_{t-|t-|t-1} \) and \( p^\text{th}_{t-|t-|t-1} \) from \( a^\text{th(d-1)}_{t-|t-|t-1} \) and \( p^\text{th(d-1)}_{t-|t-|t-1} \), it is necessary to "collapse" the set of \( M^2 \times \tau \) conditional forecasts and their associated mean squared error matrices given by (9.6) - (9.7) into a set of only \( M \times \tau \) such conditional forecasts and matrices. This is accomplished by extending an approximation taken from Harrison and Stevens (1976), whereby \( a^i_{t-|t-|t-1} \) and \( p^i_{t-|t-|t-1} \) are computed as weighted averages of \( a^\text{th}_{t-|t-|t-1} \) and 
\[
\begin{align*}
(a^\text{th}_{t-|t-|t-1} & + \left( a^\text{th}_{t-|t-|t-1} - a^\text{th(d-1)}_{t-|t-|t-1} \right) \right) \\
A^\text{th}_{t-|t-|t-1} & = \sum_{h=0}^{1} P(S_{t-1} = i, S_{t-2} = h, D_{t-1} = d|Y_{t-1}) a^\text{th(d-1)}_{t-|t-|t-1} \\
P^\text{th}_{t-|t-|t-1} & = \sum_{h=0}^{1} P(S_{t-1} = i, S_{t-2} = h, D_{t-1} = d|Y_{t-1}) p^\text{th(d-1)}_{t-|t-|t-1} \\
\end{align*}
\]
(10.1)
and
\[
\begin{align*}
A^\text{th}_{t-|t-|t-1} & = \sum_{h=0}^{1} P(S_{t-1} = i, S_{t-2} = h, D_{t-1} = d|Y_{t-1}) a^\text{th(d-1)}_{t-|t-|t-1} \\
P^\text{th}_{t-|t-|t-1} & = \sum_{h=0}^{1} P(S_{t-1} = i, S_{t-2} = h, D_{t-1} = d|Y_{t-1}) p^\text{th(d-1)}_{t-|t-|t-1} \\
\end{align*}
\]
(10.2)
where $P(S_{t-1} = i, D_{t-1} = d|Y_{t-1})$ is the inferred probability that the economy is in state $i$ at $t-1$ after having attained this state for $d$ consecutive periods.

As a consequence of (10.1) and (10.2), we do not explicitly model $\Delta Y_\tau$ in (4) as a function of $D_{t-1}$. Rather, the duration dependence exhibited by the $M \times \tau$ conditional forecasts and their mean squared error variances arises because the $M \times \tau$ updates, $d_{t-1|t-1}^{(d)}$ and $x_{t-1|t-1}^{(d)}$, from which they are iteratively computed are weighted averages of duration-dependent inferred probabilities.

Section (4.2): The DDTP Smoothing Algorithms

Once the system and transition parameters have been estimated via maximization of the log-likelihood, it remains for us, first, to obtain inferences about $S_t$ conditional on the full-sample $Y_T$ and, hence, to compute the smoothed probabilities, $\Pr(S_t = j|Y_T)$, from which we construct our interwar and post-W.W.II business cycle chronologies in Chapter 5. The smoothing algorithm with which we accomplish this task for the DDTP model is derived as follows.\(^{19}\)

\(^{19}\)Kim (1994) provides a derivation of that smoothing algorithm we apply in the case of the FTP model.
\[ P(S_{t+1} = k, S_t = j, D_t = x | Y_T) = P(S_{t+1} = k | Y_T) \times P(S_t = j, D_t = x | S_{t+1} = k, Y_T) \]
\[ \approx P(S_{t+1} = k | Y_T) \times P(S_t = j, D_t = x | S_{t+1} = k, Y_T) \]
\[ = \frac{P(S_{t+1} = k | Y_T) \times P(S_t = j, D_t = x) \times P(S_t = j, D_t = x | Y_t)}{P(S_{t+1} = k | Y_T)}, \]

(11)

where

\[ P(S_{t+1} = k | Y_t) = \sum_{j=0}^{i} \sum_{x} P(S_{t+1} = k, S_t = j, D_t = x | Y_t) \]

and

\[ P(S_t = j | Y_T) = \sum_{k=0}^{i} \sum_{x} P(S_{t+1} = k, S_t = j, D_t = x | Y_T). \]

This derivation is based upon that of a similar smoothing algorithm found in Kitagawa (1987) for the case of a general non-Gaussian state space model. As in Kitagawa's (1987) algorithm, an approximation is involved in moving from the first to the second line of equation (11). The nature of this approximation is demonstrated by the following for which \( \psi_{t+1,T} = \{y_{t+1}, \ldots, y_T\} : \)

\[ P(S_t = j, D_t = x | S_{t+1} = k, Y_t, \psi_{t+1,T}) \]
\[ = \frac{f(S_t = j, D_t = x, S_{t+1} = k, Y_t)}{f(\psi_{t+1,T} | S_{t+1} = k, Y_t)} \]
\[ = \frac{P(S_t = j, D_t = x | S_{t+1} = k, Y_t) \times f(\psi_{t+1,T} | S_{t+1} = k, S_t = j, D_t = x, Y_t)}{f(\psi_{t+1,T} | S_{t+1} = k, Y_t)}. \]

(12)

In other words, equation (11) is exact so long as

\[ f(\psi_{t+1,T} | S_{t+1} = k, S_t = j, D_t = x, Y_t) = f(\psi_{t+1,T} | S_{t+1} = k, Y_t) \]

(13)
in (12). Intuitively, we may interpret this equality to mean that if $S_{t+1}$ is known then $y_{t+1}$ contains no information about $S_t$ and $D_t$ beyond that contained in $S_{t+1}$ and $y_t$. For our state space model with regime switching, this is not the case, making our smoothing algorithm an approximation. However, this is an approximation which yields an algorithm for calculating smoothed probabilities that is significantly less involved than that found in Diebold, Lee, and Weinbach (1993).

Once we have obtained the smoothed probabilities represented by $\Pr(S_{t+1} = k, S_t = j | Y_T)$ and $\Pr(S_t = j | Y_T)$, we may in turn use these in the computation of smoothed estimates of the unobserved state vector. Extending the fixed-interval smoothing algorithm described in Harvey (1989, 154-155) to the case of a discrete state space model for which the transitions between the various states of the world are duration-dependent, we arrive at the following:

$$ a_{tt}^{k(d+1, d)} = a_{tt}^{j(d)} + \bar{P}_t \left( a_{t+1 | t}^{k(d+1)} - a_{t+1 | t}^{j(d)} \right), \quad t = T - 1, \ldots, 1, \quad (14) $$

$$ P_{tt}^{k(d+1, d)} = P_{tt}^{j(d)} + \bar{P}_t \left( P_{t+1 | t}^{k(d+1)} - P_{t+1 | t}^{j(d)} \right) \bar{P}_t', \quad (15) $$

---

20 Modifying Diebold, Lee, and Weinbachs' (1993) formulation to the case of DDTP, we would recursively compute at each $t$, $t = 1, \ldots, T$, the joint conditional probability of $(S_0, S_{-1}, S_t, S_{t-1}, D_{-1})$, $\omega = t + 2, t + 3, \ldots, T$, given by

$$ P(S_0, S_{-1}, S_t, S_{t-1}, D_{-1} = d | Y_0) = \frac{\int \left( f(y_0 | S_0, S_{-1}, D_{-1} = d) \times \Pr(S_0 | S_{-1}, D_{-1} = d) \times \right.}{\int f^*(y_0 | Y_{-1})} \left. \Pr(S_0 | S_t, S_{t-1}, D_{-1} = d | Y_{-1}) \right) \Pr(S_0, S_{-1}, S_t, S_{t-1}, D_{-1} = d | Y_0) $$

for $1 \leq d \leq \tau$. 

43
for which $\bar{P}_t^{k(d)} = P_t^{k(d)} T \left[ P_{t+1}^{k(d)} \right]^{-1}$\footnote{If $P_t^{k(d)}$ is singular for some $t$, it may be replaced in $\bar{P}_t^{k(d)}$ by its generalized inverse (Ansley and Kohn, 1982).}. Consistent with Harvey (1989), the algorithm described by (14) and (15) is initialized using the first two moments of the posterior distribution of $\alpha$, at $T$, or $\alpha^{k(d+1)}_T$ and $p^{k(d+1)}_T$, where these latter two quantities are the final product of the DDTP filtering algorithm discussed in Section (4.1).

As indicated by equations (14) and (15), the smoothing algorithm produces as its final output smoothed estimates of the first two moments of $\alpha$, conditional not simply upon the economy being in states $k$ at time $t + 1$ and $j$ at time $t$ but also upon the economy having been in state $k$ at $t + 1$ for $d + 1$ consecutive periods and state $j$ at $t$ for $d$ consecutive periods. Moving recursively back through the sample, however, at the proceeding iteration these same equations may only take as inputs estimates of the first two moments of $\alpha$, that are conditional upon $S_t = j$ and $D_t = d$. As in the case of the DDTP filtering algorithm then, before continuing to the next iteration it is necessary both to reduce the optimal number of estimates of the unobserved state vector from all possible combinations of $k$ and $j$ to all possible values of $j$ and to update the value of the duration variable from $d + 1$ to $d$. The collapsing process by which these two tasks are accomplished is given as follows:
where the weight, \( P(S_i = j, D_i = d|Y_r) \), is the inferred probability (conditional upon the full-sample, \( Y_r \)) that the economy is in state \( j \) at \( t \) after having attained this state for \( d \) consecutive periods.

The smoothed estimates of the growth rate of the common factor, \( \Delta f_{it} \), computed using equations (14) through (16.2) serve in Chapter 6 not only as a proxy for the cyclical path followed by the pre-W.W.I reference cycle (and, as such, a measure of the comovements among the pre-W.W.I coincident indicators) but also as the basis for our pre-W.W.I business cycle chronology.
APPLICATION TO INTER- AND POSTWAR COINCIDENT INDICATORS

In deriving business cycle chronologies for the inter- and postwar periods (1919.04 - 1996.09) in the U.S., we utilize several roughly coincident economic indicators that may reasonably be interpreted as providing aggregate measures of economic activity as summarized by sales and trade, income, employment, and production. For the postwar period, we select as monthly indicators total manufacturing and trade sales, 1987$, (1948.01 - 1995.08), personal income less transfer payments, 1987$, (1947.01 - 1995.09), employees on nonagricultural payrolls (1945.01 - 1996.09), and the Federal Reserve Board (FRB) index of industrial production, 1987=100, (1919.01 - 1996.09). To extend the first two of these postwar indicators into 1996, each is spliced in 1995.08 to total manufacturing and trade sales, 1992$, (1995.01 - 1996.09) and personal income less transfer payments, 1992$, (1995.01 - 1996.09). For the interwar period, we select a FRB index of department store sales (1919.01 - 1963.12), personal income (1929.01 - 1958.12), employees in nonagricultural establishments (1929.01 -1958.12), and, once again, the FRB index of

46
industrial production, 1987 = 100, (1919.01 - 1995.11). The four postwar series (through 1995), with the exception of industrial production and employees on nonagricultural payrolls, are taken from the Economic Bulletin Board; data for 1995 and 1996 are obtained from the Department of Commerce via STAT-USA. The four interwar series, also with the exception of industrial production, are taken from Moore (1961). The FRB index of industrial production for both the inter- and postwar periods is obtained from the Federal Reserve Board; employees on nonagricultural payrolls is obtained from the Bureau of Labor Statistics (BLS).

All interwar series are transformed as necessary, with each series being seasonally adjusted via the exponential moving average procedure available in RATS and with each series being deflated via a price index constructed by splicing the BLS consumer price index for urban wage earners and clerical workers (1913.01 - 1995.12), all items, 1967 = 100, to a National Industrial Conference Board cost of living index (1920.01 - 1945.12). Anticipating the analysis presented in Chapter 6, the resulting price index, which extends from 1913.01 - 1995.12, is spliced in 1913.01 to a monthly

22 The historical industrial production data used in the interwar estimations was obtained from the Federal Reserve Board in December of 1995. It has since undergone a revision that is not taken into account here.

23 All four postwar business cycle indicators (through 1995) may be downloaded from gopher://una.hh.lib.umich.edu/11/ebb. Postwar indicators extending into 1996 may be downloaded from STAT-USA (http://www.stat-usa.gov/) and the Conference Board (http://www.tcb-indicators.org/bcihome.htm) for a fee. Various postwar indicators of employment (through 1996) may also be downloaded without charge from http://stats.bls.gov:80/datahome.htm. Revised industrial production data may be obtained from the Federal Reserve Board (http://www.bog.frb.fed.us/).
index of the general price level (1860.01 - 1939.11), yielding a consumer price index covering the pre-W.W.I, interwar, and post-W.W.II periods. Finally, in accordance with statistical evidence from the Dickey-Fuller (1979) and Engle-Granger (1987) residual based tests that the series under consideration are integrated but not cointegrated at the 10 percent level or better, all series analyzed are transformed into their growth rate forms by taking 100 times the log first difference.

The log growth rates of three series covering the inter- and postwar eras — industrial production (1919.02 - 1996.09), employees in nonagricultural establishments (1929.02 - 1996.09), and personal income (1929.02 - 1996.09) -- are shown in Figures 1 - 3, respectively. As can be seen from these plots, there is a considerable amount of dispersion in the growth rates associated with the first third and the latter two-thirds of each sample. To develop a business cycle chronology from the entire time series is not possible as a result of this. In particular, if we estimate a FTP model over the full interval, 1919.04 - 1996.09, using these three series plus manufacturing and trade sales, 1948.01 - 1996.06, and department store sales, 1919.04 - 1963.12, the resulting

24 The components of this prewar index are given as follows: (1) non-agricultural, wholesale industrial prices; (2) farm prices at the farm; (3) retail food prices in 51 cities; (4) rents in 32 cities; (5) retail prices (clothing, fuel, furnishings, etc.); (6) freight and transportation costs; (7) urban and farm reality value; (8) securities, bonds, and stocks; (9) equipment and machinery; (10) hardware prices; (11) automobile prices; (12) wages (Federal Reserve Board, New York Composite).

25 Personal income data (1929.01 - 1958.12) taken from Moore (1961) is spliced in 1947.01 to personal income data (1947.01 - 1995.09) taken from the Economic Bulletin Board (EBB) to create a single, continuous income series. A single, continuous employment series is created by splicing employees in nonagricultural establishments (1929.01 -1958.12) to employees on nonagricultural payrolls (1945.01 - 1995.09) in 1945.01.
smoothed probabilities that \( S_t = 0, \quad \Pr(S_t = 0 | I_T) \), are given by Figure 4. As is evident from the shaded areas corresponding to NBER-dated recessions, these inferred probabilities may be used to develop a catalog of interwar business cycle turning points that roughly approximates that of the NBER; however, any correlation between the postwar business cycle chronology developed from these probabilities and the NBER chronology is weak, an observation which is buttressed by a quadratic probability score of 0.309 for the postwar period (1947.04 - 1996.09).\(^{26}\) To avoid the effects of variance nonstationarity in the data, we estimate both the FTP and the DDTP models over historical sub-samples for which the standard deviations and, hence, the dispersion of the growth rates, are similar. As a result, we generate three separate sets of estimations. The first covers the postwar years extending from 1947.04 to 1996.09; the second, to facilitate the dating of business cycle episodes in the 1940s, covers not only the interwar years, 1919.04 - 1939.03, but also the W.W.II and early postwar years, extending cumulatively from 1939.04 to 1948.03; the third, to provide a characterization of duration dependence in the interwar period, covers only the interwar years, 1919.04 - 1939.03. The results of two sets from estimations (1947.04 - 1996.09, 1919.03 - 1939.03), as well as the resulting chronologies, are discussed in the next two sections.

\(^{26}\) The quadratic probability score \((QPS)\) is a measure of the correlation between the inferred full-sample smoothed probabilities and the NBER business cycle chronology. A perfect fit is indicated by a \(QPS = 0\) (Diebold and Rudebusch, 1989).
Section (5.1): Estimation of the FTP and DDTP Models

Two sets of results are reported here. Tables 1 and 2 give parameter values for the FTP and DDTP models estimated using the postwar monthly coincident indicators. Tables 3 and 4 do the same for FTP and DDTP models estimated using the interwar monthly coincident indicators. For all models considered here (both FTP and DDTP), we impose an AR(2) process on equation (2) and an AR(1) process on equation (3) of the basic model given in Chapter 3.\(^{27}\) Also, for both models considered here, we estimate the first two moments of the state vector, \(\alpha_t\), as conditional upon only \(S_t\) and \(S_{t-1}\). To obtain more efficient estimates, we might include an additional lag of \(S_t\) such that \(a^h_{it-1} = E(\alpha_t|y_{t-1}, S_t = j, S_{t-1} = i, S_{t-2} = h)\). This is unnecessary, however. One advantage to transforming a model into its generalized state space form is that, regardless of the order of the AR(\(r\)) process imposed on equation (2), one need only compute the moments of \(\alpha_t\) as conditional upon \(S_t\) and its first lag, implying that each iteration need only produce \(M^2\), not \(M^{r+1}\), optimal solutions (Kim, 1994). Because the higher-order Markov processes we adopt in the DDTP model also have the effect of increasing information to the nonlinear filter, we take the simplifying step of

\(^{27}\) We impose a combination of AR(1) and AR(2) processes on equations (2) and (3) of the basic model. For the postwar period, the Schwarz criterion is minimized by imposing an AR(2) process on equation (2) and an AR(1) process on equation (3); for the interwar period, the same is minimized by imposing an AR(1) process on both equations (2) and (3).
computing $a_{t-1}^{(a)} = E(\alpha_t | y_{t-1}, S_t = j, S_{t-1} = i)$ in the FTP model and $a_{t-1}^{(d)} = E(\alpha_t | y_{t-1}, S_t = j, S_{t-1} = i, D_{t-1} = d)$ in the DDTP model.

Table 1 reports results for a FTP model estimated using the four postwar monthly coincident economic indicators. This table does present evidence in support of the hypothesis that two distinct growth phases characterize the reference cycle that is extracted from the monthly coincident indicators, with $\mu_0/(1 - \phi(L)) = -1.37162$ and $(\mu_0 + \mu_i)/(1 - \phi(L)) = 1.17611$. It also presents evidence consistent with the proposition that the two phases are characterized by certain asymmetries, with $q = \Pr[S_t = 0 | S_{t-1} = 0] = 0.84524$ and with $p = \Pr[S_t = 1 | S_{t-1} = 1] = 0.96808$. These latter two estimated parameters imply shorter expected constant durations for contractions $[(1 - q)^{-1} = 6.46159 \text{ months}]$ and longer expected constant durations for expansions $[(1 - p)^{-1} = 31.32887 \text{ months}]$.

Table 2 presents postwar estimates from a dynamic factor model for which the evolution of the states is characterized by that duration-dependent process whose conditional probability matrix is given by (8). Like Table 1, Table 2 provides evidence in support of the existence of two distinct states of the world, with the estimated contractionary mean, $\mu_0/(1 - \phi(L))$, equaling $-0.96107$ and the estimated expansionary mean, $(\mu_0 + \mu_i)/(1 - \phi(L))$, equaling $1.10884$. It also provides additional evidence of the existence of asymmetries between expansions and contractions. In
particular, for \( r = 36 \), Figure 5 indicates that the conditional probability of the economy remaining in a contraction, or \( \Pr(S_t = 0 | S_{t-1} = 0, D_{t-1} = d) \), is strongly dependent upon the number of months that the system has been in the time \( t - 1 \) state. Using as a metric an inferred probability of 0.5, the model predicts that the economy moves out of a recession after a duration of only 12 months. Postwar expansions do not exhibit such strong positive duration dependence, with \( \Pr(S_t = 1 | S_{t-1} = 1, D_{t-1} = d) > 0.95 \) even after 36 months.

Figures 6 and 8 present for each point in the sample the one-step ahead prediction probability, \( \Pr(S_t = 0 | Y_{t-1}) \), of being in a contraction, as computed from the parameters reported in Tables 1 and 2, respectively. A comparison of the two indicates that while both sets of inferred probabilities predict roughly the same cyclical turning points those derived from the duration-dependent model (Figure 8) are more decisive (or are closer to zero and one) at the turning points identified via the computed full-sample smoothed probabilities of being in a recession (as indicated by the shaded areas in the figures). This is not an observation that carries over to Figures 7 and 9, which provide time series plots of the inferred full-sample smoothed probabilities of being in a recession \( \Pr(S_t = 0 | Y_t) \) computed from the FTP and DDTP models, respectively. In these cases, both the FTP and DDTP models identify the same business cycle turning points with roughly the same degree of decisiveness. One might interpret this as prima facia evidence against duration dependence in the postwar period. However, the likelihood ratio (LR) test statistic calculated from the FTP and DDTP models is
7.41444,\(^{28}\) enabling us to reject the FTP model in favor of the DDTP model with a \(p\)-value of less than 0.025.\(^{29}\)

Tables 3 and 4, respectively, report our results for the FTP and DDTP models estimated using the four coincident indicators covering the interwar period. Here, we are once again able to conclude that two distinct growth phases characterize the unobserved reference cycle. In particular, for Table 3, the estimated value of the contractionary mean, \(\mu_c/(1-\phi(L))\), is \(-2.02969\) while the estimated value of the expansionary mean, \((\mu_e+\mu_i)/(1-\phi(L))\), is 1.08712; for Table 4,

\(^{28}\) Here, we test the null of a two-state FTP model against the alternative of a two-state DDTP model using the usual LR test. Testing the null of an \(N-1\) state model against the alternative of an \(N\) state model requires non-standard testing as a result of certain irregularities under the null. [Typically, the transition probability parameters are unidentified, while the scores with respect to the transition probability parameters and any other parameters associated with the \(Nth\) state are identically zero.] Several papers offer tests that attempt to overcome these problems. Treating the transition parameters, \(p\) and \(q\), as nuisance parameters, Garcia (1995) analytically derives the asymptotic null distribution for a LR test which may be used to compare the null of a one state linear model with the alternative of a two state Markov switching model. Davies (1987), Davidson and MacKinnon (1981), and Gallant (1977) describe tests that may be applied in the case of Markov switching models with more than two states, with the Davies test obtaining an upper bound for the significance level of the LR test statistic under a null consisting of the model with the lower number of states. Diebold and Rudebusch (1996) and Chauvet (1996) make use of Garcia’s critical values in testing the number of states characterizing a regime switching dynamic factor model of postwar U.S. business cycles. Garcia and Perron (1996) apply Davies (1987), Davidson and MacKinnon (1981), and Gallant (1977) in determining the number of regimes characterizing a simple Markov regime switching model of the ex-post real interest rate.

\(^{29}\) Similar results are obtained if we estimate the FTP and DDTP models using quarterly averages (1947.2 - 1996.3) of manufacturing and trade sales, personal income less transfer payments, employment in nonagricultural establishments, and real GDP (1987$. In particular, for \(r = 12\), we are able to reject the FTP model in favor of the DDTP model, with a LR test statistic of 14.05548 enabling us to do so at a \(p\)-value of less than 0.005. Tables 16 and 17 and Figure 25 in Appendix D summarize our results from these estimations.
\[
\frac{\mu_0}{1 - \phi(L)} = -1.78773 \quad \text{while} \quad \frac{\mu_0 + \mu_1}{1 - \phi(L)} = 1.01060. \]

We are also once again able to provide evidence of certain asymmetries between the expansionary and contractionary phases of the business cycle. In particular, in Table 3, \( q = 0.91824 \) implies an expected constant duration of 12.231 months for contractions while \( p = 0.97024 \) implies an expected constant duration of 33.602 months for expansions.

Figure 10 reveals asymmetries in the duration dependence of the interwar reference cycle that differ from those demonstrated by Figure 5 for the postwar reference cycle. Using Table 4, duration-dependent transition probabilities computed from the estimated values of \( a_0 \) and \( b_0 \) exhibit little duration dependence, with the conditional probability of remaining in a recession \( \Pr(S_t = 0|S_{t-1} = 0, D_{t-1} = d) \) exceeding 0.92 even at a 48 month horizon. However, those duration-dependent transition probabilities computed from the estimated values of \( a_1 \) and \( b_1 \) exhibit some positive duration dependence, with the conditional probability of remaining in an expansion \( \Pr(S_t = 1|S_{t-1} = 1, D_{t-1} = d) \) falling to 0.75 after 48 months.

The graphical evidence provided by Figure 5 is in agreement with the conclusions reached by others who have studied the duration dependence characterizing the postwar period. For instance, Diebold, Rudebusch, and Sichel

---

30 Similar asymmetries are found in estimations using the 1919.04 - 1948.03 sample, with the estimated contractionary mean being slightly larger (-2.07031) and with the estimated expansionary mean being slightly smaller (0.97032) for the DDTP model. Tables 18 and 19 in Appendix D summarize our results from these estimations.
(1993), using an exponential quadratic hazard function, find strong positive duration dependence in post-W.W.II contractions and no duration dependence — either positive or negative — in post-W.W.II expansions. The graphical evidence provided by Figure 10 is not, however, in agreement with the findings of those who have studied the duration dependence characterizing the pre-W.W.II period. Diebold and Rudebusch (1990), using various nonparametric tests, find their strongest evidence in support of duration dependence in pre-W.W.II expansions. Although Figure 10 reveals some positive duration dependence in expansions, we cannot interpret this finding as prima facia evidence in support of interwar duration dependence. In particular, with a LR test statistic of 2.94308, we are unable to reject the FTP model in favor of the DDTP model, with the p-value ranging between 0.20 and 0.30.

Section (5.2): Identification of Inter- and Postwar Business Cycle Turning Points

Tables 5 and 6 present the inter- and postwar business cycle chronologies identified using the inferred full-sample smoothed probabilities, \( \Pr(S_j = 0|Y_r) \), plotted in Figures 9 and 12 and in Figures 7 and 11. Two criteria are utilized in selecting these cyclical peaks and troughs. First, a business cycle peak is identified as the last period for which \( \Pr(S_j = 0|Y_r) < 0.5 \); a business cycle trough is in turn identified as the

31 Chronologies derived using the interwar, 1919.04 - 1939.03, and the interwar (plus), 1919.04 - 1948.03, samples are identical with the exception of a trough occurring in 1921 and a peak occurring in 1937. Smoothed probabilities computed from the interwar DDTP model place these turning points in 1921.09 and 1937.06. Those computed from the interwar (plus) DDTP model place them in 1921.02 and 1937.07. Our chronology incorporates the former of these cyclical turning points.
last period for which \( Pr(S_t = 0 | Y_t) \geq 0.5 \). Second, a "maturity criterion," initially described by Burns and Mitchell (1947, 57-58) as follows, is adopted:

We do not recognize a rise or fall as a specific cycle unless its duration is at least fifteen months, whether measured from peak to peak or trough to trough. Fluctuations lasting less than two years are scrutinized with special care; they are not treated as specific cycles unless they are clearly defined and in no sensible part the result of faulty adjustment for seasonal variations.

Forty-odd years after the writing of *Measuring Business Cycles* (Burns and Mitchell, 1947), Moore and Zarnowitz (1986a, 739) modify this rule, indicating that full cycles of less than a year in duration and contractions of less than six months are unlikely to be subject to classification as a business cycle episode. We adopt this revised criteria in the selection our cyclical turning points, omitting those contractions whose peak and trough dates as identified by the full-sample smoothed probabilities from the FTP model are listed as follows: 1952.04 and 1952.07, 1956.05 and 1956.07, and 1959.06 and 1959.08.32

The second and third columns of Table 5 compare our cyclical turning points to those reported by the NBER for both the inter- and postwar periods and by Romer (1994) for the interwar period alone. While the dating of our peaks and troughs roughly corresponds to that of the NBER for the postwar period, there are several differences between our interwar chronology and that of the NBER and Romer (1994).

---

32 The one exception we permit to this rule is that contraction whose peak and trough dates in the FTP model correspond to 1980.02 and 1980.07, respectively.
Most notably, our chronology omits two contractionary episodes included by the NBER and three contractionary episodes included by Romer (1994). With respect to the official NBER chronology, the relevant peak and trough dates are 1923.05 and 1924.07 as well as 1926.10 and 1927.11; with respect to Romer's (1994) chronology, they are 1923.05 and 1924.07, 1927.03 and 1927.12, and 1939.12 and 1940.03.

Beginning with the 1939-1940 episode identified by Romer (1994) alone, Figure 12 indicates that the inferred full-sample smoothed probabilities leave us no leeway in identifying either 1939.12 as a business cycle peak or 1940.03 as a business cycle trough, with \( \Pr(S_t = 0 | Y_T) \) equaling zero over this entire interval.\(^\text{33}\) Similar conclusions are reached with respect to those contractions occurring between 1923 and 1924 and between 1926 and 1927. With respect to the former, the smoothed probabilities plotted in Figure 12, \( \Pr(S_t = 0 | Y_T) \), approach 0.10 over only a three month interval extending from 1924.03 to 1924.05; with respect to the latter, they approach 0.10 in only a single month corresponding to 1927.06.\(^\text{34}\) We should point out that alternative business cycle chronologies, including those of Axe and Houghton

\(^{33}\) There is some evidence that the historical industrial production data utilized by Romer (1994) is cyclically sensitive. Adjusting the postwar FRB industrial production data to reflect the same product mix as the prewar Miron-Romer index and applying the Bry-Boschan (1971) dating algorithm to this revised postwar data, Balke and Wynne (1994) generate a postwar business cycle chronology that includes several contractionary episodes not identified by the NBER.

\(^{34}\) Similar conclusions are reached using the interwar DDTP model. From 1924.03 to 1924.05, \( \Pr(S_t = 0 | Y_T) \) never exceeds 0.04; from 1926.10 to 1927.11, \( \Pr(S_t = 0 | Y_T) \) equals 0.25 only over a four month interval extending from 1926.01 and 1926.04.
(1931), Hubbard (1936), and Ayres (1939), have also omitted the latter of these episodes, identifying it as merely a period of decelerated growth. In addition, Thorp's *Business Annals* (1926, 144-145) describes the former as only a period of "mild depression" in which 1923 saw "activity and record production recede mildly" and 1924 brought a "further decline in production to dullness" and "revived activity" in the third quarter. Such observations are buttressed by Figure 13, which plots estimates of the growth rate of the common factor, $A_{r_2}$, over the interwar, W.W.II, and early postwar years. In this case, over the two contractionary periods identified by Burns and Mitchell (1947), $A_{r_2}$ falls below 0 in only three months. The plots of $A_{r_2}$ over these two episodes contrast sharply with plots of the same over the remainder of the interwar period as well as over the postwar period (Figure 14). Here, each NBER-dated contraction, as indicated by the shaded areas in Figures 13 and 14, corresponds to a period of negative growth in $A_{r_2}$ that in absolute value approximates or exceeds the absolute value of the estimated contractionary mean.

Table 6 compares the inter- and postwar chronologies constructed via $Pr[S_t = 0|Y_r]$ for the DDTP model with those constructed via $Pr[S_t = 0|Y_r]$ for the FTP model. Given that duration dependence is not significant over the interwar period, it is not surprising that the two interwar chronologies, with the exception of that cyclical peak occurring in 1937, are identical. The postwar chronologies do, however, reveal one notable difference. As indicated by Table 6, the durations of the postwar contractions identified by the DDTP model consistently exceed the durations of those
postwar contractions identified by the FTP model. In the case of that recession occurring between 1973 and 1975, the shortfall associated with the FTP model is five months; in the case of the more recent recession occurring between 1990 and 1991, the FTP model's shortfall is three months. Measuring duration from peak to trough, the average shortfall over all nine recessionary episodes is approximately three months.
CHAPTER 6

APPLICATION TO PREWAR COINCIDENT INDICATORS

Given that we have so far not incorporated into our analysis any years that are contemporaneous with or precede W.W.I, those results reported in Section (5.1) of Chapter 5 for the interwar period can in no way be taken as definitive evidence of the absence of duration dependence in the pre-W.W.II period. Rather, a more definitive conclusion with respect to the duration dependence of prewar expansions and contractions rests with the application of the FTP and DDTP versions of our dynamic factor model to data covering the pre-W.W.I era. We face several obstacles in the completion of this exercise, the most prominent of which is a dearth of series which may be characterized as providing broad-based measures economic activity as summarized by sales and trade, income, employment, and production. In addition, that data which is available over the pre-W.W.I period is generally fragmentary and often times not particularly reliable, given its slender sampling basis (Moore and Zarnowitz, 1986a).
This is not to say, however, that long, historical series are nonexistent. Series based upon bank clearings, factory employment, and imports, as well as industrial and pig iron production, are available from the 1880s forward and are, historically, characterized by high cyclical sensitivity. It is the application of such series to our FTP and DDTP dynamic factor models which is the subject of this chapter. Our primary motivation in undertaking this exercise is not only to provide an alternative business cycle chronology by which we may evaluate Burns and Mitchells' (1947) work but also to generate various statistics through which we may arrive at conclusions regarding the duration dependence of the prewar era and the apparent duration stabilization of the postwar era.

For the pre-W.W.I period, we select as monthly indicators bank clearings outside New York City (1875.01 - 1958.12), the Miron-Romer index of industrial production (1884.02 - 1940.12), total imports (1866.07 - 1969.10), and an index of factory employment (1889.01 - 1958.12). The last of these monthly indicators is constructed by splicing the Jerome index of factory employment, 1914 = 100, (1889.01 - 1923.12) to a BLS index of factory employment (1914.06 - 1958.12) in 1914.06. We should point out that while both series provide aggregate measures of employment

---

35 Total imports here refer to "free and dutiable" imports or "imports for consumption and other."

36 In Moore (1961), the BLS index of factory employment is given in two parts -- a series extending from 1914.06 to 1920.12 for which 1923 = 100 and a series extending from 1919.01 to 1958.12 for which 1947-49 = 100. We create a single series by splicing the former of these to the latter in 1919.01.
in manufacturing the breadth of the source data from which they are computed differs significantly. In particular, the Jerome index is estimated exclusively from factory employment data covering three northeastern states -- Massachusetts, New Jersey, and New York (Jerome, 1926).[^37] The BLS index, on the other hand, is computed from estimates of full- and part-time production workers in private manufacturing industries nationwide; in 1958, the basic data for these estimates were employment schedules submitted by “approximately 44,000 cooperating manufacturing establishments” (Moore, 1961, 31).

Given the paucity of the source data used by Jerome, it is reasonable to ask how representative the Jerome index is of prewar cyclical trends in aggregate employment. To answer this, we compare annual averages from Jerome’s index with Fabricant’s annual index of wage earners employed in manufacturing (1899 - 1939). Because the Fabricant index is based on adjusted Census of Manufactures’ data for the United States in the census years (1899, 1904, 1909, 1914, 1919, 1923) and on samples collected by several states and the BLS in the intercensal years, it provides a broader based measure of pre-W.W.I employment in the U.S. (Fabricant, 1942, 331). To begin, Table 7 shows the ratio of Jerome’s index to Fabricant’s index in the census years as well as the contemporaneous correlation between Jerome’s index and Fabricant’s index over both the census and the intercensal years. These statistics,

[^37]: The sources of that data used in estimating Jerome’s index are given as follows: (1) 1889 - 1894, Massachusetts; (2) 1895 - 1903, Massachusetts and New Jersey; (3) 1904 - 1919, Massachusetts, New Jersey, and New York; (4) 1920 - 1923, Massachusetts and New York (Jerome, 1926).
combined with the graphical comparison given by Figure 15, suggest that the trends in Jerome’s index and in the country-wide totals reflected in Fabricant’s index are roughly equivalent. Table 8 gives various descriptive statistics derived from a comparison of the growth rates of Jerome’s and Fabricant’s index. While the growth rates of these series are highly correlated, the standard deviation of the growth rates associated with Fabricant’s index exceeds the standard deviation of the growth rates associated with Jerome’s index by approximately 20 percent, suggesting that Jerome’s index is perhaps less volatile than that of Fabricant.

Returning to the four monthly coincident indicators outlined above, two of these, factory employment (both Jerome’s and the BLS index) and bank clearings outside New York City, are taken from Moore (1961). The Miron-Romer index of industrial production is obtained from Miron and Romer (1990); total imports are obtained from the NBER Macrohistory Database. As in the last chapter, these prewar series are transformed as necessary, with each series being seasonally adjusted via the exponential moving average procedure available in RATS and with each series being deflated via that price index constructed by splicing the BLS consumer price index for urban wage earners and clerical workers (1913.01 - 1995.12) to the National Industrial Conference Board cost of living index (1920.01 - 1945.12) and the index of the general price level (1860.01 - 1939.11). Finally, in accordance with statistical

38 The NBER Macrohistory Database may be accessed on-line at http://www.nber.org/. The index of the general price level used to deflate our nominally-valued prewar coincident indicators may also be obtained from the same location.
evidence from the Dickey-Fuller (1979) residual based test and the Johansen rank based test (Johansen and Juselius, 1990) that the series under consideration are integrated but not cointegrated at the 10 percent level or better, all series analyzed are transformed into their growth rate forms by taking 100 times the log first difference.\footnote{Evidence from the Dickey-Fuller test indicates that the four prewar series are integrated at the 10 percent level or better; evidence from the Engle-Granger test is a bit more ambiguous. The 10 percent critical value for the Engle-Granger test, as computed using MacKinnon (1991), is -4.1775. For that cointegrating regression in which factory employment is the dependent variable and Miron-Romer industrial production, bank clearings, and total imports are the independent variables the \( t \)-statistic from a regression of the change in the residuals on past levels and lagged changes is -4.7837. For those cointegrating regressions in which Miron-Romer industrial production, bank clearings, and total imports are used alternatively as the dependent variable and factory employment is used as an independent variable, the \( t \)-statistic is less than the 10 percent critical value. On the basis of evidence from the Johansen test, however, we do not reject the null hypothesis of no cointegrating vectors. [Critical values for the Johansen test are taken from Osterwald-Lenum (1992, Table 1, 468).]}

Figures 16 - 19 plot the log growth rates of the four prewar series over an interval extending from 1885.04 to 1920.03, with the shaded areas once again corresponding to the NBER-dated recessions. Anticipating the results given later in this chapter, in only four of the ten recessionary episodes identified by Burns and Mitchell (1947) are the sample means of the smoothed estimates of the growth rate of the common factor, \( \Delta y_{rfr} \), negative. The peak and trough dates for these four episodes, as well as the corresponding sample means (\( \Delta y_{rfr} \)), are given as follows: 1893.01 to 1894.06 (\( \Delta y_{rfr} = -1.50674 \)), 1907.05 to 1908.06 (\( \Delta y_{rfr} - 1.98378 \)), 1913.01 to 1914.12 (\( \Delta y_{rfr} - 0.36712 \)), and 1918.08 to 1919.03 (\( \Delta y_{rfr} - 1.07671 \)). The pre-W.W.I reference cycle extracted using the four prewar coincident indicators contrasts sharply with that
post-W.W.II reference cycle extracted using the four postwar coincident indicators. In particular, over each of the nine NBER-dated postwar recessions the mean growth rate of $\Delta f_{ir}$ is negative. The series plotted in Figures 16 - 19 quite naturally reflect this distinction between the pre- and postwar reference cycles. For example, the constructed index of factory employment given in Figure 16 is marked by three particularly severe declines in the log growth rates (1893 - 1894, 1907 - 1908, and 1918 - 1919) and by six less severe declines; bank clearings outside New York City given in Figure 17 appear even more reflective of our estimated summary of the cyclical path followed by general business conditions, with severe declines in the log growth rates transpiring in 1893 - 1894, 1907 - 1908, 1914 - 1915, and 1918 - 1919. Table 9 compares the sample means and standard deviations of the log growth rates of the four prewar series over the four recessionary episodes for which $\Delta f_{ir} < 0$ with the means and standard deviations of the log growth rates of the same series over the six recessionary episodes for which $\Delta f_{ir} > 0$; it also lists for each of the series the $t$-statistic for testing the equality of the growth rates over the two samples. Consistent with their time series plots, both factory employment and bank clearings outside New York City exhibit significant differences in their mean growth rates, with the mean growth rate over the six less severe NBER-dated recessionary episodes actually being positive in the case of factory employment; the $t$-statistics computed from the two samples indicate that these differences are statistically significant. Perhaps more surprisingly, the differences in the mean growth rates over the two samples
corresponding to total imports and the Miron-Romer index of industrial production are also statistically significant. This is not a conclusion one would draw from a simple visual inspection of the time series plots of either of these series (Figures 18 and 19, respectively). Finally, for each of the prewar coincident indicators, Table 9 presents evidence that volatility is greater during the four NBER-dated recessionary episodes for which \( \Delta y_{it} < 0 \).

Below we estimate both the FTP and the DDTP models over a single historical sub-sample extending from 1885.04 to 1920.03. As the above analysis of the individual indicators implies and the results discussed in the next section confirm, the comovements summarized by the common factor are dominated by a subset of those four recessionary episodes for which the mean growth rates are negative and the variance or volatility is greater. The result is a picture of prewar business cycle activity that differs from that painted by Burns and Mitchell in *Measuring Business Cycles* (1947).

**Section (6.1): Estimation of the FTP and DDTP Models**

Tables 10 and 11 give parameter values for the FTP and DDTP models estimated using the four prewar monthly coincident indicators. As in the previous chapter, for both the FTP and DDTP models, we impose an AR(2) process on equation
(2) and an AR(1) process on equation (3) of the basic model. Also, as in the previous chapter, we simplify the computations significantly by estimating the first two moments of the state vector, $\alpha_r$, as conditional upon only $S_r$ and $S_{r-1}$.

Table 10 reports results from the application of the FTP model to the selected prewar monthly coincident indicators. To begin, this table presents evidence consistent with the hypothesis that two distinct growth phases characterize the extracted prewar reference cycle, with $\mu_0/(1-\phi(L)) = -11.48420$ and $(\mu_0 + \mu_1)/(1-\phi(L)) = 0.86971$. It also presents evidence consistent with the proposition that these two phases are characterized by certain asymmetries, with $q = \Pr[S_r = 0|S_{r-1} = 0] = 0.60039$ and with $p = \Pr[S_r = 1|S_{r-1} = 1] = 0.99154$. These latter two estimated parameters in turn imply much shorter expected constant durations for contractions $[(1-q)^{-1} = 2.502$ months] and much longer expected constant durations for expansions $[(1-p)^{-1} = 118.203$ months].

Table 11 presents prewar estimates from a dynamic factor model for which the evolution of the states is characterized by a duration-dependent process. Like Table 10, Table 11 provides evidence in support of the existence of two distinct growth states, with the estimated contractionary mean, $\mu_0/(1-\phi(L))$, equaling $-11.48610$ and

---

40 We once again impose a combination of AR(1) and AR(2) processes on equations (2) and (3) of the basic model. For the W.W.I and pre-W.W.I periods, the Schwarz criterion is minimized by imposing an AR(1) process on both equations (2) and (3).

67
the estimated expansionary mean, \( \left( \mu_0 + \mu_t \right)/\left(1 - \phi(L)\right) \), equaling 0.86282. It also provides additional evidence of the existence of asymmetries between the expansionary and contractionary phases. In particular, for \( \tau = 48 \), Figure 20 indicates that the conditional probability of the economy remaining in a contraction, or \( \Pr(S_t = 0|S_{t-1} = 0, D_{t-1} = d) \), declines rather dramatically as the number of months that the economy has been in the time \( t - 1 \) state increases. Using as a metric an inferred probability of 0.5, the model predicts that the economy moves out of a recession after a duration of only four months. Prewar expansions do not exhibit any such duration dependence, with \( \Pr(S_t = 1|S_{t-1} = 1, D_{t-1} = d) > 0.99 \) even after four years.

This characterization of the prewar reference cycle derived from Tables 10 and 11 contrasts sharply with that characterization of the postwar reference cycle derived from Tables 1 - 2 in the previous chapter. To begin, while the estimated expansionary means associated with the pre- and postwar periods are roughly similar (0.86282 and 1.10884, respectively, for the DDTP model), the estimated contractionary mean associated with prewar contractions exceeds the estimated contractionary mean associated with postwar contractions by an order of magnitude in absolute value (−11.48610 and −0.96107, respectively, for the DDTP model). In addition, the FTP models estimated using the pre- and postwar coincident indicators suggest that the probability of remaining in a contraction is much lower and that the unconditional expected constant duration of an expansion is much higher in the prewar period than in the postwar period. With respect to the latter of these comparisons, the FTP models
associated with the pre- and postwar indicators predict that while the expected constant duration of a postwar expansion is approximately 31 months the expected constant duration of a prewar expansion is 118 months.

The characterization of the interwar reference cycle derived from Tables 3 and 4 contrasts equally as sharply with that characterization of the W.W.I and pre-W.W.I reference cycle derived from Tables 10 and 11. In particular, Table 4 also indicates that while the estimated expansionary means associated with the pre- and interwar periods are quite similar (0.86282 and 1.01060, respectively) the estimated contractionary mean associated with pre-W.W.I contractions exceeds (in absolute value) the estimated contractionary mean associated with interwar contractions by an order of magnitude (-11.48610 and -1.78773, respectively). Table 3 in turn implies that while the prewar contractions are more severe than their interwar counterparts in terms of the estimated contractionary means they are not as prolonged, with the expected constant duration of a prewar contraction (2.5 months) exceeding the expected constant duration an interwar contraction (12.2 months) by some 80 percent. This one result is particularly striking given that our interwar sample includes Depression era data, suggesting either that the pre-W.W.I period is indeed characterized by much more severe but much shorter peak to trough declines in general business conditions or that the pre-W.W.I data underlying our estimate of the reference cycle is simply dominated by a few particularly volatile episodes, causing the filtering and smoothing algorithms to identify outliers in the growth rates in only these instances.
and to ignore other, less severe, recessionary episodes. The business cycle chronologies described below would seem to confirm that the latter of these alternatives is perhaps the correct choice.

In addition to the above outlined differences, the duration dependence in the W.W.I and pre-W.W.I reference cycle is polar to the duration dependence in the computed interwar reference cycle. In particular, seemingly consistent with Diebold and Rudebusch (1990), who apply various nonparametric tests to the official NBER chronology of business cycle turning points, Figure 10 reveals some positive duration dependence in interwar expansions. Figure 20, on the other hand, indicates that it is pre-W.W.I contractions which exhibit some positive duration dependence, with the conditional probability of the economy remaining in a contraction falling to zero within only ten months. Pre-W.W.I expansions as characterized by Figure 20 exhibit no such duration dependence, with the conditional probability of the economy remaining in an expansion equaling one over much of the plotted four year horizon.

As in the case of the computed interwar reference cycle, however, those asymmetries in the duration dependence of pre-W.W.I expansions and contractions pictured in Figure 20 are not statistically significant, with a likelihood ratio test statistic of 3.31378 implying that we cannot reject the FTP model in favor of the DDTP model ($p$-value $\geq 0.20$). Figures 21 and 22 provide graphical evidence of this. In particular, these time series plots of the one-step ahead prediction probabilities of being in a recession $[\Pr(S_t = 0|Y_{t-1})]$, as computed using the parameter values reported in
Tables 10 and 11, respectively, indicate that both the FTP and DDTP models predict the same business cycle turning points with roughly the same degree of decisiveness.\textsuperscript{41} In the case of the extracted postwar reference cycle, for which positive duration dependence in contractions is statistically significant, the one-step ahead prediction probabilities computed from the DDTP model are more decisive (or closer to zero and one) at the turning points identified by the inferred full-sample smoothed probabilities of being in a recession.

Section (6.2): Identification of Prewar Business Cycle Turning Points

Tables 12 and 13 present the prewar business cycle chronologies identified using, alternatively, the inferred full-sample smoothed probabilities, $\Pr(S_t = 0|Y_T)$, plotted in Figure 23 and the smoothed estimates of the growth rate of the common factor, $\Delta y_{it}$, plotted in Figure 24. In the case of that chronology given by Table 12, a business cycle peak is identified as the last period for which $\Pr(S_t = 0|Y_T) < 0.5$, while a business cycle trough is in turn identified as the last period for which $\Pr(S_t = 0|Y_T) > 0.5$. We do not apply a "maturity criterion" as in the last chapter for reasons to be explained shortly. In the case of that chronology given by Table 13, two alternative criteria are utilized.\textsuperscript{42} First, consistent with the definition of a contraction as

\textsuperscript{41} The shaded areas in Figures 21 and 22 correspond to the NBER-dated recessions.

\textsuperscript{42} The NBER dates both "business cycles" and "growth cycles," where a business cycle contraction is defined as a sequence of absolute declines in a series and where a growth recession is defined as a
a period of absolute decline in a series (be it factory employment, industrial production, or a summary of the cyclical path followed by the comovements among multiple aggregates), a reference cycle peak is defined as the last month for which $\Delta f_{tr} > 0$; a reference cycle trough is in turn identified as the last month for which $\Delta f_{tr} < 0$.

Second, we do apply that "maturity criterion" suggested by Moore and Zarnowitz (1986a), identifying a business cycle contraction only in those instances in which $\Delta f_{tr} < 0$ for at least six consecutive months.

Beginning with Table 12, the second and third columns compare the cyclical turning points identified via the inferred full-sample smoothed probabilities of being in a recession with those cyclical turning points reported by the Burns and Mitchell (1947) and Romer (1994) for the prewar period. As this comparison rather starkly indicates, of the ten recessionary episodes identified by Burns and Mitchell (1947) and Romer (1994), the inferred full-sample smoothed probabilities identify only two of these, where the relevant peak and trough dates are 1893.05 and 1893.09 and 1907.08 and 1907.12, respectively. For each of these, the duration of that contractionary episode identified via our mechanical dating algorithm is far shorter than the duration of the period of slow growth relative to trend. As indicated by this, "business cycles" are dated using the level or the log level of the series, while "growth cycles" are dated using the detrended series.

---

43 The official NBER chronology and Romer's (1994) chronology for the pre-W.W.I and W.W.I periods do differ slightly. In particular, Romer (1994) omits that NBER-dated recessionary episode whose peak and trough dates correspond to 1890.01 and 1891.05 and includes an additional recessionary episode whose peak and trough dates, as identified by her ex-ante rules, correspond to 1916.05 and 1917.01.
corresponding contractionary episode identified by Burns and Mitchell (1947) and Romer (1994). For instance, in the case of that recession occurring between 1893 and 1894, the peak and trough dates identified by Burns and Mitchell (1947) and Romer (1994) imply durations of seventeen months and thirteen months, respectively; in the case of that recession occurring between 1907 and 1908, they imply durations of thirteen months and eleven months, respectively. In both cases, the inferred full-sample smoothed probabilities computed from the DDTP model identify episodes with durations of only four months. Under the "maturity criterion" applied in the dating of the interwar and post-W.W.II business cycle turning points, we would omit these episodes from our chronology, effectively identifying the entire prewar period as one of an extended expansion, marked by recurrent periods of absolute decline in economic activity not subject to classification as a business cycle recession. We have not chosen to do so here, however. As Figure 23 indicates, over both four month periods, \( \Pr(S_t = 0 | Y_r) \equiv 1.0 \), which we take as a strong indicator of the occurrence of a contractionary episode within the corresponding intervals.\(^44\) The dating of these episodes is also consistent with evidence from the analysis of the individual coincident indicators, which suggests that the dominance of a few, very volatile declines in the

\(^{44}\) With respect to that NBER- and Romer-dated episode occurring between 1913 and 1914, \( \Pr(S_t = 0 | Y_r) > 0.5 \) in only a single month corresponding to 1914.08. With respect to that episode occurring between 1918 and 1919, \( \Pr(S_t = 0 | Y_r) \) never exceeds 0.01.
component series may result in smoothed probabilities identifying only periods of very sharp and very brief contractions in general business conditions, as captured by $\Delta f_{s1r}$.

The second and third columns of Table 13 compare those pre- and post-W.W.I cyclical turning points identified via estimates of the growth rate of the common factor, $\Delta f_{s1r}$, with those identified by Burns and Mitchell (1947) and Romer (1994) over the same interval. Under the alternative criteria applied here, we now identify many of the same general periods of expansion and contraction, yielding a chronology of cyclical peaks and troughs which more closely resembles those of the NBER and Romer (1994). As in Table 12, however, the durations of those postwar contractions identified by the DDTP model are almost consistently exceeded by the durations of those contractions identified by Burns and Mitchell (1947) and Romer (1994). For instance, for that episode occurring between 1893 and 1894, duration as measured by the peak and trough dates listed in column one is six months, compared with seventeen months and thirteen months for the NBER and Romer (1994), respectively. For that episode occurring between 1907 and 1908, measured duration is eight months, compared with thirteen months and eleven months for the NBER and Romer (1994). Measuring duration from peak to trough over all six recessionary episodes identified via $\Delta f_{s1r}$, the average shortfall associated with our Table 13 chronology is

45 The one exception is that post-W.W.I recession occurring between 1918 and 1919. Burns and Mitchell (1947) identify peak and trough dates in 1918.08 and 1919.03; we identify them in 1918.07 and 1919.03.
approximately seven months with respect to the NBER chronology and approximately one month with respect to Romer’s (1994) chronology.\textsuperscript{46}

Aside from these differences in duration, our chronology, as given in Table 13, omits four contractionary episodes identified by Burns and Mitchell (1947) and four contractionary episodes identified by Romer (1994). The relevant peak and trough dates are given as follows: 1887.03 and 1888.04, 1890.07 and 1891.05, 1899.06 and 1900.12, and 1902.09 and 1904.08 (NBER); 1887.02 and 1887.05, 1900.04 and 1900.12, 1903.07 and 1904.03, and 1916.05 and 1917.01 (Romer). Omission of two of these episodes is not without precedent. In particular, when analyzing the possibility of duration stabilization between the pre- and post-W.W.II eras, Diebold and Rudebusch (1992) eliminate from their analysis the 1887 to 1888 and the 1899 to 1900 episodes identified by Burns and Mitchell (1947), allowing for the possibility that these are merely growth recessions. Of the two, Diebold and Rudebusch (1992) classify the former as the most dubious, perhaps the outcome of the analysis of trend-adjusted data, and the latter as only a very mild recession at best.\textsuperscript{47} Support for such a conclusion is found in Thorp’s \textit{Business Annals} (1926, 135), which describes 1887 as a period of

\textsuperscript{46} The average duration of contractions identified using $\Delta y_{it}$ is 10.3 months; for the NBER and Romer’s (1994) chronology, average duration over the same period is 17 months and 11 months, respectively.

\textsuperscript{47} As noted by Diebold and Rudebusch (1992), the use of detrended data makes the occurrence of actual declines in economic activity fairly difficult to judge and introduces the possibility that contraction durations are biased upwards while expansion durations are biased downwards.

A third episode included in the NBER chronology but excluded from ours based upon the comovements of the four prewar series is that occurring between 1890 and 1891. In this instance, Thorp’s *Annals* (1926, 136) does identify 1890 as a period of “recession,” with “gradual curtailment [in the] last quarter,” and 1891 as a period of “depression” and “revival,” with “dullness [continuing] until August, when revival sets in;...” However, Romer (1994) does omit this particular episode from her chronology, while both Zarnowitz (1981, 503) and Fels (1959, 159), in separate studies of the reliability of the prewar dates, classify it as among the mildest of the prewar cycles.48

---

48 As computed using the DDTP model, $\Delta y_{t+\phi} < 0$ in only two consecutive months (1891.04 and 1891.05).
With respect to that recession identified as occurring between 1902 and 1904 in the
NBER chronology and between 1903 and 1904 in Romer’s (1994) chronology, 
\( \Delta f_{tr} < 0 \) in only four consecutive months spanning 1903.10 to 1904.01; within that
period the minimum value obtained by \( \Delta f_{tr} \) is \(-1.31784\). This contrasts sharply with
that contraction identified by the DDTP model as occurring between 1907.08 and
1908.04, for which \( \Delta f_{tr} = -5.05074 \) in 1907.11 and \(-5.96068\) in 1907.12. Finally,
with respect to that contraction identified by Romer (1994) alone as occurring between
1916.05 and 1917.01, \( \Delta f_{tr} \) is negative in only one month
\( (\Delta f_{tr} = -0.74138 \text{ in 1916.07}) \) and over the remainder of this interval is actually
positive.

Given the chronology detailed in Table 13, an obvious question is what the
above analysis implies with respect to the issue duration stabilization between the pre-
and post-W.W.II eras. Panel A of Table 14 is a cumulative list of those cyclical peaks
and troughs identified using the DDTP model and the duration data for expansions,
contractions, and peak-to-peak whole cycles derived from them.\(^{49}\) Panel B is a simple

\(^{49}\) The post-W.W.I dates are identified using the inferred full-sample smoothed probabilities while the
pre-W.W.I dates are identified using the smoothed estimates of the growth rate of the common factor.
Unlike the pre-W.W.I chronology, however, the inter- and postwar chronologies identified using
Pr(\( S, = 0|Y_t \)) are almost identical to those identified using \( \Delta f_{tr} \). [For the postwar period, that
chronology identified using \( \Delta f_{tr} \) most closely matches — in both the placement of cyclical peaks and
troughs and the phase durations of contractions — that chronology identified using Pr(\( S, = 0|Y_t \)) as
computed from the FTP model. The relevant peak (P) and trough (T) dates for that chronology
identified using postwar estimates of \( \Delta f_{tr} \) from the DDTP model are given as follows:
comparison of the average durations of all three over four separate samples – the postwar period (1945.02 - 1996.09), the pre-W.W.II period including the Great Depression (1893.05 - 1938.05), the pre-W.W.II period excluding the Great Depression (1893.05 - 1929.08), and the pre-W.W.II period excluding the Great Depression and W.W.I (1893.05 - 1915.01). In considering these latter two duration samples, our purpose is to exclude from the analysis those periods which may include atypical observations.

The second and third columns of Panel B compare the mean expansion, contraction, and peak-to-peak whole cycle durations computed from the NBER and Romer's (1994) chronology with the sample means of the same computed from our chronology. To begin, the pre- and postwar mean contraction durations (excluding the Great Depression) computed from our chronology would seem to suggest not only that the average length of recessions has not fallen in the postwar period but that it has actually increased slightly, from approximately 11.0 months to 12.8 months. As indicated by Panel B of the table, the NBER chronology suggests a reduction in the mean length of postwar contractions, with NBER-dated contractions falling from an average of 16.2 months in the prewar period to an average of 11.0 months in the postwar period. Romer's chronology also suggests a slight increase in the mean length of postwar contractions, with the mean postwar contraction duration rising from 10.2
months to 11.4 months on average. In addition, on the basis of duration data taken from the prewar sample excluding both the Great Depression and W.W.I, postwar mean expansion durations have actually fallen slightly, from approximately 51.3 months to 47.6 months; on the basis of that duration data taken from the prewar sample excluding only the Great Depression identical results are obtained, with postwar mean expansion durations falling from 51.1 months to 47.6 months. Both the NBER and Romer's chronology suggest dramatic increases in the mean duration of postwar expansions, with the NBER chronology suggesting a postwar increase of nearly 30 months (from 22.9 to 51.5 months for that prewar sample excluding the Great Depression) and Romer's chronology suggesting a postwar increase of approximately 20 months (from 29.3 to 51.4 months for the same prewar sample).

While these simple comparisons of pre- and postwar mean durations would seem to imply the absence of any duration stabilization in the postwar period, it would be useful to provide a more rigorous test of the hypothesis that the durations of business cycle episodes have not changed significantly over the pre- and postwar eras. We follow Diebold and Rudebusch (1992) by presenting in Table 15 the Wilcoxon rank-sum test statistics for comparing pre- and postwar contraction, expansion, and peak-to-peak whole cycle durations ($W_C$, $W_E$, and $W_{P-P}$, respectively). The rank-sum statistic itself, as applied to business cycle duration data, tests a null hypothesis of no postwar stabilization against an alternative hypothesis of postwar stabilization. The failure to reject the null implies that the population pre- and postwar duration
distribution functions are identical, or that the average rank of an observation in the prewar sample is equal to the average rank of an observation in the postwar sample.

Following the discussion in Bradley (1968) and Diebold and Rudebusch (1992), we obtain $W_c$, $W_s$, and $W_{p-p}$ as reported in Table 15 as follows: Let

\[ \{X_1, X_2, \ldots, X_n\} \text{ and } \{Y_1, Y_2, \ldots, Y_m\} \]

be two samples of pre- and postwar durations of size $n_x$ and $n_y$, respectively, where $n_x < n_y$. To construct the Wilcoxon rank-sum statistic, we first combine these two samples into the joint distribution given by

\[ \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_m\} \]

and replace the duration observations with their respective ranks within the joint sample, yielding \(\{R_1, R_2, \ldots, R_n\}\), where \(n = n_x + n_y\). In the case of a tie between observations, we replace the relevant ranks by the average of the ranks of the tied observations, so that, for \(X_i = Y_j\),

\[ R_i = R_j = \frac{\text{Rank}(X_i) + \text{Rank}(Y_j)}{2}, \quad \text{where } \text{Rank}(Y_j) = \text{Rank}(X_i) + 1. \]

Given \(\{R_1, R_2, \ldots, R_n\}\), we then compute the Wilcoxon test statistic as the sum of the ranks from the smaller duration sample, or as

\[ W = \sum_{i=1}^{n} R_i. \]

The rank-sum statistic itself, in its standardized form, is perhaps best thought of as a $t$-test for a significant change in average duration.

In Table 15, durations from the three selected prewar sample periods are each tested against a single set of postwar durations identified via the inferred full-sample
smoothed probabilities of being in a recession, \( \Pr(S_t = 0|Y_t) \), as computed from the DDTP model. For each of the pre- and postwar sample pairs, we present the duration sample sizes, the values of the Wilcoxon rank-sum tests for contractions, expansions and peak-to-peak whole cycles, and the range for the statistics' one-sided \( p \)-values.\(^{50}\)

Diebold and Rudebusch (1992), using duration data taken from the NBER chronology to compute the same test statistics, find that pre- and postwar differences in expansion and contraction durations are highly statistically significant; however, Parker and Rothman (1996), using duration data taken from Romer's revised chronology, are unable to reach such a conclusion, failing to reject the null hypothesis of no postwar duration stabilization. Consistent with Parker and Rothman (1996), we also fail to reject the null hypothesis of no postwar duration stabilization, with the \( p \)-values exceeding 10 percent for both expansions and contractions.\(^{51}\)

Also, consistent with both Parker and Rothman (1996) and Diebold and Rudebusch (1992), we fail to reject the null hypothesis of no stabilization for peak-to-peak whole cycle durations, with the \( p \)-values once again exceeding 10 percent for each of the prewar samples. In finding no evidence of pre- and postwar duration stabilization in whole-cycles but evidence of highly statistically significant differences in the pre- and postwar durations of expansions and contractions, Diebold and Rudebusch

\(^{50}\) The critical values used here are taken from Bradley (1968, Table III, 318-322).

\(^{51}\) In Parker and Rothmans (1996), the \( p \)-values for those comparisons using the 1887.02 - 1938.06 and the 1887.02 - 1929.09 prewar samples are between 5 and 10 percent.
conclude that their results reflect a reallocation of time such that expansions are longer and contractions are shorter in the postwar period. In finding no evidence of postwar duration stabilization in expansions, contractions, or peak-to-peak whole cycles, Parker and Rothman (1996, 296) argue the opposite, stating that their results "...with the revised Romer dates suggest that the evidence in favor of an unchanged distribution of peak-to-peak durations reflects the lack of any significant shift in the underlying distributions of both expansion and contraction durations." Our results would seem to be consistent with this conclusion reached by Parker and Rothman (1996).
CHAPTER 7

CONCLUDING REMARKS

Our contributions here have been threefold. First, we have applied both the FTP and DDTP dynamic factor models to inter- and postwar monthly coincident indicators, producing via the resulting smoothed probabilities a business cycle chronology covering an interval extending from 1919.04 to 1996.09. Although our mechanically selected turning points for the postwar period roughly approximate those selected by the NBER business cycle dating committee, we do find some differences between our interwar chronology and that of Burns and Mitchell (1947). In particular, aside from shifts of a few months in the dating of certain cyclical peaks and troughs, our chronology actually omits two contractionary episodes identified by Burns and Mitchell (1947), with the first of these occurring between 1923 and 1924 and the second occurring between 1926 and 1927. In both cases, estimates of the growth rate of the common factor would seem to suggest that under the selected criteria these are merely periods of decelerated growth not subject to classification as a business cycle episode.
Second, we have derived not only a filtering algorithm for solving the state space form of a dynamic factor model characterized by duration-dependent transition probabilities but also formulas for computing from this both the inferred full-sample smoothed probabilities of being in a recession and the smoothed estimates of the growth rate of the common factor component. We then apply the filtering algorithm and the former of the smoothing algorithms to our post-W.W.I monthly coincident indicators, generating results on the duration dependence characterizing the interwar and post-W.W.II periods. For the postwar period, our results are in agreement with those of Diebold, Rudebusch, and Sichel (1993), who conclude that postwar contractions exhibit marked positive duration dependence while postwar expansions exhibit no duration dependence -- positive or negative. For the interwar period, our findings of no significant duration dependence in the computed interwar reference cycle are inconsistent with Diebold and Rudebusch (1990), who find evidence of duration dependence in pre-W.W.II expansions.

Finally, we have applied our FTP and DDTP dynamic factor models and the concomitant filtering and smoothing algorithms to W.W.I and pre-W.W.I monthly coincident indicators covering an interval extending from 1885.04 to 1920.03. Three key results from this exercise warrant mention. To begin, that chronology we derive using the inferred full-sample smoothed probabilities of being in a recession omits all but two of the ten contractionary episodes identified by Burns and Mitchell (1947) as occurring over the period of estimation. That chronology we derive using the
smoothed estimates of the growth rate of the common factor identifies six of these; however, in all but one case (that contractionary episode whose peak and trough dates in our chronology correspond to 1918.07 and 1919.03, respectively), the durations of those contractionary episodes we identify are exceeded by the durations of those contractionary episodes identified by Burns and Mitchell (1947). In general, the computed reference cycle that we extract from our prewar indicators is dominated by two very sharp, very brief declines in general business conditions (those two contractionary episodes captured by the smoothed probabilities) and four relatively less pronounced declines in general business for which the mean value of $\Delta f_{\text{pr}} < 0$ but for which the minimum absolute value attained by $\Delta f_{\text{pr}}$ over the identified contractionary episode does not approach the absolute value of the estimated contractionary mean. Second, those duration-dependent transition probabilities plotted in Figure 20 indicate that it is prewar contractions -- and not prewar expansions -- that exhibit some positive duration dependence, with the estimated probability that the prewar economy remains in a contractionary state falling to zero within only ten months. Although not statistically significant, this result not only is inconsistent with Diebold and Rudebusch (1990) but also contradicts our own results for the interwar period discussed in Section (5.1) of Chapter 5. Third, in Tables 14 and 15, we address the issue of postwar duration stabilization, using our pre-, inter-, and postwar business cycle chronologies not only to compare the simple mean phase durations of expansions, contractions, and peak-to-peak whole cycles over the postwar and three selected prewar periods but also
to test formally the hypothesis of no duration stabilization in the postwar phase durations of expansions and contractions. With respect to contractions, we find -- consistent with Romer's (1994) revised chronology and inconsistent with the NBER chronology -- that the mean length of postwar contractions has increased slightly; with respect to expansions, we find -- inconsistent with both Romer's (1994) and the NBER chronology -- that the mean length of postwar expansions has perhaps decreased slightly. The Wilcoxon rank-sum tests in both cases indicate that the slight differences we see in our pre- and postwar mean durations are not statistically significant, with the p-values in each case exceeding 10 percent. We conclude from this that the postwar period has been marked by the absence of any significant shift in the durations of expansions and contractions.

Given the results outlined above, a number of avenues of research remain open. To begin, evidence from various diagnostic tests indicates that the FTP and DDTP models as we have estimated them using the pre-, inter-, and postwar monthly coincident indicators are misspecified. In particular, these tests indicate that those series representing aggregate employment -- namely, employees on nonagricultural payrolls (postwar), employees in nonagricultural establishments (interwar), and factory employment (prewar) -- are in fact lagging indicators. Stock and Watson (1989) detail the modifications to that basic dynamic factor model given by equations (1) - (3) necessary to take account of this. Second, that reference cycle extracted using our pre- and interwar monthly coincident indicators seems to indicate the presence of perhaps a
third state characterized by slow or decelerated growth. The introduction of such a
third state may not only improve our own prewar chronology by enabling us to date
certain “slow” growth episodes using smoothed probabilities as opposed to smoothed
estimates of the growth rate of the common factor but also provide a more definitive
critique of Burns and Mitchells’ (1947) work. As Diebold and Rudebusch (1992) point
out, the early (pre-1927) NBER chronology includes a number of dubious episodes that
later researchers have classified not as “business cycles” recessions but as “growth
recessions” (e.g., Fels, 1959; Zarnowitz, 1981). The introduction of a third “slow
growth” state and the computation of the corresponding probabilities of the economy
being in it might enable us to shed some light on this issue and to reclassify certain
questionable contractionary episodes on the basis of historically consistent and
mechanically applied criteria. Third, those postwar duration-dependent transition
probabilities plotted in Figure 5 are quite abrupt, with the estimated probability of
remaining in a postwar recession falling from 1 to 0 in the space of only one month.
We might smooth these probabilities by introducing into the model an additional lag of
the state variable, \( S_i \), yielding an estimate of the expected value of the unobserved
state vector whose conditionality is described by

\[
\alpha_{n,i} = E(\alpha_{i},|y_{t-1}, S_i = j, S_{i-1} = i, S_{i-1} = h).
\]

As in the case of the introduction of a

third state variable (or, \( S_i = 0,1,2 \), where \( S_i = 2 \) corresponds to a third slow growth

\[\text{Durland and McCurdy (1994) actually introduce four such lags.}\]
state), the introduction of an additional lag of the existing latent Markov variables (or \( S_t = 0,1 \)) increases the state space and the computational complexity of the model considerably. In particular, with only one lag of \( S_t \) present on the right-hand side of the conditionality operator, each iteration requires the computation of only four different combinations of 0 and 1 (i.e., \( S_t = s_t \) and \( S_{t-1} = s_{t-1} \)), namely 00, 01, 10, and 11; with two lags of \( S_t \) present, each iteration requires the computation of eight different combinations of the same (i.e., \( S_t = s_t, S_{t-1} = s_{t-1} \) and \( S_{t-2} = s_{t-2} \)), namely 000, 001, 010, 011, 100, 101, 110, and 111. An additional complication emerges because with a second lag of \( S_t \) present in the model the duration variable, \( D_{t-1} \), corresponds to the middle — and not to the final — binary (0,1) indicator in the sequence, \( S_t = s_t, S_{t-1} = s_{t-1}, S_{t-2} = s_{t-2} \). The introduction of both a third “slow growth” state and an additional lag of \( S_t \) into a duration-dependent dynamic factor model of U.S. business cycles is the subject of future research.

Finally, the framework presented here can be extended to the study of the internationalization of the business cycle. In particular, building upon Head (1997), the basic dynamic factor model given by equations (1) - (3) in Chapter 3 can be used to decompose fluctuations in employment, production, income, and sales and trade for a selected group of countries (e.g., some subset of the G7 countries or Canada and the U.S.) into components that are not only common across aggregates for a specific country (i.e., country-specific common factors) but also common across all aggregates and all countries (i.e., an international common factor). Then, building upon Filardo
and Gordon (1994), the state space of the model can be expanded so that — rather than employing a single latent Markov variable to capture the occurrence of contractionary and expansionary episodes in a single country — one employs $k + 1$ such latent Markov variables, with $k$ of these corresponding to the occurrence of booms and busts in each of $k$ different countries and with one additional latent Markov variable corresponding to the occurrence of the same in the international (or world) economy. Such a model would then produce business cycle chronologies and evidence of cyclical asymmetries and duration dependence not only for each country in the sample but also for the international economy.

The computational cost of modeling international business cycles in this way is similar in magnitude to that encountered when increasing the number of latent Markov variables carried at any one iteration of a simple Markov regime-switching model of U.S. business cycles. In this case, however, the number of feasible outcomes of $S_i = s_i$, $i = 1, \ldots, k$, in period $t$ is equal to $M^s$, where $s_i = \{s_i = j, \ldots, s_k = j\}$, $j = 0, 1$, then denotes the set of all possible states of the international economy. This in turn implies that, with the number of latent Markov variables on the right-hand side of the conditionality operator of $a_{j, i-1}$ equal to two, each iteration requires the computation of $M^{2s}$, $M = 2$, different combinations of 0 and 1, or $M^s$ different combinations of $S_i = s_i$, $S_{i+1} = s_{i+1}, \ldots, S_k = s_k$, $S_{i-1} = s_{i-1}$, $S_{i+1,1} = s_{i+1,1}, \ldots, S_{i+1, m} = s_{i+1, m}$. A study of the internationalization of the business cycle via such an expansion of the state space model is also the subject of future research.
LIST OF REFERENCES


APPENDIX A

PROOF OF IDENTIFICATION OF THE AR(2) STATE SPACE MODEL

Consider the following dynamic factor model with regime switching:

\[ y_t = \gamma \Delta f_t + \Delta v_t, \]
\[ \Delta f_t = \mu(S_t) + \phi_1 \Delta f_{t-1} + \phi_2 \Delta f_{t-2} + \eta_t, \quad \text{(A.1)} \]
\[ \Delta v_t = D_1 \Delta v_{t-1} + D_2 \Delta v_{t-2} + \varepsilon_t, \]

where \( y_t \) is an \( n \times 1 \) vector of the growth rates of \( n \) coincident indicators, \( \Delta f_t \) is the scalar common factor, and \( \Delta v_t \) is an \( n \times 1 \) vector of idiosyncratic components and where \( \eta_t \sim N(0,Q) \) and \( \varepsilon_t \sim N(0,H) \). Filtering \( y_t, \Delta v_t, \) and \( y \Delta f_t \), by \( D_1 \) and \( D_2 \) yields

\[ y_t = \left[ D_1 Ly_t + D_2 L^2 y_t \right] + \left( I - D_1 L - D_2 L^2 \right) \gamma \Delta f_t + \varepsilon_t, \]
\[ \Delta f_t = \mu(S_t) + \phi_1 \Delta f_{t-1} + \phi_2 \Delta f_{t-2} + \eta_t, \quad \text{(A.2)} \]

for which the following state space representation applies:

\[ y_t = \beta + Z \alpha_t + \xi_t, \]
\[ \alpha_t = \mu(S_t) + T \alpha_{t-1} + R \eta_t. \quad \text{(A.3)} \]

For \( n = 4 \), those system matrices given by (A.3) are specified as follows:
\[ Z = \begin{bmatrix} \gamma_1 & -\gamma_4 d_{11} & -\gamma_4 d_{12} \\ \gamma_2 & -\gamma_4 d_{21} & -\gamma_4 d_{22} \\ \gamma_3 & -\gamma_4 d_{31} & -\gamma_4 d_{32} \\ \gamma_4 & -\gamma_4 d_{41} & -\gamma_4 d_{42} \end{bmatrix}, \]

\[ T = \begin{bmatrix} \phi_1 & \phi_2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \text{and} \quad R = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \]

where \( Z \) and \( R \) are selection matrices and \( T \) is the transition matrix.

From Harvey (1989, 116), we know that if the transition equation is rewritten so that the covariance matrix of the error term is an identity matrix the state space model (A.3) is analogous to

\[ y_t = \beta + Z \alpha_t + \xi_t, \]

\[ \alpha_t = \mu(S_t) + T \alpha_{t-1} + G \eta_t, \]

(A.4)

where \( \eta_t \) is a suitably redefined disturbance term for which the mean is zero and the variance is unity and where \( G \) is a fixed \( m \times g \) matrix for which \( m \) is the dimension of the state vector. With \( \text{var}(\eta_t) \) normalized to one, this is a moot point. In particular, if we follow Harvey in setting \( G = RR' \), where \( R' \) is an \( g \times g \) matrix such that \( R'R' = Q \), the result is \( G = R \). Given this, the measurement and transition equations now comprise the following system:

\[ y_t = \beta + Z \alpha_t + \xi_t, \]

\[ \alpha_t = \mu(S_t) + T \alpha_{t-1} + G \eta_t. \]

(A.5)
To prove identification, we must first determine if the state space model given by (A.5) is minimal, or if (A.5) is (1) stable, (2) uniformly completely controllable (UCC), and (3) uniformly completely observable (UCO). The first of these conditions is satisfied by verifying that the roots of the transition matrix, $T$, are of modulus less than one, or by verifying that $T$ has all its eigenvalues strictly inside the unit circle. The second and third conditions are achieved by verifying that the respective ranks of the controllability matrix,

$$[G, TG, T^2G] = \begin{bmatrix} 1 & \phi_1 & \phi_1^2 + \phi_2 \\ 0 & 1 & \phi_1 \\ 0 & 0 & 1 \end{bmatrix}$$

and of the observability matrix,

$$\begin{bmatrix} \gamma_1 & -\gamma_1d_{11} & -\gamma_1d_{12} \\ \gamma_2 & -\gamma_2d_{21} & -\gamma_2d_{22} \\ \gamma_3 & -\gamma_3d_{31} & -\gamma_3d_{32} \\ \gamma_4 & -\gamma_4d_{41} & -\gamma_4d_{42} \\ \gamma_1(\phi_1 - d_{11}) & \gamma_1(\phi_2 - d_{12}) & 0 \\ \gamma_2(\phi_1 - d_{21}) & \gamma_2(\phi_2 - d_{22}) & 0 \\ \gamma_3(\phi_1 - d_{31}) & \gamma_3(\phi_2 - d_{32}) & 0 \\ \gamma_4(\phi_1 - d_{41}) & \gamma_4(\phi_2 - d_{42}) & 0 \\ \gamma_1(\phi_1^2 + \phi_2 - \phi_1d_{11} - d_{12}) & \gamma_1(\phi_1\phi_2 - \phi_2d_{11}) & 0 \\ \gamma_2(\phi_1^2 + \phi_2 - \phi_1d_{21} - d_{22}) & \gamma_2(\phi_1\phi_2 - \phi_2d_{21}) & 0 \\ \gamma_3(\phi_1^2 + \phi_2 - \phi_1d_{31} - d_{32}) & \gamma_3(\phi_1\phi_2 - \phi_2d_{31}) & 0 \\ \gamma_4(\phi_1^2 + \phi_2 - \phi_1d_{41} - d_{42}) & \gamma_4(\phi_1\phi_2 - \phi_2d_{41}) & 0 \end{bmatrix}$$

Conditions (1) - (3) represent the more stringent definition of a minimal state space model applied by Gevers and Wertz (1984). It would be sufficient to limit ourselves to conditions (2) and (3) in establishing that (A.5) is minimal (Burmeister, Hamilton, and Wall, 1986).
are equal to $m$, with $m = 3$. The latter of these matrices contains three linearly independent columns if we further restrict $\gamma_i \neq 0$, $i = 1, 2, 3, 4$, and $d_{j2} \neq 0$, $j = 1, 2, 3, 4$.

The motivation for our choice of a minimal state space representation lies in Gevers and Wertz (1984), who prove that two observationally equivalent structures, $S$ and $\overline{S}$, are related through an $m \times m$ nonsingular matrix $F$ for which the following equalities hold:

\begin{align*}
(i) \quad \overline{T} &= F^{-1}TF, \\
(ii) \quad \overline{G} &= F^{-1}G, \\
(iii) \quad \overline{Z} &=ZF.
\end{align*}

The two parameter pairs, $\theta$ and $\overline{\theta}$, giving rise to the observationally equivalent structures, $S$ and $\overline{S}$, are said to be indistinguishable. For (A.5) to be identified then, we must impose enough a priori structure via $Z, T, G, Q,$ and $H$ such that the only $F$ for which $S(\theta) = S(\overline{\theta})$ is an $m \times m$ identity matrix.

**Proposition:** The minimal state space model given by (A.5) is identified so long as either $\text{var}(\eta_i) = 1$ or $\gamma_1 = 1$.

**Proof:** (1) Following Burmeister, Hamilton, and Wall (1986), $F$ is taken to be the solution to the matrix equation, $F\overline{T} - TF = 0$, derived from relation $(i)$. With $T$ as given in (A.3), all solutions to this equation are represented by

\begin{equation}
-c_{x,1}(T) + Fe_k = 0, \tag{A.8}
\end{equation}

104
or by

\[
F = \begin{bmatrix}
  c_1 & c_1 p_1(T) & c_1 p_2(T) \\
  c_2 & c_2 p_1(T) & c_2 p_2(T) \\
  c_3 & c_3 p_1(T) & c_3 p_2(T)
\end{bmatrix}, \quad (A.9)
\]

where \( p_{k-1}(T) \) denotes the characteristic polynomial of the respective \((k-1)\times(k-1)\) submatrix of \( T \), beginning with the upper left hand corner; \( e_k \) denotes the \( k^{th} \) column of a \( k \times k \) identity matrix; and \( c \) is an element of the null space of \( p_3(T) \).\(^2\)

Substituting (A.9) into relation (ii), we have

\[
\begin{bmatrix}
  1 \\
  0 \\
  0
\end{bmatrix} = \begin{bmatrix}
  c_1 & c_1 p_1(T) & c_1 p_2(T) \\
  c_2 & c_2 p_1(T) & c_2 p_2(T) \\
  c_3 & c_3 p_1(T) & c_3 p_2(T)
\end{bmatrix} \times \begin{bmatrix}
  1 \\
  0 \\
  0
\end{bmatrix},
\]

implying that \( c_2 = c_3 = 0 \) and that \( c_1 = 1 \). This in turn implies that the first column of \( F \) is the first column of an identity matrix, or that

\[
c = Fe_1 = e_1. \quad (A.10)
\]

Inserting (A.10) into the remaining two columns of (A.9) yields

\[
Fe_1 p_1(T) = Fe_2, \text{ or }
\]

\[
\begin{bmatrix}
  p_1(T) \\
  0 \\
  0
\end{bmatrix} = \begin{bmatrix}
  0 \\
  1 \\
  0
\end{bmatrix},
\]

and

\[^2\text{At } n = 1, \ p_1(T) \text{ is the characteristic polynomial of the } 1 \times 1 \text{ submatrix, } \phi_1.\]
\[
F e_1 p_2(T) = F e_3, \text{ or }
\]
\[
\begin{bmatrix}
p_2(T) \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}.
\]

As such, \( F = I \) and the model is easily identified with \( \text{var}(\eta_i) \) normalized to one.

(2) Restricting \( \gamma_1 = 1 \) also forces \( F = I \) and, hence, ensures identification of our minimal state space model.\(^3\) Again substituting the solutions to \( F \) given by (A.8) and (A.9) into relation (ii), we have
\[
\begin{bmatrix}
\sigma_\eta \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
c_1 & c_1 p_1(T) & c_1 p_2(T) \\
c_2 & c_2 p_1(T) & c_2 p_2(T) \\
c_3 & c_3 p_1(T) & c_3 p_2(T)
\end{bmatrix} \times \begin{bmatrix}
\sigma_\eta \\
0 \\
0
\end{bmatrix},
\]
which implies that \( c_2 = c_3 = 0 \) and that \( c_1 \neq 0 \). Now substituting (A.8) and (A.9) into relation (iii), we have
\[
\begin{bmatrix}
\gamma_1 \\
-\gamma_1 d_{11} \\
-\gamma_1 d_{12}
\end{bmatrix} = \begin{bmatrix}
\bar{\gamma}_1 \\
-\bar{\gamma}_1 d_{11} \\
-\bar{\gamma}_1 d_{12}
\end{bmatrix} \times \begin{bmatrix}
c_1 & c_1 p_1(T) & c_1 p_2(T) \\
c_2 & c_2 p_1(T) & c_2 p_2(T) \\
c_3 & c_3 p_1(T) & c_3 p_2(T)
\end{bmatrix},
\]
which yields
\[
\begin{bmatrix}
\gamma_1 \\
* \\
*
\end{bmatrix} = \begin{bmatrix}
\bar{\gamma}_1 c_1 \\
* \\
*
\end{bmatrix},
\]

\(^3\) In this case, \( G = \begin{bmatrix}
\sigma_\eta \\
0 \\
0
\end{bmatrix} \) so that the covariance matrix of the error term is the identity matrix.
where * denotes the nonzero elements of the vector. As such, $c_1 = 1$, and the first column of $F$ represents the first column of a corresponding identity matrix, if and only if $\gamma_1 = \bar{\gamma}_1 = 1$. Inserting this result into the remaining columns of $F$ yields $F = I$.

Although not shown here, these results generalize to an AR(r) version of (A.1).
APPENDIX B

FILTERING AND ESTIMATION OF THE FTP MODEL

In this appendix, we consider the following dynamic factor model with regime switching:

\[ \Delta y_t = \gamma(L) \Delta f_t + \Delta \nu_t, \quad (B.1) \]

\[ D(L) \Delta \nu_t = \varepsilon_t, \quad (B.2) \]

\[ \phi(L) \Delta f_t = \delta_s + \eta_t. \quad (B.3) \]

Here, \( y_t \), an \((n \times 1)\) vector of the growth rates of the \( n \) coincident indicators, is comprised of two stochastic components, \( \Delta f_t \), which represents the underlying business cycle trend present in all \( n \) series, and \( \Delta \nu_t \), which encompasses those idiosyncratic fluctuations and measurement errors unique to each of the \( n \) series. In equations (B.1) - (B.3), the mean of the common factor, \( \delta_s \), is permitted to vary with the state of the world, with \( S_t = 0 \) being defined as a decelerated (or negative) growth state and \( S_t = 1 \) being defined as an accelerated (or positive) growth state. The filtering algorithm described here is for the FTP version of the state space model.
implying that it is derived under the assumption that the state transition probabilities,
\[ P(S_t = s_t | S_{t-1} = s_{t-1}) \], are constant over the entire period under consideration. A
duration dependent filtering algorithm is discussed in Appendix C and the text. The
smoothers for the both the FTP and DDTP versions of the state space model are
described in Section (4) of the text.

Section (B.1): State Space Representation

Because the model given by equations (B.1) - (B.3) allows for the existence of
alternative state of the world, we cannot simply apply a linear Kalman filtering
algorithm (Kalman 1960, Kalman and Bucy 1961) to its state space representation.
Rather, we must use a modified filtering algorithm that takes into account the inherent
nonlinearities introduced by regime switching. To that end, this section is divided into
two parts. The first briefly reviews several papers, with an eye toward placing the
algorithm we ultimately select in the context of the existing literature. The second
derives the state space representation required for the application of this filtering
algorithm.

Before continuing, it is necessary to mention one simplifying assumption. To
simplify the matrix algebra used here, we assume that \( \gamma_i(L) = \gamma_i, \, \forall \, i = 1, \ldots, n \),
ensuring that \( \Delta f_i \) enters each of the \( n \) coincident indicators only contemporaneously.
Section (B.1.1): Motivation:

Discussion of prior research is best organized on the basis of its overall contribution to the development of that filtering algorithm discussed in Section (B.2).

Harrison and Stevens (1971, 1976) introduce two separate classes of multi-process models. Although both incorporate uncertainty with respect to the correct model specification at a given point in time, they differ in their implications for the sheer number of correct models that may exist over a given period. In particular, the members of this first class assume the existence of a unique process model for all $t$, with the selected model specification representing a single choice from a set of discrete alternatives; the members of the second hypothesize that the generated model at any given point in time represents not a unique but rather a random selection from this same set. The first of these two classes is consistent with Stock and Watson (1989, 1991) specifically and with the extrinsic view of business cycles in general; the second, with its allowance for shifts in model specification, is in greater agreement with Hamilton (1988, 1989, 1993) specifically and with the intrinsic view of business cycles in general. Our focus in the remainder of this section will be on the extensions of the latter of these two model types.¹

¹ Harrison and Stevens' (1976) enduring contribution to this literature is the "collapsing" process they develop for reducing the computational burden brought on by changes in model specification. Because both the nature of the "collapsing" process as well as the magnitude of the computational burden motivating it are more easily explained in the context of a specific model, a more extensive discussion of both topics is reserved for Section (B.2).
Harrison and Stevens (1976) characterize an individual dynamic linear model as being comprised of essentially four elements - a matrix defining the manner in which the process is observed, a scalar describing some inherent characteristic governing the evolution of the system, and a scalar and a vector representing observation noise variance and disturbance noise variance, respectively; they then explicitly assume that for the discrete set of dynamic linear models available at time $t$ the process evolution described by a particular model $j$ differs from that of an alternative model $i$ by only the magnitude of these two variance components, with these magnitudes differing as a result of one of three major disturbances — outliers, level shifts, and permanent changes in slope.

One aspect of that work building upon Harrison and Stevens (1976) is the introduction of regime dependent elements into the nonrandom components of the dynamic linear model. Such a task is not easily accomplished within the framework of Harrison and Stevens (1976). Although these researchers describe the dynamic linear model in general terms, they limit their derivations and illustrations to its relatively simple local level (LL) and local linear trend (LLT) state space forms. With respect to both, changes in state are only given expression via changes in the random components. For example, in the LLT model,

\begin{align*}
y_t &= \mu_t + \varepsilon_t, \\
\mu_t &= \mu_{t-1} + \beta_t + \eta_t, \\
\beta_t &= \beta_{t-1} + \xi_t,
\end{align*}

(B.4)

111
where $\eta_t = \nu_s \mu_t$ and $\zeta_t = \varphi_s \beta_t$, the regime switching behavior of the model is restricted to the state-dependent coefficients $\nu_s$ and $\varphi_s$ and to their respective variance terms. With the general state space form, the regime switching behavior of the model is more extensive. For

$$y_t = \lambda_s + Z_s \alpha_t + \varepsilon_t,$$  \hspace{1cm} (B.5)

$$\alpha_t = \mu_s + T_s \alpha_{t-1} + R_s \eta_t,$$  \hspace{1cm} (B.6)

where $t = 1, \ldots, T$ and where $\alpha_t$ is the state vector, regime switching in the dynamic linear model is manifested not in $\varepsilon_t$ and $\eta_t$, but in the elements of the regressor matrices $Z_s$ and $T_s$ and in the nonzero intercept terms, $\mu_s$ and $\lambda_s$.

Highfield's (1990) primary contribution to the literature is the development of a modified Kalman filter for dynamic linear models having the state space form given by equations (B.5) and (B.6). The algorithm itself consists of two parts - (i) a time update generating prior distributions for the unobservable states $s_t$ (via Bayes Theorem) and model regimes $\alpha_t$ (via the Kalman filter) conditional on information available through $t-1$ and (ii) an observation update computing the posterior distributions of $s_t$ and $\alpha_t$ conditional on the observed realizations of $y_t$ through $t$.\(^2\) Highfield applies his algorithm to a simple regime shift model of the Department of Commerce (DOC) index of coincident indicators. The resulting state space form is written as follows:

\[ y_t = \theta_0 + \theta_1 s_t + Z \alpha_t, \]  
\[ \alpha_t = \mu + T \alpha_{t-1} + R \eta_t. \]  
(B.7)  
(B.8)

In (B.7), \( y_t = [AL_t, AC_t] \), where \( AL_t \) and \( AC_t \), represent, respectively the growth rates of the DOC index of leading economic indicators (DOC-LEI) and of the DOC index of coincident economic indicators (DOC-CEI), both of which are modeled as independent AR(2) processes; \( \theta_0 = [\theta_{0L}, \theta_{0C}] \) and \( \theta_1 = [\theta_{1L}, \theta_{1C} L^6] \), where the lag operator \( L^6 \) indicates that \( s_t \) enters the equation corresponding to the coincident index with a six-month lag; and \( Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \), where the absence of a subscript denotes the independence of this regressor matrix from both time and state. With the regime switching behavior of the model restricted in this manner, Highfield effectively limits the scope of regime dependency to the intercept term, \( \lambda s_t \), corresponding to the vector \( y_t \).

Highfield's purpose in constructing this regime shift model is not the computation of an index of coincident indicators but rather the development of an algorithm for identifying business cycle turning points. A logical extension of this approach would be the construction of a Bayesian dynamic linear model capable not only of generating an experimental coincident index but also of dating and predicting business cycle turning points on the basis of it. Diebold and Rudebusch (1996) consider just this. Using the dynamic factor framework attributed to Stock and Watson
(1989, 1991), they introduce a regime switching component directly into the common factor of the model given by equations (B.1) - (B.3). The result is expressed as follows:

\[ \Delta y_t = \gamma \Delta f_t + \Delta \nu_t, \quad (B.9) \]

\[ D(L) \Delta \nu_t = \varepsilon_t, \quad (B.10) \]

where the probabilistic dependence of \( \Delta f_t \) on \( p \) lags of the unobserved states, denoted \( z_t = (s_t, \ldots, s_{t-p}, \Delta f_{t-1}, \ldots, \Delta f_{t-p}) \), is written

\[ P(\Delta f_t | z_t, \theta) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left( \frac{- \left( \Delta f_t - \delta_i \right)^2 - \sum_{i=1}^{p} \phi_i \left( \Delta f_{t-i} - \delta_{t-i} \right) }{2\sigma^2} \right). \quad (B.11) \]

Using (B.11) to modify (B.3), we have

\[ \phi(L) \left( \Delta f_t - \delta_i \right) = \eta_t, \quad (B.12) \]

where the lagged scalar \( \phi(L) \) is of finite order \( p \).

Diebold and Rudebusch (1996) do not go beyond a description of the basic structure outlined in equations (B.9) - (B.12). The explanation for this lies in the difficulties inherent in constructing and maximizing the corresponding exact log likelihood. Kim (1994) provides a partial solution by first deriving an algorithm for constructing a state space approximation of the exact log likelihood, making empirical estimation of these models feasible via approximate-maximum likelihood methods, and by secondly deriving an approximation to the smoothing algorithm proposed by
Hamilton (1989, 1993), significantly reducing the computational time necessary for full-sample smoothing.

Section (B.1.2): The State Space Model

The state space-space representation to be derived from (B.1) - (B.3) consists of two parts - a measurement equation relating the observed variable $y_t$ to the elements of an unobserved state vector $\alpha_t$ and a transition equation describing the evolution of $\alpha_t$ over time. The latter of these is constructed by combining equations (B.2) and (B.3); the former is obtained by writing (B.1) as a linear combination of the elements of $\alpha_t$. Here, we begin with the derivation of the transition equation.

The transition equation corresponding to our dynamic factor model is given as follows:

$$
\begin{align*}
\begin{bmatrix}
    f_t^* \\
    u_t^* \\
    f_{t-1}
\end{bmatrix} &= 
\begin{bmatrix}
    \delta_t \\
    0 \\
    Z_t
\end{bmatrix} +
\begin{bmatrix}
    \phi^* & 0 & 0 \\
    0 & D^* & 0 \\
    Z_\xi & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    f_{t-1}^* \\
    u_{t-1}^* \\
    f_{t-2}
\end{bmatrix} +
\begin{bmatrix}
    Z_f' \\
    0 \\
    Z_\xi
\end{bmatrix}
\begin{bmatrix}
    \eta_t \\
    \varepsilon_t
\end{bmatrix},
\end{align*}
$$

(B.13)

where $Z_f = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $Z_u = \begin{bmatrix} I_n & 0 \end{bmatrix}$, $\Delta f_{t-1} = f_{t-1} - f_{t-2}$, and $\eta_t, \varepsilon_t$ are normally distributed with zero mean and covariance matrices equal to $I_n$ and $\Sigma$, respectively.

Stock and Watson (1991) augment the transition equation by this identity to facilitate the estimation of the level of the common factor, $f_{rt}$. 

---

3 Stock and Watson (1991) augment the transition equation by this identity to facilitate the estimation of the level of the common factor, $f_{rt}$. 

115
\( T \), where \( \alpha_t = \begin{bmatrix} f_t' & u_t' & f_{t-1}' \end{bmatrix} \) and \( T = \begin{bmatrix} \phi^* & 0 & 0 \\ 0 & D^* & 0 \\ Z_c & 0 & 1 \end{bmatrix} \). The derivation of each of these requires some additional explanation.

To begin, the first two elements of \( \alpha_t \) correspond to the two unobserved components into which \( \Delta y_t \) is divided - a common factor, describing any comovements across the series, and an idiosyncratic component, describing fluctuations unique to each of the series. The former of these is written as a \((q \times 1)\) vector of lagged scalars such that

\[
\begin{bmatrix} f_t' \\ \Delta f_{t-1}' \\ \cdots \\ \Delta f_{t-q+1}' \end{bmatrix} = (B.14)
\]

whereas the latter is expressed as an \((n \times 1)\) stacked vector of lagged vectors such that

\[
\begin{bmatrix} \Delta v_t' \\ \Delta v_{t-1}' \\ \cdots \\ \Delta v_{t-k+1}' \end{bmatrix} = (B.15)
\]

With respect to equation (B.15), the set of lagged vectors comprising \( u_t' \) is represented as follows:

\[
\begin{bmatrix} \Delta v_t' \\ \Delta v_{t-1}' \\ \cdots \\ \Delta v_{t-k+1}' \end{bmatrix} = \begin{bmatrix} \Delta v_t \\ \Delta v_{t-1} \\ \cdots \\ \Delta v_{t-k+1} \end{bmatrix} = (B.16)
\]

with each individual vector being of dimension \((n \times 1)\).

The derivation of \( T \) is slightly more involved. The first two of its off-diagonal elements, \( \phi^* \) and \( D^* \), denote the transition matrices taken from the state space
representation of the AR processes followed by the unobserved components \( \Delta f_i \) and 
\( \Delta \nu_i \), respectively. Beginning with the simpler of the two, the transition matrix 
corresponding to the AR(\( q \)) process followed by \( \Delta f_i \), is

\[
\phi^* = \begin{bmatrix}
\phi_1 & \phi_2 & \ldots & \phi_{(q-1)} & \phi_q \\
1 & 0 & \ldots & 0 & 0 \\
0 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & 0 & \vdots \\
0 & 0 & \ldots & 0 & 1 \\
\end{bmatrix},
\]

(B.17)

which may be simplified to

\[
\phi^* = \begin{bmatrix}
\phi_1 & \phi_2 & \ldots & \phi_{(q-1)} & \phi_q \\
I_{q-1} & 0_{(q-1)x1} \\
\end{bmatrix},
\]

(B.18)

where \( I_{q-1} \) is a \((q-1) \times (q-1)\) identity matrix and \( 0_{(q-1)x1} \) is a \((q-1) \times 1\) matrix of
zeros. The transition equation to which this matrix corresponds is expressed as
follows:

\[
f^*_i = \begin{bmatrix}
\delta_{x_i} \\
0_{(q-1)x1}
\end{bmatrix} + \phi^* \begin{bmatrix}
f^*_i \\
0_{1x(q-1)}
\end{bmatrix} \eta_i,
\]

(B.19)

where for \( q = 2 \)

\[
f^*_i = \begin{bmatrix}
\Delta f_i \\
\Delta f_{i-1}
\end{bmatrix} = \begin{bmatrix}
\delta_{x_i} \\
0
\end{bmatrix} + \begin{bmatrix}
\phi_1 & \phi_2 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
\Delta f_{i-1} \\
\Delta f_{i-2}
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix} \eta_i.
\]

(B.20)

The logic employed in arriving at the second of these off-diagonal elements, 
\( D^* \), is similar to that used above in the derivation of \( \phi^* \); the only complicating factor is
the sometimes bunglesome matrix notation involved. Given this caveat, the transition matrix corresponding to the AR\( (k) \) process followed by \( u^*_i \) is

\[
D^* = \begin{bmatrix}
    (d_{11} \ldots 0) & (d_{1(k-1)} \ldots 0) & (d_{1k} \ldots 0) \\
    (0 \ldots d_{1n'}) & (0 \ldots d_{(k-1)n'}) & (0 \ldots d_{nk'}) \\
    (1 \ldots 0) & (0 \ldots 0) & (0 \ldots 0) \\
    \vdots & \vdots & \vdots \\
    (0 \ldots 1) & (0 \ldots 0) & (0 \ldots 0) \\
    \vdots & \vdots & \vdots \\
    (0 \ldots 0) & (1 \ldots 0) & (0 \ldots 0) \\
    (0 \ldots 0) & (0 \ldots 0) & (0 \ldots 0)
\end{bmatrix}, \quad (B.21)
\]

which may be written as

\[
D^* = \begin{bmatrix}
    D_1 & \cdots & D_{(k-1)} & D_k \\
    I_{n(k-1)} & & & 0_{n(k-1)n} 
\end{bmatrix}, \quad (B.22)
\]

where \( I_{n(k-1)} \) is an \( n(k-1) \times n(k-1) \) identity matrix and \( 0_{n(k-1)n} \) is an \( n(k-1) \times n \) matrix of zeros. Following the pattern established above, the transition equation to which this matrix corresponds is expressed as follows:

\[
u^*_i = D^*u^*_{i-1} + \begin{bmatrix} I_n & 0_{n(k-1)n} \end{bmatrix} \epsilon_i, \quad (B.23)
\]

where for \( n = 2 \) and \( k = 2 \)
Given (B.13), the measurement equation may simply be expressed as linear combination of the AR(q) process corresponding to $f_i^*$, the AR(k) process corresponding to $u_i^*$, and the identity, $f_{t-1} = \Delta f_{t-1} + f_{t-2}$. In particular, the measurement equation may be written as follows:

$$y_t = \begin{bmatrix} \gamma Z_{\Delta f} & Z_{\Delta u} & 0_{n \times 1} \end{bmatrix} \begin{bmatrix} f_i^* \\ u_i^* \\ f_{t-i} \end{bmatrix} + \xi,$$  \hspace{1cm} (B.25)

with $\xi_t$ denoting a measurement error term that is uncorrelated with $\eta_t$ and $\epsilon_t$ and which is set equal to zero in empirical work. In standard notation, equations (B.13) and (B.25) appear as follows:

$$y_t = Z\alpha_t + \xi,$$  \hspace{1cm} (B.26)

$$\alpha_t = \mu_\alpha + T \alpha_{t-1} + R \zeta_t,$$  \hspace{1cm} (B.27)
where $\mu_{s_t} = \begin{pmatrix} \delta \, s_t & 0_{(q \times n_t)} \end{pmatrix}'$, $\zeta_t = \begin{pmatrix} \eta_t & \varepsilon_t \end{pmatrix}'$, $Z$ is the selection matrix for (B.26), and $R$ is the selection matrix for (B.27). With respect to those error terms present in (B.26) and (B.27), the following assumptions are made:

$$
\begin{pmatrix} \xi_t \\ \zeta_t \end{pmatrix} \approx N\left(0, \begin{pmatrix} H & 0 \\ 0 & Q \end{pmatrix}\right),
$$

(B.28)

$$
\zeta_t = \begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \approx N(0, Q),
$$

(B.29)

$$
Q = E\left(\eta_t' \eta_t, \varepsilon_t' \varepsilon_t\right) = \text{diag} \left(\sigma_{\eta_t}^2, \ldots, \sigma_{\varepsilon_t}^2\right).
$$

(B.30)

Equations (B.28) - (B.30) collectively ensure that the state space model given by (B.26) and (B.27) is representative of a general Gaussian state space model.

Section (B.2): The Modified Kalman Filter

The modified Kalman filter presented here provides a tool for recursively calculating the predicted mean, $\alpha_t$, and variance, $p_t$, of $\alpha_t$, conditional both upon those observations available through $t-1$ and upon the states operative at $t-1$ and $t$. Because the state space formulation given by (B.26) and (B.27) is consistent with the family of linear Gaussian state space models, both $\alpha_t$ and $p_t$ are derived from the first two moments of $\alpha_t$ using the relatively simple methods developed for handling...
Gaussian observations. In particular, estimates of $\alpha$, denoted $\hat{\alpha}$ for the stacked vector $[\hat{\alpha}_1', \ldots, \hat{\alpha}_t']'$, are found by taking the $E(\alpha|Y)$, where $Y_t = \{y_1, \ldots, y_t\}$.

With respect to the Kalman filter derived in this section, three assumptions are made. First, it is initially assumed that the elements of $Z$, $T$, and $Q$ and, hence, the values of $\gamma$, $\phi^*$, $D^*$, and $\sigma_k^2$, $i = 1, \ldots, n$, are known a priori. Estimation of these via maximum likelihood estimation (MLE) methods is discussed in the second part of this section. Second, it is assumed that the initial state vector, $\alpha_0'$, is $N(\alpha_0', p'_0)$, where $\alpha_0'$ and $p'_0$ are also known a priori. In the absence of any knowledge of these starting values, some methodology is required for initializing the Kalman filter. Various initialization procedures are also considered in the second part of this section. Finally, following Harvey (1989), it is assumed throughout that for $Y_t = \{y_1, \ldots, y_t\}$,

$$p(y_t|\alpha_1, \ldots, \alpha_t, Y_{t-1}) = p(y_t|\alpha_t),$$  \hspace{1cm} (B.31)

$$p(\alpha_{t+1}|\alpha_1, \ldots, \alpha_t, Y_t) = p(\alpha_{t+1}|\alpha_t).$$  \hspace{1cm} (B.32)

This third assumption establishes the Markovian nature of the state space model.

**Section (B.2.1): The Basic Filtering Algorithm**

The algorithm described below consists of two parts. The first is a Kalman filter whose output of the posterior distributions, $\alpha_{i\mu}'$ and $p_{i\mu}'$, and the conditional likelihood, $f(y_t|S_\mu, S_{i-1}, Y_{t-1})$, is used in the estimation of the common factor and the model hyperparameters. The second is a Bayesian filter whose output of the posterior
probability, \( P(s_i = i|Y_i) \), \( i = 0,1 \), is used as a means of probabilistically inferring the state of the reference cycle as measured by the common factor. Before continuing, however, we should explain the nature of the computational burden to which we previously alluded and the manner in which the filtering algorithm reduces it.

With respect to the dynamic linear models addressed in this paper, the number of past lags upon which the posterior distributions and probabilities are dependent is determined by the number of lagged dependent variables present in the model. For the state space model presented in (B.26) and (B.27), no such variables are present. However, if our state space model is generalized by the incorporation of a variable \( z_i \), denoting a vector of weakly exogenous or lagged dependent variables, the measurement equation in standard notation appears as follows:

\[
y_t = Z \alpha_t + \beta \tau_t + \xi_t.
\]  

(B.26)

with \( z_t = 1 \).\(^4\) When \( z_t = 1 \), the posterior densities may be modeled as dependent upon only two lagged values of the state variable. If \( z_t \neq 1 \) and is instead represented by an AR(\( r \)) process, as in Hamilton (1989), this default value is replaced by the order of the relevant AR process.

The number of \( r (r > 0) \) lags of the unobserved state variable in turn determines the computational burden associated with the state space model. In particular, Smith and Makov (1980) and Highland (1990) note that at each iteration, a minimum of \( M' \)

\(^4\) The transition equation is unaffected by the presence \( z_t \).
alternative model regimes is generated, one for each possible realization of \( s_{t-r}, \ldots, s_{t-1} \).

With the number of lagged dependent state variables carried at each iteration of the model equal to \( (t-r), t = r+1, \ldots, T \), the number of conditional models, or posteriors, that must be carried at any iteration of the process equals \( M' (t-r) \). As such, the number of posteriors generated by \( t = T \) iterations of the model is equivalent to \( \sum_{t=r+1}^{T} M'(t-r) \). In the absence of any lagged dependent variables, the number of posteriors generated by the \( t = T \) iterations is reduced to \( M' (T-r) \), such that at each iteration only \( M' \) posteriors need be evaluated.

Despite this reduction in the number of cases to be evaluated at each iteration, the computational burden brought on by regime switching in a dynamic linear model is still significant. For instance, with \( r = 3, \ldots, 140 \) and \( M = 2 \), the number of different conditional state space models accumulated by the time of the last iteration \( (t = 140) \) is 548. To deal with this explosion of the optimal solution, a further approximation is introduced to decrease the number of possible system histories by \( M' \) at each iteration. The technique permitting this approximation is referred to as a “collapsing” process in the literature and is discussed in some detail later.

---

5 Highfield (1990) points out that when \( r (r > 0) \) lagged dependent variables are present in the state space model, it is preferable to carry \( M'^{r+1} \) conditional models at each iteration.
Putting aside the issue of computational burden, the derivation of the Kalman filter is divided into four distinct steps. The first begins with the calculation of the one-step ahead forecast error

\[ v_t = y_t - E(y_t | y_{t-1}, S_H, S_{H-1}) , \]

(B.33)

\[ = y_t - Za_t - \beta_h , \]

for which \( a_t = E(\alpha_t | y_{t-1}, S_H, S_{H-1}) \) and for which \( S_{Ht} = s_t = 0 \) and \( S_{Ht} = s_t = 1 \). As this notation implies, the general case of \( M \) alternative states is not adopted here; rather, it is assumed that the economy exhibits discrete shifts between only two states of the world - an accelerated (or positive) growth state and a decelerated (or negative) growth state - over the course of the business cycle.

This first step is completed with the derivation of the conditional variance of the one-step ahead forecast error given in equation (B.33). Following Harvey (1989),

\[ F_t = \text{var}(v_t) , \]

(B.34)

\[ = Zp_t Z' , \]

where \( p_t = E[(\alpha_t - a_t) (\alpha_t - a_t)' | y_{t-1}, S_H, S_{H-1}] \) and where \( t = 1, \ldots, T \). Both equations (B.33) and (B.34) are required in the second step for the derivation of the posterior regime distributions corresponding to the \( 2^2 \) conditional state space models generated at each iteration. Following the Gaussian assumptions made at the beginning of this section, each posterior distribution is treated as multivariate normal with a mean vector \( a_t' \) and a corresponding covariance matrix \( p_t \), collectively denoted the
Kalman filtering updating equations. Beginning with the former of these, regression theory permits us to write

\[ a_{it}^u = a_t^u + E\left(\alpha_i, \nu_i^u\right)\left[\text{var}(\nu_i^u)\right]^{-1}\nu_i^u, \]  

(B.35)

where the linear projection of \( \alpha_i \) on \( y_i \), given \( S_\mu \) and \( S_{\mu-1} \), is set equal to the linear projection of \( \alpha_i \) on \( Y_{i-1} \), given \( S_\mu \) and \( S_{\mu-1} \), plus the linear projection of \( \alpha_i \) on \( y_i - E(y_i|Y_{i-1}, S_{\mu}, S_{\mu-1}) \). Considering the first two terms in the second component of this sum, we have

\[ E\left(\alpha_i, \nu_i^u\right)(F_i^u)^{-1} = p_i^uZ'\left(F_i^u\right)^{-1}. \]

(B.36)

where \( B_i^u \) denotes the Kalman gain. Substituting (B.36) into (B.35) yields the following:

\[ a_{it}^u = a_t^u + B_t^u\nu_i^u. \]  

(B.37)

The derivation of the variance-covariance matrix is slightly more involved. In particular, we set

\[ \text{var}(a_{it}^u) = \text{var}(a_t^u + B_t^u\nu_i^u), \]  

(B.38)

which yields

\[ p_{it}^u = p_t^u - p_t^uZ'\left(F_i^u\right)^{-1}Zp_t^u, \]

\[ = p_t^u\left(I - Z' B_i^u\right), \]  

(B.39)

for \( t = 1, \ldots, T \).
The penultimate step in the derivation of the Kalman filter is the calculation of the elements of the prior regime distribution of \( \alpha_{t+1} \), collectively denoted the prediction equations because it assumed that \( Y_t \) is observed but that \( y_{t+1} \) is not; as in the last step, each such prior distribution is assumed multivariate normal with a mean vector \( \alpha^k_{t+1} \) and a variance-covariance matrix \( p^k_{t+1} \). Beginning with the former of these, we have

\[
\begin{align*}
\alpha^k_{t+1} &= E\left( \alpha_{t+1} | Y_t, S_{t+1}, S_k \right), \\
&= \mu_k + T \alpha^k_t,
\end{align*}
\]  

(B.40)

where \( S_{t+1}, \ k = 0,1 \), is a binary indicator for which \( S_{0t+1} = s_{t+1} = 0 \) and \( S_{1t+1} = s_{t+1} = 1 \). With regard to the variance-covariance matrix, we have

\[
\begin{align*}
p^k_{t+1} &= \text{var}\left( \alpha_{t+1} | Y_t, S_{t+1}, S_k \right), \\
&= T \Sigma_{\alpha}^k + RQ\Sigma^{-1} R',
\end{align*}
\]  

(B.41)

which we may express as

\[
p^k_{t+1} = TP^k T' + RQR',
\]  

(B.42)

for \( t = 1, \ldots, T - 1 \). To initialize the filter, we assume that \( a^0_0 = a^0_0 \) and \( p^0_0 = p^0_0 \) and that both are regime dependent.

Notice that in equations (B.40) and (B.42) the mean vector \( \alpha^k_t \) and the covariance matrix \( p^k_t \) are conditional upon only \( S_k \) and not upon \( S_{t-1} \) and \( S_{k'} \) jointly. As such, we cannot simply substitute (B.37) and (B.39) into (B.40) and (B.42), respectively, to derive those expressions of mean and variance consistent with the non-Bayesian state space model but rather must modify this former set of updating
equations to limit their regime-dependence to only the current lag of the unobserved state variable. This is accomplished by “collapsing” the posterior distribution at each recursion of the model; the subsequent “collapsing” process constitutes a fourth and final step of the Kalman filter and involves reducing the $2^2$ posteriors generated at each iteration to 2 posteriors or, in the more general case of $M$ alternative states, reducing $M^2$ posteriors to $M$ posteriors. To achieve this end, Harrison and Stevens (1976) and Smith and Makov (1980) treat the current estimate of the posterior distribution of $\alpha_i$ (given $Y_t$) as a weighted average, with the weights being expressed as the posterior probabilities of specific event histories.\footnote{Smith and Makov (1980) suggest two alternative approximation procedures for minimizing the storage requirements necessitated by repeated recursions of the Kalman filter. The first of these, selection, requires the researcher to pick a single model or a small group of models at each stage, effectively limiting estimation at each iteration to a single or, alternatively, to a small number of event histories. A number of practical problems make this approach difficult to implement. In particular, it is necessary to determine not only how many (and how often) “correct” model(s) to pick but also what criteria are to be utilized in their selection. The second of these approaches is the averaging process employed in this paper; in this case, the researcher, rather than arbitrarily picking a “winner,” approximates equations (B.37) and (B.39) with a smaller mixture of Gaussian densities.}

In particular, Harrison and Stevens (1976) suggest that

$$a_{it}^I = \frac{\sum_{i=0}^1 P(S_{it}, S_{it-1} | Y_t) a_{iy}^I}{P(S_i | Y_t)} \quad (B.43)$$

and that

$$p_{iy}^I = \frac{\sum_{i=0}^1 P(S_{it}, S_{it-1} | Y_t) \left\{ p_{iy}^I + (a_{iy}^I - a_{iy}) (a_{iy}^I - a_{iy})^t \right\}}{P(S_i | Y_t)}, \quad (B.44)$$
with the posterior probability terms being estimated via the Bayesian filter comprising
the second half of this modified filtering algorithm.

Equations (B.43) and (B.44) do not, however, complete our discussion of the
first half of the filtering algorithm. The primary purpose of the four-step Kalman filter
outlined in equations (B.33) - (B.34), (B.37) - (B.39), (B.40) - (B.42), and finally
(B.43) - (B.44) is the estimation of the vector $a_{tg}$. With equation (B.43) yielding $a_{tg}^{\prime}$
and not $a_{tg}$, the $M = 2$ mixture of normal distributions described by (B.43) and
(B.44) must be further collapsed to a single, regime-independent normal distribution.
To accomplish this, the following approximations are taken from Highfield (1990):

$$a_{tg} = \sum_{j=0}^{1} \sum_{i=0}^{1} \frac{P(S_{ji}, S_{ji-1} | Y_t) p_{tg}^{\prime}}{\sum_{j=0}^{1} P(S_{ji} | Y_t)},$$

(B.45)

$$\Rightarrow a_{tg} = \sum_{j=0}^{1} \sum_{i=0}^{1} P(S_{ji}, S_{ji-1} | Y_t) a_{tg}^{\prime},$$

and

$$p_{tg} = \sum_{j=0}^{1} p_{tg}^{\prime} = \frac{\sum_{j=0}^{1} \sum_{i=0}^{1} P(S_{ji}, S_{ji-1} | Y_t) \left\{ p_{tg}^{\prime} + (a_{tg}^{\prime} - a_{tg}^{\prime}) (a_{tg}^{\prime} - a_{tg}^{\prime})^{\prime} \right\}}{\sum_{j=0}^{1} P(S_{ji} | Y_t)},$$

(B.46)

$$\Rightarrow p_{tg} = \sum_{j=0}^{1} \sum_{i=0}^{1} P(S_{ji}, S_{ji-1} | Y_t) \left\{ p_{tg}^{\prime} + (a_{tg}^{\prime} - a_{tg}^{\prime}) (a_{tg}^{\prime} - a_{tg}^{\prime})^{\prime} \right\}.$$
While the first half of this modified algorithm uses a Kalman filter to derive $f_{eq}$, the second employs a Bayesian filter to compute the posterior event distributions required not only for the completion of the "collapsing" process (equations (B.45) and (B.46)) but also for the extraction of the inferred state of the Markov regime switching component. Because our derivation follows Hamilton (1989, 1993), only a brief description of the Bayesian filter is given here. The filter itself is the outcome of the following four steps:

**Step 1:**

The posterior probabilities used in (B.43) and (B.44) are generated with recursive methods reminiscent of Kalman filtering. In particular, both the posterior state distribution, $P(S_{it}|Y_t)$, and the joint posterior state distribution, $P(S_{it}, S_{it-1}|Y_t)$, are ultimately derived from a joint prior state distribution

$$P(S_{it}, S_{it-1}|Y_{t-1}) = P(S_{it}|S_{it-1}, Y_{t-1}) \times \sum_{h=0}^{1} P(S_{it-2}, S_{it-1}|Y_{t-1}),$$

(B.47)

$$= P(S_{it}|S_{it-1}) \times \sum_{h=0}^{1} P(S_{it-2}, S_{it-1}|Y_{t-1}),$$

7 To reduce storage requirements during computation, Highfield (1990) collapses both the posterior and the prior distribution to a single unconditional normal distribution.
which is the product of the posterior state distribution, \( \sum_{n=0}^{1} P(S_{n-2}, S_{n-1}|Y_{t-1}) \), from the previous iteration and the Markov transition probability, \( P(S_{n}|S_{n-1}) \).

**Step 2:**

Using (B.47), the joint conditional density function,

\[
f(y_{t}, S_{n}, S_{n-1}|Y_{t-1}) = f(y_{t}, S_{n}, S_{n-1}, Y_{t-1}) \times P(S_{n}, S_{n-1}|Y_{t-1}), \quad \text{(B.48)}
\]

is computed via Bayes theorem, with the first term on the right-hand-side of (B.48), the conditional likelihood function, being expressed

\[
f(y_{t}, S_{n}, S_{n-1}, Y_{t-1}) = (2\pi)^{-n/2}|F_{t}^{y}|^{-1/2} \exp \left\{ -\frac{v_{t}^{y} (F_{t}^{y})^{-1} v_{t}^{y}}{2} \right\}. \quad \text{(B.49)}
\]

The right-hand-side of (B.49) is consistent with the prediction error decomposition of the likelihood generated by the Kalman filter.

**Step 3:**

Applying Bayes theorem a second time yields

\[
P(S_{n}, S_{n-1}|Y_{t}) = \frac{f(y_{t}, S_{n}, S_{n-1}|Y_{t-1})}{f(y_{t}|Y_{t-1})}, \quad \text{(B.50)}
\]

where \( P(S_{n}, S_{n-1}|Y_{t}) \) is the joint posterior state distribution and \( f(y_{t}|Y_{t-1}) \) is the marginal likelihood. The latter of these two components is written

\[
f(y_{t}|Y_{t-1}) = \sum_{j=0}^{1} \sum_{i=0}^{1} f(y_{t}, S_{n}, S_{n-1}|Y_{t-1}). \quad \text{(B.51)}
\]
Step 4:

The posterior state distribution for period $t$ is calculated from the joint posterior state distribution for the same period by integrating $S_{u-1}$ out of equation (B.50). In particular, summing over $i$ yields

$$P(S_y | Y_t) = \sum_{i=0}^{1} P(S_y, S_{u-1} | Y_t).$$

Section (B.2.2): Estimation and Initialization

Up to this point, the derivation of the two parts of the algorithm has proceeded from two assumptions. The first has made it possible to treat the coefficients parameterizing our model as known a priori; the second has made it possible to specify the initial state distribution, $\alpha_0$, as a regime dependent multivariate $N(\alpha_0, p_0)$, with $\alpha_0$ and $p_0$ also known a priori. The former of these assumptions is easily dropped at this point. The marginal likelihood function given by (B.51) provides a natural means of estimating the hyperparameters associated with the state space model. In particular, from (B.51), the state space approximation to the log likelihood is written as

$$LL = \log(f(y_T, y_{T-1}, \ldots, y_1 | Y_t)) = \sum_{t=1}^{T} \log f(y_i | Y_{t-1}), \quad (B.52)$$

with (B.52) being subsequently maximized using a modified Newton-Raphson numerical optimization procedure.

Dropping the second of these assumptions is made slightly more complex by the variety of initialization procedures available. Because we employ a two-part algorithm,
we focus our attention upon those methods available for initializing both the Kalman filter as well as its Bayesian counterpart. With respect to the former of these, two broad generalizations of the available methods are discussed here - (1) those assuming that the unknown elements of \( \alpha_0 \) are stochastic and (2) those assuming that the unknown elements of \( \alpha_0 \) are fixed (Harvey, 1989).

Beginning with the former, one way to handle the unknown elements of \( \alpha_0 \) is to assume that no genuine prior information is available about them, i.e. to assume, in the vernacular of the literature, that the distribution of \( \alpha_0 \) is diffuse or noninformative. For \( \alpha_0 \) to be representative of an "exact" diffuse prior, \( a_{00} = 0 \) and \( p_{00} = KI \), where \( K \) is a positive scalar such that \( K \rightarrow \infty \). Because the Kalman filter cannot be run with \( K = \infty \), the diffuse prior is generally approximated by setting \( K \) equal to a large finite number. Harvey (1989) points out, however, that these "large \( K \)" approximations are practical for only the simplest of models (e.g., the LL model) not only because they sometimes lead to unacceptable "rounding errors" but also because they can become cumbersome with more general state space formulations. Harvey (1989, 122-123) does show that it is possible to avoid these "large \( K \)" approximations. In particular, he proves that for the general state space model "the use of a diffuse prior is equivalent to the construction of a proper prior from the first \( m \) set of observations provided that the

\[ \text{The distribution of } \alpha_0 \text{ is diffuse because } K \rightarrow \infty \text{ implies that the variance-covariance matrix is unbounded } \left( p_{00}^{-1} = 0 \right), \text{ or that the initial distribution of } \alpha_0 \text{ does not integrate to one.} \]
model is observable." In other words, Harvey (1989) shows that it is possible to avoid imposing $P_{00} = K I$ if the first $\tau$ observations of a series are used to construct the posterior density $P(\alpha_t | Y_t)$, where $t = \tau$ is the first value of $t$ for which $\alpha_{t+1}$ and $P_{t+1}$ are defined and, hence, for which the prior distribution, $\alpha_t \approx N(\alpha_t, P_t)$, $t = \tau + 1, \tau + 2, \ldots, T$, exists.

This latter approach to the diffuse prior has the dual advantage not only of working for all state space models but also of being applicable to cases in which some elements of the state vector $\alpha_t$ are nonstationary (and, hence, diffuse) while other are stationary (and, hence, fixed or proper). Of course, once we have reached $\alpha_{t+1}$ and $P_{t+1}$ for which a proper prior distribution exists, we may continue normally with the Kalman filter. The only visible reminder of the construction of this proper prior is embedded in the log likelihood function. In particular, the regime dependent conditional likelihood function given by (B.49) would now take the form

$$f(y_t | S_t, S_{t-1}, Y_{t-1}) = (2\pi)^{(n-r)/2} |F_{t}^{-1}|^{-1/2} \exp \left\{ - \frac{1}{2} \nu_t^2 \left( (F_t)^{-1} \nu_t \right) \right\} ,$$

(B.49')

implying that

$$LL = \sum_{t=\tau+1}^T \log f(y_t | Y_{t-1}).$$

(B.52')

---

9 de Jong (1988, 1991) derives such a general algorithm for constructing a diffuse prior.
A second initialization technique involves using the log likelihood to estimate the components of \( \alpha_0 \). With this second method, we assume that \( \alpha_0 = \alpha_{00} \) and that \( p_{00} = 0 \) and subsequently start the algorithm with \( \alpha_0 \) fixed and unknown. It is then possible to estimate this element via MLE of the conditional log likelihood given by (B.52); however, such a simple initialization procedure is unlikely to be feasible in most state space models because joint MLE with respect to \( [\alpha_0 \ \theta] \), where \( \theta \) represents the hyperparameters of our state space model, complicates numerical optimization. Such complications may be avoided by concentrating \( \alpha_0 \) out of the conditional log likelihood and by then maximizing with respect to only the elements of \( \theta \). Harvey (1989) suggests two algorithms for accomplishing this.

We should point out that of these two general approaches to initializing the Kalman filter neither has provided a regime dependent initial state vector \( \alpha_i, \ i = 0,1 \).

In order to obtain this, we can analytically approximate \( \alpha'_{00} \) and \( p'_{00} \), setting

\[
\alpha'_{00} = (I - T)^{-1} \times \delta_n
\]  

(B.53)

and

\[
vec(p'_{00}) = (I - T \otimes T)vec(Q),
\]  

(B.54)

where \( S_r = i \). The result is the following log-likelihood:
Finally, an analytical representation may also be used in deriving the initial posterior probabilities. Following Hamilton (1989, 368-369), we have

\[ P(S_{i0}, S_{k-1}) = P(S_{i0}|S_{k-1}) \times P(S_{k-1}), \quad (B.55) \]

where the unconditional joint probability, \( P(S_{i0}, S_{k-1}) \), is expressed as a product of the transition probability, \( p_{il} \), and of \( P(S_{k-1}) \), the limiting probability of the Markov process. With respect to the latter of these right-hand-side elements,

\[ P(s_{-1} = 1) = \frac{1 - p_{00}}{2 - p_{00} - p_{11}}, \quad (B.56) \]

\[ P(s_{-1} = 0) = \frac{1 - p_{11}}{2 - p_{00} - p_{11}}. \quad (B.57) \]

Equation \((B.52'')\) is then subsequently rewritten

\[ LL = \log(f(y_T, y_{T-1}, \ldots, y_1|Y_0)), \]

\[ = \sum_{i=1}^{T} \log \sum_{j=0}^{1} \left\{ (2\pi)^{-1/2} |F_i|^{{-1/2}} \exp \left( -\frac{v_i^j (F_i)^{-1} v_i^j}{2} \right) \right\} \times P(S_{il} | S_{i-1}) \times \sum_{h=0}^{1} P(S_{h-2}, S_{h-1}|Y_{t-1}) \quad (B.52'''') \]

and is maximized with respect to the model hyperparameters.
APPENDIX C

FILTERING AND ESTIMATION OF THE DDTP MODEL

In this appendix, we consider the DDTP specification of that state space model summarized by equations (4') and (5') in Section (3.1). As discussed in the text, duration dependent transition probabilities are specified using the logistic functional form,

\[ P(S_t = i | S_{t-1} = i, D_{t-1} = d) = p_{i(d)} \]

\[ = \exp(a_i + b_i d) / \left(1 + \exp(a_i + b_i d)\right) \text{ if } d \leq \tau \quad (C.1) \]

\[ = \exp(a_i + b_i \tau) / \left(1 + \exp(a_i + b_i \tau)\right) \text{ if } d > \tau, \]

whereby the effects of \( \tau \)-lags of the Markov process are summarized by the integer-valued, random variable, \( D_{t-1} = d \). Although \( 1 \leq \tau \leq N \), where \( N \) is the length of the sample, we restrict \( \tau \leq 48 \) to reduce the computational complexity of the model. The first-order, duration dependent transition probabilities represented by (C.1) are introduced into Kim's (1994) modified Kalman filter using a sparse transition matrix. As mentioned in the text, the filtering algorithm resulting from this is derived under the
assumption that $r = 2$, or under the assumption that only two latent Markov variables are carried at any one iteration. The filter may be generalized to the case of $r > 2$.

Throughout this appendix we rather artificially divide our filtering algorithm into two separate components -- a Markovian filter from which the prediction and filtering probabilities are computed and a discrete Kalman filter from which estimates of first two moments of the state vector are derived.

Section (C.1): Computation of Sparse Transition Matrix and Initialization of the Filter

Initialization of our nonlinear filter requires a two-pronged approach, with the matrix of unconditional joint probabilities required by the Markovian filter being computed independently of the first two moments of $\alpha_0$ required by the Kalman filter. We begin with the more difficult of these two tasks, namely the initialization of the Markovian filter.

Section (C.1.1): Initialization of the Markovian Filter

Given a $\tau$-order system for which $d \leq \tau \forall D_{r-1} = d$, the $(2 \times \tau)$ matrix of unconditional joint probabilities represented by

$$P(S_{r-1} = s_{r-1}, \ldots, S_0 = s_0, D_{r-1} = d), \quad (C.2)$$

is the product of a $(2 \times 2)$ matrix of transition probabilities given by (8) and of a $(2 \tau \times 1)$ vector of unconditional probabilities given by
Following Durland and McCurdy (1994), we obtain the $2\tau$ elements of $\pi$ by solving a system of equations which is represented by

$$S'\pi = \pi \quad (C.4)$$

and solved subject to

$$\pi'\iota = 1, \quad (C.5)$$

where $\iota$ is a $(2\tau \times 1)$ vector of ones.

$S$ in (C.4) denotes a $(2\tau \times 2\tau)$ matrix of transition probabilities. Durland and McCurdy (1994) label this a sparse transition matrix because its only nonzero elements are those corresponding to first-order transitions between $S_t = j$ and $S_{t-1} = i$, for which $D_t = x$ and $D_{t-1} = d$ are the relevant durations.\footnote{Estimates of $\pi$ for the cases of $\tau = 32$ and $\tau = 48$ are available upon request.} In accordance with this restriction, we compute the nonzero elements of their matrix $S$ as follows:

For $x = 1$,

$$S_{d,x} = 1 - \frac{\exp(a_1 + b_1(d - \tau))}{1 + \exp(a_1 + b_1(d - \tau))} \quad \text{for } \tau < d \leq 2\tau$$

$$= 0 \quad \text{otherwise;}$$

for $1 < x \leq \tau$, 

\begin{equation}
\pi = \begin{bmatrix}
(S_0 = 0, D_0 = 1), (S_0 = 0, D_0 = 2), ..., (S_0 = 0, D_0 = \tau), \\
(S_0 = 1, D_0 = 1), ..., (S_0 = 1, D_0 = \tau)
\end{bmatrix}'.
\end{equation}
\[
S_{d, x} = \begin{cases} \frac{\exp(a_o + b_o(d))}{1 + \exp(a_o + b_o(d))} & \text{for } d = x - 1 \\ 0 & \text{otherwise} \end{cases}
\] (C.7)

for \( x = \tau + 1 \),

\[
S_{d, x} = \begin{cases} 1 - \frac{\exp(a_o + b_o(d))}{1 + \exp(a_o + b_o(d))} & \text{for } 1 < d \leq \tau \\ 0 & \text{otherwise} \end{cases}
\] (C.8)

for \( \tau + 1 < x \leq 2\tau \),

\[
S_{d, x} = \begin{cases} \frac{\exp(a_1 + b_1(d - \tau))}{1 + \exp(a_1 + b_1(d - \tau))} & \text{for } d = x - 1 \\ 0 & \text{otherwise} \end{cases}
\] (C.9)

for \( x = \tau \) and \( d = \tau \),

\[
S_{x, x} = \frac{\exp(a_o + b_o(\tau))}{1 + \exp(a_o + b_o(\tau))}.
\] (C.10)

and, finally, for \( x = 2\tau \) and \( d = 2\tau \),

\[
S_{x, x} = \frac{\exp(a_1 + b_1(\tau))}{1 + \exp(a_1 + b_1(\tau))}.
\] (C.11)

The vector \( \pi \) given by (C.3) in combination with the matrix of transition probabilities given by (8) produce a \((2 \times \tau)\) matrix of unconditional joint probabilities, the \((1,1)\) and \((2,1)\) elements of which are computed as follows:

\[
P(S_1 = 0, D_1 = 1) = \sum_{d=1}^{\tau} P(S_0 = 1, D_0 = d) \times \Pr(S_1 = 0 | S_0 = 1, D_0 = d),
\] (C.12)
\[ P(S_1 = 1, D_1 = 1) = \sum_{d=1}^{\tau} P(S_0 = 0, D_0 = d) \times \Pr(S_1 = 1 | S_0 = 0, D_0 = d). \]

(C.13)

All remaining \((i, d+1), 1 < d \leq \tau - 2\), elements of this matrix are of the form given by

\[ P(S_1 = i, D_1 = d + 1) = P(S_0 = i, D_0 = d) \times \Pr(S_1 = i | S_0 = i, D_0 = d), \]

(C.14)

implying that they are computed without any accompanying summation over \(d\).\(^2\)

**Section (C.1.2): Initialization of the Kalman Filter**

Compared with that procedure outlined above, initialization of the first two moments of \(\alpha_0\) is relatively simple, with the posterior distribution \(\alpha^{' \prime}_0 \sim N(\alpha^{\prime}_0, p^{\prime}_0)\) being made regime but not duration dependent. In particular, we set

\[ \alpha^{\prime}_0 = (I - T)^{-1} \times \mu(S_i = i) \]

(C.15)

and

\[ \text{vec}(p^{\prime}_0) = (I - T \otimes T)\text{vec}(Q), \]

(C.16)

where \(\alpha^{\prime}_0\) and \(\text{vec}(p^{\prime}_0)\) are analytical approximations of the first two moments of \(\alpha_0\)

(Harvey, 1989).

---

\(^2\) When \(d = \tau - 1\),

\[ P(S_1 = i, D_1 = \tau) = P(S_0 = i, D_0 = \tau - 1) \times \Pr(S_1 = i | S_0 = i, D_0 = \tau - 1) \]

\[ + P(S_0 = i, D_0 = \tau) \times \Pr(S_1 = i | S_0 = i, D_0 = \tau), \]

ensuring that \(\sum_{d=1}^{\tau-1} \sum_{r=0}^{\tau-1} P(S_1 = i, D_1 = d) = 1.\)
Section (C.2): Iterative Structure of the Kalman Filter

The Kalman filter presented here is best summarized in two steps --
computation of the prediction equations, \( a_{tr-1}^{(d)} \) and \( p_{tr-1}^{(d)} \), and computation of the
updating equations, \( a_{tg}^{(x)} \) and \( p_{tg}^{(x)} \). Because our nonlinear filter is similar in structure
to that of Kim (1994), we provide only a skeletal outline.

\textbf{Step 1:}

For the first iteration, Step 1 accepts as an input
\[ a_{t-1|t-1}^{(d)} \sim N(a_{t-1|t-1}, p_{t-1|t-1}) \]  
(C.17)
for all subsequent iterations, it accepts
\[ a_{t-1|t-1}^{(d)} \sim N(a_{t-1|t-1}^{(d)}, p_{t-1|t-1}^{(d)}) \]  
(C.18)
where \( a_{t-1|t-1}^{(d)} \) and \( p_{t-1|t-1}^{(d)} \) denote output from the previous iteration. From either (C.17)
or (C.18), the following prediction equations are computed:
\[ a_{tg}^{(d)} = T a_{t-1|t-1}^{(d)} \]  
(C.19)
\[ p_{tg}^{(d)} = T p_{t-1|t-1}^{(d)} T' + RQR' \]  
(C.20)
where \( T, R, \) and \( Q \) are system matrices whose elements are given in Sections (3.1).

Equations (C.19) and (C.20) are in turn used to construct the conditional
forecast error and the conditional variance of the forecast error of \( y_t \), with the former
of these being given by

141
\[ \eta_{i_{t-1}}^{j(d)} = y_{i_{t}} - Z a_{i_{t-1}}^{j(d)}, \quad \text{(C.21)} \]

and the latter being given by
\[ F_{i_{t}}^{j(d)} = Z P_{i_{t-1}}^{j(d)} Z^T + H, \quad \text{(C.22)} \]

where \( H \) and \( Z \) are also system matrices whose elements given in Section (3.1). Equations (C.21) and (C.22) are required for the computation of not only the updating equations given in Step 2 but also the prediction error decomposition of the log-likelihood given in the Section (C.3).

\textit{Step 2:}

Taking as input equations (C.19) - (C.22), we compute the updating equations as follows:

\[ a_{i_{t}}^{j(d)} = a_{i_{t-1}}^{j(d)} + K_{i_{t}}^{j(d)} \eta_{i_{t-1}}^{j(d)}, \quad \text{(C.23)} \]

\[ P_{i_{t}}^{j(d)} = \left( I - K_{i_{t}}^{j(d)} F_{i_{t}}^{j(d)} \right) P_{i_{t-1}}^{j(d)}, \quad \text{(C.24)} \]

where \( K_{i_{t}}^{j(d)} = P_{i_{t-1}}^{j(d)} Z^T \left( F_{i_{t}}^{j(d)} \right)^{-1} \). Notice that equations (C.23) and (C.24) do not provide an output that we can carry to the next iteration. In particular, not only are \( a_{i_{t}}^{j(d)} \) and \( P_{i_{t}}^{j(d)} \) jointly dependent on \( S_{t} = j \) and \( S_{t-1} = i \) but they are also functions of \( d \), the duration of state \( i \) at time \( t - 1 \). To complete Step 2, it is necessary not only to reduce the number of optimal solutions by 2, in other words, to "collapse" the \( 2^2 \) conditional forecast errors and variances to 2 such forecast errors and variances, but also to update the value of the duration variable to \( D_{t} = x \), the duration of state \( j \).
at time $t$. We accomplish both tasks by adapting a collapsing process introduced in Harrison and Stevens (1976) to the case of duration dependent inferred probabilities. In particular, we set

$$a_{rt}^{(t)} = \frac{\sum_{i=0}^{1} P(S_t = j, S_{t-1} = i, D_t = x|Y_t) a_{rt}^{(d)}}{P(S_t = j, D_t = x|Y_t)}$$

(C.25)

and

$$P_{rt}^{(x)} = \frac{\sum_{i=0}^{1} P(S_t = j, S_{t-1} = i, D_t = x|Y_t) \left[ P_{rt}^{(d)} + (a_{rt}^{(x)} - a_{rt}^{(d)})(a_{rt}^{(x)} - a_{rt}^{(d)})^T \right]}{P(S_t = j, D_t = x|Y_t)}$$

(C.26)

with the inferred conditional probabilities, $P(S_t = j, S_{t-1} = i, D_t = x|Y_t)$ and $P(S_t = j, D_t = x|Y_t)$, being computed via the nonlinear Markovian filter.

Section (C.3): Iterative Structure of the Markovian Filter

Those filtered probabilities needed to complete the collapsing process given by (C.25) and (C.26) are the final product of a nonlinear Markovian filter introduced by Durland and McCurdy (1994). We address one aspect of their filter in Section (C.1.1) -- namely, its initialization. It remains for us to discuss how that matrix of unconditional joint probabilities whose elements are described by (C.12) - (C.14) produces the necessary matrix of filtered probabilities via Durland and McCurdys'
(1994) iterative process. In doing this, we divide the nonlinear Markovian filter into four individual steps.

**Step 1:**

Taking as an input \( P(S_{t-1} = s_{t-1}, D_{t-1} = d|Y_{t-1}) \), we calculate

\[
P(S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d|Y_{t-1}) = P(S_t = s_t|S_{t-1} = s_{t-1}, D_{t-1} = d) \times P(S_{t-1} = s_{t-1}, D_{t-1} = d|Y_{t-1}),
\]

where \( P(S_t = s_t|S_{t-1} = s_{t-1}, D_{t-1} = d) \) is given by equations (6) and (7) of Section (3.2).

**Step 2:**

Using equation (C.27) from Step 1, we calculate the joint conditional density function of \( y_t, S_t, S_{t-1}, \) and \( D_{t-1} \) conditional on \( Y_{t-1} \), or

\[
f(y_t, S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d|Y_{t-1}) = f(y_t|S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d, Y_{t-1}) \times P(S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d|Y_{t-1}).
\]

Adhering to a distributional assumption of conditional normality, we have

\[
f(y_t|S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d, Y_{t-1}) = (2\pi)^{-N/2}|F_{\mu(d)}|^{-1/2} \exp\left(-\frac{1}{2} \eta_{\mu(d)} - x_{\mu(d)} - \eta_{\nu(d)}^{-1}\right),
\]

where \( \eta_{\mu(d)} \) and \( F_{\mu(d)} \) are given by equations (C.21) and (C.22). We compute the conditional likelihood of \( y_t \) by integrating out of (C.28) the effects of \( S_t, S_{t-1}, \) and \( D_{t-1} \), setting
using (C.30), the log-likelihood function is given by

\[ LL = \sum_{t=1}^{T} \log f(y_t | Y_{t-1}) \]  

(C.31)

and is subsequently maximized with respect to \( \Omega = (\mu, D, \phi, \gamma, \sigma_{\phi}^2, \sigma_{\gamma}^2, a_0, b_0, a_1, b_1) \).

**Step 3:**

The penultimate step in the FTP Markovian filter used by Kim (1994) is replaced by two separate operations in our extension to a time-varying, duration dependent Markovian filter. In the first, we update \( Y_{t-1} = \{y_1, \ldots, y_{t-1}\} \) on the right-hand side of the conditional operator of (C.27); in the second, we update the integer value of \( D_{t-1} \) on the left-hand side of the conditional operator of (C.27).

Beginning with the first of these updates, we set

\[
P(S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d | Y_t) = \frac{f(y_t, S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d | Y_{t-1})}{f(y_t | Y_{t-1})},
\]  

(C.32)

which is nothing more than a standardization of equation (C.28) using equation (C.30).

With respect to the second, three cases are considered. For \( 1 < x \leq t \),

\[
P(S_t = s_t, S_{t-1} = s_{t-1}, D_t = x | Y_t) = P(S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = x - 1 | Y_t) \quad \text{for} \ S_t = S_{t-1} \\
= 0 \quad \text{for} \ S_t \neq S_{t-1};
\]  

(C.33)
for $x = \tau,$

$$P(S_t = s_t, S_{t-1} = s_{t-1}, D_t = x|Y_t) = P(S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = x|Y_t) \quad \text{for } S_t = S_{t-1}$$

$$= 0 \quad \text{for } S_t \neq S_{t-1};$$

and, finally, for $x = 1,$

$$P(S_t = s_t, S_{t-1} = s_{t-1}, D_t = 1|Y_t)$$

$$= \sum_{d} P(S_t = s_t, S_{t-1} = s_{t-1}, D_{t-1} = d|Y_t) \quad \text{for } S_t \neq S_{t-1}$$

$$= 0 \quad \text{for } S_t = S_{t-1}. \quad \text{(C.35)}$$

**Step 4:**

In this final step, we generate the $(2 \times \tau)$ conditional probability matrix that is carried into the next iteration. In particular, from equations (C.33) - (C.35), we calculate

$$P(S_t = s_t, D_t = x|Y_t) = \sum_{s_{t-1}=0}^{1} P(S_t = s_t, S_{t-1} = s_{t-1}, D_t = x|Y_t), \quad \text{(C.36)}$$

from which we may obtain the duration-independent posterior probability,

$$P(S_t = s_t|Y_t) = \sum_{x} P(S_t = s_t, D_t = x|Y_t). \quad \text{(C.37)}$$
APPENDIX D

FIGURES, TABLES, AND ILLUSTRATIONS
Figure 1: Log Growth Rates of Industrial Production, 1987=100, 1919.02 - 1996.09
Figure 2: Log Growth Rates of Employees in Nonagricultural Establishments, 1929.02-1996.09
Figure 3: Log Growth Rates of Personal Income, Constant Dollars, 1929.02 - 1996.09
Figure 4: Inferred Probabilities that $S_t=0$ Conditional on $Y_{T}=\{y_1,...,y_T\}$,
FTP Model of Monthly Coincident Indicators.
| Estimated Parameter Values<sup>a</sup><sup>d</sup> |
|-------------------------------|----------------|
| $\sigma_1^2$ | 0.02273 |
| (0.00000) | $\phi_2$ | 0.15206 |
| $\sigma_2^2$ | 0.42479 |
| (0.00000) | $d_1$ | -0.18374 |
| $\sigma_3^2$ | 0.32729 |
| (0.00000) | $d_2$ | 0.23832 |
| $\sigma_4^2$ | 1.14490 |
| (0.00000) | $d_3$ | -0.22953 |
| $\gamma_1$ | 0.22002 |
| (0.00000) | $d_4$ | -0.09787 |
| $\gamma_2$ | 0.57862 |
| (0.00000) | $q^b$ | 1.69774 |
| $\gamma_3$ | 0.26912 |
| (0.00000) | $p$ | 3.41210 |
| $\gamma_4$ | 0.45877 |
| (0.00000) | $\mu_0^c$ | -0.90283 |
| $\phi_1$ | 0.18972 |
| (0.00031) | $\mu_1$ | 1.67697 |
| Log-Likelihood | -2196.87487 |

Table 1: Fixed Transition Probability Model: Postwar Monthly Coincident Indicators, 1947.04 - 1996.09

* Those series indexed by the subscripts 1 - 4 are employees in nonagricultural establishments, industrial production, personal income less transfer payments, and monthly manufacturing and trade sales, respectively.

$^b p = \frac{\exp(a_1)}{1 + \exp(a_1)}$ while $q = \frac{\exp(a_0)}{1 + \exp(a_0)}$, where $a_0 = 1.69774$ and $a_1 = 3.42210$.

$^c$ The estimated recession mean is given by $\frac{\mu_0}{1 - \phi_1 - \phi_2} = -1.37162$; the estimated expansion mean is given by $\frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 1.17611$.

$^d$ The $p$-values are reported in parentheses. The standard errors are computed by taking the inverse of the numerical approximation of the Hessian.
Table 2: Duration-Dependent Transition Probability Model: Postwar Monthly Coincident Indicators, 1947.04 - 1996.09

<table>
<thead>
<tr>
<th>Estimated Parameter Values(^{ae})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_i^2)</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>(\sigma_i^2)</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>(\gamma_1)</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>(\gamma_2)</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>(\gamma_3)</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>(\gamma_4)</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>(\phi_1)</td>
</tr>
<tr>
<td>(0.00001)</td>
</tr>
<tr>
<td>(\phi_2)</td>
</tr>
<tr>
<td>(0.00140)</td>
</tr>
<tr>
<td>Log-Likelihood</td>
</tr>
</tbody>
</table>

\(^a\) Those series indexed by the subscripts 1 - 4 are employees in nonagricultural establishments, industrial production, personal income less transfer payments, and monthly manufacturing and trade sales, respectively.

\(^b\) The estimated recession mean is given by \(\mu_0 = 1.10884\). The estimated expansion mean is given by \(\frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = -0.96107\); the estimated recession mean is given by \(\frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 1.10884\).

\(^c\) The \(p\)-values are reported in parentheses. Standard errors are computed by taking the inverse of the numerical approximation of the Hessian.
Figure 5: Duration-Dependent Transition Probabilities
DDTP Model of Postwar Monthly Coincident Indicators
Figure 6: Inferred Probabilities that $S_t=0$ Conditional on $Y_{t-1}=(y_1,...,y_{t-1})$, FTP Model of Postwar Monthly Coincident Indicators
Figure 7: Inferred Probabilities that $S_t=0$ Conditional on $Y_t=\{y_1,\ldots,y_T\}$.
FTP Model of Postwar Monthly Coincident Indicators
Figure 8: Inferred Probabilities that $S_t=0$ Conditional on $Y_t=(y_1,...,y_{t+1})$, DDTP Model of Postwar Monthly Coincident Indicators
Figure 9: Inferred Probabilities that $S_t = 0$ Conditional on $Y_t = \{y_1, \ldots, y_T\}$, DDTP Model of Postwar Monthly Coincident Indicators
### Estimated Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1^2$</td>
<td>1.87005</td>
<td>(0.00000)</td>
<td>$\phi_2$</td>
<td>-0.09648</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
<td>4.26876</td>
<td>(0.00000)</td>
<td>$d_1$</td>
<td>-0.08774</td>
</tr>
<tr>
<td>$\sigma_3^2$</td>
<td>0.02777</td>
<td>(0.23266)</td>
<td>$d_2$</td>
<td>0.09881</td>
</tr>
<tr>
<td>$\sigma_4^2$</td>
<td>5.44173</td>
<td>(0.00000)</td>
<td>$d_3$</td>
<td>-0.43942</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.56418</td>
<td>(0.00000)</td>
<td>$d_4$</td>
<td>0.50817</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.36236</td>
<td>(0.00052)</td>
<td>$q^b$</td>
<td>0.91824</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.48763</td>
<td>(0.00000)</td>
<td>$p$</td>
<td>0.97024</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>1.25993</td>
<td>(0.00000)</td>
<td>$\mu_0^c$</td>
<td>-1.00364</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.60200</td>
<td>(0.00029)</td>
<td>$\mu_1$</td>
<td>1.54120</td>
</tr>
</tbody>
</table>

Log-Likelihood: -1586.68495

---

Table 3: Fixed Transition Probability Model: Interwar Monthly Coincident Indicators, 1919.04 - 1939.03

* Those series indexed by the subscripts 1 - 4 are monthly department store sales, personal income, employees in nonagricultural establishments, and industrial production, respectively.

$ p = \frac{\exp(\gamma_1)}{1 + \exp(\gamma_1)} $ while $ q = \frac{\exp(\gamma_2)}{1 + \exp(\gamma_2)} $, where $ \gamma_1 = 2.41872 $ and $ \gamma_2 = 3.48444 $.

* The estimated recession mean is given by $ \frac{\mu_0}{1 - \phi_1 - \phi_2} = -2.02969 $; the estimated expansion mean is given by $ \frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 1.08712 $.

* The $ p $-values are reported in parentheses. Standard errors are computed by taking the inverse of the numerical approximation of the Hessian.
<table>
<thead>
<tr>
<th>Estimated Parameter Values&lt;sup&gt;ad&lt;/sup&gt;</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1^2$</td>
<td>1.85536</td>
<td>$d_1$</td>
</tr>
<tr>
<td>(0.00000)</td>
<td></td>
<td>(0.09504)</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
<td>4.26549</td>
<td>$d_2$</td>
</tr>
<tr>
<td>(0.00000)</td>
<td></td>
<td>0.09879</td>
</tr>
<tr>
<td>$\sigma_3^2$</td>
<td>0.03018</td>
<td>$d_3$</td>
</tr>
<tr>
<td>(0.07866)</td>
<td></td>
<td>-0.43880</td>
</tr>
<tr>
<td>$\sigma_4^2$</td>
<td>5.45402</td>
<td>$d_4$</td>
</tr>
<tr>
<td>(0.00000)</td>
<td></td>
<td>0.50673</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.58206</td>
<td>$\mu_0$</td>
</tr>
<tr>
<td>(0.00000)</td>
<td></td>
<td>-0.82157</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.37353</td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>(0.00005)</td>
<td></td>
<td>1.28600</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.49985</td>
<td>$a_0$</td>
</tr>
<tr>
<td>(0.00000)</td>
<td></td>
<td>2.90183</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>1.29202</td>
<td>$b_0$</td>
</tr>
<tr>
<td>(0.00000)</td>
<td></td>
<td>-0.09936</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.66635</td>
<td>$a_1$</td>
</tr>
<tr>
<td>(0.00000)</td>
<td></td>
<td>21.84086</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.12591</td>
<td>$b_1$</td>
</tr>
<tr>
<td>(0.09199)</td>
<td></td>
<td>-0.43283</td>
</tr>
</tbody>
</table>

Log-Likelihood -1585.21341

Table 4: Duration-Dependent Transition Probability Model: Interwar Monthly Coincident Indicators, 1919.04 - 1939.03

* Those series indexed by the subscripts 1 - 4 are monthly department store sales, personal income, employees in nonagricultural establishments, and industrial production, respectively.

$ p = \frac{\exp(a_1 + b_1 d)}{1 + \exp(a_1 + b_1 d)} \text{ while } q = \frac{\exp(a_0 + b_0 d)}{1 + \exp(a_0 + b_0 d)}, \text{ where } 1 \leq d \leq T, \ T = 48. $

$ c \text{ The estimated recession mean is given by } \frac{\mu_0}{1 - \phi_1 - \phi_2} = -1.78773; \text{ the estimated expansion mean is given by } \frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 1.01060. $

$ d \text{ The } p\text{-values are reported in parentheses. Standard errors are computed by taking the inverse of the numerical approximation of the Hessian. }$
Figure 10: Duration-Dependent Transition Probabilities
DDTP Model of Interwar Monthly Coincident Indicators
<table>
<thead>
<tr>
<th>Dates from DDTP Model</th>
<th>NBER Dates</th>
<th>Romer's Dates*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trough</td>
<td>Peak</td>
<td>Trough</td>
</tr>
<tr>
<td>1921.09</td>
<td>1920.06</td>
<td>1921.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1924.07</td>
</tr>
<tr>
<td></td>
<td>1929.08</td>
<td>1927.11</td>
</tr>
<tr>
<td>1933.03</td>
<td>1937.06</td>
<td>1933.03</td>
</tr>
<tr>
<td>1938.05</td>
<td></td>
<td>1938.06</td>
</tr>
<tr>
<td></td>
<td>1945.02</td>
<td></td>
</tr>
<tr>
<td>1945.09</td>
<td>1948.07</td>
<td>1945.10</td>
</tr>
<tr>
<td>1949.10</td>
<td>1953.03</td>
<td>1949.10</td>
</tr>
<tr>
<td>1954.07</td>
<td>1957.02</td>
<td>1954.05</td>
</tr>
<tr>
<td>1975.03</td>
<td>1980.01</td>
<td>1975.03</td>
</tr>
<tr>
<td>1980.07</td>
<td>1981.05</td>
<td>1980.07</td>
</tr>
<tr>
<td>1982.10</td>
<td>1990.05</td>
<td>1982.11</td>
</tr>
<tr>
<td>1991.05</td>
<td></td>
<td>1991.03</td>
</tr>
</tbody>
</table>

Table 5: Comparison of Inter- and Postwar Peaks and Troughs, 1919.04 - 1996.09

*Romer's chronology for the interwar years is derived from the Miron-Romer historical index of industrial production (1884 - 1940). The rules applied in establishing this chronology are discussed in Romer (1994).
### Dates from the DDTP Dynamic Factor Model

<table>
<thead>
<tr>
<th>Trough</th>
<th>Peak</th>
<th>Duration of Contraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>--------</td>
<td>----------</td>
<td>------------------------</td>
</tr>
<tr>
<td>1920.06</td>
<td>1921.09</td>
<td>15 months</td>
</tr>
<tr>
<td></td>
<td>1929.08</td>
<td>1929.08</td>
</tr>
<tr>
<td>1933.03</td>
<td>1937.06</td>
<td>43 months</td>
</tr>
<tr>
<td>1938.05</td>
<td>1937.07</td>
<td>10 months</td>
</tr>
<tr>
<td>1945.02</td>
<td>1948.07</td>
<td>7 months</td>
</tr>
<tr>
<td>1949.10</td>
<td>1953.03</td>
<td>15 months</td>
</tr>
<tr>
<td>1954.07</td>
<td>1957.02</td>
<td>16 months</td>
</tr>
<tr>
<td>1958.04</td>
<td>1960.01</td>
<td>14 months</td>
</tr>
<tr>
<td>1961.02</td>
<td>1969.10</td>
<td>13 months</td>
</tr>
<tr>
<td>1970.11</td>
<td>1974.02</td>
<td>13 months</td>
</tr>
<tr>
<td>1975.03</td>
<td>1980.01</td>
<td>13 months</td>
</tr>
<tr>
<td>1980.07</td>
<td>1981.05</td>
<td>6 months</td>
</tr>
<tr>
<td>1982.10</td>
<td>1990.05</td>
<td>17 months</td>
</tr>
<tr>
<td>1991.05</td>
<td>1990.06</td>
<td>16 months</td>
</tr>
</tbody>
</table>

### Dates from the FTP Dynamic Factor Model

<table>
<thead>
<tr>
<th>Trough</th>
<th>Peak</th>
<th>Duration of Contraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>--------</td>
<td>----------</td>
<td>------------------------</td>
</tr>
<tr>
<td>1920.06</td>
<td>1921.09</td>
<td>15 months</td>
</tr>
<tr>
<td></td>
<td>1929.08</td>
<td>1929.08</td>
</tr>
<tr>
<td>1933.03</td>
<td>1937.07</td>
<td>43 months</td>
</tr>
<tr>
<td>1938.05</td>
<td>1938.05</td>
<td>10 months</td>
</tr>
<tr>
<td>1945.02</td>
<td>1948.09</td>
<td>7 months</td>
</tr>
<tr>
<td>1949.07</td>
<td>1953.06</td>
<td>10 months</td>
</tr>
<tr>
<td>1954.06</td>
<td>1957.03</td>
<td>12 months</td>
</tr>
<tr>
<td>1958.04</td>
<td>1960.04</td>
<td>13 months</td>
</tr>
<tr>
<td>1961.01</td>
<td>1970.03</td>
<td>9 months</td>
</tr>
<tr>
<td>1970.11</td>
<td>1974.07</td>
<td>8 months</td>
</tr>
<tr>
<td>1975.03</td>
<td>1980.02</td>
<td>8 months</td>
</tr>
<tr>
<td>1980.07</td>
<td>1981.07</td>
<td>5 months</td>
</tr>
<tr>
<td>1982.11</td>
<td>1990.06</td>
<td>16 months</td>
</tr>
<tr>
<td>1991.03</td>
<td></td>
<td>9 months</td>
</tr>
</tbody>
</table>

Table 6: Inter- and Postwar Peaks and Troughs, 1919.04 - 1996.09, from the DDTP and FTP Dynamic Factor Models
Figure 11: Inferred Probability that $S_t=0$ Conditional on $Y_T=(y_1,\ldots,y_T)$
FTP Model of Interwar Monthly Coincident Indicators
Figure 12: Inferred Probabilities that $S_t=0$ Conditional on $Y_T=(y_1,...,y_T)$
DDTP Model of Interwar Monthly Coincident Indicators
Figure 13: Estimates of the Growth Rate of the Common Factor, \( \Delta f_{gt} \), DDTP Model of Interwar Coincident Indicators
Figure 14: Estimates of the Growth Rate of the Common Factor, $\Delta f_\phi$,
DDTP Model of Postwar Coincident Indicators
<table>
<thead>
<tr>
<th>Year</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1899</td>
<td>0.9937</td>
</tr>
<tr>
<td>1904</td>
<td>1.0010</td>
</tr>
<tr>
<td>1909</td>
<td>0.9972</td>
</tr>
<tr>
<td>1914</td>
<td>1.0000</td>
</tr>
<tr>
<td>1919</td>
<td>0.9860</td>
</tr>
<tr>
<td>1921</td>
<td>0.9984</td>
</tr>
<tr>
<td>1923</td>
<td>0.9773</td>
</tr>
</tbody>
</table>

Contemporaneous Correlation between Jerome and Fabricant: 0.9891

Table 7: Ratio of Jerome's Index of Factory Employment to Fabricant's Index of Wage Earners in Manufacturing Industries in Census Years

* The ratios and correlations given above are computed using the log levels of the of the two series.

<table>
<thead>
<tr>
<th>Growth Rates on an annual basis for</th>
<th>Sample Mean</th>
<th>Sample Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jerome</td>
<td>2.5180</td>
<td>8.6797</td>
</tr>
<tr>
<td>Fabricant</td>
<td>2.1707</td>
<td>7.2636</td>
</tr>
</tbody>
</table>

Contemporaneous Correlation between Jerome and Fabricant: 0.9446

Table 8: Comparison of the Growth Rates of Jerome's Index of Factory Employment and Fabricant's Index of Wage Earners in Manufacturing Industries, Summary Statistics

* The summary statistics given above are computed using 100 times the log first differences of the of the two series.
Figure 15: Fabricant's Annual Index of Wage Earners Employed in Manufacturing vs. Jerome's Index of Factory Employment, 1889 - 1923 (Log Levels)
Figure 16: Log Growth Rates of Jerome/BLS Index of Factory Employment,
1889.02-1920.03
Figure 17: Log Growth Rates of Bank Clearings Outside New York City, SA, Constant Dollars, 1885.04 - 1920.03
Figure 19: Log Growth Rates of the Miron-Romer Index of Industrial Production, 1885.04-1920.03
Table 9: Comparison of Average Growth Rates and Standard Deviations, Prewar Monthly Coincident Indicators

| Series                  | Sample 1\(^a\) | Sample 2\(^b\) | \(|t|\)       |
|-------------------------|-----------------|-----------------|-------------|
|                         | \(\bar{X}\)     | \(\sigma\)     | \(\bar{X}\) | \(\sigma\) | \(|t|\)       |
| Factory Employment      | 0.020           | 0.7226          | -0.8354     | 2.2707     | 2.8276       |
| Bank Clearings          | -0.0550         | 0.9301          | -0.7833     | 1.8288     | 2.8808       |
| Total Imports           | 0.2001          | 2.9653          | -1.5265     | 3.2589     | 3.3865       |
| Industrial Production   | -0.1549         | 1.9627          | -0.9177     | 2.5116     | 2.0279       |

\(^a\) \(\bar{X}\) denotes the average monthly growth rate of the series; \(\sigma\) denotes the average sample standard deviation of the monthly growth rates; and \(t_{\bar{X}}\) is the \(t\)-statistic for testing the equality of the growth rates across the two samples.

\(^b\) Sample 1 contains the growth rates of the four prewar series over the six NBER-dated recessionary episodes for which the mean of \(\Delta y_{it} > 0\); sample 2 contains the growth rates of the four prewar series over the four NBER-dated recessionary episodes for which the mean of \(\Delta y_{it} < 0\). The peak and trough dates for those six recessionary episodes corresponding to sample 1 are given as follows: 1887.03 to 1888.04, 1890.07 to 1891.05, 1895.12 to 1897.06, 1899.06 to 1900.12, 1902.09 to 1904.08, and 1910.01 to 1912.01. The peak and trough dates for those four recessionary episodes corresponding to sample 2 are given as follows: 1893.01 to 1894.06, 1907.05 to 1908.06, 1913.01 to 1914.12, and 1918.08 to 1919.03.
Table 10: Fixed Transition Probability Model: Prewar Monthly Coincident Indicators, 1885.04 - 1920.03

* Those series indexed by subscripts 1 - 4 are, respectively, bank clearings outside New York City, the Miron-Romer index of industrial production, the deflated value of total imports, and a constructed index of factory employment (Jerome-BLS).

\[ p = \frac{\exp(a_1)}{1 + \exp(a_1)} \] while \[ q = \frac{\exp(a_2)}{1 + \exp(a_2)} \], where \( a_0 = 0.40707 \) and \( a_1 = 4.76432 \).

* The estimated recession mean is given by \( \frac{\mu_0}{1 - \phi_1 - \phi_2} = -11.48420 \); the estimated expansion mean is given by \( \frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 0.86971 \).

** Standard errors are computed by taking the inverse of the Hessian; the numerical approximation of this is obtained by using GAUSS' MAXLIK module and by selecting scale-free BFGS as the optimization method.
Table 11: Duration-Dependent Transition Probability Model: Prewar Monthly Coincident Indicators, 1885.04 - 1920.03

* Those series indexed by subscripts 1 - 4 are, respectively, bank clearings outside New York City, the Miron-Romer index of industrial production, the deflated value of total imports, and a constructed index of factory employment (Jerome-BLS).

\[ p = \frac{\exp(a_1 + b_1 \tau)}{1 + \exp(a_1 + b_1 \tau)} \text{ while } q = \frac{\exp(a_2 + b_2 \tau)}{1 + \exp(a_2 + b_2 \tau)}, \text{ where } 1 \leq \tau \leq 48. \]

\[ \text{The estimated recession mean is given by } \frac{\mu_2}{1 - \phi_1 - \phi_2} = -11.48610; \text{ the estimated expansion mean is given by } \frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 0.86282. \]

(d) Standard errors are computed by taking the inverse of the Hessian; the numerical approximation of this is obtained by using GAUSS' MAXLIK module and by selecting scale-free BFGS as the optimization method.
Figure 20: Duration-Dependent Transition Probabilities
DDTP Model of Prewar Monthly Coincident Indicators
Figure 21: Inferred Probabilities that $S_i=0$ Conditional on $Y_{t-1}=(y_1, ..., y_{t-1})$.
FTP Model of Prewar Monthly Coincident Indicators
Figure 22: Inferred Probabilities that $S_t=0$ Conditional on $Y_{t-1} = \{y_1, \ldots, y_{t-1}\}$,
DDTP Model of Prewar Monthly Coincident Indicators
<table>
<thead>
<tr>
<th>Dates from the DDTP Model</th>
<th>NBER Dates</th>
<th>Romer's Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trough Peak</td>
<td>Trough Peak</td>
<td>Trough Peak</td>
</tr>
<tr>
<td>1893.05</td>
<td>1887.03</td>
<td>1887.02</td>
</tr>
<tr>
<td>1893.09</td>
<td>1888.04</td>
<td>1890.07</td>
</tr>
<tr>
<td></td>
<td>1891.05</td>
<td>1893.01</td>
</tr>
<tr>
<td></td>
<td>1894.06</td>
<td>1895.12</td>
</tr>
<tr>
<td></td>
<td>1897.06</td>
<td>1899.06</td>
</tr>
<tr>
<td></td>
<td>1900.12</td>
<td>1902.09</td>
</tr>
<tr>
<td></td>
<td>1904.08</td>
<td>1907.05</td>
</tr>
<tr>
<td></td>
<td>1908.06</td>
<td>1910.01</td>
</tr>
<tr>
<td></td>
<td>1912.01</td>
<td>1913.01</td>
</tr>
<tr>
<td></td>
<td>1914.12</td>
<td>1918.08</td>
</tr>
<tr>
<td></td>
<td>1919.03</td>
<td>1919.03</td>
</tr>
</tbody>
</table>

Table 12: Prewar Peaks and Troughs, 1885.04 - 1920.03, Identified via the Inferred Probabilities that $S_t = 0$ Conditional on $Y_T = \{y_1, \ldots, y_T\}$
## Table 13: Prewar Peaks and Troughs, 1885.04 - 1920.03, Identified via Estimates of the Growth Rate of the Common Factor, $\Delta f_{it}$

<table>
<thead>
<tr>
<th>Dates from the DDTP Model*</th>
<th>NBER Dates</th>
<th>Romer's Dates^b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trough Peak</td>
<td>Trough Peak</td>
<td>Trough Peak</td>
</tr>
<tr>
<td>---------------------------</td>
<td>------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1887.02</td>
<td>1887.03</td>
<td>1887.02</td>
</tr>
<tr>
<td>1893.05</td>
<td>1888.04</td>
<td>1894.02</td>
</tr>
<tr>
<td>1895.11</td>
<td>1890.07</td>
<td>1896.01</td>
</tr>
<tr>
<td>1896.11</td>
<td>1893.01</td>
<td>1897.01</td>
</tr>
<tr>
<td>1900.12</td>
<td>1895.12</td>
<td>1900.04</td>
</tr>
<tr>
<td>1902.09</td>
<td>1899.06</td>
<td>1903.07</td>
</tr>
<tr>
<td>1904.08</td>
<td>1900.12</td>
<td>1907.07</td>
</tr>
<tr>
<td>1907.05</td>
<td>1908.06</td>
<td>1908.06</td>
</tr>
<tr>
<td>1910.01</td>
<td>1911.05</td>
<td>1910.12</td>
</tr>
<tr>
<td>1912.01</td>
<td>1914.12</td>
<td>1914.06</td>
</tr>
<tr>
<td>1914.12</td>
<td>1918.07</td>
<td>1917.01</td>
</tr>
<tr>
<td>1918.08</td>
<td>1919.03</td>
<td>1918.07</td>
</tr>
</tbody>
</table>

* We identify a contraction as a period of six or more consecutive months in which $\Delta f_{it} < 0$.  
^b That algorithm used by Romer in deriving her revised chronology is described in Romer (1994).
Figure 23: Inferred Probabilities that $S_t=0$ Conditional on $Y_t=(y_1,\ldots,y_T)$, DDTP Model of Prewar Monthly Coincident Indicators
Figure 24: Smoothed Estimates of the Growth Rate of the Common Factor, $\Delta f_{it}$, DDTP Model of Coincident Indicators
Table 14: Business Cycle Peaks and Troughs from the DDTP Model and Average Phase Durations

\(^a\) Contraction durations are measured as the number of months elapsing between a business cycle peak and the subsequent trough; similarly, expansion durations are measured as the number elapsing between a business cycle trough and the subsequent business cycle peak. Peak-to-Peak durations are measured as the number of months elapsing between two consecutive business cycle peaks.

\(^b\) Mean durations for the DDTP model are computed from the duration data given in Panel A of Table 13.

\(^c\) Mean durations for the NBER chronology and Romer's (1994) chronology are taken from Table 2 of Parker and Rothman (1996).
### A. Business Cycle Peaks and Troughs and Phase Durations:

<table>
<thead>
<tr>
<th>Trough</th>
<th>Peak</th>
<th>Durations*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Contractions</td>
</tr>
<tr>
<td>1893.05</td>
<td>1893.11</td>
<td>6</td>
</tr>
<tr>
<td>1896.11</td>
<td>1907.08</td>
<td>12</td>
</tr>
<tr>
<td>1908.04</td>
<td>1910.03</td>
<td>8</td>
</tr>
<tr>
<td>1910.09</td>
<td>1913.02</td>
<td>6</td>
</tr>
<tr>
<td>1914.12</td>
<td>1918.07</td>
<td>22</td>
</tr>
<tr>
<td>1919.03</td>
<td>1920.06</td>
<td>8</td>
</tr>
<tr>
<td>1921.09</td>
<td>1929.08</td>
<td>15</td>
</tr>
<tr>
<td>1933.03</td>
<td>1937.07</td>
<td>43</td>
</tr>
<tr>
<td>1938.05</td>
<td>1945.02</td>
<td>10</td>
</tr>
<tr>
<td>1945.09</td>
<td>1948.07</td>
<td>7</td>
</tr>
<tr>
<td>1949.10</td>
<td>1953.03</td>
<td>15</td>
</tr>
<tr>
<td>1954.07</td>
<td>1957.02</td>
<td>17</td>
</tr>
<tr>
<td>1958.04</td>
<td>1960.01</td>
<td>14</td>
</tr>
<tr>
<td>1961.02</td>
<td>1969.10</td>
<td>13</td>
</tr>
<tr>
<td>1970.11</td>
<td>1974.02</td>
<td>13</td>
</tr>
<tr>
<td>1975.03</td>
<td>1980.01</td>
<td>13</td>
</tr>
<tr>
<td>1980.07</td>
<td>1981.05</td>
<td>6</td>
</tr>
<tr>
<td>1982.10</td>
<td>1990.05</td>
<td>17</td>
</tr>
<tr>
<td>1991.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### B. Comparison of Mean Expansion, Contraction, and Peak-to-Peak Durations

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean Expansion Durations</th>
<th>Mean Contraction Durations</th>
<th>Mean Peak-to-Peak Durations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NBER</td>
<td>Romer</td>
<td>DDTP Model</td>
</tr>
<tr>
<td>1887-1915</td>
<td>21.8</td>
<td>36.4</td>
<td>51.3</td>
</tr>
<tr>
<td>1887-1929</td>
<td>22.9</td>
<td>29.3</td>
<td>51.1</td>
</tr>
<tr>
<td>1887-1938</td>
<td>24.9</td>
<td>31.4</td>
<td>51.3</td>
</tr>
<tr>
<td>1945-1996</td>
<td>51.5</td>
<td>51.4</td>
<td>47.6</td>
</tr>
</tbody>
</table>

Table 14: Business Cycle Peaks and Troughs from the DDTP Model and Average Phase Durations
### Wilcoxon Rank-Sum Tests for Expansions

<table>
<thead>
<tr>
<th>Period</th>
<th>Sample Size</th>
<th>Wilcoxon Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_x$</td>
<td>$n_y$</td>
</tr>
<tr>
<td>1887-1915</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>1887-1929</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>1887-1938</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

### Wilcoxon Rank-Sum Tests for Contractions

<table>
<thead>
<tr>
<th>Period</th>
<th>Sample Size</th>
<th>Wilcoxon Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_x$</td>
<td>$n_y$</td>
</tr>
<tr>
<td>1887-1915</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>1887-1929</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>1887-1938</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

### Wilcoxon Rank-Sum Tests for Peak-to-Peak Whole Cycles

<table>
<thead>
<tr>
<th>Period</th>
<th>Sample Size</th>
<th>Wilcoxon Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_x$</td>
<td>$n_y$</td>
</tr>
<tr>
<td>1887-1915</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>1887-1929</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>1887-1938</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 15: Tests of the Null Hypothesis of No Postwar Duration Stabilization

*a* The duration data used in these Wilcoxon rank-sum tests is taken from Panel A of Table 13.

*b* The reported critical values are taken from Bradley (1968, Table III, 318-322).
<table>
<thead>
<tr>
<th>Estimated Parameter Values*&lt;sup&gt;d&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1^2$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\sigma_3^2$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\sigma_4^2$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
</tr>
<tr>
<td>(0.00052)</td>
</tr>
<tr>
<td>$\gamma_3$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_4$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\phi_1$</td>
</tr>
<tr>
<td>(0.00000)</td>
</tr>
</tbody>
</table>

Log-Likelihood -895.42643


* Those series indexed by the subscripts 1 - 4 are quarterly averages of monthly manufacturing and trade sales, personal income less transfer payments, employees in nonagricultural establishments, and real gross domestic product, respectively.

<sup>b</sup> $p = \frac{\exp(\alpha)}{1 + \exp(\alpha)}$ while $q = \frac{\exp(\alpha)}{1 + \exp(\alpha)}$, where $\alpha_0 = 0.71447$ and $\alpha_1 = 2.52859$.

<sup>c</sup> The estimated recession mean is given by $\frac{\mu_0}{1 - \phi_1 - \phi_2} = -0.98862$; the estimated expansion mean is given by $\frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 1.83255$.

<sup>d</sup> The $p$-values are reported in parentheses. Standard errors are computed by taking the inverse of the numerical approximation of the Hessian.
### Table 17: Duration-Dependent Transition Probability Model: Postwar Quarterly Coincident Indicators, 1947.II - 1996.III

| Estimated Parameter Values$^{ad}$ |  |  
|-----------------------------------|---|---|
| $\sigma_1^2$ | 1.75439 | $d_1$ | -0.03312 |
| (0.00000) | | (0.34531) |
| $\sigma_2^2$ | 0.38702 | $d_2$ | -0.18934 |
| (0.00000) | | (0.00560) |
| $\sigma_3^2$ | 0.07701 | $d_3$ | 0.59939 |
| (0.07866) | | (0.00000) |
| $\sigma_4^2$ | 0.30187 | $d_4$ | -0.15650 |
| (0.00000) | | (0.03523) |
| $\gamma_1$ | 0.69042 | $\mu_0^c$ | -0.54479 |
| (0.00000) | | (0.00001) |
| $\gamma_2$ | 0.46884 | $\mu_1$ | 1.93844 |
| (0.00000) | | (0.00000) |
| $\gamma_3$ | 0.33428 | $a_0^b$ | 62.42134 |
| (0.00000) | | (0.00000) |
| $\gamma_4$ | 0.47443 | $b_0$ | -14.96611 |
| (0.00000) | | (0.00000) |
| $\phi_1$ | 0.66929 | $a_1^b$ | 4.53633 |
| (0.00000) | | (0.00000) |
| $\phi_2$ | -0.29054 | $b_1$ | -0.18409 |
| (0.00001) | | (0.00000) |

Log-Likelihood: -888.39869

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>

Table 17: Duration-Dependent Transition Probability Model: Postwar Quarterly Coincident Indicators, 1947.II - 1996.III

* Those series indexed by the subscripts 1 - 4 are quarterly averages of monthly manufacturing and trade sales, personal income less transfer payments, employees in nonagricultural establishments, and real gross domestic product, respectively.

$^b$ $p = \frac{\exp(a_0 + b_0d)}{1 + \exp(a_0 + b_0d)}$ while $q = \frac{\exp(a_0 + b_0d)}{1 + \exp(a_0 + b_0d)}$, where $1 \leq d \leq \tau$, $\tau = 12$.

$^c$ The estimated recession mean is given by $\frac{\mu_0}{1 - \phi_1 - \phi_2} = -0.87693$; the estimated expansion mean is given by $\frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 2.24330$.

$^d$ The $p$-values are reported in parentheses. Standard errors are computed by taking the inverse of the numerical approximation of the Hessian.

188
<table>
<thead>
<tr>
<th>Estimated Parameter Values</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1^2$</td>
<td>2.68838</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
<td>3.07911</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.08749)</td>
</tr>
<tr>
<td>$\sigma_3^2$</td>
<td>0.11192</td>
</tr>
<tr>
<td>(0.00141)</td>
<td>(0.09833)</td>
</tr>
<tr>
<td>$\sigma_4^2$</td>
<td>4.72482</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.08749)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.54767</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.44605</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.48776</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>1.29287</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.54210</td>
</tr>
<tr>
<td>(0.00000)</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>-2389.28903</td>
</tr>
</tbody>
</table>

Table 18: Fixed Transition Probability Model: Interwar Monthly Coincident Indicators, 1919.04 - 1948.03

* Those series indexed by the subscripts 1 - 4 are monthly department store sales, personal income, employees in nonagricultural establishments, and industrial production, respectively.

\[ p = \frac{\exp(a_1)}{1 + \exp(a_1)} \text{ while } q = \frac{\exp(a_2)}{1 + \exp(a_2)} , \text{ where } a_0 = 2.29242 \text{ and } a_1 = 3.81444. \]

\[ \mu_0 = \frac{\mu_0^c}{1 - \phi_1 - \phi_2} = -2.21552 ; \text{ the estimated expansion mean is given by } \frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 1.03424. \]

d The p-values are reported in parentheses. Standard errors are computed by taking the inverse of the numerical approximation of the Hessian.
### Estimated Parameter Values$^{a,d}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1^2$</td>
<td>2.68358</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
<td>3.08262</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\sigma_3^2$</td>
<td>0.11045</td>
<td>(0.00146)</td>
</tr>
<tr>
<td>$\sigma_4^2$</td>
<td>4.74074</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.56158</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.45436</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.49841</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>1.31881</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.57253</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.07927</td>
<td>(0.18605)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>0.07511</td>
<td>(0.09494)</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.08519</td>
<td>(0.09779)</td>
</tr>
<tr>
<td>$d_3$</td>
<td>-0.25851</td>
<td>(0.07292)</td>
</tr>
<tr>
<td>$d_4$</td>
<td>0.42846</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>-1.04911</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>1.54081</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>$a_0$</td>
<td>2.96369</td>
<td>(0.00092)</td>
</tr>
<tr>
<td>$b_0$</td>
<td>-0.01623</td>
<td>(0.35521)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>390.81029</td>
<td>(0.00014)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-8.07421</td>
<td>(0.00016)</td>
</tr>
</tbody>
</table>

Log-Likelihood: -2387.54480

Table 19: Duration-Dependent Transition Probability Model: Interwar Monthly Coincident Indicators, 1919.04 - 1948.03

* Those series indexed by the subscripts 1 - 4 are monthly department store sales, personal income, employees in nonagricultural establishments, and industrial production, respectively.

\[ p = \frac{\exp(a_0 + b_1d)}{1 + \exp(a_1 + b_1d)} \] while \[ q = \frac{\exp(a_0 + b_0d)}{1 + \exp(a_0 + b_0d)} \], where \( 1 \leq d \leq \tau \), \( \tau = 48 \).

* The estimated recession mean is given by \( \frac{\mu_0}{1 - \phi_1 - \phi_2} = -2.07031 \); the estimated expansion mean is given by \( \frac{\mu_0 + \mu_1}{1 - \phi_1 - \phi_2} = 0.97032 \).

$^d$ The $p$-values are reported in parentheses. Standard errors are computed by taking the inverse of the numerical approximation of the Hessian.
Figure 25: Duration-Dependent Transition Probabilities
DDTP Model of Postwar Quarterly Coincident Indicators