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MODEL BASED FAULT DETECTION AND ISOLATION IN NONLINEAR DYNAMIC SYSTEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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ABSTRACT

Any complex system or process is subject to the occurrence of faults. These systems may be mechanical, chemical, electrical or some combination of these and may or may not be controlled by a computer. Faults include the complete or partial failure of actuators, sensors or other components of the system, or the occurrence of events external to the system (disturbances) that prevent its normal functioning.

The ability to detect the occurrence of any fault, and identify its cause is critical for a number of reasons. In some cases faults can lead to great loss of life and property (e.g., aircraft, nuclear power plants, etc.). Further, early detection of faults can allow timely corrective action which in many cases will greatly reduce the incidence of expensive, unexpected breakdowns (e.g., machinery in factories). Environmental considerations are also of importance as in the development of systems for monitoring automobile engine emissions.

The increasing sophistication and decreasing cost of electronic systems has made viable the use of complex algorithms for the monitoring of dynamic systems. Over the past two decades much attention has been focused on the development of diagnostic algorithms for linear dynamic systems and a number of useful results have been developed. However very little attention has been paid to nonlinear systems and most hitherto developed techniques fail when applied to systems that exhibit significantly nonlinear dynamic behavior.

This study focuses on the development of fault detection and isolation techniques for nonlinear dynamic systems. A general problem formulation that is applicable to both nonlinear and linear dynamic systems is postulated. Some of the hitherto qualitatively
understood terms and concepts are formally defined for the general nonlinear case. Necessary and sufficient conditions are provided for the solvability of the fault isolation problem. It is shown that the isolation solution exists for those systems for which the fault variables satisfy certain invertibility conditions with respect to the available set of measurements. A systematic monitoring system design procedure that depends on the construction of forward and inverse models is provided for this class of systems. The fault detection algorithm does not depend on the particular type of modeling or inversion scheme used, requiring only that such models and inverses can be derived.

New results in sliding mode state estimation that are useful for the fault isolation problem are developed and validated through experimental work.

The derived results are experimentally validated through extensive experiments in the form of 3 case studies: (i) Internal Combustion engine fault detection using nonlinear input-output identification, (ii) Vehicle steering system monitoring using continuous time sliding mode observers, and (iii) Engine air-fuel-exhaust system monitoring using discrete sliding mode observers.

The results show that the scheme developed in this study is extremely successful, and applicable to all systems that satisfy certain invertibility conditions, and for such systems is capable of isolating multiple fault occurrences even under the influence of unknown disturbances.
Dedicated to my parents
ACKNOWLEDGMENTS

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CHAPTER 1

INTRODUCTION

1. Background

Any complex system or process is subject to the occurrence of faults. These systems may be mechanical, chemical, electrical or some combination of these and may or may not be controlled by a computer. Faults include the complete or partial failure of actuators, sensors or other components of the system, or the occurrence of events external to the system (disturbances) that prevent its normal functioning.

The ability to detect the occurrence of any fault, and identify its cause is critical for a number of reasons. In some cases faults can lead to great loss of life and property (e.g., aircraft, nuclear power plants, etc.). Further, early detection of faults can allow timely corrective action which in many cases will greatly reduce the incidence of expensive, unexpected breakdowns (e.g., machinery in factories). Environmental considerations are also of importance as in the development of systems for monitoring automobile engine emissions.

One of the simplest methods of fault detection and isolation (FDI) is to utilize hardware redundancy, usually in conjunction with a voting scheme. For example, a computer may be used to monitor the outputs of a number of sensors all of which measure the same physical variable. The reading that occurs most often among the sensors is then taken as the correct one. More sophisticated schemes may assign different weights to different sensors based upon their reliability and then use some decision making scheme to estimate the correct measurement. The hardware redundancy approach is obviously
expensive and is used only in systems where the high cost is justified by the potential benefits in terms of cost or in the saving of human life.

With the increasing use of computers to control systems and processes the use of computer based approaches to FDI, that use software or analytical redundancy have become feasible. In these approaches data gathered from the system is analyzed by the computer in order to detect the presence of faults. The computer makes use of a priori knowledge about the system - usually embodied in a knowledge base or physical model - to make inferences about the presence or absence of faults from the data gathered. The focus of this work will be on model based approaches to FDI.

Model based FDI strategies use either a phenomenological model (which may be linear or nonlinear) or an empirical, input-output data based model (again linear or nonlinear) to predict the outputs of the system that is being monitored, from a knowledge of its inputs. The difference between the measured and predicted output is called a residual. Under ideal conditions, i.e., when there are no faults, no noise corrupting the measurements, no uncertainty in the model, etc., all the residuals will be zero. If a fault is present the residual is nonzero allowing the detection of the fault. When a fault is detected the residuals are further processed to isolate the location of the fault in the system.

The software algorithm that generates the residuals is called the residual generator. Though the FDI scheme has been described above as a two step process (detection and isolation), it is often combined into one step that is performed by a suitably designed residual generator. In the absence of faults the generator produces zero residuals, while in the presence of faults the generator is so constructed that the presence of specific faults give rise to specific signatures that allow the identification of the particular fault. The residual generator may operate on signals in either the time domain or in the frequency domain. Time domain residual generators fall into two main categories (i) parity equation based residual generators (PERG) and (ii) observer based residual generators (OBRG).
Frequency domain residual generators use either the Fourier transform or time-frequency distributions for signal processing.

2. Brief review of FDI schemes

Historically, the first residual generators to be developed were based on the theory of linear observers. Beard (1971) and Jones (1973) used observer eigenstructure assignment to construct a fault detection filter. Statistical testing of residual signals (innovations) of a Kalman Filter was introduced by Mehra and Peschon (1971). Clark et al. (1975) and Clark (1978) proposed the dedicated observer scheme which uses a bank of observers to detect instrument faults. Massoumnia (1986) formulated the FDI problem using a geometric approach. White and Speyer (1987) used a spectral approach for detection filter design. Park (1991) showed that the eigenstructure assignment approach to detection filter design can be treated as two independent observer design problems, one for the detection space and one for the completion space. Some frequency domain methods, for the design of residual generators have been proposed by Viswanadham and Minto (1988) and Ding and Frank (1990, 1991). The design of residual generators that operate in the frequency domain was also proposed by Olin and Rizzoni (1991) and Olin (1992). Viswanadham and Srichander (1987) and Frank and Wunnenberg (1989) used the unknown input observer scheme to design a residual generator that was robust to disturbance inputs. Min (1987) proposed a method for determining a robust reference model for OBRG designs. Frank (1990) proposed the design of post filters to make residual generators more robust to model uncertainty.

Early work on the parity equation method was performed by Potter and Suman (1977) and Mironovskii (1979). Chow and Willsky (1984), Gertler and Singer (1990) and Gertler (1990) have also proposed methods of parity equation residual generator design.

Viswanadham et al. (1987) and Gertler (1991) have demonstrated the equivalence of parity space and observer based FDI strategies.
Determining thresholds for the residuals used in any decision making scheme is an important area of research since the residuals are always nonzero, even in the absence of faults, due to model uncertainties and measurement noise. Some results have been obtained by Kosut and Walker (1984) Viswanadham et al. (1988) and Emami-Naeini et al. (1988). Clark (1989) proposed the use of adaptive thresholds. Frank (1990), Gertler (1991), Patton and Chen (1991) and Patton et al. (1989) have surveyed the different model based residual generator designs that have been proposed.

The various schemes listed above are all intended for use on linear systems and have only limited applicability to systems that have significant nonlinearities. Recently Hengy and Frank (1986), Wünnenberg (1990), Seliger and Frank (1991) and Himmelspach (1992) have all proposed residual generation schemes that employ the theory of nonlinear observers.

Another model based approach to FDI is the use of parameter identification schemes. Isermann (1989) provides a detailed survey of these methods.

3. Mathematical Preliminaries

This section presents the fundamental forms of the continuous and discrete time systems (both linear and nonlinear) considered in the rest of this work and also defines the related notation.

A linear continuous time system is represented in the state space form as follows,

\[ \dot{x} = Ax(t) + Bu(t) \]  \hfill (1)

\[ y(t) = Cx(t) + Du(t) \]  \hfill (2)

where \( x \in \mathbb{R}^{n \times 1} \) is the state vector, \( A \in \mathbb{R}^{n \times n} \) is the dynamics matrix, \( B \in \mathbb{R}^{n \times m} \) is the input distribution matrix, \( y \in \mathbb{R}^{p \times 1} \) is the vector of outputs, \( u \in \mathbb{R}^{m \times 1} \) is the vector of
inputs, $C \in \mathbb{R}^{p \times n}$ is the output distribution matrix and $D \in \mathbb{R}^{p \times m}$ is the direct term matrix. Similarly the discrete time representation is

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k) + Du(k)$$

where the system matrices remain as before.

Some of the linear FDI techniques use the Moore-Penrose right psuedo-inverse represented by the $(\cdot)^+$ and defined as follows.

$$(A^r)^+ = (A^T A)^{-1} A^T$$

The following representation of nonlinear systems is used in the following chapters.

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = h(x(t), u(t))$$

where, as before, $x \in \mathbb{R}^{n \times 1}$ is the state vector, $f \in \mathbb{R}^{n \times 1}$ is the dynamics vector, $u \in \mathbb{R}^{m \times 1}$ is the input vector, $y \in \mathbb{R}^{p \times 1}$ is the output vector, and $h \in \mathbb{R}^{p \times 1}$. Similarly in discrete time the following representation is used.

$$x(k+1) = f(x(k), u(k))$$

$$y(k) = h(x(k), u(k))$$

where the system matrices are as described for the continuous time case.

For a certain class of nonlinear systems (sometimes referred to as affine systems), the following representation is used.
\[ \dot{x}(t) = f(x(t)) + g(x)u(t) \]  
\[ y(t) = h(x(t)) \]

(10)  
(11)

Here \( g \in \mathbb{R}^{n \times m} \) while all the other variables are as defined earlier.

Following the convention of Gertler and Singer (1990) and Gertler (1991) in discussing the parity equation method for fault detection and isolation, the following input-output representation of linear discrete time systems is used.

\[ y(k) = S(z)u(k) \]

(12)

where \( y \in \mathbb{R}^{p \times 1} \) is the output vector, \( u \in \mathbb{R}^{m \times 1} \) is the input vector, and \( S(z) \in \mathbb{R}^{p \times m} \) is a transfer function matrix in terms of \( z \), the complex frequency or the shift operator \( z^{-1} \). There is some abuse of notation in the above representation, due to the combined use of the shift operator and the discrete sample time \( k \). However the notation is commonly used in the FDI literature and is well accepted. Hence the same notation will be adopted in this document also.

The derivative of a vector (such as \( f(x,u) \)), with respect to one of its arguments (say \( x \), which itself may be a vector) is denoted by \( \frac{df}{dx} \) and is defined as follows.

\[
\frac{df}{dx} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n}
\end{bmatrix}
\]

(13)
where \( f_i \) and \( x_i \) are respectively the \( i_{th} \) components of the vectors \( f \) and \( x \).

The Lie derivative is denoted by the symbol \( L_x h(x) \). Here \( x \in \mathbb{R}^{n \times 1} \), \( f \in \mathbb{R}^{n \times 1} \) is a vector function that acts on \( x \), and \( h(x) \) is a scalar function of \( x \). The Lie derivative is simply the directional derivative of \( h(x) \) in the direction of the vector field \( f \) and is computed as follows.

\[
L_x h(x) = \frac{dh}{dx} f(x)
\]  \hspace{1cm} (14)

Repeated application of the Lie derivative is denoted by the addition of a superscript \( \alpha \) in the following manner, \( L_{\alpha} h(x) \), and \( \alpha \) is the number of times the operation is performed. \( L_{\alpha} h(x) \) is defined as being just \( h(x) \).

4. Conclusion

This chapter presented an introduction to the problem of detecting and isolating faults in dynamic systems. A brief review of the literature available in the area of model based FDI was also provided. The next chapter discusses in detail the PERG, OBRG and the nonlinear residual generation schemes listed above.
CHAPTER II

REVIEW OF MODEL BASED RESIDUAL GENERATION SCHEMES

In this chapter the general principles of residual generation schemes are presented and the Parity Equation Residual Generation (PERG) and Observer Based Residual Generation (OBRG) schemes are reviewed in detail.

1. General Principles

The general form of a model based diagnostic scheme is displayed in figure 1. The inputs to the residual generator are the commanded inputs to, and the measured outputs of, the plant. The residual generator then produces as its output a vector of residuals which are then processed by a decision maker that draws inferences about the presence or absence of faults.

![Figure 1. Block diagram of a model based residual generation scheme](image)

The residual generator is usually designed such that the residual vector has a specific signature in response to a particular fault. Such residuals are called structured
residuals. One common form of structure imparted to the residuals is directionality. In this case the residual in the presence of a fault evolves in a fixed direction in the residual signal space (also called the parity space). Assuming that a residual generator generates three residual signals, $q_1(t)$, $q_2(t)$, and $q_3(t)$, in the presence of a fault, a directional residual is one in which the three residuals maintain fixed ratios with one another. In that case the residual can be plotted in the residual space as in Figure 2.

![Figure 2. Example of residual space and directional residual](image)

2. Parity Equation Residual Generation

In this section only discrete time models of systems will be discussed. Most methods presented here can be applied to continuous time systems with only minor modifications.

2.1. Generation of Primary residuals

Consider linear system represented in state space form as follows.

\[ x(k+1) = Ax(k) + Bu(k) \]  
\[ y(k) = Cx(k) + Du(k) \]
where \( x \in \mathbb{R}^{n \times 1} \) is the state vector, \( A \in \mathbb{R}^{n \times n} \) is the dynamics matrix, \( B \in \mathbb{R}^{n \times m} \) is the input distribution matrix, \( y \in \mathbb{R}^{p \times 1} \) is the vector of outputs, \( u \in \mathbb{R}^{m \times 1} \) is the vector of inputs, \( C \in \mathbb{R}^{p \times n} \) is the output distribution matrix and \( D \in \mathbb{R}^{p \times m} \) is the direct term matrix.

Then faults in the system can be modelled as follows.

\[
\begin{align*}
x(k+1) &= Ax(k) + Bu(k) + Fp(k) \\
y(k) &= Cx(k) + Du(k) + Ay(k)
\end{align*}
\]

where \( p(k) \) and \( Ay(k) \) are fault vectors and \( F \) is a constant matrix. The vector \( p \) represents actuator faults and disturbance inputs and may be of any dimension. The vector \( Ay \) represents sensor faults and is of the same dimension as the output vector i.e., \( m \).

The above system may be transformed into the input-output form by the use of the shift operator \( z \). Then,

\[
y(k) = S_u(z)u(k) + S_p(z)p(k) + Ay(k)
\]

where \( S_u(z) = C(zI - A)^{-1}B + D \) and \( S_p(z) = C(zI - A)^{-1}F \).

Then a vector of residuals can simply be defined as the difference between measured and estimated outputs (Gertler and Singer, 1990 and Gertler et al., 1991).

\[
\begin{align*}
\hat{r}_1(k) &= y(k) - \hat{y}(k) \\
&= S_u(z)u(k) + S_p(z)p(k) + Ay(k) - S_u(z)\hat{u}(k) \\
&= S_u(z)\Delta u(k) + S_p(z)p(k) + Ay(k)
\end{align*}
\]

where the \( \hat{\cdot} \) denotes estimates and \( \Delta u(k) = u(k) - \hat{u}(k) \).
Here \( r_j(k) \) is called the primary residual and must be further manipulated to produce structured residuals.

Residuals can also be generated directly from the state-space equations (Chow and Willsky, 1984).

Consider the equations (15) and (16). Then \( y(k+s) \) for any \( s > 0 \) is

\[
y(k+s) = CA^s x(k) + CA^{s-1} Bu(k+s) + \ldots + Du(k+s)
\]

Collecting the equations for \( s = 0, \ldots, n' \) and shifting by \( n' \) yields the following matrix equation,

\[
Y(k) = Rx(k-n') + QU(k)
\]

where \( Y(k) = \begin{bmatrix} y(k-n') \\ y(k-n'+1) \\ \vdots \\ y(k) \end{bmatrix} \), \( U(k) = \begin{bmatrix} u(k-n') \\ u(k-n'+1) \\ \vdots \\ u(k) \end{bmatrix} \), \( R = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix} \), and

\[
Q = \begin{bmatrix}
D & 0 & \ldots & 0 \\
CB & D & \ldots & 0 \\
CAB & CB & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots \\
CA^{n'-1} B & CA^{n'-2} B & \ldots & CB & D
\end{bmatrix}
\]

For a system with \( p \) inputs and \( m \) outputs the vector \( Y \) is of dimension \((n'+1) \times m\), \( U \) is \((n'+1) \times p\), \( R \) is \([(n'+1) \times m] \times n\) and \( Q \) is \([(n'+1) \times m] \times [(n'+1) \times p]\). In order to use equation (24) for residual generation the state variables in \( x(k-n') \) must be eliminated. This can be done by multiplying the equation by a matrix \( H \) such that \( HR = 0 \).

\[
HY(k) = HRx(k-n') + HQU(k)
\]

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Now a residual can be defined as before.

\[ r_1(k) = H\dot{Y}(k) - H\hat{Y}(k) = HQ\Delta U(k) + HRp(k) + \Delta Y(k) \]  

(26)

where ^ and \( \Delta \) have the same meaning as in equations (20 - 22) and

\[
P(k) = \begin{bmatrix}
p(k-n') \\
p(k-n'+1) \\
\vdots \\
p(k)
\end{bmatrix}
\text{ and } R = \begin{bmatrix}
D & 0 & \ldots & 0 \\
CF & D & 0 & \ldots & 0 \\
CAF & CF & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & 0 \\
CA^{n-1}F & CA^{n-2}F & \ldots & CF & D
\end{bmatrix}
\]

It should be noted that the Chow-Willsky (or time domain) and the transfer function formulations of the residual generation process are equivalent and have the same applicability to FDI problems.

2.2. Directional Residuals

Assigning the primary residual to some desired direction consists of multiplying the primary residual by a suitable matrix \( W \), in the following manner.

\[ r_2(k) = Wr_1(k) \]

(27)

The matrix \( W \) is computed by solving the following equation.

\[ W[Sd(z) Sp(z)] = d \]

(28)

\[ W[HQ \Delta Y(k)] = d \]

(29)

Equation (28) applies in the case of the input-output system representation and \( W \) is a matrix of transfer functions and equation (29) in the case of the state-space representation and \( W \) is a matrix of constants. Here, \( d \) is a matrix whose columns represent the desired directions for the faults that can occur in the system. The choice of \( d \) is made to such that the residual directions in response to different faults are separated from each other by as
large an angle as possible in the residual space. Note that all the faults can be assigned to linearly independent directions only if the number of faults is less than or equal to the number of measured outputs.

3. Observer Based Residual Generation for FDI

3.1. Introduction

Fault Identification and Isolation (FDI) methods, for linear systems, can be separated into two main classes: (i) Parity equation residual generation (PERG) methods, and (ii) Observer based residual generation (OBRG) methods. Two OBRG schemes that have been proposed recently are the Unknown Input Observer (UIO) scheme and the Detection filter scheme. This section will present the essential features of these two methods and integrate them under one general framework.

3.2. Linear Observer Based FDI Methods: The Unknown Input Observer

The problem of designing observers for linear systems with unknown inputs is one that has received much attention over the past several years. Recently it has been shown by some researchers (Viswanadham and Srichander (1987), P.M.Frank (1990)) that the unknown input observer (UIO) may be applied to the problem of diagnosing faults in linear systems.

The UIO design as proposed by Viswanadham and Srichander (1987) and, Hou and Müller (1992), consists of transforming the system equations, such that the state vector can be divided into two parts - a part that can be directly obtained from the measurements, and another part consisting of the states that have to be estimated. A reduced order observer can be designed to estimate these states, and the observer gains are so selected that they decouple the observer dynamics from the unknown input. The conditions under which such a design is possible and the design procedure are given below.

Consider the dynamic system represented by the following state space equations.
\[
\begin{align*}
\dot{x} &= Ax(t) + Bu(t) + Ed(t) + Kf(t) \\
y(t) &= Cx(t) + Gf(t)
\end{align*}
\]

Here, \( x \in \mathbb{R}^n \) is the state vector; \( u \in \mathbb{R}^p \) is a vector of known inputs; \( d(t) \in \mathbb{R}^r \) is a vector of unmeasured disturbance inputs and \( E \) is the corresponding input entry vector; \( f(t) \in \mathbb{R}^s \) is a vector of unknown fault inputs and \( K \) and \( G \) are the corresponding fault entry vectors; and \( y \in \mathbb{R}^m \) is a vector of measurements. The following observer design results in an observer that decouples the state estimates from the effect of the disturbance while maintaining sensitivity to the fault inputs.

Let us now assume that \( f(t)=0 \), i.e., there is no fault present. If \( C \) has full row rank the system (30 - 31) can then be transformed into the following form,

\[
\begin{bmatrix}
\dot{y}(t) \\
\dot{w}(t)
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
y(t) \\
w(t)
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} u(t) +
\begin{bmatrix}
E_1 \\
E_2
\end{bmatrix} d(t)
\]

Now an \((n-p)\)th order observer can be constructed to estimate \( w(t) \). This observer is given by,

\[
\dot{z}(t) = (A_{22} - LA_{12})z(t) + (A_{21} - LA_{11} + A_{22} - LA_{12} L)y(t) + (b_2 - Lb_1)u(t) + (E_2 - LE_1) d(t)
\]

where,

\[
\hat{w}(t) = z(t) + Ly(t)
\]

Since \( d(t) \) is not available for measurement the observer can be implemented, if

(i) \( E_2 - LE_1 = 0 \), and

(ii) \((A_{22} - LA_{12}) \) has stable eigenvalues.

The existence conditions for \( L \) such that the above conditions are satisfied are,
(i) \((A,C)\) is observable and

(ii) \(\text{rank}(CE) = \text{rank}(E) = r\) and \(m \geq r\). (for proof see Kudva, et al., 1980).

The eigen spectrum of \((A_{22} - LA_{12})\) must contain the invariant zeros of the triple \((A,C,E)\) (see Kudva, 1980). Then it can be shown that the error dynamic equation in the absence of any faults is,

\[
\dot{e}(t) = (A_{22} - LA_{12})e(t)
\]  \hspace{1cm} (35)

In the presence of a failure the error dynamics become

\[
\dot{e}(t) = (A_{22} - LA_{12})e(t) + [-L I]Kf(t)
\]  \hspace{1cm} (36)

where \(I\) is an identity matrix of appropriate dimension, and the error is non zero provided, \([-L I]K \neq 0\). Thus the error residual is sensitive to the fault but not to the unknown input.

The unknown input method of residual generation can also be implemented using the generalized observer scheme as described by Frank et al., in the book by Patton et al., (1989).

\[
\dot{\hat{z}}(t) = R\hat{z}(t) + Sy(t) + Ju(t)
\]  \hspace{1cm} (37)

\[
r(t) = Lf\hat{z}(t) + L_2y(t)
\]  \hspace{1cm} (38)

\[
\hat{z}(t) = T^{-1}\hat{z}(t)
\]  \hspace{1cm} (39)

Then to obtain a residual \(r(t)\) that is decoupled from the unknown input the following set of equations have to be solved.

\[
TA - RT = SC
\]  \hspace{1cm} (40)

\[
J = TB
\]  \hspace{1cm} (41)

\[
TE = 0
\]  \hspace{1cm} (42)

\[
SG = 0
\]  \hspace{1cm} (43)
These equations then lead to the following set of equations which can then be solved to give the required T, \(L_1\) and \(L_2\) matrices.

\[
L_1T + L_2C = 0 \tag{45}
\]

\[
L_2F = 0 \tag{46}
\]

\[
L_2G = 0 \tag{47}
\]

Hou and Müller (1991) extended the UIO to systems where the unknown input enters the measurement equation, as follows.

\[
y(t) = Cx(t) + Fd(t) \tag{48}
\]

where \(F\) is such that \(\text{rank} \begin{bmatrix} E \\ F \end{bmatrix} = q\).

They showed that in this case an UIO can be constructed iff:

(i) \(\text{rank} \begin{bmatrix} E & CE \\ 0 & F \end{bmatrix} - \text{rank}(F) = q\), and

(ii) \(\text{rank} \begin{bmatrix} sI_n - A & E \\ C & F \end{bmatrix} = n + q, \ \forall s \in \mathbb{C}, \text{Re}(s) \geq 0\).

Note that these conditions reduce to the ones stated earlier in the case where \(F = 0\).

Given these conditions the system can be transformed into the following form.

\[
\xi_1 = A_1\xi_1 + A_2\xi_2 + B_1u + G_1y \tag{49}
\]

\[
\xi_2 = A_3\xi_1 + A_4\xi_2 + B_2u + G_2y + \ddot{d}_1 \tag{50}
\]

\[
z_1 = C_1\xi_1 + C_2\xi_2 \tag{51}
\]

\[
z_2 = C_3\xi_1 + C_4\xi_2 + \ddot{d}_2 \tag{52}
\]
\[ \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = P_x \xi_1 \in R^n, \xi_2 \in R^n, n_1 + n_2 = n, \]
\[ \tilde{d} = \begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{bmatrix}, \tilde{d}_1 \in R^{n_1}, \tilde{d}_2 \in R^{n_2}, q_1 + q_2 = q \]

where

\[ z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = Qy = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} y \]

(Note: \( n_2 = q_1 \)).

\[ Q, P \text{ and } S \text{ are all nonsingular matrices, and are determined by the transformations required to put the system in the form of equations (49 - 52).} \]

It can be shown that \( C_2 \) must be of full column rank under conditions (i) and (ii) stated earlier. Then a nonsingular matrix \( T = \begin{bmatrix} C_2^+ \\ N \end{bmatrix} \) can be chosen, where the superscript + denotes the pseudoinverse defined as \( C_2^+ = (C_2^T C_2)^{-1} C_2^T \). Then after some algebraic manipulations we can obtain a further transformation into the following form.

\[ \xi_1 = \tilde{A}_i \xi_1 + B_i u + \tilde{G}_i y \quad \text{(53)} \]
\[ \eta = \tilde{C}_i \xi_1 \quad \text{(54)} \]

where \( \eta = N(I_{r_2} - C_2 C_2^+ Q_2) y, \quad \tilde{A}_i = A_i - A_2 C_2^+ C_1, \quad \tilde{G}_i = G_i + A_2 C_2^+ Q_1 \). If the pair \( \{ \tilde{A}_i, \tilde{C}_i \} \) is detectable an observer can be designed for this system and then since \( C_2 \) has full column rank equation (51) can be solved for \( \xi_2 \), and \( \hat{x} = P^{-1} \xi \). This state estimation is now independent of unknown inputs.

The UIO may then be used to isolate actuator faults by constructing a bank of observers, each of which treats some subset of the faulty inputs as unknown. If a fault is present, only those observers that do not treat this fault as an unknown input, produce nonzero residuals. A proper choice of the number of observers, and of the sets of faults that each one decouples from the residual, results in a unique signature corresponding to each fault.
3.3. Linear Observer Based FDI Methods: Eigenstructure Assignment (ESA)

Another method of fault detection is to design residual generators that produce what are called structured residuals, i.e., residuals that exhibits certain special properties in the presence of particular faults. The most popular form of the structured residual is a residual that maintains a fixed direction in the parity space under the influence of a specific fault. Construction of observers using eigenstructure assignment lends itself naturally to the production of such unidirectional (under the influence of a fault) residuals.

Patton(1988) and Patton and Chen (1991), demonstrate the use of eigenstructure assignment in FDI. Both left and right eigenvector assignment may be used to produce robust (to disturbances), structured residuals.

Consider the following observer constructed for the system in equation (30 - 31) with \( f(t) = 0 \).

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \quad (55)
\]

\[
\hat{y} = C\hat{x}(t) \quad (56)
\]

Define a residual vector \( r(t) \) as follows

\[
r(t) = W(y(t) - \hat{y}(t)) \quad (57)
\]

Then the response of the residual vector becomes,

\[
r(s) = WC(sI-(A-LC))^{-1}Ed(s) = G(s)d(s) \quad (58)
\]

It can then be shown (Patton and Chen, 1991) that if the rows of \( WC \) are left eigenvectors of \( (A - LC) \) or the columns of \( E \) are right eigenvectors of \( (A - LC) \), the residual \( r(t) \) maintains a fixed direction in parity space and this direction can be assigned by setting \( WCE = D \), where \( D \) is the required direction. If it is desired to decouple the disturbance, \( D \) is set to zero.
The assignability condition for left eigenvectors is that for each $\lambda_i$ the corresponding eigenvector of $(A - LC)$ must belong to the row space spanned by the rows of the matrix $C(\lambda_iI - (A - LC))^{-1}$.

For right eigenvector assignment define

$$F_\lambda = [(\lambda_1I - (A - LC))e_1...(\lambda_iI - (A - LC))e_i...(\lambda_qI - (A - LC))e_q]$$

Then the necessary and sufficient condition for assignability is

$$\text{rank}([(\lambda_1I - (A - LC))e_1...(\lambda_iI - (A - LC))e_i...(\lambda_qI - (A - LC))e_q]) = \text{rank}(CE)$$

Where $e_i$ is the $i_{th}$ column of the E matrix and the L matrix is given by,

$$L = F_\lambda(CE)^+ + K_1[Im - CE(CE)^+]$$

Where $(.)^+$ stands for the pseudoinverse defined earlier, and $K_1$ is an arbitrary matrix.

3.4. Linear Observer Based Methods: The Detection Filter

The detection filter method uses right eigenvector assignment to construct residual generators.

**Beard's Approach**

The detection filter approach to FDI was first proposed by Beard (1971).

**Definition 1:** Detectability

The fault is detectable if there exists a matrix $D$ such that,

(i) $Ce(t)$, where $e(t)$ is the error between the actual and estimated states, maintains a fixed direction in space, in the presence of the fault, and

(ii) all eigenvalues of $(A-DC)$ can be arbitrarily specified, up to conjugate symmetry.
The residual generator design is based on finding a generator vector for the fault entry vector \( E \) in equation (30)).

**Definition 2: Detection Generator**

If the conditions

(i) \( \text{rank} \left[ E (A-DC)E \ldots (A-DC)^{n-1}E \right] = k \), and

(ii) \( \text{rank} \left[ C (A-DC)E \ldots (A-DC)^{n-1}E \right] = 1 \), are satisfied, then there exists an \( n \)-vector \( g \), called the detection generator, which lies in the controllable space of \( E \), (with respect to \( [A-DC] \)) such that

\[
\begin{bmatrix}
  C \\
  CA \\
  \vdots \\
  CA^{k-2}
\end{bmatrix} g = 0 \quad (62)
\]

Then \( E \) can be expressed as,

\[
E = a_0g + a_2Ag + \ldots + a_kA^{k-1}g \quad (64)
\]

A detection generator for \( E \) can be used to generate a detection gain for \( E \), and there always exists a detection generator associated with a detection gain (for proof see Beard (1971)).

Once the detection generator has been found, the \( k \) eigenvalues of \( (A-DC) \) can be arbitrarily assigned, where \( k \) is the order of the detection generator. The matrix \( D \) is given by the solution of,

\[
DCE = p_1g + \ldots + p_kA^{k-1}g + A^kg \quad (65)
\]

The \( k \) eigenvalues of \( (A-DC) \) are then given by roots of
\[ s^k + ps^k - p_2s + p_1 = 0 \] (66)

where the \( p_i \) are scalars.

The general solution of \( D \) is,

\[ D = [p_1g + \ldots + p_kA^{k} g + A^k g]((CE)^TCE)^{-1}(CE)^T + D'(I - CE((CE)^TCE)^{-1}(CE)^T) \] (67)

\( D' \) is an arbitrary matrix, and represents the freedom left in the choice of \( D \) after having satisfied equation (65). This freedom, lets us specify arbitrarily \( q \) eigenvalues of \((A-DC')\) in addition to the \( k \) values already specified.

\[ q = \text{rank} \ M = \text{rank} \begin{bmatrix} C' \\ C'A' \\ \vdots \\ C'A^{n-1} \end{bmatrix} \] (68)

where,

\[ C' = (I - CE((CE)^TCE)^{-1}(CE)^T)C, \text{ and} \]

\[ A' = A - [p_1g + \ldots + p_kA^{k} g + A^k g]((CE)^TCE)^{-1}(CE)^T. \]

**Definition 3: Detection Space and Detection Order**

The null space of \( M \) is the detection space of \( E \), and the dimension of the detection space of \( M \) is the detection order of \( E \).

Condition 2 of detectability will be satisfied if and only if \( k+q=n \). This implies that a detection generator of order \((n-q)\) must be found to satisfy the detectability condition. Such a generator is called the maximal generator. If \((A,C)\) is observable then every \( n \) vector \( E \) has a unique maximal detection generator (for proof see Beard (1971)). The biggest drawback of this approach is that the design procedure is based on the generator vector, which is still unknown.
The Spectral Approach

The spectral approach proposed by White and Speyer (1987) considers the eigenvalue and eigenvector problem directly, without requiring the selection of a generator vector.

Assumptions:

(i) \((A,C)\) is observable.

(ii) \(CE \triangleq [Ce_1,\ldots,Ce_r]\) is rank \(r\).

(iii) \(r = m\)

(iv) The closed loop eigenvalues of \(A-DC\) are distinct.

Let \(\lambda_j,v_j\) be the \(j\)th eigenvalue eigenvector pair, \(j = 1,\ldots,n\). The eigenvectors span the error variable space and \(Cv_j \neq 0\) since the system is observable. Then the \(i\)th fault entry vector

\[ e_i = \sum_{j=1}^{n_i} \alpha_j v_j \]

(69)

where \(n_i \leq n\) is the number of nonzero of \(\alpha_j\) and \(v_j\) are those eigenvectors that combine to form \(e_i\).

The error residual in the presence of \(e_i\) can be constrained to a single direction in the error residual space if and only if \(Ce_i\) and \(Cv_j\) are collinear for all \(j = 1,\ldots,n_i\) (for proof see White and Speyer (1987)).

The detection filter problem then becomes one of solving

\[
\begin{bmatrix}
\lambda_j l - A & D \\
C & 0 \\
D & 0 \\
\end{bmatrix}
\begin{bmatrix}
v_j \\
0 \\
w_i \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
w_i \\
\end{bmatrix}
\]

(70)

where \(w \triangleq Ce_i = Cv_j, j = 1,\ldots,n_i, i = 1,\ldots,m\).
Those of equations (9) that correspond to nonzero $\alpha^j_i$ can be summed to obtain
\[ D C e_i = A e_i - \sum_{j=1}^n \alpha^j_i \lambda^j_i v^j_i \]  
(71)

Then,
\[ D = (A e_i - \sum_{j=1}^n \alpha^j_i \lambda^j_i v^j_i)(C e_i)^* + D_i(I - ((C e_i)(C e_i)^*)C \]  
(72)

where $D_i$ is an arbitrary matrix. Define,
\[ A_i = (A e_i - A_i - D_i C_i)(A^j_i)^+ \]  
(73)
\[ C_i = (I - (C e_i)(C e_i)^+)C \]  
(74)

The number of eigenvalues of $A_i - D_i C_i$ that can be arbitrarily specified by the free choice of $D_i$ is equal to $q_i = \text{rank}(M_i)$, where
\[ M_i \triangleq [(C e_i)^T, (C A e_i)^T, ..., (C A^n e_i)^T]^T \]  
(75)

The remaining $v_i \triangleq (n - q_i)$ eigenvalues of $(A_i - D_i C_i)$ are the eigenvalues associated with $e_i$.

A maximum of $k$ fault vectors $e_i, i = 1, \ldots, k$ can be assigned such that $k$ satisfies,
\[ \sum_{i=1}^k v_i = n \]  
(76)

Note that this $M_i$ and the $M$ defined in Beard's approach are identical and in both approaches the detection space and detection order of a particular fault entry vector is the same.

The main limitation of this method is that the eigenvalues and the eigenvectors of both the detection and the completion space must be specified simultaneously. Park (1991) showed that the eigenstructure assignment problem can be treated as two
independent observer design problems, one for the detection space and one for the completion space.

The Park Detection Filter

The definition of the detection space, detection order and the completion space used in the earlier approaches is also adopted by Park. However, in both Beard’s method $g$ (the generator vector) and in White and Speyer’s method the $v^j$’s, used in finding a solution for $D_i$, are unknowns. Park on the other hand uses the fault vector $e_j$ directly to find $D_i$ according to the following equation.

$$D_iCe_i = Ae_i - \lambda e_i$$  \hspace{1cm} (77)

which implies,

$$D_i = (Ae_i - \lambda e_i)(Ce_i)^+$$ \hspace{1cm} (78)

The following corollaries then follow from the definitions of the detection and completion spaces.

Corollary 1:

Any eigenvector $\omega$ of $A_i$, which is such that $C\omega$ is collinear with $Ce_i$, is in the detection space (for proof see Park, 1991).

Corollary 2:

If $v_1, ..., v_{U_i}$ are the eigenvectors associated with the unobservable eigenvalues of $(A_i, C_i)$, then $Cv_j$ is colinear with $Ce_j$ (for proof see Park, 1991).

Theorem 1:

For a given observable pair $(A, C)$, each $n \times 1$ vector $e_j$ has a unique detection space and order regardless of the choice of $\lambda_i$ (for proof see Park, 1991).
The detection filter can then be constructed by letting $D = D_i + D_c$ be the observer gain matrix, where $D_i$ is the $n \times m$ matrix defined earlier and $D_c$ is an arbitrary matrix.

\[
\dot{\hat{x}} = A\hat{x} - (D_i + D_c)(y - \hat{y})
\]  \hspace{1cm} (79)

The error equations in the presence of a fault then become,

\[
\dot{e} = (A - D_i C - D_c C) e + e_p n(t)
\]  \hspace{1cm} (80)

\[
q = C e
\]  \hspace{1cm} (81)

Without any loss of generality and retaining all the freedom in $D_c$, $D_c$ can be rewritten as,

\[
D_c = V D_j D_s (I - (C e_i)(C e_i)') \hspace{1cm} (82)
\]

Where $V$ is the matrix whose columns are the eigenvectors of $A_i$. Then $D_i \in \Re^{ui \times m}$ assigns eigenvalues in the detection space, $D_4 \in \Re^{(n-ui) \times m}$ assigns the eigenvalues in the completion space and $D_5 \in \Re^{ui \times m}$ affects eigenvectors in the completion space only.

Equation (80) can be transformed, into two decoupled sets of equations, one describing the detection space and one the completion space.

Detection space:

\[
\dot{z}_1 = (A_1 - D_j C_1)z_1 + [1 \ 0 \ldots \ 0]n(t)
\]  \hspace{1cm} (83)

\[
q_1 = C_1 z_1
\]  \hspace{1cm} (84)

Completion space:

\[
\dot{z}_2 = (A_2 - D_4(I - (C e_2)(C e_2)')C_2)z_2
\]  \hspace{1cm} (85)
\[ q_2 = C_{2z_2} \]  

(86)

where \( A_I \) and \( A_2 \) are diagonal matrices containing the eigenvalues of the detection and completion spaces respectively, \( C_I = [Cv_I, ..., Cv_{ui}] \), \( C_2 = [Cv_I, ..., Cv_{ui}] \) and \( V_U \) are the eigenvectors in the detection space and \( [v_{ui+1}, ..., v_n] \) are the eigenvectors in the completion space. Since the eigenvectors in the detection space satisfy the co-linearity property stated in corollaries 2.1 and 2.2, \( C_I \) is of rank one, revealing that the eigenstructure assignment problem is equivalent to an observer design using single output feedback. These results can then be summarised in the following theorem.

**Theorem 2:**

Any fault detection filter, for a fault event vector \( e_i \), is represented by

\[
\dot{x} = A \dot{x} + (D_i + V_U D_j)(y - \hat{y}) + D_k (I - (Ce_i)(Ce_i)^*)(y - \hat{y})
\]

(87)

\[ \hat{y} = C \hat{x} \]

(88)

Then the observer gains \( D_{dei} \) required to construct the eigenstructure of the detection space for any fault \( e_i \) is,

\[
D_{dei} = D_i + V_U D_j
\]

(89)

\[
D_j = Q \begin{bmatrix} a_{vi} & a_{vi-1} & ... & a_I & 1 \\ a_{vi-2} & a_{vi-3} & ... & 1 & 0 \\ a_I & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 1 & 0 & ... & 0 & 0 \end{bmatrix} (Ce_i)^T
\]

(90)

\[
Q^{-1} = \begin{bmatrix} a_{vi} & a_{vi-1} & ... & a_I & 1 \\ a_{vi-2} & a_{vi-3} & ... & 1 & 0 \\ a_I & 1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... \\ 1 & 0 & ... & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} c_I \\ c_I A_I \\ ... \\ c_I A_I^{ui-2} \\ A_I^{ui-1} \end{bmatrix}
\]

(91)

where \( c_I = (Ce_i)^+ CV_U \) and the \( a_i \)'s and \( a'_i \)'s satisfy
Here the \( \lambda_i \)'s are the unobservable eigenvalues of \( (A_i, C_i) \) and \( \lambda_i' \)'s are the new, desired eigenvalues of the detection space (for proof see Park, 1991). The completion space can then be constructed independently of the detection space.

3.5. Similarity of the UIO and ESA methods

The UIO and the detection filter perform virtually the same function - that of decoupling a portion of the state vector (in either the original or in some transformed coordinate system). Therefore it is only logical to think that the two methods may have some underlying similarity. Indeed, in the book by Patton et al (1989), the authors state that ESA is a special case of the UIO method. Here, an explanation is given of how the two methods relate to each other.

Recall, that in the description of the UIO scheme it was stated that the design process consists of dividing the states into two portions one of which is affected by the unknown input and another which is not. In eigenstructure assignment, when the input entry vector of the unknown input is assigned as an eigenvector of the observer, only that portion of the state which lies along this direction is affected by the input due to the well known property that the eigenvectors are orthogonal to one another.

Then in both methods only the portion of the state estimate unaffected by the unknown input - in the case of the UIO the estimated states from the reduced order observer, and in the case of eigenstructure assignment, that portion of the state that lies in the space orthogonal to the input entry vector - is used to generate residuals.

Keeping the above explanation in mind an UIO can be designed using eigenstructure assignment as follows.

(i) Assign the input entry vector \( E \) as one of the eigenvectors of the observer dynamic matrix. The eigenvalue may be picked arbitrarily.
(ii) Construct a matrix $T$ whose rows are made up of the normalized eigenvectors of the observer dynamic matrix. Then assuming the last row of $T$ contains the elements of corresponding to the eigenvector $E$, $T$ satisfies

$$TE = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \alpha \end{bmatrix}$$

where $\alpha$ is some constant.

Partition $T$ into two matrices $T_1$ and $T_2$ as follows,

$$T = T_1 + T_2$$

where $T_1 = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_{n-1} \\ 0 \end{bmatrix}$ and $T_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ \alpha \\ \alpha \end{bmatrix}$, and $t_i$ is the $i$th row of $T$.

(iii) Now the state $x$ and its estimate $\hat{x}$ may be represented as a linear combination of the eigenvectors in $T$.

$$x = \alpha_1 t_1 + \alpha_2 t_2 + \ldots + \alpha_n t_n$$

$$\hat{x} = \alpha_1 t_1 + \alpha_2 t_2 + \ldots + \alpha_n t_n$$

where the $\alpha_i$'s are constants and the $t_i$'s are the eigenvectors. Note that only the portion colinear with the eigenvector $t_n$ is affected by the unknown input and is thus at variance with the actual value of the state. Now in order to estimate the entire state correctly all we have to do is obtain the correct value of $\alpha_n$ which may be done as follows.

$$(C t_n T_1)^\top (y - \hat{y}) + t_n \hat{x} = (C t_n T_1)^\top C (x - \hat{x}) + t_n \hat{x}$$
Here the + sign represents the pseudoinverse. The above computation can easily be extended to the case where more than one input is unknown. The only constraint is that rank(Ct) = rank(Æ).

Thus we see that by means of eigenstructure assignment it is possible to estimate the entire state vector of a system even in the presence of unknown inputs, i.e., an unknown input observer can be constructed.

4. FDI Using Nonlinear Observers

The methods described in the previous section work well for linear systems. They can in some cases be extended to nonlinear systems by treating the nonlinearities either as uncertainties in the system model or as disturbances and then designing residual generators robust to these variations. However such methods fail in the presence of significant nonlinearities in the system dynamics. Hence some authors (Hengy and Frank (1986), Wünnenberg (1990), Seliger and Frank (1991) and Himmelspach (1992)) have proposed residual generation schemes that use the theory of nonlinear observers.
4.1. Nonlinear Unknown input Observers

Consider the nonlinear dynamic system represented by the following state space equations.

\[ \dot{x} = f(x) + g(x)u + p(x)d + q(x)f \]  
\[ y(x) = h(x) \]  

If certain existence conditions are fulfilled these equations can be transformed by a nonlinear transformation, \( z = T(x) \), into the following form (Krener and Respondek (1985), Zeitz (1987), Xia and Gao (1989)).

\[ \dot{z} = Az + J(y,u) + Ed + Kf \]  
\[ y = Cz \]

The nonlinear portion of this equation can then be cancelled in the observer (since the nonlinearities involve only measured outputs and known inputs), making the observer error dynamics linear. An unknown input observer can then be designed for this system following the procedure for linear systems.

However the conditions required for transforming the system equations into the form of equations (104), (105) cannot always be fulfilled even in the case when \( d \) and \( f \) are zero. If a suitable transformation exists it may be very difficult to compute, because nonlinear partial differential equations must be solved. Seliger and Frank (1991) propose an alternative technique that does not go through the two above mentioned steps of first transforming the state equation and then constructing an UIO. Instead, they use a single transformation that directly decouples the disturbance or unknown inputs. Such a transformation \( z = T(x) \) must satisfy the following equations.

\[ \frac{\partial T(x)}{\partial x} f(x) = FT(x) + \Phi \theta(h(x)) \]  

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\[ \frac{\partial T(x)}{\partial x} g(x) = \Phi_1(h(x)) \]  
(107)

\[ \frac{\partial T(x)}{\partial x} p(x) = 0 \]  
(108)

\[ \text{rank} \left( \frac{\partial T(x)}{\partial x} q(x) \right) = \text{rank}(q(x)) \]  
(109)

\[ R(T(x), h(x)) = 0 \]  
(110)

If a stable matrix \( F \) and output transformations \( \Phi_0 \) and \( \Phi_1 \) can be found such that this set of linear partial differential equations can be solved simultaneously for \( T(x) \), a robust fault detection observer can be designed for the system as follows.

\[ \dot{\hat{z}} = F \hat{z} + \Phi_0(h(x)) + \Phi_1(h(x))u \]  
(111)

\[ r = R(\hat{z}, y) \]  
(112)

Then the residual error evolves according to

\[ \dot{e} = Fe - \frac{\partial T(x)}{\partial x} q(x)f \]  
(113)

\[ r = R(T(x) + e, h(x)) \]  
(114)

It can be seen that the residual is unaffected by the unknown inputs \( d \) and is affected by the fault \( f \). The authors present a systematic method for solving the set of simultaneous partial differential equations (106 - 110). No proof for the existence of solutions is given; however the authors state that the more independent measurements available (i.e., the higher the rank of \( h(x) \)), the better the chance of obtaining a solution and that, if a solution exists it can be found using their procedure.
4.2. Component Failure Detection and Isolation

Hengy and Frank (1986) and Frank (1987) present the following scheme for detecting and isolating faults in a specific component of a complex system. Suppose that the i-th component can be represented in state space form as follows:

\[
\dot{x}_i = f_i(x_i, \ddot{x}_i, \ddot{y}_i, u_i) \tag{115}
\]
\[
y_i = c_i(x_i, u_i) \tag{116}
\]
\[
\ddot{y}_i = \ddot{c}_i(x_i, u_i) \tag{117}
\]

where \(x_i\) is a \(n_i \times 1\) state vector, \(\ddot{x}_i\) is a \(n_i \times 1\) vector of those states of the overall systems that are coupled to the states of the component through the dynamic equation (115), \(u_i\) is a \(p_i \times 1\) input vector, \(y_i\) is a \(m_i \times 1\) vector of outputs from the component and \(\ddot{y}_i\) is a \(m_i \times 1\) vector of outputs from the external system. It is assumed that the outputs of the external system are decoupled from those of the component. If all the coupled states are available through measurement i.e., \(y_i = \ddot{x}_i\), the following observer may be constructed.

\[
\dot{\hat{x}}_i = f_i(\hat{x}_i, \ddot{x}_i, \ddot{y}_i, u_i) + K_i(\hat{x}_i, \ddot{x}_i, u_i)(y_i - \ddot{y}_i) \tag{118}
\]
\[
\dot{\ddot{y}}_i = c_i(\hat{x}_i, u_i) \tag{119}
\]

A first order linearization of the above equations about the state estimate \(\hat{x}_i\) leads to the following observer error dynamics.

\[
\dot{e}_i = F_i e_i = \left( \frac{\partial f_i}{\partial x_i} - K_i \frac{\partial c_i}{\partial x_i} \right) \left| \hat{x}_i, \ddot{x}_i, e_i \right. \tag{120}
\]

The stability of the observer may be assured by proper choice of the matrix \(K_i\). Note that construction of this observer requires good knowledge of the initial conditions since convergence is assured only for small values of the state error due to the fact that the
influence of higher order terms has been neglected. A fault in the $i$th component influences the estimation error as follows,

$$
\dot{e}_i = F_i e_i + z_i(t) \quad (121)
$$

Consequently failures in the $i$th component affect the estimation error for the $i$th component only, and can be isolated.

If the coupled states are not available through measurement the observer is given by,

$$
\dot{x}_i = f_i(x_i, \dot{x}_i, u_i) + K_i(x_i, \dot{x}_i, u_i)(y_i - \hat{y}_i) + K_i(x_i, \dot{x}_i, u_i)(y_i - \hat{y}_i) \quad (122)
$$

$$
\dot{e}_i = F_i e_i + F_i \bar{e}_i + z_i(t) \quad (123)
$$

where $\bar{e}_i$ is the error in the coupling states $x_i - \hat{x}_i$ and

$$
F_i = \left( \frac{\partial f_i}{\partial x_i} - K_i \frac{\partial c_i}{\partial x_i} \right) |_{\hat{x}_i, \bar{x}_i} \quad (124)
$$

The matrix $K_i$ has to be chosen to minimize the elements of the coupling term, i.e.,

$$
K_i = \frac{\partial f_i}{\partial x_i} \left|_{\hat{x}_i, \bar{x}_i} \right. \quad \frac{\partial c_i^+}{\partial x_i} \left|_{\hat{x}_i, \bar{x}_i} \quad (125)
$$

where the $+$ sign indicates the pseudoinverse defined earlier. In some cases a compromise has to be drawn in choosing $K_i$ since the observer must remain stable and this may be very difficult to achieve. Note that terms of order higher than one can be taken into consideration in the observer design (Hengy and Frank 1987) at the expense of greater computation.
4.3. Sensor Fault Isolation using the Thau type Nonlinear Observer

Consider those nonlinear systems that can be modelled by the following state space equations.

\[ \dot{x} = Ax + Bu + f(x) \]  \hspace{1cm} (126)

\[ y = Cx \]  \hspace{1cm} (127)

Himmelspach (1992) applied the Thau type nonlinear observer to the problem of detecting and isolating sensor faults in these systems. The Thau type observer is constructed as follows.

\[ \dot{\hat{x}} = A\hat{x} + Bu + f(\hat{x}) + D(y-\hat{y}) \]  \hspace{1cm} (128)

\[ \hat{y} = C\hat{x} \]  \hspace{1cm} (129)

where \( D \) is a constant observer gain matrix.

The observer can be constructed if \((C,A)\) is observable, \( f(x) \) is continuously differentiable on a domain \( D \) of the state space, and if \( f(x) \) is locally Lipschitz on \( D \), i.e.,

\[ ||f(x) - f(z)||_2 \leq L_2f||x - z||_2, \forall x, z \in D \]  \hspace{1cm} (130)

with Lipschitz constant \( L_2f \). Then the following conditions must be satisfied to construct an asymptotic observer.

\[ \text{Real}\{\sigma(A-DC)\} < 0 \]  \hspace{1cm} (131)

\[ L_2f < \frac{\lambda(D)}{2\lambda(P)} \]  \hspace{1cm} (132)

where \( \sigma(.) \) denotes eigenvalues and \( P \) is the positive definite symmetric solution to the Lyapunov equation

\[ (A - DC)^TP + P(A - DC) = -Q \]  \hspace{1cm} (133)
where $Q$ is an arbitrary positive definite and symmetric matrix and $\lambda$ and $\lambda^*$ denote the minimum and maximum eigenvalue respectively. A similar condition to the one in equation (132) may be developed using the $\|\cdot\|_\infty$ norm rather than the $\|\cdot\|_2$ norm. Note that the Thau type observer has only locally guaranteed domain of applicability; for example it may be possible to identify only a local domain in which the Lipschitz condition is satisfied.

The sensor fault isolation problem is then solved by partitioning the measurement vector into various subsets each of which drives one observer. Those observers driven by a faulty output show a nonzero residual while those driven by outputs that do not have faults show zero residuals, allowing the use of a truth table to isolate the fault.

Himmelspach applied the above method to fault detection in an automobile powertrain/chassis angular speed sensor. He concludes that the Thau type nonlinear observer works best for small bias type faults, but does not outperform conventional linear methods for larger faults.

5. Robustness in Observer based FDI

The design of observers for FDI is based upon a number of assumptions, which may not always hold true. For instance, there always is present some error or uncertainty in the model of the plant for which the FDI scheme has been designed. Further, some input and measurement noise is inevitably present. Under these circumstances it may become hard to distinguish the nominal operation of the system from the faulty condition. Hence some modification of the above designs may be necessary to decrease the sensitivity of the FDI scheme to model uncertainty and the presence of noise, while still maintaining the ability to detect and isolate faults. It should be noted that sometimes the UIO scheme or some other disturbance decoupling scheme, where the input entry vector of the disturbance is known, is referred to as a robust design. That is not the sense in which the term robust is used here. Here the term robust is used when the FDI scheme attempts to account for the presence of input noise that enters the system in a random
manner, measurement noise, and uncertainty in the model of the system, which may arise either due to difficulty in identifying the model parameters accurately or due to the natural variability (stochastic nature) of any real life plant.

One approach to such a robust FDI scheme is to set a certain threshold magnitude, for the residuals, which has to be exceeded before a fault is recognized. Emami-Naeini et al. (1988), have proposed a method of setting such thresholds, for Kalman filter based residual generators, to account for model uncertainty. Such threshold setting schemes are sometimes referred to as passive schemes since they do not decrease the sensitivity of the residual generator to noise and uncertainty, but merely denote or define the limits outside which a fault can be distinguished from nominal operation.

Actively robust designs, on the other hand attempt to modify the design of the residual generator itself, in order to enhance the relative sensitivity of the FDI scheme to faults, in the presence of noise and uncertainty. It is such active schemes as applied to OBRG that are presented here.

5.1. The Optimal Detection Filter

In one of the earlier sections of this paper the detection filter technique of residual generation as developed by Park (1991) and Park and Rizzoni (1993) was presented. Park et al. (1993) have used the structure of the detection filter obtained, to formulate and solve the problem of designing a detection filter that is optimal in the sense of being robust to input and measurement noises.

Consider the linear time invariant system

\[ \dot{x}(t) = Ax(t) + Bu(t) + fn(t) + w(t) \]  
\[ y(t) = Cx(t) + v(t) \] (134) (135)

Where, \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^p \) is the control input vector, \( f \in \mathbb{R}^n \) is a fault event vector, \( n(t) \) is a scalar function, \( y \in \mathbb{R}^m \) is the measurement vector, and \( A, B \) and \( C \) are matrices of compatible dimension. In addition the model includes, zero-mean white
noise processes \( w \in \mathbb{R}^n \) and \( v \in \mathbb{R}^m \) which are assumed to have the following covariance matrix.

\[
E\left[ \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \begin{bmatrix} w^T(t) \\ v^T(t) \end{bmatrix} \right] = \begin{bmatrix} Q & 0_{n \times m} \\ 0_{m \times n} & R \end{bmatrix}, \quad Q \geq 0, \ R > 0
\]  
(136)

An observer for this system is given by,

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + D(y(t) - \hat{y}(t))
\]  
(137)

\[
\hat{y}(t) = C\hat{x}(t)
\]  
(138)

It was seen earlier that the observer of equations (137)-(138) is a detection filter for the system (134)-(134) with fault \( f \) if and only if,

\[
D = D_i + V_U D_j + D_k (I - (C_f)(C_f)^+) - D_W W^T
\]  
(139)

where all the terms on the right hand side are as defined earlier.

Let \( W \in \mathbb{R}^{m \times (n-1)} \) be a matrix whose columns form a basis for the column space of \((I - (C_f)(C_f)^+)\). Then the observer gain can be written as

\[
D = D_i - V_U D_j - D_k W^T
\]  
(140)

In the rest of the design \( D_i, V_U \) and \( W \) are regarded as given constant matrices where \( V_U \) and \( W^T \) have full column and row rank respectively.

The optimal estimation problem is then as follows:

**Find a gain \( D \), having the structure (140), that minimizes the covariance matrix**

\[
P = \lim_{t \to \infty} E\left\{ e^T(t) R e(t) \right\}
\]  
(141)

where \( R > 0 \) is a weighting matrix, and

\[
e(t) = x(t) - \hat{x}(t)
\]  
(142)
It is assumed that
\[ \hat{R} = C^TC + \alpha I_n \]  
(143)

The error equation (with \( n(t) = 0 \)) is
\[ e(t) = (A - DC)e(t) + w(t) - Du(t) \]  
(144)

If \((A - DC)\) is asymptotically stable, \(P\) satisfies the Lyapunov equation,
\[ (A - DC)P + P(A - DC)^T + Q + DRDT = 0 \]  
(145)

Then the cost \(J\) can be written as,
\[ J = \text{trace}(P\hat{R}) \]  
(146)

The optimization problem then can be stated as follows: Find \(D\) (\(D_j\) and \(D_k\)) that minimizes \(J\) subject to (145).

Define a lagrangian \(L\) as follows.
\[ L = \text{trace}(P\hat{R}) + \text{trace}((P(A - DC) + (A - DC)^TP + Q + DRDT)) \]  
(147)

The solution to the problem may then be found by taking the partial derivatives of \(L\) with respect to \(D_j, D_k, P,\) and \(\tilde{P}\) and the following result is obtained.
\[ D = PCTR^{-1} + \phi'(D_i - PCTR^{-1})\psi' \]  
(148)

where
\[ \phi = V_u(V^TPV_u)^{-1}V^TP, \quad \phi' = I_n - \phi \]  
(149)

\[ \psi = RW(W^TRW)^{-1}W^T, \quad \psi' = I_m - \psi \]  
(150)

and \(P\) and \(\tilde{P}\) are nonnegative matrices satisfying,
\[ 0 = AP + PAT - PCTR^{-1}CP + Q \]
\begin{align}
+ \phi(D_i - PCT^TR^{-1})\psi R\psi^T(D_i - PCT^TR^{-1})\phi^T
\tag{151}
\end{align}

\begin{align}
0 = \tilde{P} \left[ A - PCT^RC - \phi(D_i - PCT^TR^{-1})\psi^T \right]
- \left[ A - PCT^RC - \phi(D_i - PCT^TR^{-1})\psi^T \right]\tilde{P} + \tilde{R}
\tag{152}
\end{align}

The equations (151) and (152) have to be solved using some numerical procedure and the existing algorithms do not assure convergence, hence the following iterative method of solving for \( D_j \) and \( D_k \) is proposed.

First the eigenvalues of the detection space (in practice of dimension one or two) are assigned, thus fixing \( D_j \) and only \( D_k \) remains to be solved for. Then the gain \( D \) can be written as

\begin{align}
D = D_{ij} + D_k W^T
\tag{153}
\end{align}

where

\begin{align}
D_{ij} = D_i - V_D\tilde{D}_j
\tag{154}
\end{align}

is treated as a given matrix.

Now the solution for \( D_k \) can be shown to be,

\begin{align}
D_k = (PCT_W - D_{ij}RW)(WTRW)^{-1}
\tag{155}
\end{align}

where \( P \) is the solution of

\begin{align}
0 = (A - D_{ij}C)P + P(A - D_{ij}C)^T
- (PCT_W - D_{ij}RW)(WTRW)^{-1}(PCT_W - D_{ij}RW)^T + Q
\tag{156}
\end{align}

This is a standard Riccati equation and the solution can be obtained using almost any control software, e.g., MATLAB®. The cost \( J \) can then be calculated using equation (146). Thus \( J(D_j) \) can be calculated and a search for the minimum \( J \) can be made. Then the gain \( D \) is given by
\[ D^o = p_0CTR^{-1} + (D_{ij}^o - p_0CTR^{-1})\phi' \]

where the superscript \( o \) represents the optimal value.

A similar derivation can be performed for the discrete time case and the solution for that case is given below.

\[ D = APC^T(CPC^T + R)^{-1} + \phi'[D_i - APC^T(CPC^T + R)^{-1}]\psi' \]  \hfill (158)

\[ \phi = V_{\psi}(V_{\psi}^TV_{\psi})^{-1}V_{\psi}^T\tilde{P}, \quad \psi' = I_m - \psi \]  \hfill (159)

\[ \psi = (CPC^T - R)W[W^T(CPC^T + R)]^{-1}, \quad \psi' = I_m - \psi \]  \hfill (160)

and \( P, \tilde{P} \) satisfy the following equations

\[ 0 = APA^T - P - APC^T(CPC^T + R)^{-1} + Q \]  \hfill (161)

\[ + \phi'[D_i - APC^T(CPC^T + R)^{-1}]\psi'(CPC^T + R)\psi^T[D_i - APC^T(CPC^T + R)^{-1}]\phi' \]

\[ 0 = \tilde{A}^T P\tilde{A} + \tilde{P} + \tilde{A}^T \phi' \tilde{P}^T \phi \tilde{A}^T \]  \hfill (162)

\[ + A(I - PC^T(CPC^T + R)^{-1}C)\phi^T\tilde{P} \phi(I - PC^T(CPC^T + R)^{-1}C)A^T + \tilde{R} \]

where

\[ \tilde{A} = A - D_i\psi'C - APC^T(CPC^T + R)^{-1}\psiC \]  \hfill (163)

The same iterative procedure as outlined in the continuous time case may be applied to the discrete time system also without any modifications.

5.2. \( H^\infty \) Based Robust Observer Design

The \( H^\infty \) method has been extensively applied to the problem of dealing with model uncertainty in control system design. Hence the method is a logical choice in the attempt to construct robust residual generators. Viswanadham and Minto (1988) and Ding and Frank (1990) used the factorization approach to formulate the robust residual generator design problem, as an optimization problem that can be solved using the \( H^\infty \) method.

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Viswanadham and Minto (1988) consider the problem of robust residual generator design in the presence of unstructured uncertainty. Let $\mathbb{R}(s)$ denote the field of rational functions and let $\mathcal{M}(\mathbb{R}(s))$ denote the set of matrices (of arbitrary order with elements in $\mathbb{R}(s)$). Let $\mathcal{M}(S)$ be the set of matrices in $S$, where $S$ is a subset of $\mathbb{R}(s)$ containing only proper, stable rational functions. Let $P \in \mathcal{M}(\mathbb{R}(s))$ represent the plant for which the residual generator has to be designed. Let $P$ have the form $P = P_0 + \Delta P$ (additive perturbation) with the only restriction on $\Delta P$ being

$$\|\Delta P\|_\infty < \mu$$  \hspace{1cm} (164)

Then $P_0$ can be factorized into a right coprime factorization (rcf) as follows.

$$P_0 = N_0D_0^{-1} \quad N_0, D_0 \in \mathcal{M}(S)$$  \hspace{1cm} (165)

Note that two matrices $N, D$ are right coprime only if there exist $X, Y \in \mathcal{M}(S)$, such that the Aryabhatta (also known as the Bezout) identity

$$XN + YD = I$$  \hspace{1cm} (166)

is satisfied. Now an observer can be constructed as shown in Figure 1.

![Observer constructed using right coprime factorization](image)

Figure 3. Observer constructed using right coprime factorization

That the system in Figure 3 serves as an observer can be seen from the following relations.
\[ y = P_0 u = N_0 D_0^{-1} u \]  \hspace{1cm} (167)

Introducing the variable \( x \) (which is analogous to the state vector) we obtain,
\[ N_0 x = y, \quad D_0 x = u \]  \hspace{1cm} (168)

From the Aryabhatta identity (equation (2.3.4.2.3),
\[ \begin{bmatrix} X_0 & Y_0 \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} X_0 & Y_0 \end{bmatrix} \begin{bmatrix} N_0 \\ D_0 \end{bmatrix} x = x \]  \hspace{1cm} (169)

Now the set of all observers that exactly reconstruct the state of a plant \( P_0 \) is
\[ O(P) = \{ [X \ Y] = [X_0 + R \tilde{D}_0 \ Y_0 - R \tilde{N}_0 ]; \ R \in M(S), \ |Y_0 - R \tilde{N}_0 | = 0 \} \]  \hspace{1cm} (170)

Now the objective is to select the parameter \( R \) in equation (170) so as to obtain an estimate \( \hat{x} \) that minimizes the error \( e = x - \hat{x} \). The deviation of the plant \( P \) from the nominal model \( P_0 \) can be modelled by additive, multiplicative or stable factor perturbations as follows.
\[ P = P_0 + \Delta P, \]  \hspace{2cm} additive \hspace{1cm} (171)
\[ P = (I + \Delta P) P_0, \]  \hspace{2cm} multiplicative \hspace{1cm} (172)
\[ P = N D^{-1}, \quad \Delta N \]  \hspace{1cm} and \hspace{1cm} \[ D = D_0 + \Delta D, \]  \hspace{1cm} Stable factor \hspace{1cm} (173)

In each case the problem reduces to the following \( H^\infty \) optimization problem: select \( R \in M(S) \) such that,
\[ y = \min_{R \in M(S)} \left\| X_0 + R \tilde{D}_0 \right\|_\infty \]  \hspace{2cm} additive \hspace{1cm} (174)
\[ y = \min_{R \in M(S)} \left\| X_0 \tilde{D}_0^{-1} \tilde{N}_0 + R \tilde{N}_0 \right\|_\infty \]  \hspace{2cm} multiplicative \hspace{1cm} (175)
\[ y = \min_{R \in M(S)} \left\| [X_0 \ Y_0] + R \left[ \tilde{D}_0 \ - \tilde{N}_0 \right] \right\|_\infty \]  \hspace{2cm} stable factor \hspace{1cm} (176)

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The first two cases (additive and multiplicative perturbation) are standard $H^\infty$ optimization problems, and the authors describe a procedure for obtaining the solution. The third case (stable factor perturbation) is known as a two block $H^\infty$ problem and is more difficult to solve, but nearly optimal solutions can be obtained using a procedure known as $\gamma$-iteration.

Consider the following representation of a linear system with structured model uncertainty and a fault.

\[ y(s) = G_o(s)u(s) + G_d(s)d(s) + G_f(s)f(s) \]  

(177)

$G_d(s)$ represents the structure of the uncertainty, $\|d(s)\|_\infty \leq \sigma(f)$, $f(s) \in \mathbb{R}^q$ represents the fault and $G_f(s)$ is a known distribution matrix. $G_o(s)$, $G_d(s)$, $G_f(s)$ are all assumed to be stable.

Viswanadham and Minto (1988) and Ding and Frank (1990) showed that all residual generators can be parametrized so that they take the form

\[ r(s) = Q(s)\hat{M}u(s)\gamma(s) - \hat{N}u(s)u(s) \]  

(178)

where $Q(s) \in \mathbb{RH}_\infty$ denotes the parametrization matrix, and $\hat{M}u(s)$ and $\hat{N}u(s)$ form a left coprime factorization of $G_o(s)$. Therefore

\[ r(s) = Q(s)\hat{M}u(s)(G_f(s)f(s) + G_d(s)d(s)) \]  

(179)

Perfect decoupling of the fault from the uncertainty requires

\[ Q(s)\hat{M}u(s)G_f(s) = \text{diag}(t_1(s), \ldots, t_q(s)) \in \mathbb{RH}_\infty, \text{ and} \]

\[ Q(s)\hat{M}u(s)G_f(s) = 0. \]

This can be achieved iff,

\[ \text{rank } [G_f(s), G_d(s)] > \text{rank}(G_f(s)) + \text{rank}(G_d(s)) \]  

(180)
\[
\text{rank}(G_f(s)) = q = \text{number of faults} \quad (181)
\]

If these conditions cannot be met, an optimal residual generator can be designed by minimizing a performance index \( J \) such as the following.

\[
J = \frac{Q(s)G_f(s)}{Q(s)G_f(s)} \quad (182)
\]

This problem has been solved by Ding and Frank (1990, 1991) under different norms. Ding and Frank (1991) solve this using the \( H_\infty \) method.

5.3. Robust Eigenstructure Assignment

Daley and Wang (1992) represent a system with structured uncertainties acted on by disturbances, input faults and sensor faults in the following manner,

\[
x(t) = A + \Delta A(t)x(t) + B + \Delta B(t)u(t) + Q(t)v(t) + F_f(t) \quad (183)
\]

\[
y(t) = C + \Delta C(t)x(t) + \Delta y(t) \quad (184)
\]

where \( \Delta A, \Delta B, \) and \( \Delta C \) represent the structured uncertainties, \( v(t) \) is a vector of disturbance inputs, \( f(t) \) a vector of input faults and \( Q(t) \) and \( F(t) \) the corresponding distribution matrices.

An observer for this system is given by

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + K(y(t) - C\hat{x}(t)) + Bu(t) \quad (185)
\]

With this estimate the observation error vector evolves in the following manner.

\[
\dot{e}(t) = (A - KC)e(t) + E(t,K)d(t) + K\Delta y(t) \quad (186)
\]

where \( E(t,K) = [\Delta A(t),\Delta B(t),\Delta C(t),Q(t)] \), and \( d(t) = [x^T(t),u^T(t),x^T(t),v^T(t)]^T \).

Daley and Wang (1992) use the fact that once a set of desired eigenvalues (distinct from those of \( A \)) for \( A - KC \) is chosen, the observer gain matrix \( K \) can be parameterized using a parameter matrix \( Q \) as

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\[ K = W^{-1}Q \]  
(187)

and the time varying matrix \( E(t, K) \) can be parameterized as,

\[ E(t, K) = \sum_{i=1}^{l} E_i e_i(t) \]  
(188)

to arrive at the following representation of the Laplace transform of the error dynamics.

\[ e(s) = \sum_{i=1}^{n} \frac{v_i w_i^T}{s - \lambda_i} \left\{ \sum_{j=1}^{l} E_j d_j(s) - K \Delta y(s) \right\} \]  
(189)

where, \( w_i \) is the \( i \)th row of \( W \), \( v_i \) is the \( i \)th column of \( W^{-1} \), and \( d_j(s) = (s \bar{d}) (s) \).

Now the following performance function \( J \) can be defined and the problem of designing an optimum eigenstructure for the observer becomes one of minimizing \( J \) through the proper choice of the free parameter \( Q \).

\[ J(Q) = \sigma_1 \left| \sum_{i,j=1}^{n,l} v_i w_i^T E_j \right| + \frac{\sigma_2}{\sum_{i=1}^{n} v_i w_i^T K} \]  
(190)

where \( \sigma_1 \) and \( \sigma_2 \) are weights. Note that the parameter matrix \( Q \) must be chosen such that \( W^{-1} \) exists. Daley and Wang (1992) use the Nelder-Mead simplex algorithm to solve this optimization problem for a numerical example, and demonstrate improved performance over a suboptimal residual generator.

6. Conclusion

This chapter presented a survey of a representative sample of the FDI strategies and schemes that are available in the literature. A considerable amount of work has been done in the area of developing FDI techniques for linear systems. A large proportion of recent work has been focused on increasing the robustness of FDI schemes. However, only a limited amount of work has been done in the area of nonlinear systems. Many of the techniques developed only try to minimize the effect of the nonlinearity by treating it
as an uncertainty, or disturbance and then using a robust scheme that is as insensitive as possible to it. Such schemes have only restricted applicability and are difficult to apply to systems that are significantly nonlinear.
CHAPTER 3
THE NONLINEAR PARITY EQUATION RESIDUAL GENERATION SCHEME

1. Introduction

Much attention has been focused on the design of fault detection and isolation (FDI) schemes for linear dynamic systems. The schemes fall into two main types (i) Observer based residual generation (OBRG) and (ii) Parity equation based residual generation. Some representative literature on these schemes can be found in Potter and Suman (1977), Mironovskii (1979), Gertler (1991). However, these techniques are severely limited in their applicability due to their assumption of linearity. Some nonlinear FDI schemes have been proposed but suffer from restrictive requirements on the system properties and often need ad-hoc modifications that preclude general use.

This chapter presents the development of an FDI design scheme, with the following features: (i) Wide applicability: to both nonlinear & linear systems and (ii) the use of an extremely general fault model: few assumptions are made regarding the nature of the fault, which allows applicability to a variety of faults including the ones most commonly addressed in the literature - input, output and component faults.

The development presented here addresses the fundamental theory of fault isolation and presents results on the following important issues: (i) Formulation of the isolation problem (ii) Necessary and sufficient conditions for the existence of a solution to the fault isolation problem for a given system, including isolation of multiple simultaneously occurring faults (iii) Quantification of computational requirements for isolation and (iv) A Construction procedure for the FDI scheme from system models.
2. Problem Formulation

2.1. System and Fault Representation

Consider the dynamic system depicted in block diagram form in Figure 1.

![Block diagram of a general dynamic system and fault representation](image)

Fig. 1. General dynamic system and fault representation

It should be noted that all the variables listed above may be functions of time. We can write the same system representation in differential equation form as follows.

\[ \dot{x} = f(x, u, \theta) = f(x, u_o + \Delta u, \theta_o + \Delta \theta) \]  \hspace{1cm} (191)

\[ y^* = h(x, u, \theta) + \Delta y = h(x, u_o + \Delta u, \theta_o + \Delta \theta) + \Delta y \]  \hspace{1cm} (192)

The above representation is very general and can encompass almost all the different kinds of faults that can occur in a dynamic system.

Faults are usually classified into 3 types - (i) input, (ii) output and (iii) component faults. In linear system diagnostics a distinction is usually made between the first two kinds (often referred to as additive faults) and the third (referred to as multiplicative faults). However, it can be seen from (191) - (192) that in the case of nonlinear system the distinction between input and component faults disappears and hence such classifications become quite arbitrary. We will therefore omit the parameter fault vector in the rest of the study, simply noting that the input and input fault vector may be extended to include parameter fault representations. Further note that disturbance inputs are also
special cases of input faults where the nominal input value is simply zero and the disturbance forms part of the input fault vector. Thus the FDI scheme presented in this study is extremely general in applicability.

<table>
<thead>
<tr>
<th>No.</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$u_o$</td>
<td>Input vector</td>
</tr>
<tr>
<td>2.</td>
<td>$\Delta u$</td>
<td>Input fault vector</td>
</tr>
<tr>
<td>3.</td>
<td>$\theta_o$</td>
<td>Nominal parameter vector</td>
</tr>
<tr>
<td>4.</td>
<td>$\Delta \theta$</td>
<td>Parameter (component) fault vector</td>
</tr>
<tr>
<td>5.</td>
<td>$x$</td>
<td>State Vector</td>
</tr>
<tr>
<td>6.</td>
<td>$f(\cdot, \cdot)$</td>
<td>State evolution vector field</td>
</tr>
<tr>
<td>7.</td>
<td>$h(\cdot, \cdot)$</td>
<td>Output measurement function</td>
</tr>
<tr>
<td>8.</td>
<td>$y$</td>
<td>Actual output vector</td>
</tr>
<tr>
<td>9.</td>
<td>$\Delta y$</td>
<td>Output fault vector</td>
</tr>
<tr>
<td>10.</td>
<td>$y^*$</td>
<td>Measured output vector</td>
</tr>
</tbody>
</table>

Table 1. General system variables and faults

It is now possible to define a fault vector as follows.

$$d(t) = \begin{bmatrix} \Delta u(t) \\ \Delta y(t) \end{bmatrix} \quad (193)$$

Note that if $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ obviously, $d \in \mathbb{R}^{m+p}$.
It will be assumed that the system model represented by the functions \( f(\cdot, \cdot) \) and \( h(\cdot, \cdot) \) is completely known. Discussions of robustness to model variations are beyond the scope of this study and for the present will be accounted for by setting suitable thresholds on the residual vector.

2.2. FDI - Mathematical Representation and Definition

In essence, an FDI scheme is a map (linear or nonlinear) that acts upon the measurements obtained from the system under study and produces an output that is unique for each combination of faults that can occur in the system. Thus an FDI scheme can be represented as the map \( W \), such that,

\[
W: R^p \rightarrow R^q, W(y^*) = s
\]  

(194)

where \( s \in R^q \) is the output vector produced by the FDI scheme. It should be noted that,

\[
\text{rank}(W) \leq p
\]  

(195)

Now, let us consider the case when there exists a unique algebraic relationship between the inputs and the states and outputs of the system. Clearly this will not hold true in general for dynamic systems. However, the algebraic case will serve to show the development of theoretical considerations underlying fault isolation in a simple yet rigorous manner the. Once the development is complete it will be restated for the general case of dynamic systems. Therefore assume,

\[
x(t) = \Phi(t, u(t))
\]  

(196)

where \( \Phi(\cdot, \cdot) \) is a nonlinear function of the inputs and/or time.

We do not explicitly show the dependence on \( \theta \) since we intend to treat input and parameter fault detection alike and hence simply assume that the parameter vector is included in the input vector.

\[
y^* = h(x, u) + \Delta y = h(\Phi(t, u), u) + \Delta y = H(t, u_0 + \Delta u) + \Delta y
\]  

(197)
Note that,

\[ H : R^m \rightarrow R^p \text{ and rank}(H) = p \]  \hspace{1cm} (198)

which follows from the assumption of the existence of a unique solution. It can therefore be seen from (194) and (197) that the output of the FDI scheme is a composition of two maps, i.e.,

\[ s = W(H(t, u_0 + \Delta u) + \Delta y) \]  \hspace{1cm} (199)

Therefore, suppressing the time argument we can write,

\[ s = M(d) \]  \hspace{1cm} (200)

where \( M \) is the function obtained through composition of the two functions \( W \) and \( H \).

We can now put forth a definition of an FDI scheme that successfully isolates faults in a given system.

**Definition 1. FDI scheme:** An FDI scheme is a function/operator that acts upon the measurements available from a system to produce a unique output for each combination of faults (i.e., non-zero components of the fault vector \( d \)) that can occur in the system. Further, the scheme considered to be *isolating* if the elements of the vector \( s \) also track the time histories of the values of the faulty variables.

### 3. Diagnostic System - Conditions and Capabilities

In this section the conditions under which a particular fault can be diagnosed and the trade-offs involved in designing a diagnostic scheme with the ability to isolate specific combinations of faults will be discussed.

#### 3.1. Diagnosis of a single fault

Denote the \( i_{th} \) element of the fault vector by \( d_i \). Then the \( i_{th} \) fault is said to exist if \( d_i \neq 0 \) and the \( i_{th} \) fault henceforth will be denoted by \( d_i \).

The conditions under which a diagnostic scheme can detect a particular fault may be simply stated, from a physical understanding of dynamic system behavior, as follows.
Faults can be detected if they are "visible" from the measured outputs of the system. By visible, it is meant that the fault in question has a unique non-zero effect on the available measurements. The following proposition states this in more precise mathematical terms.

**Proposition 1. Isolability of a single fault:** The lone occurrence of the $i$th fault $d_i$ can be strongly isolated only if $\frac{\partial y^*}{\partial d_i} \neq 0$.

**Proof:** Note that from Definition 1 $d_i$ is isolable for $t \in [t_0, t_1]$, if the following function can be constructed.

$$w(y^*(t)) = s(t) = d_i(t) \forall t \in [t_0, t_1]$$  \hspace{2cm} (201)

From (197) we can write

$$y^*(t) = G(t, d(t)) = G(t, [d_i(t), d_i(t), \ldots, d_{i+p}(t)])$$  \hspace{2cm} (202)

$$G(t, d(t)) = H(t, u_o + \Delta u) + \Delta y$$  \hspace{2cm} (203)

From the implicit function theorem (Khalil, 1992), if and only if,

$$\frac{\partial G}{\partial d_i} \neq 0 \forall y^*(t), d_i(t), t \in [t_0, t_1]$$  \hspace{2cm} (204)

we can write,

$$d_i(t) = w(y^*(t))$$  \hspace{2cm} (205)

Which is the condition for isolability.

**Corollary 1.1:** Single output faults are always isolable.

**Proof:** This may be seen from the fact that,

$$\frac{\partial G}{\partial y} = [I_{p \times p}]$$  \hspace{2cm} (206)

**Proposition 2.** A set of $l$ simultaneously occurring faults can be isolated if and only if there exists some subset of $l+1$ component equations of
\[ y^*(t) = G(t, d_{\text{fault}}(t), d_{\text{no-fault}}) = G(t, d_{\text{fault}}(t), [0, \ldots, 0]) \]  
(207)

which taken \( l \) at a time and denoted as

\[ y^*_{\text{reduced}} = G^i_{\text{reduced}}(d_{\text{fault}}(t), [0, \ldots, 0]), i = 1, \ldots, l \]

satisfies,

\[ \text{rank} \left( \frac{\partial G^i_{\text{reduced}}}{\partial d_{\text{fault}}} \right) = l \]

(209)

**Proof:** Clearly the faults are isolable if and only if (207) can be solved uniquely.

Let us now define a new variable \( d_{nf}(t) \) and replace each of the no-fault elements in the fault vector by it. i.e., \( d(t) \) now becomes,

\[ d(t) = [d_1(t) \ldots d_l(t) \quad d_{nf}(t) \ldots d_{nf}(t)] \]

(210)

This may be done since it is assumed \( d_i(t) = d_j(t) = 0, i = l + 1, \ldots, p + m, j = l + 1, \ldots, p + m \), and we will define a new fault variable vector,

\[ d_{\text{new}}(t) = [d_1(t) \ldots d_l(t) \quad d_{nf}(t)] \]

(211)

\[ y^*(t) = G(t, d_{\text{new}}(t)) \]

(212)

and the fault isolation problem reduces to the unique solution of (212). Now (212) is a system of \( p \) equations in \( l + l \) unknowns and for a solution to exist the jacobian of \( G \) with respect to \( d_{\text{new}} \), must satisfy,

\[ \text{rank} \left( \frac{\partial G}{\partial d_{\text{new}}} \right) = l + 1 \]

(213)
Now, the Jacobian $\frac{\partial G}{\partial d_{\text{new}}}$ is a $p \times (l+1)$ matrix, and hence $l+1$ rows of the Jacobian matrix are linearly independent and the equations corresponding to these $l+1$ rows can be selected as the reduced subset and $\text{rank} \left( \frac{\partial G_{\text{reduced}}}{\partial d_{\text{new}}} \right) = l + 1$ i.e., it is non-singular.

Now, consider the following two cases: (i) only elements of the given set of variables belonging to the fault set under consideration are faulty $\Rightarrow d_{nf} = 0$ is an assumption that holds true or (ii) at least one variable not belonging to the fault set under consideration is faulty $\Rightarrow d_{nf} = 0$ does not hold.

Case (i): Under this situation the system is of full rank, the assumption $d_{nf} = 0$ holds and the system of equations can be solved uniquely.

Case (ii): In this case, the assumption $d_{nf} = 0$ does not hold and since the jacobian of the full set of equations has rank $l+1$, the set of equations must be inconsistent and have no solution. $\text{(*)}$

This correctly indicates that the assumed fault set is not the one actually present. Thus equation (209) is a sufficient condition for isolation.

However, assume there exists at least one set of $l$ equations for which equation (209) does not hold. Let us call this set $s_1$. Then, choosing that set of equations, and substituting $d_{nf} = 0$, there is an infinite number of solutions for the fault variables. Therefore assume an arbitrary value for one of the variables (choose one which appears in the equation that was removed from the complete set to obtain $s_1$) and solve for the other variables. Without loss of generality let this arbitrary variable be $d_k$. Then substituting the obtained values for the other variables and $d_{nf} = 0$ into the remaining equation, it is possible to find a solution for $d_k$ that makes the entire set of equations consistent which violates $\text{(*)}$. Therefore equation (209) is also a necessary condition and Proposition 2 is proved.
Corollary 2.1: Maximum Simultaneous multiple faults isolation: Given $p$ available measurements a maximum of $l \leq p - 1$ multiple faults that occur simultaneously can be strongly isolated.

Proof: This follows from Proposition 2 and the fact that since there are $p$ measurements a maximum of $p$ independent equations are available. Hence $l + 1 \leq p \Leftrightarrow l \leq p - 1$. ♦

Corollary 2.2: Multiple simultaneous output faults that satisfy Corollary 2.1 are always isolable. This may be seen from the fact that if a combination of any $l$ output faults, $l < p$ is denoted by $\Delta y_{new} \in \mathbb{R}^l$ then,

$$\frac{\partial \mathcal{X}_{\text{reduced}}}{\partial \Delta y_{new}} = [I_{l \times l}]$$  \hspace{1cm} (214)

4. FDI Algorithm Design For Nonlinear Dynamic Systems

4.1. Existence of an FDI design

The necessary and sufficient condition for the existence of a solution to the FDI problem for a given system making the assumption that the inputs are related algebraically to the outputs, is given in the previous section. It is clear that Propositions 1 and 2 and their associated corollaries all depend upon the existence of unique solutions to the system of equations relating the fault variables to the available measurements. Dynamic systems in which the inputs can be solved for uniquely from available outputs are said to be invertible. Thus the requirement of obtaining unique solutions may simply be stated as the requirement that the system be invertible with respect to the given set of measurements and faults. This is stated more precisely by the following proposition.

Proposition 3: A set of $l$ simultaneously occurring faults can be isolated if and only if the total number of simultaneous faults ($= \text{no. of input faults} + \text{no. of output faults}$) $l = q + r \leq p - 1$ and there exists at least $l + 1$ measurements, such that the given set of
faults have finite vector relative degree with respect to any set of these measurements taken \( l \) at a time.

*Proof:* The proof follows from the fact that the condition in Proposition 2 is the condition for the existence of unique solutions to the fault variables and the existence of finite vector relative degree \( \Leftrightarrow \) invertibility \( \Leftrightarrow \) unique solvability (Hirschorn, 1979). ♦

This result is extremely significant since it allows the application, to FDI, of results from research on inversion of nonlinear systems, an area that has received much attention over the past decade.

**Corollary 3.1:** A set of \( q \) simultaneously occurring input faults can be strongly isolated if and only if the total number of simultaneous faults (= no. of input + no. of output) \( l = q + r \leq p - 1 \) and the faulty inputs have finite vector relative degree with respect to at least some \( q+1 \) of the available non-faulty outputs taken \( q \) at a time.

*Proof:* From equation (192) it is seen that output faults enter the measurement equation algebraically. Therefore these faults are always invertible (Hirschorn, 1978). Now eliminating the measurement equations in which the output fault terms appear from the set of available measurement equations, the remaining equations arise from non-faulty output measurements and from Proposition 3 the input faults must be invertible with respect to some \( q+1 \) of the available non-faulty outputs taken \( q \) at a time. ♦

4.2. The Nonlinear Parity Equation Residual Generation (NPERG) Scheme

In the case of most nonlinear dynamic systems closed form solutions are impossible to derive. Hence numerical means of solving for the fault variables must be employed. We will now re-examine the FDI problem from this point of view and provide a systematic design procedure. We shall call this design procedure the Nonlinear Parity Equation Residual Generation Scheme (NPERG).

Assume that in the system represented in (191) and (192) that \( v \) inputs and \( w \) outputs can simultaneously be faulty and \( v + w = l \leq p - 1 \).
Looking at Proposition 2 and Proposition 3, it can be seen that the fault detection procedure may be described as the calculation of the faulty outputs and inputs from some subset of the measurements and then checking the resulting solutions for consistency. The following step by step procedure achieves this.

**Step 1:** Assume that the first \( w \) elements of the output vector and the first \( v \) elements of the input vector are faulty. Then the other \( p-w \) elements are not faulty. Referring back to Corollary 3.1 these outputs can be used to estimate the inputs that are suspected of being faulty.

Therefore construct an inverse system that estimates the \( v \) input elements, from a suitable set of \( v \) elements of the \( p-w \) outputs that are assumed to be non-faulty. Now using these input estimates and the forward model calculate all the \( w \) outputs that are assumed faulty.

This gives us estimates of all the faults, under the assumption that a particular set (the first \( v \) inputs and first \( w \) outputs) of variables are faulty.

**Step 2.** Consistency check. Using the estimated values of the assumed faulty inputs and the nominal values of the other inputs construct a forward model to calculate estimates of all the other measured outputs. If the assumed set of faults and the actual set of faults match the measured and the estimated values of the assumed non-faulty outputs (output of the forward model for this step) should match (within measurement and modelling errors) and the faults have been isolated and estimates obtained in step 1 are also the correct estimates of the faulty variables. If the values do not match the consistency check has failed.

**Step 3.** If the consistency check fails it is necessary to assume a different combination of faulty elements and repeat step 2. This is done by constructing the estimator described in step 1 and the forward model of step 2 for each possible combination of faults.

Note that the consistency check described in step 2 of the above procedure is what is usually referred to in fault detection literature as *residual generation* and each set of
inverse and forward models from step 1 and step 2 is referred to as a residual generator. Figure 2 shows the schematic of the residual generator for a particular combination of faults. In Figure 2, the term “non-faulty” means the set of variables that are assumed to be without faults for that particular residual generator and the term “fault” is the converse.

![Residual generator construction](image)

**Figure 2. Residual generator construction**

5. **Computation vs. Isolation**

The fact that we have now been able to relate the FDI design presented here to the residual generator commonly seen in the fault detection literature allows us to formalize a qualitatively understood but never quantified notion in fault isolation, *i.e.*, the tradeoff between computational effort and complexity in terms of the number of simultaneous faults that can be isolated.

**Proposition 4:** In the general case, (both input and output faults simultaneously present) the minimum number $N$ of residuals necessary for the strong isolation of faults in a system in which a maximum of $l$ faults out of a total of $n = p + m$ possible faults, can occur simultaneously is given by the following relation.
Proof: This is obvious from the fact that each possible combination of faults necessitates the use of one consistency check (or residual generator) and the total number of possible fault combinations is given by (215).

\[ N = nC_1 = \frac{n!}{l!(n-l)!} \] (215)

6. Conclusion

This chapter has presented a fundamental theory of FDI design that formalizes the hitherto somewhat ad hoc health monitoring of nonlinear dynamic systems. The NPERG design scheme presented here is extremely general, making very few assumptions about the nature of the system representation, dynamics or fault models. The existence conditions for a diagnostic solution are presented with proof and a detailed step by step construction procedure for the residual generators is given.
1. Introduction

Sliding mode control and estimation techniques are extremely suitable for application to systems that are significantly nonlinear, and exhibit good robustness properties with respect to modeling uncertainty and parameter variation. Sliding mode estimation has also been shown to be extremely effective and compares favorably with other approaches (Drakunov, 1992, Hashimoto et. al., 1990, and Misawa, 1989).

The sliding mode approach to identification and estimation in dynamic systems, consists of designing the model with discontinuous parameters and enforcing sliding modes such that the model and plant outputs coincide. Then the average values of the discontinuous parameters depend on the unknown states, parameters and disturbances and may be used for their evaluation.

The design idea may be illustrated as follows:

\[ \frac{dx}{dt} = f(x, t, a) \]  \hspace{1cm} (216)

with known state vector \( x \) and an unknown vector parameter \( a \) and disturbances.

The model equation is of form

\[ \frac{d\hat{x}}{dt} = f_0(\hat{x}, t, a_o) + V \]  \hspace{1cm} (217)

with \( \hat{x}, f_0 \) and \( a_o \) being an estimate of \( x \) and nominal values of \( f \) and \( a \). The auxiliary input \( V \) is the discontinuous function of the mismatch \( s = \hat{x} - x \):
\[ V = -M \text{sign}(s), \]
\[ M > |f_0(\dot{x}, t, a_0) - f(x, t, a)| \]

(218)

Sliding occurs after a finite time-interval with \( s = 0 \) and the average value of the signal \( V \) with slow and high frequency components depends on the unknown parameters and disturbances

\[ V_{av} = f_0(\dot{x}, t, a_0) - f(x, t, a), \quad \dot{x} = x \]

(219)

The average value \( V_{av} \) may be derived by a low-pass filter filtering out the high-frequency component. Then this function may be used to calculate the unknown parameters and disturbances directly or to find them by some search procedure (Utkin, 1992).

2. Continuous Time Sliding Mode Observer Design For Systems With inputs

Many hitherto proposed sliding observer design schemes are applicable only to systems without inputs or to systems whose inputs satisfy an extremely restrictive set of assumptions. Here a sliding mode observer design scheme is proposed, that extends an earlier design (Drakunov, 1992) to nonlinear systems with inputs, under the assumption that the derivatives of the inputs are known. This is a much less stringent condition than those applicable to the earlier proposed schemes since, in the case of most control systems the inputs are determined by the controller and thus complete knowledge of these inputs is available. It is proved that the proposed observer can be designed such that the observer error converges to zero in a finite time interval, (arbitrarily determinable by proper choice of the observer gains). It is also shown that the sliding observer can be used to estimate unknown inputs in some cases, using augmentation of the state vector.

Drakunov (1992) and Drakunov and Özgüner (1992) describe the construction of observers for linear and nonlinear systems using the theory of variable structure control. However, their observer design scheme is valid only for systems without inputs, or
systems whose input distribution vectors satisfy certain restrictive conditions. The observer design scheme is described below.

2.1. Observer Design for Systems Without Inputs

Consider the system

\[ \dot{x} = f(x) + B(x)u \]  
\[ y = h(x) \]  

where, \( x \in \mathbb{R}^n \) is the state vector, \( y \in \mathbb{R}^p \) is the output vector and \( u \in \mathbb{R}^q \) is the input vector.

If \( B(x) \equiv 0 \) an observer can be constructed for the system in equations (220) - (221) in the following manner. Define,

\[ H(x) = [h_1(x), h_2(x), \ldots, h_n(x)]^T \]  

where,

\[ h_i(x) = h(x) \]  
\[ h_i(x) = \frac{\partial h_{i-1}(x)}{\partial x} f(x) \quad i = 2, \ldots, n \]  

Note that \( h_{i,i}(x) \) is the \( i_{th} \) Lie derivative of the function \( h(x) \) along the trajectories of the system.

\[ h_{i}(x) = L_f^{-1} h(x) \]  

Then, the observer can be constructed as follows.

\[ \dot{x} = \left( \frac{\mathcal{A}(\dot{x})}{\partial x} \right)^{-1} M(\dot{x}) \text{sgn}(V(t) - H(\dot{x})) \]  
\[ V(t) = [v_1(t), \ldots, v_i(t)]^T \]  
\[ v_i(t) = y(t) \]
\[ v_i(t) = (m_i(\hat{x}) \text{sgn}(v_i(t) - h_1(\hat{x}(t))))_{eq}, \quad i = 1, \ldots, n - 1 \]  \hfill (229)

\[ M(\hat{x}) = \text{diag}(m_1(\hat{x}), \ldots, m_n(\hat{x})) \]  \hfill (230)

where, \( M \in \mathbb{R}^{n \times m} \) and the \( m_j \)’s are positive elements. The subscript 'eq' in equation (229) denotes that the signal on the right hand side of equation (229) is low pass filtered to obtain the equivalent control value. By suitable choice of \( M \) the observer converges in any prescribed finite time interval.

2.2. Design for Systems With Inputs

If \( B(x) \neq 0 \), the observer in equation (226) can be modified as follows, without affecting the convergence properties,

\[ \dot{x} = \left( \frac{\partial H(\hat{x})}{\partial \dot{x}} \right)^{-1} M(\hat{x}) \text{sgn}(V(t) - H(\hat{x})) + B(\hat{x})u \]  \hfill (231)

provided the input distribution vector \( B(\hat{x}) \) satisfies the following condition.

\[ \frac{\partial}{\partial \dot{x}} \left[ \frac{\partial H(x)}{\partial \dot{x}} B(x) \right] = 0 \quad \forall x \]  \hfill (232)

However, many systems do not satisfy equation (232). Therefore, presented below is an extension of the sliding observer scheme which allows state estimation in more general situations, given knowledge of derivatives of the inputs.

Consider once again the system in equations (220)-(221). Then,

\[ y(t) = h_1(x) \]  \hfill (233)

\[ \dot{y}(t) = \frac{\partial h_1(x)}{\partial x} \frac{\partial \dot{x}}{\partial x} + \frac{\partial h_1(x)}{\partial \dot{x}} (f(x) + B(x)u(t)) = h_2(x, u) \]  \hfill (234)

\[ \ddot{y}(t) = \frac{\partial h_2(x, u)}{\partial x} \frac{\partial \dot{x}}{\partial x} + \frac{\partial h_2(x, u)}{\partial x} \frac{\partial \dot{u}}{\partial x} + \frac{\partial h_2(x, u)}{\partial \dot{x}} \frac{\partial \dot{u}}{\partial x} + \frac{\partial h_2(x, u)}{\partial \dot{u}} \frac{\partial u}{\partial x} = h_3(x, x, u, \dot{u}) \]  \hfill (235)
Define, 

\[ X = \frac{\partial H}{\partial x} \begin{bmatrix} \text{sgn}(V(t) - \hat{H}) - \left( \frac{\partial H}{\partial u} \hat{u} + \frac{\partial H}{\partial u} \hat{u} + \ldots + \frac{\partial H}{\partial u^{n-1}} u^n \right) \end{bmatrix} \]  

(238)

where, 

\[ \hat{H} = H(\hat{x}, \hat{u}, \ldots, u^{n-1}) \]  

(239)

and the arguments have been omitted for convenience. \(V(t)\) and \(M(\hat{x})\) are as defined in equations (228) - (230). The convergence of this observer can be proved in a manner similar to Drakunov (1992), as follows.

**Proof:** Define the nonlinear transformations

\[ z = H(x, \dot{x}, \ldots, u^{n-1}) = H \]  

(240)

\[ \hat{z} = H(\hat{x}, \hat{u}, \ldots, u^{n-1}) = \hat{H} \]  

(241)

If the jacobian of the transformation \(H\) is nonsingular in a region \(X \in R^n\) of the state space is a one-to-one onto transformation. i.e. \(H\) is an injection if,

\[ \det \left( \frac{\partial H}{\partial x} \right) \neq 0 \quad \forall x \in X \]  

(242)

Note that the nonsingularity of \(H\) is the necessary and sufficient condition for observability of the the system. Define,
\[ e(t) = z(t) - \ddot{z}(t) \]  

(243)

Since \( H \) is an injection,

\[ e(t) \to 0 \iff \dot{x}(t) \to x(t) \]  

(244)

\[ \dot{\dot{z}}(t) = \frac{\partial H}{\partial \dot{x}} \dot{x} + \frac{\partial H}{\partial \dot{u}} \dot{u} + \frac{\partial H}{\partial \ddot{u}} \ddot{u} + \ldots + \frac{\partial H}{\partial \ddot{u}^{n-1}} u^n \]  

(245)

From equation (2.19) and (2.26),

\[ \dot{z}(t) = M(\dot{x}) \text{sgn}(\dot{v}'(t) - \dot{h}) \]  

(246)

\[ \dot{e}(t) = z(t) - \ddot{z}(t) = \frac{\partial H}{\partial \dot{x}} \dot{x} + \frac{\partial H}{\partial \dot{u}} \dot{u} + \frac{\partial H}{\partial \ddot{u}} \ddot{u} + \ldots + \frac{\partial H}{\partial \ddot{u}^{n-1}} u^n - M(\dot{x}) \text{sgn}(\dot{v}'(t) - \dot{h}) \]  

(247)

Therefore,

\[ \dot{e}_1(t) = h_{i+1} - m_i(\dot{x}) \text{sgn}(\dot{v}_i(t) - \dot{h}_i) \quad \text{for } i = 1, \ldots, n \]  

(248)

if \(|h_i(t)| \leq m_i(\dot{x})\), then sliding occurs in the first row of equation (248) and during sliding,

\[ e_1(t) = h_1(x) - h_1(\dot{x}) = 0 \]  

(249)

By the method of equivalent control,

\[ h_2(t) = m_i(\dot{x}) \text{sgn}(\dot{v}_i(t) - \dot{h}_i), \text{ or } \dot{v}_2 = h_2(t) \]  

(250)

Again, if \(|h_i(t)| \leq m_i(\dot{x})\), sliding occurs in the second row of equation (248) and \( e_2(t) = 0 \).

In the same manner sliding occurs in every row of equation (248) and \( e(t) = z(t) - \ddot{z}(t) = 0 \). By increasing \( m_i(\dot{x}) \) convergence can be achieved in any desired time interval.

2.3. Systems With Uncertain Parameters

Consider the following nonlinear system with parameters or slowly varying inputs that are unknown or unmeasured.

65
\[
\dot{x} = f(x) + B(x)u + g(x, \theta) \quad \text{(251)}
\]

\[
y = h(x) \quad \text{(252)}
\]

where, $\theta \in \mathbb{R}^m$ is a vector of unknown parameters or inputs.

If the $k_{th}$ derivative of $\theta(t)$, denoted by $\theta^{(k)}(t) = 0 \forall t$ (e.g. if $\theta(t)$ is a ramp input, $\theta'(t) = c$, where $c$ is a constant, and $\theta^{(2)}(t) = 0$), the above described observer structure can be modified as follows, to estimate both the states and the unknown inputs.

define an augmented state vector $x_{\text{aug}}$ as follows.

\[
x_{\text{aug}} = \begin{bmatrix} x \ T \ \theta \ \theta^1 \ \cdots \ \theta^k \end{bmatrix}^T \quad \text{(253)}
\]

Then,

\[
\dot{x}_{\text{aug}} = \begin{bmatrix} \dot{x} \ T \ \theta^1 \ \theta^2 \ \cdots \ \theta^k \ 0 \end{bmatrix}^T \quad \text{(254)}
\]

\[
y = h(x) \quad \text{(255)}
\]

Now the observer can be designed for the augmented system just as done for the original system and the augmented observer estimates both the states and the unknown parameters or inputs of the system.

3. Discrete Sliding Observer Design

As discussed in the review of sliding mode theory the design of sliding mode observers in the continuous time is well established and some applications of the same have been demonstrated. In this study the use of discrete crankangle domain powertrain models necessitates the implementation of discrete sliding estimators, which is an area that has received less attention. Hence the following discrete time sliding observer design scheme was developed.

The design of the discrete sliding estimators was performed using the delta approximation; i.e., it is assumed that the following equation holds.
\[ X(k + 1) - X(k) = \Delta T \delta X(k) \]  \hspace{1cm} (256)

where \( \delta X(k) \) approximates the derivative of the states at the sample instant \( k \) and \( \Delta T \) is the sampling interval.

The difference equation describing the system is

\[ X(k + 1) = f(X(k), U(k)) \]  \hspace{1cm} (257)

where \( U(k) \in \mathbb{R}^m \) is the vector of inputs acting on the system

\[ y = h_1(X) \]  \hspace{1cm} (258)

We define

\[ H = \begin{bmatrix} h_1 & h_2 & \ldots & h_n \end{bmatrix}^T \]  \hspace{1cm} (259)

where

\[ y(k + 1) = h_1(X(k + 1)) = h_2(X(k), U(k)) = f(h_1(X), U(k)) \]  \hspace{1cm} (260)

\[ h_3(X(k), U(k), U(k + 1)) = f(h_2(X, U(k)), U(k + 1)) \]  \hspace{1cm} (261)

\[ h_i(X, U(k), \ldots, U(k + i - 2)) = f(h_{i-1}(X, U(k), \ldots, U(k + i - 3)), U(k + i - 2)), \ i = 3, \ldots, n \]  \hspace{1cm} (262)

Then the observer can be constructed as follows:

\[ \hat{X}(k + 1) = \hat{X}(k) + \left( \frac{\partial H}{\partial \hat{X}} \right)^{-1} \left[ (L - H(\hat{X}, U(k), \ldots, U(k + n - 2))) \right] \]  \hspace{1cm} (263)

where:

\[ L = \begin{bmatrix} L_1 & L_2 & \ldots & L_n \end{bmatrix}^T \]  \hspace{1cm} (264)

\[ L_1 = h_1(\hat{X}) + M_1 \text{sign}(y - h_1(\hat{X})), \text{ and} \]

\[ L_i = h_i(\hat{X}, U(k), \ldots, U(k + i - 2)) + M_i \text{sign}(L_{i-1}(\hat{X}, U(k), \ldots, U(k + i - 2))) - h_i(\hat{X}, U(k), \ldots, U(k + i - 2)), \ i = 2, \ldots, n \]  \hspace{1cm} (265)
where $M_i > 0$ represent the gains of the observer and $L_{eq}$ are low pass filtered versions of $L$. The convergence of this observer may be proven as follows.

**Proof:** Define the non-linear transformation

$$z = H(X,U(k),...,U(k+n-2))$$ (266)

$$\hat{z} = H(\hat{X},U(k),...,U(k+n-2))$$ (267)

If the jacobian of the transformation $H$ is nonsingular in a region $X \in \mathbb{R}^n$ of the state space $H$ is a one-to-one onto transformation. i.e. $H$ is one to one if,

$$\det \left( \frac{\partial H}{\partial \dot{x}} \right) \neq 0 \quad \forall x \in X$$ (268)

Define,

$$e(k) = z(k) - \hat{z}(k)$$ (269)

Since $H$ is one to one, $e(t) \to 0 \Longleftrightarrow \hat{x}(t) \to x(t)$

Let,

$$\hat{z}(k + 1) = L$$ (270)

Then,

$$\hat{z}_1(k + 1) = L_1 = h_1(\hat{X}) + M_1 \text{sign}(y - h_1(\hat{X}))$$ (271)

Using the delta approximation,

$$\hat{z}_1(k) = \frac{\hat{z}_1(k + 1) - \hat{z}(k)}{\Delta T} = \frac{h_1(\hat{X}) + M_1 \text{sign}(y - h_1(\hat{X})) - h_1(\hat{X})}{\Delta T} = \frac{M_1}{\Delta T} \text{sign}(y - h_1(\hat{X}))$$ (27)

$$\dot{e}_1(k) = \hat{z}_1(k) - \dot{z}_1(k) = \frac{h_2(X,U(k)) - h_1(X)}{\Delta T} - \frac{M_1}{\Delta T} \text{sign}(h_1(X) - h_1(\hat{X}))$$ (273)

And,
\[ e_1(k) = z_1(k) - \hat{z}_1(k) = h_1(X) - h_1(\hat{X}) \]  

(274)

If,

\[ h_2(X, U(k)) - h_1(X) < M_1 \]  

(275)

\[ \dot{e}_1(k)e_1(k) = \left[ \frac{h_2(X, U(k)) - h_1(X)}{\Delta T} - \frac{M_1}{\Delta T} \text{sign}(h_1(X) - h_1(\hat{X})) \right](h_1(X) - h_1(\hat{X})) < 0 \]  

(276)

This is the condition for the sliding mode to be achieved in the first transformed state and \( e_1(k) \to 0 \).

By the principle of equivalent control,

\[ \dot{z}_1(k) = \left[ \frac{M_1}{\Delta T} \text{sign}(h_1(X) - h_1(\hat{X})) \right]_{eq} \]  

(277)

\[ \frac{z_1(k + 1) - z_1(k)}{\Delta T} = \left[ \frac{M_1}{\Delta T} \text{sign}(h_1(X) - h_1(\hat{X})) \right]_{eq} \]  

(278)

\[ z_1(k + 1) = \left[ h_1(X) + M_1 \text{sign}(h_1(X) - h_1(\hat{X})) \right]_{eq} = \left[ h_1(\hat{X}) + M_1 \text{sign}(h_1(X) - h_1(\hat{X})) \right]_{eq} \]  

(279)

\[ h_2(X, U(k)) = L_{eq} \]  

(280)

In a similar fashion it can be shown that

\[ h_i(X, U(k), ..., U(k + i - 2)) = L_{(i-1)eq} \]  

(281)

and that sliding occurs successively in the transformed states \( z_1, ..., z_n \).

Since \( H \) is one-to-one,

\[ \hat{z} \to z \leftrightarrow \hat{x} \to x \]  

(282)

and convergence of all the observed states is achieved.

Now assuming that the input \( U \) is constant between samples,
\[
\dot{z}(k) = \frac{\partial H(\hat{X}(k), U(k), \ldots, U(k+n-2))}{\partial \hat{X}} \dot{\hat{X}}(k)
\] (283)

Using the delta approximation to the derivatives,

\[
\hat{z}(k+1) - \hat{z}(k) = \frac{\partial H(\hat{X}(k), U(k), \ldots, U(k+n-2))}{\partial \hat{X}} (\hat{X}(k+1) - \hat{X}(k))
\] (284)

\[
\hat{X}(k+1) = \hat{X}(k) + \left(\frac{\partial H}{\partial \hat{X}}\right)^{-1} (\hat{z}(k+1) - \hat{z}(k))
\] (285)

\[
\hat{X}(k+1) = \hat{X}(k) + \left(\frac{\partial H}{\partial \hat{X}}\right)^{-1} (L - H(\hat{X}, U(k), \ldots, U(k+n-2)))
\] (286)

Which is the required observer.

4. Conclusion

This chapter presented two implementation schemes for sliding mode observers, the first for continuous and the second for discrete time system models.

The continuous time sliding observer design extends an earlier proposed design to a limited class of systems, with inputs whose derivatives are known. While, obtaining input derivatives may, in general, be difficult, in some cases, such as when controller generated commands are the only inputs entering the system, this scheme may be useful. This observer design will be applied to the diagnosis of a vehicle steering system and the results presented later in this study demonstrate its effectiveness.

This chapter also presented the development of a new discrete time sliding observer implementation and provides a proof of convergence of the observer error dynamics. The advantage of the discrete time sliding observer is that it is much less demanding computationally and is thus more suited to implementation on-board the automobile. The performance of the observer is validated later in this study by application to an automobile powertrain system.
CHAPTER 5
APPLICATION OF NONLINEAR ARMAX MODELING FOR IC ENGINE MONITORING

1. Introduction

On-board health monitoring of automobile components and subsystems has recently become an increasingly important issue for the automotive industry, due to the following reasons: (i) on-board diagnostic capabilities enhance the maintainability and repairability of vehicles (ii) compliance with increasingly stringent federal and state mandates requiring the reduction of exhaust gas emissions is facilitated by the ability to recognize and compensate for incipient faults in the fuel supply-combustion-exhaust system and (iii) the increasing capabilities of on-board computers and vehicle controllers has made feasible the implementation of extremely sophisticated diagnostic and control algorithms. Representative works of on-board diagnosis may be found in the references (Rizzoni and Min, 1991; Costin, 1991; Gertler et al., 1991, 1993).

The development of linear Parity Equation Residual Generation (PERG) schemes (Potter and Suman, 1977; Mironovskii, 1979; Chow and Willsky, 1984; Gertler and Singer, 1990; Gertler, 1991), provided a Fault Detection and Isolation (FDI) scheme, that was simpler to implement, while providing all the capabilities of, Observer Based Residual Generation (OB RG) schemes. However both the PERG and OBRG schemes are applicable only to linear systems and hence are unsuited to the problem of health monitoring of the significantly nonlinear automobile systems. Only a limited number of FDI schemes have been developed for use on nonlinear systems. Recently Hengy and Frank (1986), Seliger and Frank (1991) and Himmelspach (1992) have all proposed
residual generation schemes that employ the theory of nonlinear observers, but these schemes are complex and apply only to systems that satisfy restrictive assumptions. Gertler et al. (1993) applied the parity equation residual generation scheme to the diagnosis of systems modeled by a linear dynamic core with static input and output nonlinearities (sometimes referred to as a hybrid or Hammerstein model). The scheme was employed for monitoring an automotive engine and was shown to be sensitive to a number of sensor and actuator faults.

This chapter presents the use of the Nonlinear AutoRegressive Moving Average modeling with eXogenous inputs (NARMAX) technique of system identification to implement a newly proposed (Krishnaswami and Rizzoni, 1994) Nonlinear Parity Equation Residual Generation (NPERG) FDI scheme, for diagnosing faults in an automobile engine during execution of the United States Environmental Protection Agency (EPA) Inspection and Maintenance (IM) 240 driving cycle. The NPERG scheme uses dynamic models that can explain the relationship among different system variables so as to be able to: (i) predict what system outputs (e.g., engine speed and torque, exhaust gas oxygen sensor voltage) should be measured given a certain set of inputs (e.g., throttle position, fuel mass); and (ii) estimate the inputs that should under normal conditions have taken place to result in a measured set of outputs. This requires the development of "forward" and "inverse" models of various engine subsystems, and the reduction of these models to a computationally manageable form to permit real-time implementation, either on or off-board. The NARMAX technique is used to identify these models. The identification procedure is designed to ensure that the models obtained are valid over the range of operating conditions the engine experiences during the IM240 cycle. The NPERG scheme then uses these models to generate residuals which produce characteristic signatures in the presence of faults, allowing the development of a fault-detection logic that isolates the fault. The generated residuals are also decoupled from the unmeasured load torque input that the engine operates against (as proposed in Gertler et al., 1991 and 1993). This represents a significant advance in automobile diagnostic applications since the load torque
is always present during vehicle operation and is difficult or expensive to measure. Many hitherto proposed automotive FDI schemes simply ignore the load torque, thereby restricting their applicability to narrow ranges of vehicle operating conditions such as during engine idling, when the load torque may be ignored. Experimental results are provided to demonstrate that the NPERG scheme successfully isolates actuator and sensor faults in the engine and is robust to the unknown load torque.

This chapter is organized as follows. Section 2 briefly presents the NARMAX (Nonlinear AutoRegressive Moving Average model with eXogenous inputs) identification technique and Section 3 discusses the application of the technique to the problem of modeling the dynamic behavior of the engine. Section 4 presents the proposed residual generation and fault detection and isolation scheme. Section 5 presents experimental results used to validate the FDI scheme and Section 6 summarises the contributions of this study.

2. The Nonlinear ARMAX Method

The NARMAX (Nonlinear AutoRegressive Moving Average model with eXogenous inputs) method, proposed by Leontaritis and Billings (1985a, 1985b), is an extension of the linear ARMAX system modeling technique, which allows the development of models for nonlinear systems. Essentially, it is a discrete-time modelling technique that approximates the nonlinear function that governs the dynamics of a system, by a combination of polynomials of input and output variables. Presented below is a very brief description of the modeling technique.

Consider the following input-output, discrete-time representation of a dynamic system.

\[ y(k) = f(y(k-1), \ldots, y(k-n_y), u(k), \ldots, u(k-n_u)) \]  

where \( y \in \mathbb{R}^p \) is a vector of outputs, \( u \in \mathbb{R}^m \) is a vector of inputs, \( n_y \) and \( n_u \) are the number of delays in \( y \) and \( u \) respectively and \( f(\cdot) \) is some nonlinear function. The
NARMAX method tries to approximate the function $f(.)$ using polynomial combinations of the arguments of the function.

Denote the arguments of $f(.)$ appearing in equation (1) by the following vector.

$$P = \left[ y(k-1), \ldots, y(k-n_y), u(k), \ldots, u(k-n_u) \right]$$  \hspace{1cm} (288)

Let $P_i$ denote the $i_{th}$ element of the vector. Then the NARMAX model of the system is,

$$y(k) = \sum_{i=1}^{ny+nu+1} \theta_i P_i^1 + \sum_{j=1}^{ny+nu+1} \sum_{j=1}^{ny+nu+1} \theta_{ij} P_i P_j + (h.o.t.)$$  \hspace{1cm} (289)

where the $\theta$'s are parameters which the NARMAX method identifies and (h.o.t.) stands for higher-order terms. The series in equation (3) can contain as many terms as are necessary to achieve a desired level of accuracy. Thus the NARMAX structure is one in which nonlinear functions of the physical (measured input and output) variables form a set of basis functions, a linear combination of which is used to approximate the nonlinear function that represents the physical behavior of the system to be identified. Thus the NARMAX technique results in a linear in the parameters identification scheme.

3. Identification of Engine Models

3.1. The EPA IM240 Driving Cycle

The IM240 cycle is a driving test schedule developed for the purpose of testing compliance with federal vehicle emission standards for carbon monoxide (CO), unburned hydrocarbons (HC), and nitrogen oxides (NOx). This test is ideal for use as part of a vehicle health monitoring scheme for the following reasons: (i) the test is designed to evaluate the performance of the vehicle under real world conditions and is intended to be performed on in-use vehicles; (ii) since the EPA proposes to make the test mandatory for all vehicles, performing a simultaneous diagnostic analysis on the vehicle is an economical method of monitoring the health of the automobile subsystems. The IM240 cycle is designed to be performed under laboratory conditions, on a chassis dynamometer and is
patterned on a cycle known as the Urban Dynamometer Driving Schedule (UDDS). The UDDS itself approximates a typical morning trip in Los Angeles. Figure 1 shows the time speed plot of the IM240 cycle.

3.2. The Engine Test Cell

All the experiments for acquiring the input-output identification data and verifying the identified model were conducted on a Ford V-6 engine, mounted in an engine test cell. The engine, its associated instrumentation and the test facilities are described below.

A Ford 3.0 l V-6 engine driven by an electric dynamometer is used in the experiments. The specifications (dimensions) of the major components of the engine are tabulated in Table 5.1.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>3.0 l</td>
</tr>
<tr>
<td>Number of Cylinders</td>
<td>6</td>
</tr>
<tr>
<td>Bore</td>
<td>89.00 mm (3.50 in)</td>
</tr>
<tr>
<td>Stroke</td>
<td>80.00 mm (3.14 in)</td>
</tr>
<tr>
<td>Firing Order</td>
<td>1-4-2-5-3-6</td>
</tr>
<tr>
<td>Combustion Chamber</td>
<td>50.1-47.1 cc</td>
</tr>
<tr>
<td>Rod Connecting Length</td>
<td>140.46-140.54 mm (5.53-5.533 in.)</td>
</tr>
</tbody>
</table>

Table 5.1. Engine Specifications

Air charge temperature sensor (ACT), throttle position sensor (TPS), mass air flow sensor (MAF) are installed in the engine to measure the air charge temperature, throttle position and air mass flow rate. Two heated exhaust gas oxygen sensors (HEGO)
are located in the exhaust pipes upstream of the catalytic converter. The resultant airflow information and input from the various engine sensors are used by the engine controller (EEC-IV) to compute the required fuel flow rate necessary to maintain a prescribed air/fuel ratio for the given engine operation. This fuel flow rate is then used to calculate the needed injector pulse width from

![Graph](image)

Figure 5.1. The IM240 driving schedule

knowledge of the fuel injector calibration constant and injector opening time delay. The fuel injectors are operated in a sequential fire mode except during engine startup. The engine control strategy does not use Exhaust Gas Recirculation (EGR). An Engine Control Module (ECM) break-out box links the EEC-IV controller and the data acquisition system.

The dynamometer is controlled by a Dyne systems DYN-LOC IV dynamometer speed/torque controller in conjunction with a DTC-1 throttle controller. The throttle controller, acting on a pulsewidth modulated solenoid throttle actuator ACT-137, allows excellent regulation of throttle position by means of closed-loop feedback, permitting simultaneous load and speed control by acting as a slave to the dynamometer controller. A load cell model provides the needed measurement of average and instantaneous load...
torque and a magnetic pick up sensor, mounted above a 60-teeth gear, provides the measurement of dynamometer speed.

3.3. Experimental Work:

Since the diagnostic algorithms are intended for use in conjunction with the IM240 cycle, the engine models identified must be valid over the operating points through which the engine passes while executing the cycle. This was ensured in the following manner (Luh et al., 1994).

First the engine was made to execute the IM240 cycle using the dynamometer and throttle controllers. The IM240 cycle specifies vehicle speed throughout the cycle and hence the throttle controller was operated in the speed control mode. A load torque curve was generated from the vehicle speed using a standard formula relating vehicle parameters (inertia, drag coefficient etc.,) and speed to load, (Heywood, 1988). The cycle was then executed and the throttle opening values imposed by the controller were measured over the cycle. For generating identification data, this acquired throttle data and the designed load torque were used in designing the inputs to the engine, as described below.

The accuracy of estimation of the parameters of an unknown system is greatly affected by the excitation input. A pseudo random binary sequence (PRBS) or binary noise is frequently used for linear system identification purposes. Tulleken (1990) develops the generalized binary noise test-signal to improve the identification-experiment design and Schoukens and Pintelon (1992) suggest using multisine signals in FFT based spectrum estimation. However, Leontaritis and Billings (1987b) showed that the use of PRBS inputs to excite nonlinear systems can cause loss of identifiability. They suggest that the excitation input should be an independent Gaussian sequence for a power constraint on the input or an independent uniformly distributed sequence for an amplitude constraint on the input for a completely unknown nonlinear system. Billings et al. (1988) also suggests an input whose operating point is changed in steps over the whole amplitude range of interest and superimposed on these levels with uniformly distributed or similar
signal which has a bandwidth sufficient to excite all the modes of the dynamic subsystems within the plant.

In accordance with the method of Billings et al., (1988) it was decided that a uniformly distributed random signal was added to the throttle and load torque inputs required to take the engine through the IM240 cycle. The uniform random signals had a range of approximately 20% of the maximum values of the throttle and torque signals used in the cycle. The input signals are shown below in Figures 5.2 and 5.3.

![Figure 5.2. Throttle input (IM240 identification)](image-url)
The resulting dynamometer speed (which corresponds to vehicle speed) under the
designed inputs is shown in Figure 5.4. It should be noted that the shape of the curve
does not exactly match the IM240 cycle in Figure 5.1, due to the limitations of the
dynamometer and controllers used in the experiments. However the cycle as used in this
study is a reasonably good approximation of the actual cycle, is repeatable and serves to
adequately demonstrate the identification and fault detection concepts of this study. The
results of this study can easily be reproduced for the actual IM240 cycle given the
availability of a chassis dynamometer.

The inputs, disturbances and the measured outputs used in the FDI scheme are
listed in Table 2.
Figure 5.4. Dynamometer speed under the designed throttle and torque inputs.

The dynamometer speed sensor was assumed to be fault free. All the variables in the models, such as throttle opening, engine speed etc., were normalized (scaled) so that their range of variation is between 0 and 1. The variables used in the FDI scheme are listed in Table 5.2.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Throttle Opening ($\theta$)</td>
<td>Input</td>
</tr>
<tr>
<td>Mass Fuel ($mf$)</td>
<td>Input</td>
</tr>
<tr>
<td>Load Torque ($T_l$)</td>
<td>Unknown</td>
</tr>
<tr>
<td></td>
<td>Disturbance</td>
</tr>
<tr>
<td>Engine Speed ($\omega$)</td>
<td>Output</td>
</tr>
<tr>
<td>Mass Airflow ($ma$)</td>
<td>Output</td>
</tr>
</tbody>
</table>

Table 5.2. Variables used in the FDI scheme
4. Design of the FDI scheme

4.1. Residual Generation

The residuals are generated using the NPERG scheme proposed in (Krishnaswami and Rizzoni, 1994). It was found that the load torque affects only the engine speed output and hence only this estimate has to be decoupled from the load torque. A schematic of the residual generator is given in Figure 5.5. The $[\theta \ m_f]$ inverse model takes as inputs the measured outputs $\omega$ and $m_a$ and estimates the throttle opening $\theta$ and the fuel input $m_f$, while the $T_l$ inverse model estimates the load torque from the measured dynamometer speed and mass air flow rate $m_a$. The forward model in the diagram takes the nominal values of the command inputs $\theta$ and $m_f$ and the estimated load torque, and produces as output estimates of the engine speed $\omega$ and the mass air flow rate $m_a$. Note that the forward residual is

$$ r_f(k) = \begin{bmatrix} r_\omega & r_{ma} \end{bmatrix} = \begin{bmatrix} \omega^* - \hat{\omega} & m_a^* - \hat{m}_a \end{bmatrix} \text{ and } r_i(k) = \begin{bmatrix} r_q & r_{mf} \end{bmatrix} = \begin{bmatrix} \theta^* - \hat{\theta} & m_f^* - \hat{m}_f \end{bmatrix}, $$

the inverse residual where the '*' denotes the measured or nominal value of the variable and the '-' denotes the estimate.
Assumptions: The following assumptions are made.

(i) Only sensor and actuator faults occur.

(ii) Only one fault is present at any one instant.

Then the following reasoning allows fault isolation.

Case 1: Input ($\theta$ or $m_f$) faults. In this case the nominal or commanded value of the faulty input differs from the actual value that the engine sees. Then both the estimates of the forward model ($\hat{m}_a$ and $\hat{\omega}$) will be wrong (since the forward model uses the faulty nominal value of the input) and will hence differ from their corresponding measured values, as a result of which both elements of the forward residual ($r_f$) will be non-zero. The estimates of the inverse model ($\hat{\theta}$ and $\hat{m}_f$) are both correct, since the assumption that only one fault is present at any one time implies that the inverse model uses the correct
measured values of the outputs to estimate the inputs. Hence, only the nominal value of
the faulty input will differ from its estimate, as a result of which only one of the two
elements of the inverse residuals $r_\theta$ or $r_{mf}$ (whichever is faulty) becomes non-zero, and this
allows the input fault to be isolated.

Case 2: Sensor ($m_a$, $ma$ or $\omega$) faults. In this case the measured value of the variable
from one of the sensors differs from the actual value of the engine output. If the engine
speed sensor is faulty ($\omega$ fault), reasoning similar to that of the input fault case can be
used to see that in this case both the elements of $r_\omega$ are non-zero while only the $r_\omega$
element of the $r_f$ residual vector is non-zero, allowing the $\omega$ fault to be isolated. If the
mass air flow sensor is faulty ($m_a$ fault), however, the load torque is wrongly estimated
and hence in this case both the elements of $r_f$ are non-zero (due to the faulty estimates of
the outputs arising from usage of the wrong load torque estimate by the forward models)
and both elements of $r_\omega$ are non-zero (due to the faulty $m_a$ value used by the $\theta$ and $m_f$
inverse model). Thus the non-zero condition of all the residual elements of the $r_f$ and $r_\omega$
vectors leads to isolation of the $m_a$ fault.

<table>
<thead>
<tr>
<th>Fault Location</th>
<th>Non-zero elements of $r_f$</th>
<th>Non-zero elements of $r_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_a$ sensor</td>
<td>all</td>
<td>all</td>
</tr>
<tr>
<td>$\omega$ sensor</td>
<td>$r_\omega$</td>
<td>all</td>
</tr>
<tr>
<td>$\theta$ input</td>
<td>all</td>
<td>$r_\theta$</td>
</tr>
<tr>
<td>$mf$ input</td>
<td>all</td>
<td>$r_{mf}$</td>
</tr>
</tbody>
</table>

Table 5.3. Fault isolation logic for engine
Thus, occurrence of a fault in any one of the sensors or actuators gives rise to a unique residual signature which allows the fault to be simultaneously detected and identified. The above described FDI decision logic is summarised in Table 3 for the engine FDI residual generator.

It should be noted that in the above FDI scheme an estimate of the load torque is substituted for the actual value or measurement of the load torque used by the forward model, in order to eliminate the sensitivity to the load torque disturbance of the generated residuals. This method of decoupling is termed indirect decoupling. Krishnaswami and Rizzoni (1994) explain in greater detail the concepts of indirect and direct decoupling of residuals from unmeasured disturbances.

4.2. Thresholds

As with any modeling procedure the engine models developed by the NARMAX method are only approximations of the actual dynamics. Therefore the residuals from the residual generator are not always zero valued even under normal no-fault operation. Hence thresholds are set, which the residuals must exceed for a non-zero condition to be triggered. The thresholds were determined empirically, by running the IM240 cycle under no fault conditions and observing the range of variation of the residuals. In order to not make the fault isolation procedure very conservative a threshold is considered to have been exceeded if the residual is greater than the threshold value for at least 80% of the time window over which the diagnostics are performed. The threshold values for each of the variables used in the FDI scheme are given in Table 4. Please note that the thresholds are given as percentages of the full scale (maximum) values of the variables over the IM240 cycle.
Variable | Threshold
---|---
Mass air | 4%
Engine Speed | 4%
Throttle Opening | 1.5%
Mass Fuel | 2.5%

Table 5.4. Threshold values for the FDI scheme

Due to the non-zero threshold values, only faults that exceed a certain magnitude can be detected. The minimum magnitude that can be detected for each fault was determined through simulation and is listed in Table 5, in terms of percentage deviation from the nominal value. Smaller faults may give rise to missed detections. Increasing the accuracy of the identified models will allow setting of tighter thresholds and thus allow detection of smaller faults.

<table>
<thead>
<tr>
<th>Fault Location</th>
<th>Minimum detectable fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_a$ sensor</td>
<td>±10%</td>
</tr>
<tr>
<td>$\omega$ sensor</td>
<td>±20%</td>
</tr>
<tr>
<td>$\Theta$ input</td>
<td>±15%</td>
</tr>
<tr>
<td>$m_f$ input</td>
<td>±40%</td>
</tr>
</tbody>
</table>

Table 5.5. Minimum detectable fault magnitudes
5. Results

Shown below are the outputs of the residual generator for two faults: (i) throttle (q) calibration fault and (ii) engine speed (w) sensor calibration fault. Note that in all plots only the residuals corresponding to a portion of the cycle approximately 1 minute long are shown.

5.1. Throttle (θ) calibration fault

The throttle calibration factor was changed by 15%, i.e., the actual throttle value is 1.15 times the nominal value. The residuals generated are shown in Figs. 6 - 9. The residuals in Figs. 6, 7 and 9 lie outside the threshold regions, indicating a non-zero condition, while the residual in Figure 5.8 lies inside the threshold, which is a zero condition. From these residuals and the fault isolation logic in Table 3 the fault can be diagnosed as a throttle fault.

![Figure 5.6. Mass air residual (throttle fault)](image)
5.2. Engine speed sensor ($\omega$) calibration fault

The calibration of the engine speed sensor was changed by +20%, i.e., the measured value is 1.20 times the actual value. The resulting residuals are shown in Figs. 10-13. Note that
in Figure 5.12 the residual dips back into the threshold region for a small portion of the FDI window. As mentioned earlier, the residual will be considered to be non-zero, since it stays outside the threshold region for over 80% of the observation window. Again the fault can be correctly diagnosed as a speed sensor fault.

![Figure 5.10. Mass air flow residual (ω sensor fault)](image1)

![Figure 5.11. Mass fuel residual (ω sensor fault)](image2)
6. Conclusion

The significant contributions of this chapter may be summarised as follows.

The NARMAX system identification technique was applied to the problem of modelling the inherently nonlinear dynamic behavior of an Internal Combustion (IC) engine. The models obtained are valid throughout the wide range of operating conditions that the engine experiences during execution of the IM240 cycle.

An FDI scheme (the NPERG scheme) that takes advantage of the identified models was implemented to diagnose sensor and actuator faults. The diagnostic scheme is valid over the entire range of engine operation and further is robust to the presence of the
unmeasured load torque disturbance input. This robustness to the load torque input is of great significance since most hitherto developed automotive FDI schemes simply ignored this disturbance, greatly restricting their applicability.

The effectiveness of the scheme was demonstrated experimentally on a production Ford V-6 engine. Results of the experiments have been provided for, (i) throttle position (actuator) fault; and (ii) speed sensor (output) fault. The experiments show that the proposed FDI technique successfully isolates the faults and is robust to the load torque disturbance.

7. Future Work

This chapter considers only a subset of the engine sensors and actuators actually present in a production engine (manifold pressure sensor, exhaust gas Oxygen sensor, etc., were not considered). The faults for which the results are presented are not those that occur most commonly in real life vehicle operation. However, the examples clearly demonstrate the effectiveness of the NPERG scheme and application of the diagnostic algorithm to other fault modes is not difficult and is currently being implemented.

Some issues that have to be resolved before the abovediagnostic algorithm can be applied on a routine basis are (i) robustness of the models used to vehicle to vehicle variation and aging, and (ii) calibration of the thresholds based on the impact of the faults on emissions or driveability of the vehicle. This will allow the selection of those faults that most significantly affect vehicle performance as targets for the diagnostic algorithm. A study of this issue is already underway and some preliminary results have been published in, Soliman et. al., (1995).

8. Acknowledgement

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CHAPTER 6
CONTINUOUS TIME SLIDING MODE OBSERVERS FOR DIAGNOSTICS -
APPLICATION TO VEHICLE STEERING SYSTEMS

1. Introduction

Much attention has been recently focused on the development of improved surface transportation systems, through the integration of advances in communications, highway construction, computer and automobile technologies, and have been described as Intelligent Vehicle and Highway Systems (IVHS). The fundamental unit of any such system is the individual automobile and hence improvements in vehicle performance and safety through the use of computer based on-board control and diagnostic systems has become an important area of research. Most of these control systems (e.g., Patwardhan and Tomizuka, 1994; Venkataraman and Waldron, 1994) require the accurate knowledge of a number of vehicle states and parameters for effective functioning. However these states may be expensive to measure, or in some cases, such as tire-road friction characteristics, may be impossible to accurately determine using on-board sensors. Thus on-board state and parameter estimation is an indispensable part of any vehicle control system.

The nonlinear nature of vehicle dynamics including tire characteristics necessitate the use of a nonlinear observer design approach for the construction of accurate state estimates. Ray (1992), presented the use of an extended kalman filter to estimate vehicle states and tire forces.
The necessity to estimate vehicle states (for cost and inaccessibility reasons) and the nonlinear nature of the dynamics make it a suitable application for the sliding mode observer design.

This study presents the application of the continuous time sliding mode observer design procedure developed in the previous chapter, to the state estimation problem for a vehicle steering system. The designed state estimator is then used, following the Nonlinear Parity Equation Residual Generation (NPERG) procedure, to construct a fault detection and isolation (FDI) algorithm for the steering system. The estimator and the FDI scheme were validated through experimental testing on a Ford Taurus automobile, at the Transportation Research Center (TRC) of Ohio.

2. The Vehicle Model

A number of different vehicle models of varying complexity are available in the literature (Wong, 1978, Maalej et al., 1989 etc.). The most complex models include vehicle motions in the plane of the road (yaw about a vertical axis passing through the center of gravity of the vehicle, lateral and longitudinal velocity) and the out of plane motions (roll about a longitudinal, horizontal axis and pitch about a lateral, horizontal axis). During maneuvers involving small steering angles and low velocities the out of plane dynamics are not significant and may be neglected with little loss of accuracy. However modeling of severe maneuvers, with combined steering and braking/acceleration require the consideration of all 5 degrees of motion for the generation of accurate results.

For the purposes of this study it was decided that, in order to adequately demonstrate the concepts of observer design and fault isolation without extreme complexity, only the in plane motions (yaw, lateral and longitudinal velocity) would be considered. Accordingly the modeling, system parameter identification and experimental validation were all performed only in the region of vehicle operation in which such a model is valid.
Therefore, a simplified front wheel steered, small angle, bicycle model structure was chosen. All the tests were performed at a constant longitudinal velocity which is assumed to be known (in the tests the longitudinal velocity was computed from angular velocity measurements of the non-driven wheels). Therefore only the dependence of the vehicle lateral velocity and yaw rate on the steering input is modeled. A simplified tire force model was adopted, whereby the lateral forces of the front and rear tires are linearly related to the front and rear slip angles, through $K_f$ and $K_r$ the front and rear cornering stiffnesses. As noted earlier, this tire force model is quite accurate for the small slip angles encountered during normal (non-severe) maneuvering. Note that by non-severe maneuvers, it is meant that the vehicle lateral acceleration during all the tests was maintained below 0.2g, where $g$ is the acceleration due to gravity. The model is given below in equations (290) - (291).

Table 6.2 lists the variables used in the model and the symbols used to denote these variables. Note that the variables are specified in the body coordinate system.

\begin{align}
\dot{v} &= \left(\frac{2}{M}\right) * K_f * \left(\theta - \frac{v + r}{u}\right) - \left(\frac{2}{M}\right) * K_r * \left(\frac{1.69 * r - v}{u}\right) - r * u \tag{290} \\
\dot{r} &= \left(\frac{2}{l}\right) * K_f * \left(\theta - \frac{v + r}{u}\right) - \left(\frac{2 * 1.69}{l}\right) * K_r * \left(\frac{1.69 * r - v}{u}\right) \tag{291}
\end{align}

The mass $M$ and moment of inertia $I$ of the vehicle were obtained from data provided by the manufacturer. The front and rear cornering stiffnesses, for small steering angles on dry pavement was identified from J-turn data at various vehicle speeds. Complete details of the vehicle instrumentation, test and identification procedures may be found in Rye (1996).
VARIABLES | DESCRIPTION
--- | ---
\(u\) | vehicle longitudinal velocity
\(v\) | vehicle lateral velocity
\(r\) | vehicle yaw rate
\(K_f\) | front tire cornering stiffness (dry pavement) 60530 N/rad
\(K_r\) | rear tire cornering stiffness (dry pavement) 64656 N/rad
\(M\) | vehicle mass, 1651 Kg
\(l\) | vehicle moment of inertia, 2755 Kg/m\(^2\)
\(\theta\) | tire steering angle

Table 6.2. List of symbols used in vehicle model

3. Sliding Observer Implementation

The sliding mode observer design procedure described earlier was applied to the problem of estimating vehicle states for the purpose of fault isolation. The design of the sliding observer will be shown in some detail for one case - estimation of vehicle yaw rate, lateral and longitudinal velocities, from the yaw rate measurement.

3.1. Example - estimation of \(r\), \(v\) and \(u\) from measurement of yaw rate \(r\):

All the vehicle maneuvers in this study are performed at approximately constant vehicle longitudinal velocity. Therefore we assume that the longitudinal forces due to the engine torque applied to the wheels and the resistive forces of wind drag and tire rolling resistance are in equilibrium. Under this assumption we augment the vehicle state
equations (290) - (291) with the following differential equation for vehicle longitudinal velocity.

\[
\dot{u} = 0
\]  

(292)

Further since the measured variable is the yaw rate we can write

\[
y = h_1(x) = r
\]  

(293)

where \( x = [v \quad r \quad u]^T \) is the state vector.

Now going through the computations described in the sliding observer design procedure, we obtain

\[
h_2(x) = \frac{(4.394e1*\theta* u - 1.78e2*r + 354e1*v)}{u}
\]  

(294)

\[
h_3(x) = \frac{-(5.225e3*\theta* u - 3.377e4*r + 1.167e4*v + 3.54e1*u^2*r - 4.394e1*\theta* u^2)}{u^2}
\]  

(295)

We then compute the observability matrix (i.e., the jacobian of the transformation \( H = [h_1(x) \quad h_2(x) \quad h_3(x)]^T \)). The computation is straightforward using symbolic mathematics software such as MAPLE® and the jacobian itself will not be shown here.
However, we then compute the determinant of the jacobian for the purposes of checking the observability of the states and the determinant is given below.

\[
\det(H) = -(1.85e4* \theta f^* u - 3.14e5* r + 4.13e5* v) / u^4
\] (296)

It is clear that the determinant is a function of the vehicle operating condition as defined by the steering input and the state variables. The necessary and sufficient condition for observability is that the determinant be non-zero. Therefore, the value of the determinant is computed for various equilibrium points in the state space in the normal region of vehicle operation. The computation is done as follows. Values are assumed for the steering input and the longitudinal velocity. The corresponding steady state values of \( r \) and \( v \) are computed and then the determinant is calculated. The plot of the numerical value of the determinant is shown below in figure 6.4.

![Figure 6.4. Observability Determinant For Vehicle Range of Operation](image)

Figure 6.4. Observability Determinant For Vehicle Range of Operation
Examination of the determinant shows that the determinant is non-zero for normal vehicle operation except when the steering angle is zero. This conforms to intuition about the physics of vehicle behavior, since when there is no steering input the yaw rate dynamics are not excited and can hence provide no information about the vehicle operation. Thus when the steering input is non-zero the system is observable.

It is now necessary to choose the switching gains \( m_i(\hat{x}) \) such that \( m_i(\hat{x}) > |\tilde{h}_{i+1}(\hat{x})| \) for the region of vehicle operation. The maximum value that \( |\tilde{h}_{i-1}(x)| \forall i = 1, 2, 3 \) can be estimated from the fact that \( h_i(x) \) is the \((i - 1)_{th}\) derivative of the output (in this case the longitudinal velocity). Therefore, from knowledge of vehicle dynamics, it is chosen to make \( m_i(\hat{x}) = 100 \forall i = 1, 2, 3 \). Simulation results were used to confirm that this choice of gains performs satisfactorily.

3.2. Vehicle state demonstration: lane change maneuver estimation

The performance of the above designed observer was validated through a number of experiments. Shown here are the results for a lane change maneuver at a forward speed of approximately 25 mph (11.11 m/s). The steering input for the maneuver is shown in figure 6.5. As noted earlier the system is observable only when the steering input is non-zero, therefore the observer states are not updated until the steering input exceeds a certain threshold value \((\theta \geq 0.01 rad)\). In the case of this test this occurs approximately 2 seconds into the test. The initial conditions for all the states \((r, v, u)\) are set to be different from the measured values in order to show clearly the convergence of the observer.
Figure 6.5. Front wheel steering input for lane change maneuver

Figure 6.6. Measured (solid line) and estimated (dashed line) vehicle yaw rate
Figure 6.7. Measured (solid line) and estimated (dashed line) vehicle lateral acceleration
As can be seen from figures 6.6 - 6.8 the observer performs extremely well, converging rapidly to the actual value of the states, and then tracking the states throughout the maneuver. However, the observer is quite computationally intensive and real time implementation will require significant improvement in numerical efficiency.

4. Observer Based FDI for vehicle steering system:

The sliding observer design technique was used for constructing residual generators for steering system FDI. Table 6.3 shows the variables that are considered in the diagnostic scheme along with their classification (actuator, sensor or system parameter or component) and whether or not a measurement is available, on the experimental vehicle. The following assumptions are made for the FDI scheme implementation:

1) Faults occur singly. Since only two measured outputs are available it is possible, according to the NPERG scheme developed earlier, to isolate only single occurrences of faults. Therefore we will make the assumption that faults occur singly and note that multiple simultaneous faults will usually result in residuals that allow fault detection but not isolation.

2) The vehicle is assumed to be on dry pavement. The lateral forces generated by the tires are highly dependent on the tire/road interface conditions. Large variations in the effective coefficient of friction (0.85 to 0.2) can occur between dry roads in good weather and roads in icy conditions. Such conditions will not be considered here since the intention of this study is to demonstrate the FDI technique and its applicability to the vehicle dynamics problem and not to offer a comprehensive solution. However, the scheme can in principle be quite easily extended to include varying tire/road conditions by including the tire cornering stiffnesses as unknown parameters that have to be estimated. The estimation may be performed using the sliding mode technique and in fact is demonstrated in this study itself, although here, any significant deviation is considered to indicate a fault.
<table>
<thead>
<tr>
<th>No.</th>
<th>Variable</th>
<th>Classification</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Throttle position $\theta$</td>
<td>actuator</td>
<td>commanded</td>
</tr>
<tr>
<td>1.</td>
<td>lateral acceleration $a_x$</td>
<td>sensor</td>
<td>measured</td>
</tr>
<tr>
<td>2.</td>
<td>yaw rate $r$</td>
<td>sensor</td>
<td>measured</td>
</tr>
<tr>
<td>3.</td>
<td>$K_f$</td>
<td>parameter</td>
<td>unmeasured</td>
</tr>
<tr>
<td>4.</td>
<td>$K_r$</td>
<td>parameter</td>
<td>unmeasured</td>
</tr>
</tbody>
</table>

Table 6.3. List of variables considered in steering system FDI

Note that the lateral acceleration measured by the accelerometer on board the vehicle is not the time derivative of $v$ the lateral velocity in the body coordinate system. It is rather, the time derivative of the vehicle center of gravity relative to a coordinate system fixed to the ground and hence is actually equal to the following function of the states and steering input.

$$a_x = \left(\frac{2}{M}\right)^* K_f^* \left(\theta - \frac{v + r}{u}\right) - \left(\frac{2}{M}\right)^* K_r^* \left(\frac{1.69*r - v}{u}\right)$$

(297)

Figure 6.9 shows the observer configuration for the vehicle FDI problem. Three observers are constructed each of which uses measured or commanded (denoted by a superscript ‘*’) or nominal (denoted by a subscript ‘o’) values of three of the system variables to estimate the remaining two. Note that the vehicle longitudinal velocity is computed from measurements of non-driven wheel angular velocity. It must be noted that the choice here to estimate only two variables at a time was dictated by the fact that as greater numbers of variables are estimated by a single observer, higher order Lie derivatives have to be used. The number of terms in the Lie derivative increases rapidly.
with order and increases the computational burden of integrating the observer differential
equations. It is thus necessary to find a trade-off between the number of observers used
and the number of variables estimated using a single observer. In this study this choice
was made heuristically. However, it may be of interest in larger problems to derive a
systematic method of making this decision.

\[
\begin{align*}
\hat{r}^* & \quad \hat{v}_1 \\
K_f & \\
K_r & \\
\end{align*}
\]

Figure 6.9. Observers for vehicle FDI

\[
\begin{align*}
\hat{r}^* & \quad \hat{v}_2 \\
\theta^* & \\
K_{ro} & \\
K_f & \\
\end{align*}
\]

\[
\begin{align*}
\hat{r}^* & \quad \hat{v}_3 \\
\theta^* & \\
K_{fo} & \\
K_f & \\
\end{align*}
\]

The following residual vector $R$ is constructed.

\[
R = \begin{bmatrix}
\hat{a}_x - \hat{a}_{x1} & \theta^* - \hat{\theta} & \hat{a}_x - \hat{a}_{x2} & K_{fo} - \hat{K}_f & \hat{a}_x - \hat{a}_{x2} & K_{ro} - \hat{K}_r
\end{bmatrix}
\]

(298)
Table 6.4 lists the fault variables, indicates possible causes or sources of the fault, and indicates the pattern (zero and non-zero elements) that arises in the residual vector for each fault. Note that a zero element is denoted by a '0' and a non-zero element by a '1'.

<table>
<thead>
<tr>
<th>No.</th>
<th>Fault Variable</th>
<th>Cause</th>
<th>Residual Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>wheel steering angle $\theta$</td>
<td>actuator failure</td>
<td>[0 1 0 1 1 1]</td>
</tr>
<tr>
<td>2.</td>
<td>lateral acceleration $a_x$</td>
<td>sensor failure</td>
<td>[1 0 1 0 1 0]</td>
</tr>
<tr>
<td>3.</td>
<td>yaw rate $r$</td>
<td>sensor failure</td>
<td>[1 1 1 1 1 1]</td>
</tr>
<tr>
<td>4.</td>
<td>$K_f$ front cornering stiffness</td>
<td>blowout/incorrect inflation</td>
<td>[0 1 0 1 1 1]</td>
</tr>
<tr>
<td>5.</td>
<td>$K_r$ rear cornering stiffness</td>
<td>blowout/incorrect inflation</td>
<td>[1 1 1 1 0 1]</td>
</tr>
</tbody>
</table>

Table 6.4. Residual pattern for steering system faults

The explanation for the residual patterns is as follows. If any variable is faulty, the estimates, produced by an observer using that faulty variable as an input will differ from the measured or nominal values of the estimated states. Thus for a yaw rate fault the estimates produced by all three observers will be incorrect. Since it is assumed that only one variable is faulty, the measurements and nominal values of the other variables (apart from the yaw rate) are all correct and hence differ from the incorrect estimates giving rise to a residual in which all the elements are non-zero. It is straightforward to use similar reasoning to derive the other residual patterns.

The FDI scheme was validated experimentally through a number of tests performed at the Transportation Research Center (TRC) of Ohio. The vehicle (Ford
Taurus sedan) was initially driven at a constant velocity in a straight line. Then a step change was applied to the steering wheel resulting in the vehicle undergoing a steady turn also referred to as J-turn.

Note that the residual patterns for the $K_f$ and $\theta$ faults are the same, indicating that the two faults are indistinguishable. This is not in general true. However for the tests conducted in this study the above result holds. This can be shown as follows.

Define as the state vector $x = \begin{bmatrix} r & K_f & \theta \end{bmatrix}$. Now if we assume that the other variables are known and derive the determinant of the observability matrix corresponding to this set of states with the yaw rate measurement we obtain,

$$\det(H) = 1.466e^{-6}K_f^2v + r^2 + v^2$$

$$\det(H) = 1.466e^{-6}K_fv$$

Thus we see that the observability matrix is singular if $v = 0$. Therefore in the J-turn tests which are used to validate the FDI scheme, since the vehicle lateral velocity becomes constant, changes in $K_f$ and $\theta$ cannot be distinguished from one another.

5. Experimental Demonstration of FDI scheme

Shown here are the results of two such tests for (i) a miscalibrated yaw rate sensor and (ii) steering actuator fault.

5.1. Yaw rate sensor fault:

The yaw rate sensor calibration was changed by a factor of 0.9 thus inducing a sensor fault in the steering system. The J-turn was performed at approximately 55mph (24.4m/s). The plots show the results of the test starting with the initial straight line run and then during the turning maneuver. As noted earlier, since the system needs a non-zero steering input to be observable the observer states are updated only after the turning maneuver actually begins.
Figure 6.10. Vehicle measured (solid line) and estimated (dashed line) lateral acceleration (Observer 1)

Figure 6.11. Vehicle commanded (solid line) and estimated (dashed line) throttle input (Observer 1)
Figure 6.12. Vehicle measured (solid line) and estimated (dashed line) lateral acceleration (Observer 2)

Figure 6.13. Vehicle commanded (solid line) and estimated (dashed line) front tire cornering stiffness (Observer 2)
Figure 6.14. Vehicle nominal (solid line) and estimated (dashed) rear tire cornering stiffness (Observer 3)

Figure 6.15. Vehicle measured (solid line) and estimated (dashed line) lateral acceleration (Observer 3)
It can be seen that the all the estimates do not coincide with the measured or nominal values of the variables, thus generating the residual pattern corresponding to the yaw rate sensor fault.

5.2. Steering input fault

In this case the commanded steering input is assumed to be 0.8 times the actual steering input. The test maneuver was a J-turn performed at 75mph (33.5m/s).

Figure 6.16. Vehicle measured (solid line) and estimated (dashed line) front wheel steering angle (Observer 1)
Figure 6.17. Vehicle measured (solid line) and estimated (dashed line) lateral acceleration (Observer 1)

Figure 6.18. Vehicle measured (solid line) and estimated (dashed line) front tire cornering stiffness (Observer 2)
Figure 6.19. Vehicle measured (solid line) and estimated (dashed line) lateral acceleration (Observer 2)

Figure 6.20. Vehicle measured (solid line) and estimated (dashed line) rear tire cornering stiffness (Observer 3)
Figure 6.21. Vehicle measured (solid line) and estimated (dashed line) lateral acceleration (Observer 3)

It can be seen that the estimates of vehicle lateral acceleration from observers 2 and 3 coincide with the measured value. All other residuals are non-zero. This corresponds to the residual pattern for a steering input fault or front tire cornering stiffness deviation. As noted earlier the two cannot be distinguished without further information, in the form of extra measurements or assumptions about tire/road conditions etc.

6. Conclusion

The use of sliding mode observers for vehicle steering state estimation and FDI was presented. Vehicle yaw rate, lateral acceleration, unmeasurable but highly variable tire cornering stiffnesses and steering input estimators were constructed using the sliding mode observer design technique developed in the previous chapter. The observers were
used to construct residual generators following the NPERG residual generation scheme. The effectiveness of the observers and FDI scheme were demonstrated through a series of tests performed at the TRC of Ohio.

Some important issues that have to be considered for future implementation of the FDI scheme on-board production vehicles are (i) improvement in computational efficiency of the observer implementation (ii) extension of vehicle model to include out of plane motions (roll and pitch) and more severe maneuvers.
CHAPTER 7

APPLICATION OF DISCRETE SLIDING MODE OBSERVERS TO AUTOMOBILE POWERTRAIN FDI

1. Introduction

Control and diagnostics of automobile powertrain systems is a problem that continues to become more challenging as environmental regulations become ever more stringent. Further, the increased complexity of vehicle systems makes necessary the constant, monitoring of vehicle operation in order to detect and compensate for any faults or abnormal system behavior. In such monitoring it is important to distinguish normal variations in engine performance, due to changes in operating conditions, external disturbances, vehicle to vehicle variability, aging etc., from failures in engine components. This chapter presents a model based approach to the problem of powertrain monitoring.

This chapter is organized as follows: Section 2 describes, in brief, the identification of a model that is suited to the diagnostic problem. Section 3 presents the application discrete time sliding mode observer to the powertrain system. Section 4 presents the use of the observers for powertrain fault detection and isolation (FDI). Section 5 presents a discussion of the work performed in this study and describes current research efforts at the Center for Automotive Research (CAR) at The Ohio State University.

2. Powertrain model identification

This chapter focuses on the diagnostics of that portion of the powertrain that consists of the intake, fueling, combustion and exhaust components. This subsystem is quite complex and exhibits significantly non-linear behavior. Although engine modeling
has received much attention, most modeling has hitherto been performed from the point of view of understanding in a detailed manner the system behavior, or for designing and testing engine controllers in an off-line environment. Examples of such models may be found in Hendricks and Sorenson, (1989), and Hendricks, et. al., (1993).

Many of these models, while achieving considerable fidelity in representing real engine performance, are often much too complex to implement in real time on a production vehicle for either control or diagnostic purposes. It was therefore decided, for the purposes of this study to develop a simplified discrete crank angle domain model that would capture the essential characteristics of powertrain system behavior, while remaining simple enough to be implementable on-board a vehicle.

Towards this end a hybrid system identification approach was taken. By hybrid, is meant that the basic equations describing the system dynamics were derived from the consideration of physical principles, but these equations were then parameterized by constants (such as the discharge coefficient for the throttle) that were identified using empirical identification techniques from experimental data. This approach enables the development of reasonably accurate models that still accommodate physical intuition, which allows inferences to be made about system operation through the monitoring of these physically based constants and system variables.

The equations were first developed in the continuous time domain. They were then transformed to the continuous crank angle domain using chain rule differentiation and then discretised using the backward difference approximation to the derivative.

2.1. Air flow model

The mass flow rate of air at throttle depends on the flow condition: for unchoked flow

$$\frac{p_m}{p_a} = \left[ \frac{2}{\gamma + 1} \right]^{\gamma/(\gamma-1)}$$

(301)
\[
\dot{m}_{a,\text{th}} = \frac{C_{D,h}A_{th}p_a}{\sqrt{RT_a}} \left( \frac{p_m}{p_a} \right)^{\gamma/2} \left( \frac{2\gamma}{\gamma - 1} \left[ 1 - \left( \frac{p_m}{p_a} \right)^{(\gamma - 1)/\gamma} \right] \right)^{1/2}
\]

(302)

for choked flow

\[
p_m \leq \left[ \frac{2}{\gamma + 1} \right]^{\gamma/(\gamma - 1)}
\]

(303)

\[
\dot{m}_{a,\text{th}} = \frac{C_{D,h}A_{th}p_a}{\sqrt{RT_a}} \gamma^{1/2} \left( \frac{2}{\gamma + 1} \right)^{(\gamma + 1)/2(\gamma - 1)}
\]

(304)

Using the conservation of mass in the intake manifold, the manifold dynamics can be described as:

\[
\frac{dm_{a,m}}{dt} = \dot{m}_{a,\text{th}} - \dot{m}_{a,\text{cyl}}
\]

(305)

Using the Ideal gas law, we can write.

\[
\frac{dp_m}{dt} + \frac{\eta_v V_d \omega}{4\pi V_m} p_m = \dot{m}_{a,\text{th}} \frac{RT_a}{V_m}
\]

(306)

In the crank angle domain,

\[
\frac{\omega dp_m}{d\theta} + \frac{\eta_v V_d \omega}{4\pi V_m} p_m = \dot{m}_{a,\text{th}} \frac{RT_a}{V_m}
\]

(307)

The actual mass flow rate of air into cylinder can be computed using the volumetric efficiency (speed density equation):

\[
\dot{m}_{a,\text{cyl}} = \frac{\eta_v p_m V_d \omega}{4\pi RT_a}
\]

(308)

The volumetric efficiency itself was modeled as a polynomial function of engine speed, manifold pressure and throttle opening.
\[ n_v = a_0 + a_1 \omega + a_2 \omega^2 + a_3 P m + a_4 \alpha + a_5 \alpha^2 + a_6 \alpha^3 \]  
(309)

where the \( \alpha 's \) are empirically determined constants.

2.2. Fuel Dynamics (wall wetting model)

The dynamics of the fueling system were modeled as:

\[ \frac{d m_{ff}}{d t} = - \frac{1}{\tau_f} \dot{m}_{ff} + \dot{X} \dot{m}_{fl} \]  
(310)

\[ \dot{m}_{fe} = \frac{1}{\tau_f} \dot{m}_{ff} + (1 - X) \dot{m}_{fl} \]

Again, in the crank angle domain the model becomes,

\[ \frac{d m_{ff}}{d \theta} = - \frac{1}{\tau_f \omega} \dot{m}_{ff} + \frac{X}{\omega} \dot{m}_{fl} \]  
(311)

\[ \dot{m}_{fe} = \frac{1}{\tau_f} \dot{m}_{ff} + (1 - X) \dot{m}_{fl} \]

2.3. Air/Fuel Ratio Dynamics

Using the mass flow rate of air and fuel from equations (308) and (311), the air/fuel ratio in cylinder can be expressed as:

\[ \frac{\dot{m}_{fe}}{\dot{m}_{ac}} = \frac{1}{\tau_f} \frac{\dot{m}_{ff}}{\dot{m}_{ac}} + (1 - X) \frac{\dot{m}_{fl}}{\dot{m}_{ac}} \]  
(312)

Since \( AF = \dot{m}_{ac}/\dot{m}_{fe} \),

\[ \frac{1}{AF} = \frac{1}{\tau_f} \frac{\dot{m}_{ff}}{\dot{m}_{ac}} + (1 - X) \frac{\dot{m}_{fl}}{\dot{m}_{ac}} \]  
(313)

Introducing equivalence ratio \( \phi = \Psi AF \),

\[ \phi = \frac{1}{\tau_f} \frac{\dot{m}_{ff}}{\dot{m}_{ac}} + (1 - X) \frac{\dot{m}_{fl}}{\dot{m}_{ac}} \]  
(314)
2.4. Exhaust Transport Delay and Sensor Dynamics

The delay of the exhaust gas is the sum of cycle delay and transport delay. Therefore, sensor dynamics combined with delay can be expressed as:

\[ \tau_m \frac{d\phi_m}{dt} + \phi_m = \phi(t - t_c - t_t) \]  \hspace{1cm} (315)

In the crank angle domain,

\[ \tau_m \omega \frac{d\phi_m}{d\theta} + \phi_m = \phi(\theta - \theta_c - \theta_t) \]  \hspace{1cm} (316)

Table 6 lists the parameters in the above equations that were determined empirically.

The determination of these parameters is not a trivial task and involved the resolution of many issues, including the performance of both dynamic and static engine tests, with suitably designed inputs that excited the relevant dynamics of the engine, and the means of analysis of the data (both linear and nonlinear least squares techniques). However, the discussion of the identification procedures goes beyond the scope of this chapter and will be left for elaboration in a later publication.
<table>
<thead>
<tr>
<th>No.</th>
<th>Symbols</th>
<th>Description [Units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$P_m$</td>
<td>intake manifold pressure [Pa]</td>
</tr>
<tr>
<td>2.</td>
<td>$P_a$</td>
<td>ambient pressure [Pa]</td>
</tr>
<tr>
<td>3.</td>
<td>$T_a$</td>
<td>ambient temperature [K]</td>
</tr>
<tr>
<td>4.</td>
<td>$C_{d,th}$</td>
<td>throttle discharge coefficient</td>
</tr>
<tr>
<td>5.</td>
<td>$\dot{m}_{a,th}$</td>
<td>mass flow rate of air at throttle [Kg/sec]</td>
</tr>
<tr>
<td>6.</td>
<td>$R$</td>
<td>ideal gas constant [J/ (kg·K)]</td>
</tr>
<tr>
<td>7.</td>
<td>$\dot{m}_{a,cyl}$</td>
<td>mass flow rate of air into cylinder [kg/sec]</td>
</tr>
<tr>
<td>8.</td>
<td>$V_m$</td>
<td>intake manifold volume [m$^3$]</td>
</tr>
<tr>
<td>9.</td>
<td>$V_d$</td>
<td>displacement volume [m$^3$]</td>
</tr>
<tr>
<td>10.</td>
<td>$\eta_v$</td>
<td>volumetric efficiency</td>
</tr>
<tr>
<td>11.</td>
<td>$\dot{m}_{ff}$</td>
<td>fuel flow rate from film [kg/sec]</td>
</tr>
<tr>
<td>12.</td>
<td>$\dot{m}_{fi}$</td>
<td>fuel flow rate from injector [kg/sec]</td>
</tr>
<tr>
<td>13.</td>
<td>$\tau_f$</td>
<td>fuel evaporation time constant [sec]</td>
</tr>
<tr>
<td>14.</td>
<td>$X$</td>
<td>fraction of injected fuel enter into the film</td>
</tr>
<tr>
<td>15.</td>
<td>$\dot{m}_{fc}$</td>
<td>mass flow rate of fuel entering the cylinder</td>
</tr>
<tr>
<td>16.</td>
<td>$\tau_m$</td>
<td>fuel flow rate from film [kg/sec]</td>
</tr>
<tr>
<td>17.</td>
<td>$t_c$</td>
<td>cycle delay [/sec]</td>
</tr>
<tr>
<td>18.</td>
<td>$t_t$</td>
<td>transportation delay [/sec]</td>
</tr>
<tr>
<td>19.</td>
<td>$\phi_m$</td>
<td>measured equivalence ratio</td>
</tr>
<tr>
<td>20.</td>
<td>$\theta_s$</td>
<td>crank-angle sampling interval ($\pi/2$ radians)</td>
</tr>
</tbody>
</table>

Table 5. List of Symbols for engine model
<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$C_{d,th}$</td>
<td>Throttle discharge coefficient</td>
</tr>
<tr>
<td>2.</td>
<td>$\eta$</td>
<td>Volumetric Efficiency</td>
</tr>
<tr>
<td>3.</td>
<td>$\tau_f$</td>
<td>fuel evaporation constant</td>
</tr>
<tr>
<td>4.</td>
<td>$X$</td>
<td>direct entry fraction of fuel</td>
</tr>
<tr>
<td>5.</td>
<td>$\theta_0 + \theta_t$</td>
<td>transport+EGO sensor delay</td>
</tr>
<tr>
<td>6.</td>
<td>$\tau_m$</td>
<td>EGO sensor time constant</td>
</tr>
</tbody>
</table>

Table 6. Empirically determined engine parameters

Once the models were determined they were discretised using the backward difference approximation to the derivative to give the following equations.

\[ P_m(k) = c_1 P_m(k - 1) + c_2 W_{th}(k) \]  
\[ (317) \]

where, 
\[ c_1 = \frac{1}{\theta_0 R T_{air}(k)} \]
\[ c_2 = \frac{\theta_0 R T_{air}(k)}{V_m \omega(k)} \]

\[ W_{cyl}(k) = c_3 P_m(k), \quad c_3 = \frac{\eta V_d \omega(k)}{R T_{air}(k) 4\pi} \]  
\[ (318) \]

where, 
\[ c_4 = \frac{\tau_f w(k)}{\tau_f w(k) + \theta_s}, \quad c_5 = \frac{X \theta_s}{\tau_f w(k) + \theta_s + 1 - X} \] and 
\[ c_6 = -\frac{\tau_f w(k)}{\tau_f w(k) + \theta_s} (1 - X) \]
\[ W_f(k) = c_4 W_f(k-1) + c_5 \dot{m}_f(k) + c_6 \dot{m}_f(k-1) \] (319)

\[ \phi_s(k) = c_7 \phi_s(k-1) + c_8 \frac{W_f(k-44)}{c_3 P_m(k-44)} \] (320)

and, \( c_7 = \frac{1}{\theta_s \tau_e \omega(k)} \), \( c_8 = \frac{\theta_s}{\tau_e \omega(k)} (AF)_s \)

3. Application of discrete sliding mode observer to powertrain state estimation.

The previously developed discrete sliding mode estimator design procedure was applied to the problem of powertrain state and input estimation. Input estimation was performed through augmentation of the list of states with the input that is to be estimated. Two observers were designed as listed in table 7, to estimate different sets of variables for the purposes of residual generation for fault detection.

<table>
<thead>
<tr>
<th>No.</th>
<th>Measured Variable</th>
<th>Estimated Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( P_m )</td>
<td>( P_m ), ( \alpha )</td>
</tr>
<tr>
<td>2.</td>
<td>( \phi_s )</td>
<td>( \phi_s ), ( P_m ), ( m_f ), ( m_f )</td>
</tr>
</tbody>
</table>

Table 7. List of observers

The performance of observer number 2 is presented in figures 22 - 24. The performance of observer 1 will not be shown for lack of space. However the results are similar.
Figure 22. Measured (solid line) and estimated (dashed) $P_m$

Figure 23. Measured (solid) and estimated (dashed) $m_f$
It can be seen that the observers converge very quickly (within a few engine cycles) and track the estimated variables quite accurately. Note that the plot for $m_f$ is not shown because it is an internal variable and cannot be directly measured in an engine.

4. Observer Based Powertrain FDI

The designed observers were configured according to the Nonlinear Parity Equation Residual Generation (NPERG) procedure described in Krishnaswami and Rizzoni, (1994) as shown in figure 25. Note that in the figure '*' and '^' denote measured and estimated variables respectively. The generated residuals are $r_a$ from observer 1 and $r_{pm}$ and $r_{nfi}$ from observer 2.
For compactness of notation, the residual elements will be grouped into a single residual vector \( r = [r_a \, r_{\phi m} \, r_{m\phi}] \). A non-zero residual value will be denoted by a 1 and a zero residual will be denoted in the residual vector by a 0. However, the residuals are never exactly zero even when no faults are present in the system, since there is always some modeling uncertainty, measurement noise, etc. Therefore some non-zero threshold levels are set which have to be exceeded before the residual is considered non-zero. These thresholds are derived empirically by observing the normal deviations of the estimates from the measured variables during several tests under no fault conditions. Making the assumption that only faults occur singly and observing the residual generation structure the following fault isolation logic can be derived as listed in table 8.

In order to test the effectiveness of the FDI scheme, a number of tests were run, in which various faults were induced in the powertrain system and the data was used to generate residuals as described above. Presented here is the result of the test for a faulty fuel injector.

A fuel injector fault was induced by disabling the control input from the engine controller to the fuel injector for a portion of a test run. This corresponds to a fault where one of the engine cylinders fails to receive fuel due to a clogged injector or a problem in the engine electronics. The resulting residuals are shown in figures 26 - 28. Note that the test lasts for approximately 300 engine cycles and that the fuel injector is disabled at the 40th cycle and re-enabled at about the 130th cycle.

<table>
<thead>
<tr>
<th>Fault Location</th>
<th>signature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Throttle input</td>
<td>111</td>
</tr>
<tr>
<td>Injectors</td>
<td>001</td>
</tr>
<tr>
<td>lambda sensor</td>
<td>011</td>
</tr>
<tr>
<td>Pressure sensor</td>
<td>110</td>
</tr>
</tbody>
</table>

Table 8. Error signatures
Figure 26. Throttle position residual (fuel injector fault)

Figure 27. Manifold pressure residual (fuel injector fault)
From the figures, it can be seen that $r_\alpha$ and $r_{\text{rpm}}$ remain within the threshold levels for the entire test, while the $r_{\text{rmi}}$ residual, converges to within the threshold limits initially, goes outside the limits during the presence of the fault and then converges again to '0' once the fault disappears. Thus the residual vector during the faulty portion of the cycle correctly reads $r=[0 \ 0 \ 1]$, indicating the presence of the fuel injector fault.

6. Conclusion

A model based approach to powertrain health monitoring has been demonstrated in this study. A model suited to the diagnostic problem was derived. A design procedure for discrete sliding mode observers, for state and input estimation was presented and applied to the powertrain estimation problem. The sliding observers are used in the NPERG FDI scheme. Experimental results show that the designed observers estimate the powertrain states with good accuracy and the FDI scheme is effective in isolating both actuator and sensor faults. The discrete nature of the models also makes the scheme computationally less demanding and suitable for implementation on the vehicle on-board computer.
Current research focuses on (i) Improving the system models to further reduce the size of faults that can be isolated, (ii) Expanding the model to include more of the powertrain system, such as the evaporation canister, exhaust gas recirculation (EGR) etc., (iii) Improving the numerical implementation of the observers to decrease computational effort and achieve greater robustness.
CHAPTER 8

CONCLUSION

1. Summary

The problem, considered in this study, of detecting and isolating faults in nonlinear dynamic systems, is motivated by the need to maintain safe, reliable and efficient operation of complex systems. While much progress has been made in linear system diagnostics, the monitoring of nonlinear systems has hitherto been performed only on a limited and mostly ad hoc basis.

This study formulated the monitoring problem for the general case of nonlinear dynamic systems classified the various types of faults that may arise and proposed the Nonlinear Parity Equation Residual Generation (NPERG) scheme that is extremely general in its applicability. Further, the problems of nonlinear state estimation and nonlinear system identification, were also studied and some algorithms of particular applicability to automotive systems were developed. Finally, the proposed algorithms and techniques were validated through extensive experimental work, in the form of three automotive application case studies. The contributions of this study are detailed in the following section.

2. Contributions

The contributions of this study are as follows.

2.1. The Nonlinear Parity Equation Residual Generation (NPERG) Scheme

(i) Formulated the Fault Detection and Isolation (FDI) problem for nonlinear dynamic systems and classified faults into three basic types - actuator, sensor and component faults.
The problem formulation showed that the actuator and component faults though considered distinct in linear FDI literature, share many fundamental features and must be treated identically.

(ii) Postulated and proved the necessary and sufficient conditions for fault isolation. A significant feature of the development presented in this study is the fact that multiple simultaneous faults and unknown disturbance inputs are considered throughout the formulation. This overcomes one of the most significant drawbacks of previous schemes that either necessitate ad hoc modifications to account for the presence disturbances or multiple faults or more often simply fail under such circumstances.

(iii) Provided a systematic construction procedure for the FDI residual generation, based on the proofs for the existence of a fault isolation scheme for a given system.

(iv) Throughout the development, formalized many of the recognized, but never rigorously defined or quantified, properties and terms such as analytical redundancy, residual generator, computational effort etc.

2.2. Features of the NPERG scheme

In summary, it must be emphasised that the NPERG scheme is an extremely general scheme, with wide applicability, that overcomes most of the significant drawbacks of hitherto proposed schemes. The following is a list of the features of the NPERG scheme.

(i) Applies fault models that can encompass most faults that occur in dynamic systems including the most commonly considered actuator, sensor and component faults.

(ii) Extremely general in applicability to different types of systems, making very few assumptions on system model structure.

(iii) Applicable to systems acted upon by unmeasured disturbances. This is an extremely significant property, since in many systems disturbance inputs that are difficult or expensive to measure may be present - e.g., load torque disturbance in automobiles.
(iv) Applicable to the general case of multiple simultaneous fault occurrences. The single fault occurrence is a special case, covered by the general framework of the NPERG scheme.

(v) Provides a systematic design procedure, with a priori derivability of the computational needs.

2.3. Sliding Mode Observer Design

(i) Developed a discrete time sliding observer implementation that is much less computationally demanding than the continuous time approach. Provided a proof of convergence. Validated observer performance through application to engine powertrain system.

(ii) Extended the continuous time sliding observer systems to a limited class of systems in which the derivatives of the inputs are known. While obtaining derivatives of the inputs may, in general, be problematic, in some systems, acted upon only by inputs from a controller, it may be possible to compute the derivatives without much difficulty.

(iii) Validated observer performance by application to vehicle steering dynamics system.

2.4. Case Study I - Engine Actuator and Sensor FDI using NARMAX modeling

(i) The NPERG FDI scheme was applied to the problem of engine actuator and sensor diagnosis. The required forward and inverse models were developed by applying the Nonlinear AutoRegressive Modeling with eXogenous (NARMAX) modeling scheme.

(ii) The results demonstrated the viability of the NPERG and NARMAX schemes for the engine FDI problem and successful isolation of faults was performed, with robustness to the presence of the unmeasured load torque disturbance always present in any engine application.

2.5. Case study II - Vehicle Steering System Diagnostics

(i) Validated the performance of the proposed continuous time sliding observer implementation.
(ii) Incorporated a bank of observers into a set of residual generators based on the NPERG scheme and demonstrated viability of the scheme to diagnose steering system faults and to estimate vehicle parameters such as tire cornering stiffness.

2.6. Case Study III - Engine Powertrain Diagnostics

(i) Applied the proposed discrete time sliding observer design procedure to the engine Air-Fuel-Exhaust system. Demonstrated that the discrete sliding observer design is extremely successful for estimation of both the states and the fuel input.

(ii) Used the designed observers to powertrain system monitoring. Demonstrated excellent sensitivity to fault occurrences.

3. Future Work

Some of the issues that require addressing in order to improve the performance of FDI algorithms are listed below.

The FDI implementations in this study took what may be termed the passive approach to robustness to model uncertainty and measurement noise: i.e., the possibility of false alarms was reduced by setting thresholds on the residuals that had to be exceeded before a fault was declared. At the same time this approach trades-off some of the sensitivity to faults, resulting in some missed detections in cases in which the fault is not large enough to trigger residuals that exceed the threshold.

Also, the thresholds were set empirically, by observing the normal variation in the plant measurements and residuals and then setting bounds which resulted in an appropriate level of fault sensitivity.

Thus, two important issues for future research is the implementation of techniques that actively improve robustness and the development of a systematic procedure to establish threshold levels.

Some possible approaches for improving robustness may be to study the design of nonlinear observers with reduced sensitivity to certain errors in the system model or
measurements. Also, the incorporation of probabilistic techniques and schemes borrowed from the field of hypothesis testing and decision theory may also prove fruitful.

The setting of thresholds may be achieved through sensitivity analysis of the system to variations in system parameters and measurements. Some results and references for this may be found in Khalil (1992).

Other important and promising areas of research being pursued at the Powertrain Control And Diagnostics (PCAD) lab at the Ohio State University, are the development of nonlinear system identification techniques, linear and nonlinear observer design with limited measurement information (such as measurements from the Heated Exhaust Oxygen (HEGO) sensor, which has a switching characteristic), and incorporation of expert system or rule based reasoning in fault and control decision making.
LIST OF REFERENCES


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LIST OF MODEL USED IN NARMAX BASED ENGINE SENSOR AND ACTUATOR FDI

The forward and inverse models used in the FDI scheme for the IM240 cycle are listed below.

(i) Engine Speed $\omega$:

$$
\begin{align*}
\omega(k) &= 1.503\alpha(k-1) - 0.4456\omega(k-2) + 0.1238\omega(k-3) + 0.1397\alpha(k-1) \\
&- 0.05992\alpha(k-2) + 0.00275\alpha(k-3) + 0.15m_f(k-1) - 0.1589m_f(k-2) \\
&+ 0.01394m_f(k-3) - 0.0029141 - 0.003961T_C^2(k-2) + 0.0815\omega(k-3) \\
&- 0.8076\omega(k-1)\alpha(k-1) + 0.7116\omega(k-2)\alpha(k-1)
\end{align*}
$$

(ii) Mass air flow $m_a$:

$$
\begin{align*}
m_a(k) &= 0.7717m_a(k-1) + 0.07969m_a(k-2) - 0.06516m_a(k-3) + 0.1246\alpha(k-1) \\
&- 0.3089\alpha(k-2)m_f(k-1) + 0.3435\alpha(k-1)m_f(k-2) - 0.0711m_f^2(k-3) \\
&- 0.01973T_C^2(k-1) + 0.01921m_f(k-1)T_C(k-1)
\end{align*}
$$

(iii) Throttle opening $\alpha$:

$$
\begin{align*}
\alpha(k) &= 0.3206\alpha(k-1) + 0.3849\alpha(k-2) - 0.03272\alpha(k-3) + 0.2611m_a(k-1) \\
&- 0.1467m_a(k-2) + 0.4061\alpha^2(k-1) - 0.105\alpha^2(k-2) + 0.03041 \\
&- 0.4323\alpha(k-2)\omega(k-1) + 0.2965m_a(k-1)\omega(k-2) \\
&- 0.01082m_a(k-3)\omega(k-1)
\end{align*}
$$

(iv) Mass fuel $m_f$:

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\[ m_f(k) = 0.578 \times m_f(k-1) + 0.2 \times m_f(k-2) + 0.03183 \times m_f(k-3) + 0.5798 \times m_a(k-1) \]
\[-0.3581 \times m_a(k-2) - 1.065 \times \omega(k-1) + 0.9017 \times \omega(k-1) + 1.363 \times m_f^2(k-1)\]
\[-0.7617 \times m_f(k-2) \times \omega(k-1) + 0.2444 \times m_a(k-1) \times m_a(k-3)\]
\[-1.19 \times m_f(k-1) \times m_f(k-2) + 1.343 \times m_f(k-2) \times \omega(k-2)\]
\[-0.5419 \times m_f(k-2) \times \omega(k-3) - 0.3542 \times m_f(k-1) \times m_a(k-2)\]

(v) Load Torque \( T_L \):

\[ T_L(k) = 0.4894 \times T_L(k-1) + 0.117 \times T_L(k-2) + 0.00908 \times T_L(k-3) + 0.01618 \times \omega(k-3) \]
\[+ 0.042551 \times T_L^2(k-1) + 1.254 \times T_L(k-1) \times T_L(k-2) + 0.01878 \times \omega_{dyn}(k-3)\]
\[-0.7271 \times T_L(k-1) \times \omega(k-1) + 0.5599 \times \omega(k-1) \times \omega(k-3)\]
\[+ 0.6736 \times T_L(k-2) \times \omega(k-3) - 0.7586 \times T_L(k-2) \times \omega_{dyn}(k-1)\]
\[+ 0.6621 \times T_L(k-1) \times \omega_{dyn}(k-3) - 0.5096 \times \omega(k-2) \times \omega(k-3)\]

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