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SEMILEPTONIC DECAYS OF $D_s^+$ MESON

A Dissertation

Presented in Partial Fulfillment of the Requirements

for the Degree Doctor of Philosophy

in the Graduate School of The Ohio State University

By

Jik Lee, B.S., M.Sc

**********

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1996

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To My Parents
Abstract

Using the CLEO II detector we measure $B(D_s^+ \to \eta e^+ \nu)/B(D_s^+ \to \phi e^+ \nu) = 1.24 \pm 0.12 \pm 0.15$, $B(D_s^+ \to \eta' e^+ \nu)/B(D_s^+ \to \phi e^+ \nu) = 0.43 \pm 0.11 \pm 0.07$ and $B(D_s^+ \to \eta' e^+ \nu)/B(D_s^+ \to \eta e^+ \nu) = 0.35 \pm 0.09 \pm 0.07$. We find the ratio of vector to pseudoscalar final states, $B(D_s^+ \to \phi e^+ \nu)/B(D_s^+ \to (\eta + \eta') e^+ \nu) = 0.60 \pm 0.06 \pm 0.06$, which is similar to the ratio found in non strange $D$ decays.
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It is impossible to acknowledge all of the people who have influenced my life during my years in graduate school. I apologize to those people who are left out here.

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The members of the charm semileptonic decay analysis group had a big impact on my analysis project. In particular, the members of my paper committee (Professor Rollin Morrison, Professor Hiro Tajima, Doctor Donald Fujino and Doctor Vivek Jain) tirelessly scrutinized my analysis and provided invaluable suggestions to the analysis.

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CHAPTER I

Overview of Charm Semileptonic Decays

1.1 Introduction

One of the outstanding problems in charm semileptonic* decays has been the difficulty in computing the ratio of vector to pseudoscalar final states, $B(D \rightarrow K^* \ell^+ \nu)/B(D \rightarrow K \ell^+ \nu)$\cite{1,2} where $\ell^+ \equiv e^+$ or $\mu^+$. The experimental average for this ratio is $0.56 \pm 0.06$\cite{3,4} while most theoretical predictions range from 0.8 to 1.2\cite{5-11}. However, Ball et al. predict $0.50 \pm 0.15$ using the QCD sum rules\cite{12} and the recent ISGW2 model predicts 0.54 for the ratio\cite{13}.

It is important to repeat these measurements for the $D^+_s$, where the initial and final hadrons differ by the substitution of a light quark by a strange quark. In the $D^+_s$ sector only the $D^+_s \rightarrow \phi \ell^+ \nu$ decay has been extensively studied. The Fermilab experiment E653 has seen evidence for the remaining major

---

*As far as semileptonic decays are concerned, a lepton is meant to be either an electron or a muon, but not a tau, throughout this thesis.
$D_s^+$ semileptonic modes, $D_s^+ \rightarrow (\eta + \eta')\mu^+\nu$.\[^{[14]}\] In this thesis, the first measurement of $B(D_s^+ \rightarrow \eta \ell^+\nu)$ and $B(D_s^+ \rightarrow \eta' \ell^+\nu)$, normalized to $B(D_s^+ \rightarrow \phi \ell^+\nu)$, are presented.

1.2 Electroweak Interaction in Standard Model

In the standard model, the electromagnetic and weak interactions are unified into a single electroweak interaction by spontaneous symmetry breaking of the $SU(2)_L \otimes U(1)$ gauge group. Three generations of fermions (leptons and quarks) are assigned to be left-handed doublets and right-handed singlets (except that there exist only left-handed neutrinos). These doublets and singlets are denoted as

\[
\begin{pmatrix}
\nu_e \\
\mu \\
\tau \\
u_e' \\
\mu' \\
\tau'
\end{pmatrix}
\begin{pmatrix}
u_e \\
\mu \\
\tau \\
u_e' \\
\mu' \\
\tau'
\end{pmatrix}
\]

where the primed quark states are the eigenstates of the electroweak interaction.

The electroweak interaction eigenstates of the quarks are different from the mass eigenstates, and the former can be expressed as linear combinations (mixings) of the latter. For two generations of quarks, Cabibbo\[^{[15]}\] suggested a $2 \times 2$ orthogonal matrix parameterized by a single variable, the so-called Cabibbo angle, transforming the mass eigenstates to the weak interaction eigenstates. Kobayashi and Maskawa\[^{[16]}\] generalized Cabibbo's idea to three generations of quarks by defining a $3 \times 3$ unitary matrix parameterized by four independent variables (three angles and one phase). By convention, the three up-type quarks are unmixed and all mixings are expressed in terms of the unitary matrix $V_{CKM}$.
acting on the three down-type quarks:

$$
\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix}
= 
\begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix} \tag{1.1}
$$

where the unprimed quark states are the mass eigenstates.

One of the popular parameterizations of the Cabibbo-Kobayashi-Maskawa matrix (hereafter, the CKM matrix) is

$$
V_{CKM} = 
\begin{pmatrix}
    c_{12}c_{13} & s_{12}d_{13} & s_{13}e^{i\delta_{13}} \\
    -s_{12}c_{23} - c_{12}s_{23}d_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}d_{13}e^{i\delta_{13}} & s_{23}c_{12} \\
    s_{13}d_{23} - c_{13}s_{23}d_{13}e^{i\delta_{13}} & -c_{13}d_{23} - s_{13}s_{23}d_{13}e^{i\delta_{13}} & c_{23}c_{13}
\end{pmatrix} \tag{1.2}
$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$, with generation labels, $i, j$, and $\delta_{13}$ is the CP violating phase. In the limit $\theta_{23} = \theta_{13} = 0$, the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with $\theta_{12}$ identified with the Cabibbo angle.

The electroweak interaction Lagrangian ($L^{ew}$) incorporated with the mixing of down-type quarks in the left-handed doublets can break up into two parts describing the charged and neutral current interactions:

$$
L^{ew} = L^{cc} + L^{nc} \tag{1.3}
$$

$$
L^{cc} = \frac{g}{\sqrt{2}}(J^{\pm}_\mu W^{-\mu} + J^{\pm}_\mu W^{+\mu}) \tag{1.4}
$$

$$
L^{nc} = \frac{g}{\cos \theta_w} J^0_\mu Z^\mu + e J^e_\mu A^\mu \tag{1.5}
$$

where $L^{cc}$ and $L^{nc}$ correspond to the charged and neutral-current Lagrangians, respectively, and $\theta_w$ is the Weinberg angle. The relationship between the weak interaction coupling $g$ and the positron charge $e$ is $g = e \sin \theta_w$.

The charged current can be written as

$$
J^-_\mu = (\bar{u}_L \bar{c}_L \bar{d}_L) \gamma_\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + (\bar{\nu}_e \bar{\nu}_\mu \bar{\nu}_\tau) \gamma_\mu \begin{pmatrix} e^c_L \\ \mu^-_L \\ \tau^-_L \end{pmatrix} \tag{1.6}
$$
while the neutral current can be written as

$$J^0_\mu = \sum_f [\bar{f}_L \gamma_\mu (T^3_{L_f} - Q_f \sin^2 \theta_w) f_L + \bar{f}_R \gamma_\mu (-Q_f \sin^2 \theta_w) f_R]$$  (1.7)

where $\sum_f$ is over all fermions, $T^3_{L_f}$ is the weak isospin eigenvalue of the left-handed fermion field, and $Q_f$ is the charge of the fermion $f$. For both currents, $f_L = \frac{1-\alpha}{2} f$ ($f_R = \frac{1+\alpha}{2} f$) is the left- (right-) handed fermion field.

Since the CKM matrix is a purely theoretical input to the Standard Model, the values of the matrix elements have to be determined by appropriate experiments.

1.3 Dynamics of Charm Semileptonic Decays

The tree level Feynman diagram of the charm semileptonic decay $M \rightarrow X l^+ \nu$ is shown in Figure 1, where $M$ is a pseudoscalar meson containing a $c$ quark and $X$ is a pseudoscalar or vector meson containing the daughter $q$ quark.
from the $c$ quark decay. The dynamics of the charm semileptonic decay will be explained qualitatively with the Dalitz plot, where the population in a certain kinematic configuration is proportional to the decay rate in that configuration. The lepton energy ($E_{\ell}$) in the rest frame of the meson $M$, and the invariant mass squared of the $W^+$ boson ($q^2$) are chosen as the Dalitz variables of the charm semileptonic decay. The $q^2$ is given by

$$q^2 = m_{W^+}^2 = (p_\ell + p_\nu)^2 = (p_M - p_X)^2 = M^2 + m_X^2 - 2 M E_X$$

where $M$ is the mass of the initial meson $M$ and $p$ denotes the four momentum (for example, $p_X$ is the four momentum of the daughter meson $X$). For the simplicity, a lepton is assumed to be massless ($m_\ell = 0$, correspondingly $q^2_{\text{min}} = m_\ell^2 = 0$) in this section.

Figure 2 compares the kinematics of the charm semileptonic decay at high and low values of $q^2$ in the rest frame of the initial meson $M$. The initial meson $M$, which contains a $c$ quark and a spectator quark $\bar{q}$, is shown at the top of Figure 2. At the maximum value of $q^2$, shown in the middle in Figure 2, all of the available energy is taken up by the masses of the $W^+$ boson and the meson $X$, and therefore the $W^+$ boson is produced nearly at rest. The lepton and neutrino are then produced nearly back to back, and the daughter quark $q$ receives no recoil. This "zero-recoil" configuration, where $E_X = m_X$ and $q^2_{\text{max}} = (M - m_X)^2$, is the most favorable for the formation of the meson $X$. The motion of the daughter quark $q$ relative to the spectator quark $\bar{q}$ and gluons is then similar to that before the decay. At the minimum value of $q^2$ shown at the bottom of Figure 2, the lepton and neutrino momenta are parallel to each other, and the final state meson receives the largest recoil against them and initially moves rapidly with respect to the spectator quark $\bar{q}$. For these quarks to form a bound state, gluons must transfer a large momentum between them. As a consequence, the $q^2 = 0$ configuration is the least favorable for the formation of the meson $X$. 
Figure 2

$q^2$ configurations of charm semileptonic decay
Figure 3
A Monte Carlo simulation of the Dalitz plots for \( D_s^+ \rightarrow \eta \ell^+\nu \) and \( D_s^+ \rightarrow \eta' \ell^+\nu \) decays

The Dalitz plots for \( D_s^+ \rightarrow \eta \ell^+\nu \) and \( D_s^+ \rightarrow \eta' \ell^+\nu \) decays, each having a pseudoscalar meson in its final state, are shown in Figure 3. In the massless lepton limit \((m_\ell = 0)\), \( W^+ \) behaves like a spin-1 particle. Therefore, the decay \( D_s^+ \rightarrow \eta \, (\eta') \, W^+ \) must be a \( P \) wave process. The most characteristic feature of these Dalitz plots is that the points populate mostly low \( q^2 \). The dependence of the differential decay rate on \( q^2 \) and \(|p_X|\) explains this feature. The differential decay rate is given by:

\[
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cq}|^2 |p_X|^3}{24\pi^3} |f_+(q^2)|^2
\]

(1.9)

where \( G_F \) is the Fermi coupling constant and \( V_{cq} \) is the CKM matrix element for the \( c \rightarrow q \) process. The magnitude of the meson \( X \)'s momentum can be written as

\[
|p_X| = \sqrt{(M^2 - q^2 + m_X^2)^2 / (4M^2) - m_X^2}.
\]

(1.10)

The form factor \( f_+(q^2) \), which is in essence the overlap integral of the wavefunctions of the initial and final state mesons, represents the probability of the
formation of the meson $X$ as a function of $q^2$. The form factor must be largest at $q_{\max}^2$ and fall off as $q^2$ decreases because the $q_{\max}^2$ ($q_{\min}^2$) configuration is the most (least) favorable for the formation of the final state meson $X$. However, the $|p_x|^3$ factor enhances the decay rate at low $q^2$ where otherwise the small form factor value would lead to a small decay rate. Conversely $|p_x|$ is small near $q_{\max}^2$ so that the $|p_x|^3$ factor suppresses the decay rate at high $q^2$ even though the form factor is large in that region.

The Dalitz plot for the $D_s^+ \rightarrow \phi l^+ \nu$ decay, which has a vector meson in the final state, is shown in Figure 4. In the massless lepton limit, all $S, P$ and $D$ wave processes are allowed for the decay $D_s^+ \rightarrow \phi W^+$. The density of points in the Dalitz plot is highest near $q_{\max}^2$ because the $S$ wave process is allowed, and becomes significantly smaller toward $q_{\min}^2$. It must be noted that the $q^2$ distribution does not peak at $q_{\max}^2$ but somewhat below because at $q_{\max}^2$ the amount of phase space goes to zero.

The aspects of the dynamics that affect the lepton energy ($E_l$) distribution
are: (1) the V-A coupling, (2) the quantum numbers of the particle $X$ and (3) the $q^2$ distribution. The effects of these aspects are discussed below.

A direct consequence of the V-A coupling is that the energy spectra of the lepton and neutrino in $c$ and $b$ quark decays are different. The decays of $c \rightarrow s l^+\nu$ and $c \rightarrow d l^+\nu$ produce $s$ and $d$ quarks predominantly in the helicity state of $\lambda = -1/2$ in association with a positively charged lepton in the pure helicity state of $\lambda = +1/2$. On the other hand, the decays of $b \rightarrow c l^-\bar{\nu}$ and $b \rightarrow u l^-\bar{\nu}$ also produce $c$ and $u$ quarks predominantly in the helicity state of $\lambda = -1/2$, but they are in association with a negatively charged lepton in the pure helicity state of $\lambda = -1/2$. As a result, the collinear configuration in which the lepton recoils against the daughter quark and the neutrino, thus leading to the highest lepton energy, is forbidden for $c$ quark decays, but allowed for $b$ quark decays. Therefore, for the $c$ quark decay, the lepton energy spectrum peaks at a lower energy than the neutrino energy spectrum; the reverse is true for the $b$ quark decay.

The above argument ignores the quantum numbers of the meson $X$, but these are important in determining the lepton energy spectrum of an exclusive decay. The effect of the V-A coupling described above is manifested when $X$ is a vector meson, but is lost when $X$ is a pseudoscalar meson. These effects are best understood by relating, via Lorentz transformation, the distribution of the lepton energy $E_\ell$ (in the $M$ rest frame) to the angular distribution of the lepton in the $W^+$ rest frame.

The angle $\theta_\ell$ is defined as the polar angle of the lepton momentum in the $W^+$ rest frame with respect to the direction of the $W^+$ momentum in the $M$ rest frame. Also $p_{W^+}$ and $p_X$ are defined as the momenta of $W^+$ and the meson $X$, respectively, in the $M$ rest frame ($p_{W^+} = -p_X$). In the massless lepton limit, the lepton energy in the $W^+$ rest frame is $E_\ell^{[W]} = \sqrt{q^2}/2$. From the Lorentz
transformation from the \( W^+ \) rest frame to the \( M \) rest frame, the lepton energy can be written as

\[
E_l = \left( (E_l^{\text{max}} + E_l^{\min}) + (E_l^{\text{max}} - E_l^{\min}) \cos \theta_l \right) / 2
\]

(1.11)

where

\[
E_l^{\text{max, min}} = \left( (M^2 + q^2 - m_X^2) / 2 \pm M |p_X| / (2M) \right).
\]

(1.12)

Therefore, for a given value of \( q^2 \), a lepton moving forward (parallel to \( p_{W^+} \)) has a higher energy than the one moving backward (antiparallel to \( p_{W^+} \)).

As \( q^2 \) increases, the lepton energy tends to increase because

\[
E_l^{\text{max}} + E_l^{\min} = (M^2 + q^2 - m_X^2) / (2M).
\]

(1.13)

However, the range of the lepton energy decreases because \( E_l^{\text{max}} - E_l^{\min} = |p_X| = |p_{W^+}| \) decreases as \( q^2 \) increases, which can be seen from Equation 1.10. At \( q_{\text{max}}^2 \), \( W^+ \) and the meson \( X \) are at rest in the \( M \) rest frame, and the lepton energy is the same for all values of \( \theta_l \). Both of these features, the increase in \( E_l^{\text{max}} + E_l^{\min} \) with \( q^2 \) and the diminishing range in the lepton energy, are simple kinematic effects and can be seen from the Dalitz plots in Figure 3 and Figure 4.

Both the V-A coupling and the quantum numbers of the meson \( X \) govern the \( \cos \theta_l \) distribution. In \( c \) or \( b \) quark semileptonic decays, the daughter quark helicity is predominantly \( \lambda = -1/2 \). If this quark combines with the spectator quark to form a pseudoscalar meson \( X \), as in \( D \to \bar{K}t^+\nu \), \( D_s^+ \to \eta (\eta') t^+\nu \), \( \bar{B} \to D^{*-}\bar{\nu} \) or \( \bar{B} \to \pi^{*-}\bar{\nu} \) decays, the helicity information is lost since the helicity of the pseudoscalar meson is zero. Because the initial meson is also pseudoscalar, angular momentum conservation forces \( W^+ \) to have the helicity zero. Therefore the \( \cos \theta_l \) distribution is proportional to \( \sin^2 \theta_l \)\(^{[18]} \) independent of the value of \( q^2 \), which is shown in Figure 5(a).

If, however, the daughter and spectator quarks form a vector meson \( X \), as in \( D \to \bar{K}^*t^+\nu \), \( D_s^+ \to \phi t^+\nu \), \( \bar{B} \to D^{*+}\bar{\nu} \) or \( \bar{B} \to \rho^{*-}\bar{\nu} \) decays, then the
Figure 5
A Monte Carlo simulation of $\cos\theta_1$ and lepton energy distributions of $D_s^+ \rightarrow X\ell^+\nu$ ($X \equiv \eta, \eta'$ or $\phi$) decays. The solid (open) circles in (a) and (c) represent the distributions of the $D_s^+ \rightarrow \eta\ell^+\nu$ ($D_s^+ \rightarrow \eta'\ell^+\nu$) decay. The triangles in (b) and (d) represent the distributions of the $D_s^+ \rightarrow \phi\ell^+\nu$ decay. In (b) and (d), the solid histograms represent the $|\lambda_{W^+}| = 1$ components while the dotted histograms represent the $\lambda_{W^+} = 0$ components of the $D_s^+ \rightarrow \phi\ell^+\nu$ decay.

helicity information is not lost. It is manifested as a higher probability for the vector meson $X$ to have the helicity $\lambda_X = -1$ than $\lambda_X = +1$. Roughly speaking, a rapidly recoiling $\lambda = -1/2$ daughter quark can combine with the spectator quark to form a $\lambda_X = -1$ or $\lambda_X = 0$ meson. This description is not appropriate when the daughter quark is nonrelativistic or when $W$ is massless, as explained below.

The preference of the vector meson $X$ for $\lambda_X = -1$ to $\lambda_X = 1$ also applies to $\lambda_W$ because $W$ and the vector meson must have the same helicity to conserve the angular momentum. The predominance of $\lambda_W = -1$ over $\lambda_W = +1$ results in the different lepton energy spectra for the $c$ and $b$ quark decays. In the $c$ quark decay, the process $W^+ \rightarrow \ell^+\nu$ produces a $\lambda = +1/2$ lepton. For $\lambda_{W^+} = \pm 1$ the lepton has a $(1 \pm \cos\theta_1)^2$ distribution in the $W^+$ rest frame while it has a
\[ \sin^2\theta_\ell \] distribution for \( \lambda_{W^+} = 0 \).\(^{[18]} \) The Lorentz boost then produces a soft lepton energy spectrum for \( \lambda_{W^+} = -1 \), a hard spectrum for \( \lambda_{W^+} = +1 \) and an in-between spectrum for \( \lambda_{W^+} = 0 \). Figure 5(b) shows the \( \cos\theta_\ell \) distribution of the \( D_s^+ \rightarrow \phi \ell^+\nu \) decay along with the \( \lambda_{W^+} = 0 \) and \( |\lambda_{W^+}| = 1 \) components to the distribution. The \( |\lambda_{W^+}| = 1 \) component shows the predominance of \( \lambda_{W^+} = -1 \) over \( \lambda_{W^+} = +1 \) because there is a much larger peak at \( \cos\theta_\ell = -1 \) than at \( \cos\theta_\ell = +1 \). In the \( b \) quark decay, the process \( W^- \rightarrow \ell^-\nu \) produces a \( \lambda = -1/2 \) lepton. For \( \lambda_{W^-} = \pm 1 \) the lepton has a \((1 \mp \cos\theta_\ell)^2 \) angular distribution in the \( W^- \) rest frame while it has a \( \sin^2\theta_\ell \) distribution for \( \lambda_{W^-} = 0 \).\(^{[18]} \) The Lorentz boost then typically produces a hard lepton energy spectrum for \( \lambda_{W^-} = -1 \), a soft spectrum \( \lambda_{W^-} = +1 \) and an in-between spectrum for \( \lambda_{W^-} = 0 \).

The simplified argument given above for the decays in which the meson \( X \) is a vector meson does not hold at \( q^2 = q_{max}^2 \) or at \( q^2 = 0 \). At \( q^2 = q_{max}^2 \), the daughter vector meson \( X \) is at rest. Therefore its helicity is undefined, and \( W^+ \) and the vector meson are unpolarized. As a result, the \( \cos\theta_\ell \) distribution is uniform at high \( q^2 \). At small values of \( q^2 \), the lepton and neutrino become parallel in the \( M \) rest frame, and their combined spin projection along their direction of motion is zero. The helicity \( \pm 1 \) components are absent, and there is no lepton forward-backward asymmetry in the \( W^+ \) rest frame. The helicity of this vector meson must be zero in this configuration.

The aspects discussed above characterize the lepton energy distribution as follows: In \( D \) (\( \bar{B} \)) decays, the process \( P \rightarrow V\ell\nu \) tends to have a softer (harder) lepton energy spectrum than \( P \rightarrow P'\ell\nu \), where \( P \) and \( P' \) represent pseudoscalar mesons and \( V \) represents a vector meson. In \( D \) decays, the difference between the spectra is a consequence of the V-A enhancement of low energy leptons in \( P \rightarrow V\ell\nu \), which is more significant than the \( P \) wave effect in \( P \rightarrow P'\ell\nu \). Note that the \( P \) wave effect tends to lower the average \( q^2 \) value and consequently the average lepton energy. Figure 5(c) and (d) show the lepton energy spectra of the
$D_s^+ \rightarrow X \ell^+ \nu$ decays, and it can be noticed that the $D_s^+ \rightarrow \phi \ell^+ \nu$ decay tends to have a softer lepton energy spectrum than $D_s^+ \rightarrow \eta \ell^+ \nu$ and $D_s^+ \rightarrow \eta' \ell^+ \nu$ decays. In $\bar{B}$ decays, the difference between the spectra is a consequence of both the V-A enhancement of high energy leptons in $P \rightarrow V \ell \nu$ and the $P$ wave effect in $P \rightarrow P' \ell \nu$.

The characteristics of the Dalitz plots for $D_s^+ \rightarrow X \ell^+ \nu$ decays are summarized below. The most characteristic feature of the Dalitz plots for $D_s^+ \rightarrow \eta \ell^+ \nu$ and $D_s^+ \rightarrow \eta' \ell^+ \nu$ decays, as shown in Figure 3, is that the points populate mostly in the low $q^2$ region due to the dominating $|p_X|^3$ factor. The Dalitz plot for the $D_s^+ \rightarrow \phi \ell^+ \nu$ decay is shown in Figure 4. The $\phi$ meson moves very slowly and is nearly unpolarized near $q^2_{\text{max}}$. All three helicity states ($\lambda_\phi = +1$, 0 and $-1$) of the $\phi$ meson are present in approximately equal amounts. The $\cos \theta_\ell$ distribution is then uniform because $W^+$ is unpolarized. The form factor is largest in this region, accounting for the high density of points. As $q^2$ decreases, the $\lambda_\phi = -1$ component begins to dominate the $\lambda_\phi = +1$ component, which explains the excess of points on the left side of the Dalitz plot compared to the right side. At the lowest value of $q^2$, the lepton and neutrino are parallel in the $D_s^+$ rest frame leading to the maximum $\phi$ recoil, and both of $W^+$ and the $\phi$ meson are forced into a pure $\lambda = 0$ state. There is no asymmetry in the $\cos \theta_\ell$ distribution at $q^2 = 0$; its distribution at this edge of phase space is proportional to $\sin^2 \theta_\ell$. The depletion of points in Figure 4 at high and low lepton energies for small $q^2$ values shows the effect of the $\cos \theta_\ell$ distribution.
1.4 ISGW and ISGW2 Models

The transition matrix element for the charm semileptonic decay $M \rightarrow Xl^+\nu$ can be written as

$$T = \frac{G_F}{\sqrt{2}} V_{cq} \bar{u}_l \gamma_\mu (1 - \gamma_5) u_\nu \langle X|J_{\text{had}}^\mu|M \rangle$$

(1.14)

where $|M\rangle$ and $|X\rangle$ are the initial and final meson states while $u_l$ and $u_\nu$ are the Dirac spinors of the lepton and neutrino. In the massless lepton limit, the hadronic part of the matrix element can be written as

$$\langle X(p')|J_{\text{had}}^\mu|M(p)\rangle = f_+(q^2) (p + p')^\mu,$$

(1.15)

$$\langle X(p',\epsilon)|J_{\text{had}}^\mu|M(p)\rangle = \frac{2ie^{\mu\nu\alpha\beta}}{M + m_X} \epsilon^\nu p'_\alpha p_\beta V(q^2) - (M + m_X)\epsilon^\mu A_1(q^2)$$

$$+ \frac{\epsilon \cdot q}{M + m_X} (p + p')^\mu A_2(q^2)$$

(1.16)

where $|M(p)\rangle$ represents the state of the initial pseudoscalar meson $M$ with the four momentum $p$, and $|X(p')\rangle$ or $|X(p',\epsilon)\rangle$ represent the state of the final pseudoscalar or vector meson $X$ with the four momentum $p'$ and the polarization $\epsilon$.

In the ISGW model,[6] the form factors ($f_+(q^2)$, $V(q^2)$, $A_1(q^2)$ and $A_2(q^2)$) are assumed to have an exponential dependence on $q^2$:

$$f(q^2) \propto \exp[a \cdot (q^2 - q_{\text{max}}^2)]$$

(1.17)

Then the form factors are calculated at $q_{\text{max}}^2$, and extrapolated to the relativistic regime where the $q^2$ value is small. To calculate the form factors at $q_{\text{max}}^2$ the ISGW model assumes the final state meson $X$ to be a $q\bar{q}$ resonant state with small contributions from additional $q\bar{q}$ pairs. In addition, the ISGW model chooses the harmonic oscillator wavefunction appropriate to a Coulomb plus linear potential,

$$V(r) = -\frac{4\alpha_s}{3r} + br + c$$

(1.18)
Table 1
ISGW and ISGW2 predictions for branching fractions of the Cabibbo favored semileptonic decays of $D$ and $D_S^+$ mesons, where $D$ meson is either $D^+$ or $D^0$ meson. The branching fractions are in units of $10^{10} |V_{cs}|^2$. For $D$ meson decays, both ISGW and ISGW2 predictions are available. ISGW predictions are listed in the square brackets. For $D_S^+$ meson decays, ISGW predictions are not available. ISGW2 predicts the branching fractions for the $\eta - \eta'$ mixing angle of $-10^\circ$ and $-20^\circ$. The predictions for the $-20^\circ$ mixing angle are listed in the parentheses.

<table>
<thead>
<tr>
<th>$D \to X e^+ \nu$</th>
<th>$D_S^+ \to X e^+ \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive</td>
<td>Inclusive</td>
</tr>
<tr>
<td>$X = K$</td>
<td>$X = \eta$</td>
</tr>
<tr>
<td>16.6 [19.0]</td>
<td>5.6 (3.7)</td>
</tr>
<tr>
<td>$X = \bar{K}$</td>
<td>$X = \eta'$</td>
</tr>
<tr>
<td>10.5 [8.9]</td>
<td>2.4 (3.2)</td>
</tr>
<tr>
<td>$X = \phi$</td>
<td></td>
</tr>
<tr>
<td>5.7 [9.7]</td>
<td>4.8</td>
</tr>
</tbody>
</table>

with $\alpha_s = 0.5$, $b = 0.18$ GeV$^2$, $c = -0.84$ GeV to approximate the true wave function of the $q\bar{q}$ resonant state. The inaccuracy of extrapolating this non-relativistic calculation to low $q^2$ is corrected by comparing the calculated pion form factor with the experimental data at low $q^2$. An ad hoc factor $\kappa = 0.7$, which makes the calculated pion form factor agree with the experimental data at low $q^2$, is then applied to every form factors calculated.

The ISGW2 model[13] is not a new model, but rather an improved version of the ISGW model. Some new features in the form factors responsible for the improvement are:

1. Heavy Quark Symmetry constraints on the relations between the form factors away from $q_{\text{max}}^2$ are respected.

2. Heavy Quark Symmetry constraints on the slopes of the form factors near $q_{\text{max}}^2$ are built in.

3. More realistic shapes of the form factors, based on the experimentally measured pion form factor, are employed.
The predictions of ISGW and ISGW2 models for branching fractions of the Cabibbo favored semileptonic decays of the $D$ and $D^+_s$ mesons are listed in Table 1.
CHAPTER II

CLEO II Experiment

The data used for this thesis consist of an integrated luminosity of 3.11 fb$^{-1}$ of $e^+e^-$ collisions recorded with the CLEO II detector at the Cornell Electron Storage Ring in eight running periods between May 1990 and September 1993. It contains about 3.7 million $e^+e^-$ → $c\bar{c}$ events taken at center-of-mass energies on the $\Upsilon(4S)$ resonance and in the nearby continuum ($\sqrt{s} \sim 10.6$ GeV).

2.1 The Cornell Electron Storage Ring

The Cornell Electron Storage Ring$^{[20]}$(CESR), located at Cornell University, is an electron-positron collider with a circumference of 768 meters designed to operate with center-of-mass energies between 9.0 and 12.0 GeV/c$^2$.

A linear accelerator (LINAC), a synchrotron and a storage ring, are the main components of CESR. The main components along with the locations of the CLEO II detector and the CHESS (Cornell High Energy Synchrotron Source) facility are shown in Figure 6.
Electrons emitted from a heated filament are accelerated along a 30 meter long vacuum pipe up to 150 MeV in the LINAC. Positrons are produced by striking a tungsten plate with electrons at an intermediate point of the LINAC. A spray of electrons, positrons and X-rays emerges from the plate. The positrons are selected, accelerated in the remainder of the LINAC to the 150 MeV final energy, and injected into the synchrotron.

The synchrotron accepts the 150 MeV electrons or positrons from the LINAC and accelerates them to the energy of typically 5 GeV. For most of the circumference of the synchrotron the particles travel in vacuum between the poles.
of bending magnets, arranged in 192 3-meter long sections. At four places in the ring they pass through a 3-meter long linear accelerator. After a bunch of electrons or positrons has made 4000 revolutions around the ring in about a hundredth of a second, it has reached its maximum energy and is injected into the storage ring. The entire acceleration cycle is repeated 60 times per second for about 10 minutes, first clockwise with positrons, then counterclockwise with electrons, until the required number of electrons and positrons is built up in the storage ring.

Electrons and positrons, each in 7 evenly spaced bunches, travel in the opposite direction around the storage ring at 390 KHz. Electrostatic separators are used to prevent the electron and positron bunches from colliding at the intersection points other than at the CLEO II detector. The particles coasting around the storage ring radiate a beam of X-rays at a power of 500 KW. The lost energy due to this synchrotron radiation is restored to the circulating particles by Radio Frequency (RF) accelerating cavities operating at a frequency of 500 MHz. The technical specifications of CESR are listed in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>CESR technical specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
</tr>
<tr>
<td>Maximum energy per particle</td>
</tr>
<tr>
<td>Number of particles</td>
</tr>
<tr>
<td>rms bunch size at the interaction point</td>
</tr>
<tr>
<td>Particle circulation frequency</td>
</tr>
<tr>
<td>Maximum Luminosity</td>
</tr>
<tr>
<td>Luminosity life time</td>
</tr>
<tr>
<td>RF frequency</td>
</tr>
<tr>
<td>Average RF power (at 5.3 GeV)</td>
</tr>
</tbody>
</table>
2.2 The CLEO II Detector

The CLEO II detector,\(^{[21]}\) shown in Figures 7 and 8, is a solenoidal detector with 67 charged particle tracking layers, a time of flight detector, a CsI electromagnetic calorimeter and a muon detector. An detailed view of the detector inside the magnet coil is shown in Figures 9.

The CLEO II detector was designed to measure charged particle momentum and electromagnetic energy with excellent resolution and efficiency, as well as good charged particle identification. The following sections describe each component of the detector from inner to outer radii.

2.3 Tracking System

The CLEO II tracking system consists of three concentric cylindrical drift chambers operating under the magnetic field of 1.5 T. They are the precision tracking layer detector, the vertex detector and the main drift chamber from inner to outer radii. The precision tracking layer detector has been operated with DME since April, 1992. An equal mixture of Argon-Ethane was used before DME. The vertex detector and the main drift chamber has been operated with an equal mixture of Argon-Ethane since the beginning of the CLEO II experiment. These devices provide momentum and position measurements, as well as energy loss measurement for charged particle identification.

A sense wire with a high voltage applied and a tube or several wires shaping the electric field around the sense wire form an basic unit, a drift cell, of these tracking devices. A charged particle traveling through a gas liberates electrons, thus producing ions. The liberated electrons accelerate toward the sense wire...
Figure 7
Cross-sectional side-view of CLEO II detector
Figure 8
End view of CLEO II detector
while the ions drift toward the field shaping tube or wires. As they accelerate, they liberate more electrons and ions through the secondary ionization process. A chain of these processes leads to an avalanche of electrons very near the sense wire. The electrons and ions produce an electrical signal in the sense wire. The sense wire is instrumented with electronics designed to amplify and shape the signal. After being amplified and shaped, the signal is sent to timing and charge circuits.

The timing circuit has a capacitor that is initially maintained at a constant voltage. When the signal from the sense wire crosses a pre-set threshold, the voltage on the capacitor starts to ramp down at a constant rate. In the case that the L0 trigger (see the section of Trigger System) is satisfied, the rampdown of the voltage is stopped with the gate close, typically 430 ns after the beam
crossing. The voltage on the capacitor is then digitized to give the timing of the signal with respect to the beam crossing. The predetermined time to distance relation (the so called drift function) converts this timing to the distance from the particle's trajectory to the sense wire.

The distances of the particle's trajectory to sense wires on the trajectory are used to construct a track. A track is described by five parameters: the radius of curvature (which is proportional to the magnitude of the particle momentum in the \( r - \phi \) plane), the azimuthal and polar angles of the momentum at the point of closest approach to the beam position, the \( r - \phi \) distance of the point of closest approach and the \( z \) coordinate of the point of closest approach.

The ionization energy loss \((dE/dx)\) of a charged particle is determined by the upper 50% truncated mean of charges integrated at the charge circuits for the sense wires on the particle's trajectory. The upper 50% truncated mean of the charge distribution is the best estimator for the Gaussian part of \( dE/dx \) (governed by the Landau distribution).

### 2.3.1 The Precision Tracking Layer Detector

The Precision Tracking Layer (PTL) detector\(^{22}\) is a 6 layer tube chamber mounted on the beryllium beam pipe. A field shaping tube and a sense wire centered at the tube form a drift cell. Each layer consists of 64 tubular cells. Adjacent layers in the PTL are rotated by a half-cell in order to reduce left-right ambiguities in charged particle tracking. The resulting cell pattern of the PTL is illustrated in Figure 10 along with the cell pattern of the vertex detector.

The PTL provides a high spatial resolution to tracking measurements near the interaction point. These measurements near the interaction point improve the
Figure 10
A pie slice of the precision tracking layer detector and vertex detector

overall charged particle tracking and the localization of primary and secondary vertices. They also help to discriminate against unwanted backgrounds, such as beam-wall collisions, beam-gas collisions and cosmic ray events, while increasing acceptance of low momentum tracks and tracks close to the beam line.

2.3.2 The Vertex Detector

The Vertex Detector\textsuperscript{(23)}(VD), shown in Figure 11, is a 10 layer wire chamber with cathode strips on the inner and outer liners. It provides charged particle tracking through the use of 800 drift cells and 1280 cathode strips.
Six field wires at the corners of a hexagon and a sense wire at the center form a drift cell. The hexagonal geometry achieves close packed cells, and also provides a symmetric electric field that, when combined with the CLEO II magnetic field, produces less distortion in the electron drift path. Each of the inner 5 layers has 64 cells while each of the outer 5 layers has 96 cells. Adjacent layers are rotated by a half-cell in order to reduce left-right ambiguities in charged particle tracking. The cell pattern of the VD is illustrated in Figure 10.

Cathode strips etched onto the inner and outer liners provide z-position measurements. The inner cathode is segmented into 64 rings along the length.
of the VD, the outer cathode has 96 such rings. The rings are further divided into eight azimuthal sections. Further z-position measurements are provided by comparing charges collected at both ends of each sense wire, a procedure known as charge division. The z-position of a charged particle is a function of the ratio of charges collected at two ends, the resistivity of the sense wire and the impedances of the other circuit components. High resistivity, 3000Ω/m, sense wire is chosen so that impedances of the other circuit components have less effect on the z-position measurement.

2.3.3 The Main Drift Chamber
The main Drift chamber^{14}(DR), illustrated in Figure 12, is a 51 layer wire chamber with cathode strips on the inner and outer liners. It provides charged particle tracking measurements together with the PTL and VD, as well as energy loss ($dE/dx$) measurement for charged particle identification.

Four field wires at the corners of a rectangular, another four at the mid points of the four sides and a sense wire at the center form a drift cell. The cell pattern is shown in Figure 13. Each layer is staggered by a half-cell to reduce left-right ambiguities.

The drift chamber has a total of 12240 drift cells and 3072 cathode strips. The number of cells per layer increases from 96 for the inner most layer to 384 for the outer most layer. There are 40 axial layers and 11 stereo layers. Every fourth layer in the first 40 layers is a stereo layer while the sixth layer in the outer 11 layers is a stereo layer. The wires in the stereo layers are slanted with respect to those in the axial layers to provide z-position measurements. The slant angles of the wires in the stereo layers alternate sign from layer to layer, analogous to half-cell staggering of the axial layers in $\phi$. The magnitude of the angles increase from 3.77° for the inner most stereo layer to 6.89° for the outer most stereo layer.

The drift chamber has segmented cathode strips on the inner and outer liners for further z-position measurements. The inner cathode is segmented into 96 rings along the length of the DR, the outer cathode has 192 such rings. The rings are further divided into sixteen and eight azimuthal sections on the inner and outer cathodes, respectively.

The main drift chamber, operating at a gas gain of approximately $10^4$, provides a good $dE/dx$ measurement for charged particle identification. The $dE/dx$ measurement as a function of particle momentum is shown in Figure 14.
Figure 13
Wire position in the Central Drift Chamber
2.3.4 Performance of Tracking System

The spatial resolution of the individual tracking device is listed in Table 3. Combining the measurements of three devices, the overall tracking resolution is:

\[ (\delta p_t/p_t)^2 = (0.0011p_t)^2 + (0.0067)^2, \]

\[ \delta_\phi = 1 \text{ mrad}, \quad \delta_\theta = 4 \text{ mrad} \]

where \( p_t = p \sin \theta \) is in units of GeV/c.
Table 3
Spatial resolution of tracking System. The number in parentheses is the resolution with DME.

<table>
<thead>
<tr>
<th></th>
<th>PTL</th>
<th>VD</th>
<th>DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r - \phi$ resolution ((\mu m))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Layers</td>
<td>110 (55)</td>
<td>100</td>
<td>160</td>
</tr>
<tr>
<td>Stereo Layers</td>
<td></td>
<td></td>
<td>230</td>
</tr>
<tr>
<td>$z$ resolution (mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner Cathode</td>
<td>1.2</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Outer Cathode</td>
<td>1.2</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Stereo Layers</td>
<td></td>
<td></td>
<td>2.5</td>
</tr>
<tr>
<td>Charge Division</td>
<td></td>
<td></td>
<td>20.0</td>
</tr>
</tbody>
</table>

2.4 Time Of Flight Detector

The time of flight (TOF) detector provides the primary timing information for the CLEO II trigger system. A scintillator, a light guide and a photomultiplier tube form a basic unit, a counter, of the TOF detector. A scintillator is defined as any material that produces a light pulse after the passage of a charged particle. A light guide transports the light pulse to the surface of a photo-multiplier tube. The photo-multiplier tube converts the light pulse into an electrical signal and amplifies it. The timing of the signal with respect to the beam crossing, which is the flight time of the charged particle, is measured with a timing circuit. The brief explanation of a timing circuit has already been given in the section of Tracking System.

The barrel TOF consists of 64 counters located radially between the outside of the main drift chamber and the inside of the barrel calorimeter. A slab of Bicron BC-408 scintillator connected to two lucite light guides with 17° bend is shown in Figure 15. A photo-multiplier tube is connected to the other end of each light guide. The light guide not only transmits the light signal, but also allows the photo-multiplier tube to be located outside the return flux of the magnet.
where the magnetic field is about 100 gauss.

Each endcap TOF consists of 28 counters located between the drift chamber endplates and the endcap calorimeter. Each endcap counter, shown in Figure 16, consists of a Bicron BC-408 scintillator and a photo-multiplier tube connected directly to the scintillator with no light guide. The tube is designed to operate in a high magnetic field parallel to the tube axis.

An endview of the TOF detector is illustrated in Figure 17. The numbers shown are the addresses of the photo-multiplier tubes. The tube at the other end of each barrel counter has an even integer address. The tubes of the endcap counters on the other side are addressed from 157 through 184.

There is a difference in the flight time (proportional to $1/\beta$) of the charged particles that have equal momenta but different masses. The separation between $1/\beta$ of different particle types is illustrated in Figure 18 where $1/\beta$ measured by the barrel TOF is plotted as a function of track momentum measured by tracking devices. The time resolution of the barrel TOF for electrons in Bhabha events is about 140 psec while the resolution for pions with momentum less than 0.7 GeV/c in hadronic events is about 150 psec. The endcap TOF has a time resolution of about 270 psec for electrons in Bhabha events.

### 2.5 Electromagnetic Calorimeter

The CLEO II calorimeter[^25,^26] is a fine-segmented electromagnetic shower detector at a radius of 1 meter, covering 95% of the solid angle. It consists of 7800 thallium doped cesium iodide (CsI) crystals grouped into three sections, the barrel and two endcaps. The barrel coverage starts at a polar angle of 32°, and
Figure 15
A barrel TOF counter

Figure 16
An endcap TOF counter with a wedge-shaped scintillator
Figure 17
Endview of TOF system. The heavier lines show the divisions of the TOF trigger.

Figure 18
$1/(\beta_{TOF})$ vs track momentum
overlaps slightly with the endcap which ends at 36°.

The 6144 barrel crystals are arranged in the geometry of 48 z-rows and 128 azimuthal segments. The 256 crystals in each pair of symmetrically located z-rows have identically tapered shapes with trapezoidal cross sections. The dimensions of 24 different crystal shapes are still close to a rectangular dimension of 5 cm x 5 cm x 30 cm with at most ~ 2 cm variations in the sides of the trapezoids. These crystal dimensions are chosen so that the gaps between crystals point slightly away from the interaction point, minimizing losses in the solid angle coverage and providing the full 16 radiation lengths for photons originating from the interaction point.

The 828 crystals on each endcap are arranged in a four-fold symmetric pattern as shown in Figure 19 with their axes parallel to the beam axis. All endcap crystals have the same rectangular dimensions (5 cm x 5 cm x 30 cm) except for a few crystals with edges cut away near the inner and outer walls to facilitate stacking.

### 2.5.1 Crystal Electronics

Due to the inaccessibility of the calorimeter after the installation, a four-fold redundancy was constructed for the electronics associated with each crystal. Four photodiodes are mounted on an UVT lucite window and the window is glued to the back of each crystal. Each photodiode is connected to a separate preamplifier mounted just behind the crystal. The circuit diagram of a preamplifier is shown in Figure 20.

The 64 preamplifier signals from 16 crystals, arranged in a 4 x 4 block, are sent to a mixer/shaper (m/s) board through twisted-pair cables with a length
of about 30 ft. The m/s boards are located in modified CAMAC crates outside the detector. The circuit schematic of a m/s board is shown in Figure 21. The signals from 4 preamplifiers of each crystal are summed into a single signal. In case of a noisy diode or failed preamplifier, the channel can be excluded from the energy sum by computer with the gain automatically corrected.

The summed signal is fed into an inverting amplifier with a gain that can be varied by sixteen steps between 1 and 31/16. As with the diode or preamplifier selection in the energy sum, this gain can be also controlled by computer. The gain is adjusted in such a way to compensate for crystal to crystal variations in light output and prevent a crystal with high light output from dominating its neighbors in the trigger sum. The original set of these m/s gains was determined from a cosmic ray calibration at the beginning of the CLEO II experiment. The
calibration was replaced in September, 1993 with a new set of gains determined from a set of Bhabhas events.

The gain compensated signal from the inverting amplifier is fed into a pole-zero shaper to filter out the long tail, and integrated before the digitization. The LeCroy Fastbus Model 1885N ADC, having 96 channels per module is used to convert the analog signal from the m/s circuits to the digital signal. The output from the ADC module is digitized with 12 bits of precision and an extra bit indicating high or low range. The two ranges differ by a factor of 8 in gain, therefore, the ADC module provides an effective 15 bit dynamic range. An example of signal outputs of preamplifiers and m/s circuits are shown in Figure 22 with the ADC sampling gate. The rise (fall) times of the preamplifier and m/s outputs are about 1 (180) µsec and 2 (10) µsec, respectively.
Figure 21
Schematic of m/s circuits

Figure 22
Outputs of preamplifiers and m/s circuits
The trigger section of a m/s board sums the gain compensated signals from the 16 crystals in a 4 × 4 block and differentiates the summed signal to generate a signal with a rise time of 360 nsec and a zero-crossing of 2 or 3 μsec prior to the ADC sampling gate. This signal is fed to four discriminators, two level discriminators and two zero-crossing discriminators. Currently the outputs from the two level discriminators, one set at the threshold of 100 MeV and the other at 500 MeV, are in use for the calorimeter trigger.

2.5.2 Calibration and Monitoring of Electronics

An electronic pulser system on the m/s boards is used to test and calibrate the signal path from the preamplifiers to the ADC. Four individually selectable pulsers simultaneously pulse the test inputs of all preamplifiers on each mother board. The 68 pf capacitor on the m/s board and the 1 nf capacitor on the mother board act as a capacitive divider connected by a twisted pair cable. The system is charged by a reference voltage from a DAC on a crate controller. When the FET is turned on, the charge on the 68 pf capacitor is dumped onto the 1 nf capacitor producing a step of \( V = \frac{68 \text{ pf}}{1 \text{ nf}} \cdot V_{\text{ref}} \). The full scale output of the pulser is equivalent to approximately 8 GeV per crystal which is sufficient to saturate most ADC channels. The circuit diagram of the electronic pulser system is shown in Figure 23.

A quick ("small") electronic calibration, run approximately every two hours, monitors proper operation of the electronics. An extensive ("big") electronic calibration, run three times a week, determines gain, pedestal and its width of each channel to be used in processing the data. Pulse levels and the number of pulsings at each pulse level in both types of calibrations are listed in Table 4. A "big" calibration finds, for example, the gain of each crystal by fitting three
average ADC counts at three corresponding pulse levels to a straight line. The outputs of a typical “big” calibration for both low and high ADC ranges are shown in Figure 24.

The light-sensitive gain of crystals and photodiodes is monitored by a light flasher system. Two Xenon flash tubes each pulse 3900 crystals through a network of optical fibers. Each tube produces light pulses in six bundles of 156 1 mm diameter PCS (plastic clad silica) fibers. In turn each 1 mm fiber is connected to a bundle of 50 200 μm diameter PCS fibers. The 200 μm diameter fiber is connected at the back of a crystal through the photodiode mounting board. Light pulses are reflected by the crystal surfaces and collected by the photodiode. The schematic of the light flasher system is shown in Figure 25. Fluctuations in the intensity of light pulses make it less practical to calibrate the crystal electronics.
Distributions of pedestal, pedestal width and gain of calorimeter electronics for both low and high ADC ranges from a typical "big" calibration. Pedestal and its width are in the unit of ADC counts.

Figure 24
Table 4
Crystal electronic calibration

<table>
<thead>
<tr>
<th>Measurement</th>
<th>&quot;Small&quot; Calibration</th>
<th>&quot;Big&quot; Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestal Gain</td>
<td>Pedestal Gain</td>
<td></td>
</tr>
<tr>
<td>Low range pulse level (V)</td>
<td>0.0</td>
<td>0.2, 0.5, 0.8</td>
</tr>
<tr>
<td>High range pulse level (V)</td>
<td>0.0</td>
<td>3.0, 5.0, 7.0</td>
</tr>
<tr>
<td># of pulsings at each level</td>
<td>16</td>
<td>50</td>
</tr>
</tbody>
</table>

with the light flasher system. However, the flasher system still can indicate a failed diode when the output with the particular diode selected is considerably lower than that with all diodes selected while the electronic calibration shows little or no difference.

2.5.3 Electronics Reliability

When pedestal, pedestal width (the measure of noise) and gain of each channel in the crystal electronics fluctuate or are not in the required ranges, electronic or light pulses are used to decide whether the problems are due to failed photodiodes, preamplifiers, m/s boards or ADC modules.

When photodiodes or preamplifiers fail, the corresponding channels are turned off by computer control. The cumulative number of photodiodes or preamplifiers turned off from 1989 to 1994 is shown in Figure 26.

When the problems are on m/s boards or ADC boards, the boards are replaced. The cumulative number of m/s board failures from 1989 to 1994 is shown in Figure 26. The breakdown of types of m/s board failures is listed in Table 5.

For the period from February 1992 to September 1995, eight ADC modules
had to be replaced because of gain failures (5 times), noise (twice) and pedestal shifts (once).

2.5.4 Offline Calibration and Performance

After the online electronic calibration, two offline calibrations are performed, the relative crystal to crystal energy calibration using Bhabhas events and the absolute energy calibration using various processes involving photons.

The relative crystal to crystal energy calibration is necessary to compensate for the variations in light output of the crystals even though most of the
Figure 26
Cumulative number of failed photodiodes or preamplifiers

Figure 27
Cumulative number of m/s board failures
Table 5

m/s board failure

<table>
<thead>
<tr>
<th>type of failures</th>
<th># of m/s board failures</th>
</tr>
</thead>
<tbody>
<tr>
<td>single channel</td>
<td>14</td>
</tr>
<tr>
<td>multiple channel</td>
<td>11</td>
</tr>
<tr>
<td>short</td>
<td>20</td>
</tr>
<tr>
<td>CMOS switch</td>
<td>10</td>
</tr>
<tr>
<td>trigger</td>
<td>8</td>
</tr>
<tr>
<td>shift register</td>
<td>4</td>
</tr>
<tr>
<td>other</td>
<td>4</td>
</tr>
</tbody>
</table>

variation has already been compensated at the hardware level by adjusting the m/s gains. Bhabha events are a convenient way of calibrating the crystals since a fixed energy (the beam energy) is deposited by each electron. Also frequent calibrations are possible due to the large cross-section of Bhabha events. To avoid biases from tracking quality, Bhabha events are selected purely on calorimeter information: there must be two showers of at least 2.5 GeV each, the total remaining energy must be less than 2 GeV and the showers must be collinear to within 5 degrees in $\theta$ and 15 degrees in $\phi$.

The crystal energy $E_i$ of the $i$-th crystal is calculated from:

$$E_i = \frac{ADC_i - PED_i}{elec.gain_i \times BBCAL_i \times overall.gain},$$

where $elec.gain_i$ and $PED_i$ are the electronic gain and pedestal for $i$-th crystal in the appropriate ADC range. The constants, $BBCAL_i$, and $overall.gain$, are determined with Bhabha events. The relative energy scales, $BBCAL_i$, of all crystals are obtained by minimizing the rms width of the energy distribution. The absolute energy scale, $overall.gain$, is set by constraining the peak of the distribution to the beam energy.

Showers in the calorimeter are reconstructed from the crystal hits by a cluster finder. The cluster finder forms showers from connected regions of hits.
Table 6
Absolute energy calibrations with photons in the region of $45^\circ < \theta < 135^\circ$

<table>
<thead>
<tr>
<th>Calibration method</th>
<th>Photon energy range</th>
<th>Accuracy of calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \gamma\gamma$</td>
<td>$E_\gamma = \text{beam energy}$</td>
<td>$\pm 0.2%$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \gamma\gamma\gamma$</td>
<td>$E_\gamma &gt; 700 \text{ MeV}$</td>
<td>$\pm 0.5%$</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^-\gamma$</td>
<td>$E_\gamma &gt; 500 \text{ MeV}$</td>
<td>$\pm 0.5%$</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>$E_\gamma &lt; 2 \text{ GeV}$</td>
<td>$\pm 0.5%$</td>
</tr>
</tbody>
</table>

Table 7
Parameterization of calorimeter photon energy resolution and angular resolution ($E$ in units of GeV)

<table>
<thead>
<tr>
<th>Calorimeter section</th>
<th>$\sigma_E/E$ (%$)$</th>
<th>$\sigma_\phi$ (mr)</th>
<th>$\sigma_\theta$ (mr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrel</td>
<td>$0.35E^{-0.7k} + 1.9 - 0.1E$</td>
<td>$2.8E^{-0.8k} + 1.9$</td>
<td>$0.8\sigma_\phi \sin \theta$</td>
</tr>
<tr>
<td>Endcap</td>
<td>$0.26E^{-1.0} + 2.5$</td>
<td>$3.7E^{-0.8} + 7.3$</td>
<td>$1.4E^{-0.8} + 5.6$</td>
</tr>
</tbody>
</table>

The most energetic hit in a shower must be the one over 10 MeV and has higher energy than any immediate neighbors. No crystal in a shower can be more than 2 crystals away from another crystal in the shower. This assures that clusters that have their maxima only 2 crystals apart can be separated. The shower energy and position are calculated based on the $N$ most energetic crystals where $N$ is a logarithmic function of the cluster energy. This function varies from $N = 4$ at 25 MeV to $N = 17$ at 5 GeV. The shower energy is given by:

$$E_{\text{shower}} = C(E) \sum_i^N E_i,$$

where $C(E)$ is an energy dependent correction for the absolute energy calibration.

The absolute energy calibration requires several methods to cover the entire energy range. Table 6 lists these methods, their associated photon energy range and the accuracy of the calibration with photons in the region of $45^\circ < \theta < 135^\circ$. Back to back photons ($e^+e^- \rightarrow \gamma\gamma$) provide a calibration for pho-
Figure 28
Invariant mass of $\pi^0 \rightarrow \gamma\gamma$ candidates in four $\gamma\gamma$ momentum ranges
Figure 29
Invariant mass of $\eta \rightarrow \gamma \gamma$ candidates in four $\gamma \gamma$ momentum ranges
tons at the beam energy to about 0.2%. Photons from radiative \( \gamma (e^+e^- \rightarrow \gamma\gamma\gamma) \) events provide a calibration for photon energies above 700 MeV to about 0.5%. Photons from radiative Bhabha events provide a calibration for photon energies above 500 MeV to about ±0.5%. Photon pairs from \( \pi^0 \) decays, constrained to the \( \pi^0 \) mass, provide a calibration for photon energies below 2 GeV to about ±0.5%. The energy resolution and angular resolution of the CLEO II calorimeter can be parameterized as given in Table 7.

The invariant mass distributions of \( \pi^0 \rightarrow \gamma\gamma \) and \( \eta \rightarrow \gamma\gamma \) candidates for the four \( \gamma\gamma \) momentum ranges are shown in Figure 28 and Figure 29, respectively. The \( \pi^0 \) peak is evident in all four bins and the \( \eta \) peak appears for \( P_{\gamma\gamma} \) above 1 GeV/c. The \( \pi^0 \) mass resolution for the whole \( P_{\gamma\gamma} \) range is approximately 6 MeV/c\(^2\) while the \( \eta \) mass resolution for \( P_{\gamma\gamma} \) above 1 GeV/c is approximately 13 MeV/c\(^2\).

### 2.6 Muon Detector

The steel yoke of the CLEO II magnet serves as a return path of the magnetic flux, as well as a hadron absorber for the muon detector. Each octant of the barrel muon detector consists of three layers of 36 cm thick steel slabs and an outermost layer of 2.5 cm thick steel sheet. The layers are separated by 9 cm gaps where the units of the barrel muon detector are located. Each steel slab in front of each gap presents a minimum of 2.2 nuclear absorption lengths (16.7 cm = 1 nuclear absorption length in iron). The endcap muon detector consists of eight units, four covering each end.

A proportional counter, shown in Figure 30, consists of 8 rectangular tubes. Three sides of a tube are coated with graphite to form a cathode. A sense wire
Figure 30
Cut away view of a muon counter showing the tube structure

Figure 31
Partial cross section of a muon unit
operating at a high voltage is positioned at the center of each tube. The gas for the counter is the same Argon-Ethane gas used for the drift chamber. Copper strips on a foam board are placed perpendicular to the sense wires on the fourth (no graphite coated) side of a counter.

A set of counters and strips forms a unit, of which partial cross section is shown in Figure 31. Each unit consists of three layers of counters and strips. Each layer in a unit is divided into two multiplets of 10, 14 or 15 counters and four multiplets of either 9 or 15 strips. The structure of the units are listed in Table 8. The units in each octant of the yoke are labeled “return”, “inner” and “outer” for the inner, middle and outer gaps. Also the units in each endcap are labeled “endcap”.

Each member (counter or strip) in a multiplet is connected to its neighbor by a 100Ω resistor. Each end of a multiplet string is connected to a charge integrator. The position of a charged particle is determined from the ratio of charges collected at two ends of the multiplet, a procedure known as charge division. This method greatly reduces the amount of electronics while still provides the needed spatial resolution. The $r - \phi$ spatial resolution of the proportional counters is 2.4 cm while the $z$ spatial resolution of the copper strips is 5.3 cm.

### Table 8

<table>
<thead>
<tr>
<th>Unit location</th>
<th># of units</th>
<th># of layers per unit</th>
<th>Multiplet structure per layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>counters</td>
</tr>
<tr>
<td>Return</td>
<td>8</td>
<td>3</td>
<td>10 + 10</td>
</tr>
<tr>
<td>Inner</td>
<td>8</td>
<td>3</td>
<td>10 + 15</td>
</tr>
<tr>
<td>Outer</td>
<td>8</td>
<td>3</td>
<td>14 + 15</td>
</tr>
<tr>
<td>Endcap</td>
<td>8</td>
<td>3</td>
<td>10 + 14</td>
</tr>
</tbody>
</table>
Table 9

<table>
<thead>
<tr>
<th>Level</th>
<th>Vertex Detector</th>
<th>Drift Chamber</th>
<th>Time of Flight</th>
<th>Calorimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>L1</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>L2</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

2.7 Trigger System

Given a detector system operating at high luminosity, correct decisions must be made on which events are to be read out. The events of interest must be read out with efficient discrimination against similar but uninteresting background events. Otherwise the data acquisition system will be swamped with the background events, and the deadtime of the data acquisition system will increase. In addition, those background events read out would increase the volume of data as well as the computing time in the offline analysis. Therefore the proper design of a trigger system is essential for enriching data with the interesting events, and correspondingly increasing the sensitivity of the experiment.

The CLEO trigger system\textsuperscript{[32]} is tiered into 3 levels of detector interrogation. Table 9 shows which devices of the CLEO II detector are used in each trigger level. The first level (L0) reduces the event rate from the 2.7 MHz beam crossing rate to about 20 KHz. The second level (L1) reduces the event rate to about 25 Hz while the last level (L2) reduces the event rate to about 15 Hz. If the test at each level fails, the event is discarded and the detector is reenabled. Upon a success in the test at each level, the detector is disabled and the next level test continues. If all levels are satisfactory, the digitized detector information associated with the event is written to magnetic tape.
CHAPTER III

Analyses of Semileptonic Decays of $D_s^+$ Meson

The major semileptonic decay modes of $D_s^+$ meson are the ones with $\phi, \eta$ or $\eta'$ meson in the final state. Due to the undetected neutrino, one cannot fully reconstruct $D_s^+ \rightarrow X\ell^+\nu$ decays, where $X \equiv \phi, \eta$ or $\eta'$ and $\ell^+ \equiv e^+$ or $\mu^+$. However, there are few processes which produce both a meson $X$ and a lepton in the same jet. This $X\ell^+$ correlation is used to extract the $D_s^+ \rightarrow X\ell^+\nu$ signal by requiring that $X$ and the lepton be in the same hemisphere with respect to the thrust axis of the event. Backgrounds due to misidentified leptons and to random $X\ell^+$ combinations can be reliably estimated, and the possible contaminations from other decay modes are small. The backgrounds can be further reduced for $D_s^+ \rightarrow \eta\ell^+\nu$ and $D_s^+ \rightarrow \phi\ell^+\nu$, which have sufficient statistics, by also detecting the low energy photon from the $D_s^*+ \rightarrow D_s^+\gamma$ decay. This second method is referred as the $D_s^*$ tag method.

Yields in $D_s^+ \rightarrow \phi\ell^+\nu$, $D_s^+ \rightarrow \eta\ell^+\nu$ and $D_s^+ \rightarrow \eta'\ell^+\nu$ decay modes are measured with the non $D_s^{*+}$ tag method. From these yields $B(D_s^+ \rightarrow \eta\ell^+\nu)/B(D_s^+ \rightarrow \phi\ell^+\nu)$ and $B(D_s^+ \rightarrow \eta'\ell^+\nu)/B(D_s^+ \rightarrow \phi\ell^+\nu)$ are calculated. The $D_s^{*+}$ tag method provides clean yields in $D_s^+ \rightarrow \eta\ell^+\nu$ and $D_s^+ \rightarrow \phi\ell^+\nu$ decay modes, and therefore
another measurement of $B(D_s^+ \to \eta e^+\nu)/B(D_s^+ \to \phi e^+\nu)$. The two values of $B(D_s^+ \to \eta e^+\nu)/B(D_s^+ \to \phi e^+\nu)$ are averaged.

3.1 Event Selection

The hadronic, the so called "CLASS 10", events are use for these analyses. The criteria that each "CLASS 10" event should meet are as follows:

1. At least 3 charged tracks with good $z$ fit

2. $|x_0| < 2$ cm, $|y_0| < 2$ cm and $|z_0| < 5$ cm where $x_0, y_0$ and $z_0$ are the coordinates of the event vertex calculated with charged tracks,

3. $E_{\text{vis}} > 0.15 \cdot E_{\text{cm}}$ where $E_{\text{cm}}$ is the center of mass energy and $E_{\text{vis}}$ is the energy sum of the charged tracks (assumed to be pions) and showers in the calorimeter unmatched to the charged tracks,

4. $0.15 \cdot E_{\text{cm}} < E_{\text{cal}} < 0.9 \cdot E_{\text{cm}}$, where $E_{\text{cal}}$ is the energy sum of showers in the calorimeter.

The two $B$ mesons from an $t(4S)$ are almost at rest, therefore particles in the $t(4S)$ events tend to be spherically distributed. On the other hand, particles in the continuum events tend to form jets. The ratio of the second Fox-Wolfram moment to the zeroth moment,\[^{[33]}\] $R_2 \equiv H_2/H_0$, is an event shape parameter that efficiently discriminates the $t(4S)$ events from the continuum events. The $R_2$ distribution of the continuum events peaks toward 1 while the distribution of the $t(4S)$ events peaks at close to 0. We require that the $R_2$ value of each "CLASS 10" event be greater than 0.3. This $R_2$ cut retains 57% of the continuum events while removing 94% of the $t(4S)$ events.
The Monte Carlo events for the $D_s^+ \rightarrow X \ell^+ \nu$ decays are generated with an event generator which uses the prediction of the ISGW model[8] as input. The Monte Carlo events for the background estimations are generated with the CLEO II generic event generators. All Monte Carlo events are then passed through a full simulation of the CLEO II detector and the same event reconstruction and selection as the data.

3.2 Lepton Identification

Lepton* candidates are restricted to lie in the fiducial regions in which the lepton identification efficiencies and hadron misidentification rates are well understood.

Electron candidates are restricted to lie in the fiducial region $|\cos \theta| < 0.91$ where $\theta$ is the polar angle of the track with respect to the beam axis. Electron candidates must have momenta above 1.0 GeV/c for the non $D_s^+ \rightarrow X e^+ \nu$ tag analysis, but above 0.7 GeV/c for the $D_s^+ \rightarrow X \mu^+ \nu$ tag analysis to increase statistics. A variable “R2ELEC”, which is the likelihood of an electron candidate being a true electron, is computed with a set of measurements associated with the candidate. The set of the measurements consists of the ratio of energy to momentum, the ionization energy loss, the lateral shower shape in the calorimeter and the time of flight. To be identified as an electron, “R2ELEC” of the candidate must be greater than 3.0. Electrons from photon conversions in the detector material and Dalitz decays of $\pi^0$'s are rejected. This is accomplished by pairing an identified electron with each oppositely charged track in the event, and rejecting the electron when the electron-positron invariant mass of the pair is smaller than 0.050 GeV/c².

*It is once again meant to be either electron or muon, but not tau.
Muon candidates are restricted to lie in the fiducial regions $|\cos \theta| < 0.81$. Muon candidates in the region $|\cos \theta| < 0.61$ are required to have momenta above 1.5 GeV/c and those in the region $|\cos \theta| > 0.61$ are required to have momenta above 1.9 GeV/c. In order to be identified as a muon, not only must a charged track penetrate at least 5 nuclear absorption lengths of iron, but it also match hits in the muon chambers.

The probability of a hadron being misidentified as a lepton is measured using $\Upsilon(1S)$ events by counting how often tracks in the events satisfy the criteria for the lepton identification. The sources of leptons in the $\Upsilon(1S)$ events are almost exclusively the processes of $\Upsilon(1S) \to \ell^+\ell^-, \tau^+\tau^-$ and $q\bar{q}$. The processes of $\Upsilon(1S) \to \ell^+\ell^-$ and $\tau^+\tau^-$ can be removed by requiring that each event must have five or more tracks on top of the "CLASS 10" criteria. The contribution from the remaining lepton source, $\Upsilon(1S) \to q\bar{q}$, can be subtracted properly using the off-resonance continuum events.$^{[24]}$

The momentum dependent probabilities that a hadron will be misidentified as a lepton are given in Table 10 and Table 11. From now on, a fake lepton is meant to be a misidentified hadron.

### 3.3 $X$ Identification

The $\phi$ candidates are identified via the decay mode $\phi \to K^+K^-$ and are required to have the momenta greater than 1.0 GeV/c. Charged kaon candidates in data must have ionization energy loss and time of flight consistent with that expected for true kaons. The condition for a kaon candidate being consistent with a true kaon is that $PROB(\chi^2_K, N)$ be greater than 0.02, where $PROB(\chi^2_K, N)$ is the upper tail probability of the chi-square distribution $\chi^2_K$ for a kaon candidate.
Table 10
The probability (in %) of a hadron being misidentified as an electron as a function of the polar angle and momentum of the hadron (in GeV)

<table>
<thead>
<tr>
<th></th>
<th>$0.6 &lt; P \leq 1.0$</th>
<th>$1.0 &lt; P \leq 1.5$</th>
<th>$1.5 &lt; P \leq 2.5$</th>
<th>$2.5 &lt; P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\cos\theta</td>
<td>&lt; 0.71$</td>
<td>0.70</td>
<td>0.35</td>
</tr>
<tr>
<td>$0.71 &lt;</td>
<td>\cos\theta</td>
<td>&lt; 0.906$</td>
<td>1.40</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 11
The probability (in %) of a hadron being misidentified as a muon as a function of the hadron momentum (in GeV)

<table>
<thead>
<tr>
<th></th>
<th>$1.4 &lt; P \leq 1.6$</th>
<th>$1.6 &lt; P \leq 1.8$</th>
<th>$1.8 &lt; P \leq 2.0$</th>
<th>$2.0 &lt; P \leq 2.2$</th>
<th>$2.2 &lt; P \leq 2.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>2.4 &lt; P \leq 2.6</td>
<td>2.6 &lt; P \leq 2.8</td>
<td>2.8 &lt; P \leq 3.0</td>
<td>3.0 &lt; P \leq 3.2</td>
<td>3.2 &lt; P</td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>0.8</td>
<td>0.9</td>
<td>1.0</td>
<td>1.4</td>
<td></td>
</tr>
</tbody>
</table>

If only one of ionization energy loss or time of flight measurements is available ($N = 1$), this corresponds to $\chi^2_K < 5.41$. If both ionization energy loss and time of flight measurements are available ($N = 2$), this corresponds to $\chi^2_K < 7.82$.

Because of the small $q^2$ value associated with the decay $\phi \rightarrow K^+K^-$, the trajectories of the two kaons tend to overlap. This makes it difficult to simulate accurately the ionization energy loss of these kaons in the main drift chamber. Therefore the standard CLEO II detector simulation might not give the correct efficiency for identifying $\phi$'s. In order to avoid this problem, the momentum dependent efficiencies for identifying $\phi$'s are obtained from the data by comparing $\phi$ yields with and without requiring the kaon consistency for two kaon candidates. These efficiencies are then combined with the predicted $\phi$ momentum spectrum of Monte Carlo $D_s^+ \rightarrow \phi l^+\nu$ decays to give the total efficiency for identifying $\phi$'s.
The efficiency of identifying the \( \phi \to K^+K^- \) decay as a function of \( \phi \) momentum found using \( D_s^+ \to \phi \pi^+ \) decays in data in the decay chain of \( D_s^+ \to \phi \ell^+\nu \) and \( \phi \to K^+K^- \).

The inclusive \( \phi \) data have been tried to obtain the momentum dependent efficiencies. But the signal to background ratio of the \( \phi \) signal, especially without requiring the kaon consistency for two kaon candidates, is small due to the large combinatoric background, therefore the \( \phi \) yield obtained from fitting this signal has large uncertainties. A sample of \( \phi \)'s from the decay chain of \( D_s^+ \to \phi \pi^+ \) and \( \phi \to K^+K^- \) provides an alternative way to obtain these efficiencies. To collect this sample of \( \phi \)'s, the momentum of a \( K^+K^-\pi^+ \) combination is required to be greater than 2.5 GeV/c and the invariant mass of the combination must be consistent with the \( D_s^+ \) mass (\( \pm 3\sigma = \pm 0.024 \) GeV/c\(^2\)). Then the ratio of \( \phi \) yields with and without requiring the kaon consistency for two kaon candidates gives the efficiency at a given \( \phi \) momentum range. These momentum dependent efficiencies of identifying \( \phi \)'s, obtained from the data of \( D_s^+ \to \phi \pi^+ \) decays, are
The predicted $\phi$ momentum spectra of $D_s^+ \rightarrow \phi l^+\nu$ Monte Carlo events for both non $D_s^{*+}$ tag and $D_s^{*+}$ tag analyses are shown in Figure 33. By combining the momentum dependent efficiencies with the predicted $\phi$ momentum spectrum, we find that the total efficiency for the non $D_s^{*+}$ tag analysis is $0.846 \pm 0.025$ while the total efficiency for the $D_s^{*+}$ tag analysis is $0.838 \pm 0.025$.

The decay mode $\eta \rightarrow \gamma\gamma$ is used to select $\eta$ candidates. The momenta of $\eta$ candidates are required to be greater than 1.0 GeV/c. The photon candidates are restricted to lie in the fiducial region, $|\cos \theta| < 0.81$, where $\theta$ is the polar angle of the photon momentum with respect to the beam axis. The lateral shape of a shower is parameterized by the ratio of energy summed over 9 CsI blocks (one at the shower center plus 8 near-neighbors) to energy summed over 25 CsI blocks (the 9 explained above plus 16 next-near-neighbors), the so called "E9/E25".
The efficiency of identifying the $\eta' \rightarrow \eta\pi^+\pi^-$ decay as a function of $\eta'$ momentum found with the data and the predicted $\eta'$ momentum spectrum for $D_s^+ \rightarrow \eta'\ell^+\nu$ Monte Carlo events. The total efficiency of identifying pions in the decay chain of $D_s^+ \rightarrow \eta'\ell^+\nu$ and $\eta' \rightarrow \eta\pi^+\pi^-$ is calculated by folding two distributions.

The "E9/E25" of a photon candidate must be consistent with that expected for a true photon at 99% confidence level. It is required that $|\cos \theta_d| < 0.9$, where $\theta_d$ is the photon decay angle in the $\eta$ rest frame with respect to the $\eta$ direction in the laboratory. Backgrounds due to low momentum photons tend to peak at $|\cos \theta_d| = 1$. Since most photons are daughters of $\pi^0$ decays, any photon which, combined with another photon, has the two photon invariant mass consistent with the $\pi^0$ mass ($\pm 2.5\sigma$) and has the two photon momentum greater than 0.8 GeV/c, is vetoed in selecting the $\eta \rightarrow \gamma\gamma$ decays.

The $\eta'$ candidates are identified via the decay chain of $\eta' \rightarrow \eta\pi^+\pi^-$ and $\eta \rightarrow \gamma\gamma$. The candidates are required to have momenta greater than 1.0 GeV/c. The momenta of the $\eta$ candidates must be greater than 0.5 GeV/c. Also the invariant mass of two photons from $\eta \rightarrow \gamma\gamma$ decays must be consistent with the
The efficiency for identifying two pions in the decay chain of \( \eta' \rightarrow \eta \pi^+ \pi^- \) and \( \eta' \rightarrow \eta \pi^+ \pi^- \), calculated by folding two distributions, is 0.831 ± 0.015.

### 3.4 Non \( D_s^+ \) Tag Analysis of \( D_s^+ \rightarrow X \ell^+ \nu \) Decay

The momenta of \( X \) candidates are required to be greater than 1.0 GeV/c as explained in the previous section. This momentum cut reduces the combinatoric
Table 12
Cuts for the non $D_{s}^{+}$ tag analysis

<table>
<thead>
<tr>
<th>Type</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>lepton $X \equiv \phi, \eta, \eta'$</td>
<td>$P_{l} &gt; 1.0$ GeV/c</td>
</tr>
<tr>
<td>$\eta l^{+}$</td>
<td>$P_{X} &gt; 1.0$ GeV/c</td>
</tr>
<tr>
<td>$X l^{+}$</td>
<td>$M_{\eta l^{+}} &gt; 1.2$ GeV/c$^{2}$</td>
</tr>
<tr>
<td>$X t^{+}$</td>
<td>$M_{X t^{+}} &lt; 1.9$ GeV/c$^{2}$</td>
</tr>
<tr>
<td>$X t^{+}$</td>
<td>$2.5 &lt; P_{X t^{+}} &lt; 4.5$ GeV/c</td>
</tr>
<tr>
<td>$X t^{+}$</td>
<td>$X$ and the lepton are in the same hemisphere</td>
</tr>
</tbody>
</table>

background in identifying $X$. In order to be consistent with originating from $D_{s}^{+}$ decays, the $X t^{+}$ candidates must have invariant masses less than 1.9 GeV/c$^{2}$. For $\eta l^{+}$ candidates, the $\eta l^{+}$ invariant masses are required to be greater than 1.2 GeV/c$^{2}$ since the background due to fake leptons is large below 1.2 GeV/c$^{2}$. In addition, the $X t^{+}$ momentum is required to be between 2.5 and 4.5 GeV/c to reduce random $X t^{+}$ combinations. Finally $X$ and the lepton must lie in the same hemisphere with respect to the thrust axis. Table 12 lists the cuts used for the non $D_{s}^{+}$ tag analysis of $D_{s}^{+} \rightarrow X t^{+} \nu$.

Figure 35 shows the $K^{+}K^{-}$ invariant mass distribution for all $\phi l^{+}$ combinations which satisfy the above selection criteria. The distribution is fitted with a signal and background function. The signal function is a Gaussian function convoluted with a Breit-Wigner function. The background function is an empirical background function\cite{32} which accounts for random $K^{+}K^{-}$ combinations. The width of the Breit-Wigner function is fixed to the natural width of the $\phi$ meson,\cite{13} and the mean and sigma of the Gaussian function are fixed to the values extracted from a fit to the inclusive $\phi$ mass distribution. Only the overall normalization of the signal function is allowed to vary in the fit. The number of events found in this and the other channels are given in Table 13.
Figure 35
The $K^+K^-$ invariant mass distribution of $D_s^+ \rightarrow \phi\ell^+\nu$ candidates in the non $D_s^{*+}$ tag analysis. The curve is the fit to the invariant mass distribution.

Figure 36(a) shows the $\gamma\gamma$ invariant mass distribution for all $\eta\ell^+$ combinations which pass the above selection criteria. We fit the distribution with a signal and background function. The signal function is an asymmetric Gaussian function with different sigmas for either side of the mean to account for the detector response to photons. The background function is an exponential function which accounts for random $\gamma\gamma$ combinations. The mean and sigmas of the asymmetric Gaussian function are fixed to the values extracted from a fit to the inclusive $\eta$ mass distribution.

Figure 36(b) shows the $\eta\pi^+\pi^-$ invariant mass distribution of all $\eta'\ell^+$ combinations which pass the above selection criteria. We fit the distribution with a signal and background function. The signal function is a Gaussian function. The background function is a straight line which accounts for random $\eta\pi^+\pi^-$ combinations. The mean and sigma of the Gaussian function are fixed to the
values extracted from a fit to the inclusive $\eta'$ mass distribution.

There are two main sources of background in the sample: $X$'s accompanied by fake leptons and random $X\ell^+$ combinations. To estimate the background due to fake leptons, the events with $R_2 > 0.3$ and at least an $X$ in each are used. All charged tracks which do not satisfy the lepton identification criteria in these events are treated as leptons. The entries which pass all the cuts are multiplied by the appropriate fake probability given in Table 10 and Table 11, and summed to give the fake background.

For the range of lepton momenta considered, random $X\ell^+$ combinations come from two sources: from $e^+e^- \rightarrow c\bar{c}$ events in which an $X$ produced in the fragmentation process is combined with a lepton from a semileptonic decay of the charmed hadron in the same jet, and from $\Upsilon(4S) \rightarrow B\bar{B}$ events in which
Table 13
Summary of $D_0^+ \rightarrow \phi l^+ \nu$, $D_0^+ \rightarrow \eta l^+ \nu$ and $D_0^+ \rightarrow \eta' l^+ \nu$ yields in the non $D_0^+$ tag analysis. The errors quoted in this table are statistical only.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$D_0^+ \rightarrow \phi l^+ \nu$</th>
<th>$D_0^+ \rightarrow \eta l^+ \nu$</th>
<th>$D_0^+ \rightarrow \eta' l^+ \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidates</td>
<td>$863.6 \pm 40.5$</td>
<td>$577.6 \pm 30.3$</td>
<td>$42.3 \pm 7.9$</td>
</tr>
<tr>
<td>Fake lepton background</td>
<td>$95.3 \pm 2.2$</td>
<td>$128.8 \pm 1.6$</td>
<td>$9.0 \pm 0.4$</td>
</tr>
<tr>
<td>Continuum $c\bar{c}$ background</td>
<td>$23.1 \pm 0.8$</td>
<td>$35.1 \pm 2.0$</td>
<td>$0.0 \pm 0.0$</td>
</tr>
<tr>
<td>$B\bar{B}$ background</td>
<td>$49.6 \pm 1.8$</td>
<td>$27.6 \pm 2.5$</td>
<td>$1.9 \pm 0.2$</td>
</tr>
<tr>
<td>$D_0^+ \rightarrow \eta l^+ \nu$ feed down</td>
<td>$5.0 \pm 1.3$</td>
<td>$5.0 \pm 1.3$</td>
<td>$5.0 \pm 1.3$</td>
</tr>
</tbody>
</table>

| $D_0^+ + D^+$ | $695.6 \pm 40.6$ | $381.0 \pm 30.5$ | $31.4 \pm 7.9$ |
| $D_0^+$ fraction ($f_{D^+}$) | 0.855 | 0.926 |
| $D_0^+$ signal yield | $695.6 \pm 40.6$ | $325.6 \pm 26.1$ | $29.1 \pm 7.3$ |
| Efficiency, $\epsilon \cdot B$ (%) | 2.078 | 0.807 | 0.204 |
| Efficiency corrected yield | $16734 \pm 976$ | $20184 \pm 1616$ | $7128 \pm 1786$ |

An $X$ produced in the decay chain of one $B$ meson is combined with a lepton from a semileptonic decay of the other $B$ meson. The background from random $Xl^+$ combinations is estimated using the Monte Carlo simulation. However, the $X$ production rate from both fragmentation and $B$ meson decays is not well known. For this reason, an attempt is made to scale the Monte Carlo prediction to account for the $X$ production rate observed in the data.

In the charm continuum events, leptons come primarily from the charm semileptonic decays. Therefore the direction of a charmed hadron is close to that of the high momentum lepton. In the charm continuum events, not only the rate of $X$ production, but also the correlation between the $X$ and the charmed hadron direction is important. The agreement between the data and the Monte Carlo simulation is investigated by considering how often an $X$ is produced in the same hemisphere as a fully reconstructed $D$ meson. The $D^0$ and $D^{*+}$ mesons are reconstructed using the following decay chains: $D^{*+} \rightarrow D^0\pi^+$ and $D^0 \rightarrow K^-\pi^+$. The reconstructed $D$ mesons are required to have momenta above 2.5 GeV/c in order
Figure 37
The lepton momentum distribution of $D^+_s \rightarrow \phi \ell^+ \nu$ candidates in the non $D^+_s$ tag analysis. The solid circles represent the candidates in each lepton momentum bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background.

to account for the range of $D$ momenta which are expected to contribute leptons in the momentum range of interest. For this particular study, the $X$ momentum criterion is relaxed to 0.8 GeV/c in order to provide sufficient statistics. In this study we have assumed that the $X$ momentum distribution is well reproduced by the Monte Carlo simulation, and that it is the rate of $X$ production in the fragmentation process which contributes the greatest uncertainty. The yields of $D$ and $X$ mesons produced in the same hemisphere are obtained by fitting their invariant mass distributions. $D$ meson combinatoric backgrounds are accounted for by subtracting the number of $X$'s found in the sidebands of the $D$ meson invariant mass distribution.

In data $1.2 \pm 0.5 \eta$ mesons are found for every 1000 reconstructed $D$ mesons. This is to be compared with $1.7 \pm 0.1 \eta$ mesons per 1000 $D$ mesons.
Figure 38

The lepton momentum distributions of $D_2^+ \rightarrow \eta^+ \ell^+ \nu$ and $D_2^+ \rightarrow \eta'^+ \ell^+ \nu$ candidates in the non $D_s^+$ tag analysis. The solid circles represent the candidates in each lepton momentum bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background.
The $\phi$ momentum distribution of $D_{s}^{+} \rightarrow \phi \ell^{+} \nu$ candidates in the non $D_{s}^{+}$ tag analysis. The solid circles represent the candidates in each $\phi$ momentum bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background.

in the $e^{+}e^{-} \rightarrow c\bar{c}$ Monte Carlo sample. Therefore the Monte Carlo estimate of the charm continuum background for the $D_{s}^{+} \rightarrow \eta \ell^{+} \nu$ decay is scaled by the factor of $0.7 \pm 0.3$. The scales for $D_{s}^{+} \rightarrow \phi \ell^{+} \nu$ and $D_{s}^{+} \rightarrow \eta' \ell^{+} \nu$ decays are $1.0 \pm 0.7$ and $0.0^{+0.8}_{-0.0}$, respectively. With these scales applied, the Monte Carlo simulation predicts $23.1 \pm 0.8 \pm 15.7$ charm continuum backgrounds for $D_{s}^{+} \rightarrow \phi \ell^{+} \nu$, $35.1 \pm 2.0 \pm 14.1$ for $D_{s}^{+} \rightarrow \eta \ell^{+} \nu$ and $0.0 \pm 0.0^{+0.8}_{-0.0}$ for $D_{s}^{+} \rightarrow \eta' \ell^{+} \nu$ where the first errors are statistical and the second errors are systematic. The systematic errors originate from the uncertainties in the scales.

The background from random $X\ell^{+}$ combinations in $B\bar{B}$ events is estimated in a similar manner. In this case the directions of $X$ and the lepton are uncorrelated. For this reason it is sufficient to compare the number of $X$'s with momentum above $1.0$ GeV/c in the continuum subtracted $\Upsilon(4S)$ data with that
Figure 40
The $X$ momentum distribution of $D_{s}^{+} \rightarrow X \ell^{+} \nu$ ($X \equiv \eta$ or $\eta'$) candidates in the non $D_{s}^{++}$ tag analysis. The solid circles represent the candidates in each $X$ momentum bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background. The cut on $\eta'$ momentum, discussed in the text, has been removed to plot the corresponding distribution.

observed in the $\Upsilon(4S) B \bar{B}$ Monte Carlo sample. In data $7.4 \pm 0.8 \eta$ mesons are found for every 1000 $B \bar{B}$ events, which is to be compared with $6.4 \pm 0.1$ in the $B \bar{B}$ Monte Carlo sample. Therefore the Monte Carlo estimate of the $B \bar{B}$ background for the $D_{s}^{+} \rightarrow \eta \ell^{+} \nu$ decay is scaled by the factor of $1.1 \pm 0.1$. The scales for $D_{s}^{+} \rightarrow \phi \ell^{+} \nu$ and $D_{s}^{+} \rightarrow \eta \ell^{+} \nu$ decays are $1.0 \pm 0.1$ and $0.3 \pm 0.2$, respectively. After these scales are applied, the predicted backgrounds from $B \bar{B}$ events are $49.6 \pm 1.8 \pm 4.1$ events for $D_{s}^{+} \rightarrow \phi \ell^{+} \nu$, $27.6 \pm 2.5 \pm 3.1$ for $D_{s}^{+} \rightarrow \eta \ell^{+} \nu$ and $1.9 \pm 0.2 \pm 1.5$ for $D_{s}^{+} \rightarrow \eta' \ell^{+} \nu$. Again the systematic errors originate from the uncertainties in the scales.

For the $D_{s}^{+} \rightarrow \eta \ell^{+} \nu$ decay, the feed down from the decay $D_{s}^{+} \rightarrow \eta' \ell^{+} \nu$
Figure 41

The $P_{#phi^+}$ and $M_{#phi^+}$ distributions of $D_s^+ \to #phi #ell^+ #nu$ candidates in the non $D_s^{*+}$ tag analysis. The solid circles represent the candidates in each $P_{#phi^+}$ or $M_{#phi^+}$ bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background. The cut on $P_{#phi^+}$ or $M_{#phi^+}$, discussed in the text, has been removed to plot the corresponding distribution.

is estimated by the Monte Carlo simulation. Due to the hard momentum cuts on the $#eta$ and $#eta #ell^+$, the efficiency is found to be small (0.27%) which is about 1/8 of the reconstruction efficiency of the $D_s^+ \to #eta #ell^+ #nu$ decay. In the data sample used for this analysis, the $D_s^+ \to #eta' #ell^+ #nu$ channel has an efficiency corrected yield of 7128 ± 1786 events, which will be discussed below. Multiplying this yield by the feed down efficiency and the branching fractions of $#eta' \to #eta #pi^+ #pi^-$, $#eta' \to #eta #pi^0 #pi^0$ and $#eta \to #gamma #gamma$, the number of the $D_s^+ \to #eta' #ell^+ #nu$ feed downs is estimated to be 5.0 ± 1.3 ± 1.0. The systematic error in this estimation originates from the limited statistics of the $D_s^+ \to #eta' #ell^+ #nu$ Monte Carlo events.

The contamination from the Cabibbo suppressed $D^+ \to #eta #ell^+ #nu$ decay in the
The $P_{\eta \ell^+}$ and $M_{\eta \ell^+}$ distributions of $D_s^+ \rightarrow X \ell^+ \nu$ ($X \equiv \eta$ or $\eta'$) candidates in the non $D_s^{**}$ tag analysis. The solid circles represent the candidates in each $P_{\eta \ell^+}$ or $M_{\eta \ell^+}$ bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background. The cut on $P_{\eta \ell^+}$ or $M_{\eta \ell^+}$, discussed in the text, has been removed to plot the corresponding distribution.
$D_s^+ \rightarrow \eta \ell^+ \nu$ signal is relatively large since this analysis does not distinguish $D_s^+$ from $D^+$. The ratio of the number of $D^+ \rightarrow \eta \ell^+ \nu$ events to that of $D_s^+ \rightarrow \eta \ell^+ \nu$ events, $R^\eta \equiv N(D^+ \rightarrow \eta \ell^+ \nu)/N(D_s^+ \rightarrow \eta \ell^+ \nu)$, is estimated to be 0.17,[37] and the contamination fraction can be expressed as $1 - f^\eta_{D_s^+}$ where $f^\eta_{D_s^+} \equiv 1/(1 + R^\eta) = 0.855 \pm 0.051$. The signal yield of the $D_s^+ \rightarrow \eta \ell^+ \nu$ decay is calculated as

$$N_{\text{signal}} = (N_{\text{candidates}} - N_{\text{fake lepton}} - N_{\text{ee}} - N_{B^+} - N_{D_s^+ \rightarrow \eta \ell^+ \nu \text{ feed down}}) \times f^\eta_{D_s^+} = 325.6 \pm 26.1.$$

In the same manner the ratio of the number of $D^+ \rightarrow \eta' \ell^+ \nu$ events to that of $D_s^+ \rightarrow \eta' \ell^+ \nu$ events, $R'^\eta \equiv N(D^+ \rightarrow \eta' \ell^+ \nu)/N(D_s^+ \rightarrow \eta' \ell^+ \nu)$, is estimated to be 0.08.[37] The signal yield of the $D_s^+ \rightarrow \eta' \ell^+ \nu$ decay is calculated as

$$N_{\text{signal}} = (N_{\text{candidates}} - N_{\text{fake lepton}} - N_{\text{ee}} - N_{B^+} - N_{D_s^+ \rightarrow \eta' \ell^+ \nu \text{ feed down}}) \times f^{\eta'}_{D_s^+} = 29.1 \pm 7.3 \text{ where } f^{\eta'}_{D_s^+} \equiv 1/(1 + R'^\eta) = 0.926 \pm 0.026.$$

Table 13 gives the yields after correcting for efficiencies and the $\phi \rightarrow K^+ K^-$, $\eta \rightarrow \gamma \gamma$ and $\eta' \rightarrow \eta \pi^+ \pi^-$ branching fractions.[34]

Figure 37 and Figure 38 show the number of $\phi$'s, $\eta$'s and $\eta$'s which fall in each bin of the lepton momentum. Figure 39 and Figure 40 show the number of $\phi$'s, $\eta$'s and $\eta$'s which fall in each bin of the $\phi$, $\eta$ and $\eta'$ momentum. Figure 41 and Figure 42 show the number of $\phi$'s, $\eta$'s and $\eta$'s which fall in each bin of the $\phi \ell^+$, $\eta \ell^+$ and $\eta' \ell^+$ momentum and mass. In each figure, the combined background estimate is also shown as well as the sum of the simulated signal and the combined background estimate. The simulated signal has been normalized to the number of candidates extracted from the fit to the $K^+ K^-$, $\gamma \gamma$ and $\eta \pi^+ \pi^-$ invariant mass spectra.

To get the effective yield in the electron channel, the reduction of the muon rate relative to the electron rate due to the reduced phase space[40,41] and the efficiency loss in the electron rate due to the final state radiation[40,43] are needed to be taken into account because they are not simulated in the Monte Carlo. With these corrections applied, the ratios of branching fractions are

$$R_{\eta/\phi} = \frac{B(D_s^+ \rightarrow \eta e^+ \nu)}{B(D_s^+ \rightarrow \phi e^+ \nu)} = \frac{20733 \pm 1660}{17104 \pm 998} = 1.21 \pm 0.12 \pm 0.16, \quad (3.1)$$
\[ R_{\eta'/\phi} = \frac{B(D_s^+ \rightarrow \eta' e^+\nu)}{B(D_s^+ \rightarrow \phi e^+\nu)} = \frac{7286 \pm 1825}{17104 \pm 998} = 0.43 \pm 0.11 \pm 0.07, \]  
\[ R_{\eta'/\eta} = \frac{B(D_s^+ \rightarrow \eta e^+\nu)}{B(D_s^+ \rightarrow \eta e^+\nu)} = \frac{7286 \pm 1825}{20733 \pm 1660} = 0.35 \pm 0.09 \pm 0.07, \]

where the first error is statistical, and the second is systematic in each ratio. The sources of the systematic errors and their contributions to the above ratios will be explained in the next section.

### 3.5 Systematic Errors in Non \(D_s^{*+}\) Tag Analysis

The sources of systematic errors in the non \(D_s^{*+}\) tag analysis are the uncertainties in the followings: the estimation of fake lepton, charm continuum and \(B\bar{B}\) backgrounds, the estimation of the \(D_s^+ \rightarrow \eta' \ell^+\nu\) feed down to the \(D_s^+ \rightarrow \eta\ell^+\nu\) channel, the estimation of the \(D^+ \rightarrow \eta (\eta')\ell^+\nu\) contamination in the \(D_s^+ \rightarrow \eta (\eta')\ell^+\nu\) channel, the understanding of \(D_s^{*+}\) and \(D_s^+\) production through the fragmentation process, kaon and pion identification, \(\eta\) reconstruction, the \(\phi\) polarization in the \(D_s^+ \rightarrow \phi\ell^+\nu\) channel, fit, charged particle tracking, and the efficiency obtained using the \(D_s^+ \rightarrow X\ell^+\nu\) Monte Carlo events.

There is another method to measure the momentum dependent probability of a hadron faking a lepton, the so-called "tagging" method.\(^{[43]}\) In this method, pure samples of pions, kaons and protons (for example, pions from \(K_s \rightarrow \pi^+\pi^-\) or \(D^0 \rightarrow K^-\pi^+\), kaons from \(D^0 \rightarrow K^-\pi^+\) or \(\phi \rightarrow K^+K^-\) and protons from \(\Lambda \rightarrow p\pi^-\) decays) are selected ("tagged"), and the momentum dependent rate of each hadron species satisfying the lepton identification criteria is measured. The composition fraction (again momentum dependent) of each hadron species in data is well measured via the corresponding hadron identification. Multiplying the rate of each hadron species by the corresponding composition fraction and
summing over all species gives the probability of a hadron faking a lepton.

The backgrounds due to fake leptons in $D^+_s \rightarrow \phi l^+\nu$, $D^+_s \rightarrow \eta l^+\nu$ and $D^+_s \rightarrow \eta' l^+\nu$ channels are re-estimated with different sets of the fake probabilities measured by different “tagging” methods. These estimations differ by at most 30% from the estimation with the original set of the probabilities explained in the section of the lepton identification. This 30% difference translates to the systematic errors of 4.1% for the $D^+_s \rightarrow \phi l^+\nu$ channel, 10.1% for the $D^+_s \rightarrow \eta l^+\nu$ channel and 8.6% for the $D^+_s \rightarrow \eta' l^+\nu$ channel.

The sources of systematic error in estimating the backgrounds due to random $Xl^+$ combinations in charm continuum and $B\bar{B}$ events are the uncertainties in scaling the Monte Carlo estimates of these backgrounds. These scales are given in the previous section. The uncertainty in the scale for charm continuum events translates to a systematic error of 2.3% in the $D^+_s \rightarrow \phi l^+\nu$ channel, 3.7% in the $D^+_s \rightarrow \eta l^+\nu$ channel and 12.5% for the $D^+_s \rightarrow \eta' l^+\nu$ channel. For $B\bar{B}$ events, the uncertainty in the scale translates to a systematic error of 0.6% in the $D^+_s \rightarrow \phi l^+\nu$ channel, 0.8% in the $D^+_s \rightarrow \eta l^+\nu$ channel and 4.7% for the $D^+_s \rightarrow \eta' l^+\nu$ channel.

There is a 0.3% systematic error in estimating the the feed down of the $D^+_s \rightarrow \eta l^+\nu$ decay to the $D^+_s \rightarrow \eta l^+\nu$ decay due to the limited statistics of the $D^+_s \rightarrow \eta' l^+\nu$ Monte Carlo events. The systematic error in estimating the contamination from the Cabibbo suppressed $D^+ \rightarrow \eta l^+\nu$ ($D^+ \rightarrow \eta' l^+\nu$) decay in the $D^+_s \rightarrow \eta l^+\nu$ ($D^+_s \rightarrow \eta' l^+\nu$) channel originates from the 6.0% (2.8%) uncertainty in $f_D^2(f_{D^+_s}^2)$.  

CLEO II has measured the production ratio of $D^+_s$ mesons (almost always decaying to $D^+_s\gamma$) to “fragmentary” $D^+_s$ mesons to be $1.0 \pm 0.2$. Note that a “fragmentary” $D^+_s$ meson comes directly from the fragmentation process, not from the decay of $D^+_s \rightarrow D^+_s\gamma$. In the standard CLEO II Monte Carlo generator
Table 14
Efficiencies and various ratios of the efficiencies obtained with the four types of $D_s^+ \rightarrow X \ell^+ \nu$ Monte Carlo events in the non $D_s^+$ tag analysis. See the text for the explanation of the four types.

<table>
<thead>
<tr>
<th></th>
<th>&quot;hard&quot; $D_s^+$</th>
<th>&quot;soft&quot; $D_s^+$</th>
<th>&quot;hard&quot; $D_s^+$</th>
<th>&quot;soft&quot; $D_s^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_\phi$ (%)</td>
<td>4.414±0.072</td>
<td>3.386±0.039</td>
<td>5.104±0.063</td>
<td>4.052±0.068</td>
</tr>
<tr>
<td>$\epsilon_\eta$ (%)</td>
<td>2.086±0.037</td>
<td>1.609±0.020</td>
<td>2.555±0.030</td>
<td>2.058±0.044</td>
</tr>
<tr>
<td>$\epsilon_\eta'/\epsilon_\phi$</td>
<td>1.233±0.040</td>
<td>0.933±0.011</td>
<td>1.499±0.016</td>
<td>1.173±0.036</td>
</tr>
<tr>
<td>$\epsilon_\eta'/\epsilon_\eta$</td>
<td>0.472±0.011</td>
<td>0.475±0.008</td>
<td>0.501±0.008</td>
<td>0.508±0.014</td>
</tr>
<tr>
<td>$(\epsilon_\eta'/\epsilon_\phi)<em>\text{hard}/(\epsilon</em>\eta'/\epsilon_\phi)_\text{soft}$</td>
<td>0.994±0.029</td>
<td>0.986±0.031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\epsilon_\eta'/\epsilon_\eta)<em>\text{hard}/(\epsilon</em>\eta'/\epsilon_\eta)_\text{soft}$</td>
<td>1.014±0.041</td>
<td>1.015±0.039</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\epsilon_\eta'/\epsilon_\eta)<em>\text{hard}/(\epsilon</em>\eta'/\epsilon_\eta)_\text{soft}$</td>
<td>0.981±0.040</td>
<td>0.971±0.039</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

of continuum events, there is a parameter called the "Peterson fragmentation function" ($\epsilon_P$) that determines the momentum distribution of particles produced through the fragmentation process. It has been found that $D_s^+$ events generated with $\epsilon_P = -0.011$ and $D_s^+$ (not from $D_s^{*+}$) events generated with $\epsilon_P = -0.040$ have similar $D_s^+$ momentum spectra, and therefore this set of $\epsilon_P$ values reproduces the measured production ratio. For this reason, an equal mixture of these $D_s^+$ and $D_s^+$ (not from $D_s^{*+}$) Monte Carlo events is used for the non $D_s^{*+}$ tag analysis of the $D_s^+ \rightarrow X \ell^+ \nu$ decays. Even so, the $D_s^{*+}$ and $D_s^+$ productions through the fragmentation process are not well understood theoretically.

To compare the reconstruction efficiencies and more importantly the ratios of the efficiencies, $D_s^{*+}$ and $D_s^+$ (not from $D_s^{*+}$) Monte Carlo events with switched $\epsilon_P$ values are also generated. Therefore four types of Monte Carlo events for the $D_s^+ \rightarrow X \ell^+ \nu$ decays are used for the comparison; they are $D_s^{*+}$ with $\epsilon_P = -0.011$, $D_s^{*+}$ with $\epsilon_P = -0.040$, $D_s^+$ (not from $D_s^{*+}$) with $\epsilon_P = -0.011$ and $D_s^+$ (not
from $D_s^{*+}$) with $\varepsilon_P = -0.040$. The word "hard" is used for the Monte Carlo events generated with $\varepsilon_P = -0.011$ and the "soft" for the events generated with $\varepsilon_P = -0.040$ because $\varepsilon_P = -0.011$ produces "harder" momentum spectra for the primary particles from the fragmentation process than $\varepsilon_P = -0.040$.

Table 14 shows the efficiencies and their ratios for the four types of the Monte Carlo $D_s^+ \rightarrow X \ell^+ \nu$ events. It also shows the ratios of the "hard" to "soft" efficiency ratios. The deviation of these "hard" to "soft" ratios from unity measures the systematic errors in $R_{n/\phi}$, $R_{n'}/\phi$ or $R_{n'}/\eta$ due to the uncertainty in our understanding of $D_s^{*+}$ and $D_s^+$ production through the fragmentation process. For each of $R_{n/\phi}$, $R_{n'}/\phi$ and $R_{n'}/\eta$, there are two "hard" to "soft" ratios, one for the $D_s^{*+}$ events and the other for the $D_s^+$ (not from $D_s^{*+}$) events. The ratio which has the largest deviation from unity gives the most conservative estimation of the systematic error. Therefore the systematic errors due to this uncertainty are at most 1.5% in $R_{n/\phi}$, 1.5% in $R_{n'}/\phi$ and 3.0% in $R_{n'}/\eta$. Note that the systematic errors due to this uncertainty in $D_s^+ \rightarrow \phi \ell^+ \nu$, $D_s^+ \rightarrow \eta \ell^+ \nu$ and $D_s^+ \rightarrow \eta' \ell^+ \nu$ channels are correlated, and the errors in $R_{n/\phi}$, $R_{n'}/\phi$ and $R_{n'}/\eta$ (therefore the combined errors for two of the three channels, not for each channel) are found.

The momentum dependent efficiencies for identifying the $\phi \rightarrow K^+ K^-$ decays are obtained from a sample of $D_s^+ \rightarrow \phi \pi^+$ decays in data. There might be a systematic bias in obtaining these efficiencies because the particular decay chain of $D_s^+ \rightarrow \phi \pi^+$ and $\phi \rightarrow K^+ K^-$ is used. The source of the possible bias may be the criterion of selecting the $\pi^+$ candidates in the $D_s^+ \rightarrow \phi \pi^+$ decays. However, the kinematic restrictions required for selecting the sample of $D_s^+ \rightarrow \phi \pi^+$ decays do not cause any bias because the efficiencies are obtained as a function of $\phi$ momentum. The criterion of selecting the $\pi^+$ candidates appears in both numerator and denominator of the ratio of $\phi$ yields with and without requiring the kaon consistency for two kaon candidates. Therefore the most of the systematic bias cancels out in the first order. The residual of the bias can be
Figure 43

Ratios of the efficiencies of identifying the $\phi \rightarrow K^+K^-$ decays as a function of $\phi$ momentum found with a sample of $D^+_s \rightarrow \phi\pi^+$ decays in data. The circles represent the ratios of the efficiencies for $PROB(\chi^2_\pi, N) \geq 0.0$ to the efficiencies for $PROB(\chi^2_\pi, N) > 0.02$. The squares represent the ratios of the efficiencies for $PROB(\chi^2_\pi, N) > 0.0027$ to the efficiencies for $PROB(\chi^2_\pi, N) > 0.02$. The triangles represent the ratios of the efficiencies for $PROB(\chi^2_\pi, N) > 0.0455$ to the efficiencies for $PROB(\chi^2_\pi, N) > 0.02$.

Estimated by comparing the efficiencies obtained from different requirements for the $\pi^+$ candidates.

These efficiencies for identifying the $\phi \rightarrow K^+K^-$ decays are obtained for four different requirements of pion consistency, $PROB(\chi^2_\pi, N) \geq 0.0, > 0.0027, > 0.02, > 0.0455$. The first corresponds to no requirement for pion consistency. The rest correspond to $\chi^2_\pi < 9.0, < 5.41, < 4.0$, respectively, if only one of ionization energy loss or time of flight measurements is available ($N = 1$). If both ionization energy loss and time of flight measurements are available ($N = 2$), they correspond to $\chi^2_\pi < 11.83, < 7.82, < 6.18$, respectively.
Table 15
The break down of systematic errors (%) in $R_{\eta/\phi}$ from the non $D_s^+$ tag analysis

<table>
<thead>
<tr>
<th>Uncorrelated errors</th>
<th>$D_s^+ \to \phi \ell^+ \nu$</th>
<th>$D_s^+ \to \eta \ell^+ \nu$</th>
<th>combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c\bar{c}$ BG</td>
<td>2.3</td>
<td>3.7</td>
<td>4.4</td>
</tr>
<tr>
<td>$B\bar{B}$ BG</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>K particle ID</td>
<td>3.0</td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>$\eta$ reconstruction</td>
<td></td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>$D_s^+ \to \eta' \ell^+ \nu$ feed down</td>
<td></td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$f_{D_s^+}$</td>
<td></td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Fit</td>
<td>3.0</td>
<td>5.0</td>
<td>5.8</td>
</tr>
<tr>
<td>MC statistics</td>
<td>1.2</td>
<td>1.5</td>
<td>1.9</td>
</tr>
<tr>
<td>$\phi$ polarization</td>
<td>1.0</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Correlated errors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fake lepton BG</td>
<td>4.1</td>
<td>10.1</td>
<td>6.0</td>
</tr>
<tr>
<td>Fragmentation</td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Tracking</td>
<td>6.0</td>
<td>2.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>13.5</td>
</tr>
</tbody>
</table>

Figure 43 shows the ratios of these efficiencies obtained with different requirements for the $\pi^+$ candidates. The results of fitting the ratios to a constant are $0.996 \pm 0.024$ with $\chi^2_{\text{dof}} = 0.2$ for circles, $1.000 \pm 0.024$ with $\chi^2_{\text{dof}} = 0.1$ for squares $1.001 \pm 0.025$ with $\chi^2_{\text{dof}} = 0.1$ for triangles where $\chi^2_{\text{dof}}$ is the chi-square per degree of freedom of the fit. All fit results are close to unity, and they show that the residual of the bias is at most $(0.4 \pm 2.4)\%$.

For each criterion of selecting the $\pi^+$ candidates, the total efficiency for identifying $\phi$'s in the decay chain of $D_s^+ \to \phi \ell^+ \nu$ and $\phi \to K^+ K^-$ is calculated by combining the momentum dependent efficiencies with the predicted $\phi$ momentum spectrum of $D_s^+ \to \phi \ell^+ \nu$ Monte Carlo events. They are $0.837 \pm 0.025$, $0.843 \pm 0.024$, $0.846 \pm 0.025$ and $0.844 \pm 0.025$, respectively, in the order of $\pi^+$ selection criteria listed above. The total efficiency used to calculate the efficiency corrected...
Table 16
The break down of systematic errors (%) in \( R_{\eta'\phi} \) from the non \( D_{s}^{+} \) tag analysis

<table>
<thead>
<tr>
<th>Uncorrelated errors</th>
<th>( D_{s}^{+} \rightarrow \phi \ell^{+}\nu )</th>
<th>( D_{s}^{+} \rightarrow \eta'\ell^{+}\nu )</th>
<th>combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c\bar{c} ) BG</td>
<td>2.3</td>
<td>12.5</td>
<td>12.7</td>
</tr>
<tr>
<td>( B\bar{B} ) BG</td>
<td>0.6</td>
<td>4.7</td>
<td>4.7</td>
</tr>
<tr>
<td>( K/\pi ) particle ID</td>
<td>3.0</td>
<td>3.0</td>
<td>4.2</td>
</tr>
<tr>
<td>( \eta ) reconstruction</td>
<td></td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>( f_{D_{s}^{+}} )</td>
<td></td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>Fit</td>
<td>3.0</td>
<td>3.0</td>
<td>4.2</td>
</tr>
<tr>
<td>MC statistics</td>
<td>1.2</td>
<td>2.5</td>
<td>2.8</td>
</tr>
<tr>
<td>( \phi ) polarization</td>
<td>1.0</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Correlated errors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fake lepton BG</td>
<td>4.1</td>
<td>8.6</td>
<td>4.5</td>
</tr>
<tr>
<td>Fragmentation</td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Tracking</td>
<td>6.0</td>
<td>6.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>16.9</td>
</tr>
</tbody>
</table>

\( D_{s}^{+} \rightarrow \phi \ell^{+}\nu \) yield is 0.846 ± 0.025, which is obtained with the third \( \pi^{+} \) selection criterion. These four efficiencies are within each other's error, which is close to (but less than) 3.0%. Therefore the systematic error in determining the total efficiency of identifying \( \phi \)'s in the decay chain of \( D_{s}^{+} \rightarrow \phi \ell^{+}\nu \) and \( \phi \rightarrow K^{+}K^{-} \) is at most 3.0%.

The systematic error in reconstructing the decay of \( \eta \rightarrow \gamma\gamma \) has been studied by other members of the CLEO II collaboration. It has been found that the systematic error in the reconstruction is 5.0%.[48]

The total efficiency of identifying two pions in the decay chain of \( D_{s}^{+} \rightarrow \eta'\ell^{+}\nu \) and \( \eta' \rightarrow \eta \pi^{+}\pi^{-} \) is 0.831 ± 0.015 as explained in the previous section. The statistical error (0.015/0.831 = 1.8%) in this efficiency propagates to the signal yield in the \( D_{s}^{+} \rightarrow \eta'\ell^{+}\nu \) channel as a systematic error. But a conservative 3.0%
Table 17
The break down of systematic errors (%) in $R_{\eta'/\eta}$ from the non $D_s^+$ tag analysis

<table>
<thead>
<tr>
<th></th>
<th>$D_s^+ \rightarrow \eta l^+ \nu$</th>
<th>$D_s^+ \rightarrow \eta' l^+ \nu$</th>
<th>combined</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uncorrelated errors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c\bar{c}$ BG</td>
<td>3.7</td>
<td>12.5</td>
<td>13.0</td>
</tr>
<tr>
<td>$B\bar{B}$ BG</td>
<td>0.8</td>
<td>4.7</td>
<td>4.8</td>
</tr>
<tr>
<td>$\pi$ particle ID</td>
<td></td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$D_s^+ \rightarrow \eta' l^+ \nu$ feed down</td>
<td>0.3</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>Fit</td>
<td>5.0</td>
<td>3.0</td>
<td>5.8</td>
</tr>
<tr>
<td>MC statistics</td>
<td>1.5</td>
<td>2.5</td>
<td>2.9</td>
</tr>
<tr>
<td><strong>Correlated errors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$ reconstruction</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Fake lepton BG</td>
<td>10.1</td>
<td>8.6</td>
<td>1.5</td>
</tr>
<tr>
<td>Fragmentation</td>
<td></td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>Tracking</td>
<td>2.0</td>
<td>6.0</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>Anti correlated errors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\int_{D_s^+}$</td>
<td>6.0</td>
<td>2.8</td>
<td>8.8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td>19.3</td>
</tr>
</tbody>
</table>

Systematic error is assigned to the signal yield in the $D_s^+ \rightarrow \eta l^+ \nu$ channel due to the statistical error in this efficiency.

The measured ratio ($\Gamma_L/\Gamma_T$) of the longitudinal to transverse polarization of the final state vector meson $K^{*0}$ in the $D^+ \rightarrow K^{*0} l^+ \nu$ decay is $1.23 \pm 0.13$.\cite{3} The signal Monte Carlo events of the $D_s^+ \rightarrow \phi l^+ \nu$ decay are generated in accordance with this measured ratio. On the other hand, the measured $\Gamma_L/\Gamma_T$ of the $\phi$ in the $D_s^+ \rightarrow \phi l^+ \nu$ decay is $1.0 \pm 0.3$.\cite{4} Therefore there is a possible systematic error related to this matter. To study this systematic error, the efficiencies of $D_s^+ \rightarrow \phi l^+ \nu$ Monte Carlo events generated with different $\Gamma_L/\Gamma_T$ values are compared. The efficiencies for $\Gamma_L/\Gamma_T = 1.00, 1.23$ and $1.40$ are $(4.199 \pm 0.053)\%$, $(4.233 \pm 0.050)\%$ and $(4.254 \pm 0.055)\%$, respectively. These efficiencies vary at most by 1.0%. Therefore the systematic error due to the
uncertainty in $\Gamma_L/\Gamma_T$ is estimated to be 1.0%.

The $K^+K^-, \gamma\gamma$ and $\eta\pi^+\pi^-$ invariant mass spectra are fitted to extract the candidates and various background contributions in $D_s^+ \to \phi\ell^+\nu$, $D_s^+ \to \eta\ell^+\nu$ and $D_s^+ \to \eta'\ell^+\nu$ decays, respectively. The signal and background functions used to fit these spectra are explained in the previous section. To estimate the uncertainty in fitting each invariant mass spectrum, the mean and sigma of the signal function has been allowed to float instead of being fixed as well as the second order Chebyshev polynomial has been tried for the background function. It is concluded that the systematic errors due to the uncertainty in fitting are 3.0% for the $D_s^+ \to \phi\ell^+\nu$ decay, 5.0% in the $D_s^+ \to \eta\ell^+\nu$ decay and 3.0% for the $D_s^+ \to \eta'\ell^+\nu$ decay.

The systematic error in tracking charged particles has been studied by other members of the CLEO II collaboration. It has been found that the systematic error in tracking is 2.0% per charged track. The systematic error in the efficiency obtained using the $D_s^+ \to X\ell^+\nu$ Monte Carlo events originates from the limited statistics of the Monte Carlo events.

Table 15, Table 16 and Table 17 summarize the systematic errors for $R_{\eta/\phi}$, $R_{\eta'/\phi}$ and $R_{\eta'/\eta}$ respectively. The total systematic error in each measurement is calculated as follows: For each source of the uncorrelated errors, the errors in two decay modes are added in quadrature to give the combined error. For each source of the correlated errors, except the $\eta$ reconstruction in $R_{\eta'/\eta}$, the combined error is the absolute value of the difference between the errors in two decay modes. Different momentum cuts are used for $\eta$ mesons in $D_s^+ \to \eta\ell^+\nu$ and $D_s^+ \to \eta'\ell^+\nu$ decay modes. Therefore the systematic errors due to the uncertainty in the $\eta$ reconstruction in the two modes do not cancel out in $R_{\eta'/\eta}$. To be conservative, a 5.0% systematic error is assigned to $R_{\eta'/\eta}$ due to the uncertainty in the $\eta$ reconstruction although it must be less than 5.0%. The reason for
only the combined error, not the error in each mode, due to the uncertainty in the fragmentation being listed was explained above. For each source of the anticorrelated errors, the combined error is the linear sum of the errors in two decay modes. Finally the combined errors for all sources are added in quadrature to give the total systematic error in each measurement.

3.6 \( D_s^{*+} \) Tag Analysis of \( D_s^+ \to X\ell^+\nu \) Decay

In the \( D_s^{*+} \) tag analysis, a low energy photon from the \( D_s^{*+} \to D_s^+\gamma \) decay is searched for in coincidence with an energetic pair of a lepton and an \( X (\equiv \phi \text{ or } \eta) \). The photon is required to be in the same hemisphere as the \( X\ell^+ \) pair with respect to the thrust axis. To increase statistics the electron momentum cut is lowered to 0.7 GeV/c and the \( X\ell^+ \) momentum cut to 2.0 GeV/c. The rest of the cuts for selecting leptons, \( X \)'s and \( X\ell^+ \)'s are the same as for the non \( D_s^{*+} \) tag analysis.

The photon must lie in the fiducial region, \(|\cos \theta| < 0.71\), and have energy greater than 0.12 GeV. The “E9/E25” of a photon candidate must be consistent with that expected for a true photon. To further reject photons from \( \pi^0 \) decays, all \( \gamma\gamma \) combinations that fall within 2.5\( \sigma \) of the \( \pi^0 \) mass are vetoed. To select \( X\ell^+\gamma \) candidates which come from the \( D_s^{*+} \to D_s^+\gamma \) decays, the pseudo mass difference, \( \Delta M \equiv M_{X\ell^+\gamma} - M_{X\ell^+} \), is required to be between 0.1 and 0.2 GeV/c\(^2\), where \( M_{X\ell^+} \) and \( M_{X\ell^+\gamma} \) are the invariant masses of \( X\ell^+ \) and \( X\ell^+\gamma \) systems, respectively. Table 18 lists the cuts used for the \( D_s^{*+} \) tag analysis.

Figure 44(a) and (b) show the \( K^+K^- \) and \( \gamma\gamma \) invariant mass distributions for all \( \phi\ell^+\gamma \) and \( \eta\ell^+\gamma \) combinations which satisfying the above selection criteria. The number of candidates found in these channels are given in Table 19.
Table 18
Cuts for the $D_s^{*+}$ tag analysis

<table>
<thead>
<tr>
<th>lepton</th>
<th>$P_{\ell^+} &gt; 0.7$ GeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X \equiv \phi, \eta$</td>
<td>$P_X &gt; 1.0$ GeV/c</td>
</tr>
<tr>
<td>$\eta \ell^+$</td>
<td>$M_{\eta \ell^+} &gt; 1.2$ GeV/c$^2$</td>
</tr>
<tr>
<td>$X\ell^+$</td>
<td>$M_{X\ell^+} &lt; 1.9$ GeV/c$^2$</td>
</tr>
<tr>
<td>$X\ell^+$</td>
<td>$2.0 &lt; P_{X\ell^+} &lt; 4.5$ GeV/c</td>
</tr>
<tr>
<td>$X\ell^+$ and the lepton in the same hemisphere</td>
<td>$0.12 &lt; E_\gamma &lt; 0.60$ GeV</td>
</tr>
<tr>
<td>photon from $D_s^{*+}$</td>
<td>$\pi^0$ veto for all $P_{\pi^0}$</td>
</tr>
<tr>
<td>photon from $D_s^{*+}$</td>
<td>$X\ell^+$ and the photon in the same hemisphere</td>
</tr>
<tr>
<td>photon from $D_s^{*+}$</td>
<td>$0.1 &lt; \Delta M &lt; 0.2$ GeV/c$^2$</td>
</tr>
</tbody>
</table>

The backgrounds due to fake leptons, random $X\ell^+$ combinations and the $D_s^+ \rightarrow \eta\ell^+\nu$ feed down to $D_s^+ \rightarrow \eta\ell^+\nu$ decay are estimated by the same procedure as for the non $D_s^{*+}$ tag analysis. Also the contamination from the Cabibbo suppressed decay mode $D_s^+ \rightarrow \eta\ell^+\nu$ in the $D_s^{*+}$ signal is estimated with a Monte Carlo simulation. The contamination turns out to be small because of the requirement of the photon from the $D_s^{*+}$. These backgrounds are listed in Table 19.

The background due to the combinations of true $X\ell^+$ pairs and random photons, the so called the "random photon" background, is estimated using a Monte Carlo simulation. This Monte Carlo estimate will be corrected by two scales. The first scale is a relative correction of the "random photon" background in the "fragmentary" $D_s^+$ events with respect to the "random photon" background in the $D_s^{*+}$ events. The second scale is an overall correction for the disagreement between Monte Carlo and data estimates.

The combinations of true $X\ell^+$ pairs and random photons can occur in the events either with $D_s^{*+}$ mesons or with "fragmentary" $D_s^+$ mesons. Again note
that a "fragmentary" $D_s^+$ meson comes directly from the fragmentation process, not from the decay of $D_s^+ \rightarrow D^+_s\gamma$. CLEO II has measured the production ratio of $D_s^+$ mesons to "fragmentary" $D_s^+$ mesons to be $1.0 \pm 0.2$. Therefore the Monte Carlo estimate of the "random photon" background in the "fragmentary" $D_s^+$ events needs to be scaled by the factor of $1.0 \pm 0.2$ relative to the Monte Carlo estimate of the background in the $D_s^+$ events.

The decay chain, $D^{*+} \rightarrow D^0\pi^+$ and $D^0 \rightarrow K^-e^+\nu$, is used to find the overall correction for the disagreement between Monte Carlo and data estimates of the "random photon" background. These events are ideally suited for the test because the $K^-e^+$ pair has kinematics similar to the $X\ell^+$ pair from a $D_s^+$ meson, and any photon associated with a $K^-e^+$ pair in the same jet can not be from a $D_s^+$ meson. The $K^-e^+$ pairs are selected with the same criteria as the $X\ell^+$

Figure 44
The $K^+K^-$ and $\gamma\gamma$ invariant mass distributions of $D_s^+ \rightarrow \phi\ell^+\nu$ and $D_s^+ \rightarrow \eta\ell^+\nu$ candidates in the $D_s^+$ tag analysis. The curve on each distribution is the fit to the invariant mass distribution.
Figure 45
The lepton momentum distributions of $D_s^+ \rightarrow \phi \ell^+ \nu$ and $D_s^+ \rightarrow \eta \ell^+ \nu$ candidates in the $D_s^+$ tag analysis. The solid circles represent the candidates in each momentum bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background.
Table 19
Summary of $D^+_s \rightarrow \phi \ell^+\nu$ and $D^+_s \rightarrow \eta \ell^+\nu$ yields in the $D^+_s$ tag analysis. The errors quoted in this table are statistical only.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$D^+_s \rightarrow \phi \ell^+\nu$</th>
<th>$D^+_s \rightarrow \eta \ell^+\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidates</td>
<td>326.9 ± 27.1</td>
<td>153.4 ± 15.0</td>
</tr>
<tr>
<td>Fake lepton background</td>
<td>27.9 ± 1.2</td>
<td>22.7 ± 0.7</td>
</tr>
<tr>
<td>Continuum $c\bar{c}$ background</td>
<td>15.6 ± 0.7</td>
<td>8.5 ± 1.0</td>
</tr>
<tr>
<td>$B\bar{B}$ background</td>
<td>7.9 ± 0.7</td>
<td>3.8 ± 1.0</td>
</tr>
<tr>
<td>$D^+_s \rightarrow \eta' \ell^+\nu$ feed down</td>
<td>2.3 ± 0.6</td>
<td>1.3 ± 0.1</td>
</tr>
<tr>
<td>$D^+ \rightarrow \eta \ell^+\nu$ contamination</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_s^{*-} +$ Random photon background</td>
<td>275.5 ± 27.2</td>
<td>114.8 ± 15.1</td>
</tr>
<tr>
<td>$D_s^{<em>-}$ fraction ($f_{D_s^{</em>-}}$)</td>
<td>0.759</td>
<td>0.803</td>
</tr>
<tr>
<td>$D_s^{*-}$ signal yield</td>
<td>209.2 ± 20.6</td>
<td>92.2 ± 12.1</td>
</tr>
<tr>
<td>Efficiency, $\epsilon \cdot B$ (%)</td>
<td>1.408</td>
<td>0.474</td>
</tr>
<tr>
<td>Efficiency corrected yield</td>
<td>7431 ± 733</td>
<td>9730 ± 1276</td>
</tr>
</tbody>
</table>

pairs. The decay chain, $D_s^{*-} \rightarrow D^0\pi^+$ and $D^0 \rightarrow K^-e^+\nu$, is reconstructed by fitting the $M_{K^-e^+\pi^+} - M_{K^-e^+}$ distribution. The fit gives $1059 \pm 34 K^-e^+$ pairs in the data sample and $3038 \pm 56 K^-e^+$ pairs in the Monte Carlo sample. These reconstructed $K^-e^+$ pairs are then combined with random photons to calculate $\Delta M$. Finally the $M_{K^-e^+\pi^+} - M_{K^-e^+}$ distribution for the $K^-e^+\gamma$ combinations falling in $\Delta M < 0.5$ GeV/c$^2$ is refitted. The refit gives $226\pm16 K^-e^+$ pairs for the data sample and $676\pm26$ for the Monte Carlo sample. Therefore the probability of a true $X\ell^+$ pair being combined with a random photon is $(21.4 \pm 1.7)\%$ in the data and $(22.3 \pm 0.1)\%$ in the Monte Carlo. Thus the Monte Carlo estimates of the "random photon" background in both the $D_s^{*-}$ and "fragmentary" $D^+_s$ events are scaled by the factor of $0.96 \pm 0.08$.

With these two corrections applied, the Monte Carlo simulation predicts the ratio of the signal to the sum of the signal and the total "random photon" background, $f_{D^+_s}$, to be 0.759 for the $D^+_s \rightarrow \phi \ell^+\nu$ channel and 0.803 for the
Figure 40

The $X$ momentum distribution of $D^+_s \rightarrow X \ell^+\nu$ ($X \equiv \phi$ or $\eta$) candidates in the $D^+_s$ tag analysis. The solid circles represent the candidates in each $X$ momentum bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background.

$D^+_s \rightarrow \eta\ell^+\nu$ channel. The total “random photon” background is the sum of the “random photon” backgrounds in the $D^+_s$ and “fragmentary” $D^+_s$ events. The final signal yield is calculated as $N_{\text{signal}} = (N_{\text{observed}} - N_{\text{fake lepton}} - N_{\text{ct}} - N_{BB} - N_{D^+_s, \eta\ell^+\nu \text{ feed down}} - N_{D^+_s, \eta\ell^+\nu \text{ contamination}}) \times f_{D^+_s}$. After correcting for the detection efficiencies and for the branching fractions of $\phi \rightarrow K^+K^-$ and $\eta \rightarrow \gamma\gamma$,[38] the corrected yields are $7431 \pm 733$ for $D^+_s \rightarrow \phi\ell^+\nu$ and $9730 \pm 1276$ for $D^+_s \rightarrow \eta\ell^+\nu$. Table 19 summarizes the results for both $D^+_s \rightarrow \eta\ell^+\nu$ and $D^+_s \rightarrow \phi\ell^+\nu$ decays.

Figure 45 shows the number of $\phi$'s and $\eta$'s which fall in each lepton momentum bin. Figure 46 shows the number of $\phi$'s and $\eta$'s which fall in each bin of the $\phi$ and $\eta$ momentum. Figure 47 shows the number of $\phi$'s and $\eta$'s which fall in each bin of $\phi\ell^+$ and $\eta\ell^+$ momentum and mass. Figure 48 shows the number
Figure 47

The $P_{xt+}$ and $M_{xt+}$ distributions of $D^+_s \rightarrow Xt^+\nu$ ($X \equiv \phi$ or $\eta$) candidates in the $D^+_s$ tag analysis. The solid circles represent the candidates in each $P_{xt+}$ or $M_{xt+}$ bin. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background. The cut on $P_{xt+}$ or $M_{xt+}$, discussed in the text, has been removed to plot the corresponding distribution.
Figure 48
The $E_\gamma$ and $\Delta M$ distributions of $D_{s}^{+} \rightarrow X t^{+} \nu (X \equiv \phi$ or $\eta)$ candidates in the $D_{s}^{+}$ tag analysis. The solid circles represent the candidates in each bin of $E_\gamma$ or $\Delta M$. The solid histogram shows the sum of the simulated signal and the predicted background. The dotted histogram shows the predicted background. The cut on $\Delta M$, discussed in the text, has been removed to plot the $\Delta M$ distribution.
of $\phi$'s and $\eta$'s which fall in each bin of $E_\gamma$ and $\Delta M$ bin. The combined background estimate is also shown, as well as the sum of the simulated signal and the combined background estimate. The simulated signal has been normalized to the number of candidates extracted from the fit to the $K^+K^-$ and $\gamma\gamma$ invariant mass spectra.

Again the reduction of the muon rate relative to the electron rate due to the reduced phase space\textsuperscript{[40,41]} and the efficiency loss in the electron rate due to final state radiation\textsuperscript{[40,42]} need to be taken into account to get the effective yield in the electron channel. With these corrections applied, the ratio of branching fractions is,

$$R_{\eta/\phi} = \frac{B(D_s^+ \rightarrow \eta e^+\nu)}{B(D_s^+ \rightarrow \phi e^+\nu)} = \frac{9994 \pm 1311}{7558 \pm 746} = 1.32 \pm 0.22 \pm 0.15,$$

where the first error is statistical, and the second is systematic in each ratio. The sources of the systematic errors and their contributions to the above ratios will be explained in the next section.

3.7 Systematic Errors in $D_{s}^{*+}$ Tag Analysis

The sources of systematic errors in the $D_{s}^{*+}$ tag analysis are the uncertainties in the followings: the estimation of fake lepton, charm continuum and $B\bar{B}$ backgrounds, the estimation of the $D_s^+ \rightarrow \eta' l^+\nu$ feed down to the $D_s^+ \rightarrow \eta l^+\nu$ channel, the estimation of the $D^+ \rightarrow \eta l^+\nu$ contamination in the $D_s^+ \rightarrow \eta l^+\nu$ channel, the understanding of $D_s^{*-}$ and $D_s^*$ production through the fragmentation process, the estimation of the random photon background, kaon identification, $\eta$ reconstruction, the $\phi$ polarization in the $D_s^+ \rightarrow \phi l^+\nu$ channel, fit, charged particle tracking, and the efficiency obtained using the $D_s^+ \rightarrow X l^+\nu$ Monte Carlo events.
The backgrounds due to fake leptons in $D^+_s \to \phi \ell^+\nu$ and $D^+_s \to \eta \ell^+\nu$ channels are re-estimated with different sets of fake probabilities measured by different "tagging" methods. These estimates differ by at most 30% from the estimate with the original set of the probabilities explained in the section of the lepton identification. This 30% difference translates to a systematic error of 3.0% in the $D^+_s \to \phi \ell^+\nu$ channel and 5.9% for the $D^+_s \to \eta \ell^+\nu$ channel.

The sources of systematic error in estimating the backgrounds due to random $X\ell^+$ combinations in charm continuum and $B\bar{B}$ events are the uncertainties in scaling the Monte Carlo estimates of these backgrounds. These scales are given in the section of the non $D^+_s$ tag analysis. The uncertainty in the scale for charm continuum events translates to a systematic error of 3.8% in the $D^+_s \to \phi \ell^+\nu$ channel and 3.0% in the $D^+_s \to \eta \ell^+\nu$ channel. For $B\bar{B}$ events, the uncertainty in the scale translates to a systematic error of 0.2% in the $D^+_s \to \phi \ell^+\nu$ channel and 0.3% in the $D^+_s \to \eta \ell^+\nu$ channel.

There are systematic errors in estimating the $D^+_s \to \eta'\ell^+\nu$ feed down and $D^+ \to \eta \ell^+\nu$ contamination in the $D^+_s \to \eta \ell^+\nu$ channel due to the limited statistics of the $D^+_s \to \eta'\ell^+\nu$ and $D^+ \to \eta \ell^+\nu$ Monte Carlo events. They are 0.4% and 0.3%, respectively.

The $D^+_s$ Monte Carlo events generated with $\epsilon_P = -0.011$ are used for the $D^+_s$ tag analysis. There are two types of Monte Carlo events for the $D^+_s \to X\ell^+\nu$ decays; $D^+_s$ with $\epsilon_P = -0.011$, $D^+_s$ with $\epsilon_P = -0.040$. The word "hard" is used for the Monte Carlo events generated with $\epsilon_P = -0.011$, and "soft" for the events generated with $\epsilon_P = -0.040$ because $\epsilon_P = -0.011$ produces "harder" momentum spectra for the primary particles from the fragmentation process than $\epsilon_P = -0.040$.

Table 20 shows the efficiencies and their ratios for the Monte Carlo $D^+_s \to X\ell^+\nu$ events generated with these $\epsilon_P$ values. It also shows the ratio of the
Table 20
Efficiencies and various ratios of the efficiencies obtained with the two types of $D^+_s \rightarrow X \ell^+\nu$ Monte Carlo events in the $D^+_s$ tag analysis.

<table>
<thead>
<tr>
<th></th>
<th>&quot;hard&quot; $D^+_s$</th>
<th>&quot;soft&quot; $D^+_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_\phi$ (%)</td>
<td>2.867±0.058</td>
<td>2.186±0.034</td>
</tr>
<tr>
<td>$\epsilon_\eta$ (%)</td>
<td>1.219±0.027</td>
<td>0.935±0.016</td>
</tr>
<tr>
<td>$\epsilon_\eta/\epsilon_\phi$</td>
<td>0.425±0.013</td>
<td>0.428±0.010</td>
</tr>
<tr>
<td>$(\epsilon_\eta/\epsilon_\phi)<em>\text{hard}/(\epsilon</em>\eta/\epsilon_\phi)_\text{soft}$</td>
<td>0.994±0.038</td>
<td></td>
</tr>
</tbody>
</table>

"hard" efficiency ratio to "soft" efficiency ratio. The deviation of these "hard" to "soft" ratios from unity measures the systematic errors in $R_{\eta/\phi}$ due to the uncertainty in our understanding of $D^+_s$ production through the fragmentation process. Therefore the systematic error due to this uncertainty is 0.6% in $R_{\eta/\phi}$. However a conservative 1.5% systematic error is assigned to $R_{\eta/\phi}$ due to this uncertainty, the same as in the non $D^+_s$ tag analysis. Note that the systematic errors in $D^+_s \rightarrow \phi \ell^+\nu$ and $D^+_s \rightarrow \eta \ell^+\nu$ channels due to this uncertainty are correlated, and the error in $R_{\eta/\phi}$ (therefore the combined error for two channels, not for each channel) are found.

There is a systematic error in $f_{D^+_s}$ due to the uncertainties in two scales applied to the Monte Carlo estimate of the "random photon" background. The first scale is a relative correction of the "random photon" background in the "fragmentary" $D^+_s$ events with respect to the "random photon" background in the $D^+_s$ events. The second scale is an overall correction for the disagreement between the Monte Carlo and data estimate of the "random photon" background.

The total efficiency of identifying $\phi$'s in the decay chain of $D^{*+} \rightarrow D^+_s \gamma$, $D^+_s \rightarrow \phi \ell^+\nu$ and $\phi \rightarrow K^+K^-$ is calculated by combining the momentum dependent efficiencies with the predicted $\phi$ momentum spectrum of the $D^+_s \rightarrow \phi \ell^+\nu$. 
Monte Carlo events for the $D^+_s$ tag analysis. The momentum dependent efficiencies for identifying the $\phi \to K^+K^-$ decays are given in the section of $X$ identification.

In the order of $\pi^+$ selection criteria, as given in the section of the systematic errors in the non $D^+_s$ tag analysis, the total efficiencies are $0.830 \pm 0.024$, $0.836 \pm 0.026$, $0.838 \pm 0.025$ and $0.837 \pm 0.025$, respectively. The total efficiency used to calculate the efficiency corrected $D^+_s \to \phi \ell^+\nu$ yield is $0.838 \pm 0.025$, which is obtained with the third $\pi^+$ selection criterion. These four efficiencies are within each other's error which is close to (but less than) 3.0%. Therefore the systematic error in determining the total efficiency of identifying $\phi$'s in the decay chain of $D^+_s \to D^+_s \gamma$, $D^+_s \to \phi \ell^+\nu$ and $\phi \to K^+K^-$ is at most 3.0%.

There is a possible systematic error due to the uncertainty in $\Gamma_L/\Gamma_T$ values as explained in the section of the systematic errors in the non $D^+_s$ tag analysis. The efficiencies for $\Gamma_L/\Gamma_T = 1.00, 1.23$ and $1.40$ are $(2.848 \pm 0.063)\%$, $(2.867 \pm 0.058)\%$ and $(2.879 \pm 0.064)\%$, respectively. These efficiencies vary at most by 1.0%. Therefore the systematic error due to the uncertainty in $\Gamma_L/\Gamma_T$ is estimated to be 1.0\% in $R_{\eta/\phi}$.

The $K^+K^-$, $\gamma\gamma$ invariant mass spectra are fitted to extract the candidates and various background contributions to $D^+_s \to \phi \ell^+\nu$ and $D^+_s \to \eta \ell^+\nu$ decays, respectively. The signal and background functions used to fit these spectra are explained in the previous section. To estimate the uncertainty in fitting each invariant mass spectrum, the mean and sigma of the signal function has been allowed to float instead of being fixed as well as a second order Chebyshev polynomial has been used for the background function. It is concluded that from this exercise the systematic error due to the uncertainty in fitting is estimated to be 3.0\% for the $D^+_s \to \phi \ell^+\nu$ channel and 5.0\% for the $D^+_s \to \eta \ell^+\nu$ channel.

The systematic error related to reconstructing the decay of $\eta \to \gamma\gamma$ is 5.0\%,
the same as in the non $D_{s}^{+}$ tag analysis. The systematic error in charged particle tracking is 2.0% per charged track, the same as in the non $D_{s}^{+}$ tag analysis. The systematic error in the efficiency obtained using the $D_{s}^{+} \rightarrow X \ell^{+}\nu$ Monte Carlo events originates from the limited statistics of the Monte Carlo events.

Table 21 summarizes systematic errors in the $D_{s}^{+}$ tag analysis. For each source of uncorrelated errors, the errors in two decay modes are added in quadrature to give the combined error. For each source of the correlated errors, the combined error is the absolute value of the difference between the errors in two decay modes. The reason for the combined error, not the error in each mode, due to the uncertainty in the fragmentation being listed was explained above. The combined errors for all sources are added in quadrature to give the total systematic error in $R_{\eta/\phi}$.

3.8 Average of the Two $\mathcal{B}(D_{s}^{+} \rightarrow \eta e^{+}\nu)/\mathcal{B}(D_{s}^{+} \rightarrow \phi e^{+}\nu)$ Measurements

There are two measurements of $\mathcal{B}(D_{s}^{+} \rightarrow \eta e^{+}\nu)/\mathcal{B}(D_{s}^{+} \rightarrow \phi e^{+}\nu)$, one from the non $D_{s}^{+}$ tag analysis and the other from the $D_{s}^{+}$ tag analysis. It is found that only 21% of the non $D_{s}^{+}$ tag sample overlap with the $D_{s}^{+}$ tag sample. Considering this overlap fraction, the two measurements are consistent. The non tag measurement is statistically more significant than the tag measurement, but it has a large systematic uncertainty (6.0%) due to the $D^{+} \rightarrow \eta \ell^{+}\nu$ contamination. The tag measurement has a small systematic uncertainty (0.3%) due to this contamination. Therefore the systematic uncertainty due to the $D^{+} \rightarrow \eta \ell^{+}\nu$ contamination can be reduced by averaging the two measurements.

The symbols used for the explanation of the averaging procedure are de-
Table 21  
The break down of systematic errors (%) in $R_{\eta/\phi}$ from the $D_{s}^{+}$ tag analysis

<table>
<thead>
<tr>
<th>Uncorrelated errors</th>
<th>$D_{s}^{+} \rightarrow \phi \ell^{+}\nu$</th>
<th>$D_{s}^{+} \rightarrow \eta \ell^{+}\nu$</th>
<th>combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>c$\bar{c}$ BG</td>
<td>3.8</td>
<td>3.0</td>
<td>4.9</td>
</tr>
<tr>
<td>$B\bar{B}$ BG</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>K particle ID</td>
<td>3.0</td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>$\eta$ reconstruction</td>
<td></td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>$D_{s}^{+} \rightarrow \eta \ell^{+}\nu$ feed down</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$D_{s}^{+} \rightarrow \eta \ell^{+}\nu$ contamination</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Fit</td>
<td>3.0</td>
<td>5.0</td>
<td>5.8</td>
</tr>
<tr>
<td>MC statistics</td>
<td>1.7</td>
<td>2.1</td>
<td>2.7</td>
</tr>
<tr>
<td>$\phi$ polarization</td>
<td>1.0</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Correlated errors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fake lepton BG</td>
<td>3.0</td>
<td>5.9</td>
<td>2.9</td>
</tr>
<tr>
<td>$f_{D_{s}^{+}}$</td>
<td>2.9</td>
<td>2.2</td>
<td>0.7</td>
</tr>
<tr>
<td>Fragmentation</td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Tracking</td>
<td>6.0</td>
<td>2.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>11.3</td>
</tr>
</tbody>
</table>

Fined as follows:

$r_{1}$ : the non $D_{s}^{+}$ tag measurement

$r_{2}$ : the $D_{s}^{+}$ tag measurement

$\delta r_{1}$ : the statistical error in $r_{1}$

$\delta r_{2}$ : the statistical error in $r_{2}$

$\Delta y_{1}$ : the uncorrelated systematic error in $r_{1}$

$\Delta y_{2}$ : the uncorrelated systematic error in $r_{2}$

$\Delta y_{uc}$ : the combined systematic error of $\Delta y_{1}$ and $\Delta y_{2}$

$\Delta y_{c}$ : the correlated systematic error common to $r_{1}$ and $r_{2}$

$r$ : the weighted average of $r_{1}$ and $r_{2}$
\( \delta r \): the statistical error in \( r \)

\( \Delta r \): the total systematic error in \( r \)

The prescription for averaging the two measurements is:

**step 1.** Rearrange the non \( D_s^{*+} \) tag and \( D_s^{*+} \) tag samples into two statistically independent samples to find the statistical error \( \delta r \) in \( r \).

**step 2.** Separate the systematic errors into correlated and uncorrelated contributions in the non \( D_s^{*+} \) tag and \( D_s^{*+} \) tag samples.

**step 3.** The weighted average \( r \) is

\[
    r = \frac{(w_1 \cdot r_1 + w_2 \cdot r_2)}{(w_1 + w_2)}
\]

where the weights \( w_1 \) and \( w_2 \) for the two measurements are defined as

\[
    w_1 = \frac{1}{(\delta r_1^2 + \Delta y_1^2)}, \quad w_2 = \frac{1}{(\delta r_2^2 + \Delta y_2^2)}.
\]

**step 4.** The combined systematic error \( \Delta y_{uc} \) of \( \Delta y_1 \) and \( \Delta y_2 \) is

\[
    \Delta y_{uc} = \sqrt{(w_1 \cdot \Delta y_1)^2 + (w_2 \cdot \Delta y_2)^2}/(w_1 + w_2)
\]

**step 5.** The total systematic error \( \Delta r \) in \( r \) is

\[
    \Delta r = \sqrt{\Delta y_c^2 + \Delta y_{uc}^2}.
\]

To obtain a sample statistically uncorrelated to the \( D_s^{*+} \) tag sample, the \( X\ell^+ \) candidates that, when combined with photons, pass the cuts required for the \( D_s^{*+} \) tag analysis, are excluded from the non \( D_s^{*+} \) tag sample. The statistical error in \( B(D_s^+ \rightarrow \eta e^+\nu)/B(D_s^+ \rightarrow \phi e^+\nu) \) measured for this sample by the non \( D_s^{*+} \)
Table 22
The break down of correlated and uncorrelated systematic errors in the non $D_s^{*+}$ and $D_s^{*+}$ tag measurements of $B(D_s^+ \to \eta e^+\nu)/B(D^+_s \to \phi e^+\nu)$

<table>
<thead>
<tr>
<th>error</th>
<th>correlated</th>
<th>uncorrelated in non $D_s^{*+}$ tag</th>
<th>uncorrelated in $D_s^{*+}$ tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c\bar{c}$ BG</td>
<td>0.053</td>
<td>0.011</td>
<td>0.036</td>
</tr>
<tr>
<td>$B\bar{B}$ BG</td>
<td>0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaon particle ID</td>
<td>0.037</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$ reconstruction</td>
<td>0.062</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_s^+ \to \eta'\ell^+\nu$ feed down</td>
<td>0.004</td>
<td></td>
<td>0.004</td>
</tr>
<tr>
<td>$D^+ \to \eta\ell^+\nu$ contamination</td>
<td>0.003</td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td>Fit</td>
<td>0.072</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.023</td>
<td></td>
<td>0.028</td>
</tr>
<tr>
<td>Fragmentation</td>
<td>0.019</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$ polarization</td>
<td>0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lepton fake rate</td>
<td>0.038</td>
<td>0.062</td>
<td>0.009</td>
</tr>
<tr>
<td>$f_{D_s^{*+}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tracking</td>
<td>0.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$\Delta y_c = 0.14$</td>
<td>$\Delta y_1 = 0.10$</td>
<td>$\Delta y_2 = 0.05$</td>
</tr>
</tbody>
</table>

tag analysis is 0.15. Since this sample and the $D_s^{*+}$ tag sample are statistically uncorrelated, it is now straightforward to calculate the statistical error $\delta r$ in $r$. Recalling the statistical error of the $D_s^{*+}$ tag analysis is 0.22, the statistical error $\delta r$ is

$$
\delta r = (0.15 \cdot 0.22)/\sqrt{0.15^2 + 0.22^2} = 0.12.
$$

The correlated and uncorrelated contributions of the systematic errors to $r_1$ and $r_2$ are shown in Table 22. The weights are found to be

$$
w_1 = 1/(0.12^2 + 0.10^2) = 41, \quad w_2 = 1/(0.22^2 + 0.05^2) = 20,
$$

and the average $r$ is

$$
r = (41 \cdot 1.21 + 20 \cdot 1.32)/(41 + 20) = 1.24.
$$
The combined systematic error $\Delta y_{uw}$ of $\Delta y_1$ and $\Delta y_2$ is

$$\Delta y_{uw} = \sqrt{(41 \cdot 0.10)^2 + (20 \cdot 0.05)^2} / (41 + 20) = 0.07,$$

and the total systematic error $\Delta r$ is

$$\Delta r = \sqrt{0.14^2 + 0.07^2} = 0.15.$$

The weighted average of the two $B(D^+_s \to \eta e^+\nu)/B(D^+_s \to \phi e^+\nu)$ measurements is found to be:

$$R_{\eta/\phi} = \frac{B(D^+_s \to \eta e^+\nu)}{B(D^+_s \to \phi e^+\nu)} = 1.24 \pm 0.12 \pm 0.15. \quad (3.5)$$

### 3.9 Conclusions

Predictions and measurements for $R_{\eta/\phi}$, $R_{\eta'/\phi}$ and $R_{\eta/\phi} + R_{\eta'/\phi}$ are listed in Table 23 including our new measurements. Our measurements for $R_{\eta/\phi}$ and $R_{\eta'/\phi}$ agree well with the ISGW2 model predictions with an $\eta - \eta'$ mixing angle of $-10^\circ$. The predictions of Kamal et al. for $R_{\eta/\phi}$ and $R_{\eta'/\phi}$ are higher than our measurements. E653 reported an upper limit on $R_{\eta'/\phi}$ and a measurement of $R_{\eta/\phi} + R_{\eta'/\phi} = 3.9 \pm 1.6$ using the muon channel.

Table 24 lists predictions and measurements for the ratio of vector to pseudoscalar rates for $D^+_s$ and $D$ semileptonic decays. Our measurement of the vector to pseudoscalar ratio for the $D^+_s$ semileptonic decay, i.e., $B(D^+_s \to \phi e^+\nu)/B(D^+_s \to (\eta + \eta')e^+\nu) = 1/(R_{\eta/\phi} + R_{\eta'/\phi}) = 0.60 \pm 0.06 \pm 0.06$, agrees well with the measurement for $D$ semileptonic decays, and also with the ISGW2 model prediction.
Table 23
Summary of predictions and measurements for $R_{\eta}/\phi$, $R_{\eta'}/\phi$ and $R_{\eta}/\phi + R_{\eta'}/\phi$. ISGW2 predicts $R_{\eta}/\phi$ and $R_{\eta'}/\phi$ for the $\eta - \eta'$ mixing angle of $-10^\circ$ and $-20^\circ$. The numbers in the parentheses are for $-20^\circ$.

<table>
<thead>
<tr>
<th>Predictions and Measurements</th>
<th>$R_{\eta}/\phi$</th>
<th>$R_{\eta'}/\phi$</th>
<th>$R_{\eta}/\phi + R_{\eta'}/\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISGW2$^{[13]}$ Kamal et al.$^{[47-49]}$</td>
<td>1.17 (0.77)</td>
<td>0.50 (0.67)</td>
<td>1.67 (1.44)</td>
</tr>
<tr>
<td>E653$^{[14]}$</td>
<td>1.85 ± 0.56</td>
<td>2.22 ± 0.72</td>
<td>4.07 ± 0.91</td>
</tr>
<tr>
<td>This result</td>
<td>1.24 ± 0.12 ± 0.15</td>
<td>&lt;1.6 @90% C.L.</td>
<td>3.9 ± 1.6</td>
</tr>
</tbody>
</table>

Having measured $R_{\eta}/\phi$ and $R_{\eta'}/\phi$, the absolute branching fraction of $D^+_s \rightarrow \phi e^+\nu$ can be calculated using two assumptions. The first assumption is that three decay modes $D^+_s \rightarrow \phi e^+\nu$, $D^+_s \rightarrow \eta e^+\nu$ and $D^+_s \rightarrow \eta' e^+\nu$ almost saturate the electron channel of the $D^+_s$ semileptonic decay. This assumption is supported by the fact that $D \rightarrow \overline{K} e^+\nu$ and $D \rightarrow \overline{K}^* e^+\nu$ almost saturate the electron channel of $D$ semileptonic decay.$^{[50,51]}$ The second assumption is that $\Gamma(D^+_s \rightarrow X e^+\nu) = S \cdot \Gamma(D \rightarrow X e^+\nu)$, where $S$ is the theoretical prediction for the ratio of $D^+_s$ semileptonic decay rate to the $D$ semileptonic decay rate. The ISGW2 model predicts $S$ to be 0.80 and 0.74 for the $\eta - \eta'$ mixing angle of $-10^\circ$ and $-20^\circ$, respectively.$^{[13]}$ With these assumptions, $B(D^+_s \rightarrow \phi e^+\nu)$ can be written as

$$B(D^+_s \rightarrow \phi e^+\nu) = \frac{S \times (1 - f_{cs} - f_{misc})}{1 + R_{\eta}/\phi + R_{\eta'}/\phi} \times B(D^0 \rightarrow X e^+\nu) \times \frac{\tau_{D^+_s}}{\tau_{D^0}}$$

(3.6)

where $\tau_{D^+_s}$ and $\tau_{D^0}$ are the $D^+_s$ and $D^0$ lifetimes, respectively. The factor $f_{cs}$ is the ratio of the decay rate of all Cabibbo suppressed decay modes to the sum of $D^+_s \rightarrow \phi e^+\nu$, $D^+_s \rightarrow \eta e^+\nu$ and $D^+_s \rightarrow \eta' e^+\nu$ decay rates. The other factor $f_{misc}$ is the ratio of the decay rate of the rest of Cabibbo favored decays modes to the sum of $D^+_s \rightarrow \phi e^+\nu$, $D^+_s \rightarrow \eta e^+\nu$ and $D^+_s \rightarrow \eta' e^+\nu$ decay rates. The
Table 24

Predictions and measurements for the ratio of vector to pseudoscalar rates for $D^+_s$ and $D$ semileptonic decays. ISGW2 predicts the ratio in $D^+_s$ sector for the $\eta - \eta'$ mixing angle of $-10^\circ$ and $-20^\circ$. The number in the parenthesis is for $-20^\circ$.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>This result</th>
<th>ISGW2$^{[13]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(D^+_s \to \phi e^+\nu)/B(D^+_s \to (\eta + \eta')e^+\nu)$</td>
<td>$0.60 \pm 0.06 \pm 0.06$</td>
<td>$0.60 (0.69)$</td>
</tr>
<tr>
<td>$B(D \to K^*e^+\nu)/B(D \to Ke^+\nu)$</td>
<td>$0.56 \pm 0.06$</td>
<td>0.54</td>
</tr>
</tbody>
</table>

ISGW2 model predicts $f_{cs}$ to be $0.06 \pm 0.01$, and $f_{misc}$ to be $0.03 \pm 0.02$. Our measurements give $1/(1 + R_{\eta/\phi} + R_{\eta'/\phi}) = 0.37 \pm 0.02 \pm 0.02$. When this result is combined with CLEO II measurement of $B(D^0 \to X e^+\nu) = (6.64 \pm 0.34)\%^{[61]}$ and the world average lifetimes of $D^0$ and $D^+_s$, $^{[36]}$ Equation 3.6 gives $B(D^+_s \to \phi e^+\nu) = S \cdot (2.55 \pm 0.28)\%$. Therefore $B(D^+_s \to \phi e^+\nu)$ is calculated to be $(2.04 \pm 0.23)\%$ and $(1.88 \pm 0.21)\%$ with the ISGW2 prediction for the $\eta - \eta'$ mixing angle of $-10^\circ$ and $-20^\circ$, respectively.

The absolute branching fraction of $D^+_s \to \phi\pi^+$ which sets the scale for all $D^+_s$ hadronic branching fractions, can be extracted by combining our estimate of $B(D^+_s \to \phi e^+\nu)$ with CLEO II measurement of $B(D^+_s \to \phi e^+\nu)/B(D^+_s \to \phi\pi^+) = 0.54 \pm 0.06$. $^{[49]}$ The combination gives $B(D^+_s \to \phi\pi^+) = S \cdot (4.72 \pm 0.76)\%$, which is calculated to be $(3.77 \pm 0.61)\%$ and $(3.49 \pm 0.57)\%$ with the ISGW2 prediction for the $\eta - \eta'$ mixing angle of $-10^\circ$ and $-20^\circ$, respectively. Our estimate for $B(D^+_s \to \phi\pi^+)$ based on the ISGW2 prediction agrees with the recent and model independent CLEO II measurement of $B(D^+_s \to \phi\pi^+) = (3.59 \pm 0.91)\%$. $^{[82]}$

Richman and Burchat$^{[4]}$ use the relation

$$B(D^+_s \to \phi\pi^+) = F \cdot B(D^+ \to K^*\ell^+\nu) \cdot \frac{\Gamma(D^+_s \to \phi\pi^+)}{\Gamma(D^+_s \to \phi\ell^+\nu)} \cdot \frac{\tau_{D^+_s}}{\tau_{D^+}}$$

(3.7)

where $F$ is the theoretical prediction for $\Gamma(D^+_s \to \phi\ell^+\nu)/\Gamma(D^+ \to K^*\ell^+\nu)$. 
They predict $B(D^+_s \to \phi \pi^+) = (3.7 \pm 0.5 \pm 0.4)\%$, where the last uncertainty reflects the range of theoretical predictions for $F$.\cite{4} Their prediction agrees with our estimate for $B(D^+_s \to \phi \pi^+)$ based on the ISGW2 prediction.

The factorization hypothesis, which relates the semileptonic process $D^+_s \to (\eta, \eta')e^+\nu$ directly to the hadronic process $D^+_s \to (\eta, \eta')\rho^+$, predicts $B(D^+_s \to \eta e^+\nu)/B(D^+_s \to \eta \rho^+)$ = $B(D^+_s \to \eta' \rho^+)/B(D^+_s \to \eta' \rho^+)$\cite{47}. The combination of CLEO II measurements of $B(D^+_s \to \eta \rho^+)/B(D^+_s \to \phi \pi^+)$ and $B(D^+_s \to \eta' \rho^+)/B(D^+_s \to \phi \pi^+)$\cite{48} gives $B(D^+_s \to \eta \rho^+)/B(D^+_s \to \eta' \rho^+) = 1.20 \pm 0.35$ This prediction of the factorization hypothesis disagrees with our measurement of $B(D^+_s \to \eta e^+\nu)/B(D^+_s \to \eta e^+\nu) = 0.35 \pm 0.09 \pm 0.07$.

Our measurement of $B(D^+_s \to \eta e^+\nu)/B(D^+_s \to \eta e^+\nu)$ allows us to estimate the $\eta - \eta'$ mixing angle assuming that the ratio of the phase space integrals of the $\eta$ and $\eta'$ form factors is known. The relation between $B(D^+_s \to \eta e^+\nu)/B(D^+_s \to \eta e^+\nu)$ and the $\eta - \eta'$ mixing angle is

$$
B\left(D^+_s \to \eta e^+\nu\right)/B\left(D^+_s \to \eta e^+\nu\right) = \frac{(\sqrt{2} \tan \theta_m - 1)^2 \int p_{\eta'}^2 f_{\eta'}(q^2)^2 dq^2}{(\tan \theta_m + \sqrt{2})^2 \int p_{\eta}^2 f_{\eta}(q^2)^2 dq^2},
$$

where $\theta_m$ is the $\eta - \eta'$ mixing angle and $f_{\eta'}$ and $f_{\eta}$ are form factors for $\eta$ and $\eta'$, respectively. The ISGW2 model predicts $\int p_{\eta'}^2 f_{\eta'}(q^2)^2 dq^2/\int p_{\eta}^2 f_{\eta}(q^2)^2 dq^2 = 0.42$.\cite{13} It is expected that the model should be reliable in predicting the ratio of two integrals. Solving the quadratic equation and taking the negative answer, $\theta_m$ is found to be $(-7.1^{+4.7}_{-3.6})^\circ$.

In conclusion, we have measured $B(D^+_s \to \eta e^+\nu)/B(D^+_s \to \phi e^+\nu) = 1.24 \pm 0.12 \pm 0.15$ and $B(D^+_s \to \eta' e^+\nu)/B(D^+_s \to \phi e^+\nu) = 0.43 \pm 0.11 \pm 0.07$. These are the first measurements of the two quantities. Our value of the ratio of vector to pseudoscalar rates for the $D^+_s$ semileptonic decays agrees with that observed for the $D^+$ and $D^0$ semileptonic decays, and also with the ISGW2 model prediction. This agreement supports the determination of the $D^+_s \to \phi \pi^+$ branching ratio using models of semileptonic decays and measurements of $\Gamma(D^+_s \to \phi l^+\nu)/\Gamma(D^+_s \to \phi l^+\nu)$.
$\phi\pi^+$).
List of References

1. Note that $D^+$ and $D^0$ are not distinguished for the ratio of vector to pseudoscalar rates for $D$ mesons.

2. For all states described, the charge conjugate state is also implied.


18. The probability that $\ell^+$ and $\nu$ with a helicity of 0, +1 or −1 emerge at angles $\theta_\ell$ and $\pi - \theta_\ell$ is proportional to the square of the appropriate d-function \cite{19} where $\theta_\ell$ is the polar angle of $p_\ell$ in the $W^+$ rest frame with respect to the direction of the $W^+$.
35. The empirical background function has the form: $a \cdot (m - m_0)e^{-\beta(m-m_0)}$, where $m$ is the invariant mass of the $K^+K^-$ pair. The parameter $m_0$ is obtained from a fit to the inclusive $K^+K^-$ invariant mass distribution and is fixed in fitting the $D_s^+ \to \phi\ell^+\nu$ signal; $\alpha$ and $\beta$ are free parameters.

37. The ratio of the number of $D^+ \to \eta(\eta')\ell^+\nu$ events to the number of $D_s^+ \to \eta(\eta')\ell^+\nu$ events can be written as:

$$R_{D}^2(R_{D_s}^2) \equiv \frac{N(D^+ \to \eta(\eta')\ell^+\nu)}{N(D_{s}^+ \to \eta(\eta')\ell^+\nu)} = \frac{\sigma(D^+)}{\sigma(D_{s}^+)} \frac{\Gamma(D^+ \to \eta(\eta')\ell^+\nu)}{\Gamma(D_{s}^+ \to \eta(\eta')\ell^+\nu)} \frac{\tau_{D^+}}{\tau_{D_{s}^+}}.$$  

Using the measurement of $\frac{\sigma(D^+)}{\sigma(D_{s}^+)} = \frac{(48.8 \pm 4.6)}{(7.4 \pm 1.2)} = 6.6 \pm 1.2$,\textsuperscript{[38]} the CLEO II measurement of $B(D^+ \to K^-\pi^+\pi^+) = (9.3 \pm 1.0)\%$\textsuperscript{[38]} and the world average for $B(D_{s}^+ \to \phi\pi^+)$,\textsuperscript{[38]} $\frac{\sigma(D_{s}^+)}{\sigma(D^+)}$ is calculated to be $2.5 \pm 0.6$. ISGW2 predicts $\frac{\Gamma(D^+ \to \eta(\eta')\ell^+\nu)}{\Gamma(D_{s}^+ \to \eta(\eta')\ell^+\nu)}$ in units of $|V_{cd}|^2/|V_{cs}|^2$ to be $0.39$ ($0.38$) for $\theta_m = -10^\circ$, and $0.81$ ($0.19$) for $\theta_m = -20^\circ$ where $\theta_m$ is the $\eta$-$\eta'$ mixing angle.\textsuperscript{[13]} The unitarity constrained values for $|V_{cd}|$ and $|V_{cs}|$ and the world average lifetimes for $D^+$ and $D_{s}^+$\textsuperscript{[38]} are used. $R_{D}^2(R_{D_s}^2)$ is calculated to be $0.11 \pm 0.03$ ($0.11 \pm 0.03$) for $\theta_m = -10^\circ$ and $0.23 \pm 0.06$ ($0.06 \pm 0.01$) for $\theta_m = -20^\circ$. For $\theta_m$ ranging between $-10^\circ$ and $-20^\circ$, $R_{D}^2(R_{D_s}^2)$ is estimated to be $0.17 \pm 0.07$ ($0.08 \pm 0.03$).

38. P. Avery et al., CLEO preprint CLNS 90/992.


40. In the non $D_{s}^+$ tag analysis, leptons in the $D_{s}^+ \to \phi\ell^+\nu$ sample consist of 72% electrons and 28% muons; 62% electrons and 38% muons in the $D_{s}^+ \to \eta\ell^+\nu$ sample; and 72% electrons and 28% muons in the $D_{s}^+ \to \eta(\eta')\ell^+\nu$ sample. In the $D_{s}^*$ tag analysis, leptons in the $D_{s}^+ \to \phi\ell^+\nu$ sample consist of 85% electrons and 15% muons; and 74% electrons and 26% muons in the $D_{s}^+ \to \eta\ell^+\nu$ sample.

41. The rates of $D^+ \to \phi\mu^+\nu$, $D^+ \to \eta\mu^+\nu$ and $D^+ \to \eta(\eta')\mu^+\nu$ are 5%, 3% and 5%, respectively, lower than those of electron channels due to the reduced phase space.\textsuperscript{[7]} In the non $D_{s}^+$ tag analysis, the phase space correction to the muon sample is translated into the increase in the yield by the factor of $0.72 + 1.05 \cdot 0.28 = 1.014$ for $D_{s}^+ \to \phi\ell^+\nu$,

$$0.62 + 1.03 \cdot 0.38 = 1.011 \text{ for } D_{s}^+ \to \eta\ell^+\nu,$$

$$0.72 + 1.05 \cdot 0.28 = 1.014 \text{ for } D_{s}^+ \to \eta(\eta')\ell^+\nu.$$  

In the $D_{s}^+$ tag analysis, the phase space correction to the muon sample is translated into the increase in the yield by the factor of $0.85 + 1.05 \cdot 0.15 = 1.008$ for $D_{s}^+ \to \phi\ell^+\nu$,

$$0.74 + 1.03 \cdot 0.26 = 1.008 \text{ for } D_{s}^+ \to \eta\ell^+\nu.$$
42. D. Atwood and W.J. Marciano, Phys. Rev. D 41, 1736 (1990). Final state radiation reduces the detection efficiencies of $D^+_s \rightarrow \phi e^+\nu$, $D^+_s \rightarrow \eta e^+\nu$ and $D^+_s \rightarrow \eta' e^+\nu$ by 1.1%, 2.6% and 1.1% respectively, and has a negligible effect in the muon channels. In the non $D^{*+}$ tag analysis, the radiative correction to the electron sample is translated in to the increase in the yield by the factor of

$$\begin{align*}
1.011 \cdot 0.72 + 0.28 &= 1.008 \text{ for } D^+_s \rightarrow \phi e^+\nu, \\
1.025 \cdot 0.62 + 0.38 &= 1.016 \text{ for } D^+_s \rightarrow \eta e^+\nu, \\
1.011 \cdot 0.72 + 0.28 &= 1.008 \text{ for } D^+_s \rightarrow \eta' e^+\nu.
\end{align*}$$

In the $D^{*+}$ tag analysis, the radiative correction to the electron sample is translated in to the increase in the yield by the factor of

$$\begin{align*}
1.011 \cdot 0.85 + 0.15 &= 1.009 \text{ for } D^+_s \rightarrow \phi e^+\nu, \\
1.025 \cdot 0.74 + 0.26 &= 1.019 \text{ for } D^+_s \rightarrow \eta e^+\nu.
\end{align*}$$


47. A.N. Kamal et al., Phys. Rev. D 49, 1330 (1994). Combining the factorization hypothesis with the measurements of $B(D^+_s \rightarrow \eta\rho^+)/B(D^+_s \rightarrow \phi\pi^+)$ and $B(D^+_s \rightarrow \eta'\rho^+)/B(D^+_s \rightarrow \phi\pi^+)$, Kamal et al. predict $\frac{B(D^+_s \rightarrow \eta e^+\nu)}{B(D^+_s \rightarrow \phi\pi^+)} = 1.00 \pm 0.09 \pm 0.20$ and $\frac{B(D^+_s \rightarrow \eta' e^+\nu)}{B(D^+_s \rightarrow \phi\pi^+)} = 1.20 \pm 0.27 \pm 0.24$. The predictions of Kamal et al. and CLEO II measurement for $\frac{B(D^+_s \rightarrow \phi e^+\nu)}{B(D^+_s \rightarrow \phi\pi^+)}$ are combined to reach the predictions for $R_{\eta/\phi}$ and $R_{\eta'/\phi}$ quoted in the text.


51. Y. Kubota et al., CLEO preprint CLNS 95/1363.

52. M. Artuso et al., CLEO preprint CLNS 95/1387.