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A STUDY OF THE FORMATION OF
TIME-DEPENDENT PATTERNS IN ROTATING
CYLINDER SYSTEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

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ABSTRACT

Rotating cylinder systems have been testing platforms for new ideas of mathematical models such as symmetries, normal forms and amplitude equations as applied to pattern formation, turbulence, and chaotic flows. We have studied the properties of several time-dependent flow patterns that appear in the flow between two rotating cylinders as the rotation speeds of the inner and outer cylinders are changed. First, we investigated experimentally the effect of eccentricity on the stability of the base flow of counter-rotating cylinders. The flow between counter-rotating concentric cylinders produce either time-independent Taylor vortex flow or spiral vortices with various azimuthal wave numbers along the primary stability boundary of the base flow. The location of the crossover points between these patterns along the primary stability boundary is uniquely determined by the radius ratio of the cylinders. Eccentric cylinders break the rotational symmetry of the base flow, introducing a delay of onset of the primary instability and changes in the locations of crossover points. The locations of crossover points are found to shift towards lower rotational speeds as the eccentricity increases, in qualitative agreement with the theoretical predictions.

Second, we investigated the stability of the base flow in a Taylor-Couette geometry with both cylinders subject to in-phase and out-of-phase modulated ro-
tation with the same amplitude and frequency. The observations for in-phase mod-
ulation show that the flow is stabilized at high and low modulation frequencies and
destabilized at intermediate frequencies. The observations for out-of-phase modu-
lation show stabilization at high frequencies while for low modulation frequencies
the stability curve reaches an asymptotic value close to the one corresponding to
the onset of instability for steady Couette flow between counter-rotating cylin-
ders. We compare these results with the theoretical results from a linear stability
analysis carried out in the narrow gap limit.

Third, we studied the effect of temporal modulation on traveling waves in the
flow between eccentric Taylor-Couette and Taylor-Dean systems. Both of these
systems have broken azimuthal symmetry. The standing waves and standing-wave-
like states which are not allowed when the systems are rotationally symmetric are
observed when the control parameter is modulated at twice the critical frequency
(Hopf frequency) of the traveling waves.
Dedicated to
my family
ACKNOWLEDGEMENTS

First, I would like to express my sincere gratitude to my advisor, Dr. C. David Andereck, for his guidance and encouragement throughout the research and the writing of this dissertation.

Next, I would like to thank Dr. Innocent Mutabazi for teaching me the fundamental features of fluid dynamics and continued discussion of various subject matters related to this research. I also thank my friends and colleagues in the Nonlinear Fluid Dynamics laboratory here at the Ohio State University for their tolerance, and help. Special thanks goes to Dr. Mingming Wu, from whom I learned technical tricks for doing the rotating cylinder experiments.

I am very grateful to the first rate collaborators in the theoretical fields of my research projects. I thank Dr. Patrice Laure who provided us with theoretical background and numerical results for our eccentric-cylinder experiment. Special thanks go to Dr. J. E. Westfreid, Dr. Christiane Normand, and Dr. Ahmed Aouidef who encouraged me to do the pulsation experiment. I thank Dr. Hermann Riecke for helpful discussions on the modulation of the traveling waves research project. A special thanks is also due to the staff members of the Physics Department shops for their assistance in the construction of the experimental apparatus. I am very
grateful for the financial support received as a Graduate Research Associate and Graduate Teaching Assistant of The Ohio State University.

Also, I must acknowledge my loving wife, Jalini, for her understanding, sacrifice, and support. And I thank my son Akila and daughter Naomi, for providing me with moral support and great hope. This dissertation is dedicated to my family who stood by me during my difficult times.
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CHAPTER I

INTRODUCTION

The instabilities of centrifugal nature that lead to pattern formation play an important role in the large field of hydrodynamics instabilities. Rotating cylinder systems (especially the Taylor-Couette system) have long been test beds for novel experimental techniques which continue to unfold in organizing our knowledge of such a wide variety of flow patterns[1, 2, 3, 4]. The flow between rotating cylinders is one of the simplest geometries, yet it contains many of the nonlinearities associated with fluid flows. These systems provide simple testing platforms for new mathematical models involving symmetries, normal forms, and amplitude equations applied to pattern formation, chaotic flows, and turbulence. Many new experimental and data analysis techniques are tried and tested in this simple system before moving on to more complex geometries, partly because of the remarkably good laboratory control that is possible. The control parameters of the rotating cylinder system are the inner and outer cylinder rotational velocities, which can be controlled and changed very easily and accurately. In contrast to thermal con-
vection, the fluid properties are constant, leading to simpler and more complete analyses.

Instabilities in the flow between rotating cylinders have been studied extensively since Taylor's classic theoretical and experimental work on the problem. Much of the early work concentrated on the formation of patterns with only the inner cylinder rotating, where the first bifurcation is to time-independent toroidal Taylor vortices (Figure 1).

As indicated in the experimental surveys by Coles[3] and Andereck et al.[4], simultaneous rotation of the inner and outer cylinders gives rise to a great variety of dynamic patterns. Figure 2 shows the flow regime diagram obtained by Andereck et al. [4] for two independently rotating concentric cylinders. When the cylinders rotate in the same direction, the primary bifurcation from the base flow is always to toroidal Taylor vortex flow, neglecting the end effects. The more complex time-dependent flow patterns are observed as secondary or greater bifurcations only at high inner cylinder rotation rates. By contrast, for counter-rotating cylinders, the time-dependent flow patterns can appear as the first bifurcation. Earlier experimental and theoretical work on the flow between counter-rotating cylinders [4, 5, 6] showed that for a given radius ratio there is a unique value of outer cylinder speed, below which the primary bifurcation from the base flow is to time-independent Taylor vortices and above which it is to nonaxisymmetric spiral flow (see Figure 3).

The azimuthal wave number of the spirals increases as the outer cylinder speed increases. The locations of the crossover points (bicritical points) from Taylor vortices to spiral vortices or between spirals of different azimuthal wave numbers
Figure 1. Picture of typical Taylor vortex flow in the concentric Taylor-Couette system at $R_o = 0$ and $R_i = 266$. 
Figure 2: Flow regime diagram obtained by Andereck et al. (1986) for two independently rotating concentric cylinders.
Figure 3: Picture of typical spiral vortex flow in the concentric Taylor-Couette system at $R_o = -155$, and $R_i = 164$. 
on the primary instability boundary are uniquely determined by the radius ratio of the two cylinders [5].

A large proportion of the literature in the area of rotating flows deals with the primary instability alone, and it is only here where we might claim to have a deep understanding of the subtle interplay of nonlinearity, symmetry, and boundary conditions in establishing the patterns of the secondary flow. The present research study also is an effort to understand the primary instabilities of the flow between rotating cylinders subject to various broken symmetries and external modulations, where one can apply some of the known fundamental hydrodynamic theories and models.

In this study, several interesting variants of the fundamental problem of the flow between cylinders are being investigated. These variants may provide testing grounds for nonlinear stability analyses and may also provide the opportunities for practical applications in such diverse subjects as the geophysical sciences, lubrication technology, materials processing, etc.

This thesis is organized as follows: in the next chapter, I will discuss some of the theoretical developments of fluid mechanics and the instabilities of the flow between rotating cylinders using the concentric Taylor-Couette system as the primary system. Here, I briefly present the linear stability analysis and the weakly nonlinear analysis (the amplitude equations) for stationary and oscillatory patterns near the primary instability boundary. The rest of the thesis is divided into three major parts. First, I will present an experimental study of the formation of patterns along the stability boundary for eccentric counter-rotating cylinders. Eccentric cylinders break the rotational symmetry of the base flow, which would
in turn affect the onset of the primary instabilities. Here, I look into the nonlinear mode interactions and pattern formation occurring along the primary instability boundary. In the second part, I will present a study in which we examine the stability of the base flow between two concentric cylinders subject to in-phase and out-of-phase pulsation with zero and non-zero mean angular velocity. In the last part of the thesis, I will describe an experiment where we investigated the effect of temporal modulation on the traveling waves that appear near the threshold boundary of the flow between two rotating cylinders with broken symmetry. Here I will show that the temporal modulation not only affects the stability of the base flow but also the secondary flow appears near the primary instability boundary. Each of these three parts will again be subdivided into several chapters and subsections where I will present the theoretical background, experimental method, discussion of results, and a brief conclusion for each experiment.
CHAPTER II

THEORETICAL BACKGROUND FOR DYNAMICS OF THE FLOW BETWEEN TWO ROTATING CYLINDERS

2.1 FLUID MECHANICS

The flow between two rotating cylinders, especially the Taylor-Couette system, played a very important historical role in the study of hydrodynamic instabilities. This has been one of the two favorite systems (the other is the Rayleigh-Bénard convection cell) studied by theorists due to several advantages: the simplification arises from axial symmetry, well understood basic equations and parameters, and a body of knowledge built up over several decades. Experimentalists were attracted to this system for several reasons: a simple design geometry, easy to adjust control parameters (the inner and outer cylinder Reynolds numbers), and existence of very rich dynamics within a small range of parameter space[4]. In fluid mechanics, we assume that the fluid can be treated as a continuum and the equation which
The motion of the fluid is described by the well-known Navier-Stokes equation,
\[
\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{\nabla P}{\rho} + \nu \nabla^2 \mathbf{V} + \mathbf{f}
\]  
which expresses the conservation of momentum or Newton's second law for fluids, applied to a small volume of fluid. The two terms on the left hand side of this equation (2.1) are the total time derivative of the velocity of that fluid volume. The terms on the right side represent all of the force terms acting on the fluid element. The first term is the pressure force, the second is the viscous force, and the last represents the external force fields such as gravity, magnetic field, applied pressure gradients, etc.

The other main equation is the continuity equation,
\[
\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{V} = 0
\]
which expresses the conservation of mass of the fluid element.

These two equations plus the boundary conditions describe the behavior of the Newtonian fluid flows at constant temperature. The boundary condition that gives rise to the external driving of the flow is called the no-slip boundary condition, which states that the fluid at a solid boundary has the same velocity as the boundary surface.
2.2 THE NAVIER-STOKES EQUATION APPLIED TO THE FLOW BETWEEN TWO INDEPENDENTLY ROTATING CYLINDERS

Assuming that the fluid is incompressible ($\rho = \text{const.}$) and there are no explicit external force fields (i.e. $\mathbf{f} = 0$ or if the external force fields $\mathbf{f}$ can be incorporated into the pressure field term $p$), the equations (2.1) and (2.2) reduce to:

\[
\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{\nabla p}{\rho} + \nu \Delta \mathbf{V} \tag{2.3}
\]

\[
\nabla \cdot \mathbf{V} = 0 \tag{2.4}
\]

To solve the Navier-Stokes and continuity equations for the flow between two independently rotating coaxial cylinder system, it is convenient to write the equation in cylindrical coordinates. The equations for the three components of the equation (2.3) in cylindrical coordinates can be written as:

\[
\frac{\partial V_r}{\partial t} + (V \cdot \nabla) V_r - \frac{V_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu(\nabla^2 V_r - \frac{2}{r^2} \frac{\partial V_\theta}{\partial \theta} - \frac{V_r}{r^2})
\]

\[
\frac{\partial V_\theta}{\partial t} + (V \cdot \nabla) V_\theta + \frac{V_r V_r}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + \nu(\nabla^2 V_\theta + \frac{2}{r^2} \frac{\partial V_r}{\partial \theta} - \frac{V_\theta}{r^2})
\]

\[
\frac{\partial V_z}{\partial t} + (V \cdot \nabla) V_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 V_z
\]  

(2.5)

and the continuity equation (2.4) as:

\[
\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0
\]  

(2.6)

where $(r, \theta, z)$ are the usual cylindrical coordinates, and $(V_r, V_\theta, V_z)$ are the three velocity components. This set yields a simple solution, when the flow is purely azimuthal. This unique flow is called the base flow or Couette flow. When the flow is purely azimuthal, $\mathbf{V} = (0, V_\theta(r), 0)$ and $p = p(r)$, the equation (2.5) reduces to
The solution of the equation (2.7) is given by

\[ V_\theta = Ar + B/r \]  

(2.9)

where \( A \) and \( B \) are two constants that are determined by the no-slip boundary conditions, \( V_\theta(r = r_i) = r_i \Omega_i \) and \( V_\theta(r = r_o) = r_o \Omega_o \), at the two cylinder surfaces. Here, \( r_i, r_o \) are the inner and the outer cylinder radii and \( \Omega_i, \Omega_o \) are the inner and the outer cylinder rotational angular frequencies respectively.

Here, we define two dimensionless control parameters, the inner and the outer cylinder Reynolds numbers \( R_i \) and \( R_o \) by

\[ R_i = r_i \Omega_i d/\nu \quad \text{and} \quad R_o = r_o \Omega_o d/\nu \]  

(2.10)

where \( d = r_o - r_i \) is the gap width. The two boundary conditions give:

\[ A = \Omega_i \frac{\mu - \eta^2}{1 - \eta^2}, \quad B = \Omega_o r_i^2 \frac{1 - \mu}{1 - \eta^2} \]  

(2.11)

in terms of the radius ratio \( \eta = r_i / r_o \) and the rotation ratio \( \mu = \Omega_o / \Omega_i = \eta R_o / R_i \).

In the concentric rotating cylinders, the Couette flow is the only case where an exact analytical solution can be obtained from the Navier-Stokes equation. As the control parameters, \( R_i \) and \( R_o \), are varied, the Couette flow loses its stability to new flow states such as the Taylor vortex flow and the spiral flow. The instability
of rotating flows was first studied by Rayleigh (1880) and derived the criterion for stability known as Rayleigh's circulation criterion for an inviscid fluid. This states that a necessary and sufficient condition for stability to axisymmetric infinitesimal disturbances is that the square of the circulation increases outward from the axis of rotation, i.e., that Rayleigh's discriminant is defined by [2]

$$\Phi(r) = \frac{1}{r^3} \frac{d}{dr}(r^2 \Omega(r))^2 \geq 0.$$  \hspace{1cm} (2.12)

It can be shown more generally [2, 7] that \(\frac{d}{dr}(r V_\phi) > 0\) is a necessary and sufficient condition for stability to axisymmetric inviscid flow. This predicts instability when \(\Omega_r r_i^2 > \Omega_\phi r_o^2\) in the flow between rotating cylinders. The stability boundary for the Couette flow of inviscid fluid when the cylinders rotate in the same sense is given by the Rayleigh line \(\mu = \eta^2\). On the other hand, if the cylinders rotate in the opposite direction \((\mu < 0)\), \(\Phi\) is negative only in the part of the fluid adjoining the inner cylinder and centrifugal instabilities may occur only in this area [2, 7]. The \(\Phi\) is positive in the part of fluid adjoining to the outer cylinder. The two parts are separated by nodal surface on which \(\Omega = 0\). The presence of viscosity will only postpone the onset of instability beyond the point predicted by Rayleigh's criterion.

The above instability analysis is limited to the case of inviscid axisymmetric flows in the rotating cylinder systems. In viscous flows, the Rayleigh circulation criterion gives the necessary condition for instability to occur. More complete instability analyses are required to study more general cases. The two major types of stability analyses are the linear stability analysis, for infinitesimal disturbances, and the nonlinear stability analysis, when the perturbation has a finite amplitude.
2.3 **Linear Stability Analysis**

The mathematical investigation of the stability of a given flow with respect to an infinitesimal perturbation is carried out by substituting a known steady solution plus a perturbation into the equation of motion. The governing linearized equations of motion are obtained by neglecting all the higher order terms of the perturbations. In spite of this tremendous simplification, a general analysis of these linearized equations is still very difficult. Considerable simplifications can be realized by using a normal mode expansion which assumes that all the perturbations have the form \( \exp(st + im\theta + ikz) \), where \( m \) is the azimuthal wave number, an integer, \( k \) is a real number which is equal to the axial wave number and \( s \), which may be a complex quantity, i.e. \( s = \alpha + i\omega \).

The first linear stability analysis of the flow between two rotating cylinder systems (Taylor-Couette system) was done by G. I. Taylor[1]. This analysis was carried out for the case of a thin gap \( (d < \tau_i) \). Here, I will present the formalism used in a simple derivation of the linear stability analysis of the Taylor-Couette flow as adapted from references [2] and [7].

Let \( u = (u_r, u_\theta, u_z) \) be the perturbation to the basic flow \( V = (0, V_\theta, 0) \) and \( p = p + \bar{p} \). Assuming that the various perturbations are axisymmetric and therefore independent of azimuthal coordinate \( \theta \) (i.e. \( m = 0 \)), we could expressed them in normal modes of the form

\[
(u_r, u_\theta, u_z, \bar{p}) = (u, v, w, p')e^{st + ikz}
\]  

(2.13)

Then, the linearized equation of motion could be derived from the fundamental equations (2.5) and (2.6). After eliminating the pressure term \( p' \) and the axial
component of the velocity $w$ from the general equations, we can obtain the following equations for the amplitudes $u$ and $v$ of the radial and azimuthal components of the perturbative velocity [2]:

$$\{\nu(DD_\ast - k^2) - s\}(DD_\ast - k^2)u = 2k^2\Omega v$$  \hspace{1cm} (2.14)

and

$$\{\nu(DD_\ast - k^2) - s\}v = (D_\ast V)u$$ \hspace{1cm} (2.15)

where $D = d/dr$ and $D_\ast = d/dr + 1/r$.

This equation can be further simplified for the case of a narrow gap $d \ll r_1$ by letting $x = (r - r_1)/d$, $k = a/d$, $\sigma = sd^2/\nu$, and $\Omega(r) = \Omega_1g(x)$ and transforming $u \rightarrow \frac{24\nu^2}{\nu^4}u$. Then, the above equations reduce to the dimensionless forms:

$$(D^2 - a^2 - \sigma)(D^2 - a^2)u = (1 - (1 - \mu)x)v$$ \hspace{1cm} (2.16)

and

$$(D^2 - a^2 - \sigma)v = -Ta^2u,$$ \hspace{1cm} (2.17)

where $D = d/dx$ and $T = -\frac{4A\Omega_d}{\nu}$.

These equations must be considered together with the boundary conditions

$$u = Du = v = 0 \quad \text{at } x = 0 \text{ and } 1.$$ \hspace{1cm} (2.18)

The above equations form the basic eigenvalue problem which leads to a characteristic equation of the form

$$F(\mu, a, \sigma, T) = 0.$$ \hspace{1cm} (2.19)

These types of eigenvalue problems have been solved analytically and numerically using different methods and have been used to study the linear stability of
many hydrodynamic systems, including Taylor-Couette[2, 8]. The states near the threshold can be characterized according to the values of $\sigma$ with the largest real part, which is the fastest growing state.

Consider a system with a single control parameter $R$, and introduce the reduced control parameter $\epsilon = \frac{R - R_c}{R_c}$, assuming $R_c \neq 0$. For $\epsilon < 0$, the base flow is stable ($Re(\sigma) < 0$), and for $\epsilon = 0$, the instability sets in ($Re(\sigma) = 0$) at a wave vector $k = k_c$. For $\epsilon > 0$ there is a range of wave vectors $k_- < k < k_+$, for which the base flow is unstable. There can be two major types of instability, the stationary instability if $Im \sigma(k = k_c) = 0$, or oscillatory instability if $Im \sigma(k = k_c) = \omega_0 \neq 0$ for $\epsilon = 0$. For co-rotating cylinders ($\mu \geq 0$), the numerical results show that the primary transition from the base flow is to a stationary toroidal Taylor vortices which agrees with the experimental observations (Fig. 1).

However, the counter-rotating cylinders ($\mu < 0$) are known to produce non-axisymmetric time-dependent spiral flows as the primary instability from the base flow. Therefore, one has to carry out the non-axisymmetric perturbation analysis in this case. Using similar linear stability analyses for non-axisymmetric perturbations in the small gap limit, Krueger et al.[9] showed that for a sufficiently rapid counter-rotation of the outer cylinder, the laminar Couette flow becomes unstable first to a non-axisymmetric mode. Subsequent theoretical and experimental work on the flow between counter-rotating cylinders showed that for a given radius ratio there is a unique value of the outer cylinder speed, above which the primary bifurcation from the base flow is to a non-axisymmetric spiral flow[4, 5, 6].

The physical systems we wish to study are often quite complicated, even with the known equations and the boundary conditions. The linear stability analysis
already requires numerical evaluation and a direct analytic approach is impossible beyond threshold. The perturbation analysis and model equations described in the next section are a partial response to this situation, which will give some understanding of the spatial patterns displayed in these systems.

2.4 Amplitude Equations

The linear stability analysis reveals the basic physical mechanism leading to pattern formation near the threshold \((R_c)\). Immediately above the threshold, the perturbation will increase exponentially in time, the linear instability analysis will fail in this region, and the nonlinear analysis will become necessary. Just above the threshold, one can assume the nonlinearity is weak and apply weakly nonlinear perturbation analyses which will typically lead to a simplified description in terms of model equations such as an “amplitude equation”, whose form and parameters reflect the details of each physical system. These complex partial differential equations which describe the patterns of some physical systems can be derived from known microscopic equations using perturbative analyses (see Ref.[11] and the references therein).

The systematic derivation of the amplitude equation for the Taylor-Couette flow was first accomplished by Davey (1962)[12]. After that there were several studies that used different methods to develop amplitude equations for rotating cylinder systems [13, 14]. These derivations are fairly complicated and rather lengthy, but the reasoning is simple.
Let us consider the Taylor-Couette system with the outer cylinder at rest where the first transition from the Couette flow (base flow) is to a stationary periodic Taylor vortex \([k \neq 0, \omega_0 = 0]\) flow as we increase the control parameter (inner cylinder Reynolds number) \(R\) above \(R_c\). Near the threshold \(|\epsilon| \ll 1\), slow modulation in space and time in the band of stable solutions are likely to saturate due to non-linear effects.

This can be analyzed by writing a perturbative velocity field

\[
U(x, y, z, t) = [U_0(x) A(y, z, t)e^{ikz} + c.c] + O(\epsilon),
\]

where we have assumed a two-dimensional pattern with periodicity along the \(z\) axis with axial wave number \(k\) and no periodicity along the azimuthal direction \(y\). The dependence on the other spatial dimensions is included in \(U_0\). The \(A(y, z, t)\) is the amplitude retaining slow time and space changes and \(U_0(x)\) is the eigenfunction of the velocity field. Substituting Eqn. (2.20) with the base velocity field into the Navier-Stokes equation, and following a non-linear stability analysis near the threshold, one can obtain \([11, 14, 15]\) the amplitude equation in the following form:

\[
\tau_0 \partial A/\partial t = \epsilon A + \xi_{0z} \partial^2 A/\partial z^2 + \xi_{0y} \partial^2 A/\partial y^2 - g_0 |A|^2 A
\]  

(2.21)

where \(\tau_0\) is the inverse of the slow growth rate of the perturbation amplitude, and \(\xi_{0z}\) and \(\xi_{0y}\) are the correlation lengths in \(z\) and \(y\) directions, and \(g_0\) is a factor relating to the scale of \(A\) (saturation constant) \([10]\). The detailed properties of the individual systems are entirely contained in these real constants and can be calculated numerically for each system\([16, 17]\). When only the axial coordinate is allowed to vary, we can obtain a one-dimensional amplitude equation in the form

\[
\tau_0 \partial A/\partial t = \epsilon A + \xi_{0z} \partial^2 A/\partial z^2 - g_0 |A|^2 A
\]  

(2.22)
For an infinite system, this equation has a trivial solution \( A = 0 \), corresponding to the Couette flow. When \( \epsilon \ll 1 \), for a system with a large aspect ratio the deviation of the wave vector \( k \) from the critical wave vector \( k_c \) (wavenumber of the pattern at \( \epsilon = 0 \)) is small. Writing the spatial variation of the amplitude in the form:

\[
A = a_o \exp[\sigma t + i k z]
\]

and substituting this into equation (2.22) and neglecting nonlinear terms, we obtain the linear growth rate \( \sigma(\epsilon, k) \):

\[
\sigma = 1/\tau_0[\epsilon - \bar{k}^2 \xi_0^2].
\]

Setting \( \sigma = 0 \) and \( \epsilon_m = \bar{k}^2 \xi_0^2 \) gives marginal stability curve. When \( \epsilon < \epsilon_m \) only the circular Couette flow is a solution to the equation. At \( \epsilon_m \), the bifurcation will take place from the spatially uniform base flow. For \( \epsilon > \epsilon_m \), the steady-state spatially uniform solutions are \( a_o = [(\epsilon - \xi_0^2 \bar{k}^2)/g]^{1/2} \), which corresponds to a spatially periodic solution with a modified wave number \( k \). Further stability analyses by several groups [18, 19, 20] for various systems have shown that for solutions within the marginal stability curve \( \epsilon > \epsilon_m \), shows a phase instability, generally known as the Eckhaus instability. It has been shown that between \( \epsilon_m < \epsilon < \epsilon_E \) with \( \epsilon_E = 3\epsilon_m \), the solutions are unstable to long wavelength perturbations.

Similar expansion about threshold can be carried out for other classes of linear instabilities. For an oscillatory periodic case \( [\omega_0 \neq 0, k_0 \neq 0] \), the perturbation of the flow field can be written as

\[
U(x, y, z, t) = U_0(x)[A_R(y, z, t)e^{(ikz+i\omega t)} + A_L(y, z, t)e^{(ikz-i\omega t)}] + c.c + O(\epsilon)
\]

where \( A_R \) and \( A_L \) are right- and left-traveling wave amplitudes, respectively.
This satisfies the coupled amplitude equations known as the generalized Ginzburg-Landau equations. The one-dimensional case of this has the following form:

\[
\tau_0 (\partial_t + s_0 \partial_z) A_R = \varepsilon (1 + ic_0) A_R + \xi_0^2 (1 + ic_1) \partial_z^2 A_R - [g_0 (1 + ic_2) |A_R|^2 - g_1 (1 + ic_3) |A_L|^2] A_R, \tag{2.26}
\]

\[
\tau_0 (\partial_t - s_0 \partial_z) A_L = \varepsilon (1 + ic_0) A_L + \xi_0^2 (1 + ic_1) \partial_z^2 A_L - [g_0 (1 - ic_2) |A_L|^2 - g_1 (1 - ic_3) |A_R|^2] A_L \tag{2.27}
\]

where the coefficient \(s_0\) is the linear group velocity \(\partial \omega/\partial k|_{k = k_0}\) of the flow pattern, \(\xi_0\) is the coherence length of the perturbations, \(g_i\) are the non-linear saturation constants known as Landau constants, and the coefficients \(c_i\) are related to the dispersive properties of the oscillatory modes. Again the coefficients of the linear terms are given by the linear instability spectrum. When only one mode is present in the system, the above complex Ginzburg-Landau amplitude equation is reduced to:

\[
\tau_0 (\partial_t - s_0 \partial_z) A = \varepsilon (1 + ic_0) A + \xi_0^2 (1 + ic_1) \partial_z^2 A - g (1 + ic_3) |A|^2 A. \tag{2.28}
\]

These amplitude equations can be further simplified, when the system is spatially uniform (homogeneous), i.e., the amplitudes are only a function of time and are independent of the space coordinate. In this case, the amplitude equation (2.21) for the steady bifurcation reduces to

\[
\frac{\partial A}{\partial t} = a \varepsilon A + c |A|^2 + h.o.t \tag{2.29}
\]

and the coupled amplitude equations (2.26) and (2.27) for oscillatory bifurcations reduce to

\[
\frac{\partial A_R}{\partial t} = a \varepsilon A_R + c A_R |A_R|^2 + g A_R |A_L|^2 + h.o.t \tag{2.30}
\]
\[
\frac{\partial A_L}{\partial t} = a^* \epsilon A_L + c^* A_L|A_L|^2 + g^* A_L|A_R|^2 + h.o.t. \quad (2.31)
\]

The amplitude equations greatly simplify the mathematical analysis of pattern formation near the threshold \( \epsilon \ll 1 \). It is important to note the role of these amplitude equations as model equations. Many of the properties of nonequilibrium pattern forming systems, such as stability and competition of ideal patterns, as well as many more complex problems, such as the appearance of defects and chaos, may be addressed in the simple framework provided by these equations [11].

However, these equations are valid in a small region near the threshold, where they provide a quantitative description of the patterns' forms in real experimental systems. Far from the threshold, the results of these amplitude equations may be even qualitatively misleading. Away from the threshold, only a phase of the amplitude \( A \) may survive and the magnitude \( |A| \) of the slow modes will join the other fast modes that may saturate to a constant value.

The regions far from the threshold can be treated perturbatively using slow phase modes yielding phase equations [10, 11, 15, 21, 22, 23]. Since the experiments in this thesis dealt with instabilities near the threshold, I will not discuss the phase dynamics here.
Part 1

TIME DEPENDENT FLOW BETWEEN COUNTER-ROTATING ECCENTRIC CYLINDERS
CHAPTER III

INTRODUCTION TO ECCENTRIC CYLINDERS

3.1 HISTORICAL BACKGROUND

The classic Taylor-Couette system is certainly special owing to its symmetries. An understanding of the flows in this system is therefore incomplete unless their sensitivity to changes in the system symmetry is determined. Several geometries which bear on this issue have been studied[25], including rotating cones and spheres, tapered cylinders, horizontal cylinders with a Coriolis force, horizontal cylinders with a partially filled gap, and eccentric, but circular cylinders, the last being the subject of this part of my thesis (see Figure 6). In addition to being of fundamental theoretical and experimental interest, the problem of the stability of flow between eccentric cylinders is also important in lubrication technology. Cole[26] was the first to carry out an experimental study of the stability of the flow in this geometry. He showed that basic flow transitions occur which are similar to those found with concentric rotating cylinders. Following this, Kamal[27] and Vohr[28]
carried out experimental studies on eccentric cylinders with only the inner cylinder rotating and outer cylinder at rest. These experiments showed that eccentricity increases the stability of the base flow between the cylinders. DiPrima[29] carried out analytical solutions to the eccentric cylinder problem using a local theory based on the parallel flow assumption, which neglects the effect of the azimuthal variation of the tangential velocity. This analysis shows the flow to be unstable as the eccentricity increases which was not in agreement with Kamal and Vohr's experimental results. Later, DiPrima and Stuart[30] examined the whole flow field (non-local theory) considering the radial and tangential dependence of the flow velocity in an attempt to explain the experimental observations. This non-local stability analysis of the flow agreed well with the Kamal and Vohr's experimental results for small eccentricities and large radius ratios. Several subsequent studies have also dealt with the stability of the flow with only the inner cylinder rotating, again with similar results[31, 32].

The case where both cylinders rotate in the same direction was studied earlier by Versteegen and Jankowski[33] and later by Oikawa et al.[34]. They found that some eccentricity values have a destabilizing effect at higher outer cylinder rotational speeds. This destabilizing effect got stronger as the outer cylinder speed was increased. These studies were limited to the co-rotational cylinders or counter-rotational cylinders, where the first transition was to time-independent Taylor vortex flow. The purpose of this part of my study is to investigate the effect of eccentricity on the stability of the flow between counter-rotating cylinders, where the first transition is to the time-dependent spiral vortex flow.
Recently, Raffai and Laure\cite{35} carried out a theoretical study, where they investigated the stability of the viscous flow between not only co-rotating but also counter-rotating eccentric cylinders for the range of outer cylinder speeds. They have predicted some new behavior, especially in the counter-rotating cylinders. In the next section, I will present a brief theoretical background for the eccentric cylinder systems and I will discuss some of the results of Raffai and Laure's theoretical study in some detail.

### 3.2 Theoretical Background for the Eccentric Cylinders

Let us consider the viscous, incompressible flow between two infinite eccentric rotating cylinders. The problem is specified by several parameters: the outer and the inner cylinder angular velocities ratio $\mu = \Omega_o / \Omega_i$; the inner cylinder Reynolds number $R_i = \Omega_i r_i \bar{d} / \nu$; the outer cylinder Reynolds number $R_o = \Omega_o r_o \bar{d} / \nu$; the eccentricity defined as $e = \varepsilon / r_o$, where $\varepsilon$ is the distance between the cylinders' axes (offset) and $\bar{d} = r_o - r_i$ is the average gap width.

For small eccentricity, $e \ll \bar{d} / r_o$, one can treat this system as a geometrically perturbed classical Taylor-Couette system. Let us take the origin and the $z$-axis of the cylindrical coordinate system on and along the axis of the inner cylinder, with $\theta = 0$ passing through the axis of the outer cylinder (see Figure 5). Use $\bar{d}$, $r_i \Omega_i$, $\bar{d}^2 / \nu$, and $\rho \nu r_i \Omega_i / \bar{d}$ to scale length, velocity, time, and pressure respectively. Then the boundary, represented by the inner and outer cylinder surfaces,
is given by
\[ r = r_{in} = \frac{\eta}{1 - \eta}, \quad (3.1) \]
and
\[ r = r_{out}(\theta) = \frac{1}{1 - \eta} \{ \sqrt{1 - e^2 \sin^2 \theta} + e \cos \theta \}, \quad (3.2) \]
respectively. The no-slip boundary conditions for the dimensionless velocity field are
\[ v_r = v_z = 0, \quad v_\theta = 1 \quad \text{at} \quad r = r_{in} = \eta/(1 - \eta), \quad (3.3) \]
and
\[ v_z = 0, \quad v_r = -e \frac{\mu}{\eta} \sin \theta, \quad v_\theta = \frac{\mu}{\eta} \sqrt{1 - e^2 \sin^2 \theta} \quad \text{at} \quad r = r_{out}(\theta). \quad (3.4) \]
Note that for \( e = 0 \) we recover no-slip boundary conditions of the concentric Taylor-Couette system discussed in Chapter II.

The base flow (Couette) solution to this system, \( \mathbf{V} \equiv (0, v_\theta(r), 0) \), now satisfies [36]
\[ v_\theta |_{r=r_{in}} = 0, \quad (3.5) \]
and
\[ v_\theta |_{r=r_{out}(\theta)} = \frac{\mu}{\eta} \sqrt{1 - e^2 \sin^2 \theta} + e \cos \theta \frac{\mu(1 + \eta^2) - 2 \eta^2}{\eta(1 - \eta^2)} + e^2 \frac{\eta(1 - \mu)}{(1 - \eta^2)} (\sqrt{1 - e^2 \sin^2 \theta} + e \cos \theta)^{-1}. \quad (3.6) \]

Though there have been numerous calculations of concentric cylinder flows, the fully numerical treatment of eccentric cylinder flows has been difficult due to the increasing numerical complexity involved in solving the partial differential equations that model the eccentric cylinders with eccentricity as another parameter [30, 32, 37, 38, 39].
Figure 4: Geometry and coordinate system of the eccentric cylinder system.
Nevertheless, Raffai and Laure[35] numerically studied the effects of the eccentricity on the primary instability assuming the small eccentricity as an imperfection in the geometry of the classical concentric Taylor-Couette system. They have derived the homogeneous perturbative amplitude equation for the time-independent Taylor vortex flow and time-dependent spiral or ribbons flow near the bifurcation boundary of the flow in the eccentric cylinders system from the Navier-Stokes equations.

3.2.1 THE PERTURBATIVE AMPLITUDE EQUATIONS

The Taylor vortex type flow, breaking the rotational symmetry of the system in the azimuthal direction, does not change the form of the amplitude equation. In the eccentric cylinders, a change of $\varepsilon$ into $-\varepsilon$ is equivalent to the change of $\theta$ into $\theta + \pi$. Taking this symmetry property into account, it has been shown that the perturbative amplitude equations for the modified Taylor vortex flow near the primary transition should have the form[35, 36]

$$\frac{\partial A}{\partial t} = A(a\varepsilon + b\varepsilon^2) + cA|A|^2 + h.o.t \quad (3.7)$$

with real coefficients $a, b, c$. The perturbed Couette flow is still invariant under translations along the z-axis. According to this, the bifurcation now takes place for

$$R_c(e) = R_c - \frac{be^2}{a} + O(e^4). \quad (3.8)$$

This means that there will be a slight shift in the critical Reynolds number independent of the sign of $\varepsilon$. 
For the Hopf bifurcation to spirals and ribbons, the perturbed amplitude equations have the form

\[
\frac{\partial A_1}{\partial t} = A_1(i\omega_0 + a\epsilon + b\epsilon^2 + c|A_1|^2 + d|A_2|^2) + h.o.t \tag{3.9}
\]

\[
\frac{\partial A_2}{\partial t} = A_2(i\omega_0 + a^*\epsilon + b^*\epsilon^2 + c^*|A_2|^2 + d^*|A_1|^2) + h.o.t. \tag{3.10}
\]

Here \(|A_1| = 0, |A_2| = 0, \) and \(|A_1| = |A_2|\) represent right spirals, left spirals, and ribbons[6]. In this case, the coefficients \(a, b, c,\) and \(d\) will be complex. Again, these equations are slightly different from the amplitude equations for spiral or ribbon flow in the concentric cylinder system, mentioned in Chapter II, and contain an extra term, \(be^2A.\) This term is even in \(e,\) satisfying the condition that a change of \(e\) into \(-e\) is equivalent to a change of \(\theta\) into \(\theta + \pi.\)

The bifurcation now takes place for \(e = b_e e^2/\alpha_r,\) where subscript "r" represents the real parts of the coefficients. Therefore, the critical Reynolds number with \(e\) is given by

\[
R_c(e) = R_c - \frac{b_e}{\alpha_r}e^2 + O(e^4). \tag{3.11}
\]

The sign of the shift in the critical Reynolds number will depend on the sign of \(b_r.\)

Raffai et al.[35] pointed out that eccentricity brings out a delay \((b < 0)\) or an advance \((b > 0)\) for the bifurcation towards Taylor vortices for co-rotating cylinders \((\mu > 0).\) The results for the counter-rotating cylinders show only a little delay of the bifurcation. This delay is not the same for the Taylor vortex flow and spirals with different modes. Therefore, the location of the bicritical points between Taylor vortex flow \((m = 0)\) and spiral vortex flow \((m = 1)\) or spiral vortex flow with modes \(m\) and \(m + 1\) on the primary bifurcation curve will be sensitive to the eccentricity. As I mentioned earlier in Chapter I, for concentric cylinders, the
Figure 5: Numerically obtained primary stability boundary curves for eccentricity $e = 0.0, 0.0336, 0.0504$, and $0.0672$ for $\eta = 0.8$ by Laure (1992).
crossover points (bicritical points) from Taylor vortices to spiral vortices or between spirals of different azimuthal wave numbers are uniquely determined by the radius ratio of the two cylinders. According to the theoretical prediction by Raffai and Laure[35], eccentricity has unequal effects on different modes. As a result, the bicritical points on the primary instability curve will shift their locations as we introduce eccentricity to the system.

In general, the bicritical points between Taylor vortex flow \( m = 0 \) and spiral flow \( m = 1 \) or between spiral flows with azimuthal wave numbers \( m \) and \( m + 1 \) will shift towards the lower \( |\mu| \) as the eccentricity \( e \) increases. Figure 5 shows the numerically obtained primary stability boundary curves and the location of bicritical points on the boundary curves for eccentricity \( e = 0.0, 0.0336, 0.0504, \) and \( 0.0672 \) for a system with the same radius ratio as ours, \( \eta = 0.8 \), by Laure[40].

As we discussed earlier, the support for some of these theoretical results already existed in the experimental study by Versteegen and Jankowski[33] on the stability of viscous flow between eccentric cylinders (\( \eta = 0.5 \)) when both cylinders rotate, especially for the case of co-rotating cylinders. However, Versteegen and Jankowski's observations of counterrotating cylinders were limited to small values of the outer cylinder speed where the first bifurcation is to the time-independent Taylor vortex flow. In the next chapter, I present an experimental study of the effect of eccentricity on the primary stability boundary and the bicritical points for counter-rotating cylinders where the first bifurcation from basic Couette flow is either to time-independent Taylor vortex flow or time-dependent spiral vortex flow with different azimuthal modes. I will compare our experimental results with numerical results obtained by Laure[40] for counter-rotating eccentric cylinders.
CHAPTER IV

EXPERIMENTAL TECHNIQUES AND RESULTS

4.1 EXPERIMENTAL SETUP

The eccentric cylinder system consists of an inner cylinder made of black Delrin plastic with radius $r_i = 4.76 \, \text{cm}$, and an outer cylinder made of Plexiglas with inner radius $r_o = 5.95 \, \text{cm}$, which gives a gap size $d = r_o - r_i = 1.19 \, \text{cm}$ and a radius ratio $\eta = 0.800$. The control parameters of the system are the inner and outer cylinder Reynolds numbers, $R_i$ and $R_o$. The outer cylinder is connected to the stationary supports by means of bearings at both ends. The upper end of the inner cylinder is attached to a long shaft which hangs from a horizontally movable plate on the stationary framework (see Figure 6).

The lower end of the inner cylinder is left unattached, and is suspended $1\text{mm}$ above the bottom of the system. Eccentricity is adjusted by offsetting the axis of the inner cylinder relative to the fixed axis of the outer cylinder. The position of
Figure 6: Schematic diagram of the eccentric cylinder setup.
the axis of the inner cylinder is read to an accuracy of 0.01 cm using a micrometer attached to the stationary framework.

To maintain consistent end conditions, the upper and lower boundaries of the flow were formed by Teflon rings attached to the outer cylinder and located near the ends of the cylinder. There is a narrow gap of 0.4 cm between each of these rings and the inner cylinder that is nevertheless wide enough to allow for offsetting the inner cylinder. The length of the fluid column $L$ is 40.40 cm, giving an aspect ratio $\Gamma = \frac{L}{r_0 - r_i} = 34.0$, large enough to minimize the end effects. The cylinders are driven by Compumotor stepper motors. The motor speeds are determined by Compumotor 2100 Series indexers with a frequency accuracy of 0.02%. A PDP11/73 computer was used to control the speed settings, ramping rates, and some of the data acquisition.

The working fluid is a solution of double distilled water and 44% glycerol by weight with 1% of Kalliroscope AQ1000 added for visualization. These nearly two-dimensional ($\approx 30 \times 6 \times 0.07 \mu m$) polymeric flakes align along the streamline surfaces, reflecting light according to their orientation. Generally, the dark areas indicate flow along the observer's line of sight, while the light areas indicate flow perpendicular to the line of sight. The apparatus is kept in a temperature-controlled room so that the temperature of the working fluid is held constant to within 0.1°C.

The kinematic viscosity of the working fluid was taken from tabulated data[41]. Also, the experimentally obtained critical Reynolds number for concentric cylinders when $R_o = 0$ has been compared with reference values to check the accuracy of $\nu[5]$. The difference was less than 1%. In order to further test the procedure,
experimentally obtained critical Reynolds numbers and the locations of bicritical points on the primary stability boundary for the concentric counter-rotating cylinders were compared with the results obtained by Langford et al.[5] for the same radius ratio. The agreement is again within 1%.

4.2 DATA ACQUISITION TECHNIQUES

We have employed several data acquisition techniques to locate and visualize and also to obtain spatial and temporal characteristics of the emerging patterns. The onset of patterns has been observed in real time. However, when low ramping rates are necessary, time lapse video recording and a computer-controlled data acquisition method were used to accurately locate onset points and to obtain space and time characteristics of the patterns. One of the commonly used methods to analyze two-dimensional patterns is with space-time plots (see Figure 7). In this experiment, we have obtained space-time data using a 28 – 35mm variable focal length lens to form an image of the flow on a 1024-pixel CCD linear array that is oriented parallel to the cylinder axis. The array is interfaced through a computer automated measurement and control (CAMAC) system to a PDP computer. This system is capable of processing one line every 0.07 sec. The data are then transferred to VAX 4000-90 computer network for further data analysis. The space-time data consist of intensity maxima and minima which correspond to the centers of the vortex and inflow and outflow boundaries. An analysis of these intensity versus axial position plots produced from the space-time data yields the wavelength and the dynamics of the patterns.
Figure 7: Space-time plot for spiral vortex flow for $R_i = 133$, $R_o = 116$, and $e = 0.0$. The horizontal axis is the axial position along the cylinder axis and the vertical axis is the time. Intensity at a given point in space and time is represented by displacement along the vertical axis.
We also used a single-point light reflectance method to obtain the temporal characteristics of the instability patterns at a specific axial position. Spectra of the flow patterns were obtained by shining a low power He/Ne laser into the flow and collecting the reflected light off the Kalliroscope flakes with the photo detector. The resulting signal was digitized and then analysed using a fast Fourier transform (FFT) routine to obtain the flow frequencies.

Laser sheet illumination was another technique we employed to study the interior of the flow patterns. A thin layer of laser light has been used to illuminate the Kalliroscope flakes in a thin radial/axial cross section of the flow. Normally, the flakes perpendicular to the laser sheet are illuminated. This method was helpful for identifying the location of the instabilities in the gap and their weak time-dependent activities, which are not observable from outside.

We implemented the following method to obtain the location of the primary bifurcation boundary and the bicritical points along the boundary for each eccentricity $e$. As is common practice, we set $R_0$ to a specific value and then increased $R_i$ to a value just below the expected location of the transition from the basic Couette flow. Then several sets of frequency and space-time data were taken while increasing $R_i$ quasistatically until a fully developed flow pattern filled the entire column. We repeated this procedure for increasing $|R_o|$ in small steps until at least the spiral of azimuthal wave number $m = 2$ appeared. The CCD data has been used to find the onset of vortices and the axial wavelength and the frequency of the vortex pattern that appears. We also used single-point time series data to find the onset of vortices, especially when the primary bifurcation is to spirals, in which case the vortices may be weak. The locations of bifurcation points were
obtained by analyzing both CCD data and single-point time series data. The transition from one state to the other at the bicritical points was indicated by a sudden change in a primary frequency of the flow pattern.

4.3 RESULTS AND DISCUSSION

As we increase the inner cylinder speed slowly while keeping the outer cylinder speed constant, the base flow (Couette flow) becomes unstable and bifurcates to either time-independent Taylor vortex flow or time-dependent spiral flow. By changing the outer cylinder speed by small steps and increasing the inner cylinder speed quasistatically, we have obtained the locations of the primary transitions from the base flow to Taylor vortex flow and spiral flows with azimuthal wave numbers \( m = 1, 2 \) for several eccentricities. The transition from time-independent Taylor vortex flow to time-dependent spiral flow of azimuthal wave number \( m = 1 \) at the instability boundary is indicated by the emergence of a sharp peak \( (f \approx f_i/3) \) in the frequency spectra. The next transition along the transition boundary, from \( m = 1 \) spiral flow to \( m = 2 \) spiral flow, was indicated by the sudden appearance of a strong peak at a higher frequency value \( (f \approx 2f_i/3) \). Experimentally obtained primary instability curves for \( e = 0.0, 0.0336, 0.0504 \) and \( 0.0672 \) at various rotation speeds are shown in Figure 8; small arrows indicate the location of transition points from time-independent Taylor vortex flow to time-dependent spirals of azimuthal mode \( m = 1 \) and spirals of azimuthal mode \( m = 1 \) to \( m = 2 \).

In this experiment, we limited our observations to locating the first two bicritical points for three reasons: (i) two-mode interactions have already occurred when the \( m = 2 \) mode appears; (ii) near the \( m = 2 \) to \( m = 3 \) transition a ribbon-like flow
Figure 8: Primary instability boundary curves for eccentricity $e = 0.0$ (○), 0.0336 (□), 0.0504 (○), and 0.0672 (△). The arrows indicate the locations of bicritical points. The solid lines are to guide the eye.
overlaps the spiral, forming relatively complex flow patterns (Tagg et al.[6]); and (iii) according to the perturbation analysis the shift of bicritical points becomes smaller as the azimuthal wave number increases. Figure 9 shows the pictures of several different flow states that appear along the primary instability boundary.

For $\eta = 0.8$ and counter-rotating concentric cylinders the bifurcations to Taylor vortices and spirals have been always supercritical[6]. We have not observed any hysteresis at the first bifurcation for the eccentric cylinders case either.

Our experimental results clearly show that eccentricity is stabilizing for a considerable range of negative $\mu = \Omega_0/\Omega_1$. We also observed the shifting of bicritical points towards lower $|\mu|$, but the shift is smaller than expected according to the numerical calculations. Table I contains our experimental observations and numerical predictions by Laure[40] for the same radius ratio.

Table 1: Comparison of experimental and numerical results at bicritical points for $m = 0 \rightarrow 1$ and $m = 1 \rightarrow 2$. The experimental uncertainties are about 1% in Reynolds numbers.

<table>
<thead>
<tr>
<th>eccentricity</th>
<th>experimental</th>
<th>numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$\Omega$</td>
<td>$R_0$</td>
</tr>
<tr>
<td>$m = 0 \rightarrow 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0000</td>
<td>-0.626</td>
<td>-100</td>
</tr>
<tr>
<td>0.0336</td>
<td>-0.611</td>
<td>-99.2</td>
</tr>
<tr>
<td>0.0504</td>
<td>-0.576</td>
<td>-96.4</td>
</tr>
<tr>
<td>0.0672</td>
<td>-0.490</td>
<td>-83.1</td>
</tr>
<tr>
<td>$m = 1 \rightarrow 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0000</td>
<td>-0.730</td>
<td>-128</td>
</tr>
<tr>
<td>0.0336</td>
<td>-0.717</td>
<td>-127</td>
</tr>
<tr>
<td>0.0504</td>
<td>-0.683</td>
<td>-125</td>
</tr>
<tr>
<td>0.0672</td>
<td>-0.641</td>
<td>-124</td>
</tr>
</tbody>
</table>

The difference between experiment and theory may be due to several reasons.
Figure 9: Pictures of the (a) Taylor vortex flow state ($m=0$), (b) spiral flow state ($m=1$), (c) spiral flow state ($m=2$), and (d) ribbon flow state.
First, the experiments were performed for moderate values of $e$ whereas the perturbation calculations strictly hold only for $e \ll \frac{d}{r_o}$. Second, the numerical study assumed an infinite length system. Finally, we found a dependence of the axial wavelength on $e$ and the speed ratio $\mu$. The measured critical wavenumber, normalized to average gap size as a function of eccentricity, is given in Figure 10. The dotted lines in Figure 10 are to guide the eye. A previous experimental study by Koschmieder[42] and a theoretical study by Oikawa et al.[32] for $R_o = 0$ showed that the axial wavenumber ($k$) of time-independent Taylor vortices remained constant up to $e = 0.08$, independent of the length of the column and rapidly increased with larger $e$.

This observation is verified by our observations at $\mu = 0$, where the wavenumber remains constant $k = 3.2 \pm 0.1$ for the Taylor vortex flow independent of eccentricity. This was not the case when $\mu \neq 0(< 0)$, where the axial wavenumber depended on both eccentricity $e$ and velocity ratio $\mu$. This behaviour differs from the assumptions of Raffai and Laure [35], where the eccentricity was considered an imperfection in the concentric cylinders problem and the wavenumber was determined by the linear stability analysis for $e = 0$. Therefore the predicted axial critical wavenumber is only a slowly varying function of $\mu$ and azimuthal wave number $m[5]$. The accurate measurement of the critical wavelength of spiral vortices was hindered by the low contrast of the spiral near onset, since it forms near the inner cylinder. The inner cylinder speed has to be increased sufficiently above transition in our system to obtain a spiral pattern defined well enough to accurately measure the axial wavelength. We attempted to measure wavelengths of the spiral modes at various eccentricities, but were hindered by the appearance of mixed modes, defects, and interpenetrating spirals close to the stability boundary.
Figure 10: Critical wave number (normalized to the average gap size) $k$ for $e = 0.0$ ($\circ$), 0.0336 ($\blacksquare$) and 0.0672 ($\square$) as a function of speed ratio $\mu$. The dotted lines are to guide the eye.
4.4 Conclusion

We have found that eccentricity stabilizes the flow between counter-rotating cylinders, in agreement with the theoretical predictions of Raffai and Laure[35]. Also, our results agree with the limited experimental results of Versteegen and Jankowski[33] for counter-rotating cylinders (where the first transition is to the Taylor vortex flow). Also, we observed that the bicritical points along the primary instability boundary shift towards lower rotational speeds as eccentricity increases, qualitatively as predicted. The amount of the shift was smaller than the predicted value given by the perturbation calculation. Even though the small eccentricity shifts the location of bicritical points, it does not destroy the azimuthal symmetry of the patterns. We argue that the discrepancy between experiment and theory may be due to the finite size of the system and the limitations of the perturbation calculation, particularly regarding the axial wavelength of the patterns.
Part 2

Pulsed Flow Between Concentric Cylinders
CHAPTER V

THEORETICAL BACKGROUND FOR PULSED FLOW BETWEEN TWO CONCENTRIC ROTATING CYLINDERS

5.1 INTRODUCTION

The stability of hydrodynamical systems subject to time-dependent forcing has received increasing attention during the past several years [43, 44]. Since such flows driven by a periodic forcing are common in nature and technological applications, a knowledge of their stability properties may have important practical implications. A few examples are the blood flow in the aorta, high speed non impact printers, and oscillatory mixing machines [45]. These applications lead naturally to investigations of the effects of time modulation on the parameters that control the onset of instability. The two main prototypical experimental systems used to study modulated hydrodynamics are the convective instability in a fluid layer (Bénard convection [44, 46, 47, 48]) and the flow between two rotating cylinders (Taylor-Couette flow) [45, 49, 50, 51, 52, 53, 54, 55, 56]. These two systems have
been chosen because of their relative simplicity and because of the good control and high precision possible.

Donnelly (1964)[49] carried out the first experimental study of the Taylor-Couette flow subject to external modulation. He investigated the stability of the flow when the inner cylinder modulates while the outer cylinder is at rest. He concluded that time periodic modulation stabilized the base flow; however, he also observed "transient vortices," where vortices appear and disappear during one cycle for some frequencies, below the onset of instability for unsteady flows. This experimental result was used as the basis for theoretical studies of the stability of the time-dependent motions. Hall[52] carried out the theoretical stability analysis of the Couette flow in Donnelly's configuration for the narrow-gap case. He found that low modulation frequencies significantly affect the mean flow and lower the threshold for the onset of instability relative to the unmodulated case, while high frequencies have very little effect. Riley and Laurence[53] studied the stability of the modulated Couette flow with respect to axisymmetric disturbances, again in the narrow-gap limit, using a Galerkin expansion with time-dependent coefficients, and confirmed Hall's result that low frequency modulation destabilizes the flow, while high frequencies have very little effect on the stability of the flow.

Since these pioneering investigations, there have been several experimental and theoretical studies on Taylor-Couette flow subject to various forms of time-periodic forcing [55, 56, 57, 58, 59]. Carmi and Tustaniwskyj[55, 56] used an extension of the theory of Riley and Laurence[53], and also an energy stability theory, to study the modulated cylindrical Couette flow in the finite-gap range. They found a strong destabilizing effect in the low-frequency limit in agreement with the experimental
observations by Walsh and Donnelly[51] and by Thompson[61]. However, a later experimental study by Ahlers[62] showed a smaller threshold shift that agrees better with Hall's predictions[52]. Later Kuhlmann et al.[59] performed both a numerical simulation of the Navier-Stokes equations and an analytic Galerkin approximation with four modes for the modulated circular Couette flow. They found that the modulation weakly destabilizes the flow at low frequencies, which was in agreement with Hall[52] and Riley and Laurence[53] but in disagreement with Carmi and Tustaniwskyj[55, 56].

Most of these studies dealt with the stability of the circular Couette flow when the inner cylinder was modulated with or without a mean velocity, while the outer cylinder was at rest or rotating at a steady speed. Carmi and Tustaniwskyj[55, 56], Barenghi and Jones[57], and Wu and Swift[60] carried out theoretical analyses for additional cases where one or both cylinders were subject to modulation with or without a mean rotation. For the particular case of steady rotation of the inner cylinder and torsional oscillation of the outer cylinder, Walsh and Donnelly[58] observed that modulation stabilizes the flow, in disagreement with the prediction of Carmi and Tustaniwskyj[55, 56] A recent theoretical analysis by Murray[63] and also by Wu and Swift[60] yields much better agreement with the experimental data of Walsh and Donnelly[58].

Recently, Aouidef et al.[64, 65] carried out an experimental study of the stability of the flow between two concentric cylinders with a narrow gap subject to in-phase (i.e., both cylinders moving together) modulations, or pulsations, without a mean velocity. A linear stability analysis was performed by an implementation of the Floquet theory for solving the time-dependent differential system and a
satisfactory agreement with their experimental findings was obtained. Their ob-
servations showed destabilization of the flow in agreement with Carmi and Tustani-
wskyj's [55, 56] predictions, but the effect was much smaller than they predicted.

In this part of the thesis, we experimentally investigated the stability of the
time-dependent flow between two concentric cylinders, induced by in-phase and
also by out-of-phase pulsation of the cylinders at the same amplitude and frequency,
with or without a mean angular velocity. In the next section, we present some basic
theoretical background that is related to our study of the modulated flow between
two concentric cylinders. There, we present some of the results obtained using the
linear stability analysis and numerical calculations for both in-phase and out-of-
phase modulations in the narrow gap limit.

5.2 Theory

5.2.1 Base Flow

We consider an incompressible fluid with density \( \rho \) and the kinematic viscosity \( \nu \)
in between two concentric cylinders with the inner and outer radii \( r_i \) and \( r_o \) re-
spectively. As we discussed in Chapter II, the governing equations of the motion
of the viscous flow between two cylinders are the Navier-Stokes equation, which is
basically Newton's second law for fluid motion, and the equation for mass conserv-
vation of an incompressible fluid. Again, we utilize cylindrical polar coordinates
\((r, \theta, z)\), where \( z \)-axis is the common axis of the cylinders. Consider a situation
where the inner and the outer cylinders rotate with the angular velocities

\[ \Omega_{\text{in}}(t) = \Omega_{m1} + \Omega_{o1} \cos(\omega t) \]
\[ \Omega_{\text{out}}(t) = \Omega_{m2} + \Omega_{o2} \cos(\omega t) \]  

(5.1)

where \( \Omega_{m1} \) and \( \Omega_{m2} \) are the mean angular velocities, \( \Omega_{o1} \) and \( \Omega_{o2} \) are the amplitudes of modulations, and \( \omega \) is the common angular frequency of modulation of the two cylinders.

When \( \Omega_{\text{in}}(t), \Omega_{\text{out}}(t) \), and \( \omega \) are small, the base flow velocity field generated by the motion of one or both cylinders can be considered to be purely azimuthal as we discussed in Chapter II for the non-modulating Couette flow. Unlike non-modulating cylinders, for modulating cylinders, the base flow will be time dependent. In other words, below the onset of instabilities, the velocity components \( u_r = u_z = 0 \) and \( u_\theta = V(r, t) \). In this case, the basic velocity field \( U = (0, V(r, t), 0) \) and the pressure field \( p(r, t) \) satisfy the simplified equations of motion

\[ \frac{dV}{dt} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} - \frac{V}{r^2} \]
\[ \frac{\partial p}{\partial r} = \rho \frac{V^2}{r} \]  

(5.2)

with the no-slip boundary conditions, \( V(r_i, t) = r_i \Omega_{\text{in}}(t) \) and \( V(r_o, t) = r_o \Omega_{\text{out}}(t) \).

Without losing the generality of this problem, it is mathematically convenient to decompose the base velocity field into a steady rotation and periodic components[55, 56, 60]

\[ V(r, t) = V_s(r) + V_p(r, t). \]  

(5.3)

Substituting this into the equations of motion with the boundary conditions, we obtain the steady part of the solution[56]

\[ V_s(r) = A' r + \frac{B'}{r} \]  

(5.4)
where
\[
A' = \frac{\Omega_m r_0^2 - \Omega_m r_i^2}{r_0^2 - r_i^2}
\]
\[
B' = \frac{(\Omega_m - \Omega_m) r_i^2 r_o^2}{r_0^2 - r_i^2}
\]
which are similar to the base flow solutions obtained in Chapter II for steady rotating cylinders.

The time periodic part of the solution, which is more complicated, is given by (see references [55] and [60]):
\[
V_p = Re\{[\Omega_m( S_1 I_1(\beta r) - T_1 K_1(\beta r)) + \Omega_m(T_2 K_1(\beta r) - S_2 I_1(\beta r))]e^{iut}\} (5.5)
\]
where \( \beta = (i\omega / \nu)^{1/2} \), \( I_1 \) and \( K_1 \) are the modified Bessel functions and
\[
S_1 = r_i K_1(\beta r_o) / \Delta, \quad S_2 = r_o K_1(\beta r_i) / \Delta,
\]
\[
T_1 = r_i I_1(\beta r_o) / \Delta, \quad T_2 = r_o I_1(\beta r_i) / \Delta
\]
with \( \Delta \) defined by
\[
\Delta = I_1(\beta r_i) K_1(\beta r_o) - I_1(\beta r_o) K_1(\beta r_i).
\]

Again, as we discussed in Chapter II for the time-independent circular Couette flow, the time-dependent circular Couette flow is the only case where an exact analytical solution could be obtained from the Navier-Stokes equation for the time dependent modulated motion. To obtain the stability of the modulated base flow due to the axial and non-axial infinitesimal disturbances, the linear perturbation equations governing the motion of the flow have to be derived from the base flow solutions[53, 55]. Carmi and Tustaniwskyj[56] derived rather lengthy linear stability equations for the three-dimensional flow which include both the axial and
the azimuthal disturbances and solved them numerically to obtain the stability boundaries of the base flow.

Aouidef and Normand[66] carried out a theoretical study on a simpler case of modulation, where both cylinders modulate with the same amplitude ($|\Omega_{o1}| = |\Omega_{o2}| = |\Omega_o|$), with zero mean angular velocity ($\Omega_{m1} = \Omega_{m2} = \Omega_m = 0$) in the same direction (in-phase) or the opposite direction (out-of-phase). i.e;

$$\Omega_{in}(t) = \Omega_o \cos(\omega t)$$
$$\Omega_{out}(t) = \epsilon \Omega_o \cos(\omega t).$$

(5.6)

The parameter $\epsilon = 1$ for the in-phase modulation and $\epsilon = -1$ for the out-of-phase modulation.

Here I present a simple derivation of the governing equations as adapted from the reference [66]. First, we introduce dimensionless variables by scaling time, length and velocity by $d^2/\nu, d,$ and $r_o \Omega_o$ respectively. Assuming that the separation (gap) $d$ between the cylinders is small compared with the inner cylinder radius $r_i$ one can apply the usual thin-gap approximation to the system. Then the primary flow $U(r,t) = (0, V(r,t), 0)$ is represented by the dimensionless velocity field $U(z,t) = (0, V(z,t), 0)$, which is a solution of[53]

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}$$

(5.7)

and

$$V(0,t) = V(1,t) = \epsilon \cos \omega t.$$  

(5.8)

The pressure field is a solution of

$$\frac{\partial p}{\partial x} = V^2,$$

(5.9)
where \( x = (r - r_i)/d, \sigma = \omega d^2/\nu \), and \( V(x, t) = V(r, t)/r_i\Omega \). The solution to the equation 5.7 is given by [53, 64, 66]

\[
V = g_1(x) \cos \tau + g_2(x) \sin \tau
\]

(5.10)

where the functions \( g_1(x) \) and \( g_2(x) \) are:

\[
g_1(x) = \frac{[\cos(\gamma x) \cosh \gamma (1 - x) + e \cosh(\gamma x) \cos \gamma (1 - x)]}{[\cosh \gamma + e \cos \gamma]}
\]

(5.11)

\[
g_2(x) = \frac{[\sin(\gamma x) \sinh \gamma (1 - x) + e \sinh(\gamma x) \sin \gamma (1 - x)]}{[\cosh \gamma + e \cos \gamma]}
\]

(5.12)

The parameter \( \gamma \) is the reciprocal of the dimensionless Stokes layer thickness \( \delta = \sqrt{2\nu/\omega d^2} \) and is related to \( \sigma \) by \( \gamma = \sqrt{\sigma/2} \) [64].

To simplify the stability analysis, we introduce two time constants \( \tau_\nu \) and \( \tau_m \), where \( \tau_\nu = d^2/\nu \) is the viscous diffusion time and \( \tau_m = 1/\omega \) is the modulation period. According to the value of \( \sigma (= \tau_\nu/\tau_m) \), several different flow regimes can be characterized in this system.

### 5.2.2 Stability of Base Flow

Assume that the fluid is non-viscous and Rayleigh's stability criterion for the centrifugal instabilities remain valid instantaneously. Then, by knowing the base velocity \( V(x, t) \), the unstable regions of the flow within the gap can be obtained for various values of \( \gamma \). In the absence of viscosity, the necessary and sufficient condition for the instability to occur in the base flow is that the Rayleigh generalized discriminant, which is defined as \( \Phi(x) = V(\frac{dV}{dx}) \) in the thin gap approximation, becomes negative somewhere in the gap. Therefore, the flow can be subdivided
into different layers, where $\Phi(x) > 0$ gives the stable regions and $\Phi(x) < 0$ gives the unstable regions in which the instabilities occur. If we consider the effect of viscosity on the stability of the flow, it will only postpone the onset of instability beyond the point predicted by Rayleigh's criterion.

At low frequencies $\sigma \ll 1$, where $\tau_v$ is small compared to $\tau_m$, the modulated fluid motion has entirely diffused over the gap and the viscous wave has little phase difference across the gap, allowing for a rigid body rotation flow. At this limit, the spatial velocity profiles $g_1(x)$ and $g_2(x)$ of the base flow velocity $V$ could be expanded in power of $\sigma$[64]

$$g_1(x) = g_{10}(x) + \sigma^2 g_{12}(x) + \text{h.o.t} \quad (5.13)$$

$$g_2(x) = \sigma g_{21}(x) + \text{h.o.t.} \quad (5.14)$$

As an example, using the above expansions, the azimuthal velocity for the in-phase modulations ($\epsilon = 1$) can be expressed as

$$V(x, t) = \cos \sigma t + \gamma^2 x(1-x) \sin \sigma t + O(\gamma^4). \quad (5.15)$$

Now, the corresponding Rayleigh discriminant is given by

$$\Phi(x) = \frac{1}{2}\gamma^2 (1-2x) \sin 2\sigma t + O(\gamma^4). \quad (5.16)$$

As we discussed before, the regions where the $\Phi(x)$ is negative will be susceptible to centrifugal-type instabilities.

At high frequencies, $\sigma \gg 1$, where $\tau_v$ is larger compared to $\tau_m$, the expression for the base flow velocity field (for $\epsilon = 1$) reduces to

$$V(x, t) = \cos [\sigma t - \gamma x] e^{-\gamma x} + \cos [\sigma t - \gamma (1-x)] e^{-\gamma(1-x)} \quad (5.17)$$
and the corresponding Rayleigh discriminant is given by

\[ \Phi(x) = -\gamma(1 - \sin 2\sigma t)(e^{-2\gamma x} - e^{-2\gamma(1-x)}). \]  

(5.18)

In this case, the fluid motion will be confined to thin layers next to the inner and outer cylinder walls, which become increasingly thin as the frequency increases. The most unstable region, where \( \Phi(x) < 0 \), lies near the inner cylinder \( (x = 0) \). Therefore, the first instabilities will develop in the fluid layer near the inner cylinder. Figure 11 shows the time evolution of the base flow velocity profiles for \( \gamma = 0.5 \) and \( \gamma = 10 \) obtained from equations (5.15) and (5.17).

5.2.3 LINEAR STABILITY ANALYSIS

As we discussed in Chapter II, the stability problem is formulated by adding a perturbed flow field \( (u', p') = ([u, v, w], p') \) to the base flow field \( (U, p) = ([0, V, 0], p) \) [2]. The governing linearized equations of motion for the infinitesimal perturbation are obtained by substituting \( u = U + u', P = p + p' \) into the Equ. (5.2) and neglecting all the higher-order terms in the perturbations. These linearized equations satisfy the boundary conditions \( u = v = w = 0 \) at \( x = 0 \) and 1. Assuming that the perturbations are axially periodic with axial wavenumber \( q \)

\[ (u, v, w, p') = (\tilde{u}(x,t), \tilde{v}(x,t), \tilde{w}(x,t), \tilde{p}(x,t))e^{iqz} \]  

(5.19)

and eliminating the pressure \( \tilde{p} \) and axial velocity \( \tilde{w} \) from the linearized Navier-Stokes and continuity equations, one can obtain the governing equations for the
Figure 11: The time evolution of the base flow velocity profiles for $\gamma = 0.5$ and $\gamma = 10$ obtained from equations (5.15) and (5.17).
remaining components \( \tilde{u} \) and \( \tilde{v} \)[64]

\[
(\mathcal{L} - \frac{\partial}{\partial t})\tilde{u} = 2q^2T_a^2 V\tilde{v} \\
(\mathcal{L} - \frac{\partial}{\partial t})\tilde{v} = \frac{\partial V}{\partial x} \tilde{u}
\]  

(5.20)

where \( \mathcal{L} \equiv \frac{\partial^2}{\partial x^2} - q^2 \) and the Taylor number is defined as \( Ta = (\Omega_0 r_0 d/\nu)(d/r_1)^{1/2} \).

These velocity components satisfy the boundary condition

\[
\tilde{u} = \tilde{v} = \frac{\partial \tilde{u}}{\partial x} = 0 \quad \text{at} \quad x = 0, 1.
\]  

(5.21)

The equations (5.20) have been solved by Aouidef and Normand[66] using the Floquet theory for solving the time-dependent differential systems for both in-phase modulations [65] and out-of-phase modulations[66].

Figures 12 and 13 show the numerical results, the critical Taylor number \( Ta_c \) and wave number \( q_c \) as functions of \( \gamma \), obtained by Aouidef and Normand for both in-phase and out-of-phase modulations. In the high frequency regime (\( \gamma \gg 1 \)), the asymptotic behaviors of the critical parameters are

\[
Ta_c \approx \gamma^{3/2} \quad \text{and} \quad q_c \approx \gamma
\]  

(5.22)

for both \( \epsilon = \pm 1 \). In the low frequency regime (\( \gamma \ll 1 \)), the results show the trend towards stabilization for both \( \epsilon = \pm 1 \) with an enhancement of \( Ta_c \) more pronounced for \( \epsilon = -1 \) than for \( \epsilon = 1 \).

In the intermediate frequency range, the result shows a maximum of destabilization for both cases.

The equations (5.20) can also be solved for the limiting case \( \sigma \to 0 \) by using quasi-static the approximation[7]. A simplified system of equations can be derived
Figure 12: Numerical results for the critical Taylor number \( (T_{ac}) \) versus the frequency \( \gamma \) for in-phase modulation (\([---] \) \( \varepsilon = 1 \)) and out-of-phase modulation (\([-\cdots\cdots] \) \( \varepsilon = -1 \)) with zero mean velocity (Aouidef and Normand (1995)).
Figure 13: Numerical results for the critical wavenumber \( q_c \) versus the frequency \( \gamma \). The solid line corresponds to in-phase modulation \( \epsilon = -1 \) and the dotted line to out-of-phase modulation \( \epsilon = 1 \), with zero mean velocity (Aouidef and Normand (1995)).
in the low frequency limits by keeping the leading order terms in the expansions of the base velocity $V$ given in (5.14) and its $x$-derivatives. Using the quasi-static approximation on base velocity in which the time derivatives are neglected, a set of simplified equations[65] can be obtained from the perturbation equation (5.20). The simplified equations have the form[67]:

\[
\begin{align*}
(D^2 - q^2)^2 \ddot{u} &= 2q^2Ta^2 \ddot{v} \cos \tau g_{10}(x) \\
(D^2 - q^2)\ddot{v} &= (D(g_{10}(x))) \cos \tau + \sigma D(g_{21}(x)) \sin \tau)\ddot{u}.
\end{align*}
\] (5.23)

where $D \equiv \left(\frac{d}{dx}\right)$ and time $\tau = \sigma t$ appears as a parameter.

The solution to these equations with the boundary conditions obtained by Aouidef and Normand[66] provides the instantaneous threshold values for the control parameter $Ta$ and the wavenumber $q$ as a function of $\sigma$ for fixed time $\tau$. The solutions are

\[
Ta_c \propto \gamma^{-1},
\] (5.24)

if minimum of $Ta^2$ is reached for $\sin 2\tau = \pm 1$ and

\[
Ta_c \propto \gamma^{-2},
\] (5.25)

if either $\cos \tau = 0$ or $\sin \tau = 0$ for the in-phase modulation ($\epsilon = 1$). Similarly, for the out-of-phase modulation ($\epsilon = -1$) case

\[
Ta_c = \text{const.},
\] (5.26)

if $Ta$ reaches its lower bound when $\cos \tau = 1$ and

\[
Ta_c \propto \gamma^{-2},
\] (5.27)

if $Ta$ reaches its lower bound when $\sin \tau = 1$ when $\gamma \to 0$ during one cycle.
Other than these limiting cases, as we mentioned earlier, one has to use other methods, such as numerical analysis, to obtain the complete relationship between the critical Taylor number $Ta_c$ versus the frequency $\gamma$.

In the next Chapter, I present the experimental technique and the results for both in-phase and out-of-phase modulations of the cylinders and compare them with the theoretical predictions by Aouidef and Normand[66], which we discussed here.
CHAPTER VI

EXPERIMENTAL TECHNIQUES AND RESULTS

6.1 EXPERIMENTAL SETUP

The experimental system shown schematically in Figure 6 in Chapter II has also been used for this study. In this case the eccentricity was zero. The two main control parameters of this modulating system are the Taylor number $Ta = \frac{\Omega_0 d}{\nu} \sqrt{\frac{d}{r_t}}$, which is a function of the amplitude of modulation $\Omega_0$, and the $\gamma = \sqrt{\omega d^2/2\nu}$, which is a function of the modulation frequency $\omega$.

Again, both cylinders are driven by two Compumotor stepper motors with a rotation rate resolution of 0.001 Hz. The cylinders are modulated sinusoidally by sending the same dynamical commands (sequential change of shaft motion parameters with time delays) to both Compumotor Indexes using the PDP11/73 computer. This arrangement enabled us to modulate the cylinders' rotation rates either in-phase or out-of-phase without any major changes to the experimental
apparatus. This arrangement also made it easy to apply the modulation with non-zero mean angular velocities to the system. The working fluid is pure double distilled water or a solution of double distilled water and 44% of glycerol by weight, in either case with 1% by volume of Kalliroscope AQ1000 added for visualization.

6.1.1 Data Acquisition

The following method was applied to obtain the location of the onset of patterns for each modulation frequency $\omega$ and modulation amplitude $\Omega_0$. We kept $\omega$ at the same value and gradually increased the amplitude of modulation $\Omega_0$ in small steps until flow patterns, detected by direct visualization, appeared in the system. In common with constant rotation rate experiments, at low amplitudes only a few vortices appeared, near the end rings, and upon further increase of the amplitude by a small amount the number of vortices increased until they filled the entire length of the cylinders. The modulation amplitude at which the patterns appear all along the axis of the system was chosen as the onset value to obtain the critical Taylor number $Ta_c$.

We were unable to use the same CCD linear array camera that we used for the previous experiment due to the interference with the timing control of the Compumotor indexers, since we used the same PDP-11 computer to control both the camera and the motors. Instead, we have used a 512 x 480 pixel CCD camera connected to an image processor to acquire space-time data for the characterization of the spatial and temporal properties of the patterns. The image processor board, installed in a PC, captures a picture of the flow pattern and then a software routine
Figure 14: Transversal laser sheet visualization of the gap near $\Omega(t) = \Omega_0^\ast$ for $Ta=190$ and $g = 4.32$. 
is used to obtain the intensity along a single line parallel to the axis of the cylinders as a function of time. This system is able to process up to one line every 0.11 sec. The data are then transferred to a VAX 4000-90 computer system. An analysis of the resulting intensity versus the axial position as a function of time plots yields the wavelengths and the dynamics of the patterns in time and space. For the same pulsation frequency $\omega$, several sets of space-time data were taken while increasing $\Omega_0$. This procedure was repeated for a range of pulsation frequencies $\omega$.

Besides the observation of the vortices around the cylinder, a transverse laser sheet was used to visualize the radial structure of the vortex patterns (See Figure 14) to determine when and where the vortices form within the gap during a modulation cycle.

This technique has been used effectively to visualize the radial structure of the vortex patterns which could not be obtained from direct visualization. Another experimental technique we used to determine the velocity field inside the gap during the modulation is laser-Doppler velocimetry (LDV).

6.1.2 LASER-DOPPLER VELOCIMETRY

The LDV technique allows one to measure a velocity component at any point in the flow field without perturbing the flow [68]. Figure 15 shows the schematic diagram of the LDV system which includes a laser, optics, photodetector, and signal processor. For LDV measurement, the fluid is seeded with small particles which have a different refractive index from the rest of the fluid. In our system,
Figure 15: Schematic diagram of the LDV system.
we have used the Nobel Industries Expancel 091 microspheres with a diameter of a few microns for seeding. One can also use particles naturally present in the fluids. Measurements are made when the particles in the fluid scatter light in all directions while going through the beam crossing (see Fig. 16). This scattered light is collected in the forward scattering region, since this is where the scattered intensity is the greatest, and is focused onto the photodetector. This signal is filtered and fed to a signal processor and the output of the processor provides the velocity information for the flow passing through the crossing region. By measuring the Doppler shift of scattered laser light, which is directly proportional to the velocity, the LDV electronics determine the velocity component of the particles moving through the beam crossing region. This component can be assumed to be the same as the flow velocity at that point. The major drawback of the LDV system is that there is no simple way to visualize the flow at the same time as the data are being taken. For this reason we did the flow visualization for the same parameter values before we introduced the LDV.

To measure the azimuthal velocity component of the fluid inside the gap, we redesigned the apparatus with a plexiglass inner cylinder (see Figure 15). First, we calibrated the output signal at different positions in the gap using known velocities (using the solid rotation of the fluid when both cylinders rotate with the same angular speed). Using the LDV, we were able to study the azimuthal velocity field below the onset of instabilities, where the flow is purely azimuthal away from the end regions. First, we tested our measurements for the case where the inner cylinder pulsed with frequency $\gamma$ with zero mean velocity while the outer cylinder was stationary (see Figure 17).
Figure 16: Sketch of the beam crossing regions defines the scattering volume located in the gap between the cylinder. The diameter of the cylindrical laser beams are about $0.5mm$ and the scattering volume is about $0.07mm^2$. 

Figure 17: The plot of the maximum LDV signal strength (which is proportional to the azimuthal flow speed) versus normalized gap position, below the onset of instabilities for $\gamma = 7.2$ for various inner cylinder modulation amplitudes $\Omega_{10}$ with zero mean angular velocity ($\Omega_{m1} = 0$), with the outer cylinder stationary.
Figure 18: Plots of maximum LDV signal strength (which is proportional to the maximum azimuthal flow speed) versus normalized gap position for several frequencies of in-phase modulation with zero mean angular velocity below the onset of instabilities.
Theoretically, the first order approximation gives an exponential envelope for the amplitude of the azimuthal velocity component as a function of distance from the inner cylinder wall, with a decay constant $\gamma$ \[45\]

$$v_\phi = A(e^{-\gamma r} - i\xi + i\omega t + c.c). \quad (6.1)$$

where $A$ is a function of the amplitude of modulation. The curve fit experimental data shown in Figure 17, gives $\gamma = 7.04 \pm 0.11$ which is in reasonable agreement with the calculated value of 7.22. Figure 18 shows the variations in the maximum signal strength, which is proportional to the maximum azimuthal velocity component at different positions in the gap when both cylinders pulsate in-phase with the same frequency and the same amplitudes with zero mean velocity.

The data in Figure 18 were taken at $\gamma$ values in the range 6.7 to 10.6, for Taylor numbers below the onset of instabilities where only the azimuthal Couette flow was present. The data in Figure 18 agree with the theoretical predictions that at higher frequencies the fluid motion is confined to the thin layers (Stokes layers $\delta = 1/\gamma$) adjacent to the inner and outer cylinders and the bulk of the fluid has very little effect. Unfortunately, we could not use LDV measurements to obtain an accurate location of the onset of instability or the behaviour of the secondary flow patterns due to the time and space dependence of the flow patterns. The newer particle image velocimetry (PIV) systems are better suited for study of this kind of time-and space-dependent flow patterns, since they use a cross-section of the flow instead of just a single point.
6.2 Results and Discussion

The first case we studied was the stability of the modulated Taylor-Couette flow when both cylinders pulsed in-phase with the same amplitude and the same frequency, with no mean angular velocity. The primary experimental results, the critical Taylor number $Ta_c$ versus the frequency $\gamma$, are shown in Figure 19 where the dotted line denotes the numerical results obtained by Aouidef et al.[66], using the small gap approximation.

In Figure 20, we compare our results (open squares) for $\eta = 0.80$ with the Aouidef et al.[64] experimental results for $\eta = 0.90$ (open circles) and the Carmi et al. [56] numerical results for $\eta = 0.70$.

Our experimental results for $\eta = 0.80$ more closely resembled the shape of the theoretical curve and were similar to those of Aouidef et al. for $\eta = 0.90$. The delay of the onset in the experimental results was attributed to the wide gap size. The lowest point on the plot (the maximum instability) occurred near $\gamma = 2.0$, consistent with the theory. The value $\gamma = 2$ means that the Stokes’ boundary layers next to the inner and outer cylinders reach their maximum thickness, which is equal to half of the gap. All of the experimental and theoretical results, including ours, show that the higher and lower values of $\gamma$ have a stabilizing effect on the base flow. Figures 21, 22, and 23 show several space-time data plots obtained using the CCD camera for various values of modulation frequency. The horizontal axis is the axial position along the cylinder and the vertical axis is time.

At higher values of $\gamma$ (Figure 21), one can see that the vortex patterns persist throughout the cycle. For intermediate and lower values of $\gamma$, patterns appear
Figure 19: The critical Taylor number $T_{ac}$ versus the frequency $\gamma$ for in-phase modulations of concentric cylinders with zero mean velocity. The dotted line denotes the numerical results obtained Aouidef and Normand (1995) using the small gap approximation.
Figure 20: Comparison of our experimental results (open squares) for $\eta = 0.80$ with Aouidef et al. (1994) experimental results (open circles) for $\eta = 0.90$ and the Carmi et al. (1981) numerical results (solid squares) for $\eta = 0.70$ for in-phase modulation with zero mean velocity.
Figure 21: The space-time data for in-phase modulations at $\gamma = 4.25$ and $T_a = 197$ with zero mean velocity.
Figure 22: The space-time data for in-phase modulations at $\gamma = 2.49$ and $T_a = 176$ with zero mean velocity.
Figure 23: The space-time data for in-phase modulations at $\gamma = 1.26$ and $T_a = 226$ with zero mean velocity.
only during one part of the cycle, i.e., they are transient vortices, as illustrated by Figures 22 and 23). In between these transient patterns, some remanent images of vortices remain in the system, which we call “false vortex patterns”. It takes the Kalliroscope flakes some time to reorient along the weak shear flow remaining in the annulus and if this orientation time is large compared to the modulation time of the flow, this false pattern remains after the secondary flow has ceased.

At low frequencies, $\gamma < 1$, the gap width is small compared with the viscous diffusion length. For in-phase modulation, the motion diffuses entirely across the gap and there is little phase difference across the gap during a period of pulsation. This produces a rigid body rotation flow modulated by $\Omega(t)$. A large amplitude pulsation is needed to create a pressure and centrifugal field during part of the cycle that is sufficient to allow a bifurcation from base flow. At low and intermediate frequencies, we have observed transient vortex patterns near the onset. Transverse laser sheet illuminations show the mushroom-like patterns that are characteristic of the Taylor vortex flow developing near the inner cylinder. At low values of $\gamma$, the experimental results agreed qualitatively with the theoretical prediction where $Ta_e \propto \gamma^{-2}$ and then eventually like $\gamma^{-1}$.

At high frequencies, $\gamma \gg 1$, only thin layers of fluid (Stokes' layers) close to the walls were affected by the oscillatory motion and therefore the critical parameters were expected to become independent of the gap width. Our experimental data at high $\gamma$ nearly follows the theoretical curve, $Ta_e \propto \gamma^{3/2}$ (Figure 19). Another important observation is that there is a discontinuity in the slope of the curve $Ta_e(\gamma)$ near $\gamma = 3.0$ that has also been observed by Aouidef et al.[64]. It was pointed out that the reason for this discontinuity is the change of asymptotic
behaviour of the critical wave number \( q_c(\gamma) \) between high frequencies, \( \gamma > 4 \), where \( q_c \sim \gamma \), and low and intermediate frequencies (\( \gamma < 2 \)), where the critical wave number \( q_c \) is a constant. Figure 24 shows the critical wave numbers obtained by averaging over many sets of patterns versus the frequency \( \gamma \), with the dotted line being the numerical result in the small gap approximation.

The large error bars in the experimental data can be attributed to the larger gap size where it takes a longer time for the wavelengths of the system to adjust and there is some variation of wavelength size along the cylinder axis. In other words, the viscous diffusion time in our system with a wider gap is long compared to a narrow gap system. According to this experimental data, the critical wavelength remains nearly constant for \( 1 < \gamma < 2 \) and makes a dip near \( \gamma = 3 \), which value corresponds to the discontinuity in the slope of the critical Taylor number. The experimental data near and below \( \gamma = 1 \) did not reveal an increase in the wavenumber as predicted by the theory.

Next, we investigated the effects of in-phase modulation with the same amplitude and the same frequency when there is a mean rotation of both cylinders, \( \Omega_m \neq 0 \). This case with a non-zero mean velocity can be treated as the influence of Coriolis acceleration on the system with zero-mean angular velocity [69, 70, 71]. Here, we defined the Taylor number by \( Ta = \frac{r^2(\Omega^2 + \Omega_m^2)^{1/2}}{\nu} \sqrt{\frac{d}{r}} \) and Figure 25 shows the evolution of its critical value \( Ta_c \) versus frequency for \( \Omega_m = 1.26 \text{ rad/sec} \).

The main feature to note on this plot is again the location of the most unstable point between \( \gamma = 2.0 \) and 3.0, nearly the same as in the previous zero-mean case. The lowest value of \( Ta_c \) for this case is smaller than the zero-mean case, indicating that the Coriolis force has a destabilizing effect on the base flow of our
Figure 24: The critical axial wave vector normalized to gap width versus the frequency $\gamma$ for in-phase modulations of concentric cylinders with zero mean velocity.
Figure 25: The critical Taylor number $T_a c$ versus the frequency $\gamma$ for in-phase modulations of concentric cylinders with mean velocity $\Omega_m = 1.26 \text{ rad/sec.}$
system. Mutabazi et al. [69] pointed out that the Coriolis force has a destabilizing or stabilizing effect on the flow depending on its interaction with the shear force. Our experimental data shows that the Coriolis force enhances the centrifugal force, inducing the instability during part of the modulating cycle; its effect dominates at intermediate frequencies but is reduced at low and high frequencies.

Next we studied the configuration where the cylinders undergo out-of-phase modulation with zero mean velocity ($\Omega_m(t) = -\Omega_{out}(t), \Omega_m = 0$). Figure 26 presents the critical Taylor number $Ta_c$ versus the frequency $\gamma$. Again, the base flow is stabilized at high frequencies in agreement with the theoretical predictions. In this limit, as we stated earlier, the flow is confined to the boundary layers close to both cylinders and the critical Taylor number follows the law $Ta_c \propto \gamma^{3/2}$.

We also observed the formation of spiral-like non-axisymmetric patterns which remain in the system for almost all of the cycle. As the frequency decreases from high to moderate values, these patterns become transient vortices that appear all along the cylinder close to the inner cylinder surface for a short time interval (near $\Omega(t) = 0$) compared to the period of modulation. At low modulation frequencies, (or large modulation periods), the nature of the patterns also changes from spiral vortex-like patterns to axisymmetric Taylor vortex-like patterns. These patterns also appear near the inner cylinder wall at $|\Omega(t)| = |\Omega_0|$, but remain in the system (inner half of the gap) until $\Omega(t) \rightarrow 0$.

At low frequencies, ($\gamma < 2$) the base flow became unstable at low Taylor numbers, and the critical Taylor number approached the value for the counter-rotating cylinders with no modulation. This differs from the numerical prediction (dotted line in Figure 26), where theory predicted stabilization at low frequencies relative
Figure 26: The critical Taylor number $T_a_c$ versus the frequency $\gamma$ for out-of-phase modulations of concentric cylinders with zero mean velocity. The solid line is a power law fit to the high frequency part of the curve.
to the behavior at intermediate frequencies. The deviation of our experimental results from the theoretical predictions at low frequencies may be attributed to several factors. First, the linear analysis is based on a stability criterion that may be inappropriate at low frequencies as it defines a flow to be linearly stable if decay occurs from one to the next cycle. Therefore, during a part of a cycle, the disturbances can grow to an amplitude large enough for the secondary motions to be observed experimentally. One would then expect the experimental observation of instability to lie below the points calculated by using linear theory. Secondly, imperfections in the system can cause the experimental values to lie below the theoretical values, giving rise to negative threshold shifts. There can be several physical effects that give rise to imperfections. End effects are the most frequently discussed for the Taylor-Couette system. The Ekman pumping at the end walls tends to lower the critical Taylor number from the value it would have with infinite cylinders. However, experimental studies performed in large aspect-ratio systems have shown that the end effects often have very little effect on the bulk of the flow ($\Gamma \geq 30$)[51, 57, 58]. In our case, with $\Gamma = 34$, we expect that our choice of the onset critical Taylor number as that value for which the cells fill the system should minimize the influence of the ends, although it is still possible that a small effect remains. Even though we tried to eliminate temperature fluctuations along the cylinder by keeping the system in a controlled temperature room, fluctuations of order $10^{-2}K$ could be present in the system. These temperature variations could create convective motions affecting the bifurcation from the base flow. Another source of imperfection could come from the alignment of the cylinder axes. A related problem is very small variations in the radius of the cylinders. Either problem could give a variation of the centrifugal accelerations on the side walls[57]
leading to localized early onset of the patterns. Barenghi and Jones (1989)[57] modeled the imperfections in their theoretical study by including a source of vorticity that produces steadily oscillating transient vortices below the critical Taylor number. Finally, another issue is that both axisymmetric and non-axisymmetric patterns were observed instead of purely axisymmetric patterns near the onset as assumed in the numerical calculations. This is potentially a very important factor promoting disagreement between theoretical predictions and the experimental results.

The critical axial wave vectors obtained from space-time data for out-of-phase modulations are shown in Figure 27. The critical value exhibits a large increase near $\gamma \approx 2$ where the patterns change from those with small axisymmetric vortices to those with larger spiral-like vortices. Both the experimental and the numerical data show a sharp increase in wavenumber as $\gamma \to 0$, but the experimental values are much smaller than the numerical values obtained using a small gap approximation. The reason for this discrepancy is not yet well established and further work is needed to clarify the issue.

6.3 Conclusions

The stability of the flow between two concentric cylinders subject to in-phase and out-of-phase pulsations has been investigated. The experimental results for a system undergoing in-phase pulsations with the same amplitude and frequency are in close agreement with the experimental and theoretical findings of Aouidef
Figure 27: The critical axial wave vector normalized to gap width versus the frequency $\gamma$ for out-of-phase modulations of concentric cylinders with zero mean velocity.
et al. [64, 65]. In the régime of high and low frequencies, stabilization of the flow has been retrieved while it was found to be the most unstable for the moderate value $\gamma \simeq 2$. Especially at high frequencies the linear theory predictions appear to be in reasonable agreement with the experimental observations. Further on in the experimental investigation, we considered the effect of a Coriolis force on the whole system by imposing a mean rotation $\Omega_m = 1.26\text{rad/sec}$. In accordance with the general stability criterion, the rotation was found to induce a large shift for the critical Taylor numbers in the regions $1 < \gamma < 4$ when compared to the system with no mean rotation. The third set of investigations dealt with systems of cylinders oscillating out-of-phase with the same amplitude and frequency. In agreement with theory [80], the flow is stabilized at high frequencies with a critical Taylor number following the law $T a_c \propto \gamma^{3/2}$ but at low frequencies ($\gamma \to 0$) the critical Taylor number $T a_c$ reaches a limiting value close to the one corresponding to uniform counter-rotation, whereas theory predicts restabilization of the flow. This discrepancy was explained in the light of considerations related to the numerical method itself and to imperfections of the experimental apparatus. For $\gamma < 4$, comparisons between experimental and numerical results are less satisfactory in the case $\epsilon = -1$, especially in the low frequency regime. The reasons for this discrepancy have been discussed in the previous section.
Part 3

Temporal Modulation of Traveling Waves in the Flow Between Rotating Cylinders with Broken Symmetry
CHAPTER VII

TEMPORAL MODULATION OF TRAVELING WAVES IN THE FLOW BETWEEN ROTATING CYLINDERS

7.1 INTRODUCTION

The stability of externally modulated hydrodynamic systems has attracted great attention during the past several years. In Chapters V and VI, we have mentioned some of the recent theoretical and experimental studies. Most of these studies were limited to the system, in which the first bifurcation is to stationary spatial patterns [44, 45, 49, 51, 52, 53, 57, 55, 56, 46].

In those cases, only a mere shift of the instabilities in the parameter space has been observed under external modulations. The main features of the stationary patterns that appear in these systems when there are no modulations remain mostly the same with the modulation, since there is no strong resonance between unstable modes and the external modulations. Sometimes the shift in the instabil-
ity due to the modulation may trigger selection of other unstable stationary modes, even though there is no strong resonance. The appearance of stationary hexagonal patterns instead of parallel rolls in the temporally modulated Rayleigh-Benard system is an example of this kind of modification of patterns[73, 74].

For wave structures, however, recent studies by Riecke et al.[75] and Walgraef[76] have shown that strong resonance between an external forcing and the spatio-temporal patterns may occur for an appropriate forcing frequency. This can transform initially stable traveling waves into standing waves or quasi-periodic structures. This can also excite standing waves at an average $Re$ below $Re_c$ for the flow. This kind of behaviour has been experimentally observed in the electroconvection of nematic liquid crystals and in binary fluid convection by Rehberg et al.[77]. It is also related to the parametric excitation of surface waves (Faraday waves)[11].

In this part of the thesis, we investigate the effect of temporal modulation on the traveling wave patterns that appear near the primary bifurcation boundary of two rotating cylinder systems with broken rotational symmetry. In the next section, we present the theoretical background related to our experimental study. In the following chapter, we describe our experimental studies of the modulated Taylor-Dean and Taylor-Couette systems and compare our experimental observations with the theoretical predictions.
7.2 Theoretical Background

Consider a homogeneous, effectively one-dimensional extended system undergoing a Hopf bifurcation to traveling waves. As we discussed in Chapter II, near the onset of traveling wave patterns, where the amplitudes are small, the flow field \( U \) is described by

\[
U = A_R(T)e^{ikz+i\omega_T t}f_1(r) + A_L(T)e^{ikz-i\omega_T t}f_2(r) + \text{c.c.} + \text{h.o.t.} \tag{7.1}
\]

where \( A_R \) and \( A_L \) are the amplitude of right and left traveling wave components of frequency \( \omega_T \) and wave vector \( k \), and the \( f \)'s are the eigenvectors of the traveling waves. The \( T \) is a slow time scale of the flow.

The corresponding amplitude equations for this case can be written in the form\([11, 6, 75, 76]\)

\[
\partial_T A_R = a A_R + c A_R |A_R|^2 + g A_R |A_L|^2 + \text{h.o.t.} \tag{7.2}
\]

\[
\partial_T A_L = a^* A_L + c^* A_L |A_L|^2 + g^* A_L |A_R|^2 + \text{h.o.t.} \tag{7.3}
\]

where \( a, c, \) and \( g \) are complex coefficients which depend on the control parameters and the system parameters such as the radius ratio \( \eta \). A supercritical Hopf bifurcation occurs at \( a = 0 \) from the basic state to the traveling wave state (if \( A_R = 0 \) and \( A_L \neq 0 \) or \( A_R \neq 0 \) or \( A_L = 0 \)) or the standing wave state (if \( A_R = A_L \)) when \( c_r < 0 \). When \( g_r < 0 \), the standing waves are unstable with respect to traveling waves at the super-critical bifurcation boundary. Riecke et al.[75] and Walgraef[76] have shown that introducing uniform temporal oscillations

\[
\alpha(t) = \alpha_0 \cos(\omega_c t) \tag{7.4}
\]
with frequency \( \omega_e \), close to twice the Hopf frequency \( \omega_h \) (the frequency of traveling waves near the primary bifurcation with no modulation) to the system could linearly couple left and right traveling waves. As a result, the coupled homogeneous amplitude equations can be written in the form:

\[
\begin{align*}
\partial_T A_R &= a A_R + b A_L + c A_R |A_L|^2 + g A_R |A_L|^2 + h.o.t. \quad (7.5) \\
\partial_T A_L &= a^* A_L + b^* A_R + c^* A_L |A_L|^2 + g^* A_L |A_R|^2 + h.o.t. \quad (7.6)
\end{align*}
\]

Here, (*) indicates the complex conjugate of the coefficients. The equations (7.5) and (7.6) contain new terms with coefficient \( b \propto \alpha \omega_h/\omega_e \), which is a measure of the amplitude of modulation. The coefficient \( a = a_r + i a_i \), where \( a_r \) gives the growth rate of the waves and is proportional to the reduced control parameter, \( \frac{R - R_e}{R_e} \), with \( R \) being the Reynolds number (control parameter) of the system, and \( a_i \) represents the deviation from exact resonance, the detuning parameter \( \xi = \frac{\omega_h - \omega_e/2}{\omega_h} \). The stability analysis of the above amplitude equations around the trivial state \( A_1 = A_2 = 0 \) shows a Hopf bifurcation to traveling waves at \( a_r = 0 \) for \( b^2 < a_i^2 \) and a steady bifurcation at \( a_r^2 = b^2 - a_i^2 \) for \( b^2 > a_i^2 \) [75].

Using symmetry arguments, it has been shown that a resonant temporal forcing could excite standing waves in a one-dimensional extended system undergoing a Hopf bifurcation to traveling waves. Figure 28 shows one of the phase diagrams obtained by Riecke et al. [75] for such a system modulated with \( \omega_e = 2\omega_h \), where the locations of various transitions are given as functions of the forcing strength and the mean control parameter.

In Figure 28, along the line marked H, the basic state undergoes a Hopf bifurcation to traveling waves as well as (unstable) standing waves. This bifurcation exists already in the absence of any modulation. According to their results, for
Figure 28: A phase diagram for modulated convection at resonance $\omega_c = 2\omega_h$, obtained by Riecke et al. (1988), where the locations of various transitions are given as functions of the forcing strength (the modulation amplitude $b$) and the mean control parameter.
$a_r < 0$ the transition from the basic state of purely azimuthal flow to the phase-locked standing waves (ribbons) occurs along the line marked $S$ as the modulation amplitude increases. Along the line marked $M$ the phase-locked standing waves become unstable to the traveling waves in a secondary parity-breaking bifurcation. Also shown are the transitions of the unstable phase-locked standing waves to unstable standing waves at SW and the saddle-node bifurcation of the phase-locked standing waves along SN, neither of which are not relevant to our experimental study.

Riecke et al.[75] argued that the phase diagram somewhat similar to Figure 28 should be generically obtained by modulating any system with axial symmetry or periodicity in one dimension undergoing a supercritical Hopf bifurcation to stable traveling waves. Later, the above theory has been extended to systems with periodicity in two directions such as the concentric Taylor-Couette system[79]. The base flow of the concentric Taylor-Couette system has both $O(2)$ symmetry (periodicity and reflection along the axial direction) and $SO(2)$ symmetry (symmetry in azimuthal direction). As we mentioned in Chapter III, when the two cylinders counter rotate, this flow undergoes supercritical Hopf bifurcation to the spiral vortex flow. The spirals are periodic along the axial and azimuthal directions. It has been shown that the modulating systems with $O(2)$ and $SO(2)$-symmetry undergoing supercritical Hopf bifurcations from the base flow to stable traveling waves will not produce stable standing waves unless the rotational symmetry is broken.

For a homogeneous system with rotational symmetry (eg. the extended concentric Taylor-Couette system), the flow field of traveling waves can be expressed
as

\[ U = A_1(T)e^{ikz+im\theta+i\omega_Hf_1(r)} + A_2(T)e^{ikz-im\theta-i\omega_Hf_2(r)} + \text{c.c.} + \text{h.o.t.} \quad (7.7) \]

where \( A_1 \) and \( A_2 \) are the amplitudes of two oppositely traveling wave components.
Since this system has both translational and rotational symmetries, the amplitude equations for traveling wave states can be written in the following form:

\[ \partial_T A_1 = a A_2 + c A_1 |A_2|^2 + g A_1 |A_2|^2 + \text{h.o.t.} \quad (7.8) \]

\[ \partial_T A_2 = a^* A_2 + c^* A_2 |A_2|^2 + g^* A_2 |A_1|^2 + \text{h.o.t.} \quad (7.9) \]

where there are no \( bA_1 \) or \( b^* A_1 \) terms. The excitation of standing waves requires linear coupling of \( A_1 \) and \( A_2 \) oppositely traveling waves. It has been shown that only temporal forcing with \( \alpha e^{i\omega_Ht} \) will not produce strong coupling in this case unless the rotational symmetry of the system is also broken. In the concentric cylinders, the superposition of oppositely traveling spirals leads to waves which are standing waves along the axial direction but traveling waves in the azimuthal direction. In order to linearly couple two amplitudes to produce standing waves in both axial and azimuthal directions, the balance of forcing on the two traveling waves has to be broken. This can be achieved easily by applying eccentricity to the system, where the net increase of the energies of the two waves will be different.

The maximum effect of azimuthal forcing is achieved when the azimuthal spatial frequency of the perturbation is twice that of the waves.

In the eccentric Taylor-Couette flow the eccentricity introduces an azimuthal dependent forcing of

\[ P = \beta_0 e^{i\theta} + \text{c.c.} \quad (7.10) \]

The temporal modulation of the system with frequency \( \omega_e \) introduces periodic
forcing

\[ F = \alpha_0 e^{i\omega t} + \text{c.c.} \quad (7.11) \]

These two forcing terms allow the linear coupling of left and right traveling wave amplitudes when the frequencies \( \omega_e \) and \( \omega_h \), as well as azimuthal wave numbers \( m \) and \( n \) are related as

\[
\frac{2\omega_h}{\omega_e} = j, \quad \text{and} \quad \frac{2m}{n} = l \quad \text{where} \ j, l \text{ are integers.}
\]

The strength of this temporal and angular forcing produces the term

\[ A_2 \alpha_0 \beta_0 e^{ikz-im\theta-i\omega_h t} e^{i\omega_t} e^{in} \quad (7.12) \]

coupled with

\[ A_1 e^{ikz+im\theta+i\omega_t} \quad (7.13) \]

to excite standing waves. Now, the amplitude equations for the periodically forced spiral flow have the same form as (7.5) and (7.6). Therefore, the transition from the purely azimuthal base flow to the phase locked standing waves should occur for certain parameter values. The forcing has maximal effect if the modulation frequency \( \omega_e \) is twice the Hopf frequency \( \omega_h \), where \( j = 1 \). The azimuthal perturbation has maximal effect if \( l = 1 \), which cannot be achieved easily in our experimental system. The azimuthal perturbation of eccentric cylinders produces only \( l = 2 \). Still, without any azimuthal perturbations no linear coupling of two traveling waves occurs to produce standing waves.

In the next chapter, we present two experimental systems with broken rotational symmetry, the Taylor-Dean and the eccentric Taylor-Couette, which we have used to test the concepts we discussed here.
CHAPTER VIII

EXPERIMENTAL APPARATUS AND PROCEDURES

The Taylor-Dean and the eccentric Taylor-Couette systems break the rotational symmetry in two very different ways. In this chapter, we describe each one of these systems in detail. First, we present the experimental system with strongly broken symmetry, the Taylor-Dean system, which may be considered as a system with a single translational symmetry. Next, we present the eccentric Taylor-Couette system which represents a system with slightly broken rotational symmetry and may be considered as a system with two translational symmetries.

8.1 TAYLOR-DEAN SYSTEM

The Taylor-Dean system[81, 82] consists of two independently rotating horizontal coaxial cylinders with a partially filled gap (see Figure 29). In contrast to the completely filled concentric Taylor-Couette system, the partial filling of the gap in
Figure 29: (a) Schematic diagram of the Taylor-Dean apparatus. (b) Schematic cross section of the apparatus. The front side is the side where the observer sees the inner cylinder rotating upward as shown.
the Taylor-Dean system breaks the rotational symmetry of the flow. The rotation of the cylinders and the two free surfaces impose a pressure gradient along the azimuthal direction. As a result, the flow sufficiently far away from free surfaces is a combination of the Couette flow due to the rotation of the cylinders and the Poiseuille flow due to the azimuthal pressure gradient.

The main control parameters of this system are the inner and outer cylinder Reynolds numbers, $R_i = \Omega_i r_i d/\nu$ and $R_o = \Omega_o r_o d/\nu$. As the control parameters are varied, the base flow instabilities will change from those associated with Taylor-Couette to those associated with Dean flow. Figure 30 shows the phase diagram of the primary flow transitions in the $(R_i, R_o)$ space obtained by Mutabazi et al. [81]. The first experimental work with the Taylor-Dean system was done by Brewster and Nissan (1958)[83], who studied the threshold of the instability when the inner cylinder rotates and the outer cylinder is stationary. Since then, there have been several theoretical and experimental studies carried out on this geometry[81, 82]. More recently, Mutabazi, Normand, Peerhossaini, and Wesfreid[84] have solved the linear stability problem for axisymmetric and non-axisymmetric perturbations in the flow between two rotating cylinders with a partially filled gap. They found both stationary and traveling wave instabilities depending on the ratio of the angular velocities $\mu (= \Omega_o/\Omega_i )[81]$.

We kept the outer cylinder stationary ($R_o = 0$) for this study, while rotating the inner cylinder. In this case the first transition from the unperturbed base flow is to traveling inclined rolls as the inner cylinder speed increases, as shown in the experimental study of Mutabazi, Hegseth, Andereck and Wesfreid[82]. The transition to traveling inclined rolls is, within the experimental error, a supercritical
Figure 30: The phase diagram of the primary flow transitions in the $(R_i, R_o)$ space obtained by Mutabazi et al. (1988).
Hopf bifurcation. At onset the rolls have no preferred direction and may move either left or right along the cylinder axis. Therefore, this system can be used to test the theoretical prediction that a breaking of the time translational symmetry by a small periodic modulation of the control parameter will result in a stable standing wave pattern[75, 76].

The experimental system (Figure 29) consists of an inner cylinder made of black Delrin plastic with radius \( r_i = 4.49 \text{ cm} \) and a stationary outer cylinder made of Duran glass with radius \( r_o = 5.08 \text{ cm} \), giving a gap \( d = r_o - r_i = 0.59 \text{ cm} \) and radius ratio \( \eta = r_i/r_o = 0.883 \). Two plastic rings are attached to the inner cylinder a distance \( L = 52.4\text{ cm} \) apart, giving an aspect ratio \( \Gamma = L/d = 88 \), large enough to conceive of this as an extended system where one can neglect the end effects. In this system the filling level fraction \( n = \theta_f/2\pi \) has been fixed at 0.75 where \( \theta_f \) is the filling angle. We used two stepper motors (Compumotor A83-93) to drive the inner cylinder, one motor to produce a net rotation and the other to produce a sinusoidal modulation of the inner cylinder angular velocity. The first motor was directly connected to the inner cylinder and simply rotated with constant angular velocity. The housing of the first motor was oscillated by a push-rod arrangement mechanically coupled to the second motor (see Figure 29). The rotation of the second motor gave a net output at the inner cylinder of a constant angular velocity plus a periodic sinusoidal variation in angular velocity.

The motor speeds were controlled through Compumotor 2100 Series Indexers and could be changed either manually or through computer control. The motor speeds have a frequency accuracy of 0.02%. Normally, a PDP11/73 computer controlled the rotation speed, direction, and ramping rates. Because both the fre-
frequency and amplitude of the modulation could be varied, this introduced two new control parameters into the system, as suggested by the theory. The Reynolds number of the inner cylinder is now $R = R_i + R_m \sin(2\pi f_m t)$. The two dimensionless parameters are $R_m/R_{ic}$ and the detuning parameter $\zeta = (2f_h - f_m)/2f_h$, where $R_{ic}$ is the critical Reynolds number for the onset of the traveling roll pattern and $f_h$ is the frequency of the traveling wave patterns (Hopf frequency) in the absence of modulation. Again, the working fluid was pure double distilled water or a solution of double distilled water and 44% glycerol by weight, with 1% of Kalliroscope AQ1000 added for visualization.

We have obtained and analyzed the space-time data of the system using the same method we discussed in Chapter VI. An analysis of intensity versus axial position plots produced from the space-time data yields the wavelengths and the dynamics of the patterns.

To obtain the location of the onset of patterns for each modulation frequency $f_m$ and modulation amplitude $R_m$, we employed the following method. As is common practice, we first set the $R_i$ value below the onset of instabilities and then increased it quasistatically (keeping both amplitude $R_m$ and frequency $f_m$ of modulation at a fixed value) until a flow pattern appeared in the system. Then several sets of space-time data were taken while increasing $R_i$. We repeated this procedure with increasing amplitudes and various frequencies around twice the Hopf frequency (the frequency of the traveling rolls near onset when there is no modulation).
8.1.1 Results and Discussion

When there is no modulation ($R_m = 0$), the base flow bifurcates supercritically to a traveling roll pattern as we increase the inner cylinder speed. A typical picture of the traveling roll flow is shown in Figure 31. A space-time diagram for the traveling roll pattern near onset ($R_i = 265, R_o = 0, \text{ and } R_m = 0$) is shown in Figure 32, where the intensity along a line parallel with the axis of the cylinders was recorded every 113 ms for 58 s. The wavelength of the rolls along the cylinders is $\lambda = 0.841 \text{ cm}$.

Light sheet visualization through the gap cross section shows that the rolls exist near the outer cylinder. At $\epsilon (\equiv \frac{R_i-R_{ic}}{R_{ic}})$ slightly greater than 0.0 the pattern fills most of the working space and both left and right traveling rolls may exist with a vertical defect line between them (see Figure 31). These defect lines are not necessarily halfway along the axis of the cylinders. Such defects are inherent to traveling wave patterns[85]. The frequency of the traveling rolls (Hopf frequency) near the onset ($R_{ic} = 263$) is $0.543 \text{ Hz}$ and the wavelength $\lambda = 0.720 \text{ cm}$. Upon further increase of $R_i$, the flow undergoes a second instability to a short wavelength modulation of the traveling rolls at $R_i = 303$, and then to an incoherent pattern at about $R_i = 338[87]$.

The transition sequence changes dramatically with modulation of the inner cylinder speed. When we modulated the inner cylinder sinusoidally near the detuning parameter $\xi \approx 0$, we found standing waves rather than the traveling rolls (See Figure 33), in agreement with the theoretical results obtained by Riecke et al. [75]. Figure 34 shows a phase diagram of the primary transitions from the base flow to standing waves and the secondary transition from standing waves to trav-
Figure 31: Typical picture of traveling roll flow appeared in the Taylor-Dean system at $R_i = 265$ and $R_m = 0$. 
Figure 32: Space-time diagram of the traveling roll pattern in the Taylor-Dean system when $R_i = 265$, $R_o = 0$, and $R_m = 0$. The wavelength of the pattern is $\lambda = .712 \text{ cm}$ and the frequency $f_a = 0.543 \text{Hz}$. 
eling waves at $\xi \approx 0$ as we varied the amplitude of modulation $R_m$ and the inner cylinder Reynolds number $R_i$. Within our experimental resolution, the bifurcation from base flow to standing wave pattern appear to be a supercritical bifurcation. The interesting features to note in Figure 34 are that the standing waves can be excited at $R_i$ values much lower than the critical $R_i$ ($= 263$), when the modulation amplitude is increased above certain values ($R_m/R_{ic} > 0.05$). Also, at higher amplitude of modulation, we have observed standing waves in a large area in the parameter space above the onset boundary. The traveling waves reappear when $R_i$ increases, while keeping the modulation amplitude $R_m$ fixed, as has been similarly observed in other systems[77] and in agreement with the theoretical predictions by Riecke et al. [75, 79]. At small modulation amplitudes ($R_m/R_i < 0.05$) only the traveling roll state appears when $R_i$ is increased, also in agreement with the theoretical predictions. The space-time diagram of the standing waves for $R_m/R_{ic} = 0.30$, $R_i = 241$ and $\xi = 0 \pm 0.01$ is shown in Figure 35. During one modulation period the light intensity at a given axial position varies periodically, indicating the presence of standing wave patterns. The frequency of the standing wave pattern is half the modulation frequency.

A quantitative analysis of the standing waves was carried out using a 2-dimensional fast Fourier transformation of the patterns in time and space. This proved to be a very useful method for decomposing the standing wave patterns into their left and right traveling wave components from the original space-time CCD data. The original space-time data consists of $512 \times 512$ data points (i.e., 512 points along the axial coordinates and 512 frames along the time coordinates) of pixel intensities. First, we read this data into a 2-dimensional array of $512 \times 512$, and then carried out the fast Fourier transformation on it using fourn and rfft3.
Figure 33: Typical picture of the standing wave patterns appeared in the Taylor-Dean system at $R_i = 265$, $\xi = 0$, and $R_{ef}/R_{le} = 0.28$. 
Figure 34: Phase diagram of standing wave and traveling wave states when the detuning parameter $\xi = 0$. The plus and cross signs indicate the location of the bifurcation from the base state to standing waves and standing waves to traveling waves.
Figure 35: Space-time diagram of the standing wave state at $R_m/R_i = 0.30$, $R_i = 241$, and $\zeta = 0 \pm 0.01$. 
numerical subroutines[86] to obtain the 2-D decomposed Fourier spectra of the left and right traveling components. From the decomposition we obtained the spatial and temporal frequencies and the amplitudes of each wave component. Then, carrying out a reverse Fourier transformation separately on each component, we were able to obtained decomposed space-time plots for the left and right wave components.

Figure 36(a) and 36(b) shows the resulting right and left traveling waves obtained from the space-time data shown in Figure 35. The frequency power spectra of the decomposed right and left traveling modes, obtained using the 2-dimensional FFT, are shown in Figures 37(a) and (b). In these figures, the frequencies are scaled to the diffusive time scale $\nu/d^2 = 36$ sec. The left and right components have similar space and time characteristics. Very small differences appear in the amplitudes of the spectral peaks. This is attributable to slightly nonuniform lighting conditions.

At a still higher inner cylinder speed, the standing waves lose their stability to a traveling wave state. This transition is also supercritical within our experimental resolution. No mixed standing/traveling rolls states have been seen. The space-time diagram for the traveling wave state just above the onset at $R_i = 269$, $R_m/R_{ic} = 0.212$ for $\xi \approx 0$ is shown in Figure 38. The decomposition of this space-time data and associated power spectra are shown in Figures 39(a), (b), (c), and (d). These figures show that one traveling mode (right) dominates over the other (left), much weaker, components present in the system.

We also tested the sensitivity of the onset of the patterns and the behaviour of the patterns to a variation of the detuning parameter $\xi$. The result for
Figure 36: Space-time diagrams of (a) the right traveling wave component and (b) the left traveling wave component of the standing wave state shown in Figure 35, obtained using a 2-dimensional FFT decomposition.
Figure 37: The frequency power spectra of the decomposed (a) right and (b) left traveling modes shown in Figure 36. The frequencies are scaled to the inverse of the diffusive time scale $\nu/d^2$. 
Figure 38: Space-time diagram of the traveling wave state at $R_m/R_{rc} = 0.212$ and $R_i = 269$, just above the onset for $\xi \approx 0$. 
Figure 39: (a) and (b) are the decomposed right and left traveling components of the traveling wave state shown in Figure 38. (c) and (d) are the frequency power spectra of the decomposed right and left traveling modes.
$R_m/R_i = 0.3$, shown in Figure 40, indicates that the onset of the patterns is very sensitive to the value of the detuning parameter $\xi$ for this system. When $\xi \approx 0$ the standing wave pattern appears at $R_i$ well below $R_{ic} = 263$. As we changed $\xi$ away from $\xi = 0$ (lower or higher modulation frequencies than $2f_h$) the onset of the patterns was delayed considerably. In fact, at higher modulation frequencies ($\xi < -0.15$) patterns appeared only at supercritical $R_i(> R_{ic})$ values for $R_m/R_{ic} = 0.3$. Furthermore, for fixed $R_m$, as we move away from $\xi = 0$ the characteristics of the patterns also change, so that traveling wave patterns appear at the onset ($\xi < -0.08$ or $\xi > 0.04$ for $R_m/R_{ic} = 0.3$) rather than standing waves. Figure 41 shows the ratio of the fractional power under the left traveling wave peak to the total power, as a function of $\xi$ at $R_m/R_{ic} = 0.3$. This characteristic resonance curve also shows the disappearance of the left traveling wave as the detuning parameter $\xi$ shifts away from 0. The maximum fractional power was a little less than the expected 0.5, which can be attributed to the non-ideal lighting conditions. We also analyzed the frequencies of the two components as a function of $\xi$. Figure 42 shows the frequency ratio as a function of $\xi$, where it shows the standing wave region clearly. In this region the two wave components lock to the same frequency over a range of $\xi$.

Also, small changes of $\xi$ near 0 not only affect the primary transition to standing waves but also the secondary transition to traveling waves at higher $R_i$. Figures 43 and 44 give the phase diagrams when $\xi$ is equal to $-0.03$ and $+0.03$, which show these changes clearly. A comparison of Figure 44 and Figure 32 shows that the minimal modulation amplitude for the excitation of standing waves is actually lower for $\xi = 0.03$ than for $\xi = 0$. This can be ascribed to the dependence of the linear frequency of the waves (Hopf frequency $f_h$) on $R_i$. 
Figure 40: The onset of the primary flow transition vs the detuning parameter $\xi = \frac{2f_h - f_m}{2f_h}$ at $R_m/R_{ic} = 0.3$. Lower $\xi (< -0.2)$ or higher modulation frequencies will delay the onset of any patterns in the system relative to $R_i$ for $\xi \approx 0$. The solid line indicates the critical $R_i$ in the absence of modulation.
Figure 41: The fractional power of the secondary traveling wave vs the detuning parameter $\xi$ for $R_m/R_{ic} = 0.3$. 
Figure 42: The frequency ratio of the two traveling wave components vs the detuning parameter $\xi$ for $R_m/R_{ic} = 0.3$. 
Figure 43: Bifurcation diagram of the modulated Talor-Dean system when $\xi = -0.03$. Pluses and crosses indicate the transitions from the base flow to standing waves and to traveling waves respectively.
Figure 44: Bifurcation diagram of the modulated Taylor-Dean system when $\xi = 0.03$. Pluses and crosses indicate the transitions from the base flow to standing waves and to traveling waves respectively.
Figure 45: (a) The traveling wave state with the short wavelength modulations at $R_i = 300$. (b) and (c) are the decomposed space-time data for left and right traveling modes, respectively, of the same traveling wave state shown in (a).
Figure 46: (a) The incoherent patterns at $R_i = 340$. (b), and (c) are the decomposed space-time data for left and right traveling modes, respectively, of the incoherent pattern shown in (a).
Further increases in $R_i$ beyond the traveling wave state resulted in the emergence of short wavelength modulations near $R_i \simeq 300$ for $R_m/R_{ic} = 0.25$ and $\xi \approx 0$. The space-time data for this pattern and the decomposed left and right traveling components are shown in Figure 45. For the data in Figure 45, the wavelength of the roll pattern and short wavelength modulation are $\lambda_1 = 1.27 \pm 0.04$ and $\lambda_2 = 3.20 \pm 0.04$ respectively. Both of these wavelengths changed with $R_i$. As we further increased $R_i$, at $R_i \simeq 340$, incoherent patterns appeared in the system for $R_m/R_{ic} = 0.25$ and $\xi \approx 0$. The space-time data for this state and the decomposed left and right traveling components are shown in Figure 46. The left and right traveling components have the same wavelength $\lambda = 1.26 \pm 0.04$ and the same frequency $f = 0.550\,Hz \pm 0.011$.

8.2 **Eccentric Taylor-Couette System**

In the Taylor-Dean system, the rotational symmetry is severely broken by the air gap and can be modeled as a system with a single translational symmetry. On the other hand, the concentric Taylor-Couette system has translational symmetries in both axial and azimuthal directions. As we discussed in Chapter VII, for this system it is necessary to apply periodic forcing as well as an azimuthal perturbation in order to linearly couple the left and right traveling spirals to produce the standing waves. We applied an azimuthal perturbation to the system by offsetting the axis of the inner cylinder, thereby making the system eccentric.

The experiment was performed in the region of the $(R_o, R_i)$ parameter space where traveling waves in the form of spiral vortices occur as the primary instability. As we discussed in Part I, this happens when the cylinders are counter-rotating and
concentric \([4, 5, 6, 24]\). The spiral patterns travel in both the axial and azimuthal directions, and they break both the axial and azimuthal symmetry of the base flow. For a given radius ratio there is a unique value of the outer cylinder speed above which the primary bifurcation from the base flow is a supercritical Hopf bifurcation to the time periodic spiral flow\([5, 6]\). The azimuthal wave number of the spirals increases as the outer cylinder speed increases. The locations of the crossover points between spirals with different azimuthal wave numbers are uniquely determined by the radius ratio of the two cylinders \([5]\).

As described in Chapter VII, the standing waves would not be excited in this situation by external temporal modulation, unless the azimuthal symmetry of the basic system was also broken. The azimuthal symmetry of the Taylor-Couette system can be broken by making it slightly eccentric by, for example, moving the axis of the inner cylinder off the axis of the outer cylinder while maintaining the two axes parallel.

The same eccentric cylinders setup (shown schematically in Figure 6) used in the first experiment has been used in this experiment. The main control parameters are again the inner and outer cylinder Reynolds numbers, \(R_i = 2\pi f_i r_i \bar{d} / \nu\) and \(R_o = 2\pi f_o r_o \bar{d} / \nu\), where \(\bar{d}\) is the average gap width. As in the first experiment, the eccentricity is adjusted by offsetting the axis of the inner cylinder relative to the fixed axis of the outer cylinder. The eccentricity \(e\) is defined as \(e = \varepsilon / (r_o - r_i)\), where \(\varepsilon\) is the offset of the two cylinder axes. Again, the working fluid was pure double distilled water or a solution of double distilled water and 44% glycerol by weight with 1% of Kalliroscope AQ1000 added for visualization.
Both inner and outer cylinders are driven by two independently rotating Com- pumotor stepper motors. Since the modulation amplitude required in the eccen- tric Taylor-Couette system was much greater than for the Taylor-Dean system, we could not use the two motor push rod arrangements to produce both constant rotation and sinusoidal modulations of the inner cylinder. Therefore, we have used a single motor to produce both a constant mean rotation component and the sinu- soidal modulation of the inner cylinder rotation. This was achieved by sending control commands (a sequential change of shaft motion parameters with time de- lays) to the indexer that controlled the motor connected to the inner cylinder, using the PDP11 computer. We have tested this motion of the inner cylinder using LDV and a single point reflectance and found out that the motion is nearly sinusoidal when the number of step changes in velocity per cycle is more than 40.

To obtain the location of the onset of patterns for each modulation frequency $f_m$ and modulation amplitude $R_m$, we employed a method similar to that used in the experiment on the Taylor-Dean system. We kept the outer cylinder speed fixed at a value within the range where the bifurcation to the spiral vortices occurs. Then, at a fixed modulation amplitude $R_m$ and constant frequency $f_m$, we increased the average inner cylinder speed until the pattern appeared in the system. Then several sets of space-time data were taken while increasing $R_i$. We repeated this procedure with increasing amplitudes $R_m$ and various detuning parameters $\xi$ (i.e., frequencies around twice the Hopf frequency, where the Hopf frequency in this case is the frequency of the spiral patterns near onset in the absence of modulation).
8.2.1 Results and Discussion

When there is no modulation, the base flow bifurcates supercritically to spirals as we increase the inner cylinder speed while keeping the outer cylinder speed constant above a certain value. A typical space-time diagram for the spiral pattern near onset \((R_i = 151, R_o = 155, \text{ and } R_m = 0)\) is shown in Figure 47. Here the intensity along a line parallel with the axis of the cylinders was recorded every 0.6s for 100s. The wavelength of the spirals along the axis of the cylinder is \(\lambda = 2.07 \text{ cm} \approx 2d\). Light sheet visualization through the gap cross section shows that the spirals exist near the inner cylinder. At \(\epsilon = \frac{R_i - R_{ic}}{R_{ic}}\) slightly greater than zero the pattern fills most of the working space along the axis. An upward moving spiral exists near the top of the system and a downward moving spiral exists near the bottom. A horizontal defect line forms where the two spirals meet. These defect lines are not necessarily halfway between the top and the bottom along the axis of the cylinders. The frequency of the spirals (Hopf frequency) with azimuthal wave number \(m = 2\), as shown in Figure 47, near the onset \((R_{ic} = 151, R_o = 155 \text{ for } \epsilon = 0)\) is 0.113 Hz.

When we sinusoidally modulated the inner cylinder rotation speed, we observed wave patterns which resembled standing waves (time-dependent patterns which are stationary in space) rather than traveling waves (spirals) as the first bifurcation from the base flow. Figure 48 shows a space-time diagram of the standing-wave-like pattern near the transition to the spiral state with azimuthal wave number \(m = 2\) at \(R_i = 151, R_o = 155, \frac{R_m}{R_i} = 0.4, \epsilon = 0.126\), and the detuning parameter \(\xi \approx 0\).

Figures 49(a) and (b) show the resulting right and left traveling wave components obtained from the space-time data shown in Figure 48. The power spectra
Figure 47: Space-time diagram of the spiral pattern ($m = 2$) in the Taylor-Couette system at $R_i = 151$, $R_o = 155$, $e = 0$, and $R_m = 0$. 
Figure 48: Space-time diagram of the standing-wave-like pattern in the modulated Taylor-Couette system at $R_i = 151$, $R_o = 155$, $e = 0.126$, $R_m/R_{ic} = 0.4$, and $\xi \approx 0$. 
obtained using the 2-dimensional FFT for right and left traveling wave components are shown in Figures 49(c) and (d). These have the same frequency characteristics, but different peak amplitudes. A true standing wave would have equal amplitudes in the traveling components. Even at very high modulation amplitudes \( R_m \), one spiral component always dominates the other. Figure 50 shows the ratio of the amplitudes of the primary peaks of the two spiral components versus \( \xi \). The peak amplitude of the secondary component reaches a maximum near \( \xi = 0 \), as we expected according to the theory.

Figure 51 shows the phase diagram of \( R_m/R_{ic} \) vs \( R_i \) for \( e = 0.253 \) and \( \xi \approx 0 \). This shows that the base flow became unstable as the amplitude of modulation \( R_m \) increased, in agreement with the theoretical predictions. Also, the standing-wave-like patterns appeared only at large modulation amplitudes (for \( e = 0.126, \frac{R_m}{R_i} > 0.17 \)), when the detuning parameter was \( \xi \approx 0 \). At small modulation amplitudes (\( R_m/R_i < 0.17 \)) only the traveling roll (spiral) state appears when \( R_i \) is increased. Whether standing waves are produced also depends on the eccentricity of the system. For small eccentricity \( e \) where the rotational symmetry is slightly broken, the standing waves appeared only at higher modulation amplitudes, in agreement with the theoretical predictions.

Compared with other systems (e.g., the Taylor-Dean system discussed in the previous section or electroconvection of nematic liquid crystals and binary fluid convection [77]), in this system, the standing-wave-like patterns with two components having the same frequency appeared only in a very small region of parameter space near the resonance frequency (see Figure 52). Both the small region of parameter space and the unequal power spectra amplitudes indicate that the coupling
Figure 49: (a) and (b) are the decomposed components of the space-time data in Figure 48. (c) and (d) are the frequency power spectra of the right and left traveling modes. The frequencies are scaled to the inverse of the diffusive time scale $\nu/d^2$. 
Figure 50: The ratio of the spectral peak amplitude of the secondary spiral wave state to the spectral peak amplitude of the primary spiral wave state vs the detuning parameter $\xi$. 
Figure 51: Phase diagram of $R_m/R_{ic}$ vs $R_i$ for the eccentric Taylor Couette system with eccentricity $e = 0.253$ and detuning parameter $\xi = 0$. 
Figure 52: The frequency ratio of the two traveling wave components versus the detuning parameter $\xi$ in the eccentric Taylor-Couette system. The left and right traveling components have the same frequency only in a very small region near resonance $\xi = 0$. 
of the modulation to the fluid flow is very weak. Two factors may contribute to this. The azimuthal symmetry breaking employed in the experiment \((n = 1)\) enters the coupling between left- and right-traveling spirals only with its fourth power for spirals with \(m = 2\). Thus, quite large eccentricities are needed to give a strong effect. Motivated by this observation we have also studied the modulation of \(m = 1\)-spirals. However, no resonant excitation of standing waves was observed; instead, the patterns relax to the base flow during part of the cycle. Since the frequency of the waves is very small, they appear to follow the periodic forcing adiabatically indicating that their growth rate is of the same order as the forcing frequency. The investigation of \(m = 1\)-spirals is further complicated by the presence of other modes like axisymmetric vortices and interpenetrating spirals at nearby parameter values. A second reason for the apparent weakness of the coupling is presumably the penetration depth of the oscillation into the bulk of the fluid, as characterized by the viscous Stokes layer of width \(\delta = \sqrt{\nu/\omega}\), near the inner cylinder. The thickness of the Stoke's layer \(\delta\) at \(\xi = 0\) for both \(m = 1\) and \(2\) spiral states are 0.24cm and 0.17cm, much smaller than the size of the gap width \(d = 1.19cm\). Thus, the bulk of the fluid feels the modulation only very weakly.

When the inner cylinder speed is increased further above the onset, the standing-wave-like patterns appearing at \(\xi = 0\) near the transition boundary to \(m = 2\) spiral state lose their stability to an incoherent pattern. This may be due to the loss of stability of the basic spirals to interpenetrating spirals and wavy spirals. Even for the concentric cylinder case \((e = 0)\) we have seen some evidence of a weak secondary spiral at very high modulation amplitudes. This could be due to small imperfections in the experimental apparatus which may break the rotational symmetry sufficiently to induce a second spiral.
8.3 CONCLUSIONS

In conclusion, we have found that time-periodic modulation at close to twice the frequency of a Hopf bifurcation can induce standing waves in systems with stable traveling wave patterns. The Taylor-Dean system, with its strongly broken azimuthal symmetry, yields clear agreement with the qualitative features of the theoretical model\[75]\.

We have also found that time-periodic modulation at close to twice the Hopf frequency can induce standing-wave-like patterns in systems with traveling wave patterns in two directions. The Taylor-Couette system with counter-rotating cylinders produces spirals at onset over a large parameter range, but unless the rotational symmetry of the apparatus is broken, standing waves cannot be excited by modulation. This is in contrast with the case of one-dimensional traveling wave patterns studied in convection\[77]\, where only temporal modulation is necessary to stabilize standing wave patterns. Temporal modulation of the eccentric Taylor-Couette system induces a second traveling spiral pattern as predicted by theory. However, owing partly to weak coupling of the oscillations to the bulk of the flow, it never grows to a large enough amplitude, nor does it couple with the other traveling spiral sufficiently strongly, to produce simple standing wave patterns at onset. In the Taylor-Dean system, the inner cylinder modulation creates not only the oscillatory boundary layer influence but also an induced oscillatory azimuthal pressure gradient due to the free surfaces as well, affecting the entire pattern. In the eccentric Taylor-Couette oscillation, on the other hand, modulation creates
only a weak azimuthal pressure gradient due to the small eccentricity, and the oscil­

cillatory boundary layer influence may not be enough to excite a strong secondary

traveling component.

We have observed that the standing waves in the Taylor-Dean system lost

stability to traveling waves as we increased the inner cylinder Reynolds number $R_i$

while keeping the modulation amplitude constant in agreement with the theoreti­
cal predictions by Riecke et al.. Further increases in $R_i$ resulted in a transition
to modulated traveling waves and then to incoherent patterns. These patterns
look similar to the spatio-temporal patterns observed in other fluid dynamical

systems[88, 89, 90, 91] and might be worth looking at in more detail in the future.
CHAPTER IX

GENERAL CONCLUSION

In this study we have investigated the stability of the base flow between two rotating cylinders subject to various spatial symmetry breakings and time-dependent external forcings. The flow between two concentric rotating cylinders (the Taylor-Couette system) has often been used as a testing platform for the recently developed mathematical theories and models of nonlinear fluid dynamics. The incompressible fluids used are considered to be continuous in space and are believed to obey the Navier-Stokes equations (Newton's second law). An understanding of the instabilities that result in various flow patterns in the rotating cylinder system is really an understanding of the solution to the nonlinear equations of motion. The classical Taylor-Couette system is a very special case due to its rotational symmetry. We believe that an understanding of the flow patterns that appear in systems not only with rotational symmetries but also with broken rotational symmetries and temporal modulation is a step towards a general understanding of the nonlinear dynamics of fluid flows.
In this study we have chosen three experimental configurations that are not only mathematically important and tractable, i.e., where one can apply known mathematical models for a better understanding of the dynamics of the fluid motion, but also have practical applications in areas such as lubrication technology, geophysical science, and material processing. We have concentrated our experimental study on the patterns that appear near the primary instability boundary, where we might claim to have a better theoretical understanding of subtle interplay of nonlinearity, symmetry, and boundary conditions in establishing the patterns of the secondary flow. We have found and characterized a variety of different spatiotemporal patterns that appear near the primary bifurcation boundaries as we vary the control parameters.

First we looked at the flow between two counter-rotating cylinders with broken rotational symmetry by applying eccentricity to the system. Counter-rotating eccentric cylinders produce both time-independent Taylor-vortex flow and time-dependent spiral flow patterns along the primary instability boundary. The eccentricity changes the boundary conditions in the radial direction. We observed an increase of the stability of the base flow. This increase is not same for the Taylor vortex flow and all the spiral modes with different azimuthal wave numbers. As a result, the shift in cross-over points between different modes has been observed as the eccentricity increases. The experimental results are in qualitative agreement with the theoretical results obtained for an infinitely long system with very small eccentricity.

Second, we studied the stability of the base flow between two in-phase and out-of-phase pulsating cylinders with or without mean rotational motion. The
stabilization at both high and low pulsation frequencies and the destabilization at intermediate pulsation frequencies has been observed in the case of in-phase modulation, in agreement with theoretical predictions. Destabilization has been observed at low frequencies in the out-of-phase modulations case. This is in disagreement with the numerical results obtained assuming axisymmetric perturbations using the quasi-static approximation. We reasoned that this disagreement was due to limitations in the numerical stability analysis at low frequencies and the appearance of non-axisymmetric secondary flow patterns not accounted for in the analysis.

Last, we investigated the effect of temporal modulation on traveling waves that appear near the primary instability boundary in two rotating cylinder systems (Taylor-Dean and eccentric Taylor-Couette systems) with broken rotational symmetry. Azimuthal forcing due to broken symmetry and temporal forcing at twice the frequency excited the secondary wave pattern that coupled with primary traveling waves to produce stable standing waves or standing-wave-like patterns. The strong resonance of the external forcing and the traveling waves resulted in the onset of this quasi-static pattern much below the primary instability boundary for non-modulating case. The experimental results are in qualitative agreement with the theoretical predictions obtained using symmetry arguments for a system undergoing a Hopf bifurcation to stable traveling waves.

Throughout this research we have concentrated our investigation on time-dependent patterns that appear in the flow of eccentric Taylor-Couette and Taylor-Dean systems to understand the physical nature of the fluid flows. We have compared our experimental results with other experimental and theoretical results to
produce a coherent picture of the physical processes in these spatiotemporal patterns. A clear physical understanding of the stability of the fluid flows and the flow patterns is often difficult, but we hope that the results presented here may help to further our understanding of other, more general, fluid systems.
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