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CONTROL OF CHAOS IN THIN FILMS AT FERROMAGNETIC RESONANCE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

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1996

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To my family
ACKNOWLEDGEMENTS

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CHAPTER I

Introduction

Chaos is everywhere. It is observed in swirling winds, chemical reactions, electrical circuits and high-power lasers. The irregular and unpredictable motions of chaotic dynamics have long been mainly an academic curiosity. For practical purposes, chaos is generally something one would avoid, since it would seem to resist any type of control or utility. Surprisingly, chaos has been controlled in several situations, opening possibilities that chaotic systems may have practical uses.

This dissertation will discuss the control of chaos in thin films at ferromagnetic resonance (FMR). For more than a decade, chaotic behavior has been observed in ferromagnetic resonance. Chapter II provides a theoretical background to ferromagnetic resonance and presents a model that has been successful at predicting the behavior of ferromagnetic resonance dynamics.

Chapter III provides an introduction to chaotic dynamics and techniques used to control chaos. While many mathematical arguments are presented, it is not intended to be mathematically rigorous. It is intended to provide the necessary background to understand the theoretical and experimental results of controlling chaos.

Chapter IV describes the experimental set up used to control chaos in thin mag-
netic films. It also provides the motivations for the different techniques used.

Chapter V presents the experimental results on the control of chaos in thin films at ferromagnetic resonance. Both the stabilization of periodic orbits and the synchronization of two chaotic signals are presented.

Chapter VI analyzes the results of Chapter V. It uses a computer model to replicate the experiments and provides a physical explanation for the effectiveness of the control techniques.

Chapter VII concludes by drawing inferences from the analysis and suggests further studies in ferromagnetic resonance.
CHAPTER II

Magnetic Resonance

2.1 Dipole precession

A magnetic dipole \( \mathbf{m} \) placed in a magnetic field \( \mathbf{H}_0 \) applied in the \( z \)-direction will experience a torque of \( \mathbf{m} \times \mathbf{H}_0 \). Since the dipole has an angular momentum associated with it, the torque will produce a change of angular momentum. The equation of motion for the dipole becomes

\[
\frac{\partial \mathbf{m}}{\partial t} = -\gamma (\mathbf{m} \times \mathbf{H}_0).
\]  

(2.1)

where \( \gamma \) is the proportionality constant between the angular momentum and the magnetic moment of the dipole, known as the gyromagnetic ratio. If the equation of motion for the magnetic dipole is solved from Equation 2.1, the resulting solution describes the precession of the magnetic moment about the static magnetic field with frequency \( \omega = \gamma H_0 \). Kittel provides a derivation of this solution [1]. This motion of the dipole in a magnetic field is analogous to the precession of a top in a gravitational field. To achieve resonance in this situation, a field perpendicular to \( \mathbf{H}_0 \) must be applied that oscillates at the frequency \( \omega = \gamma H_0 \). This field usually oscillates in the
range of .5-50 GHz in most experiments and practical applications.

The magnetic resonance condition for a collection of dipoles in a magnetic material is more complicated. The local environment of each dipole is such that the field experienced by the dipoles is no longer just the applied field. The sample shape, the quantum mechanical exchange interaction, the dipole interaction, and the crystalline anisotropy energy contribute to the torques acting on each dipole. These diverse torques can be accounted for by an effective field $\mathbf{H}_{\text{eff}}$, and the macroscopic equivalent of Equation 2.1 can be expressed as

$$\frac{\partial \mathbf{M}(r)}{\partial t} = -\gamma (\mathbf{M}(r) \times \mathbf{H}_{\text{eff}}(r)).$$  \hspace{1cm} (2.2)$$

This is the basic equation of motion for ferromagnetic resonance in an insulating material. Thus, in ferromagnetic resonance, the magnetic material will absorb the maximum power when the oscillating perpendicular field is at the frequency $\omega = \gamma H_{\text{eff}}$.

A significant contribution to the effective field is the applied field. This is often called the Zeeman term of the effective field. Another significant contribution to $\mathbf{H}_{\text{eff}}$ is the shape demagnetization field, due to the dipole field of the magnetic moments in the sample which align with the external field. The shape demagnetization field opposes the applied field. Considering only the Zeeman and demagnetization fields, and assuming the sample is ellipsoidal with its principle axes aligned with the axes of a Cartesian coordinate system, the effective field can be written as

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_0 - 4\pi N_j M_j,$$  \hspace{1cm} (2.3)
where $j = x, y, z$. The components of $N$ can be calculated for various geometries, and for a sphere, $N = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$ while for a film in the $xy$-plane, $N = (0, 0, 1)$. Thus, the resonant condition for a thin magnetic film is

$$\omega = \gamma(H_0 - 4\pi M_s)$$

(2.4)

if $H_0$ is applied perpendicular to the film plane and any other material properties besides the shape demagnetization field are ignored.

### 2.2 Spinwaves

A magnetic material consists of many magnetic moments and has energy states known as spinwaves or magnons. In a ferromagnet, the ground state consists of all the magnetic moments lined up parallel to one another. The first excited state consists of one magnetic moment (often called a "spin") flipped anti-parallel to the remaining spins. This localized "flip" is a high energy state, but linear combinations of degenerate states result in spinwaves with a lower overall energy. The lowest energy configuration of the first excited state is a spinwave where all spins precess with equal phase and amplitude. Higher energy configurations of the first excited state consist of variations in the phase and/or amplitude of the moments through the film. These variations require additional energy due to the exchange interaction and dipolar interactions of the precessing moments.

Therefore, the exchange interaction and dynamic dipolar energy must be included with the applied and demagnetization field energies to determine the frequency of a
spinwave of wave vector \( \mathbf{k} \). The exchange energy can be expressed in terms of an effective field, \( H_{ex} \), which is determined by

\[
H_{ex} = D \nabla^2 M. \tag{2.5}
\]

\( D \) is known as the exchange constant. The oscillating dipole moments produce dynamic demagnetization fields oscillating at the spinwave frequency. Propagation effects can be ignored since the sample sizes are usually smaller than the wavelength of the pumping microwaves. The demagnetization fields can be found from Maxwell's equations

\[
\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{h}^d + \nabla \cdot 4\pi \mathbf{M} = 0. \tag{2.6}
\]

Since \( \nabla \times \mathbf{h}^d = 0 \), \( \mathbf{h}^d \) can be expressed as the gradient of a scalar potential, \( \Psi \), such that

\[
\nabla^2 \Psi = 4\pi \nabla \cdot \mathbf{M}. \tag{2.7}
\]

Expanding the magnetization \( \mathbf{M} \) in terms of plane waves results in

\[
\mathbf{M}(\mathbf{r}) = M_0 \hat{\mathbf{z}} + \sum_{\mathbf{k} \neq 0} \mathbf{m}_k e^{i\mathbf{k} \cdot \mathbf{r}}, \tag{2.8}
\]

separating the magnetization into a static part in the \( z \)-direction and small transverse deviations due to spinwave excitations.

The scalar potential \( \Psi \) can be expanded in terms of a spatial Fourier series

\[
\Psi = \sum_k \alpha_k e^{i\mathbf{k} \cdot \mathbf{r}}. \tag{2.9}
\]

Substitution of the expansions for \( \mathbf{M} \) and \( \Psi \) into Equation 2.7 creates the following equality.

\[
- \sum_k \alpha_k k^2 e^{i\mathbf{k} \cdot \mathbf{r}} = \sum_k 4\pi i \mathbf{m}_k \cdot \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}} \tag{2.10}
\]
Equating terms of the same order of \( k \) reveals that

\[
\alpha_k = -4\pi i \frac{m_k \cdot k}{k^2}.
\] (2.11)

So this results in the dynamic demagnetization field expressed as

\[
h^d = \sum_k h_k e^{ik \cdot r} = \sum_k -4\pi \frac{k \cdot m_k}{k^2} e^{ik \cdot r}.
\] (2.12)

The magnitude of the magnetization \( m_k \) is proportional to \( \sin \theta_k \), where \( \theta_k \) is the angle between \( k \) and the \( z \)-axis. The demagnetization energy of a spinwave is proportional to \(-m_k \cdot h_k^z\). Therefore, a \( \sin^2 \theta_k \) term appears in the spinwave dispersion relation. In addition, Equation 2.5 indicates that a \( Dk^2 \) term will enter the dispersion relation due to the exchange interaction. The dispersion relation for these spinwaves was first presented by Suhl [2] and has the form

\[
\left( \frac{\omega}{\gamma} \right)^2 = (H_0 - 4\pi N_z M_s + Dk^2)(H_0 - 4\pi N_z M_s + Dk^2 + 4\pi M_s \sin^2 \theta_k)
\] (2.13)

for a spinwave of wave vector \( k \) travelling at an angle \( \theta_k \) with respect to the \( z \)-axis. The external field is defined as \( H_0 \hat{z} \) and \( 4\pi N_z M_s \) is the shape demagnetization field and is in the \( z \)-direction. To first order in thin magnetic films, \( M_z \approx M_s \).

The \( \sin^2 \theta_k \) term in the dispersion relation indicates the existence of a spinwave band, bounded by the spinwave propagation angles \( \theta_k = 0 \) and \( \theta_k = \frac{\pi}{2} \). Figure 2.1 shows this spinwave band. For a film parallel to the \( xy \)-plane, the uniform precession frequency \( \omega = \gamma(H_0 - 4\pi M_s) \) lies at the bottom of this band, and is non-degenerate with other spinwave modes. In the case of spherical samples, the uniform precession frequency \( \omega = \gamma H_0 \) lies a third of the way down in the spinwave band, and is degenerate with a large number of spinwaves. The uniform precession in a spherical
sample easily couples to degenerate short wavelength spinwaves at the outer edges of the band. Therefore, the high power spinwave dynamics are far more complicated in spheres than thin films. Here, only the dynamics of thin films will be considered. To model the dynamics of the long wavelength spinwaves in magnetic films, it is necessary to understand the spatial forms of these spinwaves.

2.3 Spatial Forms of the Long Wavelength Normal Modes

Spinwaves with wavelengths comparable to the sample dimensions constitute standing waves, or normal modes of the sample. For spheres, these normal modes take the form of the spherical harmonics, while in thin circular films, the normal modes have the form of Bessel functions. At low microwave powers, these long wavelength modes can be excited and observed as microwave absorption peaks at various driving frequencies or applied fields (if the microwave frequency is fixed). These absorption peaks constitute the low power FMR spectrum. If the film thickness is sufficiently small, the standing modes across the film thickness may have a significant exchange energy contribution, shifting each standing wave mode to significantly higher energies. In a magnetic film, exchange energy is required to create variations in the precessing moment amplitudes through the thickness of the film, which determines the minimum energy of the the magneto-exchange branches. These groups of modes are called magneto-exchange branches. Within each exchange branch are the magnetostatic modes, separated in energy due to dynamic dipole interactions. The variations in spin orientation across the film require dipolar energy, and determines the spacing of the magnetostatic modes in each exchange branch. As one might expect, both the
Figure 2.1: The spinwave band for a ferromagnet. The frequencies corresponding to $N_z = 0, 1/3$ and $1$ are the frequencies of the uniform mode $k = 0$ for sample geometries of a needle, a sphere, and a film respectively. From reference [4].
sample geometry and boundary conditions play a significant role in the energy and spatial forms of these modes.

For thick films \( (S \gg 1\mu m) \) the dynamic dipolar interaction energy dominates and the low power ferromagnetic resonance spectrum is comprised of several magnetostatic modes with many nearly degenerate exchange branches buried within them. For very thin films \( (S \ll 1\mu m) \), the magnetostatic modes are nearly degenerate and the exchange branches are widely spaced. In the results presented in this work, the sample thickness was on the order of \( 1 \mu m \), which allows for a low power spectrum of principally magnetostatic modes and clearly identifiable exchange branches.

The boundary conditions at the air-surface and air-substrate interfaces affect the functional form of the magneto-exchange modes. For example, if the magnetic dipoles are strongly pinned at these interfaces, the \( z \)-dependence of the magneto-exchange modes will have the form \( \sin k_z z \), \( k_z = \frac{n\pi}{S} \), for \( n = 1, 2, 3, \ldots \) and where \( z = 0 \) at the air-substrate interface. If \( n \) is even, the magneto-exchange branch has no net dipole moment and will not couple to the uniform driving field. When \( n \) is odd, the corresponding magneto-exchange modes will have a net dipole moment, but the intensity of the higher order magneto-exchange modes will diminish in the magnetic resonance spectrum as \( n \) increases. If there is no pinning at the sample boundaries in the \( z \)-direction, then \( \frac{dnm}{dz} = 0 \) at the film interfaces, and the \( z \)-dependence of the magneto-exchange modes is \( \cos k_z z \), \( k_z = \frac{n\pi}{S} \), \( n = 0, 1, 2, 3, \ldots \). If \( n \neq 0 \), these modes have no net dipole moment, do not couple to the uniform driving dipole field and do not appear in the magnetic resonance spectrum. Asymmetric or partial pinning
results in a linear combination of $\sin k_z z$ and $\cos k_z z$ terms for the functional form of each magneto-exchange mode.

The functional form of the magnetostatic modes is also affected by both the sample geometry and boundary conditions across the plane of the film. Near the edge of a film sample, the demagnetization field is much smaller than it is over the rest of the film. This results in the edge dipole moments being off resonance, and effectively pins them. Since these edge spins are pinned, standing waves form across the plane of a circular film sample with the mathematical form

$$m_{in-plane}(r) \propto J_v(k_f \rho) \cos(\nu \phi) \quad (2.14)$$

where $\rho$ and $\phi$ are the in-plane coordinates, and $J_v$ is the Bessel function of order $\nu$. The in-plane wave vector is given by $k_f = x_{v,s}/a$, where $x_{v,s}$ is the $s^{th}$ zero of $J_v$ and $a$ is the radius of the sample. The domain pattern of some of the lowest order magnetostatic modes is shown in Figure 2.2. Notice that modes associated with $J_0$ have a net dipole moment, while magnetostatic modes where $\nu \neq 0$ do not have a net dipole moment. Since these modes do not appear in the magnetic resonance spectrum when a uniform driving field is used, they are sometimes referred to as the hidden modes. Combining the geometrical and pinning conditions across the film plane and through the film thickness, the mathematical form of the normal modes is

$$m(r) \propto J_v(k_f \rho) \cos(\nu \phi) \begin{Bmatrix} \cos(k_z z) \\ \sin(k_z z) \end{Bmatrix} . \quad (2.15)$$

The experimental FMR spectrum reflects the dispersion relation of these magnetostatic modes. Sparks [3] reported that the dispersion relation for the case of
Figure 2.2: The lowest order direct and lowest order hidden magnetostatic modes for a circular film
uniform magnetization across the sample thickness \((k_z = 0)\) is given by

\[
\omega / \gamma = H_0 - 4\pi M_s + 4\pi M_s \frac{k_f S}{4}
\]

(2.16)

where \(k_f\) is the component of the wave vector across the sample plane. If there some degree of pinning in the sample, resulting in non-zero values of \(k_z\), which appear as higher order exchange branches in the low power spectrum, the dispersion relation is

\[
\omega / \gamma = H_0 - 4\pi M_s + Dk^2 + 4\pi M_s \frac{2k_f S}{\pi^2}.
\]

(2.17)

These equations are often used to determine the sample thickness and pinning conditions.

### 2.4 High Power Resonance Phenomena

In a film, the resonance condition changes slightly as the power is increased, since the increase of the mode amplitude leads to a decrease in \(M_z\). As the power is increased, the mode resonances drift to lower fields. This change in the resonant condition in films as the microwave power changes leads to hysteretic and saw-toothed resonance peaks at intermediate microwave powers, and is called foldover.

An example of foldover in thin film ferromagnetic resonance is shown in Figure 2.3. Consider a film excited by a microwave field at a fixed frequency with the power large enough to observe the foldover effect. The applied field is increased and decreased through the mode resonance. As the field is increased towards resonance, the increase of the mode amplitude shifts the resonance slightly down field. As the field passes resonance, the resonance shifts back up field as \(M_z\) of the sample increases and the
mode amplitude declines. This creates an asymmetric resonance peak which is steeper on the low field side, and broader on the high field side. When the field is reduced, as the field approaches resonance, $M_z$ becomes smaller, forcing the resonance down field. As the field continues to approach resonance, the resonance field continues to shift to a lower value, creating a very broad resonance peak. Once the field passes through resonance, $M_z$ increases and the resonance field becomes higher. This results in the system being well off resonance, and the absorption of the microwave power drops abruptly. In addition to foldover, spinwaves are produced at the expense of the uniform precession through processes known as the 1st and 2nd order Suhl instabilities when the microwave power is increased beyond certain thresholds [2].

In the 1st order Suhl instability, a uniform mode, $k = 0$ spinwave is annihilated and two spinwaves at half the precession frequency and equal and opposite wave vectors are generated. The 1st order Suhl instability is observed in magnetic spheres [5, 6]. Since the uniform precession frequency for a thin film lies at the bottom of the spinwave band, there exist no spinwaves with frequencies half the precession frequency, and the 1st order Suhl instability does not occur in thin films.

The 2nd order Suhl instability involves two uniform spinwaves, $(k = 0)$ being annihilated and two spinwaves with wave vectors $k \neq 0$ and nearly degenerate in frequency with the uniform mode being created through a four wave interaction. The uniform mode is most strongly coupled in this process to spinwaves which are parallel to the bias field, $\theta_k = 0$. In bulk samples, a large number ($\sim 10^{10}$) of short wavelength spinwaves are degenerate with the uniform mode. Therefore, the spinwave dynamics
Figure 2.3: An example of the foldover in ferromagnetic resonance as the microwave power is increased. The power in b) is 12.5 dB higher than that in a). From reference [10].
in bulk samples at high microwave powers are quite complicated. In films, the uniform mode at the bottom of the band is not degenerate with any spinwaves, and is nearly degenerate with only a few long wavelength magnetostatic modes. The uniform mode is not excited in films, due to pinning at the edges of the film, but the few nearly degenerate magnetostatic modes are excited. The mode dynamics in films are much simpler and more easily modelled on a computer for this reason.

Beyond the 2nd order Suhl instability, the magnetic spin precession angle begins to oscillate due to coupling between the magnetostatic modes. Since the precession angle varies, the sample magnetization also varies. This variation is known as an auto-oscillation, and typical frequencies of these oscillations are in the .1-10 MHz range. Changes in the sample magnetization result in proportional changes in the microwave power absorbed by the sample. Therefore, auto-oscillations are observed as oscillations of the absorbed microwave power with the sample at resonance and all experimental parameters fixed.

Auto-oscillations in thin magnetic films were first reported by Wigen et al. [7] and were further analyzed by McMichael and Wigen [8]. An experimental parameter space of applied field (or equivalently, microwave frequency) versus microwave power can be constructed to create an auto-oscillation map, which indicates the experimental conditions which give rise to auto-oscillations. Auto-oscillation maps generally consist of fingerlike regions of auto-oscillation, as seen in Figure 2.4. The fingerlike patterns change when the direction of the applied field sweep is reversed, due to hysteretic effects in the magnetostatic mode resonance condition. The leftward angle
of the fingers is due to the reduction of $M_z$ as the microwave power increases. At lower powers, the hysteresis in the resonance is observed as foldover, or saw-toothed resonance peaks in the spectra. At the "finger tips", the auto-oscillations usually resemble simple sine functions. However, in the higher power regions of the auto-oscillation map, increasingly complex auto-oscillations are observed. Of particular interest to this report are the highly irregular, seemingly random auto-oscillations, known as *chaotic* auto-oscillations. Chaotic behavior will be explained in the next chapter. The following sections describe a numerical model which has accurately predicted the auto-oscillatory behavior in thin YIG films.

### 2.5 Hamiltonian of the Sample

To describe the experimental results of ferromagnetic resonance requires an understanding of the underlying dynamics of the spinwave modes. There exist two ways to determine the dynamics of the modes. One method is to use the functional forms of the modes to solve the torque equation (Equation 2.1). Alternatively, the Hamiltonian can be derived to determine the equations of motion for the modes, and this approach, first introduced by McMichael [8] will be presented here.

The sample Hamiltonian consists of four terms

$$\mathcal{H} = \mathcal{H}_{\text{static}} + \mathcal{H}_{\text{demag}} + \mathcal{H}_{dd} + \mathcal{H}_{\text{pump}},$$

(2.18)

where $\mathcal{H}_{\text{static}}$ results from the sample interaction with the static field, $\mathcal{H}_{\text{demag}}$ arises from the static shape demagnetization field, $\mathcal{H}_{dd}$ accounts for the dynamic demag-
Figure 2.4: Fingers of auto-oscillation in an experimental parameter space of microwave power and applied field. The FMR spectrum of the sample indicating the positions of the magnetostatic modes of the first branch is presented at the bottom. From reference [8].
netization fields from oscillating dipole moments, and $\mathcal{H}_{\text{pump}}$ is due to the pumping field used to excite the sample into resonance. While the $\mathcal{H}_{\text{demag}}$ and $\mathcal{H}_{\text{dd}}$ terms both account for dipole-dipole interactions, $\mathcal{H}_{\text{demag}}$ arises from fields that are essentially static, while $\mathcal{H}_{\text{dd}}$ arises from fields which oscillate with the precession frequency.

For all magnetic materials, there is a crystalline anisotropy contribution to the Hamiltonian. This term reflects the fact that the magnetic moments tend to line up in certain preferred crystalline directions. This work will discuss FMR experiments involving yttrium iron garnet (YIG) samples which have small crystalline anisotropy fields. In YIG, the small anisotropy fields result in a slight shift in the resonance frequency of the magnetostatic modes, and otherwise do not significantly affect the dynamics of the system. For this reason, no anisotropy term is included in the Hamiltonian.

To express the Hamiltonian in terms of the sample magnetization $\mathbf{M}$, it is necessary to introduce the variables $M^+ = M_z + i M_y$ and $M^- = M_z - i M_y$, which are only canonical provided $M_z \approx M_z$. The transformations

$$
M^+ = a\sqrt{2\gamma M_z - \gamma^2 a^*}
$$

$$
M_z = M_z - \gamma a^*
$$

are made to obtain equations of motion of the form

$$
\dot{a} = -i \frac{\partial \mathcal{H}}{\partial a^*}.
$$

The quantity $\gamma$ is the gyromagnetic ratio. Equations 2.19 and 2.20 constitute a classical analogue of the Holstein-Primakoff transformation [9].
Since the variable $a$ is related to the sample magnetization $M$, it therefore has a spatial dependence. The variable can be expanded into orthogonal basis functions

$$ a = \sum_i a_i m_i(r) $$

(2.22)

such that

$$ \frac{1}{V} \int_V m_i^*(r) m_j(r) = \delta_{ij} $$

(2.23)

where $V$ is the sample volume. The variables $a$ and $a^*$ correspond to boson annihilation and creation operators, appropriate to the normal modes of the system. The appropriate basis functions for the normal modes in a circular film are already known from the Bessel functions. Equations 2.21 and 2.22 can be used to determine a coupled set of equations for the mode dynamics, once the Hamiltonian is expressed in terms of $a$.

To evaluate $\mathcal{H}_{\text{static}}$, the first term in the Hamiltonian in terms of $a$, recall that the energy density of a film sample lying in the $xy$-plane in a bias field $\mathbf{H}_0 = H_0 \hat{z}$ has an energy density of $-H_0 M_z$ which transforms to $-H_0 M_z + \gamma H_0 a^* a$. Expanding $a$ into orthogonal basis functions as per Equation 2.22, integrating over the sample volume, and then dividing by the sample volume yields

$$ \mathcal{H}_{\text{static}} = -H_0 M_z + \gamma H_0 \sum_i a_i^* a_i. $$

(2.24)

The next term to evaluate is the one due to the shape demagnetization field, $\mathcal{H}_{\text{demag}}$. The shape demagnetization energy density in terms of the sample magnetization is $2\pi M_s^2$, which transforms to $2\pi M_s^2 - 4\pi M_s \gamma a^* a + 2\pi \gamma^2 a^* a^* a a$. Once again, $a$ is expanded into orthogonal functions and an integration over the sample volume
and subsequent division of the sample volume is performed. The resultant expression is

\[ \mathcal{H}_{\text{demag}} = 2\pi M_s^2 - 4\pi M_s \gamma \sum_i a_i^* a_i + 2\pi \gamma^2 \sum_{ijkl} A_{ijkl} a_j^* a_k^* a_j a_k \]  

(2.25)

where the nonlinear interaction parameter \( A_{ijkl} \) is defined as

\[ A_{ijkl} = \frac{1}{V} \int_V m_i^*(r)m_j^*(r)m_k^*(r)m_l(r). \]  

(2.26)

The dipole interaction term \( \mathcal{H}_{dd} \) is found by introducing the dipole fields \( h_d^+ = h_{d,\alpha} + i h_{d,\beta} \) and \( h_d^- = h_{d,\alpha} - i h_{d,\beta} \). The energy density due to dipole fields is then \( \frac{1}{2}(M^+ h_d^- + M^- h_d^+) \). Expanding the terms in the dipole energy density into a spatial series yields

\[ h_d^+ = \sum_i h_{d,i} m_i(r) \]  

(2.27)

\[ M^+ = \sum_i b_i^* m_i(r) \]  

(2.28)

where the expansion coefficients are related by \( h_{d,i} = D_{ij} b_j \). The matrix \( D \), which relates the dipole fields to the sample magnetization should not be confused with the exchange constant \( D \). If the orthogonal spatial functions are picked so \( D \) is diagonal and real, then through the appropriate substitution and volume normalized integration,

\[ \mathcal{H}_{dd} = \sum_i D_{ii} b_i^* b_i. \]  

(2.29)

A Hamiltonian written in terms of the canonical variables \( a \) and \( a^* \) is desired. Using Equations 2.19 and 2.28 and the orthogonality relation, \( a_i \) and \( b_i \) can be shown to be related by

\[ b_i = \frac{1}{V} \int_V dr \ a(2\gamma M_s - \gamma^2 a a^*)^{1/2} m_i^*(r). \]  

(2.30)
Expanding the radical to first order will produce a quadratic and quartic term in the Hamiltonian. The quartic term can be shown to be small compared to the quartic term arising from the shape demagnetization energy under typical experimental conditions. Therefore, \( a_i \) and \( b_i \) can be linearly related to first order, by

\[ b_i \approx \sqrt{2\gamma M_s} a_i. \] (2.31)

This result indicates that the dipole contribution to the Hamiltonian to first order is

\[ \mathcal{H}_{dd} = 2\gamma M_s \sum_i D_{ii} a_i^* a_i. \] (2.32)

The final term to be evaluated is the microwave pumping field term, \( \mathcal{H}_{\text{pump}} \). The energy density for this contribution is \( M_s \hbar_p \cos(\omega t) \). Since \( M_s = \frac{1}{2}(M^+ + M^-) \), using Equation 2.19 relating \( M^+ \) and \( M^- \) to the \( a \), one finds that the pumping field contribution to the Hamiltonian is

\[ \mathcal{H}_{\text{pump}} = \frac{1}{2} \hbar_p (a^* + a) \sqrt{2\gamma M_s - \gamma^2 a^* a} \cos(\omega t). \] (2.33)

Expanding the radical, substituting the spatial expression for \( a \), and integrating over the sample volume results, to first order, in the expression

\[ \mathcal{H}_{\text{pump}} = \hbar \sqrt{\frac{\gamma M_s}{2}} \sum_i (a_i^* I_i^* + a_i I_i) \cos(\omega t), \] (2.34)

with the parameter \( I_i \) defined to be

\[ I_i \equiv \frac{1}{V} \int_V m_i(r)dr. \] (2.35)

Cubic and higher order terms in the pumping field Hamiltonian contribution are found to be negligible near resonance where the experiments are performed.
By combining Equations 2.24, 2.25, 2.32, and 2.34 together, the complete Hamiltonian of the sample takes the form

\[ H = -H_0 M + 2\pi M^2 + \gamma \sum_i (H_0 - 4\pi M + 2M_D) a_i^* a_i + \gamma \sum_{ijkl} A_{ijkl} a_i^* a_j^* a_k a_l + h_p \frac{\gamma M}{2} \cos(\omega t) \sum_i (a_i^* I_i^* + a_i I_i). \]  

(2.36)

This includes only the terms in the Hamiltonian found to be significant under typical experimental conditions. For a full accounting of all the higher order terms in the Hamiltonian, consult Reference [10].

2.6 Equations of Motion of the Normal Modes

Once the sample Hamiltonian is established, the equations of motion for the magneto-exchange modes are found from the application of Equation 2.21. This results in the equation

\[ \frac{d a_j}{d t} = -i\gamma (H_0 - 4\pi M + D_{ij}) a_j - 4\pi \gamma^2 \sum_{klm} A_{ijkl} a_k^* a_l a_m - i h_p \sqrt{\frac{\gamma M}{2}} I_j \cos(\omega t). \]  

(2.37)

At this point, relaxation processes in the sample have not been accounted for. Sample imperfections as well as a variety of crystalline interactions drain energy from the precession of the spinwave modes, requiring a continuous input of energy to prevent the modes from damping out entirely. To account for the damping within the sample, the Landau-Lifshitz phenomenological damping mechanism

\[ \left( \frac{dM}{dt} \right)_{damp} = -\alpha M \times (M \times H) \]  

(2.38)
is employed, where $\alpha$ describes the damping strength. For the case $\mathbf{H} = (H_0 - 4\pi M_s)\hat{z}$, the $z$-component of the equation becomes

$$
\left( \frac{dM_z}{dt} \right)_{\text{damp}} = \alpha(H_0 - 4\pi M_s)(M_z^2 + M_y^2).
$$

(2.39)

This result is then transformed into the form of the canonical variables $a$ and $a^*$ to yield

$$
g \frac{daa^*}{dt} = -\alpha(H_0 - 4\pi (M_s - \gamma aa^*))aa^*(2\gamma M_s - \gamma^2 aa^*). \quad (2.40)
$$

By defining $\Gamma = \alpha(H_0 - 4\pi M_s)M_s$ and ignoring higher order terms in the expansion which are negligible under typical experimental conditions, the above expression simplifies to

$$
\frac{da}{dt} = -\gamma \Gamma a. \quad (2.41)
$$

The quantity $\Gamma$ is extracted experimentally from the half-width at half-maximum of the magneto-exchange mode resonance. Since the resonant field and frequency of each mode are related, it becomes convenient to define a resonant field for mode $j$ by

$$
H_{j\text{res}} = \frac{\omega}{\gamma} + 4\pi M_s - 2M_s D_{jj}. \quad (2.42)
$$

Inserting this definition for the resonant field and the damping considerations into Equation 2.21 produces

$$
\frac{da_j}{dt} = -i\gamma(H_0 - H_{j\text{res}} - i\Gamma)a_j - 4\pi\gamma^2 \sum_{klm} A_{jklm}a^*_ka_la_m - ih_p\sqrt{\frac{\gamma M_s}{2}} I^z \cos(\omega t). \quad (2.43)
$$

When modeling the behavior of the system numerically, a transformation to a set of slowly varying variables is made. Defining

$$
c_j = \sqrt{\frac{2\gamma}{M_s}} a_j e^{i\omega t} \quad (2.44)
$$
and ignoring terms which vary as $2\omega t$, the equation of motion for mode $j$ becomes

$$\frac{dc_j}{dt} = -i\gamma(H_0 - H_j^{res} - i\Gamma)c_j - 2\pi i\gamma M_s \sum_{klm} A_{jklm} c_k^* c_l c_m - \frac{1}{2} i\gamma h_p I_j^*.$$ (2.45)

This is the equation of motion for each mode $j$ used to model the experimental results.

The FMR signal in the model corresponding to the FMR signal measured in experiments is constructed from the evolution of the mode amplitudes

$$S(t) = -\sum I_i^* Im(c_i),$$ (2.46)

where $I_i^*$ is the coupling strength of mode $i$ to the uniform microwave field.

It is important to note the nonlinear, forcing, and dissipative terms in Equation 2.45. It is the nonlinearity and forced/dissipative character of the underlying equations of motion for spinwave dynamics which give rise to irregular, unpredictable, and seemingly random chaotic behavior.
CHAPTER III

Chaotic Phenomena and Control

3.1 Chaotic Phenomena

3.1.1 Characteristics of Chaos

While the French mathematician Poincaré realized that systems governed by Hamilton’s equations could display chaotic motion at the turn of the century, the first investigation of chaotic phenomena is often attributed to Lorenz in 1963 [11]. Lorenz attempted to model the earth’s atmosphere by simplifying equations that described a fluid filled torus in which the bottom half was heated and the top half cooled. At small temperature differences, uniform convection currents formed. However, at higher temperature differences, rapid fluid motion resulted in uneven heating of the liquid, which produced irregular and unstable convection currents. If slightly different initial conditions were used in the computer simulation, the fluid velocity and temperature gradients deviated substantially from the previous case in a short period of time. This sensitivity to initial conditions suggested to Lorenz that long range weather forecasts were impossible without impractically precise atmospheric measurements.
Whether these simulations have any significant relevance to actual weather patterns is debatable, but equations he used in this study have become known as the Lorenz equations, and have been widely studied to learn insights into chaotic phenomena.

While no mathematically precise definition exists of chaotic behavior in dynamical systems, it is generally agreed upon that dynamical systems which possess the following characteristics are chaotic.

- They are completely deterministic.
- They are sensitive to initial conditions.
- The dynamics are aperiodic.

It is important to realize that chaotic dynamics are not random but deterministic. While chaotic systems may appear random, there is no “randomness” or random variable in the underlying dynamical equations. While noise and thermal fluctuations can and do influence chaotic systems, in the absence of any random influence, the chaotic system would still possess a “random-looking” unpredictable character.

In addition to being deterministic, the chaotic systems discussed in this report require forcing and dissipation mechanisms. In the Lorenz equations, the forcing corresponds to the heating of the bottom half of the torus while the viscosity of the fluid provides the dissipation.

While not obvious from the above characteristics, the equation of motion of chaotic
system must be nonlinear. Nonlinear equations of motion can be written as

\[ \dot{x} = F(x) \]  

(3.1)

where \( x = (c_1, c_2, \ldots c_d) \) and \( F \) is a nonlinear function of the \( \{c_i\} \). Since Equation 3.1 is continuous, it is referred to as a "flow". Discontinuous equations, called "maps" also display chaos. With linear systems, where \( F \) is a linear function, the dynamics eventually become stable. In a chaotic system, it is the nonlinearity of the equations of motion which creates the perpetual instability of the system.

Chaos has been observed in a wide variety of physical systems, as well as chemical and biological systems. Both Schuster [12] and Strogatz [13] provide texts on chaotic dynamics.

### 3.1.2 Attractors

From an experimental point of view, chaotic dynamics present a challenge. How can knowledge about the underlying physics of the system be extracted from the irregular and unpredictable chaotic motion of the system variables? One way to gain insight into the physical system is to construct a phase space consisting of all the system variables and record the values of each system variable over time. A time dependent vector of the system in phase space, called the system trajectory, can be created. The points in phase space explored by the trajectory can give insights into the physical system.
To illustrate this, consider the Lorenz equations

$$
\begin{align*}
\dot{X} &= \sigma(Y - X) \\
\dot{Y} &= rX - Y - XZ \\
\dot{Z} &= XY - bZ
\end{align*}
$$

(3.2)

$r = 28$, $\sigma = 10$, $b = 8/3$

The $r$, $\sigma$, and $b$ parameter values are known to produce chaotic dynamics in the Lorenz equations. In Figure 3.1a, the $X$, $Y$, and $Z$ variables plotted over time do not appear to indicate any invariant properties of the system. However, the trajectory of the Lorenz equations in phase space shown in Figure 3.1b reveals an underlying structure in phase space, known as the attractor. It is important to realize that while the values of the variables $X$, $Y$, and $Z$ are sensitive to initial conditions, the attractor is not affected by the choice of initial conditions. To be more precise, a dynamical system's attractor consists of a bounded set of points in phase space that the dynamical system trajectory evolves toward over time. The basin of attraction of an attractor is the set of initial starting points in phase space from which system trajectories will evolve toward the attractor.

For dissipative systems, the volume of the attractor is zero. This can be illustrated by considering an arbitrary closed surface $S(t)$ and enclosing volume $V(t)$ in phase space. The points on $S$ can be thought of as initial conditions for the system trajectories. As the surface $S$ evolves into a new surface $S(t + dt)$, the new volume
Figure 3.1: The plots of $X$, $Y$, and $Z$ versus time appear erratic and irregular. When a trajectory consisting of these variables is plotted in phase space, an underlying structure, independent of the initial conditions, is apparent.
\( V(t + dt) \) can be obtained from

\[
V(t + dt) = V(t) + \int_S (\mathbf{f} \cdot \mathbf{n} dt) dA
\]

(3.3)

where \( \mathbf{n} \) is the outward normal on \( S \) and \( \mathbf{f} \) is the instantaneous velocity of points.

The volume rate of change is then given by

\[
\dot{V} = \frac{V(t + dt) - V(t)}{dt} = \int_S \mathbf{f} \cdot \mathbf{n} dA.
\]

(3.4)

Using the divergence theorem, the integral is rewritten as

\[
\dot{V} = \int_V \nabla \cdot \mathbf{f} dV.
\]

(3.5)

For the Lorenz system,

\[
\nabla \cdot \mathbf{f} = \frac{\partial}{\partial x} [\sigma (y - x)] + \frac{\partial}{\partial y} [rx - y - xz] + \frac{\partial}{\partial z} [xz - y - bz]
\]

(3.6)

\[
= -\sigma - 1 - b < 1.
\]

(3.7)

The constant divergence leads to the differential equation for \( V \)

\[
\dot{V} = -(\sigma + 1 + b)V
\]

(3.8)

which implies that the volume of phase space shrinks exponentially fast. In all dissipative systems, a collection of initial points within the basin of attraction shrinks to a bounded set of points with zero volume.

### 3.1.3 Attractor Reconstruction with a Single Variable

In many experimental situations, it is very difficult or even impossible to measure all of the system variables. Thus, the system attractor in phase space cannot be
constructed. This would appear to prohibit an experimentalist from reconstructing the system attractor from the limited data. However, if only one variable of the system $x(t)$ is measured and a vector $X(t)$ of dimension $d$ is constructed such that

$$X(t) = [x(t), x(t + \tau), x(t + 2\tau), ... x(t + (d - 1)\tau)]. \quad (3.9)$$

This trajectory explores a set of points topologically equivalent to the system attractor. This surprising result is usually attributed to Takens [14] and is very important to the field of nonlinear science, allowing chaotic attractor reconstruction in systems where all the system variables cannot be measured.

Some care must be taken in choosing $d$ and $\tau$. The appropriate choice of $d$ depends on the dimension of the attractor, $d_A$. If $d > 2d_A$, then Takens suggests the attractor in time delay space is smoothly related to the attractor in phase space. The integer dimension required to fully capture the attractor topology is often referred to as the embedding dimension. While any value of $T$ can be chosen to faithfully reconstruct the attractor, there is little gained by choosing $\tau$ such that $x(t) \approx x(t + \tau)$. For practical purposes, this choice of $\tau$ has not provided two independent coordinates. A very large $\tau$ will make $x(t)$ and $x(t + \tau)$ almost totally unrelated to each other, and the resultant trajectory will consist of unrelated coordinates, limiting the usefulness of this technique. In practice, choosing a useful value of $\tau$ requires understanding the relevant time scales of the dynamics and intuition.

An example of time delay reconstruction for the Lorenz attractor is presented in Figure 3.2. Abarbanel et al. [15] provide an extensive review of analyzing observed chaotic data.
Figure 3.2: The variable $X$ in the Lorenz equations plotted versus time and plotted in three-dimensional time delay space, with the time delay $\tau$ equal to 0.125
3.1.4 Divergence of Nearby Trajectories

To illustrate the sensitive dependence to initial conditions in chaotic dynamics, consider two numerical solutions to the Lorenz equations using slightly different initial conditions, shown in Figure 3.3. The two trajectories are seen to diverge as time progresses.

The exponential growth of the distance $\delta$ between adjacent initial starting points can be expressed as

$$ ||\delta(t)|| \sim ||\delta_0||e^{\lambda t}. $$

(3.10)

If $\lambda$ is negative, this indicates that trajectories from nearby points on the attractor converge. If $\lambda$ is positive, that means close initial starting points lead to divergent behavior, and therefore, the system is sensitive to initial conditions. The value of $\lambda$ is often referred to as the Lyapunov exponent of the system.

Actually, this is a simplified notion of the Lyapunov exponents. There exist $d$ Lyapunov exponents for a $d$ dimensional system. A $d$ dimensional sphere of initial conditions will be distorted into an ellipsoid of zero volume as the system evolves for at least one $\lambda > 0$. If $\delta_k$ is the distance between two initial starting points, where $k = 1, 2, \ldots d$ is the length of the $k^{th}$ principal axis of the ellipsoid, then the Lyapunov exponents of the system are given by

$$ \delta_k(t) = \delta_k(0)e^{\lambda_k t}. $$

(3.11)

Note, the sum of the Lyapunov exponents must be less than zero for the ellipsoid volume to approach zero. In actuality, the distortion of the $d$ dimensional sphere is
Figure 3.3: Sensitive dependence to initial conditions in the Lorenz equations. The nearly identical initial conditions in the bottom right hand corner noticeably spread apart in the upper left hand corner after $t=2.5$. In this example, the parameter values for $r$, $\sigma$, and $b$ were 28, 10, and $8/3$ respectively.
not uniform over the attractor. Therefore, $\lambda_k$ represent the average change of the ellipsoid axis as it traverses the attractor.

### 3.1.5 Periodic Orbits

In a chaotic system, there is no periodicity in the system and the system trajectory never returns to its initial starting position. In a periodic system, the trajectory will eventually return to its starting point. A simple example is a frictionless pendulum. If the variables of position and momentum are plotted in phase space, the result would be an elliptical trajectory that repeats itself after one period of the pendulum oscillation. Alternatively, just the position $x(t)$ of the pendulum bob can be measured and the two dimensional vector $\mathbf{X}(t) = [x(t), x(t+\tau)]$ can be plotted in a two-dimensional time delay space. The resulting elliptical orbit is shown in Figure 3.4. Notice that this two-dimensional space does not satisfy $d > 2d_A$ for the one dimensional elliptical attractor. But since it is known that a pendulum is uniquely described by two variables, this two dimensional time delay space is sufficient.

The dynamics of the attractor in Figure 3.4 is sometimes referred to as a period-$1$ oscillation, which consists of a fundamental period of a system and resembles one "loop" in time delay space. A period-$2$ oscillation period is twice the fundamental period of the system, and the attractor often resembles two loops in time delayed space. Extending this idea, a periodic oscillation can be referred to as a period-$n$ oscillation which is $n$ fundamental periods long and the corresponding system attractor consists of $n$ loops.
Figure 3.4: Time delay plot of period-1 oscillation, $x(t) = \sin(2\pi ft)$, where $f = 1$, $\tau = 2(1/f)$. 
Figure 3.5: Time delay plot of period-2 oscillation, \( x(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_2 t) \), where \( f_1 = 1 \), \( f_2 = .5 \), and \( \tau = .2(1/f_2) \).
3.1.6 Poincaré Maps

Poincaré maps are useful for examining periodic orbits and the instabilities in a chaotic system. Consider a \( d \) dimensional system \( \dot{x} = f(x) \). A Poincaré map is constructed by choosing a \( d - 1 \) surface of section \( S \) in a \( d \) dimensional space. This surface \( S \) is required to be transverse to the flow such that all trajectories flow through it, not parallel to it. A Poincaré map is a mapping of the points on \( S \) created by each intersection of the system trajectory. If \( x_k \in S \) indicates the \( k^{th} \) intersection, the Poincaré map is defined by

\[
x_{k+1} = P(x_k).
\]  

(3.12)

The point \( x^* \) is called a fixed point of \( P \) if \( P(x^*) = x^* \). If some point \( x_i \) is eventually mapped back to \( x_i \) by \( P \), the system trajectory for \( \dot{x} = f(x) \) is said to be a closed orbit. For chaotic systems, the surface of section consists of a large scattering of points within a finite area.

3.1.7 Stability of Orbits

The stability of a fixed point \( x^* \) on a Poincaré section can be determined by considering a small perturbation \( v_0 \) such that \( x^* + v_0 \) is in \( S \). Then, after the first return to \( S \),

\[
x^* + v_1 = P(x^* + v_0)
\]

(3.13)

\[
= P(x^*) + [DP(x^*)]v_0 + O(||v_0||^2)
\]

(3.14)
where $\mathbf{D}\mathbf{P}(\mathbf{x})$ is a matrix called the linearized Poincaré map at $\mathbf{x}^*$. Since $\mathbf{x}^* = \mathbf{P}(\mathbf{x}^*)$, Equation 3.14 becomes

$$v_1 = [\mathbf{D}\mathbf{P}(\mathbf{x}^*)]v_0$$

(3.15)

neglecting the small $O(||v_0||^2)$ term. The orbit is linearly stable if and only if the eigenvalues of $\mathbf{D}\mathbf{P}(\mathbf{x})$, $|\lambda_j| < 1$ for $j = 1, \ldots, d - 1$. In a chaotic system, the linearized Poincaré map about a fixed point always contains at least one positive eigenvalue. This is a consequence of the chaotic system's sensitivity to initial conditions. Therefore, a chaotic attractor consists of a set of unstable periodic orbits. Unstable periodic orbits are sometimes referred to as saddle orbits, since the fixed points of these orbits are saddle points.

Lathrop and Kosterlich [16] described a general procedure to extract the unstable orbits from chaotic data. The attractor is reconstructed from the data by time delayed reconstruction. The time series of the scalar variable $\{z_i\}_{i=1}^n$ and delay time $\tau$ is used to determine the set of points $\mathbf{X}_i = [z_i, z_{i+\tau}, \ldots, z_{i+(d-1)\tau}]$ on the attractor, where $d$ is the embedding dimension. Let $\epsilon > 0$ and consider the point $\mathbf{X}_i$ on the reconstructed attractor. The subsequent points $\mathbf{X}_{i+1}, \mathbf{X}_{i+2}, \ldots$ on the attractor are followed until the smallest index $k$ is found such that $||\mathbf{X}_k - \mathbf{X}_i|| < \epsilon$. This trajectory is referred to as an $(m, \epsilon)$ recurrent point for $m = k - i$. This method for extracting periodic orbits from a reconstructed attractor is sometimes called the method of close approaches. Lathrop and Kosterlich were able to extract unstable orbits from a reconstructed Belousov-Zhabotinski chemical reaction attractor and use the unstable orbits to extract further information about the nonlinear dynamics of this system.
3.2 Controlling Chaos

3.2.1 Introduction

The irregular and unstable dynamics of chaos would seem to be impossible to control without dramatically altering the system. However, chaos has been controlled in a number of physical systems \([17, 18, 19, 20, 21, 22, 23, 24]\) by applying small perturbations to a system parameter. Controlling chaos in the nonlinear dynamics literature has come to mean creating a desired outcome of a chaotic system through small parameter perturbations, generated in some way from the chaotic system itself. The next two sections will focus on two desired outcomes, stabilization of unstable orbits and the synchronization of chaotic signals.

3.2.2 Stabilization of Unstable Orbits

In 1990, Ott, Grebogi and Yorke (OGY) proposed a method with which small, discrete, time dependent perturbations to an accessible system parameter could produce periodic behavior of an otherwise chaotic system \([25]\). This method hinged upon the fact that a chaotic attractor typically has an infinite number of unstable periodic orbits embedded within its attractor and that parametric perturbations can stabilize some of these unstable periodic orbits.

The method OGY put forth is as follows. Using delay coordinates, a scalar variable
with delay $T$ can be transformed into a delay coordinate vector

$$X(t) = [z(t), z(t - T), z(t - 2T), \ldots, z(t - MT)],$$

(3.16)

where $M$ is chosen to accurately capture the degrees of freedom of the system. This continually evolving vector occasionally pierces a $M - 1$ dimensional surface of section in time delay space. For simplicity, a two dimensional surface of section will be considered, although the technique can be extended to higher dimensions. The points $\xi_1, \xi_2, \ldots, \xi_n$ indicate points on the surface of section corresponding to the $n^{th}$ piercing of the surface by the vector $X(t)$. An unstable fixed point corresponding to an unstable orbit is isolated on the surface of section. About the unstable fixed point, the experimentally determined stable and unstable eigenvalues $\lambda_s$ and $\lambda_u$ ($|\lambda_u| > 1 > |\lambda_s|$) are found. The vectors $e_s$ and $e_u$ are the experimentally determined unit vectors in the stable and unstable directions. If $\xi_F = 0$ is the unstable fixed point when the perturbation $p$ is equal to zero, when $p$ is held constant at some value $\bar{p}$, $\xi_F$ should shift to a nearby point $\xi_F(\bar{p})$. For small $\bar{p}$, the vector $g \equiv \frac{\partial \xi_F(p)}{\partial p}|_{p=0} \approx \bar{p}^{-1} \xi_F(\bar{p})$. So about the unstable fixed point $\xi_F(0) = 0$, a linear approximation can be made for the map

$$\xi_{n+1} - \xi_F(p) \approx M \cdot [\xi_n - \xi_F(p)]$$

(3.17)

where $M$ is a $2 \times 2$ matrix. Using $\xi_F(p) \equiv pg$, the $n + 1^{th}$ piercing of the surface can be predicted from the $n^{th}$ piercing from the relation

$$\xi_{n+1} \approx p_n g + [\lambda_u e_u f_u + \lambda_s e_s f_s] \cdot [\xi_n - p_n g].$$

(3.18)

In the above equation, $f_u$ and $f_s$ are basis vectors defined by $f_s \cdot e_s = f_u \cdot e_u = 1$, $f_s \cdot e_u =
\( \mathbf{f}_u \cdot \mathbf{e}_\ast = 0 \). The location of the fixed point is written as \( p_n \mathbf{g} \) to account for the fact that the perturbation \( p \) is adjusted at each piercing of the surface.

If the perturbation is chosen so \( \dot{\xi}_{n+1} \) falls on the stable manifold of \( \dot{\xi}_f(0) \), the perturbation can be set to zero and the system will approach the fixed point \( \dot{\xi}_f \) at the geometrical rate \( \lambda_e \). To accomplish this, \( p_n \) is chosen so that \( \mathbf{f}_u \cdot \dot{\xi}_{n+1} = 0 \). So for sufficiently small \( \dot{\xi}_n \), Equation 3.18 can be dotted with \( \mathbf{f}_u \) to obtain

\[
p_n = \lambda_u (\lambda_u - 1)^{-1} \frac{(\dot{\xi}_n \cdot \mathbf{f}_u)}{\mathbf{g} \cdot \mathbf{f}_u}.
\tag{3.19}
\]

This perturbation is only applied when the system pierces the surface of section in the neighborhood of the desired unstable fixed point. The size of the perturbation is allowed to vary in the range \( p_\ast > p > -p_\ast \). If Equation 3.19 results in a perturbation such that \( |p| > p_\ast \), the perturbation is set to zero. In addition, under typical experimental conditions, noise and uncertainties will require constant corrections to the system to maintain the stabilized orbit after the system falls onto the stable manifold.

The OGY method was soon implemented by Ditto, Rauseo, and Spano (DRS) to control the chaotic vibrations of a magnetoelastic ribbon [18]. The Young's modulus of the magnetoelastic ribbon used had an unusually high sensitivity to external magnetic fields. The ribbon was clamped at one end and suspended vertically. A configuration of Helmholtz coils created a vertical magnetic field of the form \( H = H_{dc} + H_{ac} \cos 2\pi ft \) where \( f \), the driving frequency was typically about 1 Hz. The d.c. and a.c. fields were approximately 0.1 Oe and 2.0 Oe respectively. From a plot of the deviation of the ribbon versus the deviation at a time \( T \) later, where \( T \) was the driving period of the ribbon \( (T = 1/f) \), a surface of section of the system dynamics was constructed.
The surface of section readily allowed for the unstable fixed point of an oscillation period equal to the a.c. driving period $T$, since any oscillation of period $T$ will have the same vertical deviation at time $T$ later. Similarly, oscillation periods of $2T$ and $4T$ can be extracted by a similar plot of the ribbon deviation versus the deviation at times $2T$ and $4T$ later.

Once the unstable fixed point was established, the unstable and stable manifolds about the unstable fixed point could be characterized. Once this was established, the d.c. magnetic field was perturbed according to the OGY method, and the ribbon oscillated periodically. A subsequent modification of the OGY technique was found to stabilize periodic orbits at kilohertz frequencies [19, 20, 21].

A drawback of the OGY technique is that identifying the proximity of the system to the unstable fixed point and computing the correct perturbation make it impractical for controlling dynamics involving time scales less than a few microseconds. However, Pyragas [26] proposed two continuous methods of chaos control which required less extensive calculations, and thus, were more amiable to the control of high frequency dynamics.

Pyragas considered a dynamical system described by ordinary differential equations which have an input available for an external force. The equations of motion for the system can be written as

\[
\begin{align*}
\frac{dy}{dt} &= P(x, y) + F(t) \\
\frac{dx}{dt} &= Q(x, y)
\end{align*}
\]  

(3.20)

where $y$ is the output variable and the vector $x$ describes remaining variables which
are not available to control or are not of interest. The input force $F(t)$ is assumed to affect only the output variable, and the functional form of $P$ and $Q$ can remain unknown. A function $y_i(t)$ is generated which has the same periodicity and waveform of an unstable periodic orbit in the system’s chaotic attractor. The difference $D(t)$ between the function $y_i(t)$ and the output of the system $y(t)$ is used to generate the control force $F(t)$:

$$F(t) = K[y_i(t) - y(t)] = KD(t)$$ (3.22)

The variable $K$ is an experimentally adjusted experimental weight of the perturbation. In addition to this external force control method, Pyragas also proposed a method of delayed feedback control, in which no external forcing agent exists. Instead, an unstable periodic orbit of period $\tau$ is targeted for stabilization by the external forcing

$$F(t) = K[y(t - \tau) - y(t)] = KD(t).$$ (3.23)

Notice that if $y(t - \tau) = y(t)$, the perturbation is zero and thus, the forcing $F(t)$ does not change the solution of the system. Once the system is stabilized onto an orbit of period $\tau$, the perturbation approaches zero. Both methods put forth by Pyragas introduce a negative feedback into the system with the proper choice of $K$. The control methods proposed by Pyragas in experiments involve simple differential circuitry, and therefore provide the opportunity to control systems with faster dynamics than those controlled by the OGY method. Experimentally, the time delay control method of Pyragas has been found to control chaotic systems at frequencies as high as 10 MHz [22, 23].
3.2.3 Synchronization of Chaos

It would appear as if synchronizing chaotic systems would be a hopeless task, since two systems with nearly identical starting points rapidly diverge. Establishing two independent chaotic systems in a laboratory to be synchronized without any coupling appears impossible, since experimental noise would create a divergence of the systems. However, it has been shown that two chaotic systems will synchronize with appropriate coupling. In many cases, this coupling may be quite weak.

The ability to synchronize coupled chaotic systems was first demonstrated by Carroll and Pecora [17, 27]. Two chaotic systems were coupled so that the behavior of the first system affects the behavior of the second, but not vice versa. The first system was referred to as the drive and the second the response. The variables of the drive were further subdivided into those that drive the response, and those that do not. This can be represented as an autonomous $n$-dimensional dynamical system

$$\dot{u} = f(u)$$  \hspace{1cm} (3.24)

divided into subsystems $u = (v, w)$, where

$$\dot{v} = g(v, w), \quad \dot{w} = h(v, w).$$  \hspace{1cm} (3.25)

Now consider an identical response system $u'$ subdivided similarly into two subsystems $u' = (v', w')$, where $v'$ in the response system is substituted by $v$ in the drive system. The following three subsystems emerge.

$$\dot{v} = g(v, w), \quad \dot{w} = h(v, w), \quad \dot{w}' = h(v, w').$$  \hspace{1cm} (3.26)
The difference $\Delta w = w' - w$ will approach zero if the Lyapunov exponents of the $w$ subsystem are all negative.

While this method has proven to be successful in experimental systems [17, 27] and has been shown to have potential applications in secure communications [28], it does present obstacles in some experimental applications. For example, any chaotic system that is not well understood analytically will create difficulties in correctly identifying the appropriate drive and response variables. Also, some experimental systems may not be decomposed into systems in a neat or obvious way. For these reasons, Pecora and Carroll’s scheme cannot be used to synchronize some chaotic systems.

A synchronization scheme that circumvents these problems was put forth by Lai and Grebogi [29]. This scheme is based on the OGY method of stabilizing unstable periodic orbits. Lai and Grebogi extended the OGY method to stabilize a chaotic trajectory of one system around a chaotic trajectory of another to achieve the synchronization of both systems. This method does not depend on the knowledge of the underlying equations of motion. It also does not involve any system decomposition. Consider two systems, A and B, which have output variable $x$ and $y$ respectively. It is assumed that the chaotic systems can be described by two-dimensional Poincaré map on a surface of section

$$x_{n+1} = F(x_n, p_0), \quad y_{n+1} = F(y_n, p),$$

(3.27)

where $p_0$ for system A is a fixed parameter value and $p$ for system B is externally controllable, but does not vary significantly from $p_0$. The linearized dynamics in the
neighborhood of the target trajectory $x_n$ is
\[ y_{n+1} - x_{n+1}(p_0) = J[y_n - x_n(p_0)] + V(\delta p_n), \]  \hspace{1cm} (3.28)
where $J$ is the $2 \times 2$ Jacobian matrix and $V$ is a two-dimensional column vector.

\[ J = D_y F(y, p)|_{y=x, p=p_0}, \quad V = D_p F(y, p)|_{y=x, p=p_0} \] \hspace{1cm} (3.29)

To stabilize $y_n$ around $x_n$, the next iteration of $y_n$ is required to fall on the stable direction at $x_{n+1}(p_0)$, which means
\[ [y_{n+1} - x_{n+1}(p_0)] \cdot f_u(n+1) = 0. \] \hspace{1cm} (3.30)

The vector $f_u(n)$ satisfies the condition $f_u(n) \cdot e_u(n) = 1$ where $e_u(n)$ is the stable direction at $x_n$.

Substitution of Equation 3.28 into Equation 3.30 yields the following expression for the parameter perturbations.
\[ \delta p_n = \frac{\{J \cdot [y_n - x_n(p_0)]\} \cdot f_u(n+1)}{-V \cdot f_u(n+1)}. \] \hspace{1cm} (3.31)

This method is not without its difficulties. Extrapolation of both $J$ and $V$ from a long chaotic time series may be difficult. The time required to calculate the perturbation may not allow high frequency dynamics to be synchronized. A $2 \times 2$ map may not accurately reflect the behavior of high dimensional systems.

A simplification of the Lai and Grebogi method has proven successful experimentally. Newell and collaborators [24] synchronized a pair of diode resonator circuits through control parameter perturbations governed by the expression
\[ \delta p(t) = \alpha[V_M(t) - V_S(t)]. \] \hspace{1cm} (3.32)
The variable $V_s$ refers to a chaotic slave signal which is induced to follow the trajectory of $V_M$, the master signal. The constant $\alpha$ is empirically determined. The control perturbation is applied to an experimental parameter of the slave system. For the diode resonator experiment, the a.c. driving voltage of the slave resonator was the parameter chosen to be perturbed. The a.c. driving voltage was not perturbed by more than 5% of its original value to maintain synchronization, indicating that the systems were only weakly coupled. Pyragas has demonstrated that this synchronization scheme also works in synchronizing chaotic systems to their prerecorded output [30]. In this situation, the prerecorded output is the master signal while the experimental system in real time becomes the slave system. A system parameter is perturbed to allow the experimental slave system to synchronize to its prerecorded trajectory. Chaotic synchronization in this manner has been demonstrated in experiments involving circuits [31, 32]. Again, the synchronized systems in these experiments were weakly coupled.

While admittedly less sophisticated than Lai and Grebogi's synchronization method, parameter perturbations determined by Equation 3.32 allow more experimental freedom. It does not involve extensive analysis of a long chaotic trajectory. Since the control perturbation is easier to calculate, it allows for the synchronization of higher speed dynamics.
3.2.4 Control of Chaotic FMR Dynamics

Since chaotic dynamics have been observed in FMR for over a decade [6, 7, 33, 34], FMR was recognized as a promising experimental system in which to attempt control the chaotic dynamics. The first experimental attempts to control chaotic oscillations involved sinusoidal external perturbations to the applied field [35, 36]. These experiments demonstrated the ability to suppress chaotic behavior through external modulations, converting chaotic oscillations to periodic oscillations with periods equal to simple integer ratios of the external modulation period. A modification of the OGY method has also led to the control of chaotic oscillations in YIG spheres [21]. Recently, a weak secondary driving of a magnetostatic mode of a thin film in resonance has led to a number of interesting effects [37]. This includes an increase in the 2nd order Suhl instability threshold, increased and decreased auto-oscillation thresholds, and extra frequency components in the auto-oscillations. In addition, the weak secondary driving can bring the film out of chaos into a periodic state.

Thin YIG films in the perpendicular resonance condition appear to be an ideal experimental system to apply different control techniques since the 1st order Suhl instability is eliminated and only a few nondegenerate magnetostatic modes are coupled through dipole-dipole forces. Knowing the spatial forms of the magnetostatic modes and the coupling between them has allowed for predictions of the onset behavior of auto-oscillations to be accurate to about 15% of the observed values [8]. In spherical samples, several degenerate spinwave modes are available to couple to the uniform precession, and the dynamics involves a large number of interacting modes. Only
qualitative models exist for auto-oscillatory behavior in spheres [38].

The next chapter describes the experimental set up used to further investigate the control of chaotic behavior in thin film FMR.
CHAPTER IV

Experimental Motivation and Set Up

4.1 The FMR Spectrometer

The experiments reported here were carried out on a microwave structure FMR spectrometer. The static magnetic field was supplied by a 4" Varian electromagnet powered by a Varian V29-1 magnet power supply. Both the magnet and its power supply were temperature regulated through water cooling. The microwave excitation field from a Hewlett-Packard 8350A sweep oscillator with a 86222B microwave plug-in module was directed down a coaxial waveguide cable to a microwave structure, onto which a circular YIG film was attached through a suction mechanism. The microwave structure delivered the excitation field to the sample perpendicular to the static magnetic field. These microwave structures are constructed by depositing gold onto a glass or quartz substrate through vacuum deposition. The microwave structure also collected the reflected radiation from the sample. Figure 4.1 shows the geometry of the microwave structure.

The reflected microwaves from the YIG sample were amplified by an Avantek C-4052T amplifier and converted to a voltage by a Hewlett-Packard 8427B crystal diode.
Figure 4.1: Geometry of the microwave structure used in the experiments
microwave detector. The crystal voltage was recorded on an oscilloscope. At low powers, a Tektronix 475 oscilloscope displayed the diode voltage versus microwave field frequency, the static magnetic field held constant. Well defined peaks were evident on the oscilloscope screen, corresponding to the magnetostatic modes of the main branch. An example of this spectrum is shown in Figure 4.2. Weak higher order magneto-exchange branches were observed, but the magnetostatic modes of these higher order branches were not resolvable, and no further investigations into the higher order branches were made. Since the microwave excitation fields from the microwave structure were not perfectly uniform, some small peaks were observed in the spectrum which corresponded to weakly excited "hidden modes".

As the microwave power was increased, the magnetostatic mode peaks in the spectrum increased in intensity. As the power was increased, resonant broadening, foldover and hysteretic resonance behavior were observed in the FMR spectrum (Figure 4.3). At higher power levels, regions of "fuzziness" in the magnetostatic mode spectrum were observed. This fuzziness indicated the crossing of the auto-oscillation threshold, at which the microwave absorption at a particular frequency and static field value is no longer constant, but possesses a time-dependent component, due to the changing precession angle of the sample moments.

To better observe the behavior of the auto-oscillation, both the static field and the microwave frequency and power were held constant at values observed to produce auto-oscillations. The time varying voltage signal from the diode was recorded by a LeCroy 9400A digital oscilloscope. This oscilloscope had two storage buffers and
Figure 4.2: Spectrum of a circular YIG film, indicating the magnetostatic modes. The field applied to the sample was 2000 Oe, and the frequency sweep is 20 MHz centered about 1.49 GHz.
Figure 4.3: Spectrum of circular YIG film at a power 30 dB higher than the spectrum in Figure 4.2, indicating resonance broadening and foldover.
had built-in software to compute the Fourier spectrum of the measured time series signal, as well as other data manipulations. The digital oscilloscope was connected to a personal computer. Installed on the personal computer was version 2.22 of the LeCroy Easywave software package which allowed for the data from the digital oscilloscope to be downloaded onto the personal computer hard disk, or be transferred to other experimental components. The FMR spectrometer set up is presented in Figure 4.4.

4.2 Sample Preparation

The magnetic film used in the experiments, YIG, is an ideal material for ferromagnetic resonance in that it has low loss characteristics and a low crystalline anisotropy. YIG is not actually a ferromagnet but a ferrimagnet. However, for the purposes of this report, it can be treated as a ferromagnet. The samples used in the experiments all originated from the same YIG film wafer provided by the Airtron Corporation [39].

The YIG film was grown by liquid phase epitaxy (LPE) on a a gadolinium galium garnet (GGG) substrate to a thickness of roughly 3.0 μm. On the underside of the GGG substrate was another YIG of lesser quality and unknown thickness.

The YIG disks used in the experiments were fabricated in the following manner. A small piece of material was separated from the wafer by creating a groove in the poor quality YIG by repeated scratchings of the surface with a diamond scribe. Unequal pressure applied to both sides of the groove cleaved the wafer along the groove. Using fine grit sandpaper, the lower quality YIG film was sanded off the GGG substrate. The remaining YIG film was cleaned with acetone, methanol, and demineralized water in
Figure 4.4: Block diagram of the ferromagnetic resonance spectrometer.
an ultrasonic cleaner. Silicon sealant was applied in circular patterns on the cleaned YIG surface. These circular patterns served as masks when the film was immersed in hot phosphoric acid to produce circular YIG patterns on the GGG substrate. The substrate was cut with a diamond wire saw to separate the YIG disks and the sealant mask was removed [40]. Further etching of the films to a thickness of about 1 μm created a spectrum consisting of well defined magnetostatic modes.

4.3 Time-Delayed Control Set Up

In previous experiments, external sinusoidal modulation of the static magnetic field has influenced chaotic auto-oscillations to behave periodically [35, 36]. In addition, periodic auto-oscillations have been totally eliminated by phase shifting the voltage produced by the FMR signal and using this voltage to generate a current in a coil placed next to a YIG film [36]. These experiments suggest that perturbations to the magnetic field can influence the behavior of a YIG film in FMR. Rather than use external suppression, the phase shifting technique was extended to control chaotic oscillations. This required two major adjustments to this phase shifting technique.

First, the inductive coil distorts the Fourier spectrum of the phase shifted FMR signal used to perturb the magnetic field. This is not a problem with periodic auto-oscillations which consist of only a single frequency component in the Fourier spectrum. However, in multiply periodic or chaotic auto-oscillations, several frequency components are significant and distortions of the signal due to the inductive coil may significantly degrade the ability to control the signal. To combat this problem, a coil
driver, designed and built by the Ohio State Physics Department Electronics Shop [41], was employed. The coil driver used the voltage of the FMR signal to generate a current in the coil, compensating for the distortions due to induction.

The second major adjustment required to extend the phase shifting technique originates from the fact that there is no phase for chaotic oscillations. Since a time delay corresponds to a phase shift in a purely sinusoidal oscillation, it was thought that a time delayed perturbation applied to a system might stabilize periodic orbits when the film would otherwise behave chaotically and might possibly eliminate the auto-oscillations totally. For this reason, a time delay control perturbation of the form

$$\delta H(t) = K[V(t - \tau) - V_{d.c.}]$$  \hspace{1cm} (4.1)

was used to perturb the static magnetic field to control the dynamics of the YIG film. The parameter $\tau$ is the delay time the FMR signal voltage $V$ is delayed before it is used to generate the perturbation $\delta H(t)$. The d.c. component of the FMR signal, $V_{d.c.}$, must be subtracted out from the control process since it would simply create a constant field that would bias the sample away from the chaotic state.

To apply time-delayed control parameter perturbations, the following additions to the FMR spectrometer were made. To perturb the static magnetic field, a 20 turn coil 1.5 cm in diameter oriented parallel to the sample plane was suspended above the sample. To generate the control perturbations in this coil, the voltage from the crystal diode detector was delayed by a time delay circuit, and then delivered to a OSU M631A coil driver, which provided current to the perturbation coil. A time
delay circuit in the coil driver created time delays in 20-30 ns intervals over the range of 290-700 ns. Even without this time delay circuit, there was a 0.1 $\mu$s intrinsic delay in delivering the perturbation to the sample through the coil driver and perturbation coil. The circuitry of the coil driver compensated for the distortions the inductive coil would introduce into the control perturbations. A resistor in series with the coil allowed for the current in the coil to be determined, providing a means to calculate the size of the control perturbations. A circuit diagram of the coil driver is presented in Figure 4.5 and a block diagram of the FMR spectrometer with the time delay control apparatus is shown in Figure 4.6.

The goal of the time delay control perturbations is to eliminate all auto-oscillations so that the magnetic moments precess at a constant angle, producing exclusively a d.c. FMR signal voltage. It may seem as if the ideal delay time would be no delay at all, but an instantaneous correction to the system. However, an intrinsic lag in the perturbation coil of 0.1 $\mu$s prevents the control perturbations from being instantaneous. The time lag is due largely to the inductive nature of the coil. Since the chaotic auto-oscillations are in the 0.5-10 MHz frequency range, this 0.1 $\mu$s lag can seriously affect any attempts to apply "instantaneous" control perturbations.

4.4 Synchronization Set Up

Since previous experiments synchronized coupled chaotic systems by decomposing the system into drive and response subsystems, this would appear to be a promising way to proceed in synchronizing FMR signals. However, since all the modes couple to the uniform microwave dipole field, there is no practical way to decompose the
Figure 4.5: Circuit diagram of the coil driver, which compensated for distortions in the feed-back loop due to the inductive coil.
Figure 4.6: Block diagram of the time delay control experimental set up.
different interacting mode amplitudes into drive and response subsystems. Instead, the synchronization method of Pyragas was used. Rather than attempt to construct two nearly identical FMR spectrometers, a FMR time series segment was recorded into a memory. This became the master signal. The real time FMR signal was then the slave signal. Perturbations of the form

\[ \delta H(t) = K[V_m(t) - V_s(t)]. \]  

(4.2)

were applied to the static magnetic field to control the magnetic film in FMR to follow its prerecorded output.

To achieve this, the following additions were made to the FMR spectrometer. A LeCroy 9100 Arbitrary Function Generator (AFG) was employed to record and play back an auto-oscillation time series segment. The Easywave software permitted signals from the digital oscilloscope to be transferred to the memory of the AFG, and also controlled the AFG remotely. The AFG output time segment loop and the real time time FMR absorption signal were connected to a differentiating circuit, which compared the two signals and computed the difference voltage. This difference voltage was fed to the OSU M631-A coil driver to create the appropriate control perturbation to the static field. The AFG time segment loop and the real time FMR signal were displayed on separate channels on the LeCroy 9400 digital oscilloscope. A function built into the oscilloscope took the difference of the two channels, and this difference was stored in one of the oscilloscope storage buffers. The Easywave software extracted the stored difference onto the personal computer disk drive. The voltage across the resistor in parallel with the perturbation coil was displayed on the
Tektronix 475 oscilloscope to allow for estimation of the perturbation field. A diagram of the apparatus used in the synchronization experiments is presented in Figure 4.7.
Synchronization Experimental Set-Up

Figure 4.7: Block diagram of the synchronization set up
CHAPTER V

Experimental Results

5.1 Period Doubling Route to Chaos

In FMR, as one approaches the chaotic state, the FMR signal undergoes a series of period doublings as the microwave power approaches the chaotic threshold. An example is shown in Figure 5.1. At a certain power, the auto-oscillations in this sample were period-1. An increase of the microwave power of approximately 2 dB resulted in a period-2 oscillation. A further increase in the microwave power of about 1-2 dB resulted in period-4 oscillations. Finally, a similar increment of power led to chaotic auto-oscillations. Because $M_z$ of the sample changes as the microwave driving power is increased, the microwave frequency had to be increased by $\sim 1$ MHz to maintain a similar resonance condition as the power was increased. Increasing the power while slightly changing the microwave frequency (or equivalently, the applied magnetic field) results in moving up a finger in the auto-oscillation map. This period doubling behavior as experimental parameters are varied is often called a period doubling bifurcation route to chaos. This route to chaos will be compared to the effects of time delayed control to stabilize periodic orbits in YIG films at resonance.
Figure 5.1: Three dimensional time delay plots of period-1 a), period-2 b), period-4 c), and chaotic d) auto-oscillations demonstrating the period doubling of auto-oscillations as the microwave power is increased.
5.2 Time Delayed Control

The time delayed control experiments in FMR were carried out with the static magnetic field at approximately 2000 Oe and the perpendicular microwave frequency oscillating at 1.0-1.4 GHz at powers of 1-20 milliwatts. The static field, microwave frequency and power were chosen so that the microwave absorption of the sample was chaotic. This generally occurred at the center of the "fingers of auto-oscillation", and towards the top of the auto-oscillation map.

By applying a control perturbation of the form

\[ \delta H(t) = K[V(t - \tau) - V_{d.c.}], \]

periodic orbits of decreasing periodicity were stabilized as \( K \) was increased for a single delay time \( \tau \). This was essentially a reversal of the period doubling route to chaos presented in Figure 5.1. With 25 experimentally available delay times within the range of 290-700 ns, only the delay time of 440 ns was able to stabilize period-4, period-2, and period-1 oscillations as the parameter \( K \) increased, as shown in Figure 5.2. In the unperturbed chaotic FMR signal Fourier spectrum (Figure 5.3), the peak frequency was found to be 1.2 MHz, corresponding to a period of 840 ns. The delay time of 440 ns is approximately half of this period. Delay times of 415 and 470 ns were also available and made it possible to stabilize periodic orbits. However, the period doubling reversal (debifurcation) all the way to period-1 could not be achieved by these delay times. Other delay times were able to stabilize periodic orbits for certain values of \( K \) as well. To a first approximation, it is believed that delay times which are half-integer multiples of the period corresponding to the peak frequency
in the unperturbed chaotic Fourier spectrum are the optimal delay times for control and stabilization the most of periodic orbits. Chapter VI will go into further detail about the effect of the delay time choice in stabilization.

In stabilizing the period-4 oscillations, the control gain $K$ was such that the maximum value of $|\delta H|$ never exceeded 0.15 Oe. As $K$ was increased to stabilize period-2 and period-1 oscillations, $|\delta H|$ never exceeded 0.30 and 0.50 Oe respectively. The static field applied to the sample was 2000 Oe and the width of the auto-oscillation finger where the oscillations were stabilized was 2-3 Oe wide.

Another example of the stabilization of orbits in a chaotic system is presented in Figure 5.4. In Figure 5.4a the FMR signal of the film in the chaotic state is shown. The application of time delay control with a delay time of roughly 470 ns with a control gain of 0.4 arbitrary units (a.u.) produced a period-4 oscillation, shown in Figure 5.4b. Further increasing the gain to 0.8 a.u. resulted in a period-2 oscillation (see Figure 5.4c). Continuing to increase the gain, a period-1 oscillation was observed, as shown Figure 5.4d, and further increase of the gain eliminated the auto-oscillations entirely (Figure 5.4e). If the gain was increased to approximately 7.5 a.u., the resulting behavior of the YIG film reverted to a chaotic state, although visually more turbulent than observed in the system without control.

The size of the perturbations to the static field in this example rarely exceeded 0.1 Oe. Compared to the value of the static field, 2000 Oe, and the width of the finger, ~ 2 Oe, the fields used to stabilize periodic orbits are much smaller than the relevant fields in the experiment. In maintaining the quiescent state, perturbation
Figure 5.2: Three-dimensional delay coordinate plots of the unperturbed chaotic signal (a), the stabilized period-4 signal (b), period-2 signal (c), and period-1 signal (d). The coordinate delay, $\tau = 50$ ns.
Figure 5.3: Fourier spectrum of the chaotic FMR signal observed in Figure 5.2a, showing the broad band characteristics and the slight peak at 1.2 MHz.
Figure 5.4: Three dimensional delay coordinate plots of an unperturbed chaotic signal (a), the stabilized period-4 oscillation (b), the stabilized period-2 oscillation (c), the stabilized period-1 oscillation (d), and the stabilized quiescent state (e).
fields no larger than 10 mOe were employed. If the polarity of the coil was reversed, effectively changing the sign of the gain, the system again became highly turbulent. When the control apparatus was switched off, the system abruptly returned to the original unperturbed chaotic state.

It should also be noted that larger gains do not necessarily indicate larger magnetic field perturbations, since the size of the perturbation depends on both the gain and the amplitude of the oscillation. For example, the perturbations required to preserve period-1 and period-2 behavior in Figure 5.2 were similar in magnitude despite the large difference in the gains required to achieve the respective periodic oscillations. This is due to the fact that the period-1 oscillations were of a much smaller amplitude.

The delay time of 470 ns corresponded roughly to the period represented by the peak in the chaotic Fourier spectrum, 1.95 MHz. Other delay times in the vicinity of 470 ns were able to produce periodic oscillations, but not quiescence. Presumably, a delay time of 235 ns (470/2) with the opposite control gain polarity would also produce the observed effects, but this time delay was below the range of delays available in the experiment. Delay times of higher order \( \frac{1}{2} \) integer multiples of 470 ns resulted in controlled periodic oscillations, but not the quiescent state.

In general, a reduction of the periodicty was observed as the control gain was increase if the time delay used was approximately a half-integer multiple of the the period corresponding to a prominent peak in the FMR signal Fourier spectrum. In addition, the controlled auto-oscillations were not limited to periods 4, 2, and 1. Controlled auto-oscillations with periodicities 6, 5, and 3 were also occasionally observed.
The delays used to achieve these oscillations were not half-integer multiples of the peak frequency period.

Time delayed control was able to stabilize periodic orbits from broader band chaotic oscillations. However, in these situations, larger control perturbations were required (0.5-1.0 Oe) and reverse period doubling routes were often not observed. Figure 5.5a presents a broad band chaotic Fourier spectrum. The broad peak of this spectrum is centered at about 2.65 MHz. A delay time of 365 ns was found to stabilize a period-2 oscillation, and Figure 5.5b shows the Fourier spectrum of the stabilized period-2 oscillation. The delay time 365 ns is roughly equal to the period of a 2.65 MHz oscillation. Other delay times available in the experiment were not able to stabilize periodic orbits as effectively as the 365 ns delay time.

5.3 Time Delayed Derivative Control

Time delayed derivative control was also investigated and found to stabilize periodic oscillations through perturbations to the film when it would otherwise behave chaotically. For this control method, the input signal was fed into a derivative amplifier before the signal was time delayed and fed to the coil driver. With the application of time delayed derivative control, the sample was observed to display period-2 behavior, and upon further increasing the control gain, period-1 behavior, as shown in Figure 5.3. However, the quiescent state was not achieved. The perturbation field amplitudes to maintain the periodic oscillations in the derivative control method were approximately 0.2-0.3 Oe. The delay time used to produce the results in Figure 5.6 was about 390 ns. The peak frequency in the chaotic Fourier spectrum was about 2.8
Figure 5.5: The Fourier spectra for the a) broad band chaotic signal and b) stabilized period-2 oscillation.
MHz, which corresponds to a period of 360 ns.

5.4 Synchronization of Chaos

In addition to stabilizing periodic orbits with time delayed control, two chaotic FMR signals were synchronized by applying perturbations of the form

\[ \delta H = K[V_m(t) - V_s(t)]. \]

(5.2)

The variable \( V_s \) refers to the "slave" FMR signal, which was the real time output of the magnetic film sample in FMR. The "master" signal \( V_m \) was a prerecorded time series segment of the sample FMR signal. The control perturbations altered the static magnetic field applied to the sample in real time. The experimental conditions that produced the slave and master signals were kept as identical as the experimental uncertainties would allow.

The results of the experiment are presented in Figure 5.7. As expected, when no continuous perturbation is applied to the sample \((K = 0)\), the prerecorded chaotic signal and the real time chaotic FMR signal have almost no correlation, as seen in Figure 5.7a. However, the systems become correlated to almost the experimental noise level when the appropriate choice of \( K \) is made in the experiment, shown in Figure 5.7b. In addition, the fields required to achieve synchronization are no larger than 1 Oe in the initial transient stage, and fields 0.1 Oe or less were required to preserve the synchronization. A two-dimensional time delay plot of the real time FMR signal attractor during synchronization is presented in Figure 5.8, which indicates the sample dynamics were chaotic during synchronization.
Figure 5.6: Two-dimensional delay coordinate plots of a) the unperturbed chaotic oscillations, b) the stabilized period-2 oscillation ($K=2.4$ au) and c) the stabilized period-1 oscillation ($K=4.2$ au) using time delay derivative control.
Figure 5.7: Correlation diagram of slave and master signal showing the correlation a) in the absence of control perturbations ($K = 0$) and b) when the two signals were synchronized.
Figure 5.8: Two dimensional attractor of the slave signal in time delay space during synchronization, indicating the dynamics were chaotic during the synchronization process.
The transient behavior of this synchronization scheme is presented in Figure 5.9. The recorded FMR signal output segment was usually 100 \( \mu s \) long and played back in a continuous loop. This created a discontinuity in the master signal every 100 \( \mu s \). This allowed for the transient behavior of the synchronization process to be observed as the signals would resynchronize after the discontinuity. The typical transient time, as indicated in the diagram, was about 10 \( \mu s \) or less. Once the signals were synchronized, they remained so to nearly the level of noise in the experiment.

To achieve synchronized signals using the Pyragas scheme, one expects that a certain threshold value of the coupling parameter \( K \) would be required. An interesting question is what effect sub-threshold values of \( K \) have on the slave system. Does the slave signal tend to be in close proximity to the master signal and incrementally becomes closer to the master as \( K \) approaches the synchronization threshold? Or do the two signals become intermittently locked together for brief periods before diverging, remaining locked permanently at the threshold \( K \) value? In the experiments, the latter seems to be the case. Results were obtained with \( K \) below the synchronization threshold. In this situation, the two signals would remain synchronized for a period of time and then diverge, only to become resynchronized at a later time, shown in Figure 5.10. Figure 5.10a shows the difference between the master and slave signals when no control perturbations were applied. In Figure 5.10b, where \( K = 2.0 \) au, below the threshold for continuous synchronization, the signals are seen to repeatedly converge and diverge. In Figure 5.10c, \( K = 3.0 \) au, the minimum threshold for the signals to be continuously locked together. The divergence at around 50 \( \mu s \) is due
Figure 5.9: The difference between the master and slave signal at the discontinuity in the master signal, indicating the transient behavior of the synchronization control.
to the discontinuity in the time series loop. Once the signals were synchronized, $K$
could be increased by a factor of 2-5 before over-corrections would destroy the syn-
chronization. In other cases, synchronization continued up to the experimental limits
of the size of $K$. Further discussion of desynchronous behavior will take place in the
following chapter.
Figure 5.10: The effect of the synchronization control perturbations when a) no control applied, b) $K = 2.0$ au, below the synchronization threshold, and c) $K = 3.0$ au, the synchronization threshold.
CHAPTER VI

Analysis

6.1 Modelling the Experiments

The numerical model developed by McMichael and Wigen described in Chapter II allows for the control experiments to be simulated on a computer. Previously, this model has been successful in predicting the onset behavior of auto-oscillations in thin films under a variety of conditions \[8, 42, 43, 44\]. It is therefore instructive to compare the model to experiments in which the film sample acted chaotically, at a higher power than the onset of auto-oscillations.

To run the model simulations, the low power spectrum of the film sample is measured in the ferromagnetic spectrometer. The linewidths and resonant positions of the magnetostatic modes in the spectrum are used to determine the input parameters for the coupled nonlinear equations that describe the mode amplitude dynamics (Equation 2.45). The eigenvalues of the Jacobian for these equations were calculated by a program written by Shields \[45\] for different values of the field and excitation power. Positive eigenvalues of the Jacobian for certain experimental conditions correspond to a linear instability in the system, indicating auto-oscillatory behavior. A complete
map of these points can be made for a range of experimental conditions similar to the auto-oscillation map in Figure 2.4.

After an auto-oscillation map was completed, points were investigated in detail by a different program which solved the coupled differential equations using a fifth order Runge-Kutta routine. This allows for the time series of the auto-oscillation point to be observed.

In the control experiments presented in this work, a control perturbation was applied to the static field. To account for these control perturbations in the model, the static magnetic field became $H \rightarrow H + \delta H(t)$ where $\delta H(t)$ is the functional form of the control perturbation.

6.2 Period Doubling in the Model

In the model, period doubling routes to chaos are observed, as witnessed in the experiments. Figure 6.1 shows a period doubling route to chaos in the first finger of the model. As the power is increased in the model in increments of 0.2-0.5 dB, the resultant auto-oscillations range from period-1 to period-2, period-4, and finally chaotic oscillations. Since $M_z$ changes with the applied microwave power, the static magnetic field must also be slightly decreased by a few tenths of an Oersted as the microwave power is increased, to maintain a similar resonance condition.
Figure 6.1: A period doubling route to chaos in the numerical model from period-1 (a), period-2 (b), period-4 (c), and chaotic oscillations (d).
6.3 Stabilization of Periodic Orbits in the Model

When time delayed control was introduced into the model as the equations of motion were behaving chaotically, periodic orbits were stabilized in a reverse period doubling sequence similar to that observed in the experiments. For example, Figure 6.3a shows the uncontrolled chaotic FMR signal in the model. Time delayed control was applied to the static field in the model of the form

\[ \delta H = K[S(t - \tau) - S_{d.c}], \]  

(6.1)

where \( S_{d.c} \) is the d.c. component of the FMR signal and \( S(t - \tau) \) was the value of the FMR signal at time \( \tau \) earlier in experimental time. A delay time of 380 ns was able to stabilize a period-8 oscillation when \( K = 2.75 \) (Figure 6.3b). Further increases of the control gain parameter \( K \) stabilized period-4, period-2, period-1 oscillations, shown in Figure 6.3. Ultimately, the quiescent state was achieved, where all auto-oscillations in the model were eliminated. When \( K \) was further increased (\( K \approx 40 \)), the model was over-corrected, and the resulting auto-oscillations were highly turbulent.

The delay time of 380 ns was roughly half the period corresponding to the peak in the chaotic Fourier spectrum at 1.5 MHz. Other delay times in the vicinity of 380 ns allowed for periodic oscillations to be controlled, but not the quiescent state. It is not clear why 380 ns and not 330 ns (half of the period associated with 1.5 MHz) is the optimal delay time, but it may be that the particular combination of significant frequencies in the chaotic Fourier spectrum makes 380 ns the optimal delay time.

The perturbation fields used in the model to stabilize periodic oscillations were never greater than 0.1 Oe. In achieving the quiescent state, the perturbation field
Figure 6.2: Three dimensional delay coordinate plots of a) the unperturbed chaotic oscillation in the model, b) the stabilized period-8 oscillation, \((K = 2.75)\) c) the stabilized period-4 oscillation \((K = 3.5)\), d) the stabilized period-2 oscillation \((K = 5.0)\), e) the stabilized period-1 oscillation \((K = 15)\), and f), the system spiraling into the quiescent state \((K = 30)\). Two lines in the period-8 plot are barely resolvable in the reproduction. The coordinate delay time \(\tau = 50\) ns.
fields approached zero after the initial transients. Generally, a transient time of 5-10 μs was required for the control to convert a chaotic auto-oscillation into a periodic or quiescent state. However, transient times as long as 40 μs were observed in the numerical simulations. It would seem as if the current system trajectory at the time the control is first applied determines, to some degree, the transient time required before stabilization.

The point at which this control was achieved lies toward the outer edge of the auto-oscillation finger associated with the (0,1) magnetostatic mode. Towards the center of this finger, the chaotic auto-oscillations could be converted into periodic oscillations by time delayed control, but the quiescent state could not be achieved. This is also observed in the experiments.

6.4 Poincaré Sections of Time Delayed Control

An important question in the analysis of stabilized periodic orbits is to ask how similar the stabilized orbits are to unstable orbits in the unperturbed attractor. In the time delayed control experiments, the perturbation approached zero only in the case of the quiescent state. In the stabilization of periodic orbits, the control perturbation did not approach zero. Therefore, the stabilized periodic orbits cannot be identical to the unstable periodic orbits in the unperturbed attractor since the experimental conditions in two cases are slightly different. Determining the similarities between the unstable orbits in the unperturbed attractor and the stabilized periodic orbits provides a means to evaluate the time delayed control technique.
Poincaré sections provide a way to measure these similarities. As the system trajectory evolves in time-delay space, the points where the trajectory pierces a surface in this space comprise a Poincaré section. By examining the Poincaré sections of the unperturbed chaotic attractor with the controlled periodic orbits, it can be determined to what extent the time delayed control stabilizes unstable orbits of the unperturbed chaotic attractor.

Figure 6.3 shows the Poincaré sections of the chaotic and stabilized periodic oscillations from the data presented in Figure 5.2. The small dots indicate the Poincaré section of the unperturbed chaotic attractor and the controlled period-4 (circles), period-2 (triangles), and period-1 (square) sections are superimposed on top. The points corresponding to the periodic section lie on or very near the region of the chaotic section. This suggests that the controlled periodic orbits are nearly identical to unstable orbits in the underlying unperturbed chaotic attractor. In the control sequence presented in Figure 5.3, the stabilized period-4 and period-2 Poincaré sections were found to overlap the unperturbed chaotic section, again suggesting that the stabilized orbits are approximately unstable orbits of the chaotic attractor. However, the period-1 oscillation appears to be significantly different from any unstable orbit of the chaotic attractor.

Similar results are also obtained in the model. In Figure 6.4, the Poincaré sections of the stabilized oscillations in the model are superimposed upon the Poincaré section of the unperturbed chaotic attractor. The stabilized period-4, period-2, and period-1 oscillation points are seen to be nearly identical to points on the unperturbed chaotic
attractor, in agreement with the experimental results. (The period-8 points are not presented here, but also lie on the chaotic attractor.)

The proximity of the stabilized periodic points to the unperturbed chaotic attractor on the Poincaré sections suggests that the control perturbations act to stabilize periodic orbits of the underlying attractor. In this sense, the periodic orbits are controlled, since small perturbations are made to the system to stabilize unstable periodic orbits, instead of overdriving the system or creating new orbits in a highly modified system.

In the quiescent state, the perturbation approaches zero since the goal of time delayed control is the quiescent state, where the precession angle of the moments are constant. While quiescence is not an unstable orbit of the periodic attractor, clearly the sample in the chaotic state is being controlled. As the moments precess about the static magnetic field, small corrections to the system keep the moments precessing at a constant angle, instead of oscillating chaotically.

6.5 The Effect of the Time Delay

It is important to understand, from both a fundamental and practical point of view, why certain time delays stabilize oscillations while others do not. While the effect of the time delay is not fully understood, analysis of the FMR signal Fourier spectrum and the time delay used in the control provides some indication of how the control process works.

Chaotic dynamics are characterized by a broadband Fourier spectrum. In the chaotic experimental FMR signal, the resulting Fourier spectrum was broad, but
Figure 6.3: Poincaré map in three-dimensional delay coordinate space of the experimental data presented in Figure 5.2. The piercings lie of the $S(t + 2\tau) = 0$ place in delay coordinate space. The points corresponding to the controlled period-4 oscillation (open circles), period-2 oscillation (open triangles), and period-1 oscillation (solid square) are superimposed upon the map of the unperturbed chaotic system (dots).
Figure 6.4: Poincaré map in three-dimensional delay coordinate space of the numerical simulations presented in Figure 5.3. The piercings lie on the $S(t + 2\tau) = -0.001$ plane in delay coordinate space. The points corresponding to the controlled period-4 oscillation (open circles), period-2 oscillation (open triangles), and period-1 oscillation (solid square) are superimposed upon the map of the unperturbed chaotic system (dots).
Table 6.1: Comparison of the period corresponding to the peak frequency in the Fourier spectrum with the optimal delay time used for control, indicating the 1/2 integer multiple relationship between the two quantities.

<table>
<thead>
<tr>
<th>Example</th>
<th>Peak Frequency Period (T)</th>
<th>Optimal Delay Time τ</th>
<th>Ratio (τ/T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period-1 AO (Exp)</td>
<td>670 ns</td>
<td>315 ns</td>
<td>.47</td>
</tr>
<tr>
<td>Figure 5.2 (Exp)</td>
<td>840 ns</td>
<td>440 ns</td>
<td>.52</td>
</tr>
<tr>
<td>Figure 5.3 (Exp)</td>
<td>510 ns</td>
<td>470 ns</td>
<td>.92</td>
</tr>
<tr>
<td>Figure 5.4 (Exp)</td>
<td>380 ns</td>
<td>365 ns</td>
<td>.96</td>
</tr>
<tr>
<td>Figure 6.2 (Mod)</td>
<td>670 ns</td>
<td>380 ns</td>
<td>.57</td>
</tr>
</tbody>
</table>

contained some peak slightly higher than the broad band background. An example of a chaotic experimental spectrum is shown is Figure 5.2. As discovered in both the experiments and computer simulations, the optimal time delay to create a debifurcation route to quiescence in the experiment corresponded to approximately half-integer multiples associated with the peak frequency in the Fourier spectrum. In addition, the stabilized periodic orbit periods were also approximately related to half-integer multiples of the time delay used in control. At lower microwave powers where the FMR signal was periodic, period-1 auto-oscillations could be totally suppressed through time delayed control with the time delay approximately a half-integer multiple of the auto-oscillation period. The perturbations to the static field generated in the time delayed control process create torques to oppose the angular variations in the precession angle. The target of the control is to eliminate all oscillations to produce a quiescent state. Ideally, the control method would contain no delay, and consist of perturbations of the form $\delta H(t) = K[S(t) - S_{dc}]$. However, due to the inductive nature of the magnetic field perturbation coil, there is a lag of about 0.1 μs before the
appropriate perturbation can be applied to the magnetic film sample. This intrinsic delay erodes the control process.

Faced with little opportunity to use an instantaneous perturbation, the next best option available would appear to be a delay time one half the period corresponding to the peak in the Fourier spectrum. Since the control perturbation creates a torque on the sample magnetic moments to oppose angular variations, the largest torques are required to correct the system when the precession angle deviates the most from the angle associated with the d.c. component of the FMR signal. Since the time delayed control perturbations are calculated from

\[ \delta H = K[S(t - \tau) - S_{d.c.}], \] (6.2)

the largest \( \delta H \) which will produce the largest torque on the moments is generated by Equation 6.2 when the precession angle deviates the greatest from the d.c. component angle. Rather than attempt to apply the control perturbation as soon as possible, the large control perturbations are delayed by half the period corresponding to the peak frequency in the Fourier spectrum. When the large control perturbation is finally applied, it is most likely that the precession angle will again greatly deviate from the d.c. component angle, and therefore, the large perturbation is applied at the most appropriate time. Similar arguments can be made for delaying the perturbation a half period when the precession angle is near the d.c. angle, since the most likely available time when a small perturbation will be appropriate is a half-period later. Since the d.c. component of the FMR signal both experimentally and numerically is calculated from the average FMR signal over a length of time, the d.c. component can change
during the control process, but it is always within the range of angular variations of the unperturbed chaotic signal. While the goal of the time delayed perturbations to eliminate all oscillations is often not achieved, periodic oscillations often result from the application of time delayed control.

It should be pointed out that delay times that are significantly different from $1/2$ integer multiples corresponding to the peak frequency in the chaotic Fourier spectrum can stabilize periodic orbits. However, a reverse period doubling is not often seen with these delay times and the stabilized oscillations may differ substantially from any possible unstable orbit in the unperturbed chaotic attractor. Also, the delay time closest to a $1/2$ integer multiple of the most dominant periodicity of the system is not always the most optimal delay time. Finally, time delayed control applied to very broad band chaos has stabilized periodic orbits. In these instances, there is no strong peak in the Fourier spectrum. This suggests there may be secondary effects occurring in the time delayed control process in addition to the explanation above.

6.6 Synchronization in the Model

The model was also able to predict the results of the synchronization experiments. To simulate the prerecording of the chaotic signal, input parameters were used in the model to describe the magnetic film sample in the chaotic state. The model was allowed to evolve for 400 $\mu$s of experimental time and every 10 ns, the output was written to a file. This file corresponded to the prerecorded master signal in the experiment. The model was then run again, only this time, perturbations to the
static field from the equation

$$\delta H(t) = K[S_m - S_*]$$

(6.3)

were applied. The master signal consisted of the data file of the initial simulation run. A random starting point from the master file was chosen and the current FMR signal in the model was compared every 10 ns of experimental time with the next entry in the saved data file. The difference between these two values was multiplied by $K$ and added to the static magnetic field.

With the appropriate choice of $K$, the numerical model synchronized to the randomly chosen time series segment in the file. Figure 6.5 shows the correlation between the stored file and the model output with no control perturbations ($K = 0$) and when $K = 35$, after the synchronization control had been applied for 10 $\mu$s to eliminate the initial transient behavior. From this figure, it can be seen that nearly perfect correlation between the two signals was achieved in the model. In addition, the attractor of the model output during synchronization reconstructed in time delay space suggests the model FMR signal was acting chaotically during the synchronization process. This is seen in Figure 6.6.

Once the control perturbations were applied in the model, synchronization was achieved after a transient period of about 5-10 $\mu$s. This is in reasonable agreement with the experimental results. Figure 6.7 shows the control perturbations used to achieve synchronization in the model. Comparing Figure 6.7a to the corresponding figure from the experiment (Figure 5.9), one notices that the control perturbations in the model have a far less noisy character than those in the experiment. While there
Figure 6.5: Correlation of the random time series segment and model output without control (a), and with $K = 35$ (b), indicating the synchronization effect in the model.
Figure 6.6: The attractor of the model output, or slave signal in time delay space.
Figure 6.7: Control perturbation in the model to produce synchronization with a) no time lag introduced in the model, and b) a 0.1μs time lag introduced, to reproduce the experimental conditions more accurately.
be approximately 30. At this value of $K$, the master and slave signals in the model were found to quickly synchronize and stay closely synchronized for 200 $\mu$s. It was assumed the two signals would remain synchronized indefinitely. When subthreshold coupling strengths were used in the model ($K < 30$), the master and slave signals did lock together intermittently before diverging at irregular intervals, as observed in the experiments. This is presented in Figure 6.8, where the difference between the master and slave signal $\Delta S$ is plotted versus time for different subthreshold $K$ values.

As the coupling strength was increased beyond the threshold by more than a factor of ten in the model, the master and slave pair were still able to synchronize and remain in synchronization. Finally, at $K = 420$, a small desynchronous burst was observed. With larger coupling strengths, the size of this burst increased and new bursts arose. At $K = 500$, irregular bursts continually interrupted synchronized behavior. These numerical results are shown in Figure 6.9.

A previous study of desynchronization in which the Pyragas method was used to attain synchronization was conducted by Heagy, Carroll, and Pecora (HCP) [46]. The numerical and experimental system used by HCP was the Rössler system. At certain values of the coupling parameter, two synchronized Rössler systems would desynchronize at some point in time, and would be highly desynchronized for a period of time before resynchronization. HCP explained these desynchronous bursts in terms of the unstable periodic orbits of the Rössler system. The unstable periodic orbits were identified in the chaotic Rössler attractor and the minimum coupling strength required for synchronization for each of these orbits was calculated. Since
Figure 6.8: Desynchronous bursts in the model as $K$ approaches the threshold for synchronization.
Figure 6.9: Desynchronization in the model for very large coupling strengths.
these minimum coupling strengths varied by a factor of three, it was possible to choose a coupling strength which would synchronize some unstable periodic orbits of the chaotic attractor, but not others. When the synchronized Rössler systems hopped onto an unstable orbit requiring a coupling larger than the one currently being used, the two systems would diverge. At some large value of the coupling strength, presumably all unstable periodic orbits would synchronize. HCP did not examine very large coupling strengths to see if the systems would desynchronize when large coupling strengths were used.

These results give a possible explanation for the bursts of desynchronization observed in the FMR experiments and the model. The observed synchronization threshold value for $K$ may actually correspond to the coupling strength required for the unstable orbit that is most difficult to synchronize. With $K$ below this value, the chance remains that the coupled systems will assume an unstable orbit that will break the synchronization. At large coupling strengths, an over-correction to the slave system when it resides on a certain unstable periodic orbit may desynchronize the systems. As larger coupling strengths are used, an increasing number of unstable periodic orbits can no longer be synchronized. While it is tempting to compare the results presented here to those of HCP, more detailed analysis is required to make a more meaningful comparison.
CHAPTER VII

Conclusion

Through perturbations to the applied magnetic field, the unpredictable oscillations in the microwave absorption of thin YIG films have been made predictable. Time delayed control has stabilized periodic orbits from chaotic dynamics through small perturbations. Using the synchronization method of Pyragas, chaotic signals at MHz frequencies have been synchronized. In this case, the perturbations required to maintain synchronization were reduced to nearly the level of noise in the experiment. These effects have been reproduced in computer simulations.

In the time delay control experiments, perturbations to the system could not be applied instantaneously to the system, due to experimental limitations. Instead, delay times equal to $1/2$ integer multiples of the peak frequency in the unperturbed chaotic Fourier spectrum were used to generate control perturbations to the magnetic field applied to the film in resonance. This had the effect of stabilizing periodic orbits. The Poincaré sections of the stabilized periodic orbits were nearly identical to points in the Poincaré section of the unperturbed chaotic attractor, suggesting the stabilized periodic orbits were nearly identical to unstable orbits of the unperturbed chaotic attractor.
The choice of delay times to achieve stabilization represents a prediction of when to apply the control. Whenever the precession angle of the moments deviates from the average value, a magnetic field perturbation is required to correct the system through a torque so that the moments precess with a constant angle. Since experimental limitations prevent instantaneous corrections to the magnetic field, a perturbation of opposite sign is delayed by $1/2$ the period represented by the peak frequency in the chaotic spectrum. (The perturbation will have the same sign for delay times equal to this period.) At this time, it is most likely that a perturbation of opposite sign will act to correct the deviations from the average precession angle. While in many cases all the variations in the precession angle cannot be eliminated, periodic orbits can be stabilized by the reduction of the angular deviations from the time delayed control. With increasing control gain, the stabilized oscillations were observed to display a reverse period doubling route (debifurcation route) to a low periodicity orbit or quiescence.

In the synchronization experiments, a chaotic FMR signal was recorded into a memory. The recorded FMR signal was played back and the difference between the real time FMR signal and prerecorded signal was used to perturb the applied magnetic field. Through the torques on the magnetic moments produced by these perturbations, the real time FMR signal was synchronized to the prerecorded signal.

There existed a window of coupling strengths between the two signals where the synchronization effect was maintained indefinitely. However, for coupling strengths outside this window, synchronization would be interrupted by desynchronous bursts.
A possible explanation for the desynchronous bursts is provided by Heagy, Carroll, and Pecora. Their work notes the existence of different coupling strengths required to achieve synchronization for the different unstable periodic orbits in the chaotic attractor. Certain values of the coupling strength may lead to synchronization when the master-slave pair traverses some unstable periodic orbits, but not others. When the synchronized signals hop onto an unstable periodic orbit where the current coupling strength is not sufficient to achieve synchronization, a desynchronous burst occurs.

The results presented in this dissertation suggest future experiments and investigations, such as the following:

- Use the method of close approaches to determine the unstable orbits in the chaotic attractor and compare these orbits to those stabilized by time delayed control.

- Analyze the desynchronous bursting in FMR at large coupling strengths experimentally by modifying the coil driver to produce larger values of $K$.

- Analyze the FMR signal as per Heagy, Carroll, and Pecora to determine the different coupling strengths required to synchronize different unstable periodic orbits. Monitor the desynchronous bursting to determine the periodic orbits which cause the master-slave pair to diverge.

- Investigate the influence of a secondary driving of one of the magnetostatic modes
on nonlinear dynamics of the system. In the previous results of secondary driving reported by Mar et al. [37] in a 10 μm thick rectangular film, the magnetostatic modes could not be easily identified, and therefore it was hard to determine which magnetostatic modes were involved in the secondary driving [47]. Since the magnetostatic modes in 1 μm thick circular films are easily identified, the effects of secondary driving could be more easily understood in the experimental system presented here.

There remains a great deal to be discovered from the control of nonlinear dynamics in FMR.
BIBLIOGRAPHY


[39] The YIG film was graciously provided by Dr. Roger Belt of Airtron Corp.


[41] The coil driver was designed and built by Chuck Rush and Paul Lennous of the Ohio State Physics Department Electronics Shop.


