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FLOW CHARACTERISTICS AND STRESS ANALYSIS
OF BULK MATERIALS IN SILOS

DISSEMINATION
Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of The Ohio State University

By
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* * * * *
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To my Late Mother

... for a wish come true
I would like to express my deepest appreciation for Professor June Key Lee for his guidance and encouragement throughout my research. I would like also to express my deepest gratitude to Professor Liang-Shih Fan for his guidance during this research and his support during my graduate studies. I would like to thank Professor George Staab for his valuable comments on my dissertation and for serving as a member of the Dissertation Committee.

I would like to thank my family back home in Egypt for the love and support they have given me all my life. My greatest thanks goes to my lovely wife "Maysaa" for being such a wonderful companion. Her kind encouragement and understanding have been an inspiration to me throughout the years of this research. Above all, I wish to thank God for His uncountable blessings on me and my family.
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Cross-sectional area of the silo.</td>
</tr>
<tr>
<td>b₁, b₂</td>
<td>Cross-sectional dimensions (length and breadth) of rectangular silos.</td>
</tr>
<tr>
<td>B</td>
<td>Constant given by Eq. (4-34).</td>
</tr>
<tr>
<td>C</td>
<td>Compressibility.</td>
</tr>
<tr>
<td>c</td>
<td>Cohesion.</td>
</tr>
<tr>
<td>D</td>
<td>Diameter of cylindrical bin.</td>
</tr>
<tr>
<td>Dh</td>
<td>Hydraulic diameter.</td>
</tr>
<tr>
<td>dh</td>
<td>Varying diameter of hopper.</td>
</tr>
<tr>
<td>G</td>
<td>Plastic potential.</td>
</tr>
<tr>
<td>h₂</td>
<td>Distance from the vertex to the top surface of the material in hopper.</td>
</tr>
<tr>
<td>K</td>
<td>Stress ratio.</td>
</tr>
<tr>
<td>Kₘ, Kₚ</td>
<td>Ratios between the principal stresses in the active and passive states, respectively.</td>
</tr>
<tr>
<td>m</td>
<td>Constant = 1 for plane strain hoppers and = 2 for conical hoppers.</td>
</tr>
<tr>
<td>N</td>
<td>Constant given by Eq. (4-28).</td>
</tr>
<tr>
<td>q</td>
<td>Shear stress.</td>
</tr>
<tr>
<td>P</td>
<td>Perimeter of the silo.</td>
</tr>
<tr>
<td>r</td>
<td>Coordinate.</td>
</tr>
<tr>
<td>S</td>
<td>Slope.</td>
</tr>
<tr>
<td>Sᵢ</td>
<td>Slope in the i-th segment.</td>
</tr>
<tr>
<td>V</td>
<td>Volume.</td>
</tr>
<tr>
<td>vᵢ</td>
<td>Velocity component.</td>
</tr>
<tr>
<td>x</td>
<td>Coordinate.</td>
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<tr>
<td>y</td>
<td>Coordinate.</td>
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z Coordinate.
z_i Depth of the i-th segment.

Greek letters:

\( \alpha \) Half angle of hopper.  
\( \Delta \) Distribution factor.  
\( \delta \) Effective angle of internal friction.  
\( \varepsilon_v \) Volumetric strain.  
\( \dot{\varepsilon}_{ij} \) Strain rate.  
\( \phi \) Angle of internal friction.  
\( \phi_w \) Angle of wall friction.  
\( \gamma \) Bulk weight density \( (\gamma = \rho g) \).  
\( \gamma_i \) Bulk density in the i-th segment.  
\( \gamma_0 \) Initial bulk density.  
\( \gamma_a \) Aerated (loose) bulk weight density.  
\( \gamma_i \) Bulk density at the beginning of the i-th segment.  
\( \gamma_{i+1} \) Bulk density at the end of the i-th segment.  
\( \gamma_p \) Packed bulk weight density.  
\( \gamma_s \) Shear strain.  
\( \gamma_w \) Working bulk weight density.  
\( \lambda \) Dimensional multiplier.  
\( \mu \) Coefficient of friction  
\( \rho \) Bulk density.  
\( \sigma \) Normal stress.  
\( \sigma_{zz} \) Vertical stress in the i-th segment.  
\( \sigma_1, \sigma_2, \sigma_3 \) Major, intermediate, and minor principal stresses, respectively.  
\( \sigma_h \) Horizontal stress component.  
\( \sigma_i \) Consolidating pressure at the beginning of the i-th segment.  
\( \sigma_{i+1} \) Consolidating pressure at the end of the i-th segment.  
\( \sigma_m \) Mean stress.  
\( \sigma_0 \) Surcharge pressure at the top of the bin.
\( \sigma_{rr} \) \quad Radial stress component in cylindrical coordinates \((r, \theta, z)\).

\( \sigma_t \) \quad Tensile strength.

\( \sigma_v \) \quad Vertical stress component.

\( \sigma_w \) \quad Pressure at wall.

\( \sigma_{z2} \) \quad Surcharge pressure at the top of the hopper.

\( \sigma_{zz} \) \quad Stress component in the z-direction.

\( \tau \) \quad Shear stress.

\( \tau_w \) \quad Shear stress at wall.

\( \omega \) \quad Angle defined by Eq. (4-29).

\( \zeta \) \quad Constant = 1 for active case and = -1 for passive case.
CHAPTER I

INTRODUCTION

Raw materials and products in granular form in industries such as food products, pharmaceutical, metals, fuel, and chemical processing are routinely handled in storage silos. A typical process in plants is to store the granular material for a period of time and then discharge it from the silo at a specified rate. The flow of granular materials discharged from silos are usually driven by gravitational forces. The performance of the silo during this process depends on how adequately the silo was designed. If the silo is designed based only on structural and/or space economy without clear understanding and knowledge of the mechanical behavior of bulk solids during the storage process, several operational and structural problems could be encountered with the silo which obstruct the handling process and may damage the silo and, eventually, lead to the need of replacing it.

1.1 Types of Silos

A typical silo consists of two parts, namely, the bin and the hopper, as shown in Figure 1.1(a). The bin is the upper part of the silo which has
vertical walls. It is the main storage area in the silo and is characterized by height-to-width ratio typically much greater than 1. The hopper is the lower part of the silo which has sloping walls. The bulk material is discharged out of the hopper through an outlet located at the bottom of the hopper. The horizontal cross-section of a silo may be circular, rectangular, square, or polygonal. In some silos, only one of these two components is utilized as a complete storage facility. For example, a silo can be composed only of a bin without a hopper as shown in Figure 1.1(b) or a hopper without a bin as shown in Figure 1.1(c).

In general, two main types of silos are recognized in the literature based on their special geometry, as shown in Figure 1.2. The first type has a cylindrical bin and a conical hopper. This type is referred to as an axisymmetrical silo where both of the stress distribution and the motion of the bulk solids are considered to be symmetrical about the center line axis of the silo. The second type is the plane strain silo in which both of the bin and the hopper have rectangular cross-sections. This type of silos is characterized by the hopper having two vertical walls and two sloping ones (wedge-shaped hopper) and an elongated rectangular outlet.

Silos may also be characterized by the type of flow of the bulk solids during the discharging process of a silo. Two flow patterns are recognized during the discharge of a silo, namely, mass flow and core (or funnel) flow patterns. A mass flow pattern is characterized by the simultaneous movement of every particle when the bulk material is being discharged from the silo, as shown in Figure 1.3(a). On the other hand, a core flow pattern is
characterized by the existence of stagnant zones where the bulk material is not moving while discharge is taking place, as shown in Figure 1.3(b). The type of flow pattern during the discharging phase of a silo strongly depends on the geometry of the hopper, the mechanical properties of the bulk material, and the interaction between the bulk material and the walls of the silo.

Advantages of silos with mass flow patterns are overwhelming compared to those with core flow patterns (Williams, 1990). In a mass flow silo, the rate of discharge and the bulk density of the discharged bulk material are more consistent than in a core flow silo. Also, most of the operating problems, which are discussed later in this Chapter, do not exist in mass flow silos. However, one disadvantage of mass flow silos is the high pressures exerted by the bulk material on the walls of the silo, especially in the hopper, during the discharging process. Also, the generated friction at the interface between the silo’s walls and the moving particles may wear away the walls especially if the stored bulk material is abrasive.

1.2 States of stress in Silos

There are three major phases to be recognized while handling bulk solids in silos that have great influence on the stress field in the bulk solids. Namely, these phases are the filling, storing, and discharging of the silo (Benink, 1989). During the filling phase, the hopper outlet is closed and the bulk solids are supplied at the top of the silo. At this stage, the bulk material tends to compress vertically and thus the vertical stress component may be considered as the major principal stress (compressive stresses are considered
positive) and the horizontal stress becomes the minor stress as shown in Figure 1.4(a). This state of stress is known as the "active state" and is also referred to as the "static state" since the bulk material inside the silo is at rest in this case.

In the storing phase where no material is coming in or out of the silo, the weight of the particles consolidates the bulk material leading to an increase in the strength and density of the bulk material. Finally, during the discharging phase, the hopper outlet is opened and the bulk material flows out of the silo. In this case, the bulk material tends to expand in the vertical direction and thus the horizontal stress becomes the major principal stress while the vertical stress becomes the minor stress, as shown in Figure 1.4(b). This state of stress is known as the "passive state" and in this case is referred to as the "dynamic state" since the bulk material is flowing. The change from active to passive stress when the discharging starts is usually accompanied by the so-called "switch stress" which produces high peak pressures on the walls starting at the hopper outlet and continues up through the hopper, as shown in Figure 1.4(c).

1.3 Operating and Structural Problems in Silos

In a recent study by Knowlton et al. (1994) on 40 bulk solids processing plants in the U.S. and Canada it was shown that 80% of these plants experienced problems in storage and handling facilities. Problems associated with silos can be classified into two groups, namely, operating (or flow) and structural problems. The operating problems are the ones that
affect the efficiency of the silo during the discharging process. The most common operating problems in silos are arching, rateholes, segregation, and flooding (Skeldon, 1991; Dick and Carson, 1989).

Arching is the ability of the bulk material to form a stable arch that spans the hopper outlet and prevents the material above it from discharging out of the silo. This is usually attributed to the cohesiveness and compressibility of the bulk material and thus called “cohesive arching”, as shown in Figure 1.5(a). For bulk materials with irregular and large particles, arching may also occur when the large particles interlock with each other forming a stable arch, as shown in Figure 1.5(b), and is called “mechanical arching” (Knott, 1993).

Another operating problem, which is also related to the cohesiveness of the bulk material, is the formation a stable vertical cavity, usually called a “rathole”, in the core of the silo. In this problem, only the central part of the stored material is able to flow out of the silo while the rest of the material is stagnant, as shown in Figure 1.5(c). The formation of a rathole dramatically reduces the storage capacity of the silo since the stagnant zones of the bulk material are never discharged from the silo, leaving only the central part of the silo for the new material to be stored in. While the use of vibrators and sledge hammers to break the rathole or the stable arch of the bulk material is a traditional way to overcome these problems, the resulting flow is usually erratic. Moreover, if the bulk material is compressible, it will gain more strength instead of collapsing as a result of hammering the walls of the silo.
Flooding is another operating problem that is usually associated with the flow of fine powders. A floodable flow is usually unstable, discontinuous, uncontrollable, and dusty. It often results from the sudden collapse of a stable arch or a rathole of the fine powder. The escaped dust which results in a floodable flow is health hazardous, annoying, and costly in terms of cleaning and equipment maintenance. In addition to these major three operating problems (arching, ratholes, and flooding), there are other operating problems that could interrupt the handling process of bulk solids in silos such as particles segregation, inconsistent flow rate of the discharging material out of the silo, and plug flow.

All these operating problems can be avoided or at least minimized if the silo is designed to operate as a mass flow silo, as mentioned earlier, and if the flow properties of the stored bulk material are measured and properly utilized during the design process of the silo. The flow properties of bulk solids cannot be taken from lists or standard handbooks since they may vary for the same material depending on the storage conditions, such as the moisture content and humidity. Therefore, actual measurements of these properties under different conditions are essential for the design of silos.

The second group of problems associated with silos is the structural problems where parts or all of the silo components are being structurally damaged due to excessive loads exerted by the bulk material on these elements. The study by Knowlton et al. (1994) shows that each year in North America alone, over 1000 silos suffer structural problems in the form of wall cracking, buckling, or even worse, a complete collapse of the silo. Most of
these problems result from misunderstanding the complex behavior of bulk solids in silos during the filling and discharging processes and can be avoided by proper modeling of the bulk solids behavior under the worst possible operating conditions.

1.4 Design of Silos

For proper and safe design of silos, four steps should be followed (Skeldon, 1991). The first step requires the measurements of the mechanical properties of the granular material under all possible handling conditions. In the second step, the geometry of the silo which is required to meet its needed storage capacity and to ensure reliable and predictable flow of the granular material during the discharging process is determined. In the third step, the wall stresses are to be determined under different operating conditions of the silo. The fourth and final step requires the utilization of the outcome of the previous three steps in the structural design of the silo. To fulfill the requirements of these design steps, it is mandatory to understand the mechanical behavior of granular materials at rest and during flowing.

The mechanics and flow behavior of bulk solids in silos has become since the early sixties the subject of researches conducted in different branches of engineering such as mechanics, civil, chemical, agricultural, mechanical, and material science. The main objectives in these researches were to introduce new theories and constitutive models for the bulk solids to simulate and predict their behavior in silos.
Concepts from soil mechanics, theory of plasticity, and continuum mechanics have been utilized in several stress analyses on bulk solids to correctly formulate the problem and determine its governing equations and parameters (Jenike, 1961; Johanson, 1964). Stress distribution in the bulk solids, especially near the walls of the silos, has received more attention for its importance to the design process (Turitzin, 1963).

Different approaches have been adopted to solve the problem such as the radial stress solution (Jenike, 1964), the method of characteristics (Savage and Yong, 1970; Horne and Nedderman, 1978), the method of differential slices (Walker, 1966; Walters, 1973) and the finite element method (Häussler and Eibl, 1984; Askari and Elwi, 1988). Nevertheless, no exact solution has yet been introduced in the literature for the stress distribution in bulk solids stored in silos because of the complex behavior of bulk solids during the different operating stages of the silo. While the method of characteristics introduces the most accurate solution for the problem, the method of differential slices is the one method which is adopted in most of the design codes to determine the wall pressure distribution in silos. This is partially because the method of differential slices provides the designer with a simple analytical form of solution for the stress field in the silo that can be carried out easily without the need to perform the lengthy numerical calculations required in other methods.
1.5 Problem Statement and Thesis Outline

As can be seen in the literature, the behavior of bulk solids stored in silos has not been yet completely understood. Unfortunately, knowledge on the stress analysis and the mechanics of flowing granular materials is more limited and fragmented in the literature compared to solid and fluid mechanics. There is a need to study and analyze the mechanical behavior of granular materials while being handled in silos in order to be able to better predict the stress field within the material and improve the efficiency and lifetime of the silo.

Though analytical methods might provide the most accurate solutions for stress distributions in bulk solids, they cannot be used universally for design purposes and standard codes of practice since closed form solutions are difficult to obtain in these analytical methods. Thus, there is a need to conduct more research on the methods of solution which are usually utilized in determining the design loads in silos, such as the method of differential slices. Also, there is a need to identify and evaluate the main parameters and variables which affect the mechanical behavior of bulk solids in silos in order to be able to adequately formulate the problem.

The objectives of this study are to investigate the flow characteristics and stress distribution of bulk solids stored in silos and to provide the designer with the appropriate guidelines and tools required for safe and efficient design of silos. Also, the study aims to investigate the effect of chemical additives on the flow properties of powders such that a reasonable balance between their chemical reactivity and flowability can be achieved.
In Chapter II, the basic concepts of the mechanics of bulk solids in silos are presented. Also, different analytical and numerical methods, available in the literature, for the stress analysis of bulk solids in silos, such as the method of characteristics and the finite element method, are introduced.

Chapter III defines the mechanical properties of bulk solids which are required to evaluate the degree of flowability of the bulk material and to correctly model the material in the stress analysis. Measurement of the flow properties of several powders using different testing equipment such as the Hosokawa powder characterization tester, the Jenike flow factor tester, and the Ajax cohesion tester are presented. Definitions of the flow properties and methodology of testing and evaluating the results are presented. The flow properties of different modified lignosulfonated hydrates are also determined to study the effect of calcium lignosulfonate on the flow properties of the modified powders. The calcium lignosulfonate is a surfactant additive which improves the chemical reactivity to sulfation of the final modified hydrate product (Jozewicz and Kirchgessner, 1989). The effect of moisture content on the flow properties of these powders are also examined.

In Chapter IV, a comprehensive study on the method of differential slices is conducted. Developments of the method in the literature are reported. The new technique of density piecewise linearization (DPL) is developed and utilized in the stress analysis of bulk solids in silos using the method of differential slices. This new technique accounts for the compressibility of the bulk material in the stress analysis. A parametric study is performed using a computer program which implements the new DPL
technique in the stress analysis. The study examines the effect of different geometrical and material parameters on the distribution of vertical and wall pressures in silos. A listing of the computer program used in the analysis is provided in Appendix A. Finally, conclusions and future work are presented in Chapter V.
Figure 1.1 Types of silos.
Figure 1.2 Axisymmetrical and plane strain silos.
Figure 1.3 Typical types of flow patterns in silos.
Figure 1.4 Trajectories of major principal stress in bulk solids for different operating phases.

(a) Active or static state during filling and storing stages.
(b) Passive or dynamic state during discharging stage.
(c) Switch stress.
(a) Cohesive arching.  
(B) Mechanical arching.  
(c) Rathole.

Figure 1.5 Operating problems in silos.
The discrete structure of bulk materials makes their mechanical behavior different from that of fluids. The mechanism of flow of bulk solids during handling operations in silos is different from that of liquids and assuming otherwise can lead to unrealistic results for the flow patterns and stress fields in the bulk solids.

There are differences in the properties of solids and of liquids that generate different mechanisms of flow for these phases (Woodcock and Mason, 1987). For example, under static conditions, bulk solids can transfer shear stresses while liquids cannot. This is why bulk solids form piles and heaps whereas liquids at rest form level surfaces. Also, many bulk solids after being consolidated retain sufficient cohesive strength and retain stable shapes under pressure like a stable dome or a stable arch but liquids, on the other hand, do not gain strength from consolidation and cannot retain stable shapes. These distinctive differences between liquids and bulk solids make it necessary to model the bulk solids as plastic solids and not as fluid continua.
There are many similarities in the discrete nature and mechanical behavior of soils and bulk solids. Consequently, constitutive relations and material modeling adopted in the more well established soil mechanics are commonly applied to bulk solids. Concepts from soil mechanics and theory of plasticity have been combined together to properly model the mechanical behavior of bulk solids in silos such as the failure criteria, yield conditions, and flow rules. These basic concepts are presented in this Chapter together with a summary of the different analytical and numerical methods that have been provided in the literature for the problem of bulk solids in silos.

2.1 General Assumptions

There are some general assumptions that are commonly proposed in formulating the problem of bulk solids in silos. Among these assumptions are the following:

1. The elastic response of the bulk material to loads is neglected and the material is assumed to deform as a homogeneous plastic continuum. This is acceptable since large deformations occur during the bulk material flowing.

2. Inertia effects are neglected. This is because the flow of bulk materials during the handling operations in silos is slow. Hence, a quasi static state can be assumed without loosing generality.

3. The effects of interparticle fluid flow are neglected and the flow of the bulk material is considered a one-phase flow. This is usually true
as long as the flow of the bulk material is driven only by the gravitational force, which is the case in most operating silos.

4. The effects of particle-particle and particle-wall collisions are neglected in the analysis. This is also due to the fact that the flow of the bulk material is slow.

These assumptions are usually associated with the continuum solid modeling approach which is covered in this Chapter. Nevertheless, there are other approaches in the literature, which are beyond the scope of this study, aiming to model the bulk solid at its micro-mechanical level (Langston et al., 1993; Adams and Briscoe, 1994).

2.2 Basic Concepts

2.2.1 Mohr-Coulomb Failure Criterion

Mohr introduced his theory for rupture in materials in 1910. According to this theory, the material fails along a plane only when a critical combination of normal and shear stresses exists on the failure plane. The relation between normal and shear stress on that failure plane may be given by:

$$\tau = f(\sigma)$$

where $\sigma$ and $\tau$ are the normal and shear stresses on the failure plane, respectively. Equation (2-1) defines a failure envelope which is generally a curved line, as shown in Figure 2-1. Coulomb defined the function $f(\sigma)$ as:

$$f(\sigma) = c + \sigma \tan \phi$$
where \( c \) is the cohesion defined as the resistance of the material to shear under zero normal load and \( \phi \) is the angle of internal friction of the material. Thus, what is known as Mohr-Coulomb failure criteria is given by:

\[
\tau = c + \sigma \tan \phi
\]

(2-3)

The Mohr-Coulomb failure envelope, Eq. (2-3), can be recognized as an upper bound for the stress combination on any plane in the material. Consider points A, B, and C in Figure 2.1. Point A represents a state of stresses on a plane along which failure will not occur since it lies under the failure envelope. On the other hand, failure will occur along a plane if the state of stresses on that plane falls on the failure envelope like point B. The state of stresses, such as point C, cannot exist since it lies above the failure envelope. Since the Mohr-Coulomb failure envelope characterizes the state of stress under which the material starts to slide, it is usually referred to as the "yield locus", \( YL \).

2.2.2 Active and Passive Failure in Coulomb Powders

A rigid-plastic powder which has a linear yield locus is called a Coulomb powder (Brown and Richards, 1970). Most of the powders have linear yield loci, although some nonlinearity appear at low compressive stresses, in some cases. An important relation between the principal stresses in Coulomb powders at failure can be concluded from the Mohr circle shown in Figure 2.2 as,

\[
\frac{(\sigma_2 - \sigma_1)}{(\sigma_1 - \sigma_t)} = \frac{1 - \sin \phi}{1 + \sin \phi}
\]

(2-4)
where $\sigma_t$ is the tensile strength of the powder. For cohesionless powders ($\sigma_t = c = 0$), Eq. (2-4) reduces to

$$\frac{\sigma_2}{\sigma_1} = \frac{1 - \sin \phi}{1 + \sin \phi} \quad (2-5)$$

A very important application of this equation is to distinguish between two extreme failure criteria, known as active and passive failures. First, the active and passive states of stress may be explained as follows. Consider a cohesionless Coulomb powder of constant density. If the powder is assembled in a large container in successive horizontal layers without disturbance, then from the symmetry of the problem there will be no shear stresses along the horizontal and vertical planes in the powder. Thus, at any point, the horizontal and vertical normal stresses are the principal stresses at that point. If the major principal stress is the horizontal stress, $\sigma_h$, then the powder is said to be in a passive state of stress. On the other hand, if the major principal stress is the vertical stress, $\sigma_v$, then the powder is in an active state of stress.

Thus, Eq. (2-5) can be written for each state as:

For the passive state ($\sigma_h > \sigma_v$): \[
\frac{\sigma_h}{\sigma_v} = \frac{1 + \sin \phi}{1 - \sin \phi} = K_p
\]

For the active state ($\sigma_v > \sigma_h$): \[
\frac{\sigma_h}{\sigma_v} = \frac{1 - \sin \phi}{1 + \sin \phi} = K_a
\]

where $K_p$ and $K_a$ are the constant ratios between the principal stresses in the passive and active states, respectively. When Mohr circle for an active or passive state of stress touches the yield locus, a state of active or passive failure takes place along the plane represented in Mohr circle by the radial
line which passes the point of intersection between the circle and the yield locus, as shown in Figure 2.3.

As mentioned before in Chapter I, during the filling of a silo an active state of stress exists since the bulk material tends to compress vertically and thus the major principal stress is the vertical stress. This case is also referred to as the "static state" of stress since the bulk material is at rest in the silo. On the other hand, during discharging from the silo, the bulk material tends to expand in the vertical direction. In that case, the horizontal stress becomes the major principal stress and a state of passive stress exists. Also, the passive state of stress is referred to as the "dynamic state" of stress since the bulk material is flowing.

### 2.2.3 Consolidation, Dilation, and Steady State Flow of Bulk Solids

When a body of bulk material is undergoing shear deformation under the effect of increasing shear load and constant normal load, its initial deformation depends on its initial state of compaction (Spencer, 1982; Jackson, 1983). Neglecting the elastic response of the bulk material, a typical shear stress-strain, \( q - \gamma_s \), characteristics of loose and dense bulk material sheared under constant normal stress is shown in Figure 2.4(a). The corresponding volumetric-shear strain response, \( \varepsilon_v - \gamma_s \), of the bulk material is shown in Figure 2.4(b). The volumetric strain \( \varepsilon_v \) is defined as \( \delta V/V \), where \( V \) is the volume of the bulk material.

For the initially loose material, shown by the dotted lines in Figure 2.4, the material compresses in volume (consolidation) and its shear resistance
increases to an asymptotic value (hardening) as the shear deformation increases. On the other hand, the response of initially dense material, shown by the solid lines in Figure 2.4, is different. As the shear strain increases, the dense material expands in volume (dilation) and its shear resistance decreases to an asymptotic value (softening). In both cases, at large shear deformation, the bulk material continues to deform, or flow, incompressibly with no change in volume which is referred to in the literature as the steady state or critical state flow (Drescher, 1991).

2.2.4 Yield Conditions and Flow Rules

The yield condition is a fundamental element in any plastic model. It defines the limiting states of stress required for plastic deformations to occur. Under steady state conditions, the yield function $F$ is expressed in terms of the principal stresses as,

$$F(\sigma_1, \sigma_2, \sigma_3) = 0$$ (2-7)

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the major, intermediate, and minor principal stresses, respectively. There are many different yield functions proposed in the literature that are found to be applicable for modeling the failure criterion of bulk solids. Among these yield functions is the Mohr-Coulomb yield condition, Eq. (2-3), which in terms of the principal stresses, takes the form:

$$\sigma_1 - \sigma_3 = (\sigma_1 + \sigma_3) \sin \phi + 2c \cos \phi$$ (2-8)

It should be noted that the failure criterion in Mohr-Coulomb condition is independent of the intermediate principal stress $\sigma_2$ and thus can be represented geometrically by the yield locus in Mohr's diagram (see Figure
2.2). In the principal stress space, the Mohr-Coulomb yield condition is represented by a pyramid with a hexagon cross-section, as shown in Figure 2.5(a). If the bulk material possesses insignificant internal friction (\(\phi = 0\)), then Eq. (2-8) reduces to:

\[
\sigma_1 - \sigma_3 = 2c
\]  

(2-9)

Equation (2-9) is known as the Tresca yield condition which is represented in the principal stress space by a prism with a regular hexagon cross-section, as shown in Figure 2.5(b).

Some soils and bulk materials show dependence of their failure criteria on all three principal stresses (Drescher, 1991). The yield condition which represents this type of failure criterion is known as the von Mises-Schleicher yield condition given by,

\[
(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 = [a(\sigma_1 + \sigma_2 + \sigma_3) - b]^2
\]  

(2-10)

where \(a\) and \(b\) are material constants related to the angle of internal friction and cohesion. The von Mises-Schleicher yield condition is represented in the principal stress space by a cone, as shown in Figure 2.6(a). For materials with insignificant internal friction (\(a \equiv 0\)), the von Mises yield condition given by,

\[
(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 = b^2
\]  

(2-11)

is applicable. The von Mises yield condition is represented in the principal stress space by a cylinder, as shown in Figure 2.6(b). It should be noted that this type of yield conditions which are dependent on all three principal
stresses cannot be represented geometrically by a yield locus and, accordingly, the use of Mohr's diagram is invalid in this case.

The relation between stresses and strain rates at failure conditions is called the flow rule where a plastic potential is defined such that the strain rates in any arbitrary direction are proportional to the derivatives of the potential with respect to the corresponding stress component. The general form of flow rules is usually given by,

\[ \dot{\varepsilon}_{ij} = \dot{\lambda} \frac{\partial G(\sigma_{ij})}{\partial \sigma_{ij}} \]  

(2-12)

where \( \dot{\varepsilon}_{ij} \) is the strain rate, \( G \) is a plastic potential, and \( \dot{\lambda} \) is a dimensional multiplier. A special form of the flow rule in Eq. (2-12) is obtained when it is assumed that the yield function \( F \) is itself a plastic potential (i.e., \( F(\sigma_{ij}) = G(\sigma_{ij}) \)) and Eq. (2-12) takes the form

\[ \dot{\varepsilon}_{ij} = \dot{\lambda} \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \]  

(2-13)

The flow rule given by Eq. (2-12) is called a non-associative flow rule, while that given by Eq. (2-13) is called an associative flow rule. The principle of normality, the principle of coaxiality, and Lévy's flow rule are different forms of associative flow rules that have been successfully used in modeling the flow behavior of bulk solids (Nedderman, 1992). The principle of normality requires the direction of the strain rate vector to be normal to the yield surface while the principle of coaxiality assumes that the principal axes of stress and strain rate are coincident (Spencer, 1982). The Lévy’s flow rule
requires that the deviatoric strain rates be proportional to the deviatoric stresses (Cleaver and Nedderman, 1993).

2.3 General Mathematical Formulation of the Problem

The plastic model for the mechanical behavior of bulk materials can be presented mathematically in the form of a set of equations as follows:

1. The yield condition,
   \[ F(\sigma_{ij}) = 0 \]  \hspace{1cm} (2-14)

2. The associative flow rule,
   \[ \dot{\varepsilon}_{ij} = \dot{\lambda} \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}} \]  \hspace{1cm} (2-15)

3. The equations of equilibrium,
   \[ \frac{\partial \sigma_{ij}}{\partial x_j} - f_i = 0 \]  \hspace{1cm} (2-16)

   where \( f_i \) is the body force in the \( i \)-th direction.

4. The continuity equation,
   \[ \frac{d\rho}{dt} + \rho \frac{\partial v_i}{\partial x_i} = 0 \]  \hspace{1cm} (2-17)

   where \( \rho \) is the bulk density of the material, \( t \) is time, and \( v_i \) is velocity component.

5. The strain rate-velocity relations,
   \[ \dot{\varepsilon}_{ij} = -\frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \]  \hspace{1cm} (2-18)
There is no closed form solution for this set of three dimensional equations, Eqs. (2-14) to (2-18). However, for the special cases of plane-strain and axisymmetric problems, a solution can be obtained either analytically or, as is mostly the case, numerically.

2.4 Solutions for the Stress Distributions in Silos

The stress field distribution in plane-strain and axisymmetric silos has been investigated by different researchers using several analytical and numerical techniques, with a varying degree of simplifying assumptions have to be made in order to be able to determine a solution for the problem. The level of accuracy of the solution depends on how suitable the assumptions are.

The method of differential slices, originally developed by Janssen (1895), has received great attention by researchers because of its simplicity and accuracy in predicting the stress distribution in bulk solids in silos, especially within the wall region. This method has been universally accepted in the design codes of practice as the principal method for determining the wall pressures required for the structural design of silos. The method of differential slice is covered in Chapter IV. Highlights on other methods of solution, such as the method of characteristics, the radial stress solution, and the finite element method are presented in the following Sections.
2.4.1 The Method of Characteristics

The method of characteristics is a numerical technique used to determine the stress distribution over a mesh of points of any desired spacing within the bulk solid which is assumed to be in a state of incipient failure at all points. Sokolovskii (1965) applied this method to his work on the stress distribution in cohesionless granular material placed between two vertical walls with two-dimensional assumptions. Other researchers followed Sokolovskii's analysis to provide solutions for the stress field in bulk solids in different types of silos (Hancock, 1970; Savage and Yong, 1970; Horne and Nedderman, 1976 and 1978; Nedderman, 1992). The main assumption in the solution by the method of characteristics is that the yield condition is obeyed at every point in the bulk material over the whole silo. The method of characteristics is introduced in this Section for the case of cohesionless bulk solids stored in two dimensional bins.

For the plane differential element shown in Figure 2.7, the equations of equilibrium in Cartesian coordinates are given by:

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0
\]

\[
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} - \rho g = 0
\]  

(2-19)

As can be seen from Figure 2.8, the state of stress in the bulk material at incipient failure under Mohr-Coulomb criterion can be completely defined by the parameters \( \sigma_m \) and \( \psi \) where \( \sigma_m \) is the mean stress defined by,

\[
\sigma_m = \frac{1}{2} (\sigma_{xx} + \sigma_{zz}) = \frac{1}{2} (\sigma_1 + \sigma_3)
\]  

(2-20)
and $\psi$ is the angle, measured counterclockwise, between the x-axis and the major principal stress direction. The stress components can thus be given by:

$$
\begin{align*}
\sigma_{xx} &= \sigma_m \left( 1 + \sin \phi \cos 2\psi \right) \\
\sigma_{zz} &= \sigma_m \left( 1 - \sin \phi \cos 2\psi \right) \\
\tau_{xz} &= \sigma_m \sin \phi \sin 2\psi
\end{align*}
$$

Substituting into the equations of equilibrium, Eqs. (2-19), gives the following set of first order partial differential equations.

$$
(1 + \sin \phi \cos 2\psi) \frac{\partial \sigma_m}{\partial x} - 2 \sigma_m \sin \phi \sin 2\psi \frac{\partial \psi}{\partial x} + \sin \phi \sin 2\psi \frac{\partial \sigma_m}{\partial z} + 2 \sigma_m \sin \phi \cos 2\psi \frac{\partial \psi}{\partial z} = 0
$$

and

$$
(1 - \sin \phi \cos 2\psi) \frac{\partial \sigma_m}{\partial z} + 2 \sigma_m \sin \phi \sin 2\psi \frac{\partial \psi}{\partial z} + \sin \phi \sin 2\psi \frac{\partial \sigma_m}{\partial x} + 2 \sigma_m \sin \phi \cos 2\psi \frac{\partial \psi}{\partial x} = \rho g
$$

Equations (2-22) and (2-23) form a set of hyperbolic equations that can be solved by the method of characteristics. There are two families of lines known as the $\alpha$-characteristics and the $\beta$-characteristics along which the set of hyperbolic equations, Eqs. (2-22) and (2-23), can be reduced to a set of ordinary differential equations given by:

$$
d\sigma_m \pm 2 \sigma_m \tan \phi \, d\beta = \rho g \left( dz \pm \tan \phi \, dx \right)
$$

where the upper sign refers to the $\beta$-characteristics and the lower sign refers to the $\alpha$-characteristics. The directions of the $\alpha$ and $\beta$-characteristics are given by:
where, again, the upper sign refers to the $\beta$-characteristics and the lower sign refers to the $\alpha$-characteristics. The angle $\chi$ is defined as,

$$2\chi = \frac{\pi}{2} - \phi$$  \hspace{1cm} (2-26)

The two families of characteristics form a mesh and they intersect each other at an angle $2\chi$, as shown in Figure 2.9. Equation (2-25) can easily be solved using a numerical technique (e.g., the finite difference method) where the boundary conditions at the starting points should be known and then by moving along the characteristics lines, the stress parameters at the points of intersection of the $\alpha$ and $\beta$ lines can be determined. The method proves to be accurate but it needs large computer's memory and time.

Horne and Nedderman (1976, 1978) extended Sokolovskii's work using the method of characteristics to determine the stress field in two dimensional bins and hoppers under conditions of active and passive states of stresses for cohesive bulk materials. Plots of their results on the wall stress distribution are shown in Figure 2.10. They concluded that the stress field is continuous in the active case and discontinuous in the passive case.

### 2.4.2 Jenike's Analysis and the Radial Stress Solution

The pioneering work of Jenike in the early 1960's on the gravity flow of bulk solids in bins and hoppers established new rules for the design of silos that are still adopted. Jenike and Johanson presented a series of studies on
the stress and velocity fields in bulk solids in plane strain and axisymmetrical silos (Jenike and Shield, 1959; Jenike, 1961, 1962, 1964a, 1964b, 1965, 1967, 1987; Johanson and Jenike, 1962, Johanson, 1964a, 1964b). In these studies, Jenike introduced the concept of effective yield locus and theory of radial stress field for the solution of stresses in conical hoppers. Jenike also introduced the concept of "flow-no flow" in mass flow silos. He produced designing graphs that are used to determine the proper outlet dimensions and angle of inclination of the hopper walls to prevent arching and to ensure mass flow patterns in the hopper. A brief summary of Jenike's analysis and methodology for stresses in bulk solids in silos is introduced below.

Jenike based his analysis on the assumption that after failure occurs in the solids inside the silo, a steady state of flow is reached in which stress and velocity at any point are unchanged with time. Also, the boundaries are assumed to be unchanged with time. It is not practical to satisfy this condition since it requires refilling the silo at the top while the bulk material is being discharged from the outlet in order to maintain the condition of constant boundaries. However, it was found that the steady state fields are very close to the real fields if the change in boundaries is small and continuous with time (Johanson, 1964a).

The steady-state flow may be characterized by continuous shear deformation without change of stresses at any point within the material. Jenike recognized that the Mohr circles which represent the steady flow of any bulk material under different consolidating conditions are tangential to a straight line that passes the origin as shown in Figure 2.11. He called this line
"the effective yield locus" (EYL) and he defined the effective angle of internal friction, $\delta$, given by

$$\sin \delta = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \quad (2-27)$$

where $\sigma_1$ and $\sigma_3$ are the principal stresses at a given point under the conditions of steady flow. The effective yield locus and the effective angle of internal friction are constant characteristics for a given material. It should be noted that for cohesionless material $\delta = \phi$, while for cohesive material $\delta > \phi$.

The effective angle of friction can be determined experimentally using the shear testing equipment described in Chapter III.

The governing equations which describe the failure criterion of bulk solids in plane-strain and axisymmetrical problems can be written in the Cartesian coordinates as follows:

1. The equations of equilibrium,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + m \left( \frac{\sigma_{xx} - \sigma_{yy}}{x} \right) = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} + m \left( \frac{\tau_{xz}}{x} - \gamma \right) = 0 \quad (2-28)$$

where $m = 0$ for plane-strain, $m=1$ for axial symmetry problems and $\sigma_0$ is the circumferential pressure in the axisymmetrical problem.

2. The equation of steady flow given by the Jenike yield function in Eq. (2-27) which can be also written as,

$$(\sigma_{xx} + \sigma_{zz}) \sin \delta - \sqrt{(\sigma_{xx} - \sigma_{zz})^2 + 4 \tau_{zz}^2} = 0 \quad (2-29)$$
3. The condition for the circumferential pressure based on the Haar von Karman hypothesis which states that the circumferential stress at failure of a Coulomb material is a principal stress and is equal to either the major or minor principal stress. Thus, the circumferential stress is given by,

\[ \sigma_{00} = \sigma_1 = \frac{1}{2} (\sigma_{xx} + \sigma_{zz}) (1 + \sin \delta) \]  

(2-30)

4. The equation of coaxiality, which is the coincidence of directions of the principal stresses and the strain rates, given by

\[ \frac{2 \tau_{xy}}{\sigma_{xx} - \sigma_{zz}} = \frac{v_{x,x} + v_{z,z}}{v_{x,x} - v_{z,z}} \]  

(2-31)

where \( v_x \) and \( v_z \) are the velocity components in the x and z-directions, respectively.

5. The equation of continuity,

\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} + \frac{v_x}{x} = 0 \]  

(2-32)

This set of partial differential equations, Eqs. (2-28) to (2-32), can be solved numerically using the method of characteristics (Jenike, 1961). It should be noted that the stress field can be determined independently of the velocity field since the stress equations, Eqs. (2-28) to (2-30), do not contain velocity terms. After determining the stress field, the velocity field can be determined by solving Eqs. (2-31) and (2-32).

Jenike proposed that the stress field in the vicinity of the hopper outlet could be reduced to a much simpler solution, called the radial stress field, in which the mean stress, \( \sigma_0 \), varies linearly with the radial distance from the apex of the hopper. Thus, the partial differential equations of equilibrium.
reduce to a pair of ordinary differential equations that can be solved combined with the boundary conditions near the hopper outlet regardless of the boundary conditions elsewhere (Jenike, 1961). The radial stress solution has been verified experimentally and numerically by several researchers (Johanson, 1964; Prakash and Rao, 1988; Wang et al., 1992).

2.4.3 The Finite Element Method

Although the finite element method has been widely applied in different engineering problems for long time, its application to the problem of bulk solids in silos is still in an early development stage. One of the main problems is the formulation of suitable constitutive equations for the bulk solids. Another problem is the frictional conditions at the interface between the walls of the silo and the bulk material. Some attempts to apply the finite element method to the problem of bulk solids in silos are briefly reviewed here.

Askari and Elwi (1988) implemented an iterative scheme to model bulk material behavior in silos. In their analysis, the bulk material is modeled as an elastic, perfectly plastic material based on the Drucker-Prager yield surface. Coulomb frictional conditions are assumed in the wall region. Results showed good agreement with the stress distribution predicted by Janssen's method of differential slices in the bin part. However, the model did not accurately predict the stress distribution in the hopper part especially under flowing conditions because of the crudeness of the iterative technique followed in the analysis.
Häussler and Eibl (1984) applied an elastic-plastic constitutive law to their model in a finite element solution. The model was completed with a viscous part to account for the fluid-like behavior assumed during the discharge of the bulk material. The model was limited to cohesionless materials and showed, wrongly, mass flow patterns under conditions which enforce the existence of funnel flow patterns. Bishara et al. (1981, 1983) developed a nonlinear finite element program, where the bulk material was modeled as an isotropic nonlinear viscoelastic continuum contained in an elastic cylinder, to predict the vertical and horizontal stresses on the cylindrical bin walls under static conditions. Vedaie (1988) extended Bishara's work to account for the stress distribution in conical hoppers under flow initiation conditions.
Figure 2.1 Mohr-Coulomb failure envelope.
Figure 2.2 Mohr circle for a generic Coulomb powder at failure.
Figure 2-3 Active and passive failures.
Figure 2.4 Variation of plastic shear strain, $\gamma_s$, with (a) shear stress, $q$, and (b) volumetric strain, $\varepsilon_v$, for loose and dense bulk material.
Figure 2.5 Types of yield conditions independent of the intermediate principal stress.
Figure 2.6 Types of yield conditions which are dependent of all three principal stresses.
Figure 2.7 Equilibrium of stresses on a differential element of bulk solid.
Figure 2.8 Parameters of the method of characteristics.
Figure 2.9  The $\alpha$ and $\beta$-characteristics.
Figure 2.10 Wall stresses in two-dimensional bins and hoppers by the method of characteristics (Horne and Nedderman, 1976, 1978).
Figure 2.11 The effective yield locus for the steady state flow (after Johanson, 1964b).
CHAPTER III

FLOW PROPERTIES OF BULK SOLIDS

The measurement of flow properties of bulk solids is a prerequisite for a qualitative stress analysis and an optimal design of installations handling bulk solids, such as silos. Problems associated with the processes of storage and handling of bulk solids can be avoided, or at least predicted, if these flow properties are determined and adequately evaluated.

The flow properties of bulk materials are not constant parameters, and may vary widely for the same material depending on different handling conditions such as moisture and temperature. Thus, a listing of these properties which may be found in standard handbooks for a given material is always misleading. It is necessary to determine the flow properties of a bulk material by direct measurement of these properties under the specific conditions that are significant for the storage and handling processes.

3.1 Experimental Work

The fundamental flow properties which characterize the mechanical behavior of bulk solids during storage and handling in silos can be categorized into two main groups.
The first one includes the properties which are related to the flowability and floodability of the bulk material. These properties are: bulk density, angle of repose, angle of fall, angle of difference, angle of spatula, dispersibility, cohesion index, and compressibility. Carr (1965) developed a standard technique to measure and evaluate these properties which has been widely adopted by other researchers (Reece, 1985; Jozewicz, 1991; Reiling, 1992). The Hosokawa powder characterization tester, shown in Figure 3.1, is used in the present work to measure the degree of flowability and floodability of the powder based on Carr's evaluation method. This tester is provided by the Hosokawa Iron Works, Ltd. (Osaka, Japan).

The second group includes the properties which are related to the failure criterion of the bulk material. These properties are: angle of internal friction, effective angle of internal friction, unconfined yield strength, and cohesive strength. The basic technique to obtain these failure properties is by shearing several sets of samples of the bulk solid under different normal loads. The corresponding normal and shear stresses give one yield locus which represents one state of consolidation for the bulk material. A set of yield loci is obtained by consolidating several sets of samples under different conditions. The failure properties can then be interpreted from the geometry of the yield loci. There are several types of shear testing equipment that can be used to measure these failure properties such as the Jenike translatory shear tester (Jenike, 1962), the rotational split-level shear tester (Peschl, 1989) and the triaxial shear tester (Kamath et al., 1993). The Jenike shear tester, shown in Figure 3.2, is considered in the literature as the standard
shear testing equipment for bulk solids (EFCE, 1989) and is being used in this work. The tester is provided by Jenike & Johanson, Inc. (North Billerica, Massachusetts).

The cohesive strength of the bulk material can be interpreted from the shear testing results. However, the Ajax cohesion tester, shown in Figure 3.3, is used in the present work in order to be able to cover a wider range of parameters, such as moisture and consolidating pressure, which might affect the cohesiveness of the bulk material. It is provided by Warren-Spring Inc. (Bolton).

Moisture content is among the most important variables that affect the flow properties of bulk solids during the storage and handling processes (Johanson, 1981; Jozewicz and Gullett, 1991). The effect of changes in the moisture content of bulk solids on the flow properties is studied in this work. Measurements of the moisture content are performed on the Mark I Moisture Analyzer provided by Denver Instrument Co. (Arvada, Colorado) The relationship between the bulk density of bulk material and the consolidating pressure is also studied in this work using the Jenike tester.

3.2 Tested Powders

Powders are perhaps the most difficult type of bulk solids that can be handled in silos and other handling equipment. The typical size of particles in powders are less than 100 μm. Because of the small particle size and the large surface area in powders, interparticle forces, such as van der Waals, electrostatic, and magnetic, play an important role in the cohesive strength of
powders and in the ability of the particles to stick and agglomerate with each others.

In this part of the research work, the flow properties of different types of calcium-based sorbent powders, such as hydrates and calcium carbonate, are evaluated. These sorbents are often used in power plants for high-sulfur coal flue-gas desulfurization. One way to improve the performance of these sorbents in SO$_2$ removal is by using additives to increase their intrinsic reactivity. While the role of additives in increasing the reactivity of sorbents is well established in the literature, another important aspect has received very little attention. That aspect is the effect of the additives on the flow properties of the sorbent powders and is explored in this Chapter.

Studies (Jozewicz and Kirchgessner, 1989; Kirchgessner and Lorrain, 1987) have shown that surfactant-modified hydrates have 15-20% more reactivity to sulfation compared to the unmodified hydrates. It was concluded in these studies that the addition of calcium lignosulfonate, an anionic surfactant, to the water of hydration improves the reactivity of the resulting modified product hydrate. Lignosulfonate is a waste product from the pulp and paper industry.

To examine the effect of the lignosulfonate surfactant on the flow properties of the modified hydrate products, four types of commercial hydrates with different mass percentages of lignosulfonate are tested. The hydrates are from the Black River quarry provided by Dravo Lime Company. The tested hydrates are: BRH (with 0.0% lignosulfonate), MH-0.5 (with 0.5% lignosulfonate), MH-1.0 (with 1.0% lignosulfonate), and MH-1.3 (with 1.3%
lignosulfonate). For comparative results of the flow properties of different calcium-based sorbents, York white pulverized limestone sorbent, CC, is also tested. The limestone is provided by Delta Carbonate Inc. Finally, a commercial baking soda powder, which is a cohesionless bulk material, is also tested in order to show the range of differences between the flow properties of cohesionless free flowing materials and cohesive hard-to-flow ones.

3.3 Definitions and Testing Procedures

In the following Sections, definitions of the flow properties of bulk solids are introduced together with the testing procedure to be followed in the measurements.

3.3.1 Group I: Flowability and Floodability Properties

As mentioned earlier, flow properties in this group are measured using the Hosokawa powder characterization tester and are evaluated according to Carr’s evaluation method (1965).

* Loose, working, and packed bulk densities: The loose (aerated) bulk density, $\gamma_a$, is the bulk density of the material when it is as loose as it can be. The packed bulk density, $\gamma_p$, is the bulk density when the material is completely packed. The procedure in measuring these densities on the Hosokawa tester is as follows.
An open cylinder is fitted on top of a coarse screen (mesh #24) and mounted on the vibrating unit of the tester as shown in Figure 3.4. The container is then filled with the powder. When the tester is set to vibrate, the powder flows through the screen and is collected in a cylindrical cup of a standard volume (100 cm$^3$). The amplitude of vibration can be controlled by changing the voltage on the rheostat on the tester. As a standard procedure, it is required to set the rheostat so that the flowing powder will heap above the edge of the cup in approximately 20-30 seconds. The powder is then leveled with the rim of the cup by, carefully, scraping the overflowing powder using a scraper. The net weight of the powder in the cup (in grams) divided by 100 is the aerated or loose bulk density of the powder (in gm/cm$^3$).

The test must then be continued to measure the packed bulk density of the same powder by placing the previously filled and leveled cup in the automatic tapping device of the tester. An extension cylinder is placed on top of the cup to allow adding more powder during the tapping process, if necessary, so that the powder will not pack below the rim of the cup. When the tapping device is turned on, it causes the filled cup to fall 180 times through a constant height. Again, after the tapping process is completed, the powder is carefully leveled with the rim of the cup and its net weight divided by 100 is the packed bulk density (in gm/cm$^3$). The working bulk density, $\gamma_w$, is defined by

$$\gamma_w = \frac{(\gamma_p - \gamma_a)^2}{\gamma_p} + \gamma_a$$  \hspace{1cm} (3-1)
The working bulk density is the one which are used as the actual density of the material in any modeling or analysis (Gaylord and Gaylord, 1984).

* Compressibility and Hausner ratio: Compressibility of a powder, $C$, is defined as

$$ C = \frac{\gamma_p - \gamma_a}{\gamma_p} \times 100 \quad (\%) $$

(3-2)

Compressibility is a very important parameter in characterizing the flow behavior of a powder. As the compressibility of a powder increases, its flowability decreases. Powders with compressibility more than 20% are not "free-flowing" materials. Powders with compressibility 40 - 50% are very hard to flow and have the tendency to block the flow and form cohesive arches across the hopper outlet.

Hausner ratio is defined as the ratio between the packed and aerated bulk densities of the powder ($i.e.$, $\gamma_p / \gamma_a$). Studies on the fluidization behavior of fine powders (Geldart et al., 1984; Rastogi et al., 1993) classify powders according to this ratio as follows:

- Powders having Hausner ratio < 1.25 are considered as "Group A" powders ($i.e.$, free-flowing easy-to-fluidize powders).
- Powders having Hausner ratio > 1.4 are considered as "Group C" powders ($i.e.$, cohesive difficult-to-fluidize powders).
- Powders having the ratio in the range 1.25 to 1.4 may have some properties of both "Group A" and "Group C".
* Angles of Repose, Fall, Difference, and Spatula: The angle of repose is the angle between the horizontal and the free surface of a powder poured freely on a horizontal surface from a given height. The angle of spatula is a rupture angle that measures the frictional behavior of the powder. Both the angles of repose and spatula provide information about the flowability of the powder. Powders with low angles of repose and spatula are more flowable. Powders with angle of spatula less than 40 degrees are "free-flowing" powders.

The angle of fall is the new angle of repose of a pile of powder that has been disturbed by a falling object. The angle of difference is the difference between the angle of repose and the angle of fall. The angles of fall and difference are important parameters in the evaluation of the floodability of a powder. A powder with low angle of fall is more likely to have an unsteady floodable flow. On the other hand, the greater the angle of difference of a powder is, the greater its potential for flooding (or fluidization) will be.

The standard procedure in measuring these angles on the Hosokawa tester is explained hereby. An arrangement of an open cylinder fitted on a 24 mesh screen and a glass funnel is mounted on the vibrating unit in the tester, as shown in Figure 3.4. A round platform is placed on a flat pan under this arrangement such that its center is aligned with the centerline of the funnel opening. The cylinder is filled with the powder. When the unit is set to vibrate, the powder flows through the screen openings and piles on the round platform forming a cone shape. The angle of the cone shaped pile is the angle
of repose of the powder. Then, a shocker (of weight 111 gm and height 7 inches) is placed on the flat pan beside the previously measured pile of powder. The 111 gm weight is repeatedly permitted to fall on the pan from the 7" height for three times. This shocking action disturbs the stable pile of powder and another angle is now measured, which is the angle of fall. Generally, the angle of fall is smaller than the angle of repose. The angle of difference is calculated by subtracting the angle of fall from the angle of repose.

To measure the angle of spatula for a powder, a 7/8-in. wide spatula is placed in the flat pan and the powder is poured inside the pan so that the spatula is completely covered with several centimeters of the powder. By carefully lifting the spatula out of the pan, a considerable quantity of powder will be left on it. The angle of the powder on the spatula should be measured in different locations along the spatula and then the average is obtained. Then, the weight of the shocker is dropped on the spatula and the angle is measured again. The average of the two angles, before and after dropping the weight, is the angle of spatula for the powder.

* **Dispersibility**: Dispersibility of a powder is defined as the ratio of the weight of the dispersed portion of a sample of the powder to the total weight of the sample when it is allowed to fall freely from a given height. Dispersibility is a measure for the potential of a powder to flood or be fluidized. The higher the dispersibility of a powder is, the more floodable it will be. A schematic diagram for dispersibility measurements on the
Hosokawa tester is shown in Figure 3.5(a). To measure the dispersibility, a sample of 10 gm. of the powder is permitted to fall freely through a glass cylinder from a height of 20". The undispersed powder is collected in a watch glass of diameter 4". Dispersibility of the powder is then calculated as

\[ \text{Disp.} = 10 \times (10 - \text{Net weight of undisturbed powder}) \% \tag{3-3} \]

* **Cohesion number**: Cohesion number is defined here as a percentage number rather than its conventional definition as the shear strength of a material under zero stresses. The cohesion number measures the level of energy required to overcome, in a specific time, the apparent cohesive forces that exist on the surfaces of the particles causing the formation of agglomerates. For the cohesion number measurement, three mesh screens (#60, #100, #200) are fitted on top of each other and mounted on the vibrating unit in the tester, as shown in Figure 3.5(b). These screens are chosen based on the average bulk density of the tested powder. A sample of 10 gm of the powder is placed on the #60 mesh screen. The unit is then set to vibrate for a period of time, \( T \), given by

\[ T = 20 + \frac{(1.6 - \gamma_w)}{0.016} \text{ (sec.)} \tag{3-4} \]

After the vibration is completed, the net weight of remaining powder on each screen is measured and the cohesion number is calculated as

\[ \text{Cohesion No.} = 10 \times (W_{60} + 0.6W_{100} + 0.2W_{200}) \% \tag{3-5} \]

where \( W_{60}, W_{100}, \) and \( W_{200} \) are the net weight of the remaining powder on the 60, 100, and 200 mesh screens, respectively.
* Degrees of Flowability and Floodability: According to Carr's evaluation method (Carr, 1965), the flow properties are sorted into two classes:

i) Properties which are measures of the flowability of the bulk material. These properties are: angle of repose, angle of spatula, compressibility, and cohesion number.

ii) Properties which are measures of floodability of the bulk material. These properties are: angle of fall, angle of difference, flowability, and dispersibility.

Each of these properties is given an index number based on its value. Summation of these indices categorizes the corresponding degree of flowability or floodability of the powder over a range of 0 to 100 points. For example, a "free-flowing" material will have a flowability index (summation of corresponding flowability properties indices) of 90-100 points, while a material with "very bad" flowability will have a flowability index of 0-19 points.

3.3.2 Group II: Failure Properties

This group of flow properties characterizes the failure criterion of the bulk solids.

* Cohesive strength: The cohesive strength of a bulk material is defined as the shearing stress on the failure plane in the material when it is sheared under zero normal stresses. The Ajax cohesion tester, shown in Figure 3.3, is used for measurements of the cohesive strength of powders. The testing procedure
is as follows. The sample container, fitted with a removable ring, is filled with the powder to cover the joint between the base and the ring. The loading bridge is then placed in position with the loading plate resting on the surface of the powder. Weights are carefully placed on the plunger top for 30 minutes to allow the density to stabilize and air to escape from the voids of the sample. After compaction, the load is removed and excess powder is scraped away from the sample container base and the sample is weighed in order to determine the bulk density of the powder. Then, the sample is subjected to a torque introduced by the floating loading cell imbedded into the sample. The torque is increased gradually until failure occurs. The counter reading at failure gives the cohesive strength of the powder through a curve that relate the counter reading to the applied torque. The test is repeated several times under different compacting loads

* Angle of internal friction, effective angle of internal friction, and unconfined yield strength: These properties define the relation between the normal and shear stresses in bulk solids at failure and steady flowing conditions. The Jenike shear tester is used to determine the combined shear and normal stresses required to cause failure along a plane within the powder mass. The relation between normal and shear stresses along the failure plan is given by:

\[ \tau = \sigma \tan \phi + c \]  

(3-6)
where $c$ is the cohesion, $\phi$ is the angle of internal friction, $\sigma$ and $\tau$ are the normal and shear stresses, respectively. The curve represented by Eq. (3-6) is the yield locus of the bulk material.

The principle of operation for the Jenike shear tester, shown in Figure 3.2, is to apply a horizontal force to a prepared sample of the powder while a constant vertical pressure is being applied to the sample. The horizontal force is gradually increased until shear failure of the sample occurs. Then, the recorded values of the applied vertical and horizontal forces at failure give a point on the yield locus. The standard procedure described by the European Federation of Chemical Engineering (EFCE, 1989) is followed in these tests.

The Jenike shearing cell, shown in Figure 3.6, consists of a base, a ring, and a shearing cover with a bracket. The inner diameter of both the base and the ring is 3.5". The cell is filled with the powder and after proper consolidation of the specimen, three levels of vertical loads are applied to the specimen. The filled cell is mounted on the Jenike tester and for each vertical load, an increasing horizontal load is applied to the bracket by a mechanically driven stem which is driven forwards at a steady rate. The stem is attached to a force transducer which records the interacting horizontal force between the stem and the cell. When shear failure is noted on the recorder by a sudden drop in the reading, the maximum horizontal force gives the shear force at failure. This procedure is repeated three times in order to obtain three points on one yield locus.

Two more yield loci are obtained by changing the consolidating procedure of the specimen. The slope of the yield locus gives the angle of
internal friction, \( \phi \). The effective angle of internal friction, \( \delta \), is obtained by drawing a tangent to the three Mohr circles which touch the end point of each yield locus. The slope of that tangent gives the angle \( \delta \). The unconfined yield strength, \( Y_u \), is equal to the diameter of the Mohr circle which passes through the origin and touches the yield locus.

3.3.3 Effect of Moisture

Measurements of the flow properties of powders with different moisture contents are conducted on the Hosokawa tester to study the effect of moisture content on the flow properties of the powders. Moisture content is defined as the weight ratio of the water contained by a sample of the powder to the original weight of the sample.

In this work, the effect of moisture on the flow properties of sorbents was examined within the range of 0% to 6% moisture content. To prepare the powders for testing, first, samples of the powders are dried (moisture content <0.5%) by placing them in an oven at a temperature of 140 °c for 24 hours. Then, in order to increase the moisture content in the powders, four to five different samples of each powder are exposed to a stream of water vapor for different periods of time to achieve reasonable distribution of moisture content values. During this operation, the samples are periodically stirred by a mixer in order to make sure the powder sample is homogeneous.
3.3.4 Effect of Consolidating Pressure on Bulk Density

The relationship between consolidating pressure and the bulk density of powders is determined experimentally in this work using the Jenike tester. The testing procedure can be described as follows. The Jenike cell, 2.5" inner diameter and 0.75" depth, is filled carefully with the powder without packing. The powder is leveled with the top of the container by scrapping of any excess powder with a spatula. The sample then is weighed to determine the "loose" bulk density of the powder by subtracting the weight of the empty container from the weight of the sample and dividing by the volume of the container. A stainless steel cover, 2.5" in diameter, is placed on top of the sample. A combination weight hanger is placed on the cover together with a dial indicator mounted on a stainless steel holder which has a counter-bored bottom that fits over the container. The indicator is calibrated such that it reads the exact height of the powder inside the container. The first indicator reading is recorded when it has stabilized and it corresponds to a vertical load of 1.5 pounds (weight of powder + weight of cover + spring resistance of the gage). Different levels of vertical loads are then applied to the sample by placing weights on the weight hanger. For each load level, the indicator reading is recorded and the corresponding new bulk density is calculated.

3.4 Results and Discussion

Results of the experimental work performed on the tested powders are shown in Figures 3-7 to 3-39. Evaluation of these results can be summarized as follows.
Figure 3.7 shows that the carbonate and all the hydrates are "Group C" powders (Hausner ratio > 1.4) which is the group of cohesive and difficult-to-fluidize powders. However, the modified hydrates are of less Hausner ratios than the BRH. The baking soda, on the other hand, is belongs to "Group A" powders since it has a Hausner ratio <1.25. Accordingly, better flow properties are expected for the baking soda than for the other tested powders.

Results of properties which characterize the degree of flowability of the dry powders (compressibility, angle of repose, angle of spatula, and cohesion number) are given in Figures 3-8 to 3-12. As shown in these Figures, the modified hydrates have better flowability properties compared to the BRH. Also, these flowability properties are, generally, improved with increasing the percentage of lignosulfonate in the modified hydrates, as can be seen in Figures 3-10 and 3-11. The modified lignosulfonated hydrates have a better degree of flowability than the BRH, as shown in Figure 3.12. Still, however, they all, including the calcium carbonate, fall in the same category of "bad flowable" powders (flowability index of 20% - 39%) according to Carr's evaluation method (Carr, 1965). The baking soda, on the other hand, has the flow properties of a "good flowable" powder (flowability index of 70% - 79%).

Results that characterize the floodability of the powders are shown in Figures 3-13 to 3-16. Poor dispersibility is noticed for the calcium carbonate compared to the hydrates, as can be seen in Figure 3.13. This recommends the use of any of the hydrates, other than the calcium carbonate, in the flue-gas desulfurization process since that good dispersibility of the sorbents is
essential in this case. Also, Figure 3.13 proves that increasing the lignosulfonate concentration in the modified hydrates improves their dispersibility. Moreover, it can be concluded from these Figures that the calcium carbonate and the baking soda are not as floodable as all of the tested hydrates. Also, increasing the percentage of lignosulfonate in the modified hydrates, increases their tendency to be fluidized, yet uncontrolled flooding flow must be expected in this case. Figure 3.16 indicates that all the tested powders have "fairly high" degree of floodability (floodability index of 60% - 79%), except for the MH-1.3 which has a "very high" degree of floodability (floodability index of 80% - 100%). Hence, unstable and floodable flow must be expected for all the tested powders.

The cohesive strength of the calcium carbonate and the hydrates versus their bulk densities are shown in Figures 3-17 to 3-21. The cohesive strength of the tested powders increases as their bulk densities increases due to compaction. Figure 3.22 shows the effect of the lignosulfonate concentration on the cohesive strength of the modified hydrates for the two cases of loose and packed samples. As the mass percentage of the lignosulfonate increases in the modified hydrates, their cohesive strength decreases. This may be because the lignosulfonate particles in the modified hydrates work as spacers between the hydrate particles (Kirchgessner and Lorrain, 1987) which, generally, reduces the interactive surface forces between the particles including the cohesive forces.

Results of the shear testing, using the Jenike shear tester, are shown in Figures 3-23 to 3-27. Several mechanical properties can be interpreted from
these Figures such as the unconfined yield strength, the flow factor, the angle of internal friction, the effective angle of friction, and the major consolidating pressure at steady state conditions. Figures 3-28 and 3-29 show the values of the measured effective angle of internal friction, \( \delta \), and angle of internal friction, \( \phi \), respectively. The effective angle of internal friction ranges between \( 37^\circ \) and \( 43^\circ \), while the angle of internal friction ranges between \( 33^\circ \) and \( 36^\circ \).

The effect of moisture content on the flow properties of the hydrates are shown in Figure 3.30 to 3-35. Presence of moisture in the tested powders changes their flow properties. Cohesion number and dispersibility, Figures 3-31 and 3-32, are among the properties that have been greatly affected by moisture. This is because of the water bonds which increases the surface cohesion in the powder and causes the particles to agglomerate. The angle of repose of the modified hydrates, Figure 3.33, increases within the range of \( 6^\circ \). The degree of flowability of the tested powders, shown in Figure 3.34, decreases by about 15% with the increase of moisture content. Also, the degree of floodability decreases by 30% with increasing the moisture content, as shown in Figure 3.35. It can be concluded from these results that the existence of moisture has a bad effect on the flow properties of the powders, however, it enhances the flooding behavior of these high floodable powders.

Finally, results of the effect of consolidating pressure on the bulk density of powders are shown in Figures 3-36 to 3-40. Because of the high compressibility of the tested powders, the working bulk density, \( \gamma_w \), is misleading and cannot be used in any material modeling or analysis. Based
on this conclusion, a new technique is suggested to represent, accurately, the bulk density of compressible bulk solids in the stress analysis in silos. In this technique, piecewise linearization of the bulk density, as a function of the consolidating pressure, is used instead of the common working density in the stress analysis presented in Chapter IV.
Figure 3.1 The Hosokawa powder characterization tester.
Figure 3.2 Jenike shear tester.
Figure 3.3 The Ajax cohesion tester.
Figure 3.4 Measurement of the angle of repose and angle of fall on the Hosokawa Tester.
(a) Dispersibility.

Figure 3.5 Dispersibility and cohesion number measurements on the Hosokawa Tester.
Figure 3.6 Jenike shearing cell.
Figure 3.7 Hausner Ratio of the tested powders.
Figure 3.8 Compressibility of the tested powders.
Figure 3.9 Angle of repose for the tested powders.
Figure 3.10 Angle of spatula for the tested powders.
Figure 3.11 Cohesion Number of the tested powders.
Figure 3.12 Degree of flowability of the tested powders.
Figure 3.13 Dispersibility of the tested powders.
Figure 3.14 Angle of fall for the tested powders.
Figure 3.15 Angle of difference for the tested powders.
Figure 3.16 Degree of floodability of the tested powders.
Moisture Content = 0.05%

Figure 3.17 Cohesive strength of the calcium carbonate.
Cohesive strength of Black River hydrate BRH (0.0% lignosulfonate).

Moisture content = 0.5%
Figure 3.19 Cohesive strength of modified hydrate MH-0.5 (0.5% lignosulfonate).
Moisture content = 0.46%

Figure 3.20 Cohesive strength of modified hydrate MH-1.0 (1.0% lignosulfonate).
Figure 3.21 Cohesive strength of modified hydrate MH-1.3 (1.3% lignosulfonate).
Figure 3.22 Effect of lignosulfonate concentration on the cohesive strength of the modified hydrates.
Moisture Content = 0.1%  

Figure 3.23 Shear test results for the calcium carbonate using Jenike shear tester.
Figure 3.24 Shear test results for the Black River Hydrate BRH (0.0% lignosulfonate) using Jenike shear tester.
Figure 3.25 Shear test results for the modified hydrate MH-0.5 (0.5% lignosulfonate) using Jenike shear tester.
Figure 3.26 Shear test results for the modified hydrate MH-1.0 (1.0% lignosulfonate) using Jenike shear tester.
Figure 3.27 Shear test results for the modified hydrate MH-1.3 (1.3% lignosulfonate) using Jenike shear tester.
Figure 3.28 Effective angle of internal friction of the tested powders.
Figure 3.29 Angle of internal friction of the tested powders.
Figure 3.30 Effect of the moisture content on compressibility.
Figure 3.31 Effect of the moisture content on cohesion number.
Figure 3.32 Effect of the moisture content on dispersibility.
Figure 3.33 Effect of the moisture content on the angle of repose.
Figure 3.34 Effect of the moisture content on the degree of flowability.
Figure 3.35 Effect of the moisture content on the degree of floodability.
Figure 3.36 Change of the bulk density of dry Black River Hydrate BRH (0.0% ligno.) with the consolidating pressure.

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<th>Bulk Density (pcf)</th>
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</table>
Figure 3.37 Change of the bulk density of dry modified hydrate MH-0.5 (0.5% ligno.) with the consolidating pressure.
Figure 3.38 Change of the bulk density of dry modified hydrate MH-1.0 (1.0% ligno.) with the consolidating pressure.
Figure 3.39 Change of the bulk density of dry modified hydrate MH-1.3 (1.3% ligno.) with the consolidating pressure.
Figure 3.40 The change of bulk density of dry calcium carbonate with the consolidating pressure.

\[ \gamma_w = 62.507 \text{pcf} \]
CHAPTER IV

THE METHOD OF DIFFERENTIAL SLICES

The first significant studies about stress distributions in bulk solids stored in bins and silos were introduced by Janssen (1895) and Airy (1897) at the end of the nineteenth Century. These studies were mainly concerned with estimating the wall pressure required for the structural design of bins and silos. Airy (1897) adopted Coulomb's work (1776) on stability of slopes in soil mechanics. Janssen (1895), on the other hand, proposed the method of differential slices to predict the wall stresses in cylindrical bins.

Different codes of practice, such as the American Concrete Institute (ACI, 1983), the Canadian Farm Building Code (1977), and the German Standards (DIN, 1964) use Janssen's approach as the basic method to calculate the wall pressures required for the structural design of silos. The method of differential slices has been extended in several studies to determine the stress field in bulk solids not only in cylindrical bins, as originally proposed by Janssen, but also in plane-strain bins and in hoppers (Walker, 1966; Walters, 1973a and 1973b; Walters and Nedderman, 1973; Enstad, 1975 and 1977).
In this Chapter, the method of differential slices is introduced. A new technique is developed and utilized in the method to account for the variation of the bulk density of compressible materials stored in silos with the vertical consolidating pressures exerted upon them. Based on this new technique, a computer program is developed to calculate the wall pressure in silos by the method of differential slices. Also, a parametric study is performed to study the effect of different parameters on the vertical and wall pressure distributions in both the bin and the hopper parts of the silo. Namely, these parameters are: the surcharge pressure, the effective angle of internal friction, the angle of wall friction, the diameter-to-depth ratio in the bin, and the angle of the hopper.

4.1 Janssen's Classical Analysis

Janssen (1895) derived an analytical solution for the stress distribution in cohesionless bulk solids stored in tall cylindrical bins. The main objective in his analysis was to estimate the upper bounds of wall pressure distributions in bins for the two extreme cases of active and passive stresses. His approach is known in the literature as "Janssen's formula" or "the method of differential slices".

To present Janssen's solution, consider the cylindrical bin with a diameter D shown in Figure 4.1. Since the geometry of the bin is of a cylinder, it is more appropriate to use the cylindrical coordinates (r, θ, z) in the analysis. From symmetry about the vertical z-axis, there is no dependency on θ (i.e., ∂/∂θ = 0). Consider a differential slice of the bulk
material of thickness $dz$ at depth $z$, as shown in Figure 4.1. Janssen made four major assumptions which enabled him to obtain a solution for the stress field in the bulk material without the need to solve the complete set of differential equations of equilibrium. These assumptions are:

(i) Stresses in the bulk solids are independent of the radial position, $r$. Hence, $\sigma_{zz}$ and $\sigma_{rr}$ are functions only of $z$.

(ii) $\sigma_{zz}$ and $\sigma_{rr}$ are principal stresses related by,

$$\sigma_{rr} = K \sigma_{zz} \tag{4-1}$$

where

$$K = K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \quad \text{in the case of active state of stress}$$

or

$$K = K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \quad \text{in the case of passive state of stress} \tag{4-2}$$

(iii) The bulk weight density of the bulk material, $\gamma = \rho g$, is constant throughout the bin.

(iv) Wall friction is fully mobilized at the interface between the bin's walls and the bulk material which is at the point of slipping in the bin.

Taking a force balance in the $z$-direction on the differential slice, yields

$$\sigma_{zz}(\pi D^2) - (\sigma_{zz} + d\sigma_{zz})(\pi D^2) + \gamma (\pi D^2) dz - 4\tau_w (\pi D) dz = 0 \tag{4-3}$$

From assumptions (i) and (iv) the wall shear stress, $\tau_w$, could be related to the horizontal stress, $\sigma_{rr}$, by the relation

$$\tau_w = \mu \sigma_w = \sigma_{rr} \tan \phi_w \tag{4-4}$$
where $\sigma_w$ is the wall pressure, $\mu$ is the wall coefficient of friction ($= \tan \phi_w$), and $\phi_w$ is the angle of wall friction.

Substituting Eqs. (4-1) and (4-4) into Eq. (4-3) gives,

$$\frac{d\sigma_{zz}}{dz} = \gamma - \frac{4\mu K}{D} \sigma_{zz} \quad (4-5)$$

Integrating Eq. (4-5) gives,

$$\ln \left( \gamma - \frac{4\mu K}{D} \sigma_{zz} \right) = -\frac{4\mu K}{D} z + C \quad (4-6)$$

where $C$ is the integration constant. Assuming the case of stress-free surface at the top of the bulk material, then the boundary condition at $z = 0$ is $\sigma_{zz} = 0$.

Thus, $C = \ln \gamma$ and $\sigma_{zz}$ can be expressed as,

$$\sigma_{zz} = \frac{\gamma D}{4 \mu K} \left( 1 - e^{-\frac{4\mu K z}{D}} \right) \quad (4-7)$$

Then, $\sigma_w$ and $\tau_w$ can be obtained from Eqs. (4-1), and (4-4), respectively as,

$$\sigma_w = \sigma_{rr} = K \sigma_{zz} = \frac{\gamma D}{4 \mu} \left( 1 - e^{-\frac{4\mu K z}{D}} \right) \quad (4-8)$$

$$\tau_w = \mu \sigma_{rr} = \frac{\gamma D}{4} \left( 1 - e^{-\frac{4\mu K z}{D}} \right) \quad (4-9)$$

From Eqs. (4-7) to (4-9), it can be seen that $\sigma_{zz}$, $\sigma_w$, and $\tau_w$ increase monotonically with increasing depth and for $z/D >> 1$, they approach the asymptotic values given by

$$\sigma_{zz} = \frac{\gamma D}{4 \mu K} \quad (4-10)$$

$$\sigma_w = \frac{\gamma D}{4 \mu} \quad (4-11)$$
\[ \tau_w = \frac{\gamma D}{4} \tag{4-12} \]

It should be noted from Eqs. (4-11) and (4-12) that the asymptotic values of \( \sigma_w \) and \( \tau_w \) at great depths are independent of the value of the stress constant \( K \). Thus, these asymptotic values are the same for the two extreme cases of active and passive stresses. On the other hand, the asymptotic value of the vertical stress, \( \sigma_{zz} \), depends on the value of \( K \) and, accordingly, the vertical pressure in the passive case is higher than that of the active case, as shown in Figure 4.2.

Janssen’s analysis was adopted by several researches (Walker, 1966; Walters, 1973; Enstad, 1975) who tried to improve the analysis by relaxing some of Janssen’s assumptions which could not be physically justified. For example, the assumption that \( \sigma_{rr} \) and \( \sigma_{zz} \) are principal stresses can not be true everywhere since if \( \sigma_{rr} \) near the wall is a principal stress then \( \tau_w \) must be zero. To relax this assumption, Walker (1966) introduced a more appropriate interpretation for the stresses near the wall. The Mohr circle shown in Figure 4.3 represents the active state of stress at a point near the wall. The wall plane is represented by point \( W \). It can be shown that

\[
K_{wa} = \frac{\sigma_{rr}}{\sigma_{zz}} = \frac{1 - \sin \phi \cos(\omega - \phi_w)}{1 + \sin \phi \cos(\omega - \phi_w)} \tag{4-13}
\]

where \( \sin \omega = \sin \phi_w / \sin \phi \). In the case of passive state of stress, \( K_{wp} \) is defined as

\[
K_{wp} = \frac{1 + \sin \phi \cos(\omega + \phi_w)}{1 - \sin \phi \cos(\omega + \phi_w)} \tag{4-14}
\]
Thus, substituting $K_{wa}$ (or $K_{wp}$) in Eqs. (4-7) to (4-9) gives the stress distribution near the wall. Walker also proposed that the vertical stress at the center of a cylindrical bin is greater than the vertical stress at the wall. He defined the distribution factor $\Delta$ given by

$$\Delta = \frac{\sigma_{zw}}{\sigma_{zav}}$$

(4-15)

where $\sigma_{zw}$ is the vertical stress near the wall and $\sigma_{zav}$ is the average vertical stress over the differential slice. Thus, the equilibrium equation, Eq. (4-5), becomes

$$\frac{d\sigma_{zav}}{dz} = \gamma - \frac{4\mu \Delta K_w}{D} \sigma_{zav}$$

(4-16)

Equation (4-16) is unsolvable since $\Delta$ is unknown. Walker listed some estimated values of $\Delta$ which were calculated by force balance equations taken at great depth below the free surface of the material. Later, Walters (1973) presented a more detailed method for the evaluation of $\Delta$ at great depth where $\Delta$ is a constant dependent on $\phi$ and $\phi_w$. It should be noted that all these analyses agree with the original Janssen formula that the wall pressure, $\sigma_w$, approaches an asymptotic value of $\gamma D/4\mu$ exponentially though they differ in the rate of approach.

4.2 Generalization of Janssen's Analysis for Stresses in Bins

Janssen's approach for the stress analysis in cohesionless bulk materials in cylindrical bins can be generalized as follows. For cohesive materials, the effective angle of friction, $\delta$, is used in the analysis instead of the angle of
friction, $\phi$. Also, to use the same form of equations for different types of bins, the hydraulic diameter, $D_h$, is used in the equilibrium equation (Eq. 4-3) instead of $D$. The hydraulic diameter is defined as,

$$D_h = \frac{4A}{P} \quad (4-17)$$

where $A$ is the cross sectional area of the silo and $P$ is the perimeter. For a circular or square silos with diameter or side length $D$, $D_h = D$. For a silo with rectangular cross-section $b_1 \times b_2$, the hydraulic diameter is given by,

$$D_h = \frac{2b_1 b_2}{b_1 + b_2} \quad (4-18)$$

If $b_1 \gg b_2$, then the silo is considered a plane strain one with hydraulic diameter, $D_h = 2b_2$ (where $b_2$ is the shorter side of the silo).

Thus, if we consider the forces acting on the general differential slice shown in Figure 4.4, the equation of equilibrium in the vertical direction may be given as,

$$\frac{d\sigma_{zz}}{dz} = \gamma - \frac{4\mu K}{D_h} \sigma_{zz} \quad (4-19)$$

where,

$$K = \frac{1 - \zeta \sin \delta}{1 + \zeta \sin \delta} \quad (4-20)$$

and $\zeta$ is a constant $= 1$ for the active case and $= -1$ for the passive case. Integrating Eq. (4-19) yields,

$$\ln \left( \gamma - \frac{4\mu K}{D_h} \sigma_{zz} \right) = -\frac{4\mu K}{D_h} z + C \quad (4-21)$$
where \( C \) is the integration constant. Assuming the general case of uniform surcharge at the top of the bulk material, then the boundary condition at \( z = 0 \) is \( \sigma_{zz} = \sigma_0 \). Thus, \( \sigma_{zz} \) can be expressed as,

\[
\sigma_{zz} = \gamma \frac{D_h}{4 \mu K} + \left( \sigma_0 - \frac{\gamma D_h}{4 \mu K} \right) e^{-\frac{4 \mu K z}{D_h}} \quad (4-22)
\]

Then, the normal and shear stresses at the wall, \( \sigma_w \) and \( \tau_w \), can be obtained by,

\[
\sigma_w = K \sigma_{zz} = \frac{\gamma D_h}{4 \mu} + \left( \sigma_0 K - \frac{\gamma D_h}{4 \mu} \right) e^{-\frac{4 \mu K z}{D_h}} \quad (4-23)
\]

\[
\tau_w = \mu \sigma_w = \frac{\gamma D_h}{4} + \left( \sigma_0 \mu K - \frac{\gamma D_h}{4} \right) e^{-\frac{4 \mu K z}{D_h}} \quad (4-24)
\]

It should be noted that the solution for the stress distributions in the bin is independent of the boundary conditions at the bottom of the bin. Also, the asymptotic values for the stresses \( \sigma_{zz}, \sigma_w, \) and \( \tau_w \) at great depths are the same as those given in the original analysis of Janssen, Eqs. (4-10) to (4-12). Thus the asymptotic values of the stresses at great depths are independent also of the boundary conditions at the top surface of the silo.

### 4.3 Stresses in Hoppers

The method of differential slices has been employed in several researches for the stress analysis of conical and plane strain hoppers (Drescher, 1991). To introduce the basic technique mostly used for stress analysis in hoppers, it is more convenient to use the coordinate system shown in Figure 4.5 where the origin of the coordinates is taken at the vertex of the
hopper. The varying diameter (or width) of the hopper, $d_h$, can be expressed as,

$$d_h = 2z \tan \alpha$$  \hspace{1cm} (4-25)

where $\alpha$ is the half angle of the hopper. Equilibrium of forces in the vertical direction on the frustum differential slice shown in Figure 4-5 yields,

$$\frac{d\sigma_{zz}}{dz} + \frac{m}{z}(\sigma_{zz} - \sigma_w - \tau_w \cot \alpha) + \gamma = 0$$  \hspace{1cm} (4-26)

where $m$ is a constant $=1$ for plane strain hoppers and $=2$ for conical hoppers. Assuming the wall friction to be fully mobilized, then the state of stress at a point near the wall can be represented by the Mohr circle shown in Figure 4.6 and Eq. (4-26) can be rewritten as,

$$\frac{d\sigma_{zz}}{dz} + mN \frac{\sigma_{zz}}{z} + \gamma = 0$$  \hspace{1cm} (4-27)

where

$$N = 1 - \frac{1}{\sin \alpha \sin \phi_w} \left[ \frac{\sin \delta \sin(\omega - \zeta \phi_w) \sin(\alpha + \phi_w)}{1 + \zeta \sin \delta \cos(\omega - \zeta \phi_w - 2\zeta \alpha)} \right];$$  \hspace{1cm} (4-28)

$$\sin \omega = \frac{\sin \phi_w}{\sin \delta}$$  \hspace{1cm} (4-29)

and $\zeta = 1$ for the active case and $= -1$ for the passive case. For most reasonable combinations of angles, the factor $mN \neq -1$ and the solution for Eq. (4-27) is given by,

$$\sigma_{zz} = -\frac{\gamma z}{mN + 1} + C z^{-mN}$$  \hspace{1cm} (4-30)
where the integration constant \( C \) can be determined from the boundary condition at the upper surface of the material in the hopper. This condition can be generally given as,

\[
\text{At } z = h_2: \quad \sigma_{zz} = \sigma_{h2}
\]

where \( h_2 \) is the distance from the origin to the upper surface of the material in the hopper and \( \sigma_{h2} \) is the uniform surcharge on that surface, as shown in Figure 4.5. Thus, the vertical stress \( \sigma_{zz} \) can be given as,

\[
\sigma_{zz} = \frac{\rho g z}{mN + 1} \left[ -1 + \left( \frac{h_2}{z} \right)^{mN+1} \right] + \sigma_{h2} \left( \frac{h_2}{z} \right)^{mN}
\]  

(4-32)

The wall stresses, \( \sigma_w \) and \( \tau_w \), are given by,

\[
\sigma_w = B \sigma_{zz} \quad ; \quad \tau_w = \sigma_w \tan \phi_w
\]  

(4-33)

where,

\[
B = (1 - N) \frac{\sin \alpha \cos \phi_w}{\sin(\alpha + \phi_w)}
\]  

(4-34)

In the case of \( mN = -1 \) and for the boundary condition given in Eq. (4-31), the solution for Eq. (4-27) is given by,

\[
\sigma_{zz} = \gamma z \ln \left( \frac{h_2}{z} \right) + \frac{z}{h_2} \sigma_{h2}
\]  

(4-35)

It should be noted that, similar to the bin case, the solution for the stress distributions in the hopper is independent of the boundary conditions at the hopper outlet.

It can be seen in Eq. (4-32) that \( \sigma_{zz} \to 0 \) as \( z \to 0 \) if \( mN < 0 \) which is usually the case with hoppers operating in the passive state. On the other
hand, $\sigma_{zz} \to \infty$ as $z \to 0$ for the condition of $mN > 0$ which is commonly the case with hoppers operating in the active state.

For illustration, consider a typical bulk material with $\gamma = 48$ pcf, $\delta = 30^\circ$, and $\phi_w = 20^\circ$ in a conical hopper with $\alpha = 10^\circ$, as shown in Figure 4.7. Substituting the values of these parameters in Eq. (4-28) yields $mN$ equals - 7.865 and 0.59 for the active and passive cases, respectively. The distributions of wall and vertical stresses in the conical hopper predicted by Eqs. (4-32) and (4-33) are shown in Figure 4.7 for both the active and passive cases. As expected, $\sigma_{zz} \to \infty$ as $z$ approaches zero at the vertex of the hopper, as shown in Figure 4.7(a) and a high level of wall pressure is being reached at the hopper outlet compared to that of the passive case, as shown in Figure 4.7(b).

In common practice, it is rarely reported that conical hoppers fail during the filling or storing phases during which the active state of stresses is expected in the hopper. Nedderman (1993) provided a reasonable physical explanation to this phenomenon by suggesting that the passive state of stresses is most likely to exist in conical hoppers even during the filling phase where the bulk material inside the hopper is being compressed by the weight of the new added material. During this compression, the material slides down the converging channel of the hopper such that it must be compressed laterally and expanded vertically. Thus, the resulting state of stresses requires that the lateral stress $\sigma_{rr}$ to be greater than the vertical stress $\sigma_{zz}$ which is the case of a passive state of stresses. Thus, an active state of stresses cannot be expected to exist in hoppers even during the filling phase.
4.4 The New Technique of Density Piecewise Linearization (DPL)

So far, in the foregoing stress analyses presented in this Chapter and in Chapter II for bulk solids in silos, the bulk density of the material is assumed to be constant throughout the silo. All granular materials have two extreme values of bulk densities, namely, aerated (loose) and packed bulk densities. The standard procedure for measuring these two densities is provided in Chapter III. For design purposes, it is always recommended to use the working bulk density in the stress analysis (Gaylord and Gaylord, 1984). The working bulk density $\gamma_w$ is defined as,

$$\gamma_w = \gamma_a + C (\gamma_p - \gamma_a)$$

(4-36)

where $C$ is the compressibility, $\gamma_a$ and $\gamma_p$ are the aerated and packed bulk density of the material, respectively.

As can be seen in Eq. (4-36), the compressibility of the material is a factor in defining the value of the $\gamma_w$. However, for highly compressible materials such as powders, the working density does not represent accurately the state of the material especially when the material is exposed to high levels of consolidating pressures such as the case of bulk solids in silos. The relation between consolidating pressures and the bulk densities of calcium carbonate, hydrate, and modified hydrates is determined experimentally as described in Chapter III and the results are provided in Figures 3-36 to 3-40. From these Figures, it is obvious that the use of $\gamma_w$ in the stress analysis in silos storing such high compressible materials is misleading and will result in incorrect values of the design pressures in the silo.
A new technique of density piecewise linearization (DPL) is proposed in this study to account for the variation of bulk density with the consolidating pressure in the stress analysis in silos using the method of differential slices. To introduce this new technique, let us first consider the simple case of bulk density varying linearly with the vertical consolidating pressure $\sigma_{zz}$ in the bin such that

$$\gamma = \gamma_0 + S \sigma_{zz}$$

(4-37)

where $S$ is a constant defining the slope of the linear relation between the density and the consolidating pressure and $\gamma_0$ is the initial bulk density. Thus, following the same procedure used in Janssen's analysis and taking the force balance in the vertical direction for the differential slice shown in Figure 4.4, the equilibrium equation can be given as,

$$\frac{d\sigma_{zz}}{dz} = \gamma_0 + \left( S - \frac{4\mu K}{D_h} \right) \sigma_{zz}$$

(4-38)

Integrating Eq. (4-38) yields,

$$\left( \frac{D_h}{SD_h - 4\mu K} \right) \ln \left[ \gamma_0 + \left( \frac{SD_h - 4\mu K}{D_h} \right) \sigma_{zz} \right] = z + C_1$$

(4-39)

where $C_1$ is the integration constant. Thus, for the boundary condition $\sigma_{zz} = \sigma_o$ at $z = 0$, $\sigma_{zz}$ can be expressed as,

$$\sigma_{zz} = \frac{\gamma_0 D_h}{4\mu K - SD_h} + \left( \sigma_o - \frac{\gamma_0 D_h}{4\mu K - SD_h} \right) e^{\left( \frac{(4\mu K - SD_h)z}{D_h} \right)}$$

(4-40)

Then, the normal and shear stresses at the wall, $\sigma_w$ and $\tau_w$, can be obtained by,

$$\sigma_w = K \sigma_{zz} \quad \text{and} \quad \tau_w = \mu \sigma_w$$

(4-41)
Now, the new technique of density piecewise linearization DPL is implemented in the foregoing analysis for linear variation of bulk density with the consolidating pressure such that it can be extended to account for any general nonlinear relation between the bulk density and the consolidating pressure.

Consider the typical $\gamma$-$\sigma_{zz}$ curve shown in Figure 4.8 for a compressible material. Using the piecewise linearization technique, the curve can be assumed to be consisting of segments within which the relation between $\gamma$ and $\sigma_{zz}$ is linear. If we consider the $i$-th segment of the curve, then Eq. (4-37) for the bulk density $^{i}\gamma$ within the $i$-th segment can be rewritten as,

$$^{i}\gamma = \gamma_i + S_i \ ^{i}\sigma_{zz}$$  \hspace{1cm} (4-42)

where $\gamma_i$ is the bulk density at the beginning of the segment, $^{i}\sigma_{zz}$ is the vertical stress distribution within the segment, and $S_i$ is the slope given by

$$S_i = \frac{\gamma_{i+1} - \gamma_i}{\sigma_{i+1} - \sigma_i}$$  \hspace{1cm} (4-43)

Thus, the vertical stress distribution in the bin which corresponds to the $i$-th segment can be given as,

$$^{i}\sigma_{zz} = \frac{\gamma_i D_h}{4 \mu K - S_i D_h} + \left(\sigma_i - \frac{\gamma_i D_h}{4 \mu K - S_i D_h}\right) e^{-\frac{(4 \mu K - S_i D_h) z}{D_h}}$$  \hspace{1cm} (4-44)

To determine the depth of the bulk material layer in the bin within which $^{i}\gamma$ can be applied, we substitute $^{i}\sigma_{zz}$ in Eq. (4-44) by $\sigma_{i+1}$ and solve for the depth $z_i$ yielding,

$$z_i = \left(\frac{D_h}{4 \mu K - S_i D_h}\right) \ln \left[\frac{(4 \mu K - S_i D_h) \sigma_i - \gamma_i D_h}{(4 \mu K - S_i D_h) \sigma_{i+1} - \gamma_i D_h}\right]$$  \hspace{1cm} (4-45)
Thus, the stress distribution in the bin can be determined by dividing the bulk material in the bin into successive layers of depths \( z_i \). In each layer, \( \sigma_{zz}, \sigma_w, \) and \( \tau_w \) can be determined from Eqs. (4-44) and (4-41), respectively.

At this stage of the study, the DPL technique cannot be extended to the stress analysis in hoppers since a closed form solution cannot be obtained for the equation of equilibrium with the bulk density taken as a function of the vertical stresses. However, in a typical silo storing a compressible material, the bulk material inside the hopper is usually packed under the effect of the weight of the material inside the bin above it. Thus, to complete the analysis, it can be assumed, without loosing generality that the material in the hopper has a uniform bulk density equals \( \gamma_p \).

A computer program (Appendix A) is specially developed in this study to implement the DPL technique in the method of differential slices. The experimental results provided in Chapter III for the \( \gamma-\sigma_{zz} \) relations of the tested five powders (Figures 3-36 to 3-40) are used to examine the effect of adopting the new DPL technique in the stress analysis using the method of differential slices.

Figure 4.9 shows the wall and vertical stress distributions in a cylindrical bin of 20 ft. diameter and 100 ft. depth storing calcium carbonate powder. The stress distributions are calculated for the case of active stresses using the DPL technique and the standard Janssen's analysis.

For the DPL analysis, the \( \gamma-\sigma_{zz} \) curve in Figure 3.40 is used to characterize the relation between the bulk density of the calcium carbonate powder and the consolidating pressure. The mechanical properties of the
calcium carbonate are taken from the measurement given in Chapter III, as shown in Figure 4.9(a). The constant bulk density used in Janssen's analysis is taken to be the working bulk density of the calcium carbonate.

As shown in Figure 4.9(b), the difference between $\sigma_{zz}$ calculated by the two methods increases as $z$ increases. The values of $\sigma_{zz}$ calculated by the DPL method is always higher than the corresponding ones which are determined by Janssen's analysis. A similar pattern is also obtained for the wall pressure distribution, as shown in Figure 4.9(a). The error in $\sigma_{zz}$ and $\sigma_w$ which results from using constant $\gamma_w$ instead of the actual variable density reaches a maximum value of 39.32% at the bottom of the bin.

A similar comparison between the two methods has been conducted for the hydrate powder BRH and the modified hydrates MH-0.5, MH-1.0, and MH-1.3. Results of the stress distributions for these powders are shown in Figures 4-10 to 4-13. Again, the mechanical properties of the powders used in both methods are taken from the measurement given in Chapter III. As in the case of the calcium carbonate powder, the same pattern of increasing difference between the stresses predicted by both methods is noticed for all of the hydrate powders. The maximum error in $\sigma_{zz}$ and $\sigma_w$ obtained at the bottom of the bin ranges between 32.922% (for the MH-1.3 hydrate, as shown in Figure 4.13) and 33.898% (for the MH-0.5 hydrate, as shown in Figure 4.11).

Based on these results, it can be concluded that for highly compressible materials such as the powders examined in this analysis, implementing the assumption of constant bulk density in Janssen's analysis underestimates the
predicted stresses and leads to remarkable errors at the bottom of the bin where maximum stresses are expected. These errors could be devastating to the structural safety of the bin if not recognized in the design of the bin.

It should be noted that the American Standard Code of Practice for design of bins (ACI, 1983) recommends the use of Janssen's analysis (with constant bulk density) to estimate the wall pressures in the bin for the active case and multiply these pressures by an overpressure correction factor (ranges between 1.35 and 2.0) to determine the values of the design pressures. Thus, in order to maintain a reasonable factor of safety for the design pressures in bins storing compressible bulk materials, it is recommended to utilize the DPL technique, developed in this study, in the analytical solution of stresses using the method of differential slices.

4.5 The Parametric Study

The stress distribution in bulk solids in silos depends on several parameters among them are:

(1) The operating phase of the silo, \(i. \ e.\), active or passive.

(2) The geometry of the silo, \(e. \ g.\), the hydraulic diameter of the bin and the slope of the hopper walls.

(3) The mechanical properties of the handled bulk material, \(e. \ g.\), bulk density and effective angle of internal friction.

(4) The boundary conditions at the top surface of the bulk material in the bin or the hopper, \(e. \ g.\), the existence of a surcharge at the top surface.
The boundary condition at the interface between the bulk material and the walls of the silo, e.g., the angle of wall friction.

One should expect that some of these parameters will have greater effect on the predicted stress distribution than the other. The main objective in this parametric study is to illustrate the effect of changing the value of each of these parameters on the stress distribution in the silo while fixing the other parameters.

In the previous Section, we illustrate how the stress distribution in silos is dramatically affected by the change of the value of the density of the bulk material. This strong effect should be expected since the forces in the stress analysis of bulk solids in silos are mainly the gravitational forces derived by the own weight of the bulk material.

The study is conducted using a typical tall silo which is composed of a cylindrical bin and a conical hopper, as shown in Figure 4.14. The bin is 20 ft. in diameter and 100 ft. in height. The hopper is 45 ft. in height and has an outlet diameter of 2 ft. which makes the half angle of the hopper, $\alpha$, to be 11.3°. To generalize the results, each of the bin and the hopper is analyzed separately. The bulk material is chosen to be the modified hydrate powder MH-1.0 since its mechanical properties are shown to be a typical representative of the other tested powders.

In the bin, the stress analysis is carried out utilizing the DPL technique and the bulk density of the MH-1.0 is taken to be a function of the vertical pressure $\sigma_{zz}$ as given in Figure 3.38. The stress analysis in the hopper is
carried out using Eqs. (4-32) to (4-34) with the bulk density taken to be uniform and equals to $\gamma_p$.

The parameters considered in this study are:

1. The effective angle of internal friction $\delta$.
2. The surcharge $\sigma_0$ for the bin or $\sigma_{h2}$ for the hopper.
3. The angle of wall friction $\phi_w$.
4. The depth-to-diameter ratio in the bin $H/D$.
5. The Half angle of the hopper $\alpha$.

Each of these parameters is changed over a range that covers all possible values of the parameter under typical operating conditions of silos. The parameters and the range of operating values are given in Table 1. The results of the study are given only for the distributions of the vertical stress $\sigma_{zz}$ and the wall pressure $\sigma_w$ since the distribution of the shear stress at the wall $\tau_w$ is typically identical to the distribution of multiplied by the factor $\mu$.

4.5.1 The Effective Angle of Internal Friction $\delta$

Variations of the distribution of the vertical and wall stresses, $\sigma_{zz}$ and $\sigma_w$, in the bin with respect to the effective angle of friction $\delta$ for both the active and passive cases are shown in Figures 4-15 and 4-16, respectively. While $\sigma_{zz}$ increases monotonically with increasing $\delta$ for the active case shown in Figure 4.15(a), the opposite is true for the passive case shown in Figure 4.15(b). However, in the passive case $\sigma_{zz}$ reach the asymptotic value at depths smaller than those in the active case. As shown in Figure 4.16(a), the
wall pressure increases monotonically with decreasing \( \delta \), opposite to the active \( \sigma_{zz} \). The value of \( \delta \) has a negligible effect on \( \sigma_w \) in the passive case from the surface down to a depth of about 25 ft. and has no effect at all throughout the rest of the bin, as shown in Figure 4.16(b).

All of these remarks are collectively summarized in Figure 4.17 which shows the maximum values of both \( \sigma_{zz} \) and \( \sigma_w \) at the bottom level of the bin for the active and passive cases. The figure confirms that the value of \( \delta \) has no effect on the maximum value of the passive \( \sigma_w \) and slight effect on the maximum values of the other stresses. Both of the passive \( \sigma_{zz} \) and the active \( \sigma_w \) appear to linearly decrease with increasing \( \delta \) with the rates of 3.42 % and 2.2 % per 1°, respectively. On the other hand, the active \( \sigma_{zz} \) appears to linearly increase with increasing \( \delta \) with a rate of 2.47 % per 1°.

Results of the stress distribution in the conical hopper are shown in Figures 4-18 and 4-19. Results of the active case show that the solution for both \( \sigma_{zz} \) and \( \sigma_w \) approach infinity at the vertex of the hopper, as shown in Figures 4-18(a) and 4-19(a), and, consequently, the values of \( \sigma_{zz} \) and \( \sigma_w \) at the hopper outlet are considerably higher than any reported measurement of the actual stresses commonly practiced at the hopper outlet. This agrees with Nedderman's conclusion (1993) that hoppers usually operate in a passive state. Results of the active case, however, will still be reported in this Section for the completion of the study.

Similar to the case of the bin, the vertical stress decrease with increasing \( \delta \) with a maximum value somewhere in the upper third of the
hopper as shown in Figure 4.18(b). The passive wall pressure appear to be independent of the value of $\delta$ as shown in Figure 4.19(b) and it reaches a maximum value of approximately 394 psf at $z = 36.5$ ft. The values of $\sigma_{zz}$ and $\sigma_w$ at the mid height and the bottom of the hopper are shown in Figure 4.20 where it can be noticed that, except for the active $\sigma_{zz}$, the other stress components are barely affected by the change of $\delta$.

Thus, it can be concluded from these results that the effective angle of internal friction is not a critical factor in the calculation of the design loads for the walls of the bin and the hopper since changes of $\delta$ appear to have a minor effect on the wall pressure throughout the bin and the hopper.

4.5.2 The Surcharge $\sigma_o$ or $\sigma_{h2}$

Results of the distribution of $\sigma_{zz}$ and $\sigma_w$ with different levels of surcharge $\sigma_o$ at the top surface of the bulk material in the bin are shown in Figures 4-21 and 4-22 for the active and passive cases. In these Figures, the effect of the $\sigma_o$ on both passive $\sigma_{zz}$ and $\sigma_w$ is shown to be vanishing very rapidly as the depth increases down to about 20 ft. under the surface of the bulk material, where $H/D = 1$. A similar approach can be also noticed for the active case but with a rather very slow rate such that even with the bin having an aspect ratio $H/D$ of 5, the value of the active stress at the bottom of the bin can be still affected by the surcharge.

Figure 4.23 shows the values of the maximum stresses predicted at the bottom of the bin. While the maximum values of $\sigma_{zz}$ and $\sigma_w$ are shown to be
independent of the surcharge value in the passive case, they linearly increase with increasing $\sigma_a$ in the active case with the rate of 1.67 % (per 100 psf of surcharge).

Figures 4-24 and 4-25 show the results for the stress distributions in the conical hopper with the surcharge $\sigma_{h2}$. Again, the stresses in the active case approach infinity at the hopper vertex and may be disregarded, from the structural design point of view. On the other hand, both $\sigma_z$ and $\sigma_w$ approach their asymptotic values near the mid height of the hopper and become independent of the boundary condition at the top of the hopper, which is the value of the surcharge, as they approach the outlet. This conclusion agrees with Jenike's radial stress theory which state that during the discharge of the hopper, the stress field near the hopper outlet is a linear function of the radial distance from the vertex of the hopper.

The critical cross section for $\sigma_w$ in the hopper is at the top surface where $\sigma_w$ becomes more than double the value of $\sigma_{h2}$. Figure 4.26 for the stresses at the mid height and bottom of the hopper confirms this conclusion since it shows that both of $\sigma_z$ and $\sigma_w$ are independent of the value of the surcharge $\sigma_{h2}$ at the top surface of the hopper.

It can be concluded from these results that the existence of a surcharge on the top surface of the bulk material in the bin and the hopper produces high localized wall pressures near the top surface especially in the hopper. This is extremely important for the structural design of a typical silo where the hopper is surplused by the bin. At the initiation of the bulk material
discharge, the hopper operates in a passive state while the bin is still operating in an active state. As was shown before, the vertical stress in the bin has higher values in the active state and reaches its maximum value at the bottom of the bin. Thus, by superimposing this maximum value of $\sigma_{zz}$ as a surcharge on the top surface of the passive hopper, high localized wall pressure, known as the "switch stress", exists at the transition plane between the bin and the hopper.

4.5.3 The Angle of Wall Friction $\phi_w$

Results of the effect of the angle of wall friction on the active and passive distributions of $\sigma_{zz}$ and $\sigma_w$ in the bin are shown in Figures 4-27 and 4-28, respectively. As has been noticed before in the previous cases, both of $\sigma_{zz}$ and $\sigma_w$, in the passive case approach their asymptotic values at a considerably high rate. It should be noted that as the value of $\phi_w$ increases, the values of $\sigma_{zz}$ and $\sigma_w$ in both active and passive cases decrease throughout the whole bin, as can be seen in the Figures. However, it is noticed that the values of $\sigma_{zz}$ and $\sigma_w$ are more sensitive to $\phi_w$ when the value of $\phi_w$ is small ($i.e.$, smooth walls) especially in the passive case. This is confirmed in Figure 4.29 which shows the nonlinearity of the relation between $\phi_w$ and the stresses at the bottom of the bin is nonlinear. Moreover, this nonlinearity appears to be decaying as $\phi_w$ increases. For example, the rate of decrease of the passive $\sigma_w$ as $\phi_w$ increases from 10 to 15 degrees is 34.4 % and it decreases to 22 % as $\phi_w$ increases from 20 to 25 degrees.
In the conical hopper, the results given in Figures 4-30 to 4-32 show that $\phi_w$ maintains the same effect as in the bin case except for the passive vertical stresses where $\sigma_{zz}$ is noticed to be increasing as $\phi_w$ increases, as shown in Figure 4.30(b).

These results for the bin and the hopper show that the $\phi_w$ is a critical factor in determining the design loads in the silo. Not only the angle of wall friction affects the values of the maximum wall pressure but also its effect on these stresses is substantially magnified as its value approaches zero. Thus, experimental measurement of $\phi_w$ should be carried out very carefully especially if it turns out that the value of $\phi_w$ is small.

### 4.5.4 The Half Angle of the Hopper $\alpha$

Four values of $\alpha$ (given in Table 1) are examined in this part of the study. The values of $\alpha$ are chosen such that they cover a wide range of hoppers varying from slightly sloped hoppers ($\alpha = 5.71^\circ$) to steep hoppers ($\alpha = 31^\circ$). In all cases, the ratio between the diameter of the hopper at the top surface and at the outlet is maintained fixed at the value of 10.

The results shown in Figures 4-33 to 4-35 indicate that, in all cases, the values of the stresses increase as $\alpha$ increases. The location of the critical cross-section in the hopper appears to be traveling up as $\alpha$ decreases. As shown in Figure 4.34(b), the critical cross-section is at about the mid height of the hopper with $\alpha = 31^\circ$ while it is located near the top surface of the
hopper with $\alpha = 5.71^\circ$. The value of the maximum wall pressure in the passive case increases by about 8% (per 1° decrease of $\alpha$).

The results show that $\alpha$ has a considerable effect on the design loads of the hopper since the maximum wall pressure remarkably increases as $\alpha$ increases. It is more economic to choose the geometry of the hopper such that its walls are slightly inclined. However, this might be a problem if there are limitations on the maximum allowable height of the hopper.

4.5.5 The Depth-to-Diameter Ratio H/D

In this part of the study, the depth of the bin is fixed at the value of 100 ft. while the diameter of the bin is being changed in order to maintain the values of the $H/D$ ratio given in Table 1. Figures 4-36 and 4-37 show the results of the distributions of $\sigma_{zz}$ and $\sigma_w$ with different $H/D$ ratios for both active and passive cases.

In the active case, the values of both $\sigma_{zz}$ and $\sigma_w$ decrease with the increasing $H/D$ ratio. The relation between the active stresses at any level in the bin and $H/D$ appears to be linear, as shown in Figure 4.38 for the two curves of active $\sigma_{zz}$ and $\sigma_w$. The same cannot be said for the passive case where the rates of change of the passive $\sigma_{zz}$ and $\sigma_w$ are much higher at $H/D = 1$ and 2.5 than at $H/D = 8$ and 10 as shown in Figure 4.38 for the two curves of passive $\sigma_{zz}$ and $\sigma_w$. Also, it should be noted that the value of the maximum passive $\sigma_w$ for $H/D = 1$, which is critical for the structural design of the bin
walls, is more than ten times the value of the maximum passive $\sigma_w$ for $H/D = 10$.

It can be concluded from these results that the design loads in tall bins with $H/D$ ratio greater than 5 are considerably smaller than the corresponding ones in short bins with $H/D$ ratio less than 5. Thus, it is more economic to choose the geometry of the bin such that the ratio $H/D$ is greater than 5 if possible.
Table 1 Parameters of the study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range of chosen values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$ (degree)</td>
<td>35 40 45 50 - -</td>
</tr>
<tr>
<td>$\sigma_o$ or $\sigma_{h2}$ (psf)</td>
<td>0 500 1000 1500 2000 3000</td>
</tr>
<tr>
<td>$\phi_w$ (degree)</td>
<td>10 15 20 25 35 -</td>
</tr>
<tr>
<td>$H/D$</td>
<td>1 2.5 5 8 10 -</td>
</tr>
<tr>
<td>$\alpha$ (degree)</td>
<td>5.71 11.3 21.8 31 - -</td>
</tr>
</tbody>
</table>
Figure 4.1 Janssen's analysis for stresses in cylindrical bins.
Figure 4.2 Janssen's distribution of (a) wall and (b) vertical stresses in a cylindrical bin.
Figure 4.3 Mohr circle for the stresses near the walls of the bin.
Figure 4.4 Equilibrium of forces in a general differential slice in a bin.
Figure 4.5 Equilibrium of forces in a differential slice in a hopper.
Figure 4.6 Mohr circle for the stresses near the walls of the hopper.
Figure 4.7 Distributions of (a) wall and (b) vertical stresses in a conical hopper.
Figure 4.8 Piecewise linearization of the $\gamma - \sigma_{zz}$ curve for compressible materials.
Properties of Calcium Carbonate

Loose Bulk Density = 46.12 pcf
Packed Bulk Density = 83.0 pcf
Working Bulk Density = 62.507 pcf
Effective Angle of Friction = 38°
Angle of Wall Friction = 15°

Figure 4.9 Pressure distributions in the carbonate stored in a cylindrical bin (300 m ft.) using Janssen's method and the Density Piecewise Linearization (DPL) technique.
Properties of Black River Hydrate (BRH)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose Bulk Density</td>
<td>18.0 pcf</td>
</tr>
<tr>
<td>Packed Bulk Density</td>
<td>39.2 pcf</td>
</tr>
<tr>
<td>Working Bulk Density</td>
<td>29.465 pcf</td>
</tr>
<tr>
<td>Effective Angle of Friction</td>
<td>37°</td>
</tr>
<tr>
<td>Angle of Wall Friction</td>
<td>15°</td>
</tr>
</tbody>
</table>

Figure 4.10 Pressure distributions in the BRH stored in a cylindrical bin (20' x 100') using Janssen's method and the Density Piecewise Linearization (DPL) technique.
Figure 4.11 Pressure distributions in the MH-0.5 stored in a cylindrical bin (20' x 100') using Janssen's method and the Density Piecewise Linearization (DPL) technique.
Figure 4.12 Pressure distributions in the MH-1.0 stored in a cylindrical bin (20' x 100') using Janssen’s method and the Density Piecewise Linearization (DPL) technique.
Properties of Modified Hydrate (MH-1.3)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose Bulk Density</td>
<td>19.517 pcf</td>
</tr>
<tr>
<td>Packed Bulk Density</td>
<td>37.354 pcf</td>
</tr>
<tr>
<td>Working Bulk Density</td>
<td>28.034 pcf</td>
</tr>
<tr>
<td>Effective Angle of Friction</td>
<td>40°</td>
</tr>
<tr>
<td>Angle of Wall Friction</td>
<td>15°</td>
</tr>
</tbody>
</table>

Figure 4.13 Pressure distributions in the MH-1.3 stored in a cylindrical bin (20' x 100') using Janssen's method and the Density Piecewise Linearization (DPL) technique.
Figure 4.14 The typical parameters used in the parametric study.
Figure 4.15 Changes of the vertical stress distribution in cylindrical bins with the effective angle of internal friction.
Figure 4.16 Changes of the wall stress distribution in cylindrical bins with the effective angle of internal friction.
Figure 4.17 Effect of the effective angle of internal friction on the stresses at the bottom of the cylindrical bin.
Figure 4.18 Changes of the vertical stress distribution in a conical hopper with the effective angle of internal friction.
Figure 4.19 Changes of the wall pressure distribution in a conical hopper with the effective angle of internal friction.
Figure 4.20 Effect of the effective angle of internal friction on the stresses in a conical hopper.
Figure 4.21 Changes of the vertical stress distribution in cylindrical bins with the surcharge at z = 0.
Figure 4.22 Changes of the wall stress distribution in cylindrical bins with the surcharge at z = 0.
Figure 4.23 Effect of the surcharge on the stresses at the bottom of the cylindrical bin.
Figure 4.24 Changes of the vertical stress distribution with the surcharge in a conical hopper.
Figure 4.25 Changes of the wall pressure distribution with the surcharge in a conical hopper.
Figure 4.26 Effect of the surcharge on the stresses in a conical hopper.
Figure 4.27 Changes of the vertical stress distribution with the angle of wall friction in a cylindrical bin.
Figure 4.28 Changes of the wall pressure distribution with the angle of wall friction in a cylindrical bin.
Figure 4.29 Effect of the angle of wall friction on the stresses at the bottom of the cylindrical bin.
Figure 4.30 Changes of the vertical stress distribution with the angle of wall friction in a conical hopper.
Figure 4.31 Changes of the wall pressure distribution with the angle of wall friction in a conical hopper.
Figure 4.32 Effect of the angle of wall friction on the stresses in a conical hopper.
Figure 4.33 Changes of the vertical stress distribution with the hopper angle in a conical hopper.
Figure 4.34 Changes of the wall pressure distribution with the hopper angle in a conical hopper.
Figure 4.35 Effect of the hopper angle $\alpha$ on the stresses in a conical hopper.
Figure 4.36 Changes of the vertical stress distribution in cylindrical bins with the depth-to-diameter ratio H/D.
Figure 4.37 Changes of the wall stress distribution in cylindrical bins with the depth-to-diameter ratio H/D.
Figure 4.38 Effect of the depth-to-diameter ratio H/D on the stresses at the bottom of the cylindrical bin.
CHAPTER V
CONCLUSIONS AND FUTURE WORK

Based on the parametric study and experimental work conducted in this research, the following conclusions and recommendations can be drawn:

A. For the stress analysis and structural design of silos:

   1. Reliable design of silos can be achieved only if the mechanical properties of the handled bulk material are accurately measured and utilized in the design.

   2. The relation between the bulk density and the consolidating pressure has to be determined from direct measurements of the variable density of the bulk material when subjected to different levels of consolidating pressures.

   3. The new technique of bulk density piecewise linearization (DPL) proposed in this study can be easily implemented in the solution of the stress field in silos using the method of differential slices. The DPL technique improves the results obtained in the stress analysis as it accurately accounts for the
variation of the bulk density of the material while being subjected to consolidating pressures in the silo.

4. For bulk materials with high compressibility, such as powders, the working bulk density cannot be used to represent the state of the material in the silo. The study shows that the maximum wall pressure predicted by the method of differential slices would be underestimated by about 30 to 40% if the working bulk density is assumed for the bulk material in the silo. Thus, it is recommended in this study that the standard codes for practice and design, such as the ACI, should explicitly state that in highly compressible materials, the bulk density cannot be assumed to be constant throughout the silo and more detailed measurements and calculations should be carried out to account for the effect of the variable density on the predicted design loads.

5. The effective angle of internal friction has no major effect on the value of the maximum wall pressure predicted in the silo. However, in bin/hopper silos it affects the value of the surcharge pressure on the hopper which equals the vertical pressure calculated at the bottom of the bin.

6. The presence of surcharge pressures on the top surface of the bulk material produces high localized wall pressures in their immediate region in the silo.
7. The critical case which should be considered for predicting the design wall pressures in the bin and the hopper is the passive case which produces higher levels of pressure than the active case. However, for a bin/hopper silo, the critical case of wall pressure in the hopper is obtained for an active bin and a passive hopper, which is the case occurred in the silo at the initiation of discharge.

8. The angle of wall friction is a critical parameter in evaluating the wall pressures in the silo. The wall pressure distribution determined in this study is shown to be highly sensitive to the changes of the value of the angle of wall friction especially when this value is small. Thus, it is recommended in this study that standard tables for values of this angle for different materials should not be used in the design of silos and actual measurement of this angle should be performed. Special care should be taken when the measurement of this angle show that it has a small value (< 10°).

9. The angle of the hopper has a considerable effect on the wall pressure distribution in the hopper. As the angle of the hopper increases (i.e., steeper hopper), the value of the maximum wall pressure increases.

10. It is more economic to choose the geometry of the bin such that it would have an aspect ratio (depth-to-diameter ratio) greater than 5. In this case, the wall pressures predicted in the bin would be considerably lower than those predicted in bins with low depth-to-diameter ratio.
B. For the flow properties of bulk materials:

1. The flow properties of cohesive powders can be improved by using chemical additives which reduce the cohesive strength of the modified powders and enhance their flowability. The results of this study show that the cohesive strength of the tested hydrates reduces as the weight percentage of the calcium lignosulfonate in the modified hydrate increases.

2. The use of low-cost additives to improve the flowability of cohesive powders could be an efficient solution to reduce some of the operating problems which are usually experienced with this type of hard-to-flow materials, such as arching, instead of the traditional solutions often followed with these problems such as hammering the hopper or the use of electric vibrators. The study shows that the use of calcium lignosulfonate, which is a waste product in the paper industry, as an additive to the hydrate powders improves the flow properties of the final modified product.

3. The hydrates have better dispersibility than the calcium carbonate powder. This is important to some industrial processes which require high dispersibility of powders, such as the flue-gas desulfurization process. Moreover, the results of this study show that the dispersibility of the modified hydrates increase as the weight percentage of the lignosulfonate in the modified hydrate increases.
4. Generally, floodable flow is to be expected when handling powders stored in silos. The results of this study show that all of the tested powders, including the baking soda which is a cohesionless material, have fairly high degree of floodability.

5. Measurement of the flow properties of bulk materials should be conducted under different operating conditions which are expected during the handling of bulk materials in silos. It is shown in this study that the flowability of the powders examined reduces by about 15% as their moisture content increases.

For future research, the following topics are suggested:

1. To study the possibility of existence of hoppers operating in an active state of stress. This may be verified experimentally by measurements of the wall pressure in model hoppers with different geometry and operating conditions.

2. To extend the density piecewise linearization technique to the solution of stresses in hoppers.

3. To examine the effect of other types of additives on the flow properties of hard-to-flow cohesive powders.

4. To utilize the flow properties determined in this study, such as the angle of internal friction and cohesion, in the development of constitutive equations.
which are suitable for the stress analysis of cohesive powders during the different operating phases of silos. The use of the finite element method is an appropriate technique to be employed in the analysis to examine and evaluate these constitutive equations.
APPENDIX A

COMPUTER PROGRAM FOR THE STRESS ANALYSIS IN BINS USING THE DPL TECHNIQUE

*TITLE: AZHAR.FOR
*Vertical and wall pressure in Bins
************************************************************************
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION Z(21), SV(21), SVJ(21), ERV(21), SW(21), SWJ(21), ERW(21)
DIMENSION ZG(9), HG(9), AG(8), RKG(8), G(9), SVGJ(9), SVG(9), SVGJ(9), HT(21)
************************************************************************
* Z(I): DEPTH WITHIN A STEP; ZG(I): HEIGHT OF A STEP;
* HG(I): TOTAL DEPTH OF A STEP; H(T): TOTAL DEPTH
* AG(I): SLOPE OF DENSITY IN A STEP; RKG(I): EXPONENT IN A STEP
* G(I) AND SVG(I): DATA OF DENSITY AND APPLIED PRESSURE
* SVGJ(I): JANSSEN VERTICAL PRESSURE AT THE BEGINNING OF A STEP
* SV(I) = vertical pressure; SW(I) = wall pressure;
* SVJ(I) and SWJ(I) = corresponding Janssen's pressures
* ERV(I) AND ERW(I) = ERROR PERCENTAGE FOR VERTICAL AND
* WALL PRESSURES W.R.T. JANSSENS STRESSES.
************************************************************************
OPEN (UNIT=1,FILE='AZHAR.INP',STATUS='OLD')
OPEN (UNIT=2,FILE='AAZHAR.OUT',STATUS='OLD')
DO 10 I=1,21
   Z(I)=0.
   HT(I)=0.
   SV(I)=0.
   SVJ(I)=0.
   ERV(I)=0.
   SW(I)=0.
   SWJ(I)=0.
10 ERW(I)=0.
   READ(1,*') D, H, EFI, EFIW, GJ
* D = bin hydraulic diameter (=2b for plane silo), H = bin height,
* FI = angle of internal friction
* FIW = effective angle of wall friction
* GJ = Working constant bulk density,

\[
\begin{align*}
FI &= EFI \times \frac{3.141592654}{180} \\
FIW &= EFIW \times \frac{3.141592654}{180} \\
AK &= \frac{1 - \sin(FI)}{1 + \sin(FI)} \\
PK &= \frac{1}{AK} \\
AN &= 4 \times \tan(FIW) \times AK / D \\
PN &= 4 \times \tan(FIW) \times PK / D
\end{align*}
\]

* AK & PK = active & passive K, AN&P=cons. active&passive exponent

WRITE(2,100)
100 FORMAT(5X,'JANSSEN VS DPL ')//
WRITE(2,95) D,H,EFI,FI,EFIW,FIW,GJ
95 FORMAT(2X,'BIN HYDRAULIC DIAMETER = ',F8.4,5X,' BIN HEIGHT = ',F8.4/2X,'ANGLE OF INTERNAL FRICTION = ',F6.3,'= ',F10.7 / 2X,'ANGLE OF WALL FRICTION = ',F6.3,'= ',F10.7 / 2X,'WORKING BULK DENSITY = ',F10.6/) DO 12 I=1,8 AG(I)=0. RKG(I)=0. DO 13 I=1,9 ZG(I)=0. HG(I)=0. G(I)=0. SVG(I)=0. 13 SVGJ(I)=0. READ(1,*) (G(I),I=1,9) READ(1,*) (SVG(I),I=1,9) SVGJ(1)=SVG(1) WRITE(2,115) 115 FORMAT(5X,DENSITY',2X,'PRESSURE',6X,'JANS VP',8X,'EQUIV. DEPTH',4X,'TOTAL DEPTH/) DO 15 I=1,8 AG(I)=(G(I+1)-G(I))/(SVG(I+1)-SVG(I)) RKG(I)=AN-AG(I) ZG(I+1)=(1/RKG(I))*LOG((RKG(I)*SVG(I)-G(I))/(RKG(I)*SVG(I+1)-

---

---

---
+ G(I))

HG(I+1)=HG(I)+ZG(I+1)

15 SVGJ(I+1)=GJ/AN+(SVG(1)-(GJ/AN))*EXP(-AN*HG(I+1))

WRITE(2,120)(G(I),SVG(I),SVGJ(I),ZG(I),HG(I),I=1,9)

120 FORMAT(2X,F10.5,2X,F10.5,2X,F14.9,2X,F14.9,2X,F14.9)

WRITE(2,110)

110 FORMAT(//3X,' depth',5X,'DEPT',5X,'J VP',5X,'V PRESS',4X,'J WP',
+ 5X,'W PRESS',2X,'WERROR'//)

DO 25 J=1,8
DO 20 I=1,11
Z(I)=10*(I-1)*ZG(J+1)/100
HT(I)=Z(I)+HG(J)
IF (HT(I) .GE. H) THEN
GO TO 40
ELSE
CONTINUE
END IF
SVJ(I)=GJ/AN+(SVGJ(J)-(GJ/AN))*EXP(-AN*Z(I))
SWJ(I)=SVJ(I)*AK
SV(I)=G(J)/RKG(J)+(SVG(J)-(G(J)/RKG(J)))*EXP(-RKG(J)*Z(I))
SW(I)=SV(I)*AK
IF (SVJ(I) .EQ. 0) THEN
ERV(I)=0
ELSE
ERV(I)=100*(SV(I)-SVJ(I))/SVJ(I)
END IF
IF (SWJ(I) .EQ. 0) THEN
ERW(I)=0
ELSE
ERW(I)=100*(SW(I)-SWJ(I))/SWJ(I)
END IF
WRITE(2,130)Z(I),HT(I),SVJ(I),SV(I),SWJ(I),SW(I),ERW(I)


20 CONTINUE
25 CONTINUE
IF (HT(11) .LT. H) THEN
GO TO 30
ELSE
CONTINUE
END IF
30 HTT=HT(11)
DO 60 I=1,10
Z(I)=I*(H-HTT)/10
HT(I)=Z(I)+HTT
SVJ(I)=GJ/AN+(SVGJ(I)-(GJ/AN))*EXP(-AN*Z(I))
SWJ(I)=SVJ(I)*AK
SV(I)=G(J)/RKG(J)+(SVG(J)-(G(J)/RKG(J)))*EXP(-RKG(J)*Z(I))
SW(I)=SV(I)*AK
ERV(I)=100*(SV(I)-SVJ(I))/SVJ(I)
ERW(I)=100*(SW(I)-SWJ(I))/SWJ(I)
60 WRITE(2,130) Z(I),HT(I),SVJ(I),SV(I),SWJ(I),SW(I),ERW(I)
GO TO 900

40 Z(I)=H-HG(J)
HT(I)=H
SVJ(I)=GJ/AN+(SVGJ(J)-(GJ/AN))*EXP(-AN*Z(I))
SWJ(I)=SVJ(I)*AK
SV(I)=G(J)/RKG(J)+(SVG(J)-(G(J)/RKG(J)))*EXP(-RKG(J)*Z(I))
SW(I)=SV(I)*AK
ERV(I)=100*(SV(I)-SVJ(I))/SVJ(I)
ERW(I)=100*(SW(I)-SWJ(I))/SWJ(I)
WRITE(2,130) Z(I),HT(I),SVJ(I),SV(I),SWJ(I),SW(I),ERW(I)
GO TO 900
900 END
LIST OF REFERENCES


