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1995
To Kim and Noah
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VITA

December 10, 1965 ...................................... Born - Coshocton, Ohio

1988 .......................................................... B.S. in Education
                          The Ohio State University
                          Columbus, Ohio

1988 - 1993 .............................................. Graduate Teaching Assistant
                          Department of Mathematics
                          The Ohio State University
                          Columbus, Ohio

1990 .......................................................... M.A. in Mathematics
                          The Ohio State University
                          Columbus, Ohio

1993 - 1994 .............................................. Graduate Research Assistant
                          The Ohio State University
                          Columbus, Ohio

1994 .......................................................... Adjunct Faculty
                          Dept. of Mathematics & Comp. Sci.
                          Wittenberg University
                          Springfield, Ohio

FIELDS OF STUDY

Major Field: Education

Studies in Mathematics Education with Drs. Douglas T. Owens and Sigrid Wagner
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CHAPTER I
PROBLEM DEFINITION

1.0 Introduction

In today's world it is difficult to escape the functional relationships inherent in our everyday lives. From stock reports to newspaper articles, functional relationships are conveyed through many representations. One of the more common representations seen is the use of graphical information. We rely on information presented in this manner to get an idea of functional relationships at a glance. Graphs allow for prediction and data analysis through a visual representation of easily recognizable patterns. With the advent of computer and calculator technology, the use of graphical representations is becoming a common practice, especially in an age where the economy is undergoing a transition from industry to information management. It is the issue of the use of technology in the learning of graphical concepts which is addressed in this study.

1.1 Need for the Study

Mathematics education has experienced a dramatic change in recent years. The adoption of the Curriculum and Evaluation Standards (NCTM, 1989) has sparked change for the mathematics classroom. One of the key changes called for in the Standards is the use of technology, and
in particular computer and calculator graphing technology. Some of the changes suggested by NCTM are:

**Increased** attention should be given to:

(a) the use of calculators and computers for concept development and as tools for learning and doing mathematics,

(b) the connections among a problem situation, its model as a function in symbolic form, and the graph of that function,

(c) using statistical methods to describe, analyze, evaluate, and make decisions,

(d) identifying and using functional relationships, and

(e) using appropriate technology for computation and exploration.

**Decreased** attention should be given to:

(a) paper-and-pencil graphing of equations by point plotting, and

(b) the graphing of functions by hand using tables of values (NCTM, 1989, pp. 126–127).

The National Council of Teachers of Mathematics (NCTM) is but one group to call for change in the curriculum regarding interpretation of graphical representations. The American Association for the Advancement of Science (1989) and The National Council of Supervisors of Mathematics (1988) also support the use of technology in the teaching and learning of mathematics. In fact, the National Research Council states, "Priorities for mathematics education must change to reflect the way computers are used in mathematics" (NRC, 1989, p. 63). The emphasis here has shifted from that of algorithm proficiency, to that of exploration with tools readily available to the mathematician or scientist. To expect
students to learn mathematics using the techniques and tools of the precomputer age is as ridiculous as expecting weather forecasters to predict the weather without the use of satellite information, relying solely on the Farmer's Almanac.

In stating reasons for the use of graphing technology in the mathematics classroom, Demana and Waits (1992) suggest that such use can help students "learn to value mathematics by solving realistic problems" (p. 95). There are many real-life problems which cannot be solved algebraically. The use of the computer and/or calculator can allow students the opportunity to explore these types of problems in a meaningful context which is not only interesting, but also relevant to their experiences. Heid (1990) states that "constant work with realistic applications increases students' sensitivity to mathematical modeling issues" (p. 196).

In a program entitled Computer-Intensive Algebra, Heid (1990) illustrates activities in which the students participate to develop mathematical understanding. In this activity, the students are given data on altitude and temperature, and asked to make sense of it using a function-fitting program to fit a linear function to the data. They then use the algebraic representation to predict the temperature at altitudes not present in the data set. This prompts the students to use the technology and then interpret the results. The time consuming task of graph generation and curve-fitting are suppressed and the students can devote more time to the real mathematical underpinnings which the teacher desires to spotlight with the activity. In discussing suggestions for using
this type of technology, Heid states that when using these devices, the teacher should be certain to choose software or other tools that will place the decision making in the hands of the students and away from the teacher. The students must command autonomy in order to construct their own conceptualizations. Another suggestion offered by Heid is that the teacher must engage the students in computer-assisted analysis of realistic problem situations. It is important for students to see some relevance in the activities and mathematics being presented. After all, it is unreasonable to expect that all students are going to be mathematicians and appreciate the "beauty" present in the mathematical structure.

In support of this idea for the use of realistic problems, Leinhardt, Zaslavsky, and Stein (1990) also state that often students who can solve graphing or function problems within the setting of the mathematics classroom cannot solve isomorphic problems within the context of a science classroom. One of the possible reasons for this difficulty posed is that within the mathematics setting, the usual progression of ideas goes from the algebraic representation to a table of generated data points to a set of plotted data points and finally to a graph. Whereas in the science context, the progression usually goes from observation to ordered pairs of data to plotted data points to a graphical representation fitting the data points, and finally to an algebraic functional representation of the relationship. It is for this reason that students must experience both progressions in each of the mathematics and science contexts. Present technology allows for both sequencing of representations to take place in both the science and mathematics environments.
While the use of technology can be of great value in developing the understanding of mathematical concepts, one must also keep in mind other factors inherent in the use of the machine which could confound these efforts. Salomon (1985) states, "one needs to become aware of the often overlooked gap between the opportunities afforded by the new information technologies and their actual impact on learning and development" (p. 207). Salomon goes on to say "that whereas computers entail many, and often unique and individually tailored, learning opportunities, their realization greatly depends on the learners' choice as to how to handle them mindfully" (p. 207).

There is some evidence to support the view that in using technology, students will lose some autonomy in the problem solving process (Reys, Bestgen, Rybolt, & Wyatt, 1980; Nachmias & Linn, 1987; Tall, 1989). Reys et al. report the use of calculators which were programmed to give incorrect answers. In this study, 36 percent of the subjects proceeded through a 7 question exercise without verbalizing any concern with regards to the accuracy of the calculator.

Nachmias and Linn (1987) report student participation in a physics experiment using Microcomputer-Based Laboratory (MBL). MBL is a means of collecting data through the use of probes and other devices and then displaying the data graphically in real-time as the experiment proceeds. In this instance, a probe was inserted into a liquid to monitor the temperature as a function of time. When the data were presented graphically, the large pixel size of the screen produced a jagged-looking graph rather than a smooth curve. Roughly one third of the students
interpreted this as a true representation of the cooling process. In other words, they thought that the liquid remained at a constant temperature for a period of time and then suddenly dropped in a discrete manner to the next lower pixel level. Even after direct instruction explaining that the jaggedness of the graph was due to software and hardware limitations, the students were still not influenced.

Tall (1989) expresses a concern for the new obstacles associated with the technological paradigm. He states that, "the 'authority of the computer' may therefore be an impediment to learning, especially in the early stages" (p. 90). This concern for the authority of the machine is justified in examining the aforementioned study by Reys et al. (1980). In exploring the estimation skills of students, the following questions were addressed:

(1) Do good estimators have confidence in their estimate, even when confronted with conflicting evidence in the form of a calculator answer?

(2) Are good estimators sensitive to calculator errors?

(3) How large must the calculator error be before good estimators question the calculator answer? (p. 223)

The calculators were programmed to give answers with increasingly larger percent errors. For the first three questions, the calculator gave an answer that was 10% above the actual answer. For the next two questions, the calculator gave an answer that was 25% above the actual answer. And for the last two questions, the calculator gave an answer
that was 50% above the actual answer. When a conflict arose and was
detected by the students, some of the common comments listed were:

- "I must have entered it wrong on the calculator; either that or I'm
thinking wrong."
- "That [student's estimate] is kind of far off — 400 off. That wouldn't
be a good estimate at all."
- "The battery could be bad, or I could have entered it wrong."
- "I don't understand. I don't understand what I'm doing wrong."
- "It doesn't look right, but if that's what the calculator says, then it's
probably right. It still doesn't look right."
- "I'd like to enter that one again."
- "Can I work this one out on paper?"
- "I'm trying to figure out why it [calculator response] is 1627."
- "It shouldn't make a difference to work it out by hand, but I don't
know."
- "The calculator is wrong — no, it couldn't be."
- "I thought my estimate would be really close, but it isn't."
- "I don't think the calculator seems reasonable." (pp. 229-230)

These comments suggest that students place a great deal of authority in
the machine. The data from this study also concur that students are
willing to ignore their own common sense and follow the lead of the
calculator.

Tall (1989) discusses avenues for future research brought about by
the new technological paradigm. Some of the questions he feels need
addressing are:
(a) How does the computer environment change the nature of mathematical concepts, the development of students' conceptualizations, and the related cognitive obstacles?

(b) How can we encourage students to participate actively in the construction of appropriate meanings, some of which will be very different in the future paradigm? (p. 91)

These are important questions which need to be addressed. There are so many benefits from the use of technology, but this should not prevent us from investigating difficulties inherent in the use of such technology.

The effectiveness of graphing technology has been examined quite extensively in the last few years, a large portion of which using MBL (Mokros, 1985; Mokros & Tinker, 1987; Brasell, 1987ab; Linn, 1988; and Nichols, 1992). Together, these studies seem to suggest that the use of graphing technology can increase the understanding of mathematics and science concepts for the students taking advantage of the machine's capabilities. Further, it is well documented in the mathematics education community that the use of calculators does not hinder the skills of students using them (Hembree & Dessart, 1986). But the point made by Reys et al. (1980) must also be taken into account. We must examine what factors, if any, may affect student understanding of mathematics concepts when immersed in a technology rich environment.

Many of the studies cited in this section have examined the use of MBL. MBL uses a method of data collection which simulates continuous data collection even though the data being collected are discrete. If one were to zoom in on a portion of the graph or change the scale, it would
become apparent that the graph is composed of discrete points
accumulated by the probes and/or other devices. It is for this reason that
the study presented here will focus on data analysis presented in a clearly
discrete manner. After all, it is not always possible to collect the data in
which one is interested, using the probes available through MBL. Surveys
or other forms of social science research rarely allow such luxuries.

There are several anticipated factors surrounding the use of
technology which may affect the student's perception of the authority of
the machine within the graphing and data analysis setting. These factors
may also interact to produce conflict which can either support the
computer as the ultimate authority or cause the student to re-examine the
computer feedback due to conflict within his/her already existing schema.
Some of these factors are: (a) algebraic rule, (b) problem situation, (c)
regression line, (d) linear affinity, (e) and the scatterplot. The
relationships among these factors are presented in Figure 1. This
theoretical model of factor relationship is meant to provide a basis for
cognitive conflict by which students may choose to challenge the authority
of the machine. It is anticipated that the use of multiple representations
will be a factor of influence for student perception of technological
authority. However, it is also expected that other factors will surface
throughout the investigation. A theoretical model for the over all
relationship of authority influences will be presented in Chapter 5. The
purpose of the current theoretical model is to serve as a motivator for
cognitive conflict so that other factors may be identified during the study.
The *algebraic rule* refers to the algebraic representation supplied by the computer as a fit for the data. The *situation* refers to the context within which the data are presented such as population verses time for New York City during the 1980's. The *regression line* refers to the graphical representation of the algebraic rule superimposed on the data plot. The *scatterplot* refers to the graphical representation of the data plotted in the Cartesian plane.

*Linear affinity* requires a more in-depth explanation. Karplus (1979), when conducting a study on student understanding of graphs without the use of technology, found that students tend to have the impression that straight lines as opposed to curves are intrinsically accurate. Thus it is conceivable that some of the students may choose to follow the linear regression model due to this misconception.
Pea (1987) suggests a taxonomy that distinguishes between "purpose functions" which affect the degree to which students are inclined to initiate mathematical thinking, and "process functions" which support the activity of mathematical thinking once the student has decided to do so. There are five general categories of process functions related to cognitive technologies. Pea identifies these categories as tools for (1) developing conceptual fluency, (2) mathematical exploration, (3) integrating different mathematical representations, (4) learning how to learn, and (5) learning problem-solving techniques.

The proposed research will concentrate on (3) and examine the role of the computer/calculator as an interactive tool for understanding the difficulties that students experience while working with the different
representations of mathematical relations: the problem situation (natural-language), the algebraic representation (equation), and the graphical representation.

1.2 Problem Statement

This study examines the perceptions of students using graphing technology regarding the authority of the technology in analyzing graphical relationships of data. Further, it probes the students' attempts to resolve conflict when there is a discrepancy between the model of curve-fitting suggested by the machine and the data presented as a scatterplot. The study addresses the following questions:

1. Will students who use graphing technology choose to believe the model of curve-fitting suggested by the machine even when that model differs significantly from the data?

2. When asked to predict within the interval of the data available, will students choose to use the computer generated model for predicting rather than the scatterplot of the data when the computer model and scatterplot differ significantly?

3. Are students who choose the computer model to predict, more confident about their prediction than those who use only the data to predict?

4. What factors surrounding the computer give authority to the machine in the eyes of the students?
1.3 Rationale for the Study

Graphing technology is becoming more commonplace in the mathematics classroom every year. The use of such technology holds many benefits for the exploration and concept development of the student. Some studies (Zehavi, Gonen, & Taizi, 1987; Dreyfus & Eisenberg, 1987) describe an increase in students' graphing ability with the use of computer software packages. Mokros and Tinker (1987), Brasell (1987b), and Thornton (1989) report an increase in performance on graphical interpretation tasks when Microcomputer-Based Laboratory is used for analyzing physical situations.

With the increased availability of data analyzing tools, one must ask, what aspects of such tools make their use so effective? One of the reasons may be that these tools allow for the use of multiple representations. Kaput (1989) gives what he considers the three core representation systems for algebra: (1) equations, (2) tables, and (3) coordinate graphs. In order to be successful problem solvers, students must be able to leap from one representation to another in the analyzation of a problem situation (Janvier, 1987). The need for inter-representational transfer suggests that these representations may be a means for exploring the use of the computer technology. This study will attempt to use these representations to identify aspects of the computer technology which might prompt students to challenge the authority of the machine. Kaput (1987) suggests that representation is made up of five facets: (1) the represented world, (2) the representing world, (3) the aspects of the represented world, (4) the aspects of the representing world doing the
representing, and (5) the correspondence between the two worlds. When conflict arises between these facets, will this conflict be sufficient to prompt students to challenge the authority of the computer and/or calculator?

The three major representations focused on by this study will be: (1) algebraic, (2) natural-language, and (3) graphical. Each of these representations address various anticipated factors as well as other factors not yet considered. Of the anticipated factors, the algebraic representation will include the algebraic rule presented by the computer for the fit of the data. The natural-language representation will include the problem situation factor which describes the context in which the data are presented. The graphical representation encompasses several of the anticipated factors: scatterplot, regression line, and linear affinity. In addition to these factors, this category of graphical representation will also include any aspect of the responses related to visual feedback of the data by the computer including the tabular form of the data.

The perspective taken in this study will center around the cognitive conflict theory. Piaget suggests that motivation for learning comes about through an imbalance experienced by the learner. This Piagetian construct of disequilibrium is the main driving force for learning. Without this imbalance, it is likely that no learning takes place.

Research (Vinner, 1983) suggests that students build their concept image in a way that is not always coherent and consistent. This construction of a concept can result in a generic understanding, which may result in errors. It is not until these errors are challenged through a
sense of disequilibrium that the student is able to gain a broader picture of the concept.

Cognitive conflict is related to the Piagetian idea of disequilibrium. Cognitive conflict arises when there exists a difference between a student's existing knowledge and the constraints of a problem situation (Behr & Harel, 1990). Behr and Harel go on to classify conflict situations into the following four categories: (1) missing a fragment, (2) violation of a rule, (3) unexpected result, and (4) matching problem components. This study will focus on the third category, unexpected result, as a means to evoke conflict in the students' minds with regard to the technology being used. The unexpected result category will be achieved through the use of differences in representations for a problem situation. It is through these differences in representation that the researcher will attempt to evoke conflict in the minds of the students.

The initial instrument gives questions for students to analyze where there is conflict between representations. These questions are designed to evoke a cognitive conflict in the minds of the students prompting them to make sense of the information being presented to them. Later, the same type of conflict will be presented during both the normal task interviews where students will be asked to predict using given data, and task interviews where a graphing calculator will give incorrect feedback to the student prompting the student to make sense of the problem situation in light of the incorrect feedback. It is suspected that the students will follow the responses of the machine giving little attempt to resolve the conflict. This suspicion is consistent with data from
a pilot study conducted during Spring quarter 1994 where only 1 out of 15 of the subjects acknowledged an error between representations for the regression line used to analyze a set of data.

Nachmias and Linn (1987) as well as Reys et al. (1980) cite evidence that students may view the computer as the ultimate authority in the problem solving process. This poses the question, will students lose autonomy due to the use of the technological paradigm?

The goal of this study is to explore, in-depth, factors involved in the student's perception of the authority of the computer and/or calculator. It will also attempt to examine how far off the machine must be before students begin to question the authority of the computer and/or calculator.

1.4 Definition of Terms

The terms in the study are defined below in alphabetical order:

**algebraic representation**: The representation of the relationship between two or more variables using a mathematical rule consisting of variables, constants, and operations.

**conflict of representations**: Any discrepancy between different means of data or pattern representation.

**graph**: A visual display showing the empirical relationship between two or more variables plotted in the Cartesian plane.

**graphical representation**: The representation used to examine the empirical relationship between two or more variables including the scatterplot, regression line, and data table.
**graphing technology:** Graphing calculators or computer software which allows the user to manipulate the graph on the screen.

**Microcomputer-Based Laboratory (MBL):** The use of probes and other data collecting devices used to collect and input data directly into the computer and/or calculator and then display the data graphically either at a later time, or in real-time as the data are being collected.

**regression line:** The least squares regression line for given data.

**scatterplot:** Graphical representation of data points.

**table:** Data listed in an array.

### 1.5 Overview of the Study

The research was conducted at Wittenberg University in Springfield, Ohio during Autumn Quarter 1994 with a pilot study being conducted in Spring 1994. Students in the study are enrolled in a first course in statistics. The course uses a computer software package (DUSA) which allows the student to do regression analysis of data as well as other statistical analyses.

The students will be given an instrument which consists of open-ended questions regarding data analysis and which examines representational preferences as well as conflict between representations. From the data collected by the instrument, students were interviewed to examine how representational conflict influences their perception of the authority of the computer and/or calculator. In addition, students were also interviewed and asked to work through tasks which required them to use the technology to make predictions about the data as well as tasks in
which the regression line was altered and shifted vertically with increasing percentage throughout the interview to see how far off from the actual regression line the machine must be before the students question the machine's authority.
CHAPTER II
REVIEW OF THE LITERATURE

2.0 Introduction

In recent years there has been a growing interest in the use of graphing technology for the teaching of mathematics. This interest has been, in part, sparked by the recommendations of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and, Everybody Counts: A Report to the Nation on the Future of Mathematics Education (NRC, 1989). The Standards not only call for the use of technology in the classroom, they also call for the integration of the mathematical concepts across the curriculum. Everybody Counts states a concern for the future development of the sciences in the United States. One of the advantages for the use of graphing technology is that it facilitates the integration of mathematics across the curriculum. Data from real-life situations can be presented and then analyzed by students. This ability to apply mathematics within a meaningful context is one of the potential benefits of the use of graphing technology.

The review of the literature is centered around the following questions.

1. What insight can be gained from research examining the use of computer or calculator manipulation of data for analysis?
2. When faced with different representations for data analysis, what does the research suggest regarding representational preferences by students?

3. How does the research suggest students will react to a conflict between common sense and the computer and/or calculator?

4. What does the research suggest with regards to the confidence of students in the infallibility of the computer and/or calculator?

2.1 Computer Manipulation of Data

When examining the research on data analysis, one of the methods currently evoking research is the use of the Microcomputer-Based Laboratory (MBL), (Brasell, 1987ab; Linn, 1988; Linn & Songer, 1988; Mokros, 1985; Mokros & Tinker, 1987; Nichols, 1992). This technology uses the computer along with probes to collect data of a phenomena to be studied. The data can then be presented in tabular, or graphical form for analysis. One advantage of this technology is that the graph of data can be displayed as it is being collected which allows for transfer between the physical event and the graphical representation.

It seems to be the consensus that the study of graphs can lead to a deeper understanding of physical and real-world concepts (Brasell, 1987ab; Goldberg & Anderson, 1989; Linn, 1988; McDermott, Rosenquist, & van Zee, 1987; Mokros & Tinker, 1987; and Omasta, 1984). In fact, Omasta and Lunetta (1988) even without the use of elaborate technology, showed that with calculators (TI-30) students in the experimental group did significantly better than students assigned to the control group with
respect to problems dealing with solutions in symbols, graphical analysis, and proportional reasoning.

There are many problems which students have with regard to graphing (Goldberg & Anderson, 1989). The research examined leads to several major categories for addressing and dealing with these problems.

- Difficulties in connecting graphs with physical concepts (McDermott, Rosenquist, & van Zee, 1987)
- Difficulties in connecting graphs with the real world (McDermott, Rosenquist, & van Zee, 1987)
- Transition between graphs and physical motion (Goldberg & Anderson, 1987; McDermott, Rosenquist, & van Zee, 1987)

The main avenue for dealing with these categories characterized in the research is Microcomputer-Based Laboratory (MBL).

There are several major problems noted by researchers (Mokros & Tinker, 1987; McDermott, Rosenquist, & van Zee, 1987; Goldberg & Anderson, 1989) which contribute to the misconceptions of the above categories. These misconceptions can be defined as follows:

- **Graph as a Picture** - students do not see a graph as a relationship between variables, but rather as one object.
- **Slope/Height Confusion** - when students are asked questions regarding rate of change of a graph, they state that the fastest change is occurring when the graph is at its highest point.
- **Graph Shape and Path of Motion Confusion** - students try to make the graph look like the physical event being observed.
These misconceptions are addressed in the research which follows.

2.11 Difficulties in Connecting Graphs with Physical Concepts

Brasell (1987a) worked with entire Physics classes which ranged from 7 to 17 students in each from seven rural schools in northern Florida. A total of 93 students were involved. Brasell used four levels of treatment: (1) Standard MBL, (2) Delayed MBL, (3) Pencil and paper, and (4) Test only. The goal was to examine the effects of the MBL approach for working with kinematic concepts, in particular, to see what the effects of the immediacy of the real-time graphing would be on the linking of graphical representations to the physical events. The Delayed MBL was introduced to see how important the simultaneity of the graph production was and to see if even a delay of 20 seconds would make a difference in the students' ability to link graph with physical concept. The results show that the Standard-MBL approach was significantly better than all of the other treatments at the .001 level. Moreover, the improvement by the MBL treatment group was accounted for almost totally by the immediacy of the graph production experienced in the Standard-MBL treatment.

Is the simultaneity of the physical event and its graphical representation the main feature which makes MBL effective? Beichner (1989) suggests that this is not enough. In doing a similar study, Beichner used re-animation along with the graphical representation so that the student saw the moving object and its graph at the same time. However, this approach was not as successful as that of Brasell (1987a). The conclusion made is that the student participation is an integral part of the
student's experience. The ability of the student to control the environment may play a vital role in the understanding of the physical event. Technology alone is not sufficient.

The application of the concepts learned by using the MBL seems to also give the students a sense of confidence in their work. As reported by Mokros (1987), a group of girls was making a velocity as a function of time graph of a cart which was accelerating, the slope of which was graphed as positive. When they were told by a teacher that the graph was incorrect, in particular, that the line should be horizontal, they argued that their graph was indeed correct and that the slope needed to be non-zero in order for the speed to go up. This demonstrates a resolution of the slope/height confusion discussed earlier.

Another implication is that in order to connect graphs to physical concepts, the student needs to see graphs of many different topics in science and mathematics (McDermott, Rosenquist, & van Zee, 1987). For example, if students are taking readings to study the effect of radius on the gravitational force on two objects whose mass are kept constant, they will observe a graph of the inverse square law. Similarly, if they now study the effects of radius on the electrostatic force of two objects whose charges are kept constant, they will also observe a graph of the inverse square law. This will illustrate the properties of relationships between variables, especially those which tend to be prevalent throughout nature. The relationship between the graph and the physical concept is stressed.

Such a transfer of relationships was examined by Linn (1988) in the Computer as a Lab Partner project, CLP, using MBL. Students involved
in the CLP study were dealing with the relationships of heat energy and temperature, however, the students gained considerable understanding of graphing and extended this to interpretation of motion graphs although they had not studied kinematics or motion within the graphing environment.

2.12 Difficulties in Connecting Graphs with the Real World

The major misconceptions associated with this category are *Graph as a Picture*, and *Graph Shape and Path of Motion Confusion*. In this area, students have difficulty making distinctions between the functional relationship of two variables and the visual stimuli received when observing the actual physical event.

Students seem to believe that the shape of the graph should resemble the shape of the physical environment involved in the experimental set up (McDermott, Rosenquist, & van Zee, 1987). If a ball is given an initial velocity on a "frictionless track" which is level, the student expects the graph of the position versus time graph to also be horizontal rather than a straight line with non zero slope. Choosing the appropriate graph to have the student explore can be important in reducing this misconception. For example, if the student used MBL to examine a velocity as a function of time graph of this same event, the graph would be "flat" reinforcing the misconception. A use of examples and nonexamples might be appropriate.

In connecting the graph with the real world experiences of the student, the Standard-MBL seems to be the most effective. Brasell
(1987b) states that the linking of the physical event with a simultaneous graphical representation of the same event may provide a similar linkage in memory. The real-time graphing capabilities of MBL can relieve the additional information-processing demands on the student and allow for linking of the real world events and the graphical representation of these events. Brasell (1987b) gives three possible reasons for the effectiveness of MBL: (a) the task exceeded the students' memory capacity, (b) the task required more effort than the students were prepared to give, or (c) students did not know how to retain the information about the event while waiting for the graph to be displayed. The immediacy of the graph production could aid students in looking at the real world differently, as relationships, rather than simply taking in visual information and remembering a picture of the event.

### 2.13 Transition Between Graphs and Physical Motion

Probably the most important skill required by science is the ability to leap back and forth between a graph and the physical event the graph describes. Brasell (1987b) built this transition into her study by having the students perform three activities: (a) familiarization activities introducing the equipment, (b) a prediction activity of what the distance and velocity graphs would look like for a complex kinematic event, and (c) a reproduction activity for a complex distance and velocity graph designed to get the students to "reenact" the event which made the graph. Students were significantly more successful in these areas after having the treatment of the Standard-MBL.
Mokros (1985) used an activity with MBL giving the students the roles of "dancer" and "choreographer." The choreographer's job was to explain to the dancer what he/she should do in order to reproduce the graph given by the teacher. This activity required the students to take the graphical representation and translate it into a series of verbal directions and therefore exhibit an understanding of the various aspects of the graph.

There is also a need for the student to be mobile in going between different types of graphs for the same event, (e.g. $x$ vs. $t$ and $v$ vs. $t$ graphs of the same situation), (McDermott, Rosenquist, & van Zee, 1987). Being confronted with different types of graphs of the same event at the same time can impress upon the students the differences in the way the information is displayed. This was observed by McDermott, Rosenquist, and van Zee (1987). Also, giving the students events which are obviously different but which produce the same visual representation on a graph can help make this distinction easier to understand. For example, different motion experiments could be posed to the students producing graphs such as the following (McDermott, Rosenquist, & van Zee, 1987): (See Figure 2)
Since the events producing the graphs are so different, the students may be concerned that the graphs are the same. This also gives reinforcement of the way information is obtained from each graph. To find the velocity from the first graph, the student must calculate the slope at a given time, to find the velocity from the second graph, the student simply reads the graph at a given time, to find the velocity from the third graph, the student must approximate the area under the curve from time 0 to the time desired.

The misconceptions expressed in this section can be cleared up by the use of MBL. Mokros and Tinker (1987), based on their study, give several reasons for the effectiveness of MBL: (1) MBL uses multiple modalities, (2) it pairs events in real-time with their symbolic representations, (3) it provides scientific experiences similar to that of scientists in actual practice, and (4) it eliminates the drudgery of graph production.
Zehavi (1988) found that software used to teach graphing concepts with middle school students in Israel resulted in a significant difference between the experimental and control groups. More notably, retention in the experimental group was evident and more so than the moderate significance achieved between the experimental and control groups due to the treatment. This gives further evidence of the effectiveness of the visual representation obtained by graphing technology.

Although it seems that the overwhelming majority of research done in the area of graphing technology in the science classroom suggests that many of the problems which students have with graphing can be aided by the current technology, one must also consider possible problems involved in the use of this method of teaching. In a nine month study of ninth grade students (Bohren, 1988), it was found that graphing software, by constructing axes for the students, can cover up missing links which the student may possess with regard to scale and what a graph really represents and in doing so make it appear to both student and teacher that the student has a solid understanding of the construction and interpretation of graphical relationships.

The national assessment also has suggestions to make with regard to graphing. Bestgen (1980) states that when students were asked for information which required the simple reading of data from the graph, performance was high, however, when the students were asked to use data from a graph to solve problems by computation or otherwise manipulation of the data, this resulted in a performance decline. "This failure to problem solve suggests that students' ability to read graphs is
superficial" (Bestgen, 1980, p. 27). According to the national assessment, Bestgen (1980) suggests five ideas which teachers should keep in mind when trying to plan for activities for classroom discussion.

- Use graphs to organize data
- Use many different types of graphs, not just line graphs
- Integrate graphing concepts with other subject areas
- Emphasize the importance of all features of graphs
- Have students work with graphs and tables used in life

2.2 Representational Preferences

With the use of data analysis software, the user has several methods of representation available. Most packages allow the user to plot the data in a scatterplot, list the data in tabular form, fit a curve or line to the data, and display an algebraic representation of the curve fitting result. As students use computer or calculator technology, the question arises as to which, if any, representational forms influence the students' confidence in the machine? Nachmias and Linn (1987) cite an incident where the limitations of the machine, in particular, pixel size of the screen led students to believe that the graphical representation presented to them was a true representation of the cooling process of a liquid.

In examining the literature on representational preferences, there is little done specifically on which representations are preferred by students. However, some insight can be gleaned from studies dealing with representational translations during the problem-solving process. Shavelson, Webb, Shemesh, and Yang (1988) examined the mental
representations students use when problem solving. They also tried to determine what translations took place as well as the accuracy of their mental representations. The study systematically varied three factors: (1) three ideal gas laws, (2) two response forms (qualitative or quantitative), and (3) five symbolic representational forms of the problem. Combinations of these three categories were taken and used and one item of each was constructed. The five symbolic forms used were: words, diagrams, tables, numbers, and graphs.

Shavelson and colleagues tried to discover which features of the problem guided the students' mental representations. When the quantitative problems were given, (i.e. asking for a numerical response) a formula or other numerical mental representation was evoked for 74% of the word problems, 77% of the diagrams, 78% of the tables, 87% of the numerical exercises, and 59% of the graphical exercises. In general, the students had a tendency to use mental representations consistent with the response demands rather than with the symbolic form of the problem. This is also seen in the qualitative problems where the major mental representation was verbal.

Webb, Gold, and Qi (1990) found similar results when looking at student performance as representations varied. They found that student performance was highest when the representational form of the problem matched the representational form of the required response (word problem \(\rightarrow\) words; algebraic \(\rightarrow\) numerical).

Another example is given where a student has learned to determine the area of a rectangle and is then given a picture of a rectangular living
room (Shavelson, Webb, & Lehman, 1986). He is asked to determine the size of rug needed to fit the room wall to wall. The student has the ability to figure out the solution of the problem, namely, the ability to calculate the area of a rectangle, as well as first hand experience with rugs and living rooms. However, until the student can link these ideas together the problem cannot be solved. In other words, the student must be able to translate between one symbol system (geometry) and other systems (verbal and iconic) before the problem can make sense to him.

Eisenberg and Dreyfus (1987) in discussing an introductory university level course on inequalities, state that while solving some 30 problems, both graphical and algebraic methods for solving the problems were presented with more emphasis given to the graphical methods. Despite the graphical emphasis presented in instruction, fewer than 5% of the students utilized the graphical approach to a solution when responding to test questions. This result is also supported by research of the Shell Centre for Mathematics Education (1985) where it was found that students tend to rely on analytic methods rather than the use of visual thinking.

This preference of analytical representation is echoed by the results of Colgan (1992) where she found that students preferred to enter numerical information and transform the equations of trigonometric functions rather than use an available "Transform" menu to apply horizontal and vertical translations, stretches or reflections to graphs of the functions. Moreover, throughout the study Colgan found that "the students demonstrated a strong, consistent preference for one input mode,
i.e., the equational format" (p. 227). It is suggested that the reason for this preference lies in the experiences of the students from earlier grades. Whatever the reason, the fact that the students did not demonstrate fluency in the use of different representational modes suggests that there might be more than computer authority at work in the minds of the students. Since the traditional algebra course spends a great deal of time writing equations of straight lines and parabolas from given points, students may perceive the algebraic representation as being more precise or accurate due to the emphasis placed on the activity by the instructor.

The interaction among the different representational forms of a problem has influence on the problem solving abilities of the student. It is also possible that these interactions may influence the students' perception of the authority of the machine when solving problems in mathematics.

2.3 Conflict with the Computer

With the obvious advantages of computer and calculator technology, what evidence is there to suggest that problems may develop within the technological paradigm? The authority given to the computer can be illustrated by a case study examining the arithmetic ability of a second-grade girl, Sigal, within a Computer-Assisted Instruction (CAI) system in Israel (Hativa, 1988). The system provided practice for topics already covered in class, but was somewhat restrictive with regards to what students were able to do within the computer environment. For example, when students were working through a vertical addition problem, they
were not permitted to use paper-and-pencil and the answer the students arrived at was to be entered digit-by-digit from right to left. Also, the correctness of the answer was determined digit-by-digit as the student typed it into the computer. If the first digit typed in was incorrect, the student was immediately given credit for an incorrect response without being able to finish working the problem. The computer also did not allow for the writing of "carry digits" above the columns even though the students were taught the use of such digits in the classroom. When given her first paper-and-pencil test, Sigal wrote carry digits when needed. When questioned about the use of carry digits, she stated that this was the method she had been taught in class, but that she thought that, "it would be fun to be able to do it the way the computer wanted it" (p. 205). When she was given her second paper-and-pencil test, Sigal did not write the carry digits as she had done on the first test. When questioned about the change, she replied that this was the right way to do the problems because, "the computer wanted it this way" (p. 205). Hativa goes on to say:

She used the same wording, which suggested that she had been so impressed with the specific way the computer enabled students to write the solutions to exercises—that is, she yielded herself so completely to the computer's authority—that she worked for 2 months until she had adjusted herself to the computer's method. (pp. 205-206)

In addition to her perception of the role of the machine, Sigal also made several errors in regrouping on the second test where she did not write the carry digits. When a conflict between the method taught in class and the
perceived computer method arose, Sigal yielded to the method of the computer.

The use of computer methods as a standard is also illustrated by Colgan (1992). In her study, the computer used decimal approximations for points common with trigonometric functions (e.g. 3.14 instead of π). This bothered the students to a point that they began to use the limitations of the technology as an excuse to suggest that exaggerated, inaccurate solutions were acceptable. In other words, the machine was given a position of authority allowing the students to use it as a crutch rather than a partner in the learning process.

Change in existing understanding of a concept to accommodate experiences with a computer is also reported by Cope and Simmons (1990). In this study, children working with Logo made erroneous adjustment in their conceptual understanding of angle in order to make sense of their experience interacting with the computer. In this case, students labeled the internal angles, actually measuring 120°, as 60° since it took a turtle command of RT 60 to form the angle. Even though a 60° and 120° angle are obviously visually different, the students still did not question their understanding, but rather simply changed their conception due to a seemingly contradictory experience involving the computer. The computer interaction seems to have been able to negate the students' previous experiences regarding angles.

As stated earlier, Nachmias and Linn (1987) also experienced a similar reaction when working with students using MBL. In this study, graphs were presented with incorrect data due to (a) graph scaling, (b)
probe setup, (c) probe calibration, and (d) probe sensitivity. On the pretest roughly 27% of the students accepted incorrect data due to graph scaling, 24% due to probe setup, 51% due to probe calibration, and 33% due to probe sensitivity. It was only after enhanced instruction in these four areas that significant gains were seen in detecting errors in data for graph scaling, probe setup, and probe calibration. Of the students accepting incorrect data due to graph scaling, many correctly identified an error in scaling, but yet still evaluated the graph as correct.

Colgan (1992) also gives an account of students' perception of the rescaling process. In this instance, the students were using a rescaling on the computer to accomplish the "zooming" effect. The visual image of the students, however, was confusing. The students spoke of the points as "moving" and of the curves as "becoming straight lines" while using the zooming option. They even commented on how a given point on a curve, could have the same coordinates, but be located on a different position in the plane, on what appeared to be a different graph.

Reys et al. (1980) give an account of student encounters with an altered calculator. As the students worked through exercises where they checked their estimates with a calculator programmed to give answers with an increasing percentage of error, only 20% of those interviewed recognized the error and verbalized that the calculator was in error on the first exercise. In addition, 36% of the subjects interviewed proceeded through all seven exercises ranging from initial calculator errors of 10% to finally errors of 50% without expressing any concern for the accuracy of the calculator answers. Some of the students waited until they had
obtained several incorrect responses before verbalizing that the machine was in error. These students responded with statements such as, "I suspected something was off on the third one, but I thought maybe I was messing up. After the second in a row, I knew I couldn't do that" (pp. 228–229).

Students used several different methods to express doubt with regards to the calculator. Among these methods were: puzzled looks, hesitancy to continue, desire to repunch keystrokes, desire to use pencil and paper to make calculations, as well as direct verbalization of their distrust. As the calculator prompted a conflict with the students' estimates, some of the common comments were:

"I must have entered it wrong on the calculator; either that or I'm thinking wrong."
"I don't understand. I don't understand what I'm doing wrong."
"It doesn't look right, but if that's what the calculator says, then it's probably right. It still doesn't look right."
"The calculator is wrong — no, it couldn't be." (pp. 229–230)

In these cases, it seems that the students are doubting their own estimates and abilities in favor of the calculator. One instance even shows a student verbalizing distrust in the calculator, but then recanting this distrust stating that the calculator certainly cannot be wrong.

Reys et al. (1980) also state some general observations and comments:

• Even good estimators were hesitant to challenge the calculator result.
• The results suggest that students perceive the calculator as infallible and that students who were able to detect and explain why the calculator answer was too high still eventually accepted the calculator answer attributing the discrepancy to an error on their part.
• The failure of good estimators to reject unreasonable answers suggests that educators must try to prepare students to be alert to unreasonable answers.

2.4 Infallibility of the Computer

In the study conducted by Reys et al. (1980), the responses of the subjects suggest that the calculator is viewed as an authority and that a mistake on the part of the calculator is almost inconceivable by the students. As indicated earlier, the students were asked to proceed through seven exercises using a calculator which was programmed to give answers which were off by increasing percentages of error as the student worked through the problems. Some of the general observations made were:

1. Males were more likely to challenge the calculator than were females.
2. Students who were categorized as good estimators were still reluctant to challenge the calculator result.
3. The students view the calculator as infallible. Even those students who were able to verbalize that there was a discrepancy between their answer and the calculator result as well as explain why the calculator result was too high, eventually accepted the calculator result giving the explanation that the mistake was on their part.

Reys et al. found that 20% of the group identified the unreasonableness of the calculator result on the first exercise whereas 36% of the students completely finished all of the problems without
voicing any concern for the accuracy of the calculator. Of those students who voiced a concern sometime before completion of the exercises, the male and female percentages differed greatly. Only 36% of the females verbalized a concern while 77% of the males voiced an error prior to reaching the final problem.

2.5 Authority

When examining students' perception of the authority of technology, it is important to explore the tendency of students to concede to sources of authority. Rigby (1985, 1986) found that students were more anti-authority than non-students. The students involved in both of these studies were adults in a tertiary school setting. The non-students were also adults of the approximate same age group as the student sample. The measures by Rigby (1985, 1986) were on general perception of authority and not specific to the classroom setting. Since the present study wishes to examine student perception of the authority of technology within the realm of the college or university as related to content knowledge, one must look elsewhere for research specific enough to apply to the current investigation.

Epistemic authority is defined as a source from which an individual tends to accept information as true or factual (Raviv, Bar-Tal, Raviv, & Peleg, 1990). Raviv, Bar-Tal, Raviv, and Peleg (1990) examined the extent to which mothers, fathers, teachers, and friends were viewed as epistemic authorities in the following nine areas: (1) School Studies, (2) Politics, (3) Science, (4) Pastime, (5) Physical Appearance, (6) Social Relations, (7)
Future Planning, (8) Values, and (9) Personal Feelings. It was found that teachers were perceived as epistemic authorities mainly in the area of Science, but also in the area of Social Knowledge during the early elementary school years.

In a later study, Bar-Tal, Raviv, Raviv, and Brosh (1991) while exploring the possible sources of epistemic authority found that children and adolescents frequently mentioned Father, Mother, Friend, and Myself as authorities. Teachers were mentioned as well. However, Teachers were mentioned mainly in the areas of School Work and Science. The findings also suggest that knowledge denoting a source's expertise, education, experience, and possession of information was the most important reason for explaining the students' choice of various external sources as epistemic authorities.

In a study with elementary through high school age students, Bar-Tal, Raviv, Raviv, and Brosh (1991) also found that students' reliance on Myself and peers as an epistemic authorities increased with age. The dependence on adults as epistemic authorities decreased with age. It is hopeful that by the time a student has reached the college or university, his/her perception of him/herself as an epistemic authority may be great enough to promote critical evaluation of information. However, in another study (Raviv, Bar-Tal, Raviv, & Abin, 1993) it was found when comparing the tendencies for perceiving professors as epistemic authorities, in areas of disciplinary knowledge, students of statistics departments had a greater tendency to perceive their professors as authorities than students of psychology departments. Since the present study was conducted in a
university statistics course, the influence of the course professor cannot be ignored.

2.6 Summary

The literature suggests that the use of technology such as MBL enables students to link the graphical representation with the physical phenomenon being represented (Beichner, 1989; Brasell, 1987ab; Linn, 1988; and Mokros & Tinker, 1987). However, the research presented in this chapter does not address the actual thoughts and beliefs of the subjects as related to their understanding of the phenomenon. The research to date focuses on achievement and does not breach the area of conceptual implications of technology use. Reys et al. (1980) documented the issue of student reliance on technology, but did not attempt to uncover the underlying reasons for student willingness to accept the authoritative answers of the machine. There is a necessity to build a theory addressing why students fail to challenge the authority of the computer or calculator, especially since reform movements in mathematics education advocate the use of computers and calculators in instruction. In particular, with emphasis being placed on the use of graphing technology (NCTM, 1989, p. 126), there needs to be research on the possible negative effects of the graphical approach of technology along with suggested measures for counteracting any of these effects.

MBL is an effective tool for linking the graphical representation of a physical phenomenon to the actual event. However, since even the MBL collected data are not truly continuous, it would be interesting to expose
students to curve fitting with obviously discrete data to see how they might react to the curve-fitting process. The study presented in this document attempts to examine the issue of student trust in the presence of discrete-data curve fitting.

It is clear that the literature suggests students are willing to accept the answers obtained by calculators even when the answers conflict with methods of estimation and common sense. With the availability of graphing technology, this perception by students could lead to the belief that mathematical ideas can be proven by a graph—proof by picture. Since each student has his/her own representational preferences, the use of multiple representations can serve as a means to introduce conflict into a problem-solving situation in a manner similar to the estimation techniques used by Reys et al. (1980).

The research in the area of epistemic authority (Bar-Tal et al., 1991; Raviv et al., 1990; & Raviv et al., 1993) examined many possible sources which could be viewed as epistemic authorities. However, the computer, calculator, or technology in general were not among the sources explored. It is important that non-human sources of authority be included in the investigation of student perception of such influential factors of knowledge acquisition.
CHAPTER III
METHODS AND PROCEDURES

3.0 Introduction

The idea for the study came about due to discussion at a SuperCalculators for the Integration of Science and Mathematics (SCISM) conference. During our time together, approximately 20 mathematics and science educators from around the country were discussing laboratory activities for use with the graphing calculator, in particular, the Hewlett Packard 48SX. On the Hewlett Packard calculators there is a \textbf{BEST fit} key for curve-fitting. The function of this key is to compare the correlation coefficients for the existing mathematical models housed in the calculator's memory and choose the one with the highest correlation coefficient. While working through one activity, the question arose: Will students simply hit the \textbf{BEST fit} key and believe the model suggested by the calculator or will they think more critically about the curve-fitting process? The group was somewhat divided on their opinions. Some stated that the students would be more critical about their analysis of the problem while others voiced concern for students whose mathematics background was somewhat limited choosing to blindly follow the machine. Since this group of respected mathematics and science educators seemed to differ in their opinion, the researcher decided that the issue warranted
further investigation. Upon reviewing the literature, the researcher found little research on this topic, and the study was devised.

The motivation for the methodology of the study stems from the research questions. Since the study attempted to identify and describe factors surrounding the use of technology in classroom teaching the primary design is qualitative. The students were given an open ended Initial Data Analysis Instrument designed to detect reactions to various representational forms of problem situations within a data analysis context. These questions allowed the investigator to determine the representations used by the subjects to solve the problem situations. In addition, the instrument was also used to give insight into the students' reaction when there is conflict between representations such as graphical line and algebraic equation. These conflicts were explored in greater depth later during interviews with the students.

Since this study attempts to probe aspects of a phenomenon which are not yet known, a qualitative approach was necessary. Qualitative research methods have their basis in anthropological investigation, primarily in cultural description. Since the introduction of the computer/calculator into the classroom poses a change in the culture of the mathematics classroom, it seems only fitting that such methods be used to describe the implications of such use of technology.

It has been suggested that an effective method for proceeding with qualitative research is to ignore the literature in the area of study until after a central list of categories has developed from the initial observations (Glaser & Strauss, 1967). The purpose for this recommendation is so that
the researcher will not focus on several pre-conceived aspects of a phenomena and thus fail to see other aspects present. The object of study here has been suggested by previous literature. So the researcher attempted to remain open to aspects of the problem under investigation other than those stated initially in Chapter 1.

In order to accentuate openness to other possible variables, the investigator was present during class and lab sessions in order to observe student interaction with the machine. This method has been suggested to accommodate the investigation of complex interaction by allowing unstructured techniques to maximize unanticipated discovery (Colgan, 1992).

3.01 The Researcher

The researcher's academic background is in mathematics and physics. His undergraduate degree was a double major in mathematics and physics and his Masters degree was in mathematics. He has been interested in the use of graphing technology for the integration of these two disciplines for approximately 5 years. In particular, he has been interested in the use of Microcomputer-Based Laboratories as described in Chapter 2. He has worked with teachers using graphing technology and has introduced these teachers to some of the advantages of curve-fitting for pattern exploration with students. The availability of calculators such as the TI-82 gives opportunity for students to investigate problems from many different perspectives such as curve-fitting and matrix applications. The technology can help show connections between these approaches and
allow students to explore questions which were not previously considered within the reach of the typical student. In addition, students can gain ownership of conjectures which they discover and be more easily motivated to prove these conjectures deductively.

The researcher has been involved with the group, Supercalculators for the Integration of Science and Mathematics (SCISM) based at the National Center for Science Teaching and Learning, for several years. This group of mathematics and science educators from around the country is primarily interested in the use of calculators similar to the Hewlett Packard 48SX and 48GX models. These machines are not only powerful graphing calculators, but are also hand-held symbol manipulators. The potential for these affordable Computer Algebra Systems gives great promise for classroom use since they are much less expensive than computer equipment and the Hewlett Packard company has recently developed even less expensive student versions of the calculators.

For this study, the Texas Instruments 82 (TI-82) graphing calculator was used. These calculators are more user friendly than their Hewlett Packard counterparts. In addition, Texas Instruments has recently developed a Calculator-Based Laboratory system (CBL) for the TI-82. This system is similar to the Microcomputer-Based Laboratory systems where the use of probes and other data collecting devices allows students to collect data and then view the data in a number of graphical formats as well as perform statistical analyses. Since the researcher's interests were in this area, it seemed appropriate to utilize this technology.
3.1 Participants

The population for the study was 33 university students enrolled in a first course in statistics (Math 107) at Wittenberg University, Springfield, Ohio, during Autumn Quarter 1994. This was a course taught using computer software named DUSA for linear regression as well as other statistical tests. The reason for the selection of this population was that it was an accessible liberal arts university. One of the uses for graphing technology is to allow students who do not necessarily plan to enter a highly rigorous mathematics field to understand mathematical concepts as they relate to the world and possibly their own field of study. The liberal arts college or university is an ideal setting for the use of such technology in this manner.

The course consisted of both classroom instruction as well as computer laboratory activities. For a large portion of the homework, the students were required to make use of the computer laboratory to complete the assignments. The computer was also used during class time to explain concepts and demonstrate the use of the software to the students through the use of an overhead LCD display unit.

The course was taught by a Wittenberg faculty member specializing in statistics. The researcher was an observer and not involved in the instruction of the course. However, the researcher was an adjunct faculty member teaching a calculus course during the study and this fact was known to the students.
3.2 Procedures

The study was conducted in an already existing curriculum using computer graphing software. One goal was to identify aspects of student perceptions which need to be addressed whenever technology is used in instruction. The instrumentation was constantly evolving. The description of the procedures will include discussion about the instruments since it would be difficult to separate the two facets of the study. Through a three-phase investigation, the research questions were addressed with increasing depth.

3.21 Phase 1: Student Selection and Initial Data Analysis Instrument

At Phase 1, the students were introduced to the concept of linear regression and then given experiences using linear regression to make predictions and solve problems. The researcher kept fieldnotes and identified 5 students on which to focus throughout the rest of the study. Overall, there were 10 volunteers from which the researcher selected 5. The volunteers were enticed by an offer of a free TI-82 graphing calculator if they remained in the study until its conclusion.

The selection of five students from the ten volunteers was made in consultation with the course instructor so that the students represented a wide spectrum of achievement. Consideration for student selection was also given to the general openness of the individual. It was important to select students who would make good subjects for interviewing. The students were selected, in part, because they would most likely talk openly about the problem-solving process during the interview part of the study.
The students consisted of three female Caucasians, one male Caucasian, and one male African-American. As it turned out, achievement for the participants covered the spectrum of the grade scale consisting of an A, A-, B, C, and D in the course giving a wide range for such a small group.

Following this identification process, the five students were given an open-ended instrument designed to reveal general aspects of this investigation—Initial Data Analysis Instrument. The instrument contained four task-oriented questions which varied the representations given to the students for making predictions based on the data. These questions can be seen in Appendix A. To help the reader, one of the questions is presented in Figure 3.
1. Data were collected on a certain population of bison giving the total population as a function of time (in years) as seen in the following scatterplot. The computer fit a regression line to the data yielding the computer model equation shown at the top of the scatterplot.

\[ y = 195.33x + 10988.49, \quad r^2 = .94 \]

What would you predict for the bison population at year 1?

How did you arrive at your prediction? Explain.

(If you need more room, please feel free to use the back of this sheet.)

Figure 3: Question 1 from Initial Data Analysis Instrument

The first question was presented with a contextual premise for the data and gave the data only in graphical form including a regression line drawn on top of the scatterplot. In addition, an algebraic representation for the regression line was included. Question 2 gave a problem in the context of the spread of AIDS with data given in tabular form only. Question 3 gave a problem in the context of radioactive decay. Only the data were given in both tabular and graphical forms and a drawn regression line was not included. Question 4 gave a problem in the context of automobile production with data given in graphical form only.
A false regression line was drawn and a false algebraic representation of the regression line, not matching the drawn regression line, was also given. The reason for the inconsistent representations was so that the representation being focused on by the student could be isolated. The open-ended responses were analyzed for content allowing the investigator to identify specific variables for further investigation in the interview part of the study.

In addition, each task was followed by a question asking the student to make an estimate of his/her certainty of the prediction on a scale of 1 to 10 with 1 being least certain and 10 being most certain. The method used to solve a particular problem and the certainty attributed to the solution were compared to see if methods incorporating the machine tended to have a higher certainty level than methods which rely on student intuition.

Following the four tasks, the Initial Data Analysis Instrument asked for responses to questions regarding the students' opinion on the advantages and disadvantages of the use of technology in the learning of mathematics. These questions provided a springboard for further investigation into the students' beliefs and attitudes toward the use of technology in the learning of mathematics during the later interviews.

3.22 Phase 2: Student Interviews 1 through 3

Individual student interviews have been suggested as one of the most valid techniques for assessing student mathematical thinking (Kaput, 1991). One of the aspects of qualitative research methods is the
use of open-ended interviews. These types of interviews afford the investigator the opportunity to explore the thinking of the subject as he/she relates connections in understanding within his/her own schema. The questions for the interviews were based on the students' responses to the Initial Data Analysis Instrument as well as responses to later statements made during interviewing. These responses were for guiding the interviews, however, and not the sole focus of the interviews. The interviews contained additional tasks similar to those of the Initial Data Analysis Instrument for the students to analyze so that the researcher could explore topics which surfaced during previous interview sessions. The researcher was continually reflecting on previously collected data. The reflection process allowed the researcher to construct interview questions which would address observations from previous sessions. The task-oriented questions were common to all students during interviewing. However, the other questions varied from student to student depending on the experiences the researcher had with the students during previous encounters. At times, a comment made by one student prompted the researcher to ask a similar question to all students. For example, the student Sue made a comment which caused the researcher to question her perception of authority for the calculator versus the computer. This aspect of the study was not anticipated. Sue's comment prompted the researcher to address the issue with all of the students in the next interview.

The task-oriented portions of the interviews were done with the students using a "think-aloud" process to get them to verbalize their thought processes. The researcher modeled this process before engaging
in the interview. During Phase 2, the students were expected to use a TI-82 graphing calculator with overhead projection unit. Therefore, the 5 students were given a TI-82 at the beginning of the study to use throughout the quarter. As an incentive to remain in the study throughout the entire quarter, the researcher told the students that if they remained they could keep the calculator at the conclusion of the research.

The researcher attempted to interview the students as many times as possible within the given time and consent constraints. Four of the students were interviewed 3 times and the other student, John, was interviewed 2 times during Phase 2 due to scheduling problems on the part of the student. The interviews attempted to explore in greater depth findings from Phase 1 and give a better picture of the aspects of the computer usage which influenced the students' perception of the authority of the machine. In addition to the planned questions, the researcher also asked other questions as new aspects of the students' perceptions became evident during the interview. This technique of using unrehearsed exploratory questions allowed the researcher to examine the students' feelings and beliefs in greater depth. The base questions, both task-oriented and philosophical, used for each student in the interviews are given in Appendix B.

3.23 Phase 3: The Final Interview—Regression Line Shift

During Phase 3, the students were interviewed one last time while completing tasks similar to those of the Initial Data Analysis Instrument.
The students were instructed to think aloud while attempting to answer questions about data. The reason for the use of the calculator instead of the software package DUSA used in class was so that the calculator could be programmed to shift regression lines with the error desired by the investigator. The purpose of Phase 3 of the investigation was to see if students would challenge the authority of the machine when given incorrect responses by the calculator.

The investigator entered a percentage shift of 0, 10, 20, and 30 as the student proceeded through the four questions and stored it in a variable. This shift was then applied to the regression line of the particular question. The shift was determined by the viewing rectangle size and thus was always visibly the same for any data set and given percentage shift. For example, if the investigator wanted to have the regression line shifted by 10%, he would enter 0.10 and store it in the appropriate variable location. The program then calculated the range for the viewing window and shifted the regression line by 10% of this range. Since the calculator can automatically scale the graph so that the data are completely contained and spaced within the viewing rectangle, the visual effect remained constant for any data set. The program for the TI-82 is given in Appendix C.

The first question of the final interview had its regression line shifted by 0%. The second question had its regression line shifted by 10%. The third question had its regression line shifted by 20%. And the fourth question had its regression line shifted by 30%. The shift was also applied to the algebraic representation of the regression line as well as the
calculated response given by the calculator due to the PREDICT command option.

To illustrate this shift, consider a 10% shift of the regression line for the following scatterplot. The graph in Figure 4 represents the data and actual regression line, while the graph in Figure 5 represents the data and a 10% shifted regression line.

![Figure 4: Scatterplot and Regression Line with No Shift](image)

![Figure 5: Scatterplot and Regression Line with 10% Shift](image)

In this example, the student might question the feedback given by the machine since the regression line is clearly above most of the data points. Whether or not the student made such observations or showed signs of
concern with regard to the responses gave insight into student perceptions.

To guard against confounding influences on the calculator's authority during the Phase 3 interview, all data sets used in the tasks exhibited obviously linear patterns. This was to prevent students from questioning the calculator based on a possible incorrect model rather than conjecturing an error on the part of the calculator.

One potential problem which needed addressing was the ability of the researcher to discern between which responses given by the students were truly questioning the authority of the machine as opposed to questioning the authority of the researcher who gave the machine to the student. In addition, the questioning of authority of the course professor as opposed to the machine in the form of either the calculator or the computer was also of concern. An attempt was made in the interviews to make this distinction between the authority of the machine and the authority of the giver of the machine.

The interviews were audiotaped as well as videotaped. The videotaping was confined to the screen image only allowing the researcher to pair verbal responses with visual responses to the calculator on the screen. The use of both audio-taping and video-taping preserved the anonymity of the students while giving visual feedback of their interaction with the machine. The use of audiotape was for ease of transcription since it allowed for the researcher to quickly rewind and type at the word processor.
In addition to the three phases of observations, the investigator also regularly attended the class to make observations and field notes on student behavior within the classroom setting. This included observations of the students while engaging in activities in the computer laboratory and interacting with the computer for homework assignments during times other than the regularly scheduled class time. Informal interviews in the form of conversations also took place during the computer sessions and at other opportune times. The data were coded and a content analysis was done on the resulting data. A comparison with other forms of data for the same students was also conducted. As the study proceeded, an ongoing analysis of the data was performed as patterns from the data were examined and future interview sessions were adjusted accordingly.

The coding of the data was broken into the following theory driven categories:

1. **Computer Relation** – ways in which the student relates and reacts to the computer.

2. **Unexpected Result** – reactions of students to differences in what they expect and the feedback given by the computer.

3. **Self-Confidence** – ways in which the student perceives him/herself with respect to the autonomy of the problem-solving process.

4. **Representational Preference** – data related to patterns of representational choice while working through tasks.
(5) **Authority Factors** – data related to the characteristics of the computer which students use to either yield or question authority.

(6) **Environment** – aspects of the classroom or laboratory environment which influence student interaction with each other or the computer.

(7) **Methods** – aspects of the instruction which influence student perceptions with regards to the computer.

Additional categories for coding were necessary as other aspects of the study, not anticipated, surfaced throughout the course of the data analysis through transcribing and reading transcripts as well as listening to and viewing tapes. These categories were:

(8) **Process** – student focused primarily on the process of solving the problem rather then the conceptual connections of mathematical ideas. This included statements of keystroking and procedures.

(9) **Experiences** – student experiences which influenced relations with the technology. This included such things as past mathematics courses, earlier schooling, life experiences, etc.

(10) **Neighboring Points** – student choice to use points in close proximity to the prediction point of interest as a basis for making a prediction.

(11) **Specific vs General** – concern for making predictions about individuals based on general trends of data.
(12) **Conflict** – instances of conflict between representations of the problem situation or other facets such as experience.

(13) **Explain Away** – ways in which the students explained unexpected results so that they could continue to accept the responses of the machine.

(14) **Outliers** – the use of accepting some points as outliers so that the students could explain the regression line's shifting without questioning the machine.

(15) **Repetition** – the use of the fact that a student had done similar problems before as a reason for certainty of an answer.

(16) **Context** – use of the context of a problem to question the response by the machine.

(17) **Accuracy** – the use of the needed accuracy as a means of selecting a problem-solving strategy.

(18) **Time Dependence** – perception by the student that points next to each other on the scatterplot could be connected to form a graph, even when the variables are not in a time dependence relationship.

The audio and video tapes were transcribed. The researcher read the transcripts as well as the field notes completely three times to develop the coding scheme. Upon the fourth reading, the data were coded according to the scheme.

Once the data were coded, the researcher then sorted the coded data into separate files on the word processor utilizing the Find command.
as well as the coded transcripts and field notes. The files were then examined for patterns and relationships between coding categories.
CHAPTER IV
DATA ANALYSIS

4.0 Introduction

To begin this chapter, it might be beneficial for the reader to take a few moments to get to know the students who participated in this study. This opportunity will help the reader to understand the background and personality of each student so that data presented in this section can be appreciated within the context of the individual. This chapter is segmented into three main parts: Student Profiles, Data Analysis, and finally Additional Observations. The data analysis section is structured around the four research questions and the final section examines some additional observations which the researcher deems relevant to the use of graphing technology in the classroom.

4.1 Student Profiles

This section gives the reader a glimpse of each of the students involved in this study as related to the purpose of investigation. This section is being presented first so that the data presented in the next section might be more fully understood.
4.11 Participant: Sue

Sue is a white female majoring in political science. She is taking the introductory statistics course in hopes of applying the ideas to her future career in politics. She has stated that she is not sure how the statistics will be used in her field, but she is certain that it will be useful somewhere along the way. Sue is involved in student athletics, but she realizes that her education and career preparation are paramount to her experiences at the university. She hopes to enter law school upon completion of her undergraduate degree.

Sue's mathematical background consists of the typical college preparatory curriculum encountered by most high school students including Algebra I, Geometry, Algebra II, and Pre-Calculus. When she came to the university, Sue was placed in the Math Workshop which is designed to bring students up to what is considered as entry level mathematics, namely College Algebra and Trigonometry. Not having performed as well on the mathematics placement exam as her high school course work would suggest, she spent some time reviewing mathematics concepts in the remedial program at the university. Sue claimed that mathematics was her strong suit in high school; however, she was still behind her peers at the university.

Sue could be characterized as hard-working and goal-focused. She is apparently from a fairly wealthy family as are many students at this particular institution. This observation is evidenced by her appearance with regard to clothing and jewelry. Further evidence of this observation
are her hobbies which include tennis and her mention of being a member of a country club back home.

As a general observation, Sue seems to have been instructed in mathematics in a rote or procedure-oriented manner. She frequently asked what "the answer" was during the interview process and seemed unconcerned about aspects of the problems presented until an offer of the "right answer" was promised by the researcher.

While explaining her solutions to problems, Sue displayed constant concern for right answers. She primarily used procedural explanations rather than thoughts which linked concepts together ending in a solution. One example of Sue's procedural explanations can be seen in her approach to the first task-oriented question of the first interview. She began by saying,

I went into STAT. It's cleared out ready. So I'll go through and enter all of the X's. And enter all of the Y's. Then I go into STAT PLOT. Turn it on. Make sure the coordinates—you know—List 1 for X and List 2 for Y. Go into PROGRAM... . View the scatterplot out of curiosity. I want to look at the equation that they have... . Get out of this and go back to the menu. Go to PREDICT. Put 1.5 and Y gives me -1.

As far as achievement is concerned, Sue received a B in the course. Much of this grade was earned through hard work rather than an apparent ability to grasp concepts quickly.

4.12 Participant: Tom

Tom is a white male majoring in biology. He is interested in the use of statistics for analyzing data in his field of study. His background
includes taking Algebra I, Algebra II, Geometry, Pre-Calculus, and Calculus in high school as well as Chemistry and Biology. At the university, he has taken two quarters of Calculus and is currently taking Physics.

Tom frequently displayed signs of confidence in the problem solving process regardless of technology use. He stated concern for the over-reliance on technology. In working through problems, he regularly went with his own intuition in coming up with a prediction based on the data presented.

Having a strong mathematics and science background, Tom seemed to be more willing to express his thoughts within the terminology and conceptual structure of analytic-based mathematics. At times he discussed graphs within the context of the variable nature of slope for curvilinear functions. Seldom did he follow the linear regression model proposed by the calculator.

Tom seemed to enjoy looking for patterns and explaining his reasoning for his solutions. This may stem from dislike of his high school experience where getting the "right" answer was stressed. He shared an high school experience with the researcher when the researcher probed into his views on technology use. The researcher asked Tom, "What sort of experiences have you had that have shaped your views of the use of technology in your own learning of mathematics"? Tom replied,

Well, I guess I kind of alluded to this a little bit. I actually had calculus in high school taught by a teacher who came from Hungary in 1956. And her whole approach to it was 'if you don't get the answer then what good is it?' And it was probably one of the most
frustrating classes I have ever taken because I understood how things worked, and I understood how to set up the formulas, but I'd make mistakes in adding up fractions. And the tests were multiple choice with no partial credit. It was very frustrating because you'd have these big integral problems and you'd put the wrong answer and she's like 'sorry' and there were only 8 problems because they were so long and you'd take a huge hit from failing to add all the fractions from the curve correctly at the bottom. Anyway, I also had it here too. I've taken Calc I and II here and it was just a much more conceptual natured—conceptual approach to the whole thing here. The whole thing was trying to discuss why the curve is changing—that type of thing. I would say that I actually did very well in Calculus here because I had had the mechanical part of it before, but I think the conceptual nature is much easier.

As far as achievement is concerned, Tom received an A- in the course. He would frequently answer questions in class and seemed to have a strong conceptual grasp on the material. At one time, the researcher asked Tom about some fairly deep philosophical questions surrounding the nature of scientific investigation as it relates to curving fitting and theory generation.

Tom displayed remarkable understanding of the implications of mathematical models for the explanation of natural phenomena. For example, since Tom was fairly advanced in his mathematics and science background, the researcher showed him how to do some curvilinear fitting of data with the TI-82 calculator. The researcher had Tom use several different models of fit and then requested that Tom graph these models superimposed on the scatterplot of the data. The researcher then asked Tom to imagine that he was Isaac Newton and he is wanting to find the universal law of gravity. The researcher then asked him, "How do you know when you have it"? To this Tom replied,
Well, it's tough—I mean a universal law is not something you find every other day of the week. I don't know—I guess I'm not really sure. In that case, I guess it would be handy if you had some kind of measure to tell you that this line is absolutely much better than another line. Then that would—you know. It's difficult, to say. And also, even if you're trying to prove something experimentally, you have other things like error and stuff—I'm not really sure.

The researcher then asked Tom how this made him feel about his trust in science in general. To which he replied,

Science is just very experimental. It has—I mean, I always felt that science had this unhealthy habit of trying to say, 'well, this is the way things are'. Every time a new journal comes out, 'look what we discovered, this is the way things are'. I mean, you try, but—and you can get lots and lots of data that says it certainly looks like this, but I suppose you never can be absolutely sure. At some point, it almost kind of doesn't matter. I mean, if you say the ball is always going to drop from the table and say that like in one little tiny place in the world that doesn't happen. Well, for most of us it's still pretty accurate.

Throughout the interviews, Tom not only produced a prediction for the problems, but also gave error estimates and limitations for his predictions. The use of such data analysis techniques was unique within the set of five students involved in the study.

4.13 Participant: Cindy

Cindy is a white female. Similar to Tom, Cindy had a fairly strong mathematics background. She had taken Algebra I, Algebra II, Geometry, Pre-Calculus, and Calculus in high school and one quarter of Calculus at the university. She also stated that she had taken introductory courses in computer programming in high school and at the university.

Cindy seemed to be procedural in her approach to mathematics. She had related some of her experiences in learning mathematics to rote
techniques and mechanical proficiency. At one point she shared an experience where she had been tutoring middle-school children as part of a community service project. She expressed extreme concern for the over reliance on technology since many of these children did not know their multiplication tables. She became very frustrated with the teacher for allowing the students to use a chart as well as calculators when they still did not know what Cindy considered basic skills.

Like Sue, Cindy also seemed to be concerned with right answers in that she frequently expressed that she liked to be exact. To illustrate this, consider Cindy's response to the following question asked by the researcher.

"Last time, you stated that you like to be exact. Can you talk about why you like to be exact and how you feel about the statement, 'Calculators and computers allow us to be exact in our answers.'" To this Cindy replied,

Well, I think just being exact is part of my nature. I don't know—I'm kind of a perfectionist about the way I do things. So I think that's why I like to be exact. There's no really like—I don't know—I mean if you're off by, you know .001 it probably doesn't matter that much, but yeah I guess calculators can help you be exact and stuff. I mean, exact quicker and stuff. Like in my computer class we were watching this video about the history of computing or something like that and they were talking about how they had to like punch all these—or do all this math by hand and how this guy spent years and years of his life trying to figure out these—I don't know—π or something and then he messed up somewhere along the way and then it just ruined like 30 years of his work or something. Then when he had the computer it did it all without a stupid mistake like that.
Despite her view of the exactness afforded by technology, Cindy also displayed indications that her mathematics background influenced her perception of data patterns. In one instance the researcher asked her to explain her hesitancy to go with a linear regression model given by the calculator. She responded,

> It looks like it was exponential or like it started off—like it was decreasing—I don't know—like it—I don't know if there is such a thing, but like I think I even wrote somewhere that it was like—it seemed like it was decreasing exponentially or something like—instead of like—you know how when you have an exponential and it curves up really fast. It seemed to do that only in the opposite way which I kind of don't think a straight line would be the right way to explain that.

Her response to this question seemed to be an indication of reliance on previous mathematics background. This might explain her overall confidence in her problem solving abilities which was observed throughout the study.

As far as achievement is concerned, Cindy received an A in the course. She was meticulous in her homework and in her study process for exams. She took the course very seriously and devoted quite a bit of time to working in the computer lab on homework and projects. Until the first interview, she did not use the TI-82. Instead she had been using DUSA and seemed more comfortable with the computer software even as the study progressed.

### 4.14 Participant: Kathy

Kathy is a white female. She feels that the statistics course will be useful in the future, however, she is not certain how she will use the ideas.
She has also tried to bypass as much mathematics as possible in her education and feels that she is not very good at mathematics.

Like Sue, Kathy is also procedural in her approach to mathematics. When explaining her thinking during interviews, she frequently, and almost exclusively, described the processes she was going through on the calculator. This procedural verbalization even included detail of what buttons she was pushing. One example of Kathy's procedural emphasis can be seen during her response to the first task-oriented question of the first interview. When asked to make a prediction about the data she began by saying,

They give a table of the X variable and the Y variable and p with a little hat on it is not given, so I'm probably going to have to enter this into the calculator. Getting to the L1 and L2 list. So at the L1, L2 I'm going to enter in the X variable: 1, 2, 3, 5, and 6. Now I want to enter the Y variables: 1, 5, 14, 55, and 91. I want to make sure there's nothing on the Y, there's nothing on the Y. So I'll go back to STAT. And I...go back to program and...go to the program regression and I want to predict the value of Y when X equals 1.5. So I'm going to press 3 for predict. And X equal to be 1.5 [Kathy entered 1.5 at the prompt.] So Y would be -1. So Y would be -1 when X equals 1.5. How would I rate my certainty of my answer? A 10 or very certain.

Kathy's view of mathematics as a system of procedures was also evident in her verbalization of the problem that she was solving. When starting a problem or answering a question, she would read the problem or question aloud. As part of her answer, she would then attach an exact phrasing from the problem statement or question. For example, when asking questions based on Kathy's responses on the Initial Data Analysis Instrument the researcher asked her to expand on her written answer to the following question:
Please list and explain what you feel are some of the advantages for the use of computer/calculator technology in the teaching and learning of mathematics and/or mathematically related fields of study.

To this question, Kathy began her response by stating, "One of the advantages of using computer/calculator technology in the teaching and learning of mathematics would be that... ." By restating the exact phrasing of the question in such detail, she may be indicating the use of a series of steps for answering questions.

It is also interesting that Kathy voiced a dislike for the use of technology even though she seemed to rely on it almost exclusively in her answers. She was convinced that a person learned more by doing a process over and over again rather than exploring different types of problems for which she had never seen a similar example performed.

Her certainty of her answers was based almost exclusively on her perception of how well she had adhered to a process for solving a problem and pushing the right buttons. For the most part, her certainty ratings were 10 on a scale of 1 to 10 with 10 being the most certain.

As far as achievement is concerned, Kathy received a C in the course. She seemed to be primarily concerned with getting through the class and not necessarily concerned with understanding the material.

4.15 Participant: John

John is an African-American male. His mathematics background consisted of Algebra I, Algebra II, Geometry, and Pre-Calculus in high school. Upon entering the university, he placed into a remedial
mathematics course and took that course during Autumn quarter of his sophomore year. He stated that he had no background with computers or graphing calculators. However, he did state that during elementary school he had been exposed to computers, but failed to keep up with the technology and now felt behind the other students around him.

John seemed to perceive himself as being at a disadvantage when it comes to technology and seemed to feel that attaining understanding of mathematics and technology in particular would be difficult for him. He stated that a person had to be intelligent in order to operate technology. In looking to clarify John's meaning on intelligence versus experience in his views of technology, the researcher posed the following question.

In your responses to the advantages and disadvantages of the use of technology, you state that the use of more advanced calculators requires more intelligence. Do you feel that you are somewhat at a disadvantage because of lack of experience with these calculators? John replied,

Yes. When I read the question, it came to mind like, the rich get richer and the poor get poorer. That's how I kind of thought of it—I think I was at a disadvantage, but I have the knowledge to work a calculator like that so all I need is somebody to show me how to do it.

The researcher then asked if John thought this was more an issue of intelligence or experience? To this John replied,

I think it's more intelligence. Well, no no, I think anyone can use it with repetition and practice, but you can't be a dummy and know how to use one of these calculators. I mean, you just can't. I mean you have to have some mathematical, conceptual—you know—you have to know something about math in order to use it.
John displayed evidence of a lack of experience with graphing technology in his initial interview where his attempts to solve problems were algebraic in nature. He did not even realize that he was allowed to use the calculator sitting on the table in front of him.

John's lack of experience with technology seemed to give him a great deal of control over his problem solving strategy. During the first interview, he looked at number patterns to try and make a prediction about the data. He tried several different strategies and revised them as he progressed toward an answer reaching what he perceived as a dead end from time to time. This behavior suggests that he felt in control of how he was to solve a problem as contrasted with Kathy who felt tied to procedures for solving particular types of problems. John appeared to be unincumbered by any preconceived notions of how he was to solve problems using the technology in a standard, procedural method.

As far as achievement is concerned, John received a D in the course. He fell behind during the beginning of the quarter and worked hard toward the end to salvage his grade. The use of technology could have played a role in his early performance in the class. At one point John told the researcher that he was scared of computers. If he felt this way he may have avoided some of his work during the beginning of the quarter which could have resulted in his falling behind.
4.2 Data Analysis

The data analysis is broken up into four main sections, one pertaining to each of the four research questions. The research questions as stated in chapter 1 are:

1. Will students who use graphing technology choose to believe the model of curve-fitting suggested by the machine even when that model differs significantly from the data?

2. When asked to predict within the interval of the data available, will students choose to use the computer generated model for predicting rather than the scatterplot of the data when the computer model and scatterplot differ significantly?

3. Are students who choose the computer model to predict, more confident about their prediction than those who use only the data to predict?

4. What factors surrounding the computer give authority to the machine in the eyes of the students?

Evidence will be presented in response to each of the research questions. For each of the research questions there will be categories consistent with the coding scheme of the data. The research question will then be addressed within the context of each of these categories as they pertain to the question.
4.3 Curve-fit Model versus Data

In this section we will examine the first research question:

- Will students who use graphing technology choose to believe the model of curve-fitting suggested by the machine even when that model differs significantly from the data?

To give the reader a brief overview of the general attitudes and beliefs of the students, some of their comments have been presented in the following list. (See Figure 6)

<table>
<thead>
<tr>
<th>Student</th>
<th>Comment</th>
</tr>
</thead>
</table>
| Sue     | "Because it's the way we've done it before and— it seems to be routine."
|         | "I become more certain everytime I do it."
|         | "I'm more comfortable with it, and therefore probably more certain..." |
| Tom     | "Some students become too dependent on the technology and make the mistake of believing the calculator instead of their own common sense."
|         | "It's a good tool, but you have to be able to back up and look at it with common sense." |
| Kathy   | "I did the problem in the way that I was taught of—for under the prediction method of putting in 3 for X. So I believe the answer is right..."
|         | "I remember how to push in the buttons and I remember what each button stands for. Whereas, I couldn't even recall precalculus..."
|         | "What gave me the confidence in my answer is how I—I guess how I arrived at my prediction...I recognized how to substitute your 1 for X and multiply 195.33 times 1 and then added the answer to 10988.49. I just recognized it as for how it was done."
| John    | "I multiplied, no I divided 200 by 5 or 5 by 200. Yeah. That's what I did. And then that sounds right. That sounds like a bank interest rate anyway, better than 40."
|         | "I picked about 3 for 1.5, let's just see how close we got." |
|         | "Well, it says it's correct. [Referring to the calculator]"

Figure 6: Student Comments for Research Question 1
There are several of the task items from the Initial Data Analysis Instrument as well as the interviews which apply to this category. From the Initial Data Analysis Instrument all four problems apply since the data in each case were curvilinear and the computer generated model was linear. In addition, the two task-oriented questions from the first interview are also pertinent to this category. Since the last three questions from the fourth interview involved regression lines which were not true to the data, they will be included in this section of the analysis as well. To begin to examine the pattern of responses to these questions, we will first consider whether or not the student chose to use the computer or calculator generated model and what level of certainty the student placed on his/her answer following the prediction. This is summarized in the following table (See Figure 7) where Y indicates that the student chose to use the computer or calculator suggested model and N indicates that the student chose not to use the computer or calculator model. The number given in the Cert. column indicates the student's stated certainty on the particular item as rated on a scale from 1 to 10 with 10 being most certain.
Student

<table>
<thead>
<tr>
<th></th>
<th>Sue</th>
<th>Tom</th>
<th>Cindy</th>
<th>Kathy</th>
<th>John</th>
</tr>
</thead>
</table>

**Initial Data Analysis Instrument—Curvilinear Data with Linear Fit**

1. Y 7 N 9 Y 8 Y 10 Y 10
2. Y 10 N 6 Y 8 Y 10 N 8
3. Y 9 N 8 Y 8 Y 10 N 8

**False Reg. Line & Eq.**
4. Y 7 N 8 N 6.5 Y 10 Y 9

**Interview 1 Task-Oriented Items—Curvilinear Data with Linear Fit**

1. Y 6 N 6.5 Y 7 Y 10 N 6
2. Y 3 N 8 N 6 Y 10 Y 9

**Interview 4 Task-Oriented Items—Regression Line Shift**

<table>
<thead>
<tr>
<th>Shift</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>N</td>
<td>7</td>
<td>4</td>
<td>6</td>
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</tbody>
</table>

**Figure 7: Student Computer Model Choice and Certainty**

For this research question we are primarily interested in the Computer Model columns in the table and not the certainty ratings. The certainty ratings will be used later to examine the third research question.

In glancing at Figure 7, Tom and Kathy stand out in their approach to solving the problems. Tom did not use the computer generated model for any of his work in these problems where the data and computer model differed, except for question 2 in the fourth interview where the regression line was shifted by 10%. But in this instance he did give a much lower
certainty rating. This was the first incident of regression line shifting for the interview and so it may not have been enough of a shift to prompt him to ignore the calculator model entirely. However, Kathy exclusively used the computer model even on the fourth question in the fourth and final interview where the regression line was above all of the data points on the scatterplot.

Similar to Kathy, Sue used the computer generated model almost exclusively invoking it on every question except for the question where the regression line was above all of the data points (question 4 of interview 4).

The behavior observed of Sue and Kathy may be linked to their view of mathematics. Both of these students approached most of the problems throughout the study in a procedural manner.

Procedural Approach: Sue and Kathy

During the second interview, Sue gave a certainty rating of 8 for her answer to a task oriented question. When asked why her rating was an 8, the student replied, "Because it's the way we've done it before and—it seems to be routine." The researcher then asked Sue to compare her certainty of this answer to her certainty of answers to previous problems to which she replied, "I become more certain everytime I do it." When questioned as to whether or not she is more certain on the latter problems than on the previous problems, she stated, "I'm more comfortable with it, and therefore probably more certain, even though I know that there is a chance this one might be incorrect, even slightly incorrect, while the other ones could have been correct, but I feel definitely more comfortable with it
and therefore more confident in it." When questioned what she meant by "it", Sue replied, "just doing these problems with the program."

Throughout the quarter Sue had been performing similar problems both in her class work and in the interviews using DUSA and the TI-82. This "routine" as she called it, may have contributed to her certainty of her answers. Especially since her view of mathematics in general seemed to be that of a set of procedures which were to be memorized and applied in a given situation according to examples consistent with a chapter section in a textbook.

The procedural approach by Kathy was even more obvious than Sue. After having the "think aloud" process demonstrated by the researcher with an example which linked concepts in mathematics together to arrive at an approach for solving a problem, Kathy then proceeded to verbalize only steps that she was performing. Her verbalizations consisted almost entirely of detailed keystroking procedures. For example, during the first interview Kathy was asked to make a prediction for the amount of money in a savings account after a given number of years based on compounded interest data. (See Interview 1, Problem 2 in Appendix B.) In response to this problem Kathy stated,

> How much would you predict is in the account after 3 years? OK, so I'll go to the STAT menu and edit my table. Time will be the X variable, and that will be: 5, 10, 15, 20, 25, and 30. [Kathy entered the time values in the L1 list.] Then the money would be: 1000, 1200, 1800, 3000, 4200, and 6000. [Kathy entered the money values into the L2 list of the calculator.] So I want to go back to the program button, go to the regression program, enter the regression program, and now to predict—3 years. [Kathy chose the Predict option from the menu.] What the money amount would be after 3 years, so X will be 3 and Y will be negative 49. [Kathy entered 3
into the calculator at the prompt and the calculator returned -49.9047619.] How much would you predict would be in the account after 3 years?

She reread the problem and then continued, "How would you rate your certainty of your answer? I want to rate my answer with a lot of certainty, 10, however, I was surprised to find that the answer was negative."

The researcher then wanted to press Kathy's concern for the negative answer to see if the context of the problem could shake her hold to her procedural approach. So the researcher then asked why she was surprised that the answer was negative? To this Kathy replied,

I guess because all of the answers that were on the Y variable for money were all positive. However, 3 is less than 5 so there is a possibility that the answer is negative. Furthermore, I did the problem in the way that I was taught of—for under the prediction method of putting in 3 for X. So I believe the answer is right -49 point—that I would have a debt of negative 49 dollars—almost 50 dollars.

One of the key indications of Kathy's step-by-step philosophy of mathematics was her statement,

I did the problem in the way that I was taught of—for under the prediction method of putting in 3 for X. So I believe the answer is right -49 point—that I would have a debt of negative 49 dollars—almost 50 dollars.

It is alarming that she not only gains confidence in her answer due to her perception of the authority of instruction, but that she perceives instruction as a process of being shown a series of keystrokes for solving problems rather than thinking about a problem and then using the tools available to explore the problem.

The button pushing view of technology enhanced mathematics is a concern we must deal with. Kathy is a prime example of how the role of
technology can be misunderstood. Further evidence of Kathy's perception of the button pushing view can be seen in the following response. She was asked if she could choose to either use or not use the calculator in mathematics which would she pick? To this Kathy replied,

I guess for me and trying to take the faster way out of math possible, I'd want the calculator because it's—you know—it's fast, it's done with, it's over with. And I remember how to push in the buttons and I remember what each button stands for. Whereas, I couldn't even recall precalculus oriented—I mean, at the time that I was learning it, I thought that it was essential that I write everything down. But that's kind of—that's not—I mean, if you made me write on the back of this sheet of paper like all of like precalculus things that I could recall from taking precalculus—having it in high school and here—I mean freshmen year, only two years ago, I couldn't even remember what it would be. Whereas the calculator does it for you. There's different buttons to kind of remind you what to do. I don't know the sine equation or the cosine equation or the different things like that.

In this example, all of Kathy's understanding of her pre-calculus experience seems to be linked to her calculator. She even states that her calculator reminds her what to do, pointing out her perception that mathematics is again a set of procedures and steps to be memorized and performed.

Some of Kathy's perception of the role of mathematics may be linked back to her experiences in mathematics at the elementary school level. She shared an experience with the researcher from her early encounters with mathematics. She had attended a Catholic school and was having trouble with her mathematics. As a result, she went to a tutor and expressed the following concern from her experience.

I don't really think that I was taught well in grade school because I was more under pressure. That I was taught well—because I had to
go to like the math man. You had to go like across the—there was like this little van and it was part of the—I went to a Catholic school, grade school. So if we had problems we had to go to like the state for tutoring. So they had a van which was outside the limits because you can't mix church and state. So this woman was just—she was just horrible because if she—she got so frustrated because you couldn't understand it that she just kind of like pressured you into it or either suckered you into it. Oh well you can get a cookie if you answer this right, and stuff like that. So was just—I just wanted to get out of there as fast as possible. So that's where I think that—that's where I come back to my math problems and math mistakes. And I feel that I have more control when I use the calculator because I know that it's right. I know that if I subtract so-and-so or add so-and-so, I don't have to check it again. I know it's right. I feel that with the TI-82's and TI-81's with the larger screens you can check it even more because you can see it on the screen versus the one line display.

Kathy's need to get the "right" answer may contribute to her belief in the computer generated model. She perceives the computer as always being right and so will not question the feedback given by the machine. Her elementary school experience may have contributed to this view of mathematics since she was rewarded for getting the "right" answer.

For Kathy, the process of making predictions based on data did not only consist of a set of steps to be memorized, but moreover consisted of only one set of steps. Approaches for the other students varied from problem to problem. One student might choose to use the scatterplot to make a prediction on one problem and turn around and use the Predict option on the next problem. Whereas Kathy exclusively used the Predict option on all problems in the study except for the final interview where the researcher specifically asked her to also use the scatterplot.

During the final interview, Kathy was asked why she had not viewed the scatterplot in any of the earlier interviews? To this she
replied, "Because—I guess because I instantly get a—I instantly get an answer in the prediction plot and I just kind of, 'well this is a math problem—got to get the answer' and I don't really take time to interpret the answer".

Kathy's reliance on only one set of procedures for solving problems is also seen in her interaction with the Initial Data Analysis Instrument where she did not make direct use of technology. She was asked to solve the following problem (See Figure 8).

Data were collected on a certain population of bison giving the total population as a function of time (in years) as seen in the following scatterplot. The computer fit a regression line to the data yielding the computer model equation shown at the top of the scatterplot.

\[ y = 195.33x + 10988.49, \quad r^2 = .94 \]

What would you predict for the bison population at year 1?

Figure 8: Initial Data Analysis Instrument Question 1

She gave a certainty of 10 for this problem and so the researcher asked her why she was so certain of her answer. To this she replied,
What gave me the confidence in my answer is how I—I guess how I arrived at my prediction. Where I substituted your 1 for X and I—it basically—it just came really simple to me. I recognized how to substitute your 1 for X and multiply 195.33 times 1 and then added the answer to 10988.49. I just recognized it as for how it was done.

The researcher asked, "So you have seen that before—something like that before"? Kathy responded, "Yeah." Even though she was not able to enter the data into her calculator or computer and go through the process she had been practicing for her analysis of data, she did the next best thing by performing the task manually from the given regression equation. The fact that she was still performing the same procedure was enough to give her certainty about her solution.

Instruction

One possible explanation for the students' willingness to believe the model of curve-fitting proposed by the computer or calculator is that other approaches were seldom presented in class. The use of linear regression to make predictions based on data was presented in approximately one and one-half class periods. During this time the instructor cautioned the students on the inappropriate use of linear regression for data patterns which were not linear. However, only two examples of such data were presented. The data for the first example were obviously periodic in nature so the fit of a straight line to the data was not considered by the students.

The second example of curvilinear data, however, was presented during an exercise in class where the students were using the algebraic representation of the regression line to interpret the pattern within the
context of the problem. The instructor typically would call on students in the class to give an interpretation of the regression line within the context of the problem expecting the students to express the meaning of the slope and the y-intercept. In this instance, however, a discussion ensued and the class came to the conclusion that the linear regression model was not appropriate for describing the data pattern. Following this conclusion, the instructor again called on students to explain the meaning of the regression line within the context of the problem even though they had stated that the line was not an appropriate model for the data.

The practice of having the students interpret the meaning of the regression line did not seem to be of much benefit. The procedural nature of the way it was done could have contributed to the students viewing the problem-solving process as a series of steps as well. The instructor would give an example of how the slope and y-intercept were interpreted in a specific example and then call on students to interpret other sets of data. However, the students would simply parrot back an answer using the same phraseology as the instructor and this was seen as an acceptable response. The rote nature of the classroom solutions may have thus contributed to the students' perception of the role of technology in mathematics as a set of procedures to be memorized.

John

The approach taken by John was quite different from the approach of other students in the study. He seemed fairly confident in his own
ability to attempt problem solving. During the first interview, he began by not using the graphing calculator provided. Later the researcher asked him about his choice not to use it, and he stated that he did not realize that he was allowed to use it. It is interesting that his first instinct was to attempt to solve the problems without the aid of the technology.

Out of all five students involved in the study, John made use of the numerical data represented in tabular form more than the others. He was algebraic in his approach to looking for patterns in the data. This can be seen in the following excerpt from the first interview. In this problem, John was asked to make a prediction for the amount of money in a savings account for which the interest was being compounded. The John began,

OK, time in years are going up by 5. And you got to think interest and all this when you think how much money is going to be there. So if it's $10000 the first year or at 5 years—no it's $1000 at five years and then 10 years later it's $1200, that's a $200 increase in 5 years so you would think the interest rate would be about—4% interest on the money. So you would have to go backwards from 5 and then go 4% of what it would be at—no that's in 5 years. It would be 4%.

The researcher interrupted, "How did you figure the 4%"? John replied,

I multiplied, no I divided 200 by 5 or 5 by 200. Yeah. That's what I did. And then that sounds right. That sounds like a bank interest rate anyway, better than 40. Of course it wouldn't be 40. Or it wouldn't be 20 or 25. It wouldn't be 25 or 40. So if I divide anything else. So 4 sounds good. 4%—4 well maybe not—yeah—no because 4 times 1000—maybe it's 4% a year. Five years that would be 20%. 20 times 1000 would be too much. So that's not 4%. Scratch that 4%—In 5 years you get $200 more so let's divide that by 5 just to see how many you get each year—how much more money you get each year. [John performs long division on the paper.] You get 40 more dollars a year. So every year you get 40 more dollars. But then you got to think about——. [John listed the numbers underneath each other vertically as if to eventually add them using column addition.] OK, 1000, 40, 40, 40, 40, 40 that
would be it, but you can't use percentage because after 1 year then if I wanted to do 4% of that it would be a different number. So that scratches. I'm making this too hard than really what it is. Let's see— It goes up 200, 600, 1200, and 1800. There's no comparison there. OK, you got to think that at zero, when you didn't have any time, but you had a certain amount of money in the bank— Let's just say $500. Just for sake—I'm just going to start—just to see what happens. Say you start off with $500 and in 5 years if you earned a 5% interest on it—just another guess—then in 5 years you would have— I'm making this harder than what it really is. I know I am. Well, I know it's not going to go up. I know it's not going to be—3 years is not going to be less than—it's not going to be 200 less than 1000 because 5 years later after 1000 you only go up 200. So I know it's going to be close to 1000 at 3 years. Just how close...

It is interesting that John is comparing the method of 5/200 to the method of 200/5 and choosing the one which he believes sounds better as an answer rather than analyzing the methods instead. It is also interesting that John saw this as a reason to disbelieve his earlier assumption since if he had calculated in his head correctly, 20% x 1000 he would have gotten 200 which would have confirmed his earlier prediction of 4% even though the 4% is incorrect.

At this point the researcher let John know that he was allowed to use the graphing calculator provided. John then proceeded to solve the problem using the graphing calculator. He entered the data into the calculator and then said,

OK, let's look at the regression equation. [John chose the regression equation option from the menu.] That doesn't tell me much. So prediction. [John then chose the predict option from the menu.] Three. [He entered 3 at the prompt and the calculator returned -49.9047619. John then went to the predict option again and entered 27 and obtained 4777.52381 from the calculator.] OK, I did prediction of 3, but since 3 wasn't on the graph— I mean on the chart, and it was -49 maybe like -50 so I probably subtract 50 from 1000 which is about $950. So it's probably $950 at 3 years. And
just to check and make sure I was doing it right I'll just put in another number, 27, which is—not halfway, but almost halfway between 25 and 30 years. And I know that 25 years is $4200 in the bank and at 30 years there's $6000 in the bank and I came up with—when I put in 27 I came up with about $477 [meaning actually $4777] so that seems about right. I feel very good about this answer.

Upon using the linear regression model for the problem and receiving a negative answer from the calculator, he proceeded to use the answer along with other information from the problem until he had an answer that he felt made more sense within the context of the problem.

After finishing this problem, he then went back to the first problem so that he could use the graphing calculator for his solution. He seemed to give some authority to the graphing calculator since he stated, "I picked about 3 for 1.5, let's just see how close we got." This statement may indicate that he views the calculator as an authority since he was going to see how close his prediction was to the calculator's prediction.

Again he proceeded to analyze the problem with the calculator, but came upon an unexpected result which prompted him to pursue other methods of predicting which were not covered in the statistics course. He began his analysis by stating,

Right now I'm entering the data into the calculator. Now that I have the data in, I will go to the program that does regression. [John went to the regression program.] I go to the prediction of 3. [He chose the prediction option and entered 3 at the prompt. The calculator returned a predicted value of Y=26.] And I get—what! 26? Oh it was a prediction of 1.5, but anyway 3 should be 14. Maybe I did that wrong. Let's view my data.
John entered the STAT menu and checked the data which were entered into the calculator and saw that they were the same as in the given data table. He then continued,

Yes. That's correct. Well, it says it's correct. Go back to regression. [He went back to the regression program and chose the predict option and entered 1.5 at the prompt. The calculator returned a predicted value of -1.] Predict at 1.5, negative 1. I don't understand what that means? Maybe you subtract 1 from Y at—maybe at 1.5 it's 4. Let's just put another one in there just to see... . [John entered 2 at the predict prompt and received 8 for a predicted value.] Two is supposed to be 5. But it says eight. I don't know. I don't understand what's going on. Oh, maybe—Oh yeah. Maybe that's a cube root— There's let's see— Cube root of 3 is 27. If it's anywhere near 27... . [He entered 3 at the predict prompt and the calculator returned 26.] Ah. So it's something that has to do with cube roots. The cube root of 1 is 1. The cube of 2... So subtracting 1 everytime I get a cube root, let's see—there's the cube root of 20, the cube root of 3 is 27. How did they get 14? Maybe that doesn't have anything to do with cube root—Let's find the cube root of 1.5. [John exited the program and used the calculator to calculate (1.5)^3 and got 3.375.] 3.375 which is what I predicted at 1.5. But I don't think the cube root has anything to do with it. I don't know. Maybe it does. I'm stuck. I don't know. Well my prediction is still 3.375."

John used the graphing calculator, but then upon reaching a number which did not make sense to him, -1, he decided to try other methods for solving the problem which were unique to his own understanding. Ultimately, he went with his solution of 3.375 rather than the calculator's response of -1. It is important to remember though that John did use the calculator model for curve-fitting on 4 of the 9 occasions where the data did not match the computer model of a linear fit suggested by the calculator.
Tom

On the other hand, Tom did not use the calculator suggested model for curve-fitting on any of the 9 problems where it was inappropriate except for problem 2 of the fourth and final interview where the regression line was shifted by 10%. The mathematics background of John may have contributed to his confidence for deviating from the suggested model and allowed him to analyze the data without the need to conform to the calculator response.

During the first interview Tom stated, "Some students become too dependent on the technology and make the mistake of believing the calculator instead of their own common sense." It appears that his view may be linked to previous experiences he has had in mathematics since following this statement the researcher asked him how he felt about the statement, "Calculators and computers are never wrong." To which Tom replied,

I would say, well, it's not as if they're not—I mean, they're correct in what they're supposed to do, but you have to know how to program them correctly in order to get information from them that's useful. I mean, like for example, if you use the graphs I mean, and say you're looking at a parabola, you could make the window weird dimensions and so that a straight line looks like it's this way... It's a good tool, but you have to be able to back up and look at it with common sense...If you just get in the habit of saying, 'it sure doesn't look right, but it says so, so I guess I'll believe it', then you're gonna end up—you know. You have to be careful of that because if you...try to believe it blindly and don't understand what it's saying then you can be fooled as to what it's telling you. What it may be telling you is not wrong, but you may not—if you're not versed in the problem you may not understand what it's telling you and you may think it's telling you something else.
From this response it appears that Tom may have had some experiences with graphing technology where scaling was changed to give different appearances to graphs. Several of the faculty members at this university use this approach of scale to point out pitfalls of the graphing calculator to their students. Since Tom had two quarters of Calculus at the university, it is possible that he had seen this technique for altering the appearance of graphs.

Experience seemed to play a role in most of the solutions to the problems for Tom. At times he alluded to concepts addressed in Calculus courses. For example, in the first interview he began by considering the rate of change of the numerical data given. The researcher gave Tom the first set of problems. He began by saying,

I guess my first thought looking at this is that this isn't a straight line obviously because the Y values are increasing much faster than the X values. So I guess my next step is to plot it and try to take a look at it and see just what it looks like. [Tom entered the data into the calculator by entering all of the X values first and then all of the Y values. He then entered the program and went to the view regression line option.] Well, I guess what I'm trying to do is decide right now if it's definitely—it still looks curved to me. If this trend continued that it would be a curved line. So, then I would use the pointer to bring it as close as possible to 1.5 and try to place it where it would look like it fit. [He moved the cursor to X=1.5212766 and Y=2.4516129.] I would say about 2.4. Then there's always that there's—well, I'm not quite at 1.2 and it's jumping a fair amount [referring to the cursor]. I would give it a 6 or 7 [meaning certainty rating].

This method of solving a problem was typical for Tom. When he did not think a pattern was linear, he would use the cursor to scroll over to the place where he felt the missing data point belonged in the pattern visually and then used the coordinates of the point to answer the question.
(See Figure 9.) He would also frequently use the cursor to place an error range on his answer as he alluded to here by saying, "Then there's always that there's—well, I'm not quite at 1.2 and it's jumping a fair amount. [Referring to the cursor.] I would give it a 6 or 7 [certainty rating]." Here he was basing his certainty rating on how much jump there was in the pixel movement on the screen.

![Figure 9: Tom's Point Insertion Technique](image)

4.4 Computer Model vs. Scatterplot

In this section we will examine how the students coped with conflicting information when the computer model for curve-fitting and the scatterplot of the data differed significantly as stated in research question 2. There are two main types of instances in deviation of computer generated model from the data scatterplot. The first is when the data were curvilinear while the curve-fit model was linear. The second was when the researcher systematically shifted the linear regression line fit to the data by increasingly larger percentages of the view screen.
We will begin with the latter since each of the five students in the study were given the same set of four task-oriented questions from which they were asked to make prediction about the data. This will allow for comparisons which can then be related to the responses on the linear fit to curvilinear data questions found earlier in the study.

Again to give the reader a general overview of the students' reactions, a list of comments by the students is presented in Figure 10.

<table>
<thead>
<tr>
<th>Student</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>• &quot;It seems like the majority of the points are below the line because of an outlier.&quot;</td>
</tr>
<tr>
<td>Tom</td>
<td>• &quot;I don’t know—when first looking at it, it's strange that the regression line is where it is. It looks to me like there's more points on one side of the line than the other.&quot; • &quot;Based on my understanding of the line, that line can't be right. But somehow the calculator thinks it is... .&quot;</td>
</tr>
<tr>
<td>Cindy</td>
<td>• &quot;You know how when you have an exponential and it curves up really fast. It seemed to do that only in the opposite way which I kind of don't think a straight line would be the right way to explain that.&quot;</td>
</tr>
<tr>
<td>Kathy</td>
<td>• &quot;It's suitable. [Referring to regression line.] I mean, it's going in the same direction and it's going through a couple of points. However, there's a lot of outliers.&quot;</td>
</tr>
<tr>
<td>John</td>
<td>• &quot;I was thinking of the regression line again and...even though they are all pretty close to the regression line, they aren't clumped together—it wasn’t hitting as many points as I would suspect for me to be 100% confident.&quot;</td>
</tr>
</tbody>
</table>

Figure 10: Student Comments for Research Question 2

4.41 Patterns from Final Interview

The fourth and final interviews show some of the typical response patterns by the students. Probably the most noticeable student with respect to scatterplot use would be Tom. Throughout the study, he tended
to make frequent use of the scatterplot for making his predictions. Even when using the calculator model of regression to make his predictions, he still would make use of the scatterplot to give error estimates by verbally stating the equivalent of error bars. This is evident in his responses to the first and second questions of the final interview.

Tom

In the first question, Tom read the problem and then entered the data by entering all of the X-coordinates [rainfall] into the L1 list and then all of the Y-coordinates [yield] into the L2 list. He stated,

All right. OK, let's do the usual here and plot it out. [Tom chose the View Regression Line option; See Figure 11.] OK, well looking at this you can see it's kind of what we've been working on where it's not a—you can see there's kind of a pattern, but it's not exactly everything on line. The regression line does a pretty good job. So, I guess what I would do is—we're trying to find a yield for 12.4 inches so find kind of where that is. [He moved the cursor along the X direction until he was close to the 12.4 and then along the Y direction until he was on the regression line, X=12.332128 and Y=27.6; See Figure 12.] Now the line says around 27, but let's say... . [Tom moved the cursor up and down vertically toward the edges of the data cloud.] I would say 27 plus or minus 4 or so because there's a lot of variables here and there's certainly a lot of other factors here besides rainfall.

Figure 11: Tom's Graphical Display
For the second question, Tom entered the data into the calculator and stated,

I'll plot these out and take a look at it. [Tom chose the View Regression Line option and then the Regression Equation option and then went back to the View Regression Line option.] OK, well this one has a correlation of .79 which is still not too bad. It's strange. I don't know—when first looking at it, it's strange that the regression line is where it is. It looks to me like there's more points on one side of the line than the other. I mean, it seems—unless there's some points that I'm not seeing... Well, it doesn't—the regression line there, the mathematical line, doesn't follow my intuitive line... I mean, I don't even know why it would be like that because it seems like as far as like least squares, there's lots more below it than above it. It seems like there's a lot more distance to the points on the bottom. Distance from the line to the point—perpendicular bars—we kind of drew in bars—that's kind of strange. Well, they're looking for 7.3 for the plasma dioxin which is the X, I would say. Is that the right way? [Referring to the X and Y coordinate set up in the calculator STATPLOT menu. Tom then moved the cursor on the screen to a point on the given regression line X=7.326383 and Y=8.0158065; See Figure 13.] It's hard to tell. It's not really in the range of the other ones. The last one I was more certain about because it had the cloud of points around it so it seemed as though there was more—you know—direct points around it and a network that applied to it. Well, this down at the end here is starting to get kind of in the range where you're basing the data down there on the data someplace else—You know, you have a couple that are a little bit closer, but most of the data is down here...
[referring to the area below the regression line] so I would be less certain of a prediction up there. If it had a similar range as the other one, it would be kind of plus or minus 2 or so. Eight plus or minus 2. But I would be less certain of this one because it seems as though there's less actual data around there and more just based on the line. Maybe 5 [certainty rating]. There might be a trend that's kind of starting to appear here that's not evident from looking at this because we don't have any data points past it or even that many that's just around it to really observe from.

![Figure 13: Tom's Cursor Movement](image)

Later on in the interview, Tom continued to use the scatterplot by scrolling around the screen with the cursor. In question 3, he first began to ignore the regression model suggested by the calculator. In order to make a prediction based on the data, he instead scrolled with the cursor to insert a point where he thought a true regression line would fall.

In question 4, Tom used the same method as in question 3 of inserting a point into the data pattern so that it matched the pattern according to his visual inspection. When asked what he would do with the calculator generated regression line, he replied, "I would chuck it." He was willing to completely ignore the calculator model and go with his visual instinct based on the scatterplot.
Tom seemed to prefer the graphical representation of the data. Throughout the study, he would choose the View Regression Line option first in solving the task oriented questions. This can be seen by looking at his response pattern during his final interview. For problem 1, he began by selecting the View Regression Line option and did not go to any other choice from the program menu. He even used the graphical representation of the regression line to make his prediction based on the calculator generated model by scrolling to the line with the cursor.

In problem 2, he chose the View Regression Line option initially. Upon concern for the asymmetrical appearance of the regression line, he then chose the Regression Equation option to investigate. But finally he went back to the View Regression Line option and scrolled over to the regression line to make his prediction. He seemed to prefer the method of scrolling to the line rather than using the Predict option. This was even more interesting since the software package DUSA used in the course did not allow scrolling on the screen, but instead primarily required the use of the Predict option for obtaining regression model coordinates.

In question 3, Tom used only the View Regression Line option scrolling to a place in the center of the data cloud to make his prediction. And in question 4, he used the same method with the exception of choosing the Regression Equation option between his graphical views of the data and regression line to investigate what he perceived as an incorrect placement of the regression line by the calculator. This view is evidenced by his response, "Based on my understanding of the line, that line can't be right. But somehow the calculator thinks it is... ." He is
confident in his understanding of the concept of regression enough to make a statement that the calculator is incorrect.

Cindy

Cindy behaved similarly in her use of the scatterplot to make predictions, however, not to the same extent as Tom. Her representational preference seemed to be algebraic. During her final interview, she chose the Regression Equation option to begin each of the four task oriented questions. She did make use of the graphical representation in each of the questions as well. The main purpose of her choice of the Regression Equation option seemed to be to check the correlation coefficient. This was a frequent practice in the previous interviews as well. The set-up of the Regression Equation output was the listing of the slope of the regression equation first, followed by the y-intercept, and finally the correlation coefficient. [See Figure 14.] However, after choosing the Regression Equation option, she would typically address the correlation coefficient first even though it was listed last on the screen. This might indicate that her primary reason for choosing the Regression Equation option was to check the correlation coefficient.
Even though Cindy chose the Regression Equation option first, she also made use of the View Regression Line option as well in each of the four questions from the final interview. In question 3 of the final interview, she used a method of predicting similar to that of Tom. She scrolled with the cursor to a point where she felt the regression line should be and used the coordinates to give her prediction. She again used the same procedure in question 4 as well since she did not trust the regression line being furnished by the calculator.

The use of inserted points was also employed by Cindy in the earlier interviews. For example, in the first interview the researcher was asking Cindy about her response to the Initial Data Analysis Instrument. The question was task-oriented and asked her to make a prediction based on a set of data. The student had outlined her methodology on the paper where she had stated that first she drew in a curved line onto the given scatterplot to fit the data pattern. She made an initial prediction of 65. After this, she entered the data into the computer program DUSA and used the Predict option to come up with an answer of 75.00969. She then
chose the 75.00969 as her solution. When questioned further about her response and why she chose the 75.00969 as her solution, she replied,

"Probably just because—the way the circles weren't drawn as dots, they were drawn as circles. [Cindy was referring to the instrument's graph using a large dot representation for the points as opposed to the representation used by DUSA which gave a much smaller representation of the points.] I didn't know if I had drawn the line through the circles the right way. [Cindy was referring to the curved line she had drawn on top of the scatterplot.] So I figured that seemed like it wasn't very—I just kind of drew it on. So I thought that by using the computer it would be closer to what it really was like if the computer had actually drawn the line—curved line."

The researcher asked if Cindy had meant a curved line or a linear regression line? To this Cindy replied, "Well, if it could have drawn the curved line, I probably would have thought that would have been more accurate, but it probably just drew a straight line, so."

The researcher asked her which she thought was more accurate, the 65 or the 75.00969? Cindy replied,

"Um—it kind of leads me to think that my guess was probably closer just because the way the line probably—it probably didn't make—it probably drew a straight line and this seems to be more curved to me and so I think mine probably might have been closer."

Cindy did seem to be more willing to use the inserted point method in the instances where the computer or calculator model of curve-fitting differed due to curvilinear data rather than a shifted regression line. The difference between scatterplot and suggested curve-fitting model due to curvilinear data will be discussed in greater detail in the next section.

In contrast to the reliance on the scatterplot by Tom and Cindy, Sue and Kathy seldom used the graphical representation. In fact, Kathy did not use any method of prediction other than the Predict option throughout
the first three interviews and laboratory observations. She used the graphical representation during the final interview only upon request by the researcher and stated that if the researcher had not asked her to do so, she would have continued to use only the Predict option.

Sue

During the final interview, Sue used solely the Predict option for the first two questions. On the third question she decided to look at the graphical representation which seemed to influence her use of the graph in question 4 as well since she exhibited concern for the regression line in question 3.

Sue also exhibited a reliance on the calculator generated model during each of the questions from previous interviews as well. Even when she used other representations in her problem-solving process, she ultimately went with the calculator model of curve-fitting in each instance except for the last question of the last interview. She seemed to express her problem-solving process in terms of the calculator stating procedures for her reasoning rather than the connection of mathematical ideas. This was also true for Kathy.

One instance of student Sue's link to the machine can be described by her response to the first problem of the first interview. The question given was (See Figure 15):
Consider the following data. Predict the value of Y when X=1.5.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
</tr>
<tr>
<td>6</td>
<td>91</td>
</tr>
</tbody>
</table>

Figure 15: Problem 1 from the First Interview

While solving this problem, Sue decided to use the Regression Equation option obtaining the following results. (See Figure 16.)

As she was verbally describing her procedure, she stated, "18X plus negative 28." It is interesting that she would choose to use this terminology rather than, "18X minus 28." During earlier informal interviews, she used a form similar to "18X minus 28." Upon examining her Initial Data Analysis Instrument which she had completed on the computer using DUSA, she expressed her answer to question 2 in the form "357.918X-405.455." This was the format used by the DUSA software. It
seems that her response form was dependent upon the output format of the technology being used. In the case of the TI-82, the coefficients of the equation are given separately as seen in Figure 39 giving reason for Sue to express her answer in the form of "18X plus negative 28."

**Outliers**

Another factor which seemed to influence Sue and Kathy with regard to the use of the computer model was the instructional emphasis by the course professor. Many times during the final interview, both of these students could explain away the shifted regression line through the use of the concept of outlier effects on regression lines. In class the professor had stressed how an outlier can affect the course of the regression line. He had demonstrated this on the computer and had discussed how the outlier was "pulling" the line toward itself. Later during the third question of the final interview Sue stated,

> If you looked at the regression line [Sue chose the View Regression Line option]—you can see there's some outliers otherwise this line would be more in the middle—like see where that is, like that whole line. . . . It would be more down there I think—but no, it's way up—It seems like the majority of the points are below the line because of an outlier.

Sue then continued to use this explanation for the shifted regression line in the fourth question as well to try to explain away the unexpected result given by the calculator.

Also in question 3 of the final interview, Kathy stated,

> The graph seems very linear. The regression line—it just seems like the regression line is above all the points in the graph. It
seems like it's not a great fit, but it's kind of like a moderate fit regression line.

The researcher then asked if the fit was suitable? To which Kathy replied,

Yeah. It's suitable. I mean, it's going in the same direction and it's going through a couple of points. However, there's a lot of outliers. I mean, considering—well, they're still close to the line, but not as close as they could be.

She then went on to use the same explanation in question 4 as well. However, for question 4 she seemed more disturbed by the graphical representation of the regression line since the line was above all of the points on the scatterplot. She stated that she was confused because the correlation coefficient was high, but the points were all below the regression line. Even with her concern, she accepted the calculator model of curve-fitting to make her prediction.

The over generalization of the influence of outliers on regression lines seemed to affect the students' ability to question the feedback given by the calculator. Since they had seen, in class, regression lines altered, in a sense by outliers, they simply attributed the unexpected placement of the line to a concept stressed in the material presented by the instructor.

4.42 Curvilinear Data and the Scatterplot

The second type of conflict with respect to the second research question can be seen in the use of a linear regression model when the data are curvilinear. As stated before, the differences in problem-solving strategies used by these two pairs of students, Tom and Cindy, and Sue and Kathy, can be seen in the earlier interviews as well. In these
interviews, the difference in computer model versus data conflict was set up to be in the form of a linear regression model versus data which were obviously curvilinear. This occurred in interview 1, questions 1 and 2 and in interview 3, question 2 as well as all 4 questions from the Initial Data Analysis Instrument.

Mathematics Background

In examining possible reasons for the use of the scatterplot and disregard of the computer generated model by Tom and Cindy, these questions from earlier interviews seem to shed some light on the thoughts of the students. Mathematics background may play a role in the confidence of the students to ignore the calculator generated model and rely primarily on intuition for inserting a point into the pattern of the data and thus make a prediction based on the scatterplot alone.

Both Tom and Cindy had considerably stronger mathematics backgrounds as compared to the other students in the study. Tom's background included Algebra I, Algebra II, Geometry, Pre-Calculus, and Calculus in high school as well as Chemistry, and Biology. And at the university, Tom also experienced two quarters of Calculus as well as Physics. Cindy's background included Algebra I, Algebra II, Geometry, Pre-Calculus, and Calculus in high school and one quarter of Calculus at the university. She had also taken introductory courses in computer programming in high school and at the university.

Evidence of the possible tie to mathematics background can be seen in the responses to interview questions by each of the students. Tom used
ideas from calculus and rates of change in particular. During the first task-oriented question of the first interview, he stated, "I guess my first thought looking at this is that this isn't a straight line obviously because the Y values are increasing much faster than the X values."

During the second task-oriented question, Tom was asked to explain why he felt that making a prediction on the graph became more uncertain depending upon the desired region on the graph? He replied,

Because the slope is a lot less here. I mean, when you're talking up here [the upper right region of the data], it's hard to say is this on the line or is this on the line or is this on the line? It's a lot less obvious. When you're down here, you're not talking too much play here. [Tom motioned to the lower left region of the data; See Figure 17.] Because you're getting—you're not—if you go down here, you're way below this point. [Tom referred to the point X=5 and Y=1000.] And we know it's not here because there's just not much room to work with.

![Figure 17: Tom's Graphical Display for Question 1 of Interview 1](image)

In this example, Tom is employing the concept of variable slope of a graph—a calculus concept. He also shared experiences from his high school background which give more insight into his use of the point insertion method for making predictions. His experience was not limited
to linear models of curve-fitting. In high school he had taken Chemistry and Physics and shared his curve-fitting background by stating,

In chemistry, and physics and stuff like that. We talked about—you know—you plot your data and try to—you do titration curves or something like that in chemistry—I realize that's not a straight line, but I mean this is just kind of—it just shows you that there's a more mathematical way of doing it rather than just completely conceptual. Except when you use linear points, you can use that to make predictions more easily and stuff like that. I noticed there's quadratic regression lines and stuff like that too. [Tom was referring to other curve-fitting models available on the TI-82.] I don't know much about them, but I just saw that they're there—I guess that's kind of the next step for beyond this....

Here Tom is revealing that he has worked with curvilinear data before in a situation where he had to go with his intuition for predicting a pattern since the technology he had access to was not helpful with curvilinear data. The other students in the study with the exception of Cindy had very little if any laboratory science background. This may have given Tom more confidence to ignore the curve-fitting model suggested by the machine when it conflicted with his perception of the data pattern. The other students, however, having lacked that experience may have been more apt to follow the model suggested by the machine.

Cindy showed similar influences due to mathematical background. On one of the task-oriented questions from the Initial Data Analysis Instrument, the researcher asked her to explain what it was about the pattern of the data that caused her to be concerned about using linear regression. She replied,

It looks like it was exponential or like it started off—like it was decreasing—I don't know if there is such a thing, but like I think I even wrote somewhere that it was like—it seemed like it was decreasing exponentially or something like—you know how when
you have an exponential and it curves up really fast. It seemed to do that only in the opposite way which I kind of don’t think a straight line would be the right way to explain that.

Later during a task-oriented question, Cindy linked the context of the problem with her mathematical background by stating, "I know that bank interest is like the more money you have, the more interest you’re going to get so I think that’s exponential or whatever." It seems that the mathematical background may have given her a means of recognizing and labeling certain types of data patterns which in turn may have given her confidence to not follow the curve-fitting model suggested by the computer.

Number of Data Points on Regression Line

Within the subset of questions involving linear regression models fit to curvilinear data, another perception of the students surfaced. Kathy became concerned about the number of points the curve-fit model passed through. During the third interview, the researcher asked Kathy the following question. (See Figure 18.)
Suppose the calculator fit a regression line and a curved line to a set of data according to the following graphs, and that the correlation coefficient for both was \( r = 0.946 \). If you were asked to predict from this data, which of the lines would you use, if any, and why?

Figure 18: Question 2 from the Third Interview
In replying to this question, Kathy stated,

> Well, I guess I would use the curved line because two of the points go exactly through—go exactly through the line. And then one of the points is almost right on the line. And I thought that that would give a more close approximation than the second graph where only one is almost close to the line. I mean, they're all close to the line, but none of them are really touching or going through the line. They're not really going through the points. The line is not going through the points exactly. So I would use the first one.

To examine her relative certainties, the researcher then asked Kathy if she had to predict from both of the models how would her certainties compare? To this Kathy replied,

> I guess for the same reason, I would use this regression line more. [Kathy was referring to the curved fit.] Or I would trust this regression line more than I would trust this one. [Kathy was referring to the linear fit.]... The curved one I might rate it as an 8 and the straight one I might rate that as a 6. Or maybe a 7. I mean, they're pretty close, but I guess that I would just—I'd be more certain with the curved regression line than I would with the straight regression line.

Since Kathy seemed to believe that the better model was the one that went through the most data points, the researcher thought it might be interesting to probe into her beliefs about connected line graphs. The desire to use such graphs had been observed by Karplus (1979). So the researcher demonstrated what was meant by a connected line graph on the scatterplot given on the written set of questions by drawing line segments between adjacent points as seen in Figure 19.
The researcher then asked Kathy which of the models given thus far: curved line, straight line, or connected line graph, would she use to make her prediction? To this she replied,

I guess this one because all of the lines are going—the points are going through the line. [Kathy was referring to the connected point graph.] It's fitting it exactly whereas this one is a little bit off toward the end. [Kathy was referring to the curved line fit.] I guess now my thought would change to this one. [Kathy was referring to the connected line graph.] And this one is more accurate than this one. [Referring to the curved line fit.] A little bit. They're both the same—they have both the same correlation coefficient, but this one looks a little bit more accurate than the first graph. [Referring to the connected point graph.]

Kathy also exhibited her feeling that, the more points which the curve-fitting model passes through the better the model, in the final interview where the regression line was being systematically shifted. Twice during this interview she referred to points which were on the
regression line. In the second question, she was describing the fit of the regression line and stated, "And the points seem to be very—very closely correlated with the line. It's actually going—it's touching about 3 lines. [Kathy was actually meaning 3 points here.] So there's a very—very close—it's a very tight line—It's very—um—the line fits the data."

In the third question, after Kathy had shown concern for the fit of the regression line, the researcher asked her if the regression line for the data was suitable? To this she replied, "Yeah. It's suitable. I mean, it's going in the same direction and it's going through a couple of points. However, there's a lot of outliers."

Sue and John also exhibited the same concern for the number of points which the regression model passed through. During the final interview, John took time before beginning the second question to re-evaluate his certainty rating for the first question. After giving some thought to his certainty, John stated,

I put 95% on this. [John was referring to changing his rating of certainty from 10 to 9.5 for the first question.]... I was thinking of the regression line again and—I mean even though they are all pretty close to the regression line, they aren't clumped together—you know—it wasn't hitting as many points as I would suspect for me to be 100% confident.

The view that the more points a curve-fit model passes through, the better may lead students to feel that connected line graphs are superior methods of data analysis since all of the points lie on the graph of the curve-fit model. Thus, this may be linked to the behavior that Karplus (1979) observed.
Context

Another observed aspect of the scatterplot versus regression model conflict involved the context of the problem. At times students would obtain an answer from the machine, but whether or not it was accepted was determined by the student’s view of how the variables involved were related in reality. During these instances, the students would frequently express the conflict between the response given by the machine and their own expectations by viewing the data analysis process as an interpretation between specific versus general predictions made with regards to the data.

For example, while using the same methods of finding a prediction for the data as in earlier questions, Tom became concerned about his answer and the certainty with which he could give it. In earlier questions using the same methods this concern did not surface. However, it appears that due to the context of the question, Tom now wished to state some limitations. The context of the question was the reduction of cholesterol levels in a sample of people. Tom used the scatterplot to arrive at his answer. (See Figure 20.) After being questioned on how he arrived at his prediction, Tom replied,

Just judging. It's not like it's a big cloud, but it's not like everything is on the line either. I guess I would say somewhere around 216, but I wouldn't—you know—it would be a very rough estimate because this isn't—these are people and this is mathematical data so you can't be absolutely sure, but you would say that—you know—they would definitely hope to see a reduction in their cholesterol level after 3 years. In general, there is a trend. Exactly how much they are reducing their cholesterol level is somewhat variable. But people are reducing. I would say there's a trend overall. [The researcher asked Tom how certain he felt about a 216 prediction?]
Well, not exactly 216. I mean, well it's an estimate. It's not—you can't say oh this will be your cholesterol level because there's too many other factors. But, I would say 6ish, 7ish, kind of in that area. I mean, you can see that overall people are reducing their level, but some people are going down a little and some people are going up a lot. There's occasional people here who like are very—194 to 207, that person went up. So, based on all the data together you can see the trend, but as for any one person you can't be too specific. You could say—you could hope for a reduction—you know of—you know of 10% or something.

Figure 20: Tom's Scatterplot of Cholesterol Data

Context—Humanness of Data

Data involving people seemed to evoke concern among the students. When people were involved often the students would invoke this idea that the data could only give a general trend and that predictions about a specific individual were not truly applicable.

For Tom, this view of the inapplicability of specific predictions for data involving people surfaced again during the final interview. The context of the question entailed the prediction of a student first year GPA in graduate school based on his/her GRE score. In response to this question, Tom stated,
Well, the correlation between GRE score and GPA it refers to here—I would guess there would be some correlation, but it’s not going to be absolute because—you know—to a certain extent how hard you’re willing to—how much time you’re willing to put in has a lot more to do with it than—I mean—it helps if you do well inherently, but that only gets you so far.

Later in the interview while addressing the same question he stated,

Because in the context of the problem, there’s a lot of other factors here. It’s not specific enough to say, oh this must be it. It depends on courses and all this other stuff. I would probably give that one a 6.

Tom seemed to have the best grasp on the nature of curve-fitting and the implications of predictions made with a certain mathematical model. However, he was also influenced by the context of the question posed to him and treated his solutions differently based on his expectations and preconceived notions of the data.

The focusing on individual pieces of data and comparing these to the overall trend suggested by the model of curve-fitting was exhibited by Cindy also. Again, this tended to happen where people were involved in the context of the question. Cindy in response to the same question addressed above by Tom stated,

Well, right now I’m looking back at the list counting to see how many people increased instead of decreased. Well, 5 out of the 20 increased instead of decreased. And it seems like a lot of them didn’t really decrease like—well it seems like some of them are pretty close in what they decreased, so—and it’s like more than 20—a lot of them seem to be more than 20—so—no actually there’s another one. [Cindy was referring to another data point where the cholesterol level increased for a total of six cases.] I guess overall it seems like a lot of them are decreasing, but this seems to make it decrease kind of substantially. I’m not really sure if that’s like as close as it—as much as they would really lose in cholesterol".
Also during the final interview, Cindy when confronted with a regression line which was shifted until it was above all of the data points, attempted to explain away the conflict by the same means of contextualizing the data involving people. When discussing her thoughts she stated, "Well, even before I started putting this stuff in I didn't think they were right. I didn't think they could really predict anyone—what they would get in their GPA from what they got on their test." Later she also stated,

Yeah. I don't think that's really true. So that's what I was first thinking. I don't know. I know a lot of people who did pretty well on their SAT's and don't have a very good GPA and the opposite too—didn't do very well on their SAT's but still have a high GPA.

When questioned about her thoughts with regard to the specific versus general nature of data analysis for the given data, Cindy replied,

Well I think there would be a general association, but I don't think one that would be so—I mean, I think in general, that like a person who scores pretty well probably have a higher GPA, but I don't think you can really say well for every point you score on the test your GPA is going to go up so much. I don't think there's that strong of a relationship.

John also exhibited the same concern for using data to generalize when people were involved in the context of the problem. In interview 2 where he was trying to make a prediction with regard to cholesterol level change based on a sample, John stated that since some people's cholesterol level went up, it was not appropriate to make any conclusions. He went on to say,

I'm just looking at this. Like one person is 233 and 3 years later it was 239. Went up 6—So I mean, you really can't draw any conclusions—It's not going down any set amount or going up any set amount.
This data suggest that the context of the question plays a role in the student's willingness to question the computer or calculator generated model of curve-fitting. When confronted with different problems which used the same methodology within the premise of the problem for data analysis, students were more apt to question generalizability when human characteristics were the basis for the unit of investigation.

**Neighboring Points**

One final practice by students was noted through the interviews. It seems that students when confronted with conflict between the answer given by the computer or calculator generated model and the scatterplot tended to revert back to the original data provided in the table to make decisions based on the data. In particular, they seemed to focus on neighboring points as a means to formulate their response to the given question.

Sue exhibited this behavior during the final interview on the fourth question where the regression line had been shifted 30% which resulted in the regression line lying above all of the data points in the scatterplot. When asked what she would do to deal with her concern over the regression line, Sue stated,

I'd look at some of the other ones directly that are close to it like the one that's 580—um, I don't know. This has 3.7 for a 570, but then a 580 on this is 3.54—you know, I don't understand this—I mean even a 690 had a 3.6. I think they all seem to be really low—I don't know—I mean that's higher than the people who had the 600. I don't think that's correct.
The researcher asked Sue how she would go about finding an answer that she thought was more correct? To this Sue replied, "Throwing out an outlier." Sue then proceeded to delete a point which she felt was an outlier, the point with the highest GRE score. After looking at the graph Sue then continued,

See, it's still high—that brought it down to a little more reasonable—a little more believable GPA. Someone with a 570 and someone with a 580 had the same thing. That's a little more reasonable to me.

The researcher asked, "Why does it seem more reasonable"? To which Sue replied, "Because it fits right in with the scores better that it was closest to—580 and 550—I mean it's right around there—that seems more reasonable to me". Sue then proceeded to ignore the curve-fitting model suggested by the calculator and stated her answer based on the method of examining the neighboring points from the data table.

Cindy also employed the use of neighboring points to make her decisions. During the second interview, Cindy was concerned about the response given by the calculator and as a result rated her certainty lower than she had previously. When questioned as to her reasoning for her low rating she stated,

Well, since the correlation wasn't—I mean, in other ones the correlation was closer to 9 or something. And this is only about 75% correlation. So—and then just the fact that when I did look over here, I was going to check and see like how much a person improved from like 233 to—and then they went to 229 and then I looked at another close one which is 258 and they increased to 276 so that just made me think, um—this person might not necessarily even decrease. I mean—the diet might not even work—so I don't know—because I'm not really sure.
In the third interview, Cindy was trying to make a prediction for the weight of an athlete from a given height based on a sample of data. After obtaining an answer from the calculator, she said,

OK, the answer I got was 211.939. And since I got a pretty reasonable answer, kind of like—looking at the other ones like the other 79 is 205 and 80 was 215 so that follows pretty close to that. There seems to be a fairly high correlation. Just that the points are kind of scattered so—I mean, it’s probably give or take some.

During the final interview, Cindy again employed the use of neighboring points to make her decision. In the first question she was asked to make a prediction for the wheat yield in bushels for 12.4 inches of rainfall based on previous data. After viewing the regression equation with correlation coefficient and the graphical representation of the regression line she stated,

It seems fair, but it’s not like a really straight line but it seems like there’s a pretty positive correlation so—I’m just going to put 12.4 into the formula [Cindy wrote the regression equation onto the paper and then plugged in the 12.4. She then proceeded to work the algebra on the calculator separate from the regression program coming up with 27.59326.] Right now I’m just going back and looking at other ones that are close to 12 inches. And it seems pretty close to the 13. I mean, it’s just a little bit smaller than the 28.5 for that. So it seems pretty reasonable that it would fit in there.

Cindy seemed to rely on this method of checking the feasibility of the calculator answer with the data from the given table. In fact, she used this procedure in the second and fourth questions from the final interview as well. For the second question she stated,

OK, then the answer is 8.0244. Now I’m going to kind of look back and see what’s close. For the 7.2 they had the answer of 7.7. This one’s a little bit—7.3 is a little bit higher so around 8 seems pretty reasonable. It seems like an OK answer.
And in the fourth question her comments were,

OK, 3.6921. [Cindy went back to the view regression line option.] Well, now I'm just kind of looking back at what they had here. [Referring to the data table.] The first one got a 580 and had a 3.54. And they had more points, but their GPA is still lower than the one that's predicted for 570. And 550 is 3.4. [Cindy moved the cursor to the position X=571.51064 and Y=3.31 and then keeping the same X coordinate, moved to Y=3.4168387 and then to Y=3.5236774. (See Figure 21.)] The thing they—the GPA they predicted seems a little bit high. Just from looking at the other points that are close to it, the 580 and the 550. I would say you need more—the score would be closer to a 3.5 or something like that”.

![Figure 21: Cindy's Cursor Movements](image)

Kathy also suggested the use of neighboring points to help finalize her decision. During the final interview, she was trying to predict the first year GPA of a graduate student who had a 570 GRE score and stated,

...There's part of me that wants to see how like compare the first year GPA with the GRE scores and see if like maybe there's differences. For example, maybe if we took only the people with a GRE score of 580 and see what ranges of the first year GPA—like what different ranges there were. And then maybe base that on the predictions in here. I mean, I guess maybe if there was an average of the scores for the 580 GRE scores—if there was an average of their GPA. And OK, say if there was an average of their GPA,—OK, let's rearrange this. Say I'm going to change the work. Like say there was a GRE score of 570 and first year GPA's range from 3.54 to 4.0 maybe I would take an average of those and then compare with 570. If the 3.697 was the average, I would predict that this was the correct.
In this instance, a GPA was given for a 580 GRE score in the table, but no GPA was given for a 570 GRE score. Kathy was trying to use information from the table to get an idea about the GPA for the desired GRE score by looking at neighboring points.

John also made use of the neighboring point strategy. During the final interview, he used this method for questions 2, 3, and 4. In question 2, he was not happy with the answer given by the calculator (8.0) and so he was asked what answer he might use to give himself a little more certainty. To this he replied,

Something closer to—well, less than 8—something less than 8. See 7.2 gives 7.7... I'll just move to different points on the thing. And the first one was 2.7. [John moved the cursor until it was over top of one of the data points to check to see if it was consistent with the data table.] —See that all the points—OK we want 7.3—What did I say? 8? Ah, that's on the line. [John moved the cursor to a point on the regression line: X=7.326383 and Y=8.0158065.] Most of these points—6.9, 6.9. I don't understand—I mean, the majority of the points aren't on the line. I mean, all these—you can't really say because when it was 6.0 you got 8.0. When the Plasma Dioxin was 6.0, then the Fat Tissue was 8.0. The number bigger than 6.0 will give you less fat and that is—it comes out different.

In question 3, John was trying to estimate engine life based on the frequency of oil changes. He was trying to predict the approximate engine life for an engine whose oil had been changed after every 18.5 hours of operation. The calculator gave him an answer of 11.61. When asked how he felt about this answer, John stated, "Let's see. I think 11 point—well—It looks pretty good—based on this data like I'm looking at close to 18. Like 17.5, lawn mower lasts 11 and a half years. That's pretty good."

Again John was making use of data points close to the desired X value.
Finally during the fourth question of the interview, John was asked to make a final prediction. At this point, the calculator had given a response of 3.69. John stated, "I would say about 3.5 maybe." The researcher then asked what he was basing his answer on since it was a number which had not come up yet in the interview. To this John replied, "Well, even though I said that there wasn't a correlation, I just looked at somebody who had a 550 had a 3.4 and I want a 570 so I think a little bit higher."

In contrast to the students documented above, Tom was the only student in the study to express the opposite view for the use of neighboring points to make predictions. During the third interview, Tom was asked to compare the methods of using a curvilinear fit to the data versus a connected line graph for the same curvilinear data set. (See Figures 22 and 23.) In his comparison, Tom stated,

"It wouldn't be bad. I mean, it would—you know—it depends how much accuracy you're trying to work out of these things. I don't think it would be that different from a curve that curved through the points rather than went at a straight point. I mean, I guess in the middle of each you would have some place where if the curve were there and kind of bending out you would get a better approximation. As to if it would be better or not, I don't know. They wouldn't be that far apart. But I think the curve would be better than the straight line connection because the curve is based on all the points together while straight line connections are based on just the two points apart and if you get one that's a little bit off or whatever, then that's going to—you know—but strange things—I mean, this is pretty—it follows pretty well. [Referring to the scatterplot.] But if one of this was just a little bit in the curve would still—probably—move pretty well through it—and it wouldn't be as affected by it. Whereas if you put straight lines you would get this strange change in the data. Like if you put a point here are you saying that the trend is like this with a strange corner in it. [Referring to a point well off of the curved pattern of the
scatterplot.] I don't think so. I think the curve gives you a better overall feel for it because it's based on more points.

Figure 22: Curved Line Fit to Curvilinear Data
The use of the neighboring point method of predicting was used both with the graphical representation in the form of the scatterplot as well as with the tabular representation. The interpretation of this method may be connected to the Specific versus General trend analysis concept as exhibited by the students and reported earlier.

4.5 Computer Model and Confidence

In this section we will examine the third research question as stated earlier:

- Are students who choose the computer model to predict, more confident about their prediction than those who use only the data to predict?
To begin let us recall the certainty responses by the students for the questions where the data and curve-fitting model were not consistent. (See Figure 24.)

<table>
<thead>
<tr>
<th>Student</th>
<th>Sue</th>
<th>Tom</th>
<th>Cindy</th>
<th>Kathy</th>
<th>John</th>
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<tbody>
<tr>
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<tr>
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<tr>
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</table>

Figure 24: Certainty Responses to Questions with Inappropriate Models

Let us consider each of the students and their stated certainties one by one. We must also be mindful of the data analysis to this point and how earlier discussions of student exhibited behavior may relate to the patterns of certainty as stated by the students in this section.
4.51 Participant: Sue

Sue's responses to the certainty of her predictions does not seem to have any clear pattern. The only time she did not use the computer or calculator suggested model for curve-fitting was in the fourth question of the final interview. Her certainty rating given for that question was 8, however, the certainty ratings given for the other questions where the computer or calculator model was used did not differ from this certainty by much with the exception of the second question from the first interview where she gave a rating of 3. We can see this from the average certainty rating for each case, where Sue did use the computer curve-fitting model, and where she did not use the computer generated model. (See Figure 25.) For Sue, she had an average certainty rating of 7.44 when the computer model was used and 8.00 for the one case where the computer model was ignored. If the certainty rating for the second question of the first interview is not included, the average certainty rating for the cases where the computer model was used becomes 8.07.

In response to the second question of the first interview, Sue became concerned when the answer she received from the calculator was negative. The problem dealt with money in a savings account and thus the context conflicted with the solution that Sue had obtained. However, in response to this conflict, Sue did not make any attempt to solve the problem in any other way. Instead she simply gave a lower certainty rating of 3. In addition, she stated,
I'm not really sure why that's a negative—I know it probably shouldn't be—so I definitely would rate my answer 3—I think it's definitely wrong, but I'm not sure where I went wrong. I think I did something wrong, but I'm not sure exactly what it is—maybe I forgot something I learned in class—yeah, I don't think it is the correct answer.

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<th>Certainty Average</th>
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<td>John</td>
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<tr>
<td></td>
<td>N</td>
<td>7.00</td>
</tr>
</tbody>
</table>

Figure 25: Student Average Certainty Ratings

4.52 Participant: Tom

Tom's pattern of certainty ratings seems to be somewhat the opposite of what the researcher would have expected. He seems to be more certain when the computer or calculator model was not used. His average certainty rating for the cases where the computer generated model was not used was 7.19 as compared to 5.00 for the one case where the computer model was employed. It is important to remember in this instance that Tom was quite confident in his abilities to solve mathematical quandaries. He had a fairly strong mathematics background and displayed some familiarity with the use of graphing.
technology prior to the statistics course where this investigation took place.

The one question where Tom did make use of the calculator generated model of curve-fitting was during the final interview where the regression line was first shifted. Tom appeared surprised by the positioning of the regression line by the calculator. However, the shift may not have been enough to give him concern to change his approach to solving the problem, but rather just enough concern to elicit a lower certainty rating. It is important to note that on the next shifted regression line, he abandoned his use of the calculator generated model for the rest of the interview and his certainty ratings remained lower than they had been in previous questions where he had dismissed the calculator model as well. This may suggest that he was relinquishing some of his certainty when even the linear regression model conflicted with his expectations. Before, on the questions where a linear regression model was fit to curvilinear data, he did not follow the suggested model, not because he thought the model was incorrect, but rather because he felt the model was inappropriate. This is a subtle difference, however, an important one since on the last two questions of the final interview, he felt that the linear regression model was incorrect and not just inappropriate.

In response to the last question of the final interview where the regression line had been shifted until it was above all of the data points on the scatterplot, Tom stated,

I mean, if that is the line that's the way that you compute a regression line, I don't know—maybe there's—There's something I'm not understanding because I don't why—Based on my
understanding of the line, that line can’t be right. But somehow the calculator thinks it is so there’s—I don’t know—something in the formula or just the way it calculated that makes it do that. But I don’t know if it’s because this data is funny.

In this statement Tom exhibited more confidence than his certainty rating of 6 would suggest. This claim can be evidenced by his statement, "Based on my understanding of the line, that line can’t be right. But somehow the calculator thinks it is—" He seemed to believe that the calculator was incorrect and that there might be a hardware limitation which could explain the unexpected answer.

4.53 Participant: Cindy

Cindy used the computer or calculator model on five of the nine questions where the data and suggested model of curve-fitting differed. It appears that her certainty rating for the computer or calculator model was higher than for the instances where she chose not to use the technology generated model. Her certainty average was 7.60 for the cases where the calculator model was employed and 4.88 for the cases where the calculator model was ignored.

We must note however, that lowest certainty ratings of 4 and 3 occurred on the last two questions of the final interview where the regression lines were shifted by 20 and 30% respectively. Since the other instances for which the calculator model was ignored occurred when the data were curvilinear and the model suggested was linear, we must be careful to note that lower trust in her own solution may be due to the
intentional altering of the line and not necessarily to her distrust of her own abilities.

4.54 Participant: Kathy

Kathy’s pattern of choice to believe the computer suggested model is the strongest of any of the students. In all instances where the data and calculator suggested model differed, she chose to believe the calculator. This even occurred for the fourth question of the final interview where the regression line had been shifted resulting in all of the data points falling below the line on the scatterplot.

Kathy’s certainty ratings for the cases where she chose to believe the model of curve-fitting suggested by the machine (all cases) were high. In fact, she rated her certainty to be 10 on all of these question except for the last two questions of the final interview for which she gave both an 8. It appears that regardless of the degree to which the answer seemed to be incorrect, Kathy blindly followed the solution given by the machine. Evidence for this observation can be further seen by Kathy’s response to the third question of the final interview. After Kathy had expressed concern for the answer she had obtained from the calculator, the researcher asked her if she would prefer to go with an answer different from the one given by the calculator? To this she replied, "I don't know what I would do. I would just leave it because I don't know what I would do." This suggests that in the event that she would become confused about a solution to a problem, rather than trying to explore other possibilities for solving it, she would simply believe the machine. This was
consistent throughout the study. Kathy would seldom deviate from her procedural steps for solving problems. She attempted to find the answer in the fastest way possible so that she could go on and not think about her solution. During one interview the researcher asked her if she had a choice to either use or not use a calculator which would she do? Kathy replied, "I guess for me and trying to take the faster way out of math possible, I'd want the calculator because it's—you know—it's fast, it's done with, it's over with." She also stated that she prefers the calculator since she can do a problem and get an answer and then go on to the next problem without having to think about the meaning of the answer. During the first interview she stated,

But I guess that I'm glad that for a test you only have a certain amount of time. Because if I had more than enough time, I would be thinking about the problem and trying to make sense of it when I just want to get the—I just want to do the equation and just have it not make any sense.

4.55 Participant: John

John seemed to find confidence in the answers obtained from the calculator for the questions where the data and suggested model of curve-fitting differed. He chose the calculator's curve-fitting model for 4 of these questions while ignoring the model on 5 of the questions. His certainty rating average for the questions where he chose the calculator model was 9.25, while his average certainty rating for the questions where he chose not to use the calculator model was 7.00. More importantly, for all of the questions where the calculator model was utilized, the certainty rating
was always a 9 or a 10. In contrast, for all of the questions where the calculator model was not invoked, the certainty ratings were always 8 or below. In fact, the certainty ratings for this case ranged from 6 to 8.

It appears that John was more certain of the answers coming from the calculator than from his own methods of data analysis. It is interesting that this pattern of belief would develop. Early in the study, John seemed very confident in his ability to solve problems. Perhaps as the quarter progressed and he experienced the infusion of technology in both his class as well as in his interviews with the researcher, he may have received confidence from the fact that authority figures around him were relying on the technology. During the course, he may also have received negative feedback that reduced his confidence in his ability. The fact that technology was being promoted in both the course as well as the research study may have had an influence on what John thought he was required to do in solving the task-oriented questions.

4.6 Authority Factors

This section will address the fourth research question as stated earlier:

What factors surrounding the computer give authority to the machine in the eyes of the students?

To begin, some of the comments from the students have been summarized in Figure 26 to give the reader an overall view of each student's perception of the machine's authority.
<table>
<thead>
<tr>
<th>Student</th>
<th>Comments</th>
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| Sue     | "Because it's the way we've done it before and— it seems to be routine."  
  • "I become more certain every time I do it."  
  • "I don't know, maybe it was programmed wrong or something—sometimes software gets those kind of things." |
| Tom     | "The computer was programmed by seniors here—I'm sure they're good, but they might not have—they might have left something out or they might not have had it take things out to really far..." |
| Cindy   | "The reason why I did start using the computer was because he [Course Professor] said we didn't need to know how to use the formula. [Referring to the formula for finding the regression line.] That's why I used the computer—Then I felt like I didn't, I didn't need to know how it worked and stuff. I just needed to know how to put them in. [Referring to data points.]" |
| Kathy   | "I still feel that you learn more by always doing it over and over again."  
  • "...the reason why it is certain is because I substituted or I entered X for 1.5 to find Y. So I entered X as 1.5 to find the answer...That's how I always do it."  
  • "...maybe I would lean more toward the TI-82 because I know that DUSA was compiled by students. So there's kind of a hesitancy because I know that it was just like college students that probably—maybe some of them didn't really take this project lightly and didn't add in the last couple of digits." |
| John    | "If he [Referring to the instructor of the course] thought that the calculator could have did it, then he would have just said everybody get calculators or use your calculator to do this and show us how to enter the data into the calculator. But the DUSA program, we took a whole class day out—to learn how to use this program that would help us throughout the rest of the term." |

Figure 26: Summary of Authority Perceptions

In examining possible aspects of technology use which might tend to influence students to forfeit some of their autonomy in the problem-solving
process, several patterns emerge. The first of these patterns is the use of repetition. With the increasing availability of technology, we must be cautious as to how we implement the technology into the classroom. The danger of mathematics being transformed into a series of procedures and repetitive processes by which students arrive at answers to modeled problems is quite real as can be seen by the following instances.

### 4.6.1 Repetition

During the second interview, Sue was asked to give her certainty rating for the given problem. She responded with a high rating and when asked why she was so certain about her solution she replied, "Because it's the way we've done it before and—it seems to be routine." She was then asked to compare her certainty for this question with her certainty for the previous questions which had been asked in the study up to this point. To this she replied, "I become more certain every time I do it." When questioned as to whether or not she was more certain on the latter problems than on the previous problems, Sue stated,

> I'm more comfortable with it, and therefore probably more certain, even though I know that there is a chance this one might be incorrect, even slightly incorrect, while the other ones could have been correct, but I feel definitely more comfortable with it and therefore more confident in it.

When asked what she meant by "it", Sue replied, "Just doing these problems with the program."

Since Sue and Kathy were similar in many of their procedural approaches to solving problems, it is not surprising that Kathy also
seemed to gain certainty through the use of repetition. In relation to her belief in the use of repetition Kathy had stated, "I still feel that you learn more by always doing it over and over again." At one point Kathy gave a certainty rating of 10 for an answer. When questioned as to why she was so certain, she replied,

By 10 or very certain, the reason why it is certain is because I substituted or I entered X for 1.5 to find Y. So I entered X as 1.5 to find the answer. When X is 1.5, Y will be -1.

Later after displaying some concern for her answer to a question, Kathy was asked if she might be less certain about her answer. She stated that she was still certain of her answer and her reasoning for keeping her certainty was, "That's how I always do it."

During the second interview she seemed to reiterate this point by saying,

See, I guess the way—I guess it's kind of like the way I'm taught is that I'm taught to do—OK, this problem set in this lesson it's taught that you do it this way. So I guess my homework kind of makes me—like the lesson of the little chapter or something makes me do it more like I'm duplicating or replicating. Like I'm constantly—like constantly like a calculator. Constantly pushing out the buttons—the little digits and getting the number. I don't get or have chance to like think about the problem or think how I would do it this way.

Kathy's strong link to the authority of the machine through the use of repetition can also be seen in the final interview where she was trying to make sense of format of the data given. Up to this point in the study all of the data had always been given in tabular form with two columns. For the second question of the final interview however, three columns were given. The first column was simply an indexing column where each of the
subjects for whom data were collected had been numbered. The second and third columns were the variables of interest for the question posed. In response to the different format of the data table, Kathy stated, "Now I'm not certain whether I should have three columns because I feel that 1 to 15 is already kind of given, but I'm not sure how the calculator will—or how the calculator will interpret the data?"

The most convincing evidence of Kathy's increased perception of authority due to repetition however, can be seen by simply reviewing her response pattern for the questions throughout the study. Until the final interview, Kathy had solved all of the problems during the study by the same series of procedures. She would enter the regression program, choose the PREDICT option from the menu, enter the X value to which the desired Y value corresponded, and receive the answer from the calculator. Her certainty ratings for all of these questions were 10. It was not until the researcher requested that she look at the graphical representation that she deviated from this pattern. It is also interesting that the only certainty ratings which were not 10 occurred during the final interview, even though ratings of 8 would probably not be considered low. The researcher made a point to ask Kathy that if he had not requested that she use the graphical representation, would she have continued to use only the PREDICT option and given certainty ratings of 10? To which Kathy replied that she would have continued in her pattern of responses.
4.62 Corroboration of Representations

The next factor which seems to influence students' authority perception of the machine is the corroboration of different representations of the computer or calculator model of curve-fitting. It was anticipated by the researcher that conflict between representations could provoke students to either challenge the machine or to bow to the authority of the machine. However, it appears that the seeming lack of conflict between representations might influence students to believe the machine.

Kathy on several occasions used the match of answers between representations as justification for believing the calculator response. During the second interview Kathy had given an answer based on her procedure of using the PREDICT option. After this the researcher encouraged her to use the graphical representation to examine the fit of the line. He then asked her, "Does the line seem to capture the general trend of the data?" Kathy replied, "Yes. The points are not that far off. I mean, there's maybe one outlier that's in the right hand upper corner, but they're basically, pretty fit toward that line." The researcher then asked her if she was still as certain about her prediction? To this she replied, "Well—216.9... . [Kathy then moved the cursor around the screen checking the coordinates of the regression line to see if they matched with the answer she had received from the PREDICT option.] Yes I feel pretty certain that they would have a 216.9."

During the final interview, Kathy again used the corroboration between representations as a reason to trust the calculator generated model of curve-fitting. On the second question where she was first
confronted with the shifting of the regression lines, she began by choosing the PREDICT option from the menu. After obtaining a predicted value of 8.02335671 from the calculator, she then used the View Regression Line option to examine the scatterplot with regression line superimposed. She scrolled around on the screen until the X coordinate was close to the desired 7.3 and then scrolled up until the Y coordinate was on the regression line. At this point the researcher asked her if the 8.02 she had obtained earlier was a good prediction. To this Kathy replied,

Oh that's very close. I mean, considering—I mean that's 7.32633, but that's very close to the regression line. Yeah. Definitely. Very good prediction—I would say it would be a ten. A very strong prediction of—it's going on the regression line, so it fits pretty much in favor of it being very certain.

Again in the third question of the final interview, Kathy exhibited the same confidence in her answer due to the matching between representations of the curve-fitting model. In this question the regression line had been shifted by 20%. Kathy began by choosing the PREDICT option and received an answer of 11.61006504. When asked about her certainty, she was more cautious than before and stated,

Yes. I feel very certain about my answer, but I'd like to check. X is 18.5—and Y is 11.61. [Kathy used the cursor to check to see the coordinates of a point on the regression line where the X value of interest would be: X=18.470745 and Y=11.66129; See Figure 27.] That's about right on the line. So I'm giving my conclusion that the answer to this prediction is very certain because it fits the regression line.
Figure 27: Kathy's Match of PREDICT Point with Graphical Line

Kathy followed the same line of reasoning for the last question as well. She began by finding an answer using the PREDICT option. [The calculator returned 3.697669565 when Kathy entered 570 at the prompt.] Kathy then said,

When X is 570—it's roughly about 3.7 for the Y. [She then chose the View Regression Line option and scrolled to the point X=568.57447 and Y=3.31 and then scrolled up from that point to the regression line with Y=3.7106452; See Figure 28.] Way down there right as I suspected. The answer to the prediction problem fits right on the regression line and fits the regression line, however, the graph with all the other data—it just seems like the other data could be a lot closer to the line, but there was that evidence that there was a high correlation.

Figure 28: Kathy's Representational Match
Ultimately she chose to believe the regression model suggested by the calculator even though the regression line was above all of the data points on the scatterplot. What the researcher observed to be the turning point for her decision was the fact that the answer she received from the PREDICT option and the answer she obtained by scrolling onto the graphical representation of the regression line were the same. She also seemed to lend more credibility to the answer since she observed the correlation coefficient to be roughly 0.8.

John also exhibited the same trust due to representational corroboration. During the final interview he had already selected the PREDICT option and obtained an answer of 8.02445671 when he decided to examine the graph. He scrolled around on the graph and stated,

I'll just move to different points on the thing. And the first one was 2.7. [John moved the cursor until it was over top of one of the data points to check to see if it was consistent with the data table.]—See that all the points—OK we want 7.3—What did I say? 8? Ah, that's on the line. [John moved the cursor to a point on the regression line: X=7.326383 and Y=8.0158065; see Figure 29.]

![Figure 29: John's Representational Match](image-url)
Following his inspection of the graphical representation, John chose the Regression Equation option from the menu. After the regression equation and correlation coefficient appeared, John decided that he could use the regression equation to calculate and answer, however, it would only give him the same answer that he received from the PREDICT option. So he did not pursue that avenue.

4.63 Instructor Influences

The next factor involved in the students' perception of the authority of the machine was the technology–instructor combination. Since the instructor made decisions as to what technology would be used in the course, some of the students' beliefs about the authority of the computer or calculator can be linked to the instructor or the instruction in class.

Sue exhibited this influence by instruction. During the third interview she was asked a question where she was given two curve-fitting models to choose from. [See Figures 30 & 31.] When asked which she would rather use, she stated, "I would use the straight one—because you can relate correlation whatever to a regression line not to a curved line—not that I know how." She was willing to use the linear model even though the data were obviously curvilinear since the only instruction for curve-fitting she had been taught was that of linear regression.
Figure 30: Curved Graph From Third Interview

Figure 31: Linear Graph From Third Interview
Another aspect of instruction which may have influenced students to follow the machine's model for curve-fitting was the instructor's choice to not have the students perform the regression calculations manually. During the earlier part of the course when mean, median, mode, and standard deviation were being presented, the instructor had the students manually calculate all of these statistics even though the students had calculators which would perform these functions automatically. Later however, when linear regression was presented, the instructor did not have the students perform these calculations manually. This inconsistency may have given a sort of magical perception of the linear regression process since the students were not expected to perform these calculations.

Tom expressed this view by stating,

Like we're talking about these linear regression lines, but we didn't go over how to do that by hand. But I'm not sure we would understand that much more about the process if we did them by hand because it's a fairly complicated formula you know.

Tom did not yield to the calculator or computer model to any great extent. In general, he seemed very confident in his own abilities. Some of this confidence appeared to stem from his mathematics background and previous instruction. He gave evidence that he had been exposed to graphing technology before and the comments he had made suggested that he had been cautioned as to the pitfalls of graphing technology as well.

Cindy also viewed the importance of understanding the regression process as a low concern due to the instructor’s difference in the presentation of linear regression as compared to the descriptive statistics
covered earlier in the course. During the early part of the course, Cindy had held off on using the computer for constructing stem and leaf plots as well as box plots even though the computer handled both with ease. When asked why she waited, Cindy stated that she wanted to do it by hand first so that she would understand it. The researcher then asked her if she felt the need to hold off on using the computer for the linear regression analysis and instead calculate the regression line by hand first. To this she stated that she started using the computer immediately for the linear regression analyses in her homework. When asked why she had not followed her earlier pattern of hand calculation she replied,

The reason why I did start using the computer was because he [Course Professor] said we didn't need to know how to use the formula. [Referring to the formula for finding the regression line.] That's why I used the computer—Then I felt like I didn't, I didn't need to know how it worked and stuff. I just needed to know how to put them in. [Referring to data points.]

Like Tom, Cindy was perceiving the instructor’s lack of emphasis on the calculation of the regression line to mean that understanding the concept behind the process was of no importance.

The instructor of the statistics course also made a point to expose the students to inappropriate uses of linear regression. Tom related one such situation during the second interview. He stated,

It's kind of like I said before, technology is great, but you can't just close your eyes to everything else and head for it. I mean, we had that thing in class where Dr. Andrews had his gas bill and his electric bill and that one was just really obvious that there was some kind of trend to it even though we won't learn how to study that kind of trend. Putting a correlation and line through that—I mean it was just stupid because it didn't tell you anything. I mean, at least with the other data, even though it was farther off the line than these are, you had a sense that it was kind of trying to get at
kind of—you're trying to—you're capturing the essence of what the
data is saying. Because some was off and certainly wasn't very
exact, but at least you felt you kind of understood what was going
on.

In this instance, the instructor seemed to be effective in getting his words
of caution across.

Again with Tom the methods of instruction seemed to help him
decide when to be concerned about the calculator generated model.
During the final interview, he expressed concern for the regression model
due to conflict with the instructor's presentation of the concept of linear
regression. The instructor had illustrated the meaning of the least
squares regression calculation by drawing in vertical line segments from
each data point to the line and discussing the minimization of the sum of
these lengths. Tom recalled this during his analysis of the second
question where the regression line had been shifted by 10%. He stated,

I don't know—I mean, I don't even know why it would be like that
because it seems like as far as like least squares, there's lots more
below it than above it. It seems like there's a lot more distance to
the points on the bottom.

The researcher said, "Distance?" To which Tom replied, "Distance from
the line to the point—perpendicular bars—we kind of drew in bars—that's
kind of strange." Here the concept presentation seemed to influence Tom's
analysis of the question prompting him to challenge the authority of the
machine.

Cindy and Tom were similar in many respects on this issue. As
stated before, they had the strongest mathematics background of the
students involved in the study. Cindy also used classroom instruction to
challenge the authority of the machine. During the first interview, she was asked if the correlation coefficient had any influence on one of her predictions? To this she replied,

Yeah, I remember I did look at that and it was high and that's why I first did put it back in the problem because I thought oh, the correlation was high. But then because of the way the picture looked, and then I thought he said in class that you couldn't use linear lines for only certain things. They could only explain certain kind of patterns. So I thought this was one of those ones that really wasn't meant to explain. So then I thought—or that it was coming in too close or something. So that's why I drew my own and went with my own.

In contrast, Sue and Kathy viewed the role of the instructor in a different light. As reported in the previous section, both Sue and Kathy seemed to use the instructor to support their use of the calculator or computer model. Kathy had stated that she was certain of her answers because she had solved the problems in the manner that she was taught. However, she failed to take note of the instructor's use of multiple representations in class and instead focused solely on the use of the PREDICT option.

Sue in like manner, viewed the instructor's role as that of a trainer who would show mathematics in a very structured and procedural way even though the instructor did not typically follow this method of teaching. Instead, the students seemed to latch onto the instances where the mathematics presented appeared procedural and ignored the other instances where the presentation of material was more conceptual. This may be in part due to the background of Sue and Kathy.
Throughout the study, both Sue and Kathy shared their experiences in mathematics in elementary and secondary school. In each case they related experiences where either they were instructed in procedural methods or that they perceived that they were being instructed in procedural methods. In contrast, Tom and Cindy related mathematics experiences where the focus of the instruction was conceptual.

John seemed to place some faith in the instructor as the person who chose the technology to be used in the course. During the second interview, John was asked a question where he needed to choose between an answer given by the TI-82 and one given by DUSA. He chose to use the DUSA answer. When asked about the influences which prompted him to choose DUSA he stated,

If he [Referring to the instructor of the course] thought that the calculator could have did it, then he would have just said everybody get calculators or use your calculator to do this and show us how to enter the data into the calculator. But the DUSA program, we took a whole class day out—to learn how to use this program that would help us throughout the rest of the term. So yeah, that did have something to do with it.

It appears that the influences of the instructor cannot be simply separated from the technology. It is important to note that even though technology may seem to be objective in its approach to problem solving, the human factors surrounding the technology play an important role as well.
4.64 Creator/Human Influence

Probably the most surprising factor involved in the authority of the machine was the perception of the persons who created the technology. The researcher had not considered this factor at the onset. However, during an informal interview with Sue, the researcher decided to pursue this avenue of questioning.

During the informal interview, Sue was sharing an experience she had had with one of her friends while working on homework. Her friend was doing her homework on the DUSA software while Sue was using the TI-82. On one of the problems they each obtained a different answer. Sue being confident in her solution had her friend check her data entry. As it turned out, Sue's friend had incorrectly entered the data. This discussion prompted the researcher to pose a question to each of the students in the next interview of what would they do if they used both DUSA and the TI-82 for a problem and came up with different answers?

In response to this question, Sue replied that she would use the answer from the TI-82. This was surprising to the researcher since he would have suspected that the students would view the computer as more accurate. When pressed as to why she would choose the TI-82, Sue stated, "DUSA's scarier." She went on to say that DUSA would have even more problems with it than the calculator would. When asked about the sorts of problems to which she was referring, she replied, "it might have a bug or something." The researcher asked where the bug might come from and the subject replied, "I don't know, maybe it was programmed wrong or something—sometimes software gets those kind of things."
Tom began with this question by suspecting that the computer might carry the number of digits out farther. However, while discussing his thoughts he voiced concern about the persons who created the software. He stated,

I don't know—it depends—there are so many factors. I mean, time would be that I would say, oh the computer must have gone bad, but the computer also depends on who programs it too. The computer was programmed by seniors here—you know—I'm sure they're good, but they might not have—they might have left something out or they might not have had it take things out to really far—it just depends on—I don't know—I don't think you can just say the calculator or the computer. The computer has more computing power, but it depends. The computer is not worth anything if the person programming it doesn't know what they're doing. So, I don't think you can say one way or the other.

Kathy also expressed concern about the creators of the DUSA software. During the second interview she was asked which piece of technology she would be more apt to believe? She replied,

But in this sense there's kind of a—maybe I would lean more toward the TI-82 because I know that DUSA was compiled by students. So there's kind of a hesitancy because I know that it was just like college students that probably—maybe some of them didn't really take this project lightly and didn't add in the last couple of digits. Or just rounded up or something. So I guess I would take it to the TI-82.

When asked who she thought wrote the regression program for the TI-82, Kathy replied, "Probably some—maybe math professor working for the company or something that just has one—probably did one formula and then put it on one and it was just duplicated or replicated."

The researcher then told Kathy that he had written the program for the TI-82 using the pre-existing functions of the calculator. Upon this
disclosure, Kathy was not concerned since she viewed the researcher as an authority since he was teaching at the university at the time.

4.7 Additional Observations

The ability to explore unanticipated aspects of a situation is one of the strengths of qualitative research. In addition to the research questions stated in chapter 1, the researcher observed other aspects of the use of graphing technology worth noting at this point. The main observation which will be addressed in this section is the danger of students to perceive graphical relationships as all being time dependent.

As stated in the review of literature, Microcomputer-Based Laboratory has become an accessible means for exploring physical phenomena. The use of probes to "instantaneously" collect data and display it in graphical form simultaneously can help students pair physical events with their abstract counterparts in the form of a graph. However, we must be careful not to give the impression to students that all data can be expressed in terms of time dependence.

The researcher first noticed this overgeneralization while working with Sue. In the third interview, Sue suggested that the scatterplot of the data given had a trend that the regression line was not fitting. (See Figure 32.) She thought that by connecting the adjacent points she could have a better representation of the pattern of the data. However, neither of the variables being examined were time and it was inappropriate to connect these points. But the researcher asked her if she would like to have the calculator connect the points by doing a line graph. She stated
that she would and the researcher showed her the selection in the STATPLOT menu which would accomplish this graph. Upon graphing the data under the new type of graph, Sue was surprised. (See Figure 33.) She had assumed that the calculator would connect the neighboring points and when the points seemed to be connected at random, she began to explore why.

![Figure 32: Sue's Regression Line](image)

![Figure 33: Sue's First Connected Line Graph](image)

After a few moments of contemplation, Sue decided that the points were connected by the order in which they occurred in the data table. Instead of at this point realizing that since neither of the variables were time that she could not use this means of analysis, she proceeded to
rearrange the data in the table so that the calculator would graph it the way she wanted. (See Figure 34.)

Figure 34: Sue's Ordered Connected Line Graph

Even though the X variable was height and the Y variable was weight, Sue still thought that this graph was appropriate for analyzing the data.

4.8 Summary

The data from this study support the concern that improper use of technology in instruction can render its potential useless. As seen in the responses of Kathy and Sue, the calculator and computer can become simply another tool for learning procedural techniques, just as pencil and paper are tools for learning skills such as the long division algorithm. However, students seem to place a greater trust in the correctness of calculators and computers. Before these technologies, students might have doubted their own procedural capabilities and used reason to check their solutions. Now that the machines have earned the trust of students, even reason may not be viewed as necessary by
students. In this respect, the improper use of technology may be a far greater danger than the rote methods used in the past.

The data presented in this chapter have illustrated some of the possible pitfalls and improper uses of technology. These instances need to be taken into consideration whenever planning the use of calculators or computers in the classroom. Chapter five will expand on these issues of classroom use and offer some suggestions for implementation of technology in ways which motivate students to think critically about mathematics.
5.0 Introduction

This chapter will address the conclusions of the study as well as suggestions for instruction and future research. The researcher will also tie the discussion to relevant research from the first two chapters. Since this study was qualitative in nature, the conclusions will be stated within the context of each individual who participated in the study. This approach is necessary since the researcher is not attempting to make generalizations to populations, but rather to document and describe behavior and perceptions of students which can then be used to generalize to theory and to motivate future research efforts. However, major patterns of behavior among students will be discussed with the caution that these patterns are not justification for generalizations toward the population.

5.1 Student Practices: Sue

Sue is a student who has potential for blindly following the machine. In most of her verbalizations of problem solutions, she focused primarily on procedural steps which she had been taught to obtain an
answer. Her lack of mathematics background appeared to play a role in her willingness to follow the answer given by the calculator even when the answer was not appropriate. For the two students in the study who had a much stronger mathematics background, the calculator model was not as easily believed. However, for Sue and the other two students, Kathy and John, whose mathematics background were lacking by comparison, the blind faith given to the machine seemed to be a greater threat.

During the final interview where regression lines were being shifted, Sue did not show any concern for the model of curve fitting suggested by the calculator until the third question where the regression line had been shifted by 20% leaving only one of the data points on one side of the regression line. To see the progression of regression line shifts experienced by Sue consider Figures 35, 36, and 37 which give a visual account of the shifting. Sue voiced concern for the third question's regression line. However, she still used the curve-fitting model suggested by the calculator to state her answer. It was not until the fourth question where all of the data fell below the regression line that she decided to ignore the calculator generated model. (See Figure 38.)
Figure 35: Question 1 Data with 0% Shift

Figure 36: Question 2 Data with 10% Shift

Figure 37: Question 3 Data with 20% Shift
Sue seemed to be very trusting of the machine and when confronted with an unexpected result such as in the question of the final interview, she would try and explain away her concern in most cases using the excuse that outliers were causing the regression line to be pulled in one direction. In general, Sue could be categorized as a person who ascribed a great deal of authority to the computer and calculator.

5.2: Student Practices: Tom

Tom appeared to be the most confident of any of the participants. Seldom would he use the calculator generated model of curve-fitting suggested by the TI-82. Most of his responses during interviews involved scrolling on the scatterplot to arrive at an answer based more on his intuition than on the numerical output of the machine. He typically placed error estimates on his answers in a manner similar to error bars on a graph. He would state an answer and then give a plus or minus range for it.
It was obvious that Tom had been instructed with graphing technology in the past. He was sensitive to pitfalls of graphical appearance which can occur when scale is manipulated on a graphing calculator. In addition, Tom had experienced several science courses including Chemistry and Physics where nonlinear curve-fitting is often used. This gave him a greater flexibility when confronted with what he perceived as incorrect regression models. Since he had placed curves on scatterplots by hand in high school, he was more willing to use the graphical representation of the data and scroll on the screen to come up with predictions.

Tom also had a well developed philosophical understanding about the nature of curve fitting and the implications for its connection to reality. He expressed that curve fitting was an attempt to model reality as best we can, but that it was by no means "the" correct model of the phenomenon being examined. He also stated that society seems to believe that science has the answers, or truth if you will, of the natural world. Tom cautioned that we need to be careful as to how much confidence we place in the scientific models theorized by our scientific community. This expression of understanding came as a great surprise to the researcher. It is unusual to find an undergraduate student with such command of philosophical issues. In general, Tom could be categorized as someone who does not concede to the authority of the machine.
5.3 Student Practices: Cindy

Like Tom, Cindy also possessed a confidence in her understanding of mathematics. However, her confidence seemed to be tied to the instructor of the course. She was willing to ignore the calculator generated model for curve fitting because the instructor had made it a point to tell the students that not all patterns can be described by linear regression models. During the first interview she stated this impression that the instructor had left her with. As a result, Cindy was willing to use the scatterplot to explore the patterns of data. She was not bound to the computer generated model, but would also scroll around the screen with the cursor looking at the coordinates of the points and on occasion choose to ignore the calculator model in favor of her intuition.

Even though Cindy would frequently use the graphical representation, she would not always choose to base her predictions on this representation. In many instances she relied on the algebraic representation. Several times she would choose the Regression Equation option from the menu and then copy the equation onto paper. After doing this she would plug the desired X coordinate for which she wanted to make a prediction into the equation, and then using the calculator figure the predicted value manually.

On occasion where she had used either the View Regression Line option or the PREDICT option from the menu, she would still check the correlation coefficient. In fact, most of the time she would choose the Regression Equation option initially so that she could view the correlation coefficient before proceeding with any other means of predicting. She
would then use the correlation coefficient to gauge the certainty of her prediction or whether or not to believe the calculator generated model.

In general, Cindy could be categorized as a person who does not typically concede to the authority of the machine, however, her certainty could be swayed by a numerical value attributed to the fit of the data through the correlation coefficient.

5.4 Student Practices: Kathy

With statements such as: "Well, if the calculator did it I would assume that it was correct."); or "Oh...well the calculator did it so I guess it's correct.", it is apparent that Kathy has very little confidence in her problem solving ability and places a great deal of confidence in the solutions obtained through the calculator. These statements occurred during the final interview where the regression line had been shifted by 20% so that only one data point lay on one side of the line. It is disturbing that she could believe the machine even when the model for curve fitting was so visually skewed. (See Figure 39.)

![Figure 39: Question 3 of Final Interview Data with 20% Shift](image-url)
Kathy verbalized concern for the regression lines beginning with the second question of the final interview. However, she followed the calculator model of curve fitting for all of the task oriented questions in the study. Her perception of mathematics as a set of procedures to be memorized gave her confidence in the machine even when she received conflicting information through multiple representations. Her lack of mathematics background made it more difficult for her to critically analyze the data patterns since this was her first experience with curve fitting.

Kathy is the epitome of the blind follower of technology. If any reason can compel teachers to use technology carefully, Kathy is that reason. Even though the technology was effective for some students, the manner by which it was implemented fell short of meeting Kathy's needs. In a sense, there was a trade-off of pencil and paper rules for learning mathematics for button-pushing rules for learning mathematics.

5.5 Student Practices: John

Initially, John began to explore the problems posed to him by the researcher through algebraic means using pencil and paper. When the researcher introduced the technology into the picture, he became less willing to approach questions through his intuition. Until this point, he had explored with pencil and paper, however inappropriately, the questions. After the technology was introduced, he thought that the researcher only wanted him to use the technology to solve the problems.
This was due in part in his mind to following the pattern set by the statistics course.

John stated that he felt inadequate in his ability to use technology as compared to the rest of the students in the class. He also stated that he felt, "behind" the other students. At times he made references to the differences between socioeconomic classes in relation to those who have access to technology and those who do not. As an African-American, he felt that he was at a disadvantage in his ability to do mathematics. He brought intelligence into his perception of who was able to use technology. He stated that only people who are intelligent can use technology. In an indirect way, he seemed to be referring to himself.

John's confidence was linked to what he believed the authority figure wanted from him. In class, he was sensitive to the instructor's leading with respect to course material as well as technology. During interviews, he was sensitive to the researcher's interests in the study. For these reasons, John used the technology and suppressed his own problem-solving strategies so as to please the one with whom he was working.

5.6 Conclusions

Based on the data of this study, it appears that students are willing to believe the model of curve-fitting suggested by the machine even when that model differs significantly from the data or scatterplot. The vast majority of students involved in this study chose to use the model suggested by the calculator or computer in most cases. The extreme case, however, was Kathy. She used the calculator model for all of her
predictions throughout the study. Moreover, she also stated that she never would have even voiced concern for the last two curve fitting models in interview 4 if it had not been for the researcher specifically asking her to use the graphical representation during that interview. She would have continued to use the PREDICT option alone throughout and as a result, would have missed the regression line shifts altogether.

Of the five students involved in the study, it seems that on the occasions where students used the computer or calculator model to predict, they were more certain of their answers than on those times where they simply used the data as a guide. For all students except for Sue, the average certainty rating for their use of the machine's model was higher than the average certainty rating for the use of the data alone. In Sue's case, if the certainty rating for the second question from the first interview is ignored, the average certainty rating for the times when the calculator model was used and the average certainty rating for the time when the calculator model was not used are essentially the same. Recall that during the second question of the first interview Sue encountered conflict between the context of the problem and the answer given by the machine. Instead of approaching the problem in a different manner, Sue chose to simply give a low rating of certainty.
The research indicates several factors which may influence students to yield to the authority of the computer or calculator. These factors are described below and are summarized as:

- Use of Repetition
- Match of Representations
- Instructor Interaction
- Mathematics Background
- Creator/Human Factor

5.61 Use of Repetition

Students were more willing to accept the calculator model if they had previously solved similar problems to those given during interviews. The class would typically solve several problems in the exact same manner. The students would then learn a set of steps or procedures for solving all problems of this type and apply those procedures by rote to other problems. As expressed in previous discussion, Kathy could be considered as the extreme case for this type of behavior.

5.62 Match of Representations

Students also lent greater credibility to answers for which they could obtain conformation through other representational forms. If for example, a student obtained 8.02 for an answer by using the PREDICT option and then 8.01 for an answer by using the graphical representation of the regression line and scrolling on the calculator screen, he/she would be more willing to believe the answer. This occurred even when students
were confronted with answers and regression lines which did not visually fit the data pattern.

5.63 Instructor Interaction

The instructor also played a role in the tendency for students to believe the computer or calculator model. When faced with an apparent contradiction, students would often relate statements made by the instructor during class. At times, the practices of the instructor also played a role in communicating unintentional over-generalizations to the students. This can be illustrated by the instructor's requirement of manual calculation of such statistics as standard deviation in contrast to no manual calculation of regression lines. Some of the students interpreted this as saying that understanding the meaning of the process behind linear regression was not important and did not need to be pursued. On another occasion, the instructor and class came to a resolution that a regression line was inappropriate for a given data set, but yet the instructor still had the students interpret the regression line within the context of the problem. This sent mixed signals to the students and reinforced the idea that the linear regression process as demonstrated in class was something to be followed on all problems.

5.64 Mathematics Background

As seen in the responses by Tom and Cindy, mathematics background may play a role in the willingness on the part of the student to challenge the results of the machine. Both Tom and Cindy had taken
calculus prior to their experience in the Introductory Statistics course. Their frequent mention of slope and rates of change indicates that the concepts presented in a typical calculus course may have influenced how they thought and communicated their ideas about the data patterns. The other students who were not willing to challenge the calculator results had comparably weaker mathematics backgrounds than Tom and Cindy.

The influence of both mathematics and science backgrounds is particularly visible from Tom. The discussion the researcher had with Tom regarding his feelings and beliefs about the use of curve fitting to model the real world speaks to the influence of his background. Recall that Tom had an extensive background in Mathematics, Chemistry, and Physics from both high school and the university. He had stated that he had worked with nonlinear curve fitting in chemistry and physics, in particular, with titration curves. He was extremely sensitive to the role of scientific models for real-world phenomena. He expressed that the models presented by the scientific community can sometimes be taken as fact rather than a model. He was concerned about this misinterpretation of scientific understanding and gave examples from his science background to support his belief that all models are incomplete. He also stated that the models generated by scientific inquiry are the best models to date, but were bound to be revised in the future. Tom’s responses are clear indications of the influence on technological authority by his mathematics and science background.
5.65 Creator/Human Factor

The last factor which seemed to influence the students in their view of the computer as an authority was the human element of the technology. Every piece of technology known to humankind throughout time has had its origin in the human mind. The students understood this and placed value judgments on the technology based on their perception of the creators of the technology. Since the computer software (DUSA) used in the course was written by seniors at the university as a project for one of their classes, the participants in this study distrusted the software. The instructor had made mention of the software's origin and without knowing it affected the students' trust in the technology. On the other hand, the students viewed the TI-82 as a product of professional activity by authorities in mathematics and thus lent more credibility to the solutions obtained from it.

The Creator/Human factor was not anticipated by the researcher. In fact, the researcher fully expected the students to trust the computer over the calculator. If it had not been for an informal interview that the researcher had with Sue in the hallway, this factor would not have come to light. It is important not to discount the human element of technology. The researcher had expected the students to view the computer or calculator as an authority devoid of human origins. This turned out to be incorrect.

The researcher attempted to bring these factors influencing student perception of technology's authority into the foreground through conflict. It was thought that conflict between: Algebraic Representation, Graphical
Representation in the form of the scatterplot and regression line, Problem Situation or Context, and Linear Affinity, would bring about the need for the students to either, (1) challenge the authority of the machine, or (2) blindly follow the solution given by the machine. The data from this study support the model of student interaction with technology shown in Figure 40. The instances where the students chose to not use the computer generated model of curve fitting were precipitated by a conflict among the five aspects of the problem: Algebraic Rule, Scatterplot, Linear Affinity, Regression Line, and Situation. On problems where there was no such conflict, the students used the computer suggested model.

![Figure 40: Theoretical Model for Representational Conflict](image-url)
The above theoretical model is related to methods of instruction which could be used to give students experiences that may help them retain autonomy in the problem solving process. The theoretical model relating the influences on student perception of the authority of technology is presented in Figure 41. The purpose of Figure 40 is not to be confused with influences on student perception. The model presented in Figure 40 is a proposed method of bringing about conflict designed to address the issues of authority presented in this document.

From a constructivist point of view, it is the disequilibrium encountered by the students which forced them to at least confront their understanding of curve fitting and data analysis. In some cases, the student chose not to attempt a restructuring of his/her scheme while in other cases he/she began restructuring during the interview process. This is not to say that no restructuring occurred for the students who chose to use the computer or calculator model when it was not appropriate. It is to say that the restructuring did not occur immediately. Even Kathy who blindly followed the lead of the machine did state concern for the feedback she was getting from the calculator. It is likely that this seed of concern will eventually grow to a point that she cannot keep from re-evaluating her understanding of linear regression.

As stated earlier, the investigation has brought to light several factors which appear to influence student perception of the authority of technology in the learning of mathematics. These factors are also related to each other. The theoretical model of the relationship among the factors of influence is presented in Figure 41.
Based on the findings, the instructor has considerable influence on the students' perception of the authority of the calculator and computer. The consistent use of examples followed by similar problems in class and on homework gave students confidence in the technology since they were solving the problems in the same manner as they had been shown in class. Instruction in this repetitive manner influenced students to view linear regression as a procedural endeavor gaining confidence through prolonged experience solving problems which had been modeled for them.

The use of repetition as justification for increased confidence also seemed to be influenced by the students' mathematical background. The students who were more willing to accept the repetitive manner of instruction had also been taught in the same manner during their previous mathematical experience. Kathy spoke at length about her previous procedural instruction. On the other hand, students like Tom and Cindy were not as apt to gain confidence due to repetitive working of
problems since they were used to critically evaluating a problem situation in their Calculus experiences.

The instructor also had influence on student perception through the Creator/Human factor. It was the instructor who found it appropriate to inform the students that the software being used for the class was written by students at the university. The fact that he felt it important to mention the origin of the software gave students reason to take note and possibly view this information as justification to question the computer. In addition, John viewed the instructor's authority as being linked to the technology since he felt that the instructor's choice of the software for the class was sufficient to have confidence in the computer rather than the calculator.

One of the more beneficial aspects of instruction which influenced student perception of technological authority was the use of multiple representations. The instructor frequently used several representations when explaining data analysis in class. Some of the students picked up on this use of different representations and applied it to their own analysis during the interviews. The confidence gained by the matching of representations gives hope that students will learn to examine mathematics from several approaches before making a judgment.

5.7 Recommendations for Instruction

There are four recommendations for instruction which can be gleaned from the data in this study. Firstly, instructors must make it a point to expose students to activities which will allow them to be
confronted by conflicting information given by the technology. In the final interview, the researcher sought to deliberately deceive the students by shifting regression lines. However, instructors need not go to such extremes in order to bring about the desired effect. The use of pathological functions can suffice. In addition, the use of pathological function can also serve as a motivator for the necessity of analytical methods of functional analysis.

For example, suppose your students have been studying periodic functions. The use of graphing technology can be misleading at times depending on the limitations of the hardware being used. One problem which illustrates this exposure to hardware limitations is the period of $f(x) = \sin(63x)$. When graphed on the standard window of a TI-82, the period appears to be about 1.7. (See Figure 42.) However, upon closer examination by zooming in on what appears to be one period, (See Figure 43) the function seems to defy the earlier conclusion. (See Figure 44.)

Figure 42: $f(x) = \sin(63x)$ Graphed on the TI-82 Standard Window
The obvious single period from the standard window has been fruitful and multiplied. This poses many questions in the minds of the students—questions which analytic methods can address.

With the availability of technology, students will question the need for other mathematical concepts as well. In Calculus, some of the researcher's students have asked why they need to learn about derivatives since they can simply graph the function on their calculator and trace to a maximum or minimum point? When approached by this argument, a typical response is to ask them to find the relative maxima and minima of
a function such as \( f(x) = 2\sin x + 0.05\sin(25x) \). When graphed on a standard window the graph appears to be periodic with period of \( \pi \). (See Figure 45.) The students trace to what appear to be the relative maxima and minima and base their answers on the visual inspection of the standard window. The researcher would then ask them to zoom in on one of the humps. (See Figure 46.) To their surprise, a new set of relative maxima and minima appear.

The use of pathological functions can illustrate the limitations of technology. No matter how sophisticated of a machine you may have, it
will never be safe from this type of error. In addition, the motivation for
the need of an analytical means for determining relative maxima and
minima comes to the forefront—why proof by picture is not acceptable.

Experiences such as these can help students find comfort in their
own ability to think critically. The limitations of the machines themselves
are sufficient to show why technology alone cannot answer all of their
questions in mathematics. It is a tool, and not Mr. Whoopie the Answer
Man.

The second recommendation evident from this study is that
instructors provide students with opportunities to fully explore interesting
questions using technology as well as other tools. In many instances,
students in this study focused on only one method of solving a problem.
The classroom atmosphere was one of the instructor presenting material
and then expecting the students to follow by example. The students were
asked to do periodic projects. However, they were shown how to approach
these projects beforehand, and there was not really a sense of student
discovery or ownership within the problem solving process.

The third recommendation for instruction is the use of data for
which neither of the coordinates is time. The availability of
Microcomputer-Based Laboratory and the new, more affordable,
Calculator-Based Laboratory (CBL) developed by Texas Instruments gives
students the opportunity to collect data with probes and display the data
in a number of ways on the calculator. However, most often this data are
in the form of velocity versus time or distance versus time, etc. The over
use of these technologies has the potential to lead students to believe that
points on a scatterplot can be connected in an ordered fashion. Many have suggested (Brasell, 1987ab; Mokros & Tinker, 1987; Nichols, 1992; Thornton, 1989; Linn, 1988) that the use of MBL and CBL can have great benefit to students' understanding of graphs and the physical events for which they represent. However, it seems important to be cautious with the way in which we utilize these technologies.

Recall Sue's interpretation of the data from the third interview. Sue was asked to predict an athlete's weight given his height based on a sample of athletes. Neither of the variables was time, however, Sue desired to connect the points anyway. Upon her first attempt she obtained a graph as seen in Figure 47.

![Sue's Connected Line Graph](image)

Sue did manage to figure out that the order of the points in the data table was the reason for the graph not being connected in the manner which she had anticipated. Rather than causing her concern for the appropriateness of this method of data analysis, she simply edited the data table and
ordered the X coordinates of the points to get the graph shown in Figure 48.

The over use of CBL or MBL could prompt students to over generalize the time-dependent relationships to data for which the connect-the-dot method is not valid. It is important to give students experiences with many types of data to avoid this misconception.

The final recommendation for instruction is the use of qualitative interpretations of data patterns. Leinhardt, Zaslavsky, and Stein (1990) state that the use of qualitative descriptions of graphs is lacking in the mathematics curriculum. Tom and Cindy made frequent use of qualitative interpretations of the data patterns. However, neither Sue, Kathy, or John described the data patterns in any way except to say that the regression was either a good or bad fit. Both Tom and Cindy had both experienced Calculus classes where graph behavior are explicitly addressed. Their mathematics background seemed to allow them to choose to ignore the calculator generated model on several occasions.
If we want students to be able to have an intuition about graphs, we must start giving them experiences which allow them to describe graphs in ways other than by simply stating a formula.

5.8 **Directions for Future Research**

The purpose of this study was to identify aspects of student perception of the calculator and computer's authority. The results suggest several avenues for further investigation.

1. The instructor seems to play an important role in the student’s perception of the authority of the technology. Future research might focus on isolating certain behaviors such as the instructor’s frequency of technology use in the classroom and the effect it may have on the student’s reliance on technology. Two classes could be taught using the same requirements of technology use for all students while varying the time devoted to technology use in class by the instructor.

2. In a similar manner, this study also suggests that the repetition of tasks by students may have an affect on the student’s perception of the machine’s authority. It would be interesting to see if giving students problems which were designed to be done in groups with access to a vast array of tools, and for which the students had never seen an example, might have an affect on their reliance on technology. How might eliminating the repetitive pattern of problems influence the student’s perception of the machine’s authority?

3. Since the students seemed to be influenced by their impression of the creators of the technology, it might be interesting to use the same
software with two classes while telling one class that the software was commercial and the other class that the software was written by their fellow students. An instrument could then be designed so that responses would detect students' willingness to believe the computer. These responses could then be compared to see if the students' belief in who created the technology had an affect on their tendency to follow the computer.

4. Another direction for research might be to examine a general measure of skepticism and see if it correlates to student behavior in the face of technological authority. Do students who are generally skeptical ignore the machine when inappropriate information is presented? How does mathematics background correlate with a measure of skepticism? Do skeptical students tend to use multiple representations when approaching a problem?

5. Additional research could also be done addressing the issue of locus of control. Since females tend to take personal responsibility for negative experiences whereas males tend to transfer responsibility to external factors such as a test, it may be that females are less willing to challenge authority as compared to males. Such factors as instructor, technology, exams, and syllabi may also hold a certain amount of control in the eyes of the students. Sex differences in relation to technology was not examined in this study, but might prove to be worth further investigation.

6. Since the instructor had influence over student perceptions according to the findings of this study, it might be interesting to survey educators
to see if they believe that students will critically examine data patterns and not routinely follow the lead of the machine. The researcher has spoken with respected educators who believe that students will indeed think critically rather than blindly follow the results of their button pushing in these situations.

7. Since the issue of time-dependent data and its relation to MBL or CBL was present in Sue's responses, it might be interesting to see if students exposed to MBL or CBL would be more likely to mistake data not linked to time as being time-dependent. For example, data sets such as SAT scores versus GPA could be constructed specifically so that the scatterplot would give the impression that a pattern existed even when the adjacent points on the graph could not be connected in a valid way. (See Figure 49.) The MBL or CBL group may be more apt to try and connect the points while the group without exposure to MBL or CBL may choose to use simple linear regression instead.
Figure 49: Example: Apparent Pattern in Data

The questions presented here can help piece together the puzzle surrounding the use of graphing technologies in the mathematics and science curricula. Just as bad practices on the part of students can develop through the use of pencil and paper, so too can the same type of practices develop through the use of graphing and other technologies. It is important to remember that the pencil was once a new piece of technology as well. We must continually question the implications of how we use new devices and not simply use them as novelties which influence the students' attitudes for a time and then fade into oblivion once the novelty effect has worn off. The appropriate use of technology in the mathematics classroom is a difficult question to address. However, the current time in which these devices are developing presents new challenges for mathematics educators and an exciting time in which to be active in this field of study.
Appendix A

Initial Data Analysis Instrument
Data Analysis

**Directions:** Answer the following questions to the best of your ability. Where you are asked to explain your methods of solving a problem, be as thorough as possible. You may use a computer or calculator to aid in your responses.
1. Data were collected on a certain population of bison giving the total population as a function of time (in years) as seen in the following scatterplot. The computer fit a regression line to the data yielding the computer model equation shown at the top of the scatterplot.

\[ y = 195.33x + 10988.49, \ r^2 = .94 \]

What would you predict for the bison population at year 1?

How did you arrive at your prediction? Explain.
(If you need more room, please feel free to use the back of this sheet.)

On a scale of 1 to 10 with 1 being the least certain and 10 being the most certain, how certain are you about your prediction? (Circle one)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncertain</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>
2. From the early years of the AIDS epidemic, the number of AIDS cases diagnosed increased according to the following data where the time is in years since 1976.

<table>
<thead>
<tr>
<th>Time (Yrs.)</th>
<th>Number of Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>158</td>
</tr>
<tr>
<td>2</td>
<td>251</td>
</tr>
<tr>
<td>5</td>
<td>997</td>
</tr>
<tr>
<td>6</td>
<td>1580</td>
</tr>
<tr>
<td>7</td>
<td>2503</td>
</tr>
</tbody>
</table>

What would you predict for the number of cases diagnosed in the 3rd year after 1976?

How did you arrive at this prediction? Explain. (If you need more room, please feel free to use the back of this sheet.)

On a scale of 1 to 10 with 1 being the least certain and 10 being the most certain, how certain are you about your prediction? (Circle one)

1 2 3 4 5 6 7 8 9 10
Uncertain Certain
3. A radioactive isotope is decaying. The number of grams present and time in minutes at which the mass was taken are given in the following scatterplot.

<table>
<thead>
<tr>
<th>Time (min.)</th>
<th>Mass (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>150.00</td>
</tr>
<tr>
<td>5</td>
<td>119.06</td>
</tr>
<tr>
<td>10</td>
<td>94.50</td>
</tr>
<tr>
<td>15</td>
<td>75.01</td>
</tr>
<tr>
<td>25</td>
<td>47.26</td>
</tr>
<tr>
<td>30</td>
<td>37.51</td>
</tr>
<tr>
<td>50</td>
<td>14.89</td>
</tr>
</tbody>
</table>

What mass would you predict to be present at 20 minutes?

How did you arrive at this prediction? Explain.
(If you need more room, please feel free to use the back of this sheet.)

On a scale of 1 to 10 with 1 being the least certain and 10 being the most certain, how certain are you about your prediction? (Circle one)

1 2 3 4 5 6 7 8 9 10
Uncertain Certain
4. A car company finds that it takes 1505 worker hours to produce the first car on the assembly line. The fifth car, however, only takes 703 worker hours to produce. The time it takes to produce the nth car on the assembly line is given in the following scatterplot. The computer has fit a line to the data and the mathematical model is given at the top of the scatterplot.

\[ y = -106.67x + 1600, \quad r^2 = .94 \]

How many worker hours would you predict it will take to produce the 8th car?

How did you arrive at this prediction? Explain.
(If you need more room, please feel free to use the back of this sheet.)

On a scale of 1 to 10 with 1 being the least certain and 10 being the most certain, how certain are you about your prediction? (Circle one)

1 2 3 4 5 6 7 8 9 10
Uncertain Certain
Please list and explain what you feel are some of the advantages for the use of computer/calculator technology in the teaching and learning of mathematics and/or mathematically related fields of study.

Please list and explain what you feel are some of the disadvantages for the use of computer/calculator technology in the teaching and learning of mathematics and/or mathematically related fields of study.
Appendix B

Interview Questions and Problems
Interview 1 Problems

1. Consider the following data. Predict the value of Y when X=1.5.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
</tr>
<tr>
<td>6</td>
<td>91</td>
</tr>
</tbody>
</table>

How would you rate your certainty of your answer?

2. Suppose if a person places a certain amount of money in a savings account the bank guarantees that the person will have the following given amount of money in the account at the end of the given time. How much would you predict is in the account at the end of 3 years?

<table>
<thead>
<tr>
<th>Time (yrs.)</th>
<th>Money ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1000.00</td>
</tr>
<tr>
<td>10</td>
<td>1200.00</td>
</tr>
<tr>
<td>15</td>
<td>1800.00</td>
</tr>
<tr>
<td>20</td>
<td>3000.00</td>
</tr>
<tr>
<td>25</td>
<td>4200.00</td>
</tr>
<tr>
<td>30</td>
<td>6000.00</td>
</tr>
</tbody>
</table>

How would you rate your certainty of your answer?
Interview 1 Questions: Sue

Introduction: Briefly describe your math background and your background with the use of graphing calculators and computer programs.

1. In the written question 2, you state that you exited the program to calculate the predicted value using the regression equation. Then you entered the program again and used the predict option from the menu to make the prediction. Can you tell me why you did both?

2. In the written question 3, you entered the data into your calculator and then viewed the scatterplot with the regression line. You also noted that the scatterplot was the same as the one given on paper. Can you elaborate on your thinking in this problem?

3. In the written questions 1 and 4, you seem less certain of your answers than in the other two questions. Why did you rate your certainty lower in these problems?

4. In answering the questions about the advantages and disadvantages of computer/calculator use, you state that computers/calculators can be scary at first. How did you feel when I first gave you the calculator to use? How do you feel now?

5. At one time you told me that you and a friend were working on your homework. Your friend was using DUSA and you were using the TI-82 and you found a discrepancy in the answers each of you came up with. Could you explain what happened?

If the problem had not been resolved, how would you have felt?

6. How do you feel about the statement, "Students should do problems by hand before they use a computer or calculator"?

7. In your written responses, you said something about hand calculations being more apt to be incorrect. How do you feel about the statement, "Computers and calculators are never wrong."?
Interview 1 Questions: Tom

Introduction: Briefly describe your math background and your background with the use of graphing calculators and computer programs.

1. In your list of disadvantages for the use of technology, you stated that some people believe the computer or calculator and not their common sense. How do you feel about the statement, "calculators and computers are never wrong"?

2. How do you feel about the statement, "Students should do problems by hand before they use a computer or calculator"?

3. In question 2 of the written instrument, you seem to be less certain of your answer than in the other questions, could you explain your thinking in this case.

4. What do you see as the role of technology in the teaching and learning of mathematics?

5. What sort of experiences have you had that have shaped your views of the use of technology in your own learning of mathematics.
Interview 1 Questions: Cindy

Introduction: Briefly describe your math background and your background with the use of graphing calculators and computer programs.

1. In question 1 of the written exercises, you used the regression line drawn on the scatterplot to make your prediction and in question 2 you used the regression equation to predict from. Could you explain why you used each of these methods on the two problems?

2. In question 3 of the written exercises, you made an initial prediction using the scatterplot and then used DUSA to make your final prediction from the regression line. Why do you think your initial prediction of 65 differed from the final prediction you gave of 75.00969.

Which of the predictions do you think is more accurate and why do feel that way?

3. In question 4 of the written exercises, you made an initial prediction, then used DUSA, and finally went back to your original prediction. What factors were there that influenced you to not go with the linear regression prediction? Did the correlation influence your decision, if so, how?

4. In what ways do you think questions 3 and 4 are similar? In what ways do you think questions 3 and 4 are different?

5. In question 4 of the written exercises, you rated your certainty lower than in the other questions. Explain why you were less certain in this question.
Interview 1 Questions: Kathy

Introduction: Briefly describe your math background and your background with the use of graphing calculators and computer programs.

1. On your written questions, you seem very certain of your answers in questions 1 and 4. What gives you the confidence in your answers? Explain.

2. On questions 2 and 3 you seem less certain. Why are you not as sure of these answers? Explain.

3. In question 3, you state, "However--what's up w/reg. line?". What were you meaning by this comment?

4. I was wondering if you might elaborate on the responses you gave to the advantages and disadvantages of the use of technology.

5. Some people have commented to me, "Computers and calculators are never wrong". How do you feel about this comment.
Interview 1 Questions: John

Introduction: Briefly describe your math background and your background with the use of graphing calculators and computer programs.

1. In the written question 1, you state that "at no time the bison population would be 10,988.49" and then that after 1 year it would have an increase of 200 bison, where did you arrive at these numbers and how did you use them to make your prediction?

2. In written question 2, you state that since 3 is closer to 2 than to 5, you would predict that the number of cases would be closer to the number of cases for the 2nd year. Could you explain your thinking in this question?

3. Also in question 2, I noticed that you did not seem to use either the computer or calculator. Could you explain why you did not go that route for solving this problem?

4. In written question 3, you said that since the mass at 15 is 75g and the mass at 25 is 47g, you used the difference in the two masses and found roughly the mass halfway between these masses. I noticed that you estimated these values, could you explain why you chose not to use the calculator to calculate the halfway point?

5. Also in question 3, could you show me on the graph how you might do the same thing you did to solve the problem numerically, but using the graph instead?

6. I noticed in written question 4 you did the calculation in your head. Is there a reason that you like to do you calculations in your head?

7. In your responses to the advantages and disadvantages of the use of technology, you state that the use of more advanced calculators requires more intelligence. Do you feel that you are somewhat at a disadvantage because of lack of experience with these calculators?
Interview 2 Problems

1. The following data represents the cholesterol level of a person and the cholesterol level of the same person 3 years after being instructed on how to reduce cholesterol levels through dieting. What would you predict would be the cholesterol level of a person, who currently has a cholesterol level of 244, 3 years after being instructed on cholesterol level reduction?

How would you rate your certainty of your prediction?

<table>
<thead>
<tr>
<th>Cholesterol</th>
<th>Cholesterol-3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>197</td>
<td>182</td>
</tr>
<tr>
<td>181</td>
<td>151</td>
</tr>
<tr>
<td>190</td>
<td>169</td>
</tr>
<tr>
<td>131</td>
<td>133</td>
</tr>
<tr>
<td>172</td>
<td>166</td>
</tr>
<tr>
<td>233</td>
<td>229</td>
</tr>
<tr>
<td>194</td>
<td>148</td>
</tr>
<tr>
<td>155</td>
<td>148</td>
</tr>
<tr>
<td>234</td>
<td>175</td>
</tr>
<tr>
<td>201</td>
<td>203</td>
</tr>
<tr>
<td>258</td>
<td>276</td>
</tr>
<tr>
<td>212</td>
<td>228</td>
</tr>
<tr>
<td>137</td>
<td>165</td>
</tr>
<tr>
<td>285</td>
<td>229</td>
</tr>
<tr>
<td>218</td>
<td>172</td>
</tr>
<tr>
<td>167</td>
<td>152</td>
</tr>
<tr>
<td>170</td>
<td>158</td>
</tr>
<tr>
<td>157</td>
<td>140</td>
</tr>
<tr>
<td>215</td>
<td>176</td>
</tr>
<tr>
<td>194</td>
<td>207</td>
</tr>
</tbody>
</table>

2. Suppose you were trying to make a prediction about a set of data and you used both DUSA and the TI-82. When working with these, DUSA gives you a regression line of $\hat{y} = 0.0513x + 14.012$ and the TI-82 gives you a regression line of $\hat{y} = 0.0523x + 14.013$. What would you do?
Interview 2 Questions: Sue

1. Suppose you were trying to make a prediction about a set of data and you used both DUSA and the TI-82. When working with these, DUSA gives you a regression line of $\hat{y} = 0.0513x + 14.012$ and the TI-82 gives you a regression line of $\hat{y} = 0.0523x + 14.013$. What would you do?

2. If you were not concerned about the discrepancy in question 1, how far off would they have to be before you were concerned?

3. If you chose to go with DUSA in question 1, why did you choose DUSA over the TI-82?

4. If you chose to go with the TI-82 in question 1, why did you choose the TI-82 over DUSA?
Interview 2 Questions: Tom

1. Suppose you were trying to make a prediction about a set of data and you used both DUSA and the TI-82. When working with these, DUSA gives you a regression line of \( \hat{y} = 0.0513x + 14.012 \) and the TI-82 gives you a regression line of \( \hat{y} = 0.0523x + 14.013 \). What would you do?

2. If you were not concerned about the discrepancy in question 1, how far off would they have to be before you were concerned?

3. If you chose to go with DUSA in question 1, why did you choose DUSA over the TI-82?

4. If you chose to go with the TI-82 in question 1, why did you choose the TI-82 over DUSA?

5. You mentioned last time that for the data which really didn't seem linear, it would be nice to use other curves to fit to the data. I want to show you how we can do this with the TI-82. Here is the set of data we used last time. [The researcher shows Tom how to perform nonlinear curve-fitting on the TI-82 and graphs several different models.]

How do you know which model is the right one for fitting the data?
Interview 2 Questions: Cindy

1. Suppose you were trying to make a prediction about a set of data and you used both DUSA and the TI-82. When working with these, DUSA gives you a regression line of $\hat{y} = 0.0513x + 14.012$ and the TI-82 gives you a regression line of $\hat{y} = 0.0523x + 14.013$. What would you do?

2. If you were not concerned about the discrepancy in question 1, how far off would they have to be before you were concerned?

3. If you chose to go with DUSA in question 1, why did you choose DUSA over the TI-82?

4. If you chose to go with the TI-82 in question 1, why did you choose the TI-82 over DUSA?

5. In our earlier discussion, you stated that at the beginning of the quarter, you didn't use the computer to work on homework for about a week until you had done some problems by hand. At that time you were doing different types of plots for data such as stem and leaf plots and box plots. Did you also hold off on using the computer for linear regression problems? Why or why not?

6. You stated before that the use of repetition is helpful to your learning. In talking about that you also stated that your experience in Calculus was difficult because the problems given didn't seem to have anything in common and so it was hard to figure out how to solve them. Some people might say that since technology takes most of the tediousness away from the student, the student can be given more varied problems to work on and use the technology to explore more complicated problems. How do you feel about this statement?

7. Last time, you stated that you like to be exact. Can you talk about why you like to be exact and how you feel about the statement, "Calculators and computers allow us to be exact in our answers".
Interview 2 Questions: Kathy

1. Suppose you were trying to make a prediction about a set of data and you used both DUSA and the TI-82. When working with these, DUSA gives you a regression line of $\hat{y} = 0.0513x + 14.012$ and the TI-82 gives you a regression line of $\hat{y} = 0.0523x + 14.013$. What would you do?

2. If you were not concerned about the discrepancy in question 1, how far off would they have to be before you were concerned?

3. If you chose to go with DUSA in question 1, why did you choose DUSA over the TI-82?

4. If you chose to go with the TI-82 in question 1, why did you choose the TI-82 over DUSA?

5. When we talked before, you shared with me your frustration when you first had to use the graphing calculator in your work in precalculus. You stated that in high school you did everything manually and as a result you understood things better. As you look at back on your experiences, how do you feel about your control of the problem-solving process as it relates to either using or not using the technology?
Interview 2 Questions: John

1. Suppose you were trying to make a prediction about a set of data and you used both DUSA and the TI-82. When working with these, DUSA gives you a regression line of $\hat{y} = 0.0513x + 14.012$ and the TI-82 gives you a regression line of $\hat{y} = 0.0523x + 14.013$. What would you do?

2. If you were not concerned about the discrepancy in question 1, how far off would they have to be before you were concerned?

3. If you chose to go with DUSA in question 1, why did you choose DUSA over the TI-82?

4. If you chose to go with the TI-82 in question 1, why did you choose the TI-82 over DUSA?

5. When talking before about using calculators and computers, you said something about intelligence being needed to work with this technology. Could you explain what you mean by intelligence.
Interview 3 Problems

1. The following data gives the height and weight of a sample of athletes. What weight would you predict for an athlete that was 79 inches tall?

How would you rate your certainty of your prediction?

<table>
<thead>
<tr>
<th>Height (Inches)</th>
<th>Weight (lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>155</td>
</tr>
<tr>
<td>74</td>
<td>183</td>
</tr>
<tr>
<td>73</td>
<td>155</td>
</tr>
<tr>
<td>78</td>
<td>205</td>
</tr>
<tr>
<td>77</td>
<td>210</td>
</tr>
<tr>
<td>72</td>
<td>190</td>
</tr>
<tr>
<td>73</td>
<td>195</td>
</tr>
<tr>
<td>74</td>
<td>205</td>
</tr>
<tr>
<td>77</td>
<td>190</td>
</tr>
<tr>
<td>77</td>
<td>195</td>
</tr>
<tr>
<td>78</td>
<td>220</td>
</tr>
<tr>
<td>79</td>
<td>205</td>
</tr>
<tr>
<td>78</td>
<td>209</td>
</tr>
<tr>
<td>80</td>
<td>215</td>
</tr>
</tbody>
</table>
2. Suppose the calculator fitted a regression line and a curved line to a set of data according to the following graphs, and that the correlation coefficient for both was \( r = 0.946 \). If you were asked to predict from this data, which of the lines would you use, if any, and why?
Interview 4 Problems

1. The following data were collected by an agriculture researcher showing annual rainfall in inches and annual yield of wheat. What would you predict for the annual wheat yield for a year with an annual rainfall of 12.4 inches?

How would you rate your certainty?

<table>
<thead>
<tr>
<th>Rainfall (inches)</th>
<th>Yield (bushels/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.7</td>
<td>28.0</td>
</tr>
<tr>
<td>19.0</td>
<td>35.1</td>
</tr>
<tr>
<td>8.2</td>
<td>23.8</td>
</tr>
<tr>
<td>11.7</td>
<td>23.0</td>
</tr>
<tr>
<td>6.9</td>
<td>20.1</td>
</tr>
<tr>
<td>13.6</td>
<td>33.0</td>
</tr>
<tr>
<td>13.0</td>
<td>28.5</td>
</tr>
<tr>
<td>15.0</td>
<td>33.7</td>
</tr>
<tr>
<td>10.3</td>
<td>23.1</td>
</tr>
<tr>
<td>16.3</td>
<td>27.3</td>
</tr>
</tbody>
</table>
2. A study of Vietnam veterans found the following levels of dioxin, measured in parts per million (ppm), in both the blood plasma and the fat tissue of each veteran. Based on the data, what would you predict for the dioxin levels in the fat tissue of a veteran who had a dioxin level of 7.3 ppm in his blood plasma.

How would you rate your certainty?

<table>
<thead>
<tr>
<th>Veteran</th>
<th>Plasma Dioxin (ppm)</th>
<th>Fat Tissue Dioxin (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.7</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>3.1</td>
<td>5.9</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>4.4</td>
</tr>
<tr>
<td>4</td>
<td>5.5</td>
<td>6.0</td>
</tr>
<tr>
<td>5</td>
<td>4.0</td>
<td>6.0</td>
</tr>
<tr>
<td>6</td>
<td>1.8</td>
<td>4.2</td>
</tr>
<tr>
<td>7</td>
<td>6.0</td>
<td>8.0</td>
</tr>
<tr>
<td>8</td>
<td>3.0</td>
<td>5.5</td>
</tr>
<tr>
<td>9</td>
<td>4.7</td>
<td>4.4</td>
</tr>
<tr>
<td>10</td>
<td>6.9</td>
<td>7.0</td>
</tr>
<tr>
<td>11</td>
<td>3.3</td>
<td>3.5</td>
</tr>
<tr>
<td>12</td>
<td>4.6</td>
<td>4.6</td>
</tr>
<tr>
<td>13</td>
<td>4.1</td>
<td>4.5</td>
</tr>
<tr>
<td>14</td>
<td>7.2</td>
<td>7.7</td>
</tr>
<tr>
<td>15</td>
<td>1.8</td>
<td>3.1</td>
</tr>
</tbody>
</table>
3. The life of a lawn mower engine can be extended by frequent oil changes. An experiment was conducted in which 20 lawn mowers were used over many years with different time intervals between oil changes. The number of hours of operation between oil changes and the number of years of service for the mower in years are given in the following table. What would you predict for the engine life of a mower whose oil was changed after every 18.5 hours of operation?

How would you rate your certainty?

<table>
<thead>
<tr>
<th>Time Between Oil Changes (Hours of Operation)</th>
<th>Engine Life (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.25</td>
<td>12.1</td>
</tr>
<tr>
<td>15.5</td>
<td>11.8</td>
</tr>
<tr>
<td>17.5</td>
<td>11.5</td>
</tr>
<tr>
<td>18.7</td>
<td>7.3</td>
</tr>
<tr>
<td>19.5</td>
<td>9.9</td>
</tr>
<tr>
<td>14.9</td>
<td>9.7</td>
</tr>
<tr>
<td>21.5</td>
<td>10.1</td>
</tr>
<tr>
<td>22.0</td>
<td>7.0</td>
</tr>
<tr>
<td>24.0</td>
<td>10.7</td>
</tr>
<tr>
<td>23.0</td>
<td>8.4</td>
</tr>
<tr>
<td>23.5</td>
<td>8.8</td>
</tr>
<tr>
<td>24.0</td>
<td>7.1</td>
</tr>
<tr>
<td>24.5</td>
<td>7.2</td>
</tr>
<tr>
<td>25.0</td>
<td>5.0</td>
</tr>
<tr>
<td>25.5</td>
<td>8.4</td>
</tr>
<tr>
<td>26.0</td>
<td>5.4</td>
</tr>
<tr>
<td>27.3</td>
<td>7.3</td>
</tr>
<tr>
<td>27.0</td>
<td>4.6</td>
</tr>
<tr>
<td>28.0</td>
<td>4.8</td>
</tr>
<tr>
<td>30.0</td>
<td>4.1</td>
</tr>
</tbody>
</table>
4. It is thought that a person's Graduate Record Exam (GRE) score can be used to predict the person's first year Grade Point Average (GPA) in graduate school. Given the following data collected from a sample of students, predict the first year GPA of a person whose GRE score was 570.

How would you rate your certainty?

<table>
<thead>
<tr>
<th>GRE Score</th>
<th>First Year GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>580</td>
<td>3.54</td>
</tr>
<tr>
<td>500</td>
<td>2.62</td>
</tr>
<tr>
<td>670</td>
<td>3.30</td>
</tr>
<tr>
<td>480</td>
<td>2.90</td>
</tr>
<tr>
<td>710</td>
<td>4.00</td>
</tr>
<tr>
<td>550</td>
<td>3.40</td>
</tr>
<tr>
<td>640</td>
<td>3.57</td>
</tr>
<tr>
<td>540</td>
<td>3.05</td>
</tr>
<tr>
<td>620</td>
<td>3.15</td>
</tr>
<tr>
<td>690</td>
<td>3.61</td>
</tr>
</tbody>
</table>
Appendix C

Regression Program for the TI-82
Lbl 0
ClrHome
0 → M
"" → Y₀
ZoomStat
LinReg(ax+b)
( Yₘₐₓ - Yₘᵟ ⁄ₙ₂ ) → R
b + F*R → B
Disp " MENU"
Disp "1: VIEW SCR PLOT"
Disp "2: VIEW REG LINE"
Disp "3: PREDICT"
Disp "4: REG EQUATION"
Disp "5: QUIT"
Input M
If M=1
Goto 1
If M=2
Goto 2
If M=3
Goto 3
If M=4
Goto 4
If M=5
Goto 5
If M=0
Goto 0
Lbl 1
DispGraph
Pause
Input
Goto 0
Lbl 2
"aX+B" → Y₀
DispGraph
Pause
Input
Goto 0
Lbl 3
"aX+B" → Y₀
ClrHome
Disp "X="
Input X
Disp "Y="
Disp Y₀(X)
Pause
Goto 0
Lbl 4
ClrHome
Disp "AX+B"
Disp "A="
Disp a
Disp "B="
Disp B
Disp "r="
Disp r
Pause
Goto 0
Lbl 5
Goto 0
Stop
BIBLIOGRAPHY

American Association for the Advancement of Science. (1989). *Project 2061: Science for all Americans*.


