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TWO-PHOTON PRODUCTION OF CHARMONIUM STATES

A Dissertation

Presented in Partial Fulfillment of the Requirements

for the Degree Doctor of Philosophy

in the Graduate School of The Ohio State University

By

Roger Fulton, B.S., M.Sc

The Ohio State University

1995

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Introduction

The exploration of the unknown leads along many tangled trails. Only after extensive journeys does the terrain become familiar. This dissertation examines a small portion of elementary particle physics called two-photon interactions. In particular it explores the production of charmonium states through photon-photon collisions.

Before exploring two-photon interactions, a survey of the surrounding land is in order. Chapter I presents a short introduction to the theory describing elementary particles and their interactions. The history of charmonium states is presented along with the history of two-photon collisions. We conclude Chapter I by presenting some previous measurements and theoretical predictions. Chapter II gives a description of the experimental apparatus: The CLEO-II detector. Chapter III describes the two-photon collision in detail, the Monte Carlo simulation and effects due to different form factors. Chapter IV describes event selection and data analysis methods. We first present a general approach and common variables. These are then applied to determine the $\eta_c$, $\chi_{c1}$ and $\chi_{c2}$ two-photon partial widths. Chapter V summarizes the results and compares them to the theoretical predictions of Chapter I.
CHAPTER I

The Standard Model.

In the present view, all matter is constructed of elementary particles known as leptons, quarks and mediators. The leptons and quarks are the basic building blocks of matter interacting through a combination of the four fundamental forces; strong, electro-magnetic, weak and gravitational. The mediators are “messengers”, traveling between particles, whose information results in the applied force. The relative strengths of the four forces are given in Table 1 along with the mediator of the force and the approximate range and strength of the force. On the scale of elementary particle interactions, gravity is by far the weakest force and can be ignored. The majority of our experimental information comes from three sources.

<table>
<thead>
<tr>
<th>Force</th>
<th>Strength</th>
<th>Theory</th>
<th>Mediator (Mass)</th>
<th>Range</th>
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<tr>
<td>Strong</td>
<td>1</td>
<td>Chromodynamics</td>
<td>Gluon (0 GeV)</td>
<td>≈ 1 fm</td>
</tr>
<tr>
<td>Electro-magnetic</td>
<td>$10^{-2}$</td>
<td>Electrodynamics</td>
<td>Photon (0 GeV)</td>
<td>∞</td>
</tr>
<tr>
<td>Weak</td>
<td>$10^{-13}$</td>
<td>Flavordynamics</td>
<td>$W$ (80 GeV)</td>
<td>$\approx 10^{-3}$ fm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z^0$ (91 GeV)</td>
<td>$\approx 10^{-3}$ fm</td>
</tr>
<tr>
<td>Gravitational</td>
<td>$10^{-42}$</td>
<td>Geometrodynamics</td>
<td>Graviton (0 GeV)</td>
<td>∞</td>
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1. Decays which are the spontaneous disintegration of a parent particle into daughter particles. The daughters properties may then provide insights into the properties of the parent.

2. Bound states which are the composite of two or more component particles held together by one or more fundamental forces. The properties of the composite may then provide insights into the properties of the components.

3. Scatters which are events where a probe particle is fired at a target particle. Most often, the probes properties are well understood while the targets properties are being investigated. Changes in the probe induced by interaction with the target may then provide insights into the properties of the target.

Using these sources we have built a picture of the elementary particles and their interactions known a the Standard Model.\[1-3\]

In many instances a property (or interaction) is common to a small set of particles. When dealing with this property we instinctively classify particles as those which need to be considered and those which can be ignored. Thus, in explaining the standard model, we inevitably form sets of particles defined by some common properties.

At the current experimental limit (r \( \sim 10^{-18}\)m) an elementary particle appears to have no internal structure, which is to say it is point-like.

An elementary particle falls into one of two categories determined by its intrinsic spin. Either it is a boson, having integral spin, or a fermion, having half integral spin. All mediators are bosons and all elementary bosons are mediators.

Fermions are further classified by the strong force. Elementary fermions which interact through the strong force are labeled quarks while those which do not
interact strongly are called leptons.

1.1 More About Leptons

All particles can be further characterized by their electric charge \((Q)\), lepton number \((L_l)\), electron number \((L_e)\), muon number \((L_{\mu})\) and tau number \((L_{\tau})\). Table 2 lists these quantities for the leptons. All leptons have a lepton number of \(\pm 1\), while all quarks and mediators have \(L_l = L_e = L_{\mu} = L_{\tau} = 0\). The lepton number \((L_l = L_e + L_{\mu} + L_{\tau})\) is redundant, but is convenient to use when the type of lepton is not important.

The numbers \(Q, L_l, L_e, L_{\mu}\) and \(L_{\tau}\) are additive quantities. For example, suppose \(L_l(sys)\) is the lepton number for oxygen-16. An oxygen-16 atom is composed of eight protons and eight neutrons in a nucleus surrounded by an electron cloud containing eight electrons. Protons and neutrons both have \(L_l(P) = L_l(N) = 0\) but, the electron has \(L_l(e^-) = 1\). As a result, oxygen-16 has \(L_{\mu} = L_{\tau} = 0\) and \(L_l(sys) = L_e(sys) = 8\), the number of electrons in the atom.

The quantum numbers listed in Table 2 arise from conservation laws. Conservation of lepton number requires that in any reaction, the sum of lepton numbers going into the reaction must equal the sum of lepton numbers coming out of the reaction. This same statement can be made for charge, electron number, muon number and tau number. These conservation laws separate leptons into three families (or generations);

\[
\begin{align*}
e, \nu_e & \quad \text{first generation} \\
\mu, \nu_{\mu} & \quad \text{second generation} \\
\tau, \nu_{\tau} & \quad \text{third generation.}
\end{align*}
\]

The electron is the most familiar of the leptons. This is primarily due to its abundance and easy access through electro-magnetic interactions. Its abundance,
Table 2
Lepton names, symbols and numbers

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<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Q</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$L_\mu$</th>
<th>$L_\tau$</th>
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<td>Leptons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e^-$</td>
<td>electron</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>electron-neutrino</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\mu^-$</td>
<td>muon</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>muon-neutrino</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\tau^-$</td>
<td>tau</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>tau-neutrino</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>Anti-leptons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e^+$</td>
<td>positron</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td>anti-electron-neutrino</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\mu^+$</td>
<td>anti-muon</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\bar{\nu}_\mu$</td>
<td>anti-muon-neutrino</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\tau^+$</td>
<td>anti-tau</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>$\bar{\nu}_\tau$</td>
<td>anti-tau-neutrino</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

in turn, is due to its electric charge and mass. The electron is the lightest of the charged leptons (see Table 3), in fact, it is the lightest charged particle. Since no other charged particles are lighter and both energy and charge must be conserved, the electron has nowhere to decay. As a result, the electron is a stable particle. The other charged leptons are not stable. They are much more massive than the electron or neutrinos and so can readily decay into an electron plus neutrinos. On the other hand, the neutrino masses are extremely small; they are believed to have zero mass. Since the neutrinos, like the electron, have nowhere to decay, they are stable particles.
Table 3
Lepton mass\cite{37} and lifetime.\cite{37}

<table>
<thead>
<tr>
<th>Lepton</th>
<th>charge</th>
<th>Mass (MeV/c²)</th>
<th>Lifetime (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>-1</td>
<td>0.51099906</td>
<td>Stable</td>
</tr>
<tr>
<td>(\nu_e)</td>
<td>0</td>
<td>&lt; 17eV/c²</td>
<td>Stable</td>
</tr>
<tr>
<td>(\mu)</td>
<td>-1</td>
<td>105.658387</td>
<td>2.19703 \times 10^{-6}</td>
</tr>
<tr>
<td>(\nu_\mu)</td>
<td>0</td>
<td>&lt; 0.27</td>
<td>Stable</td>
</tr>
<tr>
<td>(\tau)</td>
<td>-1</td>
<td>1784.1</td>
<td>0.296 \times 10^{-12}</td>
</tr>
<tr>
<td>(\nu_\tau)</td>
<td>0</td>
<td>&lt; 35</td>
<td>Stable</td>
</tr>
</tbody>
</table>

1.2 More About Quarks

Another set of properties which can be used to characterize particles is; electric charge (Q), baryon number (A), downness (D), upness (U), strangeness (S), charm (C), beauty (B) and truth (T). There are six flavors (types) of quarks determined by the flavor numbers D, U, S, C, B and T, as listed in Table 4. A quark flavor is solely determined by its one nonzero flavor number. These numbers are all 0 for leptons and mediators. Flavor is conserved in electromagnetic and strong interactions but not for weak interactions. Actually, flavor is also conserved for weak interactions involving only the \(Z^0\) mediator, but not when a \(W^{\pm}\) is involved. Quarks and anti-quarks have baryon numbers of 1/3 and -1/3, respectively, while leptons and mediators have a baryon number of 0. Like the lepton number, the baryon number is conserved in all interactions.

Quarks are extremely gregarious particles, they do not exist as isolated particles but instead form composite bound states called hadrons. Hadrons fall into two classifications, mesons and baryons, determined by the number of quarks they contain. Mesons are bound states of a quark-anti-quark pair and have a zero baryon number. They can carry an electric charge of either ±1 or 0. Baryons (anti-baryons) consist of three quarks (anti-quarks) and have a baryon number...
Table 4
Quark classification

<table>
<thead>
<tr>
<th>symbol</th>
<th>Flavor</th>
<th>Q</th>
<th>A</th>
<th>D</th>
<th>U</th>
<th>S</th>
<th>C</th>
<th>B</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>down</td>
<td>-1/3</td>
<td>1/3</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u</td>
<td>up</td>
<td>2/3</td>
<td>1/3</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>strange</td>
<td>-1/3</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>charm</td>
<td>2/3</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>beauty</td>
<td>-1/3</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>t</td>
<td>truth</td>
<td>2/3</td>
<td>1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
</tr>
</tbody>
</table>

Anti-quarks

<table>
<thead>
<tr>
<th>symbol</th>
<th>Flavor</th>
<th>Q</th>
<th>A</th>
<th>D</th>
<th>U</th>
<th>S</th>
<th>C</th>
<th>B</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>̅d</td>
<td>anti-down</td>
<td>1/3</td>
<td>-1/3</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>̅u</td>
<td>anti-up</td>
<td>-2/3</td>
<td>-1/3</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>̅s</td>
<td>anti-strange</td>
<td>1/3</td>
<td>-1/3</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>̅c</td>
<td>anti-charm</td>
<td>-2/3</td>
<td>-1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>̅b</td>
<td>anti-beauty</td>
<td>1/3</td>
<td>-1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>̅t</td>
<td>anti-truth</td>
<td>-2/3</td>
<td>-1/3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

of 1 (-1). They can carry an electric charges of ±2, ±1 or 0. States containing more than three quarks are considered bound states of hadrons. For example, a bound state of five quarks is viewed as a bound meson-baryon pair.

Each quark is endowed with one of three properties which are given the decorative name color, anti-quarks have anti-color. Color comes in three possible "hues"; red, green and blue. A combination of all three colors (or all three anti-colors) in equal amounts, or equal amounts of a color and its anti-color is termed colorless. Each quark carries one excess unit of color which combines in hadrons to form colorless states. Only colorless states are observed in nature, and so, color is conserved in all interactions.
1.3 Forces

In classical electrodynamics, a force exerted on a charged particle is the result of the charge interacting with an electric field produced by some charge distribution. The charge distribution could be as simple as a single charged particle. In quantum electrodynamics, the electric field is quantized in the form of photons. The force experienced by a particle is a result of its direct interaction with the quanta of the field. The interaction can be visualized as a continuous exchange of photons between the charged particle and the charge distribution. This is not purely a kinematic interaction. A kinematic interaction refers to a situation such as, two space men played catch in otherwise empty space. With no gravity present, the players would drift further apart at a faster rate with each toss and catch of the ball. Interactions through the exchange of photons result in both attractive and repulsive forces, therefore, the interaction must be more than “throw and catch” kinematics.

All non-contact forces can be visualized as the exchange of field quanta (mediators). The mediators for the four known forces have been listed in Table 1. Each force has its own set of mediators and a set of rules governing their interactions. The mediators consist of the photon for the electro-magnetic force, $W^\pm$ and $Z^0$ for the weak force and eight gluons for the strong force. The flow of mediators is called a current. A charged current results from $W^\pm$ exchanges, all other mediator exchanges results in a neutral current.

The electro-magnetic force can be attributed to the electric charge ($Q$), or simply charge, carried by the particle. The source of the strong force is the color charge ($Q_c$), while the weak force has the weak charges ($Q_w$ and $Q_z$) as a source. Table 5 gives a break down of the type of charge associated with each elementary particle.
Table 5
Elementary particle charges; $Q =$ electric charge in Heaviside-Lorentz units with $\hbar = c = 1$, $Q_s =$ color charge, $Q_w =$ "charged" weak charge and $Q_z =$ "neutral" weak charge.

<table>
<thead>
<tr>
<th>Fermion</th>
<th>$Q(g_e)$</th>
<th>$Q_s(g_s)$</th>
<th>$Q_w(g_w)$</th>
<th>$Q_z(g_z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-, \mu^-, \tau^-$</td>
<td>-1</td>
<td>0</td>
<td>-1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$\nu_e, \nu_\mu, \nu_\tau$</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>u, c, t</td>
<td>2/3</td>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>d, s, b</td>
<td>-1/3</td>
<td>1</td>
<td>-1/2</td>
<td>-1/2</td>
</tr>
</tbody>
</table>

Normalization is such that:

- $g_e = \sqrt{4\pi \alpha} = e =$ charge of positron
- $g_s = \sqrt{4\pi \alpha_s}$ with $\alpha_s \approx 0.22$
- $g_w = g_e / \sin \theta_w$ with $\theta_w \approx 28.7^\circ$
- $g_z = g_e / \sin \theta_w \cos \theta_w$

The quantum theory of electro-magnetic interactions is the prototype for other interactions. In light of this we next examine the electro-magnetic interaction more closely.

Quantum electrodynamics (QED) is the quantum field theory describing electro-magnetic interactions. The electro-magnetic mediator is the massless photon which couples directly to the electric charge. Interactions are represented by Feynman diagrams which, in turn, represent numbers determined by well defined rules. All QED processes can be built from the primitive vertex illustrated in Figure 1. More complicated interactions can be built by piecing together a number of these primitive vertices. For example, Figure 2 shows two such vertices pieced together to represent Bhabha scattering ($e^+ + e^- \rightarrow e^+ + e^-$).

The transition matrix for the primitive vertex of figure 1 is proportional to $g_e = e = \sqrt{4\pi \alpha}$, where $\alpha \approx 1/137$ is the fine structure constant, the energy units used are determined by setting $\hbar = c = 1$ and $e$ is the charge of the positron in Heaviside-Lorentz units. Calculating observable quantities such as the lifetime or
cross section involve, among other things, squaring the transition matrix. Thus, the primitive QED vertex introduces a factor of $\alpha$ into these calculations.

The observed particles in an interaction are represented by the *external lines* to a Feynman diagram. The internal lines represent possible interactions to get from the incoming lines to the outgoing lines. Interactions have many internal configurations for the same observed process, with the sum of all diagrams forming the amplitude matrix for the interaction. For example, any photon in an interaction can have an $e^+e^-$-loop added to it, as illustrated in figure 3. This adds a higher order term in $\alpha$ to the existing amplitude. Fortunately, the higher order terms decrease so rapidly that, for most purposes, calculations to the lowest
order in $\alpha$ gives a sufficient approximation.

As was just mentioned, photons can produce virtual $f\bar{f}$ pairs, where $f$ is a charged particle – predominantly the lightest of the charged fermions, the electron. Photons emitted by a charged particle ($q$) form a cloud of $f\bar{f}$ pairs surrounding the particle. One of the virtual pair is attracted by $q$ while the other is repelled by $q$. The result is similar to the effect observed in a dielectric medium, the charge $q$ is partial screened by the vacuum polarization, reducing the observed electromagnetic field of $q$ at a distance. The $f\bar{f}$ cloud extends outward to a distance of approximately the Compton wavelength of the electron, $\lambda_c = 2.43 \times 10^{-10}$ cm. What is commonly called the charge of $q$ is the fully screened charge. As the distance from $q$ decreases, the $f\bar{f}$ cloud is penetrated, as a result, the screening effect decreases and the effective charge increases. Since $\alpha$ is proportional to the effective charge of the positron ($e$), the value of $\alpha$ increases at small distances, which is to say, the coupling strength increases as two charges get closer together.

Quantum chromodynamics (QCD) is the quantum field theory describing strong interactions. The strong force mediators are the massless gluons which couple directly to color. Both quarks and gluons themselves carry color, leptons do not carry color and so do not participate in strong interactions. There are three colors denoted here as red, blue and green. Quarks carry an excess of a single color while gluons carry an excess of a unit of color and a unit of anti-color.
Figure 4
Primitive QCD vertices: (a) quark-quark-gluon vertex (b) three gluon vertex (c) four gluon vertex

When a quark interacts with a gluon, the color of a quark can change while flavor is unchanged. For example, a red u-quark can absorb a blue — red gluon resulting in a blue u-quark.

Since gluons couple with both quarks and gluons, there are three primitive vertices for QCD interactions. These are illustrated in figure 4

Associated with each QCD vertex is the strong coupling constant $\alpha_s$. Similar to the QED case, the quark is surrounded by a cloud of virtual $q\bar{q}$ pairs producing a color polarization of the vacuum. If this were the only factor involved, the result would again be an increasing coupling strength at small distances. But, since gluons can self couple, gluon-gluon loops also contribute to color polarization of the vacuum. The gluon-gluon loops reduce the color polarization produced by the $q\bar{q}$ loops. The result being, the effective coupling decreases at short distances. For large distances $\alpha_s$ is greater than one, but decreases to less then one for small distances. This is the basis of asymptotic freedom,[4] where bound quarks are treated as essentially free particles.

Quantum flavordynamics (QFD) is the field theory describing weak interac-
tions. The weak force mediators are the charged $W^\pm$ and neutral $Z^0$ vector bosons. Unlike the photons and gluons, these mediators have mass and thus have a limited range. Both leptons and quarks plus the weak force mediators themselves carry the weak charge. Since the $W$ also carries an electric charge, it couples with the photon through QED interactions. The "neutral" weak interactions (mediated by the $Z^0$) conserve both lepton numbers ($L_e$, $L_\mu$, $L_\tau$) and quark flavor. The "charged" weak interactions (mediated by the $W^\pm$) conserve lepton numbers but not quark flavor.

The leptons can be written as three doublets which interact through the weak force and three singlets which do not participate in weak interactions.

$$
\begin{pmatrix}
\nu_e \\
\mu \\
\tau
\end{pmatrix}_L,
\begin{pmatrix}
\nu_\mu \\
\mu \\
\tau
\end{pmatrix}_L
$$

and $e_R \mu_R \tau_R$. (1.1)

The L (R) subscript indicates that these are left-handed (right-handed) states in the mediators rest frame. The mediator rest frame is specified here because in general the handedness of a particle is not Lorentz invariant. A right-handed particle has spin and momentum vectors which tend to be aligned, while a left-handed particle has spin and momentum vectors which tend to be back-to-back. For massive particles handedness is not Lorentz invariant. If a massive particle has velocity $\vec{v}$ in one inertial system, a second inertial system can always be defined where the particle has velocity $-\vec{v}$. Since the spin does not change direction between inertial systems, the result is to reverse the handedness of the particle.

The neutrino is massless and travels at the speed of light. There is no inertial system which can reverse its velocity direction and so, the neutrino's handedness is fixed at the time of production. The neutrinos are only observed through the weak interactions, and then, only in a left-handed state, with anti-neutrinos only found in a right-handed state.

The primitive weak vertices in the lepton sector are illustrated in figure 5a for charged the weak current and figure 5b for the neutral weak current. The $W^\pm$
connect leptons within the same doublet as defined in equation 1.1. For example, an electron converts to an electron neutrino by emission of a $W^-$. The $Z^0$ and photon vertices “look” very similar although the couplings are very different. The $Z^0$ couples to everything that the photon couples to, and then some. The $l's$ of figure 5b can be either charged or neutral leptons. As a result, $Z^0$ interactions are generally masked by QED interactions, except in the neutrino sector where detection is extremely difficult.

The primitive weak vertices for the quark sector are illustrated in figure 5c for charged weak currents and figure 5d for a neutral weak current. The charged weak interaction defines three doublets and six singlets as:

$$
\begin{pmatrix}
  u \\
  d' \\
  c \\
  s' \\
  t \\
  b'
\end{pmatrix}_{L} \quad \begin{pmatrix}
  u \\
  d' \\
  c \\
  s' \\
  t \\
  b'
\end{pmatrix}_{L} \quad \begin{pmatrix}
  u_R \\
  c_R \\
  t_R \\
  d_R \\
  s_R \\
  b_R
\end{pmatrix} \quad \text{and} \quad u_R \ c_R \ t_R \ d_R \ s_R \ b_R; \quad (1.2)
$$

The $W^\pm$ connects quarks within the same doublet as defined in equation 1.2. Again, the six singlets do not participate in weak interactions. The weak interaction eigenstates $d'$, $s'$ and $b'$ are linear combinations of the strong interaction eigenstates $d$, $s$ and $b$. By convention the $u$, $c$, and $t$ quarks are unmixed with all mixing expressed in terms of the Cabibbo-Kobayashi-Maskawa matrix ($V$) operating on the $d$, $s$ and $b$ quarks:

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} = \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix} \quad (1.3)
$$

The experimental range of these matrix elements$^{[37]}$ are:

$$
\begin{pmatrix}
  0.9747-0.9759 & 0.218-0.224 & 0.002-0.007 \\
  0.218-0.224 & 0.9735-0.9751 & 0.032-0.054 \\
  0.003-0.018 & 0.030-0.054 & 0.9985-0.9995
\end{pmatrix} \quad (1.4)
$$

The off diagonal terms give the extent of the mixing. There is no such mixing of the leptons since the neutrinos are only defined through their weak interactions.
Figure 5
Primitive QFD vertices: (a) $l\bar{v} \rightarrow W$ vertex (b) $l\bar{l} \rightarrow Z^0$ vertex (c) $qq \rightarrow W$ vertex (d) $q\bar{q} \rightarrow Z^0$ vertex (e) $W^+W^- \rightarrow Z^0$ vertex (f) $W^+W^- \rightarrow W^+W^-$ vertex (g) $W^+W^- \rightarrow Z^0Z^0$ vertex (h) $W^+W^- \rightarrow \gamma\gamma$ vertex (i) $W^+W^- \rightarrow \gamma Z^0$ vertex (j) $W^+W^- \rightarrow \gamma\gamma$ vertex
1.4 Conservation Laws

A symmetry is an operation (or transformation) which, when applied to a system, results in a configuration indistinguishable from the initial configuration. According to Nöther’s theorem, every symmetry in nature implies a conservation law and every conservation law implies an existing symmetry. Thus, the following two statements have the same meaning:

Strong, electro-magnetic and weak interactions conserve linear momentum.

Strong, electro-magnetic and weak interactions are invariant under translations in space.

The first statement is in the context of a conservation law, the second is a statement of symmetry.

Besides implying a conservation law, a symmetry also implies an indeterminacy. For example, the symmetry of space translations implies that there is no absolute origin. Although calculations may be simplified for a particular choice of origin, the laws of nature are the same for any origin. By studying the symmetries of a system we gain insight into the system.

The strong, electro-magnetic and weak interactions conserve energy, linear momentum, angular momentum and charge. The corresponding symmetry for each of these conservation laws are given in Table 6. Other possible symmetries are discussed in the following paragraphs.

The space inversion operator $P$ takes a state $|\psi\rangle$ into a state $|\psi'\rangle$ where all space coordinates change sign, $P|\psi(x,y,z)\rangle = |\psi'(x',y',z')\rangle = |\psi(-x,-y,-z)\rangle$. 

Table 6
Some conservation laws and corresponding symmetries

<table>
<thead>
<tr>
<th>Conservation law</th>
<th>Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>Time Translation</td>
</tr>
<tr>
<td>Linear Momentum</td>
<td>Space Translation</td>
</tr>
<tr>
<td>Angular Momentum</td>
<td>Rotation</td>
</tr>
<tr>
<td>Charge</td>
<td>Phase Translation (Gauge Transformation)</td>
</tr>
</tbody>
</table>

For rotational invariant systems this is equivalent to a reflection through the xy-plane, \( P|\psi(x, y, z)\rangle = |\psi(-x, -y, -z)\rangle = |\psi(x, y, -z)\rangle \). Applying \( P \) twice brings the state back to itself, \( P^2|\psi\rangle = |\psi\rangle \). It follows that the eigenvalues of \( P \) are \( \pm 1 \). States with \( P = 1 \) are called symmetric states and those with \( P = -1 \) are antisymmetric states. Particles described by symmetric states are bosons, such as, mediators and mesons. Those described by antisymmetric states are fermions such as leptons, quarks and baryons.

Parity is conserved by strong and electro-magnetic interactions but not by the weak interaction. Violation of parity in a weak interaction can be seen in the decay \( \pi^+ \rightarrow \mu^+ + \nu_\mu \). For a two-body decay of a \( \pi^+ \) at rest, the decay products, \( \mu^+ \) and \( \nu_\mu \), have back-to-back momenta. And, since the \( \pi^+ \) has spin 0, the \( \mu^+ \) and \( \nu_\mu \) have opposite directed spins. Either both have their spin vectors in the direction of their momentum (right-handed) or both have opposite directed spin and momentum vectors (left-handed). The \( \mu^+ \) for this decay is always emitted in a left-handed state and so \( \nu_\mu \) is also emitted in a left-handed state. If parity were conserved, half the decays would be right-handed \( \mu^+ , \nu_\mu \) pairs. But this is not the case, in fact, parity is maximally violated in this decay. The fly in the ointment is the neutrino, or more precisely, the lack of a right-handed neutrino. Parity maps the left-handed neutrino into a nonexistent right-handed neutrino. In order to save the parity symmetry a second transformation is needed which transforms \( \nu_\mu \) into an existing particle. The most logical particle being the right-handed
anti-neutrino.

The charge conjugation operator $C$ converts each particle into its anti-particle; $C|\psi\rangle = |\bar{\psi}\rangle$. Applying $C$ twice brings the state back to itself, $C^2|\psi\rangle = |\psi\rangle$. It follows that the eigenvalues of $C$ are $\pm 1$. If $|p\rangle$ is an eigenstate of $C$, then $C|p\rangle = \pm |p\rangle = |\bar{p}\rangle$. Therefore, only particles that are their own anti-particles can be eigenstates of $C$. Like parity, $C$ is conserved for strong and electro-magnetic interactions but not for weak interactions. Charge conjugation of a left-handed neutrino results in a nonexistent left-handed anti-neutrino.

Since both $P$ and $C$ are conserved in strong and electro-magnetic interactions, the combination $CP$ is also conserved. The weak interactions $\pi^+ \rightarrow \mu^+(-1) + \nu_\mu(-1)$ is transformed into $\pi^- \rightarrow \mu^-(+1) + \bar{\nu}_\mu(+1)$ by the CP operator, where $(-1)$ indicates a left-handed particle and $(+1)$ a right-handed particle. Both decays occur at the same rate and so CP would seem to be conserved for the weak interactions, but alas, this is not the case.

Neutral kaons are produced as strong interaction eigenstates of strangeness labeled $K^0$ and $\bar{K}^0$, but decay weakly in CP eigenstates labeled $K_1$ and $K_2$. The weak eigenstates are linear combinations of the strong eigenstates; $|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$ and $|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$ with CP eigenvalues of $+1$ and $-1$ respectively,

$$CP|K_1\rangle = \frac{1}{\sqrt{2}}(CP|K^0\rangle - CP|\bar{K}^0\rangle)$$
$$= \frac{1}{\sqrt{2}}(-|\bar{K}^0\rangle + |K^0\rangle)$$
$$= (+1)|K_1\rangle$$ (1.5)

$$CP|K_2\rangle = \frac{1}{\sqrt{2}}(CP|K^0\rangle + CP|\bar{K}^0\rangle)$$
$$= \frac{1}{\sqrt{2}}(-|\bar{K}^0\rangle - |K^0\rangle)$$
$$= (-1)|K_2\rangle$$ (1.6)
The $K_1$ and $K_2$ primarily decay into $2\pi$ or $3\pi$ states. For the all neutral states this is;

$$CP|\pi^0\pi^0\rangle = (-1)^2 C|\pi^0\pi^0\rangle$$
$$= (+1)|\pi^0\pi^0\rangle$$
$$\quad \quad \quad (1.7)$$

$$CP|\pi^0\pi^0\pi^0\rangle = (-1)^3 C|\pi^0\pi^0\pi^0\rangle$$
$$= (-1)|\pi^0\pi^0\pi^0\rangle$$
$$\quad \quad \quad (1.8)$$

Similarly $CP|\pi^+\pi^-\rangle = (+1)|\pi^+\pi^-\rangle$ and $CP|\pi^+\pi^-\pi^0\rangle = (-1)|\pi^+\pi^-\pi^0\rangle$.

If CP is conserved then $K_2$ with $CP = -1$ can decay into a $3\pi$ state but never into a $2\pi$ state, and $K_1$ with $CP = +1$ can decay into a $2\pi$ state but never to a $3\pi$ state. Since the $2\pi$ decay has a larger phase space available than the $3\pi$ decay, the $K_1$ has a shorter lifetime then the $K_2$. A beam of $K^0$s decays rapidly through its $2\pi$ channel leaving, after a time, a pure $K_2$ beam. Decay of the $K_2$ beam shows the expected $3\pi$ decay channel, but also, a small ($\approx 0.3\%$) CP violating $2\pi$ decay channel.

The realization that P and CP are not conserved leads to doubts of time-reversal invariance. The *time reversal operator* $T$, as the name implies, reverses the flow of time. The $T$ operator reverses the decay process to a production process, that is, what was the decay $K_1 \rightarrow 2\pi$ becomes the production $2\pi \rightarrow K_1$ when the $T$ operator is applied. For strong and electro-magnetic interactions $T$ invariance appears to hold. This is not surprising since, according to the CPT theorem,[6] the product CPT (in any order) is a exact symmetry of any interaction. Since CP is a symmetry of strong and electro-magnetic interactions, it follows that $T$ must also be a symmetry for these interactions. Likewise, since CP is violated in some weak interactions, $T$ must also be violated. Testing $T$ invariance for weak interactions is extremely difficult. Reversing the CP violating processes requires colliding hadrons, which interact primarily through strong interactions, masking the weak effects.
1.5 Photon-Photon Scattering

The phenomenon of light scattering is attributed to quantum fluctuations which allow the photon to form virtual states of charged particle, anti-particle pairs (\( \gamma \rightarrow f^+f^- \)). The \( f^+f^- \) pair can be leptons or quarks or W's or even composites. This phenomenon was introduced earlier in explaining vacuum polarization. Once the \( f^+f^- \) pair is formed a second photon, which couples directly to charge, can then couple to the photon through the charged components of the \( f^+f^- \) pair. The second photon can also form a second virtual \( f^+f^- \) pair which then interacts with the first pair. If both \( f^+f^- \) pairs contain quarks, they can interact through the strong force. In either case at least two QED vertices exist in the interaction and so the leading term of the squared amplitude is at least of order \( \alpha^2 \).

If the total energy is insufficient to produce electron pairs (center of mass energy \( W_{\gamma\gamma} < 2m_e \)) the state reverts back to the two photon state. This interaction is purely elastic, producing only photons. Figure 6a shows the box diagram for elastic scattering of two photons. Including only the electron box diagram, the cross section below the \( e^+e^- \) threshold is given as:

$$
\frac{d\sigma}{d\theta}(\gamma\gamma \rightarrow \gamma\gamma) = \alpha^4 \left( \frac{W_{\gamma\gamma}}{2m_e} \right)^6 (3 + \cos^2 \theta)^2 (3.55 \times 10^9 \text{nb}).
$$

where \( W_{\gamma\gamma} \) is the total energy of the two-photon system. For visible light the cross section is on the order of \( 10^{-29} \text{nb} \) and rises to approximately 100 nb just below the \( e^+e^- \)-pair production threshold. Because of the extremely small cross section for the scattering of visible light, this process has never been observed.

Above the \( e^+e^- \)-threshold inelastic processes dominate. The inelastic process with the smallest \( W_{\gamma\gamma} \) energy is \( e^+e^- \)-pair production, illustrated in figure 6b. The cross section for \( \gamma\gamma \rightarrow e^+e^- \) was calculated by Breit and Wheeler\(^{(6)} \) (1934),
and later by Karplus and Neuman\textsuperscript{[8]} (1951), as;

\[ \sigma(\gamma\gamma \rightarrow e^+e^-) \approx \begin{cases} 
\alpha^2 (4.68 \times 10^{12} \text{ nb}) \sqrt{1 - \left( \frac{m_e}{W_{\gamma\gamma}} \right)^2} & W_{\gamma\gamma} \approx 2m_e \\
\alpha^2 \left( \frac{m_e}{W_{\gamma\gamma}} \right)^2 (4.68 \times 10^{12} \text{ nb}) \sqrt{2 \ln \left( \frac{W_{\gamma\gamma}}{m_e} \right) - 1} & W_{\gamma\gamma} \gg 2m_e
\end{cases} \]

(1.10)

Since intense beams of energetic photons are required to observe this phenomena, the theory went untested for many years.

Later, an intense source of virtual photons was determined to exist in the form of the strong Coulomb field surrounding the nucleus. In 1951, Wilson used the nuclear electric field to scatter photons in elastic collisions. Also in 1951, neutral pions were produced by two-photon interactions when real photons were fired at the nuclear electric field. This process, illustrated in figure 7, is known as the Primakoff effect.

Earlier (1924), Fermi had proposed that moving charged particles provide a source of virtual photons. With the advent of the high energy particle accelerator an intense source of high energy virtual photons was realized. In 1971 the first reported two-photon reactions at an $e^+e^-$ colliding beam machine were reported from Novosibirsk in the Soviet Union and shortly there after from Frascati in
Figure 7
Primakoff effect: $\pi^0$ production by a real photon collision with a virtual photon of the nuclear coulomb field.

Italy. The reaction observed was $e^+e^- \rightarrow e^+e^-\gamma \gamma \rightarrow e^+e^-e^+e^-$. Since then, colliding electron-positron beam experiments have become the premier source of two-photon interaction measurements. This study continues along these lines using virtual photons from an $e^+e^-$ colliding beam to produce $c\bar{c}$ resonances. A theoretical discussion of these interactions is presented in Chapter 3 of this thesis.

1.5.1 Two Photons Versus One Photon

Typically $e^+e^-$ colliding beam experiments are designed for the one photon annihilation process illustrated in Figure 8a. The two-photon process illustrated in Figure 8b is normally considered an extra benefit or an annoying background, depending on your interests.

In the two-photon process the electron and positron each radiating a virtual photon with the two photons interacting to form the state $\mathcal{R}$. The reaction is written as $e^+e^- \rightarrow e^+e^-\gamma \gamma \rightarrow e^+e^-\mathcal{R}$, where the notation $\gamma$ represents either real or virtual photons. The cross section for this reaction is, to first order in
Figure 8
Production of final state $\mathcal{R}$ in (a) single photon $e^+e^-$ annihilation, (b) two photon $e^+e^-$ scattering.

$\alpha$, proportional to $\alpha^4$. It increases with energy at least as $[\ln(E/m_e)^2]^2$. For example the cross section for two-photon production of lepton pairs$[^{[9]}]$ is,

$$
\sigma_{e^+e^-\rightarrow e^+e^-l^+l^-} = \frac{28\alpha^4}{27\pi m_l^2} \left( \ln \frac{s}{m_e^2} \right)^2 \ln \left( \frac{s}{m_l^2} \right); \quad l = (e, \mu).
$$

where, $\sqrt{s} = 2E$. Typically the radiated photons are nearly real and at small angles relative to the beam axis. The scattering angles are typically less than $m_e/E$. This provides two colliding beams of quasi-real photons with a small beam divergence.

Since the radiated photon energy distribution nearly follows a bremsstrahlung spectrum ($\sim \frac{1}{E}$), the state $\mathcal{R}$ has predominately low mass. Also, since the photons generally have different energies (and momenta) the energy and momentum of the state $\mathcal{R}$ is not fixed by the beam energy. Unfortunately, a great effort is required to detect even a small sample of the scattered electrons. The scattered electrons are usually lost down the beam-pipe, so the energy and momentum must be determined by fully reconstructing $\mathcal{R}$. The unequal momentum leads to $\mathcal{R}$ having a boost along the beam direction. This results in low reconstruction efficiency due to particles lost down the beam-pipe. Since the product of the photons charge conjugation eigenvalues is 1 ($C_\gamma C_\gamma = -1 \cdot -1 = 1$), $\mathcal{R}$ must have
In the one-photon process an $e^+e^-$-pair annihilate forming a single virtual photon, which in turn decays into a final state $\mathcal{R}$ of hadrons and/or leptons – the reaction being $e^+e^- \rightarrow \gamma \rightarrow \mathcal{R}$. The cross section for this reaction is, to first order of the electro-magnetic coupling constant, proportional to $\alpha^2$. The cross section also decreases as $1/E^2$ for symmetric beams with beam energy $E$. For example, the $\mu$-pair cross section is,

$$\sigma_{e^+e^-\mu\mu} = \frac{4\pi \alpha^2}{3s},$$

where $\sqrt{s} = 2E$ is the center of mass energy of the beams. Because all the beam energy goes into the reaction both energy and momentum of the state $\mathcal{R}$ is known ($E_\mathcal{R} = 2E, P_\mathcal{R} = 0$). Much of the time $\mathcal{R}$ as a whole is not the primary point of interest but instead, only an element in its decay chain is of interest. In this case not all information about $\mathcal{R}$ need be available. Since the process proceeds through a single photon having odd charge conjugation ($C_\gamma = -1$), the state $\mathcal{R}$ has $C_\mathcal{R} = -1$. A resonance such as the $\eta_c$, with $C_{\eta_c} = 1$, must then be produced as a decay product of heavier hadronic states.

Production of meson resonances, through the two-photon process, have several advantages over meson production from hadronic processes. In the two-photon process;

1. The production rate can be calculated from QED with only the $\gamma\gamma$ width as a free parameter.

2. There is only one hadronic state involved so reactions are not confused by stray information from other hadrons.

3. The quantum numbers of the final state are restricted by the two photon initial state.
1.6 Hadron Production

The two-photon process can be factored into two separate reactions. The first is the $e^+e^-$ production of two virtual photon beams with luminosity $L\gamma\gamma$, $e^+e^- \rightarrow e^+e^-\gamma\gamma$. The second is the colliding photons portion of the process, $\gamma\gamma \rightarrow \mathcal{R}$. The first factor is calculable from QED. When $\mathcal{R}$ consists of only lepton states the entire process is calculable from QED. When $\mathcal{R}$ involves hadronic states, the process is not, in general, calculable and is therefore of foremost interest.

For real or quasi-real photons the Vector-meson Dominance Model (VDM) describes the features of the process fairly well. In VDM the photons are considered as virtual vector mesons ($\rho, \omega, \phi, J/\psi$, etc.) which interact via the strong force. The VDM interaction is illustrated in Figure 9b. The two-photon interaction under the VDM model should then have the characteristics of hadronic interactions as observed in hadron-hadron scattering.

In the case of high momentum transfer (very virtual photons), where the interactions are at small distances, the photons have a point-like QED coupling
to the quark pair. The quarks can then fragment into hadrons as illustrated in Figure 9a. Since both models describe the same process, only at opposite ends of the transferred momentum spectrum, there should be a smooth transition from one picture to the other.

The cross section for the production of a resonance $\mathcal{R}$ with spin $J$ by two real photons can be written as,

$$\sigma(\gamma\gamma \to \mathcal{R}) = 8\pi(2J + 1) \frac{\Gamma_{\gamma\gamma}}{\sqrt{(W_{\gamma\gamma}^2 - M_{\mathcal{R}}^2)^2 + \Gamma_{\mathcal{R}}^2}}.$$  \hspace{1cm} (1.13)

Where $\Gamma$ is the total width, $M_{\mathcal{R}}$ the mass, $\Gamma_{\gamma\gamma}$ the $\gamma\gamma$ width of the resonance and $W_{\gamma\gamma}$ is the mass of the $\gamma\gamma$ system. For a narrow resonance this can be written in the form,

$$\sigma(\gamma\gamma \to \mathcal{R}) = 8\pi^2(2J + 1) \frac{\Gamma_{\gamma\gamma}}{M_{\mathcal{R}}} \delta(W_{\gamma\gamma}^2 - M_{\mathcal{R}}^2).$$  \hspace{1cm} (1.14)

### 1.7 Charmomium

Charmomium is a bound state of a charmed quark $c$ and its anti-quark $\bar{c}$. The charmed quark was introduced in 1964 by Hara and by Bjorken and Glashow so the quarks $(u,d,s,c)$ and leptons $(\nu_e, e, \nu_\mu, \mu)$ formed a symmetric picture. In 1970 Glashaw, Iliopoulas and Maiani gave the $c$ quark a stronger foothold by using it to explain the absence of flavor changing neutral currents in the neutral $K$ meson decay. Although the neutral current was assumed, its existence was not shown until 1973 when neutrino reactions with hadrons were observed without the production of charged leptons. This led to an enthusiastic search for the $c$ quark. In 1974 the candidate $J/\psi$ resonance was observed with a mass of 3.1 GeV. The $J$ was observed by the Brookhaven National Laboratory group through the process $J \to e^+e^-$ and the $\psi$ was observed by the SLAC (Stanford) experiment through the reverse process $e^+e^- \to \psi$. Since each group originally gave a different name to the same resonance, the names have been combined to give the $J/\psi$ today.
The $\psi$ has the same quantum numbers as the photon, $J^{PC} = 1^{--}$, $I=0$ and is the ground $^3S_1$-level of charmonium. Here the usual spectroscopic notation $^{2S+1}L_J$ is used, where $J$ is the total angular momentum, $L$ is the orbital momentum and $S$ is the spin momentum. Shortly after the $\psi$ was observed, the SLAC group observed the $\psi'$ at 3.7 GeV. This was the first radially excited $^3S_1$-state of charmonium, it also had $J^{PC} = 1^{--}$ and $I=0$.

In the mass range between the $\psi$ and $\psi'$ were found the three P-states of charmonium denoted $\chi_{c0}$, $\chi_{c1}$, $\chi_{c2}$ for the $^3P_0$, $^3P_1$, $^3P_2$ states. These states are produced in radiative decays of the $\psi'$, $\psi' \rightarrow \gamma \chi_c$. The $\chi_c$ states all have charge conjugation parity $C = +1$, G-parity = +1, and $I=0$. The lowest mass charmonium state, $^1S_0$ ground state $\eta_c$, was first misidentified at a mass of 2.830 GeV, but later measurements determined a mass of $2.9788 \pm 0.0019$ GeV with $J^{PC} = 0^{++}$.

The charmonium states $\psi$ and $\psi'$ have a hadronic decay width much smaller than expected; $\Gamma(\psi \rightarrow \text{had.}) = 76 \pm 6$ keV and $\Gamma(\psi' \rightarrow \text{had.}) = 274 \pm 32$ keV. This is only about an order of magnitude greater than their $e^+e^-$-decay width; $\Gamma(\psi \rightarrow e^+e^-) = 5.36 \pm 0.29$ keV and $\Gamma(\psi' \rightarrow e^+e^-) = 2.14 \pm 0.21$ keV. The normal hadronic decay width for a heavy meson is at least two orders of magnitude larger than the hadronic width of the $\psi$ or $\psi'$. For example, $\Gamma(\phi(1020) \rightarrow \text{Had.}) = 4.40 \pm 0.06$ MeV. This represents something new and different occurring in the $\psi$ regime, the strong interactions appear to be less strong here.

Besides the $\psi'$ state other radially excited charmonium state have also been observed. These include $\psi''(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$. The lepton-pair decay width for these states are slightly smaller than for the $\psi$ and $\psi'$ yet their total widths are at least two orders of magnitude greater. The reason for this is that the more massive states are able to produce pairs of charmed mesons while the lighter states cannot.
A charmed meson is a bound state of a $c$ quark and one other charmless quark, thus having a nonzero charm quantum number. The charm quark must therefore decay via the weak interaction resulting in decay products which contain nonzero strangeness. The lightest of these, the $D$ mesons, were observed by SLAC in 1976 through the decays $K^\pm \pi^\mp, K^{*\pm} \pi^\mp, K^{\pm} \pi^\mp \pi^\pm$. The $D^0$ (composed of $c\bar{u}$ quarks) has the lowest mass at $1864.5 \pm 0.5$ MeV. The $\psi$ and $\psi'$, as well as, $\eta_c, \chi_{c1}$ and $\chi_{c2}$ have masses that are below the $D\bar{D}$ production threshold. Since charm is conserved in both strong and electro-magnetic interactions, strong interactions proceed by the charm quarks annihilating, producing the decay products through gluons. These states are said to be Zweig suppressed, that is, they have unconnected lines in the quark flow diagram as illustrated in Figure 10b. Once above the $D\bar{D}$ production threshold (3.730 GeV) the charm quark lines can continue through the decay diagram and thus many more decays become available through the Zweig allowed processes illustrated in Figure 10a.

A charmonium state with $J^{PC} = 1^{--}$ has the same quantum numbers as the photon and so it is produced easily with $e^+e^-$ colliding beams operating at a center-of-mass energy equal to the mass of the state. The charmonium states; $\eta_c(J^{PC} = 0^{-+})$, $\chi_{c0}(J^{PC} = 0^{++})$ and $\chi_{c2}(J^{PC} = 2^{-+})$ are not produced directly
in one photon $e^+e^-$ annihilation, but instead, result from the radiative decay of the $J^{PC} = 1^{--}$ states. Because of the small number of radiative decays, branching ratio measurements have uncertainties of approximately 30%. A more productive means of producing these states at $e^+e^-$ colliders is through the two-photon interaction. This is the process studied in this thesis.

1.8 Predictions

In this section we present some predictions as to the value of the two-photon partial width for the $\mathcal{R}_c \otimes \pi$ states and compare these to experimental values. Two approaches for predicting the two-photon width of the charmonium states are presented. The first approach, labeled BCGR, employs strong QCD corrections and is based on work presented by Barbieri, Caffo, Gatto and Remiddi in references 22–24. The second approach, labeled BBL, employs factorization theorems under development by Bodwin, Braaten and Lepage. The factorization theorems result in QCD predictions presented in reference 27.

In mesons the radius of the strong interaction confinement is believed to be $R \sim 1\text{fm} = 5\text{ GeV}^{-1}$. From the uncertainty relation, quarks confined to a region $R$ have a characteristic momentum of $p \sim R^{-1}$. For mesons this is $p \sim 0.2\text{ GeV}$. For constituent quarks with a mass on the order of $\frac{1}{\Lambda}$ or less, considering the hadron as a non-relativistic system results in a very poor approximation. This is the case for light hadrons which contain $u$, $d$ and $s$ quarks. On the other hand, the $c$ quark has a mass greater than 1 GeV. In this case a non-relativistic consideration is expected to give a fairly accurate description of the charmonium system.

A theoretical estimate of the two-photon decay width $\Gamma_{\gamma\gamma}(\eta_c) \equiv \Gamma(\eta_c \rightarrow \gamma\gamma)$ can be obtained assuming a non-relativistic potential description. Since the $\eta_c$
and $J/\psi$ are both composed of $c\bar{c}$ quark pairs in a relative S-wave state, their spatial wave functions are expected to be similar at the origin.

The ratio of decay widths, $\Gamma(\eta_c \rightarrow \gamma\gamma)$ to $\Gamma(\psi \rightarrow e^+e^-)$, can then be written as,

$$\frac{\Gamma(\eta_c \rightarrow \gamma\gamma)}{\Gamma(\psi \rightarrow e^+e^-)} = \frac{4}{3} \frac{1 + D_{\gamma\gamma} \frac{\alpha_s(m_c)}{\pi}}{1 + D_{ee} \frac{\alpha_s(m_c)}{\pi}} = \frac{4}{3} \left( 1 + (D_{\gamma\gamma} - D_{ee}) \frac{\alpha_s(m_c)}{\pi} \right).$$

(1.15)

Where the expansion coefficients are given as $D_{\gamma\gamma} = -3.38$ and $D_{ee} = -5.33$. Assuming a strong coupling constant of $\alpha_s(m_c) = 0.25 \pm 0.02$ and an electron partial decay width of $\Gamma(\psi \rightarrow e^+e^-) = 5.99 \pm 0.25$ keV we estimate $\Gamma(\eta_c \rightarrow \gamma\gamma) = 9.2 \pm 0.4(\pm43\%)$ keV. The first uncertainty is due to the errors in $\alpha_s$ and $\Gamma(\psi \rightarrow e^+e^-)$. The second uncertainty is the fractional contribution of the $D_{\gamma\gamma}$ and $D_{ee}$ terms taken in quadrature.

Possible corrections to this estimate include relativistic effects, further QCD corrections, gluon condensates, and changes in the wave function due to spin-dependent forces or hyperfine mass splitting. The corrected $\Gamma(\gamma\gamma(\eta_c)$ values range from 3 keV to 15 keV. The existing measurements, listed in Table 7, range from $\Gamma(\gamma\gamma = 4$ keV to 27 keV.

The present experimental results cannot distinguish which of these corrections are important or valid. A significant improvement of these measurements should lend to a better understanding of these corrections.

We next obtain estimates of the two photon partial widths for the P-wave states $\chi_{c0}$ and $\chi_{c2}$ plus a second estimate of $\Gamma(\gamma\gamma(\eta_c)$. We express the ratio of the light-hadronic decay width $\Gamma(\mathcal{R}_{c\bar{c}} \rightarrow gg)$ to the two-photon decay width $\Gamma(\mathcal{R}_{c\bar{c}} \rightarrow \gamma\gamma)$ at next to leading order in $\alpha_s$. A light-hadron consists of u, d and s quark combinations. Infrared divergences appearing in the P-wave states
Table 7
Measurements of $\Gamma_{\gamma\gamma}(\eta_c)$.

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>$\Gamma_{\gamma\gamma}(\eta_c)$ (MeV)</th>
<th>$\eta_c$ Production</th>
<th>$\eta_c$ Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLUTO*[^41]</td>
<td>23 ± 12</td>
<td>$e^+e^- \rightarrow e^+e^-\eta_c$</td>
<td>$K^0K^{\pm}\pi^\mp$</td>
</tr>
<tr>
<td>TASSO*[^40]</td>
<td>16.1 ± 6.3 ± 6.0</td>
<td>$e^+e^- \rightarrow e^+e^-\eta_c$</td>
<td>$K^0K^{\pm}\pi^\mp$</td>
</tr>
<tr>
<td>TPC/2γ[^42]</td>
<td>6.43±5.0</td>
<td>$e^+e^- \rightarrow e^+e^-\eta_c$</td>
<td>$K^+K^-K^+K^-$</td>
</tr>
<tr>
<td>CLEO1.5[^43]</td>
<td>8.54±6.2 ± 3.9</td>
<td>$e^+e^- \rightarrow e^+e^-\eta_c$</td>
<td>$K^*K\pi$</td>
</tr>
<tr>
<td>CLEO1.5*[^43]</td>
<td>7.83±3.5 ± 2.3</td>
<td>$e^+e^- \rightarrow e^+e^-\eta_c$</td>
<td>$K^0K^{\pm}\pi^\mp$</td>
</tr>
<tr>
<td>L3[^46]</td>
<td>8.0 ± 2.3 ± 2.4</td>
<td>$e^+e^- \rightarrow e^+e^-\eta_c$</td>
<td>multiple</td>
</tr>
<tr>
<td>ARGUS[^47]</td>
<td>11.3 ± 4.2</td>
<td>$e^+e^- \rightarrow e^+e^-\eta_c$</td>
<td>multiple</td>
</tr>
<tr>
<td>R704[^44]</td>
<td>4.3±3.7 ± 2.4</td>
<td>$p\bar{p}$ annihilation</td>
<td>$\gamma\gamma$</td>
</tr>
<tr>
<td>BCGR[^26]</td>
<td>9.2 ± 0.4(±43%)</td>
<td>—</td>
<td>Theory($\psi \rightarrow e^+e^-$)</td>
</tr>
<tr>
<td>BCGR[^26]</td>
<td>4.7 ± 1.6(±32%)</td>
<td>—</td>
<td>Theory($\eta_c \rightarrow gg$)</td>
</tr>
</tbody>
</table>

are replaced by a logarithm of the bound state radius[^26] $R_c(R_{c\bar{c}})$.

The ratio of the hadronic decay width to the two-photon decay width, $B_{\gamma\gamma}(R_{c\bar{c}})$, is written as[^26];

$$B_{\gamma\gamma}(R_{c\bar{c}}) = \frac{\Gamma(R_{c\bar{c}} \rightarrow gg)}{\Gamma(R_{c\bar{c}} \rightarrow \gamma\gamma)} = A\left(\frac{\alpha_e(\mu)}{\pi}\right)^2 \frac{1 + D_{gg}(R_{c\bar{c}},\mu)\alpha_{s}(\mu)}{1 + D_{\gamma\gamma}(R_{c\bar{c}},\mu)\alpha_{s}(\mu)}$$

$$= A\left(\frac{\alpha_e(\mu)}{\pi}\right)^2 \left\{ 1 + [D_{gg}(R_{c\bar{c}},\mu) - D_{\gamma\gamma}(R_{c\bar{c}},\mu)]\alpha_{s}(\mu) \frac{1}{\pi} \right\}. \quad (1.16)$$

The coefficient for the leading order approximation is given as,

$$A = \frac{2\pi^2}{9e^4\alpha_{\text{QED}}^2}, \quad (1.17)$$

where $e_c = 2/3$ is the charge of the constituent quark and $\alpha_{\text{QED}} = 1/133.3$ is the QED coupling[^27] at the charm quark mass scale($\mu = m_c$). The coefficients $D_{gg}(R_{c\bar{c}},\mu)$ and $D_{\gamma\gamma}(R_{c\bar{c}},\mu)$ are listed in Table 8. The factor $\beta_0$ is a function of the number of active flavors, $n_f$, such that,

$$\beta_0 = 11 - \frac{2}{3}n_f. \quad (1.18)$$
Table 8
Coefficients of $B_{g\gamma}$ expansion.

<table>
<thead>
<tr>
<th>$R_{c\Sigma}$</th>
<th>$D_{g\gamma}(\mu)$</th>
<th>$D_{gg}(\mu)$</th>
<th>$D_{gg}(\mu = m_c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c$</td>
<td>$\frac{x_y}{-20}$</td>
<td>$-3.38$</td>
<td>$\beta_0 \ln(\frac{\mu}{m_c}) + 6.13 - n_f(0.427)$</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>$\frac{3}{x_y} - 28$</td>
<td>$0.18$</td>
<td>$\beta_0 \ln(\frac{\mu}{m_c}) + 9.16 - n_f(0.130 - \frac{4}{3} \ln(\mu R_c))$</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>$\frac{-16}{3}$</td>
<td>$-5.33$</td>
<td>$\beta_0 \ln(\frac{\mu}{m_c}) - 4.38 - n_f(0.149 - \frac{3}{3} \ln(\mu R_c))$</td>
</tr>
</tbody>
</table>

For charmonium states $n_f = 3$. The confinement radius, $R_c(M)$, is determined by

$$\ln(\mu R_c(M)) = \text{const} \quad \text{so that} \quad R_c(\chi_{c0}) = 2.254 \quad \text{and} \quad R_c(\chi_{c2}) = 1.646.$$ 

A light-hadron width of $\Gamma(\eta_c \rightarrow gg) = 10.3 \pm 3.6 \text{ MeV}^{[37]}$ yields an estimated two-photon width of $\Gamma(\eta_c \rightarrow \gamma\gamma) = 4.7 \pm 1.6(\pm 40\%) \text{ keV}$. We have assumed the strong coupling constant to have a value $\alpha_s(m_c) = 0.25 \pm 0.02^{[27]}$. The first uncertainty is due to the reported uncertainty in the hadron width. The second uncertainty is the fractional contribution of the $D_{g\gamma}$ and $D_{gg}$ terms taken in quadrature. The $\chi_{c2}$ hadron width of $\Gamma(\chi_{c2} \rightarrow gg) = 1.71 \pm 0.16 \text{ MeV}^{[27]}$ leads to an estimated two-photon width of $\Gamma(\chi_{c2} \rightarrow \gamma\gamma) = 1.1 \pm 0.1(\pm 41\%) \text{ keV}$. 

In the $\chi_{c0}$ case, the hadron width of $\Gamma(\chi_{c0} \rightarrow gg) = 14 \pm 5 \text{ MeV}^{[27]}$ is consistent with zero. We have little predictive power here due to the poor measurement. Circumventing the large uncertainty in $\Gamma(\chi_{c0} \rightarrow gg)$, we form the ratio $B_{g\gamma}(\chi_{c1}, \chi_{c2}) \equiv \Gamma(\chi_{c0} \rightarrow \gamma\gamma)/\Gamma(\chi_{c2} \rightarrow gg)$. Equation 1.16 still applies but we must employ the coefficient $D_{gg}(\chi_{c2}, \mu)$ in place of $D_{gg}(\chi_{c0}, \mu)$. This results in a two-photon width of $\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 1.9 \pm 0.2(\pm 21\%) \text{ keV}$. These estimates are tabulated in Table 9 under the BCGR column heading. Some previous two-photon width measurements are listed in Table 10 and Table 11 for comparison.

The second QCD approach, labeled BBL, is based on factorization theorems being developed by Bodwin, Braaten and Lepage. Through these theorems they
Table 9
Predicted $\Gamma_{\gamma\gamma}(R_{c\bar{c}})$.

<table>
<thead>
<tr>
<th>$R_{c\bar{c}}$ MeV</th>
<th>BCGR keV</th>
<th>BBL keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c$</td>
<td>10 ± 3.6$^{[37]}$</td>
<td>4.7 ± 1.6(±32%)</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>not used</td>
<td>1.9 ± 0.2(±21%)</td>
</tr>
<tr>
<td>$\chi_{c1}$</td>
<td>0.64 ± 0.10$^{[27]}$</td>
<td>—</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>1.71 ± 0.16$^{[27]}$</td>
<td>1.1 ± 0.1(±33%)</td>
</tr>
</tbody>
</table>

have predicted the following relationships between decay rates$^{[27]}$:

$$\frac{\Gamma(\chi_{c0} \rightarrow \gamma\gamma)}{\Gamma(\chi_{c2} \rightarrow gg) - \Gamma(\chi_{c1} \rightarrow gg)} \approx \frac{135}{8} e^4 \left( \frac{\alpha_{\text{qed}}}{\alpha_s(m_{c})} \right)^2, \quad (1.19)$$

$$\frac{\Gamma(\chi_{c2} \rightarrow \gamma\gamma)}{\Gamma(\chi_{c2} \rightarrow gg) - \Gamma(\chi_{c1} \rightarrow gg)} \approx \frac{9}{2} e^4 \left( \frac{\alpha_{\text{qed}}}{\alpha_s(m_{c})} \right)^2. \quad (1.20)$$

The hadron widths of $\Gamma(\chi_{c2} \rightarrow gg) = 1.71 ± 0.16$ and $\Gamma(\chi_{c1} \rightarrow gg) = 0.64 ± 0.10$ leads to the two-photon widths of $\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 3.1 ± 0.56(±36\%)$ and $\Gamma(\chi_{c2} \rightarrow \gamma\gamma) = 0.83 ± 0.15(±36\%)$. The first uncertainty is due to the uncertainties in the reported hadron widths, the second is the result of combining a 20% theoretical uncertainties due to relativistic effects and a 30% perturbative uncertainty. These results are listed in Table 9 under the BBL column heading.

Forming the ratio of the two-photon widths $\Gamma(\chi_{c0} \rightarrow \gamma\gamma)/\Gamma(\chi_{c2} \rightarrow \gamma\gamma)$ removes the light-hadron widths and their associated errors from the equation. The ratio of the BCGR equations gives,

$$\frac{\Gamma(\chi_{c0} \rightarrow \gamma\gamma)}{\Gamma(\chi_{c2} \rightarrow \gamma\gamma)} = \frac{15}{4} \cdot \frac{1 + D_{\gamma\gamma}(\chi_{c0}, \mu) \frac{\alpha_s}{\pi}}{1 + D_{\gamma\gamma}(\chi_{c2}, \mu) \frac{\alpha_s}{\pi}} = 6.8(±30\%). \quad (1.21)$$

The uncertainty in this ratio is the fractional change in its value due to the $D_{\gamma\gamma}$ terms added in quadrature.
Table 10
Measurements of $\Gamma_{\gamma\gamma}(\chi_{c0})$.

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>$\Gamma_{\gamma\gamma}(\eta_c)$ (MeV)</th>
<th>$\chi_{c0}$ Production</th>
<th>$\chi_{c0}$ Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal Ball$^{[37]}$</td>
<td>4.0 ± 2.8</td>
<td>$\psi' \rightarrow$ photons</td>
<td></td>
</tr>
<tr>
<td>BCGR$^{[26]}$</td>
<td>1.9 ± 0.2(±21%)</td>
<td></td>
<td>Theory</td>
</tr>
<tr>
<td>BBL$^{[27]}$</td>
<td>3.1 ± 0.56(±36%)</td>
<td></td>
<td>Theory</td>
</tr>
</tbody>
</table>

Table 11
Measurements of $\Gamma_{\gamma\gamma}(\chi_{c2})$.

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>$\Gamma_{\gamma\gamma}(\eta_c)$ (MeV)</th>
<th>$\chi_{c2}$ Production</th>
<th>$\chi_{c2}$ Decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal Ball$^{[48]}$</td>
<td>2.8 ± 2.0</td>
<td>$e^+e^- \rightarrow e^+e^-\chi_{c2}$</td>
<td></td>
</tr>
<tr>
<td>R704$^{[44]}$</td>
<td>2.9$^{+1.3}_{-1.0}$</td>
<td>$p\bar{p}$ annihilation</td>
<td>$\gamma\gamma$</td>
</tr>
<tr>
<td>TPC/2$\gamma$$^{[49]}$</td>
<td>3.4 ± 1.7</td>
<td>$e^+e^- \rightarrow e^+e^-\chi_{c2}$</td>
<td></td>
</tr>
<tr>
<td>CLEO1.5$^{[43]}$</td>
<td>&lt; 1.0 $CL = 95%$</td>
<td>$e^+e^- \rightarrow e^+e^-\chi_{c2}$</td>
<td></td>
</tr>
<tr>
<td>E760$^{[51]}$</td>
<td>0.321 ± 0.078</td>
<td>$p\bar{p}$ annihilation</td>
<td></td>
</tr>
<tr>
<td>CLEOII$^{[50]}$</td>
<td>1.08 ± 0.30</td>
<td>$e^+e^- \rightarrow e^+e^-\chi_{c2}$</td>
<td></td>
</tr>
<tr>
<td>BCGR$^{[26]}$</td>
<td>1.1 ± 0.1(±33%)</td>
<td></td>
<td>Theory</td>
</tr>
<tr>
<td>BBL$^{[27]}$</td>
<td>0.83 ± 0.16(±36%)</td>
<td></td>
<td>Theory</td>
</tr>
</tbody>
</table>

The BBL equations produce a ratio of,

$$\frac{\Gamma(\chi_{c0} \rightarrow \gamma\gamma)}{\Gamma(\chi_{c2} \rightarrow \gamma\gamma)} = \frac{15}{4} = 3.8(\pm 36\%).$$  \hspace{1cm} (1.22)

The uncertainty is the result of combining a 20% theoretical uncertainties due to relativistic effects and a 30% perturbative uncertainty.
CHAPTER II

The CLEO-II Experiment

2.1 The Cornell Electron Storage Ring

The beams which generate the two-photon interactions studied in this dissertation were provided by the Cornell Electron Storage Ring (CESR), located at Cornell University, Ithaca, New York. CESR is an electron-positron collider designed to operate with center-of-mass energies between 9.0 and 12.0 GeV/c\(^2\). The data used in this dissertation were taken by the CLEO-II detector in seven running periods between November 1990 and September 1993. A total integrated luminosity of 3.00 ± 0.03 fb\(^{-1}\)\(^{[38]}\) at center-of-mass energies of approximately 10.5 GeV/c\(^2\) was accumulated.

The position of the CLEO-II detector in CESR is shown in Figure 11 along with the CHESS (Cornell High Energy Synchrotron Source) facility which uses the intense synchrotron radiation produced by CESR for separate studies.

2.2 The CLEO-II Detector

The CLEO-II detector\(^{[46]}\) is a large-volume, general-purpose solenoidal detector, illustrated in Figures 12 and 13. An expanded view of the detector inside
the magnet coil is illustrated in Figures 14. CLEO-II was designed to provide measurements of charged particle momenta and electro-magnetic energies with excellent resolution and efficiency, as well as, providing good particle identification.

The following sections describe each component of the detector starting nearest the interaction point and expanding radially outward.
Figure 12
Cross-sectional side-view of CLEO-II detector elements
Figure 13
End view of CLEO-II detector
2.2.1 The CLEO-II Beryllium Beam Pipe

The beryllium beam pipe provides a strong, lightweight window between the vacuum of the interaction region of CESR and the CLEO-II detector. The beryllium section of the beam pipe is 33cm long with a radius of 3.5cm, subtending a solid angle of 98% of $4\pi$. The wall thickness is 0.5mm including a 25$\mu$m layer of silver and a 0.05$\mu$m layer of nickel deposited on the inner surface of the pipe. This material presents a total radiation length of 0.44% for normally incident particles.

To reduce the amount of synchrotron radiation reaching the detector, stainless-steel radiation mask are mounted, inside the beam pipe, to either side of the interaction point at a distance of 35cm. Further suppression is accomplished by
the layer of silver on the inner surface of the beam pipe. The number of photons passing through a layer of silver with thickness $t$ can be written as a function of the photon energy ($\nu$), and the incident angle ($\theta$) as;

$$N(\nu) = N_0(\nu) e^{-\frac{t_{\text{eff}}}{\lambda_s(\nu)}},$$

(2.1)

where $\lambda_s(\nu)$ is the photo-absorption length in silver and $t_{\text{eff}} = t \cdot \csc(\theta)$ is the effective thickness of the silver layer for a photon with incident angle $\theta$. The maximum photo-absorption length in silver is 0.11mm at the minimum of the absorption cross-section, just below the K-edge at 20 keV. The majority of synchrotron radiation reaching the beam pipe has an incident angle, at the beryllium window, of less than 10mrad. Therefore, the effective thickness ($t_{\text{eff}}$) of the silver layer for most of the synchrotron radiation satisfies $t_{\text{eff}} \geq 2.5\text{mm}$ so that,

$$\frac{t_{\text{eff}}}{\lambda_s(\nu)} \geq \frac{2.5}{0.11}.$$  (2.2)

The number of photons entering the detector satisfies the relation $N \leq N_0 e^{-\frac{t_{\text{eff}}}{\lambda_s(\nu)}} \sim N_0 \times 10^{-10}$.

### 2.2.2 The Central Tracking Detector

The Central tracking detector for CLEO-II consists of three concentric cylindrical drift chambers. From smallest to largest radii these are; the precision tracking layer detector, the vertex detector and the main drift chamber. They provide momentum and position measurements, as well as energy loss measurements used for charged particle identification.

**The Precision Tracking Layer Detector**

The Precision Tracking Layer (PTL) detector$^{[12]}$ is a high resolution tube-style cylindrical drift chamber mounted directly onto the beryllium beam pipe. By
providing high resolution tracking measurements near the interaction region, the PTL improves the overall charged particle tracking, charged particle momentum resolution and localization of primary and secondary vertices. This information helps to discriminate against unwanted backgrounds, such as beam-wall collisions, beam-gas collisions and cosmic ray events, while increasing acceptance of low momentum tracks and tracks which have small angles relative to the beam line.

The dimensions of the PTL are listed in Table 12. The active region consists of 384 tubular drift cells. Each drift cell is constructed of an aluminized polycarbonate tube, serving as a cathode, with a $15\mu$m gold plated tungsten sense wire strung down its center. The potential difference between sense wire and field tube is approximately 2000 VDC. The PTL contains an equal mixture of Argon/Ethane (Ar/C$_2$H$_6$) gas maintained just above atmospheric pressure. A charged track traveling through this gas liberates electrons, thus producing ions. The liberated electrons accelerate toward the sense wire while the ions drift toward the field tube. The electrons gathered by the sense wire produces an electronic pulse. The pulse height and timing, relative to the beam crossing, provides a measure of the minimum separation between the particles path and the sense wire.

The drift cells are arranged in six concentric layers with 64 cells forming each layer. The polycarbonate tubes are 45cm long and have radii ranging from 2.32mm for the inner layer increasing to 3.55mm for the outer layer. The PTL's active region subtends a solid angle of 96% of $4\pi$ and presents 0.32% of a radiation length of material to particles which traverse it normally. Adjacent layers in the PTL are rotated by a half-cell in order to reduce left-right ambiguities in particle tracking. The resulting cell pattern is illustrated in Figure 15.

Using the half-cell staggered properties of the PTL, the *single hit resolution*
for a cell in the \(i^{\text{th}}\)-layer is determined from the drift sum;

\[
ds_{\text{sum}}_i = d_i + 1/2((1 + \alpha)d_{i+1} + (1 - \alpha)d_{i-1})
\]

where \(d_i\) is the drift distance in the \(i^{\text{th}}\) layer and \(\alpha\) is a correction from the expanding-cell geometry of the PTL. The internal spatial resolution for the \(i^{\text{th}}\) cell is the RMS width of \(d_{\text{sum}}_i\). The internal spatial resolution as a function of \(d_i\) determined from Bhabha events is shown in Figure 16. The spatial resolution averaged over the whole drift cell, is 110 \(\mu\)m using Ar/C\(_2\)H\(_6\). The internal spatial resolution of the PTL was improved to 55\(\mu\)m, in February 1992, when the gas was changed to Dimethyl Ether (DME).

The spatial resolution of the PTL, as a function of drift distance, can be
PTL internal spatial resolution with Argon-Ethane compared to internal spatial resolution with DME.

determined by reconstructing a particle's path using all three tracking chambers and plotting the difference between the projected drift distance and the measured distance. The spatial resolution found in this way can depend on errors introduced by the track reconstruction algorithm (DUET). One such error is the misalignment of detector elements. The PTL spatial resolution, using DUET, for continuum events (events with center of mass energies between the $\Upsilon(3S')$ and $\Upsilon(4S')$ resonances) is shown in Figure 21. Consistent with the drift sum method, DME improves the spatial resolution by a factor of 2.

The efficiency of a cell is defined as the ratio of the number of "confirmed hits" in the cell to the number of "predicted hits" in the cell. We discuss two cell hit efficiencies; raw hit efficiency and DUET hit efficiency. In both cases a predicted hit occurs whenever a reconstructed track passes through the cell. The definition of a confirmed hit depends on the type of efficiency being considered.
For raw hit efficiency, a confirmed hit occurs whenever the cell is predicted to be hit and the pulse height for the cell is read out by the online system. For DUET hit efficiency, a confirmed hit occurs whenever a predicted hit is accepted, by the track reconstruction algorithm, as valid. A valid DUET hit is required to have a drift time within a given range. The range is determined from the distribution of hit residuals which can be considered as the result of two types of hits. The first are clean hits where the electronics has had time to recover from a preceding hit and negligible noise is present. In this case the residuals form an approximate Gaussian distribution. The second case is when the electronics has not recovered completely from a recent hit or is masked by noise or multiple hits. These hits tend to be scattered about the distribution. Approximately 20% to 30% of confirmed raw hits fall into this category. Since each device has a different resolution, DUET's acceptance range is device dependent. The drift time range corresponds to approximately $3\sigma$ of the device's track residuals. For the PTL this is approximately 150$\mu$m from the predicted path.

The raw hit efficiency (DUET hit efficiency) for particles with momenta normal to the beam direction, averaged over all PTL cells, is 88%(62%) using Ar/C$_2$H$_6$ compared to 90%(75%) using DME.

The Vertex Detector

The Vertex Detector[13](VD), shown in Figure 17 is a ten layer precision drift chamber. It was originally installed in 1984 as the innermost detector element of the CLEO-I detector. Owing to its reliability, tracking performance and cost effectiveness, it has become part of the CLEO-II detector. The VD extends from a radius of 8.1cm to 16.2cm with an overall length of 90.0cm and an active length of 72.2cm. The VD provides charged particle tracking through the use of 800 sense wires and 2272 field wires arranged in hexagonal cells forming 10
Figure 17
VD cut away view showing cathode pads

cylindrical layers and 1,280 cathode strip segments on the inner and outer VD liner. As illustrated in Figure 18 each sense wire marks the center of a hexagonal cell formed by six field-shaping wires; one at each corner of the hexagon. The hexagonal shape provides close packed cells. It also provides a symmetric electric field, reducing problems with the electron drift path produced by the CLEO-II magnet. The inner 5 VD-layers contain 64 cells each, while the outer 5 VD-layers have 96 cells each. Adjacent layers are rotated by a half-cell in order to reduce left-right ambiguities in particle tracking. The $r\phi$ RMS internal resolution for the VD is approx 100$\mu$m while the DUET resolution, shown in Figure 21, is slightly higher.
The sense wires are 20μm nickel-chromium, with a resistivity of 1800Ω/m. Z-positions are determined by comparing the charge flow to each end of the wires, a procedure known as charge division. Cathode strip segments etched into the inner and outer VD liner supply further Z-position measurements. The inner cathode is segmented into 64 conducting rings along the length of the VD, the outer cathode has 96 such rings. The rings are further divided into eight azimuthal sections to reduce confusion in cathode signal to sense wire signal correlations. The spatial resolution for each cathode is approximately 1200μm, with an efficiency of 83% per track for the inner cathode and 78% efficiency per track for the outer cathode. The VD's active region subtends a solid angle of 91% of 4π and presents 0.90% of a radiation length of material to particles which traverse it normally.
The CLEO-II Outer Drift Chamber

The outer Drift Chamber[14](DR), illustrated in Figure 19, is the largest charged particle tracking chamber in the CLEO-II detector. Together with the PTL and VD, it provides measurements of charged particle momentum vectors at the vertex, as well as, providing specific ionization energy loss (dE/dx) measurements used in particle identification. The DR covers the radial region from 17.8cm to 94.7cm, and has an active length of 189cm. The DR's active region subtends a solid angle of 71% of 4π. Filling this volume are 51 layers of rectangular cells formed by field wires with a sense wire marking the center of each cell. The cell pattern is shown in Figure 20. The DR contains a total of 12,240 sense wires and 36,240 field wires. There are two types of cells found in the DR, axial and stereo. Each cell measures 14mm by 14mm and are formed by 8 field
Figure 20
Wire position in the Central Drift Chamber
wires composed of either 110 micron copper-beryllium wire or 110 micron 5056-aluminum wire, each are plated with a 1.5 micron layer of gold. The aluminum is used in the first 40 layers to reduce multiple scattering. The sense wire is a 20 micron diameter gold-plated tungsten wire. Each layer is staggered by a half-cell to reduce left-right ambiguities. To provide Z-position measurements stereo layers have slanted wires with respect to the axial layers. The stereo layers slant angles alternate sign from layer to layer. The angles magnitude increase from 1.91 deg for the inner most stereo layer to 3.50 deg for the outer most layer. There are 40 axial layers of sense wires and 11 stereo layers. In the first 40 layers every fourth layer is a stereo layer, the outer 11 layers have stereo layers for every fifth layer.

Operating with a low gas gain of approximately $10^4$, the DR supplies good pulse height and dE/dx measurements used for particle identification. The dE/dx measurements as a function of particle momentum is shown in Figure 22.

The DR spatial resolution, using DUET, for continuum events is shown in Figure 21. The spatial resolution averaged over an axial cell is 190$\mu$m and for the stereo cell is 230$\mu$m. The DUET hit efficiency for particles with momenta normal to the beam direction, averaged over all DR cells, is 80%.

For further Z-position measurements the DR has segmented cathode strips at the inner and outer walls, about 0.7cm from the sense wires. With segmentation along the beam direction of 1cm, a hit on a neighboring sense wire produces an image charge covering 3 cathode segments. The spatial resolution of the DR cathodes is approximately 1800$\mu$m, with an efficiency of 76% per track for the inner cathode and 67% efficiency per track for the outer cathode.
<table>
<thead>
<tr>
<th>Physical Region(cm):</th>
<th>Beam Pipe (cm)</th>
<th>Tracking Device (stereo)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Radius</td>
<td>3.500</td>
<td>4.1</td>
</tr>
<tr>
<td>Outer Radius</td>
<td>3.552</td>
<td>7.7</td>
</tr>
<tr>
<td>Length</td>
<td>50.0</td>
<td>98.0</td>
</tr>
</tbody>
</table>

| Active Region(cm):              |               |                         |
| Inner Radius                    | 4.5           | 8.1                     |
| Outer Radius                    | 7.6           | 16.4                    |
| Length                          | 33.0          | 50.0                    |

| Number of Layers                 |               |                         |
| Solid Angle (% of 4π)            | 98            | 96                      |
| Material to Layer 1 (% of R.L.)  | 0.46          | 1.38                    |

| Resolution (μm)                  |               |                         |
| rφ                               | 110           | 100                     |
|                                  | (55)          | (230)                   |
| Inner Cathode                    | -             | 1200                    |
| Outer Cathode                    | -             | 1200                    |

| Hit Efficiency (%)               |               |                         |
| rφ                               | 88            | 93                      |
|                                  | (90)          | 80                      |
| Inner Cathode                    | -             | 78                      |
| Outer Cathode                    | -             | 83                      |

| DUET Efficiency (%)              |               |                         |
| rφ                               | 62            | 80                      |
|                                  | (75)          | 80                      |
Track Reconstruction

The goal of track reconstruction is to determine, as best as possible, the path of all ionizing particles in an event. From the curvature of the particle’s track in the magnetic field its momentum can be determined. CLEO-II uses two track reconstruction algorithms; TRIO and DUET.

TRIO is a relatively simple and fast track finding algorithm. The TRIO algorithm first searches for track-lets which are sets of three hits, one hit per layer in three consecutive layers of the tracking chambers. The alignment of track-let hits in the $r\phi$ plane must be consistent with a track originating near the interaction point. In the six layers of the PTL, track-lets can share one or two common hits, which results in some overlapping of track-lets. The VD is divided into an inner 5-layer section and an outer 5-layer section. Track-lets can share...
Drift chamber dE/dx shown as a function of momentum. The solid lines are the expected distributions for electrons, pions, kaons, protons and deuterons.

hits within a VD section. Hits forming a track-let are contained in either the inner section or the outer section but not both. A straight track with a single hit in each layer of the VD would produce 3 track-lets in each section. The first 40 layers of the DR form sets of three axial layers separated by a stereo layer. Hits forming a DR-track-let are contained within three consecutive axial layers.

A candidate track is formed from an inner most and an outer most track-let plus any other track-lets near the candidate track. The resulting set of hits are fit and accepted as a valid track if the $\chi^2$ is sufficiently small. TRIO provides tracking information for online diagnostics of the detector components. The quickness of TRIO track reconstruction is balanced against a decrease in track finding efficiency. Tracks that curl heavily in the detector are not found since
they require complicated and time consuming calculations. Wire inefficiencies, especially in the DR, can result in large decreases in the number of reconstructed tracks.

The DUET track finding algorithm processes data off-line where time can be allotted for more efficient and elaborate procedures. Some time is saved by using TRIO tracks, when available, as seed tracks for DUET. Hits near the path of the seed track are used to form a chain of hits. The chain is fit to a circle in the $r\phi$ plane and a straight line in the $rz$ plane. Chains with a small $\chi^2$ are then refit to a helix in three dimensions and saved as a valid track. With the remaining hits DUET forms a track by building a chain from track segments. The smallest segment is a link consisting of two adjacent hits. Larger track segments are created by connecting multiple links until a chain has been built. The chain is fit to a circle in the $r\phi$ plane and a straight line in the $rz$ plane. Chains with a small $\chi^2$ are then refit to a helix in three dimensions and saved as a valid track. The hit residual distribution for $\mu^+\mu^-$ events are shown in Figure 23 and the momentum distribution shown in Figure 24. An RMS momentum resolution of 61MeV/c for 5GeV/c tracks is observed.

### 2.2.3 Time-Of-Flight

The time-of-flight (TOF) system provides information used for particle identification, as well as being an integral part of the first level trigger. The TOF detector is divided into two sections, the barrel TOF and the endcap TOF, together covering 96% of the solid angle.

The barrel TOF is constructed of 64 scintillator counters a cylinder between the outside of the central drift chamber and the inside of the crystal container. One scintillator counter is shown in figure 25. The scintillation material is a 5cm thick, 10cm x 279cm slab of Bicron BC-408 plastic. The scintillation material
Figure 23
DUET hit residuals of muons from reaction $e^+e^- \rightarrow \mu\mu$ at 5.290 GeV beam energy.

Figure 24
Momentum resolution of muons from reaction $e^+e^- \rightarrow \mu\mu$ at 5.290 GeV beam energy.
Figure 25
Time-of-flight scintillators used in barrel region

Figure 26
Time-of-flight $\beta$ vs momentum
presents a radiation length of 12% in the normal direction. A photo-multiplier tube* is connected to each end of the scintillator by a lucite light pipe. The light pipe allows the photo-multiplier tubes to be located outside the return flux of the magnet where the magnetic field is about 100 gauss. The barrel TOF covers the polar angles from 36 deg to 144 deg. It has an RMS resolution of about 160ps per counter averaged over all particle types. A value for the particles mass can be determined from the measured flight time along with the tracking chamber momentum measurements. The separation of hadrons is illustrated in figure 26 where \( \frac{1}{\beta} \) determined from the barrel TOF is plotted as a function of track momentum.

The endcap TOF is located at a Z-distance of 117.5cm between the drift chamber endplates and the endcap calorimeter. It covers a radial distance from 25.9cm to 89.0cm, shown in figure 27. The solid angle coverage is 16% of \( 4\pi \). The endcap TOF is constructed of 28 wedge-shaped pieces of Bicron BC-408 plastic, shown in figure 28. Each wedge is 5cm thick and 58cm long and presents a radiation length of 12% in the normal direction. The photo-multiplier tubes are glued directly to the scintillator with the tube axis at a right angle to the length of the scintillator. The tubes operate inside the detector’s 1.5 Tesla magnetic field. The tubes† are designed to operate in a constant field of several Tesla directed along the tube axis. The endcap TOF has an RMS timing resolution of about 170ps averaged over all wedges.

2.2.4 The CLEO-II Electro-magnetic Calorimeter

The CLEO-II Electro-magnetic Calorimeter\cite{15,16} is a highly-segmented, large-volume shower detector system. It is composed of 7800 thallium doped cesium

*The photo-multiplier tube used for the barrel TOF system are Amperex XP2020.
†The photo-multiplier tubes used for the endcap TOF are Hamamatsu R2490.
Figure 27
Endview of TOF system. The heavier lines show the divisions of the TOF trigger.

Figure 28
Wedge-shaped pieces for the endcap TOF.
iodide (CsI) crystals grouped into three large sections. These are the barrel section and two endcap sections which cover a total of 95% of the solid angle. The 6144 barrel crystals are approximately $5\text{cm} \times 5\text{cm} \times 30\text{cm}$ with some variations to facilitate positioning. The barrel crystals are positioned so that the gaps between crystals point slightly away from the interaction point. This reduces losses due to photons escaping through gaps between crystals while maintaining the full 30cm (16 radiation lengths) of crystal depth for photons originating from the interaction point. The construction material around the crystals is kept at a minimum to alleviate degradation of the energy resolution. Crystals are stacked in a barrel holder constructed entirely of aluminum and providing a sealed, dry environment. A picture of the barrel holder during installation of the crystals is shown in figure 29. Figure 30 shows the components of the barrel holder. The barrel holder is 3.37m long and has an inner radius of 1.024m with a wall thickness of 1.59mm. The outer wall is at a radius of 1.425m with a thickness of 6.35mm. The ends of the barrel holder are capped with a cover annulus. Slots in the annulus allow cables and cooling tubes for the electronics to pass through the barrel holder walls. An internal lattice of 13 lateral rows of fins and 64 longitudinal fins provide compartments which hold a 2 by 4 array of snugly packed crystals in place. The fins have a thickness of 0.51mm. The fins along with mounting struts also supply structural support and rigidity to the barrel holder. The crystals are stacked in a Roman Arch configuration to distribute the weight throughout the barrel which allows the structure to withstand the 27,000 kg load of the 6144 crystals.

The endcap holder is also constructed entirely of aluminum. It is a cylindrical structure designed to hold 828 crystals in a four-fold symmetric pattern as shown in Figure 31. All endcap crystals have the same rectangular dimensions ($5\text{cm} \times 5\text{cm} \times 30\text{cm}$) with the exception of a few near the inner and outer walls, which have had edges cut away to facilitate stacking. Endcap crystals are stacked with
Figure 29
Picture of the barrel crystal holder in the process of being filled with crystals.

Figure 30
Barrel crystal holder.
their axis parallel with the beam axis. The inner cylindrical wall is at a radius of 0.3206m and has a thickness of 6.35mm. The outer wall has a radius of 0.9144m and a thickness of 9.53mm. The front cover plate is located 1.248m from the interaction point and has a thickness of 6.35mm.

Crystal Electronics

Due to the inaccessibility of the calorimeter, a four-fold redundancy was constructed for the electronics associated with each crystal. Each crystal is equipped with four photo diodes. The four diodes are mounted on an UVT lucite window and the window glued to the back of the crystal. Each diode is connected to a separate preamplifier mounted just behind the crystal. The output from each
preamplifier is connected to a mixer/shaper (m/s) circuit by twisted-pair cables with a length of about 30 ft. The m/s cards are located in rakes outside the detector with one card handling signals from 16 crystals. The m/s first sums the four diode signals. In case of a diode or preamplifier failure, the preamplifier channel can be switched off by computer control which automatically corrects the gain. The signal is then shaped, by a pole-zero filter, to eliminate the long tail of the preamplifier pulse. The m/s output pulse is digitized by an ADC and readout by the data acquisition system. Each card also differentiates and sums together all 64 signals for trigger purposes. To produce a uniform trigger throughout the calorimeter, variations between channels must be small. To account for any crystal-to-crystal variations in light output, a computer controlled fine gain compensator can increase the gain of individual channels, up to a factor of 2, with about 5% accuracy.

Shower Reconstruction

When a photon, electron or other particle interacts in the calorimeter an energy shower is produced in a number of crystals. Of the 7800 crystals, only a small fraction in any event contain energy deposits. The information stored as data consists of each crystals' portion of the energy plus noise. Shower reconstruction is an effort to determine the original photon energy and direction from these crystals.

In order to avoid reading and recording crystal information which is primarily noise, a system for identifying useful crystal information at the hardware level has been developed. The hardware module SPARXL performs this sparcification of crystal information. The first step in this process is to search for all "seed" crystals. A seed crystal is any crystal having ADC values of at least 25 counts. The LeCroy Fastbus Model 1885N, having 96 channels per card, is used to convert the analog signal from the m/s circuits to the digital signal.
above pedestal, which corresponds to about 5 MeV of energy. Units are expressed in either energy or ADC counts with a conversion factor of 0.2 MeV per ADC count. A typical pedestal is 480 counts with a width of about 3 counts. The 5 by 5 array of crystals surrounding the seed are then searched for ADC values of at least 2 counts (~0.4 MeV) above pedestal. The information from these crystals is then written to data tape.

The second step in shower reconstruction is the software process of shower cluster finding. A cluster is a group of \( n \) neighboring crystals with at least one member having more than 10 MeV of energy. The cluster member having the largest energy is called the seed crystal. All members of the cluster are contained within the 5 by 5 array of crystals physically surrounding the seed crystal. Of these only the most energetic \( n \) crystals are used to determine the energy and position of the cluster. The number of crystals to be used, varies logarithmically with the total cluster energy from 4.0 at 25 MeV (and below) to 17.0 at 4 GeV (and above). The fractional part of \( n \) represents the fractional part of the next highest energy to be used in energy and position determinations. The value of \( n \) is chosen to minimize the energy resolution which would otherwise be further degraded by noise.

The position vector for a cluster is the energy-weighted sum of the coordinates of each crystal’s geometric center. The weight factor is the ratio of the crystal energy to the cluster energy. Small energy-dependent corrections are made in the longitudinal and lateral positions. The longitudinal correction is along the length of the crystal and the lateral correction is in the plane perpendicular to the particle position vector.
Calibration and Performance

Once the calorimeter was installed into the CLEO-II detector, access to the crystals, diodes and preamplifiers became impossible without disassembling most of the detector. To insure a reliable system, extensive tests were performed on the crystals and electronics at each stage of construction. The last of these tests, before loading the crystals into the crystal holder was a cosmic ray test. The cosmic ray test checked that all diodes were working properly and that each crystal was producing a significant signal. The cosmic ray test is further described in Appendix A. As the crystals were loaded into the crystal holder a second cosmic ray calibration plus a light pulse calibration were performed to verify connections and components within the calorimeter.

To monitor electronic gain and linearity an electronic pulser system is used to inject charge at each channel's input. A short electronic calibration, performed approximately every two hours, verifies proper operation of the electronics. An extensive electronic calibration, performed weekly, determines the constants (electronic gains and slopes) to be used in processing the data.

The light-sensitive gain of crystals and photo-diodes is monitored by a light flasher system. Two Xenon flash tubes each pulse 3900 crystals through a network of optical fibers. Each tube produces light pulses in six bundles of 156 1mm diameter PCS (plastic clad silica) fibers. In turn each 1mm fiber is connected to a bundle of 50 200\(\mu\)m diameter PCS fibers. The 200\(\mu\)m diameter fiber is connected at the back of a crystal through the photo-diode mounting board. Light pulses are reflected by the crystal surfaces and collected by the photo-diode.

Fluctuations in the intensity of light pulses prohibit monitoring small changes in the absolute gain of each crystal. Instead, comparisons of gain changes for “neighboring crystals” are performed, where neighboring crystals are those flashed
Table 13
Calorimeter photon energy calibration methods in the barrel region.

<table>
<thead>
<tr>
<th>Calibration method</th>
<th>Photon energy range</th>
<th>Accuracy of calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma\gamma$</td>
<td>$\gamma = \text{beam energy}$</td>
<td>$\pm 0.2%$</td>
</tr>
<tr>
<td>$\gamma\gamma\gamma$</td>
<td>$700\text{MeV} &lt; \gamma$</td>
<td>$\pm 0.5%$</td>
</tr>
<tr>
<td>$e^+e^-\gamma$</td>
<td>$500\text{MeV} &lt; \gamma$</td>
<td>$\pm 0.5%$</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>$\gamma &lt; 2 \text{GeV}$</td>
<td>$\pm 0.5%$</td>
</tr>
</tbody>
</table>

by the same bundle of 200$\mu$m fibers. The fluctuation of pulse heights within a bundle is typically 1%. For a measurement using 25 pulses, the statistical accuracy is 0.2% for each crystal. A failed diode is indicated when the flasher calibration for that crystal changes by more then 10% while the electronic calibration shows little or no change.

Absolute energy normalization for photon clusters require several methods to cover the entire energy range. Table 13 lists these methods, their associated photon energy range and the accuracy of the calibration. Photon pairs from $\pi^0$ decays, constrained to the $\pi^0$ mass, provide a calibration for photon energies below 2 GeV to about $\pm 0.5\%$. Photons from radiative Bhabha events provide a calibration for photon energies above 500 MeV to about $\pm 0.5\%$. Photons from radiative $\gamma$ ($e^+e^- \rightarrow \gamma\gamma\gamma$) events provide a calibration for photon energies above 700 MeV to about 0.5%. Back to back photons ($e^+e^- \rightarrow \gamma\gamma$) provide a calibration for photons at the beam energy to about 0.2%.

Using Monte Carlo simulation of showering in the CLEO-II detector the photon energy resolution in the barrel/endcap is 1.5%(2.6%) at 5 GeV and 3.8%(5.0%) at 100 MeV. The barrel/endcap angular resolution in azimuth is 3 mr(9 mr) at 5.0 GeV and 11 mr(19 mr) at 100 MeV. From Monte Carlo the energy resolution and angular resolution can be parameterized as shown in Table 14. These values agree with CLEO-II data.\textsuperscript{[45]}
Figure 32
Invariant mass of pairs of calorimeter showers unmatched to tracks in four $\gamma\gamma$ ranges. (Only events with four charged tracks are used).
Table 14
Parameterization of calorimeter photon energy resolution and angular resolution determined from Monte Carlo. (E in units of GeV.)

<table>
<thead>
<tr>
<th>Calorimeter section</th>
<th>$\sigma_E/E$ (%$)$</th>
<th>$\sigma_\phi$ (mr)</th>
<th>$\sigma_\theta$ (mr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrel</td>
<td>$0.35E^{-0.75} + 1.9 - 0.1E$</td>
<td>$2.8E^{-0.5} + 1.9$</td>
<td>$0.8\sigma_\phi\sin(\theta)$</td>
</tr>
<tr>
<td>Endcap</td>
<td>$0.26E^{-1.0} + 2.5$</td>
<td>$3.7E^{-0.5} + 7.3$</td>
<td>$1.4E^{-0.5} + 5.6$</td>
</tr>
</tbody>
</table>

After calibration, the mass of pairs of calorimeter showers unmatched to charged tracks is shown in figure 32. The number of charged tracks in these events is restricted to four. The four plots correspond to the four $\gamma\gamma$ momentum bins $0 < P_{\gamma\gamma} < 0.5$, $0.5 < P_{\gamma\gamma} < 1.0$, $1.0 < P_{\gamma\gamma} < 2.0$ and $2.0 < P_{\gamma\gamma}$. The $\pi^0$ signal is clearly seen in all four plots and the $\eta$ signal appears for $P_{\gamma\gamma}$ above 1 GeV. The RMS width of the $\pi^0$ peak is approximately 5 MeV.

Electron rejection can be accomplished using the momentum from the tracking chambers along with the calorimeter energy. Figure 33 shows the ratio of shower energy to track momentum ($E/p$) for tracks from events with a 1-vs-3 topology. The sample is dominated by $\tau$-pairs, with the 1-prong side being an electron or muon and the 3-prong side consisting of hadrons. A peak near unity appears in the 1-prong side as expected for electrons while the 3-prong side shows no peak near unity.

Data acquired with a random trigger under normal colliding beam conditions shows evidence for occasional beam associated showers. Typically, for every 10 random events one shower is found in each of the polar angular ranges $0.87 < |\cos\theta| < 0.94$, $0.71 < |\cos\theta| < 0.87$, and $|\cos\theta| < 0.71$. The shower energy distributions for the barrel region are shown in figure 34. Approximately 1/5 of these showers have energy above 50 MeV, and 1/10 above 100 MeV. At low angles ($0.93 < |\cos\theta|$) approximately 1/2 of the random trigger events produce showers in the endcaps with an energy distribution similar to that of the barrel.
Figure 33
The ratio of matched shower energy to momentum (E/p) for tracks in events with a 1-vs-3 charged-track topology. The 1-prong tracks are normalized to the same number of entries as the 3-prong tracks. Only one entry is used per track. Since a track can match to multiple showers, the ratio nearest to 1.0 is taken as the E/p ratio for that track.

region. The rate of excess showers depends strongly on CESR beam conditions.

2.2.5 CLEO-II Super Conducting Magnet

The CLEO-II magnet,\cite{45} shown in Figure 35, is a super-conducting solenoid magnet. The coil has a length of 3.5m and a clear bore diameter of 2.9m. The bore size allows for the tracking drift chambers, the time-of-flight and crystal calorimeter all to be contained within the coil. An operating current of 3300 amperes maintains a constant 1.5 Tesla magnetic field. During a typical run of 1 hr the monitored field strengths are stable to about $10^{-4}$ Tesla. The field is uniform to $\pm 0.2\%$ over 95\% of the solid angle in the drift chamber.
Figure 34
Distribution of largest shower energy per event in the barrel from random trigger run under normal colliding beam conditions. The bin with 0 to 10 MeV energy, which contains 90% of all events, is suppressed because a "shower cluster" must contain at least one crystal with energy greater than 10 MeV.

The steel yoke, shown in figure 13, has an octagonal symmetry with each octant consisting of three layers of 36cm thick steel separated by 9cm gaps for the muon detectors. A forth 2.5cm thick sheet of steel covers the outermost muon chamber, providing protection and radiation shielding. The inner two steel slabs provide nearly all the magnetic flux return.

2.2.6 Muon Chambers

The yoke of the CLEO-II magnet also serves as a filter for the muon detector system. Each octant of the yoke consists of three layers of 36cm thick steel
Figure 35
CLEO-II Magnet
sheets separated by 9cm gaps where the muon detectors reside. Each layer of steel present a minimum of 2.2 nuclear absorption lengths (16.7cm = 1 nuclear absorption length in iron). The muon detector units in each octant of the yoke are labeled “Return”, “Inner”, and “Outer” for the inner, middle, and outer gaps. Also, each end of the detector is covered the “Endcap” units.

The muon detector is constructed using proportional counters with dimensions of about 5m long and 8.3cm wide. The counters, shown in Figure 36, are constructed from 8 plastic tubes having a rectangular bore approximately 9mm by 9mm. Three inner sides of the tube are coated with graphite to form a cathode and electrically isolate the anode wires. The side of the tubes with no graphite has 8cm wide copper strips perpendicular to the wires providing Z measurements. The anode wire is a 50µm diameter silver plated Cu-Be wire which is positioned in the center of the tube. The anode wires are operated at a high voltage of 2500V. The gas for the tubes is the same 50/50 argon/ethane gas used for the drift chamber. The eight anodes of each counter are ganged together giving a spatial resolution of 2.4cm per counter. The copper strips give a spatial z resolution of 2.8cm to 5.5cm.

The set of counters in a steel gap taken together is called a unit. Each unit shown in Figure 37 consists of three staggered layers of 20, 25, 29 or 24 counters (see Table 15). Two layers provide a hit efficiency of 95%, the third layer provides redundancy in case of failure.

Each layer of counters is divided into multiplets of 10 to 15 counters while the copper strips are divided into four multiplets of either 9 or 15 strips each. Each member in a multiplet is separated from its neighbor a 100Ω resistor. Determining which counter or strip was hit is accomplished through charge division. Both ends of the multiplet strings are connected to a charge integrator. The position of a hit can then be determined from the ratio of the pulse heights read from the two
Figure 36
Cut away view of the muon chamber showing the tube structure

Figure 37
Super layer of the muon detector.
ends of the multiplet. This method greatly reduces the amount of electronics needed while still providing the needed spatial resolution.

In the barrel region, a minimum momentum of 0.8 GeV/c is needed for a muon to reach the first superlayer. Reaching the second superlayer requires a minimum of 1.4 GeV/c and the third superlayer requires a minimum momentum of 1.8 GeV/c.

The muon identification efficiency is defined as the ratio of the number of muons detected by the muon system to all muon tracks reconstructed in the central detector and calorimeter. The Monte Carlo simulation for the muon system was tuned according to measured unit efficiencies during cosmic ray tests. The identification efficiency of fast muons for Monte Carlo events and $e^+e^- \rightarrow \mu^+\mu^-$ CLEO-II events agree reasonably well. The Monte Carlo simulation of lower momenta muons is assumed to give a good estimate of the muon identification efficiency. Figure 38 shows the efficiency for muons from 1 vs 3 tau Monte Carlo with a depth greater than $3\lambda_i$.

The fake rate is defined as the ratio of number of hadrons misidentified as muons to all charged hadron tracks within the muon detector geometrical acceptance and in a given momentum bin. The fake rate due to charged pions, $F_\pi$, is

### Table 15
Muon chambers

<table>
<thead>
<tr>
<th>Location</th>
<th>Number of Units</th>
<th>Number of Layers</th>
<th>Multiplet Structure counters</th>
<th>strips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>8</td>
<td>3</td>
<td>$10 + 10$</td>
<td>$9 + 15 + 15 + 15$</td>
</tr>
<tr>
<td>Inner</td>
<td>8</td>
<td>3</td>
<td>$10 + 15$</td>
<td>$15 + 15 + 15 + 15$</td>
</tr>
<tr>
<td>Outer</td>
<td>8</td>
<td>3</td>
<td>$14 + 15$</td>
<td>$15 + 15 + 15 + 15$</td>
</tr>
<tr>
<td>Endcap</td>
<td>8</td>
<td>3</td>
<td>$10 + 14$</td>
<td>$9 + 15 + 15 + 15$</td>
</tr>
</tbody>
</table>
Muon identification efficiency when requiring a penetration depth greater than $3\lambda_t$. In order to identify a muon a corresponding DUET track exist.

determined using pions resulting from the decay $K^0_s\rightarrow \pi^+\pi^-$. Requiring the $K^0_s$ decay vertex to be at least 1cm from the beam line reduces non $K^0_s$ background to a negligible amount. Figure 39 shows the resulting fake rates due to pions.

The fake rate due to charged kaons, $F_k$, is determined using kaons resulting from the decay $D^{*+}\rightarrow D^0\pi^+\rightarrow (K^-\pi^+)\pi^+$. The results are shown in figure 39.

### 2.3 CLEO-II Trigger System

Currently CESR operates with 7 bunches of particles in each beam producing about 2.7 million beam crossings per second. The time required to read out the electronics for a single event restricts the acceptance rate to a few tens of events per second. The responsibility of the trigger system is to reduce these 2.7 million beam crossings to a few choice events. This is accomplished through a three-
Figure 39
The fraction of pions and kaons which are misidentified as a muon by the muon system.
Table 16
Input Devices to the Trigger

<table>
<thead>
<tr>
<th>Level</th>
<th>Vertex Detector</th>
<th>Drift Chamber</th>
<th>Time-of-Flight</th>
<th>Calorimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>L1</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>L2</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

tiered trigger\[19\] scheme. The tiers are denoted as level-zero (L0), level-one (L1) and level-two (L2). Information from the vertex detector, drift chamber, time-of-flight and crystal calorimeter is used to determine a trigger. Table 16 shows the break down of components used in each tier. The trigger logic is illustrated in figure 40.

To generate a trigger each detector component is searched for a crude hit-pattern specified in a trigger line. A trigger line is made up of three sets of requirements for hit-patterns, one set for each trigger level. At run time the trigger system is configured according to a trigger file consisting of up to 12 trigger lines. A valid trigger occurs when any L0 requirement is met and both L1 and L2 requirements in the same trigger line are satisfied.

L0 is designed to reduce the 2.7 MHz crossing frequency to a rate of about 20 kHz which can be handled by L1. In order to handle such a high input rate, L0 must be both quick and simple. Once any L0 trigger line is satisfied, all gates to the detector are disabled and L1 interrogation begins. If an L1 trigger line is satisfied then the L2 requirements for the same line are checked. If no L1 trigger line is satisfied all trigger logic is reset and gating resumes. L1 is designed to reduce the 20 kHz rate from L0 to about 25 Hz. In turn, L2 is designed to reduce the 25 Hz from L1 to less than 8 Hz. For acceptance rates above 8 Hz the system becomes completely limited by dead-time. For lines with high trigger rates, a sampling rate of up to a factor of 1 out of 255 can also be set through L1.
event fails the L2 requirement the trigger logic is reset and gating resumed.

2.3.1 Trigger System Inputs

As mentioned earlier, the trigger system searches the detector elements for crude hit patterns. The following describes these patterns for each detector element.

Time-Of-Flight Trigger

A crude time of flight system is created by logically grouping together four adjacent scintillators counters as shown in figure 27. A logical AND of the two
ends of a barrel counter are required to generate a hit. A response from any of
the four counters results in a "hit" for that section. The grouping of counters
produces 16 trigger sections for the barrel and seven trigger sections for each
endcap. Since the time-of-flight response time is fast, this information is used by
both L0 and L1.

CsI Crystal Trigger

The barrel calorimeter contains 6144 crystals arranged with 48 along the \( z \)
direction and 128 around the \( \phi \) direction. A crude barrel calorimeter is formed
by summing the signals of 16 crystals in a \( 4z \) by \( 4\phi \) array. This divides the barrel
into 384 bins arranged in a \( 12z \) by \( 32\phi \) array as shown in figure 41.

The summed signal from each bin is then read by a high level discriminator
and a low level discriminator. If the signal satisfies the high level threshold a
high trigger bit is set. This corresponds to a minimum energy of about 500 MeV.
The low level discriminator sets a corresponding low trigger bit when more then
about 100 MeV of energy is found. Since the timing for the high level and low
level measurements are different a high trigger bit can result even though a low
trigger bit has not been set. These trigger bits are available for software triggers
if desired. For the hardware triggers the 384 bins are grouped together in \( 6z \) by
\( 4\phi \) sections. All high(low) trigger bits in a section are OR'ed together to form
a hardware high(low) trigger bit. This divides the barrel into 16 sections, \( 2z \) by
\( 8\phi \), and produces 16 high and 16 low trigger bits.

The endcap follows a similar scheme. Each endcap contains 828 crystals which
are grouped into bins. However, these bins can contain from 6 to 16 crystals.
The crystal bins are shown in figure 42 High and low software trigger bits are
determined for each bin. Then the software trigger bits are OR'ed together to
form 8 high and 8 low hardware trigger bits for each endcap. Because the high
Figure 41
Barrel crystal bins for trigger.

Figure 42
Endcap crystal bins for trigger.
trigger bits can be determined faster than the low trigger bits, only the high bits are used in L0. Both high and low bits are available for L1.

**Tracking Chambers Trigger**

There are three tracking chamber trigger processors, the Track Segment Processors (TSP), the Binary Link Tracker\(^{20,19}\) (BLT) and the Precision tracking Device (PD). Each process performs crude track finding using latched discriminator outputs from the anode to indicate a wire hit. Due to the smaller cell size in the VD it develops signals faster than the DR. Therefore, the VD information is used for L0 decisions with the DR information added for L1 decisions.

The TSP uses all 10 layers of the VD. Each layer is sectioned into groups of 4 adjacent wires with their signals ECL OR'ed together. The resulting 200 TSP bits are then searched for patterns recognized as tracks.

The TSP uses only 12 layers of the DR. These are layers 2, 18, 34 and 49, plus the two adjacent layers to each of these. The wires in these layers are ganged together in units of 4, 8, 12 and 16 wires, respectively, and a TSP bit set for each. Figure 43 shows the ganging of wires for each layer. Each bit covers \(\pi/12\) of azimuth. This information along with the output from the L0 search of the VD and the 16 barrel TOF bits (see TOF trigger system) are used as input to the correlator. The correlator classifies tracks as short, medium and long. Short tracks are those which extend through layer 3 of the DR, Medium extend through layer 35 and long tracks extend to the TOF. The total number of short, medium and long tracks is determined and compared to the tracking trigger requirements defined in the trigger file. The TSP also searches for back-to-back tracks in order to identify Bhabha scattering and \(\mu\)-pair production.

The BLT uses only DR information. Because of the cell size this information
is not available for L0 decisions but is available for L1 decisions. The BLT uses DR wires in segments called "FASTOR" bits. It attempts to link together the FASTOR bits to form tracks originating from the interaction point. The BLT counts tracks reaching layers 17 and 45. The threshold transverse momentum ($P_t$) for a track to be counted at layer 17 is 90 MeV/c, while $P_t$ for layer 45 starts at 180 MeV/c.

Either the TSP or BTL or both can be used in L1. The combination of the two increases the hadronic trigger efficiency by 5 to 10% over the TSP alone.

The BLT also uses bit manipulation algorithms to identify tracks with $P_t$ greater than 340 MeV/c and also can determine the charge of a track in many cases. A persistent background of hadronic events comes from beam interactions.
with the beam pipe or residual gas of the vacuum. Since these interactions tend
to produce tracks with positive charge, the BLT can be used to identify many of
them.

The PD uses only VD information. Hit patterns are generated using informa-
tion from single cells rather then the usual ganging of cells. The pattern of hit
cells are searched for those consistent with tracks having a $P_t$ greater than 125
MeV/c and originating from the interaction point. By requiring the track to be
consistent with the interaction point the PD is able to identify many beam-wall
events as well as cosmic rays. The PD also discriminates against clusters of hits
in a given azimuthal region, which is another signature of beam-wall interactions.
Although the VD information is available at L0-time, the time required for the
PD to make a decision is over 16μsec. Because of the relatively long processing
time the PD is only used for L2 decisions.

2.4 Data Acquisition

An event in the CLEO-II detector produces many analog signals which must
be converted into digital signals before transfer to the computer system. Other
information must pass between the detector and computer system for control
and calibration purposes. These include high voltage commands, trigger require-
ments, electronic calibration pulses and alarm signals from monitoring devices.

The data acquisition system is illustrated in figure 44. The detector electronics
is organized into an array of data crates as indicated in table 17. Associated with
each analog crate is a controller containing a fast ADC and local memory for
storage of pedestals and time constants.

When the trigger system detects an event, an interrupt signal is sent to the
on-line computer and crate controller. All the analog crates begin digitization in
Table 17
CLEO-II readout crates

<table>
<thead>
<tr>
<th>Device</th>
<th>Type</th>
<th>Number crates</th>
<th>Ready Time (msec)</th>
<th>Sparsification</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR</td>
<td>analog</td>
<td>20</td>
<td>7.2</td>
<td>T.AND.Q</td>
</tr>
<tr>
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<td>4</td>
<td>7.0</td>
<td>T.AND.Q</td>
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<td>PTL</td>
<td>analog</td>
<td>1</td>
<td>4.6</td>
<td>T.AND.Q</td>
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<td>1</td>
<td>2.4</td>
<td>T1.OR.T2.OR.Q</td>
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<tr>
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<td>2.9</td>
<td>Q1.OR.Q2</td>
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<td>none</td>
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<tr>
<td>CAMAC</td>
<td>digital</td>
<td>1</td>
<td>0.4</td>
<td>none</td>
</tr>
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<td>Calorimeter</td>
<td>fastbus</td>
<td>1</td>
<td>8.0</td>
<td>SPARXL</td>
</tr>
</tbody>
</table>

The front-end data boards process the detector signals through time-to-voltage conversion and/or charge integration with their output stored on sample-and-hold circuits. Each channel is digitized in turn by the single 12 bit ADC on the crate controller with a conversion time of $3\mu s$ per channel. The maximum digitization time for a crate for each detector element is shown in Table 17. The controller determines which channels satisfy the pedestal and timing constants and stores the valid information in data buffers for readout by the computer system. Datum for the DR, VD and PTL consists of a cell ID, a time, and a charge. For this datum to be past to the data buffers the time must fall within a specified window relative to the event time and the charge must exceed a channel-specific threshold. The datum for a TOF channel consists of four components; an address, two times and a charge. For TOF datum to be saved one of the three variables must be within the specified range. The muon system datum consists of an address, and two charges. This datum is saved if either of the two charges is above the specified minimum. The CsI calorimeter analog data is transmitted to LeCroy Fastbus ADC cards for digitization with the digitized information passed on to a sparsifier. The calorimeter presents a unique problem to sparsification. For optimum resolution of low energy photons ($\sim 50\text{MeV}$), cell energies in the edge
of the noise region must be included in the measurement. Sparsification based on a single threshold would either save excess amounts of noise or produce poor resolution for low energy measurements. To solve this problem a two-dimensional sparsification is used with two passes of sparsification. This procedure has been designed into a hardware module called SPARXL. The digitized pulse heights are read into the data RAM of SPARXL. The first pass applies a seed threshold to identify seed crystals. The 24 neighbors to the seed crystal are also identified in the first pass. The second pass compares the digitized pulse heights, identified in the preceding pass, to a lower threshold. A skim RAM holds low threshold settings allowing each cell to have a different threshold to be kept as part of the shower. After sparsification the valid address and data are stored in the output
The data crates are read sequentially by an on-line processor and the data from each event placed into one of 32 buffers located in a dual-port memory. This FIFO buffer system is read asynchronously by the on-line data-logging process which formats the data into sequential ZEBRA records and writes them to disk.

Control of the high voltage system and control of the pulse system used for electronic calibration of the detector is done through the X-BUS system. The X-BUS is an 8-bit data-bus.

2.4.1 DAQ90 Upgrade

Prior to 1991 the data acceptance rate for the CLEO-II experiment was limited, by dead-time, to approximately 8 Hz. With increasing instantaneous luminosity, thanks to the hard work of the accelerator group, and a desire to loosen trigger requirements for low-multiplicity events, such as produced in tau-pair production and two-photon interactions, it became necessary to improve the data acquisition system. This improvement came in the form of the "DAQ90" system illustrated in figure 45.

The new system retains the existing front-end analog electronics but replaces all other components in the readout chain. The analog crate controllers have been redesigned with a faster ADC and removal of on-board digital processing. The ADC word is sent to a remote dual-port RAM or "link board" where it is buffered for subsequent transfer.

An exception to this scheme is the calorimeter readout process in which digitization is accomplished by LeCroy 1885 FASTBUS ADC modules. The data are stored in the module until all channels have been digitized, at which time they are written to a remote link-board buffer. Figure 46 illustrates the buffers for the
data acquisition system.

After all data is transferred to the link buffers and providing an empty buffer is available for the next event, the experiment is re-enabled.

Raw crate data from the link buffer are read out by Motorola MVME165 microprocessor boards via the VME Subsystem Bus(VSB). Subsystem-specific sparcification algorithms are applied with the surviving data deposited in "sparse buffers". The last task to finish notifies the event-builder task that all fragments for the event are available. The event-builder constructs a full event from the fragments and stores the ZEBRA formatted event in the event buffer. An event distributor then passes the event to an event consumers, such as a software filter, upon request.
Figure 46
DAQ90 buffer system.
The system allows for expansion in case of increased data rates through the addition of multiple event builders and parallel running event consumers.
CHAPTER III

Theory of Two Photon Interactions

In chapter 1 the production of a state $\mathcal{R}$ by the interaction of two photons was introduced. In this chapter the two-photon interaction is explored further, concentrating on the reaction $e^+e^- \rightarrow e^+e^-\mathcal{R}_{c\bar{c}}$ where $\mathcal{R}_{c\bar{c}}$ is one of the $c\bar{c}$ resonances $\eta_{c}, \chi_{c0}$ or $\chi_{c2}$.

3.1 Notation

In the following sections we deal with both three-vectors and four-vectors. Three vectors are written in bold face, with the scalar product $\mathbf{r} \cdot \mathbf{t}$, of the three-vectors $\mathbf{r} = (r_1, r_2, r_3)$ and $\mathbf{t} = (t_1, t_2, t_3)$ defined as $\mathbf{r} \cdot \mathbf{t} = r_1t_1 + r_2t_2 + r_3t_3$. Four-vectors are written in italics such as, $r \equiv r^\mu \equiv (r^0, \mathbf{r})$. The metric $g^{\mu\nu}$ is defined so that the inner product $rt$ of the four-vectors $r = (r^0, \mathbf{r})$ and $t = (t^0, \mathbf{t})$ yields;

$$rt \equiv r^\mu t_\mu \equiv r^0t^0 - \mathbf{r} \cdot \mathbf{t}, \quad (3.1)$$

that is, if the contravariant four vector $t^\mu$ is defined as;

$$t^\mu \equiv (t^0, t^1, t^2, t^3), \quad (3.2)$$
then the covariant four-vector is obtained by changing the sign of the space components;
\[ t_\mu \equiv (t^0, -t^1, -t^2, -t^3) = g_{\mu\nu} t^\nu, \]
with
\[ g_{\mu\nu} = g^{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \]

3.2 Kinematics of Two-Photon Reactions

Two-photon scattering in $e^+e^-$ storage rings can be observed through the reaction $e^+e^- \rightarrow e^+e^- \mathcal{R}$. The particle system $\mathcal{R}$ produced in the two-photon reaction depicted in Figure 47 has an even C-parity. For a symmetric $e^+e^-$ collider with unpolarized beams six variables determine the $\gamma\gamma$ system. These are:

- $E$ the lepton beam energy
- $E', E''$ the scattered lepton energies
- $\theta_1, \theta_2$ the lepton scattering angles wrt the beam direction
- $\phi$ the angle between the lepton scattering planes in the $\gamma\gamma$ center of mass system ($CM$-system).

The initial electron has four-momentum denoted by $p_1 = (E, p_1 = (0, 0, p_b))$, and the initial positron has four-momentum $p_2 = (E, p_2 = (0, 0, -p_b))$, where $p_b$ is the electron beam momentum, $p_b = (E^2 - m_e^2)^{1/2} \approx E$. The scattered electron and positron have four-momentum $p_1' = (E_1', p_1')$ and $p_2' = (E_2', p_2')$ respectively, where $p_i' = (p'_{ix}, p'_{iy}, p'_{iz})$ is the three-momentum. The lepton scattering angles are determined by the relations:

\[ \theta_i = \cos^{-1}\left[ \left( \frac{p'_{ix}}{|p'|} \right) \left( \frac{p_{iz}}{p_b} \right) \right], \]

where $p_{iz}/p_b = \pm 1$ reflects the opposite directed electron and positron beams.
Two photon production of the state $\mathcal{R}$ from $e^+e^-$ scattering.
The lepton’s azimuthal angles are determined by the relations;

\[ \phi_i = \cos^{-1}\left( \frac{p'_{ix}}{|p'_{iT}|} \right), \quad (3.6) \]

where \( p'_{iT} = (p'_{ix}, p'_{iy}, 0) \) is the transverse momentum vector of the scattered lepton. The photon four-momenta, \( q_i = (\omega_i, q_i) \), can be expressed as the difference between the initial and scattered lepton four-momenta;

\[ q_i = (p_i - p'_i). \quad (3.7) \]

The positive valued Lorentz scalars;

\[ Q_i^2 = -q_i^2, \quad (3.8) \]

the negative of the squared invariant mass of the \( i^{th} \) photon, can be written in terms of the initial and scattered lepton quantities as,

\[ Q_i^2 = 2(EE'_i - p_b|p'_i| \cos \theta_i - m_
u^2). \quad (3.9) \]

By defining a normalized scattered lepton energy as \( x_i \equiv E'_i/E \), and a normalized lepton mass as \( \mu \equiv m_e/E \), equation 3.9 becomes;

\[ Q_i^2 = 2E^2 \left[ x_i - \mu^2 - \sqrt{(1 - \mu^2)(x_i^2 - \mu^2) \cos \theta_i} \right]. \quad (3.10) \]

For \( \mu \ll x_i \) this can be written as;

\[ Q_i^2 = 4EE'_i \sin^2(\theta_i/2). \quad (3.11) \]

A third Lorentz scalar \( W^2 \) is the squared invariant mass of the final-state \( \mathcal{R} \);

\[ W^2 = (q_1 + q_2)^2 \]

\[ = (\omega_1 + \omega_2)^2 - (p'_1 + p'_2)^2 \]

\[ = 2E^2 \left\{ (2 - x_1)(2 - x_2) + \mu^2 - 2 \right. \\
\left. + \sqrt{(x_1^2 - \mu^2)(x_2^2 - \mu^2) \cos \Theta} \right\}, \quad (3.13) \]
where $\omega_i$ is the photon energy $\Theta$ is the angle between the scattered leptons. In terms of the previously defined angles this is;

$$\cos \theta = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2).$$

(3.14)

For a given $W$, the minimum photon energy $\omega_{\min} = 1 - x_{\max}$ is obtained when the second photon has a maximum energy $\omega_{\max} = 1 - x_{\min}$. When this occurs $x_{\min} = \mu \approx 0$ and we have, from equation 3.12,

$$W^2 \approx 2E^2 \{ (2 - x_{\min}) (2) - 2 \}$$

$$\approx 4E^2 (1 - x_{\min})$$

$$\approx 4E^2 \omega_{\min}.$$  

(3.15)

Assuming a beam energy of 5.28 GeV, a two-photon energy of approximately the $\eta_c$ mass ($W = 3.0$ GeV $\approx m_{\eta_c}$) requires a minimum photon energy of $\omega_{\min} \approx 425$ MeV.

In the CM-system the photon momenta are;

$$\hat{q}_1 = (\hat{\omega}_1, 0, 0, \hat{q}) \quad \text{and} \quad \hat{q}_2 = (\hat{\omega}_2, 0, 0, -\hat{q}),$$

(3.16)

where '$$ implies a CM-system quantity. The magnitude of the photon momentum is;

$$\hat{q} = \sqrt{X \over W^2}.$$  

(3.17)

Here $X$ is the *Møller flux factor* defined as;

$$X \equiv (q_1 q_2)^2 - q_1^2 q_2^2$$

$$= \left( {W^2 + Q_1^2 + Q_2^2} \right) - Q_1^2 Q_2^2$$

$$= \frac{1}{4} [W^4 + 2W^2(Q_1^2 + Q_2^2) + (Q_1^2 - Q_2^2)^2].$$  

(3.18)

(3.19)

The photon energies in the CM-system are;

$$\hat{\omega}_1 = \frac{W}{2} \left( 1 - \frac{Q_1^2 - Q_2^2}{W^2} \right) \quad \text{and} \quad \hat{\omega}_2 = \frac{W}{2} \left( 1 + \frac{Q_1^2 - Q_2^2}{W^2} \right).$$

(3.20)
To determine $\phi$, the angle between the $e^+e^-$ scattering planes in the CM-system, we first define a metric tensor $T^{\mu\nu}$ for any pair of four-vectors $q$ and $p$ as:

$$T^{\mu\nu}(q,p) = g^{\mu\nu} - \frac{1}{X} \left[ (q^\mu p^\nu + q^\nu p^\mu) - q^2 p^\mu p^\nu - p^2 q^\mu q^\nu \right]. \quad (3.21)$$

This tensor projects an arbitrary four-vector $k$ into the subspace orthogonal to both $q$ and $p$, so that:

$$q^\mu k^\mu = p^\mu k^\mu = 0; \quad k^\perp = T^{\mu\nu}(q,p) k_\nu \quad (3.22)$$

Since $q_{ix} = q_{iy} = 0$ we have $\hat{p}_{ix} = -\hat{p}_{ix}'$ and $\hat{p}_{iy} = -\hat{p}_{iy}'$, as shown in figure 48, $\phi$ is the angle between the components of the initial lepton momenta in the $T(q_1,q_2)$ subspace and can be defined by the relations:

$$\cos \phi = \frac{p_{i1}p_{i2}}{\sqrt{p_{i1}^2 p_{i2}^2}}; \quad p_{i\perp} = T(q_1,q_2)p_i \quad (3.23)$$
3.3 The Feynman Amplitudes

The two-photon process $e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-\mathcal{R}$ can be factored into the photon-photon production process, $e^+e^- \rightarrow e^+e^-\gamma\gamma$, which is purely a QED process, and the photon-photon interaction process, $\gamma\gamma \rightarrow \mathcal{R}$, which incorporates any QCD effects. The Feynman diagrams forming the two-photon reaction are shown in figures 49a and 49b. Applying the Feynman rules\cite{1} the transition matrix is;

$$A = \frac{1}{q_1q_2}j_\mu M_{\mu\nu}j_\nu$$

where $j_\mu$ is the current of the $e^+ \rightarrow e^+\gamma_1$ vertex and $j_\mu$ the current of the $e^- \rightarrow e^-\gamma_2$ vertex. The currents have the form $j_\mu = ie\bar{u}(p_1,\sigma_1)\gamma\mu u(p_1',\sigma_1')$ and $j_\mu^* = -ie\bar{u}(p_2,\sigma_2)\gamma\nu(p_2',\sigma_2')$, where $e$ is the electric charge normalize so that $e^2 = 4\pi\alpha$, $u$ and $\bar{u}$ are the electron free Dirac spinors, $v$ and $\bar{v}$ are the positron free Dirac spinors and $\sigma_i$ is the spin of the associated lepton. For brevity we denote $(p_i,\sigma_i)$ by $(\sigma_i)$ and $(p_i',\sigma_i')$ by $(\sigma_i')$.
The cross section for this reaction is determined by\cite{1};
\[
d\sigma = \left| A \right|^2 \frac{(2\pi)^4 \delta^4(q_1 + q_2 - \sum j_k)}{4\sqrt{X}} d\Gamma(p'_1) d\Gamma(p'_2) \Pi_j d\Gamma(k_j),
\]
(3.25)
where \( d\Gamma(p) \equiv d^3p/2E_p(2\pi)^3 \). For a resonance with three-momentum \( k \) and energy \( \omega_1 + \omega_2 \), the phase-space volume is given as \( \Pi_j d\Gamma(k_j) = \Pi_j d^3k_j/2\epsilon_j(2\pi)^3 = d^3k/[2(\omega_1 + \omega_2)(2\pi)^3] \). The square of the amplitude matrix is written as\cite{1};
\[
\left| A \right|^2 = \frac{1}{(Q_1^2 Q_2^2)^2} \left[ j^\perp \gamma^\perp \right][j^\perp j^\perp'] \left[ M_{\mu\nu} M_{\mu\nu}' \right].
\]
(3.26)

Since the \( e^+e^- \)-beams are unpolarized we average over initial spins and sum over final spin\cite{1}:
\[
\frac{1}{2} \sum_{\sigma_1\sigma'_1} j^\perp \gamma^\perp = \frac{4\pi\alpha}{2} \sum_{\sigma_1\sigma'_1} \left[ \bar{u}(\sigma_1)\gamma^\mu u(\sigma'_1) \right] \left[ \bar{u}(\sigma_1)\gamma^\mu u(\sigma'_1) \right]^* = \frac{4\pi\alpha}{2} \text{Tr} \left[ \gamma^\mu (\not{p}_1 + m_e)\gamma^\mu (\not{p}'_1 + m_e) \right] = -\frac{4\pi\alpha}{Q_1^2} \left[ g_{\mu\nu} - 2 \left( p_1^\mu p_1^\nu + p_1^\nu p_1^\mu \right) \right] Q_1^2 = -\frac{4\pi\alpha}{Q_1^2} \left[ g_{\mu\nu} - q_1^\mu q_1^\nu + (2p_1 - q_1)\mu (2p_1 - q_1)\nu \right], \quad (3.27)
\]

similarly,
\[
\frac{1}{2} \sum_{\sigma_2\sigma'_2} j^\perp j^\perp' = \frac{4\pi\alpha}{2} \sum_{\sigma_2\sigma'_2} \left[ \bar{v}(\sigma_2)\gamma^\nu v(\sigma'_2) \right] \left[ \bar{v}(\sigma_2)\gamma^\nu v(\sigma'_2) \right]^* = -\frac{4\pi\alpha}{Q_2^2} \left[ g_{\nu\nu} - q_2^\nu q_2^\nu + (2p_2 - q_2)\nu (2p_2 - q_2)\nu \right]. \quad (3.28)
\]

The unnormalized density matrices associated with the \( e^- \rightarrow e^- \gamma_1 \) vertex and the \( e^+ \rightarrow e^+ \gamma_2 \) vertex are defined as:
\[
\rho^{\mu\nu}_1 \equiv \left( \frac{Q_1^2}{4\pi\alpha} \right) \frac{1}{2} \sum_{\sigma_1\sigma'_1} j^\perp j^\perp' \quad \text{and} \quad \rho^{\mu\nu}_2 \equiv \left( \frac{Q_2^2}{4\pi\alpha} \right) \frac{1}{2} \sum_{\sigma_2\sigma'_2} j^\perp j^\perp', \quad (3.29)
\]
thus, the density matrix \( \rho_i \) for the virtual photon generated by the \( i \)th unpolarized lepton beam, is written as\cite{21};
\[
\rho_i^{\alpha\beta} = -\left( g^{\alpha\beta} + q_i^\alpha q_i^\beta \right) \frac{Q_i^2}{Q_i^2} + \frac{2p_i - q_i}{Q_i^2} \epsilon_i^\alpha (2p_i - q_i)^\beta. \quad (3.30)
\]
The differential cross section then takes the form:

$$d\sigma = \frac{(4\pi\alpha)^2}{q_1 q_2} \rho_1^{\mu\nu} \rho_2^{\rho\sigma} M_{\mu\rho}^* M_{\nu\sigma} \frac{(2\pi)^4 \delta^4(q_1 + q_2 - k)}{4\sqrt{X}} d\Gamma(k) d\Gamma(p_1) d\Gamma(p_2).$$ \hspace{1cm} (3.31)

### 3.4 Helicity Basis

Since the photons are virtual, they have three polarization vectors. We shall denote the polarization vector of the \( j \)th photon as \( e^a(j, a) \), where \( a = 0 \) refers to scalar (S) polarization and \( a = \pm 1 \) refers to transverse (T) polarization. The choice of polarization vectors is not unique. Each polarization vector is orthogonal to the others with \( e(j, 0) \) having a time-like normalization and \( e(j, \pm 1) \) having space-like normalization. That is, they satisfy the relations;

$$q_j e(j, a) = 0 ; \quad e^*(j, a) e(j, b) = (-1)^a \delta_{ab}.$$ \hspace{1cm} (3.32)

We also define the space-like four-momentum vector in the direction of \( q_j \) as:

$$n(j, q) \equiv \frac{i}{\sqrt{q_j^2}} q_j ; \quad n^*(j, q) n(j, q) = -1.$$ \hspace{1cm} (3.33)

The polarization vectors and momentum direction vector satisfy the relation:

$$n^*(j, q) n(j, q) n^{*\mu}(j, q) n^\nu(j, q) + \sum_a e^*(j, a) e(j, a) e^{*\mu}(j, a) e^\nu(j, a) = g^{\mu\nu}.$$ \hspace{1cm} (3.34)

The scalar polarization vector \( e(j, 0) \), by definition, must be orthogonal to \( q_j \). Such a vector can be expressed in terms of \( q_1 \) and \( q_2 \) as \( e(j, 0) = \sum_{k=1,2} \lambda_k q_k \). The orthogonality condition \( e(j, 0) q_j = 0 \) requires \( \lambda_j = -(q_1 q_2 / q_j^2) \lambda_{k \neq j} \). Equation 3.32 also requires the time-like normalization \( e^*(j, 0) e(j, 0) = 1 \), which leads to the expression:

$$e(j, 0) = i \sqrt{\frac{q_j^2}{X}} \left( q_{k \neq j} + \frac{q_1 q_2}{Q_j^2} q_j \right) = i S_j.$$ \hspace{1cm} (3.35)
From this and equation 3.21 we can write the transverse projection operator:

\[ T^{\mu\nu}(q_1, q_2) = g^{\mu\nu} - (e^*\mu(j,0)e^\nu(j,0) - n^*\mu(j, q)n^\nu(j, q)) \]
\[ = -e^*\mu(j, +1)e^\nu(j, +1) - e^*\mu(j, -1)e^\nu(j, -1). \]  

(3.36)

The tensor \( T^{\mu\nu} \) satisfy the relationships:

\[ q_j^{\mu}T^{\mu\nu} = S_j^{\mu}T^{\mu\nu} = 0; \quad T^{\mu\nu}T^{\mu\nu} = 2; \quad T^{\mu\nu}T^{\mu\alpha} = -T^{\mu\alpha}. \]  

(3.37)

The transverse polarization vectors satisfy:

\[ e^\mu(j, \pm 1)e^\nu(j, \pm 1) = \frac{1}{2}\left[T^{\mu\nu}(q_1, q_2) \pm i \frac{\epsilon^{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta}{(q_1^2 q_2^2 - q_1^2 q_2^2)}\right]. \]  

(3.38)

With the usual choice of transverse polarization vectors we have,

\[ e(2, \pm 1) = e(1, \mp 1); \quad e^*(j, \pm 1) = -e(j, \mp 1), \]  

(3.39)

which implies \( e^*(2, \pm 1) = -e(1, \pm 1) \). In the CM-system the polarization vectors can take the form:

\[ e(j, \pm 1) = \pm \frac{1}{\sqrt{2}}(0, 1, \pm i, 0), \quad \text{and} \quad e(j, 0) = \frac{1}{\sqrt{-q_j^2}}(|q_j|, 0, 0, \omega_j). \]  

(3.40)

### 3.5 Helicity Basis Expansion

Integrating the differential cross section over the phase-space volume of the produced particle(s) \( \mathcal{R} \), equation 3.31 becomes

\[ d\sigma = \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} \frac{1}{2\sqrt{X}} p_1^{\mu}\bar{p}_2^{\nu}\mathcal{W}^{\mu\nu,\mu\nu}d\Gamma(p_1')d\Gamma(p_2'). \]  

(3.41)

where the absorptive part of the \( \gamma\gamma \)-forward amplitude is,

\[ \mathcal{W}^{\mu\nu,\mu\nu} = \frac{1}{2} \int M^*_{\mu\nu}M_{\mu\nu}(2\pi)^4\delta^4(q_1 + q_2 - k)d\Gamma(k). \]  

(3.42)
Lorentz invariance, T-invariance (substitution of $\mu'\nu' \leftrightarrow \mu\nu$) and gauge invariance implies,

$$ q_1^\mu \mathcal{W}^\mu{}_{\nu',\mu\nu} = q_2^\mu \mathcal{W}^\mu{}_{\nu',\mu\nu} = q_3^\mu \mathcal{W}^\mu{}_{\nu',\mu\nu} = q_4^\mu \mathcal{W}^\mu{}_{\nu',\mu\nu} = 0 . \tag{3.43} $$

Since the polarization vectors form a complete orthogonal system, $\mathcal{W}^\mu{}_{\nu',\mu\nu}$ can be expanded in terms of the polarization vectors,

$$ \mathcal{W}^\mu{}_{\nu',\mu\nu} = \sum_{ab,a'b'} (-1)^{a+b+a'+b'} e^\mu(1,a') e^{\nu'}(2,b') \mathcal{W}_{a'b',ab} e^*\mu(1,a) e^*\nu(2,b) . \tag{3.44} $$

Due to P-invariance and T-invariance

$$ \mathcal{W}_{a'b',ab} = \mathcal{W}_{-a'-b',-a-b} = \mathcal{W}_{ab,a'b'} \tag{3.45} $$

The expansion coefficients of equation 3.44 are sums of products of the polarization four-vectors. For example, the coefficient determined for $\mathcal{W}_{00,00} = \mathcal{W}_{SS}$ taking into account equation 3.39 is;

$$ C_{SS}^\mu{}_{\nu',\mu\nu} = e^\mu(1,0) e^*\mu(1,0) e^{\nu'}(2,0) e^*\nu(2,0) $$

$$ = S_1^\mu S_2^\nu S_1^{\nu'} S_2^{\nu'} \tag{3.46} $$

and in the case of $\mathcal{W}_{+,+0} = \mathcal{W}_{TS}$, taking into account equation 3.39, the coefficient is determined to be;

$$ C_{TS}^\mu{}_{\nu',\mu\nu} = [e^*\mu(1,+1) e^\mu(1,+1) + e^*\mu(1,-1) e^\mu(1,-1)] e^*\nu(2,0) e^{\nu'}(2,0) $$

$$ = T^\mu{}_{\nu'} S_2^\nu S_2^{\nu'} . \tag{3.47} $$

Following a similar procedure to determine the other coefficients, the absorptive part of the $\gamma\gamma'$-forward amplitude can be written in terms of eight independent helicity amplitudes\textsuperscript{[21]} as,

$$ \mathcal{W}^\mu{}_{\nu',\mu\nu} = T^\mu{}_{\nu'} T^{\nu'} \mathcal{W}_{TT} + S_1^\mu S_1^{\nu'} S_2^\nu S_2^{\nu'} \mathcal{W}_{SS} $$

}{
\[ + T^{\mu\nu} S_2^{\mu\nu} W_{TS} + S_1^{\mu\nu} T^{\mu\nu} W_{ST} \\
+ \frac{1}{2} [T^{\mu\nu} T^{\mu\nu} + T^{\mu\nu} S_1^{\mu\nu} - T^{\mu\nu} T^{\mu\nu}] W_{TT} \\
- [T^{\mu\nu} S_1^{\mu\nu} S_2^{\mu\nu} + T^{\mu\nu} S_1^{\mu\nu} S_2^{\mu\nu} + (\mu\nu \leftrightarrow \mu'\nu')] W_{TS} \\
+ [T^{\mu\nu} T^{\mu\nu} - T^{\mu\nu} T^{\mu\nu}] W_{TT} \\
- [T^{\mu\nu} S_1^{\mu\nu} S_2^{\mu\nu} - T^{\mu\nu} S_1^{\mu\nu} S_2^{\mu\nu} + (\mu\nu \leftrightarrow \mu'\nu')] W_{TS} \]

where,

\[ \frac{1}{2} [W_{++} + W_{--}] \equiv W_{TT} \]
\[ W_{+0,+0} \equiv W_{TS} ; \quad W_{0+} \equiv W_{ST} \]
\[ W_{00,00} \equiv W_{SS} ; \quad W_{++,-} \equiv W_{TT} \]
\[ \frac{1}{2} [W_{++} + W_{0+}] \equiv W_{TS} \]
\[ \frac{1}{2} [W_{++,-} - W_{+-,-}] \equiv W_{TT} \]
\[ \frac{1}{2} [W_{++} - W_{00}] \equiv W_{TS} . \]

The first four amplitudes \( W_{ab} \) can be expressed in terms of the cross sections \( \sigma_{ab} \) as,

\[ W_{TT} = 2\sqrt{X} \sigma_{TT} ; \quad W_{TT} = 2\sqrt{X} \sigma_{TS} ; \]
\[ W_{ST} = 2\sqrt{X} \sigma_{ST} ; \quad W_{SS} = 2\sqrt{X} \sigma_{SS} . \]

The two amplitudes \( \sigma_{ab} \) are related to the squared spin-flip amplitudes \( \tau_{TT} \) and \( \tau_{TS} \) and to maintain consistent notation the two antisymmetric amplitudes \( W_{ab}^a \) are expressed in terms of antisymmetric amplitudes \( \tau_{TT}^a \) and \( \tau_{TS}^a \) as,

\[ \sigma_{TT}^a = 2\sqrt{X} \tau_{TT}^a ; \quad \sigma_{TT}^a = 2\sqrt{X} \tau_{TS}^a ; \]
\[ \sigma_{TT}^a = 2\sqrt{X} \tau_{TT}^a ; \quad \sigma_{TT}^a = 2\sqrt{X} \tau_{TS}^a . \]

Substituting equation 3.44 into the differential cross section given in equation 3.41, taking into account 3.50 and 3.51, and summing over indices \( \mu, \nu, \mu' \)
and \( \nu' \) we obtain:

\[
\frac{d\sigma}{d\Omega} = \frac{(4\pi\alpha)^2}{q_1^2 q_2^2} [4\rho_1^{++} + 2|\rho_1^{+-} - \rho_2^{+-}|^2 \tau_T \cos 2\phi \\
+ 2\rho_1^{++} \rho_2^{00} \sigma_{TS} + 2\rho_1^{00} \rho_2^{++} \sigma_{ST} + \rho_1^{00} \rho_2^{00} \sigma_{SS} \\
- 8|\rho_1^{00} \rho_2^{++}|^2 \tau_T \cos \phi + A\tau_T^2 + B\tau_T^2] d\Gamma(p'_1) d\Gamma(p'_2),
\]

(3.52)

where \( \rho_{ij}^{ab} \) are the convolutions of the density matrices \( \rho_{ij}^{ab} \) with the expansion coefficients from equation 3.44. For unpolarized initial electrons,\(^{[21]} \) \( A = B = 0 \) and

\[
2\rho_1^{++} = 2\rho_1^{--} = \rho_1^{\mu \nu} T^{\mu \nu} = \frac{1}{X}(2p_1 q_2 - q_1 q_2)^2 + 1 + \frac{4m_e^2}{q_1^2};
\]

\[
\rho_1^{00} = \rho_1^{\mu \nu} S_{1\mu} S_{1\nu} = \frac{1}{X}(2p_1 q_2 - q_1 q_2)^2 - 1;
\]

\[
|\rho_1^{+-}| = \rho_1^{++} - 1;
\]

\[
|\rho_1^{00}| = \sqrt{(\rho_1^{00} + 1)(\rho_1^{++} - 1)}.
\]

(3.53)

The differential cross section can be written in a compact form as:

\[
\frac{d\sigma}{d\Omega} = \mathcal{L}_{ab}(q_1^2, q_2^2, W^2) \cdot \sigma_{ab}(q_1^2, q_2^2, W^2) d\Gamma(p'_1) d\Gamma(p'_2),
\]

(3.54)

where \( \mathcal{L}_{ab} \) is the \( \gamma \gamma \) luminosity function for a photon with polarization \( a \in (T, S) \) and the second photon having a polarization \( b \in (T, S) \) and where \( q^2 \) is the photon mass and \( W \) is the invariant mass of the two-photon system as defined in equation 3.12.

### 3.6 The Cross Section Formula

In most instances the experimental set up serves to simplify the cross section formula 3.52. Since CESR's electron beams are unpolarized, \( A = B = 0 \), as stated above.

The spin-flip terms containing factors of \( \tau_{ab} \) also contain a factor of \( \cos 2\phi \) or \( \cos \phi \). Unless \( \phi \) is explicitly determined these terms average to zero. Since the
CLEO-II experiment has no small angle tagging of electrons, $\phi$ is not determined and therefore, no terms containing $\tau_{ab}$ are retained.

Further simplification of the cross section formula is accomplished by considering the particle produced by the two-photon collision. We examine the production of three types of neutral resonances in the form of a pseudo-scalar, a scalar and a tensor meson.

### 3.6.1 Pseudo-scalar Resonance ($\eta_c$)

The coupling of two photons to a neutral pseudo-scalar meson is described through a single amplitude and therefore only one form factor $F^{2}_{TT0}$,

$$M_{\mu\nu}(J^{PC} = 0^{-+}) = i\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta F^{2}_{TT0},$$

Due to the form of the amplitude, a pseudo-scalar does not couple to longitudinally polarized photons. This can be seen by taking the product of equation 3.40 and the above amplitude. The only non-vanishing terms are,

$$\sigma_{TT} = \frac{1}{2} \sigma_{TT} = \frac{1}{4} F^{2}_{TT0} \frac{M\sqrt{X}}{W} \frac{\Gamma}{(W^2 - M^2)^2 + \Gamma^2 M^2},$$

where $\Gamma$ and $M$ are the total width and mass of the neutral pseudo-scalar meson.

For $\eta_c$ production, the $q^2$ dependence is modeled from the $\psi$ form factor,

$$F^{2}_{TT0} = \frac{64\pi \Gamma_{\gamma\gamma}}{M^3} \frac{1}{(1 - q_1^2/M_{\psi}^2)^2 (1 - q_2^2/M_{\psi}^2)^2} \cdot$$

where $\Gamma_{\gamma\gamma}$ is the two-photon partial width of $\eta_c$. The two-photon production cross section for the pseudo-scalar is written as,

$$\sigma_{e^+e^-e^+e^-0^{++}} = \int \mathcal{L}_{TT}^2(q_1^2, q_2^2, W^2) \cdot \sigma_{TT}(q_1^2, q_2^2, W^2) d\Gamma(p'_1)d\Gamma(p'_2).$$
3.6.2 Scalar Resonance ($\chi_{c0}$)

For the case of the scalar meson, if both photons are off-mass shell, the cross sections are given as,

$$
\sigma_{TT} = \frac{1}{2} \tau_{TT} = \frac{1}{4} F^{2}_{TT} \frac{M^2}{W \sqrt{X}} \left( \frac{\Gamma}{(W^2 - M^2)^2 + \Gamma^2 M^2} \right), \\
\sigma_{LL} = \frac{1}{4} F^{2}_{LL} \frac{M^2}{W \sqrt{X}} \left( \frac{2X q_1^2 q_2^2}{(W^2 - M^2)^2 + \Gamma^2 M^2} \right), \\
\sigma_{TL} = \sigma_{LT} = 0.
$$

When the electrons scattering angles are small, both photons are predominately near mass-shell. As a result $\sigma_{LL}$ is much smaller than $\sigma_{TT}$.

For the $\chi_{c0}$ the $q^2$ dependence is modeled from the remaining form factor,

$$
F^{2}_{TT0}(q_1^2, q_2^2) = 16\pi M \Gamma \gamma_\tau \left( \frac{1}{1 - q_1^2/M^2} \right) \left( \frac{1}{1 - q_2^2/M^2} \right).
$$

3.6.3 Tensor Resonance ($\chi_{c2}$)

For the case of the tensor meson, if at least one photon is off-mass shell, the cross sections can be written as,

$$
\sigma_{TT}(J_z = 2) = \frac{1}{4} F^{2}_{TT2} \frac{M^2}{W \sqrt{X}} \left( \frac{\Gamma}{(W^2 - M^2)^2 + \Gamma^2 M^2} \right), \\
\sigma_{TT}(J_z = 0) = \frac{1}{2} \tau_{TT} = \frac{1}{4} F^{2}_{TT0} \left( \frac{q_1^2 q_2^2}{W} \right) \left( \frac{M^2}{3W^4} \right) \left( \frac{\Gamma}{(W^2 - M^2)^2 + \Gamma^2 M^2} \right), \\
\sigma_{TL}(J_z = 1) = \frac{1}{4} (F^{eff}_{TL})^2 \frac{M^2}{W \sqrt{X}} \left( \frac{-4X q_2^2}{M^4} \right) \left( \frac{\Gamma}{(W^2 - M^2)^2 + \Gamma^2 M^2} \right),
$$

where the effective form factor $F^{eff}_{TL}$ is defined as, $F^{eff}_{TL} \equiv F_{TL} + (q_1^2 - q_2^2) F_{TL}'$.

For events with no tagging, $q^2$ is small and so $\sigma_{TL}(J_z = 1)$ can be ignored.
The contribution from $F_{T^0}$ are predicted to be very small when considering the tensor meson as a non-relativistic bound states or applying finite energy sum rules. In light of this $\sigma_{TT}(J_z = 0)$ is also ignored.

For the $\chi_{c2}$ the $q^2$ dependence is modeled from the remaining form factor,\cite{53}

$$F_{TT}^2(q_1^2, q_2^2) = 80\pi M_{\gamma\gamma} \frac{1}{(1 - q_1^2/M_\rho^2)^2} \frac{1}{(1 - q_2^2/M_\rho^2)^2}. \quad (3.66)$$

### 3.7 Monte Carlo

The total cross section integral is evaluated numerically through a Monte Carlo technique. The Monte Carlo process also produces a set of simulated events with kinematic distributions determined by the employed model.

The Monte Carlo is divided into three stages. The first stage is the $MC$-generator which simulates the scattering of the electron beams and the two-photon production of the $R$ state. The second stage is $QQ$\cite{31} where the decay of the $R$ state is simulated along with subsequent decays of short lived particles. The third stage is $CLEOG$\cite{32} which simulates the particles trek through the detector and decay of long lived particles. The CLEOG program uses $GEANT$\cite{33} routines to simulate interactions of particles with the material of the CLEO-II detector. Detailed discussions on QQ, CLEOG and GEANT can be found in the above references. We present here the first stage of the Monte Carlo process: the generation of the $R_{\eta_c} \in \{\eta_c, \chi_{c0}, \chi_{c2}\}$ states.

#### 3.7.1 The Generator

The two-photon production of a resonance is characterized by five independent variables; the relative azimuthal angle of the scattered electrons ($\phi$), their two polar angles ($\theta_1$, $\theta_2$), the energy of one of the photons ($\omega_1$) and the center of
mass energy of the two photons ($W$). The energy of the second photon ($\omega_2$) is determined from these variables.

To generate a single event each independent variable is assigned a particular value from a set of possible values according to the probability density function (p.d.f.) for that variable. The p.d.f., $\rho(x)$, for a random variable $x$ is defined so that the probability of observing $x$ between $x$ and $x + dx$ is $\rho(x)dx$.

The normalized density function $\rho(x)$ can be written as

$$\rho(x) = \lim_{N \to \infty} \frac{1}{N} \frac{dN}{dx}$$

where $N$ is the total number of observed $x$ values and $dN$ is the number of observed $x$ values in the interval $dx$.

For each variable, the MC-generator starts with a pseudo-random number $u$ from a uniform distribution on the open interval (0,1). A sequence of pseudo-random numbers appears to be random up to a point where the sequence repeats. The sequence is reproducible for the same initial value. We use the CERN function RNDM with a period of more than $10^{13}$. This allows us to generate sufficient Monte Carlo events without exceeding the sequence period.

The uniform pseudo-random number is mapped into the independent variable producing a set of random numbers with the desired range and distribution. Two methods are employed for this mapping; the inverse transform method and importance sampling method.

### 3.7.2 Inverse Transform

We wish to produce the independent variable $x$ with a probability density function $\rho(x)$ on the range $-\infty < x < \infty$. The probability that $x \leq a$ is given
by the cumulative distribution function (c.d.f.);

\[ F(a) = \int_{-\infty}^{a} \rho(x)dx. \]  \hspace{1cm} (3.68)

If \( a \) is chosen with a probability density \( \rho(a) \) then \( F(a) \) is a random variable with a uniform probability density on \([0, 1]\).

With \( F(x) = u \), where \( u \) is a random number chosen from a uniform distribution on \((0, 1)\), the desired distribution can be achieved by the inverse transformation \( x = F^{-1}(u) \).

A simple example of this method is shown by generating the relative azimuthal angle of the scattered electrons \( \phi \). The values of \( \phi \) are distributed uniformly between 0 and \( 2\pi \) so that \( \rho(\phi) = 1/2\pi \) for \( 0 < \phi < 2\pi \) and \( \rho(\phi) = 0 \) otherwise. The cumulative distribution function is given as,

\[
F(\phi) = \begin{cases} 
0 & \text{if } \phi \leq 0 \\
\int_{0}^{\phi} \frac{dx}{2\pi} & \text{if } 0 < \phi < 2\pi \\
1 & \text{if } \phi \geq 2\pi 
\end{cases}
\]  \hspace{1cm} (3.69)

For \( u \) randomly selected on the range \((0, 1)\) the inverse transformation

\[ \phi = F^{-1}(u) = 2\pi u. \]  \hspace{1cm} (3.70)

As desired \( \phi \) is uniformly distributed on \((0, 2\pi)\).

A second example is shown by generating the polar angle of the scattered electrons \( \theta \). The values of \( \theta \) are distributed as \( 1/\theta \) between some \( \theta_{\min} \) and \( \theta_{\max} \) so that \( \rho(\theta) = (\theta_{\max}/\theta_{\min})^{-1} \ln(\theta/\theta_{\min}) \) for \( \theta_{\min} < \theta < \theta_{\max} \) and \( \rho(\theta) = 0 \) otherwise. The cumulative distribution function is given as,

\[
u = F(\theta) = \begin{cases} 
0 & \text{if } \theta \leq \theta_{\min} \\
\ln(\theta/\theta_{\min})/\ln(\theta_{\max}/\theta_{\min}) & \text{if } \theta_{\min} < \theta < \theta_{\max} \\
1 & \text{if } \theta \geq \theta_{\max} 
\end{cases}
\]  \hspace{1cm} (3.71)

For \( u \) randomly selected on the range \((0, 1)\) the inverse transformation is,

\[ \theta = F^{-1}(u) = \theta_{\min}(\theta_{\max}/\theta_{\min})^{u}. \]  \hspace{1cm} (3.72)
3.7.3 Importance Sampling

Often times the inverse transformation, $F^{-1}(u)$, is too complex to be determined or does not exist. If for all values of the random variable $x$, the probability density function $\rho(x)$ can be calculated and $\rho(x)$ is well behaved, then we can apply the importance sampling method to generate $x$ with the desired probability distribution.

We start by finding an easily generated distribution with a probability density function $\rho_g(x)$, such that, for some finite $C > 1$; $C \rho_g(x) \geq \rho(x)$ for all $x$. We call $\rho_g(x)$ a normalized envelope function for $\rho(x)$. We then generate a candidate $x$ according to $\rho_g(x)$ and determine $C \rho_g(x)$ and $\rho(x)$. Choose a random number $u$ from a uniform distribution on $(0,1)$. If $uC \rho_g(x) < \rho(x)$ keep $x$, otherwise throw it away. The saved values of $x$ will produce the desired distribution.

If $C \rho_g(x)$ is chosen so that $C \rho_g(x) < \rho(x)$ for some region of $x$ values, that region will be underrepresented in the final distribution. The selection of $\rho_g(x)$ can greatly affect the acceptance efficiency. If $C \rho_g(x) \approx \rho(x)$ then most candidate $x$ values are accepted but, if $C \rho_g(x) >> \rho(x)$ few candidate $x$ values are accepted.

We note that for importance sampling, $\rho(x)$ need not be normalized to unity but the condition $C \rho_g(x) \geq \rho(x)$ for all $x$ must be satisfied.

The importance sampling method is less efficient than the inverse transform method. For each accepted value the inverse transform method uses one random number. The importance sampling method uses at least two random numbers per accepted value. Furthermore, the efficiency is just the area of the desired distribution divided by the area of the envelope distribution,

$$\epsilon \equiv \frac{\int_{-\infty}^{\infty} \rho(x) dx}{\int_{-\infty}^{\infty} C \rho_g(x) dx}. \quad (3.73)$$

For normalized distributions $\rho_g(x)$ and $\rho(x)$, the efficiency is simply $1/C$. 


3.7.4 Numerical Integration

We want to integrate a scalar function $R(x)$ over a range from $x_0$ to $x_1$. The integral can be written as the sum of contributions from minute $x$-intervals,

$$\int_{x_0}^{x_1} R(x)dx = \lim_{N \to \infty} \sum_{i=1}^{N} R(x_i) \Delta_i x$$  \hspace{1cm} (3.74)$$

$$= \lim_{N \to \infty} \sum_{i=1}^{N} \frac{R(x_i)}{\rho(x_i)} \rho(x_i) \Delta_i x$$  \hspace{1cm} (3.75)$$

Taking into account equation 3.67, the factor $\rho(x_i) \Delta_i x$ can be written as

$$\rho(x_i) \Delta_i x = \frac{\Delta_i N}{N}$$  \hspace{1cm} (3.76)$$

where $N$ is the total number of generated $x$ values and $\Delta_i N$ is the expected number of $x$ values in the interval $\Delta_i x$. We then have,

$$\int_{x_0}^{x_1} R(x)dx = \lim_{N \to \infty} \sum_{i=1}^{N} \frac{R(x_i)}{\rho(x_i)} \frac{\Delta_i N}{N}$$  \hspace{1cm} (3.77)$$

When $N$ is very large, the sum over $x$-intervals, weighted by the expected number of events in each interval, can be replaced by an unweighted sum over generated $x$ values. Equation 3.77 then becomes,

$$\int_{x_0}^{x_1} R(x)dx = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \frac{R(x_i)}{\rho(x_i)}$$  \hspace{1cm} (3.78)$$

If the $x_i$ are accepted values from the importance sampling method, then either $\rho(x_i)$ or $\rho_p(x_i)$ can be used in the above equation.

The extension of this procedure to multi-dimensions is straightforward. We want to integrate the scalar function $R(x)$ over a volume $S$, where $x$ is an $m$-dimensional vector. The probability density function is replaced by the product $\Pi_{i=1}^{m} \rho_i(x)$ so that equation 3.78 becomes,

$$\int_{S} R(x)d^m x = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} \frac{R(x_j)}{\Pi_{i=1}^{m} \rho_i(x_j)}$$  \hspace{1cm} (3.79)$$
3.7.5 Filling the Independent Variables

The line shape of the $R_{ee}$ mass distribution is modeled by a Breit-Wigner distribution with total widths of $\Gamma(\eta_e) = 10.3$ MeV, $\Gamma(\chi_{e0}) = 14.0$ MeV or $\Gamma(\chi_{e2}) = 2.00$ MeV. Starting with a random number, $u$, uniform on $(0,1)$, we generate the $W$ distribution according to,

$$W = M_{R_{ee}} + \frac{\Gamma}{2} \tan\left[\frac{\pi}{2}(2u - 1)\right]$$

(3.80)

For the other variables we use the importance sampling method for multiple dimensions.

The relative azimuthal angle of the scattered electrons $\phi$ is generated uniformly on the interval $(0, 2\pi)$ as, $\phi = 2\pi u$. The normalized envelope function is then $\rho_\phi(\phi) = 1/2\pi$.

The electron scattering angle, $\theta_i$, is sharply peaked towards zero. To generate $\theta_i$ values we use an envelope function proportional to $1/\theta_i$. Following the example of equation 3.72 we generate candidate $\theta_i$ according to

$$\theta_i = \theta_{\min} R_\theta^\theta,$$

(3.81)

where $R_\theta = \theta_{\max}/\theta_{\min}$ with $\theta_{\max} = \pi$ and $\theta_{\min} = 10^{-9}$. The normalized envelope function is then $\rho_\theta(\theta_i) = 1/\theta_i ln(R_\theta)$.

The photon energy, $\omega_1$, is also sharply peaked towards zero. To generate $\omega_1$ values we use an envelope function proportional to $1/\omega_1$. Following the example of equation 3.72 we generate candidate $\omega_1$ values according to

$$\omega_1 = \omega_{\min} R_\omega^\omega,$$

(3.82)

where $R_\omega = \omega_{\max}/\omega_{\min}$, $\omega_{\min} = M^2_{R_{ee}}/4\omega_{\max}$ and $\omega_{\max} = E_0 - m_e$. As a result the normalized envelope function is $\rho_\omega(\omega_1) = 1/\omega_1 ln(R_\omega)$.
The energy of the second photon, $\omega_2$ is $\omega_2 = E_b - E'_2$. The value of $E'_2$ is approximated as

$$E'_2 = \frac{(4E_b^2 + M_e^2 - W^2 - 4E_bE'_1)}{(4E_b - 2E'_1 + 2s_{\omega_1}(2s^2 - 1))}$$

where

$$s_{\omega_1} = \sqrt{(E_b - \omega - m_e)(E_b - \omega + m_e)}$$

and

$$s^2 = \sin^2(\frac{\theta_1 - \theta_2}{2}) + \sin(\theta_1)\sin(\theta_2)\sin^2(\frac{|\pi - \phi|}{2})$$

The $\gamma\gamma$ luminosity function, equation 3.54, is calculated with $q_1^2, q_2^2, \omega_1, \omega_2$ and $\phi$ as independent variables but we have generated $\theta_1, \theta_2, \omega_1, W$ and $\phi$. To account for the change in variables we must multiply by the appropriate Jacobian,

$$J(\theta_1, \theta_2, \omega_1, W) = 4E_b^2 s_{\omega_1} s_{\omega_2} \sin(\theta_1) \sin(\theta_2).$$

We define a ratio of the Jacobian to the product of envelop functions as

$$P(\theta_1, \theta_2, \omega_1, W) = 2\pi \frac{J(\theta_1, \theta_2, \omega_1, W)}{\rho_0(\theta_1)\rho_0(\theta_2)\rho_0(\omega_1)}$$

The factor of $2\pi$ results from the $\phi$ envelop function.

The cross section is then written as,

$$\sigma(e^+e^- \rightarrow e^+e^-\mathcal{R}) = \int_\mathcal{S} \mathcal{L}_{ab}(q_1^2, q_2^2, W^2) \cdot \mathcal{O}_{ab}(q_1^2, q_2^2, W^2)d\Gamma(p_1')d\Gamma(p_2')$$

$$= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \mathcal{L}_{ab} \cdot \sigma_{ab} \cdot P(\theta_1, \theta_2, \omega_1, W)$$

3.8 Modeling Uncertainties

In this thesis we determine the two-photon production cross section of the $\mathcal{R}_{c\bar{c}} \in \{\eta_c, \chi_{c0}, \chi_{c2}\}$ states by examining their decay through several different
Table 18

The ratio $K_{R_{c\bar{c}}} = \Gamma_{MC}(R_{c\bar{c}} \rightarrow \gamma\gamma)/\sigma_{MC}(e^+e^- \rightarrow e^+e^-R_{c\bar{c}})$ as determined from Monte Carlo with $\Gamma_{MC}(R_{c\bar{c}} \rightarrow \gamma\gamma) = 1.0$ keV.

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>$M_\rho$</th>
<th>$M_\psi$</th>
<th>$M_\Upsilon$</th>
<th>$M \rightarrow \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c$</td>
<td>570 ± 1</td>
<td>419 ± 1</td>
<td>365 ± 1</td>
<td>351 ± 1</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>1069 ± 1</td>
<td>802 ± 1</td>
<td>729 ± 1</td>
<td>712 ± 1</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>259 ± 1</td>
<td>196 ± 1</td>
<td>183 ± 1</td>
<td>180 ± 1</td>
</tr>
</tbody>
</table>

modes. The cross section is then related to the two-photon width by the factor $\Gamma_{MC}(R_{c\bar{c}} \rightarrow \gamma\gamma)/\sigma_{MC}(e^+e^- \rightarrow e^+e^-R_{c\bar{c}})$ as determined by the Monte Carlo generator. This factor is highly dependent on the parameters and form factors used in the MC-generator. Each form factor is proportional to the term,

$$1 \left(1 - q_1^2/M^2\right)^2 \left(1 - q_2^2/M^2\right)^2,$$

where $M$ is typically taken to be the mass of one of the vector mesons, such as, $\rho$, $\omega$, $\phi$, $\psi$, or $\Upsilon$ to name a few. The choice of $M$ can result in differences in $\Gamma_{MC}/\sigma_{MC}$ of 30% or more. Table 18 lists the differences in $\Gamma_{MC}/\sigma_{MC}$ resulting from four different mass values, the last of these resulting from taking the limit $M \rightarrow \infty$.

A second consideration is the radial momentum, $P_t$, of the $R_{c\bar{c}}$ states. Figure 50 shows the number of $\eta_c$ states with radial momentum less than the value $PTCUT$ normalized by the luminosity. That is, we plot

$$\frac{1}{L} \int_0^{PTCUT} N_{\eta_c}(P_t) \cdot dP_t \quad \text{vs} \quad PTCUT.$$

We see that the choice of the mass value, used in the form factor, produces the smallest differences at low $PTCUT$. 

110
Figure 50
Integrated number of $\eta_c$ as a function of transverse momentum normalized by luminosity.
3.9 Extracting the Two-Photon Partial Decay Width

The two-photon width of a $c\bar{c}$ resonance, $\Gamma_{\text{Data}}(R_{c\bar{c}} \rightarrow \gamma\gamma)$ can be related to its measured cross section, $\sigma_{\text{Data}}(e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-R_{c\bar{c}})$ by,

$$\Gamma_{\text{Data}}(R_{c\bar{c}} \rightarrow \gamma\gamma) = \Gamma_{MC}(R_{c\bar{c}} \rightarrow \gamma\gamma) \frac{\sigma_{\text{Data}}(e^+e^- \rightarrow e^+e^-R_{c\bar{c}})}{\sigma_{MC}(e^+e^- \rightarrow e^+e^-R_{c\bar{c}})}. \quad (3.92)$$

The ratio $\Gamma_{MC}/\sigma_{MC}$ is determined by the Monte Carlo generator. The measured cross section is determined by,

$$\sigma_{\text{Data}}(e^+e^- \rightarrow e^+e^-R_{c\bar{c}}) = \frac{N_{\text{obs}}}{\varepsilon} \frac{1}{B(R_{c\bar{c}} \rightarrow F_s)L}, \quad (3.93)$$

where $\varepsilon$ is the efficiency for finding the final state $F_s$ and $B(R_{c\bar{c}} \rightarrow F_s)$ is the branching fractions for the decay of $R_{c\bar{c}}$ into $F_s$. The efficiency as determined from Monte Carlo is defined to be,

$$\varepsilon = \frac{N_{MC}}{N_{\text{gen}}}, \quad (3.94)$$

where $N_{\text{gen}}$ is the initial number of generated $R_{c\bar{c}} \rightarrow F_s$ decays and $N_{MC}$ is the number of observed $R_{c\bar{c}} \rightarrow F_s$ decays.
CHAPTER IV

Analysis

4.1 Introduction

At $e^+e^-$ beam energies far enough above the charm threshold the charmonium states $\mathcal{R}_{cc} \in \{\eta_c, \chi_{c0}, \chi_{c2}\}$ can be produced through two-photon interactions. These interactions offer an ideal lab to study the two-photon vector-vector coupling to charmonium states.

In the following we determine the two-photon production cross section of the states $\mathcal{R}_{cc} \in \{\eta_c, \chi_{c0}, \chi_{c2}\}$ by examining their decay through different decay modes. The cross section is then related to the two-photon width by the factor $\mathcal{K}_{\mathcal{R}_{cc}} = \Gamma_{\text{MC}}(\mathcal{R}_{cc} \rightarrow \gamma\gamma)/\sigma_{\text{MC}}(e^+e^- \rightarrow e^+e^-\mathcal{R}_{cc})$ as determined by the Monte Carlo generator. As discussed in section 3.8, this ratio is highly dependent on the form factor used in the MC-generator.

4.2 Common Analysis Features

The decay modes studied in this dissertation have common quantities and features of interest. In this section we discuss these quantities and features.
4.2.1 Track classifications.

Reconstructed tracks from DUET are classified according to their quality and probable origin. To be classified as good, the average RMS residual for hits associated with a track (RESICD) must be less than 0.4 mm. Tracks failing this requirement are classified as junk tracks. The RESICD distribution for a sampling of events is shown in figure 51. A good track is further classified as a GOODPM track if it is consistent with originating from the primary vertex. This requires a minimum separation from the beam crossing in the $r\phi$ plane (DBCD) of less than 5 mm and in the z-direction (Z0CD) of less than 50 mm. The DBCD distribution and Z0CD distribution for a sampling of good tracks are shown in figures 52 and 53. A good track which is not a GOODPM track can result from a neutral particle traveling outside the interaction region before decaying into charged particles. A good track which has failed the GOODPM test is classified as a GOODV track.

In this dissertation we concern ourselves with decays of $\tau^\pm \rightarrow \pi^\pm + \nu_\tau$ which result in exactly four charged particles. Typically, the scattered electron and positron are not detected due to their small scattering angles. Therefore, event candidates are required to have four charged tracks observed in the detector with a net charge of zero.

4.2.2 Calorimeter Shower Classifications

The CsI calorimeter provides excellent rejection of events producing photons and electrons in the detector volume.

A cluster of energy in the electro-magnetic calorimeter is classified as either a matched shower, an isolated shower or a noise shower. The logical tree to determine the shower status is illustrated in figure 54. Showers with energy below 10 MeV need not be considered since they are removed by the sparsification
process of the online system.

All surviving showers starting out as unmatched and then migrate to other classifications based on certain selection criteria. A shower becomes matched if a drift chamber track, projected to the calorimeter surface, is within 10 cm of a crystal contained within the shower.

The electronics associated with any crystal in the remaining unmatched showers must be in good working order. Unmatched showers containing crystals with electronic channels known to be noisy, dead or shorted are identified as noise showers. Unmatched showers with an energy less than 60 MeV are also considered noise.

At this point unmatched showers separated from a matched shower by $\cos(\theta) > 0.97$ are also classified as matched.
Figure 52
Charged track impact parameter in the $r\phi$ plane.

Figure 53
Track impact parameter along the Z-axis.
Any remaining unmatched showers are said to be isolated. Thus an isolated shower has an energy greater than 50 MeV is separated from a matched showers by $\cos(\theta) < 0.97$ and has all electronic channels in working order.

### 4.2.3 Electron and Muon Rejection

We are interested in decays with no electrons, positrons or muons in the final state, therefore, events with an identified electron, positron or muon are rejected.

An electron or positron interacting with the calorimeter deposits nearly all its energy in the calorimeter. The electron(positron) energy is typically deposited in a single shower or a large shower with a few nearby smaller showers. The electron mass is extremely small compared to the momentum needed to reach the calorimeter ($m_e \ll p_{\text{min}} \sim 200 \text{ MeV}$) in CLEO's 1.5T magnetic field. Therefore, the ratio of shower energy to track momentum of an electron(positron) should be
near unity.

A single track can have multiple matched showers. Some of these showers are from other sources, such as random noise, photons from other decays or synchrotron radiation. Since most of the electrons energy is in one shower we search for that shower. The ratio of shower energy to track momentum is found for each matched shower. The ratio nearest to unity (EOVP) is assigned to the track, so that, each track has at most one EOVP value assigned to it.

Figure 33 of Chapter II shows the EOVP distributions for tracks from 1-vs-3 tau events. The 1-prong EOVP values shows a distinct peak at unity while the 3-prong side shows very little peaking near unity. A track with an EOVP satisfying $0.90 < \text{EOVP} < 1.05$ is classified as an electron or positron track.

Muon identification is accomplished through the muon chambers discussed in section 2.2.6. In the barrel region, a minimum momentum of 0.8 GeV/c is needed for a muon to pass through the inner layer of steel and reach the first superlayer. This corresponds to a penetration depth greater than 3 nuclear absorption lengths. Any track detected by the first superlayer of the muon system is classified as a muon track.

4.2.4 Extra Tracks

On occasion DUET will find multiple tracks from drift chamber hits produced by a single particle. Most often one such track best represents the particle while the other tracks gives poor or erroneous information. The routine TRKMAN identifies track multiplication and selects the single track which best represents the particle. Extra tracks fall into three classifications: ghosts, curlers and others. Events with any extra tracks identified by TRKMAN are rejected. Here we discuss the general workings of TRKMAN, the specific cuts used for identifying
extra tracks can be found in the TRKMAN reference.

A ghost pair is a set of two tracks which nearly overlap each other. This can be produced by track multiplication from a single particle or from two particles with nearly equal trajectories. We would like to resolve the former into a single track which best represents the particle while retaining both tracks in the later case.

Since DUET assigns each hit to, at most, one track, two tracks from a single particle tend to produce few hits having common layers. That is; if one ghost has a hit in the $n^{th}$ layer the other will most likely not contain a hit in that layer. On the other hand, two tracks produced by two particles tend to contain hits in the same layers.

TRKMAN compares two nearly overlapping tracks by tallying the layers containing hits on each track(HITHIT) and the layers containing hits on only one of the tracks(EXOR). A high EXOR and low HITHIT indicates the two tracks are produced by a single particle. In this case the track with the smallest number of hits in the stereo layers plus hits in the cathodes is flagged as an extra track. If a low EXOR and high HITHIT is found both tracks are kept.

Curlers can arise when a particle's radial momentum is below ~ 220 MeV with a z-momentum less than 45 MeV. The low radial momentum, in conjunction with the 1.5T magnetic field, curves the particle's $r\phi$-trajectory to such an extent, that it never extends beyond the DR's outer radius. The low z-momentum allows the particle to complete one or more spirals before reaching the REC magnets. If the z-momentum is small the particle returns near the interaction point producing further hits consistent with a particle coming from the interaction point. Multiple tracks are produced as the particle enters and again as it leaves the interaction region.
TRKMAN compares two tracks which agree with a curler hypothesis and selects one track based on transverse momentum, DBCD and Z0CD values. The curler hypothesis requires both tracks to have radial momentum below 217 MeV with a difference of less than 80 MeV/c. Tracks with like charges must have an opening angle less than 0.35 radians while opposite charged tracks must have an opening angle greater than 2.8 radians.

Track pairs are not considered as curlers if both tracks satisfy $|DBCD| < 0.0065m$, $|Z0CD - ZVPTX| < 0.03m$ and $RESICD < 0.00045m$ or if one track satisfies these conditions and has a z-momentum greater than 45 MeV/c.

If a curler produces a number of extra tracks, repeated application of the selection process results, ideally, in a single track.

Extra tracks falling into the “others” classification are found by a likelihood method. The five variables entering into the determination are RESICD, DBCD, Z0CD, NHIT and DENS. NHIT is the number of non-cathode tracking hits associated with a track and DENS is the ratio of the total number of hits associated with the track to the number of layers the track passes through. Tracks generated by Monte Carlo events determine the range of acceptable values.

A GOODV track which is not identified as an electron, positron, muon or extra track is classified as a GOLDV track. These requirements are listed in Table 20.

4.2.5 Trigger

Information from the tracking chambers, time-of-flight scintillators and CsI calorimeter is used to generate an event trigger. The details of the CLEO-II trigger system are described in Chapter II. Events are required to pass at least one of four trigger lines designed to detect events with multiple charged tracks.
Table 19

Trigger efficiencies for decay modes averaged over the $\eta_c$, $\chi_{c0}$ and $\chi_{c2}$ as determined from Monte Carlo.

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>4S2</th>
<th>4S3</th>
<th>4S4</th>
<th>4S5</th>
<th>4S6</th>
<th>4S7</th>
<th>4S8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_s^0 K^\pm \pi^\mp$</td>
<td>0.89</td>
<td>0.92</td>
<td>0.94</td>
<td>0.95</td>
<td>0.91</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>$\pi^+ \pi^- \pi^+ \pi^-$</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

The data spans seven running periods labeled $EXP_n$; $n = 2, 3, ..., 8$ with the corresponding data sets labeled $4Sn$; $n = 2, 3, ..., 8$. The first three data sets each have slightly different trigger requirements. The data sets 4S4 through 4S8 all have the same trigger configuration for the selected trigger lines. Although these triggers are the same the trigger efficiencies vary for different decay modes and resonances. The correction for variations due to different data sets and different decay modes are handled in the same manor. For a given decay mode and initial resonance, 5000 Monte Carlo events are generated for each data set. The analysis code is run on the Monte Carlo and the trigger efficiency determined for events entering the final mass plots. The trigger efficiencies for the seven run periods are labeled $\epsilon(i, mode)$; $i = 2, 3, ..., 8$. The entries into the final mass plots for real data are given a weight $W_e(i, mode) = \bar{\epsilon}(mode)/\epsilon(i, mode)$; where $\bar{\epsilon}(mode)$ is the average $\epsilon(i, mode)$ for $i = 4, 5, ..., 8$ and initial resonances* $\eta_c$, $\chi_{c0}$ and $\chi_{c2}$. Trigger efficiencies for each decay mode and data set, averaged over the relevant $R_{ce}$ states, are listed in Table 19.

*Averaging is performed over the resonances which have measured branching fractions for the decay mode of interest.
4.2.6 The Upper Tail Probability of a Chi-Squared Distribution ($P_{utp}$)

The $dE/dx$ and time-of-flight information assists identification of pions, kaons and protons. Each GOODPM track is required to have a $dE/dx$ measurement. If available, TOF information is included to aid in particle separation.

The chi-squared value, $\xi^2(pidi)$, of the $i^{th}$ track is determined by

$$\xi^2(pidi) = \sigma^2_{dE/dx}(pidi) + \sigma^2_{TOF}(pidi)$$  \hspace{1cm} (4.1)

where $\sigma(pidi)$ is the number of standard deviations between the measured value for the $i^{th}$ track and the expected peak of the distribution, assuming a particle type $pid \in \{\pi, K, p\}$.

The upper tail probability of the chi-squared distribution, $P_{utp}(pidi)$, is the area of the chi-squared distribution (with $N$ degrees of freedom) above the value $\xi^2(pidi)$, where $N$ is the number of $dE/dx$ measurements plus the number of TOF measurements. With the correct particle types selected, the ideal $P_{utp}(PID)$ distribution would be uniform on $[0,1]$.

If particle identification is used in the analysis, a consistency cut requiring $P_{utp}(pidi) > 0.001$ is applied to the track at an early stage. This reduces the number of unwanted events or track combinations. When only $dE/dx$ is measured ($N = 1$), this cut corresponds to $\xi^2(pidi) = \sigma^2_{dE/dx}(pidi) < 10.8$. If both $dE/dx$ and TOF are used ($N = 2$) then, $\xi^2(pidi) < 13.8$. A GOODPM track passing TRKMAN and the consistency cut is classified as a GOLDPM($pid$), with $pid \in \{\pi, K, p\}$.

The upper tail probability can be extended to multiple tracks of mixed particle types. For $m$ tracks with particle types $PID = (pid_1, pid_2, ..., pid_m)$ the chi-
Table 20
Track classification.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Track Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESICD ≤ 0.0004</td>
<td>good</td>
</tr>
<tr>
<td>[DBCDD] ≤ 0.005</td>
<td>*</td>
</tr>
<tr>
<td>[Z0CD] ≤ 0.05</td>
<td>*</td>
</tr>
<tr>
<td>0.9 &lt; EOVP &lt; 1.05</td>
<td>*</td>
</tr>
<tr>
<td>DPTHMU &lt; 3.0</td>
<td>*</td>
</tr>
<tr>
<td>TRKMAN</td>
<td>*</td>
</tr>
<tr>
<td>P upt &gt; 0.001</td>
<td>*</td>
</tr>
</tbody>
</table>

squared value becomes,

\[
\xi^2(PID) = \sum_{pid_i \in \{PID\}} \left[ \sigma^2_{dE/dx}(pid_i) + \sigma^2_{TOF}(pid_i) \right]
\]  \hspace{1cm} (4.2)

The upper tail probability of the chi-squared distribution, \(P_{upt}(PID)\), is the area of the chi-squared distribution (with \(N\) degrees of freedom) above the value \(\xi^2(PID)\) where \(N\) is the total number of \(dE/dx\) measurements plus TOF measurements for the \(m\) particles.

4.2.7 Getting \(N_{obs}\) from the Mass Distributions

Once all particles for the decay are identified the invariant mass \((W_{\tau\tau})\) of the candidate is determined and entered into the mass plot. To determine the number of observed events \((N_{obs})\) in a plot the invariant mass distribution is fit with a function(s) for the signal(s) and a function for the background. A minimum chi-squared fit is performed to find the best functional description of the mass plot. The error in the number of entries per histogram bin is taken as the square root of entries in that bin. The Monte Carlo produces a large signal with a small combinatoric background, since only the reaction \(e^+e^- \rightarrow e^+e^-R_{\ell\ell} \rightarrow\)
\( e^+e^- \mathcal{F}_s \) is generated. The background in data contains continuum two-photon events \((e^+e^- \rightarrow e^+e^- \mathcal{F}_s)\) with unknown parameters and processes other than two-photon interactions. No attempt is made in the Monte Carlo to simulate these sources of background. Instead the mass plot is fit using a power function for the \( \eta_c \rightarrow K_s^0 K^\pm \pi^\mp \) background, or an exponential function for the \( R_{cs} \rightarrow \pi^+\pi^-\pi^+\pi^- \) background.

The power function has the form,

\[ N_{bg} = A W_{\gamma\gamma}^m, \quad (4.3) \]

where \( W_{\gamma\gamma} \) is the histogram variable while \( A \) and \( m \) are the fit parameters. The exponential background has the form,

\[ N_{bg} = A e^{-m W_{\gamma\gamma}}, \quad (4.4) \]

again, \( W_{\gamma\gamma} \) is the histogram variable while \( A \) and \( m \) are the fit parameters.

The signal is fit to the sum of two Gaussian functions;

\[ N_s = \frac{A_1}{\sigma_1 \sqrt{2\pi}} \exp\left(-\frac{(W_{\gamma\gamma} - m_1)^2}{2\sigma_1^2}\right) + \frac{A_2}{\sigma_2 \sqrt{2\pi}} \exp\left(-\frac{(W_{\gamma\gamma} - m_2)^2}{2\sigma_2^2}\right). \quad (4.5) \]

The first Gaussian is always chosen to have the smaller of the two widths. For fitting purposes the sum of two Gaussian-s is parameterized by the six variables:

1. Total area of the two Gaussian distributions; \( A = A_1 + A_2 \)
2. Standard deviation of first Gaussian; \( \sigma = \sigma_1 \)
3. Mean of first Gaussian; \( m = m_1 \)
4. Fraction of total area contributed by second Gaussian; \( A_r = A_2/A \)
5. Ratio of widths; \( \sigma_r = \sigma_2/\sigma_1 \)
6. Difference in mean values; \( \delta m = m_1 - m_2 \)
Table 21
The ratio $K_{R_{c\bar{c}}} = 1/\sigma_{MC}$ as determined from Monte Carlo with a $\psi$ form factor.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$K_{R_{c\bar{c}}} (nb/keV)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c$</td>
<td>419 ± 1</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>802 ± 1</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>196 ± 1</td>
</tr>
</tbody>
</table>

$W_{\gamma\gamma}$ is the histogram variable while $A$, $\sigma$, $m$, $A_r$, $\sigma_r$ and $\delta m$ are the fit parameters. The data signal is fit with $A$ as free parameters. When a significant signal is present $m$ is also a free parameters. All other parameters for the data signal are fixed to values determined from Monte Carlo.

4.2.8 Extracting the Two-Photon Width.

The two-photon width of a $c\bar{c}$ resonance, $\Gamma_{Data}(R_{c\bar{c}} \rightarrow \gamma\gamma)$ can be related to its measured cross section, $\sigma_{Data}(e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-R_{c\bar{c}})$ by,

$$\Gamma_{Data}(R_{c\bar{c}} \rightarrow \gamma\gamma) = K_{R_{c\bar{c}}} \cdot \sigma_{Data}(e^+e^- \rightarrow e^+e^-R_{c\bar{c}}).$$ (4.6)

The ratio $K_{R_{c\bar{c}}} = \Gamma_{MC}/\sigma_{MC}$ is determined through the numerical evaluation of the cross section, $\sigma_{MC}$, by the Monte Carlo generator for a given two-photon width, $\Gamma_{MC}$. The two-photon width is taken to be 1 keV in all cases so that $K_{R_{c\bar{c}}} = 1/\sigma_{MC}$. Table 21 list the values determined for $K_{R_{c\bar{c}}}$ from Monte Carlo employing a $\psi$ form factor.

The cross section from data, determined by observing $N_{obs}(R_{c\bar{c}} \rightarrow F_s)$ events, is written as,

$$\sigma_{Data}(e^+e^- \rightarrow e^+e^-R_{c\bar{c}}) = \frac{N_{Data}(R_{c\bar{c}})}{L_{Data}} \Gamma_{Data}$$ (4.7)

$$= \frac{N_{Data}(R_{c\bar{c}} \rightarrow F_s)}{B(R_{c\bar{c}} \rightarrow F_s)L_{Data}}$$ (4.8)
where $\varepsilon$ is the efficiency for finding the final states $F_s$.

### 4.3 Decay Mode $\eta_c \rightarrow K_s^0 K^\pm \pi^\mp \rightarrow \pi^+ \pi^- K^\pm \pi^\mp$

We consider the production of the $\eta_c$ meson through the two-photon reaction,

\[ e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-\eta_c \rightarrow e^+e^-K_s^0 K^\pm \pi^\mp \rightarrow e^+e^-\pi^+\pi^- K^\pm \pi^\mp. \]  

The final state $e^+e^-\pi^+\pi^- K^\pm \pi^\mp$ is composed of four charged hadrons, plus the unobserved scattered electron and positron. Event candidates are required to have exactly four charged tracks with a net charge of zero. Two of the four tracks must be GOODPM tracks, while the remaining two tracks can be either GOODPM or GOODV tracks which are consistent with a $K_s^0$ hypothesis.

#### 4.3.1 Identifying the $K_s^0$

To identify the decay $K_s^0 \rightarrow \pi^+\pi^-$ we take advantage of the large $K_s^0$ decay length ($c\tau = 2.675$ cm). A $K_s^0$ candidate ($V_0$) is sought using an algorithm which searches for pairs of oppositely charged tracks intersecting at a distance, in $r-\phi$, greater than 1.0 mm from the interaction point. The interaction point is the run averaged beam position determined from the intersection of all charged tracks in the event. The invariant mass and momentum of $V_0$ is calculated using the track parameters of the two tracks evaluated at the secondary vertex assuming pion masses. Tracks originating from the interaction point are suppressed by requiring each to have an impact parameter in $r-\phi$ greater than 30% of the error in the impact parameter measurement. The angle between the $V_0$ position vector (with respect to the beam position) and momentum vector is required to be less
than $\pi/2$. The two tracks recoiling against the $K^0_s$ candidate are required to be consistent with originating from the interaction point.

The $V_0$ mass spectrum resulting from the above requirements is shown in figure 55. A fit using a Breit-Wigner function, for the signal, and a flat background is performed. The resulting $K^0_s$ mass value is $m_{K^0_s} = 497.8\pm 0.4$ MeV/c$^2$, in agreement with the accepted value $^{37}$ of 497.67 $\pm$ 0.03 MeV/c$^2$. The $V_0$ is accepted as a $K^0_s$ if its invariant mass falls within 12.0 MeV/c$^2$ of the peak value. This region contains 15,000 $\pm$ 900 reconstructed $K^0_s$ decays on a background of 7,000 $\pm$ 500. We reject events containing a second $K^0_s$ by combining the two remaining tracks assuming pion masses. The event is rejected if this invariant mass falls within 12.0 MeV/c$^2$ of the peak value. This reduces the signal peak by approximately 1000 entries. Requirements discussed in the next section reduces the number of $K^0_s$ candidates to 984 $\pm$ 38 on a background of 156 $\pm$ 8.

4.3.2 The $K^0_sK^\pm\pi^\mp$ Mass Plot

The two tracks recoiling against the $K^0_s$ are assigned $K$ and $\pi$ masses and a $K_s^0K^\pm\pi^\mp$ mass value calculated for the two possible $K\pi$ combinations.

Unwanted background is reduced by applying the requirements listed in Table 22. These cut values are determined by the algorithms discussed in section 4.3.4. Events from $e^+e^-$ annihilation are suppressed by limiting the total energy of the $K_s^0K^\pm\pi^\mp$ candidate ($E_{vis}$) is to the range $E_{vis} < 6.0$ GeV. To reduce the model uncertainty introduced by the form factor employed in the Monte Carlo generator, the net transverse momentum ($P_t$) of the four track system is required to be less than 0.2 GeV/c. Background events containing neutral particles are suppressed by requiring the total isolated shower energy ($E_{iso}$) in the calorimeter to be less than 0.5 GeV.
Figure 55

$V_0$ mass distribution. The curve is the resulting fit using a Breit-Wigner distribution and a flat background.

Entries into the mass plot are weighted using $dE/dx$ and time-of-flight information. For the 2 recoiling tracks, labeled 1 and 2, each track is required to be a GOLDPM track for the assumed particle type.

The upper tail probability of a chi-squared distribution, $P_{uTP}(PID_i)$, is determined for each of the two combinations (see section 4.2.6). The $\xi_i^2$ values of the $K\pi$ combinations are determined by

$$\xi^2(PID_1) = \sigma_{dE/dx}(K_1) + \sigma_{TOF}(K_1) + \sigma_{dE/dx}(\pi_2) + \sigma_{TOF}(\pi_2)$$  \hspace{1cm} (4.11)$$

and

$$\xi^2(PID_2) = \sigma_{dE/dx}(\pi_1) + \sigma_{TOF}(\pi_1) + \sigma_{dE/dx}(K_2) + \sigma_{TOF}(K_2).$$  \hspace{1cm} (4.12)$$

The upper tail probabilities for the two possible combinations are shown in fig-
Table 22
Requirements imposed on the candidates total energy, unmatched shower energy and transverse momentum.

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>$E_{vis}$ ≤</th>
<th>$E_{iso}$ ≤</th>
<th>$P_t$ ≤</th>
<th>$P_{UTP}$ ≥</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0 s K^± π^\mp$</td>
<td>6.0</td>
<td>0.5</td>
<td>0.2</td>
<td>0.05</td>
</tr>
<tr>
<td>$π^+ π^- π^+ π^-$</td>
<td>6.5</td>
<td>0.65</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure 56. In the ideal case, the correct choice of particle combinations produces a uniform distribution on (0,1) while the incorrect particle combinations tend to congregate near zero.

Figure 57 shows a two-dimensional histogram of $C1 = 1 - P_{utp}(PID_1)$ verses $C2 = 1 - P_{utp}(PID_2)$. We histogram $1 - P$ so that the large peaks near zero do not obscure the rest of the plot.

Those combinations with $P_{utp}(PID_i) > 0.05$ are entered into the final mass plots. We see, from figure 57, that most candidates fail the $P_{utp}$ cut while the remaining events are dominated by a single acceptable combination. For events with a single entry to the mass distribution, the retained combination is given a weight of one. For events with multiple entries the weight is determined by
\[
w_i = \frac{P_{utp}(PID_i)}{P_{utp}(PID_1) + P_{utp}(PID_2)}; \quad i = 1, 2
\]

The resulting mass spectrum for Monte Carlo events is shown in Figure 58. The mass spectrum is fit with the sum of two Gaussian distributions as described in section 4.2.7.

The mass spectrum for data is fit using two Gaussian distributions for the signal and a power function for the background. The fit has 5 free parameters; the mean, width and area of the first Gaussian plus the amplitude and exponent.
Figure 56
Upper tail probability for $K\pi$ combinations in the decay $\eta_c \rightarrow K^0_s K^{\pm}\pi^\mp$. The first bin, containing over 800 entries, have been truncated at 40 entries, allowing the structure at higher probabilities to be observed.

Figure 57
$C_1 = 1 - P_{up}(PID_1)$ vs $C_2 = 1 - P_{up}(PID_2)$ for the two possible $K\pi$ combinations in the decay $\eta_c \rightarrow K^0_s K^{\pm}\pi^\mp$. 
Figure 58
$W_{\gamma\gamma}$ mass distribution from the $\eta_c \rightarrow K_s^0 K^\pm \pi^\mp$ Monte Carlo sample. The curve is the sum of two Gaussian functions.

Figure 59
$W_{\gamma\gamma}$ mass distribution assuming $K_s^0 K^\pm \pi^\mp$ decay channel. The curve is the sum of two Gaussian functions for the signal and a power function, $N = N_0 W^x$, for the background.
of the power function. The ratio of widths and the difference in means of the two Gaussians are fixed by the Monte Carlo, as is their relative area. The resulting mass spectrum for data is shown in figure 59. We observe $54.1 \pm 12.6$ events with a mean of $2.980 \pm 0.004 \text{GeV/c}^2$. We have exceptional agreement with the the 1994, “Particle Data Group”\[^{37}\] value of $2.9788 \pm 0.0019 \text{GeV/c}^2$.

### 4.3.3 Cross Section and Two-Photon Width

The $\eta_c$ two-photon width can be related to the measured $\eta_c$ cross section by equation 4.6. The ratio $\frac{\Gamma_{\text{Data}}}{\sigma_{\text{MC}}}$ = 419 is determined by evaluating equation 3.93, when employing a $\psi$ form factor, as described in Chapter III. The two-photon width becomes,

$$\Gamma_{\text{Data}}(\eta_c \to \gamma\gamma) = (419 \text{ KeV/nb}) \cdot \sigma_{\text{Data}}(e^+e^- \to e^+e^-\eta_c). \quad (4.13)$$

The cross section is determined by the relationship,

$$\sigma_{\text{Data}}(e^+e^- \to e^+e^-\eta_c) = \frac{N_{\text{obs}}}{\varepsilon} \cdot \frac{1}{B(\eta_c \to K^0_sK^{\pm}\pi^{\mp})B(K^0_s \to \pi^+\pi^-)} \cdot C \quad (4.14)$$

where $\varepsilon$ is the efficiency for finding the decay $\eta_c \to K^0_sK^{\pm}\pi^{\mp} \to \pi^+\pi^-K^{\pm}\pi^{\mp}$.

The efficiency is determined by performing the same analysis on the Monte Carlo sample and forming the ratio,

$$\varepsilon \equiv \frac{N_{\text{MC}}}{N_{\text{gen}}}, \quad (4.15)$$

where $N_{\text{gen}}$ is the initial number of generated $\eta_c \to K^0_sK^{\pm}\pi^{\mp} \to \pi^+\pi^-K^{\pm}\pi^{\mp}$ decays and $N_{\text{MC}}$ is the area of the $K^0_sK^{\pm}\pi^{\mp}$ mass spectrum for Monte Carlo. For $N_{\text{gen}} = 26024$ events we find $N_{\text{MC}} = 3043 \pm 55 \pm 304$. The first uncertainty, $\Delta N_{\text{MC}} = \sqrt{N_{\text{MC}}}$, reflects the statistical uncertainty in fitting the mass plot of figure 58. The second uncertainty is due to systematic uncertainties in $N_{\text{MC}}$ which are considered in the next section.
The branching ratio $B(\eta_c \rightarrow K K \pi) = 0.066 \pm 0.018^{[37]}$ is the largest single source of error in our calculation. This value is also subject to change at a later date. To facilitate ease in recalculation and provide a more enduring number, we present results both with and without evaluating this branching ratio.

With an integrated luminosity$^{[38]}$ of $\mathcal{L} = 3.00 \pm 0.03$ fb$^{-1}$, we determine the product of the branching ratio and cross section to be,

$$B(\eta_c \rightarrow \pi^+ \pi^- K^0 \pi^\mp) \cdot \sigma_{\text{Data}}(e^+e^- \rightarrow e^+e^-\eta_c) = (6.7 \pm 1.6 \pm 1.1) \times 10^{-4} \text{nb.} \ (4.16)$$

The uncertainties, from left to right are statistical and systematic. The sources for systematic uncertainties are discussed in detail in the next section.

With the above branching ratios for $B(\eta_c \rightarrow K K \pi)$ the $\eta_c$ cross section is found to be,

$$\sigma_{\text{Data}}(e^+e^- \rightarrow e^+e^-\eta_c) = (1.0 \pm 0.24 \pm 0.17 \pm 0.33) \times 10^{-2} \text{nb,} \quad (4.17)$$

where the first uncertainty is the statistical uncertainties in fitting the Monte Carlo and data mass plots, the second is due to the systematic uncertainty due to our experiment and the last is the uncertainties in branching fractions. Returning to equation (4.13) we find the product of the two-photon width and $B(\eta_c \rightarrow K^0 K^\pm \pi^\mp)$ as,

$$B(\eta_c \rightarrow K K \pi) \cdot \Gamma_{\text{Data}}(\eta_c \rightarrow \gamma\gamma) = 0.28 \pm 0.066 \pm 0.048 \text{KeV.} \quad (4.18)$$

Inclusion of the branch ratio value yields a two-photon width of,

$$\Gamma_{\text{Data}}(\eta_c \rightarrow \gamma\gamma) = 4.3 \pm 1.0 \pm 0.72 \pm 1.2 \text{ KeV} \quad (4.19)$$

### 4.3.4 Systematic Checks

The $K^0 K^\pm \pi^\mp$ invariant mass plot, shown in figure 59, consists of the $\eta_c$ signal and background. In this section we examine our primary variables and quantify
the effects they have on our signal and background. We consider the statistical 
uncertainties and systematic uncertainties with respect to the four main variables:

1. \( P_t \) - the total transverse momentum of the \( K^0 K^\pm \pi^\mp \) system.

2. \( E_{vis} \) - the total energy of the \( K^0 K^\pm \pi^\mp \) system.

3. \( E_{iso} \) - the total unmatched energy deposited in the calorimeter.

4. \( P_{UTP} \) - the upper tail probability from the \( dE/dx \) and TOF for the \( K\pi \) 
candidate tracks.

The range of these variables are limited by requiring \( E_{vis} < EV\text{CUT}, E_{iso} < 
EIC\text{UT}, P_t < P\text{CUT} \) and \( P_{UTP} > XPC\text{UT} \). We would like to select the cut 
values which produce the smallest over all uncertainties.

**The Special Case of \( P\text{CUT} \)**

In section 3.8 we compared the event yield when employing a \( \rho \) form factor 
to that obtained with a \( \psi \) form factor. At the generator level we found a 20% 
difference at a large \( P\text{CUT} \) value while there was little or no difference at 
\( P\text{CUT} = 0 \). We expect a similar result from Monte Carlo after running our 
analysis. Figure 60 shows the event yield from Monte Carlo as a function of 
\( P\text{CUT} \) for five different form factors. The entry for a particular \( P\text{CUT} \) value 
was found by generating a \( K^0 K^\pm \pi^\mp \) mass plot of events with \( P_t < P\text{CUT} \) 
and fitting the signal to a double Gaussian distribution. The total area of the 
Gaussians is then scaled to the same luminosity by dividing the area by the 
appropriate value taken from Table 18†

†The \( \phi \) form factor was added after the work in chapter III. It has a \( \kappa_{\phi} \) value of 530.
Figure 60
Number of $\eta_c$ events with transverse momentum less than $P_t$ utilizing five different form factors.

The event yield for the $\rho$ and $\psi$ form factors differ by approximately 20% for $PTCUT = 2.0$ GeV/c and by approximately 10% near zero. The error was determined by the relation,

$$\% Error \equiv \frac{100}{N_{ave}} \times \sqrt{\frac{(N_{\rho} - N_{ave})^2 + (N_{\psi} - N_{ave})^2}{2}}. \quad (4.20)$$

One could suppose that the choice of form factors could be limited by normalizing the Monte Carlo luminosity to the data luminosity and determining which form factor best agrees with the data. This is not the case since the Monte Carlo luminosity depends on an assumed two-photon width. All Monte Carlo is generated with an assumed two-photon width of 1 keV. This is why different Monte Carlo samples can be normalized to the same luminosity.
\( \eta_c \) yield for the range \( P_t \leq \text{PTCUT} \). The error bars represent the statistical uncertainty when fitting the mass spectrum to a data signal plus background. The \( \eta_c \) yield from Monte Carlo, generated using a \( \rho \) form factor and a \( \psi \) form factor, is fit to the data.

An attempt was made to determine if the shape of the \( \eta_c \) yield as a function of \( \text{PTCUT} \) would lend any insight into the proper form factor. Figure 61 shows the \( \eta_c \) yield verses \( \text{PTCUT} \) for data, the \( \rho \) form factor and the \( \psi \) form factor. The Monte Carlo curves have been fit to the data. Since both nearly overlap the data, no determination can be made.

**Statistical Uncertainties and \( \text{PTCUT} \)**

We would like to select the cut values which produce the most significant signal. The fractional statistical uncertainty is given as \( \sqrt{S + B}/S \), where \( S \) is the number of signal events and \( B \) is the number of background events under the signal region. We search for the cut value which minimizes the ratio \( \sqrt{S + B}/S \).
or equivalently maximizes the ratio $S^2/(S + B)$.

Figure 61 shows that the $\eta_c$ yield in data and Monte Carlo agree in $PTCUT$ up a scale factor. To avoid prejudice introduced by fluctuations of the data, we determine the change in $S$ from Monte Carlo. The area of the Monte Carlo mass distribution within $\pm 3\sigma$ of its peak is found for $PTCUT$ values from 0.05 GeV/c to 1.8 GeV/c. The Monte Carlo is scaled by the factor $S_{data}/S_{MC}$ determined at $PTCUT = 1.8$ GeV/c. The value of $B$ is the area of the background found in data for the same range. The background function is determined by fitting the data mass plot with a double Gaussian for the signal and a power function for the background. The power function is then integrated over the region within $\pm 3\sigma$ of the Monte Carlo peak.

Figure 62(a) shows the ratio $S^2/(S + B)$ as a function of $PTCUT$. From this plot the $PTCUT$ value with greatest significant would be at or above 1.0 GeV/c. Unfortunately, figure 60 indicates that a higher $PTCUT$ produces a greater systematic error due to the form factor question. The statistical uncertainty is given by $\sqrt{S + B}/S$. At $PTCUT = 1.0$ GeV/c this is 22% while the systematic uncertainty is estimated to be 17%. At $PTCUT = 0.2$ GeV/c the statistical uncertainty increases to approximately 25% while the systematic uncertainty drops to near 10%. Since the overall uncertainty is less at $PTCUT = 0.2$ GeV/c we take this as our cut value.

The Rest of the Statistical Uncertainties

The other three cuts of interest are $EVCUT$, $EICUT$ and $XPCUT$. The ratio $S^2/(S + B)$ as a function of $EVCUT$ is shown in figures 62(b). The statistical uncertainty of our signal flattens to a minimum of approximately $100 \times \sqrt{S + B}/S = 24\%$ for $EVCUT > 4.5$ GeV. We choose a cut value above this shoulder at 6.0 GeV.
The ratio $S^2/(S + B)$ as a function of $EICUT$ is shown in figures 62(c). The statistical uncertainty of our signal flattens to a minimum for $EICUT$ above $0.25$ GeV. Cutting in the flat section above this shoulder, we choose to reject events with $E_{iso} > 0.5$ GeV. The ratio $S^2/(S + B)$ as a function of $XPCUT$ is shown in figures 62(d). The $XPCUT$ plot shows a steady decrease in the significance of our signal as we increase $XPCUT$. Cutting slightly above the maximum at 0, we reject events with $P_{UTP} < 0.05$

**More Systematic Uncertainties**

Contributions to the systematic uncertainties due to our four cuts are determined by varying their values within reasonable limits and examining the fluctuation in the two-photon width.

The $PTCUT$ value is varied from $0.1$ GeV/c to $2.0$ GeV/c in steps of $0.1$. For this range the $\eta_c$ yield in both Monte Carlo and data varies by $\sim 50\%$ with only slight fluctuations above $0.6$ GeV/c. This can be seen in figure 61. The ratio of the $\eta_c$ yield in Monte Carlo to that in data is shown in figure 63a. The plot has been normalized by the factors found in equations 4.13 and 4.17 to produce a plot of the two-photon width as a function of $PTCUT$. We estimate the systematic uncertainty in the two-photon width, due to the choice of $PTCUT$, as the RMS fluctuations from the average value,

$$\%Error \equiv \frac{100}{\Gamma_{ave}} \times \sqrt{\frac{\sum (\Gamma_i - \Gamma_{ave})^2}{N}},$$

(4.21)

where $\Gamma_i$ is the two-photon width for $PTCUT_i$. We find approximately $6\%$ uncertainty due to $PTCUT$.

The $EVCUT$ value is varied from $3.7$ GeV to $7.0$ GeV in steps of $0.3$ GeV. For this range the $\eta_c$ yield varies by nearly $20\%$ in both Monte Carlo and data.
Figure 62
Ratio of squared $\eta_c$ yield to sum of yield and background, a) For PTCUT values  
b) For EVCUT values  
c) For EICUT values  
d) For XPCUT values
Table 23
Systematic uncertainties in the \( \eta_c \rightarrow K^0 K^\pm \pi^\mp \) analysis

<table>
<thead>
<tr>
<th>Source</th>
<th>uncertainty(%)</th>
<th>Source</th>
<th>uncertainty(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTCUT</td>
<td>6</td>
<td>Tracking</td>
<td>4</td>
</tr>
<tr>
<td>EVCUT</td>
<td>3</td>
<td>Trigger</td>
<td>5</td>
</tr>
<tr>
<td>EICUT</td>
<td>4</td>
<td>Modeling(form factor)</td>
<td>10</td>
</tr>
<tr>
<td>XPCUT</td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The two-photon width varies by approximately 3% on this range, as can be seen from figure 63b.

The \( EICUT \) value is varied from 0.0 GeV to 0.4 GeV in steps of 0.05 GeV. For this range the \( \eta_c \) yield varies by nearly 30% in both Monte Carlo and data. The two-photon width varies by approximately 4% in this range, as can be seen from figure 63c.

The \( XPCUT \) value is varied from 0.0 to 0.9 in steps of 0.05. The two-photon width varies by approximately 9% in this range, as can be seen from figure 63d.

Other systematic uncertainties include a 1% per track uncertainty due to tracking\(^{[39]}\) and 5% due to trigger simulations in the Monte Carlo. The systematic uncertainties are listed in table 23.

### 4.4 The Decays \( R_{c\bar{c}} \rightarrow \pi^+\pi^-\pi^+\pi^- \)

We next examine the production of \( R_{c\bar{c}} \in \{ \eta_c, \chi_{c0}, \chi_{c2} \} \) through the reaction,

\[
e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-R_{c\bar{c}} \rightarrow e^+e^-\pi^+\pi^-\pi^+\pi^- .
\]

Event candidates are required to have exactly four reconstructed charged tracks in the detector with a net charge of zero. All four tracks must be GOLDPM tracks assuming a pion particle type. Candidate events must also satisfy the
Figure 63
Two-photon width as a function of (a) $PTCUT$, (b) $EVCUT$, (c) $EICUT$ and (d) $XPCUT$
requirements listed in table 22.

The upper tail probability of the chi-squared distribution is determined assuming all four tracks to be pions. The $\chi^2$ value for the event is given by,

$$\chi^2 = \sum_{i=1,4} \left[ \sigma_{dE/dx}(\pi_i) + \sigma_{TOP}(\pi_i) \right].$$

(4.23)

The $P_{up}(PID)$ distribution for $\pi^+\pi^-\pi^+\pi^-$ candidates is shown in figure 64. In making figure 64, the requirements listed in table 22, except for XPCUT, are applied and the $4\pi$ invariant mass restricted to the range $2.8 \text{ GeV} < M_{4\pi} < 3.7 \text{ GeV}$. Events containing one or more tracks with $P_{up}(\pi_i) < 0.003$ have also been rejected.

Candidates with $P_{up}(PID)$ greater than 0.1 are accepted into the $\pi^+\pi^-\pi^+\pi^-$ invariant mass plots. The mass distribution around the $\eta_c$ mass value is shown in figure 65. The lack of a signal leads to an upper limit for the branching fraction.
Signals for both the $\chi_{c0}$ and $\chi_{c2}$ are observed in figure 66. We are able to calculate two-photon widths for both resonances.

We first examine the $\chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^-$ and $\chi_{c2} \rightarrow \pi^+\pi^-\pi^+\pi^-$ signals. The mass distribution in figure 66 is fit with a double Gaussian for the $\chi_{c0}$ signal, a double Gaussian for the $\chi_{c2}$ signal, and an exponential function for the background. Each double Gaussian is fit to data with the area of the first Gaussian as a free parameter. The ratio of areas, for the two Gaussians forming a signal, are fixed by Monte Carlo. The width and mean of the Gaussians are also fixed by Monte Carlo when determining the event count.

The double Gaussian, for the $\chi_{c0}$ signal, has a total area of 47.2 ± 15.5. If we release the constraint on the mean of the signal, we obtain a mass of 3.413 ± 0.004 GeV/c² with the event yield increasing by 5%. This is in excellent agreement with the accepted mass value of 3.4151 ± 0.001 GeV/c².\footnote{37}

The $\chi_{c2}$ double Gaussian has a total area of 41.9 ± 13.8. Releasing the constraint on the mean of the signal results in a mass of 3.565 ± 0.007 GeV/c² with the event yield increasing by 12%. This is in good agreement with the accepted mass value of 3.55617 ± 0.00013 GeV/c².\footnote{37}

### 4.4.1 Decay Mode $\chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^-$

Concentrating first on the $\chi_{c0}$ signal, the two-photon width is determined from equation 4.6. This is written as,

$$\Gamma_{\pi^+\pi^-\pi^+\pi^-}(\chi_{c0} \rightarrow \gamma\gamma) = (802 \text{ KeV/nb}) \cdot \sigma_{\pi^+\pi^-\pi^+\pi^-}(e^+e^- \rightarrow e^+e^-\chi_{c0}),$$

\[ (4.24) \]

where the ratio $\Gamma_{MC}/\sigma_{MC} = 802$ is taken from table 18.

The efficiency for finding the decay $\chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^-$, as determined from
Figure 65

$\pi^+\pi^-\pi^+\pi^-$ mass distribution near the $\eta_c$ mass range. The curve is the sum of two Gaussian functions for the signal and an exponential for the background.

Figure 66

$\pi^+\pi^-\pi^+\pi^-$ mass distribution in the $\chi_{c0}$ and $\chi_{c2}$ mass range. The sum of two Gaussian functions is used for each signal with an exponential function for the background.
Monte Carlo, is $\varepsilon = 0.202 \pm 0.002$. The branching fraction is $B(\chi_{c0} \to \pi^+ \pi^- \pi^+ \pi^-) = 0.037 \pm 0.007$. With a luminosity of $L = 3.00 \pm 0.03$ fb$^{-1}$, the product of the $\chi_{c0}$ cross section and branching ratio is,

$$B(\chi_{c0} \to \pi^+ \pi^- \pi^+ \pi^-) \cdot \sigma_{\pi^+ \pi^- \pi^+ \pi^-}(e^+e^- \to e^+e^-\chi_{c0}) = 7.9 \pm 2.6 \pm 1.9 \text{ nb.}$$

(4.25)

The first uncertainty is statistical, while the second is due to systematics. The systematic uncertainties are discussed in detail in the next section.

Including the value for the branching ratio, the cross section is found to be,

$$\sigma_{\pi^+ \pi^- \pi^+ \pi^-}(e^+e^- \to e^+e^-\chi_{c0}) = (2.1 \pm 0.7 \pm 0.5 \pm 0.4) \times 10^{-3} \text{ nb.}$$

(4.26)

From equation 4.24 the product of the two-photon width and branching ratio is,

$$B(\chi_{c0} \to \pi^+ \pi^- \pi^+ \pi^-) \cdot \Gamma_{\pi^+ \pi^- \pi^+ \pi^-}(\chi_{c0} \to \gamma\gamma) = (6.4 \pm 2.1 \pm 1.5) \times 10^{-2} \text{ KeV,}$$

(4.27)

resulting in the two-photon width of,

$$\Gamma_{\pi^+ \pi^- \pi^+ \pi^-}(\chi_{c0} \to \gamma\gamma) = 1.7 \pm 0.6 \pm 0.4 \pm 0.3 \text{ KeV.}$$

(4.28)

The event yields for Monte Carlo and data are compared, as a function of $PTCUT$, in figure 67. The Monte Carlo curve has been fit to the data curve allowing direct comparison of the shapes of the two distributions. Considering the statistical uncertainties shown for the data points, the two distributions agree.

The two-photon width resulting from four different form factors are compared in figure 68. The error bars reflect the statistical uncertainty for events generated with the $\psi$ form factor. All four curves nearly overlap for $PTCUT < 0.5$ GeV. The three curves with form factors having a mass at or above the $\psi$ mass also overlap at large $PTCUT$ values. For $PTCUT > 0.5$ GeV the $\rho$ form factor
results in a width approximately 25% larger then the others. Considering the size of the error bars, all four distributions are in agreement.

4.4.2 Systematic Checks of \( \chi c_0 \rightarrow \pi^+ \pi^- \pi^+ \pi^- \)

We consider the systematic uncertainties with respect to the four cuts: \( PTCUT \), \( EVCUT \), \( EICUT \), and \( XPCUT \). The systematic uncertainties are found by varying the cut values around the final value used in the analysis and determining the corresponding fluctuations in our two-photon width. Before evaluating the systematic uncertainties we must determine the four cut values. The primary factor in selecting a cut value is the signal to background ratio. We would like to cut at a value where this ratio is a maximum.

Statistical Uncertainties

The procedure to determination cut values is similar to that used for the \( K^0_S K^{\pm} \pi^\mp \) analysis, found in section 4.3.4. For each cut value we determine the signal from Monte Carlo and the background from data. The signal is determined by fitting the Monte Carlo mass distribution with two Gaussian functions. The signal(\( S \)) is the area of the two Gaussians within 3\( \sigma \) of the mass peak and normalized to the data signal. The normalization factor is \( N_0(data)/N_0(MC) \), where, \( N_0(MC) \) is the largest event yield found in Monte Carlo and \( N_0(data) \) is the event yield of data at that same cut value. This occurs when \( PTCUT = 1.6 \text{ GeV/c} \).

To determining the background(\( B \)), the mass distribution from data is fit with two Gaussian functions for the signal and an exponential function for the background. The exponential function is integrated over the same region as was used to determine the signal in Monte Carlo. Figures 69a-d show the ratio \( S^2/\sqrt{(S+B)} \) as a function of the cut values \( PTCUT \), \( EVCUT \), \( EICUT \), and
$\chi_{c0}$ yield for the range $P_t \leq PTCUT$. The error bars represent the statistical uncertainty when fitting the mass spectrum to a signal plus background in data. The $\chi_{c0}$ yield from Monte Carlo, generated using a $\psi$ form factor, is also shown. The Monte Carlo has been normalized so that the average yield is the same as found in the data. Therefore, only the shapes of the distributions should be compared.

Comparison of two-photon width as a function of $PTCUT$ for four different form factors in the Monte Carlo generator. Error bars are shown for only the $\psi$ form factor entries.
Unlike the $\eta_c \rightarrow K^0 K^{\pm} \pi^\mp$ analysis, the event yield as a function of $PTCUT$ peaks at low $PTCUT$, as shown in figure 69a. We are able to maximize the significants of the signal, with respect to $PTCUT$, while minimizing the systematic error due to the choice of form factor. The $PTCUT$ value producing the maximum $S^2/\sqrt{S+B}$ should produce the most significant signal. We accept events in the region $PTCUT < 0.2$ GeV/c.

The ratio $S^2/\sqrt{S+B}$ as a function of $EVCUT$ is shown in figure 69b. We see that the distribution flattens at a maximum near $EVCUT = 5.5$ GeV. We therefore accept events with $E_{vis} < 6.5$ GeV.

For $EICUT$, shown in figure 69c, the ratio $S^2/\sqrt{S+B}$ reaches a maximum near 0.25 GeV and is approximately constant there after. Events with $E_{iso} < 0.5$ GeV are accepted.

The ratio $S^2/\sqrt{S+B}$ as a function of $XPCUT$ is shown in figure 69d. A maximum is seen just below $XPCUT = 0.1$. Events with an upper tail probability satisfying $P_{UTP} > 0.1$ are accepted in our final mass plot.

**Systematic Uncertainties**

To estimate the uncertainties produced by a cut value we vary the cut value, while holding the others fixed, and determine the variation in the calculated two-photon width.

The variations in the two-photon width as a function of $PTCUT$ is shown in figure 70a. We estimate the uncertainty in the two-photon width, due to the
Figure 69
Ratio of squared $\chi^2$ yield to sum of yield and background for the quantities: a) PTCUT, b) EVCUT, c) EICUT and d) XPCUT. Cuts placed at the maximum of each plot should produce the most significant signals.
choice of $PTCUT$, as the RMS fluctuations from the average value,

$$\%\text{Error} \equiv \frac{100}{\Gamma_{\text{ave}}} \times \sqrt{\frac{\sum_i^n (\Gamma_i - \Gamma_{\text{ave}})^2}{N}}, \quad (4.29)$$

where $\Gamma_i$ is the two-photon width for $PTCUT$. We find approximately 17% uncertainty due to $PTCUT$.

The $EVCUT$ value is varied from 3.7 GeV to 7.0 GeV in steps of 0.3 GeV. The two-photon width, shown in figure 70b, varies by approximately 10% on this range. The $EICUT$ value is varied from 0.0 GeV to 1.0 GeV in steps of 0.05 GeV. The two-photon width, shown in figure 70c, varies by approximately 4% on this range. The $XPCUT$ value is varied from 0.0 to 0.8 in steps of 0.05. Figure 70d shows the resulting distribution of points. The estimated variation in the two-photon width is approximately 10%.

The systematic uncertainties are listed in table 24. Other systematic uncertainties include a 1% per track uncertainty due to tracking$^{39}$ and 5% due to trigger simulations in the Monte Carlo. We have include estimates for these uncertainties in table 24.

\subsection*{4.4.3 Decay Mode $\chi_{c2} \rightarrow \pi^+\pi^-\pi^+\pi^-$}

The $\chi_{c2}$ signal of section 4.4 has a total area of $41.9 \pm 13.8$. For the $\chi_{c2}$, the ratio $\Gamma_{MC}/\sigma_{MC} = 196$ from table 18 results in a two-photon width determined

\begin{table}
\centering
\caption{Systematic uncertainties for $\chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^-$.}
\begin{tabular}{|c|c|}
\hline
Source & uncertainty(\%) \\
\hline
$PTCUT$ & 17 \\
$EVCUT$ & 10 \\
$EICUT$ & 4 \\
$XPCUT$ & 10 \\
Tracking & 4 \\
Trigger & 5 \\
Modeling(form factor) & 12 \\
Overall & 26 \\
\hline
\end{tabular}
\end{table}
Figure 70

The two-photon width of the $\chi_{c0}$ as a function of (a) $PTCUT$, (b) $EVCUT$, (c) $EICUT$, (d) $XPCUT$ provide estimates of uncertainties due to these cuts.
by,
\[
\Gamma_{\pi^+\pi^-\pi^+\pi^-}(\chi_{c2} \to \gamma\gamma) = (196 \text{ KeV/nb}) \cdot \sigma_{\pi^+\pi^-\pi^+\pi^-}(e^+e^- \to e^+e^-\chi_{c0}).
\] (4.30)

The efficiency for finding the decay $\chi_{c2} \to \pi^+\pi^-\pi^+\pi^-$, as determined from Monte Carlo, is $\epsilon = 0.214 \pm 0.002$. The branching fraction is $B(\chi_{c2} \to \pi^+\pi^-\pi^+\pi^-) = 0.022 \pm 0.005$. With a luminosity of $L = 3.00 \pm 0.03 \text{ fb}^{-1}$, the product of the $\chi_{c0}$ cross section and branching ratio is,
\[
B(\chi_{c2} \to \pi^+\pi^-\pi^+\pi^-) \cdot \sigma_{\pi^+\pi^-\pi^+\pi^-}(e^+e^- \to e^+e^-\chi_{c2}) = (7.5 \pm 2.5 \pm 1.4) \times 10^{-5} \text{ nb}.
\] (4.31)

The uncertainties are due to statistics and systematics. Sources of systematic uncertainties are discussed in the next section.

Including the branching ratio value, the cross section is,
\[
\sigma_{\pi^+\pi^-\pi^+\pi^-}(e^+e^- \to e^+e^-\chi_{c2}) = (3.4 \pm 1.1 \pm 0.6 \pm 0.8) \times 10^{-3} \text{ nb}. \] (4.32)

The product of the two-photon width and branching ratio is,
\[
B(\chi_{c2} \to \pi^+\pi^-\pi^+\pi^-) \cdot \Gamma_{\pi^+\pi^-\pi^+\pi^-}(\chi_{c2} \to \gamma\gamma) = (1.5 \pm 0.5 \pm 0.3) \times 10^{-2} \text{ KeV},
\] (4.33)
resulting in the two-photon width,
\[
\Gamma_{\pi^+\pi^-\pi^+\pi^-}(\chi_{c2} \to \gamma\gamma) = 0.7 \pm 0.2 \pm 0.1 \pm 0.2 \text{ KeV}.
\] (4.34)

The uncertainties, from left to right, are statistical, systematic due to the data analysis and systematic due to the branching fractions.

The signal size as a function of $PTCUT$ is shown in figure 71 for both data and Monte Carlo. The Monte Carlo plot has been scaled by the average ratio of the data to Monte Carlo event yield, so that their shapes may be directly compared.
The error bars reflect the statistical uncertainty of the data points. Considering these uncertainties the shape of the two distributions are in agreement.

The two-photon width resulting from four different form factors are compared in figure 72. The plots resulting from form factors with mass values at or above the \( \psi \) mass nearly overlap. The plot employing the \( \rho \) mass nearly agrees with the others at low \( PTCUT \) values but disagrees by approximately 25% for \( PTCUT > 0.5 \text{ GeV} \). Considering the statistical uncertainties, shown for the \( \psi \) form factor only, no real distinction can be made between the four distributions.

4.4.4 Systematic Checks of \( \chi_{c2} \rightarrow \pi^+\pi^-\pi^+\pi^- \).

We consider the systematic uncertainties with respect to the four cuts: \( PTCUT, EVCUT, EICUT, \) and \( XPCUT \). The systematic uncertainties are found by varying the cut values around the final value used in the analysis and determining the corresponding fluctuations in our two-photon width. Before evaluating the systematic uncertainties we must determine the four cut values. The primary factor in selecting a cut value is the signal to background ratio. We would like to cut at a value where this ratio is a maximum.

Statistical Uncertainties

The procedure to determine the four cut values is similar to that for the \( \chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^- \) analysis found in section 4.4.2. For each cut value we determine the signal from Monte Carlo and the background from data. The signal is determined by fitting the Monte Carlo mass distribution with two Gaussian functions. The signal(S) is the area of the two Gaussians within 3\( \sigma \) of the mass peak and normalized to the data signal. The normalization factor is \( N_0(data)/N_0(MC) \), where, \( N_0(MC) \) is the largest event yield found in Monte Carlo and \( N_0(data) \) is
\( \chi c_2 \) yield for the range \( P_t \leq PTCUT \). The error bars represent the statistical uncertainty when fitting the mass spectrum to a data signal plus background. The \( \chi c_2 \) yield from Monte Carlo, generated using a \( \psi \) form factor, is also shown.

\( \chi c_2 \) two-photon width as a function of \( PTCUT \) for four different form factors in the Monte Carlo generator. Error bars are shown for only the \( \psi \) form factor entries.
the event yield of data at that same cut value.

To determining the background(B), the mass distribution from data is fit with two Gaussian functions for the signal and an exponential function for the background. The resulting exponential function is integrated over the same region as was the Monte Carlo mass peak. Figures 73a-d show the ratio $S^2/\sqrt{(S + B)}$ as a function of the cut values $PTCUT$, $EVCUT$, $EICUT$, and $XPCUT$.

The event yield as a function of $PTCUT$ peaks at low $PTCUT$, as shown in figure 73a. We are able to maximize the significantes of the signal, with respect to $PTCUT$, while minimizing the systematic error due the choice of form factor. The $PTCUT$ value producing the maximum $S^2/\sqrt{(S + B)}$ should produce the most significant signal. We accept events in the region $PTCUT < 0.2$ GeV/c.

The ratio $S^2/\sqrt{(S + B)}$ as a function of $EVCUT$ is shown in figure 73b. We see that the distribution flattens at a maximum near an $EVCUT$ value of 5.5 GeV. We therefore accept events with $E_{vis} < 6.5$ GeV.

For $EICUT$, shown in figure 73c, the ratio $S^2/\sqrt{(S + B)}$ reaches a maximum near 0.25 GeV and is approximately constant there after. Therefore, events with $E_{iso} < 0.5$ GeV as accepted.

The ratio $S^2/\sqrt{(S + B)}$ as a function of $XPCUT$ is shown in figure 73d. A maximum is seen just below $XPCUT = 0.1$. Events with an upper tail probability satisfying $P_{UTP} > 0.1$ are accepted in our final mass plot.

**Systematic Uncertainties**

To estimate the uncertainties due to $PTCUT$, $EVCUT$, $EICUT$, and $XPCUT$ we vary each cut in turn, while holding the others fixed, and determine the variation in event yield.
Figure 73
Ratio of squared $\chi^2$ yield to sum of yield and background for the quantities: (a) PTCUT, (b) EVCUT, (c) EICUT and (d) XPCUT
Table 25
Systematic uncertainties for $\chi_c \rightarrow \pi^+\pi^-\pi^+\pi^-$. 

<table>
<thead>
<tr>
<th>Source</th>
<th>uncertainty(%)</th>
<th>Source</th>
<th>uncertainty(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PTCUT$</td>
<td>14</td>
<td>Tracking</td>
<td>4</td>
</tr>
<tr>
<td>$EVCUT$</td>
<td>16</td>
<td>Trigger</td>
<td>5</td>
</tr>
<tr>
<td>$EICUT$</td>
<td>6</td>
<td>Modeling(form factor)</td>
<td>12</td>
</tr>
<tr>
<td>$XPCUT$</td>
<td>22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The two-photon width as a function of $PTCUT$ is shown in figure 74a. We estimate the uncertainty in the two-photon width, due to the choice of $PTCUT$, as the RMS fluctuations from the average value, as defined by equation 4.29. We find approximately 14% uncertainty due to $PTCUT$.

The $EVCUT$ value is varied from 3.7 GeV to 7.0 GeV in steps of 0.3 GeV. The two-photon width, shown in figure 74b, varies by approximately 16% on this range. The $EICUT$ value is varied from 0.0 GeV to 1.0 GeV in steps of 0.05 GeV. The two-photon width, shown in figure 74c, varies by approximately 6% on this range. The $XPCUT$ value is varied from 0.0 to 0.8 in steps of 0.05. Figure 74d shows the resulting distribution of points. The estimated variation in the two-photon width is approximately 22%.

The systematic uncertainties are listed in table 25. Other systematic uncertainties include a 1% per track uncertainty due to tracking and 5% due to trigger simulations in the Monte Carlo. Estimates for these uncertainties are included in table 25.
Figure 74
The two-photon width of the $\chi_2$ as a function of (a) $PTCUT$, (b) $EVCUT$, (c) $EICUT$ and (d) $XPCUT$ provides an estimates of uncertainties due to these cuts.
4.4.5 Decay Mode $\eta_c \to \pi^+\pi^-\pi^+\pi^-$

We next turn our attention to the $\eta_c$ mass region of the $4-\pi$ mass plot shown in figure 65. From the $K^0\bar{K}^{\pm}\pi^{\mp}$ analysis of section 4.3 we expect the number of observed $\eta_c \to \pi^+\pi^-\pi^+\pi^-$ events to be:

$$N_{\text{obs}}(\pi^+\pi^-\pi^+\pi^-) = \frac{N_{\text{obs}}(\pi^+\pi^-K^{\pm}\pi^{\mp}) \cdot B(\pi^+\pi^-\pi^+\pi^-) \varepsilon(\pi^+\pi^-\pi^+\pi^-)}{B(\pi^+\pi^-K^{\pm}\pi^{\mp}) \varepsilon(\pi^+\pi^-K^{\pm}\pi^{\mp})}$$

(4.35)

The efficiency for finding the decay $\eta_c \to \pi^+\pi^-\pi^+\pi^-$ as determined from Monte Carlo is $\varepsilon = 0.16 \pm 0.01$. The branching fraction is $B(\eta_c \to \pi^+\pi^-\pi^+\pi^-) = 0.012 \pm 0.004$. We therefore expect to observe an $\eta_c$ peak of approximately 59 ± 36 events in the 4$\pi$-mass plot.

The 4$\pi$-mass distribution of figure 65 is fit with a double Gaussian for the signal and an exponential function for the background. The fit results in an area of $-3 \pm 20$ reconstructed $\eta_c$ events. The observed events are far less than the expected 59 events, but, considering the uncertainty of ±36, the two values are consistent. Due to the lack of a signal we determine an upper limit for the two-photon width of the $\eta_c$.

In determining an upper limit we search for the value $\Gamma_{CL}$ such that the probability of the “true” value falling in the range $\Gamma_{\gamma\gamma} < \Gamma_{CL}$ is $CL$. The value $CL$ is called the confidence level of the upper limit $\Gamma_{CL}$. For a $CL$ of 90% we find the value $\Gamma_{CL}$ such that 90% of the distribution’s area falls within the region $(\infty, \Gamma_{CL})$.

Similarly, from equation 4.35 we can set an upper limit for the branching fraction $B(\eta_c \to \pi^+\pi^-\pi^+\pi^-)$. Solving for $B(\pi^+\pi^-\pi^+\pi^-)$ we have,

$$B(\pi^+\pi^-\pi^+\pi^-) = \frac{N_{\text{obs}}(\pi^+\pi^-\pi^+\pi^-) \cdot B(\pi^+\pi^-K^{\pm}\pi^{\mp}) \varepsilon(\pi^+\pi^-K^{\pm}\pi^{\mp})}{N_{\text{obs}}(\pi^+\pi^-K^{\pm}\pi^{\mp}) \varepsilon(\pi^+\pi^-K^{\pm}\pi^{\mp})}$$

(4.36)

(4.37)

The upper limit on the branching fraction is determined numerically from...
the product of values from two distributions. The first distribution, for $N_{obs}$, is assumed to be a Gaussian with a mean of 0 and width of 20.

The second distribution is from the factor $A_0$. The uncertainty in $A_0$ arise from the $K^0_sK^{\pm}\pi^{\mp}$ analysis and the efficiency of finding the $\pi^{+}\pi^{-}\pi^{+}\pi^{-}$. The branching ratio for the decay $\eta_c \rightarrow \pi^{+}\pi^{-}K^{\pm}\pi^{\mp}$ is $B(\pi^{+}\pi^{-}K^{\pm}\pi^{\mp}) = 0.015\pm0.004$. The uncertainty in the efficiency, $\epsilon(\pi^{+}\pi^{-}K^{\pm}\pi^{\mp})$, are listed in Table 23. Added in quadrature, we have an uncertainty of approximately 17% in $\epsilon(\pi^{+}\pi^{-}K^{\pm}\pi^{\mp})$.

The uncertainty in the efficiency, $\epsilon(\pi^{+}\pi^{-}\pi^{+}\pi^{-})$, should arise from the same sources as those for the $\chi_{c0} \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}$ and $\chi_{c2} \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}$ analysis. These include $PTCUT$, $EVCUT$, $EICUT$ and $XPCUT$. Each should have values roughly those seen in the $\chi_{c0}$ and $\chi_{c2}$ analysis. For each source of error we take the larger of the uncertainties found in the $\chi_{c0}$ and $\chi_{c2}$ analysis. We also include a 5% uncertainty for the trigger and 4% for tracking. Added in quadrature, these uncertainties total approximately 33%.

Including all the errors listed above, the $A_0$ value is $A_0 = (2.2 \pm 1.1) \times 10^{-4}$. The $A_0$ distribution is assumed to be Gaussian with a mean of $2.2 \times 10^{-4}$ and width of $1.1 \times 10^{-4}$.

The product $X_i = N_i A_i$ is formed, where $N_i$ and $A_i$ are randomly selected values from the $N_{obs}$ and $A_0$ distributions. Since a negative $N_{obs}$ is unphysical we keep only the positive $N_i$ values. We generate $10^5$ $X$ values and order them in increasing values. We then count down $10^4$ entries from the top and take that as our value at a 90%CL. This yields an upper limit of;

$$B(\eta_c \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}) < 0.008 \text{ at } 90\% \text{ CL.}$$ (4.38)

This is consistent with previous measurements$^{37}$ of the $\eta_c \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}$ branching fraction given as $B(\eta_c \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}) = 0.012 \pm 0.004\%$.

The two-photon width is related to the measured cross section by equation 4.6.
Including the ratio $\Gamma_{MC}/\sigma_{MC} = 419$ from table 18 this is written as,

$$\Gamma_{\pi^+\pi^-\pi^+\pi^-}(\eta_c \rightarrow \gamma\gamma) = \left(419 \text{ KeV/nb}\right) \cdot \sigma_{\pi^+\pi^-\pi^+\pi^-}(e^+e^- \rightarrow e^+e^-\eta_c). \quad (4.39)$$

The cross section is given by,

$$\sigma_{\pi^+\pi^-\pi^+\pi^-}(e^+e^- \rightarrow e^+e^-\eta_c) = \frac{N_{obs}}{e \cdot B(\eta_c \rightarrow \pi^+\pi^-\pi^+\pi^-) L} \equiv N_{obs} \times L. \quad (4.40)$$

The upper limit on the cross section is determined numerically. The cross section is the product of values from two distributions. The first distribution, for $N_{obs}$, is assumed to be the positive half of a Gaussian with a mean of 0 and a width of 20. The second distribution is for the factor $L \equiv 1/eB(\eta_c \rightarrow \pi^+\pi^-\pi^+\pi^-) L$. The $L$ distribution is assumed to be the positive values of a Gaussian with a mean of $1.74 \times 10^{-4}$ and a width of $8.14 \times 10^{-5}$.

The products $\sigma_i = N_iL_i$ are formed, where $N_i$ and $L_i$ are randomly selected values from the $N_{obs}$ and $L$ distributions. We generate $10^5 \sigma_i$ values and order them in increasing values. We then count down $10^4$ entries from the top to find our $\sigma$ value at a 90% CL.

We find an upper limit, for the cross section, of,

$$\sigma_{\pi^+\pi^-\pi^+\pi^-}(e^+e^- \rightarrow e^+e^-\eta_c) < 0.0059 \text{ nb} \quad (CL = 90\%). \quad (4.41)$$

The upper limit for two-photon width is determined by multiplying the upper limit for the cross section by the factor 419 KeV/nb. The resulting upper limit for two-photon width is,

$$\Gamma_{\pi^+\pi^-\pi^+\pi^-}(\eta_c \rightarrow \gamma\gamma) < 2.4 \text{ KeV} \quad (CL = 90\%). \quad (4.42)$$
CHAPTER V

Conclusion

5.1 Summary of Results

During the period November 1990 to September 1993, CESR has produced a luminosity of over $3.00 \pm 0.03 \text{ fb}^{-1}$ for the CLEO-II detector. This is over 6 times the luminosity used for previous two-photon measurements at $e^+e^-$ colliders. We have improved on previous measurements by increasing the statistics by a factor of 4 or more and by reducing the systematic uncertainties. From this data we have determined the two-photon width for the \( \eta_c \), \( \chi_{c0} \) and \( \chi_{c2} \).

The \( \eta_c \) was searched for in the decay channels \( \eta_c \rightarrow K^0_s K^{\pm} \pi^\mp \) and \( \eta_c \rightarrow \pi^+\pi^-\pi^+\pi^- \). Examining the \( \eta_c \rightarrow K^0_s K^{\pm} \pi^\mp \) decay we determined a two-photon width of \( \Gamma_{\gamma\gamma}(\eta_c) = 4.3 \pm 1.0 \pm 0.7 \pm 1.2 \text{ keV} \). This result is shown in figure 75 along with earlier measurements. Our value is consistent with the other measurements.

Estimates for the two-photon decay width of the \( \eta_c \) derived from other decay modes were presented in section 1.8. Taking the ratio of the \( \eta_c \rightarrow \gamma\gamma \) decay width to the \( \psi \rightarrow ee \) decay width lead to an estimation of \( \Gamma_{\gamma\gamma}(\eta_c) \approx 9.2 \pm 0.4(\pm 43\%) \text{ keV} \). Our result is in agreement with this estimate.

A second estimate, presented in section 1.8, is found through the ratio of the
$\Gamma_{\gamma\gamma}(\eta_c)$ measurements by other experiments, QCD estimates from other processes and our measurement.

two-photon with to the light-hadron width, $\Gamma(\eta_c \rightarrow \gamma\gamma)/\Gamma(\eta_c \rightarrow gg)$. A light-hadron width of $\Gamma(\eta_c \rightarrow gg) = 10.3 \pm 3.6 \text{ MeV}^{[37]}$ results in a two-photon width of $\Gamma(\eta_c \rightarrow \gamma\gamma) = 4.7 \pm 1.6(\pm 40\%) \text{ keV}$. Our result agrees exceptionally well with this estimate.

Applying the results of the $K_s^0 K^+\pi^-$ analysis to the $\pi^+\pi^-\pi^+\pi^-$ analysis, we determine an upper limit for the $\eta_c \rightarrow \pi^+\pi^-\pi^+\pi^-$ branching fraction to be $B(\eta_c \rightarrow \pi^+\pi^-\pi^+\pi^-) < 0.008$ at 90% $CL$. This is consistent with previous measurements$^{[37]}$ of the $\eta_c \rightarrow \pi^+\pi^-\pi^+\pi^-$ branching fraction given as $B(\eta_c \rightarrow \pi^+\pi^-\pi^+\pi^-) = 0.012 \pm 0.004\%$.

The invariant mass plot of the $4\pi$ system, figure 65, does show an excess of $47 \pm 22$ events at approximately $30 \text{ MeV}/c^2$ above the expected mass value.$^{[37]}$ This could possibly be an $\eta_c$ signal but is over $4\sigma$ above the expected mass value. Since both the $\chi_{c0}$ and $\chi_{c2}$ mass values are within a few $\text{ MeV}/c^2$ of their reported
values\textsuperscript{[37]}, and the $K_s^0K^{\pm}\pi^{\mp}$ analysis reproduces the world averaged $\eta_c$ mass\textsuperscript{[37]} we conclude that a $4\sigma$ offset is too extreme to accept this excess as an $\eta_c$ signal.

The $\chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^-$ analysis resulted in a two-photon width of $\Gamma_{\gamma\gamma}(\chi_{c0}) = 1.7 \pm 0.6 \pm 0.4 \pm 0.3$. The QCD based equation 1.16 compares the gluon-gluon partial width to the two-photon partial width in next to leading order of $\alpha_s$. Due to the large uncertainty in the hadronic width, $\Gamma(\chi_{c0} \rightarrow gg) = 14 \pm 5$ MeV\textsuperscript{[37]} we form the ratio $\Gamma(\chi_{c0} \rightarrow \gamma\gamma)/\Gamma(\chi_{c2} \rightarrow gg)$. The resulting estimate, given in table 9, is $\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 1.9 \pm 0.2(\pm 21\%)$ keV. The first uncertainty is due to the uncertainty in the reported hadronic width. The second uncertainty is the fractional change due to the next to leading order term in the expansion 1.16.

The BBL approach of section 1.8 estimates a two-photon width of $\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 3.1 \pm 0.56(\pm 36\%)$. The first uncertainty results from the uncertainties in the hadronic widths of the $\chi_{c2}$ and $\chi_{c1}$. The second uncertainty is an estimate of errors resulting from relativistic effects and perturbative errors.

The uncertainties of both estimates are large, as are the uncertainties of our measurement. Our measurement agrees exceptionally well with the first estimate and is within $2\sigma$ of the second estimate.

The $\chi_{c2} \rightarrow \pi^+\pi^-\pi^+\pi^-$ analysis results in a two-photon width of $\Gamma_{\gamma\gamma}(\chi_{c2}) = 0.7 \pm 0.2 \pm 0.1 \pm 0.2$. This value is consistent with previous measurements as listed in table 77.

The QCD based equation 1.16 compares the gluon-gluon partial width to the two-photon partial width in next to leading order of $\alpha_s$. A value of $\Gamma(\eta_c \rightarrow gg) = 1.71 \pm 0.16$ MeV\textsuperscript{[27]} results in an estimate for the two-photon width of $\Gamma(\chi_{c2} \rightarrow \gamma\gamma) = 1.1 \pm 0.1(\pm 33\%)$ keV. The first uncertainty is due to the uncertainty in the reported hadronic width. The second uncertainty is the fractional change due to the next to leading order term in the expansion 1.16. This estimate is slightly
Table 26
Summary of $\eta_c$, $\chi_{c0}$ and $\chi_{c2}$ measurements.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Number observed</th>
<th>Branching Fraction(%)</th>
<th>$Br \cdot \Gamma_{\gamma\gamma}$ $0.01 \times$ keV</th>
<th>$\Gamma_{\gamma\gamma}$ keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c \rightarrow \pi^+\pi^-K^{\pm}\pi^{\mp}$</td>
<td>54.1 ± 12.6</td>
<td>1.5 ± 0.4</td>
<td>6.5 ± 1.5 ± 1.1</td>
<td>4.3 ± 1.0 ± 0.7 ± 1.4</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^+\pi^-$</td>
<td>−3 ± 20</td>
<td>&lt; 0.9CL = 90%</td>
<td>&lt; 2.7CL = 90%</td>
<td>&lt; 2.4CL = 90%</td>
</tr>
<tr>
<td>$\chi_{c0} \rightarrow \pi^+\pi^-\pi^+\pi^-$</td>
<td>47.2 ± 15.5</td>
<td>3.7 ± 0.7</td>
<td>6.4 ± 2.1 ± 1.5</td>
<td>1.7 ± 0.6 ± 0.4 ± 0.3</td>
</tr>
<tr>
<td>$\chi_{c2} \rightarrow \pi^+\pi^-\pi^+\pi^-$</td>
<td>41.9 ± 13.8</td>
<td>2.2 ± 0.5</td>
<td>1.5 ± 0.5 ± 0.3</td>
<td>0.7 ± 0.2 ± 0.1 ± 0.2</td>
</tr>
</tbody>
</table>

larger than our measurement but, considering the uncertainties, they are in good agreement.

The BBL approach of section 1.8 estimates a two-photon width of $\Gamma(\chi_{c2} \rightarrow \gamma\gamma) = 0.83 ± 0.16(±36\%)$. The first uncertainty results from the uncertainties in the hadronic widths of the $\chi_{c2}$ and $\chi_{c1}$. The second uncertainty is an estimate of errors resulting from relativistic effects and perturbative errors. Considering the large uncertainties, our measurement is in excellent agreement with this estimate.

A further check can be made by examining the ratio $D(0,2) = \Gamma(\chi_{c0} \rightarrow \gamma\gamma)/\Gamma(\chi_{c2} \rightarrow \gamma\gamma)$. From our data we have $D_{\text{data}}(0,2) = 2.4 ± 1.3 ± 0.64 ± 1.0$. For the KMRR equations $D_{\text{RGG}}(0,2) = 6.8(±30\%)$ and for the BBL equations $D_{\text{BBL}}(0,2) = 3.8(±36\%)$. Our value is in good agree considering the uncertainties.
Figure 76
\( \Gamma_{\gamma\gamma}(\chi_{c0}) \) determined by other measurements, determined by theory and our measurement.

Figure 77
\( \Gamma_{\gamma\gamma}(\chi_{c2}) \) determined by other measurements, determined by theory and our measurement.
5.2 Future Plans

The tremendous amount of data gathered by the CLEO-II detector is continuously growing. At the present rate the data sample should nearly doubled by the end of 1995. Further improvements can be made on these measurements through the decay modes presented in this thesis. In addition, with the increased data, many other decay modes should be visible, including those resulting in neutral particles. One large source of uncertainty is the proper choice of a form factor in the Monte Carlo. With the proper form factor known the restrictions on the transverse momentum could be lifted. This would approximately double the number of charmonium events observed in the mass plots.

There is still much to do in the study of two-photon production of charmonium states and much to learn from these processes.
Appendix A

Cosmic Ray Tests of CsI(Tl) Crystals

A.1 Introduction

Once the calorimeter was installed into the CLEO-II detector, access to the crystals, diodes and preamplifiers became impossible without disassembling most of the detector. To insure a reliable system, extensive tests were performed on the crystals and electronics at each stage of construction. The last of these tests, before loading the crystals into the crystal holder, was a cosmic ray test. This was also the first test after the 4-diodes were mounted onto the crystal and the only test, other than an actual beam test, with the complete chain of electronics: diodes, preamplifier, mixer/shaper and LeCroy Fastbus ADC. The test verified that all four photo-diodes were working and producing acceptable pulse heights.

The CsI(Tl) crystals deteriorate when they absorb water. The cosmic ray tests were conducted in the "dry-room"; a temperature and humidity controlled building temporarily constructed for storage and maintenance of the 7785 CsI(Tl) crystals. An air conditioning system maintained a cool, dry environment inside the dry-room. An inside air pressure slightly above atmospheric pressure reduced the inflow of untreated air through wall joints and access doors. Crystals were stored in 8 large lockers with a continuous flow of nitrogen gas providing a dry
storage area. Nitrogen, vented from the lockers, helped maintain a low humidity environment throughout the dry-room. Once received from the manufacturer all work on the crystals was performed inside the dry-room.

The cosmic ray test facility, illustrated in figures 78 and 79, was designed to test 64 crystals per 8 hour run. The test facility was composed of two light-tight aluminum cabinets, each holding 32 crystals, and four scintillation counters for trigger purposes. Eleven layers of drift chambers were also employed for tracking, but due to time limitations this information was not used to a great extent.

A.2 Testing Cabinets

Each cabinet contained eight sliding drawers in a $2 \times 2 \times 2$ array. The east and west cabinet walls were mounted on hinges to provide access doors. During testing the doors were fastened closed by ten quick release latches. When closed, the door pressed the drawers against the back of their slide rails, holding them snugly in place. Each drawer held four crystals, with the diode side facing front, and one preamplifier board mounted near the front of the drawer. A short "flat cable" connected the 4-diodes of a single crystal to the preamplifier board with one board handling the 16 signals from 4 crystals. A PC board, mounted in the drawer handle, served as a feed through for the preamplifier output. The output connectors on the PC boards were aligned with slots cut through the cabinet door. Compressible matting around the connectors and door frames provided a light-tight seal when the door was latched closed. The two cabinets were stacked, one upon the other, producing sixteen columns of four crystals; eight columns on the east side and eight columns on the west side.

The preamplifiers were connected to mixer/shaper(m/s) boards by 10m long twisted wire pairs. Each m/s board processed signals from 8 crystals. The m/s
Figure 78
Cosmic Ray test facility (Side view).
Scintillation Counters

Drift Chambers (Z)

Drift Chambers (Y)

Crystals

Light Tight Test Cabinets

Spare Layer

Steel Muon Filter

Figure 79
Cosmic Ray test facility (End view).
circuit summed the 4-diode signals of a crystal producing a single output pulse. The m/s circuit also contained a pole-zero shaper to eliminate the long tail of the preamplifier pulse and a circuit to drive twisted-pair cables to the ADCs. Any preamplifier channel could be disconnected at the m/s card by computer control with the gain automatically adjusted to compensate for the change. The signal from the m/s card was attenuated by a factor of 20 at the ADC and integrated over a 1μsec gate. LeCroy FastBus model 1885N ADCs, with 96 channels per card, digitized the analog pulse for readout by the computer system.

### A.3 Tracking Chambers

The tracking chambers consisted of 11 layers of tubular proportional drift cells*. Each layer was constructed of 9 aluminized cardboard rectangular tubes with dimensions 100cm x 8cm x 2.5cm. The cell anode was a 50μm gold plated tungsten wire stung down the tube center. The nine tubes were glued side-by-side forming a unit approximately 75cm wide. Each unit was coated with epoxy providing a gas-tight seal and enclosed in a box made of perforated sheet aluminum to shield against unwanted RF sources.

The potential difference between wire and tube wall was held at 2.4kV. Using a gas mixture of 90% Argon and 10% Methane, maintained just above atmospheric pressure, an average electron drift velocity of 45μm/nsec was obtained. When a particle traverses a cell, the distance between the wire and particle path was taken as the product of the average drift velocity and the drift time of the electrons.

A high pass filter, illustrated in figure 80, was used to isolate the DC high voltage of the drift tubes from the rest of the electronics while passing the pulse produced by the electron avalanche. The output pulse had a typical amplitude

*Thanks to the Los Alamos E645 experiment for providing the proportional drift chambers for the cosmic ray test facility.
of 2.0mV with an 80nsec rise-time. The drift chamber outputs were connected to LeCroy 2735B Wire Chamber Discriminator cards, mounted at the south end of the test facility, via 1.5m long micro-coaxial cables. Each card had 16 channels with 6 cards providing a total of 96 channels. The output of the LeCroy 2735B card was a differential ECL signal suitable for driving a twisted pair cable. The output duration was equal to the time over threshold where the discriminator threshold was set to 0.3mV. Twisted pair cables, approximately 10m long, connected the ECL outputs to an ECL-to-NIM converter which provided the start pulses to the TDC module. The LeCroy 1879 TDC module provided 96 TDC channels with a resolution of 2nsec. The TDC measured the time between a start pulse, generated by the the drift chamber pulse, and the arrival of the common stop pulse, generated by the trigger.

The separation ($d$) between wire and particle track was determined by,

$$d = v[t_0 - (t_{stop} - t_{start})]$$  \hspace{1cm} (A.1)

where $v$ was the average electron drift velocity and $t_0$ was the time interval between start and stop pulses if the track were to strike the wire.

The coordinate system for the test facility was defined so that the $Z$-axis was
along the length of the crystal, the Y-axis was parallel to the cabinet top and
the X-axis was vertical to the cabinet top. Measurements in the Z-direction were
provide by 6 “z-layers”; the three outer most layers above the crystals and three
outer most layers below the crystals. Y-measurements were determined by 5 “y-
layers” ; the 3 layers just above the crystals and 2 layers just below the crystals.
In case of failure a spare y-layer was positioned just below the lower two y-layers.
With the exception of the two top y-layers, which were aligned with each other,
alternate layers were offset by 2 ± 0.5cm, to reduce left-right ambiguity. The
layers were positioned manually with a accuracy of about 0.5cm. After initial
placement the wire positions were determined through software.

The y-layers were positioned so that one tube per layer covers a single crystal.
Due to limited electronics only 96 channels were available for the drift chambers.
Since the center tubes of the y-layers did not directly cover any crystals the center
tubes of the three outer most y-layers were disconnected.

The proportional drift chambers had an average hit efficiency near 90% with
a spatial resolution of approximately 1mm.

A.4 Trigger

The scintillation counters were constructed of a 90 × 33 × 0.95cm piece of
Bicron BC408 plastic with a light guide formed from ultra-violet transmitting
plastic and an Amperex XP2262B photo-multiplier tube. A 4nsec rise-time was
achieved using an in-house designed base. An operating voltage of 1.25kV pro-
duced an output pulse with an amplitude of approximately 150mV. A discrimi-
nator threshold of 35mV produced the best signal to noise ratio and a particle
detection efficiency of nearly 98%.

The trigger was divided into east and west sections, with a section consisting
of an upper counter, located about 1m above the test cabinets, and a lower counter, located 1m below the cabinets. The experiment was triggered by the coincidence of either the two east counters or the two west counters. Triggers produced by low momentum muons were suppressed by a half-inch steel sheet covering the lower counters.

A coincidence module sent a single trigger pulse to the “CSI Gate Generator”, which in turn sent an interrupt signal to the computer system, a stop pulse to the drift chamber TDCs and generated a gate pulse for the crystal pulse height integration.

The active region of the counters barely cover the array of crystals so that nearly vertical tracks were selected. The geometry restricted tracks to less than 10 degrees from vertical in the Z-direction and less than 30 degrees from vertical in the Y-direction.

A.5 Event Selection

A crystal’s average light output, resulting from high energy muons, is proportional to the length of crystal traversed. The trigger geometry allowed for many tracks which clipped the crystals, resulting in smaller pulse heights. The trigger geometry also allowed tracks with incident angles of nearly 30 degrees from normal, which resulted in pulse heights of nearly 15% above the average. The pulse height variations due to different path lengths could be removed by either dividing by the path length or using tracks with the same path length.

Originally, the drift chambers were used by the on-line system to correct for variations in pulse heights due to different path lengths. The algorithm which corrected the pulse heights was soon abandoned because of its slowness and because multiple scattering by the crystals produced poor track resolutions. In an
attempt to select tracks with the same path length through the crystals the drift chambers were used to select nearly vertical tracks. A vertical track required hits in aligned tubes for at least three out of the five y-layers, with at least one hit in the lower y-layers. The large separation between layers restricted the tracks to a small solid angle and consequently reduced statistics. A five hour run resulted in 50 to 100 events per crystal. It became evident that either a more efficient tracking algorithm or a different method of track selection was needed. Time limitations lead to a decision to pursue another means of selecting vertical tracks.

The alternate and chosen method required hits in three out of four vertically aligned crystals. A crystal hit was defined as a pulse height at least five standard deviations above the calibration pedestal. Suppression of events with multiple cosmic ray showers or coherent system noise was accomplished by rejecting events with less than 4 drift chamber hits or greater than 20 chamber hits. The event rate increased by more than a factor of 5 over the previous method.

A.6 Analysis

Crystals were tested by performing two data runs. The first run, which lasted for 6hrs, used all four diodes to determine the pulse height, PH4. This value was reported as our measurement for the cosmic ray calibration. The second run, which lasted 1hr, used only one diode to determine a single diode pulse height, PH1. The single diode pulse height was compared to PH4 to check that all four diodes were working properly.

Before analysis the ADC values were converted into DAC units, which are directly proportional to energy units. This requires electronic pedestal subtraction and gain multiplication. The DAC unit is a measure of the voltage used by the
high precision electronic pulse system. From earlier tests which used an electron beam with a well defined energy, the relation between units was determined to be $1 \text{ DAC} = 0.86 \text{ MeV}$.

The pulse height distribution for each crystal was stored as a histogram. Immediately after a run the histograms were fit to a Landau distribution with the form, $A \times \text{DENLAN} \left( \frac{x-x_p}{W} \right)$, where $A$ is the total area and DENLAN is the CERN version of the Landau density function with $x_p$ as the peak value, $W$ the width and $x$ as the fit variable. An example histogram is shown in figure 81 for a rectangular crystal. Visible in this plot are some low energy pulses resulting from particles clipping the crystal. Non-rectangular crystals populated the lower pulse height region more than rectangular crystals. Since the non-rectangular crystals had a less defined edge, particles clipped these crystals more often. The range of the fit was determined by first finding the peak value of the histogram. The range was then extended downward to the first value with less than 10% of the peak entries and extended upward to the first value with less than 5% of the peak entries. Figure 82 shows the distribution of pulse height measurements for all crystals, normalized so that crystals with the same thickness have a unit mean. The distribution have an RMS of approximately 12%.

The ratio of the 4-diode measurement to 1-diode measurement was examined to identify bad diodes. If all 4 diodes produced the same output then the ratio would be $R = PH4/PH1 = 1.0$. A factor of 1 is found instead of 4 because the selection of one diode results in the m/s board adjusted the gain accordingly. If one bad diode existed then; if the 1-diode run used the bad diode, $R$ was typically greater than 1.0, but if a good diode was chosen for the 1-diode run, $R$ was typically less than 1.0. For a diode with a pulse height less than 50% of the other three, $R$ was greater than 1.76 if the bad diode was chosen for the 1-diode measurement.

†To reduce effects from statistical fluctuation in a single bin, a five bin average was used to determining the peak-value, the 10% lower bin and the 5% upper bin.
Figure 81
Typical CsI pulse height for Cosmic rays after pedestal subtraction and gain correction. The solid curve is the fitted Landau Distribution.

Figure 82
Measured CsI pulse height for cosmic ray calibration. Pulse height of crystals with the same thickness is normalized to a unit average.
run and less than 0.88 if a good diode was chosen. When $R$ was between 0.88 and 1.3 all four diodes were assumed to be working properly. If $R$ was out of range the histograms were examined and the 1-diode pulse height checked. In many cases the 4-diode measurement agreed with other 4-diode measurements but the 1-diode measurement was larger by 15% or more, resulting in a false indicator. Crystals failing the test were moved to a new position and retested. If the crystal failed three tests the diodes were replaced and the crystal retested. Figure 83 shows the distribution of the final $R$ value determined for each crystal. The distribution has an RMS of approximately 5.0%.

The results from the cosmic ray test were also compared to the source accept­ance test. The ratio of the cosmic ray calibration to the acceptance test is shown in figure 84 where the distribution is normalized to a unit mean. The distribution has an RMS of about 6.3%.

A.7 Stability of the Results

The stability of the system was used as an indication of the reproducibility of the experiment. To test the stability of the system, one crystal remained in the same position with the same electronics throughout testing. The repeated pulse height measurement for the same crystal using the same electronics was found to be reproducible to 1%.

The resolution of the system was determined by keeping the same crystal in the test stand but with its position changed for each run. Two crystals were used for this check, one on the east side and one on the west side. The distribution of measurements for one crystal showed a spread of 2% over 65 runs with 32 different positions. The gain for each electronic channel, when testing this crystal, had a width of 4.0%. The pedestal values had a width of 2.0% of the average pulse
Figure 83
Ratio of cosmic ray calibration using 4-diodes per crystal to cosmic ray calibration using 1-diode per crystal.

Figure 84
The ratio of the acceptance test value to the cosmic ray calibration pulse height. Ratio for crystals with the same thickness is normalized to a unit mean.
height. The diode end of the second crystal was bumped approximately midway through testing. After this the pulse height dropped by approximately 30% while the resolution before and after the accident remained consistent with that found for the first crystal.

A.8 Conclusion

Measurements with the cosmic ray test facility were found to be reproducible to 1%. The resolution of the electronics was found to be approximately 2% over the interior testing period. Measurements of the average energy deposited in a CsI crystal per cosmic ray had an RMS of approximately 12%. The ratio of the source acceptance test and the cosmic ray test showed a 7% spread in values. We tested each of the crystals before installation into the CLEO-II CsI calorimeter. Each diode contributed at least 13% to the total output of the four diodes. That is, each diode produces a pulse height greater than 50% of the average diode pulse height. All crystals appear to have four working diodes.
List of References

6. A. R. Barker, *On the Formation by Photon-Photon Fusion of Resonances Decaying to \( \pi^+\pi^-\pi^0 \) and \( \eta\pi^+\pi^- \)*, Ph.D. Thesis, University of California at Santa Barbara, (1988).
18. See Appendix A in reference\cite{17}.
32. CLEO internal note, CSN 89-282(1989).
35. The routine used was PROB from the CERN Computer program library GENLIB. A short discription can be found in the CERN PROGRAM LIBRARY manual 1988.07.25 on page 6.240


