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EXAMINING THE CHANGING BELIEFS OF A HIGH SCHOOL PHYSICS TEACHER INTEGRATING MATHEMATICS THROUGH TECHNOLOGY: A CASE STUDY

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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To Those who have fulfilled my life and made this dissertation possible
ACKNOWLEDGMENTS

I truly appreciate all of the time and effort that my advisor, Dr. Patricia A. Brosnan, gave to me throughout my doctoral program. Most of all, I appreciate her belief in me and her constant encouragement. Thanks go to the other members of my advisory committee, Drs. Douglas T. Owens and Joseph C. Ferrar, for their willingness to devote much time to me.

Special thanks go to Mike Smith who so willingly participated as a subject in this study. Thanks also to Bill Williams and Mark Pifer who provided much input along the way. My appreciation goes to Worthington Christian High School and its administrators and staff for their cooperation and assistance, as well as to all those who made possible Mike's and my involvement in the Connecting Math and Science Pilot Institute.

To my husband, Lloyd, I offer my appreciation for your unshakable faith in me and your willingness to sacrifice much so that I could complete this study. Without your love, support, and encouragement, I could not have reached my goal.
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CHAPTER I
INTRODUCTION, BACKGROUND, AND RATIONALE

I have lived through a period when physics was not important for mathematics. Now we are coming back to a period where it is becoming important again . . . and that is a perfectly healthy development.

Andre Weil (cited in Horgan, 1994, p. 33)

Mathematics and science have been intertwined for centuries, with mathematics being the means to explain nature. The Greeks, in the sixth century B.C., began to understand that mathematics underlies and explains natural phenomena. The Pythagoreans were among the first to relate science and mathematics in such areas as motions of the planets and astronomy based on number theory. They "advanced two doctrines that proved later to be all-important: the first is that nature is built according to mathematical principles, and the second, that number relationships underlie, unify, and reveal the order in nature" (Kline, 1985, p. 43).

Greek ideas permeated Europe in the 1500s, with the Catholic Church's adoption of the doctrine "that the Christian God had designed the universe mathematically" (Kline, 1985, p. 45). For several centuries mathematics was definitely the language of science, but in the 1800s it began to become more abstract and analytical. "This drift away from the constraints of the real world may have had as its declaration of independence the
The emergence of non-Euclidean geometry in the first third of the nineteenth century (Dunham, 1990, pp. 245-246). Mathematics became more abstract and, to some, more "pure." However, in spite of all its purity, mathematics often remains linked to the real world. Even non-Euclidean geometry was used by Einstein in advancing his theories of relativity (Dunham, 1990).

Today, there seems to be an increasing effort to, again, link science and mathematics. Not only are mathematicians becoming more concerned about grounding their discipline in reality, but mathematics educators are becoming more concerned about mathematics curriculum that is relevant to students and teachers alike. House (1994) suggests several advantages in using the science context in the teacher-education curriculum: (1) Using such a context gives prospective teachers a new perspective from which to reconstruct important mathematics topics, (2) it allows demonstrations that can, in turn, be used with high school students, (3) the laboratory setting "stimulates discourse that models the communication that we expect teachers to engender in their own classrooms" (p. 290), and (4) using the science setting promotes the idea that mathematics can be studied concretely before being mastered abstractly.

Statement of the Problem

For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse, the rider was lost;
For want of a rider, the battle was lost;
For want of a battle, the kingdom was lost!

Author unknown
Studying a physics teacher's beliefs about mathematics can inform mathematics educators about one's beliefs in an interdisciplinary setting. Physics and mathematics teachers often teach their own respective version of mathematics, which, in many cases are two different kinds of mathematics. Mathematics that depends on instrumental understanding (i.e., "rules without reasons" (Skemp, 1978, p. 9)) is manifested and taught by some physics teachers and is inherently different from the mathematics that depends on relational understanding (i.e., "knowing both what to do and why" (Skemp, 1978, p. 9)) that many mathematics teachers teach. As a result, students who take both physics and advanced mathematics can become confused by the different kinds of mathematics that they are required to master. Unfortunately, some mathematics teachers also teach mathematics which requires the same instrumental understanding that is required by rule-based physics teachers.

Many physics teachers consider mathematics to be a tool to be used in explaining physics. Morris Kline, a mathematics historian, is one who is quite vocal on the issue of mathematics as a useful tool of physicists and engineers rather than as an abstract, self-indulgent discipline (Dunham, 1990).

Those who subscribe to the mathematics-as-a-tool idea often have little appreciation for mathematics other than for its usefulness in solving equations and in dimensional analysis. They may have very little understanding or appreciation of the beauty of mathematics inherent in graphs of functions, patterns, and data analysis. If, by understanding the technology and by using graphing calculators in one's instructional practices, a teacher gains a new appreciation of the mathematics involved, then an
investigation of one's evolving beliefs can inform both physics and mathematics educators. Also, in view of future possibilities of interdisciplinary work connecting mathematics and physics in both schools and universities, pursuing a case study of a high school physics teacher seems quite appropriate.

Purpose

The purpose of this study is to look in depth at a high school physics teacher's beliefs about mathematics, beliefs about mathematics education, and beliefs about technology. I will look at his beliefs which originally hampered the use of calculators in his instructional practices. I will also look at his beliefs as they have evolved into not only acceptance of the calculators, but endorsement and incorporation of them into his instructional practice. Although the technology that is effecting change in educational practices is not limited to calculators, the focus here will be on this item in particular.

Rationale

Recent publications promote the experiences of teachers as an acceptable, and sometimes favorable, means of educating other teachers. Ayers (1992) states candidly that "teachers can be the richest and most useful source of knowledge about teaching; those who hope to understand teaching must turn at some point to teachers themselves" (p. v).

Studying a physics teacher's beliefs as they evolve throughout his attempts to change his instructional practices can inform teachers and teacher educators as to the nature of those beliefs, the inherent difficulties in changing those beliefs, and the means of facilitating change in both the beliefs and the instructional practices of this teacher in
particular and perhaps teachers in general. By generalizing to the theories set forth in the theoretical framework of this study, teacher change can similarly be effected elsewhere.

Just how relevant are mathematics instructors' beliefs to their instructional practices? Hersh (1986) maintains that their beliefs are quite relevant: "One's conceptions of what mathematics is [emphasis in original] affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it" (p. 13).

Cooney and Wilson (1993) give an example of how teachers' beliefs affect their use of calculators in their instructional practices:

If teachers believe that graphical representations are fundamental to the teaching of functions, then technology may be viewed as an indispensable tool. On the other hand, if teachers consider graphical representations as interesting but not necessarily a central consideration, then the use of technology may be viewed as a secondary consideration. (p. 146)

Thompson (1992) discusses the scarcity of research literature on mathematics teachers' beliefs, but she says that many studies in mathematics education "have indicated that teachers' beliefs about mathematics and its teaching play a significant role in shaping the teachers' characteristic patterns of instructional behavior" (p. 131). She maintains that "the study of beliefs has emerged in recent years as an important, legitimate line of research. Its potential for making significant contributions to the field is becoming widely recognized in mathematics education" (p. 131).

Thompson (1992) posits that any study that addresses a teacher's professed views of mathematics should also include observational data of the classroom setting.
She believes that research addressing these issues is important and that case studies, in particular, would be helpful in effecting teacher change. Ernest (1988, cited in Thompson, 1992) points to two fundamental factors in teachers' beliefs that affect their instructional practices: their conceptual understanding of the nature and meaning of mathematics, and their schemas of mathematics education.

Definition of Terms

**CBL:** The Calculator-Based Laboratory System that allows students to collect data, retrieve it directly into a graphing calculator, and then generate graphs and analyze the results.

**Disequilibration:** The act of putting out of balance--involves the testing of concepts or ideas against the real world. When ideas about the world fail to fit, disequilibration occurs.

**Graphing Calculators:** Calculators that allow the user to manipulate a graph of an equation on a screen.

**Journal:** A record of experiences, ideas, or reflections.

**Paradigm:** A philosophical and theoretical framework of a scientific school or discipline.

**Physical Science:** Any of the natural sciences (e.g., physics, chemistry, astronomy) that deal primarily with nonliving materials.

**Physics:** A science that deals with matter and energy and their interactions.

**Reflective teacher:** A teacher who thoughtfully considers experiences in order to improve instructional practice.
Research Questions

A reflective teacher, who has been opposed to the use of calculators in his classroom, decides that he must readjust his thinking on the subject—what role do his beliefs play in the readjustment? This study will address issues about beliefs, namely:

- What were the physics teacher's beliefs and practices about mathematics, education, and technology prior to 1994?
- What changes in beliefs and practices occurred?
- What factors influenced this teacher's change in beliefs and practice?
- How have these changes in beliefs effected change in his instructional practice?

The physics teacher who is the subject of this case study has very tightly-held and strongly-felt beliefs and convictions about his position as an educator. Subsequent narratives provide an in-depth look at this highly reflective man and his beliefs, and at the experiences and social interactions that cause him to change his beliefs and, in turn, his instructional practice.
CHAPTER II

REVIEW OF RELATED LITERATURE

*Education in order to accomplish its ends both for the individual learner and for society must be based upon experience—which is always the actual life-experience of some individual.*

John Dewey (1938, p. 89)

There are many facets of teacher change. Teachers who want to change have decided that they need to change. Their classroom experience, social interaction with colleagues, or perhaps their participation in professional development programs have helped to build that desire to change. Rudduck (1988) posits that:

In order to commit themselves to change teachers must reflect on their own experience of schooling, higher education and teaching, and on the view of knowledge that these experiences have yielded. The teachers who can claim to 'own the problem of change' are those who recognize a potentially creative dissonance that they are prepared to confront and deal with. (p. 205)

This chapter will review literature concerning the role of experience in teacher change as well as literature concerning teacher change itself. In the concluding section of the chapter, a proposed model for teacher change will be presented that encompasses not only factors in teacher-change literature, but also factors inherent in the subsequent theoretical framework.
Teacher Change

To effectively understand the process of teacher change, one must adhere to the premise that the teacher in such a situation is not only a teacher, but also a learner. So, in addressing the review of the literature and the theoretical framework, it should be understood that the learner is being applied to the teacher as learner.

The opening quote by Dewey encompasses the essence of teacher change. Teachers who want to change are teachers who want to grow. They are teachers who do not believe that the status quo is adequate. They are teachers who are reflective and who are continually trying to do what is best for their students. Schubert and Ayers (1992) contend that "It is only reflective teachers (not those who teach by recipe, technique, or doctrine) who are able to grow continuously." It seems that a pervasive, ongoing factor of teacher change is reflection.

The Role of Experience in Teacher Change

Dewey (1916) defines reflection as the reconstruction or reorganization of experience. One needs to examine one's past experience to understand it, reorganize it, and reconstruct it. Schubert and Ayers (1992) also relate reflection to experience.

We remain convinced that conscientious teachers reflect seriously on their work. They think and feel carefully about what they do and why they do it. They use their experiences as a basis for fashioning responses to similar situations that they encounter daily. . . . They reflect on or about their experiences when they have the time, and they reflect and rebuild their orientation in the course of experience as well. (p. ix)

Millies (1992) talks about her "metamorphosis" (i.e., teacher change): "My metamorphosis was a result of my own reflection on my practice. It marked the beginning
of my fascination with the concept of teacher reflection as a possible key to teacher transformation" (p. 27). In her case, she decided to change her format of teaching. She switched from a teacher-dominated classroom to a democratic classroom, and she felt that she made great strides as a teacher by her actions of reflection and, as a result, the restructuring of her classroom.

This reflection and determination to change, in turn, promotes restructuring of instructional practices on the part of the teacher. Teachers try new ideas in the classroom to see if the changes make the kind of difference they want. Change is ongoing and often difficult.

**Teacher-Change Literature**

*Failure is instructive. The person who really thinks learns quite as much from his failures as from his successes.*

John Dewey

Not all teachers are reflective, and among those who are reflective, not all attempt to restructure their instructional practices. Not all of those who do attempt change, continually attempt it. What are some of the factors that keep teachers clinging to the status quo? A review of the literature on teacher change may provide some clues.

Ideas about the factors that affect beliefs, the difficulty of changing one's entrenched beliefs, and the effect that one's beliefs have on one's practice are prevalent in the teacher-change literature. Tobin (1987) reviewed the results of five studies on the implementation of mathematics programs and concluded that teachers' "beliefs about how students learn and what they ought to learn appear to have greatest impact" (p. 296) on
what teachers did in the classroom.

The literature posits several explanations for lack of implementation. Change instituted by others may not fit teachers' intuitive understanding of what they should be doing; school conditions such as administration, collegiality, and experimentation can hamper or enhance teachers' willingness to change; teachers' orientation toward the practical and concrete can make them either more or less receptive to change. In the foregoing components, teachers are attempting to implement changes suggested by others (Richardson, 1990). Richardson believes that one crucial component in teacher change is autonomy. Teachers should control the decision of whether or not to implement the change. In Miller's account of Hunt's change (Miller & Hunt, 1994), Hunt agrees:

The most important point made that afternoon [at a meeting to discuss a research project between university researchers and school teachers involving teacher change] was that the teachers had to feel ownership of the study. From the perspective of the university researchers, the study would not be successful if the teachers participated for the benefit of the university researchers. The intention was that the teachers participate because they want to investigate an alternative approach to learning how well their students understand the mathematics being studied in school. (p. 297)

Richardson (1990), when discussing teacher change cites both teacher-change literature and learning-to-teach literature. Learning-to-teach literature "focuses more on individual teacher's cognitions, beliefs, and other mental processes than on behaviors" (Richardson, 1990, p. 12). The two main components of teachers' ways of knowing seem to be experience and the teacher herself/himself. Richardson (1990) posits that "the improvement of the teacher-learning process requires acknowledging and building upon teachers' experiences, and promoting reflection on those experiences" (p. 12).
Richardson (1990) suggests that teacher change may best be promoted by incorporating both teacher-change literature and learning-to-teach literature. She suggests that teachers should make their own decisions about change, and that those desiring the change should "help teachers understand and be held accountable for the pedagogical and moral implications of their decisions" (p. 13). Also, there should be a major shift from the focus on changing teachers' behaviors to changing their practical knowledge and cognitions.

A third component in the teacher-change agenda should include significant and worthwhile change. Such change should be determined by practical knowledge and empirical research. Practice should be "actively embedded within theory" (Richardson, 1990, p. 16).

This idea of embedding practice within theory is reiterated by NCTM (1991) in their Professional Standards for Teaching Mathematics. These Standards were written to assist teachers in making necessary changes in their instructional practices. Major proposed shifts in classroom environment include: classrooms as mathematical communities; teachers as facilitators, stimulating classroom discourse; assignments as tasks to help students achieve mathematical power—the ability to conjecture, invent, problem-solve, reason, and communicate; and assessment as an ongoing analysis of student learning. However, included in the document is the acknowledgment that "there are many persistent obstacles to making significant changes in mathematics teaching and learning in schools" (p. 1). Among these are the beliefs, dispositions, and assumptions of students, teachers, administrators, parents, and society.
Sarason (1990) likens educational change to the change that took place several decades ago in the treatment of heart attacks. Patients who suffered heart attacks had been forced to stay in bed for several weeks, when evidence emerged that contradicted that practice and supported getting patients up and about soon after entering the hospital. Sarason said that "physicians and patients were fearful of this change because it contradicted their belief systems and customary practice" (p. 101). In the same way, Sarason argues, educational reformers "seem unable to understand what is involved in unlearning what custom, tradition, and even research have told educational personnel is right, natural, and proper" (p. 101).

The difficulty of teacher change is evidenced in case studies and personal accounts of change. Lewellen (1992) attempted to change her instructional practices in her geometry classes. The change was very difficult for her as well as for her students. In her words,

How did the students feel about this [her change in her instructional practices]? They hated it. . . . How did I feel about this? Absolutely awful. . . . I believed in what I was doing, but it was really painful to try to change the habits of a lifetime. (p. 5)

By the end of the first semester, both Lewellen and her students began to appreciate the changes, and the long-term effects proved to be beneficial for many of the students. However, the process was a difficult one.

Wood, Cobb, and Yackel (1991) reported a case study of a second-grade classroom teacher who underwent teacher change. Her changes:

did not occur as single incidents but instead consisted of gradual constructions and transformations. These periods of change were characterized by major dilemmas
and conflicts that the teacher encountered as she taught in her classroom. (p. 597)

The teacher had to reconceptualize her role in instruction, and reconceptualize her ideas about teaching and learning. Her attempt to change was aided by the research team, of which she was a member. She had an opportunity to express her concerns and to receive feedback and support as she reorganized her thinking. Interaction with others was an important component of this teacher's attempts to change.

Model

The preceding review of the related literature seems to indicate that the experiences of teachers are crucial in determining their ability and/or willingness to change. Reflecting on these experiences can cause disequilibration (i.e., a disturbance in the equilibrium of the understanding—to be discussed further in the next chapter) and can help to define teachers' beliefs which, in turn, gives teachers the understanding to implement changes in their classrooms. Teachers then reflect on their restructuring process, which again influences their beliefs. Social interaction (socialization) with reflection on that socialization can also be a large factor in influencing beliefs, thereby effecting teacher change. A model that incorporates all above-mentioned factors is presented in Figure 1.
I proposed this model as a result of my own experiences and a search of other models for teacher change. During my search, I noted that many models used reflection as a driving force. I believe that to be a true aspect of a model for teacher change, and so I incorporated such into my proposed model. Some models were cyclical, and I believe teacher change is a cyclical process, so I incorporated that thought into this model. I next developed a theoretical framework (see Chapter III) for teacher change and incorporated the theories into the cornerstones of the model, with beliefs--central to my model--as the pivot point.

In the model, socialization refers to the teacher's interaction with society, vocational community, students, and administration; disequilibration represents the disturbance of the teacher's equilibrium (to be discussed in the section about...
constructivism); *restructuring* represents the teacher's attempts to change his/her instructional practices; and *experiences* refers to the experiences the teacher has, including experiences with technology. The model is a cyclic one which places the teacher's *beliefs* as the pivotal point of all other factors and that *reflection* is the driving force between factors. Some teachers' beliefs prohibit their change; some teachers' evolving beliefs eventually require their change. How are teachers who have a desire to change able to reorganize their thinking? The next chapter addresses these questions and more.
CHAPTER III

THEORETICAL FRAMEWORK

*Nor do men put new wine into old wineskins; otherwise the wineskins burst, and the wine pours out, and the wineskins are ruined; but they put new wine into fresh wineskins, and both are preserved.*

Matthew 9:17
New American Standard Bible,
(1985, pp.842-843)

The formulation of this theoretical framework will include two components, Kuhn's (1970) theory of paradigm shifts in the midst of revolutions, and an incorporation of three learning theories: constructivism, Vygotsky's theories of learning, and anchored instruction. All incorporate the theme of changing beliefs to effect teacher (or teacher-as-learner) change. (Note that, in some cases, the process of change, in turn, affects beliefs.) First it will be established that we are, indeed, in the midst of a revolution. Then the difficulty of changing entrenched beliefs and the factors that affect that change will be examined through Kuhn's theory. Examples will be provided of those who did change beliefs and the consequences of their actions. Finally, several learning theories will be incorporated and posited as the means by which teachers who wish to change can effect that change.
The proposed theoretical framework incorporates Kuhn's theory of paradigm shifts in scientific revolutions as the basis for studying change in teachers' beliefs in the midst of a mathematics education revolution. Kuhn's theory will be shown to be an appropriate theoretical framework for teacher change. It, together with research on teacher change and with an aggregation of several appropriate learning theories, form a logical triangulation as a basis for this study.

Are We in the Midst of a Revolution?

Gillies (1992) defines revolutions as "a series of discontinuities of such magnitude as to constitute definite breaks with the past. After such episodes, one might say that there is no returning to an older order" (p. 51). Gillies maintains that many such revolutions have occurred throughout the history of mathematics--Cantor's development of transfinite set theory, among others.

Nichols (1968) discussed the revolution that was triggered by the launching of Sputnik in 1957. The revolution manifested itself in redirecting the school mathematics curriculum--this redirection came to be known as the "new math" movement. Nichols' work was one chapter in a book published by the National Council of Teachers of Mathematics (NCTM) entitled The Continuing REVOLUTION in Mathematics (1968).

Osborne and Crosswhite (1970) also called the "new math" movement a "revolution," rather than an evolution. They asserted that "some feel that what is past is only a prelude to an even greater revolution yet to come" (p. 235).

Although the actual "new math" revolution in the curriculum eventually dissipated, and the "back to basics" movement began, the effects of the revolution in mathematics and
mathematics education which were sparked by technological advances in the world are far from being over, and, in fact, seem to be escalating. The "new math" revolution involved a change in the school mathematics curriculum, but not an accompanying change in pedagogy. Whereas the present era is seeing substantial changes in both content and pedagogy.

The NCTM, in its *Professional Standards for Teaching Mathematics* (1991), state that the

Curriculum and Evaluation Standards [NCTM, 1989] implies a significant departure from the traditional practices of mathematics teaching. It suggests changes in not only what is taught but also how it is taught. Teachers and students have different roles in such classrooms and different notions about what it means to know and to do mathematics. (p. 20)

One of the assumptions presented by NCTM about the practice of mathematics teaching is that "WHAT students learn is fundamentally connected with HOW they learn it" (NCTM, 1991, p. 21). So, beliefs about mathematics and beliefs about mathematics education seem to be irrevocably intertwined.

The mathematics that today's student will see in the workplace is considerably different from the mathematics that his/her parents saw. "Computers and calculators have changed profoundly the world of mathematics. They have affected not only what mathematics is important, but also how mathematics is done" (Rheinboldt, 1985, cited in Mathematical Sciences Education Board (MSEB), 1990, p. 2).

Steen (1990) discusses the rapid growth of mathematics in the last century and its changing focus from number and shape to pattern and order. Because of computer graphics, one can not only understand with the mind, but also perceive with the eye; and
understanding of mathematics is enhanced.

Change in the practice of mathematics forces re-examination of mathematics education. . . . Students who will live and work using computers as a routine tool need to learn a different mathematics than their forefathers. Standard school practice, rooted in traditions that are several centuries old, simply cannot prepare students adequately for the mathematical needs of the twenty-first century. (Steen, 1990, p. 2)

NCTM (1989) discusses the effect that technology has had on the discipline of mathematics: "The new technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them" (p. 8). Because of NCTM's stand, they have advised that "appropriate calculators should be available to all students at all times" (p. 8), and that "students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems" (p. 8). (Note that this claim and others mentioned refer to computers, when the same claim could be made about graphics calculators which perform many of the same operations as that of computers and could, therefore, be referred to as types of mini-computers.)

Cornu (1992) claims that, if the role of the computer is that of an "electronic blackboard," in which the teacher uses it as a tool, much as he/she would use any blackboard, then it "does not upset the traditional balance in the classroom . . . [and] it will not revolutionise the classroom" (p. 27). If, however, students are encouraged to interact with the computers, then the focus changes and the teacher is no longer the imparter of knowledge, but, rather, a facilitator of learning. "Such a change would produce a revolution in most class- and lecture-rooms" (p. 27). Teachers would need to acquire new
knowledge, "radically change their present aims and emphases" (p. 27), and would be required to teach in ways that are foreign to them. "This last demand means a sacrifice of traditional security. . . . It would be foolish to underestimate the challenge this presents" (p. 27).

Kuhn's Theory

Kuhn's (1970) theory about the concept of science differed from the then prevailing concept—that of accumulated development (i.e., that individual contributions to science build upon one another.) Kuhn talked about historians of science who "ask, for example, not about the relation of Galileo's views to those of modern science, but rather about the relationship between his views and those of his group, i.e., his teachers, contemporaries, and immediate successors in the sciences" (Kuhn, 1970, p. 3). These historians "attempt to display the historical integrity of that science in its own time" (Kuhn, 1970, p. 3).

Kuhn's theory involves the belief that within the scientific community there are "accepted examples of actual scientific practice--examples which include law, theory, application, and instrumentation together--[that] provide models from which spring particular coherent traditions of scientific research" (p. 10). These models Kuhn called paradigms. In addressing the issue of changing paradigms, Kuhn acknowledged that, if the outmoded ideas of science are still, indeed, to be called science (and he believed that they are) then "science has included bodies of belief quite incompatible with the ones we hold today" (p. 2).
There have been major turning points in scientific development, initiated by men such as Copernicus and Kepler. These turning points "necessitated the community's rejection of one time-honored scientific theory in favor of another incompatible with it" (Kuhn, 1970, p. 6), thereby producing a paradigm transformation, or shift. "Such changes, together with the controversies that almost always accompany them, are the defining characteristics of scientific revolutions" (Kuhn, 1970, p. 6).

In the cases of most of the scientific revolutions, the new theories emerged only after previous theories had failed to solve the problems of the era. Debates about scientific theories occur when prevailing paradigms are first under attack and then subject to change. Four examples follow. The first is an example of a revolution in science, in which there ensued a crucial and heated debate over scientific theory—did the earth actually revolve around the sun? The next example concerns a mathematical revolution in which mathematicians contemplated the infinite. The third combines a more recent mathematical revolution involving non-Euclidean geometry with a scientific revolution involving Einstein's Theory of Relativity. The fourth example combines mathematics, science, and technology in a current revolution called chaos. So the reader has a chronological progression of examples of revolutions to consider: a revolution in science; one in mathematics; one in both science and mathematics; and one involving science, mathematics, and calculators and computers.
Examples

Perseverance

We must not hope to be mowers,
And to gather the ripe gold ears,
Unless we have first been sowers
And watered the furrows with tears.

It is not just as we take it,
This mystical world of ours,
Life's field will yield as we make it
A harvest of thorns or of flowers.

Johann Wolfgang von Goethe

Copernican revolution

Kline (1985) reported a mathematician's view of the Copernican Revolution. In the second century A.D., Ptolemy, adhering to the belief that heavenly bodies move in circular paths, and that they move with a constant velocity, developed his theories of motion about the earth. Although he was aware of Aristarchus' heliocentric theory (in which the sun is stationary and the earth moves about the sun), he dismissed the theory as unsound because of his belief that the motion of an object is proportional to its mass--hence objects would be flying off of the earth. (Aristarchus' theory had little acceptance at all because: "The fact that Earth dwellers could not feel the rotation and revolution of the Earth and the belief that the Earth was the natural center of the universe countered Aristarchus' scheme" (pp. 58-59).)

Ptolemaic theory evolved to a stage requiring complex mathematics--seventy-seven circles within circles (somewhat like the moon revolving about the earth as the earth revolves about the sun). His theories were accepted as truth for about fifteen hundred
years, until the complexity of the mathematics drove Copernicus to look for another theory—something that would be mathematically simpler. His belief was that God created the heavens with mathematical simplicity and harmony, and he found that with the sun as the center, he could reduce the number of circles required from seventy-seven to thirty-four.

Because of the beauty of the mathematics and because of the entrenched societal beliefs about a stationary earth, "for a long time only mathematicians supported Copernicus" (p. 72). In fact, the Catholic church condemned Copernicanism and called it a heresy "more scandalous, more detestable, and more pernicious to Christianity than any contained in . . . all other heretics put together" (cited in Kline, 1985, p. 73).

To many, Copernicanism required more than just a change of thought about the heavens, it seemed to require a change of thought about the whole of man's existence. The beliefs were so entrenched that society refused to even consider that Copernicus could be correct. Antagonism to Copernicus' theories was so strong that, in the seventeenth century--decades after Copernicus' death--the Church made anti-Copernicanism an official Church doctrine.

Kepler, another mathematician who appreciated simplicity in mathematics, "sacrificed his most beloved mathematical hypotheses when he saw that they did not fit observational data" (Kline, 1985, p. 75). He spent years searching for answers and following false trails until he finally concluded that the motions of the planets were not circular, but rather were elliptical. Two other laws then fell into place, and Kepler's laws of planetary motion are the ones we use today. "Despite the weighty scientific arguments
against a moving Earth, despite the religious and philosophical conservatism, and despite the affront to common sense, the new theory gradually won acceptance" (p. 82).

This paradigm shift required by the Copernican Revolution involved mathematics that was tied to the natural sciences and the real world. At that time mathematics was studied mainly for its ability to explain and represent nature.

**Cantor and the Infinite**

Mathematics, itself, in the nineteenth century, shifted away from explaining and representing nature to a trend toward abstractness. The problem of infinitely large and infinitesimally small quantities was introduced by both Newton and Leibniz in the 1600s. Galileo, in 1638, showed a one-to-one correspondence between the natural numbers and their squares. But, how could there be a one-to-one correspondence between a set of numbers and another set of numbers which intuitively seems to be smaller?

For over two centuries, this seeming contradiction blighted any attempt to contemplate infinity in a precise sense. Leibniz went as far as to suggest that we should only ever consider finite sets, that the contradiction arose because the natural numbers are infinite. (Stewart & Tall, 1977, p. 230)

As the nineteenth century progressed, mathematicians began struggling with the idea that the rationals and the irrationals were not dispensed equally among the set of real numbers. Dunham (1990) gives us the story: "Mathematicians were coming to realize that some of the most important, and fundamental, questions of the calculus rested upon profound properties of sets" (p. 252).

Cantor devised a means for comparing the sizes of infinite sets and gave this definition:
Two sets \(M\) and \(N\) are equivalent... if it is possible to put them, by some law, in such a relation to one another that to every element of each one of them corresponds one and only one element of the other. (Cited in Dunham, 1990, p. 253)

He then "introduced a new 'transfinite' cardinal number \([\aleph_0]\) to represent the number of items in a denumerable set" (p. 255). (A denumerable set is one which can be put into one-to-one correspondence with the natural numbers.) Dunham muses, "One imagines many of his contemporaries shaking their heads in pity at the poor fellow who touted such absurd ideas" (p. 255). Dunham even calls Cantor's ideas "revolutionary" (p. 257).

Cantor continued to explore the infinite and discovered (and proved) that the set of real numbers was non-denumerable, which means that it had a different cardinal number from that of the natural numbers, integers, and rationals. When looking for a cardinal number greater than that of the reals, Cantor spent several years trying to prove something for which his original intuition was wrong. He found that the points of a square and the points of the unit interval \((0,1)\) have the same cardinality, to which he exclaimed, "I see it but I do not believe it!" (Cited in Dunham, 1990, p. 273).

As Cantor worked and his understanding advanced, "Cantor eventually overcame even his own discomfort with the actual infinite to support his transfinite numbers" (Gillies, 1992, p. 64). So, his beliefs about the infinite changed as he worked with the infinite.
Non-Euclidean Geometry

One of the most revolutionary ideas in both science and mathematics has been a product of the nineteenth century—non-Euclidean geometry. For centuries mathematicians had attempted to prove Euclid's parallel postulate. Playfair's postulate, a simpler postulate which is logically equivalent to Euclid's parallel postulate (and is, therefore, called The Euclidean Parallel Postulate) states: For every line $l$ and for every point $P$ that does not lie on $l$ there exists a unique line $m$ through $P$ that is parallel to $l$ (Greenberg, 1993). Although the postulate may seem obvious, it was shown to be impossible to prove from Euclid's nine other postulates (Kline, 1985).

Some mathematicians began to realize that the intuitiveness of the parallel postulate was based on experience. Perhaps other geometries that included a contradiction of the parallel postulate could also be shown to be valid logical structures. Gauss, Lobatchevsky, and Bolyai derived their own versions of such a geometry—what came to be called, hyperbolic geometry. With their geometry, given a line $l$ and a point $P$, there are an infinite number of parallels to $l$ through $P$. Although Euclid's parallel postulate fails, all of his other postulates remain valid.

When one considers the counterintuitive nature of hyperbolic geometry, one can understand why the ideas were revolutionary. Some dismissed the geometry as worthless, and others regarded it as a curiosity. "Almost all mathematicians maintained that the geometry of physical space, the geometry, [emphasis in original] must be Euclidean" (Kline, 1985, p. 155).
Gauss also considered the surface of a sphere to be a geometry and described the properties of that geometry. Using great circles to be the "lines" of the geometry, all lines would intersect twice. This form of geometry was called "double elliptic" geometry.

Riemann, a student of Gauss, investigated space as it behaves locally and used an approach called "differential geometric" (Kline, 1985). This form of geometry was to be used later by Einstein in his theory of relativity.

As mathematicians were slowly accepting non-Euclidean geometry and its implications about the physical world, physicists began to question their own long-held beliefs about the universe. Experiments were being made by scientists which did not confirm these entrenched beliefs. As a result, physicists began the task of overhauling some of their science. At the turn of the century Einstein entered the scene and utilized his professor's--Minkowski's--ideas about a four-dimensional space-time world. Drawing on ideas of Riemann's differential geometry, Einstein utilized a four-dimensional curved space-time in his general theory of relativity (Kline, 1985).

As with previously-mentioned revolutionary ideas, Einstein's theory is not universally accepted even today. Kline (1985) flatly states:

In spite of astonishing and dramatic verifications of the theory, many people find its four-dimensional, non-Euclidean universe totally unpalatable. No one can visualize a four-dimensional, non-Euclidean world, but those who insist on visualizing the concepts with which science and mathematics now deal are still in the dark ages of their intellectual development. . . . To insist that each step in a chain, even of geometrical reasoning, be meaningful to the senses is to rob mathematics and science of two thousand years of development. (p. 179)
Chaos

We are now in the midst of still another revolution that encompasses mathematics and physics and utilizes computers and calculators. Many scientists are redefining their thinking about the world in which we live by the theory of chaos. Newton gave us his laws of motion and indicated by them that there was complete predictability of bodies in motion within the universe—we only needed to know enough about the present situation. The first time that these ideas of complete predictability were seriously challenged was in the middle of the nineteenth century with the kinetic theory of gases—the theory that matter is made up of atoms that are in continuous random motion (Taylor, 1989). At the end of the nineteenth century, Poincaré discussed a receptacle filled with gas in which innumerable molecules, moving at high speeds, randomly flash through the receptacle, colliding with walls or each other.

If a molecule deviated right or left from its trajectory, by a very small quantity, comparable to the radius of action of the gaseous molecules, it would avoid a collision or sustain it under different conditions, and that would vary the direction of its velocity after the impact, perhaps by ninety degrees or by a hundred and eighty degrees. (Poincaré, 1988, p. 1363)

Poincaré during the nineteenth century anticipated some things that could be applied in the next century to chaos theory. Newton had described gravitational interaction of two bodies, and in the nineteenth century mathematicians attempted to extend Newton's equations to three bodies. In so doing, they found no stable solution to their investigations. Poincaré reasoned that we could not prove stability of our solar system, because Newton could not establish predictability for three bodies. Poincaré concluded that:
It may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible." (Taylor, 1989)

For many years after Poincaré's death, the study of dynamical systems (i.e., the branch of science and mathematics that attempts to understand processes in motion) was practically nonexistent in mathematics (Gleick, 1987).

In 1960, Lorenz created simulated weather on his computer. By inputting some rather simple mathematical equations that expressed the relationships between temperature and pressure, or between pressure and wind speed, Lorenz believed that he could predict the weather. He reasoned that, since those equations were repeatedly calculated with mechanical efficiency, predictability should result. One day Lorenz wanted to repeat a certain sequence and examine it, so he decided to run the pattern again by beginning in the middle of one of his simulations. He inputted the values and left to get a cup of coffee. About an hour later, he returned to find that the simulated weather had begun along the same path as before, but it soon digressed and began a completely different pattern. He realized that his computer stored six decimal places, while his printout gave only three. Assuming that the difference was inconsequential, Lorenz had entered the shorter numbers, and the seemingly minuscule change had made a significant difference. From this discovery, a new theory would develop--chaos (Gleick, 1987; Taylor, 1989).

Lorenz's experience with the simulated weather gave rise to the idea that aberrations as seemingly insignificant as the flap of a butterfly's wings can make monumental changes in the weather--a phenomenon known as the Butterfly Effect. This Butterfly Effect was based on the same premise that Poincaré had used--sensitive
dependence on initial conditions (Gleick, 1987). The phenomenon arises from the nonlinearity of the equations describing the motions. Nonlinear relationships were an enigma to scientists. To get a simple understanding, scientists prefer to leave out those nonlinear relationships. Nonlinearity makes systems much more complicated (Gleick, 1987; Taylor, 1989).

However, chaotic processes often give rise to order. Lorenz's work with the geometry of chaotic dynamics attracted little attention until 1977, when Shaw and Scott were working with an analog computer and used Lorenz's weather equations (Taylor, 1989). The solution to the equations was chaotic and yet controlled. The image resembled a butterfly's wings or an owl's mask, and the boundary of it was ordered, but inside the boundary lay unpredictability. The image, called the Lorenz Attractor, "revealed the fine structure hidden within a disorderly stream of data" (Gleick, 1987, p. 29). On the small scale, chaotic dynamical systems behave randomly; however on the larger scale, order emerges.

Using a mathematical process called \textit{iteration} (i.e., repeating a procedure over and over using the output of the previous operation as the input for the next), is one way to generate and to begin to understand chaos. Computers and calculators can easily carry out the iterations--a process which would be time-consuming and practically impossible with paper and pencil. Regardless of the initial value of $x$, iterating a simple function like $\sqrt{x}$ eventually yields the number \textit{one}, which remains unchanged under subsequent iterations. However, iterations of the logistics function, $c \times (1 - x)$ vary greatly depending on the value of $c$ for an initial value of $0 < x < 1$. Some orbits tend to 0, $1/3$, \ldots
or 1/2. Some orbits approach the period two cycle .5130456... and .799456..., and
others approach a period three cycle, four cycle, or eight cycle. Still others never develop
any pattern. So even a very simple dynamical system can exhibit complicated behavior,
and the behavior changes as the initial condition changes (Devaney, 1990).

As illustrated above, nonlinearity, sensitive dependence on initial conditions, and
constrained randomness, are the heart of the theory of chaos. In mathematics, fascinating
computer graphics images can be generated by iterating fairly simple functions. Scientists
observe chaos in many areas such as medicine, engineering, and meteorology. However,
the ideas of chaos are quite revolutionary. Order and predictability had been the mainstay
of science for centuries, and chaotic behavior had been dismissed as being outside of the
framework. Unpredictability within structure; suggestions of structure within apparently
random behavior; a transformation in a way of thinking; a paradigm shift—all of these
describe the theory of chaos (Gleick, 1987). With the revolution have come the inevitable
obstacles and resistance.

At first, the claims made on behalf of chaos sounded wild and unscientific. And
chaos relied on mathematics that seemed unconventional and difficult... Work
fell between disciplines—for example, too abstract for physicists yet too
experimental for mathematicians. To some the difficulty of communicating the
new ideas and the ferocious resistance from traditional quarters showed how
revolutionary the new science was. Shallow ideas can be assimilated; ideas that
require people to reorganize their picture of the world provoke hostility. (Gleick,
1987, p. 38)

The verdict is still out on this revolution. Some say that the new ideas are just
games and will not last. Others have embraced revolutionary beliefs and have forged
ahead in new areas. Whatever the outcome, Kuhn's (1970) theory of scientific revolutions
has a direct bearing on the theory of chaos (Gleick, 1987). For those who adhere to the Newtonian idea of predictability of bodies of motion, thinking has to be redefined, and so paradigm shifts need to occur in order to embrace chaos theory.

**Conclusion**

In the preceding examples, revolutions occurred because long-standing, entrenched paradigms had to give way to new paradigms. The switch was neither easily nor quickly made in any of the cases, and the reconstruction resulted in changes in "some of the field's most elementary theoretical generalizations as well as many of its paradigm methods and applications" (Kuhn, 1970, p. 85). At some point in each of the above-mentioned cases, the society was divided into two camps--one which sought to defend the old paradigm and one which sought to institute the new one.

How, then, are scientists brought to make this transposition [paradigm shift]? Part of the answer is that they are very often not. Copernicanism made few converts for almost a century after Copernicus' death. The transfer of allegiance from paradigm to paradigm is a conversion experience that cannot be forced. Conversions will occur a few at a time until, after the last holdouts have died, the whole profession will again be practicing under a single, but now a different, paradigm" (Kuhn, 1970, pp. 150-152).

Kuhn's entire theory of scientific revolutions is incorporated into the proposed model for teacher change. One's beliefs (encompassed in one's paradigm) must be changed in order for the person to embrace the revolutionary ideas. One's own beliefs must be shaken to such an extent that disequilibration occurs in that individual. When that disequilibration occurs, one strives for equilibrium. That equilibrium can be realized by socialization and by experience with the revolutionary ideas themselves.
Society's views often play a major role in affecting beliefs. *Socialization* with those who adhere to the new ideas can facilitate a change in beliefs in a *reflective* person to the extent that his/her equilibrium is restored by embracing those new ideas.

Also, *experience* with the revolutionary ideas can bring about change in beliefs. As mentioned before, Cantor's beliefs about the infinite changed as he worked with the infinite. Equilibrium can be restored by *experience*.

All of the above can be accomplished by *reflection* on those experiences, *socialization*, and *disequilibration*. Beliefs can then be changed (paradigm shifts can be made) and the result will be a *restructuring* which manifests itself in action. The actual process of *restructuring* enables one to become involved in more experiences, which in turn affects beliefs. The process is cyclical.

In some way, scientists and mathematicians who have dared to persist in investigating revolutionary ideas have constructed for themselves new understandings of the ideas they were studying. The disparities they saw between paradigms of their day and their own emerging beliefs drove them to strive for equilibrium. This drive for order, called *equlibration*, is a cornerstone of Piaget's theory (Eggen & Kauchak, 1992).

**Learning Theories**

Von Glasersfeld (1990) discusses Piaget's theory that: "Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject" (p. 22). Piaget believed that people developed schemas, or conceptual structures, about their world. He believed that underlying all of human existence was the desire for equilibration—"the drive for order and predictability" (Eggen
Humans are motivated by a desire to understand their world and to order and categorize their experiences so that they fit or relate to what they already know.

When one's scheme of his/her world (paradigm) does not lead to the expected result, perturbation results (von Glasersfeld, 1990). This perturbation is labeled \textit{disequilibration} in the proposed model for teacher change. \textit{Disequilibration} is a word taken from Piaget's \textit{equilibration}, but was chosen because of its meaning--putting out of balance.

According to Piaget's theory, the processes of fitting and relating are called \textit{assimilation} and \textit{accommodation}. When one's equilibrium is disturbed by an unfamiliar experience/situation, one must either assimilate the experience into an existing schema or accommodate the experience by forming a new schema. In either case, after the adaptation ("process of adjusting our schemata" (Eggen & Kauchak, 1992, p. 38)) has taken place, the person's equilibrium is restored.

Both assimilation and accommodation are necessary for personal development. If one only assimilated new information, no growth would take place, because the schema would never be restructured. However, if one constantly restructured and never assimilated, the learner would remain in a confused mental state (e.g., the "culture shock" experienced in a foreign land) (Eggen & Kauchak, 1992).

Mathematics teachers who undergo teacher change must construct their own understanding of mathematics and their own understanding of mathematics education. Their beliefs, which are entrenched, become perturbed when confronted with beliefs
inconsistent with their own—they experience cognitive conflict. Their equilibrium becomes disturbed, and they seek to restore that equilibrium. The process is ongoing and difficult as they strive for equilibrium in their world. The question is, "How do they restore that equilibrium?" Vygotsky (discussed in the next section) best answers that question.

Simon and Schifter (1991) conducted the Educational Leaders in Mathematics (ELM) Project at Mount Holyoke College and addressed the issue of teacher change. Their theoretical framework for ELM included a constructivist view of learning as a major component. Two of the key ideas stressed throughout the project were:

1. Teachers must be encouraged to examine the nature of mathematics and the process of learning mathematics as a basis for deciding how to teach mathematics.

2. Teachers' learning can be viewed in much the same way as mathematics students' learning. (p. 312)

One of their key components that drove the program was the idea of teachers constructing their own understanding of teaching.

Simon and Schifter determined that ELM was successful:

Almost all participants in the project adopted new strategies in their mathematics teaching. More importantly, a significant number of these teachers came to base their instructional decisions on a view of learning as construction. (p. 328)

They believed that, by constructing their own understanding of mathematics teaching and learning, the teachers' changes would be lasting. They also believed that the teachers experienced empowerment as they developed their own theories of learning and used those theories as the basis of instructional decisions.
Along with the construction of one's knowledge is the social construction of knowledge by the group to which the individual belongs. "Groups of any size and function work to develop 'taken-to-be-shared meanings' (Cobb et al., 1991, cited in Simon & Schifter, 1991), in order to communicate and progress towards the goals of the group" (Simon & Schifter, 1991, p. 310). With teachers, this group could be colleagues, administrators, or a research team. It is quite helpful for teachers to have some sort of active group to provide support and feedback as the teacher reflects on his/her beliefs and experiences. This socialization as a means to promote change is one of the factors in the model for teacher change and is a cornerstone of Vygotsky's theory.

**Vygotsky's Theory**

Like Kuhn with his theories of scientific development, so is Vygotsky with his theories of human development. "Vygotsky rejects the concept of linear development and incorporates into his conceptualization both evolutionary and revolutionary [emphasis added] change. The recognition of these two interrelated forms of development is for him a necessary component of scientific thought" (John-Steiner & Souberman, 1978, p. 122).

Vygotsky's approach to human development was based on a two-state psychological transformation.

Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interspsychological), and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals. (Vygotsky, 1978, p. 57)
From a Vygotskian perspective, humans develop and change by interacting with others and learning to make use of a culture's tools, both physical and psychological. So the constructions that humans make in their minds originate in interchanges with people. The transformation from interpsychological to intrapsychological takes place within a person's "zone of proximal development (ZPD)." Vygotsky described this zone as the "distance between the [person's] actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers [emphasis in original]" (Vygotsky, 1978, p. 86).

When novices and more capable peers team together, the experts assist the novices in their ZPD. Novices then internalize that which they previously did not know or understand. "The experts' information is 'appropriated' in a unique way by the novices according to their own past experiences [emphasis added]" (Martin, 1993, p. 74). So the novice does not mimic the understanding of the expert but interprets, transforms, and internalizes knowledge according to his/her own understanding based on his/her own experiences.

Because the teacher is also a learner, then the expert is also a novice, but that should not confuse the issue. Martin (1993) discusses teachers' ZPD in learning new teaching strategies. She says that teachers' ZPD should be applied to the "conditions under which an adult learns new things" (p. 81). Teachers need to be reflective individuals to effectively implement change, and they also need interaction with others, specifically colleagues and students (Martin, 1993).
Vygotsky's view of how learners construct knowledge is different from Piaget's theories of assimilation and accommodation.

The Vygotskian view is that complex thinking arises from the particular patterns of information exchange in which individuals engage, and which are signified as meaningful by the society that surrounds the individual. Patterns acquire meaning to individuals as they participate and partake in a set of practices and as they learn to make use of a culture's tools. In interaction with objects and others, individuals come to organize their thought. (Martin, 1993, p. 73)

Reflection is also a key component of Vygotsky's theory—it is "a critical component of higher mental functioning . . . [and is] promoted and supported by the cultural patterns of interchange in which an individual engages" (Martin, 1993, p. 73). Recall that reflection is the driving force in the proposed model for teacher change and that the social interchange referred to is highlighted in the model as socialization.

Since much of teacher change is revolutionary, teachers need reinforcement to deter the resistance which often follows (Martin, 1993). Teachers need time and encouragement to develop and reflect on new ideas. "Without time and support for constructive interaction, there is no chance that the teacher will appropriate the new information" (Martin, 1993, p. 84). The time-and-support issue discussed above leads naturally into the final theory that should be incorporated in a learning theory for teacher change—that of anchored instruction.

Anchored Instruction

The major goal of anchored instruction is to overcome the inert knowledge problem . . . by creating environments that permit sustained exploration by students and teachers and enable them to understand the kinds of problems and opportunities that experts in various areas encounter and the knowledge that these experts use as tools. (The Cognition and Technology Group at Vanderbilt, 1990, p. 2).
The ideas of anchored instruction have their roots in theories of Dewey and Hanson, who believed that experts are immersed in their areas of expertise and, consequently can assimilate new ideas relevant to their fields with relative ease. Whereas, novices receive the same new information disjointly and out of context and so cannot assimilate it with the same ease or understanding afforded those already immersed in the relevant area (The Cognition and Technology Group at Vanderbilt, 1990).

Situated cognition is a similar idea in which "knowledge is situated, being in part a product of the activity, context, and culture in which it is developed and used" (Brown, Collins, & Duguid, 1989, p. 32). Anchored instruction is related to and based upon situated cognition in that instruction is anchored in situations which simulate apprenticeships—processes of enculturation. To those who espouse the views of situated cognition, learning is such a process of enculturation. "When authentic activities are transferred to the classroom, their context is inevitably transmuted; they become classroom tasks and part of the school culture" (Brown, Collins, & Duguid, 1989).

Brown, Collins, and Duguid (1989) contend that learners should begin "with a task embedded in a familiar activity" and should be permitted to "generate their own solution paths" (p. 38). They should enculturate through their activities and use the language of the culture to discuss, reflect, and collaborate. Because of the cultural aspect of situated cognition, social interaction is a natural part of the learning process.

The contention of anchored instruction is that students learn best when they are immersed (anchored/situated) in the setting, much as an apprentice is immersed in the task at hand; and that such situations generally involve collaboration of sorts. Sustained
thinking about a topic aids in understanding and retention of relevant material, and enculturation is often accompanied by social interaction.

Extensive inservice professional development programs can provide the sustained/anchored instruction necessary to facilitate teacher change. Teachers spend time collaborating with other teachers in educational settings. Also, if teacher researchers team with university researchers to conduct studies in their own classrooms, they can receive intensive support and encouragement throughout their attempts to restructure their classrooms. The principles of anchored instruction provide yet another means to promote teacher change. These principles are encompassed in the *experiences* factor of the model for teacher change. Anchored instruction is, simply put, extensive experience.

**Conclusion**

Recall the cyclic model for teacher change which incorporated the factors *beliefs*, *disequilibration*, *socialization*, *experiences*, and *restructuring* connected by the driving force of *reflection*. Teachers' *beliefs* play a major role in their willingness and ability to change their instructional practices. Kuhn's theory of paradigm shifts gave a rationale for the difficulty of changing entrenched *beliefs* in the midst of revolutions and gave plausible factors involved in developing those *beliefs*. Copernicus, Cantor, and Lorenz served as examples of individuals who persisted in understanding their world (natural or mathematical, or both), even when their persistence led to discoveries that were counter to their own intuitive *beliefs*. Their evolving *beliefs* resulted in formulated theories that, in turn, led to ridicule and disbelief by their contemporaries.
Review of the teacher-change literature reiterated Kuhn's theory in showing how difficult revolutionary change is, and that change in practice is brought about by change in beliefs—beliefs mainly about learning and teaching. As shown in the model for teacher change, these beliefs, in turn, are brought about by reflection following socialization, experiences, and disequilibration. Disequilibration is the cornerstone of Piaget's theory of constructivism. Social interaction and experiences are major factors in both Vygotsky's theories and in anchored instruction. When teachers are faced with disequilibrium in their understanding of teaching and learning, they strive for equilibrium. By sustained experiences with new ideas and assistance/encouragement of colleagues and others, teachers restore that equilibrium by constructing (through reflection) new understandings and therefore new beliefs. Their new beliefs then manifest themselves in a restructuring of the classroom strategies.

Changing teachers' beliefs and the beliefs of those involved (i.e., parents, students, administration, etc.) can be a slow and arduous process. Skemp (1978) noted that one of four factors contributing to the difficulty of teachers changing their instructional practices could be: "The great psychological difficulty for teachers of accommodating (restructuring) their existing and longstanding schemas [emphasis in original], even for the minority who know they need to, want to do so, and have time for study" (p. 13).

According to Thompson (1992):

We should not take lightly the task of helping teachers change their practices and conceptions. Attempts to increase teachers' knowledge by demonstrating and presenting information about pedagogical techniques have not produced the desired results. Indeed, the research reviewed here suggests that teachers' conceptions of mathematics, of how it should be taught, and of how children learn
it are deeply rooted. Research would caution us against underestimating the robustness of those conceptions and practices. (p. 143)
CHAPTER IV
BACKGROUND, DESIGN, AND METHODOLOGY

By recognizing that bringing about changes in what goes on in mathematics classrooms depends on individual teachers changing their approaches to teaching and that these approaches, in turn, are influenced by teachers' conceptions, mathematics educators have acknowledged the importance of this line of research.

Alba Thompson (1992, p. 128)

This case study is about the beliefs of a single physics teacher as he struggles to implement calculators into his instructional practice. The questions, as delineated in Chapter I, are basically "what" and "how" questions. Although "what" questions may be answered by using a variety of research techniques, "how" questions are usually best answered by case studies, experiments, or histories. Of those three research methods, case studies are preferred when "examining contemporary events, but when the relevant behaviors cannot be manipulated" (Yin, 1994, p. 8). As mentioned in an earlier chapter, Thompson (1992) suggests that case studies about teachers' beliefs would be helpful in effecting teacher change.

Case studies have traditionally been maligned by many researchers as a legitimate means of study. Yin (1994) suggests that reasons for this disdain include: (1) the use of sloppy evidence and biased views on the part of some investigators, and (2) the fear that
generalizations cannot be made from a single case. Yin's answer to the first problem is, of course, careful, well-planned investigations; while his answer to the second problem is that case studies are generalizable to theory rather than to populations. "In this sense, the case study, like the experiment, does not represent a 'sample,' and the investigator's goal is to expand and generalize theories (analytic generalization) and not to enumerate frequencies (statistical generalization)" (p. 10).

The Researcher's Story

The second research role is the researcher as a learner. It is important to have this sense of self from the beginning.

(Glesne & Peshkin, 1992, p. 36)

To set the stage, it would be helpful for the reader to know something about me and why I chose to conduct this study. My story as a teacher began over 30 years ago when I was in an undergraduate program in a college of education. I learned traditional ways to teach mathematics, and I then proceeded to teach in those ways. We all know the routine: Go over the previous day's homework, lecture on new material, provide examples, assign homework on the new material. Lesson plans were fairly uncomplicated and the textbook was usually closely followed. Drill and practice were the accepted norms of assignments--students who could not compute could not be expected to use computations to solve the contrived word problems.

The beginning of my own teacher change began when I attended a Calculator and Computer Precalculus institute (C²PC). I began to see the ways in which calculators could enhance students' understanding of mathematics, and I obtained the accompanying
Demana/Waits textbooks for my precalculus students. Encouraged by the success of my students and their enthusiasm with the calculators, I continued to attend professional development programs in which calculators were being demonstrated and I became involved in calculus reform as well. I taught classes at The Ohio State University (OSU) which were calculator-based as well as ones that were computer-based.

At the time of my initial C^2PC institute, I was teaching mathematics, including precalculus and advanced placement (AP) calculus at Worthington Christian High School (WCS). Mike Smith, the subject of this case study, was (and still is) the physics teacher at WCS. (The names of people and places have not been changed. Permission has been given by all concerned to use actual names. The reason for this usage will be explained later in this chapter.) Mike was basically against the use of calculators in the classroom—he viewed them as a crutch. Mike and I had relatively few discussions about the calculator. I simply went about teaching my classes with the calculator, and he taught his without. Mike and I were colleagues, and we respected each other's views.

My own attempts at change have made me very much aware of the inherent difficulties of accomplishing such change. It has also made me very interested in the process of teacher change. This case study has evolved from that interest. In January of 1994, I felt that I would like to study two teachers with opposing views of calculator usage in the classroom. Because of my interest in physics and physics teachers' concepts of mathematics, I wanted to study two teachers who were both excellent physics teachers. I phoned one of them and found that she had moved out of the area, and then I phoned Mike. When he told me that he would like to investigate the possibilities of calculator
usage in his instructional practices, I became really excited. Here was a teacher who wanted to investigate change—a teacher who had once been adamantly opposed to calculators. What had caused him to want to change? How difficult would such a change be? What would it take to cause him to make a one-hundred eighty degree turn? The questions seemed endless, and I was quite interested in pursuing them. That was the beginning of Mike's change as a teacher, and that was the beginning of this case study.

When I was a teacher at WCS, Mike and I were senior class co-advisors. As such, we chaperoned retreats, parties, and senior trips, and we were in charge of graduation. So, we had chances to talk and to get to know each other as professionals and as colleagues. Because of this collegiality, Mike did not hesitate when I asked if I might study his attempts at change. Also, because of my successful teaching experience at WCS, the superintendent/principal (Taylor Smith) was quite willing to allow me to conduct my study there. Other members of the faculty, including the assistant principal (Tom Angel), the precalculus/calculus teacher (Mark Pifer), and the chairman of the science department (Bill Williams) have been very helpful and very encouraging as I have been collecting data through interviews, observations, and questions.

The Subject, Mike Smith

Mike Smith is a high school physics teacher at Worthington Christian High School (WCS) in Worthington, Ohio. He received his bachelor's degree from The Ohio State University in Physical Science Education with a dual major in chemistry and physics and a minor in calculus. He then earned fifty hours of graduate credit in the Department of Food Science and Nutrition at The Ohio State University.
By his own admission, Mike's understanding of mathematics is weak, and so was his understanding of physics until he was forced to teach it several years ago (in Marysville High School). He preferred chemistry, but when he constructed his own understanding of physics, it became fun for him; and he has enjoyed teaching it ever since.

Mike is a very reflective teacher, and he wants his students to learn concepts rather than computations only. It is for this reason that Mike was against the use of calculators in his classroom. His method of teaching physics depended highly on dimensional analysis—solving problems computationally by rectifying the dimensions so that the desired answer has the required dimensions, kilograms times meters per second squared, for example. So he viewed the calculator as a crutch which bypasses the concept involved.

When I left WCS, Mark Pifer took over my precalculus and calculus classes. He inherited the Demana/Waits textbooks and also began to get enthusiastic about the enhancement of mathematics through the use of graphics calculators. As time went on, Mark and Mike found themselves at opposite ends of the calculator-usage spectrum. When I asked Mark, in an interview, about his relationship with Mike, he said, "I've been working with him and against him for the last several years."

Mark told me that Mike was weak in mathematical concepts and so used different methods from what Mark used to explain similar concepts. As a result, their students were in constant confusion, and Mike and Mark did not understand each other's ways. Mark's precalculus and calculus students then infiltrated Mike's physics classes with their programmable calculators, and Mark claimed that that was what "threatened Mike the most." Mike's students got their answers too easily and could not explain to him how they
got them.

When I asked Mark to tell me about Mike's change of belief towards calculators, he offered to write down the details. Mark wrote the following story about Mike's "metamorphosis."

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**Mark's Story About Mike**

For many years (I have been at WCS for 16!) the science and mathematics departments did "their own things." Our conversations were casual but sometimes interesting in what was being taught in each discipline. We rarely ever discussed how a topic was being taught. As Mike moved more heavily into the teaching of physics, he became frustrated with mathematics questions the kids were asking him. He would explain a physics concept which was math related using some technique which he understood and which always gave him the correct answer. The problem was that the students did not understand what he was doing and when they questioned him, he was unable to explain the logic and the process in terms they understood based on their exposure (or lack of exposure) to math. Then he started coming to me and asking me questions like: 1) "In what course is this concept taught?" or 2) "At what point in the year is this concept taught?" or 3) "How do you teach this concept to the students?" He expressed to me that he felt he was having to spend too much of his time teaching math instead of physics. Each of us began to realize how important it was that students be exposed to certain math instruction prior to the application of that concept in the physics class. We discussed the juggling of chapters in each discipline so that the concepts would coincide better, however, we never did this because of the interruption to the continuity of each course. What we finally settled on was the establishment of prerequisites for science courses. This way we could be sure that a student coming into physics would have accumulated the necessary tools for success there. The trimester is a fantastic way to accomplish this goal! [The trimester idea will be discussed more fully in the next section of this chapter.]

The next phase of his "metamorphosis" was the realization that his own mathematics background was too weak to accomplish the science goals he had set for himself. One year I had a precalculus class which met in his classroom during his free period. He would sit at his desk working on paperwork while I was teaching the math. I think this was when he began to realize that I was giving the students the insight they were lacking in his explanations. He then started asking the how questions and discovered that my use of the graphics calculator to illustrate and explain was an essential part of the explanation. Initially, Mike had been very reticent to permit too much use of calculators because he wanted the students to prove to him that they understood how to do the problem. He then
began to realize that the students could not get the correct answers from the calculator if they didn't understand the problem well enough to interpret the graph. Even students who wrote programs to simplify their work had to have a working understanding of the problem or the programs they were inventing would not work. (It was interesting when some students took programs that other students had written but still couldn't get correct answers all the time because they did not understand the logic of the program or how to interpret the answers the student-written programs were giving.) I think I finally convinced him that the calculator was not a crutch, but a valuable tool in the overall understanding of the math concepts which would translate directly into physics.

Finally he made the statement to me: "I've got to get one of those calculators. If I get one, will you teach me how to use it?" BREAKTHROUGH!!! Once he got his hands on one and began to see what it could do, there was no holding him back. He signed up for a TI-85 institute at Columbus State [Community College] and came back even more excited. [This will be discussed in the rendition of the pilot study.] Next came the recent summer program and the expenditure of funds for equipment. Now, I feel that in some ways he is getting more mileage out of the technology than I am in the math department.

The Setting: Worthington Christian High School

Something has to be said about the school setting, because WCS has definitely contributed to Mike's desire to change. WCS is a highly respected school--respected both in academics and in sports. Many of the graduates of WCS go on to higher education in some of the nation's most prestigious colleges and universities. One of the reasons for the success of WCS is the background, education, and commitment of the faculty and administration. Another reason is WCS's commitment to academic excellence. When I, as a faculty member, asked if I could go to the C²PC institute in 1989, there was no hesitation. I was encouraged to attend, to implement the ideas, and to purchase the textbooks. Teachers are encouraged to be innovative and to grow professionally. When the NCTM regional convention was held in Columbus a couple of years ago, the mathematics teachers at all levels (elementary through secondary) were encouraged to
attend, and many of them did.

This school year (1994-95), WCS is trying the trimester approach to education. Each subject is taught in a two-hour block of time for a twelve-week period, for a total of one-hundred twenty classroom hours per subject. Students take two or three subjects per trimester, and teachers teach a normal load of two subjects per trimester. In this way, a teacher's workload is reduced, with fewer papers to grade, fewer students to teach, and fewer lessons for which to prepare. Teachers are instructed not to lecture for two solid hours, but to try different methods of instructional practices. Mike accepted this challenge, and his successes in trying to implement change are due in part to the encouragement and innovativeness of the school in which he teaches.

**Addressing Possible Limitations**

Bogdan and Biklen (1992) caution that research subjects' identities should be protected—that anonymity is the norm in research. However, Shulman (1990) says that "Many teachers no longer wish to remain hidden behind a cloak of anonymity; they prefer to be credited and recognized for their contributions" (p. 11). Shulman claims that the anonymity tradition began when subjects were seen as "powerless and in need of protection" (p. 14), but that today researchers must "treat teachers as professional colleagues who deserve as much recognition as the traditional scholar" (p. 14). There may still be cases in which it is preferable to offer aliases for teachers because of risks of disapproval by peers and administrators, but in this case, it is my belief that Mike's story should be told with full recognition of who he is and where he works. It is, after all, his story, and he fully concurred with divulging his identity.
Another possible limitation could be the study of a single case. Yin (1994) suggests that a case that represents a "unique case" (p. 39) provides rationale for the single case study. Mike's situation is such a unique case. The factors involved are: 1) a physics teacher who was against the use of calculators in his classroom, 2) the teacher's decision to participate in a professional development program which used calculators, 3) the teacher's subsequent change in his stance against calculator usage, 4) his decision to participate (as a team with the researcher) in another professional development program, 5) his administration's complete support and encouragement of his change, and 6) his school's unique trimester program which facilitates his change. The combination of the factors prescribes the appropriateness of the single-case-study design.

Patton (1990) and Denzin (1978) posit four types of triangulation: 1) data triangulation, 2) investigator triangulation—the use of several different researchers, 3) theory triangulation, and 4) methodological triangulation. I am triangulating the data (see Data Collection section) and the theories (see Chapter III). The triangulation of researchers and methods is beyond the scope of this dissertation.

A concern related by Bogdan and Biklen (1992) is that of conducting a study with people you know. They claim that people who know you well may not feel free to relate to you as a researcher, as a neutral observer; and, you, the researcher, in turn, may not feel comfortable with such a role. Because some may claim that these conditions exist in this study, these concerns need to be addressed.

Each situation is unique, and I believe that WCS is unique in that it is a Christian school. Employees of WCS are also members of a local church. Church members think
nothing of seeing their peers perform different roles. Some members become elders of the
church or trustees of the church or members of the school board. As such, they are
afforded the respect that comes with the title. A few years ago, Tom Angel was a
gometry teacher at WCS. Now he is the assistant principal, and the faculty treats him
differently. He is respected for his role and his office and his work among the people.

I have been away from WCS for several years, and people know that I have
pursued higher education. They respect my background and my education, and they have
told me so. They seem to have no difficulty relating to me as a researcher, and Mike, in
particular, treats me as an expert in the field of mathematics education. He has confessed
to me several times that he feels very comfortable talking to me and asking me questions,
knowing that he can trust me and trust my judgment. Mike also knows that I am as
interested in learning as he is—that I want to learn from him. Glesne and Peshkin (1992)
note that one of the research roles is the "researcher as learner" (p. 36). They say that
"the relationship you and your others develop should be marked by reciprocity, trust,
mutual respect, and learning" (p. 36), and I believe that those with whom I worked and I
have developed all of the qualities mentioned. Mark Pifer said it well when he mentioned
to me in passing one day that I used to be a fish in the pond and then I returned to study
the fish. As a participant observer, I believe that I gained the access I needed to
adequately study, learn, and relate Mike's story.

Design of the Pilot Study

Bogdan and Biklen (1992) suggest that some fieldwork and data collection be
done before the actual proposal is written. They believe that qualitative research is best
accomplished by making detailed plans after spending some time at the setting with the participants. Anselm Strauss (1987) definitively states that "No proposal should be written without preliminary data collection and analysis" (p. 286). Although the actual details and analysis of the pilot study will be discussed in a later chapter, the design of it follows.

The pilot study and the ensuing data collection and analysis took place during the early months of 1994. I mentioned the way in which I decided who and what to study, and the timing seemed right to begin the study prior to the T³ (Teachers Teaching with Technology) institute which was held in February 1994 and which Mike and I both attended.

Before the institute, I interviewed Mike (see Appendix A for interview protocol used). I audiotaped Mike's interview, which lasted about an hour and fifteen minutes, and I transcribed some parts of it. The week following Mike's interview, I attended his physics class and videotaped his teaching. The day I observed him, Mike was discussing homework problems, rather than teaching new material.

Mike then attended the T³ institute, and I went to the sessions with him and made some field notes. A few days after the institute I audio taped him as he and I talked in an unstructured interview about what he felt he had gained from attending the institute. I then left my tape recorder with him, and he used it to make a journal, making comments into it as he reflected on relevant issues. As with the audiotapes of the first interview, I transcribed parts of both his journal and our discussions.
Design of the Study

My design for the study was based on theoretical assumptions (delineated in Chapter III) and on what Bogdan and Biklen (1992) call "data-collection traditions (such as participant observation, unstructured interviewing, and document analysis" (p. 58). As suggested by Bogdan and Biklen, data analysis was an ongoing part of my research, and a full account of my procedures were best accomplished in retrospect. However, I did have a plan, and that plan began with the pilot study and continued with ensuing data collection and analysis.

Time Frame

This study began in January of 1994 and continued through the end of the second trimester (February 1995), when Mike finished teaching his physics class. It began with Mike's initial desire to change that was precipitated by his purchase of a TI-85 graphics calculator and his attendance at the T3 institute.

Data Collection

Yin (1994) suggests that increase in construct validity can be accomplished by: (1) the use of "multiple sources of evidence" that converge, (2) establishing a "chain of evidence," and (3) having the case study "reviewed by key informants" (p. 34). Use of multiple-data-collection methods is commonly called triangulation (Glesne & Peshkin, 1992). As a participant observer, I, of course, took field notes whenever I was at WCS. I spent many days with Mike, talking and observing. We attended the T3 institute together and the Connecting Math and Science (CMS) week-long professional development institute together. After the CMS institute in August, I was in weekly contact with Mike--
either through telephone calls, or observations, or meetings—until his physics class began in November 1994. We planned workshops together in which we introduced the CBL unit to other educators, and so we spent time reworking many of the CBL experiments that we learned at the CMS institute.

After the second trimester began at WCS, I initially attended Mike's twelve-week long physics class on a daily basis. During that time, I made field notes and videotaped his teaching (see time-line section). Towards the end of January 1995, when I felt classroom observations were no longer necessary, I began collecting data through informal discussions with Mike as we prepared for two major convention presentations. My data collection ended towards the end of February, when I asked Mike to read the last two chapters of this dissertation to perform a member check.

Besides relying on my own observations of Mike, I talked mainly with Mark and Bill (chairman of the science department) to get their input as to what changes they saw in Mike. This is a second source of evidence, and the ongoing discussions aided in developing a chain of evidence.

Because of Mike's limited time to put anything in writing, I encouraged him to make use of the audio cassette player. He complied as well as he could considering time constraints and provided me with a journal via audio tape. His journal provided yet another source of evidence, and aided in developing a chain.

From time to time, I mentioned to Mike, Mark, or Bill my interpretations of what had been said to me by them, and I asked for their input as to my understanding. In this way, I asked for a review of my understandings by key informants (commonly referred to
as *member checks*). When I finished writing, I provided Mike with a draft copy for his review. I felt that I should provide Mike's alternate views, if there were any, along with my own when I reported my findings. Mike read the draft copy, and he said he agreed with my interpretations of occurrences. The only question he had was why I had waited until the end of my study to allow him to read what I had written. I told him that I did not want to influence his beliefs by showing him what I had written. Anytime that I provided suggestions to him that might have influenced him—which I did intentionally at times that I deemed appropriate—I noted that in context.

**Time Line**

I began daily data collection on Monday, November 21, 1994. I observed Mike's physics class and videotaped his teaching every day (except test days, and other days when the class was not meeting) until January 17. The class met from 1:00 p.m. until 3:00 p.m., Monday through Friday for twelve weeks. Interviews with Mike consisted of semi-structured and unstructured interviews and informal conversations. Mike also recorded a journal at his convenience, and at the end of the study, he recorded a journal in response to some direct questions.

**Data Analysis**

In order to generalize to theory, I coded the data. After I collected data, I coded it along the lines of the proposed model: \( D \) for disequilibration that Mike experienced, \( I \) for any social interaction that influenced his beliefs about teaching, \( R \) for restructuring his instructional practices in any way, and \( E \) for any experiences that Mike had that influenced his beliefs. Mike's beliefs were coded according to their type—\( MB \) for beliefs about
mathematics, *EdB* for beliefs about education, and *TB* for beliefs about technology. The two other codes I used were in response to my research questions--*F* for factors that influenced him to change his beliefs, and *O* for obstacles to his beliefs. Coding the data facilitated ease of analysis and also aids in addressing issues about reliability.

Bogdan and Biklen (1992) posit that qualitative researchers do not view *reliability* with reference to replicating a study. They say that qualitative researchers are more concerned with the accuracy and comprehensiveness of their data than with replication by another researcher, since the latter would be impossible. Also, since researchers bring different backgrounds to the settings, they may be looking for different things, collecting different data, and reaching different conclusions. Bogdan and Biklen conclude that reliability is in question only if two different studies of a single setting "yielded contradictory or incompatible results" (p. 48). Coding the data can aid in alleviating any concerns about reliability.

It is the researcher's task to analyze and make sense of the data and "to transform it from its acquired form . . . into a form that communicates the promise of a study's findings" (Glesne & Peshkin, 1992, p. 145). According to Glesne and Peshkin (1992), in order to accomplish this task, the researcher's interpretations must be trustworthy. Time spent in the field and time spent building relationships with those in the field contribute to trustworthiness of one's data. Spending a year in the field with daily contact for much of the time should lend credibility to the data that I collected.

Glesne and Peshkin (1992) mention that triangulated findings are also an important factor that contributes to trustworthiness. As mentioned above, I did triangulate the
data—in methods used in collecting (interviews, journal, field notes, videotapes, artifacts—e.g., student tests), and also in interviews with Bill and Mark.

Another factor Glesne and Peshkin add is that of "continual alertness to your own biases, your own subjectivity" (1922, p. 147). Collecting data from several different sources and with several different methods, and having Mike corroborate the data is a good check on this. The collection of data from colleagues of Mike's and the subjection of the data to the scrutiny of my dissertation committee should help guard against my biases distorting the data. Lincoln and Guba (1985) suggest such scrutiny by others.

Another important aspect of demonstrating the trustworthiness of the data is to understand and include in the analysis the limitations of the study. Detailing such limitations aids readers in knowing how to interpret the findings (Glesne & Peshkin, 1992).

In summary, using the code mentioned above and my own interpretations, subject to review by others, I reported the data according to major developments of Mike's beliefs and practices. The first narrative is composed of the pilot study, in which Mike attended a two-day professional development program involving the TI graphing calculator. As a result of that institute, Mike's uneasiness about the use of calculators in his physics classes lessened considerably. Disequilibrium occurred--his beliefs about how his classroom should operate were thrown out of balance by his experiences at the institute. He reflected over the experiences he encountered and discussed them with colleagues and with me--this is described in the model as socialization. Over the course of the following few months, his beliefs began to change, but no substantial change occurred until he
participated in the summer CMS professional development institute.

The next narrative takes place during the week of the CMS institute, a time when Mike was immersed in experiences--anchored instruction--involving the use of the CBL and the TI-82. Again disequilibrium occurred, and as a result, he began restructuring his instructional practice in his classes. The period immediately following the CMS institute--when Mike began to restructure his instructional activities--developed as the third narrative. The fourth and final narrative occurred as Mike taught his physics class using the CBL and the TI-82. This was the time to validate his apparent evolving beliefs by observation of his actual classroom practices.

Conclusion

Bogdan and Biklen (1992) give several characteristics of qualitative research, all of which were inherent in this study. The direct source of data was the natural setting of WCS and the main instrument for analysis was the researcher's insight. This is a descriptive study, with data being in the form of words and pictures, and with prime importance being given to process rather than product. The data was analyzed inductively--there were no preconceived hypotheses to test. Finally, since participant perspectives are of utmost importance, I undertook participant observation. I was more than simply an outsider looking neutrally at a setting.

According to Glesne and Peshkin (1992), "participant observation ranges across a continuum from mostly observation to mostly participation" (p. 40), and I would place myself on the "participant as observer" side of the continuum. I was interested in the CBL and calculator usage in the classroom, and Mike and I conducted workshops to
demonstrate such usage. We worked together in some ways, and, yet, in other ways, I was simply an observer. Glesne and Peshkin describe the paradox that develops as one moves along the continuum: "The more you function as a member of the everyday world of the researched, the more you risk losing the eye of the uninvolved outsider; yet, the more you participate, the greater your opportunity to learn" (p. 40).

However, simply declaring oneself a participant observer does not ensure that status. Sharp (1989, cited in Millroy, 1992) points out that "it is the 'locals' who must confer the status of 'participant' on the researcher. This is not a status that one can confer upon oneself. . . . In many circumstances, the researcher is not given this status" (Millroy, 1992, pp. 83-84). Mike's continued comments during the study (e.g., asking me for advice both during class and in informal conversations, looking to me for help, participating in workshops with me, acknowledging that I was a great help mathematically to him) led me to believe that he did indeed confer the status of participant upon me.

I am personally interested in teacher change and am committed to doing research which can help to facilitate that change in others. By doing an in-depth case study of a highly reflective teacher, and by reporting an analysis of his beliefs as he progressed through change, I am hoping that others will be affected as positively as I was.
CHAPTER V

MIKE'S STORY

*Perhaps the most valuable results of all education is the ability to make yourself do the thing you have to do, when it ought to be done, whether you like it or not.*

Thomas Henry Huxley, *Technical Education*,
(cited in Gallian, 1986, p. 333)

Storytelling as Inquiry: An Introduction

I have chosen to tell Mike's story as exactly that—-a story. I believe that storytelling as inquiry is an authentic and scholarly way in which to portray the data. By reading the story, the reader can identify with characters and can vicariously partake of the events of the story. Before launching into Mike's story, I thought it best to give some background about storytelling as inquiry.

Van Maanen (1988) speaks in detail of three forms of ethnographic writing: (1) reality tales, (2) confessional tales, and (3) impressionist tales. Of the first, he says, "A single author typically narrates the realist tale in a dispassionate, third-person voice" (p. 45). He posits four conventions that are generally followed in narrating the realist tale. One is that of the experiential author(ity), in which a credentialed author attempts to remove hints of subjectivity by removing the "I" (the observer) from the report. The narrative reads as statements of the people studied. Another convention is that of a
documentary style, in which the author focuses on minute, mundane details of everyday life. A third convention is that the narrative must closely reproduce the native's point of view, and the fourth convention asserts interpretive omnipotence—"self-reflection and doubt [of the ethnographer] are hardly central matters in realist tales" (p. 51).

The second form of ethnographic writing suggested by Van Maanen is that of the confessional tales. Van Maanen claims that "the distinguishing characteristics of confessional tales are their highly personalized styles and their self-absorbed mandates" (p. 73). Instead of saying, "The teacher does X," as is reported in realist tales, "I saw teacher do X" is the norm of confessional tales. Confessionals portray the ethnographer as either a student of the observed, who desires to learn, or as an interpreter (Clifford, 1983; Van Maanen & Kolb, 1985; Geertz, 1973), and the account includes autobiographical details of the fieldworker being altered by the experience. Conventions of confessionals include: personalized authority, autobiographical accounts, and end results of fieldworker "supporting whatever realist writing the author may have done and displayed elsewhere" (p. 79).

The third narrative account that Van Maanen describes is that of impressionist tales. When the field experience is expressed and "told in the first person as a tightly focused, vibrant, exact, but necessarily imaginative rendering of fieldwork, an impressionist tale of the field results" (p. 102). The tales represented are a "collage of brief images" (p. 121) that the researcher noted at the time of the experience. Conventions include: dramatic recall recounted roughly in the order in which the events occurred; involvement of interesting characters in the story, rather than a dispassionate
recall of facts; "standards are not disciplinary but literary ones. . . [whose objective is to] keep the audience alert and interested" (p. 106).

In describing the different forms of ethnographic writing, Van Maanen points to the current revisions taking place in the conventions involved in each of them. The forms are evolutions and will probably continue to evolve in the future. My own thoughts as to how best I can portray Mike's story is a merger of confessional and impressionist tales. My rendition is not entirely confessional; although I do interject some confessional autobiographical accounts into it. Neither is my portrayal of Mike's story entirely dramatic; although it is not a dispassionate recall of facts. I have hoped to write it in a style that will keep the audience alert and interested.

I enjoy reading. I especially enjoy reading stories—both fiction and nonfiction. As I read about characters' lives, I vicariously live them, too.

Characters in stories are said to be compelling by virtue of our capacity for 'identification' or because in their ensemble they represent the cast of characters that we, the readers, carry unconsciously within us. (Bruner, 1986, p.4)

As a teacher, I have a couple of favorite films I have seen--ones that I have viewed over and over, each time gleaning new insights into the teaching role. I think that I am not unique in the application of film and story characters to my life. Teachers look to films and biographies/autobiographies because "the lives of other teachers help them understand themselves better" (Schubert, 1992, p. 141).

Reading autobiographies and biographies, seeing films about teachers and their struggles, identifying with them "not through an omniscient eye . . . but through the filter of the consciousness of protagonists in the story" (Bruner, 1986, p. 4)--this helps
teachers to reflect and grow. This offers us inspiration and example.

I have always been a reflective person. I think and analyze and redo a hundred times in my head. I have always been a strong advocate of learning by experiences. I believe that my life’s experiences make me who I am today. Reading has enhanced my experiences—by reading about others and their experiences, I can learn (and do learn) much about myself.

I am a firm believer that teachers’ experiences and their reflection on those experiences are crucial to the education process. If reform is to happen, it must happen first with teachers who are themselves committed to change and who take ownership of their change by reflecting on their own experiences. The biographical approach is an important component of the process of such change (Rudduck, 1988).

Storytelling is a natural way for people to relate their experiences. Since it has been a natural vehicle for learning throughout history and throughout cultures, should it be surprising that it can be a useful tool for learning to teach in the twentieth century? Since it is possible to live through a situation and not learn anything significant from it, one effect of telling the story is to wrest meaning from the experience (Mattingly, 1991). It is possible that merely relating the facts in a non-narrative form could produce the same results, but it is not likely. Mattingly addressed this very issue in a study and found that in the narrative form, actors are identified and accounts focus on people rather than on facts. This can hamper those who would rather not focus on individuals, but it should be helpful for those whose practice (like teachers) requires them to focus on individuals.
Storytelling is also being increasingly used in research studies of educational experience. This line of research, called *narrative inquiry*, is based on the premise that teachers, learners, and researchers are all both storytellers and characters in one another's stories. Collaborative stories between researcher and teacher have been created from a merging of their individual stories (Connelly & Clandinin, 1990).

Another purpose of telling a story is to cause readers to reflect on their own practices (Clandinin & Connelly, 1991). Through reading about others' experiences, readers can "question their own stories, raise their own questions about practices, and see in the narrative accounts stories of their own stories" (p. 277).

Reason and Hawkins (1988) posit two paths of inquiry: Explanation, in which the inquirer stands back, analyzes the situation, and relates what s/he has seen; and expression, in which the inquirer "partake[s] deeply of experience" (p. 80). In the latter, storytelling is an excellent way in which to relate that experience.

The outcome of a co-operative inquiry is often deeply personal and practical, as well as theoretical. The theory and the practice can often be well grounded and expressed in the personal or collective story. . . .Thus science can learn to tell good stories, and then explanation and expression become married, and the progeny are theories born of story, and stories born of theory. (Reason & Hawkins, 1988, pp. 100-101.)

To this end, it is almost impossible for a participant observer who has partaken of the experience of which s/he is writing to simply write about it as a scribe (Clandinin & Connelly, 1991; Connelly & Clandinin, 1990). Mike's story merges with my story, and the two are difficult to separate. There will be times, of necessity, in which I relate
my participation with Mike, and my reflections of the experience in which I also grew as an educator.

The Setup

Mike's story will be told roughly in the same sequence that it unfolded to me. I begin with the pilot study—entitled Narrative One—during which I initially interviewed Mike. Then I observed his teaching of a physics class. Shortly thereafter we attended the Teachers Teaching with Technology (T³) Regional Convention. We then had a few conversations in which we talked about what Mike had learned. Narrative One describes Mike's beliefs as they evolved throughout the time frame surrounding the initial interview and the T³ convention.

In the first narrative, I will separate Mike's basic beliefs into categories. The narratives following the pilot study will be the chronological unfolding of his beliefs. No longer separated into categories, his changing beliefs will be depicted as they occurred. In the final chapter, I will again, reassemble his beliefs into categories so that we can examine them for their change and the factors that contributed to that change.
One of the stumbling blocks that Mike had to the use of calculators in his classroom was that he insisted that he wanted his students to understand the mathematics that they used in physics. During an initial interview with Mike, he gave me some insight into his drive to help his students understand.

I, as a child, always wondered why. I'd walk through the fields and I'd see something and I wondered why. And so, that's probably the key driving force in me that has made me go to science, because I want to know why. The manipulation of these equations helps me understand the whys... I'm not an engineer-type who is trying to get to an answer because I want to apply it. I'm more of a philosopher-type who just is curious and likes answers. But I get really bored with the applications. Now, I can see an engineer would be much more keyed on getting to a very quick answer in many, many ways... But I'm always more of a big picture type person...

I remember sitting in college--this was an exam question. This professor--we had gotten done talking about angles, coefficients of friction, centrifugal forces, all these type of things, OK; and he had a problem where he had a record player. He told us the diameter of this record player. He told us the revolutions per minute it was going, told us the weight of the record arm, the head, the needle, the angle of the slope on each groove; and he went through all this information, and the question was: How many grooves were on this record? The answer's one. It's a continuous groove--that's the only way a record works. Marsha, I jumped into every equation, put in all these values, was doing all these calculations; and the thing the guy was trying to teach is: Step back, look at the problem, and apply the points that are important, and ignore the others. Well, because I was memorizing physics at that time--I didn't understand physics--I went into my equations. If I had had one of these babies [TI-85--Texas Instrument's graphing programmable calculator], with all the equations, I would have rolled through them and tried to find one I had all the variables for; and I never would have stepped back to say, "What's happening?"... Maybe it's because in science you never know all of the variables, and in math you usually know them, that I'm less likely to jump into a total dependency on an equation; because there's always assumptions I'm making in science, because there's some things I can't fit into my [pause] there's too much to know.
Beliefs About Mathematics

But, just what was the mathematics that Mike wanted his students to understand? He claimed that by manipulating equations, students understood a relationship and derived from that some understanding of what was happening with the physics. When I pressed him for an explanation of what his idea of "understanding the mathematics" meant, he related several different examples with explanations. His simplest example:

Why do two objects fall at the same rate, even though they have different masses? If you think through some logic, initially you say, "OK, the heavier object's got a greater force, because mass times gravity is force, and so, this object's going to have a greater force, therefore, it should fall faster--it's got a greater force." Well then you take the next principle you're dealing with and say, "Well, what makes an object move?" And you say, "OK, well, mass considered with the force that's acting on it will determine what kind of acceleration is produced. So even though there's a greater force, there's a greater mass; and, therefore, it's going to take a greater force to achieve the same acceleration." Well, you set that out in an equation, and suddenly the masses are cancelling out on both sides, and you've got it. . . . Objects always fall at the same rate independent of mass because the mass is unimportant. The masses cancel out. . . . That's what I mean by [understanding] the math. Not so much that they can manipulate equations, but the manipulation of the equation gets them to that next level. And if they don't know how to manipulate equations, they can't get to that next level. And I find that kids do not manipulate equations unless they have to.

When I observed Mike's teaching, his entire focus on the physics problems he was explaining seemed to be dimensional analysis. As Mike described it later to me:

[It's] getting the units to work out and helping that be a pathway to where they [the students] want to go. I find a total resistance in kids to do this if it's just a plug and chug. "Give me the values, put me in the calculator and hit solve." And I just come unglued. I can't stand it.
Through all of this, was Mike still achieving only instrumental understanding—in both physics and mathematics—from his students (as well as from himself)? It seemed that the level of understanding Mike desired was still at the procedural level, even though he claimed he wanted it at the relational level.

Beliefs About Technology

Because of Mike's firm belief that his students needed to understand the mathematical equations, his main resistance to the use of calculators came from the key he calls the "solver key." (With the TI-85, it is possible to input a formula, such as the ideal gas law, \( PV = nRT \), and solve for any one of the variables, by plugging in the known values and using the solver key.) Mike mentioned this key by name several times in our conversations, and he mentioned it indirectly even more times.

As far as this calculator [TI-85] right now—there's one button that I think should be outlawed—it's the solver button. With the exception of that button, there's a lot of things that I'm really excited about right now. I'm still not sure I like that no-brain button that says, "Hit me." And you can argue that, "Well, you've gotta put the equation in the right form before you can use the solver button, so it's not a no brainer; you've gotta know what you're doing." But it's still... [voice trails off]

He reiterated this in a later conversation:

If TI-85 would take the solver key off that, then I'd be happy. That becomes my most fearful key right now.

Another fear of Mike's was alleviated at the T³ conference. Previously, calculators had not been a problem for Mike, because he did not understand their power. His students were programming answers on tests and sharing calculators. Mike did not understand that, even though the screen was cleared, students could
access the information. Now that he was no longer ignorant of the power of the technology, his fear was that students could too easily cheat on his tests. He learned, however, that through a link to the computer, students could transfer their files to computers before the test and retrieve their files after the test. They would then have cleared memory for the test, but they would not lose all of their programs. That seemed to remove one of Mike's obstacles to the technology.

Notice in the above statements, that Mike was not totally against the calculator. That was not always the case. By Mike's own admission, until about a year and a half ago, he was really quite against calculator usage in his classrooms. In fact, in our first interview he said:

Mark is the one that pushes me the hardest on the thing [calculator]. "You've gotta let them use it. You've gotta let them use it." And I'm the one that fights him like crazy. He thinks I'm in the dark ages. I use it. You know. I've got my TI-85. . . . Because I see sometimes it's such a great tool--especially the more laborious type of calculations . . . and maybe it's because I'm a visual person.

When Mike returned from the technology conference, he talked to me about his conversation with the Worthington Christian School (WCS) chemistry teacher. When he showed her the solver button, her reply, according to Mike, was that she did not allow the TI calculators to be used in her chemistry classes.

So her way of dealing with this is to outlaw--and I thought, "OK, now, that's where I was two years ago." I really was. TI-81 . . . I didn't want you to use it. In fact, I told a group of kids one year (I don't know if you were around then), "Go buy a cheap calculator, 'cause on test day, that TI is not in here. You're using the other one. And I don't care if you want to do your homework on the TI-85 (or TI-81 at that time), but you're not going to use it on the test." And I listened to her [the chemistry teacher], and I thought, "Boy, there I am two years ago." She was very strong about it. . . . I think a lot of her problem is--
she doesn't know what it does. And she doesn't have the time to learn what it
does. Therefore, outlaw it, rather than deal with it. Time is a big enemy. That
two-day conference I went to has gotten me to the point where I'm not afraid of

When I asked Mike what had caused him to change his mind about the
calculators, he said, "Why I'm entertaining this thing? I've been badgered into it by my
students!!" Mike also mentioned team-teaching in our first interview:

Mark keeps saying to me, "Mike we've gotta team teach. Because with your
understanding of the physics that's going on, and my understanding of the math
description of it, we could take our kids even faster and farther." That's why
I'm going to the seminar. I mean, honestly, it's because I understand what he's
saying is right, but I'm not willing to compromise the physics for the math.

According to Mark, after Mike began to see some of the concepts that Mark
was teaching with the calculator, Mike finally decided that he needed to have a
calculator; and he bought a TI-85 for the department with school funds. He would get
frustrated trying to use it as he "faced his own lack of knowledge" (Mark's words) and
would ask Mark for help. Mark finally told him that he needed to go to the T³
technology workshop, where he could learn how to use the calculator. Mike originally
planned to go only on Friday, but he returned for the Saturday sessions, and, Mark
remarked, "Now he's like a horse pulling me!" According to Mark, Mike went from, "I
won't allow it in my classroom," to "Every kid needs one and needs to know what to
do." Mark claimed that it was a remarkable 180 degree turn in less than a year's time.

Mark told me another event that occurred which influenced Mike. One of
Mike's students, Joe, worked on a science project that required the use of the
calculator. As Joe's advisor, Mike saw the data that Joe was able to pull from his
calculator, and it inspired him.

As for Mike's fears that some students will become calculator-dependent, Mark said that they probably would have become dependent on something else, anyhow—like formulas. He believes that calculator-dependent students are learning to cope, and using a calculator as a crutch might be the only way for them to learn the material. He cited one student who used the calculator in such a way, and, therefore, was a source of frustration to Mike in his physics class. When that student went to college, he went into computer programming, which was obviously where his strengths lay.

Beliefs About Education

Mike was a traditional teacher in his lectures and homework review. He was an imparter of knowledge, rather than a facilitator. He attended a seminar for the state of Ohio model curriculum for science education at the high school level. Their motto for the seminar was, "Less is more" (i.e., it is better to teach fewer facts and deal with deeper thinking on those facts.) They wanted teachers to get involved more in the discovery method of learning. Mike agreed with much of what was said, but the conflict came when he felt that his students were being cheated because they had not covered as much of the material as they should have due to time constraints.

Mike was also a traditional teacher in his evaluation of his students. The calculator remained a problem for him, in part, because he did not understand it well enough to know what questions to ask to assess his students' knowledge of physics and still allow them the freedom to use their calculators. When I suggested to him that he could ask why questions, he said that time was also a factor in that decision. As all
teachers know, it takes much more time to grade open-ended questions than to grade multiple-choice questions, or questions with simple answers. Mike was grappling with issues that many practitioners face and prioritize. Beliefs about which issues should receive priority in education have caused many educators to maintain the status quo.

Time was also a key issue in Mike's lack of ability with the calculator:

I haven't had the time I wanted to play a lot with the calculator. I think one of the things that keeps striking me is: Until I learn how to use that thing, I really am not equipped to make a good decision on whether I think it's of value. The more I use the calculator, I realize that you've got to have some understanding before the calculator can really do you any good, and maybe that's something that they're [students are] going to learn after they try to use it as a crutch; and as they fail, come back around to deciding there's only one way to learn it, and that is to really dig in and learn it. It seems like the individuals differ. Some very quickly use it in the right way. I see others, though, that keep relying on things like that solver key [notice his objectionable key, again!] and somebody else's program that they've linked to their calculator and copied as an excuse not to learn. I guess that's my greatest fear. I'm kind of excited to continue exploring the use of the thing and then try to build it into the course, so that maybe I can show them the right way to use it, so they're not looking so hard for the wrong way to use it. Maybe it's like anything else--it's the fear of the unknown that keeps it off guard a little. I still feel there's times, though, that well, in short, I wish that that solver key wasn't there. So much of the other functions that it does I feel good about, but it's that solver key I think that bothers me.

We seemed to have come full circle--back at the solver key, which was Mike's biggest obstacle with the TI-85. In his mind, that key inhibited his students' understanding of the mathematics and therefore of the physics.

However, in this equilibration process (i.e., the process of trying to restore balance to one's thinking process, see Chapter III) there was an interaction in which the technology was affecting Mike's beliefs. This interaction is an occurrence that happens in the midst of revolutions as one's emerging beliefs undergo change. Copernicus's belief in the heliocentric theory of our planetary system was not an instantaneous belief.
He originally believed, as did everyone else at that time, in the Ptolemaic geocentric theory. It was only through much time spent on observing, making measurements, and using mathematical reasoning, that he came to the conclusion that the earth rotated on its axis and about the sun (Kline, 1985).

**Technology's Effect on Beliefs**

In our first interview, I asked Mike how to solve a problem involving a baseball hit at an angle. He used his method, which he understood, but which I believe is difficult to understand (see Figure 2). His method was a decomposition of the problem into several parts. In the first drawing (the one on top), Mike decomposed the vector into its components. Then, using the component parts, he drew the next graph (directly below the first). This part of the problem was modeled after a ball that had been thrown straight up into the air. For this Mike used a speed-versus-time graph (but called it a *velocity-versus-time graph*) in which the magnitude of the acceleration (change in speed divided by change in time) was the slope of the graph. Knowing that the rate of acceleration was 32 feet per second (the rate of acceleration due to gravity), Mike calculated the time it took for the ball to reach its maximum height, at which time the speed would be zero. The minimum value in this graph is the time when the ball reaches its maximum height, which, in itself, seems to be confusing.

Mike reasoned that the distance the ball had traveled was the area under the graph. In explaining his rationale, Mike first showed me that, with a constant speed, the area under the curve would be the area of a rectangle, speed times time, because
A major league baseball player hits a ball at an angle of elevation of 35° when the ball is 3 feet off the ground. The ball is hit with an initial velocity of 103 ft/sec in the direction of a 10-foot fence that is 300 feet from home plate. Assume that gravity is the only force affecting the path of the ball; disregard air resistance. Is the hit a home run (i.e., does it clear the fence?)

\[ V_y = 103 \sin 35° = 57.0 \text{ ft/sec} \]
\[ V_x = 103 \cos 35° = 84.37 \text{ ft/sec} \]

\[ t = \frac{300}{84.37} = 3.54 \text{ sec} \]

\[ d = \frac{1}{2} \cdot 32 \cdot (3.54)^2 = 185 \text{ ft} \]

\[ \text{Time falling} = 3.54 - 1.85 = 1.71 \text{ sec} \]

\[ \text{Distance above ground} = 54.75 - 46.97 = 7.77 \text{ ft or 10.37 ft above ground.} \]

\[ \text{Height} \]

**Figure 2: Mike's Worksheet**
distance is equal to speed times time. He then generalized to the area under the
triangle. The area of each triangle is $A = \frac{1}{2} bh$, where $b$ is the time and $h$ is the speed-
-acceleration multiplied by time. So $A = \frac{1}{2} tat = \frac{1}{2} at^2$. Mike's contention was that
students need not memorize this formula, since they need only to calculate the area, and
the answer would fall out.

The next thing that Mike did was to calculate the time it took for the ball to
travel the horizontal distance of 300 feet, moving with a speed of 84.4 feet per second.
Using the rectangle (because $d = r \times t$, so $A = b \times h$, which implies that $300/84.4 = b = t$).
He determined that the time should be 3.6 seconds, and he then used that time in
the rest of the problem. He finished the problem with several more steps, which I will
not elaborate, but the main point of the illustration is to show the reader the extent to
which Mike decomposed and fragmented the mathematics. First he dealt with vectors,
and then he drew two speed-versus-time graphs, keeping the vector notation. He
understood his method, but it seemed tedious and fragmented to me.

I believe there are several inherent difficulties with this graph, some of which I
mentioned above. Another issue would be the difficulty of precalculus students using
this graph with a varying acceleration. However, Mike insisted that this graph (with its
modifications) was the only way he had ever been able to understand projectile motion
problems. He said that when he first saw this method, the projectile motion problems
became totally clear to him.

The most serious problem that I have with Mike's use of the speed-versus-time
graph, is that the problems he sought to solve clearly dealt with velocity, rather than
And Mike called it *velocity*, when in actuality it was the absolute value of velocity--speed--that he was graphing. This seemed to confuse many issues and reduce mathematics to a system full of unconnected and unrelated terms, formulas, and facts.

I suggested another solution to the velocity problem in Figure 1, using parametric equations and the TI-85 (see Figures 3-5). In this method, the graph actually follows the path of the ball; therefore, one can visually watch what is happening and ascertain whether the ball actually clears the fence. Figures 3-5 are three computer-generated printouts of a TI-82 graph. Using the trace key (see Figure 5), I traced to an appropriate place on the graph. The small square on Figure 5 is the trace cursor, and the coordinates of the trace cursor are noted at the bottom of the screen. The coordinates show that, after the ball reaches a horizontal distance of 300.36667 feet, it is still 10.54 feet in the air; therefore, the ball will clear a ten-foot fence. Figure 4, which shows a close-up view of Figure 3, visually indicates that the ball clears the fence.

When Mike saw the graph that I had generated, he said that he did not understand the mathematics involved. We talked about some of the difficulties involved in using his methods of teaching physics to calculus-bound students. In fact, both Mark and I had tried for years to get Mike to change the way he taught these problems. I had one calculus student who, using Mike's graph like an algorithm, used a varying acceleration for the slope of a velocity linear function. He obviously had no conceptual understanding of Mike's method. That student insisted on using Mike's method for almost the entire year of calculus--he just could not understand why he
Figure 3: Path of ball and fence

Figure 4: Close-up view of path of ball and fence

Figure 5: Close-up view of path of ball and fence, and coordinates of trace cursor

T = 3.56
X = 300.36667  Y = 10.541408
could not use it with a varying acceleration. The algorithmic procedure that he had learned had become entrenched.

Mike was beginning to see the need to improve his understanding of mathematics. After the technology conference, he told me that he believed my use of the calculator in the above-mentioned velocity problem was a legitimate use of the mathematics applied to physics. He said that he would never have used such graphs because it would have taken too long to hand draw them, but he thinks that the calculator-generated graphs are a valid way of approaching the physics.

In an interview after the conference, Mike talked about his experience in talking to the chemistry teacher.

I felt like I was on the other side when I was talking to her [chemistry teacher], because I was trying to get her to understand that I don't use a slide-rule anymore. You know? . . . There was a time that I thought it was a more honorable way of getting a solution. And I've now put it aside for this [calculator].

Technology not only had affected Mike's beliefs about its own legitimacy, but also his beliefs about mathematics and the role it plays in physics.

But as I was in the seminar the other day—especially with the CBL [Calculator-Based Laboratory]—and you talk about collecting real-live data and then taking that data and making some conclusions: Are there some relationships here; can we summarize that relationship in a mathematical way; can we now use that mathematical thing to make some predictions; are there some limits on the mathematics? That's real science. I mean, that really gets out of the theory of playing science and getting into science. If I got a piece of equipment that allows a kid to do a very simple lab but get real significant data out of it and then be able to use some mathematical tools to see what I got, that excited me. So to me, that CBL may be the key for making me want to learn this sucker. And the learning of that may be what's preventing me from wanting to use it at least in some respects. . . . That CBL is, to me, the key hooking the math to the science. Because when this thing can do what it can do as far as collecting
data, and then you feed the data into the Texas Instrument, and I come out with a real line plot and have to say, "That's not a linear relationship. What kind of a relationship is that? What kind of mathematics can summarize that?" And that's really what mathematics is doing for me in science. It's taking a thousand data points and putting it into a very nice summary. ... That's real science.

He seemed to be viewing mathematics as more than just algebraic manipulations, but it was not clear exactly what he thought mathematics was. I doubt that he himself, could have verbalized it at that point.

Conclusion

So the process of equilibration continued. Mike was constructing his own ideas of mathematics, education, and technology; and the technology, itself, was playing a part in his construction. Mike was caught in the middle of a technological scientific/mathematical revolution with allegiances to both sides of the issues. He was slowly beginning to see the advantages that technology had to offer his students, but he had to weigh each advantage against his beliefs about the disadvantages. The following summer, Mike and I were to attend the "Connecting Mathematics and Science (CMS) Pilot Institute." He had a few months to reflect on what he had learned before an intense week-long plunge into technology.
Narrative Two: The CMS Institute

In the spring of 1994, Mike and I applied to be participants in the Connecting Math and Science Pilot Institute to be held for one week during the summer. The expectation of most of the participants was that their application would be accepted as a two-person team consisting of one mathematics teacher and one science teacher. Our application was accepted for the institute that was being held in Pennsylvania, and Mike and I drove there together on August 14.

The Trip

During our five-hour trip, we talked much about mathematics, science, and teaching. We talked about our expectations of the week-long institute of which we were to be a part. Mike's summer had been so busy, that he had considered not going to the institute, even though he had been excited about it a few months before. He had some apprehensions about it in general, and he definitely still had apprehensions about that solver key on the TI-85.

Part of Mike's motivating force was the change in routine he was to face when school started again. WCS, who likes to be on the cutting edge, was changing to the trimester system for the 1994-95 school year. Mike was to have only two classes each trimester. The first trimester he was to teach Investigative Physical Science, which is a requirement for all freshmen; and a course in marine biology, which included a trip to the ocean for scuba diving. Each course would meet two hours per day, five days per
week, for twelve weeks. That gives each student 120 hours of class time for each class s/he takes. The advantages are that teachers have more blocks of time in which to teach, and they have fewer preparations per trimester. Also, students have fewer classes for which to prepare and, therefore, more time to prepare for each class.

The trimester system, with its two-hour classes, does not lend itself well to the lecture method of teaching. In fact, teachers at WCS were explicitly told not to lecture for two hours. To reinforce that mandate, teachers’ lesson plans had to be quite detailed and full of alternative teaching methods. Mike was hoping that what he would learn at the CMS institute would help him in planning his two-hour physics classes in the second trimester of the school year.

The First Day

Mike kept an audio-cassette journal for me during the institute, and he reported into it on Monday morning, Tuesday and Wednesday evenings, and Friday morning. We had had a brief opening session of the institute upon our arrival Sunday evening, and Mike reflected on things that were said.

[He was thinking about] how imperative it was that I become familiar with the instrument [TI-82], even if I don't use it totally in the class--simply because, again, knowing what it can do helps me know what the students are bringing in the classes. Stories about sabotage between students--between what you can put on that computer, how you can mask information you have on it--just told me again and again that, if I'm going to be effective as a teacher, I'm gonna have to know what it is that the students have in their hands.

He had mixed feelings about taking the time to become familiar with both the TI-82 and the TI-85 for many reasons. Part of his hesitation was his conjecture that as soon as he took the time to master this technology, another new piece of equipment
would arise, and he would have to take more time to learn again. However, he decided that he did not have any options at this time, because his students were all too familiar with the calculators in question.

Notice, too, that Mike's main driving force at this time was his belief that he needed to learn the technology so that he would know at least as much about the calculators as his students did. It does not appear that he yet believed that it could be a useful teaching tool in the hands of a knowledgeable teacher. The disequilibration that had brought him to this point seemed to center around the fact that his students were using something he believed he should be familiar with, and his resistance to calculator use in his classroom was being challenged. Also, his status as a teacher was being challenged. Students could not easily take advantage of someone who understood what the technology could do.

**Day Two**

On our trip to Pennsylvania, Mike had confided in me that he was intimidated by some mathematicians who had an air of superiority. He felt that his own knowledge of mathematics was inadequate, and yet he preferred not to ask mathematicians about the mathematics he did not understand. He felt comfortable with me, and he assured me that he would be asking many mathematical questions during the week.

That feeling of mathematical inadequacy came out in Mike's Tuesday evening report in his journal.

My reaction after a second day is that I'm realizing that the power of that calculator intimidates me. A tremendous amount of math appears to be necessary to know—so that you can answer the questions that the calculator is
asking you—decisions that you need to make that it prompts you for direction, that, quite frankly, it's like I'm afraid it's going to reveal too much about what I don't know. And it does it at such a rate that it never allows you to recover. Some functions I'm becoming more familiar with, and I feel that maybe they're not as scary as I thought. There is a tremendous amount of knowledge needed to operate it. It's not a matter of just a black box.

Mike liked the visual aspects of the calculator. He admitted that, as he saw more and more of the functions that he had never before understood, those functions were becoming less scary, and he was becoming more comfortable with them. He did reiterate his feelings of inadequacy.

I guess as a result of today, I'm starting to realize that a lot of my limitations of that calculator are basically me not wanting to reveal my weaknesses in math, possibly, rather than fear of the calculator itself. Although it's amazing to listen to the war stories from other teachers and what exactly students have pulled on them when they really know how to use that box in a deceptive way. So we'll see how I feel by the end of the week.

Although his comfort with the calculator was beginning to escalate, he continued his assault on the solver key of the TI-85. (It should be noted here that, because the statistical analysis in which we were going to be involved during the institute was better accomplished on the TI-82 than on the TI-85, we would be using the TI-82 exclusively during the week.) Mike did mention that, because the TI-82 did not have a solver key, he found that he preferred that calculator over the TI-85. Note that he still did not give any calculator his full endorsement.

**Day Three**

Today we were assigned to groups for a project. Teams were placed in a group together, so Mike and I were placed in a group with another man called Jim. Since Jim's interest lay more in science than in mathematics, Jim and Mike made the decisions
about what our project was to be. As Jim and Mike were running different experiments with the CBL unit, Mike was really becoming enamored with the technology as a learning tool. At one point in the afternoon, someone came by and casually asked, "What's going on here, Mike?" Mike's response was, "I'm learning math and it's been exciting!" That was a real breakthrough.

That evening Mike elaborated in his journal.

I'm excited today. That work with the CBL unit was incredible. The ability to take the data and use the calculator to find mathematical fits--to use the concept of derivatives and integrals in order to answer questions about the data and see it verify accurately a lot of the textbook situations for realistically very little cost make me see how that CBL unit is just a tremendous aspect to have in your science department. [Pause] I'm doing better about my math. It's interesting to see how much math I feel I've been learning this week, and, of course, a lot of it's been through your help. But the ability to visualize and then go back and reinforce so quickly on that screen after you've shown me something and then, with trial and error, very rapidly being able to see how the functions vary--how you can manipulate functions--how certain functions do represent the data better than others--is teaching me a lot of math in a conceptual way.

That entry in Mike's journal was exciting to me. Mike was beginning to see that there is more to mathematics than algebraic manipulations.

Continuing on with his journal:

I still don't know if I could ever manually manipulate some of those equations--the way I've watched you take many, many equations and work with them on paper and derive things and look for the form you want--um--I still need my hands on a piece of paper and a pen to do that--I feel at this point. But at least the conceptual part is not as bad now. . . . The calculator's got rid of a big fear area for me. Again, I think the TI-82 isn't as much of a black box as the TI-85, and I'm really liking this machine. . . . Putting that combination of the CBL and the TI-82 together with a math teacher/science teacher I see as a tremendous tool to teach math and show how math is the language of science. And that's something we've said, but I've never seen it verified in the way it has these last couple of days. So--a great day today, and I'm eagerly looking forward now to tomorrow.
So it appeared that Mike was beginning to change his beliefs about what mathematics was and how it was used in science. He noted that he had said that mathematics is the language of science, but he was seeing that verified in new ways. He was beginning to appreciate mathematics. As time would go on, Mike would experience disequilibrium about what mathematics really was. When we would work together, he would use the word, mathematics, differently from when he was teaching his class. I never confronted him on his beliefs about mathematics until the study ended, because I wanted him to make mathematical discoveries on his own. It was really difficult, at times, not to talk to him about it, but I was determined to allow him to determine for himself what mathematics was all about.

**Day Four**

Mike and Jim really worked on our project and, although unintentionally, effectively left me out. (I heard from another mathematics teacher who was teamed with two science teachers that it was not an uncommon occurrence for the mathematics person to be left out!) I really did not mind (too much), since Mike was apparently getting so much out of this conference. Mike and Jim came up with an experiment in which we measured the pressure emitted inside a closed beaker partially filled with water into which an effervescent antacid tablet was dropped. The relationship was a linear one, which was mathematically rather uninteresting to me. I was, quite frankly, unimpressed and bored. Then they decided to test the reaction for different temperatures of water. As the temperature was increased, the reaction rate increased. Now it was getting more mathematically (as well as scientifically) interesting.
Mike next mentioned to me that he remembered that reaction rates were supposed to double with each temperature increase of ten degrees Celsius. Now they had my attention. I decided that I wanted to verify that rule. So, I figured out how the equation for such a rule would look: \( y = m_0(2^{0.1})^x \), in which \( m_0 \) represents a designated original reaction rate and \( x \) represents the change in temperature—in Celsius—from that temperature which produced the designated original reaction rate. To illustrate from our data, our original reaction rate, based on the slope of our linear regression equation, was 0.0093. Our change in temperature for that rate, was zero, since we had designated that rate to be our original rate. Hence, we obtained the point, (0, 0.0093). Our next two points were (7, 0.163) and (13, 0.0221). I plotted those three points on the TI-82 and calculated the exponential regression equation: \( y = 0.0096(1.069)^x \). Since \( 2^{0.1} \) is approximately 1.07, the regression equation closely resembled our calculated theoretical equation, \( y = 0.0093(1.07)^x \).

Neither science teacher on our team was as excited as I was, but Mike was clearly more interested than was Jim. They were simply more interested in the scientific findings than in the mathematical ones, and neither knew how I had come up with my equation. They both just accepted it. Mike did say that at some future time he wanted to know how I calculated the equation, but he was just too overwhelmed with new mathematical ideas at this point to pursue it.

That evening Mike entered the following into his journal:

I feel like I've crossed over the fear of the programming. It's, through your help, becoming easier and easier for me to understand it. The math that interprets the data is not as bad as I thought. It's scary, because I haven't used
it for awhile, but after you use it, you find that certain patterns repeat themselves often and the things that vary are very minor and they're doable—especially with that CBL. I think if I had to hand plot all of this data—guessing $k$ value $[\text{in } e^{kt}]$ would be hard, and I'd be a little bit more fearful of it. But I'm seeing now that it's very easy to collect the data with the CBL... the TI-82 allows me to take that data and just manipulate it time after time after time. And even if I'm making wrong guesses initially, I can go back, and before long I'm on target. . . . So the old concept of worksheets and do it again and again and again is happening but doing it at a much higher rate—probably much more efficient. I love what I've learned from this.

As far as his wanting his students to manipulate the variables, well, Mike was making a slight change in that area, but he still was very much into his students' understanding of algebraic manipulations.

I still feel that way with the math . . . where the manipulation of the variables isn't something that you do it once and you're done. But after you've done it enough that it becomes understandable, then the use of the tools of this calculator are tremendous. I guess in short what I'm looking at is: I really like the use of the calculator to help reinforce concepts that are there and help even teach concepts that aren't quite there. My battle again will be how to keep it from becoming the little black box that a student uses to avoid learning—-to escape the understanding.

During the institute Mike developed an appreciation of the TI-82 and what it could do, but he still did not like the TI-85. To him it was complicated and overwhelming, and he felt students would not try to understand, but would just rely on the calculator.

So, I'm liking the 82 a lot better than the 85 because it has less in it, and thereby allows me to have the student less computer-dependent, but being that cooperative with the computer--the calculator—and maybe I like that cooperative aspect. It makes you feel like you're more part of the solution rather than the spectator watching it go on. Overall I'm sure that familiarity is a big factor on whether I use it.
A large concern of Mike's was whether the WCS would provide the money for him to be able to implement technology in his classrooms.

If I get back and see what this tool can do and find that finances are not going to be made available, then it's going to frustrate me, and it's one of those things like it would be better not to know the TI-82 than to know it and then not be able to use it. So, I'm hoping that the frustration level doesn't prevent me from getting innovative. And I've found myself at times, quite frankly, getting a catalog from somebody and literally throwing it away—not even looking at it, because it's no sense—the school's not going to spend money, so why should I get my focus on something I can't have. And this sometimes avoids the use of technology in your classroom because of that.

The Ride Home

Mike and I talked at length on the way home. He had learned much at the institute and was excited about what he had learned. He liked the TI-82 because it had no solve key on it. He was beginning to understand that there is more to the mathematics in physics than merely dimensional analysis. He talked about the growth and decay graphs, which he still did not understand. In fact, he had never before heard of Newton's Law of Cooling, which is a function that exhibits exponential decay. He was beginning to understand linear-motion graphs and was looking at positive and negative acceleration and velocity in new ways. He was seeing a need to understand more mathematics. In our project he wanted to understand the mathematics that we used. He wanted a copy of my notes. He also indicated that he wanted us to present a workshop at the 1994-1995 Association of Christian Schools International (ACSI) Teachers Convention, which was to meet November 3-4, at the Columbus Convention Center.
Mike indicated to me that he might not use the technology extensively in his classes because of the amount of work that it would take and because he just was not sure in what ways it could benefit his students. He seemed excited about what he had learned, but he also seemed uncertain about how to implement it all. He also was unsure about what really was important mathematics for students. Should students know how to manipulate algebraic equations? Should they know how to long divide? Should they be able to compute simple problems without the benefit of the calculator? How much of all of the above should be his concern as a physics teacher? He had much to ponder as he began the school year.
I phoned Mike on August 29, which was shortly after the school year started. He told me, "I keep checking my ID to see if it's really me!" He was becoming a real convert to the use of the graphing calculator. His teaching load consisted of a ninth-grade Investigative Physical Science (IPS) class, which was a required introduction-to-science class, and a marine-biology class. He had planned on using the new technology in his marine biology class, but he had been unsure of how to implement any of what he had learned in his IPS class. During our telephone conversation, Mike told me about the lesson he had taught his IPS class. He wanted to teach them conversions between centimeters and inches and then between the Celsius temperature scale and the Fahrenheit one. He had students take measurements in both centimeters and inches. They recorded their data in lists on the calculator and graphed their results. The equation proved to be linear with the slope equal to 2.54 (when the inches were assumed to be the independent variable). The discussion then centered around the slope and its implications for unit conversion. (See Appendix B for reproductions of Mike's lesson plans/activity sheets for the day.)

The Celsius/Fahrenheit conversion equation, in which the Fahrenheit temperature scale was the independent variable, resulted in \( C = \frac{5}{9}F - 17.7 \). Mike had shown his students the y-intercept of approximately -18 and the discussion ensued as to
why the function was an accurate description of the conversion.

I asked Mike if he had ever attempted to give students a visual description of the conversion of units. He said in previous classes he simply had given them the formulas. He was excited about the new way of teaching conversions, because students could participate in deriving the formulas and in interpreting the graphical models. Mike had already begun using the graphing calculator in innovative ways to facilitate student understanding.

September

Approximately a week later I visited Mike's IPS class. They were discussing the Periodic Table of the Elements. The day before, Mike had given them several colored rectangles with notches and holes cut into them. The colors were various shades of red, yellow, and blue. Students were to discover patterns in the rectangles and place them together so that an obvious overall pattern emerged. By doing this, students could tell how a missing piece should look. Students worked in groups trying to discover patterns, and when they finished they had experienced some of the beauty of how the periodic table was developed.

I was amazed. I asked Mike if he had ever done anything like the above activity before. He said that he never had. His mode of teaching was generally straight lecture. I then asked him where he got the idea. He had seen the manipulatives somewhere and had decided that using them would be beneficial to his class. Then he reflected on our week at the CMS Institute. He remembered how well our team worked together, and he reflected on how valuable it was to be able to interact with others--he had
helped me with the science, and I had helped him with the mathematics. He decided that he would try a similar approach in his class by putting his students in groups. He also liked the discovery approach we had taken during the institute, and he wanted to experiment with that. So, this highly reflective teacher had attempted to teach his students, not from the theory he had learned from a textbook, but rather from the practical things he had learned in his experience. I do not believe he even realized how much mathematics he was utilizing as he taught his students to learn by discovering patterns. I thought that some day, when the time was right, I might mention it to him.

So, what major changes had Mike made up to this point? He said when we returned from the CMS Institute he had a difficult time "getting back to reality." He had found a new toy and he wanted to get comfortable with it. The way he put it was, "Throw me some raw hamburger and leave me alone!" The major change as Mike saw it was:

I'm seeing the calculator as a tool to get where I want to go rather than as an obstacle to keep me from going where I want to go.

He saw his courses going to more and more hands-on work. I saw that, too, as I observed his IPS class a few more times. Where he would have normally given students a worksheet to take home and complete, he would now allow students to work in groups during class. Part of that was the directive from the administration not to spend the entire two hours of class lecturing. However, teachers can get around that by having students do their assignments during class. I do not believe that Mike was putting his students in groups to follow the administrative mandate, but rather,
because—from his own experience—he believed it was what was best for his students.

I interviewed Bill Williams who is the Chair of the Science Department at WCS and also the administrative assistant in charge of curriculum development K-12 for all disciplines. Bill has a master's degree in Christian school administration and sixteen years of experience as a teacher and administrator. He has worked side-by-side with Mike for eight years. They have team-taught a course in scientific research and were currently team-teaching the course in marine biology, so they had much interaction.

Bill said that Mike had moved 180 degrees from his beliefs of a few years ago. At that time, Mike had believed that students should not use calculators at all. They needed to understand the *whys*. Mike had been frustrated with Mark Pifer, because Mike had believed that students had not been taught to think—only to rely on the calculator. About two years ago, Mike had slightly altered his position in that he still required students to do the paper-and-pencil computations, but they could then use their calculators. This year Bill had seen a change in Mike. He said that Mike had "come to the point that the calculator is here to stay and he needs to use it." He said that Mike had come back from the CMS Institute "thrilled," in fact "to say he was excited was probably an understatement." He told everyone he could about the TI and the CBL, and he had ordered a classroom set for his own classes. He was encouraging his students to bring in the TI-82, and he was even using the calculator in his ninth grade class. In that class, Mike was no longer teaching any conversions, but rather using the calculator to help students determine the conversions. Bill also mentioned the presentations that Mike had lined up for us to give at conferences.
Immediately following our return from the CMS Institute, Mike had submitted our names to present at the ACSI Teachers Convention. We were accepted, and we met several times to run experiments and determine our plan of action. At the end of September, we met for such a time. I wanted to run the motion detector and work with distance-versus-time plots. Mike insisted that he was not very interested in that kind of data—it was not physics, it was mathematics. Finally, I said that at the CMS Institute, it was the physics teachers, rather than the mathematics teachers, who could answer the velocity and acceleration questions on the worksheets. Vectors were involved, and principles of physics, and many of us who were mathematics teachers had difficulty coming up with correct solutions to the questions. From that, I had assumed that projectile motion was more of a physics topic than a mathematics topic. At this point in time, I perceived that Mike thought what we were dealing with was too theoretical and too mathematical and not practical enough—especially when we got into negative velocities. He rarely, if ever, used negative velocities, and therefore did not deem them to be necessary to his physics classes.

**October**

In Mike's marine biology class, some students asked Mike if it was true that every seventh wave that came in onto the beach was larger than the others. Mike answered with the graphing calculator! He showed them that wavelengths are sinusoidal and they have different frequencies, depending on what caused them. He put several sine functions with different frequencies together and showed them that there was a pattern in which one of the waves would be larger than the others, and that the
pattern was a repeating one. I was excited that he had thought about answering the question with a graph, especially since he had avoided sine functions in previous courses that he had taught!

Mike and I met again to plan for our presentation. We bounced a ball and recorded the data. The program we used reverses the data so that the graph looks like several, connected, concave-down parabolas. Connecting piecewise quadratic functions seemed to be the way to model the data. That evening Mike phoned. He said we both knew that the data was not sinusoidal, but he had decided to use a cosine function multiplied by an exponential function (to account for the damping), and he had moved the x-axis down so that the minimum values could be zero. I suggested that he use the absolute value of the cosine function. This seemed to model the data well for a small interval. It broke down because the frequencies were also decreasing. I tried modifying our graph using an exponential for the decreasing frequencies, but, since we were not dealing with actual sinusoidal data, the results were not accurate. However, I thought Mike's ingenuity in this was great. To think, just a short time before, he had been afraid of trigonometric functions! Now he was using them in various situations.

In early October, a friend of Mike's, also a physics teacher, came to see Mike. As Mike was demonstrating some of the things he had learned on the calculator, his friend remarked to him, "How long have you had that thing [TI-82 ]? You move around on that thing faster than our calculus teacher, and he has been using it for a long time!" As Mike related the above conversation to me, he added, "I really feel good about starting to find my way around this machine." He also admitted to me that he is
being forced to relearn physics because the tool [TI-82 ] allows him to visualize and
makes him want to know.

In the middle of the month, I asked Mike to assist me in teaching my graduate-
level mathematics methods class at Ohio State. I wanted to demonstrate to my class an
effective way to teach parametric equations by using the CBL.

The experiment involved using two TI-82 s, two sonic motion detectors, and
two CBLs--one to record motion on an \( x \) versus \( time \) plane and the other to record
motion on a \( y \) versus \( time \) plane. We then put the \( x \) and \( y \) data together and plotted an
\( x \) versus \( y \) graph (see Figures 6-9). Mike and I had never quite been able to
conceptualize the \( x \) versus \( time \) and \( y \) versus \( time \) graphs (even though we had spent a
great deal of time in trying). The basic reason for this was that, in our minds, we had
switched the \( x \) and \( y \) motion detectors. So, when we looked at an \( x \) versus \( time \) graph,
we thought we were looking at a \( y \) versus \( time \) graph and vice versa. The result of all
of this for me was that I felt very inadequate in presenting the material to my students.

I decided to plunge ahead with the lesson, knowing that it was potentially bad
for me, but it was good for them. As feared, students did ask what was going on, and I
did not know. Mike tried to explain the graphs, but he, too, got bogged down. The
next class period, my students and I talked about the lesson. I asked one of the
students who appeared to understand to explain. He explained that I was switching the
\( x \) and \( y \) motion detectors, and he showed the rest of the class the correct way to
interpret the data. I was unconvinced, but I went home that evening determined to
understand. As I sat with the data, I had an Aha experience--I finally understood.
Figure 6: Photograph of Mike and a colleague performing a CBL laboratory experiment using parametric equations
Figure 7: X data versus time

Figure 8: Y data versus time

Figure 9: Curve graphed parametrically
Once I placed the data with its correct interpretation, the rest came easily.

As I reflected on that lesson, I still felt quite inadequate as a teacher. Then I read about, a teacher who had taken many risks (Schifter & Fosnot, 1993). Her paradox was similar to the one Mike and I were facing—how can one not fully understand the mathematics one is teaching, often set problems for students that s/he herself/himself cannot solve, and yet remain an effective mathematics teacher?

According to Schifter and Fosnot, the resolution of the paradox lay in several factors, one of which was: The teacher helped students pose their own questions, helping them think them through, and used "her own ignorance to help model this process for them" (p. 141) by helping them listen to others' suggestions and ideas. Also, by using the mathematics that she did understand, she could guide them into uncharted waters because she recognized some of the landmarks.

Why is it so difficult to remember that teachers do not need to be omniscient? Mike and I talked about that. It is really difficult to take risks as a teacher, because so many believe that the good teacher knows all the answers and can explain them effectively to his/her students. We teachers ourselves often believe it. Even when we say we do not, we show by our actions that we do. Mike mentioned on several occasions that, when his students knew more about the calculator than he did, he felt a lack of control.

Mike recorded his feelings about control in his journal.

There was this part of you that when you first started teaching that, I don’t know, somewhere deep within me, I had this belief that--you know--the teacher always knows everything--the teacher’s never wrong. And I don’t know if that’s
when I was growing up, I always looked at the teachers with that kind of admiration, or if it's just my own ego problem. But I think one of the things that I really enjoy as being a teacher of fifteen years plus is that I'm no longer in that insecure position anymore, because, number one I am very comfortable with the subject, and second, I thought I had reached the point where I understood I don't know all the answers, and that's not a big deal. That's one of those times it's an opportunity to let the student learn as you learn. But the calculator was a new technology that totally put me back into that insecure moment of being a new teacher again. Here I was a teacher for ten years and kids were walking in with this machine that did things that I had no idea how they did what they did--I had no idea what its capacities were, and I felt very insecure.

He recounted a story in which students had shared answers during a test via the calculator, and Mike had been oblivious to it all, because he did not understand the capacity of the calculator. He then continued:

And so, it suddenly hit me that a lot of my problem with this calculator may be the fact that I fear not being in control. I fear not being in a position that I'm controlling what's going on, because I don't know its limitations. And part of the difference that's happening right now, is even though the kids know the calculator, they don't know the CBL, and that puts me back in control again. I've got the ability to teach them, because they don't know as much as I do. So maybe part of my willingness to jump into this is the newness of the CBL putting me just as knowledgeable as they, and second, the CBL has forced me to find out that the graphing calculators aren't as scary as I thought they were.

Now Mike was back in control, because he knew much more about the CBL than his students did. He knew what it could do, and he knew how to use it appropriately to help his students. Along with that came knowledge of the calculator. Since he was teaching science classes, rather than mathematics classes, he did not mind telling his students that his knowledge of mathematics was weak in some areas. Neither did it bother him that he did not know all about the CBL. What did bother him was for him to feel that he was not in control of his classroom. It is indeed, a fine line
that teachers walk in this area when the teachers are risk takers.

When we prepared for our ACSI presentation, Mike decided he wanted to begin the presentation by telling the participants that he wanted to use an instrument that did not need batteries, was not expensive, was not intimidating, and could be used by students without fear of cheating—the slide rule. He told me, "I want them to know that I came into this kicking." Change is difficult for us all, and we need to recognize that.

Mark's Reactions

In early December, I talked to Mark Pifer (recall that he is a WCS mathematics teacher—see Chapter IV). He was complaining—he said that he used to "bug" Mike about the calculator and Mike did not want to hear it. Now Mike was "bugging" Mark about the CBL, and Mark did not want to hear it. Mark said that he was not ready to use the CBL—it would be too time-consuming to make the change.

We talked about why Mark thought that Mike had changed. One thing Mark mentioned was Mike's willingness. He said that many teachers were not willing to change. He had often seen teachers attend a half-day inservice professional development meeting and return only to resume their old ways. I thought the entire conversation was ironic. Mark was talking about Mike's willingness to change to calculators, and Mark had made that change. However, Mark was not willing to take the next step and make yet another change. His reactions are another testimony of the inherent difficulties involved in teacher change. We are involved in revolutionary changes in education, and such changes require much from teachers who are willing to
make those changes.

The Time Factor

As in Mark's case, one of the major obstacles to teacher change is the teacher's lack of time to do what is necessary to implement change. (Note that I will temporarily chronologically diverge in this narrative for purposes of continuity of ideas.) Shortly after beginning his physics class in the second trimester, Mike addressed the time factor in his journal as being a great stumbling block for himself:

I've spent a lot of time with this machine. I guess that's the biggest drawback. And it's a temporary drawback because it's something new and it takes a lot of time when you're just learning it. That's probably one of the reasons I held off--not learning it, because it just wasn't something that was going to happen overnight. It was going to only happen if I took some concentrated time--spent time with it and got comfortable with it. And I didn't have the time, and I didn't see the need to do it because I felt I was successful without the technology--without the device. Not knowing the machine, I had no idea how I could use it to teach, and I was using a different way of teaching; and the machine, I felt was interfering with that different method. And I kept resisting it thinking that my method was better because I had been successful ten years ago with it, and I really didn't see the need to change.

He talked about the students being more responsive to the calculator than to other methods, and he speculated as to why that situation existed. Maybe it was because they had grown up in a technological era and an era of instantaneous gratification. Whatever the reasons, as he prepared for his physics classes, he was more willing to spend time on lessons and experiments which were calculator enhanced. He said that his students were enjoying doing their laboratory assignments and that was unusual. He continued:

The same subject now taught through the calculator seems to maintain their interest. . . . Because of their response it's made it easier for me to put the hours
in and to try things that are a little bit different. . . . So it's been different, and yet it's starting to produce.

The preceding paragraph foreshadowed what would happen in Mike's physics class the following trimester of school. As I begin the next narrative, the time frame will revert back to the beginning of the second trimester.
Narrative Four: Mike's Physics Class

Armed with one trimester behind him—with all of the experiences and increased knowledge of the CBL and TI-82 had brought—Mike was ready to teach his physics class. The background of the students in that class overall was rather weak. Some were taking precalculus concurrently with physics. Two were taking calculus. The class had nine students. Six of them were seated two at a table, and the remaining three were at another table. One of those students floated between a couple of tables during the trimester. Mike did not assign seats, nor groups. Most students simply placed themselves with a lab partner and kept that partner for the entire trimester. The only exception to this was the student who floated between tables.

Vector Addition

Mike began his physics class by talking about vectors. He told them about three ways of adding vectors. One way is the parallelogram method, in which the resultant force vector is equal to the diagonal of a parallelogram of which the two force vectors are sides. A second way is called the analytic method, but Mike called it "the mathematical way." In the analytic method, vectors can be decomposed into their horizontal (x) and vertical (y) components and expressed as rectangular, rather than polar coordinates. Addition of vectors can then be accomplished by summing x components and summing y components. The third way is the tail-to-tip method, in which, after one vector is drawn, the following vectors are added by placing the tail of
the next vector at the tip of the preceding vector, and the arrow drawn from the tail of
the first vector to the tip of the last vector is the resultant of the vectors (Giancoli,
1985; Williams, Trinklein, & Metcalfe, 1984).

Mike's preferred means of adding vectors is the mathematical way. He told the
students that if they did not like mathematics, the other two ways were great methods!
(Note that he was experiencing difficulty in letting go of his mathematics-as-
manipulations ideas.) He demonstrated the geometrical ways of adding vectors to his
students. Then we all went into a large multi-purpose room and he proceeded to
involve the students in adding vectors of large magnitudes by representing them with
rope on the floor. He had a directional compass, and a protractor, and they measured
vectors such as: magnitude 15 feet, and direction 30° east of north. They repeated the
vector addition with the same set of vectors added in a different order. No one seemed
to notice that the two westerly-directed vectors were not parallel. So when the end
resultants were different, rather than trace their source of error, the students just
acknowledged that there had been an error in measurement.

My thoughts about the above description were that mathematics could have
been utilized in analyzing the source of error. By simply making sure that all same-
direction vectors were parallel, error could have been kept at a minimum. However,
since Mike never did acknowledge that the geometrical way was, indeed, a
mathematical way, I doubted that he had formally made mathematical connections in
the geometrical addition. When we later briefly talked about the source of error in the
demonstration, Mike intuitively knew what I was talking about. However, I doubt that
he ever would have used it as a point of discussion in his classes, because he, himself, had not made the mathematical connections.

As the class continued, we reassembled into the classroom, and Mike continued his lecture by adding several more vectors geometrically by drawing them on the board. He even made mention of drawing one vector parallel to the y-axis. So, he was even using mathematical terminology as he described his "nonmathematical" method! As Mike left the room momentarily, I overheard the students talking about the "math way" with the sines and cosines. Either his students had adopted his terminology, or they had the same faulty beliefs that he did about what constituted mathematics. If the latter was the case, then Mike was reinforcing that belief. In either case, the situation was not conducive to the students' development of mathematical power. After Mike returned to the room, students asked if they needed protractors, to which Mike replied, "No, not if you use the math method," again reinforcing the premise that the graphical method was not mathematical. He drew a few more vectors and concluded by saying, "I call this the graphical [method], because it's drawing."

**Modeling Data: Regression Equations**

Mike began his class today with a story about himself. He used to envy his wife, because she could stay home all day with the children. Then one day she got sick and Mike had to stay at home and do all of the work that his wife normally did. At the end of the day he was glad to go back to work.

[It was] her way of making me appreciate what she did for me and for the family. Your homework last night was kind of directed towards that. I want you to really appreciate the twentieth century, OK? This is a lesson I would not
have taught you last year, because...um...quite frankly, I didn't like the graphing calculator. It was something I did not use at all. And now what I'm doing is I'm changing my thinking a whole lot and this one lesson is a tremendous...um...boost in my confidence in this calculator.

Their homework the previous night had consisted of collections of data that did not always have a linear relationship, but that could be made into a linear relationship by redefining the variables. For example, data which fit the model $y = x^2$ could be made into a linear relationship by making the independent variable $x^2$ instead of $x$.

In many situations, changing the relationship into a linear one facilitates graphical interpretations. Mike showed his students the simplistic beauty of a linear graph. He used an example: Pressure = 40(temperature) + 3. With such a relationship, one could graph temperature on the x-axis and pressure on the y-axis.

Suppose you wanted to predict the temperature if the pressure is 3.5 atmospheres. You would go up to 3.5, go over to where that intercepts the graph, and then go down to find the answer. That's the graphical way to do it. It's pretty easy. You don't need to know any math. You just need a rule—over, down.

Again, Mike was clinging to his algebraic definition of mathematics.

Changing all nonlinear relationships into linear ones can be a tedious process. Mike wanted them to experience a way that people used to interpret data before the advent of calculators that performed statistical analyses. He likened it to going to California—one could go by car or by airplane. Either way one would get to California. Going by airplane would be the most expedient, but one loses all of the beauty of the trip across the country. Going by car would allow one to take in all of the countryside, but it could become very tedious. Mike saw the calculator as the expedient method of
data analysis. He spent the rest of the class period showing students how to calculate regression equations and obtain a printout of their solutions and of their graphs.

At one point in his lecture, he looked at me:

Mike: If I start teaching how to use this thing [calculator], is that called, 'endorsement'?

Observer: Yes, scary isn't it?

M: Yes!

The tool he was so against just a few months ago, he now was not only endorsing, but he was also integrating it into his teaching.

**Velocity and Acceleration: Day One**

The topic for the day was an introduction into projectile motion. Mike began by telling the students that he is a visual learner, and the only way he could understand velocity problems was by the visual method. He told them, "The visual that I'm talking about is graphs. The graphs really help me."

He began by talking about velocity, but then he told them that, since at this point direction was not an issue, they would really be talking about speed. However, throughout the entire lesson, he continued to refer to the speed as the velocity. I found this to be confusing. We later talked about it.

Mike then drew the graph (see Figure 10) on the board and labeled the x-axis $t$ for *time* and the y-axis $D$ for *distance*. 
He indicated that the velocity was the change in distance divided by the change in time, or the slope of the graph.

If I gave you some data and you made a distance/time plot of it, you could get some information from this graph. You could get velocity by finding the slope. For a greater velocity, the slope is steeper. For a slower velocity, the slope is more gradual.

Suppose I wanted to know how far I have gone at any time during the experiment? The graph is useful for that also. I want to know how far I am at time 5. Go over to 5 on time. Take it up until it intersects our line—which represents the whole experiment. Come across and read our value off. That is the distance we are at 5 seconds. So we don't have to use any math. The graph does it for us.

The last two sentences were an incredible statement. It was a reinforcement of the statement he had made in the previously described lesson—that with the graphical method, one does not need to know any mathematics. Even after Mike and I had spent a couple of months interpreting graphs, he still did not consider graphical analysis to be mathematics. I decided at that time not to mention any of my concerns to Mike. I hoped that, with time, perhaps he would perceive mathematics differently.

Mike then went into his lecture about velocity-versus-time graphs. (I mentioned his method of doing these in the pilot study.) At one point, when describing
a ball being thrown into the air, he admitted that the velocity graph should actually become negative. However, since he wanted to stay in the first quadrant, he simply told the students to use the absolute value of the velocity (speed), but to continue to call it velocity. After class, he said that he knew that he had slipped back into his old methods, but that he just did not feel comfortable with any other method. It was the way he had been able to understand the problems. He also mentioned that he had helped many students throughout the years, because they just did not seem to understand the equation method that was presented in the textbooks. They generally preferred his method.

I pointed out to him that for most of his students, his method was just that—a method. It did not have any more meaning for them than an equation did. Mike understood why the slope of the line was acceleration and why the area under the graph represented distance, because he had computed it himself. But the students just accepted it and used it. I felt that they really did not understand it. That seemed to be a revelation to him. He then seemed more open to other methods.

I also mentioned to him that I thought it was very confusing to students to be dealing with speed and calling it velocity—at least one student voiced that confusion at the end of class. I said that an important issue in mathematics is the ability of students to make connections within their mathematics (NCTM, 1989). By interchanging vectors and scalars, Mike was causing his students to fragment their mathematics. I told him if he continued to use a graph that modeled speed, students would end up with even more fragmented and confused mathematics when it later became necessary to
separate velocity into its component vectors. He had not viewed it that way. 

In order to show him the confusion that could result from his method of teaching, I gave him a question from the 1992 AP Calculus Examination (1992) and I asked him to solve it that evening. The question involved a particle moving along the x-axis on a time interval. The velocity was given by a quadratic function, and the students were asked to find the total distance traveled by the particle and the average velocity of the particle over the interval. The questions emphasized the need to understand the difference between distance and displacement, since the two were different in this problem. The problem also involved a nonlinear velocity and so a varying acceleration. When Mike returned the following day, he acknowledged that his method would not be a practical means for solving the problem. He, himself, had to solve it using other means.

**Velocity and Acceleration: Day Two**

This day was a struggle for Mike. As with many teachers going through change, especially revolutionary change, his tendency was to cling to his old habits. Change is difficult—we must never underestimate just how difficult it is. Mike had thought about what I had said. Since my reasoning had been reinforced by the confusion of at least one student, Mike decided that he would plunge into uncharted waters. He began his lesson with these words:

What I did yesterday—it works—and it's probably the way I'll continue doing it for myself. But I'm going to consider looking at it a little different for you guys. . . . What confused a student, and also has confused others in the past is my ability to take one problem and break it down into stages and look at each stage independent and lump them all back together. I'm being told that may not be
real helpful from a mathematics standpoint.

Mike then discussed a problem: A car moves with a velocity of 30 m/s. Driver puts on his brakes and so has a negative acceleration of 5 m/s$^2$. How long does it take for the car to stop? He talked about the negative slope. What did the negative slope tell about velocity? They decided it meant that the velocity was decreasing. After solving the problem using his velocity-versus-time graph, Mike showed his students an equation for the problem. He said that some students might prefer equations, and he told them,

The more I do physics, the more comfortable I am with equations. When I first started, I needed more of a concrete thing to hang onto.

Then he said,

I had an initial velocity and I had a slope. The velocity was negative and the slope was positive. What does that mean? I'm going to draw a picture and you tell me what's happening.

As he drew a figure similar to Figure 11, he told them that he had shown an initial velocity of 5 m/s and gravity made it slow down.

![Figure 11: Velocity versus time](image-url)
As Mike continued drawing the graph through the x-axis into the fourth quadrant, he
asked what was happening. Students mulled over the question, and one student said
that it was speeding up, to which Mike replied:

What do you mean, it's speeding up? I'm below zero. I'm negative. How could
I be speeding up? That was the transition I had a terrible time making. It is
going 5 meters per second here [pointed to graph at appropriate point]. It's
going slower [each time Mike pointed to graph]. It's going slower. It's going
slower. At this point [graph was intersecting the x-axis], it's velocity is 0. Now
it just went into the negative [Mike again pointed to graph each time]. It's now
going -1. It's now going -2. It's now going -5. It's now going -10. It doesn't
mean it's going slower. It means it's going in a different direction.

When we define signs--positive and negative--in physics--and I assume
in math, too--what we're doing right now, and remember what velocity is.
Velocity is a vector. It's a magnitude and a direction. And so the change of
sign here doesn't mean it got smaller. It simply means that velocity was defined
in one direction and acceleration was defined in the other direction and they're
opposing each other.

After a brief discussion of what a ball is doing as it travels up into the air and
returns to earth, Mike continued:

I tried to play around with the graphing calculator, and this is where I'm going
to get myself in big trouble real quick. I said, OK, I was going to graph this
and try to look at it. I need an equation. So y = 5--which was my initial
velocity--plus my acceleration times my time. [He put 5 - 9.8x into the Y,
place on the calculator.] My acceleration is 9.8. Why did I subtract it?
Because it's in a negative direction, so I've got 5 - 9.8 times x, which is my
time.

He then graphed the function.

Next, Mike told the students to imagine a radar gun that was aimed at him. It
said he was going 20 miles per hour, but it did not care whether he was going north or
south. He then plotted the absolute value of the function y = 5 - 9.8x (see Figure 12.)
Mike talked about taking the lower triangle and rotating it up. He told his students that this particular graph works all right for some things, but not for others. If one wants to look at a continuous process, the first function is better. He then talked about distance and displacement. Next he did what I found to be an incredible thing. He showed his students the integral key on the calculator—he called it the area-finding key. The calculator shaded in the area between the graph and the x-axis on the [0,2] interval Mike had set and then gave the numerical answer for the area. Students were obviously impressed, and so was I! Mike had found the key in his experimenting and had made use of it as a great teaching tool. That tool would help these students as they moved into calculus.

The next step was to move from a constant acceleration into a varying acceleration. He graphed the function $y = 5 - \sin x$, and then (generalizing again, that area represented distance) used the area finder to compute the distance over the
interval he had set. He told his students that acceleration varies in the real world. The key was that the students had to have an equation to integrate. They had to do some thinking before they could use the calculator. Mike was coming to the realization that the calculator, to be used properly, was not just a mindless tool.

He then started thinking and explaining to his students.

What I'm doing right now is realizing with [the student's] questions yesterday that by me taking absolute values, I'm causing some people some grief. And I've always caused the math department some grief. So what I'm trying to do now is I'm going to rethink my thinking. It's still the same, but rather than flipping that triangle on top, I'm going to let it go underneath. . . . If I ever get into some problems that I can't solve this way, you may see me jump back into my old method and then come back out and then I'll figure out how I could have done it the right way. So school isn't just for the students. . . . [He looked at the student who had been confused yesterday.] Was any of that worthwhile?

Student: Yeah, that helped me out a lot.

Mike told his students about his experience with tennis. He had taught himself how to play, and as a consequence, he had developed some bad habits. However, his tennis game was quite good, and he had a powerful serve. When he got on the freshman tennis team at Ohio State, his coach told him to change the way he held his racquet. It was quite difficult for him to change, and after he made the change he never got his game back.

I was self-taught. I had bad habits, and because I had bad habits, they were very well entrenched. Well, I'm self-taught in physics. I hated it in high school. I hated it in college, and I had to teach myself physics when I went to Marysville and had to start teaching it. So, I'm scared to death that by changing my grip right now, I may get very insecure with you guys. So, if I cry a lot, that's my problem. . . . So, let's hope it doesn't mess me up.
Notice how real Mike was with his students. He was always honest with them and did not try to cover up any weaknesses. With his confession, he was making himself vulnerable to his students, and that is not an easy thing for a teacher—especially for one who wants to maintain control of the classroom. For a teacher like Mike, who is well respected by students and colleagues, this kind of vulnerability can be an asset. Students have told me that they appreciate a teacher who does not profess to know all and who is not afraid to admit weaknesses and mistakes. However, acknowledging weakness is still a difficult thing.

The next problem that Mike and the students tackled was confusing for the students because they had a difficult time grasping exactly what was happening. The problem was from their textbook:

A stone is released from a balloon while the balloon is ascending at the rate of 5.0 m/s and when the balloon is 353 m above the ground. What time is required for the stone to reach the ground? (Williams, Trinklein, & Metcalfe, 1984, p. 56)

Students had a difficult time understanding that because the balloon was ascending when the stone was released, the stone would also initially ascend. Mike began the problem by drawing his velocity-versus-time graph and then using the equation:

\[ D = \frac{1}{2} at^2 + v_0 t + D_0. \]

When he inserted a positive 5 for the initial velocity, a student questioned him:

Student: Positive 5?

Mike: Yeah, because it starts out at positive 5. If you guys are going to make me be consistent mathematically, we're going to be consistent. It may kill me, but we're gonna do it."
He made some computations and showed them that the large triangle below the x-axis had an area of -\((353 + 1.28)\), or -354.28. He concluded that the triangle would have the same area if he had used absolute value—speed—in order to get distance. So the stone went upward a distance of 1.28 meters and then went in a negative direction a distance of 354.28 meters. He then used the -354.28 in his formula to find that the time for the downward distance was 8.5 seconds. Adding that to the .51 seconds that the stone initially went upward, gave 9.01 seconds.

Mike could have achieved the same results from using displacement of 353 meters. Substituting 353 meters in for original distance from the ground—\(D_0\), gives 9.01 seconds. We talked about that after class. Our discussion and its results are at the end of this section.

After finding the final time, Mike recapped the problem for his students:

Can you visualize what's happening? We release this stone. The stone doesn't fall immediately. If it fell immediately, we would have got a graph that looked like this [drew graph]. . . . If we let loose of this thing--I want to go to the positive axis so bad it's killing me! [Mike drew correct velocity-versus-time graph.]

Mike showed his frustration at trying to change an entrenched habit. Intuitively, he knew that what he was doing was the best way to teach, but he also knew that his own experience had taught him that he did not understand the concepts when he had learned them that way. He had only understood when he fragmented the problem and looked at each incident separately—as speed, rather than velocity in this instance. The disequilibrium that was occurring was obvious, not only to me, but also to his students and to himself. He was voicing it quite emphatically.
Since Mike was now using the formula for projectile motion, he was dealing
with a quadratic equation. He told his students that he does not like quadratic
equations, but, since his students "wanted to go that route," he had "stayed with them."
However, he did not have the quadratic formula for solving quadratic equations
programmed into his calculator. He asked the class:

How many of you have the quadratic equation built into your little calculator?
How many of you don't? Well, I don't and I want it!"

I linked my calculator to his with the link cable, and gave him my program for solving
quadratic equations. He told his students to link with someone and get the program, so
they could learn how to link.

Mike then resumed his place at the front of the class and looked at me. I was
observing from the back of the room.

M: Mrs. Nicol, are we better mathematically today?

O: Yes.

M: So, we're OK mathematically. The thing that causes all of us grief in
doing it this way is this concept of negative meaning--how can you have
a negative number that's faster, bigger. It's because we still think of
negatives in a counting sense. . . . We're not thinking of it in terms of
direction. . . . We're dealing with vectors. It's painful right now for me
as much as it is for you. I want to flip those things [graphs] on top [to
the first quadrant] and go to the positive areas and just intuitively make
the changes. But I'm gonna try to stick with this until it becomes less
painful. OK?

He then told students to take some problems and try to put them into their
calculators. He tried a problem in which the initial velocity was 30 m/s and the final
velocity was 60 m/s. The object traveled for 3 seconds. What was the acceleration?
Students found the slope. What was the distance? They put the equation for the velocity-versus-time graph in their calculators and then found the distance by using the area-finding key.

Mike said to them:

I kind of like doing it that way. That's kind of an easy solution, isn't it? Once you get the graph and you go in and use the graph—I mean that's just another way to pick the values out. You've gotta know what you're looking for in the graph.

He seemed to be talking to himself as much as to his students. He was admitting that the calculator was not at much at odds with students' understanding as he had thought. In fact, students still had to understand in order to find the answers. They still had to know what they were looking for.

He assigned several problems for the students to work on their own. Not yet convinced that there might be a better method of solution, he told his students, "On every one of those [assigned problems], I want a velocity-versus-time drawing."

After class, he asked me how I thought the class went. He mentioned that some students were having great difficulty in understanding that the stone released from an ascending balloon would go in an upward direction initially. I showed him how I would have solved the problem. I used the distance-versus-time equation:

\[ D = \frac{1}{2}(-9.8)x^2 + 5x + 353 \]

Plotting it in a window [-1,10] by [-10,400] showed the graph of the problem situation (see Figure 13). Then using a window of [-1,2] by [352,355] showed that the stone, indeed does initially go upward (see Figure 14). We
Figure 13: Stone released from ascending balloon

Figure 14: Different Window of Stone Released from Ascending Balloon
then talked about how one could solve problems using the function and the graphical analysis of it. Mike had still more to ponder over the weekend for Monday, when he would next teach his class.

**Velocity and Acceleration: Day Three**

Mike had reflected much over the weekend. He began his class by saying:

In the old days, I only knew one way to do a [projectile motion] problem. And it made my life so much simpler, because there was only one way. I always attacked the problems the same way--never did them any different. Life was so nice. Then I learned a second way and then I've learned a third way. And now when we come and approach a problem, I never know when I walk in front of you guys which approach is more successful. So what I've tried to do lately is let you see it from a multiple standpoint. But you guys, most of the time you would really like it if there was one approach you always did--never varied [students say, "Yes," in the background]. Made life so straightforward. But [Mike looks up in the air and then back at students] it doesn't always work that way.

Mike then made an analogy to Biblical principles. There are many Biblical principles, but one has to know how to use them in daily living. The underlying principles are the same as in Biblical times, but we have to apply them to situations that the Bible never spoke about. He continued:

That's really what physics does. I have all these physical relationships and tools and equations and truths, but every problem is a little different. What works in one problem may not work real well in another, because you have a different starting point, they've left out a different value. So, I wish that we could kind of look at what I'm teaching as different tools that are in our tool box. . . .The big problem I'm having right now with method number 3 [graphing-calculator method] that's hitting me is doing everything from an equation base, but doing the equation with the graph. And that wasn't something I did before this graphing calculator--because it took too long to do the graph. If I actually did it using graph methods, the time was not worth the effort, and what would happen was we would all lose our interest.
Mike then drew sketches of three graphs: (1) A distance-versus-time plot, (2) a velocity-versus-time plot, and (3) an acceleration-versus-time plot. These sketches were his *method number one*. Method number two was the method in which one uses only equations, with no graphs. Method number three was the graphing-calculator method. He talked about the methods and then talked about his method number one. From the velocity-versus-time plot, he could get almost all of the information that he really ever wanted to have.

This [velocity-versus-time graph] became my camp. That became the curve that I liked the most. . . . This particular plot, up until this trimester, would have been the only one I would have looked at. Now what is happening to me this trimester is this distance-versus-time graph becomes interesting to me, because for once in my life I can graph it quickly, because of the graphing calculator. OK? And so, another tool that I've got in my toolbox that I never had before is the graphing calculator. . . .

[Method three] is really not method three, but is just a continuation of method one, but allows me to do some things that I never tried before because I didn't have the time to do it.

Mike next went into a review of the balloon problem of the Friday before. He first reviewed the velocity-versus-time graph that he had used to solve the problem in the previous class period.

My number one preferred approach would have been looking at this graph, and number one, I probably would have flipped that big triangle [showing negative velocity] up on top, because I don't like that negative area. It bothers me. The area of that triangle is going to be identical to the same triangle flipped up on top. [Mike drew a red one on top to represent speed. The rest of the graph is blue. He then talked about speed and velocity and the absolute value of velocity being speed.]. . . . The reason I like it is just because I like working with things in that first quadrant. It makes me more comfortable working there, but both are the same and the blue [velocity] one is more correct. It *is* the correct one--it's not *more* correct.

The nice thing I guess about this [velocity-versus-time graph] is that this area is a positive area [points to blue triangle in first quadrant] and this area is a
negative area [points to blue triangle in fourth quadrant], and these two areas are going to cancel each other out. Right? . . . In reality it [the stone] went up and then canceled itself out coming back down. So in reality that really is what's happening. This is truth. This positive area is canceled out by this negative area and what's left is the real height of 353 meters. . . . So my displacement is 353.

Mike seemed to be convincing himself that the velocity-versus-time graph was really a better graph to use than the speed-versus-time graph. In his entire discussion, he had placed both graphs on the board and referred to each one individually. It seemed to be very difficult to let go of the one he had used all of his life.

He again talked about distance and displacement and the difference between the two concepts. Then he showed how to do the same problem using only the formula:

\[ D = \frac{1}{2}at^2 + v_o t + D_o. \]

Substituting 0 in for \( D \), -9.8 in for \( a \), 5 in for \( v_o \), and 353 in for \( D_o \) and using the quadratic formula, he arrived at \( t = 9.01 \).

Simple, to the point. I'm not positive if I have all the beauty of the understanding of what's going on in the whole path. I know when the path is done--9.01. In the other problem [method number one] I forced myself to analyze what was going on. In this problem, I simply get a quick answer. . . .

Now I'm going to go into a method I don't know anything about--method number three, which is really method number one revisited.

Mike put the above function into the Y key on the graphing calculator.

I didn't use this method before, because it was hard for me to do this. . . . [Mike graphed the function on the calculator.] Now I was able to actually do a distance-versus-time plot which I wouldn't have done before because I didn't have time.

Mike then discussed ways in which one could determine the time it took for the stone to hit the ground. His first method used zooming in by placing a box around an area and instructing the calculator to enlarge just that area inside the box. He then
traced the graph until he reached the x-intercept, and he got 9.01.

What did we get by the math? 9.01. That wasn't bad, was it? Now is *this* cheating? I don't know [laughed]. It seems too easy—it's gotta be. But it's really legal. We drew a graph, and we had to know that—on the graph what point we were going for, didn't we? We had to know it's where it hit the axis. So this is a legal mathematical tool.

Mike then showed another way to get the answer. He turned off the axes and graphed the original function along with the function $y = 0$ (the x-axis). He then instructed the calculator to find the intersection of the two graphs by using an *intersection* key. The answer, again, was 9.01. He mentioned that a student would have to know the two equations in order to solve the problem that way.

A student suggested that he use the *root-finder* key, to which Mike responded:

Help me through it. [Mike used the root-finder.] Oh, look at this. Educate the teacher. So, we've got another way to get that intersection. Again this is graphical tools—right? These are graphical tools.

Next, Mike wanted to take a look at the maximum of the function. He commented that the graph did not look like it went upward from the y-intercept. It looked level for awhile and then began descending. He, again, used the *zoom box* feature.

At this point in my life, if I do something that's slower or archaic, and there's a better way to do it, don't be afraid to tell me.

Again, Mike was making himself vulnerable to his students. Teachers who consider themselves to be omniscient authoritarians cannot allow weaknesses to show. Teachers who consider themselves to be facilitators and guides, *can*. 

Mike next posed some different problems for the students to consider. One of them was: "What if the problem asked, how far does it [the stone] rise?" He used the \textit{maximum key} to find the answer and compared the answer to the one he had obtained using method number one.

He then posed another problem:

If I want to find how fast it was going--um--I haven't tried this. If I go to the x-axis and ask for a slope at that point, would it give me a final velocity? Try it?

Mike went back to his original window and used the \textit{dy/dx} key, calling it the \textit{slope-finding key}, to calculate the slope at the x-intercept.

I haven't tried this before, so if it doesn't work out, we'll talk another day on it. Because I'm sure it \textit{should} work out from the theory. I just--I haven't tried this before. . . [Got -83--the same answer they obtained before.] It looks like it works pretty good.

Mike then showed students how to solve another problem using the graphing calculator. The students took a break, but some students remained in the room. Mike talked to me and to a few of the students.

Mike: I started thinking about that this weekend and thought, "Let's do it a different way. Might as well start using all these crazy keys." I'm still not happy with the answer--am I cheating? What do you think?

Student: I don't think you're cheating.

M: Because--

S: Because it's technology.

Another student: It's cheating. [Mike laughed.]

S: No, it's not. I don't think it's cheating. I mean, because still have to know the equations. It's not--your calculator just does it.
M: Not only that—I guess the thing that tells me I'm not cheating is I've still got to know what to get.

S: Yeah.

M: I mean, even with the graph up there, it doesn't give me the answers unless I know what to look for. But it seems so easy. Doesn't it?

I told Mike that I thought it was great that he had used the $dy/dx$ key to find final velocity. I had forgotten about that key. (Unfortunately, he did not explain to his students why he had used that key—why slope of distance-versus-time at a point gave velocity. Again, he had generalized, but had not made the connections for his students. Sometimes we teachers get so involved in the lesson that we forget to make those connections.) Mike seemed surprised that I had forgotten that key, and said that he liked that key a lot. He told me that the reason he was emphasizing the graphing calculator so much was that he thought about the labs that the students were going to be doing. They would be collecting data and graphing the data. The labs would look like the graphs we had been doing in class. So students would be going from textbook problems to real-world problems with the same graph, and they would need to know how to analyze them.

I felt that Mike made giant leaps that day in his beliefs about the graphing calculator, and about education. His struggles were obvious, but so was his determination. He was using multiple solution strategies, and engaging in communication with his students as each one helped the other. The classroom, educationally, looked like one that could be used as a model for teachers of mathematics!
Using Multiple Methods: A Journal Entry

Using the multiple solution strategies did pose a dilemma for Mike. He recorded it in his journal.

Now I do find that when I get stressed--when I get pushed into a corner, I automatically go right back to my old method, and I've gotta wrestle this one out. Because I've got a couple of students, that when I show them another method for solving it, it seems to really confuse them. They want desperately to have one method that works all the time, and they really don't like it when I pull out another method. Um. I'm still evaluating whether or not the multiple approach to a problem helps them or at least forces them to think through it. I'm finding that a lot of my fears at least on the calculator have not happened.

Critical Analysis of Data

Students were involved in a laboratory assignment in which a paper tape was inserted under a clapper, and a disk of carbon paper was placed face down between the clapper and the paper tape. Each time the clapper struck it would make a dot on the tape. Using a timer, students pulled the paper tape and measured the distance between two dots on the tape. This time required for a single vibration was the period of the timer. They then used the period in measuring the speed and acceleration of a falling object (see Figure 15). Students recorded their data into lists in the calculator and plotted velocity-versus-time graphs of the data. By graphing their data quickly and accurately and then obtaining print outs of their graphs, they were able to identify sources of error rather quickly (see Appendix C for a student's laboratory write-up).

Mike told me that prior to this trimester, he would have had the students hand plot some of their data, but replotting guesses of their sources of error would have been "out of the question." With the calculator's capabilities to find regression
Figure 15: Photograph of ticker-tape laboratory experiment
equations of students' input data, students can, for example, test if their calibrated time was incorrect. So, they can use "known time" values and replot with those values. If the latter gives a better value for gravity, then students know their main source of error was the error in their calibration of time. Replotting by hand was too time-consuming, so sources of error were not easily found. All that students could do was guess.

Another group of students apparently had skipped one of their dots on the paper tape, because the dot apparently did not print. So they recalibrated, assuming that their one large distance between two dots was really two separate distances. A replot of their data on the calculator gave a more realistic graph, which showed that they had found their probable source of error.

Mike told me that, before students had graphed their data on the TI, he did not understand why their data was so different from acceleration due to gravity. As students showed him their graphs, their sources of error became apparent to him, and he could help them through the process of analyzing their data and finding their sources of error. He was excited about the use of the calculator in this laboratory, because students' analyses were so good and their discussions were so in depth. He told me that he was really pleased, "especially with this calculator that does things so quickly and makes hard copies. . . . By the end of the period I found a tool that I didn't know I could do."

Mike recounted this event in his journal. He talked about graphically interpreting data in his engineering classes fifteen years ago. He had enjoyed doing it, because he felt it was a good way to be able to summarize data and look at
relationships, and because he is very visual and needed to actually see the relationships. However he had quit using that approach because it was too time-consuming. He then talked about using the graphing calculator:

The graphing calculator, I'm finding now, has that ability now to allow me to quickly take data, visually look at it, and then because I can see the graph, I can then identify points that are not in line with the other points. I can then go back and start looking at my experimental results and say, "OK, now point number 3 is off, let's look at it again. Let's see what's going on." And my kids have been able to do this.

One of my emphases in lab reports is an analysis of error, and to estimate causes and try to prove that to be the culprit. And now with this calculator, because we can easily get that graph, can easily see those points that are off and then go back and reexamine them—the kids now find it almost like it is an enjoyable experience rather than a misery. So I guess my focus is: this new tool gives me another visual way of teaching. So rather than fight it and them using it to prevent my way of teaching, I've found that I can use it as their way of learning. And it seems at this moment to be very successful.

Finding Gravity with the CBL

There is an experiment in which the experimenter bounces a ball, and the sonic motion detector determines the distance at certain time intervals. The data is retrieved through the CBL into the TI-82, and a distance-versus-time plot is given. Since the motion detector is placed above the bouncing ball, the time at which the ball hits the floor is the time when the ball is farthest away from the motion detector. In order to compensate for this when graphing, the program reverses the data in the distance list so that the graph shows a distance of 0 when the ball hits the floor. I had never analyzed the graph in any detail, and had simply worked with the data that we had been given. Mike asked me if I had ever noticed that the graph was upside down. I told him that I had not noticed. He then proceeded to tell me about the program and that it had
reversed the data. So this man who, just a few short months ago, had been intimidated by the calculator, was now investigating the programs.

A student had been absent from class the previous day, when Mike had shown them the bouncing ball experiment. (In the experiment, Mike determined the regression equation for one of the parabolas and, from that equation, determined the acceleration due to gravity.) That student came in the next morning to ask Mike what had transpired during class, and Mike showed her the experiment. However, since he showed it to her on a linoleum floor in his office (the floor in the physics classroom was carpeted), apparently the motion detector picked up the glare from the linoleum and the experiment gave them unlikely values. Mike did not understand at the time that the floor could make a difference, so he was frustrated that his values were not even close to being accurate. The next day he told the students that he would lecture during the entire class because he was afraid to continue with the laboratory. He explained what had happened and that he had gotten such inaccurate values. He said he would redo the experiment the next day after he got "the bugs out of it."

He talked about his frustration with the experiment in his journal:

I've probably had some of my greatest frustrations with this machine. Because I'm learning it. Case in point [Mike recreated the scene described above in which his experiment did not work for the student.] . . . I spent over an hour and a half on something that should have taken me five minutes only to find out that the whole reason that the experiment wasn't working is that the prep room has a hard tile floor and was giving me too many echoes, and I was just by chance in the other room that had a carpeted floor that gave me perfect values. Well, I'm learning that I've got to know this machine or it just frustrates you to death. Now to be quite honest with you, it might be something now in the future when I get a non-ideal value, that's part of the kids' problem. They've got to work out what's causing the echoes, what's causing the interference--how
can they eliminate these problems, because I think that's part of the scientific method also--to be able to identify variables and to control those that are not part of the experimental design and to evaluate your results and make sure you're not picking up extraneous data points rather than real variations due to some scientific principle that you're trying to discover. But again the kids have to be comfortable with the equipment before you can just turn them loose and say, "Resolve this problem."

Because of the frustration with the experiment, he went into his lecture mode. So he first showed his students a video and then told them: "We're going to get into the math, which I know is what you really desire." The tone seemed to be quite sarcastic, which is unfortunate. Mike had admitted to me that he was not fond of mathematics in his college classes, and so he apparently assumed that his students had the same feelings that he did. Actually, Mike seems to enjoy mathematics at times. I believe that he still was under the impression that mathematics is simply algebraic manipulations.

Towards the end of the class period an incident occurred which showed me that Mike was really changing his beliefs and these changes were manifesting themselves in his classroom. At one point Mike had divided a positive force by a negative acceleration due to gravity and gotten a negative mass. A student told him that she did not understand how one could have a negative mass. Mike said, "Hear what she's doing to me? She's absolutely right. If I want to do it mathematically correct..." and he corrected his problem so that the force was also negative, making the mass positive. He was beginning to think in terms of -9.8 for acceleration due to gravity, and that was forcing him to use vectors for force also. His mathematics was beginning to become connected. The next day, during a chemistry class, he told his students that he was
"worrying more about signs now than ever before" in his life.

Chemistry Class and Mathematics

One day I observed Mike's advanced chemistry class to see how he used technology there. One of the problems they got into was a problem in which they were solving for time. The equation was: \(40(4.18)(T - 90) = 60(4.18)(T - 20)\). Solving for \(T\), gives a time of -120. The answer was obviously incorrect. Mike mentioned again that he is becoming conscious of signs, whereas he was not prior to this trimester. He was confused on this problem. When they substituted \(90 - T\) for \(T - 90\), they got a more realistic answer for time of 48. A student mentioned that, perhaps they should be using the absolute value of \(90 - T\) (which would have been fine, because they were simply finding a temperature difference), but that made the problem really difficult to algebraically manipulate.

After class I showed Mike, that, by using the absolute value of \(T - 90\) and of \(T - 20\), one could graph each side of the equation as a separate function and find the intersection of the functions. Using a window of \([-200, 200]\) by \([-10, 50000]\), the resulting graphs showed an intersection in the second quadrant, which could be rejected because the graphs did not represent the problem situation; and an intersection in the first quadrant, which would be the acceptable solution. He had not thought of using the calculator in this way, and he seemed to be interested.

That same problem, which, without the absolute-value signs, produced a rather simple equation to solve algebraically, was difficult to solve for some of the students--even those who had already taken precalculus. Mike and I talked about this difficulty
after class. He said that that was why he was confused. He wanted to know if he was hurting his students by not "forcing them to do the math." I told him about the NCTM Standards (1989) and their stand on calculators. I said that if students could not algebraically manipulate equations after having completed a precalculus course, should he use his chemistry-teaching time to teach algebra? Perhaps he should go on to other things. He just looked at me. Then he said that he wanted to give them "one last try at it."

That afternoon, in Mike's physics class, he was going over problems in review for a test. Since the class had now established that they would assume downward direction to be negative, then acceleration due to gravity was $-9.8 \text{ m/s}^2$. At one point during the class, Mike forgot to include the negative sign, and a student asked him if it should be included. Mike replied, "Yes. I am nonsensitive to signs." Again, this incident reiterates the difficulty of a teacher changing paradigms.

**Maximum Velocity and Acceleration: What Does it Mean?**

Mike ran a laboratory experiment in which a rubber band was attached to both ends of a cart, and also attached to two ends of a runway on which the cart was riding. Pulling on the rubber bands enabled the cart to move back and forth between the two ends of the runway. Using a sonic motion detector attached to the CBL, one could measure a varying distance versus time, as the motion was being damped by friction. A program instructing the CBL to calculate the first derivative produced approximate velocity-versus-time data. The class then looked at the graph exhibited by the data. Mike noted that a maximum velocity was at a minimum on the damped sinusoidal
graph. There was obvious confusion here—not only for Mike!

After class I told him about an AP Calculus question from the 1992 AP Calculus exam in which students were instructed to find the minimum acceleration—over a time interval—of the particle moving along the x-axis. Since the velocity of the particle was given by a quadratic equation, the acceleration was linear, with the minimum being a negative number, and the maximum being positive. Confusing the mathematical terminology with the physical intuitive interpretation can cause problems. Physically, since negative numbers simply indicate direction, it intuitively seemed to both of us as if the minimum acceleration would occur when there was no acceleration—when acceleration was 0. However, a look at the graph clearly showed that the minimum of the linear function was a negative number. For a physics teacher who was not used to dealing with the positive and negative signs, this could be confusing indeed.

For a mathematics teacher—well I had never questioned the problem when I had initially solved it. It seemed quite clear. However, as I began to try to understand the underlying physics concepts, the sign issue became confusing for a while. I understood Mike's dilemma. We both pondered this one, each trying to fit the pieces together and construct our own understanding.

**Trigonometry**

In finding the solution to a physics problem, the students came up with the equation: \( 0 = 130 \sin \theta - 80.6 \cos \theta - 45 \). Mike asked them to solve it in their calculators. He told them, "You won't have to have a clue about how to do the math [students graphed the function and solved for the root] and that bugs me."
After students gave the calculator-produced answer, Mike asked, "Can someone tell me how to do this based on algebra?" After several incorrect guesses (at first glance, I could not solve it algebraically, either), Mike told the students to substitute $\sqrt{1 - \cos^2 \theta}$ for $\sin \theta$. Getting a radical equation, he next isolated the radical and squared both sides of the equation, which left him with a quadratic equation in cosine $\theta$. The whole process was tedious and time-consuming. When Mike had finished, he asked:

Isn't there a sense of reward now that I went through all that garbage and got the right answer? I personally like to do it the old-fashioned, crank-it-out way. [He then looked at me.] Math teacher, which way would you do it?

I replied unequivocally that I would use the calculator. I later told Mike that doing it the algebraic way involved knowing the mathematical *trick* of using the substitution. If one did not think of the *trick*, then one would not be able to solve the problem. The other, more obvious, reason to use the calculator was to efficiently produce an answer to a physics problem. I still do not understand why a physics teacher would want to devote a relatively enormous amount of time to algebraic manipulations.

**More on Algebraic Manipulations**

Mike generated an equation: $(37 + m) \cdot 0.03 = 233 - 60 - 9.8 m$. He decided that it would be just as easy to solve the problem algebraically as it would be to graph it on the calculator. His reasoning was that, in order to graph the equation, students would have to set the equation equal to zero, and so would have to solve much of it anyhow. Because he was not used to manipulating equations for calculator usage, he did not see some of the ways in which to handle the equation. I ascertained that he was going to
multiply the left-hand side through and then subtract the left-hand side from the right-hand side and combine like terms. An easier way would be to simply subtract the left-hand side from the right-hand side, producing: \[ 0 = 233 - 60 - 9.8 \, m - (37 + \, m) \cdot 0.03, \] and then input the entire expression into the calculator.

Another way to graphically solve the problem on the calculator was to assume each side was an independent function, graph each function, and find their intersection. During a break, I mentioned the latter way to Mike. He liked it and told his students about it. They tried it and liked it, too. Mike said that he never would have thought of doing it that way. Recall that I had shown him something similar after his chemistry class, but that had been almost a month before this class. I think it is good for us all—teachers and researchers—to keep in mind that the mere mention of a new idea does not assure its recall and/or use at a later time. Ideas of anchoring instruction and immersing oneself in a learning situation could be applied in experiences like this one.

After the problem above had been solved both by hand and by the use of technology, Mike told his students that he and his wife had been talking the evening before. Many years ago, Mike's wife had worked at a fast-food restaurant. At that time, society was not technologically advanced, so she had to figure tax and change in her head. He then said to the students, "It's scary when you think where we are today." It seems to be a real problem with which teachers have to grapple. Should we teach concepts at the expense of computation? Or should we teach computation at the expense of more interesting mathematics? How important is computation and algebraic manipulation in the overall understanding of physics and mathematics? In our
technological age, these are not easy questions to answer.

Writing CBL Programs

Mike enjoyed working with the CBL so much that he began to modify laboratory experiments in order to incorporate technology into them. The students did a laboratory experiment, without using the CBL, in which they were measuring torque with a meter stick and weights. Mike was curious if he could use the CBL to execute the same experiment. He wrote a program for the experiment, but he could not get it to run. He asked me to check it for error. When I did, I found that he had put values into a list that he had later removed by using another command. With a simple modification, the program worked well—again we had helped each other. It was exciting for us both. I was pleased because I was finally beginning to understand how the CBL program commands operated; he was happy because he had written his own laboratory experiment using technology. It was a big moment.

 Velocity and Acceleration Revisited: Projectile Motion

Mike began the lesson with a review of a ball being thrown into the air. Since the ball exhibited both positive and negative direction, the velocity-versus-time graph would involve a linear function which intersected the x-axis. However, Mike had reverted to his standard speed-versus-time graph, so that the function was an absolute-value function. Mike looked at me, and then he decided to extend the line below the x-axis—but he made the line dashed (see Figure 16.) He pointed to the dashed line and told the students that if they were dealing with velocity they should use the dashed line \(v\) in the Figure, but if they were dealing with rate, they should use the line with the
positive slope ($s$ in the Figure).

![Graph with positive slope](image)

**Figure 16: Velocity and speed compared**

After a brief time, Mike said, "Mrs. Nicol is going to convict me, so it's coming off," and he erased the portion of the graph labeled $s$.

Since I had never said a word to him about the graph in question—I was simply observing—I think that Mike was actually convicting himself. He was still struggling with using something that had always worked for him versus teaching his students a physically and mathematically correct approach. Seeing me triggered in him the realization that his old way of dealing with the problem could cause his students mathematical confusion.

The lesson for the day centered around two-dimensional motion in a curved path, so vectors were definitely involved. The vector for the initial velocity needed to be broken down into its horizontal and vertical components. In order to accurately describe the situation, Mike decided to show our experiment using parametric equations. We attempted to draw the letter $B$ using two motion detectors and two
CBLs. When the drawings repeatedly failed to reproduce accurately, rather than quitting, Mike suggested that we try another program and manipulate it so it could work with our two dimensions by obtaining data separately on our two calculators and then linking the calculators together. When we were convinced that our separate graphs would produce the desired result when combined, we instructed the calculator to plot the vertical-versus-horizontal data on the overhead viewscreen. The \( B \) printed on the screen, and the reaction of the students was great--lots of "oo"s and "ah"s. Our reaction was similar! We experimented with several more letters for the students and had them guess which letters we had drawn by analyzing each component separately. The students' guesses were quite accurate.

Then Mike began lecturing and reverted to his old method of teaching projectile motion. He used his favorite velocity-versus-time graph, but first calculated the initial velocity by using trigonometric calculations. I had mentioned to him that he could use the parametric-equation mode on the TI-82, and the graph would model the motion. He asked me to show him how to do it, and we decided that it would be best for him to use it the following day with the students, rather than that day.

**Projectile Motion: Day Two**

Mike showed his students three different ways to solve the problems. He first showed them the parametric equations using the parametric-equation mode on the calculator. They analyzed the graph for many different possibilities, maximum height, hang time, etc. Mike's comment at that time was:

This [using parametric-equation mode on the calculator] is a great way to
answer the question [i.e., to solve the problem]—we're not going to do it that way, but it's a great way to answer the question. We'll do it looking at x and y components. I'm wrestling with this right now. You might be the last group to see me do it this way [velocity-versus-time graph]. We'll do it the old-fashioned way. You might not be the last group. I'm still wrestling with this.

Next Mike separated the horizontal and vertical components and using each as a different function, graphed them separately in the function mode. They looked at what was happening in each direction.

Finally he again showed the students his velocity-versus-time graph. He said, "My favorite graph is a velocity-versus-time graph," (to which the students groaned). He went through the entire problem using his favorite graph and then a debate ensued with his students about the merits of the calculator versus the crank-it-out method, after which the following conversation took place:

M: Let's go back to the graph on the calculator.
S: Going back to the real world here.
M: Going back to the real world? Thanks!

After he was finished with the lesson, Mike assigned some problems and told the students to solve them using three different methods: (1) parametric equations on the calculator with print-outs of their graphs and solutions, (2) vertical and horizontal functions graphed in function mode on the calculator, with print-outs of their graphs and solutions, and (3) "the old-fashioned, grind it out with algebra" method. Mike told them:

Do all three. See which one works out best. I've got a feeling that the parametric really does work out best, but there's still something in me that says, "That's too easy—it must be cheating."
The next day, Mike was solving some problems for the students, using the parametric mode on the calculator. At the end of solving the problem, Mike then showed the students the algebraic way with his velocity-versus-time graph. He then remarked:

I really don't care which way you solve it. [He talked about the fact that they still needed to know that the solution involved vectors in the horizontal and vertical direction.] I guess either way you have to know what you're doing. It's really a whole lot nicer with this one [calculator-generated graphs using parametric equations].

He then solved another problem using the TI-82 parametric mode, and when he was finished he said, "It's a neat tool." His experience with the calculator had helped him to discover the usefulness of it as a teaching tool. As he solved problems with it, he came to discover that one needed to understand some concepts in order to use the tool to analyze the graphs. It not only was more expedient than his "old-fashioned" way, maybe it was also just as effective.

**The Pendulum Problem**

Mike conducted an experiment with his students in which they tried to determine what affected the period of a pendulum. The equation that models the data should be: $T = 2\pi \sqrt{\frac{l}{g}}$, in which $l$ represents the length of the pendulum, $g$ is the acceleration due to gravity, and $T$ represents the period. The students collected data and plotted points on a graph. The regression equation was a power function, $T = a \times b^x$, in which Mike had helped the students to determine that $a = 2\pi \sqrt{\frac{1}{g}}$, $x = l$, and $b = 0.5$. Mike told me that he had remembered a problem in which I had changed $e^x$ to $b^k$, where $k = \frac{x}{\ln b}$, and he surmised that he could also work with a
power function and manipulate it to become the above pendulum equation.

The experiment was a success, and some students calculated values of $b = 0.48$ on their regression equations. Mike liked the experiment, because it gave the students a chance to collect the data themselves rather than use the text. He told me that he thought one should spend the time necessary to allow students to collect data themselves, even though it might take an extra hour or so. It seemed to make physics more real when students could actually demonstrate what the textbook said.

**Time Management in the Classroom**

Mike mentioned the time-management problem in two different journal entries. One entry was made in early December, at the beginning of the school's second trimester. At that time he had said:

I wish we would have had a little bit more time [to complete a laboratory experiment]. . . . Of course the one thing that's happening is that the number of chapters I'm looking at covering this year has dropped drastically. And I'm not talking just one or two chapters. I'm talking about possibly cutting it in half if I continue going at the rate I'm going. And so I'm facing the dilemma of which is of a higher priority--allowing them to develop this critical approach to data and laboratories, or whether I need to maintain the survey status of this class where I'm giving them an exposure to various areas of physics. At the moment I've kind of given up the survey approach into the analytical approach. I'm not sure if that's a good decision or not, but it's the direction I'm heading, and I'll continue thinking that through. And a lot of it again is because of this new tool.

Towards the end of that trimester, Mike was still struggling with time management in his classroom. He recorded another journal entry:

I guess the biggest problem I'm facing right now is: all these type of processes in class take much longer than a quick put the values in an equation and kick out an answer. And so I find myself running into a time restraint. And that's why I keep oscillating back and forth between --you know, my old approach of straight, what I used to call plug and chug--put the numbers in the equation and
kick out an answer--into more of an intuitive approach. I don't have time to do the whole course that way [using the intuitive approach--with the calculator], so I've either got to leave out sections or I've got to become more efficient about what I'm doing. But yet what I've taught I've felt very good about. And I haven't resolved that time management right now. I wish I had the kids for two hours a day for all year, not just for 60 days.

It is a dilemma that all of us in education face. Sometimes less is more. In Mike's own field, science education, the recommendation is to teach less material but more in depth: "To ensure the scientific literacy of all students, curricula must be changed to reduce the sheer amount of material covered" (American Association for the Advancement of Science, 1989, p. 5).

On Assessment

Mike's approach to assessing his students has changed during this past year. He told me that in previous years, students would want to use their graphing calculators to find answers to problems, but Mike would not allow it. He wanted answers shown by algebraic means. According to him, he was much more concerned with the answers then, and the answers had to be found his way. Now, he is beginning to see that the answers are not always the most important part of assessment of students' understanding. He allows students to use their graphing calculators. He asks questions that require them to find minimums and maximums--questions he never would have thought of asking in previous classes. Since his students still need to determine the equations in order to solve the problems, Mike believes that he is still testing for student understanding. I believe that, too.
The thing that I wanted to see him address on his own (without probing from me) was that previously Mike believed that algebraic manipulations were an important part of his physics class. That was Mike's concept of understanding the mathematics, and he wanted his students to be able to understand the mathematics. Now graphical analysis is also accepted by him. Did that mean that algebraic manipulations were no longer as important to him as they used to be?

Of his own admission, Mike is "much more open to alternate solutions" than he used to be. On the students' first test in Mike's physics class (see Appendix D), Mike asked questions that he never would have asked before, and he accepted solution strategies that he never would have accepted before. Students had the option of solving problems in whatever way they chose, and even some students who were not particularly fond of the graphing calculator chose to analyze calculator-generated graphs.

A problem on another test involved a quick data collection experiment, in which Mike struck a tuning fork and shared the data with his students via the link cable. Each student was then required to model the data with a regression equation. This was an entirely new kind of problem that Mike had addressed, and there was much mathematics involved. I saw in this problem a deviation from the mathematics-as-a-tool idea in that the mathematics was an integral part of this entire experiment.

Titrations and the Logistics Curve

I had never heard of a titration until we went to the CMS Institute last summer. Since Mike really likes chemistry, he was excited about the ease with which the CBL
collected data for a titration. Several of us mathematics teachers sat in confusion as our CMS leader talked about the beauty of a titration experiment with the CBL. What was this titration? According to Random House Dictionary (1994), to titrate means "to ascertain the quantity of a given constituent by adding a liquid reagent of known strength and measuring the volume necessary to convert the constituent through a given reaction." What we were doing involved acids and bases and their pHs.

Mike has run several titration experiments with his chemistry students. He decided that he would also run the experiment for our presentation at the Teachers Teaching with Technology (T³) National Convention in Utah. The graph that was produced on the calculator from the data looked like a logistics curve, but the mathematics involved in the experiment seemed to be minimal—give a sophisticated guess as to where the middle of the upward swing of the curve was and find the coordinates. We both decided that this experiment was basically a chemistry experiment, and that it would not be very interesting to mathematics teachers.

Shortly before our trip to Utah in February, Mike reflected on what I had said about the logistics curve. He started to become interested in finding out if there was actually more mathematics in the data than we had determined. He asked Mark about the logistics curve, and then Mark asked me. We studied it separately and pooled our resources. The graph of the data certainly looked like a logistics curve and, with some modifications (adding $cx$ to the equation for the logistics curve to obtain $y = cx + k/(1 + e^{bx})$, where $b < 0$), Mike found that he could produce a regression equation that could model the curve. Next, using the derivative-finding key on the
calculator, Mike found the inflection point of the logistics curve. This was the point that he had previously simply guessed. Now Mike's chemistry experiment also involved interesting mathematics, and Mike had been the one who had pursued that mathematics!

Mike recorded into his journal some of his musings about the logistics curve and its implications for his changing beliefs.

[When] I [was] trying to figure out the pH, you kept using the term, it's a logistics equation--a logistics relationship--well, I had no idea what that was. With Mark Pifer's books and his definitions he looked up for me, and as I read logistics, the description of logistics is really what is going on in a titration. . . . You know, it excited me when I was able to match that curve--the logistics curve. You still are more dogmatic than I am as far as why this is happening. [I had told him that I wanted to know why we had to modify the logistics curve. What did that mean as far as the titration was concerned? What was actually happening mathematically in a titration?] When I added that value of slope at the beginning of the curve in order to get it to match the actual data, I was very content with the fact that it did match, and I wasn't really worried about the fact that mathematically I shouldn't have added that value--it worked. And, and I guess what I was looking for at that moment, is I was looking for an equation, so that we could take the second derivative and find that point of inflection, rather than just guessing at where I thought it looked like it was on my graph.

As we worked on our experiments for the T3 convention presentations, Mike commented, "You're going to teach me some math in all this--do you know that?" He seems to be much more interested in finding out the mathematical implications of his findings than ever before. And this interest has helped him in his relationship with Mark Pifer. Mike also commented about that in his journal.

The other thing that has been kind of interesting, I've probably had more school conversations, academic conversations, with Mark Pifer in the last few weeks than I've probably had in all the years put together, because for the first time, I'm not afraid, I guess, to ask him questions about things I don't know, because
I need to know the solutions. I'm not really worried if I'm revealing that I'm--uh--rusty in these areas. And I'm finding that he is very responsive because he's just as interested in finding real-life applications of the stuff he's teaching and very willing to look for things that are not just straight book problems but real-life problems. So that's been encouraging. So I may even be affecting his concept of math in the sense that he's coming out of his absolute tried and true methods and looking into things that are a little bit more abstract--i.e., he hasn't worried too much about the logistics curve until now, and now it becomes something he looks and finds and tries to figure out. So there's even a neat relationship between the math/science department on a personal level.

**What Exactly is Mathematics?**

The previous journal entry was a part of a response to a question that I had asked Mike. I had observed Mike talking in class about simple machines--machines that do not need a power source, such as levers, pulleys and inclined planes. He illustrated on the board a first-class lever--one in which the fulcrum of a lever is located between the effort and the resistance. He placed three, 100-gram weights on one end of the lever and one, 100-gram weight on the other end. He then asked the question, "How do you do the mathematics of this?" He elaborated somewhat by asking the students, "If I wanted to get this [the lever] to balance, where do I put the three, 100-gram weights?" He then proceeded to determine the torque (product of a force and the length of its torque arm) of each end and balanced the lever by making the torques equal.

After showing the students second-class levers and third-class levers, Mike told them that "The math is the same on all three of these." Again, he seemed to elaborate by telling them that they needed to take into account the resistance arm and the effort arm, and that the efficiency was based on what they got out and what they put in. So,
what did Mike actually mean by his statement that "The math is all the same"? Exactly what was he calling math?

Up until now, I had kept quiet about what my beliefs were as to what mathematics was all about. I did not want to inject my beliefs at all. Several times I almost had to bite my tongue, because I really wanted to tell him that mathematics was so much more than algebraic manipulations. However, I wanted Mike to come to that realization himself after having worked with me, with the TI-82, and with the mathematics for the past year.

After that class, I felt that it was time to ask the question of Mike directly. I wanted to know exactly what he was referring to when he talked in class about the mathematics of the problem. Had his perception changed any over the past year? If so, what (in his opinion) had influenced his perception? Since I wanted to give him time to think about the answers, I wrote these questions on a piece of paper and asked him to respond to them later in his journal.

We talked briefly about the questions. Mike's immediate response was, "Math to me is algebra." When I probed further and asked if it still meant that to him, he thought about it and responded that it meant more to him than that now, but that mathematics had always been just algebra. He noted that he is a creature of habits, and now he is beginning to change a lot of his habits, but it is neither an easy nor quick process. Mike talked about how much he liked the calculator and its ability to provide graphs. He said that it made the abstract mathematics concrete, and that he now could understand concepts that he had never understood before. He talked again about how
he had always skipped the section on simple harmonic motion because he had not
understood the graphs of the trigonometric functions. But this time in his teaching, he
had zeroed in on them, because he now understood them and they made the abstract
mathematics much more concrete. On one occasion, Mike pointedly said, "I like
graphics!"

Another question we pursued briefly was whether he still considered a
geometric approach to addition of vectors to be a nonmathematical approach. He just
stared at me and, after a few seconds, responded that he would have to call the
geometrical approach a mathematical approach, too. He indicated that when he had
geometrically added vectors in drafting, he never had considered that that was
mathematical. After all, to him, mathematics was algebra, and geometric solution
strategies did not involve algebra and so therefore were not mathematical. It seemed as
though he had just had a revelation into his own thinking processes and his own
perception of mathematics. I was anxious to receive and transcribe the journal entry
that would address some of these issues.

Mike did not get around to recording a journal entry for about a week, but the
entry was well worth the wait. He began by telling me that the day he had taught about
simple machines he had fallen back into old patterns, because it was a subject that he
had taught for many years, and also because the trimester was almost over and he felt
the need to move more quickly through material. He then reiterated that the old
patterns involved thinking about mathematics as algebraic manipulations to get a
solution. Then he said:
If I am then pressed by you or we get into areas that I'm not as comfortable with, then I find that my concept of math has really been stretched into looking at relationships. Rather than the math being an algebraic manipulation, it has now become more of a summation—a relationship of—of—um—patterns or of tendencies and trends as a result of the various factors that are interacting with whatever problem it is we're talking about. . . . So then, I'm starting to see a relationship—what is actually occurring in a much broader sense, more than just a solution for a problem. It's now trends and tendencies and the ability to predict and manipulate, because I'm seeing a bigger picture. And so, in that respect, the math is showing me what's really happening—not just a solution.

It's hard for me to just look at an equation and have this feel for what's happening. You have this ability that, you know, you're comfortable enough with different equations and you see different patterns. You know it's gonna rise sharply, it's gonna dip, it's gonna level off, it's gonna reach a certain asymptote—those aren't things that I'm comfortable with. But yet I'm finding that as I use the calculator, I'm doing the same type of stuff. I'm starting to visualize, and therefore that math now is becoming more of a general pattern—um—to the point that I actually get—um—preoccupied with coming up with that curve, and therefore I know what it's gonna do.

At that point he talked about the logistics curve and the sinusoid. He reiterated the fact that he avoided the section about sine waves, because he was not comfortable with the graphs. Then he went on to say:

Now because we've been playing with that type of wave so much with the CBL, I'm very comfortable with what's going on, and it is not scary. And although a solution is not hard to get, I'm no longer hung up on a solution. I'm content with looking at the relationship and what's happening—again a big-picture concept.

So, yeah, I'd say my math has taken a turn from an algebraic solution, I've conquered the problem, into more courage to dwell into the unknown and the theoretical and say, "What if?" and "How come?" and "What if we vary something, what can we expect on the other side?" And that's a big trend, and I like it. I mean, I feel as a teacher it's a much better approach for me to show the kids a big-picture concept because—um—you're not always—you know, especially as an engineer or someone that's into research—not always going to have the ability to get the final answer without at least having a road map to where you're going. And that's what those graphs become are just those road maps that point you to the right direction, and they'll show you inflections and changes and variations and stuff. . . . So I'd say, yes, my definition of a math solution has broadened considerably.
He mentioned again that the old patterns would probably emerge when he was
teaching on the more familiar material.

You're probably going to see the greatest effect [of his new beliefs] in those
areas that are not taught by me for fifteen years and therefore have got the old
thought processes. Chemistry is going to probably benefit the greatest from this
[new thinking about mathematics], because I haven't taught it for many years,
and if I start teaching it next year, I will probably want to see things from this
aspect.

In a previous study, I had asked several university professors of mathematics
what mathematics meant to them, and I got a variety of answers (Nicol, 1994). One
professor told me that the process of doing mathematics was the process of reasoning,
and that "mathematicians are extraordinarily good at drawing logical conclusions from
illogical axioms." Another professor told me that mathematics was "trying to describe
life as we see it. . . . trying to learn new things about the phenomena by using
mathematics." A third professor basically skirted the issue and talked about engineers,
statisticians, etc. and asked rhetorically if they were using "mathematics or physics or
whatever." When I asked one of the professors, "What is math to you?" he simply
stated, "Oh, that's absolutely impossible--far too broad a question."

I personally, preferred Mike's newly formed answer. I believe that he has
captured the essence of what it means to do mathematics. And he did not acquire his
views from a textbook, or from a course, or from a person, but rather from a year of
working with mathematics in a variety of settings. He used a tool to help him visualize
the abstract, and, using that tool, he looked at mathematics in many different ways, and
through many different lenses.
Conclusion

After a year of experience with technology and with teaching, professional development conferences, peer interaction, disequilibration, and much reflection on all of the above, Mike has changed his beliefs in many areas. Not only does he profess to have changed his beliefs, these changed beliefs have manifested themselves in his practice. He has changed the way he views mathematics, calculators, and educational practices. In the next chapter, I will address these changes specifically and the implications of these changes for educators.
CHAPTER VI

CONCLUSIONS

Solving a problem is similar to building a house. We must collect the right material, but collecting the material is not enough; a heap of stones is not yet a house. To construct the house or the solution, we must put together the parts and organize them into a purposeful whole.

George Polya (cited in Gallian, 1986, p. 120)

The Research Questions

Mike's story has been told as it unfolded to me. However, I began this study with more than a story in mind. I began with a list of questions about Mike's beliefs--questions that I hoped to be able to answer at the conclusion of the year-long study. Because the beliefs of a teacher drive his/her practice (Hersh, 1986; Cooney & Wilson, 1993; Thompson, 1992), an examination of Mike's beliefs were the focus of the research questions. For convenience to the reader, the research questions are restated:

- What were the physics teacher's beliefs and practices about mathematics, education, and technology prior to 1994?
- What changes in beliefs and practices occurred?
- What factors influenced this teacher's change in beliefs and practice?
- How have these changes in beliefs effected change in his instructional practice?
In this chapter I will answer the questions based on data that I collected over the past year. Wherever possible, data collection sources will be provided in parenthetical material to establish the chain of evidence mentioned in Chapter IV.

What Were Mike's Beliefs and Practices About Mathematics,

What Changes Occurred, and How Did His Change in Beliefs

Effect Change in His Instructional Practice?

Mike believed that mathematics consisted of algebraic manipulations, and so solutions were of prime importance. However, since Mike wanted his students to "understand their mathematics," he insisted that students perform the computations themselves. According to Mike's beliefs, solving equations and using dimensional analysis was the essence of the mathematics in his physics classroom. Mathematics was a tool of physics, but was certainly not an integral part of it (interview, February 1994; videotape, March 7, 1994; journal February 1995). Even the geometry used by his students was not considered by Mike to be mathematics (field notes, November 22, 1994).

Mike's mathematics was fragmented, and so the mathematics that he taught was also fragmented. It was unconnected in vector addition (field notes, November 22, 1994) and unconnected especially in his teaching of motion. By using speed instead of velocity, but calling it velocity, Mike portrayed to his students a disconnected mathematics that used vectors only when convenient and scalars at all other times (field notes and videotapes, November 29, 1994; interview February 1994).
Most graphs were considered to be nice, but not essential (e.g., the distance-versus-time graph), and so graphical analysis was almost nonexistent (informal conversations). Mike did use the velocity-versus-time graph for its slope (acceleration) and its area (distance), but he never made the connections to nonlinear velocities (field notes and videotapes, November 29, 1994; informal conversations). In fact, even though Mike explained the basis of his method to his students, it seemed to me that some of his students rapidly forgot the connections and just took the method as a procedure thereby developing no conceptual understanding of his method (observations of students during class and my own experience with a student of mine, see Narrative One).

By the end of the study, Mike's beliefs about mathematics had changed dramatically. He began to see mathematics as trends, patterns, and relationships. Graphs not only became important to him, but they became an integral part of the physics that he taught (field notes, see Narrative Four). Graphical analysis became a priority, and mathematics became the language of his physics rather than a useful procedural tool. He began enjoying mathematics, and his fear of it diminished (journal, February 1995).

Single solution strategies were no longer necessary or even desired. He not only showed students multiple ways to arrive at their answers, he also accepted various solutions (artifacts, student tests). He began to appreciate the fact that different students understand mathematics in different ways; and he attempted to teach them accordingly (field notes and videotapes, see Narrative Four).
Mike had struggled through his dilemma of whether to show his students multiple solution strategies, because it tended to confuse students when they did not have one single method to learn (journal, December 1994). However, he also wanted his students to understand conceptual underpinnings of the problems. With time, he came to realize that students learned those underpinnings better when presented with multiple ways to view a problem (observations).

Algebraic manipulations and dimensional analysis still had their place, but they were no longer the sole use of mathematics in Mike's classes. In fact, algebraic manipulations often gave way to graphical analysis (e.g., finding a maximum, or a root of an equation) (field notes and videotapes, see Narrative Four). In previous physics classes, Mike's use of dimensional analysis was an attempt to solve problems that he did not understand well. He reasoned that if the final dimensions were correct, the solution was probably correct. With his new understanding of the mathematics, and also of the physics, he no longer needed to rely on dimensional analysis. He could solve the problems without it—and could use dimensional analysis to check his solutions (informal conversations). Because students were looking at problems in depth, rather than superficially surveying them, they too were able to analyze graphs and otherwise solve problems, and so Mike's classroom use of dimensional analysis diminished considerably.

Mathematics is often viewed as consisting of discrete strands like computation, problem solving, and geometry. Ideally mathematical ideas should be seen as being related to one another, related among different mathematical topics, and used in other
disciplines (NCTM, 1989). Mike's fragmented mathematics became much more connected, as he connected algebraic and graphical representations of problems. Even his favorite unconnected method of using the speed-versus-time graph eventually gave way to the velocity-versus-time graph along with the distance-versus-time and acceleration-versus-time graphs (field notes and videotapes, December 1994; see Narrative Four). By using the latter three graphs, he made powerful connections for his students. Not only did he make connections for them within the solutions of projectile motion, but he also paved the way for his students to connect the mathematics they learned in physics class with the mathematics they learned in calculus class.

I asked a university professor of mathematics, Frank Demana, to describe mathematics in one word. His answer was, "fun." He went on to say, "If everybody thought math was fun, we would not have the problems we have today. Instead we have parents running around wearing 'My worst subject' like a badge on their chest when you tell them you teach mathematics--like a badge of honor. It shouldn't be" (Nicol, 1994). In college, Mike admittedly did not like mathematics. That dislike continued into his teaching and was even made evident sometimes by his remarks. Mike's comment at the CMS Institute, that he was learning mathematics and it was exciting (field notes, August 1994), and his increasing interest in finding graphical models for his data (see Narrative Four) were indications to me that Mike was beginning to see mathematics as "fun."
What Were Mike's Beliefs and Practices About Education, What Changes Occurred, and How Did His Change in Beliefs Effect Change in His Instructional Practice?

**Instructional Practice**

Mike's main mode of teaching had always been lecture (informal conversations). Mike was in control of his classroom, and he was the disseminator of information. As he began teaching with the calculator and with other manipulatives, he began to allow his students to discover patterns and relationships by working together to solve problems (informal conversation, September 1994).

**Assessment**

Mike had always used multiple-choice and short-answer types of tests, because of their ease in grading (interview, February 1995; informal conversations). Solutions derived by the calculator were to be used to check work, but were not permitted as actual answers on the tests. His tests became much more influenced by the calculator method of learning. Questions were asked that required more detailed answers. Calculator solutions were not only accepted, they were expected. Computer printouts of graphs became the norm in both tests and in laboratory write-ups. Since he had shown students multiple methods, he accepted various solutions. Ease in grading became secondary in his ideas of ideal evaluation (observation and field notes, see Narrative Four; informal conversation).
Content

Prior to use of calculators in his classroom, Mike's idea of his physics class was a survey type of class. Students would learn a little about many topics in physics. As he progressed through this school year, his ideas changed. He began to consider the possibility that in-depth learning might be more important than breadth of topics (interview, February 1994; journal, December 1994). The Project 2061 (American Association for the Advancement of Science, 1993) reform vision for science literacy includes the idea that "depth of useful learning, rather than sheer coverage and marginal content" be accented in the schools. So his beliefs about what he should be teaching seemed to be lining up with those expressed in national standards.

What were Mike's Beliefs and Practices about Technology,

What Changes Occurred, and How Did These Change in Beliefs

Effect Change in His Instructional Practice?

Initially, Mike was against the use of graphing calculators in his classroom, because he believed that students used calculators as a crutch to avoid learning and to escape understanding (interview, February 1994; journal, August 1994; observations, see Chapter IV). He did not accept students' solutions when they were obtained using the calculator (informal conversations), and he, himself, did not use it in class at all (classroom talk). The solver key on the TI-85 was a fearful key to him, because he believed that by using the key, students would avoid doing the dimensional analysis and algebraic manipulations that he believed were an essential component of his physics class (informal conversations, journal; interview, February 1994; journal, February
As Mike became more familiar with the calculator, and as he began using it to do statistical calculations and analyze graphs, he began to see the calculator as a valuable tool rather than as a crutch for students (unstructured interview, September 7, 1994; observations, see Narratives 2, 3, 4). He liked the use of the calculator to "help reinforce concepts that are there and help even teach concepts that aren't quite there" (journal, August 1994). He also liked the calculator's ability to quickly summarize data, plot a graph, and enable students to look at the relationships (journal, December 1994). Graphs, with their analyses that he had used in engineering classes years before and had appreciated because of the graphs' visibility, had been too cumbersome and tedious to pursue. The efficiency of the calculator and the calculator's ability to bypass tedious paper-and-pencil calculations renewed his interest in graphical analysis (journal, December 1994). He found out that many of his fears about the calculator (mentioned above) did not materialize (journal, December 1994).

Mike's change in beliefs towards the calculator manifested itself greatly in the classroom, to the extent that he was teaching his students how to use a calculator, and he was using it almost daily in his instructional practices (observations and field notes, see Narratives 3 and 4; journal, December 1994). Because of his extensive use of the calculator in his classroom, he felt that he was forced to relearn some of his physics (informal conversation, October 1994). He noted that, because the calculator allowed him to visualize quickly, it made him want to know. One example of this was Mike's pronounced interest in harmonic motion, a subject that he had neglected to teach in
previous physics classes because of his lack of understanding of sinusoids (informal conversations, observations, and field notes, see Narrative Four).

His antagonism about the solver key diminished as he came to use the calculator for discovering trends and relationships and was no longer viewing the calculator as simply a crutch for solving equations (informal conversations). In fact, in his physics classes he began using other keys (e.g., integral key, derivative key, root-finding key) to find quick solutions, recognizing that these solutions were not the main concepts of the mathematics and physics that he was trying to reinforce (field notes, observations, and videotape, see Narrative Four).

Mike became such an advocate of the graphing calculator that his colleagues noticed and mentioned it often (informal conversations; interview with Bill, September 21, 1994; Mark's account of Mike's change, September 21, 1994). Bill Williams, after our presentation at the ACSI convention, jokingly wrote on our evaluation that he had lost his friend to the graphing calculator!

What Factors Influenced Mike's Change in Beliefs and Practice?

In order to adequately address this question, I again refer to you the model for teacher change that I proposed in Chapter IV (see Figure 17). I now refer to this as a model (rather than a proposed model), and I will show how it has modeled Mike's change.
Recall that one of the theories I was triangulating was Kuhn's theory of scientific revolutions involving paradigm shifts. Since paradigm shifts are basically one's system of beliefs, then Kuhn's theory is represented by beliefs in the model. Vygotsky's theory of capable-peer assistance in one's Zone of Proximal Development (ZPD) is represented by socialization, and the theory of Anchored Instruction (or learning in environments that permit sustained exploration) is represented by experiences. The constructivist learning theory is also prevalent in teacher change. The disequilibrium espoused by constructivists (von Glasersfeld, 1990) and by Kuhn (1970) is a precursor to that change. The model is a cyclic one, in that all of the factors affect one another, and the driving force is reflection.
According to Kuhn, when people undergo revolutionary change, they are faced with many obstacles to that change. First I will discuss those obstacles, and then I will discuss factors that facilitated change. The facilitating factors are incorporated in the model for teacher change, and show the interaction of the theories in the theoretical framework.

What Obstacles Inhibited Mike's Change in Beliefs?

_The transfer of allegiance from paradigm to paradigm is a conversion experience that cannot be forced._

(Kuhn, 1970, p. 151)

NCTM (1989) assumes for grades 9-12 that "Scientific calculators with graphing capabilities will be available to all students at all times" (p. 124). Advantages to students are numerous. Without having to spend time in tedious calculations and point plotting, students have more time to develop mathematical concepts. The graphing calculator also fosters in-depth analyses of functions--not only the traditional ones, but also the more complex which have been heretofore beyond the scope of high school mathematics. Real-world problems can be modeled and examined (Eckert, Kitchen, Nichols, & Vonder Embse, 1989).

So, if there are numerous benefits to the use of graphing calculators in the secondary schools, why would a reflective teacher like Mike hesitate to incorporate them into his instructional practices? Because the beliefs of a teacher drive his/her practice (Hersh, 1986; Cooney & Wilson, 1993; Thompson, 1992), beliefs were the focus of the research questions, and the pivotal point of the model for teacher change.
Instrumental Beliefs About Mathematics

When Kuhn (1970) discusses paradigm shifts in scientific revolutions, he notes that proponents of competing paradigms employ different standards or different definitions of science. Tharpe, Brown, & FitzSimmons (1994) noted that there was a high correlation between teachers' beliefs about mathematics and their use of the calculator in their instructional practices. Their results indicated that teachers whose understanding of mathematics was instrumental tended to believe that calculators do not enhance, and could even hinder, instruction. Whereas, teachers who had a relational understanding of mathematics tended to believe that calculators should be an integral part of mathematics and science instruction.

Mike's paradigm was definitely that of the former. His belief that mathematics was simply a sequence of algebraic manipulations placed him in the instrumental category. Holding that belief, he thought the calculator would not enhance, and would probably hinder, mathematics and science instruction in his classes. As a result, he disallowed the calculator's use in his classroom, particularly on tests.

Time

Time was an enormous obstacle for Mike. When Mike talked about a colleague (Interview, March 1994), he noted that much of her problem with the graphing calculator was that she did not understand its capabilities, and she did not have the time to learn. So rather than dealing with the problem, she outlawed the calculators. With so many demands on a teacher's time, sometimes it is much easier and much less time-consuming to maintain the status quo. Mike mentioned in his journal (February 1994)
that he had not had the time he needed to actually use the calculator so that he could feel competent making a decision about its value.

Another issue about time was the time it took to grade work that involved the use of a graphing calculator. Questions needed to be asked differently--no longer would simple numeric solutions be adequate. Open-ended questions and questions that ask, "Why," are appropriate kinds of questions to assess students' understanding, but they are quite time-consuming to grade. All of the rhetoric about the value of asking good questions often gets lost in the day-to-day routine of an already overworked teacher.

Another time issue is the one of time spent on a particular concept. Using technology adequately requires more time in development of the lesson than straight lecture does. Mike struggled (and still struggles) with the "less is more" idea of Project 2061 (American Association for the Advancement of Science, 1993). It is an area of cognitive conflict (i.e., disequilibration) for him. Often, teachers struggle with this problem--when to take the time to develop a concept, and when to "cover the material." Which is better for the students? Teaching with technology is time consuming, and teachers must make the decision of whether using the technology is worth the time it takes to develop the concepts. In my own case, I firmly believe that using technology in instructional practices is worth the time. However, when I have been pushed by administrators to "cover the material," I regret that I have forsaken the technology and have resorted to lecture. Colleagues of mine have had similar experiences. For me, personally, it has caused much internal struggle--it is not an easily
dismissed problem with a simple solution.

Still another issue of time is the longevity aspect. Even after Mike had attended the T³ convention in February 1994 and had shown an inclination to change (see Narrative One), he did not make the changes in his instructional practices. By the following summer, Mike was not even sure he wanted to attend the CMS institute, and he had considered cancelling his involvement in it (informal conversation, August 1994). His two-day participation in professional development several months before was not enough time with the graphing calculator to allow him to feel competent to make a decision regarding its use. He had not had enough time working with it to realize its potential in his classroom, and so his interest waned.

The Problem of Control

Being in control of a classroom can be a delicate balance for a teacher. For those who prefer the authoritarian/omniscient role, the lines seem to be rather clear. However, for those who believe they should facilitate and lead (like a coach) rather than dominate, the lines can blur. Although Mike assumed the role of lecturer, his heart was to be a coach (observations, informal conversations). He always wanted what was best for his students (observations, informal conversations), and his reason for even investigating the possibility of calculator usage was that his students had "badgered" him into it (unstructured interview, March 7, 1994).

Mike's idea of being in control was to lead, rather than follow. Because Mike was comfortable with the subject material that he taught, and because he felt that he had overcome the inherent belief that "teachers always know everything," he had come
to a place of security in his teaching (journal, December 1994). However, the graphing calculator put him back into the insecure feeling of being a new teacher. (At this point, it appeared that disequilibrium was beginning to affect him as he considered his teaching role.) His students were coming to him far more knowledgeable about the calculator than he was. Several years before, he had been deceived by students who were passing answers back and forth using the memory in their calculators, and he did not want to be caught unaware again (journal, December 1994).

**Fear**

Because his understanding of mathematics was limited, Mike did not want to reveal that weakness. After two days in our CMS professional development institute, Mike was feeling quite inadequate with his mathematical knowledge (journal, 1994). Although, at that point, he still had an instrumental belief about mathematics as a sequence of algebraic manipulations, he was starting to internalize the fact that mathematics involves much more. (At this point, I believe he was beginning to experience disequilibrium in his beliefs about mathematics.) He noted in his journal (August 1994) that a tremendous amount of mathematics appeared to be necessary to know if one wanted to adequately use the graphing calculator in the classroom.

Mike was also afraid to change his entire method of teaching. Using the graphing calculator was a different way of teaching and a different way of thinking. As he taught his physics class, he was facing new challenges every day. Also, he was self-taught in physics and by changing the way he was teaching, he was afraid he could also become insecure about his subject matter. He admitted to his physics class that he was
delving into the unknown, and it could make him feel insecure (videotape, observations, December 1994, see Narrative Four).

Several times during the course of Mike’s physics class, he mentioned that he thought he might be cheating by using the calculator (videotapes, see Narrative Four). He feared that by bypassing some algebraic manipulations, he was making the mathematics too easy. Again, he was reverting to his instrumental definition of mathematics, but in his state of disequilibrium, he also believed that his students were developing holistic mathematical concepts by using his new approach. His instrumental definition and newly emerging relational understanding were causing underlying conflict in his belief system. The conflict seemed to continue to emerge in his statements, like, "Is this cheating?"

Entrenched Habits

It is difficult to change the habits of a lifetime. Often, when Mike was teaching material that he had taught for years, he reverted back to his former methods of teaching. He mentioned that he falls into old patterns when he is pushed for time, under pressure, frustrated with the technology, and/or teaching familiar material (journal, February 9, 1995; observation, videotape, see Narrative Four). It takes energy to consciously change habits, and teachers involved in revolutionary change must constantly change much of what they are doing. It is not an easy task, and it is often frustrating.

One of the most outstanding examples of Mike’s difficulty to change entrenched habits was the one in which he continually reverted back to his speed-versus-time
graph. Ignoring the negative velocity seemed to be an ongoing problem, and change seemed to be a constant challenge. Part of the difficulty in his change was that he was struggling with the concept, himself. By his own experience, he had not understood the concepts when he had learned them the traditional way and had only understood them when he had been taught the speed-versus-time method. Because of his experience, he assumed all students would learn his method better. Many students, after having been taught the traditional way by others, would come to him for explanations. When students would then seemingly understand his explanation, his beliefs that his method was best was reinforced. However, he initially failed to realize that he had fragmented the mathematics, and thus had confused some students later; that many students did not have his conceptual understanding and simply used his method algorithmically; that his method would not work with AP calculus questions because they asked questions involving negative and varying velocities; and that not all students learn in the same way, and so some students may prefer the traditional distance-versus-time graph (and, in fact, many of his students did).

As time went on, Mike began to understand the problems inherent in teaching his speed-versus-time method, and disequilibrium set in. When my study ended, Mike had not fully resolved the cognitive conflict. The habits and beliefs were deeply entrenched, and he needed more time.

**School Finances**

Although finances were not a problem for Mike, in that he ordered and obtained all of the technological equipment he wanted and needed, and in that he was financed in
his professional development endeavors, he did voice some concerns. In his journal entry (August 1994), towards the end of the CMS institute, Mike mentioned that he was not at all sure that the school would purchase the equipment he felt he needed to implement the changes he wanted to make. He said that it would be better not to know what the graphing calculator could do than to know it and then not be able to use it because of budget constraints. He also mentioned that previously he had thrown away catalogs, because he had known that the school had no money for new items. His concerns were not unique. Many teachers have voiced similar concerns to me.

**What Factors Contributed to Change?**

**Socialization**

*There is no getting around the primacy of personal contact* [emphasis in original]. *Teachers need to . . . have one-to-one and group opportunities to receive and give help, and more simply to converse* [emphasis in original] *about the meaning of change.*

(Fullan, 1982, p. 121)

**Students.** Students badgered Mike into using the calculator, and students encouraged his continued use of it. Mike had witnessed one of his student's use of the graphing calculator in his science project and it had inspired him (Mark's observations). Also, Mike's students seemed to be interested in using the calculator, and they enjoyed printing out their graphical results on the computer. When given the option of using any method on a test, the calculator-based method was the preferred one (informal conversations, observations, artifacts, see Narrative Four; journal, December 1994). All of those considerations inspired Mike, and he wrote in his journal (December 1994)
that his students' reactions made worthwhile the time he had spent preparing things that were different.

Also, it was a student who initially forced Mike to reconsider his speed-versus-time graph. One of his students was quite confused, and I tried to help Mike see that perhaps her confusion lay in his use of speed instead of velocity. So the following day Mike gave a calculator-based explanation using the velocity-versus-time graph with the area-finding key (videotape, November 22, 1994). The student told him that the new explanation greatly helped her to understand. Mike then told the class that he was rethinking his thinking about his speed-versus-time graph. He admitted that he was causing students, as well as the WCS mathematics department, some grief. However, Mark and I had been trying for years to get him to change his method. It was not until he had understood the other method better and he had seen another way to present the material using the graphing calculator, that he finally decided to change. Both the student and I had talked to him about the problem, so socialization with both of us had been a factor; and experience with the technology had also been a factor. The experience factor will be discussed in the next section.

Encouragement from me. Mike has mentioned on numerous occasions that I have been a tremendous help to him as he has attempted his change. I have tried to be encouraging and helpful, and I was present for the majority of his physics classes, especially at the beginning when I attended every class.

Our teamwork at CMS was helpful to both of us, as each one tried to utilize the other's subject matter. His knowledge of mathematics was inadequate for his comfort
level in presenting experiments with the CBL, and my knowledge of physics was similar to his knowledge of mathematics. The presentations that we did at different conventions, neither of us wanted to do alone. I have learned much about physics this year as I sat in on Mike's physics classes and worked with him preparing for presentations. I know that he, too, has learned much about mathematics.

As situations presented themselves, I tried to guide him into mathematically correct ways. I helped him use the calculator in ways that were new to him by showing him how to solve equations graphically, by giving him an AP calculus question to show him the confusion that could result from his speed-versus-time method, by showing him how I would have solved problems, and by just being in class to help him when he had questions and to be "his conscience" (e.g., when he told the class that I was going to convict him, so he was going to erase his speed-versus-time graph) (observations, informal conversations, videotapes, see Narrative Four).

Teachers need support from colleagues, and they need help with problems that they are having. Vygotsky talked about capable peers helping others in their ZPD. I believe that teachers have a ZPD in what they have taken ownership of and are willing to change. Mark Pifer was willing to change his practice to incorporate graphing calculators, but, due to time constraints, was not willing to take the time to change again to incorporate the CBL. At the time, he felt that what he was doing in class had worked for him and was quite adequate. If teachers do not take ownership of the changes they are being asked to make, then peer assistance will be of little value.

In Chapter III, I talked about Martin (1993) and her belief that teachers' ZPD
should be applied to the "conditions under which an adult learns new things" (p. 81). I would argue that, in the case of teacher change, ownership is one of the critical conditions under which the teacher learns new things relative to change. Castle and Aichele (1994) maintain that:

True professional development, in the sense of resulting in meaningful and long-lasting qualitative change in a teacher's thinking and approaches to educating, is an autonomous activity chosen by a teacher in search of better ways of knowing and teaching mathematics. . . . Externally imposed professional development activities, although well intentioned, are doomed to failure. (p. 3)

Since Mike had taken ownership of his change, he was willing and able to listen to my suggestions. As I mentioned in the section above, never before had he been willing to change his method of teaching velocity problems. This year, as he began to see reasons for the change and ways to incorporate the change, his ZPD changed from years past, and he rethought his method and changed his lesson. Not only did he change one lesson, but he also changed his thinking on presenting multiple solution strategies to his students and on accepting varying solution strategies on students' work. His ZPD, and so, what he was able to accept from others, had broadened considerably.

**Working with Mark.** Watching Mark's precalculus class in action was a contributing factor in getting Mike to even want to change. And Mark was the one who encouraged Mike to go to the T³ meeting in February 1994. Mark was a definite positive influence in effecting Mike's change.

As time went on, Mike felt more comfortable about asking Mark questions about mathematics, especially after I had finished my data collection. When I was no
longer immediately available, Mike sought Mark's expertise. Together they worked on
the logistics graph, each assisting the other. Mike finally was no longer afraid to ask
Mark questions about things he did not know, because Mike needed to know the
answers. And Mike thought that Mark was becoming more willing to look at real-life
applications of the mathematics he was teaching. Mathematics and science were
connecting in the department as well as in the classroom.

Experiences

Mike had many experiences that led him to continue to pursue change. The
experience on the first day of T³ in February 1994 caused him to return on Saturday.
His original plan was to probably stay only one day (informal conversation). However,
as I mentioned before, even his experience at that conference did not produce in him
lasting change. Only the week-long CMS institute did that.

At the CMS institute, we were with others of like mind for 40 hours in class
and numerous hours out of class. This is my idea of what the anchored instruction
concept entails. Our instruction on the CBL and on the TI-82 was anchored in real-life
applications. Being exposed to that much experience at one time was very helpful for
us and very instrumental in changing Mike's paradigm. Kuhn (1970) talks about the
paradigm shift in scientific revolutions:

Just because it is a transition between incommensurables, the transition between
competing paradigms cannot be made a step at a time, forced by logic and
neutral experience. Like the gestalt switch, it must occur all at once (though
not necessarily in an instant) or not at all. (p. 150)
During the week of the institute, Mike worked with the technology and became comfortable with it. He was seeing the ways in which mathematics was an integral part of physics rather than simply a tool of physics. As a result, he became more interested in understanding the underlying mathematics. He began to view the calculator as a tool to get him where he wanted to be (journal, observation, see Narrative Two). The experience with the calculator reduced his fears of the calculator and of the mathematics. He then became interested in programming—both the TI-82 and the CBL.

By the end of the week, Mike's instruction on the TI-82 and the CBL had been anchored. His beliefs about mathematics had changed, and his beliefs about the calculator had changed. Going along with the findings of Tharpe, Brown, and FitzSimmons (1994), since he now had begun to have a relational view of mathematics, he was more likely to see calculators as an integral part of instruction. In one week, Mike's paradigm had made a definite shift (Bill William's interview; Mark Pifer's interview; observation; informal conversation).

That shift in paradigms then manifested itself in Mike's practice. Because of his experience at CMS institute, Mike put his Investigative Physical Science (IPS) class into groups and had them discover patterns, leading to a discussion of the Periodic Table of the Elements (informal conversation, see Narrative Three). Immediately he began to find ways to incorporate technology into his instructional strategies. He introduced metric conversions to his IPS class through linear graphs produced on the graphing calculator. His experience with sinusoids gave him confidence in working
with sinusoids, so that when a marine-biology student asked about the "seven wave theory," (i.e., that every seventh wave is larger than the others) Mike answered him with the graphing calculator (informal conversation, see Narrative Three). Prior to the CMS institute, Mike had not felt comfortable enough working with sinusoids to even mention them in his physics classes.

Mike's experience with parametric equations during the CMS institute and then in preparing for our presentations gave Mike the confidence to teach projectile motion using parametric equations--something he never would have tried before (informal conversations, observation, see Narrative Four). He began experimenting with graph shifts and began developing his own programs and experiments (informal conversations, see Narrative Four). He even investigated graphs like the logistics curve. And the velocity-versus-time graph, although difficult in coming, eventually made its way into Mike's physics class. The changes in Mike had, indeed, been revolutionary.

The Model, the Theories, and the Study

In Chapter III, I established that we are indeed in the midst of a revolution in mathematics education. Documents like the *Curriculum and Evaluation Standards* (NCTM, 1989) and the *Professional Standards for Teaching Mathematics* (NCTM, 1991) advocate sweeping reform of mathematics education. This revolutionary change does not come easily. Habits developed over years of teaching are deeply entrenched, but many of them must be changed if reform is to happen.
Change in our educational system can only happen through teachers who are willing to make that change. Mike Smith is such a teacher. In Chapter II, I discussed views of Schubert and Ayers (1992) and Dewey (1916) who believe that reflection is absolutely necessary for the professional development of teachers. Thus, reflection is the driving force between the factors involved in the model for teacher change. Mike is not only a willing teacher, he is also a very reflective teacher. I spent many hours in conversation with Mike during the past year, and I observed his reflection throughout.

I watched Mike go through many periods of disequilibration. He began with his students and Mark pushing him in a direction that he did not want to go. Following that lead, he pondered many such issues as: If he is to be an effective teacher, should he be familiar with the graphing calculator that students brought into his classroom? Learning how to use the calculator is time consuming—is it really worth the time it takes? Will use of the calculator reveal his inadequacy in mathematics? Does use of the calculator cause calculator dependence? Should students know how to manipulate algebraic equations, and should that be his concern as a physics teacher? Is calculator usage cheating? Should Mike teach students a method that made sense to him, or should he teach them a mathematically correct approach? Should he allow a variety of solution strategies? Should physics be a survey course, or should he spend the time developing concepts with the calculator and CBL?

Through his disequilibration, he was striving for equilibrium. One factor that influenced his change in beliefs involved socialization, in which conversation with others helped and encouraged him. Conversations with mathematicians were a great
benefit to him, as they conversed with him in his ZPD. Another factor influencing his change was his anchored instruction with the technology during the week-long CMS institute. Intensive experience with the technology as an instructional aid caused him to experience a shift in paradigms, which in turn, manifested itself in the restructuring of his instructional practices. Although his shift in paradigms was quick and was dramatic, his beliefs within his paradigm changed as he continued with his socialization and experiences. As a result, he continued to restructure his practice. Throughout all of it, he continued to experience disequilibration, which resulted in a new construction of beliefs. The factors of the model continue to influence each other, as his beliefs remain pivotal to all. The process, of course, is not finished and hopefully will continue as Mike continues to seek and to reflect.

Implications

I believe that there are some major implications for the mathematics education community. First are implications for professional development. Mike did not experience change until he took ownership of his change, and that ownership came as a result of his extensive experience with the technology and its implications for mathematics education. Inservice activities that are imposed on teachers and are accomplished in one or two days are not likely to facilitate any lasting change. Teachers should request inservice activities that interest them and should be anchored in instruction for several days, so that they can begin to feel comfortable with the changes.
Many of the teachers who attended the CMS institute with us did not implement the technology after they returned to their classrooms. They were too overworked and did not have the support and encouragement that they needed. Mike and I worked together and were committed to change; and Mike was supported financially and otherwise by the WCS administration. Financial support, administrative support, and peer support are all important factors in influencing teacher change. I reiterate that Mike told me several times that he really appreciated my support and assistance, and he said that he learned a great amount of mathematics through my work with him. I can also say that I grew as an educator as I worked with Mike and watched him through this process. I learned much about teachers and the change process, and also, incidentally, about physics! It was definitely a collaborative effort. I was a participant observer, with an emphasis on participant.

Since we are in the midst of revolutionary change, principles of change in the midst of revolutions hold. Change is dependent on paradigm shifts, which might never come. Entrenched habits remain difficult to break; traditions often present obstacles to change; and peer and society views are often antagonistic and hard to overcome. Change is slow and arduous even after a paradigm shift. We should not expect instantaneous results.

Implications also exist for teachers who have an instrumental view of mathematics rather than a relational view. Unless this view changes in a shift of paradigms, use of technology as an integral part of instruction will probably not occur. In Mike's case, his beliefs about mathematics had to radically change before he saw the
advantage of using the graphing calculator in his classroom. Even after his paradigm shifted, he continued to voice old beliefs to his students and exhibit old beliefs in the classroom, while his new beliefs were obviously driving his instructional practices. It was a testimony to the disequilibration in his thinking. Teachers must be given the benefit of the doubt, and those who work with teachers in transition must look beneath the exterior to find if paradigm shifts have indeed occurred.

Finally, a major obstacle for Mike was the time factor. This factor manifested itself in numerous ways, and those who advocate teacher change must be aware of the extent that time can hinder the change process. Teachers not only need time with the new ideas (or, in this case, instructional aids), but they also need time to implement change. Change can be time consuming, frustrating, difficult, overwhelming, but it can also be extremely satisfying and rewarding. Those who work with teachers in transition can reap the benefits of a teacher who has blossomed. I have worked with such a teacher and have been rewarded many times.

Recommendations for Further Research

This has been a single case study and as such provides an in-depth look at a single teacher. As I mentioned in the beginning of this dissertation, the circumstances of this particular case study were unique. However, questions arise from the study. The mathematics and physics connection was strong in this study. The CBL had a great influence on Mike's willingness to change. He saw a tool that was very appealing in its ability to retrieve data quickly and transfer the data to a graphing calculator. He and I worked well together, because of my mathematical background and his physics
background. However, I do not think that I would have pursued the CBL alone. Nor do I think that Mike would have gleaned much about it from a mathematical standpoint without assistance from a mathematician.

Presentations that we have seen have been made by teams of mathematics and science teachers. Being a mathematics teacher who is basically unfamiliar with laboratory equipment and with physics implications, I believe that I would have made minimal use of the CBL in a high school mathematics class. I would not have known how to set up the equipment, nor would I have known how to make the necessary mathematics and science connections for my students. Mike has told me that he would have felt very uncomfortable teaching some of the mathematical concepts without my help.

Since many educators are advocating the interdisciplinary approach, especially in mathematics and science, studies should be made on the feasibility of this, using the CBL and a graphing calculator. Given mathematics teachers' limited science backgrounds and science teachers' limited mathematics backgrounds, are those connections possible without collaborative efforts, and should they be? Perhaps collaboration is a key ingredient. It was for us. Encouraging mathematics and science teachers to collaborate and thereby facilitate change in each other would be a practical way to effect change in education.

I looked at Mike's change through the lens of a mathematics educator. Another recommendation would be to look at similar change through the lens of a physics educator. What, if any, significant changes occurred in Mike's beliefs about physics
and/or physics education? It is neither within the scope of this study nor within my area of expertise to address such issues.

As I look back over the past year, other questions arise. Would Mike have changed his views of mathematics had he had the same introduction to the calculator alone (without the CBL)? I doubt that he would have expended nearly the effort that he did. The CBL was the instigating tool. Would his enthusiasm have waned had he not been granted the funds to purchase the equipment? Would he have continued to incorporate the CBL if he had only been permitted to buy one CBL and one TI-82 for demonstration purposes? Would that have made a difference in the students' reactions and in Mike's enthusiasm?

Would Mike have made his radical change had he had a stranger as the researcher? Did our good relationship, which had already been established, make him feel more comfortable? Did his comfort level manifest itself in his freedom to ask me questions? (Recall that he had not previously felt that freedom with Mark Pifer, a colleague of his.) Was our friendship an asset in this study?

A case study which has been researched by a participant observer can run the risk of experimenter bias. Although I have attempted to report any influences that I might have had on Mike's beliefs, so that the readers can reach their own conclusions, it could be in the interest of the education community to conduct other similar studies without participant observation. I would suggest that a study be conducted on pairs of mathematics and physics educators who are attempting to incorporate graphing calculators into their classrooms. Given similar conditions of a week-long professional
development institute, and then collaboration between the mathematics and physics teachers, a study could be conducted to determine if beliefs changed. Perhaps the mathematics teachers would have relational understanding of mathematics and would have been successfully using the graphing calculator in the classroom; and the physics teachers would have instrumental understanding of mathematics and would have little knowledge of the calculator. Determination could be made if the physics teachers' beliefs changed significantly when paired with mathematics teachers who were not otherwise involved in the study. In such a study, the researcher would simply be an observer, so that experimenter bias could be kept at a minimum.

A next step could be a larger, quantitative study in which an instrument could be devised to determine beliefs, both before and after the study. Perhaps teams of mathematics and physics teachers could be paired together to determine their influence on each other as they collaborated on year-long courses of precalculus and physics using the graphing calculators. Again, I would want teachers to participate in a week-long professional development institute. Possibilities could include having a control group who did not participate in the institute to determine the institute's effect on the beliefs.

A recommendation for study would be a two-year study of someone in change. As I mentioned previously, one of the motivations for this case study was the uniqueness of the circumstances surrounding the study. A reflective physics teacher who had been against the use of calculators in his classroom decided to participate in an institute and try to incorporate the calculators in his classroom--with his
administration's encouragement and financial support. It seemed logical for me to become a participant observer for the duration of a high school physics class. It would have been interesting to have continued the research for another year to document continuing change.

What will Mike do in the next trimester? He will be teaching another physics class. How will he teach it differently? Will he revert back to any old ways or will he continue to press on? Will my absence cause any problems (i.e., we will not be working together, and I will not be there to encourage him to press on). There is at least one reason to believe he will continue to press on. Mike had a student assistant who typed all of his laboratory experiments, complete with CBL and TI-82 laboratory directions. He then placed the plans in a binder and is reusing them this trimester. By recording all of the information from last trimester, he has considerably reduced the amount of work he will have to spend in the future.

He will also be teaching an advanced physics class to students who have already had calculus. Most of those students already own the TI-85 calculator. Will he begin using that calculator or will he insist that students use the TI-82? (Recall that a student could borrow one of his TI-82s, since he owns a classroom set of them.) With a two-year perspective, one could look at how the change process continues, once it has begun.

Questions seem endless. Many teachers are in some process of change. Studies of them can inform educators as they attempt to read the stories and reflect on their own lives or the lives of those they teach.
Conclusion

No one can say how many people will be influenced because one teacher decided to expend a great amount of time and energy to accomplish revolutionary change. Certainly his students will reap the benefits. It also appears that many of his colleagues will also gain from his new insights. I know I certainly have grown through working with him, and hopefully the teachers whom we have taught together through professional development conventions have also changed and have influenced others.

Mike Smith is a reflective teacher who continues to base his teaching practice on what is best for his students. Because his students continued to walk into his physics classroom with a piece of equipment (the calculator) that Mike did not understand, Mike finally decided that he needed to understand the equipment in order to make an adequate decision regarding its use in his classroom. With each continuing step, he encountered and conquered obstacles in his path, but he persevered through uncharted waters. Because he emerged victorious, so do we.

*THE ROAD NOT TAKEN*

*Robert Frost*

*Two roads diverged in a yellow wood,*  
*And sorry I could not travel both*  
*And be one traveler, long I stood*  
*And looked down one as far as I could*  
*To where it bent in the undergrowth;*

*Then took the other, as just as fair,*  
*And having perhaps the better claim,*  
*Because it was grassy and wanted wear;*  
*Though as for that the passing there*  
*Had worn them really about the same,*
And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I--
I took the one less traveled by,
And that has made all the difference.
APPENDIX A

UNSTRUCTURED INTERVIEW QUESTIONS
Give your background.

Tell me about your teaching methods.

How does mathematics enter into your teaching of physics?

Tell me about your views of technology in the classroom.

I'm going to give you two physics problems. How would you explain these problems to your students?

1. A person throws a ball upward into the air with an initial velocity of 15.0 m/s. Calculate (a) how high it goes, and (b) how long the ball is in the air before it comes back to his hand. We are not concerned here with the throwing action, but only with the motion of the ball after it leaves the thrower's hand. Disregard air resistance.

2. A major league baseball player hits a ball at an angle of elevation of 35° when the ball is 3 feet off the ground. The ball is hit with an initial velocity of 103 ft/sec in the direction of a 10-foot fence that is 300 feet from home plate. Assume that gravity is the only force affecting the path of the ball; disregard air resistance. Is the hit a home run (i.e., does it clear the fence?)
APPENDIX B

LESSON PLANS OF UNIT CONVERSIONS
Activity #1

Get into your groups and measure the length, width and thickness of your IPS. Book. Make these measurements both in inches and centimeters. (Please use decimals, not fractions.)

<table>
<thead>
<tr>
<th></th>
<th>Inches</th>
<th>cm</th>
<th>cm/Inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Width</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Questions:

1) Which is longer, a cm or an inch?

2) Can you develop a mathematical equation to convert from cm → inches and from inches → cm?

3) Using the attached graph paper, graph your measurements. Placing cm on the y-axis and inches on the x-axis. How can this graph be used?
Activity 2

In a lab using two thermometers (one in °F and the other in °C) the following data was collected.

<table>
<thead>
<tr>
<th>Temp*</th>
<th>°F</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°</td>
<td>-18</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>66</td>
</tr>
<tr>
<td>5</td>
<td>212</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>250</td>
<td>121</td>
</tr>
</tbody>
</table>

1) Using graph paper plot this data. Put °F on the x-axis and °C on the y-axis.

2) Using the graph, convert the following temperatures:
   a) 37°F = °C
   b) 90°C = °F

3) Derive a linear equation using the slope intercept form.

4) Using your equation from question #3 convert question #2 values.
APPENDIX C

STUDENT'S LABORATORY WRITE-UP: TICKER TAPE
Determining the Acceleration of Gravity

I. Purpose:
The purpose of this lab was to prove that by dropping different objects, consisting of differing weights, and by studying their increase in velocity, we can calculate the acceleration of gravity.

II. Procedure:
1. Set up lab
2. Mark the recording timer with a form of identification so that you can use the timer that you have calibrated.
3. Fasten the recording timer to the table using a C-clamp and connect it to the wall outlet.
4. Make sure you have plenty of room to pull a long section of paper tape through the timer without hitting any obstacles.
5. Insert the end of the paper between the clapper and the carbon disk, making sure that the tape can move freely.
6. Turn on timer and make sure that it is working properly and by pulling a short piece of paper through and checking to make sure that it is marking dots onto the paper tape.
7. Turn the timer off and insert a clean piece of paper.
8. Turn on the timer and pull the paper steadily through the timer for approx. 2 seconds, using a clock to measure the time accurately.
9. Turn off the timer and mark the first and last dots on the paper.
10. Count the number of dots, starting with the second dot (ignore any variation in the spaces between dots; the spaces between dots represent equal time intervals regardless of variations in the spaces).
11. Repeat this procedure 3-4 times and make sure you have at least three trials that are in good agreement with each other.
12. Record your data and figure the average period between dots.
13. Cut a piece of paper that is at least 20 cm longer than the distance between the table top and the floor.
14. Pass the end of the tape through the timer and fasten a 200-g mass to the end of the tape.
15. Hold the mass at a convenient level near the top of the table.
16. Start the time and let the mass fall to the floor.
17. Stop the timer when the mass hits the floor.
18. Repeat steps #13-17 for masses of 300 grams and 400 grams.
19. Using the tape for the 200-g mass, label the second third dots A and B, respectively.
20. Count 4 dots from B and label the next two dots C and D, respectively.
21. Repeat this procedure for the remainder of the dots that were produced before the mass hit the ground.
22. Measure the distance between A and B, between C and D, etc.
23. Record this data on your data sheet.
24. Repeat steps #19-23 for the masses of 300-g and 400-g
### Data Table #1

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Current Read (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0177</td>
</tr>
<tr>
<td>2</td>
<td>0.0180</td>
</tr>
<tr>
<td>3</td>
<td>0.0174</td>
</tr>
<tr>
<td>Average</td>
<td>0.0177</td>
</tr>
</tbody>
</table>

### Data Table #2

<table>
<thead>
<tr>
<th>Trial</th>
<th>A, B, C, D, E, F, G, H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90.9 cm, 7.9 cm, 3.9 cm</td>
</tr>
<tr>
<td>2</td>
<td>90.9 cm, 7.9 cm, 3.9 cm</td>
</tr>
<tr>
<td>3</td>
<td>90.9 cm, 7.9 cm, 3.9 cm</td>
</tr>
</tbody>
</table>

### Data Table #3

<table>
<thead>
<tr>
<th>Trial</th>
<th>Speed</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{in}$</td>
<td>$V_{en}$</td>
</tr>
<tr>
<td>1</td>
<td>1.43</td>
<td>1.45</td>
</tr>
<tr>
<td>2</td>
<td>1.43</td>
<td>1.45</td>
</tr>
<tr>
<td>3</td>
<td>1.43</td>
<td>1.45</td>
</tr>
</tbody>
</table>
IV. Discussion:
In this lab, we were to measure the acceleration of gravity. The theoretical answer is 9.8 meters/second^2. Our actual answers differed from the theoretical answer by a significant margin.

Our actual answer for the 200-g mass was 6.9 meters/second^2. This is a DA of 2.9 m/s^2 and a DR of 29.5% error. Our actual answer for the 300-g mass was 7.3 meters/second^2. This is a DA of 2.5 m/s^2 and a DR of 25.5% error. Our actual answer for the 400-g mass was 6.6 meters/second^2. This is a DA of 3.2 m/s^2 and a DR of 32.7% error. These are rather large percentage errors but they can be accounted for.

First, in dealing with the 29.5% error in the 200-g mass, this can be accounted for by first of all, the problem of friction. Our apparatus was set up such that there was a string that was attached to our paper, running over a wooden board. The friction caused by the string on the wood, could have thrown off our data points. Also, we were dealing with an average time that we had calculated to be .0175 seconds. The theoretical time is .0166 seconds. When we graphed the picture of our velocity of the 200-g mass with the theoretical time of .0166, we ended up with 7.7 meters/second^2. This is a DA of 2.1 m/s^2 and a DR of 21.4% error. This is a better value than the value gained using the time of .0175 seconds. (See graphs/linear reg. #1&2).

The 300-g mass had a percentage error of 25.5%. Friction could be blamed but most likely, the friction was minimal with a heavier weight, that the values wouldn't have been thrown off that much. I feel that the error here definitely comes from our error in the actual time we calculated to be .0175 seconds. When I graphed the velocity of the 300-g mass with the theoretical time (.0166 seconds), the value became 8.14 m/s^2 instead of the 7.32 m/s^2. This is a DA of 1.66 m/s^2 and a DR of 16.9% error. This is much less of an error than the 25.5% error that we received using our actual time. (See graphs/linear reg. #3&4).

This 400-g mass had a percentage error of 32.7%. This is an enormous error and I knew that friction would not be to blame in this error. When I graphed the velocity of the 400-g mass against the theoretical time (.0166) the value was 7.36 meters/second^2. This is a DA of 2.4 m/s^2 and a DR of 24.8% error. This is a better value than using our actual time but it is still a rather large error. (See graphs/linear reg. #5&6). After studying the graph, I wondered if we had missed a point that fell between points A and B. When I held the paper strip up to the light, I could see an imprint between A and B where the clapper had hit the paper but had not made a mark. Then we calculated the velocity of the 400-g mass weight by dividing the distance between the dots (A and B) in half. We then graphed our actual time with that velocity and we ended up with a value of 7.5 meters/second^2. This is a DA of 2.3 m/s^2 and a DR of 23.5% error. Since that was still a large percentage error, we decided to calculate this same velocity but using the theoretical time of .0166 seconds. After graphing this we received a value of 7.07 m/s^2 which is a DA of 2.1 m/s^2 and a DR of 21.7% error. This proved to me that there was a point between A and B which made our first DA equal to 32.7% and that our actual time of .0175 seconds was wrong. When we changed the time to .0166 and found the point that was missing between A and B, the DR dropped to 21.7% error. (See graphs/linear reg. #7&8).

The percentage errors were still present, even after we had made some adjustments to the equations, but I feel that these errors were due to "human error." Using such a "crude" set-up, averaging the time period between dots, using a faulty recording timer, not keeping accurate time, etc., can all add to the percentage errors that are still present. If we had accurate recording pieces and an accurate time-keeping piece (and if we weren't human), these percentage errors would have been lower. Still, we were able to calculate the acceleration of gravity but the calculations were not precise.
200-gram weight with actual time period of 0.0178 s.
#2

200-gram weight with theoretical time fit.

LinReg

\[ y = ax + b \]

\[ a = 770.4388022 \]

\[ b = 5.822013423 \]

\[ r = 0.9991988014 \]
300-gram weight with actual time period 0.0175 s.

LinReg

\[ y = ax + b \]

\[ a = 732.6788111 \]
\[ b = 27.15221477 \]
\[ r = 0.9788947604 \]
LinReg

\[ y = ax + b \]

\[ a = 814.2826339 \]
\[ b = 28.62442953 \]
\[ r = 0.9788935646 \]

300- gram weight with theoretical time & 0156.5 s.
LinReg

y = ax + b

a = 662.9306488
b = 33.97919463
r = .9711912784
LinReg

\[ y = ax + b \]

\[ a = 736.7517048 \]
\[ b = 35.82134228 \]
\[ r = 0.9711899864 \]

400-gram weight with theoretical time vs. circles.
LinReg
\[ y = ax + b \]
\[ a = 750.7108127 \]
\[ b = 15.70299838 \]
\[ r = .9876844342 \]

400-gram weight with actual time period \( \theta = 0.0175\) s.
and point \( = 1(\theta - b) \) changed.
LinReg
\[ y = ax + b \]
\[ a = 767.5009791 \]
\[ b = 15.56961185 \]
\[ r = 0.9900270948 \]
APPENDIX D

PHYSICS TEST
PHYSICS TEST

1) On the problem \( V_0 = 2.0 \text{ m/s} \)
\[ a = 4 \text{ m/s}^2 \]
\[ t = 3 \text{ sec} \]

**Draw**

a) distance vs time plot
b) velocity vs time plot
c) acceleration vs time plot

**Give**

a) distance equation for this problem
b) velocity equation
c) calculate \( V_f = ? \)
d) change of distance

2) For an object thrown up at \( V_o = 35 \text{ m/s} \)

**Draw**

a) distance equation
b) velocity equation
c) maximum height
d) time to reach ground
e) time to be at 25 m above the ground

3) I am in a car going 13 m/s when I see a fallen tree 40 m in front of me, and I apply the brakes 1/2 second later. The brakes cause an acceleration of -2.0 m/s\(^2\).

a) Will I get stopped in time? (Prove your answer.)

b) If not, calculate the velocity with which I strike the tree.
LIST OF REFERENCES


