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Microwave absorber analysis, design and measurement

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The Ohio State University, 1994
Microwave Absorber Analysis, Design and Measurement

A Dissertation
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of the Ohio State University

by

Jiahn-Rong J. Gau

* * * * *
The Ohio State University
1994

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DEDICATION

To My Parents, Ho-hsien and Phonchien
ACKNOWLEDGMENTS

I would like to express my sincere appreciation to my advisor, Prof. Dennie Burnside, for his guidance and insight throughout my 6 years of graduate study at OSU/ESL. Especially, I feel fortunate to have the opportunity to learn electromagnetics by using the compact range chamber that his research group developed. I would like to thank Prof. Munk and Prof. Newman for being on my committee and for their comments.

I owe a debt of gratitude to Chang-Fa who developed the PMM code. I would like thank Mark Beals, Jiti Gupta and Li-Chung for the image processing programs. In addition, a special thanks should be given to Harry Shamansky for the wonderful computer power he has installed at ESL and for his help in all kinds of software problems. I have to thank my Taiwanese (Chinese) friends at ESL, Teh-Hong, Chang-Fa, Wei-Hong, Ling-Miao, Jo-Yu, Li-Chung, Chi-Chih, Huan-Wan, Ming-Wang and Shi-Tseng for their participation in daily discussions on various topics. I also enjoy the frequent conversation that I had about American culture and EM research with my officemate Mimi Hsu.

When I reminisce on my graduate study several years from now, I will definitely think of my good friends in the Taiwanese Student Association at OSU. We share a common concern toward our mother land—Taiwan. Many of them are so sincere and caring that one cannot but make friends with them for life. That was how I met my dear wife, Ho-hsien. Finally but not the least, I have to thank my parents-in-law
for their unconditional care of our 9-month-old daughter, Phonchien. Without their help, we could not have finished our Ph.D. studies with such ease.

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CHAPTER I
Introduction

Compact ranges have been extensively used for radar cross section (RCS) and antenna measurements. In these anechoic chambers, wedge and pyramidal absorbers are normally mounted as a continuous wall to attenuate stray signals resulting from chamber confinement [1]. For measurements at low signal levels, absorber scattering properties and placement become a critical design issue [2]. In addition, a significant amount of chamber construction cost is related to absorber due to the large volume of materials needed and the enlarged chamber size for accommodating absorber placement. Consequently, high performance absorbers with minimal size are required to improve measurement quality and reduce construction cost. This dissertation will attempt to address the theoretical analyses, design issues and experimental verifications associated with the development of high performance absorber products.

To characterize absorber wall performance in an anechoic chamber, it is more appropriate to model an absorber structure as an infinitely periodic structure than a finite sample. Yang et.al. [3, 4, 5] developed the periodic moment method (PMM) for singly and doubly periodic structures with lossy dielectric bodies and applied it to accurately determine the scattering performance of various absorber materials. Absorber performance was shown to be significantly improved by using specially contoured geometries such as curved, serrated and multilayered tapers. Their study has demonstrated the possibility to design a new generation of high performance absorbers, which are, in some cases, as good as conventional absorbers twice the
size. However, due to the tremendous computation requirements, the PMM solution is not an efficient tool for design purposes.

In most cases, absorber performance becomes worse at lower frequencies. Thus, absorber designs tend to focus mainly on the low frequency region. When the wavelength is much larger than the period, the periodic modulation of material properties can be homogenized using the effective material concept [6]. In this case, the periodic structure is approximated by a stratified layered medium, and can then be solved by using a transmission line formalism. This type of analysis will be referred to as a transmission line approximation (TLA). Effective material constants for static cases have been extensively studied [7, 8]. Based on the static results and a coupled mode formalism, Kuester et.al. [9, 10, 11] showed that the fundamental mode in a periodic structure can be modeled as the scattering mechanisms associated with the effective medium. Their TLA formulation is valid for wide angle incidence in the principal planes, where the two polarizations are decoupled. In this dissertation, the TLA method is generalized to arbitrary angle of incidence with arbitrary polarization by using a set of polarization coupled differential equations. The generalized formulation will reduce to Kuester's formulation in the principal planes.

The TLA method is so efficient that it can be used as an absorber design tool. However, the limitations of this method must be examined using the PMM method to ensure a proper design. This low frequency technique considers only the fundamental mode and may become inaccurate due to higher order mechanisms as the frequency or the dielectric constant increases. Even though there are some limitations, the TLA method can still be applied to many absorber designs in that it can provide at least a rough estimation of absorber performance.

A design procedure combining the merits of the TLA and PMM methods is presented for both wedge and pyramidal absorbers. The procedure contains three
major steps. Firstly, the most appropriate absorber material is chosen based on
the evaluation of scattering properties versus the dielectric constants for a typical
absorber geometry at critical design frequencies. Secondly, a conventional optimiza-
tion scheme [12] is incorporated with the TLA method to optimize the curved taper
geometry. Finally, the TLA designs must be verified by the PMM solution. In some
cases, the TLA prediction is so erroneous that the PMM must be used for the final
iteration. The designs of 40° wedge and pyramidal absorber will be used as design
examples to illustrate the application of the proposed design procedure.

The scattering performance of a new absorber design needs to be critically
evaluated through various measurements. The absorber measurements presented in
this study were performed in the OSU/ESL compact range [13, 14]. Normally, a
finite size absorber panel mounted on a metal plate of the same size is measured.
Therefore, in addition to the mounting errors caused by the support structure, there
are end effect errors from the panel edges as one intends to obtain the absorber wall
performance. The basic scattering properties of such edge effects are studied using
the uniform theory of diffraction (UTD) for a finite slab [15]. This edge effect error
can impact absorber measurements throughout the frequency spectrum and becomes
very significant for high performance absorbers for which the reflection coefficient is
relatively small. To remove these error terms, the inverse synthetic aperture radar
(ISAR) technique [16, 17] is applied to construct an image from the frequency and
angle domain data. In the image domain, the error terms can be properly identified
and then removed using an image filtering algorithm. The corrected image can be
reconstructed back to the frequency and angle domains, and the resulting data will
better represent the true absorber wall performance.

As will be shown in the measurements of 6° curved pyramidal and wedge ab-
sorbers, there are some discrepancies between the measured and designed data. Such
an error is mainly caused by cutting errors and improper material doping. One way to improve absorber performance is to enhance manufacturing quality. The other alternative is to use a Chebyshev absorber design. As opposed to the conventional uniform height absorber configuration, this multilevel design divides absorber elements into different heights based on the Chebyshev transformer concept [18, 19] to destroy the coherence among the specularly scattered fields from absorber elements. The resulting response can be written as the original response multiplied by the additional Chebyshev reduction factor. With this in mind, a design procedure will be presented and applied to several absorber structures. As will be illustrated by the measured and calculated data, the Chebyshev pattern can improve absorber performance by more than 10 dB with a height increase equal to only a fraction of the original dimension.

During this research effort, several high performance commercial absorber materials have become available. They were designed based on the same principles as developed in this dissertation, but without the PMM verification. Two of these new materials were evaluated in the OSU/ESL compact range and the measured data was processed using the image editing procedure described previously. The evaluation and Chebyshev modification of these materials involved the major aspects of this dissertation. Therefore, this dissertation serves as a nice summary of the current progress of absorber research and products.

The rest of this dissertation is organized as follows. In Chapter 2, the PMM method is briefly summarized and the TLA method is developed to deal with the scattering of an infinite absorber wall for arbitrary incidence angle and polarization. The TLA limitations are then examined by the PMM method through various taper structures and dielectric properties in Chapter 3. Based on these theoretical developments, an absorber design procedure combining the TLA and PMM methods is
described in Chapter 4 with the application to the 40'' wedge and pyramid designs. Chapter 5 is devoted to the image editing techniques to remove absorber measurement errors. Measured results for several designed absorber structures are presented. The Chebyshev transformer concept is applied to absorber designs in Chapter 6 to significantly improve absorber performance. Chapter 7 describes the evaluation and Chebyshev modification of two commercially available high performance absorbers. This dissertation is concluded by Chapter 8 with some suggested future research topics. Appendices A, B, C and D supplement the formulation of the TLA method described in Chapter 2.
CHAPTER II

PMM and TLA Analyses for Wedge and Pyramidal Absorber Structures

2.1 Introduction

The PMM and TLA solutions introduced in this chapter utilize Floquet's theorem [20, 21] to analyze the scattered fields from an infinite wall of periodic wedge or pyramidal absorber structures with a time harmonic ($e^{j\omega t}$) plane wave incidence. For a periodic structure, Floquet's theorem states that the field quantities can be written in the form of transversely propagating plane waves modulated by periodic vector functions which have the structure's periodicity. A PMM solution combines the periodic Green's function and the moment method technique [22] to solve the integral equation of the unknown induced currents only inside a chosen reference element in the periodic structure. Yang et.al. [3, 4, 5] developed this PMM code for the scattering of dielectric absorber covered wall. In this chapter, this method is briefly summarized.

The TLA method homogenizes material distribution in the periodic plane by applying an effective material concept and then simplifies the original problem into the equivalent transmission line problem which is nonuniform along the direction normal to the periodic plane. Using a waveguide analogy to derive frequency dependent effective materials, Bucci et.al. [23] implemented the TLA analysis for wedge absorbers at normal incidence. However, it is not obvious how to extend this approach to pyramidal cases and wide angle scattering. In fact, the homogenization process for the static case has been extensively studied for many years [7, 8]. For
a singly periodic slab structure which can be applied to wedge absorbers, there is a closed form effective constant; whereas, for a doubly periodic cylinder structure which can be applied to pyramidal absorbers, there is no closed form formula but some variational bounds reported. Based on this knowledge, Kuester and his colleagues [9, 10, 11] developed a more general scheme for wide angle scattering of both wedge and pyramidal absorbers. In their TLA method, they assumed that the periodic structure can be homogenized by the static effective material properties and then extended to low frequencies where the wavelength is considerably larger than the period. Using an asymptotic expansion theory [10] for a uniform array of cylinders, they showed that the average of the periodic functions can be described as a plane wave propagating in a homogeneous but anisotropic effective medium. To include a tapered geometry in their analysis, they derived a coupled mode formalism and argued that only the “local” fundamental mode is needed for low frequency applications [9]. They further assumed that the two orthogonal polarizations are so weakly coupled that the TLA formulation can be written as a normal transmission line formula, in which each polarization is governed by two first order differential equations [11]. Note that this assumption basically limits their methods to the principal incidence planes where two polarization cases are decoupled. Finally, a Riccati differential equation was derived to solve for the reflection coefficient of the equivalent anisotropic effective medium.

In this study, the TLA analysis is derived using a more direct approach. By plugging the Floquet’s fields into the Maxwell’s equations, one can formulate the original problem in terms of periodic vector functions, which represent the static solution in the low frequency limit. Furthermore, a formulation in terms of the averaged periodic functions can be obtained by invoking the averaging procedure. Applying the static effective parameters as proposed by Kuester, the averaged fields
can be represented in a form similar to a transmission line problem. For the most general case, the averaged fields are governed by a set of four coupled differential equations, in which the two orthogonal polarizations are coupled. The formulation reduces to Kuester's for the principal incidence planes where the two polarizations are decoupled. The coupled equations are solved by a finite difference method detailed in Appendix D. To provide a clearer transmission line analogy, the decoupled case can be modeled as a stratified multilayer medium [24]. Both reflection and transmission coefficients are given in the calculations. The calculations involved in the TLA analysis are much simpler than the PMM method. Thus, the TLA analysis is suitable for optimization of absorber designs in the low frequency region, which is usually of utmost design interest.

2.2 Absorber Geometry and Coordinate System

A finite section of a pyramidal absorber panel and the associated coordinate system are shown in Figure 1. The absorber layer is doubly periodic along the $\hat{x}$

![Figure 1: A finite section of an infinite wall of pyramidal absorber and the associated coordinate system.](image)
and \( \hat{z} \) directions with the inter-element distances equal to \( D_x \) and \( D_z \), respectively. The unit vector, \( \hat{y} \), is normal to the periodic plane. The permittivity distribution of the absorber material is denoted by \( \varepsilon(r) \), and the permeability is represented by \( \mu(r) \). The following derivation will focus on dielectric bodies for practical reasons with emphasis on the current progress of the PMM code. The periodic material distribution of the absorber structure can be explicitly described by

\[
\varepsilon(r + p) = \varepsilon(r) \quad (2.1)
\]

\[
\mu(r) = \mu_0 \quad (2.2)
\]

where \( r \) is the position vector and \( p \) is a primitive translation vector of this periodic structure with the following form

\[
p = \hat{x}mD_x + \hat{z}nD_z. \quad (2.3)
\]

where \( m \) and \( n \) are any integers. The periodic structure can be built by starting with an element and duplicating it at every position which is multiple periods away from the original position. Note that a wedge absorber as shown in Figure 2 is singly periodic and can be considered a special case of the doubly periodic structure with the material distribution along the \( \hat{z} \) direction being uniform.

As shown in Figures 1 and 2, the spherical coordinate system \( \hat{r} - \hat{\theta} - \hat{\phi} \) has the following relationship with the Cartesian one such that

\[
\hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \cos \theta + \hat{z} \sin \theta \sin \phi \quad (2.4)
\]

\[
\hat{\theta} = \hat{x} \cos \theta \cos \phi - \hat{y} \sin \theta + \hat{z} \cos \theta \sin \phi \quad (2.5)
\]

\[
\hat{\phi} = -\hat{x} \sin \phi + \hat{z} \cos \phi \quad (2.6)
\]

A plane wave incident in the angular direction \((\theta, \phi)\) with an arbitrary polarization can be written as

\[
E^i = (\hat{\theta}E^i_\theta + \hat{\phi}E^i_\phi) e^{-jk_0\delta i \cdot r} \quad (2.7)
\]
where the unit incident direction vector \( \hat{s}^i \) is expressed as

\[
\hat{s}^i = s_x^i \hat{x} + s_y^i \hat{y} + s_z^i \hat{z}
\]  
(2.8)

or

\[
\hat{s}^i = -\hat{x} \sin \theta \cos \phi - \hat{y} \cos \theta - \hat{z} \sin \theta \sin \phi
\]  
(2.9)

Note that \( \hat{s}^i = -\hat{r} \). The incident magnetic field, \( \mathbf{H}^i \) can be found by

\[
\mathbf{H}^i = \frac{1}{Z_0} \hat{s}^i \times \mathbf{E}^i
\]  
(2.10)

where \( Z_0 \) is the intrinsic impedance of free space. It is a common procedure to decompose the incident wave into the \( \theta \) and \( \phi \) polarizations. One can also refer to \( \theta \) polarization as transverse magnetic (TM) polarization and \( \phi \) polarization as transverse electric (TE). By transverse, it is referred to as being with respect to the incidence plane which contains the incident direction and the normal to the periodic plane.

![Diagram](image)

Figure 2: A finite section of an infinite wall of wedge absorber and the associated coordinate system.
2.3 Floquet’s Theorem

Floquet’s theorem is the basic principle underlying the theory of wave propagation in periodic structures [20, 21, 25]. The application of Floquet’s theorem to the doubly periodic structure, under a plane wave excitation of the form $e^{-j\kappa \hat{s} \cdot \mathbf{r}}$, ensures the total fields to be described as

$$E(r) = e^{-j\kappa \hat{s}_t \cdot \mathbf{r}} E^p(r) \quad \text{and} \quad (2.11)$$
$$H(r) = e^{-j\kappa \hat{s}_t \cdot \mathbf{r}} H^p(r) \quad (2.12)$$

where $s_t$ is the transverse-to-$y$ component of $\hat{s}$ which is expressed in Equation (2.9). The total fields can be viewed as the transversely propagating plane wave modulated by the periodic vector functions, $E^p$ and $H^p$, which have the same periodicity as the absorber structure; i.e.,

$$E^p(r + p) = E^p(r) \quad \text{and} \quad (2.13)$$
$$H^p(r + p) = H^p(r) \quad (2.14)$$

where $p$ is a primitive translation vector as defined in Equation (2.3). It is important to note that the expressions in Equations (2.11) and (2.12) can be equivalently written as

$$E(r + p) = e^{-j\kappa \hat{s}_t \cdot \mathbf{p}} E(r) \quad \text{and} \quad (2.15)$$
$$H(r + p) = e^{-j\kappa \hat{s}_t \cdot \mathbf{p}} H(r) . \quad (2.16)$$

These equivalent forms mean that the fields at two positions which are separated by a period vector, $p$, can be related by the phase factor $e^{-j\kappa \hat{s}_t \cdot \mathbf{p}}$. Therefore, under a known plane wave excitation, the knowledge about the fields over a reference area is sufficient for the whole structure.
2.4 Periodic Moment Method (PMM)

The PMM solution employs the moment method technique to solve the integral equation for the unknown induced currents inside a chosen element in the periodic structure. The integral equation is formulated based on the volume equivalence theorem and the periodic Green's function associated with the given periodic geometry.

2.4.1 Volume Equivalence Theorem

Let us consider a scattering problem in an unbounded region as shown in Figure 3 for an inhomogeneous dielectric material whose constitutive parameters are specified as $\mu_0$ and $\varepsilon(r)$. The incident fields $E^i$ are generated by the impressed source currents $(J^i, M^i)$ without the presence of the material body. The total fields $E$ can be decomposed into the sum of the incident fields $E^i$ and the scattered fields $E^s$ such that

$$ E = E^i + E^s $$

(2.17)
From the volume equivalence theorem[26], the scattered fields can be obtained from the free space radiation of the equivalent current sources $J^s$ which exist only in the material region and are given by

$$ J^s(r) = j\omega (\epsilon(r) - \epsilon_0) \mathbf{E} $$  \hspace{1cm} (2.18)

No magnetic current is induced because the material is non-magnetic. Note that the total fields inside the material body can be represented by the induced currents and the scattered fields $\mathbf{E}^s$ can be expressed in terms of induced currents via the free space dyadic Green's functions [26]. Therefore, Equation (2.17) can be formulated in terms of the unknown induced currents $J^s$, which can then be solved by using a numerical technique.

### 2.4.2 Periodic Green's Function

Since the total fields, under a plane wave excitation, have the periodicity property as shown in Equations (2.15) and (2.16) and the material distributions are also periodic, the induced currents which are expressed in Equations (2.18) have the following periodic properties:

$$ J^s(r + p) = J^s(r) e^{-jk_0 s_t \cdot p} $$  \hspace{1cm} (2.19)

where $s_t$ is the transverse-to-$y$ component of $\hat{s}_l$ and $p$ is any primitive translation vector of the underlined periodic structure. Similarly for the scattered fields $\mathbf{E}^s$, one obtains that

$$ \mathbf{E}^s(r + p) = \mathbf{E}^s(r) e^{-jk_0 \hat{s}_l \cdot p} $$  \hspace{1cm} (2.20)

The above relationship implies that, given an incident plane wave, one can relate the field quantities and induced currents at any element of a periodic structure to those
Figure 4: A doubly periodic array of electric delta current sources.

at the other element simply by a phase shift. Let us pick one of the elements and refer to it as the reference element. Accordingly, the enforcement of the continuity of the total fields only needs to be done in the reference element since all the associated fields have a common phase shift with respect to the reference element. As a result, one can focus on solving for the induced currents only at the reference element instead of the whole periodic structure, which is a great saving in memory resources. However, the associated Green’s function has to take into account the plane wave incident direction and the structure’s periodicity. As illustrated in Figure 4, the periodic Green’s function, \( \tilde{G}^p_e (r, r') \), is defined as the radiated field at \( r \) of the array of delta electric currents with the reference element located at \( r' \). The volume current for the \((m, n)\) element differs from the reference element by a phase factor.
described by Floquet's theorem such that

\[ dJ_{mn}(r) = \hat{u} \delta[r - (r' + \hat{x}mD_x + \hat{z}nD_z)]e^{-jk_0(s_zmD_x + s_znD_z)} \tag{2.21} \]

where \( \hat{u} \) is the unit polarization vector of the current array. Note that the periodic Green's function can be formulated either in the spatial domain or spectral domain. For faster convergence, the spectral domain representation is chosen and written as [3, 5, 27]

\[ G^e_p(r, r') = \frac{Z_0}{2D_xD_z} \sum_{i=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} e^{-jk_0\hat{r}_\pm \cdot (r-r')} \frac{\hat{r}_\pm \times \hat{r}_\pm \times}{r_y} \tag{2.22} \]

where

\[ k_0 = \omega \sqrt{\mu_0 \varepsilon_0}, \text{ the free space propagation constant} \tag{2.23} \]

\[ \hat{r}_\pm = \hat{x}(s_x + \frac{i\lambda_0}{D_x}) \pm \hat{y}r_y + \hat{z}(s_z + \frac{i\lambda_0}{D_z}), \text{ for } y \geq y' \tag{2.24} \]

\[ \lambda_0 = \text{ the free space wavelength}, \text{ and} \tag{2.25} \]

\[ r_y = \begin{cases} \sqrt{1 - (s_x + \frac{i\lambda_0}{D_x})^2 - (s_z + \frac{i\lambda_0}{D_z})^2}, & (s_x + \frac{i\lambda_0}{D_x})^2 + (s_z + \frac{i\lambda_0}{D_z})^2 \leq 1 \\ -j\sqrt{(s_x + \frac{i\lambda_0}{D_x})^2 + (s_z + \frac{i\lambda_0}{D_z})^2 - 1}, & \text{otherwise} \end{cases} \tag{2.26} \]

The periodic Green's functions are written as the summation of infinite number of Floquet modes whose phase dependencies are determined by \( \hat{r}_\pm \). The Floquet mode will propagate along the \( y \) direction when \( r_y \) is real but decays when \( r_y \) is imaginary. The fundamental mode which corresponds to \( i = 0 \) and \( l = 0 \) represents the specular reflection and transmission off the periodic structure. The propagation directions of this mode can be predicted by Snell's law as if the structure were a flat layered medium. When the wavelength is smaller than twice the period it becomes possible to have grating lobes which propagate along different directions than the specular mechanisms.

From Green's theorem, the scattered field can then be written in terms of the integration of the periodic Green's function and the current density distribution at
the reference element such that

\[ E^s(r) = \int_{ref} \bar{G}_e(r, r') J^s(r') dv' \]  

(2.27)

### 2.4.3 Moment Method Solution

Combining the expression of the scattered field in Equation (2.27) and the volume equivalence theorem described by Equations (2.17)–(2.18) leads to the integral equations for the induced currents as follows:

\[ \frac{J^s(r)}{j\omega(\varepsilon(r) - \varepsilon_0)} - \int_{ref} \bar{G}_e(r, r') J^s(r') dv' = E^i(r) \]  

(2.28)

Note that the integration is evaluated only at the reference element. The moment method [22] is used to solve this integral equation. In this method, the unknown quantities, \( J^s \), are expanded by a finite set of vector basis functions such that

\[ J^s(r) = \sum_{l=1}^{L} \alpha_l b_l(r) \]  

(2.29)

where \( L \) is the total number of modes representing \( J^s \); and the \( \alpha_l \)'s are the associated expansion coefficients for the induced electric currents. Note that each vector basis function \( b_n \) can span either the entire or sub-sectional domain. To deal with three-dimensional (3-D) geometries, a set of sub-sectional basis functions is preferred to better model the object's geometric properties. In common practice, one divides the reference element into a finite number of cells, say \( N \). For each cell, three vector basis functions are needed to represent an arbitrary polarization, and they have a nonzero constant value in that cell but zero elsewhere. More explicitly, the basis functions for cell \( n \) are chosen as

\[ b_n^l(r) = \begin{cases} \frac{\hat{q}_n}{V_n} & \text{if } r \text{ in cell } n, \text{ and} \\ 0 & \text{elsewhere} \end{cases} \]  

(2.30)
where \( q \) stands for \( x, y, \) or \( z \) and \( V_n \) is the volume of the \( n^{th} \) cell. Therefore, to emphasize the polarization components, the unknown currents can be rewritten as

\[
J^q(r) = \sum_{n=1}^{N} \alpha_n^q b_n^q(r) + \alpha_n^y b_n^y(r) + \alpha_n^z b_n^z(r)
\]

(2.31)

In most cases, the current expansion in Equations (2.31) is not the exact solution; and the plugging of this expansion into the integral equations will not ensure the equality. To numerically solve this problem, one needs another set of test functions to force the residue to zero. The simplest and practical approach for a general 3-D geometry is the so-called point matching method[22] which enforces the equalities to be held at the centers of the divided cells. As a result, one obtains that

\[
\frac{1}{j\omega(e(r_m) - \varepsilon_0)}(\alpha_n^x b_n^x(r_m) + \alpha_n^y b_n^y(r_m) + \alpha_n^z b_n^z(r_m))
\]

\[
- \sum_{n=1}^{N} \int_{ref} G_e(r_m, r') \cdot (\alpha_n^x b_n^x(r') + \alpha_n^y b_n^y(r') + \alpha_n^z b_n^z(r')) dv = E^i(r_m)
\]

(2.32)

for \( m = 1, 2 \ldots, N \), with \( N \) being the total number of cells in the reference element. Therefore, the integral equations are transformed into a system of simultaneous algebraic equations for the coefficients. Note that there are \( 3 \times N \) equations since three polarizations have to be equated. After solving the algebraic equations, the induced currents are known and then can be used to evaluate the fields at any position via the periodic Green's functions. One should note that, for the reflection and transmission coefficients, only the fundamental mode and grating lobes are needed because the evanescent modes do not contribute to the far field.

As demonstrated by Yang et.al [3], the PMM program requires enormous memory and computation resources such that it usually has to be run on a modern high speed workstation or a supercomputer. However, this numerically exact solution is a valuable tool to ensure a proper absorber design and examine some other approximate methods, such as the TLA analysis.
2.4.4 Hybrid Approach

To ease the tremendous memory and computation requirement involving a PMM calculation, a hybrid approach [3] can be used to calculate the scattering effect of adding various flat base layers to a periodic structure. As shown in Figure 5, the reflection and transmission coefficients for the front and back of the tapered section are first calculated by PMM. One may readily calculate the coefficients for the front of the added base multilayer by a multilayer program [24]. Both results can then be combined to give the total reflection and transmission coefficients such that

\[
R_{tot} = R_1 + \frac{T_1 T_2 R_3 e^{-2 j k_0 d \cos \theta}}{1 - R_2 R_3 e^{-2 j k_0 d \cos \theta}}, \quad \text{and} \quad (2.33)
\]

\[
T_{tot} = \frac{T_1 T_3}{1 - R_2 R_3 e^{-2 j k_0 d \cos \theta}}. \quad (2.34)
\]

The above formulas are valid only for the cases where no grating lobes occur. This limitation is not critical since the absorber design is usually performed for the low frequency region without any grating lobe mechanisms. The PMM results shown in later chapters will be obtained by either the pure PMM calculation or the hybrid method whichever is more appropriate.

2.5 TLA analysis

The TLA analysis is derived in this section to approximate the complex scattering problem to a transmission line problem which is nonuniform only in the direction normal to the periodic plane. Since the derivation is quite involved, the major steps are outlined as follows:

- Demonstrate that the averaged periodic vector functions are associated with the specular reflection and transmission mechanisms of a periodic structure.
Figure 5: The hybrid approach for calculating absorber scattering.
• Formulate the original problem in terms of the averaged fields, which is detailed in Appendix A.

• Introduce the effective material concept to homogenize the material distribution in the periodic plane and then simplify the averaged field formulation.

• A set of 4 coupled differential equations is derived to fully describe the homogenized problem for any angle of incidence and arbitrary polarization. The details are shown in Appendix C

• The coupled differential equations can be solved by a finite difference solution as described in Appendix D. For some decoupled cases, the homogenized problem can be modeled as a stratified multilayer medium of the effective material properties.

Note that, for the sake of generality, magnetic properties are included in the derivation described in Appendices A, C and D; while, only dielectric properties are described in this Chapter for practical reasons.

2.5.1 Averaged Fields and Specular Mechanisms

Floquet’s theorem states that the fields associated with a transversely periodic structure can be written as a transversely propagating plane wave modulated by periodic functions of the structure’s periodicity such that

\[ E(r) = e^{-j\mathbf{k}_0 \mathbf{S}_r \cdot \mathbf{r}} \mathbf{E}^p(r), \quad \text{and} \]
\[ \mathbf{H}(r) = e^{-j\mathbf{k}_0 \mathbf{S}_r \cdot \mathbf{r}} \mathbf{H}^p(r). \]

Let us consider the averaging operation defined as

\[ (f(x, y, z))_{av} = \frac{1}{D_x D_z} \int_{C_x}^{C_x+D_x} \int_{C_z}^{C_z+D_z} f(x, y, z) dxdz \]
where \( C_x \) and \( C_z \) are any constants and \( D_x \) and \( D_z \) are the periods in the \( x \) and \( z \) directions, respectively. For a periodic function \( f(x, y, z) \), it can be represented as a Fourier series such that

\[
f(x, y, z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f_{mn}(y) e^{-j2\pi \left( \frac{m}{D_x} + \frac{n}{D_z} \right)}.
\]

(2.38)

One should note that an averaging operation of a periodic function over a period area retains only the constant term, i.e.

\[
(f(x, y, z))_{av} = f_{00}(y).
\]

(2.39)

Note that the averaged term is a function of \( y \).

By carefully examining the periodic Green's functions in Equations (2.22), one can find the average of the periodic functions \( \mathbf{E}^p(r) \) in the following form:

\[
\begin{align*}
(\mathbf{E}^p)_{av}(y) &= \begin{cases} 
\mathbf{E}_> e^{-jko|s_y|^2u} + \mathbf{E}_< e^{-jko|s_y|^2u}, & y \geq y_{\max} \\
\mathbf{E}_+^+(y)e^{-jko|s_y|^2u} + \mathbf{E}_-^-(y)e^{jko|s_y|^2u} + \mathbf{E}_+^i e^{-jko|s_y|^2u}, & y_{\min} \leq y \leq y_{\max} \\
\mathbf{E}_< e^{jko|s_y|^2u} + \mathbf{E}_+^i e^{-jko|s_y|^2u}, & y \leq y_{\min}
\end{cases}
\end{align*}
\]

(2.40)

where \( y_{\min} \) and \( y_{\max} \) are the lower and upper bounds of the material body in the \( y \) direction. Similar results can be found for the \( \mathbf{H} \) fields. The explicit expressions for the averaged fields are not needed at this point. Note that \( \mathbf{E}_> \) and \( \mathbf{E}_< \) are two constant vectors. For an incident wave from \( y = \infty \) \( (s_y < 0) \) \( \mathbf{E}_> \) is associated with the reflected field and \( \mathbf{E}_< \) is with the transmitted field. Inside the material region the averaged field can be decomposed into the positively and negatively going waves. More importantly, the averaged fields outside the material region correspond to the desired reflected and transmitted fields in the specular direction. Such observation implies that, as far as the specular far field mechanisms are concerned, one does not need to find the complicated field behavior but only the averaged fields of the periodic functions \( \mathbf{E}^p \) and \( \mathbf{H}^p \).
2.5.2 Formulation in Terms of $E^p$ and $H^p$

Let us focus on the formulation of $E^p$ and $H^p$ instead of $E$ and $H$. Plugging the Floquet expressions (2.35) and (2.36) into the source free Maxwell Equations, one obtains that

$$\nabla \times (e^{-jk_0S_tr}E^p(r)) = -j\omega \mu_0 e^{-jk_0S_tr}H^p(r), \text{ and}$$

$$\nabla \times (e^{-jk_0S_tr}H^p(r)) = -j\omega \epsilon(r) e^{-jk_0S_tr}E^p(r) \text{ (2.42)}$$

It is important to note that the periodic functions $E^p$ and $H^p$ approach the static solutions as the frequency decreases to zero. After some manipulation, which is detailed in Appendix A, the scattering problem for dielectric bodies can be formulated in terms of the averaged functions $(E^p)_{av}$ and $(H^p)_{av}$ such that

$$-\frac{d}{dy}(E^p_{tav} - jk_0\hat{y} \times s_t \times (E^p_y)_{av} + j\omega \mu_0 \hat{y} \times (H^p_t)_{av}) = 0$$

$$-\frac{d}{dy}(H^p_{tav} - jk_0\hat{y} \times s_t \times (H^p_y)_{av} - j\omega \hat{y} \times (E^p_t)_{av}) = 0$$

$$-jk_0s_t \times (E^p_{tav})_{av} + j\omega \mu_0 (H^p_{yav})_{av} = 0$$

$$-jk_0s_t \times (H^p_{tav})_{av} + j\omega (eE^p)_{av} = 0$$

Unfortunately, these equations are numerically no easier than the original expressions because the detailed field behavior has to be known to evaluate the terms such as $(eE^p)_{av}$. In other words, the knowledge of the evanescent waves which are apparently smoothed out in the averaging process is still needed in solving Equations (2.43)–(2.46). This conclusion is not unexpected since the induced currents in material bodies depend on the cells’ mutual interaction in which evanescent modes can play a significant role. However, this formulation provides a new perspective to look at the mechanisms associated with the reflection and transmission of a periodic structure. At low frequencies where the wavelength is much larger than the structure’s periodicity, some approximations can be made to decouple the material
properties and the desired averaged fields and the original scattering problem may then be transformed to an easier one.

2.5.3 Effective Material

As discussed in the last section, the coupling between the material properties and the periodic functions prevents one from formulating the averaged fields in a simple form. When the wavelength of the incident wave is much larger than the periods, as depicted in Figure 6, a periodically distributed material can be approximated by a uniform effective material. For a periodic structure which is uniform along the $y$ direction, the effective permittivity tensor $\bar{\varepsilon}$ is defined by

$$
(\varepsilon(x, z)E(x, z))_{av} = \bar{\varepsilon} \ (E)_{av} .
$$

The effective tensor is a function of material properties, material geometric distribution and the frequency. This definition is similar to the one used for $\varepsilon$ and $\mu$ to represent the material's macroscopic properties which are the averaged effects of the polarization and magnetization of molecules or atoms. The ratio of wavelength to the atoms' distance is around $10^3$ for optical waves. The effective material formulas are usually found for the static case [7, 8] and extended to low frequencies when the
wavelength is significantly larger than the period. Recall that the periodic functions \( E^P \) and \( H^P \) approach the field solutions for the static case in the low frequency limit. Therefore, the effective material concept in (2.47) can be applied to \( E^P \) such that

\[
(e(x,z)E^P(x,z))_{av} = \bar{\varepsilon} \cdot (E^P)_{av}.
\] (2.48)

For a slab structure as shown in 7 with thickness \( t \) being infinite, the effective constant \( \bar{\varepsilon} \) for the static case can be derived by noting that the tangential components of \( E \) and the normal components of \( \varepsilon E \) have to be continuous. With the slab's width equal to \( d \), the effective constants for a slab structure can be described exactly by

\[
\bar{\varepsilon} = \hat{x}^x \varepsilon^x + \hat{y}^y \varepsilon^y + \hat{z}^z \varepsilon^z
\] (2.49)

where

\[
\varepsilon^x = \frac{1}{((1-g)\varepsilon_0^{-1} + g\varepsilon^{-1})^{-1}}
\] (2.50)

\[
\varepsilon^y = (1-g)\varepsilon_0 + g\varepsilon
\] (2.51)

\[
\varepsilon^z = \varepsilon_y.
\] (2.52)

Note that \( g \equiv \frac{d}{D_x} \) is the length fraction of the absorber material with respect to the period. Note that \( g \) also means the volume fraction for such a singly periodic structure. This slab structure can be used to model the taper section of wedge absorbers.

For a square rod structure as shown in Figure 8 with \( D_x = D_z \) and the thickness \( t \) being infinite, the effective material constant in the \( y \) direction can be expressed in the similar form as the slab case such that

\[
\varepsilon^e_y = (1 - g^2)\varepsilon_0 + g^2 \varepsilon
\] (2.53)
Figure 7: A singly periodic array of material rectangular cylinder.

where $g$ again is the length fraction while $g^2$ is the volume fraction for a doubly periodic structure. In the transverse direction, the effective constants are more involved in that the boundary conditions cannot be easily applied for the material-to-air interfaces. It was found by Kuester and Holloway [28] that, among the available variational bounds, the one proposed by Hashin and Shtrikman (HS) [8] best agrees with some available numerical results in the literature. The HS formula for the transverse effective constants are given by [8]

$$
\varepsilon_t^e = \varepsilon_0 [1 + g^2 \frac{2(\varepsilon - \varepsilon_0)}{(1 + g^2)\varepsilon_0 + (1 - g^2)\varepsilon}] .
$$

This square rod structure can be used to model pyramidal absorbers which have equal periods for both periodic directions in most cases.

As verified by the PMM analysis in Appendix B, these effective material formulas provide a good approximation to decompose the averaged field and the dielectric constant as long as the wavelength is sufficiently larger than the period.
2.5.4 TLA formulation

In order to apply the effective material concept to pyramidal and wedge structures which have a tapered distribution along the y direction, one needs to assume that the effective material constant at any y position depends only on, $g(y)$, the material ratio at that specific height. As a result, the effective material expression in (2.48) can be generalized to the following form:

$$(\epsilon(x,y,z)E^P(x,y,z))_{av} = \bar{\epsilon} (g(y)) (E^P)_{av}(y). \quad (2.55)$$

With this assumption, the formulation in terms of $E^P$ and $H^P$ in Equations (2.43)-(2.46) can be further simplified as

$$-\frac{d}{dy}(E^P)_{av} - jk_0\hat{y} \times (s_t \times (E^P)_{av}) + j\omega \mu_0 \hat{y} \times (H^P)_{av} = 0 \quad (2.56)$$

$$-\frac{d}{dy}(H^P)_{av} - jk_0\hat{y} \times (s_t \times (H^P)_{av}) - j\omega \hat{y} \times \bar{\epsilon} (E^P)_{av} = 0 \quad (2.57)$$

$$-jk_0s_t \times (E^P)_{av} + j\omega \mu_0 (H^P)_{av} = 0 \quad (2.58)$$

$$-jk_0s_t \times (H^P)_{av} - j\omega \epsilon (E^P)_{av} = 0 \quad (2.59)$$
Since Equations (2.58) and (2.59) provide the expressions for the $y$ components in terms of the transverse ones, the scattering problem can be formulated in terms of the transverse components such that

$$\frac{d}{dy}(\mathbf{E}_t^P)_{av} + j\frac{\omega \mu_0 \varepsilon_0}{\varepsilon_y^0} \times (s_t \times (\mathbf{H}_t^P)_{av}) + j\omega \mu_0 \mathbf{y} \times (\mathbf{H}_t^P)_{av} = 0$$ \hspace{0.5cm} (2.60)

and

$$\frac{d}{dy}(\mathbf{H}_t^P)_{av} - j\omega \varepsilon \mathbf{y} \times (s_t \times (\mathbf{E}_t^P)_{av}) - j\omega \mathbf{y} \times \varepsilon \varepsilon_t^0 (\mathbf{E}_t^P)_{av} = 0$$ \hspace{0.5cm} (2.61)

As detailed in the Appendix C, the above formulation can be written in a form where both the TM ($\hat{\theta}$) and TE ($\hat{\phi}$) components are clearly expressed. For this purpose, the transverse fields are denoted as

$$(\mathbf{E}_t^P)_{av} = \hat{\rho} \hat{E}_\rho + \hat{\phi} \hat{E}_\phi$$ \hspace{0.5cm} (2.62)

and

$$(\mathbf{H}_t^P)_{av} = \hat{\rho} \hat{H}_\rho + \hat{\phi} \hat{H}_\phi$$ \hspace{0.5cm} (2.63)

where the unit vectors, $\hat{\rho}$ and $\hat{\phi}$, are defined as

$$\hat{\rho} = -\hat{x} \sin \phi + \hat{z} \cos \phi$$ \hspace{0.5cm} (2.64)

and

$$\hat{\phi} = \hat{x} \cos \phi + \hat{z} \sin \phi$$ \hspace{0.5cm} (2.65)

It is easy to show that

$$s_t = -\hat{\rho} \sin \theta.$$ \hspace{0.5cm} (2.66)

Therefore, $\hat{\rho}$ is in the plane of incidence. ($\hat{E}_\rho, \hat{H}_\phi$) are associated with the TM polarization; and ($\hat{E}_\phi, \hat{H}_\rho$) with the TE case. As derived in Appendix C, by plugging the above notations into Equations (2.60) and (2.61), one can describe the scattering problem by a set of coupled first order differential equations such that

$$\frac{d\hat{E}_\rho}{dy} = j\omega \varepsilon \varepsilon_t (y) \hat{H}_\phi(y) + j\omega \mu (y) \hat{H}_\rho(y)$$ \hspace{0.5cm} (2.67)

and

$$\frac{d\hat{H}_\phi}{dy} = j\omega \varepsilon \varepsilon_t (y) \hat{E}_\rho(y) + j\omega \mu (y) \hat{E}_\phi(y)$$ \hspace{0.5cm} (2.68)
\[ \frac{d\tilde{E}_\phi}{dy} = -j\omega\mu_{te}(y)\tilde{H}_\rho(y) - j\omega\mu_c(y)\tilde{H}_\phi(y) \text{ , and} \]
\[ \frac{d\tilde{H}_\rho}{dy} = -j\omega\epsilon_{te}(y)\tilde{E}_\phi(y) - j\omega\epsilon_c(y)\tilde{E}_\rho(y) \]

where

\[ \mu_{tm}(y) = \mu_o - \frac{\mu_o\epsilon_o\sin^2 \theta}{\epsilon_r(y)} \]
\[ \epsilon_{tm}(y) = \epsilon_r(y)\cos^2 \phi + \epsilon_i(y)\sin^2 \phi \]
\[ \mu_{te}(y) = \mu_o \]
\[ \epsilon_{te}(y) = \epsilon_r(y)\sin^2 \phi + \epsilon_i(y)\cos^2 \phi - \epsilon_o\sin^2 \theta \]
\[ \mu_c(y) = 0 \text{ , and} \]
\[ \epsilon_c(y) = (\epsilon_r(y) - \epsilon_i(y))\sin \phi \cos \phi . \]

Note that \( \epsilon_r(y) \)'s are \( \epsilon_o \) outside material region and are the effective constants given in Section 2.5.3 inside the material region. The TM and TE cases are coupled together via the coupling coefficient, \( \epsilon_c \). As a result, for either a TM or TE wave incidence, both co- and cross-polarized signals are generally scattered from the periodic structure. The coupled differential equations are solved numerically by the finite difference method as outlined in Appendix D for both the reflection and transmission coefficients. This solution represents the most general case for the TLA method in that the incident angle and polarization can be arbitrary.

If the coupling parameter, \( \epsilon_c \), is zero, the formulation can be decomposed into two independent problems; namely, the TM and TE cases. Equations (2.67) and (2.68) are associated with the TM case, and Equations (2.69) and (2.70) for TE case. These expressions are in fact the governing equations for propagation in a nonuniform transmission line or normal incidence scattering on a slab of nonuniform material in which the material properties have been transformed to \( \epsilon_{tm} \) or \( \epsilon_{te} \). In this case, the derived problem can be modeled as a stratified multilayer medium, as
illustrated in Figure 9, which can then be solved for both the reflection and transmission coefficients [24]. Let us examine when the scattering problem can be decoupled. The coupling coefficient described in Equation (2.76) is associated with the product of the difference between $e_r^e$ and $e_t^e$ and trigonometric functions of $\phi$. It is obvious that $e_c$ is zero if the plane wave is incident in the principal planes, where $\phi$ is an integral multiple of $90^\circ$. For a pyramidal structure with square cross-section, $e_r^e = e_t^e$ due to the $x - z$ symmetry, the TM and TE cases can always be decoupled for a pyramidal structure. On the other hand, $e_r^e \neq e_t^e$ for a wedge structure and thus the TM and TE polarizations are coupled for a wedge case except for the principal incidence planes.

![Figure 9: Transmission line approximation.](image)

2.6 Summary

Two analytical methods, namely the PMM and TLA, have been summarized and developed, respectively. The PMM method is a numerically exact solution but
requires tremendous computation resources. The scattering from a periodic structure is greatly simplified by the TLA method which homogenizes the periodic material distribution by the static effective material properties. The TLA formulation developed in this chapter represents the most general case in which the two orthogonal polarizations are coupled and cross-polarized fields can be evaluated. Obviously the TLA analysis would start to fail when frequency becomes so high that grating lobe mechanisms occur. In many cases, absorber designs become more critical in the low frequency region where absorber electrical dimension would be too small to effectively absorb incident energy. If the errors associated with the TLA approximation are insignificant, the TLA method should provide a very effective tool for designing absorber structures. The TLA limitations will be examined via comparison with the PMM method in the next Chapter.
CHAPTER III
Comparison of the PMM and TLA Analyses

3.1 Introduction

The accuracy and limitations of the TLA method are examined by the PMM analysis in this chapter. Both methods have been implemented for wedge and pyramidal absorbers with various taper geometries which, as illustrated in Figure 10, can have straight, curved, serrated and multi-layer structures. The taper section is made of dielectric absorber material for practical reasons but can be backed by stratified layers of general material properties, such as a ferrite or a metal layer. The TLA and

![Diagram of taper structures](image)

Figure 10: Various taper structures implemented in the PMM and TLA analyses.
PMM comparison will focus on the taper section where the TLA method transforms the original problem to an effective homogenized medium. The scattering properties associated with absorber structures will be discussed based on both the frequency and time domain responses. Note that all the time domain responses shown in this chapter will be obtained by a Fourier transform with a band-pass Kaiser Bessel window [29].

Since the TLA method considers only the lowest propagating mode, this method should be limited by the interaction from higher order mechanisms. Basically, these mode coupling effects can be characterized by two physical mechanisms; i.e., the free space grating lobe and the guided wave interaction. Their basic properties are first discussed to better explain the discrepancy observed in the calculated examples. Both wedge and pyramidal absorbers are used to evaluate the TLA limitation with respect to absorber geometry, material properties, polarization and angle of incidence. To control these variables for clear comparison, the dielectric constant is fixed throughout the frequency band in most calculated cases. The material effect will be discussed more thoroughly in a later section. In the following results, curved tapers are not evaluated since they will be addressed in Chapter 4 where the optimization of curved taper geometry is the main topic.

3.2 Basic Properties of Higher Order Mechanisms

The basic scattering properties of higher order mechanisms associated with a periodic dielectric structure are discussed in this section. Based on these discussions, the comparison between the TLA and PMM methods in the later sections can be better understood.
As shown in Chapter 2, the scattered fields of a periodic structure can be written as an integral of the periodic Green's function, which is an infinite sum of space harmonics in free space. In evaluating the impedance matrix in a moment method calculation, the contribution from the propagating modes and a finite number of significant cutoff modes must be included to account for mode interaction. Thus, all the Floquet modes are coupled together and not a single mode can fully represent the complicated scattering mechanism. As an approximation, the TLA method retains the lowest propagating mode and assumes the coupling from all the other modes is negligible. In the low frequency limit, only the fundamental mode is above cutoff and the coupling from the cutoff modes may be insignificant due to the exponential field decay. As a result, the TLA method should provide a good approximation at low frequencies but starts to fail as the frequency increases. When the frequency is near the cutoff frequency of the next mode, the strong interaction between the two lowest modes may occur, and the TLA approximation can become inaccurate.

3.2.1 Free Space Grating Lobe

One of the clear indications of the higher order interaction is the free space grating lobe. The onset of these modes can be determined by the periodicity and the incident direction but is independent of material properties. Recall the formulation of the periodic Green's function in Equation (2.22). With the incident \( k \) vector equal to \( k_0(\hat{x}s_x + \hat{y}s_y + \hat{z}s_z) \), the grating lobe can be excited in a doubly periodic structure if the following inequality is held:

\[
(s_x + \frac{i\lambda_0}{D_x})^2 + (s_z + \frac{l\lambda_0}{D_z})^2 \leq 1.
\] (3.1)

The onset of the \((i, l)\) propagating mode occurs when the above equality is held. The propagating mode next to the fundamental mode \((i = l = 0)\) is the case with either
$i$ or $l$ equal to 0. Consequently, one can focus on the case with $l = 0$ to find the limitation of the TLA method, and the result is also applicable to a singly periodic case. In this case, the onset wavelength for the first grating lobe is determined by

$$(s_x \pm \frac{\lambda_0}{D_x})^2 + s_z^2 = 1.$$  (3.2)

This equation represents two unit circles with origins at $(\pm \frac{\lambda_0}{D_x}, 0)$ on the $s_x$ and $s_z$ plane. The trajectory is shown in Figure 11 for the case with $\frac{\lambda_0}{D_x} = 1.5$. The central unit circle is related to the incident wave and the far field scattered waves. An incident wave with its $(s_x, s_z)$ value lying in the shaded area will excite the grating lobe mechanism. From the graphic display, one can easily observe that there is no free space grating lobe if $\frac{\lambda_0}{D_x} > 2$. Given a $(s_x, s_z)$ value, the onset (cutoff) of higher mode occurs when the $i = 1$ (or $-1$) circle just passes that point. For convenience, let us focus on the incidence at $\phi = 0$; i.e., $s_z = 0$. The onset wavelength can then

![Figure 11: Grating lobe circles with $\frac{\lambda_0}{D_x} = 1.5$.](image)
be simplified from Equation (3.2) as

$$\frac{\lambda_o}{D_x} = 1 + s_x, \text{ for } 0 \leq s_x \leq 1.$$  \hspace{1cm} (3.3)

At normal incidence \((s_x = 0)\), the onset grating lobe occurs at \(\lambda_o = D_x\) or \(f_o = \frac{c}{2D_x}\); at grazing incidence \((s_x = 1)\), it occurs at \(\lambda_o = 2D_x\) or \(f_o = \frac{c}{2D_x}\). As \(s_x\) is closer to 0, \(\frac{\lambda_o}{D_x}\) becomes smaller or the onset frequency becomes larger. In other words, the grating lobe starts at lower frequencies at wider incident angles. This indicates that the TLA calculation should have better performance at normal incidence than oblique incidence in terms of grating lobe interaction.

The grating lobe mechanism does not affect the fundamental mode abruptly without any transition. In fact, the coupling between these two modes should exist for all frequencies but become strong as the frequency approaches the grating lobe onset frequency. Around this frequency, the grating lobe basically propagates on the periodic plane along the direction of periodic modulation. Although this grazing scattering does not propagate in the specular reflection direction, it may strongly interact with the fundamental mode by a re-radiation process [30], which is illustrated in Figure 12 for a wedge geometry. To explain this phenomenon, one can multiply \(k_o(\frac{2\pi}{\lambda_o})\) on the both sides of Equation (3.3) and arrange it as

$$- k_o + \frac{2\pi}{D_x} = k_o s_x .$$  \hspace{1cm} (3.4)

The left hand side is the first modulated propagation phase constant (wavenumber) of the grazing scattered wave. The modulation parameter \(\frac{2\pi}{D_x}\) comes from the periodic distribution of dielectric properties. The right hand side is the transverse wavenumber of the incident wave in the \(z\) direction. The above equality states that the modulated wave satisfies the phase matching condition. Consequently, the scattered wave along the periodic plane from each element will be coherent in the specular
direction and a significant re-radiation should occur. By such a re-radiation process, the grating lobe can strongly interact with the fundamental mode at its onset.

As illustrated in the Figure 12 for a wedge configuration, one of the possible grating lobe re-radiations can occur as a form of the interaction of two edges via the edge diffracted fields. Relative to the specularly edge diffracted wave, the time delay of the re-radiated wave between two adjacent edges can be found as

$$t_{\text{delay}} = (1 + \sin \theta)D_x/c,$$  \hspace{1cm} (3.5)

where $c$ is the light speed. Since the re-radiation can happen between any two edges along the grazing propagation, there should be infinitely many re-radiated waves with a constant time spacing of $t_{\text{delay}}$ in time domain. The re-radiated fields should decay with the increase of order since the larger order re-radiated waves

![Figure 12: Interaction between the grating lobe and fundamental mode occurs as a re-radiation process.](image)
experience more edge disturbance. One should note that, the time delay $t_{\text{delay}}$ can become larger if the re-radiation happens inside the taper section where the wave propagates with less speed. This time domain phenomenon will be illustrated in a later calculation example.

### 3.2.2 Guided Wave Interaction

In the previous discussion, the onset of the grating lobe interaction was interpreted as the re-radiation of the traveling wave which propagates along the periodic plane. The laterally propagating wave acts as a free space plane wave with its wavenumber modulated by the periodicity. The re-radiation becomes significant when the phase matching condition is satisfied.

However, the effect of material property has not been taken into account. Let us consider the dielectric layer with alternating bars as shown in Figure 13. The relative dielectric properties are denoted as $\varepsilon_1$ and $\varepsilon_2$. The length fractions are denoted as $g_1$ and $g_2$ with $g_1 + g_2 = 1$. Bertoni et al. [30] studied the use of this type of periodic dielectric layer as a frequency selective surface at millimeter wave frequencies. They found that, for the $\phi\phi$ polarization ($E||\hat{z}$), guided waves can be induced and interact with the lowest propagating mode. The re-radiation of the guided wave can strongly couple to the specularly reflected wave if it satisfies the phase matching condition.

By replacing the free space wavenumber, $k_0$, with the wavenumber of a guided wave, $k_x$, the phase matching condition in Equation (3.4) can be written as

$$-k_x + \frac{2\pi}{D_x} = k_0 s_x .$$  \hspace{1cm} (3.6)

For this $\phi\phi$ case, the $k_x$ values can be estimated as those in a layer of uniform dielectric constant equal to the average value in the periodic layer [30]. Recall that
this average permittivity is exactly the same as the static effective dielectric constant for this polarization, as described in Equation (2.52)

The guided wave interaction is verified here by a calculated example with $\epsilon_1 = 2.33, d_1 = 0.75cm$ and $\epsilon_2 = 1, d_2 = 0.25cm$. The period ($D_x$) of the periodic layer and the thickness ($h$) are chosen to be $1cm$ and $1.713cm$, respectively. Note that the height is chosen to have significant guided wave interaction. The PMM and TLA calculated results for $45^\circ$ angle of incidence are shown in Figure 14. These two calculated results agree very well in the low frequency portion. However, the TLA method cannot predict the higher order mechanisms which are clearly seen above $14.5 \text{ GHz}$. Among these interactive terms, the free space grating lobe in this configuration at $\frac{\lambda_0}{D_x} = 1 + \sin 45^\circ = 0.59$ or at $17.4 \text{ GHz}$. The two spikes, which

![Figure 13: A periodic dielectric layer with two alternating bars.](image)

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occur before the grating lobe mechanism, indicate an interaction of the fundamental mode and the guided waves. The frequencies of the spikes can be predicted by the phase matching condition in Equation (3.6), which is solved graphically as follows [30]. Based on the characteristic wave equation for a dielectric slab [26], a computer program was written to find the possible $k_x$ values given a certain $k_0$ value. With $\varepsilon_r = 2(2.33 \times \frac{3}{4} + 1 \times \frac{1}{2})$, the propagation constant of guided wave is plotted horizontally in Figure 15 versus the free space wavenumber, which is plotted vertically. Note that both $k_x$ and $k_0$ are normalized by multiplication of $\frac{\omega}{2\pi}10^9$. The guided modes start at $k_0 = \frac{n\pi}{h\sqrt{\varepsilon_r - 1}}$ from the $45^\circ$ line for $n = 0, 1, 2, \ldots$. To graphically solve the phase matching equation, the modulated wavenumber, $-k_x + \frac{2\pi}{D_z}$, is plotted versus the free space wavenumber in Figure 16. The dashed line corresponds

![Reflection Coeff. in dB](image)

**Figure 14**: Comparison of the PMM and TLA calculated results for a periodic dielectric layer. TE($\phi\phi$) case. $\phi = 180^\circ$, $\theta = 45^\circ$, $\epsilon_1 = 2.33$, $d_1 = 0.75cm$, $\epsilon_2 = 1$, $d_2 = 0.25cm$, $D_z = 1cm$, $h = 1.713cm$. 

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to $-k_0 + \frac{2\pi}{D_x}$, which is related to the free space grating lobe. The dot-dash lines correspond to the transverse wavenumber, $k_0 s_x$, along the $\hat{a}$ direction for several incident angles. The intersecting points indicate where the guided wave or grating lobe can effectively couple to the reflected wave based on the phase matching condition. In this case, the guided wave interaction for the 45° case happens at about 14.6 GHz and 16 GHz, and free space grating lobe at 17.4 GHz. One should note that these are very close to the resonance positions found in the reflection coefficient shown in Figure 14.

Figure 15: Normalized dispersion curves for the first four guided modes in a uniform layer of relative dielectric constant, $\varepsilon_r = 2$. Layer thickness is $h = 1.713cm$. 

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Figure 16: Normalized transverse wavenumbers of the incident plane wave at several incident angles and dispersion curves for the first space harmonics of the first four guided modes. $\varepsilon_r = 2$, $h = 1.713 \text{cm}$, $D_x = 1 \text{cm}$. 
Some observations can be drawn from the above results. From Figure 16, the guided wave interaction occurs at lower frequencies as the incident plane wave becomes closer to the grazing direction. The guided wave interaction always happens before the free space grating lobe since the relative dielectric constant of the dielectric slab is greater than 1. As mentioned in the explanation for Figure 15, the guided modes start at $k_0 = \frac{n\pi}{h\sqrt{\varepsilon_r - 1}}$ from the 45° line. This formula indicates that higher dielectric constant can initiate the guided wave interaction at a lower frequency, which may affect the application of the TLA method for a highly doped material.

The guided wave interaction can be greatly reduced if the dielectric material is lossy. The theoretical calculation is repeated for the previous dielectric layer with $\varepsilon_1 = 2.33$ changed to $\varepsilon_r = 2.33 - j0.2$. The calculated results are shown in Figure 17. As compared with Figure 14, the guided wave interaction is significantly damped by the dielectric loss. It is common that absorber materials are moderately or highly lossy. Therefore, the guided wave interaction may not be as strong as the lossless case, at least for the taper section close to base area. However, as discussed above, the guided wave tends to affect the fundamental mode at lower frequencies for higher permittivity materials. It will be shown through calculation examples that the higher order interaction degrades the TLA performance for large relative dielectric constant materials.

Similar high order mechanisms can be found by PMM calculations for the $\theta \theta$ case in a singly periodic structure and for both polarizations in a doubly periodic structure. However, this issue is not pursued further because the above $\phi \phi$ case is a typical case and has clearly indicated those mechanisms. In summary, the periodic modulation of the dielectric property can limit the validity of the TLA to a lower frequency than predicted by the onset of the grating lobe. For higher dielectric constants, the guided wave interaction can occur at lower frequencies, which means
the TLA method is more accurate for applications to materials with lower dielectric constant values.

3.3 Wedge Absorbers

Due to the asymmetry in the periodic plane, the scattered fields from wedge absorbers are polarization dependent. Cross-polarized fields can be induced by an oblique plane wave incidence which does not lie in the $x-y$ or $y-z$ plane. Both frequency and time domain responses will be used to analyze the associated scattering mechanisms. The phase reference for the time domain processing will be placed at the wedge valley.
3.3.1 Straight Taper

A 40" metal-backed straight wedge with an 8" period and a 10" base is used as the first test sample. For comparison purposes, some minor changes in geometry will be made as indicated. The relative dielectric constant of an absorber material is usually frequency dependent. A typical value \( \varepsilon_r = 2 - j1.5 \) is chosen and fixed to emphasize the effect of the absorber geometry.

Normal Incidence

At normal incidence, the incident electric field is parallel to the edge lines for the TE or \( \phi\phi \) polarization; and, is perpendicular to the edges for the TM or \( \theta\theta \) polarization. The frequency and time domain responses calculated by PMM and TLA methods are shown in Figure 18 for the \( \phi\phi \) case. The agreement between these two methods is very good except for the high frequency portion. Although the onset of the grating lobe occurs at \( \lambda_0 = D_\pi \) or at 1500 MHz, the effect of higher order mechanisms becomes significant at around 750 MHz. In the time domain response, the peak at -5 ns is identified as the edge scattered field, which is usually the dominant mechanism when the polarization is parallel to the edge. The second peak is a combination of the scattered fields from the wedge valley and the metal backing. From the low frequency ripple, one can conclude that the valley scattering is more significant at lower frequencies where the incident wave can penetrate the absorber more effectively. The smaller ripple amplitude in the PMM time domain results indicates that the re-radiation mechanism for this scattering case is not very significant in this frequency range evaluated.

The calculated frequency and time domain results for the \( \theta\theta \) polarization are shown in Figure 19. The agreement is very good throughout the frequency band except for some insignificant higher order interactions. In the time domain, both
Figure 18: Comparison of the PMM and TLA results at normal incidence for a 40" metal-backed straight wedge wall with $D_x = 8''$ and $\varepsilon_r = 2 - j1.5$. $\phi\phi$ polarization.
Figure 19: Comparison of the PMM and TLA results at normal incidence for a 40" metal-backed straight wedge wall with $D_w = 8"$ and $\varepsilon_r = 2 - j1.5$. $\theta\theta$ polarization.
calculated results agree very well with respect to the major mechanisms although, the TLA result fails to predict the insignificant coupling from the higher order modes, which can be seen in the PMM results around 10–15 ns. As compared with the time domain response in Figure 18, the average edge scattered fields are about 10 dB weaker than the φφ polarization. Based on the effective material concept, the θθ case has a better impedance transition from free space to wedge section. On the other hand, the base scattering becomes more significant because more energy penetrates and is seen by the valley. Such a polarization characteristic is found to be generally true for wedge absorbers at least in the low frequency limit.

Oblique Incidence in the Principal Planes

To better illustrate higher order mechanisms, let us examine a wide angle case with the angle of incidence equal to (θ = 45°, φ = 0°). The calculated results for both φφ and θθ polarizations are shown in Figure 20 and 21, respectively. As can be easily seen, the PMM results include more mechanisms than the TLA results which only estimates the fundamental specular term. The onset of the grating lobe occurs when λ₀ is equal to 13.7\(\text{m}(≈ 1.71 \times 8\text{m})\) or at about 860 MHz. In fact, the effects of higher order terms are clearly seen before the onset frequency as a dip in the frequency response. This is because the coupling of the fundamental mode and the grating lobe become strong as the frequency approaches the cutoff(onset) frequency. The other possible explanation is the guided wave interaction which was shown to happen before the free space grating lobe. These re-radiated waves repeatedly radiate into the specular direction for every certain time interval. This phenomenon is shown as the decaying ripples in the time domain responses of both Figures 20 and 21. Some scattering mechanisms apparently come from the area between the edge and valley. The time difference between the small ripple peaks is about 1.3 ns
Figure 20: Comparison of the PMM and TLA results at ($\theta = 45^\circ, \phi = 0^\circ$) incidence for a 40" metal-backed straight wedge wall with $D_x = 8"$ and $\varepsilon_r = 2 - j1.5$. $\phi\phi$ polarization.
Figure 21: Comparison of the PMM and TLA results at \((\theta = 45^\circ, \phi = 0^\circ)\) incidence for a 40" metal-backed straight wedge wall with \(D_x = 8''\) and \(\varepsilon_r = 2 - j1.5\). \(\theta\theta\) polarization.
around 10 ns. Based on Equation (3.5), the time delay between the fundamental specular wave and the re-radiated edge diffracted wave can be estimated as

\[ 8'' \times (1 + \sin 45^\circ)/c \approx 1.1\text{ns} \]  

(3.7)

The PMM result shows a little longer delay because the re-radiation mechanism is not limited to the edge area and can be excited inside the dielectric layer, where waves travel with less speed. As a result, the average time delay is larger than what is expected based on the edge diffraction model shown in Figure 12.

Oblique Incidence in Non-Principal Planes

For oblique incidence in non-principal planes, \((\phi \neq 0^\circ, 90^\circ)\), cross-polarized fields can be excited from a wedge absorber. The PMM and TLA calculated results with the angle of incidence equal to \((\phi = 45^\circ, \theta = 45^\circ)\) are shown in Figures 22 and 23 for both the co- and cross-polarized fields. In this case, the base height is only 2'' and no metal layer is added because the hybrid approach presented in Section 2.4.4 does not consider the cross-polarization components. For testing the TLA limitations on transforming the taper section to an equivalent uniform material, the configuration of the backing layer is not of concern. Note that the polarization is specified in the way that \(\theta \phi\), for example, means that the incident wave is \(\phi\) polarized and the receiving polarization is \(\theta\) polarized. At \((\phi = 45^\circ, \theta = 45^\circ)\), \(s_x = \cos 45^\circ \cos 45^\circ = 0.5\) and \(s_z = \cos 45^\circ \sin 45^\circ = 0.5\) and the onset grating lobe can be determined by Equation (3.2) as

\[ (-0.5 + \frac{\lambda_o}{8''})^2 + 0.5^2 = 1 \]  

(3.8)

The first grating lobe occurs when \(\lambda_o\) is equal to 11'' or at 1.1 GHz. The grating lobe is excited at a higher frequency than the previous \((\phi = 0^\circ)\) case because the incoming
Figure 22: Comparison of the PMM and TLA results for both the $\phi\phi$ and $\theta\phi$ cases at $(\theta = 45^\circ, \phi = 45^\circ)$ incidence for a $32''$ straight wedge. $D_x = 8''$ and $\varepsilon_r = 2 - j1.5$. 
Figure 23: Comparison of the PMM and TLA results for both the $\theta\theta$ and $\phi\theta$ cases at $(\theta = 45^\circ, \phi = 45^\circ)$ incidence for a 32″ straight wedge. $D_x = 8^\circ$ and $\varepsilon_r = 2 - j1.5$. 
wave strikes the absorber farther away from the direction of periodic variation. As a result, the agreement between the two methods seems better than the \((\phi = 0^\circ, \theta = 45^\circ)\) case. The close agreement validates the formulation of the polarization-coupled transmission line equations developed in Section 2.5.4. It is interesting to note that, at \(\phi = 45^\circ\), the cross-polarized fields are of about the same magnitude order as the co-polarized ones. The cross-polarized component is usually not considered in designing microwave absorbers due to the analytic complexity and less interest in making cross-polarization measurements.

### 3.3.2 Multi-layer Wedge Absorbers

There was a design effort in using the multi-layer wedge absorbers to improve absorber performance [3]. Due to high manufacturing cost and limited improvement, this configuration is not of as much importance as once thought. Nevertheless, this example is presented to show the application of the TLA method to this complex taper structure. In the design, the material doping gradually increases from the outer to the inner sections. This transition is intended to have a better impedance matching from the free space to the backing layer. In fact, from the TLA point of view, a curved contour of a highly doped uniform material can easily serve this purpose. Besides, the material changes in a multi-sectional contour produce several scattering centers which require a great deal of effort in optimizing these values.

For applying the TLA method to a multi-layer structure, one needs to generalize the effective material formulas described in Equations (2.50)-(2.52) for a two-phase configuration. Iteratively applying the two-phase formula, one can generalize it for an \(N\) multi-phase case such that

\[
\varepsilon^e_x = \left( \sum_{i=1}^{N} g_i \varepsilon_i^{-1} + (1 - \sum_{i=1}^{N} g_i) \varepsilon_o^{-1} \right)^{-1}
\]

(3.9)
Figure 24: Geometry for a 4-section 40" wedge absorber. The relative dielectric constants for the 4 layers are 1.5 – j0.5, 2 – j1, 2.5 – j2 and 2.5 – j3 from top to bottom.

\[
e_{y}^{x} = \sum_{i=1}^{N} g_{i}e_{i} + (1 - \sum_{i=1}^{N} g_{i})e_{o}, \quad \text{and} \quad e_{z}^{x} = e_{y}^{x}. \tag{3.10}
\]

A 4-layer 40" straight wedge is used for theoretical comparison. The geometry and material distribution are shown in Figure 24. The relative dielectric constants become gradually larger toward the bottom of the taper section. These values are assumed to be 1.5 – j0.5, 2 – j1, 2.5 – j2 and 2.5 – j3 for the calculation. The PMM and TLA calculated results at normal incidence are shown in Figure 25 for both co-polarized cases. In general, very good agreement is achieved. The multi-layer geometry basically does not affect the application the TLA analysis. The guided wave interaction can be seen at around 1000 MHz in the \(\theta\theta\) results. As
compared with the previous uniform case, this coupling term has been moved to a lower frequency due to the increased dielectric constant.

### 3.3.3 Serrated Wedge

Serrated wedge absorbers were designed to minimize the strong edge diffraction for the polarization parallel to the edge [3]. This design was found to be effective if the absorber height is more than 1.5 wavelengths. Due to difficulty in cutting tiny edges, the serration design is not as attractive as others. However, there is a theoretical interest to examine if the TLA method can be applied to such a non-gently tapered structure. In this section, an 18° serrated wedge design with a 3° period is used as the test case. As shown in Figure 26, there are 4 serrations on
each side with an offset of 3\". The serrated fin length is 8\" and the base is 0.1\". To show the transmission data, the serrated wedge in calculation is not backed with a metal layer. The dielectric constant used in the calculation was measured by an HP slotted line [31] and is shown in Figure 27.

The calculated reflection and transmission coefficients are compared in Figure 28 for the $\phi\phi$ polarization. The agreement is very good for both scattered fields. Note that this design starts to be effective above 1 GHz but rises up very quickly at the low end, which is characteristic of this polarization for a serrated wedge. The good agreement indicates the interaction between the serrations is principally along the specular transmission and reflection directions for the polarization parallel to the edges. The dominant terms come from the edge diffracted fields which can be successfully modeled by the TLA formulation. The calculated results for the $\theta\theta$ polarization are shown in Figure 29. The agreement for the reflection coefficient is good only in the average sense. The significant ripple in the PMM calculation
Figure 27: Measured dielectric constant of a commercial absorber material.

Figure 28: Comparison of the PMM and TLA results at normal incidence for an 18" serrated wedge. φφ polarization.
Figure 29: Comparison of the PMM and TLA results at normal incidence for an 18" serrated wedge. θθ polarization.

indicates that the higher order interactions are very strong for the perpendicular polarization. The interaction between serrations seems to have a significant component propagating along the lateral direction and create strong re-radiated waves. The large transmission field is a direct result of the smooth impedance transition for this polarization. Metal backing is needed for this wedge design to block out external electromagnetic waves.

The above results illustrate that the TLA formulation can be applied to an non-gently tapered structure but with some caution. For the polarization parallel to the edge, the edge diffraction is dominant and can be well predicted by the TLA method. However, when the electric field is perpendicular to the edge, the incident wave can penetrate more and the interaction between the serrations cannot be predicted as
well by the TLA formulation such that the agreement is good only in an average sense. In both polarizations, the agreement associated with the transmission results is very good.
3.4 Pyramidal Absorbers

The pyramidal absorbers considered here have the same period in the x and z directions. Due to this symmetry, the scattered fields are polarization independent at normal incidence and, in terms of the TLA point of view, no cross-polarized fields should be induced. In this section, only a straight taper is considered since, from the previous wedge studies, it is useful in terms of evaluating the TLA limitations. The pyramidal absorber under test is a 40\" metal-backed pyramid with a 10\"-thick base. The relative dielectric constant is $\varepsilon_r = 2 - j1.5$ and the inter-element distance is 8\".

3.4.1 Normal Incidence

The frequency and time domain responses of the calculated reflection coefficients from 100 to 1000 MHz are shown in Figure 30 for the normal incidence case. As can be easily seen, these two methods agree very well except at the high frequency end. The discrepancy found above 800 MHz is actually insignificant since the reflection coefficient is below -55 dB and there is no clear indication of higher order interactions. Besides, the PMM calculation could have a convergence problem in the high frequency portion because, to avoid exceeding computer resources, the segment size used for high frequencies is equal to $0.15 \frac{\lambda_0}{\sqrt{|\varepsilon_r|}}$. The time domain plot shows a relatively small scattered field from the front but large fields from the backing layer, which is the opposite to the wedge cases as shown in Figures 18 and 19. This is because the tip cross section in a pyramid makes a better transition from free space than the edge area in a wedge case. In addition, with a pyramid shape, less energy is attenuated due to less material such that the valley and backing layers become the dominant mechanisms if the material is not highly lossy. One can observe that...
Figure 30: Comparison of the PMM and TLA results at normal incidence for a 40" metal-backed straight pyramid wall with $D_x = D_z = 8''$ and $\varepsilon_r = 2 - j1.5$. 
the reflection level drops in a much faster rate than the wedge case above 450 MHz. Note that a pyramidal absorber has a good performance when it is electrically large, say 1.5 wavelength, to allow both the impedance transition and attenuation to be effective together.

3.4.2 Oblique Incidence at Principal Planes

The higher order mechanisms become more significant at wider angles of incidence. Let us examine the case with the incident direction equal to \((\theta = 45^\circ, \phi = 0^\circ)\). Both the frequency and time domain responses are shown in Figures 31 and 32 for the \(\phi\phi\) and \(\theta\theta\) polarizations, respectively. The scattered fields from a pyramid become polarization dependent at an oblique angle of incidence. For both polarizations, the TLA results agree very well with the PMM calculation except for the high frequency portion. Recall that the first grating occurs at 860 MHz and its interference with the fundamental mode is definitely seen from the nearby fluctuation. In the time domain, the TLA results do not have the small ripple for the late times which is caused by the higher order terms. However, the re-radiation mechanism is not as clear as the wedge cases. This is probably because the re-radiated mechanisms for a pyramid may come from different depths while the re-radiated edge diffracted fields for a wedge case is the most significant one. As compared with the previous \(\theta = 0^\circ\) case, the scattering mechanisms from the tip and backing layer are more comparable for this case. This indicates that the impedance matching at the tip area for the 45\(^\circ\) case is not as effective as the 0\(^\circ\) case and results a stronger tip scattered field. It is interesting to note that the first peak positions are about 1 ns apart between these two polarizations. In Figure 31 for the \(\phi\phi\) case, the first peak is at -3.3 ns which is about right at the tip position with the 45\(^\circ\) factor taken into account. However, for the \(\theta\theta\) case, the scattering seems to come from a place 1 ns deeper, which is
Figure 31: Comparison of the PMM and TLA results at \((\theta = 45^\circ, \phi = 0^\circ)\) incidence for a 40" metal-backed straight pyramid wall with \(D_x = D_z = 8\) and \(\varepsilon_r = 2 - j 1.5\). \(\phi\phi\) polarization.
Figure 32: Comparison of the PMM and TLA results at \((\theta = 45^\circ, \phi = 0^\circ)\) incidence for a 40" metal-backed straight pyramid wall with \(D_x = D_z = 8''\) and \(\varepsilon_r = 2 - j1.5\). \(\theta\theta\) polarization.
Figure 33: Comparison of the PMM and TLA results versus $\phi$ angle at 200 MHz for a 40\" straight pyramid. $\theta = 45^\circ$, $D_x = D_z = 8\"$ and $\varepsilon_r = 2 - j2$.

equivalently $8.5\" (= 1 \times 6/\cos 45^\circ)$ deep from the tip. This observation is different than the conventional interpretation of a pyramid scattering in which the tip and valley are considered the primary scattering centers.

3.4.3 $\phi$ Independence in the Scattering from Pyramid

As predicted by the TLA formulation in Section 2.5.4, the scattered fields are independent of the $\phi$ angle in the low frequency limit. To verify this property, the scattered fields of the 40\" pyramid without metal backing are calculated with $\varepsilon_r = 2 - j2$ at 200 MHz. The $\theta$ angle is fixed at $45^\circ$, and the $\phi$ angle is changed from $0^\circ$ to $90^\circ$ with a step size of $5^\circ$. The co-polarized results are compared in Figure 33. The TLA results are a constant due to the $x - z$ symmetry in the effective dielectric constant. The PMM results are quite stable with the $\phi$ angle and the variation is no
Figure 34: PMM calculated co- and cross-polarized results versus $\phi$ angle at 200 MHz for a 40' straight pyramid. $\theta = 45^\circ$, $D_x = D_z = 8''$ and $\varepsilon_r = 2 - j2$.

more than 0.3 dB. The stable co-polarized fields verify the TLA formulation using the coupled differential equations to describe the scattering of a periodic structure in the low frequency limit. To further verify the TLA theory, one can examine the cross-polarized result which is predicted as zero in the TLA calculation. The PMM results are shown in Figure 34 for both the co- and cross-polarized cases. The maximum cross-polarized fields occur at around $\phi = 22.5^\circ$ and $67.5^\circ$ and are almost 40 dB below the co-polarized signals. Such a low cross-polarized level again verifies the polarization coupled formulation in the generalized TLA theory.
3.5 Material Effect

In this section, the TLA limitation in terms of material properties is studied by using contour plots of the error value versus the dielectric constant. The error parameter associated with the PMM and TLA results is defined as:

\[ dB\left(\left|\frac{\text{TLA} - \text{PMM}}{\text{PMM}}\right|\right). \]  

(3.12)

The transmission coefficient is used to perform the error analysis. The reason for not using the reflection data is because, in most absorber structures, there are nulls in the reflection curves due to cancelation of mechanisms at different locations. The small values associated with a null are not suitable for the division operation as defined above. On the other hand, the transmission data shows a smooth result since the transmission path lengths from different absorber segments are about the same. In the following results, the segment size in the PMM result is chosen to be \(0.1\frac{\lambda_0}{\sqrt{\epsilon_r}}\). Proper convergence has been assured by comparing with the results using a \(0.08\frac{\lambda_0}{\sqrt{\epsilon_r}}\) segment size.

The error analysis for the 40° straight wedge without metal backing is performed at 200 MHz. The dielectric values range from \((1.5 - j0)\) to \((8. - j8)\) with a step size of 0.5 unit. The frequency is chosen to be well below the onset of grating lobe so that the material effects can be emphasized. The results for polarization parallel and perpendicular to the edge are shown in Figures 35 and 36, respectively. In both cases, one can clearly see the agreement between these two methods degrades as the magnitude of the dielectric constant increases. The discrepancy is mainly due to the higher order interaction which becomes more significant as the dielectric constant increases.
Figure 35: Error contour for the 40º straight wedge with respect to relative dielectric constants for normal incidence at 200 MHz. φφ polarization.
Figure 36: Error contour for the 40° straight wedge with respect to relative dielectric constants for normal incidence at 200 MHz. θθ polarization.
Next, the error analysis is applied to the 40° straight pyramidal absorber at 200 MHz for normal incidence. The contour plot is shown in Figure 37 and similar conclusions to the wedge cases can be drawn.

In fact, the reflected fields have very different scattering mechanisms than the transmitted fields, especially in terms of the phase paths for different “scattering centers”. The reflected fields from different heights have different paths for either the lowest propagating mode or the re-radiating modes; while, the transmitted fields from different heights tend to have the same path for either modes. As a result, the accuracy in predicting various scattering mechanisms is more critical for the reflected
fields. In other words, the error analysis using the transmission data may underestimate the problem one can have in the reflected data. However, the contour plots for the transmission data illustrate that care has to be exercised in an application of the TLA formulation to absorber materials with large relative dielectric constants. It is felt that the -25 dB contour line can be used as the criterion. For all the three configurations evaluated, this criterion roughly corresponds to $|\varepsilon_r| = 5$.

To examine the material effects to reflected fields, let us use some calculated examples to demonstrate the problem caused by large dielectric constant materials. The relative dielectric constant of the 40° straight wedge calculated in Section 3.3.1 is increased to $\varepsilon_r = 6 - j4.5$. The frequency and time domain responses for both methods are shown in Figures 38 and 39, respectively, for the $\phi\phi$ and $\theta\theta$ cases. As compared with the $\varepsilon_r = 2 - j1.5$ case in Figures 18 and 19, the TLA performance for the $\phi\phi$ case is about as good but the dip pattern at the high frequency end occurs at a lower frequency in the more lossy case. This phenomenon can be explained as the strong guided wave interaction for high dielectric material. From the time domain response, the high dielectric constant creates a very dominant edge diffraction term and the lossiness prevents the transmitted energy from scattering back from the base. The dominance of the edge diffraction can explain why the TLA analysis works very well. For the $\theta\theta$ case, the better impedance transition allows more transmitted energy to be reflected back such that an accumulated error results in passing through the taper section. In addition, the guided wave interaction can occur at lower frequencies for a higher dielectric constant. Therefore, the agreement is not as good as the lightly doped case. Note that, for the $\phi\phi$ case, the reflection level for the $\varepsilon_r = 6 - j4.5$ case is about 10 dB higher than the $2 - j1.5$ case; whereas, for the $\theta\theta$ case, the high doping case is lower at low frequencies end but higher at the high
end. This comparison shows that increasing material doping does not necessarily improve wedge absorber performance.

For the pyramid case, the 40° straight pyramid is revisited with the dielectric constant being changed to \( \varepsilon_r = 2 - j10 \). This dielectric value is not a realistic value for ordinary absorber materials but can be used to illustrate the TLA limitation in calculating highly doped material. The frequency and time domain results are shown in Figure 40. As can be easily observed, the agreement is not as good as the lightly doped case. The discrepancy between these two calculated results begins from 100 MHz. The higher order re-radiation mechanism can be seen at 0 ns in the time domain for the PMM result, which is not shown in the TLA calculation. Note that, due to the time delay in penetrating the high dielectric material, the valley scattering shows up at about 2.5 ns instead of 0 ns. This example illustrates the TLA calculation can be inaccurate for heavily doped materials.

3.6 Summary

The accuracy of the TLA analysis has been examined by the PMM method. Several calculated examples for both wedge and pyramidal absorbers were presented with various geometries and material properties. Co- and cross-polarized results were evaluated to verify the polarization-coupled TLA formulation developed in Chapter 2. In general, the TLA analysis provides accurate results when the higher order interaction is not significant and provides an averaged or approximate level otherwise. Two mechanisms i.e., the free space grating lobe and guided wave interaction, were used to explain the discrepancy between the two analytic methods. The onset frequency of the grating lobe is determined by the periodicity of the structure and ranges from \( \frac{\varepsilon}{2D_x} \) to \( \frac{c}{D_x} \) as the angle of incidence changes from 0° (normal) to
Figure 38: Comparison of the PMM and TLA results at normal incidence for a 40\" metal-backed straight wedge wall with $D_2 = 8''$ and $\varepsilon_r = 6 - j4.5$. $\phi\phi$ polarization.
Figure 39: Comparison of the PMM and TLA results at normal incidence for a 40" metal-backed straight wedge wall with $D_2 = 8''$ and $\varepsilon_r = 6 - j4.5$. $\theta \theta$ polarization.
Figure 40: Comparison of the PMM and TLA results at normal incidence for a 40” metal-backed straight pyramid wall with $D_x = D_z = 8”$ and $\varepsilon_r = 2 - j10$. 
90° (grazing). However, the guided wave interaction always occurs at lower frequencies than the free space grating lobe. In the study of lossless dielectric layers, the guided wave interaction is found to happen at lower frequencies for higher dielectric constant materials. From the contour plots for transmitted fields, it was found that the TLA calculation can be inaccurate if |\( \varepsilon_r \) | is greater than 5. These high order mechanisms can be interpreted as a re-radiation process in the time domain. The associated time delay is a function of the angle of incidence, the periodicity and material properties.

The polarization of the incident plane wave was found to have profound impact on the TLA accuracy. For the wedge case with the polarization parallel to the edge, the TLA prediction is most accurate due to dominance of the edge diffracted field. For the other configurations, the agreement is not as good because the incident wave can effectively penetrate and induce more mechanisms. The most illustrious example was the serrated wedge in which the agreement for the perpendicular-to-edge polarization is much worse than the parallel-to-edge case.

In addition to the theoretical aspects, the major difference between the PMM and TLA analyses is computation time. A typical PMM calculation for the wedge absorber as shown in Figure 18 requires about 30 minutes of CPU time on a modern workstation; whereas, the pyramidal absorber such as shown in Figure 30 can cost hours of CPU time. For both cases, the TLA calculation was completed in seconds. The extreme efficiency of the TLA method makes it possible to apply an optimization scheme to refine absorber designs, which will be described in the next chapter.
CHAPTER IV
Absorber Design

4.1 Introduction

As shown in the last chapter, the TLA method provides an accurate analysis of periodic absorber structures for low frequencies where the interaction between the higher order mechanisms and the fundamental mode is not significant. Basically, the wavelength needs to be at least twice as large as the period, and the material must be lightly doped, say $|\varepsilon_r| < 5$. Such limitations are not critical in many cases since absorber performance is normally satisfactory at the high frequency region. Therefore, the TLA analysis is very efficient and can be incorporated with a numerical optimization procedure [12] to finalize design parameters. However, for heavily doped materials, the TLA analysis can be so inaccurate that the PMM method must be used. Therefore, the combination of the TLA and PMM methods should provide a complete tool to design absorber structures.

Absorber design is a very complicated procedure in that it involves many factors such as material properties, absorber dimensions and geometry, incident direction and polarization, performance requirements and production costs. In principle, all these factors can be incorporated into an objective function and then optimized numerically. However, as the number of variables increases, it becomes increasingly difficult to define an appropriate objective function and the numerical stability and efficiency may deteriorate. Therefore, the number of variables needs to be minimized based on some realistic constraints. For example, absorber materials are normally
made of inexpensive impregnating foam doped with carbon to reduce production cost. Although a variety of material properties can be obtained by controlling the foam density and carbon doping level, only a small number of standard absorber materials can be chosen for the sake of controlling quality and cost. Consequently, one is not allowed to freely optimize dielectric constant values. It is practical to optimize the other parameters with a chosen material property. The other major production cost lies in the labor required to cut and glue absorber elements. Thus, one would like to have as simple a taper transition region as possible, which means that the multilayered and serrated taper structures are not considered in most cases. The optimization procedure will then focus on a uniform taper section with straight or curved contours.

Based on the above constraints, absorber designs are simplified to the choice of absorber material and the optimization of absorber geometry. In terms of the performance requirement for a microwave absorber, one would like to have the reflection level as low as possible over a very broad frequency band over as wide an angle of incidence as possible. In an anechoic chamber, the operating frequency band is usually very wide; while, major stray signals impinge on absorber walls normally at a narrow angle region. For example, the end wall absorber materials should be optimized for normal incidence since the feed antenna’s illumination basically propagates in the direction normal to the wall. As a result, the optimization procedure should be performed over a certain frequency band at a fixed angle of incidence.

The flow chart of the absorber design procedure for simple transition tapers is shown in Figure 41. The design starts with specifying performance requirements, which are an integral part of a chamber system design. One should then choose the appropriate absorber material for either the wedge or pyramidal absorbers. The
Design Starts

Design Requirements

Choose among Available Materials

Optimize Absorber Shape Using TLA

Is TLA Inaccurate?

Yes

Satisfied?

Yes

PMM Verification

No

PMM Only Design

Yes

Satisfied?

Yes

Higher Requirement?

Yes

No

Done

No

Exhaust Possible Initial Conditions?

Yes

PMM Verification

No

Use New Materials, or Use Other Designs, or Change Requirement to Restart Design.

Figure 41: Flow chart of the absorber design procedure
choice of an absorber material is based on the evaluation of absorber scattering versus the dielectric properties for a typical absorber geometry at critical frequencies. Although such an evaluation does not consider all the conditions, it provides a useful guideline for selecting appropriate absorber materials. The absorber transition shape can then be determined by incorporating the TLA analysis with a conventional optimization scheme such as the Fletcher-Powell algorithm [12]. Note that the numerical search for optimal values may rest on a local minimum depending on the objective function and initial guess values. Normally, one should iterate the optimization cycle until a reasonable range of parameters is tested. In some cases, the TLA evaluation is so inaccurate that the PMM method must be used for the final iteration, which is computationally intensive but accurate. If the optimized result is not acceptable, one has to consider using a new material, modifying the requirement or using other designs. One might find the initial requirement is so lenient that one can choose to shorten the absorber dimension or maintain a wide safety margin.

In this chapter, the proposed procedure will be applied to the design of 40" wedge and pyramidal absorbers with metal backing. The goals are to study if one can obtain a -40 dB reflection level at normal incidence for both absorbers above 300 MHz, where the absorber is only one wavelength long. As will be shown later, the design goal will be very difficult to achieve for wedge material. The multilevel Chebyshev design described in Chapter 6 will be used to provide a significant reduction factor so that the design goal can be easily achieved with a comfortable safety margin.

To better understand the basic scattering mechanisms, the effective material concept is first applied to study the polarization effects of wedge and pyramidal absorbers from the impedance matching point of view.
4.2 Polarization Effects in Wedge and Pyramidal Absorbers

As shown in the previous chapter, the polarization difference for the scattered fields from a wedge absorber is very significant; while, the scattering from a pyramidal absorber is polarization independent for normal incidence. In this section, the polarization effects are studied by examining the functional behavior of the effective material parameter. Such an approach is based on the success of TLA method in predicting the low frequency scattering. As absorbers taper from the edge or tip area to the valley, the length ratio of the material portion to the period gradually increases from 0 to 1. Recall that static effective material formulas described in Section 2.5.3 are given in terms of the dielectric constant and the material length fraction. By examining the functional behavior of the effective material with respect to the length fraction, one can obtain some insight into the scattering characteristics of tapered absorber structures.

With the relative dielectric constant equal to $3 - j2$, the effective parameters for three configurations are shown in Figure 42. The solid line is the effective value for the singly periodic case with the polarization parallel to the edge; and the dotted line is with the polarization perpendicular to the edge; and the dash line is for the doubly periodic case in which both orthogonal directions have the same effective quantities. One should note that the singly periodic geometry corresponds to a wedge absorber, and the doubly periodic case is related to a pyramidal absorber. In Figure 42, $g = 0$ corresponds to the edge or tip position and $g = 1$ corresponds to the junction of the taper section and the base layer. Based on the slopes at $g = 0$, the pyramid case has the smoothest transition from free space to the absorber material; while, the parallel to the wedge case has the worst transition. In fact, the functional behavior of the perpendicular to the wedge case is very similar to the pyramid case. This
Figure 42: The effective dielectric constants for three configurations with $\varepsilon_r = 3 - j2$. 
observation is consistent with the calculated results presented in Chapter 3 in that
the edge diffracted fields were found to be strong for the parallel to the wedge case
and relatively weak in both the perpendicular to the wedge and pyramidal cases.
Due to this polarization issue, the design of wedge absorbers is more complicated
than pyramidal absorbers. The smooth transition of the effective material properties
also explains why pyramidal absorbers usually have better performance than wedge
absorbers at normal incidence given the same absorber dimension and dielectric
properties.

In this case, the relative dielectric constant is changed to $\varepsilon_r = 2 - j10$, and the
effective permittivties are shown in Figure 43. The similarity between the perpen­
dicular to the wedge case and the pyramid case is very obvious both in the real and
imaginary parts. The overshoot of the real part at $g \approx 0.9$ has been verified by the
PMM calculation in Appendix B. The slope variations in the imaginary part around
$g = 0$ indicates that there is a strong edge diffracted field for the parallel polarization
case. It is interesting to note that in both Figures 42 and 43, the imaginary parts
of the effective permittivties are between 0 to $-0.25$ up to about $g = 0.5$ for the
two smoothest configurations. Therefore, the first half of the taper section does not
have much effective loss and acts as a low loss dielectric material. Consequently,
there is a trade off between the impedance matching and the wave attenuation. In
the very low frequency region where the electrical absorber height is less than, say
$\frac{1}{2} \lambda_o$, the normal pointed taper geometry may not have enough attenuation within
a small electrical dimension. The smooth transition obtained from shaping requires
a moderately long absorber dimension to have sufficient absorption. Liu et.al. [32]
reported that tip-truncated pyramids can have better performance than a pointed
tapered pyramid in the low frequency portion, where the absorber height is less than
Figure 43: The effective dielectric constants for three configurations with \( \varepsilon_r = 2 - j10 \).
0.5\lambda_0. For example [32], an 8' truncated pyramid has been designed for applications from 30 MHz-200 MHz.

In summary, the design of a wedge absorber is more involved than a pyramidal one because of the non-isotropic dielectric property. Based on the similarity of the effective permittivities, a wedge design for the polarization perpendicular to the edge is similar to that of the pyramidal absorber at normal incidence.

4.3 Material Effects in Wedge and Pyramidal Absorbers

The dielectric constant, \( \varepsilon_r \) \( (\varepsilon_r' - \varepsilon_r'') \), of an absorber material certainly has a significant impact on absorber performance. The material effects are studied using the TLA method to evaluate the reflection levels versus dielectric constants. The result of this evaluation is to provide a guide line in choosing among available absorber materials.

Five typical commercial absorber materials are assumed to be the available materials for the 40" absorber designs. Their dielectric constants were measured using an HP slotted line [31] and are shown in Figure 44. They are labeled as A-E which vary from small to large dielectric constants. The rise in the magnitude at lower frequencies of both real and imaginary parts is typical for this type of material. At higher frequencies, the dielectric constants approach a constant value. In an absorber design procedure, once a certain type of material is chosen, the design focuses on shaping. One can use the TLA method to study material effects and provide a guideline for choosing among available materials or manufacturing new materials to achieve better scattering performance.

In this section, contour plots are used to study the functional behavior of the absorber scattering with respect to material properties. Let us examine the 40"
Figure 44: The dielectric constants of four typical absorber materials.
metal-backed wedge and pyramidal absorbers with a 30") taper section and a 10") base for normal incidence. The operating frequency is chosen to be 300 MHz where the absorber height is about one wavelength. Conventionally, to reach a -40 dB reflection coefficient, one requires a pyramid of 2\( \lambda_o \) length at the lowest operating frequency. Based on our experience, it is possible to design a pyramid of only 1 \( \lambda_o \) length to meet this requirement [3]. It would be desirable to obtain such a low reflection from wedge materials, too. Therefore, 300 MHz (\( \lambda_o \approx 40\)") is the critical design frequency for the 40") long absorbers.

(a) Wedge Case

The contour plot of the reflection coefficient for the wedge case is shown in the Figure 45 for the \( \phi \phi \) polarization. Clearly, there are minimal reflection values occurring at about \( \text{Im}(\varepsilon_r) = -\varepsilon_r' = -1.5 \). The contour lines are principally parallel to the real \( \varepsilon_r \) axis, which indicates the reflection coefficient is more sensitive to the change in the imaginary part than the real one. Also, moderate loss is needed to attenuate the incident wave to minimize scattered fields from the metal backing. However, further increasing \( \varepsilon_r' \) does not improve the absorber performance. This is because the edge diffraction term is dominant for this polarization as \( \varepsilon_r' \) increases. The optimal dielectric constant values are the result of the tradeoff between the edge diffraction and base reflection terms. For this parallel polarization case, the imaginary part should be between -1 and -2 for this \( \lambda_o \) long absorber dimension.

For the \( \theta \theta \) polarization, the contour lines for the reflection coefficient are shown in Figure 46. As one should expect, the functional behavior is very different from the previous \( \phi \phi \) case. For this polarization, the contour plot suggests that a higher dielectric constant is needed to have optimal performance. Minimum reflection occurs at about \( \varepsilon_r = 2 - j2.5 \) and \( \varepsilon_r = 7.5 - j4 \). For this polarization, the effective smooth
Figure 45: Reflection coefficient of a 40" metal-backed straight wedge at 300 MHz with \( \phi \phi \) polarization (electric field is parallel to the edge).
Figure 46: Reflection coefficient of a 40" metal-backed straight wedge at 300 MHz with \( \theta \theta \) polarization (electric field is perpendicular to the edge).
transition from free space to the absorber requires lossy material to attenuate the incident wave. Even though the edge diffraction term is not as dominant as the $\phi\phi$ polarization, it can become significant as the permittivity increases and may degrade the absorber reflection performance. The dB level at the minimum reflection area is lower than that for the $\phi\phi$ polarization by roughly 10 dB. Therefore, the $\phi\phi$ polarization is usually considered as the worst case in designing wedge absorbers. In a wedge absorber design, a compromise must be exercised between these two polarizations. As a result, the desired dielectric constant should be around $2 - j2$, which is about the average value between the optimal values for the two polarization conditions.

(b) Pyramid Case

From the effective material concept as discussed in the previous section, the functional behavior of a pyramidal absorber should be somewhat similar to the wedge case with the polarization perpendicular to the edge (or $\theta\theta$ polarization). The previous wedge geometry is changed to a pyramid configuration, and the TLA calculated reflection coefficient is shown in Figure 47. This contour plot suggests that higher permittivity than the wedge case is needed to improve the pyramidal absorber performance. This phenomenon is expected since the sharp tip provides the best impedance transition but the reduced absorber material has to be compensated by increased doping. The minimum reflection occurs at about $\varepsilon_r = 5 - j5$ or $4.5 - j7.5$. From the contour line behavior, the loss tangent of $\varepsilon_r$ should be greater than 1. The dB level around these optimal positions is at least 10 dB lower than the perpendicular to the wedge case. Therefore, pyramidal absorber performs better than wedge absorber. As a result, most areas of an anechoic chamber are usually covered by pyramidal absorbers. As studied by Burnside [2], wedge absorber is useful to line the
Figure 47: Reflection coefficient of a 40" metal-backed straight pyramid at 300 MHz.
target zone areas with edge lines parallel to the transmitted wave propagation. The periodicity for a pyramidal absorber in the wave direction can create strong specular reflections from the side faces of the pyramids. In this case, the wedge absorbers create only a forward scattered field, which does not significantly perturb the target zone field. In fact, the wedge specular reflection from the long straight edges tends to skirt around the target zone and is normally absorbed by the end wall materials.

The above contour plots may be affected by the absorber geometry. However, the general description about material requirements for the three configurations are found to be true for absorber heights around one wavelength, which is the critical design issue addressed here. Based on the above discussion, one can have a better idea in choosing among available absorber materials. Let us take as an example using the five commercial materials shown in Figure 44. It should be obvious based on the previous results that the E material should be chosen for the pyramidal shape since it has an appropriate high dielectric loss. On the other hand, the B material should be chosen for the wedge material. In fact, B material is not really suitable for the perpendicular case since the real part is too high and the imaginary part is not lossy enough. However, this choice is a compromise between the two polarization conditions. Recall that one is always limited to certain dielectric constant values. Therefore, one must make appropriate tradeoffs as done here.

4.4 Numerical Optimization

As demonstrated by Yang et.al. [3], a curved taper contour may have significant improvement over a straight taper. This design feature can be understood from the impedance matching point of view. As shown in Figure 48, the sharper tip or edge
areas make the transition from the free space to the absorber smoother and the material's lossiness can effectively absorb the incident wave energy because of the wider valley areas. However, the determination of the curved shape is a computationally intensive procedure using the PMM method. To efficiently finalize the taper geometry, the TLA method is incorporated here using an numerical optimization algorithm developed by Fletcher and Powell [12].

4.4.1 Taper Contour Specified by a Polynomial Curve

The taper contour is modeled as a fourth-order polynomial as illustrated in Figure 49. Due to the symmetry of absorbers, only a single curve needs to be considered for both wedge and pyramidal absorbers. The curved geometry is rotated 90°, and the coordinates \((u, v)\) are used to describe the polynomial function. A fourth order polynomial is chosen and the point \((0, 0)\) is assumed to be at the absorber's
A fourth order polynomial, \( v = a_4 u^4 + a_3 u^3 + a_2 u^2 + a_1 u + a_0 \), is used to model the curved geometry.

edge or tip. As a result, the polynomial can be expressed as

\[
v = a_4 u^4 + a_3 u^3 + a_2 u^2 + a_1 u + a_0
\]  

(4.1)

where the \( a_i \)'s are the coefficients of the polynomial. To prevent the curved shape from becoming impractical, the polynomial curve is forced to intersect with the straight line at the junction point \( j \). By specifying five geometric parameters (the tip width, the taper section length(\( h_t \)), the distance(\( h_u \)) from the the tip or edge to the junction point \( j \), the slope \( \frac{dv}{du} \) at the tip or edge (\( s_0 \)), and the slope at the taper end(\( s_e \)) one can numerically solve for the coefficients, \( a_i \)'s, given a fixed total absorber height. In this dissertation, only the pointed geometry is considered; i.e. the tip width is zero. Note that a non-zero tip width can produce significant edge diffracted fields at high frequencies and normally is not preferred. Therefore, four independent variables are needed for a general curved shape. For a complementary geometry, \( s_o = s_e \) and \( h_u = \frac{h_t}{2} \), the number of variables is reduced to 2. For a straight contour, only one variable is needed.

Figure 49: A fourth order polynomial, \( v = a_4 u^4 + a_3 u^3 + a_2 u^2 + a_1 u + a_0 \), is used to model the curved geometry.
4.4.2 Objective Function

To apply an optimization procedure, one has to define the objective function to be optimized. By such a definition, a designer basically determines the desired features associated with the design object. In terms of the performance requirements, one would like to have the reflection level as low as possible over a broad frequency band and as wide an angle of incidence as possible. In an anechoic chamber, major stray signals impinge on an absorber wall normally at a narrow angle region. Besides, one should not use too complicated an objective function for the sake of numerical stability. Thus, the following absorber designs will be performed at a fixed incidence angle over a specified frequency band. The reflected fields from an absorber panel depend on many factors, such as material properties, absorber geometries, source frequency, incident direction and polarization. In general, a reflection coefficient can be denoted as a function of $n$ variables and $q$ parameters with the following form:

$$r(a_1, \ldots, a_n; |\theta|)$$  \hspace{1cm} (4.2)

where $X_j$'s are the variables which can be changed during the optimization process and the parameters $p^j$'s are fixed during the design. One possible definition of the objective function, $\Psi$, is described as

$$\Psi(x_1, \ldots, x_n) = \int_{f_{min}}^{f_{max}} dB(r(x_1, \ldots, x_n; f, p_1, \ldots, p_q))df$$  \hspace{1cm} (4.3)

where $dB = 20 \log$ which is the decibel function, and the integration interval $[f_{min}, f_{max}]$ specifies the frequency band to be used for the optimization. Note that the frequency dependence is singled out in the parameter list and will be the integration variable in evaluating the objective function. In many cases, as seen in some plots in the previous chapter, a reflection coefficient curve can have nulls whose levels are too low to be emphasized. Therefore, a threshold is needed to eliminate
the null effect by forcing the $dB$ values to be equal to the threshold values if the reflection level is below the threshold. In general, the threshold curve is a function of frequency and will be denoted as $\chi(f)$. With $\chi(f)$ being incorporated, the objective function in Equation (4.3) can be rewritten as

$$
\Psi(x_1, \ldots, x_n) = \int_{f_{min}}^{f_{max}} A(x_1, \ldots, x_n; f, p_2, \ldots, p_q) \, df
$$

(4.4)

where

$$
A(x_1, \ldots, x_n; f, p_2, \ldots, p_q) = \begin{cases} 
dB(\Gamma(x_1, \ldots, x_n; f, p_2, \ldots, p_q)) & \text{if} \quad dB(\Gamma(f)) > \chi(f) \\
\chi(f) & \text{otherwise}
\end{cases}
$$

(4.5)

By specifying the threshold curve, $\chi(f)$, designers can effectively control the optimization results. In the following designs, $\chi(f)$ will be defined at three points; i.e., $f_{min}$, $\frac{1}{2}(f_{min} + f_{max})$ and $f_{max}$ and a parabolic threshold curve is then assigned accordingly.

### 4.5 40" Wedge Design

In this section, the optimization procedure will be used to design a 40" metal backed wedge absorber for normal incidence. As discussed in Section 4.3, the B material should be chosen based on the trade off between the two polarizations. The inter-element distance is chosen to be 8". Note that the free space grating lobe diffraction begins to occur where $\lambda = 2D_x = 16''$ or $f = 750$ MHz. This frequency is 2.5 times higher than the lowest operating frequency and thus the reflection level should be so low that the grating lobe effect does not create significant levels. The design goal is to study if a -40 dB reflection level can be reached for both polarizations above 300 MHz. The TLA calculated results for a straight wedge with a 10"-thick base is shown in Figure 50, and this result will be used as the reference case to
Figure 50: The TLA calculated reflection coefficients at normal incidence for a metal backed 40° straight wedge made of material B. The taper section height is 30" and the inter-element distance is 8".

compare with the new designs. Note that, the reflection level is about -32 dB for the φφ case and -38 dB for the θθ case at 600 MHz, where the total absorber height (40") is about twice the wavelength. The reflection levels at 300 MHz are about -24 dB for both cases. Apparently, to achieve the desired requirement is going to be very challenging. Let us start with the straight contour case.

### 4.5.1 Straight Contour

The optimization is performed by changing the proportion between the transition section height and the base thickness without modifying the straight contour
shape. The optimization is first performed by considering only the reflection coefficient for parallel to the wedge ($\phi\phi$) polarization. The initial guess value and the associated parameters are described by

\[ h_t : 30'' \]

\[ f_{\text{min}}, f_{\text{max}}, f_{\text{step}} \text{ (MHz)} : 250, 450, 10 \]

\[ \chi_{\text{min}}, \chi_{\text{middle}}, \chi_{\text{max}} \text{ (dB)} : -40, -40, -40 . \]

Note that this design will be referred to as #W1. By applying the FP scheme to optimize the single variable $h_t$, one finds that the base thickness is forced to be zero (or $h_t = 40''$). The optimized results are compared with the reference straight wedge in Figure 51. Note that the $\phi\phi$ polarization has been improved but the $\theta\theta$ case becomes much worse. This phenomenon is due to the polarization difference associated with wedge absorber. The edge diffraction term is so dominant in the $\phi\phi$ case that a longer taper section is needed to make a smaller wedge angle to minimize the diffracted field. However, the lack of the base region makes the $\theta\theta$ reflection level much higher than the reference case. Recall that, for the $\theta\theta$ polarization, the better impedance matching allows more energy to be transmitted into the base layer; as a result, this polarization requires a thicker base to absorb the incident energy. If only the $\theta\theta$ case is considered in optimization, the optimal taper section tends to be around 25''. However, not much improvement for the $\theta\theta$ case results from this optimization, and the $\phi\phi$ case becomes worse as expected. From the above examples, one can conclude that changing the proportion is not enough to dramatically change the wedge absorber performance and both polarizations have to be considered together.
4.5.2 Curved Contour

The above dilemma can be partially resolved if the transition contour is curved. Using a curved shaping, the edge becomes sharper to minimize the wedge diffracted field. Thus, one does not need as long a taper section for the $\phi\phi$ polarization and can have a thicker base to improve the $\theta\theta$ polarization. In the following design, the absorber transition geometry is defined by the polynomial function as described in Equation (4.1). As suggested in the previous case, the objective function in Equation (4.4) is defined as the integration of the average of $\phi\phi$ and $\theta\theta$ reflection coefficients. With the following inputs to the FP algorithm:

$h_l, h_u : 30'', 20''$

![Graph showing comparison of reflection performance for optimized and reference wedge absorbers. The optimization is done without changing the straight shape and by only considering the $\phi\phi$ case.](image)
Figure 52: Comparison of the reflection levels of the optimized curved wedge #W2 and the reference straight wedge.

\( s_0, s_e : 0.05, 0.05 \)

\( f_{\text{min}}, f_{\text{max}}, f_{\text{step}} \text{ (MHz)} : 250, 450, 10 \)

\( X_{\text{min}}, X_{\text{middle}}, X_{\text{max}} \text{ (dB)} : -40, -40, -40 . \)

The results, which will be referred to as wedge design #W2, are given by

\( h_l, h_u : 27.55^\prime, 20.90^\prime \)

\( s_0, s_e : 0.059, 0.0 . \)

Note that there are 4 design variables in this optimization. The reflection performance is compared with the reference straight wedge in Figure 52. For both polarizations, the designed performance is considerably better than the straight wedge.
above 300 MHz. For the $\phi \phi$ case, the reflection coefficient of #W2 is below -40 dB above 320 MHz, which is very close to our design goal. However, the $\theta \theta$ case merely reaches -30 dB at 300 MHz. One should note that the perpendicular polarization usually has better performance than the parallel case but not for this design. This interesting phenomenon is caused by the inappropriate dielectric properties of material B. Recall that material B was chosen because its functional behavior around 300 MHz falls between the desired curves for both polarizations. However, this material is more appropriate for the $\phi \phi$ case but does not have enough loss to effectively attenuate the incident energy for the $\theta \theta$ case. This wedge design will be revisited later with a more appropriate material.

To verify design #W2 and identify scattering mechanisms, the PMM and TLA calculated frequency and time domain responses are compared in Figures 54 and 55.
for the $\phi\phi$ and $\theta\theta$ polarizations, respectively. The agreement is about as good as the straight case, which was discussed in the last chapter. To accurately model the transition curvature, one needs a finer segment size in a curved section than a straight one to have proper PMM convergence. Based on numerous calculated examples, it was found that the curved taper does not have a significant impact on the TLA accuracy. Therefore, the TLA limitations found by using the other type of transition in the last chapter can also be applied to the curved transition case.

From the time domain plot in Figure 54 for the $\phi\phi$ case, one can easily observe the edge diffracted term has been significantly reduced due to the curved shape. A distinctive scattering mechanism apparently comes from the middle of the taper section. This feature is not found in a straight taper where the edge and valley are usually the two dominant scattering mechanisms. In this case, the transition curve makes the most distinctive impedance change happen around the inflection point instead of the edge area. The good reflection performance for this polarization is achieved because there is not a single dominant scattering center and effective cancelation is optimized among the mechanisms of similar amplitudes. For the $\theta\theta$ polarization shown in Figure 55, the dominant metal scattering term at about 3.5 ns clearly indicates B material does not have enough loss. One may, at the expense of the transition section length, increase the base thickness from the designed 12.45" to about 15" without much improvement but with significant degradation for the $\phi\phi$ polarization. This #W2 wedge design illustrates the importance of choosing an absorber material in the first place and the difficulty in optimizing absorber performance for a wedge absorber.
Figure 54: Comparison of the PMM and TLA calculated results of the wedge design 
\#W2 at normal incidence. \(\phi\phi\) polarization.
Figure 55: Comparison of the PMM and TLA calculated results of the wedge design #W2 at normal incidence. θθ polarization.
4.5.3 Change Material

Material B was found to have insufficient loss based on the optimized results in the previous section. The dielectric constant $\varepsilon_r$ is equal to $2.66 - j1.79$ at 300 MHz. To increase the loss tangent, one can either decrease the real part or increase the imaginary part. From the contour plot shown in Figure 45 for the parallel to the wedge case, the imaginary part should be limited to -2 so that the wedge edge diffraction is not too significant. Therefore, a new material assumed to have $\varepsilon_r = 2 - j2$ at 300 MHz is used for the design. Note that one would tend to believe that a larger real part is more heavily doped and introduces more loss. However, the propagation constant, which is proportional to $\sqrt{\varepsilon_r}$, shows the contrary such that, for example,

$$\text{Im}(\sqrt{2-j2}) = -0.64 \quad \text{and}$$

$$\text{Im}(\sqrt{3-j2}) = -0.55 .$$

(4.6) (4.7)

The dielectric constant is generated by the following formulas:

$$\varepsilon'_r = \frac{0.3}{f(\text{GHz})} + 1 , \text{and}$$

$$-\varepsilon''_r = \frac{-0.6}{f(\text{GHz})} .$$

(4.8) (4.9)

With the following inputs to the FP algorithm:

$h_t, h_u : 28.5", 22"$

$s_\alpha, s_e : 0.07, 0.01$

$f_{\text{min}}, f_{\text{max}}, f_{\text{step}} (\text{MHz}) : 250, 450, 10$

$\chi_{\text{min}}, \chi_{\text{middle}}, \chi_{\text{max}} (\text{dB}) : -40, -40, -40$ .

The results, which will be referred to as wedge design #W3, are given by

105
The reflection performance and the geometry are shown in Figures 56 and 57, respectively. As compared with Figure 52, the increased material loss improves the parallel polarization around 300 MHz but deteriorates the parallel polarization performance. However, the balance in the reflection levels for these two cases illustrates the trade off made in choosing the optimized material. This design provides a -35 dB reflection level above 300 MHz. The optimization procedure has been performed iteratively without finding any solution which can satisfy the -40 dB design goal. It seems that it is extremely difficult to achieve the -40 dB reflection requirement with a uniform material. To further verify this argument, the calculated reflection contours
for design #W3 versus dielectric constants are shown in Figure 58 and 59 for both polarizations. By comparing these two contours, the polarization difference can be observed in that the parallel polarization needs a material of small imaginary part; while the perpendicular polarization prefers a larger imaginary part. This finding is similar to that observed in Section 4.3 for the straight wedge case. One may conclude that it is not feasible to obtain -40 dB reflection levels from a wavelength long wedge composed of a uniform dielectric material. One must either change the design requirement or attempt other design changes.

4.5.4 New Wedge Requirement

The previous designs focus on the frequencies around 300 MHz and the reflection response is not close to the design goal. The limited improvement at the low end of
Figure 58: Reflection coefficient of the wedge design #W3 versus dielectric constants for normal incidence at 300 MHz for the parallel to the wedge case.
Figure 59: Reflection coefficient of the wedge design #W3 versus dielectric constants for normal incidence at 300 MHz for the perpendicular to the wedge case.
the frequency band was accomplished at the expense of the midband performance. If the design requirement can be loosen so that the lowest operating frequency is 360 MHz, not 300 MHz, it is possible to reach -40 dB level for both polarizations using the assumed material. The inputs to the FP algorithm are described by

\[ h_t, h_u : 27.5", 20.5" \]

\[ s_0, s_e : 0.05, 0.01 \]

\[ f_{\text{min}}, f_{\text{max}}, f_{\text{step}} \text{ (MHz)} : 320, 500, 10 \]

\[ \chi_{\text{min}}, \chi_{\text{middle}}, \chi_{\text{max}} \text{ (dB)} : -43, -43, -43 \].

The results, which will be referred to as wedge design #W4, are given by

\[ h_t, h_u : 26.96", 20.56" \]

\[ s_0, s_e : 0.057, 0.023 \].

The reflection performance is shown in Figure 60 where one should observe the -40 dB reflection levels above 350 MHz for both polarizations. Note that such improvement is basically obtained by removing the 300 MHz requirement. The resulting curved shape is shown in Figure 61. One can notice that the edge is thinner than that of the design #W3. The designs #W3 and #W4 are two typical patterns for this wedge case. Design #W3 focuses on the very low end of the frequency band but sacrifices the midband performance; while, design #W4 emphasizes midband frequencies. The choice of these two should depend on the chamber design requirements.
Figure 60: TLA calculated results for the design #W4 with the assumed material.

Figure 61: Geometry of the curved wedge design #W4.
4.5.5 Multilayered Base Design

It was shown that using a single uniform material, one cannot satisfy both polarization design requirements. Recall that design #W2 does not have good performance for perpendicular polarization due to the strong base scattering resulting from insufficient doping of material B. One can replace part of the base section with more heavily doped materials to reduce the base scattering. Note that the taper section should not be changed in that a good impedance matching has been achieved for the $\phi\phi$ polarization. In addition, the manufacturing cost will not increase considerably since it is relatively easy to implement a multilayered flat base. A three layered base design related to design #W2 is shown in Figure 62. Note that this design will be referred to as #W5. The taper section and first base section are made of material B; and the second base section is made of material C; and the bottom
Figure 63: Comparison of the reflection levels of curved wedge design #W2 with a 3 layered flat base and a uniform base.

As one can observe, the perpendicular polarization ($\theta\theta$) case has been improved by about 8 dB around 300 MHz; while, the parallel polarization ($\phi\phi$) case, which was not a problem, has very similar performance. This design #W5 is better than design #W3 in that the reflection levels quickly drop below -40 dB above 350 MHz. In addition, it is also more desirable than design #W4 since the reflection level at 300 MHz is kept as low as -34 dB instead of -28 dB. Although the challenging design goal is not completely achieved, this 3-layered base design illustrates it is feasible to deal with the polarization difference associated with a wedge design by using a multilayered base which has gradually increased doping toward the metal backing. The
presented design is limited by the dielectric constant behavior of the five available materials assumed here.

Even though it might be possible to achieve the -40 dB requirement if more appropriate materials are used, the designed performance will be marginal. In Chapter 6, a multilevel Chebyshev design will be applied to improve the low frequency performance of the wedge design. The wedge elements will be decomposed into several layers so that the scattered fields from different levels interfere destructively for a lower reflection level.
4.6 40" Pyramid Design

As discussed in Section 4.3, the E material was chosen for the pyramid design. The inter-element distance for both $x$ and $z$ directions is chosen to be 8" using similar arguments as the wedge case. The application of the FP algorithm to a pyramidal absorber design is principally the same as the wedge case. For the optimization procedure performed for normal incidence, either the $\phi \phi$ or $\theta \theta$ case is sufficient since both cases have the same reflection behavior. However, since the E material has large dielectric constants, the TLA results may be so inaccurate that the PMM method must be used. The design goal is to reach -40 dB reflection level above 300 MHz. Recall that this design goal was very challenging for the wedge case. It should be more achievable for pyramids because of a better impedance transition and simpler polarization properties.

4.6.1 Straight Contour

Unlike the wedge case, a pyramidal absorber can be designed to have decent performance with a straight contour. The first design is performed by simply adjusting the proportion between the taper section length and the base thickness. The input parameters for design #P1 are as follows:

$h_t : 31"$

$f_{min}, f_{max}, f_{step} \text{ (MHz)} : 250, 500, 10$

$X_{min}, X_{middle}, X_{max} \text{ (dB)} : -43, -43, -43.$

The only one output parameter of the FP scheme is given as

$h_t : 30.71"$
The base thickness was then determined to be 9.29". Note that this optimized geometry is very close to that used in the calculation examples presented in Chapter 3. To verify this design, both the PMM and TLA calculations are compared in Figure 64. As one can observe, these two calculations do not agree well above 250 MHz and have more than a 5 dB envelope difference. Such a discrepancy is caused by the higher order interaction associated with the large dielectric constants of material E. The sharp null patterns produced by the fundamental mode scattering mechanisms in the TLA optimization are filled in by the higher order terms. However, the PMM calculated results happen to show a more desirable response with the reflection levels below -40 dB above 300 MHz. As compared with the wedge case, this straight
pyramid has much better performance. The reason for this improvement is mainly because both polarizations are identical for normal incidence, which allows a better choice of absorbing materials.

Among the five available materials, the absorber designs based on the four less doped materials can be well predicted by the TLA method. However, the scattering performance is not as good as this E material and is not shown here. Due to the significant discrepancy, the TLA method is not suitable for designs using heavily doped materials. Consequently, the PMM method is more appropriate. However, the PMM calculation becomes less efficient with increasing dielectric constants. For example, it takes about ten CPU hours in a modern workstation to calculate a frequency scan for design #P1 from 260 MHz to 440 MHz with a step size of 20 MHz. Therefore, it is more practical to design either a straight or complementary contour whose number of design variables is no larger than two. Besides, the TLA optimized results should be used as a starting point for the PMM design. For the straight pyramid design #P1, one can vary the taper section length with a step of 0.5" to search for the most desirable PMM responses. Since 300 MHz is most critical, the PMM solutions for various taper lengths were evaluated only from 260 to 440 MHz and are shown in Figure 65. As can be observed, the increase in the taper length creates a deeper null around 280 MHz but a larger level around 340 MHz. It is clear that the result with 31.21" has a balanced response between these two bands. Therefore, the 31.21" is chosen as the #P2 design for a straight contour, which provides the best overall scattering performance based on the PMM evaluation. One can also emphasize the response at 300 MHz and choose a value close to 31.71". In any case, this design result shows that the PMM iteration is feasible since the starting value is reasonably defined based on the TLA estimations.
Figure 65: Comparison of the PMM calculated results for the 40'' straight pyramids with various taper section lengths.
4.6.2 Curved Taper

Although the TLA method is not accurate for designs using the E material, it can still provide a rough idea about what the optimal values should be for a general curved design, which involves four optimization variables. The input parameters to the FP algorithm for the curved design #P3 are given as

\[ h_t, h_u : 31.4" , 17.3" \]

\[ s_o, s_u : 0.130 , 0.007 \]

\[ f_{min}, f_{max}, f_{step} \text{ (MHz)} : 250, 550, 10 \]

\[ \chi_{min}, \chi_{middle}, \chi_{max} \text{ (dB)} : -47, -47, -47 \]

The output parameters of the FP scheme are given as

\[ h_t, h_u : 31.33", 17.29" \]

\[ s_o, s_u : 0.118, 0.01 \]

The optimized curved pyramid geometry is shown in Figure 66. The PMM and TLA calculated reflection results are shown in Figure 67. As one can observe, the prediction discrepancy caused by the large dielectric constants is significant. Unlike the straight pyramid design #P1, the PMM results show a higher reflection level than the TLA prediction. The optimized null shape around 300 MHz is filled by the higher order terms such that the scattering performance of #P3 is about the same as the straight design #P2. Since the PMM finalization of the four variables needs tremendous computer resources, that design approach was not pursued. In the next section, a complementary curved design is used to illustrate the improvement made by using a curved contour.
Figure 66: Geometry of the curved pyramid design #P3.

Figure 67: Comparison of the PMM and TLA calculated results for the curved pyramid design (#P3) at normal incidence.
4.6.3 Complementary Contour

To maximize material usage, a complementary geometry is usually preferred since a single cut through a foam block can make two identical absorber elements. In terms of the polynomial expression shown in Figure 49, this means that the intersecting point of curved and straight contours must be at the middle point; and the slopes $s_o$ and $s_e$ are equal. With these two constraints, the number of optimization variables decreases from four to two. By specifying the taper length and the slope, one can determine the complete absorber geometry. Since the TLA method is not accurate, the PMM design is applied for this case. For practical reasons, the optimization of the two design variables will be divided into two steps. Firstly, the contour is assumed to be straight; then, the complementary curvature will be optimized based on the optimized straight one. Note that the straight contour is a special case of a complementary one with the slopes equal to $\frac{Dz}{2h_l}$. Design #P2 has been shown to provide the best performance for straight contours and is used as the output of the first step. The PMM calculated results with slope values ($s_o = s_e$) equal to 0.11 and 0.09 are shown in Figure 68 for comparison with the straight design #P2. Note that the straight contour for design #P2 corresponds to a complementary shape with slope values equal to 0.128 ($\frac{4}{31.2\pi}$). From Figure 68, the decrease in the slope values or a thinner tip results in a larger reflection level around 300 MHz but a much smaller level around 400 MHz. Recall that similar tradeoffs between these two bands were observed in the wedge designs #W3 and #W4. To maintain good responses for both regions, one should design a complementary curve based on a straight pyramid which has a null around 300 MHz. From Figure 65, the response associated with the 31.71° design has the desired behavior. Therefore, the PMM evaluation with varying slopes was performed on this straight pyramid and the results are shown in Figure 69. As one can observe, the design with the slope equal
Figure 68: Comparison of the PMM calculated results for design #P2 and the complementarily curved modifications for normal incidence.
Figure 69: Comparison of the PMM calculated results for the 40" straight pyramid with 31.71" taper length and the complementarily curved modifications for normal incidence.
to 0.11 has optimal performance across 300 to 400 MHz. Note that one can obtain a very low reflection level above 350 MHz using an even smaller slope such as the 0.09 case, but, the reflection behavior around 300 MHz is sacrificed. Nevertheless, if one was willing to make the absorber slightly longer than 40'' this would be the best choice.

In summary, the slope value for this complementary 40'' pyramid design is optimized to be 0.11 with a 31.71'' taper length. This design will be referred to as #P4 and its geometry is shown in Figure 70. Design #P4 can achieve a -44 dB reflection coefficient above 300 MHz and a reasonable fall off at higher frequencies. One can notice that the curved pyramid design is very versatile in that various design requirements can be achieved by adjusting the design variables. The proposed design procedure provides an accurate and efficient tool to determine the design parameters.
4.7 Summary

An absorber design procedure using both the PMM and TLA methods has been presented with the application to the designs of 40" wedge and pyramidal absorbers for normal incidence. For the sake of controlling manufacturing cost, absorber designs are normally restricted by a limited choice of dielectric materials and simple transition sections. The design procedure basically can be divided into two steps. Firstly, an absorber material is chosen based on the functional behavior of absorber scattered fields versus dielectric constants. The second step is to optimize the transition curve by incorporating the Fletcher-Powell optimization scheme with the TLA method. The optimized results must then be verified by the PMM method especially when the chosen material has large dielectric constants or when the optimized levels are very small.

Five commercially available materials are used for the design of 40" wedge and pyramidal absorbers, which are to reach a -40 dB reflection coefficient above 300 MHz for normal incidence. Such a goal is very difficult to achieve for a wedge material due to the polarization diversity in which the parallel to the wedge polarization requires a lightly doped material to reduce edge diffraction; while, the perpendicular polarization needs more lossy material to attenuate the penetrating wave. The curved shape presented here is one of the solutions in that it can narrow the edge and widen the valley. However, the chosen B material was found to be too lightly doped for the perpendicular polarization. A hypothetical but realizable material was suggested to further relieve the polarization problem. Unfortunately, the improvement was limited and one can conclude that a wedge design using a single material cannot achieve the desired goal. Without significantly increasing the manufacturing cost, a multilayered base was optimized for this application. In this design, more lossy
materials were used to replace part of the base section so that there is enough loss for perpendicular polarization without affecting the good parallel polarization response. The three-layered base design was shown to be very close to the design goal even though the choice of absorber materials is limited to the five commercial materials.

For the pyramid design at normal incidence, a lossy material is needed to compensate for the reduced material resulting from the pointed geometry. The E material was found to be appropriately doped. Unfortunately, the TLA optimized results could not be directly used for this case because it has significant errors as described earlier for heavily doped materials. In this case, the TLA optimization provides only a rough estimation of the design variables, and the PMM method must be used for further refinement. The application of the PMM method is practically limited to a small number of iterations since it requires tremendous computational resources. Nevertheless, it was used to design an optimized straight pyramid that achieved a -43 dB reflection coefficient. Such high performance for a pyramid was obtained due to the pointed shape and the chosen lossy material. Next, a complementary curved design was proposed to provide a cost effective pyramid design with lower reflection levels than the straight material for the higher frequencies. The final optimized complementary design was shown to achieve a -44 dB reflection coefficient above 300 MHz and quickly drops off to -50 dB above 440 MHz.

In summary, the TLA based optimization scheme is a very effective design tool in that it is very efficient and one can resort to complex design constraints without exceeding limited resources. However, if a heavily doped material is chosen or an extremely small reflection level is optimized, one must be concerned that the higher order mechanisms might dominate the results and are neglected in the TLA solution. Thus, one must use PMM to find the optimal values based on the TLA rough estimations. From these findings, it becomes very clearly that the TLA
and PMM are necessary and tend to compliment each other. Although the designs presented in this chapter focused on the normal incidence, the proposed procedure can be applied to the other incidence angle.

One should note that the above designs focused on the conventional absorber configuration where absorber elements are of the same height. In Chapter 6, multi-level Chebyshev designs will be shown to provide a significant reduction factor due to the destructive interference among different levels. This new design concept is very effective for both pyramid and wedge absorbers and should particularly help the unsatisfactory wedge design presented in this chapter.
CHAPTER V
Absorber Measurement and Data Processing

The scattering performance of a new absorber design needs to be critically evaluated through various measurements. The absorber measurements presented in this study were performed in the OSU/ESL compact range [13, 14] in which the primary-fed parabolic reflector transforms the feed antenna’s spherical wave into a plane wave illumination to simulate far field conditions. To characterize absorber performance, far field measurements are more suitable than near field ones since near field results are a function of the distance between the source and absorber panel. Besides, far field results are more comparable with the theoretical results which are based on a plane wave illumination.

As one attempts to compare theoretical predictions and measured results of absorber, discrepancies often occur. There are two major reasons for this phenomenon. Firstly, the theoretical absorber designs do not take into account some realistic quality control problems in making microwave absorbers, such as the dielectric property variation and cutting inaccuracy. Secondly, measured results can be corrupted by various error terms, such as mounting error and the so called end effect errors. In a measurement chamber, absorber panels are mounted to form a continuous wall in most areas for better shielding and absorbing effects. Consequently, to evaluate chamber performance, one has to determine the reflection properties of an absorber wall. Note that this requirement is consistent with the PMM and TLA calculations.
which are developed for an infinite wall. Unfortunately, absorber is normally evaluated experimentally using a finite sample. As a result, absorber measurements are corrupted by end effect errors. Basic characteristics for this edge effect are studied using the uniform geometric theory of diffraction (UTD) [15].

In this research work, no attempt has been made to deal with the absorber manufacturing issues. Instead, the main effort has been to apply inverse synthetic aperture radar (ISAR) image processing techniques to identify various scattering mechanisms associated with absorber measurements [16, 17]. Based on this diagnostic tool, the problems with material geometry and properties can be evaluated. Furthermore, measurement errors can be isolated using image filtering or gating. The corrected image is then transformed back to the frequency and angle domains and the resulting data better represents the actual absorber performance.

5.1 Measurement Setup

The reflection coefficient measurements of various absorber structures were performed in the OSU/ESL compact range [13, 14]. A schematic of this measurement chamber is shown in Figure 71. Note that the parabolic reflector is terminated with an elliptic rolled edge to minimize the edge diffracted fields from the reflector junction [14]. The result of the improved edge treatment is to increase the usable quiet zone or, equivalently, to reduce the reflector size and its cost. The quiet zone of for this reflector and edge treatment is about 6' at 1 GHz and 10' at 10 GHz [14].

The test target is normally mounted on an ogive-shaped pedestal and rotated in the horizontal plane for RCS versus azimuth angle measurements. The radar system is a low cost pulsed-IF design capable of making very accurate magnitude and phase measurements [33]. The synthesizer in the transceiver system is so fast.
Figure 71: The side view and top view of the OSU/ESL compact range

\[ \hat{D} : \text{DOWN\_RANGE AXIS} \]
\[ \hat{C} : \text{CROSS\_RANGE AXIS} \]
\[ \hat{V} : \text{VERTICAL\_RANGE AXIS} \]
that one can take frequency sweeps, while the target is being rotated. Finally, the recorded raw data can be sent to a high speed workstation for a real time or future data processing and storage.

For these measurements, the absorber panels were glued on an aluminum plate of equal size. As shown in Figure 72 for a 2' square case, the metal plate is attached to the pedestal via a T-shaped rod. A stepping motor in the back provides the automated elevation angle adjustment through computer control. The counter weight is used to balance the plate and absorber weight for smooth elevation and azimuthal movements. Note that this metal-backed absorber configuration is consistent with

Figure 72: Side view of the absorber measurement setup.
how absorbers are normally mounted in an anechoic chamber with a metal enclosure. However, the ISAR images will show that, there are end effect errors caused by the finite size sample and mounting errors caused by the support structures. These errors will be removed by image filtering techniques.

5.2 Data Calibration

The measured raw data needs to be normalized against a sphere or the flat plate in order to calibrate out the radar system response. The equations to obtain the reflection coefficient is given by

$$\text{reflection coefficient} = \frac{(\text{absorber panel}) - (\text{background})}{(\text{sphere}) - (\text{background})} \cdot \frac{(\text{sphere RCS})}{(\text{plate RCS})}$$  (5.1)

when the sphere measurement is used as the reference; or

$$\text{reflection coefficient} = \frac{(\text{absorber panel}) - (\text{background})}{(\text{plate})}$$  (5.2)

when the mounting plate is used as the reference. In both cases, the absorber results are indirectly or directly normalized against the plate at normal incidence so that the reflection coefficient can be obtained. When the mounting plate surface is slightly bent or contaminated by glue, the plate measurement can be inaccurate especially for high frequencies. In this case, the first calibration method using a reference sphere becomes more appropriate. In the calibration, the background measurements are taken with the absorber panel and the T-shaped support structure removed. After the background subtraction, there are mounting errors associated with the support structure. One should note that the reflection measurement for a finite sample is an approximation to the reflection properties for an absorber wall. The finite size of the absorber panel introduces the end effects. These end effects are not desired in characterizing absorber performance since absorbers are usually mounted as a
continuous wall in an anechoic chamber. In addition, the end effects are not included in the PMM and TLA solutions both of which are based on the Floquet theorem for an infinitely periodic structure. Therefore, the end effects should be removed or minimized so that they do not dominate the absorber scattering. As shown in a later section, one can apply image processing techniques to remove the error terms and obtain the desired reflected fields.

5.3 End Effects

The basic properties associated with the end effects are first studied using a UTD solution of a thin dielectric slab of finite width. The UTD solution will be generalized qualitatively to a thick absorber panel.

5.3.1 UTD Solution for a Thin Dielectric Slab

A heuristic UTD solution has been developed by Burnside et.al. [15] for a thin dielectric slab. Since the UTD solution decomposes the total field into the reflected field from the slab width and the edge diffracted field from the two end points, it can provide some insight into the end effects. The total field for a thin slab at normal incidence can be written as [15]

\[ E_t = -\frac{e^{-j\pi/4}}{2\sqrt{2\pi k}}(2(1 - T(f)) - 2j\Gamma(f)kW) \]

where \( W \) is the slab width and \( T \) and \( \Gamma \) are the transmission and reflection coefficients, respectively, for the infinitely wide case. The term with \( (1 - T) \) describes the edge diffracted fields from the two ends; and the term with \( \Gamma kW \) provides the reflected field from the slab width. To emphasize their difference, these two terms are denoted as

\[ E_d = \frac{A(1 - T(f))}{\sqrt{k}} \]

\[ (5.4) \]
where $A$ is a constant equal to $-e^{-j\pi/4}$. If the reflection and transmission coefficients are a constant, the edge diffracted fields decrease with frequency while the reflected fields increase. In this case, the end effect errors decrease by 6 dB as the frequency doubles. However, the reflection coefficient, especially for absorber materials, also decreases with frequency and thus, there is no guarantee that end effects become smaller with frequency.

Let us consider a quarter wavelength slab with the relative dielectric constant equal to 4. For this case, the transmission coefficient, $T$, is $-1$ and the reflection coefficient, $\Gamma$ is exactly zero. This means that, if the slab is infinitely wide, there would be no reflection but complete transmission for normal incidence. However, the end effects due to the slab truncation produce nonzero backscattered fields. In this case, the total field comes from only the edge diffracted fields such that

$$E_t = E_d = \frac{2A}{\sqrt{k}}. \quad (5.7)$$

One can define the reflection coefficient by normalizing the slab total field against a perfectly conducting strip of the same size. For a strip more than 1 $\lambda$ wide, its scattered field can be obtained by Equation (5.5) with $\Gamma = -1$ such that

$$|E_t| = AW\sqrt{k}. \quad (5.8)$$

Thus, the UTD reflection coefficient can be obtained as

$$\frac{E_t(\text{slab})}{E_t(\text{PEC})} = \frac{2A}{\sqrt{k}} \frac{\lambda}{AW\sqrt{k}} = \frac{\lambda}{\pi W}. \quad (5.9)$$

This UTD result is compared with the moment method solution [34] in Figure 173. As discussed above, the end effect errors decrease linearly with frequency. These
two solutions basically have the same trend with frequency. The discrepancy is due to the fact that the heuristic UTD solution [15] assumes a very thin slab, but the actual slab thickness is quarter wavelength. In other words, the slab thickness for the UTD solution needs to be so thin that the diffracted field can be attributed to a single point at each end. With this in mind, it is clear that the observed scattered fields are associated with the end effect errors.

Since absorber material is usually metal backed, it is of interest to examine a metal backed slab. In this case, $T = 0$ and $\Gamma \neq 0$. Thus, based on Equations (5.4) and (5.5), the scattering components can be described by

\begin{align*}
E_d &= \frac{A}{\sqrt{k}}, \text{ and} \\
E_r &= A\Gamma(f)W\sqrt{k}. 
\end{align*}

(5.10) \hspace{2cm} (5.11)

![Figure 73: Reflection coefficient of a dielectric slab versus width. $\varepsilon_r = 4$ and $d = 0.25\lambda_o$.](image)

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The measurement of the reflected field is accurate only if the reflected field is dominant; i.e.

\[ \frac{E_r}{E_d} \gg 1, \text{or}, \Gamma(f)Wk \gg 1. \]  

(5.12)

Based on this criteria, the edge effects become more serious for low reflection levels. Increasing the sample size can reduce the edge effects because the edge diffraction does not increase with width while the reflected field does. From the above equation, one would be tempted to conclude that, given a fixed sample width, the end effect becomes less significant with frequency. However, the reflection coefficient, \( \Gamma \), normally also decreases with frequency in most absorber cases. Consequently, the end effect errors can be significant throughout the frequency spectrum.

5.3.2 Thick Slab

For an absorber measurement, the absorber is usually electrically thick. The above derivation for a thin slab cannot be applied rigorously to a thick geometry. In addition, it is difficult to include the absorber taper effect. However, scattered fields can still be symbolically decomposed into the edge diffracted field from the ends and the reflected field such that

\[ E_d = A\frac{D(f)}{\sqrt{k}}, \text{ and} \]

\[ E_r = A\Gamma(f)W\sqrt{k} \tag{5.14} \]

where \( f \) represents frequency. Note that the quantity \( D(f) \) is introduced to represent the thickness effect experienced by the edge diffracted wave. Again the measurement of the reflected field can be accurate if \( \frac{E_r}{E_d} \gg 1; \) i.e.

\[ \frac{\Gamma(f)Wk}{D(f)} \gg 1. \]  

(5.15)

This expression is exactly the same as Equation (5.12) except for the thickness factor \( D(f) \). Since absorber materials are lossy, the edge diffracted field is attenuated when
propagating along or through absorber sides. Thus, the magnitude of \( D(f) \) should be smaller than 1. This observation suggests that, given the same reflection level and slab width, the end effect errors for a thick absorber case are relatively smaller than the thin slab case.

5.3.3 Absorber Panel

The above discussions have focused on 2D cases. In an absorber measurement, absorber materials are usually mounted on a square metal plate and there are both horizontal and vertical edges. For a square panel of side length \( W \), the previous 2D expression in Equations (5.13) and (5.14) can be transformed to the 3D case by multiplying the appropriate conversion factor, \( \sqrt{\frac{2}{\lambda}} W \). With \( D_h \) and \( D_v \) denoting the thickness factors for the horizontal and vertical edges, respectively, one can represent the scattering mechanisms by

\[
E_d = \frac{A(D_v(f) + D_h(f))}{\sqrt{k}} \sqrt{\frac{2}{\lambda}} W = \frac{A}{\sqrt{\pi}}(D_v(f) + D_h(f)) W
\]

and

\[
E_r = \frac{A \Gamma(f)}{W} \sqrt{\frac{2}{\lambda}} W = 2 A \sqrt{\frac{\Gamma(f) W^2}{\lambda}}.
\]

The end effects are insignificant if \( \frac{E_r}{E_d} \gg 1 \); i.e.

\[
\frac{E_r}{E_d} = \frac{\Gamma(f) W k}{(D_v(f) + D_h(f))} \gg 1.
\]

As compared with the 2D case in Equation (5.15), the basic difference is the added diffraction term, \( D_R \), from the horizontal top and bottom edges. Note that \( \Gamma(f) \) can be calculated by the PMM or TLA methods. However, it is a complicated matter to evaluate \( D(f) \) for a general absorber structure. Since both \( D(f) \) and \( \Gamma(f) \) decrease with frequency, there is no guarantee that the edge effects become insignificant with
frequency although the wavenumber, $k$, is in the numerator. From Equation (5.18),
doubling the sample dimension can reduce the edge effect error by 6 dB. This fact
can also be understood by noting that the reflected field increases 12 dB by the area
but the edge diffracted field increases only 6 dB by the periphery length. There is
a limit, however, to the panel size that one can practically handle. Consequently,
absorber measurements are normally corrupted by end effect errors. Due to the
complicated nature of the end effect errors, ISAR image techniques become a useful
tool to determine their significance.

5.4 ISAR Image Techniques

Based on the back projection algorithm [16, 17], the calibrated frequency and
angle scan data can be used to generate ISAR image which can be used to identify
scattering mechanisms associated with the absorber target. The frequency sweep
data at each rotation angle is transformed to a time domain response by using the
Fast Fourier transform. Based on the rotation angle and time position, the time
domain data is projected to the image spatial domain and integrated to obtain a
coherent summation. If there is any stationary scattering center associated with the
measured target, the image value will peak up at that position due to the coherent
summation. Therefore, from an ISAR image, one can identify various scattering
mechanisms associated with the absorber and support geometry. Thus, this tech­
nique provides a diagnostic tool to examine the absorber manufacturing problems
such as improper material doping and cutting problems. In addition, based on
Fourier Transform's invertible property, one can edit the ISAR image to remove
or minimize the unwanted scattering mechanisms, such as mounting error and end
effects, and then transform the corrected image back to the frequency and angle
domains. The resulting data should then much better represent the true absorber performance.

5.4.1 Diagnostic Applications

A series of measurements for a 6" curved pyramidal absorber is used to demonstrate the diagnostic capabilities of this ISAR image technique. The curved geometry, as shown in Figure 74, was designed based only on the PMM method [3]. This

\[ \text{Figure 74: The geometry of a 6" curved pyramidal absorber.} \]

6" pyramid was used as a scale model for a 40" pyramid to test if \(-40 \text{ dB reflection level at normal incidence can be achieved with a wavelength thick material.} \)

Note that 6" corresponds to a \(\lambda\) at 2 GHz. The measurements were performed using a 24" square panel, and a 6" sphere located at the rotation center was used as
the reference. The ISAR image based on measured data taken from 2-8 GHz and ±15° angular span is shown in Figure 75. Note that the plate position is about at −12.5" in the down range direction. Some scattering mechanisms associated with the mounting structures are indicated in the image. As can be easily seen, the image stripe slightly behind the plate position is the most dominant term associated with the absorber panel. Based on its uniform amplitude and the time delay, one can conclude that this image stripe results from the wave penetrating the absorber and scattered back from the mounting plate. The wave penetration is so strong that multiple bounces between the plate and valley are seen in the image area behind the plate position. These observations indicate that the absorber material does not have enough loss to effectively attenuate the incident wave. A good absorber design should minimize the base scattering with enough material absorption. As a result, there is a serious dielectric constant discrepancy between the material made and the one used in the theoretical design. Recall that this 6" pyramid is a scale model for a 40" pyramid which will be applied above 300 MHz. As shown in Chapter 4 (Figure 44), absorber dielectric constant at 300 MHz is significantly larger than that at 2 GHz. For this reason, the 6" scale model does not have enough loss. Consequently, one should be very cautious about the dielectric constant change when scaling absorber dimension. The second reason for this improper material doping is that the absorber company did not choose more heavily doped options. As shown in Figure 76, there was, for example, another available material which has a higher loss tangent than the 6" pyramid material. As discussed in Chapter 4, the more lossy material should provide better performance at normal incidence, especially for a pyramid structure. The TLA calculated results for this 6" pyramid with these two different materials are compared in Figure 77. As can be easily seen, there is more than 10 dB improvement by using the more lossy material.
Tomographic Processing, HAMMING down range, HAMMING cross range
Original Freq. (GHz): Fmin = 2.002  Fmax = 8.002  DeltaF = 0.020
Used Freq. (GHz):  Flo = 2.002  Fhi = 8.002
Original Asp. (deg.): Tmin = -14.00  Tmax = 14.00  DeltaT = 0.20
Used Asp. (deg.):  Tlo = -14.00  Thi = 14.00
Far Field, Tbistatic = 0 deg.

Figure 75: The ISAR image for a 24" square panel of the 6" curved pyramidal absorber.
Figure 76: Comparison of the dielectric constants for the 6" curved pyramid and the other commercially available material.
The end effect errors are not significant in the previous ISAR image since the base reflected field from the plate is predominant. To reduce the base scattering, additional absorber material, 2.25" thick, was added to the 6" pyramid’s base to form a 8.25" pyramid. The corresponding ISAR image is shown in Figure 78 with the same amplitude scale as the 6" one. One can notice that the base plate image is not uniform and edge effects are apparent. In addition, the strong multiple bounces between the plate and the valley are significantly reduced due to the added 2.25" base. From the ISAR image, the valley scattered fields are most dominant, which is characteristic of a pyramidal absorber made of a low loss material. The edge diffracted fields from the two ends have stronger amplitude density than the direct penetration. Note that the four panel edge lines were surrounded by a row of 7.5" pyramids to taper off the shadowing effect across the plate terminations. The 7.5"
Tomographic Processing, HAMMING down range, HAMMING cross range

Original Freq. (GHz): Fmin = 2.002  Fmax = 8.002  DeltaF = 0.020
Used Freq. (GHz): Flo = 2.002  Fhi = 8.002

Original Asp. (deg): Tmin = -14.00  Tmax = 14.00  DeltaT = 0.20
Used Asp. (deg): Tlo = -14.00  Thi = 14.00

Far Field, Tbistatic = 0 deg.

Figure 78: The ISAR image for a 24\" square panel of the 8.25\" curved pyramidal absorber. The image gating to extract the desired signal is indicated by the dotted lines.

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pyramids were made with the 6" curved pyramids plus a 1.5" base. Since the 1.5" base was placed behind the plate, the end effects corresponding to the two side edges are imaged behind the plate as shown in Figure 78. Although this arrangement reduces the scattered fields from the edge lines, the scattered fields from the added material itself introduces the other error terms. With the image editing algorithm implemented as shown in the next section, the flush mounted configuration is more desirable.

5.4.2 Image Editing

Once the error terms are identified, they can be gated out or attenuated in the image domain. The corrected image can then be transformed back to the frequency and angle domain to better represent absorber performance. The algorithm based on ISAR image techniques to remove the mounting errors and end effect errors is summarized in Figure 79. Note that the frequency extrapolation is useful to reduce the end band Gibb's phenomenon in transforming the filtered image [17]. The key step in this algorithm is to identify and isolate error mechanisms. Either the PMM or TLA calculation may help interpret the images provided the absorber materials and shape are accurately determined. Once the measured image is corrected, the true absorber performance is obtained using the reconstructed frequency and angle data. This image editing procedure has been implemented by extending the RIDE computer code developed by Gupta and Beals [35].

(a) 8.25" Curved Pyramid

Let us take the previous 8.25" pyramid as an example. To gate out the mounting error, a rectangular gate is imposed to extract out absorber panel response as indicated in Figure 78. The two image spots corresponding to the end effects are
Take Freq & Angle Data

Extrapolate Frequency Data if Necessary

Generate ISAR Images

Identify Scattering Mechanisms

Remove or Attenuate Error Mechanisms in Images

Reconstruct Freq & Angle Data from the Corrected ISAR Image

Figure 79: The flow chart for the ISAR image editing technique.
Figure 80: Comparison of the image processed and original reflection response.

attenuated by 8 dB so that these error terms are not dominant in the base scattering and are about of the same magnitude order as the base scattering term. The reconstructed frequency response is then compared with the original data in Figure 80. The image processed response basically passes through the average of the original response. At some frequencies, the amplitude variation is more than 5 dB. In fact, the image processing effect for this absorber structure is not very significant because the absorber scattering is relatively strong itself.

(b) 18” Straight Pyramid
The next example is an 18” straight pyramidal absorber. This pyramid has a 2” base and 6” period, and a 24” square panel was measured. The ISAR image corresponding
to the 2-8 GHz frequency band and ±15° angle span is shown in Figure 81. As one can observe, the end effects and the mounting error terms are more significant than the absorber material itself mainly due to the large absorber dimension. In this case, the image processing technique must be applied to obtain a correct reflection level. As indicated in Figure 81, a rectangular gate was applied to extract the absorber panel data. Other than the two obvious end effects that correspond to the two side edges, the image stripe between them at −12.5" is mainly contributed by the top and bottom edges of the panel. The following arguments are used to support this observation. (1) If the line mechanisms were signals penetrating absorber and scattered back from the metal plate, they should be imaged at a position behind the plate due to propagation time delay in dielectric absorber materials. (2) The four edge lines of the absorber panel should all have end effects. Each of the two distinctive spots represents a line integration along the side edge of the panel. As a result, they have stronger image intensity than the top and bottom edges. (3) One might argue that the stripe mechanism results from the valley scattering. If this were true, the incident wave should be able to penetrate the thin 2" base and bounce back significantly from the mounting plate. However, there is no such base scattering found behind the plate. Based on the above arguments, the end effects for the two side edges are attenuated by 25 dB using the rectangular gates indicated in Fig. 81. For top and bottom edges, an 8 dB attenuation gate is applied so that this error mechanism is not stronger than the rest of absorber scattering. The resulting frequency response for this corrected result is compared with the original data in Fig. 82. As one can observe, the removal of mounting error does not have significant impact for this material; while, The end effect errors are very significant and cause 10 to 15 dB error across the frequency band. The corrected reflection coefficient around 2 GHz is about -50 dB, which is what one should expect for this material.
Figure 81: The ISAR image for a 24" square panel of 18" straight pyramids. The image gating to extract the desired signal is indicated by the dashed lines.
This example demonstrates that the ISAR image processing techniques become very important for high performance absorbers with low reflection levels.

5.5 Measurement Comparison for a 6'' Curved Wedge

The measurement techniques described in this Chapter were applied to a 6'' curved wedge design. As part of the design effort for the 40'' curved wedge absorbers, a 6'' scaled one was tested to examine the design and manufacture aspects. The 6'' length was chosen so that the absorber dimension is a wavelength at 2 GHz, which is the lowest operating frequency for the normal feed antenna setup in the OSU/ESL compact range. The 6'' absorber was cut out of 8'' commercial wedges. The dielectric
property of this absorber material was measured in a slotted line [31] and is shown in Figure 83.

The original goal was to design a 6" wedge absorber with metal backing to have a -40 dB reflection coefficient at 2 GHz for normal incidence. The optimization scheme discussed in Chapter 4 was applied. There are basically two types of designs whose geometries and PMM calculated frequency responses are shown in Figure 84. The polarization parallel to the edge is used here since this is usually the worst case. The main difference between these two designs is the choice of the frequency band in optimization. For the first one, the midband from 2.8 GHz to 4.3 GHz was used for optimization and it results in very thin edge design. The sharp edge has very small diffracted fields at frequencies above 3 GHz. However, the lack of absorber material deteriorates the low frequency performance because significant energy can penetrate and come back from the metal backing plate. For the second design, the low band from 1.7 GHz to 2.5 GHz is emphasized in the optimization design procedure. The better low frequency performance is achieved by having edge and valley scattered fields destructively interact and creating a null centered around 2.5 GHz. However, the strong edge diffraction does not drop as fast as the first case. Recall that such tradeoff has been observed in the 40" wedge designs #W3 and #W4, which were presented in Chapter 4. Neither of these designs meets the -40 dB requirement at 2 GHz. The optimization process tends to show that it is very difficult to reach -40 dB reflection level for one wavelength thick wedge absorber, at least for the uniform material case. Between these two basic choices, the first design was chosen because of its fast drop to the -40 dB requirement at 3 GHz and very good high frequency performance.

A 24" square panel was manually cut out of 8" blue wedge commercial materials. There were 20 elements and each 24" element was constructed from 3 small
Figure 83: Measured dielectric constant for a commercial wedge which is used for the 6\textdegree curved wedge design.
Figure 84: Two 6" curved wedge designs.
wedges of 8\" length. With large amount of cutting and very thin wedge edges, there could be measurement errors due to geometry inaccuracies. This absorber panel was measured and processed using the above-mentioned image techniques.

(a) Polarization Parallel to the Wedge Edges
The ISAR images corresponding to the 1.5-8.5 GHz band and ±15° is shown in Figure 85. Note that the measured data was calibrated against the mounting plate measurement so that 0 inch down range location corresponds to the plate position. The extrapolation scheme was applied to obtain the 1.5-2 GHz data to avoid end band Gibb's phenomenon around 2 GHz. The most dominant mechanism in the image comes from the wedge edge diffraction. In this case, the end effects should be insignificant. A rectangular pass gate was applied to gate out the absorber panel and the end effects are trimmed down by 5 dB. The reconstructed frequency responses are compared with the original result in Figure 86. As expected, the panel end effects in this wedge case do not have significant impact on the response. The error terms do no significantly change the frequency behavior because they are relatively small compared to the absorber reflected signals from the panel area. Finally, the measured and PMM calculated results are compared in Figure 87. The measured one is always higher than the calculated one. The agreement is good only in the general trend. From the time domain response, the measured edge diffraction term is larger than the calculated one, which indicates the edge thickness is wider than designed. In fact, the absorber material is too flimsy to accurately cut the very thin edge curve. Such a realistic problem was not considered in the numerical optimization. For the same reason, the designed fast drop to -40 dB at 3 GHz is shifted to about 3.6 GHz. The general agreement in the response indicates that one should be able to realize the designed absorber performance if the absorber geometry quality
Figure 85: The ISAR image for a 24” square panel of the 6” curved wedge absorber. Polarization is parallel to wedge edges.
Figure 86: Processed reflection coefficients at normal incidence for the 6" curved wedge with polarization parallel to the edge.
Figure 87: Comparison of the measured and PMM calculated reflection coefficients at normal incidence for the 6° curved wedge with polarization parallel to the edge.
control in absorber making is improved. Even so this new wedge still performs much better than the conventional wedge materials.

(b) Polarization Perpendicular to the Wedge Edges
The ISAR images corresponding to the 1.5-8.5 GHz frequency band and ±15° angular span around normal incidence is shown in Figure 88. Similar image processing as the parallel polarization was used. The most dominant mechanism associated with the absorber panel comes from the wedge edge diffraction which is located 6" before the plate. From the image, the end effects are insignificant but the mounting errors must be removed from the image. A rectangular pass gate was applied to gate out the absorber panel, and the end effects are trimmed down by 5 dB. The reconstructed frequency responses are compared with the original result in Figure 89. As expected, the panel end effects in this case do not have significant impact on the response. The mounting error in this polarization is very significant in that the reflection level is reduced 10 dB after the mounting error is removed. Although it is possible to use bulky absorber materials to cover the support structure, it would take more effort in mounting and dismounting the absorber panel. Therefore, the image processing technique is preferred. The measured and PMM calculated results are compared in Figure 90. The agreement is generally good except at the low frequency end of the band. From the time domain response, the measured edge diffraction term is larger than the calculated one, which indicates the same cutting problem as found in the parallel polarization result. The erroneous wedge edge curve incidentally creates a null shape around 2.4 GHz, which is not predicted by design. Because of a slightly thicker edge, the measured high frequency result is about 4 dB higher than the theoretical calculation. Considering the various absorber manufacturing problems, the agreement found in this measurement set is satisfactory.
Tomographic Processing, HAMMING down range, HAMMING cross range

Original Freq (GHz) \( F_{\text{min}} = 2.002 \), \( F_{\text{max}} = 8.502 \), \( \Delta F = 0.020 \)

Used Freq (GHz) \( F_{\text{lo}} = 1.502 \), \( F_{\text{hi}} = 8.502 \) (MCM extrapolation)

Original Asp (deg) \( T_{\text{min}} = -15.00 \), \( T_{\text{max}} = 15.00 \), \( \Delta T = 0.25 \)

Used Asp (deg) \( T_{\text{lo}} = -15.00 \), \( T_{\text{hi}} = 15.00 \)

Far Field, T bistatic = 0 deg

Figure 88: The ISAR image for a 24" square panel of the 6" curved wedge absorber. Polarization is perpendicular to wedge edges.
Figure 89: Processed reflection coefficients at normal incidence for the 6" curved wedge with polarization perpendicular to the edge.
Figure 90: Comparison of the measured and PMM calculated reflection coefficients at normal incidence for the 6\textdegree curved wedge with polarization perpendicular to the edge.
5.6 Summary

Absorber measurements are corrupted by the mounting errors and the end effect errors. The mounting errors due to the support structure are usually recognized and removed. However, the end effect errors due to the finite panel size are not normally removed. This error mechanism can be significant throughout the frequency spectrum, especially for high performance absorbers such as the 18" pyramid. To properly determine absorber performance, ISAR image processing techniques have been introduced to remove the error terms. As illustrated by the measured data of 6" pyramids, ISAR images also provide diagnostic information about material doping. With these applications, the image processing techniques will become more important as new high performance absorbers are developed.

These absorber measurement techniques were then applied to the optimized 6" curved wedge measurements. Considering the inaccuracies in making the 24" square panel, the agreement between the measured and theoretical results was generally good. It is felt that, once the absorber quality control is improved, one can accurately and effectively design high performance absorbers using the TLA and PMM analyses; and experimentally verify the designs by using the advanced image processing techniques presented here.
CHAPTER VI
Chebyshev Absorber Design

Under plane wave illumination, the scattered fields from an infinite absorber wall are coherent in the specular direction. Such a coherence property produces a strong specular return and theoretically zero scattered field otherwise provided grating lobe phenomenon does not occur. To reduce the dominant specular scattering, one can partially destroy the strong coherence by dividing absorber elements into different levels. The scattered fields from these levels have unequal path lengths so that they can effectively cancel one another to achieve a lower reflection level. This multilevel design idea is exactly the same as that for multisection impedance transformers which have been well publicized [18, 19]. Among numerous choices, the Chebyshev impedance transformer was found to be optimized in terms of the bandwidth and reflection level tradeoff. Therefore, the Chebyshev configuration is adopted for this multilevel design.

The other desirable aspect of Chebyshev absorber is that the addition of the Chebyshev pattern is basically independent of the original absorber design, which has been addressed in Chapter 4. The resulting reflected field for the Chebyshev absorber is the product of the Chebyshev factor and the original absorber performance. As discussed in the 6″ curved wedge case, the measured results for a periodic absorber structure are usually worse than the designed performance. The discrepancy results from the cutting error and inhomogeneity in the absorber materials. The optimization at the low frequency end is achieved with several mechanisms interacting
destructively. As a result, the cutting error and material inhomogeneity can easily corrupt the designed frequency response. In this case, the Chebyshev steps can provide another reduction factor for the critical low frequency band. Measured and calculated results for both wedge and pyramidal cases will be presented to validate this new absorber structure.

6.1 Chebyshev Transformer and Its Application to Multilevel Absorber Design

The concept of an impedance transformer is briefly summarized and applied to a multi-level absorber design. The Chebyshev type design will be emphasized because of its desirable characteristics.

6.1.1 Multisection Impedance Transformers

In microwave circuits, an impedance transformer is used to match two different impedance loads or transmission line sections so that the reflected field caused by the impedance mismatch is minimized. Multiple matching sections are usually needed for wideband applications. A systematic synthesis method for equal step transformers was developed based on the small reflection analysis in [18, 19]. As shown in Figure 91 for an \( N \)-section(step) impedance transformer, the reflection coefficients, denoted as \( \Gamma_0, \Gamma_1, \ldots, \Gamma_N \) are assumed to be small so that the higher order interaction among sections can be neglected. As a result, the total reflection for a matching transformer with equal separation, \( d \), can be described by

\[
\Gamma(\psi) = \Gamma_0 + \Gamma_1 e^{-2j\psi} + \Gamma_2 e^{-4j\psi} + \cdots + \Gamma_N e^{-2jN\psi} \tag{6.1}
\]

where \( \psi = kd = \frac{2\pi}{\lambda}d \) and \( \lambda \) is the effective wavelength for the propagating wave. By properly choosing the step and adjusting \( \Gamma_n \)'s values, one can minimize the
Figure 91: Analogy between a multisection impedance transformer and a multilevel absorber design.
total reflection coefficient. Such a reflection reduction scheme can be applied to the absorber design as illustrated in Figure 91. If the scattered field from each element is essentially the same, one can then divide the absorber elements into different height levels. The number of elements in each level is proportional to the reflection coefficient at that level. In other words, the number of absorber elements in each layer, $M_n$, can be represented by

$$M_n = M_t \frac{\Gamma_n}{\Gamma_0 + \Gamma_1 + \cdots + \Gamma_N} = M_t \tilde{\Gamma}_n, \quad n = 0, 1, \ldots, N$$

(6.2)

where $M_t$ is the total number of elements in a unit panel and $\tilde{\Gamma}_n$ as defined above will be referred to as the normalized reflection coefficient at level $n$. By noting that

$$\Gamma(0) = \Gamma_0 + \Gamma_1 + \cdots + \Gamma_N,$$

one can obtain that

$$\Gamma(\psi) = \Gamma(0) \cdot \tilde{\Gamma}(\psi)$$

(6.3)

where

$$\tilde{\Gamma}(\psi) = \tilde{\Gamma}_0 e^{-2j\psi} + \tilde{\Gamma}_1 e^{-4j\psi} + \cdots + \tilde{\Gamma}_N e^{-2jN\psi}.$$  

(6.4)

For a multilevel absorber case, $\Gamma(0)$ represents the original absorber reflection coefficient because the level step size is zero. The normalized reflection coefficient, $\tilde{\Gamma}(\psi)$, provides the additional reduction factor due to the multilevel configuration. The following discussion will be focused on this normalized factor.

The height difference can be simply implemented by adding absorber blocks in the base area. Therefore, the absorber base area does not have the same step patterns as the taper sections. The base scattered fields from different levels do not have the designed multilevel pattern as the taper section. For a well designed absorber, the base scattering should not be a significant mechanism. As a result, the multilevel design should provide significant reduction for the major part of reflection mechanisms. To fit in the small reflection model, the interaction between different
levels should be negligible, which is true at least for angle of incidence closer to normal incidence. As will be shown later, this model can be applied to wide angle cases if the angle factor is taken into account.

Several properties associated with the small reflection model are worthy of being mentioned. The frequency response, \( \tilde{T}(\psi) \) is periodic with a period determined by

\[
kd = \frac{2\pi}{\lambda} \cdot d = \pi \quad \text{or} \quad f_p = c/2d
\]

(6.5)

where \( c \) is the free space speed of light. There is no multilevel effect or \( \tilde{T}(\psi) = 1 \) when \( \psi = 0, \pi, 2\pi \cdot \cdot \cdot \) or equivalently when \( f = 0, f_p, 2f_p \cdot \cdot \cdot \). The design effort here is on the first band \( f = (0, f_p) \) since the higher bands could be dealt with smaller step size if they were the design focus. The center frequency of the design band, \( f_c \), is at \( \frac{f_p}{2} \). The level step size, \( d \), is equal to the quarter wavelength at the center frequency; i.e. \( d = \frac{\lambda}{4} \). For this reason, an equal length impedance transformer is also called a multi-section quarter wave transformer.

6.1.2 Synthesis

To synthesize the total reflection response described in Equation (6.4), one can assume the transformer is made symmetric with \( \bar{T}_0 = \bar{T}_N, \bar{T}_1 = \bar{T}_{N-1} \cdot \cdot \cdot \), the reflection coefficient can then be written as

\[
\tilde{T}(\psi) = 2e^{-jN\psi}[\bar{T}_0 \cos N\psi + \bar{T}_1 \cos(N-2)\psi + \cdot \cdot + \bar{T}_n \cos(N-2n)\psi + \cdot \cdot + \bar{T}_l]
\]

(6.6)

where \( \bar{T}_l = \frac{1}{2} \bar{T}_{\frac{N}{2}} \) for \( N \) is even; and \( \bar{T}_l = \bar{T}_{\frac{N-1}{2}} \cos \psi \) for \( N \) is odd. The above representation is a Fourier Cosine series which can be used to synthesize any desired reflection coefficient response by properly choosing \( \bar{T}_n \)'s and using a large enough \( N \) value.
6.1.3 Chebyshev Transformer

There are two most commonly used synthesis responses; i.e. the binomial and Chebyshev responses. The binomial one has a maximum flat response around the center frequency but has a very narrow bandwidth given a reflection coefficient limit. On the other hand, the Chebyshev transformer has a wider passband width at the expense of the reflection ripple inside the passband [18, 19]. For the absorber application, the bandwidth is a more important factor than the passband amplitude. It is not practical to design a multilevel absorber structure with zero reflection coefficient because the inhomogeneity among elements can produce residual scattered field. Usually one would like to have absorber performance below a certain threshold for as wide a frequency band as possible. Consequently, the Chebyshev configuration is a better choice.

An N-section Chebyshev transformer is implemented by synthesizing the total reflection response in terms of the \( N \)th order Chebyshev polynomial, \( C_N(x) \). The first two order of Chebyshev polynomials are given by [19]

\[
C_1(x) = x, \quad \text{and} \\
C_2(x) = 2x^2 - 1. 
\]

(6.7) \hspace{1cm} (6.8)

A Higher order one can be found using the following recurrence formula :

\[
C_n(x) = 2xC_{n-1}(x) - C_{n-2}(x) .
\]

(6.9)

It can be shown that an \( N \)th order Chebyshev polynomial is also an \( N \)th order polynomial.

These polynomials have some useful features as follows :

1. \( |C_n(x)| < 1 \) if \( |x| < 1 \); and the function value oscillates between \( \pm 1 \) with equal ripple. This region will be mapped to the passband.
2. \(|C_n(x)| > 1\) if \(|x| > 1\); and \(|C_n(x)|\) increases faster with \(x\) as \(n\) increases. This region will be mapped to outside the passband.

3. \(C_n(\cos \psi) = \cos n\psi\) and \(C_n(\cosh \psi) = \cosh n\psi\).

One can synthesize the cosine series representation of the reflection coefficient in Equation (6.6) using the Chebyshev polynomial \(C_N(a \cos \psi)\) since it is a function of the power of \(\cos \psi\) and \(\cos^n \psi\) can be expanded by \(\cos(n - 2m)\psi\), for \(n = 0, 1, \ldots N\) and \(m = 0, 1, \ldots n/2\). To utilize the first two features, a \(\psi_m\) value is assigned so that \(a \cos \psi_m\) can be mapped to 1 and \(a \cos(\pi - \psi_m)\) to -1. Then, the constant \(a\) is found to be \(\sec \psi_m\) and \(\psi_m\) corresponds to the lowest passband frequency. Therefore, the frequency response has an equal ripple in the passband from \(\psi = \psi_m\) to \(\pi - \psi_m\). Outside the passband, the frequency response rises up at a faster rate for a larger order. As a result, the reflection coefficient in (6.6) is synthesized for an \(N\)-section transformer as

\[
\tilde{\Gamma}(\psi) = \tilde{\Gamma}_m e^{-jN\psi} C_N(\sec \psi_m \cos \psi). \tag{6.10}
\]

The constant \(\tilde{\Gamma}_m\) is the passband ripple amplitude, which can be determined by letting \(\psi = \psi_m\) in Equation (6.10) such that

\[
\tilde{\Gamma}_m = |\tilde{\Gamma}(\psi_m)|. \tag{6.11}
\]

By noting that \(\tilde{\Gamma}(0) = 1\), one can establish the relationship between \(\tilde{\Gamma}_m\) and \(\psi_m\) such that

\[
\tilde{\Gamma}(0) = 1 = \tilde{\Gamma}_m C_N(\sec \psi_m). \tag{6.12}
\]

Provided that either one of these two parameters is specified, the normalized reflection coefficients can be determined for a given \(N\) value. Let us take \(N = 3\) as an example. Using the recurrence formula in Equation (6.9), the expression for \(\tilde{\Gamma}(\psi)\) in
Equation (6.10) can be explicitly written as

\[ \bar{\Gamma}(\psi) = \bar{\Gamma}_m e^{-jN\psi}[\sec^3 \psi_m \cos 3\psi + (3 \sec^3 \psi_m - 3 \sec \psi_m) \cos \psi]. \]  

(6.13)

By comparing with the cosine series expression in Equation (6.6), one can show that

\[ \bar{\Gamma}_0 = \bar{\Gamma}_3 = \frac{\bar{\Gamma}_m}{2} \sec^3 \psi_m, \text{ and} \]

\[ \bar{\Gamma}_1 = \bar{\Gamma}_2 = \frac{\bar{\Gamma}_m}{2}(3 \sec^3 \psi_m - 3 \sec \psi_m). \]

(6.14)

(6.15)

The lowest passband frequency, \( f_m \), is related to the center frequency by the value, \( \psi_m \), such that

\[ \frac{f_m}{\psi_m} = \frac{f_c}{0.5\pi} \quad \text{or} \quad f_m = \frac{2f_c}{\pi} \psi_m. \]

(6.16)

The passband width normalized against the center frequency can be determined by

\[ \frac{\Delta f}{f_c} = \frac{(\pi - \psi_m) - \psi_m}{0.5\pi} = 2 - \frac{4\psi_m}{\pi}. \]

(6.17)

It is important to note that, based on Equation (6.12), when the ripple level, \( \bar{\Gamma}_m \), decreases, \( \psi_m \) approaches \( \pi \); and thus, the bandwidth shrinks. So there is a tradeoff between the bandwidth and reflection level.

### 6.1.4 Design Procedures

There are four design parameters involving the Chebyshev absorber; i.e. number of sections \( N \), center frequency of passband \( f_c \) or equivalently step size \( d \), lowest frequency for the passband \( \psi_m \) and the passband ripple amplitude \( \bar{\Gamma}_m \). In fact, \( \psi_m \) and \( \bar{\Gamma}_m \) are correlated by Equation (6.12). Therefore, there are only three independent design variables. The general design considerations are to have

1. Small matching section length \( N \times d \) so that the overall absorber height dimension can be confined.
2. Small number of sections \((N)\). This is to reduce the effort of making different absorber levels and minimize the rising rate of the frequency response outside the passband.

3. Low passband ripple amplitude \((\tilde{T}_m)\) to reduce absorber reflection levels.

4. Wide passband width or small \(\psi_m\) value for a wideband absorber application.

These requirements are mutually conflicting, and one must make tradeoffs among these factors. For example, bandwidth becomes narrower for lower ripple amplitude. The other constraint is that the number of absorber elements is discrete. Therefore, the normalized reflection coefficients need to be adjusted so that an integral number of elements can be chosen.

Let us use the 6\(^{th}\) curved wedge absorber to explain the design procedure. The reflection coefficients for both polarizations are again shown in Figure 92. The design goal was to reach \(-40\) dB reflection coefficient above \(2\) GHz. It is obvious that these optimized results do not meet this requirement, especially for the polarization parallel to the edges. One can improve the performance by using the Chebyshev design provided that the lower end of the passband is lower than \(2\) GHz. The passband ripple level should be around \(-15\) dB so that the reflection coefficient for the parallel polarization case can possibly reach the \(-40\) dB goal. Based on \(f_m = 2\) GHz and \(\tilde{T}_m = -15\) dB, one can calculate the other relevant parameters as shown in Table 1. Recall that the Chebyshev factor is a periodic function with a period equal to \(2f_c\). As a result, \(\tilde{T}(0) = \tilde{T}(2f_c) = 1\); and hence, there is no reflection reduction due to the Chebyshev effect at \(0, 2f_c, 4f_c \ldots\). As can be seen in Table 1, both the bandwidth and the total matching section length increases as \(N\) increases. For the \(N = 1\) case, the bandwidth is so narrow that the highest passband frequency is 2.51 GHz, which is not acceptable. For the \(N = 2\) case,
the bandwidth is nearly four times than the $N = 1$ case but still quite marginal. The bandwidth can be further doubled when $N = 3$ is chosen. In addition, the total section length increases only about 10% from the $N = 2$ case. The bandwidth for the $N = 3$ case is sufficient since, as shown in Figure 92, the original reflection levels for the highest passband frequency (6.62 GHz) and the period frequency ($2f_c = 8.62$ GHz) are all below -45 dB. Even so, one may choose an even larger number of sections to implement the Chebyshev absorber but the bandwidth increase rate is not as significant as the lower $N$ cases and the improvement seems to be excessive. More importantly, the number of sections should be kept small to reduce the production cost. As a result, $N = 3$ is the optimal choice based on the tradeoff among the various design factors.
Table 1: Parameters for Chebyshev transformers when $f_m = 2$ GHz and $\tilde{\Gamma}_m = -15$ dB.

<table>
<thead>
<tr>
<th>N, number of sections</th>
<th>$f_c$(GHz) center frequency</th>
<th>d (inch) step length</th>
<th>Nd (inch) total length</th>
<th>$2(f_c - f_m)$(GHz) bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.26</td>
<td>1.31</td>
<td>1.31</td>
<td>0.51</td>
</tr>
<tr>
<td>2</td>
<td>3.18</td>
<td>0.93</td>
<td>1.86</td>
<td>2.35</td>
</tr>
<tr>
<td>3</td>
<td>4.31</td>
<td>0.69</td>
<td>2.06</td>
<td>4.62</td>
</tr>
<tr>
<td>4</td>
<td>5.52</td>
<td>0.54</td>
<td>2.14</td>
<td>7.03</td>
</tr>
<tr>
<td>5</td>
<td>6.76</td>
<td>0.44</td>
<td>2.18</td>
<td>9.52</td>
</tr>
</tbody>
</table>

Table 2: Parameters for a 3-step Chebyshev transformer when $N = 3$, $f_m = 2$ GHz and $\tilde{\Gamma}_m = -15$ dB.

<table>
<thead>
<tr>
<th>N, number of sections</th>
<th>$f_c$(GHz) center frequency</th>
<th>d (inch) step length</th>
<th>Nd (inch) total length</th>
<th>$2(f_c - f_m)$(GHz) bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.31</td>
<td>0.69</td>
<td>2.06</td>
<td>4.62</td>
</tr>
</tbody>
</table>

6.2 Chebyshev Design for 6\" Curved Wedge

The Chebyshev transformer concept is applied to the 6\" curved wedge, whose optimization procedure was discussed in Section 5.5. The basic principles about its Chebyshev design have been addressed as an example in the last section. The optimal choice for $N$ was found to be 3. The other Chebyshev parameters based on $\tilde{\Gamma}_m = -15$ dB, $f_m = 2$ GHz are summarized in Table 2. The total length is 2.06\", which is close to 2\". Therefore, 2\" is chosen and thus, $d = \frac{2}{3}$\". The other change is to have a safety margin in the passband width so that $f_m$ is chosen to be 1.8 GHz. In this case, the normalized reflection coefficients for different levels can be obtained.
Figure 93: Chebyshev response based on $N = 3, d = \frac{2}{3}''$ and $\tilde{\Gamma}_n = 0.25$ for $n = 0,1,2,3$.

by Equations (6.14) and (6.15) such that

$$\tilde{\Gamma}_0 = \tilde{\Gamma}_3 = 0.246; \quad \tilde{\Gamma}_1 = \tilde{\Gamma}_2 = 0.254$$

Note that, the wedge element width was equal to 1.2'', thus, a 24'' panel has 20 wedge elements. Since the number of elements in each level is proportional to the reflection coefficient, one can round off the numbers and choose 0.25 or 5 elements for each level. The Chebyshev response for this design is plotted in Figure 93. Note that the lowest passband frequency is slightly shifted from 1.8 GHz to 1.76 GHz. The ripple amplitude in the passband increases from -15 dB for the $f_m = 2$ GHz case to -11.3 dB for the current design. The degradation in this parameter is due to the increased bandwidth. The bandwidth factor is more important in that absorber performance at 2 GHz is most critical and some safety margin is needed to avoid any
Table 3: Parameters for a 3-step Chebyshev transformer when \( N = 3, Nd = 2'' \) and \( \bar{\Gamma}_n = 0.25, \quad n = 0, 1, 2, 3 \).

<table>
<thead>
<tr>
<th># of sections</th>
<th>( f_m ) (GHz)</th>
<th>( f_c ) (GHz)</th>
<th>( d ) (inch)</th>
<th>( Nd ) (inch)</th>
<th>( 2(f_c - f_m) ) (GHz)</th>
<th>( \bar{\Gamma}_m ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.76</td>
<td>4.5</td>
<td>0.667</td>
<td>2.00</td>
<td>5.48</td>
<td>-11.3</td>
</tr>
</tbody>
</table>

Figure 94: Chebyshev layout for the 6'' curved wedge design.

Slight passband shift due to possible construction inaccuracy. Also from Figure 93, the fact that 2 GHz is around the null area ensures this Chebyshev design.

The parameters for the final design are listed in Table 3. The Chebyshev layout is shown in Figure 94, which is generated by the block diagram used in the moment method calculation. Note that the 1'' base layer is not included in design but was added as a base to glue all the wedge elements together. Based on the moment method calculation and later measured results, it is found that the additional layer does not affect the desired Chebyshev response.
6.2.1 Measured Results for Normal Incidence

The measured results of this Chebyshev wedge design was processed using the image processing techniques as described in Section 5.4. The 2-18 GHz frequency scan data was extrapolated to the 1.5-18.5 GHz band at each angle. To better characterize the low frequency mechanisms, only the lower portion of frequency band is used to generate ISAR images when determining the image gating parameters.

(a) Polarization Parallel to the Wedge Edges

The ISAR image corresponding to the 2-8 GHz frequency band and ±15° around normal incidence is shown in Figure 95. Based on the 2-8 GHz ISAR image, a rectangular pass gate can be used to isolate absorber signals from the mounting errors. And the end effects were reduced by 10 dB. This image filtering information is applied to the ISAR images generated by the 1.5-18.5 GHz data. Therefore, the corrected 2-18 GHz data for the broadside direction is reconstructed from the filtered wideband ISAR images. The frequency responses are compared with the original ones without Chebyshev steps in Figure 96 for polarization parallel to the wedge edges. The Chebyshev design clearly improves the reflection response by 15 dB for low frequencies and the -40 dB level is achieved for all the frequency band. The Chebyshev pattern can be noted by the closer agreement between these two results at 9 and 18 GHz, where the periodic Chebyshev factor becomes 1. To illustrate the multilevel effect, the original reflection response multiplied by the Chebyshev factor is compared with the measured Chebyshev absorber result in Figure 97. As one can observe, the agreement is very good given the fact that the level junction and base scattered fields are not properly considered in the Chebyshev pattern.
Tomographic Processing, HAMMING down range, HAMMING cross range

Original Freq. (GHz): $F_{\text{min}} = 2.002$  $F_{\text{max}} = 8.002$  $\Delta F = 0.020$

Used Freq. (GHz):  $F_{lo} = 2.002$  $F_{hi} = 8.002$

Original Asp. (deg.):  $T_{\text{min}} = -15.00$  $T_{\text{max}} = 15.00$  $\Delta T = 0.25$

Used Asp. (deg.):  $T_{lo} = -15.00$  $T_{hi} = 15.00$

Far Field, $T_{\text{bistatic}} = 0$ deg

Figure 95: The ISAR image for a 24" panel of 6" Chebyshev curved wedges with polarization parallel to the wedge edges.
Figure 96: Comparison of the measured results for the 6\" Chebyshev and original curved wedge with polarization parallel to the wedge edges.
Figure 97: Comparison of the 6" Chebyshev measured results and original curved wedge results multiplied by the Chebyshev factor. Polarization parallel to the wedge edges.
(b) Polarization Perpendicular to the Edges

Similar processing was applied to the other polarization which is perpendicular to the wedge edges. The ISAR image corresponding to the 2-8 GHz frequency band and ±15° around normal incidence is shown in Figure 98. A rectangular pass gate can be used to isolate absorber signals from the mounting errors. And the end effects should be trimmed down by 10 dB. This image filtering information is applied to the ISAR images generated by 1.5-18.5 GHz data. Therefore, the corrected 2-18 GHz data for broadside direction is reconstructed from the filtered wideband ISAR images. The frequency response is compared with the original one without Chebyshev steps in Figure 99. The Chebyshev design clearly improves the reflection level by 10 dB for low frequencies and the -40 dB design goal is achieved for the full frequency band. The Chebyshev pattern can be noted by the close agreement between these two results at 9 and 18 GHz, where the periodic Chebyshev factor becomes 1. To illustrate the multilevel effect, the original reflection response multiplied by the Chebyshev factor is compared with the measured Chebyshev absorber result in Figure 100. Good agreement can again be observed.

6.2.2 Wide Angle Performance

The previous Chebyshev design was developed for normal incidence. The wide angle performance for a Chebyshev absorber is described using the 6° curved wedge as an example. For specular reflection at a wide angle, θ, the Chebyshev step is effectively shortened by a factor \( \cos(\theta) \). By taking into account the angle effect, one can still use Equation (6.4) to model the Chebyshev effect at wide angles such that

\[
\bar{\Gamma}(\psi) = \bar{\Gamma}_0 + \bar{\Gamma}_1 e^{-2j\psi} + \bar{\Gamma}_2 e^{-4j\psi} + \cdots + \bar{\Gamma}_N e^{-2jN\psi}.
\]  

(6.19)
Tomographic Processing, HAMMING down range, HAMMING cross range

Original Freq. (GHz): $F_{\text{min}} = 2.002$  $F_{\text{max}} = 8.002$  $\Delta F = 0.020$

Used Freq. (GHz): $F_{\text{lo}} = 2.002$  $F_{\text{hi}} = 8.002$

Original Asp. (deg): $T_{\text{min}} = -15.00$  $T_{\text{max}} = 15.00$  $\Delta T = 0.25$

Used Asp (deg): $T_{\text{lo}} = -15.00$  $T_{\text{hi}} = 15.00$

Far Field, $T_{\text{static}} = 0$ deg

Figure 98: The ISAR image for a 24" panel of 6" Chebyshev curved wedges with polarization perpendicular to the wedge edges.
Figure 99: Comparison of the measured results for the 6" Chebyshev and original curved wedge with polarization perpendicular to the edges.
Figure 100: Comparison of the 6" Chebyshev measured results and original curved wedge results multiplied by the Chebyshev factor. Polarization perpendicular to the edges.
where $\psi = \frac{2\pi}{\lambda} d \cos \theta$. The reflection level for a Chebyshev pattern can then be obtained by

$$\Gamma(\psi) = \Gamma(0) \cdot \tilde{\Gamma}(\psi)$$

(6.20)

Note that $\Gamma(0)$ is the original specular reflection response at $\theta$ without a Chebyshev pattern.

A 2D moment method [34] was used to model a 24" panel of 6" curved wedge with and without the Chebyshev pattern. The typical block diagram used was shown in Figure 94. The calculated reflection levels versus $\theta$ at 2 GHz are shown in Figure 101 for the polarization parallel to the wedge edges. Note that the dotted line is obtained by using Equation (6.20) and the other two results are obtained by the moment method. The small reflection model in Equation (6.19) for wide angle

Figure 101: Comparison of the calculated results for the 6" curved wedge with and without Chebyshev pattern. Polarization is parallel to the wedge edges.
Table 4: Parameters for a 3-step Chebyshev transformer when $N = 3$, $f_m = 2$ GHz and $d = \frac{2}{3}$.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$f_m$(GHz)</th>
<th>$f_c$(GHz)</th>
<th>d (inch)</th>
<th>Nd (inch)</th>
<th>$2(f_c - f_m)$(GHz)</th>
<th>$\tilde{f}_m$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of sections</td>
<td>lowest pass frequency</td>
<td>center frequency</td>
<td>step length</td>
<td>total length</td>
<td>pass bandwidth</td>
<td>ripple amplitude</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4.5</td>
<td>0.667</td>
<td>2.00</td>
<td>5.</td>
<td>-13.95</td>
</tr>
</tbody>
</table>

incidence is validated by the close agreement between the two Chebyshev responses. As shown in Figure 101, the Chebyshev absorber has better performance than the original one. The improvement decreases with the angle of incidence due to the shortened effective step size. Therefore, the Chebyshev design is best suited for the normal incidence case, such as the end wall materials in a compact range.

6.3 Chebyshev Design for 6" Curved Pyramid

The Chebyshev transformer concept is applied to improve the scattering performance of the 6" curved pyramid. Recall that this curved pyramid does not function as designed because of the material scaling problem and low material doping. Since the design frequency range is the same as the wedge case, the discussions in Section 6.1.4 about how to choose the Chebyshev parameters can still be applied. The number of steps is then chosen as 3 and the step size as $\frac{2}{3}$. As shown in the wedge case, the Chebyshev factor functions very closely to expected. Therefore, it should be feasible to have the lowest Chebyshev passband frequency ($f_m$) right at 2 GHz so that the ripple amplitude can be reduced. Consequently, the Chebyshev parameters associated with this 6" curved pyramid are listed in Table 4. In this case, the
Figure 102: Chebyshev level pattern for the 6'' curved pyramid case.

Normalized reflection coefficients for different levels are found as

$$\tilde{\Gamma}_0 = \tilde{\Gamma}_3 = 0.223 ; \tilde{\Gamma}_1 = \tilde{\Gamma}_2 = 0.277 \ .$$

(6.21)

Since the period for the 6'' pyramid panel is \(\frac{324}{3}\), a 24'' square panel has a total of 324(18 x 18) elements. For levels 0 and 3, each has 72(\(\approx 324 \times 0.223\)) elements and the other two levels have 90(\(\approx 324 \times 0.277\)) elements for each. The layout pattern for this design is shown in Figure 102. Note that level 0 is the highest one while level 3 is the lowest. This pattern was chosen to limit the maximum junction height difference between two adjacent elements to one step. There was a concern about the possible strong valley scattered fields from a large height discontinuity. The drawback, however, lies in the tedious cutting and gluing. One may choose the more simple pattern as shown in Figure 103, in which every level has a rectangular area.
Figure 103: Rectangular level pattern for the 6'' Chebyshev pyramid.
It is easy to make this pattern and it should be a better choice for implementing the Chebyshev design in a big area, such as the end wall of a compact range.

The ISAR image based on the 2-8 GHz band and ±15° around normal incidence is shown in Figure 104 for vertical polarization. One can roughly observe the Chebyshev level pattern by the tip diffraction terms. The image processing technique was applied to remove the mounting error and end effects. The reconstructed frequency response at normal incidence is shown in Figure 105 and compared with the 8.25" pyramid. Note that the Chebyshev effect improves the reflection level by more than 10 dB for the low frequency end. As a result, the design requirement to have a -40 dB reflection coefficient above 2 GHz is achieved with a 5 dB safety margin. The Chebyshev effect for the high frequency portion is not obvious. For reflection signals lower than -70 dB, the high frequency reflected field can be dominated by a random scattering effect due to the nonuniform dielectric property of absorber materials [36]. In this case, the absorber signals do not behave as designed by the theoretical methods, in which the smooth surface and uniform material properties are assumed.

Recall that the original 6" pyramid has poor performance mainly because of improper dielectric constants. This Chebyshev design can greatly improve its reflection behavior by simply stacking up more base material. Therefore, the Chebyshev design provides an easy and practical way to modify unsatisfactory absorber panels which might have already been purchased for a chamber.

6.4 Chebyshev Design for 40" Wedge and Pyramidal Absorbers

In Chapter 4, several 40" wedge and pyramidal absorber designs were presented in an attempt to achieve a -40 dB reflection level above 300 MHz. It was concluded
Figure 104: The ISAR image for a 24" panel of 6" Chebyshev curved pyramids.
Figure 105: Comparison of the measured results for the 6" Chebyshev and non-Chebyshev curved pyramids.
that the curved wedge W2 design with a three-layered base has best performance but did not meet the design requirement. The reflection level at 300 MHz could only reach -34 dB. On the other hand, the complementary curved pyramid design reached a -44 dB reflection coefficient above 300 MHz. In this section, the Chebyshev designs for these 40" materials are presented to particularly help the wedge material. The performance improvement will be shown using the theoretical calculations, which should be realizable based on the agreement between the measured and predicted data for the 6" Chebyshev absorbers.

The Chebyshev design for the 6" curved wedge can be directly applied by scaling the step size with a scale factor of \(6\frac{2}{3}\). Therefore, the step size is \(4.4" (= \frac{2}{3} \times 6\frac{2}{3})\). However, one may choose 4" to obtain an integral number and reduce the overall step size. The feasibility of this choice should be decided as follows based on the corresponding Chebyshev parameters. With a 4" step size, the center frequency \(f_c\) is determined to be 750 MHz. To include 300 MHz, the lowest passband frequency, \(f_m\), is chosen as 300 MHz. Based on these choices, the normalized reflection coefficients can be found as

\[
\vec{\Gamma}_0 = \vec{\Gamma}_3 = 0.246 ; \quad \vec{\Gamma}_1 = \vec{\Gamma}_2 = 0.254 .
\]  

(6.22)

Note that these numbers are close to 0.25, which can lead to an easy Chebyshev configuration. Therefore, the normalized reflection coefficients are forced to be 0.25. The corresponding Chebyshev parameters are listed in Table 5. Note that the critical 300 MHz is inside the passband. And, the -11.31 passband ripple level is satisfactory since the wedge reflection level can be reduced down to -45 dB and provides a 5 dB safety margin. Therefore, this Chebyshev design should provide the desired performance improvement.

The Chebyshev improvement for the 40" wedge is shown in Figures 106 and 107, respectively, for the parallel and perpendicular to the wedge polarizations. It should
Table 5: Parameters for a 3-step Chebyshev transformer when $N = 3$, $f_m = 300$ MHz and $d = 4''$.

<table>
<thead>
<tr>
<th># of sections</th>
<th>$f_m$</th>
<th>$f_c$</th>
<th>d</th>
<th>Nd</th>
<th>$2(f_c - f_m)$</th>
<th>$\bar{\Gamma}_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>294 MHz</td>
<td>750 MHz</td>
<td>4''</td>
<td>12''</td>
<td>900 MHz</td>
<td>-11.31 dB</td>
</tr>
</tbody>
</table>

Figure 106: Comparison of the theoretical results for the 40" Chebyshev and non-Chebyshev curved wedge design #W5. Parallel to the wedge polarization.
Figure 107: Comparison of the theoretical results for the 40° Chebyshev and non-Chebyshev curved wedge design #W5. Perpendicular to the wedge polarization.
be clear that the Chebyshev design will achieve the design goal with at least a 5 dB safety margin. Although it might be excessive to use the Chebyshev design to improve the 40" pyramid, the larger than 10 dB reflection reduction is very attractive and the 12" increase in dimension is acceptable in a large chamber. The theoretically predicted improvement is shown in Figure 108. As one can observe, this Chebyshev design potentially can achieve a -55 dB reflection coefficient for normal incidence.

6.5 Summary

The Chebyshev absorber design has been presented for both wedge and pyramidal absorbers. The design procedure is very similar to that for a microwave matching
circuit except that the reflection coefficient at each level is proportional to the number of absorber elements. The reflected fields for a Chebyshev absorber wall can be written as the product of the original response and the Chebyshev reduction factor. Therefore, the design of Chebyshev steps is independent of the original absorber design. This feature implies that, using a Chebyshev pattern, one can greatly improve absorber performance over unsatisfactory absorber materials which may already be manufactured or purchased. The measured data for the 6" wedge and pyramidal absorbers has demonstrated more than 10 dB improvement using the Chebyshev transformer approach. Similar improvement can be predicted by adding 12" Chebyshev steps to the 40" wedge and pyramidal absorber designs, which were presented in Chapter 4. Recall that a 10 dB absorber reflection coefficient reduction normally requires material twice as thick. On the other hand, the Chebyshev approach only changes the thickness by a small factor. As shown in the moment method calculation for 6" curved wedge, the Chebyshev effect decreases with the bistatic angle of incidence due to the shrinking effective step size. Therefore, the Chebyshev multilevel design is most suited for normal incidence.
CHAPTER VII
High Performance Absorbers

7.1 Introduction

During this absorber study, several high performance commercial absorber materials have become available. They were designed based on the same principles as described in Chapter 4 but without PMM verifications. Two of these new materials were evaluated in the OSU/ESL compact range and the measured data was processed using the image editing procedure described in Chapter 5. Both materials were designed for a compact antenna measurement range which will be operated from 1 GHz to 100 GHz. The first one is the 30" straight pyramids for the side walls; and, the second one is the 23" curved pyramids for the end wall. Since the end effect measurement errors are significant for these high performance absorbers, it is important to apply the image processing techniques to obtain the actual performance for an absorber wall. The measured data is verified by the theoretical calculations. As will be shown later, the 23" material does not meet the design requirement due to the limitations of the TLA analysis. The Chebyshev design described in the last chapter was then applied to significantly improve its performance so that the required low reflection levels are achieved. The improvement has been verified experimentally using measurements of a 6' square panel.
7.2 30" Straight Pyramid

This 30" straight absorber was optimized for a 45° bistatic specular reflection and required to be at least -45 dB above 1 GHz. The absorber has a 4.6" base and a 4" period. The absorber panel size was increased from normally a 2' to a 4' square to reduce end effect errors by 6 dB. As shown in Figure 109, a receiving antenna was placed close to the side wall of the compact range to form a 90° angle with respect the incident plane wave. The distance from the receiving antenna to the

![Figure 109: Compact range setup to measure 45° bistatic specular reflection.](image)
pedestal center was about 216" or 18'. In fact, this bistatic setup provides near field measurements. At 1 GHz, the distance between the receiving antenna and absorber sample is larger than the far field range, which is determined by $\frac{D^2}{\lambda} = \frac{4f^2}{\pi^2} = 16'$. Thus, the measured reflection level at 1 GHz should be accurate. In calibration, the measured absorber data is normalized against the plate response to obtain the reflection coefficient.

(a) Horizontal Polarization

For the horizontal polarization, the ISAR image corresponding to the 0.4-6.4 GHz band and ±15° around the 45° specular direction is shown in Fig. 110. In this image, the mounting errors associated with the support structure are most dominant and are gated out using a rectangular gate. As one can see, the edge effects imaged at the plate position are more significant than the absorber material itself. Note that the plate position is slightly off the 0" position. This is because two additional attenuation pads were inserted in the receiving cable for the plate measurement to increase the measurement dynamic range. Using similar arguments to the 18" pyramid case, which was described in Section 5.4.2, the stripe area at plate position mainly comes from top and bottom end effects. To reduce these terms, one can simply apply a rectangular gate to attenuate its level by about -15 dB. The reconstructed data with and without the edge effects removed are compared with the TLA calculation as shown in Fig. 111. From the time domain response, the edge effects are very dominant in the original data. The -15 dB attenuation gating may not be sufficient to completely remove the edge effect, but further attenuation does not change the frequency behavior around 1 GHz. For this absorber design, the end effect errors are more serious at the high frequency end of the band. Based on the measured and calculated results, this new material reaches -50 dB above 1 GHz, which is 5 dB
Tomographic Processing, HAMMING down range, HAMMING cross range

Original Freq. (GHz): \( F_{\text{min}} = 0.402 \quad F_{\text{max}} = 6.402 \quad \Delta F = 0.010 \)

Used Freq. (GHz): \( F_{\text{lo}} = 0.402 \quad F_{\text{hi}} = 6.402 \)

Original Asp. (deg.): \( T_{\text{min}} = -60.00 \quad T_{\text{max}} = -30.00 \quad \Delta T = 0.25 \)

Used Asp. (deg.): \( T_{\text{lo}} = -60.00 \quad T_{\text{hi}} = -30.00 \)

\( R_t = 393700 \text{ inches} \quad R_r = 209.9996 \text{ inches} \)

\( T_{\text{ bistatic}} = 90 \text{ degrees} \)

Plate Position

Figure 110: ISAR image for the bistatic reflected fields for a 4' panel of 30'' straight pyramids. Horizontal polarization.

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Figure 111: Comparison of the image processed and TLA calculated results for the 30° straight pyramid wall at a 45° bistatic angle.
better than the design requirement. The reason for the level difference between the measured and calculated results for the high frequency region is twofold. Firstly, the TLA method does not consider higher order mechanisms which become more significant at high frequencies. For example, a grating lobe effect may occur around 3.5 GHz, which is observed in the measured data but not the TLA calculation. Secondly, the random scattering effect due to material nonuniformity prevents the measured reflection levels from continuously dropping with frequency. Normally a -75 dB reflection coefficient is what has been observed experimentally for absorber materials. Based on the above discussions, the image processed result is fairly close to the theoretical prediction.

(a) Vertical Polarization

At a 45° bistatic angle, the vertically polarized wave excites different responses than the horizontal one for a pyramid. Similar image processing was applied to the vertically polarized results and the ISAR image is shown in Figure 112. The end effects due to the panel truncation are more significant than the absorber material itself. Note that the end effects from the two side edges are not as obvious as the previous absorber images. The reasons are twofold. Firstly, in this 45° bistatic configuration, the edge diffracted fields from the side edges are shadowed by the whole absorber panel either in the transmit or receive direction; while, the top and bottom edges are seen by both the transmit and receive antennas. Secondly, the vertically polarized incident wave can propagate on the top and bottom edge surfaces of the panel with less attenuation than on the two sides. To reduce the end effect errors, one can simply apply a rectangular gate to attenuate its level by 15 dB. The reconstructed data with and without the edge effects removed are compared with the TLA calculation as shown in Fig. 113. From the time domain response, the
Tomographic Processing, HAMMING down range, HAMMING cross range

Original Freq (GHz) Fmin = 0.402 Fmax = 6.402 DeltaF = 0.010

Used Freq (GHz) Flo = 0.402 Fhi = 6.402

Original Asp (deg) Tmin = -60.00 Tmax = -30.00 DeltaT = 0.25

Used Asp (deg) Tlo = -60.00 Thi = -30.00

Rt = 393700 inche Rr = 209.9996 inche

Tbistatic = 90 degrees

Figure 112: ISAR image for the bistatic reflected fields for a 4' panel of 30" straight pyramids. Vertical polarization.
Figure 113: Comparison of the image processed and TLA calculated results for the 30° straight pyramid wall at 45° incidence. Vertical polarization.
edge effects are very dominant in the original data. The -15 dB attenuation gating may not be sufficient to completely remove the edge effect, but further attenuation does not change the frequency behavior around 1 GHz. For this polarization, the end effect errors are more serious at the low and mid frequency bands. The removal of end effects is especially critical to represent the absorber performance around 1 GHz, which determines if this design achieves the –45 dB criterion. The corrected measured data shows a near -50 dB reflection level at 1 GHz.

In summary, this 30" straight pyramid has been verified to achieve almost -50 dB reflection level at a 45° bistatic angle above 1 GHz. Note that at 1 GHz this material is equivalently 2.5 \( \lambda \) long. As compared with the conventional 18" pyramid case, which has -50 dB reflection level at normal incidence with 3 \( \lambda \) length, one should realize this new material has much better performance than the conventional materials. The success of this design lies in the accuracy of the TLA analysis for this absorber configurations. The PMM and TLA calculated results are compared in Figures 114 and 115 for the horizontal and vertical polarizations, respectively. These two methods agree very well at least for frequencies below 1.3 GHz. Note that the PMM calculation at higher frequencies were not performed due to computer memory constraints. On the other hand, the TLA calculation becomes inaccurate around this frequency region. Thus, the above comparisons focus on the low frequency end, which is satisfactory since the lowest design frequency (1 GHz) is included.
Figure 114: Comparison of the PMM and TLA calculated results for the 30° straight pyramid wall at 45° incidence. Horizontal(θθ) polarization.
Figure 115: Comparison of the PMM and TLA calculated results for the 30\(^\circ\) straight pyramid wall at 45\(^\circ\) incidence. Vertical(\(\phi\phi\)) polarization.
7.3 23\" Curved Pyramid

The next example is a high performance 23\" absorber which was required to exceed a -55 dB reflection coefficient for normal incidence above 1 GHz. The curved pyramid used to meet this requirement has a 4.22\" base and a 6\" period. Note that at 1 GHz, the absorber dimension is only about 1.9 \( \lambda \). Recall that the 18\" pyramid, as shown in Figure 82, merely reaches -50 dB level with 3 \( \lambda_0 \) length at 2 GHz. Thus, this new material is expected to have much better performance than the conventional one. To reduce the significant end effects observed in the 2\' case, a 4\' square panel was measured and normalized against the plate response to obtain the measured reflection coefficient levels. For horizontal polarization, the ISAR image corresponding to the 0.5-6 GHz frequency band and \( \pm 15^\circ \) angular span is shown in Fig. 116. Note that the two end effect image spots are clearly seen roughly at the plate down range position. From the image, the top and bottom edge terms are not more dominant than the absorber mechanisms for this polarization. Thus, only the two most dominant end effects are attenuated by 15 dB. As shown in Fig. 117, the end effects have a significant impact on the response around 1 GHz, which is the critical design area. The fully corrected result has a similar reflection level as the PMM calculated result, which verifies the image processing technique. Around 1 GHz, the envelope levels are about -52 dB, which does not achieve the design requirement but still has much better performance than conventional materials. Similar processing has been applied to the vertically polarized results. The measured results for both polarizations are compared in Figure 118 with end effect errors removed. As one can easily observe, both polarizations have very similar frequency behavior, which is expected for normal incidence.
Figure 116: ISAR image of the reflected fields for a 4' panel of 30" straight pyramids. Horizontal polarization.
Figure 117: Comparison of the measured and PMM calculated results for the 23° curved pyramid wall at normal incidence. Horizontal polarization.
Figure 118: Comparison of the measured results for the 23" curved pyramid absorber wall at normal incidence for both vertical and horizontal polarizations.
Figure 119: Comparison of PMM and TLA calculated results for the 23\textsuperscript{rd} curved pyramid wall at normal incidence.

In summary, this end wall material does not meet the -55 dB reflection requirement. This design problem is caused by the limitations of the TLA analysis. The PMM and TLA calculated results are shown in Figure 119. As can be easily observed, there is a very significant discrepancy between these two calculated results. The TLA method predicts an extremely low level (-80 dB) above 0.9 GHz; while, the PMM result has reflection levels around -52 dB, which has been verified by the measurements. This phenomenon is caused by: (1) the period of this absorber is 6", which means the higher order interaction terms can be significant above 1 GHz according to the $\lambda = 2D_x$ criterion; (2) the absorber material is heavily doped, which can worsen the higher order interactions as discussed in Chapter 3; (3) the
higher order mechanisms become relatively significant especially when the fundamental mode contribution is optimized by the TLA analysis to an extremely low level such as done here. One should note that the discrepancy caused by first two reasons has been observed previously. However, none of the comparisons has shown such a significant error as this 23" material. Therefore, the third reason should be the major cause in this case. This observation confirms that, as pointed out in Chapter 4, the optimized design based on the TLA method must be verified by a PMM calculation. In the next section, the Chebyshev concept will be applied to improve this material so that the design requirement can be realized.
7.4 23'' Chebyshev Curved Pyramid

As shown in the last section, the 23'' curved pyramid design did not achieve the required -55 dB reflection level around 1 GHz. Thus, the Chebyshev transformer design is applied to improve its performance. The design procedure is very similar to the 6'' curved pyramids, which was described in Section 6.3. The lowest design frequency was 2 GHz for the 6'' pyramids; while, it is 1 GHz in this 23'' case. Therefore, the Chebyshev parameters can be roughly scaled by a factor of 2 so that the step size should be \( \frac{4''}{3} \). However, to have a safety margin around 1 GHz, a 1.5'' step size is used to shift the passband toward the low frequency end. Recall that the step size is equal to a quarter wavelength of the passband center frequency. Thus, the center frequency is 2 GHz. Note that the Chebyshev factor is periodic with a period equal to twice the center frequency; i.e. 4 GHz. From Figure 118, the reflection level for the 23'' pyramid around 4 GHz is certainly below -60 dB. Thus, the passband is wide enough to meet the -55 dB requirement.

A 6' square panel was set up to further reduce end effect errors since a very low reflection level was expected from the Chebyshev design. Note that commercial absorbers usually come in 2' square pieces. Thus, there are 9 pieces of 2' absorbers on the 6' plate. The normalized reflection coefficients for the 6'' Chebyshev pyramid case in Equation (6.21) was given by

\[
\tilde{\Gamma}_0 = \tilde{\Gamma}_3 = 0.223 \approx \frac{2}{9}, \quad \tilde{\Gamma}_1 = \tilde{\Gamma}_2 = 0.277 \approx \frac{2.5}{9}.
\]  

These factors can fit a 6' square panel very nicely due to the factor 9 in the denominator. The resulting layout is shown in Figure 120. Note that level 0 is the highest and the level 3 is the lowest. The Chebyshev parameters for this design are summarized in Table 6.
Figure 120: Chebyshev layout for a 6' panel of 23'' curved pyramids.
Table 6: Parameters for a 3-step Chebyshev transformer when $N = 3$, $f_m = 0.9$ GHz and $d = 1.5''$.

<table>
<thead>
<tr>
<th>N sections</th>
<th>$f_m$ lowest pass frequency</th>
<th>$f_c$ center frequency</th>
<th>d step length</th>
<th>Nd total length</th>
<th>$2(f_c - f_m)$ pass bandwidth</th>
<th>$\tilde{\Gamma}_m$ ripple amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.9 GHz</td>
<td>2 GHz</td>
<td>1.5''</td>
<td>4.5''</td>
<td>2.2 GHz</td>
<td>-14.24 dB</td>
</tr>
</tbody>
</table>

The ISAR image corresponding to the 0.5-6 GHz band and ±15° angle span is shown in Figure 121 for vertical polarization. The mounting errors have already been gated out, and the end effect errors are the most dominant mechanisms associated with the absorber panel. The image stripe between the two side edges mainly comes from the top and bottom edges as stated earlier. Therefore, the side edges were attenuated by 20 dB and the horizontal edges were attenuated by 10 dB so that this line mechanism is approximately uniform and does not dominate the absorber scattering levels. The reconstructed responses with and without the edge effects removed are compared in Figure 122. As one can see from the time domain plot, the end effect errors are very serious. Their presence affects the reflection level throughout the frequency spectrum, especially around 1 GHz. The corrected reflection levels are almost -60 dB above 1 GHz, which well satisfies the required -55 dB level. The measured results for the Chebyshev pattern are compared with the original one in Figure 123. Both from the time and frequency domains, one can clearly see the improvement made by the destructive interference among different Chebyshev levels. Due to the periodicity of the Chebyshev factor, the Chebyshev response is about the same level as the original one around 4 GHz. To further examine the multilevel effect, the original response multiplied by the Chebyshev factor, $\tilde{\Gamma}$, is compared with the Chebyshev response in Figure 124. These two results do
Figure 121: The ISAR image for a 6\' panel of 23\" Chebyshev curved pyramids. Vertical polarization.
Figure 122: Comparison of the measured results at normal incidence for the 23° Chebyshev curved pyramids to illustrate the end effect errors. Vertical polarization.
Figure 123: Comparison of the measured results at normal incidence for the 23" curved pyramids to illustrate the Chebyshev improvement. Vertical polarization.
Figure 124: Comparison of the measured Chebyshev result and the original result multiplied by the Chebyshev factor. Vertical polarization.

not agree well in the passband. For example, notable differences can be observed at 1.1 and 2 GHz where the expected Chebyshev null responses are measured as peaks. Recall though that the Chebyshev design neglects the scattered field from the level junctions, which may corrupt the designed Chebyshev pattern for very low reflection levels. In addition, the higher order interactions can play a role in the above comparison. Since the 23" pyramid has a 6" period, the grating lobe interaction should be relatively significant at 2 GHz as compared with the very low scattered field from the fundamental mode. Such a high order interaction occurs as a reradiation process among absorber elements. This mechanism at each level does not scatter exactly proportional to the number of elements due to the small number of elements at each level. Therefore, some residual terms are not completely nulled
based on the Chebyshev design. However, the major portion of the scattered fields is proportional to the number of elements. As a result, a 10 dB improvement due to the Chebyshev pattern has still been observed and shown in Figure 123. One should note that it is very difficult to obtain such improvement in an absorber design based on the TLA optimization. As shown in the 23'' case, the higher order mechanisms, which are not included in the TLA method, may dominate the low reflection levels. The Chebyshev design has been verified to provide an independent reduction factor which is valid for both fundamental mode and higher order mechanisms.

The horizontally polarized results are not shown here since they are very similar to the vertical case. In summary, the scattering performance of the 23'' pyramids has been improved by 10 dB using the Chebyshev modification so that the design requirement is actually exceeded. Note that such 10 dB improvement usually requires twice the absorber dimension; while, the added three Chebyshev steps are merely a fifth of the 23'' height. In addition, the Chebyshev improvement at reflection levels around -55 dB is also very valuable in that the higher order mechanisms, which dictate the reflection level, cannot be optimized using the TLA analysis.

7.5 Summary

Three high performance absorbers have been verified experimentally and theoretically. Both the 30'' and 23'' pyramids were designed based on the TLA analysis and optimized for frequencies above 1 GHz. The 30'' side wall materials were verified to achieve the required -45 dB reflection level at a 45° bistatic angle. However, the 23'' end wall materials did not meet the -55 dB level for normal incidence due to the inaccuracies of the TLA analysis. The TLA analysis does not consider higher order mechanisms, which become relatively significant when the fundamental mode
mechanism is optimized to an extremely low level. On the other hand, the measured reflection levels were accurately predicted by the PMM calculations although the desired performance was not achieved. The Chebyshev transformer concept came to the rescue of this absorber product. By simply adding three 1.5" steps to form a Chebyshev pattern, it was shown that the absorber reflection performance was improved by nearly by 10 dB and a close to -60 dB reflection level was achieved above 1 GHz. For this 23" material, such a low reflection level is very difficult to be designed by a numerical optimization because the dominant higher order mechanisms are not included in the TLA method. These measured examples also demonstrate that the image editing techniques must be applied to remove end effect errors to obtain true absorber performance. The good agreement between the measured and calculated results indicates that the quality control over the dielectric properties and taper contour cutting has been improved for these commercial materials. With significant progress in the absorber manufacturing process and the theoretical and experimental techniques, more high performance absorbers will soon be developed to improve measurement quality in new or existing anechoic chambers.
CHAPTER VIII
Conclusions

Research progress associated with the analysis, design and measurement techniques for microwave absorbers has been presented in this dissertation. Based on these developments and the enhanced quality control in commercial materials, one may expect new high performance absorbers will soon be commercially available. These new absorber products should lead to better measurement quality and/or reduced chamber cost.

The TLA method, which simplifies a periodic structure to a stratified layered medium, has been developed based on the averaging properties of periodic vector functions and the effective material concept. The most general expression was derived as four first order differential equations with the TE and TM polarizations coupled. This formulation provides both the co- and cross-polarized scattered fields associated with the fundamental mode for an arbitrary incidence angle. The TLA and PMM methods were implemented for both wedge and pyramidal absorbers with various absorber taper types, such as straight, curved, serrated and multilayered. Very good agreement was observed between these two methods as long as the higher order mechanisms were not significant. Basically, these higher order interactions become stronger with an increase in frequency, off-normal incidence angles and larger dielectric constants. The first two factors are related to the onset of the free space grating lobe; whereas, the large dielectric constant can enhance the guided mode interaction. Based on numerous calculations, it was found that the TLA predictions
may become inaccurate if the wavelength is smaller than twice the period or the magnitude of the dielectric constant is larger than 5. These limitations may not be critical in some cases depending on the specific absorber design requirements. The main advantage of the TLA analysis is its efficiency. Roughly speaking, the TLA calculation needs only 1 sec CPU time for a typical frequency scan; while, the PMM may take hours of CPU time even for a lightly doped pyramid case.

The polarization difference for wedge scattering is very notable from the effective material point of view. For normal incidence, the parallel to the wedge case is dominated by the edge diffraction; while, the perpendicular polarization can better penetrate to the base, which is very similar to the pyramid case. From the contour plots of the scattered field versus the dielectric constant, the basic polarization dilemma is further verified. Parallel polarization requires a lightly doped material to reduce edge diffraction; while, perpendicular polarization requires a heavily doped material to attenuate the penetrating energy. Such a polarization diversity makes it difficult to choose a wedge material. This phenomenon is one of the reasons why a pyramid normally has much better performance than a wedge absorber for normal incidence.

An absorber design procedure combining the TLA efficiency and PMM accuracy has been presented for 40° wedge and pyramidal absorber cases. To minimize the production cost, only straight or curved transition contours using uniform materials were considered. To simulate the realistic situation, only five typical absorber materials can be chosen for the design. Based on the contour plots in the \((\varepsilon'_r, \varepsilon''_r)\) domain for the 40° straight absorber at 300 MHz, a moderately doped material was chosen for the wedge case, and the most heavily doped material for the pyramid case. The TLA based optimization was then used to determine the taper geometry. For the wedge case, it is very difficult to achieve the -40 dB reflection level above 300 MHz.
due to the polarization difference. It was also shown that by curving the wedge, one may reduce the edge diffraction for the parallel polarization case and, at the same time, allow a thicker base area for the perpendicular case. Such improvement is still limited by the polarization issue. The best results came from a 3-layered base design in which the taper section was basically optimized for the parallel polarization case; while, the base was replaced by heavily doped materials to attenuate the penetrating wave for the perpendicular polarization case. Unfortunately, this design still did not achieve the design requirement. In this wedge case, the TLA was found to provide reasonably accurate results since the material for the taper section was moderately doped. However, the approximation became too crude for the pyramid using the heavily doped material E. The TLA optimization provided a rough estimation, but the PMM was used for the final iteration. Both straight and complementary curved designs were shown to achieve the -40 dB reflection level with the complementary case providing the best overall frequency behavior. It was also found that the slope value in a curved geometry is a useful parameter for a designer to control the absorber performance for various design requirements. This feature has been demonstrated for both the wedge and pyramidal absorbers.

The previously-mentioned absorber designs were based on conventional absorber layouts in which all the absorber elements are positioned to the same height. The Chebyshev transformer concept was then applied to destroy the coherency among the absorber elements so that an additional reduction factor was obtained. The resulting Chebyshev response is then shown to be equal to the original response multiplied by the Chebyshev factor. There are several distinctive features associated with this new absorber design concept. (1) The Chebyshev factor is independent of the original absorber response, which means this design can be used to improve an existing material; (2) this design is robust in that the reduction factor does not depend on
material properties which may be changed, for example, due to humidity variation; (3) the cancelation among different layers works for both the fundamental and some higher order mechanisms, which is important for very low reflection levels; (4) a height increase of only a small fraction of the original dimension, typically from \( \frac{1}{5} \) to \( \frac{1}{3} \), can improve absorber performance by more than 10 dB, which normally requires twice the dimension. These desirable aspects have been verified by several measurements of commercially available materials.

One of the major aspects of this dissertation has been to use ISAR image editing techniques to remove measurement errors, especially the end effect errors. Based on the UTD formulation, these error terms may be significant throughout the frequency band, especially for high performance absorbers. The measurements of a 2' square panel of 18" straight pyramids showed that these errors dominate the measured response by more than 10 dB. Even though edge effect errors can be reduced 6 dB by doubling the panel side length, there is a practical limit to the panel size. The image editing procedure is necessary for critical evaluation of high performance absorbers, such as those shown in Chapter 7.

Two commercially available high performance pyramidal absorbers, which were designed based on the TLA analysis without PMM verifications, were evaluated experimentally. With the measurement errors properly removed, the 30" side wall straight pyramids were verified to reach a -45 dB reflection coefficient at a 45° bistatic angle for both polarizations. Such a design success was based on the accurate TLA predictions for this moderately doped material. On the other hand, the 23" curved pyramids did not meet the -55 dB design requirement for normal incidence. In this case, the TLA calculations were not accurate due to the high dielectric constants. In addition, the optimized TLA levels were extremely low such that the higher order mechanisms dictate the true reflection levels. This absorber product was able to
meet the desired design goal by using the Chebyshev concept. In fact, it reached reflection levels close to -60 dB, which is a significant improvement over conventional materials.

Some future research topics are suggested below:

1. The TLA method can be applied to evaluate chamber performance in terms of absorber effects. This issue should become more important as one attempts to design high performance measurement chambers.

2. Improve the PMM efficiency in terms of computation time and memory requirements.

3. The absorber design techniques developed here can be applied to the low frequency EMC/EMI chambers that operate typically from 30 MHz to 1 GHz.

4. Although magnetic materials are normally expensive, very heavy and inaccessible, it may be of interest to study the application of these materials to absorber designs using the same basic theoretical and experimental techniques.
APPENDIX A
Formulation in Terms of (E<sub>av</sub>) and (H<sub>av</sub>)

The formulation in terms of the averaged fields (E<sub>av</sub>) and (H<sub>av</sub>) is derived in this Appendix. Although only the dielectric material was concerned in Chapter 2, the derivation can be easily extended to magnetic materials. For the sake of generality, the following derivation includes the magnetic properties.

Two of Maxwell Equations (2.41)-(2.42) are given by

\[ \nabla \times (e^{-jk_0s_1 \cdot r} E(r)) = -j\omega (r)e^{-jk_0s_1 \cdot r} H(r) , \text{ and } \]
\[ \nabla \times (e^{-jk_0s_1 \cdot r} H(r)) = -j\omega (r)e^{-jk_0s_1 \cdot r} E(r) . \]  

(A.1)  (A.2)

Using the vector identity which is given by

\[ \nabla \times (AB) = A\nabla \times B + \nabla A \times B \]  

(A.3)

and eliminating the common term \( e^{-jk_0s_1 \cdot r} \), one can transform Equations (A.1) - (A.2) to the equations for \( E(r) \) and \( H(r) \) such that

\[ \nabla \times E(r) - jk_0s_1 \times E(r) = -j\omega (r) H(r) , \text{ and } \]
\[ \nabla \times H(r) - jk_0s_1 \times H(r) = j\omega (r) E(r) . \]  

(A.4)  (A.5)

To decompose the fields into \( \hat{y} \) and transverse to \( \hat{y} \) components, it is necessary to introduce the decomposition of the delta operation which is given by

\[ \nabla \times B = \nabla_y B_t + \nabla_t \times B_y + \nabla_r \times B_t \]  

(A.6)
where $\nabla_y \equiv \hat{y} \frac{\partial}{\partial y}$ and $\nabla_t \equiv \hat{x} \frac{\partial}{\partial x} + \hat{z} \frac{\partial}{\partial z}$ and $B_t$ means the transverse portion of a vector function $B$. By applying this identity to Equations (A.4)–(A.5) one obtains that

$$\nabla_y \times E_t^p(r) + \nabla_t \times E_y^p(r) + \nabla_t \times E_z^p(r) - j k_0 s_t \times E^p(r) = -j \omega \mu(r) H^p(r) \quad (A.7)$$

and

$$\nabla_y \times H_t^p(r) + \nabla_t \times H_y^p(r) + \nabla_t \times H_z^p(r) - j k_0 s_t \times H^p(r) = j \omega \epsilon(r) E^p(r) \quad (A.8)$$

The next step is to take the averaging operation as defined in (2.37). By using the Fourier series to represent any periodic functions, $B_t^p$ and $B_y^p$, and taking the average, one can prove that

$$(\nabla \times B_t^p)_{av} = 0 , \quad \text{and}$$

$$(\nabla \times B_y^p)_{av} = 0 . \quad (A.9)$$

Since $y$ is independent of $x$ and $z$, the delta operation, $\nabla_y$, can be interchanged with the averaging operation which is performed on the $x$ and $z$ coordinates such that

$$(\nabla_y \times B_t^p)_{av} = \nabla_y \times (B_t^p)_{av} \quad (A.10)$$

Using the above identities (A.9) to (A.11) and taking the average on the two sides of Equations (A.7) and (A.8), one can obtains the following equations:

$$\nabla_y \times (E_t^p)_{av} - j k_0 s_t \times (E^p)_{av} = -j \omega (\mu H^p)_{av} , \quad \text{and}$$

$$\nabla_y \times (H_t^p)_{av} - j k_0 s_t \times (H^p)_{av} = -j \omega (\epsilon E^p)_{av} \quad (A.12)$$

Note that the $y$ dependence of the averaged fields in the above equations is not expressed for notational economy. One should notice that every term in Equations (A.12) and (A.13) can be decomposed into the $\hat{y}$ and transverse-to-$\hat{y}$ components. To enforce the equality for the transverse components, one can apply $\hat{y} \times$
on both sides of Equations (A.12) and (A.13) and then invoke the vector identity,
\[ \hat{y} \times \nabla \times \mathbf{B}_t^0(\hat{y}) = -\frac{d}{dy} \mathbf{B}_t^0, \]
to obtain that
\[ \frac{d}{dy} \mathbf{E}_t^0(\hat{y}) - jk_0 \hat{y} \times s_t \times \mathbf{E}_t^0(\hat{y}) + j\omega \hat{y} \times \mathbf{H}_t^0(\hat{y}) = 0, \quad \text{and} \quad (A.14) \]
\[ \frac{d}{dy} \mathbf{H}_t^0(\hat{y}) - jk_0 \hat{y} \times s_t \times \mathbf{H}_t^0(\hat{y}) - j\omega \hat{y} \times \mathbf{E}_t^0(\hat{y}) = 0. \quad (A.15) \]

By enforcing the equality for the \( \hat{y} \) portion, one obtains that
\[ -jk_0 s_t \times \mathbf{E}_t^0(\hat{y}) + j\omega (\mu \mathbf{H}_t^0(\hat{y}) = 0, \quad \text{and} \quad (A.16) \]
\[ -jk_0 s_t \times \mathbf{H}_t^0(\hat{y}) + j\omega (\varepsilon \mathbf{E}_t^0(\hat{y}) = 0. \quad (A.17) \]

The formulation in terms of \( (\mathbf{E})_{av} \) and \( (\mathbf{H})_{av} \) has been derived. With \( \mu = \mu_0 \),
Equations (A.14)–(A.17) reduce to Equations (2.43)–(2.46).
APPENDIX B
PMM Verification of Effective Material Constants

The effective material constants for both singly and doubly periodic structures were given in Section 2.5.3. A significant simplification in formulating the scattering mechanisms for a periodic structure was achieved using these effective constants. Although the effective material constants should depend on frequency, the exact solutions, if possible, and some variational bounds have usually been found only for static cases. At relatively low frequencies, such an approximation should be valid. The numerical results are normally obtained by a finite element method solution for the static case [28]. In fact, the reflection and transmission coefficients obtained by the PMM solution for the desired periodic structure of finite thickness can provide the information for effective constants. If the effective constants of the uniform layer are valid, one can find the effective material constants based on the PMM calculated coefficients. The procedure is similar to how one calculates the material properties using measured reflection and transmission data [37]. For the time being, the PMM method is implemented only for dielectric materials; therefore, the effective dielectric constant of non-magnetic materials is studied. Note that one only needs either the reflection or transmission coefficient to solve for the effective dielectric parameters. Due to the intrinsic function behavior, the dielectric constant is less sensitive to the inaccuracy of the transmission data than the reflection one in a wideband application [31]. In addition, the numerical results for the transmission case converges faster than the reflection one since the induced fields from each volume
cell tend to be in phase in the transmission direction. Consequently, the transmission information is used to analyze the effective constants.

As shown in Figure 7, a periodic array of rectangular dielectric cylinder characterized by \( \varepsilon = (2 - j1.5)\varepsilon_0 \) is considered. The PMM calculations are performed at normal incidence with the length fraction, \( g \), varying from 0.05 to 0.95 with a step size of 0.05. The calculated transmission results are then used to evaluate the effective dielectric constants. With \( t = 1'' \) and \( D_x = 2'' \), the numerical results for both polarizations at 600, 1800, and 3000 MHz are compared with the exact solution for the static case in Figure 125 and 126. As expected, the calculated results converge towards the static case as the frequency decreases, which verifies the use of the PMM calculation for this purpose. From these plots, one can observe that, as the frequency increases or the ratio of the period to wavelength becomes larger, the static results begin to deviate from the PMM ones which take the frequency effects into account.

For a doubly periodic structure, no exact solutions exist but some variational bounds for the transverse effective constants are known for the static case. As a result, the PMM solution can then be used as a reference to examine these bounds. Since the HS formula in Equation (2.54) has been considered as the best approximation [28] for a square rod structure, only this formula is tested here. The PMM calculation is performed in the same fashion as the previous case. With \( \varepsilon = (2 - j10)\varepsilon_0 \), \( t = 10'' \) and \( D_x = D_z = 8'' \), the numerical results for 100, 200 and 300 MHz are compared with the HS approximation in Figure 127. Note that the result at 100 MHz can be used as a reference solution since its wavelength is 15 times as large as the period. The HS formula agrees with the reference solution very well and correctly predicts the overshoot of the real part near \( g = 0.9 \), which is larger than the material's real part. The frequency variation of the effective value is another error
mechanism in applying the TLA method. In general, the HS formula provides a very good approximation to homogenize the pyramidal absorbers whose cross section is a square.
Figure 125: Comparison of the exact and PMM calculated effective dielectric constants for a singly periodic rectangular structure. $\varepsilon = (2 - j1.5)\varepsilon_0$. TE polarization.
Figure 126: Comparison of the exact and PMM calculated effective dielectric constants for a singly periodic rectangular structure. $\varepsilon = (2 - j1.5)\varepsilon_0$. TM polarization.
Figure 127: Comparison of the HS formula and PMM calculated effective dielectric constants for a doubly periodic rectangular block. $\epsilon = (2 - j10)\epsilon_0$. 

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APPENDIX C
Derivation of The Coupled First Order Differential Equations for The Transmission Line Approximation

The derivation from Equations (2.60) and (2.61) for the averaged fields to Equations (2.67)-(2.70) for the coupled first order differential equations is described here for the general case. Equations (2.60) and (2.61) are rewritten with the magnetic properties such that

\[-\frac{d}{dy} (E_t^P)_{av} + j \frac{\omega \mu_0 e_0}{\varepsilon_y} \times (s_t \times (H_t^P)_{av})) + j \omega \hat{y} \times \frac{-\varepsilon}{\varepsilon_t} \cdot (H_t^P)_{av} = 0 \]  
(C.1)

and

\[-\frac{d}{dy} (H_t^P)_{av} - j \frac{\omega \mu_0 e_0}{\mu_y} \times (s_t \times (E_t^P)_{av})) - j \omega \hat{y} \times \frac{-\varepsilon}{\varepsilon_t} \cdot (E_t^P)_{av} = 0 \]  
(C.2)

To decompose the problem into the TM and TE cases, it is convenient to use cylindrical coordinates (\(\hat{\rho}, \hat{\phi}\)) which are given by

\[\hat{\rho} = \hat{x} \cos \phi + \hat{z} \sin \phi , \text{ and} \]
\[\hat{\phi} = -\hat{x} \sin \phi + \hat{z} \cos \phi \]  
(C.3)

or equivalently

\[\hat{x} = \hat{\rho} \cos \phi - \hat{\phi} \sin \phi , \text{ and} \]
\[\hat{z} = \hat{\rho} \sin \phi + \hat{\phi} \cos \phi . \]  
(C.4)

Recall that the unit direction vector for a (\(\theta, \phi\)) incidence is defined by (2.9) such that

\[\hat{s}^i = -\hat{x} \sin \theta \cos \phi - \hat{y} \cos \theta - \hat{z} \sin \theta \sin \phi . \]  
(C.7)
The transverse to \( \hat{y} \) component of \( \dot{s}^t \) can be described by

\[
s_t = \dot{s}^t - \dot{y} (\dot{s}^t \cdot \hat{y}) = -\dot{\rho} \sin \theta . \tag{C.8}
\]

Note that \( \dot{\rho} \) and \( \dot{\phi} \) are both transverse to \( \dot{y} \) and are, respectively, parallel and perpendicular to the incidence plane. The transverse averaged fields can be denoted by

\[
(E^p_t)_{av} = \dot{\rho} \vec{E}_\rho + \dot{\phi} \vec{E}_\phi , \quad \text{and} \tag{C.9}
\]
\[
(H^p_t)_{av} = \dot{\rho} \vec{H}_\rho + \dot{\phi} \vec{H}_\phi . \tag{C.10}
\]

By using the cyclic properties of unit vectors which are given by

\[
\hat{y} \times \dot{\phi} = \dot{\rho} , \quad \text{and} \tag{C.11}
\]
\[
\hat{y} \times \dot{\rho} = -\dot{\phi} , \tag{C.12}
\]

and using Equation (C.8), one finds that

\[
\hat{y} \times (s_t \times (s_t \times (E^p_t)_{av})) = -\dot{\rho} \vec{E}_\phi \sin^2 \theta , \quad \text{and} \tag{C.13}
\]
\[
\hat{y} \times (s_t \times (s_t \times (H^p_t)_{av})) = -\dot{\rho} \vec{H}_\phi \sin^2 \theta . \tag{C.14}
\]

By using the equalities in (C.5)-(C.6) and (C.9)-(C.14), one can transform the dyadic term, \( \hat{y} \times \vec{e}_t \cdot (E^p_t)_{av} \) in Equation (C.2) into the cylindrical expressions by the following operation:

\[
\begin{align*}
\hat{y} \times \vec{e}_t \cdot (E^p_t)_{av} & = \hat{y} \times [(\hat{\rho} \vec{e}_\rho + \hat{\phi} \vec{e}_\phi)] \\
& = \hat{y} \times [\hat{\rho} \vec{e}_\rho (\vec{E}_\rho \cos \phi - \vec{E}_\phi \sin \phi) + \hat{\phi} \vec{e}_\phi (\vec{E}_\rho \sin \phi + \vec{E}_\phi \cos \phi)] \\
& = \dot{\rho} [\vec{E}_\rho (e^2 - e^2 \sin^2 \phi) \sin \phi \cos \phi + \vec{E}_\phi (e^2 \sin^2 \phi + e^2 \cos^2 \phi)] \\
& - \dot{\phi} [\vec{E}_\rho (e^2 \cos^2 \phi + e^2 \sin^2 \phi) + \vec{E}_\phi (e^2 - e^2 \sin^2 \phi) \sin \phi \cos \phi] . \tag{C.15}
\end{align*}
\]
A dual expression for the dyadic term associated with the magnetic field in Equation (C.1) is given by

\[ \dot{\mathbf{y}} \times \mu_0 \cdot (\mathbf{H}_y^2)_{av} \]

\[ = \hat{\rho} [\hat{H}_\rho (\mu_2^e - \mu_2^t) \sin \phi \cos \phi + \hat{H}_\phi (\mu_2^e \sin^2 \phi + \mu_2^t \cos^2 \phi)] \]

\[ - \hat{\phi} [\hat{H}_\rho (\mu_2^e \cos^2 \phi + \mu_2^t \sin^2 \phi) + \hat{H}_\phi (\mu_2^e - \mu_2^t) \sin \phi \cos \phi] \]  

(C.16)

By plugging the cylindrical expressions in Equations (C.9)-(C.10), (C.13)-(C.16) into Equations (C.1) and (C.2), one can obtain two equations in terms of cylindrical components. By equating the \( \hat{\rho} \) and \( \hat{\phi} \) components for each equation, one obtains a set of four coupled differential equations described by

\[ \frac{d\tilde{E}_\rho}{dy} = j\omega \mu_{tm} \tilde{H}_\phi + j\omega \mu_c \tilde{H}_\rho \]  

(C.17)

\[ \frac{d\tilde{H}_\phi}{dy} = j\omega \epsilon_{tm} \tilde{E}_\rho + j\omega \epsilon_c \tilde{E}_\phi \]  

(C.18)

\[ \frac{d\tilde{E}_\phi}{dy} = -j\omega \mu_{te} \tilde{H}_\rho - j\omega \mu_c \tilde{H}_\phi \]  

and

(C.19)

\[ \frac{d\tilde{H}_\rho}{dy} = -j\omega \epsilon_{te} \tilde{E}_\phi - j\omega \epsilon_c \tilde{E}_\rho \]  

(C.20)

where

\[ \mu_{tm}(y) = \mu_2^e(y) \sin^2 \phi + \mu_2^t(y) \cos^2 \phi - \frac{\mu_0 \epsilon_0 \sin^2 \theta}{\epsilon_0^e(y)} \]  

(C.21)

\[ \epsilon_{tm}(y) = \epsilon_2^e(y) \cos^2 \phi + \epsilon_2^t(y) \sin^2 \phi \]  

(C.22)

\[ \mu_{te}(y) = \mu_2^e(y) \cos^2 \phi + \mu_2^t(y) \sin^2 \phi \]  

(C.23)

\[ \epsilon_{te}(y) = \epsilon_2^e(y) \sin^2 \phi + \epsilon_2^t(y) \cos^2 \phi - \frac{\mu_0 \epsilon_0 \sin^2 \theta}{\epsilon_0^e(y)} \]  

(C.24)

\[ \mu_c(y) = (\mu_2^e(y) - \mu_2^t(y)) \sin \phi \cos \phi \]  

and

(C.25)

\[ \epsilon_c(y) = (\epsilon_2^e(y) - \epsilon_2^t(y)) \sin \phi \cos \phi \]  

(C.26)

Note that Equations (C.17)-(C.18) are principally associated with the TM case; and, Equations (C.19)-(C.20) are with the TE case. These two cases are coupled.
via the coupling coefficients, $\mu_c$ and $\epsilon_c$. With $\mu_x(y) = \mu_y(y) = \mu_z(y) = \mu_0$, the above equations reduce to the dielectric case as described by Equations (2.67)-(2.70) in Chapter II.
APPENDIX D

A Finite Difference Solution for the Coupled First Order Differential Equations

The finite difference (FD) solution for the coupled first order differential equations derived in Appendix C is presented in this appendix. For convenience, the coupled equations are rewritten here as

\[
\frac{d\tilde{E}_\phi}{dy} = j\omega\mu_{tm}\tilde{H}_\phi + j\omega\mu_c\tilde{H}_\rho \\
\frac{d\tilde{H}_\phi}{dy} = j\omega\epsilon_{tm}\tilde{E}_\rho + j\omega\epsilon_c\tilde{E}_\phi \\
\frac{d\tilde{E}_\rho}{dy} = -j\omega\mu_{te}\tilde{H}_\rho - j\omega\mu_c\tilde{H}_\phi \\
\frac{d\tilde{H}_\rho}{dy} = -j\omega\epsilon_{te}\tilde{E}_\phi - j\omega\epsilon_c\tilde{E}_\rho
\]

As illustrated in Figure 128, the solution divides the homogenized material into \( N \) electrically small sections in the \( y \) direction and uses the FD formulation to approximate the differential equations. Combined with appropriate boundary conditions, the differential equations can be converted to a banded matrix equation in terms of the field variables for all the divided sections. The obvious choice for the field variables is \((E_\rho, H_\phi, E_\phi, H_\rho)\); however, to ensure numerical stability, one needs to normalize the variables so that they have about the same magnitude order. By recognizing \( E = \sqrt{\frac{\mu_0}{\epsilon_0}} H = Z_0 H \) in free space, the set of variables \((E_\rho, Z_0 H_\phi, E_\phi, Z_0 H_\rho)\) is chosen. As a result, the differential equations in (D.1)-(D.3) can be formulated as

\[
\frac{d\tilde{E}_\rho}{dy} = jk_0\mu_{tm}^r(Z_0\tilde{H}_\phi) + jk_0\mu_c^r(Z_0\tilde{H}_\rho)
\]

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\[
\frac{d(Z_0 \tilde{H}_\phi)}{dy} = j k_0 \varepsilon_\text{r}_\text{m} \tilde{E}_\rho + j k_0 \varepsilon_\text{r}_\text{c} \tilde{E}_\phi \quad \text{(D.6)}
\]

\[
\frac{d\tilde{E}_\phi}{dy} = -j k_0 \mu_\text{r}_\text{c} (Z_0 \tilde{H}_\rho) - j k_0 \mu_\text{r}_\text{m} (Z_0 \tilde{H}_\phi) \quad \text{, and} \quad \text{(D.7)}
\]

\[
\frac{d(Z_0 \tilde{H}_\rho)}{dy} = -j k_0 \varepsilon_\text{r}_\text{c} \tilde{E}_\phi - j k_0 \varepsilon_\text{r}_\text{m} \tilde{E}_\rho \quad \text{(D.8)}
\]

Note that \( k_0(= \omega \sqrt{\mu_0 \varepsilon_0}) \) is the free space wave number, and the magnetic and dielectric constants have been normalized against \( \mu_0 \) and \( \varepsilon_0 \), respectively; i.e.

\[
\mu_s^r = \frac{\mu_s}{\mu_0} \quad \text{and} \quad \text{(D.9)}
\]

\[
\varepsilon_s^r = \frac{\varepsilon_s}{\varepsilon_0} \quad \text{(D.10)}
\]

where the superscript \( r \) means 'relative' and the subscript \( s \) can stand for \( \text{tm, te} \) or \( \text{c} \). All the material constants are converted to these dimensionless quantities, which are easier to handle in a computer program.

The derivative in Equation (D.5) for the layer between \( y_n \) and \( y_{n+1} \) can be approximated by the FD expression such that

\[
\tilde{E}_\rho(n+1) - \tilde{E}_\rho(n) = (y_{n+1} - y_n) \left[ j \omega \mu_\text{r}_\text{m} \left( \frac{y_n + y_{n+1}}{2} \right) \tilde{H}_\phi \left( \frac{y_n + y_{n+1}}{2} \right) 
\right.
\]

\[
+ j \omega \mu_\text{r}_\text{c} \left( \frac{y_n + y_{n+1}}{2} \right) \tilde{H}_\rho \left( \frac{y_n + y_{n+1}}{2} \right) \right] . \quad \text{(D.11)}
\]

Since the layer thickness is assumed to be electrically small, the field quantities at the middle of the layer can be represented by the average of two adjacent field quantities such that

\[
\tilde{H}_\phi \left( \frac{y_n + y_{n+1}}{2} \right) = \frac{1}{2} \left[ \tilde{H}_\phi(n) + \tilde{H}_\phi(n + 1) \right] , \quad \text{and} \quad \text{(D.12)}
\]

\[
\tilde{H}_\rho \left( \frac{y_n + y_{n+1}}{2} \right) = \frac{1}{2} \left[ \tilde{H}_\rho(n) + \tilde{H}_\rho(n + 1) \right] . \quad \text{(D.13)}
\]

To simplify notation, \( \mu_\text{r}_\text{r}_\text{m} \) and \( \mu_\text{r}_\text{c} \) at the middle of the layer are shortened as

\[
\mu_\text{r}_\text{r}_\text{m} \left( \frac{y_n + y_{n+1}}{2} \right) \triangleq \mu_\text{r}_\text{r}_\text{m}(n) , \quad \text{and} \quad \text{(D.14)}
\]

\[
\mu_\text{r}_\text{c} \left( \frac{y_n + y_{n+1}}{2} \right) \triangleq \mu_\text{r}_\text{c}(n) . \quad \text{(D.15)}
\]
Furthermore, \( y_{n+1} - y_n \) can be expressed as the layer thickness \( \Delta d(n) \) by

\[
y_{n+1} - y_n = -\Delta d(n)
\]

(D.16)

Note that \( \Delta d(n) \) can be different for each layer depending on the variation of material properties. With the approximations and definitions in Equations (D.12)–(D.16), Equation (D.11) can be described by the field variables such that

\[
\begin{align*}
\tilde{E}_\rho(n) - \tilde{E}_\rho(n + 1) &= -\frac{jk_0 \mu_{im}(n) \Delta d(n)}{2} \left[ Z_0 \tilde{H}_\phi(n) + Z_0 \tilde{H}_\rho(n + 1) \right] \\
&\quad - \frac{jk_0 \mu_{e}(n) \Delta d(n)}{2} \left[ Z_0 \tilde{H}_\rho(n) + Z_0 \tilde{H}_\rho(n + 1) \right] = 0
\end{align*}
\]

(D.17)

Similar procedures and definitions can be applied to the other three differential equations (D.6)–(D.8) and the associated FD formulation is given by

\[
Z_0 \tilde{H}_\phi(n) - Z_0 \tilde{H}_\phi(n + 1) - \frac{jk_0 \varepsilon_{im}(n) \Delta d(n)}{2} \left[ \tilde{E}_\rho(n) + \tilde{E}_\rho(n + 1) \right]
\]
\[- \frac{j \omega_0 e^c_c(n) \Delta d(n)}{2} \left[ \tilde{E}_\phi(n) + \tilde{E}_\phi(n + 1) \right] = 0 \]  

\[ \tilde{E}_\phi(n) - \tilde{E}_\phi(n + 1) + \frac{j k_0 \mu e(n) \Delta d(n)}{2} \left[ Z_0 \tilde{H}_\rho(n) + Z_0 \tilde{H}_\rho(n + 1) \right] \]

\[ + \frac{j k_0 \mu e(n) \Delta d(n)}{2} \left[ Z_0 \tilde{H}_\phi(n) + Z_0 \tilde{H}_\phi(n + 1) \right] = 0 \]

and

\[ Z_0 \tilde{H}_\rho(n) - Z_0 \tilde{H}_\rho(n + 1) + \frac{j k_0 e(n) \Delta d(n)}{2} \left[ \tilde{E}_\rho(n) + \tilde{E}_\rho(n + 1) \right] \]

\[ + \frac{j k_0 e(n) \Delta d(n)}{2} \left[ \tilde{E}_\rho(n) + \tilde{E}_\rho(n + 1) \right] = 0 \]

To simplify the notation, \( a(n) \) is defined as

\[ a(n) \Delta = \frac{j k_0 \Delta d(n)}{2} \]  

\[ (D.21) \]

One can then form a 4 x 8 matrix equation to describe wave mechanisms for the layer between \( y_n \) and \( y_{n+1} \) such that

\[ \left[ L_{ij}(n) \right] \left[ S_j(n) \right] = \left[ 0 \right] \]

or

\[ \begin{array}{cccccccc}
1 & 0 & -a e_{im} & -a e_c & -1 & 0 & -a e_{im} & -a e_c \\
0 & 1 & a e_{im} & a e_c & 0 & -1 & a e_{im} & a e_c \\
a e_{im} & -a e_c & 1 & 0 & -a e_{im} & -a e_c & -1 & 0 \\
a e_c & a e_{es} & 0 & 1 & a e_c & a e_{es} & 0 & -1 \\
\end{array} \]

\[ \begin{bmatrix}
\tilde{E}_\rho(n) \\
\tilde{E}_\phi(n) \\
Z_0 \tilde{H}_\phi(n) \\
Z_0 \tilde{H}_\rho(n) \\
\tilde{E}_\rho(n + 1) \\
\tilde{E}_\phi(n + 1) \\
Z_0 \tilde{H}_\phi(n + 1) \\
Z_0 \tilde{H}_\rho(n + 1)
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \]  

\[ (D.22) \]
where $[L_{ij}(n)]$ is the localized $4 \times 8$ matrix and $[S_j(n)]$ is the column matrix for the 8 field variables. Note that the index $(n)$ has been eliminated for notational economy in the explicit expression for $[L_{ij}(n)]$.

For a region of uniform material, like a base layer, the TM and TE cases are decoupled because $\mu_c = \varepsilon_c = 0$. The wave behavior can then be described by the positively and negatively going plane waves for either polarization. As a result, it is not necessary to divide a uniform section into small subsections, which can save memory and computation time. Let us take the TM case as an example, the field quantities can be described by the plane waves as [38]

\begin{align}
\tilde{E}_\rho(n) & = A^+ e^{-jk_{tm}(n)yn} + A^- e^{jk_{tm}(n)yn} \quad \text{(D.23)} \\
Z_0 \tilde{H}_\phi(n) & = -\frac{A^+}{Z_{tm}(n)} e^{-jk_{tm}(n)yn} + \frac{A^-}{Z_{tm}(n)} e^{jk_{tm}(n)yn} \quad \text{(D.24)} \\
\tilde{E}_\rho(n + 1) & = A^+ e^{-jk_{tm}(n)yn+1} + A^- e^{jk_{tm}(n)yn+1} \quad \text{and} \quad \text{(D.25)} \\
Z_0 \tilde{H}_\phi(n + 1) & = -\frac{A^+}{Z_{tm}(n)} e^{-jk_{tm}(n)yn+1} + \frac{A^-}{Z_{tm}(n)} e^{jk_{tm}(n)yn+1} \quad \text{(D.26)}
\end{align}

where

\begin{align}
k_{tm}(n) & = k_0 \sqrt{\mu_{tm}(n)\varepsilon_{tm}(n)} \quad , \quad \text{and} \quad \text{(D.27)} \\
Z_{tm}(n) & = \sqrt{\frac{\mu_{tm}(n)}{\varepsilon_{tm}(n)}} \quad . \quad \text{(D.28)}
\end{align}

From (D.23) and (D.24), $A^+$ and $A^-$ can be described by

\begin{align}
A^+ & = \frac{1}{2} \left[ \tilde{E}_\rho(n) - Z_{tm}(n)Z_0 \tilde{H}_\phi(n) \right] e^{jk_{tm}(n)yn} \quad , \quad \text{and} \quad \text{(D.29)} \\
A^- & = \frac{1}{2} \left[ \tilde{E}_\rho(n) + Z_{tm}(n)Z_0 \tilde{H}_\phi(n) \right] e^{-jk_{tm}(n)yn} \quad \text{(D.30)}
\end{align}

Substituting (D.29) and (D.30) into (D.25) and (D.26), one obtains that

\begin{align}
\tilde{E}_\rho(n) \cos k_{tm}(n)\Delta d(n) - (Z_0 \tilde{H}_\phi(n))Z_{tm}(n) \cdot j \sin k_{tm}(n)\Delta d(n) - \tilde{E}_\rho(n + 1) = 0 \quad \text{(D.31)}
\end{align}

and

\begin{align}
- \tilde{E}_\rho(n) \frac{j \sin k_{tm}(n)\Delta d(n)}{Z_{tm}(n)} + (Z_0 \tilde{H}_\phi(n)) \cos k_{tm}(n)\Delta d(n) - Z_0 \tilde{H}_\phi(n + 1) = 0 \quad \text{(D.32)}
\end{align}

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where $\Delta d(n) = y_n - y_{n+1}$. Similar results for the TE case are given here as

$$\tilde{E}_\phi(n) \cos k_{te}(n) \Delta d(n) + (Z_0 \tilde{H}_0(n)) Z_{te}^*(n) j \sin k_{te}(n) \Delta d(n) - \tilde{E}_\phi(n+1) = 0 \quad (D.33)$$

and

$$\tilde{E}_\phi(n) \frac{j \sin k_{te}(n) \Delta d(n)}{Z_{te}^*(n)} + (Z_0 \tilde{H}_0(n)) \cos k_{te}(n) \Delta d(n) - (Z_0 \tilde{H}_0(n + 1)) = 0 \quad (D.34)$$

where

$$k_{te}(n) = k_0 \sqrt{\mu_{te}(n) \varepsilon_{te}(n)} , \quad \text{and}$$

$$Z_{te}^*(n) = \sqrt{\frac{\mu_{te}(n)}{\varepsilon_{te}(n)}} . \quad (D.35)$$

The localized matrix for this uniform layer can be written as

$$[L_{ij}(n)] = \begin{bmatrix}
\cos k_{tm} \Delta d & 0 & -j Z_{tm}^* \sin k_{tm} \Delta d & 0 & -1 & 0 & 0 & 0 \\
0 & \cos k_{te} \Delta d & 0 & j Z_{te}^* \sin k_{te} \Delta d & 0 & -1 & 0 & 0 \\
-j \sin k_{tm} \Delta d & 0 & \cos k_{tm} \Delta d & 0 & 0 & 0 & -1 & 0 \\
0 & -j \sin k_{te} \Delta d & 0 & \cos k_{te} \Delta d & 0 & 0 & 0 & -1
\end{bmatrix} \quad (D.37)$$

where the index $(n)$ has been eliminated on the right hand side for notational economy.

One can cascade the $N$ localized matrix to form a $(4N) \times (4N + 4)$ matrix equation. However, the number of variables is more than the number of equations by 4. This problem can be dealt with by imposing the boundary conditions at the top and bottom interface. For the TM case, by matching the $E$ and $H$ fields at $y_0$, one can show that

$$\frac{\tilde{E}_\rho(0)}{Z_{0tm}^*} + Z_0 \tilde{H}_\rho(0) = \frac{2\tilde{E}_i}{Z_{0tm}^*} , \quad \text{and}$$

$$\tilde{E}_\phi(0) - Z_{0te}^* Z_0 \tilde{H}_\rho(0) = 0 . \quad (D.38)$$

(D.39)
where $\tilde{E}_p^i$ is the incident electric field for the TM case, and $Z_{0_{\text{tm}}} (= \cos \theta)$ and $Z_{0_{\text{te}}} (= \frac{1}{\cos \theta})$ are the normalized free space impedances for the TM and TE cases, respectively. By matching the boundary conditions at $y_N$, one obtains that

\begin{align}
\tilde{E}_p(N) - Z_{0_{\text{tm}}} \tilde{H}_\phi(N) &= 0, \quad \text{and} \quad (D.40) \\
\tilde{E}_\phi(N) + Z_{0_{\text{te}}} \tilde{H}_\rho(N) &= 0. \quad (D.41)
\end{align}

Similarly for the TE incidence, one acquires four additional equations by matching the boundary conditions such that

\begin{align}
\tilde{E}_p(0) + Z_{0_{\text{tm}}} Z_0 \tilde{H}_\phi(0) &= 0 \quad (D.42) \\
\frac{\tilde{E}_\phi(0)}{Z_{0_{\text{te}}}} - Z_0 \tilde{H}_\rho(0) &= \frac{2\tilde{E}_\phi^i}{Z_{0_{\text{te}}}} \quad (D.43) \\
\tilde{E}_p(N) - Z_{0_{\text{tm}}} Z_{0_{\text{te}}} \tilde{H}_\phi(N) &= 0, \quad \text{and} \quad (D.44) \\
\tilde{E}_\phi(N) + Z_{0_{\text{te}}} Z_{0_{\text{te}}} \tilde{H}_\rho(N) &= 0 \quad (D.45)
\end{align}

where $\tilde{E}_\phi^i$ is the incident electric field for the TE case.

Combining these boundary conditions and cascading the localized matrix for each layer, one obtains the banded matrix equation as illustrated in Figure 129. The mapping between the localized matrix and the whole cascaded one is determined by

\begin{align}
S_1(n) &= \tilde{E}_p(n); \quad S_2(n) = \tilde{E}_\phi(n) \quad (D.46) \\
S_3(n) &= Z_0 \tilde{H}_\phi(n); \quad S_4(n) = Z_0 \tilde{H}_\rho(n), \quad \text{and} \quad (D.47) \\
L_{ij}(n) &= C_2+4i,4j+1 \quad (D.48)
\end{align}

The boundary elements for the TM case are given by

\begin{align}
C_{11} &= \frac{1}{Z_{0_{\text{tm}}}}; \quad C_{13} = C_{22} = 1; \quad C_{24} = -Z_{0_{\text{tm}}} \quad (D.49) \\
C_{4N+3,4N+1} &= C_{4N+4,4N+2} = 1 \quad (D.50) \\
C_{4N+3,4N+3} &= -Z_{0_{\text{tm}}}; \quad C_{4N+4,4N+4} = Z_{0_{\text{te}}}, \quad \text{and} \quad (D.51) \\
F_1 &= \frac{2}{Z_{0_{\text{tm}}}}. \quad (D.52)
\end{align}
\[
(C_{ij}) (S_j) = (F_i)
\]

Figure 129: The schematic of a banded matrix equation.
For the TE case,

\[ C_{11} = 1; \quad C_{13} = Z_{0te}; \quad C_{22} = \frac{1}{Z_{0te}^r}; \quad C_{24} = -1 \quad (D.53) \]

\[ C_{4N+3,4N+1} = C_{4N+4,4N+2} = 1 \quad (D.54) \]

\[ C_{4N+3,4N+4} = Z_{0te}^r; \quad C_{4N+4,4N+4} = Z_{0te}^r \quad , \text{and} \quad (D.55) \]

\[ F_2 = \frac{2}{Z_{0te}} \quad (D.56) \]

In the computer implementation, only the banded portion is stored to save memory.

A banded matrix solver based on the Crout method is used to efficiently solve the equation.

The reflection coefficient can be related to the solved field variables by the following formulas:

for the TM case,

\[ \Gamma_{\theta \theta} = 1 - \tilde{E}_\rho(0) \quad , \text{and} \quad (D.57) \]

\[ \Gamma_{\phi \theta} = \frac{\tilde{E}_\phi(0)}{Z_{0te}^r} \quad ; \quad (D.58) \]

for the TE case,

\[ \Gamma_{\phi \phi} = \tilde{E}_\phi(0) - 1 \quad , \text{and} \quad (D.59) \]

\[ \Gamma_{\theta \phi} = \frac{\tilde{E}_\rho(0)}{Z_{0tm}^r} \quad ; \quad (D.60) \]

The transmission coefficient must be obtained by normalizing the field variables at \( y_N \) against the free space transmission. The TM case results are given by

\[ T_{\theta \theta} = \tilde{E}_\rho(N)e^{jk_0(y_0-y_N)\cos\theta} \quad , \text{and} \quad (D.61) \]

\[ T_{\phi \theta} = -\frac{\tilde{E}_\phi(N)}{Z_{0te}^r}e^{jk_0(y_0-y_N)\cos\theta} \quad . \quad (D.62) \]

For the TE case,

\[ T_{\phi \phi} = \tilde{E}_\phi(N)e^{jk_0(y_0-y_N)\cos\theta} \quad , \text{and} \quad (D.63) \]

\[ T_{\theta \phi} = \frac{\tilde{E}_\rho(N)}{Z_{0tm}^r}e^{jk_0(y_0-y_N)\cos\theta} \quad . \quad (D.64) \]
Note that, in the expressions for the reflection and transmission coefficients, the first subscript is associated with the receive polarization and the second one with the incident polarization. The co-polarized results need not be divided by the incident field because the transverse incident electric field has been set to unity. On the other hand, the cross-polarized fields have to be normalized properly such that the projection from the electric field to its transverse component is taken into account.

This FD solution completes the theoretical development for the TLA analysis. Various comparisons of the TLA and PMM results are presented in Chapter III.
BIBLIOGRAPHY


