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An atomic beam magnetic resonance study of a superconductor's magnetic vortex lattice

Dapore-Schwartz, Samuel, Ph.D.
The Ohio State University, 1994
AN ATOMIC BEAM MAGNETIC RESONANCE STUDY OF A SUPERCONDUCTOR'S MAGNETIC VORTEX LATTICE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Samuel Dapore-Schwartz, B.A., M.S.

* * * * *

The Ohio State University

1994

Dissertation Committee:
Dr. Gregory P. Lafyatis
Dr. Thomas R. Lemberger
Dr. Charles Pennington

Approved by

Adviser

Co-Adviser

Department of Physics
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Samuel Dapore-Schwartz

1994
To Renée,

who continues to inspire me

to through her strength, dedication, and brilliance
This thesis represents the work of many people, and would not have been accomplished without the support of many more. First, I would like to thank my advisor, Greg Lafyatis, for his encouragement, insight, and clearheadedness throughout the seven years we have worked together. Second, I owe a deep debt of gratitude to my colleague Nathan Woodard. Nate and I spent many all-nighters together working on this experiment, and it would not have been completed without his tireless effort or his unflagging friendship.

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VITA

October 19, 1965 ..................................................... Born - Battle Creek, MI

1983-1987 ............................................................ National Merit Scholar
The University of Chicago
Chicago, IL

1987 ................................................................. B.A. Physics
The University of Chicago
Chicago, IL

1987, 1993-present ................................................. Graduate Teaching Assistant
Department of Physics
The Ohio State University
Columbus, OH

1988, 1992-1993 .................................................... Graduate Research Assistant
Department of Physics
The Ohio State University
Columbus, OH

1988-1992 ............................................................ National Science Foundation Graduate Research Fellow
Department of Physics
The Ohio State University
Columbus, OH

1990 ................................................................. M.S. Physics
The Ohio State University
Columbus, OH

Publications


Fields of Study
Major Field: Physics
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CHAPTER I

Introduction

Since the discovery of high-temperature superconductors, there has been a great deal of scientific interest in their magnetic properties. In particular, there has been a strong effort to study the Abrikosov magnetic vortex lattice in these materials. In this dissertation, we will introduce a new experimental tool for the study of the spatial and temporal properties of the magnetic vortex lattice in type-II superconductors. Although still in its infancy, we feel that this new technique, which we call the “Atomic Beam Magnetic Resonance Probe” (ABMR), will prove to be widely applicable to the search for understanding of magnetic vortex lattices in a wide variety of materials.

In ABMR, an atomic beam of potassium skims over the surface of a superconducting sample that has been prepared in the Abrikosov state, as shown in Figure 1.1. If the magnetic field produced by the vortex lattice has a spatial period $d$, then a potassium atoms passing through this field at velocity $v$ will encounter a magnetic field periodic in time with frequency

$$f = \frac{v}{d}. \quad (1.1)$$

If this frequency is equal to the resonance frequency $f_0$ of a magnetically-coupled two-level atomic transition, then atoms will be driven from one state to the other.
Figure 1.1: The basic idea behind the atomic beam magnetic resonance probe. An atom traveling at velocity $v$ will make a transition from the initial state (total angular-momentum quantum number $F = 1$) to the final state ($F = 2$) if the frequency at which it passes the vortices equals the resonance frequency $f_0$ of the transition.
By analyzing the transition probability as a function of the atoms' velocity, we can determine the spatial periodicity of the magnetic vortex lattice, the degree of disorder in the lattice structure, and the velocity (if any) at which the vortices drift in the presence of an applied current. The ABMR technique idea was originated by Truman Brown and his thesis advisor, John King, at MIT in 1969[8]. Our implementation of the ABMR technique was inspired by their work, but differs in important ways. The differences in experimental design turn out to have serious implications for the interpretation of their results.

Our experimental technique is capable of measuring one of the basic structural properties of the magnetic vortex lattice—the vortex-vortex correlation function. We will show in a later section (Section 4.1) that the transition probability for atoms moving at a particular velocity \( v \) is proportional to the Fourier transform of the magnetic field squared at wavenumber \( q = \omega_0/v \), where \( \omega_0 \) is the resonance frequency (in radians per second) of the magnetic transition. This quantity turns out to be the Fourier transform of the magnetic field auto-correlation function, which is closely related to the vortex-vortex correlation function of the underlying lattice. In other words, our tool effectively measures the vortex lattice correlation function directly.

Atomic beam magnetic resonance also provides a new means with which to measure one of the most important microscopic parameters of a superconductor—the magnetic penetration depth. The magnetic field in the vicinity of a magnetic vortex depends strongly on the penetration depth. Generally, the larger the penetration depth, the broader and weaker the vortex field. We will show in Chapter IV that the
size of the ABMR signal at a particular velocity and applied field can be calculated as a function of the magnetic penetration depth alone. This means that by comparing the observed size of ABMR signals to theoretical calculations, we can in principle directly measure the absolute magnetic penetration depth.

Before we go on to discuss our experiment in more detail, we will try to place the ABMR technique in the context of several other probes of magnetic vortex structure. The oldest method for directly studying the structure of the vortex lattice is Bitter pattern decoration[11]. In this technique, small ($\sim 500$ Å) particles of a magnetic material such as nickel or cobalt are allowed to settle on the surface of a superconductor that has been prepared in the Abrikosov state. The particles are attracted to the strong magnetic fields near the vortex cores and stick to the surface. Later, the sample is removed from the decoration apparatus and a carbon mold is made of the surface. The mold is then examined in an electron microscope. The locations of the vortex cores show up as dots in the micrograph.

The Bitter decoration technique provided the first direct observation of Abrikosov's proposed flux lattice, and confirmed his prediction that the lattice would be triangular. In recent years, the technique has been instrumental in studying the structural properties of vortex lattice in high-$T_c$ materials[13, 28, 5, 6, 14].

Bitter decoration has the inherent feature of producing a direct image of the vortex lattice at the surface of virtually any material. The images produced can be analyzed to extract information about the correlation function of the lattice in two dimensions. On the other hand, the technique has several disadvantages: To begin
with, the experiment can only perform one measurement at a time, since the sample has to be removed from the apparatus and examined before another measurement. Furthermore, the decoration process itself takes some seconds to perform, so that the Bitter pattern reveals only those magnetic features that remained fixed for that period of time. Bitter decoration cannot determine much about the dynamics of the vortex lattice.

There have been several new techniques developed in the past few years that also provide images of the flux lattice. First of all, the electronic structure of individual vortices and the vortex lattice as a whole has been imaged by scanning tunneling microscopy of surfaces of 2H – NbSe$_2$[20]. The images obtained in this fashion are breathtaking, and yielded rich spatial and spectroscopic information about the vortices in this material. However, due to the stringent demands that scanning tunneling microscopy makes upon the quality and cleanliness of the surface, the technique has not been applied to any other materials.

Another class of scanning techniques involve scanning magnetic probes, in particular, scanning Hall probes[15] have been able to directly image individual vortices in high-$T_c$ materials, and map the dependence of the vortex shape on temperature (and therefore magnetic penetration depth). Other magnetic probes include scanning SQUIDS[39] and magnetic force microscopy[42]. Although these techniques can be applied to any sample with a reasonably smooth surface, they are still not capable of studying the dynamics of the vortex lattice in real time. Their spatial resolution is limited to about 0.5 μm.
Perhaps the most dramatic direct imaging technique is electron holography[27, 16, 17, 7]. In this technique, a beam of nearly monoenergetic electrons is split in two. One beam passes through a thin sample of YBa$_2$Cu$_3$O$_{7-x}$ (YBCO) or niobium and then recombines with the second beam. The electrons interfere due to Aharonov-Bohm phase shifts introduced by the magnetic fields in the sample, and the interference pattern produces a moving picture (at 30 frames per second) of the vortex lattice. This technique is very powerful, providing real time information on the structure and motion of the vortex lattice, as well as the diameter of individual vortices in niobium. It's main disadvantage, beyond its enormous expense, is the fact that the superconducting sample has to be thin enough to allow the electrons to pass through unhindered.

All the above techniques attempt to image the vortex lattice directly. ABMR, on the other hand, measures structural information in Fourier transform space. There is one other well-established tool for studying vortex lattice structure in Fourier space—small-angle neutron scattering. In neutron scattering, a mono-energetic beam of neutrons passes through a sample parallel to an applied magnetic field. The neutrons are diffracted by the lattice, being deflected from their original path by a momentum shift corresponding to a reciprocal lattice vector. Neutron scattering is capable of providing information about the geometry, orientation, lattice spacing, and degree of disorder present in the vortex lattice.

Small-angle neutron scattering has been used recently to obtain experimental evidence of flux-lattice melting in high-$T_c$ materials[10] and in niobium[24]. Since the
form factor for neutron scattering depends on the magnetic field profile of the vortex lattice, the variation in scattered intensity with temperature also gives information about the temperature-dependence (but not the absolute value of) the magnetic penetration depth[41].

While very powerful, small-angle neutron scattering suffers from the common difficulty of poor time resolution. Since the scattering rate in these experiments is small, data must be accumulated over a long period, which precludes the real-time observation of vortex dynamics. Also, neutron diffraction requires large, difficult to obtain, single-crystal targets in order to obtain usable scattering signals.

In our opinion, although still under development, the atomic beam magnetic resonance probe will become a valuable research tool in the study of magnetic vortex lattices. The technique has a combination of advantages that allow it to provide complementary and possibly unique information about vortex structures. For instance:

- We can observe the spatial structure of the vortex lattice in any material in real time.

- We do not need the large single-crystal samples and long integration times required for neutron diffraction studies.

- We can take many measurements in succession on the same sample, unlike Bitter decoration experiments that require the sample to be removed from the apparatus and studied in an electron microscope after every measurement. This allows
us to make direct comparisons between the structures of the vortex lattices and different magnetic fields and temperatures.

- We can study any material, in any thickness, at any temperature. We do not need a specially prepared surface such as those required for scanning-tunneling microscopy and electron holography.

- Since we determine the spatial structure of the vortex lattice by measuring the velocity of atoms that make hyperfine transitions while passing over it, we can also measure the drift velocity of the vortex lattice by observing changes in the velocities of features of the data when transport currents when we apply a transport current.

- Finally, we are able to predict the magnetic field seen by the atom as a function of the sample’s magnetic penetration depth and the atom’s response to the field, allowing us to make direct, absolute measurements of our sample's magnetic penetration depth.

On the other hand, our technique has limitations of its own. In particular, ABMR works in one dimension and in Fourier transform space. It does not provide information about the angular orientation of the reciprocal lattice vectors of the vortex lattice, such as that provided by neutron scattering measurements. ABMR certainly does not image the vortex lattice directly, and therefore cannot provide the same type of spatial information that is probed by Bitter decoration, STM, and electron holography.
Figure 1.2: Energy levels of potassium. The scale is extremely distorted. The magnetic, optical pumping, and detection transitions are indicated by arrows.
Before going into the detailed discussion of the experiment, we will outline the basic structure of our experiment. First, an atomic beam of potassium atoms is prepared in an initial hyperfine state, then allowed to pass over a niobium sample. The fraction of atoms that made transitions to the final hyperfine state is then measured by a detector as a function of their velocity. Figure 1.2 shows the energy level structure of potassium. The atomic beam is prepared in the initial \( F = 1 \) state by optical pumping using a laser tuned to the \( 4s^2 S_{1/2}, F = 2 \rightarrow 4p^2 P_{1/2}, F' = 2 \) near-infrared transition at 770 nm. As the atoms pass over the sample, they make magnetic-dipole hyperfine transitions (which we will also refer to as nuclear magnetic resonance or NMR transitions) from the \( F = 1 \) to \( F = 2 \) hyperfine states of the \( 4s^2 S_{1/2} \) electronic ground state. Finally, the atoms are detected in the \( F = 2 \) hyperfine state by their resonance fluorescence when they are illuminated by a second laser tuned to the \( 4s^2 S_{1/2}, F = 2 \rightarrow 4p^2 P_{3/2}, F'' = 3 \) near-infrared transition at 767 nm. The atomic velocity is determined by the Doppler-shift of the laser radiation in the atoms' moving frame of reference.

The magnetic structures probed by our experiment have length scales of about 1 \( \mu \)m. To see this, let's examine the frequency and velocities of the atoms we are using as a probe. The resonance frequency of the NMR transition between \( F = 1 \) and \( F = 2 \) in the electronic ground state of \( ^{39}\text{K} \) is 462 MHz. Our potassium atoms originate in a thermal atomic beam oven at a temperature between 300 and 400 degrees C. Their typical thermal velocities range from 300 m/s to 1200 m/s. The period \( d \) probed by an atom moving at 500 m/s will be 1.1 \( \mu \)m. By comparison, a vortex lattice
produced by an average magnetic field of 20 Gauss will have a fundamental period of
\[ d = \sqrt{\phi_0 / B} = 1.0 \, \mu m. \]
We would therefore expect the atomic beam probe to be most sensitive to the vortex structures produced by magnetic fields ranging from about 5 Gauss to about 30 Gauss.

In Chapter II of this dissertation, we discuss the experimental apparatus in detail. In Chapter III, we present the experimental results we obtained for niobium. In particular, we present measurements of the spatial structure of the vortex lattice as a function of magnetic field, and examine how temperature, transport currents, and long-range structure affect our results. In Chapter IV, we examine the theory of atomic excitation by the spatially-varying magnetic fields produced by a vortex lattice, and use this theory to make predictions for the size and velocity-dependence of our signal that we compare directly to experimental data. We are also able to directly determine the temperature-dependence of the magnetic penetration depth from the temperature-dependence of our transition strength. In Chapter V, we compare our results to the original work of Brown and King (who used a somewhat different apparatus to conduct similar experiments to ours) and are able to suggest solutions to some long-standing difficulties involving the interpretation of their results. Finally, in Chapter VI, we summarize our results and point the way to some further applications of the atomic beam probe technique.
CHAPTER II

The Apparatus

The experiment consists of an atomic beam line with initial state preparation, sample, final state selection, and detection regions, as shown in Figure 2.1. The atomic beam originates in a thermal effusive source (oven) and crosses an optical pumping laser beam at right angles for Doppler-free state preparation, as will be discussed in Section 2.1. The laser’s wavelength is 770nm, resonant with the $4s^2S_{1/2} \rightarrow 2 \rightarrow 4p^2P_{1/2}, F = 2$ electronic transition of potassium-39 ($^{39}$K). The remaining isotopes ($^{40}$K and $^{41}$K, at .01% and 6% relative abundance, respectively) are not optically pumped. The laser is frequency-stabilized and servo-locked to the transition frequency, as will be discussed in Section 2.1. The beam can blocked at this point by a rotating flap, which is usually done except when collecting data. The oven/optical pumping region of the experiment is differentially pumped by its own 30 liter/sec ion pump.

A movable hot-wire ionization detector can be lowered into place at the end of the detector chamber. As discussed in Section 2.8.6, the hot wire detector ionizes all the potassium atoms that hit it, producing a current proportional to the atomic beam intensity. This current can be measured on an electrometer. We align the atomic beam line by monitoring the current on the hot wire detector as we position...
Figure 2.1: Experimental layout. The atomic beam travels from left to right. The oven/first pumping chamber, the cryostat, and the detector chamber all can be isolated by gate valves and independently evacuated. The oven and detector chambers are each pumped by a 30 liter/sec ion pump, while the cryostat is cryogenically pumped by activated charcoal.
the various pieces of the apparatus.

After being optically pumped out of the $F = 2$ levels, the atomic beam enters the cryostat, where it passes within 1.8 μm of the sample surface in the presence of an applied magnetic field. The sample itself is described in Section 2.5, and the cryostat and sample stage are described in Section 2.6. The cryostat is attached to a movable platform, allowing alignment of the sample with the atomic beam. The sample is mounted on a variable-temperature stage attached to a liquid helium cryostat and can be warmed from around 5 K to well above its transition temperature of $\approx 9$ K.

The atoms make transitions from the $F = 1$ to $F = 2$ levels of the atomic ground state while passing the sample, resulting in a mix of both levels when leaving the sample region. A Stern-Gerlach "bending" magnet separates the $F = 1$ atoms (and the $m_F = -2$, as well) from the remaining $F = 2$ atoms (called "signal"), and deflects the signal atoms into the detector. The bending magnet is described in Section 2.7.

Before entering the detector proper, which is described in Section 2.8, the beam crosses a second Doppler-free 770nm laser beam which can be modulated by a chopper. This effectively turns the signal off and on by optically pumping the excited atoms back into the $F = 1$ level, allowing us to use phase-sensitive detection to enhance our sensitivity.

The atoms then enter the detector chamber, in which the quantity of atoms in the $F = 2$ state is measured as a function of their velocity. The atoms are detected by their fluorescence as they cross a laser tuned to the $4s^2 S_{1/2}, F = 2, m_F = 2 \rightarrow 4p^2 P_{3/2}, F' = 3, m'_F = 3$ transition of $^{39}$K. The fluorescence light is collected and sent
to a photomultiplier tube (PMT), and the PMT current is measured and recorded as
signal. The atomic beam intersects the laser beam at a five-degree angle as shown in
Figure 2.8. This nearly head-on intersection causes the laser frequency in the atoms’
rest frame to be Doppler shifted in proportion to the atoms’ velocity. Therefore, we
can measure the atomic velocity by measuring the Doppler shift. We discuss how this
is done in detail in Section 2.8.5.

The detector’s sensitivity is enhanced by enabling each atom to emit many photons
as it crosses the detection laser beam, using a “cycling” transition. To do this, the
detection laser is $\sigma^+$ polarized so that it will only drive $\Delta m_F = +1$ transitions, such
as the cycling transition from $m_F = 2$ in the ground state to $m'_F = 3$ in the excited
state. Atoms in this excited state can only decay back into the $m_F = 2$ from which
they came—therefore, the atoms can be excited again and again until they leave the
laser beam.

The oven, cryostat, Stern-Gerlach magnet, and the entire detector assembly, includ-
ing optics and its own ion pump, are connected by flexible vacuum bellows and
mounted on movable platforms. These allow us to precisely position each of the four
locations where the beam must pass through a narrow opening (that is, the oven
nozzle, the sample, the Stern-Gerlach magnet gap, and the intersection of the atomic
beam with the detection laser).

A typical data run involves preparing the sample in the desired manner, opening
the atomic beam, and then scanning the detector magnet field while phase-sensitively
recording the PMT current on a lock-in amplifier. Since the deflection angle of the
beam exiting the Stern-Gerlach magnet depends on velocity, it is necessary to scan the Stern-Gerlach field synchronously with the detector magnet field so as to keep the proper velocity class aimed at the detector. The magnet currents and synchronization are controlled by a IBM-compatible 486 computer running ASYST data acquisition and control software[3], which also records the data. The computer interface is an IBM Data Acquisition and Control Adapter card.

2.1 Lasers and Optics

Both the atomic state preparation and detection rely upon electronic excitation using laser light. The initial atomic state is prepared in the $F = 1$ state using optical pumping to depopulate the $F = 2$ state. Detecting NMR transitions to the $F = 2, m_F = 2$ level as a function of atomic velocity relies upon measuring the fluorescence from atoms interacting with a counter-propagating laser beam and keeping track of the atoms' Doppler shift. Both procedures require continuous-wave laser light with a narrow linewidth (preferably less than the 6 MHz natural linewidth of the potassium electronic excited state) and a stable, known frequency. For optical pumping, the laser must be precisely tuned to the correct atomic transition—frequency fluctuations larger than the excited state linewidth severely affect the results of the optical pumping process, as we will discuss in Section 2.4. The velocity resolution of the detector is also directly set by the frequency width of the laser light unless it is less than the excited state linewidth. Therefore, narrow linewidth and stable frequency are essential.
We use STC quantum well semiconductor diode lasers, from a special batch which operate in the 767-770 nm region at room temperature. We have also been able to hit the 770 nm line using a standard 780 nm device cooled to -20 °C, and we intend to use these devices in the future, since the supply of 770 nm lasers is limited.

We mount our lasers at the end of a copper rod which can be inserted into a standard Newport Optics five-axis lens positioner, which holds a (Brand?) collimating lens very close to the laser's face. The copper rod is attached to a thermo-electric cooler by silver-bearing epoxy (for good thermal contact), and contains a resistive heater and a thermistor to measure temperature. Fluctuations in the temperature of the copper rod are regulated by a feedback circuit to be less than 10 mK. The thermo-electric cooler is a "pyramid" type made by epoxying together two standard Melcor coolers. The coolers are powered by a 2 Amp, 15 Volt regulated current supply. They also provide electrical isolation from their heatsink, which supports the rod assembly. Isolation is necessary since a short between the laser mount and ground will invariably destroy the laser. The heat sink, lens positioner, and locking optics are all attached to an aluminum plate, which is vibrationally isolated from the optics table by Sorbothane pads. The whole assembly is kept covered by a lucite box, to minimize temperature fluctuations and vibration, and to keep out dust. The laser is powered by a variable current supply. A large inductor is connected in series with the laser diode to reduce current fluctuations (which might damage the laser).

\(^{1}\)STC Optical Devices
Bixham Rd.
Paignton, Devon
United Kingdom
A special “shorting box” connects the laser supply to the laser, in which a 100 kΩ potentiometer is wired across the laser diode, which is kept shorted when the laser is not in operation. A nine-conductor shielded cable connects the shorting box to the laser controller using multi-pin Amp connectors.

We control the lasers’ frequency using the diffraction grating feedback locking technique. This technique is described in detail by MacAdam et al.[25] and the broader application of diode lasers to atomic physics is outlined in an excellent review article by Wieman and Hollberg[38]. Grating locking involves using a diffraction grating as one element of an external resonant cavity coupled to the diode laser’s output mode. Figure 2.2 shows the two commonest locking schemes. The optical pumping laser uses grazing incidence locking, while the detector laser uses the Littrow scheme. In both cases, coarse tuning is done using an 80-pitch screw, while a piezo-electric actuator provides electronic fine control over the laser frequency. An additional 80-pitch screw controls the alignment of the grating (or tuning mirror) out of the plane of reflection. This adjustment is necessary to keep the feedback light coupled directly into the diode’s front facet.

The laser output must be focused very precisely to optical infinity to ensure exact coupling of the returning feedback light into the laser. We focus our lasers using a 4.5 mm focal length $f = 1$ collimating lens. The lenses are mounted in Newport Optics five-axis (three translation, two tilt) lens positioners. To properly focus the output beam, a spot is located about 20 feet directly in front of the laser (with the grating removed) and a white card placed there. The horizontal and then the
Figure 2.2: Littrow (a) and grazing incidence (b) locking schemes: In the Littrow configuration, the negative first order diffraction from the grating is directed back into the diode, forming an external resonant cavity. Rotating the grating changes the wavelength of light that couples back into the diode, tuning the laser. A beamsplitter couples light out of the cavity. The grazing incidence scheme is similar, but the negative first order light reflects off a mirror, which is rotated to tune the laser. The specular reflection from the grating provides coupling out of the cavity.
vertical translation and tilt controls are adjusted together, maintaining the beam location directly in front of the laser. The controls are adjusted until the laser spot is symmetrical. The axial translation is then adjusted until an ideal spot shape (a TEM00 mode with a narrow, bright central fringe with only dim side fringes) is achieved.

The grating is then installed and aligned using the coarse controls until the negative first-order diffraction peak is reflected directly back into the lens. The output beam is then directed into a grating monochromator equipped with a video camera to monitor the laser's spectrum over a 10 nm region, encompassing many of the laser's longitudinal modes. The grating tilt and laser focus are now carefully adjusted until the spectrum retains only a single mode and is smoothly tunable as seen on the monochromator monitor. A very sensitive test of the grating alignment is made possible by using a ramp-generator to continuously scan the grating piezo back and forth.

With the laser current set just below the laser's free-running threshold, we monitor the laser output power (using the video camera or an infrared viewer) while adjusting the grating's vertical tilt. When the tilt is correct, the laser will blink on and off. By steadily reducing the laser current while adjusting the tilt, one can achieve optimal alignment. Finally, some of the light is sent through a potassium vapor cell, and the grating position and laser current are adjusted until fluorescence is seen.

The grating locking technique both tunes the laser and narrows its spectral linewidth. We can tune our lasers over a 20 to 30 Å range, which we can coarsely control by varying the temperature of the laser. The tuning is not continuous, but "hops"
from mode to mode of the free-running laser diode cavity. These modes can be tuned by varying the laser current; therefore, to reach the desired frequency, one has to precisely set the laser temperature and current as well as the grating position.

The linewidth of a free-running diode laser is about 30 MHz, which is too broad compared to the natural linewidth of the excited states for our purposes. Using a 300 MHz Fabry-Perot cavity with a finesse of 100, we were able to see that grating locking reduced this linewidth below 3 MHz, but only on time scales short compared to the vibrational noise present in our optics alignment. Vibrations were responsible for increasing the linewidth of the locked lasers to 10 to 15 MHz. In addition, the center frequency was prone to drift, presumably in response to small changes in temperature. We therefore needed to reduce the effects of vibrational noise and to keep the laser frequency locked to the desired atomic transitions. We accomplished this through the use of a double feedback loop, which locked the laser frequency to a Fabry-Perot cavity fringe, and in turn controlled the length of this locking cavity so as to keep the laser locked to the transition frequency.

Before getting into the details of the servo-locking arrangement, we should discuss how we determined that the laser frequency indeed matched the correct transition frequency. At the coarse level, we attached a CCD closed circuit video camera (a black and white surveillance type that works well in the near-infrared) to the output of a grating spectrometer. This enabled us to tell when the laser was functioning in a single-mode manner (meaning that the grating locking optics were correctly aligned), and more importantly, when the laser was tuned close to the correct wavelength. An
Figure 2.3: Saturated absorption: When the laser is tuned to the exact atomic resonance frequency, both beams interact with the same atomic velocity class. The power beam saturates the atomic transition, giving rise to a "Lamb dip" in the absorption signal. If the power beam is modulated by a chopper or AOM, the Lamb dip signal can be isolated by phase-sensitive detection.

infrared viewer lets us see the resonance fluorescence in a potassium vapor cell when the laser is tuned within the Doppler profile of an atomic transition. Unfortunately, this profile is much broader than the entire hyperfine structure of potassium, so we needed to use Doppler-free saturation spectroscopy to resolve the individual hyperfine absorption lines.

We use saturated absorption, the technique most widely used for Doppler-free spectroscopy in vapor cells. The essence of the technique is to direct two counter-propagating beams from the same laser through a potassium vapor cell, as shown in Figure 2.3. At a given laser frequency $\omega_L$, each beam will interact with atoms moving with velocity $v$ (in the direction of the beam) so that $\omega_L + kv = \omega_b$, where $k =$
Figure 2.4: (a) Probe beam transmission as a function of laser frequency: The Lamb dip appears as a sharp “bump” in the transmitted signal at the exact resonance frequency. (b) The saturated absorption spectrum for the $^2S_{1/2} \rightarrow ^2P_{3/2}$ (detection) transition in $^{39}$K. The pumping beam is chopped and the probe beam transmission measured by phase-sensitive detection. The locking point, to which the laser frequency is stabilized, is indicated by the arrow. Only the ground state hyperfine structure is resolved. The “upside-down” peak in between the two Lamb dips is a cross-over resonance.
$2\pi/\lambda$ is the wavenumber of the laser light, and $\omega_0$ is the atomic resonance frequency. The beams will interact with atoms moving with the same speed, but in opposite directions, except when $v = 0$. When $v = 0$, $\omega_L = \omega_0$, and both beams interact with the same velocity class of atoms. The “pump” beam will saturate the atomic transition, so that more of the “probe” beam will be transmitted than for nearby laser frequencies where $\omega_L \neq \omega_0$. If one measures the probe beam transmission as a function of frequency, one will see a curve like that shown in Figure 2.1, with a narrow spike of increased transmission at the bottom of a Doppler broadened absorption valley.

For our saturated absorption reference, we use a cylindrical pyrex cell into which some potassium metal has been evaporated under vacuum. We keep our cell heated to increase the potassium vapor pressure using thermocouple wire powered by a Variac. We also make special heated end caps to keep the windows through which the laser beams pass warmer than the rest of the cell and therefore free of condensed metal. The transmitted probe beam is monitored by a PIN photodiode with built-in operational amplifier—typically a United Detector Technologies UDT-155D or an EG&G Photon Devices EG&G4000B. The power beam can be modulated—either in amplitude by a chopper or in frequency by an acousto-optic modulator—to allow phase-sensitive detection. This improves our Doppler-free spectroscopy by eliminating the Doppler-broadened part of the transmission signal. Since the Lamb dip is the only part of the probe beam’s transmission that depends on the behavior of the power beam, it will be the only part of the signal that varies in phase with the power beam modulation. Therefore, phase-sensitive detection isolates the Doppler-free signal from the Doppler-
broadened background.

Figure 2.5 outlines the servo-locking setup that we used to stabilize the lasers' frequencies and lock them to the desired atomic transition. There are two nested feedback loops: the outer loop locks the laser's frequency to an atomic transition, while the faster inner loop stabilizes the laser's frequency over short time scales by locking the laser frequency directly to an interference fringe in a resonant Fabry-Perot cavity. The outer feedback loop control the laser's frequency indirectly by controlling the resonant frequency of the cavity. We needed to use the second inner loop because mechanical vibrations in the laser optics caused rapid fluctuations in the laser frequency that often exceeded the width of the atomic spectral features to which we locked the laser frequency. As a result, a single-loop servo-locking arrangement was too unstable to use in practice.

The inner feedback loop of the servo-locking arrangement locks the laser's frequency to a resonance fringe of a tunable Fabry-Perot cavity. The light transmitted through this cavity is monitored by a photodiode, whose output signal is sent to a servo-controller (described in Appendix A. The cavity and photodiode are kept under an opaque box to minimize extraneous light signals. The output of the servo controller is sent to a summing amplifier that controls the laser frequency via the grating (or feedback mirror) piezo. The summing amplifier also has a variable ramp generator that can be used to sweep the laser frequency across the atomic transition when it is not servo-locked.

Our Fabry-Perot cavities are 25 mm long and have a 10-GHz free-spectral range.
Figure 2.5: Laser servo-locking setup.
The fringes can be very narrow when the cavity is properly aligned. On the other hand, the same frequency fluctuations that made direct locking to an atomic spectral feature unstable also make locking to a narrow Fabry-Perot interference fringe unstable. We therefore resorted to deliberately misaligning the Fabry-Perot cavity to obtain a fringe several hundred MHz wide but with a steep slope on one side to allow stable locking.

The ability of the servo-electronics to respond to vibrations and other fluctuations in the laser frequency is limited by the speed with which this inner servo-loop can change the laser’s frequency. This limitation is a fundamentally mechanical one, since the piezo must physically move the mass of the grating or mirror mount. In practice, the loop resonates at around 200 Hz, and it cannot compensate for fluctuations at higher frequencies.

The purpose of our frequency locking arrangement is to lock the laser frequency to an atomic transition, not to a Fabry-Perot cavity fringe. The outer feedback loop works to control the length of the cavity (thereby controlling its resonance frequency) to keep the laser frequency locked to a spectral feature of potassium.

The spectral feature to which the laser is locked is obtained by modulating the pump beam of our saturated absorption setup, either in amplitude by a chopper or in frequency by an acousto-optic modulator. When the power beam is turned on and off by a chopper, as is done for the detector laser frequency reference, the transmitted portion of the probe beam will increase and decrease in phase, provided that the laser is tuned to a “Lamb-dip” spectral feature. The output of a phase-sensitive detector
as the laser frequency is scanned will therefore show only the Lamb dips. This signal is insensitive to background light and other sources of noise and can serve as a stable reference to which to servo-lock the laser frequency. We use a servo-locking device which controls the laser’s frequency so as to keep the phase-sensitive detector’s output about halfway up the side of the Lamb dip, which keeps the laser frequency locked about 3 MHz to the blue of the line center, which is sufficient for the purposes of velocity measurement in the detector.

We need to lock the pumping laser frequency exactly to line center to obtain sufficiently good optical pumping to run the experiment. We found as a result that we could not use the same amplitude modulation technique that we used for the detector laser. If we could obtain a signal that was proportional to the frequency derivative of the Lamb dip signal, on the other hand, we could use a servo-locking scheme that controlled the laser frequency to keep this derivative equal to zero, which would lock the laser exactly to the peak of the Lamb dip feature.

To obtain such a frequency-derivative signal, we rely on frequency modulation of the pump beam in saturated absorption. We then demodulate the transmitted probe beam signal in phase with the frequency modulation to obtain a signal that is proportional to the frequency derivative of the Doppler-free spectrum.

To do this, we pass the power beam for the pumping laser frequency saturated absorption cell through an acousto-optic modulator driven by a 55 MHz signal (which is exactly equal to the hyperfine splitting of the $4p^2P_{1/2}$ excited state of $^{39}K$). The modulator is aligned so as to maximize diffraction into the negatively shifted beam.
so that the power beam’s frequency is 55 MHz less than the probe beam’s. The 55 MHz is in turn frequency modulated at 20 kHz by about ±3 MHz (i.e. half the transition linewidth). The probe beam does not pass through the modulator; therefore, a phase-sensitive detector monitoring the probe transmission signal will see variations in the Lamb dip signal with power-beam frequency. These variations depend on the derivative of the Lamb dip with respect to the power beam frequency, and so will pass rapidly through zero at line center.

We still have to account for the -55 MHz shift of the power beam’s frequency. Since both beams are not at the same frequency, they will not interact with the atoms moving at the same velocity when \( \omega_L = \omega_0 \), but instead when \( \omega_L - \Delta \omega/2 = \omega_0 \), or when the laser is shifted by 27.5 MHz from the resonance line. To get around this problem, we employ what is known as a “cross-over” resonance, which occurs -27.5 MHz from the resonance line. A cross-over resonance occurs in saturated absorption spectrum when light from the two beams interacts with the same atomic population, but by driving different transitions. For instance, the probe beam may excite atoms in such a velocity class into the \( F' = 2 \) excited state hyperfine level, while the power beam excites atoms into the \( F' = 1 \) excited level. Since the atomic ground state population is still largely depleted by the power beam, there will be a dip in probe beam absorption at this laser frequency, that occurs halfway between the two transition frequencies. In the case of the pumping transition, this happens -27.5 MHz from the \( F = 2 \rightarrow F' = 2 \) pumping transition. Therefore, if we servo-lock the laser frequency to the line-center of the cross-over transition as seen by the saturated absorption
setup using the -55 MHz-shifted power beam, the two frequency shifts cancel out and
the laser frequency locks exactly to the pumping transition frequency.

2.2 The Vacuum System

The vacuum system consists of three chambers: the oven chamber (that includes the
first optical pumping region), the cryostat, and the detector chamber. The three can
be isolated from one another by closing gate valves, and each can be independently
pumped down by attaching a small portable diffusion pump system to an external
vacuum valve.

The oven and detector chambers are each kept under high vacuum by a 30 liter/sec
ion pump. The cryostat does not have a pump; however, activated charcoal placed
on the helium and nitrogen dewars cryogenically pump the cryostat chamber. Liquid
nitrogen traps are filled in the oven and detector chambers whenever the oven is in
operation to help reduce the background pressure as much as possible. Ionization
guages are mounted in the detector and cryostat chambers, but not in the oven
chamber.

The oven chamber is routinely baked out to 200 °C when the oven is loaded (as
is described in Section 2.3), and the inside surface is quickly coated with a layer of
potassium metal after the oven has been turned on. The potassium acts as a getter
when the chamber is cold. Typical pressures in the oven chamber are estimated to
be in the low 10^{-7} torr range when the oven is cold. Under the hottest operating
conditions (400 °C), the stainless steel in the oven begins to outgas hydrogen (despite
our quite successful efforts to stop it) and the pressure can rise into the high 10^{-7}
torr range.

There is no provision to bake out the cryostat chamber. Instead, the liquid nitrogen shroud is kept full whenever the cryostat is under vacuum. The gate valve to either the detector or the oven chambers is also left open so that hydrogen gas can be pumped by the ion pumps. The typical pressure in the cryostat with the nitrogen dewar filled is in the low $10^{-7}$ torr range. When the liquid helium dewar is filled, this pressure drops into the low $10^{-8}$ torr range.

During operation, hydrogen gas from the hot oven will accumulate on the cryogenic surfaces in the detector. This can be a problem for two reasons: First, the hydrogen can condense on the sample and block the passage of potassium atoms. Second, hydrogen condensed on the sample stage can boil off in sufficient quantity when the stage is heated to completely block the potassium beam. We installed a 1 cm diameter differential pumping aperture between the oven and cryostat chambers to reduce the flow of gas from the oven into the cryostat. We also made sure that activated charcoal was placed on the unheated surfaces near the sample stage.

The detector chamber is also designed to be baked out. A mild bakeout is essential in the detector because of the large surface area that is covered by anodized aluminum light baffles. This bakeout cannot be allowed to exceed 100 °C, however, because the optical windows in the detector chamber are sealed by Viton rubber O-rings.

### 2.3 Atomic Beam Source

The atomic beam originates in an effusive oven source at a maximum temperature of about 400°C, corresponding to a vapor pressure of about 3 torr\[29\] and an atomic
number density of $4.3 \times 10^{16}$ cm$^{-3}$. The most probable speed in the oven is given by
\[ v = \sqrt{\frac{2k_BT}{m}} = 535 \text{ m/s} \] at this temperature, where $k_B$ is Boltzmann's constant and $m = 6.47 \times 10^{-26}$ kg is the mass of a $^{39}$K atom. The most probable speed in the beam is about 20% larger.

The oven consists of a reservoir which holds about 3 gm of potassium metal, with a nozzle measuring 1 mm diameter by 1 cm long. The oven is heated by a cartridge heater manufactured by ARI Industries which is wound around and brazed directly onto the stainless steel oven (see Figure 2.6). Note that the heater is wrapped with a tighter pitch at the nozzle end of the oven, to ensure that the nozzle is the warmest point of the oven, which prevents metal from condensing in and clogging the nozzle. The oven temperature is measured by a Type-K thermocouple attached to the nozzle end. The oven is supported by two 0.25 inch diameter by 6 inch long stainless steel rods which provide good thermal isolation from the vacuum jacket.

Before the oven is loaded with potassium, it is baked out in air at 400°C for 1 to 2 hours to create a barrier layer of chromium oxides on the surface of the stainless steel, which serves to prevent hydrogen gas trapped in the steel during its manufacturing process from outgassing when the oven is heated. Without this treatment, the oven produces a hydrogen vapor pressure in the oven vacuum chamber of about $10^{-4}$ torr at operating temperatures, which reduces the mean free path for the atomic beam to about 1 cm, effectively turning it off. After this treatment, the hydrogen pressure is less than $10^{-6}$ torr during operation. The whole assembly is then wrapped by about 3 layers of thin molybdenum foil, which acts as a radiant heat shield. We
Figure 2.6: The potassium oven. The inset shows the nozzle and the loading hole with its copper seal.
chose molybdenum for its low permeability to hydrogen[29, pg. 134].

The oven is loaded with liquid potassium metal in an argon atmosphere. A five gram ampoule is heated along with a glass pipette by a hot plate inside a glove box. When the metal is melted, the warm pipette is used with a rubber dropper bulb to transfer the liquid to several cold pipettes which are wrapped with Nichrome wire heaters. These pipettes are inserted through a small threaded hole in the nozzle end of the oven, which is held nozzle up and heated to about 100°C by its own heater. A vinyl hose is attached to the end of the pipette, the other end of which connects to a plastic squeeze bottle that is compressed to keep positive pressure on the contents of the pipette. The pipette heater is hooked up to a Variac and turned on. When the metal melts, we squeeze the bottle hard, forcing the metal into the oven. After about three grams have been transferred in this manner, the oven is sealed by a specially made OFHC copper washer crushed against the oven by a number 6 cap screw threaded into the loading hole. The underside of the screw's cap is smoothed on a lathe to make a good sealing surface, and the oven surface around the hole is sanded smooth before the loading begins.

Once sealed, the oven is cooled down to room temperature by pouring liquid nitrogen over it. It is then removed from the argon atmosphere and quickly inserted into the vacuum chamber, which is also kept under argon. The cryostat and detector sections are kept sealed off and under vacuum during the procedure. A vacuum seal is made, and the oven section is evacuated by an external, movable pumping station and baked out at 200°C for two hours, at which time the ion pump can be turned on and
the external pump disconnected. The oven is also turned on at low power at this time
to remove water vapor from the oven and dissolved gases from the potassium metal.
The pressure during bakeout can sometimes rise as high as $10^{-3}$ torr in the portable
diffusion pump, before falling back below $10^{-5}$ torr in the oven chamber. The oven is
kept at about $100^\circ$C when not in use, so that solid metal doesn't condense and block
the nozzle as the oven cools down.

The oven vacuum chamber is connected to the optical pumping chamber by a
flexible bellows, and is mounted on a translation/rotation stage. The chamber itself
is clamped into a home-made gimbal mount, centered on the oven nozzle, with mi-
crometer driven motions, which can be locked in place by set screws. This gimbal
stage is bolted to a standard steel machining translation stage, mounted sideways to
give vertical and side-to-side displacement. There is no longitudinal adjustment.

### 2.4 Initial State Preparation

Coming from the oven at 400 °C, the atomic beam populates all eight hyperfine
levels of the electronic $4s^2 S_{1/2}$ ground state equally. For our experiment, however, it
is necessary to prepare the beam in the three $F = 1$ magnetic hyperfine states only.
This is done by optical pumping.

To accomplish this, the optical pumping laser is tuned to the 770 nm $4s^2 S_{1/2},$
$F = 2 \rightarrow 4p^2 p_{1/2}, F' = 2$ transition. Atoms excited out of the five $F = 2$ ground
state levels will be able to decay into either the $F = 2$ or $F = 1$ levels by emitting
a photon. However, while atoms returning to $F = 2$ will be re-excited, atoms falling
into $F = 1$ will be 462 MHz out of resonance, and will not absorb any more light.
Therefore, atoms will accumulate in the $F = 1$ levels until the $F = 2$ levels are empty.

In practice, the pumping laser crosses the atomic beam at right angles and is retro-reflected back for a total of two passes. The beam has a spot diameter of about 5 mm and a total power of 1.5 mW. The atomic beam is therefore illuminated by laser light at about the transition's saturation intensity of 1.3 mW/cm$^2$ over a distance of about 1 cm. For 500 m/s atoms, this takes 20 µsec.

The evolution of the populations of the eight hyperfine states while they are illuminated by laser light are governed by rate equations. The detailed behavior depends on laser frequency, intensity, polarization, and on the particular atomic states involved. The detailed behavior can be studied using numerical simulations, which are described in Appendix B. Much of the qualitative behavior can be understood without the aid of sophisticated analysis, however.

Our experiment’s ultimate sensitivity depends on the fraction of the initial $F = 2$ population that remains after the beam is optically pumped, since these remaining atoms are indistinguishable from atoms excited into $F = 2$ by the sample. There are both theoretical and practical lower limits to the fraction of the original $F = 2$ population that will remain after optical pumping.

The theoretical optical pumping limit arises from the fact that atoms can be pumped from $F = 2$ to $F = 1$ and back from $F = 1$ to $F = 2$. Although the probability that an atom will be pumped from $F = 2$ to $F = 1$ in a time $t$ greatly exceeds the probability of an $F = 1$ being pumped back into $F = 2$, the latter rate is not zero. Therefore, the atomic population will eventually reach a steady state where
the rates at which atoms are pumped between the two levels are equal. In this steady state, a finite population of atoms will remain in $F = 2$.

This finite remaining population can be estimated if we assume that the atom behaves like a three-level system with identical transition moments between two ground state levels and a single excited state. In this case, the ultimate population will have been obtained when the rate at which atoms are pumped from the $F = 2$ to the $F = 1$ levels equals the rate at which atoms are pumped back from the $F = 1$ to the $F = 2$ levels. This will happen when

$$P_1 R_{1 \rightarrow 2} = P_2 R_{2 \rightarrow 1},$$

(2.1)

where $P_i$ is the population of the $i$th ground state level and $R_{i \rightarrow j}$ is the normalized transition rate from the $i$th to the $j$th ground state level. This means that the relative population of the $F = 1$ and $F = 2$ levels is

$$\frac{P_2}{P_1} = \frac{R_{1 \rightarrow 2}}{R_{2 \rightarrow 1}}.$$  

(2.2)

The transition rate

$$R_{i \rightarrow j} = \frac{\Gamma^2 I_0/2}{\delta^2 + \Gamma^2 (1 + 2I_0)},$$

(2.3)

where $\Gamma = 1/\tau$, $\tau$ is the lifetime of the excited state, $I_0 = 1/2(\Delta E_0)^2/(\hbar \Gamma^2)$ is the dimensionless intensity, and $\delta = \omega_L - \omega_0$ is the detuning (in radians/sec) of the laser from the resonance for the $i \rightarrow j$ transition. Here, $d$ is the electric dipole moment and $E_0$ is the laser electric field.

In the limit of low laser intensity, the remaining $F = 2$ population will be smallest when the laser is tuned exactly on resonance, and will only depend on the linewidth
and hyperfine splitting of the excited and ground states, respectively:

\[
\frac{P_2}{P_1} \approx \frac{\Gamma^2}{4\omega_{HFS}^2}.
\]  

(2.1)

With \( \Gamma = 2\pi \times 6 \text{ MHz} \) and \( \omega_{HFS} = 2\pi \times 462 \text{ MHz} \), we find an best possible optical pumping ratio of \( \frac{P_2}{P_1} = 4.2 \times 10^{-5} \).

The evolution of the populations during optical pumping will be affected in two ways by increasing the laser intensity: A positive effect is that the rate of optical pumping increases linearly with the laser intensity, meaning that a bright laser will empty out the \( F = 2 \) levels faster than a dim laser. The unfortunate side effect is that the pumping transition is \textit{power broadened} by the laser light, thus increasing the ultimate ratio of \( F = 2 \) to \( F = 1 \) atoms. A good compromise is to use a laser beam whose intensity varies as the atoms pass through it: starting out bright, and then fading gradually to get the best possible pumping.

We chose the \( 4s^2S_{1/2}, F = 2 \rightarrow 4p^2P_{1/2}, F' = 2 \) transition simply because it worked best, as determined by our numerical simulations. Its major advantage over the \( 4s^2S_{1/2}, F = 2 \rightarrow 4p^2P_{3/2}, F' = 2 \) transition, is that the latter’s resonance frequency is within a transition linewidth of the transition to \( F' = 3 \), (the same one used in the detector). Although this would be convenient in terms of the number of lasers needed to run the experiment, atoms which are excited to \( F' = 3 \) \textit{cannot} decay to \( F = 1 \) in the ground state. This unfortunate effect slows the optical pumping down by a factor of about 2.

A possible disadvantage of the transition we do use is that linearly polarized light, as a result of conservation of momentum, will fail to excite atoms out of the
$F = 2, \ m_F = 0$ levels into the $F' = 2, \ m_{F'} = 0$ state. Likewise, right-circularly polarized light will fail to excite atoms out of the $m_F = 2$ level in the ground state, and left-circularly polarized light will strand atoms in the $m_F = -2$ state. Although these atoms can still be excited to the $F' = 1$ level, this transition is 55 MHz from resonance, and would therefore be driven very slowly. The solution we found was to use laser light that contains a mix of all polarizations. We inserted a pseudodepolarizer (a device that radically alters the polarization of light passing through it over lateral distance scales of about a wavelength) into the pumping laser beam before it passed through the chamber to implement this solution.

Figure 2.7 illustrates the results of the numerical simulations described in the appendix. In brief, the simulation integrates the rate equations for the populations of the various atomic states, keeping track of excitation, stimulated emission, spontaneous emission and fluorescence. The details of this calculation are discussed in Appendix B. Figure 2.7a shows the population in the $F = 2, \ m_F = 2$ ground state level as a function of laser detuning from “line center” after pumping for 20 μsec in light with intensities $I = 0.1I_0, I_0, \text{ and } 10I_0$. The experiment operated at $I \approx I_0 = 1.3 \text{ mW/cm}^2$ for this transition. Note that the populations depend strongly on detuning at low intensities. Also note the effect of power-broadening: at larger intensity, the population doesn’t depend as strongly on detuning, but is larger. Figure 2.7b shows the time evolution of the $m_F = 2$ population with the laser tuned exactly at the pumping resonance, for the same pumping laser intensities. Note that at higher intensity, the population initially decreases faster, but comes to equilibrium at
Figure 2.7: Results of numerical simulations of optical pumping. In (a), the population in the $m_F = 2$ hyperfine level is plotted as a function of pumping laser frequency from line center of the $4s \frac{1}{2} S_{1/2} \rightarrow 4p \frac{3}{2} P_{1/2}$ transition of $^{39}$K for three pumping laser intensities. The saturation intensity $I_{sat} \approx 1.3 \text{ mW/cm}^2$. In (b), the evolution of the $m_F = 2$ state is plotted as a function of time for the same three laser intensities at the optimum laser detuning indicated by the arrow in (a).
a larger ultimate value, than at lower intensity.

In practice, we are unable to achieve the 1 part in $10^4$ pumping efficiency predicted by theory, but instead have achieved at best 5 parts in $10^3$. We believe there are two reasons for this failure: First, the interaction time of the atoms in the pumping laser is limited by the 1 cm width of the pumping beam. This is borne out by the observation that the background level was seen to be greater at higher atomic velocities, where the atoms spend less time in the pumping beam. Second, there is still some frequency noise in the pumping laser. This frequency noise leads directly to fluctuations in the flux of unpumped $F = 2$ atoms in the detector. These fluctuations form the dominant source of noise in the experiment.

In conclusion, we should be able to reach a pumping efficiency approaching the theoretical limit by doing two things: First, we should use a more powerful laser and spreading the beam out so that atoms are pumped at the same laser intensity for a longer time. Second, we need to improve the spectral characteristics of the laser so that its frequency deviates by much less than the width of the atomic transition from the line center.

2.5 The Sample

The sample was truly the heart of the apparatus, and the focus of considerable effort. The sample itself is a 1000 Å thick niobium film, sputter-deposited on a 0.5 mm thick GaAs substrate which was prepared in a special manner before the niobium was deposited.

To begin with, the substrate was cut into strips measuring 1.5 mm long (along
the atomic beam axis) by 25 mm wide. To accomplish this, a 3 inch wafer of GaAs, polished on both sides, was mounted in a special jig on a standard mill platform. Care was taken to mount the wafer with one of its cleavage planes as nearly parallel as possible to the travel direction of the mill. The mill was also thoroughly cleaned before the wafer was mounted. A diamond-tipped scribe was mounted in the mill chuck, using a spring-loaded holder, so that the tip made gentle contact with the GaAs surface. A 1.5 mm by 25 mm rectangular grid was then scribed on the wafer. Once this was done, the wafer was removed from the mounting jig and cleaved using a simple device consisting of a roughly 25 mm square by 6 mm thick lucite block with a 1.5 mm deep slot, just wide enough for the wafer to fit inside. After the wafer was cleaved into 25 mm strips, these could be cleaved to the desired 1.5 mm length by just inserting the 25 mm end into the slot and breaking off a strip. With some practice, it is easy to achieve a 90% success rate using this technique. This technique has the advantage of inexpensively producing identical substrates with clean edges on the unscribed side.

After the strips are cleaved, they are cleaned by a five-stage process in an ultrasonic bath cleaner, using the following chemicals: trichloroethylene, acetone, methyl alcohol, double-distilled demineralized water, sulfuric acid (5% by volume in distilled water), and finally two more distilled water rinses. The strips are rinsed in polyethylene beakers containing the chemicals, and are handled at all times using vinyl gloves and Teflon tweezers. They are blown dry by filtered dry nitrogen between each set of rinses (except after the acid rinse, for safety), and are kept covered.
Immediately after cleaning, the strips are mounted on a specially constructed aluminum jig for deposition. The jig is designed to be easily mounted in both the sputtering apparatus and a standard high vacuum evaporator. Each strip is examined under an optical microscope, and rejected if any scratches or pits are found in the central region. The strips are mounted with the unscribed surface up, usually in batches of about twenty.

Figure 2.8 shows the stages of the sample deposition process. In order to provide low-resistance contacts for high-current measurements, gold contact pads about 1000 Å thick are laid down on both ends of the strips by evaporation in high vacuum. The mounting jig has provision for a 15 mm wide stainless-steel mask to be laid down directly above the central region of the strips, which are mounted side-by-side on the jig. This mask covers the entire strip except for 5 mm at either end. The mask also serves to hold the strips in place. The jig is placed in the evaporator and a 1000 Å layer of gold is deposited on the ends, leaving the center bare.

The jig is then removed from the evaporator, and a second pair of masks are laid down that cover the last 2 mm of both ends of the strips. The center mask is then removed, and the jig is immediately placed in the sputtering apparatus. A 1000 Å niobium film is then deposited at 500 Å per minute, overlapping the gold for 2 mm at each end, and making excellent electrical contact with it. The finished samples are taken out of the sputtering chamber and immediately stored in a dry-box and kept covered.

Electrical connections to the sample for four-wire resistance measurements were
Figure 2.8: The sample deposition process. After cleaning, a mask is placed over the center of the sample (a) and 1000 Å of gold is deposited on the exposed ends (b). Then the ends are masked and the center (c) exposed for sputtering of 1000 Å of niobium (d). An overlap of niobium on gold ensures a good electrical contact.
made using very fine (AWG 40 gauge) copper wire and indium press-pads. The outer “current” leads were attached to the gold pads, while the inner “voltage” leads were attached directly to the surface of the niobium film. No additional contact resistance could be measured across the “current” connections.

We found that the gold contact pads were necessary to prevent heating of the sample when large transport currents are applied. Niobium forms a roughly 30Å oxide layer when exposed to air. This layer causes considerable contact resistance. Samples prepared in a manner otherwise identical to what we have described, but with the current leads connected directly to the niobium surface, were heated above \( T_c \) by transport current as small as 15 mA. In contrast, we were able to pass currents as large as 150 mA through the sample using the gold pads without any observable heating.

The samples in the batch that will be discussed in detail exhibit a sharp resistive transition at 8.9 K. We were unable to drive these samples normal with DC currents as large as 100 mA, indicating that the critical current density \( J_c \) was at least 67,000 Amps/cm². No evidence of magneto-resistance was seen with DC currents this large in fields up to 100 Gauss. The sample’s normal state resistance was about 20 ohms at room temperature, with a residual resistance ratio of about 5. This indicates that the sample is “dirty”, that is, the electron mean-free-path is fairly short. We estimate that the grain size in the film is about 20 to 50 Å[18], and expect that that mean-free-path would be approximately this length, as well.
2.6 Sample Mount and Cryostat

The sample is mounted on a variable-temperature platform at the end of a homebuilt liquid helium cryostat. The sample is attached to a removable gold-plated OFHC copper block by GE varnish. A “gate”, which serves to restrict passage of the atomic beam to within 1.8 μm of the sample surface, is held against the surface by a spring-loaded mount. This entire sample assembly can be attached to the top of the variable-temperature platform by four 0-80 thread screws, and good thermal contact is insured by polishing and gold-plating both surfaces and using Apiezon-N grease.

The gate is constructed by evaporating 1.8 μm of SiO onto the surface of a 1.5 mm long by 10 mm wide GaAs strip prepared in a fashion identical (except for the width) to that of the sample substrates. The SiO is deposited in exactly the same manner as the gold contact pads were deposited on the sample substrates. The finished gate consists of two SiO pads raised above a smooth GaAs surface. The gate is mounted with these pads in contact with the sample surface, as shown in Figure 2.9, and held in place by a spring assembly that also serves to completely shield the region downstream of the sample from the atomic beam. This assembly attaches to the sample mounting block with four 0-80 screws to form the complete sample assembly. The assembly consisting of the sample, gate, copper mounting block and spring-loaded gate holder is shown in Figure 2.10.

The helium cryostat (shown in Figure 2.11) is an optical type, with a cold finger extending into vacuum. The inner helium dewar is shielded by an outer liquid nitrogen dewar which is kept filled when the system is under vacuum. A copper shroud is
Figure 2.9: The gate assembly. The atomic beam passes under the gate and over the sample.
Figure 2.10: The sample mount assembly. The sample is attached to a copper block that has been gold-plated. The gate is attached to a small aluminum piece, then this assembly is pressed against the sample surface by beryllium-copper springs that are held in an aluminum block. An aluminum cap holds the springs in place.
Figure 2.11: The helium cryostat with liquid nitrogen shield and shroud.
soldered (using low melting-temperature solder) to the underside of the stainless steel nitrogen dewar. This shroud shields the cold finger and the sample assembly, and is penetrated by two small (0.25 inch) diameter holes to allow the atomic beam to pass.

The design of the variable-temperature platform is driven by the observation that the bottom of the helium cryostat can move millimeters when the dewar is filled, making atomic beam alignment impossible. Figure 2.12 shows the variable-temperature stage assembly. To prevent the sample from moving when the helium dewar is filled, the variable-temperature stage is thermally linked to the helium dewar, but mechanically attached to the nitrogen shroud, which doesn’t move. This is accomplished by using flexible thermal links made of copper wire bundles, and thermally insulating mounts made of G-10 fiberglass. The variable-temperature stage is thermally connected to the liquid helium dewar by a “weak-link”, made of a thin bundle of copper wires. The variable-temperature stage is mounted via a G-10 block that in turn is mounted to an intermediate “cold” stage. The cold stage is thermally connected to the dewar by a “strong” thermal link, made of a thick bundle of copper wires. The cold stage is mounted to the bottom of the nitrogen shroud by a thin-walled G-10 tube. The intermediate stage serves to collect heat that flows up this tube from the 77 K nitrogen shroud, preventing the sample stage from becoming too warm. The extra heat load from this mounting assembly decreased the lifetime of the helium dewar to about 16 hours from 30 hours without the sample mounting hardware in place.

The flexible links are attached to the dewar at the upper end and to the two stages at the lower end by squeeze connectors. These squeeze connectors are thin split
Figure 2.12: The variable-temperature sample stage assembly. The sample/gate assembly of Figure 2.10 is bolted onto a variable-temperature stage equipped with heater and thermometer. The variable-temperature stage is mounted using springs to a G-10 block. The G-10 block is mounted using springs to a cold stage. The cold stage is rigidly mounted to the outer liquid nitrogen-temperature shroud by a thin-walled G-10 tube. The sample stage is kept in contact with the liquid-helium cold-finger by a thin flexible copper cable, while the cold stage is connected to the cold finger by a thicker copper cable. The heater, thermometer, and sample wiring are not shown.
rings of gold-plated OFHC copper which slide snugly over gold-plated OFHC copper cylinders. A nylon ring slides tightly over this assembly, contracting when cooled to cryogenic temperatures, crushing the thin split ring against the cylinder and making an excellent removable thermal link that is easy to assemble and disassemble.

The variable-temperature stage is equipped with a heater and a thermometer. The heater consisted of a 100 ohm constantan wire counterwound and epoxied to the stage with Stycast. The thermometer was a 100 ohm Allen-Bradley carbon resistor. The thermometer was calibrated using the expression found in White[37, pg.127], using room temperature, 77 K and 4.2 K as reference points. A fourth point was obtained by checking the superconducting transition temperature of the niobium film (8.9 K) against that of another sample from the same batch that was measured using a Lakeshore Cryotronics silicon diode thermometer. The sample's temperature with no applied thermal load (i.e. no heater current) was 5.05 K. With a 150 mA heater current, the stage reaches about 15 K, well above the transition temperature of niobium.

Resistance measurements on the sample were taken while the experiment was running by using a four-wire technique. A 100 kHz AC current of 10 μA RMS was coupled into the outer (gold) contacts using a transformer. This allowed a DC current of up to 150 mA to be applied at the same time if desired. The voltage between the inner pads was measured using a lock-in amplifier, with a sensitivity of about 1 μV, giving an overall sensitivity of 0.1 Ω. This sensitivity was limited by noise picked up by the long wires leading from the lockin amp to the cryostat.
We found that aligning the system was an important and difficult experimental problem. The problem was made more difficult by the small sample opening through which the atomic beam had to pass in order to align anything else. Since the sample tends to become blocked by condensed potassium metal after a few hours exposure to the oven, we needed to provide a second, larger opening to use for alignment and calibration when the sample itself was not required. We milled a 1 mm square by 1 cm long groove out of the top surface of the G-10 piece that separates the sample stage from the cold stage. This groove serves as an alternate path for atoms to travel through the sample region. The groove allows many more atoms to pass through into the downstream regions of the apparatus, and also prolongs the lifetime of the sample.

The bias magnetic field was applied to the sample by a pair of Helmholtz coils wound directly on the outside of the stainless-steel vacuum can. The coils are capable of producing a field as large as 200 Gauss when powered by a 10 Amp supply. The entire region including the coils is contained inside a triple-layer magnetic shield. The inner layer is made of 0.010 inches of a high-permeability alloy (Ad-Vance Magnetics AD-MU-80, with a peak permeability of 350,000 and a saturation induction of 7500 Gauss), while the outer two layers are made of 0.025 inches of low-permeability, high saturation alloy (AD-MU-00, with a peak permeability of 9000 and a saturation induction of 22,000 Gauss). This arrangement allows the shield to screen large fields effectively. The shield is shown in Figure 2.11. The shield is open at the top where the can attaches to the cryostat housing and is penetrated by four 1.5 inch diameter
vacuum pipes. The shield is capable of screening out the largest stray fields from the bending magnet to less than 0.3 Gauss.

2.7 The Bending Magnet

When we first envisioned the experiment, the atoms remaining in the initial $F = 1$ state passed through the detector along with the $F = 2$ signal we wished to detect. Unfortunately, these $F = 1$ atoms' transition frequency was only 462 MHz higher than the transition frequency of the $F = 2$ signal. This meant that the $F = 1$ background population traveling 350 m/s faster than the $F = 2$ signal would also be resonant with the laser. These fast background atoms did not “cycle”, being optically pumped into the $F = 2$ level after emitting only one or two photons. Nevertheless, we found that they contributed a background signal about as third as large as the “unpumped signal” seen when the pumping laser was blocked.

Figure 2.13 shows a calculation of the expected fluorescence signal seen in the detector under typical conditions. Although the the cycling transition emits many more photons on resonance, the lengthy interaction time (relative to an excited state lifetime) allows atoms in the $F = 1$ levels to emit photons over a frequency range extending far from resonance. When integrated over the Doppler profile of the beam, the total photon emission from the $F = 1$ levels is about a third of that from the unpumped population. This effect was seen experimentally: Figure 2.14 shows the relatively large fluorescence background seen even when all the atoms were pumped into the $F = 1$ levels.

We remove this large background fluorescence by physically separating the $F = 1$
Figure 2.13: (a) The expected fluorescence signal calculated as a function of detector laser frequency at a typical laser intensity (ten times saturation) and transit time (10 μsec). The dashed line is the unpumped signal (all levels equally populated); the solid line is the background signal seen when the $F=2$ levels are depopulated. (b) The total fluorescence integrated over laser detuning for the same two situations, plotted as a running integral. The integration approximates convolution with the Doppler profile of the atomic beam, which is over 1 GHz wide.
Figure 2.14: The fluorescence as function of detector laser frequency as seen in the detector for pumped ($F = 1$ only) and unpumped ($F = 1$ and $F = 2$) atomic populations. The pumping laser was successively blocked and then allowed to illuminate the atomic beam.
atoms from the $F = 2$ atoms, using a Stern-Gerlach or “bending” magnet. Atoms in the four hyperfine states with electron spin $m_J = +1/2$ Zeeman shift to higher energies in increasing magnetic fields, while atoms with $m_J = -1/2$ (including those in all three $F = 1$ levels) do the opposite. If a beam of atoms including both electronic spin states passes through a region with a strong gradient in magnetic field perpendicular to its direction of motion, it will split into two beams. Those atoms having positive electronic spin will be deflected in the direction of decreasing field, while those with negative electronic spin will be deflected the other way. A bending magnet consists of an electromagnet with specially shaped pole pieces designed to produce a very strong magnetic field gradient through which an atomic beam must pass.

The angular deflection $\gamma$ of a spin-1/2 atom in a magnetic field gradient perpendicular to its direction of motion is given by

$$\gamma = \frac{\mu_B l}{m v^2} \frac{\partial B}{\partial z},$$

where $\mu_B$ is the Bohr magneton, $l$ is the length of the magnet along the beam direction, $m$ is the mass of the atom, $v$ is its velocity, and $\partial B/\partial z$ is the perpendicular magnetic field gradient.

In our apparatus, we constructed a bending magnet and installed it immediately downstream from the cryostat region. We then arranged the position of the detector so that the atoms with positive electronic spin (those in the four $F = 2$ hyperfine levels with $m_F \geq -1$) would be deflected into the detector, and the remaining atoms would be discarded.

Our bending magnet is of the “two-wire” design discussed by Ramsey[31] and
Figure 2.15: The bending magnet pole pieces in cross-section. The pole-pieces follow the "two-wire" design, with wires located 3 mm apart. The atomic beam should ideally travel through the hatched region, where the field gradient is most uniform.
elaborated upon by Vanier and Audoin[36]. The pole pieces are designed to produce a magnetic field that approximates the field produced by two parallel wires carrying current in opposite directions. Figure 2.15 illustrates the design of the pole pieces of our magnet. The equipotential surfaces are cylinders which intersect both wires: the field lines are also circular. Vanier and Audoin state that the most uniform field gradient occurs at a distance of $0.1a$ above the top of lower pole piece, where $a$ is the equivalent wire spacing. The value of the gradient at this location is $\frac{\partial B}{\partial z} = -0.4B_c/a$, where $B_c$ is the field at the apex of the lower pole piece.

The equivalent wire spacing in our magnet is 3.0 mm, while the gap at the beam axis is 0.9 mm high. The gap extends along the beam axis for 10 cm, with a additional 1 cm long bevel at each end to allow the field to taper off gradually. The peak field at the pole pieces is approximately 1 Tesla, giving a field gradient of $\frac{\partial B}{\partial z} = -0.27$ Tesla/mm at the beam axis. With this field gradient, a potassium atom moving 1500 m/s will be deflected 3.3 mrad after 10 cm travel.

The magnet’s pole pieces are constructed from Vanadium Permendur. They were machined on a computerized wire cutting machine (a wire EDM) by Nicholson Tool and Die of Columbus. The pole pieces are bolted onto a soft gray iron yoke inside the vacuum system. The yoke makes a vacuum seal and passes outside the vacuum system. Thirteen turns of AWG #2 welding cable are wrapped around the yoke and connected to a programmable 165 Amp current supply, which is controlled by a voltage from the computer. Fields of less than 1 Tesla at the bending magnet gap are approximately proportional to the applied current.
Since we need to deflect atoms of different velocities by the same angle as we scan the detector, we program the computer to produce a current which varies with the detector velocity as \( I(v) = Av^2 + Bv + C \). The constants \( A \), \( B \), and \( C \) are determined by scanning the magnet currents with the detector set to different velocities and fitting the peak velocity vs. current data to this form. Figure 2.16 shows how this is done.

Care must be taken that the magnet is returned to its "zero" state (zero field at zero current) after each run. This can be done by applying an alternating current which decays in time to zero. Since the programmable supply can only produce positive currents, we wired a second supply in parallel to the programmable supply, but with the leads reversed. This supply produces a constant current, which is canceled by the programmed current of the first supply in the zero state. We can therefore produce a negative current in the magnet coils by reducing the programmed current below the fixed current.

The magnet is arranged so as to deflect atoms with \( m_J = +1/2 \) (those we wish to detect) upward by 9 mrad. The detector itself is moved into this position after the atomic beam has been aligned with the rest of the experiment.

The bending magnet, its vacuum can, and its coils are enclosed in a magnetic shield constructed of plate iron. This shield is penetrated only by the atomic beam line at both ends of the magnet and by the magnet's power-supply cables.

The shield and magnet together sit on three screws that thread through a base-plate that in turn is mounted to a machinist's translation/rotation stage. This ar-
Figure 2.16: Synchronizing the bending magnet current to the detector velocity. (a) The detector velocity is fixed at several values and the bending magnet control voltage at which the largest signal occurs is determined. This voltage is fitted to a quadratic function of the detector velocity, after which the bending magnet current can be swept synchronously with the detector velocity, as shown in (b). Note that the small, broad peak to the right of the sharp peak in the 255 m/s curve is due to fast-moving $^{41}$K atoms, while the “hump” on the rising side of (b) is due to $F = 1$ atoms that have the same resonant frequency as the $F = 2$ atoms we intend to detect at the detector magnetic field corresponding to that velocity.
rangement gives us total control of the magnet's position: translation both vertically and horizontally across the atomic beam axis, vertical tilt and horizontal tilt. Each adjustment is either equipped with a vernier or a dial indicator so that the magnet's position can be recorded and returned to later.

2.8 The Detector

The detector is designed to detect how many atoms made the transition from the $F' = 1$ hyperfine state to the $F' = 2$ state when they passed over the sample as a function of their velocity. To do this properly, the detector must be as sensitive as possible, while remaining as insensitive as possible to all sources of noise and background signal.

The basic detection scheme is outlined in Figure 2.8. The atomic beam is directed by the bending magnet to cross a counter-propagating laser beam at a five degree angle. The beams cross at one focus of an elliptical light collector. Atoms resonant with the laser light absorb light from the laser and re-emit it in all directions. This light is collected by the elliptical reflector and focused into a light pipe, which takes it outside the vacuum system to a photomultiplier tube (PMT). The PMT current is measured by the computer and recorded.

The detector relies upon the Doppler shift to determine the atoms' velocities. Since the atoms are traveling in the opposite direction as the detector laser beam, they will see the laser frequency as being higher than its frequency in the lab by an amount proportional their velocity. To determine the atoms' velocity, therefore, all we need to do is determine their Doppler shift. The simplest way to do this would
Figure 2.17: Layout of detector.
be to sweep the laser frequency and record the fluorescence signal as a function of frequency.

Unfortunately, we have to sweep the current in the Stern-Gerlach magnet in synchrony with the detector velocity. This would require that we be able to reproducibly sweep the laser’s frequency over a specific interval determined in relation to an atomic spectral feature. While we can keep a laser tuned to a specific frequency for extended periods of time, it would be much more difficult to return the laser to a locking point after sweeping its frequency. Rather than attempt this, we decided to use a different approach.

Our velocity selection scheme uses the Zeeman shift of the atoms in a magnetic field to cancel out the Doppler shift, allowing the laser to remain locked to the transition while the detector velocity is being swept. Since the Zeeman shift of the cycling transition is proportional to the magnetic field, all we need to do is sweep two magnet currents in synchrony to control the detector velocity.

An atom moving with velocity $v$ will see the detector laser Doppler shifted by $\Delta \nu_D = v / \lambda = 1.30 \text{ MHz/(m/s)}$ for the $\lambda = 767 \text{ nm}$ transition. The Zeeman shift of the cycling transition $m_F = 2 \rightarrow m'_F = 3$ is Zeeman shifted by $+\mu_B = 1.40 \text{ MHz/Gauss}$ in a magnetic field applied parallel to the laser axis. Therefore, by fixing the frequency of the detector laser at the zero-field resonance and changing the magnetic field applied to the detector, we can tune the velocity of $m_F = 2$ atoms resonant with the detector laser at a rate of $1.07 \text{ (m/s)/Gauss}$.

The detector magnet is a pair of water-cooled coils in a quasi-Helmholtz geometry
with $dB/dz = 0$ at the intersection of the atomic and laser beams. The coils produce a 294 Gauss field at a current of 1 Amp. Our power supply is capable of producing 3.32 Amps for a maximum field of 975 Gauss and a maximum detectable velocity of 1043 m/s.

The detector’s sensitivity is enhanced by using a “cycling” transition. The detector laser beam is circularly polarized in the $\sigma^+$ sense, so that it only drives optical transitions that increase the magnetic quantum number $m_F$ by one. Atoms initially in the $m_F = 2$ state will therefore only be excited into the $m_F' = 3$ excited state. However, atoms in that state can only decay back into the original $m_F = 2$ state. This means that an atom in this initial state can “cycle”, absorbing $\sigma^+$ photons over and over as long as it remains illuminated by the laser. A fully saturated potassium atom will emit a photon every 52 nsec. Such an atom traveling at 500 m/s that is illuminated over a 1 cm long region can emit as many as 380 photons in 20 $\mu$s.

Two important sources of background signal are scattered light from the laser and resonance fluorescence from the $^{41}$K isotope that makes up 6% all naturally occurring potassium. Great care is taken to reduce the amount of scattered laser light that is mistakenly detected by means of anodized aluminum baffles. Nevertheless, some laser light is still detected. Also, even with perfect optical pumping, six percent of the atomic beam is composed of the $^{41}$K isotope. At the appropriate laser frequency, this isotope will contribute a fluorescence background six percent as large as the total fluorescence in the absence of optical pumping.

We use phase sensitive detection to reduce the detector’s sensitivity to these two
background sources. To accomplish this, a second pumping laser beam is sent across the atomic beam between the bending magnet and the light collector. This beam is turned off and on by a chopper to modulate the $F = 2$ population, while the PMT current is measured by a lock-in amplifier. Since the second pumping beam only affects $^{39}\text{K}$ atoms, only signal atoms will be turned on and off, and the phase-sensitive signal seen by the lock-in amplifier will be insensitive to both scattered light and $^{41}\text{K}$.

There are three major noise sources remaining in the system: optical pumping inefficiency and fluctuations, shot noise from the discreet arrival times of fluorescence photons, and finally, shot noise from the arrival times of atoms.

The primary source of both background signal and noise is currently the efficiency of the first optical pumping. An average background of 0.5% of the unpumped signal is still left over, and this background fluctuates by another 0.5% or so as a result of fluctuations in the pumping laser frequency. The background light from unpumped $F = 2$ atoms, $^{41}\text{K}$ atoms, and scattered light all contribute to the second remaining noise source: shot noise caused by the random arrival times of individual photons at the PMT. Finally, even if all the background noise sources were eliminated, the experiment would still be sensitive to shot noise caused by the random arrival times of atoms in the detector.

2.8.1 The second pumping region

Phase-sensitive detection of the fluorescence signal requires that a second pumping beam cross the atomic beam before it enters the detector. This output beam is
chopped and serves as the reference for phase-sensitive detection. About half the output of the pumping laser is sent through a chopper and then into the detector vacuum chamber, through an anti-reflection coated window, where it makes a single pass across the atomic beam and exits through a second, identical window. This region of the detector, like the rest, is lined with anodized aluminum baffles to reduce the amount of scattered laser light that enters the light collector. An additional pair of 1.5 inch diameter anodized cylinders provide critical light shielding by preventing any lines of sight from the windows to the light collector itself. It is especially important to reduce the amount of light scattered from the second pumping beam since this light is turned on and off by the chopper and will show up as a background signal in phase-sensitive detection.

The vacuum chamber housing the second pumping region also contains the exit window for the detector laser beam and a small mirror mounted to a vacuum translation stage that can be lowered into the atomic beam line to allow inspection of the bending magnet opening. The entire second pumping region is surrounded by a magnetic shield to prevent Zeeman shifts of the pumping transition frequencies from reducing the optical pumping efficiency. The shield is constructed in the same three-layer fashion as the sample shield. We observed that the installation of this shield improved the optical pumping efficiency of the second pumping beam from 80% to 98%.

The variable delay between the second pumping region and the detector for different velocities will introduce a varying phase shift into the signal we are attempting to
observe with phase-sensitive detection. We need to choose a chopping frequency slow enough to minimize this variable phase shift. We chose to chop the second pumping beam at 147 Hz, or with a period of 6.8 msec. The second pumping beam crosses the atomic beam about 24 cm upstream from the light collector. The slowest atoms present (in significant quantities) in our thermal atomic beam move at about 300 m/s. These atoms will take 0.8 msec to traverse the distance to the collector, leading to a maximum phase shift of 0.12 rad, or 6.5 degrees. This phase shift will not significantly affect our results.

2.8.2 Detector laser optics

The detector laser must enter the detector, intersect the atomic beam, and leave again while scattering as little light as possible into the light collector. Furthermore, it must do this while the detector itself is translated up and down during alignment of the experiment. This would be quite difficult to accomplish using mirrors to steer the beam into the detector.

A good solution to this problem is to use a flexible optical fiber to carry light from the detector laser output to the detector. We used a single-mode optical fiber about two meters long. A single-mode fiber has the added bonus of acting as an excellent “spatial filter” for the laser. The fiber's output transverse mode is nearly Gaussian. This, we found out, greatly aided in the task of reducing the scattered light inside the detector chamber. Our fiber has a 5 micron core and a nominal operating wavelength of 780 nm, and was manufactured by 3M Corp.

The greatest benefit of using a fiber was that it allowed us to mount all the
critical detector laser optics directly on the movable detector platform. Once they were aligned, only very minor adjustments were necessary to minimize the scattered light signal in the detector, even after weeks had elapsed, and no adjustment was needed at all as the detector platform was raised and lowered during the alignment procedures.

Light is coupled into and out of the fiber by two identical 20 power microscope objectives. These are mounted in Newport Optics model fiber couplers that secure the fiber and allow its position and tilt to be controlled in relation to the incoming laser beam. The position and direction of the incoming laser beam were controlled by two mirrors mounted on the optics table, and the spot size of the incoming laser beam was adjusted by a lens to about 1 mm diameter, so as to produce the optimal size focus to couple into the fiber. The output coupler was mounted directly on the movable detector platform. After being coupled out of the fiber, the light was steered through the detector by means of the two mirrors, and focused to a point on the upstream (from the atomic beam point of view) of the light collector by the lens. The spot size in the detection region is estimated to be about 1 mm diameter. Typically, about 0.2 mW of laser power is coupled out of the fiber, giving a rough intensity in the detector of 25 mW/cm², or 20 times the saturation intensity of the transition.

Unfortunately, the process of coupling light into the fiber also created a serious problem for the frequency control of the detector laser. The fiber coupler focuses the beam at normal incidence onto the perfectly flat fiber end. This ensures the optimum coupling of light into the TEM-00 mode of the fiber, but also resembles a “cat’s eye”
retro-reflector, in which light is focused at normal incidence at the surface of a mirror. Light reflected off the fiber end will therefore couple very well back into the laser itself, forming a resonant cavity and messing up the optical feedback locking that control the laser frequency. The best coupling implies the greatest amount of retro-reflected light that can prevent the laser from operating at the correct frequency.

We solved this problem by inserting two Faraday effect optical isolators between the laser output and the fiber coupler. The isolators were Hoya Optics model M-500 with 20 dB isolation, for a combined isolation of 40 dB. With both isolators in place, the detector laser was no longer disturbed by optical feedback from the fiber coupler.

The polarization optics consisted of a 1 inch polarizing cube beamsplitter and a λ/4 plate. The polarizer was mounted so that it could tilt up-and-down or left-and-right across the beam, while the λ/4 plate could also be rotated about the beam axis. These were mounted after the second mirror but before the lens and the vacuum window so that the beam first passed through the polarizer, then the λ/4 plate. The two polarizing elements were adjusted to produce circularly polarized light. This adjustment was done by inserting a mirror between the λ/4 plate and the lens that reflected the laser light back on itself. The light would pass through the λ/4 plate a second time and, if the λ/4 plate and polarizer were adjusted correctly, would be polarized at right angles to the incoming laser light. The reflected light would therefore not be allowed back through the polarizing beamsplitter. Therefore, the circular polarization could be set correctly by minimizing the power returning from the mirror. We were able to achieve extinction ratios of better than one in one
2.8.3 Light collection

There were two major technical challenges involved in measuring the fluorescence signal from atoms in the detector. First, we needed to build a collector that could detect as many fluorescence photons as possible with as much sensitivity as possible. Second, we needed to illuminate the atomic beam while allowing as little laser light as possible to scatter into the light collector. We ultimately achieved an estimated ratio of scattered to incident light of one photon in $4.2 \times 10^8$, while we collected an estimated one in 100 fluorescence photons.

The first requirement on the detector was high sensitivity and efficiency. Ideally, we would like to be able to detect each atom that passes through the detector, using the "photon-burst" technique in which we detect the burst of atoms emitted as a cycling atom passes through the detector. In practice, our detection scheme is not yet sensitive enough to detect single atoms, so we rely upon an average light signal converted into a current by a photomultiplier tube.

Figure 2.18 shows the collection optics, consisting of an elliptical cylinder collector, a glass light pipe, and a photomultiplier tube (PMT). The collector is machined from OFHC copper and gold-plated. It is 1.00 cm thick, with a semi-major axis of 2.70 cm and a semi-minor axis of 1.91 cm. The atomic and laser beams enter and exit through two 1.00 cm diameter holes centered on one focus of the ellipse. The light pipe terminates at the second focus. It is also 1 cm in diameter. We estimate that 25% of the fluorescence light reaches the far end of the light pipe.
Figure 2.18: The light collector. The elliptical cylinder focuses the fluorescence from atoms passing through the detector onto the end of the glass light pipe. Total internal reflection traps all the photons that enter the light pipe as they propagate to its end, which butts up against the photocathode of a photo-multiplier tube.
The light pipe makes a vacuum seal and leaves the detector chamber and enters the PMT housing, ending where it butts against the photocathode of the PMT. The PMT has a semi-transparent photocathode, a quantum efficiency of 2%, and is suitable for photon counting. Since this photocathode actually lines the front surface of the PMT, we are able to collect all the incident photons, giving an overall collection efficiency of 1%. By cooling the PMT to about -10 °C, we are able to reduce the PMT’s dark current at operating voltage to about 0.5 nA. By comparison, the typical scattered light background is 1.5 nA.

Figure 2.19 shows the arrangement of baffles and the route taken by the detector laser when passing through the detector chamber. The points labelled “A”, “B”, and “C” are the dominant sources of scattered light: “A” is forward scattering off the anti-reflection coated entrance window (and the lens and other optics behind it). “B” is scattering from the edge of the 2 mm diameter aperture through which the beam is focused. “C” is scattering off the two apertures on either side of the collector itself.

All interior surfaces except for the collector itself are anodized aluminum.

When light passes through a surface such as a window (or reflects from a mirror), the scattering due to surface imperfections is primarily in the forward direction[33]. That is, most of the light that is scattered travels in very nearly the same direction as the original beam. This means that most of the effort involved in reducing scattered light must center on reducing the amount of light scattered from “A”, “B”, and “C” that enters the light collector, while we didn’t need to worry too much about light.

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2We found that, although PMT’s exist with quantum efficiencies as high as 15% at our wavelength, these tubes have a recessed photocathode that would reduce the fraction of light leaving the light pipe that actually hits the photocathode by more than the loss due to low quantum efficiency.
Figure 2.19: The arrangement of the detector laser beam and baffles. The labels A, B, and C indicate possible sources of scattered light. The arrangement prevents light scattered from A or B from hitting the reflective portions of the light collector. Light scattered from C must first have scattered from B. The exit window D primarily scatters light in the forward direction, out of the chamber.
scattered from the exit window.

We found that, by focusing the laser beam through the small aperture at “B”, we were able to reduce the scattered light flux by a factor of 10 over any other method we tried[40]. We believe this has a simple explanation: focusing the beam at the center of the aperture allows us to use a smaller aperture. A smaller aperture reduces the solid angle of “outside world” visible from the collector, therefore restricting the amount of diffuse scattering that can find its way into the collector. A smaller aperture also has a smaller perimeter that can scatter and diffract diffusely scattered light into the collector.

The second crucial technique involved the placement of the baffles at the collector itself. These baffles are about 2 mm smaller in diameter than the collector openings, and are located in such a manner that there is no line of sight between any reflective surface and the aperture at “B”. (Consequently, there is also no line of sight between any reflective surface and the entrance window.) This means that all the light scattered into the collector must have scattered off the baffles themselves, after first scattering from an earlier surface, all of which are black.

2.8.4 Effects of hyperfine structure on photon emission in the detector

We have discussed the detector up to this point using the assumption that the detection scheme involved a simple, two level atomic system excited by laser light. In reality, atoms enter the detector in any of four hyperfine levels, each of which can be excited by circularly polarized light to one of four excited-state hyperfine levels. Of
these, the three states with $m'_F \neq 3$ can decay by spontaneous emission into more than one ground state hyperfine level. The frequency of each of the four transitions depends differently on magnetic field, as do the excitation and emission amplitudes. The transition frequencies will alter the apparent velocity of atoms emitting photons from each of the hyperfine states, while the average number of photons emitted by an atom in the detector will depend on the atom's state, its velocity, and the magnetic field in the detector.

Figure 2.20 (a) shows the Zeeman-shifted transition frequencies for the four transitions driven by the detector laser. The Zeeman shift is positive and linear with the same slope at high fields for all four states, but the three states with $m_F \neq 2$ have their curves shifted to higher frequencies. Figure 2.20 (b) shows the effect of these different Zeeman shifts on the detector's velocity measurement. The true velocity of atoms in each of the four states is plotted as a function of the detector's "nominal velocity". By "nominal velocity", we mean the velocity of $m_F = 2$ atoms resonant with the detector laser at a particular magnetic field. Therefore there is no velocity shift for the $m_F = 2$ state; the others are really traveling faster than they appear.

These other hyperfine states can, in our detector, also cycle many times. This is because the probability that an atom excited by the detector laser from one of these state will decay into any other state approaches zero as the magnetic field in the detector gets larger. Figure 2.21 shows the transition moments as a function of the detector field. These transition moments correspond to the probability $P_{n_F}$ that an atom initially excited to the $m'_F$ state will decay back into the original $m_F = m'_F - 1$
Figure 2.20: (a) The Zeeman frequency shifts of the four $\Delta m_F = +1$ transitions that take place in the detector, as a function of detector magnetic field. (b) The resulting velocity corrections: The true atomic velocities for atoms in each of the four states are plotted as a function of the apparent velocity (the velocity of $m_F = 2$ atoms).
Figure 2.21: (a) The relative transition amplitudes squared (moments) of the four detector transitions as a function of detector magnetic field. The cycling transition’s relative moment is always unity, but the moments of the other three transitions increase with field. (b) The average number of photons emitted by atoms in the four detector states as a function of magnetic field. The number of photons emitted by an $m_F = 2$ atom is limited by that atom’s transit time; the average numbers emitted by atoms in the remaining states are determined by the probability of optical pumping each time a photon is absorbed and spontaneously emitted.
state. Since the population remaining in state $m_F$ after $n$ photons have been absorbed is $(P_{m_F})^n$, the population must decay exponentially in time, with the average number of photons absorbed by an atom in any of the three non-cycling states before optical pumping occurs just being

$$\tilde{n} = -\frac{1}{\ln(P_{m_F})}. \quad (2.6)$$

In contrast, atoms in the cycling state can absorb photons forever without being optically pumped. However, these atoms are only illuminated by the detector for a short time. (This short time is still long enough so that an atom in any other state will have been optically pumped before leaving the detector.) Hence, the number of photons emitted by a cycling atom will fall off as $1/n$. Figure 2.21 (b) shows the average number of emitted photons $\tilde{n}$ for the three non-cycling states as a function of detector field as well as the number that can be emitted by an atom in the cycling state while traversing a 1 cm detector at the velocity corresponding to that field. The average number of photons emitted by the non-cycling atoms increase with field, but never exceed the number emitted by cycling atoms at the corresponding velocity. Figure 2.22 shows the ratio of photons emitted by atoms in each state divided by the total number emitted. This normalized emission allows one to compare an observed signal directly to a control made by measuring the fluorescence from an unpumped atomic beam.

2.8.5 Calibration

In our experiment, we measure excitation probability as a function of atomic velocity. For a given data point, the detector magnet’s field determines the frequency at which
Figure 2.22: The fraction of the total fluorescence emitted by atoms in each hyperfine state as a function of detector magnetic field and velocity.

atoms resonate with the detector laser. This frequency determines the velocity of atoms that will be Doppler-shifted into resonance. The velocity ultimately determines the vortex spacing that we probe. We therefore need to pay particular attention to the detector magnet’s calibration to ensure that we understand the meaning of the measurements we are taking.

The detector magnet is controlled by a drive voltage generated by the computer. We will now discuss how we calibrate the magnetic field in terms of this drive voltage. The velocity of resonant \( m_F = 2 \) atoms in the detector is given by \( v = \mu_B \lambda B \), where \( \mu_B \lambda = 1.07 \text{(m/s)}/\text{Gauss} \), so long as the laser is tuned to the \( B = 0 \) resonance frequency of the \( F = 2 \rightarrow F' = 3 \). The detector magnet produces 294 Gauss/Amp.
as measured *in situ* with the magnetic shield in place. Finally, the magnet power supply produces 0.711 Amps/Volt when driven by the computer controller, so that the detector field varies with the control voltage as $dB/dV = 209$ Gauss/Volt.

Unfortunately, there are several sources of uncertainty about drive voltage that corresponds to $B = 0$. First, the iron magnetic shield (intended to keep fringing fields produced by the detector magnet out of the rest of the experiment) produces a remanent field of about 6 Gauss when the magnet current is turned off. Secondly, the power supply has a current offset of about -0.3 Amps.

We can independently check our calibration by making use of the hyperfine structure of $^{39}$K itself. Figure 2.23 shows a calibration run taken during the same experimental session in which we took the data we will discuss in the next chapter. In this run, the first pumping beam was blocked. The atomic beam passed through the 1 mm square “big slit” instead of over the sample. The second pumping beam was chopped, and phase-sensitive detection was used to detect the signal. When the second pumping beam is on, it pumps all the $F = 2$ atoms down into the $F = 1$ level, effectively turning the signal off. The only exception to this rule comes at a detector field of 380 Gauss, at which the $F = 1, m_F = 1 \rightarrow F' = 2, m'_F = 1$ transition has the same frequency as the cycling transition. Since this transition has $\Delta m_F = 0$, it cannot be driven by circularly polarized light. However, the magnetic field is tilted by 5 degrees with respect to the laser axis, so that the light has an 8.7% component parallel to the magnetic field. Since the laser intensity is 25 times the saturation intensity for the cycling transition, this transition will effectively see light at twice its saturation
Figure 2.23: An unpumped calibration run. The atomic beam was sent through the “big slit” in the sample region without first being optically pumped into the $F = 1$ state. The second pumping beam was then chopped and the phase-sensitive signal recorded by a lock-in amp. Positive signal means that there was more fluorescence when the second pumping laser was off than when it was on. The notch occurs at $B = 380$ Gauss, at which field the $F = 0, m_F = 1 \rightarrow F' = 2, m_F' = 1$ transition has the same resonant frequency as the $m_F = 2 \rightarrow m_F' = 3$ cycling transition.
intensity, meaning that transitions will be driven at close to the maximum rate of once every 52 nsec. In other words, every atom in the $F = 1$, $m_F = 1$ hyperfine state will have time to make about 470 transitions while in the detector. Furthermore, after making this transition to the $F' = 2$, $m'_{F'} = 1$ excited state hyperfine level, the atoms have a 33% probability of being optically pumped directly into the $m_F = 2$ level, where they will continue to absorb and re-emit photons forever.

This means that, at a magnetic field of 380 Gauss, one third of the atoms pumped into the $F = 1$ state by the second pumping laser will emit just as many photons as they would have if left in the $F = 2$ state, and the phase-sensitive detector signal will be reduced to 2/3 of its expected value at this field, producing the distinctive notch. The notch occurs at a detector magnet control voltage of 2.17 Volts, so that the final calibration of the detector magnetic field is

$$B = A \ast V - B_0,$$

(2.7)

where $A = 209$ Gauss/Volt and $B_0 = 75.3$ Gauss. The detector control voltage is scanned from 1 Volt to 5 Volts, resulting in a velocity scan range from 144 m/s to 1043 m/s.

### 2.8.6 The hot wire detector

A hot-wire ionization detector (HWD) is installed at the end of the detector chamber. The HWD detects essentially all the potassium atoms which fall on its active surface by ionizing them[31]. The ion current is measured using an electrometer. The HWD serves to measure the atomic beam current that reaches the end of the experiment
during many alignment procedures.

Our HWD is shown in Figure 2.24. The hot wire in our detector is actually a 15 \( \mu m \) thick by 2 mm wide tungsten foil mounted inside a cylindrical ion collector. A larger, grounded cylindrical shield surrounds the assembly. Both cylinders are open along one side to allow the atomic beam to hit the foil. The whole assembly is mounted by a rigid rod to a vacuum translation stage that allows the HWD to be lowered into the atomic beam path and removed when not needed.

The foil is heated by an AC current supplied by a Variac. After the foil has been baked out at high temperature for a few hours, the current is reduced so as to maximize the ionization current. The optimal current for the foil currently in use is 1.0 A. The foil is kept at a potential of +9 V with respect to the collector by an alkaline battery. Both the foil and collector float with respect to the grounded shield and the rest of the experiment. This requires the electrometer input to float as well.

Care must be taken to minimize the background current of the device. We meticulously cleaned (using abrasives) all the insulating components of the HWD assembly. Only Teflon insulated wires and cables were used for electrical connections. All electrical connections that could be soldered were (including the connections to the 9 volt battery). The circuit had a background current of about \( 10^{-13} \) amps, corresponding to \( 10^6 \) atoms per second. By blocking and unblocking the atomic beam source, we were able to see atom currents as small as \( 10^{-4} \) atoms per second.
Figure 2.24: The hot wire ionization detector. The tungsten foil is 15 μm thick by 2 mm wide. It is heated by AC current. The ion current is measured between the foil and the collector.
CHAPTER III

Results

The goal of the experiment is to prove in principle that an atomic beam can be used to study the magnetic vortex lattice at the surface of a superconductor. Beyond this, we intend to demonstrate the technique’s utility as a measurement tool. To accomplish the proof of principle, we need to demonstrate that we can detect signals that are caused by magnetic vortex lines. We will further demonstrate the utility of the atomic beam probe by extracting sensible information about the spatial structure of vortex lattice, the temperature dependence of the magnetic penetration depth, and the transport properties of the superconductor in a magnetic field.

We can begin to show that we detect signals caused by magnetic vortices by showing that we detect signals only when the sample is in its superconducting state in the presence of a magnetic field. Furthermore, this signal should depend in a simple manner upon the strength of the magnetic field and upon the temperature of the sample. We found that our signal depended on both of these parameters in a manner consistent with a disordered arrangement magnetic vortices.

Upon a more detailed examination of the dependence of the signal on magnetic field, we were able to determine that the signal was almost certainly caused by a disordered arrangement of magnetic vortices, each containing one flux quantum $\phi_0 = \ldots$
\(\frac{hc}{2e} = 20.7 \text{ Gauss (\mu m)^2}\). Examining how the signal strength varied with sample temperature, we were able to determine that the London penetration depth of our sample appeared to have a temperature dependence similar to that reported elsewhere for niobium, and we were able to estimate the zero-temperature bulk penetration depth, finding a value that agreed well with the accepted value of 44 nm. By observing the behavior of the magnetic vortex signal in the presence of a DC transport current, we deduced a lower bound for the sample’s critical current. Finally, in Chapter V, we will be able to suggest answers to some lingering questions about the results of the similar experiment performed by Brown and King in 1969\[8, 9\].

3.1 Experimental Procedure

The majority of the data that will be discussed was taken on September 2 and 9, 1993. We were also able to run the experiment at other times, confirming that our results were reproducible using different niobium samples. We followed the same basic experimental procedure on each occasion.

After the experiment was prepared in the manner described in the previous chapter, the cryostat was positioned so that the atomic beam passed through the sample slit. A typical data set consisted of three runs: First, we took a calibration run with the first optical pumping beam blocked. This served as a reference for the intensity and velocity distribution of the atomic beam as it passed through the detector. Second, we took a “warm” control run with the first optical pumping beam in place and the sample raised above the transition temperature of niobium (8.9 K for our sample). This run served as a baseline control, telling us what proportion of the atoms
in our initially random distribution of hyperfine states remained in the $F = 2$ levels after being illuminated by the optical pumping beam. Finally, we applied the desired magnetic field to the sample and then turned off the heater, allowing it to cool to 5 K in the presence of the applied field. (This is called "field cooling".) We then took a "data" run.

The data in all three runs were taken using phase-sensitive detection, with a $10^6$ V/A current preamplifier at the input of the lock-in amp. The lock-in's time constant was set at 0.1 sec, while 1000 data points were taken at a rate of 10 per second. Each data point therefore corresponds to a time interval equal to one time constant. The unpumped calibration runs were usually taken at a voltage gain of 100 (100 mV sensitivity scale on the lock-in), while the warm control and cold data runs were usually taken at a voltage gain of 1000 (10 mV sensitivity). Therefore, one volt of signal on a data run corresponds to 1 nA RMS current coming out of the photomultiplier tube. Since the PMT operates at a current gain of $5 \times 10^5$, this corresponds to 12,500 photons RMS detected per second.

The results of a typical set of runs are shown in Figure 3.1. The unpumped calibration run shows the atomic beam's velocity distribution (including the notch discussed in Section 2.8.5). The warm run shows the small fraction of this initial signal that remains after optical pumping. The cold run shows the signal after the sample was field-cooled at $B = 8.9$ Gauss. The signal is about 10% of the height of the unpumped signal, and is clearly narrower than the thermal velocity distribution. It appears to be centered at a velocity of about 725 m/s. At the zero-field transition
Figure 3.1: Raw results for a typical set of three runs. The unpumped calibration run at the top shows the velocity distribution of the 403 K atomic beam. The warm run ($T > T_c$) below shows the remainder after optical pumping, while the cold ($T = 5$ K) run shows the raw signal as a function of velocity at an applied field of 8.9 Gauss.
frequency of 462 MHz, this would correspond to an intervortex spacing of 1.57 μm.

The nominal intervortex spacing is given by $d = \sqrt{\phi_0/B} = 1.52 \mu m$ at $B = 8.9$ Gauss.

The raw data are reduced by subtracting the warm control run and normalizing to the unpumped calibration velocity distribution. First, we smooth the unpumped calibration data using a running mean filter with a width of 30 data points, giving a smooth normal velocity distribution $N(v)$. Then we fit the warm control curve to the same form as the normalization curve by $W(v) = A \ast N(v) + B$, where $A$ and $B$ are least-squares fitting parameters. Finally, we smooth the cold data $C(v)$ using the same filter at a width of 20 points, subtract from it the warm fit and divide the result by the normalization curve to get the normalized result

$$P(v) = \frac{C(v) - (A \ast N(v) + B)}{N(v)}.$$ (3.1)

The velocity scale is converted to intervortex spacing by $d = v/f(B_{samp})$, where $f(B_{samp})$ is the mean NMR transition frequency at the applied field $B_{samp}$. The dependence of $f(B_{samp})$ on $B_{samp}$ is discussed in Section 4.2.

Figure 3.2 shows the result of this procedure when applied to the raw data shown in Figure 3.1, at a field of 8.9 Gauss and a temperature of 5 K. This result can be roughly interpreted as the probability that an atom moving at velocity $v$ makes a transition from the $F = 1$ to the $F = 2$ hyperfine states.

### 3.2 Dependence on Applied Magnetic Field

We performed this procedure at each of the following applied magnetic fields: 0 Gauss, 8.9 Gauss, 1.8 Gauss, 4.5 Gauss, 8.9 Gauss, 13.3 Gauss, 17.8 Gauss, 26.7 Gauss, 35.6
Figure 3.2: Reduced data at $B = 8.9$ Gauss. The cold data curve in Figure 3.1 is smoothed, a fit to the warm data subtracted from it, and the result divided by the smoothed, unpumped normalization curve to get a normalized result. The distance scale is derived from the measured velocity using the mean NMR transition frequency for an 8.9 Gauss applied field. The vertical line indicates the nominal intervortex spacing $d = \sqrt{\phi_0 / B}$. 
Gauss, 44.5 Gauss, 66.7 Gauss, 89 Gauss, 133 Gauss, and 178 Gauss. The results are shown in Figures 3.3 through 3.15. All runs were taken with the sample field-cooled to 5 K.

The most striking feature of these data is the dependence of the peak location on magnetic field. For all the runs in which the peak was located within the main body of the thermal velocity distribution (\(v\) between about 400 and 1000 m/s, or \(B\) between 4.5 and 35.6 Gauss), the peak signal was located very close to the nominal intervortex spacing (as indicated by the vertical lines). This indicates that a large fraction of the signal came from magnetic fluctuations with a scale length corresponding to the
Figure 3.4: $B = 0.9$ Gauss, $T = 5$ K.
Figure 3.5: $B = 1.8$ Gauss, $T = 5$ K.
Figure 3.6: $B = 4.5$ Gauss, $T = 5$ K. The vertical line indicates the nominal inter-vortex spacing.
Figure 3.7: $B = 13.3$ Gauss, $T = 5$ K. The vertical line indicates the nominal intervortex spacing.
Figure 3.8: $B = 17.8$ Gauss, $T = 5$ K. The vertical line indicates the nominal intervortex spacing.
Figure 3.9: $B = 26.7$ Gauss, $T = 5$ K. The vertical line indicates the nominal intervortex spacing.
Figure 3.10: $B = 35.6$ Gauss, $T = 5$ K. The vertical line indicates the nominal intervortex spacing.
Figure 3.11: $B = 44.5$ Gauss, $T = 5$ K.
Figure 3.12: $B = 66.7$ Gauss, $T = 5$ K.
Figure 3.13: $B = 89$ Gauss, $T = 5$ K.
Figure 3.14: $B = 133$ Gauss, $T = 5$ K.
Figure 3.15: $B = 178$ Gauss, $T = 5$ K.
expected separation of the vortices in an Abrikosov flux lattice. It is reasonable to conclude that these fluctuations were in fact caused by vortices.

The strength of the signal is also observed to vary with field. At $B = 0$, there is no observable structure, and the signal strength is at most 0.5% of the unpumped signal. As the field increases, the signal quickly grows and develops structure, leveling off at about 9% of the unpumped signal in the range of fields from 4.5 to 13.3 Gauss. As the field is increased further, the signal begins to decrease steadily, with a relative strength of only 2% at 89 Gauss and less than 1% at 178 Gauss, the highest field we were able to generate.

The smallest field for which the nominal intervortex spacing lay within our range of observation was 4.5 Gauss. Nevertheless, the $B = 1.8$ Gauss data shown in Figure 3.5 display a fairly clear peak at an intervortex spacing of about 1.6 $\mu$m. This is close to one half of the nominal intervortex spacing of 3.39 $\mu$m, and could possibly be interpreted as arising from the second spatial harmonic of the magnetic field that has a nominal period of 3.39 $\mu$m.

Most of the curves display some rapid oscillation at a velocity of about 400 m/s. We believe that this behavior does not arise out of the magnetic vortex lattice, but is instead caused by the notch discussed in Section 2.8.5. This notch occurs at a magnetic field of 380 Gauss, corresponding to a detector velocity of 407 m/s. Furthermore, the notch occurs in a region of the velocity distribution in which there are very few atoms to begin with. Normalizing the data to a velocity distribution that contains this sharp feature can easily introduce rapid oscillations into the signal at
that point. These oscillations will only be made larger when the signal is normalized to the small fraction of atoms traveling at that velocity. The alternative, in which the normalization is fit to a smooth curve lacking the notch, would introduce an artificial drop in signal strength at the notch velocity. The best way to deal with the situation for now is to treat any sharp features in the normalized data curves at velocities around 400 m/s as systematic error, and rely on the general trend of the data when making interpretations.

3.3 Temperature Dependence

The magnetic field at the center of a vortex in the bulk of a superconductor will be roughly \( B \approx \phi_0 / (\pi \lambda^2) \), where \( \lambda = \lambda(T) \) is the magnetic penetration depth. Therefore, the size of the signal should decrease with increasing penetration depth roughly as \( 1/\lambda(T)^4 \). Since \( \lambda(T) \) diverges as \( T \) approaches \( T_c \), the signal should decrease in a predictable manner to zero as the sample temperature is raised to \( T_c \).

To test this hypothesis, we conducted a series of runs at a constant field and different temperatures. First, the usual calibration and control runs were taken. Then we field-cooled the sample at an applied field \( B = 8.9 \) Gauss to 5 K. We took five data runs at temperatures of 5.05 K, 6.03 K, 7.31 K, 8.25 K, and 8.88 K, without warming it above \( T_c = 8.90 \) K or changing the magnetic field. Figure 3.16 shows the results. The signal does in fact decrease to zero as the temperature approaches \( T_c \).

We will use this temperature dependence to study the magnetic penetration depth in Section 4.5.
Figure 3.16: Dependence of signal on temperature at $B = 8.9$ Gauss. (a) Reduced data runs taken at temperatures of 5.05 K, 6.03 K, 7.31 K, 8.25 K, and 8.88 K. The vertical line indicates the nominal intervortex spacing of 1.52 $\mu$m. (b) The height of the curves at the nominal intervortex spacing plotted as a function of temperature. The dashed line indicates the transition temperature $T_c$. 
3.4 Transport Currents, Changing Applied Fields, and Critical Currents

We carried out several tests that attempted to probe the transport properties of our sample. In particular, we studied the spatial signal in the presence of transport current densities, examined the response of the signal at a fixed spacing in real time as we changed the applied magnetic field, and examined the spatial signal before and after changing the transport current.

Our attempt to observe a significant change in the vortex signal when a transport current was applied was unsuccessful. When a transport current is applied to a superconductor in the mixed state, each vortex is subjected to a Lorentz force perpendicular to both the current and the vortex axis of

$$F_{tor} = \frac{J \times \Phi_0}{c},$$

where $J$ is the current density and $\Phi_0$ has magnitude $\phi_0$ and points along the vortex axis[31] (in CGS units). This driving force is opposed by a viscous drag force caused by dissipative normal currents in the vortex core and by pinning forces that attempt to bind vortices to local features (such as defects) in the material. In the absence of pinning, or when the current density exceeds the critical current density necessary to “depin” vortices, the vortices drift at a velocity

$$v = \frac{J \phi_0}{\eta c},$$

where $\eta$ is a coefficient of viscosity given by

$$\eta \approx \frac{\phi_0 H_{c2}}{\rho_n c^2}.$$
Here, $H_{c2}$ is the upper critical field and $\rho_n$ is the normal state resistivity. Combining these expressions, we get

$$v = \frac{J\rho_n c}{H_{c2}}. \quad (3.5)$$

We were able to run a 100 mA current through the sample after field cooling in a 8.9 Gauss field. This current produced a current density of $J = 6.7 \times 10^4$ Amps/cm$^2 = 2.0 \times 10^{14}$ statamps/cm$^2$. Estimating $H_{c2} \approx 1000$ Oe and $\rho_n \approx 4 \times 10^{-6} \Omega cm = 4.7 \times 10^{-18}$ stat$\Omega$cm, we can estimate the drift velocity for free vortices as $v \approx 240$ m/s, which should be clearly visible as a velocity shift in our measurements. Figure 3.17 shows the results of our measurements. We were able to observe only minute differences between the signals observed at currents of 0, ±100 mA, or -100 mA, in applied fields of both 4.5 Gauss and 8.9 Gauss. We were unable to observe any magnetoresistance using our four-wire resistance probe.

We believe the small differences we observed are most likely caused by Majorana transitions between the various $F = 2$ levels that were driven by the sudden change in local magnetic field at the edges of the sample due to the transport current. This interpretation of the results is supported by the fact that small, identical details are present, at the same velocity, in runs with either ±100 mA transport current but are absent when no current is applied. We will discuss the effects of Majorana transitions in detail in Section 4.2.2 and Chapter V.

These results suggest that the vortices are prevented from moving by pinning forces. Therefore, the average pinning force per vortex must be at least as large as the Lorentz force. In other words, the sample's critical current $J_c > J_{app} = 6.7 \times 10^4$
Figure 3.17: Effect of transport currents on signal. The sample was field-cooled at 4.5 Gauss and 8.9 Gauss. At each field, three runs were taken in succession at currents of 0, +100 mA, and -100 mA.
Amperes/cm².

Some further support for this interpretation is given by an additional experiment: One possible effect of pinning forces would be to reduce the average drift velocity so much that we would be unable to observe it using our technique. However, if we first field-cooled the sample in some non-zero field and then turned the field off, we might be able to observe the motion of vortices out of the sample after we turned on a transport current. In other words, if the average drift velocity were as small as 1 mm/sec, we should observe our signal to disappear entirely after about 1.5 seconds! We carried out this experiment by first field cooling the sample to 8.9 Gauss, setting the detector at the peak velocity of 660 m/s, and then collecting data for 20 seconds without scanning the detector. After data collection began, we turned off the external field and then turned on the transport current.

Figure 3.18 shows the results of one such run. The signal at the peak velocity did not change after the current was turned on, suggesting that the vortex lattice remained fixed in the presence of the current. However, the signal clearly and suddenly increased when the external field was turned off, indicating that the magnetic field in the vicinity of the sample changed, perhaps because the vortices were forced into a critical state when the field was turned off. Similar changes were observed in the signal at fixed velocity when the external field was suddenly changed in a variety of ways, including being increased from zero, being changed from one positive value to another, and even being reversed.

The critical state of a superconductor in the mixed state can be qualitatively
Figure 3.18: Effect of turning off magnetic field and turning on transport current at fixed velocity. The sample was field-cooled at 8.9 Gauss and data were collected for 20 seconds. The external field was turned off at 5 seconds and a 100 mA transport current was turned on at 10 seconds. These data were not filtered or normalized.
explained in terms of the Bean critical state model[4, 34]. Simply put, the Bean model notes that for a superconducting slab of finite thickness but infinite extent parallel to an applied field that has been changed since the slab entered the superconducting state, the magnetic field inside the slab will change with slope

\[
\frac{dB}{dx} = \frac{4\pi J_c}{c},
\]

so that the field at the edges of the sample matches the external field. Although the critical state of a thin film is more complicated, the physics remains qualitatively the same. When the external field is increased or decreased, the flux density inside the sample no longer matches the field outside the sample. In order for this situation to continue, a current must flow around the edges of the sample sufficient to generate this difference in magnetic field. This current would normally flow within a penetration depth of the sample edge, creating an extremely large current density. But if \( J_c \) is locally exceeded, vortices will move so as to reduce the current density. When \( J \) has been reduced to \( J_c \), the vortices will again become pinned, and the flux distribution will remain fixed. Hence, when we suddenly turned off the magnetic field in Figure 3.18, flux immediately began to leave the edges of the sample until the critical state was reached.

The major physical insight of the Bean model is that the slope of the current density is approximately proportional to the critical current density in the critical state; that is

\[
\frac{dB}{dx} \approx \frac{4\pi J_c}{c}.
\]

If the critical current were in fact equal to the maximum current density we applied
to the sample, the slope $dB/dx$ would equal $8.4 \times 10^4$ Gauss/cm (as we will show in Section 4.2.2). If we turned off an initial field of 10 Gauss, the critical state would only extend 1.2 $\mu$m into the sample. In other words, all the flux would remain trapped in the sample, and the field would drop abruptly at both edges. We will show in Section 4.2.2 that this abrupt field shape can produce a characteristic excitation signal that closely resembles the experimental data.

To explore the critical state further, we prepared the sample by first field-cooling it in an 8.9 Gauss applied field, then turning the field off while keeping the sample cold. In Figure 3.19, we compare the result of a run taken after the sample was prepared in this manner to a run taken by field-cooling the sample at 8.9 Gauss in the usual fashion. The solid line shows the data taken in the critical state. The two obvious effects are that (i) the signal at large spacings increases significantly, and (ii) that the signal develops more pronounced oscillations. The larger signal at large vortex spacings indicates that the mean inter-vortex spacing increased, as would be expected when vortices left the sample. The oscillatory behavior is once again probably the result of Majorana transitions among the various $F = 2$ levels. The Majorana transitions would in this case be driven by the magnetic field gradient $dB/dx = 4\pi J_c/c$ along the direction of the atom’s travel.
Figure 3.19: Effect of turning off an 8.9 Gauss field at 5 K. (a) The signal after the sample was field-cooled in an 8.9 Gauss field (dashed line) and the signal after the field was reduced to zero while the sample remained cold (solid line). (b) The difference between the zero-field and 8.9 Gauss signals in (a).
CHAPTER IV

Theory and Modeling

We have undertaken extensive theoretical and numerical analysis of our experimental system. By doing so, we hope to better understand the physics of the interaction between the potassium atoms and the vortex lattice and to extract as much useful quantitative information as we can from our results. First, we will discuss the excitation process for a two-level magnetic dipole-coupled atom traveling through a spatially-varying magnetic field. Then we will discuss the effects of the multiplicity of states, including mixing between $F = 2$ levels. We will then discuss analytic and Monte Carlo numerical modeling of the signal caused by a vortex lattice. Finally, we will use these models to study the signal’s dependence on penetration depth, magnetic field, and vortex lattice structure.

4.1  Excitation of a Two-level Atom by a Spatially-varying Magnetic Field

The simplest model for our experimental system is of a two-level atom passing through a spatially-varying magnetic field. The atom has levels $g$ and $e$ (corresponding to the $F = 1$ and $F = 2$ hyperfine states in $^{39}\text{K}$), coupled by their interaction with the magnetic field, as shown schematically in Figure 4.1. The system’s state at time $t$ is
Figure 4.1: The two-level atom. The states $|g\rangle$ and $|e\rangle$ correspond to the $F = 1$ and $F = 2$ levels of the real potassium atom, and are separated by $\omega_0 = 2\pi \times 462$ MHz.
The system is described by the Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_B$, where

$$\mathcal{H}_t = \hbar \begin{pmatrix} \omega_0 & 0 \\ 0 & 0 \end{pmatrix},$$

with $\omega_0$ being the system’s transition frequency, and

$$\mathcal{H}_B = \frac{\mu_B}{\hbar}(L + 2S) \cdot B,$$

with $\mu_B$ being the Bohr magneton. ($\mu_B/\hbar = 1.40$ MHz/Gauss.) In the $4s^2S_{1/2}$ state of potassium, $L = 0$, so the coupling is just

$$\mathcal{H}_B = 2(\mu_B/\hbar) S \cdot B = 2\mu_B B_z(t) \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix},$$

where $M$ is the dimensionless angular-momentum part of the interaction matrix element $\langle g | \mathcal{H}_B | e \rangle$. We are assuming that $M$ is real since the system is driven by a real field $B(t)$.

$B_z = B(t)$ is determined by the atom’s speed and path through the spatially varying field $B(x)$ using the fact that $x = x_0 + vt$. In one dimension, this simplifies to $B(t) = B(x_0 + vt)$.

The atom obeys the time-dependent Schroedinger equation

$$\frac{d\Psi(t)}{dt} = -\frac{i}{\hbar}\mathcal{H}(t)\Psi(t).$$

In terms of the state amplitudes $c_g$ and $c_e$, this becomes

$$\frac{d}{dt} \begin{pmatrix} c_e(t) \\ c_g(t) \end{pmatrix} = -i \left[ \begin{pmatrix} \omega_0 & 0 \\ 0 & 0 \end{pmatrix} + 2\frac{\mu_B}{\hbar} B(t) \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} \right].$$
The initial conditions are that

\[ c_e(-\infty) = 0, \quad (4.7) \]
\[ c_g(-\infty) = 1. \quad (4.8) \]

This pair of coupled equations can be solved directly for a given \( B(t) \) numerically. However, except under unusual circumstances, we can obtain a better insight into the underlying structure of \( B(x) \) if we use time-dependent perturbation theory (TDPT) instead. The TDPT equations of motion are

\[ i\hbar \frac{dc_i}{dt} = \mathcal{H}_{Bij} e^{i\omega_0 t}, \quad (4.9) \]

where \( i \) stands for \( e \) or \( g \) while \( j \) stands for the opposite. In particular,

\[
\begin{align*}
 i\hbar \frac{dc_e}{dt} &= 2\frac{\mu_B}{\hbar} B(t) e^{i\omega_0 t} \\
i\hbar \frac{dc_g}{dt} &= 2\frac{\mu_B}{\hbar} B(t) e^{-i\omega_0 t}.
\end{align*}
\]

(4.10)

We can integrate these directly, and given the initial condition that the atom begins in state \( g \), we get an expression for the final excited state amplitude of

\[ c_e(+\infty) = -2i \frac{\mu_B}{\hbar} M \int_{-\infty}^{\infty} B(t) e^{i\omega_0 t} dt, \quad (4.11) \]

which is proportional to the Fourier component of the driving field \( B(t) \) at the transition frequency \( \omega_0 \). The transition probability is given by

\[ P_e(t) = |c_e(t)|^2 = 4 \left( \frac{\mu_B}{\hbar} \right)^2 |M|^2 \left| \int_{-\infty}^{+\infty} B(t) e^{i\omega_0 t} dt \right|^2. \quad (4.12) \]

So far we have determined the excitation probability in TDPT given that we know the time-dependence of the magnetic field \( B(t) \). We can express the excitation
probability in terms of the spatially-varying field \( B(x) \) and the atomic velocity \( v \) using the fact that \( B(t) = B(x_0 + vt) \). The integral in Equation 4.12 becomes

\[
\int_{-\infty}^{+\infty} B(t)e^{i\omega_0 t} dt = \frac{1}{v^2} \int_{-\infty}^{+\infty} B(x)e^{iq(v)x} dx,
\]

where \( q(v) = \omega_0/v \). We can now re-express the transition probability in terms of the spatial Fourier transform of the field \( B(q) \) as

\[
P_e = \frac{4}{v^2} \left( \frac{\mu B}{h} \right)^2 \left| \int_{-\infty}^{+\infty} B(x)e^{iq(v)x} dx \right|^2 \equiv \frac{4}{v^2} \left( \frac{\mu B}{h} \right)^2 |B(q)|^2.
\]

The spatial Fourier transform \( B(q) \) is intimately related to the autocorrelation of \( B(x) \) by the “Wiener-Khinchin Theorem”[30]. The autocorrelation of a real function \( B(x) \) is defined to be

\[
\text{Corr}(B(x), B(x)) \equiv \int_{-\infty}^{+\infty} B(x + \chi)B(\chi)d\chi.
\]

Now, the modulus-square of \( B(q) \) is

\[
|B(q)|^2 = \int_{-\infty}^{+\infty} B(x)e^{iqx} dx \int_{-\infty}^{+\infty} B(x')e^{-iqx'} dx' = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} B(x)B(x')e^{iq(x-x')} dx dx'.
\]

We can change variables, letting \( x'' = x - x' \). Now,

\[
|B(q)|^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} B(x)B(x-x'')e^{iqx} dx dx'' \quad (4.17)
\]

\[
= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} B(x + \chi)B(\chi)e^{iqx} \quad (4.18)
\]

\[
= \int_{-\infty}^{+\infty} \text{Corr}(B(x), B(x))e^{iqx} dx. \quad (4.19)
\]

In other words, \( |B(q)|^2 \) is the Fourier transform of the autocorrelation function of \( B(x) \)! The autocorrelation function of the field is directly related (in Appendix C) to the vortex-vortex correlation function of the lattice.
Time-dependent perturbation theory is valid only while certain conditions hold true—namely, when $P_e \ll 1$. Although this will remain true for most atoms passing above our sample, simulations (to be discussed later) show that the atoms passing closest to the sample surface will become too excited for TDPT to remain valid. The result is that the excited state population becomes saturated, and is no longer proportional to $|B(q)|^2$. Fortunately, we can make a weaker approximation (the rate equation approximation) that allows us to benefit from the simplicity of the TDPT result but still account for the effects of saturation.

The rate equation approximation relies upon the fact that the typical arrangement of vortices (and therefore the typical magnetic field) is disordered or broadband—that is, it does not contain any strongly periodic signals. This in turn means that the phase of the driving term in the Hamiltonian does not remain coherent for long, and the excitation probability may be approximated to increase at a rate that is proportional to $|B(q)|^2$ and to the population difference $P_g - P_e$. Therefore, the populations of the excited and ground states satisfy

$$\frac{dP_e}{dt} \approx 4 \left( \frac{\mu_B}{\hbar} \right)^2 \left| \int_{-\infty}^{\infty} B(t') e^{i\omega t'} \mathcal{L}(t' - t) dt' \right|^2 (P_g - P_e),$$  
(4.20)

$$\frac{dP_g}{dt} \approx 4 \left( \frac{\mu_B}{\hbar} \right)^2 \left| \int_{-\infty}^{\infty} B(t') e^{i\omega t'} \mathcal{L}(t' - t) dt' \right|^2 (P_e - P_g),$$  
(4.21)

where $\mathcal{L}(t' - t)$ is some gentle localizing function (such as a Gaussian) that restricts the effective integration range to the coherence time for the broadband signal.

Define the local transition rate

$$R(t) \equiv 4 \left( \frac{\mu_B}{\hbar} \right)^2 \left| \int_{-\infty}^{\infty} B(t') e^{i\omega t'} \mathcal{L}(t' - t) dt' \right|^2$$  
(4.22)
and assume that $R$ varies only slowly with time (or position). Then

\[
\begin{align*}
\frac{dP_e}{dt} &= R(P_g - P_e), \\
\frac{dP_g}{dt} &= R(P_e - P_g),
\end{align*}
\]

or, since $P_g = 1 - P_e$,

\[
\frac{dP_e}{dt} = R(t)(1 - 2P_e).
\]

We can rewrite this in terms of the distance $x$ the atom has traveled at velocity $v$ over a sample of length $\ell$ as

\[
\frac{dP_e}{dx} = \frac{1}{v} R(x)(1 - 2P_e(x)).
\]

When $P_e(0) = 0$, this has the solution

\[
\ln(1 - 2P_e(\ell)) = \frac{1}{v} \int_0^\ell R(x) \, dx.
\]

The right-hand side of this equation is equal to the result we obtained in TDPT (Equation 4.14). Therefore, all we have to do to calculate the saturated signal in the rate equation approximation is calculate the transition probability $P_{TDPT}$ and then solve Equation 4.26 for

\[
P_e = \frac{1}{2} [1 - \exp(-2P_{TDPT})].
\]

This solution has slope $R = vP_{TDPT}/\ell$ at $t = 0$ (like TDPT) and approaches a limiting value of $1/2$ as $t \to \infty$ (saturation).

### 4.2 Excitation in the multi-level atom

Unlike the simple two-level system we have just described, the $^{39}K$ electronic ground state has eight hyperfine levels. Magnetic dipole transitions are allowed between each
pair of levels with $m_F$ values separated by not more than 1. The levels are all Zeeman shifted in the presence of a magnetic field, and the transition moments of the various transitions also change with magnetic field. As a result, the transition frequency and moment of the "average" NMR transition must be calculated as a function of the sample field before we can interpret (or simulate) an experimental run.

The multiplicity of hyperfine states in potassium does more than just shift the NMR transition frequency. It is also possible (and, in fact, almost certain) that magnetic dipole transitions may be driven between neighboring members of the same hyperfine multiplet—between, for example, $F = 2$, $m_F = 1$ and $F = 2$, $m_F = 2$. Such transitions between closely-spaced energy levels are usually called "Majorana transitions". These transitions naturally have quite small transition frequencies, approaching zero at $B = 0$. Majorana transitions can be driven either resonantly by long-wavelength magnetic fluctuations or non-adiabatically by rapidly changing fields. These transitions will have important consequences on our understanding and interpretation of our experimental results.

4.2.1 Zeeman shifts, transition moments, and the "average transition"

Figure 4.2 schematically indicates the nine transitions from $F = 1$ into $F = 2$. The $M_F = -2$ state cannot pass through the Stern-Gerlach magnet. The dashed lines correspond to the Zeeman shifted levels in a weak magnetic field. Figure 4.3 shows the angular-momentum matrix elements squared for each of the nine transitions in the same range of fields. In the next section (4.2.2), we will show that,
Figure 4.2: NMR transitions in $^{39}K$. The arrows indicate the nine magnetic dipole transitions into the five $F = 2$ states. The curved arrows indicate Majorana/low frequency transitions within $F = 2$. The dashed lines indicate the Zeeman shifts in a magnetic field parallel to the positive $z$-axis.

especially at low fields, non-adiabatic Majorana transitions are likely to occur at a rate sufficient to thoroughly mix the populations of the five $F = 2$ states as the atom passes over the surface. Even in the absence of such mixing, the excitation spectra are sufficiently broad that the sum over individual states with their individual transition frequencies and transition moments can be approximated by a single weighted average transition frequency. If the angular-momentum matrix element for the transition from $F = 1$, $m_F = i$ to $F = 2$, $m_F = j$ is given by $M_{ij}$ and its transition frequency by $f_{ij}$, then the weighted average transition frequency is given by

$$f = \frac{\sum_{i=-1}^{1} \sum_{j=0}^{i+1} |M_{ij}|^2 f_{ij}}{\sum_{i=-1}^{1} \sum_{j=0}^{i+1} |M_{ij}|^2}. \quad (4.28)$$
Figure 4.3: Zeeman shifts for $^{39}\text{K}$. The transition frequencies of the nine $F = 1 \rightarrow F = 2$ hyperfine transitions in the potassium ground state are plotted as a function of magnetic field.
Figure 4.4: NMR transition strengths as a function of magnetic field. The transition strengths plotted are just the angular-momentum part of the NMR matrix elements squared.
Figure 4.5: Weighted average NMR transition frequency as a function of magnetic field.

The transition moment of the "average transition" is $M_{12}$, the transition into the $m_F = 2$ state. The average transition frequency is plotted as a function of field in Figure 4.5. We will use this transition frequency in the numerical modeling to calculate approximate transition probabilities for a two-level system. We have already used this transition frequency in Chapter III to convert our measured velocity scale into intervortex $d = v/f_{\text{mean}}(B)$.

There are two major effects of the multiplicity of hyperfine states on the excitation probability in the rate equation approximation. The first is simply a result of optical pumping from $F = 2$ to $F = 1$. When the atomic beam leaves the oven at 400° C, the populations of the eight hyperfine levels are essentially equal (462 MHz =
2.22 \times 10^{-8} \text{ K}). After being optically pumped, the five \( F = 2 \) levels are empty, and the entire population is distributed evenly among the three \( F = 1 \) levels. Therefore, each \( F = 1 \) levels starts out with an initial population of \( 8/3 \), rather than 1 in the two-level system.

The second effect is upon the rate equation system itself. Since the atoms are (we claim) thoroughly mixed among the \( F = 2 \) levels, we can approximate the complicated evolution of the eight populations by a simple two-"level" system in which populations make transitions between the \( F = 1 \) and \( F = 2 \) manifolds collectively. In other words, atoms will be excited from the three \( F = 1 \) states to the five \( F = 2 \) states at a rate

\[
\frac{dP_{F=2}}{dt} = R(P_{F=1} - P_{F=2}),
\]

and be distributed equally among all of them. In steady state (complete saturation), the collective \( F = 1 \) and \( F = 2 \) populations will be equal, and the population in each individual \( F = 1 \) state will be 3/10 of the total, rather than 1/2 in the two-level system.

When we combine these two effects, we find that the saturated population must start out by increasing \( 8/3 \) times faster than the saturated population in the two-level system, but must saturate at a value of \( 3/10 \). The modified solution is therefore

\[
P_e = \frac{3}{10} \left[ 1 - \exp\left(-\frac{80}{9} P_{TDPT}\right) \right].
\]

4.2.2 Transitions among the \( F = 2 \) levels

The interpretation and modeling of our results is made much more interesting by the fact that there are non-zero magnetic-dipole matrix elements between each neighbor-
ing pair of $F = 2$ states. Under the proper conditions, Majorana transitions can occur between these neighboring $F = 2$ states. We have already discussed some of the consequences of this “mixing” of levels in the context of the “average” NMR transition between $F = 1$ and $F = 2$ (in Section 4.2.1) and when discussing the experimental results for runs in which the sample was forced into a critical state (in which magnetic field gradients were present) by abruptly changing the value of the applied magnetic field (Section 3.4). In this section, we will discuss both non-adiabatic and resonant Majorana transitions and some of their experimental consequences.

First, let’s reconsider the two-level system of Figure 4.1. In the normal analysis, the magnetic-dipole coupling energy $2\mu_B B(t)$ is assumed to be much smaller than the transition energy $\hbar \omega_0$. However, in the case of two neighboring $F = 2$ states, $\omega_0$ can be arbitrarily small as $B$ approaches zero. Therefore, it is possible for the coupling energy to be large compared to the transition energy. In such a case, the simple resonant treatment (both TDPT and the rate equation approximation) is seen to break down if the adiabatic condition

$$\left| \frac{2\mu_B}{\hbar} \frac{dB}{dt} \right| \ll (\omega_0)^2$$

is not satisfied.

Now consider an atom traveling at 500 m/s 0.5 μm above a vortex lattice. Let the average magnetic field of the lattice be 10 Gauss, corresponding to a lattice spacing of about 1.5 μm. For a simple square lattice, the field 0.5 μm up oscillates with an amplitude of $B_0 = 4.5$ Gauss and period of $d = 1.5$ μm. The peak value of $\left| dB/dx \right|$ will be $\left| dB/dx \right|_{max} = 2\pi B_0/d = 1.9 \times 10^7$ Gauss/m. The maximum of $2\mu_B/\hbar \left| dB/dt \right|$
will be \(|dB/dt|_{\text{max}} = 2\mu_B/\hbar v|dB/dx|_{\text{max}} = 2.7 \times 10^{16} \text{ s}^{-2}\). On the other hand, for the \(m_F = 1 \rightarrow 2\) transition, \(f_0 = 6.7 \text{ MHz}\), so that \((\omega_0)^2 = 1.8 \times 10^{15} \text{ s}^{-2}\). Therefore, at this fairly typical field, the adiabatic condition is violated by an order of magnitude, and the \(F = 2\) populations will almost certainly mix. By the same argument, we can show that mixing will occur under most conditions in which transitions from \(F = 1\) to \(F = 2\) occur—that is, close to the sample.

The amplitude of an oscillating field (or Fourier component) with wavelength \(\lambda\) falls off exponentially with height above the surface as \(B_0 \sim \exp(-2\pi z/\lambda)\). Therefore, we can expect that both the signal and the non-adiabatic transitions will decrease in strength rapidly with height. However, extremely long-range field fluctuations (with \(\lambda \gg z\) will remain at full strength within the entire 1.8 \(\mu\)m space between the sample surface and the gate.

Under normal field-cooling preparation, the average field over the entire sample surface is constant—that is, there are no long-range field fluctuations. However, we conducted experiments where we purposefully produced long-range changes in the average field. Specifically, we performed experiments in the critical state and in the presence of large transport current densities. In both situations, the local average field can change significantly in the region above and at the edges of the sample. These long-range average field fluctuations can resonantly drive Majorana transitions among the \(F = 2\) levels, and we believe the interestingly oscillatory signals we observed under these conditions are in fact caused by these transitions.

Let’s consider a sample of length \(\ell = 1.5 \text{ mm}\) that was initially field-cooled at
$B_0 = 2$ Gauss, after which the field was turned off. If the film has a critical current density equal to the largest current density we supplied in the lab ($J_c = 2 \times 10^{14}$ statamps/cm²), corresponding to a field gradient of $8.4 \times 10^4$ Gauss/cm. Using the Bean model, we estimate the length over which the field will decrease to zero from 10 Gauss to be $1.2 \mu$m. In other words, the critical state does not extend more than one lattice spacing into the material, and the field can be thought of as turning off immediately at the edge of the sample. In this case, we can write the transition probability as a function of velocity in TDPT as

$$P(v) = \frac{4}{v^2} \left( \frac{\mu B}{h} \right)^2 \left| \int_0^\ell B_0 \exp \left( \frac{i \omega_0 x}{v} \right) dx \right|^2$$

$$= 16|\mathcal{M}|^2 \frac{\mu_0^2 B_0^2}{\omega_0^2} \sin^2 \left( \frac{\omega_0 \ell}{2v} \right).$$

At a field of 2 Gauss, $\omega_0/2\pi = 1.4$ MHz, and the matrix element $|\mathcal{M}|^2 = .25$ for the $m_F = 1 \rightarrow 2$ transition. The result is shown in Figure 4.6.

The prominent features of this result are the constant amplitude and the period that decreases as $v$. Although this is only a simplified model of the conditions that are likely to be found in our sample in the critical state, it is still useful to make a direct comparison to the experimental results found under the same circumstances. Figure 4.7 shows the results of this procedure carried out at a field of 1.9 Gauss. The striking feature of this experimental run is the similarity of the oscillation periods to those of the calculation. Since the oscillation period is proportional to $\omega_0/v$, it is interesting to compare these results to the 4.5 Gauss results described in Figure 3.19. We would expect the of the 4.5 Gauss oscillations to be 42% as long as the 1.9 Gauss period, since $\omega_0 \sim B$. The oscillations seen at that field are in fact about twice as
Figure 4.6: Resonant Majorana transitions at 2 Gauss. The results of a simple calculation of the response of the $F = 2 : m_F = 1 \rightarrow 2$ transition to a sample that had been field-cooled at 2 Gauss and then had the field turned off while cold. The effective driving field in the atom’s frame of reference is a square pulse 1.5 mm long traveling at velocity $v$. 
rapid as the 1.9 Gauss oscillations.

In Figure 4.7, the highest peaks seem to be about equal in height to the original field-cooled baseline peak of about 0.02. This raises the question of how large we should expect the oscillatory signal to be. The result of the model shown in Figure 4.6 suggests that a large fraction of the population present in, say the $F = 2$, $m_F = 1$ state could be transferred into the $m_F = 2$ state via Majorana transitions. In order for us to observe this signal, atoms would need to be initially present in the $m_F = 1$ state, and in greater numbers than were initially present in the $m_F = 2$ state. These could come from the initial 0.5% remnant from optical pumping, but not in sufficient quantity to produce a 2% signal. The most likely source is simply normal, resonant excitation from the $F = 1$ levels. All that would be necessary is for the population of the $m_F = 1$ level to increase faster than the population of $m_F = 2$. This would produce a population difference that would in turn allow resonant Majorana transitions to transfer this excess population into the $m_F = 2$ state. The process could run continuously as the atom traversed the sample, causing the two populations to equilibriate at velocities where the Majorana transition was strong, but maintain their population difference ($P_1 > P_2$) at velocities where the Majorana transition was weak. Since the field that drives the Majorana transitions extends throughout the region between the sample and the gate, this process would be particularly effective moderately far from the surface, where the $F = 2$ populations are significant, but neither saturated nor non-adiabatically mixed.

We believe that the puzzling results reported by Brown and King[8, 9] were caused
Figure 4.7: Effect of turning off a 1.9 Gauss field while the sample was cold. The sample was field-cooled at 1.9 Gauss, then the field was turned off, as was done in Figure 3.19. The data display oscillations with increasing period at high velocity. The spacing of the oscillatory bumps is very similar to that predicted by theory.
in large part by resonant Majorana transitions. We will discuss this theory in detail in Chapter V.

4.3 Analytic Modeling of the Excitation Signal

We will now examine the relationship between the structure of the vortex lattice and the excitation signal. We have already seen that the excitation signal in TDPT is proportional to the Fourier transform of the magnetic field autocorrelation function. We will now go on to describe the relationship between the magnetic field autocorrelation function. We will then examine the consequences for some trial lattice types. We will be using TDPT for a two-level atom throughout this section. We will use the term "vortex lattice" to describe any collection of vortices on a two-dimensional surface, whether or not it is really lattice-like in structure.

4.3.1 The relationship between excitation and vortex lattice correlation function

In Appendix C, we derive the following expression for the magnetic field above the surface of an isotropic superconducting film in the mixed state:

\[ B(q, z) = \frac{\phi_0}{\lambda^2 \alpha} \frac{(\hat{z} - i \hat{q})}{(q \coth(\alpha d/2) + \alpha)} e^{-q^2} \sum_i e^{i \mathbf{q} \cdot \mathbf{R}_i}, \]

where \( \mathbf{R}_i \) is the position of the \( i \)th vortex, \( \mathbf{q} \) is the horizontal wavevector of the atom. \( \lambda \) is the London penetration depth, \( \alpha^2 = q^2 + 1/\lambda^2 \), and \( d \) is the thickness of the film (Equation C.26). This expression lets us calculate the two-dimensional spatial Fourier transform of the magnetic field at a height \( z \) above the surface. This is not yet the expression we need in order to calculate the transition probability. We need
the one-dimensional Fourier transform of the field along a particular path at height 
$z$ and transverse position $y$

$$B(q_r, y, z) = \frac{\phi_0}{2\pi \lambda^2} \int_{-\infty}^{\infty} dq_y e^{iq_y y} \frac{e^{-qz}}{\alpha(q \coth(\alpha d/2) + \alpha)} \sum_i e^{i \mathbf{q} \cdot \mathbf{R}_i}.$$  \hspace{1cm} (4.35)

where $q^2 = q_x^2 + q_y^2$. I have kept only one component of the field, since dipole transitions
are driven by just one component of the field.

We can now insert this expression into Equation 4.14 to get the transition proba­
bility

$$P_r(q_r, y, z) = \frac{4|M|^2}{\nu^2} \left( \frac{\mu_B}{h} \right)^2 \frac{\phi_0^2}{4\pi^2 \lambda^4} \times \left| \int_{-\infty}^{\infty} dq_y \frac{e^{-2qz}}{\alpha^2(q \coth(\alpha d/2) + \alpha)^2} \exp(i \mathbf{q} \cdot \mathbf{R}_i) \right|^2.$$ \hspace{1cm} (4.36)

for this arrangement of vortices. This equation expresses the transition probability
directly in terms of the vortex lattice, as specified by the set of points $\{R_i\}$.

This expression can be simplified considerably if we make the assumption that
all transverse locations $y$ are equivalent. Then we can assert that, since $P_r(q_r, y, z)$
doesn’t depend on $y$, we can replace it by its average value

$$P_r(q_r, z) = \frac{4|M|^2}{\nu^2} \left( \frac{\mu_B}{h} \right)^2 \frac{\phi_0^2}{4\pi^2 \lambda^4} \frac{1}{\Delta y} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dq_y \int_{-\infty}^{\infty} dq_y' \times \frac{e^{i(q_y - q_y') \cdot \mathbf{r}}}{\alpha \alpha'(q \coth(\alpha d/2) + \alpha)(q' \coth(\alpha' d/2) + \alpha')} \times \sum_{i,i'} e^{i \mathbf{q} \cdot \mathbf{R}_i}.$$ \hspace{1cm} (4.37)

$$= \frac{4|M|^2}{\nu^2} \left( \frac{\mu_B}{h} \right)^2 \frac{\phi_0^2}{2\pi \lambda^4} \frac{1}{\Delta y} \times \int_{-\infty}^{\infty} \frac{e^{-2qz}}{\alpha^2(q \coth(\alpha d/2) + \alpha)^2} \left| \sum_i \exp(i \mathbf{q} \cdot \mathbf{R}_i) \right|^2.$$ \hspace{1cm} (4.38)
since
\[ \int_{-\infty}^{\infty} dy e^{i(q_y - q'_y)y} = 2\pi \delta(q_y - q'_y). \]

We defined \( q' \) and \( \alpha' \) in the same manner as we did \( q \) and \( \alpha \), but using \( q'_y \) instead of \( q_y \). \( \Delta y \) is the transverse width of the vortex lattice.

We have two choices in how to proceed from here. First, we can specify a specific arrangement of vortices \( \{ R_i \} \) and crank out the transition probability numerically. This approach is equivalent to one in which we add up the fields of individual vortices in real space along the atom's path, Fourier transform to get \( B(q_x) \), and plug that result into Equation 4.14 to get \( P(v) \). The latter method is conceptually straightforward, and is in fact the basis of the Monte Carlo technique we will discuss in Section 4.4. The other way is to make a guess as to the functional form of the vortex lattice factor

\[ L(q) \equiv \left| \sum_i \exp(iq \cdot R_i) \right|^2 \]  

(4.39)

and try to work out an analytic expression for \( P(v) \). We will follow the analytic path in this section.

Let's examine \( L(q) \) in detail. We can immediately split \( L \) into two terms, one of which is entirely independent of the vortex lattice structure

\[ L(q) = \sum_{i,i'} \exp(iq \cdot (R_i - R_{i'})) \]

(4.40)

\[ = N + \sum_{i \neq i'} \exp(iq \cdot (R_i - R_{i'})), \]

(4.41)

where \( N = B \Delta x \Delta y / \phi_0 \) is the total number of vortices in the lattice of length \( \Delta x \) and width \( \Delta y \). We will let \( n = B / \phi_0 \) be the two-dimensional density of vortices.
The second term in the above expression is the Fourier transform of the vortex-vortex correlation function. To see how this is so, let’s make the assumption that the vortex lattice is homogeneous—that is, that each vortex will reside in a local environment that is very similar to the local environment of every other vortex. This means that we can replace the sum over \( i' \) in Equation 4.41 by a factor of \( N \). It also means that we should replace the sum over \( i \) by a two-dimensional integral over \( r \), weighted by a function \( f(r) \) that specifies the probability, given a vortex at the origin, of finding another vortex at \( r \). This is the definition of the vortex-vortex correlation function. Now, the vortex factor becomes

\[
L(q) = N \left[ 1 + \int d^2 r f(r) \exp(iq \cdot r) \right].
\]

I can rewrite

\[
f(r) \equiv \bar{f} + g(r),
\]

where \( \bar{f} = (\int d^2 r f(r))/\Delta x \Delta y \) is the average value of \( f \) and \( g \) averages to zero over the lattice. We will call \( g(r) \) the “non-uniform” correlation function, since it corresponds to the deviation of the correlation function from that of an ideal, uniform, uncorrelated lattice. Now, \( \int d^2 r \bar{f} \exp(iq \cdot r) = \bar{f} \delta^2(q) \), where \( \delta^2(q) \) is the two-dimensional Dirac delta-function. This term can be ignored for all finite atomic velocities, leaving us with

\[
L(q) = N \left[ 1 + \int d^2 r g(r) \exp(iq \cdot r) \right].
\]

The non-uniform correlation function \( g(r) \) must satisfy certain requirements: First, \( g(0) = -1 \), since two vortices cannot lie at the same location. Second, as we mentioned, \( g \) must have an average value of zero. Finally, we generally prefer that \( g \) vanish
far from the origin. This is mathematically convenient in that it allows the integral in Equation 4.44 to converge, but it also reflects the fact that no vortex lattice is perfectly ordered.

4.3.2 Excitation by an ideal gas of vortices

The simplest model for a vortex lattice is one in which there is no correlation at all! This is the case we will refer to as the “ideal vortex gas”. In such a gas, the vortices are distributed randomly, with a constant average density \( n = B/\phi_0 \). (The vortices are not, however, assumed to be moving randomly as well!) The ideal vortex gas has little resemblance to reality, since the mutual repulsion between real vortices prevents them from lying too close to one another.

The ideal gas vortex-vortex correlation function \( f(r) \) is constant (and equal to one) everywhere. Therefore, \( g(r) = 0 \), and

\[
P_e(q, z) = \frac{B}{\phi_0} \Delta x \frac{4|M|^2}{\nu^2} \left( \frac{\mu_B}{\hbar} \right)^2 \frac{\phi_0^2}{2\pi \lambda^4} \int_{-\infty}^{\infty} dq_0 \frac{e^{-2qz}}{\alpha^2(q coth(\alpha d/2) + \alpha)^2}.
\]

The transition probability for an ideal vortex gas increases linearly with the magnetic field (as long as we ignore the magnetic field dependence of the transition frequency \( \omega_0(B) \)), but its shape (as a function of velocity) only depends on the atom’s height above the sample surface and the material’s London penetration depth. Figure 4.8 shows the TDPT signal strength as a function of velocity for an atom passing 0.2 \( \mu \)m above the surface of a 0.1 \( \mu \)m thick superconductor with \( \lambda = 113 \) nm, at fields of 5 Gauss and 10 Gauss. The atom traveled a total distance of 1.5 mm.

The curves have their maxima at \( 4\pi \) times the height above the surface. This
Figure 4.8: Analytic results for an ideal vortex gas. This result is for an atom passing 0.2 μm above a 0.1 μm thick sample with London penetration depth $\lambda = 113$ nm for a distance of 1.5 mm, at fields of 5 and 10 Gauss.
can be explained by a combination of two effects: At the low velocities corresponding to short wavelengths, the magnetic field fluctuations decay very rapidly with height and are therefore very small if the wavelength $2\pi d < z$. At high velocities, the atom simply doesn't spend enough time above the lattice for the excitation probability to become large.

### 4.3.3 Isotropic correlation, the isotropic glass and the “powder” spectrum

The second simplest class of vortex lattices we can study analytically are the isotropic lattices, in which the correlation function $g(\mathbf{r}) = g(r) \neq 0$ is independent of the direction of $\mathbf{r}$, but is not zero. For such a lattice, we can further simplify the vortex factor $L(\mathbf{q})$ by converting the two-dimensional integral over $\mathbf{r}$ into a radial integral

$$L(\mathbf{q}) = N \left[ 1 + \int d^2g(\mathbf{r}) \exp(i\mathbf{q} \cdot \mathbf{r}) \right]$$

$$= N \left[ 1 + \int_0^{2\pi} d\phi \int_0^{\infty} r \, dr e^{iqr \cos(\phi)} \right]$$

$$= N \left[ 1 + 2\pi \int_0^{\infty} r \, dr J_0(qr) g(r) \right].$$

An isotropic lattice correlation function must satisfy three conditions: First, $g(0) = -1$, meaning that vortices cannot lie on top of one another. Second, $g \to 0$ as $r \to \infty$, meaning that the correlation extends over a limited distance. Finally, the average of $g$ must be zero, or

$$\int_0^{\infty} r \, dr g(r) = 0.$$

Depending on its microscopic structure, an isotropic lattice can be either “glass-like” or “powder-like”. An isotropic glass does not display strongly correlated be-
behavior with $r$, but allows for mutual repulsion of vortices. It represents a highly disordered lattice of interacting vortices, and avoids the unphysically close proximity of vortices seen in the ideal gas. Typically, we would expect a glass-like correlation function to have a peak at the nominal inter-vortex spacing and to die off quickly after that.

A simple glass-like non-uniform correlation function has the form

$$g_{gl} = a e^{-\alpha r} - b e^{-\beta r}.$$  

(4.49)

This clearly dies off at large $r$. We find that we can produce a peak at a nominal spacing of $r_0$ and satisfy the remaining two conditions on $g(r)$ by taking

$$b = a + 1,$$  

(4.50)

$$\alpha = \frac{3 \ln(1 + 1/a)}{2 r_0 \sqrt{1 + 1/a - 1}},$$  

(4.51)

$$\beta = \alpha \sqrt{1 + 1/a}.$$  

(4.52)

The remaining free parameter $a$ describes the strength of the correlation—generally, a larger value of $a$ means a higher, narrower peak in $g(r)$. Figure 4.9 shows $g(r)$ as a function of $r/r_0$ for values of $a = 0.005$ and $a = 5$. The limiting case as $a \to 0$ is

$$g(r) = \left\{ \begin{array}{ll} -1 & r = 0 \\ 0 & r > 0 \end{array} \right.,$$  

(4.53)

This is equivalent to $g = 0$, or the ideal gas limit.

Unfortunately, inserting this expression for $g(r)$ into the full expression for $P_r(v)$ resulted in a numerical integral that was too difficult to evaluate. However, we are still able to gain some insight into the behavior of the vortex glass by examining the
Figure 4.9: The non-uniform correlation function $g(r)$ for the isotropic vortex glass, for $a = 0.005$ and 5. $g(r)$ approaches the ideal gas limit of $g = 0$ as the parameter $a \to 0$. 
lattice factor $L(q)$. This factor becomes

$$L(q) = N \left[ 1 + 2\pi \int_0^\infty r \, dr (ae^{-\alpha r} - be^{-\beta r}) \right]$$

for the isotropic glass. Figure 4.10 plots the dimensionless correlation kernel $K(d) = L(2\pi/d)/N$ for four values of $\alpha$: 0.005, 0.05, 0.5, and 5. As we expect, the kernel approaches the ideal gas limit of $K = 1$ as $\alpha \to 0$. The effect of increasing the strength of the correlation is to remove an ever greater piece of the original uniform ideal gas kernel. This means that increasing the correlation strength should generally decrease the strength of the broad-band signal. This model does not account for the effects of periodic order on the signal at a specific vortex spacing, however. The extremely correlated limit ($\alpha \to \infty$), in the glass case, corresponds to a single delta-function ring at $r_0$, with the region with $r < r_0$ emptied of vortices and an ideal vortex gas in the region $r > 0$.

A good way to include periodicity in an analytic model is to simply add more rings corresponding to the locations of vortices in a periodic lattice. This idea is reminiscent of the powder-sample X-ray diffraction measurements, in which the sample is crystalline in small pieces, but the pieces are all randomly oriented with respect to one another. We will do the same thing in two-dimensions for what we will call the “powder” correlation function. When considering the correlation function of a powdered crystalline sample, one can be certain that, given a vortex at the origin, one will always find vortices at specific distances, but will have no idea of what direction in which to find them. Therefore, the powder correlation function $g(r)$ will be isotropic, consisting of delta-function rings located at the radii of all the lattice
Figure 4.10: The dimensionless correlation kernel for the isotropic vortex glass at four values of \( \alpha \).

Figure 4.10: The dimensionless correlation kernel for the isotropic vortex glass at four values of \( \alpha \). The ideal gas limit is a constant value of 1, when \( \alpha = 0 \).

Vectors of the Abrikosov lattice out to some maximum crystallite radius. Beyond this distance, the non-uniform correlation function will return to the ideal gas value of 0.

Figure 4.11 shows how we construct the rings for the isotropic powder correlation function. The \( i \)th ring out contains \( N_i \) vortices at a distance \( R_i \) from the origin. The vortex lattice can be divided into polygonal zones by linking the vortices of the \( i \)th ring with straight lines. The area of the \( i \)th zone \( A_i \) is the area between the \( i \)th and \((i - 1)\)st polygons. Table 4.1 gives the values of \( N_i, A_i, \) and \( R_i \) for lattice vectors out to five lattice spacings from the origin.

Now we can write out the form of the non-uniform correlation function

\[
g(r) = \sum_{i=1}^{i_{\text{max}}} \frac{N_i}{A_i} r \delta(r - R_i) - \begin{cases} 
1 & r \leq R_{i_{\text{max}}} \\
0 & r > R_{i_{\text{max}}} 
\end{cases}, \tag{4.55}
\]
Figure 4.11: The vortex ring construction for a triangular vortex lattice. The rings lie at the radii of the lattice vectors. Each successive polygon encloses a new lattice zone. Beyond a maximum radius, the lattice becomes uncorrelated.
Table 4.1: Radii, occupation number, and occupied area for Abrikosov lattice vectors and zones out to five lattice spacing from the origin in real space. The \( i \)th Lattice vector occurs \( R_i \) lattice spacings from the origin. A total of \( N_i \) vortices lie at this distance. The zone between the \( i \)th and \((i - 1)\)st polygon has an area \( A_i \).
where \( R_{r_{\text{max}}} \) is the radius of the last ring before the correlation function becomes uniform. The correlation kernel becomes

\[
K(q) = \int_0^\infty r \, dr \, g(r) J_0(qr)
\]

\[
= 1 + 2\pi \left[ \sum_{i=1}^{r_{\text{max}}} \frac{N_i}{A_i} J_0(q R_i) - \int_0^\infty r \, dr \, J_0(qr) \right]
\]

\[
= 1 + 2\pi \left[ \sum_{i=1}^{r_{\text{max}}} \frac{N_i}{A_i} J_0(q R_i) - \frac{R_{r_{\text{max}}}}{q} J_1(R_{r_{\text{max}}} q) \right]
\]

integral evaluated numerically. The results are shown in Figure 4.12 for 5 rings (out to three lattice spacings) and 11 rings (out to five lattice spacings), for a 0.1 \( \mu \)m thick film with penetration depth \( \lambda = 100 \) nm in an applied field of 20 Gauss \( (r_0 = 1.02 \mu \text{m}) \). Both curves indicate a strong peak at \( r = r_0 \), with additional structure at \( r_0/2 \) and \( 2r_0 \). The structure becomes more complicated as we add more rings, but the overall size and shape of the excitation signal is preserved.

### 4.3.4 The hexatic vortex glass

A vortex structure that is commonly reported in Bitter decoration experiments performed on High-\( T_c \) superconductors is the hexatic glass phase\[13, 28, 5, 6\]. The hexatic vortex glass is a disordered arrangement of vortices that retains long-range orientational order while keeping only short-range positional order. Such an arrangement is clearly not isotropic, and so the formalism we have employed in the previous two cases will no longer suffice.
Figure 4.12: The excitation signal in TDPT for the "powder" correlation function, at \( B = 20 \) Gauss, \( \lambda = 100 \) nm, and a film thickness of \( 0.1 \) \( \mu m \). The two curves represent correlation rings for a triangular lattice extending to 3 and 5 lattice spacings, respectively. The lattice spacing is \( r_0 = 1.02 \) \( \mu m \).
Instead, we will have to include hexagonal symmetry in our correlation function. We can put this into a form similar to the form we used to describe the isotropic glass:

\[ g(r, \theta) = ar \cos^2[3(\theta - \theta_0)]e^{-\alpha r} - e^{-ibr}, \]  

(4.59)

where \( \theta_0 \) specifies the orientation of the hexagonal symmetry. This function must satisfy the same conditions as \( g(r) \) did for the isotropic glass, and we would like the peak correlation to occur at \( r = r_0 \). These conditions are met by taking

\[ \beta = \frac{\alpha^{3/2}}{a^{1/2}} \]  

(4.60)

and then solving

\[ \beta e^{-br_0} + ae^{-\alpha r_0} - \alpha r_0 e^{-\alpha r_0} = 0 \]  

(4.61)

numerically for \( \alpha r_0 \) as a function of \( a \).

The correlation kernel can now be found to be

\[ K(q, \theta_0) = 1 + 2\pi \int_0^\infty r dr \left\{ -e^{-br}J_0(qr) + \frac{rae^{-\alpha r}}{2} \left[ J_0(qr) - \cos(6\theta_0)J_0(qr) \right] \right\}. \]  

(4.62)

The presence of the Bessel function \( J_0(qr) \) is a direct effect of the hexagonal symmetry.

Once again, since this kernel does not exist in closed form, we did not evaluate the transition probability as a function of velocity. Figure 4.13 shows the hexatic kernel for \( a = 1 \), at orientation angles \( \theta_0 = 0, \pi/12, \) and \( \pi/6 \). Figure 4.14 shows the same results at \( a = 0.5 \). These results contrast with the isotropic glass results of Figure 4.10 in that the kernel can now rise above one for \( \theta_0 = \pi/3 \), which corresponds
Figure 4.13: Correlation kernel $K(r = 2\pi/q, \theta_0)$ for hexatic vortex glass with $a = 1.0$. The kernel was evaluated at three orientation angles $\theta_0 = 0, \pi/6, \text{and } \pi/3$.

to the [11] direction of the lattice. There is a clear dependence on direction, with the $\theta_0 = 0$ curve lying lowest. The parameter $a$ no longer has the same meaning as it did in the isotropic glass. Instead, increasing $a$ reduces the kernel's excursion from the ideal gas constant value of 1. A larger value of $a$ also compresses the kernel toward shorter $r$. The kernel shares the same limits of $K(r, \theta_0 \to 1$ at $r = 0$ and $r \to \infty$.

The single common feature of all the analytic models we've discussed is that the excitation probability was always proportional to the magnetic field. Although we have made no attempt to directly compare the analytic results to experiment, we might expect some difficulty in reproducing the experimental results in which the signal strength rapidly decreased as the applied field increased above the vicinity of
Figure 4.14: Correlation kernel $K(r = 2\pi/q, \theta_0)$ for hexatic vortex glass with $a = 0.5$.

40 Gauss.

### 4.4 Monte Carlo Simulations

So far, we have attempted to model the behavior of an atom passing over a vortex lattice analytically, beginning by assuming a particular functional form for the vortex lattice correlation function and working back toward the excitation probability. The insight we’ve gained into the interpretation of our experimental results has necessarily been limited by the tractability of our model correlation functions.

An alternative approach that we hope will yield a better understanding of the experiment is to begin with a trial lattice, calculate the magnetic field directly, and then calculate the transition probability directly using the formalism outlined in Sec-
tions 4.1 and 4.2. This technique has the advantage that we can use any model lattice we like to perform the calculation. We can also include some of the more detailed systematics of our experimental setup that we had to leave out of the analytic models. The disadvantage is that we have to begin the process by creating a realistic vortex lattice from scratch, as it were, rather than simply picking a likely analytic form for the correlation function. We will find that the Monte Carlo approach is nevertheless quite powerful.

4.4.1 The basis of the Monte Carlo technique

The basic Monte Carlo technique begins with a set of two-dimensional points corresponding to the locations of vortices at the superconductor surface. We then calculate the magnetic field from each of the vortices at 1024 equally spaced points along a path that passes above the surface. In Appendix C we show that the $z$-component of the magnetic field at a point a height $z$ above and distance $r$ from the axis of a vortex at the surface of a superconductor is (Equation C.29):

$$B_z(r,z) = \frac{\phi_0}{2\pi\lambda^2} \int_0^\infty q dq \frac{J_0(qr) \exp(-qz)}{\alpha(q \coth(\alpha d/2) + \alpha)}.$$

In practice, the numerical integral makes this very time-consuming to compute, so we cheat by computing the field beforehand over a fine mesh of points in the $r$-$z$ plane and then use linear interpolation to calculate the field at specific locations. The mesh extends to a radius of 10 $\mu$m from the vortex axis and from $z = 0.01$ $\mu$m to 2.0 $\mu$m. For points lying outside this radius, we use a different cheat: Far from the point where the vortex core emerges from the sample surface, the vortex field closely
resembles the field of magnetic monopole of charge \( g = \phi_0/2\pi \) located a distance \( \delta z(\lambda) \) below the surface. We’ve found that in the range of penetration depths 30 nm < \( \lambda < 300 \) nm, \( \delta z \) satisfies the empirical formula

\[
\delta z(\text{nm}) \approx 0.200 (\lambda(\text{nm}))^{1.451} + 7.717.
\]  

(4.63)

as determined by least-squares fitting. This expression for the field enables us to quickly calculate the fields at large distances from the core, which helps reduce the error of the simulation at high fields.

In general, the path is parallel to the \( x \)-axis horizontally, but tilted vertically so that it leaves the vicinity of the sample at a different height above the surface than the height at which it entered. Rather than compute the fields of every vortex in the sample for each point along the trajectory, we instead ignore all the vortices outside of a window centered on the atom’s location. This window is typically 18 \( \mu \text{m} \) long (along the vortex path) by 6 \( \mu \text{m} \) wide. The error introduced by this windowing is quite small for low fields, but becomes bothersome at higher fields.

Once we have calculated the field \( B(x) \) along such a path, we take its fast-Fourier transform to get the function \( B(q) \), which we then insert into Equation 4.14 to get the transition probability \( P_{TDPT} \) in time-dependent perturbation theory. If we like, we can now “saturate” this probability using either the two-level rate-equation approximation of Equation 4.27 or the eight-level rate equation approximation of Equation 4.30.

We now repeat this process over a large number of paths (usually 1000), randomly selecting the transverse location, entrance height, and exit height each time. This
allowance for tilted trajectories is one of the systematics we did not include in our analytic models. Finally, we compute the average transition probability as a function of velocity.

A second systematic that we need to consider when attempting to compare the Monte Carlo results to experiment is the tendency of potassium atoms to stick to metal surfaces if they get too close. (This potassium eventually closes up the gap between sample and gate entirely, which is the primary means by which samples die!) The attraction between an alkali atom and a metal surface is caused by the van der Waals interaction, which has the form

\[ E_{VDW} = -\frac{C_3}{z^3}, \tag{4.64} \]

where \( z \) is the height above the surface and \( C_3 \) is the van der Waals coefficient for the particular atom/metal system. The value of \( C_3 \) can be computed from the plasma frequency of the metal and the energies and oscillator strengths of the low-lying electronic states of the atom. For potassium on niobium, the value is \( C_3 = 13.34 \times 10^{-40} \text{ Jm}^3 \).

The atom’s height as a function of time satisfies

\[ \frac{d^2z}{dt^2} = -\frac{3C_3}{mz^4}, \tag{4.65} \]

where \( m = 6.47 \times 10^{-26} \text{ kg} \) is the mass of a \(^{39}\text{K} \) atom. Initially, \( z(t = 0) = z_i \) is the atom’s entrance height, and \( v_x(0) = v(z_i - z_f)/\ell \), where \( z_f \) would have been the atom’s exit height in the absence of the van der Waals interaction, and \( \ell \) is the length of the sample. We can solve this differential equation numerically for a given \( z_i, z_f, \) and \( v \)
to determine how long the atom will take to hit the surface \((t\) for which \(z(t) = 0\)). If \(t\) is less than \(L/v\), the time it takes the atom to cross the surface horizontally, then the atom sticks.

Our procedure is to determine, for each of a large set of initial and final heights, the particular cutoff velocity \(v_c(z_i, z_f)\) below which atoms traveling along that trajectory will stick to the surface. Then, when computing the average transition probability \(P_r(v)\) for a set of trajectories, we include in the average for each trajectory only those velocities above \(v_c\). The trajectories themselves are not modified. Figure 4.15 illustrates the dependence of \(v_c\) on \(z_i\) and \(z_f\). The cutoff velocities are lowest where one or both ends of the trajectory come close to the surface. The effect of the van der Waals interaction is to remove atoms close to the sample (those with the largest transition probabilities), especially at the low velocities corresponding to short vortex spacings. This effect can severely limit our ability to work at short lattice spacings. The best way to overcome this difficulty is to use a shorter sample.

4.4.2 Monte Carlo for an ideal gas—comparison to analytic results

We are fortunate that there is one class of vortex lattice for which we know both the functional form of the correlation function and the arrangement of vortices. This is the ideal vortex gas of Section 4.3.2. The vortices in an ideal gas are located randomly with no correlation whatsoever. Therefore, we can generate an ideal vortex gas on a rectangular surface by using a random number generator to choose the \(x\)- and \(y\)-coordinates of the positions of as many vortices are needed to generate an average
Figure 4.15: Cutoff velocities for van der Waals interaction above the sample. Atoms entering the sample at height $z_i$ and leaving (before the interaction is considered) at $z_f$ will stick to the surface if traveling slower than $v_c$. The van der Waals coefficient $C_3 = 13.34 \times 10^{-49}$ Jm$^3$. The sample is 1.5 mm long. The contour lines are labelled by the cutoff velocity in m/s.
Figure 4.16: Monte Carlo results for an ideal vortex gas. The results of the Monte Carlo (rough curves) are compared to the analytic model (smooth curves). The results are TDPT at fields of 5 and 10 Gauss, at a fixed height of 0.2 μm above the sample, for 1.5 mm length, with $\lambda = 113$ nm.

Field $B$: that is, $N = B/\phi_0 A$, where $A$ is the area of the rectangle. We have done this for fields of 5 and 10 Gauss and calculated the Monte Carlo signal in TDPT (no saturation). Figure 4.16 shows the results, and compares them to the results of the analytic model. 1000 trajectories were averaged, all at a constant height of 0.2 μm, without the van der Waals cutoff. The magnetic penetration depth was $\lambda = 113$ nm.

The Monte Carlo and analytic models agree perfectly. We can conclude that the Monte Carlo simulation will return an accurate prediction for the excitation signal of an atom passing over a specific vortex lattice, to the extent that our approximations (TDPT or rate-equation) for the excitation process itself remain valid.
4.4.3 Hexatic correlation in the Monte Carlo model

Under most conditions, experimentally observed vortex lattices have resembled an equilaterally triangular lattice with some degree of disorder. This disorder is cumulative, meaning that the correlation of the lattice, no matter how strong at short distances, eventually dies away over long distances. In many cases (in particular, the hexatic glasses observed in High-$T_c$ materials), the lattice’s orientational order has a much longer range than its positional order, meaning that most vortices’ nearest neighbors tend to lie in the same direction over large areas of the sample, but that locally the spacing between nearest neighbors varies significantly from vortex to vortex.

We have found it fairly easy to mimic this behavior when generating a trial vortex lattice to use in the Monte Carlo simulation. The technique is simple. First, we start with a single vortex in the lower left corner $(x_{11}, y_{11})$ of the rectangle we intend to tile with vortices. We then begin building a vortex row in the $x$-direction by finding the location one lattice spacing $a$ to the right $(x_{11} + a, y_{11})$. We then add a Gaussian deviate[30, pg.288] of full-width at half-maximum (FWHM) $\delta a$ to determine the location of the second vortex:

$$
\begin{align*}
    x_{21} &= x_{11} + a + G(\delta a) \\
    y_{21} &= y_{11} + G(\delta a),
\end{align*}
$$

(4.66)

where $G(x)$ is a Gaussian deviate of FWHM $x$. That is, $G(x)$ is generated pseudo-
randomly so that the probability that $G(x) = y$ is

$$p(y) dy = \frac{1}{\sqrt{2\pi}} e^{-y^2 / 2x^2} dy.$$  \hfill (4.67)

We fill out the remainder of the bottom row in the same fashion, with

\[
\begin{align*}
x_{n1} &= x_{n-11} + a + G(\delta a) \\
y_{n1} &= y_{n-11} + G(\delta a).
\end{align*}
\hfill (4.68)
\]

Now we begin the second row by choosing $(x_{12}, y_{12})$ to be a Gaussian deviate away from a point a single lattice spacing along a 60 degree line up and to the right from the original point:

\[
\begin{align*}
x_{12} &= x_{11} + a/2 + G(\delta a) \\
y_{12} &= y_{11} + \frac{\sqrt{3}a}{2} + G(\delta a).
\end{align*}
\hfill (4.69)
\]

We complete the second row by finding a position midway between one lattice spacing directly right of each vortex's left-hand neighbor, and one lattice spacing 60 degrees up and to the right of its "downstairs" neighbor. That is,

\[
\begin{align*}
x_{n2} &= x_{n-12} + x_{n1} + \frac{3a}{4} + G(\delta a) \\
y_{n2} &= y_{n-12} + y_{n1} + \frac{\sqrt{3}a}{4} + G(\delta a).
\end{align*}
\hfill (4.70)
\]

In general,

\[
\begin{align*}
x_{nm} &= x_{n-1m} + (-1)^m x_{n,m-1} + \frac{3a}{4} + G(\delta a) \\
y_{nm} &= y_{n-1m} + y_{n,m-1} + \frac{\sqrt{3}a}{4} + G(\delta a),
\end{align*}
\hfill (4.71)
\]
where we remember to start odd-numbered rows out 60 degrees up and to the left of the start of the row below.

The result is a vortex lattice that is continually prevented from becoming a perfect hexagonal crystal by the random deviations added to the position of each vortex. These deviations are cumulative, so that there is no long range positional order. But each vortex’s nearest neighbors lie roughly in a hexagon with one point directly to the right. This “fake” hexatic vortex glass differs from that seen in nature in the way the disorder is characterized. Bishop et al.[5] point out that the hexatic glass in Bi2Sr1.9Ca0.9Cu2O8+δ (BSSCCO) is disordered by the presence of dislocation pairs with opposite Burger’s vectors, the frequency of which increases as the vortices spread farther apart in weaker magnetic fields. Such a dislocation pair consists of a vortex with seven nearest neighbors next to another vortex with only five nearest neighbors. Our method of simulating an hexatic glass, on the other hand, mandates that all the vortices have six nearest neighbors.

Figure 4.17 shows sections of four vortex lattices that each produce an average field of 10 Gauss. The four cases are an ideal gas, a highly disordered hexatic glass, a weakly disordered hexatic glass, and a perfect triangular crystal. In the $ba/a = 7\%$ case, the hexagonal orientational symmetry is clearly visible, while the positional order quickly decays with distance. Both the 7% and 15% lattices bear a strong visual resemblance to the decoration patterns reported in BSSCCO[14] at strong and weak fields, respectively.

Figure 4.18 shows the results of running the Monte Carlo simulation on each of
Figure 4.17: Hexatic vortex lattices with varying disorder at $B = 10$ Gauss. The lattices range from an ideal gas $\delta a/a = \infty$ through two intermediate levels of disorder $\delta a/a = .15$ and $\delta a/a = .07$ to a perfect crystal $\delta a/a = 0$. 

---

**ideal gas**

$\delta a/a = 15%$

$\delta a/a = 7%$

$\delta a/a = 0$
the four lattices in Figure 4.17. These runs were all performed at a penetration depth of $\lambda = 113$ nm for a 1.5 mm long lattice. We included eight-level saturation and van der Walls cutoff velocities and considered 1000 randomly selected trajectories whose entrance and exit heights were between 0.05 $\mu$m and 1.8 $\mu$m above the sample surface. This represents the most complete simulation of our actual experimental situation that we are able to perform at this time. We believe that these simulation results are of sufficient quality to compare directly to experimental data.

The results of the ideal gas and perfect crystal simulations clearly do not resemble our experimental data. The ideal gas excitation signal continues to increase all the way out to the highest velocities we calculated, and remains featureless. The perfect crystal excitation signal consists of a set of sharp peaks centered on successive fractions of one lattice spacing (that is, at $a$, $a/2$, $a/3$, ... ) corresponding to higher harmonics of the fundamental period of the lattice. The signal quickly falls to zero (as a Lorentzian) at longer vortex spacings.

The results for the two hexatic glasses resemble the experimental results much more closely. The better visual fit appears to be the $\delta a/a = 15\%$ run. The effects of decreasing the degree of disorder in the lattice appear to be two-fold: First, a peak appears at the lattice spacing $a$ (1.44 $\mu$m in this case) and becomes narrower and taller. Additional peaks also begin to appear at the higher harmonics. Second, the broad signal at long vortex spacings becomes smaller as the order increases. These long-wavelength field fluctuations are caused by long-range fluctuations in the vortex density, and therefore get smaller as the lattice’s long-range order increases.
Figure 4.18: Full Monte Carlo simulations run using the four 10 Gauss lattices shown in Figure 4.17: (a) ideal gas, (b) δa/a = 15%, (c) δa/a = 7%, (d) perfect crystal. These runs were taken at a penetration depth of λ = 113 nm, by averaging 1000 tilted trajectories and including eight-level saturation and van der Waals cutoff velocities.
We can get a sense for this by visually examining the lattices themselves: the ideal gas lattice is characterized by large regions virtually devoid of vortices as well as by densely packed regions. In the 15% and 7% glasses, the empty areas are much smaller, and the density of vortices becomes much more uniform. The perfect crystal is absolutely uniform—the only fluctuations in its vortex density are delta-functions spaced periodically with period $a$.

Our data are not consistent with either an ideal gas or a perfect crystalline lattice. They are consistent with the hexatic vortex glass simulations that we have presented here, and in particular support a disorder parameter of $\delta a/a \approx 15\%$. However, we have no way of determining the degree of orientational symmetry of the arrangement of vortices in our sample from data taken along a single axis. Our vortex lattices could just as well be isotropic glasses or “powders” consisting of regions of hexatic or even rectangular symmetry with random orientation. Our data do not, however, seem to support the notion of a powdered sample of randomly-oriented perfect crystallites. We will proceed under the assumption that a hexatic vortex glass, though not necessarily correct, is nevertheless a sufficiently good description of our experimental system to yield meaningful results that can be compared to experiment.

4.5 Measurement of Magnetic Penetration Depth

Since the strength of the excitation signal depends strongly on the temperature-dependent magnetic penetration depth, we are now capable of estimating the temperature-dependence of the penetration depth by comparing the results of our full Monte Carlo simulation to the experimental data for the decay of signal strength.
with temperature we presented in Section 3.3.

We will begin by running the full Monte Carlo simulation on a vortex lattice at the same magnetic field that we took our measurements (8.9 Gauss) and $\delta a/a = 15\%$ disorder for a wide range of penetration depths. We will then try to extract a simple empirical formula for the signal strength as a function of penetration depth. Figure 4.19 shows the signal strength $P_c(d = 1.52 \mu m)$ as a function of $\lambda$ at 8.9 Gauss, as calculated using the full Monte Carlo simulation. We least-squares fit this to the form

$$ P_c = \frac{a^4}{(\lambda^2 + b^2)^2}, \quad (4.72) $$

where $a$ and $b$ are fitting parameters. The solid curve displays the fit, with $a = 129$ nm and $b = 227$ nm.

To compare the simulations to our data, we will make use of the fact that $\lambda(T)$ in niobium obeys the empirical "two-fluid" approximation

$$ \lambda(T) = \lambda(0) \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-1/2}, \quad (4.73) $$

quite well, where $\lambda(0)$ is the bulk zero-temperature penetration depth[21]. The commonly accepted value for $\lambda(0)$ in pure niobium metal is $\lambda(0) = 44$ nm[1], while microstrip resonator measurements on a sputtered thin film[23] yielded a higher value of $\lambda(0) = 86.0$ nm. We performed a least-squares fit of our data to this form of $\lambda(T)$, using the form of Equation 4.72 for the excitation strength. The zero-temperature penetration depth $\lambda(0)$ was one free parameter, the overall height of the signal was another. Figure 4.20 shows the results of the fit. Our best fit value for the zero-temperature bulk penetration depth is $\lambda(0) = 142 \pm 13$ nm. The scaling factor for
Figure 4.19: Monte Carlo simulation dependence of signal on penetration depth. A full Monte Carlo simulation was carried out for a hexatic lattice with 20% disorder at 8.9 Gauss at the penetration depths indicated by the boxes. The solid line is a least-squares fit to these results.
the overall height was 1.04. This value was sufficiently close to unity to give us confidence that we can make absolute penetration depth measurements—that is, direct measurements of \( \lambda(T) \) without any free parameters.

The data points showing the measured penetration depth in Figure 4.20 were obtained by dividing the raw signal strength data by the scale factor of 1.04 and then inverting the relation of Equation 4.72 to get the penetration depth. This measurement gives us almost knowledge of the penetration depth. The ability to measure the temperature dependence of the penetration depth without needing a reference measurement carried out by another method is a major advantage of the atomic beam technique.

Because of saturation, the exact dependence of signal strength on penetration depth may depend on our choice of the set of tilted trajectories that we include in our average over the sample surface. In particular, if atoms are prevented by some obstruction from passing close to the surface, the fraction of fully-saturated atoms will decrease. This may affect not only the size of the signal, but its temperature dependence as well, possibly leading to an erroneously large estimate of the penetration depth. This possibility is worth investigating since potassium is deposited on the sample as the experiment runs, gradually building up a layer that blocks atoms from passing near the surface. Eventually, the potassium fully obstructs the sample slit.

We believe that a significant amount of potassium was deposited between the time that we took the original 10 Gauss runs at 9:44 pm and when we took the temperature-dependence runs at 10 Gauss between 11:17 and 11:40 pm. The sample
Figure 4.20: (a) Fit of numerical results for temperature dependence of signal to experimental data. The boxes are experimental data, while the curve is the fit. The best fit value for the zero-temperature bulk penetration depth was $\lambda(0) = 142 \pm 13$ nm. (b) The solid curve shows the temperature dependence of $\lambda(T)$ using $\lambda(0) = 142$ nm. The boxes are absolute experimental measurements obtained by inverting the empirical relationship between $P_e$ and $\lambda$. The dashed curves show the statistical error in $\lambda(0)$. 
Figure 4.21: Effect of potassium deposition on 8.9 Gauss data at 5 K.

was exposed to the potassium source for at least 50 minutes during this period. Figure 4.21 compares the run taken at 9:44 pm to the run taken at 11:26 pm under identical conditions (10 Gauss field-cooled, at 5 K). The signal from the 11:26 pm run is about 30% smaller than the 9:44 pm signal, under identical conditions.

With this in mind, we performed a second set of calculations in which we raised the lower bound on the height of the atomic trajectories from 0.05 μm to 0.2 μm. When we analyzed the results of this run, we found a zero-temperature bulk penetration depth of $\lambda(0) = 139 \pm 13$ nm, which is not significantly different from the value we obtained when we included the entire width of the gate.

The one remaining question concerns the effect of varying amounts of disorder on the estimated magnetic penetration depth. We ran two additional series of calcula-
tions at disorders of \( ba/a = 10\% \) and 20\%. The 10\% run yielded a zero-temperature bulk penetration depth of \( \lambda(0) = 145 \pm 13 \) nm; the 20\% run yielded \( \lambda(0) = 143 \pm 13 \) nm. These results also do not differ significantly from the 15\% result. We conclude that our data on the temperature-dependence of the signal strength are consistent with a value of \( \lambda(0) = 142 \pm 13 \) nm, and that this value can be used with impunity in our Monte Carlo simulation to predict the excitation signals for a variety of vortex lattices.

This value of \( \lambda(0) \) is over three times larger than the accepted value of 44 nm for bulk niobium\[1\]. However, Langely \textit{et al.} report a measurement of \( \lambda(0) = 86 \) nm in a sputtered niobium film, using a microstrip resonator technique to measure the temperature-dependence of \( \lambda \). We plan to carry out corroborative penetration depth measurements on sputtered films similar to our samples using a different technique.

We can use this result for the penetration depth to estimate the mean-free-path \( \ell \) of the electrons in our material. BCS theory\[34\] gives the result

\[
\lambda_{eff}(\ell, T) = \lambda_L(T) \left(1 + \frac{\xi_0}{\ell}\right)^{1/2}
\]

for the effective penetration depth in the dirty limit \( (\ell \ll \xi_0) \). For niobium, \( \xi_0 = 38 \) nm\[1\], so we get an estimate for the mean-free-path of \( \ell = 4 \) nm. This is approximately equal to our estimate of the grain size in sputtered niobium films.

To corroborate these penetration depth measurements, we recently (August, 1994) performed a two-coil relative penetration depth measurement on a sputtered niobium film grown for this purpose. The two-coil measurement was performed by Eric Ulm using apparatus of his construction\[35\]. The result of the measurement was an ex-
traooplated value of $\lambda(0) = 140 \pm 20$ nm, in perfect agreement with our results.

This measurement lends strong support to our conclusion that we are, in fact, measuring the absolute penetration depth of our sample. Since this technique potentially has important applications in the field of penetration depth measurement, we should briefly examine the technique in the context of the techniques already in existence.

There are two basic types of penetration depth measurement—relative and absolute. Relative penetration depth measurement techniques include the two-coil technique, micro-strip resonator techniques[23], small-angle neutron diffraction[41], and others. These techniques are all subject to uncertainty about the absolute magnitude of the penetration depth being measured. This difficulty is ordinarily worked around by assuming a particular functional form for the temperature-dependence of $\lambda$ and then performing a fit of this form to the data. This method of analysis works well when one is certain of the functional form in question, but becomes more problematic when one tries to experimentally determine the functional form itself. Unfortunately, the goal of much of the work currently under way in high-$T_c$ superconductivity research is to do just that, and from the result, to infer the symmetry of the order parameter (i.e. whether $s$-wave or $d$-wave models of high-$T_c$ superconductivity are more accurate).

The alternative is to attempt to measure the penetration depth absolutely, without any recourse to theoretical models of its temperature-dependence. The most successful method of performing absolute penetration depth measurements is muon spin rotation[19] ($\mu$SR). In $\mu$SR, a beam of energetic spin-polarized positive muons
are stopped in a superconducting sample where they decay, emitting a positron and two neutrinos. The positron is emitted preferentially along their final spin polarization direction. The muon spin polarization axis precesses in a known fashion in the magnetic field, so by determining the emission direction of the positron and measuring the time elapsed before it was emitted, one can learn about the magnitude of the magnetic field inside the superconductor at the point where the muon came to rest. In a uniform magnetic field, the muons would all precess in phase and the distribution of emitted positrons would be very asymmetric. In contrast, a sample containing a vortex lattice with spacing large compared to the penetration depth will quickly begin to emit positrons symmetrically, as the muons in different magnetic fields rotate out of phase with one another. By analyzing the decay of asymmetry in the distribution of emitted positrons over time, one can extract the absolute penetration depth of the material. This technique can achieve powerful results, but is difficult, time-consuming, and expensive.

A second, direct method for measuring the penetration depth is through low-field DC magnetization measurements[22]. In this method, a bulk sample is first cooled in zero field and then slowly warmed up in the presence of a small applied field, typically around 1 Oe. The negative DC magnetic susceptibility is seen to decrease as the temperature rises, indicating a deeper penetration of the field into the sample. This technique is quite capable of extracting the absolute penetration depth in the Meissner state, but can only be applied to bulk samples large enough to have a measurable macroscopic magnetic moment.
Of all the techniques we have mentioned, the ABMR penetration depth measurements most closely resemble neutron diffraction. In both techniques, the magnitude of the signal decreases in a well-defined way with increasing penetration depth. The crucial difference between the techniques is that, in ABMR, we are able to calculate the expected size of our excitation signal ahead of time. In neutron scattering, on the other hand, one can only determine the fractional change in scattering intensity (the form factor) for a given change in the penetration depth. Therefore, neutron scattering is only capable of relative penetration depth measurements, while ABMR is capable of absolute measurements.

The ABMR technique differs from $\mu$SR in that, while $\mu$SR extracts information from the overall variation of magnetic field within the sample, ABMR determines the magnitude of the fluctuations as a function of scale length. This may result in a more complete description of the magnetic field near the surface of the material than the description of the field inside the material provided by $\mu$SR measurements. In practical terms, our experiment is much simpler, faster, and cheaper than the $\mu$SR experiment, and does not require access to a particle accelerator.

### 4.6 The Dependence of Signal on Magnetic Field

The remaining test of our Monte Carlo method is to compare its predictions for the size and shape of the excitation signal to experiment at a variety of magnetic fields. We ran a series of simulations in which we varied the lattice spacing and disorder of hexatic glasses. We then compared the results directly to experimental data taken at the same field.
We used the nominal intervortex spacing \( a = \sqrt{\phi_0/B} \) as the lattice spacing for several of the magnetic fields \( B \) we used in the experiment. For each field, we ran simulations with disorder parameter values of \( \delta a/a = 7\%, 10\%, \) and \( 15\% \). We then chose the value of \( \delta a/a \) for which the Monte Carlo results most resembled the experimental data. In all cases, we used the experimentally determined value of \( \lambda(0) = 142 \) nm. We used the "full" Monte Carlo simulation method, with van der Waals cutoffs imposed on a \( 0.5 \mu m \) to \( 1.8 \mu m \) height window.

Figures 4.22 through 4.30 show the results of these calculations and compare them directly to experiment. In the top graph of each figure, the solid line shows the calculation while the dashed line shows the experimental data taken at that field. The lower graph shows the Monte Carlo result for the full velocity range we calculated (50 m/s to 2000 m/s). There are no free parameters beyond the choice of \( \delta a/a \).

There is one source of error in the numerical results that needs to be discussed in more detail. At higher magnetic fields, the Monte Carlo signal at low fields does not approach zero exponentially as predicted by the analytic theory, but instead remains finite or even diverges as \( v \) approaches zero. This error, although annoying, is easy to understand and in principle correct.

The error is caused by the measures we took to reduce the number of vortices for which we calculate the magnetic fields at each point along an atomic trajectory. Rather than calculate the field contributed by each vortex in the entire lattice, we instead calculated the fields produced just by vortices within a \( 60 \mu m \) long by \( 20 \mu m \) wide window centered on the atom's position, ignoring the contributions of the rest
Figure 4.22: Monte Carlo results at $B = 0.9$ Gauss, with $\delta a/a = 10\%$. For the next several figures, the top graph will directly compare the Monte Carlo results to the experimental data taken at the same magnetic field. The dashed line shows the experimental data. The bottom figure shows the Monte Carlo result for the full range of vortex spacings that we calculated. There are no free parameters.
Figure 4.23: Monte Carlo results at $B = 1.9$ Gauss, with $\delta a/a = 15\%$. 
Figure 4.24: Monte Carlo results at $B = 4.5$ Gauss, with $\delta a/a = 15\%$. 
Figure 4.25: Monte Carlo results at $B = 8.9$ Gauss, with $\delta a/a = 15\%$. 
Figure 4.26: Monte Carlo results at $B = 13.4$ Gauss, with $\delta a/a = 15\%$. 
Figure 4.27: Monte Carlo results at $B = 17.8$ Gauss, with $\delta a/a = 10%$. 
Figure 4.28: Monte Carlo results at $B = 26.7$ Gauss, with $\delta a/a = 10\%$. 
Figure 4.29: Monte Carlo results at $B = 35.6$ Gauss, with $\delta a/a = 7\%$. 
Figure 4.30: Monte Carlo results at $B = 44.5$ Gauss, with $\delta a/a = 5\%$. 
of the lattice. As the atom moves along the lattice, therefore, new vortices will enter this window and old vortices will leave as the window passes over them. This process will cause small but sudden changes in the total field at the atom’s position, which will eventually lead to a broad-band signal at low velocities in the simulation. The problem gets worse as either the disorder or the number of vortices increases. The fluctuations are most bothersome when we include saturation due to the fact that their effect on each atom is small but is caused by vortices located a large horizontal distance from the atom. The effect therefore doesn’t decay rapidly with height. The signals don’t saturate, and are present even for atoms which are not excited by the vortex fields.

The simplest solution to the problem is to include every vortex field at every point along the atoms’ trajectories. This solution rapidly becomes prohibitive in terms of computer time, since the number of vortices increases as the magnetic field squared. In order to test this hypothesis, however, we did perform one run in which we included all the vortices in a much larger (200 μm by 200 μm) window. Figure 1.01 compares the results of Monte Carlo simulations done at $B = 89$ Gauss and $\delta a/a = 15\%$ with the normal window to results obtained under identical conditions but using the larger window. The large-window signal is clearly smaller at low velocities, but matches the small-window signal at high velocities.

When we choose the appropriate amount of disorder for different magnetic fields, the theory fits the experiment remarkably well. This is especially true in the range of magnetic fields from 4.5 Gauss to 17.8 Gauss. The size, location, and shape of the
Figure 1.31: Effect of vortex windowing on low-velocity signal. These runs were taken on the same vortex lattice using a 60 by 20 \(\mu m\) vortex window and a 200 by 200 \(\mu m\) vortex window. \(B = 89\) Gauss and \(\delta a/a = 15\%\).
calculated peaks all agree closely with the experimental results. The simulations with \( \delta a/a = 15\% \) fit the experimental data best in all cases except at 4.5 Gauss, where \( \delta a/a = 20\% \) is better.

At the lowest two fields \((B = 0.9 \text{ and } 1.9 \text{ Gauss})\) for which we carried ran simulations, the results agree very well with experiment at low velocities. However, the experimental data fall off at high velocities after peaking at a vortex spacing of about 1.6 \( \mu m \) in both cases, while the Monte Carlo results continue to rise. The peaks are in roughly the correct position for the second harmonic of the vortex spacing at \( B = 1.9 \) Gauss and the third harmonic at \( B = 0.9 \) Gauss. The Monte Carlo simulations do indicate the presence of harmonics at the low fields, but begin climbing toward the peak corresponding to the next lower harmonic (or the fundamental at 1.9 Gauss) where the experimental data does not. We have no good explanation for this discrepancy.

At higher fields, the peak of the Monte Carlo results tends to lie at a velocity where we do not have good experimental data (at or below 450 m/s). The Monte Carlo simulation can produce a signal that falls off with increasing velocity in a way that resembles the experimental results at higher velocities if we use ever smaller values of the disorder parameter \( \delta a/a \) at higher fields. However, better-ordered lattices tend naturally to produce strong, narrow peaks at low velocities. We do not see any evidence of such low-velocity peaks in our experimental data.

There are several possible explanations for this discrepancy: First, there simply aren't very many atoms in that part of the thermal velocity distribution (see Figure 3.1). Second, these peaks would lie at the same general velocity as the "notch"
Figure 4.32: Sensitivity to long-range order at $B = 26.7$ Gauss. The solid curves are the Monte Carlo results at $\delta a/a = 7\%, 10\%$, and $15\%$. The dashed curve shows the experimental data at the same field.

produced by fluorescence of $F' = 1$ atoms in the detector (see Figure 2.23). Finally, the NMR resonance frequencies for the nine $F' = 1 \rightarrow F = 2$ transitions spread apart rapidly as the magnetic field increases, as shown in Figure 4.3. The Monte Carlo simulation calculated the transition probability for a single "average" transition, based upon the likelihood of strong non-adiabatic mixing between the $F = 2$ states. The simulation did not account for the fact that narrow signals would become smeared out as the transition frequencies making up the average spread apart. We believe that the Monte Carlo results would be no longer be inconsistent with experiment if the spread of transition frequencies were taken into account.

The Monte Carlo simulations indicate that our experiment provides a sensitive test
of the long-range order of a vortex-lattice. This sensitivity is illustrated in Figure 4.32, where we compare the results of Monte Carlo runs taken at $B = 26.7$ Gauss at $\delta a/a$ values of 7%, 10% and 15%. The behavior at high velocities, where the vortex spacing being probed is much bigger than the nominal intervortex spacing shows a clear dependence on disorder: the high-velocity signal increases uniformly with increasing disorder. We were able, in this case, to determine that the disorder was closer to 10% than to either 7% or 15%.

The fact that the disorder parameter $\delta a/a$ needs to decrease at higher fields comes as no surprise. Other researchers have seen a strong negative dependence of disorder on increasing magnetic field. In Bitter decoration experiments on BSSCO single crystals, Grier et al.[14] found that the translational and orientational order for a hexatic glass both decreased rapidly as the magnetic field decreased from 69 Gauss to 8 Gauss.

This effect can be qualitatively understood by considering the strength of the vortex-vortex interaction. For an isotropic bulk material like niobium, the interaction energy per unit length of two vortex lines a distance $r$ apart is[31]

$$E_{\text{bulk}} = \frac{\phi_0^2}{8\pi^2\lambda^2} K_0(r/\lambda),$$

where $K_0$ is the zeroth-order Hankel function of imaginary argument. This energy decays as $\sqrt{\lambda/r}e^{-r/\lambda}$ for $r \gg \lambda$.

The contribution to the interaction energy from the vortex tips in a slab or film do not fall off as quickly. This interaction is entirely due to the energy of the magnetic fields in free space. For vortices spaced more than a penetration depth apart, these
fields resemble the fields produced by magnetic monopoles, and the interaction energy falls off with increasing separation as

\[ E_{\text{tips}} = 2 \left( \frac{\phi_0}{2\pi} \right)^2 \left( \frac{1}{r} \right). \]  \hspace{1cm} (4.76)

The tip energy will eventually dominate over the bulk interaction energy at large separation, provided the vortices do not bend.

In either case, if two neighboring vortices are moved sufficiently far apart, the interaction energy will always become less than the local fluctuations in energy due to pinning, and the flux line will become pinned to a random site independently of the vortices around it. Therefore, the distribution of vortices will become increasingly random at smaller magnetic fields. By determining the manner in which order increases with magnetic field, we should be able to learn a great deal about the microscopic pinning mechanism in these and other materials.

The generally excellent agreement between our experimental results and the predictions of the detailed Monte Carlo calculations gives us great confidence that our basic understanding of the ABMR experimental system is correct. The signals that we see appear to be caused by a disordered lattice of flux lines, each containing one quantum of flux. The dependence of the signal on temperature supports the conclusion that the magnetic vortices live in a material with a magnetic penetration depth \( \lambda(T) \) that behaves consistently with the empirical two-fluid model.
This experiment was inspired by the original work of Truman Brown and John King who, in 1969, performed an experiment that was similar in many respects to our own\cite{9, 8}. They reported seeing excitation signals that were consistent with excitation by magnetic vortices in a superconductor. That is, their signals disappeared when the sample was warmed above $T_c$ and had structure at velocities consistent with the lattice spacing in their applied fields.

In addition, they reported the presence of fine, sharp structures under certain conditions that shifted in velocity in the presence of transport currents. However, their measured velocities were much higher than can be accounted for even in the case of pure flux flow without pinning, where $J_c = 0$. Furthermore, the direction of their observed velocity shifts were often the reverse of the calculated Lorentz force. In one case, the velocity was observed to shift in the same direction no matter which way the transport current flowed! One final point that struck us as unusual after we had attempted for many months to observe any signal was that their excitation probability for atoms passing through a 5 $\mu$m slit was as high as 80%, much higher.
Figure 5.1: Brown and King’s experimental setup[9]. Their caption read, “Top view of the apparatus.” Note that both the initial and final state selection was accomplished using Stern-Gerlach magnets. (These are labelled “A magnet” and “B magnet” on the diagram.)

than either our experiment observes or our simulations predict, even though we use a slit only 1.8 μm high. These apparent inconsistencies prompted us to reexamine their experiment in detail.

Figure 5.1 is a reproduction of their depiction of their experimental setup. The overall layout of their apparatus was similar to ours, with an potassium source, initial and final state selection, a sample with a small slit allowing the atomic beam to pass only very close to the surface, and a velocity-sensitive detector. Their sample was a cold-rolled vanadium foil epoxied to the outside of a 2 cm radius cylinder (rather than being flat like ours) and was 125 μm thick by 1.0 cm long (along the beam axis)
by 1.8 cm wide. The atoms were restricted by a knife edge to pass less than 5 μm from the sample surface.

The major differences between Brown and King's experiment and our own is in the method of state selection and velocity discrimination. Continuous-wave, tunable lasers with which to optically pump and detect atoms were not available in 1969, so they used the established state selection technique of the time: Stern-Gerlach bending magnets. After leaving the beam source, the potassium atoms would encounter an "A magnet" that allowed to pass only atoms with negative electronic spin (those with \( m_J = -1/2 \), including those in \( F = 1 \) at low fields as well as the \( m_F = -2 \) state). After being excited by the sample fields, the atoms then had to pass a second "B magnet" that functioned in exactly the same manner as our Stern-Gerlach bending magnet, letting through only atoms with \( m_J = +1/2 \) (\( F = 2, \ m_F = -1, 0, 1, 2 \)).

Their velocity-sensitive detection relied on time-of-flight: the beam was pulsed by a rotating chopper before the "A magnet" and detected after the "B magnet" by a hot-wire detector equipped with an electron multiplier to allow single-atom detection. The atoms' arrival times were binned by multichannel analyzer to record the signal as a function of velocity.

Figures 5.2 and 5.3 show their experimental data. Figure 5.2 compares the results they obtained in the presence of a superconducting sample in an applied field to their normalization curve, obtained by exciting the atoms with an external RF field. The data curve is about 80% as high as the normalization curve, and displays fine peaks occurring at equally-spaced arrival times (corresponding to velocities: \( \ldots, \frac{v}{n}, \frac{v}{n+1}, \ldots \)).
Figure 5.2: Results of Brown and King's experiment[9]. Their original caption read. "Accumulated counts versus arrival times of atoms which have undergone transitions. In the upper curve transitions were induced by an oscillating field in order to determine the velocity distribution of the atoms passing the foil. In the lower curve transitions were induced by the magnetic field outside the superconducting foil at 4.2 K and in an external field of 2.6 Gauss. Each curve represents a counting interval of 1 min." Note the large signal (80%!) and the characteristic wiggles, here spaced at roughly equal arrival times.
Figure 5.3: Effect of transport currents on Brown and King’s results[9]. Their original caption reads, “Relative transition probability versus velocity for various current conditions. The field direction was the same as that indicated in the insert of Fig. 1.” (Their Fig. 1 is our Figure 5.1.) The wiggles appear to shift when transport currents are applied.
Figure 5.3 shows the effect of an applied transport current density of 27 Amps/cm$^2$ on the signals, here displayed as a function of velocity. These data were taken using an alternating current, with data collected at the two extremes and the nodes of the cycle. The peaks appear to be displaced by ±50 m/s or so when the current reversed, relative to their location in the absence of current. The velocity shifts are in the opposite direction to the Lorentz force. Brown and King suggested that the signal was due to trapped vortices aligned opposing the applied field.

Curves bearing this distinctive fine structure were only obtained when the sample was prepared in a particular manner before the data was taken, and only then about 1/4 of the time. In the remainder of the runs, they obtained a single broad curve that did not shift when current was applied, as shown in Figure 5.4. This particular data set has a peak located at about 700 m/s, corresponding to a vortex spacing of 1.5 μm. The nominal intervortex spacing at 8.2 Gauss is 1.59 μm. In the cases where the narrow peaks were observed, the sample was prepared by cooling through $T_c$ in the presence of a large transport current, while simultaneously reducing the applied field from an initial $\approx 10$ Gauss to its final value of one to three Gauss.

The magnitude of the velocity shifts was difficult to explain consistently in terms of standard flux-flow theory. Figure 5.5 shows the size of the observed velocity shifts as a function of current density. The slope $\frac{dv}{dJ}$ of velocity with applied current density is about 1000 times larger than the slope

$$\frac{dv}{dJ} = \frac{\rho_v}{cH_{c2}}$$ (5.1)
Figure 5.4: Broad peak at 8.2 Gauss[8. Fig. 9]. The original caption read. "Broad peak presumably from disordered lattice." The peak is located at about 750 m/s. and is independent of current.
Figure 5.5: Velocity shifts as a function of applied current density[9]. The original caption read, "Constant velocity shift versus current density." The motion was believed to begin at a finite current density because of pinning.

given by the standard theory[34], where $\rho_n$ is the normal-state resistivity of the material.

One possible explanation for this discrepancy between theory and experiment is that the flux-flow resistivity is in fact 1000 times smaller than that given by the usual expression

$$\rho_f = \rho_n \frac{B}{H_{c2}},$$

(5.2)

perhaps because the current density avoids the vortex cores. Another possible explanation is that trapped anti-vortices would be unstable and could move much faster as a result. In fact, Brown points out in his conclusions[8] that the vortices should move "more than 2 cm in the time it takes the beam pulse to pass the sample". Un-
Figure 5.6: The velocity sometimes shifted in the same direction regardless of the direction of the applied current [8, Fig. 16]. (There was no original caption.)

Fortunately, this implies that trapped flux would pass completely out of the 1 cm long sample in the time it took a single chopper pulse to traverse the sample. Such flux would not remain trapped for long! Also, this hypothesis cannot explain the effect seen in Figure 5.6 in which the velocity shift was in the same direction regardless of the direction of the applied current.

We believe we have found an alternative explanation for these results that does not rely on flux-flow or a well-ordered lattice. Furthermore, we have been able to experimentally confirm our hypothesis.

The key to our understanding of Brown and King’s results lies in their method of initial atomic state preparation. Recall that their “A magnet” allowed all four...
hyperfine states with negative electronic spin to pass. In particular, the $F = 2, m_F = -2$ state passed over the sample. In the low fields near the sample region, this particular state is only separated by one or two MHz from the remaining $F = 2$ levels, and is quite capable of making resonant Majorana transitions into those levels if any long-range ($\sim 1 - 10$ mm) field variations are present. In contrast, our optical pumping state preparation allows only atoms in the three $F = 1$ levels to traverse the sample.

In Section 4.2.2, we discussed resonant Majorana transitions between closely-spaced hyperfine states, and found that a large fraction of the population of two neighboring $F = 2$ levels could be exchanged if: (a) there was a long-range variation in the magnetic field, such as that produced when the external field was turned off after our samples were field-cooled, and (b) there was a large population difference between the levels.

This is exactly the situation that Brown and King encountered. Their state preparation left the $m_F = -2$ hyperfine state fully populated and the $F = 2, m_F = -1, 0, 1,$ and 2 levels unoccupied. Their sample preparation technique almost certainly left flux trapped in the sample, meaning that the field over the sample was greater than the field on either side. Furthermore, the transport currents present when the sample was cooled probably produced long-range variations in the local field across the sample itself. Either of these conditions could easily lead to signals that look very much like they reported.

Resonant Majorana transitions also offer an explanation of the velocity shifts in
the peak locations when transport currents were applied. The current necessary to produce the 27 Amps/cm² current density of Figure 5.3 was about 340 mA. This current flowed along a cylindrical arc of radius 2.0 cm. This current would produce a magnetic field similar to the current a 4 Amps current would have produced at the center of the cylinder: \( B = \frac{\mu_0 I}{r} = 2.5 \text{ Gauss} \) oriented transversely, along the atom’s path. The component of this field parallel to the applied field of 2.6 Gauss will change by as much as \( 2B_{\text{trans}} \sin(0.5\text{cm}/2.0\text{cm}) = 1.2 \text{ Gauss} \) in a 1 cm distance.

The one factor we cannot include in our modeling is the fact that the Zeeman shift of the hyperfine states increases with increasing field. This means that the transition frequency of the resonant Majorana transition that produces the narrow peaks will change significantly (\( \delta f/f \sim 10\% \)) when a transport current is applied to the sample. This will certainly cause the peaks to shift, in ways that depend on the current and the particular (unknown) field configuration that was produced when the sample was cooled.

We have obtained convincing experimental evidence that resonant Majorana transitions do in fact occur, and can transfer a large fraction of the atomic population between \( F = 1 \) hyperfine states. To obtain this evidence, we decided to prepare the sample in a critical state as we discussed in Section 3.4 and then deliberately spoil the optical pumping so as to leave a significant population in the \( F = 2 \) levels. We hoped to obtain a significant population difference between \( m_F = 2 \) and \( m_F = 1 \), so that we would see a clear resonant Majorana signature.

Figure 5.7 shows the results of one such run. The sample was first field-cooled
Figure 5.7: Experimental resonant Majorana transitions at 1.9 Gauss. The sample was prepared by field cooling in a 1.9 Gauss field, then the field was removed. The “good pumping” curve was taken with the optical pumping set up in the usual fashion. The “bad pumping” curve was taken with the retro-reflector blocked, so that the pumping laser only made one pass across the potassium beam. (The apparent divergence at 0.7 μm is an artifact of the data analysis.)
in a 1.9 Gauss field. Then the field was removed, and a scan taken with "good" optical pumping (the usual set-up where the pumping laser is retro-reflected so as to cross the atomic beam twice). We then blocked the retro-reflector and repeated the scan, without doing anything to the sample. The results are striking: the signal with good optical pumping is small and featureless, while the signal with the retro-reflector blocked displays characteristic oscillations whose spacing decreases as the velocity decreases, just as we calculated in Section 4.2.2. These peaks reach over 30% of the unpumped signal strength, much larger than any other signals we have observed. The data were fully reproducible, with the oscillations disappearing when the retro-reflected beam was again allowed to illuminate the beam, and reappearing when it was again blocked.

These data strongly support the conclusion that the oscillatory structure seen by Brown and King was not in fact caused by the Fourier components of a well-ordered vortex lattice as they claimed, but instead was produced by resonant low-frequency transitions between the fully-populated $m_F = -2$ state and the remaining $F = 2$ states. Since these transitions are driven by long wavelength field fluctuations ($\sim 1$ mm), atoms passing at all distances from the sample would be affected equally, leading to the exceptionally large transition probabilities they measured. The large fractional Zeeman shift at low fields can also account for the anomalously large velocity shifts seen in the peaks in the presence of transport currents. Finally, we were able both to predict and experimentally observe similar large, oscillatory features when we prepared our sample in a manner similar to the way theirs was prepared.
We have introduced a new experimental tool for the study of the magnetic flux lattice in superconductors—the atomic beam magnetic resonance probe. In our technique, two-level atoms traveling over the surface of a superconductor at velocity \( v \) will make transitions from one state to the other if the magnetic field produced by the vortex has a spatial periodic component with period \( d = \frac{v}{f_0} \), where \( f_0 \) is the resonance frequency of the transition. By recording the transition probability as a function of atomic velocity, we can directly analyze the periodic structure of the magnetic fields above the lattice. This structure immediately reflects the periodic structure of the vortex lattice itself.

We have demonstrated this technique by using it to study the vortex lattice in a niobium film. In this study, we examined the dependence of the vortex structure on the applied magnetic field, the dependence of the signal strength on temperature, the effect of transport current on vortex structure, and the effects of long-range field variations on the observed signal. We have made detailed comparisons of these results to theoretical models of the system, and found excellent agreement between experiment and theory.

We performed measurements on vortex lattice produced by field-cooling the sam-
ple in the presence of bias fields ranging from 0 to 178 Gauss. We found that the experiment, as expected, was particularly good at probing the structures produced by magnetic fields between 5 and 30 Gauss. In this range of magnetic fields, the data consistently showed a broad peak in transition probability centered at the expected lattice spacing of $d = \sqrt{\phi_0/B}$. The results resembled the predictions of the detailed theoretical calculations in size, shape, and location of the transition probability peak. From these results, we are able to confidently conclude that:

- Our experiment does in fact respond to magnetic field fluctuations that are produced by superconducting niobium in the presence of an applied magnetic field.

- The magnetic field fluctuations are produced by a disordered lattice of vortex lines, each containing one quantum $\phi_0$ of flux.

- The degree of disorder present in the vortex lattice decreases with increasing field, an effect that is observed using other techniques than our own[14].

When we applied a constant magnetic field and varied the temperature of our sample, we found that the size of the peak transition probability decreased to zero as the temperature approached $T_c$. This decay of transition probability is similar to that seen in neutron diffraction studies[41], and can be used to study the divergence of the magnetic penetration depth $\lambda(T)$ as $T$ approaches $T_c$. By comparing our data for the peak transition probability as a function of temperature to our detailed model, we were able to conclude that:
• The temperature dependence of the penetration depth in our niobium sample is consistent with the empirical "two-fluid" model.

• The data were consistent with a bulk zero-temperature penetration depth of \( \lambda(0) = 142 \text{ nm} \). This value is somewhat larger than the values obtained by other researchers for sputtered niobium films, but is not inconsistent with an electron mean-free-path equal to the estimated grain size of our films. This value is also not inconsistent with independent penetration depth measurements made on similar niobium films grown in the same apparatus as the films used in our experiment.

In contrast to several common methods used for penetration depth measurement, the ABMR technique is capable of measuring the absolute penetration depth of our thin film sample. ABMR appears to be the only technique capable of performing absolute penetration depth measurements on thin films. The technique could therefore become a source of important knowledge about the magnetic properties of thin films.

We were unable to see any significant effects when we applied transport currents to our sample, even when flux flow in the absence of pinning would have resulted in a drift velocity of over 200 m/s. From this negative result, we were able to conclude that our film had a critical current density \( J_c \) in excess of 67,000 Amps/cm\(^2\), a result consistent with strong pinning forces produced by dense grain boundaries in the sputtered film.

When the critical current density is this high, flux will remain completely trapped in the film, even after the external bias field is turned off. When we did this, we
observed that a characteristic pattern of oscillations was superimposed on the already present transition probability vs. velocity peak. We proposed that this oscillatory pattern was produced by the response of resonant, low frequency Majorana transitions among the $F = 2$ magnetic hyperfine states to the sudden step-like changes in the local magnetic field at either end of the sample caused by the trapped flux. We calculated the transition probability vs. velocity response of such a transition and found an oscillatory pattern that closely resembled the pattern we observed experimentally.

Finally, we suggested a possible explanation for the puzzling results of Brown and King's original experiment. When they prepared their sample in a certain manner, they observed oscillatory signals that they believed were caused by a well-ordered lattice. The locations of the peaks in this oscillatory signal appeared to shift in velocity when a transport current was applied to the sample. Brown and King naturally interpreted this velocity shift to reflect the motion of the vortex lattice itself, although the apparent velocity of the lattice was about 1000 times larger than the maximum flux flow velocity predicted by theory.

We suggested that the oscillatory signals were caused by the same resonant Majorana transitions we observed in our own experiment, and that the velocity shifts of the peaks were in fact caused by the comparatively large Zeeman shifts of the resonance frequencies caused by the magnetic fields produced by the transport current itself.

We noted that in their experimental setup, the $F = 2, m_F = -2$ magnetic hyperfine state remained fully populated after the atoms’ initial state was prepared. Under the conditions they described, these $m_F = -2$ atoms could make resonant
Majorana transitions into the other four $F = 2$ states in large numbers, and then be detected as signal. This signal would display the characteristic pattern of oscillations that we calculated, and would closely resemble the oscillatory signal that they in fact observed. The strong relative Zeeman shift of the spacing between the $F = 2$ magnetic hyperfine levels implies that the velocities of the oscillatory peaks could easily shift in much the same manner that their peaks shifted when a transport current was applied to their sample. This velocity shift would not be due to the motion of magnetic vortices (which would have to move 1000 times faster than the predictions of flux flow theory). Instead, the transport currents would produce magnetic fields that would shift the resonant frequencies of the Majorana transitions, causing the peaks to shift as well.

When we deliberately reduced the optical pumping efficiency of our apparatus, we observed oscillatory signals closely resembling the results of our calculations as well as the signals observed by Brown and King. These signals were three times as large as any signals we observed under normal conditions. We believe that this observation provides strong experimental evidence that Brown and King’s observations were the result of Majorana transitions, and were not caused by a highly ordered flux lattice.

The atomic beam magnetic resonance technique has great potential to further our understanding of the magnetic properties of high-$T_c$ and other type-II superconductors. The technique is fast, versatile, and powerful.

- We are able to measure the nominal intervortex spacing and penetration depth of our samples and determine the approximate degree of disorder present in the
vortex lattice.

• We are currently capable of detecting changes in the vortex lattice structure on time scales of 25 msec. With additional technical improvements, we should be able to measure changes in transition probability as small as 0.01% in a time as short as 100 μsec.

• The only requirements that our technique makes upon the sample is a moderately flat surface (RMS roughness less than about 50 nm).

• We can operate over a range of temperatures, and even change the temperature and observe the effects on the transition probability in real time.

The immediate application of this technique beyond this proof of principle is to high-$T_c$ superconductivity. Preparations are under way to study single crystals of YBCO. We intend to make real-time measurements of flux-lattice melting, as well as to observe the flux chains reported by Bishop’s group[12] by Bitter decoration when a tilted field is applied to the sample. We also would like to measure the temperature-dependence of the penetration depth at low temperatures in an attempt to understand more about the symmetry of the order parameter in this material. The technique is applicable to other high-$T_c$ materials such as BSSCO.

Atomic beam resonance techniques might become useful in applications beyond the study of superconductivity. In general, any spatially-varying field that can be generated in a vacuum can be probed by an atom or molecule with the right coupling and a transition frequency appropriate for the scale length of the variations one wishes
to probe. For instance, magnetic domains at the surface of ferromagnetic materials would be good candidates for study. Finally, molecules with permanent electric dipole moments would be able to probe spatially varying electric fields.

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Appendix A

Laser servo-locking electronics

Figure A.1 shows a circuit diagram of the servo-controller used to lock the laser frequency to the side of a Fabry-Perot cavity interference fringe. The circuit amplifies the difference between the output signal from the cavity photodiode and the set-point voltage. This difference signal is sent to an integrator, whose output will remain constant if the difference is zero and will act through negative feedback on the laser frequency to reduce the difference signal to zero if it deviates.

The op-amps are a Motorola MC34004 quad premium op-amp, with the exception of the simple follower on the setpoint voltage, which uses an OP77. The resistors are all 1%. The values of the capacitors can be adjusted to give the best performance for a particular laser.

Figure A.2 is a circuit diagram of the saturated-absorption servo-controller. In this circuit, the output of the saturated-absorption photodiode and the AC reference signal from the chopper/AOM are mixed by an Analog Devices AD734 four-quadrant multiplier. The low-frequency portion of the resulting signal corresponds to the locking signal, and is compared to a setpoint, amplified, integrated, and fed back to the Fabry-Perot cavity controller to lock the laser frequency to a saturated-absorption feature. The values shown are good for the pumping laser setup, which has an mod-
Figure A.1: The cavity servo-controller.
ulation frequency of 20 kHz. The detector laser is chopped at a frequency of about 1 kHz, so different values are needed for some of the capacitors. The monitor output allows us view the locking signal on an oscilloscope and tune the laser via the cavity controller to within the locking capture range of the feature to which we wish to lock the laser frequency.
Figure A.2: The saturated absorption servo-controller.
Appendix B

Detailed calculation of optical pumping evolution

The detailed frequency- and time-dependence of the hyperfine state populations under optical pumping was calculated numerically. This appendix will outline the basic steps involved in the calculation.

In any optical pumping process, an atom is excited from one of several low-energy states into an excited level and then spontaneously decays into a different, desired low-energy state. In our case, the low-lying levels are the eight hyperfine states of the $^{39}$K electronic ground state. Atoms are excited out of these levels by resonant laser light to the eight hyperfine levels of the first excited electronic state: $4p^2P_{1/2}$, from which they decay by spontaneous emission back to the electronic ground state. We desire to empty out the ground state $F = 2$ states by pumping them into the $F = 1$ states. This is best done via resonant excitation into the excited state’s $F' = 2$ states, which decay more rapidly into the ground state’s $F = 1$ states than do the excited $F' = 1$ states.

There are three optical processes involved in the absorption and re-emission of light by atoms: resonant absorption, resonant stimulated emission, and spontaneous emission. Absorption and stimulated emission are reversible phenomena, and will not result in a net population transfer between ground state hyperfine levels. Spontaneous
emission, which is random and irreversible, is necessary for optical pumping to occur. It follows that spontaneous emission must compete with stimulated emission in the optical pumping process, reducing the excited state population and therefore the optical pumping rate.

The calculation of the time evolution of the atomic populations begins by specifying the initial populations of the ground state atoms, the laser frequency, laser intensity, laser polarization, atomic transition linewidth, and external magnetic field. The electric dipole transition amplitudes between all pairs of ground and excited states are calculated by diagonalizing the appropriate hyperfine Hamiltonians both for the polarized incident light and unpolarized spontaneous emission in the magnetic field.

Now the evolution of all sixteen levels can be calculated from the following sets of rate equations. Let the indices $g$ and $e$ refer to the ground and excited hyperfine levels respectively. $P_g$ and $P_e$ refer to the populations of the respective ground and excited states. $B_{ge}$ is the (frequency-, intensity-, and polarization-dependent) rate of absorption from state $i$ to state $j$, while $B_{eg}$ is the rate of stimulated emission from the excited state $j$ to state $i$. Finally, let $A_{ji}$ be the rate of spontaneous emission from state $j$ into state $i$. The evolution of the ground state population $P_g$ is now given by

$$\frac{dP_g}{dt} = - \sum_i B_{gi} P_i + \sum_e B_{eg} P_e + \sum_e A_{eg} P_j.$$  \hspace{1cm} (B.1)

where the sum is over the excited states $j$. The excited state population $P_e$ satisfies

$$\frac{dP_e}{dt} = \sum_g B_{ge} P_i - \sum_g B_{eg} P_e - \sum_g A_{eg} P_e.$$  \hspace{1cm} (B.2)
The spontaneous emission rate is given by the angular part of the dipole transition amplitude squared divided by the radiative excited state lifetime $\tau = 26 \text{ nsec}$:

$$A_{eg} = \frac{1}{\tau} \sum_{\hat{\varepsilon}} |\langle j|\hat{\varepsilon}||i\rangle|^2,$$  \hspace{1cm} (B.3)

where the sum over $\hat{\varepsilon}$ refers to all possible polarizations and wave-vectors of the emitted photon.

The stimulated emission rate is equal to the resonant absorption rate given by[32]

$$B_{ji} = B_{ij} = \frac{\Gamma}{\delta^2 + (\Gamma^2/4)(1 + 2I/I_{sat})},$$  \hspace{1cm} (B.4)

where $I$ is the laser intensity, $I_{sat} = 1.3 \text{ mW/cm}^2$ is the transition's saturation intensity, $\Gamma = 1/\tau$ is the transition decay rate or linewidth, and $\delta = \omega_L - \omega_{eg}$ is the detuning of the laser frequency $\omega_L$ from the transition frequency $\omega_{eg}$. 
Appendix C

Dependence of magnetic field on bulk penetration depth and vortex lattice correlation function

We can calculate the magnetic field above the surface of a superconducting slab of finite thickness $2d$, after M. Cristina Marchetti's calculation for a semi-infinite slab[26]. We assume that the vortices are straight and normal to the surface, and that the superconductor is isotropic. The geometry is illustrated in Figure C.1. The solution for the field in a slab extending from $z = -d$ to $d$ will be symmetric under reflection over the $x$-$y$ plane, so we can solve the problem for $z > 0$ with appropriate boundary conditions at $z = 0$ and $z = d$. The local magnetic induction must satisfy Maxwell's equations in free space and the London equations in the superconductor. If we designate the location of the $i$th vortex by $\mathbf{R}_i$, we can write the following equations for the local magnetic induction $\mathbf{h} = \mathbf{h}(\mathbf{r})$ above the surface ($z > d$) and inside the superconductor ($0 < z < d$):

$$\nabla \cdot \mathbf{h} = 0,$$  \hspace{1cm} (C.1)

$$\nabla \times \mathbf{h} = 0,$$  \hspace{1cm} (C.2)

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Figure C.1: The geometry of the superconducting slab in a perpendicular magnetic field for which we solve the London-Maxwell equations.
where $\lambda = \lambda(T)$ is the temperature dependent bulk penetration depth for the isotropic superconductor and $r = (r_\perp, z)$.

The boundary conditions at $z = d$ are that (i) $h$ be continuous across the boundary, and that (ii) there be no currents normal to the surface. The latter may be expressed

$$[\hat{z} \cdot (\nabla \times h)]_{z=d^-} = 0. \quad (C.5)$$

At $z = 0$, these two conditions also apply, with the added restriction that that transverse field $h_\perp = h - (h \cdot \hat{z})\hat{z} = 0$.

We can express the solution to the equations directly in terms of the partial Fourier transform of the local induction,

$$h(q_\perp, z) = \int dr_\perp \exp(iq_\perp \cdot r_\perp) h(r). \quad (C.6)$$

The various derivative operators in the field equations can now be expressed as a sum of algebraic terms in $q_\perp$ and derivative terms in $z$: The divergence equation ($\nabla \cdot h = 0$) becomes

$$\left( i q_\perp + z \frac{\partial}{\partial z} \right) \cdot h(q_\perp, z) = 0. \quad (C.7)$$

The curl equation for the induction above the surface becomes

$$\left( i q_\perp + z \frac{\partial}{\partial z} \right) \times h(q_\perp, z) = 0. \quad (C.8)$$
The double curl equation (Equation C.4) for $0 < z < d$ becomes

$$h(q_\perp, z) + \lambda^2 \left( q_\perp^2 + \frac{\partial^2}{\partial z^2} \right) h(q_\perp, z) = \phi_0 \sum_i \exp(-i q_\perp \cdot R_i) \hat{z}. \quad (C.9)$$

Finally, the boundary conditions $[\hat{z} \cdot (\nabla \times h)] = 0$ at $z = d$ and $z = 0$ become simply

$$q_\perp \times h_\perp = 0, \quad (C.10)$$

or more simply,

$$h_\perp \parallel \hat{q}_\perp. \quad (C.11)$$

Now I can propose trial solutions in the two regions: $z > d$ and $0 < z < d$.

First, since the transverse induction satisfies Laplace’s equation and is parallel to the transverse wavevector at both boundaries, it follows that $h_\perp(q_\perp, z) = h_\perp(q_\perp, z)\hat{q}_\perp$ everywhere. We can now write $h = h_\perp \hat{q}_\perp + h_z \hat{z}$. In the region $z > 0$, the fields satisfy $\nabla^2 h = 0$. Solutions will be exponential in $z$ and periodic in the $x-y$ plane, so we may write

$$h(q_\perp, z) = h(z) e^{-q_\perp(z-d)}, \quad (C.12)$$

where we discard the exponentially increasing solution since we require the induction to be finite at large $z$. Here,

$$h(q_\perp, z) = h_\perp(q_\perp, z)\hat{q}_\perp + h_z(q_\perp, z) \hat{z}. \quad (C.13)$$

Now, the divergence equation $\nabla \cdot h = 0$ requires

$$ih_\perp - h_z = 0; \quad z > d. \quad (C.14)$$
For $0 < z < d$, we can rewrite the double curl equality (Equation C.9) as

$$\frac{\partial^2 h}{\partial z^2} + \alpha^2 h = \frac{\phi_0}{\lambda^2} \sum_i \exp(iq_\perp \cdot R_i) \hat{z},$$  \hspace{1cm} (C.15)$$

where $\alpha \equiv \sqrt{q_\perp^2 + 1/\lambda^2}$. This equation is homogeneous for $h_\perp(q_\perp, z)$, but inhomogeneous for $h_z(q_\perp, z)$. Remembering that our boundary conditions require $h_\perp(q_\perp, 0) = 0$, we can choose solutions to the homogeneous equations of the following form:

$$h_\perp = h_{\perp 1} \sinh(\alpha z) + h_{\perp 2} \cosh(\alpha z)$$  \hspace{1cm} (C.16)$$

$$h_z = h_{z 1} \sinh(\alpha z) + h_{z 2} \cosh(\alpha z).$$  \hspace{1cm} (C.17)$$

The particular solution for the $z$-component may be obtained from the solutions of the homogeneous equation and the driving term[2, pg. 479]:

$$h_p(q_\perp, z) = \frac{\phi_0}{\lambda^2 \alpha^2} \sum_i e^{i q_\perp \cdot R_i}.$$  \hspace{1cm} (C.18)$$

As we did for $z > d$, we can now use the divergence equation (Equation C.7) to relate the transverse and $z$-components of the field:

$$h_\perp(q_\perp, z) = \frac{-1}{iq_\perp} \frac{\partial}{\partial z} (h_{z 1} \sinh(\alpha z) + h_{z 2} \cosh(\alpha z))$$

$$= \frac{-\alpha}{iq_\perp} (h_{z 1} \cosh(\alpha z) + h_{z 2} \sinh(\alpha z)), $$  \hspace{1cm} (C.19)$$

since the particular solution $h_p(q_\perp)$ does not depend on $z$. But we also have an expression (Equation C.16) for $h_\perp$, which requires that

$$h_{\perp 1} = \frac{-\alpha}{iq_\perp} h_{z 2},$$

$$h_{\perp 2} = \frac{-\alpha}{iq_\perp} h_{z 1}. $$  \hspace{1cm} (C.20)$$
Remembering our boundary condition \(h_{\perp}(q_{\perp},0) = 0\) requires that we discard the \(\cosh(\alpha z)\) solution for \(h_{\perp}\), which is non-zero at \(z = 0\). So \(h_{\perp 2} = h_{z1} = 0\). We can now write an expression for the induction inside the superconductor:

\[
h = h_{\perp 1}(q_{\perp}) \sinh(\alpha z) \hat{q}_{\perp} + (h_{z2}(q_{\perp}) \cosh(\alpha z) + h_p(q_{\perp})) \hat{z}. \tag{C.21}
\]

Using the continuity of \(h\) at \(z = d\), above the superconductor we may write

\[
h_{\perp}(q_{\perp}, d) = -i h_z(q_{\perp}, d)
= -i \left( \frac{\phi_0}{\lambda^2 \alpha^2} \sum_i \exp(i q_{\perp} \cdot R_i) + h_{z2} \cosh \alpha d \right). \tag{C.22}
\]

But inside the superconductor,

\[
h_{z2} = \frac{i q_{\perp}}{\alpha} \frac{h_{\perp}}{\sinh \alpha d}. \tag{C.23}
\]

So, we arrive at

\[
h_{\perp} = -i \left( \frac{\phi_0}{\lambda^2 \alpha^2} \sum_i \exp(i q_{\perp} \cdot R_i) - \frac{i q_{\perp}}{\alpha} h_{\perp} \coth \alpha d \right), \tag{C.24}
\]

or,

\[
h_{\perp} = -i \phi_0 \frac{\sum_i \exp(i q_{\perp} \cdot R_i)}{\lambda^2 \alpha q_{\perp} \coth(\alpha d) + \alpha}. \tag{C.25}
\]

Now we can write the complete expression for the fields above and below the surface:

\[
z > d:\]

\[
h(q_{\perp}, z) = \frac{\phi_0}{\lambda^2 \alpha} \frac{(\hat{z} - i \hat{q}_{\perp})}{q_{\perp} \coth(\alpha d) + \alpha} e^{-q_{\perp} (z - d)} \sum_i e^{i q_{\perp} \cdot R_i}, \tag{C.26}
\]

\[
0 < z < d:\]

\[
h(q_{\perp}, z) = \frac{\phi_0}{\lambda^2} \sum_i \exp(i q_{\perp} \cdot R_i)
\times \left[ \hat{z} + \frac{q_{\perp} \cosh(\alpha z)}{\alpha^2 \sinh(\alpha d)} \hat{z} - \frac{i q_{\perp}}{\alpha} \sinh(\alpha z) \hat{q}_{\perp} \right]. \tag{C.27}
\]
From this result, it is simple to calculate the field of an isolated vortex above the surface in real space by an inverse fourier transform of Equation C.26. If we consider an isolated vortex at $r_\perp = 0$, the factor $\sum_q \exp(iq \cdot R_i)$ reduces to 1. Then the magnetic field as a function of position is given by

$$h_z(r_\perp, z) = \phi_0 \frac{1}{(2\pi)^2 \lambda^2} \int_0^{2\pi} d\phi \int_0^\infty q_\perp dq_\perp$$

$$\times \frac{\exp(iq_\perp r_\perp \cos(\phi)) \exp(-q_\perp z)}{\alpha (q_\perp \coth(\alpha \bar{d}) + \alpha)}$$

$$= \frac{\phi_0}{2\pi \lambda^2} \int_0^\infty q_\perp dq_\perp \frac{J_0(q_\perp r_\perp) \exp(-q_\perp z)}{\alpha (q_\perp \coth(\alpha \bar{d}) + \alpha)} \quad \text{(C.28)}$$

$$h_r(r_\perp, z) = \frac{\phi_0}{2\pi \lambda^2} \int_0^\infty q_\perp dq_\perp \frac{J_1(q_\perp r_\perp) \exp(-q_\perp z)}{\alpha (q_\perp \coth(\alpha \bar{d}) + \alpha)} \quad \text{(C.29)}$$

where $J_0$ and $J_1$ are the zeroth- and first-order Bessel functions of the first kind.
Appendix D

Individual Contributions to this Work

I find myself in the embarrassing position of having claimed more credit than was my due.

Major Charles Emerson Winchester III

This dissertation represents the work of many individuals. In this appendix, I will attempt to clarify which work was performed by which individuals, where possible.

The inspiration for this work came from the original idea of Brown and King.

My advisor, Dr. Jergory Lafyatis, was responsible for conceiving of the experiment in its present form (i.e. using lasers for state selection and velocity-sensitive detection), as well as for convincing the Midwest Superconductivity Consortium to provide the capital and operating expenses (not to mention salaries for two graduate students) for the project. He was also instrumental throughout the construction, data taking, and subsequent data analysis for keeping the work moving forward on target.

My own major contributions to the experiment include the lasers, optics, frequency-locking schemes, and frequency-locking electronics, the design and construction of the sample and gates, the cryogenic instrumentation and wiring (including the four-wire resistance measurement, thermometer, and heater), the oven, the
designs of all the magnets in the experiment, including the Stern-Gerlach bending magnet, and the hot-wire detector. I am responsible for the data collection and analysis, and for the idea of using the temperature dependent signal to determine magnetic penetration depth. I am also responsible for the theoretical analysis and numerical simulation, much of which was inspired by discussions with my advisor.

My fellow student, Nathan Woodard, was responsible for a major portion of the remainder of the experiment. In particular, he designed the cryostat, the sample stage, and the detector. The complexity and sensitivity of the detector are a tribute to Nate's genius in experimental design. Nate was responsible for the construction of the Stern-Gerlach magnet. He was also responsible for much of the supporting structure under the experiment. In particular, he designed the gimbal mount for the oven and the system of position and tilt adjustments that allows the atomic beam path to be aligned.

We owe a particular debt to Edward Harris, who sputtered the niobium films (and built the apparatus to do it).

Eric Ulm performed the corroborative measurement of the temperature-dependent penetration depth using his two-coil magnetic screening probe. He also performed the calibration check for $T_c$ in the original samples.

Tom Barrett and Nate Woodard are responsible for the physical design of the laser mounts. Tom also wrote most of the ASYST data acquisition software, and was the first person in our lab to successfully use grating locking to control the frequency of a diode laser.
Harald Hauglin participated in the experimental runs in September, 1993, and has since been closely involved in the ongoing development of the experiment.

A good deal of grunt work was cheerfully carried out by several undergraduate students working in our lab over the years. These students are Carolyn James, Lolita Zukowski (who worked with single-mode fibers before I ever did), Diana Dibaradino, Timothy Holcomb, Eric Hufstedler (who built what will become the single-atom sensitive replacement for the PMT), Jeff Fox, and Lara Keefer.
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