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The effects of "The Geometric Supposers": Spatial ability, van Hiele levels, and achievement

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The Ohio State University, 1994
THE EFFECTS OF *THE GEOMETRIC SUPPOSERs*:
SPATIAL ABILITY, VAN HIELE LEVELS, AND ACHIEVEMENT

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Eileen Marie Smyser, B.S., M.A.

*****

The Ohio State University

1994

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Advisor
College of Education
To My Husband, Mike
and
To My Daughters, Cheri, Laura, Emily, Margie, and Jennifer
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FIELDS OF STUDY

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CHAPTER I
PROBLEM DEFINITION

1.0 Introduction

Since the mid-80s, when software designed for geometry classes became available, researchers and teachers have been interested in the effects of its use in the classroom. This study first examines the relationship between students learning geometry with the aid of *The Geometric Supposer* software series and the students' spatial visualization ability, van Hiele level, and achievement. Second, the effects of a series of lessons using *The Geometric Supposer* software series on students' spatial visualization ability is explored. Thirdly, the study explores the relationships between the students' spatial visualization ability, van Hiele level, and achievement. Knowledge of these relationships will help build a more complete understanding of the consequences of using educational software in teaching and learning geometry.

1.1 Nature of the Problem

Spatial ability is one of the three primary factors of mathematics aptitude which Guilford (1967) factored into two independent components: space visualization and spatial orientation. Because of geometry's visual nature, spatial ability has been linked with geometry achievement. In particular, "spatial orientation skill appears to be used in specific and identifiable ways in the solution of mathematics problems"
Smith (1964) concluded that the spatial factor is greater than the numerical factor as a basis for aptitude for mathematics. Every geometry course taught calls on logical reasoning and spatial ability (Mitchelmore, 1974). While Sherman (1979) found spatial visualization measured in ninth grade statistically significant in predicting the geometry grade for girls only, Sowder (1974) found the School Mathematics Study Group (SMSG) Paper Folding Test measured at the beginning of tenth grade geometry to be a good predictor for end-of-the-year geometry achievement for both girls and boys. Rolen (1985) found a significant positive correlation between spatial skills, measured at the end of the year, and final grade in geometry.

Spatial ability can be significantly enhanced by instruction in the primary grades (Miller, Boismier, & Hooks, 1969; Bishop, 1973; Miller, Kelly, & Kelly, 1988). Instruction in spatial visualization tasks significantly affects the students' spatial visualization performance in the middle school grades (Brinkman, 1966; Wolfe, 1970; Ben-Chaim, Lappan, & Housang, 1988). Ben-Chaim, Lappan, and Housang found the effects were retained after one year and that the seventh grade classes gained more than the fifth, sixth, or eighth grades. This suggests that the optimal time for teaching spatial visualization tasks may be in the seventh grade.

Levels of geometric thinking were first characterized by van Hiele (1957). The five discrete levels have a hierarchic arrangement through which the student moves sequentially (Denis, 1987; Senk, 1989), but not all at the same rate. The van Hiele model was formulated to describe the geometric thinking of elementary and secondary students (Burger & Shaughnessy, 1986b; Fuys, Geddes & Tischler, 1985; Usiskin, 1982). There is no developmental timetable determining progress through
the levels. Instruction rather than maturation appears to be the most significant factor contributing to progression through the levels (van Hiele, 1986).

Dina van Hiele (1957) developed a series of 70 lessons which successfully moved secondary students from the first to the third van Hiele level. Since then instruction was developed and tested in the classroom (Wirszup, 1976) and in a laboratory setting (Fuys, Geddes, & Tischler, 1985) which raised the van Hiele levels of thinking for elementary and middle school students. Usiskin (1982) and Bobango (1987) found the van Hiele levels were good predictors of performance on tests of standard geometry content. The higher a student's van Hiele level the greater likelihood of attaining higher achievement on geometry content tests. Both van Hiele level and spatial ability were shown to be positively correlated with achievement. Furthermore, Naraine (1989) found a significant correlation between van Hiele levels and spatial visualization among college students, with $r = .34$ indicating a moderate relationship.

Since instructional software was first introduced as a tool in the classroom to enhance teaching and learning of mathematics, theorists and practitioners have been attempting to uncover the full implications of its use. The National Council of Teachers of Mathematics (NCTM) recommends in the Standards (NCTM, 1989) that every mathematics classroom have a computer for demonstration purposes, and that every student have access to a computer. One specific recommendation for the high school geometry classroom is for students to use interactive software which provides experimentation with geometric figures to discover relationships by induction followed by a "deductive argument verifying their discovery" (NCTM, p. 159). The Geometric Supposer series is interactive software well suited for this purpose, and is one of the geometry software packages available for classroom use.
In a year long study of the effects of *The Geometric Supposer*, Yerushalmy (1987) found that on the mid-term and final exams, "students working with the SUPPOSER performed as well as, or better than, their non-SUPPOSER counterparts (p. 66). Building on this and other research, Yerushalmy and Chazan (1990) later identified three obstacles students encounter in interpreting geometric diagrams. Students using the *Supposer* were successful in overcoming the obstacles. They were able to approaching diagrams with more flexibility, focus on different parts of the diagram, look for more non-standard diagrams for a given problem, change their point of view, and think of sequences of related diagrams as motions. All of these characteristics appear to indicate improved spatial abilities for the *Supposer* users.

In another study, *The Geometric Supposer*, in phase-based instruction, was used in a 20-day treatment. The *Supposer* "had a significant effect on raising...student's van Hiele levels of thought, more so from level 1 to level 2 than for any other levels" (Bobango, 1987, P. 2566). Since the van Hiele levels are good predictors of performance on tests of standard geometry content, the Supposers acting through the van Hiele levels could affect success on geometry content tests.

A full year of laboratory site research near Boston established the *Supposer* "approach to teaching geometry was more fun for most students, enhanced their skill and confidence in thinking mathematically, and, for some students, increased their knowledge of geometry more than their traditional approach" (Wiske, Niguidula, & Shepard, 1988, p. 29).

Clearly, the benefits to students who use the *Supposer* software are supported by the research. *Supposer* use was found to be significantly correlated with van Hiele level and achievement. Since *Supposer* use raises students' van Hiele level (Bobango, 1987), and there is a positive correlation between van Hiele levels and spatial
visualization (Naraine, 1989), this suggests a connection between the use of the
Supposers and spatial visualization ability. However, the Supposers' effect on spatial
ability has not been established.

Investigating the possible correlation between The Geometric Supposer series
and spatial visualization would provide further data regarding the use and effects of
this particular instructional software in the classroom. At the same time, analyzing
the relationships between the software, van Hiele level, and achievement would be
helpful.

The van Hiele model of thinking claims a student's level of geometric
understanding is a function of learning and instruction rather than of development
(van Hiele, 1986). The data support this. Spatial visualization is a cognitive factor
which can also be enhanced by instruction (Brinkman, 1966; Evans & Pezdek, 1980;
Lord, 1985; Wolfe, 1970). Both van Hiele level and spatial visualization are
predictors of success on standard content geometry tests.

The Geometric Supposer series is an instructional aid which has been shown
to raise a student's van Hiele level and improve test scores on geometry content.
Previous research suggests a possible relationship between Supposer use and an
increase in student's spatial visualization, thereby increasing the student's probability
of success on tests of achievement in geometry.

In Figure 1, previous studies have established significant correlations between
instruction, spatial abilities, van Hiele levels, and geometry achievement. These
studies have involved a wide variety of classes, age groups, and instructional methods
and have not looked at all of these relationships at one time for a larger overall
picture.
The present research investigates these same relationships except that it is with the same subjects at the same time, using the Supposer software incorporated with instruction during their study of high school geometry. First, the study will examine the effects of using the software on spatial visualization ability, van Hiele level, and achievement. These are visually depicted in Figure 2.

Figure 1. Model of correlations between instruction, spatial abilities, van Hiele levels, and geometry achievement established in previous research.

Figure 2. A model of the first phase of the study investigating the effects of Supposer use on spatial visualization ability, van Hiele levels, and achievement.
Second, the associations between spatial visualization, van Hiele level, and achievement were examined, as shown in Figure 3. In the spring, when the subjects were well into the school year, the pair-wise correlations between spatial visualization, van Hiele level, and geometry achievement were analyzed.

![Figure 3. A model of the correlations investigated in the study.](image)

The present research looks at all of these relationships at the same time with the same subjects. This information may help in designing instructional strategies which will improve the teaching and learning of geometry.

1.2 Problem Statement

This study seeks to determine (a) the effects of using *The Geometric Supposer* series on spatial visualization ability, van Hiele level, and achievement, and (b) the relationships between cognitive style, maturity of geometric thinking, and achievement (spatial visualization, van Hiele level, and test performance).
The following questions will be investigated:

1. Will using *Supposer* software increase a student's spatial visualization ability?
2. Will using the *Supposer* software increase a student's van Hiele level.
3. Will using the *Supposer* software increase student achievement?
4. What relationships exist among spatial visualization ability, van Hiele levels, and achievement?
5. Will using the *Supposer* software in a series of spatial visualization lessons increase a student's spatial visualization ability?

### 1.3 Definitions

Below is an alphabetical listing of definitions for terms and concepts that have significant meaning for this study. Both constitutive and operational definitions in the context used are given.

**Achievement (ACH):** A quantitative measure of knowledge gained. Beginning achievement is measured by the grade assigned for the first nine-weeks grading period and ending achievement by the score on the end-of-course test.

**The Geometric Supposers:** A series of four software programs in which the user, without the use of a straightedge and compass, can discover geometric concepts by induction. Using the software, students collect numerical and visual data and then form and test conjectures, find examples to support or contradict the conjectures, and develop logical arguments. *The Geometric Supposers* used were: *Geometric preSupposer: Points & Lines*, *The Geometric Supposer: Triangles*, *The Geometric Supposer: Quadrilaterals*, and *The Geometric Supposer: Circles*. 
Phase I: Long-term *Supposer* use, integrated with textbook material, from October through March.

Phase II: One week of spatial visualization lessons using the *Supposers* occurring in May.

Spatial Visualization Ability (SVA): The ability of a student to mentally manipulate, rotate, twist, or invert a geometric figure without verbal or numerical symbols (McGee, 1979), measured by The Card Rotations Test.

Van Hiele Level (VHL): One of five hierarchical levels of geometric maturity indicating the sophistication of the student's thinking, evaluated by the Van Hiele Geometry Test. The levels are: Level 1—Visual, Level 2—Descriptive, Level 3—Theoretical, Level 4—Formal Logic, and Level 5—Nature of Logical Laws.

1.4 Hypotheses

The descriptive research of the first phase of the study investigated the relationships between using computer designed lessons integrated with the text material and the students' SVA, VHL, and ACH.

The following hypotheses were tested:

H1. There will be no difference in spatial visualization scores (SVA) between students using the Supposer Series and students not using the software.

H2. There will be no difference in van Hiele levels (VHL) between students using the Supposer Series and students not using the software.

H3. There will be no difference in achievement (ACH) between students using the Supposer Series and students not using the software.
The relationships between the subject variables SVA, VHL, and ACH were analyzed in a correlational study at the end of Phase I, in the spring.

The following three hypotheses were tested:

H4. Spatial visualization ability (SVA) is positively correlated with achievement.

H5. Spatial visualization ability (SVA) is positively correlated with van Hiele level (VHL).

H6. Van Hiele level (VHL) is positively correlated with achievement (ACH).

In an experimental design for the second phase of the study, the causal relationship of a week of specially designed lessons on the subjects' spatial visualization ability (SVA) was tested.

The general hypothesis was:

H7. Students participating in a series of spatial visualization lessons, using the Supposer software, will have higher spatial visualization ability (SVA) than students who do not receive the series of lessons.

1.5 Educational Significance of the Study

Geometry relies heavily on pictures or diagrams and deductive reasoning; mental transformations of geometric figures is customary. In designing instruction, the students' ability to read and work with diagrams is an important consideration along with the students' level of geometric reasoning. Instruction must be matched with the abilities of the students or misunderstandings might occur (van Hiele, 1986). More recently, the availability of software designed for geometry classes, as well as
the recommendation for using it (NCTM, 1989), introduced another instructional option for consideration. Why should inquiry-based software be used? How can it be integrated with the material? What are the results of its use? What changes will occur? These are a few of a substantial number of questions raised about using educational software in high school geometry classes which continue to be addressed and answered by teachers and researchers alike.

Although, studies have begun to uncover the effects of using the Supposer software in teaching geometry, the relationship between Supposer use and student's spatial visualization ability has not been studied. Since spatial visualization ability plays a role in promoting geometry learning (Fennema & Sherman, 1977; Schonberger, 1976), and software such as the Supposer series is becoming more common in high school geometry classes, it is important to understand their association.

The relationship between using The Geometric Supposer series and students' spatial visualization ability, as well as the relationship with van Hiele level and overall mathematics achievement, is important for making instructional decisions. It is hoped that these findings will help to more fully understand the implications of using The Geometric Supposer software in order that it be utilized to its fullest potential in the classroom.

1.6 Summary

We are now in the midst of mathematics reform; the way mathematics is approached and taught is being critically examined. The emphasis on concepts and abstract mathematics in the 1960s and the "back to basics" movement in the 1970s which narrowed the curriculum, characterize the history of school mathematics in this
country. A movement with an emphasis in one direction was replaced with another in the opposite direction. The persons in the mathematics education community have allowed themselves to be "manipulated into false choices between the old and the new in mathematics, skills and concepts, the concrete and the abstract, intuition and formalism, structure and problem solving, induction and deduction" (National Advisory Committee on Mathematics Education, 1975, p. 3).

At this point in time, the National Council of Teachers of Mathematics (NCTM) published An Agenda for Action (NCTM, 1980) recommending a more balanced approach to curriculum and instruction for school mathematics in the 1980s. These were too general making them rather difficult to put into practice. Specific guidelines for curriculum and instruction were not published by NCTM until the release of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989).

As curriculum and instruction in mathematics cycled through various reforms, there was little evidence of their effect on teaching and learning in the geometry classroom. The traditional manner in which geometry was taught and learned has remained much the same in most classrooms. Conventionally, the content of high school geometry is organized around students' learning to prove the theorems of Euclidean geometry using deductive reasoning. The teacher passes the information along to the students in a traditional didactic presentation. Although this pattern of instruction has changed little during the past 50 years (Usiskin, 1987), the recent introduction of educational software for geometry classrooms has the potential to affect widespread change. The response from the few classrooms in which the software is used now is promising. These are detailed in Chapter II.
The first software for teaching high school geometry, *The Geometric Supposer* series, was first available in the mid-80s. Research has shown the success and value of the *Supposer* series to promote change in the manner in which geometry is taught and learned, a change closely aligned with the recommendations of NCTM. Several other interactive, discovery-based computer programs were developed since, and the NCTM recommends that high school geometry students use this type of interactive software to experiment with geometric shapes and inductively discover relationships (NCTM, 1989).

The introduction of educational technology has often been a catalyst for change and has stimulated research. The effects of using the *Supposer* series on student behavior have generally focused on mathematical behaviors, attitudes, geometry content, and learning behaviors, while teacher behavior studies have concentrated on similar categories, namely, mathematical behaviors, attitudes, geometry content, and teaching behaviors. The cognitive effects have received little attention.

Both spatial ability and van Hiele level of geometric thinking are considered important cognitive factors in teaching and learning geometry. The effects of using the *Supposer* series on spatial visualization, van Hiele level, and achievement, and the interrelationships between these factors should help to provide meaningful data supporting an alternative to the traditionally taught geometry course and address the cognitive needs of the students.
CHAPTER II
REVIEW OF THE LITERATURE

2.0 Introduction

Above the entrance of Plato's lecture room at his famous Academy in Athens, was the inscription, "Let no one destitute of geometry enter my doors". As in ancient Greece, the importance of geometry continues today and it remains an integral part of our school mathematics curriculum.

The National Council of Supervisors of Mathematics (NCSM, 1978, 1989) lists geometry as one of 10 basic skill areas. The National Council of Teachers of Mathematics (NCTM, 1980) believes students need to learn geometric concepts to function effectively in our three-dimensional world. Furthermore, the NCTM's Standards (1989) recommend specific curriculum standards for geometry at every level of the K-12 mathematics curriculum.

Gearhart (1975) surveyed high school mathematics teachers and found 86% felt geometry to be an important part of the college-preparatory mathematics curriculum. Geometry plays a significant role in preparing students for further study in mathematics, particularly calculus where "Many of the ideas needed for understanding college calculus are based on the traditional secondary school geometry course..." (Balomenos, Ferrini-Mundy, & Dick, 1987, p. 195). Rolen (1985) found the final grade in high school geometry to be a good predictor of performance in college calculus I.
After 1870, demonstrative geometry became a part of an American high school education when several major universities made it an entrance requirement (Jones & Coxford, 1970). Not long after, it was recommended that informal instruction of concrete geometry begin as early as kindergarten and elementary school in preparation for the formal secondary geometry course (Committee of Ten, 1894; National Committee of Fifteen, 1912). Smith (1927) expressed similar views in the 1927 yearbook of the National Council of Teachers of Mathematics (NCTM). In spite of these recommendations, Beatley (1933) reported that the Committee on Geometry of the NCTM found our students were getting only a meager introduction to geometry in junior high school. A third report of the Committee on Geometry (Beatley, 1935) included an opinion survey of 101 teachers from eight states who supported the gradual introduction of informal geometry in grade school, with the transitional study in ninth grade leading to the systematic study in tenth grade. This view was consistently reinforced in the 1940, 1973, and 1985 NCTM yearbooks, as well as in the 1987 yearbook which was devoted entirely to geometry K-12.

Continuing to support this viewpoint, NCTM (1989) recommends that "High school geometry should build on the strong conceptual foundation students develop in the new K-8 programs" (p. 157). Geometry is envisioned as an integral part of each level of the K-12 mathematics curriculum and part of the core curriculum, grades 9-12, providing necessary preparation for further study of mathematics.

In spite of the recommendations and curricular reforms which have taken place, geometry appears to be "the most troubled and controversial topic in school mathematics today" (Fey, 1984, p. 31). Usiskin (1987) points to an outdated curriculum and poor student performance as two major problems (Usiskin, 1987). In the third National Assessment of Educational Progress (Carpenter, Lindquist,
Matthews, & Silver, 1983), just 52% of 17-year-olds had taken at least one semester of high school geometry and fewer than 45% could compute the measure of the third angle of a triangle given the measures of the other two angles (NAEP, 1983). Out of all the geometry students in 13 high schools, 99 classes in all, Senk (1985) judged only 31% of the students competent in proof. Data from the Second International Mathematics Study (McKnight et al., 1987) showed our high school seniors answered 31% of the geometry questions correctly, 11 percentage points below the median, and ranked 12th out of 16 countries in geometry achievement (pp. 23-24). Usiskin (1987) feels, "Geometry is too important in the real world and in mathematics to be a frill at the elementary school level or a province of only half of all secondary school students" (p. 30).

This chapter will look at what the literature provides for influencing and improving geometry instruction and learning. Specifically, what role spatial abilities play, how knowledge of the van Hiele levels of geometric thought is influencing what is taught and how it is learned, and the potential of microcomputer software, namely the Geometric Supposer series, for improving geometry instruction and student learning.

2.1 Spatial Ability

"Historically, spatial abilities have been of interest ever since Galton (1883) began his systematic psychological inquiry" (Bishop, 1980, p. 257). In 1938, Thurston (1968) classified spatial-visual aptitude as a primary mental ability, when attempting to clarify the structure of human intelligence. By 1957, the analysis by Michael, Guilford, Fruchter, and Zimmerman (1957) showed the deficiency of "spatial ability" as a single construct for the wide range of spatial tasks which had been
developed. Consequently, individual spatial skills within spatial ability were being identified and distinguished.

Two of the major factors differentiated within "spatial ability" are: spatial visualization and spatial orientation (Connor & Serbin, 1980; Guilford, 1967; McGee, 1979a, 1979b; Zimmermann, 1954). The distinction between these two distinct spatial skills is in identifying whether the object is mentally moved or altered, or whether the perceptual perspective of the subject viewing the object is moved or changed.

McGee (1979b) stated that spatial visualization involves "the ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object" (p. 893). The subject mentally manipulates visual stimuli that are presented pictorially (Fennema, 1975; McGee, 1982). Tests for spatial visualization require a transformation of a mental object (Kersh & Cook, 1979). While mentally moving the object in space is required in spatial visualization tasks, it is not for spatial orientation activities.

The perceptual perspective of the subject viewing the object is moved or changed in spatial orientation tasks. McGee (1979b) felt that spatial orientation tasks "involve the comprehension of the arrangement of elements within a visual stimulus pattern and the aptitude to remain unconfused by the changing orientation in which a spatial configuration may be presented" (p. 909). The subject mentally readjusts his or her perspective, reseeing the object or seeing it from a different angle, but not mentally moving it (Connor & Serbin, 1980; McGee, 1979a, 1979b). The relationship between the subjects' position and the various parts of the object must be recognized and understood.
A factor analysis by Connor and Serbin (1980) and a review of factor analytic studies (McGee, 1979a) support the existence of McGee's (1979b) two distinct spatial abilities. According to Carroll (1974), spatial visualization and spatial orientation require short-term memory.

Tartre (1990) suggests "that spatial orientation skill appears to be used in specific and identifiable ways in the solution of mathematics problems" (p. 227) and spatial skills may be related to organizing thought to help make sense of new material and connecting it to previous knowledge structures. Because spatial ability has been linked as a component of mathematical ability (Krutetskii, 1976; Smith, 1964; Suydam & Weaver, 1977; Werdelin, 1961) and is logically related to mathematics (Fennema, 1975), it has been the focus of much research.

**Spatial Ability-Based Research**

Studies have shown that general spatial ability is significantly correlated with mathematics achievement (Fennema & Sherman, 1978; Lean, 1984; Moses, 1980; Parker, 1989; Shieh, 1985; Smith, 1964; Teles, 1989; Watt, 1990; Weidemann, 1991). Spatial visualization ability is positively related to achievement in mathematics (Burnett, Lane, & Dratt, 1979; Fennema & Sherman, 1977; Guay & McDaniel, 1977; Moses, 1980; Munn, 1991; Schonberger, 1976; Sherman, 1979) and mathematics problem solving (Battista, 1990; Fennema & Tartre, 1985; Pribyl, 1988). Fennema and Sherman (1977) found verbal ability and spatial visualization ability correlated nearly the same with mathematics achievement. Among learning disabled students, aged 6-15, whenever visual-spatial scores were higher than verbal, then arithmetic scores were higher than reading and spelling (Park, 1988).

Fennema (1975) found spatial visualization ability very important for learning mathematics in the primary grades, and Balomenos, Ferrini-Mundy, and Dick (1987)
and Smith (1964) pointed out that spatial visualization ability becomes increasingly important as students continue taking mathematics courses up through calculus.

Spatial visualization ability has been significantly correlated with van Hiele level (Naraine, 1989) and the cognitive style, field dependence-independence (Naraine, 1989; Oleksiw, 1989). Lim (1987) found a significant correlation between spatial visualization ability and right brain dominance.

Specially designed instruction have been successful in significantly raising spatial visualization scores (Brinkman, 1966; Evans & Pezdek, 1980; Lord, 1985; Siegel & Schadler, 1977; Truax, 1988; Wolfe, 1970). However, spatial ability was not improved when students learned Logo turtle graphics (Luckow, 1985), studied solid geometry (Ranucci, 1952), or constructed solids (Cohen, 1959).

Some research has focused on gender differences in spatial ability. Where differences have been observed, the studies showed males scoring higher than females on spatial skills tests (Bagdady, 1990; Battista, 1990; Fennema & Sherman, 1977; Lim, 1987; Mitchelmore, 1974; Vrbancic, 1989), and there was a gender difference in the use of spatial visualization skills (Fennema, 1983). Weidemann (1991) looked at gender differences of students with high spatial visualization scores and found the female group had significantly higher geometry achievement than the male group. Low spatial visualization scores were more debilitating in problem solving for girls than for boys (Fennema & Tartre, 1985). Higher spatial ability for females was a factor for choosing to take more mathematics courses (Stallings, 1979).

Cultural differences in spatial ability have been observed and are increasingly important as our classrooms become more multicultural. Japanese middle school students scored higher than their American counterparts on spatial relations (Iben, 1988). The development of spatial abilities in Navajo children on a reservation was
much the same as that among their counterparts in a nearby city (Cohen, 1985), and there was very little difference between the spatial scores of American students and North American Indians and Eskimos (Kleinfeld, 1973). Davison (1988) found spatial ability scores of Americans higher than Puerto Ricans, and scores in Western cultures were higher than Africans, and citizens of other developing nations of comparable age and education (Smith, 1970).

Investigating the role of spatial ability has not been confined to mathematics education research only. The wide range includes research such as in the relationship of spatial skills to college biology achievement (Sutherland, 1991), functional hemispheric asymmetry (Vrbancic, 1989), decision making performance of master level business students (Ruf, 1990), computer programming (Webb, 1985), and library skills (Eaton, 1990).

2.2 The van Hiele Model of Learning in Geometry

In 1957 with joint doctoral dissertations, Pierre Marie van Hiele and his wife Dina van Hiele-Geldof introduced their model of geometric thought. With Dina's death shortly afterward, the work to clarify, refine, and amend the theory was done by Pierre (1986). The van Hiele model has three main components: insight, phases of learning, and thought levels (Hoffer, 1983).

Many of the ideas for the insight and structure component were "borrowed from Gestalt theory" (van Hiele, 1986, p. 5). "Insight exists when a person acts in a new situation adequately and with intention. The Gestalt psychologist and I say the same thing with different words" (p. 24). To gain insight into a geometry problem, a student must first perceive a structure. In most classrooms, students learn a language
to accompany the structure, although it is possible for students to construct visual structures in their minds and be able to continue the structure without using language.

The second component of the van Hiele model, the phases of learning, describes the stages through which students progress in order to attain the next higher level of thinking. Van Hiele (1986) identifies five stages in this learning process and gives an example of the stages in the study of the rhombus:

1. In the first stage, that of information, pupils get acquainted with the working domain. (For example) A certain figure is demonstrated, it is called "rhombus." The pupils are shown other geometrical figures and are asked if they also are rhombuses.

2. In the second stage, that of guided orientation, they are guided by tasks (given by the teacher, or made by themselves) with different relations of the network that has to be formed. (At this stage) The rhombus is folded on its axes of symmetry. Something is noticed about the diagonals and the angles.

3. In the third stage, that of explicitation, they become conscious of the relations, they try to express them in words, they learn the technical language accompanying the subject matter. (At this stage) The pupils exchange their ideas about the properties of a rhombus.

4. In the fourth stage, that of free orientation, they learn by general tasks to find their own way in the network of relations. (At this stage) Some vertices and sides of a rhombus are given by position. The whole rhombus has to be constructed.

5. In the fifth stage, that of integration, they build an overview of all they have learned of the subject, of the newly formed network of relations now at their disposal. (At this stage) The properties of a rhombus are summed up and memorized (pp. 53-54).
The transition from one level to the next is a learning "process that has to be done by the pupils themselves" (p. 62). Teachers can give guidance to the students during this complicated exercise. "The transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program" (p. 50).

The teachers' choice of lessons and activities is critical in the transition from one level to the next. In this manner, teachers help students find ways to ascend to the next higher level. During this transition, van Hiele considers discussion to be the most important part of the teaching-learning process and without learning a new language, the transition is impossible.

The third component of the van Hiele model grew out of the concern the van Hiele's felt when their geometry students repeatedly encountered difficulties with parts of the subject matter even after being given various explanations. Their joint interest in wanting to improve teaching outcomes led to the development of a theoretical model involving five levels of geometric thinking.

It is the levels of thinking that have generated most of the attention and research. They are described briefly by van Hiele (1986):

- **First Level**: the visual level
- **Second level**: the descriptive level
- **Third level**: the theoretical level; with logical relations, geometry generated according to Euclid
- **Fourth level**: formal logic; a study of the laws of logic
- **Fifth level**: the nature of logical laws (p. 53)

The theory of the van Hiele levels of geometric thinking had its origin in Piaget. As van Hiele stated, "an important part of the roots of my work can be found in the theories of Piaget" (1986, p. 5). However, there exist important differences. The psychology of Piaget was one of development not learning, it differentiated only two levels, it did not recognize the importance of language in progressing from one
level to the next, and in Piaget's theory, the higher structure was primary with the mathematical structure always defining the whole structure. Even though there were important differences, a paper by Piaget mentioning levels led to van Hiele's theory of cognitive levels in geometry.

Hoffer (1981) describes the van Hiele levels of learning in geometry in the following manner:

**Level 1:** Recognition. The student learns some vocabulary and recognizes a shape as a whole. For example, at this level a student will recognize a picture of a rectangle but likely will not be aware of many properties of rectangles.

**Level 2:** Analysis. The student analyzes properties of figures. At this level a student may realize that the opposite sides and possibly even the diagonals of a rectangle are congruent but will not notice how rectangles relate to squares or right triangles.

**Level 3:** Ordering. The student logically orders figures and understands interrelationships between figures and the importance of accurate definitions. At this level a student will understand why every square is a rectangle but may not be able to explain, for example, why the diagonals of a rectangle are congruent.

**Level 4:** Deduction. The student understands the significance of deduction and the role of postulates, theorems, and proof. At this level a student will be able to use the SAS postulate to prove statements about rectangles but not understand why it is necessary to postulate the SAS condition and how the SAS postulate connects the distance and angle measures.

**Level 5:** Rigor. The student understands the importance of precision in dealing with foundations and interrelationships between structures. This most advanced level is rarely reached by high school students. At this level a student
understands, for example, how the parallel postulate (Euclidean) relates to the existence of rectangles and that in non-Euclidean geometry rectangles do not exist (pp. 13-14).

Since his initial work, van Hiele (1986) has focused his attention and in-depth descriptions to Levels 1 through 4, for it is at the lower levels that most geometry students function. It is more likely that a thorough understanding of the lower levels will lead to improving the teaching and learning of geometry. The higher levels are easily over-valued and have only theoretical value.

The main characteristics of the levels are:

1. The levels have a hierarchic arrangement through which the person moves sequentially.
2. Moving from one level to the next is more a result of a learning process rather than a result of age or maturation.
3. The learning process which leads to a higher level is distinguished by various phases of learning.
4. Each level has a unique language, set of symbols, and network of relations joining these symbols.
5. What appears in an explicit manner at one level was intrinsic at the preceding level.
6. A person reasoning at a higher level cannot be understood by another person at a lower level.
7. Material taught above a person's level may be reduced to a lower level by that person.

The levels of thinking grew out of van Hiele's interest in improving instruction in geometry, however the same "levels" approach can be used in the teaching and learning of other topics as well (van Hiele, 1986).
Between 1960 and 1964, the Soviets were quick to organize research on the van Hiele levels of development, verifying and validating the assertions and principles in the van Hiele model. As reported by Wirszup (1976), the Soviets developed a new 10-year geometry curriculum for grades 1 through 10, gradually introduced since 1969. "The new curriculum is clearly the most radical change in Russian mathematics education in nearly a century" (Wirszup, 1976, p. 96).

It wasn't until the mid-seventies that the Americans' attention was drawn to both the van Hiele levels and the Soviet studies. Isaak Wirszup's lecture entitled "Some Breakthroughs in the Psychology of Learning and Teaching Geometry", given at the Closing General Session of the NCTM Annual Meeting in 1974, introduced a new approach to instruction based on the van Hiele levels and the successes of the new model Soviet curriculum for grades 1 through 10. Since that time, there has been considerable interest and research in the van Hiele model, which has affected curriculum design, text preparation, teaching methods, and planning of activities and materials.

Van Hiele-Based Research

Even though Wirszup (1976) introduced the van Hiele theory to the American mathematics educators in the mid-seventies, research lagged in response. In the early 80's, the topic was gaining more recognition and interest. Carpenter (1980) claimed that if the van Hiele model of thinking was correct, it would have a profound effect in changing instruction in geometry. As a result of the growing interest in the United States, three projects were funded by the National Science Foundation, Research in Science Education Program, during 1980-83. These federally funded investigations of the van Hiele model were directed by William Burger, Oregon State University, Dorothy Geddes, Brooklyn College, and Zalman Usiskin, University of Chicago.
Burger and Shaughnessy (1986a) interviewed 45 students, of average and above ability in grades K-college, from 5 school districts in Oregon, Michigan, and Ohio on triangle and quadrilateral concepts. The research was conducted to develop an interview procedure to disclose the student's van Hiele level, to determine the usefulness of the van Hiele levels in describing students' thinking on specific geometry tasks, and to operationally characterize the levels in terms of specific tasks. An interview script and analysis packet are included in the final report (Burger & Shaughnessy, 1986a) which can be easily administered by teachers and researchers.

Research at Brooklyn College (Fuys, Geddes, Lovett, & Tischler, 1988) had four research objectives. First, several of the van Hiele's writings were translated into English, from which a detailed model of the levels was formulated. Three assessment instructional modules were developed and used in clinical interviews with 32 sixth and ninth graders to characterize their thinking in geometry. Pierre van Hiele himself conferred with the project staff and validated the module-based clinical interviews for assessing the students' level of thinking. Videotaped interviews were used also with 8 preservice and 5 inservice teachers to determine if teachers could be trained to recognize the van Hiele levels in students' thinking and in geometry curriculum materials. Finally, three widely used commercial textbook K-8 series were analyzed in view of the van Hiele model.

The Usiskin research team (1982) collected data from approximately 2,700 geometry students in 13 high schools in Florida, Maine, Michigan, Illinois, and California, with students representing a broad diversity of cultural, socioeconomic, and educational backgrounds. The project investigated if entering geometry students could be assigned to one of the van Hiele levels and, if this was feasible, what changes in these levels take place throughout a year-long study of geometry. Other research
questions were: How are the levels related to achievement? What is the predictive nature of entering van Hiele levels, and to what extent is there a match between the level of teaching and the level of the student and how does this differ across different educational and socioeconomic environments?

Two instruments were developed within the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project at the University of Illinois (Usiskin, 1982), the CDASSG Proof Test to assess proof-writing achievement and The Van Hiele Geometry Test to test the van Hiele theory, its descriptive and predictive nature. It is the latter test which gained widespread recognition in the United States. Usiskin and Senk (1990) commented, "The test has been more widely used than we would ever have imagined... (and) has almost universally been used to determine van Hiele levels for a set of individuals" (p. 242).

Starting with these three rather lengthy research projects, the van Hiele model of thinking has become an important issue in American mathematics education. The researchers have explored the van Hiele level theory and its usefulness in improving education, particularly in teaching geometry.

The Soviets' research on the van Hiele levels of development was the first to verify and validate the assertions and principles of the van Hieles' (Wirszup, 1976). Subsequent research, largely in the United States, started on a similar path in the 80's. Focusing on validating important characteristics of the van Hiele model was one major thrust of research, along with investigating the predictive quality of the levels, use of the theory in instruction and textbooks, and the applicability the level theory to other areas.

Researchers validated the hierarchical nature of the levels through which the student moves sequentially (Burger & Shaughnessy, 1986b; Denis, 1987; de Villiers
& Njisane, 1987; Fuys et al., 1988; Mason & Schell, 1989; Mayberry 1983; Wilson, 1989). Although the hierarchy of the levels is supported, research fails to confirm the discreteness of the levels (Burger & Shaughnessy, 1986b; Fuys et al., 1988; Kieren & Olson, 1983; Senk, 1985; Usiskin, 1982).

Usiskin (1982), using the 25-item multiple choice Van Hiele Geometry Test, was able to classify only 68% to 92% of students into a van Hiele level, the percentages varying with the criteria used. Burger and Shaughnessy (1986b), while using the responses on clinical interview tasks to assess a student's van Hiele level, detected students in transition between levels. Some students "oscillated from one level to another on the same task under probing from the interviewer" (p. 45). Observation of transitional thinking was also observed by Fuys et al. (1988) during interview and instruction modules. These observations might suggest the continuous nature of the levels, however, is it continuity in learning or continuity in teaching that is being observed (Hoffer, 1983)?

Not all students are at the same van Hiele level on all concepts (Denis, 1987; Fuys et al., 1988; Gutierrez & Jaime, 1987; Mayberry, 1983). Although, Fuys et al. (1988) found each student's level seemed to be consistent across topics after instruction.

Establishing a student's level of reasoning plays an important part in van Hiele model research. Assessment criteria for determining the level are based on either the performance on a written test or the level of thinking shown during interview activities. When a student does not fit a particular level, the solution varies. Students who cannot be assigned to one van Hiele level and appear to be in transition between two levels, for example Level 1 and Level 2, are either excluded from the study, assigned to Level 1-2, or designated to one level by a consensus of the evaluators.
Because of the difficulty of assigning some students to one particular level, levels which do not appear to be discrete, Gutierrez, Jaime and Fortuny (1991) have proposed an alternative method for analyzing the van Hiele level of students' geometric thinking. Rather than assigning students to a specific level, a clearer more accurate picture would be to determine the degree of acquisition within each van Hiele level. This yields a profile of the student's geometric thinking across the van Hiele levels. Students' reasoning processes are complex; students "do not behave in a simple, linear manner, which the assignment of one single level would lead us to expect" (p. 250). In assessing the performance on a written and manipulative test, Gutierrez, Jaime, and Fortuny (1991) felt characterizing the geometric thinking associated with each van Hiele level more fully represented the students' geometrical reasoning than the traditional assignment of one level.

The usefulness of characterizing the geometric thinking of students and teachers has been explored (Bobango, 1987; Burger & Shaughnessy, 1986b; Denis, 1987; de Villiers & Njisane, 1987; Fuys et al., 1988; Kay, 1985; Lowry, 1987; Mayberry, 1983; Senk, 1985, 1989; Usiskin, 1982). The van Hiele level of high school geometry students at the beginning of the year can be used to predict performance later in the year, on end-of-the year geometry achievement (Johnson, 1988) and proof achievement (Senk, 1985; Usiskin, 1982). Research has established correlations between a student's final grade in geometry and van Hiele level at the end of the year (Corley, 1990), as well as after 20 days of phase-based instruction (Bobango, 1987). The significant positive correlation between van Hiele level and proof achievement was reported by Bobango (1987), Senk (1989), and Usiskin (1982), but not found by Braconne and Dionne (1987). Investigating the relationship between Piagetian stages and van Hiele levels, Denis (1987) found a significant
difference in the means of the van Hiele levels of students at the concrete and formal operational stages.

Van Hiele (1986) believes that progression through the levels is more dependent on instruction rather than maturation. Not only have studies shown instruction significantly increases the van Hiele levels of 9th- and 10th-grade geometry students (Bobango, 1987; Corley, 1990; Fuys, 1985; Johnson, 1988; Usiskin, 1982), but 4th- through 8th-grade students as well (Fuys, 1985; Kieren & Olson, 1983; Ludwig & Kieren, 1985; Mason, 1989; Yusuf, 1990).

Materials and methodology have been designed to match the levels and promote growth through the levels (Burger, 1985; Burger & Shaughnessy, 1986b; Fuys, Geddes, & Tischler, 1985; Geddes, Fuys, Lovett, & Tischler, 1982; Shaughnessy & Burger, 1985). In the late 50's, Dina van Hiele-Geldof developed a series of 70 lessons which could lead students from the first level to the third level (van Hiele & van Hiele-Geldof, 1958). During the 60's, the Soviet Union used the van Hiele theory to implement changes in the geometry curriculum, beginning in the elementary grades and continuing through the secondary school. In the Netherlands, van Hiele (1986) has written mathematics textbooks in the levels of thinking framework.

The match between the level of instruction and the level of the student is important for each level has its own, unique language, set of symbols, and network of relations. The van Hiele model contains recommendations for textbook change in accordance with the levels. If a student is at one level and the instruction and textbook are at a higher level, the student will not be able to follow the thought processes being used and might completely misunderstand the material (van Hiele, 1986).
Preservice elementary teachers usually act at the first or second van Hiele level (Matos, 1985; Mayberry, 1983; Gutierrez & Jaime, 1987). Burger and Shaughnessy (1986a) and Fuys (1985) found mainly Level 1 thinking for students in K-8. After some instruction, many 6th-grade students moved up to Level 2 or Level 3 (Fuys, 1985).

In an analysis of van Hiele levels of thinking of geometry material in three major U.S. textbook series for grades K-8, Fuys et al. (1988) found most geometry material at Level 1. The small amount at Level 2 had in most cases been reduced to Level 1 by the use of rote exercises. These texts provide very little opportunity for students to progress to a higher level even though some 6th-grade students are capable of Level 2 and Level 3 thinking (Fuys, 1985).

A unified 7th-grade mathematics program, using a text series adapting exercises to the van Hiele phases for promoting growth through the levels, promoted thinking leading to Level 2 (Joyce, 1984).

The majority of U.S. students enter 10th-grade geometry at Level 1 or Level 2 (Burger & Shaughnessy, 1986a; Shaughnessy & Burger, 1985; Usiskin, 1982; Wirszup, 1976). Traditional geometry texts are written with exercises starting at Level 1 and jumping to Level 4 (Usiskin, 1982). The use of Level 4 reasoning among secondary and post secondary school students is nearly absent (Burger & Shaughnessy, 1986a; Usiskin, 1982). As a result, students do not understand axiomatics or proof for which Level 4 reasoning is needed. About 40% of high school students do study proof (Usiskin, 1982). A student entering the year at Level 2 has a 50-50 chance of success with proofs by the end of the year, entering at Level 3 there is a good likelihood of success (Senk, 1989; Usiskin, 1982).
The effects on attitude and achievement of students using a textbook consistent with the van Hiele theory, in which proof was delayed until the last half of the year, were compared with students using a more traditional textbook which emphasized proof throughout (Han, 1986). At year's end, the students using the traditional textbook were significantly better in proof-writing achievement and had a more positive attitude toward proof, even finding geometry easier than the students using the text conforming with the van Hiele theory.

The applicability of the van Hiele theory to transformation geometry was investigated, by first establishing levels for transformational geometry, corresponding to the van Hiele levels for Euclidean geometry. The levels for transformation geometry were found valid, the levels formed a hierarchy (Molina, 1990; Nasser, 1989), and correlation between the van Hiele levels in traditional geometry and transformation geometry was found (Nasser, 1989).

An association between the van Hiele levels of geometric thinking and geometry learning in Logo has been the focus of some research. A theory of the relationship between levels of Logo use and the van Hiele levels of learning was researched by Ludwig (1986) who found the Logo/geometry behavioral changes observed could be interpreted within the framework of this theory. Olson, Kieren, and Ludwig (1987) found the van Hiele levels a convenient way of analyzing the relationship between geometry and Logo and to describe levels in Logo activities and language use. The level structure was used to analyze concept learning and reasoning abilities in a Logo environment (Kieren & Olson, 1983). Logo instruction and materials were developed based on the van Hiele model with activities corresponding to the van Hiele phases of learning (Kieren, 1984; Lehrer, Randle, & Sancilio, 1987).
Logo instruction and activities were found to significantly increase van Hiele levels (Ludwig & Kieren, 1985; Mason, 1989; Scally, 1986; Yusuf, 1990).

Not only has the van Hiele model been applied to other areas in mathematics, but to other disciplines as well, for instance chemistry (Crowley, 1987) and an economics textbook (van Hiele, 1988).

2.3 The Geometric Supposers

The Geometric Supposer (1985) software series, published by Sunburst Communications, Inc. and designed by Judah L. Schwartz, Michal Yerushalmy, and the Center for Learning Technology at Education Development Center (EDC), is a tool for investigating and reinforcing geometric concepts and relationships. The Supposer series is recommended for grades 5 through 12 and is designed for a wide range of ability levels. Wherever used, the "Supposers provide an environment which encourages geometric curiosity among computer novices as well as experts" (Mathis & Hanfling, 1986, p. 45).

Presently, The Geometric Supposer series is composed of four microcomputer programs. Titles in the series are: The Geometric preSupposer: Points and Lines, The Geometric Supposer: Triangles, The Geometric Supposer: Quadrilaterals, and The Geometric Supposer: Circles. The Geometric preSupposer was developed for use primarily at the elementary and middle school level to help students define and classify geometric figures, and acquire basic concepts and relationships such as measurement, congruence, similarity, and parallelism. It is at the elementary and middle school level where students take an intuitive and exploratory approach in learning geometry, an approach complimented by the design of the Supposer software series, allowing its effective integration in the lessons. The remaining three in the
series, *The Supposer: Triangles, Quadrilaterals, and Circles*, are appropriate, but not exclusively used, in secondary geometry courses.

*The Geometric Supposer* is available from Sunburst Communications, Inc.© on either a 3.5 inch or 5.25 inch data disk for Apple®, IBM PC®, IBM PS/2®, Tandy 1000®, and Macintosh® computer systems. Both the single-user version and the lab pack come with a teacher's guide. Other materials designed to facilitate use of the software include booklets and manuals suggesting ways to integrate the software with widely used textbooks along with unit outlines, activities, and reproducible blackline masters.

The *Supposer* series has been generally well received and acknowledged as a software tool for teaching geometry. Two in the series, *The Geometric preSupposer* and *The Geometric Supposer: Quadrilaterals*, were recognized in the top six educational software titles chosen by the 1987 Classroom Computer Learning Software Awards. The *Supposers* have also received recognition by being on a list of "best" software packages relevant to mathematics instruction (Dickey, 1988).

The *Supposers* can be used either by the teacher for a class presentation or in a laboratory setting with the students working individually or in small groups. In either situation, lessons can be designed to introduce new concepts, for students to invent or discover theorems and definitions, to explore problems, to answer questions, and generally extend their knowledge of geometry. Students use this software in an exploratory manner to hypothesize, using induction, about relationships and patterns they find in geometric figures. The *Supposers* can be integrated in Euclidean geometry courses to be used in "laboratory sessions where students investigate empirically the characteristics of geometric constructions" (Yerushalmy & Chazan, 1990, p. 201).
Compass and straightedge constructions are possible with the *Supposers* incorporating construction of segments, perpendiculars, parallels, angle bisectors, and inscribed and circumscribed circles. In addition, numerical measurements of lengths, angle measure, area, circumference can be made and recorded separately or coupled by sum, difference, product, or ratio. Students begin with a shape, assemble a geometric construction, and take measurements of the resulting diagram.

The constructions, measurements, and arithmetic operations on the data are recorded as a procedure and can be replicated on a new figure using the REPEAT function, thereby allowing the students to make and test their conjectures about properties and relationships on new and different shapes in a class of figures. A student discovering a relationship in one particular example can immediately explore the validity of that relationship in many equivalent cases without the cumbersome use of pencil and paper. Conjectures can be explored quickly and accurately. Possible misleading visual cues from sketches are also avoided.

REPORT, SAVE, and LOAD options are included in the more recent versions of *The Geometric Supposers* for the Macintosh and IBM systems. Using REPORT, printed material can be written below the current figure. SAVE and LOAD allows constructions, measurements, and the written report saved to a data disk file, which can all be retrieved for later work.

Further options and improvements for the software series are being considered and studied (Yerushalmy, 1990). The developments are directed toward easier handling of large amounts of data and for facilitating conjecture making.

*The Geometric Supposer-Based Research*

Starting in the 1984-85 school year, *The Geometric Supposer* was used initially in a variety of Boston area middle and high school settings where its authors,
Schwartz, Yerushalmy, and the Educational Technology Center (ETC) staff, closely collaborated with the classroom teachers to insure the successful integration of the software series. The ETC Geometry Group provided *Supposer* problems and solutions to use in the classes. Later in the school year, as the classroom teachers became more adept with the software, most teachers modified the problems and worksheets for their classes.

The nearby site enabled the authors of the series and ETC Geometry Group to conduct research concerning the integration of the *Supposer* in typical 10th grade Euclidean geometry classes. This research at the Laboratory Sites in Boston, during the five year period from 1984-1989, included year-long studies of *Supposer* classes, some compared with traditionally taught classes, and studies of smaller instructional units, approximately one month in length.

In the first of two year-long studies with *Supposer* and comparison classes, Yerushalmy (1986) investigated the inductive skills of students in generalizing conjectures from empirical data, and identified the skills and reasoning processes involved in developing convincing arguments. The second study the following year, (Yerushalmy, Chazan, & Gordon, 1987), explored the implementation of the guided-inquiry approach using *The Geometric Supposer*, assessed student learning, and examined the implementation of the software in terms of classroom dynamics and from the teachers' and students' perspective.

During the third year at the Laboratory Sites, the year-long research focused on the effect of the written materials used during the year in directing student learning with the *Supposers* (Yerushalmy, Chazan, & Gordon, 1988). In a shorter study that year, Chazan (1988) studied four *Supposer* classes working with a unit on similarity,
followed the next year by working with five Supposer classes with a unit on proof (Chazan, 1989).

Further studies (Yerushalmy, 1990; Yerushalmy & Chazan, 1990) used data gleaned from all or part of the five year NIE-funded ETC project, a project producing the major portion of the Supposer research. The Supposer-related studies from the ETC Laboratory Sites and the few disassociated from the five-year project deal with a variety of issues concerning the integration of the software in the classroom.

The research focuses on (a) issues of implementation and management best suited for teaching with The Geometric Supposer, (b) changes in beliefs, attitudes, and roles of both the teachers and the students using The Geometric Supposer, (c) assessing student outcomes when using The Geometric Supposer, and (d) observing student thought processes while using The Geometric Supposer.

Students are generally not familiar with inductive work (Yerushalmy, 1986). Supposer-based research has identified practical and successful strategies for introducing student inquiry in the classroom (Yerushalmy et al., 1988; Wiske & Houde, 1988, Wiske, Niguidula, & Shepard, 1988). To effectively teach with the Supposers, well-designed worksheets and appropriate problems are essential prerequisites (Yerushalmy et al., 1988). Characteristics of good problems to use with the Supposer differ from traditional textbook problems and must leave room for student initiative and creativity, neither specifying too much nor too little instruction.

Worksheets and activities are available in manuals accompanying the Supposer software (Schwartz, Yerushalmy, & ETC, 1985) as well as from other manuals attainable from Sunburst Communications, Inc. Additional resources include material used in the teaching experiments at the ETC Laboratory Sites (Chazan, 1990; Schwartz & Yerushalmy, 1987; Wiske et al., 1988; Yerushalmy et al., 1987, 1988;
Yerushalmy & Houde, 1986) and from teachers sharing their classroom activities (Driscoll & Confrey, 1985; Olsen & White, 1989; Troutner, 1988). Also, Sunburst Communications, Inc. sponsors The Geometric Supposer Society open to users of the software to encourage the exchange of ideas.

Lessons in ready-to-use form, including homework exercises, were developed by the Project to Increase Mastery of Mathematics and Science (PIMMS) for The Geometric preSupposer and The Geometric Supposer: Triangles and Quadrilaterals (Cetorelli, DeMeo, Grant, Hastings, & Kopij, 1988; Cetorelli, Kopij, & Llorda, 1987, 1989). A description of the approach to teach with the Supposer and recommendations for teachers and students using the software are included with the lessons.

Using the Supposer to incorporate inductive teaching in a traditional geometry course requires new skills and significant changes in the teacher's familiar role as well as the students' (ETC Third Year Report, 1986; Lampert, 1988a, 1988b; Wiske & Houde, 1988; Wiske et al., 1988; Yerushalmy, 1986; Yerushalmy et al., 1987, 1988). Teachers need to understand and model problem-solving strategies for students to emulate, provide students with direction and assistance in synthesizing the data, and connect student findings with curricular objectives (Yerushalmy et al., 1987). Specific skills are needed in lab management and supervision, in leading productive discussions, and in assessment (Wiske et al., 1988).

Teaching with the Supposer challenges the customary beliefs and attitudes of many teachers (Wiske et al., 1988; Yerushalmy et al., 1987). Characteristically, The Geometric Supposer assumes students can inductively discover much of the geometry in a 10th-grade course, sometimes in a different sequence from that intended, and learn facts well without being taught. Students exploring with the Supposer are
capable of making conjectures or asking questions for which the teacher might not always know the answer or might even give an incorrect answer. Teachers are challenged to change when working with the software, and students are, too.

An inquiry approach demands students to take more responsibility for learning. One student interviewed said,

It's different. It's like abstract thinking. It's different than anything else you've done. ... Maybe a little harder than I expected. We have to think about everything that you learn, instead of just having a teacher teach you, memorize, and just do it. You have to think about it yourself (Yerushalmy et al., 1987, p. 48).

The student is characterizing learning with the Supposer by contrasting "teach and memorize" with "learn and think".

As work with the Supposer progressed through the year, students' appreciation of data decreased, while their appreciation of data as a source of ideas increased (Yerushalmy, 1986). There was a gradual shift in focus from data collection to making conjectures, and finally, to formal proof (Yerushalmy et al., 1987).

The findings concerning the facility with which students make conjectures is mixed. High ability students observed during the 1984-85 school year made conjectures easily (Yerushalmy, 1986), while students from all levels of ability participating at the ETC lab sites the following year had much greater difficulty in making conjectures (Yerushalmy et al., 1987).

Compared to the control group, students working with the software were more likely to use non-conventional methods of analysis, had more arguments to support their conjectures, recognized the need for formal proof, gave significantly more arguments supporting their generalizations, and on the midterm and final exams,
performed as well as or better than those not working with the *Supposer* (Yerushalmy et al., 1987).

McCoy (1990), also looking at achievement, compared students using the *Supposer* one hour every two weeks throughout the year in 10th grade geometry class to geometry students not using the software. The students using the *Supposer* were significantly better on total geometry achievement, applications, and higher-level items, but remained statistically the same on lower level items.

Researchers and teachers involved in earlier ETC projects felt another important difference, between students learning geometry with the *Supposer* and those who did not, involved students' use of diagrams. In the earliest study, Yerushalmy (1986) noted "Diagrams seemed to accompany the thinking process of group A (*Supposer*) students...there were numerous free-hand drawings" (pp. 95-96). These opinions from earlier studies led Yerushalmy and Chazan (1990) to reexamine classroom observations, student papers, written tests, and data relating to students' use of diagrams from previous ETC research.

Yerushalmy and Chazan (1990) identified three learning obstacles which traditionally beset students when interpreting geometric diagrams. They are: (a) Understanding "that a diagram may include characteristics not shared by all members of the class and (being) able to envision mentally other possible configurations of the diagram" (p. 210)., (b) Having the ability to focus on different parts of a diagram and changing their point of view., and (c) Using "auxiliary lines to look for previously learned connected facts" (p. 213). In their post-hoc analysis, Yerushalmy and Chazan (1990) found students using the *Supposer* overcame each of the three obstacles and "understood diagrams and their limitations better than students from the traditional classrooms portrayed in the research literature" (p. 216). The
Supposer students' approach to diagrams was flexible; they could focus on particular parts of the diagrams, add auxiliary lines to generate conjectures, look for non-standard diagrams, create and examine sequences of related diagrams, and used these skills to analyze problems and prove their conjectures.

In the first of three shorter instructional studies, Bobango (1987) used the Supposer in phase-based instruction, for a 20-day period. The data revealed the Supposer "had a significant effect on raising...student's van Hiele levels of thought, more so from level 1 to level 2 than for any other levels" (p. 2566).

In the second, a unit on similarity was constructed for use with the Supposer (Chazan, 1988). The lab setting enabled Chazan to directly examine student thought processes in understanding similarity, providing a greater understanding of students' difficulties with the concept.

Chazan (1989) studied five geometry classes during a unit on proof to explore students' conceptions of a mathematical way of knowing whether geometrical statements are true. Two widely held problematic conceptions were found. First, a large number of students felt a deductive proof was "proof" for only the example illustrated in the accompanying diagram. Second, when working with the Supposer, a sizable number of students were confident that measuring examples was sufficient to reach a conclusion safe from any counterexamples. When students worked the entire year with the Supposer, they relied less on data for proof, and valued more the need for formal proof (Yerushalmy et al., 1987).

Students working with the Supposer generate large quantities of empirical information, visual and numerical. To what empirical information students are most likely to pay attention and how students make use of the information generated was analyzed by Yerushalmy (1990), using ETC Laboratory Sites data collected from
This provided background for design changes for a new version of the software which "could possibly direct students to form better criteria more rapidly for the generalization of data into hypothesis in geometry" (p. 31). Pilot studies were conducted (Yerushalmy, 1990) and field experiments are now in progress using a few additional tools developed for the new version of the Supposer.

With one tool, the multiple on-screen windows, students were able to make easier visual comparisons having four figures on the screen at once, resulting in a greater number of students identifying a general phenomenon of a class of figures better than with the old screen layout. This increased the possibility of forming a hypothesis from the data.

A second tool provides a method to observe and define a locus. It enables the user to move a point, in many cases a vertex, on a geometrically defined track, along with repeating on the new figure all or part of the constructions made on the original figure. A family of figures is defined by the movement, making it possible to identify common properties for the family of figures and arrive at a conjecture for the locus. Both of these tools were developed to give students a greater ability to identify visual generalizations.

A collection of options for the REPEAT algorithm has been created, also. As a student carries out constructions, a list is automatically created which appears on the screen whenever a REPEAT manipulation is requested. The student repeats the entire block or can define sub-blocks for subsequent constructions, with each command being highlighted as it is completed on the screen.

As previously shown (Yerushalmy et al., 1987), these tools will need written materials and teacher support for the additional complexity they will bring to the
software. Generalization techniques for greater success in making conjectures will need to be taught.

2.4 Summary

Since Allendoerfer (1969) wrote, "It is easy to find fault with the traditional course in geometry, but sound advice on how to remedy these difficulties is hard to come by" (p. 165), there has been a plethora of recommendations, largely based on research, for changes in the geometry curriculum (Conference Board of the Mathematical Sciences, 1983; Driscoll, 1986; Freudenthal, 1973; Mathematical Sciences Education Board, 1990; NCTM, 1989; Suydam, 1985; Usiskin, 1982, 1987; van Hiele, 1986; Wirszup, 1984).

Freudenthal (1973) feels one of the main reasons for the lack of success of traditional Euclidean geometry is the excessive emphasis on logico-deductive reasoning and lack of attention to the spatial abilities which are essential prerequisites for understanding and mastery of geometry concepts. The Soviets took this position in revising their geometry curriculum, concentrating on the visuo-spatial aspects first, beginning in the early school years, before introducing the logico-deductive reasoning much later in a formal geometry course (Wirszup, 1976).

The new Soviet geometry curriculum and methods of instruction were heavily influenced by the van Hiele model of learning. Following intensive research and experimentation on the levels of development, which verified the validity of van Hiele's assertions and principles, the Soviets established "a single sequence in the formation of mathematical concepts for the entire eight-year school, beginning with the first grade" (Wirszup, 1976, p. 91). Special attention was given to texts, workbooks, visual aids, models, films, and slides to aid students through the levels.
The Soviets found films, in particular, to be "an effective means not only of developing geometric concepts but also of forming spatial conceptions" (Wirszup, 1976, p. 92).

Spatial abilities have a complex role in understanding geometry. Smith (1964) asserted that "the spatial factor appears to have a greater claim for consideration as an essential basis for aptitude for math" (p. 126). Fischbein (cited in Hershkowitz, 1990) claimed "that what one cannot imagine visually is difficult to realize mentally" (p. 103).

Similarly, here in our country, the research and recommendations for revising the geometry curriculum are following much the same path. The van Hiele theory was verified for classifying students at different levels by descriptions of their behavior (Usiskin, 1982). Furthermore, these classifications of students' levels could be used as predictors of achievement in a traditional high school geometry course. For those students found "below level", effective activities can be given to students prior to a formal course in geometry to better ensure their success (Fuys, Geddes, & Tischler, 1988).

The computer, with its interactive capacity, provides strong visual elements creating learning situations with the potential to enhance visual skills, facilitate learning specific geometric concepts, and develop thinking processes. "There is much room for using computers in geometry. The power of graphics packages makes it much easier for students to get a visual sense of geometric concepts and transformations" (Conference Board of the Mathematical Sciences, 1983, p. 4).

The importance of spatial abilities, van Hiele level theory, and the use of interactive software in geometry instruction are evident in the recommendations for
curriculum and instructional changes. One of the leaders in the curriculum changes for school mathematics is the NCTM.

Specific objectives for geometry, K-12, published by the NCTM (1989), include developing geometry and spatial sense at all grade levels in the primary and in the middle school grades in preparation for secondary geometry. The van Hiele theory has been interwoven throughout these recommendations and "serve[s] as an excellent blueprint for the incorporation of the van Hiele theory" (Teppo, 1991, p. 214).

The NCTM (1989) further recommends that geometry "students should first use an interactive computer software package that allows experimentation with figures and relations..." (p. 159) to strengthen the relationship between inductive and deductive reasoning. *The Geometric Supposer* software is one of several computer-based systems which are powerful tools enabling students to make geometric constructions and explore the results. Teaching and learning geometry in a computer environment has stimulated new directions in research and instruction.

Research involving spatial abilities, van Hiele level theory, and interactive computer software has been the basis for many of the changes and recommendations for making geometry understanding more accessible to students. Geometry is an important part of the mathematics curriculum. Today, as reported in the National Research Council's (1989) *Everybody Counts*, "over 75 percent of all jobs require proficiency in simple algebra and geometry, either as a prerequisite to a training program or as part of a licensure examination" (p. 4) and more than 3 out of 4 university degree programs require mathematics courses. Knowledge of geometry was important for Plato's students as well as for our students today.
CHAPTER III

METHODOLOGY

3.0 Introduction

The purpose of this research was to investigate (a) the long term effects of using *The Geometric Supposer* software on spatial visualization ability, van Hiele level, and achievement when the software is incorporated with the lessons in the text, (b) the relationships between cognitive style (spatial visualization), maturity of geometric thinking (van Hiele level), and achievement (test performance), and (c) the effects of a series of spatial visualization lessons, using the *Supposer* software, on students' spatial ability.

The study was conducted in two phases. The subjects in two intact geometry classes participated throughout the entire study. In the first phase, from October through March, a nonequivalent control group design was used. Both classes were given pre- and posttests. The class designated as the treatment group used *Supposer*-based materials during the 5-month phase while the other class, acting as the control group, did not. This allowed the researcher to investigate the long-term effects of *Supposer* use and the relationships between spatial visualization ability, van Hiele level, and achievement, (a) and (b) above.

The second phase occurred in May and examined the effects of a full week of spatial visualization lessons on the subjects' spatial ability. Subjects from both classes
were randomly assigned to work with the lessons during either the first or second week they were given. By using a 2×2 factorial design, the main effects for the spatial visualization lessons and for previous Supposer use on the students' spatial ability were examined as well as the interaction between these two independent variables.

This chapter describes the sample, procedures, equipment, instrumentation, data collection, statistical analyses, and limitations of the study.

3.1 The Sample

This research was conducted during the 1991-1992 school year in the high school of an upper-middle class city school district in suburban Columbus, Ohio. High school enrollment was approximately 1,800. The school was selected because of its proximity to The Ohio State University, the availability of the school's computer lab and computers for classroom use, and the willingness of one of the geometry teachers, Mrs. B, to participate in the study and learn to work with the software.

This was the first year for the high school to change the sequence of mathematics courses from Algebra I-Algebra II-Geometry to Algebra I-Geometry-Algebra II. As a result, there were twice as many geometry students. Classes were a combination of 10th and 11th-grade students, coming from either Algebra I or Algebra II classes the previous year, and a few 12th-grade students who had repeated Algebra II. With the exception of four honors geometry classes, the majority of students taking geometry signed up for the regular-level geometry. Mrs. B taught two of the regular level geometry classes.

The final sample of 39 subjects was comprised of all students enrolled in Mrs. B's two geometry classes throughout the entire year. The sample represented a heterogeneous mix of ability and grade levels.
There were 16 subjects in the treatment group which included eleven 10th-grade, two 11th-grade, and three 12th-grade students. The control group was made up of seventeen 10th-grade, four 11th-grade, and two 12th-grade for a total of 23 students.

Although calculators were generally used in the mathematics classes in the high school, computers were not. The mathematics office had two Apple® computers on carts. The office also had a small software library, where one of the programs was *The Geometric Supposer: Triangles*. As far as Mrs. B knew, this was the first time computers were used in any of the geometry classes, whether for whole class demonstration or for student use.

Mrs. B had been teaching high school mathematics, Algebra I, Algebra II, and Geometry, for 10 years; this was her 5th year at the present high school. Mrs. B had never before used software programs or computer demonstrations in her classes and did not have experience teaching a class in a computer laboratory setting.

For the initial lab sessions, these circumstances required the researcher to be present in the computer lab to help Mrs. B with student questions and any potential hardware or software problems. When a whole-class presentation was necessary on one occasion, the researcher conducted the lesson, again as one who was "more familiar with the software." With the exception of this particular lesson, all the other lessons in the computer lab and the classroom were taught by Mrs. B during the first phase of the study.

### 3.2 Procedures of the Study

Prior to the present research, a 2-week pilot study, conducted in May of 1991 with two 7th-grade classes, revealed some important outcomes affecting planning for
this study regarding the instruments, the spatial lessons, and working with students in a computer lab setting. Several tests were used to measure the spatial visualization of the subjects working with the Supposers. The Card Rotations Test appeared to be the most appropriate and was used in the present study. This was an opportunity for the researcher to field test a series of spatial lessons and make changes and minor modifications. Moreover, the pilot study afforded the researcher valuable experience working with students who had little or no previous training in a computer lab setting and with students new to the Supposer software. The choice of instruments, refinement of spatial lessons, and experience with students using the software were useful prerequisite knowledge for both phases of the present study.

The first phase began with pretests, the week of October 14-18, and ended with posttesting the last day of school before Spring Break, on March 27th. The two tests given to all subjects in both classes at the beginning and at the end of the 5-month period were the Card Rotations Test (CRT) and the van Hiele Test (VHT). Since the subjects were unaware of being in a study, Mrs. B conducted all testing during the first phase. A written script was provided by the researcher, and Mrs. B used a watch with a second hand for accurate timing. Subjects absent for testing were given the test individually within a few class days.

As well as administering tests, Mrs. B taught all lessons in the classroom and all but one in the Macintosh® computer lab. This was a precaution taken to eliminate subjects' awareness of being in a study which might influence the results. Using the Supposer software in the treatment class was given a low profile in a further attempt to reduce a change in the subjects' behavior not due to the treatment. Mrs. B just wanted to "try it out". The researcher assisted in the computer lab for the first two labs, as someone who could help with all their questions, until Mrs. B and her
students became self-sufficient with the software. When the researcher later led a class demonstration, it was as someone who was more familiar than Mrs. B was with the software. The research was conducted in as natural a setting as feasible, and the researcher felt that the actions taken to preserve these conditions controlled as much as possible any extraneous variance due to the Hawthorne effect, subjects' knowledge of being in an experiment or feeling they were receiving special attention.

Throughout the five month period of the first phase, the class designated as the treatment group used Geometric Supposer-based activities while the other class, the control group, did not. Mrs. B either prepared the Supposer-based activities and worksheets with some assistance from the researcher or used ready-made copyable problem sheets from Sunburst manuals. The treatment group worked in the lab and in the classroom with the Supposer-based materials a total of 24 times during the first phase of the study. As the subjects completed the worksheets, Mrs. B visually checked for completion of data and acceptable conjectures before taking the papers. The Supposer activities were designed for 55 minute class periods and integrated with the textbook material. On two occasions, the Sunburst worksheet, which had two problems, was divided into two. The subjects spent two half class periods completing these activities, on successive days.

Both classes used the same textbook, Geometry (Jurgensen, Brown, & Jurgensen, 1985), and covered the same material daily. Assignments were for the most part identical. When different, either the treatment students in the Supposer lab did not have time to complete formulating and writing conjectures or Mrs. B did not have time to discuss the students' conjectures before the end of the lab period. In either case, the students' assignment was to complete the data sheets and be prepared to discuss their conjectures the following day.
Initially, the subjects used *The Geometric Supposer: Triangles* in the Macintosh® lab while completing the chapters on triangles. Quadrilaterals followed triangles, and since *The Geometric Supposer: Quadrilaterals* software was available for the Apple® computers only at that time, not the Macintosh®, the students did not work individually with computers. Instead, a whole class review of quadrilaterals with an Apple® IIIE and *The Supposer: Quadrilaterals* was presented by the researcher.

Similar polygons were studied next in the sequence of topics from the text and subjects once again worked with *The Geometric Supposer: Triangles* in the Macintosh® lab. With the completion of similar polygons, the first half of the school year ended, and the students took mid-term exams.

After mid-term exams the treatment group worked in another Macintosh® computer lab similar to the first. The subjects used *The Geometric Supposer: Triangles* to investigate the relationships of the sides in special right triangles, the 30-60-90 and the 45-45-90, and the sine and tangent ratios in any right triangle. Working again in pairs, the subjects shared the lab with the yearbook staff who used the tables in the center of the room, the subjects used the computers along the walls. This arrangement of sharing the lab was possible for the first few weeks before the yearbook staff started using the computers.

For the remainder of the time, February through March, the subjects worked with Apple® II microcomputers in the classroom in groups of 2-3. *The Supposer: Circles* was used with the chapter on circles, and *The Supposer: Quadrilaterals* with areas of polygons. These two chapters of text material were completed by spring break and ended the first phase of the study.
The second phase occurred in mid-May. Subjects from both classes were randomly selected to receive spatial instruction either the week of May 11 or the week of May 18. Approximately half of the treatment and control groups took part in the series of spatial lessons with the *Supposer* materials the first week. These lessons were conducted by the researcher. The other half remained in the classroom with Mrs. B to learn geometric constructions with compass and straightedge. For the second week, the halves switched places. By the end of the 2-week period, all students had covered the geometric constructions and worked through the spatial lessons.

The spatial lessons were developed in the spirit of Bruner and Piaget. Initially, subjects used cardboard polygons to practice translations, rotations, and reflections. With a basic understanding of these transformations, the subjects continued with geoboards and progressed to paper-and-pencil exercises before working with the *Supposers* on Apple® II microcomputers.

The subjects worked again in pairs with the *Supposer* materials. When there was an odd number of subjects, the student without a partner chose to work either alone or with another group. The subjects used *The Geometric preSupposer: Points and Lines*, and *The Geometric Supposer: Triangles and Quadrilaterals*. The reflect and repeat options were utilized to simulate rotations and reflections.

The researcher gave brief introductions to the exercises with an Apple® II microcomputer, LCD, and overhead projector to clarify the lessons and familiarize the subjects with the software. This was done in the classroom which was connected through a doorway to the computer area.
At the end of the first week, all subjects in all four classes, two with the researcher and two with Mrs. B, were given the Card Rotations test for posttest scores. No tests were given at the conclusion of the second week.

In the one week of classes before reviewing for final exams, Mrs. B completed the chapter on geometric constructions. The final exam was used for the achievement posttest score. Two subjects from the control group and two from the treatment group did not take the final exam. They were seniors who had at least a C average and, by school policy, were exempt from exams.

**Software Programs and Equipment Used in the Study**

*The Geometric Supposer* (1985) is a series of four microcomputer programs each dealing with a broad section of the geometry curriculum. The first program in the series, *The Geometric PreSupposer: Points and Lines*, was designed to prepare students for high school geometry and the remaining three, *The Geometric Supposer: Triangles, Quadrilaterals, and Circles*, for a secondary geometry course. The software series enabled students to take a more dynamic role in learning Euclidean geometry, supplementing the more traditional classes consisting of review of homework, lecture of new material, and subsequent assignment. With the *Supposer* software, students were able to explore actively open-ended problems and develop and test conjectures in a classroom environment where problem solving, mathematical reasoning, and communication were indispensable.

During the study, all four software programs in *The Geometric Supposer* series were used. These programs, their accompanying teacher guides, and a Liquid Crystal Display (LCD) were borrowed from The Ohio State University for use during both phases of the study. The researcher purchased a soft-cover book from NCTM, *How to Use Conjecturing and Microcomputers to Teach Geometry* (Chazan &
Houde, 1989), and two resource manuals from Sunburst Communications, *Supposer Solutions: Making the Most of Your Classroom Computer* (Yerushalmy, Butcher, & Chazan, 1992) and *Geometry Problems and Projects: Circles* (Yerushalmy & Houde, 1988). These were read and discussed by the researcher and Mrs. B. Two videotapes to help teachers become aware of the software's potential in the classroom, *Geometric Supposer* and *Supposer in the Classroom*, were loaned from Sunburst Communications and viewed by the researcher and the teacher. Copies were made by school library personnel and placed in the library for future reference.

A Macintosh® computer lab and the geometry classroom were used during the first phase of the study. From October through mid-January, the computer lab was available and used by the treatment class when working with the *Supposers*. The lab was equipped with thirteen computers and also had a chalkboard, tables, and chairs which were used during class discussions and follow-up activities. In the Macintosh® computer lab, the subjects worked in groups of 1-2 using *The Geometric Supposer: Triangles*. Here the researcher lead a whole class review of quadrilaterals using *The Geometric Supposer: Quadrilaterals*, an Apple® computer with LCD and overhead projector.

From mid-January through March, when the computer lab no longer had open time, the treatment group used another similar Macintosh® computer lab for several weeks before working in the geometry classroom for the remainder of Phase I. For classroom work, six Apple® computers on carts were brought in from the math department office and science classrooms. The subjects worked in groups of 2-3 using *The Geometric Supposer: Circles and Quadrilaterals* software.

The Guidance Department's classroom and adjoining computer area were utilized for the second phase of the study in May. Three additional Apple®
computers on carts were brought in from the math department office and physics classroom so there would be no more than two students per computer. The subjects worked with *The Geometric preSupposer: Points and Lines, The Geometric Supposer: Triangles, and The Geometric Supposer: Quadrilaterals* software programs. The researcher gave whole class demonstrations with the same software using an Apple® computer, overhead projector, and LCD unit.

### 3.3 Instrumentation

The two paper-and-pencil tests administered at the beginning and end of the first phase of the study were the Van Hiele Test (Usiskin, 1982) and the Card Rotations Test (French, Ekstrom, & Price, 1963). At the conclusion of the second phase of the study in May, an alternate form of the Card Rotations Test (Ekstrom, French, Harman, & Dermen, 1976a) was administered. The Geometry End-of-Course Test (CTB/McGraw-Hill Inc., 1986) was incorporated as part of the students' final exam given at the end of the school year.

**The Van Hiele Test (VHT)**

As part of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project directed by Usiskin (1982), a test was developed to test the van Hiele theory. This 35 minute, multiple-choice test was used to determine the van Hiele level of the subjects in the project. The 25 items on the test are divided into 5 sets of 5 items each. The first set of 5 items deals with Level 1 reasoning, the second set with Level 2, the third set with Level 3, the fourth with Level 4, and the fifth with Level 5. Since the existence of Level 5 is questionable (van Hiele, 1986) and secondary geometry students rarely exhibit reasoning at this level (Usiskin, 1982), only the first 4 van Hiele Levels were considered for this study.
Students were assigned to levels according to what Usiskin (1982) calls the "modified van Hiele theory, 3 of 5 criterion" (p. 80). The word, modified, indicates Level 5 is not included. The 3 of 5 criterion, which reduces the chance of a Type I error, refers to a subject answering at least 3 of the 5 items correctly at a particular level. A subject is assigned a van Hiele Level N if the subject passes the 3 of 5 criterion at that level, as well as all levels below, but none above Level N. The Kuder-Richardson Formula 20 reliability coefficients for the five parts of the VHT reported in the Usiskin study were .39, .55, .56, .30, and .26 respectively.

**Card Rotations Test (CRT)**

Spatial orientation requires mental rotation of shapes using short-term memory (Carroll, 1974). The figure is seen as a whole. The Card Rotations Test (French, Ekstrom, & Price, 1963; Ekstrom, French, Harman, & Dermen, 1976a), based on Thurstone's Cards, was developed by Educational Testing Service to measure the spatial orientation factor. Each item on the test gives a drawing of an irregular shape followed by drawings of the same shape rotated to different positions and some also turned over. The subject indicates whether the shape has been only turned around or if it has been both turned around and turned over.

Researchers do not uniformly agree about either the two terms, spatial orientation and spatial visualization, or the classification of tasks for the two major types of spatial skills. ETS in 1976 "originally classified...the Card Rotation Test as an (spatial) orientation test" (Tartre, 1990, p. 217), and defined spatial orientation as "the ability to perceive spatial patterns or to maintain orientation with respect to objects in space" (Manual for Kit of Factor-Referenced Cognitive Tests, 1976, p. 149). Based on definitions set forth by Connor and Serbin (1980), Kersh and Cook (1979), and McGee (1979), Tartre and others have since used the Card Rotations
Test as an indicator of spatial visualization skills in which all of a pictorial representation is mentally moved or altered by the subject. In this research, the Card Rotations Test is used as a spatial visualization test.

The earlier form of the Card Rotations Test (French, Ekstrom, & Price, 1963) has 14 items. Each of the 14 shapes is followed by eight of the same shape for a total of 112 responses. Subjects are given 4 minutes for the test. The score on the test is the number of items answered correctly minus the number answered incorrectly. There is no penalty for items left unanswered. Therefore, the range of possible scores is from -112 to +112.

The later form of the Card Rotations Test (Ekstrom, French, Harman, & Dermen, 1976a) has two 10-item parts for a total of 20 items on the test. Each of the 20 shapes is followed by eight of the same shape for a total of 160 responses. The subject is given 3 minutes for each part. With scoring the same as the earlier form, the possible scores range from -160 to +160.

The test is suitable for grades 8-16. The kit manual reports a split-half reliability of .86 for males and .89 for females. The sample was made up of 11th- and 12th-grade suburban students.

**Geometry End-of-Course Test**

The Geometry End-of-Course Test, published by CTB/McGraw Hill (1986), is a basic skills achievement test representing the most commonly taught curricula in a one-year geometry course. The 45-minute test contains 42 multiple choice items which measure the knowledge and understanding of basic concepts such as congruency, similarity, angle relationships, special properties of polygons, and finding perimeters, areas, and volumes of 2 and 3 dimensional figures. The test is scored by counting the number of correct responses to the items.
Based on a sample of 5675 secondary students given the test at the completion of a full year of geometry (CTB/McGraw-Hill, Inc., 1986), the KR-20 reliability estimate was computed at .822 with a standard errors of measurement (SEM) of 2.83. The scores ranged from 4 to 40; the mean was 21.78 with a standard deviation of 6.70 and 0.27 estimate of skewness.

The final exam given to the subjects in this study included the 42-item Geometry End-of-Course Test and 13 additional multiple-choice questions, added by Mrs. B, for a total of 55 items. The content areas of the 13 exam questions added to the standardized test included circles, constructions, solids, and coordinate geometry. These topics were either not sufficiently covered in the 42-item test or not covered at all. With the 13 additional items, Mrs. B felt the 55-item test was a comprehensive representation of the curriculum taught. The time allotted for the exam was 90 minutes. The student's score was the number of items that had correct responses with no penalty given for incorrect responses.

3.4 Statistical Analyses

An unequivalent control group design with two entact geometry classes was used for Phase 1. Analysis of the data was done using the SAS statistical analysis computer program. To determine if significant differences existed between control and treatment groups on the dependent variables, spatial visualization and achievement, an analysis of covariance was performed to test the hypotheses. Pretest scores were used as the covariates. Because of the ordinal nature of the van Hiele levels, the Kruskal-Wallis test of significance was used to compare the two groups using the van Hiele level pre- and posttest data.
Further statistical analysis was done with the posttest data, for which control and treatment groups were combined. Three correlation coefficients were calculated to determine the degree of relationship between van Hiele level, spatial visualization ability, and achievement. Due to the ordinal nature of the van Hiele levels, Spearman rank correlations were used in two of the analyses which used the van Hiele level data, and a Pearson product-moment correlation was performed with the spatial visualization ability and achievement pair. Data collected in the spring were used for these correlations.

The design for Phase II, in which the subjects from both groups were randomly assigned to either the first or second week of spatial lessons, was a 2×2 factorial design. A multi-factor analysis of covariance was used to analyze the results using the F-ratio to test the difference in the means. The October spatial visualization scores were used for the covariate.

3.5 Limitations

By using two intact classrooms for a prolonged period of time, in this case lasting through 5 months of the school year, there existed limitations beyond the control of the researcher dealing with attendance and school policy. These limitations affected the number of subjects used in the study as well as the number of students for which there were complete data sets.

The beginning of the year enrollments in the treatment and control classes were 21 and 23 respectively. Before the end of the first 6-weeks, two students dropped out of the treatment class, and two more at the end of the first grading period. In addition, one subject in the treatment class moved out of the district during
the treatment period. Overall, the treatment class lost five students due to attrition; the control class lost none.

Two subjects initially in honors geometry transferred, one to each the treatment and control class, after the beginning of Phase I. These two were excluded from the study. The total enrollment in Mrs. B’s two classes started at 44 in the beginning of the year and ended with 41. Since the two transfer students were excluded, the total number of subjects in the study was 39 (See Table 1).

Table 1

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</tr>
<tr>
<td>Transferred in from Honors Geometry</td>
</tr>
<tr>
<td>End-of-Year Enrollment</td>
</tr>
<tr>
<td>Transfer Subjects Excluded from Study</td>
</tr>
<tr>
<td>Subjects in the Study</td>
</tr>
</tbody>
</table>

Five subjects who were initially enrolled in the treatment class dropped out and were excluded from the study. To determine if these drop-outs were significantly different from those in the study, a t-test was performed using the first 6-weeks grade
for a comparison between drop-outs and those completing the year (See Table 2). Just 3 of the 5 were in class the entire first grading period and had received a 6-weeks grade.

Table 2

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>39</td>
<td>78.72</td>
<td>12.50</td>
</tr>
<tr>
<td>Drop-outs</td>
<td>3</td>
<td>78.33</td>
<td>24.95</td>
</tr>
</tbody>
</table>

\( t = -0.0479, \ p = .9620 \)

The mean grade for the 39 subjects in the study was 78.72 (\(SD = 12.5\)) compared to 78.33 (\(SD = 24.95\)) for the 3 drop-outs. At \(\alpha = .05\) and \(df = 40,\) the critical value of the \(t\) ratio was 0.304. The calculated value of \(t\) was -0.0479, with \(p = .9620.\) Therefore, the three drop-outs were not significantly different from the subjects in the study.

Complete data were not collected from all 39 subjects for two reasons. First, there are no prespatial scores for 3 subjects, 2 in the control and 1 in the treatment group. These subjects did not write their names on the Card Rotations Test and afterward could not confidently identify which test was theirs. Originally there were 3 from the treatment group who could not identify their test, but 2 of these were drop-outs and had been eliminated from the study. Second, posttest achievement scores are missing for 4 subjects, 2 in the control and 2 in the treatment group. These
subjects were seniors who had maintained a minimum grade in the course and by school policy were exempt from taking the final exam.

Selection of the classroom teacher had some limitations. The teacher chosen was unfamiliar with *The Geometric Supposer* series and integrating software of any kind in her classes. Using the *Supposers* in the classroom requires new implementation and management skills, changes in the teacher's attitude and role in the classroom, and modeling inductive discovery and synthesizing data. The classroom teacher used in this study had not acquired the skills and changes in attitude and role customarily brought about by teaching with the *Supposer*. On one hand, the teacher's lack of familiarity was a limitation for implementing the software in the treatment class, but, on the other hand, it was necessary to guarantee that the control class would be taught geometry in a traditional deductive manner following the style of the textbook.

If a veteran teacher with *Supposer* use had been chosen, the method of teaching would be contaminated and affect how the control class was taught. Therefore, in order to keep the teaching of the two classes as distinct as possible, a teacher using somewhat traditional teaching methods and unfamiliar with the software and its inductive, conjecturing methods was used in the study.

There were several criteria for the selection of the achievement test for the end-of-course exam. *SCANTRON®* forms were customarily used for final exam answer sheets and scored by school office staff. This required a multiple choice test. Furthermore, this eliminated having the subjects construct formal proofs and execute constructions with straight edge and compass. Both topics were covered extensively in both treatment and control classes.
The test was to be a comprehensive representation of the curriculum taught during the school year and, yet, stay within the scope of activities presented in the control class. As a result, some aspects of behavior associated with *Supposer* activities in the treatment class were missing, mainly inductive thinking and conjecturing. Both Mrs. B and the researcher felt the 42 multiple choice item Geometry End-of-Course Test by CTB/McGraw Hill (1986) supplemented with 13 additional items was suitable within the limitations.
CHAPTER IV

STATISTICAL ANALYSIS AND RESULTS

4.0 Introduction

The first section discusses the data analysis and results which answer the questions regarding the long term use of The Geometric Supposer software and the effect on students' spatial visualization ability, van Hiele levels of thought, and achievement in standard content knowledge. The second section addresses the relationships between the subjects' spatial visualization ability, van Hiele level, and score on the end-of-the-year exam. And the third presents the results that a series of spatial lessons using The Geometric Supposer has on the subjects' spatial visualization ability.

Three testing instruments were used in this study. Their descriptions and reliability's were reported in Chapter III. The reliability of the Card Rotations Test was established by Educational Testing Service, The van Hiele Test by Usiskin, and the Geometry End-of Course Test by CTB/McGraw-Hill.

The internal consistency reliability's of the three testing instruments used were established for the sample in this study. The Cronbach Coefficient Alpha's are in Table 3. Reliability's were found for pretest and posttest except for the Geometry End-of Course Test which was used for posttesting only.
Table 3

*Cronbach Coefficient Alpha's for Testing Instruments Established for the Sample in this Study*

<table>
<thead>
<tr>
<th>Test</th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card Rotations Test (Part I)</td>
<td>.77</td>
<td>.73</td>
</tr>
<tr>
<td>Van Hiele Test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 1</td>
<td>.30</td>
<td>.36</td>
</tr>
<tr>
<td>Level 2</td>
<td>.44</td>
<td>.57</td>
</tr>
<tr>
<td>Level 3</td>
<td>.37</td>
<td>.46</td>
</tr>
<tr>
<td>Level 4</td>
<td>.18</td>
<td>.30</td>
</tr>
<tr>
<td>Geometry End-of-Course Test</td>
<td></td>
<td>.81</td>
</tr>
</tbody>
</table>

4.1 First Phase - Analysis of Subject Variables (SVA, VHL, ACH)

The descriptive research of the first phase of the study investigated the relationships between using computer-designed lessons integrated with the text material and the subjects' spatial visualization ability, van Hiele level of geometric understanding, and end-of-course achievement.
The following null hypotheses were tested:

H1. There will be no difference in spatial visualization ability (SVA) scores between students using the 
Supposer series and students not using the software.

H2. There will be no difference in van Hiele levels (VHL) between students using the Supposer series and students not 
using the software.

H3. There will be no difference in achievement (ACH) between 
students using the Supposer series and students not using the 
software.

**Subjects' Spatial Visualization Ability**

Do students in the treatment class using the Supposer software integrated with the text material achieve significantly greater change in spatial ability scores compared to students in the control class who do not use the software? To answer this question an analysis of covariance was used. The independent variable is use or non-use of the software, and the dependent variable is the spatial posttest, with the spatial pretest as the covariate.

The same spatial visualization test was given immediately before and at the end of the treatment period, mid-October and late March. Possible scores on this test were from -112 to 112 for each subject. Actual scores ranged from 14 to 108 on the pretest and from 24 to 112 on the posttest. Table 4 contains a summary of the descriptive statistics for the two groups of subjects.
Table 4

*Means and Standard Deviations for Each Group on Pre- and Posttest Spatial Visualization*

<table>
<thead>
<tr>
<th>Test</th>
<th>Treatment</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Covariate (Pretest)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>( M^a )</td>
<td>70.47</td>
<td>64.81</td>
</tr>
<tr>
<td>( SD )</td>
<td>23.53</td>
<td>25.85</td>
</tr>
</tbody>
</table>

| **Dependent Variable (Posttest)** |       |          |
| \( n \)                        | 15    | 21       |
| \( M^a \)                      | 83.00 | 81.57    |
| \( SD \)                       | 14.23 | 21.99    |
| \( M^b \)                      | 81.39 | 82.72    |

*Unadjusted means

*Adjusted means

There were spatial visualization scores for 15 subjects in the treatment group and 21 in the control group. The means for the treatment group was 70.47 (\( SD = 23.53 \)) compared to the control group means of 64.81 (\( SD = 25.85 \)). The posttest means for both groups were an increase from the pretest, with the control group showing the greater increase. On the posttest, the means of the treatment group was 83.00 (\( SD = 14.23 \)), and 81.57 (\( SD = 21.99 \)) for the control group. The treatment
and control groups' posttest means, when adjusted for the covariate, were 81.39 and 82.72 respectively. Even though the treatment group had a higher average score on both the pre- and posttest, the control group's adjusted means was higher than the treatment group's when adjusted for the pretest scores.

An analysis of covariance was used to determine if there was any significant difference between the mean scores of those using the Supposer software in the treatment group and those not using the software in the control group. The source table for this analysis of covariance is presented in Table 5.

Table 5

*ANCOVA Comparison on Spatial Visualization*

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>15.15</td>
<td>1</td>
<td>15.15</td>
<td>0.07</td>
</tr>
<tr>
<td>Pre-Spatial</td>
<td>4999.81</td>
<td>1</td>
<td>4999.81</td>
<td>21.98***</td>
</tr>
<tr>
<td>Error</td>
<td>7505.33</td>
<td>33</td>
<td>227.43</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12523.00</td>
<td>35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

***p < .001

The pretest accounted for a significant amount of the variation in the posttest scores (p < .001). Spatial visualization scores at the beginning of the treatment period were a significantly related to scores on the spatial visualization test at the end of the treatment period.
Using an analysis of covariance, the null hypothesis was tested at \( \alpha = .05 \). The calculated \( F \) was 0.07. The critical value of \( F(1,33) \) was 4.14. Therefore, the null hypothesis could not be rejected. There is no significant difference between the posttest mean scores of the two groups. There was no significant difference in spatial visualization scores between subjects using the *Supposer* series and subjects not using the software.

**Subjects' van Hiele Levels of Thought**

Do subjects in the treatment class using the *Supposer* software integrated with the text material achieve significantly greater change in van Hiele levels compared to subjects in the control class who do not use the software?

To answer this question a Kruskal-Wallis test was performed on the data. The subjects were sorted by group and change in van Hiele level from pre- to posttest. Loss of level did not occur for any of the subjects in this study. After posttesting, all subjects either remained at the same level or were assigned to a level one or two above their pretest level.

Table 6 shows the number of subjects remaining at the same level or gaining one or two levels by the end of the treatment period, along with group totals and corresponding percentages.
Table 6
Frequencies and Percentages for Change in Van Hiele Levels by Group

<table>
<thead>
<tr>
<th>VHL</th>
<th>Treatment</th>
<th>Control</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency %</td>
<td>Frequency %</td>
<td>Frequency %</td>
</tr>
<tr>
<td>No Gain</td>
<td>7 44</td>
<td>14 61</td>
<td>21 54</td>
</tr>
<tr>
<td>Gained 1 Level</td>
<td>7 44</td>
<td>8 35</td>
<td>15 38</td>
</tr>
<tr>
<td>Gained 2 Levels</td>
<td>2 13</td>
<td>1 4</td>
<td>3 8</td>
</tr>
<tr>
<td>Total</td>
<td>16</td>
<td>23</td>
<td>39</td>
</tr>
</tbody>
</table>

Table 6 data reveals a greater percentage of the subjects in the treatment group gaining one or two levels from pretest to posttest. Subjects gaining at least one van Hiele level, compared to those with no gain, shows more than half of the subjects, 9 of the 16 or 56%, in the treatment group showed a gain in levels, while the opposite situation occurred in the control group. Fewer than half of the subjects, 9 of the 23 or 39%, gained one or two levels in the control group. Overall, 46% of the 39 subjects in the study showed a gain from pretest to posttest. The 3 subjects gaining two levels all began at Level 1 and were assigned Level 3 on the posttest.
The frequency counts in Table 6 show number of levels gained, 0, 1, or 2, for all subjects in each of the two the groups, treatment and control, independent of the subjects' initial van Hiele level on the pretest. At the beginning of the study, subjects were determined to be at either van Hiele Level 1, 2, or 3. For a clearer view of where subject gains occurred relative to their beginning van Hiele levels, the data was organized by pretest van Hiele level in Table 7.

Table 7

*Relationship Between Group and Change by Pretest van Hiele Level*

<table>
<thead>
<tr>
<th>VHL</th>
<th>Group</th>
<th>Treatment</th>
<th>Control</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gain No Gain</td>
<td>Gain No Gain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6 0</td>
<td>7 7</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3 4</td>
<td>1 5</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0 3</td>
<td>1 2</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>9 7 9 14</td>
<td>39</td>
<td></td>
</tr>
</tbody>
</table>

Of the 20 subjects beginning at Level 1, all 6 in the treatment group showed a gain, whereas just half of the control group, 7 of the 14, gained at least one level. At
Level 2 the same trend continued as at Level 1, where the treatment group had a
greater portion of subjects gaining compared to the control group. Thirteen subjects
had a beginning Level 2. In the treatment group, 3 of the 7 showed a gain, compared
to only 1 of the 6 in the control group. The only gain in the group of 6 subjects at
Level 3 was shown by 1 subject in the control group.

Tables 6 and 7 do not indicate individual subjects' change and the number of
van Hiele levels gained. For a comparison of subjects' pre- and posttest van Hiele
levels by group, see Table 15 in Appendix A.

A Kruskal-Wallis test statistic was calculated to test the hypothesis, H2. The
procedure requires converting all changes in van Hiele levels to ranks. In Table 8, the
sum of the ranks is given for the treatment and control groups, the expected sums and
standard deviations if there is no difference between the two groups, and the mean
scores of the groups.

The sum of the treatment group ranks is 36 above, and the control group sum
is 36 below, what could be expected under the null hypothesis. The mean rank of the
subjects in the treatment group is 22.25 compared to 18.43 for the control group.
The Kruskal-Wallis test was performed to determine if the higher treatment group
scores were significantly different from those of the control group.
Table 8

*Relationship between Rank Change in van Hiele Levels by Group*

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Sum of Scores</th>
<th>Expected Scores under H₀</th>
<th>SD under H₀</th>
<th>Mean Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>16</td>
<td>356.0</td>
<td>320.0</td>
<td>31.07</td>
<td>22.25</td>
</tr>
<tr>
<td>Control</td>
<td>23</td>
<td>424.0</td>
<td>460.0</td>
<td>31.07</td>
<td>18.43</td>
</tr>
</tbody>
</table>

\[ \chi^2 = 1.3424 \]

The Kruskal-Wallis test (Chi-square approximation) of the data in Table 8, with \[ \chi^2 = 1.3424, df = 1, \text{ and } p = .2466 \], indicates no significant difference between treatment and control groups on subjects' rank change in van Hiele levels from pretest to posttest. There is no significant relationship between the treatment and gain in van Hiele level. Although the data show a higher sum of the ranks in the treatment group, the difference is not great enough to reject the null hypothesis. The results of the analysis show no significant difference in gain of van Hiele levels between the students who did use the *Supposer* series and those students who did not use the software.

**Subjects' Achievement**

Do students in the treatment class using the *Supposer* software integrated with the text material achieve significantly higher end-of-course achievement scores compared to students in the control class who do not use the software? To answer this question an analysis of covariance was used. The independent variable was use
or non-use of the software, and the dependent variable was the end-of-course posttest. The pretest achievement score, the covariate, was the first 9-weeks grade.

The treatment period began one week after the end of the first 9-weeks grading period in mid-October. The range in scores was from 55 to 99. The posttest score was the end-of-course exam given in early June. Possible scores were from 0 to 55 with the actual scores ranging from 19 to 46. Table 9 contains a summary of the descriptive statistics for the two groups of subjects.

Table 9
 Means and Standard Deviations for Each Group on Pre- and Posttest Achievement

<table>
<thead>
<tr>
<th>Test</th>
<th>Treatment</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pretest)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>M</td>
<td>79.86</td>
<td>77.19</td>
</tr>
<tr>
<td>SD</td>
<td>12.75</td>
<td>12.37</td>
</tr>
<tr>
<td>Dependent Variable (Posttest)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>M&lt;sup&gt;a&lt;/sup&gt;</td>
<td>34.50</td>
<td>31.24</td>
</tr>
<tr>
<td>SD</td>
<td>6.10</td>
<td>7.25</td>
</tr>
<tr>
<td>M&lt;sup&gt;b&lt;/sup&gt;</td>
<td>33.94</td>
<td>31.61</td>
</tr>
</tbody>
</table>

<sup>a</sup>Unadjusted means.

<sup>b</sup>Adjusted means.
There were pretest and posttest achievement scores for 14 subjects in the treatment group and 21 in the control group. The pretest mean for the treatment group was 79.86 ($SD = 12.75$) compared to the control group mean of 77.19 ($SD = 12.37$). The treatment group mean was higher than the control group mean, on the pretest as well as on the posttest. Posttest means were 34.50 ($SD = 6.10$) for the treatment group and 31.24 ($SD = 7.25$) for the control group. The treatment and control group posttest means, when adjusted by the covariate, were 33.94 and 31.61 respectively. After statistically controlling for initial achievement differences between the groups, the mean of the treatment group remained greater than the control groups. The subjects in the treatment group did as well as or better than their counterparts in the control group on end-of-course geometry achievement.

An analysis of covariance was used to determine if there was any significant difference between the mean scores of those using the Supposer software in the treatment group and those not using the software in the control group. Table 10 is the source table for this analysis of covariance.
**Table 10**

*ANCOVA Comparison on Achievement*

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>44.94</td>
<td>1</td>
<td>44.94</td>
<td>1.60</td>
</tr>
<tr>
<td>Pre-Achievement</td>
<td>636.85</td>
<td>1</td>
<td>636.85</td>
<td>22.73***</td>
</tr>
<tr>
<td>Error</td>
<td>896.46</td>
<td>32</td>
<td>28.01</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1622.69</td>
<td>34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

***p < .0001

The pretest accounted for a significant amount of the variation in the posttest scores (p < .0001). The first 6-weeks grade at the beginning of the treatment period was significantly related to scores on the final exam given in June.

Using an analysis of covariance, the null hypothesis was tested at \( \alpha = .05 \). The calculated \( F \) was 1.60. The critical value of \( F(1,32) \) was 4.15. The data supports the null hypothesis of no significant difference between the posttest mean scores of the two groups. Although the treatment group had a higher adjusted mean, the difference was not great enough to reject the null hypothesis. There was no significant difference in achievement between subjects using the *Supposer* series and subjects not using the software.
4.2 Correlations of Subject Variables (SVA, VHL, ACH)

The relationships between the subject variables SVA, VHL, and ACH were analyzed in a correlational study using data obtained in the spring. All 39 subjects took the spatial visualization and van Hiele level tests, while just 35 subjects took the end-of-course achievement test. Four seniors elected to exercise their option for the school policy regarding seniors with a minimum grade average in the course and did not take the final exam.

The following three hypotheses were tested:

H4. Spatial visualization ability (SVA) is positively correlated with achievement (ACH).

H5. Spatial visualization ability (SVA) is positively correlated with van Hiele level (VHL).

H6. Van Hiele level (VHL) is positively correlated with achievement (ACH).

The means and standard deviations for posttest spatial visualization ability and achievement used in these correlations are listed in Table 14 in Appendix A. Table 15 in Appendix A, which presents the subjects' changes in level from pretest to posttest relative to their beginning van Hiele level, includes the posttest van Hiele levels for the two groups used in the correlational analysis.

Table 11 shows correlations between the March scores of spatial visualization ability and van Hiele level, and the end-of-the-year achievement for the entire sample.
Table 11

*Correlation Coefficients Between Spatial Visualization Ability (SVA), Van Hiele Level (VHL), and Achievement (ACH)*

<table>
<thead>
<tr>
<th></th>
<th>VHL</th>
<th>ACH</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVA</td>
<td>.0890\textsuperscript{a}</td>
<td>.0407\textsuperscript{b}</td>
</tr>
<tr>
<td>VHL</td>
<td>.4483\textsuperscript{b**}</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Four subjects did not take the end-of-course achievement exam.

\(a_n = 39, b_n = 35\)

\(**p < .01\)

Because of the ordinal nature of the van Hiele levels, Spearman rank correlations were calculated to determine if there were significant correlations between VHL and SVA and between VHL and ACH. In the first pair (VHL and SVA), the Spearman rank correlation, \(r_s = .0890\), indicates a negligible relationship in the sample between the subjects' van Hiele levels and spatial visualization scores. Knowing the subject's score on one test gives little or no indication of the subject's score on the other test.

In the second pair (VHL and ACH), there is a moderate association in the sample between achievement and van Hiele levels where \(r_s = .4483\) which is significant at the .01 level (\(p = .0069\)). This tells us that a subject with a high van
Hiele level will probably have an above-average score on the achievement test, and subjects who are at the lowest van Hiele Level 1 are more likely to have a below-average achievement score.

Since scores on the achievement and spatial visualization test are interval data, a Pearson product-moment correlation coefficient was computed to determine their relationship. The calculated Pearson \( r \) of \(-.0407\) with \( p = .8165 \) shows a negligible relationship in the sample between a subject's achievement score and spatial visualization ability score. Knowledge of one score yields little if any information of the other score.

4.3 Second Phase - Analysis of Subject Variable (SVA)

The subjects in the treatment and control classes participated in a week of spatial lessons during either the third or fourth week in May. The subjects were randomly assigned to one of the weeks. The treatment group received the lessons first, and the control group the following week. Posttesting was at the end of the first week of lessons when half of the subjects had participated in the lessons. As a result, there were four groups of subjects sorted by group during the first phase of the research, October through March, and by group during this second phase in May: (a) subjects who used the software earlier in the year and during the third week in May, (b) subjects who used the software earlier in the year only, (c) subjects who used the software during the third week in May only, and (d) subjects who did not use the software at any time during the year.
The general hypothesis was:

H7. Students participating in a series of spatial visualization lessons, using the Supposer software, will have higher spatial visualization ability (SVA) than students who do not receive the series of lessons.

Using a $2 \times 2$ factorial analysis of covariance in an experimental design for the second phase of the study, the causal relationship of a week of specially designed lessons on the subjects' spatial visualization ability (SVA) was tested. The two independent variables are group during the first phase of the research and group for the spatial lessons during the second phase. For the nonmanipulated variable, group for first phase, 15 and 21 subjects were in the treatment and control groups respectively. These subjects were randomly assigned to either the first or second week of the spatial lessons, representing the two levels of the manipulated independent variable. The dependent variable, score on posttest, was score on the spatial visualization test given at the end of the third week in May, with the October spatial pretest score as the covariate.

The range of possible scores on the pretest was from -112 to 112 for each subject. Actual scores ranged from 14 to 108. A different form was used for the posttest which had a possible range from -160 to 160 for each subject. The range for actual posttest scores was from 35 to 158. Table 12 lists the means and standard deviations for each of the four groups.
Table 12

Means and Standard Deviations for Each Group on Pre- and Posttest Spatial Visualization

<table>
<thead>
<tr>
<th>Test</th>
<th>T₁ T₂</th>
<th>T₁ C₂</th>
<th>C₁ T₂</th>
<th>C₁ C₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>8</td>
<td>7</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Covariate (October Spatial)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mᵃ</td>
<td>74.25</td>
<td>66.14</td>
<td>67.55</td>
<td>61.80</td>
</tr>
<tr>
<td>SD</td>
<td>23.74</td>
<td>24.36</td>
<td>24.55</td>
<td>28.20</td>
</tr>
<tr>
<td>Dependent Variable (May Spatial)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mᵃ</td>
<td>138.00</td>
<td>125.86</td>
<td>129.45</td>
<td>122.50</td>
</tr>
<tr>
<td>SD</td>
<td>20.05</td>
<td>14.09</td>
<td>30.94</td>
<td>38.39</td>
</tr>
<tr>
<td>Mᵇ</td>
<td>132.19</td>
<td>126.70</td>
<td>129.14</td>
<td>126.90</td>
</tr>
</tbody>
</table>

Note. T₁T₂ - **Supposer** Treatment Group Phase I and Spatial Treatment Group Phase II.
T₁C₂ - **Supposer** Treatment Group Phase I and Spatial Control Group Phase II.
C₁T₂ - **Supposer** Control Group Phase I and Spatial Treatment Group Phase II.
C₁C₂ - **Supposer** Control Group Phase I and Spatial Control Group Phase II.
aUnadjusted means.
bAdjusted means.
There were pretest and posttest spatial visualization scores for 36 subjects. Fifteen had used the *Supposer* software earlier during the first phase, 21 had not. Eight of the 15, and 11 of the 21, participated in the spatial lessons during the first week. These were the subjects in the treatment group for this phase of the study. The remaining 17, the control group, took the spatial lessons the second week. The pretest mean for the group using the software during both phases was 74.25 ($SD = 23.74$), which was the highest of the four means. The next highest mean 67.55 ($SD = 24.55$) was of the group using the software during Phase II only. The means for the group using the software during Phase I only was 66.14 ($SD = 24.36$), and the lowest mean was from the group not using the software during either phase, $M = 61.80$ ($SD = 28.20$).

The rank order, from highest to lowest, of the posttest means was the same as for the pretest. The means ranged from 138.00 ($SD = 20.05$) held by the group which used the software during both phases to the lowest, 122.50 ($SD = 38.39$), from the group who did not use the software at any time during the year. The other two were 129.45 ($SD = 30.94$) for the treatment group in the second phase only, and 125.86 ($SD = 14.09$) for the treatment group during Phase I only.

The posttest means of the four groups, when adjusted for the covariate, were 132.19, 126.70, 129.14, and 126.90 with the highest belonging to the group using the software for both phases. Next, was the spatial treatment group using the software for the first time in Phase II with 129.14. The two groups remaining had similar adjusted means, 126.90 and 126.70. These adjusted means belonged respectively to the group which did not use the software during either phase and to the group using
the software during Phase I only. The two groups which did not use the software during Phase II had the lowest adjusted means on the spatial visualization posttest.

The combined effect of using *The Geometric Supposer* series for both Phase I and Phase II is evident in Table 12. The adjusted means of the group of subjects in both the *Supposer* and spatial treatment groups ($T_1 T_2$) was higher than the other three groups.

An analysis of covariance was used to determine if there was any significant difference between the mean scores of the four groups of subjects using the October spatial pretest as the covariate. The data for the $2 \times 2$ factorial analysis of covariance are in Table 13.

Table 13

*Factorial Analysis of Covariance Summary Table*

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Supposer</em> Treatment 1 ($T_1$)</td>
<td>17.33</td>
<td>1</td>
<td>17.33</td>
<td>0.04</td>
</tr>
<tr>
<td>Spatial Treatment 2 ($T_2$)</td>
<td>127.70</td>
<td>1</td>
<td>127.70</td>
<td>0.31</td>
</tr>
<tr>
<td>$T_1 \times T_2$</td>
<td>23.02</td>
<td>1</td>
<td>23.02</td>
<td>0.06</td>
</tr>
<tr>
<td>Pre-Spatial</td>
<td>13932.78</td>
<td>1</td>
<td>13932.78</td>
<td>33.46****</td>
</tr>
<tr>
<td>Error</td>
<td>12909.30</td>
<td>31</td>
<td>416.43</td>
<td></td>
</tr>
</tbody>
</table>

****$p < .0001$

The pretest accounted for a significant amount of the variation in the posttest scores ($p < .0001$). October spatial visualization scores used for the covariate were
significantly related to scores on the spatial visualization test at the end of the treatment period.

There was no evidence of an interaction between the two treatments. There is no indication of an effect of *Supposer* Treatment 1 and Spatial Treatment 2. The presence or absence of $T_1$ did not have any significant effect on the May spatial visualization scores. The presence or absence of $T_2$ did not have any significant effect on the May spatial visualization scores. The three, Treatment 1, Treatment 2, and the interaction did not account for much of the variance in the posttest scores.
CHAPTER V
DISCUSSION AND RECOMMENDATIONS

5.0 Introduction

This study took place during the school year at a suburban high school and involved two regular level geometry classes, one the treatment class, the other the control. There were 39 subjects in the final sample. Pretests were given in October and the final posttest was completed at the end of the school year in June. The research consisted of two phases.

During Phase I, from October through March, the classroom teacher integrated *The Geometric Supposer* series with the text material for the treatment group. Analyzing posttest data, with the pretest as the covariate, the effects of using *The Geometric Supposer* software during this time on spatial visualization ability, van Hiele levels, and achievement were determined.

Three correlations were performed on the posttest data. The relationships between spatial visualization ability with achievement, spatial visualization ability with van Hiele level, and van Hiele level with achievement were investigated.

Phase II in May consisted of a full week of spatial visualization lessons using the *Supposer* software. Four groups were formed by whether the subjects used the software during both phases of the study, during just one phase, or not at all. A 2×2 (Group × Group) factorial analysis of covariance was performed on the data from the
four groups of subjects to determine the effects of the treatment on subjects' spatial visualization ability.

5.1 Phase I

At the beginning of the study, *The Geometric Supposer* series was introduced to both subjects and teacher. This was the first time the classroom teacher had integrated software with the text material in any of her classes, and it was the first time for the geometry classes to use the software. The balancing of the benefits and limitations and the choice of a teacher having no prior experience with the *Supposer* software are discussed in Chapter III. Choosing a teacher whose method of teaching geometry was more in the traditional manner and strongly aligned with the textbook method of presentation, was to help guarantee that the control class would be taught in this manner throughout Phase I. Occasionally, Mrs. B found herself, quite unintentionally, employing inductive thinking in the control class lessons, which she was using in the treatment class. The control group was to some extent being contaminated, and this may have affected the outcomes.

It is interesting to note that the treatment class after using the *Supposers* scored as well as or better, but not significantly better, than the control class on the spatial visualization ability test, the van Hiele level test, and end-of-year achievement test of content knowledge. All three null hypotheses, H1, H2, and H3, were accepted.

A teacher need not be reluctant to use geometry software, such as *The Geometric Supposer* series, fearing the class might not do as well or feeling that using the software will take too much time and all the material will not be covered by the end of the year. In this study, the treatment class using the software did as well as or
better than the class which was taught in the traditional manner and the progress through the textbook was the same as the control class. The subjects in both treatment and control groups progressed well throughout the year in learning geometry.

For future research with this or similar software, it is recommended that subjects receive more intense exposure during the treatment period. Using the software as an everyday practice in whole class activities and with student hands-on discovery and conjecturing is possible. Provided subjects have computer access throughout the day, assignments can involve using the software as well. Studying the effects of using the software the entire year while maximizing the time and use of the software is recommended for future research.

The standard deviations for treatment and control groups on the spatial pretest were much the same, 23.53 and 25.58 respectively. On the posttest, the variability in the scores of the treatment group ($SD = 14.23$) was much less than that of the control group ($SD = 21.99$). The treatment group scores were clustered closer to its mean of 83.00, and the control group scores were spread farther from its mean of 81.57.

The treatment subjects' rank change in van Hiele levels from pretest to posttest was higher as well as their mean rank score, and the control groups lower, than what could be expected under the null hypothesis. The subjects in the treatment group exhibited a greater rank change but was not significantly different statistically from the control subjects' change (See Table 8). Even though the difference was not significant, there are some trends worth noting in the data.

On the pretest, 33 of the 39 subjects were assigned levels below Level 3. Twenty-six of the 39 subjects were still classified at levels below Level 3 on the posttest. This is consistent with other finding where most geometry students are
reasoning at levels below Level 3 before the end of the year (Burger & Shaughnessy, 1986a; Usiskin, 1982; Wiarszup, 1976). It is possible that there was a ceiling effect on van Hiele levels contributing to the acceptance of the null hypothesis, H2.

Compared to the control group, not only did a greater portion of subjects in the treatment group gain one or two levels (See Table 6), but all treatment subjects pretested at Level 1 and nearly half of the treatment subjects initially at Level 2 gained at least one van Hiele level (See Table 7). Further research is recommended to investigate the possibility of significant differences at the lower van Hiele levels. A sufficient number of subjects should be used to guarantee a good estimate of the probability when testing for significant changes in ranks using the Kruskal-Wallis test statistic.

The treatment group's adjusted mean of the end-of-the-year achievement was higher but not significantly higher than that of the control group's. The group using the Supposer series integrated with the textbook material did as well as or better on the control class on the end-of-the-year general achievement exam. The knowledge and understanding of basic geometric concepts learned throughout the year was not statistically different between the two groups.

It is recommended that replications of this research use a larger number of subjects including both honors and regular geometry classes, and maximize the use of the software during the treatment period. Future research might explore using alternatives to Usiskin's (1982) van Hiele test for assigning subjects to van Hiele levels. Interview techniques with higher reliability and validity might be used such as those developed by Fuys, Geddes, Lovett, and Tischler (1988) and Shaughnessy and Burger (1985) and discussed in Chapter II.
5.2 The Correlations

Taking two at a time, spatial visualization ability, van Hiele level, and achievement were analyzed in three different correlations. The relationship between each pair was determined. The results, both parametric and nonparametric, of the correlational analysis are in Table 10.

In this study no significant correlation was found between spatial visualization ability and achievement. The data did not support the hypothesis (H4) of a positive correlation between these two subject variables which the researcher expected to find.

Spatial visualization ability is one of at least two distinct spatial skills (McGee, 1979a) which becomes increasingly important in school mathematics courses (Balomenos, Ferrini-Mundy, & Dick, 1987; Smith, 1964). Previous research has found a positive relationship between spatial visualization ability and achievement in mathematics (Burnett, Lane, & Dratt, 1979; Fennema & Sherman, 1977; Guay & McDaniel, 1977; Moses, 1980; Munn, 1991; Schonberger, 1976; Sherman, 1979). Why was that relationship not corroborated in this study?

Some degree of spatial skill is necessary for geometry achievement. Hoffer (1981) lists spatial skills as just one of five skills essential in learning geometry. Spatial skills encompasses at least two factors, spatial visualization and spatial orientation ability. Spatial visualization is necessary but not sufficient. Furthermore, Hoffer claims some students "get by" in geometry classes by memorizing without developing an understanding of geometry.

By looking closely at the data, there were several subjects with low spatial visualization ability scores and high achievement scores. By contrast, there were a few low achievers with spatial visualization ability scores well above the mean. The
classroom teacher characterized the former subjects as conscientious, hard working students, and recognized in the latter subjects a lack of effort and motivation. Some high achievers, by their diligent study habits, apparently compensated for low spatial visualization ability, while ability alone does not guarantee high achievement scores. A more in-depth look at such subjects is warranted.

Additional study is needed to identify which skills contribute to achievement and an understanding of geometry, how they contribute, and what part each plays. The standardized achievement test in this study should be examined for the extent of spatial visualization ability involved in answering each question.

There was no significant correlation found between spatial visualization ability and van Hiele level. The data did not support the hypothesis (H5) of a positive correlation between these two subject variables.

Although no previous research investigating the relationship between van Hiele levels and spatial visualization ability was found for subjects in grades K-12, the characteristics of the van Hiele model and previous research results on spatial visualization imply the possibility of a correlation. My research design just did not have the strength needed to satisfactorily answer this question. The relationship between spatial visualization ability and van Hiele level is still open. Further study using high school geometry students is recommended in this area.

This study found a significant correlation between van Hiele level and end-of-the-year achievement, thus supporting the hypothesis (H6) of a positive correlation between the two variables. Subjects reasoning at higher van Hiele levels scored significantly higher on the final exam compared with subjects at lower van Hiele levels. Subjects with low achievement scores were more likely to be at lower van Hiele levels of geometric thinking.
Studies have shown a positive correlation between van Hiele level and geometry proofs and general achievement (Bobango, 1987; Usiskin, 1982). The van Hiele model was formulated to describe the levels of geometric thinking of elementary and secondary students and the phases of the learning process (Burger & Shaughnessy, 1986; Fuys, Geddes, & Tischler, 1985; Usiskin, 1982; van Hiele, 1986). Students passing sequentially through the levels progress from simply recognizing geometric shapes to constructing formal deductive proofs and understand geometry as a mathematical system of axioms, undefined terms, definitions, and theorems. Appropriate instruction and experience help students attain higher levels.

Since van Hiele level at the beginning of a secondary geometry course can predict scores on the final achievement test at the end of the school year, suitable instruction should occur in the primary and middle grades. Once in a formal geometry class, instruction should be matched with the van Hiele levels of the students and help them progress to higher levels.

5.3 Phase II

The analysis (See Table 12) of the causal relationship of a week of specially designed lessons using the Supposer series revealed no significant difference in the spatial visualization scores of the groups. Though the data failed to reject the null hypothesis H7, there are some data worth noting in the means and standard deviation table (See Table 11).

The scores of the subjects who used the Supposer software during the first phase of the study had posttest spatial visualization scores with considerably smaller standard deviation than the control group in March for Phase I (See Table 1). The subjects with the more tightly clustered scores on the March posttest exhibited the
same trend on the May spatial posttest for Phase II (See Table 11). The standard deviations of the two groups made up of the Phase I treatment group were 14.09 and 20.05 compared to 30.94 and 38.39 for the control subjects in Phase I. The Phase II treatment did not seem to effect this clustering, as it was observed in the May posttest data also.

The subjects were randomly assigned to either the first or second week of instruction using the *Supposer* software. This created four groups. The means of the pretest spatial visualization scores for the groups were 61.80, 66.14, 67.55, and 74.25, where there was approximately a 20% increase from the lowest to the highest means. The group with the highest pretest means was the treatment group for both phases of the study; the lowest means belonged to the control group in both phases.

For future research, it is recommended that subjects be blocked high, medium, or low based on their pretest spatial visualization scores and then randomly assign to the first or second week of classes. Blocking would help guarantee a more even distribution of scores among groups, thereby preventing any large variation in the means of the groups.

The week of spatial lessons was given near the end of the school year. The subjects' work was not graded, and it did not become part of their final 9-weeks grade. Future research might explore the possibility of placing the lessons nearer the beginning of the year, grading the subjects' work, and having the grade included as part of the 9-weeks grade.

The Phase II lessons used *Supposer* software available at the time. The computer activities in this study involved a sequence of key strokes to imitate rotations, flips, and slides. This required a little learning time and expertise. Shortly after this study was conducted, *The Geometric superSupposer* was released for
market. The new *superSupposer* software has quite a few new features including a full complement of transformations. Using this or similarly capable software would avoid the time and distraction spent creating the transformations. For future research involving transformational activities, it is recommended that software designed for "easy" transformations be used rather than trying to adapt less capable software for the task.
APPENDIX A

TABLES 14 THROUGH 15 REFERRED TO IN CHAPTER IV
Table 14  
*Means and Standard Deviations for Posttests Spatial Visualization Ability (SVA) and Achievement (ACH)*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Visualization Ability</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>39</td>
</tr>
<tr>
<td>$M$</td>
<td>82.21</td>
</tr>
<tr>
<td>$SD$</td>
<td>19.07</td>
</tr>
<tr>
<td>Achievement</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>35</td>
</tr>
<tr>
<td>$M$</td>
<td>32.54</td>
</tr>
<tr>
<td>$SD$</td>
<td>6.91</td>
</tr>
</tbody>
</table>
Table 15

Comparison of Students’ Pre- and Posttest Van Hiele Levels

<table>
<thead>
<tr>
<th>Pretest VHL</th>
<th>Posttest VHL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>(0,7)</td>
</tr>
<tr>
<td>2</td>
<td>(4,5)</td>
</tr>
<tr>
<td>3</td>
<td>(3,2)</td>
</tr>
</tbody>
</table>

*Note.* The ordered pairs indicate the number of subjects (Treatment, Control) whose van Hiele levels changed or remained the same from pretest to posttest.
APPENDIX B

SPATIAL VISUALIZATION LESSON PLANS
LESSON 1

**Topic:** Transformations

**Objectives:**
1. Students will learn vocabulary associated with transformations, building on their previous knowledge.
2. Students will be able to identify and create rotations and reflections of figures with hand-held triangles, geoboards, and pencil and paper.

**Materials:**
Large hand-held cardboard triangles; geoboards with geoboards; overhead projector with markers, transparencies, and transparent geoboards, and student worksheets #1, #2, #3 and their transparencies.

**Procedure:**
1. Briefly give the students an overview of the week. Tell the students about today's lesson.
2. Using hand-held triangles, geoboards, transparencies, and plastic pieces on the overhead, demonstrate slides, reflections, and rotations. Encourage student participation and determine what students already know about transformations. Allow students to use non technical terminology.
3. Students in pairs will use geoboards to challenge each other to create and identify reflections and rotations.
4. Solicit facts and list on the board what slides, reflections, and rotations do and do not have in common.

5. Students complete worksheets #1 and #2 in class. Select students to draw solutions on the transparencies.

Closure:

1. Ask students to review the vocabulary and demonstrate reflections and rotations with different materials. Accept the students' terminology but rephrase the responses in more precise terms.

2. Ask students to explain their strategies in determining whether the image is a rotation or a reflection of the original figure. Refer to lists on the board showing similarities and differences in the transformations.

3. Assignment - Distribute worksheet #3.

4. Students place completed worksheets in appropriate boxes before leaving.
GIVEN THE FIRST FIGURE ON THE LEFT, DESCRIBE THE ROTATION OF THE SECOND FIGURE. IS IT 1/4, 1/2, OR 3/4 CLOCKWISE OR COUNTERCLOCKWISE TURN FROM THE FIRST FIGURE?

1. 

2. 

3. 

4.
Given the first figure, draw the second figure using the description of the rotation.

1.

1/4 turn clockwise

2.

1/2 turn counterclockwise

3.

1/4 turn counterclockwise

4.

3/4 turn clockwise
1. Reflect this figure in the vertical line. Is the image turned around or turned over from the original?

2. Reflect this figure in both lines to get 3 images. Number the 3 images and describe which images are turned around from the original and which are turned over from the original figure.

3. Reflect this figure in 4 lines to get 7 images. Number the 7 images and describe which images are turned around from the original and which are turned over from the original figure.
LESSON 2

**Topic:** Transformations

**Objectives:**
1. Students will review how to identify and create rotations and reflections of figures with hand-held triangles, geoboards, and pencil and paper, while recalling the vocabulary associated with these transformations.
2. Students will become hands-on familiar with *The Geometric preSupposer: Points and Lines* software and will reflect points and segments in two intersecting axes.

**Materials:**

**Procedure:**
1. Actively involve students in reviewing transformation vocabulary along with visually demonstrating reflections and rotations with hand-held triangles and plastic pieces and geoboards on the overhead projector.
2. Go over assignment from Lesson 1 using the transparency of worksheet #3. Discuss any questions.
3. Give a brief overview of the lesson.

4. Give student instructions for computer lab work before entering the lab.
   a. Students will work in pairs and take turns at the keyboard.
   b. Each student will complete his/her own worksheets.
   c. On entering the lab, pick up worksheets from the numbered boxes, quick reference cards, and one Supposer disk for each pair of students.
   d. Insert the program in the disk drive, turn on the computer, and press N to begin.
   e. Refer to the quick reference card as necessary.
   f. At the completion of lab time, students will turn off the computers, return the disks to the box, and prepare for closing discussion.

5. Demonstrate, with the LCD, the keys and functions of the program. Point out the similarity between the geoboards and the grid function on the computer. Draw and label two axes and reflect a point and a line segment once. Select a triangle, construct a line of reflection, and reflect the triangle. Encourage student participation.

6. Give the students sufficient time to complete worksheets #4 and #5.

7. Allow 5-10 minutes at the end of the lab for closing discussion.

**Closure:**

Whole class discussion of students' findings.

1. Have students discuss solutions to any problems they might have incurred.
2. Ask students for properties of a reflection in relation to the original figure and list them on the board. Be sure to identify lines of symmetry on the worksheets.
3. If time permits, discuss and list the properties of a rotation.
4. Students place completed worksheets in appropriate boxes before leaving.
1. Record below the reflections of a point in 2 lines.
   On the second graph, once the initial point, D, is in place, position a moveable point, E, where you think the reflection is going to be, then reflect the initial point. If you were correct, the letter E will change to F. Continue in the same manner for the remaining pts.

2. Record below the reflections of a line segment in 2 lines.
1. Reflect a right, acute, obtuse, and an isosceles triangle in one line. Record these reflections below.

2. Reflect a random, parallelogram, trapezoid, and a kite quadrilateral in one line. Record these reflections below.
LESSON 3

Topic: Transformations

Objectives:
1. Students will become hands-on familiar with *The Geometric Supposer: Triangles* software, and will be able to reflect a right triangle in one of its sides, to obtain at least three images, and repeat the procedure with other right and obtuse triangles.
2. Students will be able to identify images as reflections or rotations of the original triangle.

Materials:

Procedure:
1. In appropriately marked boxes, have worksheet #6 and quick reference cards for students to pick-up as they walk in.
2. Using the LCD, demonstrate the keys and functions of *The Geometric Supposer: Triangles* software, referring to the reference card. Reflect a triangle in one of its sides, choosing the vertex of an acute angle resulting in several images of the original
triangle. Show both a clockwise and counterclockwise movement. Use the REPEAT key for other triangles.

3. Demonstrate a potential problem (incurred by choosing the vertex of an obtuse angle) resulting in overlapping figures. Ask students for possible solutions.

4. Restate student expectations in the lab.

5. Give the students sufficient time to complete worksheet #6.

6. Allow 5-10 minutes at the end of the period for closing discussion.

Closure:

Whole class discussion of students' findings.

1. Give the students an opportunity to discuss what they learned about the options in the program and any problems incurred, allowing the students to suggest possible solutions.

2. Ask students to explain their strategies in determining whether the image is a rotation or a reflection of the original figure and list similarities and differences of the two transformations. Have students identify lines of symmetry and points of rotation.

3. Students place completed worksheets in appropriate boxes before leaving.
Using the Supposer

1. New triangle - 1 Right
   2 Label - 3 Reflect (Reflect the triangle 3 times!)
   R Repeat (On another right triangle, and another, etc.)
   If computer BEEPS, immediately press ESC for Main Menu, followed
   by S (scale change), then continue with the reflections.

   Reflect the triangle using one of the sides as the line of reflection.
   Reflect 3 times so that you have the original triangle and 3 images.
   Draw what appears on the monitor and describe which images are
   reflections and which are rotations of the original triangle.
   Use R Repeat on other right triangles.

2. New triangle - 1 Obtuse
   2 Label - 3 Reflect (Reflect the triangle 3 times!)
   R Repeat (On another acute triangle, and another, etc.)
   If computer BEEPS, press ESC for Main Menu, press S scale change.

   Reflect the triangle using one of the sides as the line of reflection.
   Reflect 3 times so that you have the original triangle and 3 images.
   Draw what appears on the monitor and describe which images are
   reflections and which are rotations of the original triangle.
   Use R Repeat on other obtuse triangles.
Lesson 4

Topic:
Transformations

Objectives:
1. Students, building on the previous lab session, will devise a Supposer procedure whereby the triangle's images are all reflections or rotations of the original figure.
2. After using this procedure for a variety of triangles, students will make conjectures concerning the properties of products of reflections.
3. Students will become hands-on familiar with The Geometric Supposer: Quadrilaterals software, and will be able to reflect a parallelogram in one of its sides, to obtain at least three images, and repeat the procedure with other parallelograms and trapezoids.
4. Students will be able to identify images as reflections or rotations of the original parallelogram or trapezoid.

Materials:
Copies of The Geometric Supposer: Triangles and Quadrilaterals quick reference cards. Six Apple® II microcomputers, liquid crystal display (LCD), The Geometric Supposer: Triangles and Quadrilaterals lab packs, and worksheet #7, #8, and #9.

Procedure:
1. In appropriately marked boxes, have worksheets #7, #8, #9, and quick reference cards for students to pick-up as they walk in.
2. Give a brief overview of the lesson.
3. Ask students to study the two reference cards and look for similarities and differences between the two programs.
4. To the extent necessary, reiterate student expectations, and instructions regarding computer and software use.
5. Allow approximately 10 minutes at the end of the period for closing discussion.

Closure:
Whole class discussion of students' findings.
1. Have one student from each pair explain his/her conjectures from worksheets #7 and #8. Identify lines of symmetry and points of rotation.
2. Let students compare similarities and differences between groups and summarize the findings.
3. Ask students to explain their strategies in determining whether the image is a rotation or a reflection of the original figure.
4. Students place completed worksheets in appropriate boxes before leaving.
1. After reflecting a triangle 3-4 times using one of the sides as the line of reflection, which segments would you erase in order to have only the original triangle and those images which are turned around from the original triangle?

**Erase segments ________________________________**

2. Now erase the segments. Were you correct? If so, draw what appears on the monitor and then use the repeat key for other triangles.

Conjectures:_________________________________
1. After reflecting a triangle 3-4 times using one of the sides as the line of reflection, which segments would you erase in order to have only the original triangle and those images which are turned over from the original triangle?

Erase segments ____________________________

2. Now erase the segments. Were you correct? If so, draw what appears on the monitor and then use the repeat key for other triangles.

Conjectures: ____________________________________
                                                 ________________________
                                                 ________________________
                                                 ________________________
                                                 ________________________
WORKSHEET # 9 Using the Supposer

1. **New quadrilateral - 1 Parallelogram - 1 Random**
   - 2 Label - 3 Reflect (Reflect the parallelogram 3 times!)
   - R Repeat (On another random parallelogram, and another, etc.)
   - If computer BEEPS, press ESC for Main Menu, press S scale change.

Reflect the quadrilateral using one of the sides as the line of reflection. Reflect 3 times so that you have the original quadrilateral and 3 images. Draw what appears on the monitor and describe which images are reflections and which are rotations of the original quadrilateral.

Use R Repeat on other random parallelograms.

2. **New quadrilateral - 1 Trapezoid - 1 Random**
   - 2 Label - 3 Reflect (Reflect the trapezoid 3 times!)
   - R Repeat (On another random trapezoid, and another, etc.)
   - If computer BEEPS, press ESC for Main Menu, press S scale change.

Reflect the trapezoid using one of the sides as the line of reflection. Reflect 3 times so that you have the original trapezoid and 3 images. Draw what appears on the monitor and describe which images are reflections and which are rotations of the original trapezoid ABCD.

Use R Repeat on other random trapezoids.
LESSON 5

Topic:
Transformations

Objectives:
1. Students, building on the previous lab session, will devise a Supposer procedure whereby the quadrilateral's images are all reflections or rotations of the original figure.
2. After using this procedure for a variety of quadrilaterals, students will make conjectures concerning the properties of products of reflections.
3. Students will be able to identify whether triangles formed by a diagonal of a parallelogram, rectangle, and kite are reflections or rotations of each other.

Materials:

Procedure:
1. In appropriately marked boxes, have worksheets #10, #11, #12, and quick reference cards for students to pick-up as they walk in.
2. Give a brief overview of the lesson.
3. If necessary, reiterate student expectations and instructions regarding computer and software use.
4. Allow approximately 20 minutes at the end of the period for closing discussion.
Closure:
Whole class discussion of students' findings.
1. Have one student from each pair explain his/her conjectures from worksheets #10 and #11. Identify lines of symmetry and points of rotation.
2. Let students compare similarities and differences between groups and summarize the groups findings.
3. Use a transparency of worksheet #12 to visually assist in the discussion of the students' findings.
4. Have students review properties of reflections and rotations and strategies to determining whether an image is a rotation or a reflection of the original figure.
5. Students place completed worksheets in appropriate boxes before leaving.
CHALLENGE!

1. After reflecting a quadrilateral 3-4 times using one of the sides as the line of reflection, which segments would you erase in order to have only the original quadrilateral and those images which are turned around from the original quadrilateral?

   Erase segments ____________________________

2. Now erase the segments. Were you correct? If so, draw what appears on the monitor and then use the repeat key for other quadrilaterals.

Conjectures:

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________
CHALLENGE!

1. After reflecting a quadrilateral 3-4 times using one of the sides as the line of reflection, which segments would you erase in order to have only the original quadrilateral and those images which are turned over from the original quadrilateral?
   
   Erase segments ____________________________

2. Now erase the segments. Were you correct? If so, draw what appears on the monitor and then use the repeat key for other quadrilaterals.

Conjectures: ____________________________________________
WORKSHEET # 12   Using the Supposer

1. New quadrilateral - 1 Parallelogram - 1 Random
   1 Draw - 1 Segment (diagonal AC)
   R Repeat for a rectangle and a kite shape.

The diagonal divides the quadrilateral into 2 triangles. Draw what appears on the monitor and determine whether one triangle can be described as a reflection or rotation of the other. Indicate the line of symmetry or the point of rotation.

2. Complete each of the following statements by using the word REFLECTION or ROTATION.

   Triangle (1) to (2) is a _________
   Triangle (1) to (3) is a _________
   Triangle (1) to (4) is a _________
   Triangle (4) to (2) is a _________
   Triangle (4) to (3) is a _________
   Triangle (3) to (2) is a _________
BIBLIOGRAPHY


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Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. In J. L. Martin & D. A. Bradbard (Eds.), *Space and geometry: Papers from a research workshop* (pp. 75-97). Columbus, OH: ERIC Center for Science, Mathematics and Environmental Education.


