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Dexterous manipulation using multifingered systems

Chung, Wen-Yeuan, Ph.D.

The Ohio State University, 1994
DEXTEROUS MANIPULATION USING MULTIFINGERED SYSTEMS

DISSE TATION

Presented in Partial Fulfillment of the Requirement for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

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CHAPTER I
INTRODUCTION

1.1 Human Hands and Robot Hands

The study of robot hands has been an active research topic for more than 15 years. The potential applications fall into two categories, which are in industry and in prosthetics. In industry, a robot hand is expected to work as a versatile gripper, and to manipulate objects dexterously. In prosthetics, the ultimate objective is for artificial hands to match the performance of human hands.

The work in this dissertation focuses on the dexterous manipulation by using multifingered systems. All of sensing, grasping, and manipulation problems are considered. The proposed strategies in this work are integrated with the results of other researches to fulfill the ultimate goal -- dexterous manipulation.

In designing an artificial hand or robot hand, we usually study human fingers as a first step. The human hand has five fingers and 22 degrees of freedom (Kato and Sadamoto, 1987). There are various superficial sensors which can respond to touch, pressure, and temperature (Herrick, 1927). The eyes also help us in understanding the environment that our hands are encountering. Most of all, the brain of a human being, and even those of other creatures, can analyze and integrate the information collected from the sensors. Finally, commands are send out to move the joints, and tasks, such as
manipulating an object, are executed.

Most of the grippers attached at the end-effector of robot arms are parallel jaw grippers so far. Each gripper can usually work for only some specific functions. Various kinds of parallel jaw grippers can be found in the reference Kato and Sadamoto, 1987. During the past 10 years, several robot hands or artificial hands have been designed and tested. Some of those developed in Japan are also discussed by Kato and Sadamoto (1987). In the United States, the best known dexterous robot hands used for research purposes are the Stanford/JPL hand (Mason and Salisbury, 1985) and the Utah/MIT hand (Jacobsen et al., 1984; Speeter, 1990b). The DIGITS system (Coe, 1989) is also now being constructed at The Ohio State University for research purposes. Two fingers have been built and operated to study power grasping (Mirza, 1992).

1.2 Theoretical Background and Literature Review

A robot hand is a hybrid serial and parallel mechanism. The kinematics and dexterity of the multifingered system are investigated in Hunt et al., 1991, and McAree et al., 1991. Based on the dexterity of the robot hand, the robot hand can be used to grasp or even manipulate the objects. For manipulation tasks, the process can be divided into several parts. These are recognition of the environment and the grasped object, the choice of the contact points, the calculation of the distributed forces among the fingers, and the control of the system.

Understanding of the environment and the grasped object can be treated as the first step in manipulation tasks. Contact positions can be detected from the change of the positions of the finger joints (Kaneko and Tanie, 1990; Siegel, 1991). The hardness, elasticity or plasticity characteristics, and solidity of the
grasped object can be obtained by using tactile sensors (Stansfield, 1991b). Sliding motion can also be sensed by mounting an accelerometer on the skin (Howe et al., 1988; Howe and Cutkosky, 1989). Furthermore, the techniques of computer vision, and the data collected from tactile sensors were combined together to reconstruct the geometry of the grasped object (Allen 1987; Allen and Michelman, 1990). Besides tactile and vision sensors, force sensors, especially six-axis force sensors, were also proposed. The data collected from the sensors can be used to reconstruct the contact information (Salisbury, 1984; Bicchi and Dario, 1988; Voiorl et al., 1989; and Bicchi, 1990), and to rebuild the geometric shape of the object (Tsujimura and Yabuta, 1992). A suspension-shell mechanism was also proposed to detect the contact information (Okada and Rembold, 1992).

When the geometry of the grasped object has been recognized, the choice of proper contact points becomes the next issue. Two categories of grasp, form closure and force closure, are usually used when discussing stability. Force closure is defined as a grasp in which it is possible to generate the contact forces to resist an external force, while form closure implies contact forces that can equilibrate any external force by virtue of the grasp geometry (Lakshminarayana, 1978; Nguyen, 1988 and 1989; Al-Fahed et al., 1992; and Trinkle, 1992). On the other hand, mobility and stability analysis of a finger system can be found in Salisbury and Roth, 1983. The ways that human beings grasp objects were also investigated in Cutkosky, 1985 and 1989. These works can help us to choose the contact points and decide if the finger system can be used to grasp an object stably.

If the finger system can satisfy the requirement of form closure, then finding the required contact force at each finger becomes the next issue. An object can
be held stably by at least two fingers (Abel et al., 1985). Three frictional contacts might be the simplest finger system that can satisfy the form closure requirement. Some researchers, such as in Holzmann and McCarthy, 1985; Yoshikawa and Nagai, 1991; and Demmel and Lafferriere, 1989, worked on the force distribution problem for three-finger cases. Ji and Roth (1988) found the interaction forces, that is, pairs of contact forces with the same magnitude and opposite direction along the line joining both contact points, with minimum friction angles directly. Mukherjee (1992) took both equilibrium and interaction forces into consideration when finding the solution with minimum friction angle. Among these works, the grasping forces in the null space, i.e. the interaction forces, are usually used to modify the solution.

As for the four- or more-finger cases, a matrix method (Salisbury and Roth, 1983) may be used to solve the set of equilibrium equations. Since some constraints, such as friction constraints, are nonlinear, linearization techniques (Kerr and Roth, 1986) might be used to simplify the problem. Linear programming is also used for the optimization process (Cheng and Orin, 1990; Badreldin and Seireg, 1990). A quadratic optimization technique (Nahon and Angeles, 1992) can be used to improve the continuity of the force or torque curves. Similar problems were solved for the walking machine (Klein and Kittivatcharapong, 1990) with friction constraints. Finding the distributed force by minimizing the internal force (Nakamura et al., 1989) and building the stress model at the contact point (Sinha and Abel, 1992) were also worked out.

In order to operate robot hands successfully, the system should be intelligently controlled. One way to control the system is to use a teleoperation technique (Burdea and Zhuang, 1991a and 1991b). Soft finger and compliance control were also proposed and tested (Cutkosky and Kao, 1989; Akella and
Cutkosky, 1989; Akella et al., 1991; Kao and Cutkosky, 1992). A way to control the Stanford/JPL hand was illustrated in Salisbury and Craig, 1982, and Mason and Salisbury, 1985. Hybrid control was also proposed in Speeter, 1990a. For manipulation tasks, the dynamics of the fingers and in particular the grasped object have to be considered together. Therefore, the computed torque method has been proposed to control multifingered systems or multiple cooperating robot systems (Li et al., 1989 and Cole et al., 1992) and also has been tested experimentally (Hsu and Su, 1992 and Hollerbach et al., 1992).

Although the robot hand problem has been worked on for about 15 years, the applications are still limited to simple object handling tasks. Some examples can be found in Okada 1982, Fearing 1986a and 1986b, Hong et al. 1990, Howe et al. 1990, Stansfield 1991a and 1991b, Tedford 1990, and Trinkle and Paul 1990.

1.3 Objective and Preview

Despite all the studies described in the previous section, many problems still remain unsolved. The focuses of this work are on three major topics: 1. test of force sensor and analysis of ambiguity problem, 2. finding distributed force by optimizing friction angles, and 3. dexterous manipulation. A brief description of these topics and preview of each chapter are listed in the following.

A six-axis force sensor has been designed (Coe, 1989) in order to detect the contact information. However, the sensor has never been tested, and the ambiguity problem in reconstructing the contact information deterred the real application of this sensor. Therefore, a matrix which relates output voltages of the strain gage bridges to the applied wrench and the way to reconstruct the contact information efficiently and correctly should be worked out.
When the object is grasped by the fingers, the friction angles, which are the angles between applied forces and contact normals, must be within the constraints to avoid the slipping motion. Thus, the second objective is to find the distributed forces efficiently so that the wrench can be equilibrated and the friction angles are within the criteria. Moreover, the proposed method should also be applicable to the systems with any number of fingers.

The last objective is to let the multifingered system manipulate the object dexterously and stably. During the manipulation, one requirement is that the object should follow the desired trajectory. The other requirement is that the fingers and the object should contact well and no slipping motion occurs. Based on these two requirements, the commanded joint torque will then be determined and commanded to operate the system.

In Chapter II, the design of the DIGITS system is introduced. Some basic equations and parameters, such as the Jacobian matrix, are derived. Six-axis force sensors are analyzed and tested to get the mapping matrix between the output voltages of the bridges and the resultant wrenches.

In Chapter III, the methods of reconstructing the applied wrench by using a six-axis force/torque sensor are discussed. Various types of contact, such as soft point and soft line contacts, are investigated. The ambiguity problem in reconstructing the wrench for soft point contact has been shown to be nonexistent if the boundary surface is convex.

In Chapter IV, an efficient way of finding the force distribution by optimizing the contact friction angles, which are the angles between the contact forces and contact normals, for multifingered grasping systems is developed. Three steps, projection, residual wrench equilibrium, and optimization, are proposed to solve the force distribution problem. Examples are presented to show how the
proposed method works. The results are also compared to those obtained by using Lagrange multipliers.

In Chapter V, the method of finding the distributed forces described in Chapter IV is combined with the computed torque method to find the torques to be commanded at finger joints for multifingered systems. In this way, slip can be avoided when the object is grasped or manipulated. A history-based method is also proposed to improve the smoothness of the input torque commands. The force feedback is used to compensate for the uncertainty of the system. The strategy used to move the finger, which moved beyond the boundaries of its working volume, to a new position is also proposed. The proposed method is also applicable to three dimensional manipulation.

In Chapter VI, a method to update the data needed for simulation is described. The rolling motion between the fingers and the grasped object is also considered. Two examples, which are three dimensional manipulation tasks, are shown.

The summary of this work, the recommendation, and potential future work are also listed in Chapter VII.
CHAPTER II
 BASIC STRUCTURE

2.1 Introduction

A robot hand named DIGITS is being built at The Ohio State University. This system has four fingers, as shown in Figure 2.1. All of the four fingers can be arranged in different configuration for different operations or purposes. Each finger has three revolute joints. Brushless DC motors were chosen as the actuators. Thus, there are 12 degrees of freedom in the whole four-finger system. Within each finger, the first and the second joints are orthogonal and intersect. The third revolute joint connects the proximate and distal links and is parallel to the second joint. The lengths of the proximal and distal links are similar to get the maximum possible working volume. Optical encoders are also attached to the brushless motors to detect the angular velocities and positions of the finger joints.

The shape of the distal link is a hemisphere joined to a cylinder with the same radius. A six-axis force sensor, which consists of six strain gage bridges, is mounted within the fingertip to reconstruct the contact location, force, and moment even for soft contacts. Recently, a force sensor has also been designed and attached on the proximal link to get contact information on that link.
Figure 2.1 Illustration of DIGITS Robot Hand
2.2 Kinematic Problem and Jacobian Matrix

The kinematic equations, which relate the position, rate and force at the fingertip and the angular positions, rates, and torque at joints, for single finger will be extensively used and will be derived first. The geometry and coordinate frames used to describe one finger, which is similar to that of DIGITS system, are shown in Figure 2.2. The lengths of the proximal and distal links are respectively \( P_1 \) and \( P_2 \). \( \theta_i \) is the position of the \( i \)th finger joint. The coordinate frames \( X_0Y_0Z_0 \) and \( X_1Y_1Z_1 \) are respectively fixed on the palm and the first revolute joint. The coordinate frame \( X_3Y_3Z_3 \) is fixed on the distal link and the origin point is at the center of the hemisphere. If \( P_3 \) is the coordinate of point \( P \) on coordinate frame \( X_3Y_3Z_3 \) and \( P_0 \) is the coordinate of the same point on coordinate frame \( X_0Y_0Z_0 \), \( P_3 \) and \( P_0 \) can be related by

\[
P_0 = \begin{bmatrix}
  C\theta_1 C(\theta_2 + \theta_3) & -C\theta_1 S(\theta_2 + \theta_3) & S\theta_1 \\
  S\theta_1 C(\theta_2 + \theta_3) & -S\theta_1 S(\theta_2 + \theta_3) & -C\theta_1 \\
  S(\theta_2 + \theta_3) & C(\theta_2 + \theta_3) & 0
\end{bmatrix}
\begin{bmatrix}
P_3 + P_1 \\
S\theta_2 \\
S(\theta_2 + \theta_3)
\end{bmatrix}
\begin{bmatrix}
C\theta_1 C(\theta_2 + \theta_3) \\
S\theta_1 C\theta_2 \\
S(\theta_2 + \theta_3)
\end{bmatrix}
\]

where \( C \) : cosine function
\[ S \] : sine function.

The parameters written in boldface, such as \( P_3 \) and \( P_0 \), are either vectors or matrices.
The detailed shape of the distal link is shown in Figure 2.3. u and v are two angles used to represent the contact positions on the hemisphere. If point P is at the hemisphere side, \( P_3 \) can be expressed as

\[
P_3 = \begin{bmatrix} R \cdot u \\ R \cdot c \cdot u \cdot v \\ R \cdot c \cdot u \cdot v \end{bmatrix}
\]

If point P is at the cylindrical side, L and v are two parameters to represent the contact position. \( P_3 \) becomes

\[
P_3 = \begin{bmatrix} -L \\ R \cdot v \\ R \cdot v \end{bmatrix}
\]
In Figure 2.3, O is the center of the fingertip and the origin of the coordinate frame $X_3Y_3Z_3$. If the position of the point O on coordinate frame $X_0Y_0Z_0$ is known and is expressed as $O_0: (x_0, y_0, z_0)^T$, the positions of the finger joints can be found from

$$\theta_1 = \tan^{-1} \left( \frac{y_0}{x_0} \right)$$

$$\theta_2 = \beta - \gamma$$
$$\sin \theta_3 = \frac{\sin \gamma}{P_2} \sqrt{x_0^2 + y_0^2 + z_0^2}$$

or

$$\theta_2 = \beta + \gamma$$
$$\sin \theta_3 = -\frac{\sin \gamma}{P_2} \sqrt{x_0^2 + y_0^2 + z_0^2}$$

(2.4)

where

$$\beta = \tan^{-1} \left( \frac{z_0}{\sqrt{x_0^2 + y_0^2}} \right)$$

$$\cos \gamma = \frac{x_0^2 + y_0^2 + z_0^2 + P_1^2 - P_2^2}{2P_1 \sqrt{x_0^2 + y_0^2 + z_0^2}}$$

Figure 2.4 Illustration of Finding Positions of Finger Joints
The reason for the two sets of solutions is that two solutions are possible for a given position of the origin point O. One has the third joint bent upwards and the other has it bent downwards, as shown in Figure 2.4.

The rate kinematic equations relating the rate at the fingertip and the angular velocities at the joints can be divided into two parts. The first sub-problem is to find the rate at the fingertip if the angular velocity of each joint is known. Let P be any point on the distal link, and its position in coordinate frame $X_0Y_0Z_0$ be $(x_p, y_p, z_p)$. The relationship between the angular velocities of the finger joints and the velocities at point P can be written in the form (Craig 1989):

$$\begin{bmatrix} \omega_x & \omega_y & \omega_z & v_x & v_y & v_z \end{bmatrix}^T = J\begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T$$

(2.5)

The linear velocity $\omega$ and angular velocity $v$ of point P are based on the coordinate frame $X_0Y_0Z_0$. $J$ is a Jacobian matrix and

$$J = \begin{bmatrix}
0 & S\theta_1 & S\theta_1 \\
0 & -C\theta_1 & -C\theta_1 \\
1 & 0 & 0 \\
-x_p & -z_pC\theta_1 & Pp(C\theta_1S\theta_2 - z_pC\theta_1) \\
x_p & -z_pS\theta_1 & PpS\theta_1S\theta_2 - z_pS\theta_1 \\
0 & y_pS\theta_1 + x_pC\theta_1 & -P_xC\theta_1 + y_pS\theta_1 + x_pC\theta_1
\end{bmatrix}$$

(2.6)

The other sub-problem is to find the joint rates given the linear velocity at the finger tip. If the contact is a frictional point contact, the Jacobian matrix can be decomposed into two parts and can be expressed as

$$J = \begin{bmatrix}
J_w \\
J_v
\end{bmatrix}$$

(2.7)

where
The relationship between the linear velocity at the finger tip and the angular velocities of the finger joints can then be related:

\[
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix} = J_v^{-1} \begin{bmatrix}
v_x \\
v_y \\
v_z
\end{bmatrix}
\] (2.8)

On the other hand, the wrench applied at the fingertip and the torque at finger joints can also be related by Jacobian matrix. When a wrench \( W \), force \( F \) and torque \( T \), are acting at the contact point \( P \), the commanded torque at the finger joints has to equilibrate the wrench. These two terms can also be related by the equation (Craig 1989)

\[
\begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix}^T = J^T \begin{bmatrix}
T_x \\
T_y \\
T_z \\
F_x \\
F_y \\
F_z
\end{bmatrix}^T
\] (2.9)

where \( \tau_j \) is the torque of \( j \)th joint and \( J \) is the same Jacobian matrix as in Equation 2.6. Furthermore, if frictional hard point contact is the case, only force \( F \) can be applied at the contact point. The relationship becomes

\[
\begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix}^T = J_v^T \begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}^T \quad \text{or} \quad \begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}^T = (J_v^T)^{-1} \begin{bmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix}^T
\] (2.10)
2.3 Six-Axis Force Sensors

Force sensors are necessary for force feedback control. A six axis force sensor mounted within the fingertip is intended to get the resultant wrench at some specified point. The resultant wrench can then be used to reconstruct the contact location, force and moment for even soft fingers.

The sensor and the cap which have been designed by Coe in 1989 are respectively shown in Figure 2.5 and Figure 2.6. The geometry of the cap is similar to that of a human being's finger and is a hemisphere joined to a cylinder. This sensor when assembled with the cap becomes the distal link. The theoretical sensitivity, which relates output voltages of strain gage bridges to the resultant wrench, of this sensor will be studied.

Figure 2.5 Design of Six-Axis Force Sensors
The six-axis force sensor is made of aluminum. Six full strain gage bridges have been cemented to the sensor. Each bridge consists of four strain gages. As shown in Figure 2.7, the $X_3$ axis is defined as being along the axial direction. The bridges located at the first section are used to detect the moments along the $Y_3$ and $Z_3$ directions, which are $M_y$ and $M_z$. Forces along the $Y_3$ and $Z_3$ directions, $F_y$ and $F_z$, are detected by the bridges located on the second section. The third section is especially designed to measure $F_x$, the force along the axial or $X_3$ direction. The fourth section is used to detect $M_x$, the moment along the axial direction. The point $O$ is again the center of the hemisphere when the cap and the sensor are assembled together. The distance between point $O$ and the right most point of the sensor is 0.18".

Figure 2.6 Design of Cap

Figure 2.7 Position of Wheatstone Bridges
The theoretical relationship between the applied loads and the output voltages of the bridges will be briefly described. A similar calculation has been done in Coe 1989, but some mistakes should be corrected. The output voltage, $e$, of a full bridge is related to the strain sensed by the gage and can be expressed as

$$e = V \cdot G_f \cdot K_c \frac{\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4}{4}$$

(2.11)

where $V$ is the supply voltage to the bridge, $G_f$ is the gage factor, actually 2.05 (the EA series made by Measurements Group), and $K_c$ is the gain of the strain gage conditioner. $\varepsilon_i$ is the strain sensed at each strain gage. The positions of the gages have been properly designed so that $\varepsilon_1 = -\varepsilon_2 = \varepsilon_3 = -\varepsilon_4 = \varepsilon$, therefore, Equation 2.11 can be rewritten as

$$e = V \cdot G_f \cdot K_c \cdot \varepsilon$$

(2.12)

The EA-13-062TV-350 strain gages are chosen to sense $M_x$. Since the primary axes of the strain gages are at an angle of 45° with respect to the principal shear directions, the gages will sense a normal stress of equal magnitude to the shear stress. Therefore, the sensitivity of sensing the axial torque, $M_x$, can be expressed as

$$e = V \cdot G_f \cdot K_c \cdot \varepsilon = V \cdot G_f \cdot K_c \cdot \frac{\tau \cdot (1+\nu)}{E} = V \cdot G_f \cdot K_c \cdot \frac{(1+\nu)}{E} \cdot \frac{M_x \cdot r}{J}$$

(2.13)

$J$ is the polar moment of inertia and $J = \frac{\pi r^4}{2}$. $r$ is the radius of the cylinder to detect $M_x$ and is 0.125". The Young's modulus, $E$, of the aluminum is
$1.03 \times 10^7$ psi, and the Poisson's ratio, $v$, is 0.334. Thus, the sensitivity can be found:

$$\frac{e}{M_x} = 8.654 \times 10^{-5} \cdot V \cdot K_c \text{ volt/(lb\cdot in)}$$

(2.14)

A thin ring was designed to detect the component $F_x$. When a tension force is applied to a thin ring, the equivalent moments, $M_x$, will result as shown in Figure 2.8. For a circular thin ring, the moment $M_x$ is $0.182 \cdot r \cdot F_x$ (Young, 1989). $r$ is the radius of the inner ring and is 0.175". Four EA-13-060CD-350 strain gages were glued at the inner and the outer sides of the ring. The relationship between the output voltage and $F_x$ can be expressed in the form

$$e = V \cdot G_r \cdot K_c \frac{M_x c}{EI}$$

(2.15)

In which $I$ is the moment of inertia and $I = \frac{bh^3}{12}$. The inner diameter and the outer diameter can be treated as 0.35" and 0.5", thus, the height, $h$, of the beam is then 0.075". The width, $b$, of the beam is 0.25", as shown in Figure 2.5. The output voltage for per unit load can then be calculated:

$$\frac{e}{F_x} = 2.704 \times 10^{-5} \cdot V \cdot K_c \text{ volt/lb}$$

(2.16)
A beam with square cross section, as shown in Figure 2.5, was intended to detect the force and moment along the \( Y_3 \) and \( Z_3 \) direction. The EA-13-060PB-350 strain gages were cemented at all of the four sides to sense \( M_y \) and \( M_z \). A similar formula to Equation (2.15) can be used to express the relationship between the output voltage and \( M_y \) or \( M_z \). The equation can be written in the form

\[
e = V \cdot G_f \cdot K_c \cdot \frac{M_{y,z}c}{EI}
\]  
(2.17)

By plugging in the dimension of the square cross section, width = height = 0.2", the sensitivity can be obtained:

\[
\frac{e}{M_{y,z}} = 1.493 \times 10^{-4} \cdot V \cdot K_c \quad \text{volt/(lb.in)}
\]  
(2.18)
Four EA-13-062TV-350 strain gages were also cemented on each side of the square beam to detect the transverse forces, $F_y$ and $F_z$. There are two separate grids in each strain gage. The spacing of the two grids is 0.005", and the width of each grid is 0.055", as shown in Figure 2.12. The height, $h$, of the beam is 0.2". Thus the area sensed by each grid is from 0.0125$h$ to 0.2875$h$ away from the central line. The average shear stress sensed by the strain gages, due to the effect of the $F_y$ or $F_z$, can then be found.

$$
\tau_{\text{ave}} = \frac{1}{0.2875h - 0.0125h} \int_{0.0125h}^{0.2875h} \frac{F_{y,z} Q}{lb} dy_1
$$

$$
= \frac{1}{0.275h} \int_{0.0125h}^{0.2875h} \frac{F_{y,z} ((h/2)^2 - y_1^2) \cdot b/2}{lb} dy_1 = 1.327 \frac{F_{y,z}}{b \cdot h} \quad (2.19)
$$

Since the strain gages used here are the same as those used for detecting $M_x$, a similar formula to Equation (2.13) can be applied to this case. The equation becomes

$$
e = V \cdot G_f \cdot K_z \cdot \frac{\tau_{\text{ave}} \cdot (1 + \gamma)}{E} \quad (2.20)
$$

The output voltage for per unit load can then be found:

$$
\frac{e}{F_{y,z}} = 8.808 \times 10^{-6} \cdot V \cdot K_c \quad \text{volt/lb} \quad (2.21)
$$
2.4 Test of Six-Axis Force Sensor

Although theoretical sensitivities have been derived, the manufacturing error and stress concentration will result in some discrepancies and the cross-talk between the bridges usually occurs. Therefore, the force sensor should be tested to build the matrix which relates the output voltages to the resultant wrench. The setup of the test, analysis of the data, and the results will be described in this Section. The sensitivity derived from experimental data will also be compared with the theoretical one described in previous Section.

Instead of using the cap shown in Figure 2.6, the cap shown in Figure 2.9 was especially designed for test purposes. This cap is made to be a cylinder and has six grooves. By hanging the weight at different positions, we can collect data to find out the relationship between the output voltages and $F_y$ as well as $M_z$, or $F_z$ as well as $M_y$.

![Figure 2.9 Design of Cap Used for Testing Sensitivity](image)
In each test, the applied weight was increased from 0 lb to 4 lb. The output voltages for all of the six bridges were recorded. The output voltages versus the applied weights were plotted to find the slopes. Figure 2.10 shows an example in which the weight is hung at the left most position of the grooves. The vertical axis represents the output voltage of the bridge which detects \( M_y \), and the horizontal axis represents the load which is toward the \( Z_3 \) direction. The slope was found to be 766 mv/lb. Similar plots were made and slopes were determined for the other bridges. The same procedures were also executed by hanging the weight at the other five positions.

![Figure 2.10 Output Voltage versus Applied Force](image)

When the slopes at all the six hanging positions have been collected, the plot as shown in Figure 2.11 can be made. In Figure 2.11, point O is the same point as in Figure 2.7. The points marked by "x" represent the positions of the grooves. The value for \( M_y \) at point \( g \) came from the slope found out from
Figure 2.10. The other values were obtained by hanging the weight at the other five grooves. These six points were correlated to draw a line $M_y$ in Figure 2.11. Thus, the values shown on the line represents the output voltage of the bridge, which detects $M_y$, by hanging 1 lb at different positions. Similar lines, $F_y$ and $F_z$, were also plotted in Figure 2.11. If points e and f are considered, the distance between these two points is 1 inch. The difference of the wrenches applied at points e and f is just 1 lb in moment along the $Y_3$ direction. Therefore, the slope of each line on Finger 2.11 represents only the effect of the moment $M_y$ on each bridge.

In Figure 2.11, point B is the center of the strain gages for detecting $M_y$, and is 0.23" away from point O. It is evident that the line $M_y$ passes through point B. The reason is that the values of $M_y$ sensed by the strain gages is zero if the load is hung at this position. Furthermore, the sensitivity of the bridge which detects $M_y$ can be found from the slope of line $M_y$ and is $911\, \text{mv}/(\text{lb in})$.

The line $F_y$ was obtained by following the same procedure. The values of the points on line $F_y$ are almost zero around point C. This point is the center of the strain gages for sensing $F_y$, and is 0.50" away from point O. This fact means the output voltage is almost zero if only $F_z$ is applied. However, the bridge detecting $F_y$ was designed to only be sensitive to the transverse force $F_y$. The output voltage should be zero even though $M_y$ is applied. The main reason is that the strain gages were not positioned perfectly. When the strain gages are cemented, as shown in Figure 2.12, so that the central line of the strain gages is deviated from the principal axis by $\xi$ degrees. If $\sigma$ is the normal stress due to the moment $M_y$, the normal stresses along the directions of both grids are
Figure 2.11 Output Voltage versus $F_z$ and $M_y$

slope from Figure 2.10

$F_z$

$65$ mv/lb

$F_y$

$20$ mv/lb

$60$ mv/lb

$55$ mv/lb

$0$ mv/lb

$-20$ mv/lb

$-40$ mv/lb

$-800$ mv/lb

$0$ mv/lb

$-400$ mv/lb

$0.5''$

$-1''$

$-0.5''$

$-1.5''$

$M_y$

$-800$ mv/lb

$-400$ mv/lb

$0$ mv/lb

$400$ mv/lb

$800$ mv/lb
\[
\sigma_1 = \frac{\sigma}{2} + \frac{\sigma}{2}\cos(2\xi + 90^\circ)
\]
\[
\sigma_2 = \frac{\sigma}{2} + \frac{\sigma}{2}\cos(2\xi - 90^\circ)
\] (2.22)

The difference of the strains sensed by both strain gages can then be derived:

\[
\varepsilon_1 - \varepsilon_2 = \frac{1}{E}(\sigma_1 - \gamma\sigma_2) - \frac{1}{E}(\sigma_2 - \gamma\sigma_1) = \frac{1}{E}(1 + \gamma)(\sigma_1 - \sigma_2)
\]

\[
= \frac{1}{E}(1 + \gamma)\frac{\sigma}{2}(\cos(2\xi + 90^\circ) - \cos(2\xi - 90^\circ))
\] (2.23)

Apparently, \(\varepsilon_1 - \varepsilon_2 = 0\) when \(\xi\) is zero. However, the strain effect can not be eliminated whenever \(\xi\) is not zero. A crosstalk effect will then result and should be considered when the force sensor is put into use. The crosstalk effect can be read from the slope of the line \(F_y\). It was found that the output voltage of the bridge detecting \(F_y\) increased by 30.2 mv when extra unit of \(M_y\) was applied.

Figure 2.12 Illustration of Applied Stress and EA-13-062TV-350 Strain Gage

The results for the bridge detecting \(F_z\) were also shown in Figure 2.11, and the line \(F_z\) was also drawn. As was discussed in the previous paragraph, the
value at point C represents the case in which only a transverse force was
applied. Thus, the sensitivity for $F_z$ should be read from the line $F_z$ at point C.
From Figure 2.11, the sensitivity for $F_z$ is 60.6 mv/lb. The slope of the line is 7.1
mv/(lb in), which represents the crosstalk value due to the effect of $M_y$.

By use of the same processes shown in Figure 2.11, the output voltages
for all the six bridges due to the effect of $F_z$ and $M_y$ can be obtained.
Furthermore, the weight was also hung toward the negative $Z_3$ direction to
check the coincidence. Similarly, the sensitivity due to the effect of $F_y$ and $M_z$
can also be obtained by hanging the weight along the $Y_3$ direction. The
sensitivity for $F_y$ was obtained and is 60.5 mv/lb, and the sensitivity for $M_z$ was
917 mv/(lb in).

There are two holes on the cap shown in Figure 2.9. The wires connected
to the weight could be placed through these holes to apply the weight along the
axial direction. A similar plot to Figure 2.10 can then be drawn. The sensitivity of
$F_x$ could also be read from the slope and was 129 mv/lb.

![Figure 2.13 Design of the Device for Testing $M_x$](image-url)
The device shown in Figure 2.13 was used to collect data for $M_x$. The weight hung at either position would result in different values of $M_x$, and the resultant force and moment along the $Y_3$ or $Z_3$ axes would be the same. By comparing these two sets of data, the effect due to $M_x$ can then be found. The sensitivity for $M_x$ was also found to be -519 mv/(lb in).

The sensitivities for the six bridges due to the principal effects are listed on Table 2.1. Both the theoretical values and experimental results were shown for comparison. Although the values of $V$ were around 10 volt, and those for $K_c$ were around 600, they were not the same for each strain gage conditioner used in the test. The discrepancies are also shown. The sensitivities for moments are very close for both experimental results and theoretical values. The error for $F_x$ might be that the shape of the sensor is not a thin ring, and the stiffness of the sensor would be greater than that of a real thin ring. Thus, the real sensitivity is smaller than the one we predicted. The reasons that the real sensitivities for $F_y$ and $F_z$ are higher than those predicted might be stress concentration or other factors.
Table 2.1 Sensitivities of Six-Axis Force Sensor

<table>
<thead>
<tr>
<th></th>
<th>Experimental Results</th>
<th>Theoretical Values</th>
<th>Discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mv/lb or mv/(lb in)</td>
<td>v/lb or v/(lb in)</td>
<td>%</td>
</tr>
<tr>
<td>$F_x$</td>
<td>129.0</td>
<td>$2.201 \times 10^{-5} \cdot V \cdot K_c$</td>
<td>$2.704 \times 10^{-5} \cdot V \cdot K_c$</td>
</tr>
<tr>
<td>$F_y$</td>
<td>60.5</td>
<td>$1.034 \times 10^{-5} \cdot V \cdot K_c$</td>
<td>$8.808 \times 10^{-6} \cdot V \cdot K_c$</td>
</tr>
<tr>
<td>$F_z$</td>
<td>60.6</td>
<td>$1.032 \times 10^{-5} \cdot V \cdot K_c$</td>
<td>$8.808 \times 10^{-6} \cdot V \cdot K_c$</td>
</tr>
<tr>
<td>$M_x$</td>
<td>-519.0</td>
<td>-$8.873 \times 10^{-5} \cdot V \cdot K_c$</td>
<td>-$8.654 \times 10^{-5} \cdot V \cdot K_c$</td>
</tr>
<tr>
<td>$M_y$</td>
<td>911.0</td>
<td>$1.566 \times 10^{-4} \cdot V \cdot K_c$</td>
<td>$1.493 \times 10^{-4} \cdot V \cdot K_c$</td>
</tr>
<tr>
<td>$M_z$</td>
<td>917.0</td>
<td>$1.566 \times 10^{-4} \cdot V \cdot K_c$</td>
<td>$1.493 \times 10^{-4} \cdot V \cdot K_c$</td>
</tr>
</tbody>
</table>
In real applications, a 6 by 6 matrix, which relates the output voltages of the six bridges to the applied wrenches has to be built. As is shown in Figure 2.11, the output voltages were not the same if the same load was applied at different positions. However, the effects of the moment, which are represented by the slopes of the lines, would remain the same. This means that the output voltages with respect to the transverse forces, \( F_y \) and \( F_z \), will change when the selected origin point is changed. For example, the sensitivity for \( F_z \) is 60.6 mv/lb if the origin is selected at point C, it will become 57.1 mv/lb if the origin is at point O. The equation that relates the output voltages and applied wrenches can be expressed in the form

\[
C \begin{bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta F_z \\ \Delta M_x \\ \Delta M_y \\ \Delta M_z \end{bmatrix}^T = \begin{bmatrix} \Delta v_1 \\ \Delta v_2 \\ \Delta v_3 \\ \Delta v_4 \\ \Delta v_5 \\ \Delta v_6 \end{bmatrix}^T 
\]  

(2.24)

\( \Delta F \) and \( \Delta M \) represent the increments of the applied wrench, and \( \Delta v_i \) is the change of the output voltage of each bridge. The units are in volt for output voltage, lb and lb in are respectively the units of force and moment. \( C \) is the 6 by 6 matrix which relates these two vectors. If the origin is chosen at point O, the matrix \( C \) was found to be

\[
C = \begin{bmatrix}
22.011 & 1.348 & -4.180 & 0.000 & 5.119 & 6.228 \\
0.171 & 10.407 & -2.580 & 0.342 & 5.161 & 0.120 \\
0.000 & -3.492 & 9.726 & 0.000 & 1.209 & -6.984 \\
0.000 & 0.000 & 0.000 & -88.734 & 0.000 & 0.000 \\
0.000 & 0.129 & -36.008 & -0.344 & 156.579 & 1.839 \\
0.649 & 36.026 & -0.103 & 0.000 & 0.888 & 156.641
\end{bmatrix} \times 10^{-6} \cdot V \cdot K_c 
\]  

(2.25)
Each row in matrix C represents the output voltages of the six bridges when unit force or moment is applied. The six bridges are principally used to measure $\Delta F_x$, $\Delta F_y$, $\Delta F_z$, $\Delta M_x$, $\Delta M_y$, and $\Delta M_z$ in sequence. The values of the first and the fourth columns were obtained directly from the slopes of voltage versus load figures, as shown in Figure 2.10. The values of the second and third columns were read from the lines, as shown in Figure 2.11, at point O. The slopes of the lines in Figure 2.11 became the values in the fifth and the sixth column.

On the other hand, if another point, instead of point O, is chosen as the origin point, the matrix can be easily modified. If we take point Q, as shown in Figure 2.11, as an example, the point is located at the right side of point O and the distance is $L_Q$. If $C_i$ represents each the ith column of the 6 by 6 matrix in Equation (2.25), the matrix C becomes

$$C = \begin{bmatrix} C_1 & C_2 + C_6 \cdot L_Q & C_3 - C_5 \cdot L_Q & C_4 & C_5 & C_6 \end{bmatrix} \times 10^{-6} \cdot V \cdot K_c$$

(2.26)

The matrix can be easily built when the origin point is decided and the specifications of the strain gage conditioner are known. On the other hand, the applied wrench at the chosen origin point can easily be found to be

$$\begin{bmatrix} \Delta F_x & \Delta F_y & \Delta F_z & \Delta M_x & \Delta M_y & \Delta M_z \end{bmatrix}^T = C^{-1} \begin{bmatrix} \Delta V_1 & \Delta V_2 & \Delta V_3 & \Delta V_4 & \Delta V_5 & \Delta V_6 \end{bmatrix}^T$$

(2.27)

If the values of $V$ and $K_c$ are taken from the conditioner used for experiment and the origin point is chosen at point O, then $C^{-1}$ is also found to be
The wrench found by using Equation (2.27) can be used to reconstruct the contact information which includes contact position, force, and moment. The details of the strategies used will be discussed in the next Chapter.

2.5 Conclusion

The basic equations for the single finger have been developed. The six-axis force sensor was analyzed and tested. The matrix which relates the output voltages of the bridges and the applied wrenches for this sensor was built. This matrix can be used directly, with minor modifications, for future application in the DIGITS system.
CHAPTER III
WRENCH RECONSTRUCTION

3.1 Introduction

The intrinsic six-axis force/torque sensor has received much attention recently. It can be mounted within the fingertip and be used to reconstruct the applied wrench. A fingertip with a spherical shape was suggested to reconstruct the contact location for frictional point contact (Salisbury 1984; Bicchi and Dario, 1988). The force/torque sensor was also used to rebuild the geometric shape of the object (Tsujimura and Yabuta, 1992). The methods of reconstructing the wrench applied on a soft fingertip with ellipsoidal geometry was discussed in references Bicchi, 1990 and Voirol et al., 1989. However, the ambiguity for the soft finger (Voirol et al., 1989) has not been resolved if the fingertip is combined with two or more ellipsoidal elements.

The design of the sensor and the experimental results have been discussed in the preceding chapter. In this chapter, the methods of reconstructing the applied wrench by using the data collected from the six-axis force/torque sensor will be discussed. Since a method to reconstruct the wrench has been well developed by Bicchi (1990) and Voirol et al. (1989), the ambiguity problem (Chung and Waldron, 1993a) becomes the focus of this work. Whether the data from a six-axis force/torque sensor can be used to reconstruct the applied wrench for various types of contact, such as soft point and soft line contacts, correctly are investigated.
3.2 Sensors Mounted within a Plane

The simplest contact problem is when the contact surface is a plane. The category of the contact may be point contact, line contact, or surface contact. The problem discussed in this Section is how to reconstruct the wrench applied on a plane by using the data collected from a six-axis sensor. The point, line, and planar contacts will all be discussed.

3.2.1 Point contact

A pushing force, $F$, with any arbitrary direction as well as a moment, $T$, along the normal direction, as shown in Figure 3.1, can be applied to a plane in a soft point contact case. The resultant wrench, $F_0$ and $T_0$, is collected from the sensor mounted at point $O$. These two sets of wrenches, $(F_0, T_0)$ and $(F, T_k)$, can be related as follows

$$F_0 = F$$

$$T_0 = r \times F + T_k$$

(3.1)

The position $r(x, y)$ and moment $T$ can be solved for giving

$$x = -\frac{T_{0y}}{F_{0z}} \quad y = \frac{T_{0x}}{F_{0z}} \quad T = T_{0z} - xF_{0y} + yF_{0x}$$

(3.2)

If an ambiguity does exist, there may be other sets of wrenches, $F'$ and $T'$, which can result in the same data at point $O$ as the previous wrench, $F$ and $T$, does. These two applied wrenches must then be related by

$$F_0 = F = F'$$

$$T_0 = r \times F + T_k = r' \times F' + T'k$$

(3.3)
After calculation, we conclude that $F = F'$, $x = x'$, $y = y'$, and $T = T'$. Therefore, the solution is unique and the ambiguity does not exist for this case.

![Figure 3.1 Planar Soft Point Contact](image)

Either the frictionless or frictional contact case can be regarded as a special case of the soft contact. Therefore, the data from a six-axis sensor mounted within a plane is adequate to reconstruct the wrench for various point contacts correctly.

### 3.2.2 Line and surface contact

When a plane mounted on a force sensor is subjected to a line contact, the wrench applied at point $A$, as shown in Figure 3.2, can be expressed as $(F_x, F_y, F_z, T_1, 0, T_3)$ for frictional contact or $(0, 0, F_z, T_1, 0, 0)$ for frictionless contact. The vector of $T_1$ lies on the plane and is perpendicular to the contact line. $T_3$ is along the normal vector of the contact plane.

If $\ell$ is the distance between point $A$ and any arbitrary point $C$ on the line, the wrench applied at point $C$ will be $(F_x, F_y, F_z, T_1 + \ell F_z, 0, T_3 ')$. Therefore, there must be a unique point $B$ along the line so that the wrench applied at this point is $(F_x, F_y, F_z, 0, 0, T_3 ')$. The methods of finding the position and the applied wrench at point $B$ is exactly the same as in the case of soft point contact.
Furthermore, point B is equivalent to the common point in Equation (7) of reference Salisbury (1984).

![Figure 3.2 Planar Line Soft Contact](image)

There are an infinite number of solutions in this case, since six equations are used to solve for seven unknowns. Any line passing through point B could be a solution. One way to get the right solution is to find the other common point by perturbing the applied wrench without moving the line contact as in Salisbury (1984). The other one is to attach tactile sensors, located along dashed lines shown in Figure 3.2, to the plane. Therefore, the other point(s) on the line can be sensed and the line can then be inferred.

With regard to planar contact, although the point at which the torque along the horizontal directions is zero can be found, the contact position cannot be obtained by using the data from the force sensor. Touch sensors become necessary to find the contact point information, and the wrench can then be inferred.
3.3 Sensors Mounted within the Ellipsoidal Fingertip

Most of the contact surfaces, especially those of fingers, are ellipsoids or consists of several ellipsoids. The problem of reconstructing the wrenches applied to an ellipsoidal fingertip will be discussed. The cases of line and surface contact seldom occur. In addition, the wrenches for these cases cannot be reconstructed by using the data from the six axis force/torque sensors only, as shown in previous section. Various point contacts will be the main issues left to be discussed.

The method of reconstructing the wrench for a fingertip with only a cylindrical or spherical shape has been developed by Bicchi (1990) and Voirol et al. (1989). The modified solution will be described briefly as follows:

In the spherical case, as shown in Figure 3.3(a), let $F_0$ and $T_0$ be the measured resultant force and moment at the center of the sphere. $F$ and $T$ are the applied force and moment at the contact point respectively. $u$ is the unit
vector parallel to the direction of the radial vector. The values of \( T \) and \( u \) were solved for to give

\[
T = \text{sign}(F_r \cdot T_0) \sqrt{\frac{1}{2} \left( \sqrt{(F_r^2 - T_0^2)^2 + 4(F_r \cdot T_0)^2} - (F_r^2 - T_0^2) \right)}
\]  

(3.4)

\[
u = \frac{F_r \times T_0 + T T_0 - \sqrt{T^2 + F_r^2 - T_0^2} F_r}{T^2 + F_r^2}
\]  

(3.5)

where \( F_r = r F_0 \).

If the fingertip has a cylindrical shape, as shown in Figure 3.3(b), \( F' \) and \( T' \) are the applied force and moment at the contact point. \( u' \) is the unit vector in the direction of the applied rubbing moment, lying in the \( y, z \) plane. \( L \) is the distance between the contact point and the origin along the symmetry axis, that is, the \( x \) axis. The values of \( T' \), \( u' \), and \( L \) were also solved for giving

\[
u_y = \frac{F_{rz} T_{0x} - F_{ry} \sqrt{(F_{rz}^2 + F_{ry}^2) - T_{0x}^2}}{F_{rz}^2 + F_{ry}^2}
\]  

\[
u_z = \frac{-F_{ry} T_{0x} - F_{rz} \sqrt{(F_{rz}^2 + F_{ry}^2) - T_{0x}^2}}{F_{rz}^2 + F_{ry}^2}
\]  

(3.6)

\[T' = \frac{F_{ry} (T_{0y} - (F_{rx} \cdot u_z)) + F_{rz} (T_{0z} + (F_{rx} \cdot u_y))}{F_{ry} u_y + F_{rz} u_z}
\]  

(3.7)

\[L = \frac{F_{rx} - (T_{0y} u_z) + (T_{0z} u_y)}{F_{ry} u_y + F_{rz} u_z}
\]  

(3.8)

where \( F_r = r F_0 \).
3.4 Discussion of the Ambiguity Problem

The equations listed in the previous section can be used to reconstruct the contact force for either a single sphere or a single cylinder. However, when the finger shape is a hemisphere joined to a cylinder or composed of several parts with ellipsoidal shape, an ambiguity may occur. This means that we may get a set of solutions from each part respectively, and two or more sets of solutions can be reconstructed if one set of $F_0$ and $T_0$ is given. This problem will be investigated in this Section.

3.4.1 Hard contact

The contact may be either hard point contact or soft point contact. For the hard contact case, only pure forces can be applied at the contact point. The contact does not transfer moments. The line of action, which is defined as the line passing through the contact point and along the direction of applied force, can be reconstructed exactly (Salisbury, 1984; Tsujimura and Yabuta, 1992). If the boundary surface is convex, there will be just two intersection points between the line of action and the boundary. However, one of the solutions is a force that pulls away from the fingertip, and therefore should be discarded. The other solution then becomes unique and no ambiguity occurs in this case.

3.4.2 Soft contact

In the case of soft contact, a moment along the normal direction, $u$, can be applied in addition to the force. When ambiguity occurs, the resultant forces and moments should be the same. If two sets of wrenches - $(F, Tu)$ acting at position $v$ and $(F', T'u')$ acting at position $v'$ - result in the same wrench $(F_0, T_0)$ at the point where the sensor is mounted, then

$$F_0 = F = F'$$
After rearrangement, we get

\[(v - v') \times F = T'u' - Tu\]  \hspace{1cm} (3.10)

Dot multiplying both sides of Equation (3.10) by \((v - v')\) gives

\[(v - v') \cdot (T'u' - Tu) = 0\]

That is

\[T' = \frac{Tu \cdot (v - v')}{u' \cdot (v - v')}\]  \hspace{1cm} (3.11)

Dot multiply both sides of Equation (3.10) by \(F\) and substitute \(T'\) by using Equation (3.11), then

\[T\left(\frac{u \cdot (v - v')}{u' \cdot (v - v')} u' - u\right) \cdot F = 0\]

That is

\[F \cdot u = -K(F \cdot u')\]  \hspace{1cm} (3.12)

where

\[K = \frac{u \cdot (v - v')}{u' \cdot (v' - v)}\]  \hspace{1cm} (3.13)

Both \(F_r \cdot u\) and \(F_r \cdot u'\) must be negative since the applied forces always push toward the fingertip. Therefore, \(K\) has to be negative in order to satisfy Equation (3.12) should an ambiguity exist. On the other hand, no ambiguity exists if \(K\) is positive. Several cases will be discussed as follows:
(a) Both \( \mathbf{u} \cdot (\mathbf{v} - \mathbf{v}') \) and \( \mathbf{u}' \cdot (\mathbf{v}' - \mathbf{v}) \) are positive. If this is true for any two points on the boundary surface of the body, then \( K \) is positive and no ambiguity occurs. The geometry of the boundary surface which satisfies this condition is shown in Figure 3.4. Points A and B are any two points on the surface of the object. \( \mathbf{v} \) and \( \mathbf{u} \) are the position vector and the normal vector of point A, respectively, and \( \mathbf{v}' \) and \( \mathbf{u}' \) are the position vector and normal vector of any other point B. The tangential plane is perpendicular to the normal vector at point A. If \( \mathbf{u} \cdot (\mathbf{v} - \mathbf{v}') \) is negative for any other points with respect to point A, then all of the points should lie on one side of the tangential plane or support plane. This is the characteristic of a convex surface (Howard, 1972; Pogorelov, 1973). Therefore, we can conclude that no ambiguity is present if and only if the boundary surface is smooth and convex.

![Figure 3.4 Geometry for Case (a)](image-url)
(b) Both \( u \cdot (v-v') \) and \( u' \cdot (v'-v) \) are negative for all points. The geometry which satisfies this condition is shown in Figure 3.5. The forces are applied on the interior surface which is convex. No ambiguity occurs in this case, either.

(c) The signs of \( u \cdot (v-v') \) and \( u' \cdot (v'-v) \) are different. When the boundary surface is not convex, \( u \cdot (v-v') \) and \( u' \cdot (v'-v) \) will have different signs in some sectors. This case can arise when the surface is not convex as shown in Figure 3.6. \( K \) is negative in both the sectors BC and DE, drawn in bold.
lines, with respect to the point A. Therefore, ambiguity is proved to occur in this case.

(d) One of the values of $u \cdot (v - v')$ and $u' \cdot (v' - v)$ is zero. As shown in Figure 3.6, the pair of point A and either of points B, C, D, and E has this characteristic. Although this fact leaves Equation (3.12) unsatisfied, this case arises only when the surface is not convex. Therefore, as in case (c), an ambiguity will exist.

(e) The values of both $u \cdot (v - v')$ and $u' \cdot (v' - v)$ are zero. There are two cases which satisfy this condition. The first one is shown in Figure 3.7(a), in which an ambiguity exists since the surface is not convex. The other case is shown in Figure 3.7(b). The existence of an ambiguity can be judged by looking at Equation (3.10) by changing $u'$ to $u$. This equation can be satisfied only when the vector $u$ is perpendicular to $F$. However, this is not possible physically. Ambiguity then does not exist in this case. What is shown in Figure 3.7(b) can also be regarded as a special case of Figure 3.4.

Based on the discussion of the above cases, it is evident that ambiguity does not exist for soft point contact if the boundary surface, whether the exterior
or interior one, is convex. Furthermore, it should be noted that the condition for ambiguity to be nonexistent in soft point contact is the same as in the case of hard point contact.

3.5 Fingertip for DIGITS

The ambiguity analysis and techniques to reconstruct the applied wrench for individual ellipsoidal contact surface can then be integrated to analyze any contact surfaces. The geometry of the fingertip for the DIGITS system is shown in Figure 3.8. A hemisphere is joined to a cylinder with the same radius, \( r \). Since the boundary surface on which the force is applied is convex, there is no ambiguity.

\[
L = \frac{\Delta_2}{(F_{r_y}^2 + F_{r_z}^2)\sqrt{\Delta_1}}
\]

(3.14)
where \[ \Delta_1 = F_{yx}^2 + F_{zx}^2 - T_{0x}^2 \] (3.15)

and

\[ \Delta_2 = F_{rx}(F_{rx}^2 + F_{rz}^2) + T_{0x}(F_{ry}T_{0y} + F_{rz}T_{0z}) + (F_{rz}T_{0y} - F_{ry}T_{0z})\sqrt{\Delta_1} \] (3.16)

For the cylinder side, \( T_{0x} \) equals \( \sqrt{F_{ry}^2 + F_{rz}^2} \sin \eta \), where \( \eta \) is the angle between \( F' \) and \( u' \). Thus \( \Delta_1 \) must be positive for the cylinder side. Furthermore, if both \( \Delta_1 \) and \( \Delta_2 \) are positive, then \( L \) is positive, and the answer is on the cylinder side. Otherwise the answer is on the hemisphere side. The strategy is also summarized in Figure 3.9.

![Figure 3.9 Flowchart of Finding Contact Wrench](image)

3.6 Examples

The method to calculate the resultant wrench at the origin point from the output voltages of the six-axis force sensor has been discussed in the previous Chapter. The resultant wrench can then be used to reconstruct the contact information by using the strategies described in this Chapter. Two numerical
examples, which illustrate the way to reconstruct the contact information by
giving output voltages of strain gage bridges, are listed below.

Example 1: The shape of the fingertip is similar to the one shown in Figure 3.8
and the radius is 0.405". A force about 1 lb is applied toward the negative Y
direction on the cylindrical side. The increments of the output voltages are read
to be

\[
(0.013 \ -0.060 \ -0.007 \ -0.011 \ -0.006 \ 0.434)
\]

The units are volt. The objective is to find the contact force, contact position, and
applied moment.

Solution: By using Equation (2.27) and the value listed in Equation (2.28), the
increment of the applied wrench is found to be

\[
F_\theta: (-0.0334 \ -0.9897 \ 0.0248) \quad T_\theta: (0.0212 \ 0.0049 \ 0.7010)
\]

The units for \(F_\theta\) and \(T_\theta\) are respectively lb and lb in.

By using Equations (3.15) and (3.16), \(\Delta_1 = 0.1603 > 0.0\) and \(\Delta_2 =
0.1105 > 0.0\). Thus, the force is applied to the cylindrical segment. By
using the Equations (3.6), (3.7), and (3.8), we calculate the applied force
to be \((-0.0334\text{lb}, -0.9897\text{lb}, 0.0248\text{lb})\). The contact position is at
\((-0.6950", 0.4048", 0.0113")\), and the applied moment is \(-0.0120\text{ lb in}\).

Example 2: The same fingertip as in Example 1 is used for test. An unknown
force is applied to the hemispherical surface. The increments of the output
voltages are read to be

\[
(-0.056 \ 0.024 \ -0.013 \ -0.012 \ 0.039 \ 0.076)
\]

Solution: By using Equation (2.27) and the value listed in Equation (2.28), the
increment of the applied wrench is found to be

\[
F_\theta: (-0.4800 \ -0.3670 \ -0.0982) \quad T_\theta: (0.0231 \ 0.0200 \ 0.0003)
\]
By using Equations (3.15) and (3.16), $\Delta_1 = 0.0231 > 0.0$ and $\Delta_2 = -0.0047 < 0.0$. Thus, the force is applied on the hemispherical segment.

By using the Equations (3.4) and (3.5), we compute the applied force to be (-0.4800 lb, -0.3670 lb, -0.0982 lb). The contact position is at (0.3215", -0.2457", 0.0163"), and the applied moment is 0.0062 lb in.

3.7 Conclusion

The data collected from a six-axis force/torque sensor can be used to reconstruct the wrench for various point contacts. Touch sensors are necessary for the cases of line and planar contacts. The ambiguity problem for soft and/or hard point contact does not occur if the boundary surface on which the forces are applied is convex. In the case of geometries composed of several surface segments, it is suggested that we first decide to which segment the force is applied before reconstructing the contact information.

Strategies that can be used to reconstruct the wrenches correctly and efficiently for the force sensors used in DIGITS system are proposed. Examples of reconstruction of contact information from the output voltages of the sensor bridges are also given.
CHAPTER IV
FORCE DISTRIBUTION

4.1 Introduction

For multifingered systems, the fingers have to be coordinated to grasp or even manipulate an object. A method of finding the appropriate distributed force among the fingers so that the object can be grasped stably becomes an important issue. This problem has received much attention recently. Since most researchers assumed that the environment was totally known, the magnitudes of the grasping forces or joint torques were chosen as objectives. Furthermore, these functions were usually linearized to make linear programming algorithms, such as the Simplex method, applicable. However, a problem of chatter in the grasping forces or joint torques calculated by linear programming algorithms occurs. This phenomenon is undesirable in a real-time control input.

The problem we are dealing with is that the environment and geometry of the workpiece is at least partly unknown. Thus, grasping an object stably without slipping becomes the main concern. As a result, the maximum friction angle will be the object function to be minimized. This is obviously a non-linear problem.

In this Chapter, the problems which are described by force equilibrium equations and constraint equations will be discussed. Three processes, projection, solving for a residual wrench, and optimization, are proposed to solve the problem (Chung and Waldron, 1993b). A general way to decompose
finger force into equilibrium force and interaction force is also presented. Examples for three-fingered and four-fingered cases are also shown. Finally, the grasp model for quasistatic passive grasping problems is introduced.

4.2 Problem Distribution

The description of the grasping problem is introduced first. When the trajectory of the grasped object is given, the wrench, force \( \mathbf{R} \) and moment \( \mathbf{M} \), of the object can be found by using Euler's equations. As shown in Figure 4.1, the origin point of coordinate is fixed at the mass center of the object. The wrench can be found as

\[
\mathbf{R} = \frac{d}{dt}(m \mathbf{v}) + \omega \times m \mathbf{v} - \mathbf{w} \tag{4.1}
\]

\[
\mathbf{M} = \frac{d}{dt}(\mathbf{H}) + \omega \times \mathbf{H} \tag{4.2}
\]

where \( m \) is the mass and \( \mathbf{w} \) is the weight vector of the object. While \( \mathbf{H} \) is the angular momentum of the body, \( \mathbf{v} \) and \( \omega \) are the linear velocity and angular velocity of the body, respectively.

![Figure 4.1 The Grasped Object and the Fingers](image-url)
In order to let the hand grasp the object stably, the forces applied by the fingers, as in Figure 4.1, should equilibrate the wrench in Equations (4.1) and (4.2). Thus, the equilibrium equations between the grasped object and fingers can be written as

\[
\sum_{i=1}^{n} F_i = R \tag{4.3}
\]

\[
\sum_{i=1}^{n} r_i \times F_i + T_i = M \tag{4.4}
\]

where \( F_i \) : force applied by the ith finger
\( r_i \) : position of contact point of the ith finger
\( T_i \) : moment applied by the ith finger. This term does not exist if the contact is frictional hard point contact.
\( n \) : total number of grasping fingers.

In addition to the equilibrium equations, the distribution forces, \( F_i \), should also satisfy some constraints. The first one is the joint torque constraint

\[
\tau_{ij_{\text{min}}} \leq \tau_{ij} \leq \tau_{ij_{\text{max}}} \tag{4.5}
\]

where \( \tau_{ij} \) is the torque at the jth joint of the ith finger. \( \tau_{ij_{\text{min}}} \) and \( \tau_{ij_{\text{max}}} \) are the torque limitation at the jth joint of the ith finger. In order to avoid slippage, friction has to be considered, too. That constraint can be written as

\[
u F_i \cdot n_i \geq \sqrt{F_i^2 - (F_i \cdot n_i)^2} \tag{4.6}
\]

where \( u \) is the friction coefficient and \( n_i \) is the unit vector in the normal direction for the ith contact point. Besides, interaction forces must be considered when grasping a fragile object. The constraint equation is
where $\mathbf{u}_{ij}$ is the unit vector along the line connecting point $i$ and point $j$, and $f_{ij}$ is the limitation of interaction force which will not break the grasped object.

### 4.3 Strategy of Solving Problems

The distributed forces, $\mathbf{F}_i$, and torques, $\mathbf{T}_i$, among the fingers should satisfy Equations (4.3) and (4.4) and the other constraints. The proposed method focuses on frictional point contact, and only contact forces, $\mathbf{F}_i$, are considered. How to find the distributed forces can be divided into three processes. The first process is to project the wrench onto the space spanned by the Plücker coordinates of the contact normals and to find the force component along each normal direction. The force field method provided by Kumar and Waldron in 1988 can then be used to find the forces which generate the residual wrench calculated in the prior step. By combining the forces found in these two steps, we will get the equilibrium forces which satisfy Equations (4.3) and (4.4). However, if the result does not satisfy the constraints expressed in Inequalities (4.5), (4.6), and (4.7), a homogeneous solution should be added in order to modify the answer. If the answer cannot be found after these three steps, the answer may not exist. It will then be necessary to find another set of grasping points.

All of these three steps will be described in more detail in the following.

#### 4.3.1 Projection

The friction angle is defined as the angle between the line of action of the contact force and the contact normal. It will be smaller if the finger force component along the normal direction can equilibrate a larger wrench, or if the force component along the tangential direction can be made as small as
possible. The largest wrench which the normal finger forces can take is the projection of the wrench on the subspace spanned by all of the normal components.

For hard contact fingers, the Equations (4.3) and (4.4) can be rewritten as follows:

\[ G F = W = \{R, M\}^T \quad (4.8) \]
\[ F = \{F_1, F_{1m}, F_{2n}, F_{2m}, \ldots, F_{nm}, F_{nm}\}^T \]
\[ G = [g_1, g_2, \ldots, g_n] \quad g_i = \begin{bmatrix} n_i & l_i & m_i \\ r_i \times n_i & r_i \times l_i & r_i \times m_i \end{bmatrix} \]

in which \( r_i \) is the position vector of the \( i \)th contact point in the body coordinate \( xyz \). \( l_i \) is the unit vectors being perpendicular to \( n_i \) and \( m_i = n_i \times l_i \). These equations can also be rearranged into the form

\[ [G_n, G_f][F_n] = W \quad (4.9) \]

where

\[ F_n = \{F_1, F_{2n}, \ldots, F_{nm}\}^T \quad F_f = \{F_{1f}, F_{1mf}, F_{2f}, F_{2mf}, \ldots, F_{nmf}\}^T \]
\[ G_n = [g_{1n}, g_{2n}, \ldots, g_{mn}] \quad G_f = [g_{1f}, g_{2f}, \ldots, g_{mf}] \]
\[ g_{in} = \begin{bmatrix} n_i \\ r_i \times n_i \end{bmatrix} \quad g_{if} = \begin{bmatrix} l_i & m_i \\ r_i \times l_i & r_i \times m_i \end{bmatrix} \]

If \( W_n = G_n F_n \) is the projection of \( W \) on the subspace spanned by \( g_{in} \), then \( W_{res} = W - G_n F_n \) should be orthogonal to \( g_{in} \). That is

\[ G_n^T (W - G_n F_n) = 0. \]
$F_n$ can then be found to be

$$F_n = \left( G_n^T G_n \right)^{-1} G_n^T W \quad (4.10)$$

Because $(G_n^T G_n)$ is an $n$ by $n$ symmetric matrix, the calculations and inverse operation will not be too cumbersome. However, $F_n$ may be negative. This would mean that the finger(s) would pull away from the object. Two methods are suggested to solve this problem. One is to let the finger(s) with negative $F_n$ be inactive. The other is to set a positive value to replace the negative $F_n$. Finally, the residual wrench can be found to be

$$W_{\text{res}} = W - G_n F_n \quad (4.11)$$

4.3.2 Solve for the Residual Wrench

The methods used to find the forces which equilibrate the residual wrench will be introduced in this Section. These forces combined with $F_n$ could satisfy the force equilibrium equations.

The residual wrench can be equilibrated by the tangential forces. The equilibrium equation is written as

$$G_t F_t = W_{\text{res}} \quad (4.12)$$

$F_t$ can be solved by using Pseudo-Inverse and can be expressed as

$$F_t = G_t^T \left( G_t G_t^T \right)^{-1} W_{\text{res}} \quad (4.13)$$

The equilibrium forces $F_i$ are then

$$F_i = F_{in} n_i + F_{in} l_i + F_{in} m_i \quad (4.14)$$
Furthermore the residual wrench can also be equilibrated by all of the force components, that is

$$ \mathbf{G} \mathbf{F}_{\text{res}} = \mathbf{W}_{\text{res}} $$  \hspace{1cm} (4.15)

and

$$ \mathbf{F}_{\text{res}} = \mathbf{G}^T \left( \mathbf{G} \mathbf{G}^T \right)^{-1} \mathbf{W}_{\text{res}} $$  \hspace{1cm} (4.16)

The other method to solve Equation (4.15) is by using the force field theory, as in Kumar and Waldron, 1988. \( \mathbf{F}_{\text{res}} \) can be solved as

$$ \mathbf{F}_{\text{res}} = \mathbf{hL}u + \mathbf{Lu} \times (\mathbf{n}_t - \rho_n) $$  \hspace{1cm} (4.17)

The equilibrium forces \( \mathbf{F}_i \) can then be obtained by combining Equations (4.10) and (4.17) or (4.16). That is

$$ \mathbf{F}_i = \mathbf{F}_{\text{in}} \mathbf{n}_t + \mathbf{F}_{\text{tres}} $$  \hspace{1cm} (4.18)

All of these three methods would give us the minimum norm, or the least squares solution, for the vector \( \mathbf{F}_{\text{res}} \) or \( \mathbf{F}_i \). This implies that the direction of the total forces would be close to those of the normal vectors. Thus, the result in Equation (4.14) or (4.18) can be a good initial point for the optimization process which will be discussed in the next subsection.

As explained in Kumar and Waldron, 1988, and Appendix A, both Equations (4.16) and (4.17) will give us the same answers of \( \mathbf{F}_{\text{res}} \). However, since a pseudo-inverse is a time consuming operation, to find \( \mathbf{F}_{\text{tres}} \) by using Equation (4.17) should be the most efficient of all of the three methods discussed and is used in this work.
4.3.3 Optimization Process

The forces derived in prior sections can satisfy Equation (4.8), but do not always satisfy the constraint equations. Thus, a homogeneous solution in the null space is suggested to modify the distribution of contact forces.

In Equation (4.8), six equations are used to find the $3n$ unknowns. Thus, the space dimension of the homogeneous solution is $(3n-6)$. On the other hand, the homogeneous solution is the one which satisfies $GF = 0$. Thus, any interaction force pair can be one of the homogeneous solutions. The interaction force pair is defined as pairs of contact forces with the same magnitude and opposite direction along the line joining both contact points. For $n$ finger cases, there are $C_n^2$ interaction force pairs which can be used to generate the space of the homogeneous solutions. Especially, for the three and four finger cases, the linearly independent interaction force pairs can just be bases for the space of homogeneous solutions, because $C_n^2 = 3n - 6$ for $n=3$ and $n=4$ (Sreenivasan and Waldron, 1992).

The interaction force pairs for three and four finger cases are shown in Figure 4.2. The forces acting in the three finger case can be modified as

$$
F_{11} = F_1 + k_1 u_{12} + k_2 u_{13}
$$

$$
F_{12} = F_2 - k_1 u_{12} + k_3 u_{23}
$$

$$
F_{13} = F_3 - k_2 u_{13} - k_3 u_{23}
$$

(4.19)

Similarly, the forces acting in the four finger case can be expressed as

$$
F_{11} = F_1 + k_1 u_{12} + k_2 u_{13} + k_3 u_{14}
$$

$$
F_{12} = F_2 - k_1 u_{12} + k_4 u_{23} + k_5 u_{24}
$$
\[
F_{i3} = F_3 - k_2u_{i3} - k_4u_{23} + k_6u_{34}
\]

\[
F_{i4} = F_4 - k_3u_{i4} - k_5u_{24} - k_6u_{34}
\]  \hspace{1cm} (4.20)
where \( F_{i} \) is the total force on the \( i \)th finger. By adjusting the values of \( k_1, k_2, \ldots, k_c \), we may get an answer which satisfies the constraints and does not violate the force equilibrium equations.

\[
F_{i1} = F_1 - k_2u_{i1} - k_4u_{21} + k_6u_{31}
\]

\[
F_{i2} = F_2 - k_3u_{i2} - k_5u_{22} - k_6u_{32}
\]

\[
F_{i3} = F_3 - k_2u_{i3} - k_4u_{23} + k_6u_{34}
\]

\[
F_{i4} = F_4 - k_3u_{i4} - k_5u_{24} - k_6u_{34}
\]

where \( F_{i} \) is the total force on the \( i \)th finger. By adjusting the values of \( k_1, k_2, \ldots, k_c \), we may get an answer which satisfies the constraints and does not violate the force equilibrium equations.

One way to do the optimization process is the external gradient method. This method is proposed to transform an answer which violates some of the constraints into one which just satisfies these constraints in one step. However, this method is only applicable to linear constraints and the final answer is not guaranteed to satisfy the other constraints. This method will be explained briefly as follows:

Let \( h_i(k_1,k_2,\ldots,k_c) \) be the constraints which are violated with the error being \( e_i \). If \( \Delta k \) in Equation (4.19) or (4.20) is

\[
\Delta k = (\lambda_1h_1 + \lambda_2h_2 + \ldots + \lambda_mh_m)
\]  \hspace{1cm} (4.21)
where \( M \) is the total number of violated constraints. The object is to find \( \Delta k \) which can eliminate \( c_i \). Then, for the \( i \)th violated constraint, we get

\[
-e_i = \sum_{j=1}^{c_i} \Delta k_j \frac{\partial h_i}{\partial k_j} = \nabla h_i[\nabla h_1, \nabla h_2, \ldots, \nabla h_M][\lambda_1, \lambda_2, \ldots, \lambda_M]^T
\]  

(4.22)

By combining all violated constraints we get

\[
\begin{bmatrix}
\nabla h_1 \\
\nabla h_2 \\
\vdots \\
\nabla h_M
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_M
\end{bmatrix}
=
\begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_M
\end{bmatrix}
\]  

(4.23)

\( \lambda_i \) can be obtained by solving Equation (4.23). And the previously violated constraints will be satisfied.

However, if \( M \) is larger than 6 for four finger cases, or larger than 3 for three finger cases, Equation (4.23) will become unsolvable for \( \lambda_1, \lambda_2, \ldots, \lambda_M \), and this method will fail.

The gradient search method is also suggested for use in the optimization process. An objective function should first be properly defined, and then the gradient search algorithm might be used to maximize the function value. However, this method is not intended to change the answer to satisfy the constraints in one step, as was the prior method, and iterative calculation is necessary.

The objective function to be maximized was defined to be

\[
G_{\text{obj}}(k_1, k_2, \ldots, k_{c_2}) = \min(\cos \theta_1, \cos \theta_2, \ldots, \cos \theta_n)
\]  

(4.24)

where \( \cos \theta_i = \frac{F_{ui} \cdot n_i}{|F_{ui}|} \).
The objective function is not a linear function, the gradient of the function may be obtained by a numerical method. That is

\[
\frac{\partial G}{\partial k_j} = \frac{G(k_1, k_2, \ldots, k_j, k_{j+1}, k_{c_j}) - G(k_1, k_2, \ldots, k_{j-1}, k_{c_j})}{\delta k_j}
\]  

(4.25)

And the change of the function value around the prior point can be approximated as

\[
\Delta G = \Delta k V_G = \sigma \frac{V_G}{|V_G|^2} V_G
\]

(4.26)

where

\[
\Delta k = \sigma \frac{V_G}{|V_G|^2}
\]

(4.27)

This means that, by adjusting the value of the step size \( \sigma \), we can anticipate the change of the function value.

Two methods of finding the value of \( \sigma \) are proposed. One is to use different values of \( \sigma \). A larger value of \( \sigma \) is suggested in the early iterations, and smaller values later on. The other method is to find the best value of \( \sigma \) by a one dimensional optimization search technology, such as the quadratic method.

Since the objective function is defined as in Equation (4.24), the inequality constraint (4.6) can be satisfied automatically. The other constraints, expressed by Inequalities (4.5) and (4.7), can be considered by using the external penalty strategy. Let the constraint functions be

\[
h_j \leq 0 \quad j = 1, 2, \ldots, M
\]

(4.28)
where $M$ is again the total number of constraint functions. The objective function in Equation (4.24) can then be modified to

$$
\bar{G}_{\text{obj}} = G_{\text{obj}} - \sum_{i=1}^{M} \text{factor} \cdot (\max(o, h_j))^2
$$

(4.29)

where $\max(o, h_j) = \begin{cases} 
0 & \text{if } h_j \leq 0 \\
h_j & \text{if } h_j > 0
\end{cases}$.

When the value of $\text{factor}$ is very large, the constraint functions $h_1, h_2, \ldots, h_M$ will be totally satisfied (Walsh, 1975). As a result, the distributed forces found by using all of these three steps can satisfy Equation (4.3) and (4.4) as well as the constraints.

**4.4 Examples**

Two examples will be presented in this Section to illustrate how the proposed strategies work. One is a three-fingered case and the other one is a four-fingered case. The rates of convergence of different optimum processes can also be seen in this example.

**Example 1:** The data in this example was adapted from Mukherjee, 1992. Three fingers were used to grasp a ball, as shown in Figure 4.3. The position and the unit normal vector of each finger were

<table>
<thead>
<tr>
<th>Position</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.866, 0.000, 0.000)</td>
<td>(-0.866, 0.000, 0.500)</td>
</tr>
<tr>
<td>(-0.433, 0.750, 0.000)</td>
<td>(-0.433, -0.750, 0.500)</td>
</tr>
<tr>
<td>(-0.433, -0.750, 0.000)</td>
<td>(0.433, 0.750, 0.500)</td>
</tr>
</tbody>
</table>

And the wrench to be generated was $R(0.0, 0.0, 5.0)$ and $M(0.0, -0.5, 0.0)$. 
Solution: The projected normal force was found to be $F_n : (3.487, 3.256, 3.256)$. And the residual wrench was $W_{\text{res}} : (0.2, 0.0, 0.0, 0.0, -0.4, 0.0)$.

The equilibrium forces, without adding the homogeneous solutions, were

- $F_1 : (-2.953, 0.000, 2.052)$
- $F_2 : (1.477, -2.442, 1.474)$
- $F_3 : (1.477, 2.442, 1.474)$.

The cosine values were $\cos(\theta_1) = 0.9965$, $\cos(\theta_2) = \cos(\theta_3) = 0.9981$. The smallest value, 0.9965, was even larger than the value 0.9943 which was the best answer found in Mukherjee, 1992.

When the optimum process was applied, the results became

- $F_{1} : (-3.0750, 0.0000, 2.0516)$
- $F_{2} : (1.5375, -2.4773, 1.4742)$
- $F_{3} : (1.5375, 2.4773, 1.4742)$,

and the cosine values are $\cos(\theta_1) = 0.9979$, $\cos(\theta_2) = \cos(\theta_3) = 0.9981$. 

Figure 4.3 Illustration of the Three-Finger Grasping Example
After the optimization technique was applied, the values of all the \( \cos(\theta_i) \)'s were almost the same. This can be explained when looking at Equation (4.19). If \( F_i \) is the force with the smallest \( \cos(\theta_i) \) value, then \( k_1, k_2, .., k_{c_2} \) can be adjusted to increase \( \cos(\theta_i) \). This process can be repeated and will terminate only when all of the values of \( \cos(\theta_i) \) are the same.

Example 2: Four fingers were used to grasp an object. The position and the unit normal vector of each finger were

<table>
<thead>
<tr>
<th>Position</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.200, 1.000, 0.200)</td>
<td>(0.300, -0.699, 0.649)</td>
</tr>
<tr>
<td>(-1.000, -0.100, -0.200)</td>
<td>(0.851, -0.400, -0.340)</td>
</tr>
<tr>
<td>(-0.200, -0.900, 0.000)</td>
<td>(0.400, 0.901, -0.170)</td>
</tr>
<tr>
<td>(1.000, 0.000, 0.000)</td>
<td>(-0.800, 0.480, 0.360)</td>
</tr>
</tbody>
</table>

The wrench to be generated was \( R(3.0, 2.0, 10.0) \) and \( M(1.0, 1.0, -1.0) \). The contact configuration and the wrench to be generated are also shown in Figure 4.4. The force constraints, instead of the joint torque constraints, were the absolute values of \( F_{in} \), \( F_{by} \), and \( F_{ux} \) to within 10 units.

Solution: The projected normal forces were \( F_n \): \( (6.002, 1.000, 4.392, 2.051) \), where the second normal force was negative, and was reset to be 1.0. The second normal force is negative, because the Z component of the second contact normal is negative and the force to be generated is 10 units along the positive Z direction. The cosine values before the optimization iteration were \( \cos(\theta_1) = 0.989 \), \( \cos(\theta_2) = -0.041 \), \( \cos(\theta_3) = 0.743 \), and \( \cos(\theta_4) = 0.953 \). Obviously, the projection method did not give us a good answer in this example because one of the values of \( F_{in} \) was negative. Therefore an optimization process was definitely necessary.
Figure 4.4 Illustration of the Four-Finger Grasping Example

The result of iteration with changing step size, \( \sigma \), is shown in Figure 4.5. It shows that good answers can be obtained in just 10 iterations. Furthermore, it was estimated that it took 1.24 ms CPU time for each iteration on a VAX 8550 computer. The results for the grasping forces after 50 iterations were

\[
\begin{align*}
F_{t1} & : (4.3878, -1.8236, 3.9962) \\
F_{t2} & : (8.7293, -4.4450, 2.8797) \\
F_{t3} & : (-0.1534, 8.0800, 2.4267) \\
F_{t4} & : (-9.9637, 0.1886, 0.6974).
\end{align*}
\]

The cosine values were \( \cos(\theta_1) = 0.8349 \), \( \cos(\theta_2) = 0.8057 \), \( \cos(\theta_3) = 0.8061 \), and \( \cos(\theta_4) = 0.8321 \). Obviously, the grasping forces were all within the constraints. The values of \( \cos(\theta_i) \) were very close, and the
minimum value increased to a much larger value than it had been before iteration.

The result of iteration by using a one dimensional quadratic search to find the value of $\sigma$ is also shown in Figure 4.5 for comparison. This method also gives a good answer after 10 iterations.

![Figure 4.5 Iteration Result of Four-Fingered Case](image)

Each iteration does not guarantee a better result than the prior one if the method of changing $\sigma$ is used. Thus, the curve showed some vibration in Figure 4.5. This phenomenon can be reduced or even eliminated if the value of $\sigma$ is defined appropriately. The curve of iteration results derived from using quadratic search kept improving and increased rapidly in the early iterations because the best point in each gradient direction was supposed to be used. Although the number of iterations may be less than that obtained by using changing $\sigma$, this method takes about 2.83 ms CPU time for each iteration and has a tendency to reach the local maximum. Therefore, the method of changing
will be a better choice based on the consideration of time consumed and the quality of the answer.

4.5 Decomposition of Finger Forces

In grasping a fragile object, the interaction forces should be within the allowance. One way is to limit the force component along the lines connecting contact points, as shown in Equation (4.7). The other way is to decompose the distributed forces into equilibrium force and interaction force and then set the constraints for the magnitude of the interaction forces. The general method of decomposing the finger force into an equilibrium force and an interaction force component will be described in this section. Although only the cases of three and four fingers will be described in this section, this method can be applied to cases with any number of fingers. On the other hand, other definitions of manipulating force and grasping force were provided in Yoshikawa and Nagai, 1991. The decomposition of finger force into both terms has been worked out up to only the three-fingered case.

The equilibrium forces without any component of interaction force can be described by the equation

\[
(F_{mi} - F_{mj}) \cdot u_{ij} = 0 \quad (4.30)
\]

For three fingers, the relationship among the total forces, equilibrium force, and interaction forces is similar to Equation (4.19) and can be written:

\[
\begin{align*}
F_{t1} &= F_{m1} + k_1u_{12} + k_2u_{13} \\
F_{t2} &= F_{m2} - k_1u_{12} + k_3u_{23} \\
F_{t3} &= F_{m3} - k_2u_{13} - k_3u_{23} \quad (4.31)
\end{align*}
\]
where $F_{mi}$ and $(F_{ti} - F_{mi})$ are the equilibrium force and interaction force of the ith finger, respectively. By substituting Equation (4.31) into Equation (4.30), we can get
\[
\begin{bmatrix}
2 & u_{12} \cdot u_{13} & -u_{12} \cdot u_{23} & \cdots & -u_{12} \cdot u_{24} & 0 \\
-u_{12} \cdot u_{13} & 2 & u_{13} \cdot u_{14} & \cdots & -u_{12} \cdot u_{34} & 0 \\
-u_{12} \cdot u_{14} & u_{13} \cdot u_{14} & 2 & \cdots & -u_{12} \cdot u_{23} & 0 \\
0 & u_{13} \cdot u_{23} & -u_{13} \cdot u_{23} & \cdots & 2 & 0 \\
0 & -u_{14} \cdot u_{24} & u_{14} \cdot u_{24} & \cdots & u_{23} \cdot u_{24} & 2 \\
0 & -u_{34} \cdot u_{34} & u_{14} \cdot u_{34} & \cdots & u_{23} \cdot u_{34} & 2
\end{bmatrix}
\begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4 \\
k_5 \\
k_6
\end{bmatrix}
= \begin{bmatrix}
(F_{11} - F_{12}) \cdot u_{12} \\
(F_{11} - F_{13}) \cdot u_{13} \\
(F_{11} - F_{14}) \cdot u_{14} \\
(F_{12} - F_{13}) \cdot u_{23} \\
(F_{12} - F_{14}) \cdot u_{24} \\
(F_{13} - F_{14}) \cdot u_{34}
\end{bmatrix}
\tag{4.32}
\]
Thus, the values of $k_i$ can be found by solving Equation (4.32) and the interaction force can also be obtained.

Similar Equations to (4.30) and (4.31) can be derived for the four-fingered cases. The values of $k_1, \ldots, k_6$ can also be found by solving
\[
\begin{bmatrix}
2 & u_{12} \cdot u_{13} & \cdots & -u_{12} \cdot u_{24} & 0 \\
-u_{12} \cdot u_{13} & 2 & \cdots & -u_{12} \cdot u_{34} & 0 \\
-u_{12} \cdot u_{14} & u_{13} \cdot u_{14} & \cdots & -u_{12} \cdot u_{23} & 0 \\
0 & u_{13} \cdot u_{23} & \cdots & 2 & 0 \\
0 & -u_{14} \cdot u_{24} & \cdots & u_{23} \cdot u_{24} & 2 \\
0 & -u_{34} \cdot u_{34} & \cdots & u_{23} \cdot u_{34} & 2
\end{bmatrix}
\begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4 \\
k_5 \\
k_6
\end{bmatrix}
= \begin{bmatrix}
(F_{11} - F_{12}) \cdot u_{12} \\
(F_{11} - F_{13}) \cdot u_{13} \\
(F_{11} - F_{14}) \cdot u_{14} \\
(F_{12} - F_{13}) \cdot u_{23} \\
(F_{12} - F_{14}) \cdot u_{24} \\
(F_{13} - F_{14}) \cdot u_{34}
\end{bmatrix}
\tag{4.33}
\]

4.6 Quasistatic Grasp Model

A way to find the distributed forces by optimizing friction angles to avoid slipping motion was discussed in Section 4.3. In fact, the grasp can also be modeled as if the object is supported by several springs and dampers, as shown in Figure 4.6. $k_i$ and $B_i$ represents the spring constants and damping
coefficients along three principal directions for the ith contact. The original wrench, \( W \), can be equilibrated by the original distributed forces, \( F_1, F_2, \ldots, F_n \). The distributed forces consists of equilibrium forces and interaction forces, and can be found by optimizing friction angles. \( W_d \) is the disturbance force and torque applied to the object. This may happen when the object encounters some obstacle.

\[
\begin{aligned}
&k_1, B_1, k_2, B_2, k_3, B_3, k_4, B_4, k_5, B_5, \\
&W + W_d, \\
&F_n + f_n, F_1 + f_1, F_2 + f_2, \\
&F_3 + f_3
\end{aligned}
\]

**Figure 4.6 Illustration of Quasistatic Grasp Model**

When the distributed forces are found and applied, the system becomes a quasistatic passive grasping problem. The disturbance wrench, \( W_d \), will result in extra acceleration of the object and deformation of springs and dampers. The extra contact force, \( f_e \), and \( \delta x_e \), the deformation of the springs and dampers, can be related by

\[
f_e = \begin{bmatrix}
k_{1x} \delta x_{c1x} + B_{1x} \dot{x}_{c1x} \\
k_{1y} \delta x_{c1y} + B_{1y} \dot{x}_{c1y} \\
k_{1z} \delta x_{c1z} + B_{1z} \dot{x}_{c1z} \\
\vdots \\
k_{ex} \delta x_{cex} + B_{ex} \dot{x}_{cex} \\
k_{ey} \delta x_{cey} + B_{ey} \dot{x}_{cey} \\
k_{ez} \delta x_{cez} + B_{ez} \dot{x}_{cez}
\end{bmatrix} = K \delta x_e + B \dot{x}_e
\]

(4.34)
Furthermore, the relationship between $\delta X_e$, the extra displacement of the object, and $\delta x_e$ is

$$G^T \delta X_e = \delta x_e$$

$$G^T \dot{X}_e = \dot{x}_e$$

(4.35)

where $G$ is again the grasp matrix. Finally, by using Equations (4.34) and (4.35), the dynamic equation of the grasped object becomes

$$W_d = M\ddot{X}_e + GB^T\dot{X}_e + GKG^T\delta X_e$$

(4.36)

The system described in Equation (4.36) is similar to a vehicle suspension system. If the disturbance wrench is within the allowance, the object will oscillate among the springs and be grasped stably. If the disturbance or extra wrench exceeds the allowance, slipping motion will occur and the object will eventually disengage from the springs. The criterion of the allowance depends on the distributed forces, contact configuration, spring constants, and damping coefficients and needs further study.

4.7 Conclusion

The problem of the force distribution using the friction angle as the objective function has been solved. The projection method has been shown to
provide a good initial point for the optimization process. A solution close to the best friction angle can be found very efficiently by using the proposed method.
CHAPTER V
DEXTEROUS MANIPULATION

5.1 Introduction

The ultimate goal of using multifingered systems is to manipulate the object. The flowchart of using a multifingered system to manipulate an object can be illustrated by Figure 5.1. $X_o$ is used to represent the position and orientation of the object, and is a 6 by 1 vector. The orientation of the object coordinate frame xyz with respect to the global coordinate frame XYZ is represented by the Euler angles, $\{\phi, \theta, \psi\}$. The grasped object is intended to move along the desired trajectory by giving $X_d$, $\dot{X}_d$, and $\ddot{X}_d$. $X_d$ is also a 6 by 1 vector and consists of the desired position and orientation. $\dot{X}_d$ and $\ddot{X}_d$ are respectively the first and second time derivatives of $X_d$. The difference between the desired trajectory and the actual trajectory, $X_d$ and $X_o$, are used to find the commanded torques at the finger joints.

![Figure 5.1 Overview of Simulation Work](image)

Figure 5.1 Overview of Simulation Work
In this work, the computed torque method is combined with the techniques developed by Kumar and Waldron (1988) as well as the one described in Chapter IV to find the input torques of the finger joints efficiently. Therefore, the friction angles are optimized so that the object can be grasped stably while it is manipulated along the desired trajectory. A history-based strategy is also proposed and used to increase the smoothness of the commanded input torque functions. The proposed method can be efficiently used to find the input torques, and is applicable for real-time application.

5.2 Euler Angles

In three dimensional manipulation, the angular velocity of the object on the coordinate frame \( \{x, y, z\} \) fixed on the object, cannot be used directly to derive the orientation of the object by integration. Therefore, Euler angles, \( \{\phi, \theta, \psi\} \), are usually used and are adopted to represent the object's orientation so that the method can be applied to three dimensional tasks. The coordinate frame \( \{x, y, z\} \) is fixed to the object as shown in Figure 5.2. The global coordinate frame \( \{X, Y, Z\} \) is fixed to the palm. When \( \psi = \theta = \phi = 0 \), the coordinate frame \( \{x, y, z\} \) is coincident with \( \{X, Y, Z\} \). The coordinate frame \( \{x, y, z\} \) is then moved to the coordinate frame \( \{x', y', z'\} \) by rotating an angle \( \psi \) with respect to the \( Z \) axis. The coordinate frame \( \{x'', y'', z''\} \) is got by rotating the coordinate frame \( \{x', y', z'\} \) with respect to the axis \( y' \). The rotation angle is \( \theta \). Finally, the coordinate frame \( \{x'', y'', z''\} \) is rotated by an angle \( \phi \) with respect to the \( x'' \) axis to get the coordinate frame \( \{x, y, z\} \). Based on these rotations, the coordinate frame \( \{x, y, z\} \) and the global coordinate frame \( \{X, Y, Z\} \) can be related as

\[
\{X, Y, Z\}^T = T\{x, y, z\}^T
\]
where $T$ is the rotation matrix and

$$
T = \begin{bmatrix}
C\psi C\theta & S\phi S\theta C\psi + S\psi C\phi & C\phi S\theta C\psi + S\psi S\phi \\
S\psi C\theta & S\phi S\theta S\psi - C\psi C\phi & C\phi S\theta S\psi - C\psi S\phi \\
-S\theta & S\phi C\theta & C\phi C\theta
\end{bmatrix}.
$$

C and $S$ respectively represent the cosine and sine functions. Furthermore, the parameters, such as $T$, which are written in boldface are either vectors or matrices.

![Figure 5.2 Global Frame and Object Frame](image)

The changing rates of the Euler angles and the angular velocity $\omega$ of the object can be related by the expression

$$
\{\omega_x, \omega_y, \omega_z\}^T = T_w \{\phi, \dot{\phi}, \psi\}^T \tag{5.2}
$$

where $T_w = \begin{bmatrix} 1 & 0 & -S\theta \\ 0 & C\phi & C\theta S\phi \\ 0 & -S\phi & C\theta C\phi \end{bmatrix}$.

Similarly, the second time derivatives of the Euler angles can be related to the angular acceleration of the object by
\[
\{\alpha_x, \alpha_y, \alpha_z\}^T = T_w \{\dot{\phi}, \dot{\theta}, \dot{\psi}\}^T + S(\phi, \theta, \psi)
\] (5.3)

where

\[
S(\psi, \dot{\theta}, \dot{\phi}) = \begin{bmatrix}
-\psi \dot{\theta} C \theta \\
-\psi \dot{\theta} S \theta S \phi - \dot{\theta} \phi S \phi + \psi \phi C \theta C \phi \\
-\psi \dot{\theta} S \theta C \phi - \dot{\theta} \phi C \phi - \psi \phi C \theta S \phi
\end{bmatrix}.
\]

These three Equations, (5.1)-(5.3), provide the relationship between the coordinate frame \{x, y, z\} and the global coordinate frame \{X, Y, Z\}. Thus, the three parameters, \(\phi\), \(\theta\), and \(\psi\) can then be integrated directly. On the other hand, the wrist of a robot arm usually consists of three revolute joints, which intersect and are perpendicular to each other. It is noted that the motions of these three revolute joints are similar to the rotations represented by the Euler angles.

### 5.3 Modified Computed Torque Method

For manipulation tasks, the dynamics of the fingers and, in particular, the grasped object have to be considered together. Therefore, the computed torque method has been proposed by Li et al. (1989) and Cole et al. (1992) to control multifingered systems or multiple cooperating robot systems.

The computed torque method has been modified (Chung and Waldron, 1993c) so that the friction angles can be optimized while the object is moved along the desired trajectory. Furthermore, the Euler angles are also adopted in deriving the equations and will be described in this Section.

The dynamic equation relating the motion of the object and the contact forces can be expressed in the form
\[
M_0 \ddot{X}_o + H_o = GF \quad \text{or}
\]
\[
G^+ (M_0 \ddot{X}_o + H_o) + F_I = F
\]

where
\[
M_0 = \begin{bmatrix} M & 0_{3 \times 3} \\ 0_{3 \times 3} & I_T \end{bmatrix}, \quad M = \begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix}, \quad H_o = \begin{bmatrix} T_w \dot{\xi} \times m \dot{x} \\ T_w \dot{\xi} \times I_T \dot{\xi} \end{bmatrix} + \begin{bmatrix} T^T m \\ 0 \end{bmatrix}, \quad \ddot{X}_o = \{\ddot{x}, \ddot{\xi}\}^T,
\]
x = \{x, y, z\}, and \( \xi = \{\phi, \theta, \psi\} \). \( I \) is the inertia matrix, \( m \) and \( m \) are the mass and the weight vector of the object respectively. \( 0_{p \times q} \) represents a \( p \) by \( q \) zero submatrix. \( F \) is the vector of the forces applied by the fingers and should equilibrate the wrench \( (M_0 \ddot{X}_o + H_o) \). \( G \) is the grasp matrix described in Chapter IV. \( G^+ \) is the pseudoinverse of matrix \( G \), and equals \( G^T (GG^T)^{-1} \). \( F_I \) is the vector of the interaction forces, which are the pairs of contact forces with the same magnitude and opposite direction along the line joining two contact points, and \( GF_I = 0 \).

On the other hand, the dynamic equations of the finger links can be expressed in the form of

\[
A(\theta) \ddot{\theta} + N(\theta, \dot{\theta}) + g(\theta) + J^T TF = \tau
\]

where
- \( A \): inertia matrix of links
- \( N \): Coriolis and centrifugal effects
- \( g \): gravitation effect
- \( \theta \): joint position
- \( J \): Jacobian matrix
- \( \tau \): joint torque.
If there is no sliding, the velocities of the contact points on both the fingertip and the object should be the same. Thus

\[ \mathbf{J}(\theta)\dot{\theta} = \mathbf{T}\mathbf{g}^T\mathbf{T}_e\mathbf{X}_o \]  

(5.6)

where \( \mathbf{X}_o = [\dot{x}, \dot{\xi}]^T \) and \( \mathbf{T}_e = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & T_w \end{bmatrix} \). \( \mathbf{I}_3 \) represents the 3 by 3 identity submatrix. The kinematic relationship of the accelerations can then be expressed in the form

\[ \ddot{\theta} = \mathbf{J}^+(\mathbf{T}\mathbf{g}^T\mathbf{T}_e\mathbf{X}_o + \mathbf{L}(\mathbf{X}_o, \dot{\mathbf{X}}_o - \dot{\mathbf{X}}_o) - \mathbf{J}\dot{\theta}) + \ddot{\theta}_h \]  

(5.7)

where

\[ \mathbf{L}(\mathbf{X}_o, \dot{\mathbf{X}}_o) = \mathbf{T}\mathbf{g}^T\mathbf{T}_e\mathbf{X}_o + \mathbf{T}\mathbf{g}^T\left[ \begin{array}{c} \mathbf{0}_{3\times1} \\ \mathbf{S}(\dot{\phi}, \dot{\theta}, \psi) \end{array} \right] \]

\( \ddot{\theta}_h \) is the homogeneous solution in the null space of matrix \( \mathbf{J} \). The superscript "+" represents the pseudoinverse matrix and \( \mathbf{J}^+ \) equals \( \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1} \).

By substituting Equations (5.4) and (5.7) into Equation (5.5), we get

\[ \tau = (\mathbf{M}_h + \overline{\mathbf{M}}_0)\ddot{\mathbf{X}}_o + \mathbf{H}_h + \mathbf{J}^T\mathbf{F} + \mathbf{A}(\theta)\ddot{\theta}_h \]  

(5.8)

where

\[ \mathbf{M}_h = \mathbf{A}(\theta)\mathbf{J}^+\mathbf{T}\mathbf{g}^T\mathbf{T}_e \]

\[ \overline{\mathbf{M}}_0 = \mathbf{J}^T\mathbf{g}^+\mathbf{M}_0 \]

\[ \mathbf{H}_h = \mathbf{H}_F + \mathbf{J}^T\mathbf{g}^+\mathbf{H}_0 \]

\[ \mathbf{H}_F = \mathbf{A}(\theta)\mathbf{J}^+(\mathbf{L}(\mathbf{X}_o, \dot{\mathbf{X}}_o) - \mathbf{J}\dot{\theta}) + \mathbf{N}(\theta, \dot{\theta}) + \mathbf{g}(\theta). \]
The purpose of the control strategy is to make the position error converge to zero. Thus the input torque $\tau_i$ can be set to be

$$\tau_i = \tau_F + J^T T \bar{F}$$

(5.9)

where $\tau_F = M_h E_r + H_F$

$$E_r = \ddot{X}_d + k_v (\dot{X}_d - \dot{X}_o) + k_p (X_d - X_o)$$

$$\bar{F} = G^+ (M_0 E_r + H_o) + F^d$$

(5.10)

$X$ and $X_d$ are respectively the actual and desired trajectories. $F^d$ is the vector of desired interaction forces. $k_p$ and $k_v$ are the gain vectors for position and velocity feedback control, respectively. By substituting Equation (5.9) into Equation (5.8) and neglecting the term $\ddot{\theta}_h$, we get

$$\left( M_h + M_0 \right) (\ddot{E} + k_v \dot{E} + k_p E) + J^T T (F^d - F_i) = 0$$

(5.11)

where $E = X_d - X_o$ and $F_i$ is the actual vector of interaction forces. Multiplying both sides of Equation (5.11) by $GT^T (J^T)^+$, we can get

$$\ddot{E} + k_v \dot{E} + k_p E = 0$$

(5.12)

This means that $E$ will converge to zero and $F_i$ will also converge to $F^d$.

The term $\bar{F}$ in Equation (5.10) is the vector of distributed forces intended to be applied to the object by the fingers. If $\bar{F}$ can be found by optimizing the friction angles to avoid slipping, the fingers can grasp and manipulate the object stably. The processes described in Chapter IV can be used to find the distributed force.
Figure 5.3 Block Diagram
The methods proposed in this Section and Chapter IV can be combined together to fulfill the manipulation task. The block diagram is shown in Figure 5.3. \( \mathbf{X}_d \) is the desired trajectory of the object. \( \dot{\mathbf{X}}_d \) and \( \ddot{\mathbf{X}}_d \) are respectively the first time derivative, and the second time derivative of \( \mathbf{X}_d \). \( \theta \) and \( \dot{\theta} \) are the actual position and angular velocity of the joints. They can be measured by the sensors mounted on the actuators. \( \Delta V \) are the output voltages of the data detected by the six-axis force sensors. These data can be used to reconstruct the contact position, force, and moment in the force reconstruction block by the use of the technique described in Chapter II and Chapter III.

5.4 Force Feedback

The block diagram of Figure 5.3 can be considered to be a high level control and is proposed to move the object along the desired trajectory. However, the actual forces applied to the object may be different from the commanded ones due to some uncertainties, such as compliance, joint friction, and imperfect modeling of the fingers. Thus, force feedback becomes necessary in order to manipulate the object successfully. The feedback control of the contact forces is at a low level and should be executed more frequently than that shown in Figure 5.3. Figure 5.4 shows the proposed force-feedback strategy during the ith interval. \( \mathbf{F}_d \) and \( \mathbf{F}_a \) are respectively the desired and the actual contact forces between the fingertips and the object. \( \mathbf{F}_a \) can be reconstructed from the data collected by the six-axis force sensors. \( \tau_{ij} \) represents the commanded joint torque at the jth force feedback during the ith interval. The controller can be any combination of proportional, integral, and differential control, or the Robbins-Monro method (Wu and Paul, 1982) can be
used. The summation of $\Delta \tau$ and the previous commanded torque $\tau_{i(j-1)}$ becomes the present commanded torque $\tau_{ij}$.

$$\begin{align*}
F_d & \quad \text{Controller} \quad J_T \quad \Delta \tau \quad \tau_{ij} \quad \text{Forward} \\
& \quad \text{Dynamics} \quad + \quad + \quad \tau_{(j-1)} \quad F_a
\end{align*}$$

Figure 5.4 Block Diagram of Force Feedback

Furthermore, the object may slip away from the fingers even after the friction angles have been optimized because the environment is at least unknown. Thus, the position of the mass center has to be updated at each interval. The line passing through the mass center, and along the gravity direction can be derived from the contact information as shown in Appendix B. Thus, the mass center can be updated by finding the intersection point of two such lines in the two most recent consecutive intervals.

5.5 A History-Based Strategy

During the continuous manipulation, chatter phenomenon of the commanded joint torque sometimes occurs. A history-based method is then proposed to increase the smoothness of commanded joint torques. There are three cases of continuous manipulation: a fine manipulation in which one seeks to manipulate the object by moving the fingers alone, gross manipulation of the object by moving the arm or wrist while the relative position between the object and the fingers is kept the same, and combined manipulation of the object by moving the fingers and the arm simultaneously.

For the first case, the shaded area of the block diagram shown in Figure 5.3 can be modified to the one shown in Figure 5.5 from the second interval. $\tau_{ip}$
is the input torque in the previous interval, and \( \tau_f \) is the torque needed to move the fingers. The difference between these two terms can be converted to the wrench \( W \) applied to the object. The residual wrench, \( W_{\text{res}} \), will then be the difference between the present wrench \((M_0E_r + H_o)\) and \( W \). Since the differences of the wrench \((M_0E_r + H_o)\) and the joint positions of the fingers between two consecutive intervals are very small, \( W_{\text{res}} \) is small as well. The force field method can then be adopted to find the residual forces. The resultant set of distributed forces would be an appropriate initial point for an optimization process, and a small step size, \( \sigma \), should be large enough and is suggested to be used after the second interval. Thus, the curves of the input torques will be smoother than those obtained when only the procedures shown in Figure 5.3 are followed.

![Figure 5.5 Modified History-Based Block Diagram](image)

The second case can be treated as a special case of the first. The joint positions of the fingers are fixed. Therefore, this is a much easier case than the first. Finally, the strategy used for the first case can be applied to the third by considering the dynamics of the arm and fingers together.
5.6 Regrasping

When an object is manipulated by the fingers, some finger may reach the boundary of its working volume, approach a singularity, or assume a bad contact position. Therefore, this finger has to be moved to a new position and the object has to be regrasped.

The strategy used for a system with four fingers is shown in Figure 5.6. There are three modes. One is to manipulate the object by using the four fingers. Another is to let three fingers manipulate the object while the other finger is moved to a new position. The other is to let three fingers hold the object stationary until the loose finger again contacts the object. The algorithm used to interchange between these three modes are illustrated in Figure 5.6. The proposed strategy is for frictional point contact between the fingertip and the object. Thus at least three fingers are needed to manipulate or hold the object. If the contact between the fingertip and the object is soft, then the third mode can be modified to let two fingers manipulate the object, while the other two fingers move to new positions.

Figure 5.6 Flow Chart of Regrasping
The trajectory of any finger which is out of contact should be defined carefully. All of the conditions of system limitation, interference with the other fingers or the object, singularity, and the time needed for the motion have to be considered. Any strategies which are applicable in controlling robot arms can be used to control the out-of-contact finger.

5.7 Compliance Control

In addition to the computed torque method described in Section 5.3, active compliance control might be one of the alternative choices to control the multifingered systems. Equation (4.36) describes the dynamic equation of the quasistatic passive grasping model and it also shows that the object will be constrained by the fingers and will settle into the equilibrium position if the disturbance wrench is within the allowance. By using this character, we can let $W_s$, the wrench applied by the fingers, be given by (Hogan, 1985):

$$ W_s = K_s(X_d - X_o) + B_s(\dot{X}_d - \dot{X}_o) $$

(5.13)

For manipulation tasks, the effective spring constants, $K_s$, and damping coefficients, $B_s$, are determined by the characteristics of the grasped object, geometry at the contact points, the characteristics of the fingers and systems (Kao, 1990), and the gains of the controllers. If the wrench shown in Equation (5.13) is applied to the object, the movement of the object might follow the desired trajectory, $X_d$ and $\dot{X}_d$.

The commanded joint torque can then be found by using Equations (5.5), (5.6), and (5.13):

$$ \tau = H_p + M_p\ddot{X}_o + J^T\tau $$

(5.14)
where \( \bar{F} = G^T\left[K_s(X_d - X_w) + B_s(\dot{X}_d - \dot{X}_w)\right] + F_i \)

Similarly, the desired distributed force, \( \bar{F} \), can also be found by using the method described in Chapter IV to optimize the friction angles.

Although the concept of active compliance have been used in manipulation by robot arms (Hogan, 1985), the availability and stability obtained by applying this method to multifingered systems need further study.

5.8 Conclusion

The computed torque method has been combined successfully with a force allocation rule, which optimizes the friction angles. The strategies proposed are applicable to three dimensional manipulation tasks. The use of force feedback is also proposed to compensate for uncertainty. Since either the projection or history-based method can provide a proper initial point, the friction angles can be satisfactorily modified by iterating no more than ten times, as in the examples shown in the previous Chapter. Therefore, the torques to be commanded at the joints can be found very efficiently.
CHAPTER VI
SIMULATION OF DEXTEROUS MANIPULATION

6.1 Introduction

Since the hardware of DIGITS system is not available yet, the strategies described in the previous chapters are tested by running simulation examples. The flowchart shown in Figure 5.1 is modified to the one shown in Figure 6.1 for simulation work. The actual trajectory can be found using forward dynamics. The contact information can be updated by using the method proposed by Montana, 1988. Furthermore, a method for finding the contact forces was also derived. The details will be described in the following Sections.

The simulation program has been written using the C language. X Window is also adopted for animation so that the program is portable and can be run on any UNIX environment. Two manipulation examples using the DIGITS system will be presented in this Chapter.

Figure 6.1 Overview of Simulation Work
6.2 Forward Dynamics

In real application, most of the data can be detected by using sensors. These data include the positions of the object and the finger joints, contact positions at the fingertip and grasped object, and the contact forces. In DIGITS system, the positions of finger joints can be decoded from the data collected by the optical encoders. The encoders are mounted on the shafts of the brushless motors. The intrinsic six-axis force sensors can be used to collect the data, and the contact force and the contact position can then be reconstructed. To update the position of the object, the line passing through the mass center, and along the gravity direction, can be derived from the contact information as in Appendix B. The mass center can then be updated by finding the intersection point of two such lines in the two most consecutive intervals.

As to the simulation, the data have to be updated by calculation. The methods to update these data in computer simulation will be described. In Chapter V, the dynamics of the grasped object and the fingers are described respectively in Equation (5.4) and (5.5).

\[
\mathbf{M}_o \ddot{\mathbf{X}}_o + \mathbf{H}_o = \mathbf{GF} \tag{5.4}
\]

\[
\mathbf{A}(\theta) \ddot{\theta} + \mathbf{N}(\theta, \dot{\theta}) + g(\theta) + \mathbf{J}^T \mathbf{T} \mathbf{F} = \tau \tag{5.5}
\]

The kinematic relationship between the acceleration at the contact points and the joint accelerations can also be written in the form

\[
\ddot{\theta} = \mathbf{J}^T (\mathbf{T} \mathbf{G}^T \mathbf{T}_c \ddot{\mathbf{X}}_o + \mathbf{L}(\mathbf{X}_o, \dot{\mathbf{X}}_o) - \dot{\mathbf{J}} \dot{\theta}) + \ddot{\theta}_h \tag{5.7}
\]
When the input torque has been determined, we can substitute from Equation (5.7) and (5.5) into Equation (5.4). If the term \( \dot{\theta}_h \) is neglected, we can get the following Equation:

\[
[\mathbf{M}_0 + \mathbf{G}^T (\mathbf{J}^T)^+ \mathbf{A} \mathbf{J}^+ \mathbf{G}^T \mathbf{T}_e] \ddot{\mathbf{x}}_o = \mathbf{G}^T (\mathbf{J}^T)^+ \left[ \tau - \mathbf{N} - \mathbf{g} + \mathbf{A} \mathbf{J}^+ (\dot{\mathbf{J}} \dot{\theta} - \mathbf{L}) \right] - \mathbf{H}_o
\]  

(6.1)

In Equation (6.1), the summation within the bracket at the left hand side is a 6 by 6 matrix. The terms at the right hand side become a 6 by 1 vector. \( \ddot{\mathbf{x}}_o \) can then be obtained uniquely by solving the Equation. By integrating \( \ddot{\mathbf{x}}_o \), we can compute the velocity, \( \dot{\mathbf{x}}_o \), and the position, \( \mathbf{x}_o \), of the object.

The angular accelerations, \( \ddot{\theta} \), of the finger joints can be found by substituting \( \ddot{\mathbf{x}}_o \) into Equation (5.7). Then the angular velocities, \( \dot{\theta} \), and the positions, \( \theta \), of the finger joints can then be updated by integration from \( \ddot{\theta} \). On the other hand, \( \dot{\theta} \) can also be calculated from \( \ddot{\mathbf{x}}_o \) by using Equation (5.6), and \( \theta \) can then be computed by integrating \( \dot{\theta} \).

The real contact forces should be updated for the force feedback if the system is not perfectly modeled. From the Equations (5.5) and (5.7), the contact force can be expressed as

\[
\mathbf{F} = \mathbf{T}^T (\mathbf{J} \mathbf{A}^{-1} \mathbf{J}^T)^{-1} (\mathbf{a}_o - \mathbf{a})
\]  

(6.2)

where

\[
\mathbf{a}_o = \mathbf{J} \mathbf{A}^{-1} \left( \tau - \mathbf{N}(\theta, \dot{\theta}) - \mathbf{g}(\theta) \right) + \dot{\mathbf{J}} \dot{\theta}
\]

\[
\mathbf{a} = \mathbf{T} \mathbf{G}^T \mathbf{T}_e \ddot{\mathbf{x}}_o + \mathbf{L}(\mathbf{x}_o, \dot{\mathbf{x}}_o)
\]
The term \( a_o \) represents the acceleration at the fingertips with the commanded torque, \( \tau \), when the fingers do not contact or grasp any object. While \( a \) represents the acceleration at the contact points of the grasped object. Both \( a_o \) and \( a \) can be solved after the values of \( \tau \) and \( \dot{X}_o \) are obtained. The real contact forces can also be determined by using Equation (6.2).

### 6.3 Updating Contact Information

The contact position on both grasped object and fingertips will change due to the rolling motion in manipulation process. The force sensor can be used to update the contact position in real application. Similarly, a method to update contact positions at fingertips and the grasped object for simulation tests need to be developed. Montana proposed a method to update the contact point position on two contacting bodies in 1988. This method was based on the theory of differential geometry. The contact position on the surface of the each body can be represented by two parameters, such as \( u \) and \( v \) in Figure 6.13. Two coordinate frames are attached to two contact bodies. The origins of these two frames are at the contact point and the \( z \) axes are along the direction of the contact normal. The angle between the two \( x \) axes is represented by \( \psi \).

A block diagram of this method is shown in Figure 6.2. \( v_z \), the relative velocity along the normal direction, is zero since the two bodies are in contact. The other five parameters, \( v_x \), \( v_y \), \( \omega_x \), \( \omega_y \), and \( \omega_z \), are the inputs for this method. If these are given, all of \( u_1 \), \( v_1 \), \( u_2 \), \( v_2 \), and \( \alpha \) can then be updated.
For any grasped rigid body, the linear velocity at the ith contact point can be found by using the grasp matrix, and can be related by the equation

\[ V_{oi} = G_i^T T_e \dot{X}_o \] (6.3)

The angular velocity, \( \omega_{oi} \), is the same as that of the body. For the ith contact point on the fingertips, both the linear and angular velocities can be found if the Jacobian matrix and the angular or linear velocity at each joint are given. The relationship can be written

\[
\begin{bmatrix}
V_{\text{fi}} \\
\omega_{\text{fi}}
\end{bmatrix} = J \dot{\theta}
\] (6.4)

Thus, the relative velocity, \( \{v_x, v_y, v_z\} \), and angular velocity, \( \{\omega_x, \omega_y, \omega_z\} \), at the contact points are the difference between \( \{V_{oi}, \omega_{oi}\} \) and \( \{V_{fi}, \omega_{fi}\} \). The contact positions can then be updated by using Montana's Equations.

### 6.4 Simulation Example I

The results of simulation of coordination of four fingers to manipulate an object will be presented. As in the DIGITS hand, there are three revolute joints on each finger. The first and the second joints are orthogonal and intersect at the point represented by the small solid circle shown in Figure 6.3. The
positions of the intersection points for all of the four fingers are arranged symmetrically at (0.75, 0.0, 0.0), (-0.75, 0.0, 0.0), (0.0, 0.75, 0.0), and (0.0, -0.75, 0.0), where the lengths or distances are in inches. The third revolute joint connects the proximal and distal links and is parallel to the second joint. The lengths of the proximal and distal links are both 2.125 inches. The shape of the distal link is a hemisphere attached to a cylinder. The radius of the cylinder or the hemisphere is 0.375 inch. The effects of gravity and centrifugal force on the links were neglected. The inertia matrix for each finger was assumed to be constant and was

\[
A = \begin{bmatrix}
140.0 & 1.0 & 1.5 \\
1.0 & 7.0 & 2.0 \\
1.5 & 2.0 & 3.5
\end{bmatrix} \text{ (lb.in}^2)\]

The system was assumed not to be modeled perfectly. The inertia matrixes used to find the commanded torques were set to be 0.5A. The force feedback routine was executed one time during each interval to compensate for the uncertainty.

The geometry of the grasped object was assumed to be a polyhedron with six faces. The weight of the object was 7.0 lb. The initial positions of the contact points and the directions of the contact normals, with respect to the coordinate frame, xyz, fixed to the object, were

<table>
<thead>
<tr>
<th>Position</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00 0.08 0.15</td>
<td>-0.95 -0.24 -0.20</td>
</tr>
<tr>
<td>-2.00 0.20 0.00</td>
<td>0.90 0.32 -0.30</td>
</tr>
<tr>
<td>-0.15 1.75 0.15</td>
<td>-0.00 -0.98 -0.20</td>
</tr>
<tr>
<td>-0.15 -2.00 0.15</td>
<td>-0.20 0.98 0.00</td>
</tr>
</tbody>
</table>
In this manipulation movement, the palm of the hand was kept stationary. The object was then manipulated by the four fingers to move along the desired trajectory, \( X_d, \dot{X}_d, \) and \( \ddot{X}_d \). The origin of the object fixed coordinate frame, \( \{x, y, z\} \), was moved from position \((0.375, -0.375, -2.0)\) to position \((-0.375, 0.375, -1.5)\) with respect to the global coordinate frame, \( \{X, Y, Z\} \), which was fixed to the palm. Meanwhile, the Euler angles were changed from \((0.0, -5.0, 10.0)\) to \((-5.0, 0.0, -5.0)\), (the units are expressed in degrees). This movement was completed in three seconds and the object would then be kept stationary at the final position. The absolute values of all the joint torques were set to be within 18.0 lb in and the calculation interval was 20 ms.

Three dimensional animation pictures, as shown in Figure 6.3, were plotted on the computer screen during the simulation process. The positions of the object and fingers, the values of the input torques, and the friction angles were all displayed on the monitor. Furthermore, the program was designed so that the animation can be viewed from different directions. Four different views of the object and the fingers at the original position are shown in Figure 6.4. Figure 6.5 also shows the positions of the object and the fingers at different instants.

The other results of the simulation are shown in Figures 6.6, 6.7, and 6.8. Figures 6.6 and 6.7 show the desired and actual trajectories, and the position and orientation, of the grasped body. Since the movement of the object followed the desired trajectory very well, the dashed lines are overlaid by the solid lines and can hardly be seen. The cosine values of the friction angles for all of the four contacts are shown in Figure 6.8. Most of the cosine values are greater than 0.80 during the whole manipulation. For the optimum unconstrained solution, all the cosine values should be equal, as was proven by Mukherjee...
However, the results turn out that some of the cosine values are very close to the smallest if some additional constraint, such as joint torque capacity, is included. This fact is shown in Figure 6.8. The results were obtained by iterating ten times in the optimization process. It took only 5.2 ms to find the distributed forces by iterating ten times if the program was running on a 80486 (50Mhz) personal computer. This is probably quick enough for real-time control.

The commanded torques of the finger joints, the joint positions, contact point position at the fingertip, and the contact point position of the first finger on the polyhedral object are shown in Figure 6.9. The input torques are all within the allowable, 18.0 lb in and the curves are smooth. Similar results for the other three fingers are shown in Figure 6.10, Figure 6.11, and Figure 6.12.

The algorithms used in the previous examples are meant to optimize friction angles. However, we can appropriately set a threshold value for the cosine values, such as 0.80, if the friction coefficient between the fingers and the object is known. The threshold value must be greater than the cosine of the physical friction angle. This means that the iteration will terminate when all of the cosine values of the friction angles are greater than 0.80. Using this technique, the number of iterations might decrease and the computation time would then be reduced.
Figure 6.3 Animation Picture of Example I
Figure 6.4 Initial Position Viewed from Different Directions
Figure 6.5 Animation Picture at Different Instants
Figure 6.6 Actual and Desired Trajectory

Figure 6.7 Actual and Desired Orientation
Actual Cosine Values

![Graph showing actual cosine values over time.](image)

Figure 6.8 Cosine Values of Friction Angles
Figure 6.9 Simulation Results of 1st Finger
Figure 6.10 Simulation Results of 2nd Finger
Figure 6.11 Simulation Results of 3rd Finger
Torque of 4th Finger

(a) Commanded Torque

(b) Position of Finger Joints

(c) Contact Position on Object

(d) Contact Position on Fingertip

Figure 6.12 Simulation Results of 4th Finger
6.5 Simulation Example II

The same parameters of the system used in the previous example were used for this manipulation. The system was again assumed not to be exactly modeled. The inertia matrices used to find the commanded torques were set to be 0.2A. The force feedback routine was also executed one time during each interval to compensate for the uncertainty. The mobility ranges of the three joints were set to be respectively

\[-45.0^\circ < \theta_1 < 45.0^\circ\]
\[-180.0^\circ < \theta_2 < 10.0^\circ\]
\[-110.0^\circ < \theta_3 < 70.0^\circ.\]

The determinants of the Jacobian matrix were also required to be larger than 2.0 in$^3$ to avoid singularity.

The geometry of the grasped object was a sphere with radius 1.5 inches. The weight of the object was 5.0 lb. u and v in Figure 6.13 were the two angles to represent the contact point positions on the sphere, and on the hemispherical
100

fingertip. The four original positions of contact points on the ball were (-18.0°, -18.0°), (-18.0°, 85.0°), (-18.0°, 188.0°), and (-18.0°, 290.0°). The characters E, N, W, and S were patched onto the ball at positions (0.0°, 0.0°), (0.0°, 90.0°), (0.0°, 180.0°), and (0.0°, 270.0°) to help us understand the motion of grasped object.

In this movement, the palm of the hand was also kept stationary. The object was then manipulated to move along the desired trajectory, $X_d$, $\dot{X}_d$, and $\ddot{X}_d$. The desired position and orientation of the object coordinate frame $xyz$ with respect to the global coordinate frame $XYZ$ were

$$x(t)=0.0, \quad y(t)=0.0, \quad z(t)=-2.0+0.2\sin(12t)$$

$$\phi(t)=0.0, \quad \theta(t)=0.0, \quad \psi(t)=3.0t.$$  

The angles were represented in degrees, and the positions are represented in inches. The absolute values of all the joint torques were set to be within 18.0 lb in and the calculation was executed every 20 ms.

During manipulation, the object was intended to be contacted by all the four fingers. However, any finger which moved beyond the boundary of the working volume had to be moved to a new position and could not contact the object during this movement. Thus, only three fingers were used to manipulate the object at some times.

When the grasped object is a sphere, the vector $r \times n$ in the grasp matrix is 0 if the origin is at the center of the sphere. This means that the contact forces along the normal direction cannot equilibrate the external torque at the sphere center. In this examples, the external force was intended to be equilibrated by the forces along the normal direction. The contact forces along the normal directions and the external force can then be related by
If only three fingers are used to manipulate the object, \( N \) becomes a 3 by 3 matrix. The force along the normal direction can be found to be

\[
F_n = N^{-1}R
\]

(6.6)

When the object is grasped by four or more fingers, \( N \) is a 3 by \( n \) matrix and \( n > 3 \). \( F_n \) can be found by using a Pseudo-inverse and

\[
F_n = N^T(\!N\!N^T)^{-1}R
\]

(6.7)

Frames from a three dimensional animation, as shown in Figure 6.14, were also plotted on the computer screen during the simulation process. The cosine value of the first finger did not appear in Figure 6.14 because this finger was not in contact with the object at this moment. Four frames, which show the positions of the object and the fingers at different instants, are shown in Figure 6.15. The rotation motion along the Z axis can be easily understood by looking at the changes of the xyz coordinate frame and the character S.

The results of the simulation are shown in Figures 6.16, 6.17, 6.18, and 6.19. Figures 6.16 and 6.17 show the position and orientation of the grasped body. The dashed and solid lines represent the desired and actual trajectories. Apparently, the actual trajectory followed the desired one very well. The cosine values of the friction angles for all of the four contacts are shown in Figure 6.18. The cosine values were all larger than 0.996. The results were obtained by iterating ten times in the optimization process. Figure 6.19 shows the active
mode of each finger. When \( N \) was \( k \), the \( k \)th finger was not contacting the object and only the other three fingers were used to manipulate the sphere. The object was manipulated by all four fingers when \( N \) is 0.

The data for the first finger are shown in Figure 6.20. Solid, dashed, and dot-dashed lines represent the first, second and third joints for the commanded torque and joint position. The commanded torques were all within the allowable, 18.0 lb in. The curves are smooth except when a finger left or came back into contact with the object. The contact positions on the object and the hemispherical fingertip are also shown in Figure 6.20. Similar results for the other three fingers can be seen in Figures 6.21, 6.22, and 6.23.
Figure 6.14 Animation Picture of Example II
Figure 6.15 Animation Picture at Different Instants

TIME(sec): 0.04000

TIME(sec): 5.02000

TIME(sec): 10.02000

TIME(sec): 15.02000
Figure 6.16 Actual and Desired Trajectory

Figure 6.17 Actual and Desired Orientation
Figure 6.18 Cosine Values of Friction Angles

Figure 6.19 Active Modes of the Fingers
Figure 6.20 Simulation Results of 1st Finger
Figure 6.21 Simulation Results of 2nd Finger
Figure 6.22 Simulation Results of 3rd Finger
Figure 6.23 Simulation Results of 4th Finger
6.6 Conclusion

In this Chapter, the proposed strategies have been successfully tested by running computer simulations. The movements of the object follow the desired trajectory very well even though the fingers are assumed to not be perfectly modeled. The friction angles are satisfactorily modified by iterating just ten times. Since it take only 5.2 ms on 80486 personal computer to find the commanded torque by iterating 10 times, it is expected that the proposed methods can be used for real-time control even when personal computer type hardware is used for coordination.

This work is for three dimensional manipulation, it is believed that any complicated movement of an object can be simulated, as long as the desired trajectory is defined. Although the simulation example focused on the DIGITS system, this work should be applicable to other multifingered systems with minor modifications. In the future, the methods proposed here will be tested on the DIGITS robot hand.
CHAPTER VII
SUMMARY AND RECOMMENDATIONS

7.1 Summary

This research has been concerned about manipulation by using robot hands and has three major aspects. They are (i) reconstruction of the contact information, (ii) calculation of distributed forces, and (iii) the control strategies.

We began with the test of the six-axis force sensor. The mapping matrix between the output signal and the resultant wrench was built by analyzing the test data. The ambiguity problem for soft point contact has been shown to be nonexistent if the surface of the fingertip is convex. By integrating the ambiguity analysis and the equations derived by the other researchers, the contact information: contact position, force, and moment can be correctly reconstructed.

The contact forces applied by the fingers were solved for by optimizing the contact friction angles. Three steps: projection, residual wrench equilibrium, and optimization were used to solve the problem. All of the pseudoinverse method, the velocity field method, and the force field method can be used to solve the distributed forces which equilibrate the residual wrench. The relationship between these three methods was discussed. The first and the second steps can provide a good initial point for the optimization process. Therefore, a solution close to the best friction angles can be found efficiently.

In order to manipulate the grasped object, the computed torque method has been combined successfully with a force allocation rule which optimizes the
friction angles. All of the force feedback, regrasping, and rolling motion were considered. The method proposed was based on three dimensional manipulation and can be applied to any tasks as long as the desired trajectory is defined. The simulation examples showed that the object was moved along the desired trajectory very well and the friction angles were well optimized. The time needed to find the commanded torque is only 5.2 ms when running on an 80486 (50 MHz) personal computer for the four finger case. It is probably quick enough for real-time control.

The work developed in this research was based on the model of the DIGITS system. However, the proposed strategies and techniques can be applied to other multifingered systems or even to a walking machine with minor modification.

7.2 Recommendations

The geometry of the six-axis force sensor appears complicated due to the thin ring. The main purpose for this design is to increase sensitivity to loads along the axial direction. However, the sensor is not easy to manufacture and this design should be avoided if cost is a major concern. An alternative design is to manufacture the sensor as a tube or hollow cylinder. A hole drilled along the radial direction can then cause a stress concentration and the sensitivity along the axial direction can be increased. On the other hand, the strain gages cannot be positioned perfectly. Thus, the mapping matrix between the output voltages and the resultant wrench will not be the same for different sensors. It is recommended that similar tests to those described in Chapter II should be conducted in order to obtain the mapping matrix for each sensor.

In finding the distributed forces, iteration is suggested to get a better result.
The number of iterations can be reduced if some criteria for friction angles has been set. The criteria can be defined based on the material and roughness of fingertips and the grasped object. On the other hand, instead of using an arbitrary positive value to replace the negative normal force in projection step, the value might be well defined by considering the grasp configuration. Furthermore, regrasping can be an alternative choice. The way to find the new grasping point properly needs further study.

In simulation examples, the inertia matrix was assumed to be constant, and the friction as well as the gravity effects were neglected. In real applications, all of these terms should be considered. The inertia matrix can be efficiently updated at each position (Lilly and Orin, 1991). The other effects should be investigated experimentally or may be compensated for by force feedback control.

The effect of compliance between fingers and the grasped object has great potential in future research. The active compliance alone or combined with the computed torque method might be applied to control the multifingered systems. On the other hand, sliding motion is a major concern for both grasping and manipulation tasks. The onset of the sliding motion (Howe et al. 1988) can be affected by distributed forces, the wrench to be equilibrated, contact configuration, disturbance wrench, and compliance between fingers and the grasped object. This topic can also be extensively studied and tested.

The speeds and performances of computers are improving significantly. If we can take the advantage of this fact, computing time can be decreased. Parallel calculation is also suggested to save computing time by integrating several CPUs together. The tactile and vision sensors may also be added to increase the sensing ability of the system. Finally, the techniques of artificial
intelligence and neural networks may be adopted to improve the performance of the system.
APPENDIX A

PSEUDO-INVERSE AND VELOCITY FIELD METHOD

In solving the linear equations, two vectors, \( X \) and \( Y \), are usually related by a matrix, \( A \). The relationship between \( A \), \( X \), and \( Y \) can be expressed in the form

\[
AX = Y \tag{A.1}
\]

The problem is to solve \( X \) by giving \( A \) and \( Y \). If \( A \) is an \( m \) by \( n \) matrix and \( m < n \), this is an underdetermined problem. There will be an infinite number of solutions of \( X \) which can satisfy Equation (A.1).

If the norm of \( X \) is defined as

\[
\|X\| = (X^T N X)^{1/2} \tag{A.2}
\]

where \( N \) is an \( n \) by \( n \) symmetric positive definite matrix, there exists a unique solution for \( X \) such that \( \|X\| \) can be minimized. The solution can be found to be (Rao and Mitra, 1971)

\[
X = PY \tag{A.3}
\]

where

\[
P = N^{-1} A^T (AN^{-1} A^T) \tag{A.4}
\]
On the other hand, there exists an infinite number of homogeneous solutions $X_h$, such that $AX_h = 0$. $X_h$ can be found in the form

$$X_h = (I - PA)Z$$

(A.5)

in which $Z$ is an arbitrary vector with $n$ elements. By adding the solution shown in Equation (A.3) and the homogeneous solutions, we can get the general solution. The expression of the general solution of $X$ is

$$X = PY + (I - PA)Z$$

(A.6)

For the multifingered system, the object is grasped by the fingers. The contact forces $F$ and the external wrench $W$ can be related by the grasp matrix $G$, and is expressed in the form

$$GF = W$$

(A.7)

If $x$ represents the displacement vector at the contact points, and $K$ is the symmetric positive definite matrix of the spring constants. The potential energy, $E$, stored in the object due to the deformation becomes

$$E = \frac{1}{2} x^T K x$$

(A.8)

Furthermore, $F$, $x$, and $K$ can also be related by

$$F = Kx$$

(A.9)

Substituting Equation (A.9) into Equation (A.7), we get

$$GKx = W$$

(A.10)

If the objective is to minimize the potential energy, that is to minimize $x^T K x$, we can compare Equations (A.10) and (A.8) to the Equations (A.1) and (A.2). $x$ can be found to be
\( x = K^{-1}(GK)^T(GK)K^{-1}(GK)^T)W = G^T(GKG^T)W \)  \( (A.11) \)

By using Equation (A.9), \( F \) can be found to be

\[ F = KG^T(GKG^T)^{-1}W \]  \( (A.12) \)

When there are not any interaction forces, the grasped object is not squeezed. We can treat the object as a rigid body, and the velocity field can be applied. The small increment of the displacements of the object, \( X_o \), and those at any contact points, \( x \), can also be related by the grasp matrix. The relationship is

\[ G^T X_o = x \]  \( (A.13) \)

Where \( X_o \) and \( x \) are both 6 by 1 vectors and consist of linear and angular displacements. By combining Equations (A.9) and (A.13), we get

\[ F = KG^T X_o \]  \( (A.14) \)

Substitution of Equation (A.14) into Equation (A.7), gives

\[ GKG^T X_o = W \]  \( (A.15) \)

In Equation (A.15), \( GKG^T \) is a 6 by 6 matrix, and both \( X_o \) and \( W \) are 6 by 1 vectors. Therefore, \( X_o \) can be uniquely solved for and

\[ X_o = (GKG^T)^{-1}W \]  \( (A.16) \)

The contact forces \( F \) can then be found out to be

\[ F = KG^T(GKG^T)^{-1}W \]  \( (A.17) \)
Evidently, Equations (A.12) and (A.17) are exactly the same mathematically. During the derivation of Equation (A.12), the objective is to minimize the potential energy. Physically, this can be done if and only if there are no interaction forces which squeeze the object. Equation (A.13) can then be applied directly. Therefore, we can conclude that the general pseudo-inverse method and the velocity field method are the same mathematically and physically.

If the spring constants are the same along all the different directions at all the contact points, then $K = kI$. Equation (A.12) or (A.17) becomes

$$F = G^T(GG^T)^{-1}W$$  \hspace{1cm} (A.18)

And Equation (A.14) becomes

$$F = G^T(kX_o)$$  \hspace{1cm} (A.19)

By comparing Equations (A.18) and (A.19), we can get

$$kX_o = (GG^T)^{-1}W$$  \hspace{1cm} (A.20)

If we let the external wrench be $W = (Q, T)^T$, and let $kX_o = (V_k, w_k)^T$, it can be shown that

$$w_k = \frac{1}{n} (T - \bar{r} \times Q)$$

$$V_k = \frac{Q}{n} - \frac{1}{n} (T - \bar{r} \times Q) \times \bar{r}$$  \hspace{1cm} (A.21)

where
\[ I = \begin{bmatrix} y^2 + z^2 & -xy & -xz \\ -xy & x^2 + z^2 & -yz \\ -xz & -yz & x^2 + y^2 \end{bmatrix} \]

\[ \bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i \quad \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \quad \bar{z} = \frac{1}{n} \sum_{i=1}^{n} z_i \]

\[ \bar{x}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x}) \quad \bar{y}^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y}) \quad \bar{z}^2 = \frac{1}{n} \sum_{i=1}^{n} (z_i - \bar{z}) \]

\[ \bar{y}z = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})(z_i - \bar{z}) \quad \bar{x}z = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(z_i - \bar{z}) \]

\[ \bar{xy} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \]

If we look at Equation (A.19), the contact force at the ith contact point, \( F_i \), can also be expressed in the form

\[ F_i = V_k + w_k \times r_i \quad (A.22) \]

In Section 3.2 of Chapter 4, \( F_i \) is expressed in Equation (4.17) as

\[ F_i = hL u + L u \times (r_i - \rho_n) \quad (4.17) \]

Since the derivation is based on the force field method, as in Equation (A.19), the value of \( F_i \) should be the same when obtained by using both Equations (A.22) and (4.17). The parameters in these two equations can also be derived and related to give

\[ w_k = L u = \frac{1}{n} (T - \bar{r} \times Q) \]

\[ V_k = hL u - L u \times \rho_n = \frac{Q}{n} - \frac{1}{n} (T - \bar{r} \times Q) \times \bar{r} \quad (A.23) \]
where
\[ u = \frac{1}{nL} (T - \bar{r} \times Q) \]
\[ L = \frac{1}{n} (T - \bar{r} \times Q) \]
\[ h = \frac{1}{nL} (\mathbf{u} \cdot \mathbf{Q}) \]
\[ \rho_n = \frac{1}{nL} (\mathbf{u} \times \mathbf{Q}) - (\mathbf{u} \cdot \bar{r}) \mathbf{u} + \bar{r} \]

Evidently, the force field method can provide the same result as by using the velocity field method if \( K = kI \). However, it should be noted that the force field method can be applied to get the minimum energy only when \( K = kI \), that is the stiffness are the same along all the directions at all the contact points. For the general cases, the velocity field or the general pseudo-inverse method should be applied in order to minimize the potential energy.
APPENDIX B

FINDING THE MASS CENTER

As is shown in Figure B.1, several fingers are coordinated to grasp an object. The equilibrium equations between the fingers and the grasped object are the same as in Equations (4.3) and (4.4). They are

\[ \sum_{i=1}^{n} F_i = R \]  
\[ \sum_{i=1}^{n} F_i \times \mathbf{r}_i + T_i = M \]

(C.1)  
(C.2)

If the object is stationary or the terms in Equations (4.1) and (4.2), except for the weight, are negligible, then the equilibrium equations become

\[ \sum_{i=1}^{n} F_i = -w \]  

(B.3)
As in Figure B.1, let \( r_0 \) be the position of the mass center with respect to the coordinate frame XYZ, and

\[
\mathbf{r}_i = \mathbf{r}_i - r_0 \tag{B.5}
\]

By substituting Equations (B.5) and (B.3) into Equation (B.4), we get

\[
\sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i + T_i = 0
\tag{B.4}
\]

\[
\mathbf{r}_0 \times \mathbf{w} = \left( \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i + T_i \right) \tag{B.6}
\]

If \( \mathbf{r}_{on} \) is the vector which satisfies Equation (B.6) and is perpendicular to \( \mathbf{w} \), we may get \( \mathbf{r}_{on} \) by cross multiplying both sides of Equation (B.6) by \( \mathbf{w} \). Thus, \( \mathbf{r}_{on} \) can be expressed as

\[
\mathbf{r}_{on} = \left( \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{F}_i + T_i \right) \times \mathbf{w} \tag{B.7}
\]

The vectors \( \mathbf{r}_i, \mathbf{F}_i, \) and \( T_i \) can be reconstructed from the six-axis force sensors and \( \mathbf{w} \) can be obtained from Equation (B.3). Thus, the line of action \( L_w \), as in Figure B.1, which is defined as the line passing through the mass center with the direction of \( \mathbf{w} \), can be found from \( \mathbf{r}_{on} \) and \( \mathbf{w} \). The mass center is then the intersection point of two or more different lines of action.
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