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A study of compressible mixing layers using laser-based diagnostic techniques

Elliott, Gregory Scott, Ph.D.
The Ohio State University, 1993
A STUDY OF COMPRESSIBLE MIXING LAYERS
USING LASER BASED DIAGNOSTIC TECHNIQUES

A DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Gregory Scott Elliott, B.S., M.S.

* * * * *

The Ohio State University
1993

Dissertation Committee:
L.A. Kennedy
J.W. Rich
M. Samimy
K. Vafai

Approved by

[Signature]
Advisor
Department of Mechanical Engineering
To My Wife
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March 23, 1964 ............................................. Born - Newark, Ohio

1984 - 1987 .................................................. Co-op Engineer, Ford Motor Company

1987 ............................................................ B.S.M.E., The Ohio State University

1989 ............................................................ M.S., The Ohio State University

1987 - Present .............................................. Graduate research associate, Department of Mechanical Engineering, The Ohio State University
Publications

Journal Articles


Conference Proceedings


FIELDS OF STUDY

Major Field: Mechanical Engineering

Areas of Interest: Fluid Mechanics, Compressible Flow, Turbulence, and Laser Diagnostics
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NOMENCLATURE

Roman symbols

\( a \)  
Speed of sound

\( A_r(I/I_0) \)  
Frequency shift in terms of the intensity ratio from the iodine absorption profile

AARL  
Aeronautical and Astronautical Research Laboratory

\( b_A \)  
\( y \) intercept from linear regression of iodine absorption filter

\( b_t \)  
\( y \) intercept from linear regression of camera comparison

\( b_{vel} \)  
Thickness between \( U^* = 0.1 \) and 0.9 and \( U^* \)

\( b \)  
Visual thickness of the mixing layer

\( C_0 \)  
Incompressible growth rate coefficient

\([db/dx]_i\)  
Incompressible growth rate

\([db/dx]_c\)  
Compressible growth rate

\( d_p \)  
Scattering particle diameter

\( dx_s \)  
Streamwise distance between the pressure probes

\( dz_s \)  
Spanwise distance between the pressure probes

\( D \)  
Exit diameter of axisymmetric jet

\( f_0 \)  
Illuminating laser frequency
\( f(u_1, u_2, u_3, \ldots) \) Measured property as a function of the independent variables for error analysis

\( f - f_0 \) Frequency difference from line center

\( f_0 - f_{\text{min}} \) Half-width of the scattered spectrum

\( f - f_{\text{avg}} \) Frequency fluctuation of laser

\( G/G_{\text{max}} \) Power spectrum from pressure fluctuations nondimensionalized by peak

\( I(x,y) \) Velocity discriminating (or filtered) intensity adjusted to the same dynamic range as the reference camera \( (I_0) \)

\( I'(x,y) \) Collected light intensity from velocity discriminating camera

\( I_j(x,y) \) Instantaneous intensity in the image \( j \)

\( I^*_j(x,y) \) Instantaneous intensity subtracted from the local mean

\( I_{\text{max}}(x) \) Maximum intensity of averaged streamwise image at each \( x \) location

\( I_0(x,y) \) Intensity of unfiltered incident beam or image (reference camera)

\( I_{\text{RMS}}(x,y) \) RMS intensity fluctuation

\( I_{\text{vel}}(x,y) \) Processed image with intensity proportional to velocity

\( k \) Boltzmann’s constant

\( k_s \) Observed unit light wave vector

\( k_0 \) Incident unit light wave vector

\( k_r \) Absorption coefficient

\( K \) Scattered wave vector

\( L_{\text{cell}} \) Molecular cell length

\( L_p \) Distance between the centroids of the two peak correlation levels

\( L_{\text{ps}} \) Planar structure length
mA  Slope from linear regression of iodine absorption filter
mI  Slope from linear regression of camera comparison
M   Molecular weight of the scattering molecule
n   Number of images used for statistics
N   (Number of images) x (number of reference lines)
P   Pressure
P12  Iodine vapor partial pressure
PN2  Nitrogen gas partial pressure
P0   Stagnation pressure
r   Radial distance from jet axis
ru   Velocity ratio U2/U1
R   Radius of axisymmetric jet
R(xr,yp)  Molecular Rayleigh scattering profile
Reθ   Reynolds number based on momentum thickness
RMS  Root mean square
Rs   Two dimensional spatial correlation
Rxt   Space-time correlation calculated from initial and delayed images
Ruv  Correlation coefficient
s   Density ratio (ρ2/ρ1)
T   Temperature
T0   Stagnation temperature
Tcell  Cell temperature

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$T_{12}$  Coldest point in the cell in the side arm

$u$  Streamwise velocity fluctuation

$u_1, u_2, u_3, \ldots$  Independent variables for error analysis

$U$  Streamwise mean velocity

$U^*$  Nondimensional streamwise velocity $(U-U_2)/(U_1-U_2)$

$U_c$  Theoretical convective velocity of the large scale structures

$U_{cp}$  Convective velocity of the large scale structures measured by pressure correlations

$v$  Lateral velocity fluctuation

$V$  Flow velocity vector

$V$  Magnitude of flow velocity

$V_0$  Most probable molecular velocity

$x$  Streamwise distance measured from end of splitter plate

$x_0$  Streamwise location of the virtual origin

$x_c$  Nondimensional convection distance

$x_f$  Nondimensional frequency

$x_{f,\text{max}}$  Nondimensional frequency half-width of molecular Rayleigh scattering profile

$x_{\text{ref}}$  Streamwise coordinate of origin used for image correlations

$X_0$  Streamwise position of the first probe

$y$  Lateral distance measured from the surface of the splitter plate

$y^*$  Nondimensional lateral distance $(y-y_0)/b$

$\hat{y}$  Lateral location of any given point in the shear layer to be calculated
\( y_c \) Lateral location where the measured convective velocity equals the theoretical
\( y_0 \) Lateral location of maximum intensity in the average image
\( y_{0.5} \) Lateral location where \( U = (U_1 + U_2)/2 \)
\( y_p \) Ratio of the collision frequency to the acoustic spatial frequency
\( y_{ref} \) Lateral coordinate of origin used for image correlations
\( y_w \) Radial distance from center of disk
\( z \) Spanwise distance measured from the center of splitter plate

**Greek symbols**

\( \alpha \) Collision frequency
\( \beta \) Spanwise angle of large scale structure
\( \delta \) Boundary layer thickness
\( \delta_w \) Vorticity thickness
\( \Delta f_D \) Magnitude of Doppler shift
\( \Delta u_1, \Delta u_2, \ldots \) Uncertainty of independent variables
\( \Delta u_r \) Total uncertainty of dependent variable
\( \Delta V \) Total uncertainty of velocity
\( \gamma \) Intermittency
\( \lambda \) Wavelength of laser
\( \theta \) Momentum thickness
\( \theta_D, \gamma_D, \phi_D \) Angles used to calculate Doppler shift (Figure 4)
\( \theta_{s,0} \) Angle between the incident wave vector and the scattered wave vector
\( \rho \) Density
\( \sigma_u \) Standard deviation of streamwise velocity fluctuations
\( \sigma_v \) Standard deviation of lateral velocity fluctuations
\( \tau \) Time delay between the two successive images
\( \tau_p \) Time delay of the peak in the pressure fluctuation space-time correlation
\( \tau_s \) Shear stress

Subscripts

1 High speed free stream
2 Low speed free stream
d Delayed image
i Initial image
i/d Initial or Delayed image
CHAPTER I.
INTRODUCTION

Compressible free mixing layers have undergone much study in recent years due to interest in an air-breathing hypersonic vehicle. The scramjet (supersonic combustion ramjet) has been proposed as a possible engine to mix and combust the fuel and oxidizer at supersonic speeds. There are many possible configurations of the scramjet [Waltrup, 1987]. Common in all of these proposed configurations is the need to understand how a fuel-rich flow will mix and combust with a supersonic oxidizer flow. It has been known for many years that both incompressible and compressible mixing layers contain highly organized large scale structures which grow as they convect downstream. These large scale structures play an important role in entraining fluid from the free streams into the mixing region. Although much is known about the global characteristics of the mixing layer (i.e. growth rate, velocity profiles, etc.), the formation of these large scale structures, their development, and their effect on the global characteristics of the shear layer are not well understood, particularly when compressibility becomes important. One goal of the present study is to obtain more detailed information about these large
scale structures in compressible mixing layers.

Many techniques are available for studying these large scale structures, including intrusive techniques which introduce probes into the flow field (i.e. pressure transducers, hot-wires, etc.) and nonintrusive techniques such as laser Doppler velocimetry, schlieren photography, and laser sheet lighting techniques in which the scattering or fluorescence from seeded particles or the fluid molecules is collected. Images acquired with laser sheet lighting techniques have generally provided qualitative information, but research is currently underway to develop two-dimensional imaging techniques that yield quantitative information. Some of these techniques use an iodine molecular filter as a central part of the imaging system to either eliminate unwanted background light scattered off of windows and walls or attenuate the scattering from the flow field as a function of its frequency content in order to measure velocity [Komine and Brosnan, 1991, Komine et al., 1991, Meyers and Komine, 1991, Meyers et al., 1992, Miles and Lempert, 1990, Miles et al., 1991, and Miles et al., 1992]. Further development of these techniques is needed before they can be practically applied to flow fields such as the compressible shear layer.

There are two main goals in the present research. The first is to study the characteristics of large scale structures in compressible free shear layers and the effects of compressibility on the structures. The investigation builds upon our previous laser Doppler velocimetry (LDV) and pressure fluctuation measurements which provided statistical quantities to characterize the compressible mixing layers. In order to investigate the large scale structures independently, laser sheet lighting techniques are
developed to not only provide qualitative flow visualizations, but to also provide quantitative information in a two-dimensional plane. This is the second main goal of this research. Following is a background of both compressible free shear layers and the laser sheet lighting techniques utilizing the iodine molecular filter, followed by a presentation of the results and discussion.
CHAPTER II.

BACKGROUND

2.1. FREE MIXING LAYERS

Although many different configurations could be used for studying compressible free shear layers, the planar free shear layer depicted in Figure 1 seems to be the simplest geometry to study large scale structures and their characteristics [Brown and Roshko, 1974]. Two parallel streams of air, with at least one supersonic stream in the compressible case, combine downstream of the splitter plate to form a shear layer which grows in thickness with increasing downstream distance. The subscripts 1 and 2 indicate the higher and lower velocities free streams respectively. The spatial coordinates are \( x \), the streamwise distance from the splitter plate tip, \( y \) the lateral distance from the splitter plate, and \( z \), the spanwise distance with the origin in the center of the tunnel. The shear layer region is defined as the region of fluid containing a higher velocity than the lower speed free stream and a velocity deficit relative to the high speed free stream. The thickness of the mixing layer has been measured in terms
of many different quantities [Brown and Roshko, 1974], similar to the boundary layer. The rate at which these different thickness measures grow with increasing downstream distance is defined as the growth rate of the mixing layer.

In the streamwise direction, there are two regions of interest, the developing region and the fully developed region. The fully developed region is characterized by the following attributes as described by Mehta and Westphal [1986]:

1. The boundary layer from the splitter plate has been fully engulfed in the shear layer.
2. The mixing layer grows linearly.
3. Both mean and turbulence profiles collapse onto a single curve.

This fully developed region of the shear layer has been studied more than the developing region, where these characteristics are not yet achieved.

Birch and Eggers [1972] reported that the compressible free mixing layer experiences a significantly smaller growth rate than its incompressible counterpart. The original notion was that the density gradient across the free shear layer was the cause of this significant decrease in growth rate [Birch and Eggers, 1972], but this was disputed by detailed experimental investigations of heterogeneous incompressible free shear layers conducted by Brown and Roshko [1974]. Recent experimental results by Papamoschou and Roshko [1988] and theoretical results by Bogdanoff [1983] show that compressibility is the main cause of lower growth rates in compressible shear layers. The convective Mach number, $M_c$, a Mach number with respect to a frame of reference
traveling with the average large scale structures in the flow, has been identified as a compressibility parameter. The convective Mach number seems to correlate the effect of compressibility on the shear layer growth rate normalized by its incompressible counterpart. If the two streams of a compressible free mixing layer have the same specific heats ratio, the convective Mach numbers of the two streams become the same and are given by

\[ M_c = \frac{(U_1 - U_c)}{a_1} \quad (1) \]

where \( U_c \) is the convective velocity of the large scale structures propagating with the flow and is given by

\[ U_c = \frac{(a_1 U_2 + a_2 U_1)}{(a_1 + a_2)} \quad (2) \]

where \( U_1 \) and \( U_2 \) are the free-stream velocities and \( a_1 \) and \( a_2 \) are the speeds of sound of the faster and slower streams, respectively [Papamoschou and Roshko, 1988]. Equation 2 is derived by assuming that there is a stagnation point with respect to a frame of reference moving with the large scale structures, and that streamlines from the two free streams stagnate isentropically at the stagnation point. Recently this assumption has been slightly modified to include cases where there could be losses when this process is not isentropic [Papamoschou, 1989], but in general the isentropic convective Mach
number is used. Since the work of Papamoschou and Roshko [1988], the convective Mach number has been confirmed as the quantity correlating the decrease in growth rate due to compressibility effects [Samimy and Elliott, 1990, Elliott and Samimy, 1990, and Messersmith et al., 1988].

The convective Mach number has also been used to investigate the effect of compressibility on other characteristics of free shear layers. From laser Doppler velocimetry (LDV) measurements in the compressible mixing layer, it was found that as the convective Mach number is increased, both the level and the lateral extent of the turbulence levels are decreased [Samimy and Elliott, 1990 and Elliott and Samimy, 1990]. The turbulence fluctuations have been measured by other investigators studying the compressible shear layer with LDV [Goebel and Dutton, 1991, Barre et al., 1992, Bonnet and Debiisschop, 1992, and Gruber et al., 1992]. Although there are some discrepancies, as in some of the turbulence measurements by Goebel and Dutton [1991], the general trends of decreasing turbulence quantities with increasing $M_c$ has been confirmed by many investigators [Samimy and Elliott, 1990, Elliott and Samimy, 1990, Barre et al., 1992, and Bonnet and Debiisschop, 1992]. A more detailed discussion of the turbulence measurements in compressible mixing layers is given later when the LDV measurements are presented.

One question that the measurements of the turbulence quantities do not fully answer is what is the character of turbulent structures within the mixing layer. Hussain [1986] defines coherent structures in turbulent flows as "a connected turbulent field mass with instantaneously phase-correlated vorticity over its spatial extent"; in other words,
a region of large scale vorticity which has distinct boundaries from the surrounding random turbulent motion. Large scale structures in incompressible planar free shear layers have been studied extensively in the past and only recently have compressible cases received some attention. Brown and Roshko [1974] were among the first to discover large scale coherent structures in an incompressible shear layer. The structures were detected by using gases of different density on either side of the splitter plate and collecting spark shadowgraphs. These structures have been called Brown and Roshko type or roller type structures, having distinct core and braid regions as depicted in Figure 2 [Bernal and Roshko, 1986]. The core regions of these structures were found to interact and pair as they convect downstream. Their dynamics have been reported by many investigators for the incompressible case [Hussain and Clark, 1981, and Hernan and Jimenez, 1982]. Bernal and Roshko [1986] further investigated these roller structures using both schlieren photography and laser-induced florescence in a water tunnel. With plan view schlieren photographs they found that the structures have well defined streaks indicating the presence of streamwise vortices intertwined on the larger spanwise rollers as shown in Figure 2. These streamwise vorticity structures were also observed by Jimenez [1983].

Once these large scale structures were identified in incompressible shear layers, the mechanism of formation of these large scale structures was investigated. The initiation of the large scale structures has been related to Kelvin-Holmhlotz instability waves starting from the tip of the splitter plate [Bernal and Roshko, 1986]. This instability wave is generally associated with upstream disturbances which are amplified
at the most unstable wavelengths as derived from linear stability theory [e.g. Moser and Rogers, 1991]. Winant and Browand [1974] describe the initiation of roller structures from a constant-vorticity layer between two parallel streams. From the initial small amplitude wave, Winant and Browand [1974] suggest that the perturbations induce vertical velocities in this layer which causes the vorticity-containing region to become periodically fatter and thinner. The vortical regions are eventually pinched off and form the well defined roller structures as seen in Figure 2. Many experimentalists in subsonic mixing layers have investigated the effects of forcing [Oster and Wygnanski, 1982], incoming boundary layer state [Browand and Latigo, 1979], and changes to the splitter plate [Dziomba and Fiedler, 1985] on the growth rate of the mixing layer. Structure formation was investigated with numerical simulations by Grinstein et al. [1986].

In general the large scale structures do not simply convect downstream, but evolve as they interact with each other and the free streams while convecting downstream. Figure 3 presents three categories of the evolution of large scale structures as presented by Hussain [1986] and as suggested by many subsonic investigators [Moore and Saffman, 1975, Hernan and Jimenez, 1982, and Hussain and Clark, 1981]. First there is entrainment of free stream fluid (Figure 3a) [Hernan and Jimenez, 1982 and Hussain and Clark, 1981] which causes the large scale structure to grow due to the increase in structure mass. The second process is called pairing [Figure 3b]. The two vortices approach each other and begin to rotate around a common origin as they combine into a single structure [Hernan and Jimenez, 1982]. This can occur with participation of the entire structure or a fraction of a structure
In some forced mixing layers, a multiple of three or more structures have been seen to pair [Ho and Haung, 1982]. Tearing (Figure 3c) is defined as a process in which a structure is torn into two or more parts that become independent structures [Hussain, 1986]. Although one might consider cascading as a process experienced by coherent structures, Hussain [1986] suggests that coherent structures evolve into other coherent structures and are disintegrated into eddies of different scales. This exclusion of cascading as a coherent structure process is contested by other investigators [Moser and Rogers, 1991].

Characteristics of the incompressible large scale structures have been measured by many investigators. First, Brown and Roshko [1974] measured the average distance between structures to be about 2.9 times the vorticity thickness and the lifetime (defined as the time until a structure amalgamates with another) to be about 4.3 times the vorticity thickness. The convection speed of these structures is generally quoted between 0.5 and 0.6 of the average of the free stream velocities [Brown and Roshko, 1974 and Hussain and Clark, 1981]. The convective velocity was also found to change with location within the mixing layer [Hussain and Clark, 1981]. This difference in convective velocity governs the amalgamation of adjacent structures, i.e. faster structures catch up and pair with slower structures. The growth of these large scale structures causes the peak frequency in the power spectrum to decrease with increasing streamwise distance [Hussain and Clark, 1981].

Although there is a wealth of information about the structures in the incompressible mixing layer, studies of the structures and their evolution in the
compressible case are much less common. In the compressible planar free shear layer, Papamoschou and Roshko [1988] showed the presence of large scale structures using schlieren photography. The presence of large scale structures has been confirmed by other investigators [Clemens and Mungal, 1992a, Bonnet and Chaput, 1986, Elliott et al., 1992, and Messersmith et al., 1991]. Due to the spanwise averaging of schlieren photography, planar visualization techniques are more suited to studying structure characteristics, particularly as the flow field becomes more three dimensional. Investigating the structures in a supersonic wake behind a flat plate, Bonnet and Chaput [1986] used a laser planogram technique by introducing ether into one of the boundary layers, thereby making it possible to visualize the structures in the wake. They compared the large scale structures found in their visualizations with earlier correlation data obtained with hot-wire anemometry. Clemens and Mungal [1992a] collected the light scattered from condensed ethanol to investigate the change in structure characteristics for a range of convective Mach numbers in the fully developed region. The existence of distinct core and braid regions was found for convective Mach numbers less than 0.50, and there was an increase in three dimensionality for higher convective Mach numbers. Messersmith et al. [1991] also used condensed ethanol particles for visualizations of the shear layer. Two dimensional covariance fields were calculated for the purpose of investigating structure size, angle, and eccentricity. Planar laser induced fluorescence from nitric oxide seeded into the flow was used by Clemens et al. [1991] to investigate the scalar mixing of the structures. Statistical characteristics of mixture fractions such as mean, RMS fluctuations, and PDF’s were computed. For the lower
convective Mach number ($M_c = 0.28$) the RMS fluctuations were about 15% higher and the PDFs were typically broader than the higher convective Mach number case ($M_c = 0.62$). This suggests the larger scale structures of the $M_c = 0.28$ shear layer are more dominant than those in the $M_c = 0.62$ shear layer.

Another way that experimentalists have investigated large scale structures in compressible mixing layers is through the use of high frequency response pressure transducers. Using multiple probes, correlations can be made in space and time. Shau and Dolling [1990] measured the frequency content and space-time correlations from fluctuating pitot pressures (probes aligned with the flow direction) in the developing region of the shear layer at a convective Mach number of 0.38. They found that structures are inclined approximately 35-50 degrees from the flow direction in the higher Mach number side of the shear layer and tend toward 90 degrees toward the lower Mach number stream. The structures in this region of the shear layer appeared to have a spacing of about 1.5-1.8 local shear layer thicknesses. More about pressure fluctuation results will be discussed when measurements taken in the current mixing layer are presented.

Since one motivation for research into compressible mixing layers is the application to mixing fuel and air in combustion systems, before leaving the background section ways of enhancing this mixing process should be reviewed. One of the first attempts at enhancing mixing was by Papmoschou [1992], who attempted to introduce streamwise vorticity through a "saw tooth" splitter plate and also by using small vortex generators at the end of the splitter plate. Schlieren pictures showed little if any effect
on the shear layer growth rate. Various experiments have been preformed in which a shock wave is sent into the shear layer, but the effects were always shown to be local to the point of impingement and did not result in a significant increase in growth rate after the flow returned to a fully developed profile [Papamoschou, 1992]. Even when an object such as a cylinder is placed spanwise into the mixing region, creating a bow shock on the supersonic side, it had relatively little effect on the growth rate in the fully developed region [Samimy et al., 1992a]. Clemens and Mungal [1992b] introduced an oblique shock slightly upstream of the splitter plate that increased the surface area between the mixing layer and either the free stream by setting up stable indentations in the shear layer, but the size of the large scale structures or the thickness of the mixing region did not seem to increase. One technique that was investigated by McLaughlin et al. [1992] was to introduce a periodic disturbance through glow discharge excitation in a low Reynolds number mixing layer. Preliminary results seem to indicate the ability to control the formation of the large scale structures, but the effect on the growth rate and ability to control the structure formation at high convective Mach numbers or high Reynolds numbers is still uncertain. Yu et al. [1993] investigated passive control of the large scale structures in a compressible mixing layer using a cavity which would amplify the acoustic noise at a resonant frequency associated with the wave length of the structures. Although this was shown to be effective in an axisymmetric jet it is uncertain if it would be as effective in the two dimensional case.

Turning our attention to computational results, both stability analyses and direct numerical simulations have been used to study the compressible free shear layer.
Stability analyses by Ragab and Wu [1989] indicated that the normalized maximum amplification rate of disturbances showed similar dependence on convective Mach number as the experimental growth rate. They also showed, along with Sandham and Reynolds [1991], that the oblique instability waves have larger growth rates than the two dimensional waves at higher convective Mach numbers. This would suggest that at higher convective Mach numbers the large scale structures become more three dimensional. Sandham and Reynolds [1991] proposed the empirical relation given by

\[ \beta = \cos^{-1}\left(\frac{0.6}{M_c}\right) \]  

where \( \beta \) is the spanwise angle of the most amplified wave.

Linear stability analysis was used by Shin and Ferziger [1991] to investigate a compressible reacting mixing layer. They found that three groups of unstable modes exist, one group moving with the average speed of the two flows, and two other groups that are faster and slower than this central group. Heat release was found to stabilize the central mode and destabilize the faster and slower modes. Also, unstable modes were found to be two-dimensional for large heat release, even in highly compressible mixing layers. The effect of the chemical reaction during the instability was found to be unimportant [Shin and Ferziger, 1991]. Planche and Reynolds [1992] confirmed these results and extended the analysis of the compressible reacting mixing layer using direct numerical simulations. They showed that two independent co-layers exist for
reacting mixing layers and are preserved for a relatively long period of time during the transition of the mixing layer to turbulence.

While the stability analyses discussed thus far neglect the effect of physical boundaries, other investigations have considered compressible mixing layers growing within a rectangular channel [Guirguis et al., 1987, Tam and Hu, 1989, Soetrisno et al., 1989, Morris and Giridharan 1991, and Lu and Wu, 1991]. Guirguis et al. [1987] used an inviscid vortex sheet and a flux-corrected transport algorithm to convect the conserved quantities. Contours of density, mass fraction, pressure, vorticity and mixing showed similar organized structures as seen in shadowgraph images of subsonic mixing layers. Further studies by Guirguis [1988] showed that the convective Mach number played an important role in describing the mixing characteristics of the compressible mixing layer. When the convective Mach number becomes greater than unity, interactions between the mixing layer instabilities and the channel acoustics produce new instability modes [Tam and Hu, 1989, Soetrisno et al., 1989, and Lu and Wu, 1991]. By properly selecting the aspect ratio of the channel, Morris and Giridharan [1991] suggested that the instability growth rates could be maximized. Although the inviscid linear instability analysis shows general trends rate that are in agreement with experimental results (particularly growth), one should realize that nonlinear and viscous effects are still important in describing the behavior of compressible mixing layers.

Two dimensional direct numerical simulations by Lele [1989] showed the evolution of vorticity contours for a low Reynolds number compressible mixing layer including roll up and pairing of large scale structures. For a convective Mach number
of $M_e = 0.8$ eddy shocklets were present, but were not found to exist in three dimensional studies [Sandham and Reynolds, 1991]. Three dimensional direct numerical simulations with random initial conditions reported by Sandham and Reynolds [1991] have demonstrated the onset of oblique structures as compressibility increases. Structures represented by three dimensional pressure contours appear to be oblique in both the spanwise and streamwise directions. Large-eddy simulations of an $M_e = 0.4$ mixing layer by Ragab et al. [1992] have shown temporal evolutions of the compressible planar shear layer which indicated the presence of streaks of streamwise vorticity interacting with the familiar spanwise vorticity structures. Although the large scale structures were dominated by Kelvin-Helmholtz rollers, the presence of nonuniform spanwise vorticity caused the structures to pair in localized regions, creating A-shaped structures.

2.2. FILTERED RAYLEIGH/MIE SCATTERING TECHNIQUES

Visualization techniques have been used by investigators to get a qualitative sense of the flow field. Supersonic flows have traditionally been visualized using schlieren, shadowgraph, or interferometry techniques. Unfortunately, these techniques make it difficult to evaluate details of the turbulent flow field due to the inherent line-of-sight averaging of the density effects. Laser sheet lighting techniques, however, make it
possible to view the flow field in a thin plane. The collected signal can be from the fluorescence of molecules naturally present in the flow or artificially seeded [e.g. Clemens et al., 1991, and Hanson et al., 1990]. In scattering techniques, Mie scattering [Bonnet and Chaput, 1986, Messersmith et al., 1991, and Clemens and Mungal, 1991, and Komine et al., 1991], or Rayleigh scattering [Miles, et al., 1991, Smith et al., 1991, and Fourguette, et al., 1990] from small particles or molecules within the flow field is collected. One problem often encountered in scattering techniques is the presence of background noise from the light scattered off of walls or windows. This situation is especially troublesome in Rayleigh scattering techniques because of the low level of the scattered light intensity. One way to eliminate this background noise is through the use of a molecular absorption filter placed in front of the collecting optics. The molecular absorption filter is simply an optical cell containing a gas that absorbs at the frequency of the incident light. Molecular filters have not only been used to eliminate background noise, but also to obtain velocity and temperature measurements using the frequency discrimination characteristics of an absorption filter. The broad category of techniques that collect scattering in the Rayleigh or Mie scattering regime and use molecular absorption filters to modify the collected signal (by eliminating background scattering or velocity discrimination) will therefore be termed filtered Rayleigh/Mie scattering (FRMS) techniques in the following discussions. Following is a description of the use of these molecular filters in flow visualization systems.

One of the first uses of the molecular filter was by Shimizu et al. [1983] who used them as blocking filters in atmospheric measurements of temperature and pressure.
They proposed the use of molecular filters to eliminate the Mie scattering from particles in the atmosphere which have a much narrower linewidth than the temperature broadened signal of molecules. Miles et al. [1991] extended the use of molecular iodine filters to flow visualizations in which a Nd:YAG laser at 532 nm was used as the light source, a technique they called filtered Rayleigh scattering (FRS). In FRS flow visualizations, there are two main sources of scattering. The first source is the Rayleigh\Mie scattering from particles or molecules in the flow field, which possess a Doppler shift because of the flow velocity. The magnitude of the Doppler shift is given by

\[ \Delta f_D = (k_\lambda - k_0) \cdot V \]  

where \( k_\lambda \) is the observed unit light wave vector, \( k_0 \) is the incident unit light wave vector, \( V \) is the flow velocity vector, and \( \lambda \) is the wavelength of the illuminating light. This equation can be written in terms of the angles shown in Figure 4 as

\[ \Delta f_D = \frac{V}{\lambda} [\cos(\phi_D)\cos(\gamma_D) - \cos(\theta_D)] \]  

The second main source of collected scattering is unwanted scattering from the windows and walls bounding the test section, which experiences no Doppler shift. Figure 5a shows these two components of the collected scattering in the frequency domain. In the
figure, the illuminating laser light appears with a narrow linewidth. The scattering collected from a particle in the illuminated plane is Doppler shifted away from the laser wavelength. This situation is also illustrated in Figure 5b, where a hypothetical absorption profile has been superposed onto Figure 5a. For background suppression, an optically thick filter at low pressure (dominated by temperature broadening), similar to that of Figure 5b is used. Miles et al. [1991] used this type of filter to collect the scattering from condensed ice particles marking the structures of a supersonic boundary layer. Not only were they able to cut out background light, but also demonstrated that the average velocity in a supersonic flow can be measured by tuning the laser frequency, thereby moving the Rayleigh scattered spectrum in and out of the absorption filter. Shirley and Winter [1993] have presented a velocity and density measurement system using a molecular filter similar to FRS. The proposed system would utilize an anamorphical optical system which allows the optical rays from different scattering angles, thus different Doppler shifts, to be recorded separately. This promises to eliminate the need to tune the laser or to use multiple cameras, but at a cost of obtaining measurements only along a line instead of in a two dimensional plane.

Komine et al. [1991] used an iodine molecular filter whose absorption profile exhibited more gradual slopes on either side of the actual molecular iodine transition frequency (relative to the sharp cut-off slopes of the thermally-broadened profile) to obtain velocity measurements in a seeded jet using a multiple camera system. The seeded particles provided scattering in the Mie scattering regime. The role of the filter in this case was not to suppress the background noise, but rather to attenuate the
collected intensity as a function of frequency, so that the amount of Doppler shift present in the signal could be obtained. The zero velocity shift condition (laser frequency) was tuned to the 50% absorption position as illustrated in Figure 6. A positive Doppler shift resulted in a relative increase in intensity and a negative Doppler shift resulted in a decrease in the recorded intensity. A second camera with no filter was used to normalize the discriminated signal, allowing the frequency of the narrow linewidth Mie scattering to be determined from the known absorption profile. This process allows the Doppler shift, and thus the velocity, to be determined at the locations of the seeded particles. The current FRMS imaging system will use similar ideas presented here for both background suppression and velocity measurement.
CHAPTER III.

EXPERIMENTAL ARRANGEMENT

Following is a description of the high Reynolds number dual-stream wind tunnel at the Aeronautical and Astronautical Research Laboratory (AARL) of The Ohio State University used in the current experiments. This wind tunnel has been used in investigations of reattaching shear layers [Samimy and Abu-Hijleh, 1988] and is currently also being used to investigate the effect of expansion on supersonic boundary layers [Arnette et al., 1993a and Samimy et al., 1994]. A jet facility which was used for feasibility studies of the optical techniques and also for underexpanded jet studies [Arnette et al., 1993b] is also described. Also, the LDV and pressure fluctuation measurements systems are discussed. Finally, the optical systems that have been used in schlieren photography and the FRMS techniques are presented.
3.1. COMpressible Planar Free mixing Layer Facility

A schematic of the air delivery system for the dual stream supersonic blowdown facility is given in Figure 7. At AARL, the air is provided by two four-stage compressors and stored in two storage tanks with 42.5 m³ capacity at a maximum of 164 MPa. Dessicant dryers reduce the moisture content of the air to almost immeasurable levels. The first gate valve is used to set the mass flow rate for both streams and the second globe valve is used to properly divide the flow between the supersonic and subsonic streams. A pneumatic-controlled ball valve is used to turn the flow on and off. This system allows the stagnation pressure of the top flow to be maintained within ±2% of the set point.

A photograph of the dual stream supersonic blowdown wind tunnel is shown in Figure 8 and an internal schematic is shown in Figure 9. Air is introduced into the settling chamber through a pipe with radial holes followed by a screen and flow straightener assembly. A 3.18 mm thick steel splitter plate with a machined angle on the subsonic side of approximately 1° over a 125 mm length and a flat profile on the supersonic side separates the two flows. The trailing edge of the splitter plate has a thickness of about 0.5 mm. The upper stream is always supersonic with two interchangeable converging-diverging nozzles with nominal Mach numbers of 2 and 3. The bottom stream has a converging nozzle and is always operated subsonically. The streams merge downstream of the splitter plate to form a planar shear layer in the 152.4 x 152.4 mm test section. Optical access is provided by windows in the side, top, and
bottom walls, with possible side viewing areas 80 mm high and 450 mm long, and top and bottom viewing areas 30 mm wide and 300 mm long. All glass windows were set in frames allowing them to be easily replaced when scratched or damaged. For flow visualizations requiring moist air, the bottom free stream can be open to ambient air in the stagnation chamber.

Pressure measurements are made from static pressure taps on the top, bottom, and a removable plexiglass side panel which can replace the window section. The side panel contained static pressure taps in the shear layer region. An access plug can be removed from the top panel to place a pitot pressure probe within the supersonic and subsonic free streams. Pressure data was measured sequentially by a 48 port scanivalve and saved on a Harris H800 mainframe computer. Flow conditions (i.e. stagnation pressure settings) were selected for each case by adjusting the top and bottom control valves to achieve the most constant streamwise pressure profile. The streamwise variation of the static pressure within the test section was less than 6% for all cases. It should be noted that due to the high settling times of the low static pressure in the higher Mach number case only a few measurements were made. Therefore the uniformity of the shear layer was also checked using schlieren photographs to make sure there were no strong shock or expansion waves which would distort the mixing layer.
3.2. AXISYMMETRIC JET FACILITY

For the development and feasibility studies of the employed FRMS techniques, an axisymmetric jet facility which is shown in Figure 10 was utilized. This facility was used by Arnette et al. [1993b] in the study of the structure of high Reynolds number supersonic jets. The jet facility has two advantages. First, the jet can be run continuously so that adjustments can be made to the optical system while the flow is running. Secondly, the supersonic jet can be heated to avoid condensation which usually results when the supersonic air mixes with the moist ambient air. Thus experiments can be performed using molecular Rayleigh scattering. Jet nozzles are available from converging to a converging-diverging Mach 2 nozzle, all with a throat diameter of 19 mm. The stagnation chamber can be run up to 1.72 MPa, resulting in an equivalent Mach number (Mach number which would result in an isentropic expansion for the given pressure ratio) of 2.5. The air supply and control was tapped from the existing system used for the supersonic tunnel discussed above. More information on this facility is given by Arnette [1992].

3.3. FLUCTUATING PRESSURE MEASUREMENT SYSTEM

Endevco fast response pressure transducers (model 8514-20) were mounted perpendicular to the flow direction as shown in Figure 11. The pressure sensitive
diaphragm is 0.7 mm in diameter. These transducers have a sensitivity from 0 to 20 psi and an estimated frequency response better than 60 kHz, which is higher than the expected fluctuations from large scale structures of the mixing layer cases studied. The probes could be separated in the spanwise direction from 3.5 to 40 mm or in the streamwise direction from 3.5 to 140 mm. Most measurements were made in the subsonic portion of the flow to avoid the formation of shockwaves, although it is believed that the signal from large scale structures is not significantly altered by weak shocks. An Ectron model 563F signal conditioner provides the excitation voltage and amplification of the output signal from each transducer. The signal was filtered with two analog filters (Krone-Hite model 3202 and Ithaco 4302) at a high-pass cut-off of 800 Hz and a low-pass cut-off of 63 kHz. The filtered data was recorded and reduced on a Masscomp series 5520 computer with an analog-to-digital converter that allowed a sampling rate of 250 kHz per channel. The data was analyzed in 100 blocks of 1024 samples per block. In addition, the data could be immediately analyzed by a model 660B Nicolet Scientific Corporation dual-channel FFT analyzer to verify the computations. More information about this system is given by Reeder [1991] which gives a more complete accounting of the fluctuating pressure measurements than is presented here.
3.4. LASER DOPPLER VELOCIMETRY SYSTEM

A two-component coincident laser Doppler velocimetry system was used to collect velocity data in the current experiments. The laser was a Spectra-Physics model 2020 5-watt argon-ion laser. The two beams used were the blue (488 nm) and green (512.5 nm), which are the most powerful of all the frequencies. The components of the laser were aligned $\pm 45^\circ$ relative to the mean flow direction to reduce the velocity measured and thus to improve the frequency counter clock resolution. A TSI 9182-3A frequency shifter set at 40 MHz was used to reduce fringe bias and enable different flow directions to be detected. The measurement volume diameter and length are 0.13 mm and 0.90 mm respectively, as set by the transmitting optics.

The receiving optics were set in a 10-degree off-axis forward-scatter mode to reduce the measurement volume. Two TSI 9162 photomultiplier tubes recorded the intensity, which was converted to velocity by two TSI 1900C frequency counters. Transmitting and receiving optics were placed on an optical table which was controlled by a stepper motor. The transmitting and receiving optics were mounted to the rigid optical table, which maintained proper alignment. The data was stored and analyzed using a personal computer in samples of 2048 points. The results are corrected for velocity bias by using the inverse of total instantaneous velocity as a weighting factor, which has been shown to be a suitable technique for non-zero flow fields [Petrie et al., 1988].

Both top and bottom streams were seeded with atomized silicone oil introduced
into the settling chamber. Due to the high stagnation pressures the LDV seed generator (Model 9306 TSI) had to be enclosed in a pressure vessel since it is only designed for 413 kPa. The particles used to measure the velocity are less than a micron in diameter [Samimy and Abu-Hijleh, 1989] and have been shown to sufficiently represent the instantaneous velocity in the mixing layer. [Samimy and Lele, 1991]

3.5. PULSED LASER AND VISUALIZATION SYSTEM

The light source used in the FRMS experiments is a Quanta Ray GCR-4 frequency doubled (532 nm) Nd:YAG laser which is injection seeded to provide a narrow line width (≈ 50 MHz). The injection seeder has been modified to enable tuning of approximately 100 GHz by applying a bias voltage from a digital to analog converter board on a personal computer. Another feature added to the laser is a double-pulse option which allows two pulses to be generated, one slightly delayed from the other with a controllable interval from 15 μs to 200 μs. The delay was measured using a photodiode connected to an HP 54501 digital oscilloscope. The Nd:YAG laser has a pulse width of 9 ns and a maximum pulse rate of 10 Hz. The laser can provide a maximum energy of 660 mJ/pulse when frequency doubled. The beam from the laser is directed to the test section through a set of prisms, and the sheet of light is made by a combination of spherical and cylindrical lenses as shown in Figure 12. The thickness of the sheet is controlled with a 1 m focal length spherical lens which produces a beam
waist about 60 mm long and a sheet thickness of approximately 0.1 mm. The cylindrical lens expands the beam into a sheet and controls the width of the sheet, which varies depending on the focal length and distance of the cylindrical lens from the test section. TTL outputs on the laser enable the laser pulse to be synced with the CCD cameras.

Two different recording systems are available to capture and store images. The first is an ITT 8-bit double intensified CCD camera with an RS-170 video output which can be stored on a Panasonic S-VHS recorder. It has a 483 by 378 array with a luminous gain adjustable from 0 - 10^6 and a threshold sensitivity of 5.0 x 10^-8 foot-candle. The second system is a Princeton Instruments intensified CCD camera with a 14-bit digital output stored on a 486 personal computer. Cooling provided to this camera decreases the background signal, enabling it to have a sensitivity on the order of 1 to 100 counts/photoelectron. It has a 576 by 384 array which can be binned or disabled at selected areas decreasing the frame rate of 2.2 seconds/image for the full array. Since the camera has a frame rate much slower than the laser pulse rate, the camera controller does not send a pulse to gate the camera until the previous image has been stored on the computer. Two of the Princeton Instrument systems are available. Both camera systems, ITT and Princeton Instruments, are synchronized with the laser through a pulse generator which controls the delay and pulse width. Figure 12 shows the complete laser and Princeton Instrument system when operated in the double-pulsed mode. The single-pulse configuration is similar with the double pulse generator removed from the laser.
Figure 13 shows a schematic of the schlieren photography system used for confirming the straightness of the shear layer and also for initial studies of the existence of large scale structures in the mixing layer. A Xenon model 437B lamp provided a light pulse of 20 ns at a peak power of 500 kW. The parabolic mirrors are approximately 250 mm in diameter providing a full view of the flow in a single image. The collimated beam was passed through the test section and focused on a frosted glass plate. The images were recorded onto S-VHS tape using the ITT intensified CCD camera. It should be noted that the sensitivity of schlieren systems as controlled by the knife edge can dramatically affect the appearance of the flow field. The knife edge was varied throughout the run and images containing the best visualization of the structures are presented.
CHAPTER IV.

EXPERIMENTAL RESULTS AND DISCUSSION

The following section presents the results obtained in this study of compressible mixing layers. First is a presentation of the incoming and mean flow conditions of the mixing layers. Next is a discussion of mean and turbulent measurements made using the previously discussed LDV system. Although some of these measurements have been presented previously [Elliott, 1989], a few of the LDV results are presented to serve as a base line for the mean and turbulent quantities measured in this study. This is followed by a presentation of multi-point fluctuating pressure measurements. The main body of the experimental results section will present both qualitative and quantitative flow visualizations, including a discussion of the molecular filter based diagnostic systems employed.
4.1. INCOMING AND FREE STREAM FLOW RESULTS

Two perfectly expanded compressible free shear layers were investigated by changing the top supersonic nozzle and setting the bottom flow so as to equalize the pressure across the shear layer. A third convective Mach number was achieved by running the supersonic stream in an under expanded condition and making measurements upstream of where the expansion waves reflect back into the mixing layer. This underexpanded case was only investigated using LDV. The convective Mach numbers for these cases have been determined to be 0.51 (Case 1), 0.64 (Case 2), and 0.86 (Case 3) with incoming flow parameters given in Table 1 and free stream flow parameters given in Table 2. Note that the higher supersonic Mach number for the $M_c = 0.64$ mixing layer relative to the $M_c = 0.51$ mixing layer is due to the fact that the flow has passed through an expansion fan at the tip of the splitter plate.

For both incoming flow conditions the turbulent boundary layers have been found to be fully developed. Figure 14 presents the profiles of the mean streamwise velocity and turbulence intensity (standard deviation of velocity fluctuations divided by the velocity), and the correlation coefficient, $R_{uv}$, for the supersonic turbulent boundary layers. The turbulence intensity profile with approximately 7.5% maximum intensity level at $y/\delta = 0.3$ is similar to both subsonic and supersonic turbulent boundary layer profiles reported by many other investigators [Samimy et al., 1986, and Smits et al., 1988]. The correlation coefficient profile is similar to subsonic and supersonic boundary layer results. The magnitudes of the correlation coefficient in the lower part
of the boundary layer agree with other supersonic boundary layers, but are approximately 15% lower than subsonic values [Samimy et al., 1986, Smits et al., 1988]. The decrease in the correlation coefficient in the upper half of the boundary layer for increasing Mach numbers has been reported earlier [Smits et al., 1988]. Work is currently underway to investigate at these boundary layers in more detail [Arnette et al., 1993a].

4.2. LASER DOPPLER VELOCIMETRY MEASUREMENTS

One of the global mixing layer characteristics that is significantly affected by compressibility is the normalized growth rate. As mentioned previously, Papamoschou and Roshko [1988] proposed the convective Mach number (as defined in Eqn. 1) as a parameter that correlates the reduction of normalized growth rate, which is defined as the growth rate of the mixing layer normalized by the growth rate of an incompressible mixing layer having the same velocity and density ratios. The incompressible growth rate, \([db/dx]\), given by Papamoschou and Roshko [1988], is of the form

\[
\left[\frac{db}{dx}\right] = C_0 \frac{(1-r_U)(1+s'^2)}{1+r_U s'^2}
\]

where \(r_U\) is the velocity ratio \((U_2/U_1)\), \(s\) is the density ratio \((\rho_2/\rho_1)\), and \(C_0\) is a
coefficient which depends on the measurement technique used to define the thickness \( b \). For thicknesses based on pitot measurements \( C_0 = 0.17 \). For visual thickness based on schlieren measurements \( C_0 = 0.14 \) as proposed by Papamoschou and Roshko [1988] and used by Clemens [1991]. Goebel and Dutton [1991] used \( C_0 = 0.083 \) for thicknesses based on velocity profiles where the thickness is measured from the velocity profiles between \( U^* = 0.1 \) and 0.9 and \( U^* \) is given by

\[
U^* = \frac{U - U_1}{U_1 - U_2}
\]  

(7)

Figure 15 shows the normalized growth rate as a function of convective Mach number from various experiments. As can be seen there is a significant amount of scatter in these results, but the trend of decreasing growth rate with increasing convective Mach number is evident, as noted by several investigators [Goebel and Dutton, 1991, Messersmith et al., 1988, and Clemens and Mungal, 1992a]. It is possible that this scatter is due to the selection of \( C_0 \). This is supported in the results of the present study by the differences in the nondimensional growth rate for the same convective Mach number and experimental arrangement using different techniques to define the thickness. It should be noted that using the same \( C_0 \) for the LDV results as for the visual or pitot thickness would bring the points measured with LDV much closer to those measured with visual or pitot thicknesses. The scatter could also be due to differences in thickness definitions and experimental uncertainties. A point worth noting is that in
defining thicknesses based on velocity or pitot pressure profiles, the small vertical
gradients of the measured parameter near the edges of the mixing layer make it difficult
to accurately locate the position where the velocity or pitot pressure reaches a certain
value. Other thickness definitions such as the momentum thickness defined by

\[ \theta = \int_{-\infty}^{+\infty} \frac{\rho}{\rho_1} U^*(1 - U^*) dy \] (8)

or the vorticity thickness defined by Brown and Roshko [1974] as

\[ \delta_\omega = \frac{U_1 - U_2}{(\partial U/\partial y)_{\text{max}}} \] (9)

use a more significant portion of the measured velocity profile to define the thickness.
Figure 16 shows the normalized vorticity thickness growth rate as a function of
convective Mach number from various experiments as compiled by Bogdanoff [1983].
To nondimensionalize the compressible vorticity growth rate Bogdanoff [1983] used the
relation

\[ \frac{d\delta_\omega}{dx - x_0} = 0.181 \left[ \frac{(U_1 - U_2)}{2(U_1 + U_2)} \right] \] (10)
which is the incompressible vorticity growth rate as given by Brown and Roshko [1974], where $x_0$ is the streamwise location of the virtual origin. Figure 16 again displays the trend of decreasing growth rate with increasing convective Mach number, with the present results showing good agreement with those of other investigators. As noted by Bogdanoff [1983], the incompressible growth rate does not contain the effect of freestream density ratio which would decrease the displayed normalized growth rates by an additional 20%.

As mentioned in the background, both incompressible and compressible mixing layers are separated into at least two streamwise regions, the developing region and the fully developed (or linear growth rate) region. One basic question is where does the fully developed region start. Figure 17 shows the mean velocity profiles for a convective Mach number of 0.51 at various downstream locations. The lateral scale is non-dimensionalized by the vorticity thickness ($\delta_\omega$) and $y_{0.5}$, which is the lateral location where $U = (U_1 + U_2)/2$. As can be seen, this vorticity thickness similarity parameter collapses the velocity profiles for $x \geq 120$ mm, although the subsonic side may collapse earlier. Although many investigators have used the collapse of the mean velocity profiles as a sufficient condition to define the beginning of the fully developed region, it will be seen that the collapse of the turbulence profiles is delayed until further downstream.

Another condition that must be met according to Mehta and Westphal [1986] is that the turbulence profiles must collapse. Figures 18 - 20 show the development of the streamwise and lateral turbulence intensities and the Reynolds stress, respectively.
Turbulence intensity is defined as the standard deviation (\(\sigma\)) of either the streamwise or lateral velocity fluctuations normalized by the velocity difference of the two free streams. From Figure 18 it can be seen that the streamwise turbulence intensity decreases from approximately 19% to 16.5% as the flow becomes fully developed. Also it is interesting to note that the lateral extent of the turbulence seems to decrease as the shear layer proceeds to a fully developed state, especially for the lower velocity side. Figure 19 shows the same decreasing trends in the lateral extent as the mixing layer approaches self similarity, but the maximum levels are approximately constant as the flow becomes fully developed. Incompressible shear layers, such as the unforced cases of Oster and Wygananski [1982] have similar trends in turbulence level and lateral extent as the flow becomes fully developed. The Reynolds stress profiles (Figure 20) show a decrease in level and lateral extent of the turbulence (mostly large-scale fluctuations) within the shear layer as it becomes fully developed. This is similar to the subsonic case of Hussain and Husain [1980] who attribute this to the breakdown of large-scale structures following roll-up and pairing processes.

All of these turbulence profiles seem to collapse for \(x \geq 150\) mm for \(M_c = 0.51\) thus indicating that the fully developed region is further downstream than would be indicated by the collapse of the mean velocity profiles only. Similar profiles have been taken for the \(M_c = 0.86\) case which show that this mixing layer is fully developed for \(x \geq 180\) mm. Some investigators have reported the beginning of the fully developed region in terms of the number of incoming boundary layer thicknesses measured from the splitter plate tip. For the current experiments the initiation of the fully developed
region in terms of the incoming boundary layer momentum thickness for the $M_c = 0.51$ and 0.86 cases are $x/\theta = 242$ and 486, respectively. The location of self-similarity has been proposed to be anywhere from 350 to 1000 momentum thicknesses for incompressible mixing layers [Birch and Eggers, 1972, Mehta and Westphal, 1986, and Oster and Wygananski, 1982]. Chinzei et al. [1986] suggested that the self-similar location moves farther downstream as the velocity gradient across the shear layer increases. Goebel and Dutton [1991] reported that the turbulence profiles collapsed approximately 600-1750 momentum thicknesses from the splitter plate tip for a convective Mach number from 0.20 - 0.99, apparently increasing with increasing velocity gradient. The uncertainty of the initiation of the self-similar region has been suggested as a cause of scatter in the experimental determinations of the spread rate parameter and growth rate [Birch and Eggers, 1972].

Figure 21 shows the mean velocity profiles in the fully developed region for three convective Mach numbers. It can be seen that the vorticity thickness based similarity parameter not only collapses the profiles for various streamwise locations, but also for the convective Mach numbers studied in this current investigation. Figure 22 gives the mean velocity profiles from other investigators using the vorticity based similarity parameter. Since the profiles collapse, only one curve is plotted from the current experiments representing the average of all fully developed streamwise locations for that convective Mach number. One can fit a hyperbolic tangent curve to these profiles which is given by
The collapse of mean velocity profiles with different convective Mach numbers and even different investigators indicate that the vorticity thickness is the correct scaling parameter to determine the effects of convective Mach number on the turbulence characteristics.

Figure 23 shows the streamwise turbulence fluctuation profiles for the present experiments and the incompressible results of Oster and Wygnanski [1982] in the self-similar region. Compressibility is seen to reduce the maximum fluctuations, which occur around \((y-y_{0.5})/\delta_w = 0\) for both compressible and incompressible flows. The maximum fluctuations for incompressible flows, shown in Figure 23, is about 18.5%, very close to values reported by other investigators [Mehta et al., 1987 and Wygnanski and Fiedler, 1970]. The maximum drops to 16.5% for \(M_c = 0.51\), 15% for \(M_c = 0.64\) and 13% for \(M_c = 0.86\). In any given lateral location, especially in the high speed side of the flow, the intensity level increases as the convective Mach number decreases and it seems that the incompressible results define the upper limit of the fluctuations. Another way of viewing this is to say that the lateral extent of the streamwise turbulence decreases with increasing convective Mach number (i.e. less of the thickness of the shear layer is highly turbulent as the convective Mach number increases). Figure 24 shows the lateral turbulence fluctuation with trends similar to those of the streamwise turbulence fluctuations. The lateral turbulence fluctuations of
the incompressible case are much higher throughout the shear layer than the compressible cases presented here. A check with incompressible turbulence measurements from Mehta and Westphal [1986] confirms the turbulence levels presented by Oster and Wygnanski [1982].

Figure 25 shows the Reynolds stress distributions for the present experiments and Oster and Wygnanski's [1982] incompressible results for a velocity ratio of 0.6. The difference between the incompressible and compressible results are similar to those of the lateral turbulence intensity case. The maximum Reynolds stress value for \( M_c = 0.51 \) is about 40% lower than the incompressible result, with the more compressible results decreasing further. Figures 23 - 25 show a general trend that the level and lateral extent of both small and large scale fluctuations decrease with increasing convective Mach number in the high speed side of the flow. The maximum values of these turbulence results are plotted on Figure 26. It is interesting to note that the maximum levels of the turbulence fluctuations and Reynolds stress decrease almost linearly with increasing \( M_c \). Only a few turbulence fluctuation studies exist for compressible mixing layers that permit a comparison of the trends of decreasing level and extent of turbulence fluctuations in other facilities and at higher convective Mach numbers. Similar trends have been reported by Barre et al. [1992] and Bonnet and Debisschop [1992]. Goebel and Dutton [1991] investigated compressible mixing layers with convective Mach numbers ranging from 0.20 - 0.99 using LDV. Although the trends for lateral turbulence intensity and Reynolds shear stress are similar to those presented here, the streamwise turbulence intensity is relatively constant for convective
Mach numbers greater than 0.46. This discrepancy might be due to the fact that their test section is much smaller, making the mixing layer more sensitive to the effects of wall boundary layers and reflecting waves.

It may seem surprising that the turbulence properties in the self-similar region do not collapse, especially the Reynolds stress since it can be calculated (with some approximations) from the mean velocity profiles which do collapse relatively well. Lumley [1986] investigated a similar paradox in thermal mixing layers with the relationship between the temperature profile and the heat flux (mathematically similar to the velocity profile and Reynolds stress). He found that a very small fourth order component of temperature variance has a surprisingly large effect on the maximum value of the heat flux. In a supersonic boundary layer with large distributed surface injection, Fernandez and Zukoski [1969] calculated the shear stress distribution from the velocity profile. The analysis is similar for the present two-stream planar shear layer. With a frame of reference moving with the bottom stream at a velocity $U_2$, the two-dimensional continuity and momentum equations in a zero pressure gradient self similar region are given as:

$$\frac{\partial (\rho U^*)}{\partial x} + \frac{\partial (\rho V^*)}{\partial y} = 0$$ (12)

$$\frac{\partial (\rho U^{*2})}{\partial x} + \frac{\partial (\rho V^* U^*)}{\partial y} = 0$$ (13)
Integrating both equations and combining them with the assumptions used by Fernandez and Zukoski [1969] for a self similar flow field [that in the self similar region \( \rho(U - U_2)/\rho_1(U_1 - U_2) = f(\dot{y}/\theta) \) and that the entrainment rate is approximately equal to the momentum thickness growth rate \( (d\theta/dx) \)], an equation is obtained for the shear stress

\[
\frac{\tau_s(\dot{y})}{\rho_1(U_1 - U_2)} = \frac{d\theta}{dx} [U^* - U^* \int_0^{\dot{y}_0} \frac{\rho}{\rho_1} U^* d(\frac{\dot{y}}{\theta}) + \int_0^{\dot{y}_0} \frac{\rho}{\rho_1} U^* d(\frac{\dot{y}}{\theta})]
\]  

(14)

where \( \dot{y} \) is the lateral location of any given point in the shear layer to be calculated.

Figure 27 shows the shear stress calculated from the velocity profiles for the three convective Mach number cases. The trend that the level and extent of the shear stress decrease with increasing convective Mach number is consistent with the direct Reynolds stress measurements (Figure 25). Investigating equation 14, the shear stress basically equals the momentum thickness growth rate multiplied by a velocity dependent integral quantity which collapses relatively well for different convective Mach numbers. Thus the observed decreasing trends are due mainly to the decrease in \( d\theta/dx \) as the convective Mach number is increased. In an earlier paper [Samimy and Elliott, 1990], it was found that the local momentum thickness in the shear layer can be used to collapse the velocity profiles only if it was multiplied by a convective Mach number dependent parameter. This parameter is believed to cause the trend seen in the calculated shear stress.
In order to have a little more direct comparison of the measured Reynolds stress of Figure 25 and the calculated Reynolds stress shown in Figure 27, the measured results in Figure 25 were multiplied by the density ratio, $\rho/\rho_1$. The mean density profile was determined from the measured mean velocity, static pressure, and stagnation temperature. Figure 28 shows the results. Comparing the results in Figures 27 and 28, good agreement in both profile shape and stress levels is found. It should be noted that in compressible flow the Reynolds stress has two other terms for a two dimensional flow besides the one shown in Figure 28. The other two terms represent density-velocity fluctuation correlations. Following Morkovin’s hypothesis that the effects of density fluctuations on turbulence are negligible if the root-mean-square density fluctuation is small relative to the mean density, these two terms have been generally ignored [Bradshaw, 1977]. Bradshaw [1977] suggests a Mach number of 5 as the upper limit of applicability of Morkovin’s hypothesis for boundary layer and wake flows and 1.5 for jets and mixing layers. A favorable comparison of the results in Figures 27 and 28 seems to indicate that Morkovin’s hypothesis is valid in this type of flow also.

One of the concerns associated with LDV in high speed flows is whether the LDV seed particles will follow the relatively high frequency fluctuations associated with the large eddies. A good comparison between results in Figures 27 and 28 is also a good indication that the silicone oil particles of approximately 1 $\mu$m diameter are following the large scale structures. In fact, recent direct numerical simulations of particles with inertia in compressible shear layers strongly support these findings.

Flatness for the u component is given in Figure 29. The flatness indicates whether there are high probabilities in the PDF profile far away from the mean. Thus it indicates the probability of large fluctuations, relative to the mean. A skewed tail, or the existence of different high probability modes within the PDF profile will result in a high flatness. As usually defined, the value of flatness for a normal distribution is 3. The u component of flatness shows peaks at the edges of the shear layer with a value of approximately 3 in the center and at both free streams. Incompressible shear layers have the same general shape, but peak on the high velocity side at about 8 and on the low velocity side at about 22. Two things are observed from the u component of flatness in the present results. First, if the two peaks indicate the edges of the shear layer, the thickness decreases with increasing convective Mach number. Second, the level of the peak flatness on the high speed (supersonic) side of the shear layer generally decreases with increasing convective Mach number. The decrease could be due to a suppression of the perturbations from large scale structures which extend into the free stream and engulf fluid at the edges of the shear layer. This is in agreement with the smaller growth and entrainment rates observed at higher convective Mach numbers. The v component of flatness shows similar characteristics as those described above for the u component. It should be noted that the higher order moments (skewness and flatness) results are based on a low sample size (2048). Averaging profiles obtained at various streamwise locations might improve this situation.

A brief review of LDV results has been given here, and will be referred to in
the rest of the dissertation as the results concerning the large scale structures in the mixing layer are characterized. A more complete sample of the LDV results can be found in other papers [Samimy and Elliott, 1990], [Elliott and Samimy, 1990] and thesis of Elliott [1989].

4.3. PRESSURE FLUCTUATION MEASUREMENTS

As part of the study of compressible mixing layers at the AARL, there have been extensive pressure fluctuation measurements made using the system described earlier (Section 3.3). Recall that the probes are aligned perpendicular to the flow direction, unlike the pressure fluctuation measurements made by Shau and Dolling [1990] who measured total pressure fluctuations to evaluate the lateral structure angles. A few results of the pressure fluctuation study are presented here, and given in further detail in Elliott et al. [1990], Reeder [1991], and Samimy et al. [1992]. Most measurements were made below the sonic line of the mixing layer to avoid creating shock waves off of the probes. Power spectra, coherence, and two-point space-time correlations were calculated from the measured static pressure fluctuations using the arrangement given in Figure 11. When these probes are aligned in the streamwise direction, one would expect the downstream probe to be affected by the wake of the upstream probe. This effect should become less significant as the separation is increased. As a check, the power spectra was measured at the second probe with and without the first probe in
place. Little if any change was encountered for the cases presented here.

Figures 30 and 31 give the power spectra for the $M_c = 0.51$ and 0.86 mixing layers respectively taken at the bottom edge, $(y-y_c)/\delta_w = -0.47$ of the mixing layer ($y_c$ is the point where the measured convective velocity equals the theoretical as calculated by equation 2) at different streamwise locations. Each spectrum has been normalized by the maximum of that particular spectrum. For the lower convective Mach number the peak is between 9 - 13 kHz and decreases in frequency towards the fully developed region to 4 - 7 kHz. Results are approximately the same in the middle of the mixing layer, but are slightly more broadband. The spectrum for the $M_c = 0.86$ mixing layer again shows a decrease in frequency of the peak from 5 - 15 kHz in the developing region to 2 - 10 kHz in the fully developed region. Note that the portion of the spectrum containing significant energy is much broader for the higher convective Mach number than for the lower convective Mach number. This signifies less temporal organization of the structures at $M_c = 0.86$ relative to $M_c = 0.51$.

The coherence profiles are given for both convective Mach numbers in Figure 32 for the $M_c = 0.51$ and 0.86 mixing layers with the probes located approximately in the middle of the mixing layer. $X_s$ is the streamwise position of the first probe and $dx_s$ is the distance between the two probes. At this streamwise location the vorticity thickness is 17 mm and 24 mm for $M_c = 0.51$ and 0.86 respectively. When $dx_s/\delta_w = 0.5$, the coherence levels are comparable for the two cases. The coherence level drops by about 50% after increasing the probe separation to $dx_s/\delta_w = 4.7$ and 1 for $M_c = 0.51$ and 0.86, respectively. This indicates the existence of a tremendous compressibility
effect on the streamwise evolution of large scale structures. For greater streamwise separations, the breadth of the coherence peak decreases. Blackwelder and Kovasznay [1972] showed this same trend in turbulent boundary layers, interpreting this trend as resulting from only larger scale structures being preserved for larger streamwise separations between the two probes. Figure 33 is a plot of the streamwise space-time correlations of the $M_c = 0.51$ and 0.86 mixing layers for different streamwise separations of the two probes in the middle of the mixing layer. These space-time correlation measurements show compressibility effects similar to the coherence. The secondary negative peaks seen for the lower convective Mach number case (Figure 33a) indicate that the large scale structures are more well defined in comparison with the higher convective Mach number. The results at other lateral locations show this same effect.

Knowing the distance between the two probes, $d_x$, and the time delay of the peak, $\tau_p$, the convective velocity can be calculated from the pressure fluctuation measurements as

$$U_{cp} = \frac{d_x}{\tau_p}$$  \hspace{1cm} (15)

One convenient way to present the convective velocity is to plot it versus the local mean velocity. Figure 34 shows the nondimensional convective velocities at different lateral locations plotted against the mean velocity measured with LDV at that location. If the convective velocity exactly matches the local velocity, the point falls on the line where
U_{cp}^* (the nondimensional convective velocity measured from the space-time correlations) is equal to U^*. The M_e = 0.86 points fall near this line suggesting that the structures convect with the local mean velocity. The lateral variation of average convective velocity has been demonstrated for both compressible [Ikawa and Kubota, 1975] and incompressible mixing layers [Jones et al., 1973 and Hussain and Clark, 1981]. The M_e = 0.51 mixing layer is characterized by higher convective velocities than the local mean velocities, particularly towards the low speed side of the mixing layer. This is probably due to the fact that the large scale structures travel at varying convective velocities, but the pressure transducers are sensitive to more than a single point. A more detailed analysis of the convective velocity has been given by Reeder [1991], who calculated the PDF's of the convective velocities associated with individual large scale structures that by conditionally-sampling only the largest pressure fluctuations. Three things were discovered [Samimy et al., 1992]:

1. Even at the middle of the mixing layer where the average U_{cp} is equal to the theoretical U_c, structures have a wide range of convective velocities.

2. Both the average U_{cp} and the U_{cp} PDF change with lateral location in the mixing layer.

3. The lateral variation becomes larger as the compressibility is increased, which is thought to indicate a decrease in large scale organization.

Figures 35 and 36 present space-time correlations, which were not normalized by RMS, of the two probes separated in the spanwise direction for both convective Mach numbers. For the space-time correlation at the edge of the M_e = 0.51 mixing
layer (Figure 35) the peaks show no delay, which indicates that the structures are two dimensional. As expected, as the distance between the probes increases the peak decreases in magnitude. The same trends are seen at other lateral locations. The $M_c = 0.86$ spanwise space-time correlations (Figure 36) are quite different, with peaks at non-zero time shifts depending on the lateral location in the mixing layer and spanwise distance between the probes. Two correlations at lateral locations of $(y-y_c)/\delta_o = -0.17$ and 0.21 for a spanwise separation between the probes of $dz_o/\delta_o = 0.27$ are shown in Figure 36. Assuming that the delay is caused by spanwise oblique structure orientation, an average spanwise structure angle, $\beta$, can be calculated by

$$\beta = \tan^{-1}(U_c \ast \tau_p/dz_o)$$

(16)

Resulting spanwise structure angles from 0 to $\pm 75^\circ$ have been measured in the $M_c = 0.86$ mixing layer and are reported in detail in Reeder [1991]. The significance of these measurements will be discussed shortly.

### 4.4. QUANTITATIVE AND QUALITATIVE FLOW VISUALIZATIONS

Although the LDV measurements and fluctuating pressure measurements give an indication that significant changes in large scale organization are incurred between $M_c = 0.51$ and 0.86, very little detailed information concerning the topology and evolution
of the large scale structures is available. Flow visualizations are useful for obtaining such information. For qualitative visualizations, both schlieren and laser sheet lighting techniques were utilized. Quantitative visualizations were made using both double-pulsed measurements and a molecular filter based technique to obtain two-dimensional, instantaneous velocity measurements.

4.4.1. SCHLIEREN PHOTOGRAPHY

One of the most widely used flow visualization techniques in supersonic flows is schlieren photography. Although there are obvious disadvantages due to the spanwise averaging of the technique, general conclusions can be made about the flow field. Particularly useful is the confirmation that the flow field is free of strong shock or expansion waves so that the compressible mixing layer develops in a constant pressure environment. Figures 37 and 38 give an example of instantaneous (15 ns exposure time) schlieren photographs taken with the system described previously for convective Mach numbers of 0.51 and 0.86 respectively. The appearance of the recorded images are highly dependent on the knife-edge location. Therefore, two images with different knife edge locations are presented for each case. Images 37a and 37b are overall views of the $M_c = 0.51$ mixing layer. As can be seen, the shear layer is straight, which is confirmed by pressure measurements. Images 37c and 37d are schlieren photographs of a closer upstream view, which shows the boundary layer of the supersonic stream being engulfed as the shear layer grows downstream. Images 37e and 37f are schlieren
photographs taken downstream in the fully-developed region of the mixing layer. One can see relatively well-organized large scale structures inclined in the streamwise direction. These structures were first observed in the compressible mixing layer by Papamoschou and Roshko [1988] and have since been observed by other investigators using schlieren photography [Elliott and Samimy, 1990, Clemens and Mungal, 1991]. It is difficult to say exactly what individual structures look like, due to the spanwise averaging of the schlieren technique, but they appear similar to those seen in incompressible shear layers. Figure 38 gives schlieren photographs for the $M_e = 0.86$ mixing layer with the same three views described above. In general the characteristics are the same. As previously noted, schlieren photographs suffer not only from the inherent spanwise averaging, but also the appearance of the flow is a strong function of the knife edge setting. This is probably the reason that Goebel and Dutton [1991] did not find the large scale organization seen here and by other investigators.

4.4.2. MOLECULAR ABSORPTION FILTER ISSUES

A vital component of the new class of FRMS optical techniques used in the current investigation is the molecular absorption filter. The filter is a cylindrical optical cell containing a gas which absorbs light within the tuning range of the laser. Before describing FRMS techniques and presenting the results, some important characteristics of the iodine molecular filter as it applies to the techniques used here will be described.
The visible absorption lines of the B-X electronic band for iodine are between 500 and 650 nm. Details of this spectrum have been studied extensively by other investigators [Hiller and Hanson, 1990, Capelle and Broida, 1973, Leroy, 1970, McNaught, 1980, Tellinghuisen, 1973, and Tellinghuisen, 1978]. One of the important aspects of this frequency range is that it includes the wavelengths of argon ion (\( \lambda = 514.5 \text{ nm} \)) and frequency doubled Nd:YAG (\( \lambda = 532 \text{ nm} \)) lasers. The Quanta Ray GCR-4 frequency-doubled Nd:YAG laser used in the present study has an injection seeder which allows the laser frequency to be scaled with an internal potentiometer for coarse adjustments (through a range of approximately 100 GHz), or by applying a bias voltage from 0 to +10 VDC (through a range of approximately 10 GHz) for fine adjustments. Using a beam splitter a small percentage of the laser beam is separated, directed through the iodine cell at room temperature, and focused onto a photodiode. Figure 39 is a plot of the absorption lines from scanning the injection seeder using the coarse adjustment. The frequency is not shown on the abscissa since the adjustments were relatively coarse and the profile is given for line identification purposes over approximately 40 GHz of the total scanning range. The accurate absorption spectrum over the full scanning range of the injection seed has been given by Miles et al. [1992] from measurements and modeling, although the positive frequency direction is reversed in their paper. Absorption lines used in the present study are labeled similar to Miles et al. [1992] and given in Figure 39.

It is important to realize that in practice these absorption lines are not discrete lines, but have frequency-domain profiles dependent on the thermodynamic state of the
molecule in the absorption cell. Each line is a combination of hyperfine lines which interact as they broaden together, creating a single strong absorption line as described by Miles et al. [1991]. The absorption profile is given in general by Beer’s Law [Davis and McFarlane, 1977], which is given by

\[ I = I_0 e^{-k L_{\text{cell}}} \]  \hspace{1cm} (17)

where \( I_0 \) is the intensity of the incident beam, \( L_{\text{cell}} \) is the length of the molecular cell, and \( k \) is the absorption coefficient, which is a function of frequency and usually has units of cm\(^{-1}\). There are three basic processes, as cited by Mitchell and Zemansky [1934], that modify the absorption profiles of the iodine filters in the present study:

1. Natural broadening due to the finite lifetime of the excited state.
2. Doppler effect (temperature) broadening due to the random motion of the atoms.
3. Lorentz or pressure broadening due to inter-molecular collisions.

Natural broadening is only a function of the chemical composition of the absorbing molecule. The relative importance of temperature and pressure broadening, however, is governed by the thermodynamic state of the absorbing gas. For instance, if the cell is maintained at moderate temperatures and low pressures the absorption profile is dominated by temperature broadening and the absorption coefficient has a Gaussian profile [Miles et al., 1991]. Operated in the optically thick regime this profile has very
sharp slopes at the edges as shown in the sketch given in Figure 40a. For an absorption cell dominated by pressure broadening the absorption coefficient is given by a Lorentzian profile [Miles et al., 1991]. This tends to make the slopes at the edges of the absorption profile more gradual as shown schematically in Figure 40b. These are the predominate effects governing the absorption profiles used in the current experiments. The reader is directed to other references for a more detailed discussion of absorption line broadening characteristics [Mitchell and Zemansky, 1934, Penner, 1959, and Davis and McFarlane, 1977].

Before describing the iodine optical cell used in the following studies, a discussion of what must be controlled to accurately set the absorption profile is in order. There are three variables which must be easily adjustable:

1. The number density of the diatomic iodine vapor in the cell.
2. The temperature of the diatomic iodine vapor in the cell.
3. The partial pressure of the foreign gas added to the cell for pressure broadened cases.

Figure 41 gives a schematic of the iodine cell used in the present studies. The cell is similar to that used by Miles et al. [1992]. The filter is a glass cylinder 9.0 cm in diameter and has a length of 26 cm. Iodine vapor is formed in the cell by placing a small amount of iodine crystals in the cell and evacuating the cell. Since the iodine vapor pressure is a function of the coldest point in the cell, the lowest temperature is controlled in the side arm which is enclosed in a water jacket. The cell temperature
(T_{\text{cell}}) is raised above the ambient temperature so that no iodine crystallizes on the windows. The cell temperature is controlled using an Omega closed loop controller and heated using insulated electrical heat tape. \( T_{\text{cell}} \) is normally maintained at about \( 98 \pm 1 \) °C. This uncertainty does not significantly affect the profile since the temperature broadening is relatively unaffected by this slight temperature change. The coldest point in the cell is set in the side arm (T_{12}), which is cooled in a circulating water jacket and maintained within \( \pm 0.1 \)°C by a VWR circulation water bath. This method of controlling the iodine vapor in the an absorption cell is not unique and has been used previously by other investigators [Tellinghuisen, 1973, and Miles et al. 1992].

The partial pressure of the iodine in the cell can be calculated using the equation found in the TRC Thermodynamic Tables [1975] given by

\[
\log_{10}(P_{I_2}) = 9.75715 - \frac{2867.028}{(254.180 + T_{I_2})}
\]  

(18)

where \( P_{I_2} \) is in torr and \( T_{I_2} \) is given in degrees Celsius. Once the cell and side arm temperatures had stabilized (after approximately 2 hours), the side arm valve was closed to set the partial pressure of the iodine vapor in the cell. For more information on measuring and controlling the vapor pressure in similar systems, the reader is directed to Nesmeyanov [1963].

Figure 42 gives a schematic of the system used to add the foreign gas for the
pressure broadened profiles. The procedure is to first set the partial pressure of the iodine and close the side arm valve. Next the cold finger is placed in a solution of dry ice and acetone which almost completely freezes out the iodine in the cell. Nitrogen gas is then added to the desired pressure as measured by an MKS baratron gage to within ± 0.1 torr. After closing the nitrogen valve on the cell, the cold finger is reheated and maintained at $T_{\text{cell}}$. This scheme was found to be extremely repeatable as no differences were detected in the absorption profiles of filters with the same set-points which were prepared at different times.

To measure the absorption profiles, a system was designed so that the absorption profiles could be measured with the same instruments to be used in the flow visualizations. Figure 43 gives a schematic of the system used. The laser frequency was controlled by a bias voltage applied to the injection seeder of the Nd:YAG laser from a CyberResearch CYDAC-02 DAC board on a 486 personal computer, providing a tuning range from 0 - 10 VDC with an uncertainty of ± 1 LSB. The beam was attenuated by using only the back reflection from a prism and then splitting this beam into three separate beams which were focused onto a screen. The first beam passed through a confocal etalon with a 150 MHz free spectral range (FSR). It should be noted that the finesse of the confocal etalon can be increased by mode matching, but the FSR is increased to 300 MHz. The second beam was used as a reference beam passing directly to the screen. The third beam passed through the iodine cell before it was focused onto the screen. The Princeton Instruments intensified CCD cameras were used to collect the intensity of the three beams onto a single image which was recorded on
the 486 personal computer. A Spectrum Basic (programming language used by Princeton Instruments) program was written to change the frequency using the Digital to analog converter between image acquisitions. At each laser frequency an image containing the average intensity of 25 pulses was saved. The greatest advantage of this system is that the profiles are automatically adjusted for the linewidth of the laser and any irregularities associated with the cameras.

Figure 44 presents optically thick profiles of the iodine lines important in the optical techniques used in the present compressible mixing layer experiments. The profiles were taken with the cell operated at $T_{cell} = 85^\circ$ C, $T_{12} = 30^\circ$ C. These profiles were taken at Princeton University during a visit with the same system used by Miles et al. [1992]. The two lines used the most often are lines A (18788.4 cm$^{-1}$) and C (18787.8 cm$^{-1}$) since they have the strongest absorption wells and plateau on at least one side. Figure 45 shows line A profiles taken for $T_{cell} = 95^\circ$ C and different side arm temperatures which, as mentioned previously, changes the partial pressure of the iodine vapor. The partial pressure increases from 0.3 to 3.0 torr for this range of temperatures for line A. $I_o$ is the intensity of the reference beam and I is the intensity of the beam after it passes through the absorption filter. These profiles are characterized as having a relatively sharp edge to the right since temperature broadening dominates the profile. This characteristic makes temperature broadened filters useful in background suppression techniques such as FRS. As the temperature in the side arm is increased the absorption profile not only becomes broader, but weaker absorption lines become more prevalent. Also, the absorption increases away from line center as the number density of the iodine
vapor increases, which has been seen by other investigators [Miles et al., 1992]. Although the absorption profiles appear to totally absorb at line center, this notion is inaccurate. For instance the minimum transmission ($I/I_0$) for a similar cell reported by Miles et al. [1992] is approximately $10^3$.

Figure 46 presents line A absorption profiles that are pressure broadened using nitrogen gas introduced at the indicated partial pressures ($P_{N_2}$). Six profiles are presented ranging from no nitrogen present to a nitrogen partial pressure of 110 torr. Pressure broadening effects become observable at a nitrogen pressure of 1 torr. As expected the slope becomes more gradual for increasing pressures due to the Lorentzian behavior of the profile. By varying the pressure, the range from total absorption to maximum transmittance can be adjusted. Thus the pressure broadened filter can be used in place of the optically thin filter used in DGV by Komine and Brosan [1991] and Komine et al. [1991]. It should be noted that the effect of adding the nitrogen is different depending on the absorption line used. The following sections show how both temperature and pressure broadened filters were used in optical diagnostic techniques.

4.4.3. FRS BACKGROUND SUPPRESSION MEASUREMENTS

This section presents images taken using FRS for background suppression. As an example, Figure 47 shows the effect of using an absorption filter in a flow visualization of the jet used to evaluate the feasibility of optical techniques in this study. The views are cross sections of a jet operated at an underexpanded condition with a pressure ratio corresponding to an isentropic Mach number of 1.88. The air flow to the
jet for this case has been heated upstream of the stagnation chamber to prevent condensation in the shear layer. Since the Rayleigh scattering signal is from air molecules the signal strength is very weak. Image 47a shows an image without an absorption filter. The bright dots from dust particles in the ambient air saturate the camera and the Rayleigh scattering signal is significantly suppressed (confined to the lower bits of the 14-bit Princeton Instruments Camera). With the FRS technique, an iodine filter is placed in front of the camera and the laser is tuned so that the unshifted signal (from scattering off the jet nozzle or the dust particles in ambient air) is suppressed. Image 47b presents a FRS image of this same flow using a temperature broadened filter (similar to those in Figure 45) with a side arm temperature of 55 °C. As seen in Image 47b the background noise is almost totally absorbed, and only the Rayleigh scattering signal from the jet is collected. This preliminary experiment shows the effectiveness of the iodine molecular filter for background suppression. In presenting flow visualizations of the compressible mixing layer obtained using the FRS technique for background suppression, issues concerning the scattering medium are discussed before the collected images are presented.

4.4.3.1. Concerns About Condensed Water Particles

The first flow visualizations that will be presented use condensation particles as the scattering medium. These particles are formed due to the cold temperatures found in supersonic flows. Two condensation particle marking techniques are differentiated by the nature of introduction and the portions of the flow marked. Clemens and Mungal
[1991] called them the product formation and passive scalar methods. In the product formation method (Figure 48a), moisture present in the warmer subsonic flow condenses and forms small particles upon mixing with the colder supersonic flow. These particles basically mark the mixed fluid. In the passive scalar method (Figure 48b) the particles are formed upstream in the supersonic nozzle and mark the colder supersonic flow. The particle number density is reduced when the particle containing flow is mixed with the warmer subsonic flow in the shear layer. Although other investigators have used ethanol vapor [Clemens and Mungal, 1991 and Messersmith et al., 1991], in the results presented here the existing water vapor in the air has been used to provide condensation and subsequent formation of particles. For the product formation results, water vapor present in the ambient air is introduced in the subsonic side of the flow which is similar to supersonic jet experiments by other investigators [Fourguette et al., 1990, Samimy et al., 1993 and Arnette et al. 1993b]. The passive scalar method, however, uses the small amount of moisture still present in the compressed air after drying, similar to the experiments by Miles et al. [1991] and Smith et al. [1991]. The thermodynamic conditions allowing particles to form in the supersonic side for the $M_c = 0.86$ mixing layer, but not for the $M_c = 0.51$ case, are discussed in detail by other investigators who looked at the onset of condensation in supersonic nozzles [Wegner and Mack, 1958, Wegner and Pouring, 1964 and Wegner and Stein, 1968].

Two concerns important in interpreting the visualizations based on product formation are the size of the particles formed and the response time of the formation of these particles. The formation of these particles has been studied in investigations of
the onset of condensation in supersonic nozzles [Wegner and Mack, 1958, Wegner and Pouring, 1964 and Wegner and Stein, 1968]. The formation of ice particles in supersonic flows has been shown to involve a process of condensation or sublimation, followed by nucleation into ice clusters. The diameter of these particles formed in supersonic nozzles (passive scalar formation) has been found from light scattering measurements to be from 40 to 120 Å [Wegner and Stein, 1968]. This estimate of particle diameter seems reasonable for the cases studied here since the scattering seems to be between the Rayleigh scattering regime ($d_p/\lambda < 0.1$ where $d_p$ is the particle diameter and $\lambda$ is the laser light wavelength) and the Mie scattering regime. Evidence of this arises not only from the low signal levels, but also by the significant dependence of the collected signal on the polarization direction of the laser. It should be noted, however, that the line width of the scattering from condensation particles is the same as the illuminating light source [Miles et al. 1991]. These particles should be able to follow the large structures in the flow field according to computational work by Samimy and Lele [1991].

The second and much more difficult issue is to obtain an estimate for the formation time scale of these ice particles. Again we turn to the supersonic nozzle experiments to obtain an estimate of the time scale required for particle formation. Initially it was believed that nucleation was caused by condensation on impurities such as dust particles in the flow. Experiments have shown, however, that the number density of particles in the nozzle is too high to be accounted for by impurities in the supply [Wegner and Mack, 1958]. Two other possibilities were proposed: small particles
formed from impurities with a higher vapor pressure, or self nucleation of the condensed vapor itself [Wegner and Mack, 1958]. The time scale for condensation of the vapor to reach saturation has been measured experimentally to be from 10 to 40 μs [Wegner and Pouring, 1964], but the onset of nucleation can occur in much shorter times [Wegner and Mack, 1958] and is dependent on the sensitivity of the imaging system. An important result from analyzing different nozzles is that the condensation rate increases as the streamwise temperature gradients in the nozzle increase. Although these measurements give an idea of the time scale of formation of particles in the supersonic nozzles, formation and destruction of condensed particles or ice particles in the product formation technique is fundamentally different.

Recent experiments conducted by Arnette [1993b] on the underexpanded axisymmetric jet give a view of the formation and destruction of ice particles. Figure 49 shows an instantaneous view of an underexpanded axisymmetric jet issuing from a Mach 1.5 converging-diverging nozzle expanding into ambient air. The stagnation pressure for the images is 1.74 MPa, which would result in a Mach 2.5 jet for an isentropic expansion to ambient pressure. Figure 49 is a plan view obtained by passing a horizontal sheet through the center of the horizontal jet. The bright region in the center of the jet indicates the downstream location of the Mach disk. When the jet encounters the lower ambient pressure at the nozzle exit, it is accelerated through an expansion fan as the first step in the pressure equilibration process. As the flow accelerates, the temperature and pressure of the flow decrease. Decreasing the temperature of the flow moves the thermodynamic state of the air/water vapor mixture
closer to saturation, but decreasing the pressure has the opposite effect. The rate of
temperature drop per unit drop in pressure for the expansion is greater than that for the
air/water vapor saturation curve. As a result, saturation is achieved. It has been shown
that in the case of water condensation in supersonic expansions, the actual condensation
occurs as a rapid collapse of a super-saturated state [Wegner and Mack, 1958]. The
dark-to-bright intensity boundary indicates this collapse of the super-saturated state.
This situation corresponds to the formation of the scattering particles in the scaler
transport method. The water particles are then seen to vaporize almost instantaneously
upon encountering the large temperature gradients across the Mach disk.

The bright regions on the top and bottom of the Mach disk indicate the jet shear
layer. The signal in the shear layer is fundamentally different from that in the shock
cell. Condensation in the shear layer results when the moist ambient fluid mixes with
the cold jet fluid. The condensation in the shear layer is much closer to an equilibrium
process, although there still may be significant departure. The condensation in the shear
layer is an example of the product formation method. The passive scalar condensation
is confined within the cylindrical intercepting shock, again due to the temperature
gradient across the shock. Jet cross-sections at the exit show that product formation
from the shear layer occurs very near the jet exit, although the intensity of the product
formation near the jet exit is masked by the bright scalar transport condensation in
Figure 49. The almost instantaneous particle formation (at the jet exit) and vaporization
(after the Mach disk) indicate that the particles should have sufficient response time to
mark changes in the mixing layer.
4.4.3.2. Product Formation and Passive Scalar Images

Flow visualizations are useful for qualifying the characteristics of the mixing layer and the large scale structures therein. The different views taken of the mixing layer are shown in Figure 50. This figure gives the camera and light sheet arrangement used to visualize the flow field in streamwise (X-Y view), plan (X-Z view), and spanwise (Y-Z view) planes. A combination of prisms, spherical lenses, and cylindrical lenses were used to construct a sheet of light approximately 0.1 mm thick in the test section. It should be kept in mind that the Nd:YAG laser has a pulse width of 9 ns, which is sufficient to effectively freeze the flow field, but successive images presented in this section are uncorrelated due to very low repetition rate of the laser (10 Hz). In presenting the following flow visualization results, the developing region is considered before the fully developed region for both convective Mach numbers. Recall that LDV measurements have shown that for both convective Mach numbers the developing region extends up to about 120 mm. The fully developed region begins at approximately 150 mm from the splitter plate. It should be kept in mind that hundreds of images have been taken for each case and only a few are presented here. Most of the visualization images shown are from the product formation technique, although a few passive scalar images of the $M_c = 0.86$ mixing layer are presented. The flow direction in all streamwise and plan view images is from right to left. For the plan view images a new lateral length scale is used to indicate the level in which the sheet passes through the shear layer, $y^*$ which is given by
\[ y^* = \frac{(y-y_0)}{b} \] (19)

where \( b \) is the visual thickness of the mixing layer and \( y_0 \) is the point of maximum intensity in the average of the side view images.

Figure 51 shows streamwise (X-Y) views of the developing region for \( M_c = 0.51 \) at different instances in time. The Gaussian variation across the shear layer has been reduced by dividing each pixel by \( I_{\text{max}}(x) \), where \( I_{\text{max}}(x) \) is the maximum intensity in the averaged image at each streamwise location. The product formation starts essentially immediately after the splitter plate tip. A substantial number of the images for this lower compressibility case (i.e. Images 51a - 51d) show "roller- type" large scale structures with distinct braid and core regions similar to those observed by Brown and Roshko [1974], and further described by Bernal and Roshko [1986] in incompressible shear layers. These Brown and Roshko type structures develop and grow as they convect downstream. Classically, for incompressible shear layers the development of these structures has been attributed to the Kelvin Helmholtz instability as discussed previously. In Images 51a and 51c one can see the wave like nature of the initial part of the compressible mixing layer. At some instances in time (Image 51e), the large scale motions display little similarity to the well defined rollers of Images 51a-51d.

Another view that provides information about the developing region is the plan (X-Z) view shown in Figure 52, which gives plan view images acquired in the middle
of the shear layer. The product formation seems to begin almost immediately after the end of the splitter plate with streamwise streaks present in the first 30 mm of the flow. These streaks do not seem to exhibit a stationary spanwise location and seem to break up as they are engulfed by other structures in the shear layer further downstream. The streaks resemble streamwise vortices observed in incompressible shear layers [Jimenez, 1983 and Bernal and Roshko, 1986]. Bernal and Roshko[1986] attribute these streaks to upstream conditions that are amplified in the shear layer and remain spanwise stationary. Recent flow visualizations of compressible boundary layers by Samimy et al. [1994] show the presence of elongated streamwise structures which may be the instigators of these streaks. Constant vorticity contours from large-eddy simulations by Ragab et al. [1992] give indications of the development of streamwise vorticity in an $M_e = 0.4$ shear layer. The initial temporal evaluations indicate the presence of streamwise streaks similar to those shown here, but the evolution of the large scale spanwise structure is less two dimensional than indicated in the present results. This could be due to the fact that different markers of flow characteristics are being used in the experimental and computational investigations.

Although little two dimensionality is observed in plan views in the middle of the developing shear layer, two dimensionality is more evident in plan views located slightly lower ($y^* = -0.4$) shown in Images a and b of Figure 53 and higher ($y^* = 0.4$) shown in Images 53c and 53d. Since the shear layer is growing very rapidly, the initial part of the shear layer (0 to 30 mm) is below or above the sheet and not seen in Figure 53. The latter portion of the image becomes more representative of the middle of the shear
layer, where the two dimensionality is masked by lack of unmixed fluid from either side.

Figure 54 presents two spanwise views in the developing region of the shear layer for each of 3 different streamwise locations. Streamwise streaks in the plan view now appear as dots that grow larger as they convect downstream from $x = 10$ mm (Images 54a and 54b) to $x = 25$ mm (Images 54c and 54d). By $X = 50$ mm (Images 54e and 54f) the dots appear to interact with each other, being connected in some regions of the flow, and the shear layer is more uniform, but wavy across the span. Streamwise vortices have been shown to be present in the initial region of underexpanded supersonic jets [Amette et al, 1993b]. The streamwise vortices observed in underexpanded jets are believed to be a manifestation of a Taylor-Gortler type instability, and are formed due to streamline curvature. In the constant pressure case presented here, there is no streamline curvature. As a result, if the bright streaks can be identified with streamwise vortices, the mechanism for their formation is fundamentally different than that in underexpanded jets. The streaks could possibly be attributed to streamwise streaks in the incoming boundary layer which act as a source of instabilities that grow down stream [Arnette et al. 1993a and Samimy et al., 1994].

The fully developed region for $M_c = 0.51$ exhibits many of the same characteristics as the upstream examples. Figure 55 shows streamwise (X-Y) views of the $M_c = 0.51$ case in the fully developed region. Again the flow field is dominated by rollers with distinct core and braid regions of the Brown and Roshko type, but their presence is not as distinct in all the images (i.e. Image 55d). Similar structures were
also observed by Clemens [1991] for convective Mach numbers from 0.28 to 0.50, becoming more distorted and less organized for higher convective Mach numbers. It can be seen that for horizontal lines drawn at the edges of the shear layer in these images the intensity would be very intermittent as alternating regions containing mixed fluid and free stream fluid would be encountered. This corresponds to the high levels of flatness seen in the LDV results (Figure 29). Another observation to note is the occurrence of smaller scales imbedded within the core region suggesting a high level of mixing.

Plan views (X-Z) in the fully developed region of the $M_c = 0.51$ flow are shown in Figures 56 - 58. Two dimensionality is seen for the majority of the planar cuts made through the middle of the shear layer, as is the case in Figure 56 and 57. Sometimes however the two dimensionality is masked by the smaller scales in the middle of the mixing layer. Towards the top edge of the mixing layer ($y^* = 0.4$), shown in Images 58a and 58b, the two-dimensionality of these roller type structures is not as evident. This indicates that the top edge of the mixing layer is slightly more irregular. Towards the bottom of the shear layer ($y^* = -0.4$), however, the two dimensionality of the large scale roller structures is much more observable (Images 58c and 58d).

Spanwise views in the fully developed region are shown at a location 250 mm downstream of the trailing edge of the splitter plate in Figure 59. These are five representative images showing very different characteristics. Images 59a and 59b show the spanwise view cutting through the braid region, and Images 59d and 59e show
instantaneous spanwise cut through the core region of large scale structures. As expected, when a core passes the laser sheet the amount of mixed fluid causing the product formation is greater, thus appearing thicker than the small amount of entrained fluid in the braid region. Even at this streamwise location, significant undulations are present, illustrating the existence of streamwise structures embedded in the spanwise structures.

As mentioned earlier, ice particles from the water vapor present in the supply air were used as the scattering medium for this case. For the higher convective Mach number, the thermodynamic properties of the top stream are such that the minute amount of residual moisture in the supersonic flow condenses. Thus there is always some passive scalar signal present for \( M_c = 0.86 \) case. Product formation is introduced by supplying moist ambient air to the bottom stream, with condensation resulting upon mixing of the subsonic air with the cold supersonic air. As a result, when the low speed side is supplied with ambient air rather than the dry compressed air, both signals are present, although the passive scalar signal marking the top stream is generally much weaker than the product formation marking the mixing region.

Figure 60 presents streamwise views in the developing region of the \( M_c = 0.86 \) case. Again, the images are normalized by the maximum average intensity at each \( x \) location to compensate for the Gaussian nature of the light sheet. The mixing region marked by the product formation is separated from the passive scalar signal in the top freestream. This delay in product formation could be caused by different thermodynamic conditions than the lower convective Mach number (i.e. lower moisture
content in the ambient air), but is most likely due to reduced level of mixing at the higher compressibility level. The product formation in this case gives no indication of the distinct core and braid regions seen before in the $M_c = 0.51$ case. Also, the wavy nature of the mixing layer is absent, which seems to be the precursor to well organized roller type structures.

Figure 61 gives an example of the plan (X-Z) view for the developing region with the sheet passing through the middle of the mixing layer. Although no clear organization is seen, one thing that is observable is the delay of product formation until approximately 25 mm downstream, which was seen also in the side view. Another difference from the lower convective Mach number case is that there is no indication of the streamwise streaks. This could be an important mechanism in the observed trend of decreasing growth rate with increasing convective Mach number.

Spanwise views confirmed this delay in initiation of the product formation in the shear layer as presented for the developing region in Figure 62. Images 62a and 62b are at a location of $x = 10$ mm. The horizontal line in the center of the image is a reflection off the splitter plate tip which is not fully absorbed by the filter. There is no product formation signal at this location for the reasons given above, but the passive scalar signal can be seen in which strong undulations of the warmer boundary layer are clearly evident. Images 62c and 62d show spanwise views at $x = 30$ mm where the product formation signal is quite observable. The dark region between the product formation and passive scalar signal can still be seen. The product formation signal is uniformly wavy and does not have the discrete dots caused by streamwise streaks as
seen for the $M_c = 0.51$ case. The wavy product formation signal continues to grow downstream as seen in Images 62e and 62f, which are located at $x = 55$ mm.

Figure 63 presents the streamwise (X-Y) product formation images in the fully developed region for the $M_c = 0.86$ case. Although large structures are seen (Image 63b), they are typically much less organized than in the lower convective Mach number mixing layer. There is little visual evidence of core and braid regions for the higher convective Mach number case. This trend has also been observed by Clemens and Mungal [1991] for a compressible mixing layer with a convective Mach number of 0.79. The inspection of many images confirms that there are less frequent excursions of large scale structures into the supersonic freestream (Images 63a, 63c-63d) relative to the $M_c = 0.51$ case. This is probably a cause of the reduced level of the LDV flatness profile for the high speed side (Figure 29). Figure 64 shows side views of passive scalar images in the same fully developed region for the $M_c = 0.86$ mixing layer. Again there are a few images which show large scale structures (Images 64a and 64b), but in general the shear layer appears relatively unorganized. Like the lower convective Mach number case, there are smaller scales embedded within the larger ones.

Figure 65 shows product formation plan views for the fully developed region in the middle of the mixing layer. The images shown contain much more mixed fluid than the lower convective Mach number, also the structures appear much more three dimensional. Figures, 66 and 67 present product formation plan views for the fully developed region at $y^* = -0.4$ and 0.4, respectively. These product formation images are taken from the double-pulsed images to be presented later which can not effectively
use the absorption filter since both pulses are not injection locked, therefore the bottom window causes horizontal streaks in the images. The structures seen in plan views of both the bottom ($y^* = -0.4$) and top ($y^*=0.4$) edges of the shear layer appear to be highly three dimensional and some structures appear oblique having angles of approximately $\pm 45^\circ$.

Sandham and Reynolds [1991] predicted the presence of oblique structures from linear stability analysis and numerical simulations for $M_c = 0.80$. Their linear stability analysis showed that as the convective Mach number increases, the angle for the most amplified wave increases. Also, in low Reynolds number numerical simulations using random initial disturbances, Sandham and Reynolds [1991] showed contours of zero lateral velocity. They showed that as the convective Mach number increases from 0.3 to 1.05, the spanwise structures go from a two dimensional orientation to an oblique orientation. At $M_c = 0.80$ the structures appear to be in transition with an angle of approximately $45^\circ$. This would indicate that for the $M_c = 0.86$ mixing layer, this state of transition exists, and at higher convective Mach numbers the oblique structures seen experimentally would be more well defined. Also, as previously discussed, experimental two point space-time correlation measurements of pressure fluctuations for the $M_c = 0.86$ mixing layer suggested that large scale structures become oblique [Samimy et al., 1992].

Figure 68 contains passive scalar plan view images with the light sheet passing through the middle of the $M_c = 0.86$ mixing layer in the fully developed region. Images a and b of Figure 69 are passive scalar plan view images of the lower edge ($y^*$
= -0.3) of the shear layer and Images 69c and 69d are at the upper edge (y* = 0.3). The characteristics of these passive scalar plan view images are similar to the product formation plan view images, with three dimensional structures which appear oblique in some frames. Two types of spanwise oblique structures seem to typically occur for the passive scalar images. First there are images, such as Images 68a-68c, where an entire large scale structure appears with oblique orientation in the shear layer. Another type of obliquity is seen in Images 68d and 68e, and 69c and d which show oblique branching from a streamwise core.

Spanwise images using the product formation method are shown in Figure 70 in the fully developed region at 225 mm from the splitter plate. The spanwise views indicate again that the flow is highly three dimensional. None of the images show any two dimensionality or distinct braid and core characteristics.

4.4.3.3. Statistics of Single-pulsed Images

Although in general the single-pulsed FRS images presented here are qualitative in nature, simple statistics such as average, RMS, and intermittency can be computed. These statistics were calculated for the side views and are based on 400 instantaneous images. Also, space correlations were calculated to identify typical characteristics of structures in the mixing layer.

Images of the average and RMS intensity fluctuations for $M_e=0.51$ and $M_e=0.86$ are given in Figure 71. The flow direction in these and all other images is right to left. Each column of the average and RMS images has been nondimensionalized by the
maximum average in that column, $I_{\text{max}}(x)$, which as stated previously, occurs at a lateral location $y_0$. This is done to remove the effects of the Gaussian intensity distribution of the interrogating laser sheet. The average profiles across the mixing layer at various $x$ locations are given in Figure 72 for both convective Mach number cases. The average profiles collapse when $y^*$ (Eqn. 19) is used as the nondimensional parameter and appear Gaussian in shape. In this parameter the visual thickness $b$ is calculated as the distance between the bottom and top edges of the shear layer as defined by 15% of $I_{\text{max}}$. Figure 73 gives the variation of the visual thickness with streamwise location for both convective Mach numbers with the lines representing linear curve fits to the data. The slope for the $M_c = 0.51$ and $M_c = 0.86$ mixing layers is 0.098 and 0.061 respectively.

Returning to the RMS images in Figure 71 it can be seen that not only are they laterally wider than the average image, but that they also exhibit a bimodal shape. This bimodal shape is characterized by peaks occurring both above and below the center of the mixing layer for both convective Mach numbers. By looking at $I_{\text{RMS}}/I_{\text{max}}$ profiles at various downstream locations, this trend becomes more obvious as plotted in Figure 74. Although the turbulence intensity measured using LDV (Figures 23 and 24) did not show this bimodal profile the trends of decreasing level and extent with increasing Mach number are seen from the visual $I_{\text{RMS}}/I_{\text{max}}$ also. It is interesting to note that the peaks and valleys for both convective Mach numbers occur at the same $y^*$. For the higher convective Mach number, the $I_{\text{RMS}}/I_{\text{max}}$ peak on the high speed side is slightly lower than the low speed side. Although this could be a result of the condensation process [Messersmith et al., 1991], the reduction of the peak RMS on the supersonic side for
Mc=0.86 relative to Mc=0.51 is most likely due to less frequent excursions of large scale structures into the supersonic freestream at the higher compressibility level. This is strongly supported by the instantaneous images (Figure 63) and by previous measurements of the flatness (Figure 29) discussed earlier.

The bimodal RMS shape has been seen in temperature [Fiedler, 1974] and mixture fraction measurements [Masutani and Bowman, 1986] in incompressible mixing layers and in product formation measurements in compressible mixing layers [Messersmith et al., 1991]. Clemens et al. [1991] observed the same bimodal shape in mixture fraction RMS results from laser induced fluorescence studies of compressible shear layers. Reduction of the peak on the high speed side with increasing Mc was observed, but peaks were not as distinct. This is probably a result of the different flow marking techniques (fluorescence versus scattering from condensed particles). It has been suggested that the mechanism of the bimodal shape is the presence of large scale structures which have a uniform scalar concentration [Fiedler, 1974] and that the reduction of the peak on the higher speed side at higher convective Mach numbers is due to a decreasing occurrence of these structures [Clemens et al., 1991].

Another useful statistic is the intermittency, \( \gamma \), which is defined as the fraction of time a point in the flow field spends in turbulent fluid. For hot-wire measurements the intermittency is generally defined in terms of a fraction of measurements made above or below a set threshold fluctuation level. Similarly, an intermittency like quantity can be calculated from the product formation images. Since product formation signal marks regions of mixed fluid, it will be assumed that if the intensity is above
20% of $I_{\text{max}}(x)$ that the point in the image is in turbulent fluid. It should be noted that although this might exclude some of the turbulent fluid, the trends should be valid. The images containing the intermittency are given in Figure 75 for the $M_c = 0.51$ and $M_c = 0.86$ cases with profiles at various streamwise locations given in Figure 76. The lower convective Mach number shows a profile similar to incompressible free shear layers. For the higher convective Mach number the profile on the low speed side is the same, but on the high speed side the profile is considerably fuller and then drops off to zero with a sharper slope. This fuller profile is caused by the reduced frequency of large scale structures extending in the supersonic free stream. This finding is consistent with individual images and also with LDV results.

One statistical quantity that can give an idea of the nature of the large scale structures in a mixing layer is the two dimensional spatial-correlation. Miles and Lempert [1990] calculated the two dimensional spatial-correlation in studying a supersonic boundary layer using the passive scalar imaging technique. The equations defining this correlation are given by Miles and Lempert [1990] as

\begin{equation}
I_k^*(x,y) = I_k(x,y) - \frac{1}{n} \sum_{j=1}^{n} I_j(x,y) \tag{20}
\end{equation}

\begin{equation}
I_{\text{RMS}}(x,y) = \left[ \frac{1}{n} \sum_{j=1}^{n} (I_j^*(x,y))^2 \right]^{1/2} \tag{21}
\end{equation}
\[
R_s(x_{ref,ref},dx,dy) = \frac{1}{n} \sum_{j=1}^{n} \left[ I_j^* (x_{ref,y_{ref}}) I_j^* (x_{ref+dx,y_{ref+dy}}) \right] \]

where \( x_{ref} \) and \( y_{ref} \) are the coordinates of the origin in the image, \( I_j(x,y) \) is the instantaneous intensity in the image, \( n \) is the number of images, \( I_j^* (x,y) \) is the instantaneous intensity fluctuation about the local mean in the image, \( I_{RMS}(x,y) \) is the RMS intensity fluctuation in the image, and \( R_s(x_{ref,ref},dx,dy) \) is the two dimensional spatial correlation. It should be noted that this correlation expression is nondimensionalized such that \( R_s(x_{ref,ref},0,dy) = 1.00 \) and that the low direction in the figures is left to right.

Figures 77 and 78 show the two-dimensional spatial correlations in the streamwise plane for the \( M_c = 0.51 \) and 0.86 mixing layers, respectively, for \( x_{ref} = 200 \) mm and at different lateral locations. 400 images were used in these calculations. For both convective Mach numbers the contours of constant correlation are basically circular around the reference point in the lower portions \( (y* = -0.51) \) to the middle of the mixing layer. For the lower convective Mach number, as the reference point is moved towards the supersonic side, the contours of appreciable correlation extend farther down into the mixing layer. This is probably due to the fact that when there is a large scale structure towards the high speed side of the \( M_c = 0.51 \) mixing layer, it is robust enough to be "sensed" in the lower speed side. However, the structures in the \( M_c = 0.86 \) mixing layer are not coherent enough to have such an effect. Another characteristic is that the contours become slightly angled and elongated in the streamwise direction as
the reference point is moved to the supersonic side of the mixing layer. This increased angle and elongation of the contours on the supersonic side is much more exaggerated for the higher convective Mach number mixing layer (Figure 78c and d). This characteristic probably indicates the degree of shearing that is imposed on large scale structures by the high speed free stream as they convect at a lower velocity.

4.4.3.4. Structure Descriptions

At this point it is appropriate to pause in the presentation of data and explanation of the results and draw some conclusions from the single-pulsed instantaneous FRS images and the resulting statistics. Figure 2 gives a schematic of the roller type structures reported by many investigators for a description of the large scale structures seen in incompressible and compressible mixing layers [Bernal and Roshko, 1986, and Clemens and Mungal, 1991] and seen in the $M_c = 0.51$ mixing layer in the present study. For the $M_c = 0.86$ mixing layer the structures are not nearly so well defined. A few characteristics for this case have been discussed in viewing the instantaneous images:

1. The structures do not typically extend far into the high speed freestream.
2. The degree of large scale organization in compressible mixing layers is less than that found in incompressible mixing layers.
3. Structures are more three dimensional (sometimes oblique).
4. The large scale structures are stretched and angled towards the high speed side of the mixing layer.
Figure 79 gives a possible schematic for structures in the high convective Mach number case. This sketch of large scale structures emphasizes their unorganized nature and inability to extend into the supersonic stream as compared to the low speed side. Also as seen in the intermittency, the mixing layer is fuller as shown in Figure 79, seldom having thin regions of mixed fluid as seen in the braid region of the roller type structures. It is unclear from the images presented, however, if there are well defined vorticity regions. Furthermore no comment can be made on the lifetime and evolution of the large scale structures. Insight to these questions will be obtained by the double-pulsed and FRMS measurements to follow.

4.4.3.5. Concerns About Molecular Rayleigh Scattering

The second medium used for the flow visualizations is Rayleigh scattering from the air molecules present in the flow. As previously mentioned, molecular Rayleigh scattering techniques offer the possibility of obtaining flow field measurements without seeding the flow field. Typically the density is measured since the scattered intensity is proportional to the number density of the gas molecules. Molecular filter based techniques (such as FRS) offer the possibility of obtaining velocity, temperature and density information. Extracting velocity, temperature, and density information from the Rayleigh scattering mandates the resolution of the Rayleigh scattering spectrum in the frequency domain. The shape of the scattering spectrum is determined by the thermal and acoustic motion of the scattering molecules. As discussed by Miles et al. [1991], the shape of the scattered spectrum is a function of the $y_p$ parameter, which is the ratio
of the collision frequency to the acoustic spatial frequency (or the ratio of the acoustic 
wave length to the mean free path), i.e.

\[ y_p = \frac{\alpha}{KV_0} \]  

(23)

where \( \alpha \) is the collision frequency, and is given by

\[ \alpha = \frac{P}{\mu} \]  

(24)

where \( P \) is the pressure and \( \mu \) is the viscosity of the gas. \( K \) is the scattered (observed) 
wave vector, given by

\[ K = \frac{4\pi}{\lambda} \sin \left( \frac{\theta_{s-o}}{2} \right) \]  

(25)

where \( \theta_{s-o} \) is the angle between the incident wave vector and the scattered wave vector. 
\( V_0 \) is the most probable molecular velocity, given by
where $k$ is Boltzman's constant, $T$ is the temperature of the gas, and $M$ is the molecular weight of the scattering molecule [Miles et al., 1991. The $y_p$ parameter can be written for air [Shimitzu et al., 1983] as

$$y_p = 0.2308 \frac{T[K] + 110.4}{T^2[K]} \frac{P[atm] \lambda[nm]}{\sin(\theta_{s-o}/2)}$$

Figure 80 gives the molecular Rayleigh scattering profile, $R(x_f,y_p)$, for various $y_p$ parameters calculated from the model given by Yip and Nelkin [1964] and normalized such that

$$\int_{-\infty}^{\infty} R(x_f,y_p) \, dx = 1$$

The abscissa is given in terms of a nondimensionalized frequency given by

$$x_f = \frac{2\pi}{KV_0} |f-f_0|$$
where \((f - f_0)\) is the frequency difference from line center. It is recognized that for \(y_p\) values greater than one, Brillouin peaks become significant, but at lower \(y_p\) values \((y_p < 0.1)\) the scattering is essentially Gaussian [Shirley and Winters, 1993]. From the presented profiles it is seen that the scattered spectrum is confined almost entirely within \(|x_f| \leq 1.5\). The given equations describing the Rayleigh spectrum have been taken from more detailed references on the spectral content of Rayleigh scattered signals. [Miles et al., 1991, Shimitzu et al., 1983, Yip and Nelkin, 1964 and Seasholtz, R.G., 1991]

Figure 81 presents a schematic of the spectral distribution of the molecular Rayleigh scattered light relative to an absorption filter used for background suppression in a FRS system. The laser is tuned to just inside the right edge of the absorption profiles to cut all of the background scattering. The spectrum of scattered light from a point in the flow is Doppler shifted out of the absorption profile. In order for the measured signal to be directly proportional to the density, the entire Rayleigh scattering profile must be shifted out of the absorption profile. This condition is met if the Doppler shift arising from the flow velocity is greater than the half-width of the scattered spectrum, i.e.

\[
\Delta f_D \geq f_0 - f_{\text{min}} = \frac{x_f \text{max} K V_0}{2\pi}
\]  

(30)

where \(f_0 - f_{\text{min}}\) is the half-width of the scattered spectrum and \(x_f \text{max}\) is the nondimensional
frequency half-width. As is the case depicted in Figure 81, this condition is not completely satisfied in the $M_c = 0.51$ shear layer investigated in this study, and as a result, true density measurements can not be obtained. In general, a recorded signal level for points that are shifted out of the absorption profile is directly proportional to the density. Obtaining the instantaneous velocity for the flow at each point in the illuminated region is a matter of determining the center frequency of the scattered spectrum.

In the present study, molecular Rayleigh scattering images were taken for the $M_c = 0.51$ case only. Molecular Rayleigh scattering images for the $M_c = 0.86$ case could not be acquired since there is always a passive scalar signal in the supersonic freestream which dominates the molecular scattering, as discussed previously. Therefore only the $M_c = 0.51$ case was free of ice particles. Table 3 gives the calculated $y_p$ parameters, Doppler shift (with angles of 38° between the streamwise flow direction and the camera axis, 90° between the streamwise flow direction and the incident wave vector for the streamwise view camera orientation given in Figure 50), and spectral half-width ($f_0 - f_{\text{min}}$) calculated with Eqn. 30 assuming $x_{f \text{ max}} = 1.5$, for the upper and lower streams for the $M_c = 0.51$ mixing layer. For these free stream conditions the $y_p$ parameter is such that the Rayleigh scattering spectrum is approximately Gaussian. Unfortunately, the profile of the Rayleigh scattering is never fully shifted out of the filter, therefore the intensity is a function of the temperature, density, and velocity of the flow field. Although the intensities recorded in the images contain all of these properties, they are not separable using only one camera. The molecular Rayleigh scattering images
presented are useful for qualitative flow visualizations, however. Although it has been established that the present results are not sensitive only to density, it is still instructive to examine the mean intensity profiles quantitatively.

4.4.3.6. Molecular Rayleigh Scattering Images

Figure 82 gives an example of an image with no flow in the tunnel and the filter operated at the temperature broadened conditions with $T_{12} = 35^\circ$ C. The frequency of the laser is at the edge of the filter so that the background is attenuated, but the temperature broadened signal from the still ambient air inside the tunnel partially passes. Figure 83 are streamwise images in the developing (Image 83a) and fully developed (Image 83b) regions. These images look much like the passive scalar images of the higher convective Mach number. For the image taken in the fully developed region (Image 83b) a large scale roller structure is presented. This image shows the structures streamwise density gradient as well as the lateral density gradient. An average background image has been subtracted, and thus the lower subsonic stream (which is barely shifted out of the filter) is almost totally attenuated. Unfortunately, the streamwise viewing area is limited due to the large power density required to obtain an acceptable signal to noise ratio.

Figure 84 presents the mean profile of the measured intensities for the developing region. Since in these results only the center of the sheet is considered, a constant threshold was set in order to disregard pixels that were inhabited by contaminating particles in the flow field. A total of 141 images were processed in calculating an
average image. The displayed profile represents the mean of the seven adjacent pixel columns in the center of the average image. Each column correspond to a streamwise region of 0.25 mm extent, and the central column of the seven averaged together is located 56 mm downstream from the splitter plate (developing region). Figure 84 shows the average intensity normalized by the value in the top free stream. The mean intensity of the supersonic free stream is used as the nondimensionalizing value because intensity variations in the upper portions of the shear layer are more indicative of density variations, since the majority of the Rayleigh scattering spectrum is shifted out of the absorption filter. The mean density is also plotted at this location (nondimensionalized by the supersonic density), as calculated previously from the LDV velocity profile [Samimy and Elliott, 1990 and Elliott and Samimy, 1990]. The intensity profile follows the basic trend of the density in the upper portions of the shear layer, but deviates upon approaching the center of the density profile. This is because, as the mean velocity decreases at lower lateral positions, more of the Rayleigh scattering profile is cut-off by the iodine filter. Velocity becomes the most prominent attenuation factor in the low speed side of the shear layer since the lateral gradients of mean density and temperature are very small and the scattering is not shifted out of the absorption profile.

Figure 85 shows the mean profile of the intensities at a location in the fully developed region (x = 180 mm). The profile represents the average of 121 instantaneous images. Again seven pixel columns are averaged together, normalized by the supersonic freestream intensity, and then plotted with the calculated density profile. Similar to the upstream case (Figure 84), the intensity profile follows the density
contour for the upper side of the shear layer and then deviates due to the increasing effect of signal attenuation resulting from the smaller mean Doppler shift on the low speed side of the shear layer. Unfortunately the deviation caused by the portion of the signal unshifted out of the filter cannot be solved easily. Two possible solutions are to scan the laser during the run, or use multiple cameras and filters as proposed by Miles and Lempert [1990].

4.4.4. DOUBLE-PULSED MEASUREMENTS

Single instantaneous images and the statistics calculated from them are good for making general observations about large scale structures, but it is difficult to draw conclusions about the evolution of the large scale structures. Questions about the formation, destruction, and evolution of the large scale structures can only be answered if the time history of the structures can be studied. Although it would be more desirable if one could take images of a full time history of structures in the mixing layer, with current techniques this is not possible. Double-pulsed images, however, can be taken by multiple Q-switching the laser (as discussed in Section 3.5) and using two intensified cameras in the arrangement shown in Figure 86. The goal is to study the evolution of the structures and calculate their convective velocity. The time delay for each double-pulsed image set to follow is presented in terms of a nondimensional convection distance given by
\[ x_c = \frac{U_c \tau}{b} \]  

where \( \tau \) is the time between the two successive images, \( b \) is the visual thickness defined earlier, and \( U_c \) is the theoretical convective velocity given by Equation 2. This nondimensional parameter has been used by other investigators in studying space-time correlations [Bonnet and Chaput, 1986]. The theoretical convective velocity and \( b \) (in the fully developed region, \( x \approx 190 \text{ mm} \)), for \( M_e = 0.51 \) are 338 m/s and 18.2 mm and for \( M_e = 0.86 \) are 428 m/s and 16.3 mm, respectively. Table 4 gives the actual time delays, the nondimensional convective distance, and actual distance in mm that a structure would travel if moving at the free stream velocities or the theoretical \( U_c \) for the employed time delays.

Figure 87 and 88 shows the double-pulsed streamwise images for a convective Mach number of 0.51 for the developing region. Again these images have been normalized by \( I_{\text{max}}(x) \). As seen in the single-pulsed images, there are roller type structures which appear to be simply convecting downstream. Also, Image 87b offers a clue on the formation of these roller type structures from an initial wave. The structure labeled c in this figure shows the start of the wave distortion, another structure d in this same image shows how a distorted wave evolves into a roller type structure. Although the complete evolution of one structure is not seen, we can use these two structures and others to draw a schematic of the evolution of the roller type structures from a distorted Kelvin-Helmholtz wave as seen traveling with a frame of reference moving at \( M_e \) (Figure 89). Image 89a shows a typical Kelvin-Helmholtz wave that is
distorted by the lower velocity on the bottom and higher velocity on the top. In order to roll-up, the point that was in the bottom (point 2) must travel to the top of the mixing layer and accelerate, and the top point should do the reverse (point 1) as indicated in Figure 89d. Similar processes have been described by other investigators in incompressible mixing layers [e.g. Winant and Browand, 1974, Grinstein et al., 1986].

Figure 90 and 91 give the double-pulsed images for a convective Mach number of 0.86 in the developing region. The decrease in organization of the large scale structures is again seen in these double-pulsed images. Note that it is much more difficult to follow characteristics from the initial to the delayed image and the large scale structures sometimes change totally (Image 90b). Features in the shear layer can be seen to convect downstream. Even when there are wave like structures (center of Image 90a), they tend to convect instead of evolve into roller type structures as seen in the lower convective Mach number mixing layer.

The double-pulsed images in the fully develop region for the $M_c = 0.51$ mixing layer are seen in Figures 92-97 for three different time delays. For all of the time delays there are recognizable large scale structures which grow and evolve as they convect downstream. Four basic categories of processes have been used to describe the evolution of these large scale structures within the flow field: pairing (complete, partial, and fractional), tearing, normal growth, and cascading. Examples of these processes have been discussed previously in conjunction with Figure 3. Image 92a shows a structure to the left of the image which appears to form from portions of two different structures. It seems that if a major portion of a structure is slightly high in the shear
layer and the structure proceeding it is located towards the low speed side the two structures have a chance to pair. This pairing process may be occurring in Image 97b where two roller type structures (one outside of the first frame) are distorting and combining together. At other instances in time the roller type structures seem to only grow as they convect downstream (Image 93a) without much change in physical appearance even at the longest delay (Images 96a and 96b). Even large scale structures that do not have distinct core and braid regions remain correlated as they convect downstream, as in Images 93b and 94b. Tearing processes in general are not as obvious for the structures in the $M_c = 0.51$ convective Mach number, but in Image 95a the structure seems to be torn apart by the shearing action of the free streams.

In the developing region there were indications of waves which develop into roller type structures. Although there are no thin waves in the fully developed region there are times in which a fuller wave like structure experiences a roll-up process (Images 92b and 94a). When the developed structure is followed by a second structure, as in Image 92b, the braid region is stretched further. If this region contains streamwise vorticity, the magnitude of the streamwise vorticity would increase due to vortex stretching. Brown and Roshko [1974] suggest a lifetime of the roller type structures to be $4.3\delta_c$. For the $M_c = 0.51$ mixing layer this would be an $x_c \approx 4.4$ which is larger than measured here, but it appears that the structures are still well defined for the largest delay associated with $x_c = 1.71$.

Double-pulsed side view images in the fully developed region are given in Figures 98 - 103 for the $M_c = 0.86$ mixing layer for the same dimensionless time delays
as the lower convective Mach number case. Just as in the single-pulsed instantaneous images, there is little indication of roller type large scale structures and the structures are relatively unorganized. For the first time delay (Figure 98 and 99; \(x_c = 0.64\)) the structures can be followed from the initial to the delayed image. For the second time delay (Figure 100 and 101; \(x_c = 1.18\)) portions of the mixing layer can be followed, particularly on the low speed side, but large scale structures change shape becoming almost unrecognizable between successive images. The delayed image of Figure 100a shows what appear to be a roller type structure, but this is uncommon and such structures are torn apart rapidly. By the last time delay (Figures 102 and 103; \(x_c = 1.71\)), the structures undergo complete change and only a few portions of the mixing layer can be followed from the initial to delayed image. For the incompressible shear layer using the lifetime criteria of Brown and Roshko [1974] a structure would have a lifetime associated with a delay of \(x_c \approx 6.1\) which is much greater than the \(x_c = 1.71\) by which most structures are unidentifiable. One characteristic that is common for the higher convective Mach number case is the tearing process which is seen in structures of Figures 98a, 99b and 103b, where the top portion of a large scale structure goes faster than the portion of the structure on the low speed side, thus stretching or tearing it apart. Another characteristic seen particularly on the low speed side of the mixing layer. Sometimes structures seem to "pull" low speed free stream fluid (characterized by a region of no signal in the bottom of the mixing layer) into the mixing layer. The entrained fluid is then stretched as it experience the local acceleration (Images 98a, 102a, 103a). This low speed fluid entrainment process is depicted in Figure 104.
Generally the large scale structures seen in the $M_c = 0.86$ mixing layer appear to be traveling with the local velocity. This could be due to the fact that the structures are less organized, and it is less difficult to follow undulations on the top and bottom of the mixing layer which convect closer to the free stream velocities. For the few images that structure roll-up appears to be taking place (Image 100a), the structure appears to move at the theoretical convective velocity. This would support the theoretical arguments made in calculating $U_c$ [Papamouschou and Roshko, 1986].

In order to study the evolution of the mixing layer, structure space-time correlations between initial and delayed images have been calculated for both convective Mach numbers at various time delays. Traditionally, space-time correlations are computed with multiple probes where data is collected for long time sequences at a few spatial points. The correlations presented here, however, are different in that data is collected at only two instants in time, but in a two-dimensional image. Each of the presented correlations are based on 400 pairs of initial and delayed images. Before the correlations were calculated, average and RMS fluctuation images were generated for both the 400 initial images and the 400 delayed images. The appropriate average image was then subtracted from each of the initial and delayed images, so that the correlations are of the fluctuations from the local mean. The correlations were nondimensionalized by the RMS fluctuation at the two pixel locations being correlated (the RMS at the reference location was taken from the initial RMS image and the RMS at the shifted location was taken from the delayed RMS image). In the initial fluctuation image, a vertical line is taken at a streamwise location $x_0$ as shown in Figure 105. The signal in
this vertical line is then correlated with the corresponding signal in the delayed image at streamwise locations denoted by \(x_0 + dx\). The resulting correlation is then averaged over the 400 image pairs in the ensemble for the given time delay. To reduce the dependence on the selection of the reference line location \((x_0)\), the correlations for 4 different \(x_0\) locations spaced through the initial images were averaged together.

The expression for the space-time correlation just described is similar to that used for spatial correlations calculated earlier for single-pulsed images (Eqns. 20 - 22) and is given by

\[
I^*_{k,(q,q)}(x,y) = I_{k,(q,q)}(x,y) - \frac{1}{n} \sum_{j=1}^{n} I_{j,(q,q)}(x,y) \tag{32}
\]

\[
I_{RMS,(q,q)}(x,y) = \left[ \frac{1}{n} \sum_{j=1}^{n} \left(I^*_{j,(q,q)}(x,y)\right)^2 \right]^{1/2} \tag{33}
\]

\[
R_{xy}(x_{ref}, dx, y, \tau) = \frac{1}{n} \sum_{j=1}^{n} \left[I^*_{j,(x_{ref},y)} I^*_{j,(x_{ref} + dx, y)}\right] \tag{34}
\]

where \(\tau\) is the time delay between the initial and delayed images, \(I_{j,(q,q)}(x,y)\) is the instantaneous intensity in the initial or delayed image, \(n\) is the number of image pairs for the given time delay, \(I^*_{j,(q,q)}(x,y)\) is the instantaneous intensity fluctuation from the local mean in the initial or delayed image, \(I_{RMS,(q,q)}(x,y)\) is the RMS intensity fluctuation.
in the initial or delayed RMS image, and \( R_s(x_{\text{ref}}, dx, y, \tau) \) is the space-time correlation calculated from the initial and delayed image. It should be noted that this correlation expression is nondimensionalized such that \( R_s(x, 0, y, 0) = 1.00 \) for \( \tau = 0.0 \). One should note that since the lines are not always located at the center of a structure the correlation contours do not exactly show what a typical structure looks like, but how a vertical line of points change with time. The streamwise direction in these correlation plots is from left to right.

For a given ensemble, averaging the two-dimensional correlation resulting from the various reference locations (\( x_{\text{ref}} \)) allowed average correlation contours to be constructed. To indicate that the number of images taken is sufficient to describe the level and shape of correlation contours Figure 106 shows four different sets of contours of the \( M_c = 0.86 \) mixing layer for different numbers of lines taken where \( N = (\text{number of images}) \times (\text{number of reference lines in one image}) \). For \( N \) less than 400 the number of lines in a single image is one. As seen by this figure the shape and level of the correlation contours remains approximately the same for \( N > 400 \). For the contours to follow \( N \) is taken to be 1600. It also should be noted that unrelated images were correlated and the maximum correlation coefficient was found to be less than 0.1.

Figures 107 and 108 give the correlation contours for three non-dimensional convective distances, \( x_c \) (i.e. time delays), for \( M_c = 0.51 \) and \( M_c = 0.86 \), respectively. For all the contours presented, the points of highest correlation occur between the spatial shifts expected from the lower and higher speed free stream velocities and the employed time delay. A striking characteristic of the correlation contours presented here (Figures
107 and 108) is that two maxima are found to occur, one near each edge of the mixing layer. The probable explanation lies in the nature of the turbulent mixing layer. Near the edges of the mixing region, large fluctuations occur about the mean (indicated by the RMS images of Figure 71) which are associated with the passage of large scale structures. The fluctuations associated with the large scale structures appear to be remaining well correlated for the employed time delays. The higher correlations at the edges of the mixing layer relative to the center are thought to be a result of a wider range of scales contributing to the fluctuations at the center (i.e. smaller scales on the average relative to the edges) and correlation levels tend to increase with the scale of the motions. In the $M_c=0.51$ correlation contour (Figure 107), the two regions of maximum correlation seem to convect with the local velocity, so that the region near the low speed edge travels less for the given time shift than does the region near the high speed edge. The center between the two maxima appears to be traveling with the theoretical convection velocity. If one imagines an axis between the two regions of maximum correlation, it appears that the axis rotates and stretches between the two time shifts. The axis rotation gives an indication of the vorticity associated with the large scale structures. It is difficult to identify the specific cause of the axis stretching, contributing factors could be: 1) a structure 'tearing' process and 2) the reference lines for the correlations are not chosen a priori to fall within a structure so that reference lines which don't fall in a structure travel near the local free stream velocity.

Figure 108 shows the correlation contours for $M_c=0.86$. Figures 108a-c are for the same $x_c$ as the $M_c=0.51$ contours of Figures 107a-c, respectively. Again the center
between the two maxima convects at the theoretical $U_c$, with the maxima convecting towards the free stream velocity. The correlation level has dropped for $M_c=0.86$, suggesting that the large scale structures are less correlated for $M_c=0.86$ relative to $M_c=0.51$. This is supported by instantaneous images at $x_c = 1.71$, where it is much more difficult to follow one structure from the initial to the delayed image, particularly events in the high speed side of the shear layer. Assuming that the highest correlation levels are associated with the large scale coherent motions, the "average" structure appears to be stretched, particularly on the high speed edge of the mixing layer where the structure appears to be tilted and elongated in the streamwise direction.

Figure 109 gives the decrease of correlation level at different lateral points in the shear layer for both convective Mach numbers with increasing $x_c$ (time delays). It can be seen that the levels decrease much more quickly for the higher convective Mach number indicating that the structure's life time is decreased for the higher convective Mach number case. This agrees with the results of multi-point correlations of fluctuating pressures (Figure 33) where it was found that the streamwise correlation levels decreased much more rapidly with increasing probe separation for $M_c=0.86$ than for $M_c=0.51$ [Samimy et al., 1992b]. Figure 110 gives the increase in distance between the centroids of the two peak correlation levels ($L_p$) with increasing $x_c$. From this figure one can observe that $L_p/b$ increases 20% for $M_c = 0.51$ and 41% for $M_c = 0.86$ over the convective distances investigated. This may be an indication of the greater importance of the tearing processes for higher convective Mach numbers, suggesting a reduced interdependence of the structure between the high and low speed sides of the
mixing layer.

Figures 111 - 116 are plan view double-pulsed images for the $M_e = 0.51$ mixing layer for a characteristic convection distance of 1.18. As in the single-pulsed results, two dimensional structures are again seen which convect downstream with some change in structure shape, but remain mostly two dimensional. These two dimensional structures convect faster or slower than the theoretical $U_e$ depending on where the sheet passes laterally in the mixing layer. At the bottom of the shear layer ($y^* = -0.3$) there are times in which a large portion of the image is unmixed fluid (Image 112a), with a small part of a structure convecting downstream. Other instances show a large portion of mixed fluid, but aspects of the structures are difficult to follow (Image 112b). This is particularly true for the middle of the mixing layer ($y^* = 0$) where only the images with well defined two dimensional structures can be followed (Figure 113). Other plan view images in the middle of the shear layer show little correlation from the initial to delayed images (Image 114b). The high speed side of the $M_e = 0.51$ ($y^* = 0.3$) shows the same characteristics as discussed above (Figures 115 and 116).

The plan view double-pulsed images for the $M_e = 0.86$ mixing layer are shown in Figures 117 - 122 for a characteristic convective distance of 1.18. The much more three-dimensional and at times oblique structures are again seen and convect downstream. These oblique structures convect slightly faster or slower than the theoretical $U_e$ depending on where the sheet passes laterally in the mixing layer. The oblique structures generally convect downstream with a constant angle, but change shape, particularly in the middle (Image 120a) and top (Image 121a) portions of the
mixing layer. Plan views through the middle of the mixing layer (Figures 119-120) have no large regions of unmixed fluid. A streamwise branching type structure is seen in Figure 119a which remains relatively unchanged as it convects downstream. The plan view double-pulsed images cutting through the top of the mixing layer seem to change the most from the initial to delayed images.

Similar to the side view cases, the space-time correlation contours were calculated for the plan views and are given in Figures 123 and 124 for the $M_c=0.51$ and $0.86$ mixing layers respectively. Correlation contours are based on 300 images and the contours are smoothed in the graphics package. In general, the centroid of the contours moves a distance associated with the local mean velocity, except for the middle ($y^*=0$) plan view of the $M_c = 0.51$ mixing layer where the shift is slightly higher and closer to the distance associated with the theoretical convective velocity. This was also observed in the fluctuation pressure measurement calculations of the convective velocity (Figure 34). One difference between the two convective Mach numbers is the planar structure length ($L_{pa}$) which is defined as the length of the correlation contour measured along the x direction between the 0.55 contour. For the $M_c = 0.51$ mixing layer $L_{pa}/b \approx 0.68$ and for the higher convective Mach number case $M_c = 0.86$, $L_{pa}/b \approx 0.34$. This is most likely due to the fact that the two-dimensional roller structures remain more coherent and have a longer streamwise extent than the less organized structures for the $M_c = 0.86$ mixing layer. It may be surprising that for the higher convective Mach number case the contours are not oblique, but since the oblique structure angle can be either positive or negative [Samimy et al., 1992] and moves in the streamwise direction
the contours are elliptical as shown in Figures 123 and 124.

4.4.5. MOLECULAR FILTERED IMAGE VELOCIMETRY

The single and double-pulsed images shown above give a good indication of the qualitative characteristics and evolution of the large scale structures, but it is desirable to have a knowledge of the flowfield properties such as pressure, temperature, density, and velocity. Systems have been proposed to obtain images containing instantaneous measures of these properties using a molecular absorption filter similar to that employed in the FRS background suppression measurements discussed above. The goal of the present study is to obtain images containing velocity information and evaluate the feasibility of the technique. First, preliminary measurements obtained in a perfectly expanded Mach 2 axisymmetric jet are discussed. This work enabled us to develop the procedures for calculating the velocity. Next, the accuracy of the technique is evaluated experimentally and theoretically. The experiments are designed so that the tangential velocity was measured on a solid disk which was rotating at a known RPM. A theoretical prediction of the uncertainty associated with the velocity measurements arising from the uncertainties associated with the various components of the measuring system is presented and discussed. Finally, measurements made in the $M_c = 0.51$ and $0.86$ planar mixing layers with two different views are presented.

4.4.5.1. General Procedure

The procedure for obtaining the images with intensity proportional to velocity
presented here is somewhat similar to that employed in the DGV technique developed by Komine et al. [1991] and Meyers [1992]. Figure 125 gives a schematic of the basic idea. Two cameras are used in this technique. One camera has a pressure broadened molecular filter to attenuate the scattered intensity as a function of frequency, hereafter referred to as the velocity discriminating camera with a collected light intensity $I'(x,y)$. A second camera with no filter is used to record the unattenuated scattered intensity, hereafter referred to as the reference camera with intensity $I_0(x,y)$. By properly normalizing the velocity discriminated intensities by the corresponding reference intensities, the Doppler shift (and therefore velocity) is determined at each pixel location. It should be noted that the intensity fluctuations arising from fluctuations in scattering source number density or laser intensity will be present in the readings of both cameras, so that the resulting velocity measurements are not degraded. Instead of using a beam splitter and dividing the scattered signal between the cameras as is done in DGV, each camera is focused onto the light sheet. A grid placed in the laser sheet is recorded with each camera so that the images from the two cameras can be aligned properly in post-processing.

The profile seen schematically in Figure 125 is an optically thick pressure broadened profile whose frequency range can be controlled, depending on the iodine absorption line selected and the partial pressure of the nitrogen added to the cell. This is different from DGV which uses optically thin profiles which offer very limited control of the profile slope and don’t offer the absorption breadth needed for supersonic flows. In the present studies of shear layers, the frequency shift is always positive. The laser
frequency is set at a point of maximum absorption and the Doppler shift (given by Eqn. 5) places the scattered light in the sloping region of the absorption profile (Figure 125). Therefore, after normalizing the frequency discriminated signal by the reference camera the intensity becomes a function of Doppler shift (or velocity).

Although the idea of obtaining images containing velocity information using molecular filters seems straightforward, there are a number of steps needed to obtain these images. The procedure used in this study to obtain an image containing velocity measurements is given as follows.

1. Select filter properties ($P_{N2}$ and $T_D$) and laser frequency that will give approximately linear transmission over the expected velocity range.

2. Set-up an optical arrangement which is sensitive to the velocity component to be measured.

3. Arrange cameras to have approximately the same view.

4. Set gain on both cameras (without the molecular filter) and run the wind tunnel and laser, obtaining simultaneous images to compare the dynamic ranges of the two cameras.

5. Place grid in field of view of cameras to allow image alignment in post processing.

6. Check filter profile by splitting a partial beam, directing it onto the grid, and obtaining images at various frequencies.

7. Obtain simultaneous images of flow field.

These are the basic steps used to obtain the raw data for the velocity images. A few points need to be made. First, when setting the camera gains, the gain for the camera
which will have the filter was set slightly higher to allow for the absorption due to the cell windows and uniform absorption. Though not strictly necessary, this step insured that the filtered signal would still be appreciable. The filter profile check was useful for making sure that the profile was the same as previously measured.

Once the images are taken, a fair amount of image processing needs to be done to obtain an image containing velocity information. The required steps are:

1. Calculate the functional relationship between the normalized intensity \( I/I_0 \) and the velocity for the given optical arrangement using Equation 4.

2. Using the grid image, calculate the amount of shift required to line up each pixel of the reference camera with the velocity discriminating camera.

3. From the aligned images with no filter (step 4 above) the intensities are compared such that an equation is obtained which maps the intensities recorded by the velocity discriminating camera onto the dynamic range of the reference camera.

4. Create an image which is the filtered intensity divided by the reference intensity after it is aligned and adjusted for intensity for each instantaneous image.

5. Create the velocity image, \( I_{vel}(x,y) \), by applying the known relationship between \( I/I_0 \) and the Doppler shift to the \( I/I_0(x,y) \) values contained in the image.

These are the basic image processing steps taken to obtain the velocity image from the raw data. To adjust the intensity of the velocity discriminating camera in terms of the reference camera (step 3), the aligned images acquired without the absorption filter are compared. A program samples the intensity at a pixel location from one camera, places the intensity in the appropriate bin (bin widths were 100 counts out of a dynamic range of 16384), and then places the intensity from the corresponding
location in the second image in a corresponding bin. All of the intensities in each bin were then averaged to obtain a functional relationship between the readouts of the two cameras at the employed gain settings. The functional form was always linear and can be written as

\[ I = m_I I' + b_I \]  

(35)

where \( I' \) is the intensity from the velocity discriminating camera, \( I \) is the velocity discriminating intensity adjusted to the same dynamic range as the reference camera, and \( m_I \) and \( b_I \) are the constants found from a linear curve fit comparing the two cameras. Using this expression, intensities in the reference camera are then transformed so that the dynamic range of the filtered and reference cameras are the same. After the images are aligned and this intensity correction step is performed, the calculation of the normalized image \( I/I_0 \) is straightforward.

The final two steps of the process are to apply the given filter profile and calculate the velocity. Once the intensity ratio \( (I/I_0(x,y)) \) is calculated for each pixel, a computer program searches the pressure broadened filter profile (calculated in terms of the velocity) for the appropriate velocity for that intensity ratio. The entire process can be expressed by a combination of the Doppler shift equation (Eqn. 5), the intensity correction equation, and the filter profile \( \Delta f_D = A_I(I/I_0) \), where \( A_I(I/I_0) \) is the frequency shift in terms of the intensity ratio as given by the absorption profile. The combined equation can be written as
$$V = \frac{\lambda A_f \left( m_f (I') + b_1 \right)}{\cos(\phi_D) \cos(y_D) + \cos(\theta_D)}$$

(36)

More will be said about this equation when the sources of error are discussed below.

4.4.5.2. Preliminary Velocity Measurements of an Axisymmetric Jet

Preliminary velocity measurements were made in a Mach 2 perfectly expanded supersonic jet to investigate the feasibility of the technique. The light sheet was oriented oblique to the jet axis, with an included angle of 45° ± 1°. The filtered camera’s axis was oriented at 45° ± 1° relative to the jet axis and perpendicular to the laser light sheet, intersecting the light sheet at the jet axis. The jet axis, camera axis, and light sheet axis were coplanar. This orientation results in the measurement system being sensitive only to the streamwise velocity $V_x$ (and independent of $V_y$ and $V_z$) at the intersection point of the jet axis, camera axis, and light sheet axis. This orientation is depicted in Figure 126. Note that although the velocity measured is the streamwise velocity, the orientation of the laser sheet with respect to the jet is oblique and not the normally desired perpendicular jet cross section. Figure 127 gives the relation between the velocity discriminating camera and the reference camera with the appropriate linear curve fit. As can be seen the relation between the two cameras is extremely linear. A pressure broadened profile with 70 torr of nitrogen, $T_{12} = 45^\circ C$, and $T_{cell} = 85^\circ C$ was used. This profile is given in Figure 46. Assuming an isentropic expansion with a
constant ratio of specific heats equal to 1.4, the highest velocity (jet centerline velocity) should be approximately 492 m/s. This velocity with the given orientation yields a Doppler shift of 1.3 GHz. Because the velocity is measured only at those positions where condensation is present, and the condensation forms only in the shear layer surrounding the potential core of the jet, it was expected that all the realized velocities would be less than the calculated maximum isentropic value. Figure 128 shows the functional relationship between the streamwise velocity and the normalized intensity ratio for the utilized absorption profile and optical arrangement. This graph was obtained from the coupling of Equation 5 and the absorption profile of Figure 46 through the illuminating laser's frequency, $f_0$.

As a first step, the average velocity image was calculated for the perfectly expanded Mach 2 jet by averaging 150 instantaneous velocity images. Since the velocity is calculated with the scattering from the condensation particles, velocities are obtained only in the shear layer region of the jet as mentioned earlier. Each pixel location had to display significant signal in at least 20 of the 150 images for inclusion in the average. The result is presented in Figure 129, where the gray level of the image is directly proportional to the streamwise velocity. Three velocity profiles are presented in Figure 130. The profiles are for horizontal lines 8.4 mm above the jet centerline, the centerline, and 8.4 mm below the jet centerline in Figures 130a, 130b, and 130c, respectively.

As discussed previously, the laser sheet was oriented 45° away from the jet axis. The center of the light sheet intersected the jet axis at $x/D = 5.75$ ($x = 110$ mm). The
right side of the jet in Figures 129 and 130 is further downstream than the left side. The average trend of increasing velocity with decreasing radial distance from the center of the jet is present in Figure 130. The maximum average velocity is well below the calculated free stream velocity of 492 m/s (which corresponds to $I/I_0 = 0.63$). This indicates the inability of the moist, ambient air to penetrate into the jet core by this $x/D$ as is evidence in Figure 129. The maximum velocities found around the inner core of the average image do not exceed approximately 380 m/s, and examination of all of the horizontal profiles showed no significant difference in the maximum values on either side of the jet.

Examination of the horizontal velocity profiles in Figure 130 gives rise to several observations. The horizontal velocity gradient on the left (upstream) side of the contours is greater than that found on the downstream side. Further, the profiles appear to be skewed to the downstream side of the contour. These trends reflect the decay of the jet velocity profile and growth of the shear layer with increasing downstream distance, which has a top-hat shape at the nozzle exit. Another interesting observation in Figure 130 is that the maximum velocities on either side of the jet do not occur at the smallest $r/R$ for which values are reported, as would be expected. Although no definitive explanation can be given, two possible reasons are cited. It is possible that the only time an appreciable signal is present at the inner radii is when a significant amount of low velocity fluid is entrained into the core region of the jet, thus giving rise to velocity biasing. It is also possible that the instantaneous core fluctuations coupled with the relatively small number of images used to compute the average gave rise to
poor averages at these radial locations.

Two instantaneous non-filtered images (image from the reference camera) are presented in Image a of Figures 131 and 132 and the corresponding velocity images are presented in Images 131b and 132b. The horizontal velocity profiles through the center of the jet are presented in Figures 133. Figures 133a and 133b correspond to the Images 131b and 132b, respectively. The reference camera (Images 131a and 132a) marks the region in which the velocity is calculated. The gray levels of Figures 131b and 132b are directly proportional to the streamwise velocity. The instantaneous images illustrate the significant fluctuation of the core region. The horizontal velocity profiles of Figure 133 again seem reasonable, displaying both decreasing velocity with increasing radial distance and more severe velocity gradients on the upstream side of the jet. In general, the non-stationary nature of the instantaneous velocity maps clearly illustrates the turbulent nature of the jet shear layer.

4.4.5.3. Technique Validation from Rotating Disk Measurements

Now that the feasibility of the technique was demonstrated with the preliminary investigation of the axisymmetric jet, the next step was to evaluate the accuracy of the technique. Figure 134 shows the orientation of the rotating disk experiment and the optical arrangement to measure the tangent velocity. The disk was an 8.0 cm diameter black disk which was rotated with a high speed motor controlled by a variable transformer which could be controlled from 0 to 20,000 RPM. The RPM was measured using a timing light and a photodiode which recorded the pulse rate on an HP 54501
digital oscilloscope. The partial laser beam split from the Nd:YAG pulsed laser was expanded into a sheet and focused onto the disk creating a vertical line through the disk center. For the velocity discriminating camera, Figure 135 shows the intensity as a function of the reference camera intensity. Figure 136 is the iodine profile used in the disk experiment which is line A of Figure 44. The cell was operated at $T_{I2} = 45^\circ$ C and $P_{N2} = 5$ torr. For the optical configuration given in Figure 134 the absorption profile has an intensity velocity profile as given in Figure 137 with the laser frequency corresponding to zero velocity.

Figure 138 shows the average tangential velocity profiles, both measured and calculated for three different RPM settings. The averages are calculated from 100 instantaneous images. The symbols are used to show the measured velocity and the velocity calculated for each case from the measured RPM is shown by a line. Through a majority of the profiles the error seems to be about $\pm 10$ m/s. The velocity of the center is not given since the beam was deflected by a bolt that held the disk to the rotating shaft. The velocity profiles from 6 instantaneous images are given in Figure 139. For these images the scatter seems to be approximately $\pm 25$ m/s independent of the average velocity. The RMS was calculated to be approximately 15 m/s for all locations. Similar to the method presented above, Meyers and Komine [1991] used a rotating disk to roughly calibrate the DGV and reported approximately the same estimates of the accuracy.

Although this system seemed to be sufficient for validation of the technique the resulting errors are higher than expected. Unfortunately, the disk system itself is not
free from errors. First, the RPM of the motor was found to fluctuate approximately ±
10%. One difficulty inherent in the disk experiment, was the vibration of the disk
encountered when running at these high RPM's. This caused the intensity of the laser
sheet to vary differently for each camera, which degraded the measurements since this
changed the intensity ratio artificially. Also, irregularities (bumps, scratches, particles
etc.) on the disk would have this same effect.

4.4.5.4. Error Analysis

An analysis of the uncertainties associated with each component of the
measurement system can give a reading on the uncertainty associated with the velocity
measurement. As discussed in many texts on error analysis, the absolute error can be
calculated if the property measured can be written as a function of the independent
variables. Beckwith et al. [1982] defines the overall uncertainty as

\[
\Delta u_f = \sqrt{\left( \Delta u_1 \frac{\partial f}{\partial u_1} \right)^2 + \left( \Delta u_2 \frac{\partial f}{\partial u_2} \right)^2 + \left( \Delta u_3 \frac{\partial f}{\partial u_3} \right)^2 + \ldots} \tag{37}
\]

where \( f(u_1, u_2, u_3, \ldots) \) is the property measured as a function of the independent variables
which have corresponding errors of \( \Delta u_1, \Delta u_2, \Delta u_3, \ldots \). Before the absolute error can be
written for the molecular filter velocity technique, the frequency shift from the filter
profile \( [\Delta f_d = \Delta f(I/I_0)] \) must be expressed. For most of the images taken, a filter
profile and laser frequency are selected so that the expected frequency shift occurs in
the linear region of the filter profile. Therefore, for the uncertainty analysis, the frequency profile is approximated by

\[ \Delta f_D = f(I/I_0) = m_A(I/I_0) + b_A \]  

(38)

where \( m_A \) and \( b_A \) are determined from a linear curve fit. One important thing to recognize is that \( b_A \) is a function of the laser frequency, \( f_0 \), therefore any fluctuation in laser frequency will change this quantity. The uncertainty is calculated by applying equation 36 to equation 37, with the resulting equation given by:

\[ \Delta V = \sqrt{\left( \frac{\partial V}{\partial \phi_D} \right)^2 + \left( \frac{\partial V}{\partial \gamma_D} \right)^2 + \left( \frac{\partial V}{\partial \theta_D} \right)^2 + \left( \frac{\partial V}{\partial \phi_0} \right)^2 + \left( \frac{\partial V}{\partial \gamma_0} \right)^2 + \left( \frac{\partial V}{\partial \theta_0} \right)^2 + \left( \frac{\partial V}{\partial b_A} \right)^2 + \left( \frac{\partial V}{\partial b_0} \right)^2} \]  

(39)

The left three terms are termed systematic errors since they are the same for each reading, and the last four terms are random errors which can change with each reading [Doeblin, 1975]. In practice the systematic error can be eliminated with calibration or modeling.

Before evaluating the uncertainty, the error of each independent variable must be estimated. For the optical arrangement, the errors in measuring the angles \((\phi_D, \gamma_D, \theta_D)\) are assumed to be \( \pm 3.0^\circ \). Sources of this inaccuracy are the inability to accurately measure the angles, the slight expansion of the laser sheet, the viewing angle...
(field of view) of the camera, and the aperture of the camera lens \( f\# = 16 \) giving an angle of 1.8°). Although a larger \( f\# \) would be desirable, this would decrease the signal level below desirable levels. For the intensities \( I' \) and \( I_0 \), the error depends on the cameras used. For the Princeton Instruments cameras this variation is quoted as ± 2.3% of the reading. To check this, a camera was focused on an object illuminated by ambient sun light. Assuming that the illumination is constant, the intensity variation was measured to be about ± 2%, which will be used in the subsequent calculations.

The frequency variation of the laser is incorporated in \( b_A \) from the linear curve fit of the filter profile. Quanta Ray, the manufacturer of the GCR-4 Nd:YAG laser quotes a fluctuation less than 10 MHz when the laser is seeded. There is a way to quantify this fluctuation in laser frequency. This was done by setting the laser at the 50% absorption point of a temperature broadened filter (Figure 45) and recording instantaneous images with the same setup used to measure the absorption profiles (Figure 43). A change in intensity ratio \( (I/I_0) \) is a direct result of a fluctuation in frequency. Figure 140 gives a trace of the variation in normalized intensity at a single pixel location for 100 instantaneous images. The RMS of the laser frequency is measured to be 10.1 MHz. One should note that since the frequency fluctuation measurements were made using the Princeton Instruments CCD camera the fluctuations recorded here also include errors resulting from the cameras. Therefore the uncertainty is estimated to be twice the RMS which is 20.2 MHz.

The absolute error is calculated for the two optical arrangements shown in Figure 141 which were used to obtain velocity measurements in the mixing layer. The oblique
view of Figure 141a (similar to the jet feasibility study) was used in the $M_c = 0.86$ planar mixing layer and the streamwise view (Figure 141b) was used for the $M_c = 0.51$ mixing layer. The constants for the curve fits used in the inter-camera intensity correction (Eqn. 35) and in modeling the filter profile (Eqn. 38) are given in Table 5. The individual terms that comprise the absolute error are given in Appendix C. Looking at these terms it is seen that the error is a function of the intensities recorded by the velocity discriminating and reference cameras. The absolute error is given in Figures 142 and 143 for the $M_c = 0.51$ and $M_c = 0.86$ mixing layer optical arrangements respectively in terms of the velocity discriminating camera intensity $I'$, and the intensity ratio $(I/I_0)$. Since it will be shown that $I'$ turned out to be equal to $I$ for the $M_c = 0.86$ mixing layer, the error is only a function of the intensity ratio. The estimated total error for the technique for the conditions given above is approximately ±12%.

In order to reduce the total absolute error each component should be examined. Table 6 contains the error for each of the independent variables. First, better methods of measuring the angles would reduce this error. The largest error in the angles is mostly caused by the lens through the viewing angle and aperture. It may be possible to increase the f# if the signal level permits and longer focal length lenses can reduce the viewing angle. The best method, however, is to take the lens effects into account when reducing the raw data by using different angles to compute the Doppler shift at each pixel. Cameras with a smaller intensity fluctuation would also help, although the accuracy to be gained is relatively small for what would certainly be a large capital cost.
Another source of error is associated with the laser frequency fluctuations present in the $\Delta b_\lambda(\partial V/\partial b_\lambda)$ term. Although there is current work to reduce this uncertainty in Nd:YAG lasers, a more practical solution is to monitor the frequency fluctuation for each instantaneous image. By using an additional detector (such as a photodiode), this fluctuation in frequency could be recorded with each image by splitting off a very small portion of the laser beam and passing it through a separate molecular filter so that the nominal laser frequency would be set at the 50% absorption point. This is similar to the measurements recorded in Figure 140. Incorporating these improvements, it appears that the error could be reduced to about ± 3%.

4.4.5.5. $M_e = 0.86$ Mixing Layer Velocity Measurements

Figure 141a gives a basic schematic of the oblique view optical arrangement of the $M_e = 0.86$ mixing layer with the employed angles given in Figure 144. As indicated in Figure 144, in this configuration the system is sensitive to the streamwise velocity. A comparison of the intensities recorded by the two cameras without the molecular filter in place is shown in Figure 145. The plot of Figure 145 shows that the relationship between the cameras is not only linear, but for this case the gains were set so that the counts were almost identical. One problem encountered when gathering images in this configuration was an increased background signal from walls and windows relative to other optical configurations. This background signal would be recorded in the reference image, but not the filtered image since non-shifted signals are attenuated. The level of this signal approached 50% of the signal scattered from the
flow field in some locations. Left uncorrected, this would result in the calculated velocities being artificially low since the intensities recorded in the reference image used to normalize the velocity-discriminated image would be artificially high.

One solution to the problem of significant background scattering is shown in Figure 146 which illustrates a concept for filtered background suppression and velocity discrimination (FBSVD). This method requires two cameras and two filters with different characteristics, but both operated in the optically thick regime. The first filter (Figure 146a) is used as the velocity discriminator, but it should be noted that zero and negative Doppler-shift scattering is attenuated. This is the pressure broadened filter which has the characteristic of increasing recorded intensity as the Doppler shift increases. As discussed previously, the recorded intensities must be normalized with an image that is not velocity-discriminated in order to discern the frequency content of the scattering. For the reference camera, a steep thermally-broadened absorption filter that totally passes all Doppler shifted scattering and fully absorbs all background scattering is used (Figure 146b). Figure 147 shows the absorption profiles of line C (Figure 44) used for the FBSVD technique for both the frequency discriminating camera and the reference camera. The velocity discriminating profile in terms of transmission versus velocity for the given optical configuration is given in Figure 148.

Figure 149 gives the average velocity image for the oblique view of the $M_c = 0.86$ mixing layer in the fully developed region using the product formation technique. The average was only calculated at points where the signal level was above 500 counts.
to avoid problems at low signal levels. As expected, the velocity is higher at the top of the mixing layer and falls off toward the bottom. Profiles at five spanwise locations are given in Figure 150 and nondimensionally in Figure 151. For the shear layer of the jet experiment, the highest and lowest velocities measured in the shear layer were slightly lower and higher than the free stream velocities, respectively. The high speed side for this case however is approximately the expected free stream velocity of 600 m/s. This is probably due to the fact that there is a small amount of passive scalar signal in the supersonic free stream, allowing free stream measurements. The lower velocity side of the shear layer is again at a slightly higher velocity than expected, for the same reasons as given previously in the jet results. Although this velocity bias caused by the marking ice particles is present, the collapse of the nondimensional profiles in Figure 151 with previously measured LDV profiles at roughly the same position is quite good.

The best use of this data for furthering our understanding of the compressible mixing layer is to investigate instantaneous velocity content of the structures in the mixing layer. Figures 152 and 153 show instantaneous images from the reference camera and the corresponding calculated velocity image. One characteristic that is noticeable is that the velocity within the mixing layer seems to roughly vary with the local mean velocity. There are images, however, that show the high speed velocity extending into the lower portions of the mixing layer (Images 153a and 153b). Although this oblique view has the advantage of measuring the actual streamwise velocity, the disadvantage is that individual structures are not recognizable since the
sheet is at a 45° angle to the flow direction, and the flow has been shown to be three dimensional.

4.4.5.6. $M_e = 0.51$ Mixing Layer Velocity Measurements

The best view of the nominally two-dimensional structures that populate the mixing layer is the streamwise (X-Y) view which was employed for the $M_e = 0.51$ mixing layer. Figure 141b gives the basic schematic of the position of the cameras in this view with the vectors and angles shown in Figure 154. It should be noted that the velocity being measured is not the streamwise component only, but also has lateral and spanwise components. The gain settings for the two cameras gave an intensity correction plot as shown in Figure 155. The filter conditions used in this set of experiments were the same as those used in the disk experiments with $T_{I2} = 45°$ C and $P_{N2} = 5$ torr. For the employed optical arrangement, the velocity-transmission profile is given in Figure 156. It should be noted that the background signal for this optical arrangement was relatively low ($\approx 1\%$ of the scattered signal) so a filter was not used in front of the reference camera.

The average velocity image is given in Figure 157. In calculating the velocity signals, pixels with readouts below 500 counts were again disregarded to avoid errors at low light levels. Thresholding the signal at this level was probably slightly conservative as will be seen in the instantaneous images. As expected, the velocity image is brighter at the high speed side of the mixing layer and gradually decreases in brightness towards the lower edge. Assuming the lateral and spanwise velocity
components are small in the mean, the streamwise velocity profiles can be calculated by projecting the measured velocity component onto the x-axis. Figure 158 gives 4 profiles through different portions of the mixing layer. As was the case for the jet feasibility study, the measured velocity is slightly higher on the low speed side (240 m/s compared to 173 m/s) and the high speed side of the mixing layer is low (400 m/s compared to 492 m/s). Again this is probably due to the product formation signal only marking the mixed region, not the molecular filter velocimetry technique.

Instantaneous images of the reference camera and velocity are given in Figures 159 - 161. Many of the images in Figures 159 and 160 show the roller type large scale structures. Towards the top of the structures' core region the velocity is high as indicated by the brighter portions on the images. The velocity decreases gradually toward the bottom of the structures. Within the structures there are slight variations, but the bulk of the core region goes with this decreasing trend. This can be seen clearly in Figure 162 which is a velocity contour map of the structure shown in Image 159b. The velocity variation in the structures indicate that they contribute to the shear stress and do not just convect with a uniform velocity. Also, the velocity variations are necessary for tearing to occur. The braid regions in these images seem to show a high amount of shear as evidenced by the rapid decrease of velocity (as indicated by the intensity) across a thin spacial region. Figure 161 shows some images of structures that are much less organized than the well defined roller type structures which are more common in the $M_e = 0.51$ mixing layer. There appears to be more locations experiencing extreme shear in these structures which have an apparent mixture of high
and low speed fluid throughout. Some images (Images 161a and 161b) seem to indicate structures that were found commonly in the higher convective Mach number case where the high speed side of the shear layer seems to be stretching the lower speed fluid into the mixing layer. This is a very different entrainment mechanism than that typically attributed to the large roller type structures.
CHAPTER V.

CONCLUSIONS

High Reynolds number compressible mixing layers with convective Mach numbers \( (M_c) \) of 0.51 and 0.86 have been investigated. Profiles of mean velocities and turbulence statistics measured with laser Doppler velocimetry were reviewed for both the developing and fully developed regions of the mixing layer. In the developing region, the profiles of the measured turbulence quantities displayed decreasing levels with increasing streamwise distance. This continued until the establishment of the fully developed profiles. This evolution of the turbulence profiles is similar to that found in incompressible mixing layers. In the fully developed region, the decrease in growth rate found by other investigators to occur with increasing convective Mach number was confirmed. Additionally, the fully developed profiles of the turbulence quantities were shown to decrease in both level and lateral extent with increasing convective Mach number. This trend was confirmed by the shear stress profiles calculated from the measured mean velocity profiles. Although these LDV results provide much insight into the effect of compressibility level on the global characteristics and turbulence statistics
of the compressible mixing layer, information about the large scale structures is limited.

As a first step in obtaining information about the large scale structures in compressible mixing layers, static pressure fluctuations were measured at two points in the free shear layer using high frequency response transducers. The power spectra of the pressure fluctuations obtained at various probe locations show that the peak frequency decreases with increasing streamwise distance for both convective Mach numbers. This reflects the fact that the large scale structures, which represent the more energetic low frequency portions of the spectra, have longer lifetimes than smaller scale motions, interact with each other and evolve as they proceed downstream. The $M_c = 0.86$ mixing layer displays much broader peaks, suggesting that large scale structures are less organized than those of the $M_c = 0.51$ mixing layer. This is particularly evident in the streamwise coherence, which decreases much more rapidly for the higher convective Mach number case with increasing probe separation, suggesting that the structures within the flow are not as well preserved as those of the $M_c = 0.51$ mixing layer. A similar trend is present in the streamwise space-time correlations. The convective velocity calculated from the space-time correlations was shown to vary laterally within the mixing layer. For the $M_c = 0.86$ mixing layer the convective velocity varied approximately with the mean velocity of the mixing layer. One of the most dramatic results from the pressure fluctuation measurements were the angles calculated from the spanwise space-time correlations. For the $M_c = 0.51$ mixing layer the structures appear to be two dimensional, but for the $M_c = 0.86$ mixing layer the emergence of oblique angles was apparent. This trend has been seen previously in the
theoretical and computational results of other investigators.

Using a molecular filter for background suppression in the FRS technique, planar images of the mixing layers were obtained. The characteristics of the iodine molecular filter were investigated, including the effects of varying iodine number density and pressure broadening effects from the addition of nitrogen.

Condensed ice particles were used as the scattering medium and images were taken with both product formation (marking the mixing region) and passive scalar (marking the high speed freestream) techniques. Visualizations for the lower convective Mach number, \( M_c = 0.51 \), show organized large scale structures that are largely two dimensional, with core and braid regions similar to those seen in incompressible shear layers. Plan views show streamwise streaks in the developing region that resemble streamwise vortices observed in incompressible shear layers. For the higher convective Mach number, \( M_c = 0.86 \), large scale structures are apparent, but occur much more randomly in space and time. At this higher convective Mach number, large scale structure activity is much more pronounced near the low speed edge of the mixing layer as protrusions of large scale structures into the high speed freestream are seldom found. In accord with the spanwise pressure correlations results, evidence of spanwise obliqueness was present in the plan view visualizations for the higher convective Mach number.

Statistical quantities such as average, RMS, intermittency, and spatial correlations were calculated from the instantaneous images. The visual growth rate calculated from averaged images shows the same decrease with increasing convective
Mach number as observed in the LDV measurements. The RMS intensity profiles from product formation images possess a bimodal shape due to the essentially constant scalar concentration within the large scale structures. A decrease in level and lateral extent in the RMS was again found to occur with increasing convective Mach number. Intermittency profiles were fuller for the $M_c = 0.86$ mixing layer, again indicating the reduced excursions of the large scale structures into the free stream at the higher convective Mach number. Spatial correlations centered at various lateral locations indicate that the high speed side of the "average structure" is inclined in the streamwise direction with the orientation expected from the mean shear, particularly for the higher convective Mach number case.

Images were also obtained using Rayleigh scattering from air molecules for the $M_c = 0.51$ mixing layer using the filtered Rayleigh scattering technique. The resulting images are similar in appearance to the scalar transport images. Approximate density profiles were acquired at the convective Mach numbers investigated, but true density profiles were unattainable. This is due to the inability to fully Doppler shift the increased spectral width of the molecular Rayleigh scattering out of the absorption filter (while keeping the laser frequency within the absorption well) at the Mach numbers investigated.

In order to investigate the evolution of large scale structures, double-pulsed images were taken for both streamwise and plan views. For the $M_c = 0.51$ mixing layer, images in the developing region showed the formation of the roller structures to consist of the "roll-up" of waves that emanated from the splitter plate. Further
downstream the large scale structures undergo pairing and tearing processes similar to those seen in incompressible mixing layers. For the higher convective Mach number the structures are much less visually well-defined and seem to have a significantly shorter lifetime. Correlations between the initial and delayed images were calculated to quantify the reduction in large scale organization and structure lifetime in the $M_c = 0.86$ mixing layer relative to $M_c = 0.51$. Also, the correlations indicated what appeared to be a structure "stretching" in the higher convective Mach number case, which again probably reflects their reduced lifetime.

The feasibility of molecular filter-based velocity imaging techniques was investigated. Pressure broadening was used to "tune" the absorption profile of the filter to a given flow field/ optical arrangement. It was shown that two filters could be used: one as the velocity-discriminating filter, the other to eliminate unwanted background light from the reference camera. Two different configurations were used to investigate the $M_c = 0.51$ and 0.86 mixing layers. Uncertainty analysis showed that the error was conservatively $\pm 12\%$ for the current measurements, but improvements to reduce the error to $\pm 3\%$ are suggested. Images taken of the roller type structures in the $M_c = 0.51$ mixing layer show a variation in velocity within the structure similar to the lateral distribution of mean velocity. For less organized structures the velocity within the structure did not display this well-defined distribution, but instead variations were much more irregular. Molecular filter-based velocimetry techniques were shown to be feasible for the study of high speed flows, but additional work is needed to reduce the error associated with these techniques.
APPENDIX A

TABLES
TABLE 1. Incoming flow conditions

<table>
<thead>
<tr>
<th>Source</th>
<th>$P_0$ (kPa)</th>
<th>$T_0$ (K)</th>
<th>$M_1$</th>
<th>$U_1$ (m/s)</th>
<th>$\delta$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>$Re_\theta$</th>
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</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>314</td>
<td>291</td>
<td>1.83</td>
<td>492</td>
<td>8</td>
<td>0.62</td>
<td>27700</td>
</tr>
<tr>
<td>Case 2</td>
<td>314</td>
<td>291</td>
<td>1.83</td>
<td>492</td>
<td>8</td>
<td>0.62</td>
<td>27700</td>
</tr>
<tr>
<td>Case 3</td>
<td>722</td>
<td>276</td>
<td>3.01</td>
<td>597</td>
<td>9.2</td>
<td>0.37</td>
<td>24700</td>
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TABLE 2. Mean flow parameters

<table>
<thead>
<tr>
<th>Source</th>
<th>$M_c$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$U_2/U_1$</th>
<th>$\rho_2/\rho_1$</th>
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<tr>
<td>Case 1</td>
<td>0.51</td>
<td>1.80</td>
<td>0.51</td>
<td>0.36</td>
<td>0.64</td>
</tr>
<tr>
<td>Case 2</td>
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<td>1.96</td>
<td>0.37</td>
<td>0.25</td>
<td>0.58</td>
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<tr>
<td>Case 3</td>
<td>0.86</td>
<td>3.01</td>
<td>0.45</td>
<td>0.25</td>
<td>0.37</td>
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### TABLE 3. Molecular Rayleigh Scattering Parameters for $M_c = 0.51$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$y_p$</th>
<th>$\Delta f_D$ (GHz)</th>
<th>$f - f_0$ (GHz)</th>
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<tr>
<td>1.8</td>
<td>1.34</td>
<td>0.74</td>
<td>1.24</td>
</tr>
<tr>
<td>0.51</td>
<td>0.74</td>
<td>0.25</td>
<td>1.5</td>
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### TABLE 4. Double-pulse time delays

<table>
<thead>
<tr>
<th>$M_c$</th>
<th>$x_c$</th>
<th>$\tau$ [\mu s]</th>
<th>$U_1 \tau$ [mm]</th>
<th>$U_2 \tau$ [mm]</th>
<th>$U_c \tau$ [mm]</th>
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<tr>
<td>0.51 (dev.)</td>
<td>1.4</td>
<td>38.1</td>
<td>18.7</td>
<td>6.74</td>
<td>12.9</td>
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<tr>
<td>0.51</td>
<td>0.64</td>
<td>34.7</td>
<td>16.2</td>
<td>5.72</td>
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<td>0.51</td>
<td>1.18</td>
<td>63.7</td>
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<td>0.51</td>
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<td>92.6</td>
<td>43.3</td>
<td>15.3</td>
<td>31.2</td>
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<tr>
<td>0.86 (dev.)</td>
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<td>30.0</td>
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<td>4.48</td>
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<tr>
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<td>0.64</td>
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<td>10.7</td>
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<td>26.9</td>
<td>6.75</td>
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<tr>
<td>0.86</td>
<td>1.71</td>
<td>65</td>
<td>38.8</td>
<td>9.75</td>
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TABLE 5. Curve fits for camera conversion and filter profile

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<th>$M_e = 0.51$</th>
<th>$M_e = 0.86$</th>
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<td>0</td>
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<td>$m_A$</td>
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<td>$1.80 \times 10^9$</td>
</tr>
<tr>
<td>$b_A$</td>
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<td>$1.92 \times 10^8$</td>
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TABLE 6. Percent error for various components

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<tr>
<th>Sources of Error</th>
<th>Maximum % error in Vel.: $M_e = 0.51$</th>
<th>Maximum % error in Vel.: $M_e = 0.86$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I'$</td>
<td>1.17</td>
<td>1.70</td>
</tr>
<tr>
<td>$I_0$</td>
<td>1.20</td>
<td>1.70</td>
</tr>
<tr>
<td>$\phi_D$</td>
<td>1.74</td>
<td>0</td>
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<tr>
<td>$\gamma_D$</td>
<td>0</td>
<td>2.18</td>
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<tr>
<td>$\theta_D$</td>
<td>4.36</td>
<td>2.21</td>
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<tr>
<td>$b_A$</td>
<td>6.17</td>
<td>5.41</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$3.57 \times 10^{-6}$</td>
<td>$3.57 \times 10^{-6}$</td>
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</table>
Figure 1. Schematic of the planar compressible mixing layer.

Growth rate $= \frac{db}{dx}$
Figure 2. Conceptual model of the large scale rollers and accompanying streamwise structures derived from studies of incompressible mixing layers (as suggested by Bernal and Roshko[1986]).
Figure 3. Schematics of various large scale structure evolution processes.
Notes:
1) $\theta_D$ and $\gamma_D$ are in the plane defined by the vectors $\vec{k}_0$ and $\vec{V}$.
2) $\phi_D$ is the angle between $\vec{k}_s$ and the plane defined by the vectors $\vec{k}_0$ and $\vec{V}$.

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Figure 6. Schematic of the optically thin molecular filter employed in Doppler Global Velocimetry.
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Figure 9. Internal configuration of the two stream supersonic wind tunnel.
Figure 10. Photograph of the free jet facility at the AARL.
Figure 11. Experimental setup for the two-point fluctuating pressure measurements.
Figure 12. Schematic of the camera and laser sheet configuration for double-pulse visualizations.
Figure 13. Top view schematic of the optical configuration for schlieren photography.
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Figure 16. Normalized vorticity thickness growth rate versus convective Mach number for various experiments.
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Figure 20. Reynolds stress profiles at various streamwise locations for $M_c = 0.51$. 
Figure 21. Dimensionless mean velocity profiles at various streamwise locations in the fully developed region for three convective Mach numbers.
Figure 22. Mean velocity profiles in the fully developed region from various experiments nondimensionalized with vorticity thickness.
Figure 23. Fully developed streamwise turbulence intensity profiles for various levels of compressibility.
Figure 24. Fully developed lateral turbulence intensity profiles for various levels of compressibility.
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Figure 28. Fully developed Reynolds stress profile multiplied by the mean density profile.
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Figure 30. Streamwise evolution of the power spectrum of the pressure fluctuations for $M_c = 0.51$. 
Figure 31. Streamwise evolution of the power spectrum of the pressure fluctuations for $M_c = 0.86$. 
Figure 32. Streamwise coherence for various probe separations in the fully developed region at $X_s = 162$ mm and $(y - y_c)/\delta_w = 0$ for a) $M_c = 0.51$ and b) $M_c = 0.86$. 
Figure 33. Streamwise space-time correlations for various probe separations in the fully developed region at $X_s = 162$ mm and $(y - y_c)/\delta_x = 0$ for a) $M_c = 0.51$ and b) $M_c = 0.86$. 
Figure 34. Dimensionless convective velocity versus mean velocity at the same lateral location for $M_c = 0.51$ and $0.86$. 
Figure 35. Spanwise space-time correlations for various probe separations at $X_s = 150$ mm and $(y - y_c)/\delta_o = -0.227$ for $M_c = 0.51$. 
Figure 36. Spanwise space-time correlations at various lateral locations with $X_s = 150$ mm and $dz_s = 6.5$ mm for $M_c = 0.86$. 
Figure 37. Full view (a and b), upstream (c and d), and downstream (e and f) schlieren photographs of the $M_c = 0.51$ mixing layer.
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Protrusions of large scale structures into the supersonic freestream are strongly attenuated.

Well-defined protrusions of large scale structures into the subsonic freestream.

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APPENDIX C

UNCERTAINTY ANALYSIS FOR PLANAR VELOCIMETRY TECHNIQUE
The expression for the velocity measurement is given by

\[ V = \frac{\lambda m_A m_I \left( \frac{I'}{I_o} \right) + m_A b_I \left( \frac{1}{I_o} \right) + b_A}{(\cos \phi_D \cos \gamma_D - \cos \theta_D)} \]  

(40)

The corresponding uncertainty in the velocity measurement is given by

\[ \Delta V = \sqrt{\left( \frac{\partial V}{\partial \phi_D} \right)^2 + \left( \frac{\partial V}{\partial \gamma_D} \right)^2 + \left( \frac{\partial V}{\partial \theta_D} \right)^2 + \left( \frac{\partial V}{\partial I_I} \right)^2 + \left( \frac{\partial V}{\partial I_o} \right)^2 + \left( \frac{\partial \phi_D}{\partial \phi_D} \right)^2} \]  

(41)

where the included partial derivatives are given by

\[ \frac{\partial V}{\partial \phi_D} = \frac{\lambda m_A m_I \left( \frac{I'}{I_o} \right) + m_A b_I \left( \frac{1}{I_o} \right) + b_A}{(\cos \phi_D \cos \gamma_D - \cos \theta_D)^2} \cdot (\cos \gamma_D \sin \phi_D) \]  

(42)

\[ \frac{\partial V}{\partial \gamma_D} = \frac{\lambda m_A m_I \left( \frac{I'}{I_o} \right) + m_A b_I \left( \frac{1}{I_o} \right) + b_A}{(\cos \phi_D \cos \gamma_D - \cos \theta_D)^2} \cdot (\cos \phi_D \sin \gamma_D) \]  

(43)

\[ \frac{\partial V}{\partial \theta_D} = \frac{\lambda m_A m_I \left( \frac{I'}{I_o} \right) + m_A b_I \left( \frac{1}{I_o} \right) + b_A}{(\cos \phi_D \cos \gamma_D - \cos \theta_D)^2} \cdot (\sin \theta_D) \]  

(44)

\[ \frac{\partial V}{\partial I_I} = \frac{m_A m_I \lambda}{I_o (\cos \phi_D \cos \gamma_D - \cos \theta_D)} \]  

(45)

\[ \frac{\partial V}{\partial I_o} = \frac{-\lambda m_A [m_I I' + b_I]}{(I_o)^2 (\cos \phi_D \cos \gamma_D - \cos \theta_D)} \]  

(46)
\[
\frac{\partial V}{\partial b_A} = \frac{\lambda}{(\cos \phi_D \cos \gamma_D - \cos \theta_D)},
\]

and

\[
\frac{\partial V}{\partial \lambda} = \frac{m_A m_I \left( \frac{I'}{I_0} \right) + m_A b_I \left( \frac{1}{I_0} \right) + b_A}{(\cos \phi_D \cos \gamma_D - \cos \theta_D)}.
\]

The estimated uncertainties as discussed in the text are given by

\[
\Delta \phi_D = \pm 3^\circ,
\]

\[
\Delta \gamma_D = \pm 3^\circ,
\]

\[
\Delta \theta_D = \pm 3^\circ,
\]

\[
\Delta I' = \pm 0.02 I',
\]

\[
\Delta I_o = \pm 0.02 I_o,
\]

and

\[
\Delta \lambda = \pm 1.9 \times 10^{-14} m.
\]
LIST OF REFERENCES


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