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Vibro-acoustic analysis of computer disk drive components with emphasis on electro-mechanical noise sources

Lee, Ming-ran, Ph.D.
The Ohio State University, 1993
VIBRO-ACOUSTIC ANALYSIS OF
COMPUTER DISK DRIVE COMPONENTS WITH EMPHASIS ON
ELECTRO-MECHANICAL NOISE SOURCES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Ming-ran Lee, B.S.M.E., M.S.M.E.

* * * *

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1993

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Adviser
Department of Mechanical Engineering
To my parents
ACKNOWLEDGMENTS

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>VITA</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xvi</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I</strong> INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Problem Formulation</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Objectives</td>
<td>4</td>
</tr>
<tr>
<td>REFERENCES FOR CHAPTER I</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>II</strong> NOISE SOURCE: ELECTROMAGNETIC TORQUE</td>
<td>8</td>
</tr>
<tr>
<td>PULSATIONS</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>8</td>
</tr>
<tr>
<td>2.2 Literature Review</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Experimental Investigation</td>
<td>12</td>
</tr>
<tr>
<td>2.4 Analytical Formulation</td>
<td>14</td>
</tr>
<tr>
<td>2.4.1 Model I [Phase Variable Formulation]</td>
<td>18</td>
</tr>
<tr>
<td>2.4.2 Model II [Fourier Series Representation]</td>
<td>23</td>
</tr>
<tr>
<td>2.5 Results and Concluding Remarks</td>
<td>33</td>
</tr>
</tbody>
</table>
REFERENCES FOR CHAPTER II ....................................................... 39

III NOISE SOURCE: STEADY STATE ANALYSIS OF A TIME VARYING BRUSHLESS D.C. MOTOR BY USING THE GALERKIN'S METHOD ................................................................. 41

3.1 Introduction ................................................................................... 41

3.2 Problem Formulation .................................................................... 43

3.2.1 Fourier Coefficients of Inductance and Back E.M.F. ..... 43

3.2.2 Scope and Objectives .............................................................. 48

3.3 Linear Time Varying (LTV) Model: Equivalent Electrical Circuits without Mechanical System Dynamics ........................... 48

3.3.1 3-DOF LTV Model ................................................................ 49

3.3.2 Reduced LTV Model .............................................................. 61

3.4 Non-Linear Time Varying (NLTV) Model: Equivalent Electric Circuits with Mechanical System Dynamics ....................... 66

3.4.1 4-DOF NLTV Model .............................................................. 69

3.4.2 Reduced NLTV Model ......................................................... 71

3.5 Conclusion ..................................................................................... 74

REFERENCES FOR CHAPTER III ..................................................... 79

IV RADIATOR: VIBRO-ACOUSTIC CHARACTERISTICS OF A COMPUTER DISK ................................................................. 80

4.1 Introduction ................................................................................... 80

4.2 Literature Review .......................................................................... 81

4.3 Problem Formulation .................................................................... 84

4.4 Modal Representation .................................................................... 85

4.4.1 Exact and Approximate Solutions of a Stationary Disk ... 85

4.4.2 Experimental Results and Analytical Predictions ............. 90

4.5 Modal Radiation Efficiency Formulations ................................. 94

4.5.1 Far-field Approach ............................................................... 94

4.5.2 Impedance Approach ........................................................... 96

4.5.3 Rigid Body Modes ............................................................... 102

4.6 Modal Coupling effects ............................................................... 102
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Overview of objectives and methods.</td>
<td>5</td>
</tr>
<tr>
<td>2.1</td>
<td>Measured dominant pure tones between 1.3 and 6.1 KHz and corresponding free field sound pressure levels (in relative decibels).</td>
<td>16</td>
</tr>
<tr>
<td>2.2</td>
<td>Harmonics of motor torque (as yielded by Model II).</td>
<td>31</td>
</tr>
<tr>
<td>2.3</td>
<td>Data set used for analysis.</td>
<td>34</td>
</tr>
<tr>
<td>3.1</td>
<td>Harmonic groups.</td>
<td>51</td>
</tr>
<tr>
<td>3.2</td>
<td>Comparison of mean angular velocity</td>
<td>75</td>
</tr>
<tr>
<td>4.1</td>
<td>Dimensions and material properties of the sample computer disk.</td>
<td>86</td>
</tr>
<tr>
<td>4.2</td>
<td>Selected natural frequencies of a disk.</td>
<td>91</td>
</tr>
<tr>
<td>4.3</td>
<td>Predicted modal radiation efficiencies of a stationary disk.</td>
<td>100</td>
</tr>
<tr>
<td>4.4</td>
<td>Dimensions and material properties of the scaled-up steel disk.</td>
<td>102</td>
</tr>
<tr>
<td>4.5</td>
<td>Radiation efficiency of a rotating disk at $\Omega_r/2\pi = 72$ Hz.</td>
<td>122</td>
</tr>
<tr>
<td>5.1</td>
<td>Bearing parameters for example disk drive.</td>
<td>132</td>
</tr>
<tr>
<td>5.2</td>
<td>Bearing stiffnesses.</td>
<td>132</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Problem formulation of the computer disk drive; (a) key components, (b) source-path-radiator network examined by this study.</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Experimental set-up.</td>
<td>13</td>
</tr>
<tr>
<td>2.2</td>
<td>Measured spectra (in relative decibels); (a) structural acceleration on the casing, (b) free field sound pressure level with casing cover, (c) free field sound pressure level without casing cover</td>
<td>15</td>
</tr>
<tr>
<td>2.3</td>
<td>Schematic of a 3-phase, 2-pole brushless d.c. motor (BDCM). (a) the current inverters and phase circuits and (b) the cross-sectional view of rotor and stator.</td>
<td>19</td>
</tr>
<tr>
<td>2.4</td>
<td>Air-gap flux density distribution modulated by open stator slots.</td>
<td>22</td>
</tr>
<tr>
<td>2.5</td>
<td>Motor torque pulsations generated by the PWM switching: (a) 120° discrete type switching logic. (b) 180° continuous type switching logic.</td>
<td>24</td>
</tr>
<tr>
<td>2.6</td>
<td>Assumed trapezoidal rotor flux density distribution and the quasi-square line-to-neutral phase voltage.</td>
<td>26</td>
</tr>
<tr>
<td>2.7</td>
<td>PWM pulse train of period τ.</td>
<td>32</td>
</tr>
<tr>
<td>2.8</td>
<td>Normalized rotor angular velocity fluctuation.</td>
<td>35</td>
</tr>
<tr>
<td>2.9</td>
<td>Motor torque pulsation $T_e(f)$ predictions (in relative decibels) by using Model I (---) and Model II (○).</td>
<td>36</td>
</tr>
</tbody>
</table>
2.10 Comparison of measured and predicted pure tones given $N_p \Omega_s/2\pi$ =72 Hz, $N_p$=8 and $N_s$=9. Also refer to Table 2.1 and Figures 2.2 and 2.9 ................................................................. 38

3.1 Coordinate system of a rotor and a stator with eccentricity $e$. ............ 44

3.2 Flow chart of numerical scheme for the Galerkin's method. ................. 58

3.3 Comparison of the steady-state phase current of the LTV model with combined effects; (a) time domain (b) frequency domain. Key: solid line: the numerical integration; circles: the Galerkin's method. ......... 59

3.4 Comparison of the steady-state torque of the LTV model with combined effects; (a) time domain (b) frequency domain. Key: solid line: the numerical integration; dotted line and circles: the Galerkin's method. ................................................................. 60

3.5 Steady-state torque of the LTV model with eccentricity effect; (a) time domain (b) frequency domain. Key: solid line: the numerical integration; circles: the Galerkin's method. .......................... 62

3.6 Steady-state torque of the LTV model with the saturation effect; (a) time domain (b) frequency domain. Key: solid line: the numerical integration, dotted line and circles: the Galerkin's method. .............. 63

3.7 Steady-state torque of the LTV model with the open stator slot effect; (a) time domain (b) frequency domain. Key: solid line: the numerical integration, dotted line and circles: the Galerkin's method. 64

3.8 Comparison of the steady-state phase current of the LTV model using the Galerkin's method; (a) time domain (b) frequency domain. Key: solid line: 3-DOF model, dotted line and circles: reduced model. ................................................................. 67

3.9 Comparison of the steady-state torque of the LTV model using the Galerkin's method; (a) time domain (b) frequency domain. Key: solid line: 3-DOF model, dotted line and circles: reduced model. .... 68

3.10 Steady-state phase current of the NLTV model; (a) time domain (b) frequency domain. Key: solid line: the numerical integration, dotted line and circles: the Galerkin's method. ............................................. 72
3.11 Steady-state spectra of the NLTV model; (a) the angular velocity, (b) the torque. Key: solid line: the numerical integration, circles: the Galerkin's method. ................................................................. 73

3.12 Steady-state phase current of the NLTV model using the Galerkin's method; (a) time domain (b) frequency domain. Key: solid line: 4-DOF model, dotted line and circles: the reduced model. ................. 76

3.13 Steady-state spectra of the NLTV model using the Galerkin's method; (a) the angular velocity, (b) the torque. Key: solid line: the 4-DOF model, circles: the reduced model. ........................................... 77

4.1 Schematic of computer disk radiating into a hemispherical free field. 87

4.2 Comparison of approximated and exact vibration modes of a 3.5" computer disk, empty symbols: exact solution; solid line: polynomial approximation. Key: ——— and oooo (0,0) mode; ••••• and □□□□ (1,0) mode. ......................................................... 92

4.3 Predicted vibration modes. Key: ——— (a) (0,0) mode, (b) (1,0) mode; — — — (a) (0,1) mode, (b) (1,1) mode; ••••• (a) (0,2) mode, (b) (1,2) mode. ................................................................. 93

4.4 Definition of annuli for the impedance approach used to calculate the mutual radiation impedance. ................................................................. 98

4.5 Contribution of sound power from each annulus by using the impedance approach. (a) (0,0) mode. (b) (1,0) mode. ......................... 101

4.6 Radiation efficiency for rigid body modes of a single stationary disk; (a) translating piston mode, (b) rocking piston mode. Key: ——— analytical formulation; oooo BEMAP program. ................................. 103

4.7 Selected self terms of radiated sound power for multi-modal excitation. Key: ——— (a) (0,0) mode, (b) (1,0) mode; — — — (a) (0,2) mode, (b) (1,2) mode; ••••• (a) (0,4) mode, (b) (1,4) mode. 107

4.8 Selected mutual terms of radiated sound power between elastic modes. Key: ——— (0,0) and (1,0) modes; — — — (0,2) and (1,2) modes; ••••• (0,4) and (1,4) modes. ................................. 108

xii
4.9 Selected mutual terms of radiated sound power between rigid translating piston mode and elastic modes. Key: ----------(-1,0) and (0,0) modes; ; ----------(-1,0) and (1,0) modes. ........................................ 109

4.10 Selected mutual terms of radiated sound power between rigid rocking mode and elastic modes. Key:   (-1,1) and (0,1) modes; (-1,-1) and (1,1) modes ............................................... 110

4.11 Radiation efficiency for multi-mode excitation. ........................................ 111

4.12 Radiation efficiency of a disk under a concentric harmonic excitation; (a) axial force excitation, (b) moment excitation. .......... 114

4.13 Normalized natural frequency of a spinning disk. Key: ----------- (a) (0,0) mode, (b) (1,0) mode; --- (a) (0,1) mode, (b) (1,1) mode; · · · · (a) (0,2) mode, (b) (1,2) mode. ................................................ 116

4.14 Comparison of normalized modal radiation efficiency, empty symbols: formulation for stationary cases; solid line: formulation including source rotation effect. Key: ----------- and oooo (a) (0,0) mode, (b) (1,0) mode; --- and △△△△ (a) (0,2) mode, (b) (1,2) mode; · · · · and △△△△ (a) (0,4) mode, (b) (1,4) mode. ..................... 119

4.15 Normalized modal radiation efficiency vs. normalized rotational speed. Key: ----------- (a) (0,0) mode, (b) (1,0) mode; --- (a) (0,2) mode, (b) (1,2) mode; · · · · (a) (0,4) mode, (b) (1,4) mode. 120

5.1 Schematic of the disk drive system. ................................................................. 128

5.2 Mobility transfer function of axisymmetric modes (without the effect of bearing clearance); (a) rigid translating mode, (b) (0,0) mode, (c) (1,0) mode. ................................................................. 138

5.3 Mobility transfer functions of asymmetric modes (without the effect of bearing clearance); (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. ................................................................. 139

5.4 Mobility transfer functions of axisymmetric modes (with the effect of bearing clearance); (a) rigid translating mode, (b) (0,0) mode, (c) (1,0) mode. ................................................................. 140
5.5 Mobility transfer functions of asymmetric modes (with the effect of bearing clearance); (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. ................................................................. 141

5.6 Sound power spectra; (a) predicted by using mobility function of Figures 5.2 and 5.3, (b) predicted by using mobility function of Figures 5.4 and 5.5, (c) measured data. ........................................ 143

5.7 Normalized natural frequencies of disk elastic modes as functions of (a) thickness $h$ and (b) outer radius $b$. .......................................................... 145

5.8 Effect of disk thickness on normalized modal radiation efficiencies. Key: —— (a) (0,0) mode, (b) (1,0) mode; —— (a) (0,1) mode, (b) (1,1) mode. .................................................. 146

5.9 Effect of disk radius on normalized modal radiation efficiencies. Key: —— (a) (0,0) mode, (b) (1,0) mode; —— (a) (0,1) mode, (b) (1,1) mode. .................................................. 147

5.10 Effect of disk radius on self-modal sound power for asymmetric modes; (a) rigid translating mode, (b) (0,0) mode, (c) (1,0) mode. Key: —— 0.75$b^*$; — 1.125$b^*$; · · · 1.5$b^*$. ........................................ 148

5.11 Effect of disk radius on self-modal sound power for asymmetric modes; (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: —— 0.75$b^*$; — 1.125$b^*$; · · · 1.5$b^*$. ........................................ 149

5.12 Effect of disk radius on coupled-modal sound power for axisymmetric modes; (a) (-1,0) and (0,0) modes, (b) (-1,0) and (1,0) modes, (c) (0,0) and (1,0) modes. Key: —— 0.75$b^*$; — 1.125$b^*$; · · · 1.5$b^*$. ........................................ 150

5.13 Effect of disk radius on coupled-modal sound power for asymmetric modes; (a) (-1,1) and (0,1) modes, (b) (-1,1) and (1,1) modes, (c) (0,1) and (1,1) modes. Key: —— 0.75$b^*$; — 1.125$b^*$; · · · 1.5$b^*$. ........................................ 151

5.14 Effect of disk thickness on mobility transfer functions; (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: —— 0.2$h^*$; — 1.1$h^*$; · · · 2$h^*$. .................................................. 152

xv
5.15 Effect of disk radius on mobility transfer functions; (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: — 0.75b*; — — 1.125b*; · · · 1.5b*. 

5.16 Effect of disk thickness on sound power spectra; (a) 0.2h*, (b) 1.1h*, (c) 2h*. 

5.17 Effect of disk radius on sound power spectra; (a) 0.75b*, (b) 1.125b*, (c) 1.5b*. 

5.18 Effect of bearing stiffness on mobility transfer function of rigid translating mode; (a) 0.1K_b, (b) K_b, (c) 10K_b. 

5.19 Effect of bearing stiffness on mobility functions (asymmetrical modes); (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: — 0.1K_b; — K_b; · · · 0.1K_b. 

6.1 Proposed disk drive acoustic model for future studies.
LIST OF SYMBOLS

Note: bold symbols are used for matrices and vectors.

\( a \) \hspace{1cm} \text{inner radius of disk (m)}
\( b \) \hspace{1cm} \text{outer radius of disk (m)}
\( B \) \hspace{1cm} \text{rotor flux density distribution (weber/m\(^2\))}
\( B \) \hspace{1cm} \text{magnitude of rotor flux density (weber/m\(^2\))}
\( \hat{B} \) \hspace{1cm} \text{modulated rotor flux density distribution (weber/m\(^2\))}
\( B_m \) \hspace{1cm} \text{angular viscous damping coefficient (N-m-s/rad)}
\( c \) \hspace{1cm} \text{speed of sound in the air (m/sec)}
\( c_{mn,s}, \bar{c}_{mn,s} \) \hspace{1cm} \text{coefficient of approximate modal function of the disk (m,n) mode}
\( d \) \hspace{1cm} \text{distance between the source and observation points (m)}
\( D \) \hspace{1cm} \text{plate flexural rigidity (N-m)}
\( D_g \) \hspace{1cm} \text{mean diameter of air gap (m)}
\( \text{DIAG}\{ \} \) \hspace{1cm} \text{diagonal matrix}
\( e \) \hspace{1cm} \text{eccentricity (m)}
\( E \) \hspace{1cm} \text{Young's modulus (N/m\(^2\))}
\( g \) \hspace{1cm} \text{air gap (m)}
\( g \) \hspace{1cm} \text{mean air gap (m)}
\( h \) \hspace{1cm} \text{disk thickness (m)}
\( I, i_k \) \hspace{1cm} \text{stator phase current (A), } k=a,b,c
\( I_n \) \hspace{1cm} \text{modified Bessel function of the first kind}
\( J \) \hspace{1cm} \text{rotational inertia (kg-m\(^2\)/rad)}
\( J_n \) \hspace{1cm} \text{Bessel function of the first kind}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Jacobian matrix</td>
</tr>
<tr>
<td>K</td>
<td>Fourier coefficient</td>
</tr>
<tr>
<td>$K_b$</td>
<td>bearing stiffness matrix (N/m)</td>
</tr>
<tr>
<td>$K_c$</td>
<td>casing dynamic stiffness matrix (N/m)</td>
</tr>
<tr>
<td>$k_{mn}$</td>
<td>acoustic wave number of disk (m,n) mode (rad/m)</td>
</tr>
<tr>
<td>$K_n$</td>
<td>modified Bessel function of the second kind</td>
</tr>
<tr>
<td>$KE$</td>
<td>kinetic energy (w)</td>
</tr>
<tr>
<td>$\bar{L}$</td>
<td>mean value of self inductance (H)</td>
</tr>
<tr>
<td>L</td>
<td>stator inductance matrix (H)</td>
</tr>
<tr>
<td>$L_{ij}$</td>
<td>inductance between i-th and j-th stator phases (H)</td>
</tr>
<tr>
<td>$L_r$</td>
<td>axial length of rotor (m)</td>
</tr>
<tr>
<td>m</td>
<td>radial modal index of the disk</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>mean value of mutual inductance (H)</td>
</tr>
<tr>
<td>$M_r$</td>
<td>Mach's number toward to observation point</td>
</tr>
<tr>
<td>$M_d$</td>
<td>mass matrix of disk (kg)</td>
</tr>
<tr>
<td>$M_r$</td>
<td>mass matrix of rotor (kg)</td>
</tr>
<tr>
<td>$M_s$</td>
<td>mass matrix of stator (kg)</td>
</tr>
<tr>
<td>$M_{rd}$</td>
<td>coupled mass matrix of rotor and disk (kg)</td>
</tr>
<tr>
<td>n</td>
<td>tangential modal index of the disk</td>
</tr>
<tr>
<td>$N$</td>
<td>the highest term used in disk approximate modal function</td>
</tr>
<tr>
<td>$N_a$</td>
<td>number of discretized annuli</td>
</tr>
<tr>
<td>$N_s$</td>
<td>number of stator slots</td>
</tr>
<tr>
<td>$N_e$</td>
<td>number of turns per phase winding</td>
</tr>
<tr>
<td>$N_p$</td>
<td>number of poles on rotor</td>
</tr>
<tr>
<td>$P_{mn}$</td>
<td>modal sound pressure of disk (m,n) mode (Pa)</td>
</tr>
<tr>
<td>$PE$</td>
<td>potential energy (w)</td>
</tr>
<tr>
<td>$\Delta PE$</td>
<td>increase of potential energy associated with disk rotation (w)</td>
</tr>
<tr>
<td>$r_d$</td>
<td>modal participation vector of disk</td>
</tr>
<tr>
<td>$r_r$</td>
<td>displacement vector of rotor (m)</td>
</tr>
</tbody>
</table>
\( r_{o.s}, r_{i.s} \) outer and inner radius of s-th annulus (m)

\( \bar{r}_s \) mean radius of s-th annulus (m)

\( r_s \) displacement vector of stator (m)

\( R_r \) diameter of rotor (m)

\( R_s \) diameter of stator (m)

\( R \) stator phase winding resistance matrix (\( \Omega \))

\( t \) observation time (sec)

\( t' \) retarded time (sec)

\( T \) collocation matrix

\( T \) total motor torque (N-m)

\( \bar{T} \) mean value of motor torque (N-m)

\( T_e \) oscillating component of motor torque (N-m)

\( V \) line-to-neutral stator phase voltage vector (v)

\( W \) total air gap energy (W)

\( W_f \) stored energy in magnetic field (W)

\( W_{rad} \) radiated sound power (W)

\( x \) slot width (m)

\( y \) slot gap (m)

\( Y_n \) Bessel function of the second kind

\( Z_{mn,ij} \) modal radiation impedance between i-th and j-th annuli

\( \beta_{mn} \) elastic wave number of disk (m,n) mode (rad/m)

\( \delta \) load angle (rad)

\( \delta(x), \delta(y) \) delta functions

\( \Delta \) ratio of duration time in PWM scheme

\( \eta_{mn} \) participation factor of disk (m,n)

\( \Gamma \) PWM pulse train

\( \lambda \) total air gap flux linkage vector (weber)
\[ \lambda_r \] air gap flux linkage contributed by rotor (weber)
\[ \nu \] Poisson's ratio
\[ \mu \] permeance function
\[ \rho_o \] air density (kg/m\(^3\))
\[ \rho \] disk density (kg/m\(^3\))
\[ \omega \] angular frequency of harmonic torque excitation (rad/sec)
\[ \omega_c \] angular frequency of PWM control scheme (rad/sec)
\[ \omega_{c,nn} \] angular natural frequency of casing elastic mode (rad/sec)
\[ \omega_{nn} \] angular natural frequency of disk elastic mode (rad/sec)
\[ \bar{\Omega}_{r} \] mean value of angular velocity of rotor (rad/sec)
\[ \Pi \] modal sound power matrix (w)
\[ \Pi^{ij} \] modal sound power matrix for disk modes with i and j nodal circles (w)
\[ \Pi_{i,n_1}^{j,n_2} \] coupled modal sound power associated with (i,n\(_1\)) and (j,n\(_2\)) modes (w)
\[ \Pi_{mn} \] modal radiated sound power of disk
\[ \Pi_{mn,ref} \] modal reference sound power
\[ \psi_{nn} \] disk natural mode
\[ \psi_{c,nn} \] casing natural mode
\[ \phi_k \] phase angle of Fourier expansion (rad)
\[ \sigma \] sound radiation efficiency
\[ \sigma_{mn} \] modal sound radiation efficiency
\[ \sigma_s \] ratio of outer radius to inner radius of s-th annuli

Superscript
\[ E \] harmonic associated with electric defect
$I$ harmonic associated with imbalance

$M$ harmonic associated with misalignment

$H$ Hermitian operation

$T$ transpose operation

$*$ complex conjugate

Subscript

$E$ harmonic of back e.m.f.

$I$ harmonic of phase current

$V$ harmonic of line-to-neutral phase voltage
CHAPTER I

INTRODUCTION

1.1 MOTIVATION

Vibroacoustic characteristics of compact electromechanical devices are not well understood [1.1-7]. One specific example is the brushless dc motor (BDCM) which is used in a wide variety of industrial, office and consumer products such as robots, cassette recorders, computer disk drives, printers, fans, etc. Recently, the noise problems of devices driven by BDCM have attracted considerable attention, of which the computer disk drive is of particular interest [1.2,3,6,7]. The disk drive may be a major source of noise in personal (desk-top or lap-top) computers and work stations as demonstrated by four experimental case studies [1.2,3,6]. The overall sound power radiated by a hard disk drive may be not very high but prominent tones over the higher frequency range are perceived to be objectionable. As evident from the literature, only experimental investigations of computer disk drive acoustic behavior have been carried out. The mechanisms of noise sources which generate such high frequency tones are yet to be explored in depth. Further, the vibroacoustic characteristics of a computer disk and coupling between the radiator(s) and source(s) are poorly understood. In order to predict the acoustic performance of a computer disk drive and develop design guidelines for reduced noise, a complete understanding of each component which contributes to structure-borne and radiated sound is needed. Hence, it is desirable to
develop mathematical models for disk drive noise source, radiator and path(s), with emphasis on the generation of high frequency pure tones.

1.2 PROBLEM FORMULATION

The main focus of this research is to develop mathematical models of key components of a compact rotor-disk system driven by a brushless d.c. motor (BDCM) as illustrated in Figure 1.1(a). Special emphasis will be placed on acquiring an understanding of the mechanisms responsible for the generation, transmission and radiation of pure tones over the high frequency range, say from 1 to 6.5 KHz. Accordingly, a narrow band approach including modal analysis will be employed. A 3.5" computer disk drive, in the idling mode of operation, is used as the primary example case. Nevertheless, mathematical models are expected to be generic and accordingly should be conceptually applicable to other rotating disks and electromechanical devices.

A typical personal computer (PC) hard disk drive consists of several printed circuit boards, a casing and a multi-disk-spindle assembly. To simplify the problem, only a single disk driven by an in-hub BDCM which is attached to elastic mounts will be considered. Refer to Figure 1.1(b) for the source-path-radiator network assumed for this study. The only noise source mechanism considered here is the torque pulsations of the motor. Other mechanisms such as bearing defects and aerodynamic noise are excluded. Also the sound is assumed to be radiated by a single disk directly into a hemispherical free field. The casing is modeled as an equivalent spring in the overall dynamic model of the system. Consequently, the acoustic cavity modes of
Figure 1.1 Problem formulation of the computer disk drive; (a) key components, (b) source-path-radiator network examined by this study.
casing are ignored. More specific problem formulations and associated literature reviews will be included in each chapter.

1.3 OBJECTIVES

Chief objectives of this study are to resolve mathematical modeling issues associated with key components of the disk drive acoustic model. Narrow band and modal base formulation are used to construct models. An overview of tasks and methodologies employed is given in Table 1.1. Specific objectives are as follows; these are grouped in accordance with Figure 1.1 and Table 1.1. Note that each chapter focuses on a specific component and is written in a self sufficient journal style.

a. Noise Source: Electromechanical torque pulsation of BDCM (Chapters II and III)

(i) Develop a mathematical model to predict frequency contents of torque pulsation associated with several mechanical and electrical defects. Simplify the model to predict Fourier coefficients of torque harmonics over the frequency range of interest and identify the cause associated with each harmonic group. Compare results from the proposed analytical and the numerical techniques with the measured sound and vibration spectra on a narrow band basis from 1 to 6.5 KHz.

(ii) Apply the Galerkin's method to refine the BDCM model. Develop a new computational scheme to yield Fourier coefficients and examine effects associated with inductance harmonics and angular velocity fluctuation.
Table 1.1 Overview of objectives and methods.

<table>
<thead>
<tr>
<th>Objectives/Methods</th>
<th>Tasks</th>
<th>Analytical Methods</th>
<th>Computational Methods</th>
<th>Supporting Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise Source: Electromechanical torque pulsations of BDCM</td>
<td>Brushless d.c. motor model</td>
<td>Fourier series expansion</td>
<td>State space model (Numerical integration, FFT)</td>
<td>• Sound measurement</td>
</tr>
<tr>
<td></td>
<td>Frequency domain analysis</td>
<td>Galerkin's method (Numerical harmonic balance)</td>
<td>Newton-Raphson method</td>
<td>• Casing vibration measurement</td>
</tr>
<tr>
<td>Radiator: Annular disk sound radiation characteristics</td>
<td>Modal analysis</td>
<td>Rayleigh-Ritz method</td>
<td>Modal testing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modal radiation efficiency for stationary and rotating disks</td>
<td>• Far-field approach (Green's function)</td>
<td>Boundary element method (BEMAP)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modal coupling effect</td>
<td>Modal base formulation</td>
<td>Boundary element method (BEMAP)</td>
<td></td>
</tr>
<tr>
<td>Path: Mobility transfer function and overall vibroacoustic models</td>
<td>Path (mobility transfer function)</td>
<td>• Normal mode expansion</td>
<td>Finite element method (ANSYS)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Overall model and parametric studies</td>
<td>Narrow band approach</td>
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<td></td>
</tr>
</tbody>
</table>
b. Radiator: Annular disk sound radiation characteristics (Chapter IV)

(i) Derive approximate modal functions of the annular disk. Validate theory by comparing results with exact eigensolutions and modal experiments.

(ii) Derive new formulations for modal radiation efficiency in terms of power series of the wave number based on the approximate modal functions. Compare predictions with numerical results obtained from a boundary element program. Examine effect of the source rotation on modal radiation efficiency.

(iii) Investigate the effect of modal coupling on radiated sound and predict radiated sound given a surface velocity distribution.

c. Path: Mobility transfer function and overall vibroacoustic models (Chapter V)

(i) Develop a path model in terms of modal mobility transfer functions.

(ii) Construct an overall narrow band model by integrating source, radiator and path models. Conduct parametric studies to demonstrate the effects of disk dimensions and bearing stiffness on vibroacoustic characteristics.
REFERENCES FOR CHAPTER I


CHAPTER II

NOISE SOURCE: ELECTROMAGNETIC TORQUE PULSATIONS

2.1 INTRODUCTION

The disk drive is a major source of noise in personal (desk-top or lap-top) computers and work stations as demonstrated by three experimental case histories [2.1-3]. Luttrell and Dunens examined various noise sources of two disk drive prototypes and reduced manufacturing related variations in sound levels [2.1]. Prasad et al. measured the sound power of a disk drive in rotation and seeking modes. Several intense pure tones were noted, especially in the seeking operation [2.2]. According to them, radiated noise during the seeking mode was related to the design parameters of the stepper motor. In a recent experimental investigation, Woldemar and Kumano have found that tonal noise is amplified by motor or structural dynamics [2.3]. They successfully reduced the prominent high frequency tones by inserting foam material between the disk drive base and electronic circuit board, and by adding a varnish material to the motor coils. Based on these studies and other experimental data available to the authors [2.4], it is evident that the source mechanisms of such high frequency pure tones are poorly understood. Also, disk drive acoustic behavior is yet to be characterized analytically. To remedy this situation, a comprehensive theoretical and experimental investigation has been undertaken. This study presents preliminary results with an emphasis on the modeling of the electromechanical source of noise.
radiated by an idling disk drive. Main focus is on noise associated with motor torque pulsations; other noise source mechanisms are beyond the scope of this study.

2.2 LITERATURE REVIEW

The modeling of brushless d.c. motor (BDCM) is an essential step in study vibro-acoustic characteristics of disk drive systems. Of interest here is the prediction of motor torque pulsations, since torque oscillations seem to be the primary sources of high frequency tones radiated from disk drive systems. A considerable of published literature on the modeling of BDCM/synchronous motor is available [2.5-22]. Chief mathematical approaches employed by researchers include d-q axis theory, phase variable formulation, and Fourier series representation [2.10]. Here, a review of the literature relevant to our problem is documented.

D-q axis theory has been widely used in modeling of 3 phase a.c. machines. If the inductances of a motor are sinusoidal functions of rotor angle, this approach can transform the equivalent circuits into a reference d-q frame with constant inductances. Krause and Lipo [2.5,22] analyzed six-step and pulse-width-modulated (PWM) type of rectifier-inverter reluctance-synchronous motor drive systems by transforming rectifier, inverter and phase variables to the d-q reference axes rotating in synchronous with the fundamental frequency of inverter output voltage. Harmonics due to the rectifier are neglected and the Fourier series expansion of a pulse train was used to express PWM inverter operation with fixed switching rate. However, the harmonics components of rectifier output voltage were ignored. The analytical expression of a three-phase machine showed that output torque contained harmonics of fundamental frequency
equal to six times the electrical angular velocity. Takeda, et al. [2.8] developed a
generalized model in the d-q reference frame, in which both damper windings and
compensating windings were included. Spee and Wallace [2.19] simplified the exact
three-phase model in order to simulate BDCM performance. Non-sinusoidal winding
inductances and back e.m.f. were approximated by sinusoidal functions with the same
peak values such that the system can be formulated in the d-q reference frame. They
showed that most performance features can be adequately predicted by using this
simplified model.

Several researchers have modeled BDCM by using phase variable representation,
since it is the simplest formulation and it can be simulated by using a digital computer
even with additional control units. Demerdash and Nehl [2.6] proposed a fourteen
order state space model for aerospace actuation. In their model, all switching
transistors and diodes were represented by nonlinear resistances. The discrete-time
solution was used to simulate dynamics of drive and control system. Jahns [2.12]
found the torque pulsation at low-speed operation can be minimized by designing rotor
magnet pole arcs as close as 180 degrees as possible or employing rotor speed
feedback compensation. Funabiki et al. [2.13,14] dealt with torque pulsations of
voltage-source-inverter-fed BDCM by including the performance of the converter. The
back e.m.f. of each phase was expressed by sinusoidal functions and inductances were
assumed to be invariant. A computer simulation was performed by dividing the system
into six operational modes according to the behavior of the converter and inverter. The
results showed significant effects of non-ideal d.c. voltage on torque harmonics. A
linearly-varying mutual inductance between stator phases was assumed by Bolton et al.
[2.15] in their study. The influences of stator slots and rotor skew angle were also
included in their formulation. Pickup and Russell [2.17] showed that flux linkage spectra of the state phase contained sidebands during the oscillatory operation. The major feature of their approach, compared with other studies, is that the Jacobi expansion was used to analytically formulate the torque equation with oscillating rotor angular velocity in the Fourier series form. Hart and Bonwick [2.18] presented a harmonic model which included the harmonic effects caused by either power system harmonics or inductance harmonics. Pillay and Krishnan [2.20] simulated the dynamics performance of BDCM with trapezoidal back e.m.f. and quasi-square phase current. Dynamic equations of BDCM were expressed in the state-space form with assumption of constant self and mutual inductances. Hysteresis and PWM current controllers were used to examine the system performance. Murai et al. [2.21] also simulated a miniature BDCM by phase variable approach. However, equations were separated into two modes based on the switching operation. Two types of switching strategies were proposed — namely, overlapping method and PWM chopping method — to reduce the overall torque pulsation level.

The advantage of the Fourier series approach is that results can be obtained directly in the frequency domain. Besides, all kinds periodic back e.m.f., voltage or current waveforms can be easily represented analytically. For instance, Slemon and Gumaste [2.7] used Fourier series expansion of the line-to-line voltage waveform to analyze the dynamics of the permanent magnet synchronous motor with current source inverter. Four different combinations of the rotor damper winding were employed in simulations. Optimal control strategies were developed for torque-speed profiles under two types of operations. Bolton and Ashen [2.10] presented the influence of input current harmonics on torque ripples by describing current and m.m.f. waveforms in the
Fourier series forms. Torque pulsations were then investigated directly in the frequency domain. Le-Huy et al. [2.16] used the same approach to study disk-type BDCM for two-phase and three-phase feeding schemes. Fourier series approach also can be applied in the d-q reference frame, as given by Goodman [2.9]. In his model, all harmonics of the winding inductance were taken into account. Kaufman and Plunkett [2.11] also used Fourier series approach in the d-q reference frame. Arbitrary line-to-neutral voltage was investigated, and the steady state torque pulsation was evaluated by using the phase equations in rotor reference frame.

None of the studies reported here could be applied directly to our problem. Also, most of the available analyses or models are time domain or lower frequency oriented. For the acoustic study, one obviously needs a higher frequency model.

2.3 EXPERIMENTAL INVESTIGATION

A 3.5" personal computer (PC) hard disk drive idling at a constant speed \( \Omega_r \) (rad/s) is selected as an example case. A stack of disks is driven by an in-hub brushless d.c. motor (BDCM). The drive is driven by a typical PC power supply and is tested under two different conditions; with casing cover and without casing cover. First, the drive is evaluated without any other PC components being present. Second, the casing cover is removed from the disk drive and the in-hub motor is mounted on a rigid base built specifically for the experiment. The drive, as shown in Figure 2.1, is suspended freely in a mini-anechoic chamber for sound pressure and structure acceleration measurements on a narrow band basis.
Figure 2.1 Experimental set-up.
In-situ intensity measurements, though not reported here, have been found to be similar to the free field data. Figure 2.2(a) shows a typical acceleration spectrum measured on the casing cover near the motor mounts. Numerous pure tones are clearly present over a wide range of frequencies. Figures 2.2(b) and (c) illustrate free field sound pressure spectra at 0.3 m above the disk with and without the casing cover, respectively. It is apparent that the noise below 1.3 KHz is inhibited or attenuated by the casing. However, several prominent pure tones with side band modulations are seen between 1.3 and 6.1 KHz even without the casing cover. Results for two disk drives (A and B) idling at 72 Hz are summarized in Table 2.1. Variations or uncertainties in the measured data under virtually the same conditions are observed. Frequency components and their amplitudes seem to be dependent on the electrical circuit control mechanisms, motor design parameters, thermal conditions, bearing characteristics, casing dynamics, assembly conditions and manufacturing variations.

2.4 ANALYTICAL FORMULATION

The objective of this study is to model the electromechanical noise source and to explain the emission of high frequency tones. Based on the experimental investigation and initial calculations, the following assumptions are made to simplify the analytical formulation of the idling disk: 1. BDCM is the only source through rotor torque pulsation. 2. Structure-borne paths are not included in the analysis. 3. Disk dynamics, casing vibration and aerodynamic excitation are ignored. 4. Thermal conditions do not affect the acoustic behavior of the source. 5. Input d.c. voltage source is ideal and ripple free. 6. Current inverters are ideal switches. 7. Motor
Figure 2.2 Measured spectra (in relative decibels); (a) structural acceleration on the casing, (b) free field sound pressure level with casing cover, (c) free field sound pressure level without casing cover.
Table 2.1 Measured dominant pure tones between 1.3 and 6.1 KHz and corresponding free field sound pressure levels (in relative decibels).

<table>
<thead>
<tr>
<th></th>
<th>Drive A (with cover)</th>
<th>Drive B (with cover)</th>
<th>Drive B (without cover)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set A1*</td>
<td>Set A2</td>
<td>Set B1*</td>
</tr>
<tr>
<td>f (Hz) dB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1725</td>
<td>0</td>
<td>1725</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2225</td>
<td>0</td>
<td>2208</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2800</td>
<td>4</td>
<td>2840</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3450</td>
<td>1</td>
<td>3160</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5175</td>
<td>3</td>
<td>4312</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5475</td>
<td>5</td>
<td>5176</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5750</td>
<td>7</td>
<td>5464</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6050</td>
<td>-1</td>
<td>5752</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Data provided by the manufacturer.
phase variables are balanced, i.e. each is offset by an electrical angle equal to $2\pi/3$. 8. Rotor magneto-motive force (m.m.f.) wave is trapezoidal [2.5,6]. 9. Torsional dynamics of the rotor can be given in terms of moment of inertia $J$ and damping coefficient $B_{mr}$.

Three mathematical approaches have been employed previously to predict motor torque pulsation $T_d(t)$. However, all of the available models are limited to the lower harmonics of $\Omega_r$ or to just a few higher harmonics as shown in the previous section. Accordingly, two new mathematical models are proposed to explain the disk drive acoustic behavior. In our approach the d-q axis theory is not used since it does not lead to high frequency response prediction in a convenient manner.

In this study, two new mathematical models have been developed to predict harmonics of torque pulsations. The Phase Variable Formulation (Model I) retains the governing equations in the state space form. The inverter switching operation is simulated numerically and the induced voltage (back e.m.f.) is represented in terms of a piece-wise linear function of rotor angular position. This model employs essentially a numerical integration approach to solve the governing equations in time domain. An FFT algorithm is then used to predict the torque pulsation spectrum. The Fourier Series Representation (Model II) yields an analytical expression for the steady state torque harmonics by expressing line-to-neutral phase voltage and back e.m.f. in terms of Fourier series. Fourier coefficients of phase current are obtained by using phase voltage equations. Consequently, the steady state torque harmonics are calculated directly by substituting Fourier coefficients of key variables into the electromagnetic torque equation. Model I provides more accurate predictions, however considerable
computation time is needed for numerical solutions. Model II is obviously more efficient since it calculates the output torque directly in the frequency domain.

2.4.1 Model I [Phase Variable Formulation]

A typical 3-phase \((a, b,\) and \(c\) with Y connection) BDCM includes 6 pairs of current inverters as shown in Figure 2.3. Each phase circuit consists of a winding resistance \(R\), inductance \(L\) and back e.m.f. \(E\). The input d.c. voltage is converted to nonsinusoidal a.c. voltage via an inverter switching logic and its amplitude is controlled using the pulse-width-modulation algorithm. [2.5,20] Phase voltage equations of the BDCM are given as follows where a bold letter implies a matrix/vector, \(V\) is the phase voltage, \(I\) is the phase current, \(\lambda_r\) is the air gap flux linkage contributed by rotor, and \(\lambda\) is the total air gap flux linkage.

\[
V = RI + \dot{\lambda} \tag{2.1}
\]

\[
\dot{\lambda} = L\dot{I} + \lambda_r \tag{2.2}
\]

\[
L = \begin{bmatrix}
L_{aa} (\theta_r) & L_{ab} (\theta_r) & L_{ac} (\theta_r) \\
L_{ba} (\theta_r) & L_{bb} (\theta_r) & L_{bc} (\theta_r) \\
L_{ca} (\theta_r) & L_{cb} (\theta_r) & L_{cc} (\theta_r)
\end{bmatrix} \tag{2.3}
\]

\[
R = \begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix} \tag{2.4}
\]

Using Equations (2.1-4), one can obtain the governing equation of the electromagnetic circuits as given below by Equation (2.5).
Figure 2.3 Schematic of a 3-phase, 2-pole brushless d.c. motor (BDCM). (a) the current inverter and phase circuits and (b) the cross-sectional view of rotor and stator.
\[ \dot{I} = L^{-1}[\dot{L} + R]I + L^{-1}[V - E] \] (2.5)

Rotor dynamic equation is derived by relating the electromagnetic torque to rotational inertia and viscous damping effects as shown by Equation (2.6) where \( \theta_r \) is the angular position of rotor, \( \bar{T}_e \) and \( T_e \) are mean and fluctuating components of the motor output torque.

\[
\begin{bmatrix}
\dot{\theta}_r \\
\dot{\Omega}_r
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & B_m/J
\end{bmatrix}
\begin{bmatrix}
\theta_r \\
\Omega_r
\end{bmatrix} +
\begin{bmatrix}
0 \\
(\bar{T}_e + T_e)/J
\end{bmatrix} \tag{2.6}
\]

Next, the output electromagnetic torque of a \( N_p \)-pole motor is derived by differentiating the stored energy in the air gap magnetic field with respect to rotor angular position. [2.22]

\[
\bar{T} + T_e = \frac{d}{d\theta_r} \left[ W_f(I, \theta_r) \right] \tag{2.7}
\]

\[
W_f(I, \theta_r) = \frac{N_p}{2} \left[ \frac{1}{2} IT \left[ d\theta_r \right] L(\theta_r) I + I^T \lambda \right] \tag{2.8}
\]

The equation of electromagnetic torque is thus expressed by

\[
\bar{T} + T_e = \frac{N_p}{2} \left[ \frac{1}{2} IT \left[ \frac{d}{d\theta_r} L(\theta_r) \right] I + \frac{1}{\Omega_r} E^T I \right] \tag{2.9}
\]

in which back e.m.f. \( E \) can be written by [2.15]

\[
E(\theta_r) = \frac{1}{2} N_s \Omega_r D_s L_s B(\theta_r) \tag{2.10}
\]
where \( N_g \) is the number of turns per phase winding, \( D_g \) is the mean air gap diameter, \( L_r \) is the axial length of rotor, and \( B \) is the air gap flux density. The trapezoidal back e.m.f. \( E \) and quasi-square phase voltage \( V \) are expressed in terms of piece-wise linear curves and the expressions for \( L \) include the effects of dynamic imbalance, rotor misalignment, winding harmonics and magnetic saturation; these will be discussed later as a part of Model II. The spectral contents associated with the number of stator slots are formulated by differentiating the stored energy in the air gap field associated with the modulated rotor flux distribution with respect to the rotor angle [2.15]. Assume that trapezoidal rotor flux density is modulated by stator slots as shown in Figure 2.4. Then, the stored energy between any slot can be expressed by

\[
\Delta W_k(\theta_r) = \frac{D_g g L_r}{2} \int_{\alpha_1}^{\alpha_2} \frac{1}{2\mu_0} \left(1 + \frac{x}{y}\right) B^2_r(\theta_r) d\theta_r
\]

\[
\alpha_1 = \theta_r + \frac{x}{D_g} + \frac{2k\pi}{n} \quad , \quad \alpha_2 = \theta_r + \frac{x + 2y}{D_g} + \frac{2k\pi}{n}
\]

where, \( x \) is the slot width and \( y \) is the gap of slot opening. The total air gap energy \( W \) is thus the summation of stored energy over \( n \) slots.

\[
W(\theta_r) = \sum_{k=1}^{n} \Delta W_k(\theta_r)
\]

Consequently, the torque pulsation due to open stator slots (\( T_z \)) is

\[
T_z = \frac{dW}{d\theta_r}
\]
Figure 2.4 Air-gap flux density distribution modulated by open stator slots.
Since this analytical differentiation expression causes significant numerical errors, the
effect of $T_z$ was not considered in the computer simulation of Model I. It will be
proven later by using Model II (see Section 2.3) that the fundamental frequency of $T_z$ is
given by the product of slot number and rotor pole pair number. Therefore, such
torque harmonics can be predicted even without a computer simulation model. Model
I is solved by using the direct time domain integration techniques. [2.10,15,21] An FFT
algorithm is then used to determine the torque pulsation spectrum $T_z(f)$ where $f$ is the
frequency in Hz.

The pulse-width-modulation (PWM) switching control algorithm is studied
separately from this model in order to simplify the overall problem. It is seen that the
switching operation induces another family of harmonics with the fundamental
frequency equal to the carrier frequency. Figure 2.5 shows typical simulation results of
an 8-pole, 3-phase BDCM with constant inductances operated under a fixed-rate PWM
switching logic. The fixed PWM switching rate was selected to be 24 times the
electrical frequency ($N_p \Omega_r/2$); therefore, the dominant tone is at $96 \Omega_r$ which is the
same as the carrier frequency.

2.4.2 Model II [Fourier Series Representation]

For a three-phase machine, the phase voltage $V_a$ and back e.m.f. $E_a$ of the first
phase (a) can be expressed by

$$E_a(\theta_r) = \sum_{q, odd}^\infty K_{E,a} N_p / 2 \exp\left( j q \frac{N_p}{2} \theta_r \right)$$  \hspace{1cm} (2.14)
Figure 2.5 Motor torque pulsations generated by the PWM switching: (a) 120° discrete type switching logic. (b) 180° continuous type switching logic.
\[ V_a(\theta_r) = \sum_{q,odd} K_{V,qN_p/2} \exp \left( jq \frac{N_p}{2} (\theta_r - \delta) \right) \]  

(2.15)

where \( \delta \) represents the load angle which is the phase lead angle of the stator m.m.f. wave relative to the rotor m.m.f. wave. For assumed trapezoidal back e.m.f. \( B \) and quasi-square line-to-neutral phase voltage \( V \) functions of the rotor angular position \( \theta_r \) as shown in Figure 2.6, the Fourier coefficients can be expressed as follows

\[ K_{E,qN_p/2} = \frac{4N_p \Omega_r D_2 L_r B}{q^2 \beta \pi} \sin \left( \frac{q \beta}{2} \right) \]  

(2.16)

\[ K_{V,qN_p/2} = \frac{2V_{dc}}{q \pi} \cos \left( \frac{q \pi}{2} \right) \]  

(2.17)

Since phase variables are assumed to be balanced, corresponding formulations for phases \( b \) and \( c \) are obtained by substituting \( \theta_r \) with \( \theta_r - (4\pi/3N_p) \) and \( \theta_r + (4\pi/3N_p) \), respectively, where \( N_p \) is the number of rotor poles. Fourier coefficients for each phase current can now be calculated by employing Equations (2.1-5). In order to simplify this formulation, only mean values of self (\( \bar{L} \)) and mutual (\( \bar{M} \)) inductances are included, i.e. only those harmonics of current which are related to the harmonics of input voltage and back e.m.f. are considered in the calculation of \( T_a(t) \). In addition to the mean inductance term, rotor imbalance and drive misalignment effects are introduced through \( L' \) and \( M' \), respectively, as shown below where \( K_L \) represents the Fourier coefficient of self inductance. Similarly, electrical defects of the inductance are modeled via the \( L^E \) term as given below. Therefore the total self inductance of phase a is
Figure 2.6 Assumed trapezoidal rotor flux density distribution and the quasi-square line-to-neutral phase voltage.
\[ L_{ab}(\theta_r) = \bar{L} + L'(\theta_r) + L^M(\theta_r) + L^E(\theta_r) \]  
(2.18)

\[ L'(\theta_r) = \sum_{s=1}^{\infty} K_{L,s}^I \exp(js\theta_r) \]  
(2.19)

\[ L^M(\theta_r) = \sum_{s=1}^{\infty} K_{L,2s}^M \exp(j2s\theta_r) \]  
(2.20)

\[ L^E(\theta_r) = \sum_{s=1}^{\infty} K_{L,N_p,s}^E \exp(jN_p s\theta_r) \]  
(2.21)

Self inductances for phases \( b \) and \( c \) are expressed using the substitution of \( \theta_r \) as defined earlier for \( E_a \) and \( V_a \). Similarly, the mutual inductance \( L_{bc} \) between phases \( b \) and \( c \), i.e. flux link of state phase \( b \) due to current of state phase \( c \), is derived as follows where \( K_M \) is the Fourier coefficient associated with mutual inductance.

\[ L_{bc}(\theta_r) = \bar{M} + \sum_{s=1}^{\infty} \left[ K_{M,s}^I \exp(js\theta_r) + K_{M,2s}^M \exp(j2s\theta_r) + K_{M,N_p,s}^E \exp(jN_p s\theta_r) \right] \]  
(2.22)

Next, \( L_{ac}(\theta_r) \) and \( L_{ab}(\theta_r) \) are obtained by substituting \( \theta_r \) with \( \theta_r-(4\pi/3N_p) \) and \( \theta_r+(4\pi/3N_p) \), respectively. Substituting Equations (2.14-22) into Equation (2.9), the torque pulsation \( T_p(t) \) can be written explicitly in the Fourier series form. After a considerable analytical manipulation, the final expression of output torque pulsations is found to be as follows:

\[ T_p(t) = \frac{3}{2\Omega} \sum_{q,s, odd} K_{i,qN_p/2} K_{E,sN_p/2} \left\{ A_1 \cos \left[ \frac{N_p}{2} (s+q)\theta_r + \phi_{qN_p/2} \right] + A_2 \cos \left[ \frac{N_p}{2} (s-q)\theta_r - \phi_{qN_p/2} \right] \right\} \]
\[-\frac{3}{4} \sum_{t,q,s} tK_{L,t}K_{L,sNp/2}K_{L,qNp/2} \left\{ B_1 \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s + q \right) \theta_r + \left( \phi_sNp/2 + \phi_qNp/2 \right) \right] \right. \\
\quad \left. + B_2 \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s - q \right) \theta_r - \left( \phi_sNp/2 + \phi_qNp/2 \right) \right] \right. \\
\quad \left. + B_3 \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s - q \right) \theta_r - \left( \phi_sNp/2 - \phi_qNp/2 \right) \right] \right. \\
\quad \left. + B_4 \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s + q \right) \theta_r - \left( \phi_sNp/2 - \phi_qNp/2 \right) \right] \right. \right\}

\[-\frac{1}{4} \sum_{t,q,s} tK_{M,t}K_{L,sNp/2}K_{L,qNp/2} \left\{ \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s + q \right) \theta_r + \left( t - \frac{N_p}{2} q \right) \frac{2\pi}{3} + \left( \phi_sNp/2 + \phi_qNp/2 \right) \right] \right. \\
\quad \left. + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s - q \right) \theta_r + \left( t + \frac{N_p}{2} q \right) \frac{2\pi}{3} - \left( \phi_sNp/2 + \phi_qNp/2 \right) \right] \right. \\
\quad \left. + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s - q \right) \theta_r + \left( t + \frac{N_p}{2} q \right) \frac{2\pi}{3} + \left( \phi_sNp/2 - \phi_qNp/2 \right) \right] \right. \\
\quad \left. + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s + q \right) \theta_r + \left( t - \frac{N_p}{2} q \right) \frac{2\pi}{3} - \left( \phi_sNp/2 - \phi_qNp/2 \right) \right] \right. \right\}

\[-\frac{1}{4} \sum_{t,q,s} tK_{M,t}K_{L,sNp/2}K_{L,qNp/2} \left\{ \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s + q \right) \theta_r - \left( t - \frac{N_p}{2} q \right) \frac{2\pi}{3} + \left( \phi_sNp/2 + \phi_qNp/2 \right) \right] \right. \\
\quad \left. + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s - q \right) \theta_r - \left( t - \frac{N_p}{2} q \right) \frac{2\pi}{3} - \left( \phi_sNp/2 + \phi_qNp/2 \right) \right] \right. \right\}
\[ + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s - q \right) \theta_r - \left( t + \frac{N_p}{2} q \right) \frac{2\pi}{3} + \left( \phi_{sN_p/2} - \phi_{qN_p/2} \right) \right] \]

\[ + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s + q \right) \theta_r - \left( t - \frac{N_p}{2} q \right) \frac{2\pi}{3} - \left( \phi_{sN_p/2} - \phi_{qN_p/2} \right) \right] \right) \}

\[ - \frac{1}{4} \sum_{t,q,s} tK_{M,t}K_{1,sN_p/2}K_{1,qN_p/2} \left\{ \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s + q \right) \theta_r - (s - q) \frac{N_p\pi}{3} + \left( \phi_{sN_p/2} + \phi_{qN_p/2} \right) \right] \right. \]

\[ + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s - q \right) \theta_r + (s - q) \frac{N_p\pi}{3} - \left( \phi_{sN_p/2} + \phi_{qN_p/2} \right) \right] \]

\[ + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t + s - q \right) \theta_r - (s + q) \frac{N_p\pi}{3} + \left( \phi_{sN_p/2} - \phi_{qN_p/2} \right) \right] \}

\[ + \sin \left[ \frac{N_p}{2} \left( \frac{2}{N_p} t - s + q \right) \theta_r + (s + q) \frac{N_p\pi}{3} - \left( \phi_{sN_p/2} - \phi_{qN_p/2} \right) \right] \right) \right) \right) \right) \right) \right) \right) \right) (2.23) \]

where, \( s \) and \( q \) are odd number, \( \phi_i \) is the phase angle of Fourier expansions of current, and

\[ A_1 = \begin{cases} 1 & s + q = 3k \\ 0 & \text{otherwise} \end{cases} \quad A_2 = \begin{cases} 1 & |s + q| = 3k \\ 0 & \text{otherwise} \end{cases} \]

\[ B_1 = \begin{cases} 1 & t + s + q = 3k \\ 0 & \text{otherwise} \end{cases} \quad B_2 = \begin{cases} 1 & |t - s - q| = 3k \\ 0 & \text{otherwise} \end{cases} \]

\[ B_3 = \begin{cases} 1 & |t + s - q| = 3k \\ 0 & \text{otherwise} \end{cases} \quad B_4 = \begin{cases} 1 & |t - s + q| = 3k \\ 0 & \text{otherwise} \end{cases} \]
Torque $T_\tau(t)$ associated with the fluctuating flux is derived by substituting Fourier series of machine variables into Equations (2.11-13). The final expression is given below where $K_{B,q,N_p,N_q/2}$ is the Fourier coefficient of rotor flux density distribution and $g$ is the air gap [2.15].

$$T_\tau = \frac{N_p D_s L_s g}{4\mu_0} \left(1 + \frac{x}{y}\right)^2 \sum_{q,odd} K_{B,q,N_p,N_q/2}^2 \left\{ \sin \left[ qN_s \left( \frac{N_p \theta_r + x + 2y}{D_g} \right) \right] - \sin \left[ qN_s \left( \frac{N_p \theta_r + x}{D_g} \right) \right] \right\}$$

(2.24)

To simulate the PWM control algorithm operation, the phase voltage in Model II is multiplied with a pulse train $\Gamma(t)$ of unit magnitude [2.5]. In order to simplify this phenomenon, the period $\tau=2\pi/\omega_c$ of pulse train is assumed to be fixed as shown in Figure 2.7. Consequently, the Fourier series of the pulse train is expressed as follows

$$\Gamma(t) = \epsilon + \sum_{k=1}^{\infty} \frac{1}{k\pi} \left\{ \sin(2k\pi)\cos(k\omega_c t) + [1 - \cos(2k\pi)] \sin(k\omega_c t) \right\}$$

(2.25)

Finally, the spectral contents of torque consist of four groups, as expressed below functionally and identified clearly in Table 2.2.

$$T_\tau = \sum_{q=1}^{\infty} \sum_{\alpha=0}^{\infty} F_1 \left[ \left( 3q \frac{N_p}{2} \Omega_r \pm \alpha \omega_c \right) t \right] + \sum_{s,odd}^{\infty} F_2 \left( s \frac{N_p}{2} N_s \Omega_r t \right)$$

$$+ \sum_{q=1}^{\infty} \sum_{k=1}^{\infty} \sum_{s=k,2k,N_p k}^{\infty} F_3 \left[ \left( q \frac{N_p}{2} \Omega_r \pm s \Omega_r \pm \alpha \omega_c \right) t \right]$$

(2.26)
Table 2.2 Harmonics of motor torque (as yielded by Model II).

<table>
<thead>
<tr>
<th>Harmonic Group</th>
<th>Frequency Contents</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Pulse-Width-Modulation (PWM)</td>
<td>(i) harmonics of $3N_p\Omega_r$</td>
<td>dynamic interactions between $E$ and $I$</td>
</tr>
<tr>
<td></td>
<td>(ii) odd harmonics of $0.5N_pN_p\Omega_r$</td>
<td>open stator slot</td>
</tr>
<tr>
<td></td>
<td>(iii) $sN_p\Omega_r \pm m\Omega_r$</td>
<td>inductance variation</td>
</tr>
<tr>
<td></td>
<td>where $s$ is an integer and $m=q, 2q, N_pq; q=1, 2, 3, \ldots$</td>
<td></td>
</tr>
<tr>
<td>With Pulse-Width-Modulation (PWM)</td>
<td>(i) $3qN_p\Omega_r \pm a\omega_c$</td>
<td>dynamic interactions between $E$ and $I$ with side bands generated by PWM</td>
</tr>
<tr>
<td></td>
<td>where $q=1, 2, 3, \ldots$ and $a=0, 1, 2, \ldots$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) odd harmonics of $0.5N_pN_p\Omega_r$</td>
<td>open stator slot</td>
</tr>
<tr>
<td></td>
<td>(iii) $sN_p\Omega_r \pm m\Omega_r \pm a\omega_c$</td>
<td>inductance variation with side bands generated by PWM</td>
</tr>
<tr>
<td></td>
<td>where $s$ is an integer, $a=0, 1, 2, \ldots$ and $m=q, 2q, N_pq; q=1, 2, 3, \ldots$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(vi) harmonics of $\omega_c$</td>
<td>carrier frequency $\omega_c$.</td>
</tr>
</tbody>
</table>
Figure 2.7 PWM pulse train of period $\tau$. 
The rotor angular velocity $\Omega_r$ has been assumed to be constant in our analyses. In practice, $\Omega_r$ also contains ripples due to the pulsating torque. To include this effect, $\Omega_r$ needs to be decomposed into a mean angular velocity and an oscillating speed with the amplitude equal to the zero-to-peak value of given velocity ripple. Then, the Jacobi expansion has to be applied to the above torque formulation. Since the pulsating amplitude in the steady state is much smaller compared to the mean angular velocity as shown in Figure 2.8, it can be ignored.

2.5 RESULTS AND CONCLUDING REMARKS

The numerical data set used to illustrate the application of both models is given in Table 2.3. Note that several motor parameters are estimated by trial and error to match the given operating conditions of the sample disk drive. Since the PWM control scheme is not known sufficiently, it is not included in subsequent calculations. Only the first three harmonics of $L^J$, $L^M$ and $L^E$ terms in Equations (2.18-22) are included in the predictions yielded by Model II.

Torque pulsation spectra $T(f)$ as predicted by Models I and II for the sample case are shown in Figure 2.9. The most dominant peak corresponds to $3N_p\Omega_r$ and it has been taken as the reference (0 dB) for both spectra. Only those peaks whose amplitudes are greater than -50 dB are retained for analysis. Even though the frequency range of interest is restricted, spectral predications are given from 0 to 10 KHz. Both models match with each other reasonably well. Discrepancies are due to the fact that Model II represents an analytical approximation to the solution; also, numerical solution and FFT algorithm errors are present in Model I.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of rotor magnetic poles</td>
<td>$N_p$</td>
<td>8</td>
</tr>
<tr>
<td>mean rotor rotation speed</td>
<td>$\Omega_r$</td>
<td>72 Hz</td>
</tr>
<tr>
<td>number of stator slots</td>
<td>$N_s$</td>
<td>9</td>
</tr>
<tr>
<td>number of turns per phase windings</td>
<td>$N_e$</td>
<td>141</td>
</tr>
<tr>
<td>phase resistance</td>
<td>$R$</td>
<td>2.2 $\Omega$</td>
</tr>
<tr>
<td>mean self inductance</td>
<td>$\bar{L}$</td>
<td>0.000147 H</td>
</tr>
<tr>
<td>mean mutual inductance</td>
<td>$\bar{M}$</td>
<td>0.00005 H</td>
</tr>
<tr>
<td>axial length of rotor</td>
<td>$L_r$</td>
<td>0.104 m</td>
</tr>
<tr>
<td>mean diameter of air gap</td>
<td>$D_o$</td>
<td>0.0181 m</td>
</tr>
<tr>
<td>air gap</td>
<td>$g$</td>
<td>0.00191 m</td>
</tr>
<tr>
<td>width of slot opening</td>
<td>$x$</td>
<td>0.00104 m</td>
</tr>
<tr>
<td>width of slot face</td>
<td>$y$</td>
<td>0.00513 m</td>
</tr>
<tr>
<td>amplitude of d.c. voltage source</td>
<td>$V_{dc}$</td>
<td>12 v</td>
</tr>
</tbody>
</table>
Figure 2.8 Normalized rotor angular velocity fluctuation.
Figure 2.9 Motor torque pulsation $T_e(f)$ predictions (in relative decibels) by using Model I ( ) and Model II ( o ).
Measured and predicted pure tones are compared in Figure 2.10. Even though a number of simplifications have been made in our noise source model, most (if not all) of the pure tones are predicted consistently either by Model I or II. Various harmonics of $\Omega_r$ as discussed earlier are also identified in Figure 2.10. Chief among these are $3N_p$, $6N_p$ and $9N_p$ from group (i), $0.5N_pN_p$ from group (ii), and $2N_p+2$, $3N_p-2$, $4N_p-2$, $7N_p+1$, $8N_p-4$, $8N_p-2$ and $10N_p+4$ from group (iii). Given these results and especially an encouraging correlation between theory and experiment, one can conclude that the frequencies associated with motor torque pulsation coincide with the frequencies of pure tones. Nonetheless, the models presented here are incapable of explaining all of the available experimental data and other empirical observations [2.4] since noise transmission paths and radiating surface(s) are yet to be included in our theory. Further analytical work will consider these and our models are expected to resolve a few perplexing aspects of the disk drive acoustic behavior. Our formulations can also be extended to examine the noise source characteristics of other 3-phase electrical machines and motion control devices used in a variety of consumer and industrial products.
Figure 2.10 Comparison of measured and predicted pure tones given $N_p\Omega_r/2\pi=72$ Hz, $N_p=8$ and $N_s=9$. Also refer to Table 2.1 and Figures 2.2 and 2.9.
REFERENCES FOR CHAPTER II


2.4 Personal conversations with computer disk drive engineers (1990).


3.1 INTRODUCTION

The eccentricity, open stator slots, and the magnetic saturation are key factors influencing motor torque pulsations [3.1-6]. Eccentricity effects have been studied for the diagnosis and monitoring of induction motors [3.1,2]. Different approaches have been proposed to examine the brushless d.c. motor (BDCM) open stator slots effects. For instance, Jufer [3.3] and Cassat [3.4] used conformal mapping to take into account the variation of the permeance function associated with open stator slots. Bolton [3.5] assumed that open stator slots have zero permeance, so that the stored air gap energy is calculated by summing the stored energy between each slot. Yang [3.6] has developed a formulation of the induction motor to take into account all possible effects; however, the emphasis was only on the frequency contents. It is obvious that most studies have focused primarily on a single parameter [3.1-5] or only on frequency contents [3.6]. A complete study of the combined effect of all parameters on a BDCM is yet to be carried out.

In Chapter II, the torque pulsation spectrum of BDCM was obtained based on two different models, namely the state space model (Phase Variable Formulation) and the Fourier series model (Fourier Series Representation). The state space model yields
the time history of torque by using a numerical integration scheme. Subsequently, the
torque spectrum is obtained by applying an FFT algorithm to the steady-state time
trace. The numerical integration is time consuming since small integration steps are
essential to yield a high frequency range of spectrum. Though the Fourier series model
is an analytical frequency domain model, it includes only the effects associated with the
trapezoidal rotor flux density distribution and quasi-square phase voltage to determine
the Fourier coefficients of phase currents. The overall emphasis was on the frequency
contents rather than the magnitude of the torque spectrum. Furthermore, the
fluctuation of angular velocity had to be assumed negligible, which may not be true for
some cases, to ensure a tractable analytical formulation. It is obvious that both
approaches have some limitations. An efficient frequency domain analytical or semi-
analytical technique is needed. It should be capable of predicting the magnitude of the
torque spectrum and yet must include all the inductance harmonics in phase currents.

The Galerkin's method (also referred to as the harmonic balance method) has
been widely used in solving nonlinear dynamic equations [3.8-11]. The technique
consists of assuming a trigonometric representation for the periodic steady-state
solution, substitution in the governing ordinary differential equations and balancing the
terms corresponding to coefficients of the same frequency. This results in non-linear
algebraic equations which are functions of the unknown Fourier coefficients in the
trigonometric series. The scheme is quite powerful, as it can handle a wide variety of
non-linearities, which may be weak or strong, and smooth or piece-wise continuous.
Its ability to handle complex effects coupled with the fact that solutions are obtained
directly in the frequency domain, makes it more efficient than the direct numerical
integration method. Most of the applications of this technique, in the literature, have
been restricted to simple nonlinear electrical circuits or mechanical systems of small dimension. A comprehensive study of systems with time varying (parametric) and nonlinear properties, such as a BDCM, is yet to be attempted.

3.2 PROBLEM FORMULATION

3.2.1 Fourier Coefficients of Inductance and Back E.M.F.

The BDCM is essentially a nonlinear time varying system as shown in Chapter II. The inductances and back e.m.f. of equivalent electrical circuit model of a BDCM are affected by mechanical and electrical defects, such as the eccentricity between the rotor and the stator, the magnetic saturation, and open stator slot modulation. All of these effects can be analytically incorporated into the BDCM model by including the variation of permeance associated with each cause.

The coordinates of a stator and a rotor with eccentricity is shown in Figure 3.1. The air gap between the stator and the rotor is

\[
g(\theta_r, \theta_s) = R_r - e \cos(\theta_s - \theta_r) + \sqrt{R_s^2 - e^2 \sin^2(\theta_s - \theta_r)}^2,
\]

where \( R_r \) and \( R_s \) are the radii of the rotor and the stator, \( e \) is the eccentricity, and \( \theta_r \) and \( \theta_s \) are angles of rotor position and stator. Assuming that the air gap is much smaller than the mean air gap radius, the permeance function which is inversely proportional to the air gap is represented by

\[
\mu(\theta_r, \theta_s) = \frac{\mu_\infty}{g} \frac{1}{1+\zeta \cos(\theta_s - \theta_r)},
\]
Figure 3.1 Coordinate system of a rotor and a stator with eccentricity $e$. 
where $g$ is the mean air gap and $\zeta = \frac{e}{g}$ is the relative eccentricity. Equation 3.2 can be also represented in terms of a Fourier series as [3.1,2]

$$\mu(\theta_r, \theta_s) = \frac{\mu_o}{g} \sum_{q=0}^{\infty} K_{e,q} \cos[q(\theta_s - \theta_r)];$$

$$K_{e,q} = \begin{cases} \frac{1}{\sqrt{1-\zeta^2}} & \text{for } q = 0 \\ \frac{2(\sqrt{1-\zeta^2} - 1)^q}{\zeta^q \sqrt{1-\zeta^2}} & \text{otherwise} \end{cases}$$

(3.3a,b)

The permeance function associated with the open stator slots is obtained by using the conformal mapping [3.3]. It is given below

$$\mu(\theta_s) = \frac{\mu_o}{g} \left\{ 1 - \beta \left[ 1 + \cos \left( \frac{2\pi r_g}{s} \theta_s \right) \right] \right\};$$

$$\beta = \frac{(1-u)^2}{2(1+u)^2}; \quad u = \frac{x}{2g^2} + \sqrt{\left( \frac{x}{2g} \right)^2 + 1};$$

$$S = \frac{\sqrt{g}}{\beta}; \quad \gamma = \frac{4}{\pi} \left[ \frac{x}{2g} \tan^{-1} \left( \frac{x}{2g} \right) - \log \left( 1 + \left( \frac{x}{2g} \right)^2 \right) \right]$$

(3.4a-e)

where $r_g = (R+r)/2$ is the mean air gap radius and $x$ is the slot width. Again, the permeance function is represented in terms of a Fourier series as follows
\[ \mu(\theta_r) = \frac{\mu_0}{\delta} \sum_{q=0}^{\infty} K_{S,q} \cos(\text{q}N_s \theta_s) \]

\[ K_{S,q} = \begin{cases} 
1 - \frac{xSN_s}{2 \pi r_g} & \text{for } q = 0 \\
(-1)^q \frac{2}{q \pi} \left[ \frac{x \sin \left( \frac{qSN_s}{2r_g} \right)}{\left( \frac{qSN_s}{2\pi r_g} \right)^2} \right] & \text{otherwise}
\end{cases} \quad (3.5a,b) \]

where \( N_s \) is the number of stator slots. The flux density associated with the rotor permanent magnets without the effects of open stator slots and eccentricity are obtained by using the equivalent current approach and Fourier analysis [3.7]. The predicted Fourier coefficients for radial magnetization are shown in Equation (3.6).

\[ B(\theta_r, \theta_s) = \sum_{q=\text{odd}}^{\infty} K_{B,q} \cos \left( \frac{N_p}{2} q(\theta_s - \theta_r) \right) \]

\[ K_{B,q} = \frac{2N_p k_m B_r}{\pi(a^{2q} - b^{2q})^2} \sin \left( \frac{q\alpha}{2} \right) \alpha^{-1} \left[ \frac{C_2^{1+q} - C_1^{1+q}}{(1 + q)} - b^{2n} \chi \right] \]

\[ \chi = \begin{cases} 
\frac{C_2^{1+q} - C_1^{1+q}}{1+q} & \text{for } q \neq 1 \\
\ln \left( \frac{C_2}{C_1} \right) & \text{for } q = 1
\end{cases} \quad (3.6a-c) \]
where $N_p$ is the number of poles of the rotor. To include the modulation effects of eccentricity and the stator slots, the Fourier series of the rotor flux density is multiplied by the permanence wave form as shown in Equations 3.3 and 3.5.

$$\tilde{B}(\theta_r, \theta_s) = \sum_{q,odd} \sum_{u=1}^{\infty} \sum_{l=1}^{\infty} K_{B,qN_p/2} K_{e,u} K_{S,N_l} \cos\left(\frac{qN_p}{2}(\theta_s - \theta_r)\right) \cos\left[u(\theta_s - \theta_r)\right] \cos\left(N_u \theta_s\right)$$

(3.7)

Assuming the concentrated stator winding, one obtains the back e.m.f. by substituting Equation (3.7) into Equation (2.10) [3.5].

Since the harmonics of inductances associated with the magnetic saturation depends on the saturation curve of the magnetic materials, no generic formulation is available. However, these effects are introduced as a Fourier series with known harmonic frequencies. The self and mutual inductances for concentrated stator windings are obtained by using the same procedure used in Chapter II and are shown below:

$$L_{ua}(\theta_r) = L\left[1 + \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} K_{E,qN_p} K_{e,s} \cos(qN_p \theta_r) \cos(s \theta_r)\right]$$

(3.8)

$$L_{bc}(\theta_r) = M\left[1 + \sum_{q=1}^{\infty} \sum_{s=1}^{\infty} K_{E,qN_p} K_{e,s} \cos(qN_p \theta_r) \cos(s \theta_r)\right]$$

(3.9)

Other components follow the same phase relationship indicated in Chapter II.
3.2.2 Scope and Objectives

The main objective of this chapter is to develop a steady-state frequency analysis scheme by using the Galerkin's method for a non-linear time varying BDCM model. It should include predicted inductance and back e.m.f. harmonics associated with eccentricity, open stator slots and magnetic saturation, and it must yield both magnitudes and frequency contents of the torque spectrum. Comparison of various effects acting one at a time versus combined or simultaneous action is also included to illustrate the salient feature. The Galerkin's method is first applied to a 3-degree-of-freedom BDCM model without the mechanical system, based on the assumption of a constant angular velocity. The model is essentially equivalent to three first-order electrical circuits with time varying parameters. A detailed derivation of the method is included to demonstrate a step-by-step procedure of the methodology. The model is then reduced to a single equivalent circuit by assuming simple phase relationship among different stator phase currents. The next step would be to include the mechanical system dynamics in the motor model, taking into account the fluctuation of the rotor angular velocity.

3.3 LINEAR TIME VARYING (LTV) MODEL: EQUIVALENT ELECTRICAL CIRCUITS WITHOUT MECHANICAL SYSTEM DYNAMICS

A general theory of the Galerkin's method for non-linear dynamic equations has been developed in an earlier paper by Urabe and Reiter [3.8]. Generic formulations for both first and second order system were derived and applied to several examples by using the iterative Newton method. The methodology was later extended to multi-
excitation problems with incommensurable frequencies by Chua and Ushida [3.9,10]. Ling and Wu [3.11] increased the computational efficiency by employing the Broyden method and the fast Fourier transform (FFT) in the numerical scheme. The Galerkin's method developed in the following sections lies somewhere between the two approaches developed in Chapter II. It includes all inductance harmonics in calculating phase currents in the frequency domain. Nevertheless, it is a semi-analytical scheme and the mechanical system dynamics can be included in this model.

3.3.1 3-DOF LTV Model

Assuming that the angular velocity fluctuation is negligible, the mechanical system dynamics can then be decoupled from the equivalent circuit. Consequently, we need to apply the Galerkin's method to only the equivalent circuit of Equation (2.5). This equation is rewritten here in the spatial domain as shown in Equation (3.10), so that the fundamental frequency will be normalized to one, no matter what the angular velocity is. The significant of the system analysis in spatial domain will become apparent when the mechanical system dynamics is included.

\[
L(\theta_r)\left\{\frac{d\theta_r}{dt} \frac{d}{d\theta_r} I(\theta_r)\right\} + \left\{\frac{d\theta_r}{dt} \frac{d}{d\theta_r} L(\theta_r) + R\right\} I(\theta_r) = V(\theta_r) - E(\theta_r)
\] (3.10)

Since the rotor angular velocity \(\Omega_r\) is assumed constant, the above equation is further simplified as

\[
\Omega_r L I' + \left[\Omega_r L' + R\right] I = V - E,
\] (3.11)
where superscript ' is the derivative with respective to $\theta_r$. The following trigonometric expansion is assumed for each phase current

$$i_s(\theta_r) = K_{i,0}^s + \sum_{n}^N \left[ K_{i,2n}^s \cos(f_n \theta_r) + K_{i,2n-1}^s \sin(f_n \theta_r) \right] ; \ s = a, b, c \tag{3.12}$$

The harmonic indices $f_n$ include all harmonic contributions due to the phase voltage $V$, back e.m.f. $E$, and inductances $L$. Here, $N$ represents the total number of harmonics included. In this study, only 3 side bands around any dominant harmonic are selected. The harmonic groups associated with various system variables are listed in Table 3.1. Equation (3.12) is differentiated with respect to $\theta_r$ to obtain the following expression for the derivative of the phase current $I'$.

$$i'_s(\theta_r) = \sum_{n}^N f_n \left[ K_{i,2n-1}^s \cos(f_n \theta_r) - K_{i,2n}^s \sin(f_n \theta_r) \right] ; \ s = a, b, c \tag{3.13}$$

Further, the first and second terms of the left hand side of Equation (3.12) are expanded into a trigonometric series, as shown below:

$$\Gamma = \Omega_r \mathbf{L} \mathbf{I}' = \gamma_0 + \sum_{n=1}^N \left[ \gamma_{2n} \cos(f_n \theta_r) + \gamma_{2n-1} \sin(f_n \theta_r) \right] \tag{3.14}$$

$$\Lambda = \left\{ \Omega_r \mathbf{L}' + \mathbf{R} \right\} \mathbf{I} = \alpha_0 + \sum_{n=1}^N \left[ \alpha_{2n} \cos(f_n \theta_r) + \alpha_{2n-1} \sin(f_n \theta_r) \right] \tag{3.15}$$

where,

$$\Gamma = [\Gamma_a \ \Gamma_b \ \Gamma_c]^T ; \quad \Lambda = [\Lambda_a \ \Lambda_b \ \Lambda_c]^T$$
<table>
<thead>
<tr>
<th>Parameter/Variable</th>
<th>Harmonic Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eccentricity</td>
<td>$k\Omega_r ; k \in [0,3]$</td>
</tr>
<tr>
<td>Magnetic saturation</td>
<td>$kN_p\Omega_r$</td>
</tr>
<tr>
<td>Open stator slot</td>
<td>$kN_s\Omega_r$</td>
</tr>
<tr>
<td>Inductance</td>
<td>$(N_p k_1 \pm k_2)\Omega_r ; k_1, k_2 \in [0,3]$</td>
</tr>
<tr>
<td>Phase voltage</td>
<td>$\frac{N_p}{2}(2k-1)\Omega_r$</td>
</tr>
<tr>
<td>Back e.m.f.</td>
<td>$\left[\frac{N_p}{2}(2k_1-1)\pm N_s k_2 \pm k_3\right]\Omega_r ;$</td>
</tr>
<tr>
<td></td>
<td>$k_1 \in [1,13] ; k_2, k_3 \in [0,3]$</td>
</tr>
<tr>
<td>Phase current</td>
<td>$\left[\frac{N_p}{2}(2k_1-1)\pm N_s k_2 \pm N_p k_3 \pm k_4\right]\Omega_r ;$</td>
</tr>
<tr>
<td></td>
<td>$k_1 \in [1,13] ; k_2, k_3, k_4 \in [0,3]$</td>
</tr>
<tr>
<td>Rotor angular velocity &amp; Torque</td>
<td>$[N_p k_1 \pm N_s k_2 \pm N_p k_3 \pm k_4]\Omega_r ;$</td>
</tr>
<tr>
<td></td>
<td>$k_1 \in [1,13] ; k_2, k_3, k_4 \in [0,3]$</td>
</tr>
</tbody>
</table>
\[ \gamma_n = \begin{bmatrix} \gamma_n^a & \gamma_n^b & \gamma_n^c \end{bmatrix}^T \quad ; \quad \alpha_n = \begin{bmatrix} \alpha_n^a & \alpha_n^b & \alpha_n^c \end{bmatrix}^T \] (3.16a-d)

\( V \) and \( E \) are also written in terms of a Fourier series, with harmonic indices \( f_n \) listed in Table 3.1. An arrangement of coefficients of like terms on both sides of Equation (3.11), corresponding to the harmonic index \( f_n \) yields \( 3(2N+1) \) equations.

\[ \Re_n^s \equiv K_{V,n}^s - K_{E,n}^s - \gamma_n^s - \alpha_n^s = 0 \quad ; \quad s = a, b, c \quad n = 0, 1, \ldots, 2N \] (3.17)

Since the terms \( \gamma_n^s \) and \( \alpha_n^s \) in Equation (3.17) are implicit functions of phase current coefficients \( K_{I,n}^s \), the above equations are non-linear. Hence, an iterative Newton-Raphson method is employed to solve them, as shown below:

\[ J\{\Delta K_s\} = K_V - K_E - \alpha - \gamma \] (3.18)

The procedure is initiated by assuming a set of trial values for the phase current coefficients \( K_{I,n}^s \). The current vector \( I \) is sampled \( M \) times in the \( \theta_r \) domain over \([0, 2\pi]\), where \( M \) is chosen such that spectral aliasing is avoided \((M \geq 2f_N)\). This leads to the rectangular collocation matrix \( T \) of dimension \( M(2N+1) \), as shown below:

\[
T = \begin{bmatrix}
1 & \sin(f_1\theta_{r,1}) & \cos(f_1\theta_{r,1}) & \sin(f_2\theta_{r,1}) & \cos(f_2\theta_{r,1}) & \ldots & \cos(f_N\theta_{r,1}) \\
1 & \sin(f_1\theta_{r,2}) & \cos(f_1\theta_{r,2}) & \sin(f_2\theta_{r,2}) & \cos(f_2\theta_{r,2}) & \ldots & \cos(f_N\theta_{r,2}) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & \sin(f_1\theta_{r,M}) & \cos(f_1\theta_{r,M}) & \sin(f_2\theta_{r,M}) & \cos(f_2\theta_{r,M}) & \ldots & \cos(f_N\theta_{r,M})
\end{bmatrix}
\] (3.19)

where, \( \theta_{r,m} = \frac{2\pi(m-1)}{M} \).
The phase currents at selected rotor positions are given by

\[
\begin{bmatrix}
I_s(\theta_{r,1}) \\
I_s(\theta_{r,2}) \\
\vdots \\
I_s(\theta_{r,M})
\end{bmatrix} = \mathbf{T} \begin{bmatrix}
K^s_{I,0} \\
K^s_{I,1} \\
\vdots \\
K^s_{I,2N}
\end{bmatrix} ; \quad s = a, b, c \tag{3.20}
\]

Similarly, the derivative of phase currents with respect to \( \theta_r \) can be expressed as

\[
\begin{bmatrix}
I'_s(\theta_{r,1}) \\
I'_s(\theta_{r,2}) \\
\vdots \\
I'_s(\theta_{r,M})
\end{bmatrix} = \mathbf{T}_v \begin{bmatrix}
K^s_{I,0} \\
K^s_{I,1} \\
\vdots \\
K^s_{I,2N}
\end{bmatrix} ; \quad s = a, b, c \tag{3.21}
\]

where,

\[
\mathbf{T}_v = \begin{bmatrix}
0 & f_1 \cos(f_1 \theta_{r,1}) & -f_1 \sin(f_1 \theta_{r,1}) & \cdots & -f_N \sin(f_N \theta_{r,1}) \\
0 & f_1 \cos(s_1 \theta_{r,2}) & -f_1 \sin(s_1 \theta_{r,2}) & \cdots & -f_N \sin(s_N \theta_{r,2}) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & f_1 \cos(s_1 \theta_{r,M}) & -s_1 \sin(s_1 \theta_{r,M}) & \cdots & -s_N \sin(s_N \theta_{r,M})
\end{bmatrix} \tag{3.22}
\]

The voltages of \( \Gamma \) and \( \Lambda \) in Equations (3.14) and (3.15) are also calculated at selected rotor positions as shown below:

\[
\begin{bmatrix}
\Gamma_s(\theta_{r,1}) \\
\vdots \\
\Gamma_s(\theta_{r,M})
\end{bmatrix} = \mathbf{T} \begin{bmatrix}
\sum_{q=a,b,c} \Omega_r L_{sq}(\theta_{r,1}) \hat{\gamma}_q(\theta_{r,1}) \\
\vdots \\
\sum_{q=a,b,c} \Omega_r L_{sq}(\theta_{r,M}) \hat{\gamma}_q(\theta_{r,M})
\end{bmatrix} = \mathbf{T} \begin{bmatrix}
\gamma^s_0 \\
\vdots \\
\gamma^s_{2N}
\end{bmatrix} ; \quad s = a, b, c \tag{3.23}
\]
\[
\begin{bmatrix}
\Lambda_s(\theta_{r,1}) \\
\vdots \\
\Lambda_s(\theta_{r,M})
\end{bmatrix} = \begin{bmatrix}
\sum_{q=a,b,c} \left[ \Omega_q L_{sq}^r(\theta_{r,1}) + R \delta_{sq} \right] i_q(\theta_{r,1}) \\
\vdots \\
\sum_{q=a,b,c} \left[ \Omega_q L_{sq}^r(\theta_{r,M}) + R \delta_{sq} \right] i_q(\theta_{r,M})
\end{bmatrix} = T \begin{bmatrix}
\alpha_0^s \\
\vdots \\
\alpha_{2N}^s
\end{bmatrix}; \ s = a, b, c \quad (3.24)
\]

From the above equations one obtains \( \alpha_n \) and \( \gamma_n \) as follows.

\[
\begin{bmatrix}
\gamma_0^s \\
\vdots \\
\gamma_{2N}^s
\end{bmatrix} = \left( T^T T \right)^{-1} T^T \begin{bmatrix}
\Gamma_s(\theta_{r,1}) \\
\vdots \\
\Gamma_s(\theta_{r,M})
\end{bmatrix} \quad (3.25)
\]

\[
\begin{bmatrix}
\alpha_0^s \\
\vdots \\
\alpha_{2N}^s
\end{bmatrix} = \left( T^T T \right)^{-1} T^T \begin{bmatrix}
\alpha_s(\theta_{r,1}) \\
\vdots \\
\alpha_s(\theta_{r,M})
\end{bmatrix} \quad (3.26)
\]

The Jacobian matrix \( J \), of dimension \( 3(2N+1) \) by \( 3(2N+1) \), consists of elements \( J_{mn}^{sq} \) defined by

\[
J_{mn}^{sq} = \frac{\partial \mathcal{R}_m^s}{\partial K_{l,n}^{q}}; \ m, n = 0, 1, \ldots, 2N; \ s, q = a, b, c \quad (3.27)
\]

Since \( \mathbf{E} \) and \( \mathbf{V} \) are not functions of phase current, Equation (3.27) is given by

\[
J_{mn}^{sq} = -\frac{\partial \alpha_m^s}{\partial K_{l,n}^q} - \frac{\partial \gamma_m^s}{\partial K_{l,n}^q} \quad (3.28)
\]
To determine the terms in the above equation, the following procedure needs to be followed. Taking derivative of Equation (3.25) on both sides with respect to $\theta_r$,

$$\frac{\partial}{\partial \theta_r} \begin{bmatrix} \gamma_0^s \\ \vdots \\ \gamma_{2N}^s \end{bmatrix} = \left[ T^T T \right]^{-1} T^T \frac{\partial}{\partial \theta_r} \begin{bmatrix} \Gamma_s(\theta_{r,1}) \\ \vdots \\ \Gamma_s(\theta_{r,M}) \end{bmatrix}$$  \hspace{1cm} (3.29)

From Equations (3.23) one can see that

$$\frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,n}^q} \bigg|_{\theta_r = \theta_{r,k}} = \Omega_r \frac{\partial}{\partial l'_q} \left[ \sum_{x=a,b,c} L_{xx} l'_x \right] \frac{\partial l'_q(\theta_r)}{\partial K_{f,n}^q} \bigg|_{\theta_r = \theta_{r,k}}$$  \hspace{1cm} (3.30)

Expressing in the matrix form and incorporating the collocation matrix, we have

$$\begin{bmatrix} \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,0}^q} \bigg|_{\theta_r = \theta_{r,1}} & \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,1}^q} \bigg|_{\theta_r = \theta_{r,1}} & \cdots & \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,2N}^q} \bigg|_{\theta_r = \theta_{r,1}} \\ \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,0}^q} \bigg|_{\theta_r = \theta_{r,2}} & \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,1}^q} \bigg|_{\theta_r = \theta_{r,2}} & \cdots & \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,2N}^q} \bigg|_{\theta_r = \theta_{r,2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,0}^q} \bigg|_{\theta_r = \theta_{r,M}} & \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,1}^q} \bigg|_{\theta_r = \theta_{r,M}} & \cdots & \frac{\partial \Gamma_s(\theta_r)}{\partial K_{f,2N}^q} \bigg|_{\theta_r = \theta_{r,M}} \end{bmatrix} = Q_{sq}^I T_v$$

$$Q_{sq}^I = \Omega_r \text{DIAG} \left[ L_{sq}(\theta_{r,1}) \ L_{sq}(\theta_{r,2}) \ \cdots \ L_{sq}(\theta_{r,M}) \right]$$  \hspace{1cm} (3.31a,b)

By incorporating Equation (3.31) into Equation (3.29), we obtain the following equation.
\[
\begin{bmatrix}
\frac{\partial \gamma_0^q}{\partial K_{L,0}} & \frac{\partial \gamma_0^q}{\partial K_{L,1}} & \cdots & \frac{\partial \gamma_0^q}{\partial K_{L,2N}} \\
\frac{\partial \gamma_1^q}{\partial K_{L,0}} & \frac{\partial \gamma_1^q}{\partial K_{L,1}} & \cdots & \frac{\partial \gamma_1^q}{\partial K_{L,2N}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \gamma_{2N}^q}{\partial K_{L,0}} & \frac{\partial \gamma_{2N}^q}{\partial K_{L,1}} & \cdots & \frac{\partial \gamma_{2N}^q}{\partial K_{L,2N}}
\end{bmatrix} = \{T^T T\}^{-1} T^T Q_{sq}^1 T_v
\] (3.32)

Applying the same procedure to Equation (3.26), we obtain the following equation

\[
\begin{bmatrix}
\frac{\partial \alpha_0^q}{\partial K_{L,0}} & \frac{\partial \alpha_0^q}{\partial K_{L,1}} & \cdots & \frac{\partial \alpha_0^q}{\partial K_{L,2N}} \\
\frac{\partial \alpha_1^q}{\partial K_{L,0}} & \frac{\partial \alpha_1^q}{\partial K_{L,1}} & \cdots & \frac{\partial \alpha_1^q}{\partial K_{L,2N}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \alpha_{2N}^q}{\partial K_{L,0}} & \frac{\partial \alpha_{2N}^q}{\partial K_{L,1}} & \cdots & \frac{\partial \alpha_{2N}^q}{\partial K_{L,2N}}
\end{bmatrix} = \{T^T T\}^{-1} T^T Q_{sq}^2 T
\]

\[Q_{sq}^2 = \text{Diag}[\Omega_r L_{sq}'(\theta_{r,1}) + \delta_{sq} R, \Omega_r L_{sq}'(\theta_{r,2}) + \delta_{sq} R, \ldots, \Omega_r L_{sq}'(\theta_{r,M}) + \delta_{sq} R] \] (3.33a,b)

Now, we have the necessary formulations needed to assemble the Jacobian matrix. Since only the three-phase BDCM is considered in this study, we can partition the matrix \(J\) into the following sub-matrices.

\[
J = \begin{bmatrix}
J^{aa} & J^{ab} & J^{ac} \\
J^{ba} & J^{bb} & J^{bc} \\
J^{ca} & J^{cb} & J^{cc}
\end{bmatrix}
\] (3.34a)

\[
J^{ij} = \{T^T T\}^{-1} T^T \{Q_{ij}^1 T_v + Q_{ij}^2 T\} \quad ; \quad i, j = a, b, c
\] (3.34b)
Since the formulation is linear, the Jacobian is independent of the state variables (phase currents) and hence is evaluated only once in the process of iteration. The flow chart of the numerical scheme is shown in Figure 3.2. The computational steps are as follows:

(i) Calculate the collocation matrices $T$ and $T_y$ from Equations (3.19) and (3.22).

(ii) Obtain the Jacobian matrix from Equations (3.34).

(iii) Assume an initial value of Fourier coefficients of phase currents.

(iv) Find residues of Equation (3.17), by using Equations (3.23) to (3.26) with the assumed values of phase currents in accordance with step (iii).

(v) Modified the assumed Fourier coefficients of phase currents by finding corrections using Equations (3.18).

(vi) Repeat steps (iv) to (v) until the corrections are within the tolerance.

In Chapter II, the motor torque has been formulated analytically since only the simplified harmonics of phase currents are included. Nonetheless, the final expression is complicated and tedious. If all of the harmonic effects are included as presented here, the explicit formulation of torque becomes untractable. Therefore, the Discrete Fourier Transform (DFT) algorithm is employed to find the Fourier coefficients of torque.

The phase current (phase $a$) and torque are obtained by using both numerical integration and the Galerkin's method, given the same governing equations. Results are compared in time and frequency domains as shown in Figures 3.3 and 3.4. The Galerkin's method matches well with numerical integration especially at the dominant harmonics. However, some of the harmonics in the higher frequency ranges have
Governing Equations

Solving Steady-State Solutions:
  a. Write $I$, $V$ and $E$ in the form of truncated trigonometric series.
  b. Calculate the collocation matrices.

Harmonic Balances:
  a. Use series expressions for $I$, $E$ and $V$
     in governing equations.
  b. Equate terms with like harmonics.

Newton Iteration Procedures:
  a. Obtain Jacobian matrix.
  b. Assume initial values of Fourier coefficients.
  c. Calculate residues.
  d. Modify the assumed Fourier coefficients.
  e. Iterate step c and d.

Figure 3.2 Flow chart of numerical scheme for the Galerkin's method.
Figure 3.3 Comparison of the steady-state phase current of the LTV model with combined effects; (a) time domain (b) frequency domain. Key: solid line: the numerical integration; circles: the Galerkin's method.
Figure 3.4 Comparison of the steady-state torque of the LTV model with combined effects; (a) time domain (b) frequency domain. Key: solid line: the numerical integration; dotted line and circles: the Galerkin's method.
discrepancies, possibly due to the intrinsic numerical damping of the numerical integration scheme and the errors associated with FFT algorithm. Each parameter is also studied separately to illustrate its effect on the electromagnetic torque pulsations. Figures 3.5 to 3.7 compare the torque of the system calculated using both methods and considering separately the effects of eccentricity, saturation and open stator slots, respectively. One can see from these results that dominant harmonics are those having harmonic frequencies equal to \(24k\Omega_r\), which has already been observed in Chapter II. Among the various factors, open stator slots have the most significant effects, which is observed by comparing Figures 3.4 and 3.7. If only some dominant harmonics are of interest, then most of the harmonics need not to be included for this sample case. It can significantly reduce the computational time associated with the Galerkin's method but would be the same for the numerical integration. This is the most important advantage that the Galerkin's method has over the numerical integration.

3.3.2 Reduced LTV Model

The numerical computation of harmonic balance method on 3-DOF BDCM model involves a Jacobian matrix of \(3(2N+1)\) dimension. To evaluate a large number of harmonics, such as 100 used in the sample case, the computing time is enormous. Since it was observed in the previous section that phase currents of the three stator phase have almost the same wave form with a phase shift which is equal to \(4\pi/3N_p\), the governing equations can be further reduced to a single DOF system. This implies a reduction of the size of the Jacobian matrix and consequently the computation time.
Figure 3.5 Steady-state torque of the LTV model with eccentricity effect; (a) time domain (b) frequency domain. Key: solid line: the numerical integration; circles: the Galerkin's method.
Figure 3.6 Steady-state torque of the LTV model with the saturation effect; (a) time domain (b) frequency domain. Key: solid line: the numerical integration, dotted line and circles: the Galerkin's method.
Figure 3.7 Steady-state torque of the LTV model with the open stator slot effect; (a) time domain (b) frequency domain. Key: solid line: the numerical integration, dotted line and circles: the Galerkin's method.
The currents of phase b and c are the same as phase a with a phase shift as follows.

\[ i_b[\theta_r] = i_a[\theta_r - \phi] \quad ; \quad i_c[\theta_r] = i_a[\theta_r + \phi] \quad (3.35a,b) \]

where, \( \phi = \frac{4\pi}{3N_p} \).

The governing equation of the reduced model becomes

\[ (\Omega_r L_{aa} + R)i_a + \Omega_r L_{ab}i_b + \Omega_r L_{ac}i_c + \Omega_r L_{ab}i_a + \Omega_r L_{ac}i_a + \Omega_r L_{ac}i_a = v_a - \Omega_r \hat{e}_a \quad (3.36) \]

Accordingly, the trigonometric collocation matrix of phase \( b \) with respect to the Fourier coefficients of phase \( a \) is

\[
T_b = \begin{bmatrix}
1 & \sin[f_1(\theta_{r,1} - \phi)] & \cos[f_1(\theta_{r,1} - \phi)] & \cdots & \sin[f_N(\theta_{r,1} - \phi)] \\
1 & \sin[f_1(\theta_{r,2} - \phi)] & \cos[f_1(\theta_{r,2} - \phi)] & \cdots & \sin[f_N(\theta_{r,2} - \phi)] \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & \sin[f_1(\theta_{r,M} - \phi)] & \cos[f_1(\theta_{r,M} - \phi)] & \cdots & \sin[f_N(\theta_{r,M} - \phi)]
\end{bmatrix} \quad (3.37)
\]

The collocation matrix \( T_c \) for phase \( c \) is obtained by substituting \(-\phi\) with \(+\phi\) in the above equation. Similarly, the collocation matrices for the derivative of phase currents \( T_{v,b} \) and \( T_{v,c} \) are defined. Following the same approach demonstrated in developing the Jacobian matrix for the 3-DOF model, the Jacobian matrix for the reduced model is found as

\[
J = \left[ T^T T \right]^{-1} T^T \left\{ \sum_{s=a,b,c} Q_{as}^1 T_{v,s} + Q_{as}^2 T_s \right\} \quad (3.38)
\]
The dimension of the Jacobian matrix is now reduced to one third of the 3-DOF model, correspondingly, the computer time is decreased by about one half for the 3-DOF model. Comparisons between reduced model and 3-DOF model are shown in Figures 3.8 and 3.9. Excellent agreement is obvious for most of the harmonics. Therefore, very little accuracy is lost when the reduced model is employed. Nevertheless, a significant improvement in computational efficiency is observed.

3.4 NON-LINEAR TIME VARYING (NLTV) MODEL: EQUIVALENT ELECTRICAL CIRCUITS WITH MECHANICAL SYSTEM DYNAMICS

In previous sections, the mechanical dynamics was not included in the model, because the angular velocity of the rotor was assumed to be constant. For the sample case, verified by using the numerical integration scheme, as shown in Chapter II, the angular velocity fluctuation is indeed negligible. Nevertheless, in other applications, especially for large motors, the fluctuation of rotor velocity may have a significant effect on the output torque. For this reason, the proposed Galerkin's method is extended to include the mechanical system dynamics to make the methodology more general.

The mechanical system is assumed to be represented by an equivalent rotational inertia and a viscous damping as in Chapter II. Consequently, an additional degree of freedom, the angular velocity of the rotor, is added to the governing equations. Also, the system becomes a nonlinear time-varying system. This means that Jacobian matrix needs to be evaluated now for every iteration. Both 3-DOF and reduced models with mechanical system are developed in the following sections.
Figure 3.8 Comparison of the steady-state phase current of the LTV model using the Galerkin's method; (a) time domain (b) frequency domain. Key: solid line: 3-DOF model, dotted line and circles: reduced model.
Figure 3.9 Comparison of the steady-state torque of the LTV model using the Galerkin's method; (a) time domain (b) frequency domain. Key: solid line: 3-DOF model, dotted line and circles: reduced model.
3.4.1 4-DOF NLTV Model

Equations (2.5), (2.6) and (2.9) are rewritten here in spatial domain as

\[ \Omega_r \left[ L'r' + IL' \right] + RI = V - \Omega_r \hat{E} \]  \hspace{1cm} (3.39)
\[ J \Omega_r \Omega'_r + B_m \Omega_r = \hat{E}^\top I + \frac{1}{2} I^\top L'I \]  \hspace{1cm} (3.40)

Since the mean angular velocity is not known, the collocation matrices will change in every iteration if the state variables are expressed in time domain. Therefore, a spatial domain representation, the same one used in the previous section, is used to represent phase currents and angular velocity. This ensures that the normalized period will be \(2\pi\). The steady-state currents and angular velocity in the spatial domain are expressed as

\[ I = K'_{1,0} + \sum_{s=1}^{N} K'_{1,2s-1} \sin(f_s \theta_r) + K'_{1,2s} \cos(f_s \theta_r) \]  \hspace{1cm} (3.41)
\[ \Omega_r = K_{\omega,0} + \sum_{s=1}^{N} K_{\omega,2s-1} \sin(f_s \theta_r) + K_{\omega,2s} \cos(f_s \theta_r) \]  \hspace{1cm} (3.42)

Note that phase current and angular velocity could have different families of harmonic frequencies as listed in Table 3.1. This means that the dimensions of collocation matrices for the phase current and angular velocity are not the same. To avoid the complexity, all harmonics of fundamental frequency are included in both Fourier series in the numerical examples, which is true in the sample case if all effects are included. Define the Fourier coefficient vector, which has dimension of \(4 \times (2N+1)\) by \(1\) as
Following the procedure developed in the previous sections, the iterative Newton-Raphson method is obtained. The Jacobian matrix is divided into sub matrices as follows.

\[
J = \begin{bmatrix}
J_{aa} & J_{ab} & J_{ac} & J_{a\Omega} \\
J_{ba} & J_{bb} & J_{bc} & J_{b\Omega} \\
J_{ca} & J_{cb} & J_{cc} & J_{c\Omega} \\
J_{\Omega a} & J_{\Omega b} & J_{\Omega c} & J_{\Omega\Omega}
\end{bmatrix},
\]

(3.44)

where,

\[
J^T_{ii} = \begin{cases}
\{T^T T\}^{-1} T^T \{Q_{ij}^3 T + Q_{ij}^4 T_v\} & \text{for } i, j = a, b, c \\
\{T^T T\}^{-1} T^T Q_{ij}^5 T & \text{for } i = a, b, c; j = \Omega \\
\{T^T T\}^{-1} T^T Q_{ij}^6 T & \text{for } i = \Omega; j = a, b, c \\
\{T^T T\}^{-1} T^T \{Q_{ij}^7 T + Q_{ij}^8 T_v\} & \text{for } i, j = \Omega
\end{cases}
\]

\[
Q_{ij}^3 = \text{DIAG} \left[ \Omega_r(\theta_1) L_{ij}'(\theta_1) \ldots \Omega_r(\theta_M) L_{ij}'(\theta_M) \right]
\]

\[
Q_{ij}^4 = \text{DIAG} \left[ \Omega_r(\theta_1) L_{ij}(\theta_1) \ldots \Omega_r(\theta_M) L_{ij}(\theta_M) \right]
\]

\[
Q_{ij}^5 = \text{DIAG} \left[ \sum_{s=a,b,c} \{i_s(\theta_1) L_{is}(\theta_1)\}' + \hat{e}_i(\theta_1) \ldots \sum_{s=a,b,c} \{i_s(\theta_M) L_{is}(\theta_M)\}' + \hat{e}_i(\theta_M) \right]
\]

\[
Q_{ij}^6 = \text{DIAG} \left[ - \sum_{s=a,b,c} i_s(\theta_1) L_{js}'(\theta_1) - \hat{\varepsilon}_j(\theta_1) \ldots - \sum_{s=a,b,c} i_s(\theta_M) L_{js}'(\theta_M) - \hat{\varepsilon}_j(\theta_M) \right]
\]
\[
Q^7 = \text{DIAG}[J\Omega_r(\theta_1) + B_m \ldots J\Omega_r(\theta_M) + B_m]
\]

\[
Q^8 = \text{DIAG}[J\Omega_r(\theta_1) \ldots J\Omega_r(\theta_M)]
\]

Since the Jacobian matrix needs to be evaluated for every iteration, it takes a very long time to complete if a large number of harmonics are included. Figures 3.10 and 3.11 compared results obtained from the Galerkin's method versus the numerical integration. Comparing results from LTV model, Figures 3.3 and 3.4, it is apparent that the effect of angular velocity fluctuation is indeed insignificant for the sample case.

### 3.4.2 Reduced NLTV Model

To make computer code of the system manageable, only the reduced model of the equivalent electrical circuit for BDCM is used. The governing equations of the reduced model with the mechanical system are shown as follows.

\[
\begin{align*}
[L_{aa} \Omega_r i'_a + L_{ab} \Omega_r i'_b + L_{ac} \Omega_r i'_c + [(L'_{aa} \Omega_r + R) i_a + L'_{ab} \Omega_r i_b + L'_{ac} \Omega_r i_c ]] = v_a - \Omega_r \hat{e}_a & \quad (3.46) \\
J\Omega_r \Omega_r' + B_m \Omega_r = \hat{E}^T \mathbf{1} + \frac{1}{2} \mathbf{1}^T \mathbf{L}^T \mathbf{L} \mathbf{1} & \quad (3.47)
\end{align*}
\]

where,

\[
i_b(\theta_r) = i_a \left( \theta_r - \frac{4\pi}{3N_p} \right) \quad ; \quad i_c(\theta_r) = i_a \left( \theta_r + \frac{4\pi}{3N_p} \right)
\]

Similar to the procedures used in earlier sections, the Jacobian matrix and linear equations associated with the Newton-Raphson method applied to Equation (3.43) is represented by
Figure 3.10 Steady-state phase current of the NLTV model; (a) time domain (b) frequency domain. Key: solid line: the numerical integration, dotted line and circles: the Galerkin's method.
Figure 3.11 Steady-state spectra of the NLTV model; (a) the angular velocity, (b) the torque. Key: solid line: the numerical integration, circles: the Galerkin's method.
\[ J = \begin{bmatrix} J^ii & J^{i\Omega} \\ J^{\Omega i} & J^{\Omega\Omega} \end{bmatrix} \]  

(3.49)

where,

\[ J^ii = \left\{ T^T T \right\}^{-1} T^T \left\{ \sum_{i=a,b,c} Q^3_{ai} T_i + Q^4_{ai} T_{v,i} \right\} \]

\[ J^{i\Omega} = \left\{ T^T T \right\}^{-1} T^T Q^5_{a} T \]

\[ J^{\Omega i} = \left\{ T^T T \right\}^{-1} T^T \left\{ \sum_{i=a,b,c} Q^6_{i} T_i \right\} \]

\[ J^{\Omega\Omega} = \left\{ T^T T \right\}^{-1} T^T \left\{ Q^7 T + Q^8 T_v \right\} \]  

(3.50a-b)

Comparisons of 3-DOF model and reduced model with mechanical system dynamics are shown in Figures 3.12 and 3.13. Mean steady-state rotor angular velocities from numerical integration and the Galerkin's method are compared in Table 3.2.

3.5 CONCLUSION

The Galerkin's method has been successfully applied to both LTV and NLTV BDCM models, which include non-trapezoidal rotor flux density distribution and effects of dynamic eccentricity, open stator slots and magnetic saturation, to predict Fourier coefficients of motor torque pulsation. This is the first study on the application of the Galerkin's method to BDCM. The results from the Galerkin's method and numerical integration match within a reasonable accuracy. The harmonics associated
Table 3.2 Comparison of mean angular velocity.

<table>
<thead>
<tr>
<th>Harmonic group included</th>
<th>$\bar{\Omega}_r$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical Integration</td>
<td>Harmonic Balance Method</td>
</tr>
<tr>
<td>Full Model</td>
<td>Reduced Model</td>
</tr>
<tr>
<td>Eccentricity, saturation and open state slots.</td>
<td>72.0 74.5 75.0</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>75.3 77.9 75.8</td>
</tr>
<tr>
<td>Saturation</td>
<td>74.0 76.8 77.2</td>
</tr>
<tr>
<td>Open stator slots</td>
<td>75.7 78.4 79.0</td>
</tr>
</tbody>
</table>
Figure 3.12 Steady-state phase current of the NLTV model using the Galerkin's method; (a) time domain (b) frequency domain. Key: solid line: 4-DOF model, dotted line and circles: the reduced model.
Figure 3.13 Steady-state spectra of the NLTV model using the Galerkin's method; (a) the angular velocity, (b) the torque. Key: solid line: the 4-DOF model, circles: the reduced model.
with the rotor flux density distribution have the most significant effects on the torque spectrum, which has already been identified in Chapter II. Due to complex interactions among various variables or parameters, it is very difficult to isolate the sole effect of each factor. Nevertheless, open stator slots seem to dominate the spectrum for this sample case. The methodology also includes fluctuating effects of the rotor, which is not possible by using conventional frequency domain analysis. Though, the effects of rotor angular velocity pulsation are not significant for the sample case used in this study, it may be important in other cases. The methodology also has been used to study in a reduced BDCM model which employs a simple phase relationship among phase current wave forms. This method significantly reduces the dimension of the Jacobian matrix, and consequently the computation time.
REFERENCES FOR CHAPTER III


CHAPTER IV

RADIATOR: VIBRO-ACOUSTIC CHARACTERISTICS
OF A COMPUTER DISK

4.1 INTRODUCTION

In Chapters II and III, the pulsating torque of the brushless d.c. motor was found to be the primary source of tonal noise radiation and several mathematical models of the noise source mechanism were developed. Next, the attention is focused on the vibro-acoustic characteristics of a computer disk which is considered to be the radiator in this study. While considerable research efforts have been devoted to understand dynamic and stability issues of spinning disks and head-disk interfaces [4.1], virtually no attention has been focused on the acoustic radiation characteristics of a computer disk. This study attempts to fill this void by developing modal radiation efficiency formulations of a single disk for both elastic deformation and rigid body modes. The mathematical formulations are then expanded to include multi-modal excitation case by using the modal base approach. The resulting analysis also considers the effect of the source rotation on natural frequencies and modal radiation efficiencies. The theory developed here may be applied to other rotating disks including gear bodies and circular saws.
4.2 LITERATURE REVIEW

The sound radiation characteristics of vibrating circular/annular plates have been studied extensively, especially when the plate vibrates in its natural mode or given specific velocity distribution. However, literature on annular disk acoustics, especially its modal radiation properties, is rather sparse.

Sound power radiated from a planar radiator is typically calculated by employing two approaches; the far-field sound pressure expression and the surface/near-field intensity expression [4.14]. The first approach essentially integrates the far-field Green's function over the surface of a radiator. The second approach usually transforms the time domain problem into the wave number domain. The task of finding the sound power then becomes how to adequately evaluate the complicated integrals involved by using numerical, analytical or asymptotic expansion techniques.

Early studies of circular or annular planar radiators focused on plates vibrating in the translating-type or rocking-type rigid body modes [4.2-8]. For instance, Gladwell [4.2] analyzed a rigid circular piston by using the Hankel transform to map the velocity potential into wave number space. Asymptotic results for the far-field velocity potential were found and the radiation impedances were derived in terms of the power series expansions of Bessel functions. Self and mutual impedance methods have also been employed in studies of sound radiation due to rigid-translating motion of circular and annular plates using different approaches [4.3-8]. These include the surface pressure distribution method (near-field approach) [4.3], the Bouwkamp's integral [4.4-6], in which the impedance is obtained by integration of the square of directivity function, and the Fourier transform of a generalized impulse response [4.7,8]. In
general, these mathematical techniques are complex and the final formulae are often expressed in terms of Bessel, Struve and hypergeometric functions. Bouwkamp [4.5] transformed Merriweather's final expression [4.3], which consists of the summation of an infinite series of hypergeometric functions, into a finite integral of trigonometric functions to yield a form suitable for numerical calculation. Mechel [4.6] used a power-based definition of the radiation impedance to derive radiation impedances of a sphere, a cylinder and other planar radiators of different geometry. The influence of mutual radiation of an array of sources is also included in his paper.

Greenspan [4.9] extended the theory of piston radiation to radiators with special axisymmetric velocity distributions. Solutions for sound fields, especially along symmetry axis, are provided for a few classical boundary conditions. The general solution for the velocity distribution which can be represented by a power series of the square of radius is also derived. Investigation of radiation characteristics of planar radiators vibrating in their elastic modes has been conducted both analytically and experimentally [4.10-18]. Holographs of near-field sound pressure of a clamped circular plate vibrating in its natural modes are presented in an early paper [4.10]. Together with the vibration measurement, the modal radiation efficiency is calculated. Predicted and measured near-field sound intensity flow of a clamped circular plate in resonant vibration are also available in Krishnappa and McDougall's study [4.11]. Theoretical prediction in their work is based on Rayleigh's integral and finite-difference approximation and measurements were carried out by using a two-microphone intensity probe. Energy recirculating zones are found to be increased with the number of nodal circles. Levine and Leppington [4.12] analyzed modal radiated sound power using an exact integral representation for frequencies above the modal coincidence. Asymptotic solutions for radiation efficiency of a clamped circular plate vibrating at high frequency
modes are derived. Similar analyses on beams and rectangular panels were performed by Levine in an early paper [4.13]. Williams [4.14] derived a power series representation of sound power in terms of wave number for planar radiator of arbitrary geometry. Others have performed related research on the sound radiation dealing with the effects of distributed load on rectangular plates [4.16], internal damping and viscous/thermal losses [4.17] and arbitrary boundary conditions [4.18].

Modal coupling effects are relatively important for accurate prediction of radiated sound power when more than one of the radiator's natural modes are excited, but the literature on this subject is very sparse and has considered only cases of one-dimensional beams [4.19-21]. Keltie and Peng [4.19] found that the coupling between two natural modes is as important as the individual mode when both natural frequencies are much lower than the excitation frequency. Cunefare [4.20,21] developed a quadratic expression for the radiation efficiency of a beam under multimodal excitation using a far-field intensity integration technique. It has been shown that the minimization of radiation efficiency becomes essentially an eigenvalue type problem in a form identical to the Rayleigh's quotient employed in the discipline of vibrations.

The effect of source motion on the sound radiation from circular or annular plates is obviously of interest in many applications but literature on this topic deals primarily with aerodynamic sources [4.22-24]. Typically, sources are assumed to be either classical point sources (monopole, dipole and quadrupole) or a combination of them to simplify the mathematical problems [4.25-26]. Majority of approaches employ the far-field Green's function of a moving source evaluated at the retarded time. For instance, Lowson [4.25] extended Lighthill's theory [4.22] for a point force in uniform rectilinear motion into a general expression which takes into account the acceleration of
source motion. Morfey and Tanna [4.26] employed Lowson's result to study the point force in a circular motion. Overall sound power is evaluated by calculating the autocorrelation of sound pressure. Levine [4.27] used a different approach to derive an expression for the instantaneous sound power radiated by a point source in arbitrary motion; the formula essentially is the same as the one yielded by Lowson's result. A point monopole-dipole combination is investigated to study interactive effects. For the moving surface, analysis is based on extension of the Kirchhoff's formula [4.28-30]. One way to apply the Kirchhoff's theory is to assume a hypothetical surface to substitute the surface of a moving radiator [4.28,29]. The sound field is the same as the real field outside the hypothetical surface and is arbitrary inside of it. A generalized function theorem is then employed to represent the discontinuities across the hypothetical surface. The other approach includes the turbulence effects associated with the moving surface [4.30].

4.3 PROBLEM FORMULATION

Prior publications on the sound radiation from annular disks reveal that several key issues including the modal radiation efficiency and acoustic coupling effects associated with multi-modal excitation are yet to be examined. Also, the effect of structural sound source rotation has not been studied. Considering the noise problem of rotating computer disks, it is obvious that such a formulation is needed. It is apparent that an in-depth study of radiation characteristics of a computer disk is essential to understand the noise problem.

Typically several disks, separated by spacing collars, are stacked on a single spindle. In practice, the sound is radiated by the rotating disk stack into an irregular
cavity enclosed by a flexible casing. In this section, only a single stationary disk is considered, as shown in Figure 4.1. It is further assumed to be mounted flush with an infinite rigid baffle, and sound is radiated into an acoustic free field to make the problem general. The acoustical loading of the disk structure by the surrounding medium (air) over the frequency range of interest is ignored.

For a stationary disk, the compression exerted by the collars simulates the clamped boundary condition at the inner edge; the outer edge is essentially free. A typical 3.5" computer hard disk is used as a sample case in this study. The dimensions and material properties of the disk is listed in Table 4.1.

4.4 MODAL REPRESENTATION

4.4.1 Exact and Approximate Solutions of a Stationary Disk

Natural frequencies \( \omega_{mn} \) of a computer disk for transverse flexural motion are determined by applying the classical thin plate theory [4.31] where \( m \) and \( n \) denote the number of nodal circles and diameters, respectively. The governing equation of plate flexure is as follow:

\[
D \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \right]^2 w + \rho h \frac{\partial^2 w}{\partial t^2} = F(r, \varphi, t); \tag{4.1a}
\]

\[
D = \frac{Eh^3}{12(1-\nu^2)}. \tag{4.1b}
\]
Table 4.1 Dimensions and material properties of the sample computer disk.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer radius (m)</td>
<td>47.5E-3</td>
</tr>
<tr>
<td>Inner radius (m)</td>
<td>16.0E-3</td>
</tr>
<tr>
<td>Thickness (m)</td>
<td>0.81E-3</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>2800</td>
</tr>
<tr>
<td>Young's modulus (N/m²)</td>
<td>6.19E10</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Figure 4.1 Schematic of computer disk radiating into a hemispherical free field.
For a stationary disk, the compression exerted by the collars simulates the clamped boundary condition at the inner edge; the outer edge is essentially free. Consequently the boundary conditions at the inner edge \( r=a \) are

\[
\begin{align*}
  w &= 0 ; \\
  \frac{\partial w}{\partial r} &= 0
\end{align*}
\]

(4.2,3)

and at the outer edge \( r=b \) are

\[
\begin{align*}
  \frac{\partial^2 w}{\partial r^2} + \nu \left( \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} \right) &= 0 \\
  \frac{\partial}{\partial r} \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} \right) + \frac{1-\nu}{r^2} \frac{\partial^2}{\partial \phi^2} \left( \frac{\partial w}{\partial r} \frac{w}{r} \right) &= 0.
\end{align*}
\]

(4.4)

The eigensolution of equation (4.1) consists of various Bessel functions and it is expressed by [4.31]

\[
\psi_{mn}(r, \phi, t) = e^{i\omega_{mn}t} \cos(n\phi) \left[ c_1 J_n(\beta_{mn}r) + c_2 Y_n(\beta_{mn}r) + c_3 I_n(\beta_{mn}r) + c_4 K_n(\beta_{mn}r) \right]
\]

(4.6)

where \( \beta_{mn} = \sqrt{\frac{\omega_{mn}^2 \rho h}{D}} \).

Eigensolution are determined by solving the characteristic equation which is obtained by substituting Equation (4.6) into Equations (4.2-5). Since exact solutions of vibration modes are expressed in terms of the Bessel functions, it is very complicated, if not impossible, to derive the analytical formulation of radiation efficiency. Therefore, a polynomial approximation for natural mode \( \psi_{mn} \) is employed here to avoid this.
problem. The approximate modal function consists of trial functions which satisfies the geometric boundary conditions at \( r=a \). It is represented by

\[
\psi_{mn}(r, \varphi) = \cos(n\varphi) \sum_{s=2}^{N(m)} c_{mn,s}(r-a)^s
\]  

The number of tried function is decided by the modal index \( m \). In this study, the highest term of polynomial \( N \) is assigned to be 8. The potential energy (\( PE \)) and the kinetic energy (\( KE \)) are obtained by substituting the approximate modal function into the following equations: \[4.32\]

\[
PE = \frac{D}{2} \int_a^b \left( \nabla^2 \psi_{mn} \right)^2 - 2(1-\nu) \left[ \frac{\partial^2 \psi_{mn}}{\partial r^2} \left( \frac{1}{r} \frac{\partial \psi_{mn}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi_{mn}}{\partial \varphi^2} \right) - \left( \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi_{mn}}{\partial \varphi} \right) \right)^2 \right] r dr d\varphi
\]

\[
KE = \frac{\rho}{2} \int_a^b \left( \frac{\partial \psi_{mn}}{\partial t} \right)^2 r dr d\varphi
\]

A set of linear equations as shown in Equations (4.10-12) is then obtained by applying the energy principle \[4.31-33\].

\[
\begin{bmatrix}
\Lambda_{22} & \Lambda_{23} & \cdots & \Lambda_{2N} \\
\Lambda_{32} & \Lambda_{33} & \cdots & \Lambda_{3N} \\
\vdots & \vdots & \ddots & \vdots \\
\Lambda_{N2} & \Lambda_{N3} & \cdots & \Lambda_{NN}
\end{bmatrix}
\begin{bmatrix}
c_{mn,2} \\
c_{mn,3} \\
\vdots \\
c_{mn,N}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]  

(4.10a)

\[
\Lambda_{ij} = ij(i-1)(j-1)X_{i-j}^{i+j-4} + \nu ij(i+j-2)X_{i-j}^{i+j-3} + \left[ ij \left[ 2(1-\nu)n^2 + 1 \right] - \nu n^2 (i^2 + j^2 - i - j) \right]
\]

\[
X_{i-j}^{i+j-2} - (3-2\nu)n^2 (i+j)X_{i-j}^{i+j-1} + n^2 [n^2 + 2(1-\nu)]X_{i-j}^{i+j} - \beta_{nm} X_{i-j}^{i+j}
\]  

(4.10b)
Equation (4.10) yields the approximate natural frequencies and the coefficients of associated modal functions. The approximate polynomial mode shapes and exact solutions for two axisymmetric modes of the sample disk are shown in Figure 4.2. Excellent agreement is evident.

4.4.2 Experimental Results and Analytical Predictions

Impulse modal experiments on a stationary disk have been conducted to validate the analytical results. The Young’s modulus is estimated by comparing the predicted first natural frequency of a free-free disk with the experiment result. A single computer disk is mounted on a rigid shaft between the spacing collars used in the disk drive to simulate the real boundary conditions. Analytical predictions of natural frequencies match the measured values as shown in Table 4.2. Selected mode shapes obtained from the approximated modal functions are illustrated in Figures 4.3. For natural modes with the same number of nodal circles, the variation of mode shape in the radial direction is not very significant.
Table 4.2 Selected natural frequencies of a disk.

<table>
<thead>
<tr>
<th>Case</th>
<th>Prediction</th>
<th>Experiment</th>
<th>( \omega_{mn}/2\pi ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-Free</td>
<td>408</td>
<td>682</td>
<td>997</td>
</tr>
<tr>
<td>Clamped-Free</td>
<td>613</td>
<td>711</td>
<td>1130</td>
</tr>
<tr>
<td>(m,n)</td>
<td>(0,1)</td>
<td>(0,2)</td>
<td>(0,3)</td>
</tr>
</tbody>
</table>

(m,n)=(number of nodal circles, number of nodal diameters)
Figure 4.2 Comparison of approximated and exact vibration modes of a 3.5" computer disk, empty symbols: exact solution; solid line: polynomial approximation. Key: —— and oooo (0,0) mode; ····· and oooo (1,0) mode.
Figure 4.3 Predicted vibration modes. Key: ——— (a) (0,0) mode, (b) (1,0) mode; ——— (a) (0,1) mode, (b) (1,1) mode; ——— (a) (0,2) mode, (b) (1,2) mode.
4.5 MODAL RADIATION EFFICIENCY FORMULATIONS

As described in Section 4.4, the polynomial approximation matches the exact modal function almost perfectly. Consequently, we can use the approximated modal functions to derive the formulation for radiated sound without losing any accuracy. The object here is to develop formulation for radiated sound from an annular disk on the modal base. The modal radiation efficiency of a single stationary disk is formulated first to analyze acoustic characteristics of the disk vibrating in its rigid body modes or elastic deformation modes. Two methods, namely Far-field Approach and Impedance Approach, are developed to formulate the modal radiated sound power $\Pi_{mn}$.

4.5.1 Far-field Approach

This approach essentially employs the free field Green's function and geometric far-field assumption to evaluate radiated sound. Sound pressure for Figure 4.1 is given by employing the Raleigh's integral [4.14] as

$$P_{mn}(R, \theta, \phi) = \frac{j \omega_{mn} \rho_a}{2\pi} \int_0^{2\pi} \int_a^b \psi_{mn}(r, \varphi) e^{-j km d} r dr d\phi,$$

where $d = R - r \cos(\varphi - \phi) \sin \theta$ is the distance between the source and observation points. The approximated modal function, Equation (4.6), is rewritten in terms of the dimensionless radius coordinate as shown in Equation (4.12).

$$\psi_{mn}(r, \varphi) = \cos(n\varphi) \sum_{s=0}^{N} c_{mn,s} \left( \frac{r}{b} \right)^s$$

(4.12a)
\[ \bar{c}_{mn,j} = \left( \frac{b}{a} \right)^j \sum_{i=j}^{N} (-1)^{i+j} \frac{i!}{(i-j)!j!} d^i c_{mn,j} \]  

(4.12b)

With geometric far-field assumption, \( R \gg r \), the far-field modal sound pressure \( P_{mn} \) is obtained by substituting the approximated modal function, equation (4.12), into equation (4.11). It is expressed by

\[ P_{mn}(R, \theta, \phi) = \frac{j^{n+1} e^{-jkr}}{R} \rho_o c k_{mn} \cos(n \phi) \sum_{s=0}^{N} \bar{c}_{mn,s} \int_{\theta} r^{s+1} J_n(k_{mn} r \sin \theta) dr \]  

(4.13)

Since the far-field impedance is the characteristic impedance \( \rho_o c \), the modal sound power \( \Pi_{mn} \) is obtained by integrating the far-field sound intensity over a hemispherical surface in the far field. By expressing Bessel function of the first kind in terms of a power series, the modal sound power is represented in terms of a convergent power series of modal wave number \( k_{mn} \).

\[ \Pi_{mn} = \frac{R^2}{2 \rho_o c} \int_0^{\pi/2} \int_0^{2\pi} P_{mn}(R, \phi, \theta) P_{mn}(R, \phi, \theta)^* \sin \theta d\theta d\phi \]

\[ = \frac{\rho_o c \pi b^2}{\epsilon_n} (k_{mn} b)^{2(n+1)} \sum_{s=0}^{\infty} A_{mn,s} (k_{mn} b)^{2s}, \]  

(4.14a)

where,

\[ \epsilon_n = \begin{cases} 1 & \text{if } n = 0 \\ 2 & \text{otherwise} \end{cases} \]

\[ A_{mn,s} = \frac{(s+n)!^2 (-1)^s}{(2s+2n+1)!} \sum_{i,j_1}^{N} \bar{c}_{mn,i} \bar{c}_{mn,j_1} \sum_{q=0}^{s} q!(s-q)!(n+q)!(s+n-q)! \frac{(1-\lambda_1)(1-\lambda_2)}{\lambda_1 \lambda_2}; \]
\[
\lambda_1 = t_1 + 2q + n + 2 \quad ; \quad \lambda_2 = t_2 + 2(s - q) + n + 2 \quad ; \quad \alpha = \frac{a}{b}.
\] (4.14b-f)

and * denotes complex conjugate of a function. The coefficients \( A_{\mu n,s} \) of this power series representation are clearly the functions of the geometry of the disk. Since the radiated sound is represented in terms of a convergent series of the wave number, it is convenient to relax the accuracy of predictions and implement it in a computer code by limiting the number of terms included in the series. The corresponding reference power is

\[
\Pi_{nm,ref} = \frac{\rho_0 c \pi b^2}{\varepsilon_n} \sum_{l_1, l_2=0}^{N} \frac{1 - \alpha_{l_1+l_2+2}}{l_1 + l_2 + 2}
\] (4.15)

Consequently, the modal radiation efficiency \( \sigma_{nm} \) is obtained by

\[
\sigma_{nm} = \frac{\Pi_{nm}}{\Pi_{nm,ref}}
\] (4.16)

### 4.5.2 Impedance Approach

In this approach, the disk is discretized into several concentric annuli and each annulus is assumed to have an uniformly distributed velocity in the radial direction. The velocity distribution of the \( s \)-th annulus surface of \( r_{o,s} \) and \( r_{i,s} \) as the outer and inner radii is

\[
\Psi_{nm,s}(\varphi) = \Psi_{nm}(\bar{r}_s) \cos(n \varphi)
\]

\[
\bar{r}_s = \frac{r_{o,s} + r_{i,s}}{2}
\] (4.17a,b)
The self-radiation impedance of a single annulus with rigid body motion is available in the literature \[4.4\]. Using this concept, the self-radiation impedance of \(s\)-th annulus given the velocity distribution of Equation (4.17) is \[4.34\]

\[
Z_{mn,ss} = \frac{2\pi \rho c k_{mn}}{e_n} \int_0^{k_{mn}} \frac{f_n^2(\xi)\xi d\xi}{\sqrt{k_{mn}^2 - \xi^2}}
\]

\[
f_n(\xi) = \int_{r,s}^\infty I_n(\xi r) rdr
\] (4.18a,b)

The final expansion is again represented in terms of a convergent series of the wave number as shown in Equation (4.19).

\[
Z_{mn,ss} = \frac{\pi}{e_n} \rho c (k_{mn}r_{o,s})^{2(n+1)} \sum_{\nu=0}^{\infty} B_\nu (k_{mn}r_{o,s})^{2\nu}
\] (4.19a)

where,

\[
B_\nu = \frac{[(t+n)]^2(-1)\nu}{(2t+2n+1)!} \sum_{q=0}^{\nu} \frac{(1-\alpha_1)(1-\alpha_2)}{q!(t-q)!(n+q)!(n+q)!} \beta_1 \beta_2
\]

\[
\beta_1 = 2q + n + 2 ; \quad \beta_2 = 2(t-q) + n + 2 ; \quad \alpha_1 = \frac{r_{i,s}}{r_{o,s}}
\] (4.19b-e)

Mutual-radiation impedance between two non-consecutive concentric annuli is derived in terms of self-radiation impedances. Given three consecutive annuli (number 1, 2 and 3 in Figure 4.4), the mutual-radiation impedance between 1 and 3 is determined by the following expression

\[
Z_{mn,13} = \frac{1}{2}(Z_{mn,66} + Z_{mn,22} - Z_{mn,55} - Z_{mn,44})
\] (4.20)
Figure 4.4 Definition of annuli for the impedance approach used to calculate the mutual radiation impedance.
where annulus 4 contains annuli 1 and 2, annulus 5 contains annuli 3 and 4, and annulus 6 consists of annuli 1, 2, and 3. With the formulation of self and mutual radiation, the sound power radiated from each annulus is formulated by

$$\Pi_{mn,s} = Z_{mn,ss} \psi_{mn}^2(\bar{r}_s) + 2 \sum_{l=1, l\neq s}^{N_a} Z_{mn,st} \psi_{mn}(\bar{r}_s) \psi_{mn}(\bar{r}_l),$$  \hspace{1cm} (4.21)

where $N_a$ is the total number of annuli. The total modal sound radiated from the disk is then obtained by summing the power emitted by each annulus.

$$\Pi_{mn} = \left[ \begin{array}{ccc} \psi_{mn}(\bar{r}_1) \\ \psi_{mn}(\bar{r}_2) \\ \vdots \\ \psi_{mn}(\bar{r}_{N_a}) \end{array} \right]^T \left[ \begin{array}{cccc} Z_{mn,11} & Z_{mn,12} & \cdots & Z_{mn,1N_a} \\ Z_{mn,21} & Z_{mn,22} & \cdots & Z_{mn,2N_a} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{mn,N_a,1} & Z_{mn,N_a,2} & \cdots & Z_{mn,N_a,N_a} \end{array} \right] \left[ \begin{array}{c} \psi_{mn}(\bar{r}_1) \\ \psi_{mn}(\bar{r}_2) \\ \vdots \\ \psi_{mn}(\bar{r}_{N_a}) \end{array} \right],$$ \hspace{1cm} (4.22)

Predicted radiation efficiencies of selected vibration modes by both approaches are compared with numerical results yielded by a boundary element program BEMAP [4.35] in Table 4.3. The annular disk was discretized into 30 concentric annuli and 48 quadratic elements were used in the boundary element model. Excellent agreement is found between the two analytical approaches. Analytical predictions match the numerical results very well for those modes which have smaller number of nodal diameters. Apparently, as the number of nodal diameters increased, a higher number of elements is required by the boundary element model to yield accurate results. Figure 4.5 illustrates the contribution of sound power from each annulus for two axisymmetric modes.
Table 4.3 Predicted modal radiation efficiencies of a stationary disk.

<table>
<thead>
<tr>
<th>Modal Index (m,n)</th>
<th>Natural Frequency (Hz)</th>
<th>Far-field Approach</th>
<th>Impedance Approach</th>
<th>BEMAP [4.35]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>604</td>
<td>0.842E-1</td>
<td>0.842E-1</td>
<td>0.842E-1</td>
</tr>
<tr>
<td>(0,1)</td>
<td>602</td>
<td>0.272E-2</td>
<td>0.272E-2</td>
<td>0.272E-2</td>
</tr>
<tr>
<td>(0,2)</td>
<td>713</td>
<td>0.749E-4</td>
<td>0.749E-4</td>
<td>0.739E-4</td>
</tr>
<tr>
<td>(0,3)</td>
<td>1120</td>
<td>0.187E-4</td>
<td>0.187E-4</td>
<td>0.170E-4</td>
</tr>
<tr>
<td>(0,4)</td>
<td>1820</td>
<td>0.223E-4</td>
<td>0.222E-4</td>
<td>0.212E-4</td>
</tr>
<tr>
<td>(0,5)</td>
<td>2750</td>
<td>0.485E-4</td>
<td>0.484E-4</td>
<td>0.454E-4</td>
</tr>
<tr>
<td>(0,6)</td>
<td>3870</td>
<td>0.138E-3</td>
<td>0.138E-3</td>
<td>0.145E-3</td>
</tr>
<tr>
<td>(0,7)</td>
<td>5180</td>
<td>0.457E-3</td>
<td>0.456E-3</td>
<td>0.491E-3</td>
</tr>
<tr>
<td>(0,8)</td>
<td>6670</td>
<td>0.164E-2</td>
<td>0.163E-2</td>
<td>0.174E-2</td>
</tr>
<tr>
<td>(0,9)</td>
<td>8330</td>
<td>0.608E-2</td>
<td>0.606E-2</td>
<td>0.632E-2</td>
</tr>
<tr>
<td>(1,0)</td>
<td>3890</td>
<td>0.330</td>
<td>0.329</td>
<td>0.335</td>
</tr>
<tr>
<td>(1,1)</td>
<td>4050</td>
<td>0.322</td>
<td>0.322</td>
<td>0.325</td>
</tr>
<tr>
<td>(1,2)</td>
<td>4540</td>
<td>0.111</td>
<td>0.112</td>
<td>0.112</td>
</tr>
<tr>
<td>(1,3)</td>
<td>5390</td>
<td>0.403E-1</td>
<td>0.405E-1</td>
<td>0.400E-1</td>
</tr>
<tr>
<td>(1,4)</td>
<td>6620</td>
<td>0.281E-1</td>
<td>0.283E-1</td>
<td>0.274E-1</td>
</tr>
<tr>
<td>(1,5)</td>
<td>8220</td>
<td>0.452E-1</td>
<td>0.455E-1</td>
<td>0.433E-1</td>
</tr>
</tbody>
</table>

m: number of nodal circles, n: number of nodal diameters
Figure 4.5 Contribution of sound power from each annulus by using the impedance approach. (a) (0,0) mode. (b) (1,0) mode.
4.5.3 Rigid Body Modes

Two rigid body modes, the rigid translating and rigid rocking modes, are also included. To distinguish them from the elastic modes, the radial modal index $m$ is intentionally assigned the value -1 for these modes. The corresponding tangential modal index $n$ (number of nodal diameters) are 0 for the rigid translating mode and 1 for rigid rocking mode, respectively. The radiation efficiency of these two rigid modes can be derived directly by using either the far-field approach or the impedance approach. The normalized surface velocity functions are represented by

$$\psi_{-1,0} = \frac{1}{\sqrt{(b^2-a^2)}\pi}$$  \hfill (4.23)

$$\psi_{-1,1} = \frac{2}{\sqrt{(b^4-a^4)}\pi} r \cos(\varphi)$$  \hfill (4.24)

Figure 4.6 shows the calculated radiation efficiencies of both modes for the example case.

4.6 MODAL COUPLING EFFECTS

4.6.1 Modal Base Formulation

If several structural modes are excited simultaneously, coupling effects between the acoustic field generated by different modes need to be included. From the far-field formulation, it can be shown easily that non-zero sound power due to coupled mode exists only when the coupled modes have the same number of nodal diameters.
Figure 4.6  Radiation efficiency for rigid body modes of a single stationary disk; (a) translating piston mode, (b) rocking piston mode. Key: — analytical formulation; oooo BEMAP program.
Assume that the disk is excited by a harmonic force at frequency $\omega$ and the velocity distribution can be expressed in terms of elastic and rigid body modes $\psi$ normalized such that their norms are equal to one, so that

$$w = \eta^T \psi$$

$$\psi = \begin{bmatrix} \psi_{-1,0} & \psi_{-1,1} & \psi_{00} & \psi_{01} & \ldots & \psi_{0n} & \psi_{10} & \psi_{11} & \ldots & \psi_{mn} \end{bmatrix}^T$$

$$\eta = \begin{bmatrix} \eta_{-1,0} & \eta_{-1,1} & \eta_{00} & \eta_{01} & \ldots & \eta_{0n} & \eta_{10} & \eta_{11} & \ldots & \eta_{nn} \end{bmatrix}^T \quad (4.25a,b,c)$$

Here, the normalized elastic modes are grouped by the number of nodal circles, and the same number of natural modes are selected for each group. In addition to rigid body modes, $(m+1)$ groups with $(n+1)$ natural modes in each one are included in the formulation. If some of the selected natural modes are not excited, the corresponding participating factors are numerically set to be zero. The far-field pressure is then expressed by

$$P = \eta^T P;$$

$$P = \frac{jk\rho_o c}{2\pi R} \int_0^b \int_0^{2\pi} \psi e^{-jkd} r dr d\phi. \quad (4.26a,b)$$

Consequently, the sound radiation efficiency is

$$\sigma = \frac{\eta^H \Pi \eta}{2\rho_o c \eta^H G \eta},$$

$$\Pi = \frac{R^2}{2\rho_o c} \int_0^{\pi/2} \int_0^{2\pi} P^H P \sin \theta d\theta d\phi, \quad G = \int_0^{2\pi} \int_a^b \psi^T \psi r dr d\phi = \begin{bmatrix} I_{2n} & M^T \\ M & I_{(m+1)} \end{bmatrix},$$

$$M^T = \begin{bmatrix} \langle \psi_{-1,0}, \psi_{00} \rangle & 0 & 0 & \cdots & \langle \psi_{-1,0}, \psi_{m0} \rangle & 0 & 0 & \cdots & 0 \\ 0 & \langle \psi_{-1,1}, \psi_{01} \rangle & 0 & \cdots & 0 & \langle \psi_{-1,1}, \psi_{m1} \rangle & 0 & \cdots & 0 \end{bmatrix}.$$
where $I_n$ is unit diagonal matrix of dimension $n$.

For an arbitrary plate, the modal sound power matrix $\Pi$ is a fully occupied matrix, and each component needs to be evaluated numerically. For the annular disk, it consists of $(m+2)$ by $(m+2)$ sub matrices and is expressed by

$$\Pi = \begin{bmatrix} \Pi^{-1,-1} & \Pi^{-1,0} & \cdots & \Pi^{-1,m} \\ \Pi^{0,-1} & \Pi^{0,0} & \cdots & \Pi^{0,m} \\ \vdots & \vdots & \ddots & \vdots \\ \Pi^{m,-1} & \Pi^{m,0} & \cdots & \Pi^{mm} \end{bmatrix},$$

where,\n
$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{0} & \Pi_{ij}^{1} & \cdots & \Pi_{ij}^{n} \\ \Pi_{ij}^{0} & \Pi_{ij}^{1} & \cdots & \Pi_{ij}^{n} \\ \vdots & \vdots & \ddots & \vdots \\ \Pi_{ij}^{0} & \Pi_{ij}^{1} & \cdots & \Pi_{ij}^{n} \end{bmatrix},$$

$$\Pi_{jn}^{in} = \frac{R^2}{2\rho_0 c} \int_0^{\pi/2} \int_0^{2\pi} P_{in} P_{jn}^* \sin \theta \vartheta d\vartheta d\phi. \quad (4.28a-c)$$

It is evident from Equation 4.28c that only those modes which have the same number of nodal diameters, i.e. $m_1 = m_2$, contribute to the radiated sound. Therefore, each sub-matrix $\Pi_{ij}$ is a diagonal matrix expressed by

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{0} & 0 & \cdots & 0 \\ 0 & \Pi_{ij}^{1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Pi_{ij}^{n} \end{bmatrix},$$

for $i, j \neq -1$.\n
\[
\left\langle \psi_{m_{1n}}, \psi_{m_{2n}} \right\rangle = \int_0^{2\pi} \int_0^b \psi_{m_{1n}} \psi_{m_{2n}} r \vartheta dr d\vartheta, \quad (4.27a-e)\n\]
For $i = j$, the sub-matrices represent the sound power associated with the individual elastic or rigid body modes. Other sub-matrices are the contributions due to modal coupling effects between different modes. Each component of these sub-matrices is formulated in terms of a wave number power series as developed in the previous section. Figures 4.7-10 show selected self (single mode) and coupled (multiple mode) terms of radiated sound power of the sample disk for excitation frequency up to 10 KHz. From these results, it is observed that self radiated sound power spectra associated with elastic modes which have the same number of nodal circles (index $m$) essential have the similar patterns, except that the humps shift to higher frequencies as the number of nodal diameters (index $n$) increased. For sound power associated with modal coupling, similar phenomena is observed for the selected natural modes, and all of them have positive contribution to the over-all radiated sound power. However, if a higher number of nodal circles is included, the negative effects, i.e. sound cancellation in the modal domain, can also exist.

Figure 4.11 shows typical results for the disk vibrating with the following surface velocity distribution.

\[
\psi = \begin{bmatrix} \psi_{00} & \psi_{01} & \psi_{02} & \psi_{10} & \psi_{11} & \psi_{12} \end{bmatrix}^T \quad (4.30a)
\]

\[
\eta = [0.5 \ 1 \ 0.2 \ 0.3 \ 0.8 \ 0.4]^T \quad (4.30b)
\]
Figure 4.7 Selected self-terms of radiated sound power for multi-modal excitation. Key: ——— (a) (0,0) mode, (b) (1,0) mode; ——— (a) (0,2) mode, (b) (1,2) mode; ——— (a) (0,4) mode, (b) (1,4) mode.
Figure 4.8  Selected mutual terms of radiated sound power between elastic modes. Key: (0,0) and (1,0) modes; (0,2) and (1,2) modes; (0,4) and (1,4) modes.
Figure 4.9  Selected mutual terms of radiated sound power between rigid translating piston mode and elastic modes. Key: —— (-1,0) and (0,0) modes; ; ——— (-1,0) and (1,0) modes.
Figure 4.10 Selected mutual terms of radiated sound power between rigid rocking mode and elastic modes. Key: \(-1,1\) and \((0,1)\) modes; \(-1,-1\) and \((1,1)\) modes.
Figure 4.11 Radiation efficiency for multi-mode excitation. Key: —— analytical formulation; oooo BEMAP program.
4.6.2 Radiation Efficiency for a Disk under Concentrated Harmonic Moment or Force Excitations

The transverse motion of a computer disk can be excited by the sinusoidal axial force and bending moment due to a coupling between bearing stiffness and structural impedance at the junction between casing cover and spindle shaft. Radiation characteristics of the disk under a combined elastic deformation and rigid body motion associated with such types of excitation are examined next.

For a concentrated harmonic force excitation in the axial direction distributed along the inner edge of disk, the response of disk is a combination of the rigid translating mode and axisymmetric elastic modes. The deformation of the disk is represented by

\[ w = [\eta_{-1,0} \quad \eta_{00} \quad \cdots \quad \eta_{m_0}]^T [\psi_{-1,0} \quad \psi_{00} \quad \cdots \quad \psi_{m_0}] \]  \hspace{1cm} \text{(4.31)}

Assumed the amplitude of the rigid piston mode is equal to \( w_0 \), then the participating factors of elastic modes are obtained by using Equation (4.1) and normal mode expansion method. They are expressed by

\[ \eta_{m_0} = \frac{2 \pi \omega^2 w_0}{(\omega_{m_0}^2 - \omega^2) \sqrt{(b^2 - a^2)}} \sum_{s=0}^{N} \bar{c}_{m_0,s} b^{s+2} \frac{(1 - \alpha^{s+2})}{(s+2)} \]  \hspace{1cm} \text{(4.32)}

where \( \omega \) is the excitation frequency. The radiation efficiency is thus obtained by substituting Equation (4.32) into Equation (4.29).
For a concentrated sinusoidal moment excitation at the center, only the rigid rocking mode and the natural modes with one nodal diameter are excited. The response of the disk is

\[ w = \begin{bmatrix} \eta_{-1,1} & \eta_{01} & \cdots & \eta_{m1} & \psi_{-1,1} & \psi_{01} & \cdots & \psi_{m1} \end{bmatrix}^T \] (4.33)

Again, the participating factors of elastic modes are represented in terms of the amplitude of the rigid rocking mode and are written by

\[ \eta_{ml} = \frac{2\pi \omega^2 w_o}{(\omega_{ml}^2 - \omega^2)\sqrt{b^4 - d^4}\pi} \sum_{s=0}^{N} \tilde{c}_{ml,s} b^{s+3} \left(1 - \alpha^{s+3}\right) \] (4.34)

Figure 4.12 shows analytical results of radiation efficiency for excitation frequency up to 10 KHz.

### 4.7 SOURCE ROTATION EFFECTS

#### 4.7.1 Natural Frequencies of a Rotating Disk

If the disk is rotating at \( \Omega_r \) rad/s, the potential energy expression, Equation (4.8), must be modified to include the additional potential energy \( \Delta PE \) associated with a rotation-induced in-plane stress [4.36].

\[ \Delta PE = \frac{1}{2} \left( \frac{3 + \nu}{8} \right) \Omega_r^2 \rho h \int_0^{2\pi} \int_0^b \left[ a^2 + b^2 - r^2 - \left( \frac{ab}{r} \right)^2 \left( \frac{\partial w}{\partial r} \right) \right] dr \]
Figure 4.12 Radiation efficiency of a disk under a concentric harmonic excitation; (a) axial force excitation, (b) moment excitation.
Consequently, the contribution of the in-plane stress to the coefficients of linear equation set, Equation (4.10), is derived as

\[
\Delta \lambda_{ij} = \left( \frac{3+\nu}{8D} \right) \Omega_r^2 \rho h \left[ i \left[ \left( a^2 + \frac{b^2}{r^2} \right) X_{1}^{i+j-2} - X_{3}^{i+j-2} - (ab)^2 X_{-1}^{i+j-2} \right] + \right. \\
\left. \frac{3+\nu}{3+\nu} \right] \left[ (a^2 + b^2) X_{-1}^{i+j} + (ab)^2 X_{3}^{i+j} \right] \right];
\]

\[ X_3^j = X_3^{i+3} + 3aX_3^{i+2} + 3a^2X_3^{i+1} + a^3X_3^j \]  \hspace{1cm} (4.36b)

Selected natural frequencies of the rotating disk with the rotational speed up to 100 Hz are shown in Figure 4.13. It is seen that the increase of natural frequency is proportional approximately to $\Omega_r^2$, which has also been observed by Ramaiah [4.36].

### 4.7.2 Rotation Effects on Modal Radiation Efficiency

To account for the rotational motion of sound source, an expression for the Green's function of a moving sound source is used instead of the stationary one in deriving the far-field sound pressure of the disk [4.27]. Integrating the Green's function over the surface of the disk, we obtain the sound pressure in the far-field where $[\cdot]'$ denotes evaluation at retarded time $t'$.

\[
P_{mn} (R, \Theta, \phi, t) = \frac{\rho_o}{2\pi R} 2\pi \int_0^a \left[ \left( \frac{\dot{w}_{mn}}{(1-M_r)^2} + \frac{w_{mn} \dot{M}_r}{(1-M_r)^3} \right) r dr d\phi \right]' \quad \text{(4.37)}
\]
Figure 4.13 Normalized natural frequency of a spinning disk. Key: ——— (a) (0,0) mode, (b) (1,0) mode; ——— (a) (0,1) mode, (b) (1,1) mode; ——— (a) (0,2) mode, (b) (1,2) mode.
Here, Mach's number is in the direction toward the observation position. For observation in the far-field, it can be written by

\[ M_r = \frac{\Omega_r r}{c} \sin \theta \sin (\phi - \varphi - \Omega_r t') \] (4.38)

By taking the Fourier transform on the sound pressure in time domain, the spectrum of pressure is obtained as follows.

\[ P_{nn}(R, \Theta, \Phi) = \int_0^\infty P_{nn} e^{j\omega t} dt \] (4.39)

The relationship between the observation time \( t \) and the retarded time \( t' \) is as follows in spherical coordinates.

\[ t' = t - \frac{1}{c} [R - r \cos (\phi - \varphi - \Omega_r t') \sin \theta] \] (4.40)

\[ dt = [1 - M_r] \, dt' \] (4.41)

The sound pressure is evaluated by substituting Equations (4.37-38,40-41) into Equation (4.39) and letting the reference frame rotate synchronously with the disk. Assume that the following subsonic speed condition is valid.

\[ \frac{\Omega_r b}{c} << 1 \] (4.42)

Now we can incorporate the following approximations in evaluating the integral equation.

\[ (1 - M_r)^{-1} = 1 + M_r \quad ; \quad (1 - M_r)^{-2} = 1 + 2M_r \] (4.43a,b)
The final expression of sound pressure in the far-field, as given below, is obtained after a tedious mathematical manipulation.

\[
P_{nn} = \frac{(-j)^n \rho_o c \pi}{2 R} \sum_{s=0}^{\infty} \sum_{q=0}^{N} c_{mn,q} \frac{(-1)^q (k_{mn} \sin \theta)^{2s+n}}{s!(s+n)!2^{2s+n}} \left\{ f^n 2k_{mn} \cos(n \phi) Y_{2s+n+q+1} \right\}
\]

\[
-k_{mn} \left( \frac{\Omega_r}{c} \right) \sin(n \phi) \left[ \frac{k_{mn} \sin^2 \theta}{2(s+n+1)} Y_{2s+n+q+3} + \frac{2(s+n)}{k_{mn}} Y_{2s+n+q+1} \right]
\]

\[
+ \left( \frac{\Omega_r}{c} \right)^2 \cos(n \phi) \left[ \frac{k_{mn} \sin^2 \theta}{2(s+n+1)} Y_{2s+n+q+3} - \frac{2(s+n)}{k_{mn}} Y_{2s+n+q+1} \right]
\]

\[
+ \left( \frac{\Omega_r}{c} \right)^3 \sin(n \phi) \left[ \frac{k_{mn}^2 \sin^4 \theta}{4(s+n+1)(s+n+2)} Y_{2s+n+q+5} - \frac{4(s+n)(s+n-1)}{2k_{mn}^2} Y_{2s+n+q+1} \right]; \quad (4.44a)
\]

\[
Y_t = \int_a^b r' dr = \frac{b^{t+1} - a^{t+1}}{t+1} \quad (4.44b)
\]

Figure 4.14 shows normalized radiation efficiencies obtained by using the above expression and Equation (4.14) derived early for the stationary case for rotational speed up to 100 Hz. Both expressions yield almost the same results. Therefore, the effect of source rotation on \( \sigma_{mn} \) can be ignored in this case for engineering purposes. An increase in modal radiation efficiency can however be taken into account simply by considering an increase in natural frequencies and the change of mode shapes without losing any accuracy. Figures 4.15 shows clearly a linear relationship between the normalized radiation efficiency and the square of the normalized speed \( (\Omega_r/\omega_{mn})^2 \). A simple empirical equation has been found for the sample case which expresses this relationship adequately.
Figure 4.14 Comparison of normalized modal radiation efficiency, empty symbols: formulation for stationary cases; solid line: formulation including source rotation effect. Key: — — — and △△△△ (a) (0,0) mode, (b) (1,0) mode; — — — and △△△△ (a) (0,2) mode, (b) (1,2) mode; — — — and △△△△ (a) (0,4) mode, (b) (1,4) mode.
Figure 4.15 Normalized modal radiation efficiency vs. normalized rotational speed.
Key: ——— (a) (0,0) mode, (b) (1,0) mode; ——— (a) (0,2) mode, (b) (1,2) mode; ——— (a) (0,4) mode, (b) (1,4) mode.
A quantitative comparison between analytical results and approximations for a disk rotating at 72 Hz is listed in Table 4.4.

4.8 CONCLUDING REMARKS

The proposed methodology has yielded new analytical formulations for modal radiation efficiencies of a computer disk. From the sample case, it is obvious that the modal coupling effects on the radiated sound are indeed significant when multi modes are excited. The modal base analysis on sound radiation developed in this study yields a basic formulation to predict the sound radiation of a annular disk radiator. The formulations which are expressed in terms of power series of the wave number are convenient since they can be computer coded easily. Source rotation has little effects on the natural frequencies of the disk within the rotating speed of interest but the increase in modal radiation efficiencies is much more significant when the disk is rotating. Nevertheless, the effect of source rotation is still not significant from the engineering viewpoint. Therefore, the formulation to predict modal radiation efficiencies developed for the stationary case can be used to obtain the radiation efficiency of a rotating disk without losing much accuracy.
Table 4.4 Radiation efficiency of a rotating disk at $\Omega_r/2\pi = 72$ Hz.

<table>
<thead>
<tr>
<th>Modal Index (m,n)</th>
<th>Natural Frequency (Hz)</th>
<th>Rotating Disk (Equation 4.44)</th>
<th>Stationary Disk (Equation 4.14)</th>
<th>Approximate Formulation (Equation 4.45)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>608</td>
<td>0.854E-1</td>
<td>0.853E-1</td>
<td>0.853E-1</td>
</tr>
<tr>
<td>(0,1)</td>
<td>608</td>
<td>0.278E-2</td>
<td>0.282E-2</td>
<td>0.275E-2</td>
</tr>
<tr>
<td>(0,2)</td>
<td>724</td>
<td>0.814E-4</td>
<td>0.815E-4</td>
<td>0.811E-4</td>
</tr>
<tr>
<td>(0,3)</td>
<td>1130</td>
<td>0.205E-4</td>
<td>0.203E-4</td>
<td>0.204E-4</td>
</tr>
<tr>
<td>(0,4)</td>
<td>1830</td>
<td>0.239E-4</td>
<td>0.237E-4</td>
<td>0.238E-4</td>
</tr>
<tr>
<td>(0,5)</td>
<td>2760</td>
<td>0.512E-4</td>
<td>0.508E-4</td>
<td>0.509E-4</td>
</tr>
<tr>
<td>(0,6)</td>
<td>3880</td>
<td>0.144E-3</td>
<td>0.143E-3</td>
<td>0.143E-3</td>
</tr>
<tr>
<td>(0,7)</td>
<td>5190</td>
<td>0.472E-3</td>
<td>0.470E-3</td>
<td>0.470E-3</td>
</tr>
<tr>
<td>(0,8)</td>
<td>6680</td>
<td>0.168E-2</td>
<td>0.168E-2</td>
<td>0.168E-2</td>
</tr>
<tr>
<td>(0,9)</td>
<td>8340</td>
<td>0.620E-2</td>
<td>0.619E-2</td>
<td>0.619E-2</td>
</tr>
<tr>
<td>(1,0)</td>
<td>3890</td>
<td>0.330</td>
<td>0.329</td>
<td>0.330</td>
</tr>
<tr>
<td>(1,1)</td>
<td>4050</td>
<td>0.323</td>
<td>0.323</td>
<td>0.322</td>
</tr>
<tr>
<td>(1,2)</td>
<td>4540</td>
<td>0.112</td>
<td>0.112</td>
<td>0.112</td>
</tr>
<tr>
<td>(1,3)</td>
<td>5400</td>
<td>0.410E-1</td>
<td>0.409E-1</td>
<td>0.409E-1</td>
</tr>
<tr>
<td>(1,4)</td>
<td>6630</td>
<td>0.287E-1</td>
<td>0.287E-1</td>
<td>0.286E-1</td>
</tr>
<tr>
<td>(1,5)</td>
<td>8230</td>
<td>0.463E-1</td>
<td>0.462E-1</td>
<td>0.462E-1</td>
</tr>
</tbody>
</table>

m: number of nodal circles, n: number of nodal diameters
REFERENCES FOR CHAPTER IV


CHAPTER V

PATH: MOBILITY TRANSFER FUNCTION AND
OVERALL VIBRO-AcouSTIC MODEL

5.1 INTRODUCTION

The focus of this chapter is on the development of mobility transfer functions which couple the BDCM source and the annular disk radiator models developed earlier. The major objective is to formulate mathematical path transfer functions and then to construct an overall vibro-acoustic model of the disk drive by integrating the source, path and radiator models to yield the radiated sound power spectrum. Parametric studies such as the effects of disk dimensions and bearing stiffness are also conducted and their role in influencing the modal radiation characteristics, mobility function and radiated sound is determined.

5.2 MOBILITY TRANSFER FUNCTION

The mobility function by definition is the complex quotient of the structural velocity to excitation force. Since the sound radiation characteristics of the radiator is formulated in terms of its natural and rigid body modes and the excitation is the pulsating torque of the BDCM, one need to develop transfer functions to couple the disk vibration modes and harmonic torque excitation. Therefore, the mobility transfer function as developed in this study is defined as the complex quotient of velocity
participation factors of disk vibration modes to a sinusoidal electromechanical torque excitation.

5.2.1 Problem Formulation

The schematic and coordinate system of the disk-motor assembly is shown in Figure 5.1. Several assumptions are made in developing the path model. First, the spindle (to which the disk is fixed on) is assumed to be rigid since the lowest torsional mode of the spindle is beyond the frequency range of interest. The same assumption is made to the stator of the in-hub BDCM. Second, the casing, which is not included in disk sound radiation, is assumed to be flexible in this study and is modeled by two equivalent dynamic stiffness matrices. Third, the bearing dynamics is assumed to be modeled by a stiffness matrix in order to couple the rotor and the stator. Finally, the coupling between the electromagnetic torque and the disk flexural motion is assumed to exist via two paths: (i) equivalent stiffness matrices of the bearing and the casing, and (ii) the clearance within the bearing. The first path induces indirectly moment and axial force excitation on the rotor. The inertial forces associated with the rigid translating and rocking modes then excite the disk natural modes. In the second path, the bearing clearance is assumed to cause an additional direct moment excitation on the disk.

5.2.2 Equivalent Stiffness Matrices of the Casing and the Bearing

The equivalent stiffness matrix of dimension 6 for the casing cover, as shown in Figure 5.1, is defined by
Figure 5.1 Schematic of the disk drive system.
Consider a harmonic bending moment excitation of unit amplitude at location \((x_o, y_o)\)

\[ M_x = e^{j\omega t} \delta(x-x_o)\delta(y-y_o). \]  

(5.2)

The flexural motion of the casing cover under this excitation is then represented in terms of its normal modes as follows [5.1]

\[ w(x, y) = e^{j\omega t} \sum_k \frac{F_k(x_o, y_o)}{\omega^2_k - \omega^2} \psi_{c,k}(x, y), \]  

(5.3)

\[ F_k(x_o, y_o) = -\delta_{y} \psi_{c,k}(x, y) \bigg|_{x=x_o, y=y_o}. \]  

(5.4)

where \(\psi_{c,k}\) is the natural mode of the casing. The dynamic influence coefficient at the driving point is thus expressed by

\[ K_{c,M_{\mu_z}} = K_{c,F_{\theta_y}} = \frac{1}{\sum_k \frac{1}{(\omega^2 - \omega^2_k)} \frac{\partial \psi_{c,k}(x_o, y_o)}{\partial x} \psi_{c,k}(x_o, y_o)}. \]  

(5.5a)

By using the same procedure, other components of the stiffness matrix are determined as follows:
Values of other diagonal components of the stiffness matrix are assumed to be very large, and the off-diagonal components are assumed to be zero. Since the geometry of casing is very complicated, the numerical values of modal functions $\psi_{c,k}(x_o, y_o)$, $\frac{\partial \psi_{c,k}(x_o, y_o)}{\partial x}$ and $\frac{\partial \psi_{c,k}(x_o, y_o)}{\partial y}$ are obtained from the finite element analysis [5.3].
Assuming $\theta_c$ as the angle between the spindle and the normal vector on the surface of the casing cover, the equivalent stiffness with respective to the spindle coordinate becomes

$$\mathbf{K}_c = \mathbf{B}^{-1} \mathbf{K}_e,$$  \hspace{1cm} (5.6a)

$$\mathbf{B} = \begin{bmatrix}
1 & 0 & \sin \theta_c & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
\sin \theta_c & 0 & \cos \theta_c & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & \sin \theta_c \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \sin \theta_c & 0 & \cos \theta_c
\end{bmatrix}. \hspace{1cm} (5.6b)$$

A bearing stiffness matrix [5.2] of dimension 6, which has zero values for the 6th row and column, is used in the model to couple the flexural motion with the excitation. The parameters of the ball bearings used in the sample disk drive are listed in Table 5.1, and the corresponding stiffnesses are given in Table 5.2.

5.2.3 Governing Equations

The Lagrange's equations are used to derive the governing dynamic equations of the system. The displacement vectors at locations $a$ and $b$, which are defined as the joints between the stator and the casing as shown in Figure 5.1, are expressed by

$$\mathbf{r}_{r,a} = (I + A)\mathbf{r}_r \hspace{1cm}; \hspace{1cm} \mathbf{r}_{r,b} = (I - A)\mathbf{r}_r$$

$$\mathbf{r}_{s,a} = (I + A)\mathbf{r}_s \hspace{1cm}; \hspace{1cm} \mathbf{r}_{s,b} = (I - A)\mathbf{r}_s \hspace{1cm} (5.7a-d)$$
Table 5.1 Bearing parameters for example disk drive. Type: Ball bearings.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of balls</td>
<td>7</td>
</tr>
<tr>
<td>Contact angle</td>
<td>8°, 47′, 53″</td>
</tr>
<tr>
<td>Preload (kg)</td>
<td>21</td>
</tr>
<tr>
<td>Ball diameter (m)</td>
<td>2.778E-3</td>
</tr>
<tr>
<td>Path diameter (m)</td>
<td>10.5E-3</td>
</tr>
<tr>
<td>radius of inner groove (m)</td>
<td>1.51E-3</td>
</tr>
<tr>
<td>Radius of outer groove (m)</td>
<td>1.57E-3</td>
</tr>
<tr>
<td>Radial clearance (m)</td>
<td>0.007E-3</td>
</tr>
<tr>
<td>Load-deflection constant (N/m^3/2)</td>
<td>8.5E7</td>
</tr>
</tbody>
</table>

Table 5.2 Bearing stiffnesses as computed.

<table>
<thead>
<tr>
<th>Stiffness term(s)</th>
<th>Value (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{b,F_x u_x}$</td>
<td>0.20197E6</td>
</tr>
<tr>
<td>$K_{b,F_y u_y}$</td>
<td>0.61573E8</td>
</tr>
<tr>
<td>$K_{b,M_x \theta_x}$</td>
<td>0.84856E3</td>
</tr>
<tr>
<td>$K_{b,M_y \theta_y}$</td>
<td>-0.5797E4</td>
</tr>
<tr>
<td>$K_{b,M_y u_y}$</td>
<td>0.5797E4</td>
</tr>
</tbody>
</table>
where \( \mathbf{r}_r \) and \( \mathbf{r}_s \) are displacement vectors at the mid points of the rotor and the stator and defined by

\[
\mathbf{r}_r = \begin{bmatrix}
    u_{r,x} & u_{r,y} & u_{r,z} & \theta_{r,x} & \theta_{r,y} & \theta_{r,z}
\end{bmatrix}^T,
\]

\[
\mathbf{r}_s = \begin{bmatrix}
    u_{s,x} & u_{s,y} & u_{s,z} & \theta_{s,x} & \theta_{s,y} & \theta_{s,z}
\end{bmatrix}^T.
\]

The potential energy \((PE)\) of the system consists of strain energy associated with the casing elasticity and the bearing stiffness matrices as well as the potential energy associated with the disk flexure. The total potential energy \((PE)\) and the kinetic energy \((KE)\) are expressed by

\[
PE = \frac{1}{2} \left\{ [(I+A)(\mathbf{r}_r - \mathbf{r}_s)]^T \mathbf{K}_{b,a} [(I+A)(\mathbf{r}_r - \mathbf{r}_s)] + [(I-A)(\mathbf{r}_r - \mathbf{r}_s)]^T \mathbf{K}_{b,b} [(I-A)(\mathbf{r}_r - \mathbf{r}_s)] \right\}
\]

\[
\left\{ [(I+A)\mathbf{r}_s]^T \mathbf{K}_{c,a} [(I+A)\mathbf{r}_s] + [(I-A)\mathbf{r}_s]^T \mathbf{K}_{c,b} [(I-A)\mathbf{r}_s] + \mathbf{q}_d^T \mathbf{K}_d \mathbf{q}_d \right\} \quad (5.8)
\]

\[
KE = \frac{1}{2} \left\{ \mathbf{q}_s^T \mathbf{M}_s \mathbf{q}_s + \mathbf{q}_r^T \mathbf{M}_r \mathbf{q}_r + \mathbf{q}_d^T \mathbf{M}_d \mathbf{q}_d + \mathbf{q}_r^T \mathbf{M}_d \mathbf{q}_r + \mathbf{q}_d^T \mathbf{M}_d \mathbf{q}_d \right\} \quad (5.9)
\]

where the energy associated with the casing and bearings are expressed in terms of physical coordinates and the potential energy of the disk is represented in the modal domain by using truncated natural modes. It should be noted that the last two terms in
Equation (5.9) are associated with the coupling between the rigid body modes and the elastic modes of the disk. Since the rigid rocking motions can exist in both x and y directions, both $\cos$ and $\sin$ modes of disk flexure are included and these are denoted by subscript $c$ and $s$, respectively. In Chapter IV, it has been shown that the elastic modes are excited by the inertial forces associated with the rigid body motion, and thus only $(m,0)$ and $(m,1)$ modes would be excited. Therefore, only these modes need to be included in the displacement participation vector of disk as shown below.

$$r_d = \frac{1}{\omega} \begin{bmatrix} \eta_{00} & \eta_{01,c} & \eta_{01,s} & \eta_{10} & \cdots & \eta_{m0} & \eta_{m1,c} & \eta_{m1,s} \end{bmatrix}^T$$  (5.10)

The modal stiffness matrix $K_d$ and the modal mass matrix $M_d$ of the disk are then expressed by

$$K_d = \rho h \text{DIAG} \left[ \omega_{00}^2 \quad \omega_{01}^2 \quad \omega_{10}^2 \quad \cdots \quad \omega_{m0}^2 \quad \omega_{ml}^2 \right]$$  (5.11)

$$M_d = \rho h \text{DIAG} \left[ 1 \quad 1 \quad \cdots \quad 1 \right]_{3(m+1) \times 3(m+1)}$$  (5.12)

where subscript $3(m+1) \times 3(m+1)$ denotes the dimension of the matrix. The mass matrix $M_{dr}$ of the size $3(m+1)$ by 6 is associated with the coupling between the rigid body motion and elastic modes of the disk. By employing normalized modal functions for rigid body modes defined earlier in Chapter IV, $M_{dr}$ is expressed by
where,

\[ A_p = \sqrt{\pi (b^2 - a^2)} \quad ; \quad A_R = \sqrt{\frac{\pi}{4} (b^4 - a^4)} \quad ; \]

\[ \langle \psi_{m0}, \psi_{-1,0} \rangle = \int_0^{2\pi} \int_0^a \psi_{m0}\psi_{-1,0} rhd\rho d\varphi = \frac{2\pi\hbar}{A_p} \sum_{q=0}^{N} \tilde{c}_{m0,q} \frac{(b^{q+2} - a^{q+2})}{q+2} \quad ; \]

\[ \langle \psi_{ml,c}, \psi_{-1,l,c} \rangle = \frac{\pi\hbar}{A_R} \sum_{q=0}^{N} \tilde{c}_{ml,q} \frac{(b^{q+3} - a^{q+3})}{q+3} \quad . \quad (5.13b-e) \]

Applying the Lagrange's equations of motion, governing equations are derived as

\[ \left[ (K_{b,a} + K_{b,b}) + 2 AT(K_{b,a} - K_{b,b}) + A^T(K_{b,a} + K_{b,b})A \right]r_s \]

\[ - \left[ (K_{c,a} + K_{c,b}) + 2 AT(K_{c,a} - K_{c,b}) + A^T(K_{c,a} - K_{c,b})A \right]r_s \]

\[ - \left[ (K_{b,a} + K_{b,b}) + A^T(K_{b,a} - K_{b,b}) + (K_{b,a} - K_{b,b})A + A^T(K_{b,a} + K_{b,b})A \right]r = -F, \]

\[ M_r \ddot{r}_r + M_{dr} \ddot{r}_d + \left[ (K_{b,a} + K_{b,b}) + 2 AT(K_{b,a} - K_{b,b}) + A^T(K_{b,a} + K_{b,b})A \right]r_r \]
\[
-\left[(K_{b,a} + K_{b,b}) + A^T(K_{b,a} - K_{b,b}) + (K_{b,a} - K_{b,b})A + A^T(K_{b,a} + K_{b,b})A\right]r_s = F,
\]

\[
M_d\ddot{r}_d + M_{dr}\ddot{r}_r + K_d r_d = 0. \quad (5.14-16)
\]

If the equivalent casing stiffness matrices at the top and the bottom are identical and that both bearings are the same, one can simplify Equations (5.14) and (5.15) as follows:

\[
M_s\ddot{r}_s + 2\left[K_b + K_c + A^T(K_b + K_c)A\right]r_s - 2\left[K_b + A^T K_b A\right]r_r = -F, \quad (5.17)
\]

\[
M_r\ddot{r}_r + M_{dr}\ddot{r}_d + 2\left[K_b + 2A^T K_b A\right]r_r - 2\left[K_b + 2A^T K_b A\right]r_s = F. \quad (5.18)
\]

If the effect of the bearing clearance is not included, the generalized force vector is simply a torsional excitation expressed by

\[
F = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1\right]^T e^{j\omega t}. \quad (5.19)
\]

If the bearing clearance causes misalignment between the rotor and stator, it should induce an additional direct moment excitation on the disk. Then \(F\) is modified as

\[
F = \left[0 \quad 0 \quad 0 \quad \theta_b \cos \varphi_r \quad \theta_b \sin \varphi_r \quad 1\right]^T e^{j\omega t}, \quad (5.20)
\]

where \(\theta_b\) is the angle between the normal vector of the disk and the stator, \(\varphi_r\) is the angle between the projection of the normal vector on the X-Y plane and X axis. The displacement of the disk under a harmonic excitation is thus obtained by solving Equations (5.16-20). Consequently the participation factors of the disk rigid body and elastic modes and mobility functions are obtained by

\[
\eta_{-1,0} = M_{-1,0} = \omega A_{\theta_{r,z}} \quad ; \quad \eta_{-1,lc} = M_{-1,lc} = \omega A_{\theta_{r,x}} \quad ; \quad \eta_{-1,ls} = M_{-1,ls} = \omega A_{\theta_{r,y}}
\]
\[ \eta = M = \omega r_d. \] (5.21a-d)

For the sample case, only (0,0), (0,1), (1,0) and (1,1) modes are excited within the frequency range of interest. Since the casing is very stiff, the resulting rigid body motion is very small. Mobility transfer functions for a pure torsional excitation as given by Equation (5.19) are shown in Figures 5.2 and 5.3. It is seen that the amplitudes of mobility functions of elastic modes are very small compared to those of the rigid body modes, and that the frequencies of the peaks in these mobility spectra correspond to the natural frequencies of the casing and the disk. Since the casing geometry is complicated, there are about 50 modes within the frequency range of interest. Consequently, high modal density is observed in mobility transfer function spectra. If Equation (5.20) is employed as the excitation, the induced direct moment excitation dominates over excitations via other paths. While the mobility functions of the axisymmetric modes are almost the same as the previous case, the resonant frequencies of mobility functions of the rigid rocking modes and the non-axisymmetric elastic modes are primary corresponding to the disk elastic modes as shown in Figures 5.4 and 5.5.

5.3 OVERALL VIBRO-ACOUSTIC MODEL

Now the overall vibro-acoustic model can be constructed by combining the source, radiator and mobility function models. From the radiator model, the sound power is as follows; refer to Chapter IV.

\[ W_{rad}(\omega) = \eta^T(\omega)\Pi(\omega)\eta(\omega). \] (5.22a)
Figure 5.2 Mobility transfer function of axisymmetric modes (without the effect of bearing clearance); (a) rigid translating mode, (b) (0,0) mode, (c) (1,0) mode.
Figure 5.3 Mobility transfer functions of asymmetric modes (without the effect of bearing clearance); (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode.
Figure 5.4 Mobility transfer functions of axisymmetric modes (with the effect of bearing clearance); (a) rigid translating mode, (b) (0,0) mode, (c) (1,0) mode.
Figure 5.5 Mobility transfer functions of asymmetric modes (with the effect of bearing clearance); (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode.
where,

\[ \eta(i) = \begin{bmatrix} \eta_{-1,0} & \eta_{-1,1,0} & \eta_{-1,1,s} & \eta_{00} & \eta_{01,0} & \eta_{01,1} & \cdots & \eta_{ml,s} \end{bmatrix}^T, \]

\[ \Pi(i) = \begin{bmatrix} \Pi^{-1,-1} & \Pi^{-1,0} & \cdots & \Pi^{-1,m} \\ \Pi^{0,-1} & \Pi^{0,0} & \cdots & \Pi^{0,m} \\ \vdots & \vdots & \ddots & \vdots \\ \Pi^{m,-1} & \Pi^{m,0} & \cdots & \Pi^{m,m} \end{bmatrix}, \]

\[ \Pi_{ij} = \begin{bmatrix} \Pi_{00} & 0 & 0 \\ 0 & \Pi_{11} & 0 \\ 0 & 0 & \Pi_{11} \end{bmatrix}. \]

Incorporating the mobility function spectrum developed in the previous section, one can express participation factor vector in terms of the torque excitation \( T_e \) as

\[ \eta(i) = T_e(\omega)M(\omega), \]

\[ M(\omega) = \begin{bmatrix} M_{-1,0} & M_{-1,1,0} & M_{-1,1,s} & M_{00} & M_{01,0} & M_{01,1} & \cdots & M_{ml,s} \end{bmatrix}^T. \]

Consequently, the sound power spectrum is represented by

\[ W_{rad}(\omega) = T_e^2(\omega) [M(\omega)^T \Pi(\omega)M(\omega)]. \]

Predicted sound power spectra are shown in Figure 5.6.

### 5.4 PARAMETRIC STUDIES

The sound radiated from a disk drive is affected by various parameters of the overall vibro-acoustic model. For the sake of illustration, a few parametric studies are
Figure 5.6 Sound power spectra; (a) predicted by using mobility function of Figures 5.2 and 5.3, (b) predicted by using mobility function of Figures 5.4 and 5.5.
presented on the effect of disk geometry such as thickness \( h \) and outer radius \( b \). In addition to sound radiation characteristics of the disk, the mobility functions are also affected by the disk dimensions. Limited parametric studies are conducted by varying the thickness from \( 0.2h^* \) to \( 2h^* \) and the outer radius from \( 0.75b^* \) to \( 1.5b^* \), where \( h^* \) and \( b^* \) are the current values of disk thickness and outer radius. Figure 5.7 shows changes in natural frequencies of \((0,0), (0,1), (1,0)\) and \((1,1)\) modes; results are normalized with respect to their natural frequencies at \( h^* \) and \( b^* \). Since natural frequency of the disk is proportional to the square root of the bending flexural rigidity, eigenvalues are linear functions of the disk thickness \( h \) as shown in Figure 5.7(a). The normalized natural frequencies of the disk vary parabolically as a function of the outer disk radius \( b \) as illustrated in Figure 5.7(b).

The modal radiation efficiencies of \((0,0)\) and \((0,1)\) modes, as shown in Figure 5.8(a), are found to increase monotonically as \( h \) increases. On the other hand, local maxima are seen in Figure 5.8(b) for \((1,0)\) and \((1,1)\) modes over the selected range of the disk thickness. It is also observed that the modal radiation efficiency of \((0,1)\) mode is most sensitive to \( h \) since \( \sigma_{01} \) increases about 13 times when \( h^* \) is doubled. Similar phenomena are found for \( \sigma_{mm} \) as a function of \( b \), as shown in Figure 5.9. Modal sound power of two rigid body modes and four elastic modes for selected values of outer radius \((0.75b^*, 1.125b^* \text{ and } 1.5b^*)\) are shown in Figures 5.10 to 5.13. Inspection of these curves shows that humps shift to lower frequencies as \( b \) increases. It is also seen that the magnitudes of these humps increase as \( b \) increases, except for the second humps associated with the rigid body modes. Mobility transfer functions for selected values of \( h \) and \( b \) are shown in Figures 5.14-15, note that the effect of bearing
Figure 5.7 Normalized natural frequencies of disk elastic modes as functions of (a) thickness $h$ and (b) outer radius $b$. 

Figure 5.8 Effect of disk thickness on normalized modal radiation efficiencies. Key: (a) (0,0) mode, (b) (1,0) mode; (a) (0,1) mode, (b) (1,1) mode.
Figure 5.9 Effect of disk radius on normalized modal radiation efficiencies. Key: 
--- (a) (0,0) mode, (b) (1,0) mode; --- (a) (0,1) mode, (b) (1,1) mode.
Figure 5.10 Effect of disk radius on self-modal sound power for asymmetric modes; (a) rigid translating mode, (b) (0,0) mode, (c) (1,0) mode. Key: —— $0.75b^*$; — $1.125b^*$; —— $1.5b^*$. 
Figure 5.11 Effect of disk radius on self-modal sound power for asymmetric modes; (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: —— 0.75$b^*$; —— 1.125$b^*$; ····· 1.5$b^*$.
Figure 5.12 Effect of disk radius on coupled-modal sound power for axisymmetric modes; (a) (-1,0) and (0,0) modes, (b) (-1,0) and (1,0) modes, (c) (0,0) and (1,0) modes. Key: —— 0.75b*; —— 1.125b*; ··· 1.5b*. 
Figure 5.13 Effect of disk radius on coupled-modal sound power for asymmetric modes; (a) (-1,1) and (0,1) modes, (b) (-1,1) and (1,1) modes, (c) (0,1) and (1,1) modes. Key: —— 0.75$b^*$; —— 1.125$b^*$; ···· 1.5$b^*$. 
Figure 5.14 Effect of disk thickness on mobility transfer functions; (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: — 0.2$h^*$; — 1.1$h^*$; — 2$h^*$.
Figure 5.15 Effect of disk radius on mobility transfer functions; (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: \( \ldots 0.75b^* \); \( - - 1.125b^* \); \( \ldots 1.5b^* \).
clearance is included here. The resonant frequencies move up as $h$ increases or $b$ decreases, which obviously corresponds to shifts in disk natural frequencies. Finally the radiated sound power is shown in Figures 5.16 and 5.17. Observe that the natural modes of the disk affect radiated sound significantly. The effect of bearing stiffness on mobility transfer functions are illustrated in Figures 5.18 and 5.19. For axisymmetric modes, the magnitudes of mobility functions in the low frequency range decrease as the bearing stiffness is increased as shown in Figure 5.18. For asymmetric modes, the bearing stiffness dictates the natural frequency of the disk assembly, i.e. rigid rocking mode which affects the peak frequency in the low frequency range.

5.5 CONCLUDING REMARKS

The mathematical models on path mobility transfer functions and the overall vibro-acoustic characteristics, as developed in this chapter, have made several contributions. First, the coupling mechanism between the electromagnetic excitation and the flexural motion of the disk has been understood. Non-ideal mounting between the stator of the in-hub BDCM and the casing and the bearing clearances are believed to be two main causes which result in the moment and the axial force excitations on the disk. Mobility function results indicate that the rigid body modes dominate over the elastic modes when the bearing clearance is not included. If the bearing clearance is taken into consideration, the peak frequencies in the mobility functions for elastic modes correspond to the natural frequencies of the disk. Second, a new overall vibro-acoustic model of a computer hard disk drive has been developed by combining source, radiator and path models to yield the radiated sound power spectrum associated with
Figure 5.16 Effect of disk thickness on sound power spectra; (a) $0.2h^*$, (b) $1.1h^*$, (c) $2h^*$. 
Figure 5.17 Effect of disk radius on sound power spectra; (a) $0.75b^*$, (b) $1.125b^*$, (c) $1.5b^*$. 
Figure 5.18 Effect of bearing stiffness on mobility transfer function of rigid translating mode; (a) 0.1$K_b$, (b) $K_b$, (c) 10$K_b$. 
Figure 5.19 Effect of bearing stiffness on mobility functions (asymmetrical modes); (a) rigid rocking mode, (b) (0,1) mode, (c) (1,1) mode. Key: — 0.1K_b; — K_b; —— 0.1K_b.
the pulsating motor torque. Note that predictions are yet to be compared with measured data. Finally, effects of disk geometry and bearing stiffness have been explained. It is seen that the disk dimensions have a significant effect on both modal sound power and mobility functions.
REFERENCES FOR CHAPTER V


CHAPTER VI
CONCLUSION

6.1 SUMMARY

This study has resolved several modeling issues regarding vibroacoustic characteristics of a compact disk-rotor system driven by a brushless d.c. motor (BDCM) through the example case of a 3.5" personal computer hard disk drive. A comprehensive study of key components has led to the development of a general vibroacoustic narrow band model of a computer disk drive which is constructed by integrating the motor torque pulsation (source), disk sound radiation characteristics (radiator), and mobility transfer functions (path). Each component is treated in a general manner and hence proposed techniques can be applied to other electromechanical devices. The overall vibroacoustic model has yielded promising results, especially in the prediction of several prominent high frequency tones associated with motor torque pulsations over 1-6.5 KHz.

Two different mathematical models, State Space Model and Fourier Series Representation, have been developed for BDCM to predict the torque spectrum associated with inverter switching logic, pulse width modulation control scheme, eccentricity, and magnetic saturation. Results of the state space model are obtained by using the numerical integration method followed by an FFT scheme. The Fourier series expansion is employed separately to derive analytically the Fourier coefficients of
torque pulsations. Further, the Galerkin's method, essentially a quasi-analytical harmonc balance method, is employed to the complex non-linear, time-varying BDCM model by converting the dual domain problem (time and spatial domains) into a single spatial domain formulation. This has led to an efficient computational scheme which predicts the Fourier coefficients of torque pulsations directly including various effects associated with inductance harmonics. The mechanical system dynamics is also incorporated into the computational scheme by expressing the unknown angular velocity in terms of a Fourier series.

For the annular disk, a generic modal radiation formulation has been developed by using a polynomial approximation of disk eigen-functions. Modal sound power associated with the elastic and rigid body modes is expressed in terms of a power series of the wave number, which is computationally efficient. This leads to the prediction of sound radiation given an arbitrary velocity distribution. Additionally, the effect of source rotation is investigated by employing the Green's function of a moving point sound source. It has been demonstrated that the effect of source rotation on modal radiation efficiency is rather insignificant and it can be computed directly by the modal radiation efficiency of a stationary disk via an empirical formulation. New mobility transfer function, which is defined as the quotient of the modal participation factors of disk natural and rigid body modes to the harmonic torque excitation amplitude, is derived to couple the source and radiator models. Consequently, an overall model has been developed by combining torque pulsation, modal sound power and path mobility models. Selected parametric studies have been conducted to demonstrate the effects of disk dimensions and bearing stiffnesses on modal radiation efficiency, modal sound
power, mobility transfer function and radiated sound. It is observed that the natural modes of disk have a significant effect on the sound radiation.

6.2 ACCOMPLISHMENTS

Though the emphasis of this study has on the computer hard disk drive, the analytical framework of BDCM model and annular-disk sound radiation can be extended conceptually to other devices such as laser printers, circular saws, gear bodies, fans, etc. Various research issues associated with this work are diverse and originate from different disciplines such as electromechanical theory, structural dynamics and acoustics. In addition to analytical modeling issues, the methodology employed in this study included direct time domain numerical integration, Fourier analysis, Galerkin's method, boundary and finite element calculations, modal testing and sound measurements. To incorporate all of these in the context of a single study itself is a salient contribution. Specific contributions to the state-of-art are as follows:

a. Noise Source Models:

(i) Developed two new mathematical BDCM models to predict high frequency harmonics of electromechanical torque pulsation associated with mechanical and electrical defects.

(ii) Develop a new computation scheme by using the Galerkin's method to calculate the Fourier coefficients of the steady-state torque including complex modulation effects associated with inductance harmonics and mechanical system dynamics. It is believed to be the first study which has incorporated the unknown angular velocity into the computational scheme.
b. Radiator Models:

(i) Developed new modal radiation efficiency and sound power formulations by using a polynomial approximation of annular disk eigen-functions. Modal sound power associated with the coupling effect between elastic and rigid body modes of an annular disk has been investigated for the first time. The proposed formulation, expressed in terms of convergent power series of the wave number, is capable of predicting radiated sound associated with an arbitrary disk velocity distribution and yields modal sound information. The effect of disk rotation on modal radiation efficiency has also been clarified.

(ii) Developed a specialized impedance approach by discretizing the annular disk into concentric annuli. This yields valuable sound radiation information in the physical domain that can be used to solve structure-borne and radiated noise problems of annular/circular plates.

c. Path and Overall Vibroacoustic Models:

(i) Developed a new mobility transfer function to couple the source and the radiator models.

(ii) Constructed an overall vibroacoustic model of the disk drive to predict radiated sound power associated with electromechanical sources over a narrow band basis.

6.3 FUTURE RESEARCH TOPICS

Due to time constraints and the comprehensive nature of research issues, this study has not been able to fully explore all aspects of the problem. Also, a number of
simplifications have been made in the overall model in order to make the analytical modeling tractable. Consequently, some discrepancies exist between predictions and measurements. This implies some of models need to be refined, and other source mechanisms and paths should be included as suggested in Figure 6.1. Nonetheless, proposed models and methodology have shown promising results and provided a suitable foundation for further research. The following specific research issues regarding the disk drive noise problem have been identified and proposed for further studies:

a. Modeling of the excitation and path mechanisms associated with the bearing defects.

b. Modeling of the axial force excitation of BDCM.

c. Experimental validation on sound radiation characteristics of a single disk.

d. Radiation characteristics of the multi-disk system, which includes mutual-radiation impedances between different disks and coupling effects between the sound radiation and the wave guides in between two disks.

e. Interactions between sound radiated from the disk and the acoustic cavity modes of the casing cover.

f. Effect of thermal conditions on torque pulsation source and sound radiation characteristics of the disk.
Figure 6.1 Proposed disk drive acoustic model for future studies.
BIBLIOGRAPHY


