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Discrete-time model regulation control for systems with uncertain dynamics: Applications to electromechanical and electropneumatic systems

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The Ohio State University, 1993
DISCRETE-TIME MODEL REGULATION CONTROL FOR SYSTEMS WITH UNCERTAIN DYNAMICS: Applications to Electromechanical and Electropneumatic Systems

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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To My Family
ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

$A_p$  Cylinder cross section area

$A_{v+}, A_{v-}$  Cross areas of valve orifice

$B$  Viscous damping coefficient

$\hat{B}$  Nominal viscous damping coefficient

$B_{eq}$  Equivalent viscous damping of DC motor

$C$  Transfer function of feedback controller

$C_1, C_2$  Transfer function of feedback controllers of two-degree-of-freedom system

$C_{d+}, C_{d-}$  Coefficients for effective orifice area

$C_f$  Feedforward control signal

$C_{m+}, C_{m-}$  Parameters for mass flow rate through nozzle

$c_p$  Specific heat for constant pressure

$c_v$  Specific heat for constant volume

$d$  External disturbance or time proportioning control

$D$  Time proportioning control signal

$D_o$  Operating point of time proportioning control signal

$f$  Net force on the piston

$F$  Transfer function of low-pass sensor filter or force

$F_f$  Friction force

$\hat{G}$  Transfer function of nominal model

$\hat{G}^{-1}$  Pseudo inverse transfer function of nominal model
\( \delta G \quad \) Model uncertainty

\( \hat{G}_p \quad \) Nominal transfer function of pressure response model

\( \hat{G}_q \quad \) Nominal transfer function of DC motor position model

\( \hat{G}_v \quad \) Nominal transfer function of DC motor velocity model

\( \hat{G}_x \quad \) Nominal transfer function of piston position response

\( \hat{G}_x^{-1} \quad \) Pseudo inverse transfer function of \( \hat{G}_x \)

\( \mathcal{G} \quad \) Set of possible control plants

\( H \quad \) Closed-loop transfer function of Model Regulation Control

\( H_e \quad \) Closed-loop transfer function of two-degree-of-freedom system

\( H_v \quad \) Closed-loop transfer function of DC motor velocity model regulation

\( I \quad \) Identity

\( \bar{I} \quad \) Product of \( \hat{G} \) and \( \hat{G}^{-1} \)

\( J_{eq} \quad \) Equivalent rotary moment of inertia of DC motor

\( \hat{j} \quad \) Nominal rotary moment of inertia of DC motor

\( k \quad \) Steady state gain

\( \hat{k} \quad \) Nominal steady state gain

\( K_e \quad \) Back emf constant of DC motor

\( K_G \quad \) Nominal gain of DC motor velocity transfer function

\( k_{psd} \quad \) Steady state gain of pressure response model

\( K_s \quad \) Spring constant

\( \hat{K}_s \quad \) Nominal spring constant

\( K_i \quad \) Torque constant of DC motor

\( L \quad \) Transfer function of total low-pass filter

\( L_a \quad \) Armature inductance of DC motor

\( M \quad \) Total mass of moving part

\( \hat{M} \quad \) Nominal total mass
\begin{itemize}
  \item $\dot{m}$ \hspace{2em} Net mass flow rate
  \item $\dot{m}_v$ \hspace{2em} Mass flow rate through valve orifice to cylinder chamber
  \item $\dot{m}_a$ \hspace{2em} Mass flow rate through valve orifice to atmosphere
  \item $p_i$ \hspace{2em} Variation of left chamber pressure
  \item $P_i$ \hspace{2em} Pressure in the left cylinder chamber
  \item $\dot{p}_i, \dot{P}_i$ \hspace{2em} Time derivative of left chamber pressure
  \item $P_{i,o}$ \hspace{2em} Operating point of left chamber pressure
  \item $p_2$ \hspace{2em} Variation of right chamber pressure
  \item $P_2$ \hspace{2em} Pressure in the right cylinder chamber
  \item $P_{2,o}$ \hspace{2em} Operating point of right chamber pressure
  \item $P_a$ \hspace{2em} Atmospheric pressure
  \item $P_s$ \hspace{2em} Supply pressure
  \item $Q$ \hspace{2em} Angular position or heat transfer
  \item $\dot{Q}$ \hspace{2em} Angular velocity
  \item $R$ \hspace{2em} Reference input or gas constant
  \item $R_a$ \hspace{2em} Armature resistance of DC motor
  \item $s$ \hspace{2em} Variable of Laplace Transform
  \item $S$ \hspace{2em} Sensitivity function or piston stroke
  \item $S_n$ \hspace{2em} Nominal sensitivity function
  \item $\bar{S}$ \hspace{2em} Maximal allowable sensitivity
  \item $T$ \hspace{2em} Sampling time
  \item $T_i$ \hspace{2em} Temperature in the left cylinder chamber
  \item $T_a$ \hspace{2em} Temperature of the atmosphere
  \item $T_s$ \hspace{2em} Temperature of the supply gas
  \item $U$ \hspace{2em} Control input
  \item $\dot{U}$ \hspace{2em} Nominal control input
\end{itemize}
$V$  Total volume of cylinder chamber
$v_1$  Variation of left chamber volume
$V_l$  Volume of left cylinder chamber
$V_{1,o}$  Operating point of left chamber volume
$v_2$  Variation of right chamber volume
$V_2$  Volume of right cylinder chamber
$V_{2,o}$  Operating point of right chamber volume
$x$  Linear position
$\dot{x}$  Linear velocity
$\ddot{x}$  Linear acceleration
$X$  Piston position
$X_o$  Operating point of piston position
$Y$  Control output
$z$  Variable of $Z$-transform
$\hat{z}_p$  Nominal pole of DC motor velocity transfer function
$z_z$  Zero of transfer function
$\hat{z}_z$  Nominal zero of DC motor position transfer function
$\alpha$  Factor of payload variation
$\gamma$  Specific heat ratio
$\Delta$  Normalized model uncertainty
$\zeta$  Damping ratio
$\hat{\zeta}$  Nominal damping ratio
$\mu$  Model Regulation compensation
$\omega$  Frequency
$\omega_a$  Critical design frequency
$\omega_c$  Gain crossover frequency
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<td>$\omega_n$</td>
<td>Natural frequency</td>
</tr>
<tr>
<td>$\hat{\omega}_n$</td>
<td>Nominal natural frequency</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of gas</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>Time constant of pressure response model</td>
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<tr>
<td>$\phi_c$</td>
<td>Phase angle of $L(z)\Delta(z)$ at $\omega_c$</td>
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<td>$\phi_d$</td>
<td>Delay angle</td>
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CHAPTER I
INTRODUCTION

In many control applications, the controllers are called upon to perform with existing machines or devices. These systems are designed and built in advance without taking the control problem into account. The physical limitations and the best performance of these systems are decided at the designing and building stage of the product. From the control engineer's point of view, the dynamic characteristics of the control plant is described by the nominal model. The nominal model can be analytically derived form the physical parameters if the system is not too complicated. Otherwise, the nominal model can be experimentally identified under the intended operating condition. Either way, the obtained nominal model represents the physical meaning of the control plant and implies the designated performance of the designed system. Therefore, this dissertation is concentrated in regulating the systems with uncertain dynamics to their nominal model so that their specified dynamic characteristics can be preserved.

Digital control is chosen as the element for the controller implementation for the Model Regulation Control. Due to the great advance in the electronics technology and the rapidly growing popularity of microcomputers, using the computer as a control element has become not only available but also affordable. Besides, a computer-controlled system can provide more flexibility and computational ability than analog control. Any inverse transfer function of the nominal model can be easily executed by digital computers. This is critical
for the Model Regulation Control because it employs the inverse of the nominal transfer function in the control algorithm. The following analysis and synthesis are based on discrete-time control theories. This approach is different from translating the designed analog controller into digital a controller. If one views the digital controller as the approximation of the analog controller, the full potential of computer control is not used. At best the results are only as good as those obtained with analog control (Astrom and Wittenmark, 1984).

The unique benefit of introducing feedback control to existing systems is its ability to deal with the uncertain dynamics. Model uncertainty and response to external disturbances are the two reasons why the plant subject to a given input does not behave completely the same as the nominal model does. Model uncertainty is originated from the variation of the physical parameters, the change of the operating conditions and the unmodelled complexity or nonlinearity of the plant. In the case of external disturbance, the output is not only the response to the input but also to the disturbance which is neither controllable by the input signal nor dependent on the state variables of the plant. The first application of the Model Regulation Control in this dissertation is a DC servomotor system; the payload variation of the system makes a good example of the model uncertainty caused by the physical parameter variation. The second application is a pneumatic system which demonstrates a different kind of model uncertainty. Due to the highly nonlinear dynamic characteristics of the system, the linearized nominal model obtained is only accurate at the nominal operating condition. Those tasks that demand a large change in operating conditions will face nonlinear type model uncertainty inferior performances. Mechanical systems generally suffer from friction forces for motion control.
1.1 BACKGROUND

Feedback control for systems with uncertain dynamics is one of the most studied areas in the control community. As a matter of fact, the existence of uncertain dynamics is the only reason for the introduction of feedback to the system. If it is not for the uncertainty, open-loop control can produce equal or even better output. More specifically, the functions of feedback control are to produce robust performance and maintain stability against variations of the internal parameters and unpredictable external disturbances. The numerous research efforts in general can be considered as finding the transparent relations between the uncertain dynamics and the desired performance so that the controller can be solved. The differences among the existing approaches are the representation of the uncertainty, the specification of the desired performance and the mathematical tools they use to perform analysis and synthesis.

Several different types of description of the uncertain models have been proposed. Bounded norm uncertain systems was introduced for the approach to multivariable systems by Donati and Carlucci (1975). In the singular value approach, Doyle (1979) and Safonov (1979) used the maximum singular value to represent the upper bound of the uncertainty. The general model structure was proposed by Kouset (1982) and Lunze (1986). Element-by-element bounds was used in the robust stability study by Kantors and Andres (1983). Conic sectors was used by Safonov and Athans (1977) on the stability margin of LQG-regulators. It is also suitable for nonlinear systems as shown by Noldus (1982). Thompson et al. (1983) have described the use of conic sectors for sampled-data systems. The relation between the bound of time domain response and the bound of frequency domain response is given by Krishnan and Cruickshanks (1977).
Sensitivity analysis represents a standard treatment of control systems subject to uncertain dynamics. Bode (1945) introduced the differential sensitivity function to provide an analytical measure for system accuracy improvement. Summaries of the tools and results of sensitivity analysis of dynamic systems are given in the book edited by Cruz (1973) and Frank (1978). Horowitz (1963) extended the sensitivity to be a function of frequency to develop quantitative feedback design method. In this method, the open-loop transfer function and its tolerances are transformed into the closed-loop counterparts using the Nichols chart point by point in the frequency domain. Extension of this method to nonlinear, time varying or non-minimum phase plants were also developed (Horowitz, 1976, 1975; Horowitz and Sidi, 1978). Multivariable control problems are tackled by Horowitz (1979, 1982). Nwokah (1984) applied the quantitative feedback design method to time domain problems. Optimal approach of this method appeared in the papers by Horowitz and Sidi (1978) and by Thompson and Nwokah (1990). Real applications of this design method to hydraulic systems and flight control have been reported by Graser and Tolle (1983) and by Walke et al. (1984), respectively. This approach makes the relation between the plant uncertainties, the controller parameters and the closed-loop performance explicit. The weak point of this approach is the lack of systematic methods for choosing the loop transfer function. The graphical synthesis process on a Nichols chart can easily become unmanageable when dealing with multivariable systems.

Kalman (1964) first demonstrated that optimal LQ state feedback laws for SISO systems have infinite gain margins and 60 degrees of phase margin. Safonov and Athans (1977) showed that this gain and phase margin can be extended to MIMO systems. These are the early reported robustness properties of LQR control; however, when the full state
feedback is not available, these desirable robustness properties do not exist. Later Doyle and Stein (1979) recovered the desirable properties of the optimal LQ state feedback control law by suitable design of the Kalman filter. LQR controllers (Doyle and Stein, 1981; Athans, 1986; Stein and Athans, 1987) are popular due to their ability to handle MIMO systems and the existing solution for the controller. The shortcoming of this approach is that there is no systematic way to specify reasonable performance tradeoffs and robust stability constraints.

The $H^\infty$ approach was initiated by Zames (1981), who introduced the weighted $H^\infty$ norm for a special case system with a single right-half s-plane plant zero. Zames and Francis (1983) also solved the $H^\infty$ optimal sensitivity design problem for SISO systems. Solutions to the MIMO $H^\infty$ optimal sensitivity design problem were published by Chang and Pearson (1984) and Francis et al. (1984). Robust stabilization for MIMO systems was presented by Vidyasagar and Kimura (1986) and Glover (1986). The $H^2$ optimal sensitivity solution was presented by Youla and Bongiorno (1985). This is the combination of their earlier works on Wiener-Hopf design of multivariable systems (Youla et al., 1976) and on optimal $H^2$ sensitivity design.

There are also good books that summarize the control problem for uncertain dynamics. A short list includes Ashworth (1982), Vidyasagar (1985), Francis (1987) and Lunze (1989).

Digital control theory and controller design methods can be found in Astrom and Wittenmark (1984); Phillips and Nagle (1984), Franklin et al. (1990).
1.2 RESEARCH OBJECTIVE AND SCOPE

The objective of this research is to develop a discrete-time Model Regulation Control approach for control systems with uncertain dynamics. This approach regulates the input/output behavior of the actual control plant to its nominal model by generating the necessary compensation for the uncertain dynamics. The effectiveness of this approach on cancelling both linear and nonlinear types of uncertain dynamics will be demonstrated through applications on a DC servomotor system and a pneumatic system.

The theoretical solution of the Model Regulation Control is proven to be an exact solution and it could totally cancel the uncertain dynamics if it were implementable. For the actual implementation of the control algorithm, practical limitations should be examined and a realizable approximation for the Model Regulation Control should be found. Accordingly, the approach will be developed by the following four stages:

(1) Solution of the desired compensation for the model uncertainty and external disturbance.
(2) Investigation of the discrete-time implementation of the solved compensation.
(3) Analytical study of the sensitivity reduction, disturbance rejection and stability of the proposed approach.
(4) Derivation of design procedures for achieving specified sensitivity and robust stability.

Model uncertainties are usually induced by inherent complexity of the system or by application purposes. The difference between the actual control plant and the nominal
model is either the mismatch of the model structure or the unknown parametric values. For mechanical systems, linearization of system nonlinearity or simplified friction model are the cases of the former and payload variation is an example of the latter. In this research, a DC servomotor system and a pneumatic system are chosen as applications of the proposed approach because each of them features typical yet different kinds of model uncertainty.

(1) The DC servomotor is a well modeled linear system. By intentionally varying the load inertia, the resulting model uncertainty is the unknown value of system parameters. This kind of model uncertainty can be categorized as interval parametric variation of a known linear model structure.

(2) The governing equations of pneumatic valves and cylinders are highly nonlinear due to the on-off action of the valve and the compressibility of gas. The discrepancy between the actual control plant and the linearized nominal model is caused by the continuous motion of the piston and increases largely for wide stroke motion. Thus the nonlinearity of the actual control plant becomes a control problem when the application calls for large stroke.

(3) Both of the two example systems feature friction problems. The friction torque on the DC servomotor system is noticeable; the dry friction force which is of the same order as the needed driving force for the pneumatic system causes rough low speed motion and poor positioning accuracy. Generally, the friction force is composed of stiction, Coulomb friction and viscous friction. The model structure of stiction is nonlinear and is the most difficult to compensate. The
Coulomb friction can be treated as constant disturbance as long as the sign of the velocity is not changed. The viscous friction is a linear model uncertainty with an unknown parameter.

The proposed Model Regulation Control approach is aimed at regulating the input/output behavior of the control plant to its nominal model so that desirable dynamic performance can be achieved in spite of the existence of significant uncertain dynamics. Focused on this intention, the evaluation of the effectiveness of this approach should be based on comparing the closeness of the compensated system to the nominal model. In a qualitative sense, the results of this research should be the predictable reduction of sensitivity to the uncertain model and rejection of disturbances by the proposed approach. Quantitatively, specific measures like sensitivity function, locations of dominant poles, frequency response and time domain response that characterize the behavior of the compensated system are to be compared with those of the nominal system.

1.3 DISSERTATION OUTLINE

The rest of the dissertation is organized as follows. Chapter 2 introduces the original idea of the model Regulation Control. The uncertain dynamics faced consists of model uncertainty and external disturbance. Exact solution of the necessary compensation for the model uncertainty and external disturbance is solved. The discrete-time implementation of the solved control algorithm is then studied. The proposed Model Regulation Control is realized by using a time delayed signal, pseudo inverse dynamics and
a low-pass sensor filter. The analytical study of the compensated system includes sensitivity reduction, disturbance rejection and robust stability.

The design problem of the Model Regulation Control is studied in Chapter 3. The design problem essentially is to find the controller that can achieve the desired closed-loop performance in the face of uncertain dynamics and at the same time guarantee robust stability. These requirements are formulated by sensitivity specification in the frequency domain and the robust stability condition. The most important design factor of the Model Regulation Control is the sampling time. Synthesis steps for finding the necessary sampling time to meet the design requirements are developed. Graphical interpretation of the synthesis method is also given. At the end of this chapter, a second order control system subject to parameter variation is given as an example to demonstrate the design process.

The application of the Model Regulation Control to a DC servomotor system is studied in Chapter 4. The uncertain dynamics under consideration are the payload variation which is linear and the nonlinear friction torque. The boundary of payload variation for robust stability is found by using Nyquist criterion. Tasks such as velocity control, position control and tracking control are analytically studied and verified by experimental results.

Chapter 5 and 6 investigate the application of the Model Regulation Control to a pneumatic control system. The highly nonlinear dynamics of the pneumatic system is first studied. The governing equations of the dynamic system include the nozzle equation of the
gas flow through the valve orifice, the energy equation for the total mass flow rate and the perfect gas law for the chamber pressure. Also in Chapter 5, the pressure response of the pneumatic system with a discrete-time input signal is modeled as a first order transfer function by experimental identification. The position response is also derived as a linear model. Chapter 6 uses the identified linear models as the nominal models for the Model Regulation Control. The control problem faced is the uncertain dynamics caused by the nonlinearities in the system. The objective of the pressure model regulation application is to make the pressure response, with an unknown volume change, identical to the response of the linear nominal model. For the position response model regulation, the objective is to obtain behavior the same as the linear nominal model for long stroke motion.

Finally, Chapter 7 presents a summary of the overall results and contributions of this dissertation. Some topics of future research that may enhance these results are also discussed.
CHAPTER II
THE MODEL REGULATION CONTROL

The discrete-time Model Regulation approach for control systems with uncertain dynamics is developed in this chapter. This approach regulates the input/output behavior of the actual control plant to its nominal model by generating the necessary compensation for the uncertain dynamics. The uncertain dynamics considered in this research consists of model uncertainty and external disturbance. The functions of the Model Regulation Control in the overall control structure are to largely reduce the system's sensitivity to the model uncertainty and to reject the effect of the external disturbance so that other controllers in the structure, either feedback or feedforward, can be designed solely for the purpose of assigning a desirable dynamic performance.

It is well known that a feedback control system with two degrees of freedom can achieve two fundamental design objectives (Horowitz, 1963):

1. to obtain desirable closed-loop dynamic characteristics $H_c$;
2. to reduce the sensitivity function $S$ of the system to uncertain dynamics.

From this point of view, the Model Regulation Control is a discrete-time, two-degree-of-freedom feedback controller that targets the control plant’s nominal model as the desired closed-loop transfer function and at the same time ensures low sensitivity of the system to
model uncertainty and external disturbances. Design techniques for the first objective are well developed and proven to be successful when the mathematical model of the control plant is known. They are most effective in shaping the dynamic characteristics of the given model into desired command input response. Typical examples are pole-zero cancellation, phase-lead/-lag and PID for feedback controller design and inverse dynamics techniques for feedforward controller design. The second design objective has dominated the field of feedback control research for the past thirty years. The numerous design methods proposed can be seen in general as systematic efforts to find a controller with constant gain that can yield low sensitivity in the necessary frequency range. The differences among them are the types of uncertain dynamics they are able to deal with and the tools they use for analysis and synthesis.

In Section 2.1, a theoretical solution of the desired compensation for the uncertain dynamics is derived. For the actual implementation of the Model Regulation Control algorithm, several practical limitations are examined in Section 2.2. Realizable approximation in discrete-time form is discussed. The proposed Model Regulation Control is implemented by using a time delayed signal, pseudo inverse dynamics and a low-pass sensor filter. The analytical study of the compensated system includes sensitivity reduction (Section 2.3), disturbance rejection (Section 2.4) and robust stability (Section 2.5)
2.1 COMPENSATION FOR UNCERTAIN DYNAMICS

As shown in Figure 2.1, the Input/Output behavior of a real system is

\[ Y = [\hat{G}(z) + \delta G(z)][U + d] \]  

(2.1)

where \( \hat{G}(z) \) is the nominal model, \( \delta G(z) \) is the model uncertainty and \( d \) is the external disturbance. If a compensation \( \mu \) is introduced to entirely cancel the effect of \( \delta G(z) \) and \( d \), then ideally one has

\[ \frac{Y}{U}(z) = \hat{G}(z) \]  

(2.2)

which means the observed Input/Output behavior is regulated to the nominal model. It is noticed that the desired compensation is

\[ \mu = U - \hat{U} \]  

(2.3)

For formulating a control algorithm, \( \mu \) needs to be expressed in terms of accessible signals and known system model. Substitution of Equations (2.2) and (2.3) into Equation (2.1) yields

\[ \mu = U - \hat{G}^{-1}(z)Y \]  

(2.4)

where \( \hat{G}^{-1}(z) \) is the direct inverse of the nominal model \( \hat{G}(z) \) and
Figure 2.1  Compensation for Model Uncertainty and External Disturbance
\[
\hat{G}^{-1}(z)\hat{G}(z) = I
\]  

Note that the first term in Equation (2.4) is the actual input to the system and the second term is computed from the actual output via inverse dynamics of the nominal model. One can consider the second term as the estimated input from the actual output and the nominal model. What Equation (2.4) implies is that the dynamic effects of model uncertainty and external disturbance are reflected by the difference between the actual input and the estimated input; furthermore, the discrepancy is exactly the compensation needed and can be directly used for regulating the observed Input/Output behavior.

2.2 DISCRETE-TIME IMPLEMENTATION FOR THE MODEL REGULATION CONTROL

So far, the solved compensation is an exact solution that can completely cancel the dynamic effects of model uncertainty and external disturbance. However, there are restrictions which make realization of the exact solution impossible. One can identify "causality", "unacceptable zeros", and "sensor noise" as the major issues which degrade the perfect theoretical performance of the Model Regulation Control in real implementation. These issues will be discussed based on a computer-controlled scheme since the inverse dynamics of the nominal model is most conveniently executed digitally.

The discrete-time transfer functions of their corresponding nominal models are usually obtained from the continuous time transfer functions through Z-transformations. Here the A/D converter is assumed to be an ideal sampler. The D/A converter is also
assumed to have the characteristics of a zero-order-hold. Therefore, the $Z$-transfer function of the sampled system $\hat{G}(s)$ is

$$\hat{G}(z) = Z \left[ \frac{1 - e^{-Ts}}{s} \hat{G}(s) \right] \quad (2.6)$$

where $T$ is the sampling time. Since the nominal model $\hat{G}(s)$ is based on the physical control plant, its denominator should always be a polynomial of $s$ with higher order than the numerator. Therefore, the denominator of the discrete-time transfer function should also be a higher order polynomial of $z$ than the numerator.

2.2.1 Causality

The control algorithm (2.4) involves using the inverse transfer function $\hat{G}^{-1}(z)$ of the nominal model. Since the nominal model $\hat{G}(z)$ is always causal, the implementation of $\hat{G}^{-1}(z)$ to estimate the nominal input would be a noncausal event. Therefore, at least one step of time delay is needed to make the algorithm causal. The compensation then becomes

$$\mu = z^{-1} \left[ U - \hat{G}^{-1}(z)Y \right] \quad (2.7)$$

This new control algorithm (2.7) can be interpreted as computing the compensation for the uncertain dynamics of the present sampling step, from the known input and output of the previous sampling steps. Although it is an approximation of algorithm (2.4), it still can cancel most of the uncertain dynamics of the present sampling step if the uncertainties do
not change too fast relative to the sampling rate. Similar ideas of using time delayed signals as approximation for present step dynamics have been found in several other investigators' works. The time delay controller by Youcef-Toumi and Ito (1987,1990) was developed in state space form. The unknown dynamic terms are estimated by the difference between the derivative of the state variables and the known dynamics of the previous observation. This scheme has been applied to the control of magnetic bearings by Youcef-Toumi and Reddy (1990). Hsia and Gao (1990) employed a time delay technique in robotic control to bypass the computation of nonlinear terms such as centrifugal, Coriolis and gravity forces which are required in the computed-torque algorithm. The mass matrix of the robot dynamic model was also simplified to a diagonal matrix in their time delay algorithm.

2.2.2 Unacceptable Zeros

The zeros of the nominal model become the poles of the inverse transfer function in Equation (2.4). If any of these zeros is nonminimum phase or very close to the unit circle in the $z$-plane, the inverse system will either be unstable or generate oscillating control signals. Both of these two cases are unacceptable. A stable pseudo inverse transfer function is, therefore, needed to replace the use of the direct inverse model. The product of the pseudo inverse transfer function and the nominal model is like a desirable low-pass filter. That is

$$\tilde{G}^{-1}(z)\hat{G}(z) = \tilde{I}(z)$$

(2.8)

where $\tilde{G}^{-1}(z)$ is the pseudo inverse transfer function and $\tilde{I}(z)$ is the desirable low-pass filter. $\tilde{I}(z)$ can be designed to stay very close to the unit vector $I$ in a specified low
frequency range. The bandwidth of \( \tilde{f}(z) \) will affect the ability of the model regulation algorithm to cancel the uncertain dynamics. Existing methods for the design of the pseudo inverse transfer function \( \tilde{G}^{-1}(z) \) have been presented by Menq and Chen (1992), Menq and Xia (1990), and Tomizuka (1987).

2.2.3 Sensor Noise

Another practical issue is the high frequency sensor noise associated with the measured output signal. In this case, low-pass filters are usually used to filter out the high frequency components of the measured output. There are standard methods for the design of such low-pass filters and their implementation on digital computers is straightforward. Here, \( F(z) \) denotes the transfer function of the designed low-pass filter. The use of low-pass filters will introduce more complicated transfer functions in the loop of the model regulation algorithm and further degrade its effectiveness in sensitivity reduction and disturbance rejection.

2.2.4 The Equivalent General Two-Degree-of-Freedom System

Figure 2.2 shows a block diagram of the discrete-time implementation for the Model Regulation Control when there are unacceptable zeros in the nominal model. The compensated system is

\[
H(z) = \frac{Y(z)}{U(z)}
\]  
(2.9)
Figure 2.2  Discrete-Time Implementation of the Model Regulation Control
Notice that $L(z)$ is the product of $\tilde{I}(z)$ and $F(z)$. One has

$$L(z) = \tilde{I}(z)F(z) = \hat{G}^{-1}(z)\hat{G}(z)F(z)$$

(2.10)

which can be considered as the overall low-pass filtering effect of the implemented loop. When there is no unacceptable zeros in the nominal model, the pseudo inverse system becomes the direct inverse of the nominal model. That is

$$\hat{G}^{-1}(z) = \hat{G}^{-1}(z)$$

(2.11)

and

$$\tilde{I}(z) = I$$

(2.12)

Then one has

$$L(z) = F(z)$$

(2.13)

The general configuration of a two-degree-of-freedom feedback control system is sketched in Figure 2.3. The closed-loop transfer function of this system is

$$H_c = \frac{C_1G}{1 + C_1C_2G}$$

(2.14)
Figure 2.3 General Configuration of Two Degree-of-Freedom Feedback Control System
The transfer functions $C_1$ and $C_2$ are the two degrees of freedom available for the designer to achieve the two design objectives -- $H_c$ and $S$. The sensitivity function of the closed-loop system in Figure 2.3 is

$$S = \frac{1}{1 + C_1 C_2 G}$$  \hspace{1cm} (2.15)

and the closed-loop transfer function can be expressed as

$$H_c = C_1 S$$ \hspace{1cm} (2.16)

Therefore, $C_1$ and $C_2$ are fixed once $S$ and $H_c$ are specified by the design requirements.

To find the equivalence of the Model Regulation Control in the general two-degree-of-freedom configuration, the closed-loop transfer function of the system in Figure 2.2 can be written as

$$H_c(z) = \frac{1}{1 - z^{-1} L} \frac{G}{1 + \frac{z^{-1} L}{\hat{G}^{-1} G}}$$ \hspace{1cm} (2.17)

The equivalent two-degree-of-freedom feedback controller is
One can say that the Model Regulation Control is a special case of the two-degree-of-freedom feedback controller. Nevertheless, its distinguishing feature is the use of the inverse transfer function of the nominal model in the feedback loop. This makes the nominal model the target closed-loop transfer function. Not only does the nominal model have the physical meaning of the control plant, it also has the intended dynamic characteristics of the design of the plant. For many practical systems operated under different conditions or payloads, such as motors, engines, robots, etc., Model Regulation Control can ensure robust performance and at the same time preserve their specified dynamic characteristics. Another advantage of the Model Regulation Control is in the direct approach in discrete-time form. Any inverse transfer function of the nominal model can be realized by digital computation without difficulty. The causality problem is solved by adopting one-step time delay. The approximated solution for nonminimum phase zeros can be easily included in the control algorithm.

Based on the above restrictions on implementation, time delayed signals have been introduced and a pseudo inverse transfer function of the nominal model has been used when it is necessary. In the following sections, the Model Regulation Control will be studied to determine if the regulation effect is still valid or to what extent it reduces the model uncertainty and rejects the disturbance for the compensated system. The stability of the compensated system should also be checked since time delay generally endangers system stability.

\[
C_1 = \frac{1}{1 - z^{-1}L} \quad (2.18)
\]

\[
C_2 = z^{-1}L\hat{G}^{-1} \quad (2.19)
\]
2.3 SENSITIVITY REDUCTION

One function of the Model Regulation Control is to reduce the sensitivity of the system to the model uncertainty. The regulation effect is measured by the sensitivity function defined as

\[ S = \left| \frac{(H - \hat{G})/\hat{G}}{\delta G/\hat{G}} \right| \]  

(2.20)

The sensitivity function is an analytical tool that compares the regulated result with the nominal model. This comparison clearly indicates the reduction of model uncertainty.

The actual control plant with model uncertainty is described by a family of possible plants. Let \( \mathcal{G} \) be the set of all the possible control plants, thus

\[ \mathcal{G} = \{ G(z) \mid G(z) = \hat{G}(z) + \delta G(z) \}, \quad \text{and} \quad \hat{G}(z) \in \mathcal{G} \]  

(2.21)

The uncertain model \( \delta G(z) \) indicates the difference between the actual control plant and the nominal model \( \hat{G}(z) \). In order to quantitatively express the size of the model uncertainty, a measure \( \Delta(z) \) can be defined as

\[ \Delta(z) = \hat{G}^{-1}(z)\delta G(z) \]  

(2.22)
which is the ratio of the uncertain model over the nominal model. It can be considered to be the normalized size of the uncertain model. Now, the transfer function of the uncompensated system becomes

\[ G(z) = \hat{G}(z) + \hat{\delta} G(z) = \hat{G}(z)[1 + \Delta(z)] \quad (2.23) \]

On the other hand, the transfer function of the compensated system is

\[ H(z) = \hat{G}(z) \frac{1 + \Delta(z)}{1 + z^{-1}L(z)\Delta(z)} = \hat{G}(z)[1 + \Delta'(z)] \quad (2.24) \]

where

\[ \Delta'(z) = \Delta(z) \frac{1 - z^{-1}L(z)}{1 + z^{-1}L(z)\Delta(z)} \quad (2.25) \]

This result relates the normalized model uncertainty of the compensated system by a factor to that of the uncompensated system. It is desired that becomes as small as possible relative to . When equals zero, the compensated system is the nominal model; however, the terms \( z^{-1} \) representing one step of time delay and the low-pass filter cause not to equal zero. Nevertheless, reduction on model uncertainty can still be expected when is smaller than . This is best illustrated in the frequency domain by assessing the sensitivity function \( S(\omega) \):
$S(\omega) = \left| \frac{\Delta'(j\omega)}{\Delta_j(j\omega)} \right| = \left| \frac{1 - e^{-j\omega T}L(j\omega)}{1 + e^{-j\omega T}L(j\omega)\Delta(j\omega)} \right|$ (2.26)

where $T$ is the sampling period and $\omega$ is the frequency. This measure indicates the reduction ratio on the uncertain model achieved by the Model Regulation Control. The model uncertainty is reduced when $S(\omega)$ is less than one and is totally canceled when $S(\omega)$ equals to zero. Equation (2.26) is now examined for the entire frequency range.

First of all, at $\omega = 0$,

$$e^{-j\omega T}\bigg|_{\omega=0} = 1 \quad \text{and} \quad L(j\omega)|_{\omega=0} = 1$$ (2.27)

Thus

$$S(0) = 0$$ (2.28)

The model regulation effect is perfect at this point.

Secondly, in the low frequency range well below the cut-off frequency of $L(z)$,

$$L(j\omega) = 1$$ (2.29)

and
\[ e^{-j\omega T} = 1 + j\phi_d \]  
\[ \phi_d = -\omega T \]

where \( \phi_d \) is the delay angle

Thus, if \( \Delta(j\omega) << 1 \), from Equation (2.26), one has

\[ S(\omega) = \omega T \]

That is, the sensitivity function linearly increases as \( \omega \) increases in the low frequency range. The slope of the increase is determined by the sampling time \( T \).

Next, in the higher frequency range near the cut-off frequency of \( L(z) \), \( \phi_d \) is not negligible and \( |L(j\omega)| \) starts to decrease. The sensitivity function is more strongly dependent on the natural of the uncertain model and thus not guaranteed to be less than one.

Finally, when frequency is very high, \( |L(j\omega)| \) is zero so that

\[ S(\omega) = \frac{|1 - 0|}{|1 + 0|} = 1 \]

The compensated system is the same as the uncompensated system at this point.
An example is used to illustrate the sensitivity reduction achieved by the Model Regulation Control. Figure 2.4 shows the Bode plot of the nominal model and the Bode plots of its associated uncompensated as well as compensated systems. It is seen that the frequency response of the compensated system using the Model Regulation Control is very close to that of the nominal model in the low frequency range. Figure 2.5 plots the sensitivity function \( S(\omega) \) of this example. It is seen that \( S(\omega) \) is very small and increases linearly when \( \omega \) is small. The sampling time of this example is 0.05 sec therefore the highest frequency of the Bode plot is 10 Hz. This frequency is relatively high when it is compared with the bandwidth of the given example.

2.4 DISTURBANCE REJECTION

The transfer function of the uncompensated system due to external disturbance is

\[
\frac{Y}{d}(z) = G(z) \tag{2.34}
\]

and the transfer function of the compensated system is

\[
\frac{Y}{d}(z) = G(z) \frac{1 - z^{-1}L(z)}{1 + z^{-1}L(z)\Delta(z)} \tag{2.35}
\]

The two transfer functions can be compared from the frequency response point of view as two filters which process the disturbance. Although the transfer function \( G(z) \) of the uncompensated system is able to attenuate the high frequency components of the
Figure 2.4  Bode Plots of a Nominal Model and Its Associated Uncompensated and Compensated Systems
Figure 2.5  Sensitivity Function of the Model Regulation Approach
disturbance due to its physical bandwidth, all low frequency components of the disturbance are transmitted. On the other hand, one can see the transfer function of the compensated system provides further attenuation of the disturbance than the uncompensated system if the term \([1 - z^{-1}L(z)]/[1 + z^{-1}L(z)\Delta(z)]\) is less than one. This term is the same as the sensitivity function in Equation (2.26). Thus from the previous study on the sensitivity function, the Model Regulation Control guarantees the zero-frequency disturbance is totally cancelled since \(S(0) = 0\). The low frequency components of the disturbance are also largely rejected since \(S(\omega)\) is very small in the low frequency range.

2.5 STABILITY CONDITIONS

The stability of the compensated system can be studied by means of the Nyquist criterion. Since the nominal model does not have any unstable poles, the characteristic equation of Equation (2.24) is simplified to

\[
1 + e^{-j\omega T}L(j\omega)\Delta(j\omega) = 0
\]  

(2.36)

Figure 2.6 shows a typical Nyquist plot of \(L(j\omega)\Delta(j\omega)\) for a member \(G(z) \in \mathcal{G}\). Since the total low-pass filter \(L(j\omega)\) reduces the magnitude of \(L(j\omega)\Delta(j\omega)\) to zero when frequency approaches infinity, the Nyquist plot of \(L(j\omega)\Delta(j\omega)\) goes to the origin. Thus, the phase margin technique, which is simpler than the general Nyquist criterion, can be applied. At gain crossover frequency \(\omega_c\), \(L(j\omega)\Delta(j\omega)\) has phase angle \(\phi_c\) while the delay angle \(\phi_d\) induced by time delay is \(-\omega_c T\). One then easily obtains the stability condition:
Figure 2.6 Nyquist Plot of the Filtered Model Uncertainty
The compensated system is stable if

\[ \phi_e - \omega_e T > -\pi \]  

(2.37)

For all the members of \( G \),

the compensated system is robustly stable if

\[ \phi_e(G) - \omega_e(G)T > -\pi \quad \text{for} \quad \forall G \in G \]  

(2.38)

2.6 SUMMARY

A discrete-time Model Regulation Control has been developed for digital implementation. The objective of this controller is to regulate the Input/Output behavior of the actual control plant to its nominal model in spite of significant uncertain dynamics. The possible limitations, such as causality, unacceptable zeros and sensor noise, for the real implementation of the Model Regulation Control have been discussed. The performance of the Model Regulation Control has been analytically studied in terms of sensitivity reduction and disturbance rejection. Low sensitivity is guaranteed in the low frequency range. The stability condition of the closed-loop system has been derived based on the Nyquist criterion. The critical design factor for this approach is to increase the sampling rate to extend the bandwidth for acceptable sensitivity reduction and disturbance rejection and also to improve the stability margin.
CHAPTER III
DESIGN OF MODEL REGULATION CONTROL

The previous chapter has discussed the original idea and the actual implementation of the Model Regulation Control. Analytical studies on its performance in terms of sensitivity reduction and disturbance rejection have been done. Robust stability conditions for the closed-loop system have also been given. For the Model Regulation Control approach to be useful in any control application, a design process is necessary. First of all, the design problem should be laid out to formulate the control requirements of the expected system. Two fundamental requirements considered in this chapter are the sensitivity specification and robust stability. Secondly, synthesis steps should be developed for determining the parameters of the regulation algorithm that can meet the design requirements. The most important factor of the sensitivity performance and stability property is the sampling time. A frequency domain synthesis method is developed for finding the necessary sampling time for the Model Regulation Control implementation. This method is modified from Horowitz's "Quantitative Feedback Design Method" (1963) to accommodate the discrete-time form of the transfer functions. A second order system is used as an example to demonstrate the design procedure. All the physical parameters of the system, which are the spring constant, mass and the viscous damping coefficient, are allowed to vary in given ranges.
3.1 DESIGN REQUIREMENT FORMULATION

In general, the control design problem is to find the controller that can achieve the desired closed-loop performance in the face of uncertain dynamics and at the same time guarantee robust stability. The specification of the desired performance should reflect the actual need of the system's function but also facilitate the mathematical manipulations in the synthesis process. For the Model Regulation Control, its sensitivity function has been studied as a function of frequency. The performance requirement used in this chapter will be the sensitivity boundary specified in the frequency domain. On the other hand, the robust stability is a condition that must be satisfied. Any chance for the system to go unstable is not tolerable. In this chapter, the Nyquist criterion is the tool used in the synthesis process since it provides the phase margin that is explicitly related to the sampling time.

3.1.1 Sensitivity Specification

The analytical studies in Chapter 2 reveal that the performances of sensitivity reduction and disturbance rejection in the frequency domain can be separated into three ranges. From zero to a certain frequency, the sensitivity can be kept lower than one; in the middle to high frequency section, the sensitivity raises above one; for extremely high frequencies, the sensitivity approaches one. The width of the low frequency range for low sensitivity is determined by the actual design of the Model Regulation Controller. Since most control systems are operated in the frequency range from zero to some extent, the requirement on the Model Regulation Control is to limit the sensitivity under a specified upper bound within the important low frequency range $[0, \omega_a]$. That is, as shown in Figure 3.1,
The choice of the maximal allowable sensitivity $\bar{S}$ and the low frequency range $\omega_a$ should reflect the demand of the actual application of the system.

### 3.1.2 Robust Stability

The stability of the system should be guaranteed for the entire set $\mathcal{G}$ of all the possible control plants. This design stage requires the knowledge of the range of the model uncertainty. That means $\mathcal{G}$ should be a known set but the actual control plant $G \in \mathcal{G}$ is uncertain. The robust stability requirement is formulated as

$$\Phi_c - \omega_c T > -\pi \quad \text{for} \quad \forall G \in \mathcal{G}$$  \hspace{1cm} (3.2)

where $\Phi_c$ is the phase angle of the filtered model uncertainty $L(z)\Delta(z)$ at its crossover frequency $\omega_c$:

$$\Phi_c = \angle L(j\omega_c)\Delta(j\omega_c)$$  \hspace{1cm} (3.3)

$$|L(j\omega_c)\Delta(j\omega_c)| = 1$$  \hspace{1cm} (3.4)

The phase angle $\Phi_c$ and the gain crossover frequency $\omega_c$ are functions of the the model uncertainty $\Delta(z)$ and the low-pass filter $L(z)$. 
Figure 3.1 Frequency Domain Sensitivity Requirement
3.2 FREQUENCY DOMAIN SYNTHESIS

3.2.1 Sensitivity Function and Nominal Sensitivity Function

The sensitivity function definition of Equation (2.20) is

\[ S = \left| \frac{(H - \hat{G})/\hat{G}}{\delta G/\hat{G}} \right| \]  

(3.5)

One can consider this to be the sensitivity of the closed-loop system with respect to the nominal model \( \hat{G} \). For the Model Regulation Control approach, the explicit result is

\[ S = \left| \frac{1 - z^{-1}L(z)}{1 + z^{-1}L(z)\Delta(z)} \right| \]  

(3.6)

Here, an alternative sensitivity function defined with respect to the uncertain model \( G \) and the closed-loop response \( H \) is given as

\[ S_n = \left| \frac{(H - \hat{G})/H}{\delta G/G} \right| \]  

(3.7)

The explicit result of this sensitivity function for the Model Regulation Control is

\[ S_n = |1 - z^{-1}L(z)| \]  

(3.8)
$S_n$ is called the nominal sensitivity function since the uncertain model $\Delta(z)$ does not appear in Equation (3.7). The purpose for introducing $S_n$ is to take the advantage of its simple formulation so that the effects of the parameters $T, F(z), \tilde{I}(z)$ and $L(z)$ can be studied without knowing the model of the control plant. However, achieving small $S_n$ does not guarantee small $S$ which is the true measure of the performance of the closed-loop system. This can be seen from the following relationship between $S$ and $S_n$:

$$S = \frac{S_n}{|1+z^{-1}L(z)\Delta(z)|}$$  \hspace{1cm} (3.9)

The nominal sensitivity functions associated with the four possible algorithms for the discrete-time implementation of the Model Regulation Control are:

(1) Simple one-step delay algorithm

$$S_n = |1-z^{-1}|$$  \hspace{1cm} (3.10)

(2) One-step delay and low-pass sensor filter algorithm

$$S_n = |1-z^{-1}F(z)|$$  \hspace{1cm} (3.11)

(3) One-step delay and pseudo inverse algorithm

$$S_n = |1-z^{-1}\tilde{I}(z)|$$  \hspace{1cm} (3.12)
(4) One-step delay, pseudo inverse and low-pass sensor filter algorithm

\[ S_n = |1 - z^{-1} \bar{I}(z) F(z)| \]  \hspace{1cm} (3.13)
\[ = |1 - z^{-1} L(z)| \]  \hspace{1cm} (3.14)

The nominal sensitivity function of the first algorithm is plotted against frequency in Figure 3.2. One can see that the increase in sampling rate can extend the width of the frequency range for low sensitivity. This shows the importance of the choice of sampling time for the potential of sensitivity reduction. The Nyquist plots of the above four algorithms are shown in Figure 3.3. The low pass sensor filter \( F(z) \) is an example of a fourth order Butterworth filter with cutoff frequency at 3 Hz. The pseudo inverse \( \bar{I}(z) \) is for the compensation of a nonminimum phase zero at \( z_2 = 5.0 \). The sampling time used is 0.05 seconds.

3.2.2 Synthesis for Sampling Time

The dominant design factor for the Model Regulation Control is the sampling time \( T \). The study on sensitivity reduction, disturbance rejection and stability found the performance of the compensated system is heavily influenced by the delay angle \( \phi_d \). Small delay angle is favorable for both the model regulation performance and the stability margin. Equations (2.27-33) imply that sensitivity reduction on model uncertainty is guaranteed when \( \phi_d \) is negligible. The stability conditions also require \( \phi_d \) not to exceed the phase margin of the filtered uncertain dynamics. The delay angle is determined by the frequency
Figure 3.2 Nominal Sensitivity Function of Simple One-Step Delay Algorithm
Figure 3.3 Nyquist Plots of Nominal Sensitivity Functions
\( \omega \) and the sampling time \( T \). The bandwidth for acceptable sensitivity reduction and disturbance rejection is proportionally extended when the sampling rate is increased. From this point of view, it is desirable to implement the sampling rate as fast as possible. The possible limitations on the sampling rate come from the computational device, the data acquisition device, the sensor response time and the hardware of the control plant. Faster sampling speed requires more investment on the whole control system. From the economic point of view, it is a waste to use a sampling rate faster than necessary. Therefore, the synthesis process here is for finding the slowest sampling rate that can satisfy the design requirements of Section 3.1. The implementation of the Model Regulation Control in Figure 2.2 also uses low-pass filter \( F(z) \) and pseudo inverse \( \hat{G}^{-1}(z) \). Their respective functions in the control algorithm are to suppress sensor noise and to approximate the inverse of the unacceptable zeros. Their effects on the sensitivity and stability of the closed-loop system are not significant compared with the sampling time. Thus the design of \( F(z) \) and \( \hat{G}^{-1}(z) \) will not include consideration for sensitivity specification and robust stability. They will be designed independently solely for their own purposes.

Since the sensitivity function of the Model Regulation Control increases monotonously from zero frequency in the low frequency range, one can assume that if the sensitivity at \( \omega_a \) is less than or equal to \( S \), the sensitivity at frequencies lower than \( \omega_a \) will all be smaller than \( S \). Therefore, the sensitivity specification only needs to be considered at one frequency point \( \omega_a \). That is to find the sampling time so that

\[
S(\omega_a) = \frac{S_a(\omega_a)}{1 + e^{-i\omega_a T}(j\omega_a)F(j\omega_a)\Delta(j\omega_a)} \leq S
\]  

(3.14)
or

$$S_s(\omega_a) \leq \min_{G \in \mathcal{G}} |1 + e^{-j\omega_a T} \tilde{I}(j \omega_a) F(j \omega_a) \Delta(j \omega)| \cdot \bar{S}$$  \hspace{1cm} (3.15)

An assumption is that \( \Delta(j \omega) \) can be calculated at any frequency based on the knowledge of \( \mathcal{G} \). However, before the sampling time is determined, \( e^{-j\omega_a T} \) can not be calculated and \( \tilde{I}(z) \) is not designed. Therefore, instead of \( \min_{G \in \mathcal{G}} |1 + e^{-j\omega_a T} \tilde{I}(j \omega_a) F(j \omega_a) \Delta(j \omega)| \), one can only calculate \( \min_{G \in \mathcal{G}} |1 + F(j \omega_a) \Delta(j \omega)| \) at this stage. The complex value of \( 1 + F(j \omega_a) \Delta(j \omega) \) for all the uncertain models will form a region in the complex plane as plotted in Figure 3.4. The minimal value will be the closest distance from the shaded region to the origin. The condition on the nominal sensitivity is

$$S_s(\omega_a) \leq \min_{G \in \mathcal{G}} |1 + F(j \omega_a) \Delta(j \omega)| \cdot \bar{S}$$  \hspace{1cm} (3.16)

Then the needed sampling time can be determined form

$$|1 - e^{-j\omega_a T} F(j \omega_a)| = S_s(\omega_a)$$  \hspace{1cm} (3.17)
Figure 3.4 Graphical Interpretation of Sampling Time Synthesis

\[ m_1 = \min_{\omega} |1 + F(j\omega)\Delta(j\omega)| \]

\[ m_2 = S_n(j\omega_a) \]
To find the needed sampling time for robust stability requires the calculation of the crossover frequency $\omega_c$ and the phase angle $\phi_c$ of $F(j\omega)\Delta(j\omega)$ for all the possible plants. Then the largest sampling time allowed is

$$T \leq \max_{\omega \in G} \{T|\omega_c T = \pi + \phi_c\} \quad (3.18)$$

After the sampling time is determined, the $z$-transfer function of the nominal model can be obtained by using Equation (2.6). Should there be any unacceptable zero, the pseudo inverse transfer function $\tilde{G}^{-1}(z)$ needs to be designed so that the total low-pass filter $L(z)$ is

$$L(z) = \tilde{G}^{-1}(z)\hat{G}(z)F(z) \quad (3.19)$$

The assessment of the actual sensitivity of the closed-loop system at $\omega_a$ now can be made by calculating

$$S(\omega_a) = \frac{|1 - e^{-j\omega_a T} L(j\omega_a)|}{|1 + e^{-j\omega_a T} L(j\omega_a)\Delta(j\omega_a)|} \quad (3.20)$$

The current design is satisfactory if
The robust stability can be verified by solving the characteristic equation

\[ 1 + z^{-1}L(z)\Delta(z) = 0 \]  

(3.22)

for the whole \( \mathcal{G} \) set to see if any pole is out of the unit circle.

If Equation (3.21) or the robust stability condition is not satisfied, one needs to choose a sampling time smaller than the current design and redesign \( \tilde{G}^{-1}(z) \). Then the calculations of Equations (3.19-22) should be repeated.

3.3 DESIGN EXAMPLE

A second order system is given as an example to demonstrate the design process of the Model Regulation Control. The plant is a typical mass-spring-damper mechanical system with the governing equation

\[ M\ddot{x} = F - B\dot{x} - K_s x \]  

(3.23)

where \( M \) is the total mass of the moving part, \( B \) is the viscous damper and \( K_s \) is the spring constant. The transfer function of the displacement to the input force is
\[ G(s) = \frac{X(s)}{F(s)} = \frac{k}{\frac{1}{\omega_n^2} s^2 + \frac{2\zeta}{\omega_n} s + 1} \]  

(3.24)

where \( k \) is the steady state gain, \( \omega_n \) is the natural frequency and \( \zeta \) is the damping ratio.

\[ k = \frac{1}{K_s} \]  

(3.25)

\[ \omega_n = \sqrt{\frac{K_s}{M}} \]  

(3.26)

\[ \zeta = \frac{B}{2\sqrt{MK_s}} \]  

(3.27)

The nominal values of the physical parameters are: \( \dot{K}_s = 1, \dot{M} = 0.0517, \dot{B} = 0.3215 \). This set of nominal parameters will yield a nominal transfer function of the plant \( \dot{G}(s) \) with nominal coefficients at \( \dot{k} = 1, \dot{\omega}_n = 0.7 \text{ Hz} \) and \( \dot{\zeta} = 0.707 \). The three physical parameters are allowed to vary independently in the following ranges to form the set \( \mathcal{G} \):

\[ K_s = [0.4, 2.5] \]  

(3.28)

\[ M = [0.03, 0.07] \]  

(3.29)

\[ B = [0.15, 0.45] \]  

(3.30)

The resulting variation range of the transfer function coefficients are:

\[ k = [0.4, 2.5] \]  

(3.31)
\[ \omega_a = [0.38, 1.45] \text{ Hz} \quad (3.32) \]
\[ \zeta = [0.179, 2.054] \quad (3.33) \]

However, their variations are not independent due to the relations of Equations (3.25-27). There is not any physical plant in \( \mathcal{G} \) that can reach the extreme values of these coefficients at the same time. Therefore the search of \( \min_{\mathcal{G}} |1 + F(j \omega_a) \Delta(j \omega_a)| \) will be performed in the physical parameter space rather than the coefficient space. Assume the cutoff frequency \( \omega_f \) of the low-pass filter at 30 Hz is adequate to filter out the sensor noise. Here, \( F(s) \) is a continuous fourth order Butterworth filter. The sensitivity specification is

\[ S(\omega) \leq \bar{S} = 0.5 \quad \text{for} \quad \omega = [0, 2] \text{ Hz} \quad (3.34) \]

Figure 3.5 shows the distribution of \( 1 + F(j \omega_a) \Delta(j \omega_a) \) after the search in \( \mathcal{G} \) is complete. The minimal magnitude is 0.6845. The maximal nominal sensitivity allowed is 0.3423 by the following calculation:

\[ S_n(\omega_a) \leq \min_{\mathcal{G}} |1 + F(j \omega_a) \Delta(j \omega_a)| \cdot \bar{S} \quad (3.35) \]

Then the largest sampling time allowed is obtained by solving

\[ |1 - e^{-j\omega_a T} F(j \omega_a)| = S_n(\omega_a) \quad (3.36) \]
Figure 3.5 Distribution of $1 + F(j\omega)\Delta(j\omega)$ at $\omega_a$.
It is found that $T$ smaller than 0.0135 seconds will satisfy Equation (3.36). However, this is done without including $e^{-j\omega_a T}$ in the search for the minimal magnitude (Equation (3.35)).

The effect of $e^{-j\omega_a T}$ on the shaded region in Figure 3.4 is a clockwise rotation around the point 1 with an angle $\phi_d = \omega_a T$. In this case the minimal magnitude of $1 + e^{-j\omega_a T} F(j\omega_a) \Delta(j\omega_a)$ will be even smaller than the minimal magnitude of $1 + F(j\omega_a) \Delta(j\omega_a)$. One can also easily obtain the $z$-transfer function of the nominal model and find the nominal zero is -0.9724. This indicates the pseudo inverse $\tilde{G}^{-1}(z)$ of the nominal model is needed since this zero is very close to the unit circle. The effect of using $\tilde{G}^{-1}(z)$ will cause more rotation of the shaded area, but more importantly, $S_n(\omega_a)$ will also become larger. Therefore, sampling time smaller than 0.0135 seconds will be used to take these effects into account.

To satisfy the robust stability requirement, the crossover frequencies $\omega_c$ of $F(j\omega)\Delta(j\omega)$ for every possible plant in $\mathcal{G}$ are found. The associated phase angles $\phi_c$ at $\omega_c$ are also calculated. Figure 3.6 plots the phase angles against the crossover frequencies. The worst case should be the one with the largest crossover frequency and the smallest phase angle. A straight line can be drawn starting from $-180^\circ$ towards the right upper direction. The straight line with the largest slope without contact with any point can be used to calculate the largest sampling time for robust stability. In this example, the contact is at the point $\omega_c = 22.3313 \text{ rad/sec}$ and $\phi_c = -19.992^\circ = 0.345 \text{ rad}$. The sampling time that satisfies the stability condition is

$$T < \frac{\pi - 0.345}{22.3313} = 0.125 \text{ sec} \quad (3.37)$$
Figure 3.6  Phase Angle and Crossover Frequency of $F(j\omega)\Delta(j\omega)$
The demand on the sampling speed in this case by the robust stability requirement is much less than that by the sensitivity specification. Here, the sampling time is chosen to be $T = 0.005$ sec to anticipate the effects of the pseudo inverse system in the next step. The discrete-time transfer function of the nominal model is

$$G(z) = \frac{0.0002393(z + 0.9897)}{z^2 - 1.9689z + 0.9694}$$

(3.38)

The zero at -0.9897 is too close to the unit circle to be used for direct inverse. The pseudo inverse system designed is

$$\tilde{G}^{-1}(z) = \frac{0.0007884(z^5 - 1.580z^4 + 0.7060z^3 + 0.1603z^2 - 1.034z + 0.7490)}{z^4}. \quad (3.39)$$

The discrete-time low-pass sensor filter is

$$F(z) = \frac{0.01571z^4 + 0.06282z^3 + 0.09423z^2 + 0.06282z + 0.01571}{z^4 - 1.6896z^3 + 1.4020z^2 - 0.5486z + 0.08756} \quad (3.40)$$

and the total low-pass filter is
\[ L(z) = G^{-1}(z) \hat{G}(z) F(z) \]
\[ = 10^{-3}(2.963z^8 + 15.94z^7 + 36.75z^6 + 50.64z^5 + 52.40z^4 + 46.24z^3 + 31.28z^2 + 12.83z + 2.266) \]
\[ \frac{z^8 - 1.690z^7 + 1.402z^6 - 0.5486z^5 + 0.08757z^4}{z^8 - 1.690z^7 + 1.402z^6 - 0.5486z^5 + 0.08757z^4} \]
(3.41)

The minimal magnitude of \(1 + z^{-1}L(z)\Delta(z)\) at \(\omega_a\) is found by searching in the physical parameter space again. The distribution is plotted in Figure 3.7. The necessary value is

\[ \min_{\mathcal{G}\in\mathcal{G}} |1 + z^{-1}L(z)\Delta(z)|_{z=e^{-\mu T}} = 0.6772 \]
(3.42)

The nominal sensitivity for this design is also calculated as

\[ S(\omega_a) = |1 - z^{-1}L(z)|_{z=e^{-\mu T}} = 0.3503 \]
(3.43)

Therefore the resulting sensitivity function at \(\omega_a\) is

\[ S(\omega_a) = \frac{S_n(\omega_a)}{\min_{\mathcal{G}\in\mathcal{G}} |1 + z^{-1}L(z)\Delta(z)|_{z=e^{-\mu T}}} = 0.5172 \]
(3.44)

This result is slightly over the sensitivity specification \(\bar{S} = 0.5\). One can improve the sensitivity by further decreasing the sampling time. The robust stability is checked by solving the characteristic equation \(1 + z^{-1}L(z)\Delta(z) = 0\) for all the possible plants. The system is still stable since all the poles are within the unit circle.
Figure 3.8 shows the Bode plots of the uncompensated systems. The Bode plots of the same systems with Model Regulation Control are plotted in Figure 3.9. The three cases which are compared with the nominal model are:

Case A  Control plant that has the highest amplitude peak
Case B  Control plant that has the lowest steady state gain and the widest bandwidth
Case C  Control plant that has the highest steady state gain and the narrowest bandwidth

Their sensitivity to the change of the physical parameters is largely reduced as required by the design objective.

3.4 SUMMARY

The design problem of the Model Regulation Control has been studied. The model uncertainty is caused by bounded but unknown variations in the parameters of the control plant. The sensitivity of the closed-loop system is required to stay below an upper bound in the low frequency range. Also, the closed-loop system should always be stable for all the possible control plants.

The most important design factor for Model Regulation Control is the sampling time. Synthesis steps, modified from the Quantitative Feedback Design Method, have been developed for finding the necessary sampling time. Graphical interpretation of the
synthesis process provides the geometrical view of the relationship among the model uncertainty, sensitivity specification and the sampling time. Because of the transparency of the synthesis process, one can easily make adjustments to the sensitivity specification or the bounds of parameter variation to compromise with limitations of the real implementation.

The synthesis process has been demonstrated through the Model Regulation Control design of a second order control system.
Figure 3.7 Distribution of $1 + z^{-1} L(z) \Delta(z)$ at $\omega_a$
Figure 3.8

Bode Plot of the Uncompensated Systems

Phase (degree)

Frequency (Hz)

Amplitude (dB)
Figure 3.9  Bode Plot of the Compensated Systems
4.1 INTRODUCTION

In this chapter, the Model Regulation Control is applied to a DC servomotor to regulate the uncertain dynamics of the system. The uncertain dynamics under consideration are the payload variation which is linear and the nonlinear friction torque. The applications on velocity, position and tracking control will demonstrate robust transient response as well as accurate steady state response against the uncertain dynamics. Analytical studies and experimental results prove that both linear and nonlinear uncertain dynamics are compensated by the Model Regulation Control.

DC motors has been widely used in industrial applications for decades. Recent advance in electronics techniques has made it possible to implement more sophisticated control strategies for DC motors to perform more demanding tasks. The most common schemes for DC motor control appearing in recent literature are adaptive control (Tamura et al., 1991), variable structure control (Lim et al., 1991) and robust feedback control (Ohishi et al., 1987; Umeno and Hori, 1991). The first two control methods are considerably more complicated compared with constant gain feedback controllers. Their practical applications will be limited to those systems or tasks for which constant gain feedback controller is inadequate. The feedback controller by Ohishi et al. employed a deadbeat observer to estimate the unmodelled external torque. Load-insensitive positioning is achieved. The
two-degree-of-freedom controller by Umeno and Hori yielded robust steady-state speed response against inertia variation and disturbance torque. Position control is not reported.

In Section 4.2, the description of the DC servomotor control system is given. The nominal model of the control plant is derived and the sources of uncertain dynamics are discussed. The actual experimental set-up is also described. The velocity control of the DC servomotor against unknown payload and friction torque is studied in Section 4.3. Nyquist criterion is carried out to find the stability property. Analytical results are presented in terms of Bode plots and pole locations. Experimental results of step response are compared with simulated nominal response. Section 4.4 studies the application of the discrete-time Model Regulation Control to the position response of the DC servomotor. Analytical and experimental results are also presented. In Section 4.5, a trajectory control application of the Model Regulation Control is tested to demonstrate its overall performance.

4.2 SYSTEM DESCRIPTION

The dynamic characteristics of a DC servomotor are determined by the armature circuit of the motor. Without considering friction torque on the shaft, the motor dynamics are linear and can be considered as a third order transfer function

\[
G(s) = \frac{Q}{U}(s) = \frac{1}{s\left\{\frac{J_{eq}}{K_t} s + \frac{B_{eq}}{K_t}(R_a + L_a s) + K_T\right\}}
\]  

(4.1)
The variables and the parameter values of the actual motor being used in the experiment are listed in Table 4.1. Normally, the inductance is relatively small and can be neglected so that the order of the system can be reduced to second order as

\[
G(s) = \frac{1}{s \left\{ R_s \left( \frac{J_{eq}}{K_t} s + \frac{B_{eq}}{K_t} \right) + K_t \right\}}
\]  \hspace{1cm} (4.2)

The uncertain dynamics in this system are the friction torques and the payload variation. The friction torques are composed of stiction, Coulomb type friction and viscous friction. Their dynamic effects are shown by the experimental results to be nonlinear and very significant compared to the motor's capacity. They are inherent complexities of the hardware which are very difficult to be included in the nominal model. On the other hand, payload variation does not add complexity to the nominal model but changes the parametric values of the system. One can use a scalar \( \alpha \) to represent payload variation

\[
J_{eq} = \alpha \cdot \hat{J}
\]  \hspace{1cm} (4.3)

where \( \hat{J} \) is the nominal inertia. The quantity of the uncertain dynamics caused by the unknown payload variation will be represented by Bode plots, pole locations and experimental response. The dynamic effect of the unmodelled friction torque will be seen in experimental results.
Table 4.1 Variables and Parameters of the DC Servomotor System

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>Load angular position</td>
<td></td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>Load angular velocity</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>Armature voltage</td>
<td></td>
</tr>
<tr>
<td>$R_a$</td>
<td>Armature resistance</td>
<td>3.96 ohms</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Armature inductance</td>
<td>8.15 mH</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Torque constant</td>
<td>0.1133 N-m/amp</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Back emf constant</td>
<td>0.1133 Volt/rad/sec</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Factor of payload variation</td>
<td></td>
</tr>
<tr>
<td>$j$</td>
<td>Nominal rotary moment of inertia</td>
<td>$7.451 \times 10^{-4}$ N-m-sec²</td>
</tr>
<tr>
<td>$J_{eq}$</td>
<td>Equivalent rotary moment of inertia</td>
<td></td>
</tr>
<tr>
<td>$B_{eq}$</td>
<td>Equivalent viscous damping</td>
<td>$6.744 \times 10^{-5}$ N-m/rad/sec</td>
</tr>
<tr>
<td>$T$</td>
<td>Sampling period</td>
<td>5 msec</td>
</tr>
</tbody>
</table>
Figure 4.1 The DC Servomotor Experimental System
Figure 4.1 is the schematic diagram of the whole experimental system. The DC servomotor being controlled is made by Electro-Craft Corporation (model E-586). The back emf of the armature circuit is used as the tachometer. This analog voltage represents the angular velocity of the motor shaft. The potentiometer (Bourns, model 35305-1-203) sends the analog voltage for angular position measurement. The experimental procedure and the control algorithm are written in FORTRAN and executed by the microcomputer (Intel 86/310). The measured angular velocity and position are received by the ADC input board (iSBX311). The control signal is sent to the motor from the DAC output board (iSBX328). The motor control signal goes through the amplifier (Torque System, model PA-3087) to drive the motor.

4.3 VELOCITY MODEL REGULATION

The discrete-time nominal transfer function for velocity response, from armature voltage to shaft velocity, is described as a first order transfer function:

$$\hat{G}_v(z) = \frac{K_\theta}{z - z_p}$$  \hspace{1cm} (4.4)

where the gain is

$$K_\theta = 0.1897 \text{ rad/sec/Volt}$$  \hspace{1cm} (4.5)

and the nominal pole location is at
\[ \hat{p}_p = 0.9785 \]  

(4.6)

The sampling time is

\[ T = 5 \text{ msec} \]  

(4.7)

The block diagram for the implementation of the model Regulation Control is shown in Figure 4.2. The inverse nominal model \( \hat{G}_v^{-1}(z) \) is used to calculate the estimated input. \( H_v(z) \) represents the transfer function after the Model Regulation Control is used. Note that a low-pass filter \( F(z) \) is used to cut off the noise introduced by the tachometer. It is designed as a fourth order digital filter with cut-off frequency at 30 Hz. One has

\[
F(z) = \frac{0.01571z^4 + 0.06282z^3 + 0.09424z^2 + 0.06282z + 0.01571}{z^4 - 1.6896z^3 + 1.4020z^2 - 0.5486z + 0.08756}
\]

(4.8)

Thus the closed-loop transfer function becomes

\[
H_v(z) = \frac{\hat{Q}}{\hat{U}} = \hat{G}_v(z) \frac{1 + \Delta(z)}{1 + z^{-1}F(z)\Delta(z)}
\]

(4.9)

where \( \Delta(z) \) is defined in Equation (2.22).
Figure 4.2 Block Diagram for Velocity Model Regulation

\[ G_v(z) = \frac{K_G}{z - z_p} \]
Figure 4.3 presents the Nyquist plots of $\Delta(z)$ for different values of $\alpha$. It represents the model uncertainty caused by the payload variation. The unmodelled dynamic effect of friction torque is not included. When $\alpha = 1$, the equivalent inertia equals the nominal value; therefore, $\Delta(z)$ is zero. For $\alpha < 1$, $\Delta(z)$ increases as $\alpha$ goes to zero. For $\alpha > 1$, $\Delta(z)$ becomes larger but never goes outside the unit circle as $\alpha$ increases to infinity. The characteristic equation

$$1 + z^{-1}F(z)\Delta(z) = 0$$

(4.10)

is examined to determine the stability boundary. Since the Nyquist plot of $\Delta(z)$ always stays within the unit circle when the scalar $\alpha$ is greater than one, instability will only happen for the cases of $\alpha < 1$. Figure 4.4 presents the Nyquist plots of $F(z)\Delta(z)$ and $z^{-1}F(z)\Delta(z)$ for the critical value of payload variation ($\alpha = 0.476$). In this case, the crossover frequency $\omega_c$ of $F(z)\Delta(z)$ is 23.5 Hz and the phase margin is 42.3 degrees. The delay angle $\phi_d = -\omega_c T$ is also 42.3 degrees to make the closed-loop system marginally stable. For any payload variation smaller than this case, the system is predicted to be unstable under the Model Regulation Control. Thus the system is robustly stable for $\alpha \geq 0.476$.

Figure 4.5 compares the locations of the uncompensated pole $z_p$ and the dominant pole $z_d$ of $H_v(z)$. The variation of $z_p$ according to the change of payload shows the range of model uncertainty. The closeness of $z_d$ to the nominal pole location shows strong model regulation effect achieved for the payload variation. Figure 4.6 presents the Bode plots of $G_v(z)$ with different values of $\alpha$ ($\alpha = 1, 0.6, 2.0$). They can be compared with
Figure 4.3 Nyquist Plot of $\Delta(z)$ under Payload Variation
Figure 4.4 Nyquist Plots of $F(z)\Delta(z)$ and $z^{-1}F(z)\Delta(z)$ when Payload Variation Is at the Critical Value
Figure 4.5 Variation of the Pole Location of the Uncompensated and Compensated Systems
Figure 4.6 Bode Plots of the Uncompensated System $G_v(z)$
Figure 4.7 Bode Plots of the Compensated System $H_v(z)$
their corresponding compensated transfer functions of \( H_c(z) \) in Figure 4.7. The nominal model is accurately followed in the low frequency range as predicted in Section 2.3.

Experimentally, the effectiveness of the Model Regulation Controller is demonstrated by step response tests. Figure 4.8 plots the results of the uncompensated systems to the command input of 20 rad/sec. Since the velocity model is a first order system, the dynamic effect of pole variation can be seen in the difference of time constants. Besides, strong friction torques cause the significant steady state error. The step responses of the compensated systems are plotted in Figure 4.9. All three different payloads have the same time constant and zero steady state error as well. To further study the compensation of nonlinear dynamics, the system with nominal payload is tested under different command inputs. It is believed that a true linear system should have identical normalized responses under different step inputs. The normalized step responses of the uncompensated systems and the compensated systems are shown in Figures 10 and 11, respectively. Poor performance of the uncompensated systems is caused by the nonlinear friction torques. The identical normalized responses of the compensated systems indicate that the nonlinear model uncertainty has been canceled.
<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Velocity (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>2.60</td>
</tr>
</tbody>
</table>

**Figure 4.8** Velocity Step Response of the Uncompensated Systems

**Figure 4.9** Velocity Step Response of the Compensated Systems
Figure 4.10 Normalized Velocity Step Response of the Uncompensated Systems
(Reference Input $R = 10, 20, 30$ rad/sec)

Figure 4.11 Normalized Velocity Step Response of the Compensated Systems
(Reference Input $R = 10, 20, 30$ rad/sec)
4.4 POSITION MODEL REGULATION

The discrete-time position transfer function of the nominal model, from armature voltage to shaft position, is a second order model:

\[
\hat{G}_q(z) = \frac{K_\delta(z - \hat{\delta}_z)}{(z-1)(z-\hat{\delta}_p)}
\] (4.11)

where the gain is

\[
K_\delta = 0.0004759 \text{ rad/Volt}
\] (4.12)

and the nominal zero is

\[
\hat{\delta}_z = -0.9928
\] (4.13)

The implementation of the Model Regulation Control is shown in Figure 4.12. Here, \(H_q(z)\) represents the transfer function of the regulated system. Since the zero of the nominal model is located at \(-0.99277\) which is an unacceptable zero for digital implementation, a pseudo inverse system \(\hat{G}_q^{-1}(z)\) is used to replace the inverse nominal model in this case. One has
Figure 4.12 Block Diagram for Position Model Regulation
\[
\bar{G}_q^{-1}(z) = \frac{-4.810z^5 + 28.88z^4 - 23.86z^3 - 23.72z^2 + 28.19z - 4.673}{0.02744z^4}
\]

(4.14)

The transfer function of the compensated system is

\[
H_q(z) = \frac{Q(z)}{U} = \frac{\hat{G}_q(z)}{1 + z^{-1}L(z)\Delta(z)}
\]

(4.15)

where

\[
L(z) = \frac{10^4(-0.3596z^8 - 0.3472z^7 + 5.074z^6 + 17.64z^5 + 25.12z^4 + 17.53z^3 + 5.000z^2 - 0.3528z - 0.3545)}{0.02744z^8 - 0.04636z^7 + 0.03847z^6 - 0.01505z^5 + 0.002403z^4}
\]

(4.16)

is the total low-pass filter defined in Equation (2.10). The stability property of the system is studied by examining the Nyquist criterion of the characteristic equation

\[
1 + z^{-1}L(z)\Delta(z) = 0
\]

(4.17)

Following a procedure similar to that of Section 4.3, it is found that the compensated system is robustly stable for \(\alpha \geq 0.495\).
Figure 4.13 and Figure 4.14 illustrate the Bode plots of $G_q(z)$ and $H_q(z)$, respectively. One can see that the transfer functions of the uncompensated systems are different from the nominal transfer function while those of the compensated systems are robust to payload variation in the low frequency range as expected.

The step responses of the shaft position are performed with a feedback loop and a compensator

$$C(z) = \frac{0.5664z - 0.5654}{z - 0.9989}$$

(4.18)

This compensator is designed following standard procedure based on the nominal model. Since the nominal gain is rather high for the current limitation of the motor, a phase-lag compensator is used to reduce the bandwidth to avoid actuator saturation. The phase margin of $C(z)\hat{G}_q(z)$ is 45 degrees and the crossover frequency is 0.6 Hz. The computer simulation which predicts this designed response is shown in Figure 4.15. Friction torques are not included in the simulation model. Figure 4.15 also compares experimental results of the uncompensated system with different payloads to the predicted result. The dynamic effect of changing payload is observed in the difference of rise time; the effect of friction torques causes large steady state error. After the Model Regulation Control is applied, the experimental results are compared to the predicted result in Figure 4.16. The improvements on both rise time and steady state error are obvious. The slower rise time and larger overshoot of the $\alpha = 2.0$ case are due to motor current saturation. The nonlinearity of friction torque is shown in Figure 4.17 with normalized responses in the cases without Model Regulation Control. All the responses are with nominal payload. It is
Figure 4.13 Bode Plots of the Uncompensated System $G_q(z)$
Figure 4.14 Bode Plots of the Compensated System $H_q(z)$
Figure 4.15 Position Step Response of the Uncompensated Systems

Figure 4.16 Position Step Response of the Compensated Systems
Figure 4.17 Normalized Position Step Response of the Uncompensated Systems
(Reference Input $R = 1.5, 3.0, 4.5$ rad)

Figure 4.18 Normalized Position Step Response of the Compensated Systems
(Reference Input $R = 1.5, 3.0, 4.5$ rad)
obvious that the friction torque is very strong compared with the needed driving torque for the nominal linear model. Figure 4.18 shows that a significant part of the nonlinear model uncertainty has been canceled by the Model Regulation Controller.

4.5 TRACKING CONTROL

The block diagram of the implementation for tracking control is shown in Figure 4.19. The pseudo inverse system with preview steps $z^2\hat{G}_q^{-1}(z)$ is used as a feedforward controller in this case. It is designed to approximate the inverse of the nominal model. Since the trajectory is known, the feedforward control $C_f$ can be calculated in advance using $z^2\hat{G}_q^{-1}(z)$. A PD feedback controller $C(z)$ is designed to give the nominal closed-loop transfer function a natural frequency at 20 rad/sec and damping ratio at 0.707. One has

$$C(z) = \frac{135.2z - 124.8}{z} \quad (4.19)$$

The motor is controlled to follow a triangular trajectory. The length of the trajectory is 6.0 seconds and the extreme displacement is 1.0 radian. This trajectory is designed not to call for actuating actions which exceed the motor capability for the nominal payload.

Since the feedforward controller is designed based on the nominal model, the tracking performance is very sensitive to the uncertain dynamics. Figure 4.20 compares the tracking outputs. When the Model Regulation Control is not applied, the slow start of the position output is due to the effect of friction torques. The compensated system closely
Figure 4.19 Block Diagram for DC Servomotor Tracking Control

\[ G_q(z) = \frac{K_G(z-z_1)}{(z-1)(z-z_p)} \]
Figure 4.20 Comparison of DC Servomotor Tracking Performance
Figure 4.21 Comparison of DC Servomotor Tracking Errors
follows the desired trajectory. The associated tracking errors are plotted in Figure 4.21. The Model Regulation Controller significantly improves the tracking error. The main part of the error is caused by friction torques.

4.6 SUMMARY

In this chapter, the Model Regulation Control has been applied to a DC servomotor for velocity, position and tracking control. The uncertain dynamics faced are the payload variation and nonlinear friction torque. The boundary of payload variation for robust stability has been found using the Nyquist criterion. Strong model regulation effect payload variation has been illustrated by pole locations and Bode plots. Experimental results have demonstrated that both robust transient response and accurate steady state response are achieved.
CHAPTER V
DYNAMICS AND MODELING OF
THE PNEUMATIC CONTROL SYSTEM

5.1 INTRODUCTION

Pneumatic systems are commonly used in industry as the actuators of various mechanical systems for force or motion output. Compared with alternative actuating systems, such as hydraulic and electromechanical actuators, pneumatic actuators have advantages in their low cost, convenient energy storage and supply, simple adaptability, and easy maintenance. At the same time, the great advance of electronic technology has made complex signal processing and sophisticated control algorithm implementation available. The possibility of achieving fast and economical actuation by using pneumatic actuators has also stimulated many innovated pneumatic components to accommodate electronic control signals and measurement transducers. As a result of this trend, integrated electropneumatic systems are expected to extend their applications among manufacturing, robotic, biomedical, automotive and aerospace systems.

Currently, pneumatic positioning systems use only point-to-point or on-off type open loop control. Continuous fine motion cannot be achieved unless control schemes that can deal with the control problems of pneumatic positioning systems are developed. The challenging control issue is the huge uncertain dynamics caused by nonlinear gas flow through the valve orifice, compressibility of gas, varying operating states along the stroke,
and nonlinear friction force. A linearized model is only accurate within a small region around the operating volume and the operating pressure. Performance for large pressure change or long stroke motion is yet to be improved. In this part of the dissertation, the discrete-time Model Regulation Control is employed to generate the necessary compensation for the uncertain dynamics. A linearized transfer function derived at one chosen operating condition will be used as the nominal model for controller design. When the system is called upon for large stroke motion, the control problem faced is the nonlinear uncertain dynamics caused by the nonlinear process of gas and the high friction force. The highly nonlinear pneumatic system is expected to be regulated to the nominal model by the Model Regulation Control so that the pressure response and position response of the system behave like the responses predicted by the linear nominal model.

There have been several reported experimental results on the pneumatic positioning control problem. Noritsugu (1986, 1987) has studied the performance of on-off pneumatic valves under PWM control for motion control applications. Reliable low velocity motion is achieved and accurate positioning is obtained by shutting off control action when the piston moves within the tolerance of the desired position. This position control method belongs to the on-off type control scheme. Araki and Yamamoto (1991) used Model Reference Adaptive Control with a servovalve for pneumatic position control. The effect of the order of the reference model is studied. In some cases, poor performance caused by nonminimum phase zero is also reported. Lai and Ou (1992) employed a Time Delay Control algorithm on a servovalve controlled, pneumatic positioning system. Uncertain dynamics such as gas nonlinearity, friction force and payload variation were effectively
compensated. It should be pointed out that the cost of a pneumatic servovalve is much higher than an on-off solenoid valve.

In this chapter, the description of the pneumatic system is given. The governing equations of the dynamic system include the nozzle equation for gas flow through the valve orifice, the energy equation for the total mass flow and the perfect gas law for the chamber pressure. The discontinuity caused by operation of the on-off valve is discussed. The linear nominal model for pressure response is experimentally identified. The transfer function for position response is also derived.

5.2 SYSTEM DESCRIPTION

Figure 5.1 illustrates a physical model for the pneumatic positioning control system. The main component of the system is a pneumatic cylinder with a double end piston. The linear motion of the piston is driven by the difference in pressure between both sides of the cylinder chamber. The pressure of the right chamber is kept around a constant value by a pressure regulator. The pressure of the left chamber is varied by the gas flow in or out of the chamber. The gas flow is controlled by a two-position, 3-way, on-off solenoid valve. When the valve is at the ON position, gas from an up-stream high pressure supply charges into the left chamber to cause a pressure rise. When the valve is at the OFF position, chamber gas discharges into the atmosphere and reduces the chamber pressure. The variables of the system are defined in Table 5.1. The parameters of the system and their values are listed in Table 5.2.
Figure 5.1 Model of the Pneumatic Position Control System
Table 5.1 Variables of the Pneumatic System

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>Piston position</td>
<td>$X = X_o + x$</td>
</tr>
<tr>
<td>$X_o$</td>
<td>Operating point</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>Variation of piston position</td>
<td></td>
</tr>
<tr>
<td>$V_1$</td>
<td>Left chamber volume</td>
<td>$V_1 = V_{1,o} + v_1$</td>
</tr>
<tr>
<td>$V_{1,o}$</td>
<td>Operating point</td>
<td></td>
</tr>
<tr>
<td>$v_1$</td>
<td>Variation of left chamber volume</td>
<td></td>
</tr>
<tr>
<td>$V_2$</td>
<td>Right chamber volume</td>
<td>$V_2 = V_{2,o} + v_2$</td>
</tr>
<tr>
<td>$V_{2,o}$</td>
<td>Operating point</td>
<td></td>
</tr>
<tr>
<td>$v_2$</td>
<td>Variation of right chamber volume</td>
<td></td>
</tr>
<tr>
<td>$P_1$</td>
<td>Left chamber pressure</td>
<td>$P_1 = P_{1,o} + p_1$</td>
</tr>
<tr>
<td>$P_{1,o}$</td>
<td>Operating point</td>
<td></td>
</tr>
<tr>
<td>$p_1$</td>
<td>Variation of left chamber pressure</td>
<td></td>
</tr>
<tr>
<td>$P_2$</td>
<td>Right chamber pressure</td>
<td>$P_2 = P_{2,o} + p_2$</td>
</tr>
<tr>
<td>$P_{2,o}$</td>
<td>Operating point</td>
<td></td>
</tr>
<tr>
<td>$p_2$</td>
<td>Variation of right chamber pressure</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Total control effort</td>
<td>$D = D_o + d$</td>
</tr>
<tr>
<td>$D_o$</td>
<td>Operating point</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>Control effort</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2 Parameters of the Pneumatic System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>Total mass</td>
<td>1.124 kg</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Supply pressure</td>
<td>583.955 kPa</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Atmospheric pressure</td>
<td>101.325 kPa</td>
</tr>
<tr>
<td>$S$</td>
<td>Piston stroke</td>
<td>20 in, 0.508 m</td>
</tr>
<tr>
<td>$A_p$</td>
<td>Cylinder cross section area</td>
<td>$1.9 \times 10^{-3}$ m$^2$</td>
</tr>
<tr>
<td>$V$</td>
<td>Total chamber volume</td>
<td>$V = V_1 + V_2 = A_p \cdot S$</td>
</tr>
<tr>
<td>$k_{pd}$</td>
<td>Pressure response steady state gain</td>
<td>1320 kPa</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Pressure response time constant</td>
<td>0.55 sec</td>
</tr>
<tr>
<td>$B$</td>
<td>Viscous damping coefficient</td>
<td>$1200 \frac{N}{m\cdot sec}$</td>
</tr>
<tr>
<td>$F_f$</td>
<td>Friction force</td>
<td>value unknown</td>
</tr>
</tbody>
</table>
Due to the operation of the on-off valve, the gas flow at the valve port is discontinuous when the electrical signal switches between ON and OFF. The gas flow rate is discussed separately for both the charging (ON) duration and the discharging (OFF) duration. Assuming the fluid properties are uniform across the valve orifice, the mass flow is considered to be one-dimensional, compressible and isentropic. The mass flow rate for the charging duration is (McCloy and Martin, 1980):

\[ m_+ = C_{m+} A_+ P_s C_{m+}/\sqrt{T_s} \]  \hspace{1cm} (5.1)

where \( P_s \) is the supply pressure and \( T_s \) is the temperature of the supply gas. The mass flow parameter \( C_{m+} \) is a function of the chamber pressure \( P_l \):

\[
C_{m+} = \begin{cases} 
\frac{1}{\sqrt{R(\gamma - 1)}} \left[ \left( \frac{P_l}{P_s} \right)^{\frac{2}{\gamma}} - \left( \frac{P_l}{P_s} \right)^{\frac{\gamma+1}{\gamma}} \right], & \frac{P_l}{P_s} \geq 0.528 \\
\left( \frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \sqrt{\frac{2\gamma}{R(\gamma + 1)}} = 0.0405, & \frac{P_l}{P_s} < 0.528 
\end{cases} \]  \hspace{1cm} (5.2)

In Equation (5.2), \( R \) is the gas constant \((R = 287 \text{ m}^2/\text{s}^2 \cdot \text{K})\) and \( \gamma \) is the specific heat ratio \( (\gamma = c_p/c_v = 1.4 \text{ for air}) \). The geometric cross-sectional area of the valve's input port is \( A_+ \). It is modified by the empirical coefficient \( C_{d+} \) for effective orifice area (Andersen, 1967) which is approximated by three first-degree polynomials for three pressure ranges:
Similarly, for the discharging duration, the mass flow rate from the chamber to the atmosphere is:

$$\dot{m}_e = -C_{d-}A_{v-}P_1C_{m-}/\sqrt{T_1}$$  \hspace{1cm} (5.4)

where $T_1$ is the temperature in the cylinder chamber and the mass flow parameter $C_{m-}$ is a function of the atmospheric pressure $P_a$ and the chamber pressure $P_1$:

$$C_{m-} = \begin{cases} \sqrt{\frac{2\gamma}{R(\gamma-1)}} \left[ \left( \frac{P_a}{P_1} \right)^2 - \left( \frac{P_a}{P_1} \right)^{\gamma+1} \right], & P_a \geq 0.528 \\ \left( \frac{2}{\gamma+1} \right)^{1-1} \sqrt{\frac{2\gamma}{R(\gamma+1)}} = 0.0405, & P_a < 0.528 \end{cases}$$  \hspace{1cm} (5.5)

and the effective orifice area coefficient $C_{d-}$ to modify the valve's geometric release port cross-sectional area $A_{v-}$ is:
When the gas is charging into the left chamber, the pressure dynamics, which is a function of the chamber volume \( V_i \), pressure \( P_i \) and temperatures \( T_2 \) and \( T_1 \), is governed by the energy equation:

\[
c_p(T_2 - T_r) \dot{m}_+ - P_i \frac{dV_i}{dt} + \frac{dQ}{dt} = \frac{d}{dt} [\rho V_i c_v (T_i - T_r)] \tag{5.7}
\]

where \( c_p \) is the specific heat at constant pressure, \( c_v \) is the specific heat at constant volume. \( T_r \) is a reference temperature, \( Q \) is the heat transferred to the system and \( \rho \) is the density of the gas. According to the perfect gas law,

\[
\rho = \frac{P_i}{RT_i} \tag{5.8}
\]

Substituting Equation (5.8) into Equation (5.7), the energy equation becomes
It is assumed that the thermofluid process is isentropic so that the heat transfer rate is zero, i.e.

\[
\frac{dQ}{dt} = 0 \tag{5.10}
\]

The volume is also assumed to be fixed at the operating point. That is

\[
V_i = V_{i, o} \tag{5.11}
\]

\[
\frac{dV_i}{dt} = 0 \tag{5.12}
\]

The temperature variation in the cylinder chamber can be considered negligible. This is a widely used assumption (Shearer, 1956, 1957; Andersen, 1967) for pneumatic control systems. Therefore \( T_i \) is a constant in Equation (5.9). Substituting Equations (5.10-12) into Equation (5.9), yields

\[
c_p(T_s - T_r)\dot{m}_s - P_1 \frac{dV_i}{dt} + \frac{dQ}{dt} = \frac{d}{dt}\left[ \frac{P_i}{RT_i} V_i c_v(T_i - T_r) \right] \tag{5.9}
\]

Similarly, for the discharging process, the energy equation can be simplified to become
where $T_a$ is the temperature of the atmosphere. One can also assume that the temperature in the chamber is the same to the temperature of the supply gas and the atmospheric temperature. That is

$$T_i = T_s \quad (5.15)$$

$$T_i = T_a \quad (5.16)$$

Equations (5.13) and (5.14) now can be further simplified to become

$$\dot{P}_1 = \dot{p}_1 = \begin{cases} 
\frac{\gamma RT_1}{V_{1,0}} \dot{m}_+, & \text{charging} \\
\frac{\gamma RT_1}{V_{1,0}} \dot{m}_-, & \text{discharging}
\end{cases} \quad (5.17)$$

This model is not simple enough for controller design but serves well as an analytical tool. Equation (5.17) illustrates that the pressure response for either charging or discharging can be linearized as a first order system for small pressure changes at constant volume. It also indicates the speed (time constant) of the pressure response is affected by the orifice area ($A_{v+}$, $A_{v-}$), outside pressure ($P_s$, $P_a$), temperature, operating pressure and chamber volume.

Some attention should be paid to the operation of the on-off valve which controls the gas flow in or out of the left chamber. Since the solenoid-driven, spring.returned valve
has only two fixed port positions, direct proportional control effort cannot be realized. Time Proportioning Control signal (Figure 5.2) is thus used to operate the valve to approximate proportional control effort. That is, the valve is controlled by an ON/OFF signal but the percent of ON-time is adjustable from 0 to 100% of the fixed sampling time $T$. The control effort $D$ calculated by the controller is proportional to the time duration of the ON-signal between the sampling steps. $D$ is dimensionless and is scaled from -1 (0% time ON) to +1 (100% time ON). Another similar technique for "smoothly" adjusting the opening of an on-off valve is Pulse-Width-Modulation PWM control by which the width of the ON-pulse is adjusted according to the comparison of the analog control signal and the triangular carrier wave. Lai (1989) has conducted an analytical study of the effect of PWM mode on pneumatic systems and derived a state space model of a pneumatic cylinder based on the nozzle equation (McCloy and Martin, 1980), Perry's orifice characterization (Andersen, 1967), and the perfect gas law. The accuracy of this model is verified by consistent simulation and experimental results. This model works very well for dynamic simulation but provides little help for the development of the controller due to its nonlinearity and complexity.
Figure 5.2 Time Proportioning Control Signal for the On-Off Solenoid Valve
5.3 THE LINEAR NOMINAL MODEL

Due to the compressibility of gas, the mass flow dynamics through the valve port is highly nonlinear and complicated (Equations (5.1 - 6)); the pressure response in the cylinder chamber is nonlinear and dependent on the operating pressure and operating volume (Equation (5.15)); and the ratio of nonlinear mechanical friction to the actuating force is high. Regardless of all the nonlinearities, a linearized transfer function can be derived at any chosen operating condition.

5.3.1 The Pressure Response Nominal Model

A first order transfer function is experimentally obtained to relate the dynamic responses of the Time Proportioning Control signal and pressure. This transfer function is obtained by using experimental identification and linearization techniques. The resulting nominal model is

\[ \hat{G}_p(s) = \frac{\hat{p}_i}{d} = \frac{k_{pd}}{\tau_p s + 1} \]  \hspace{1cm} (5.18)

where \( k_{pd} \) is the calibrated steady state gain and \( \tau_p \) is the identified time constant. Their nominal values are

\[ k_{pd} = 1320 \text{ kPa} \]  \hspace{1cm} (5.19)
\[ \tau_p = 0.55 \text{ sec} \]  \hspace{1cm} (5.20)

for the chosen operation pressure
\[ P_{I,0} = 170 \text{ kPa} \quad (5.21) \]

and operating volume

\[ V_{I,0} = 2.68 \times 10^{-4} \text{ m}^3 \quad (5.22) \]

While this first order transfer function model is generally true for any volume size, the time constant and the steady state gain vary noticeably when the volume is different than the nominal volume. Therefore, the uncertain dynamics for fixed volume pressure control can be considered as time-invariant parameter uncertainty.

The steady state calibration was performed for different values of \( D \) to calculate the steady state gain. Figure 5.3 is the calibration curve for steady state pressure versus \( D \). One can see that the steady state pressures under open-loop control are not consistent even at the operating condition; therefore, the steady state gain is a statistical average rather than a reliable figure. Ideally, the valve's open-time and close-time is the same as the ON-time and OFF-time of the Time Proportioning Control signal. In reality, the solenoid has some response time (about 5 msec) in opening the port and the return spring also takes time (about 35 msec) to shut the port. This is the reason why the valve was only operated in the range of \( D = -0.8 \) to \( D = -0.4 \). Sending a \( D \) value out of this range causes valve stalling.
Figure 5.3 Calibration Curve for Pressure Response
The step response test at the operating pressure and operating volume was performed to identify the time constant. Figure 5.4 shows the step responses at several different fixed volumes. One can expect that the larger the volume is the slower the pressure response will be. Table 5.3 lists the identified time constant for each tested volume size.

<table>
<thead>
<tr>
<th>Volume ($10^{-4} \text{m}^3$)</th>
<th>0.78</th>
<th>1.73</th>
<th>2.68</th>
<th>3.63</th>
<th>4.58</th>
<th>5.53</th>
<th>6.48</th>
<th>7.43</th>
<th>8.38</th>
<th>9.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Constant (sec)</td>
<td>0.25</td>
<td>0.33</td>
<td>0.55</td>
<td>0.90</td>
<td>1.10</td>
<td>1.53</td>
<td>1.94</td>
<td>2.45</td>
<td>2.65</td>
<td>2.92</td>
</tr>
</tbody>
</table>
Figure 5.4  Step Pressure Response of the Uncompensated Systems for Different Volumes ($V_i = 1.73, 2.68, 4.58, 8.38 \times 10^{-4} \text{ m}^3$)
5.3.2 The Position Response Nominal Model

A linear model is analytically obtained for the piston motion at the operating point of the states $P_{1,o}$, $P_{2,o}$, $X_o$, $V_{1,o}$, and $V_{2,o}$. The motion of the piston is considered to be caused by the net force $f$ where

$$f = P_1A_p - P_2A_p - F_f$$  \hspace{1cm} (5.23)

The friction force $F_f$ which consists of stiction and Coulomb friction was proven by experimental results to have a very strong effect on the motion performance. Nevertheless, $F_f$ is not included in the nominal model because it is very difficult to identify the stiction force and the Coulomb coefficient. By choosing the same operating pressure for both sides of the piston, one has the following equation for the nominal model:

$$f = p_1A_p - p_2A_p$$ \hspace{1cm} (5.24)

The right side pressure $P_2$ is kept at the operating point $P_{2,o}$ by a pressure regulator. The change of volume when the piston is in motion will cause $P_2$ to change temporarily before being regulated back to $P_{2,o}$. This pressure change is observed to be closely related to the velocity of the piston; consequently, it is approximately modeled as an air damper:

$$p_2A_p = B\dot{x}$$ \hspace{1cm} (5.25)

where
\[ B = 1200 \, \frac{N}{m/sec} \] (5.26)

The model of the piston motion now becomes

\[ M \ddot{x} = p_1A_p - B \dot{x} \] (5.27)

or

\[ \frac{x}{p_1} = \frac{A_p}{Ms^2 + Bs} \] (5.28)

Multiplying the transfer function of pressure response and the transfer function of piston motion, the total nominal transfer function becomes

\[ \hat{G}_x(s) = \frac{x}{d} = \hat{G}_p(s) \cdot \frac{x}{p_1} = \frac{k_{pd}A_p}{(\tau_p s + 1)(Ms^2 + Bs)} \] (5.29)

The parameter values for this nominal model are fixed to the identified values at the operating condition. Thus the uncertain dynamics of the position control problem can be considered as state-dependent parameter variation plus the unmodelled friction force.
5.4 SUMMARY

The governing equations of the pneumatic position control system have been listed as the analytical model. At the operating condition $P_{1,o}$ and $V_{1,o}$, the pressure change of the left chamber can be linearized as first order for either charging or discharging. The time constant of this response is affected by the valve's orifice area, outside pressure, temperature, operating pressure and chamber volume. The operation of the on-off valve has been discussed. Time Proportioning signal is the solution for applying different "amounts" of control effort to the valve. The speed limitation of the chosen valve has been found. The response of the pressure to the control effort $D$ is described by a first order transfer function. The values of the time constant and the steady state gain have been experimentally identified. The nominal model of the position response has been derived without consideration of the friction force. The pressure change of the right chamber is modelled as a viscous damping force since it is closely related to the velocity of the piston. The resulting nominal models are the basis for the Model Regulation Control in the next chapter. The problem for the pressure control is the time-invariant parameter uncertainty when the chamber volume is unknown. The problem faced by the position control is the state-dependent parameter variation plus the nonlinear friction force.
CHAPTER VI
PRESSURE AND POSITION CONTROL
OF THE PNEUMATIC SYSTEM

The linear nominal models for the pressure response and the position response have been derived in Chapter 5. The values of their parameters are obtained for the operating volume and operating pressure. Based on the analytical study of the governing equations, the main sources of the uncertain dynamics of the system are the changes in chamber volume and pressure. The time constant of the nominal model is affected by the chamber volume; the steady state gain is affected by the operating pressure. In this chapter, the Model Regulation Control is applied to the pressure and position control problems. Stability due to uncertain dynamics is studied first. The bound of volume change for robust stability is found. Computer simulation is used to predict responses using the nominal models. Experimental results are then compared with the predicted responses to demonstrate that the nonlinear uncertainty has been compensated.

6.1 EXPERIMENTAL SETUP

The complete experimental system is shown in Figure 6.1. The main component is a 20-inch-stroke, 2-inch-bore pneumatic cylinder (ARO 0420-2201-200) with a double end piston. The left port of the cylinder is connected to a three-way, two-position solenoid valve (ARO CAT33S-012-D). An accumulator is connected to the other side of the valve to
Figure 6.1 The Pneumatic Experimental System
provide steady supply pressure $P_s$ which is set by a pressure regulator (Wilkerson R00-02-000). The right port of the cylinder is connected to another pressure regulator which sets the right chamber pressure $P_{2,c}$. The control routines and the experimental administration reside on an IBM compatible 486/33 microcomputer. The interface boards used are the National Instruments Lab-PC Input/Output board for D/A and A/D conversion and Industrial Computer Source 5312 encoder board for digital measurement signals. The pressure in the left chamber is measured by a pressure transducer (OMEGA PX176-100S5V). Its analog measurement signal is sent to the ADC port of the Lab-PC board. The position of the piston is measured by a linear displacement transducer system (MTS TTS-RC-U0200, MTS EOM-C15) which generates quadrature encoder pulses. This signal is received by the encoder board. The control signal to the solenoid valve is from the programmed clock output on the Lab-PC Input/Output board.

6.2 PRESSURE RESPONSE MODEL REGULATION

The linearized nominal model for pressure response is:

$$\hat{G}_p(z) = \frac{1.1471 \times 10^5}{z - 0.9131} \quad (6.1)$$

This transfer function is obtained by identification experiments with the piston fixed at the operating position $X_o$ (nominal volume $V_{1,\sigma} = 2.68 \times 10^{-4} \text{ m}^3$) for a small change of pressure around the operating pressure ($P_{1,\sigma} = 170 \text{ kPa}$). The Model Regulation Controller is applied to the pressure response experiment as in the block diagram shown in Figure 6.2. Since there is not any unacceptable zero in the nominal model, direct inverse transfer
Figure 6.2  Block Diagram for Pressure Response Model Regulation
function $\hat{G}_p^{-1}(z)$ is used. Also, the measurement of the pressure does not contain serious sensor noise. Thus $F(z)$ is not included and $L(z)$ equals one. The sampling rate is limited to 20 Hz (sampling time $T = 50$ msec) because of the valve's operating response time.

Analytical study of stability is based on the identified time constants with respect to different chamber volumes. Assuming for each fixed chamber volume there is a first order dynamic model, the model uncertainty $\Delta(z)$ defined in Equation (2.22) can be calculated. The stability condition discussed in Section 2.5 can be applied to find the boundary on volume change for robust stability. The Nyquist plots of $\Delta(j\omega)$ for several different volume sizes are shown in Figure 6.3. The magnitude of $\Delta(j\omega)$ is never greater than one for those volumes bigger than the nominal volume. The closed-loop system will always be stable for these volume sizes. For those volumes smaller than the nominal model, it is possible for $\Delta(j\omega)$ to encircle the -1 point when it goes out of the unit circle. Using Equation (2.37), the stability condition for pressure control loop can be predicted. Figure 6.4 shows the pressure response is unstable at $V_i = 0.78 \times 10^{-4}$ m$^3$ ($\tau_p = 0.25$ sec). Figure 6.5 shows the pressure response at $V_i = 1.73 \times 10^{-4}$ m$^3$ ($\tau_p = 0.33$ sec) is stable but oscillation is observed because its Nyquist plot of $\Delta(j\omega)$ is near the unit circle.

Figure 6.6 shows the compensated pressure responses for volume sizes $V_i = 2.68$, 4.58, and $8.38 \times 10^{-4}$ m$^3$ at which the control loop is stable. In the figure, the regulated pressure responses from actual measurement are compared with the response predicted by the nominal model. The command input is 20 kPa. It can be seen that the Model Regulation Controller has made these responses closely follow the linear nominal model. Figure 6.7 compares the actual pressure response at different initial pressures ($P_i = 170$, 100, and 70 kPa).
250, 340 kPa) with the nominal response when the Model Regulation Controller is turned off. The command input is also 20 kPa. The model uncertainty caused by a large change in the pressure initial condition is evident. Since the outside supply pressure is fixed, steady state gain becomes smaller when the initial pressure is higher. The compensated pressure responses at the initial pressures $P_i = 170, 250, 340$ kPa are plotted in Figure 6.8. One can see the pressure responses are the same as the predicted nominal response. For different pressure command inputs ($R = 20, 50, 100$ kPa), the pressure response can also be regulated to the linear nominal model. This is shown in Figure 6.9. The above results verify that the pressure response is regulated to the linear nominal model for the entire cylinder stroke and for large pressure ranges.
Figure 6.3 Nyquist Plots of $\Delta(j\omega)$ for Different Volumes
Figure 6.4 Pressure Response for $V_i = 0.78 \times 10^{-4}$ m$^3$
Figure 6.5 Pressure Response for $V_1 = 1.73 \times 10^{-4} \text{ m}^3$
Figure 6.6  Pressure Response of the Compensated Systems for Different Volumes
\( V_i = 2.68, 4.58, 8.38 \times 10^{-4} \text{ m}^3 \)
Figure 6.7 Pressure Response of the Uncompensated Systems at Different Initial Pressures ($P_i = 170, 250, 340$ kPa)
Figure 6.8 Pressure Response of the Compensated Systems at Different Initial Pressures ($P_i = 170, 250, 340$ kPa)
Figure 6.9  Pressure Response of the Compensated Systems for Different Command Inputs ($R = 20, 50, 100$ kPa)
6.3 POSITION RESPONSE MODEL REGULATION

The uncertain dynamics becomes a more difficult problem for position control since the chamber volume changes continuously and the pressure must rapidly cover a large range to drive the piston. Besides, the stiction force and the Coulomb friction force are of the same order as the net force needed for the piston's motion.

The identified nominal model for position is:

\[ \hat{G}_x(z) = \frac{0.004443(z + 0.0006582)(z + 1.0429)}{z(z-1)(z-0.9131)} \]  (6.2)

Figure 6.10 shows the implementation of the Model Regulation Control algorithm in the position control experiment. Since one zero of the nominal model is located at -1.0429 which is a nonminimum phase zero, a pseudo inverse transfer function

\[ \hat{G}_x^{-1}(z) = \frac{41.4464z^5 - 63.2351z^4 + 27.9571z^3 + 6.6646z^2 - 41.9173z + 29.0842}{z^4 + 0.0006582z^3} \]  (6.3)

is used to approximate the inverse of the nominal model. There is no need to use a low-pass sensor filter since the linear displacement transducer used in the experiment for position measurement is relatively noise free for the whole sampled frequency range of the control loop. The overall low-pass filter for the implemented loop is:
Figure 6.10 Block Diagram for Position Response Model Regulation
\[ L(z) = \hat{G}_x^{-1}(z) \hat{G}_x(z) = \frac{0.1841z^4 + 0.2634z^3 + 0.1669z^2 + 0.2380z + 0.1476}{z^4} \]

(6.4)

The nominal step response of the piston position is determined with an outside loop and a feedback compensator \( C(z) \):

\[ C(z) = \frac{0.02441(z - 0.9131)}{z - 0.9703} \]

(6.5)

This compensator is obtained with a pole-zero cancellation design based on the nominal model \( \hat{G}_x(z) \). The nominal pole, which is associated with the pressure response time constant \( z = 0.9131 \), is cancelled by \( C(z) \) to yield a nominal closed-loop system with a pair of dominant poles at -0.3 in the \( s \)-plane. The computer simulated response of this system is shown in Figure 6.11. The command input is 200 mm. Friction forces are not included in the simulation model. Figure 6.11 also compares the experimental response of the uncompensated system with the response of the compensated system. The initial position in the experiments is at the operating position \( X_{i,0} = 141 \text{ mm} \) which is associated with the operating chamber volume. For the uncompensated system, the late start of motion after the command input is applied is caused by the stiction force. The low response velocity is caused by both the Coulomb friction and the increase in chamber volume. Poor positioning accuracy is also observed. For the compensated system, the Model Regulation Controller makes the position response behave very close to the linear nominal response by cancelling the friction force and the gas nonlinearity. Position
Figure 6.11 Position Response at Nominal Initial Position ($X_{I_o} = 141$ mm)
responses were also tested for different sizes of the step command ($R = 50, 100, 200, 300 \text{ mm}$). The normalized responses obtained from these tests are plotted in Figure 6.12. It is believed that a true linear system would have identical normalized responses no matter what the value of the step command input is. The results in Figure 6.12 indicate that the Model Regulation Controller is capable of compensating large nonlinear model uncertainty. Figure 6.13 plots the position responses for different step commands ($R = 50, 100, -100, -200 \text{ mm}$) with initial position at $X_i = 341 \text{ mm}$. It shows that the pneumatic positioning system behaves like a linear system even for initial positions far away from the nominal position and for both motion directions.

6.4 DISCUSSION

The pressure and position control problems of a highly nonlinear pneumatic system have been studied. After the linear nominal model is fixed, the challenging control issue is the huge uncertain dynamics caused by the nonlinearities in the system. The Model Regulation Controller has been employed to generate the necessary compensation for the uncertain dynamics. The stability condition of the pressure control loop has been investigated. The pressure responses for different volumes and initial pressures can be controlled to be almost identical to the response of the nominal model. The position output is regulated to behave linearly for almost the entire stroke. Compared with other investigators' results, this approach employs lower cost equipment and a simpler controller yet accomplishes accurate pressure output and large stroke position output.
Figure 6.12 Normalized Position Response for Different Step Commands

\(X_{1,c} = 141\) mm, \(R = 50, 100, 200, 300\) mm
Figure 6.13 Position Response at Different Initial Positions for Different Step Commands ($X_i = 341$ mm, $R = 50, 100, -100, -200$ mm)
7.1 SUMMARY

This research was intended to develop a Model Regulation Control for systems with uncertain dynamics. The function of the controller is to regulate the input/output behavior of the actual control plant to its nominal model. The actual implementation of the model regulator was studied. The algorithm involves using inverse dynamics, a time-delayed signal and a low-pass sensor filter. Pseudo inverse dynamics is used when the nominal model is nonminimum phase. The performance of the Model Regulation Control was analytically studied. Low sensitivity to the uncertain model and good disturbance rejection are guaranteed in the low frequency range. The width of the frequency range for acceptable performance is dictated by the sampling time. Stability of the closed-loop system was studied by using Nyquist criterion.

The design problem of the model Regulation Control was studied. The design requirements are the sensitivity bound specified in the frequency domain and the robust stability for all the possible plants. The most important design parameter is the sampling time. Synthesis steps were developed for finding the necessary sampling time to meet the design requirements.
The Model Regulation Control was applied to a DC servomotor for velocity, position and tracking control tasks. The control problems faced were the payload variation and nonlinear friction torque. Analytical study and experimental results verified that both robust transient response and accurate steady state response were achieved.

The application of the Model Regulation Control to a pneumatic system was studied. The nonlinear governing equations of the system were listed for analytical study. Experimental identification at the nominal operating condition yielded the linear nominal model for the pressure response. The linear nominal model for position response was derived without considering the friction force. After the linear nominal model was fixed, the control problem was the huge uncertain dynamics caused by the nonlinearity of the system. The Model Regulation Control was employed to generate the necessary compensation for the uncertain dynamics. The pressure response for different volumes and initial pressures was controlled to be almost identical to the response of the nominal model. The position output was regulated to behave linearly for almost the entire stroke.

7.2 CONTRIBUTIONS

The function of the Model Regulation Control is to regulate the input/output behavior of the actual control plant to its nominal model. The reason that this objective can be achieved is because the inverse transfer function of the nominal model is used in the Model Regulation Control algorithm. The digital implementation makes the computation of the algorithm convenient for any linear nominal model which is not always possible in the analog case. Unlike other feedback control approaches, the transfer functions of the Model
Regulation Control are fixed. The design process is basically to determine the necessary sampling speed instead of finding the transfer function of the controllers. The effort and cost for designing complicated controllers are shifted to the cost of implementation of the Model Regulation Control algorithm with fast sampling speed. This makes the Model Regulation Control very attractive to the motion control applications for many mechanical systems because the significant frequency range for these applications are much lower than the speed of the average electronic devices used for data acquisition and computation. The maximum conversion rate of the Input/Output board for the pneumatic system (National Instruments Lab-PC) is 62.5 kHz. On the 486/33 PC, the computational time for the control algorithm is less than 5 msec without paying special attention to the efficiency of the codes. Both of the speeds are adequate for mechanical systems. From practical point of view, the prices of these devices are affordable and they are already being used on many control systems. Thus, there is really no extra cost for using the Model Regulation Control unless the demand of sampling speed can not be satisfied by the common electronic devices.

The technical contributions of this dissertation are specified as follows:

(1) The model regulation approach is an effective algorithm to regulate the wide varying input/output behavior of the control plant to its nominal model. The success of this attempt will preserve the intended dynamic characteristics of the control plant and also benefit the design of the succeeding controllers to eventually achieve desired overall performance of the system.
(2) The theoretical analysis presents the knowledge about the sensitivity reduction, disturbance rejection and stability of the proposed approach. The study illustrates the effects of sampling rate, pseudo inverse dynamics and low-pass sensor filter on the compensated system. Better performance for sensitivity reduction and disturbance rejection can be achieved based on this information. Stability bounds can be found by Nyquist criterion based on the obtained characteristic equation.

(3) The derived design procedures provide a systematic way to determine the sampling time, pseudo inverse transfer function and the cut-off frequency of the low-pass sensor filter subject to the specified sensitivity and robust stability requirements.

(4) The application of the model regulation approach to the DC servomotor verifies its ability to regulate linear model uncertainty caused by parameter variation. The application of the approach to pneumatic systems compensates the nonlinear model uncertainty caused by linearization.

7.3 FUTURE RESEARCH

(1) The Model Regulation Control developed so far is for single-input/single-output (SISO) systems. The extension of this method to multi-input/multi-output (MIMO) systems can widen application of the Model Regulation Control to more practical systems. The difficulty of the MIMO case is the that the exact solution of the inverse transfer function of the nominal model does not always exist. Pseudo inverse solutions will be needed when the number of outputs is greater than the number of
inputs. Moreover, the stability condition and the sensitivity function become more complicated or even unmanageable.

(2) The capability of generating the compensation for nonlinear uncertain dynamics has been demonstrated in this dissertation. The nonlinearity of the pneumatic system has been treated as time-varying linear uncertainty. This is mainly because frequency domain analysis and design were used throughout the dissertation. A more rigorous approach for explicit nonlinear analysis can be investigated by using time domain techniques in the future. The analysis for stability and sensitivity of this approach will accordingly involve studying the bounds of the time functions of the nominal system and the uncertainties.

(3) The Model Regulation Control uses the inverse transfer function of the nominal model in the feedback. The inverse model is also commonly used as feedforward control for trajectory-following applications. The combination of the Model Regulation Control and the feedforward controller will be a unified approach for robust tracking control problems. Since the sensitivity function of the closed-loop system can be guaranteed by the Model Regulation Control in the frequency domain, the frequency components of the command trajectory should be studied first to find the frequency range needed for good sensitivity.

(4) The sampling speed of the pneumatic system studied is restricted by the response time of the on-off valve. Since the sampling speed dominates the performance of the
Model Regulation Control system, the current achievement of the pneumatic system is expected to be enhanced when a faster servovalve is installed.

(5) Due to the digital implementation required by the Model Reference Control, computational speed may be a potential problem for systems demanding extraordinary performance. Therefore the integration of the control system with more powerful electronics technology, such as a Digital Signal Processor (DSP) and Digital Control Processor (DCP), is worth investigating.
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