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Identification of coherent structures in the turbulent wake of a circular cylinder with comparison to linear instability modes

Gieseke, Thomas James, Ph.D.
The Ohio State University, 1993

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Identification of Coherent Structures in the Turbulent Wake of a Circular Cylinder with Comparison to Linear Instability Modes

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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1993

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To Karen M. Johnston

my very best friend
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h^A  eigenvector of the adjoint to the stability problem
l_w  hot-wire length
l_t  wave length of the characteristic large structures
L_x  length scale in coordinate direction x or based on variable x
L  matrix of correlations defined \( L_{jk} = x_{ji}x_{ki} \)
M, R, S  matrices from stability eigenvalue problem
n_{modes}  number of stability modes in an eigenfunction expansion
n_{sf}  number of Chebychev polynomials in a Chebychev expansion
n_x, n_y, n_z  number of grid locations in the streamwise, cross-wake, and spanwise directions
n_y  number of grid locations in the normal direction
n_z  number of grid locations in the spanwise direction
N_s  number of digital samples
q  grid stretching parameter
q_i(t)  arbitrary function of time
\tilde{q}_i(f)  Fourier transform of \( q_i(t) \)
Q_i  cooling velocity of the \( i^{th} \) hot-wire in a multiple-wire array
r'  position vector
R_{xy}  correlation between variable x and variable y
Re_b  Reynolds number based on wake half width, \( (U_\infty b)/(\nu) \)
Re_d  Reynolds number based on cylinder diameter
\( Re_w \) Reynolds number based on the wake half width and defect velocity

\( Re_{\theta^*} \) Reynolds number based on wake momentum thickness and eddy viscosity, \((U_{\infty}\theta^*)/\varepsilon_0)\)

\( s \) expression for the contribution to a velocity field at a specific wavenumber by both \( u \) and \( v \)

\( t \) time

\( u, v, w \) streamwise, normal, and spanwise perturbation velocity components

\( \hat{u}, \hat{v}, \hat{w} \) streamwise, normal, and spanwise velocities

\( u \) perturbation velocity vector

\( \hat{u} \) velocity vector

\( u_i \) perturbation velocity vector at location \( i \)

\( u_c \) condition vector

\( u_e \) estimated vector

\( -\rho \vec{u} \vec{v} \) Reynolds stress

\( U, V, W \) streamwise, normal, and spanwise mean velocity

\( U \) mean velocity vector

\( U_{\infty} \) free-stream velocity

\( U_1 \) defect velocity, \( U_{\infty} - U \)

\( U_{1\text{max}} \) maximum defect velocity

\( \hat{v} \) normal velocity component of an instability mode

\( x_j \) vector of realizations of the \( j^{th} \) variable
$x, y, z$ streamwise, normal, and spanwise coordinate directions

$x^*$ Estimate for streamwise distance from Taylor's hypothesis $rU_\infty/b$

$y_j$ vector of realizations of the $j^{th}$ variable

$\alpha$ streamwise wave number

$\hat{\alpha}$ streamwise wave number of the secondary instability, $\alpha_F/2$

$\alpha_F$ streamwise wave number of the primary instability fundamental mode

$\beta$ spanwise wave number

$\tilde{\beta}$ ratio of the cross-stream length scale to the wake half width

$\Delta_x$ separation in the coordinate direction $x$ or spacing of the variable $x$

$\epsilon$ small coefficient

$\epsilon_T$ total error of a linearly estimated function

$\epsilon_0$ eddy viscosity

$\eta$ nondimensionalized similarity coordinate

$\theta$ flow angle defined by $\tan^{-1}(v/u)$

$\theta^*$ wake momentum thickness

$\theta_w$ an angle associated with hot-wire probe geometries

$\lambda$ matrix eigenvalue $1/\sigma$

$\lambda_x$ Taylor micro-scale based on variable $x$

$\nu$ molecular kinematic viscosity

$\rho$ density
\( \sigma_x \) standard deviation of variable \( x \)

\( \sigma \) matrix eigenvalue

\( \tau \) temporal separation

\( \phi \) flow angle defined by \( \tan^{-1}(w/(u^2 + v^2)^{1/2}) \)

\( \psi \) flow angle defined by \( \tan^{-1}(v/u) \)

\( \omega \) complex frequency of instability modes

\( \omega_x \) vorticity component in coordinate direction \( x \)

\( \Phi_i \) Chebychev polynomial

\( \Omega_w \) an angle associated with hot-wire probe geometries
CHAPTER I
INTRODUCTION

The wake of a circular cylinder in a uniform cross-flow is a commonly occurring turbulent flow and has been studied extensively by many researchers. The distinctive two-dimensional turbulent vortices formed immediately downstream of the body disappear rapidly as they advect downstream. Far downstream, the wake develops into a self-similar turbulent flow containing very different structures. The far-wake structures are born of some unknown mechanism that is related to the mean flow and/or the local turbulent characteristics. There does not appear to be any direct relationship between the near-wake vortices and the far-wake structures. The purpose of this study is to experimentally identify the coherent structures in the far-wake and to investigate the possibility that the stability characteristics of the mean velocity profile may determine the character of the far-wake structures. Similar comparisons for forced wakes can be found in the literature, but no study to date has compared the structures in unforced turbulent wakes to the structures predicted by linear instability theory.

Identifying coherent structures in forced flows is relatively direct due to the phase information introduced by the forcing. However, identifying structures in unforced turbulent flows is much more difficult and can become somewhat subjective due to
the random phase jitter inherent to turbulent flows. It is generally understood that coherent structures are fluid motions that have a well correlated spatial structure. The spatial structure can often be observed in flow visualizations and if the structure is a commonly occurring flow event, its signature can be found in the space-time correlation function. Extracting these structures from the correlation functions can be done mathematically using either the proper orthogonal decomposition (Lumley, 1965) or the stochastic estimation technique (Adrian, 1976). Both methods have been criticized because they use time averaged quantities, thus averaging the effects of many different types of eddies into one coherent structure. Therefore, for the structures suggested by these methods to be considered physically realistic, the shape of the space-time correlation functions must be dominated by the effects of a single type of structure.

The method chosen to determine the three-dimensional shape of far-wake structures in this study is the stochastic estimation technique. The procedure is implemented by first selecting an event. An event is one or more velocity signatures in the turbulent field that are considered indicators of the presence of an important flow structure. What constitutes such events is determined from analysis of the statistical properties of the flow. Stochastic estimation is then used to predict the most likely state of the velocity field surrounding the given events. These velocity fields can be considered representative of the coherent structures associated with the events.

The stochastic estimation procedure can be extended for use in evaluating the influence of the estimated coherent structures on instantaneous velocity fields. The
instantaneous fields can be estimated by measuring velocities at a few locations in space and using stochastic estimation with these real conditions to estimate true velocity fields. These fields can then be examined to determine whether or not typical coherent structures exist in actual velocity fields. The fields can also be used as simulated turbulent velocity fields for numerical analysis.

Even if the large scale motions in wake turbulence can be described in terms of the interaction of a few dominant structures, their origins, dynamics, and interactions remain unknown. One theory is that the structures are the result of the growth and decay of stability modes of the mean velocity profile.

Many studies have been conducted to test the hypothesis stated above. Most have been improvements or variations of the work by Hollingdale (1940). The common approach used in these studies is to introduce perturbations into a wake and to compare the shape and growth of the naturally selected instability mode to the results of linear stability calculations. The results clearly show that the turbulent wake is receptive to external disturbances. However, it has also been shown that the wake (and other flows such as the turbulent shear layer) can be dramatically changed by the introduction of these types of disturbances. This leads to some criticisms about these studies and questions about the governing mechanisms of free shear flow turbulence. The main criticism is that phase correlated periodic disturbances of significant amplitude do not exist in the turbulent wake. By introducing them, it is possible that the original flow is changed and the resulting structures and dynamics do not represent the structures and dynamics that would exist without the forcing. If this
is the case and the forcing changes the turbulent dynamics of the wake, then it would appear that some of the fundamental characteristics of the flow are determined by external conditions rather than by the flow geometry. This would make the results of all studies of turbulent structures in the wake somewhat ambiguous. Fortunately there is enough agreement among studies to believe that some structures are independent of the facilities and other uncontrollable influences.

The difficulties in interpreting the results of forced wake studies are often overlooked because of some qualitative observations of growth and decay of structures in the wake that would be consistent with the mean flow stability mechanism. One popular understanding of the role of stability in the turbulent wake is that two-dimensional waves grow as a result of the mean flow instability. The background turbulence has a damping effect, thus lowering the effective Reynolds number. A secondary sub-harmonic instability grows out of instability of the new periodic base flow. All of the motions generated in this way are short-lived because as they propagate downstream, the stability characteristics of the wake change and they become damped. New modes grow in their place, taking energy from the mean flow while the decaying modes return energy back to the mean through the action of the Reynolds stresses.

A second and less commonly accepted theory, is that the large scale motions are the result of an instability of the turbulent-irrotational interface. Although this theory was convincingly argued by Townsend (1966) and Grant (1957), it has received little attention in the modern literature.
This study is designed to address three major questions: 1) are the structures estimated by stochastic estimation equivalent to those estimated by other methods, 2) do those structures appear in instantaneous velocity fields, and 3) how do they compare to the instability modes of the far-wake mean velocity profile?

This study focuses on the wake at $Re_d = 5080$ at a downstream position of $x/d = 100$ which is in the range that is consistent with the bulk of similar published research. The work represented in this dissertation has been organized in the following fashion:

- The body of literature related to coherent structures and stability characteristics of turbulent wakes has been reviewed. A need for studies that integrate experimental techniques and stability analysis to the analysis of an unforced wake has been identified.

- A detailed experimental analysis of a turbulent wake at $Re_d=5080$ has been conducted providing the data required to conduct such an analysis.

- A statistical analysis of the wake has been conducted to document the various scales and parameters relevant to the wake dynamics.

- The stochastic estimation technique has been implemented to estimate the coherent structures associated with statistically significant events.

- Fully three-dimensional turbulent velocity fields have been estimated using a combination of stochastic estimation and experimentally measured velocity histories.
• The primary and secondary instability problems have been solved using the parameters determined in the experimental portion of this study. The mode shapes have been directly compared to the stochastically estimated structures to look for some relationship between the instability modes and coherent structures.

• A system of modes has been selected from the stability results and were projected onto the estimated turbulent velocity fields. The presence of stability modes in an unforced turbulent wake was then investigated by considering the results of this projection.
CHAPTER II
LITERATURE REVIEW

Many studies of the mechanisms controlling turbulent development and sustenance in the wake of a circular cylinder have been conducted over the past century. From these studies, a great deal of information about the structures in both the near and far-wakes has been gathered. The near-wake structure has been well established and a few dominant structures have been found. The far-wake structure has not been characterized as thoroughly. Some structures have been identified and dynamic models have been formulated but a need still exists to more clearly identify the role of the far-wake structures in the turbulence dynamics.

2.1 Wake Formation

In studies by Strouhal (1878), Benard (1908), and Von Karman and Rubach (1912), the vortex street shed behind circular cylinders was identified as the cause of fluctuating forces and aeroacoustic tones. The nature of the shedding goes through a series of characteristic phases as the Reynolds number is increased from zero. Below a $Re_d$ of 40, the near-wake flow is laminar and steady. Steady two-dimensional vortices are generated immediately behind the cylinder, and far downstream, periodic two-dimensional vortices are formed out of instabilities of the mean flow. Above $Re_d=40$, 

7
the near-wake vortices become unstable due to strong interactions between them and results in the periodic shedding observed up to Reynolds numbers of $5 \times 10^6$. For $Re_d$ above 200, increasing three-dimensionality in the near-wake vortices ultimately leads to shedding of fully turbulent vortices. The nature and frequency of the shedding as a function of Reynolds number has been carefully documented by many researchers, most recently by Williamson (1988;1989).

For $400 < Re_d < 5 \times 10^4$, which is the range of interest in this study, the vortex shedding acts in a predictable manner. As shown by the conditionally averaged structures in the wake of a two-dimensional body with a backward D cross-section (Davies, 1974), the near-wake flow can be accurately described as a sequence of alternately signed turbulent vortices advecting downstream. In several studies of conditionally averaged flow fields in the near wake (Cantwell and Coles, 1983; Perry, 1984a;1984b; Perry and Steiner, 1987) it was found that the most important part of these vortices is the saddle points in between them. In these regions, both turbulence transport and production are a maximum. Others studies have associated these saddle-point regions with braid-like structures of $\omega_z$ interconnecting the larger nominally two-dimensional spanwise vortices. Flow visualization studies by Williamson (1989), have shown that the onset of three-dimensionality in laminar two-dimensional vortices is associated with these braids. Similarly, Hussain and Hayakawa (1987) showed, using conditional averages of near-wake vorticity, that the large rollers connected by braids is the predominant organization even at high $Re_d$ (of the order of $1.3 \times 10^4$). Antonia, Brown, and Bisset (1987) showed the existence of these structures much farther downstream.
of the cylinder. At $x/d = 400$, they found from two-dimensional conditional averages, a distribution of saddle-points in the $x - y$ plane. The saddle-points appeared in a staggered configuration across the wake centerline.

As the wake develops in the downstream direction, the structures initially generated at the body undergo some reorganization and quickly disappear. No studies of turbulent far-wakes have shown convincing evidence of large two-dimensional structures or organized braid-like structures. Cimbala, Nagib, and Roshko (1988) showed that the initial vortex street cannot be visualized beyond $x/d = 50$. This agrees with the results of Kovasznay (1949) and Roshko (1953). To study this decay of the near-wake structures in detail, Hayakawa and Hussain (1989) extended their studies of the evolution of rollers and ribs (braids) in the intermediate region of the wake, $20 < x/d < 40$. They observed that even near the cylinder, the spanwise extent of the large rollers was very small. The correlation function decayed to 40% by 0.95$b$ and 0.3$b$ at $x/d=20$ and 40 respectively. By considering the $\omega_y$ distributions, they found pairs of oppositely signed vorticity, which looked like counter-rotating eddies or kinks in the spanwise rollers. The lack of periodicity in the spanwise kinks that would be expected to accompany the growth of spanwise instabilities led the authors to conclude that spanwise instability could not be the driving force behind the onset of three-dimensionality in the near-wake vortices.

The results of these studies describe the flow characteristics in the formation region of the turbulent wake. They are important to the far-wake studies because they describe the initial conditions for the far-wake. If the far-wake structures are left
over from the decay of the initial vortex street, it is expected that far-wake structures
should resemble those generated by the breakdown of the near-wake vortices.

2.2 Far-Wake Structures

Far downstream of the initial vortex street decay, the flow takes on a significantly
different character. Beyond $x/d=50$ there is no sign of the vortex street in the velocity
spectra or in flow visualizations. New structures dominate the flow and some new
mechanisms develop to sustain the turbulence.

From a statistical standpoint, the turbulent wake has received a great deal of at­
tention in the literature. Although the statistical approach does not provide much
information about flow structures, it serves as a method to evaluate phenomenolog­
ical studies, to guide pattern recognition studies, and to aid in coherent structure
interpretation.

The methodology for traditional statistical analysis of the turbulent far-wake can
be found in Schlichting (1955). This statistical analysis will be outlined in this study
as an introduction to relevant equations and parameters governing the turbulent far­
wake. Figure 1 illustrates the turbulent wake problem and some parameters. The
flow is assumed to be developing very slowly in $x$ so that the components of velocity
can be expressed

\[
\begin{align*}
\bar{u}(x,y,z) & = U(x,y) + u(x,y,z) \\
\bar{v}(x,y,z) & = V(x,y) + v(x,y,z) \\
\bar{w}(x,y,z) & = w(x,y,z)
\end{align*}
\] (2.1)
The local defect velocity $U_1$ is defined by

$$U_1(x, y) = U_\infty - U(x, y)$$ (2.2)

Tennekes and Lumley (1972) define two velocity scales, the freestream velocity $U_\infty$ and the maximum velocity defect $U_{1\text{max}}$. Far downstream, the defect velocity is very small and $U(x, y)$ is of the order of $U_\infty$. The cross-stream length scale of the mean flow $L_y$, which is related to the mixing length, is assumed proportional to the wake width $b$. The streamwise length scale $L_x$ of the mean flow is much larger than the cross-stream scales so $\partial/\partial y \gg \partial/\partial x$. From the continuity equation, it can be seen that $V \ll U$. Based on the scaling above, and assuming that molecular viscosity is negligible when compared to the turbulence effects, the steady streamwise momentum
The equation can be reduced to

\[ U_\infty \frac{\partial U}{\partial x} = -\frac{\partial (\overline{uv})}{\partial y} \quad (2.3) \]

\[ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (2.4) \]

The growth of the length scale and velocity scale can be approximated using Prantl’s mixing length theory by which the growth of the wake thickness is related to the cross-stream velocity fluctuations, i.e.

\[ \frac{Db}{Dt} = U \frac{\partial b}{\partial x} + V \frac{\partial b}{\partial y} = U_\infty \frac{db}{dx} \sim v \sim L_y \frac{\partial U}{\partial y} \quad (2.5) \]

If the mean value of \( \partial U/\partial y \) is assumed to scale as \( U_{1max}/b \), the growth of the wake is found to be proportional to the velocity defect,

\[ \frac{db}{dx} \sim \frac{L_y U_{1max}}{bU_\infty} \quad (2.6) \]

The drag per unit length of the cylinder,

\[ D = \theta^* \rho U_\infty^2 \quad (2.7) \]

is related to the momentum thickness of the wake,

\[ \theta^* = \int_{-\infty}^{\infty} \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty}\right) dy \quad (2.8) \]

For very small defect wakes, Equation 2.8 can be approximated by

\[ \theta^* \sim \frac{U_{1max}}{U_\infty} b \quad (2.9) \]

The drag is then proportional to the velocity defect

\[ \frac{U_{1max}}{U_\infty} \approx \frac{C_D}{2b} \quad (2.10) \]
and using Equation 2.6 and the relationship \( L_y/b = \tilde{\beta} \), a differential equation for the wake growth rate is obtained

\[
2b \frac{db}{dx} \sim \tilde{\beta} C_D \tag{2.11}
\]

Solving this equation shows that the growth of the wake width and the decay of the velocity scales scale with the square root of the downstream distance,

\[
b \sim x^{1/2} \tag{2.12}
\]

\[
U_{1\text{max}} \sim x^{-1/2} \tag{2.13}
\]

Using an eddy viscosity model, the Reynolds stress can be expressed as

\[
-\rho \overline{u'v'} = -\rho \varepsilon_0 \frac{\partial U}{\partial y} \tag{2.14}
\]

which when substituted into Equation 2.3 provides a differential equation for the streamwise velocity component:

\[
U_\infty \frac{\partial U_1}{\partial x} = \varepsilon_0 \frac{\partial^2 U}{\partial y^2} \tag{2.15}
\]

This problem can be solved using a similarity solution with

\[
\eta = y \sqrt{\frac{U_\infty}{\varepsilon_0 x}} \tag{2.16}
\]

\[
U_1(x, y) = U_\infty C \left( \frac{x}{L_y} \right)^{-1/2} g(\eta) \tag{2.17}
\]

Substituting into Equation 2.15 a linear differential equation is obtained,

\[
g'' + \frac{1}{2} \eta g' + \frac{1}{2} g = 0 \tag{2.18}
\]
It has the solution

\[ g(\eta) = \exp\left(-\frac{1}{4}\eta^2\right) \]

\[ U_1(x, y) = U_\infty C \left(\frac{x}{d}\right)^{1/2} \exp\left(-\frac{1}{4}\eta^2\right) \]  
\[ C = \frac{C_D}{4\sqrt{\pi}} \left[\frac{U_\infty d}{\varepsilon_0}\right] \]

The Reynolds stress can be found directly from

\[ g_{12}(\eta) = \frac{-\bar{u}v}{\sigma_u^2} = \frac{\varepsilon_0}{\sigma_u^2} \frac{\partial U}{\partial y} \]  

(2.20)

Wygnanski, Champagne, and Marasli (1986) suggested a slightly different exponential distribution,

\[ U_1(\eta) = U_\infty - U_{1\max} \exp(-0.637\eta^2 - 0.056\eta^4) \]  

(2.21)

The \( \eta^4 \) term allows the accurate prediction of the mean velocity near the wake edge which would otherwise be overestimated.

The term \textit{far}, as used in the previous discussion, is a relative term. Traditionally, \textit{far} indicates the region downstream of the generating body beyond where the velocity statistics, properly normalized by the local scales, are functions of the inhomogeneous direction only. At this point, the flow is considered self-similar. When determining if self-similarity is reached in the turbulent wake, the choice of the criteria used is important, since each one implies something about the dynamics of the flow. If the wake becomes self-similar in some way, then any descriptions or processes applicable at one downstream location are applicable at all locations. If the statistical description...
changes with downstream location it is possible that the turbulence mechanism is also changing.

Self-similarity is reached at different downstream locations for different velocity statistics (Sreenivasan, 1981; Sreenivasan and Narasimha, 1982; Narasimha and Prabhu, 1972). The mean profile becomes self-similar fairly quickly, before \( x/d = 100 \). The self-similarity of the turbulence intensity profile is evaluated by the relative difference between wake center-line value and the maximum value. This measure does not approach a constant value until \( x/d \approx 400 \). Another measure of self-similarity is the invariance of the shape of turbulence intensity profile with streamwise distance from the cylinder. The profile shape gradually changes until far downstream, particularly in the tails of the profile (Uberoi and Freymuth, 1969).

It is not clear whether the wake ever reaches a fully developed state that is independent of the laboratory conditions and the wake generating body. However, it appears that beyond \( x/d = 100 \) the role of the initial vortex street is negligible, the mean profile is established, the Reynolds stress profiles are approaching self-similarity, and the turbulence velocity scales have decayed such that they are small relative to the mean velocities (less than 10%). The turbulence mechanisms beyond this initial region, \( x/d > 100 \), have been studied in the literature, for the most part, in two ways, descriptively based on observations and basic measurements, and in terms of the stability properties of the mean profile.

Townsend (1956) and Grant (1957) provided the benchmarks for far-wake phenomenological studies. Grant, after noting the structure in the transverse velocity
correlations measured by Townsend, extended Townsend's measurements to include spanwise and normal velocity component correlations at a downstream location of $x/d=533$ and $Re_d=1300$. Based on the structure of the space-time correlation tensor, Grant proposed that the far-wake contained pairs of counter-rotating eddies, rotating in the plane of the wake with axes inclined with the mean shear as shown in Figure 2.

The existence of these structures is supported by other studies designed to extract the dominant structures from the correlation functions. A method for mathematically extracting the dominant eddy structure from correlation data was proposed by Lumley (1965), and termed *proper orthogonal decomposition* (POD). An application
of Karhunen-Loeve decomposition (Loeve, 1955), the coherent structures are defined as the eigenvectors of the correlation tensor. Payne (1966) and Payne and Lumley (1967) used the correlation data of Grant to compute the first POD mode in the turbulent wake. The suggested structure was similar to the double-rollers originally proposed by Grant, but was calculated in a more mathematically rigorous way. Aubry (1992) computed the first three POD modes using the correlation data measured in the present study, and found the first mode to be identical to Payne and Lumley's double-rollers.

Grant's large eddy model has no $v$ component of velocity. The $R_{uu}$ distribution was attributed to the action of separate structures, groups of jets, moving fluid away from the center of the wake as shown in Figure 2. The jets were believed to be the result of a stress relief mechanism similar to that found in strained homogeneous turbulence (Batchelor and Proudman, 1954). If a bulge occurs at the interface between the turbulent wake core and the irrotational mean flow, the turbulent stresses will act as a nonuniform pressure distribution along the boundary, and low speed turbulent fluid will tend to be ejected into the high speed irrotational mean flow. The ejecting action also acts to further contort the turbulent boundary and create additional bulges (on both sides the wake) which in turn develop more jets.

A similar analysis (Townsend, 1956;1966) predicts the jet structures based on the stability of turbulent-irrotational interface. If a bump were to appear in the interface, pressure variations across the bump would cause it to grow as a Kelvin-Helmholtz type of instability. Townsend argues that the action of the Reynolds stress and energy
exchange among the turbulent scales gives the turbulent portion some visco-elastic properties. This property makes it possible for oscillations to occur in the interface bumps. The effect is that energy can be stored in the large turbulent motions which can then be returned to the mean flow. From a detailed analysis, Townsend concludes that such oscillations can account for the spacing and numbers of jets within the jet groups observed in experiments. He also concludes that the Kelvin-Helmholtz instabilities at the interface are strongly amplified and mean velocity gradients should not change the basic characteristic of this mechanism.

Townsend (1970; 1978) suggested that the growth of jet groups is the main entrainment mechanism in the wake. He described the process as a cycle where bulges in the turbulent interface grow and develop into jet groups. They then roll over and engulf irrotational fluid. In this way, energy is extracted from the mean flow causing the wake width to grow.

In this type of model, the turbulence-irrotational interface plays the dominant role in the entrainment process. LaRue and Libby (1974; 1976; 1978) used temperature as a passive scalar to study entrainment at the interface. Using two cold-wires, they were able to describe the shape of typical turbulent bulges. No periodicity or grouping patterns were seen in the distribution of the bulges. Details of the interface angles show a much steeper trailing edge than leading edge, 132° and 66° respectively, and both have relatively large standard deviations, 23° and 28°. It was found that the downstream edge contains overhangs roughly 25% of the time indicating that engulfment of irrotational fluid is a frequent and significant mechanism of entrainment.
The spanwise distribution of bulges was studied by Barsoum, Kawai, Keffer (1978) in the wake of a slightly heated circular cylinder at $Re_d=4200$ and $x/d=96$. They measured the space-time correlations for intermittency and temperature and concluded that the spanwise scale of bulges was roughly half of its streamwise scale. No periodicity was found in either the spanwise or streamwise directions indicating a random distribution of very narrow, strongly three-dimensional bulges.

Keffer (1965) investigated the cross-wake distribution of jets in a strained turbulent wake. He found jets similar to those seen by Grant but could not find any evidence of a preferred distribution of eddies across the wake center. He also found that the increased strain resulted in the appearance of additional jet groups. This observation supports the idea that jets are stress relieving motions in the wake.

The relationship between jets and cross-wake rollers was not made clear in these studies. To further study the rollers, Townsend (1978) studied a wake at $Re_d=8000$ and $x/d=170$ by looking for specific patterns in experimental data. Time histories of streamwise velocities measured with a hot-wire rake, aligned with either the inhomogeneous or spanwise direction, were processed using a pattern recognition technique. Perturbation velocity patterns consistent with double-roller eddies were compared to the experimental data to search for matches with the pattern. Matches were found with patterns existing in the experimental data and between 15% and 20% of the turbulent energy was contained in these matched eddies.

A similar study was conducted by Mumford (1983) at $x/d=178$ and $Re_d=7000$ to find turbulent structures and examine their distributions. Using an eight probe rake,
time histories of the streamwise velocity component were measured and compared to the two-dimensional exponential velocity pattern given by Grant (1957). Pairs of rotating eddies consistent with retardation of the flow between the rollers was found to be more dominant than eddies rotating in the opposite direction. They also noted that approximately 30% of the eddies extend across the wake centerline and that they sometimes appear in small clusters.

A series of pattern recognition studies (Ferrè and Giralt, 1989; Ferrè, Mumford, Savill, and Giralt, 1990) were conducted to study the distribution and roles of large eddies in the turbulent wake. The procedure was designed to recognize repetitive trends from any type of stochastic signal. They found that in several types of wakes, the double-rollers are the most dominant structure. These eddies were found to contain up to 40% of the turbulent energy and it was hypothesized that entrainment in the wake occurs primarily between successive pairs of these rollers. The large percentage of captured energy was attributed to the nature of the pattern recognition technique.

Wlezien (1981) conducted a detailed flow visualization and pattern recognition study of a turbulent wake at $Re_d=5000$. Flow visualizations were generated using the smoke-wire visualization technique to illustrate the turbulent wake development from 10 to 180 diameters downstream of the cylinder. The study clearly showed that the disappearance of the Von Karman vortex street was the result of a break-down process rather than one of decay. The two-dimensional vortices were replaced by a field of turbulence with bulges extending into the irrotational fluid. These observations were
compared to reconstructed low wave-number velocity fields obtained from a rake of hot wires. At $x/d=10$, the results showed a distorted turbulent vortex street. This pattern decayed rapidly and although at $x/d=40$ what looked like a quasi-period vortex street could be seen, the onset of three-dimensionality makes this vortex-street interpretation very unlikely. Conditionally averaged streamwise velocity fields showed periodic structures in the near-wake and isolated structures beyond $x/d=40$.

Despite the extensive studies that describe the character of far-wake eddies, the mechanisms of wake turbulence have not been clearly illustrated. The description of the far-wake as a sequence of turbulent jets relieving the internal stresses of the wake core is an attractive view of the turbulence mechanism. However, it does not address the role of counter-rotating eddies, there is no clear connection between jets and other turbulent scales, and no approach has been found to accurately predict the shape and distribution of turbulent interface bulges. An alternative approach to the analysis of the far-wake is to investigate the stability characteristics of the mean velocity profile.

2.3 Far-Wake Stability

Hollingdale (1940) was the first to study the instability of small deficit wake with a Gaussian mean velocity profile using an experimental approach. He treated the far-wake as an inviscid parallel flow problem, to which linear instability calculations are well suited. Hollingdale attempted to match the results of his calculations with experimentally measured stability characteristics. Although his results were not conclusive, his work was the prototype for a myriad of studies that followed over the next 50 years.
Taneda (1959) was the first to attribute the growth and decay of interface bulges and large eddies in the turbulent wake to the changing stability characteristics of the mean velocity profile. As the wake develops in the streamwise direction, the turbulence diffuses into the irrotational bounding flow which is associated with an increase in the wake width and a decrease in the wake defect. As found from linear stability theory, this development causes a shift in the unstable eigenvalues to progressively lower frequencies. Therefore, disturbances of a given wave-length are only amplified in a small downstream region of the wake. They grow in that region and then decay beyond it while other disturbances are amplified.

Sato and Kuriki (1961) and Sato and Saito (1975; 1978) studied the stability properties of a laminar wake by introducing acoustic disturbances and tracking their growth and shape as they developed downstream. Comparisons with theoretical calculations indicated that the instability of the laminar wake can be predicted by linear inviscid stability calculations. The shape of the most amplified mode agreed with that of the calculated antisymmetric (in $v$), Orr-Sommerfeld mode. However, some asymmetry in the inhomogeneous direction was observed in the turbulence intensity profile. The cause of this asymmetry was unclear. Wygnanski et al. (1986) attributed it to contributions of the symmetric mode, while others attributed it to asymmetric secondary sub-harmonic instabilities (Flemming, 1987; Corke, Ghassemi, and Krull, 1990).

Disturbances in the far-wake can be treated as growing temporally or spatially. Mattingly and Criminale (1972) compared spatial and temporal stability results.
They concluded that where growth rates are large, in the near-wake, spatial calculations must be used. In the far-wake, temporal calculations can be used and the group velocity can be used to express an equivalence with spatial calculations (Gaster, 1962).

Cimbala, Nagib, and Roshko (1988) conducted a series of turbulent wake experiments and found support for Taneda’s original hypothesis that wake instabilities must be the driving mechanism in wake turbulence. Using smoke-wire flow-visualizations, large scale antisymmetric structures were observed in the wake of a cylinder at $Re_d=2200$ and $x/d=200$. They appeared in groups and were described as a secondary vortex street arising from the mean flow instability. The instability mechanism was more apparent when the wake of a porous plate was examined. Because the near-wake flow contained no spanwise vortices, the second vortex street could not have been a remnant of the near-wake structure. Additional evidence for the stability mechanism was found in the exponential growth and decay of disturbances. At $Re_d=150$, frequencies other than the shedding frequency were found to grow and decay in a manner consistent with linear stability theory. Although the sub-harmonic of the dominant frequency was not observed in the wake at high Reynolds numbers, at low Reynolds numbers the wake developed three-dimensional cells which were attributed to a sub-harmonic resonance with oblique and two-dimensional modes.

Wygnanski, Champagne, and Marasli (1986) looked for evidence of the stability mechanism in both forced and un-forced turbulent wakes. Spectral analysis of the streamwise velocity slightly outside the wake ($\eta = 3$) showed a peak in the velocity
spectrum near the most amplified two-dimensional stability mode as computed from
the Rayleigh equation. The response of the wake to external forcing was examined by
introducing a sinuous disturbance at the most amplified frequency for the downstream
region of interest. The phase locked distribution of $u^2$ agreed roughly with the most
unstable Rayleigh mode but with a slight asymmetry in $y$ similar to that observed
by Sato (1961). They attributed the asymmetry to the effects caused by the probe
holder and the symmetric instability mode. Natural instability modes were examined
by band pass filtering the velocity signal. No asymmetry was found and there were
large differences with the forced instability mode. The symmetric mode was again
cited for the discrepancies and the possibility of secondary instabilities suggested.
Flow visualization supported the idea that symmetric modes which are unstable but
less amplified than the antisymmetric modes were present.

The results of all of these studies previously cited agreed closely with theoret­
ical predictions, however, growth rates determined by experimental methods are
consistently smaller than those predicted by inviscid theory. In a continuation of
their earlier work, Marasli, Champagne, and Wygnanski (1991) found that by using
an eddy viscosity model and viscous stability calculations they could obtain better
agreement with experimental results than has been obtained by other calculations.
They used $sech^{1/2}(A\eta + B\eta^3)$ as a mean profile and similarity arguments to eval­
uate the eddy viscosity $\varepsilon_0$. They found $\varepsilon_0 = 5.0 \times 10^{-4} m^2/s^2$ giving an effective
$Re_{\theta^*} = U_\infty \theta^*/\varepsilon_0 = 31$. They noted that for a Gaussian profile, $\varepsilon_0$ is a constant
across the wake, which makes the eddy viscosity model reasonably accurate. The
damping effect of the eddy viscosity can help explain the over-estimated growth rates of earlier viscous stability calculations.

In most of these studies, the unstable modes were externally forced. The effects of this forcing was tested by Marasli, Champagne, and Wygnanski (1992). The wake half width was considered as a function of forcing amplitude and downstream position. At a fixed frequency of forcing, the non-dimensional wake length scale grew with downstream distance up to a critical location which corresponded to a local Strouhal number of 0.22. This was followed by a relaxation of the wake to a smaller scale and a slower wake growth rate than would be found if the wake was unforced. Before the critical location, the coherent Reynolds stress was significantly larger than it would have been in the unforced case. After this point, the stress dropped and eventually changed sign. This indicated a change in the direction of energy transfer from the mean to the perturbation, then from the perturbation to the mean, or in other words, a change from an amplified wave to a damped wave. Far downstream, no return to the original profile was observed, however this would be expected if the far-wake structure was independent of its upstream conditions. Because the wake did not return to its natural behavior, the character of the wake must have been changed by the high amplitude forcing. This raises doubts as to the applicability of forced wake results to the unforced turbulent wake because in the forced case the flow has been fundamentally changed.

In a study of a shear layer, Gaster, Kit, and Wygnanski (1985) introduced a two-dimensional disturbance into a shear layer and its growth was observed. The
results were found to agree with stability calculations provided that the correct mean profile was used. If fact, the results were very sensitive to subtleties in the mean velocity profile. This illustrates that care must be taken when computing eigenmodes from stability calculations for predictions of experimental modes. This also suggests possible mechanisms for the asymmetry observed by Sato (1961) and for the onset of three dimensionality. It has been observed in studies of axisymmetric jets that it is possible to strongly modify the mean profile by introducing groups of helical waves with the appropriate phasing (Strange and Crighton, 1985; Cohen and Wygnanski, 1987; Long and Petersen, 1992). Similar distortions have been observed in shear layers by Oster and Wygnanski (1982). It is likely that the introduction of disturbances, natural or otherwise, has a similar nonlinear effect in the wake.

The importance of secondary instabilities in the transition process of shear flows has been demonstrated in a number of stability studies (Kelly, 1967; Klebanoff, Tidstrom, Sargent, 1962; Craik, 1971; Herbert, 1983; 1984; 1988). As the primary instability mode grows and reaches a saturated state, the mean flow must be redefined for further stability analysis. It is most accurately described as a mean flow with a periodic component superimposed. This flow has its own stability characteristics. Harmonic resonances are the most likely mechanism by which secondary disturbances grow. In particular, the sub-harmonic resonance is believed to be dominant as has been found in plain Poiseuille flow and in boundary layers.

In a numerical study, Flemming (1987) computed the secondary instability characteristics for a Gaussian mean profile with an antisymmetric two-dimensional periodic
disturbance superimposed. He found that with a large amplitude primary mode, the most unstable secondary mode was an oblique wave with a shape having some asymmetric properties in the inhomogeneous direction. He suggested that the asymmetries found in the computed mode shapes could account for those measured by Sato and others.

Corke, Krull, and Ghassemi (1990;1992) performed experiments to investigate whether or not the secondary instability calculation could predict the mode shapes observed in a forced wake. Two dimensional and oblique wave disturbances were introduced into the wake of a two-dimensional streamlined body. The oblique waves were always introduced as in-phase pairs at a frequency exactly equal to the first sub-harmonic of the two-dimensional disturbance. This forcing configuration was consistent with the mode triad found by Craik (1971) in the transition of a boundary layer. They found that the resulting resonant mode was in good agreement with both the computational results of Flemming (1987) and the previous experimental results of Sato and Kuriki (1961). This suggests that the sub-harmonic secondary instability cause the onset of three-dimensionality in the laminar wake.

From the information presented in this review of the literature, a picture of the development and sustenance in the turbulent wake has been developed. In the near-wake, two-dimensional vortices are formed from the roll-up and interaction between the two separated shear layers. These near-wake structures rapidly deteriorate due to the onset of three-dimensionality. After a period of reorganization, the turbulent self-similar wake develops. From a coherent structure viewpoint, the far-wake resembles a
distribution of double-rollers and/or jets, that may be remnants of near-wake vortices or may be generated out of the far-wake instabilities. The receptivity of the far-wake to perturbations is fairly accurately predicted by viscous linear stability calculations using an eddy viscosity model. It has not been shown that stability modes exist in unforced turbulent wakes or that the double-roller eddies can be explained in terms of instability modes.
CHAPTER III

METHODOLOGY

3.1 Experimental Approach

The experiments described in this section were designed to document the characteristics of a turbulent wake at $Re_d=5080$. This $Re_d$ was selected as a typical value, consistent with similar published research. It is in the $Re_d$ regime associated with laminar separation at the cylinder and fully turbulent vortex street generation. It is also an appropriate value given the facility and equipment limitations. The variables that were measured are the mean and perturbation velocities. Single-point measurements were made to investigate the variation of statistical properties with the three coordinate directions. Multiple-point measurements were made to look at the cross-variable statistics as functions of spatial separation. The data reduction procedures were designed to extract physical variables from measured data and compute statistical information.

3.1.1 Facilities

All of the experiments described here were conducted in the subsonic wind-tunnel at the Ohio State University, Department of Mechanical Engineering. The tunnel, illustrated in Figure 3, was an open-return type driven by a 20HP DC motor. The
inlet consisted of a bell-mouth followed by a plastic-tube honey-comb, three steel wire screens, and a settling chamber. A 9 to 1 contraction lead to the test section. An optimal contraction profile and careful fairing of tunnel seams helped eliminate any flow separation. Beyond the test section, a diffuser lead to the axial fan. The fan and motor speed were controlled by an accurate feedback control system capable of maintaining the fan speed to within 0.1% of the desired value. The freestream r.m.s. turbulence level, as computed with no low frequency cut-off, was approximately 0.2% over the velocity range 4.5m/s to 20.0m/s. A low frequency (less than 1Hz) component in the mean flow rate effected the flow measurements. This fluctuation added 0.2% to the uncertainty in freestream velocity measurements at the lowest speed increasing
to 0.4% at the highest speed. Because the drift was a much slower process than the slowest time-scales in the turbulent flow of interest, this component of noise could be removed with digital filtering techniques.

The test section had the internal dimensions 0.46m wide by 0.61m high by 3.66m long. As shown in Figure 4, holes were drilled in the tunnel walls so that cylinders could be mounted horizontally in the test section. The walls were constructed of Plexiglass and could be removed to gain access to the experimental set-ups.

Two precision traversing mechanisms were mounted on tracks on the top of the test section. Both accessed the inside of the tunnel by 1.27cm diameter rods that passed through a slot in the top of the test section. The slot was sealed by rubber flanges to prevent leakage. The upstream traversing mechanism moved its support rod vertically using a lead screw arrangement attached to a stepper-motor. Cross-stream motion was afforded by a small sliding carriage on a track mounted horizontally inside the test section at the end of the support rod. A drum driven cable was attached to the carriage and controlled by another stepper motor. The second traversing mechanism accessed the tunnel 25cm downstream of the first. It was only capable of vertical motion using a lead screw mechanism. Additional access to the test section was achieved using fixed supports mounted through holes in the bottom of the test section.

A Masscomp 55-S-05 computer with a 12 bit, 16 channel A/D interface and two D/A channels were used to control the traversing mechanisms and to acquire data from various instruments. The A/D interface was capable of acquiring data at a rate
Figure 4: Test-section schematic
of 333kHz, but the maximum data sampling rate in these experiments was 10kHz. Fourth order Butterworth analogue filters with a cut-off frequency of 6.2kHz were used as anti-aliasing filters for the data acquisition system. Sampling of multiple channels was conducted in bursts at the maximum sampling rate resulting in a delay between channel samples of 3.33 microseconds. The resolution of both the A/D and the D/A systems was 4.88mV per bit which compares to a typical r.m.s. of a fluctuating quantity being measured of 0.24V. Measurements sampled from the A/D board were temporarily stored on the Masscomp’s hard disk drive until they could be transferred to another system for tape storage and processing. The Masscomp was strictly dedicated to data acquisition and no analysis was conducted using it. Data reduction was conducted using a DEC 3100 and Silicon Graphics workstations.

The tunnel was instrumented with monitoring equipment to determine the testing conditions. The freestream velocity was measured using a pitot tube and a Validyne pressure transducer, Model P304D-1-N-1-22-S4. The temperature was measured using a linear thermistor array connected to an electronic thermometer. A rack of specially made constant temperature anemometers were mounted with the tunnel instruments. Sixteen independent circuits were available, each with its own variable overheat setting, frequency compensation, and analogue pre-linearizer. The signal-to-noise ratio was found to be greater than 500 for the anemometer/linearizer system.

Three triple-wire probes were used to take flow measurements. They can be described in terms of the physical parameters which are defined in Figure 5. The geometrical information for two identical probes manufactured by Auspex Corp. is
Figure 5: Hot-wire geometrical parameters
$d_w = 5 \mu m$, $l_w = 1.3 mm$, $\theta_w = 45.0^\circ$, and $\Omega_w = 0.0, 120.0, \text{ and } 240.0$ for wires 1, 2, and 3 respectively. The third probe was manufactured by TSI inc. with the following geometrical parameters $d_w = 51 \mu m$, $l_w = 1.67 mm$, $\theta_w = 45.0^\circ$, and $\Omega_w = 0.0, 180.0, \text{ and } 90.0$ for wires 1, 2, and 3 respectively.

Two cylinder models were used in the wake experiments, Figure 3.1.1. The first was a 0.635 cm diameter steel rod spanning the width of the tunnel test section. It mounted in the holes marked A in Figure 4. End plates were attached to the cylinder near the tunnel walls with a $10^\circ$ angle to the mean flow. When installed, the distance between the endplates was 35 cm, giving a length $L$ to diameter $d$ ratio of approximately 55. The second model was a 1.27 cm diameter hollow brass rod which was mounted in the holes marked B. The end-plates for this model were attached parallel to the mean flow and the length to diameter ratio was approximately 27. Only one model was installed at any given time.

The coordinate system in the test section has its origin at the center of the cylinder model. The streamwise direction is $x$, the vertical direction is $y$, and $z$ is the spanwise direction completing the orthonormal set.

### 3.1.2 Experiments

All of the experiments were conducted in the wake of the circular cylinder models at a constant freestream velocity. The instrumentation always consisted of the thermistor measuring the air temperature within the test section, the pitot tube measuring the freestream velocity, and one or more hot-wire probes in the turbulent wake mounted on the traversing mechanisms and supports. The outputs of these instruments were
Figure 6: Cylinder models, with end-plates
directly connected to the A/D channels of the Masscomp computer so that they could be digitally sampled. Each experiment can be classified by the freestream velocity, wake generating model, probes used, probe locations in the turbulent wake, number of digital samples \( N_s \), and digital sampling frequency \( f_s \).

The simplest of the experiments was the single-point measurements. A single probe was attached to the \( y - z \) traversing mechanism. It was positioned at some location in the wake, and the anemometer outputs, freestream velocity, and temperature were sampled at a constant rate. The consecutive realizations of the unprocessed variables were collected and these time-histories were stored on computer disk for later processing. The probe was repositioned and the sampling was repeated. The positions where measurements were taken are referred to as the experimental grid. The single point experiments are summarized in Table 1. Multiple-point experiments are an extension of the single-point experiments which were designed to measure spatially separated cross-variable statistics. One probe was mounted to the \( y - z \) traversing mechanism and moved over an experimental grid. Additional probes were mounted on the second traversing mechanism and/or to rigid supports. Data was recorded for each of the desired probe spacings. The multiple-probe experiments are also summarized in Table 1.

Before, after, and occasionally during each of the experiments, the hot-wires were calibrated against the freestream velocity to provide a function relating the anemometer output to cooling velocity \( Q \). This was conducted by placing the hot-wire probe in the freestream well out of the turbulent wake and recording the output of the
Table 1: Summary of experiments

<table>
<thead>
<tr>
<th>Exp</th>
<th>Model</th>
<th>$U_\infty$ (m/s)</th>
<th>$N_s$</th>
<th>$f_s$ (kHz)</th>
<th>Probe</th>
<th>Grid</th>
</tr>
</thead>
</table>
| 1   | 1     | 11.6            | 66k   | 10         | Auspex #1 | $x/d=50,(100i, i=1,4)$  
|     |       |                 |       |            |       | $y/d=-16.0+0.508j, j=0.63$  
|     |       |                 |       |            |       | $z/d=0.0$ |
| 2   | 1     | 11.6            | 66k   | 10         | Auspex #2 | $x/d=50,(100i, i=1,4)$  
|     |       |                 |       |            |       | $y/d=-16.0+0.508j, j=0.63$  
|     |       |                 |       |            |       | $z/d=0.0$ |
| 3   | 1     | 11.6            | 66k   | 10         | Auspex #1 | $x/d=100$  
|     |       |                 |       |            |       | $y/d=0.0, z/d=0.0$  
|     |       |                 |       |            |       | Auspex #2 | $x/d=100+1.2i, i=0.16$  
|     |       |                 |       |            |       | $y/d=0.0, z/d=1.2$ |
| 4   | 1     | 11.6            | 66k   | 10         | Auspex #1 | $x/d=100, z/d=0$  
|     |       |                 |       |            |       | $y/d=-9.60+1.20i, i=0.16$  
|     |       |                 |       |            |       | Auspex #2 | $x/d=101.2, z/d=1.2$  
|     |       |                 |       |            |       | $y/d=-9.60+1.20i, i=0.16$ |
| 5   | 1     | 11.7            | 66k   | 10         | Auspex #1 | $x/d=100, z/d=0.0$  
|     |       |                 |       |            |       | $y/d=-9.60+1.20j, j=0.16$  
|     |       |                 |       |            |       | $z/d=1.20k, k=0.8$ |
| 6   | 2     | 5.24            | 66k   | 10         | Auspex #1 | $x/d=16$  
|     |       |                 |       |            |       | $y/d=-0.60+0.60i, i=0.9$  
|     |       |                 |       |            |       | $z/d=0.60j, j=0.14$  
|     |       |                 |       |            |       | Auspex #2 | $y/d=-1.20, z/d=0.0$ |
| 7   | 2†    | 5.24            | 66k   | 10         | Auspex #1 | $x/d=16$  
|     |       |                 |       |            |       | $y/d=-0.60+0.60i, i=0.9$  
|     |       |                 |       |            |       | $z/d=0.60j, j=0.14$  
|     |       |                 |       |            |       | Auspex #2 | $y/d=-1.20, z/d=0.0$ |
| 8   | 1     | 11.7            | 66k   | 10         | Auspex #1 | $x/d=100$  
|     |       |                 |       |            |       | $y/d=-9.60+1.2i, i=0.16$  
|     |       |                 |       |            |       | $z/d=2.4j, j=0.4$  
|     |       |                 |       |            |       | Auspex #2 | $x/d=100, z/d=0$  
|     |       |                 |       |            |       | $y/d=(0.2.4)$  
|     |       |                 |       |            |       | TSI | $y/d=-2.4, z/d=0.0$ |

† end plates removed from the model
anemometers as the freestream was varied over a range of velocities commensurate with the velocities in the wake.

### 3.1.3 Data Reduction

The only variables of interest in this study were the mean and perturbation velocities. The physical quantities measured from the experiments were the anemometer output voltages, temperature, and freestream velocity from the pitot tube.

Using the calibration data for a given experiment, the anemometer outputs could be converted to cooling velocities. A polynomial was fit to the calibration data, which typically had an error of less than one percent, and the fit was evaluated at every voltage allowed by the A/D converter, 4096 values. With all the voltage-velocity pairs pre-computed, the conversion could be conducted using a table lookup. Because the hot-wires were sensitive to both velocity and temperature fluctuations, the cooling velocities were corrected for drifts in the mean temperature before additional processing was conducted.

Extracting the velocity components from triple-wire probes is very complicated. The procedure used is given by Gieseke (1990) and Gieseke and Guezenne (1993) and makes use of variable transformations originally used by Lekakis (1988) and Lekakis, Adrian, and Jones (1989). Details of the procedure are quite lengthy and are discussed in depth by Gieseke and Guezenne (1993). The procedure can be broken into three parts: an angular calibration, solution tabulation, and table lookup.

The angular calibration was conducted by attaching the probe to a specialized positioning mechanism and tabulating the response of each wire in the hot-wire array
to a range of flow angles and velocities. For the calibration used in this study the range was, $-30^\circ < \phi < 30^\circ$, $-30^\circ < \psi < 30^\circ$, and $6m/s < U_\infty < 15m/s$. The variables $\phi$ and $\psi$ are the flow angles with respect to the probe axes. Each variable was divided into 5 levels resulting in a total of 125 calibration points. A response model, having the general form

$$Q_i \approx U_\infty f_i (\phi, \psi)$$

(3.1)

was fit to the data for each wire. Jorgensen's response model (Jorgensen, 1971) was used for Equation 3.1 because of its accuracy and wide use. Tabulations were made of cooling velocities and flow angles by evaluating Equation 3.1. Using these tabulations, variable manipulation, and inversion equations, physical velocities were determined from experimentally measured cooling velocities.

Single-point statistical information was computed using the extracted velocities. Means, deviations, and Reynolds stresses of the velocity components were computed in the standard way and will be presented in Chapter 4.1 as functions of the nondimensionalized coordinates for comparison with previously published results. Spectra of the velocity components were computed using an FFT algorithm. The autocorrelation function was used to estimate turbulent scales and to determine the ideal sampling rate for the various anemometry experiments. Joint probability density functions of velocities were used to estimate the contribution of different events to turbulence processes. Event scales and spacings in time were also computed.

Two-point statistics were computed from the multiple point experiments. Joint probability density functions were computed between various velocity statistics. The
velocity components, flow angles, and Reynolds stresses have been considered. By considering the joint probability densities of quantities separated spatially, the contribution to statistical properties by spatially separated events could be determined. The contribution to the Reynolds stress by events separated in time was also considered to determine preferred event spacings.

For this study, the most important two-point statistic was the correlation function. If we wish to compute the correlation between two variables, say $q_1(t)$ and $q_2(t)$, the correlation function is defined (Flannery, Teukolsky, and Vetterling, 1988),

$$R_{q_1q_2}(t) = \int_{-\infty}^{\infty} q_1(\tau + t)q_2(\tau) d\tau$$

(3.2)

Noting that this is related to the convolution,

$$q_1 * q_2 = \int_{-\infty}^{\infty} q_1(\tau)q_2(t - \tau) d\tau$$

(3.3)

and from the convolution theorem and the transform pairs,

$$q_1(t) \leftrightarrow \hat{q}_1(f), \quad q_2(t) \leftrightarrow \hat{q}_2(f), \quad q_1 * q_2 \leftrightarrow \hat{q}_1 \hat{q}_2$$

(3.4)

the correlation can be expressed as

$$R_{q_1q_2}(t) \leftrightarrow \hat{q}_1(f)\hat{q}_2^*(f)$$

(3.5)

To calculate the correlation between two signals, they are first transformed to the spectral domain using an FFT algorithm, the product in Equation 3.5 is evaluated, and then the inverse Fourier transform of that quantity is computed. The correlation function is computed for all possible time separations if the entire time record
is used in one calculation. By doing so, the correlations are computed for very large
time off-sets, much larger than the useful range. To avoid calculation of these unused
correlations, the original time records were broken into short blocks and the correla­tions were computed for each of the smaller data sets. All the results were averaged
together to compute the correlations for a restricted range of time leads and lags.

The space-time correlation tensor was computed using the data of Experiments 3
through 7 as described above. Most of the analysis will focus on the results of Ex­periment 5. In this case, the correlations were computed for each combination of
positions of the two probes. The first probe, termed the reference probe, was located
at the reference location, \( y_r \) and \( z_r \). The second probe, termed the mapping probe,
was located at the grid locations \( y \) and \( z \). A total of 1024 temporal separations were
considered for the computed correlations, which corresponded to dividing the original
data into 64 blocks. The experiments were designed such that a number of redun­
dant measurements would be taken. Four redundancies were present and they were
averaged together to reduce the uncertainty in the measurements. Considering the
redundancies and the original blocks of data, the final correlation function was the
average of 256 separate measurements and the complete functions were expressed

\[
R_{ij}(y_r|\tau, y, z)
\]  

(3.6)

The correlation functions of Equation 3.6 depend strongly on \( y_r \).

Experiment 8 was used solely for simultaneous time records of the velocity vectors
at three points in space. These records were used in the pseudo-dynamic reconstruc­
tion technique.
3.2 Stochastic Estimation

Conditional and phase averaging are useful means to estimate fully three-dimensional velocity fields from coarse spatial measurements. Stochastic estimation was originally proposed by Adrian (1976) to estimate conditionally averaged structures without having to pre-select an averaging condition. This work was followed by many related studies testing and applying the theory (Adrian, 1979; Adrian et al. 1987; Guezennec, 1989).

By expressing the conditionally averaged velocity field as a function of the conditions, and then determining the function using space-time velocity correlations, conditionally averaged structures could be determined accurately. The procedure is very robust and has been extended for use in more complicated applications. The procedure can be summarized by the statement, given velocities at some coarsely distributed points in space and time, the velocities at all other points in space and time can be estimated as functions of the known velocities. The procedure is an application of multiple regression analysis to the stochastic space and time dependent variables of turbulent flow.

3.2.1 General Formulation

The procedure is built from the premise, given a set of $N_s$ realizations of $(N + 1)$ stochastic variables, one of the variables $y$ can be estimated by some function of the others $(x_1, \cdots, x_n)$ having the form,

$$y \approx F(x_1, \cdots, x_n) = a_j f_j(x_1, \cdots, x_n)$$ \hspace{1cm} (3.7)
The basis functions $f_j$ and the number of them can be chosen arbitrarily. The functions evaluated at realization $i$ will be termed $f_{ji}$. The constants $a_j$ must be determined such that the function $F$ predicts $y$ as best as is possible on the available statistical sample from an error minimization standpoint. The total error is defined as the sum of the square of the difference between $F$ and $y$,

$$
\epsilon_T = \sum_{i=1}^{N_s} (y_i - a_j f_{ji})^2 \quad (3.8)
$$

For convenience and clarity, in all the equations which follow, the summations will be implicit, and the summation signs will be dropped. This error can be minimized with respect to each of the coefficients by taking the partial derivatives of $\epsilon_T$ with respect to each $a_j$ and setting the result equal to zero,

$$
\frac{\partial \epsilon_T}{\partial a_j} = (-2y_i f_{ji} + 2a_k f_{ki} f_{ji}) = 0 \quad (3.9)
$$

A set of simultaneous equations is generated that can be solved for the coefficients,

$$
a_k f_{ki} f_{ji} = y_i f_{ji} \quad (3.10)
$$

If it is assumed that $F$ is continuous and smooth, it can be expanded in a polynomial series. In its simplest form, the linear estimate can be expressed as

$$
F(x_1, \cdots, x_n) \approx a_0 + a_1 x_1 + \cdots + a_n x_n \quad (3.11)
$$

With this formulation, the simultaneous equations become,

$$
a_k x_{ki} x_{ji} = y_i x_{ji} \quad (3.12)
$$
The accuracy of the prediction can be evaluated by considering how much of the variance in $y$ is predicted by $F$, $\sigma^2_y$. By manipulating the preceding equations, this can be evaluated by computing

$$\sigma^2_y = \frac{(a_j x_{ji} y_i)}{N_s} \quad (3.13)$$

The effects of using various predicting variables on the accuracy of the estimation can be determined quantitatively using this formula. The improvement to an estimation afforded by increasing the number of estimating variables can also be determined by comparing the errors of the predictions.

If more than one variable is to be predicted by the same set of basis functions, the regression formula for each variable can be computed independently and the results combined to form a single matrix problem. If $P$ variables $y_p$ are to be predicted from $N$ other variables $x_n$ both measured at $N_s$ realizations, the regression equations can be written

$$y_{li} = a_{lj} x_{ji} \quad (3.14)$$

To evaluate the coefficients, the set of simultaneous equations

$$a_{lk} x_{ki} x_{ji} = y_{li} x_{ji} \quad (3.15)$$

must be solved. This can be expressed as a matrix problem by defining $L_{jk} = x_{ki} x_{ji}$ and $D_{jl} = y_{li} x_{ji}$ which yields

$$AL = D \quad (3.16)$$

This problem can be solved for all of the coefficients,

$$A = L^{-1}D \quad (3.17)$$
Once the coefficients are computed, the variables $y$ can be predicted from measured values of the variables $x$ using a matrix multiplication,

$$A x \approx y \quad (3.18)$$

### 3.2.2 Application to Turbulence Prediction

The variables under consideration in this study are the space and time dependent perturbation velocities $u(\vec{r}, \tau)$. Applications of the stochastic estimation procedure that make use of these variables can be used to extract information about the coherent structures in the turbulent wake.

The simplest application of the stochastic estimation is the estimated single-point conditional average. For this calculation, the three components of velocity are specified at a single point in space. This velocity vector is called the *condition* and noted $u_e$. Near this condition, the estimated flow field $u_e(\vec{r}, \tau)$ is the most likely state of the field given the condition. It can be considered the coherent structure associated with that condition and is constructed using the formulas

$$
\begin{align*}
    u_e(\vec{r}, \tau) &= a_{11}(\vec{r}, \tau) u + a_{12}(\vec{r}, \tau) v + a_{13}(\vec{r}, \tau) w \\
    v_e(\vec{r}, \tau) &= a_{21}(\vec{r}, \tau) u + a_{22}(\vec{r}, \tau) v + a_{23}(\vec{r}, \tau) w \\
    w_e(\vec{r}, \tau) &= a_{31}(\vec{r}, \tau) u + a_{32}(\vec{r}, \tau) v + a_{33}(\vec{r}, \tau) w
\end{align*}
$$

or more concisely by

$$u_e(\vec{r}, \tau) = A(\vec{r}, \tau) u_e \quad (3.19)$$

Comparing this formula to Equation 3.18, the estimated variable vector is

$$y(\vec{r}, \tau) = [u_e, v_e, w_e]^T(\vec{r}, \tau) \quad (3.21)$$
and the predicting variables are

\[ \mathbf{x} = [u_c, v_c, w_c]^T \]  \hspace{1cm} (3.22)

In these expressions, the regression coefficients have been presented as functions of space and time. This extension of the equations of Section 3.2.1 simply requires that the regression analysis be conducted for each point in space and time where the reconstruction formulas are to be applied. To construct the matrix of coefficients \( \mathbf{A}(\mathbf{r}, \tau) \), the correlation tensor is required between the velocities at the condition location and each location where the reconstructed velocities are required. As presented in Section 4.5, these correlations are available.

Extending the reconstruction technique to estimate the most likely velocity field given the velocities at multiple points in space requires the redefinition of \( u_c \) and computation of the required coefficients. For a two-point reconstruction, \( u_c \) is defined as

\[ u_c = [u_1, v_1, w_1, u_2, v_2, w_2]^T \]  \hspace{1cm} (3.23)

where \( u_1 \) is the condition at the first spatial point and \( u_2 \) is the condition at the second spatial point. The matrix of coefficients can be computed from the space-time correlation tensor using Equation 3.15.

Similarly, the field estimation can be conducted using multiple points in space and multiple phases in time. Mathematically, there is no distinction between space and time dependence. However, by considering multiple conditions in time, the equivalence between fixing multiple conditions with small separations and fixing condition values and their derivatives can be seen. For example, an estimation of the streamwise
velocity component at a single point in space \((y_0, z_0)\) and time \((\tau)\) can be estimated based on the streamwise velocity at one point in space \((y_r, z_r)\) and at two times \(u_c(\tau, y_r, z_r)\) and \(u_c(\tau + \Delta\tau, y_r, z_r)\). The regression formula is given by

\[
u_c(\tau, x_0, y_0) = a_1 u_c(\tau, y_r, z_r) + a_2 u_c(\tau + \Delta\tau, y_r, z_r)
\]  (3.24)

This can be rewritten, dropping the spatial dependence, as

\[
u_c = b_1 (u_c(\tau) + u_c(\tau + \Delta\tau)) + b_2 (u_c(\tau) - u_c(\tau + \Delta\tau))
\]  (3.25)

The summed terms represent the time average of the conditions, and the difference terms represent the rate of change of the condition. Increasing the number of conditions in time has the same effect as imposing increasingly higher order derivatives. The effects of this in terms of the accuracy of the estimated velocity field will be considered in Section 5.1 in terms of the estimated variances given in Equation 3.13.

The most general application of the procedure is to estimate a three-dimensional velocity field given velocities at points distributed in space and time. For this analysis, the formulation does not change. The condition vector is defined as

\[
u_c = [u_1, v_1, w_1, u_2, \ldots, u_3, \ldots]^T
\]  (3.26)

Each condition velocity vector is given at some location in space and at some point in time with respect to a known reference. The required coefficients can be computed from the available space-time correlation tensor.

In order to estimate a velocity field, the conditions must be explicitly given. Single and two point conditions can be fixed by considering the statistical properties of the
flow. The joint probability density functions of velocity statistics has been used to select the conditions following the Quadrant technique used by Wallace, Eckelmann, and Brodkey (1972) and Willmarth and Lu (1973). This and other statistical techniques used to establish conditions are described in Section 5.2. By appropriately selecting the conditions, information can be gained about the flow structures associated with statistically significant events. For multiple-condition reconstructions, that is, several conditions in space and time, specifying the conditions based on statistical arguments is practically infeasible. A useful alternative is to measure the conditions using a coarsely distributed array of hot-wire probes. The velocity field in areas between the probes can be reconstructed using the stochastic estimation procedure. These fields represent estimates of instantaneous turbulent velocity fields given specific measured realizations at selected points. This technique will be termed the pseudo-dynamic reconstruction technique. Whether or not conditionally averaged structures be found in real flows can be addressed by making use of these fields.

3.3 Modal Field Decomposition

The pseudo-dynamic reconstruction technique described briefly in the preceding section is a useful method to generate fully three-dimensional velocity fields from a very coarse distribution of simultaneous velocity measurements. These fields must be analyzed to extract useful information about the structures of the turbulence. The integration of the reconstructing and analyzing turbulent velocity fields which was conducted in this study involved a large number of interrelated steps which are schematically illustrated in Figure 3.3.
Figure 7: Flow decomposition steps
3.3.1 Field Reconstruction

Currently, there are no data sets available that provide three-dimensional perturbation velocity fields in the turbulent wake. However, it is possible to estimate such velocity fields using the stochastic estimation procedure and velocities measured at a few locations in space. The wake experiment, described in detail in Section 3.1, provided all of the necessary data for the procedures of this section.

Using three triple-wire probes, velocity time histories (condition signals) were measured simultaneously at three points in the $y - z$ plane, Experiment 8. Short time histories of the condition signals were used to make a condition vector Equation 3.26, for use in Equation 3.20 to estimates velocities in the $y - z$ plane. For the reconstructions in this study, seven equally spaced samples in time were considered: three lagging samples, three leading samples, and one sample at the time reference. The time increment was selected to minimize the estimation error. As discussed in Section 5.2, this is roughly equivalent to the physical grid spacing, $\Delta r U_\infty / b = 0.41$. Because only the large scale turbulent motions are of interest here, the high frequency components of the condition signals were removed using a digital filter. The cut-off frequency was selected such that the smallest temporal scale was larger than the condition spacing in $y$ and $z$: $L_{cut} = 2U_\infty / (b f_{cut})$.

The temporal evolution of the velocities on this transverse two-dimensional grid was computed by using the time histories of the conditions. At each point in time, the $y - z$ velocity field was estimated using the local velocity time histories. The condition record was then advanced by one time increment and the next plane of velocities
estimated. This advancement of the conditions and estimation of the velocities was repeated until the end of the available time record was reached.

Using Taylor's hypothesis and assuming frozen turbulence, short time sequences of velocities in the $y - z$ plane can be considered three-dimensional velocity fields. The validation of Taylor's hypothesis will be justified in Section 4.5. The evolution of quasi-instantaneous fields can be examined by observing the flow dynamics as the condition vector is advanced in time. No streamwise spatial information is available, but because the lifetime of large structures is longer than several of their characteristic wave-lengths, much of the structure dynamics can be captured in this way. This approach to examine flow dynamics is where pseudo-dynamic reconstruction gets its name. Similar reconstructions of scalar functions have been used by Bellin, et al. (1992) and termed pseudo-flow visualization.

Estimating velocity fields in this way eliminates the need to measure the entire three-dimensional flow field. From very coarse spatial measurements, the entire flow field can be estimated.

Four methods of analysis have been implemented to extract significant characteristics from these velocity fields. The fields have been decomposed in terms of their symmetry properties, Section 6.2, decomposed into Fourier space, Sections 3.3.2 and 6.3, qualitatively studied, Section 6.1, and numerically compared to linear instability modes, Sections 3.7 and 7.4.
3.3.2 Field Decomposition

The velocity fields estimated in this way are defined in physical space. It is much more convenient to compare them to stability modes if they are recast in an alternate form using Fourier and Chebychev decomposition.

The fields were originally evaluated on a uniform grid in the inhomogeneous direction. To express the velocities as a Chebychev series, the velocities were computed at the Chebychev collocation points on a stretched grid using cubic spline interpolation. Integration yields the new expressions for the streamwise velocity component,

\[ u(x, y, z) = \sum_{n=0}^{n_f - 1} A_n(x, z) \Phi_n( \eta ) \tag{3.27} \]

with \( \eta = \tanh(qy) \). A similar expression can be found for the normal and spanwise components of velocity. Since all the Chebychev coefficients are real, they can be expressed as a real Fourier expansion in either the streamwise or spanwise direction. The spanwise direction was chosen here, giving

\[ u(x, y, z) = \sum_{n=0}^{n_f - 1} \sum_{m=0}^{n_z/2} A_{mn}(x) \Phi_n( \eta ) e^{i\beta_m z} \tag{3.28} \]

In this expression the coefficients are all complex quantities except the \( \beta = 0 \) and \( \beta = \beta_{\text{max}} \) coefficients. In this transformation, a set of spanwise spatial wave numbers are defined which when normalized by the wake half width are

\[ \beta_m = m\Delta_\beta = m \left( \frac{2\pi}{n_z} \right) \left( \frac{b}{\Delta_x} \right), \quad m = 0, \frac{n_z}{2} \tag{3.29} \]

The \( x \) dependency can also be expressed as a Fourier expansion

\[ u(x, y, z) = \sum_{n=0}^{n_f - 1} \sum_{m=0}^{n_z/2} \sum_{l=-n_z/2}^{n_z/2} A_{nml} \Phi_n(y) e^{i\beta_m z + io_l w} \tag{3.30} \]
The set of streamwise wave numbers is given by

$$\alpha_l = l \Delta \alpha = l \left( \frac{2\pi}{n_t} \right) \left( \frac{b}{U_\infty \Delta t} \right), \quad l = -n_t/2, n_t/2$$  \hspace{1cm} (3.31)

All three velocity components were expanded in this way. The energy in the turbulent motions as a function of $\alpha$ and $\beta$ can be computed by integrating the energy contributions as a function of $\alpha$ and $\beta$ over $y$.

To express the velocity fields in a form that is identical to the eigenmodes of the stability analysis, $u$ and $v$ were recast as,

$$u(x, y, z) = \sum_{n=0}^{n_s - 1} \sum_{m=0}^{n_s/2} \sum_{l=0}^{n_t/2} \left( A_{nm}^+ e^{i\alpha_l x} + A_{nm}^- e^{-i\alpha_l x} \right) \Phi_n(\eta) e^{i\beta_m z}$$  \hspace{1cm} (3.32)

$$v(x, y, z) = \sum_{n=0}^{n_s - 1} \sum_{m=0}^{n_s/2} \sum_{l=0}^{n_t/2} \left( B_{nm}^+ e^{i\alpha_l x} + B_{nm}^- e^{-i\alpha_l x} \right) \Phi_n(\eta) e^{i\beta_m z}$$  \hspace{1cm} (3.33)

The coefficients $A_{nm}^+, A_{nm}^-, B_{nm}^+, \text{ and } B_{nm}^-$ are all complex quantities. For a given $\alpha$ and $\beta$, a single complex vector is defined that expresses the contribution to the velocity field by that wave number pair,

$$s_{ml} = \begin{bmatrix} A_{0ml}^+ \cdots A_{(n_s - 1)ml}^+ & A_{0ml}^- \cdots A_{(n_s - 1)ml}^- \\ B_{0ml}^+ \cdots B_{(n_s - 1)ml}^+ & B_{0ml}^- \cdots B_{(n_s - 1)ml}^- \end{bmatrix}^T$$  \hspace{1cm} (3.34)

This can be directly compared to the stability analysis eigenmodes associated with the same wave number pair.

### 3.4 Primary Instability Analysis

If the turbulent wake can be described as a slowly developing parallel flow as given in Section 2.2, the stability characteristics of its mean profile should be predictable
by linear stability theory. If the turbulent fluctuations are initially ignored, the Orr-Sommerfeld equation governs the stability properties of the viscous flow to small disturbances

\[
[D^2 - \alpha^2 - \beta^2 - iRe_b(\alpha U - \omega)(D^2 - \alpha^2 - \beta^2) + i\alpha Re_b D^2 U] v = 0
\] (3.35)

\[v \to 0 \quad Dv \to 0 \quad \text{as } y \to \infty \quad \text{or } y \to -\infty \] (3.36)

\[v(x, y, z, t) = v(y)e^{i\alpha x + i\beta z + \sigma t} \] (3.37)

A solution for the temporal instability problem of a Gaussian mean velocity profile, Equation 2.19, is given in detail by Flemming (1987). He used a pseudo-spectral technique that involved expanding the eigenfunctions using Chebychev polynomials

\[v = Re \left\{ \hat{v}(y)e^{i\alpha(x-ct)} \right\} \] (3.38)

\[\hat{v}(y) \approx \sum_{n=1}^{n_{xf}} a_n \Phi_n(\eta) \] (3.39)

After stretching the grid using \(\eta = tanh(qy)\), recursion formulas were used for the Chebychev polynomial derivatives and they were substituted into the Orr-Sommerfeld equation. This gave an eigenvalue problem for the wave speed \(c\). The computer codes used in that study were written by M. Flemming and Dr. L. Hultgren. They are installed on a Convex mini-supercomputer at the NASA Lewis Research Center and have been made available for additional computations. All of the primary instability calculations conducted in this study have been done using those codes. The wake defect and half width were fixed as experimentally determined values and 64 Chebychev polynomials were used.
The viscous problem can be extended to account for the effects of turbulence by defining a turbulence Reynolds number based on the an effective eddy viscosity model (Marasli, Champagne, and Wygnanski, 1991). Using a Gaussian mean profile and the fully developed flow equations of Section 2.2, the eddy viscosity can be expressed in terms of the mean flow. The eigenvalue problem solved using this definition of the Reynolds number takes the damping effects of background turbulence into effect. Some additional comments on this turbulence Reynolds number can be found in Chapter VII.

Inviscid, viscous, and turbulence modeled cases have been solved for a range of streamwise wave number perturbations commensurate with the physical reconstruction grid. The amplitude and frequency of the most amplified mode was recorded as a function wave number and the eigenfunctions were saved for use in secondary calculations.

3.5 Secondary Instability Analysis

Based on the analysis formulated in Section 3.4 and the results of Section 7.2, the instability of the inflectional parallel mean flow profile causes the amplification of periodic two-dimensional disturbances. When these disturbances grow to the point where they are saturated, a periodic base flow is established. This flow is vulnerable to additional instabilities through the mechanism of secondary subharmonic three-dimensional instability.

The new instability problem is considered in a frame of reference moving with the primary instability disturbance. As with the primary instability solution, the
equations and computational details are given by Flemming (1987) and the computer
codes were written and provided by Flemming and Hultgren.

The problem is formulated by conducting a triple decomposition of the velocities

\[ \tilde{u} = U + A_p u_1 + \epsilon_s u_2 \]  

Here \( U \) is the mean velocity, \( u_1 \) is the primary instability mode, and \( u_2 \) is the second­
ary instability. Substituting these components into the Navier Stokes equation,
two instability problems are defined. If the amplitude of the primary instability is
finite, and the mode shape is known from the calculation of Section 3.4, an eigenvalue
problem can be written for the secondary instability. The secondary mode shape is
assumed to have the form

\[ u_2(x, y, z, t) = e^{\sigma t + i\beta x} \sum_{n=-\infty}^{\infty} \hat{u}_n e^{i\hat{n}x} \]  

(3.41)

\[ \hat{n} = \frac{\alpha_F}{2} \]  

(3.42)

for sub-harmonic resonance, \( n \) will be limited to \( \pm 1 \). Harmonic resonance will not be
considered. The primary instability can also be expressed as a Fourier series but the
shape assumption in Equation 3.38 restricts the primary instability wave numbers to
the positive and negative first harmonic. The expansion of the disturbance equation
results in four coupled differential equations. These are solved using a Chebychev
collocation technique.

By assuming \( A_p = 0 \) and \( \epsilon_s << 1 \), the primary instability problem is recovered.
However, due to the nature of the solution procedure, both the Orr-Sommerfeld modes
and the Squire modes are allowed. In the primary instability calculation of Section 3.4,
only the Orr-Sommerfeld modes can be computed. For both cases, the secondary instability problem reduces to a single eigenvalue problem

\[(R - \sigma S)h = 0\]  \hspace{1cm} (3.43)

the solution of which is found by rewriting the problem

\[Mh = \lambda h\] \hspace{1cm} (3.44)

\[M = R^{-1}S\] \hspace{1cm} (3.45)

\[\lambda = \frac{1}{\sigma}\] \hspace{1cm} (3.46)

As noted in Section 3.3.2, the turbulent velocity fields were broken down into the contributions by various combinations of wave number disturbances. For direct comparison with stability modes, the secondary instability problem has been solved for each of these wave number combinations. The amplitude of the primary instability was set at small values (see Section 7.2).

3.6 Mode System Selection

As a spectral code, the secondary instability program produces all the eigenvalues and eigenmodes afforded by the formulation. Most of the eigenvalues fall on a continuous curve that may not represent physical solutions. Other eigenvalues are separate from the continuous curve and are considered discrete physical modes. These modes are classified as either Orr-Sommerfeld modes if they contain a \(v\) component of velocity or Squire modes if they do not. Although all of the modes could be used as a large set of basis functions for the modal decomposition, a subset of the modes that is
considered the physical modes must be selected. The discrete modes that could be identified in the eigenspectra for disturbance wave numbers in the range expected to be the most important in the wake were selected as the subset of modes. This was the range where the dominant primary instability mode was amplified.

Not only must the mode shapes be computed for every wave number combination, they must also be ordered by mode shape. This ensures that there is no confusion as to which coefficient is associated with which physical mode when the modal decomposition is conducted. Eigenvalue ordering by mode shape is not generally conducted by spectral codes and the eigenspectra results are most always scrambled.

An algorithm was used to track the selected modes through the solution space as the stability parameters were slowly varied. The process is as follows:

- The secondary instability problem was solved over the range of streamwise and spanwise wave number disturbances defined in the Fourier expansion of the velocity fields, Equations 3.29 and 3.31.

- A set of discrete physical modes was interactively selected from the eigenvalue spectrum computed for a single characteristic wave number pair.

- An algorithm designed to track particles in digital images was used to track all the desired eigenvalues from solution spectrum to solution spectrum (Guezennec et al., 1992).

- A table of solutions was generated that contained the eigenvalue of each mode as a function of the streamwise and spanwise disturbance wave numbers.
• The shape of each of the selected modes was also recorded as a function of the wave numbers.

Each stability mode is expressed in the same form as the recast velocity fields, Equation 3.34. The \( j^{th} \) mode computed for the \( l^{th} \) discrete streamwise wave number and the \( m^{th} \) discrete spanwise wave number will be termed \( h_{mij} \).

### 3.7 Modal Decomposition

The velocity field expressed in Equation 3.34 can be expressed as a sum of the instability modes selected following the procedure in Section 3.6.

\[
\sum_{m=0}^{n_x/2} \sum_{l=0}^{n_z/2} s_{ml} = \sum_{m=0}^{n_x/2} \sum_{l=0}^{n_z/2} c_j h_{mij} \tag{3.47}
\]

Considering the velocities at a single wave number pair,

\[
s = c_j h_j \tag{3.48}
\]

In order to determine the coefficients \( c_j \) a set of functions must be found that are orthogonal to the eigenvectors. This is required because the original eigenvectors are not mutually orthogonal, i.e.

\[
h_m^T h_n \neq 0 \tag{3.49}
\]

The adjoint of the original matrix eigenvalue problem can be solved to compute the necessary functions,

\[
M^T h^A = \lambda h^A \tag{3.50}
\]
The eigenvalues of this problem are the same as those of the original one, but the eigenvectors are not. These eigenvectors can be used in the orthogonality condition

\[(\lambda_n - \lambda_m) h_m^T h_n = 0 \quad (3.51)\]

\[h_m^T h_n = \begin{cases} 0 & n \neq m \\ C & n = m \end{cases} \quad (3.52)\]

With both the original eigenvectors and the adjoint eigenvectors, the coefficients of the expansion can be determined by taking the inner product of the expanded velocity field with each of the adjoint eigenvectors

\[h_p^T s = h_p^T c_j h_j \quad (3.53)\]

\[c_j = \frac{h_j^T s}{h_j^T h_j} \quad (3.54)\]

Because of the non-orthogonality of the eigenvectors, the coefficients do not directly relate to the energy contained in a specific mode. The total energy predicted by a truncated expansion with \(n_{\text{modes}}\) terms is given by

\[[s^*]^T s \approx E(n_{\text{modes}}) = (a_j h_j)^T (a_p h_p)^* \quad (3.55)\]

The contribution to the total Energy by individual modes in the expansion can be computed by considering the energy predicted by increasing order expansions. The one term expansion, \(s \approx a_1 h_1\), predicts the energy as, \(E(1) = (a_1 a_1^*)(h_1^T h_1^*)\). This is assumed to be the energy contributed by the first mode. The energy contributed by the combination of the first and second modes is given by \(E(2)\). The difference between the two estimations, \(E(2) - E(1)\), represents an estimate for the energy contributed by the second mode. This procedure can be repeated until the energy
contributed by each mode is estimated. The difficulties of this portion of the study were not satisfactorily resolved and will be discussed in Section 7.4
CHAPTER IV
STATISTICAL ANALYSIS

4.1 Statistical Profiles

The results presented in this section focus on the downstream development of the turbulent statistics of a wake at $Re_d = 5080$ measured using triple hot-wire probes. The important statistics of the wake were calculated from the single and two point experiments outlined in Section 3.1. The conditions, model, and experimental equipment are the same as those used in the correlation measurement and pseudo-dynamic reconstruction experiments described in Sections 4.5 and 6.1. The following results primarily serve to document the conditions and scales associated with those experiments.

Mean velocity profiles are shown in Figure 8 for five distances downstream of the cylinder. The average velocities have been normalized using the free-stream velocity and the downstream position. The $y$ coordinate has been normalized by estimates for the wake half width $b$. The value of $b$ was determined by fitting a Gaussian profile to each profile, matching the point of one half of the maximum defect to the experimental value. The amplitude of the defect is taken as the maximum of the measured velocity defect, $U_{1\text{max}}$. These Gaussian profiles are also shown in Figure 8 for comparison with
the measured profiles. The profiles reach a self-similar shape near the cylinder. At all stations, the profiles are roughly Gaussian with some slight deviations near the profile tails. The normalized maximum defect is approximately 0.91 at 50, 100, and 200 diameters downstream increasing to 1.0 at 300 and 400 diameters. This increase is the result of inaccuracies of mean measurements where the defect is very small.

The two parameters, $b$ and $U_{1\text{max}}$, of the Gaussian profile are considered the outer scales of the wake. Their variation with streamwise position are predicted as given in Section 2.2. The experimentally determined values are shown in Figure 9 with linear fits to the data. The half width follows the linear prediction well over the range considered. The velocity defect, presented as a fraction of the free-stream velocity, was not as accurately measured due to the small defect velocities far downstream of the cylinder. These errors result in the poor agreement with the linear fit which would be expected based on the theory.

The $\sigma_u$ and $\sigma_v$ turbulence intensities are shown in Figure 10 and the $\sigma_w$ is shown in Figure 12. These quantities are normalized by the free-stream velocity and the downstream position and are plotted against $y/b$. The $uv$ correlations are shown in Figure 11. The quantity $\overline{uv}$ is shown normalized by the square of the free-stream velocity and the downstream position. Self-similar behavior is achieved by approximately $x/d = 200$ for the turbulence intensities and is reached by $x/d = 100$ for $\overline{uv}$. The shapes of the statistical profiles are qualitatively constant beyond $x/d = 100$. The non-zero tails of the $\sigma_u$ curves are believed to be the result of unremovable noise in the experimental equipment used.
Figure 8: Normalized mean streamwise velocity profiles for $Re_d = 5080$ at
a) $x/d=50(\times\cdots\times)$, 100(+ --- +), and 200(* --- --- *) and
b) $x/d=200(\times\cdots\times)$, 300(+ --- +), and 400(* --- --- *)

Figure 9: Streamwise development of a) wake half-width and b) wake defect. The
data measured using Auspex probe 1 is represented by *, and that measured using
probe 2 by $\times$
Figure 10: Profiles of fluctuating velocity at $x/d=50(\times--\times), 100(+\cdots\cdots+), 200(\ast-\cdots\ast), 300(-----),$ and $400(\circ-\cdots\circ)$ of the a) streamwise component and b) normal component.

Figure 11: Profiles of $\overline{uv}$ at $x/d=50(\times--\times), 100(+\cdots\cdots+), 200(\ast-\cdots\ast), 300(-----),$ and $400(\circ-\cdots\circ)$
Figure 12: Fluctuating spanwise velocities at $x/d = 50(\times - - - \times), 100(+ \cdots \cdots +), 200(* - - - - *), 300(---), and 400(o - - o)$

The turbulent eddy viscosity, $\varepsilon_0 = -\overline{w}\overline{v}/(\partial \overline{U}/\partial y)$, and turbulence Reynolds numbers were determined using the data in these figures. The eddy viscosity was computed using the Gaussian profile for the mean velocity and the values of $\overline{w}\overline{v}$ given in Figure 11. Following the analysis given in Section 2.2, $\varepsilon_0$ was assumed constant over the wake width. It was computed as the average of the values measured between $y/b = -2$ and $y/b = 2$ (the approximate turbulent portion of the wake). Over this range, the computed values remain relatively constant (±15%). Because both $\overline{w}\overline{v}$ and $(\partial \overline{U}/\partial y)$ decay as $(x/d)^{-1}$, $\varepsilon_0$ is expected to remain constant in the fully developed far-wake. In these experiments, $\varepsilon_0$ reached an asymptotic value of $8.0 \times 10^{-4} m^2/s^2$ for $x/d$ greater than 200. At $x/d = 100$, the value was found to be slightly higher, $9.3 \times 10^{-4} m^2/s^2$. 
The wake Reynolds number which is based on the local outer-scales and is constant in the far wake is defined as $Re_w = U_{1max}b/\nu$. For this experiment $Re_w = 1425$. A turbulence Reynolds number is defined as $Re_{\omega t} = U_{1max}b/\varepsilon_0$ which accounts for the damping effects of the turbulence. In this case, $Re_{\omega t} = 22.3$.

The auto-correlations and spectra of the three velocity components were used to estimate velocity scales of the wake turbulence. Spectra of the velocity components at approximately the location of peak $\sigma_u$ are shown in Figures 13 and 14. The frequencies have been scaled by the free-stream velocity and an estimate for the wake half width based on the streamwise position and the cylinder diameter. This is a local Strouhal number for the turbulent fluctuations, $St_b = 2\pi f(d/U_\infty)(x/d)^{1/2}$. There is a slight peak in the spectra at $St_b \approx 6$ to 7, at all downstream locations in both the $v$ and $w$ components of velocity. Using Figure 10 to better estimate $b$ and $U_{1max}$, the non-dimensional wave numbers have been calculated to be between 1.5 and 1.75. These are very close to the wave numbers predicted by inviscid linear stability theory (see Section 7.1). The peaks are not clearly defined and more accurately should be described as a band of frequencies. The wave-lengths of motions within this band are approximately $4b$ which would be commensurate with any motions that scale with half of the total size of the wake.

The normalized auto-correlations of the velocities are shown in Figures 15 and 16. The time axis has been converted to an estimate for normalized convection distance using the free-stream velocity, the diameter, and the distance downstream. Two turbulent scales are defined from these correlations, the Taylor micro-scale $\lambda$ and
the integral scale $L$. Both of these scales are nondimensionalized as the temporal correlations are, i.e. $L = L_t(U_\infty/d)(x/d)^{-1/2}$. The Taylor micro-scale is defined as the intersection of a parabola fit to the curvature of the auto-correlations at $\Delta t = 0$ with the $x$ axis. For these measurements, the parabola is fit to the first four points of the correlation function using a least squares estimate. The Integral scale is defined as the $x$ dimension of a box with a height equal to the maximum auto-correlation (in this case 1.0) that has the same area as the area between the correlation curve and the $x$ axis. To calculate this area from the correlation data, the correlation function was integrated up to either the second zero crossing or when the auto-correlation reached an asymptotic value. The streamwise variation of these two scales for the three velocity components are tabulated in Table 2. These are the nondimensional scales, $L = L_t(U_\infty/d)(x/d)^{-1/2}$. The variation of the streamwise velocity scales is plotted in Figure 17. All of the scales are varying nonlinearly in the downstream direction. This suggest that although the outer scales of the wake reach asymptotic behavior quickly, the turbulence is evolving much differently and may not reach an self-similar state until very far downstream of the wake generating body if at all.

4.2 Event Identification

The time-average statistics computed in the preceding sections do not provide any information about typical or significant instantaneous flow motions or coherent structures. In order to identify coherent turbulent structures, some quantity must be defined that is indicative of their presence. From an experimental standpoint, this must be a measurable quantity, the realization of which during the course of an experiment
Figure 13: Spectra of velocities at, from the top curve to the bottom curve, $x/d=50, 100, 200, 300,$ and $400$ for a) streamwise component and b) normal component.

Figure 14: Spectra of the spanwise velocity component at, from top to bottom, $x/d=50, 100, 200, 300,$ and $400$. 
Figure 15: Auto-correlation coefficients of the a) streamwise velocity component and b) normal velocity component at $x/d=50$ (top), $100$ ( ), $200$ ( ), $300$ ( ), and $400$ (bottom).

Figure 16: Auto-correlation coefficients of the spanwise velocity component at $x/d=50$ (top), $100$ ( ), $200$ ( ), $300$ ( ), and $400$ (bottom).
Table 2: Nondimensional turbulent scales

<table>
<thead>
<tr>
<th>$x/d$</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_u$</td>
<td>0.1499</td>
<td>0.1305</td>
<td>0.1031</td>
<td>0.0925</td>
<td>0.0854</td>
</tr>
<tr>
<td>$L_u$</td>
<td>0.2564</td>
<td>0.2512</td>
<td>0.2454</td>
<td>0.1959</td>
<td>0.1829</td>
</tr>
<tr>
<td>$\lambda_v$</td>
<td>0.0992</td>
<td>0.0748</td>
<td>0.0629</td>
<td>0.0567</td>
<td>0.0517</td>
</tr>
<tr>
<td>$L_v$</td>
<td>0.1054</td>
<td>0.0835</td>
<td>0.0788</td>
<td>0.0818</td>
<td>0.0735</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>0.0995</td>
<td>0.0788</td>
<td>0.0650</td>
<td>0.0572</td>
<td>0.0539</td>
</tr>
<tr>
<td>$L_w$</td>
<td>0.0990</td>
<td>0.0899</td>
<td>0.0827</td>
<td>0.0744</td>
<td>0.0725</td>
</tr>
</tbody>
</table>

Figure 17: Streamwise variation of a) $\lambda_u$ and b) $L_u$
should indicate the occurrence of the coherent structure of interest. Such a realization is termed an *event*.

The quantity or quantities that constitute an event can be determined based on physical arguments and consideration of the statistical properties of the turbulence. The structures that contribute most to the Reynolds stress are those that contribute to turbulence transport. The *Quadrant* technique used by Wallace, Eckelmann, and Brodkey (1972) and Willmarth and Lu (1973) in a turbulent boundary layer, considers both the frequency of occurrence of velocity fluctuations within a certain range of \((u,v)\) and the relative contribution of those velocities to the Reynolds stress. Very large but infrequent perturbations make little contribution to the turbulent statistics as do very frequent but small fluctuations.

Two-dimensional joint probability density functions (PDFs) of \(u\) and \(v\), \(P(u,v)\), at several points within the turbulent wake are shown in Figure 18. Similarly, two-dimensional joint PDFs of \(u\) and \(v\) weighted by \(uv\), \(P(u,v)uv\), are shown in Figure 19. The shape of the same statistical functions on the opposite side of the wake obey the symmetry of the problem and can be inferred from the shown figures. The data used to compute these functions, and all others in this section, is taken from Experiment 5.

From these figures, the combinations of \(u\) and \(v\) that make the largest contribution to \(uv\) can be found. The combinations can be classified by the quadrant in which the \((u,v)\) vector is found. The quadrants are identified counter-clockwise from the top right quadrant and velocities falling into those quadrants will be termed \(Q1\), \(Q2\), \(Q3\), and \(Q4\) motions. Peaks in \(P(u,v)uv\) at \(y/b = -0.898\) are found for \((u,v)\) in the
Figure 18: Two-dimensional joint probability density functions of $u$ and $v$ evaluated at a) $y/b = -2.25$ b) $y/b = -1.35$ c) $y/b = -0.898$ d) $y/b = 0.0$
Figure 19: Two-dimensional joint probability density functions of $u$ and $v$ weighted by $uv$ evaluated at a) $y/b = -2.25$ b) $y/b = -1.35$ c) $y/b = -0.898$ d) $y/b = 0.0$
first and third quadrants. The first quadrant events are motions that act to entrain high speed fluid from the irrotational outer flow and the third quadrant events are those that act to eject low speed turbulent fluid from the wake center. Although the first quadrant peak has a larger amplitude, the third quadrant event makes a larger contribution to the Reynolds stress at this point by a factor of 1.2. The frequencies of occurrence of the two types of motions are roughly equivalent. Based on the shape of the distributions, the ejecting motions appear to be more intense than the entraining motions and have a wider range of intensity and direction.

Because $w$ is uncorrelated with $u$ and $v$ at zero spatial and temporal separation and because the flow is homogeneous in the spanwise direction, $w$ can be expected to be normally distributed about zero with a constant standard deviation for any value of $uv$. The joint PDF of $w$ and $uv$, shown in Figure 20, supports this argument.

The discussion above is based on information at a single point in space and time. No information about the relative locations (in space or time) of various types of events can be found in this way. To get this kind of information, events can be defined by quantities measured simultaneously at two locations. To determine the importance of combinations of events that contribute to the Reynolds stress, the PDF of velocities at one point is determined using data conditionally sampled based on the occurrence of turbulence producing events at another point. The joint PDF of $u$ and $v$ weighted by $uv$ at $y/b = -0.898$ given the realization of each of the four quadrant events at $y/b = 0.898$ is shown in Figure 21. For any event on the top side of the wake, first and third quadrant events are expected on the bottom side.
The same information can be summarized more concisely by considering the joint PDF of the velocity directions. Figure 22 shows the joint PDFs of the flow angle, $\theta = \tan^{-1}(v/u)$, at $(y/b)_1 = 0.898$ and $(y/b)_2 = -0.898$ weighted by the contributions to the Reynolds stress at each point. Peaks in this function, $P(\theta_1, \theta_2)uw_1uw_2$, indicate what combinations of events are most likely to contribute to the Reynolds stress at the two points simultaneously at the physical separation prescribed. For cross-wake separation, there is a slight preference for third quadrant events on the bottom of the wake associated with second quadrant events on the top. This is an ejection motion on both sides of the wake occurring simultaneously. This is followed in importance by entrainment-ejection combinations and then by simultaneous entrainments. For the case where the conditions are considered with significant spanwise separation as
Figure 21: Two-dimensional joint probability density functions of $u$ and $v$ weighted by $uv$ at $y/b = -0.898$ given the condition at $y/b = 0.898$ is in quadrant a) two b) one c) three d) four
Figure 22: Two-dimensional joint probability density functions of $\theta_1$ and $\theta_2$ where points 1 and 2 are at locations $(y_1, z_1, y_2, z_2) =$ a) $(0.898, 0.0, -0.898, 0.0)$ b) $(0.898, 1.80, -0.898, 0.0)$

well as cross-wake separation, there is roughly an equal contribution by $Q_1$ and $Q_3$ events on the bottom side of the wake given a $Q_2$ event on the top and by $Q_3$ events given a $Q_4$ event on the top.

The plots in Figure 22 are divided into sixteen cells and each cell is associated with one combination of events. The integral of $P(\theta_1, \theta_2)uv_1uv_2$ in each cell is a value that indicates the relative contribution of the combination of events associated with that cell to the Reynolds stress for no temporal offset between the two events. In other words, each of the sixteen values represents the relative contribution to $\overline{uv_1uv_2}$ by that combination of events. The same measures of event combination significance can be generated for events occurring at non-zero time offsets. By evaluating the contributions of each cell for many time separations at a fixed spatial separation,
Table 3: Key to quadrant combination figures

<table>
<thead>
<tr>
<th>*</th>
<th>first quadrant event at first point</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>second quadrant event at first point</td>
</tr>
<tr>
<td>+</td>
<td>third quadrant event at first point</td>
</tr>
<tr>
<td>o</td>
<td>fourth quadrant event at first point</td>
</tr>
<tr>
<td>------</td>
<td>first quadrant event at second point</td>
</tr>
<tr>
<td>---</td>
<td>second quadrant event at second point</td>
</tr>
<tr>
<td>---</td>
<td>third quadrant event at second point</td>
</tr>
<tr>
<td>------</td>
<td>fourth quadrant event at second point</td>
</tr>
</tbody>
</table>

A set of functions, which are the sixteen contributors to the temporal correlation between \( uv_1(t) \) and \( uv_2(t + \tau) \), can be generated. These contributions will be noted \( C_{ij}(\tau) \) where

\[
C_{ij}(\tau) = \int_{cell_{ij}} P(\theta_1(t), \theta_2(t + \tau)) \frac{(uv_1(t)uv_2(t + \tau))}{(\sigma_u \sigma_v)^2} dA
\]

Figures 23 to 26 show plots of these quantities for six spatial separations. The \( y \) scale in these figures is somewhat arbitrary and is dimensionless. The amplitudes can be compared within a group of curves associated with a certain physical measurement, but comparisons between measurements should only be conducted qualitatively. Both the most significant combination of events and their relative distribution in time can be established for a given spatial separation from these figures. The symbols in the figures indicate the event type at the first point and the line type notes the event type at the second point. The convention is given in Table 3.

Several combinations of events are suggested by this analysis. For two points separated in time only at \( y/b = -0.898 \), ejection motions (Q3) are followed and preceded in time by motions of the opposite sign in \( v \) and the same sign in \( u \) (Q2) at
Figure 23: Two-point event levels with temporal separation only at
a) \( y/b = 0.898 \) b) \( y/b = 0.898 \) c) \( y/b = 0 \) d) \( y/b = 0 \)
See Table 3 for key.
Figure 24: Two-point event levels with temporal separation only at
a) $y/b = -0.898$ b) $y/b = -0.898$ c) $y/b = -1.79$ d) $y/b = -1.79$
See Table 3 for key
Figure 25: Two-point event levels with point one at $z/b=0.0$ and point two at $z/b=0.449$ both points are at $y=-0.898$. The events at $z/b=0.449$ are a) Q1  b) Q2  c) Q3  d) Q4. See Table 3 for key.
Figure 26: Two-point event levels with point one at $y/b=-0.898$ and point two at $y/b=0.898$ both points are at $z/b=0.0$. Events at $y/b=0.898$ are a) Q1 b) Q2 c) Q3 d) Q4. See Table 3 for key
small separations, \((\tau U_\infty / b) \approx 0.8\). This sequence becomes more pronounced further from the wake centerline. At \(y/b = -1.79\) the Q3-Q2 combination is the dominant event sequence but occurs at a smaller spacing, \((\tau U_\infty / b) \approx 0.5\). Repeating Q3-Q2 combinations at the spacing shown is the equivalent to decelerated flow that is periodic in \(v\) at a normalized angular frequency of \(\alpha = 4\) at \(y/b = -0.898\), and \(\alpha = 6\) at \(y/b = -1.79\). At larger separations, \((\tau U_\infty / b) > 1.5\), Q3 motions are followed and preceded by entrainments (Q1). The typical spacing of Q1 and Q3 events cannot be determined from this analysis. The Q1-Q3 curve rises from 0 at no separation to a maximum beyond \(\tau U_\infty / b = 2\) where the curve is flat. This suggests a lack of periodicity and significant jitter in the typical spacing of these events.

All phase information between events measured by probes separated in the spanwise direction disappears with separation. For a spacing of \(z/b = 0.449\), the Q1-Q4 and Q3-Q4 combinations can be seen but they do not appear as clearly as they do for no separation. At larger spanwise separations, no distinguishable combinations of events could be found.

From the flow angle PDF measured with one probe at \(y/b = 0.898\) and one at \(y/b = -0.898\) (Figure 22), ejections on both sides of the wake were found to be the most significant combination of events. Figure 26 shows that a larger contribution to the Reynolds stress comes from ejections with a temporal separation, \((\tau U_\infty / b) \approx 1.3\), than from simultaneous events.
4.3 Event Statistics

The original definition of an event developed when considering the joint PDF of $u$ and $v$, was that an event is a combination of $u$ and $v$ that contributes significantly to the $\overline{uv}$ at a single point. This definition can be extended to include information about the event properties also defined at a single point. Every event has a length (duration in time), a peak $uv$ level, and an average $uv$ intensity. These properties of Q1, Q2, Q3, and Q4 events can be estimated using single point measurements of $u$ and $v$.

By breaking a continuous $uv$ signal into a sequence of time intervals, with each interval beginning and ending at a zero crossing of $u$ or $v$, a series of events of finite length with $(u, v)$ in a single quadrant are defined. Each event is described by its quadrant, length, peak $uv$ value, average $uv$ value, and the product of its length and its average $uv$ value. The midpoint of each event in time serves as a marker for the location of that event. Cumulative histograms of event properties give an indication of the typical properties for each event type. The histograms, weighted by the contribution to the Reynolds stress, of event lengths and intensities indicate the properties of the events which are the most significant in terms of turbulent transport.

Because the definition of event length given above depends on zero-crossings of the $u$ and $v$ signals, the signals were low-pass filtered to remove the spurious zero-crossings caused by the small scales which are superimposed on the larger motions of interest. A normalized cut-off frequency of $2\pi fb/U_\infty = 3.66$ was selected for this analysis (see Section 6.3 for details about this cut-off frequency selection).
Figures 27 and 28 show the event statistics at $y/b = -0.898$. These curves include the fraction of events and the fraction of contribution to the Reynolds stress by events having a length less than or equal to $(\tau U_\infty/b)$, having an peak intensity less than $(uv/\sigma_u \sigma_v)_{\text{peak}}$, having an average intensity less than $(uv/\sigma_u \sigma_v)_{\text{ave}}$, and having an Reynolds stress contribution less than $(\tau U_\infty/b)(uv/\sigma_u \sigma_v)_{\text{ave}}$. All four events types have been considered. These statistics are sensitive to the filtering frequency but the relative lengths and importance of events does not change with the cut-off frequency.

Determining the typical event properties from these plots is somewhat subjective. From a purely statistical standpoint, the median, or the value where $N/N_T = 0.5$, is a reasonable approximation of the typical properties. Events having the property value associated with the point of inflection in the $uv/\overline{uv}$ curves are those that make the largest contribution to $\overline{uv}$. However, because the distributions are heavily skewed, most of $\overline{uv}$ is generated by events that are larger and more intense than either of these measures predict.

Both entrainment and ejection events have approximately the same length with a median length and a length associated with maximum $\overline{uv}$ contribution of 1.0. However, approximately 55% of the $\overline{uv}$ is generated by events having a length between 1.0 and 3.0. From the standpoint of significance, the larger motions should be considered the most important scales of the flow. Motions in this range of scale would span roughly from the wake centerline to the irrotational boundary.

The peak value of the Reynolds stress is significantly larger in the ejection events than in the entrainment events. Approximately 80% of the entraining events have a
Figure 27: Cumulative histograms at $y/b = -0.898$ of the a) fraction of events with a length less than $(\tau U_{\infty}/b)$ b) fraction of an event types total contribution to $\bar{u}\bar{v}$ by events with a length less than $(\tau U_{\infty}/b)$ c) fraction of events with an absolute value peak intensity less than $(uv/\sigma_u \sigma_v)_{peak}$ d) fraction of an event types total contribution to $\bar{u}\bar{v}$ by events with an absolute value peak intensity less than $(uv/\sigma_u \sigma_v)_{peak}$. See Table 3 for symbol key.
Figure 28: Cumulative histograms at \( y/b = -0.898 \) of the a) fraction of total number of events with an absolute value mean intensity less than \((uv/\sigma_u\sigma_v)_{ave}\) b) fraction of an event types total contribution to \(\bar{w}\) by events with a mean intensity less that \((uv/\sigma_u\sigma_v)_{ave}\) c) fraction of total number of events with an absolute value of their total contribution to \((T U_\infty/b)(\bar{w}/\sigma_u\sigma_v)\) less than \((\tau U_\infty/b)(uv/\sigma_u\sigma_v)_{ave}\), where \((T U_\infty/b)\) is the total length of events of that type, \((\tau U_\infty/b)\) is the event length, and \((uv/\sigma_u\sigma_v)_{ave}\) is the event mean intensity. d) fraction of an event types total contribution to \((T U_\infty/b)(\bar{w}/\sigma_u\sigma_v)\) by events with a contribution less than \((\tau U_\infty/b)(uv/\sigma_u\sigma_v)_{ave}\). See Table 3 for symbol key.
peak intensities less than $(uv/\sigma_u\sigma_v)_{peak} = 2.5$ while only 55% of the ejecting events are below the same level. The median peak value is quite low, less than 1.0, and the value of the maximum contribution to $\overline{uv}$ is slightly higher, 1.2 to 1.5. But, as with the lengths, most of the $\overline{uv}$ is generated by much more intense structures. For ejections, 50% of their contribution comes from events having a peak value between 1.5 and 4 and for entrainments 55% of their contribution comes from events having a peak value between 1.0 and 2.5. The differences in average value and total contribution to $\overline{uv}$ are less pronounced between the two types of events, but the trend is the same.

Information about the relationship between events is found by considering the spacing between significant events. A significant event will be defined as one having an amplitude, either peak or total $\overline{uv}$ contribution, greater than some threshold. Reasonable threshold values can be selected using the event property data of the preceding figures. The threshold was chosen such that most of the Reynolds stress contributing events were considered and smaller less significant events were ignored. A threshold of 0.2 in the quantity $(\tau U_\infty/b)(uv/\sigma_u\sigma_v)_{ave}$ removed 50% of the events found in the filtered data while retaining 90% of the Reynolds stress contribution.

The most significant spacing between two types of events is determined by considering the contribution to the Reynolds stress by one type of event event as a function of how far it follows another type of event. In this analysis, the two types of events must immediately follow each other. There cannot be any significant events in between them. This statistic is similar to the sixteen correlation contributors $C_{ij}(\tau)$ given earlier in this section, but discrete events have been considered instead of a
continuous distribution of velocities. This processing clarifies some elements of the relationships between events by eliminating the low amplitude velocity fluctuations which reduce the continuous correlations. Each point in Figures 29 to 31, represents the relative contribution to the Reynolds stress by events at one point (of the type given by the symbols) following events at another point (of the type given by the line type) by the distance shown on the $x$ axis. The locations of peaks in the curves represent the spacing of event combinations that are strong contributors to the Reynolds stress.

For temporal separation only at $y/b = -0.898$, the event combinations suggested by this analysis are, in the approximate order of significance, Q3-Q3, Q1-Q3, Q1-Q1, Q3-Q2, and Q1-Q4 at spacings of $(\tau U_\infty/b) \approx 3.5, 2.8, 3.5, 1.5,$ and $1.8$ respectively. This order of significance must be considered fairly loosely. The plotted levels only depend on the amplitude of the second event given that the first event is above a certain threshold. Although the level of the second event may be small, the event generally preceding it may be large and vice versa. These results agree with the previous results but the ejection spacing is shown more clearly.

For events measured at a spanwise separation, very large peaks appear for Q1-Q1 and Q3-Q3 event combinations. This was a result of slight phase jitter in individual structures across their span and so some comparisons were made between each event and itself. Otherwise, there is little difference with the set of events determined for no spanwise separation. For events at very large spanwise separation, there does not appear to be any preferred distribution of motions. For events measured on both sides
Figure 29: The total contribution to \((TU_\infty/b)(\bar{uv}/\sigma_u\sigma_v)\) at \((y/b, z/b) = (-0.898, 0.0)\) by events occurring at the same point with a separation \((\tau U_\infty/b)\). a) a first quadrant event detection followed by events in the other quadrants b) a second quadrant event detection c) a third quadrant event detection d) a fourth quadrant event detection. See Table 3 for key.
Figure 30: The total contribution to \((TU_{\infty}/b)(\overline{uv}/\sigma_u \sigma_v)\) at \((y/b, z/b) = (-0.898, 0.0)\) by events at \((y/b, z/b) = (-0.898, 0.449)\) with a separation \((\tau U_{\infty}/b)\) a) a first quadrant event detection followed by events in the other quadrants b) a second quadrant event detection c) a third quadrant event detection d) a fourth quadrant event detection. See Table 3 for key.
Figure 31: The total contribution to \((T_{U_{\infty}/b})(\overline{uv}/\sigma_u\sigma_v)\) at \((y/b, z/b) = (-0.898, 0.0)\) by events at \((y/b, z/b) = (0.898, 0.0)\) with a separation \((\tau_{U_{\infty}/b})\). a) a first quadrant event detection followed by events in the other quadrants b) a second quadrant event detection c) a third quadrant event detection d) a fourth quadrant event detection. See Table 3 for key.
of the wake center-line, there is a very strong peak for ejections following ejections by around $\tau U_\infty / b = 1.5$. There is also evidence of entrainments following entrainments at $\tau U_\infty / b = 1.5$ and ejections following entrainments at $\tau U_\infty / b = 2.0$.

These results agree well with those of Wlezien (1981). Wlezien considered the distribution of $u$ velocity in the $y-t$ plane. Histograms of the spacing and size of areas of peak $u$ were used to identify the turbulent scales. At $x/d = 80$, the spacing was found to be roughly $\tau U_\infty / b = 1.0$ and the streamwise length scale was slightly smaller at 0.8. These values were the median values and they agree well with the median values of the present study.

4.4 Summary of Events

In the preceding two sections, several methods (PDFs, conditional correlations, and event spacings) were used to estimate the combination of velocities distributed in time and space that are associated with large contributions to the Reynolds stress. Based on all of these results, many events have been identified. Each event is identified by the number of velocity conditions required for its definition, the quadrant of each of those velocities, and the locations in space and time of those velocities. Table 4 lists the principal identified events. Of these events, numbers 1, 4, and 5 are the most significant. Each represents a sequence of ejection jets staggered across the wake center-line.
Table 4: Summary of identified events

<table>
<thead>
<tr>
<th>Event #</th>
<th># of velocity conditions</th>
<th>Condition quadrants</th>
<th>Condition locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0 -0.898 0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0 -0.898 0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0 -0.898 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>2.5 -0.898 0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>0 -0.898 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3.5 -0.898 0</td>
</tr>
<tr>
<td>5</td>
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<td>3</td>
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<td></td>
<td></td>
<td>2</td>
<td>1.5 0.898 0</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td>0 -0.898 0</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>1.5 0.898 0</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>0 -0.898 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>2.0 0.898 0</td>
</tr>
</tbody>
</table>

4.5 Correlation Measurements

The results of the correlation measurements described in Section 3.1.2 are presented here as sets of contour plots. Figures 32 through Figures 40 document the \( x/d = 100 \) \( Re_d = 5080 \) experiment (Experiment 5). The data set contains all nine components of the cross-correlation tensor as a function of 9 spanwise separations, 17 normal separations, 1024 temporal offsets, and 17 reference probe locations in the \( y \) direction. The complete data set cannot be presented in this document. The subset chosen illustrates the significant structure and wealth of information available in the data.

Each figure shows one element of the cross-correlation tensor in three planes and for two reference probe locations. The temporal separation between the reference and
the mapping probes $\tau$ is positive for the mapping probe signal leading the reference probe signal. In these plots, the reference point is at $y/b = -0.898$, $z/b = 0$, and $\tau U_\infty/b = 0$ for the three figures on the left side of each page and at $y/b = 0.0$, $z/b = 0$, and $\tau U_\infty/b = 0$ for the three figures on the right side of each page. In each plot, the contour levels are scaled such that there are fifteen levels including one level at 0 (the lowest positive contour). Solid contours represent positive quantities and dashed contours negative ones. The contour spacings are tabulated in Table 5.

Several elements of the wake structures can be inferred directly from these contour plots. The strong three-dimensionality of the dominant structures can be seen in all spanwise correlations. The $y-t$ correlations show an inclined structure in the direction of mean shear. This is very strongly seen in the $u v$ correlations.

To the author's knowledge, these measurements are the most complete correlation measurements in the turbulent far-wake conducted to date. Such detailed measurements were not required for the event reconstruction or the pseudo-dynamic reconstruction portions of this study. For those calculation, only three reference probe locations were required, $y_r/b = -0.898$, $y_r/b = 0.0$, and $y_r/b = 0.898$. The full 17 reference locations were needed by Dr. N. Aubry of the City College of CUNY to calculate proper orthogonal modes from the wake data.

Additional correlation measurements at $x/d = 16$ (Experiments 6 and 7) were conducted and although elements of the results may be questionable due to unresolvable experimental errors (large flow angles), they illustrate the spanwise structure in the intermediate wake. Figure 41 shows one $z-t$ plane of the $v v$ correlation at $y/b = 0.25$
Table 5: Contour spacings for $x/d = 100$ correlations

<table>
<thead>
<tr>
<th></th>
<th>$y_r/b = -0.898$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y$-$z$ $y$-$t$ $z$-$t$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$&lt; u_1u_2 &gt;$</td>
<td>0.1450 0.1339 0.1532</td>
<td>0.1403 0.1322 0.1322</td>
<td></td>
</tr>
<tr>
<td>$&lt; u_1v_2 &gt;$</td>
<td>0.0478 0.0438 0.0467</td>
<td>0.0107 0.0166 0.0108</td>
<td></td>
</tr>
<tr>
<td>$&lt; u_1w_2 &gt;$</td>
<td>0.0320 0.0319 0.0317</td>
<td>0.0193 0.0224 0.0264</td>
<td></td>
</tr>
<tr>
<td>$&lt; v_1u_2 &gt;$</td>
<td>0.0470 0.0398 0.0467</td>
<td>0.0479 0.0604 0.0206</td>
<td></td>
</tr>
<tr>
<td>$&lt; v_1v_2 &gt;$</td>
<td>0.0872 0.0944 0.1006</td>
<td>0.1327 0.1241 0.1241</td>
<td></td>
</tr>
<tr>
<td>$&lt; v_1w_2 &gt;$</td>
<td>0.0181 0.0199 0.0216</td>
<td>0.0123 0.0166 0.0125</td>
<td></td>
</tr>
<tr>
<td>$&lt; w_1u_2 &gt;$</td>
<td>0.0321 0.0249 0.0170</td>
<td>0.0192 0.0229 0.0265</td>
<td></td>
</tr>
<tr>
<td>$&lt; w_1v_2 &gt;$</td>
<td>0.0178 0.0170 0.0136</td>
<td>0.0225 0.0197 0.0118</td>
<td></td>
</tr>
<tr>
<td>$&lt; w_1w_2 &gt;$</td>
<td>0.1199 0.1199 0.1377</td>
<td>0.1707 0.1476 0.1476</td>
<td></td>
</tr>
</tbody>
</table>

for the $x/d = 16$ case. Figure 41 shows a plot of the same variable in the same plane except it was measured with the endplates removed from the model. Oblique shedding or vortex bending and spanwise variation of the shedding frequency are reduced by using endplates. The far downstream effects of endplates and the differences between endplates was not investigated here. However, the increase in two-dimensionality of the dominant structures associated with the use of endplates is clear. This implies that the cylinder ends do not strongly influence the flow near the center of the cylinder. Also, no three-dimensionality is introduced to the near-wake structures by the nature of the shedding. These are sufficient reasons to use the endplates and they have been used in all of the experiments described in this document.

The validity of Taylor's hypothesis was verified using two-point space-time correlation measurements (Experiments 3 and 4). These measurements were taken at $x/d = 100$ and under the same conditions as the correlation measurements were.
Figure 42 shows the temporal correlation coefficient for the $u$ component of velocity at $y/b = 1.347$ and $z/b = 0$ and the $u$ component at $y/b = 1.347$, $z/b = 0.449$, and $\Delta x/b = -0.449, 0, 0.898, 1.796, 2.694,$ and $3.592$. The spanwise separation was required to prevent probe interference.

The lack of significant decay in the correlation peaks with streamwise displacement indicates a very strong relationship between streamwise and temporal separation. The advection velocity at this cross-stream location was inferred from the location of the largest streamwise separation as approximately $0.98U_\infty$. It can be clearly seen that there is some significant structure for separations in $x$ and $z$ of the same order. This makes discerning spatial structure and advection effects difficult. Uncertainty in measurements of the free-stream velocity adds additional unreliability to the advection velocity measurements. Temporal correlations are also shown for $u$ measured with one probe located at $z/b = 0$ and one positioned at $z/b = 0.449$ and $\Delta x/b = 0.898$ for the probes at the same $y/b$ locations. Correlations for four vertical positions are shown, $y/b = 1.347, 0.0, -0.898,$ and $-1.796$. There appears to be some slight variation in the convection velocity with $y$ position, but the effects of the spatial structure confuse the interpretation of these results. Because the estimated convection velocity is very close to the free-stream value, the free-stream velocity will be used as a reasonable approximation of the advection velocity (for representational purposes only) in the remainder of this study. Using this advection velocity, the relationship between the streamwise coordinate and time suggested by Taylor's hypothesis will be approximated as $x^* = -(\tau U_\infty/b)$. 
Figure 32: Contours of $uu$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0$, $z/b = 0$, $y/b = -0.898$, and $y/b = 0.0$
Figure 33: Contours of $uv$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0$, $z/b = 0$, $y/b = -0.898$, and $y/b = 0.898$
Figure 34: Contours of uw correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0.898$, $z/b = 0.898$, $y/b = -0.898$, and $y/b = 0$.
Figure 35: Contours of $\nu u$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0$, $z/b = 0$, $y/b = -0.898$, and $y/b = 0.898$. 
Figure 36: Contours of $vv$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0$, $z/b = 0$, $y/b = -0.898$, and $y/b = 0$.
Figure 37: Contours of $vw$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty /b = 0$, $\tau U_\infty /b = 0$, $z/b = 0.898$, $z/b = 0.898$, $y/b = -0.898$, and $y/b = 0.898$
Figure 38: Contours of $wu$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and the right side for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0.898$, $z/b = 0.898$, $y/b = -0.898$, and $y/b = 0$
Figure 39: Contours of $uv$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0.898$, $z/b = 0.898$, $y/b = -0.898$, and $y/b = 0.898$.
Figure 40: Contours of $ww$ correlations at $x/d=100$ and $Re_d = 5080$. The plots on the left were measured for $y_r/b = -0.898$ and on the right for $y_r/b = 0.0$. The out-of-page plane is, from left to right, top to bottom, $\tau U_\infty/b = 0$, $\tau U_\infty/b = 0$, $z/b = 0$, $z/b = 0$, $y/b = -0.898$, and $y/b = 0$. 
Figure 41: Contours of $uv$ correlations at $x/d = 16$ and $y/b = -0.737$ in the $x-z$ plane with endplates installed (left) and with the endplates removed (right)
Figure 42: Temporal correlations at $y/b = 0.898$ of $u$ for probes separated in $z$ by $\Delta z/b = 0.449$ and for several $x$ separations a) the streamwise separation for curves from left to right is $\Delta x/b = -0.449, 0, 0.898, 1.796, 2.694, \text{ and } 3.592$ b) streamwise separation is fixed at $\Delta x/b = 0.898$ with vertical position $y/b = 1.347(- - - -), 0.0(-----), -0.898(---), \text{ and } -1.796(\cdots\cdots)$
CHAPTER V

STOCHASTIC ESTIMATION

5.1 Errors

Using the detailed correlation measurements at $x/d = 100$, the stochastic estimation procedure was employed to estimate conditionally averaged structures and to reconstruct three-dimensional turbulent velocity fields using the pseudo-dynamic reconstruction technique. In each reconstructed velocity field, the points at which the velocities were specified or measured are termed the condition locations. At all other points in space, the velocities were computed based on the regression functions determined using the space-time correlation function. A single-point estimate is the velocity field constructed based on velocities specified at one point. A two-point estimate is the velocity field constructed based on velocities specified at two points, etc.

Inherent to all stochastically estimated fields is an element of error that varies with separation between the condition location and points within the reconstructed fields. This error is rigorously defined from the error minimization procedure as given in Section 3.2. The measure of error considered here is the fraction of the turbulence fluctuations $(u^2 + v^2 + w^2)^{1/2}$ not predicted by the regression function. The error
of single-point estimations as a function of $y$ for six condition locations is shown in Figure 43. Each curve represents the error associated with estimating the velocities at a given $y$ location for a condition imposed at the point the error drops to zero. There does not appear to be a significantly optimal condition location. The error distributions in the $y - z$ plane are shown as well for selected condition locations. The contour spacings for these and all following two-dimensional error plots is 0.0667. In the cases where the condition is within the turbulent part of the wake, the error increases very rapidly with separation in both $y$ and $z$. However, when the condition is near the edge of the wake, motions on the opposite side of the wake from where the condition is imposed are well predicted. Also, the error decays very slowly with spanwise separation. This suggests the possibility that structures with large spanwise scales are found in the outer part of the wake.

The very high errors predicted by the regression formulas may be somewhat misleading. The estimating function behaves like the correlations from which it is constructed and the correlations decay very rapidly with separation. As noted by Townsend (1954), the correlation between two variables at a separation $r$ contains the integrated effect of all motions having a scale greater than or equal to $r$. The stochastic estimation function is affected by the same integrating effects. The velocities estimated at a separation $r$ represent the superposition of many typical structures with scales $r$ or greater. The true error of the estimation at a separation $r$ should be computed based on the variance attributable to motions having scales that influence the correlations at that separation rather than the total variance caused by all motions.
Figure 43: Estimation error as a function of condition location a) $y$ variation of error for several condition locations b) error in the $y - z$ plane for the condition at $y/b = -1.796$ c) $y/b = -0.898$ d) $y/b = 0.0$
Figure 44: Fraction of $u$ energy in motions having a normalized frequency less than $2\pi fb/U_\infty$ at $y/b = 0.898$

The error of the function can then be thought of as the sum of two components: the variance of those scales that cannot be resolved due to grid spacing, and the phase jitter and shape variations in the motions that contribute to the correlation. It should also be pointed out that the errors associated with conventional ensemble averaging, although rarely calculated, similar, i.e. the ensemble averaged velocities rapidly decay to zero away from the condition location (Adrian, 1979).

By considering the fraction of energy in motions with a frequency less than a given level, Figure 44, an estimate for the component of error associated with the unresolvable energy as a function of spatial separation can be obtained. The grid spacing in $y$ corresponds to a maximum resolvable normalized wave number of 6.99, at two spacings the maximum wave number is 3.50, etc. The error at $y/b = 0$
for a condition imposed at \( y/b = -0.898 \), as taken from the error distribution of Figure 43, is 85%. However, approximately 20% of the total energy is contained in motions having a scale smaller than that which can be resolved at that separation. By considering only the resolvable portion of the energy, the true error is 81%.

It is possible to make multi-point estimations by fixing two or more conditions in time to reduce the estimation errors. Figure 45 shows the errors of a two-point estimate as a function of \( y \). Each curve in part \( a \) of the figure represents the error of a different temporal separation of the conditions. By normalizing the error to 0 to 1 for each spatial separation, the effects of the spacing can be seen more clearly (part \( b \)). The optimal temporal offset scales roughly with the physical separation. For the smallest \( y \) separation considered, the optimal temporal offset is \( \tau U_\infty/b = 0.412 \), which compares to \( \Delta_y/b = 0.449 \). This optimal spacing increases to \( \tau U_\infty/b = 1.2 \) at \( \Delta_y/b = 0.898 \) and reaches a maximum of \( \tau U_\infty/b = 1.5 \) at \( \Delta_y/b = 1.796 \).

Figure 45 also shows the effect of increasing the number of conditions in time. A single-point estimate was considered with the condition location at \( y/b = -0.898 \). Conditions were alternately added at \( \pm 0.412 \) for each increase in the number of conditions \( N_c \). As with the spacing, the errors are considered in ways. The variation of error with \( y \) separation is shown in part \( c \) of the figure and the normalized errors are shown in part \( d \) to better observe the effects of adding additional conditions. For all separations, the error decays with increasing \( N_c \).

Errors can also be reduced by placing more than one condition in span. The errors for two-point estimations for several separations in span are shown in Figure 46. This
type of two-point estimation does not provide significantly better estimates than the single-point. This is expected due to the relatively rapid decay of spanwise correlation in the velocity components.

Placing two conditions separated in the normal direction offers the best improvements in reducing estimation errors as is illustrated in Figure 47. By imposing conditions at $y/b = \pm 0.898$, the error at $y/b = 0$ is reduced from 85% in the single-point estimation to 80%.

In the pseudo-dynamic reconstruction portion of this study, estimations using conditions at three points in space and multiple points in time were considered. The four distributions of conditions in the $y - z$ plane that were investigated are shown in Figure 48.

When reconstructing a velocity field using the stochastic estimation, the error acts as a windowing function, producing a field that decays with the distance from the condition location. This effect is also seen in conventional ensemble averaging. This decay is non-physical and represents the uncertainty introduced by varying scales, jitter, and the conglomeration of many types of structures. To create fields that have a more physical appearance, the estimated velocities can be scaled by the inverse of the error and the true standard deviation of the velocities being estimated.

Compensating for the error in this way is only an ad hoc correction and is not supported by the mathematics or error minimization of the stochastic estimation. In fact, when the error of the estimated fields without error compensation and fields estimated with error compensation are compared, the error is increased significantly.
by the compensation. However, the error compensation does not change the spatial structure of the estimated fields, it only rescales the velocity magnitudes so that the estimated fields have the correct statistical properties.

5.2 Event Field Estimation

The stochastic estimation technique was used to estimate the flow fields associated with each of the events identified in Table 4. In each of the Figures 49 through 55, the velocity field estimates are illustrated by four vector plots. There is one plot of \((u, v)\) in the \(x^* - y\) plane, and three plots of \((u, w)\) in the \(x^* - z\) plane for different vertical positions. The cylinder is located far upstream, at \(x^* << 0\) and \(y/b = 0\), and is aligned with the \(z\) axis. The cylinder diameter is \(d = 0.37b\). Here the local streamwise distance has been estimated using Taylor's hypothesis, i.e. \(x^* = -\tau U_{\infty}/b\).

Event 1 is a single-point ejection event. In the \(y - z\) plane, it has a very simple jet structure moving fluid from the turbulent core outward toward the irrotational outer flow over a streamwise length of \(2.0b\). A \(Q3 - Q2\) event sequence can be seen in the reconstruction. It appears that this is part of the ejection structure and does not represent an independent event. The three-dimensional structure is a pair of double-rollers, inclined in the direction of mean shear. The angle of inclination of these eddies is approximately \(55^\circ\) from the vertical and inclined downstream. This angle must represent an average orientation of these eddies because the mean shear acts to stretch them and incline them further in the streamwise direction. If at some time, a hypothetical vortex structure had its axis aligned with the \(y\) coordinate axis, the structure would be inclined to \(55^\circ\) from vertical after travelling a streamwise distance
Figure 45: Estimation error as a function of number and spacing of conditions.

a) $y$ variation of error for a two-point in time estimate for the temporal spacings $\tau U_{\infty}/b = 0(o---o)$, $0.092(o----o)$, $0.183(o---o)$, $0.367(o\cdots\cdots o)$, $0.550(+\cdots\cdots +)$, $0.733(+---+)$, $1.100(+---+)$, and $1.467(+\cdots\cdots +)$

b) error variation as a function of condition spacing for $y$ separations of $0.449(o\cdots\cdots o)$, $0.898(o---o)$, $1.347(o\cdots\cdots o)$, and $1.796(o\cdots\cdots o)$

c) $y$ variation of error for multiple point in time estimates with 1, 2, 3, 4, 5, 6, and 7 conditions. The symbolic representation follows the same sequence as in part a of this figure

d) error variation as a function of number of conditions for several $y$ separations (see b for the legend)
Figure 46: Error of a two-point in space estimate as a function of condition spacing in the spanwise direction a) $y$ variation of error for several condition separations b) error in the $y-z$ plane for $z_r/b = 0.0$ and $z_r/b = 0.898$ c) $z_r/b = 0.0$ and $z_r/b = 1.796$ d) $z_r/b = 0.0$ and $z_r/b = 3.592$. The conditions are located where the error functions are zero.
Figure 47: Error of a two-point in space estimate as a function of condition spacing in the normal direction a) $y$ variation of error for several condition separations b) error in the $y-z$ plane for $y_r/b = -0.898$ and $y_r/b = 0.898$
Figure 48: Error of three-point in space estimates. The conditions are located where the error functions are zero.
of approximately $25b$. This distance is estimated by considering the action of the mean shear in the vicinity of $x/d = 100$.

In Event 1, the double-rollers bracket the jet structure. It is possible that the jet is a result of a pumping action of the rollers or the rollers could be generated by a roll-up induced by the jet. Event 2 is identical to Event 1 except the sign of all velocities have been changed. This event is an entrainment jet with the same double-roller structure and scale. The major limitation of the stochastic estimation can be clearly seen by comparing Events 1 and 2. The statistical analysis of Section 4.3 showed significant differences between these two types of events. However, the differences are lost in the averaging nature of the stochastic estimation technique.

The double-rollers are very similar to the ones originally presented by Grant (1957). Grant proposed that the double-rollers were independent of the jet structures and represented an entirely different structure. He formulated his model based on inspection of the space-time correlation tensor, so it should not be surprising that the stochastic estimation yields a similar model. Using the proper-orthogonal decomposition, Payne and Lumley (1966) extracted similar eddies from Grant’s correlation data. Aubry (1992) also extracted similar structures from the present data set.

The remaining five structures are two-point estimates and show complex interactions between the structures associated with the imposed conditions. Events 3 and 4 show the interactions between events occurring in succession on one side of the wake. The entrainment-ejection sequence of Event 3 shows that the event spacing dictated from the statistical analysis corresponds to the center to center spacing of eddies if
they were placed in sequence as the figure shows. The angle in the $x^* - z$ plane defined by the eddy centers is roughly $58^\circ$ inclined from the $z$ axis. Because of the direction of structure inclination in the $x^* - y$ plane, the downstream eddy is more strongly effected by the upstream eddy. This is even more true in Event 4 where only the most upstream jet has a clearly identifiable vortex structure.

Event 4 shows the interaction between two jets at large streamwise spacing. A third jet is clearly indicated at the midpoint of the jets on the opposite side of the wake. The velocities at the $y$ location of the conditions has a wave-like structure in $v$ and is decelerated in $u$ in the $z/b = 0$ plane. The opposite is observed at $z/b = \pm 2$ where the flow is accelerated. The spanwise structure is not a simple combination of double-rollers. There is a general movement of fluid toward $z = 0$ that turns and moves upstream with the wavy $v$ pattern discussed. At the position of the most upstream jet, a double-roller is found.

Events 5, 6, and 7 show the interaction of structures that span the wake width. Event 5 shows two ejections in a staggered configuration in $x^* - y$. Structurally, this combination is similar to Event 4. It appears from these figures that this type of staggered configuration of jets, which were observed by Grant and Townsend, is supported by the structure of the space-time correlation functions. Event 6 shows the same combination with opposite sign to illustrate staggered entrainments. The staggered arrangement of ejections and entrainments shown in Event 7 does not show strong interactions between the structures and double rollers are seen on both sides of the wake center.
By comparing the single-point estimates, Events 1 and 2, with the two-point estimates, one realizes that viewing conditions in isolation can be grossly misleading and represent an oversimplification of the turbulent fields. It is the generalization of this concept of using multiple conditions to get more realistic fields which is embodied in the pseudo-dynamic reconstruction described in the following chapter.

Of all of the events considered, Events 1, 4, and 5 appear to be the most significant. By all statistical measures, the staggered jet sequence is predicted as the dominant structure in the wake.
Figure 49: Event 1 velocities at a) y/b=-0.898, b) y/b=0.0, c) y/b=0.898, and d) z/b=0
Figure 50: Event 2 velocities at a) y/b=-0.898, b) y/b=0.0, c) y/b=0.898, and d) z/b=0
Figure 51: Event 3 velocities at a) y/b=0.898, b) y/b=0.0, c) y/b=0.898, and d) z/b=0
Figure 52: Event 4 velocities at a) $y/b=-0.898$, b) $y/b=0.0$, c) $y/b=0.898$, and d) $z/b=0$
Figure 53: Event 5 velocities at a) $y/b=-0.898$, b) $y/b=0.0$, c) $y/b=0.898$, and d) $z/b=0$
Figure 54: Event 6 velocities at a) $y/b$ = -0.898, b) $y/b$ = 0.0, c) $y/b$ = 0.898, and d) $z/b$ = 0
Figure 55: Event 7 velocities at a) $y/b = -0.898$, b) $y/b = 0.0$, c) $y/b = 0.898$, and d) $z/b = 0$
CHAPTER VI

PSEUDO-DYNAMIC RECONSTRUCTION

The events identified and estimated in the preceding sections suggest certain types of fluid motions. Whether or not those motions are realistic is somewhat questionable given that they are computed using time averaged statistics. The Pseudo-dynamic reconstruction technique, outlined in Sections 3.2 and 3.3.1, was used to construct time histories of estimated flow fields so that qualitative comparisons could be made between the estimated structures and velocity fields reconstructed from experimentally measured data sequences.

6.1 Reconstructed Fields

Although the reconstructed fields will be considered as actual three-dimensional velocity fields in the following analysis, the fields are only estimates of the actual instantaneous flow. Because the conditions used to compute the fields are distributed in space, the velocities between them represent interpolated values based on the imposed conditions and the structure of the correlation functions. These velocities should be reasonable estimates of the instantaneous velocities. The streamwise evolution is measured directly from the temporal evolution of the condition signals, and therefore is the actual instantaneous evolution of the flow. The most questionable element of
the reconstruction is how realistic the estimated spanwise structure is. The accuracy of the estimation decays rapidly in span, but it is assumed here that the sequence of conditions and the influence of large $w$ components in the conditions result in realistic instantaneous spanwise structures.

The required time traces of conditions for the reconstructions were measured using three triple-wire probes positioned within the turbulent wake. The locations of these probes were selected based on the error distributions in Figure 48. Two cases were selected for analysis. For the first case, the probes were aligned along the $y$ axis with one at the wake center, one $0.898b$ above the center, and one $0.898b$ below the center. The second case was similar to the first except the probe at $y/b = 0$ was shifted $1.80b$ the positive $z$ direction. It was hoped that this triangular configuration would capture the large scales in the cross-wake direction and provide some information about the structure and spacing in span that does not come only from the correlation data but also from the actual instantaneous measured flow structure.

The condition spacing in time was determined by considering the error of a two point in time estimate as a function of condition spacing. Based on the results shown in Figure 45, the optimal spacing in time was commensurate with the reconstruction grid spacing in $y$, $\Delta r U_\infty / b = 0.41$. Although little improvement in the total reconstruction accuracy is achieved by increasing the number of conditions in time, the relative improvement is significant. Seven conditions in time at each probe location were used so that the length of the time window in the condition signal was roughly equivalent to the large motions being considered.
Because scales smaller than the reconstruction grid cannot be resolved using the stochastic estimation technique, the condition signals were low pass filtered to extract only those scales large enough to be resolved on the grid. The extracted velocity signals were filtered at 400Hz, \(2\pi f(b/U_\infty) = \alpha = 3.66\). This frequency is half the Nyquist frequency defined using the reconstruction grid spacing and 81\% of the turbulent energy is retained in the filtered signals. A comparison of the spectra of the filtered and the unfiltered streamwise velocity data at \(y/b = -0.898\) is shown in Figure 56.

The reconstructions that follow are estimates of the time evolution of a single \(y - z\) plane of data. They can also be considered estimates of three-dimensional velocity fields if the \(x^* - t\) relationship is valid. In that case, the velocity patterns can be considered three-dimensional flow patterns and a direct comparison to typical isolated structures can be made.

In Figures 57 and 58, the reconstruction for probes aligned vertically is shown (Case 1). A small part of the data is illustrated with six vector plots, three crosscuts in the \(x^* - y\) plane and three in the \(x^* - z\) plane. Time has been converted to the streamwise direction using Taylor's hypothesis and the free-stream velocity as the advection speed. The effects of estimation errors can be seen as a rapid decay of the velocity vector magnitudes with increased distance from the condition locations. As discussed in Section 5.1, this decay is not physical but rather represents the uncertainty in estimating the velocity fields.

Figures 59 and 60 show the same segment of data except the velocities have been rescaled by compensating for the error. The large structures can be seen much more
clearly when the velocities are rescaled in this way. Figure 61 shows $y - z$ planes of velocities at selected $x^*$ locations. This small sub-set of data illustrates the richness of structure reconstructed using the pseudo-dynamic reconstruction technique. A brief inspection clearly shows that the wake is a conglomeration of three-dimensional structures rather than some contorted two-dimensional or periodic patterns like a secondary Von-Karman vortex street, as has been postulated in some studies. At $x^*/d = 32$, $z/b = 0$, and $y/b = 0$ in Figure 59, there appears to be a small span-wise vortex at the wake center-line. However, careful inspection of Figure 61 clearly shows that this is only one view of a much more complex three-dimensional structure having scales in all directions of equivalent sizes. Many such misinterpretations can be made from this data. This example illustrates the danger of speculating about
the three-dimensional structure of turbulent flows from two-dimensional information which is commonly done in experiments using hot-wire rakes.

This data segment was arbitrarily selected from a much larger data set, but it shows the significance of many of the statistics used in single and two point analyses. At the far left of Figure 59 are a pair of cross-wake ejections that have strong symmetric properties. Four symmetric streamwise rollers can be seen in Figure 61 at $x^*/b = 28.0$. Structurally, these eddies look much like the double roller structures found in the event reconstruction. Of course, the pseudo-dynamic reconstruction and the stochastic estimation of coherent structures are generated from the same correlation data and a certain degree of agreement can be expected from that alone.

Between $x^*/b = 35$ and $x^*/b = 40$ at $z/b = 0$, a sequence of three jets is observed. They alternate on opposite sides of the wake at a streamwise spacing of roughly $1.5b$. On the top side of the wake there is a $Q2 - Q3 - Q2$ sequence. This event looks very much like Event 4, which implies that the suggested structure and spacing in the jet groups is physical and not an artifact of the statistical procedures. Within the jet group, the most upstream jet is the longest, $Lx = 2.8b$, and it has a very well defined double roller structure in span. The other two are somewhat smaller and do not have as clear of a spanwise structure. It should be kept in mind that these fields are moving to the right and so events at the right precede events to their left in time. Between $x^*/b = 40$ and $x^*/b = 45$, very large spanwise motions at $y = 0$ are associated with rollers spanning the entire wake width. The diameter of these eddies is approximately $2.0b$. At $x/b = 45$ and $z/b = -0.898$, an extended sequence of jets
is observed which appears to be coupled with the large rollers. This reconstruction is further documented with four more consecutive time segments in the following eight figures, Figures 62 through 69.

Throughout these fields, many jet groups can be found. The three-dimensional structure appears to have a simple form. Jets appear for some as yet undetermined reason on both sides of the wake in groups of three or more. The ejected fluid is fed by spanwise motion that converges at the axis of streamwise alignment of the jets. This axis is roughly aligned with the mean flow but varies to around ±30°. It appears that only one set of rollers is associated with any given jet sequence and they appear with the most upstream jet in the group.

One segment of the reconstruction generated using probes in the triangular configuration is shown in Figures 70 and 71. Many of the types of structures found in the previously considered data set and the types predicted from the statistical analysis are found in this data set. There appears to be no significant differences between both arrangements of conditions. At several locations, wave-like motions are seen that are either locally accelerated or decelerated flow a with periodic v component. The wavelength of the periodic component, as estimated at \( x^*/b = 60, y/b = 0, \) and \( z/b = 0, \) is roughly 2.8b which gives a normalized wave number of 2.2. Jets and entrainments are also seen at the expected spacings and with the associated cross-wake vortices. Inspection of many velocity fields indicates that normal vorticity can appear either in double-rollers or in isolated vortices. There does not appear to be any preference for a z-symmetric configuration.
Figure 57: Pseudo-dynamic reconstruction Case 1 without error compensation in the $x^* - y$ plane at spanwise positions $z/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 58: Pseudo-dynamic reconstruction Case 1 without error compensation in the $x^* - z$ plane at vertical positions $y/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 59: Pseudo-dynamic reconstruction Case 1 (part 1) with error compensation in the $x^* - y$ plane at spanwise positions $z/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 60: Pseudo-dynamic reconstruction Case 1 (part 1) with error compensation in the $x^* - z$ plane at vertical positions $y/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 61: Velocities in the $y - z$ plane for Case 1 at $x/b$ at (from left to right, top to bottom) 28.0, 32.1, 36.2, 40.4, 44.5, and 48.6
Figure 62: Pseudo-dynamic reconstruction Case 1 (part 2) with error compensation in the $x^* - y$ plane at spanwise positions $z/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 63: Pseudo-dynamic reconstruction Case 1 (part 2) with error compensation in the $x^*-z$ plane at vertical positions $y/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 64: Pseudo-dynamic reconstruction Case 1 (part 3) with error compensation in the $x^* - y$ plane at spanwise positions $z/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 65: Pseudo-dynamic reconstruction Case 1 (part 3) with error compensation in the $x^* - z$ plane at vertical positions $y/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 66: Pseudo-dynamic reconstruction Case 1 (part 4) with error compensation in the $x^* - y$ plane at spanwise positions $z/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 67: Pseudo-dynamic reconstruction Case 1 (part 4) with error compensation in the $x^*-z$ plane at vertical positions $y/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 68: Pseudo-dynamic reconstruction Case 1 (part 5) with error compensation in the $x^* - y$ plane at spanwise positions $z/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 69: Pseudo-dynamic reconstruction Case 1 (part 5) with error compensation in the $x^*-z$ plane at vertical positions $y/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 70: Pseudo-dynamic reconstruction Case 4 with error compensation in the $x^* - y$ plane at spanwise positions $z/b$ of a) 0.898 b) 0.0 c) -0.898
Figure 71: Pseudo-dynamic reconstruction Case 4 with error compensation in the $x^* - z$ plane at vertical positions $y/b$ of a) 0.898 b) 0.0 c) -0.898
6.2 Symmetry Analysis

The reconstructed velocity fields can be analyzed more objectively by considering some of their statistical properties. The energy in each \( y - z \) plane of reconstructed velocities can be described in terms of contributions by symmetric and antisymmetric motions in both the \( y \) and \( z \) directions. The \( y = 0 \) and \( z = 0 \) lines divide the \( y - z \) plane into four quadrants as illustrated in Figure 72. Each point in the positive \( y \), positive \( z \) quadrant has corresponding points in each of the other three quadrants when reflected about the \( y \) and \( z \) axes. The velocities at these four points can be expressed as \( y \)-symmetric \( z \)-symmetric \((S_y - S_z)\), \( y \)-symmetric \( z \)-antisymmetric \((S_y - A_z)\), \( y \)-antisymmetric \( z \)-symmetric \((A_y - S_z)\), and \( y \)-antisymmetric \( z \)-antisymmetric \((A_y - A_z)\) components. For the streamwise component of velocity, these quantities are

\[
\begin{align*}
    u_{S_y-S_z}(y, z) & = \frac{(u(y, z) + u(y, -z) + u(-y, -z) + u(-y, z))}{4} \quad (6.1) \\
    u_{A_y-S_z}(y, z) & = \frac{(-u(y, z) + u(y, -z) + u(-y, -z) - u(-y, z))}{4} \quad (6.2) \\
    u_{S_y-A_z}(y, z) & = \frac{(u(y, z) + u(y, -z) - u(-y, -z) - u(-y, z))}{4} \quad (6.3) \\
    u_{A_y-A_z}(y, z) & = \frac{(-u(y, z) + u(y, -z) - u(-y, -z) + u(-y, z))}{4} \quad (6.4)
\end{align*}
\]

For \( v \) and \( w \), the equations are formulated such that symmetric \( v \) motions do not cross the \( z = 0 \) axis and symmetric \( w \) motions do not cross the \( y = 0 \) axis. These symmetries are defined so that a set of four streamwise rollers, as illustrated in Figure 72, is a symmetric configuration for all components in all directions. This configuration is that which would be expected from a \( y - z \) cross-cut of inclined double-rollers.
All of the turbulent velocities in the reconstructed fields were broken down into four components based on the four different motion symmetries. Each quantity was spatially integrated over individual $y - z$ planes to give the relative energy contributions by symmetric and antisymmetric motions as a function of time.

Spectra of these quantities are shown in Figure 73 for Case 1 without error compensation and in Figure 74 for Case 1 with error compensation. Results for Case 2 with error compensation are given in Figure 75. The relative contributions to the total energy of each component is tabulated in Table 6.

From the tabulated data, it can be inferred that most of the $u$ energy is in $y$-symmetric and $z$-symmetric motions while most of the $v$ energy is in $y$-antisymmetric motions and the $w$ energy is in $y$-antisymmetric and $z$-antisymmetric motions. This is consistent with the staggered jet sequence pattern which has $y$-symmetric $u$ motions.
The staggering results in an antisymmetric $v$ component. The $w$ component would be expected to be $z$-symmetric if the jets were aligned in the streamwise direction. Slight misalignments contribute to the antisymmetric property.

The spectra of the velocity components divided by symmetry properties for Case 1 without error compensation show some periodic motions in the $v$ component of velocity at a streamwise wave numbers of $\alpha \approx 4.0$ for $y$-antisymmetric motions and at $\sim 3.0$ for $y$-symmetric motions. A slight peak in $y$-antisymmetric and $z$-antisymmetric $u$ motions at $\alpha \approx 2.8$ can also be seen. From the event types and spacings in Section 5.2, $y$-symmetric $v$ motions would be expected at a normalized frequency associated with a staggered configuration of ejections of 4.0. Even though the conditions have been filtered at $\alpha = 3.66$, this frequency was not removed from the signals because it depends on the phase of motions measured at two points in space and no spatial filter has been implemented. The spectra for Case 1 with error compensation shows some additional peaks. The $u$ component shows a peak for $y$-symmetric and $z$-symmetric motions at $\alpha \approx 2.0$ and the $w$ component has a peak for $y$-symmetric and $z$-antisymmetric at $\alpha \approx 4.0$. Only the peaks in $v$ clearly appear in the spectra of Case 2 with error compensation.

Results of this analysis are difficult to interpret because the nature of the analysis includes the effects of structure location within the volume considered and the symmetry properties of those structures.
Figure 73: Spectra for Case 1 without error compensation of the time evolution of the four components to the turbulent energy by symmetry, a) y-symmetric z-symmetric b) y-symmetric z-antisymmetric c) y-antisymmetric z-symmetric d) y-antisymmetric z-antisymmetric u (-----), v (-- -- --), and w (·····)
Figure 74: Spectra for Case 1 with error compensation of the time evolution of the four components to the turbulent energy by symmetry, a) $y$-symmetric $z$-symmetric b) $y$-symmetric $z$-antisymmetric c) $y$-antisymmetric $z$-symmetric d) $y$-antisymmetric $z$-antisymmetric $u$ (-----), $v$ (---), and $w$ (-----)
Figure 75: Spectra for Case 2 with error compensation of the time evolution of the four components to the turbulent energy by symmetry, a) \(y\)-symmetric \(z\)-symmetric b) \(y\)-symmetric \(z\)-antisymmetric c) \(y\)-antisymmetric \(z\)-symmetric d) \(y\)-antisymmetric \(z\)-antisymmetric \(u\) (\(---\)), \(v\) (\(-\cdots\cdots\)), and \(w\) (\(--\cdots\cdots\))
Table 6: Percentage contributions to turbulent energies by symmetry properties for three cases

<table>
<thead>
<tr>
<th></th>
<th>% ((S_y - S_z))</th>
<th>% ((S_y - A_z))</th>
<th>% ((A_y - S_z))</th>
<th>% ((A_y - A_z))</th>
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<tbody>
<tr>
<td>u (no comp.)</td>
<td>47.0190</td>
<td>10.1583</td>
<td>35.0415</td>
<td>7.7812</td>
</tr>
<tr>
<td>v (no comp.)</td>
<td>15.9906</td>
<td>23.4118</td>
<td>40.6711</td>
<td>19.9265</td>
</tr>
<tr>
<td>w (no comp.)</td>
<td>12.9443</td>
<td>28.3156</td>
<td>21.3258</td>
<td>37.4142</td>
</tr>
<tr>
<td>total (no comp.)</td>
<td>31.0477</td>
<td>18.1281</td>
<td>31.5997</td>
<td>19.2245</td>
</tr>
<tr>
<td>u (Case 1)</td>
<td>55.1632</td>
<td>9.6236</td>
<td>30.2213</td>
<td>4.9919</td>
</tr>
<tr>
<td>v (Case 1)</td>
<td>13.6116</td>
<td>23.3046</td>
<td>30.6679</td>
<td>32.4159</td>
</tr>
<tr>
<td>w (Case 1)</td>
<td>11.4556</td>
<td>30.7724</td>
<td>18.4542</td>
<td>39.3178</td>
</tr>
<tr>
<td>total (Case 1)</td>
<td>27.6170</td>
<td>21.1951</td>
<td>25.9872</td>
<td>25.2007</td>
</tr>
<tr>
<td>u (Case 2)</td>
<td>47.2542</td>
<td>7.8347</td>
<td>38.4516</td>
<td>6.4595</td>
</tr>
<tr>
<td>v (Case 2)</td>
<td>18.9892</td>
<td>24.0628</td>
<td>27.3121</td>
<td>29.6360</td>
</tr>
<tr>
<td>w (Case 2)</td>
<td>12.2129</td>
<td>27.5407</td>
<td>13.6169</td>
<td>46.6296</td>
</tr>
<tr>
<td>total (Case 2)</td>
<td>26.7504</td>
<td>19.4560</td>
<td>26.3426</td>
<td>27.4510</td>
</tr>
</tbody>
</table>

6.3 Spectral Analysis

Although no periodicity is observed in turbulent velocities in the far wake, the structure scales and their preferred spacings are associated with certain streamwise and spanwise wave numbers. To see if there are several preferred combinations of wave numbers or a preferred wave angle in the \(x - z\) plane that would not appear in one-dimensional spectral analysis, the reconstructed velocity fields were decomposed in terms of their Fourier components as described in Section 3.3.2. The fields were broken into blocks of data 64 planes in length with successive blocks overlapping by 32 planes and were truncated in span to 16 points by eliminating the \(z/b = 3.58\) data. A total of 256 blocks were considered. Two-dimensional spectra of all three velocity components were calculated at each of the 17 \(y\) locations. These results were
integrated over $y$ and averaged over all available blocks of data. The results of the two-dimensional Fourier decomposition are shown in Figures 78 and 77 for Case 1 with and without error compensation and in Figure 78 for Case 2 with error compensation. The two-dimensional contributions to the spectra have been removed from these figures. The spectra of the $\alpha = 0$ and the $\beta = 0$ contributions are plotted in Figure 79 for Case 1 with no error compensation, in Figure 80 for Case 1 with error compensation and in Figure 81 for Case 2 with error compensation. The total energy of the same three cases was divided 66.8%, 60.8%, and 65.3% in three-dimensional motions, 17.5%, 17.9%, and 15.3% in two-dimensional $\alpha = 0$ motions, and 15.7%, 21.3%, and 19.4% in two-dimensional $\beta = 0$ motions.

For Case 1 without error compensation, there is a relatively strong peak in the $v$ spectra for $\alpha = 1.9$ at $\beta = 0.9$. This peak shifts to a lower value, $\alpha \approx 1.2$, for $\beta > 1.5$. At the larger $\beta$ the peak is very slight and may not represent any physical behavior of the wake. When the error is compensated, a peak arises in the $w$ component at $\alpha = 1.2$ and $\beta = 0.9$. Other than these very small peaks, the three-dimensional energy is well distributed over the range of wave numbers considered. No preferred distinct wave angle or wave combinations is indicated.

The two-dimensional energy shows a very strong preference for $v$ fluctuations at $\alpha = 1.9$ in all of the cases considered. The normalized wave-length of this disturbance is 3.3 which is approximately the spacing of ejections and entrainments at a fixed location in $y$. The two peaks in the $w$ spectra in Figure 80 do not appear to be physical because they do not appear in any of the other spectra.
Figure 76: Two-dimensional spectra of the velocity components integrated over $y$ for Case 1 with no error compensation, a) $u$ energy b) $v$ energy c) $w$ energy d) total turbulent energy
Figure 77: Two-dimensional spectra of the velocity components integrated over $y$ for Case 1 with error compensation a) $u$ energy b) $v$ energy c) $w$ energy d) total turbulent energy
Figure 78: Two-dimensional spectra of the velocity components integrated over y for Case 2 with error compensation a) u energy b) v energy c) w energy d) total turbulent energy
Figure 79: Spectra of the two-dimensional energy of $u(\circ)$, $v(\cdot)$, $w(\times)$, and total (*) for Case 1 with no error compensation a) $\alpha = 0$ b) $\beta = 0$

Figure 80: Spectra of the two-dimensional energy of $u(\circ)$, $v(\cdot)$, $w(\times)$, and total (*) for Case 1 with error compensation a) $\alpha = 0$ b) $\beta = 0$
Figure 81: Spectra of the two-dimensional energy of $u(\circ)$, $v(\oplus)$, $w(\times)$, and total (*) for Case 2 with error compensation a) $\alpha = 0$ b) $\beta = 0$
The analysis of the preceding chapters indicates that there are preferred scales associated with coherent motions in the turbulent wake. If those structures are generated through the action of the mean flow instabilities, one would expect reasonable agreement with the structure and frequencies predicted by stability theory. The stability calculations presented in this chapter have been designed to complement the event identification and the analysis of the reconstructed fields.

A Gaussian mean profile was selected as a base flow. This profile was chosen for consistency with published calculations and due to its reasonable agreement with experimental profiles. The experimental conditions of Experiment 5 were used to establish the stability calculation parameters. Direct comparison with the results of the experimental portion of this study was then possible. The wake deficit $\lambda$ and half width over cylinder diameter $b/d$ were taken from Figure 9 as 0.105 and 2.67 respectively resulting in a $Re_\omega = 1425$. Given the evidence that stability calculations that neglect turbulence over-predict the disturbance growth rates observed in experiments (Marasli, Champagne, and Wygnanski, 1990), the turbulent Reynolds number which accounts for the damping effects of turbulence, was considered the most representative of the flow and was used in most of these calculations $Re_{\omega t} = 22.3$. 

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Using an eddy viscosity to compute a turbulence Reynolds number for stability calculations suffers from the same shortcomings as attempting to use the mean velocity profile to describe the turbulent wake stability properties; i.e. it must be assumed that the unstable perturbations are not influenced by the other large motions present in the turbulent flow. Also, most of the turbulent motions must be contained in incoherent background motions that only act to diffuse the large coherent motions in the wake. Of course turbulence is a strongly non-linear phenomenon and motions interact with each other, but in order to discuss the possibility that instability modes play a role in the turbulent dynamics, the assumptions above must be made. Because the turbulent wake grows as predicted by the turbulence Reynolds number and not the viscous Reynolds number, the wake must be governed by the turbulence characteristics and not some viscous decay of many independent scales. This means that some method must be used to account for the action of the turbulence. Use of the eddy viscosity is the most reasonable technique to take the effects of the turbulent diffusivity into account.

7.1 Primary Instability

Primary instability analysis was used to determine the behavior of the dominant two-dimensional mode of instability in terms of its growth rate, frequency, and shape. The dominant mode is identified as the mode having the largest peak amplification rate within a wide range of disturbance wave numbers. The temporal eigenvalue spectra for four $Re_\omega$ (22.3, 178, 1425, and 91200) are shown in Figure 82. A total of 64 spectral functions were used in each calculation. The grid stretching parameter $q$
Figure 82: Eigenvalue spectra of the viscous Orr-Sommerfeld equation at $Re_\omega$ of
a) 22.3 b) 178 c) 1430 d) 91200
Figure 83: a) Growth rate of the antisymmetric primary instability mode as a function of streamwise wave number at four Reynolds numbers: $R_w = 22.3(*)$, $R_w = 178(\circ)$, $R_w = 1430(\times)$, and $R_w = 91200(+)$. The solid lines are parabolic fits to the computational data. b) Eigenvalue of the antisymmetric ($\circ$) and symmetric ($+$) primary instability mode at $Re_\omega = 22.3$ as a function of streamwise wave number. The wave numbers run upward from 0.1586 by steps of 0.0793.

was set to 0.4, 0.6, 0.8, and 0.8, and the streamwise disturbance wave number $\alpha$ was set at 0.6383, 0.7819, 0.8062, and 0.8049 for the four $Re_\omega$ respectively. The $\alpha$'s were selected as those associated with the most amplified disturbance. There is only one amplified mode that appears in all four of the cases. This is an antisymmetric in $v$ mode of instability and is considered the dominant primary mode. The symmetry is inferred based on the symmetry convention outlined in Section 6.2.

Figure 83 shows the growth rate of this mode as a function of $\alpha$ for the same set of $Re_\omega$. Parabolas are fit to the data to estimate the most amplified streamwise disturbance and the neutrally stable frequency. The disturbance wave numbers of
the most amplified disturbances are noted above and the neutrally stable points are approximately 1.20, 1.51, 1.55, and 1.56 at the four \( Re_w \) considered.

The eigenvalues of the dominant Orr-Sommerfeld mode and the next most amplified mode (separated from the continuous spectra) are plotted as a function of increasing \( \alpha \) in Figure 83. As with the previous calculations, \( q = 0.4, Re_w = 22.3 \) and the number of spectral functions was 64. The minimum \( \alpha \) is 0.1586 and it increases by steps of 0.0793. The trajectory of the eigenvalues follows the arrows in the figure. These trajectories of the eigenvalues were computed using an algorithm originally designed to track particles through a sequence of images (Guezennece et al., 1993). The eigenvalue spectra were computed for each \( \alpha \). The \( \sigma_i, \sigma_r \) pairs of the spectra were treated as coordinates of particles and were followed as \( \alpha \) was varied. The trajectory of any eigenvalue could be tracked through the sequence of solutions and the most significant ones were easily selected from the resulting set of solutions.

The stream function of the dominant two-dimensional disturbance is shown in Figures 84 and 85. For all \( \alpha \), the modes are cross-stream rollers. The tops and bottoms of which are stretched into the outer flow. This distortion is consistent with the mode's amplification rate and the sign of the Reynolds stress in the wake. For amplified modes, the modes shape is such that high speed outer fluid is transported inward to the wake center and low speed fluid is transported outward. In terms of the energy transfer, (Stuart, 1958) the rate of energy increase of an instability in a volume \( V \) is proportional to

\[
\frac{\partial E}{\partial t} \propto - \int_V A \bar{w} (\partial U / \partial y) \, dV + \text{dissipation} \tag{7.1}
\]
Figure 84: Streamfunction of the dominant primary antisymmetric eigenfunctions for a) $\alpha = 0.714$, near the most amplified point and b) $\alpha = 0.234$ amplified at low frequency.
Figure 85: Streamfunction of the dominant primary antisymmetric eigenfunctions for a) $\alpha = 1.190$, near the neutral stability point and b) $\alpha = 3.570$ heavily damped

Here the sign of $\overline{uv}$ determines the direction of energy transfer. Two-dimensional streamlines must be inclined upstream to maintain the proper sign of $\overline{uv}$ even when slightly damped to absorb the energy lost through dissipation.

The shape of amplified two-dimensional modes resembles a sequence of jets and entrainments. The motions span the entire wake width. Entrainments on one side of the wake turn and become ejections on the opposite side. Near the neutrally stable point, the mode shape changes in accordance with the energy transfer. The streamfunction becomes nearly circular and its contribution to the Reynolds stress balances the dissipative forces. At much higher $\alpha$, the mode distorts by stretching downstream. Here the mode is damped by both the dissipative effects and the direction of energy transfer associated with the sign of $\overline{uv}$. 
For all $\alpha$, the cross-stream scale of the disturbance is commensurate with its streamwise scale. The amplified modes have a cross-wake size that is significantly larger than the turbulent portion of the wake. This is a result of the artificially large viscosity assumed by the eddy viscosity approach, which lowers the effective $Re_\omega$.

### 7.2 Secondary Instability

The dominant mode of the preceding section was used as a periodic disturbance to the mean flow and secondary instabilities were computed. For all secondary calculations, the subharmonic instabilities were considered using the procedures and codes described in Flemming (1983). As in those calculations, 256 spectral functions were used. Here, the grid stretching parameter was fixed at 0.4 and the only $Re_\omega$ considered was 22.3. The remaining three parameters, $\alpha$, $\beta$, and $A$ were varied over a range consistent with the measurement grid of the correlation experiment and reconstructed velocity fields.

Figure 86 shows a direct comparison between a primary calculation at $\alpha = 0.714$ and a secondary calculation for $\alpha = 2\alpha_F$, $\beta = 0$, and $A=0.012$. The very small amplitude of the primary disturbance, results in eigenvalue spectra that are very nearly the equivalent of the primary instability results. Seven discrete modes are noted in the figure. All future references to modes by number will be referring to this figure. Four Orr-Sommerfeld modes (1, 2, 5 and 6) and three Squire modes (3, 4 and 7) are seen. The shapes of the Orr-Sommerfeld modes are shown in Figures 87 and 88 for $\beta = 0$ and $\alpha = 0.952$. They are of increasing order in $y$ with Modes 1 and 5 odd in $y$ and 2 and 6 even. Mode 1 is nearly identical to the dominant primary mode.
Figure 86: Comparison between eigenvalue spectra computed using the secondary (+) and primary (o) stability codes at $R_\omega = 22.3$, $\alpha = 0.714$. Seven discrete modes are identified.

For non-zero $\beta$, the secondary modes take on a different character. Figures 89 through 92 show all seven modes at $\beta = 0.874$ and $\alpha = 0.952$. These are all superpositions of the $\pm \alpha$ mode pairs. Both the Orr-Sommerfeld and the Squire modes have a cellular structure of cross-stream vorticity in the $x - z$ plane. The Orr-Sommerfeld modes have a complicated structure involving $v$ as is seen in the $x - y$ plane crosscuts. When both positive and negative wave number disturbances are considered, none of the three-dimensional modes resemble distorted vortex streets. They appear to be cross-stream rollers containing complex fluid motions. For Mode 1 these are sequences of jets and entrainments.

As noted by Flemming and Hultgren, the role of the primary disturbance amplitude in the secondary calculations is small when the amplitudes are sufficiently small.
Figure 87: Shapes of secondary eigenfunctions for $\alpha = 0.714$ and $\beta = 0$

a) Mode 1  b) Mode 2
Figure 88: Shapes of secondary eigenfunctions for $\alpha = 0.714$ and $\beta = 0$

a) Mode 5  b) Mode 6
Figure 89: Shape of Mode 1 for $\alpha = 0.714$ and $\beta = 0.874$ a) top view at $y/b = 0.898$ b) side view at $z/b = 0$
Figure 90: Mode shape for $\alpha = 0.714$ and $\beta = 0.874$, side view at $z/b = 0$ for
a) Mode 2  b) Mode 5
Figure 91: Shapes (contours of $u$ velocity) of Squire modes for $\alpha = 0.714$ and $\beta = 0.874$ a) Mode 3 b) Mode 4
Figure 92: Shapes of a) Mode 6 (Orr-Sommerfeld mode with \( v \) near 0 at low wave numbers) and b) Mode 7 (contours of \( u \) velocity) for \( \alpha = 0.714 \) and \( \beta = 0.874 \)
(A < 0.15). They note, however, that there is some effects on the mode shapes for even very small amplitudes. Figure 93 shows the variation of $\sigma_r$ with $\beta$ for several disturbance amplitudes. As expected, at the low amplitudes considered for a $Re_\omega$ of 22.3, where motions are strongly damped, all growth rates decay with $\beta$, with the exception of Mode 5. This behavior is consistent with the behavior of lower order modes at higher $Re_\omega$.

The primary goal of the secondary instability calculations was to determined the mode shapes of a set of significant discrete modes as a function of $\alpha$ and $\beta$. These modes were to be used as a set of basis functions for the modal decomposition procedure described in Section 3.7. To determine this set of basis functions, first an assumption must be made about the amplitude of the primary disturbance. The amplitude is defined relative to the maximum velocity defect of the wake, $A = u'/U_{1_{\text{max}}}$. It must be somewhere between 0 (which reduces the problem to the primary instability calculation) and the amplitude that would arise if all of the two-dimensional energy was in the dominant primary mode. From the spectral analysis of the pseudo-dynamic velocity fields, approximately 19% of the total turbulent energy is found in two-dimensional motions. The peak streamwise velocity r.m.s for the wake flow being studied is $0.42\,m/s$. Therefore, the r.m.s contribution by all two-dimensional modes is roughly $0.18\,m/s$ and the sum of the normalized amplitudes of all two-dimensional motions is 0.15. This energy is distributed over a wide band of streamwise wave numbers and it is not clear how much is contained in primary modes of instability, secondary modes of instability, and motions arising from other sources. From the analysis of
Figure 93: Growth rates of five discrete secondary modes at $\alpha_F = 1.36$ as a function of spanwise wave number $\beta$ for four primary mode amplitudes: a) $A=0.0$ b) $A=0.03$ c) $A=0.06$ d) $A=0.12$
Flemming (1983), the growth rates and mode shapes of the most amplified primary and secondary instabilities have some differences but are roughly equivalent for small amplitude sinusoidal disturbances, \( A << 0.15 \). Two-dimensional disturbances in this flow are well within this range so it becomes somewhat arbitrary what is called a primary mode of instability and what is called a secondary.

It has been assumed that all of the two-dimensional energy is contained in the dominant mode of primary instability distributed over a band of streamwise wave numbers. The distribution of energy by wave number is given by the \( \beta = 0 \) spectrum of the \( u \) component of velocity (see Figures 79 through 81). The disturbance amplitude was divided among eight wave numbers, the maximum amplitude from which was 0.066 at \( \alpha_p = 0.468 \). This decayed to approximately 0.01 for \( \alpha_p > 0.936 \) and then dropped to 0 above \( \alpha_p > 3.3 \). These amplitudes are a discrete approximation to the actual continuous distribution of energy. Because the amplitude given by this distribution is small, it should not significantly effect the results and it was used as a reasonable estimate for \( A(\alpha_p) \).

The set of \( \alpha \) and \( \beta \) for which modes shapes were calculated was determined from the correlation measurement grid used in the pseudo-dynamic reconstruction. Fourier analysis of the velocity fields in the \( x^* - z \) plane defined a set of streamwise and spanwise wave numbers (Equations 3.29 and 3.31). From the experimental grid spacing, \( \Delta_\alpha = 0.238 \) and \( \Delta_\beta = 0.874 \). Finer gradations were used when required for tracking the eigenvalues. Results of the secondary instability calculations for these wave numbers are shown in Figures 94 through 96. Trajectories of eigenvalues for changing
\( \alpha \) or \( \beta \) are seen in Figure 94. For the \( \alpha \) tracking, the tracks initiate at the mode number labels at \( \alpha = 0.238 \) and progress as \( \alpha \) is increased by 0.238 steps. The value of \( \beta \) is fixed at 0.0. For the \( \beta \) tracking, the tracks initiate at the label numbers at \( \beta = 0 \) and increase by steps of 0.874. Here \( \alpha \) is fixed at 0.714. Modes 1 through 4 remain separate from the continuous eigenvalue spectrum for the range of wave numbers considered. The other modes do not separate from the continuous spectrum until \( \alpha \) is sufficiently large.

The growth rates of each mode are seen in Figures 95 and 96. Each set of curves documents the dependence of \( \sigma_r \) on \( \alpha \) for \( \beta \) varied from 0 to 6.992. At each wave number combination, both the stability problem eigenvectors and eigenvectors of the adjoint problem are computed.

### 7.3 Comparison with Coherent Structures

Both the two-dimensional primary and three-dimensional secondary instability modes have very distinctive structures. As a first test of their influence in the turbulent wake, a qualitative comparison with the structures calculated from the stochastic estimation and proposed in the literature has been conducted. The very small component of two-dimensional energy found in the pseudo-dynamic reconstruction suggests that two-dimensional modes do not play a large role in dictating the shape of turbulent structures. However, the eigenmodes of three-dimensional primary and secondary instability might be more important.

Because the three-dimensional primary modes have the same shape as the two-dimensional primary modes in the \( x - y \) plane, the two-dimensional modes can be
Figure 94: Eigenvalue trajectories with changing wave numbers of seven discrete modes a) $\beta = 0$ and $\alpha$ is varied from 0.238 by steps of 0.238 up to 7.616 b) $\alpha = 0.714$ and $\beta$ is varied from 0.0 to 6.99 by steps of 0.874

considered for comparison with the estimated coherent structures. The spanwise structure of the three-dimensional modes is cells of $\omega_z$ with centers aligned with the inclined streamlines of the two-dimensional modes. This vortical structure is the same for all modes, both primary and secondary, as it is the result of the combination of waves in $x$ and $z$. The spanwise and streamwise periodicity is dictated by this assumed wave structure.

For amplified and neutrally stable two-dimensional modes, the structures are inclined upstream. This is consistent with the Reynolds stress and the closed streamlines of two-dimensional flow. This suggests that the unstable mode eddies are inclined in the opposite direction as the coherent structures determined by the stochastic estimation procedure (also Grant, 1956; Payne and Lumley, 1967). The shape of the
Figure 95: Growth rates of Modes 1 through 4 as a function of \( \alpha \) and \( \beta \). The value of \( \beta \) is varied from 0.0 to 6.99 by steps of 0.874. The amplitude of the primary disturbance is a function of \( \alpha \). a) Mode 1 b) Mode 2 c) Mode 3 d) Mode 4
Figure 96: Growth rates of Modes 5 through 7 as a function of $\alpha$ and $\beta$. The value of $\beta$ is varied from 0.0 to 6.99 by steps of 0.874. The amplitude of the primary disturbance is a function of $\alpha$. a) Mode 5 b) Mode 6 c) Mode 7
correlation function in the $x-y$ plane would also be inclined upstream (opposite to the
direction that has been measured). Damped modes are inclined in the downstream
direction but the $uv$ contribution has a sign opposite to that of $\overline{uv}$.

The dominant stability mode has a structure that spans the entire wake width.
This results in no phase difference between events occurring on opposite sides of the
wake. No evidence was found for this type of motion in the statistical analysis of this
study.

In addition to the structural differences, the predicted frequency of the stability
modes was much lower than any peaks in the experimental data spectra. The effects
of Reynolds number selection cannot account for the discrepancy. The scales of
the coherent structures are significantly smaller than the wave lengths of typical
instability waves.

Despite the bad qualitative match between stability modes and the coherent struc­
tures measured, the role of mean flow instabilities should not be discounted categor­
ically for the turbulent wake. The possibility exists that the mean flow instabilities
could amplify low energy low frequency disturbances. The structures generated in
this way could then deform through the action of vortex stretching and tilting. How­
ever, in such a scenario, the link between instabilities and turbulent structures would
be very indirect and much weaker than is sometimes postulated.
7.4 Stability Mode Projection

If the stability modes of the mean velocity profile exist in the turbulent wake in their undistorted form, a projection of the modes onto three-dimensional velocity fields would show the frequency and type of modes that were present in the flow. With the set of primary and secondary eigenvector solutions computed in the stability analysis, it is possible to attempt such a projection onto the velocity fields computed using the pseudo-dynamic reconstruction technique.

The decomposition was conducted for the pseudo-dynamic reconstruction with error compensation generated using three vertically aligned probes. Results for the other cases were similar to these and do not add any additional useful information to this analysis. The fields were decomposed as described in Section 3.7 using blocks 64 planes in length. A total of 256 blocks were considered, each one overlapping the preceding block by 32 planes.

Modes 1 through 7 described in this Chapter were used as a set of basis functions. Their shapes were computed at the streamwise and spanwise wave numbers dictated by the field decomposition. However, only positive $\beta$ and $\alpha$ waves were explicitly computed. The required $-\alpha$ waves were considered the complex conjugates of the $\alpha$ modes. Projection of the modes onto the fields gave the complex coefficients as a function of time, $\alpha$, and $\beta$. The magnitude of these coefficients were integrated over time to look for preferred wave angles by the different modes. The results of this analysis are shown in Figures 97 through 99. The $y$ axis of these figures is not an energy scale but a coefficient amplitude scale and has no dimensions. Comparisons
between curves associated with different modes should not be made on anything other than a qualitative level because the levels do not represent the mode energies.

The only peak in the spectra for the four low order modes (1 through 4) occurs at $\beta = 0$ and $\alpha = 2.0$ for Mode 1. This peak is not nearly as pronounced in any of the other mode spectra.

The higher order modes are found in a band of frequencies covering the central portion of the range of frequencies considered. It is unlikely that these higher order modes play a significant role in the turbulent dynamics because they are always more strongly damped than the other modes. The modes are increasingly damped for both increasing $\alpha$ and $\beta$ with a frequency of maximum amplification of $\alpha = 0$ and $\beta = 0$. Therefore the peaks in the spectra which do not occur at $\alpha = \beta = 0$ are not associated with more amplified wave numbers of these disturbances.

Due to difficulties in computing the energies associated with each mode, those results will not be presented any further in detail. Preliminary analysis indicates that the low order modes do not contribute significantly to the turbulent energy. Because this does not agree with the most fundamental component of the stability mechanism theory, i.e. that the dominant energy containing motions grow and decay by the action of the mean flow instability, either the procedure is flawed or the most unstable modes are not the most influential motions in the turbulent wake. Considering the indications of the other results of this study, the later is most likely the case.
Figure 97: Two-dimensional spectra of the mode coefficient amplitudes for Case 2 with error compensation, a) Mode 1 b) Mode 2 c) Mode 3 and d) Mode 4
Figure 98: Two-dimensional spectra of the mode coefficient amplitudes for Case 2 with error compensation, a) Mode 5 b) Mode 6 and c) Mode 7
Figure 99: Spectra of coefficient amplitudes for Case 2 with error compensation for $\beta = 0$ of a) Modes 1(––––○), 2(––––+), 3(○–––––○), and 4(+––––+).

b) Modes 5(×––––×), 6(∗––––∗), and 7(×––––×).
8.1 Conclusions

The aim of this study was to identify the coherent structures and their dynamics in the turbulent wake and investigate the possibility that mean flow instabilities are the dominant driving force generating those structures. In general the results of this study are self-consistent and in reasonable agreement with other published results. However, no evidence was found to support the mean flow instability hypothesis and the results support a model of the dynamics that would be consistent with periodic groups of jets as the dominant wake structure.

The mean wake statistics given in Section 4.1 agree well with published results and it appears that at $x/d = 100$ the wake has reached a reasonably self-similar state. The spectral peak associated with vortex shedding has disappeared and the mean flow profile is roughly Gaussian. Experimental errors are consistent with the limitations of the experimental equipment and uncertainties associated with specific details of these facilities (noise, freestream fluctuations, model characteristics, etc.). Detailed measurements of the full space-time correlation tensor were conducted and represent a rich source of information for studying the structures in the turbulent far-wake.
The dominant events and scales are very important elements in the description of turbulence. Ejections of low speed fluid from the turbulent core were identified as the dominant far-wake event and were the largest contributor to the Reynolds stress. These motions were typically larger and more intense than entrainment motions. Based on these and other cited differences between these two fundamental types of motions, it is likely that the structure of entrainments is significantly different from that of ejections. Several estimates for the scales of these events were made using different techniques. They were all in agreement, suggesting that the dominant large streamwise scale of the wake is roughly half of the total width of the turbulent portion of the wake, or $2b$.

The structures estimated using the stochastic estimation technique and single point statistics look like the large double-rollers postulated by Grant (1957). One peculiarity of these structures is that they would appear to have a much longer scale in $x$ than they would in $z$ if the region of high $uv$ was considered alone. If double-rollers/jets are associated with bulges in the turbulence interface, it would be expected that the region of positive $v$ would dictate the shape of the bulge. Based on the estimated double-roller ejection-jet structure, the turbulent bulges would be longer than they are wide by a factor of roughly 2.5. This appears to be what was observed by Barsoum, Kawall, and Keffer (1978). Based on the diameter of the eddies and their expected spacing in $x$, the actual aspect ratio of typical motions is less elongated and is roughly 1.6.
Isolated events did not appear to be important in the wake. Typical combinations and spacings of events tend to support a jet sequence structure. This structure bears little resemblance to a sequence of cross-wake rollers due to the interactive effects of the jets. A simple model developed from the stochastic estimation of such jet groups, is a sequence of jets alternating on opposite sides of the wake at a spacing of $1.5b$. At the most upstream jet in the sequence, large vortex structures containing $\omega_y$ and inclined in the direction of mean shear are found.

The pseudo-dynamic reconstruction technique appears to be a valuable flow reconstruction tool. The relationship between error and qualitative behavior could not be addressed well without the availability of full three-dimensional instantaneous velocity data to use as a comparison. Despite the uncertainties, some three-dimensional patterns predicted by the event identification and reconstruction were clearly observed. Rollers, jet sequences, and entrainments were all observed in the estimated velocity fields. Staggered jet arrangements three or four jets in length were observed most frequently. The groupings were narrow and were roughly aligned with the direction of mean flow. The observed jet spacing agreed with that predicted from the statistical analysis and the quasi-periodic $v$ component appeared to occur at approximately the peak in the $y$-antisymmetric, $z$-symmetric, $v$ component spectra computed from the pseudo-dynamic reconstruction fields, Figure 56. Entraining motions were also observed but were dispersed throughout the field without particular grouping. However, some motions were seen with a wave-like structure in $v$ for the accelerated flow.
The stability analysis conducted was primarily designed to generate the set of eigenmodes required to decompose the pseudo-dynamic reconstruction fields into contributions by stability modes. In the course of generating these modes, the stability characteristics of the wake were well documented. Growth rates and mode shapes calculated were in agreement with those found in the literature. Very slight peaks in the $v$ and $w$ spectra were found in the experiments at the wave numbers predicted by inviscid instability theory ($\alpha \approx 1.5$) but that was the only supporting evidence for the mean flow instability mechanism. The predicted mode shapes and wave numbers based on the mean wake profile did not agree well with those suggested by the reconstructed structures or event spacing. No periodicity in $x$ or $z$ was observed in the pseudo-dynamic reconstruction or in correlation measurements. In addition, the dominant instability mode was $y$-antisymmetric $u$ and $v$. Motions with these symmetries were not found in the experiment.

The mean flow stability theory does not adequately predict the structures observed in the far-wake. If it is accepted that stability modes of the mean velocity profile do not dominate the wake dynamics, then some other theory must be developed that would be consistent with the structures estimated here, observations made in the literature, and general properties of the flow. The non-linear behavior of combinations of disturbances could account for the discrepancies.

It has been suggested (Townsend, 1966) that jet groups could grow out of a Kelvin-Helmholtz type of instability occurring near the turbulent-irrotational interface. A similar process may be observed at the steepening of the inflectional region of the
mean profile that would be generated if a portion of the wake were decelerated. Instabilities of this sort would grow at a much faster rate and at a higher frequency than the instabilities of the mean flow because they would scale with the dimensions of the local inflectional profile. Such a mechanism could account for the observed periodicity in $v$ in generally decelerated parts of the wake. However, a great deal of information about the intermittent region of the wake would need to be obtained before these ideas could be reasonably argued.

8.2 Future Research

The results of this research suggest that many aspects of the wake structure need to be studied in more depth in order to formulate models about the wake dynamics. From the one and two point statistics, there appears to be distinct differences between ejection jets and entrainments. These differences could not be resolved in this study using the stochastic estimation technique due to the averaging nature of the correlation functions used. One possible way to overcome this limitation is by consider the statistics used in Figures 23 through 26. These plots represent the sixteen contributors to the $uv_1(t)$ $uv_2(t + \tau)$ correlations sorted based on the type of events that are present in the flow. This study has shown that there is one dominant state in the wake turbulence, decelerated flow with periodic jets, and possible another state, accelerated entraining flow. Conditional correlation functions could be calculated based on these two states. The condition signal would be taken as the low-pass filtered $(u,v)$ velocity signal measured at the reference probe location. The flow at that point could be classified by using the quadrant technique and conditional
velocity correlations could be built accordingly. The stochastic estimation could then be used to reconstruct the three-dimensional velocity field associated with each state of the turbulence. The result would be a hybrid between conditional averaging and stochastic estimation that keeps the flexibility of stochastic estimation but respects the possibility of multiple states of the turbulent field. The only clear drawback of this approach would be the large data sets required to obtain converged conditional correlation functions. This approach could easily be extended to study other regions of the flow where there could be multiple states. This would be particularly useful to study the structure of turbulent bulges and the turbulent-irrotational interface.

Because there is no evidence to support the theory of linear stability driving the wake motions in a natural wake, the experiments cited in the literature that use forcing of instability modes must be considered for what they are, forced flows. It would be very useful to further support this assertion by conducting a set of experiments similar to those presented here where the dominant instability mode of the wake has been excited. It would be particularly interesting to look far downstream of the region where the wake was receptive to the disturbances and see if the structure of the wake has been changed by the forcing. Such a study would also address the issue of how important initial conditions are to the structure in the far-wake.

The other element of this study that would benefit from additional study would be a careful analysis of the turbulent/irrotational interface. The stress relief jet mechanism proposed by Grant and the interface instability proposed by Townsend are both tied to the structure and behavior of bulges of turbulent fluid. Flow visualization
and temperature marking have been useful in studying the interface structure in the past. The emerging technology of particle image velocimetry would be particularly well suited for these types of studies. Observing the nature by which particles from the outer flow are entrained into the turbulent part of the wake would be very helpful in further formulating a model of the wake turbulence.

A more direct use of PIV and one of more immediate interest is to use PIV to measure true instantaneous velocity fields. These fields could be used to evaluate the pseudo-dynamic reconstruction technique and could be used to directly identify the wake structures and observe their dynamics.
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