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Effects of error correction strategies on long division performance of students with learning problems

Drevno, Gregg E., Ph.D.
The Ohio State University, 1993

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EFFECTS OF ERROR CORRECTION STRATEGIES ON LONG DIVISION PERFORMANCE OF STUDENTS WITH LEARNING PROBLEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

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* * * * *

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To my mother, Gene A. Drevno,
and in memory of my father, Harold Drevno
ACKNOWLEDGMENTS

A number of people have assisted me throughout this endeavor and deserve recognition. First and foremost, I acknowledge the intellectual support and personal understanding of my adviser, Professor Timothy E. Heron. I thank him for his sage counsel, patient criticism and constant kindness. He is a worthy model as a scholar, special educator, friend, and person. Professor Heron has made a meaningful difference in my life. My other dissertation committee members, Dr. Ralph Gardner III, and Dr. Daryl Siedentop, each offered stimulating comments and criticism that greatly assisted me in the formulation and refinement of the ideas guiding this study.

I also want to thank Phil Ward and Marie C. Ward for their support and friendship throughout graduate school. They have been an endless source of encouragement and ideas. I want to thank Stacy Martz for collecting procedural data and commenting on the dissertation. A special thanks is extended to Barbara Nagy for her contributions, both intellectual and emotional. Also, I want to thank Michael Thomasgard for his encouragement and comments regarding this study, and his friendship.

A special thanks goes to the staff and teachers at South High School. In many respects they made this study possible. They allowed me to conduct research with their students, and responded with dispatch to my many requests. School based research happens because of the behaviors and attitudes shown by the fine staff and teachers at South High.
Finally, I owe a profound debt and gratitude to my family. I thank my mother for her love, humor, and belief in me. I am grateful to my brothers, sisters and their families. Their love, encouragement and humor have long been a source of strength for me.
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CHAPTER I
INTRODUCTION

Because of mounting dissatisfaction with student academic achievement, parents, professionals, and politicians have called for unprecedented changes in public education. Some would argue that many students who graduate high school, and certainly students who “drop-out” of school, are not prepared to live in a world where the ability to read, write, and do mathematics is essential to earn a wage and to be a productive member of society.

The demand for educational reform has been heard. The federal government, various educational organizations, and higher education organizations have responded in several ways. In particular, during the last 20 years surveys have been conducted and committees have been convened to identify the purpose of education (Gallup, 1982, 1986; National Commission on Excellence in Education, 1983). Studies have also sought to discern effective ways to manage and operate schools (e.g., Brandt, 1986; Chubb & Moe, 1990; Murphy, 1992), ascertain efficient forms of instruction (e.g., Good & Biddle, 1988) and curricula materials (e.g., Scheid, 1990; Stanic, 1986), and institute efficacious methods of training professional educators (e.g., National Commission on excellence in Education, 1983, National Council of Teachers of Mathematics [NCTM], 1991).
Mathematics education has especially undergone several revisions in the last quarter century (e.g., D'Augustine & Smith, 1990). Presently, the "consensus" among math educators, the broader community of professionals, employers, and parents is that the quality of school mathematics needs to be significantly improved (NCTM, 1991). The National Council of Teachers of Mathematics (NCTM) and the National Council of Supervisors of Mathematics (1988) have set down several standards regarding math curriculum and instruction. NCTM stipulated that elementary and secondary teachers need to be proficient in several areas. The areas covered included (a) selecting and presenting appropriate mathematics materials, (b) structuring opportunities for students to engage in a variety of mathematical activities (e.g., problem solving), (c) directing activities in the classroom regardless of the number of students, (d) teaching students how to use computers and other mathematics related technologies, and (e) assisting students to expand and generalize their knowledge about mathematics.

As stated in the National Council of Supervisors of Mathematics (1988) publication Twelve Components of Essential Mathematics, these math educators have isolated what they consider to be important in teaching mathematics to students. Four of the components relate directly to the instruction of math computational skills: problem solving, communicating mathematical ideas, applying mathematics to everyday situations, and focusing on appropriate computational skills. Each of these components stress the importance having students generalize upon and maintain that which they acquired in the instructional setting.
Recommendations for changes in mathematics instruction also have been proposed by others (e.g., Case, Harris, & Graham, 1992; Cawley & Parmar, 1991; Englemann & Carnine, 1982; Heshusius, 1991; Mercer & Miller, 1992; Silbert, Carnine, & Smith, 1990). A common factor among the above suggestions is that instruction should be tailored to the characteristics and needs of the learner. The ultimate objective of instruction is to enable students to use mathematics to solve practical problems (Scheid, 1990). Indeed the manner in which students are assessed and taught mathematics differs according to the model and theory behind a method of instruction. A method of instruction is distinguished, however, by the degree of “measurable success” it has in teaching students mathematics.

Although the education reform movement is active, widespread, and according to some, cyclical (Powell, Farrar, & Cohen, 1985), Good and Biddle (1988) warned against making hasty decisions in the acceptance of new curricular programs and forms of teaching. They observed that instruction is complicated, and that students come to school with varying backgrounds and learning abilities. In sum, there are no simple answers to teaching students mathematics. Good and Biddle advocated for changes in education and in particular math education based upon empirical data. To act otherwise could lead to costly and potentially counterproductive results.

Under the present conditions for reforms in mathematics education, Cooney, Grouws and Jones (1988) identified three different directions research in math instruction is likely to proceed “(a) analyses of classroom discourse; (b) analyses of effective sequences of moves for teaching concepts, generalizations, and skills; and (c) development of strategies for using the sequences of moves in teaching education programs” (p. 253). It would seem
by implication that a fourth direction for research would concern math education in special education. This seems particularly pertinent given the legal and education foundation of special education (e.g., least restrictive environment), recommended changes in special education (e.g., Jenkins, Pious, & Jewell, 1990; Reynolds, 1988; Reynolds, Wang, & Walberg, 1987; Stainback & Stainback, 1984; Wang, Reynolds, & Walberg, 1986), and adopted standards in mathematics instruction (NCTM, 1991). Mercer and Mercer (1993) have emphasized the importance for special and general educators “to work together to ensure that the instructional reforms are sensitive to the unique learning and emotional needs of student with learning problems” (p. 274). In part, the larger question facing special and regular educators is what to teach and how to teach it to students with disabilities.

Answers to the above questions have been addressed. Pereira and Winton (1991), for example, reviewed 20 years of behavioral research on teaching and remediating mathematics describes some of this activity. Their descriptive summary of over 55 studies showed that a variety of procedures have been used successfully (e.g., reinforcement and modeling), and that most investigations addressed the “consequent component” of the “three-term contingency.” The most often used type of consequent procedure was reinforcement; in comparison, few studies addressed the area of “error correction.” Pereire and Winton commented that “maintenance and generalization” continue to be understudied, although during the last 10 years the number of investigations reporting on these areas has increased steadily. In addition, they observed that although the number of studies using “self-instruction” (SI) in mathematics has grown recently, details regarding the methods and procedures used have been vague, and the analysis of
components that constitute SI have been less than complete. In addition, Pereire and Winton pointed out the need for research in mathematics instruction that assesses the "long-term effectiveness of various behavioral procedures" (p. 32), and that include more non-elementary aged students with and without disabilities.

Several authorities in special education (Cawley, Fitzmaurice-Hayes, & Shaw, 1988; Cawley, Miller, & School, 1987; Hammill & Bartel, 1990; Kelly, Gersten, & Carnine, 1990; Scheid, 1990) have provided evidence that students' difficulties in mathematics often stem from or are compounded by faulty instructional practices. In part, poor instruction includes inaccurate identification and response to students' errors. Although "error analysis" and to a limited extent "error correction strategies" are used by teachers, their relative effects on student achievement has not been well studied (Barbetta, 1991; Barbetta, Heron & Heward, 1993a; Barbetta, Heward, & Bradley, 1993b; Englehardt, 1977; Koorland, 1986; Pereira & Winton, 1991, Silbert, Carnine, & Stein, 1990).

The need for effective instruction in mathematics is particularly important given Cawley and Miller's (1986) claim that, on average, learning disabled students only make one year of progress for every two years of instruction. Coupled with McLeod and Armstrong's (1982) finding that a large number of students whose primary learning disability was reading also had significant disabilities in mathematics adds to the teacher's resolve. Their discovery stands in contrast with reported national averages on the relative lower incidence of math disabilities to reading disabilities among learning disabled students (Garnett & Fleischner, 1987).
Clearly, there is also a need to investigate ways to teach and remEDIATE specific mathematical skills (e.g., basic facts, computational skills, application skills). This is particularly important with those mathematics operations that involve algorithms and problem solving; operations that are functional, and operations basic to more advanced mathematics. In particular, computational division is a skill that based upon an algorithm and is often difficult for students to learn (e.g., Silbert, Carnine, & Stein, 1990; Kalin, 1982; Laing & Meyer, 1982).

There is a great need for education to improve the methods and materials used to instruct mathematics. A renewal in this responsibility has been proposed by recent national standards that describe and measure mathematical literacy in terms of a students conceptual and computational understanding of mathematics (NCTM, 1991). In the event that students have difficulty acquiring computational skills, effective forms of instruction need to be instituted that examine the entire instructional protocols ranging from task presentation to material preparation to feedback mechanism. Error correction is an important component of a comprehensive analysis.

GLOSSARY OF TERMS

The following is a list of terms and definitions used in the study.

ACCURACY -- The number of correct problems solved correctly divided by the total number attempted, multiplied by 100. In the present study, responses will be measured in terms of percentage and rate correct.

ALGORITHM -- The special process or set of rules for solving a certain type of mathematics problem.
AT RISK STUDENTS -- Students not currently identified with a disability but because of physical, social, or psychological factors have a greater risk for school failure than other students in general education.

CONVENTIONAL DIVISION ALGORITHM -- A rule for solving a division problem, whereby the student engages in several mathematics operations (i.e., estimation, multiplication, subtraction) to produce the correct response.

DIGITS PER MINUTE -- To calculate the digits per minute, digits written were counted and divided by the amount of time allocated. For example, 99 digits written during the 3 minutes was calculated as 33 digits per minute.

DIVIDEND -- A quantity that is to be divided by another quantity. It corresponds to the product in a multiplication problem. In the problem 6 6 6 , 6 6 is the dividend.

DIVISION -- The inverse operation of multiplication. The result of dividing one number (the dividend) by another (the divisor) is called the quotient. Division is a mathematical operation for finding a missing factor.

DIVISOR -- The quantity by which the dividend is to be divided. It is written in front of the division sign. In the problem 3 6 6 , 3 is the divisor.

ERROR - An incorrect digit response or digit misplaced on the division problem (i.e., written response to a problem).

ERROR ANALYSIS -- The analysis and classification of types of errors made by a student in response to a long division problem. The following serve as a model for analyzing the potential types of errors students can make in a division problem.

(a) Regrouping - student does not regroup (multiply or subtract).

(b) Process substitution - student changes the process of one or more of the computation steps and creates a different algorithm that results in an
incorrect answer.
(c) Omission - students leaves out a step in the process or leaves out a part of the correct response or leaves out digits.
(d) Directional - student computation is correct, but the steps are performed in the wrong direction and/or order.
(e) Placement - digits or numbers are written in the wrong place.
(f) Random response - the error lacks logical quality, no apparent relationship between problem-solving process and the problem.
ERROR CORRECTION -- The presentation of feedback from the experimenter to students about the accuracy of their written response or application of an algorithm to a long division problem.
FLUENCY -- Refers to the accuracy of response with respect to the speed at which those response were made. Fluency is described as an instructional outcome, and is measured by correct rate.
LONG DIVISION -- Refers to a computation where the steps to solving the problem are carried out above and below the division sign. In contrast, short division refers to reporting the quotient only.
QUOTIENT -- The quantity resulting from the division of one quantity by another. In the problem 6166, 11 is the quotient.
REMAINDER -- The remainder in the division of integers is the part of the dividend left when it does not contain the divisor an exact (integral) number of times. In the problem 17 divided by 5, the quotient is 3, with a remainder of 2.
SHORT DIVISION -- A division problem of relative ease such that the quotient can be calculated directly.

SPECIFIC LEARNING DISABILITY -- A disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations and reasoning.

PROBLEMS CORRECT -- The number of division problems completed accurately in terms of digits and the placement of digits within the problem.

TELL PLUS SHOW -- A type of error correction strategy whereby the experimenter shows the student in writing how to solve a division problem.

TELL PLUS WRITE -- A type of error correction strategy whereby the students write the correct responses (digits) to a division problem according to the experimenter's verbal direction.

WHOLE NUMBER -- A subgroup of integers that includes zero and all counting numbers (0, 1, 2, 3, 4 ...).

STATEMENT OF THE PROBLEM

This study compared the effects of error correction strategies on the acquisition and generality of long division of whole numbers with students with students at-risk for school failure and students with specific learning disabilities. The study evaluated two error correction strategies administered individually to students by the experimenter immediately after the student made an error on a long division problem. Specifically, the study analyzed whether a "Tell Plus Show" or a "Tell Plus Write" error correction strategy more efficiently and effectively corrected a student's long division errors.
In the Tell Plus Show error correction strategy, the experimenter showed students how to do the long division problem. The Tell Plus Show error correction strategy took place immediately after students completed a problem on which they made an error. The students watched and listened as the experimenter solved a long division problem on a worksheet. The experimenter read a script to the student specifying the steps to solve the problem. The script told the student the steps the experimenter was followed as he solved the long division problem. Immediately after the experimenter modeled the correct steps to solve the long division problem in writing, the student used a calculator to check the experimenter's response to the long division problem (i.e., the answer or quotient).

In comparison, with the "Tell Plus Write error correction strategy" the student solved in writing a long division problem on a worksheet according to the experimenter's verbal directions. The Tell Plus Write error correction strategy took place immediately after students completed a problem on which they made an error. In this strategy, the experimenter read from a script the steps to solving the problem. The experimenter provided the solution to the problem at a pace commensurate with the students writing the digits (numbers) that constitute the correct response. For example, the second step to solve the division problem was not presented until students had responded correctly in writing to the first step.

The primary dependent variable was the rate and percentage of digits correct on end-of-session tests of long division problems. These end-of-session tests followed instruction during which error correction strategies were administered or available if the student made an error on a long division problem. Also, maintenance of the long division skills was assessed during
next-session, weekly, and comprehensive tests.

A measure of generality also examined students’ performance on subsequent types of long division problems. An analysis was conducted on the amount of time (sessions) required to “master” a presented type of division problem, and the kinds of errors made by the student across the various of types of long division problems.

Student opinion and preferences of error correction strategies were measured by using an exit interview on which students were asked individually which strategy they preferred and believed to be more effective. Likewise, the teacher’s opinion regarding the error correction strategies was collected after the study was completed.

RESEARCH QUESTIONS

The specific research questions studied in this experiment are listed below.

1. Will a Tell Plus Show or Tell Plus Write error correction strategy result in a higher number and percentage of digits written correctly on end-of-session tests?

2. Will a Tell Plus Show or Tell Plus Write error correction strategy produce increased generality responses on next-session tests?

3. Will a Tell Plus Show or Tell Plus Write error correction strategy result in a higher of percentage of long division problems solved correctly on worksheet problems?

4. Will either of the two error correction strategies result in more long division problems solved correctly on the long division problem immediately after error correction?
5. Will either error correction strategy result in a reduction in the types of errors made by students on end-of-session tests?

6. What are students’ preferences and opinions regarding the two error correction strategies?

7. Did teacher’s report any significant differences in the students’ performance on long division problems?
CHAPTER II
REVIEW OF LITERATURE

This chapter reviews the rationale for and description of mathematics instruction with students with specific learning disabilities. Theories of learning disabilities, cognitive science and applied behavior analysis separate explanations of learning disabilities. In addition, two instructional approaches used with learning disabled students are considered. At this juncture in the chapter, research that concerns error analysis and error correction strategies is presented. Also, research on mathematics interventions with learning disabled students, and research that concerns long division instruction is discussed. Finally, the social validity and generality of behavior change procedures, and future research in error correction strategies related to interventions used in mathematics instructions are considered.

Rationale for Mathematics Instruction for Students with Specific Learning Disabilities (SLD)

A review of the special and general education literature's shows that relative to other academic areas (i.e., reading) less research attention has been paid to instructional methods in mathematics for students with learning disabilities (e.g., Mercer & Mercer, 1993; Fleischner & Marzola, 1988; Rivera & Smith, 1988). The explanations are varied regarding this apparent discrepancy. Bender (1992) attributes the paucity of research to several
mathematics and students without a mathematics disabilities (e.g., Bender, 1985; Blankenship, 1978, 1985; Gajar, 1979; Tifiletti, Frith & Armstrong, 1984).

Finally, the system used in special education to identify students for services may have an influence on the type and quantity of research studies in various academic content areas. According to Bender (1992), when students are assessed for special education, more attention is placed on the assessment of their reading and language arts abilities than their mathematics skills. Of the 7 areas for eligibility in learning disabilities 5 of them relate to reading and the language arts. This disproportionate emphasis can be traced to several factors: number, variety and quality of commercially made mathematics assessment instruments, reason for referral, and methods used to qualify a student for learning disabilities. In relation to other academic areas, there are relatively fewer tests of mathematics and many of these tests have deficiencies (Wallace, Larsen, & Elksnin, 1992). In addition, the systems procedure for establishing a discrepancy between achievement and measured ability often focuses on discrepancies in reading and measured ability (Bender, 1992). According to Fleischner and Mazola’s (1988) most children referred for special education were suspected of having a learning disability in reading. In comparison, relatively fewer children were referred for special education for deficits in mathematics.

The reasons cited previously may only provide a partial explanation for the under emphasis of mathematics instruction in special education. The greater number of students referred and placed in special education because of disabilities in reading and language arts may be a true reflection of the incidence of learning disabilities in these areas. The factors behind differences in incidence rates in the areas of reading and mathematics among
learning disabled students have yet to be completely examined. Nonetheless, the role and importance of mathematics instruction in special education should be noted. A need for additional research into the area of mathematics instruction for the learning disabled is warranted.

Clearly, the ability to compute and solve mathematics problems is important. Students face many situations in and out of school where they must use mathematics. Their use of mathematics is not limited to basic computational arithmetic, problem solving from a worksheet, or textbook exercises, but also includes practical and everyday living situations (e.g., personal finance, health, and home repair) (Blankenship, 1988; Smith, 1989).

Most educators are concerned about students under achievement in mathematics. They are disturbed by the decline in students' test scores on state wide examinations (e.g., Brown, Carpenter, Kouba, Linquist, Silver, & Swafford, 1988; Dossey, 1988; Kouba, Brown, Carpenter, Linquist, Silver & Swafford, 1988; National Center for Education Statistics [NAEP], 1990; Pereira & Winton, 1991), and complaints from the market place about students' difficulties in doing basic mathematics (e.g., Scheid, 1990; Smith, 1989). In special education, student difficulties in mathematics are perhaps more acute. McLeod and Armstrong (1982) found, for example, that a significant percentage of high school aged students with learning disabilities students could perform only third or fourth grade level arithmetic. Some have questioned whether a complete overhaul of mathematics curriculum for special education students is necessary (Smith, 1989).

As mentioned earlier, an overhaul in the content and manner that mathematics is taught in general education has been undertaken (NCTM, 1991). Unfortunately, the National Council of Teachers of Mathematics
Curriculum and Evaluation Standards for School Mathematics (1991) does not provide specific comments and discussion regarding mathematics curriculum and instructional reforms for students with disabilities and particularly students with Specific Learning Disabilities. This is especially problematic given the recent changes in special education and specifically the Regular Education Initiative (REI). REI's proposal that most students with learning disabilities will be educated in the general education classroom implies that the general education teacher will have to meet the needs of students with disabilities. Consideration for the special learning needs of students' with learning disabilities does not appear to be included in the rationale for NCTM Standards. That is, the central purpose of the Standards was founded on the argument that students were not learning mathematics at a level commensurate with their potential and in step with the demands of the market place, and more generally, society. In order for students to improve their acquisition and performance in mathematics, NCTM stipulated that education must not only change the math curriculum, but also change the manner in which students are taught mathematics (NCTM, 1991; D'Augustine & Smith, 1990).

This seeming inattention to the learning requirements of students in special education also extends to the National Assessment of Educational Progress (NAEP). This nationwide assessment of students' skills in mathematics seeks to “Provide information on the educational performance of United States youth and to measure changes in performance over time” (Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988, p. 14). The performance of students with disabilities was not reported on in the Results from the Fourth Mathematics Assessment of the National Assessment of
Educational Performance (Lindquist, 1989). It is unclear as to whether or not students with disabilities even participated in this nationwide testing. Fortunately, the NAEP reported on the performance differences between genders, and on the achievements of students from minority backgrounds with the purpose of addressing their specific needs in mathematics education (Johnson, 1989; Meyer, 1989).

Mathematics Instruction with SLD Students

Several commercial programs have been developed for students with special needs. Briefly, these programs have sought to improve student achievement in mathematics. The strategies used to accomplish this objective differ among the programs. In part, these differences can be attributed to the model of learning and instruction used within the particular program (e.g., DISTAR, 1978). Also, these programs were either specifically developed for students with special needs, for Project Math (Cawley, Fitzmaurice, Goodstein, Lepore, Sedlak, & Althaus, 1976) were considered supplemental, or a part of a math basal series, for example, SRA Mathematics Program.

In light of the many commercial math programs available to special education, Polloway, Payne, Patton, and Payne (1985) reported that most special education teachers use basal math textbooks. Although the stated objectives, structure, and methods of evaluating student achievement varied among the instructional programs, the common objective among all the programs was to change and improve the students mathematics skills.

Rivera and Bryant (1992) outlined several challenges that face the special education teacher in mathematics instruction. These challenges were "to (a) target where and why mathematical breakdowns occur, (b) identify an appropriate curriculum, and (c) develop a variety of remedial activities that
include opportunities for students to explore, practice, and predict and reason about mathematical concepts and skills" (p. 71).

The points described by Rivera and Bryant require some degree of individualized instruction and skill on the part of the teacher to identify and respond appropriately to student's mathematics difficulties. Missing from their "list of challenges" was a statement regarding the circumstances under which mathematics instruction would be conducted. That is, the significant and pervasive impact of the Regular Education Initiative on the instruction of students with disabilities was not addressed (e.g., Stainback & Stainback, 1984; Will, 1986). In contrast, Cawley and Parmar (1991) identify some of the important considerations that need to be presented before placing a student with learning disabilities in a general education math classroom. They indicate that in order for the student with learning disabilities to achieve and succeed in the regular education math classroom, school personnel need to determine the extent of the student's math difficulties and his or her need for (a) instructional adjustments, or (b) curricula and instructional adjustments. In addition, the student and the general education teacher must be committed to the instructional and/or curricula modifications. In other words, the student and teacher must be convinced that the modification is worthwhile (Cawley & Parmar, 1991).

There has been much criticism and debate regarding the manner in which students with learning difficulties should be taught mathematics. Some individuals have criticized the stress placed on accuracy rather than mastery (Mercer & Mercer, 1993). Still, others have said that computational skills have been overemphasized to the neglect of teaching math problem solving skills (e.g., Case, Harris, & Graham, 1992; Cawley & Parmar, 1991; Cawley & Miller,
1989; D'Augustine & Smith, 1990; Mastropieri, Scruggs, & Shah, 1991; Mercer & Miller, 1992). Some have even questioned the appropriateness of the standard mathematics textbook (Englemann, Carnine, & Steeley, 1991; Scheid, 1990), and traditional methods of instruction (e.g., Carnine, 1989; Cawley, Fitzmaurice-Hayes, & Shaw, 1988; Cawley, Miller, & School, 1987; Kelly, Gersten, & Carnine, 1990; Scheid, 1990). Finally, concern has been raised by the amount of time used for mathematics instruction (Carpenter, 1985), and the apparent acceptance of low levels of achievement in mathematics among students with learning disabilities (Deshler, Schumaker, Alley, Warner, & Clark, 1982; Warner, Schumaker, Alley & Deshler, 1980).

A common thread underlying the above list of criticisms and call for change in mathematics instruction is that all students, including students with math deficits-learning disabilities, can learn mathematics. Under appropriate circumstances students with math deficits can acquire not only accuracy in solving math problems but also display mastery of skills within the content area (Bartel, 1990; Wallace & Larson, 1979).

Theories of Learning Disabilities

There are numerous and different explanations regarding the causes of learning disabilities. The theoretical perspectives on learning disabilities range from a medical model (e.g., Cruickshank, 1985) to a model that incorporates certain societal and political values in its interpretation (e.g., Heshuisus, 1991; Poplin, 1988).

Although there are several theories of learning disabilities, it is generally explained from a cognitive science or applied behavior analytic perspective. The predominant influence of these two approaches is clearly seen in the legal definition of learning disabilities (Federal Register, 1977),
and the manner in which students are evaluated for eligibility for special education services. Essentially, the federal government definition requires an assessment of those psychological processes (e.g., auditory & visual perception) and/or environmental factors that may interfere with the student's learning. Placement of a student in this type of special education program is generally based upon (a) the elimination of non-related causes for the student's difficulties in school (e.g., deafness, mental retardation, social disadvantage), (b) a review of the student's record of school achievement, (c) behavioral observations of the student in an academic setting, and (d) a quantified discrepancy between the student's estimated psychological processes (i.e., cognitive and spatial abilities) and his or her estimated level or school achievement.

**Cognitive Science**

The cognitive sciences or constructionist view of learning disabilities attributes math difficulties to deficits in various cognitive constructs (e.g., memory, processing deficits). These cognitive deficits can be exacerbated by inappropriate instruction or instruction that does not address the student's difficulties (Cawley & Parmar, 1991). Fleischner and Garnett (1983) proposed that students with learning disabilities in mathematics exhibit difficulties in "processing" math problems. Specifically, the complexity of the problem is beyond the information-processing capabilities of the student. The student is not discriminating adequately, processing and responding to the math problem. For example, Case et al. (1982) attributes some learning disabled student's processing difficulties to presenting math problems that over-tax his or her "central processing capacity." Central processing capacity refers to a construct that represents the student's ability to manage and solve a given
math problem. To remediate this problem, the teacher would adjust the difficulty level of the problem, but with the intention of developing the student's capacity to solve more difficult problems.

Cawley, Fizmaurice-Hayes, and Shaw (1988) conceptualized cognitive functioning as a three-way interaction between the instructional content (e.g., computation, word problems), mediating factors (e.g., short- and long-term memory), and cognitive processes (e.g., analysis, & evaluation). At some point in the interaction there is a breakdown in processing that prevents the student solving a mathematics problem. While constructionistic theories on learning disabilities do not view the learner as incapable of learning mathematics (e.g., Kirby & Becker, 1991; Scheid, 1990), the student with disabilities is considered not to have developed specific cognitive functions (Cawley et al., 1988). Thus, it is the teacher's or instructional designer's responsibility to provide a mathematics program that develops the student's cognitive abilities. Cawley and Parmer (1991), for example, established a set of guidelines for teaching the learning disabled student: (a) use standardized test data with caution; (b) prevent the student from "habituating incorrect routines;" (c) set standards such that the student is correct more often than he or she is incorrect; in other words, make the learning activity as successful as possible for the student; and (d) "offer the [student] alternative ways to acquire and to demonstrate proficiency with the skills, concepts, and problem-solving components of mathematics" (p. 428).

**Applied Behavior Analysis**

Applied behavior analysis conceptualizes learning disabilities in terms of "maladaptive behavior" (Neeper & Lahey, 1988) and views the learner's deficits as a function of his or her learning history or acquisition of
"inappropriate responses to instructional stimuli" (Koorland, 1986, p. 298). In particular, Blankenship (1985), Blankenship and Baumgartner (1982) discuss learning problems in mathematics from a behavioral perspective. They describe a student's learning problems in terms of accuracy, rate, inconsistency (maintenance), and application (generality). Blankenship (1985) like Koorland (1988) attributes student's mathematical difficulties to inadequate instruction, poor stimulus control, poor schedules of reinforcement, and limited opportunities to respond to math problems.

Unlike cognitive science, applied behavior analysis focuses on the interaction between the environment and behavior. It avoids the use of hypothetical constructs to explain behavior (Malott, Whaley & Malott, 1993). Applied behavior analysis is based upon certain principles and fundamental concepts, and attributes behavior to events (Cooper, Heron, & Heward, 1987). Behavior is considered lawful and viewed as movement through time that makes some measurable change in the environment (Johnston & Pennypacker, 1980). Behavior is assumed to be interrelated to the events or stimuli that precede and follow it. This interrelationship is called the three-term contingency (Cooper et al., 1987; Sulzer-Azaroff & Mayer, 1991). The events or stimuli that precede behavior or a response are called antecedent stimuli. Antecedent stimuli serve as a signal that a response is or is not likely to be reinforced. A reinforcer or reinforcement is any event or stimuli that follows a behavior or response and maintains the future probability that the response will be emitted again under similar circumstances (Cooper et al. 1987). "All applied behavior analysis procedures involve manipulation of one or more components of the three-term contingency" (Cooper et al., 1987, p. 30).
In addition to the fundamental concepts and principles of behavior, applied behavior analysis is founded upon certain defining attributes. Bear, Wolf, and Risley (1968) delineated seven defining characteristics. First, applied behavior analysis seeks to address problems that are of pressing concern for the individual. In the present context, this could mean increasing the accuracy and rate at which a student solves story math problems. Second, difficulties are defined in terms of behavior; behavior of the individual or learner that can be measured. This criterion of measurement implies two things: (a) the student’s behavior is measured, and (b) the student’s behavior is measured directly. In other words, the behavior targeted for change is measured directly and “continuously.” Third, applied behavior analysis attempts to establish a functional relationship between a change in the environment and change in behavior. An example of this third characteristic would be to increase a student’s accuracy and rate in doing story math problems by imposing a certain kind of error correction feedback.

The fourth characteristic of applied behavior analysis is that the procedure used to change behavior is communicated clearly and effectively. The methods are described in such way that they could be replicated by another person. In other words, the intervention is “technological” (Baer et al., 1968). For example, the teacher would need to know when and how to deliver an error correction strategy.

The fifth characteristic of applied behavior analysis calls for procedures to be “conceptually systematic” (Baer et al., 1968). The point being that methods of behavior change stem from principles of behavior. According to Baer et al. by conceptualizing a problem or behavior change project in terms
of principles of behavior, the science is more likely to be advanced and accepted.

As noted earlier applied behavior analysis seeks to modify behaviors that are important and socially significant. What constitutes a socially significant behavior varies according to the context and the individual. For example, for a ninth grade student learning how to solve algebraic word problems independently might be the most pressing problem. Thus, the sixth characteristic of behavior analysis is to make modifications that are "practical" and "effective."

The final defining characteristic of applied behavior analysis is for the "new" behavior to occur in settings and contexts other than the place of instruction or training (Baer et al., 1968). For example, the student exhibits the skill to solve algebraic word problems in other classrooms, under different circumstances (e.g., examination), and with different types of problems (e.g., word problems that incorporate geometrical figures).

Blankenship (1985) describes many of the advantages that applied behavior analysis offers to students who have learning disabilities or difficulties in mathematics. According to Blankenship, perhaps the most prominent benefit is its system for targeting or identifying the student's difficulty, instituting an intervention, measuring the effect of the procedure (e.g., error correction strategy), and thereby making instructional decisions. Clearly, it is a data-based approach to instruction that enables the teacher to demonstrate what has occurred for the good of the learner.

Instructional Approaches

Within special education there are several models and theories of instruction. Direct Instruction (DI) (Englemann & Carnine, 1982) and holistic
education (e.g., Heshusius, 1982; Poplin, 1988) are two approaches to instruction that represent opposite sides of the continuum in terms of philosophy and critical features of instruction. In many respects, each is the antithesis of the other. These two orientations have different assumptions regarding the rationale, methods of instruction, and measurement of student achievement. Each has a distinct view on what the teacher's role and function is during instruction. The following sections will describe each of these instructional orientations.

**Direct Instruction.** The term Direct Instruction refers to specific practices and methods of teaching and classroom management. In this context, direct instruction refers to teachers who display a particular combination of behaviors during classroom instruction. For example, these teachers have prepared lesson plans, carry-out their lesson plans, maintain and monitor students engaged time on-task, provides students with ample opportunities to respond in classroom, and provide students with positive and corrective feedback (Anderson, Evertson, & Brophy, 1979; Rosenshine, 1979).

The term direct instruction is also used by others (Engleman & Carnine, 1982) to describe and name a particular approach to mastery-learning (Becker, 1986). Direct Instruction as used by Engleman and Carnine (1982) and Gersten, Carnine, and White (1984) names an approach to instruction and curriculum design based upon "a comprehensive analysis of instructional variables that include factors that the teacher is ultimately responsible for" (Silbert Carnine, & Stein, 1990, p. 2).

The term direct instruction was coined by Rosenshine and Furst (1973) based upon a number of large-scale research studies that sought to identify those instructional variables that were associated with higher student
achievement. This line of research has come to be referred to as "teacher effects" research (Brophy & Good, 1986). A basic finding of this area of research has been that teachers who take an active and directive role during instruction generally have students who achieve higher test scores and grades than students whose teachers were not directive.

Although there is overlap between "teacher effects" research (e.g., Brophy & Good, 1986; Rosenshine & Stevens, 1986) and Direct Instruction (Englemann & Carnine, 1982), there are several differences between the two approaches. First, each evolved from different circumstances. The objective of teacher effects research was to identify characteristics common among teachers considered effective in their instruction and management of students (Rosenshine & Furst, 1973; Rosenshine, 1979). In contrast, Direct Instruction originated from the compensatory education movement and specifically Project Follow Through (Becker, 1986). Its aim was to accelerate and maintain the learning gains of students from disadvantaged backgrounds.

Also, Direct Instruction involves a specified curriculum and instructional design (Gersten, 1985) that can be tailored to individual or groups of students. The form of direct instruction described by Rosenshine and Stevens (1984) does not contain a developed curriculum. Rather, the emphasis of the direct instruction model is on how the teacher interacts and directs students during instruction regardless of the curriculum.

The theory or concept behind each form of direct instruction is different. Direct Instruction (Englemann & Carnine, 1982) is a model of mastery learning that has integrated aspects of Bruner's (1966) theory of instructional design, instituted principles and practices of applied behavior analysis, and used what is referred to as a "logical analysis" to determine the critical features of content
area (Becker, 1986). In comparison, direct instruction as described by Rosenshine (1979), Rosenshine and Stevens (1984), and others (e.g., Brophy & Good, 1986) is not considered behavioral in the applied behavior analytic sense of the word. Within the direct instruction model the objectives are different, and the activities of the learner may be conceptualized in terms of cognitive constructs.

The Direct Instruction model developed by Engleman and Carnine (1982) and others (e.g., Gersten, Carnine, & White, 1984; Silbert et al. 1990) has been used and researched extensively in special education. Given the focus of this proposed dissertation, that is, instructional strategies with students with learning disabilities, and the emphasis of the Direct Instruction model on instructional design, the remainder of this section will present the basic assumptions and presentation principles of the Direct Instruction model of instructional design.

The Direct Instruction model assumes that for the most part students success or failure depends upon the kind and quality of instruction delivered. When a child fails to learn in school, certain factors were not present in the environment. Teachers, therefore, must present, monitor, and individualize instruction to ensure that a student acquires the academic skills or subject matter. Thus, a primary focus of Direct Instruction is on the antecedent variables or stimuli that set the occasion for learning to occur (Gersten, Carnine, & White, 1984).

Direct Instruction also makes the assumption that students learn from examples (Carnine, 1983). Students must receive an ample number and variety of examples and nonexamples of the given skill before they can adequately generalize and discriminate between untrained examples and
nonexamples of the targeted response. For example, it is not likely that students will learn to solve addition problems after solving only a few of such problems. Rather, students are more likely to acquire and generalize the addition skill after they have responded correctly to a variety of addition problems. Thus, the instructional goal of acquiring addition computational skills depends upon the manner (i.e., teacher presentation) and form (i.e., curriculum materials) by which the computation problem are presented.

A third assumption of Direct Instruction is that the instructional communication must specify not only the concept or skill to be acquired, but also communicate the concept or skill at a level understandable to the learner (Carnine, 1983). The notion of being “understandable” means that communication must embody features or characteristics of the skill that learners can execute, and thus demonstrate their level of acquisition or ability to conduct the skill or understand the concept.

This third assumption involves what Englemann and Carnine (1982) call classifying “knowledge” according to its features or characteristics, and establishing instructional communications that serve to elicit from learners a demonstration of “any cognitive operation.” In the Direct Instruction model, a cognitive operation refers to learners making either simple discriminations or engaging in “complex operations” (Carnine, 1983, p. 23). The reference to executing a cognitive operation refers to the instructional communication setting the occasion for learners to do overtly, as much as possible, that which they are doing privately or covertly. The purpose for this “overt display” is to allow the teacher or instructor to monitor students’ acquisition of the skill or concept, and thereby change, refine, or maintain the manner and content of communication. According to Englemann and Carnine (1982) and Becker
(1986) students' overt actions enable them to receive feedback regarding their performance and thereby "extract the critical features" about the skill or concept. In other words, as much as possible, the teacher is enabled to observe learners "do" that which is normally unseen or seldom monitored, and "shape" students responses through verbal or other feedback. An example of a student displaying covert behavior would be to have the student overtly carry out the steps to a long division algorithm according to the teacher's scripted directions. The script would include providing the student with immediate feedback regarding the accuracy of his or her response.

The classification of knowledge is a key component of the Direct Instruction model. Carnine (1983), Englemann and Carnine (1982) categorize knowledge in terms of (a) basic forms, (b) joining forms, and (c) complex forms. Each of these categories of knowledge reflect skills or concepts of differing complexity, and in turn require a different kind of instructional communication. Basic forms represent simple concepts or skills "that cannot be clarified through verbal explanations" (p. 23). For example, students are taught the concept of quantity by showing them that regardless of the order of eight individual marbles, the total number of marbles remains the same.

The second category, joining forms, represent relationships between two or more basic forms concepts (e.g., The boys had eight marbles between them). The third category, complex forms involve complex concepts or skills to which learners make multiple responses. Complex forms "join together facts into systems of related facts and join together rules into problem-solving routines" (Becker, 1986, p. 209). For example, a student carries out and verbalizes the steps to a long division problem. In sum, conceptualizing
knowledge in terms of its structure or complexity enables the teacher to
sequence instruction such that students make the necessary discriminations
and generalization to acquire and master a given skill or concept.

Along with the above assumptions, the Direct Instruction (DI) model
institutes several key principles of instructional design. According to Becker
(1986), these key principles have been found to be efficient and effective in
setting the occasion for the teacher to develop and present lessons “that will
lead the student to make appropriate generalizations across the widest
possible range of applications” (p. 220). These principles include the small
group teacher directed instruction, specification of objective, construction,
sequence and pacing of lessons, methods of assessment, procedures for
correcting students errors, and the selection or construction of instructional
materials. The following discussion will use examples from mathematics to
illustrate these key principles of the DI model of instruction.

The DI specification for small group teacher directed instruction
operationalizes many of the assumptions of the model. First, the teacher is in
a better position to respond immediately to students' responses, and more
specifically correct student's mistakes. In addition, students have more
opportunities to respond to a teacher's directions and questions in a small
group of students. Also, with a small group of students the teacher can vary
the pace of instruction in relation to students performance more easily.
Finally, working with a small group of students better enables the teacher to
monitor and provide immediate feedback to the student.

Within the DI model, the instructional program must specify the long-
term objectives of the program. The question of what will the student know
upon completion of the program or unit of study must be stated at the
beginning (Silbert et al., 1990). In addition, the teacher or designer must specify the students' achievement in terms of observable and measurable behaviors. Silbert et al. (1990) describe these behaviors as performance indicators in which the type, accuracy, and rate criteria are specified. For example, students will have demonstrated the competency to solve single digit divisor and dividend long division problems, when they can solve at least 18 of 20 problems in two minutes.

As noted earlier, an objective of DI is to use examples to teach the target skill that will then allow the student to generalize the target response to other examples. In part, this method of instruction attempts to avoid the memorization of facts or rules. Rather, the student is taught a rule or strategy to follow when presented with certain situations. For example, in a long division problem the student is taught to view the problem as a series of steps and as such to indicate (e.g., through marks made on the problem) when individual steps have been completed and the problem has been solved.

The objectives of any instructional program are most likely to be accomplished if the student has the necessary prerequisite skills to attain the stated objective. In other words, the student must have the necessary entry skills to proceed from one component of a skill to another. The acquisition of these skills is especially important if the student is eventually to acquire a strategy for solving a mathematics problem or engaging in other academic skills.

The need for pre-skill training when instituting the DI model of instruction has empirical support (Kameenui & Carnine, 1986). In the DI model, the teacher or designer must sequence the components of a skill or "pre-skills" (Silbert et al., 1990) in such a way that the student acquires the
skill. The intent of teaching pre-skills or components of a skill is to (a) ensure the student learns all of the components of the skill, and (b) identify and address those pre-skills that cause the student problems (Becker, 1986).

Combined with pre-skills identification and instruction, the teacher or designer must also be sensitive to the sequence in which the skills are taught. The objective is to determine the "optimum order for sequencing new information and skills" (Silbert et al. 1990, p. 4) and thereby help to ensure learning. According to Becker (1986), there are three rules or guidelines to follow when sequencing skills "(a) teach the preskills for a given strategy before teaching the strategy (b) teach easy skills before more difficult skills, (c) separate in time those concepts, skills and operations likely to be confused" (p. 222). In sum, the guidelines account for the eventual objective, ensure student success and thereby motivate or reinforce the student, and seek to prevent learning conditions that might be aversive to the student.

Another key principle of DI programming, the teacher must monitor student performance routinely (Carnine, 1983). Within the DI model students' performance can be monitored in several different ways, for example, students' written work and oral responses. More specific to this principle, however, monitoring should occur on a regular basis, and result in data that can aid the teacher in making decisions regarding the students' instructional needs. In other words, monitoring should yield information regarding the students' mastery of a particular skill. Carnine (1983) considers Precision Teaching an example of a system for evaluating the rate and accuracy at which students demonstrate mastering of a skill or subject area. The Precision Teaching measurement system can also serve to motivate or reinforce students for their achievements (Carnine, 1983).
As noted earlier, a central characteristic of the DI model is to prevent as much as possible student failure during instruction. In the event that the student makes an error, the model calls for immediate correction of the student’s error. Although the error correction strategy used can take several different forms, the general message communicated to the student is that he or she made an error followed with the teacher providing the correct response. The teacher will then direct the student to repeat the correct response. As a part of this error correction procedure, the DI model requires the teacher to diagnosis the cause of the error (Carnine, 1983). Specifically, the teacher needs to determine if the student was inattentive during instruction, in which case the teacher will work “on increasing the students’ motivation to attend” (Carnine, p. 39). In comparison, if the teacher determines that the error was caused by a lack of knowledge, then the teacher must identify and correct the skill deficit.

Another principle of DI is to provide the student with ample opportunities for engagement in the academic activity or skill. This is also referred to as academic engaged time (Rosenshine & Berliner, 1978). Simply put, students learn by doing, by being engaged in the targeted skill or skills. Carnine states “if students are expected to learn to read or solve computational problems, they must be engaged in reading-related or math related activities” (p. 41). It is apparent that academic engaged time alone will not lead to learning. Within the DI model, however, it plays a critical role in students acquiring skills.

Finally, according to Silbert et al. (1990) “the quality of mathematics instructional [or any instructional] materials is an important factor in determining not only how quickly some students will learn new skills but also
whether some students learn certain skills at all" (p. 15). Within the DI model this is especially critical given the assumptions and principles of presentation. Thus, Silbert et al. (1990) have recommended that consumers (teachers or designers) of instructional programs evaluate the prospective program for its (a) emphasis on teaching students problem solving, (b) presentation or sequence of skills from easiest to most difficult, (c) examples that represent what the student has been taught, and (d) opportunities for students to practice and review skills. Although there are curriculum materials developed according the Direct Instructional model, for example DISTAR (1975) commercially made materials can be adapted for use.

In summary, Direct Instruction is one approach to mastery learning. It adheres to certain assumptions about the student and the teacher's responsibility during instruction. Direct Instruction is diagnostic and prescriptive in arranging its instructional material, and attempts to be efficient and humane in leading students to the acquisition, generalization, and maintenance of new learning (Silbert, et al., 1989). Originally, the model was developed for students from disadvantaged backgrounds, however, it has also been used successfully with learners of different backgrounds and ability levels (Becker, 1986). Finally, Direct Instruction is based upon a system of instruction that involves three analyses: analysis of behavior, analysis of knowledge systems, and analysis of communication or stimuli, that addresses cognitive learning (Englemann & Carnine, 1982).

Holism. Holism is defined as "a theory that the universe and especially living nature is correctly seen in terms of interacting wholes ... that are more than the mere sum of elementary particles" (Webster's New Collegiate Dictionary, 1979). Those who apply holism to education make the argument
that learning occurs through a person's interactions with others and the environment. Specifically, holistic education makes certain assumptions about the learner: a learner creates meaning and purpose from their interactions, a learner is self-organizing and self-regulating (Heshuisus, 1991; Poplin 1988). Holistic education views learning as "ongoing and continual" (McNutt, 1984, p. 315) and the learner as being an active and reactive participant in the learning process. The holistic educator conceives learning as being generated within the learner (Kronick, 1988). In addition, some who support a holistic theory of learning view it more broadly as a paradigm (e.g., Heshusius, 1989). Poplin (1989) has expanded the assumptions of holistic education to include structuralists values and constructivist beliefs, and renamed it "holistic constructivism." Holistic constructivism or holism is nonreductionistic and nonmechanistic in its analysis and explanation of the learner, and instruction (Heshusius, 1986; Poplin, 1988).

In education, the theory of holism has been applied primarily to the areas of reading and language arts instruction and called the "whole language approach." The theory has also been used in special education and in particular learning disabilities (e.g., Heshusius, 1984; Poplin, 1988, 1984; Kronick, 1988). Those who support a holistic concept of education and special education have argued against the exclusive use of traditional scientific or empirical methods of investigation, and methods of instruction that stem from such inquiry (Heshusius, 1982; Poplin, 1987; McNutt, 1984). Instead, holistic educators have advocated for a type of inquiry "whereby variables are viewed and treated within the context of all the interaction that form a particular experience" (Poplin, 1984, p. 290). An ethnographic
approach would be a primary method of inquiry used by holistic educators (Heshusius, 1991).

Holistists associate learning to Piaget's "growing spiral." (McNutt, 1984). Like a spiral, initially the base of a learner's knowledge or skills is small and immature. Through interactions with others in the form of instruction or other social exchanges, the learner expands the breadth of his or her knowledge and skills. These interactions have a transformative effect on the learner (Poplin, 1988). In other words, students transform that which they know (i.e., scheme) with new experiences or knowledge. As inferred by the term, the concept of transformation is not additive or incremental (Poplin, 1988). The latter concepts according to Poplin (1988) and others (e.g., Heshusius, 1988; Greene, 1984) are linear and mechanistic. They fail to address the learner's human qualities and characteristics adequately, and contexts that lead to learning.

The characteristics and qualities of the learner are considered part biological, and a result of the learner's learning history within a society and culture. According to Neal (1984) "The child comes into the world learning and expecting things to make sense" (p. 309). The implication is that humans seek out meaning and purpose in their environment. When meaning or purpose is not forthcoming the learner functions in a state or stage of disequilibrium (Poplin, 1988). Furthermore, the learner cannot be forced into a stage of equilibrium; holists make the assumption that a learner cannot be forced to learn. Learning occurs or the "spiral of knowledge" expands when the learner's passions are elevated, the purpose for learning is authentic and conditions are encouraging and positive (Heshusius, 1991; Poplin, 1988). According to Poplin, learners, especially young children and adolescents, are
preoccupied with passions such as love and fear. These emotions are expressed by the learner toward that which is being taught. Passions can serve to set the occasion for the learner to engage or disengage from the learning situation.

When in a stage of "disequilibrium" a learner can essentially act in two ways: (a) transcend the impasse through interactions with others or events in the environment (e.g., an encouraging and responsive teacher), or (b) withdraw from the situation, or maintain a status quo. The first possibility would lead to a transformation in the learner; learning would have occurred. In comparison, the second possibility results when learning activities do not support the learner's potential. The structure of the activities or instruction do not interact with the learner in such a way as to change his or her knowledge, as a whole. (Poplin, 1988). The second possibility occurs frequently with learners with learning disabilities (Poplin, 1984, 1988; Quinn, 1984; Kronick, 1988). The holistic definition of a learning disabled person implies the notion of a limited or non-dynamic interaction between the learner's existing structures of knowledge and the structure of the new learning experience.

According to Kronick (1988) "The LD student has not learned sufficient shared meanings nor perceived the implicit or assumed information that is part of the schemata" (p. 24).

In addition to learner qualities and characteristics, the context in which learning occurs is critical. According to Poplin (1988) and Neal (1984), context refers to the interchange between the learner's learning history, preferences, interests, and level of enthusiasm, and the history and characteristics of the teacher or parent. In turn, the holistic educator adheres to certain principles in teaching the learner (McNutt, 1984; Poplin, 1988). For
example, teachers facilitate learning through encouragement, and by observing the learner engaged in learning systematically (e.g., reading and writing). The teachers' observations of the learner lead to a sensitivity for the learner's passions, level of knowledge, and selection of things to learn. This notion of "facilitator" is illustrated by Neal's (1984) definition of a holistic teacher: "The holistic teacher is a child who has somehow traversed the system and has managed to grow up still a learner" (p. 313). The implication is that the adult like the child is curious, intuitive, and learns in trusting and encouraging circumstances.

Another principle adhered to by the holistic educator is an appreciation for the learner's errors. A learner's errors are considered necessary, and viewed as opportunities for the learner to see that their current understanding or knowledge is incomplete and in need of reconstruction (Poplin, 1988). Heshusius (1991) describes errors as "intrinsic, natural, valuable, and welcome part of authentic learning ... [a source of emerging understanding]" (p. 325). For example, a student may learn from his errors in solving math story problems that words are associated with numbers and types of computation. Poplin added that the learner should not be penalized for making an error, and Coles (1984) commented that a learner does not necessarily need to be immediately informed of his/her error. Informing the learner of an error depends upon such factors as "the learner's personality and other elements of the situation" (p. 327).

Another principle adhered to by holistic educators is to provide learners with instructional materials (e.g., books), assignments, and activities that match the learners "personal history" and level of interest. (Poplin, 1988). According to McNutt (1984) and Doherty-Hale (1984) this will likely require the
teacher to develop their own materials, use reading books that reflect the students own life experiences, and to involve the learner in shared writing activities (i.e., co-authoring). Finally, the assessment of student's performance is based upon a student's engagement in "real learning activities" (Heshusius, 1986). For example, Jochum (1993) spoke of an "authentic assessment of writing" through a portfolio of a students writings. The student's performance would be based upon the process involved and product yielded from the writing activities.

McNutt (1984) further demonstrates the use of holistic principles of education in language arts instruction: (a) provide learners with good models; (b) reinforce and encourage students for learning and trying; (c) teachers and parents must enjoy their work with learners, and (d) teachers should generally assume the role of facilitator so as to assist the learner in the learning process.

McNutt (1984) delineates five basic components of a holistic language arts program. Each component seeks to engage the learner in reading and writing that is tailored to their interest and ability level. Unlike other holistic education theorists (e.g., Heshusius 1991; Poplin, 1988), McNutt describes and illustrates the use of skill building activities (strategy lessons) in response to students' errors made during a learning activity. According to McNutt, strategy lessons should not be a part of a students "daily language arts program." Rather, the strategy lessons (e.g., teaching a grammar skill) should be used when more direct and natural approaches do not result in learning. The skills strategy program advocated by McNutt resembles strategies discussed by Englemann and Carnine (1982). That is, the teacher explains and demonstrates the skill, and the student practices the skill and receives feedback from the teacher regarding his or her performance.
In summary, holistic education is based upon the theory of holism. The holistic educator places a great deal of emphasis on matching instructional activities (e.g., language arts) with students' level of interest and passion for the activity or topic. The learner is considered an active and able participant in deciding what he or she should learn.

**Error Correction Strategies**

Direct Instruction and Holistic education acknowledge and address student errors. An "error" refers to an incorrect response made by a student to some question or on some activity (e.g., solving computational problems). Each approach to education takes a different position on when and how a student's error should be addressed. Their differences stem from their respective philosophies and positions on instruction.

In addition to the general manner in which student errors are treated in Direct Instruction and holistic education, there has been a recent flurry of studies and instructional methods textbooks that consider different error correction strategies (e.g., Barbetta, Heron & Heward, 1992a; Barbetta, Heward, & Bradley, 1993b; Drevno, Kimball, Possi, Heward, Gardner, & Barbetta, 1993; Mercer & Mercer, 1989). An error correction strategy refers to a method for providing feedback to the learner with respect to the accuracy, completeness, or precision of a prior response. Generally, an error correction strategy is delivered by the teacher or instructor after the learner makes a response. The necessity for considering and addressing student errors goes without argument. The form, frequency, and timing of error correction strategies is a point of contention.
The following is a brief overview of the various kinds of error correction strategies that have been used with learners. Most of the studies reviewed have dealt with the area of reading.

As mentioned earlier, during the 1970s a number of large scale research studies were conducted to describe the relationship between teacher behavior and student learning (Anderson, Evertson, & Brophy, 1979; Rosenshine, 1979; Stallings & Kaskowitz, 1974). These studies are generally referred to as "teacher effectiveness," or the more neutral term "teacher effects" research (Brophy & Good, 1986). Many of these studies demonstrated a positive correlation between the way teachers conduct themselves in an instructional setting and the learner's performance.

From this body of research findings, Rosenshine and Stevens (1984) proposed a general model of group instruction that included demonstration, practice, and feedback. The aspect of the model of particular interest is student "feedback." Rosenshine and Stevens recommended effective methods of delivering feedback following a student's correct response, incorrect response, or non-response. In general, if the student was correct, the teacher was to ask the student another question or in some way maintain the interaction. In contrast, if the student was incorrect or failed to respond, the teacher was to prompt or direct the student to the correct response. Rosenshine and Stevens (1986) stressed that by responding constructively to the student's error (e.g., prompting to reread a word), the student is less likely to continue making the error and thus acquire the target skill or knowledge.

Through a naturalistic study and correlation findings, Good and Grouws (1979) also identify various teacher behaviors and instructional factors associated with student performance. In this case, the content area being
taught was mathematics at the elementary level. Corrective feedback or error correction essentially involved reteaching the concept to the student. This corrective process occurred while the student or students were doing their work. This potentially prevented students from practicing mistakes (Good, Grouws, & Ebmeier, 1983).

Kulhavy (1977) was more specific in describing the rationale behind corrective feedback or error correction. The learner was informed that his or her response was or was not accurate. Furthermore, error correction not only informed the learner of his or her error, but should ideally prompted the learner to make the correct response. Barbetta et al. (1993a, 1993b) and Drevno et al. (1993) reported a noticeable and positive difference in students’ performance when they participated actively by making the accurate response, rather than when they participated passively in an error correction procedure.

In addition to the rationale for error correction discussed in the teacher effectiveness literature (Brophy & Good, 1986), certain groups or types of learners have a tendency to make more errors and respond differently to their errors. Jenkins and Larson (1979) speculated, for example, that students with reading disabilities may be more “tolerant of [reading] errors because they have learned to tolerate a good deal of ambiguity or anomaly in reading situations” (p. 146). Blank (1973) observed that learners with disabilities generally have a long history of responding incorrectly to school related tasks. Furthermore, their errors are qualitatively and quantitatively different from most other learners. In light of these differences, without corrective feedback (error correction) special learners may gain a false sense of their performance (McGee, 1970). Specifically, instruction should develop functional repertoires
of behavior and not inaccurate responses (Skinner, 1968). The latter is clearly counterproductive and inefficient (Martin & Pear, 1988). In plain terms, the learner needs to see the distinctions between the response that is correct from the response that is incorrect (Vargas, 1977).

Because of environmental factors or other disadvantages, some learners require instruction that attends systematically to their errors so as not to exacerbate their deficits (Bereiter & Englemann, 1966; Carnine, 1980; Englemann & Carnine, 1982). In addition, because of cultural differences some groups of learners may avoid or fail to request such assistance (e.g., Doyle, 1986; McDonald, 1976). Thus, it is incumbent upon the teacher to be alert for and respond accordingly to these deficits or differences (Silbert, Carnine, Stein, 1990). In this regard, Stallings and Kaskowitz (1979) reported noticeable gains in reading achievement within a group of students with a history of reading problems when instruction included positive error correction.

The term "error correction," refers to "immediate feedback given after the child makes an error for the purpose of correcting that error" (Grimes, 1981, p. 17). Carter (1984) delineates three kinds of delayed feedback: (a) feedback presented after each item or response, but after a brief period of time; (b) feedback presented immediately after a learner responds to a series of items or makes a number of responses; and (c) feedback presented within hours or days following the learners' responses. Immediate and delayed error corrections procedures are used in instructional settings.

Collins, Carnine, and Gersten (1987) describe three different ways that corrective feedback can be delivered to a student: minimal feedback, basic feedback, and elaborated feedback. Minimal feedback, or general corrective
feedback, involves the teacher telling or informing the student whether or not his or her response is correct. To correct an error, the student makes additional responses until the correct response is made. In other words, the learner applies the process of trial-and-error to arrive at the correct response.

Basic feedback, or specific corrective feedback, is the second level of corrective feedback. This type of feedback involves the teacher informing the student if his or her response is correct or incorrect. In the case of the latter, the teacher supplies the student with the correct answer.

Elaborated feedback constitutes the third type of feedback. In this form of feedback, the teacher indicates to the student that his or her response is correct or incorrect. The teacher then prompts the student to arrive at the correct response.

The next section of this chapter will discuss examples of research studies that have addressed error correction particularly in the academic areas of mathematics and spelling. Also, error analysis will be described and its incorporation with error correction strategies will be illustrated. Sections of this chapter will cover research on interventions used in mathematics instruction, and about long division instruction. The final section will consider issues concerning the social validity of error correction procedures in mathematics instruction.

Error Correction Research

The research in the area of error correction has grown during the last ten years (e.g., Allington, 1980; Barbetta et al., 1993ab; Carnine, 1980; Dalrymple & Feldman, 1992; Drevno et al., 1993; Kearney & Drabman, 1993; Singh & Singh, 1985). Error correction feedback strategies have been used with various academic and social behaviors. Interestingly, much of the
research in the area of error correction concerns oral reading errors. Therefore, the following review of studies illustrates the use of error correction feedback strategies with oral reading errors.

Through a naturalistic study of grade 1 and 2 students and teachers, Allington (1980) assessed for differences between the "point" and "direction" of error corrections made by teachers during oral reading groups. "Point" referred to the point in time that an error correction was delivered. "Direction" referred to the content of the corrective comment rendered by the teacher. The study showed that "poor readers" were interrupted more often than "good readers" after making a reading error. While data was not presented regarding the point in time when error correction was delivered, Allington reported statistical data that suggested that poor readers were corrected more frequently regardless of the reading error (e.g., graphic phonic, phonemic). Allington recommended that future research address those variables that occasion a teacher to provide corrective feedback. In this regard, a more systematic and perhaps individualized approach to error correction might be warranted.

Carnine (1980) took a systematic approach to studying two common methods of correcting beginning readers' oral reading errors. The methods studied were phonic and whole-word. Carnine hypothesized that learner's who were acquiring reading skills would perform better following an error correction strategy that directed them to sound out the word (phonic), than a strategy that modeled the correct reading of the word (whole-word). The latter strategy was followed by the learner repeating the word. Nine pre-school children (4- and 5-year olds) were pre-trained on letter sound relationships and following basic directions. Learners were then randomly assigned to one
of three treatment groups. Learners met in a group with the teacher for training and error correction. A multiple baseline design across the two correction procedures was used. After training was completed, learners met individually with the teacher and were tested on unfamiliar words. The results of the study essentially showed that the phonic error correction strategy was superior to the whole word strategy. Further, the results suggested that error correction strategies should match the strategies used during instruction.

Jenkins and Larson (1979) took a direct approach in studying various error correction strategies used in the classroom. Based upon observations of classroom teachers, they identified five types of error correction procedures used by remedial-reading teachers to correct a student’s oral-word recognition error. Through an alternating treatment design with “word supply” serving as the control, Jenkins and Larson implemented four other correction strategies with five junior high students. All of the students were classified as having a learning disability. In the “word supply” error correction procedure, the teacher supplied the student with the correct word. According to Jenkins and Larson, word supply was treated as a control because of its pervasive use across all grade levels as an error correction strategy. The four other procedures were extensions of word supply and were delivered by the teacher. The procedures required the student (1) to repeat correctly the erred word and then to reread the sentence it came from (sentence repeat); or (2) to repeat correctly the erred word, and at the end of the page, reread from a list any and all erred words (“end of page review”); or (3) to repeat correctly the erred word and then to define the word for the teacher, followed by reading and defining from a list the word(s) erred on the page (“word meaning”); or (4) to repeat correctly the erred word. Immediately after the reading session the
student read the erred word or words from index cards ("drill"). The drill phase ended when the student correctly read any of the words previously erred during the reading of the passage.

In addition to the error correction procedures, a "no correction" phase was administered. That is, students' reading errors were not corrected. Two dependent measures were taken one day following an error correction procedure. These measures consisted of the student reading from a list of erred words and sentences containing the erred words. Jenkins and Larson (1979) reported that students' performance on the four extensions of the word supply procedure was superior to that of the word supply and no correction procedures. In addition, performance on word supply was similar to the no correction procedure. Overall, however, students performed best under the drill error correction procedure.

Jenkins and Larson (1979) raised the issue of efficiency and effectiveness of error correction procedures in a classroom setting. While all of the extensions of the word supply procedure were effective, error correction requires time and effort on the part of the teacher and the student. This can be especially true for some of the above procedures. However, in the long term these more intensive procedures (e.g., drill) might be of greater benefit to the student. To resolve these issues of efficiency and effectiveness, Jenkins and Larson recommend that teachers apply, on a trial basis, various error correction strategies with their students. Thereby, the teacher is likely to identify effective and efficient error corrective procedures.

O'Shea, Munson, and O'Shea (1984) systematically replicated Jenkins and Larson's (1979) study in an effort to establish whether another form of error correction, "phrase drill," is more effective than "word drill" in correcting
oral reading errors. The phrase drill error correction procedure drilled the erred word within the context of a phrase (O'Shea et al., 1984). In an alternating treatment design, five elementary aged students (7 to 11 years of age) with learning disabilities were presented the error correction procedures and “word supply” procedure used by Jenkins and Larson (1979). In that regard, word supply served as a control. Students met individually with the experimenter and an observer and read orally from a passage. At the point the student erred in reading a word, the experimenter prompted the student to reread the word. If the student continued to err on the word or did not know the word, the experimenter supplied the word. The experimenter and observer separately recorded the erred word. These erred words were presented to the students immediately following the oral reading. These words or phrases were presented on flash cards in isolation (O'Shea et al., 1984). The dependent measures were reading accuracy and fluency, and were recorded the day following the passage reading. Accuracy was based upon words read correctly that had formerly been read incorrectly. This was measured with words and phrases presented in isolation (flash cards) and in context (reading passage). Fluency, or words read correctly per minute, was measured from the students' oral rereading of the previous day's passage.

Based upon the students' performance on the next day's test, O'Shea et al. (1984) reported that the word drill and the phrase drill error corrective procedures were equally effective with words presented in isolation (flash cards). However, in context the phrase drill procedure was shown to be superior to the word drill procedure. That is, the students made fewer errors on next day rereading of the passage under the phrase drill phase than the word drill phase. Furthermore, through inspection of graphs depicting
in response to the positive findings of Jenkins and Larson (1979) and to the conflicting results of Meyer (1982), and Rose, McEntire, and Dowdy (1982) regarding error correction procedures, Singh and Singh (1985) compared the effects of two error correction strategies on oral reading errors. Unlike Meyer's and Rose et al. studies, the students in the Singh and Singh study were four adolescents who had developmental disabilities. Through an alternating treatment design which included a baseline, two error corrective strategies, "word supply" and "word analysis," were administered; a "no error corrective" feedback strategy was also administered. Students met individually with the experimenter and read from passages in a reading book written at a level slightly above the students' reading level. Two types of errors were recorded. The first was an "uncorrected reading error," and the second type was a "self-corrected reading error." The former was responded to by the experimenter during the error corrective feedback strategy. Self-correction represented errors made by the student to which the student spontaneously corrected. The results of the study showed that the number of oral reading errors decreased under the word supply and word analysis procedures over the control and no corrective feedback strategy. However, based upon visual display of data, Singh and Singh observed that the word analysis procedure was superior to the word supply procedure. Their findings parallel that of Carnine (1980) and his use of word analysis error corrective strategies with preschool aged students. Singh and Singh (1985) recommended that error corrective procedures should not just correct an error in the rote sense of the term, but...
rather provide the learner with a strategy to decipher subsequent words that are troublesome.

As noted earlier, teachers seem to use certain oral reading error correction strategies that are less effective than other approaches (e.g., Jenkins and Parson, 1979; Singh, 1990). In an effort to determine the relative benefits of certain error correction strategies, Singh (1990) used an alternating treatment design to compare the differential effects of “word supply,” “sentence repeat,” and “no-corrective feedback.” Three adolescent-aged students with developmental disabilities read passages from books that were written slightly above their current reading level. In addition to alternating the presentation of the three procedures during three daily sessions, the experimenter and observer recorded the words missed and the words spontaneously corrected by the student. The students’ “retention” of the words learned from a read passage was measured one week following their initial reading of the passage. Singh reported that students’ retention of words under the sentence repeat condition was superior to their performance under the word supply condition. Both procedures were more effective than the no-correction strategy. Singh (1990) observed that learners’ improved performance under the sentence repeat procedure might be related to the contextual cues present in the sentence coupled with the “drill” like nature of sentence repeat. In this regard, Singh (1989, 1990), like others (Carnine, 1980; Perkins, 1988), emphasized the need for teachers to use error corrective feedback strategies that are functional beyond training.

Espin and Deno (1989) also compared various error correction strategies. Through an alternating treatment design they studied the effects of modeling and prompting in the correction of sight word reading errors. The
students who participated in their study were in elementary school; all had learning disabilities. Students were divided into two groups based upon their grade level: grade 2 (Group 1) and grades 4 and 5 (Group 2). The dependent measure was words read correctly on a daily posttraining test. In the study, students read from words that were judged to be within their reading level, but not known by the student. In the model and prompt error correction procedures, students met individually with the experimenter and were presented words on flash cards. The individual training session consisted of two practice trials with the word on flash cards, followed by posttesting over the words. The words used during each session were randomly assigned to either the model or prompt error correction procedure. In the prompting condition, if the student did not know the word, the experimenter prompted by sounding out a portion of the word based upon the number of syllables in the word. A second prompt was delivered if the student did not respond; if the student still did not respond, the next word in the deck of flash cards was presented. In the modeling condition, if the student did not know the word, the experimenter modeled the word with the student repeating the word.

Espin and Deno (1989) reported that students in both groups were able to read more words that had been modeled than prompted. This direction in findings continued to be in place at one and three month follow-up maintenance checks.

A limitation of Espin and Deno's (1989) findings was that no measure of generalized outcome was taken. The list of words used during error correction was also administered at follow-up. Despite the limitations, there appears to be a positive relationship between modeled error corrective feedback and student retention of learned sight words. In addition, error
correction that involves modeling can be more time efficient than one that involves prompting.

In an effort to identify and analyze components of error correction, Barbetta et al. (1993a) addressed the role of student participation in error correction. Through an alternating treatment design, students either made an active response during error correction or no response during error correction. The students who participated in the study all had developmental disabilities. Students met on an individual basis with the experimenter and received sight word instruction. Error correction involved the teacher modeling the correct word by saying the word and showing the word on a flash card to the student. In the error correction procedure itself, an active response included the student repeating the word. In the no response condition, the student visually attended to the word card. Both error correction procedures were administered immediately after a student erred. The results of the study showed that students read more words correctly following the active student response condition on a test that followed instruction, a test administered one day following instruction, and a test administered two weeks after the last day of instruction. This final test involved the learner reading the word within the context of a sentence. Based upon these results it would seem important for teachers to provide learners with the opportunity to practice the correct response during an error correction procedure (Barbetta et al., 1993a).

As noted earlier, it is counterproductive in any instructional setting for learners to make errors repeatedly (Martin & Pear, 1988). Effective error correction strategies should be instituted as immediately and directly as possible. This is particularly true among certain groups of students or individuals with disabilities who require more time and practice to acquire
behavioral repertoires. To this end, a number of researchers have applied "positive practice overcorrection" (e.g., Foxx & Jones, 1978; Singh, Singh, & Winton, 1984; Singh & Singh, 1986; Singh, 1987), or what is also known as "directed rehearsal" procedures (Dalrymple & Feldman, 1992; Lenz, Singh, & Hewett, 1991) to academic and verbal behavior.

Much of the research covering positive practice overcorrection or directed rehearsal has been with oral reading and spelling errors. In particular, Singh et al. have done a considerable amount of research in the use of positive practice overcorrection procedures with students with developmental disabilities. For example, Singh (1987) compared overcorrection procedures used in small group and large group instruction. In both situations when a student made an error, the teacher directed the student to repeat the word correctly five times. The student was then to reread the sentence in which the word was contained. The results of the study showed "There were no differences in student performance regardless of the format used for training" (Singh, 1987, p. 173). However, students performed better under both training formats than the students in the no-correction control group. In addition, Singh reported that the teacher preferred the use of overcorrection in a group setting. Singh commented that further research needs to be conducted in the use of overcorrection procedures in group settings.

In a 1984 study conducted by Singh et al., three error correction strategies were implemented with oral reading errors using an alternating treatment design. The strategies were positive practice overcorrection, positive practice overcorrection with positive reinforcement, and no corrective feedback. The results of the study showed that the use of both forms of
positive practice overcorrection resulted in fewer oral reading errors than the no corrective feedback strategy. The positive practice overcorrection and positive reinforcement were more effective than the overcorrection procedure without reinforcement.

Singh and Singh (1986), through an alternating treatment design, compared the use of positive practice overcorrection, word drill, and no corrective feedback. The purpose of the study was to ascertain whether corrective feedback or no corrective feedback positively effected students' retention of corrected words. The measure of "retention" was taken one day following instruction. The results of the study showed that both positive practice overcorrection plus positive reinforcement and word drill were superior to the no corrective feedback strategy. In addition, positive practice overcorrection plus reinforcement was superior to word drill. However, both procedures "Facilitated retention of error words measure one day later" (p. 124). Singh and Singh (1986) commented that teachers were positively disposed to overcorrection procedures; teachers found them to be efficient and effective.

Error Analysis

In part, the previous discussion was concerned with how teachers can apply effective error correction procedures within the classroom. An important portion of the error correction process involves the teacher recognizing that the student has made an error. This is also known as "error analysis."

According to Skinner (1966), claiming that an error has been made requires that a judgment be passed about the accuracy of a response. In many instructional settings, a judgment is made at the point in time when the student makes a response. Many of the aforementioned oral reading error
correction studies involved the experimenter or teacher making "on the spot" discriminations regarding the presence or absence of an error. There are other situations, however, when a teacher's judgment is made long after a student's response. This is due to practical considerations (e.g., time, resources, class size), or because the teacher needs to analyze the error itself. That is to say, the teacher needs to determine where the student is making his/her error (e.g., steps in a division problem), and if there is a pattern to the errors. In addition, the teacher needs to determine the nature of the student's error, such as inattentiveness, lack of pre-entry behaviors (i.e., prerequisite skills), or faulty instruction.

The following section considers error analysis applied to students' response to mathematics problems. As mentioned above, this form of student evaluation can be applied to most, if not all, academic subject areas taught in school. Because the focus of this dissertation is on whole number computation-long division, the discussion regarding error analysis will be confined to its use in mathematics.

**Error Analysis in Math Instruction**

According to Silbert et al. (1990), the first step in analyzing a student's mathematical errors is to determine the cause of the error. This process is called error analysis and is important for several reasons. As noted earlier, identifying and responding to a student's error makes it less likely that the student will spend time acquiring faulty algorithms and mathematical skills (Bereiter & Englemann, 1966; Martin & Pear, 1988). In addition, this kind of analysis can lead to procedures that prevent errors, or prevent the problem from reoccurring. In sum, error analysis has the potential to promote achievement.
Unlike quantitative analysis of student difficulties, "error analysis [EA] strives to identify the nature of errors a learner may commit [in solving a mathematics problem]" (Knifong, 1985, p. 1708). Howell and Kaplan (1980) conceptualized error analysis as a kind of task analysis wherein the math problem (stimulus) contains the requirements of the task, and the response contains the student's behavior. Through a task analysis a teacher can ascertain what the student does and does not know (Howell & Kaplan, 1980). This process makes the assumption that students' errors can be discriminated into patterns and thereby can be categorized.

Descriptions and classifications of students' mathematical errors can be found in several different places (e.g., Ashlock, 1990; Brown & Burton, 1978; Englehardt, 1977; Enright, 1983; Howell & Kaplan, 1980; Roberts, 1968). The purpose for classifying students' errors has varied from establishing prescriptive means to eliminate errors (Ashlock, 1990) to formulating elaborate theories (e.g., Brown & Burton, 1978; Englehardt, 1977). With some exceptions (e.g., DeCorte & Verschaffel, 1985; Easterday, 1980) most error analysis research has focused on categorizing students' whole number computational errors on written work (Ashlock, 1990). According to Howell, Fox, and Morehead (1993), Roberts (1968) and Enright's (1983) extension of Roberts's error analysis research are the error patterns most frequently used. Specifically, Roberts identified the following categories of errors: (a) wrong operation, (b) obvious computational error, (c) defective algorithm, and (d) random response. Enright identified seven other categories of computational errors. The seven errors were (a) regrouping errors, (b) process substitution errors, (c) omission errors, (d) directional errors, (e) placement errors, (f) attention to sign errors, and (g) guessing errors.
EA research in mathematics is relatively extensive and dates back well over sixty-years (e.g., Brueckner 1930; Buswell & John, 1926; Cox 1975; Engelhardt; 1977; Enright, 1983; Grossnickle, 1935; Guiler 1946; Inskeep, 1977; Roberts, 1968; Moore et al., 1993; Mercer & Mercer, 1993; Wilson, 1939). Essentially, this type of diagnostic research has focused on what students did incorrectly in solving math computational problems, and provided a means of assessing what effect instruction and instructional materials had on students' performance. For example, Buswell and John (1926) analyzed students' errors as a means to establish more effective forms of instruction and to categorize patterns of errors. Specifically, they instituted and compared three different approaches to diagnosing students' difficulties in solving computational problems. The methods included (a) studying the eye movements "exhibited by students when adding columns of digits, (b) recording the time consumed by individual students during the processes of addition, subtraction, multiplication, and division and of comparing the records thus secured, and (c) skillfully questioning and observing the student as [s]he performs the four fundamental operations of arithmetic" (p. 4).

Buswell and John's (1926) third approach continues to be used (Howell, Fox, & Moorehead, 1993; Enright, 1983). Specifically, they analyzed students' daily classroom math work, and performance on items on a diagnostic test. In addition, Buswell and John observed students solve computational problems. This analysis and diagnosis of students' difficulties yielded a list of methods ("habits") used by students when solving problems from the four fundamental operations of arithmetic. According to Buswell and John, such a catalogue makes "it comparatively easy for a teacher to identify and classify the methods that work in the case of a given pupil" (p. 131).
Clearly, diagnosing students' computational difficulties continues to be relevant. It enables a teacher to determine what and how best to address students' problems in acquiring math skills (Hammill & Bartel, 1990). Also, it has the potential to (a) circumvent teachers' frustration with quantitative or standardized methods of assessment, (b) provide teachers with a means of individualizing instruction, (c) eliminate vague terms (e.g., "math phobia") and ineffective methods of instruction (Radatz, 1979). Further, EA enables teachers to explore plausible reasons for the student's difficulties (Romberg, 1980).

With the exception of applying a theory to explain students' computational errors (Brown & Burton, 1978), the procedures for conducting an error analysis and generating categories of errors are fairly standard. Based upon Ashlock's (1990) suggestions regarding error pattern analysis, Howell, Fox and Moorehead (1993) recommended that the following steps be followed when conducting this kind of an analysis.

1. Collect an adequate behavior sample by having the student work several problems of each type in which you are interested.
2. Encourage the student to work and talk aloud about what (s)he is doing, but do nothing to influence his/her responses.
3. Record all responses the student makes, including comments.
4. Look for patterns in the response.
5. Look for exceptions to any apparent pattern.
6. List the patterns you have identified as assumed causes for the student's computational difficulties.
7. Interview the student. Ask him or her to tell you how (s)he worked the problem to confirm suspected patterns.
Although error analysis and error patterns are generated from a student's work, as pointed out by Mercer and Mercer (1993) "It is helpful to examine some of the research regarding types of error made by many students of different grades" (p.240).

Roberts (1968), for example, identified error patterns or "failure strategies" in the math computational assignments completed by a group of third grade students. These students were divided into four quartiles based upon student's "grade-level score." As noted earlier, his analysis of their responses to math problems yielded four major categories of error patterns or failure strategies: wrong operation, computational error, defective algorithm (list subdivisions), and random response. The defective algorithm was subdivided into 3 major categories: inappropriate reversals, grouping errors, and mixed operations.

Roberts analysis also showed trends in the type and frequency of errors made by the students. There was a tendency for all students to make errors in basic addition and multiplication facts. In all quartile groups, however, "weakness in algorithmic techniques" was a prominent problem (Roberts, p. 443).

Cox (1975) compared computational error patterns made by approximately 700 elementary and middle school aged students in general and special education. The special education population student made up approximately 25% of the total sample of students. In her study, Cox sought to identify and describe commonplace computational errors made on addition, subtraction, multiplication, and division problems. Student math problems were analyzed according to the following categories: systematic, random, and careless errors, and no error, and incomplete response. Systematic error was
defined as an error that is made in a specific algorithmic computation repeatedly. Like Roberts (1968) results, Cox found a declining trend with each grade level in errors made in addition and subtraction skills. In addition, she revealed that students in special education made a high percentage of systematic errors for each of the math operations (e.g., division) relative to their general education counterparts. Finally, at one year follow-up approximately 25% of the students continued to make the same or some other systematic error. (Cox, 1975). Unfortunately, follow-up data were not reported for special education students. It was unclear whether or not the students in special education were included in this final segment of the study.

Diagnostic interviewing is another way to analyze students' errors. This procedure has the potential advantage of clarifying for a student and teacher the students written response. Generally, the interviewer or teacher seeks to observe and question the student but not necessarily offer correction (Ashlock, 1990). Lankford (1974) interviewed 176 grade 7 students to obtain verbalization of their "pattern of thinking" or "computational strategy" as they solved math operations that involved whole numbers and fractions. The results of his individual interviews with students yielded two groups of "computers": good and poor computers. "Good computers" were described as students who were fluent with math facts and in the use of algorithms, but who were not affected negatively by the way problems were presented (i.e., vertical or horizontal arrangement). In comparison, "poor computers" were students who were less accurate with math facts, had trouble with fractions, and who had difficulty with conventional operational algorithms. Interviewing students about their math work may also have the added benefit of informing
teachers of the alignment between what is taught in class and student performance (Lankford, 1974).

Although there are many benefits to conducting a qualitative analysis of students' computational error patterns, classes of errors can be vague and lack precision. Englehardt (1977) concluded this about two of Roberts' error classes: random response and defective algorithm. He argued that the former was likely to become "a catch-all for errors which [did] not fit into the other error classes, and that the major subdivisions in the latter (i.e., inappropriate reversals, grouping errors, and mixed operations) "merited consideration as separate error types" (p. 149). To test his conclusion, Englehardt applied his extension of Roberts's classes of error patterns to 84 computational problems completed by 198 general education students in grades 4 and 5.

The results yielded eight types of errors: basic fact error, defective algorithm, grouping error, inappropriate error, inappropriate operation, incomplete algorithm, identity errors, and zero errors. Missing from the list were the terms "random" and "careless" errors. Englehardt acknowledged that such errors might have occurred among some of the students' work, but decided against using these terms because of their broad meaning. All students were subject to making "basic fact errors." In comparison, defective algorithm errors distinguished students who performed poorly from students who performed well on the various computational problems. Finally, "examination of the distribution of errors revealed that neither grade level, [gender,] ethnicity nor urban/rural school district membership suggested the types of errors which [students] committed." (p.154).
In contrast to Englehardt's findings, Lepore (1979) discovered differences in computational errors made between a group of students with educable mentally disabled and learning disabilities. However, Lepore descriptions of students was incomplete and vague, thereby made it difficult to conclude that students with LD tend to make certain types of errors different from students with educable mental disabilities. Clearly, additional study is warranted in this area.

In a more recent study of error patterns displayed by special education students, Miller and Milam (1987) pointed out that a given category of error (e.g., wrong operation) can represent a variety of specific kinds of errors. Specifically, 213 students with learning disabilities were administered a 92 item written computation test over a 4 day period of time. Analysis revealed that students error responses varied greatly and particularly on two types: a multiplication and division problem. Students' response on these two items revealed that there were "98 different answers to the multiplication item, and 93 different answers to the division item" (p. 120). Miller and Milam reported that a preponderance of the errors were errors in a particular component of the problem. The entire problem was not necessarily incorrect. For example, on long division problems, a great many of the students erred by omitting the remainder. Based upon their finding, Miller and Milam stressed the importance for teachers to conduct error analysis as a means of identifying and remediating that aspect of the math operation that is difficult for the student. Furthermore, error analysis can assist the teacher in identifying those students who lack readiness for a particular type of math operation or lack prerequisite skills.
Finally, Kelly, Gersten, and Carnine (1990) investigated the relationship between the curriculum design of a math program (i.e., organization and presentation of math problems) and the error patterns of high school aged students with and without learning disabilities. Essentially, these researchers found that students in special education, and low achieving students in general education made significantly fewer errors on math problems (i.e., fractions) when the teacher’s instruction and the curriculum adhered to certain principles. That is, the teacher instituted various components of effective math instruction as described by Good and Grouws (1979). For example, students were assigned seatwork only after demonstrating the ability to work independent of the teacher, and students were given daily feedback regarding their performance. In comparison, the curriculum (a) stressed systematic practice, (b) separated math concepts from other concepts that could be easily confused, and (c) provided students with a sufficient number of exemplars. Kelly et al. concluded by reemphasizing the positive relationship between the design of curriculum materials and student errors, and the importance of minimizing the likelihood of such errors with materials that adhere to the above three curriculum design variables.

Although the return on the use of EA appears to be high, it is not a panacea. Once a pattern of errors is identified, the teacher must institute instructional measures to reverse the student’s incorrect response. Clearly, a teacher’s knowledge of methods of teaching a particular subject area would be called upon. This demand would be especially great among teachers of students with learning disabilities. The following sections will provide a brief overview of error analysis and error correction procedures used in various subject areas.
Error Analysis and Error Correction

Kameenui, Carnine, Darch and Stein (1986) compared two approaches to the development phase of mathematics instruction. "Development" refers to a process whereby the teacher works directly with students in teaching a math skill (Good et al., 1983). The intent of development is to provide students with an opportunity to interact with the teacher regarding the math operation and concept under instruction (Zahn, 1966). The two approaches were a basal program and direct instruction; the direct instruction procedure was based upon the findings of Project Follow Through (Kameenui et al., 1986). Essentially, these two approaches differed in terms of three instructional features. The basal approach used manipulatives or objects, discussions, and "a personalized general correction procedure" (p. 634) during instruction. The personalized error correction procedure involved the students reexamining their work to discover how and why their errors were made. In contrast, the direct instruction approach involved the teacher demonstrating math concepts and operations to be performed. In turn, the student responded by solving math problems on worksheets. In addition, when a student erred the teacher "immediately interjected with either a model of how to answer correctly or some type of instructional prompt based on previously taught information" (Kameenui et al., 1986, p. 635).

In three separate studies, Kameenui et al. (1986) compared the effectiveness of a basal program with a direct instruction program during a development phase of mathematics instruction. Students in all three studies were assigned randomly to one or the other of the two programs. The students in the first study were 26 low performing first graders. During the
development phase students in both groups were taught the concept of subtraction, but through separate strategies as mentioned earlier. For example, students in the basal or "comparison group" used objects and pictures taken from a basal math program; in addition, students engaged in discussion with the teacher. Students in the direct instruction group read the math related problem, and then drew their response. The dependent measures were students' performance on work completed two posttraining tests that involved "numerical subtraction problems and picture problems" (p. 639). Through group analysis, Kameenui et al. (1986) reported that students in the direct instruction group performed better on the posttraining test measures than the basal group. Kameenui et al. explained that development that took an explicit strategy (direct instruction) rather than a non-explicit strategy (basal approach) seemed to be of more benefit to the low performing first grade student.

The second study systematically replicated the first study with a group of relatively advanced students and with more complex forms of arithmetic. Twenty-seven students in grades 2 and 3 participated in the study. Through a basal or direct instruction approach, the development phase involved three different fraction skills. Both groups used the same demonstration and practice examples. The direct instruction group was taught rules and terminology related directly to fractions. In contrast, the basal group participated in activities involving fraction skills. In addition, different error correction strategies were used with each group. Students in the direct instruction were prompted to the correct response by way of being asked questions about rules related to fractions. In contrast, the students in the basal group were initially directed to determine, on their own, the reason for their
error. If this failed, students were prompted to the correct response via paper and pencil tasks and/or discussion with the teacher. The result of this study showed that students in the direct instruction group achieved higher performance scores on developmental activities than the basal group.

The third study was another systematic replication of the first study. However, the third study involved twenty-four older-aged elementary students and used a different math skill during the development phase (Kameenui et al., 1986). Division was the math skill taught. Both groups of students were provided strategy training with this math operation along with worksheets and corrective feedback. In the direct instruction group, the teacher first reviewed or instructed basic multiplication skills for three training sessions. Using a chalk board, the teacher modeled the steps to solving the multiplication problems. The teacher did the same for division problems. This was followed by students solving division problems on worksheets. In contrast, the teacher presented the concept of division from a basal mathematics program to the basal group. A multiplication review was not conducted with the basal group. Thereby, the basal group had three extra days of practicing division during the development phase.

In this third study, Kameenui et al. (1986) reported that statistically, the two groups differed significantly in their performance on probes (tests) taken during training. The average mean performance was higher for the direct instruction group than for the basal group. In contrast, posttraining testing taken three times in the days and weeks following training showed no statistical difference between the two groups. Kameenui et al. (1986) commented, however, that students who received direct instruction during the
development phase, achieved similar levels of performance as the basal group but in a shorter period of time.

Despite the lack of dependent measure data on the error correction procedures used in the Kameenui et al. (1986) studies, it seems apparent that the separate error correction procedures had an influence on students' performance. Future research should consider comparing the different effects of "personalized" forms of error correction. In addition, it would be informative to compare the type of error correction used in direction instruction (Silbert, 1990) to a personalized, general correction procedure as described by Kameenui et al. (1986).

In another study, Van Houten and Rolider (1989) researched the variables that affect the use of flash cards in an instructional setting. They used flash cards to teach basic addition and subtraction facts in a drill fashion to second and third grade students. All of the students were in general education, but displayed deficits in basic arithmetic skills as reported by their teacher. The variables addressed in the study essentially concerned error correction. The objective was to ascertain what effect different methods of presentation and feedback had on the students’ acquisition of basic math facts.

In the first study, Van Houten and Rolider (1989) compared sequential with rapid re-presentation of erred math facts. Specifically, in the sequential condition, after the student erred on the math fact, the teacher provided the correct response and placed the flash card at the back of the deck of cards. In contrast, in the rapid re-presentation condition after the student erred on the math fact, the teacher provided the correct response and then placed the card immediately behind the next card to be presented to the learner. Results of
the study showed that students achieved criterion, that is learned all of the math facts in fewer sessions, under the rapid re-presentation condition.

According to Van Houten and Rolider (1989), "another way to reduce the likelihood of errors is to follow them with a decelerating consequence such as a mild reprimand" (p. 113). To this end, they implemented two error correction procedures following a student's error in stating a math fact. The first procedure, "correction," involved the experimenter saying "No" in a neutral sounding voice following the student's error. The second procedure, "correction plus firm reprimand," involved the experimenter putting her index finger to her lips and saying the word "no" in a firm tone of voice in response to a student's error. The results showed that under the correction plus firm reprimand condition, students acquired math facts in fewer sessions than those students under the correction condition.

In the final study, Van Houten and Rolider (1989) simultaneously administered the rapid re-presentation procedure, correction plus firm reprimand procedure, and a knee-to-knee teaching procedure. The purpose of the study was to identify a potential treatment package to be used in instructional settings. Overall, the procedures used in this experiment resulted in learners acquiring math facts in fewer sessions than a group of students who received the following procedures simultaneously: sequential presentation, correction, and desk-between. Further, students interviewed following the study did not complain about the procedures, but rather discussed the positive results they achieved.
Research Studies on Mathematics Interventions with Learning Disabled Students

Within the last few years, special education has begun to survey the various types of research conducted in the area of math with mildly disabled students (Fulk, 1992). In part, the discipline is attempting to focus on subjects that in the past may have received less research attention than other areas. In addition, it is fair to say that the impetus to emphasize math instruction stems from pressures to serve students with learning disabilities in the "mainstream" (e.g., Regular Education Initiative), and to conform to recent recommendations for the adoption of new curriculum standards in math education (NCTM, 1991).

According to Lessen, Dudzinski, Karsh, and Van Acker (1989), Lovitt (1989), Mastropieri, Scruggs and Shiah (1991), Pereira and Winton (1991), and Fulk (1992), instructional-intervention type research has the potential to address the learning needs of LD students who have math disabilities. Intervention type research is generally behaviorally oriented, short-term, targets a particular skill for change, and measures students achievement continuously. In addition, interventions seek to effect changes that extend to non-intervened upon skills, settings or situations.

A review of several sources (e.g., Lovitt, 1989; Mastropieri, Scruggs & Shiah, 1991; Pereira & Winton, 1991; Fulk, 1992) showed that research based intervention studies have either manipulated reinforcement contingencies or implemented some instructional procedure.
Reinforcement contingencies. In two separate studies, Smith and Lovitt (1975) examined the effect of tangible and activity reinforcers on students’ accuracy in solving basic computational problems, and on students’ fluency in solving basic computational problems.

In the first study with elementary aged LD students, students were told that if they improved their performance (i.e., level of accuracy) on math problems that were difficult for them they would earn points that could be exchanged for toys. Although students' level of accuracy on math problems increased under these conditions, the results of the study did not clearly show if tangible reinforcers alone had a positive affect on students' performance in solving math problems. The fact that students were taught how to solve computational math problems and given opportunities to practice them would confound the conclusion that their improved performance was solely the result of being rewarded.

In Smith and Lovitt’s second study, with a group of elementary aged LD students, the objective was to see what effect tangible rewards would have on the rate or fluency at which students solved basic computational problems. In addition, students were given a choice of the kind of reward they could earn for increasing the rate at which they solved math problems. The results showed that students' rate of performance increased when such responding was reinforced. Also, students' rate of responding was partially affected by the type of reward to be made available to them for increased responding.

Instructional procedures. An instructional procedure that has been studied for some time is the demonstration and permanent model (D & PM) technique (e.g., Smith & Lovitt, 1975; Blankenship, 1978). A number of studies have used it to teach basic math computational skills to students (e.g.,
Smith & Lovitt, 1975; Blankenship, 1978; Sugai & Smith, 1986; Rivera & Smith, 1988). The D & PM involves the teacher demonstrating and explaining the algorithm to solving a particular type of math problem to a student. Afterwards, the teacher leaves the solved problem (permanent model) with the student who is to refer to it when solving subsequent problems.

Blankenship (1978) applied the D & PM technique with elementary aged LD students in solving subtraction problems that involved “trading” (borrowing). Students' permanent model of subtraction illustrated trading through the traditional “decomposition method.” That is, trading or borrowing only occurred in the minuend. Although D & PM had a positive effect on acquisition of subtraction skills, it had a limited effect on students' performance on more difficult subtraction problems.

Sugai and Smith (1986) also applied the D & PM technique with LD students in grades 3, 4 and 5. Unlike Blankenship, they taught students how to subtract using the “equal additions method” (trading or borrowing involved increasing by the same amount digits in the subtrahend and minuend). The purpose for teaching this method of trading (borrowing) was to analyze its effect on students acquisition of subtraction skills. The results of the study showed that students acquired subtraction skills using the “equal additions method” in a relatively short amount of time, and that all students generalized subtraction skills acquired under the D & PM technique to more difficult problems.

The demonstration and permanent model technique has also been used to teach long division skills. Rivera and Smith (1988) used this instructional strategy with 8 middle school students with learning disabilities. Prior to the introduction of this modeling procedure the students displayed
less than 10% accuracy in solving division problems. In comparison, all students were fairly proficient in solving the other basic operations: addition, subtraction, and multiplication.

Based upon other research findings, Rivera and Smith reasoned that because of the complexity and steps involved in solving division problems, immediately following the teacher's demonstration of how to solve a division problem students solved another division problem and verbalize key words associated with the operation. The students' verbalization of key words were prompted by the teacher. The purpose of the verbalization was to assist students in learning the algorithm, and secondarily, to monitor their accuracy in imitating the teacher. Thus, if the student was inaccurate in solving an example of the model division problem another example was given. This form of error correction sought to prevent the student from similar errors when solving division problems independent of the teacher.

Rivera and Smith's results showed that in a relatively short amount of time (i.e., 2 to 9 days) the D & PM technique could help students who were already proficient in three of the basic operations learn how to solve different types division problems. Further, these same students demonstrated maintenance of division skills after the formal study was completed.

Other intervention based studies have applied instructional techniques that required the student to make some written and verbal response to increase the fluency multiplication facts. Skinner, Turco, Beatty and Rasavage (1989) had 4th grade students with disabilities, employ the "cover, copy and compare" (CCC) technique. Essentially, the CCC technique required the student to write on a sheet of paper the answer to a basic multiplication problem, and then to immediately check his/her response with an answer key
on the same sheet of paper. The procedure provided the student with immediate feedback and specifically, when necessary, error correction feedback. Based upon a multiple-baseline design across students, the CCC technique had a positive effect on students rate and precision in solving basic multiplication fact problems.

Lombardo and Drabman (1985) also had students make written and verbal responses when learning multiplication facts. Specifically, they implemented a three phase study with intermediate grade level students with LD who were not making progress in acquiring multiplication facts. The first phase of the study consisted of collecting baseline data about the students performance in solving multiplication facts. The second and third phases involved instituting different error correction procedures: “Write” and “Write-Say,” respectively. Students administered these procedures only when they made a error on a worksheet of multiplication problems. The teacher monitored students for compliance with the respective procedures.

During the “Write” error correction procedure (second phase), immediately after a student wrote his/her answer to a problem on a worksheet, he/she checked its accuracy by “uncovering” the correct response printed on the worksheet. If the student’s response was correct he/she moved to the next problem. In comparison, if a student’s response was incorrect, the student wrote the problem and its answer five times.

During the “Write-Say” error correction procedure (third phase), Lombardo and Drabman had students recite the problem and its answer as he/she wrote the answer. Although results of the study showed that students acquisition of multiplication facts improved under both procedures, all students showed particular improvement under the Write-Say procedure.
Lambardo and Drabman attributed the success of the Write and Write-Say procedures over more traditional instructional procedures (e.g., practice without immediate feedback) to the "active" response made by students, and to the "immediate performance feedback" provided to students. They also speculated that under these instructional procedures students' performance may have reflected their "desire" to learn facts so as to avoid having to engage in either error correction procedure. Further, they attributed the relative difference between the two procedures and relatively improved performance under the Write-Say procedure to the "quantity" of active responses made by students.

Although intervention based research or single-subject type research in the area of math has been shown to be effective, overall it has been particularly under- and perhaps poorly implemented. For example, in their review of 10-years of special education research articles, Lessen, Dudzinski, Karsh, and Van Acker (1989) found fewer than 25 studies conducted on instructional interventions in math. In Pereira and Winton's (1991) inspection of 55 published studies that implemented an intervention or single-subject type design. They found that many of the studies provided incomplete descriptions of students, did not collect teacher or experimenter integrity type data, and that relatively few of the studies programmed for generalized outcomes.

Although there have been problems in the experimental methods used in the single-subject type research design in math, there are many examples of studies that have yielded positive outcomes. In addition, several factors contribute to the increased and perhaps renewed interest in this form of research and particularly its procedures (Cf. Lovitt & Curtiss, 1968; Lovitt &
Esveldt, 1970). First, this type of procedure is fairly easy to institute and yields results that can be measured relatively continuously (Blankenship, 1987) and thereby facilitates accountability. Second, these techniques have been proven to be effective in students' learning and mastery of mathematics (Fulk, 1992). Third, because of the relative ease and effectiveness of these various procedures, they can facilitate students' academic achievement in various settings and circumstances.

In summary, the findings from intervention based research can assist students in acquiring and mastering various mathematics skills. In many respects, the techniques examined by this line of research can help learning disabled students develop adequate foundation skills (e.g., basic computational skills) (Wallace & McLoughlin, 1979). This may be especially true among such students at the secondary level who can take more responsibility for their own learning.

**Generality and Maintenance**

A primary objective of error correction procedure is to prevent students from continuing to make errors. The efficacy of such a procedure is dependent upon the durability of the results in "non-instructional" settings and circumstances (Stokes & Baer, 1977). This is accomplished by the teacher or instructional designer who programs instruction so that the learner maintains and transfers the skill or knowledge to other situations and contexts. There are several instructional strategies that have incorporated error correction procedures (e.g., self-monitoring, computer assisted instruction, and peer tutoring). Each of these strategies can be used by students independent of the teacher, and in different settings and contexts. In addition, these particular strategies meet the defining characteristics of maintenance and generality of
behavior change (Stokes & Baer, 1977), and the need for procedures that are socially valid (Wolf, 1978).

Social Validity

Like generality, another major characteristic of applied behavior analysis is to study behaviors that are "of immediate importance to the student or individual receiving the procedure" (Cooper et al., 1987, p. 5). The use of error correction procedures is obviously not exempt from this characteristic. Because the procedure itself involves a response from a student or individual. Therefore, the student has the potential to counteract the error correction procedure, thereby potentially lessening or limiting the outcome of error correction. This lack of student compliance can also extend to the person administering the error correction procedure. Thus, the procedure itself must be considered socially valid by those who are participating in it. Schwartz and Baer (1991) refer to social validity as the degree of viability and acceptability of a behavioral procedure or intervention by the student or individual and other individuals involved. This has direct implications on the generalized outcome of corrective procedures.

It would seem incumbent on the teacher or other individual delivering error correction procedures to design or use procedures that are effective and acceptable in some measurable way to the student. In part, this may mean the teacher needs to anticipate precipitating factors of countercontrol (Martin & Pear, 1988). In other words, the teacher would need to assess for signs of countercontrol, as well as institute procedures that are individualized and minimize the negative effects of countercontrol. In simple terms, the error correction procedure should be functional without being detrimentally aversive.
CHAPTER III
METHOD

This chapter describes the method used to conduct the study and includes a description of subjects and setting, experimenter, definitions and measurement of dependent and independent variables, interobserver agreement, social validity measures, materials, experimental design, and procedures.

Subjects and Setting

Five male high school students in grade 9 who were identified as either having specific learning disabilities (SLD) in math, or considered to be "at-risk" for academic failure participated in this study. Students with the learning disabilities classification received special education services through a state-approved program. In comparison, students with the "at-risk" classification were enrolled in the school's deterrent program. This program monitored students' performance and attendance in school, and offered students assistance in completing their classwork and in locating and maintaining after school employment.

Several criteria were used to select students. First, the 5 youngsters who participated in the study ranged in age from 15 years to 17 years. The 3 students who received special education services obtained a portion of their instruction in general education classes. The 2 students who were classified as "at-risk" were enrolled in general education classes. Second, the students
in special education met the requirements for the Specific Learning Disabilities (SLD) program as specified by the Rules for the Education of Handicapped Children Handbook (Ohio Department of Education, 1982).

Next, the students were nominated by their special education or general education teacher. Specifically, students who participated in the study (a) had received instruction in long division, but continued to make calculation errors with these problems, and (b) would benefit from additional instruction in long division. Fourth, the teacher nominated students who displayed mathematical computational deficits relative to their age peers in general education. The determination of relative deficits was based in part upon the students' grades in mathematics, and their test scores on the California Test of Basic Skills. Fifth, students were selected based upon their performance on the Key Math Diagnostic Arithmetic Test-Revised test (Key Math-R) (Connolly, 1988), and the ENRIGHT© Diagnostic Inventory of Basic Arithmetic (DIBAS) (Enright, 1983). These tests served to: (a) obtain current standardized mathematics test scores, (b) determine whether the student has the necessary entry skills to perform long division problems, and (c) further document and establish the students' level of performance on long division.

If available the student's Wechsler Intelligence Scale for Children-Revised (WISC-R) test scores had to fall within the average range (i.e., 85 to 115). Table 1 shows the students' scores on standardized assessment instruments, the California Test of Basic Skills, current grade in mathematics, and their personal identifying information (age, gender, grade level, level of special education). To protect the students' identity they were given fictitious names which appear in Table 1.
Table 1

Student Characteristics

<table>
<thead>
<tr>
<th>Student</th>
<th>Gender</th>
<th>Age</th>
<th>Ethnicity</th>
<th>SES</th>
<th>Grade</th>
<th>IQ</th>
<th>Overall Academic Ach.</th>
<th>Math Academic Ach.</th>
<th>Math Computation Academic Ach.</th>
<th>Grade in Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>male</td>
<td>17-0</td>
<td>white</td>
<td>low</td>
<td>9</td>
<td>N/A</td>
<td>711</td>
<td>55</td>
<td>55</td>
<td>D</td>
</tr>
<tr>
<td>2a</td>
<td>male</td>
<td>16-9</td>
<td>white</td>
<td>low</td>
<td>9</td>
<td>87</td>
<td>N/A</td>
<td>68</td>
<td>59</td>
<td>F</td>
</tr>
<tr>
<td>3b</td>
<td>male</td>
<td>15-4</td>
<td>black</td>
<td>low</td>
<td>9</td>
<td>85</td>
<td>N/A</td>
<td>55</td>
<td>55</td>
<td>F</td>
</tr>
<tr>
<td>4b</td>
<td>male</td>
<td>16-2</td>
<td>black</td>
<td>low</td>
<td>9</td>
<td>85</td>
<td>499</td>
<td>55</td>
<td>55</td>
<td>C</td>
</tr>
<tr>
<td>5b</td>
<td>male</td>
<td>16-6</td>
<td>black</td>
<td>low</td>
<td>9</td>
<td>90</td>
<td>711</td>
<td>55</td>
<td>55</td>
<td>F</td>
</tr>
</tbody>
</table>

1. Students attended a urban high school.
2. Name(s) of test(s) used California Basic Skills Test- Math subtest (expressed in standard score).
3. Name(s) of test(s) used Key Math-Revised total test score (expressed in standard score).
4. Name(s) of test(s) used Key Math-Revised computational test score (expressed in standard score).
5. Average grade in math for the present school year.
N/A = not available.  a not in special education.  b enrolled in a learning disabilities resource classroom
Finally, in compliance with state and federal regulations, the experimenter obtained written permission from the students' parents to participate in the study. To protect the anonymity of the parents and students, school personnel obtained written permission. The letter and permission form were furnished by the experimenter (see Appendix A).

Most of the study was conducted in the learning disabilities classroom at a large table located at the back of the room. On two successive days, when the classroom was being painted, sessions were held in a conference room in the library. The table used in the library was the same size as the table used in the classroom.

**Experimenter and Observers**

The experimenter was a doctoral candidate in the third year of his Ph. D. program in special education and applied behavior analysis.

The experimenter had 20 years experience in special education. During his years of service, he had held various positions including volunteer, classroom aide, residence hall advisor, substitute teacher, school psychologist, and educational consultant.

The experimenter had two years of experience conducting single subject research. He had carried out studies with pre-school aged students in special education in the area of social skills, and with students in special and general education that involved error correction strategies.

The two observers in the study each had backgrounds in special education. The first observer collected procedural reliability and was a doctoral student in special education with emphasis in applied behavior analysis. The second observer was completing an undergraduate degree in special education with emphasis in developmental disabilities. The second
observer (a) checked students session tests and worksheets for accuracy, (b) completed error analysis on session tests, and (c) completed checks of the experimenters accuracy in presenting the erred problems on the error correction worksheet. Finally, the second observer administered social validity measures to students. Both observers were trained in completing their respective roles prior to the study. In addition, the observers were naive to the purpose of the study.

**Definition and Measurement of the Dependent Variables**

This section describes the dependent variables measured during the experiment and includes a description of rate and percentage of digits correct, percentage of digits correct, mean number of math problems solved correctly subsequent to error correction, type and number of errors (error analysis), maintenance tests, generality tests, experimenters accuracy in scoring students’ permanent products, and social validity measures.

**Rate and percentage of digits correct.** The primary dependent variable was the rate and percentage of digits correct on a specific type of long division problem administered by timed tests during each session (session tests). Rate of digits correct referred to the number of digits written correctly by students during session tests. Digits correct were divided by 3 minutes which referred to the number of minutes the students were given to complete the test. Only digits that constituted the answer were counted. Specifically, the total number of digits that represented the quotient, subtrahend, and a remainder were summed together. For example, in the problem 2+22217, the quotient is 11108 (digits = 5), the subtrahend was 22208 (digits = 6), and remainder is 1 (digits = 1). The total number of digits is 12. If a student wrote above a divisor to denote that a number was being regrouped, that digit was not counted. In
the event that a student wrote digits to the side of the problem (e.g., multiplies the quotient and divisor, or estimates the number of times the divisor goes into the dividend), these digits were not counted in the analysis.

Percentage of digits correct. Percentage of digits correct were determined based upon students' responses on worksheets, maintenance tests, and the comprehensive test administered after the experiment was completed. Percentage of digits correct were determined by taking the total number of digits written correctly and dividing it by the total number of digits written correctly and the total number of digits written incorrectly during a timed test. This ratio was then multiplied by 100. The percentage of digits correct were determined based upon number of problems attempted during the timed tests. Problems not attempted by the student, or problems the student did not mark, were not included in determining the percentage of digits correct.

Mean number of math problems solved correctly on worksheets immediately after error correction. To evaluate the mean number of long division problems solved correctly immediately after error correction, that is the next-problem after error correction, the second observer inspected worksheets and error correction sheets. For each problem that a student erred on (i.e., digit(s) incorrect) and received error correction, the second observer examined the next-problem on the worksheet and the error correction sheet. She examined the next-problem on the worksheet (that followed the erred problem) to see if the student got the problem correct. Then she confirmed whether or not the student received error correction on the erred problem by examining the error correction sheet. This second observer recorded her findings on a separate recording form.
The type and number of errors made on end-of-session tests.

Students' end-of-session tests were evaluated for three types of errors: estimation, computation, and omission errors. Enright's (1983) system for analyzing and categorizing long division errors was used. The second observer conducted this analysis and quantified and recorded the type of errors students made.

Maintenance measures. Maintenance referred to the students completing a response correctly after training or instruction was completed. In the present experiment, students' maintenance of a specific type or types of long division problems (e.g., one digit divisor and one digit quotient; no remainder; or two digit divisor; one or two digit quotient) were assessed immediately before and after the student completed a worksheet of long division problems. In addition, approximately one week following the last end-of-session test students were administered a comprehensive test of long division.

As mentioned earlier, session tests were administered immediately prior to and following the administration of a worksheet. The test taken prior to the administration of a worksheet was referred to as "next-session-test." The test taken after the completion of a worksheet was called an "end-of-session test." Students were not administered a next-session-test on the first day of instruction. Rather, the student was administered a worksheet on a particular type of long division problem. Their assignment on this worksheet was based upon either pre-experimental testing, or performance on a previous worksheet of a particular type of long division problem.
As mentioned earlier, approximately one week after students completed their final end-of-same test, a comprehensive test of division problems of the type previously completed by the student was administered. This comprehensive test consisted of an alternative set of division problems from ENRIGHT® Diagnostic Inventory of Basic Arithmetic (DIBAS) (Enright, 1983). The comprehensive test was administered according to the procedures for the other maintenance tests (i.e., next-session, end-of-session), save for students were given 25 minutes to complete the comprehensive test.

Generality measures. Stimulus and response generality were measured during the study. Stimulus generality occurred when students responded to an event or a stimulus situation that was similar but not identical to an event or stimulus situation present during instruction. In this study students displayed stimulus generality when they exhibited a targeted response (solving long division problems) under different conditions and with a different set of long division problems. In this experiment, students had available to them either error correction procedure only while they completed a worksheet that contained a specific type of long division problem (e.g., one digit divisor and one digit quotient; no remainder; or two digit divisor; one or two digit quotient). Response generality was measured when the student completed session tests of a different set of long division problems.

Response generality was also measured during the experiment. Response generality referred to responses that were functionally related to the taught or targeted response. Although these responses were not directly taught or targeted, the responses were nonetheless emitted in those situations or settings under which the targeted behavior was emitted. In the present
study, response generality was recorded by measuring the rate and percentage of digits correct on timed tests administered to students prior to (next-session-test) and following (end-of-session test) the administration of a different long division worksheet. In each of these testing situations the student was required to solve long division problems not encountered on the worksheet within a specified period of time.

**Accuracy checks on math data scoring.** The experimenter’s accuracy in scoring students’ worksheets was checked by the “second observer.” This observer scored independently (correct and incorrect) all of the worksheets and session tests completed by the students.

**Social validity.** Social validity referred to the students’ and/or teachers’ acceptance and satisfaction with a particular intervention or procedure. The teachers, whose students participated in the study, anonymously completed a questionnaire about the experiment. The students completed an questionnaire during an interview with the second observer.

**Interobserver Agreement**

Two observers assisted in conducting the study. The observers were trained separately by the experimenter. The first observer (primary observer) recorded the activities of the experimenter and students during the experimental sessions. This observer completed a checklist as sessions were conducted. Using this checklist, she recorded how accurate the experimenter was collecting data on the students’ performance, and measuring the students’ compliance with directions. This observer was naive to the study’s objective, and to the particular conditions in the study. Given that this observer was present during the study, she may have made assumptions and drawn conclusions on her own about the experiment.
The primary observer collected procedural agreement measures on 30% of all sessions for all students. On a checklist, the primary observer recorded the experimenter's compliance with the sessions activities and specifically, accurate execution of error correction procedures (see Appendix B).

The second observer was trained to score the students' "permanent products": long division worksheets, session tests and comprehensive test. In addition, the second observer was trained how to analyze and categorize student's errors. This observer was not informed of the independent and dependent variables used in the study. When the second observer checked the students' responses to the long division problems, she compared each digit in the students' answer with correct digit responses on an answer key. The answer key referred to a worksheet on which the long division problems were solved by the experimenter. Answer keys were also used by the experimenter to assess the students' accuracy when completing problems on their worksheets, and as a reference when either error correction procedure was administered.

The second observer scored and recorded on students' worksheets, session tests, and comprehensive tests. Digits were not circled on the students' problem if the digits were identical to digits on the same problem on the answer key. In contrast, digits were circled on the students' problem if they were not the same as digits on the answer key. At the top of the students' worksheet, the observer recorded the number of correct digits, incorrect digits, and the total number of digits written by the student. In turn, the second observer, computed the rate and percentage of digits correct, and recorded scores on a data collection sheet. As said earlier, this observer recorded
whether or not a worksheet problem that immediately followed an error correction procedure was correct or incorrect. This was referred to as "next-problem correct."

The second observer also completed accuracy checks on the experimenter's administration of the independent variable, (i.e., the error correction procedures). On all of the error correction worksheets, the second observer compared the problem(s) completed by the student with the answers made by the experimenter (Tell Plus Show error correction). In addition, on all of the worksheets on which the Tell Plus Write error correction procedure was delivered, the second observer assessed the students' response with the correct answers on the answer key. The second observer applied the same method for scoring or comparing digits on the student worksheets when conducting accuracy checks. The second observer recorded the accuracy data on separate recording forms.

Finally, the second observer completed an error analysis on student's end-of-session tests. This error analysis identified patterns or types of errors a student made on his division problems. Students' errors were analyzed and categorized according to three of Enright's (1983) categories of errors: estimation, computational, and omission errors.

**Definition and Measurement of the Independent Variables**

The independent variables in the study consisted of (a) an experimenter modeled error correction procedure (Tell Plus Show error correction), and (b) experimenter directed error correction procedure (Tell Plus Write error correction). The presentation of the error correction procedures were alternated randomly each session. In other words, the Tell Plus Show error correction procedure was used on some sessions, and the
Tell Plus Write error correction procedure was used on other sessions. A random drawing determined the sequence.

In addition, the error correction procedures were presented by the experimenter from a script, and a record was kept on the students' compliance in carrying out the steps in each error correction procedure. The administration of the two error correction procedures is described in the procedures section.

Materials

The materials and equipment used in the study are listed and described below.

Writing supplies. Students wrote with an erasable medium point black ink pen. An erasable ink pen allowed students to make corrections and provided a more readable and reliable mark than pencils. In addition, writing in ink is important for backup data purposes.

Student permanent products. Long division worksheets, and maintenance tests, were printed on the computer on 21.7 cm x 28 cm white paper. At the top of each of these worksheets, or tests, there was a place to record the student's name, date, session number, type of error correction procedure, elapsed time, the function of the sheet of long division problems (e.g., worksheet, session test, error correction sheet), and a space to report digits (in)correct, and "next problem correct" (completed on only the worksheet). The second observer was not told what the symbols at the top of the session tests, worksheet, error correction sheet represented. She was shown, however, how to read and record data related to the "symbols"

Long division worksheets. Each worksheet had 42 problems. There was a different worksheet for each type of long division problem (e.g., one
digit divisor and one digit quotient; no remainder; or two digit divisor; one or
two digit quotient). A worksheet had 6 rows of 7 problems. There were five
different forms of the same long division problem worksheet. The order of
problems was different on each form of a long division worksheet. The
purpose of the different forms was to prevent students from memorizing the
answers to the long division problems (see Appendix C).

**Session tests.** Session tests consisted of different problems but the
same type of long division problem (e.g., two digit divisor; one or two digit
quotient) as used on worksheets. A session test had 6 rows of 7 problems
printed by computer on each sheet of paper. The session test contained 42
problems. Essentially, the format of session test was identical to the
worksheet (see Appendix D).

**Comprehensive maintenance test.** The comprehensive test consisted
of the alternate or second form of the DIRAS (Enright, 1983). The number of
problems on a comprehensive test depended upon the number of types of
long division problems administered to a given student during the study. The
format of the comprehensive test was similar to the format used on session
tests.

**Worksheet answer key.** A worksheet of long division problems was
solved by the experimenter and checked by the second observer for each type
of long division problems administered to students. There were answer keys
for each form of a worksheet of long division problems. The answer key
showed each problem solved in adherence with the standard or conventional
long division algorithm. In other words, the solved problem showed not only
the quotient, but also the steps (estimation, multiplication, and subtraction)
leading to the quotient (answer). Identical answer keys were used by the experimenter and both observers in their respective roles (see Appendix E).

**Error correction worksheet.** An unmarked worksheet identical to the kind used by the student was used during the administration of error correction procedures (see Appendix F).

**Data sheets and graphs.** A set of individual data recording forms for students' performance on session tests, worksheets, and comprehensive tests was used by the experimenter. The forms provided a space to record the students' name, session date, type of error correction procedure, performance on the worksheet, and session tests. Finally, the basic data reported on the recording form was expressed in terms of rate, number and percentage of digits correct on long division problems.

A second data recording form was kept on students' computational errors. For each student, a record was kept on the type and frequency of errors a student made during a given session of long division problems.

Students' individual performance on session tests and worksheets was recorded on separate graphs. A graph was constructed for session tests to show rate of digits correct and incorrect. A second graph was kept on the number of correct and incorrect digits on worksheets of long division problems. The point when a new or different type of long division problem was administered was indicated on each of the students' graphs. In addition, on each graph the number of sessions spent on a given type of long division problem was reported.

Data for students' individual recording form and graphs were updated after each instructional session. Data forms were used by the experimenter and second observer. The recording forms used by the second observer were
identical to the form used by the experimenter. The recording forms had a place to write: student’s name, date, session number, type of long division problem under instruction, and error correction strategy in effect. The second observer also reduced and recorded data from first observer’s procedural reliability observations.

Math textbooks. Several different math textbooks were used to establish a balanced representation of long division problems. The problems were categorized by type of long division problem. The experimenter used a computer (Macintosh SE/30) to generate worksheet problems and maintenance tests problems. These problems were printed on white bound paper (21.7 cm x 28 cm).

Multiplication fact chart. Two commercially made multiplication tables (American Teaching Aids) were used. One table displayed facts 2 through 6 (e.g., 3x12, 6x9), and the second table showed facts 7 through 12 (e.g., 10x12, 7x4). Each chart measured 21.7 cm x 28 cm.

Timers. A Radio Shack brand Micronta Dual TimerR was used by the experimenter during the session tests, worksheets, maintenance, and comprehensive tests. The first observer was able to see and read the timer throughout a session.

Calculator. A Texas Instrument Model TI - 1795+ was used by students to check problems solved on worksheets used during either error correction procedure. Prior to the study, students were taught how to use the calculator to proof long division problems. The experimenter monitored the students accuracy in checking answers to long division problems solved on worksheets used during error correction. Students were to be retrained in the use of the
calculator should they make an error in proofing long division problems on worksheets used during error correction.

Experimental Design

The experiment extended across an 8-week period, 5 days per week. The length of time or number of sessions a particular type of long division problem was taught under the error correction conditions depended upon student performance, and/or graphic analysis of individual student responses. That is, students were advanced to the next highest type of long division problem after they achieved at least 70% of the digits correct on the type of problem currently being used with them. This was determined based upon their performance on end-of-session tests.

An alternating treatment design was used to compare the differential effects of Tell Plus Show, and Tell Plus Write error correction strategies on the acquisition, generalization and maintenance of long division algorithm. The assignment of a particular type of long division problem was based upon either student pre-testing and estimated level of acquisition (skill level) on a type of long division problem, or student achievement on worksheets of a particular type of long division problem. Each session (day) a different form of a worksheet was administered to the student. In other words, each day students solved comparable problems to the problems solved during the previous session. The purpose of the different forms was to prevent students from memorizing the answers to the long division problems.

The form of the worksheet used during a given session was subject to either error correction procedure. The presentation of the Tell Plus Show or Tell Plus Write error corrections conditions was counterbalanced across daily sessions.
PROCEDURE

The following section describes the procedures used in the study. The section includes a description of general experimental procedures, student assessment, and pre-experimental training. In addition, this section discusses generality and maintenance testing, session instruction procedures, and the administration of error correction strategies.

General Experimental Procedures

Initially, the experimenter sat next to student only when the student completed a worksheet of long division problems. In comparison, when the student engaged in maintenance-type testing, the experimenter sat across from the student. Initially, this same seating arrangement was also in place for the first observer. Within a few session of conducting the study, however, the experimenter and first observer agreed that to move from one seat to another was awkward, especially for the experimenter. Thus, the experimenter remained seated next to the student throughout the session. In contrast, the first observer remained seated across from the student and experimenter and in readable view of students computing division problems.

Students had access to a multiplication fact table when they completed worksheets of long division problems. The tables were available so that measures of long division could be obtained irrespective of students' knowledge of memorized multiplication facts.

Consideration was given to allow students to use a calculator while completing the worksheet of long division problems. Indeed, there would be advantages to allow students to use an electronic calculator to solve the worksheet of long division problems. For example, a calculator offers speed and accuracy in obtaining multiplication products. The overriding
disadvantage in allowing the student to use a calculator is that students would arrive at the answer (e.g., divisor) without actually carrying out the long division problem. In other words, the use of a calculator could make it difficult to determine what effect error correction strategies had on students' acquisition of long division skills.

The experimenter administered an error correction strategy immediately after a student made an error on a long division problem on a worksheet. The error correction strategy was presented on a second unmarked worksheet of identical problems. On this unmarked worksheet, the experimenter either modeled for the student ("Tell Plus Show"), or verbally directed ("Tell Plus Write") the student through the steps to solve the erred problem. Essentially, the student was given another chance to solve the problem on which he erred. In the "Tell Plus Show" error correction strategy, the student watched and listened to experimenter as he solved the problem. In the "Tell Plus Write" error correction strategy, the student solved the problem according to the experimenter's directions. When administering each error correction strategy, the experimenter provided all the steps to solving the problem one direction at a time. In other words, the experimenter stated the steps to solving the division problem one step at a time. In addition, depending upon the error correction strategy in use, the experimenter wrote or had the student write the response (i.e., number). The experimenter did not move on to the next step to solving the division problem until the current step was completed accurately. A step was completed when the student or experimenter was finished writing the correct number.
Immediately following administration of either error correction strategy, the student was directed to check the accuracy of the answer (quotient) on the second worksheet. The student used a hand-held calculator to multiply the quotient by the divisor (and when appropriate add the remainder) and to see if the product equaled the dividend. The student stated and showed to the experimenter that the calculated result agreed or disagreed with the answer on his error correction worksheet (second worksheet). The purpose of the check with the calculator was to give the student another opportunity to respond to the division problem.

The second observer scored and checked students' permanent products (e.g., long division problem worksheets, session and maintenance tests) one to three times a week.

After the study was completed, the second observer administered to the students a social validity measure. The teachers completed anonymously and independently a social validity measure mailed to them. They mailed their completed questionnaires to the experimenter (see Appendix G).

Student Assessment

During the week prior to the experiment, students were administered the Key Math Diagnostic Arithmetic Test-Revised test (Key Math-R) (Connolly, 1988), and the ENRIGHT R Diagnostic Inventory of Basic Arithmetic (DIBAS) (Enright, 1983). The Key Math-R, and DIBAS were administered on separate days of the week.

The DIBAS assessed students' skills computing whole numbers long division problems. On the DIBAS, there are 22 subtests each consisting of a different type of division problems or subtests (e.g., 2-digit number with zero in the ones place by a 1-digit number, with no remainder, and 4-digit number by
a 3-digit number, with remainder). The DIBAS has two forms (primary and alternate) of the same subtest of a particular type of division problem. Each form of a subtest contains 5 different problems.

Students were assigned long division problems based upon their performance on the DIBAS division of whole numbers subtest. More specifically, students were assigned the type of division problem on which he achieved an accuracy range of between 0 to 50%.

Pre-Experimental Student Training

The day or two prior to the start of the study, the experimenter reviewed with students the symbols and basic terminology associated with long division: division equation sign (÷), long division symbol (infeld), dividend, divisor, quotient, remainder; and how to solve long division problems. Students were told that long division problems on worksheets and session and maintenance tests would be expressed with the long division symbol (infeld).

Also during this session, the error correction strategies were explained to students. The experimenter read a description of each error correction strategy, and demonstrated the error correction strategy with the student. The students were told that either the “modeling” (Tell Plus Show error correction) or “verbal direction strategy” (Tell Plus Write error correction) would be used with them on any given day. When the student made an error on a worksheet problem, either error correction strategy was administered. The error correction strategy to be administered was based upon a random drawing conducted just before meeting with the student for that session.
The experimenter administered the Tell Plus Show strategy by saying: "Your answer is not correct (right). Let me show you how to do it. Watch and listen to me as I solve the division problem on this worksheet (error correction worksheet). When we are finished you will check the answer with a calculator."

In comparison, the experimenter administered the Tell Plus Write strategy by saying: "Your answer is not correct (right). Let me tell you how to do it correctly. As I solve the problem aloud, on this worksheet (error correction worksheet) you'll write in the number for each step to the division problem. When we are finished you'll check the answer with a calculator." (See Appendix H for examples of the error correction scripts).

Also, on the day or two days before the beginning of the study, students were informed of the purpose behind the seating arrangement used during the study. The student was told that the experimenter needed to be in clear view and close proximity in order to detect an error and to deliver the appropriate procedure.

The experimenter also showed and reviewed with students how to use a multiplication fact table and an electronic calculator. Students were told that they would not have access to the multiplication fact table and the calculator during the session tests (next-session, end-of-session), and comprehensive test.

Session Tests

Session tests were administered prior to (next-session test) and following (end-of-session test) the administration of a long division problem worksheet. Specifically, with the exception of the first day of instruction on a specific type of long division problem, students were administered a next-
session test prior to being administered a long division problem worksheet. Similarly, after the student completed a long division problem worksheet, an end-of-session test was administered. These session tests consisted of the type of long division problem solved by students on the long division worksheet. The problems on the long division worksheet were not the same as the problems administered on the maintenance tests.

**Maintenance Tests**

Students were to be administered a maintenance test one week following the completion of a specific type of long division problem (e.g., one digit divisor and one digit quotient; no remainder; or two digit divisor; one or two digit quotient). "Completion" referred to a student achieving on two consecutive end-of-session tests at least 70% of the "digits correct." The maintenance test was to be a form of the worksheet on which error correction procedures were administered. Because of student attendance and performance this type of maintenance test was not administered.

**Session Instruction Procedure**

Once a long division worksheet was administered, or after the next-session test was completed and a worksheet was administered, error correction procedures were available for use. The student was given 12 minutes to complete a worksheet of long division problems. The type of error correction strategy to be used during a given session was predetermined. In other words, students did not know which error correction strategy was in effect for the session until they made an error.

Students' execution of long division problems on the worksheet were directly observed by the experimenter. The experimenter had an answer key before him for the particular worksheet of problems that the student was
solving. The experimenter was able to observe on the answer key where the student made the error(s) on his worksheet problem. The experimenter instituted the predetermined error correction procedure immediately after the student completed the long division problem on which he made an error.
CHAPTER IV
RESULTS

This chapter describes the results of the study. The chapter includes Interobserver Agreement (IOA) and procedural reliability data for the independent and dependent variables. In addition, Students' data are presented for the rate of digits correct during session tests, and the number and percentage of digits solved correctly on worksheets. Also, the mean number and percentage of digits correct on the next problem following error correction is reported. Further, the number and percentage of types of errors made on long division problems on end-of-session tests is reported. Finally, for comprehensive tests the number and percentage of digits correct, and the number and percentage types of errors is reported.

The number of sessions varied among Students. With the exception of Student 1, there was a significant problem with student attendance. The attendance record for Student 4 and Student 5 was particularly poor; they were absent for several weeks at a time. Thus, Student 4 and Student 5 after session 8 and 7 respectively were no longer administered the error correction procedure. Rather, when they attended school they were administered individually a probe that was administered like a session test (i.e., 3-minute timed test). The purpose of the probe was to measure these Students' performance on the type of long division problem under which they last received error correction.
INTEROBSERVER AGREEMENT SCORES

Interobserver agreement (IOA) measures were obtained for the independent variables (procedural IOA), and the dependent variables (i.e., rate of digits correct during session tests, number and percentage of digits solved correctly on comprehensive tests, and number and percentage of types of errors made on long division problems, and number and percentage next problem correct). The agreement scores were calculated by dividing the number of agreements (e.g., correct or incorrect digits) by the total number of agreements plus disagreements (total number of student responses) and multiplying by 100.

PROCEDURAL RELIABILITY

Procedural reliability checks were taken each week during the study. The first observer completed a checklist identical to the checklist used by the experimenter. The number of items completed on any given checklist depended upon whether or not the student was administered a next-session test. The checklist contained 35 items if a session included a next-session test, worksheet problems, and an end-of-session test. In the event, however, that the student was not administered a next-session test, the checklist contained 24 items. Further, the experimenter completed fewer items on the checklist when the student did not make any errors (30 items), or the student did not receive a next-session test and error correction (19 items).

Procedural reliability checks yielded a mean of 97.64% with a range of 70.8% to 100% of the procedures followed (see Table 2). During the course of the study there were 3 sessions when the experimenter did not completely follow procedures. During one session the introductory remarks about long division were not read to a student, and there was a delay in the presentation
Table 2

Procedural Interobserver Agreement for Next-Session, End-of-Session, and Worksheet Sessions.

<table>
<thead>
<tr>
<th>Session</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(17/24)</td>
<td>70.8</td>
</tr>
<tr>
<td>2</td>
<td>(35/35)</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>(35/35)</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>(35/35)</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>(24/24)</td>
<td>94.3</td>
</tr>
<tr>
<td>6</td>
<td>(33/35)</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>(35/35)</td>
<td>94.3</td>
</tr>
<tr>
<td>8</td>
<td>(35/35)</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>(35/35)</td>
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<tr>
<td>10</td>
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<td>11</td>
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<tr>
<td>14</td>
<td>(35/35)</td>
<td>100</td>
</tr>
<tr>
<td>15</td>
<td>(34/35)</td>
<td>97.14</td>
</tr>
<tr>
<td>16</td>
<td>(35/35)</td>
<td>100</td>
</tr>
</tbody>
</table>

Mean (31/34) 97.64%
Range (17 to 35) 70.8% - 100%
of an error correction procedure. On another session, and during the error correction procedure, the experimenter failed to tell the student twice to compare an addend with the divisor. During a third session, the experimenter placed the end-of-session test face up instead of face down. The student was supposed to have this test presented face down, and to have the test instruction read to him, and then to be directed to turn the test over and begin work by the experimenter.

RATE AND PERCENTAGE OF DIGITS CORRECT
ON END-OF-SESSION TESTS

Students' performance on an end-of-session test was described relative to either the TPS or TPW error correction procedure. Students' results on the end-of-session tests were analyzed based upon the type of error correction procedure that was administered during the session.

The second observer checked the accuracy of students' written responses on the end-of-session tests. This observer checked and scored all of the Students' tests. In addition, this observer conducted the error analysis on Students' solutions to long division problems.

Student 1. Student 1 participated in 35 sessions. The sessions were almost evenly divided between the TPS (17) and TPW (18) error correction procedures. Figure 1 shows Student 1's rate of digits correct on the timed (3 minutes) end-of-session test. The arrows indicate when Student 1 was administered a different and more demanding type of long division problem. For example 3 digit divided by 2 digit number without remainder preceded a 3 digit divided by 2 digit number with a remainder. The transition to more difficult problems occurred when Student 1 achieved a percent correct of at least 70% or higher on two consecutive sessions. On average, he spent 7
sessions solving a particular type of long division problem. The number of
sessions ranged from 4 to 11 sessions. During the study, he was
administered 4 types of long division problems.

Under the TPS error correction condition, Student 1 averaged 8.54
digits correct per minute, with a range from 0 digits to 22 digits correct. In
comparison, under the TPW error correction condition, he averaged 9.96
digits correct per minute, with a range from 1 digit to 36.3 digits correct.
Inspection of Table 3 shows Student 1’s percentage of digits correct session
by session relative to the TPS and TPW error correction procedures. On
average his mean rate of digits correct was 9.96 under TPW condition as
compared to 8.54 under TPS condition.

Student 2. Student 2 participated in 10 sessions. Five sessions were
administered to him while he was placed in in-school suspension, which
followed 6 days out-of-school suspension. The sessions were almost evenly
divided between the TPS (6) and TPW (4) procedures. Session 8 ended,
however, before the end-of-session test could be administered. The school
was on a shortened schedule and the class period ended early.

Figure 2 shows Student 2’s rate of digits correct on the timed (3
minutes) end-of-session test. The arrows on this figure indicate when Student
2 was administered a different and more demanding type of long division
problem. For example, 3 digit divided by 2 digit number without remainder
preceded 3 digit divided by 2 digit number with a remainder. This transition
occurred when Student 2 achieved a percent correct of at least 70% or higher
on two consecutive sessions. On average, he spent 3 sessions solving a
particular type of long division problem. The number of sessions ranged from
Figure 1. Rate of digits correct on end of session test. Arrows indicate when a change in the type of long division problem occurred.
Table 3

Student 1's Number and Percentage of Digits Correct on End-of-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>End-of-Session Test Frequency (3 Min.)</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-2</td>
<td>TPS</td>
<td>8/12a</td>
<td>66%</td>
</tr>
<tr>
<td>2</td>
<td>D-2</td>
<td>TPW</td>
<td>34/88</td>
<td>38.6%</td>
</tr>
<tr>
<td>3</td>
<td>D-2</td>
<td>TPS</td>
<td>63/76</td>
<td>82.8%</td>
</tr>
<tr>
<td>4</td>
<td>D-2</td>
<td>TPW</td>
<td>109/140</td>
<td>77.8%</td>
</tr>
<tr>
<td>5</td>
<td>D-8</td>
<td>TPW</td>
<td>14/29</td>
<td>48%</td>
</tr>
<tr>
<td>6</td>
<td>D-8</td>
<td>TPS</td>
<td>37/53</td>
<td>69.8%</td>
</tr>
<tr>
<td>7</td>
<td>D-8</td>
<td>TPS</td>
<td>66/72</td>
<td>91.6%</td>
</tr>
<tr>
<td>8</td>
<td>D-8</td>
<td>TPW</td>
<td>48/48</td>
<td>100%</td>
</tr>
<tr>
<td>9</td>
<td>D-10</td>
<td>TPW</td>
<td>5/20</td>
<td>25%</td>
</tr>
<tr>
<td>10</td>
<td>D-10</td>
<td>TPS</td>
<td>3/9</td>
<td>33%</td>
</tr>
</tbody>
</table>
## Table 3 cont

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>End-of-Session Test Frequency (3 Min.)</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>D-10</td>
<td>TPS</td>
<td>29/50</td>
<td>58%</td>
</tr>
<tr>
<td>12</td>
<td>D-10</td>
<td>TPW</td>
<td>4/10</td>
<td>40%</td>
</tr>
<tr>
<td>13</td>
<td>D-10</td>
<td>TPS</td>
<td>40/55</td>
<td>72.7%</td>
</tr>
<tr>
<td>14</td>
<td>D-10</td>
<td>TPW</td>
<td>13/36</td>
<td>36%</td>
</tr>
<tr>
<td>15</td>
<td>D-10</td>
<td>TPW</td>
<td>22/45</td>
<td>48.8%</td>
</tr>
<tr>
<td>16</td>
<td>D-10</td>
<td>TPS</td>
<td>34/56</td>
<td>60.7%</td>
</tr>
<tr>
<td>17</td>
<td>D-10</td>
<td>TPW</td>
<td>35/35</td>
<td>100%</td>
</tr>
<tr>
<td>18</td>
<td>D-10</td>
<td>TPS</td>
<td>30/36</td>
<td>83%</td>
</tr>
<tr>
<td>19</td>
<td>D-15</td>
<td>TPS</td>
<td>12/29</td>
<td>41%</td>
</tr>
<tr>
<td>20</td>
<td>D-15</td>
<td>TPW</td>
<td>46/65</td>
<td>70.7%</td>
</tr>
<tr>
<td>Session Number</td>
<td>Type of Division Problem</td>
<td>Type of Error Correction Procedure</td>
<td>End-of-Session Test Frequency (3 Min.)</td>
<td>End-of-Session Test Percent</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------------</td>
<td>------------------------------------</td>
<td>----------------------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>21</td>
<td>D-15</td>
<td>TPW</td>
<td>39/51</td>
<td>76.4%</td>
</tr>
<tr>
<td>22</td>
<td>D-15</td>
<td>TPS</td>
<td>34/65</td>
<td>52%</td>
</tr>
<tr>
<td>23</td>
<td>D-15</td>
<td>TPS</td>
<td>48/63</td>
<td>76%</td>
</tr>
<tr>
<td>24</td>
<td>D-15</td>
<td>TPW</td>
<td>49/71</td>
<td>69%</td>
</tr>
<tr>
<td>25</td>
<td>D-16</td>
<td>TPS</td>
<td>6/52</td>
<td>11.5%</td>
</tr>
<tr>
<td>26</td>
<td>D-16</td>
<td>TPW</td>
<td>20/64</td>
<td>31%</td>
</tr>
<tr>
<td>27</td>
<td>D-16</td>
<td>TPW</td>
<td>3/20</td>
<td>15%</td>
</tr>
<tr>
<td>28</td>
<td>D-16</td>
<td>TPS</td>
<td>0/38</td>
<td>0%</td>
</tr>
<tr>
<td>29</td>
<td>D-16</td>
<td>TPW</td>
<td>12/32</td>
<td>37.5%</td>
</tr>
<tr>
<td>30</td>
<td>D-16</td>
<td>TPS</td>
<td>1/9</td>
<td>11%</td>
</tr>
<tr>
<td>31</td>
<td>D-16</td>
<td>TPW</td>
<td>25/50</td>
<td>50%</td>
</tr>
<tr>
<td>32</td>
<td>D-16</td>
<td>TPW</td>
<td>50/56</td>
<td>89.2%</td>
</tr>
</tbody>
</table>
(Table 3 cont)

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>End-of-Session Test Frequency (3 Min.)</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>D-16</td>
<td>TPS</td>
<td>3/13</td>
<td>23%</td>
</tr>
<tr>
<td>34</td>
<td>D-16</td>
<td>TPW</td>
<td>10/24</td>
<td>41.6%</td>
</tr>
<tr>
<td>35</td>
<td>D-16</td>
<td>TPS</td>
<td>22/22</td>
<td>100%</td>
</tr>
</tbody>
</table>

Range<sup>b</sup>  
- TPS = 0 to 22 digits  
- TPW = 1 to 36 digits

Grand Mean<sup>b</sup>  
- TPS = 8.54  
- TPW = 9.96  
- TPS = 61.40%  
- TPW = 60.85%

<sup>a</sup> Number of digits correct by the total number digits written.  
<sup>b</sup> Range and Grand Mean based upon digits correct per minute.
Figure 2. Rate of digits correct on end-of-session test. Arrows indicate when a change in the type of long division problem occurred.
2 to 6 sessions. During the study he was administered 3 types of long division problems.

Under the TPS error correction condition, Student 2 averaged 6.16 digits correct per minute, with a range from 3.6 digits to 14 digits correct. In comparison, under the TPW error correction condition, he averaged 6.25 digits correct per minute, with a range from 0 digits to 14.3 digits correct. In addition, inspection of Table 4 shows Student 2's number of digits correct session by session relative to the TPS and TPW error correction procedures. On average his rate and percentage of digits correct was higher under the TPW (74%) condition than the TPS (61%) condition.

**Student 3.** Student 3 participated in 9 sessions. Three of the sessions used TPS and six of the sessions applied TPW error correction procedures. Figure 3 shows Student 3's rate of digits correct on the timed (3 minutes) end-of-session test. The arrows on this figure indicate when he was administered a different and more demanding type of long division problem. For example, 3 digit divided by 2 digit number without remainder preceded 3 digit divided by 2 digit number with a remainder. This transition occurred when Student 3 achieved a percent correct of approximately 70% or higher on two consecutive sessions. He spent as few as 1 day and as many as 4 days on a particular type of division problem. During the study, he was administered 3 types of long division problems.

Under the TPS error correction condition, Student 3 averaged 9.42 digits correct per minute, with a range from 1.6 digits to 24 digits correct. In comparison, under the TPW error correction condition, he averaged 13.81 digits correct per minute, with a range from 0 digits to 22 digits correct. Furthermore, Table 5 shows Student 3's number of digits correct session by
Table 4

Student 2’s Number and Percent of Digits Correct on End-of-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>End-of-Session Test Frequency (3 Min.)</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-16</td>
<td>TPW</td>
<td>0/10a</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>D-16</td>
<td>TPS</td>
<td>15/22</td>
<td>68.1%</td>
</tr>
<tr>
<td>3</td>
<td>D-16</td>
<td>TPW</td>
<td>43/54</td>
<td>79.6%</td>
</tr>
<tr>
<td>4</td>
<td>D-16</td>
<td>TPS</td>
<td>11/32</td>
<td>34%</td>
</tr>
<tr>
<td>5</td>
<td>D-16</td>
<td>TPS</td>
<td>25/35</td>
<td>71.4%</td>
</tr>
<tr>
<td>6</td>
<td>D-16</td>
<td>TPS</td>
<td>42/52</td>
<td>80.7%</td>
</tr>
<tr>
<td>7</td>
<td>D-18</td>
<td>TPW</td>
<td>20/22</td>
<td>90.9%</td>
</tr>
<tr>
<td>8</td>
<td>D-19</td>
<td>TPS</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>9</td>
<td>D-19</td>
<td>TPS</td>
<td>18/40</td>
<td>45%</td>
</tr>
<tr>
<td>10</td>
<td>D-19</td>
<td>TPW</td>
<td>12/15</td>
<td>80%</td>
</tr>
</tbody>
</table>
End-of-Session Test
Frequency
(3 Min.)

End-of-Session Test
Percent

<table>
<thead>
<tr>
<th></th>
<th>End-of-Session Test Frequency (3 Min.)</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range(^c)</td>
<td>TPS = 3.6 to 14 digits</td>
<td>TPS = 61.3%</td>
</tr>
<tr>
<td></td>
<td>TPW = 0 to 14.3 digits</td>
<td>TPW = 74.2%</td>
</tr>
<tr>
<td>Grand Mean</td>
<td>TPS = 6.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPW = 6.25</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Number of digits correct over the total number digits written. \(^b\) End-of-Session test not administered. \(^c\) Range and Grand Mean based upon digits correct per minute.
Figure 3. Rate of digits correct on end-of-session test. Arrows indicate when a change in the type of long division problem occurred.
Table 5

Student 3's Number and Percentage of Digits Correct on End-of-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>End-of-Session Test Frequency (3min.)</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-1</td>
<td>TPW</td>
<td>66/72a</td>
<td>91.6%</td>
</tr>
<tr>
<td>2</td>
<td>D-1</td>
<td>TPW</td>
<td>56/63</td>
<td>88.8%</td>
</tr>
<tr>
<td>3</td>
<td>D-1</td>
<td>TPW</td>
<td>63/76</td>
<td>82.8%</td>
</tr>
<tr>
<td>4</td>
<td>D-1</td>
<td>TPW</td>
<td>40/42</td>
<td>95%</td>
</tr>
<tr>
<td>5</td>
<td>D-16</td>
<td>TPS</td>
<td>72/77</td>
<td>93.5%</td>
</tr>
<tr>
<td>6</td>
<td>D-16</td>
<td>TPS</td>
<td>8/12</td>
<td>66.6%</td>
</tr>
<tr>
<td>7</td>
<td>D-16</td>
<td>TPS</td>
<td>5/20</td>
<td>25%</td>
</tr>
<tr>
<td>8</td>
<td>D-16</td>
<td>TPW</td>
<td>24/52</td>
<td>46%</td>
</tr>
<tr>
<td>9</td>
<td>D-17</td>
<td>TPW</td>
<td>0/10</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>End-of-Session Test Frequency (3min.)</td>
<td>End-of-Session Test Percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>---------------------------------------</td>
<td>----------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Range</strong>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>TPS = 1.6 to 24 digits</td>
<td>TPS = 77.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPW = 0 to 22 digits</td>
<td>TPW = 79%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Grand Mean</strong>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>TPS = 9.42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPW = 13.81</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Number of digits correct over the number of total digits attempted.

<sup>b</sup> Range and Grand Mean based upon digits correct per minute.
session on the TPS and TPW error correction procedures. His mean percentage of digits correct was slightly higher under the TPW condition (79%) than the TPS condition (77.9%). However, the TPS procedure was used with only one type of long division problem. In comparison, TPW procedure was used with each of the types of division problems administered to Student 3.

**Student 4.** Student 4 participated in 7 sessions of which 4 of the sessions instituted TPS and 3 of the sessions applied TPW error correction procedures. Figure 4 shows Student 4’s rate of digits correct on the timed (3 minutes) end-of-session test. The arrows on this figure indicate when Student 5 was administered a different and more demanding type of long division problem. This transition occurred when Student 4 achieved a percent correct of approximately 70% or higher on two consecutive sessions. Student 4 received instruction on only one-type of long division problem.

Under the TPS error correction condition, Student 4 averaged 14.90 digits correct per minute, with a range from 5 to 27.33 digits correct. In comparison, under the TPW error correction condition, he averaged 22.44 digits correct per minute, with a range from 14 to 27.66 digits correct. Furthermore, examination of Table 6 shows that on average the percentage of digits correct was higher under the TPW error correction procedure (83.8%) than the TPS error correction procedure (78%). Student 4 received error correction procedures for only 7 sessions.

**Student 5.** Student 5 participated in 7 sessions of which 4 of the sessions used TPS and 3 of the sessions applied TPW error correction procedures. Figure 5 shows Student 5’s rate of digits correct on the timed (3 minutes) end-of-session test. The arrows on this figure indicate when Student
Figure 4. Rate of digits correct on end-of-session test. Arrows indicate when a change in the type of long division problem occurred.
Table 6

Student 4's Number and Percentage of Digits Correct on End-of-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>End-of-Session Test Frequency (3min.)</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-15</td>
<td>TPS</td>
<td>15/29&lt;sup&gt;a&lt;/sup&gt;</td>
<td>51.7%</td>
</tr>
<tr>
<td>2</td>
<td>D-15</td>
<td>TPW</td>
<td>42/51</td>
<td>82%</td>
</tr>
<tr>
<td>3</td>
<td>D-15</td>
<td>TPS</td>
<td>22/41</td>
<td>53.6%</td>
</tr>
<tr>
<td>4</td>
<td>D-15</td>
<td>TPS</td>
<td>60/74</td>
<td>81%</td>
</tr>
<tr>
<td>5</td>
<td>D-15</td>
<td>TPW</td>
<td>77/93</td>
<td>82.7%</td>
</tr>
<tr>
<td>6</td>
<td>D-15</td>
<td>TPW</td>
<td>83/97</td>
<td>85.5%</td>
</tr>
<tr>
<td>7</td>
<td>D-15</td>
<td>TPS</td>
<td>82/84</td>
<td>97.6%</td>
</tr>
</tbody>
</table>

Range<sup>b</sup>  
TPS = 5 to 27.33  
TPW = 14 to 27.66  

Grand Mean<sup>b</sup>  
TPS = 14.90  
TPW = 22.44  

<sup>a</sup> Number of digits correct over the total number digits written.  
<sup>b</sup> Range and Grand Mean based upon digits correct per minute.
Figure 5. Rate of digits correct on end-of-session test. Arrows indicate when a change in the type of long division problem occurred.
Student 5 was administered a different and more demanding type of long division problem. For example, 3 digit divided by 1 digit number without remainder preceded 2 digit divided by 2 digit number with a remainder. This transition occurred when Student 5 achieved a percent correct of approximately 70% or higher on two consecutive sessions. During the study, he was administered 3 types of long division problems. Two types of long division problems were administered while he participated in the error correction procedure. The third and more advanced type of division problem was administered on the probe math sheet. He was given this more advanced type of problem because of his performance on the previous type of division problems.

Under the TPS error correction condition, Student 5 averaged 11.91 digits correct per minute, with a range from 5 to 15.66 digits correct. In comparison, under the TPW error correction condition he averaged 18.22 digits correct per minute, with a range from 15 to 22 digits correct. Furthermore, the average percentage of digits correct on the end-of-session test was higher during sessions when the TPW error correction (96.4%) was administered rather than the TPS procedure (84.1%) (see Table 7). Student 5 received error correction procedures for 7 sessions.

Table 8 summarizes the Students' mean number of digits correct per minute on end-of-session tests under the TPS and TPW error correction conditions. For this same test, Table 8 also shows the group's mean number of digits correct per minute on end-of-session tests for the respective error correction conditions.
### Table 7

**Student 5's Number and Percentage of Digits Correct on End-of-Session Tests.**

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>End-of-Session Test Frequency</th>
<th>End-of-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-2</td>
<td>TPS</td>
<td>36/45&lt;sup&gt;a&lt;/sup&gt;</td>
<td>80%</td>
</tr>
<tr>
<td>2</td>
<td>D-2</td>
<td>TPS</td>
<td>47/47</td>
<td>100%</td>
</tr>
<tr>
<td>3</td>
<td>D-2</td>
<td>TPW</td>
<td>53/56</td>
<td>94.6%</td>
</tr>
<tr>
<td>4</td>
<td>D-2</td>
<td>TPW</td>
<td>45/48</td>
<td>93.7%</td>
</tr>
<tr>
<td>5</td>
<td>D-8</td>
<td>TPS</td>
<td>45/56</td>
<td>80%</td>
</tr>
<tr>
<td>6</td>
<td>D-8</td>
<td>TPW</td>
<td>66/66</td>
<td>100%</td>
</tr>
<tr>
<td>7</td>
<td>D-8</td>
<td>TPS</td>
<td>15/22</td>
<td>68%</td>
</tr>
</tbody>
</table>

**Range<sup>b</sup>**

- TPS = 5 to 15.6
- TPW = 15 to 22

**Grand<sup>b</sup> Mean**

- TPS = 11.91
- TPW = 18.22

End-of-Session Test Percent

- TPS = 84.1%
- TPW = 96.4%

---

<sup>a</sup> Number of digits correct over the total number digits written.  
<sup>b</sup> Range and Grand Mean based upon digits correct per minute.
Table 8

<table>
<thead>
<tr>
<th>Student</th>
<th>TPS</th>
<th>TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>8.54</td>
<td>9.96</td>
</tr>
<tr>
<td></td>
<td>(61.4%)</td>
<td>(60.8%)</td>
</tr>
<tr>
<td>Student 2</td>
<td>6.16</td>
<td>6.25</td>
</tr>
<tr>
<td></td>
<td>(61.3%)</td>
<td>(74.2%)</td>
</tr>
<tr>
<td>Student 3</td>
<td>9.42</td>
<td>13.81</td>
</tr>
<tr>
<td></td>
<td>(77.9%)</td>
<td>(79%)</td>
</tr>
<tr>
<td>Student 4</td>
<td>14.9</td>
<td>22.4</td>
</tr>
<tr>
<td></td>
<td>(78%)</td>
<td>(83.8%)</td>
</tr>
<tr>
<td>Student 5</td>
<td>11.91</td>
<td>18.22</td>
</tr>
<tr>
<td></td>
<td>(84.1%)</td>
<td>(96.4%)</td>
</tr>
<tr>
<td>Grand Mean</td>
<td>10.1</td>
<td>14.12</td>
</tr>
<tr>
<td></td>
<td>(68.1%)</td>
<td>(71.7%)</td>
</tr>
</tbody>
</table>

a Mean percent of total digits correct on end-of-session tests.
RATE AND PERCENTAGE OF DIGITS CORRECT
ON NEXT-SESSION TESTS

Students' performance on next-session tests is described relative to either the TPS or TPW error correction procedure. Students' results on next-session tests were analyzed based upon the type of error correction procedure that was administered during the immediately previous session. For example, if during session 6 a student received the TPW error correction procedure, for session 7 his results on the next-session test were analyzed relative to the TPW error correction procedure.

Students' next-session tests were scored for accuracy by the second observer.

Student 1. As noted earlier Student 1 participated in 35 sessions. The type of error correction procedure administered was almost evenly represented across sessions: TPS (17) and TPW (18). The total number of next-session tests, however, was 32: TPS (16) and TPW (16). This total reflects the fact that next-session tests were not administered on the first session of the study, and on two subsequent sessions when a change was made in the type of division problem administered to Student 1.

Figure 6 shows Student 1's number of digits correct on the timed (3 minutes) next-of-session test. The arrows on this figure indicate when Student 1 was administered a different and more demanding type of long division problem. As before, this transition occurred when Student 1 achieved a percent correct of at least 70% or higher on two consecutive sessions. On average he spent 3 sessions solving a particular type of long division problem. The number of sessions ranged from 2 to 6 sessions. During the study he was administered 3 types of long division problems.
Under the TPS error correction condition, Student 1 averaged 11.24 digits correct per minute, with a range from 0 digits to 39 digits correct. In comparison, under the TPW error correction condition, he averaged 9.99 digits correct per minute, with a range from 0 digits to 31 digits correct. In addition, a review of Table 9 shows the number and percentage of digits solved correctly by Student 1 under the TPW and TPS error correction condition for next-session tests. Under the TPS condition, his percentage of digits correct averaged 66.3, with a range from a low of 0% to a high of 100% of digits correct. In contrast, under the TPW condition, his percentage of digits correct averaged 60.5, with a range from a low of 0% to a high of 100% of digits. Overall, his number of digits correct and percent of digits correct on long division problems was slightly lower under the TPW condition (60.5%) than the TPS condition (66.3%).

Student 2. Because a next-session test was not administered on the first session of the study, Student 2 participated in 9 next-session tests. The sessions were almost evenly divided between the TPS (6) and TPW (3) procedures.

Figure 7 shows Student 2's rate of digits correct on the timed (3 minutes) next-session test. The arrows on this figure indicate when Student 2 was administered a different and more demanding type of long division problem. For example 3 digit divided by 2 digit number without remainder preceded 3 digit divided by 2 digit number with a remainder. As before, this transition occurred when Student 2 achieved a percent correct of at least 70% or higher on two consecutive sessions. On average he spent 3 sessions solving a particular type of long division problem. The number of sessions
Figure 6. Rate of digits correct on next-session test. Arrows indicate when a change in the type of long division problem occurred.
Table 9

Student 1's Number and Percentage of Digits Correct on Next-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Next-Session Test Frequency (3 min.)</th>
<th>Next-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>D-2</td>
<td>TPSb</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>D-2</td>
<td>TPW</td>
<td>52/129c</td>
<td>40%</td>
</tr>
<tr>
<td>3</td>
<td>D-2</td>
<td>TPS</td>
<td>93/93</td>
<td>100%</td>
</tr>
<tr>
<td>4</td>
<td>D-2</td>
<td>TPW</td>
<td>118/123</td>
<td>95.9%</td>
</tr>
<tr>
<td>5</td>
<td>D-8</td>
<td>TPW</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>D-8</td>
<td>TPS</td>
<td>18/40</td>
<td>45%</td>
</tr>
<tr>
<td>7</td>
<td>D-8</td>
<td>TPS</td>
<td>45/48</td>
<td>93.7%</td>
</tr>
<tr>
<td>8</td>
<td>D-8</td>
<td>TPW</td>
<td>112/112</td>
<td>100%</td>
</tr>
<tr>
<td>9</td>
<td>D-10</td>
<td>TPW</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>10</td>
<td>D-10</td>
<td>TPS</td>
<td>0/0c</td>
<td>0%</td>
</tr>
<tr>
<td>11</td>
<td>D-10</td>
<td>TPS</td>
<td>0/25d</td>
<td>0%</td>
</tr>
<tr>
<td>Session Number</td>
<td>Type of Division Problem</td>
<td>Type of Error Correction Procedure</td>
<td>Next-Session Test Frequency (3min.)</td>
<td>Next-Session Test Percent</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------------</td>
<td>------------------------------------</td>
<td>------------------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>12</td>
<td>D-10</td>
<td>TPW</td>
<td>0/25d</td>
<td>0%</td>
</tr>
<tr>
<td>13</td>
<td>D-10</td>
<td>TPS</td>
<td>30/63</td>
<td>47.6%</td>
</tr>
<tr>
<td>14</td>
<td>D-10</td>
<td>TPW</td>
<td>53/68</td>
<td>77.9%</td>
</tr>
<tr>
<td>15</td>
<td>D-10</td>
<td>TPW</td>
<td>35/51</td>
<td>68.6%</td>
</tr>
<tr>
<td>16</td>
<td>D-10</td>
<td>TPS</td>
<td>14/37</td>
<td>37.8%</td>
</tr>
<tr>
<td>17</td>
<td>D-10</td>
<td>TPW</td>
<td>57/60</td>
<td>95%</td>
</tr>
<tr>
<td>18</td>
<td>D-10</td>
<td>TPS</td>
<td>25/35</td>
<td>71%</td>
</tr>
<tr>
<td>19</td>
<td>D-15</td>
<td>TPS</td>
<td>10/25</td>
<td>40%</td>
</tr>
<tr>
<td>20</td>
<td>D-15</td>
<td>TPW</td>
<td>14/24</td>
<td>58%</td>
</tr>
<tr>
<td>21</td>
<td>D-15</td>
<td>TPW</td>
<td>41/46</td>
<td>89%</td>
</tr>
<tr>
<td>22</td>
<td>D-15</td>
<td>TPS</td>
<td>52/72</td>
<td>72%</td>
</tr>
<tr>
<td>23</td>
<td>D-15</td>
<td>TPS</td>
<td>32/50</td>
<td>64%</td>
</tr>
<tr>
<td>Session Number</td>
<td>Type of Division Problem</td>
<td>Type of Error Correction Procedure</td>
<td>Next-Session Test (3min.)</td>
<td>Next-Session Test</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------------</td>
<td>-----------------------------------</td>
<td>--------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>24</td>
<td>D-15</td>
<td>TPW</td>
<td>30/47</td>
<td>63.8%</td>
</tr>
<tr>
<td>25</td>
<td>D-16</td>
<td>TPS</td>
<td>11/28</td>
<td>39%</td>
</tr>
<tr>
<td>26</td>
<td>D-16</td>
<td>TPW</td>
<td>6/10</td>
<td>60%</td>
</tr>
<tr>
<td>27</td>
<td>D-16</td>
<td>TPW</td>
<td>14/66</td>
<td>21%</td>
</tr>
<tr>
<td>28</td>
<td>D-16</td>
<td>TPS</td>
<td>11/22</td>
<td>50%</td>
</tr>
<tr>
<td>29</td>
<td>D-16</td>
<td>TPW</td>
<td>5/22</td>
<td>41%</td>
</tr>
<tr>
<td>30</td>
<td>D-16</td>
<td>TPS</td>
<td>0/22d</td>
<td>0%</td>
</tr>
<tr>
<td>31</td>
<td>D-16</td>
<td>TPW</td>
<td>6/32</td>
<td>18.7%</td>
</tr>
<tr>
<td>32</td>
<td>D-16</td>
<td>TPW</td>
<td>30/42</td>
<td>71%</td>
</tr>
</tbody>
</table>
(Table 9 cont)

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Next-Session Test (3min.)</th>
<th>Next-Session Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>D-16</td>
<td>TPS</td>
<td>1/12</td>
<td>8.8%</td>
</tr>
<tr>
<td>34</td>
<td>D-16</td>
<td>TPW</td>
<td>0/14</td>
<td>0%</td>
</tr>
<tr>
<td>35</td>
<td>D-16</td>
<td>TPS</td>
<td>30/40</td>
<td>75%</td>
</tr>
</tbody>
</table>

Range

| Grand | Mean | TPS = 0 to 39 | TPW = 0 to 31 | TPS = 66.3% | TPW = 60.5% |

* Range and Grand Mean based upon digits correct per minute.

\[ a \] Next-session test not administered. \[ b \] Next-session test results are associated with the type of error correction procedure administered the previous session. For example, the results of session 2's next-session test are analyzed as products of the type of error correction procedure administered during session 1. \[ c \] Student did not attempt any problems. \[ d \] Number of digits correct by total digits written by student.
ranged from 2 to 6 sessions. During the study he was administered 3 types of long division problems.

Under the TPS error correction condition, Student 2 averaged 9.95 digits correct per minute, with a range from 0 digits to 16.66 digits correct on next session tests. In comparison, under the TPW error correction condition, he averaged 13 digits correct per minute, with a range from 5 digits to 20 digits correct. Under the TPS condition, his percentage of digits correct averaged 82.5, with a range from a low of 0% to a high of 98% of digits correct. In contrast, under the TPW condition, his percentage of digits correct averaged 79, with a range from a low of 0% to a high of 90% of digits. For Student 2 the mean number of digits correct on next-session tests was slightly better during the TPW condition than the TPS condition (see Table 10).

Student 3. Student 3 participated in 8 next-session tests. For 3 of the next-session tests, the TPS error correction procedure was administered during the previous session. In comparison, for 4 next-session tests the TPW error correction procedure was administered during the previous session.

Figure 8 shows Student 3's rate of digits correct on the timed (3 minutes) next-of-session test. Changes in the type of division problems are shown by the arrows on this figure. Student 3 was administered 3 types of long division problems during the course of the study. He spent 4 sessions on each of two types of long division problems, and 1 session on a third type of long division problem.

Under the TPS error correction condition, on next-session tests Student 3 averaged 8.2 digits correct per minute, with a range from 3.6 digits to 12 digits correct. In comparison, under the TPW error correction condition, on
Figure 7. Rate of digits correct on next-session test. Arrows indicate when a change in the type of long division problem occurred.
<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Next-Session Test Frequency (3min.)</th>
<th>Next-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>D-16</td>
<td>TPW&lt;sup&gt;b&lt;/sup&gt;</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>D-16</td>
<td>TPS</td>
<td>42/58&lt;sup&gt;c&lt;/sup&gt;</td>
<td>72%</td>
</tr>
<tr>
<td>3</td>
<td>D-16</td>
<td>TPW</td>
<td>26/32</td>
<td>81%</td>
</tr>
<tr>
<td>4</td>
<td>D-16</td>
<td>TPS</td>
<td>60/66</td>
<td>90.9%</td>
</tr>
<tr>
<td>5</td>
<td>D-16</td>
<td>TPS</td>
<td>50/54</td>
<td>92.5%</td>
</tr>
<tr>
<td>6</td>
<td>D-16</td>
<td>TPS</td>
<td>38/46</td>
<td>82.6%</td>
</tr>
<tr>
<td>7</td>
<td>D-18</td>
<td>TPW</td>
<td>50/51</td>
<td>98%</td>
</tr>
<tr>
<td>8</td>
<td>D-19</td>
<td>TPS</td>
<td>15/24</td>
<td>62.5%</td>
</tr>
<tr>
<td>9</td>
<td>D-19</td>
<td>TPS</td>
<td>16/16</td>
<td>100%</td>
</tr>
<tr>
<td>10</td>
<td>D-19</td>
<td>TPW</td>
<td>0/19</td>
<td>0%</td>
</tr>
</tbody>
</table>
(Table 10 cont)

<table>
<thead>
<tr>
<th></th>
<th>Next-Session Test Frequency (3min.)</th>
<th>Next-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
<td>TPS = 0 to 16.6</td>
<td>TPS = 0 to 98.03%</td>
</tr>
<tr>
<td></td>
<td>TPW = 5 to 20</td>
<td>TPW = 0 to 100%</td>
</tr>
<tr>
<td></td>
<td>TPS = 9.95</td>
<td>TPS = 82.56%</td>
</tr>
<tr>
<td><strong>Grand Mean</strong></td>
<td>TPW = 13</td>
<td>TPW = 79.05%</td>
</tr>
</tbody>
</table>

*a Next-session test not administered.  
*b Next-session test results associated w/correction procedure administered the previous session.

*c Number of digits correct by total number of digits attempted by student.  
*d Range and Grand Mean based upon digits correct per minute.
Figure 8. Rate of digits correct on next-session test. Arrows indicate when a change in the type of long division problem occurred.
next-session tests he averaged 13.8 digits correct per minute, with a range from 3 digits to 22.6 digits correct. In addition, Table 11 shows that Student 3's percent and number of digits correct was higher for next-session tests that immediately followed sessions where the TPW error correction procedure was administered. Under the TPS error correction condition, Student 3's percentage of digits correct averaged 89.1, with a range from a low of 28.57% to a high of 72% of digits correct. In contrast, under the TPW condition, his percentage of digits correct averaged 78.70 with a range from a low of 84% to a high of 89.7% of digits. Specifically, under the TPW error correction condition, his mean number of digits correct on next-session tests was 13.8 digits correct. In contrast, under the TPS error correction condition, his mean number of digits correct on next-session tests was 8.2 (see Table 11).

Student 4. Student 4 participated in 6 next-session tests. All of this next-session tests consisted of the same type of long division problem. Figure 9 shows Student 4's rate of digits correct on the timed (3 minutes) next-session test.

Student 4's results on next-session tests are placed under either the TPS or TPW condition. This was based upon the type of error correction procedure that immediately preceded the given next-session test. In Student 4's case, four next-session tests immediately followed the TPS condition and 3 next-session tests immediately followed the TPW condition.

Under the TPS error correction condition, on next-session tests Student 4 averaged 15.1 digits correct per minute, with a range from 5.6 digits to 20.3 digits correct. In comparison, under the TPW error correction condition, on next-session tests he averaged 24.9 digits correct per minute, with a range from 7.3 digits to 45.3 digits correct. Further, under the TPS error correction
Table 11
Student 3's Number and Percentage of Digits Correct on Next-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Next-Session Test Frequency (3min.)</th>
<th>Next-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>D-1</td>
<td>TPWb</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>D-1</td>
<td>TPW</td>
<td>61/68b</td>
<td>89.7%</td>
</tr>
<tr>
<td>3</td>
<td>D-1</td>
<td>TPW</td>
<td>52/60</td>
<td>86.6%</td>
</tr>
<tr>
<td>4</td>
<td>D-1</td>
<td>TPW</td>
<td>68/78</td>
<td>87%</td>
</tr>
<tr>
<td>5</td>
<td>D-16</td>
<td>TPS</td>
<td>16/22</td>
<td>72.7%</td>
</tr>
<tr>
<td>6</td>
<td>D-16</td>
<td>TPS</td>
<td>11/28</td>
<td>39.3%</td>
</tr>
<tr>
<td>7</td>
<td>D-16</td>
<td>TPS</td>
<td>36/52</td>
<td>69%</td>
</tr>
<tr>
<td>8</td>
<td>D-16</td>
<td>TPW</td>
<td>27/32</td>
<td>84%</td>
</tr>
<tr>
<td>9</td>
<td>D-17</td>
<td>TPW</td>
<td>10/35</td>
<td>28.5%</td>
</tr>
<tr>
<td>Session Number</td>
<td>Type of Division Problem</td>
<td>Type of Error Correction Procedure</td>
<td>Next-Session Test Frequency (3min.)</td>
<td>Next-Session Test Percent</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------------</td>
<td>-----------------------------------</td>
<td>-----------------------------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>Range&lt;sup&gt;d&lt;/sup&gt;</td>
<td></td>
<td>TPS = 3.6 to 12 TPW = 3 to 22.6</td>
<td>TPS = 28.57 to 72% TPW = 84 to 89.7%</td>
<td></td>
</tr>
<tr>
<td>Grand Mean&lt;sup&gt;d&lt;/sup&gt;</td>
<td></td>
<td>TPS = 8.2 TPW = 13.8</td>
<td>TPS = 89.1% TPW = 78.7%</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Next-session test not administered. <sup>b</sup> Next-session test results associated w/correction procedure administered the previous session. <sup>c</sup> Number of digits correct by total number of digits attempted by student. <sup>d</sup> Range and Grand Mean based upon digits correct per minute.
Figure 9. Rate of digits correct on next-session test. Arrows indicate when a change in the type of long division problem occurred.
condition, Student 4's percentage of digits correct averaged 71.9, with a range from a low of 41.46% to a high of 86.56% of digits correct. In contrast, under the TPW condition, his percentage of digits correct averaged 90.3 with a range from a low of 53.6% to a high of 93% of digits. Finally, his mean percent of digits correct on next-session tests was marginally higher under the TPW condition (90%) than his mean percent under the TPS condition (71%) (see Table 12).

**Student 5.** Student 5 participated in four next-session tests. Two of these next-session tests were preceded by sessions in which the TPS error correction procedure was in place. In comparison, the TPW error correction procedure preceded two next-session tests.

Figure 10 shows Student 5's rate of digits correct on the timed (3 minutes) next-of-session test. The arrows on this figure indicate when Student 5 was administered a different and more demanding type of long division problem. This transition occurred when he achieved a percent correct of at least 70% or higher on two consecutive sessions. During the study he was administered 3 types of long division problems. Because student 5 was only administered 4 next-session tests, three of which were under the TPS error correction condition a mean score on next-session tests was not generated. Inspection of Table 13, however, shows that his percentage of digits correct was slightly higher under the TPW condition than the TPS condition.

Table 14 summarizes the Students' mean number of digits correct per minute on next-session tests under the TPS and TPW error correction conditions. This Table also shows the group's mean number of digits correct per minute on next-session tests for the TPS and TPW conditions.
Table 12

Student 4's Number and Percentage of Digits Correct on Next-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Next-Session Test Frequency (3min.)</th>
<th>Next-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>D-15</td>
<td>TPSb</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>D-15</td>
<td>TPW</td>
<td>17/41c</td>
<td>41.5%</td>
</tr>
<tr>
<td>3</td>
<td>D-15</td>
<td>TPS</td>
<td>22/41</td>
<td>53.6%</td>
</tr>
<tr>
<td>4</td>
<td>D-15</td>
<td>TPS</td>
<td>61/81</td>
<td>75%</td>
</tr>
<tr>
<td>5</td>
<td>D-15</td>
<td>TPW</td>
<td>58/67</td>
<td>86.5%</td>
</tr>
<tr>
<td>6</td>
<td>D-15</td>
<td>TPW</td>
<td>67/72</td>
<td>93%</td>
</tr>
<tr>
<td>7</td>
<td>D-15</td>
<td>TPS</td>
<td>136/136</td>
<td>100%</td>
</tr>
</tbody>
</table>

Range:

<table>
<thead>
<tr>
<th></th>
<th>TPS = 5.6 to 20.3</th>
<th>TPS = 41.46 to 86.56%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TPW = 7.3 to 45.3</td>
<td>TPW = 53.65 to 93.05%</td>
</tr>
</tbody>
</table>

Grand Mean:

<table>
<thead>
<tr>
<th></th>
<th>TPS = 15.1</th>
<th>TPS = 71.9%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TPW = 24.9</td>
<td>TPW = 90.3%</td>
</tr>
</tbody>
</table>

Notes:

a Next-session test not administered. b Next-session test results associated w/correction procedure administered the previous session. c Number of digits correct by total number of digits attempted by student. d Range and Grand Mean based upon digits correct per minute.
Figure 10. Rate of digits correct on next-session test. Arrows indicate when a change in the type of long division problem occurred.
Table 13

Student 5's Number and Percentage of Digits Correct on Next-Session Tests.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Next-Session Test Frequency (3min.)</th>
<th>Next-Session Test Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>D-2</td>
<td>TPSb</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>D-2</td>
<td>TPS</td>
<td>22/32c</td>
<td>68.7%</td>
</tr>
<tr>
<td>3</td>
<td>D-2</td>
<td>TPW</td>
<td>55/56</td>
<td>98%</td>
</tr>
<tr>
<td>4</td>
<td>D-2</td>
<td>TPW</td>
<td>65/66</td>
<td>98.4%</td>
</tr>
<tr>
<td>5</td>
<td>D-3</td>
<td>TPS</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>D-11</td>
<td>TPW</td>
<td>72/81</td>
<td>88.8%</td>
</tr>
<tr>
<td>7</td>
<td>D-18</td>
<td>TPS</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Range</td>
<td>Next-Session Test Frequency (3min.)</td>
<td>Next-Session Test Percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------</td>
<td>--------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPS = 7.33 to 22</td>
<td>TPS = 68.75. to 98%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grand Mean</td>
<td>TPW e</td>
<td>TPW e</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPS = 16.55</td>
<td>TPS = 88.1%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

^a Next-session test not administered.  ^b Next-session test results associated w/correction procedure administered the previous session error.  ^c Number digits correct by total number of digits written by student.  ^d Range and Grand Mean based upon digits correct per minute.  ^e Only 1 next-session test followed TPW session.
Table 14

**Individual Students and Group Means for Number of Digits Correct Per Minute on Next-Session Tests.**

<table>
<thead>
<tr>
<th></th>
<th>TPS</th>
<th>TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>11.2</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>(66.3%)\textsuperscript{a}</td>
<td>(60.5%)</td>
</tr>
<tr>
<td>Student 2</td>
<td>9.9</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>(82.5%)</td>
<td>(79%)</td>
</tr>
<tr>
<td>Student 3</td>
<td>8.2</td>
<td>13.8</td>
</tr>
<tr>
<td></td>
<td>(89.1%)</td>
<td>(78.7%)</td>
</tr>
<tr>
<td>Student 4</td>
<td>15.1</td>
<td>24.9</td>
</tr>
<tr>
<td></td>
<td>(71.9%)</td>
<td>(90.3%)</td>
</tr>
<tr>
<td>Student 5</td>
<td>16.5</td>
<td>21.6</td>
</tr>
<tr>
<td></td>
<td>(88.1%)</td>
<td>(98.4%)\textsuperscript{b}</td>
</tr>
<tr>
<td><strong>Grand Mean</strong></td>
<td><strong>12.1</strong></td>
<td><strong>15</strong></td>
</tr>
<tr>
<td></td>
<td><strong>(73.2%)</strong></td>
<td><strong>(71.7%)</strong></td>
</tr>
</tbody>
</table>

\textsuperscript{a} Mean percent of total digits correct on end-of-session tests.

\textsuperscript{b} This score not factored into Grand Mean because only 1 next-session test followed a session in which TPW was used.
LONG DIVISION WORKSHEET PROBLEMS

Number and Percentage of Digits Correct on Worksheets

This section reports on students’ performance in solving long division problems on worksheets. Each worksheet contained 42 long division problems. The total number of digits necessary to solve a worksheet of division problems varied according to the type of division problem presented on the worksheet. For example, division problems with a remainder yielded more digits in its solution than division problems without a remainder.

Students were given 10-minutes to solve a worksheet of division problems. It was during the administration of worksheet problems that students errors were corrected. Thus, the rate at which students solved these problems (i.e., digits correct) cannot be established. Instead the percentage and number of digits correct and total number of digits written by students is presented.

The second observer checked each of the students worksheet problems for accuracy. This observer reported that on three long division problems solved by three different students, error correction was delivered when it was not necessary. The students’ individual responses were correct and were counted as correct.

Student 1. The 35 sessions that Student 1 participated in were almost evenly divided between the TPS (17) and TPW (18) error correction procedures. Figure 11 shows Student 1’s performance on worksheets expressed as a percentage of digits correct across sessions. The arrows on this figure indicate when he was administered a different and more demanding type of long division problem. On average, he spent 7 sessions solving a particular type of long division problem. The number of sessions
ranged from 4 to 11 sessions. During the study he was administered 4 types of long division problems.

Table 15 shows that under the TPS error correction condition, Student 1's grand mean was 55.5 digits correct out of an average of 80.6 digits attempted per worksheet of long division problems (68%). Further, the range of digits correct was from 0 to 164 digits, with his performance on session 19 yielding 0 digits correct.

In comparison, under the TPW error correction condition he averaged 55.5 digits correct out of an average of 78 digits attempted on worksheets of long division problems (71%). The range of digits correct was from 5 to 145. The mean percent of digits correct was higher under the TPW condition (71%) than the TPS condition (68%) (see Table 15).

Student 2. Figure 12 shows Student 2's performance on worksheet problems. The TPS error correction procedure was used during 6 of the sessions. In contrast, the TPW procedure was used only during 4 sessions. It was not needed during session 7 because Student 2 did not make any errors on the worksheet problems. The arrows on Figure 12 show when he was given a different and more demanding type of long division problem. Over the course of the study he was administered three different types of division problems.

Table 16 shows that under the TPS error correction condition Student 2 averaged 47 digits correct out of an average of 76 digits attempted on worksheets of long division problems. The range of digits correct was from 0 to 104 digits, with his performance on session 9 yielding 0 digits correct.
Figure 11. Percentage of digits correct on worksheet problems across sessions. Arrows indicate when a change in the type of long division problem occurred.
Table 15

Student 1's Number and Percentage of Digits Correct on Worksheet and Number of Next-Problem Correct After Error Correction.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Worksheet Problems Frequency</th>
<th>Worksheet Problems Percent</th>
<th>Next-Problem Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-2</td>
<td>TPS</td>
<td>4/22(^a)</td>
<td>18(^b)%</td>
<td>1/4(^c)</td>
</tr>
<tr>
<td>2</td>
<td>D-2</td>
<td>TPW</td>
<td>114/170</td>
<td>67%</td>
<td>4/5</td>
</tr>
<tr>
<td>3</td>
<td>D-2</td>
<td>TPS</td>
<td>137/151</td>
<td>90.7%</td>
<td>5/6</td>
</tr>
<tr>
<td>4</td>
<td>D-2</td>
<td>TPW</td>
<td>94/117</td>
<td>80%</td>
<td>5/5</td>
</tr>
<tr>
<td>5</td>
<td>D-8</td>
<td>TPW</td>
<td>10/45</td>
<td>22%</td>
<td>2/7</td>
</tr>
<tr>
<td>6</td>
<td>D-8</td>
<td>TPS</td>
<td>31/96</td>
<td>32%</td>
<td>2/4</td>
</tr>
<tr>
<td>7</td>
<td>D-8</td>
<td>TPS</td>
<td>162/168</td>
<td>96%</td>
<td>2/3</td>
</tr>
<tr>
<td>8</td>
<td>D-8</td>
<td>TPW</td>
<td>133/152</td>
<td>87%</td>
<td>3/4</td>
</tr>
<tr>
<td>9</td>
<td>D-10</td>
<td>TPW</td>
<td>5/20</td>
<td>25%</td>
<td>1/3</td>
</tr>
<tr>
<td>10</td>
<td>D-10</td>
<td>TPS</td>
<td>33/71</td>
<td>46%</td>
<td>1/3</td>
</tr>
<tr>
<td>11</td>
<td>D-10</td>
<td>TPS</td>
<td>29/68</td>
<td>42.6%</td>
<td>3/5</td>
</tr>
<tr>
<td>Session Number</td>
<td>Type of Division Problem</td>
<td>Type of Error Correction Procedure</td>
<td>Worksheet Frequency Correct</td>
<td>Worksheet Percent Correct</td>
<td>Next-Problem Correct</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------------</td>
<td>------------------------------------</td>
<td>-----------------------------</td>
<td>--------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>12</td>
<td>D-10</td>
<td>TPW</td>
<td>40/74</td>
<td>54%</td>
<td>3/4</td>
</tr>
<tr>
<td>13</td>
<td>D-10</td>
<td>TPS</td>
<td>65/85</td>
<td>76%</td>
<td>2/4</td>
</tr>
<tr>
<td>14</td>
<td>D-10</td>
<td>TPW</td>
<td>145/158</td>
<td>91.7%</td>
<td>2/2</td>
</tr>
<tr>
<td>15</td>
<td>D-10</td>
<td>TPW</td>
<td>95/104</td>
<td>91%</td>
<td>2/3</td>
</tr>
<tr>
<td>16</td>
<td>D-10</td>
<td>TPS</td>
<td>164/177</td>
<td>92.6%</td>
<td>1/2</td>
</tr>
<tr>
<td>17</td>
<td>D-10</td>
<td>TPW</td>
<td>113/131</td>
<td>86%</td>
<td>1/2</td>
</tr>
<tr>
<td>18</td>
<td>D-10</td>
<td>TPS</td>
<td>102/105</td>
<td>97%</td>
<td>1/1</td>
</tr>
<tr>
<td>19</td>
<td>D-15</td>
<td>TPS</td>
<td>0/25</td>
<td>0%</td>
<td>0/4</td>
</tr>
<tr>
<td>20</td>
<td>D-15</td>
<td>TPW</td>
<td>51/65</td>
<td>78%</td>
<td>4/4</td>
</tr>
<tr>
<td>21</td>
<td>D-15</td>
<td>TPW</td>
<td>45/68</td>
<td>66%</td>
<td>2/2</td>
</tr>
<tr>
<td>22</td>
<td>D-15</td>
<td>TPS</td>
<td>58/82</td>
<td>70.7%</td>
<td>4/5</td>
</tr>
<tr>
<td>23</td>
<td>D-15</td>
<td>TPS</td>
<td>55/75</td>
<td>73%</td>
<td>2/2</td>
</tr>
</tbody>
</table>
(Table 15 cont)

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Worksheet Frequency Correct</th>
<th>Worksheet Percent Correct</th>
<th>Next-Problem Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>D-15</td>
<td>TPW</td>
<td>50/74</td>
<td>67.5%</td>
<td>3/3</td>
</tr>
<tr>
<td>25</td>
<td>D-16</td>
<td>TPS</td>
<td>11/46</td>
<td>23.9%</td>
<td>0/3</td>
</tr>
<tr>
<td>26</td>
<td>D-16</td>
<td>TPW</td>
<td>16/42</td>
<td>38%</td>
<td>0/3</td>
</tr>
<tr>
<td>27</td>
<td>D-16</td>
<td>TPW</td>
<td>18/42</td>
<td>42.8%</td>
<td>1/1</td>
</tr>
<tr>
<td>28</td>
<td>D-16</td>
<td>TPS</td>
<td>28/66</td>
<td>42%</td>
<td>2/3</td>
</tr>
<tr>
<td>29</td>
<td>D-16</td>
<td>TPW</td>
<td>16/46</td>
<td>34.7%</td>
<td>2/2</td>
</tr>
<tr>
<td>30</td>
<td>D-16</td>
<td>TPS</td>
<td>22/47</td>
<td>46.8%</td>
<td>1/1</td>
</tr>
<tr>
<td>31</td>
<td>D-16</td>
<td>TPW</td>
<td>13/32</td>
<td>40.6%</td>
<td>0/2</td>
</tr>
<tr>
<td>32</td>
<td>D-16</td>
<td>TPW</td>
<td>16/22</td>
<td>72.7%</td>
<td>1/1</td>
</tr>
<tr>
<td>33</td>
<td>D-16</td>
<td>TPS</td>
<td>8/33</td>
<td>24%</td>
<td>0/3</td>
</tr>
<tr>
<td>34</td>
<td>D-16</td>
<td>TPW</td>
<td>25/42</td>
<td>59%</td>
<td>1/2</td>
</tr>
<tr>
<td>35</td>
<td>D-16</td>
<td>TPS</td>
<td>35/54</td>
<td>64%</td>
<td>1/2</td>
</tr>
</tbody>
</table>
(Table 15 cont)

<table>
<thead>
<tr>
<th></th>
<th>Worksheet Problems Frequency</th>
<th>Worksheet Problems Percent</th>
<th>Next-Problem Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
<td>TPS = 0 to 164</td>
<td>TPS = 28/55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPW = 5 to 145</td>
<td>TPW = 37/55</td>
<td></td>
</tr>
<tr>
<td><strong>Grand Mean</strong></td>
<td>TPS = 55.5 to 80.6</td>
<td>TPS = 68.85%</td>
<td>TPS = 50%</td>
</tr>
<tr>
<td></td>
<td>TPW = 56 to 78</td>
<td>TPW = 71.15%</td>
<td>TPW = 67.2%</td>
</tr>
</tbody>
</table>

*a* Number of digits correct over total number of digits written by student.

*b* Percentage of digits correct on worksheet problems.

*c* Next problem correct over total number of next problems corrected.
Figure 12. Percentage of digits correct on worksheet problems across sessions. Arrows indicate when a change in the type of long division problem occurred.
<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Worksheet Problems Frequency</th>
<th>Worksheet Problems Percent</th>
<th>Next Problem Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-16</td>
<td>TPW</td>
<td>24/64&lt;sup&gt;a&lt;/sup&gt;</td>
<td>37.5%&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0/2&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>2</td>
<td>D-16</td>
<td>TPS</td>
<td>35/54</td>
<td>64.8%</td>
<td>2/2</td>
</tr>
<tr>
<td>3</td>
<td>D-16</td>
<td>TPW</td>
<td>48/71</td>
<td>67.6%</td>
<td>1/1</td>
</tr>
<tr>
<td>4</td>
<td>D-16</td>
<td>TPS</td>
<td>35/66</td>
<td>53%</td>
<td>2/3</td>
</tr>
<tr>
<td>5</td>
<td>D-16</td>
<td>TPS</td>
<td>104/106</td>
<td>98%</td>
<td>1/1</td>
</tr>
<tr>
<td>6</td>
<td>D-16</td>
<td>TPS</td>
<td>62/101</td>
<td>61.8%</td>
<td>1/3</td>
</tr>
<tr>
<td>7</td>
<td>D-18</td>
<td>TPW</td>
<td>54/54</td>
<td>100%</td>
<td>***</td>
</tr>
<tr>
<td>8</td>
<td>D-19</td>
<td>TPS</td>
<td>47/71</td>
<td>66%</td>
<td>1/1</td>
</tr>
<tr>
<td>9</td>
<td>D-19</td>
<td>TPS</td>
<td>0/59</td>
<td>0%</td>
<td>0/2</td>
</tr>
<tr>
<td>10</td>
<td>D-19</td>
<td>TPW</td>
<td>13/24</td>
<td>54%</td>
<td>1/1</td>
</tr>
<tr>
<td></td>
<td>Worksheet Problems Frequency</td>
<td>Worksheet Problems Percent</td>
<td>Next Problem Correct</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>-----------------------------</td>
<td>---------------------------</td>
<td>---------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Range</strong></td>
<td>TPS = 0 to 104</td>
<td>TPS = 61.9%</td>
<td>TPS = 7/12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPW = 13 to 54</td>
<td>TPW = 65.2%</td>
<td>TPW = 2/4</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Grand Mean</strong></td>
<td>TPS = 47/76</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TPW = 34.7/53.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a Number of digits correct over the total number of digits written by the student. b Percentage of digits correct on worksheets. c Number of next problems correct over the total number of next problems that followed error correction.
In comparison, under the TPW error correction condition he averaged 34.7 digits correct out of an average of 53.2 digits attempted on worksheets of long division problems. The range of digits correct was from 13 to 54 digits. The mean percent of digits correct was higher under the TPW condition (65%) than the TPS condition (61.9%) (see Table 16).

**Student 3.** Figure 13 shows Student 3's performance on worksheet problems. The TPS error correction procedure was used during 3 sessions. In comparison, the TPW error correction procedure was used during 6 sessions and not applied during session 4 because Student 3 did not make any errors on the worksheet problems. The arrows on figure 13 show when Student 3 was given a different and more demanding type of long division problems. During the course of the study Student 3 was administered 3 different types of division problems.

Under the TPS error correction condition Student 3 averaged 43 digits correct out of an average of 58 digits attempted on worksheets of long division problems. The range of digits correct was from 16 to 68 digits (see Table 17). In comparison, under the TPW error correction condition Student 3 averaged 123.8 digits correct out of an average of 132 digits attempted. The range of digits correct was from 20 to 249 digits. The mean percent of digits correct was higher under the TPW condition (93.7%) than the TPS condition (74%) (see Table 17).

**Student 4.** Figure 14 displays Student 4's performance on worksheet problems. The TPS error correction procedure was used during 4 sessions, and the TPW error correction procedure was used during 3 sessions. The arrows on Figure 14 show when Student 4 was given a different and more
Figure 13. Percentage of digits correct on worksheet problems across sessions. Arrows indicate when a change in the type of long division problem occurred.
Table 17

Student 3's Number and Percentage of Digits Correct on Worksheet and Number of Next-Problem Correct after Error Correction.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Worksheet Problems Frequency</th>
<th>Worksheet Problems Percent</th>
<th>Next-Problem Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-1</td>
<td>TPW</td>
<td>140/162a</td>
<td>86%</td>
<td>2/2b</td>
</tr>
<tr>
<td>2</td>
<td>D-1</td>
<td>TPW</td>
<td>118/129</td>
<td>91.4%</td>
<td>3/3</td>
</tr>
<tr>
<td>3</td>
<td>D-1</td>
<td>TPW</td>
<td>162/163</td>
<td>99%</td>
<td>1/1</td>
</tr>
<tr>
<td>4</td>
<td>D-1</td>
<td>TPW</td>
<td>249/249</td>
<td>100%</td>
<td>c</td>
</tr>
<tr>
<td>5</td>
<td>D-16</td>
<td>TPS</td>
<td>16/20</td>
<td>80%</td>
<td>0/1</td>
</tr>
<tr>
<td>6</td>
<td>D-16</td>
<td>TPS</td>
<td>45/67</td>
<td>67%</td>
<td>2/2</td>
</tr>
<tr>
<td>7</td>
<td>D-18</td>
<td>TPS</td>
<td>68/87</td>
<td>79%</td>
<td>2/2</td>
</tr>
<tr>
<td>8</td>
<td>D-16</td>
<td>TPW</td>
<td>54/66</td>
<td>81.8%</td>
<td>0/1</td>
</tr>
<tr>
<td>9</td>
<td>D-17</td>
<td>TPW</td>
<td>20/24</td>
<td>83.3%</td>
<td>1/1</td>
</tr>
<tr>
<td>Range</td>
<td>Worksheet Problems Frequency</td>
<td>Worksheet Problems Percent</td>
<td>Next-Problem Correct</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>-----------------------------</td>
<td>----------------------------</td>
<td>---------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPS = 16 to 68</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPW = 20 to 249</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPS = 43/58</td>
<td>TPS = 74.1%</td>
<td>TPS = 4/5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPW = 123.8/132</td>
<td>TPW = 93.7%</td>
<td>TPW = 6/8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a Number of digits correct over the total number of digits written by the student.
b Number of next problems correct over the total number of next problems that followed the that received an error correction procedure. c Student made no errors on division problems.
Figure 14. Percentage of digits correct on worksheet problems across sessions. Arrows indicate when a change in the type of long division problem occurred.
demanding type of long division problems. During the course of the study he was administered 2 different types of division problems.

Under the TPS error correction condition Student 4 averaged 88.25 digits correct out of an average of 122 digits attempted on worksheets of long division problems. The range of digits correct was from 26 to 137 digits (see table 18). In comparison, under the TPW error correction condition Student 4 averaged 77.6 digits correct out of an average of 103 digits attempted. The range of digits correct was from 67 to 144 digits. The mean percent of digits correct was higher under the TPW condition (75.3%) than the TPS condition (72.3%) (see Table 18).

Student 5. Figure 15 shows Student 5's performance on worksheet problems. The TPS error correction procedure was used during 4 sessions. In comparison, the TPW error correction procedure was used during 3 sessions. The arrows on Figure 15 show when Student 5 was given a different and more demanding type of long division problems. During the course of the study Student 5 was administered 3 different types of division problems.

Under the TPS error correction condition Student 5 averaged 61 digits correct out of an average of 80.2 digits attempted on worksheets of long division problems. The range of digits correct was from 2 to 112 digits (see Table 19). In comparison, under the TPW error correction condition Student 5 averaged 161.6 digits correct out of an average of 168 digits attempted. The range of digits correct was from 130 to 199 digits. The mean percent of digits correct was higher under the TPW condition (96%) than the TPS condition (76.3%) (see Table 19).
Table 18

Student 4's Number and Percentage of Digits Correct on Worksheets and Number and Percentage of Next-Problems After Error Correction.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Worksheet Problems Frequency</th>
<th>Worksheet Problems Percent</th>
<th>Next Problem Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-15</td>
<td>TPS</td>
<td>26/66&lt;sup&gt;a&lt;/sup&gt;</td>
<td>39.4%</td>
<td>2/5&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>2</td>
<td>D-15</td>
<td>TPW</td>
<td>22/67</td>
<td>32.8%</td>
<td>2/5</td>
</tr>
<tr>
<td>3</td>
<td>D-15</td>
<td>TPS</td>
<td>54/95</td>
<td>56.8%</td>
<td>2/4</td>
</tr>
<tr>
<td>4</td>
<td>D-15</td>
<td>TPS</td>
<td>136/148</td>
<td>91.8%</td>
<td>1/2</td>
</tr>
<tr>
<td>5</td>
<td>D-15</td>
<td>TPW</td>
<td>144/158</td>
<td>91%</td>
<td>3/3</td>
</tr>
<tr>
<td>6</td>
<td>D-15</td>
<td>TPW</td>
<td>67/84</td>
<td>79.7%</td>
<td>6/6</td>
</tr>
<tr>
<td>7</td>
<td>D-15</td>
<td>TPS</td>
<td>137/179</td>
<td>76.5%</td>
<td>5/5</td>
</tr>
</tbody>
</table>

Range

TPS = 26 to 137
TPW = 67 to 144

Grand Mean

TPS = 88.25/122
TPW = 77.66/103
TPS = 72.33%
TPW = 75.39%
TPS = 11/17
TPW = 11/14
(Table 18 cont)

a  Number of digits correct over total number of digits written by student.
b  Percentage of digits correct on worksheet problems.
c  Next-problem correct over total number of next-problems corrected.
Figure 15. Percentage of digits correct on worksheet problems across sessions. Arrows indicate when a change in the type of long division problem occurred.
Table 19

Student 5's Number and Percentage of Digits Correct on Worksheets and Number and Percentage of Next-Problems After Error Correction.

<table>
<thead>
<tr>
<th>Session Number</th>
<th>Type of Division Problem</th>
<th>Type of Error Correction Procedure</th>
<th>Worksheet Problem Frequency</th>
<th>Worksheet Problem Percent</th>
<th>Next Problem Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D-2</td>
<td>TPS</td>
<td>40/62&lt;sup&gt;a&lt;/sup&gt;</td>
<td>64.4%</td>
<td>2/5&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>2</td>
<td>D-2</td>
<td>TPS</td>
<td>112/122</td>
<td>91.8%</td>
<td>3/3</td>
</tr>
<tr>
<td>3</td>
<td>D-2</td>
<td>TPW</td>
<td>156/159</td>
<td>98%</td>
<td>3/3</td>
</tr>
<tr>
<td>4</td>
<td>D-2</td>
<td>TPW</td>
<td>199/201</td>
<td>99%</td>
<td>1/1</td>
</tr>
<tr>
<td>5</td>
<td>D-3</td>
<td>TPS</td>
<td>91/104</td>
<td>87%</td>
<td>1/2</td>
</tr>
<tr>
<td>6</td>
<td>D-11</td>
<td>TPW</td>
<td>130/145</td>
<td>89.6%</td>
<td>2/2</td>
</tr>
<tr>
<td>7</td>
<td>D-18</td>
<td>TPS</td>
<td>2/33</td>
<td>6%</td>
<td>0/2</td>
</tr>
</tbody>
</table>

Range

TPS = 2 to 112  
TPW = 130 to 199

Grand Mean

TPS = 61/80.2  
TPW = 161.6/168

TPS = 76.3%  
TPW = 96%  
TPW = 6/6
(Table 19 cont)

a. Number of digits correct over total number of digits written by student.
b. Percentage of digits correct on worksheet problems
c. Next-problem correct over total number of next-problems corrected.
Table 20 summarizes the Students' mean number of digits correct on worksheet problems under the TPS and TPW error correction conditions.

**NEXT PROBLEM CORRECT**

The “next-problem” correct data are based upon student achievement on worksheet problems that immediately follows the problem on which they received an error correction procedure. Students' performance on this “next-problem” is reported in terms of number and mean percent of next problems correct over total number of next problems. The second observer scored Students worksheets problems for “next-problem” correct.

**Student 1.** Table 15 shows Student 1's performance on the next problem following the problem on which he received error correction. On sessions that received the TPS error correction procedure an average 28 out of 55 of the next problems were solved correctly. In other words, 50% of his “next problems” were solved correctly.

In comparison, during sessions when the TPW error correction procedure was in place, an average 37 out of 55 of the next problems were solved correctly (i.e., 68%).

**Student 2.** Table 16 shows Student 2's performance on “next problems.” On sessions that received the TPS error correction procedure an average of 7 out of 12 of the next problems were solved correctly (i.e., 58%).

In comparison, during sessions when the TPW error correction was in place, an average 2 out of 4 of the next problems were solved correctly (i.e., 50%).

**Student 3.** Table 17 shows Student 3’s performance on “next problems.” On sessions that received the TPS error correction procedure an average 4 out of 5 of the next problems were solved correctly (i.e., 80%).
Table 20

Individual Students and Group Means for Number of Digits Correct on Worksheet Problems.

<table>
<thead>
<tr>
<th></th>
<th>TPS</th>
<th>TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>55.5/80.6&lt;sup&gt;a&lt;/sup&gt;</td>
<td>55.5/78</td>
</tr>
<tr>
<td></td>
<td>(68.8%)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>(71.1%)</td>
</tr>
<tr>
<td>Student 2</td>
<td>47/76</td>
<td>34.7/53.2</td>
</tr>
<tr>
<td></td>
<td>(61.9%)</td>
<td>(65.2%)</td>
</tr>
<tr>
<td>Student 3</td>
<td>43/58</td>
<td>123/132</td>
</tr>
<tr>
<td></td>
<td>(74.1%)</td>
<td>(93.7%)</td>
</tr>
<tr>
<td>Student 4</td>
<td>88.2/122</td>
<td>77.6/103</td>
</tr>
<tr>
<td></td>
<td>(72.3%)</td>
<td>(75.3%)</td>
</tr>
<tr>
<td>Student 5</td>
<td>61/80.2</td>
<td>161.6/168</td>
</tr>
<tr>
<td></td>
<td>(76.3%)</td>
<td>(96%)</td>
</tr>
<tr>
<td>Grand Mean</td>
<td>58.9/83.2</td>
<td>90.2/106.8</td>
</tr>
<tr>
<td></td>
<td>(70.7%)</td>
<td>(84.4%)</td>
</tr>
</tbody>
</table>

<sup>a</sup> Mean number of digits correct over the mean number of total digits written by student.

<sup>b</sup> Mean percent of total digits correct on worksheets.
In comparison, under TPW error correction conditions an average 6 out of 8 of the next problems were solved correctly (i.e., 75%).

Student 4. On sessions that received the TPS error correction procedure an average 11 out of 17 next problems were solved correctly (i.e., 65%). By comparison, when TPW procedure was used an average of 11 out of 14 of the next problems were solved correctly (i.e., 79%) (see Table 18).

Student 5. During sessions when the TPS error correction procedure was used an average 6 out of 12 of the next problems were solved correctly (i.e., 50%). In comparison, when the TPW procedure was in place, an average 6 out of 6 of the next problems were solved correctly (i.e., 50%) (see Table 19).

The Students' performance on "next-problems" is summarized in Table 21. Included in this summarization is the group's mean for number of next-problem correct for the TPS and TPW conditions.

ERROR ANALYSIS

This section presents a summary of the types of errors students made in solving long division problems. The purpose of the error analysis was to determine what effect the error correction procedures had on students' performance in solving long division problems. Students' responses to end-of-session tests were analyzed for three kinds of errors: computational, estimation, and omission. Computational errors included errors of subtraction, addition, and multiplication. Estimation errors involved errors of over- and under-estimation of a quotient. That is, the product of the quotient multiplied by the divisor must equal or approximates the partial or complete dividend.
Table 21

**Individual Students and Group Means for Number of Next-Problems Correct on Worksheets After Error Correction.**

<table>
<thead>
<tr>
<th></th>
<th>TPS</th>
<th>TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student 1</strong></td>
<td>28/55(^a) (50%)(^b)</td>
<td>37/55 (67.2%)</td>
</tr>
<tr>
<td><strong>Student 2</strong></td>
<td>7/12 (58%)</td>
<td>2/4 (50%)</td>
</tr>
<tr>
<td><strong>Student 3</strong></td>
<td>4/5 (80%)</td>
<td>6/8 (75%)</td>
</tr>
<tr>
<td><strong>Student 4</strong></td>
<td>11/17 (64%)</td>
<td>11/14 (78%)</td>
</tr>
<tr>
<td><strong>Student 5</strong></td>
<td>6/12 (50%)</td>
<td>6/6 (100%)</td>
</tr>
<tr>
<td><strong>Grand Mean</strong></td>
<td>11.2/20.2 (55.4%)</td>
<td>12.4/17.4 (71.2%)</td>
</tr>
</tbody>
</table>

\(^a\) Mean number of next problems correct over total number of next problems.

\(^b\) Mean percent of next problems solved correct over total number of next problems.
Omission errors referred to the omission of a step in the process or a part of the correct response or of digits.

The second observer conducted the error analysis of Students' responses to long division problems on end-of-session tests.

Error analyses completed on students' end-of-session tests are reported in text and tables. Student 1's error analysis results are also reported in a Figure. The other students did not participate in a sufficient number of sessions to justify Figures for their individual error analyses.

**Student 1.** Figure 16 shows separate cumulative graphs of the type of errors Student 1 made under the TPS and TPW error correction procedures during end-of-session tests. Under both procedures the type of long division problems increased in complexity across sessions.

The graph in the upper tier (see Figure 16) shows that under the TPS error correction procedure there was an accelerating trend in estimation and computation errors across sessions. On the same graph, the line that represents omission errors was relatively flat. In other words, under the TPS error correction procedures the number of omission errors did not increase substantially across sessions.

The graph in the lower tier (see Figure 16) shows that under the TPW procedure there was an accelerating trend in estimation errors. In comparison, computation and omission errors showed negligible change under the TPW error correction procedure.
STUDENT 1

Tell Plus Show

Cumulative Number of Errors

Sessions

Tell Plus Write

Cumulative Number of Errors

Sessions

Figure 16. Student 1’s cumulative number of three types of errors on end of session tests across conditions.
Table 22 shows the results of the error analysis completed on Student 1's end-of-sessions tests. It shows the number and type of errors made on each of the end-of-session test during TPS and TPW error correction procedures.

**Student 2.** Inspection of Table 23 shows the number and type of errors Student 2 made on end-of-session tests. This error analysis data are categorized according to the type of error correction procedure implemented during the session that end-of-session test was administered.

**Student 3.** Inspection of Table 24 shows the number and type of errors Student 3 made on end-of-session tests. This error analysis data are categorized according to the type of error correction procedure implemented during the session that end-of-session test was administered.

**Student 4.** Inspection of Table 25 shows the number and type of errors Student 4 made on end-of-session tests. This error analysis data are categorized according to the type of error correction procedure implemented during the session that end-of-session test was administered.

**Student 5.** Inspection of Table 26 shows the number and type of errors Student 5 made on end-of-session tests. This error analysis data are categorized according to the type of error correction procedure implemented during the session that end-of-session test was administered.

**COMPREHENSIVE TEST**

Approximately one-week after the completion of the study students were individually administered a second form of the **ENRIGHT Diagnostic Inventory of Basic Arithmetic** (DIBAS) (Enright, 1983). The DIBAS served as a measure of students' maintenance of long division skills. The comprehensive test was administered like a session test, except for students were given up to
Table 22

Student 1's Type and Number of Errors Made on End-of-Session Tests During Sessions in which TPS or TPW Error Correction Procedures was used.

<table>
<thead>
<tr>
<th>Session</th>
<th>Estimation Errors TPS/TPW</th>
<th>Computational Errors TPS/TPW</th>
<th>Omission Errors TPS/TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0/-</td>
<td>0/-</td>
<td>4/-</td>
</tr>
<tr>
<td>2</td>
<td>-/0</td>
<td>-/0</td>
<td>-/18</td>
</tr>
<tr>
<td>3</td>
<td>2/-</td>
<td>0/-</td>
<td>1/-</td>
</tr>
<tr>
<td>4</td>
<td>-/3</td>
<td>-/3</td>
<td>-/0</td>
</tr>
<tr>
<td>5</td>
<td>-/2</td>
<td>-/0</td>
<td>-/1</td>
</tr>
<tr>
<td>6</td>
<td>1/-</td>
<td>1/-</td>
<td>1/-</td>
</tr>
<tr>
<td>7</td>
<td>0/-</td>
<td>0/-</td>
<td>0/-</td>
</tr>
<tr>
<td>8</td>
<td>-/0</td>
<td>-/0</td>
<td>-/2</td>
</tr>
<tr>
<td>9</td>
<td>-/3</td>
<td>-/1</td>
<td>-/0</td>
</tr>
<tr>
<td>10</td>
<td>1/-</td>
<td>1/-</td>
<td>0/-</td>
</tr>
<tr>
<td>11</td>
<td>3/-</td>
<td>3/-</td>
<td>0/-</td>
</tr>
<tr>
<td>12</td>
<td>-/1</td>
<td>-/2</td>
<td>-/0</td>
</tr>
<tr>
<td>13</td>
<td>3/-</td>
<td>3/-</td>
<td>0/-</td>
</tr>
<tr>
<td>14</td>
<td>-/2</td>
<td>-/2</td>
<td>-/1</td>
</tr>
<tr>
<td>15</td>
<td>-/3</td>
<td>-/2</td>
<td>-/0</td>
</tr>
<tr>
<td>16</td>
<td>1/0</td>
<td>0/-</td>
<td>2/-</td>
</tr>
<tr>
<td>17</td>
<td>0/-</td>
<td>0/-</td>
<td>0/-</td>
</tr>
<tr>
<td>18</td>
<td>0/-</td>
<td>1/-</td>
<td>1/-</td>
</tr>
<tr>
<td>Session</td>
<td>Estimation Errors TPS/TPW</td>
<td>Computational Errors TPS/TPW</td>
<td>Omission Errors TPS/TPW</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------------</td>
<td>-------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>19</td>
<td>3/-</td>
<td>3/-</td>
<td>0/-</td>
</tr>
<tr>
<td>20</td>
<td>-/1</td>
<td>-/3</td>
<td>-/0</td>
</tr>
<tr>
<td>21</td>
<td>-/2</td>
<td>-/0</td>
<td>-/2</td>
</tr>
<tr>
<td>22</td>
<td>4/-</td>
<td>4/-</td>
<td>0/-</td>
</tr>
<tr>
<td>23</td>
<td>3/-</td>
<td>0/-</td>
<td>0/-</td>
</tr>
<tr>
<td>24</td>
<td>-/4</td>
<td>-/1</td>
<td>-/0</td>
</tr>
<tr>
<td>25</td>
<td>1/-</td>
<td>1/-</td>
<td>1/-</td>
</tr>
<tr>
<td>26</td>
<td>-/2</td>
<td>-/0</td>
<td>-/0</td>
</tr>
<tr>
<td>27</td>
<td>-/2</td>
<td>-/0</td>
<td>-/0</td>
</tr>
<tr>
<td>28</td>
<td>5/-</td>
<td>4/-</td>
<td>1/-</td>
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<td>29</td>
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<td>-/1</td>
<td>-/1</td>
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<tr>
<td>30</td>
<td>0/-</td>
<td>0/-</td>
<td>0/-</td>
</tr>
<tr>
<td>31</td>
<td>-/1</td>
<td>-/0</td>
<td>-/1</td>
</tr>
<tr>
<td>32</td>
<td>-/0</td>
<td>-/0</td>
<td>-/1</td>
</tr>
<tr>
<td>33</td>
<td>1/-</td>
<td>1/-</td>
<td>0/-</td>
</tr>
<tr>
<td>34</td>
<td>-/1</td>
<td>-/1</td>
<td>-/1</td>
</tr>
<tr>
<td>35</td>
<td>0/-</td>
<td>0/-</td>
<td>0/-</td>
</tr>
<tr>
<td></td>
<td>Estimation Errors</td>
<td>Computational Errors</td>
<td>Omission Errors</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------------</td>
<td>----------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Total</td>
<td>TPS = 28</td>
<td>TPS = 22</td>
<td>TPS = 11</td>
</tr>
<tr>
<td></td>
<td>TPW = 29</td>
<td>TPW = 12</td>
<td>TPW = 28</td>
</tr>
<tr>
<td>Mean</td>
<td>TPS = 1.5</td>
<td>TPS = 1.2</td>
<td>TPS = 61</td>
</tr>
<tr>
<td></td>
<td>TPW = 1.7</td>
<td>TPW = .70</td>
<td>TPW = 1.64</td>
</tr>
</tbody>
</table>
Table 23

**Student 2's Type and Number of Errors Made on End-of-Session Tests During Sessions in which TPS or TPW Error Correction Procedures was used.**

<table>
<thead>
<tr>
<th>Session</th>
<th>Estimation Errors TPS/TPW</th>
<th>Computational Errors TPS/TPW</th>
<th>Omission Errors TPS/TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-/0</td>
<td>-/0</td>
<td>-/6</td>
</tr>
<tr>
<td>2</td>
<td>-/1</td>
<td>-/0</td>
<td>-/7</td>
</tr>
<tr>
<td>3</td>
<td>-/0</td>
<td>-/1</td>
<td>-/1</td>
</tr>
<tr>
<td>4</td>
<td>-/0</td>
<td>-/0</td>
<td>-/3</td>
</tr>
<tr>
<td>5</td>
<td>1/-</td>
<td>1/-</td>
<td>1/-</td>
</tr>
<tr>
<td>6</td>
<td>0/-</td>
<td>1/-</td>
<td>1/-</td>
</tr>
<tr>
<td>7</td>
<td>0/-</td>
<td>1/-</td>
<td>3/-</td>
</tr>
<tr>
<td>8</td>
<td>-/0</td>
<td>-/0</td>
<td>-/1</td>
</tr>
<tr>
<td>9</td>
<td>-/0</td>
<td>-/0</td>
<td>-/0</td>
</tr>
</tbody>
</table>

**Total**

TPS = 1/-

TPW = -/1

TPS = 3/-

TPW = -/1

TPS = 5/-

TPW = -/17
Table 24 1

Student 3's Type and Number of Errors Made on End-of-Session Tests During Sessions in which TPS or TPW Error Correction Procedures was used.

<table>
<thead>
<tr>
<th>Session</th>
<th>Estimation Errors TPS/TPW</th>
<th>Computational Errors TPS/TPW</th>
<th>Omission Errors TPS/TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-/1</td>
<td>-/0</td>
<td>-/0</td>
</tr>
<tr>
<td>2</td>
<td>0/-</td>
<td>1/-</td>
<td>0/-</td>
</tr>
<tr>
<td>3</td>
<td>-/1</td>
<td>-/1</td>
<td>-/2</td>
</tr>
<tr>
<td>4</td>
<td>2/-</td>
<td>2/-</td>
<td>0/-</td>
</tr>
<tr>
<td>5</td>
<td>1/-</td>
<td>0/-</td>
<td>1/-</td>
</tr>
<tr>
<td>6</td>
<td>0/-</td>
<td>0/-</td>
<td>1/-</td>
</tr>
<tr>
<td>7</td>
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<td>-/0</td>
<td>-/0</td>
</tr>
<tr>
<td>8</td>
<td>0/-</td>
<td>0/-</td>
<td>0/-</td>
</tr>
<tr>
<td>9</td>
<td>2/-</td>
<td>0/-</td>
<td>1/-</td>
</tr>
<tr>
<td>10</td>
<td>-/0</td>
<td>-/1</td>
<td>-/0</td>
</tr>
</tbody>
</table>

Total

TPS = 5/-
TPW = -/2

TPS = 3/-
TPW = -/2

TPS = 3/-
TPW = -/2
Table 25

Student 4's Type and Number of Errors Made on End-of-Session Tests During Sessions in which TPS or TPW Error Correction Procedures was used.

<table>
<thead>
<tr>
<th>Session</th>
<th>Estimation Errors TPS/TPW</th>
<th>Computational Errors TPS/TPW</th>
<th>Omission Errors TPS/TPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2/-</td>
<td>1/-</td>
<td>1/-</td>
</tr>
<tr>
<td>2</td>
<td>-/1</td>
<td>-/0</td>
<td>-/1</td>
</tr>
<tr>
<td>3</td>
<td>2/-</td>
<td>1/-</td>
<td>2/-</td>
</tr>
<tr>
<td>4</td>
<td>1/-</td>
<td>1/-</td>
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</tr>
<tr>
<td>5</td>
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<td>-/0</td>
<td>-/0</td>
</tr>
<tr>
<td>6</td>
<td>-/1</td>
<td>-/0</td>
<td>-/0</td>
</tr>
<tr>
<td>7</td>
<td>0/-</td>
<td>1/-</td>
<td>0/-</td>
</tr>
</tbody>
</table>

Total: TPS = 5/-
TPW = -/4
TPS = 4/-
TPW = -/0
TPS = 3/-
TPW = -/1
Table 26

Student 5's Type and Number of Errors Made on End-of-Session Tests During Sessions in which TPS or TPW Error Correction Procedures was used.

<table>
<thead>
<tr>
<th>Session</th>
<th>Estimation Errors</th>
<th>Computational Errors</th>
<th>Omission Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TPS/TPW</td>
<td>TPS/TPW</td>
<td>TPS/TPW</td>
</tr>
<tr>
<td>1</td>
<td>1/-</td>
<td>2/-</td>
<td>2/-</td>
</tr>
<tr>
<td>2</td>
<td>0/-</td>
<td>0/-</td>
<td>0/-</td>
</tr>
<tr>
<td>3</td>
<td>-/0</td>
<td>-/0</td>
<td>-/3</td>
</tr>
<tr>
<td>4</td>
<td>-/1</td>
<td>-/0</td>
<td>-/2</td>
</tr>
<tr>
<td>5</td>
<td>1/-</td>
<td>1/-</td>
<td>1/-</td>
</tr>
<tr>
<td>6</td>
<td>-/0</td>
<td>-/0</td>
<td>-/0</td>
</tr>
<tr>
<td>7</td>
<td>0/-</td>
<td>0/-</td>
<td>1/-</td>
</tr>
<tr>
<td>Total</td>
<td>TPS = 2/-</td>
<td>TPS = 3/-</td>
<td>TPS = 4/-</td>
</tr>
<tr>
<td></td>
<td>TPW = -/1</td>
<td>TPW = -/0</td>
<td>TPW = -/5</td>
</tr>
</tbody>
</table>
25 minutes to complete the problems. Only Student 1 and Student 2 took this maintenance test.

On this maintenance measure of long division skills, Student 1 achieved 447 digits correct out of 570 digits written by him in response to the division problems. His percentage of digits correct was 78%. Student 2 achieved 105 digits correct out of 115 digits written by him in response to the division problems. His percentage of digits correct was 91%.

MAINTENANCE PROBES

As mentioned earlier, Student 4 and 5's attendance at school prevented them for participating in the study beyond sessions 8 and 7 respectively. Therefore, when they attended school they completed maintenance probes. The probes were administered like a session test (i.e., 3-minute timed test) and consisted of the type of division problem on which they last received error correction procedures. The second observer completed the accuracy check on Students' responses to probes.

Student 4 was administered 2 probes: 4 weeks and 6 weeks after his last end-of-session test. On the first probe, Student 4 obtained 82 out of 97 digits correct (84.53%), and at a rate of 27.33 digits correct per minute. On the second probe, Student 4 achieved 21 out of 36 digits correct (58%), and at a rate of 7 digits correct per minute.

Student 5 was also administered two maintenance probes. He completed his first probe 2 weeks after his last end-of-session test. He achieved 19 out of 34 digits correct (55.33%), and at a rate of 6.33 digits correct per minute. He took the second probe 4 weeks after his last end-of-session test. He obtained 20 out of 30 digits correct (66.66%), and at a rate of 6.66 digits correct per minute.
SOCIAL VALIDITY MEASURES

Social validity was assessed by measuring students and teachers' satisfaction with the experimental intervention. This measurement was taken through the administration of opinion questionnaires.

Student Evaluation

Approximately 1 week after the study was completed 4 of the 5 students were interviewed individually by the second observer (interviewer). One of the students did not attend the student evaluation.

Prior to the interview, students were told they would be asked questions about the math study. The interview was conducted in the classroom where the study took place. Only the student and interviewer were present in the classroom; the interviewer asked questions from the Student interview form, and encouraged students to be candid (see Appendix G).

The questions asked students to: describe the procedures (question 1), distinguish the procedure (error correction strategy) (question 2) that they believed helped them the most to learn long division, indicate how the preferred procedure helped them to learn long division (question 3), and indicate which procedure they preferred and why (question 4).

Question 1 asked students to describe the study. According to the interviewer each of the students provided an accurate description of the study. Specifically, each of the students described the difference between the TPS and TPW error correction procedures. For example, one student said that the [experimenter] showed me how many times the number [divisor] goes into the number [dividend]. On some days he [experimenter] wrote out the answers [numbers] and some days I wrote down the answers [numbers].
Question 2 asked students to indicate which method of error correction they thought helped them the most. Each of the students said they preferred the Tell Plus Write procedure. Unfortunately, the students did not elaborate on their responses. In turn, question 3 asked students to describe how the Tell Plus Write procedure helped them to learn long division. Students either said they did not know how it helped them or gave the following explanations: "It [TPW] was easier to comprehend and made more sense," and "It helped [me] to learn the process better by actually doing the problem."

Finally, question 4 asked students to indicate which procedure they preferred or liked the most. All of the students said that they preferred TPW, and only two students told why they preferred TPW. One student said that "it helped him to understand where he made a mistake." The second student said "I feel that this program [referring to the TPW procedure] will help me in the regular math class."

Teacher Evaluations

The three teachers, whose students participated in the study, responded anonymously and individually to a 10-item questionnaire (see Appendix G).

Question 1 asked teachers to describe or explain how the experimenter helped their students to learn long division. Two of their teachers said they did not know the specifics of study. The third teacher gave the following description: "Students were shown how to work long division problems. When the students were asked to work problems on their own, error correction was used to provide immediate feedback, so that students did not practice incorrect procedures before correction."
Question 2 asked the teachers if any of the students discuss the study with them, and if so, what kinds of comments did the student(s) make about the study? All of the teachers said “no.”

Question 3 asked the teachers if they observed any improvement in the students’ math performance, and if so to describe such improvement. Two teachers said yes and stated the following: “As we continued our usual math exercises in class they performed at a higher level,” and “... especially in one student’s performance. He completed year-end assessment with 68% accuracy. Pre-assessment level was only 34%. He also completed the assessment within a single class period. Pre-assessment took up two class periods.”

Question 4 asked whether or not during the course of the study were any of the students assigned math work that involved long division. Also, teachers were questioned as to whether or not they noticed any changes in their students’ performance. Again, two teachers said yes and one of the teachers made the following comment: “Students completed such problems with 60-90% accuracy and were able to complete them in 5 to 15 minutes less time than previously required.”

Question 5 asked teachers if they had observed any changes in the way that students responded to being corrected when they made an error in solving a math long division problem. One of the teachers said “Before the study they did not accept being corrected, I’m certain they has been a long line of such corrections. During and after the study they appeared to be much more objective and not personalize the corrections.” Another teacher commented “The students were more willing to correct the error.”
Question 6 asked the teachers whether or not they had noticed any changes in the rate or speed at which students completed their math long division assignments? Only two of the teachers responded to this question, and both teachers said that their students' rate had increased. One of teachers qualified his/her comment by saying “There was a slight improvement, the biggest improvement was the overall concept of long division was firmly planted in their mind.”

Question 7 asked the teachers to rate the school performance or achievement level of those students from their class who participated in the study. Three of the students were rated as “very below average.” One student was rated as “below average” and one student was rated as an “average” student.

Question 8 asked teachers if in any way the study was an imposition upon them or otherwise interfered with their students instruction. All of the teachers responded “no” to this question. Also, two teachers made comments. “In fact, [the study] provided an opportunity for students to receive more intense instruction in an area that is difficult for students involved in this study.” “As a matter of fact, often when students are singled out for extra help they are embarrassed as fellow students tend to make fun, etc. Such was not the case!”

All of the teachers said “yes” to Question 9 which asked if they would be willing to participate in a similar study in the future.

Question 10 asked teachers to make additional comments. One teacher remarked that the success of an educational program depends not only on educational methods, but also the teachers attitude toward the students. Another teacher said “I would prefer to arrange a more private
setting for the researcher and student. I was concerned that classroom activity might be distracting for you and/or the student.” The third teacher commented “I would hope that someone would continue to work with out students in studies such as yours. Much of the one to one instruction or practice is very valuable to our students.”
CHAPTER V
DISCUSSION

This study compared the effects of a “Tell Plus Show” (TPS) and a “Tell Plus Write” (TPW) error correction strategy on the math performance of at risk students and students with learning disabilities. Specifically, the study sought to determine the effects of these error correction strategies on high school students’ rate and percentage of digits solved correctly on long division problems. The students' performance was also analyzed for the number and percentage of digits correct, and the response generality of division problems across time and settings. In addition, students and teachers were questioned about their preferences with respect to the intervention. The results are discussed in relation to the research questions posed by this study. Further, this chapter addresses the limitations of the study, implications for practice, and suggests areas for future research. The last section of this chapter summarizes the study.

RESEARCH QUESTION ONE
Will a Tell Plus Show or Tell Plus Write error correction strategy result in a higher number and percentage of digits written correctly on end-of-session tests?

A functional relationship in favor of the TPW condition was not demonstrated because a clear fractionation of data paths did not occur due to the variability in student performance. Still, it was observed for all students that
the mean number of digits correct on end-of-session tests was slightly higher during the Tell Plus Write error correction strategy. These data are merely suggestive, not conclusive, of an experimental effect.

Overall, the group’s mean number of digits correct per minute on end-of-session tests was 14.1 digits under the TPW condition, and 10.1 digits under the TPS condition. On average, the group of students solved 5 more digits correct per minute under the TPW condition than the TPS condition (see Table 8). Stated another way, students solved about one-half of a problem more per minute correct under TPW than they did under TPS. Extrapolating these data to a regularly scheduled 30 to 50 minute math class might mean that students would solve 15 to 25 more problems correctly per day or 75 to 125 more problems correctly per week. The cumulative effect of .5 problem correct per minute under the TPW might be of substantial benefit to students, especially low achieving ones, when examined in terms of active student research literature (Barbetta et al. 1993a; Barbetta, 1991; Drevno et al., 1993).

However, when drawing comparisons between TPW and TPS error correction as measured on end-of-session tests, wholesale adoption of TPW must be tempered by considering that the total number of sessions that individual students participated in differed. Except for Student 1, who received the TPS error correction strategy during 17 sessions and the TPW error correction strategy during 18 sessions, Students 2 to 5 individually participated in no more than 9 end-of-session tests. Further, the number of sessions that each of these students received TPS and TPW was different: Student 2 (TPS = 5; TPW = 4), Student 3 (TPS = 3; TPW = 6), Student 4 (TPS = 4; TPW = 3), Student 5 (TPS = 4; TPW = 3).
In comparing Student 1’s performance, for example, on end-of-session tests when a varied type of long division problem was administered, some differences were observed in the number of digits correct. During sessions 1 and 25 when a new type of long division problem was introduced for the first time and the TPS error correction strategy was in place, the end-of-session tests showed that student 1 scored 8 out of 12 digits correct, and 6 out of 52 digits correct, respectively. In contrast, during sessions 5, 9, and 20 when a new type of division problem was introduced and the TPW error correction strategy was used, the end-of-session tests showed that student 1 scored 14 out of 29, 5 out of 20, and 46 out of 65 digits correct, respectively. In terms of total number and percent of digits correct under the TPS condition, he achieved a total of 14 out of 64 digits correct or 21.8% of total digits correct. Under TPW conditions, he scored a total of 65 out of 114 digits correct or 57% of total digits correct. In other words, when a new type of division problem was introduced, under the TPW conditions, Student 1 achieved more digits correct per minute on end-of-session tests than on the same test during TPS conditions (see Table 3).

Summary. Although the data do not show a clear functional relationship in favor of TPW or TPS, there is a close parallel to findings reported by other researchers with respect to active versus passive responding (e.g., Barbetta, 1991; Barbetta et al., 1993a). Taken as a whole the extant literature shows that when students actively make responses—(write items, say items, perform demonstrations)—performance increases relative to passive conditions where students may have merely listened to the teacher. The TPW condition—the analog to active responding in these studies—did produce a gain, albeit not a significant one at the level of proof.
RESEARCH QUESTION TWO

Will a Tell Plus Show or Tell Plus Write error correction strategy produce increased generality responses on next-session test?

Next-session tests were administered prior to worksheets and end-of-session tests, and contained different exemplars of division problems from those presented during the previous sessions. Similar to the end-of-session tests, the next-session tests sought to establish the generality of students' performance from one session to the next session. Each of the students participated in a different number of next-session tests and thus was exposed to a different number of error correction strategies: Student 1 (TPS = 16; TPW = 16), Student 2 (TPS = 6; TPW = 3), Student 3 (TPS = 3; TPW = 5), Student 4 (TPS = 4; TPW = 3), and Student 5 (TPS = 3; TPW = 1). The overall mean number of digits correct for Students 1 to 4 on next-session tests was slightly higher during the TPW conditions (15.03), than the TPS conditions (11.12). Student 5 was not included in this analysis because only one of his next-session tests was under the TPW error correction condition. However, his performance (rate of digits correct per minute) on the lone next-session test under the TPW condition was higher than two of the three next-session tests administered under TPS conditions (see Table 14).

As with end-of-session tests, Students 2 to 5 participated in too few sessions to make any firm conclusions about their individual performances. Overall, however, Students 2 and 5 earned more digits correct on next-session tests under TPW error correction conditions than during TPS conditions. Further, for Students 2 to 4, under the TPW condition their separate and respective mean scores for next-session tests (13.0 and 24.9) were higher than their separate and respective mean scores under the TPS condition (9.95 and
15.1). This difference was also seen in Students 2 and 4's separate and respective performances on end-of-session tests under the TPW (6.25 and 22.4) and under the TPS condition (6.16 and 14.9).

On average, however, Student 1 earned more digits correct per minute on next-session tests under the TPS (11.23) condition than under the TPW condition (9.99). These next-session tests mean scores were dissimilar to mean scores of corresponding conditions on end-of-session tests (TPW = 9.96; TPS = 8.54).

Summary. Baer, Wolf, and Risley (1968) stated that generalization must be programmed, not merely lamented. In this study, generalization was designed into the day-to-day operation by having students participate in next-session tests that assessed their ability to solve different, but similar, math problems. Overall, the data from this study, although not conclusive, do suggest that the TPW condition set the occasion for generalized responses.

RESEARCH QUESTION THREE
Will a Tell Plus Show or Tell Plus Write error correction strategy result in a higher of percentage of long division problems solved correctly on worksheet problems?

For all students more digits were solved correctly on worksheet problems under the TPW than the TPS condition (see Tables 15 to 20).

As a group, students achieved a grand total of 451 out of 534 digits correct under the TPW condition (84%), versus 294 out of 416 digits correct under TPS condition (70%). This means that the students responded with 157 more correct digits under TPW condition than under the TPS condition. On average, the group responded with 32 more digits correct under the TPW condition than the TPS condition. This represents a 34% increase in the
number of digits correct, and a 22% increase in the total number of digits written by the students (see Table 20).

**Summary.** On session worksheet problems, the type of error correction procedure played some role in the total number and percentage of digits correct. As a group, the students achieved a higher number and percentage of digits correct under the TPW than the TPS condition. Student participation distinguishes these two error correction procedures. Under the TPW condition, students observed and then responded in writing to the experimenter's direction, rather than observing the experimenter make the correct response.

The role and importance of student participation during error correction has been studied by others, and has been shown to have a decidedly positive effect on student performance (e.g., Barbetta et al., 1993a; Drevno et al., 1993; Jenkins & Larson, 1979; Kearney & Drabman, 1993; Lambardo & Drabman, 1987). For example, Barbetta et al. (1993a) found that every student in her study read more words correctly during instruction, same-day tests, next-day tests, and maintenance and generality tests. By comparison, in the present study, students achieved a one-third higher level of digits correct, and a one-fourth higher level of digits produced, essentially corroborating the main finding of Barbetta et al. (1993a). A distinction between the TPW and TPS conditions is that under the former, the student actually practices responding to the problem correctly, whereas with TPS the student is a passive participant.
RESEARCH QUESTION FOUR

Will either of the two error correction strategies result in more long division problems solved correctly on the long division problem immediately after error correction?

Under the TPW condition, as a group the students were accurate on 71% of the problems that immediately followed the problem on which they received error correction. That is, 62 out of 87 of the "next-problems" were solved correctly during the TPW error correction condition. By comparison, under the TPS condition out of a total of 101 next-problems, the students solved 56 of them correctly; that is, 54% of the next-problems were solved accurately (see Table 21).

There was some variability among students in their achievement on "next-problems." Separately Students 1, 4, and 5 had a higher number and percent of next-problems correct during the TPW condition than during the TPS condition. In comparison, Students 2 and 3 achieved individually a slightly higher number and percent of next-problems correct during the TPS condition than during the TPW condition.

Summary. These data, although mixed, suggest that TPW condition was more effective with setting the occasion for a student to produce a correct response on a next problem after an error. Even setting aside the data for Student 1, who had the overwhelming majority of next-problem trials, two of the four students (Students 4 and 5) did better under TPW than TPS.
RESEARCH QUESTION FIVE

Will either error correction strategy result in a reduction in the types of errors made by students on end-of-session tests?

Error analysis was completed by the second observer who evaluated each long division problem for three kinds of errors: computation, omission, and estimation.

The attendance of Students 2 to 5 prevented a definitive statement regarding the effects of the two error correction strategies on the reduction of particular kinds of errors. However, an informal analysis of their performance (see Tables 23 to 26) shows a tendency for each of the them to make relatively fewer computational errors during the TPW condition than the TPS condition. This phenomenon was also observed in Student 1's error analysis results (see Table 22 and Figure 16). Student 1 made approximately half as many computational type errors during the TPW condition than under the TPS condition. This pattern of errors could be attributed to the saliency of multiplication facts during the study. That is, students had access to multiplication tables when solving long division problems on worksheets. In addition, during the TPW error correction condition, students listened and then wrote the product of the multiplication fact as stated by the experimenter. In other words, under TPW, the students made an "active response" versus observing the experimenter make the response on the long division problem, as was the case under the TPS condition. Although the findings from the present study do not show that under TPW conditions certain kinds of errors are more readily reduced or eliminated than during TPS conditions, the pattern of errors suggest that students' direct participation in correcting their own errors had an influence on their performance. As mentioned earlier, research on effective
classroom instruction has shown a positive relationship between student achievement and student participation in an academic task (e.g., Gettinger, 1986; Greenwood et al., 1984).

The pattern of omission errors for all students on end-of-session tests was similar regardless of the error correction strategy. Most omission errors tended to have occurred during the initial sessions of the study. The lower frequency of such errors later in the study probably reflected the general effect of error correction. That is, repeated modeling of omitted steps (e.g., recording remainders) may have set the occasion for students to complete that portion of the algorithm previously omitted.

Unlike computation and omission errors, estimation errors reflect a more abstract process and were less directly responded to during error correction conditions. In neither error correction procedure did the experimenter demonstrate estimation. During error correction, the experimenter always stated the correct quotient. This may explain why Student 1 showed a general increase in estimation type errors across sessions under both error correction procedures. For Student 1, estimation played a more central role in the long division problems as the type of long division problem increased in complexity (e.g., 3 digit by 2 digit division with a zero in the 10's place of the dividend and a remainder).

Summary. Error analysis of students' responses on end-of-session tests showed a pattern in the types of errors they made across sessions. Students made most of their errors in the areas of computation and estimation, and made relatively fewer omission type errors. For Student 1, his cumulative number of estimation type errors was similar for both error correction conditions. In comparison, his cumulative number of computation errors was relatively higher.
under TPS conditions than TPW conditions. This difference in the number of computation type errors may reflect the active nature of the TPW condition.

**RESEARCH QUESTION SIX**

*What are students' preferences and opinions regarding the two error correction strategies?*

Students were asked four questions about the study. All of the students preferred the Tell Plus Write (TPW) over the Tell Plus Show (TPS) error correction strategy. Only two students told how the TPW procedure helped them learn long division "It [TPW] was easier to comprehend and made more sense," and "It [TPW] helped [me] to learn the process better by actually doing the problem." Further, while all the Students preferred the TPW over the TPS procedure, only two students stated why they preferred it: "It helped me to understand where [I] made a mistake." The second student said "I feel that this program [referring to the TPW procedure] will help me in the regular math class."

**RESEARCH QUESTION SEVEN**

*Did teacher's report any significant differences in their students' performance on long division problems?*

All of the teachers said that none of the students in the study spoke with them about the details of the experiment. Two of the teachers reported, however, that their students appreciated and gained confidence from the individual attention in math. These same two teachers noted measurable improvements in their respective students' achievement in math during and after the study. For example, one teacher observed positive changes in the students' routine class work in math. Specifically, this teacher saw an improvement in the students' performance on long division problems. The other teacher saw
improvements in a student's performance on a required math assessment that took place after the study. Specifically, the student was able to complete the assessment in less time, and achieved a high percentage of items correct than on the previous assessment.

Only one of the teachers claimed to have any knowledge regarding the error correction strategies investigated in this study. Two of the teachers stated, however, that their students were more willing to receive correction and to correct their errors on their math work. One of the teachers attributed this change in the students to the “objective manner” in which error correction was delivered during the study. According to this teacher, the students did not “personalize the corrections.”

LIMITATIONS

There were several factors that limited this study: student characteristics, computational math operations, the error correction strategies, classroom conditions, and the time of the school day.

Student Characteristics

The subjects in this study were 5 ninth grade male students with identified learning disabilities or who were considered at-risk for school failure. The students attended an inner city high school, and each of the students had a long school history of earning less than average grades in math and other academic subjects. In addition, in some form or another school attendance was a problem for each of the students. Further, all of the students were enrolled in special education classes designed to meet their individual needs. What is not known is the generality of effects of the TPS and TPW conditions under different situations, with different students (e.g., female students), and with students learning long division for the first time.
Computational Math Operations

Generally, students learn the long division operation during the elementary grades. High school students seldom receive instruction or practice with this operation on a consistent basis (Bitter, Hatfield, Edwards, 1989; Lampert, 1992; Reys, Suydam, Lindquist, 1989). Although long division computation involves skills necessary for more advanced math (Lampert, 1992), it is not known to what extent this procedure would have been effective if used with a more grade-appropriate math operation (e.g., algebra). Results could have been effected by student unwillingness to practice long division, even though their teacher believed that it was necessary for them to do so.

Error Correction Strategies

The two kinds of error correction strategies required students to observe or observe and write the correct digits to a long division problem. Students were not explicitly shown where they had made their errors during the execution of the algorithm, nor were they told the type of error they made (e.g., estimation). Attention to specific operations (e.g., estimation) within the algorithm was not addressed. The relative effects of using an error correction strategy on a specific type of error made in the long division problem (e.g., omission error) are not known.

Classroom Setting and Time of School Year

This study took place in an urban high school resource classroom for students with learning problems. While the other students in the class were engaged in various activities, students in the study worked individually with the experimenter at a table at the back of the classroom. The outcome of this study might have been different had it been held in another type of classroom and with students with different backgrounds.
The study began at the start of the spring term of school. The results of the experiment might have been different had it been held during another time of the school year.

**Time of School Day**

Students met with experimenter each day between 7:30 and 10:00 A.M. Although students never complained about participating in the study, the results might have been different had sessions been held later in the day. In addition, individual sessions lasted for approximately 15 minutes. Thus, the effect of extending the amount of time students spent working division problems and receiving remediation (i.e., error correction) is not known.

**IMPLICATIONS FOR CLASSROOM PRACTICE**

Implications can be considered for preservice teachers and inservice teachers. At the preservice level, the results of this study—and others—underscore the importance of teaching emerging practitioners about the importance of active student response (ASR) methodologies as an integral component of any instructional program. Clearly, the extant literature demonstrates that when students are active participants in response making that performance is enhanced. Accordingly, preservice teachers who receive consistent instruction prior to entering the profession will have conceivably received (a) the conceptual training upon which ASR is based, (b) practicum opportunities to engage in ASR activities through classroom simulations, clinical applications, or field-based practica, and (c) the chance to practice ASR techniques during student teaching to further develop their skills. In short, they should be better prepared to integrate this concept into their daily instructional routine. Students, especially those with special needs, will be the beneficiary.
At the inservice level, training directed toward improving ASR with students could be "translated" into a variety of formats. For instance, computer assisted instructional programming could be designed that would permit the student to progress through lessons, only after making an active response to a stimulus item. Likewise, classroom, cross-age, small group, or 1:1 peer tutoring programs could be implemented that replicate the procedures used in this study or others (e.g., Barbetta et al., 1993) so as to improve the acquisition, maintenance, or generalization of skills. A body of literature demonstrating the effectiveness of tutoring approaches (e.g., Heward, Courson, & Narayan, 1986; Maheady, Sacca, & Harper, 1988) could be extended by investigating more completing the effects of ASR during the error correction component of the tutoring exchange.

Finally, special and general education teachers state that a Tell Plus Write strategy is not intrusive and is relatively easy to implement. Data derived from the literature on consultation effectiveness supports the premise that when teachers perceive a particular program as being beneficial to their students, and they do not perceive the cost as being excessive, the probability of implementation improves (Heron & Harris, 1993).

SUGGESTIONS FOR ADDITIONAL RESEARCH

An important aspect of effective classroom instruction is the manner in which students' learning errors are addressed. This statement implies that the teacher engages in constructive exchanges with the learner to prevent further errors from occurring. The "ingredients of this exchange" have been the focus of this study. This dissertation examined the difference between a student's active versus passive involvement in error correction. It sought to extend the findings of previous research in the area (e.g., Barbetta et al., 1993ab; & Drevno
et al., 1993) and in particular assess the effect of these instructional strategies on more complex academic tasks (i.e., long division). In addition, this study concerned an academic content area (math) and subjects (secondary level students with learning problems) who are of increasing interest and in need of further study.

Although the results of the present study only suggest that active student involvement in error correction procedures had a positive influence on student achievement, the results do provide several ideas for future studies. First, the research questions in this study have important implications for all learners, especially learners who have deficits in basic math operations. The long division algorithm is the most complicated of whole number math operations (Mercer & Mercer, 1993). The concept behind division and types of division (partition & measurement) occur in the real world (Irons, 1981). Further, the long division algorithm is analogous to and has a role in the acquisition of more advanced math operations (e.g., algebra). Thus, the present study should be replicated with adjustments in its methodology. These changes would include conducting the study at a time during the school year when student attendance was not a problem. Another modification would be to inform students of the kind of error they had made. Providing prescriptive and differential feedback may help to distinguish different levels of performance under the active versus passive conditions. In other words, telling the student what his or her error was, coupled with active involvement from the student, may set the occasion for the student to draw distinctions more efficiently and effectively between accurate and inaccurate responses.
Second, it would be informative to assess the traditional classroom approach (i.e., teacher demonstration) with the kind of approach used in this study to remediate student math errors. This proposed study would seek to compare the effectiveness and efficiency of the teacher providing short periods of individualized error correction with the more traditional approach or several different students during math class. The more traditional approach is the situation where the teacher lectures and demonstrates to students how to solve a math problem. The comparison between these two approaches of error correction would seem especially appropriate in the secondary school setting where methods of instruction are being seriously studied (NCTM, 1991). Third, students with learning problems are not always receptive to remedial instruction (Webster, 1984). Researchers should consider investigating creative and yet effective methods of error correction that require active participation. Computers and other electromechanical devices might have the potential to counteract student resistance, while increasing student involvement in remedial instruction.

Fourth, the area of error correction with higher forms of math needs to be investigated. Unlike basic computational math where the interresponse time during error correction is relatively short and feedback specific, error correction for more advanced mathematics operations is complex and time consuming. The latter area is particularly problematic given the importance of multiple exemplars for students who have difficulty generalizing a math operation from one problem to the next problem (Becker, 1986).

Fifth, the type of active student response applied in this study involved a teacher (experimenter) and the student solving math problems on paper. Computers offer another possible way of actively involving students in the error correction process. The computer has the advantage of providing students with
specific feedback about the kind(s) of errors they made on a math problem when such individualized instruction is not possible directly from the teacher.

Finally, the problem of student attendance could be addressed by replicating the study with several students over the course of the school year. That is, instead of applying the error correction strategies with several students during one quarter, the procedure could be staggered, allowing for the sequential introduction of the error correction strategies. This approach would enable the experimenter to conduct several systematic replications rather than simultaneous replications.

SUMMARY

This study compared the effects of two academic error correction strategies on the accuracy of 5 high school students' responses to long division problems, and their responses to similar problems in "noninstructional" settings. These experimenter administered error correction strategies were called "Tell Plus Show" and "Tell Plus Write." The respective procedures differed in the degree to which the student participated in correcting their errors on long division problems. Under Tell Plus Show, students watched and listened as the experimenter solved the problem on which they erred. Under Tell Plus Write, students followed the experimenters verbal direction and wrote the solution to erred problems. The basic difference between these error correction strategies was the level of student activity in the error correction process.

The 5 ninth grade students who participated in the study were selected based upon (a) teacher nomination, (b) their enrollment in either special education classes (learning disabilities) or a school-based program for academically at-risk students, (c) their academic record and scores on statewide proficiency testing, and (e) their performance on achievement tests
administered by the experimenter. In sum, all of the students' academic achievement was below expectation, particularly in math computation.

The students' responses were analyzed for rate and percentage of digits solved correctly on long division problems. In addition, student performance was evaluated for the number and percentage of digits correct, and the response generality of division problems across time and settings. In addition, students and teachers were questioned about their preferences with respect to the intervention.

A functional effect was not demonstrated for either error correction procedure. Overall, however, students performed slightly better under the Tell Plus Write procedure than the Tell Plus Show procedure. In addition, all of the students preferred the Tell Plus Write over the Tell Plus Show error correction procedure. Finally, the findings and implications were discussed with respect to extant literature.
REFERENCES


Heron, T. E., & Harris, K. C. (1993). The educational consultant: Helping professionals, parents, and mainstreamed students (3rd ed.). Austin, TX: Pro-Ed.


APPENDIX A
PERMISSION LETTER
March 22, 1993

Dear Parents:

We request your permission for your student, _______________ to take part in a study which will be conducted by a student from The Ohio State University, Gregg Drevno. Mr. Drevno will be testing students from my math class before he begins a program aimed at improving student skills in the process of long division.

If you approve, please sign below and have your son/daughter return this form to me. If you have any questions, feel free to write or phone me (365-5541). Thanks for your cooperation.

Mrs. Donna White
APPENDIX B
PROCEDURAL RELIABILITY
Procedural Reliability

Student: 

Date_ 

Time begins/ends_

Type of division problem_

Reliability Check of (check one): worksheet (), next-session test (), end-of-session test ().

Circle type of error correction procedure admin: Tell Plus Show /

Please write "yes" or "no" next to numbered or lettered items to indicate whether or not the event occurred during the session. Written comments/observations are welcomed too.

1. General instructions regarding worksheet and tests are read to student.

2. Depending upon where the student is in their achievement in solving a particular type of division problem, a session will either begin with a next-session test (see #3), or a worksheet of division problems (see #4).

3. Experimenter sits next to student throughout session.

   a. Experimenter said "Before you do a worksheet of division problems, I want you to take a next-session test.

   b. Next-session test is placed faced down in front of the student.

   c. Student is told "Here is the next-session test, it has series of division problems for you to solve. When I say "Go," do as many problems correctly as you can in 3-minutes.

   d. "Don't worry if you don't finish all of the problems. Just do your best work.

   e. "Do the problems in order. Do not skip any problems."
f. "Stop immediately when the timer beeps. Do not write even one more number or answer."

g. "Are you ready?" "Please turn over the sheet."

h. "Ready? and Go." Timer started on the word "Go."

i. When timer beeps: "Stop writing on your test." Experimenter (E) tells student to stop if he fails to do so at the sound of the beeper. Experimenter records elapsed time_________.

j. Experimenter collects next-session test, records on next-session test elapsed time and administers directions for worksheet (see #4).

4. Worksheet is placed face up and in front of the student.

5. "Here are 42 division problems. Please solve each problem in the order in which they are presented on the worksheet. Feel free to use the (show student) "multiplication fact sheet" to solve the division problems."

6. "As you know, I'll keep a watch over your progress in doing the division problems. If you should make a mistake, we will work together on a second worksheet to figure out how to do the division problem correctly. After we solve the division problem, you will use this calculator (show student) to check the division problem that we solved together."

7. "Are you ready?"

8. "Ready and begin." Experimenter set the timer for 10-minutes. (OF NOTE: There is the possibility that the student will not make an error on his worksheet, and therefore not receive error correction.)

9. Experimenter administers error correction strategy according to the respective script (i.e., script for either Tell Plus Show or Tell Plus Write)
a. Error correction strategy was delivered immediately after the student completed a problem on which he made an error.

(Tally number of times strategy was delivered correctly__________)
(Tally number of times strategy was not delivered correctly__________)

b. Strategy was delivered on a second worksheet of identical division problems.

c. Immediately after each administration of the error correction strategy, student uses a calculator to check the division problem used during the strategy.

(Tally the number of times student used calculator_______________)
(Tally the number of times student did not use calculator__________)

d. Student is directed to resume worksheet problems after showing answer displayed on calculator to the experimenter.

e. Immediately after the 10-minute period has elapsed the experimenter collects the worksheet, calculator, and multiplication fact sheet.

10. Experimenter administers end-of-session test immediately after the timer alarm rings and materials (e.g., worksheet, multiplication fact table) are put away.

a. End-of-session test placed faced down in front of the student.

b. Student is told “Here is the end-of-session test, it has a series of division problems for you to solve. When I say “Go,” do as many problem as you can; try your best to answer as many problems correctly as you can in 3-minutes.
c. “Don’t worry if you do not finish all of the problems. Just do your best work.

d. “Do the problems in order. Do not skip any problems.”

e. “Stop immediately when the timer beeps. Do not write even one more number or answer.”

f. “Are you ready?” Please turn over the sheet.”

g. “Ready? and Go.” Timer starts on the word “Go.”

h. When timer beeps: “Please, stop writing on your test and turn it over.”

Experimenter tells student to stop if he fails to do so at the beeper.

i. Experimenter records elapsed time _________.

j. Experimenter collects faced down “end-of-session test,” thanks and dismisses student.
APPENDIX C
WORKSHEET
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<th>Digit Corr.</th>
<th>Incorrect</th>
<th>Elapsed Time</th>
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### Session 0

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 72|92 | 45|55 | 33|79 | 30|45 | 11|27 | 23|54 | 23|55 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34|65 | 49|89 | 22|49 | 21|44 | 11|74 | 91|93 | 89|91 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12|15 | 24|37 | 78|88 | 11|98 | 71|72 | 92|95 | 55|57 |
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APPENDIX E

WORKSHEET ANSWER KEY
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<td>Next-, End-Session, Work-, EC-sheet, Answer-sheet</td>
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| 22/23 | 76/89 | 36/71 | 63/75 | 29/57 | 12/17 | 89/91 |
| 12/15 | 24/37 | 78/88 | 10/90 | 76/78 | 92/99 | 55/56 |
| 15/78 | 67/89 | 35/71 | 23/25 | 29/59 | 12/17 | 89/91 |
| 16/67 | 55/56 | 23/35 | 22/25 | 21/22 | 67/68 | 59/60 |
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APPENDIX F
ERROR CORRECTION WORKSHEET
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<td>10/18</td>
<td>17/35</td>
<td>28/39</td>
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</tbody>
</table>
APPENDIX G
SOCIAL VALIDITY QUESTIONNAIRE
1. When Mr. Drevno worked with you on long division problems, he used two different ways to respond to you when you did not get the correct answer. On some days he would do one thing when you did not get the correct answer and on other days he would do something else. Tell me what he did?
[If student doesn’t describe two procedures accurately read him the following
"This is what he did: On some days, if you made a mistake on a long division problem, he corrected your mistake(s) by redoing the problem himself. He showed you the correct way of solving the problem. While he worked the problem, you were to watch him solve the problem. After he was finished, you resumed work on your own worksheet.
On other days, if you made a mistake on a long division problem, he corrected your mistake(s) by telling you how to solve the problem. He verbally directed you through the steps to solving the long division problem. As he spoke, you wrote in the answers to the problem. After he was finished, you resumed work on your own worksheet.]

2. Which method of correcting mistakes on your long division problems do you think ("Tell Plus Show" or "Tell Plus Write") helped you learn long division better?
(Stating "Tell Plus Show" or "Tell Plus Write" in the following question depends upon the response to question #2)

3. How did the ("Tell Plus Write" or "Tell Plus Show") help you learn long division better?

4. Did you like one of the ways that Mr. Drevno corrected your mistakes better than the other? Which one? Why did you like the ("Tell Plus Show" or "Tell Plus Write") better?
Dear Teacher,

As you know, I recently completed a math study with some of your students. Specifically, I worked with them in learning to solve long division problems. The following questionnaire requests your opinion and reaction to various aspects of the study.

In the event that you make comments about students, please distinguish them by letters; for example, student A has never received special help in math, or student C has been in my class this entire year. After you complete this questionnaire, please return it in the enclosed addressed and stamped envelope. Thank you for taking the time to respond to this questionnaire.

Gregg

1. As best you can, please describe or explain what I did to help your students learn math long division?

2. Did any student discuss the study with you? If so, what kinds of comments did the student(s) make about the study?

3. During the course of the study did you observe any improvement in the students' math performance? If yes, please describe.

4. During the course of the study were any of the students assigned math work that involved long division? Did you notice any changes in their performance? If yes, please describe.
5. Had you observed any changes in the way that students responded to being corrected when they made an error in solving a math long division problem? If yes, please describe.

6. Had you noticed any changes in the rate or speed at which students completed math long division assignments? If yes, please describe.

7. How would you rate the school performance or achievement level of the students from your class who participated in the study? Please write “student A,” “student B,” etc. below the descriptor that describes the student to whom your are rating.

very below average below average average above average excellent

8. In any way did the study impose upon your classroom routine or otherwise interfere with the students instruction.

9. Would you be willing to participate in a similar study in the future?

10. Please feel free to make additional comments.
APPENDIX H
ERROR CORRECTION SCRIPT

243
Tell Plus Show strategy this is what the experimenter will say and do.

Step 1. This problem is wrong. I will tell and show you how it should be done.

(Turn over the student's worksheet and pull out error correction worksheet on which the problem has not been solved. The experimenter reads the following script and includes in the script the numbers from the problem. The following script was written for 2-digit number by a 2-digit number, with Remainder).

Step 2. The problem reads xx divide by xx.

Step 3. I estimate that xx goes into xx, x times. I write x over the x in the number x. (x in the quotient over x in the dividend).

Step 4. Next, I multiply xx times xx which = x. I write x under x.

Step 5. Now, I subtract x - x = x.

Step 6. I compare this answer or difference with the divisor (x). I am comparing to see if the number (difference) is less than xx (divisor).

Step 7. I see that the difference is less than the divisor (x).

Step 8. I write this difference as a remainder.

Step 9. The answer to the problem reads x (state the divisor) goes into x (state the dividend) x times (state the quotient) with a Remainder of x (state this figure).

Step 8. Check the quotient (answer) with this calculator. Be sure to add the remainder, and to show me your answer.

Step 9. The answer to the problem reads x (state the divisor) goes into x (state the dividend) x times (state the quotient) with a Remainder of x (state this figure).

Step 10. Check the quotient (answer) with this calculator. Be sure to add the remainder, and to show me your answer.
Tell Plus Write strategy this is what the experimenter will say and do.

Step 1. This problem is wrong. I will tell you how it should be done. I speak, you will do the write the numbers. (Turn over the student's worksheet and pull error correction worksheet on which the problem has not been solved. The experimenter reads the following script and includes in the script the numbers from the problem. The following script was written for 2 digit number by a 2-digit number, with Remainder).

Step 2. The problem reads xx divide by xx.

Step 3. I estimate that xx goes into xx, x times. Write x over the x in the number x. (x in the quotient over x in the dividend).

Step 4. Next, I multiply xx times xx which = x. Write x under x.

Step 5. Now, Subtract x - x = x.

Step 6. I compare this answer or difference with the divisor (x). I am comparing to see if the number (difference) is less than xx (divisor).

Step 7. I see that the difference is less than the divisor (x).

Step 8. Write this difference as a remainder.

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Step 10. Check the quotient (answer) with this calculator. Be sure to add the remainder, and to show me your answer.