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Plane strain finite element analysis of sheet forming operations including bending effects

Cho, Uk Youn, Ph.D.

The Ohio State University, 1993

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PLANE STRAIN FINITE ELEMENT ANALYSIS
OF SHEET FORMING OPERATIONS
INCLUDING BENDING EFFECTS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

by

Uk Youn Cho, B.S., M.S.

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1993
To My Ju In Gong
An improved finite element method suitable for the plane-strain analysis of sheet metal forming operations is presented. The method incorporates a computationally efficient shell model and a consistent frictional contact algorithm through an implicit updated Lagrangian formulation. The workpiece material model is rigid-visco-plastic with a choice of power law hardening and plastic normal anisotropy, capable of modeling a wide variety of sheet metals. A simplified nonlinear incremental shell theory is employed along with an optional reduced integration through the thickness for computational efficiency, while retaining the advantages of the kinematic model containing the bending effects. Complex tool geometry can be handled by discrete data points, by primitives (lines and arcs), or by analytical functions.

The capabilities of the method are demonstrated through verification problems and comparisons with experimental data, benchmark results and published data for several practical problems of sheet metal forming industry. The problems include stretching and/or deep drawing operations, simulation of automobile panel section and brake bending operation.
As an independent investigation from the first portion of the dissertation, measured data from a set of simple bending experiments of two types of aluminum are presented and analyzed. Generated data from the experiments include strain histories (loading and unloading), springback information (springback angle and strains), and friction coefficients. As a by-product, a simple way of estimating the friction coefficient (Coulomb's law) during a bending operation is proposed and demonstrated.
ACKNOWLEDGMENTS

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NOMENCLATURE

A_1, A_2 unit vectors in the reference configuration
a_1, n unit vectors in the current configuration
N normal vector to the mid-surface in the reference configuration
u, w incremental in-plane and out-of-plane displacements in shell coordinate
\Delta u, \Delta w incremental in-plane and out-of-plane displacements in local coordinate
R mid-surface position vector
\mathbf{R} position vector of a material point
z thickness coordinate in the reference configuration
Z thickness coordinate in the current configuration
K_D tangent stiffness matrix due to inertia effects
\mathbf{F}_D nodal force vector due to inertia effects
K_E tangent stiffness matrix due to the contact forces
\mathbf{F}_E nodal force vector due to the contact forces
K_I tangent stiffness matrix due to internal reactions
\mathbf{F}_I nodal force vector due to internal reactions
r average anisotropy or "r-value"
M yield surface shape factor
\sigma_0 initial yield stress
K material strength parameter
\( a_0 \) pre-strain of a material
\( n \) strain hardening exponent
\( m \) strain rate sensitivity index
\( \gamma \) base strain rate
\( \dot{Q}_j \) generalized local nodal accelerations
\( \Delta Q_k \) incremental generalized displacements
\( \phi_j \) linear shape functions in the local axial direction
\( \psi_k \) Hermit cubic shape functions in the thickness direction
\( \Delta u \) incremental displacement vector
\( g_v \) vertical component of the gap vector
\( G_{\alpha\beta} \) metric tensor at the reference configuration in time \( t=to \)
\( g_{\alpha\beta} \) metric tensor at the current configuration in time \( t=to+\Delta t \)
\( k_1 \) local curvature given in terms of the angle \( \theta \) measured from the local \( x \)-axis
\( \bar{k}_1 \) bending curvature in the current configuration
\( k_{1} \) current apparent curvature defined in shell coordinates as \( k_1 = \zeta_1 \lambda_1^2 \bar{k}_1 \)
\( s \) shell reference coordinate in the axial direction
\( dS \) infinitesimal element length in the current configuration
\( dS_0 \) infinitesimal element length in the reference configuration
\( \Delta t \) incremental time step
\( S_a \) principal components of the symmetric 2nd Piola-Kirchhoff stress tensor
\( \delta E_a \) variation of the incremental Lagrangian strain tensor
\( V_0 \) element volume in its reference state
\( f_{c}^{a} \) contact traction vector

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$\mathbf{d}u_a$ virtual displacement

$A_0$ element surface area in the reference configuration

$\Delta \epsilon_i$ principal incremental true strains

$\Delta \epsilon_i^m$ membrane component of the incremental true strain

$\Delta \epsilon_i^b$ bending component of the incremental true strain

$\Delta E_i$ principal incremental Lagrangian strains

$\Delta E_i^m$ stretching component of the incremental Lagrangian strain

$\Delta E_i^b$ bending component of the incremental Lagrangian strain

$\theta$ angle between the shell and local coordinates

$\Delta \theta$ incremental rotation angle

$\lambda_1^0$ incremental stretch ratio on the mid-surface

$\lambda_1$ incremental stretch ratio at an arbitrary point
1.1 General Overview

The numerical modeling of sheet metal forming operations is a complex problem involving large rotations and large plastic deformations as well as interface interaction phenomena. Each of these aspects requires an accurate solution methodology if the simulation results are to be useful for an optimized part design and/or to minimize expensive die try-outs. While the full three-dimensional models are becoming more feasible with the use of vector and parallel processing hardware, simpler models requiring less processing time and therefore less expensive hardware will continue to be valuable analysis tools for quite some time. While processing times for 3D models are frequently measured in CPU hours on super computers, simplified 2D finite element models are available these days which can analyze a part in minutes on a personal computer.

Clearly, the amount and usefulness of the generated results of any finite element model will be commensurate with the computational effort, but in light of access and resource limitations for the typical industrial user, the super computer solution will not always prove to be preferable or even feasible. In an attempt to strike a balance between the need for short computation times and modeling capabilities the work outlined in this research aims at the development of a finite element method resulting in rapid computing
times while retaining the capability to model key features of the sheet metal forming process.

A major research issue has been the numerical stability and convergence problems associated with the frictional contact and material softening. While numerical stability and convergence problems have always been a major concern for developers and users of implicit finite element codes, it appears that a general consensus is building toward identifying the contact algorithm as the major factor responsible for the degradation of the convergence characteristics. Another issue is improving the computational speed of the simulation code. Optimal method of including bending effects is also an issue to reduce simulation costs.

1.2 Literature Review

Sheet forming simulation process using finite element method can be categorized into three basic approaches, namely membrane theory, bending theory and continuum approach. The process also deals with the interfacial phenomena between the tooling which usually assumed to be rigid and the deformable sheet body. Thus this review consists of those four main areas. Detailed reviews on finite element analysis of sheet forming area can be found in Lee et al.[1988].
1.2.1 Contact Problem

Tools and the deformable sheet body interferes each other during the forming process. Contact problem thus arises as a key issue during the deformation. Which parts of the body are in contact with the tools and how to deal with the contact condition numerically are the major issue in the simulation process. Contact area and contact force distribution are not known a priori so that it may be necessary to assume the contact geometry and let forces to satisfy equilibrium equations at a certain stage of time step. After reaching equilibrium, it is also necessary to check to see if the assumed contact geometry matches to the geometry at the equilibrium state. The friction condition between tools and the deformable sheet is another parameter which plays a role in the contact problem.

In the earlier finite element applications, Lagrange multiplier method is introduced by Chen et al.[1971] and Hughes et al.[1976] to deal with unknown contact parameters. The unknown parameters are added into the system of equation due to the contact constraint conditions, thus resulting in a large system of equations. The system of equations due to the increase in dimension gives the indefinite tangent operator and has zero diagonal entries, thus causes some difficulties in the solution stage. The penalty method to overcome such numerical difficulties is introduced by Kikuchi et al.[1984] and Cheng and Kikuchi [1985]. This penalty method also has some drawback in the penalty parameter selection and the solution is highly depend on the parameter chosen.

Nour-omid and Wriggers[1986] suggests the partitioning scheme in Lagrange method to overcome those drawbacks. Wriggers et al.[1985] suggest the penalty and augmented Lagrangian formulation method to overcome the increase in size of the system of equations. This method enforces contact condition in an average sense over the element boundary while direct penalty or Lagrangian parameter method enforces contact
constraints directly into the nodes. Chang, Saleeb and Shyu [1987] presented a consistent mixed finite element method for solving two-dimensional contact problem. A perturbed Lagrangian variation principle were used to derive stiffness equations for contact elements and pressure function was assumed in such a way that all non-contact modes in deformation must be excluded. Dalin and Onate [1989] used penalty function approach to model the contact condition in such a way that mesh normal vector is used to impose non-penetration condition. Pourboghrat and Stelson [1988] propose a simple model that predicts some of the nonuniformities in the punch-sheet contact region on pressbrake bending.

Another popular approach is the direct method. This method is a trial and error correction type method in which contact geometry is assumed first in a certain step and contact force distribution is assumed and updated geometry is obtained through equilibrium iteration. This way contact geometry can keep updated iteratively until the assumed geometry and that of equilibrium state match. Wang and his coworkers [1970, 1978, 1982, 1986] calculated the contact forces through the projection scheme of the penetrated nodes onto the surface of tools using interface conditions. Kim, Oh and Kobayashi [1978, 1980] suggested a linearized approach for the tool surface to describe the geometrical boundary condition. This method was extended to sheet forming application of general shapes by Toh and Kobayashi [1983, 1985].

Onate et al.[1983, 1990] treat the friction condition in such a way that a node in contact with the tool slips along the tangential direction which is the average of the direction of the element adjacent to that node. Thus the stiffness matrices at each node are assembled in a friction coordinate system which gives the forces automatically in those direction. Tool normal description was used for the calculation of the contact force by Tang [1976, 1989]. Choudhry and Lee [1989] also used the tool normal description.
Keum et al. [1990] suggested a mesh normal approach in calculating normal vector to the tool surface. They utilized the analytic method, B-spline method and piecewise linear method to describe an arbitrarily-shaped tool surface. This method is also implemented in the author's research as a tool library. Later Saran et al. [1990, 1991] extend the mesh normal based theory through consistent linearization of the full set of governing relation including frictional contact equations. Adding the contact forces to the additional nodal degrees of freedom, governing equations becomes large like Lagrangian method but the implementation gives some promising results on the forming with irregular shape tools.

1.2.2 Membrane Theory

Wang and Budiansky [1978] developed a membrane based finite element program for the simulation of sheet forming operations. Rate form of elastic-plastic constitutive model was formulated involving the rate of the Lagrangian strain tensor and the Jaumann rate of the second Piola-Kirchhoff stress tensor. Wang and Wenner [1978] first implemented elastic-viscoplastic analysis of sheet forming operation utilizing nonlinear membrane shell, normal anisotropy and work hardening constitutive model, and friction model. Finite element method and an iterative method were compared for plane strain stretch with a flat bottom punch. Both methods gave nearly identical results in the plane strain stretch case. Wang and Wenner [1982] compared the power hardening law with saturation model which predicted different results at large strains.

Kim and Kobayashi [1978] reported a similar membrane theory based on rigid-plastic model through which axisymmetric sheet forming processes such as the hydrostatic bulge test, the punch stretching, and the deep drawing of a circular sheet with
a hemispherical punch were analyzed. Tang [1981] employed an updated Lagrangian membrane formulation and two nodded linear elements for the elastic-plastic analysis of flanged hole forming. Wenner [1983] presented an analysis of the work hardening and sheet springback in plane strain draw operation by adopting a simple force balance concept of the membrane theory.

Rebelo and Kobayashi [1980] analyzed axisymmetric punch stretching of strain rate sensitive sheet metal. Park, Oh, and Altan [1987] adopted the approach and further expanded to simulate axisymmetric punch stretching and hydrostatic bulge forming based on the membrane theory and the Hill's normal anisotropy flow rule. The results showed that the rate sensitivity affects the deformation as expected. To get sound products and to maintain the superplasticity, the control of the strain rate is a key factor.

Chandra [1986] presented a general finite element analysis method with elastic-viscoplastic material model and updated Lagrangian description of motion. Massoni et al. [1986] presented a finite element analysis of quasi-static large strain and large displacement response of nonlinear elastic-viscoplastic membrane formulation. Some of the terms usually neglected in the computation of tangent operator were found to have an impact on the rate of convergence in the nonlinear iteration. Gavriushin and Zienkiewicz [1986] presented a general computational procedure capable of predicting the thickness change during forming operation. Numerical process inversion procedure was devised to arrive at a certain initial thickness from the final thickness. Superplastic forming simulation shows in good agreement with the experimental results. Duncan and Sowerby [1987] in a review paper discussed some of the important areas where mathematical modeling of the deformation processes would assist in the designing of tooling and development of technique in the sheet metal industry.
Chung and Wagoner [1987] suggested a non-linear finite element modeling technique for the simulation of plastic forming processes. The work devised several techniques based on an incremental approach to the trial solution which allowed a valid final convergent solution to be obtained. Onate, Kleiber and Saractbar [1988] suggested axisymmetric viscous voided membrane formulation for void-containing sheet metals. The effect of void porosity on the hemispherical stretching of a circular sheet was presented.

Nakamachi [1988] presented an updated Lagrangian type three dimensional finite element model, based on the elastic-plastic (J2 flow theory) membrane shell theory utilizing triangular membrane, constant strain elements. A linear relationship between the Eulerian increment of Lagrange strain and the Jaumann increment of Cauchy stress was suggested and slip-stick model was adopted for the friction condition. Yoon, Rao and Kikuchi [1989] presented numerical solutions of axisymmetric sheet stretching employing an experimentally determined stress-strain curve and measured coefficient of friction along the punch-sheet interface. Experimental results on mild steel, aluminum and brass with a hemispherical steel punch were compared with the simulation favorably. Bellet, Massoni and Chenot [1990] suggested a numerical model for solving either elastic-plastic, elastic-viscoplastic or purely viscoplastic deformation of thin sheets using a membrane approach. Contact algorithm is presented and allows for application to cold stretching and deep-drawing problems and to the superplastic forming of thin sheet.

Wenner [1990] developed numerically integrable elementary solutions to the membrane equilibrium equations and a pre-integrated version of the plasticity equations. The method was applied to the cylindrical punch stretch and draw problems proposed as OSU benchmarks by Lee, Wagoner and Nakamachi [1990].
Germain, Chung and Wagoner [1987, 1989] formulated a finite element program to simulate a general three-dimensional sheet stretching operation. The program is based on an anisotropic, rigid-viscoplastic material model and utilizes triangular, plane stress elements utilizing a membrane approximation. Special algorithms for dealing with die contact condition, material unloading and Coulomb friction have been developed. Kim and Wagoner [1987, 1991] aimed at developing a numerical method to solve coupled thermoplasticity problems related to sheet forming processes. Extension to the previous work of Germain et al.[1989] was made to solve the coupled deformation and heat transfer problem utilizing Bishop's step-wise decoupled method. The role of temperature gradients is revealed by examination of the simulation results such as hemispherical punch-stretching and square punch-stretching operations.

Guo et al.[1990] developed inverse finite element approach which is to perform a non-linear analysis to determine the positions of the nodes in the initial flat blank and the strains in the final drawn workpiece without considering the path dependent incremental process of plasticity and contact. Bellet, Massoni and Chenot [1990] presented a numerical model for solving either elastic-plastic, elastic-viscoplastic or purely viscoplastic deformation of thin sheets using a three dimensional membrane mechanical approach.

Keum et al.[1990] developed a plane strain simulation code based on the rigid-plastic membrane model adopting a quasi-rigid material law and direct nodal projection external to Newton-Raphson iteration at each time step. Shortcoming of the code lied in the difficulty of obtaining convergence, especially under draw-in conditions. Saran and Samuelsson [1990] developed a 2-D formulation for numerical simulation of complex forming processes with generally shaped tools. To increase efficiency and accuracy, adaptive remeshing techniques for sheet forming operation were investigated. Saran,
Keum and Wagoner [1991] developed section analysis code utilizing membrane-based consistent linearization of the full set of governing relations including frictional contact equations through which earlier shortcoming of draw-in simulation was improved. Rigid-viscoplastic material model with Hill's normal anisotropy yield condition and rate sensitive hardening law was adopted, along with the Coulomb friction model.

Sukhomlinov, Engelsberg and Davydov [1992] presented the rigid-viscoplastic finite element method for the analysis of axisymmetric sheet metal forming processes such as punch stretching, deep drawing and bulging. The flow theory associated with Hill's quadratic yield criterion for transversely isotropic sheets, membrane theory of shells and Coulomb's frictional contact law are included in the algorithm with an incremental procedure including Lagrangian formulation.

1.2.3 Bending Theory

Lubahn and Sachs [1950] analyzed the stresses and strains in plastically bent parts for a hypothetical metal with no strain-hardening and described the method to treat any actual metal. The solution was obtained by successive approximations. The result shows the movement of neutral axis toward the compression surface. Gotoh [1974] included the bending effect through a shell formulation to study the hydraulic bulging of an orthotropic plate. Later Gotoh and Ishise [1978] and Gotoh [1980] presented a rigid-plastic finite element method to simulate flange deformation in the deep-drawing process. Onate and Zienkiewicz [1983] presented a finite element method with viscoplastic shell formulation for axisymmetric and general 3D stamping problems utilizing the analogy between viscoplastic flow and incompressible elasticity. Triantafyllidis [1980, 1981, 1982] discussed the bending effects in general. Wrinkling effects due to compressive circumferential stresses in the cup flange and high curvature effects on the deformation of
a ductile plate were addressed as main issues. Levy [1984] presented empirically derived equations capable of predicting springback for various die geometry with different angles, hold down conditions, plan view curvatures and clearances between punch and die.

Triantafyllidis et al.[1986] used penalty finite element method to treat contact at the tool interface to model the drawbeads.

Chu [1986] developed analysis method on the double-bend technique which involves a rather complex bending-unbending procedure. The study also addresses the influence on springback prediction of possible early occurrences of reversed plastic deformation.

Majlessi and Lee [1987] suggested the simplified finite element method for the axisymmetric sheet metal forming processes applying deformation theory of plasticity. Elastic stress-strain relationship is used to initiate the solution. Ni [1988] developed an analytical technique to include the bending and draw-in effects in plane strain and axisymmetric metal forming. The technique is based on a lumped-parameter model in connection with the finite difference scheme and idealized nonlinear springs to account for the draw-in effect. The bending and draw-in effects were found to be quite significant, especially for cases in which punch radius to thickness ratio $R/t$ is less than ten.

Wang and Tang [1988] presented an axisymmetric and plane-strain finite element analyses of stretch/draw of sheet metal over a die corner radius. The study shows the relative significance of including the bending effects in various simple forming operations. Hambrecht et al.[1989] made comparison of bending, membrane and continuum finite elements and found that membrane theory is adequate for the axisymmetric hemispherical punch stretching, however the membrane theory has disadvantages for plane strain case with sharp corner.
Lee et al. [1988, 1989, 1990] introduced a two-dimensional bending model, including dynamic and strain rate effects. Comparative simulation shows excellent agreement with the analytical, experimental and other numerical solutions.

Yang, Shim and Chung [1990] investigated the effect of bending through the comparison of the membrane analysis and the shell-bending analysis for stretching and deep drawing. Degenerated shell element, which has several layers, was employed to take into account the change of the material properties in the thickness direction. Comparison of the result to the experiment showed that the bending effect is not appreciable for both in the load-displacement curve and in the strain distribution in the stretch-dominant process. However, in deep drawing, bending affects the strain distribution considerably. Thus bending must be taken into account to obtain more exact information on the strain distribution for the deep drawing process. Onata and Saracibar [1990] presented the possibility of the viscous voided shell approach for deriving bending-membrane finite elements for sheet metal forming problems. The elements can be selectively utilized according to the nature of deformation.

Karafillis and Boyce [1992] conducted springback studies for the sheet metal forming process using finite element analysis for two types of steel covering one low-yield and one high-yield. The results were utilized for tool design to eliminate the part shape error and to replace current experimental trial and error type die design procedures.

1.2.4 Other Theories

Cavendish et al. [1986, 1988] developed a new approach, the differential equation on a manifold (DEM) method, which discretizes the more fundamental equilibrium equation in non-rate form, thus guaranteeing that equilibrium is satisfied for all size of
time steps. The method is a variant of the mixed finite element method in which displacements, stresses and effective strain are given independent finite element representations. The constitutive equations are discretized by collocation to form a system of ordinary differential equations which are coupled with the discrete system of non-linear algebraic equations from equilibrium equations and then integrated using available numerical software. The new approach proved to be 6 to 26 times as fast as the old incremental method in hydrostatic bulging, plane strain punch stretching and hemispherical punch stretching case. Hall et al.[1989] later noticed the lack of robustness in the DEM approach.

Kim and Stelson [1988] used finite element method to evaluate an algorithm to identify the material characteristics of a pressbrake-bent sheet from force displacement measurements. Two dimensional solid element and the shadow node concept were used to represent the geometry and the frictional contact between tools and sheet.

Lee and Yang [1990] suggested energy method for an analysis of hemispherical punch stretching. Through comparative study with finite element solutions and corresponding experiments, the simple upper bound approach, based on kinematically admissible velocity fields and optimization with respect to some assumed parameters in the total deformation energy, can be effectively utilized for the analysis of axisymmetric sheet forming processes. Yang et al.[1990] developed a method of initial guess generation and contact treatment in the framework of the rigid-plastic finite element method.

developed a numerical formulation combining the micro and macroscopic models to predict the limit curve of aluminum sheets.

1.3 Research Objectives and Scope

In recent years, a new forming process has gained increasing acceptance, particularly in the automobile industry, where it is being used to produce outer body panels of superior quality. This process is generally referred to as stretch-drawing and uses a multi-action press equipped with die cushions. This allows the blank to be pre-stretched over the die before the actual drawing process begins, thereby permitting a closer control of the process parameters, resulting in a better precision and repeatability while producing a more favorable strain state in the finished part.

The illustrations in Fig. 1.1 and 1.2 depict the four essential stages of the stretch-drawing process. In the stretch-drawing process outlined in Fig. 1.1 and 1.2, the blank is first confined around its periphery in order to restrict the metal flow into the die cavity. This stage, which is completed at the end of Phase II in the illustrations above, is usually called the binder wrap or setting stage. Virtually all stretching or drawing processes involve the confinement of the workpiece in the periphery in order to gain better control over the deformation process.

If the sheet metal flow into the die cavity is unrestricted or the confining pressure in the binder area is too low, the sheet will wrinkle and the formed part will generally not be of an acceptable quality. Since it is virtually impossible to remove wrinkles through subsequent processes, it is imperative to avoid wrinkle formation during the drawing process itself through an accurate control of binder (confining) pressure and friction conditions in the blank periphery.
Phase I:

The blank is loosely placed on the lower binder ring and deforms under its own weight.

(a) Stretch-Draw Process, Phase I

Phase II:

The upper binder ring descends and confines the blank periphery between upper and lower binder rings.

(b) Stretch-Draw Process, Phase II

Fig. 1.1 Stretch-draw Forming Process during Phase I and II
Phase III:

Both binder rings descend together against the die cushion pressure, stretching the blank over the lower die.

Phase IV:

The upper die closes and performs the actual drawing process.

Fig. 1.2: Stretch-draw forming processes during Phase III and IV
An accurate model for the numerical simulation of this stage has to include bending effects and an elasto-plastic material model, as well as a contact algorithm capable of handling the complex double-contact conditions occurring in the binder area. The binder wrap stage analysis, dealing with the above phenomena is not addressed here.

The primary focus of this research is the analysis of the actual forming stage by assuming that an appropriate binder wrap has been completed and that phases III and IV are combined into one action. The binder restraint condition (either pure stretch or stretch-draw) must be known to begin the analysis. For the stretch-draw analysis, the binder hold down force must be converted to an equivalent binder restraint force to be applied along the tangent of the sheet under the binder.

The main objectives of this research are;

a) to develop an efficient finite element model including bending and rate-sensitivity effects.

b) to provide a set of reliable experimental data for a future use, which include strain histories (loading and unloading, springback) under a simple bending operation.

The research objectives are pursued in remaining chapters in the following fashion.

Chapter II consists of the theoretical development of the governing equations. To reduce computing times without abandoning the shell model, the number of degrees of freedom per node has been reduced from four to three. This reduction results from a simplification in the updated Lagrangian description of motion.

Chapter III includes finite element formulation and computer implementation of the theory. The basic finite element formulation is an extension of the work published by Choudhry and Lee [1990]. The feasibility of a consistent contact formulation, proposed
and implemented by Saran and Wagoner [1991] for a membrane line element, is investigated. Also, the use of a reduced quadrature rule is explored for through-the-thickness integration, which further simplifies the analysis and yields promising results for the adopted rigid visco-plastic material assumption.

Chapter IV consists of numerical experiments including patch tests, comparison with the OSU benchmark problem, formability test, plane strain strip draw, some springback considerations in the plane strain draw with flat bottom punch and brake bending simulation. Chapter V consists of simulations conducted for industry scale problems. Measured data for actual parts are compared with the simulation results.

Chapter VI provides experimental data through bending experiments and some mechanics considerations. These data can be a useful resource for a future investigation. A direct comparison with the developed finite element model is not carried out simply because the current model neglects the elastic effects. Some limited comparisons with other models are made.

Chapter VII presents summary and conclusions of the present study and recommendations for the future work in utilizing the developed theory and simulation code.
CHAPTER  
II  
GOVERNING EQUATIONS  

2.1 Basic Assumptions  

Sheet forming processes have been analyzed through plane strain finite element analysis. Thin sheet is considered as a deformable geometric surface, thus the geometry to be analyzed can be considered as a cross section along which major principle strains are anticipated.  

Sheet thickness in forming operation is usually very small compared to other dimensions, thus thin shell theory can be applied. Therefore, plane stress condition holds in the thickness direction. Next, Kirchhoff's assumption holds. In other words, straight lines normal to the middle surface are assumed to remain normal after deformation. That is to say, shear deformation has been neglected.  

Updated Lagrangian formulation based on non-linear thin shell theory is adopted. Shell coordinate system is employed to describe the motion in the reference and current configuration and transformation to local coordinate system is made for the formulation of element tangent stiffness matrices and element force vectors. Transformation to a global coordinate system is made for the assemblage of the local element stiffness matrices and force vectors and solution of the system of equations.
The variation of the derivative of the incremental displacement vector in axial direction in shell coordinate is assumed to be very small comparing to that of rotation term. This assumption reduces the degree of freedom of the system of equilibrium equations and results in great savings in computing time without losing the accuracy of the solution.

Incremental deformation theory assumption holds by enforcing a proportional strain path during each time increment and total deformation theory of plasticity functions within each time step (for details, see Chung and Wagoner [1987]).

Material is characterized as rigid-viscoplastic, holding Hill's normal anisotropy yield criterion. The power hardening law is adopted for the constitutive model.

To model contact friction between sheet and tools, modified Coulomb's law is utilized. Here, punch and die are considered as rigid bodies.

Newton-Raphson algorithm is used to solve the linearized system of equilibrium equations within a time step. Incremental updating scheme is utilized here for the time stepping from the referential configuration \( t = t_0 \) to the current configuration \( t = t_0 + \Delta t \). The detailed theory is discussed in the forthcoming sections for the plane strain problem.

2.2 Updated Lagrangian Description of Motion

The updated Lagrangian description is based on an incremental motion from the known reference configuration at time \( t = t_0 \) to the current configuration at time \( t = t_0 + \Delta t \), where \( \Delta t \) is an incremental time step (Fig.2.1).

In the reference configuration, the position vector \( \mathbf{R} \) of a material point is related to the mid-surface position vector \( \mathbf{R} \) by
Fig. 2.1: Shell geometry and coordinate systems defined at times $t = t_0$ and $t = t_0 + \Delta t$
\[ R(s, z) = R(s) + zN(s) \]  \hspace{1cm} (2-1)

where \( s \) is the shell reference coordinate in the axial direction, \( z \) is the thickness coordinate, and \( N \) is the normal vector to the mid-surface.

Similarly, in the current configuration, a material point position vector \( \mathbf{r} \) is related to the mid-surface position vector \( \mathbf{R} \) by

\[ \mathbf{r}(s, z) = \mathbf{r}(s) + z\mathbf{n}(s) \]  \hspace{1cm} (2-2)

where \( z \) is the current thickness coordinate. The incremental displacement vector \( \Delta \mathbf{u} \) specifies the translation of the reference mid-surface position vector:

\[ \mathbf{r}(s) = \mathbf{R}(s) + \Delta \mathbf{u}(s) \]  \hspace{1cm} (2-3)

The tangent and normal vectors in the current configuration can be expressed in terms of the incremental displacements \( (u, w) \) in the referential shell coordinate as

\[ \mathbf{a}_1 = (1 + u_s - k_1 w)\mathbf{A}_1 + (w_s + k_1 u)\mathbf{N} \]  \hspace{1cm} (2-4)

\[ \mathbf{n} = \mathbf{a}_2 = \sqrt{|\mathbf{a}_1|/|\mathbf{a}_2|} [-(w_s + k_1 u)\mathbf{A}_1 + (1 + u_s - k_1 w)\mathbf{N}] \]

where \( \mathbf{A}_1 \) and \( \mathbf{N} = \mathbf{A}_2 \) are unit vectors in the reference configuration and \( k_1 \) is the local curvature given in terms of the angle \( \theta \) measured from the local \( x \)-axis

\[ k_1 = \mathbf{N} \cdot \mathbf{A}_{1s} = \frac{d\theta}{ds} \]  \hspace{1cm} (2-5)

Using the definitions of the derivatives of the unit vectors in the reference configuration with respect to the shell coordinate and assuming that \( u_{ss} \) is negligible, the
derivative of the tangent vector in the current configuration with respect to the shell coordinates can be expressed as,

\[ a_{1,n} = -\{(k_{1,s}w + k_{1}w_{s}) + (w_{n} + k_{1}u)k_{1}\}A_{1} \]

\[ + \{(1 + u_{n} - k_{1}w)k_{1} + (w_{n} + k_{1}u)k_{1}\}N \]

Then, the incremental stretch ratio \( ^{0}\lambda_{1} \) on the mid-surface can be defined as

\[ ^{0}\lambda_{1} = \sqrt{a_{1} \cdot a_{1} = \sqrt{(1 + u_{n} - k_{1}w)^{2} + (w_{n} + k_{1}u)^{2}}} \]

The current curvature \( \bar{k}_{1} \) can then be derived from Eqns. (2-4) and (2-6) as

\[ \bar{k}_{1} = \frac{1}{6\lambda_{1}^{3}} \{(1 + u_{n} - k_{1}w)(w_{n} + k_{1}u) + k_{1}\} \]

\[ + (k_{1,s}w + k_{1}w_{s})(w_{n} + k_{1}u) + k_{1}^{0}\lambda_{1}^{2} \}

2.3 Definition of Incremental Strains

According to the description of motion, the tangent base vectors in the current configuration can be expressed by

\[ g_{1} = (1 - zE_{1})a_{1} + z_{n} n \]

\[ g_{2} = \frac{\partial z}{\partial z} \]

where \( a_{1} \) and \( n \) are unit vectors in the current configuration.
The corresponding metric tensor is given by,

\[ g_{\alpha\beta} = \begin{bmatrix} \lambda_1 (1 - \frac{z_1}{z_0})^2 + \frac{z_3}{z_0} (z_3) (z_2) \\ (z_3) (z_2) \\ (z_2)^2 \end{bmatrix} \] (2-10)

while the metric tensor at the reference configuration in time \( t = t_0 \) can be written as,

\[ G_{\alpha\beta} = \begin{bmatrix} (1 - \frac{z_k}{z_0})^2 + (z_3)^2 + (z_3) (z_2) \\ (z_3) (z_2) \\ 1 \end{bmatrix} \] (2-11)

Assuming the shell to be thin, i.e. by neglecting the through-the-thickness shear deformation, only the diagonal terms of the metric tensors are taken into account. Defining the incremental stretch ratio at an arbitrary point as \( \lambda = \frac{dS}{dS_0} \), where \( dS \) and \( dS_0 \) are the infinitesimal element lengths in the current and in the reference configuration as:

\[ dS^2 = dr \cdot dr = g_{\alpha\beta} ds^\alpha ds^\beta \] (2-12a)

\[ dS_0^2 = dR \cdot dR = G_{\alpha\beta} ds^\alpha ds^\beta \] (2-12b)

one arrives at the expression:

\[ (g_{\alpha\beta} - \lambda^2 G_{\alpha\beta}) ds^\alpha ds^\beta = 0 \] (2-13)

where \( g_{ab} \) and \( G_{ab} \) are the metric tensors of the current and reference configuration, respectively. The solutions to the eigenvalue problem of equation (2-13) are the principal stretch ratios:

\[ \lambda_1 = \frac{\lambda_1}{(1 - z_k)} \quad \text{and} \quad \lambda_2 = \frac{dz}{dz} \]

(2-14a & b)
where $0 \lambda_1$ is defined in equation (2-7).

For plane strain problems, the major incremental true strains can therefore be defined as:

$$
\Delta \varepsilon_1 = \frac{1}{2} \ln \left\{ \frac{G_{11}}{G_{11}} \right\} = \ln \left\{ 0 \lambda_1 \left( 1 - \frac{z k_1}{\lambda_1} \right) \right\} - \ln(1 - zk_1) \quad (2-15a)
$$

$$
\Delta \varepsilon_2 = 0 \quad (2-15b)
$$

$$
\Delta \varepsilon_3 = -\Delta \varepsilon_1 \quad (2-15c)
$$

where $k_1$ is the current apparent curvature which is defined in shell coordinates as $k_1 = 0 \lambda_1^2 \bar{k}_1$.

The major incremental Lagrangian strain in the longitudinal direction is expressed as:

$$
\Delta E_1 = \frac{1}{2} \left[ 0 \lambda_1^2 (1 - \bar{k}_1)^2 - (1 - zk_1)^2 \right] \quad (2-16a)
$$

$$
\equiv \frac{1}{2} (0 \lambda_1^2 - 1) - (zk_1 - zk_1) \quad (2-16b)
$$

Equation (2-16b) is an approximation of (2-16a) obtained by excluding second order terms in the thickness variable.

In order to facilitate the finite element formulation, the kinematic quantities derived above in shell coordinates are transformed into the local element coordinate system by a linear transformation $(u, w) = T (\Delta u, \Delta w)$, where $(\Delta u, \Delta w)$ is the incremental displacement in the local projected element coordinate $(x, z)_{\text{local}}$. 
The mid-surface stretch ratio, in terms of the local variables can then be written as:

\[ 0 \lambda_1 = \cos \theta \left[ (1 + \Delta u')^2 + (\Delta w' + Z')^2 \right]^{0.5} \]  

(2-17)

where ' denotes d/dx, \( \theta \) is the angle between the shell and local coordinates, and 
\( Z' = \tan(\theta) \).

The current apparent curvature \( k_1 \) is transformed into local coordinates as:

\[
k_1 = \cos^2 \theta \left\{ \frac{(1 + \Delta u')(\Delta w'' + Z'')}{\left[ (1 + \Delta u')^2 + (\Delta w' + Z')^2 \right]^{0.5}} \right\} \]

(2-18a)

\[
= \cos^2 \theta (\Delta w'' + Z'') \left\{ 1 - \frac{1}{2}(\Delta w' + Z')^2 \right\} \]

(2-18b)

Here, equation (2-18b) is an approximation of (2-18a) by a binomial expansion.

The curvature in the reference configuration can be expressed as:

\[ k_1 = \cos^2 \theta \ Z'' \]  

(2-19)

Thus, the incremental true strain \( \Delta \varepsilon_1 \) from Eqn.(2-15a) can be written as

\[ \Delta \varepsilon_1 = \Delta \varepsilon_i^m + \Delta \varepsilon_i^b \]  

(2-20a)

where

\[ \Delta \varepsilon_i^m = \ln \left\{ \cos \theta \left[ (1 + \Delta u')^2 + (\Delta w' + Z')^2 \right]^{1/2} \right\} \]  

(2-20b)

\[ \Delta \varepsilon_i^b = \ln \left\{ 1 - \frac{1}{2}(\Delta w'' + Z'') \left\{ 1 - 2\Delta u' + 3(\Delta u')^2 - 1.5(\Delta w' + Z')^2 \right\} \right\} \]

- \ln \left\{ 1 - \cos^3 \theta \cdot Z'' \right\} \]

(2-20c)

Likewise, the incremental Lagrangian strain in the axial direction can be derived from Eqn.(2-16) using equations (2-17) and (2-18) as
\[ \Delta E_i = \Delta E_i^m + \Delta E_i^b \]  

(2-21)

where \( \Delta E_i^m \) is the stretching component and \( \Delta E_i^b \) is the bending component of the incremental Lagrangian strain. These two components can be written as:

\[ \Delta E_i^m = \cos^2 \theta \left\{ (\Delta u' + \Delta w' Z') + 0.5[(\Delta u')^2 + (\Delta w')^2] \right\} \]  

(2-22a)

\[ \Delta E_i^b = z \frac{\cos^3 \theta}{z} \frac{(1 + \Delta u')(\Delta w'' + Z'')}{\left[ (1 + \Delta u')^2 + (\Delta w' + Z')^2 \right]^{1/2}} - z \cos^3 \theta Z'' \]  

(2-22b)

The strain expressions typically employed in a membrane finite element method can be recovered by neglecting the curvature terms in the above expressions:

\[ \Delta \varepsilon_i^m = \ln^0 \lambda_i \quad \text{and} \quad \Delta E_i^m = \frac{1}{2} (\lambda_i^0 - 1) \]  

(2-23a,b)

Both incremental true strain formulation and incremental Lagrangian strain formulation were implemented in the simulation program. The simulation results were highly depend on the problem to be solved, in other words, some case true strain formulation worked while Lagrangian strain formulation failed to converge after several steps. Thus, care should be taken in choosing the type of formulation. Generally incremental Lagrangian strain formulation gives better convergence.
2.4 Reduction of the Number of Nodal Unknowns

As mentioned in the derivation of equation (2-6), in a full bending theory the strain expressions will contain the second derivative of the incremental in-plane displacement. Therefore, for a compatible finite element approximation, the generalized nodal displacement vector would be comprised of the four quantities \( \{ \Delta u, \frac{d\Delta u}{dx}, \Delta w, \frac{d\Delta w}{dx} \} \). Here \( \frac{d\Delta w}{dx} \) refers to the incremental nodal rotation while no physical meaning can be attached to the \( \frac{d\Delta u}{dx} \) degree of freedom. However, if we assume that the incremental displacement \( \Delta u \) varies linearly within an element, the quantity \( \frac{d\Delta u}{dx} \) does not need to be considered as a nodal degree of freedom.

Hence, the number of nodal degrees of freedom can easily be reduced from four to three. Obviously, this reduction in number of nodal unknowns necessitates some changes in the formulation of the finite element equations. It will be shown that the accuracy of this model is comparable to the full four degree of freedom model while leading to considerable savings in computing times for the presented example problems.
3.1 Virtual Work Principle

The principle of virtual work can be stated as,

\[
\int_{V_0} \rho \ddot{u}_a (\delta u_a) dV + \int_{V_0} S_a (\delta E_a) dV = \int_{A_0} t^a_u (\delta u_a) dA \tag{3-1}
\]

where the left hand side is the internal virtual work done by the inertia and internal forces, while the right hand side represents the external virtual work done by the tractions. \( S_a \) is the principal components of the symmetric 2nd Piola-Kirchhoff stress tensor, \( \delta E_a \) is the variation of the incremental Lagrangian strain tensor, and \( V_0 \) is the element volume in its reference state. \( t^a_u \) is the contact traction vector, \( \delta u_a \) is a virtual displacement, and \( A_0 \) is the element surface area in the reference configuration.

In cases involving plane strain, it is sufficient to set \( a = 1 \) and it is further assumed that the through-the-thickness stress \( S_3 \) be negligible due to the thin shell theory adapted here. Thus plane stress condition holds in the thickness direction.
3.2 Constitutive Equation

Hill's new yield criterion [1979] is,

\[
2f(\sigma) = F[|\sigma_1 - \sigma_2|^M + G|\sigma_3 - \sigma_1|^M + H|\sigma_1 - \sigma_2|^M + A|2\sigma_1 - \sigma_2|^M + B|2\sigma_2 - \sigma_1 - \sigma_3|^M + C|2\sigma_3 - \sigma_1 - \sigma_2|^M = 1
\]  

(3-2)

Applying inplane isotropy and plane stress conditions to (3-2) results,

\[
2f(\sigma) = H[\sigma_1 - \sigma_2]^M + C[\sigma_1 + \sigma_2]^M
\]  

(3-3)

Normal anisotropy condition and tension test result,

\[
2(1+2r)\bar{\sigma}^M = (1+2r)[|\sigma_1 - \sigma_2|^M + |\sigma_1 + \sigma_2|^M]
\]  

(3-4)

Associated flow rule gives,

\[
\frac{d\varepsilon_1}{(1+2r)[|\sigma_1 - \sigma_2|^{M-1} + |\sigma_1 + \sigma_2|^{M-1}]} = \frac{d\varepsilon_2}{-(1+2r)[|\sigma_1 - \sigma_2|^{M-1} + |\sigma_1 + \sigma_2|^{M-1}]} = \frac{d\bar{\varepsilon}}{(1+r)\bar{\sigma}^{M-1}}
\]  

(3-5)

Rewriting (3-4), effective stress can be expressed as,

\[
\bar{\sigma} = \left[\frac{1}{2(1+2r)}\left\{(1+2r)[|\sigma_1 - \sigma_2|^M + |\sigma_1 + \sigma_2|^M]\right\}^{1/M}\right]
\]  

(3-6)

Manipulation of (3-5) gives,

\[
\frac{d\varepsilon_1 + d\varepsilon_2}{|\sigma_1 - \sigma_2|^{M-1}} = \frac{d\varepsilon_1 - d\varepsilon_2}{(1+2r)[|\sigma_1 - \sigma_2|^{M-1} + |\sigma_1 + \sigma_2|^{M-1}]} = \frac{d\bar{\varepsilon}}{(1+r)\bar{\sigma}^{M-1}}
\]  

(3-7)

(3-6) and (3-7) yield the effective strain definition as,

\[
d\bar{\varepsilon} = \frac{1}{2}[2(1+2r)]^{1/M}\left\{d\varepsilon_1 + d\varepsilon_2 \right\}^{M-1} + (1+2r)\left\{d\varepsilon_1 - d\varepsilon_2 \right\}^{M-1}
\]  

(3-8)
Applying plane strain condition \( d\varepsilon_z = 0 \) to (3-8), assuming deformation theory of plasticity gives,

\[
\Delta \varepsilon = C_1 (1 + C_2) \frac{M}{M-1} |\Delta \varepsilon_1| \tag{3-9}
\]

where the constants \( C_1 \) and \( C_2 \) can be expressed in terms of the two anisotropy parameters \( r \), the average anisotropy or "r-value", and \( M \), the yield surface shape factor.

\[
C_1 = \frac{1}{2} \left\{ 2(1 + r) \right\}^{\frac{1}{M-1}} \quad \text{and} \quad C_2 = \left\{ 1 + 2r \right\}^{\frac{1}{M-1}} \tag{3-10a,b}
\]

Assuming the material response to be rigid visco-plastic, the incompressibility condition can be applied using the natural strains. Integration of the incompressibility condition yields an expression for the current thickness coordinate \( z \), viz.,

\[
z = \frac{1}{k_1} \left[ \left( 1 - \left\{ 1 - \frac{k_1}{\gamma_k z} (2z - k_1 z^2) \right\}^{1/2} \right) \right] \tag{3-11}
\]

The principal components of the second Piola-Kirchhoff stress can be related to the principal components of the Cauchy stress by a standard transformation. This allows us to use the true stress-strain relation in the constitutive equation while using the second Piola-Kirchhoff stress and Lagrangian strain in the virtual work principle.

A power law hardening relation with strain rate sensitivity of the form

\[
\sigma = \sigma_0 + K (\varepsilon_0 + \varepsilon_0 + \Delta \varepsilon)^n \left( \frac{\varepsilon}{\gamma} \right)^m \tag{3-12}
\]
is utilized in the constitutive model. Here $\sigma_0$, $K$, $a_0$, $n$, $m$ and $\gamma$ denote the yield stress, material strength parameter, pre-strain, strain hardening exponent, strain rate sensitivity index and base strain rate. Obviously, depending on the choice of constitutive parameters, this expression can accommodate the Hollomon, Swift and Ludwik hardening models and consequently is capable of accurately modeling a wide range of sheet metals.

3.3 Finite Element Formulation

The acceleration vector and incremental displacement vector are approximated using standard finite element shape functions:

$$\ddot{\mathbf{U}} = \sum_{j=1}^{n} \phi_j \ddot{Q}_j \quad \text{and} \quad \Delta \mathbf{u} = \sum_{k=1}^{n} \psi_k \Delta Q_k$$

(3-13)

Here $\ddot{Q}_j$ and $\Delta Q_k$ are the generalized local nodal accelerations and incremental displacements respectively, $\phi_j$ and $\psi_k$ are shape functions which are linear in the local axial direction and Hermit cubic for the out-of-plane displacement, resulting in three degrees of freedom at a node. Substituting (3-13) into (3-1) and factoring out the variation of $\Delta Q_k$ yields

$$\int_{V_0} \rho \ddot{Q}_j \psi_k \psi_k \, dV + \int_{V_0} S_a \frac{\partial (\Delta e_a)}{\partial (\Delta Q_k)} \, dV = \int_{A_0} \gamma_k \, dA$$

(3-14)
For sheet nodes in contact with the tool surface, the following geometric constraint equation must be satisfied:

\[ \int_{A_0} g_v dA = 0 \]  \hspace{1cm} (3-15)

Here \( g_v \) can be the vertical or normal component of the gap vector and will be discussed in a later section. Equations (3-14) and (3-15) form the basic equation system which needs to be solved iteratively during each time step.

Linearization of (3-14) and (3-15) by Taylor series expansion results in a set of linear algebraic equations for each node

\[ (K_I + K_E + K_D)\delta Q = F_E - F_I - F_D \]  \hspace{1cm} (3-16)

\( K_D \) and \( F_D \) are the tangent stiffness matrix and force vector due to inertia effects, \( K_E \) and \( F_E \) are the tangent stiffness matrix and nodal force vector due to the contact forces, and \( K_I \) and \( F_I \) are the tangent stiffness matrix and force vector due to internal reactions.

\[
(K_I)_{ij} = \int_{V_0} \left\{ S_\alpha \left( \frac{\partial^2 (\Delta E_\alpha)}{\partial \Delta Q^k \partial \Delta Q^l} + \frac{\partial \Delta E_\beta}{\partial \Delta Q^k} \frac{\partial S_\alpha}{\partial \Delta E_\beta} \frac{\partial \Delta E_\alpha}{\partial \Delta Q^l} \right) \right\} dV \]  \hspace{1cm} (3-17a)

\[
(K_E)_{ij} = \int_{V_0} \phi_k \left( \frac{\partial f_q}{\partial \Delta Q_j} \right)_{\Delta Q^{j-1}} dV \]  \hspace{1cm} (3-17b)

\[
(K_D)_{ij} = \frac{4\rho}{\Delta t^2} \int_{V_0} \phi_k \psi_j dV \]  \hspace{1cm} (3-17c)

\[
(F_I)_k = \int_{A_0} \left\{ S_\alpha \frac{\partial \Delta E_\alpha}{\partial \Delta Q^k} \right\}_{\Delta Q^{k-1}} dA \]  \hspace{1cm} (3-17d)

\[
(F_E)_k = \int_{A_0} \left\{ \phi_k f_q \right\}_{\Delta Q^{j-1}} dA \]  \hspace{1cm} (3-17e)

\[
(F_D)_k = \frac{4}{\Delta t^2} \Delta Q_j^{j-1} - \frac{4}{\Delta t} \ddot{Q}_j - \dot{Q}_j \int_{V_0} \left\{ \phi_k \phi_j \right\} dV \]  \hspace{1cm} (3-17f)
The detailed expressions for stiffness matrices and force vectors given above are expressed in terms of the element coordinates. A transformation from local to global coordinates is therefore necessary before assembly in the global stiffness matrix. The details can be found in Appendix A.

Numerical integrations are carried out for both longitudinal and through-the-thickness directions during the element stiffness and force vector calculations. The through-the-thickness integration is related to the bending terms. As a way of providing an approximation to the complete bending kinematics, the through-the-thickness integration is reduced to a single point quadrature rule. This measure results in significantly reduced computing times, while retaining the key features of the bending model, as will be demonstrated in the discussed examples.

3.4 Coordinate Transformations

Since the current formulation so far dealt exclusively with quantities defined in local coordinates, but the generalized displacements (total and incremental) have to be expressed in terms of the global coordinate system to permit the assembly process, it is necessary to convert the global nodal quantities into the local quantities during the element stiffness and force calculations. While this is a trivial task for the actual nodal displacements, the third nodal unknown $dw/dx$ is properly defined as the tangent to the deflection curve rather than the actual rotation of the node. In a large deformation setting, care has therefore to be taken in order to properly add the incremental rotations at
the nodes and to avoid numerical problems associated with the element becoming vertical and the tangent becoming infinite.

To illustrate the problem, consider an element (IJ) in the reference configuration (cf. Fig.2.1) at time \( t = t_0 \), which deforms to the current configuration (ij) at time \( t = t_0 + \Delta t \). Let \( \alpha \) be the angle between local and global coordinates and the subscripts \( g \) and \( l \) denote the global and local quantities, respectively. Let \( \theta_1 \) and \( \theta_2 \) be the angle of rotation in the reference and current configurations (cf. Fig.3.2), respectively. Because of the large rotation considered here, \( (w_{,x})_g \) is not equal to \( \theta_1 \), but rather

\[
(w_{,x})_g = \tan \theta_1 \tag{3-18}
\]

Also, from Fig.3.2 it can be seen that the nodal rotation can be defined as

\[
(w_{,x})_l = \tan(\theta_1 - \alpha) \tag{3-19}
\]

Therefore, the relation between local and global slopes can be written as

\[
\frac{(w_{,x})_l}{(w_{,x})_g} = \frac{\tan[\tan^{-1}(w_{,x})_g - \alpha]}{(w_{,x})_g} \tag{3-20}
\]

In general, the resulting transformation matrix for the generalized displacements at a node is

\[
T = \begin{bmatrix}
\cos \alpha, \sin \alpha, & 0 \\
-sin \alpha, \cos \alpha, & 0 \\
0, 0, (w_{,x})_l / (w_{,x})_g
\end{bmatrix} \tag{3-21}
\]
Clearly, the transformation of the incremental displacement components $\Delta u$ and $\Delta w$ follows the usual procedure, and the only issue needing further study is the form of the expression for $T_{33}$ governing the transformation of the rotational degree of freedom. Referring to Fig. 3.1, the incremental rotation $\Delta \theta$ is

$$\Delta \theta = \theta_2 - \theta_1$$  \hspace{1cm} (3-22)$$

and remains unchanged regardless of the choice of coordinates. Here $\theta_2$ can be found to be

$$\theta_2 = \tan^{-1}\left[(w_{,x})_g + (\Delta w_{,x})_g\right]$$  \hspace{1cm} (3-23)$$

However, the incremental quantity $\Delta w_{,x}$ changes from global to local coordinate with the angle $\alpha$. The angle $\theta_1$ is a function of $(w_{,x})_g$ only in the reference configuration, while $\theta_2$ is a function of $(w_{,x})_g$ and $(\Delta w_{,x})_g$. But from Fig. 3.1, one can write

$$(\Delta w_{,x})_\perp = \tan[\tan^{-1}\{(w_{,x})_g + (\Delta w_{,x})_g\} - \alpha] - \tan[\tan^{-1}(w_{,x})_g - \alpha]$$  \hspace{1cm} (3-24)$$
Fig. 3.1: Detailed Shell Coordinate Transformation
Finally, the transformation matrix for the generalized incremental displacement quantity $\Delta u$ for a node is

$$
T = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & (\Delta w_{r_x})_i / (\Delta w_{r_x})_j
\end{bmatrix}
$$

Equations (3-21) and (3-25) are the required transformation matrices for the generalized total and incremental displacements at a node from global to local coordinates. Note that the resulting transformation matrices are not orthogonal and the use of the proper inverses rather than the transpose matrices is necessary.

### 3.5 Contact Constraints

For the contact constraints referring to equation (3-15), the gap between the nodes in the sheet and the tool surface must be taken into account. Two different formulations based on the vertical gap $g_v$ and the normal gap $g_n$ will be considered here. For all contacting nodes, the vertical gap between the tool surface and the nodes should be zero to meet geometric compatibility [Saran and Wagoner, 1991]. Therefore,

$$g_v = z^p - z^s = 0$$

where $z^s$ and $z^p = z(x)$ are the $z$-coordinates of a sheet node and the point on the tool surface vertically above or below the node during the current time step. Both quantities are not known at the current iteration. Thus, it is necessary to calculate them iteratively.
after a linearization. Linearization of the gap equation at the (i)th iteration results in an expression of the form:

\[
\begin{bmatrix}
S_x \cdot [-1,0,0] \\
\delta u \\
\delta w \\
\delta w_x \\
\delta p
\end{bmatrix} = (z^\alpha - z^\beta)_{i-1}
\quad (3-27)
\]

where \( S_x \) is the slope of the tool at the contact node and \( \delta u, \delta w, \delta w_x, \delta p \) are global incremental nodal quantities. Note that \( S_x \) is only related to the tool surface geometry, but not to the shape of the sheet.

The constraint equation becomes more complex if a normal gap is employed, which may be defined in the following way:

\[
g_n = n \cdot (x^p - x^s) = 0
\quad (3-28)
\]

Here \( x^s \) and \( x^p \) are position vectors of a sheet node and its nearest point on the tool surface respectively. The nodal positions are unknown quantities, hence an iterative procedure must be employed. Consider a series expansion of the expression for the normal gap using the quantities known in the current equilibrium iteration:

\[
(g_n)_{i-1} + (n_x + n_z S_x) \delta u + n_z \delta w \\
- [(x^P - x^s) + w_x (z^P - z^s)] / D^3 \cdot \delta w_x = 0
\quad (3-29)
\]
In this expression \( n_x \) and \( n_z \) are the \( x \) and \( z \) components of the sheet normal vector \( \mathbf{n} \) and \( i \) is an iteration counter. The coordinates \((x^P, z^P)\) specify the unknown position of a point on the tool surface nearest to the sheet node \((x^S, z^S)\) and \( D = \sqrt{1 + w_x^2} \).

If the punch is assumed to move in the vertical (global \( z \)) direction only, the vertical gap formulation is convenient to apply. However, for a general motion of the tools, the normal gap formulation must be considered.

### 3.6 Consistent Contact Formulation

In areas where the sheet is in contact with the forming tools, external contact forces will be transmitted through the contacting nodes. The model used in the present research is a modified Coulomb friction law proposed by Oden and Pires [1983]. The essence of the model is that the force vector due to external contact forces in equation (3-17e) can be integrated at a node and written as

\[
F_z = p\Gamma \quad \text{and} \quad \Gamma = \mathbf{n} + \mu\mathbf{f}\mathbf{t} \tag{3-30a,b}
\]

where \( p \) is the nodal contact pressure, \( f \) is a smoothing function, \( \mu \) is the friction coefficient, and \( \mathbf{n} \) and \( \mathbf{t} \) are unit normal and tangent vectors at the contact node. No sticking is considered in this model, i.e. the nodes are permitted to slide continuously over the tool surface. The relation between contact forces and node motion is governed by the smoothing function. Two types of smoothing functions \( f \) can be selected in the current formulation: a bilinear ramp function or a hyperbolic tangent function of the tangential displacement \((\Delta u_t)\) at the node,
\[ f(\Delta u_i) = \tanh(31 \Delta u_i / \delta) \] (3-31)

where \( \delta \) is tolerance input value, and essentially prescribes how far the model differs from the traditional stick-slip type algorithm.

The tangent stiffness due to the external force, \( K_E \), can be found by taking the derivative of the contact force with respect to the generalized coordinate variables leading to the following matrix form:

\[
K_e = \begin{bmatrix}
p \frac{\partial \Gamma_x}{\partial x}, & p \frac{\partial \Gamma_x}{\partial z}, & p \frac{\partial \Gamma_z}{\partial z}, & \Gamma_x \\
p \frac{\partial \Gamma_z}{\partial x}, & p \frac{\partial \Gamma_z}{\partial z}, & p \frac{\partial \Gamma_z}{\partial z}, & \Gamma_z \\
0 & 0 & 0 & 0 \\
C
\end{bmatrix}
\]

(3-32)

where \( \Gamma_x \) and \( \Gamma_z \) are \( x \) and \( z \) components of the vector \( \Gamma \) in (3-30b).

Here \( C \) contains the contact constraint equation derived from equation (3-27) or (3-29) as discussed earlier. Note that because no external moments are applied by means of the tool contact, the entries in the third row in eqn. (3-32) are all zero.
CHAPTER
IV
NUMERICAL EXPERIMENTS

In order to verify the capability of the presented formulation, several numerical simulations are presented. The code is programmed in ANSI-standard FORTRAN 77 and was developed and run on a DEC VAX 8550 system of the College of Engineering at The Ohio State University. The standard FORTRAN implementation permits the execution of the code on any hardware or operating system platform and was an important consideration during the development phase.

The computing times for all the problems presented here are in the range of 4 to 12 CPU minutes. This represents a reduction of computing times to approximately one half to one third of the previous version based on four nodal degrees of freedom with full integration.

4.1 Patch Test

A cantilevered beam with end loads (axial and transversal direction) was considered in the first set of patch tests with elastic material law by setting the strain hardening index \( n=1 \). Six test cases were examined; horizontal, vertical and inclined beams having the same length and axial and transverse loads each.
The results are in good agreement with elementary beam solution, regardless of the inclination of the beam, as long as the tip deflection is small.

Another patch test was performed for the cylinder wrap around problem. Sheet is wrapped around a large radius cylinder like Fig.4.1 (a). The strain distribution on the sheet can be found analytically due to simple geometry and must be uniform.

The geometry of the tool and sheet is;

\[ R = 200\text{mm}, l_0 = 60\text{mm}(\text{original sheet length}), t = 1\text{mm}(\text{original sheet thickness}) \]

Boundary conditions due to symmetry of the geometry should be;

\[ u = \frac{dw}{dx} = 0 \quad \text{at } x = 0 \quad \text{and} \quad u = (\frac{dw}{dx})_1 = 0 \quad \text{at } x = 60\text{mm} \]

However, in the simulation \( u = w = 0 \) at \( x = 60\text{mm} \) can be applied as an end boundary condition due to incremental nature of the scheme.

Under the assumption of small sheet thickness, total punch travel can be found from the geometry to be;

\[ PH = R - \sqrt{R^2 - l_0^2} = 9.21216\text{mm} \]

From the geometry, final length of the sheet, \( l \), should be;

\[ l = R \times \sin^{-1}(\frac{l_0}{R}) = 60.9385\text{mm} \]

Thus, midsurface strain can be found to be;

\[ \varepsilon_{\text{mid}} = \ln(\frac{l}{l_0}) = 1.5521\times 10^{-2} \]

Surface strains for top and bottom can also be found by using constant curvature as;

\[ \varepsilon_{\text{top}} = \ln[\frac{1}{l_0}(1 + \frac{t}{2R})] = 1.8018\times 10^{-2} \]

\[ \varepsilon_{\text{bottom}} = \ln[\frac{1}{l_0}(1 - \frac{t}{2R})] = 1.3018\times 10^{-2} \]
Fig. 4.1: Wrapping Around the Cylinder - (a) Geometry, (b) Strain Distribution
Fig. 4.1 (b) shows the strain distribution by the simulation which are in good agreement with the analytical calculations. Slight kink at the end of the sheet in the result can be explained that improper boundary condition is imposed on the last node due to incremental solution scheme.

4.2 Comparison with OSU Benchmark Test

The first problem considered was introduced by Lee et al. [1990] and served as a benchmark test of several programs suitable for the analysis of sheet metal forming processes. An interesting comparison between the results of these programs was made for simple plane strain and axisymmetric problem geometries involving stretching and drawing problems.

The problem geometry is shown in Fig. 4.2 and 4.3. For stretching, the sheet is rigidly clamped at the die radius corner, in the drawing cases a restraining force is applied to the sheet periphery over the flat portion of the die. The dimensions were given to be \( R_p = 50.8 \text{mm}, \ R_d = 6.35 \text{mm}, \ C_d = R_0 = 59.18 \text{mm} \) for the stretch problem and \( R_0 = 80 \text{mm} \) for the draw case and the sheet thickness was \( h = 1.0 \text{mm} \).

The material properties used are as follows:

- Anisotropy & Yield Function: \( r = 1.0, M = 2.0 \)
- Hardening Law: \( \bar{\sigma} = K(a_0 + \bar{\varepsilon})^m \left( \frac{\bar{\varepsilon}}{Y} \right)^n \)

where \( K = 589 \text{ MPa}, a_0 = 0.0001, m = 0.0, n = 0.216, \gamma = 1.0 \)
Fig. 4.2  Geometry of Benchmark Problem for Plane Strain Stretching and Drawing with Cylindrical Punch.
Fig. 4.3: Tool Geometry for Benchmark Plane Strain Stretch/Draw
The results for this plane strain stretching case are compared with the elementary solution obtained by Wenner [1990] and the analytical solution based on membrane theory for the frictionless case in Fig. 4.4. The dotted line represents the analytical solution without friction. A good agreement is found between Wenner's and the present results.

The second problem considered is a drawing operation using the same geometry and material properties as in the previous problem. The results are compared in Figs. 4.5 and 4.6 with Wenner's solution and results obtained by Saran and Wagoner [1991] using a membrane-based finite element program. Shown in Fig. 4.5 are the strain distributions at a punch height of 20 mm, and all programs are seen to yield very comparable results. The results obtained with the bending model using the three nodal DOF model and one point integration are closer to the membrane solution than the fully integrated bending model. The punch height of 40 mm is attained only by the reduced one point integration scheme for the bending code, as shown in Fig. 4.6. Punch force vs. displacement curve is shown in Fig. 4.7. The other two versions of the bending code, i.e. the three and four nodal DOF models with full integration experience convergence problems at a punch height of approximately 34 mm and fail.
Fig. 4.4: True Strain Distributions for Benchmark Plane Strain Stretch at Punch Height = 20mm
Fig. 4.5: True Strain Distributions for Benchmark Plane Strain Draw with $f=0.3$ at Punch Height $= 20\text{mm}$
Fig. 4.6: True Strain Distributions for Benchmark Plane Strain Draw with \( f = 0.3 \) at Punch Height = 40mm
Fig. 4.7: Punch Force-Displacement Curve for Benchmark Plane Strain Draw
4.3 Cylindrical Punch Stretch

The problem geometry is same as the Benchmark problem shown in Fig.4.2 with the slight changes in dimension. The test case was for stretching only. The dimensions of the tools were given to be $R_p=50.8\text{mm}$, $R_d=6.35\text{mm}$, $C_d=59.18\text{mm}$ and $R_0=62.5\text{mm}$ for the case. The sheet thickness was $h=1.0\text{mm}$ and the width was $60\text{mm}$. The material properties of the AKDQ steel used are as follows:

Anisotropy &

Yield Function

\[ r = 1.95, \quad M = 2.0 \]

Hardening Law:

\[ \bar{\sigma} = K(a_0 + \bar{\varepsilon})^m \left( \frac{\bar{\varepsilon}}{\gamma} \right) \]

where

\[ K = 617.6 \text{ MPa}, \quad a_0 = -0.000846, \]

\[ m = 0.017165, \quad n = 0.263, \quad \gamma = 1.57018 \]

Young' modulus

\[ E = 206000 \text{ Mpa} \]

The results for this plane strain stretching case are compared with the formability test results by Saunders et al.[1991] in Fig.4.8. The dotted line represents the current solution. Even though the experiment was for cylindrical punch stretch with the plane strain condition, the measured strain distribution look like axisymmetric results. The width to thickness ratio was big enough so that plane strain condition could be assumed, however the results resembles the axisymmetric results as the punch travel increases. The current solution catches the peak to some extents and further investigation is necessary to get the fair solution. Punch force-displacement curve is shown in Fig.4.9.
Fig. 4.8: Strain Distributions for the Cylindrical Punch Stretch
Fig. 4.9: Punch Force-Displacement Curve for the Cylindrical Punch Plane Strain Stretch Problem
While in the previous problem the results of the current model are compared with results of other programs and an analytical solution, the present problem was solved in an effort to compare numerical results with experimental data. The experimental data was obtained by Wang and Wagoner [1990] and involved the deep drawing of metal strips using a flat bottom punch (see Fig. 4.10). The material properties are as follows:

- Anisotropy & Yield Function: \( r = 1.7, M = 2.0 \)
- Hardening Law: \( \bar{\sigma} = K(a_0 + \bar{\varepsilon})^m \left( \frac{\bar{\varepsilon}}{\gamma} \right)^n \)

where \( K=598 \) MPa, \( a_0=0.0001 \),
\( m=0.013, n=0.23, \gamma=1.0 \)

The thickness of the strips was 1mm and a friction coefficient of 0.2 was chosen. The geometry has the dimensions \( R_p=7.14 \) mm, \( R_d=6.35 \) mm, \( C_p=30.96 \) mm, \( C_d=59.35 \) mm, \( R_0=75 \) mm and the test set up is shown in Fig. 4.11. The blank holder pressure of 200 psi was converted to a constant draw bead force of 74.2 kN/m.

This geometry presents some difficulties to obtain a convergent solution because of the possible loss of contact in the flat region of the punch. As the punch advances, the sheet over the flat region of the punch tends to bend, confining the contact region between punch and sheet to a few nodes near the punch corner. This separation can be observed during an actual experiment and was modeled accurately by the bending model employed here. The code has restarting capability so that time stepping can be controlled through restart option and simulation can, thus, reach more punch travel with controlling on the input parameters.
Fig. 4.10: Plane Strain Strip Stretching and Drawing Geometry with Flat Bottom Punch
Fig. 4.11: Tool Setup for Plane Strain Strip Draw with Flat Bottom Punch
Fig. 4.12  Engineering Strain Distributions for Strip Plane Strain Draw with $f=0.2$, BHF=200psi, PH=15mm at the Mid Surface
Fig. 4.13  Engineering Strain Distributions for Strip Plane Strain Draw with $f=0.2$, BHF=300psi, PH=15mm at the Mid Surface
The resulting strain distributions are compared with experimental data by Wang and Wagoner [1990] in Fig.4.12 and Fig.4.13 for different blank holder pressures. The bending result clearly indicates the peak strain points accurately near the die and punch corners and is in good agreement with experimental measurements.

4.5 Plane Strain Draw and Springback

In typical sheet metal forming operations, the sheet conforms closely to the tool surfaces during the stamping process. Upon removal of the press load, however, the residual elastic stresses present in the sheet will force the deformed sheet to partially recover from the forming process and change its shape. This shape change due to elastic recovery is termed springback. A very simple illustration of springback occurring in the deep-drawing of a channel section is shown in Fig.4.14. The solid line indicates the shape of the channel at the end of the press stroke, i.e. at bottom dead center, while the shaded lines indicate the part shape after the partial relaxation due to springback.

![Diagram of springback in cup or channel forming](image)

**Fig.4.14: Springback in Cup or Channel Forming**

It can be seen that the actual part shape deviates from the desired shape in several respects. Beside the corner radii being too large, the wall angle will not be as steep as
desired and the cup or channel will not have the desired height or width. Also, it is often found that instead of being straight, the walls have a slight curvature - this phenomenon is commonly termed side-wall curl. Each one of these springback features is detrimental to the quality and the ease of assembly of the formed parts and it is therefore essential for the die designer to control part springback.

The test case for this investigation, a plane strain draw case with flat bottomed punch, was chosen from the examples discussed by Tang [1987]. A view of the geometry is shown in Fig. 4.15. Only the right half of the symmetric view is shown.

The initial sheet thickness was 1 mm and the sheet material was assumed to be a typical mild steel with the following material parameters:

- Anisotropy & Yield Function: \( r = 1.0, M = 2.0 \)
- Hardening Law: \( \bar{\sigma} = K(a_0 + \bar{\varepsilon})^m \left( \frac{\bar{\varepsilon}}{\gamma} \right)^n \)

where

- \( K = 482.6 \text{ MPa}, a_0 = 0.0001, \)
- \( m = 0.0, n = 0.2, \gamma = 1.0 \)

and

- Young's Modulus: \( E = 207000 \text{ MPa} \)
- Poisson's Ratio: \( \nu = 0.3 \)
- Yield Strength: \( \sigma_y = 106 \text{ MPa} \)

The strain distributions at different punch heights for a restraining force of 250 kN/m are shown in Fig. 4.16, which indicate the presence of appreciable bending strains around the die corner.

From the figure, it is noted that current coordinate representation gives more meaningful information on the strain distribution of deep drawing problem.
Fig. 4.15: Geometry Change along the Punch Travel for Springback Test Case
Fig. 4.16: True Strain Distributions for the Geometry of Springback Test Case
Original coordinate representation can not give proper position information of peak strain, while current coordinate representation gives the exact position of peak strain which is at the die corner toward the free region. Thus, it is better to get the strain output in terms of current coordinate system for deep drawing problem.

4.6 Brake Bending Simulation

Brake bending or air bending is quite popular forming process in industry. The process is very simple but it gives good measure on sheet bendability. A 1mm thick flat sheet is placed over the die and deformed with cylindrical punch into a V-shaped channel. The tool geometry and sheet setting is shown in Fig.4.17. The dimensions are \( R_p = R_d = 10 \text{mm}, \ C_d = 50 \text{mm} \) and \( R_0 = 60 \text{mm} \). The material properties of the sheet are same as those of benchmark case by Lee et al.[1990]. Friction coefficient was assumed to be 0.1.

The deformed geometry at punch height of 10, 20, 30 and 40mm are shown in Fig.4.18. Along the punch movement, many part of sheet are moved into the die so that many part deform elastically. Thus lots of springback is expected on this model. Comparison was made with old version of the code[Choudhry, 1990], however, the deformed geometry were exactly the same for both implementations.

Punch force distribution is plotted against the punch displacement in Fig.4.19. Oscillations in the punch force are observed due to the dynamic effect of the formulation. Toward the end of simulation lots of oscillation occurred due to convergence problem and program stopped. Thus, further investigation is necessary on this problem.
Fig. 4.17: Tool Geometry for the Brake Bending Operation
Fig. 4.18: Deformed Sheet Geometry along the Punch Displacement for the Brake Bending Operation
Fig. 4.19: Punch Force-Displacement Curve for the Brake Bending Operation
Punch force rises steadily in the earlier stage and becomes constant. This is due to the dragging-in of the sheet into the die cavity without further plastic deformation. In this stage, most span of the sheet deform elastically. This brake bending is like almost pure bending operation, so that membrane simulation can not catch the result at all.
5.1 Inclined Sheet Stretching

An automotive inner panel stamping operation is considered here to demonstrate the applicability of the code to complex industrial problems. An initially inclined ("tipped") tool and sheet geometry is shown in Fig. 5.1 along with the Simulation results and measured data.

The material properties for AKDQ steel sheet are as follows:

Anisotropy & Yield Function:

\[ r = 1.46, \quad M = 2.0 \]

Hardening law:

\[ \sigma = K(a_0 + \bar{e})^{n \left( \frac{\bar{e}}{\gamma} \right)^m} \]

where

\[ K = 503.4 \text{ MPa}, \quad a_0 = 0.38, \]

\[ m = 0.0038, \quad n = 0.196, \quad \gamma = 1.0 \]

The measured thinning strains were provided by N.M. Wang through a private communication. The membrane simulation was performed by Keum et al. [1990]. All results are in good agreement with the measured data.
Fig. 5.1: Geometry and Thinning Strain Distributions for an Automotive Sheet Panel Stretch Operation at PH=15.84mm
Fig. 5.2: Punch Force-Displacement Curve for SECBB
Since the thinning is an average engineering quantity and largely due to stretching, the bending results are very similar to the membrane results. Punch force vs. punch displacement curve is shown in Fig.5.2. The result indicates the applicability of the current formulation to the simulation of industrial forming operations.

Several verification computations reveal that the equilibrium iterations (Newton-Raphson method) fails to converge when the force-displacement curve starts to decrease. This phenomenon, which is commonly called softening behavior, occurs when the strain reaches the n-value. At that stage the positive definiteness of the stiffness matrix is lost and consequently any implicit method using Newton-Raphson iteration methods will suffer from serious convergence problems.

5.2 Forming with Tools having Arbitrary Geometry

Saran et al. [1990] suggested a concave tool to demonstrate the applicability of the simulation code to the industrial application. The problem designed is that both sides have different draw-in conditions and tools are horizontally set. Tools setting and sheet blank are shown in Fig.5.3. The material properties for AKDQ steel sheet are as follows:

Anisotropy & Yield Function: \( r = 1.46, M = 2.0 \)

Hardening law: \( \bar{\sigma} = K(a_0 + \bar{\varepsilon})^n \left\{ \frac{\bar{\varepsilon}}{\gamma} \right\}^m \)

where

\( K = 503.4 \) MPa, \( a_0 = 0.38, \)

\( m = 0.0038, n = 0.196, \gamma = 1.0 \)

First simulation is made for stretch case in which both side of tools are clamped. The results are in Fig.5.3 along with tool setups.
Fig. 5.3: True Strain Distribution for Plane Strain Stretch with Complex Tool Geometry
Even though successful simulation was made for stretch case, draw case gives convergence problem in the early stage due to unstable boundary conditions given and reverse drawing process occurrence during punch travel. Further investigation is necessary for the draw problem with this geometry.
CHAPTER VI
BENDING EXPERIMENTS

6.1 Introduction

Sheet metal forming operation requires lots of tryout before doing the actual operation. This leads us to reduce the relating costs by replacing the die tryout to computer simulation. Computer simulation, however, requires lots of verification process before applying it to the actual forming process simulation. To do such verification, experimental data or exact theoretical solutions are required. For the bending code which is currently under developed and refined stage, bending experiments consisting of air bend operation have been carried out. The air bend operation is one of major processes in the sheet forming operation and it is simple to set up an experiment.

To verify a simulation code, such data as strain histories and springback information is crucial. Some people did the bending experiment, but mostly restricted to punch force-displacement comparison. A simplified springback calculation method was suggested by Ragnupathi et al.[1983] using simplified theory in bending. This method is worth to be compared with experiment and simulation. However, no strain history data are available for the comparison of simulation.
Thus, this bending experiments were carried out to provide all the necessary information to verify simulation codes. The bending experiments are very simple in experimental point of view, but it is extremely difficult to simulate the process with FEM code. The process has a point or rather line contact throughout the process which results severe numerical instability. This process has been simulated for different tools and sheet dimensions by Choudhry et al.[1989] but it lacks comparison with other verifiable data because no data was available on the specific simulation. Springback simulation was carried out by Lee et al.[1990a] but it also requires further comparative study with verifiable data.

An attempt was made to simulate the bending experiments with the code developed in this research. However, due to the nature of rigid-plastic nature of the code, serious convergence problem occurred during the simulation. Thus comparison were made with other simulations including new elastic-viscoplastic formulation of the bending code [Hambrecht, 1993].

This bending experiments provide material properties of the test specimen, strain histories, springback information, calculation method for Coulomb friction coefficient from the bend tests, bend geometry and some mechanics consideration etc.

6.2 Test Material

In order to exaggerate the problems with elastic deformation in the bending operation, two high strength aluminum alloys were chosen. In the following, they are referred to as material A and B.
The alloys were

Material A : aluminum alloy 2024 T3 (t=2.5mm)
Material B : aluminum alloy 7075 T7651 (t=6.35mm)

Bend test specimen, 25.4mm wide, were cut from a 2.5mm thick sheet (material A) and from a 6.35mm thick cold drawn bar (material B). Also standard tensile test specimen were cut from the same materials.

The tensile tests, performed in a MTS tensile testing machine, with a strain rate of 10^{-3}/s provided the following effective stress strain curves.

Material A \( \bar{\sigma} = 40 + 840\varepsilon^{0.25} \) MPa \( (6-1) \)
Material B \( \bar{\sigma} = 837\varepsilon^{0.11} \) MPa \( (6-2) \)

The functions for the hardening law (equations 6-1 and 6-2) were not appropriate for the BEND simulation code [Wang 1993] so that those are regenerated through curve fitting as follows.

Material A \( \bar{\sigma} = 879.5(0.0001 + \varepsilon)^{0.2} \) MPa \( (6-3) \)
Material B \( \bar{\sigma} = 836(0.00001 + \varepsilon)^{0.11} \) MPa \( (6-4) \)

For elastic calculations, standard values for Young's modulus and Poisson's ratio can be used for both materials.

\( E = 87 \) GPa \( \nu = 0.33 \)

With these data, fairly large elastic strains are involved. Roughly, the calculated yield stresses for the two materials are

Material A \( Y \sim 305 \) MPa
Material B \( Y \sim 475 \) MPa
giving the yield strains

Material A $\varepsilon_y \sim 0.35\%$
Material B $\varepsilon_y \sim 0.75\%$

At a total strain of 0.1(10%), a strain that could become exceeded in the bend tests, the elastic part of the strain for the materials is as large as

Material A $\varepsilon_e \sim 0.65\%$
Material B $\varepsilon_e \sim 0.75\%$

Thus in the following bend test, a relatively large portion of the deformation is elastic. To have a better comparison of the simulation to the experiment, it is necessary to utilize the bending simulation program with elastic-plastic or elastic-viscoplastic finite element model.

6.3 Bending Tool

A suitable bending tool was borrowed from the department of Materials Science and Engineering. It consists of an open frame with two fixed 1" round bars with a distance of 4". It was used as a die and a special adapter to the MTS machine holding a round steel pin of 1/2" diameter was used as a punch. The main dimensions and condition of the tool are sketched in Fig. 6.1.

Real industrial bending operation has the die opening to thickness ratio (W/t value) of 4 ~ 8. However in this bending experiment W/t values are:

Material A $W/t = 30$
Material B $W/t = 12$
Max Punch Stroke = 2"

Dry Friction Condition

Fig. 6.1: Bend Test Schematics and Dimensions
Thus this test does not resemble the actual bending operation, however it gives some trend of the bending operation along with valuable data base on strain history and springback. This big $W/t$ value means that many part of the sheet will bend elastically so that lots of springback can be expected. Actual bending application requires small springback so that $W/t$ value and the clearance, which is half of die opening minus punch radius, need to be as small as possible. In this experiment, however, the intention is that springback is to be big enough, thus many part of the sheet deforms elastically.

The bending ratios ($R/t$ ratio which determines sheet bendability) with this tool are:

- Material A $R/t = 1.88$
- Material B $R/t = 0.75$

Recommended bending ratio in the sheet forming industry is for the soft material which has low $n$ value ranging between one to two and for hard material it is larger than two. For material A the ratio looks reasonable. However for material B, the bending ratio is too small, so that earlier failure due to localized bending was expected.

The main reason for choosing this tool was that at a bending angle of 90 degree the limited ductility of material B would lead to fracture, which actually occurred on one of the specimens.

6.4 Preparation of the Test Specimen

All surfaces of the test specimens were carefully sanded and polished using fine emery paper. The final polishing was carried out in the longitudinal direction. This
treatment ensures that there are no scratches on the surface perpendicular to the bending strain direction. Such scratches are known to affect the strain distribution dramatically.

Some of the specimens were prepared for measurement of the strain distribution by means of a square photo grid. The intention was to use a fine grid with a grid size of 0.5mm, but problems with the photoresist available in combination with the aluminum material (too sensitive to the exposure time and variations in photoresist thickness) made it necessary to use a 2.5mm grid. Kepro Bench Top Coater (BTC-101) and related coating equipment in MSE department were used for photo grid coating.

The rest of the specimens were equipped with 3 high strain strain gages (M&M, EP-08-062AK-120 type) with a measuring length of 3.2mm. This particular type of strain gage has approximately strain limits of 20% for both tension and compression. They were glued in such a way that one gage was placed in the center of the bend specimen on the outside (bottom surface) and the two others displaced 5.5mm from the center on each side of the specimen. Only 3 strain gages were used for a specimen because the recording device has only 4 channels for the input.

The strain gages were calibrated in a separate tensile test. The calibrated results are shown in Fig.6.4 giving good linear relationship between voltage readout and measured strains. This strain gage calibration chart was obtained by relating the two measured data (Fig.6.2 and 6.3) which come from the extensometer calibration and the strain gage readout in relation to the extensometer readout.

Test specimens were named in such a way that A and B denote different material types. S and G stand for strain gauged and grid coated specimens respectively. Summary of the test specimen used are in Table 6.1.
Fig. 6.2: Strain Gage - Extensometer Calibration Chart

\[ Y = 0.032237 + 9.17249 X \]
Fig. 6.3: Extensometer Calibration Chart

\[ Y = 0.001537 + 0.19423 X \]
Fig. 6.4 Strain Gage Calibration Chart
Table 6.1 Summary of Aluminum Alloy Test Specimen

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>2024T3</td>
<td>7075-T7651</td>
</tr>
<tr>
<td>Thickness</td>
<td>2.5 mm</td>
<td>6.35 mm</td>
</tr>
<tr>
<td>Width</td>
<td>25.4 mm</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>Length</td>
<td>200 mm</td>
<td>200 mm</td>
</tr>
<tr>
<td>Hardening Law</td>
<td>$\bar{\sigma} = 40 + 840\varepsilon^{0.205}$</td>
<td>$\bar{\sigma} = 837\varepsilon^{0.11}$</td>
</tr>
<tr>
<td>No. of Specimen</td>
<td>2 Strain Gaged (AS1, AS2)</td>
<td>2 Strain Gaged (BS1, BS2)</td>
</tr>
<tr>
<td></td>
<td>3 Grid Coated (AG1, AG2, AG3)</td>
<td>2 Grid Coated (BG1, BG2)</td>
</tr>
</tbody>
</table>
6.5 Test Equipment

The bend tests were performed in a MTS test machine equipped with a 5 metric ton load cell at the Strength of Material Lab in the Engineering Mechanics Department. The signals from the MTS machine (load and stroke) and from the strain gages were sampled and stored on a digital oscilloscope (Nicolet 4094 type having 4 channel digital recorder) through 10 channel strain gage conditioner. One channel for punch displacement and other three channels for strain output were used for the recording of the strain gauged specimen.

For gridded specimen, two channels were used for recording (one for punch force and the other for punch displacement). The load-stroke curve was also monitored on an ink pen plotter simultaneously and stored on an IBM PC using the L4000 digital recorder as an A/D converter. Equipment sketch is shown in Fig.6.5.

6.6 Test Performance

The bend tests were performed in the following way.

A low punch speed of 0.1mm/s was chosen so that the maximum strain rate during bending was the same as during the tensile tests.

After a punch stroke of around 10mm, the punch was stopped and the punch force kept constant and a photo was taken with a die objective (Fig.6.6 and 6.8). Then the punch travel was reversed and new photo was taken when the punch force was zero after complete springback (Fig.6.7 and 6.9).
Fig. 6.5: Equipment Sketch and Line up
Fig. 6.6: Bend Test Loading Process (PH=38mm, Material A)

Fig. 6.7: Bend Test after Springback (PH=38mm, Material A)
Fig. 6.8: Bend Test Loading Process (PH=22mm, Material B)

Fig. 6.9: Bend Test after Springback (PH=22mm, Material B)
This procedure was repeated at different punch strokes until a bend angle of approximately 90 degree was obtained after springback. This angle could not be reached with the thick specimens (material B) because the material was too brittle. Some specimens were bent to a lower final angle. They were used for the evaluation of the strain distribution.

6.7 Friction During Bending

Friction between the sheet and the die has an influence on the bending force which makes it possible to calculate the Coulomb friction coefficient from our bend tests. When the stroke of the MTS machine was reversed in order to obtain springback data, the load dropped immediately to a lower value even before a return stroke could be detected. This drop is due to the fact that the friction force changes its direction. During bending, more material is drawn into the die and the sheet material is sliding over the die corner. When the stroke of the punch is reversed, the elastic energy stored in the bend forces the bend to spring back and the material to move out of the die. The friction force changes direction at that time.

As a first approximation, it is assumed that this change occurs with zero movements which were identified as $F_1$ and $F_2$ in the punch force-displacement curves like Fig.6.10 and 6.11. Thus the form of the bend is unchanged as well as the bending moment in the bend.
Fig. 6.10: Punch Force-Displacement Curve for Material A
Fig. 6.11: Punch Force-Displacement Curve for Material B
The bending moment in the middle of the bend at the punch can be calculated from
\[ M = N \cdot x + \mu N \cdot y \]  \hspace{1cm} (6-5)
where \( x \) is the tangential distance of the punch contact point from the die contact point and \( y \) is the normal distance from the die contact point to the punch contact point as defined in Fig.6.12.

According to the following Fig.6.12, subscript 1 in the punch force vector refers to bending and subscript 2 to springback. The term \( \mu y \) is much smaller than \( x \), so let us neglect it. Therefore the contact force \( N \) can be regarded as constant just at the load speed reversed. Then the two following equations can be derived.
\[ F_1 = 2(N \cos \frac{\alpha}{2} + \mu N \sin \frac{\alpha}{2}) \]  \hspace{1cm} (6-6)
\[ F_2 = 2(N \cos \frac{\alpha}{2} - \mu N \sin \frac{\alpha}{2}) \]  \hspace{1cm} (6-7)
where \( \alpha \) is the bend angle before springback.

The sum and the difference of the equations results in the following friction coefficient form
\[ \mu = \frac{F_1 - F_2}{F_1 + F_2} \cot \frac{\alpha}{2} \]  \hspace{1cm} (6-8)

More accurate calculation was carried out by considering the geometry of the bend specimen. From the Fig.6.12, the term \( x \) and \( y \) can be calculated by observing the geometry as:
\[ x = \sqrt{a^2 + b^2} \cos \beta \quad \text{and} \quad y = \sqrt{a^2 + b^2} \sin \beta \]  \hspace{1cm} (6-9a,b)
LOADING

\[ \alpha = \text{Bend Angle} \]
\[ \Delta \alpha = \text{Springback Angle} \]
(Changes in Bend Angle)

UNLOADING STARTED

Fig. 6.12: Definition of Bend Angle and Springback Angle and Direction Changes in Friction Force Before and After Unloading Started
where,
\[
\begin{align*}
a &= c - r \cdot \sin \frac{\alpha}{2} , \\
b &= h + t - r \cdot \left(1 - \cos \frac{\alpha}{2}\right) \\
\text{and} \quad \beta &= \frac{\alpha}{2} - \tan^{-1} \left\{ \frac{b}{a} \right\}
\end{align*}
\]

Here, \(c\) is a half of the die center-to-center distance, \(h\) is punch travel height, \(t\) equals to sheet thickness and \(r\) is die radius.

From Fig. 6.12 moment and force equilibrium gives three equations with three unknowns \(N_1, N_2\) and \(\mu\). Solving for \(\mu\) gives the quadratic equation of the form:

\[
\mu^2 + \frac{F_1 + F_2}{F_1 - F_2} \left(\frac{x}{y} - \cot \frac{\alpha}{2}\right) \mu - \frac{x}{y} \cot \frac{\alpha}{2} = 0
\]

which gives \(\mu\) as:

\[
\mu = -\frac{1}{2} \left(\frac{F_1 + F_2}{F_1 - F_2}\right) \left(\frac{x}{y} - \cot \frac{\alpha}{2}\right) + \frac{1}{2} \sqrt{\left(\frac{F_1 + F_2}{F_1 - F_2}\right)^2 \left(\frac{x}{y} - \cot \frac{\alpha}{2}\right)^2 + 4 \frac{x}{y} \cot \frac{\alpha}{2}}
\]

Assumption of small "\(\mu y\)" on the equation (6-11) results in the same equation of (6-8). Thus, the approximate equation (6-8) also can be used for a quick calculation of the Coulomb friction coefficient. Detail calculation was carried out using both equations (6-8) and (6-12) and the results are compared on tables 6.4 and 6.5 for each material respectively. The table shows similar Coulomb friction coefficient values between the two methods. Even though the results from the accurate equation (6-12) gave slightly higher value of friction coefficient which could be more accurate, equation (6-8) is more practical in real field applications because the equation includes only one geometric parameter \(\alpha\) and it can easily be found by the on-site measurement or by the calculation of the deformed geometry.
The friction data from these simple experiments are consistent. For each type of aluminum alloy and surface condition, the friction coefficient is astonishingly constant and the difference between a naturally oxidized surface and a gridded surface (with exposed and developed photoresist) is revealed. Gridded surface results in higher friction coefficients in both materials. This is due to the surface treatment during photoresist coating process.

In these experiments, the contact pressure is very low as well as the sliding velocity. Under these conditions, an unlubricated clean but oxidized aluminum surface in contact with a steel punch seems to generate a friction coefficient of $\mu \approx 0.18$. Details are on Tables 6.2 and 6.3. Much more data is needed to determine the $\mu$ value with good accuracy.

From the tables 6.2 and 6.3, it is noted that higher friction coefficient is obtained in material A comparing to material B. The reason is because material A has more elastic deformation so that comparatively high springback forces are acting on the die surface during the process. This elastic forces result more secure contact with the die, thus causing more friction during the deformation process. Therefore, the case of material A reveals higher Coulomb coefficient of friction comparing to the case of material B.
Table 6.2 Friction Coefficients ($\mu$) for Material A

<table>
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<tr>
<th>Test Specimen</th>
<th>Angle (deg)</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$\mu$ by (6-12)</th>
<th>$\mu$ by (6-8)</th>
<th>Remarks</th>
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<tr>
<td>AS1</td>
<td>34.5</td>
<td>32.0</td>
<td>29.0</td>
<td>0.16</td>
<td>0.16</td>
<td>Natural</td>
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<tr>
<td></td>
<td>57.3</td>
<td>35.0</td>
<td>26.8</td>
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<td>Surface</td>
</tr>
<tr>
<td></td>
<td>86.4</td>
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<td>21.4</td>
<td>0.20</td>
<td>0.18</td>
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<tr>
<td></td>
<td>111.0</td>
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<td>13.3</td>
<td>0.19</td>
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</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
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<td>0.17</td>
<td></td>
</tr>
<tr>
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<td>0.14</td>
<td></td>
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<tr>
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<td></td>
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<tr>
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<td></td>
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<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Test Specimen</td>
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<td>F₁</td>
<td>F₂</td>
<td>μ by (6-12)</td>
<td>μ by (6-8)</td>
<td>Remarks</td>
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<tr>
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<td>55.2</td>
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<td>54.0</td>
<td>0.18</td>
<td>0.17</td>
<td>Gridded Surface</td>
</tr>
<tr>
<td>BG2</td>
<td>66.3</td>
<td>60.2</td>
<td>46.0</td>
<td>0.22</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td>0.20</td>
<td>0.19</td>
<td></td>
</tr>
</tbody>
</table>
6.8 Results and Discussion

All original test data are saved on diskettes. Punch force displacement curves for material A and B are shown in Fig.6.10 and 6.11 respectively. Here, punch force is the load divided by the width of the specimen. From the typical curve of material A (Fig.6.10), it is observed that the sharp drop of punch force immediate after unloading ($F_1$ and $F_2$ in Fig.6.10) exists even before punch movement detected. These data was used to estimate the friction as discussed before.

Definition of bend angle and springback angle used is on Fig.6.12. This definition gives actual angle of the bent while Ragnupathi et al.[1983] used one half of the actual bend angle.

Bend angles before springback and after springback are plotted for punch stroke on Fig.6.13 and 6.14 respectively. Those angles were measured from photographs (e.g. Fig.6.6) with the microscope. Both results for material A and B coincide well as expected because the bend angle before springback largely depends on the geometry of the tool setting (Fig.6.13). From the Fig.6.13, it is noted that geometric nonlinearity comes into play after certain punch height of around 20mm and bend angle of around 45 degree. However in Fig.6.14, one can recognize the dependency of both material and geometry appears on bend angle after springback to punch displacement curve especially for material A. If the curve is extrapolated to zero bend angle, complete elastic unloading punch displacement can be found; e.g., upto around 6mm punch travel for material A and upto around 3mm punch travel for material B, deformation is completely elastic (see Fig.6.14). Punch displacement vs. bent angle and springback angle data are shown in Table 6.4 and 6.5 for material A and B respectively.
Fig. 6.13: Bend Angle before Unloading vs Punch Displacement Curve
Fig. 6.14: Bend Angle after Unloading vs. Punch Displacement Curve
Table 6.4 Punch Displacement vs. Bend Angle and Springback Angle Data for Material A

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Punch Displ. (mm)</th>
<th>Angle before Unloading (deg)</th>
<th>Angle after Unloading (deg)</th>
<th>Springback Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS1</td>
<td>12.4</td>
<td>34.5</td>
<td>14.9</td>
<td>19.6</td>
</tr>
<tr>
<td></td>
<td>34.0</td>
<td>86.4</td>
<td>61.8</td>
<td>24.6</td>
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<tr>
<td></td>
<td>46.7</td>
<td>111.0</td>
<td>82.5</td>
<td>28.5</td>
</tr>
<tr>
<td>AS2</td>
<td>6.1</td>
<td>17.1</td>
<td>1.5</td>
<td>15.6</td>
</tr>
<tr>
<td></td>
<td>15.6</td>
<td>43.0</td>
<td>22.7</td>
<td>20.3</td>
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<td></td>
<td>25.1</td>
<td>66.7</td>
<td>44.2</td>
<td>22.5</td>
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<tr>
<td></td>
<td>28.3</td>
<td>74.0</td>
<td>50.8</td>
<td>23.2</td>
</tr>
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<td></td>
<td>37.8</td>
<td>93.8</td>
<td>68.2</td>
<td>25.6</td>
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<tr>
<td></td>
<td>44.2</td>
<td>105.1</td>
<td>78.1</td>
<td>27.0</td>
</tr>
<tr>
<td></td>
<td>50.5</td>
<td>116.0</td>
<td>86.3</td>
<td>29.7</td>
</tr>
<tr>
<td>AG1</td>
<td>8.5</td>
<td>24.1</td>
<td>5.7</td>
<td>18.4</td>
</tr>
<tr>
<td></td>
<td>18.8</td>
<td>51.3</td>
<td>30.1</td>
<td>21.2</td>
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<td></td>
<td>30.2</td>
<td>78.5</td>
<td>54.9</td>
<td>23.6</td>
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<tr>
<td></td>
<td>42.9</td>
<td>102.0</td>
<td>75.3</td>
<td>26.7</td>
</tr>
<tr>
<td>AG2</td>
<td>31.5</td>
<td>81.1</td>
<td>57.3</td>
<td>23.8</td>
</tr>
<tr>
<td>AG3</td>
<td>18.8</td>
<td>51.0</td>
<td>30.0</td>
<td>21.0</td>
</tr>
</tbody>
</table>
Table 6.5 Punch Displacement vs. Bend Angle and Springback Angle Data for Material B

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Punch Displ. (mm)</th>
<th>Angle before Unloading (deg)</th>
<th>Angle after Unloading (deg)</th>
<th>Springback Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS1</td>
<td>9.4</td>
<td>25.8</td>
<td>14.4</td>
<td>11.4</td>
</tr>
<tr>
<td></td>
<td>14.0</td>
<td>37.6</td>
<td>25.1</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>21.2</td>
<td>56.8</td>
<td>43.8</td>
<td>13.0</td>
</tr>
<tr>
<td>BS2</td>
<td>15.7</td>
<td>44.1</td>
<td>31.5</td>
<td>12.6</td>
</tr>
<tr>
<td></td>
<td>22.0</td>
<td>57.1</td>
<td>44.1</td>
<td>13.0</td>
</tr>
<tr>
<td></td>
<td>28.4</td>
<td>71.1</td>
<td>58.3</td>
<td>12.8</td>
</tr>
<tr>
<td>BG1</td>
<td>18.3</td>
<td>48.8</td>
<td>36.0</td>
<td>12.8</td>
</tr>
<tr>
<td>BG2</td>
<td>25.9</td>
<td>66.3</td>
<td>53.3</td>
<td>13.0</td>
</tr>
</tbody>
</table>
In the process of bending operation, usually it is important to know how far the punch should go to get desired bend angle which is the angle after springback. Thus for manufacturing point of view, the correlation between bend angle after springback and punch displacement (see Fig.6.14) is more useful.

Springback displacement vs. punch displacement curve is shown on Fig.6.15. Bend angle before and after springback curve and springback angle vs. bend angle curve are shown on Fig.6.16 and 6.17 respectively. To make a final product of bend angle 60 degree, for example, material A requires to bend upto 85 degree while material B requires only 73 degree bending (see Fig.6.16) with this die setting. For each material, different test sets were plotted together to show good agreement between the different tests. From Fig.6.17 material B shows relatively constant springback angle regardless of bend angle after certain bend angle (about 30 degree) while material A shows nonlinear relationship. This is because many part of the material flow into the die cavity and more span has elastic bending energy which will work as a springback energy. Thus it results more springback in material A. The reason is that the material B is soft material (lower n value) than material A. If we can move the punch further for material B, then we can see the nonlinear behavior like material A. However we cannot go that far because of localized bending induced fracture of material B. This gives an general idea of choosing material in bending operation. It is better to choose lower n valued material to have more accurate product control over the desired bend angle (Fig.6.17). To make a complete plot, it is necessary to have the data in the elastic region which will vary linearly.
Fig. 6.15: Springback Displacement vs. Punch Displacement Curve
Fig.6.16: Bend Angle Curve before and after Springback
Fig. 6.17: Springback Angle vs. Bend Angle before Unloading Curve
Bending strains are plotted for each punch displacement on Fig.6.18, 6.19 and 6.20 at the center, bottom off center and top off center respectively for the different specimens (AS1 and AS2) of material A. Quite good agreement is observed for the different tests.

Bending strain histories are also plotted for material B on Fig.6.22, 6.23 and 6.24. Strain gages were broken too early for one of the specimen (BS1) of material B. This was due to localized plastic deformation of the material around punch center due to the properties of the material. Thus the next specimen (BS2) was displaced 0.3" from the punch center so that we were able to get usable data. Comparison is made for the bottom and top off center strains on Fig.6.21 and 6.25 for each material which shows quite similar patterns, thus it can be claimed that strain gages were correctly positioned so that the readings are quite accurate.

Springback strain vs. bend angle before unloading curves are shown on Fig.6.26. Roughly maximum 1% of springback strain is observed for material A. However more data are needed to assess the case of material B. Also more data are needed for small bend angle (below 20 degree) to see the complete behavior of the materials which can be assumed to vary linearly to some angle. Similar behavior was observed on springback strain vs. true bend strain curves (Fig.6.27).

Typical photographs showing bend shapes before and after springback are shown on Fig.6.6 through 6.9 for material A and B respectively. Bend shapes for both materials before springback along with the punch displacement are plotted on Fig.6.28 and 6.29 which were digitized from the photographs. It can be seen that many parts of the specimen especially for material A deforms elastically thus large springback is expected.
Fig. 6.18: True Strain Distribution at the Punch Center Bottom Surface for Material A
Fig. 6.19: True Strain Distribution at the 5.5mm Offcenter of the Bottom Surface for Material A
Fig. 6.20: True Strain Distribution at the 5.5mm Offcenter of the Top Surface for Material A
Fig. 6.21: Comparison of True Strains of the Top and Bottom Surfaces for Material A
Fig. 6.22: True Strain Distribution at the 7.62mm Offcenter Bottom Surface for Material B
Fig. 6.23: True Strain Distribution at the 13.12mm Offset of the Bottom Surface for Material B
Fig. 6.24: True Strain Distribution at the 13.12mm Offcenter of the Top Surface for Material B
Fig. 6.25: Comparison of True Strains of the Top and Bottom Surfaces for Material B at 13.12mm Offcenter
Fig. 6.26: Springback Strain vs. Bend Angle Curve
Fig. 6.27: Springback Strain vs. True Strain Curve
Fig. 6.28: Bend Shape for Material A Digitized from the Photograph before Unloading
Fig. 6.29: Bend Shape for Material B Digitized from the Photograph before Unloading
Finally strain distributions from the typical square grid measurement for material A after springback are shown with strain gage readings for material A on Fig.6.30. Strain gage readings were taken before unloading occurs. If we consider the springback strain of about 1%, both measurements agree well. Photoresist coating on the specimen was too brittle after the test that the grids were torn off near the punch center. Thus not many reliable data were obtained. Preliminary elastic-plastic bending simulation results [Hambrecht, 1993] show that the elastic-plastic boundary for the material A is located at around 25mm from the punch center and this result coincide well with the grid readings (Fig.6.30).

To see some mechanics of the specimen through the thickness, linear strain distribution through the thickness is assumed and the strain distribution were plotted at different punch heights along with the strain after springback for material A and B (Fig.6.31 and 6.36). The enlarged graph(Fig.6.32) of the Fig.6.31 around the mid surface clearly shows the shifting of the neutral axis during the punch progresses and unloading process. From the true strain distribution, effective strain distribution can be found as Fig.6.33 and 6.38 for material A and B respectively. Effective stress distributions calculated by using the hardening law of each material are shown in Fig.6.34 and 6.39 for material A and B. Now axial stress distributions can be found as Fig.6.35 and 6.40 using true and effective stress relationship and residual stress distributions after springback are shown in Fig.6.36 and 6.41.
Fig. 6.30: True Strain Distributions by the Square Grid Measurement for Material A at PH=43mm
Fig. 6.31: True Strain Distribution through the Thickness for Material A at the 5.5mm Offcenter from the Punch Center.
Fig. 6.32: Detail of True Strain Distribution around Mid-Surface for Material A, Loading and Unloading Processes
Fig. 6.33: Effective Strain Distribution through the Thickness for Material A at the 5.5mm Offcenter from the Punch Center
Fig. 6.34: Effective Stress Distribution through the Thickness for Material A at the 5.5mm Offcenter from the Punch Center
Fig. 6.35: Axial Stress Distribution Through the Thickness for Material A at the 5.5mm Offcenter from the Punch Center.
Fig. 6.36: Residual Stress Distribution for Material A after Springback at PH=35mm
Fig. 6.37: True Strain Distribution through the Thickness for Material B at the 13.12mm Offcenter from the Punch Center
Fig. 6.38: Effective Strain Distribution through the Thickness for Material B at the 13.12mm Offcenter from the Punch Center
Fig. 6.39: Effective Stress Distribution through the Thickness for Material B at the 13.12mm Offcenter from the Punch Center
Fig. 6.40: Axial Stress Distribution Through the Thickness for Material B at the 13.12mm Offcenter from the Punch Center
Fig. 6.41: Residual Stress Distribution for Material B after Springback at PH=22mm
From Figs.6.36 and 6.41, large tensile residual stresses around top surface are observed. If we see the stress-strain relationship (Fig.6.42) during loading and unloading, some part of the material may experience plastic unloading especially near the top surface around center region.

Point B in Fig.6.42 was the strain state when the material was in loading state, while point C was the strain state after a complete springback. Thus, stress at point D is the residual stress after springback and this stress is very close to the plastic unloading stress in case of no Baushinger effect considered. If the Baushinger effect is considered, the stress state should be changed so as the equilibrium state also be changed accordingly. Therefore, the Baushinger effect should be considered to assess the unloading, that is, springback process more precisely (see Fig.6.42).
if Baushinger Effect, then Plastic Unloading

Fig. 6.42: Loading and Unloading Process in True Stress-Strain Curve for Material A
6.9 Comparison with Simulation Results

Comparisons with some other available simulation results are made for the springback data on the bending experiments with the case of material A. Fig.6.43 through 6.47 show comparisons of the experimental results with simulation results by the SHEETB elastic-viscoplastic FEM code [Hambrecht, 1993] and by the BEND code which was developed by C.T. Wang at the Engineering Research Center in the Ohio State University. Punch force-displacement comparison is made on Fig.6.43 which shows good agreement between experiments and the elastic-viscoplastic FEM simulation. Fig.6.44 shows comparison on the punch displacement vs. bend angle before springback. Punch displacement vs. bend angle after springback is shown in Fig.6.45, punch displacement vs. springback angle is shown in Fig.6.46 and Angle before springback vs. springback angle is shown in Fig.6.47. All the results show that elastic-viscoplastic FEM formulation gives very good predictions in springback simulation.

Fig.6.45 is valuable in industry point of view because it is critical in the process control that how much punch should travel to make the desired bend angle. The simulation can, thus, gives some preliminary information to do the actual tryout of the bend operation. The BEND results show some deviation in early stage and last stage of bending operation, however, general trend matches with the experimental results.

This formulation with rigid-viscoplastic FEM failed to get convergent solution due to the severe contact condition and large elastic deformation involved in the experiments. This large elastic deformation gives kind of numerical buckling in the compressive region. This is the area which still needed to be investigated. It is recommended to utilize elastic-viscoplastic FEM formulation to do a complete simulation.
Fig. 6.43: Punch Force-Displacement Comparison for Material A
Fig. 6.44: Comparison with Simulation Results for Material A on Punch Displacement vs. Bend Angle before Springback Curve
Fig. 6.45  Comparison with Simulation Results for Material A on Punch Displacement vs. Bend Angle after Springback
Fig. 6.46: Comparison with Simulation Results for Material A on Punch Displacement vs. Springback Angle
Fig. 6.47: Comparison with Simulation Results for Material A on Bend Angle before Springback vs. Springback Angle
Detailed springback simulation is still going on with elastic-viscoplastic Finite Element model using the same thin shell formulation of this research. Some of the preliminary results from the elastic-plastic bending simulation [Hambrecht, 1993] are compared with experimental data as well as the BEND simulation results [Wang, 1993]. As shown in the comparison, elastic-viscoplastic formulation gives quite good predictions in the springback simulation.

Quick preliminary comparison was made on the strain distribution and general trends were in good agreement with elastic-viscoplastic FEM formulation, while the results from BEND simulation deviate a lot with the experiments. The author can not provide complete comparison on the strain distribution at this point because the elastic-viscoplastic FEM simulation is still going on. Generally speaking, simplified approach like BEND simulation gives good prediction in geometry, however, it fails to get reasonable strain distribution. Thus, to get good prediction on strain distribution, FEM simulation is required.
CHAPTER VII

SUMMARY AND CONCLUSIONS

A plane strain finite element simulation code has been developed for the simulation of the sheet forming processes, such as, stretch-drawing, deep drawing, flanging and brake bending. Updated Lagrangian formulation based on fully non-linear thin shell theory has been utilized to model the plane strain deformation of a sheet. Thus large strains and large rotations can be modeled by the theory. Both incremental true strain and incremental Lagrangian strain formulations are implemented. The reduction of the number of degrees of freedom leads to significant savings in computing time without losing the accuracy of the solution.

Implemented constitutive model can handle the Hollomon, Swift and Ludwik hardening models and consequently is capable of modeling a wide range of sheet metals.

Both normal gap and vertical gap constraints were considered to check contact condition. To handle more general case of problems especially deep drawing case with narrow die gap, normal gap consideration is desired. However, further investigation is necessary to model properly with the normal gap constraints. A linearized normal gap constraint equation is proposed.
To verify the accuracy of the program, several patch tests through comparison with analytic solution have been made. Extensive comparisons have been made with measured data and other available numerical data. The results show a significant bending strain around a sharp die corner, which cannot be corrected by simply adding t/2R to a membrane solution.

Two cases of industrial sheet forming operation were analyzed and compared with the measured data, which are in good agreement. Further improvements in convergence are necessary to apply the current model to more complex forming operations.

Bending experiments have been carried out and analyzed to provide verifiable data on the strain distribution, springback calculation, geometric changes and material properties. Two kinds of test specimen were used to get enough data on strain distribution; namely strain gauged specimen and grid coated specimen for two types of high strength aluminum.

MTS machine was used for the bending experiments along with a strain gage conditioner, a digital oscilloscope, a digital recorder for A/D converting purpose and an IBM PC. Because of shortage in number of recording channels (only four were available), only three strain gages were used in each strain-gauged specimen.

To get the springback information, punch travel was stopped at certain height and photo was taken to get the geometry and unloading was done by reversing the punch travel until complete unloading took place. Then another photo was taken at the fully unloaded stage. This process was repeated until the bend angle reached to around 90 degree. Later those angles from photos were measured using microscope to get proper springback angle information.
Punch force-displacement curves were obtained by combining force history and displacement history from the digital oscilloscope. Those time histories have some oscillations due to the short sampling time chosen in the oscilloscope. Thus, filtering has been done within the oscilloscope and later using IBM PC software.

Friction coefficients between the specimen and die were evaluated by observing the punch force-displacement curve. Those friction data calculated from the experiments are consistent. Under these kind of low contact pressure and low sliding velocity, dry Coulomb friction coefficient, \( \mu \), is roughly 0.18 on the unlubricated aluminum surface in contact with a steel punch and is fairly consistent. This simple bending experiments, therefore, can be used as a tool for a rough estimation of Coulomb friction coefficient on this type of process.

Evaluated friction coefficient shows some dependency on the material behavior; that is, more elastic deformation within the die region causes more friction forces, thus Coulomb friction coefficients turn out to be bigger. Case of material A produces higher friction coefficient comparing to material B due to large elastic deformation within the die region. This is reasonable because large elastic deformation forces to secure the contact during springback, thus more friction results.

To see the complete springback behavior in the early stage, it is necessary to have more experiments on early step. However those can be assumed to vary linearly so that the figures such as Figs.6.13 - 6.17 can be utilized to assess the bend angle and springback information. Springback results were compared with other simulation data which show fair agreement.

To make a good control over the desired bend angle, it is recommended to choose the material of lower strain hardening index (n) value. Also, to reduce the springback, it
is necessary to choose a smaller die width to thickness ratio, $W/t$ value, so that entire
deformation within die region has to be plastic.

Through this simple bending experiments, it is noted that the Baushinger effect
should be considered during the springback unloading process to have a precise
assessment on the material behavior. Plastic unloading will occur if the Baushinger effect
is considered during the springback process in the punch side of the material.

The strain histories and the springback information will provide good resources to
evaluate the developing software especially for the bending code because the operation in
brake bending is mostly pure bending.

To a further development of the theory and program, the following implementations
are recommended;

i) Detailed springback analysis by utilizing fully elastic-viscoplastic or elastic-plastic
formulation to handle proper unloading and multi-action press operation.

ii) Extension of the theory to three dimension.

iii) Intensive study on the pure bending problem which gives lots of convergence
problem.

iv) Optimization of the code such as vectorization and parallel processing.

v) Post processing of the code to have a graphing capability.

vi) Proper contact modeling technique is still needed to be investigated.

vii) On the bending experiments to get more accurate assessment on the friction
coefficient (Coulomb type), more parametric studies including experiments are
recommended.
BIBLIOGRAPHY


APPENDIX

A

ELEMENT STIFFNESS MATRIX AND FORCE VECTOR
DUE TO INTERNAL REACTIONS

The stiffness matrix and force vector due to internal reactions, \( K_l \) and \( F_l \) are given as,

\[
K_l = \int_{V_0} \left[ N^T (B^T DB + S_l A) N \right] dV_0 \tag{A-1}
\]

and

\[
F_l = \int_{A_0} \left[ N^T B^T S_l \right] dA_0 \tag{A-2}
\]

where,

\[
B_j = \frac{\partial \Delta E_i}{\partial \Delta Q_j} \quad \text{and} \quad A_{jk} = \frac{\partial^2 (\Delta E_i)}{\partial \Delta Q_j \partial \Delta Q_k} \quad (j=1,3; \ k=1,3) \tag{A-3a,c}
\]

\[
D = \frac{\partial S_l}{\partial \sigma} \frac{\partial \sigma}{\partial \epsilon} \frac{\partial \epsilon}{\partial \Delta E_i} + \frac{\partial S_l}{\partial g_{ii}} \frac{\partial g_{ii}}{\partial \Delta E_i}
\]

where, \( \Delta Q = \begin{bmatrix} \frac{d\Delta u}{dx} \frac{d\Delta w}{dx} \frac{d^2 \Delta w}{dx^2} \end{bmatrix}^T \), and \( N \) is a \((3x6)\) matrix containing the first and second derivatives of the interpolation functions.
\[ \mathbf{B} \text{ is a (1x3) vector defined as,} \]

\[
\mathbf{B} = \cos^2 \theta \left[ \frac{\partial \varepsilon}{\partial \Delta \mathbf{Q}} - \frac{z}{\Delta \mathbf{Q}} \frac{\partial \lambda_i}{\partial \Delta \mathbf{Q}} - \lambda_i \frac{\partial z}{\partial \Delta \mathbf{Q}} \right] = \cos^2 \theta (\gamma - \zeta k - \lambda_i \omega) \quad (A-4)
\]

where \( \varepsilon \) is membrane strain term, \( k_i \) is apparent bending strain term and \( z \) is current thickness coordinate.

\[
\varepsilon = \Delta u' + \Delta w' Z' + \frac{1}{2} \left\{ (\Delta u')^2 + (\Delta w')^2 \right\} \quad (A-5)
\]

\[
k_i = (\Delta w'' + Z'') \left\{ 1 - \frac{1}{2} (\Delta w' + Z')^2 \right\} \quad (A-6)
\]

Here ' and " denote 1st and 2nd derivatives with respect to the local x-coordinate and \( Z' = \tan \theta \).

\[
\gamma = \{ 1 + \Delta u', \Delta w' + Z', 0 \}^T \quad (A-7)
\]

\[
\pi = \left\{ 0, - (\Delta w'' + Z'')(\Delta w' + Z'), 1 - \frac{1}{2} (\Delta w' + Z')^2 \right\}^T \quad (A-8)
\]

\[
\omega = \frac{\partial z}{\partial \kappa} \frac{\partial \kappa}{\partial \Delta \mathbf{Q}} + \frac{\partial z}{\partial \lambda_i} \frac{\partial \lambda_i}{\partial \Delta \mathbf{Q}} = \frac{Z^2}{2(1-zk)} \Psi - \frac{z(2-zk)}{2(1-zk)} \Phi \quad (A-9)
\]

and

\[
\Psi = \left\{ \begin{array}{c} (\Delta w'' + Z'')(-2 + 6 \Delta u') \\ -3(\Delta w'' + Z'')(\Delta w' + Z') \\ 1 - 2 \Delta u' + 3(\Delta u')^2 - 1.5(\Delta w' + Z')^2 \end{array} \right\} \quad (A-10)
\]

\[
\Phi = \cos \theta \left\{ \begin{array}{c} 1 + \Delta u' \\ \Delta w' + Z' \end{array} \right\} \quad (A-11)
\]

\( \overline{k} \), a current curvature in the current coordinate is defined as,

\[
\overline{k} = (\Delta w'' + Z'') \left\{ 1 - 2 \Delta u' + 3(\Delta u')^2 - 1.5(\Delta w' + Z')^2 \right\} \quad (A-12)
\]
A, a (3x3) matrix, is defined as follows by neglecting the 2nd derivatives of the thickness with respect to the local incremental displacements,

\[
A \equiv \cos^2 \theta \left[ \frac{\partial^2 \varepsilon}{\partial \Delta Q \partial \Delta Q} - Z \frac{\partial^2 k_i}{\partial \Delta Q \partial \Delta Q} - 2 \frac{\partial k_i}{\partial \Delta Q} \frac{\partial z}{\partial \Delta Q} \right] \equiv \cos^2 \theta \left[ \Gamma - z \Pi - 2 \pi \omega \right] \quad (A-13)
\]

where

\[
\Gamma = \begin{bmatrix}
1, & 0, & 0 \\
0, & 1, & 0 \\
0, & 0, & 0
\end{bmatrix}
\]  
(A-14)

\[
\Pi = \begin{bmatrix}
0, & - (\Delta w'' + Z''), & - (\Delta w' + Z') \\
0, & - (\Delta w' + Z'), & 0
\end{bmatrix}
\]  
(A-15)

In case of dynamic analysis, the element mass matrix, \( M \) is obtained from the diagonal components of consistent mass matrix, thus only diagonal terms exists.

\[
M = \frac{m}{148} \begin{bmatrix}
35, & 39, & 74 \times l^2 \\
39, & 39, & 74 \times l^2
\end{bmatrix}^T
\]  
(A-16)

where \( l \) is the length of each element at the current configuration.

Thus, Stiffness matrix due to inertia force is:

\[
K_p = \frac{4}{\Delta t^2} M = \frac{1}{\Delta t^2} \frac{m}{37} \begin{bmatrix}
35, & 39, & 74 \times l^2 \\
39, & 39, & 74 \times l^2
\end{bmatrix}^T
\]  
(A-17)