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The effects of embedded grains on ionized plasmas

Kingdon, James B., Ph.D.
The Ohio State University, 1993
THE EFFECTS OF EMBEDDED GRAINS
ON IONIZED PLASMAS

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

James B. Kingdon, B.S.

******

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For my Parents
ACKNOWLEDGEMENTS

I am indebted to my advisor, Dr. Gary J. Ferland, for his suggestion of this dissertation topic and his guidance and support throughout. I thank R. M. Wagner and Ray Bertram at the Perkins telescope for their assistance in obtaining the observational data used in this work, and Martin Sawey for supplying me with his work prior to publication. Finally, I thank my parents for their unstinted support throughout my graduate career.
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Introduction

We begin with a general overview of dust in gaseous nebulae. This introduction shall be brief, and is intended to provide a general background of the current state of knowledge in this area. We shall only discuss certain aspects of this problem. In particular, this review will concentrate on the evidence for grains in nebulae, the location of these grains, and some of the effects that they will have on the emission-line spectrum.

1 Evidence for Dust

The existence of dust in gaseous nebulae has been known for several decades now. In H II regions, the presence of dust is often obvious on direct photographic images. In addition, scattered light due to dust makes up an appreciable fraction of the optical continuum in these objects (cf. O'Dell and Hubbard, 1965). The situation is not as simple for planetary nebulae (PNs), however. Although a few claims have been made for a dust-scattered continuum in these objects, all have proved to be erroneous. It is currently believed that such a scattered component exists, but that it is so weak that it is extremely difficult to distinguish from observational
uncertainties in the continuum measurement. A few nebulae have faint optical halos, which have been theorized to be caused by light scattered from dust, but these conjectures remain uncertain.

The first convincing evidence for the presence of dust in PNs came from their infrared (IR) emission. In their study of the PN NGC 7027, Gillett, Low, and Stein (1967) found that the 10\(\mu\)m emission from this object was much greater than could be accounted for by atomic processes. The excess emission was later interpreted by Krishna Swamy and O'Dell (1968) to be due to graphite particles heated by Ly\(\alpha\) photons. Further IR studies have shown this excess to be a common trait of PNs.

Although the IR emission is still the most direct evidence for the existence of dust in gaseous nebulae, there are several other indications of its presence. We shall discuss each of these below, including some problems with them.

(1) **Destruction of Resonance Lines** Resonance lines, which are optically thick, will be scattered many times within a nebula. If dust exists in the region of the nebula where these lines are produced, destruction of the line photons via heating of dust grains can occur. This effect is most pronounced in the ultraviolet (UV), since typical grains have higher opacities in this region of the spectrum. The resonance line C IV \(\lambda 1549\) is observed to be too weak (based on theory) relative to [C III] \(\lambda 1909\) in many PNs. This is believed to be caused by destruction of 1549 onto grains. For objects in which C\(^+3\) exists in substantial amounts, a similar ratio, C IV
\( \lambda 1549 \) / C III \( \lambda 2297 \), can be used to measure the optical depth in dust (Seaton, 1983). Although this seems to be a fairly good indicator of the presence of dust in PNs, care must be taken when deriving dust optical depths in this manner due to some uncertainties in the atomic parameters for the carbon lines.

(2) Depletion of Condensable Elements Several studies have found Fe, Mg, Si, and other elements to be depleted in gaseous nebulae, compared to solar values. These same elements are expected to condense onto grains during the late stages of evolution of PN precursors. Comparison of the amount of depletion with that found in the general interstellar medium (ISM) can put restraints on the nature of the dust grains responsible.

(3) Asymmetries in Line Profiles If we consider a spherical, expanding shell with a uniform distribution of gas, each line profile would have a red and blue component. If dust is present within the gas, the red profile, which comes from the far side of the nebula, should suffer more extinction than the blue. Such profile asymmetries have been observed in several PNs. However, a non-homogeneous distribution of gas within the object could mimic this effect. Moreover, for an inclined, spheroidal nebula, a non-uniform distribution of external dust could also cause such an asymmetry (Hicks et al., 1976).
2 Location of the Dust

One question critical to our understanding of dust in ionized plasmas is its location within the nebula. The answer to this question is by no means simple, nor is there probably a single answer. There is evidence that at least some grains exist in the outer, or neutral regions, based on the fact that certain spectral features due to grains are observed only outside the ionized region in some nebulae. However, it is also clear that some dust is well-mixed with the ionized gas. This is evidenced by the destruction of C IV $\lambda$1549, which is produced far within the "H II region", as well as the depletion of condensable elements. We list a few other indicators below.

(1) High-resolution imaging of many nebulae have found the IR emission (due to dust) to be similar (although not necessarily coincident) in distribution to the H$^+$ zone as determined from radio measurements. One must be careful here, however, as the resolution of such observations is often not good enough to clearly place the IR emission within the ionized gas.

(2) Scattering by grains within the ionized region, although usually too small to detect in the optical continuum, can cause polarization. Polarization attributed to this effect has been observed in the [O III] lines of several PNs (cf. Leroy et al., 1986).

(3) For most PNs, it is found that the IR luminosity makes up most of the total luminosity emitted. Calculations for the central star luminosities also give
values similar to the emitted IR luminosity. In symbols, $L_{TOT} \sim L_{IR} \sim L_\ast$. Thus, almost all of the incident stellar luminosity is eventually reprocessed by dust into IR radiation. Inside the H$^+$ region, the radiation consists primarily of photons whose energies are greater than 13.6 eV, whereas in the outer, neutral H region, the emergent photons have energies less than 13.6 eV. As discussed above, due to the increased dust opacity in the UV, dust must exist within the H$^+$ region in order for $L_{IR}/L_\ast \sim 1$.

3 The Effects of Dust on Gaseous Nebulae

The presence of dust within the ionized gas implies that grains will play a role in modifying the nebular physics that determines the observed emission-line spectrum. It is generally agreed that dust has little effect on the ionization structure of gaseous nebulae. This is based on a combination of photoionization models and the relatively small optical depths in dust derived from the UV carbon lines as described above. This matter is not completely settled, however. For example, it is conceivable that dust could modify the radiation field by preferentially absorbing high-energy photons. Even the measured dust optical depths would certainly reduce the size of the ionized zone. Grains would also play a role in the dynamical evolution of PNs. Because grains are generally charged by a combination of ionization and recombination effects, they are coupled to the gas. It is believed that
radiation pressure from the central star may cause the dust to drag the gas away, resulting in a central cavity. Furthermore, through photoelectric emission and collisional cooling, dust grains will affect the nebula's heating/cooling balance. As mentioned above, dust grains will also destroy resonantly-trapped line photons such as H Ly$\alpha$, which will alter the emitted spectrum.

One line which is of great importance in regard to this last point is He I $\lambda$10830 ($2^3S - 2^3P$), which has long been observed to be weaker than predicted by theory. Dust has been suggested as a possible explanation, but previous studies have not proved conclusive. The resolution of this discrepancy is critical for an understanding of the importance of collisional effects in He I, which theory predicts could lower the calculated He/H abundance by at least 10% in PN. This, in turn, has important implications for studies of stellar evolution, galactic chemical evolution, and the primordial He abundance, all of which require very accurate determinations of the He/H ratio. An accurate determination of He$^+$ is also important for the abundances of other elements, since the ratio He$^{++}$/He$^+$ is often used as an ionization correction factor.

4 This Dissertation

In Chap. 1, we calculate newly-determined charge exchange rate coefficients for several ions of Al and Ca onto neutral H. These calculations employ the
Landau-Zener approximation. In Chap. 2, we use these results to assist in the determination of the depletions of Al and Ca in two nebulae. Our results confirm the existence of dust in the ionized region. In Chap. 3, we examine several properties of dust with radius in a model nebula. We also determine the extent to which internal dust can resolve the $\lambda$10830 discrepancy. In Chap. 4, we study the effect of telluric absorption on the 10830 intensity. We derive new theoretical collision-to-recombination correction factors in Chap. 5, and compare these with observations of two PNs. Finally, we summarize our results in Chap. 6.
1 Introduction

Since abundance determinations in interstellar medium objects require detailed modelling of all physical processes occurring in gaseous nebulae, it is important to have an accurate and complete atomic data base from which to work. One important process which has only been incorporated into abundance calculations during the last two decades is the phenomenon of charge exchange recombination and ionization. This process is described by the equation

\[ X^{+n} + Y \rightarrow X^{+(n-1)} + Y^+ \]  \hspace{1cm} (1)

where \( X \) and \( Y \) are any two ions. In most cases of astrophysical interest, \( Y \) is an abundant ion, usually H or He. The equation with the arrow pointing as above describes charge transfer recombination, while the reverse reaction is referred to as charge transfer ionization.
Although charge exchange reactions involving singly-charged ions with neutral hydrogen have been studied for several decades (cf. Chamberlain, 1956 for examples involving O\(^+\)), reactions with multiply-charged ions can be much more important. This is because reactions with singly-charged ions require a close matching of the ionization potentials of the two atomic species present. Reactions with multiply-charged ions, on the other hand, do not have such a requirement. The presence of Coulomb repulsion in these reactions insures that the potential curves describing the reactions will cross at some point (assuming the reaction is exothermic). Charge transfer recombination of multiply-charged ions with neutral hydrogen has been shown to be an important process in determining the structure of nebulae (cf. Steigman, 1975), mainly through its effect on the degree of ionization. Models including charge exchange calculations have led to more self-consistent nebular models (cf. Péquignot et al., 1978). In general, however, very few reactions have been examined experimentally, and full quantal treatments (cf. Butler et al., 1980) are equally rare. However, estimates based on the Landau-Zener approximation are often sufficiently accurate for most purposes, especially when the rate coefficients are large (Butler and Dalgarno, 1980). In this chapter, we present rate coefficients for reactions involving ions of Al and Ca with H, using the Landau-Zener method.
In §2.1, we discuss the Landau-Zener approximation and the theory of charge exchange in detail. In §2.2, we use this formalism to derive charge transfer rate coefficients for several ions of Al and Ca with neutral H. These results are incorporated into a photoionization code and will allow us to improve the ionization balance of these elements. In the next chapter, we will utilize this to calculate Al and Ca depletions in two nebulae.

2 Charge Transfer Calculations

2.1 Landau-Zener Theory

Our discussion of the Landau-Zener theory will follow that presented by Butler and Dalgarno (1980). As discussed in the introduction, we consider reactions of the type $X^+ + H \rightarrow X^{+(n-1)} + H^+$, where $n > 1$. We will refer to the left side of this reaction as the entrance, or incoming channel, and to the right side as the outgoing, or exit channel. The incoming and exit channels for this reaction are described by two potential curves, which have a separation $\Delta E$ at infinity. $\Delta E$ is simply the heat of reaction. Although there are complicated short-range potentials, the incoming channel can reasonably be described as a constant potential for these calculations. Similarly, the potential in the exit channel is basically a Coulomb potential. Due to the Coulomb repulsion in the exit channel, the two diabatic potential curves (i.e.
those curves which are calculated by assuming that the wavefunction of the "active" electron remains constant throughout the reaction) will cross at an internuclear distance $R_x$ for exothermic reactions. In reality, however, due to a rapid change in the wavefunction of the active electron, the curves do not intersect and there is an avoided crossing of the adiabatic potential curves at $R_x$. Fig. 1, adapted from Dalgarno (1978), illustrates the general situation. If polarizability and repulsive core interaction in the incoming channel can be neglected, the point of this avoided crossing is simply given by $R_x=(n-1)/\Delta E$ in atomic units.

Let us denote the probability that the system undergoes a transition from one curve to another at $R_x$ by $p$. Then since the system passes $R_x$ twice during the collision, the probability that the transition occurs once out of two passes is simply $P=2p(1-p)$ (Zener, 1932; Bates, 1960). The reaction cross-section is then obtained by integrating $2\pi R p_0 P$ over all possible impact parameters. The factor $p_0$ is the probability that the reaction proceeds initially along the incoming channel. We will discuss its evaluation in the next subsection. Finally, the reaction rate coefficient is determined by integrating the cross-section over a Maxwellian velocity distribution.

The Landau-Zener approximation asserts that the probability $p$ is given by $p=e^{-w}$, where

$$w = \frac{\pi^2[\Delta U(R_x)]^2}{\hbar v[d/dr(H_{11}-H_{22})]_{R_x}} \quad (2)$$

(Bates and Moiseiwitsch, 1954). Here $\Delta U$ is the separation of the two potential
Figure 1: Diabatic and adiabatic potential curves
curves, \( v \) is the radial component of the relative velocity of the collision, and \( H_{11} \) and \( H_{22} \) are the incoming and exit potential curves in the absence of a change in the wavefunction of the active electron. Neglecting polarizability and core repulsion, the derivative in brackets in the denominator of Eq.(2) is simply equal to \((n - 1)/R_x^2\), in atomic units. Again neglecting polarizability, the reaction cross-section is given by

\[
Q(v) = 4\pi R_x^2 p_0 \int_1^\infty (1 - e^{-wx})e^{-wx}x^{-3} \, dx
\]

(3)

and the rate coefficient \( \alpha \) is then determined from

\[
\alpha = \int_0^\infty vQ(v)f(v)dv
\]

(4)

where \( f(v) \) is a Maxwellian velocity distribution given by

\[
f(v) = \frac{4}{\sqrt{\pi}} \left( \frac{m}{2kT} \right)^{3/2} v^2 e^{-mv^2/2kT}
\]

(5)

### 2.2 Calculations

As detailed in the previous subsection, the necessary parameters for calculation if we ignore polarizability and core repulsion are \( \Delta E \) and \( \Delta U(R_x) \). The former is determined from examination of the appropriate energy levels of H and \( X^{+(n-1)} \). For the latter, we use the asymptotic formula given by Butler and Dalgarno (1980),

\[
\Delta U(R_x) = 27.21 R_x^2 e^{-R_x} eV
\]

(6)

where \( R_x \) is measured in units of \( a_0 \). From the work of the above reference, it is found that Eq.(6) seriously fails when \( R_x \) is small, and results in very small
cross-sections. However, a full quantal calculation also leads to small cross-sections for these cases, so that we are left with the fortunate result that Eq.(6) is least accurate for those reactions for which charge transfer is least important.

We have calculated charge transfer rate coefficients for neutral H onto X = Al++, Al+, Al+, Al+, Ca++, Ca+, and Ca++. In our work, we have assumed that all ions considered are initially in their ground state. This assumption is quite reasonable for conditions in gaseous nebulae. The energy level wavenumbers needed for calculation of $\Delta E$ were obtained from Bashkin and Stoner (1975) or Moore (1949) for Al, and from Sugar and Corliss (1985) for Ca. Rate coefficients are generally appreciable only for $7a_0 \leq R_x \leq 13a_0$ (Neufeld and Dalgarno, 1987). We initially considered all possible exit channels within this range, but then continued outside of it in both directions until the values became negligible. We have taken fine structure of the levels into account, but any implied increase in accuracy by doing so is probably illusory, due to the uncertainty of the approximation as a whole. The factor $p_0$ in Eq.(3) was obtained by considering what fraction of the reactant states are also product states. The possible molecular states were determined by application of the selection rules from Herzberg (1950). The calculation of the cross-sections and rate coefficients was done using software described by Bienstock (1983). The final rate coefficients were obtained by summing the individual rate coefficients for each exit channel.
We shall provide an example to illustrate the above formalism. Consider the reaction \( \text{Al}^{+3} + \text{H} \rightarrow \text{Al}^{++} + \text{H}^+ \). From Moore (1949) and Bashkin and Stoner (1975), the ground state wavenumbers for H and Al\(^{++}\) are 109678.758 cm\(^{-1}\) and 229445.71 cm\(^{-1}\), respectively. These two ions are the ones between which the electron moves. The difference in these two wavenumbers is \( \Delta = 119766.95 \text{ cm}^{-1} \).

The only levels in Al\(^{++}\) which are accessible to the electron are those whose wavenumbers are less than this number. In this case, there are three levels (ignoring fine structure for the moment) that satisfy this requirement: \(^2\text{D}\) (3809.59 cm\(^{-1}\)), \(^2\text{P}^0\) (65967.19 cm\(^{-1}\)), and the ground state \(^2\text{S}\) (119766.95 cm\(^{-1}\)). The numbers given in parentheses are the differences between the actual wavenumbers for each level and \( \Delta \) given above. If we convert each of these wavenumber differences into electron volts and then into crossing distances in bohrs, we obtain \( R_x = 115.3, 6.66, \) and 3.67, respectively for these three levels. Note that all of these fall outside the range 7-13 bohr given above. Let us focus on the reaction in which the electron transfers into the \(^2\text{P}^0\) level with a crossing distance of 6.66 bohr. We can determine \( \Delta U(R_x) \) from Eq.(6). The only remaining parameter is \( p_0 \). The two reactant states, H and Al\(^{+3}\), have ground state configurations \(^2\text{S}\) and \(^1\text{S}\), respectively. There are two possible molecular states, \(^2\Sigma^+\). For the product states, H\(^+\) (\(^1\text{S}\)) and Al\(^{++}\) (\(^2\text{P}^0\)), there are six possible molecular states, \(^2\Sigma^+\), and two sets each of \(^2\Pi\) and \(^2\Delta\). Then \( p_0 \) is simply equal to the fraction of reactant states that are also product states. In
this case, \( p_0 = 1 \).

Table 1 presents the results (rate coefficients in \( \text{cm}^3/\text{s} \)) for a range of temperatures (corresponding to the velocity in Eq. (2)). We have arbitrarily set to zero any value less than \( 1 \times 10^{-25} \text{cm}^3/\text{s} \). The results for Al\(^{+2}\) and Al\(^{+3}\) show that charge transfer with H is generally negligible for these ions. However, in these cases, charge transfer may occur by emission of a photon, i.e. \( X^{+n} + H \rightarrow X^{+(n-1)} + H^+ + h\nu \). Rate coefficients for this radiative charge transfer are generally about \( 1 \times 10^{-14} \) (Dalgarno, 1978). The results for Al\(^{+4}\) were complicated by the lack of Is coupling schemes for some levels, which prevented accurate determination of \( p_0 \). Likewise, for Al\(^{+5}\), wavenumbers were not available for some levels. In the situation where several exit channels for a reaction have the same symmetry, simply summing the individual rate coefficients can lead to an overestimate of the total result. This is the case for Ca\(^{+4}\).

3 Conclusions

We have presented charge transfer rate coefficients for ions of Ca and Al with neutral H. The results indicate that charge transfer can be important for Al\(^{+4}\), Al\(^{+5}\), Ca\(^{+3}\), Ca\(^{+4}\), and Ca\(^{+5}\), while Al\(^{++}\) and Al\(^{+3}\) may proceed by radiative charge transfer. Although it is uncertain how accurate the Landau-Zener approximation is, full quantal calculations show that it is reasonably accurate in the case where the
### TABLE 1

Rate coefficients (cm$^3$/s)

<table>
<thead>
<tr>
<th>Ion</th>
<th>$10^3$ K</th>
<th>$10^{3.5}$ K</th>
<th>$10^4$ K</th>
<th>$10^{4.5}$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^a$Al$^{+2}$ + H $\rightarrow$ Al$^+$ + H$^+$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$^a$Al$^{+3}$ + H $\rightarrow$ Al$^{+2}$ + H$^+$</td>
<td>7.68(-22)</td>
<td>5.95(-17)</td>
<td>7.11(-14)</td>
<td>8.18(-12)</td>
</tr>
<tr>
<td>$^b$Al$^{+4}$ + H $\rightarrow$ Al$^{+3}$ + H$^+$</td>
<td>5.76(-10)</td>
<td>6.30(-10)</td>
<td>7.89(-10)</td>
<td>2.06(-9)</td>
</tr>
<tr>
<td>$^c$Al$^{+5}$ + H $\rightarrow$ Al$^{+4}$ + H$^+$</td>
<td>7.73(-10)</td>
<td>1.08(-9)</td>
<td>1.43(-9)</td>
<td>2.40(-9)</td>
</tr>
<tr>
<td>Ca$^{+3}$ + H $\rightarrow$ Ca$^{+2}$ + H$^+$</td>
<td>3.85(-12)</td>
<td>3.18(-11)</td>
<td>2.88(-10)</td>
<td>1.61(-9)</td>
</tr>
<tr>
<td>$^d$Ca$^{+4}$ + H $\rightarrow$ Ca$^{+3}$ + H$^+$</td>
<td>1.67(-9)</td>
<td>3.23(-9)</td>
<td>5.99(-9)</td>
<td>1.07(-8)</td>
</tr>
<tr>
<td>Ca$^{+5}$ + H $\rightarrow$ Ca$^{+4}$ + H$^+$</td>
<td>9.71(-11)</td>
<td>1.00(-10)</td>
<td>1.03(-10)</td>
<td>1.04(-10)</td>
</tr>
</tbody>
</table>

$^a$ Radiative charge transfer may be the most rapid

$^b$ Results are less accurate due to lack of l-s coupling schemes for some levels

$^c$ Results are less accurate due to lack of wavenumbers for some levels

$^d$ Rates may be overestimated due to several channels having the same symmetry
rate coefficients are large. In any case, these results are a first step in constructing more accurate models of the ionization equilibrium of Ca and Al, and are a definite improvement over models which do not include charge transfer at all. As mentioned in the introduction, these data will assist us in our determination of the depletion of these elements in the next chapter.
Chapter II

The Depletion of Al and Ca in Gaseous Nebulae

1 Introduction

Two of the most important questions in nebular studies are what amount of dust exists in gaseous nebulae, and where the dust is located. Although grains might be expected to survive for long periods in neutral, well-shielded regions, it is not known how long they survive in the more hostile ionized region (Draine and Salpeter, 1979). As discussed in the general introduction, the depletion of condensable elements onto grains provides indirect evidence for the existence of dust.

Furthermore, if ionization states of a refractory element which are known to exist within the H⁺ region of a nebula are depleted, then there is clear proof that at least some fraction of the total dust exists in the ionized zone.

Before beginning our calculations, we shall briefly review the topic of depletion both in PNs and the general ISM. The general method of determining elemental depletions is to observe spectral lines of the element in question. One then usually
forms line intensity ratios of these lines, usually with respect to Hβ. If the spectral lines of the element are known to arise from well-understood processes for which accurate atomic data are available (such as radiative recombination or collisional excitation), these line ratios can be converted into ionic abundance ratios, i.e. $N(X^+)/N(H^+)$, where $X$ represents the particular element being studied and $i$ is the stage of ionization. These ionic abundances are then converted into total abundances, either by using ionization correction factors (ICFs) or a photoionization model. Finally, one compares these gas-phase abundances to solar or cosmic values to determine the relative amount of depletion. Clearly, the accuracy of the final results depends on the quality of the observations and atomic data.

There exist several determinations of heavy-element depletions in the general ISM. A good overview is given by Mathis (1990) for O, N, Zn, P, Mg, Cl, Fe, Cr, Si, Ca, Ti, and Al. There are substantial differences in the amounts of the depletion of these elements as well as in how these depletions depend on the mean gas density. Turner (1991) gives depletions relative to solar for many of these elements. His work divides the ISM into four categories: diffuse warm, diffuse cold, dense warm, and dense cold. Certain elements, such as Ca, Ti, and Al have very strong dependences on the category. More recent results based on UV observations with the Hubble telescope's Goddard High-Resolution Spectrograph are summarized by Cardelli et al. (1991).
There are somewhat fewer measurements of depletions for PNs. Shields (1983) provides a review of the state of affairs up to 1983. He discusses results for Fe, Ca, Si, and Mg, and finds that in general the gas-phase abundances of these elements are usually between one and two orders of magnitude lower than solar. Mg is especially interesting, as it appears to show a gradient in high-excitation planetaries, in the sense that the amount of depletion increases as one moves further out in the nebula. Middlemass (1988) found a nearly solar abundance of this element in the low-excitation PN IC 418, and a depletion of roughly half an order of magnitude in IC 4997.

Two elements which are strongly depleted in the ISM but have not received much attention in PNs are Al and Ca. This is largely due to the lack of strong spectral lines of these elements in the optical (Al has none, Ca has only two weak lines at λ5309 and λ6087, the latter of which is sometimes blended with a line due to [Fe VII]) and the depletion itself, which further weakens these lines. Based on [Ca V] λλ 5309, 6087, Shields et al. (1981) found a depletion of roughly one order of magnitude for this element in the PN NGC 2440. Similarly, Keyes et al. (1990) estimate a depletion of roughly a factor of five for NGC 7027 from the λ5309 intensity. Both of these are much less than that found in the ISM. Pwa et al. (1984, 1986) were able to determine Al depletions in the PNs NGC 6543 and BD+30°3639 by making UV observations with the International Ultraviolet Explorer (IUE).
Their technique consisted of measuring absorption lines from the central stars of these objects. Their results suggest a depletion of an order of magnitude for the latter nebula and an upper limit of two orders of magnitude for the former.

In this chapter, we shall determine depletions of Al and Ca in a model PN. Our Al calculations will use recent IUE observations of Al II λλ 2660, 2669 in the well-studied PN NGC 7027. We shall then utilize a photoionization model to determine the ionization correction. For Ca, we use the fact that the strong calcium doublet [Ca II] λλ 7291, 7324 is not observed in PNs or H II regions. Using published high-resolution observations and photoionization models, we determine a lower limit to the Ca depletion in NGC 7027 and the Orion nebula.

2 Al and Ca Depletion

2.1 Al Depletion in NGC 7027

In order to determine the depletion of Al in NGC 7027, we have used the IUE observations reported by Keenan et al. (1992). These authors were able to make the first detection of the intercombination line Al II λλ2660, 2669. They obtain a flux of $6.6 \times 10^{-14}$ erg/cm$^2$/s for 2660 from LWR 2571, and a flux of $9.2 \times 10^{-14}$ erg/cm$^2$/s for 2669 from LWR 2571. The sum of these two lines relative to He II λ2511 (average flux of $1.6 \times 10^{-13}$ erg/cm$^2$/s) from the same exposure is $\sim 0.55$, corrected
for reddening using the extinction curve of Seaton (1979). Then using the UV data of Keyes et al. (1990), we derived a reddening-corrected intensity for the sum of the two Al lines of \( \sim 0.0574 \) relative to H\( \beta \) by scaling to the He \( \text{II} \) line. Now, since H\( \beta \) is produced through recombination while the Al lines are produced by collisional processes, we can relate the relative intensities of both 2670 and H\( \beta \) to the Al\(^+\) and H\(^+\) abundances via the equation

\[
\frac{I(2670)}{I(H\beta)} = \left( \frac{8.629 \times 10^{-8} \Omega}{T^{0.5} g_i} e^{-h\nu/kT} \alpha_{H\beta}^{\text{eff}} \right) \left( \frac{\nu_{2670}}{\nu_{H\beta}} \right) \left( \frac{N(\text{Al}^+)}{N(\text{H}^+)} \right)
\]  

(7)

where \( \Omega \) is the effective collision strength for the 2670 pair, taken from Tayal et al. (1984), \( g_i \) is the statistical weight of the lower level (here simply equal to 1), and \( \alpha_{H\beta}^{\text{eff}} \) is the recombination coefficient, extrapolated to a temperature of 14,000K from Osterbrock (1989). The density required for collisional de-excitation is sufficiently large (\( N_{\text{crit}} \sim 1 \times 10^{10} \)) that we can safely ignore it compared to radiative de-excitation. We have also made use of the nebular approximation, i.e. that essentially all of the Al\(^+\) is in the ground state. Putting in the appropriate numbers gives

\[
\frac{I(2670)}{I(H\beta)} = 4.14 \times 10^{5} \frac{N(\text{Al}^+)}{N(\text{H}^+)}
\]  

(8)

Using the above line intensity ratio, this becomes \( N(\text{Al}^+)/N(\text{H}^+) = 1.39 \times 10^{-7} \).

In order to convert this into a total, rather than ionic abundance, we will need to determine the fraction of Al that is in the form Al\(^+\). We shall accomplish this by use of a photoionization model. Since we shall be making frequent use of such
models throughout this work, we shall briefly discuss the photoionization code and the particular model used below.

**CLOUDY photoionization code** The code that we shall use in this and subsequent photoionization calculations is CLOUDY. We list some of its basic parameters and input here; a much more complete discussion is provided by Ferland (1992). CLOUDY is a self-consistent photoionization model, in that both the ionization and energy balance are maintained at each point in the calculations. CLOUDY divides the modeled nebula into a number of thin concentric shells or zones whose thicknesses are varied to insure that the physical parameters of the model (density, temperature, etc.) are kept essentially constant throughout the zone. All published atomic processes are included in the computations. In addition, the charge exchange rate coefficients derived in Chapter 1 have been incorporated into the code. CLOUDY requires only three sets of input: (1) the shape and intensity of the incident continuum, (2) the chemical composition of the gas, and (3) the geometry of the gas (including the run of density as a function of radius). One can specify these parameters in numerous ways, and many types of geometries, abundances, and continua are possible. CLOUDY’s output includes the relative intensities of several hundred emission lines, plus data on ionization fractions, optical depths, temperatures of different zones, etc. Grains have recently been incorporated into CLOUDY; this topic will be discussed further below.
The Meudon Model, or Paris PN The particular photoinization model that we shall use here is the Meudon PN, first introduced at the Paris Conference on Model Nebulae (Péquignot, 1986). Since we shall make use of this model often, we again provide a brief discussion here. The model is designed to approximate NGC 7027. Although it is a fairly simple representation of a rather complex object, it should be sufficient for our purposes. The model parameters are listed in Table 2. The abundances listed there, and throughout this paper, are given by number, not by mass. The model considers the gas to be ionized by a blackbody with a temperature of 150,000°K and a radius of $10^{10}$ cm. The inner face of the ionized gas is assumed to be a distance of $10^{17}$ cm. from the star. A constant density is assumed throughout the gas. In addition, we have included grains, with a constant dust-to-gas ratio taken to be equal to that found in the general ISM. CLOUDY assumes two species of grains: graphites and silicates. All understood properties of grains, including their effect on the energy balance, are considered. CLOUDY uses a Mathis, Rumpl, and Nordsieck (1977) power-law distribution of grain sizes for the optical properties of the grains, whereas other properties, such as temperature and potential, are calculated using a mean grain size. The optical constants used here are from Martin and Rouleau (1990), with the IR opacities for the silicate component taken from the unpublished work by Volk. Again, a more complete discussion can be obtained from Ferland (1992).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackbody temperature</td>
<td>150,000°K</td>
</tr>
<tr>
<td>Blackbody radius</td>
<td>10^{10}\text{cm}</td>
</tr>
<tr>
<td>Inner radius</td>
<td>10^{17}\text{cm}</td>
</tr>
<tr>
<td>Log(H density)</td>
<td>3.4771213</td>
</tr>
<tr>
<td>Log(He/H)</td>
<td>-1</td>
</tr>
<tr>
<td>Log(C/H)</td>
<td>-3.523</td>
</tr>
<tr>
<td>Log(N/H)</td>
<td>-4</td>
</tr>
<tr>
<td>Log(O/H)</td>
<td>-3.222</td>
</tr>
<tr>
<td>Log(Ne/H)</td>
<td>-3.824</td>
</tr>
<tr>
<td>Log(Mg/H)</td>
<td>-4.523</td>
</tr>
<tr>
<td>Log(Al/H)</td>
<td>-7</td>
</tr>
<tr>
<td>Log(Si/H)</td>
<td>-4.523</td>
</tr>
<tr>
<td>Log(S/H)</td>
<td>-4.824</td>
</tr>
<tr>
<td>Log(Ar/H)</td>
<td>-9</td>
</tr>
<tr>
<td>Log(Ca/H)</td>
<td>-7</td>
</tr>
<tr>
<td>Log(Fe/H)</td>
<td>-7</td>
</tr>
</tbody>
</table>
Results We ran the Paris PN model discussed above and found $\frac{N(\text{Al}^+)/N(\text{Al})}{= 0.33}$. Then, from the numbers given above, we obtain $\text{Al}/\text{H} \approx 4.1 \times 10^{-7}$, a depletion of roughly an order of magnitude compared to the solar value of $2.9 \times 10^{-6}$ (Grevesse and Anders, 1989).

2.2 Ca Depletion in NGC 7027

The calcium doublet [Ca II] $\lambda\lambda 7291,7324$ ($^2S - ^2D$) is expected to be quite strong in PNs, if we assume its abundance is solar (our photoionization model predicts an intensity relative to $\text{H}\beta$ of roughly 1.2 for solar Ca). However, it has not been detected in any nebular spectrum, even those done at high resolution or with long exposure times. This fact implies that Ca must be depleted and allows us to set an upper limit on the gas-phase abundance of Ca.

We used the observations of Keyes et al. (1990) for NGC 7027. This study involved detection of very weak features, and is thus well-suited for our calculations. We determined the detection limit of these observations by finding an appropriately weak line, and then considering this to be an upper limit to the 7306 intensity. Our criteria for choosing the line were (1) that it be well-identified, to avoid the possibility of ghosts or spurious detections; and (2) that it be relatively close in wavelength to 7306, to avoid problems with reddening and signal to noise. With this in mind, we chose the He I $\lambda 7500$ line, which, using the extinction curve of Seaton
(1979), gives \( I(\lambda 7500)/I(H\beta) = 2.85 \times 10^{-4} \). We take this to be an upper limit to the intensity of \([\text{Ca II}] \lambda 7306\).

We then ran the Meudon model, varying the relative Ca abundance until we arrived at the above limit to the intensity of the \([\text{Ca II}] \lambda 7306\) line. This produced an abundance (by number) of \( \text{Ca}/\text{H} \leq 5 \times 10^{-10} \), implying a depletion of more than three orders of magnitudes with respect to the solar value of \( 2.3 \times 10^{-6} \) (Grevesse and Anders, 1989).

2.3 Ca Depletion in Orion

We now wish to examine the grain depletions in an H II region for comparison. Our method is exactly the same as in the previous subsection. Our photoionization model is that of Baldwin et al. (1991). The geometry is again spherical. We used a constant pressure and dust-to-gas ratio (again set equal to that of the general ISM) throughout the nebula. The grains used here are more typical of those in Orion.

The major difference between these grains and those used in the previous subsection are that the Orion grains have a lack of small grains and a larger mean grain size. CLOUDY accomplishes this by increasing the smallest grain size in the Mathis, Rumpl, and Nordsieck (1977) distribution from \( 0.0025 \mu m \) to \( 0.03 \mu m \). Table 3 lists the parameters of the model. A more detailed description can be found in the above reference.
<table>
<thead>
<tr>
<th>Orion Model Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log((\phi(H))) (cm(^{-2}) s(^{-1}))</td>
<td>13</td>
</tr>
<tr>
<td>Turbulence (km/s)</td>
<td>8</td>
</tr>
<tr>
<td>Log(H density) (cm(^{-3}))</td>
<td>4</td>
</tr>
<tr>
<td>He/H</td>
<td>0.10</td>
</tr>
<tr>
<td>C/H</td>
<td>3.2(-4)</td>
</tr>
<tr>
<td>N/H</td>
<td>5.0(-5)</td>
</tr>
<tr>
<td>O/H</td>
<td>4.0(-4)</td>
</tr>
<tr>
<td>Ne/H</td>
<td>8.0(-5)</td>
</tr>
<tr>
<td>Mg/H</td>
<td>4.2(-5)</td>
</tr>
<tr>
<td>Al/H</td>
<td>2.7(-7)</td>
</tr>
<tr>
<td>Si/H</td>
<td>4.3(-6)</td>
</tr>
<tr>
<td>S/H</td>
<td>2.0(-5)</td>
</tr>
<tr>
<td>Ar/H</td>
<td>3.7(-6)</td>
</tr>
<tr>
<td>Fe/H</td>
<td>5.0(-6)</td>
</tr>
</tbody>
</table>
Our observations were taken from Osterbrock et al. (1992), which include many faint lines. For our weak line, we used [Ni II] λ7412 which gave a result analogous to that determined for NGC 7027 of $I(\lambda 7306)/I(\text{H}\beta) \leq 2.65 \times 10^{-4}$. Our photoionization model then gave $\text{Ca/H} \leq 1 \times 10^{-9}$ for Orion, suggesting a depletion of about 3 orders of magnitude relative to solar.

It is interesting to examine the effect on the values from this and the last subsection if we remove the presence of grains from our calculations. This will cause the code to ignore the thermal and opacity effects of grains, while preserving the depleted abundances of the elements. For both NGC 7027 and Orion, we found that removing grains lowered the above upper limits to the Ca abundance by a factor of 2.

3 Conclusions and Discussions

The abundances of Al and Ca relative to H determined in the last section are depleted by one and three orders of magnitude from solar values, respectively. This clearly demonstrates not only that dust is present in gaseous nebulae, but more importantly, that it is mixed with the ionized gas. The derived Al abundance is similar to that obtained by Pwa et al. (1984, 1986) for two PNs by using interstellar absorption features in the UV. We have thus extended to three the number of PNs for which Al depletions have been measured. All of these suggest that this element
is $\sim 6$ times more abundant than in the general ISM, but several times less abundant than solar. The fact that the grains in PNs seem to be as efficient as those in the ISM in depleting Fe and Ca (but see the following), but less efficient in depleting Al may provide a clue to the nature of these grains.

The measured depletion of Ca requires additional discussion. As discussed in the introduction to this chapter, previous estimates of the gas-phase Ca abundance in PNs obtained depletions of only one order of magnitude, far less than the value determined here. These analyses relied on the intensity of $[\text{Ca V}] \lambda 5309$, which is roughly a factor of 0.003 smaller than that of $\text{H}\beta$ in NGC 7027 (Keyes et al., 1990). This reference estimated an observational uncertainty of $\sim 20\%$ for this line. It is also expected that since only one ionization stage of Ca was observed, the ICF will be quite large (for this particular reference, an ICF of 5.9 was used) and thus very uncertain. In addition, our inclusion of charge exchange will affect the relative amounts of each ion of Ca. We note here, however, that charge exchange does not directly affect $\lambda 5309$, since the value of $R_\infty$ for the line’s upper level is significantly less than 7$a_0$, and thus the rate coefficient is vanishingly small. In an attempt to reconcile this difference, we have again used the Paris PN in an attempt to match the Keyes et al. (1990) observed intensity of $[\text{Ca V}] \lambda 5309$. Since Ca V should be produced in the same region as Ne IV, we have tried to vary the Ca abundance relative to Ne in order to get a good match for both the $\lambda 5309$ line and the $[\text{Ne IV}]$
lines centered around 4720Å. We found that the Ne abundance listed in Table 2
gives a nearly perfect match to the observed [Ne IV] λ4720 intensity. In order to
match the λ5309 line, we needed to increase the Ca abundance to Ca/H \sim 9 \times 10^{-7}.
This is actually a factor of 2 greater than what Keyes et al. (1990) determined, and
implies only a depletion of roughly a factor of 3 from solar values.

One possible explanation for this seeming discrepancy is that Ca possesses a
"depletion gradient", with the inner parts of the nebula, represented by [Ca V]
having an essentially solar Ca abundance, whereas the outer zones, represented by
Ca II, showing a strong depletion. This is precisely the situation suggested for Mg
by Péquignot and Stasinska (1980), who found a cosmic abundance of Mg in
NGC 7027 based on [Mg V], but a depletion of roughly one order of magnitude
based on Mg II. Similar results were obtained for Mg by Harrington and Marrioni
(1981) for the PNs NGC 2165 and NGC 2440 and by Shields et al. (1981) for
NGC 2440. Péquignot and Stasinska (1980) attributed their findings to a selective
destruction of metallic magnesium grains by radiation heating during an early phase
in the nebular expansion. We should note here, however, that these results for Mg
have been weakened by two factors: (1) new collision strengths for [Mg V] from
Mendoza and Zeippen (1987), which reduce the gradient, and (2) interstellar
absorption of the Mg II λ2800 line, which severely limits its use in abundance
calculations (cf. Clegg et al. 1987). For Ca, one can also imagine that spallation
would tend to destroy dust grains existing in the inner regions of the nebula.

Confirmation of these results for other nebulae would be useful, but difficult due to
the aforementioned lack of strong Ca lines.
Chapter III

The Effect of Dust on Nebular Temperatures, Dynamics, and He I Line Transfer

1 Introduction

In the previous chapter, we have seen that Al and Ca, as well as other condensable elements, are strongly depleted in both PNs and H II regions. This provides strong, indirect evidence that dust exists in the H\textsuperscript{+} zone in these objects. Clearly, this dust will have an effect on various physical conditions within the nebula. In this chapter, we shall examine the effects of dust grains on the energy balance, dynamics, and radiative transfer within PNs.

As discussed in the general introduction, one effect of dust grains will be to absorb line photons within the nebula. This effect is most pronounced among lines which have a large optical depth, such as resonance lines. The absorption is not strongly dependent on the physical conditions in the nebula, since the probability of absorption by dust is simply related to the number of scatterings.
An astrophysically important line which should be affected by this phenomenon is the pseudo-resonance line He I $\lambda$10830 ($2^3S-2^3P$). Because of the metastability of the $2^3S$ level in He I, a substantial fraction of this ion will be in the $2^3S$ state. This in turn implies a large optical depth for any lines for which $2^3S$ is the lower level of the transition. Before calculating the extent to which destruction by grains affects this line, we first provide a brief discussion of the importance of this line, and previous work on this topic.

The population of the $2^3S$ level must be known with precision in order to derive He abundances for nebulae. This is because collisions from this level can significantly alter the line intensities calculated from recombination theory (We shall give a much more thorough discussion of the importance of collisional effects in Chapter 5.). The main observational diagnostic for the $2^3S$ level population is the $\lambda$10830 to $\lambda$5876 ratio, since the former is produced primarily through collision excitation from $2^3S$ and the latter through recombination. It has been known for several decades now (Osterbrock 1964; O'Dell 1965; Capriotti 1967; Robbins 1968) that the observed intensity of 10830 is less than predicted by standard theory. These early studies suffered both from inaccuracies in the atomic data and in the observed line measurements (Scrimger 1984). Drake and Robbins (1972), using better atomic data, found "reasonable" agreement (within a factor of 3) between theory and observation for seven out of eleven PNs, but their use of Capriotti's
photodestruction rates limits the accuracy of their results (Clegg and Harrington 1989). More recently, Peimbert and Torres-Peimbert (1987b) found a range in the ratio of the observed to the predicted value of $N(2^3S)/N(He^+)$ (denoted by $\gamma$) of 0.47–0.62 for six PNs, based on the observed intensity of the 10830/5876 and the 7065/5876 ratios. From this and earlier computations (Peimbert & Torres-Peimbert, 1987a), they conclude that some previously unconsidered mechanism must be depopulating the $2^3S$ state. Similarly, Péquignot, Baluteau, and Gruenwald (1988) found the predicted intensity of 10830 to be a factor of 1.5 times larger than the observed intensity in NGC 7027 and also suggested that a moderate depopulation of $2^3S$ was occurring. Peimbert & Torres-Peimbert (1987b) find that this latter result corresponds to a value $\gamma = 0.51$ according to their formalism.

Despite much recent work on the theoretical side, this discrepancy remains an outstanding problem in nebular physics. The necessary atomic parameters are now thought to be sufficiently accurate, and newly examined or updated depopulation mechanisms such as collisional ionization (Clegg 1987), Ly$\alpha$ photoionization (Clegg and Harrington 1989), and charge exchange (Baldwin et al. 1991) have proven negligible or insufficient.

A rather different approach is to assume that the predicted level population for $2^3S$ is correct, but that some other effect weakens the 10830 intensity. One proposed mechanism, suggested independently by Persson (1970) and Robbins
(1970), is that 10830 is partially destroyed by dust grains within the nebula.

Persson found that for reasonable dust parameters, this effect could completely explain the discrepancy. More recently, LeVan and Rudy (1983) found that the size of the discrepancy increased with the planetary's electron density, and showed that a simple dust model could account for the observed correlation. However, the importance of dust in resolving this problem remains unsettled. Peimbert and Torres-Peimbert (1987a, 1987b) have argued that dust is insufficient to explain the discrepancy based on dust optical depths determined from the C IV λ1549 intensity and C(Hβ) extinction parameter.

In this chapter, we again examine the effect of dust on the intensity of the 10830 line. In §2, we present a simplified formalism for determining the size of this effect, and give results for a small sample of PNs. In §3, we calculate the effect of dust and the electron density on the 10830/5876 line ratio and the IR excess by using the Meudon photoionization model. In addition, we use this model to examine various grain properties and effects as a function of nebular radius. Finally, we discuss our results in §4.
2 A Dust Correction Factor for the 10830 Intensity

2.1 General Procedure

In this section we present an approximate formalism to determine the effect of dust on 10830. In the following subsection, we apply this formalism to a small sample of PNs, and show that it agrees well with more extensive numerical calculations.

The population of the $2^3P$ state of He I is found by equating all processes which populate the state and all processes that depopulate it. This gives

$$N_e(N_p \alpha_{2^3P} + N_{2^3S}q_{2^3S,2^3P}) = N_{2^3P}(N_e q_{2^3P,2^3S} + A_{2^3P,2^3S} \epsilon)$$

(9)

where $\alpha_{2^3P}$ is the recombination rate coefficient to $2^3P$, the $q_{ij}$ are the collisional rate coefficients, and $\epsilon$ is the escape probability. For the electron densities present in PNs, collisional de-excitation from $2^3P$ is very small compared to radiative decay. Thus, we ignore the first term on the right-hand side of Eq.(9) (This is valid for $N_e \ll 10^{13}$).

For a nebula in which dust is present, the escape probability in Eq.(9) is modified to contain an extra term (c.f. Netzer et al., 1985). This gives

$$\epsilon' = \epsilon + \delta$$

(10)

where $\delta$ represents the destruction of the 10830 photon by dust. We shall discuss
the exact form of $\delta$ later on.

The emitted intensity of $\lambda 10830$ is given by

$$j_{10830} = \left(\frac{h \nu_{10830}}{4\pi}\right) N_{2} A_{2^3P,2^3S} \epsilon$$

(11)

Substituting Eq.(9), modified as discussed above into Eq.(11), we obtain

$$j_{10830} = \left(\frac{h \nu_{10830}}{4\pi}\right) N_{e} (N_{P} \alpha_{2^3P} + N_{2^3S} q_{2^3S,2^3P}) \left(\frac{\epsilon}{\epsilon + \delta}\right)$$

(12)

Note that in the absence of dust, the last term in parenthesis in Eq.(12) would be equal to unity. Thus, the effect of dust is to reduce the $\lambda 10830$ line intensity by a factor of $\epsilon/(\epsilon + \delta)$. Therefore, it is this ratio, hereafter designated by $f$, that we need to calculate.

We define a normalized destruction probability which is given by

$$\delta = (1 - \epsilon)f F(\beta)$$

(13)

where $\beta$ is simply the ratio of continuum to total (continuum plus line) opacity, i.e.

$$\beta = \frac{k_{C}}{k_{L} + k_{C}}$$

(14)

with $k$ the absorption coefficient in cm$^{-1}$. $F(\beta)$ is a known, tabulated function of $\beta$ (c.f. Hummer, 1968, Table 1). The continuum opacity due to embedded grains can be conveniently rewritten as $\sigma_{C} N_{H}$, with $\sigma_{C}$ the cross-section for continuum opacity (Draine and Lee, 1984) and $N_{H}$ the total hydrogen density. Likewise, the line
absorption coefficient for 10830 is

\[ k_L = N_{23S} \left( \frac{\pi e^2 f_{10830}}{m_e c \Delta \nu_D} \right) \]  

(15)

Note that this is the normalized or mean line absorption coefficient, following Hummer, rather than the line center value. The population of \( N_{23S} \) is then determined by the results of Clegg (1987) to be

\[ N_{23S} = \left( \frac{5.79 \times 10^{-9} T^{-1.18}}{1 + 3110 T^{-0.51} N_e^{-1}} \right) N_{He^+} \]

(16)

Thus, the factor \( f \) is seen to be a simple function of temperature, density, and optical depth.

2.2 Calculations and Results

Although the above formalism can easily be incorporated into a photoionization model, for this dissertation we shall develop a formalism which will allow us to determine \( f \) for a series of PNs for which the input parameters are known. We shall calculate a more rigorous model later in this section, in which depth effects are taken into account. This model will be shown to agree well with the simplified technique discussed here.

Our approximation consists of considering the entire 10830-producing zone to be represented by a single point, defined by an average \( T \) and \( N_e \), and located at one half the total optical depth of 10830. This choice of optical depth greatly
simplifies the calculation of the escape probability, since $e$ will be the same in both inward and outward directions. We have considered each member of the 10830 triplet separately, with the relative intensities determined by the statistical weights. The escape probability $e(r)$ was calculated by the approximate formulae for the $K_2$ function presented by Hummer (1981). We have taken $\sigma_C = 1 \times 10^{-22}$ cm$^2$ for the general ISM from Draine and Lee (1984). Finally, we take $N_{He^+} = N_{He} = 0.1 N_H$.

We have calculated $f$ as described above for several PNs listed in Clegg and Harrington (1989). The pertinent information is given in Table 4 and is taken directly from the above source, with the exception of the Meudon Model, which will be discussed shortly. Columns 1, 2, & 3 list the nebula's name, average electron temperature (in units of $10^4$ K), and average electron density, respectively. Column 4 gives the total optical depth in 10830. Note again that this is the mean optical depth, and is related to the line center optical depth by $\tau_{\text{mean}} = \sqrt{\tau_0}$. The dust-to-gas ratio by mass as a fraction of the interstellar medium dust to gas ratio is listed in column 5. This number multiplies the continuum cross-section $\sigma_C$ given for the ISM above. The values in column 5 were obtained by dividing the values $R$ in Table 2 of Clegg and Harrington (1989) by 0.007 (cf. Savage and Mathis, 1979). The results of the calculations are presented in column 6. We see that the effect of dust on 10830 is only a few percent for IC 418, NCC 7662, and IC 3568, using the listed
# TABLE 4

<table>
<thead>
<tr>
<th>Name</th>
<th>(N_e)</th>
<th>(T_e)</th>
<th>(\tau(10830))</th>
<th>Dust/Gas</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>IC 418</td>
<td>1.45(4)</td>
<td>0.85</td>
<td>373.2</td>
<td>0.1886</td>
<td>0.93</td>
</tr>
<tr>
<td>NGC 7662</td>
<td>2.47(3)</td>
<td>1.31</td>
<td>68.2</td>
<td>0.0871</td>
<td>0.98</td>
</tr>
<tr>
<td>IC 3568</td>
<td>2.63(3)</td>
<td>1.16</td>
<td>58.2</td>
<td>0.3571</td>
<td>0.96</td>
</tr>
<tr>
<td>DDDM-1</td>
<td>6.61(3)</td>
<td>1.21</td>
<td>480.8</td>
<td>0.2143</td>
<td>0.82</td>
</tr>
<tr>
<td>Vy2-2</td>
<td>3.21(5)</td>
<td>1.03</td>
<td>2592.2</td>
<td>0.1429</td>
<td>0.66</td>
</tr>
<tr>
<td>Meudon</td>
<td>3.34(3)</td>
<td>1.18</td>
<td>148.6</td>
<td>1.0000</td>
<td>0.76</td>
</tr>
</tbody>
</table>
parameters. However, the effect is appreciable for DDDM-1 and Vy2-2. The latter is an extremely dense, compact nebula.

We now wish to test the accuracy of our approximation. To this end, we have run a photoionization model for the Meudon PN, which was described in detail in Chapter 2 and whose basic input parameters are listed in Table 2. The "Cloudy" code incorporates the formalism described in the previous subsection, but integrates the results over a self-consistent ionization and thermal structure. The temperature and optical depth listed in Table 4 for Meudon are the results of this model. We determined $f$ by running Cloudy with an amount of grains equal to the ISM, and then again with the same parameters but with the destruction probability set to zero. We then simply took the ratio of the emitted 10830 intensity for the two cases. A spherical geometry was assumed in the model. The result of the above procedure was $f=0.76$, which exactly matches the value in column 6 of Table 4. Thus, we are confident that our approximation and the results for the nebulae are valid.

3 Photoionization Results

In this section, we wish to further examine the effects of dust on certain observable quantities in PNs. For this purpose, we have again used the Meudon Model. We have run several models in which we varied both the dust-to-gas ratio and the hydrogen density, $N_H$, while keeping all other parameters constant. Although this
changes the physical structure of the nebula, it is instructive for giving one a feel for how the nebular quantities vary.

We have examined two quantities: the line ratio He I 10830/5876 and the IR luminosity relative to Hβ. Here we take the IR luminosity to be the total luminosity reradiated by grains. The first of these two quantities is the most common observational diagnostic for the population of the 2^3 level as discussed in section 1, while the second is considered to be an important diagnostic for the determination of dust content (cf. Natta and Panagia 1976). The results of the modeling are presented graphically in Fig. 2. Fig. 2(a) shows the variation of the 10830/5876 ratio as a function of the dust-to-gas ratio (again expressed relative to the ISM) for several different values of \( N_H \). In all models, the total hydrogen density, as well as the dust-to-gas ratio, is assumed to be constant throughout the nebula. Fig. 2(b) shows the same relation for the ratio \( L(\text{IR})/L(\text{H}\beta) \). These results are discussed in the next section.

It is also interesting to examine what effect dust has on the general structure of a PN. We have run the Meudon model with a dust-to-gas ratio equal to that of the ISM and have plotted several quantities as a function of nebular radius. Similar studies have been conducted by Baldwin et al. (1991) and Borkowski and Harrington (1991). In these models, we have assumed a grain composition comparable to the ISM (Martin and Rouleau, 1990); i.e., both graphite and silicates
Figure 2: (a) The 10830/5876 intensity ratio is plotted as a function of the dust-to-gas ratio for several values of the hydrogen density. Plotted densities are as follows: solid line, $10^3$; dotted line, $3.34 \times 10^4$; short dash, $10^4$; long dash, $5 \times 10^4$; dot and dash, $10^5$. All densities are in cm$^{-3}$. (b) The ratio $L(\text{IR})/L(\text{H}\beta)$ is plotted for several values of the hydrogen density. The densities for each line are the same as in (a).
are included. Figs. 3(a) & (b) show the ionization structure of helium and oxygen, respectively, for comparison. Fig. 3(c) shows the grain temperature. In this and the next two figures, graphite and silicates are represented by solid and dotted lines, respectively. Fig. 3(d) shows the grain drift velocity and Fig. 3(e) the grain potential. Finally, the fraction of heating (solid line) and cooling (dotted line) of the gas due to grains is shown in Fig. 3(f). These results are discussed in the next section.

4 Discussion and Conclusions

We shall now analyze and assess the results of the two previous sections. We begin with the photoionization models of §3.

The behavior of the ratio 10830/5876 as shown in Fig. 2(a) is quite straightforward. Since 10830 is largely produced by collisions, one expects that an increase in density would result in an increase in the 10830 line intensity. Although 5876 is also collisionally enhanced, the effect is significantly less. Thus we expect 10830/5876 to increase with density as predicted.

Likewise, as discussed previously, because the optical depth in 10830 can be large in PNs, the line can be significantly destroyed by dust grains. Since 5876 is unaffected by dust, the 10830/5876 ratio should decrease with increasing dust. However, as Fig. 2(a) shows, this decrease is very gradual, and the effects of dust
Figure 3: Several parameters are plotted vs. nebular radius. (a) The He ionization structure (b) The O ionization structure (c) Grain temperature. The solid line is for graphite; the dotted line for silicates (d) Grain drift velocity. Lines as in (c) (e) Grain potential. Lines as in (c) (f) The fraction of nebular heating and cooling due to grains. The solid line shows the heating; the dotted line the cooling. Both grain types are included.
only become significant for dust-to-gas ratios comparable to the ISM. We emphasize that the behavior depicted in Fig. 2(a) is completely different from the results of section 2, which give the error in the calculated 10830 intensity for a given amount of dust if the dust is not included in the line transfer.

In Fig. 2(b), we see that the ratio $L(\text{IR})/L(\text{H}_\beta)$ increases with increasing dust content for any density. This is expected since an increase in the amount of dust will clearly increase the net heating of dust grains, and thus their IR emission.

The behavior of $L(\text{IR})/L(\text{H}_\beta)$ with density needs some clarification. It appears from Fig. 2(b) that for a given dust-to-gas ratio, $L(\text{IR})/L(\text{H}_\beta)$ increases with increasing density, then decreases with density, with a peak at roughly $1 \times 10^4$. This is actually due to a change in the geometry of the model with density. At low densities, the model is spherical, then becomes a thick shell, and finally becomes plane-parallel for large densities. This causes the hydrogen column densities to exhibit a rise and fall with density. If we constrain our model to be plane-parallel for all densities, $L(\text{IR})/L(\text{H}_\beta)$ is a monotonically decreasing function of density. This is due to a decrease of column density with electron density, which in turn causes a decrease in the dust optical depth. In effect, the greater the density, the less important dust becomes compared to gas in absorbing radiation.

The grain temperature, as depicted in Fig. 3(c) is around 75–125 K throughout the model, with a pronounced drop occurring for both grain types as the
neutral edge of the nebula is encountered. The steady fall in temperature is due to
the attenuation of the incident ionizing continuum, which is a primary grain heating
source.

The grain drift velocity depicted in Fig. 3(d) is the result of the interplay
between the radiative acceleration imparted to the dust by the radiation field and
the drag forces within the nebula. From this figure, it can be seen that over a
typical PN lifetime, the grains will drift about $10^{16}$ cm relative to the gas, a
negligible amount compared to the nebular size. Thus, neglecting the possible
destruction of some grain types by the passage of the ionization front or other
mechanisms, the grains should effectively "stay put" relative to the gas. Note that
the close coupling between gas and dust is considered to be the cause of the
observed central cavity in PNs, as discussed in the general introduction. The grain
potential results from a combination of ionization and recombination effects. As
seen in Fig. 3(e), the potential of both grain types drops steadily until the neutral
edge is encountered. This results from the attenuation of the incident continuum,
which causes photoejection of electrons from the grain surface. Near the center of
the nebula, photoejection of electrons by the incident radiation field outweighs the
capture of electrons, and the grains tend to be positively charged, whereas farther
out in the nebula, the situation is reversed, and the grains are negatively charged
(cf. Osterbrock, 1989).
Grains heat a nebula by providing electrons to the gas via photoionization and cool the nebula by collisions with electrons. As Fig. 3(f) shows, the fraction of total heating due to grains (both graphites and silicates) drops with radius, again due to the decrease in photoejection caused by the attenuation of the incident radiation. Grain photoionization can be seen to be a major heating source in the inner ionized region of the nebula. This is because the gas photoelectric opacity is diminished (the gas is highly ionized) so that grains are by far the dominant opacity source. Since the cooling depends largely on the average electron kinetic temperature, one would expect it to show less variation with radius, as is the case here.

Next, we wish to discuss the results of §2. As Table 4 clearly shows, the effect of including dust in the line transfer of 10830 is significant for PNs with large optical depths in 10830. We illustrate the importance of the dust content in Table 5, where we list $f$ for the Meudon Model for several values of the dust-to-gas ratio. Columns 1 & 2 list the average electron temperature and mean optical depth in 10830, respectively (These were determined from photoionization models). Column 3 gives the dust-to-gas ratio relative to the ISM, and column 4 gives the values of $f$. An average electron density of $3.34 \times 10^3$ was used throughout. We have included the Meudon result from Table 4 for reference. As is evident from the table, $f$ is a strong function of the dust content. We also note that the electron temperature is a weak function of dust content.
TABLE 5

<table>
<thead>
<tr>
<th>$T_e$</th>
<th>$\tau(10830)$</th>
<th>Dust/Gas</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.11</td>
<td>218.0</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>1.14</td>
<td>195.0</td>
<td>0.30</td>
<td>0.89</td>
</tr>
<tr>
<td>1.18</td>
<td>148.6</td>
<td>1.00</td>
<td>0.76</td>
</tr>
<tr>
<td>1.21</td>
<td>96.0</td>
<td>3.00</td>
<td>0.65</td>
</tr>
</tbody>
</table>

The results of Table 5 are important since the dust content of PN is not agreed upon and is a topic of some debate. Early attempts to determine dust mass relied on middle-IR observations which missed large, cool grains and therefore greatly underestimated the amount of dust (cf. Balick, 1978). Even with careful far-IR observations, the lack of knowledge concerning the grains' size distribution, composition, and emissivity causes the amount of dust to be underestimated by a factor of $\sim 2$ (cf. Natta and Panagia, 1976). This suggests that the $f$ values given in Table 4 may in fact be only lower limits.

Finally, we return to the question of how much of the observed observational to predicted discrepancy in the 10830 intensity is due to dust. Peimbert and Torres-Peimbert (1987b) found an observed to predicted 10830 intensity of $\gamma = 0.55$
for the well-studied PN NGC 7027, while the value deduced by Péquignot, Baluteau, and Gruenwald (1988) was 0.51. To the extent that the Meudon Model is an accurate representation of this object, we see from Table 5 that we would require a dust content of greater than three times that found in the ISM in order for dust alone to account for the discrepancy. However, with a dust-to-gas ratio equal to that of the ISM, the 10830 discrepancy for this object would be cut in half, with the observed intensity being only a factor of 1.3 less than that predicted.

In summary, we have found that taking dust effects into account when considering the He I λ10830 line transfer can significantly decrease the intensity of this line, but its role in resolving the 10830 discrepancy is hampered by serious uncertainties in nebular dust content. In the next chapter, we shall discuss yet another mechanism which can reduce the observed intensity of this line.
Chapter IV

The Effect of Telluric Absorption on the Intensity of He I λ10830

1 Introduction

In the last chapter, we examined the extent to which internal dust could solve the He I λ10830 discrepancy in PNs. We found that while the effect of dust is probably non-negligible and may in fact go some way towards resolving the discrepancy, it is by itself insufficient to solve the problem. As discussed previously, most attempts to solve this problem have relied on examining potential depopulation mechanisms for the 2^3S state, and have been unsuccessful. These attempts have therefore focused on errors in the theoretical 10830 intensity.

With these thoughts in mind, we wish to examine this problem from an entirely different angle. The presence of internal dust in nebulae has the effect of reducing the observed intensity of 10830. Similarly, we might ask whether there are other effects which might affect the observed intensity of 10830. Specifically, this
chapter will address the question of what effect telluric absorption might have on reducing the line intensity of 10830. Our approach will be entirely theoretical, since an observational assessment would require extremely high-resolution data due to both the close spacing of the members of the 10830 multiplet and the narrowness of the individual telluric lines.

2 Calculations

2.1 General Procedure

The red and near-IR spectrum of astronomical objects is broken up by absorption in the Earth’s atmosphere, primarily due to the molecules H$_2$O, O$_2$, and CO$_2$. This effect in low-resolution spectra is most noticeable through the presence of several strong absorption bands. However, at extremely high resolution, these bands can be seen to consist of myriad sharp individual lines, each corresponding to a particular transition in the parent molecule (cf. Delbouille et al. 1973). Some weak bands which have a small equivalent width and thus may not be readily apparent on low resolution spectra may in fact contain strong lines. Although isolated weak lines may not be apparent at low resolution, if such a line falls near a line in the object spectrum, it can substantially affect the latter’s measured flux. The effect can be dramatic in sharp-lines objects such as H II regions or planetary nebulae. It is the
purpose of this chapter to determine specifically the size and nature of this effect.

Due to the Earth's orbital motion, lines in the object spectrum will shift back and forth with respect to any telluric lines, with the precise magnitude of the shift determined by the object's coordinates and heliocentric radial velocity (cf. Spinrad 1966; Davidson and Kinman 1985). We computed the geocentric radial velocity of the well-studied planetary nebula NGC 7027 during the period April 15 – December 7 (a reasonable observing season), and thus determined the exact wavelength of 10830 for any date within this range.

The flux removed from the 10830 emission line by a single telluric line is given by

\[ A = \int_{-\infty}^{\infty} P(\lambda)[1 - e^{-\tau_\lambda}]d\lambda \]  (17)

where \( \tau_\lambda \) is the optical depth in the telluric line, and the 10830 profile \( P(\lambda) \) is normalized accordingly. The details of the telluric line and 10830 profiles are presented in the next two sections.

2.2 Telluric Lines

For the wavelength range involved in this problem (roughly 10828.07 – 10832.12 Å), only four telluric lines need to be considered. All four are due to water vapor. The wavelengths and other necessary parameters, as determined from sources discussed below, are listed in Table 6.
The telluric line profiles are largely the result of collisional broadening by air, and thus to a good approximation, can be modeled as Lorentzians. For this case, the value of $\tau_\lambda$ in Eq.(17) is given by (cf. Goody 1964)

$$\tau_\lambda = \frac{S a \Gamma}{2\pi[(\Delta \lambda)^2 + \Gamma^2/4]} \quad (18)$$

where $S$ is the line strength (given here in units of Å/(g·cm⁻²)), $\Gamma$ is the Lorentzian full-width at half-maximum (FWHM), $a$ is the total column density of water vapor, and $\Delta \lambda$ is the distance from the rest wavelength of the line.
The column density $a$ is dependent on the air mass and the amount of precipitable water vapor. This clearly is equivalent to assuming a given set of observing conditions. To this end, we chose to perform the calculations using conditions applicable to Kitt Peak National Observatory. The chosen air mass was 1.016, roughly the air mass of NGC 7027 at transit from Kitt Peak. Very accurate water vapor data exist for Kitt Peak (cf. Wallace and Livingston 1984 & Wallace et al. 1984). Rather than use a monthly mean, we chose to use an average water vapor content over the observing period of 12 mm precipitable water, which corresponds to a column density of 1.2 g-cm$^{-2}$ at unit air mass. It must be stressed the results presented here would be increased for observations made through higher air masses, and either increased or decreased for higher or lower water vapor content, respectively. Specifically, we found that increasing or decreasing the column density by a factor of two resulted in a $\sim$32% increase or decrease in the maximum effect.

The FWHM $\Gamma$ were obtained from a mixture of sources. The data for the two redmost lines in Table 6 come from the ATMOS linelist (see Brown et al. 1987 for a description). Note that this list gives half-width at half-maximum (HWHM), and that the $\Gamma$ are functions of both pressure and temperature. The pressure chosen was that corresponding to the altitude of Kitt Peak, and the chosen temperature was 280 K. The values for the remaining two lines, neither of which appeared in the ATMOS linelist, were determined from a scaling of observed and predicted values
from Table III of Breckenridge and Hall (1973, hereafter B&H). Note that the values printed in this table are actually half-width at half-maximum (HWHM), and that they are already adjusted for Kitt Peak's altitude.

The line strength $S$ is effectively a measurement of a line's equivalent width per column density; it therefore corresponds to the slope along the linear part of the curve of growth. For the two lines listed in the ATMOS linelist, the values given are the same as the predicted values given in B&H, Table III. We chose to use the predicted values of B&H rather than the observed for two reasons. First, the predicted line strength given for 10833.982 agrees well with observed equivalent widths for this line determined by Livingston and Holweger (1982). Second, the composite telluric line spectrum used in B&H corresponds to 21 mm of precipitable water in the path, which is sufficiently high so that the four telluric lines considered are probably beyond the linear section of the curve of growth. That this might be the case is demonstrated by the large discrepancies between the observed and predicted values listed for this parameter. For the line 10830.338, a value was determined by scaling the observed values in B&H.

It is somewhat difficult to evaluate the accuracy of the chosen values for $\Gamma$ and $S$, as neither the ATMOS linelist nor the list of B&H give error estimates. Based on the discussion of the ATMOS linelist (Brown et al. 1987), we would expect errors of 5% - 20%, with the bluemost two lines considered to be the least accurate.
However, these two lines contribute the least to the overall absorption, and we estimate errors of 5% - 10% in our results due to this.

Extra telluric lines needed for the determination of heliocentric radial velocity "danger zones" to be discussed in section 3 below are presented in Table 7. These were taken from Table III of B&H, with the predicted values used for $S$ and $\Gamma$.

### 2.3 He I Lines

The expansion of planetary nebulae results in the familiar double-peaked line profile. The profile itself is a convolution between this expansion and the mechanisms broadening the line, and is given by (cf. Osterbrock 1989)

$$P(\Delta \lambda) = \text{const.} \int_{-\infty}^{\infty} E(\Delta L) e^{-m^2(\Delta L - \Delta \lambda)^2/2kT\lambda^3} d(\Delta L)$$

where $E(\Delta L)$ is the distribution function of the emission coefficient in the line per unit wavelength shift $\Delta L$ for an ion of mass $m$ in a nebula with temperature $T$.

Since He I $\lambda$ 10830 is actually a triplet, we would need to consider three separate lines for a static nebula. However, the nebular expansion mentioned above causes each member of the triplet to be split into two components, so that there are effectively six He lines to consider. In the following discussion, we will use the term "member" to refer to one of the three lines in the 10830 triplet, whereas we will use "component" to refer to either of the two lines that each "member" is split into by
## TABLE 7

**Telluric Line Parameters**

<table>
<thead>
<tr>
<th>Wavelength $^a$</th>
<th>Line strength $^b$</th>
<th>FWHM $^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10818.723</td>
<td>0.0020</td>
<td>0.0960</td>
</tr>
<tr>
<td>10821.819</td>
<td>0.0017</td>
<td>0.0679</td>
</tr>
<tr>
<td>10822.125</td>
<td>0.0176</td>
<td>0.1359</td>
</tr>
<tr>
<td>10823.813</td>
<td>0.0022</td>
<td>0.0797</td>
</tr>
<tr>
<td>10837.381</td>
<td>0.0052</td>
<td>0.1128</td>
</tr>
<tr>
<td>10837.539</td>
<td>0.0062</td>
<td>0.0940</td>
</tr>
<tr>
<td>10838.034</td>
<td>0.0867</td>
<td>0.1198</td>
</tr>
<tr>
<td>10840.810</td>
<td>0.0600</td>
<td>0.1340</td>
</tr>
</tbody>
</table>

$^a$ In Å

$^b$ In Å/(g⋅cm$^{-2}$)

$^c$ In Å
the expansion.

For these calculations, we used a nebular temperature of 12500°K (Péquignot & Baluteau 1988) and an expansion velocity of 21.5 km/s. For each date, we shifted the He I line from its previously determined geocentric position, with the amount of the shift determined by the chosen expansion velocity. The relative intensity of each member of the multiplet was assumed to be proportional to its statistical weight. For each member of the multiplet, the total flux was assumed to be divided equally between its components (i.e., each is considered to represent 50% of the total flux in the line). We would expect that the results for a given date would be changed somewhat for any deviation from this symmetric model, and we will examine the effects of these deviations in the next section.

If the line is broadened solely by thermal Doppler effects, then the distribution function $E(\Delta L)$ in Eq.(19) becomes a simple sum of delta functions for each component, and the resultant profile is two Gaussians. In the discussion that follows, this particular model for He I $\lambda10830$ will be referred to as Model 1.

It might be thought that this Doppler-broadened profile would be reasonable for a PN line. However, Osterbrock et al. (1966) found from high resolution observations of several lines (not including 10830) in several PNs that the profiles were not well-fit by Gaussians, with the measured FWHM always larger than that predicted from pure Doppler broadening. They attributed this to mass motions
within the objects. They found empirically that a distribution function that is triangular with its peak at line center gave a good fit to the line profiles. Thus, we choose such a distribution function, centered at \( \Delta L = 0.7761 \), and falling to zero at \( \Delta L = 0.3072 \) & 1.245. Note that the width of this distribution function has been chosen rather arbitrarily, as we have no high resolution observations of 10830 in NGC 7027 to compare with. However, the profile seems reasonable, and will be useful to compare with the profile of Model 1. This more realistic model of the 10830 line will be referred to as Model 2. The profiles of Models 1 & 2 (the latter of which has to be numerically integrated) were then substituted into Eq.(17) with the afore-mentioned normalization of 50%.

3 Results

The results of the calculations of telluric line absorption of the He I \( \lambda 10830 \) flux are given in Figures 4 & 5, for He line Models 1 & 2, respectively. The data are presented as percentage of the \( \lambda 10830 \) flux removed, as a function of day of the year, with day 1 corresponding to January 1. Several points are worth commenting on.

First, as is readily apparent from the graphs, this effect is highly time-dependent, with a minimum \(~1.0\%\) and a maximum \(~20 - 25\%\). This time dependence makes it somewhat awkward to correct 10830 for telluric absorption since the dates of observation for all objects involved must be known.
Figure 4: The percentage of He I $\lambda$10830 removed by telluric absorption for NGC 7027, as a function of day of the year. The graph corresponds to Model 1 of the text.
Figure 5: Same as Figure 4, but for Model 2 of the text
Second, there is very little difference in the results for Models 1 & 2. This makes us confident that the effect is not highly model-dependent, and that the numbers presented here should be fairly accurate for any reasonable PN He line profile.

Third, it may be seen that the range of values is smaller for Model 2 than for Model 1, with the former having a smaller maximum effect but a larger minimum. This is the result of two factors. First, since the broader He profiles of Model 2 cover a wider range of wavelength, they are more likely to overlap with a telluric line. However, this broadening also means that a smaller fraction of the total line flux is contained within a given wavelength range, thus reducing the amount of flux that the much narrower telluric lines can remove.

At this point we return to our assumption that the flux is divided equally between the two components of the He line. For Model 1, we recomputed the amount of absorption assuming that one of the components contained twice as much of the total flux as the other. The minimum values changed only slightly, but the maxima changed to ~18% and ~34% for the case where the blue component and the red component had the larger percentage of the total flux, respectively. This change is primarily due to the fact that the blue component never quite reaches the wavelength of the strongest telluric line, $\lambda$10832.109, in this PN, and so contributes less to the total absorption at maximum. Since we would not expect a stronger
deviation from symmetry than this two-to-one ratio, we can consider the above
values rough lower and upper limits to the actual maximum effect.

We now wish to examine how this phenomenon has affected determinations of
the $2^3S$ level population. As stated previously, Peimbert and Torres-Peimbert
(1987b) and Péquignot et al. (1988) found that this level population was predicted
to have a value roughly twice that found through their observations. However, both
of these papers use the observations of Scrimger (1984) for their value of the 10830
intensity in NGC 7027. Scrimger's observations of this object were made on Nov.
4-5, which is close to the date of maximum telluric absorption, as seen in Figs. 4 &
5. The magnitude of the absorption on the above dates is approximately 24% and
20% for Models 1 & 2, respectively. This would reduce the stated discrepancy to a
factor of $\sim$1.4. Note that this value is based on the above models, which assume
specific observing conditions. It is impossible to determine the exact reduction in
the discrepancy without knowing what the observing conditions were for Scrimger's
data. Still, it is doubtful that the data was obtained at significantly drier conditions
than those assumed in the models. Assuming a water vapor column density of only
1.3 mm, roughly the minimum for Kitt Peak, we still find a reduction of the
discrepancy to a factor of $\sim$1.75. On the other hand, assuming a water vapor
column density of 32 mm, roughly the maximum at Kitt Peak, the discrepancy
would fall to a factor of only $\sim$1.25.
Although the primary purpose of this chapter is to determine the importance of telluric absorption of 10830 as a means of explaining the discrepancy between its theoretical and observed intensity, it is clear that the results given here have broader ramifications. Specifically, this effect must be considered for any observations of He I λ10830 in relatively narrow-line objects if accurate fluxes or intensity ratios are to be obtained. This is especially important not only because this line is reasonably strong in many objects, but also because it is often used to calibrate optical and IR spectra (cf. Scrimger 1984). From Figs. 4 & 5, it is clear that, for some times of the year, optical/IR line ratios calculated by this method could be greatly in error if the observations were taken more than several days apart.

To assist in the evaluation of this effect in other objects, we present in Table 8 a series of heliocentric radial velocity “danger zones” for a range of He I λ10830 widths. In order to make this table useful and yet applicable to a wide host of emission-line objects, we have assumed in these calculations a 10830 profile as in Model 1 (i.e., thermal Doppler broadening only). In addition, we have assumed that each member of the triplet has only a single component; the extension to double components is straightforward. Since, as was stated in Section 2.1, the effect is dependent on the object’s coordinates, we assume that all objects lie exactly on the ecliptic; thus the numbers in Table 8 can be considered to be upper limits. In order to be as general as possible, we have taken an airmass of 1.0, while keeping our
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*Entries are heliocentric radial velocity ranges in km/s*

*b* Doppler width of He line in km/s
TABLE 8 (Continued)

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</table>

*a Entries are heliocentric radial velocity ranges in km/s

*b Doppler width of He line in km/s
previous value of 12 mm precipitable water vapor. The heliocentric radial velocity range which we can cover is dependent on two factors: the wavelength range for which reasonably accurate telluric line data exists, and the intrinsic width of the He line profile. These two considerations led us to chose a range of -100 km/s to 100 km/s, which should include many important astrophysical objects.

The entries in the table give the range of heliocentric radial velocities (in km/s) for which a given percentage of flux, indicated by the columns, is removed from the 10830 line. These percentages represent the maximum amount of flux removed during the year. Therefore, at certain times of the year, the amount of flux removed may be much less. Thus, for example, an object that has a Doppler FWHM of 15 km/s for 10830, and has a heliocentric radial velocity of 10 km/s will have a maximum of 20–25% of its 10830 flux removed, at some time of the year. Due to numerical difficulties in evaluating the absorption integral, we expect the FWHM in the range 15-35 km/s will have the lowest accuracy, estimated to be ~20%. As Table 8 indicates, objects having 10830 FWHM of greater than 250 km/s will not suffer any appreciable telluric absorption, at least within the stated range of radial velocities, since their width is many times greater than the telluric line widths. We wish to emphasize that Table 8 is meant to be used as a guide for 10830 absorption, and that any detailed examination of this effect in a particular object must include the necessary parameters for that object, including profile shape, object
co-ordinates, and observing conditions (air mass, amount of water vapor, etc.).

4 Conclusions

We have examined the degree to which telluric absorption affects the observed flux of He I λ10830 in the planetary nebula NGC 7027, in the hopes of explaining the long-standing discrepancy between its observed and predicted intensity. We have found that the amount of absorption is highly dependent on the date of observation, ranging from \(~1\%\) to \(~25\%\), with a small dependence on the modeling of the 10830 line profile. We have found that telluric absorption had a significant effect on published observations. Our best estimate is that the intensity of 10830 determined in these observations was reduced by \(~22\%\), although, depending on the amount of water vapor present during the observations, this value could range from \(~10\%-30\%. Since Peimbert and Torres-Peimbert (1987b) find \(\gamma=0.55\) for this object, it is clear that even at maximum, telluric absorption is not sufficient to explain this discrepancy. However, it is also clear that this effect does contribute towards this end, and therefore must be considered in any future analysis of this problem. In particular, we find that the value of \(\gamma\) stated above should be increased to roughly 0.70.

Since the telluric absorption considered in this chapter and the dust effects considered in the previous one are independent, they can be combined to examine
the total reduction of the 10830 intensity. Our average value of 22% for telluric effects compared with the results of Table 5 show that a dust content equal to that of the ISM would result in a total reduction of the observed value of $\gamma$ of $0.70 \times 0.76 = 0.53$ for NGC 7027. Thus, the combination of these two effects would completely resolve the 10830 discrepancy.

In any discussion of the observed intensity of 10830, it is also important to realize that this line lies in a wavelength regime that is often near the tail of detector sensitivity, both for optical and IR detectors. The observational uncertainties can thus be appreciable, a fact which is usually ignored with respect to this problem. As an example, we note that recent observations of NGC 7027 by Rudy et al. (1992) found a value of $10830/\text{H}\beta$ equal to 1.66, a value greater than that predicted by theory (we derive a value of $\sim 1.30$ for this object, ignoring dust). This can be compared with the value 1.05 found by Scrimger (1984), which was used as the basis of the claim that the $2^3$ level is underpopulated. Although Scrimger's observations were made near the maximum of telluric absorption and the Rudy et al. observations were made near the minimum, this effect is insufficient to explain the observational discrepancy. Careful observations of 10830 are essential to resolve this problem.

Improvements upon this present study would require either more accurate parameters for the telluric lines and/or extremely high spectral resolution.
observations of 10830. The former is not expected soon and the latter will be
difficult, at least for faint objects.
Chapter V

An Observational Study of Collisional Effects in He I

1 Introduction

In the two preceding chapters, we have seen that the apparent discrepancy between the observed and predicted intensity of He I \( \lambda 10830 \) can be resolved for at least one object by a combination of dust effects, telluric absorption, and observational uncertainties. As alluded to in the last chapter, this discrepancy is important in that it called into question our knowledge of the processes populating the \( 2^3S \) level. Although the resolution of the 10830 problem is a significant advance in this area, there remain other indications that the \( 2^3S \) level is underpopulated. This is important, since the metastability of this level implies that collisions from it can substantially affect the intensity of several He I emission lines. We give a brief overview of the history of the collisional problem and the current state of affairs below.
Because helium abundances must be known to within a few percent for studies of galactic chemical evolution and primordial nucleosynthesis, the metastability of the $2^3S$ level of He I has been a topic of concern for decades. Cox and Daltabuit (1971) determined that collisional excitation of $\lambda\lambda 5876$ and $\lambda 4471$ could be important in gaseous nebulae. Their results were contested on theoretical grounds by Brocklehurst (1972) and on observational grounds by Peimbert and Torres-Peimbert (1971) and Barker (1978), who suggested that their collisional rate coefficients were too large by a factor of 3. More recently, Ferland (1986), using the results of an 11-state quantal calculation by Berrington et al. (1985, hereafter BBFK), showed that while these new rate coefficients were roughly a factor of 2 lower than Cox and Daltabuit’s rates at low temperatures ($\sim 5000$ K), they approached the Cox and Daltabuit values at high temperatures ($\sim 20,000$ K). Based on observations of the well-studied planetary nebula NGC 7027, Péquignot et al. (1988) determined that the BBFK rates were too large by a factor of 1.5–2.5. This result was largely confirmed by newer quantal calculations involving a 19-state R-matrix computation up to $n=4$ (Berrington and Kingston, 1987, hereafter BK). Both Peimbert and Torres-Peimbert (1987a, hereafter PTP1) and Clegg (1987) derived collision-to-recombination (C/R) correction factors for several lines using these results. Clegg showed for a sample of PNs that these corrections lowered the calculated He abundances by an average value of 10%.
There remains some uncertainty in these collisional rate coefficients, however. PTP1 found that for a sample of PNs, the He abundance determined from 6678 was discrepant with abundances determined from other lines. They then assumed that the C/R correction factors were multiplied by a factor $\gamma$, and proceeded to determine this factor by requiring that the 6678 abundance match that obtained from other lines. Their $\gamma$'s ranged from 0.25–0.45. In another paper, Peimbert and Torres-Peimbert (1987b, hereafter PTP2) found a discrepancy in the observed vs. predicted ratios of $\lambda 10830/\lambda 5876$ and $\lambda 7065/\lambda 5876$ for several PNs. Here they define $\gamma$ as the ratio of collisional plus radiative de-excitation of the $2^3S$ level to the total depopulation rate. They then varied the two parameters $\tau(3889)$ and $\gamma$ to obtain the best fit to these two line ratios. Their average value of $\gamma$ was 0.54. They attributed this discrepancy to an unknown mechanism depopulating the $2^3S$ level. Although several possible mechanisms have been explored, none have proven successful.

Although 10830 is the line most affected by collisions from $2^3S$, other lines will also be affected by this process, as detailed above. Since these lines are relatively free of the problems encountered with 10830, they provide an important alternate check on the population of the $2^3S$ state. In particular, since the collisional component of a line's intensity is directly proportional to the population of $2^3S$, any factor $\gamma$ multiplying the theoretical level population of this state will also be
multiplied by the C/R factor for each line. Thus, a comparison between the observed relative line intensities and theory should yield the value of $\gamma$, which in turn will tell us to what extent (if any) the $2^3S$ level is underpopulated. Recently, Sawey and Berrington (1993) have completed the next step in their He atom calculations. This involves a 29-state quantal calculation including levels up to $n=5$. In light of these new results, we wish to re-examine the C/R factors for other He I lines. To this end, we have obtained long-slit CCD spectra of two PNs, NGC 7027 and NGC 7026. The first of these is a hot, dense object, and is predicted to have a relatively large collisional enhancement. The second object is cooler and much less dense, and should therefore show only small collisional effects. We shall use the relative He I line intensities from this object to derive empirically the depopulation factor, $\gamma$. We shall also closely examine the errors encountered in each step of the reduction process to determine the uncertainties in the final results.

In §2, we discuss our observations. Data reduction, including reddening and telluric corrections, is discussed in §3. We present new formulae for the C/R factors in §4 based on the new data, and calculate the value of $\gamma$ from a comparison of these numbers with our observations. Finally, we discuss our results in §5.
2 Observations

We obtained long-slit CCD spectra of the PNs NGC 7027 and NGC 7026 on the nights of July 23, 26-28, and October 3-4, 1990. The observations were made with the 1.8m Perkins Telescope of the Ohio State and Ohio Wesleyan Universities at Lowell Observatory. The arrangement consists of a Texas Instruments 4849 virtual phase CCD with format 580 × 390 pixels, mounted on a Boller and Chivens spectrograph. For both objects, the slit was oriented in an east-west position through the center of the nebula. The slit length was 5 arcseconds (3 unvignetted) and the width was approximately 2 arcseconds. Our spectra of NGC 7026 included both lobes. Our data cover the wavelength range $\lambda 3850$–$\lambda 9650$, which required five separate grating tilts. The spectral resolution was $\sim 4.5\text{Å}$ FWHM, sampled with 2 arcsecond pixels. In addition, we took two lower resolution spectra ($\sim 9\text{Å}$ FWHM) covering the ranges $\lambda 4600$–$\lambda 7100$ and $\lambda 6650$–$\lambda 9150$ in order to connect the higher resolution data. Since we wish to measure the intensities of both strong and weak features, it was essential to insure linearity of the detector. Tests have shown that the device is linear to within a percent provided a standard 20 millisecond preflash is applied. It was also necessary to obtain several exposures of different lengths at each grating tilt. This resulted in saturation of some strong lines on exposures where faint lines were present. These saturated features were noted and avoided during measurements.
In addition to our objects, we also obtained for each night several bias frames, quartz lamp flat field exposures, FeNe calibration arcs, and twilight sky frames (or dome flats if we were unable to observe the sky). Exposures of standard stars for flux calibration were made using a 5 arcsecond slit width. The stars used were BD 40°4032, BD 28°4211, and Wolf 1346. The proximity of the two nebulae allowed us to use the same standard stars for both objects. We generally observed each of the first two stars listed for each object. However, the faintness of these stars in the red forced us to use only one of the stars per object in this region. The last star was used only for the redmost spectra.

3 Data Reductions

3.1 Basic Reductions

The spectra were reduced using the standard IRAF long-slit data reduction procedures. First, the average bias was determined from the overscan region of the CCD chip and subtracted from all frames. The result of this procedure on the bias frames themselves yielded the preflash. All bias frames were medianed and the result subtracted from the remaining frames. In general, we took at least three exposures of the objects and calibration frames for each exposure length. Frames of a given exposure length were then medianed to remove the effects of cosmic rays.
The pixel-to-pixel variations were removed by use of the quartz flat fields. Although our objects cover only a small portion of the total slit length, we still corrected for any possible variation in the response along the slit perpendicular to the dispersion direction. This was accomplished using the dome flats or sky exposures where available. Next, we extracted the objects, standard stars, and FeNe frames to produce one-dimensional images. For the objects, the entire nebula falling within the slit was extracted and binned together. Sky subtraction of the objects and stars was done during this step by fitting a simple polynomial to the background on either side of the object profile. We then used the FeNe spectra to calibrate the frames in wavelength. In general, wavelengths were accurate to less than 1 Å; however, there were somewhat greater errors in the red, due to a lack of lines for calibration. The wavelength correction for λ ≤ 4200 was quite poor, with the bluemost line off by nearly 30 Å. However, since there was no difficulty in identifying the lines, and we are not concerned with any kinematic data, this does not present a problem. The data were then extinction calibrated using a standard extinction curve derived for the telescope. Since both of our objects and the standard stars were observed at very low airmass, we do not expect that this step will cause any significant errors in the final relative line intensities. Finally, the spectra were flux-calibrated using the standard stars.
The above procedure resulted in a set of one-dimensional flux- and wavelength-calibrated spectra. We measured the intensities of all observable lines using an interactive IRAF package. In cases where two or more lines were blended, each line was fit individually by varying the input wavelength and width. Lines which were known to be blended, but in which the blend could not be resolved, were treated as a single line. In general, the deblending package works quite well, except in the case of a very weak line blended with a strong one. If a line was measured on several different exposures for a given grating tilt, the final value was obtained by weighting the different measurements according to the exposure time. Finally, all measured line intensities were determined with respect to H$\beta$. This process involved scaling all the spectra to the bluest spectrum containing H$\beta$. The scale factors were determined by comparing the strongest few lines in common between two spectral ranges, avoiding blends and lines affected by telluric absorption. The resulting intensities, relative to a value of H$\beta$=100.00, are given in Table 9. Here column 1 lists the line identification, including unresolved blends, column 2 gives the rest wavelength (or a rough value in the case of blended lines), and column 3 lists the relative intensities, with the value for NGC 7027 followed by that for NGC 7026.
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<th>Line ID</th>
<th>Wavelength (Å)</th>
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<th>$F_{\lambda}/F_{H\beta}$</th>
<th>% Tell. Abs.</th>
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TABLE 9 (cont.)

Line Intensities (Relative to Hβ = 100.00)

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**TABLE 9 (cont.)**  
Line Intensities (Relative to H\(\beta = 100.00\))

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TABLE 9 (cont.)

Line Intensities (Relative to $\text{H} \beta = 100.00$)

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<th>$F_\lambda/F_{\text{H} \beta}$</th>
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TABLE 9 (cont.)

Line Intensities (Relative to Hβ = 100.00)

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**TABLE 9 (cont.)**

Line Intensities (Relative to Hβ = 100.00)

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**TABLE 9 (cont.)**

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<th>Line ID</th>
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<th>% Tell. Abs.</th>
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<td>[S III]</td>
<td>9069.0</td>
<td>126.02/231.69</td>
<td>25.46/56.30</td>
<td>37/25</td>
</tr>
<tr>
<td>Pa9</td>
<td>9229.0</td>
<td>14.46/11.52</td>
<td>2.83/2.73</td>
<td>10/8</td>
</tr>
<tr>
<td>[S III]</td>
<td>9530.9</td>
<td>572.90/506.71</td>
<td>106.56/114.52</td>
<td>30/30</td>
</tr>
<tr>
<td>Pa8</td>
<td>9546.0</td>
<td>21.35/7.70</td>
<td>3.95/1.73</td>
<td>30/30</td>
</tr>
</tbody>
</table>
3.2 Extinction Corrections

Before we can compare our observations with theory, we must first determine the amount of interstellar reddening for each object. This requires a determination of both the shape, or wavelength dependence of the extinction curve, and the total amount of extinction.

For the behavior of the extinction curve with wavelength, we have utilized the formalism of Cardelli, Clayton, and Mathis (1989). They present a simple equation for the relative amount of extinction, expressed as \( A(\lambda)/A_\nu \), as a function of a single parameter, \( R_\nu \), the ratio of total to selective extinction. We use \( R_\nu = 3.1 \) for both objects.

In order to determine the total amount of extinction \( A_\nu \), one would ideally like to use two lines which are widely separated in wavelength and whose intensity ratio is well-known from theory. Unfortunately, although our data contain both H8 and Pa8, the former is strongly blended with He I \( \lambda 3889 \), while the latter is strongly affected by telluric absorption (see next subsection). Thus, we are forced to follow the usual route and use H lines whose relative intensity is derived from recombination theory. We used a method similar to that of Osterbrock et al. (1992) and determined \( A_\nu \) from an average of the ratios \( H\beta/H\alpha \) and \( (\text{Pa13} + \text{Pa12} + \text{Pa11})/H\gamma \). This choice has several advantages. First, these two ratios are complementary in that the former covers a relatively small wavelength range but is
accurately measured, whereas the latter is less accurately measured but covers the largest wavelength range possible from our data (the three Paschen lines redward of Pa11 all have non-negligible telluric absorption, while the Balmer lines blueward of Hγ have potential blending problems). Second, Mathis (1983) has shown that for dusty nebulae (as NGC 7027 is), determining the reddening by using Hβ/Hα alone could lead to serious errors for \( \lambda \geq 8000 \) Å. He attributes this to the differing dust albedos at the wavelengths of these two lines. The Paschen-line to Hγ ratio should be only slightly affected by this phenomenon. In any case, since the He lines of interest to us all have wavelengths less than 8000 Å, this should not be a concern.

We have determined the theoretical intensities of the Hβ/Hα and (Pa13 + Pa12 + Pa11)/Hγ ratios from the data of Hummer and Storey (1987) assuming a temperature of 10⁴K and a density of 10⁴ cm⁻¹. The precise values of \( N_e \) and \( T_e \) used are not critical to the final result, as these ratios are not strongly dependent on these parameters. Table 10 shows the results of this method. For NGC 7026, we obtain \( A_V = 2.22 \). We note that the final average result for NGC 7027 of \( A_V = 2.51 \) is significantly lower than that derived by other investigators (e.g. \( A_V = 3 \), Atherton et al., 1979). This discrepancy may reflect the uncertainty involved in connecting our different wavelength ranges. We will use the value derived here since it is self-consistent with our data. In any event, our final, reddening-corrected line intensities agree well with other observations of this object. These intensities are
listed in column 4 of Table 9, again relative to H\(\beta = 100.00\).

### 3.3 Corrections for Telluric Absorption

As discussed in Chapter 4, telluric absorption can significantly affect the observed intensities of lines in PNs. We have attempted to roughly correct for this effect, using essentially the same technique as that used for 10830 in the preceding chapter. For each spectral line measured in our objects, we determined the rest wavelengths. Then, based on the date of observation, and the object's coordinates and heliocentric radial velocity, we computed the geocentrically-shifted wavelengths. We then searched for all significant telluric lines in the vicinity of each line using the high-resolution solar spectra by Delbouille et al. (1973) and the compilation by Park et al. (1981). We discuss the evaluation of the telluric parameters below.

The line strength \(S\) was determined from Park et al. (1981) by scaling their values (expressed in different units) to those determined for 10830. The column density of precipitable water vapor was again determined from monthly means of data taken from Kitt Peak (cf. Wallace et al., 1984 and Wallace and Livingston, 1984), which should be reasonably similar to the conditions at the Perkins Telescope in Flagstaff. The airmasses were taken from the observing logs. Since very little data exist for the telluric line FWHM, we have assumed a reasonable value of \(\Gamma = 0.134\) Å for all lines.
# TABLE 10

## Determination of Amount of Extinction

<table>
<thead>
<tr>
<th>Line Ratios</th>
<th>Observed</th>
<th>Intrinsic</th>
<th>$A_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NGC 7027</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F(\text{H}\alpha)/F(\text{H}\beta)$</td>
<td>6.65</td>
<td>2.85</td>
<td>2.66</td>
</tr>
<tr>
<td>$F(\text{Pa}11)/F(\text{H}\gamma)$</td>
<td>0.1836</td>
<td>0.0294</td>
<td>2.25</td>
</tr>
<tr>
<td>$F(\text{Pa}12)/F(\text{H}\gamma)$</td>
<td>0.1318</td>
<td>0.0226</td>
<td>2.27</td>
</tr>
<tr>
<td>$F(\text{Pa}13)/F(\text{H}\gamma)$</td>
<td>0.1279</td>
<td>0.0178</td>
<td>2.57</td>
</tr>
<tr>
<td>$F(\text{Pa}11 + \text{Pa}12 + \text{Pa}13)/F(\text{H}\gamma)$</td>
<td>Mean</td>
<td>...</td>
<td>2.36</td>
</tr>
<tr>
<td>Final Average</td>
<td></td>
<td></td>
<td>2.51</td>
</tr>
<tr>
<td><strong>NGC 7026</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F(\text{H}\alpha)/F(\text{H}\beta)$</td>
<td>6.13</td>
<td>2.85</td>
<td>2.40</td>
</tr>
<tr>
<td>$F(\text{Pa}11)/F(\text{H}\gamma)$</td>
<td>0.1373</td>
<td>0.0294</td>
<td>1.96</td>
</tr>
<tr>
<td>$F(\text{Pa}12)/F(\text{H}\gamma)$</td>
<td>0.1068</td>
<td>0.0226</td>
<td>2.00</td>
</tr>
<tr>
<td>$F(\text{Pa}13)/F(\text{H}\gamma)$</td>
<td>0.0933</td>
<td>0.0178</td>
<td>2.16</td>
</tr>
<tr>
<td>$F(\text{Pa}11 + \text{Pa}12 + \text{Pa}13)/F(\text{H}\gamma)$</td>
<td>Mean</td>
<td>...</td>
<td>2.04</td>
</tr>
<tr>
<td>Final Average</td>
<td></td>
<td></td>
<td>2.22</td>
</tr>
</tbody>
</table>
Next it is necessary to determine the object line profiles. Due to the well-known expansion effect, lines in PNs are split into two components. Although the amount of splitting differs for each ion, with the higher stages of ionization showing less splitting, we used "average" expansion velocities (usually determined from [O III] lines) for each object. We took $V_{\text{exp}} = 18 \text{ km/s}$ for NGC 7027 and $V_{\text{exp}} = 42 \text{ km/s}$ for NGC 7026 from the compilation by Acker et al. (1982). Each line was then split into two components from its geocentrically-shifted position. As in Chapter 4, both components were assumed to be thermally-broadened Gaussians, with the total flux divided equally between the two. No account was made for fine structure (i.e. doublets, triplets, etc. were considered to be single lines). This essentially corresponds to Model 1 of Chapter 4.

For each spectral line in our objects, we calculated the integral in Eq.(17) for all nearby telluric lines, then summed the results. The percentage of telluric absorption, if different from zero, is given in column 5 of Table 9, again with the data for NGC 7027 first.

As discussed in Chapter 4, we do not expect that our results are of very high accuracy due to the lack of data for the telluric lines. However, the values given in Table 9 are useful for indicating which lines are affected by telluric absorption, and the relative importance of this effect. One might consider making an estimate of the accuracy by examining a line such as Pa8 that is affected. By using recombination
theory, one could determine an intensity by scaling from other, unaffected lines. In theory, comparing this scaled value and the observed value could empirically determine the amount of absorption. In practice, however, the observational uncertainties are such as to render this method useless.

Finally, we wish to emphasize again the importance of this effect for narrow-lined objects. In a study of the Orion nebula, Osterbrock et al. (1990) noted that Pa10 was anomalously weaker than Pa11, but attributed this to some unknown observational error. Although our data due not show such an effect due to the observational uncertainties, our calculations show that Pa10 should be strongly affected by telluric absorption, whereas Pa11 is only weakly so. Any nebular diagnostics using lines that are strongly affected by this phenomenon must therefore be viewed with an appropriate amount of skepticism.

4 Calculations

In this section, we shall determine the accuracy of the standard C/R correction factors by comparing our observed He I line intensities with those predicted by theory. We detail our calculations step by step below.
4.1 Physical Parameters

In order to calculate both the recombination intensities and the C/R correction factors, it is necessary to know both the temperature and density in the He\(^+\) zone in which these lines are produced. We begin our discussion with the density.

There exist several diagnostic line ratios in the optical for the determination of electron density, \(N_e\). A useful paper showing the variation of \(N_e\) for several lines ratios is Stanghellini and Kaler (1989, hereafter SK). These authors also give the range of applicability for the various line ratios. Of the four line ratios discussed in this paper, we have three present in our data: [Cl III] \(\lambda 5517-5537\), [S II] \(\lambda 6717-6730\), and [Ar IV] \(\lambda 4711-4740\). For NGC 7027, our [Cl III] ratio comes very close to the upper density limit for this ion, and is thus unreliable (this fact was noted by SK). This is also the case for the [S II] ratio. We are left then with the [Ar IV] ratio. As mentioned by SK, this ratio is very useful for high density PNs. Our data imply \(N_e \approx 2.5 \times 10^4\). Although this value is in good agreement with other determinations for this nebula listed in SK, we consider it to be a lower limit for the actual density due to blending of the [Ar IV] \(\lambda 4711\) line with He I \(\lambda 4713\). The observations of Keyes et al. (1990) show that the He I line has approximately 20% of the total intensity of the blend. If we modify 4711 for this effect, we obtain \(N_e \approx 3 \times 10^4\). We shall use this density in our subsequent calculations.
For NGC 7026, all three diagnostic ratios give reliable results. Both the [Cl III] ratio and the [S II] ratio give $N_e \sim 4 \times 10^3$, while the [Ar IV] ratio gives a very similar result of $N_e \sim 3 \times 10^3$. These results compare well with other listed values. We shall use the [S II] and [Cl III] value.

The temperature in the He$^+$ zone, $T_e(\text{He}^+)$, can be determined from $T_e(\text{He}^+) = T_e(\text{O}^{++})$, where the latter temperature is that in the zone producing the O III lines. Kaler (1986) provides a simple formula for determining this temperature, based on atomic data from the compilation by Mendoza (1983). This formula is a function of three parameters, $T_e$, $N_e$, and the ratio of the sum of [O III] $\lambda 5007$ and $\lambda 4959$ to [O III] $\lambda 4363$. However, the results are only weakly dependent on the values of the first two parameters, and we thus use $T_e = 10^4$ and the $N_e$ values determined above.

For NGC 7027, using our observed [O III] ratio, we obtain $T_e \sim 14,500$. This value is larger than most other determinations as listed in Kaler (1986). If we again make use of the Keyes et al. (1990) results, corrected for reddening as described above, we obtain a slightly lower value of $T_e \sim 14,000$. We believe that the cause of these high values is due to a problem in the observations. The lines [O III] $\lambda \lambda 4959, 5007$ are the two strongest lines in the optical for this object, which, in addition, is the brightest PN known. For our data, this meant that it was necessary to use extremely short integration times in order to avoid saturation of these lines.
Therefore, we were not able to measure these lines with as much accuracy as desired. Similarly, Keyes et al. (1990) take their intensities for these lines from photometric measurements, whereas [O III] $\lambda 4363$ comes from long-slit data. One way around this difficulty is to make use of an empirical formula derived by Kaler (1986),

$$T_e[O III] = 9700 + 58I(\lambda 4686),$$

where the line intensity of He II $\lambda 4686$ is expressed with respect to H$\beta$=100. Since 4686 is a strong line in NGC 7027, but not so strong as to suffer the problems discussed above with the [O III] lines, this formula should produce better results for this nebula. Indeed, our observations yield $T_e \sim 12,500$ using this method, which agrees better with other observations, as well as with the photoionization code results of Péquignot et al. (1988). We thus adopt this value.

For NGC 7026, the two [O III] lines are better determined, and thus we would not expect a similar problem as that found with NGC 7027. For this object, the [O III] lines give $T_e \sim 9500$, while the 4686 formula gives $T_e \sim 10,500$. We choose the lower value.

In summary, our adopted parameters are $N_e = 3 \times 10^4$ and $T_e = 12,500$ for NGC 7027 and $N_e = 4 \times 10^3$ and $T_e = 9500$ for NGC 7026.

### 4.2 Observational Data and Errors

We are now ready to begin with our empirical calculation of the depopulation factor $\gamma$. We shall consider nine He I lines for this purpose. These lines are $\lambda 3889$, $\lambda 4026$, $\lambda 4121$, $\lambda 4387$, $\lambda 4471$, $\lambda 4713$, $\lambda 4718$, $\lambda 5876$, and $\lambda 6678$. The results of our measurements are shown in Table 4.1. For each line, we have determined the observed intensity and the expected intensity based on the adopted parameters. The difference between these two intensities is due to the depopulation factor $\gamma$. We then use a least-squares fitting procedure to determine the best estimate of $\gamma$ for each line. The parameters $N_e$ and $T_e$ are assumed to be constant over the region of interest.

The results of this analysis are presented in Table 4.2, where we list the line, the observed intensity, the expected intensity, the difference, and the standard deviation of the measurement. We find that the results for the different lines are in good agreement, with a standard deviation of less than 5%.

The final step is to use these values of $\gamma$ to infer the electron density and temperature of the nebula. This is done by solving the empirical formulae for $N_e$ and $T_e$ in terms of $\gamma$. The results are shown in Table 4.3, where we list the line, the observed intensity, the expected intensity, the difference, and the standard deviation of the measurement. We find that the results for the different lines are in good agreement, with a standard deviation of less than 5%.
100

\(\lambda 4387, \lambda 4471, \lambda 4922, \lambda 5876, \lambda 6678, \lambda 7065,\) and \(\lambda 7281.\) We have already listed the intensities of these lines relative to H\(\beta.\) In the following, we shall attempt to determine the various errors in these intensities.

4.2.1 Scaling Errors

Unfortunately, it is extremely difficult to accurately determine errors in these relative intensities, since small errors can occur in virtually every step of the data reductions. We will consider two sources of error here. First, if a line was measured on more than one exposure, we determined an error in its intensity by comparing the intensity on the different exposures. Second, we determined errors due to the scaling described above by examining the intrinsic error in each scaling (done by comparing the scale factors determined from each strong line used in the calculation of the average scale factor for that particular grating tilt), and then propagating these errors through each subsequent spectral range. For the wavelength ranges \(\lambda 5007-\lambda 6118, \lambda 6300-\lambda 7281, \lambda 7320-\lambda 8467,\) and \(\lambda 8502-\lambda 9650,\) these scaling errors are 8\%, 12\%, 15\%, and 16\% for NGC 7027 and 5\%, 8\%, 10\%, and 20\% for NGC 7026.

4.2.2 Reddening Errors

We have determined the errors in our dereddening procedure by considering the observational uncertainties in the H lines used in our calculation of \(A_V\) in the last
section. These errors were then propagated throughout the reddening computations. In general, these errors amounted to roughly 10% on average. The percentage error is smaller for lines near the reference H/β, and larger far away from it.

4.2.3 Removal of Blends

Two of the lines that we wish to consider are strongly blended with other lines: 3889 with H8 3889, and 4026 with He II 4026. Our deblending technique involved determining the intensity of H8 and He II 4026 by using our observations and recombination theory. For H8, we used the observed intensities of Hδ, Hγ, Hβ, and Hα. Using the recombination intensities of Hummer and Storey (1987), we derived an average line intensity for the ratio H8/Hβ. This intensity, which amounts to roughly 50% of the total intensity of the blend for both nebulae, was then subtracted from the blend to yield He I 3889 relative to Hβ.

Likewise, in order to eliminate the He II 4026 line from the He I line at this wavelength, we used the He II lines λ4199 (for NGC 7027 only), λ4541, λ4686, and λ5411. We again made use of the relative recombination intensities by Hummer and Storey (1987). The resulting average intensity was roughly 25% of the blend for NGC 7027, but only 4% of the blend for NGC 7026, due to its lower abundance of He II.
Our errors were determined by propagating the observational uncertainties in the lines used in determining the deblending. As can be expected, the deblending process increased the percentage errors in both of these lines. We should note that we did not consider the errors in the recombination intensities. All our calculations assumed $N_e = 10^4$ and $T_e = 10^4$. Although we could have interpolated from the Hummer and Storey tables, these values change by only a few percent over the range of interest in these parameters. These errors are sufficiently small compared to the total observational errors that they can safely be neglected.

4.2.4 Final Observational Intensities and Errors

In order to examine the He I lines without resorting to a photoionization model, it is necessary to express their intensities relative to a given He I line. We choose the line 4922 as our reference line. This line has the advantage of being reasonably well measured, and also having a very small collisional enhancement. Another advantage of using this reference line will be seen later on.

In Tables 11 & 12, we list the final intensities of the He I lines, relative to $I(\lambda 4922) = 1.00$, for NGC 7027 and NGC 7026, respectively. In both tables, column 1 gives the wavelength and transition for each line. Column 2 gives the observed intensities, along with their errors (given in parentheses). Most of the errors are at the 10% level. The intensities of 5876, 7065, and 7281 in NGC 7027
and of 7281 in NGC 7026 have been tentatively corrected for telluric absorption. No account has been made of this correction in the listed errors.

4.3 Theoretical C/R Factors

In this section, we will derive theoretical C/R factors to compare with the empirical factors calculated above. Since this will involve the calculation of new formulae, we wish to first briefly discuss their derivation. More detailed explanations are given by Clegg (1987) and PTP1.

For any given line, the ratio of the collisional component to that arising from recombination is given by

\[
\frac{C}{R} = \frac{N_{2^3S}k_{\text{eff}}}{N_{\text{He}^+}\alpha_{\text{eff}}}
\]  

(20)

where \(N_{2^3S}\) and \(N_{\text{He}^+}\) are the densities of the \(2^3\)S state and \(\text{He}^+\), respectively, \(\alpha_{\text{eff}}\) is the effective recombination coefficient for the line, and \(k_{\text{eff}}\) is the effective collisional rate coefficient, including the appropriate branching ratios. Note that for each line, \(k_{\text{eff}}\) may consist of a series of terms.

If we neglect photoionization (see Clegg and Harrington, 1989, for a discussion of this effect), the ratio of the densities in Eq.(20) can be written

\[
\frac{N_{2^3S}}{N_{\text{He}^+}} = \frac{N_e\alpha_B}{A_{21} + N_e q_{\text{tot}}}
\]  

(21)

where \(N_e\) is the electron density, \(\alpha_B\) is recombination coefficient to all triplet levels,
and \( A_{21} \) is the radiative decay rate from \( 2^3S \) to \( 1^1S \). The term \( q_{\text{tot}} \) consists of collisional transfers from \( 2^3S \) to all singlet levels plus collisional ionization.

We used \( A_{21} = 1.13 \times 10^{-4} \text{s}^{-1} \) (Hata and Grant, 1981). We then used power-law fits to the other parameters, which are generally valid over the range \( t_4 = 0.8-2.0 \) (Here and subsequently, we define \( t_4 = T/10,000 \)). For \( \alpha_B \), using the tables in Osterbrock (1989), we obtained \( \alpha_B = 2.03 \times 10^{-13} t_4^{-0.69} \text{cm}^3/\text{s} \). Values for \( \alpha_{\text{eff}} \) were taken from Smits (1991) for all lines except 4387 and 7281, for which the earlier Brocklehurst (1972) data were used. We assumed a temperature of \( t_4 = 1.0 \) for these calculations. We obtained the collisional ionization part of \( q_{\text{tot}} \) from Table 1 of Clegg (1987).

Recently, Sawey and Berrington (1993) have completed a 29-state quantal calculation of He I, including states up to \( n=5 \). We have made power-law fits to the collision strengths from their data. Analogously to Clegg (1987), we used the full rates up to \( n=4 \), but took 50% of the values for \( n=5 \). The new data has allowed us to improve upon previously calculated formulae, such as those of Clegg (1987) and PTP1, as well as to derive new C/R factors for 4026, 4387, 4922, and 7281.

We present the new C/R formulae below. In order to calculate the appropriate branching ratios, we have made use of data from the ongoing Opacity Project. The formulae contain only significant terms; in general, terms comprising less than 1% of the total were ignored. As mentioned by Clegg (1987), these formulae are only
strictly correct in objects for which photoionization of the \(2^3S\) state is negligible. In all of the formulae, the denominator \(D\) is equal to \((1 + 3130t_4^{-0.50}N_e^{-1})\).

For 3889, we used the collision terms to \(3^3P, 4^3S, 4^3D, 5^3S,\) and \(5^3D\). This gives

\[
\frac{C}{R}(3889) = (10.06t_4^{-1.01}e^{-3.699/t_4} + 1.77t_4^{-0.88}e^{-4.379/t_4} + 0.89t_4^{-0.19}e^{-4.545/t_4} + 0.55t_4^{-1.14}e^{-4.818/t_4} + 0.42t_4^{-0.45}e^{-4.900/t_4})/D
\]

(22)

For 4026, only the \(5^3D\) term contributes, yielding

\[
\frac{C}{R}(4026) = 6.82t_4^{-0.15}e^{-4.900/t_4}/D
\]

(23)

Likewise, for 4387, only the \(5^1D\) term is appreciable.

\[
\frac{C}{R}(4387) = 3.98t_4^{-0.60}e^{-4.900/t_4}/D
\]

(24)

For 4471, we used the \(4^3D\) term, along with the new \(5^3P\) and \(5^3F\) terms to obtain

\[
\frac{C}{R}(4471) = (7.00t_4^{0.17}e^{-4.545/t_4} + 0.23t_4^{-0.53}e^{-4.884/t_4} + 0.99t_4^{-0.43}e^{-4.901/t_4})/D
\]

(25)

For 4922, we used the \(4^1D\) and \(5^1F\) terms to obtain

\[
\frac{C}{R}(4922) = (3.89t_4^{-0.34}e^{-4.545/t_4} + 0.32t_4^{-0.75}e^{-4.901/t_4})/D
\]

(26)

For 5876, we considered the \(3^3D, 4^3F,\) and \(5^3F\) terms which yielded

\[
\frac{C}{R}(5876) = (6.99t_4^{0.10}e^{-3.776/t_4} + 1.73t_4^{-0.12}e^{-4.545/t_4} + 0.63t_4^{-0.31}e^{-4.901/t_4})/D
\]

(27)
Analogously, for 6678, only the $3^1D$, $4^1F$, and $5^1F$ terms were appreciable.

$$\frac{C}{R}(6678) = (3.23t_4^{-0.52}e^{-3.776/t_4} + 0.52t_4^{-0.49}e^{-4.545/t_4} + 0.20t_4^{-0.64}e^{-4.901/t_4})/D \quad (28)$$

For 7065, we used only the $3^3S$ and $3^3P$ terms to obtain

$$\frac{C}{R}(7065) = (38.09t_4^{-1.11}e^{-3.364/t_4} + 2.80t_4^{-1.08}e^{-3.699/t_4})/D \quad (29)$$

Finally, for 7281, only the $3^1S$ term was appreciable, giving

$$\frac{C}{R}(7281) = 32.33t_4^{-1.52}e^{-3.698/t_4}/D \quad (30)$$

We list the theoretical C/R factors and their uncertainties in column 3 of Tables 11 & 12. The uncertainties in the C/R factors were obtained by considering how a 50% change in $N_e$ and a 10% change in $T_e$ would affect the results. These percentages were determined by comparing values of $N_e$ and $T_e$ as determined by other observers for these objects. An inspection of these values shows that the uncertainties can be rather large. In particular, the percentage uncertainties are significantly greater for NGC 7026 than for NGC 7027. This is due to the fact that for high density objects such as NGC 7027, the collisional effects are almost independent of density. Thus, the uncertainties in C/R for NGC 7026 reflect the uncertainties in both $N_e$ and $T_e$, while those for NGC 7027 only reflect the uncertainties in $T_e$. 
4.4 Recombination Intensities

Since the total observed intensity of each line we are considering is a sum of the recombination plus collisional component, we must determine the former in order to measure C/R. For this we used the data of Smits (1991) for all lines except 4387 and 7281, which Smits does not list. The recombination intensities for these two lines were taken from the earlier tables by Brocklehurst (1972). With the exception of lines in the series $2^3P - n^3S$, the data of Smits is in excellent agreement with that of Brocklehurst. It was necessary to interpolate these tables to our adopted densities and temperatures. The interpolation in the density was done linearly, as the line intensities are not strongly dependent on this parameter. For the temperature, we assumed an equation of the form $I(N_e, T_e, n', n) = A(N_e, n', n)T_4^\beta$, where $n'$ and $n$ denote the upper and lower levels of the transition, $T_4$ is the temperature in units of $10^4$, and $A$ and $\beta$ are parameters to be solved for. Note that we assumed Case B for the singlet lines.

We determined errors in the recombination intensities analogously to those for the theoretical C/R values, that is, by considering how a 50% change in $N_e$ and a 10% change in $T_e$ would change our results. In general, the errors are less than a few percent, although the error in 3889 is somewhat larger due to a stronger temperature dependence of this line. Our results are given in column 4 of Tables 11 & 12.
TABLE 11

Determination of $\gamma$ for NGC 7027

<table>
<thead>
<tr>
<th>Line ID</th>
<th>Observed</th>
<th>C/R (Theor.)</th>
<th>Rec.</th>
<th>Self-Abs.</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3889</td>
<td>9.80</td>
<td>0.46</td>
<td>8.74</td>
<td>0.83</td>
<td>1.06</td>
</tr>
<tr>
<td>$2^3S-2^3P$</td>
<td>(2.34)</td>
<td>(0.13)</td>
<td>(0.44)</td>
<td>(0.24)</td>
<td>(1.57)</td>
</tr>
<tr>
<td>4026</td>
<td>2.16</td>
<td>0.12</td>
<td>1.83</td>
<td>1.00</td>
<td>19.00</td>
</tr>
<tr>
<td>$2^3P-5^3D$</td>
<td>(0.24)</td>
<td>(0.05)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(133.63)</td>
</tr>
<tr>
<td>4387</td>
<td>0.46</td>
<td>0.06</td>
<td>0.47</td>
<td>1.00</td>
<td>0.33</td>
</tr>
<tr>
<td>$2^1P-5^1D$</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.002)</td>
<td>(0.01)</td>
<td>(1.72)</td>
</tr>
<tr>
<td>4471</td>
<td>3.76</td>
<td>0.20</td>
<td>3.72</td>
<td>1.01</td>
<td>0.09</td>
</tr>
<tr>
<td>$2^3P-4^3D$</td>
<td>(0.15)</td>
<td>(0.09)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>4922</td>
<td>1.00</td>
<td>0.09</td>
<td>1.00</td>
<td>1.01</td>
<td>—</td>
</tr>
<tr>
<td>$2^1P-4^1D$</td>
<td>(0.00)</td>
<td>(0.04)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>—</td>
</tr>
<tr>
<td>5876</td>
<td>12.14</td>
<td>0.37</td>
<td>9.68</td>
<td>1.01</td>
<td>0.96</td>
</tr>
<tr>
<td>$2^3P-3^3D$</td>
<td>(1.80)</td>
<td>(0.14)</td>
<td>(0.13)</td>
<td>(0.02)</td>
<td>(0.92)</td>
</tr>
<tr>
<td>6678</td>
<td>3.45</td>
<td>0.14</td>
<td>2.76</td>
<td>1.01</td>
<td>8.33</td>
</tr>
<tr>
<td>$2^1P-3^1D$</td>
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<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(18.07)</td>
</tr>
<tr>
<td>7065</td>
<td>9.32</td>
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<td>1.93</td>
<td>1.50</td>
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</tr>
<tr>
<td>$2^3P-3^3S$</td>
<td>(1.69)</td>
<td>(0.47)</td>
<td>(0.12)</td>
<td>(0.77)</td>
<td>(1.15)</td>
</tr>
<tr>
<td>7281</td>
<td>1.36</td>
<td>1.18</td>
<td>0.56</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$2^1P-3^1S$</td>
<td>(0.26)</td>
<td>(0.26)</td>
<td>(0.03)</td>
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</tr>
</tbody>
</table>
### TABLE 12

**Determination of \( \gamma \) for NGC 7026**

<table>
<thead>
<tr>
<th>Line ID</th>
<th>Observed</th>
<th>C/R (Theor.)</th>
<th>Rec.</th>
<th>Self-Abs.</th>
<th>( \gamma )</th>
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<tbody>
<tr>
<td>3889</td>
<td>7.62</td>
<td>0.14</td>
<td>7.79</td>
<td>0.85</td>
<td>1.33</td>
</tr>
<tr>
<td>( 2^3S - 2^3P )</td>
<td>(1.34)</td>
<td>(0.08)</td>
<td>(0.26)</td>
<td>(0.14)</td>
<td>(2.50)</td>
</tr>
<tr>
<td>4026</td>
<td>2.53</td>
<td>0.02</td>
<td>1.80</td>
<td>1.00</td>
<td>—</td>
</tr>
<tr>
<td>( 2^3P - 5^3D )</td>
<td>(0.22)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>—</td>
</tr>
<tr>
<td>4387</td>
<td>0.68</td>
<td>0.01</td>
<td>0.47</td>
<td>1.00</td>
<td>-23.00</td>
</tr>
<tr>
<td>( 2^1P - 5^1D )</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.002)</td>
<td>(0.01)</td>
<td>(34.50)</td>
</tr>
<tr>
<td>4471</td>
<td>3.57</td>
<td>0.04</td>
<td>3.70</td>
<td>1.00</td>
<td>-1.50</td>
</tr>
<tr>
<td>( 2^3P - 4^3D )</td>
<td>(0.20)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(4.24)</td>
</tr>
<tr>
<td>4922</td>
<td>1.00</td>
<td>0.02</td>
<td>1.00</td>
<td>1.01</td>
<td>—</td>
</tr>
<tr>
<td>( 2^1P - 4^1D )</td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>—</td>
</tr>
<tr>
<td>5876</td>
<td>11.01</td>
<td>0.08</td>
<td>9.94</td>
<td>1.01</td>
<td>1.83</td>
</tr>
<tr>
<td>( 2^3P - 3^3D )</td>
<td>(1.09)</td>
<td>(0.06)</td>
<td>(0.14)</td>
<td>(0.01)</td>
<td>(2.59)</td>
</tr>
<tr>
<td>6678</td>
<td>3.06</td>
<td>0.04</td>
<td>2.85</td>
<td>1.01</td>
<td>3.50</td>
</tr>
<tr>
<td>( 2^1P - 3^1D )</td>
<td>(0.38)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(8.75)</td>
</tr>
<tr>
<td>7065</td>
<td>3.75</td>
<td>0.68</td>
<td>1.69</td>
<td>1.45</td>
<td>0.85</td>
</tr>
<tr>
<td>( 2^3P - 3^3S )</td>
<td>(0.59)</td>
<td>(0.34)</td>
<td>(0.07)</td>
<td>(0.42)</td>
<td>(0.92)</td>
</tr>
<tr>
<td>7281</td>
<td>0.73</td>
<td>0.44</td>
<td>0.50</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>( 2^1P - 3^1S )</td>
<td>(0.17)</td>
<td>(0.21)</td>
<td>(0.02)</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
4.5 Correction for Self-Absorption

Another effect of the metastability of the $2^3S$ level is that lines resulting from transitions $2^3S-n^3P$ can have large optical depths. If such lines are scattered many times, it is possible for them to be converted to other lines via resonance fluorescence. A well-known example is 3889 ($2^3S-3^3P$), which can be converted to an infrared line at 4.3μ ($3^3S-3^3P$) plus 7065. Thus 3889 is weakened as a result of this process, while 7065 is strengthened. Similar processes resulting from absorption of higher $2^3S-n^3P$ lines can produce lesser effects in other He I lines such as 5876.

The usual method of correcting line intensities for this effect is to use tables to determine $\tau(3889)$, the optical depth in 3889, based on line intensity ratios. One then uses this value and the tables to correct all lines by comparing their intensity at this optical depth to that for $\tau(3889) = 0.0$.

One set of tables commonly used are those of Robbins (1968). He lists several line intensity ratios as a function of $\tau(3889)$ and $V_{exp}/V_{ther}$, the nebular expansion velocity divided by the thermal velocity, for $N_e = 10^4$ cm$^{-3}$ and $T_e = 10^4$ K & $2\times10^4$ K. Unfortunately, these tables are rather old, but the values for $\tau(3889) = 0.0$ are reasonably close to those given by Smits (1991).

At this point, we are faced with a major problem. The Robbins tables show how the recombination line intensity ratios vary, whereas the observed line ratios contain both recombination plus collisional components. In order to use the tables,
then, one would have to divide each line by a factor $1 + C/R$. However, it is the $C/R$ factors that we are attempting to verify. Any assumption for these values would result in a circular argument. To make matters worse, the lines which are most affected by self-absorption and are therefore the best to measure this effect are also strongly collisionally enhanced. In theory, since the “true” $C/R$ factors are simply $\gamma$ times the theoretical values, if two lines have the same value for $C/R$, then the effect of collisions would effectively drop out of the calculations. Unfortunately, as Tables 11 & 12 show, the only line ratios which satisfy this requirement are useless for measuring the self-absorption.

We are thus forced to try an alternate approach. For any observed line ratio, $I_{\text{obs}}^1/I_{\text{obs}}^2$, the recombination ratio, which is used in the Robbins tables, can be written as

$$\frac{I_{\text{rec}}^1}{I_{\text{rec}}^2} = \frac{I_{\text{obs}}^1}{I_{\text{obs}}^2} \times \frac{1 + \gamma(C/R)_2}{1 + \gamma(C/R)_1} \quad (31)$$

Therefore, by using two separate line ratios, we can determine what value of $\gamma$ will result in the same measured optical depth $\tau(3889)$. This is essentially the method used by PTP2.

For NGC 7027, we took $V_{\exp}/V_{\text{ther}} = 3$ and interpolated to our adopted $T_\ast$ and $N_\ast$. Although the line ratios $3889/4471$ and $7065/4471$ are the preferred choice for measuring self-absorption, we have avoided the latter because of the uncertain telluric correction. Although this correction is relatively small, 7065 depends
strongly on optical depth, so that even a minor error in the intensity can result in a substantial error in the derived optical depth and corresponding self-absorption correction. We therefore choose 3889/4471 and 3889/5876. Since 5876 depends only weakly on optical depth, the 2% telluric correction should have essentially no effect. These two line ratios predicted an optical depth of $\tau(3889) = 6.22 \pm 11.79$ for a value of the depopulation factor $\gamma$ of $1.17 \pm 0.89$. This optical depth was then used to correct all triplet lines.

For NGC 7026, we used $V_{exp}/V_{ther} = 5$, although the actual value is somewhat higher. In this case, the absence of telluric absorption in 7065 allowed us to use 3889/4471 and 7065/4471 as our two ratios. These gave $\tau(3889) = 8.31 \pm 8.29$ for $\gamma = 0.82 \pm 0.31$. Again, the mean value was used to correct all triplet lines.

We determined the errors in these self-absorption corrections in several steps. First, we combined the observational uncertainties in the lines and the uncertainties in the $C/R$ factors to derive the errors in the 3889/4471, 3889/5876, and 7065/4471 ratios used to determine $\tau(3889)$. Second, we used the tables to translate these uncertainties into uncertainties in the adopted mean optical depths. Finally, we again used the tables to translate these errors into uncertainties in resulting corrections. Although the uncertainties in $\tau(3889)$ obtained by this method are quite substantial, the weakness of the self-absorption effect on most lines insures that the resulting errors in the correction factors are relatively small.
Self-absorption is also present in the singlet lines. This effect is due to absorptions in $1^1S-n^1P$, in which the upper level can decay to other states, thus enhancing the lines formed by those transitions. Robbins and Bernat (1973) have compiled a similar set of tables for this effect, listing line intensity ratios as a function of $V_{exp}/V_{ther}$ and the optical depth in 584 ($1^1S-2^1P$). We are again faced with the same problem as with the triplets, that is, the self-absorption effects are coupled with the collisional effects. Unfortunately, since 7281 is the only observed line for which the self-absorption is appreciable, and this line is strongly affected by telluric absorption in both objects, we have no accurate way to measure the amount of self-absorption. Since Robbins and Bernat (1973) state that our remaining singlet lines should show almost no self-absorption effects, we discard 7281 from our calculations and adopt "reasonable" corrections for the other singlet lines.

The self-absorption correction factors and their accompanying uncertainties are listed in column 5 of Tables 11 & 12. These corrections are to be multiplied by the recombination intensities, or divided into the observational intensities.

Although our measurement of self-absorption in the triplets has allowed us to determine $\gamma$, we will make use of these corrections to determine $\gamma$ individually for each line in the next section.
4.6 Depopulation of the $2^3S$ State

We are now ready to determine whether the $2^3S$ level is depopulated and to what extent by deriving the depopulation factor $\gamma$ from our data. First, we divide the observed intensity ratios by the recombination intensity ratios corrected for self-absorption. This gives us, for each line ratio, an equation of the form

$$\frac{1 + \gamma(C/R)_1}{1 + \gamma(C/R)_2} = \text{const.}$$

from which $\gamma$ is readily determined.

The results, along with the accompanying uncertainties, are listed in column 6 of Tables 11 & 12. Obviously, negative values have no meaning and only reflect the uncertainties. We shall discuss these values and the anomalously high values in the next section. Note that the value of $\gamma$ derived from 4026 and 4387 shows the largest discrepancy with theory for both objects and has the largest percentage errors for NGC 7027. These lines suffer from several errors: they are the weakest lines included in the derivation, they have the smallest predicted C/R values, and the closeness of their C/R values with that of the reference line is also a problem (see the next section). Furthermore, the theoretical C/R values for these lines are especially uncertain, since the only available collisional rates involve the uncertain $n=5$ levels. It is expected that collisions to the $n=6$ levels could make a significant increase to the C/R values for these lines. We thus reject 4026 and 4387 from our
determination of an average value of $\gamma$. If we do a simple average of the remaining values, we obtain $\gamma = 2.36 \pm 3.64$ for NGC 7027 and $\gamma = 1.20 \pm 2.08$ for NGC 7026. The averages obtained by this and other methods are listed in Table 13 for convenience. A more realistic average can be calculated by taking into account the individual uncertainties in the lines. We calculate averages in which the individual $\gamma$'s are weighted in inverse proportion to their errors. These results are listed as "Weighted Average 1" in Table 13. Another major source of error involves lines in which the collision enhancement is small, and the C/R value is close to that of our reference line, 4922. In order to correct for this effect, we weight each line by a factor discussed in the next section. These results are listed as "Weighted Average 2" in Table 13. Finally, we weight each line by a combination of the two error sources. We choose this "doubly-weighted" average as our best result, which gives $\gamma = 1.21 \pm 0.70$ for NGC 7027 and $\gamma = 1.16 \pm 1.00$ for NGC 7026.

5 Discussion and Conclusions

A cursory glance at column 6 of Tables 11 & 12 reveals both a large scatter in the results and appreciable uncertainties. We shall discuss the reasons for this below, beginning first with the anomalous values of $\gamma$.

It is clear from an examination of Eq.(32) that when $(C/R)_1$ and $(C/R)_2$ are very close, the value of $\gamma$ is poorly determined. Indeed, in the limit in which $(C/R)_1$
## TABLE 13

Average values of $\gamma$

<table>
<thead>
<tr>
<th></th>
<th>NGC 7027</th>
<th>NGC 7026</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Self-Absorption</td>
<td>$1.17 \pm 0.89$</td>
<td>$0.82 \pm 0.31$</td>
</tr>
<tr>
<td>Straight Average</td>
<td>$2.36 \pm 3.64$</td>
<td>$1.20 \pm 2.08$</td>
</tr>
<tr>
<td>Weighted Average 1</td>
<td>$1.99 \pm 2.37$</td>
<td>$1.25 \pm 1.55$</td>
</tr>
<tr>
<td>Weighted Average 2</td>
<td>$1.14 \pm 0.70$</td>
<td>$1.15 \pm 1.24$</td>
</tr>
<tr>
<td>Weighted Average 3</td>
<td>$1.21 \pm 0.70$</td>
<td>$1.16 \pm 1.00$</td>
</tr>
</tbody>
</table>
equals \((C/R)_2\), the right-hand-side of the equation goes to 1 and we can say nothing about \(\gamma\). This is the situation for 4026 in NGC 7026. If we solve Eq.(32) for \(\gamma\), we get

\[
\gamma = \frac{c - 1}{(C/R)_1 - c(C/R)_2}
\]  

(33)

where \(c\) denotes the constant. It is clear that for certain values of \((C/R)_1\), \((C/R)_2\), and \(c\), the derived \(\gamma\) can be negative. In practice, when \((C/R)_1\) and \((C/R)_2\) are very close, this can occur for very slight observational errors in determining \(c\). This is the case for 4387 and 4471 in NGC 7026. Further, from Eq.(33) it can be seen that as \((C/R)_1/(C/R)_2\) approaches \(c\), \(\gamma\) goes asymptotically to \(\pm \infty\). Again, in practice, this effect is more pronounced for \((C/R)_1\) and \((C/R)_2\) very close in value. This effect results in the anomalously large (both positive and negative) values in column 6 of Tables 11 & 12. This is one of the main reasons that we chose 4922 as a reference line rather than a stronger, but more collisionally-affected line such as 5876. Since the \(C/R\) value for 4922 is small, the above effect will only affect lines with similarly small collisional enhancement. These lines will have greater intrinsic error anyway, as discussed below. Had we chosen the more commonly-used 4471 as our reference line, stronger lines such as 5876 and 3889 would have been overestimated due to this problem. Although one obvious way to avoid this effect would be to use 7065 as a reference line, since its \(C/R\) value is significantly above the other lines, the problems of uncertain self-absorption and telluric effects in NGC 7027 have negated this
choice. In order to correct our average for this problem, we have weighted all lines by a factor \((1 - c(C/R)_a/(C/R)_b)^2\). This factor goes to zero when the denominator in Eq.(33) is zero, and approaches 1 for lines which are strongly collisionally enhanced.

Column 6 shows that even for lines not affected by the above, the accompanying errors can be appreciable. Although these uncertainties result from each step in the procedure as described above, we shall focus here on the two main sources of error.

First, as is evident from column 3, the uncertainties in the theoretical C/R values are significant. As discussed previously and as can be seen from comparing Tables 11 & 12, these errors can be reduced by including only high density \((N_e\geq2\times10^4)\) objects in the analysis. However, the exponential dependence of the C/R factors on \(T_e\) can still cause appreciable errors. This is clearly seen in Fig. 2 of PTP1, which shows the behavior of \(1 + C/R\) as a function of temperature for several lines.

Second, although we “observe” the quantity \(1 + C/R\), it is the value \(C/R\) that is needed to determine \(\gamma\). This can result in a significant increase in the relative error for lines with small to moderate collisional enhancement. To demonstrate this more clearly, let us assume for simplicity that we have a fictitious reference line with zero collisional enhancement. Eq.(32) then becomes

\[
\text{const.} = 1 + \gamma(C/R)_1
\]
If a line whose collisional enhancement we wish to measure has $C/R = 0.5$, the percentage error in the constant will increase by a factor of $1.5/0.5 = 3$ when the 1 is subtracted. Thus, even an observational uncertainty as low as 10% will be increased to 30% in this step. Mathematically, the problem results from trying to measure $C/R$ from the quantity $1 + C/R$, where often $C/R \ll 1$. Physically, this translates into the common sense result that the smaller the collisional effect, the more uncertain its measurement. This can be seen in our data by comparing the entries in column 6 between Tables 11 & 12. The percentage errors are much greater in NGC 7026 due to the relative weakness of collisional effects in this object. This was expected from the onset; we have included this object solely to demonstrate the uncertainties acquired in the determination of $\gamma$.

Similarly, we can see that our other method for determining $\gamma$, by self-absorption, also results in appreciable errors. This is largely due to the observational errors in the line ratios used.

It is difficult to imagine how one could substantially decrease the uncertainties obtained in this study, but we make some suggestions below. First, it is essential to observe only high density objects, but even these will result in appreciable uncertainties due to the strong temperature dependence of the collisional effects. Second, the preceding discussion makes it obvious that only strongly collisionally-enhanced lines should be considered, and that these lines should be
observed with respect to a line showing very small enhancement. Unfortunately, the
lines most affected by collisions suffer from a host of other problems. The line which
is the most enhanced by collisions, 10830, can be substantially affected by dust and
telluric absorption, and is also somewhat of an observational challenge. Both 7065
and 3889 suffer from uncertainties in self-absorption, with 3889 having the
additional problem of blending with H8, and 7065 subject to possible telluric
absorption. The singlet line 7281 also has non-negligible self-absorption and telluric
effects. In addition, unlike 6678, there is a significant difference between the Case A
and Case B recombination intensity. Our data suggest that \( \gamma \) is most accurately
determined from 7065, but any observations of this line should be carefully planned
to avoid telluric contamination. We note here that our best average for NGC 7027 is
slightly weighted toward 7065, which has an uncertain telluric correction. This
uncertainty will affect the derived \( \gamma \). If we had made no correction for telluric
absorption, for example, our best average would drop to 1.10 ± 0.65.

Finally, we wish to remark on the discrepancy between our results and those
of PTP2. As mentioned previously, PTP2 used the line ratios 10830/5876 and
7065/5876 to derive \( \gamma \) from a consideration of self-absorption. We have already
discussed the inherent problems with the 10830 line. In addition, the \textit{recombination}
intensity for 7065 was underestimated by Brocklehurst (1972). The new data from
Smits (1991) is roughly a factor of 1.4 higher. Thus, previous determinations of the
collisional enhancement of this line have been overestimated by the same factor. This would cause the derived $\gamma$ for this line to be only 70% of its actual value.

In summary, our best average $\gamma$ for NGC 7027 implies that there is no depopulation of the $2^3S$ state, although the appreciable errors make this result somewhat uncertain. However, we argue that any attempts to observationally determine $\gamma$ will suffer from similar uncertainties. We therefore feel that there is no compelling observational evidence for any depopulation of the $2^3S$ level. This is supported by the failure to discover any such mechanisms theoretically. Thus, we suggest that measured He I line intensities should be corrected for collisions by using the formulae in this dissertation with $\gamma = 1$. 
Chapter VI

Summary and Conclusions

Since we have discussed our conclusions for each individual chapter already, we shall summarize the most important points here. We divide our conclusions into three main areas, although there is some degree of overlap among them.

1 Dust in Gaseous Nebulae

We have calculated charge exchange rate coefficients using the Landau-Zener approximation for reactions involving several ions of Al and Ca with neutral H. Although these rates are probably only accurate to within a factor of 3 or so, they should provide an improvement in determining the ionization structure of these elements. Based on observations and photoionization models, we have shown that Al and Ca exhibit significant depletions relative to solar values in both PNs and H II regions. Since both these elements are expected to be condensable onto grains, the calculated depletions lend further support to the presence of dust within gaseous nebulae. In particular, they imply that at least some fraction of the dust grains lie
within the ionized zone.

Our calculation for Al is significant in that it represents the first depletion of this element measured from Al emission lines. We suggest that further IUE observations of Al II λ2660,2669 be made for other PNs to verify our results, and to increase the number of objects for which Al depletions are measured.

The three order of magnitude depletion of Ca obtained here suggests the possibility of a depletion gradient in this element. Whether or not such a gradient exists will depend on future improvements in the atomic database such as dielectronic recombination rate coefficients.

We also note that our calculations suggest that the grains present in PNs are as efficient as those in the general ISM in depleting Ca, but not as efficient in depleting Al. Similar comparisons for other condensable elements may put important constraints on the nature of dust grains in these objects.

The existence of dust in the ionized zone of gaseous nebulae has important ramifications for the study of these objects. We have calculated a photoionization model of a standard test case PN including dust effects. We cite two of our more important finding here.

First, an examination of grain drift velocity shows that dust grains move a negligible amount with respect to the gas during a typical PN lifetime. This implies that such dynamical effects do not play a role in removing the grains from the
ionized region. Second, we have shown that for a dust-to-gas ratio equal to that of the general ISM, dust cooling through recombination amounts to roughly 10% of the total cooling rate. More importantly, grain heating by photoejection can amount to a significant fraction of the total heating rate, especially in the innermost regions of the nebula. Therefore, grain heating and cooling must be included in any accurate modeling of dusty nebulae.

We have also examined the effects of dust on radiative line transfer within nebulae. This process can be very important for resonance lines. We have presented a simple formalism for the calculation of dust destruction effects, based on the work of Hummer. Application of this formalism to the important line He I λ10830 has shown that this line can be substantially weakened by dust effects in optically thick objects.

Any study of dust effects is hampered by an uncertainty in the values of the dust-to-gas ratio for gaseous nebulae. While IRAS (infrared astronomical satellite) observations now provide accurate measurements of the total IR emission due to dust in these objects, translating this information into a dust mass involves accurate modeling of the grain size distribution, grain composition, and grain emissivity as a function of wavelength, all of which remain somewhat uncertain.

A recent study which has important implications for this problem is that of Mallik and Peimbert (1988). From an examination of a large number of PNs whose
distances were determined independently of any statistical distance scale, they found that the filling factor $\epsilon$ for these objects is generally much smaller than has previously been assumed, implying that statistical distances scales have been underestimated ($\epsilon \propto 1/d$). The mass of ionized gas is directly proportional to $\epsilon$, and thus inversely proportional to the distance $d$. The traditional method of determining the dust mass of an object utilizes the far-IR flux and the equation

$$M_{\text{dust}} = \frac{4a\rho d^2 F_{\nu}}{3Q_{\nu}B_{\nu}(T_{\text{D}})}$$

(35)

where $a$ is the grain size, $\rho$ the grain density, $F_{\nu}$ the flux at the observed wavelength, $Q_{\nu}$ the grain emissivity at the observed wavelength, and $B_{\nu}$ the Planck function at the dust temperature $T_D$. Therefore, the dust-to-gas ratio scales as the cube of the distance. Mallik and Peimbert recompute dust-to-gas ratios for several nebulae using their new values of $\epsilon$ and find an average value of $5.2 \times 10^{-3}$, which is roughly three quarters the value for the general ISM. For the one object in our Table 4 considered by Mallik and Peimbert, IC 418, their dust-to-gas ratio is roughly 2.5 times that which we used. If we recalculate the function $f$ for this new value, we now obtain 0.86, a reduction of 7%. This value is certainly significant. It is imperative that further work along the lines of Mallik and Peimbert be conducted to verify their results. If correct, dust will play an even more substantial role in gaseous nebulae than is currently believed.
Collisional Effects in He I

We have examined the possible depopulation of the $2^3S$ state of He I, and the implications of such a depopulation for collisional effects in the He I spectrum in two separate ways. First, we attempted to resolve the discrepancy in the observed vs. predicted intensity of He I $\lambda 10830$, and secondly, we determined empirically the value of the depopulation factor $\gamma$. Note that both of these are aspects of the same basic problem; we have only considered them separately since the 10830 discrepancy has a long history of its own, and is often discussed in the literature without regard to the larger problem.

For the 10830 problem, we have found that a combination of telluric absorption and line destruction by internal dust can resolve the discrepancy for NGC 7027, and suggest that these same effects can explain the discrepancy in other objects. Unfortunately, both these processes involve significant uncertainties, due to the lack of accurate parameters for telluric absorption and the uncertain dust-to-gas ratio for the dust effects. We have also emphasized the observational problems in measuring the 10830 line intensity. We therefore suggest that in order to verify our findings, this line should be observed using improved detectors with high sensitivity in this wavelength region. Further, such observations should be carefully planned in advance to insure minimum telluric effects on the observing dates. One should also attempt to observe relatively dust-free objects, or objects known to have small
Based on observations of several optical He I lines in NGC 7027, we obtain a best average value for the depopulation factor $\gamma$ of $\sim 1.20$. We emphasize, however, that the uncertainties acquired in observationally determining this factor are substantial and difficult to reduce. We would suggest observations of several very dense, $(N_e \geq 2 \times 10^4)$ hot PNs, focusing on the 7065 line and planned to avoid telluric contamination as the best way to determine $\gamma$ with a minimum uncertainty. However, we feel that the value of $\gamma$ derived here, the large uncertainties inherent in this observational procedure, and the failure to theoretically discover any additional depopulation mechanisms, suggest that the $2^3S$ state is not depopulated beyond well-established radiative and collisional processes. This implies that the He I collisional data derived by Sawey and Berrington (1993) and parametrized in this dissertation must be used without any reduction factors, and that He abundances in PNs and H II regions must be substantially revised downward. This in turn has serious implications for observational determinations of the primordial He abundance, which require abundance accuracies to within a few percent.

3 Telluric Absorption

We have seen that telluric absorption can significantly weaken the intensity of the He I $\lambda 10830$ line. Additionally, although our treatment was somewhat less rigorous,
an examination of column 5 of Table 9 shows that this effect is important for a host of other lines as well. We wish to emphasize again that traditional techniques of correcting for telluric absorption are largely cosmetic; they remove the telluric bands, but give no information about whether individual telluric lines have reduced the intensity of object lines. Although a full correction of telluric effects requires detailed knowledge of several poorly-known parameters, telluric line wavelengths are sufficiently accurate to ascertain whether these lines interfere with any lines of interest in a given object. Conversely, if one is interested in a given line or lines, it should be possible to plan one's observing dates to avoid or minimize these effects. We also note that telluric effects, while predominant in the IR, are not restricted to this wavelength region. Column 5 of Table 9 shows that He I λ5876 suffered minor absorption in NGC 7027 (Telluric absorption of this line was also noted by Davidson and Kinman, 1985). While minor telluric interference such as that for 5876 can be reasonably ignored in many cases, it is extremely important for any high-precision spectroscopy. It is therefore essential that future observers take telluric absorption into account when planning or analyzing observations.
List of References


_________ 1992, Hazy, OSU Internal Report 92.01


