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Money demand in the United States

Lee, Choong-Lyul, Ph.D.
The Ohio State University, 1992
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1992
To My Parents
ACKNOWLEDGEMENTS

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# TABLE OF CONTENTS

**DEDICATION** ................................................ ii

**ACKNOWLEDGEMENT** ............................................ iii

**VITA** ................................................................... iv

**LIST OF TABLES** ............................................. vii

**LIST OF FIGURES** ........................................... ix

**CHAPTER**

I  **Introduction** ........................................ 1

II  **Methodologies** ....................................... 10

   2.1  **Partial Adjustment Model** ......................... 10

       2.1.1  **Real Adjustment Model** ....................... 11

       2.1.2  **Nominal Adjustment Model** .................... 12

       2.1.3  **Price Adjustment Model** ...................... 13

   2.2  **Cointegration Approach** ........................... 15

       2.2.1  **Integrated Time Series and Unit Root** ..... 15

       2.2.2  **Cointegration Theory** ........................ 19

           (a)  **Residual Based Tests**  ....................... 20

           (b)  **Johansen's Trace Test and Maximal Eigenvalue Test**  24

           (c)  **Stock and Watson's Common Trend Tests and Dynamic OLS estimator and Dynamic GLS Estimator**  27

       2.2.3  **Error-Correction Model** ....................... 32

           (a)  **Partial Adjustment Model and Univariate Error-Correction Model**  32

           (b)  **Vector Error-Correction Model** ............. 45

   2.3  **Structural Break Test** ............................ 49
III Empirical Results ........................................ 53

3.1 The Money Demand in 1953 - 1991 .......................... 54

3.1.1 Unit Root Tests ...................................... 55
3.1.2 Cointegration Tests ................................. 56
3.1.3 Error-Correction Model and the Short-run
Money Demand Function ................................. 62

3.2 Partial Adjustment and Error-correction Model ........... 66

3.2.1 Real Adjustment Model and Nominal Adjustment .......... 67
3.3.2 Error-correction Model .............................. 70

3.3 The Money Demand Function in the 1970s .................... 72

3.3.1 Missing Money in the 1970s ........................... 73
3.3.2 Cointegration Tests and the Long Run Coefficients .... 75
3.3.3 Error-Correction Model and the Dynamic Coefficients ... 82

IV Conclusion .................................................. 84

Tables .......................................................... 87
Figures .......................................................... 111
Bibliography .................................................... 126
<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Unit Root Tests 1: Augmented Dickey-Fuller Tests</td>
</tr>
<tr>
<td></td>
<td>(1953 - 1991:2)</td>
</tr>
<tr>
<td></td>
<td>87</td>
</tr>
<tr>
<td>2.</td>
<td>Unit Root Tests 2: Phillips and Perron Tests</td>
</tr>
<tr>
<td></td>
<td>(1953 - 1991:2)</td>
</tr>
<tr>
<td></td>
<td>88</td>
</tr>
<tr>
<td>3.</td>
<td>Residual Based Cointegration Tests</td>
</tr>
<tr>
<td></td>
<td>89</td>
</tr>
<tr>
<td>4.</td>
<td>Stock and Watson’s Cointegration Test Statistics</td>
</tr>
<tr>
<td></td>
<td>89</td>
</tr>
<tr>
<td>5.</td>
<td>C(K, s) Statistics for Lag Determinations for VAR Model</td>
</tr>
<tr>
<td></td>
<td>90</td>
</tr>
<tr>
<td>6.</td>
<td>Trace Test and Maximal Eigenvalue Test Statistics</td>
</tr>
<tr>
<td></td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>92</td>
</tr>
<tr>
<td>8.</td>
<td>ADF Test Statistics on the Residuals of Estimated Cointegrating Equation</td>
</tr>
<tr>
<td></td>
<td>94</td>
</tr>
<tr>
<td>9.</td>
<td>Estimated Vector Error-correction Model</td>
</tr>
<tr>
<td></td>
<td>95</td>
</tr>
<tr>
<td>10.</td>
<td>Granger Causality Test</td>
</tr>
<tr>
<td></td>
<td>96</td>
</tr>
<tr>
<td>11.</td>
<td>Neutrality Test</td>
</tr>
<tr>
<td></td>
<td>97</td>
</tr>
<tr>
<td>12.</td>
<td>General Unrestricted ECM</td>
</tr>
<tr>
<td></td>
<td>98</td>
</tr>
<tr>
<td>13.</td>
<td>Chow Test for a Structural Break</td>
</tr>
<tr>
<td></td>
<td>98</td>
</tr>
<tr>
<td>14.</td>
<td>Non-linear Least Squares Estimators of Real Adjustment Model and Nominal Adjustment Models</td>
</tr>
<tr>
<td></td>
<td>99</td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>15.</td>
<td>Non-linear Least Squares Estimators of Cointegrating Vector in Generalized ECM</td>
</tr>
<tr>
<td>16.</td>
<td>Parameter Constancy Tests for PAMs</td>
</tr>
<tr>
<td>17.</td>
<td>Estimates of Parsimonious ECMs</td>
</tr>
<tr>
<td>18.</td>
<td>Parameter Constancy Tests for ECMs</td>
</tr>
<tr>
<td>19.</td>
<td>Real Adjustment Model Estimates in the 1970s</td>
</tr>
<tr>
<td>22.</td>
<td>Structural Break Test Statistics (1953 - 1980)</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

FIGURE

1. Residuals of Cointegration Equation .......................... 111
2. Actual and Fitted Values of Real Money Balance by ECM (3.2.1) ......................................................... 112
3. Actual and Fitted Values of Real Money Balance by Real Adjustment Model ........................................... 113
4. Actual and Fitted Values of Real Money Balance by Nominal Adjustment Model ....................................... 114
5. Actual and Fitted Values of Real Money Balance by Error-Correction Model I ......................................... 115
6. Actual and Fitted Values of Real Money Balance by Error-Correction Model II ....................................... 116
7. Dynamic Simulation of Real Adjustment Model in the 1970s. ............................................................... 117
8. Sequence of Cointegration Test Statistics ...................... 118
9. Residuals of Cointegration Equations ............................. 119
10. Wald Test Statistics for Structural Break of Residuals of Cointegration Equations .................................. 120
11. Sequence of Cointegration Test Statistics With a Dummy Intercept ............................................................ 121
12. Sequence of Cointegration Coefficients:
   (Equation 4) .................................. 122

13. Sequence of Cointegration Coefficients:
   (Equation 8) .................................. 123

14. Sequence of Coefficients of ECM:
   (Equation 4) .................................. 124

15. Sequence of Coefficients of ECM:
   (Equation 8) .................................. 125
Chapter I. Introduction

Since econometric and statistical methods were applied in economics, much empirical work has been done to estimate and to specify the demand for money function. As economic theory suggested, the demand for money function is expressed as a stable function of a scale variable and opportunity costs of holding money, such as a GNP and an interest rate, respectively. It is quite natural that a rich tradition exists on it, because the demand for money is known to be a critical component in the formulation of monetary policy and a stable demand for money function is regarded as a prerequisite for the use of monetary policy.

By the early 1970's, this attempt looked quite successful\(^1\) and the demand for money function in the United States seemed to be well established as a relatively simple and stable equation with a sound...
theoretical foundation. However, after the mid-1970s, a conventional money demand function such as Goldfeld's equation began to significantly and consistently over-forecast the money demand and showed the so-called 'missing money' problem. Several major changes in a conventional money demand equation were also noticed and a very unstable and unpredictable velocity was observed through the 1980s. Since then, voluminous research has been done to solve this 'missing money' puzzle and to estimate a proper money demand function. Some of those efforts are well surveyed in the literature such as Judd and Scadding (1982) or Goldfeld and Sichel (1990).

2. The term 'stability' relates to (a) the constancy over time of the estimated parameters in an statistical model and (b) predictability of money balance by a small set of key economic variables.

3. Goldfeld said "For ten quarter periods (after 1974) ... (Goldfeld's) equation over-predicts by an average of $13 billion in 1972 price. The root mean square error is 6.3% and the error for 1976 is a whopping 9.8 percent, or nearly $30 billion in current price" (Goldfeld 1976). Judd and Scadding described it as 'the observed instability in the demand for money after 1974'. (Judd and Scadding 1982). See Section III to find some empirical evidence on these arguments.

4. The 'conventional demand functions' mean the money demand functions estimated before 1973. They are based on a partial adjustment model and a long-run theoretical money demand function. See Section 2.1.
However, despite all these efforts and important achievements in this field, some important and interesting questions remain yet unsolved and challenge economists.

First, it is not certain that, empirically, the money demand has a stable long-run relationship with a few economic variables as economic theory asserts. Even though Friedman and his followers (Friedman 1959, Meltzer 1963, Friedman and Schwartz 1982, and Lucas 1988) have consistently insisted for the last three decades that the long-run relationship of the demand for money is quite stable, empirically it is still a controversial issue. Therefore, it would be valuable to examine the stability of a long-run money demand function and, if possible, to find it.

Second, in addition to this stability problem of a long run money demand relationship, that of a short-run one is even worse. Since the short-run monetary policy is conducted based on the short-run money demand function, its stability is quite an important issue. However, presently, it is believed to be very unstable so that a reliable prediction of money balance based on it is impossible (Goldfeld and Sichel 1990, Hafer 1986). Therefore, it would be an important work to search for a stable short-run money demand function. Even though several dynamic adjustment models have been suggested in the money demand literature, there has been no comprehensive comparison among these models. Different dynamic adjustment model gives a different
form of money demand function. It would be valuable to compare them and find a suitable one.

Third, historically, the money demand function was noticed to be unstable in the mid-1970s for the first time. Considering that several changes in the U.S. economy occurred in the mid-1970s such as a double digit inflation and high unemployment rate, it may be important to investigate how the money demand function changes and what kind of break happened in the 1970s.

Fourth, it is an interesting work to estimate and to test hypotheses on the coefficients of the money demand function. Different theories indicate different functional forms and different elasticities of the money demand function with respect to the key macroeconomic variables, such as national income or interest rates. Testing these hypotheses would be an indirect way of testing specific theories concerning the money demand function.

This dissertation attempts to solve the above problems and to estimate a correctly specified U.S. money demand function of M1. For this empirical investigation, recent developments in econometric techniques, especially in time series analysis, will be employed: (1)

---

5. For example, Baumol's inventory theory indicates that

\[ m_t^d = a \cdot r_t^{-1/2} \cdot y_t^{1/2}, \]

where \( m_t^d \) is a demand for real balances, \( r_t \) is an interest rate, and \( y_t \) is an income. The elasticity of real balances with respect to the interest rate is \(-1/2\) (Baumol 1952).
cointegration theory, (2) an error-correction model and (3) a structural break test.

In the conventional approach, two different equations are specified to build a single estimation equation. For example, Goldfeld's model (Goldfeld 1973) is based on the two different model: (1) a long-run money demand function and (2) a dynamic adjustment model. Since it is expressed as a single reduced form equation, these two specified models cannot be estimated and tested separately. Therefore, if either one is mis-specified, then the whole money demand function becomes mis-specified. It is more robust that both models be treated and estimated separately, if possible.

A cointegration approach and an error-correction model, stemming from Engle and Granger (1987) and Davidson et al. (1978) and Hendry (1980), become increasingly popular in recent years in many macroeconomic model building and testing. Since many macroeconomic variables are now regarded as non-stationary, the traditional statistical tests such as a t-test or a F-test on these variables may lead to false inferences and consequently, standard regression theory

---

should not be applied.\(^7\)

A cointegration approach gives a solution to this problem and enables us to analyze the relationship among those integrated economic variables. In addition, it enables to estimate and to test a long-run model and a short-run dynamic adjustment model, separately. A long-run equilibrium relationship among integrated variables can be found by applying a cointegration test and long-run equilibrium coefficients can be derived by estimating a cointegrating vector. An error-correction model is another way to analyze the relationship of integrated variables. Since it is a quite general type of a dynamic adjustment model, the application of an error-correction model to money demand seems to be a good extension, considering that most previous models are based on a restrictive partial adjustment model.\(^8\)

To examine a possible structural break of the money demand function in the 1970s, three tests; (1) sup Wald test, (2) mean Wald test, and (3) exp-mean Wald test, suggested by Andrews et al. (1992) and Bai et al. (1992) are performed. In addition to these tests, a

---

7. To find asymptotic distributions of t-distribution or F-test statistics on the coefficient of the integrated regression and cointegration equation, see Phillips (1985) and Phillips and Ouliaris (1991).

8. To see the relationship between a partial adjustment model and an error-correction model, see Section 2.2.3 and Nickell (1985).
confidence interval for a possible structural break date is also constructed.

There have been some attempts to analyze a money demand function by way of a cointegration theory and an error-correction model in recent years. As a simple univariate model, Engle and Granger (1987) tested the cointegration relationship between real M1 and GNP and real M2 and GNP in their introductory paper to a cointegration theory. More generally, Miller (1991) applied Engle-Granger's augmented Dickey-Fuller (ADF) test methods and the vector error-correction model to estimate and analyze the monetary dynamic of M2 through 1959-1987.

Hafer and Jansen (1991) estimated long-run elasticities of demand for money (M2) during 1915-1988 with respect to real GNP and commercial paper rate through a cointegration equation. They applied Johansen’s maximum likelihood estimation (MLE) method. Dickey et al. (1991) also investigated the M1 demand by cointegration methods. They used Engle and Granger's ADF test, Johansen's MLE method and Stock and Watson's common trend test and presented weak evidence that real money balances are cointegrated with real GNP and a treasury bill rate.

In addition, Boughton (1991) estimated both the long-run and short-run demand for money functions (M2) of the five large industrial countries by a cointegration theory and an error-correction model and showed that they have been stable despite many shocks to those economies through the 1970s and 1980s.
Baba, Hendry and Starr (1992) estimated the demand for money (M1) in the U.S. from 1960 - 1988 by applying an error-correction model. Their estimation is mainly based on a long-lag univariate error-correction model and Hendry's 'general to specific' specification method. They tried to explain the volatility of M1 money demand in the 1970s and 1980s, by including a few other variables such as a holding period risk of a bond and learning curve weighted yields on newly introduced instruments in M1 and non-transaction M2. In the conclusion of their paper, they claimed that they specified a stable money demand function of M1 despite many shocks in real and financial sector during last 20 years.

Obviously, all the above mentioned works are quite valuable and important developments with respect to money demand estimation. However, most of them are not about M1 money demand, which was controversial and regarded as unstable for the past fifteen years, but about M2 money demand. In this dissertation, the demand for narrow money, M1, is extensively examined and several cointegration tests are performed on the M1 money demand equation.

The cointegration relationship among the money demand and some macroeconomic variables will be systematically investigated and several cointegration test and estimation methods will be employed to generalize the previous empirical results. In addition, an error-correction model is applied to find a proper dynamic adjustment model
and a short-run money demand function. Furthermore, by performing a sequence of structural break tests, a possible 'break' timing of the cointegration equation in the 1970s is estimated and a money demand function with this break is estimated.

This dissertation consists of four Chapters. Chapter I serves as an introduction and Chapter II reviews some established and recently developed econometric methodologies for these empirical investigations. In Section 2.1, a conventional partial adjustment model is presented and in Section 2.2, a cointegration approach including a cointegration theory and an error-correction model is introduced and structural break tests are reviewed.

Chapter III presents empirical results of this investigation. First, Section 3.1 shows the estimation and test results of money demand of United States (1953 - 1991:2). Several available cointegration tests are performed and a long-run model is estimated. In addition, a short-run money demand function is also specified based on the long-run model. In Section 3.2, three dynamic adjustment models, such as a real adjustment model, a nominal adjustment model and an error-correction model are examined and compared with each other. In Section 3.3, a money demand in 1970s is revisited and 'missing money' problem is examined. The last Chapter summarizes some conclusions.
Chapter II Methodology

This chapter reviews econometric methodologies for this empirical research. First, a conventional money demand function based on a partial adjustment model is reviewed in Section 2.1 and later, a new approach such as cointegration theory and an error-correction model is introduced in Section 2.2. Finally, a structural break test is discussed in Section 2.3.

2.1 Partial Adjustment Model

Many previous empirical models of a money demand including Goldfeld's were built upon two equations: (1) a long-run money demand function supported by economic theories (Baumol 1952, Tobin 1956) and (2) a short-run dynamic adjustment model suggested by Chow (1966).

A long-run money demand function provided by economic theories is usually specified as below:

\[ M_t = \frac{M}{P} = a_0 + a_1 Y_t + a_2 r_t + e_t \]

where \( Y_t \) is a scale variable such as GNP or national income and \( r_t \) is a vector of opportunity costs of holding money such as interest rates.
Usually, $a_1$ and $a_2$ are regarded as positive and negative, respectively.

As for a short-run dynamic adjustment model, the below partial adjustment model suggested by Chow (1966) is applied, provided that there is a quadratic cost of adjustment function:

\[(Y_t - Y_{t-1}) = b_1 (Y^*_t - Y_{t-1}) + e_t\]

where $Y^*_t$ is an equilibrium value of $Y_t$ and $b_1$ is an adjustment coefficient and $0 < b_1 < 1$. Based on these two equations, a long-run money demand function and a short-run dynamic adjustment model, several money demand models have been suggested in the literature. In this dissertation, three models are considered: (1) a real adjustment model, (2) a nominal adjustment model and (3) a price adjustment model.

2.1.1 Real Adjustment Model

One of the first applications of a partial adjustment model in the money demand literature is Goldfeld’s real adjustment model (Goldfeld 1974). In his model, economic agents are assumed to adjust their real balances to a long-run equilibrium value through the partial adjustment mechanism and a following adjustment model is suggested:

\[(m_t - m_{t-1}) = \alpha_1 (m^*_t - m_{t-1}) + e_t\]
where \( m_t = \left( \frac{M}{P} \right)_t \) is a real balance. With the standard long-run money demand function as equation (2.1.1), an estimated money demand function has the following form:

\[
(2.1.4) \quad m_t = a_0 + a_1 y_t + a_2 r_t + a_3 m_{t-1} + e_t
\]

and this was later refereed to as a real adjustment model.

To estimate equation (2.1.4) in Goldfeld's model (1973), first an ordinary least squares method was used and later the Cochrane-Orcutt iteration method was used because strong serial correlation was detected. His estimation result looked very nice: (1) the sign of each coefficient is reasonable and (2) back-forecasts based on dynamic simulations were very close to actual values. However, soon after 1974, several problem showed up in his equation: first it began to consistently over-forecast the money demand and showed the 'missing money' problem and second, coefficients on his estimated equation appeared very unstable through the 1970s.

2.1.2 Nominal Adjustment Model

Soon after Goldfeld's real adjustment model show a problem of over-forecasting in mid-1970s, it began to be criticized and a flourishing debate had arisen on this issue. Afterwards, several new
models were introduced and one of them was a nominal adjustment model
(Goldfeld 1976, Judd and Scadding 1982).

The key difference between those two models lies in the assumption
that which variable, nominal or real balance, actually adjusts to the
disequilibrium gap in the economy. In a nominal adjustment model,
Goldfeld argued that people do not adjust their real balance fully and
instantaneously to price level change and suggested a nominal
adjustment model as below:

\[ M_t - M_{t-1} = \alpha_2 (M_t^* - M_{t-1}) \]

According to this, economic agents adjust not their real balances but
their nominal balances to an equilibrium value. In a microeconomic
sense, this model is more reasonable than a real adjustment model
because economic agents may have a money illusion in the short period
and they determine only their nominal balance. With a standard form of
a long-run money demand function like equation (2.1.1), a final
estimation equation is of the form:

\[ m_t = \left( \frac{M_t}{P_t} \right)_t = a_0 + a_1 y_t + a_2 r_t + a_3 \left( \frac{M_{t-1}}{P_t} \right) + \epsilon_t \]

2.1.3 Price Adjustment Model
In the price adjustment model, it is assumed that price level adjusts to the monetary disequilibrium. This model looks more reasonable than previous two models in macroeconomic sense. In the national economy, money supply is controlled by the Federal Reserve Board and is fixed at a certain point. Therefore, with a given money stock, it is only a price level that adjusts to the macroeconomic disequilibrium. From this idea, the following model is introduced (Laidler 1982):

\[(2.1.7) \quad (P_t - P_{t-1}) = \alpha \cdot (\bar{P}_t - P_{t-1})\]

where \(\bar{P}_t = \frac{M_t}{m_t}\) is an equilibrium price level. With a long-run money demand function in equation (2.1.1), the reduced form becomes:

\[(2.1.8) \quad m_t = a_0 + a_1 y_t + a_2 r_t + a_3 \frac{M_t}{P_{t-1}} + \varepsilon_t\]

As seen in this section, all these models are simple applications of a partial adjustment model. As Thornton (1985) indicated, these three models cannot be directly compared because they are non-nested models and are also built upon different assumption of error terms. They can be judged only based on their statistical properties such as parameter stability or the theory compatibility of estimated
coefficients. However, empirical examination do not give any dominance to one particular model of them (Thornton 1985).

2.2 Cointegration Approach.

Recently, economists have realized these kinds of estimating and testing methods may have some statistical problems. After being shown by Nelson and Plosser (1982), it has become now almost a stylized fact that many macroeconomic variables, including a real balance and GNP, are not stationary and have unit root, even though this is still debatable. Since a money demand function contains those integrated series, standard regression theories cannot be applied, either to estimate a proper money demand function nor to make correct inference on estimated parameters (Phillips 1985). Fortunately, in recent years, many new econometric theories have been developed, especially about the unit root and cointegration, making it possible to solve the above problems. In this section, these approaches are reviewed.

2.2.1 Integrated Time Series and Unit Root

9. To know the recent argument of this issue, see Dejong and Whiteman (1991)
An integrated time series is defined as follows: when an economic time series has invertible, ARMA representation after differencing \(d\) times, it is called an integrated series of order \(d\) or a series with \(d\) unit roots and it is denoted \(x_t \sim I(d)\) (Engle and Granger 1987).

This definition is quite general. Most literature has primarily concentrated on \(I(1)\) series or \(I(0)\) series because most economic time series variables appear to be \(I(1)\) or \(I(0)\).\(^{10}\) There are substantial differences between \(I(0)\) series and \(I(1)\) series both in data appearance and in statistical inference.\(^{11}\) For example, the unconditional variance of \(I(1)\) series is unbounded as time increases when that of \(I(0)\) is not. For this reason, a standard asymptotic theory cannot be applied to an \(I(1)\) series in a large sample inference.

Since many macroeconomic series are considered \(I(1)\) series, each variable in the money demand equation should be tested first whether it has a unit root or not. In this dissertation, Dickey-Fuller test (ADF(a)), and augmented Dickey-Fuller (ADF(t)) test by Said and Dickey and Z(a) and Z(t) test by Phillips and Perron are employed to test the unit roots of economic variables.

\(^{10}\) For a higher order integration theory or cointegration theory, see Stock and Watson (1991) and Davidson (1991).

\(^{11}\) To know more on the difference between stationary series and non-stationary series, see Dickey et al. (1986).
Estimation equations in the ADF tests are of the forms \cite{Fuller1975} and \cite{SaidDickey1984}:

\begin{align*}
(2.2.1) \quad x_t &= a_2 x_{t-1} + \varepsilon_{1,t} \\
(2.2.2) \quad \Delta x_t &= d_1 x_{t-1} + d_2 \Delta x_{t-1} + \ldots + d_{p+1} \Delta x_{t-p} + \varepsilon_{2,t}
\end{align*}

and the null hypotheses are $H_0 : a_2 = 1$ in the equation (2.2.1) and $H_0 : a_2 = 0$ in the equation (2.2.2). Under the null hypothesis, $x_t$ has a unit root and $x_t$ is I(0) series. In this test, a general case such that the error term, $\varepsilon_{1,t}$, is not only i.i.d. $N(0, \sigma^2)$ but ARMA $(p, q)$ is considered.

The usual $t$-statistic does not follow the conventional $t$-distribution under the null hypothesis but converges to a non-standard distribution \cite{Phillips1988}. Therefore, conventional statistical tables cannot be used to find significance levels of these $t$-statistics and Monte-Carlo simulations based on an asymptotic distribution have been utilized. In addition, more general cases in which $x_t$ has a

\[T\text{-statistic converges to } (\sigma/2\sigma_u)(w(1)^2 - \int_0^1 w(r)^2 dr); \text{ where } \sigma^2_u = \lim_{T\to\infty} T^{-1} \sum_{t=1}^T E(u_t), \sigma^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^T E(u_t)^2, \sigma^2 = \sum_{t=1}^T u_t \text{ and } w(r) \text{ is a standard Wiener process.}\]
intercept or a linear time trend can be also considered with a slight modification of equation (2.2.1) (Fuller 1976, Dickey and Fuller 1981).

To select the number of lags in equation (2.2.2), a standard lag selection test statistics such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are used.

Phillips-Perron’s two tests, Z(a) or Z(t), are also more generalized and modified forms of Dickey-Fuller tests. In their tests, more general situations such that e_{it} is not only an i.i.d. process but also a temporally depended and heteroskedastically distributed process, are addressed. Their tests use the same tabulated critical values as Dickey and Fuller’s. Their estimation equations are:

\[(2.2.1)' \quad X_t = b_0 + a_2 X_{t-1} + e_{2,t} \]

\[(2.2.3) \quad X_t = c_0 + c_1 t + a_3 X_{t-1} + e_{3,t} \]

13. Phillips’s assumptions about error terms are (1) \( E(e_t) = 0 \) for all \( t \); (2) \( \sup E|e_t|^\beta < \infty \) for some \( \beta > 2 \), \( \varepsilon > 0 \); (3) as \( T \to \infty \), \( \sigma = \lim E(T^{-1} S_T^2) \) exists and \( \sigma^2 > 0 \), where \( S_T = e_1 + e_2 + \ldots + e_T \); (4) \( \{e_t\} \) is strong mixing with mixing coefficients \( \alpha_m \) that satisfy \( \sum \alpha_m^{1-2/\beta} < \infty \), where the sum is over \( m = 1, \ldots, \infty \). If it is assumed that \( e_{it} \) is not an i.i.d. process but satisfy only the above conditions, then the asymptotic distributions of test statistics in equation (2.1) depends on some nuisance parameters. For this reason, Phillips and Perron made a simple transformations to the test statistics to get rid of this nuisance parameter dependence and called them \( Z(a) \) or \( Z(t) \) statistics. For more about this in detail, see Phillips and Perron (1987).
and test statistics are:

(2.2.4) \[ Z(a_2) = T \cdot (a_2^{-1}) - \lambda_2 / m_1, \]

(2.2.5) \[ Z(t_{a_2}) = (S_2 / \sigma_{t12}) \cdot t_{a_2} - \lambda_2 / (\sigma_{t12} \cdot m_0^{-1/2}), \]

(2.2.6) \[ Z(a_3) = T \cdot (a_3^{-1}) - \lambda_3 / m, \]

(2.2.7) \[ Z(t_{a_3}) = (S_3 / \sigma_{t13}) \cdot t_{a_3} - \lambda_3 / (\sigma_{t13} \cdot m^{-1/2}), \]

where \( m_0 = T^{-2} \Sigma x_t^2, m_1 = T^{-2} \Sigma (X_t - \bar{X}_t)^2, m_2 = T^{-3/2} \Sigma x_t, \)

\( m_3 = T^{-5/2} \Sigma t \cdot x_t, \quad \lambda_i = (1/2) \cdot (\sigma_{tli}^2 - s_i^2), \quad i = 2, 3, \)

\( M = (1 - T^{-2}) \cdot m_0 - 12 \cdot m_3 + 12 \cdot (1 + T^{-1}) \cdot m_3 \cdot m_2 - (4 + 6 \cdot T^{-1} + 2 \cdot T^{-2}) \cdot m_3^2, \)

\( \sigma_{tli}^2 = T^{-1} \cdot \Sigma_{t=1}^T e_i^2 + 2 \cdot T^{-1} \cdot \Sigma_{s=1}^T \Sigma_{t=s+1}^T e_i \cdot e_i, i = 2, 3, \)

for some choice of lag window such as \( w_s = 1 - s/(1+1), \) and \( e_i, t \) and \( s_i, \) for \( i = 2, 3, \) are residuals and standard errors of equations (2.2.1)' and (2.2.3) respectively.

2.2.2 Cointegration Theory

Cointegration refers to a certain kind of relationship among integrated time series variables. When a linear combination of some
integrated variables is stationary, those variables are called "cointegrated" and a coefficient vector of the linear combination is called a cointegrating vector (Engle and Granger 1987). A cointegrating equation can have an important meaning in economic theory. For instance, if \( x_t \) and \( y_t \) are cointegrated, then there exists a stationary time series \( z_t \), as a linear combination of \( x_t \) and \( y_t \). Therefore, the linear relationship of \( x_t \) and \( y_t \) can be interpreted as a long-run equilibrium relationship and a cointegration test can be utilized to find a long-run money demand and to test the stability of a money demand function. In addition, the long-run elasticity of money demand is derived by estimating a cointegrating vector.

In this dissertation, several cointegration tests are performed on a money demand equation: (1) residual based tests such as Engle and Granger's ADF test and Phillips and Ouliaris's Z(a) test and Z(t) test, (2) Johansen's trace test and maximal eigenvalue test based on a likelihood ratio test principle and (3) Stock and Watson's common trend tests. To find the cointegrating vector, several estimators will be estimated: (1) a simple ordinary least squares estimator (OLS) (2) Johansen's estimator and (3) Stock and Watson's dynamic ordinary least squares (OLS) estimators and dynamic generalized least squares (GLS) estimators.

(a). Residual Based Tests: ADF Test and Z(t) and Z(a) tests
Residual based tests begin with the definition of the cointegration equation $Z_t = \alpha X_t$, where $X_t$ is a vector of integrated time series. If the variables in $X_t$ are cointegrated, then their linear combination $Z_t$ should be $I(0)$ or stationary by the definition of cointegration. Conversely, the stationarity of $Z_t$ implies the cointegration of $X_t$ and the cointegration test becomes the unit root test of $Z_t$. Actually, it is another application of unit root tests.

In these residual based tests, the residuals of cointegration equation must be known first and in order for them to be derived, a cointegrating vector must be previously estimated. Therefore, these tests consist of two estimation steps and are called 'two step' tests. In the first step, a possible consistent cointegrating vector is estimated and residuals of the cointegrating equation are derived based on the estimated cointegrating vector. In the next step, a unit root test is performed on the estimated residuals. If the unit root on the estimated residuals is rejected and the estimated residuals are regarded as stationary, then those integrated economic variables are said to be cointegrated and to have a stable long-run relationship with each other.

In the residual based test, the estimation equations have the forms of:
(2.2.8) \[ X_{1t} = a_0 + \sum_{i=2}^{n} a_i X_{it} + u_t, \]

(2.2.9) \[ \Delta \hat{u}_t = b_0 \cdot \hat{u}_{t-1} + \sum_{i=1}^{P} b_i \cdot \Delta \hat{u}_{t-i} + e_t \]

(2.2.10) \[ u_t = c_1 \cdot u_{t-1} + e_t \]

where \( X_t = (X_{1t}, X_{2t}, \ldots, X_{nt}) \) is a vector of \( n \) integrated time series, \( \hat{u}_t \) is an estimated residual of the possible cointegrating equation (2.2.8), \( \Delta \) means a difference. At the first step, equation (2.2.8) and \( \hat{u}_t \), a possible cointegration equation and residual of cointegration equation respectively, are estimated by an OLS method as suggested by Stock (1987). The OLS estimator has some desirable properties such as super-consistency, as shown in Stock (1987), and ease of computation. However, it is shown by small sample simulation studies that an OLS estimator has a bigger small sample bias than other consistent estimators (Ahn and Reinsel 1991, Phillips and Hansen 1992, Stock and Watson 1991). 14

14. By a simulation study of a small sample, Ahn and Reinsel (1990, 1992) compared the OLS estimator with their reduced rank estimator and Phillips and Hansen (1992) compared the OLS estimator and several other cointegrating vector estimator such as non-linear least squares estimator from error-correction model, or their instrumental variable

(Footnote continues on next page)
In the second step, equation (2.2.9) or (2.2.10) is estimated and a unit root test is performed on $\hat{u}_t$. In the Engle and Granger's ADF test, the usual t-test is performed for $H_0: b_0 = 0$ and in Phillips and Ouliaris tests, $Z(t)$ and $Z(a)$ test statistics are derived for testing $H_0: c_1 = 1$ in equation (2.2.10). As in the unit root tests of Section 2.2.1, the test statistics follow neither a conventional t-distribution nor a normal distribution asymptotically but a function of Brownian motion. Based on their asymptotic distributions, critical values are derived by simulations and they are reported in Engle and Yoo (1987), Blangiewicz and Charemza (1990), Phillips and Ouliaris (1991). If the

(Footnote continued from previous page)

estimator. Stock and Watson compare the OLS and some other estimator of cointegrating vector such as Johansen's estimator, their dynamic OLS and dynamic GLS estimators and etc.

15. The asymptotic distributions of ADF statistics and $Z(t)$ and $Z(a)$ statistics in a residual based test are:

\[
\begin{align*}
\text{ADF} & = \int_0^1 R \, ds \\
Z(a) & = \int_0^1 R \, dR \\
Z(t) & = \int_0^1 R \, ds
\end{align*}
\]

where $R(r) = Q(r)/(\int_0^1 Q^2)^{1/2}$, $S(r) = Q(r)/(\kappa^2)$, $\dot{Q} = W'_1(r) - \int_0^1 W_1 W_2 W_2(r)$ and $\kappa = (1, -\int_0^1 W_1 W_2 W_2(r))$. 
null hypothesis of unit root in these tests is not rejected, it is assumed that integrated series $X_t$ are not cointegrated.

(b). Johansen's Trace Test and Maximal Eigenvalue Test.

Johansen's approach (Johansen 1988, 1992, Johansen and Juselius 1990) begins with the vector autoregressive (VAR) model given by:

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \ldots + \Pi_k X_{t-k} + a_0 + e_t$$

which can also be expressed as follows

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{k-1} \Delta X_{t-k-1} + \Pi X_{t-k} + a_0 + e_t$$

where $X_t$ are $n$ integrated time series, $e_t$ are i.i.d. $n$ dimensional Gaussian random variable with mean 0 and variance matrix, $\Gamma_i$

$$\Gamma_i = -(I - \Pi_1 - \Pi_2 - \ldots - \Pi_i), i = 1, 2, \ldots, k-1$$ and $\Pi = -(I - \Pi_1 - \Pi_2 - \ldots - \Pi_k)$.

In this approach attention is paid to the rank of an impact matrix $\Pi$. As shown in Johansen (1992) and Johansen and Juselius (1990), for example, if the rank of $\Pi$ is 0, then the impact matrix $\Pi$ is the null matrix and equation (2.2.12) corresponds to a traditional
differenced VAR. However, if the matrix $\Pi$ has a full rank, then the vector process $X_t$ is stationary. If the rank of matrix $\Pi$ is $r < n$, then it can be expressed: $\Pi = A \cdot B'$ for suitable $(n \times r)$ matrices $A$ and $B$, and it can be also interpreted as an error-correction model in Engle and Granger's terminology (Engle and Granger 1987). Therefore, by Granger's representation theorem, it follows that there are $r$ cointegrating vectors among $n$ vector time series. In other words, the linear combinations given by $B'X_t$ are stationary even though $X_t$ itself non-stationary, meaning that $X_t$ becomes cointegrated with cointegrating vectors $B'$. For this reason, the cointegration test is the same as the test of rank of matrix $\Pi$.

From this idea, Johansen suggested a likelihood ratio test be utilized to test the rank of matrix $\Pi$, which was developed previously (Anderson 1951). The likelihood ratio test statistic for the null hypothesis that there are at most $r$ cointegrating vectors, called the trace test statistic, is given by:

\[
(2.2.13) \quad \text{Trace Test Statistic} = -T \cdot \sum_{i=r+1}^{n} \ln (1 - \lambda_i)
\]

16. To see more about an error-correction model and the relationship of cointegration theory and an error-correction model, see Section 2.2.3 or Engle and Granger (1987).
where \( \lambda_{r+1}, \ldots, \lambda_n \) are the \((n-r)\) smallest squared partial canonical correlations between \( X_{t-k} \) and \( \Delta X_t \) upon \( \Delta X_{t-1}, \ldots, \Delta X_{t-k-1} \). 17 Johansen and Juselius introduced another test statistic, known as a maximal eigenvalue test statistic, for testing \( r \) cointegrating vectors versus \( r+1 \) cointegrating vectors. It is of the form:

\[
(2.2.14) \quad \text{Maximal Eigenvalue Test Statistic} = -T \cdot \ln (1 - \lambda_r)
\]

Asymptotic distributions of these two statistics are known as a function of a Brownian motion. 18 Like ADF test statistics, small sample distributions of these two statistics are not known yet, so that critical values of these tests are made by simulations and are tabulated in Johansen (1988, 1992), and Johansen and Juselius (1990).

These tests have a big advantage over the previous residual based tests. In a residual based test, it is tested only whether a

---

17. The partial canonical correlation between \( X_{t-k} \) and \( \Delta X_t \) upon \( \Delta X_{t-1}, \ldots, \Delta X_{t-k-1} \) is a canonical correlation between conditional residuals of \( X_{t-k} \) upon \( \Delta X_{t-1}, \ldots, \Delta X_{t-k-1} \) and conditional residuals of \( \Delta X_t \) upon \( \Delta X_{t-1}, \ldots, \Delta X_{t-k-1} \).

18. Asymptotic distribution of a partial canonical correlation matrix is a stochastic matrix of:

\[
\int_0^1 (dU) F'(\int_0^1 FF'dt)^{-1} \int_0^1 F' dU'
\]

where \( U \) is an \( n \)-dimensional Brownian motion. If an intercept is included, \( F(t) = U(t) - \int_0^t B(t) dt \) and if a linear time trend is included, then \( F(t) = t^{-1/2} \).
cointegration relationship holds or not. But, these tests make it possible to test how many cointegrating vectors exist. For this reason, these tests, along with Stock and Watson's common trend tests, are called multivariate cointegrating tests (Dickey et al. 1991).

(c). Stock and Watson's Common Trend Tests and Dynamic OLS and Dynamic GLS Estimator.

Stock and Watson's approach is different in some sense from those of residual based tests. They extended the Beveridge-Nelson decomposition of the univariate model to the multivariate model, and argued that a cointegrated model can be represented as a reduced number of common stochastic trends plus transitory or stationary components (Stock and Watson 1989, Hylleberg and Mizon 1989). They developed some test statistics for testing \((n-k)\) common stochastic trends versus \((n-m)\) common stochastic trends. However, these test are the same as the cointegration tests because \((n-k)\) common stochastic trends among \(n\) integrated series mean \(k\) cointegrating vectors.

In their tests, the following multivariate regression equation is considered:

\[
X_t = A_1 X_{t-1} + e_t
\]
where $X_t$ is a vector of $n$ integrated series, $A_1$ is a linear coefficient matrix and $e_t$ is i.i.d. with mean 0 and variance matrix $\Sigma$. If $X_t$ is not cointegrated, then the coefficient matrix $A_1$ will have $n$ real unit roots. If $X_t$ has $(n-k)$ linearly independent cointegrating vectors, then $A_1$ will have $k$ real unit roots, and the real part of the $k$-th eigenvalue of $A_1$ should be close 1 and the real part of the $(K+1)$-th eigenvalue should be significantly different from 1. So they suggested the following test statistics to test for $k$ versus $m$ common trends:

\begin{equation}
q_f(k, m) = T(\text{real}(\hat{\lambda_f}) - 1)
\end{equation}

where $\hat{\lambda_f}$ is the $m+1$-th eigenvalue of $A_1$ and the null hypothesis is $k$ common trends. Besides this, they suggested several modified test statistics similar to the above one, which can be applied in more general cases, such that $X_t$ has intercepts or trends.

As an estimator of matrix $A_1$, an OLS estimator is suggested. However, since the error term $e_t$ is not guaranteed to satisfy the above conditions of i.i.d. with mean 0 and variance matrix $\Sigma$, they suggested $X_t$ be prefiltered by a finite order vector autoregressive model as follows:

\begin{equation}
P(L) \Delta X_t = \eta_t
\end{equation}
where $\eta_t$ is i.i.d. with mean 0 and variance 1. When $\xi_t = \Pi(L)X_t'$, their test statistic becomes:

$$(2.2.18) \quad q_f(k, m) = T(\text{real}(\lambda_{m+1}) - 1)$$

where $\lambda_{m+1}$ is the $m+1$-th eigenvalue of the first sample autocorrelation matrix formed using $\xi_t$. Like other cointegration tests, their critical values are estimated by simulations and they are tabulated in Stock and Watson (1989).

Since one of my interests in this dissertation is to find the possible change of a money demand function in the 1970s, a sequence of cointegration tests are performed through the 1970s and these test statistics are examined (i.e. cointegration tests are performed with the data from 1953:1 to each different period and the change of their test statistics is examined).

To estimate the cointegrating vectors, Stock and Watson suggested dynamic ordinary least squares (OLS) estimators and dynamic generalized least squares (GLS) estimators. As indicated in Stock and Watson (1991), these estimators have good statistical properties such as asymptotical efficiency and relatively ease of computation. Moreover, standard statistical test theory can be applied to these estimates.
asymptotically because these estimates belong to the locally asymptotic mixed normal (LAMN) family.\textsuperscript{19}

The non-stationary but cointegrated series $X_t = (X_{1t}, X_{2t})$ can be written in triangular representation as follows (Phillips 1991):

\begin{align*}
(2.2.19) & \quad \Delta X_{1t} = u_{1t} \\
(2.2.20) & \quad X_{2t} = \Theta X_{1t} + u_{2t}
\end{align*}

where $u_t = (u_{1t}, u_{2t})$ is a Gaussian stationary process. In this system of equations, equation (2.2.20) are cointegration equations and $\Theta$ is a cointegrating vector. Since $E[u_{2t} \mid \{ u_{1t} \}] = E[u_{2t} \mid \{ \Delta X_{1t} \}] = d(L) \Delta X_{1t}$, equation (2.2.20) becomes:

\begin{align*}
(2.2.21) & \quad X_{2t} = \Theta X_{1t} + d(L) \Delta X_{1t} + \tilde{u}_{2t}
\end{align*}

where $d(L)$ is a two sided polynomial of lag operator $L$ such that $d(L) = \sum_{i=-k}^{k} \sum_{j} L^j$ where $k$ is the number of leads and lags, $\tilde{u}_{2t}$ is $u_{2t} - E[u_{2t} \mid \{ u_{1t} \}]$ and $\tilde{u}_{2t}$ is independent of $u_{1t}$. Therefore, the system of equations (2.2.19) and (2.2.20) can be rewritten as follows:

(2.2.22) \[ \Delta x_{1t} = c_{11} (L) \varepsilon_{1t} \]

(2.2.23) \[ x_{2t} = \theta x_{1t} + d(L) \Delta x_{1t} + c_{22}(L) \varepsilon_{2t} \]

where \( \varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}) \) is NIID \((0, \Sigma_\varepsilon)\) and \( \Sigma_\varepsilon = \text{diag}(\Sigma_{11}, \Sigma_{22}) \).

Equation (2.2.23) is a standard regression equation and it can be estimated by a feasible GLS method and as a by-product, cointegrating vector \( \theta \) is estimated. It is also suggested in Stock and Watson (1991) to estimate equation (2.2.23) by an OLS method because as shown in Stock and Watson (1991), the OLS estimator of \( \theta \) is asymptotically the same as the GLS estimator of \( \theta \). These estimators are called a dynamic OLS estimator and a dynamic GLS estimator respectively to be distinguished from conventional OLS estimator and GLS estimator.

Since one of the issues in the demand for money in the 1970s is to examine the change in the long-run coefficients of the money demand function, a recursive least squares method is used to estimate cointegrating vectors: a cointegrating vector is estimated with data from 1953:1 to certain period and adding observation one at a time, then a cointegrating vector is re-calculated and their movements throughout the sample period are examined. As Hendry indicated, it can be a powerful tool to examine the parameter constancy of the model (Hendry 1987).
2.2.3 Partial Adjustment Model and Error-correction Model

As a long-run money demand function is estimated based on a cointegration theory described in Section 2.2.2, a dynamic property of a money demand function is analyzed by applying an error-correction model (ECM). In the error-correction model, it is assumed that a present dependent variable depends on the last period disequilibrium gaps or the last period forecasting errors as well as the present and past values of the independent variables. In this section, a univariate error-correction model is introduced and the relationship of error-correction model and conventional partial adjustment model is derived. Later, a general vector error-correction model is examined.

(a). Partial Adjustment Model and Univariate Error-correction Model

There are several reasons for using a dynamic adjustment model when estimating the money demand function. First, it is reasonable to assume that economic agents cannot adjust their money balances to their desirable levels instantly during the short-run. Once it is accepted that there are some adjustment costs for changing money balances, it
seems quite natural to use a dynamic adjustment model to estimate the money demand function.

Second, a certain type of a dynamic adjustment model should be used to link a long-run equilibrium with a short-run disequilibrium state because usually, quarterly or monthly data are used to estimate the money demand function. As Laidler (1990) indicated, an actual economy adjusts to various shocks so that quarterly or monthly economic data usually tend to fluctuate around a long-run relationship over time.

As indicated in Section 2.1, one of the most popular dynamic adjustment model in economics is a partial adjustment model (PAM) and it is of the form:

\[ Y_t - Y_{t-1} = \Delta Y_t = b_1(Y^* - Y_{t-1}) + \epsilon_t \]  

(2.1.2)

where \( Y^*_t \) is a long-run equilibrium value of \( Y_t \), and \( 0 < b_1 < 1 \). If the long-run equilibrium relationship \( Y^*_t = \sum_{i=1}^{n} a_i X_{it} \) is substituted into equation (2.1.2), it becomes as below when a constant term \( b_0 \) is included:

\[
(2.2.24) \quad Y_t = b_0 + b_1 \sum_{i=1}^{n} a_i X_{it} + (1 - b_1) Y_{t-1} \\
= c_0 + \sum_{i=1}^{n} c_i X_{it} + c_{n+1} Y_{t-1} + \epsilon_t
\]
where \( b_1 a_i = c_i \), and \( (1 - b_1) = c_{n+1} \). If \( q \) lags are allowed, then it is:

\[
Y_t = b_0 + \sum_{i=1}^{q} \sum_{j=1}^{n} b_{ij} a_{it} + \sum_{j=1}^{n} (1 - b_{1j}) Y_{t-j} + e_t
\]  

(2.2.25)

It was Chow (1967) who first applied this partial adjustment model to estimate the demand for money function. Since then, it became usual to apply a PAM to estimate the money demand function until the 'missing money' problem showed up in the mid-1970s (Goldfeld and Sichel 1990, Gordon 1984, Judd and Scadding 1982).

An error-correction model (ECM) is different from a PAM as equation (2.2.24) or equation (2.2.25) and has the form of:

\[
(Y_t - Y_{t-1}) = \Delta Y_t = b_0 + b_1 (\sum_{i=1}^{n} a_i X_{it-1} - Y_{t-1}) + \sum_{i=1}^{n} b_i \Delta X_{it} + e_t
\]  

(2.2.26)

where \( Y^*_t = \sum_{i=1}^{n} a_i X_{it} \) is a long-run equilibrium value of \( Y_t \). When \( q \) lags are allowed, it becomes:

\[
(Y_t - Y_{t-1}) = \Delta Y_t = b_0 + b_1 (\sum_{i=1}^{n} a_i X_{it-1} - Y_{t-1}) + \sum_{j=0}^{q} \sum_{i=1}^{n} b_{ij} \Delta X_{it-j} + \sum_{j=0}^{q} c_j \Delta Y_{t-j-1} + e_t
\]  

(2.2.27)
This ECM has become increasingly popular in recent years as a unit root theory and a cointegration approach were introduced in econometrics and their relationship with ECM was known (Hendry 1991, Hendry and Ericsson 1991, Engle and Granger 1987).

Both an ECM and a PAM are derived from a cost minimizing behavior wherein the costs of disequilibrium are balanced against adjustment costs. Following Nickell (1985), the general cost function has a quadratic form such as:

\[
L = \sum_{i=0}^{T} \{ \lambda (Y_{t+i} - Y^*)^2 + (Y_{t+i} - Y_{t+i-1})^2 \}
\]

(2.2.28) where \( Y^* \) is the 'desired' or equilibrium value of \( Y_t \). Therefore, cost minimizing with respect to \( Y_t \) gives a second order difference equation as follows:

\[
\alpha Y_{t+i+1} - (1 + \lambda + \alpha) Y_{t+i} + Y_{t+i-1} = -\lambda Y^*_t
\]

(2.2.29)

and after some manipulation, it becomes:

\[
\Delta Y_t = (1-\mu) \{(1-\alpha\mu) \sum (\alpha\mu)^i Y^*_t - Y_{t-1}\}
\]

(2.2.30)

20. This derivation is similar to that of Stephen Nickell (1985) but it is more general. To see more in detail, look at Nickell (1985).
where $\mu$ is a stable root of difference equation (2.2.29). Therefore, to estimate this equation, a stochastic processes generating the $Y_{t+i}$, a sequences of expected values of $Y_{t+i}$, should be specified. For this purpose, several simple but general cases of $Y_{t+i}$ are considered below. If the actual sequences of target values is given by $\tilde{Y}_{t+i}$ where $\tilde{Y}_{t+i}$ are random variables and $i > 0$, then $Y_{t+i}$ is defined by $Y_{t+i} = E_t[\tilde{Y}_{t+i}]$.

(i) $Y_t$ follows a random walk.

If $Y_t$ follows a random walk with a drift, $\tilde{Y}_t$ is expressed as below.

\begin{equation}
(2.2.31) 
\tilde{Y}_t = \varrho_0 + \tilde{Y}_{t-1} + e_t \quad \text{or} \quad \Delta \tilde{Y}_t = \varrho_0 + e_t
\end{equation}

Therefore $Y_{t+i} = \varrho_0 i + Y_{t-1}$ for all $i > 0$ and if substituted into equation (2.2.30), then the observable form of below equation is derived:

\begin{equation}
(2.2.32) 
\Delta Y_t = \frac{(1 - \mu) \varrho_0}{(1 - \varrho \mu)} + (1-\mu) \Delta Y_{t} + (1-\mu)(Y_{t} - Y_{t-1})
\end{equation}
where $\Delta Y^*_t = Y^*_t - Y^*_{t-1}$. Obviously, equation (2.2.32) is in a form of a partial adjustment model.

(ii) $\ddot{Y}_t$ follows ARIMA($p$, 1, 0) with drift

Then $\ddot{Y}_t$ is of the form:

\[
(2.2.33) \quad \Delta \ddot{Y}_t = q_0 + \sum_{i=0}^{p} \phi_i \Delta \ddot{Y}_{t-i} + e_t
\]

and the equation determining $Y^*_t$ is written as below:

\[
(2.2.34) \quad Y^*_t = Y^*_{t-1} + \sum_{i=0}^{p-2} \phi_i \Delta Y^*_{t-i} + f_1 q_0
\]

If it is substituted into equation (2.2.30), then the observable decision rule becomes, after some manipulations, of the form:

\[
(2.2.35) \quad \Delta Y_t = (1 - \mu)(1 - \alpha \mu) \{\sum_i (\alpha \mu)^i \phi_i \} \sigma_0 + \sum_{i=0}^{p-2} [(1 - \mu)(1 - (1 - \alpha \mu) \{\sum_i (\alpha \mu)^i \phi_i \}] \Delta Y^*_{t-i} + (1 - \mu)(Y^*_{t-1} - Y^*_{t-1})
\]

and it is clearly a form of ECM.

(iii) $\dddot{Y}_t$ follows ARIMA(0, 1, q)
If $\tilde{Y}_t$ is ARIMA(0, 1, q), it is of the form:

$$\Delta \tilde{Y}_t = e_t + \sum_{i=0}^{q} \theta_i e_{t-i}$$  \hspace{1cm} (2.2.36)$$

and $\tilde{Y}_{t+s} = \tilde{Y}_{t-1}$. After substituting it into equation (2.2.36) and manipulating it, the decision rule is derived as below:

$$\Delta Y_t = \sum_{i=0}^{q} \theta_i \Delta Y_t + \sum_{i=0}^{q-2} (1-\mu) \Delta Y^*_t + \sum_{i=0}^{q-2} (1-\mu) (1-\sum_{i=0}^{q} \theta_i) (Y^*_t - Y_{t-1})$$  \hspace{1cm} (2.2.37)$$

and it is a form of ECM.

From these simple examples, several things can be observed. First, several forms of ECMs are derived from a dynamic cost minimization. Second, the stochastic process of $Y_t$ should be specified first to specify and to make observable form of ECM. Third, a PAM is a special case of an ECM. If the stochastic process of $Y_t$ is not a simple random walk but a more complicated stochastic process such that $\Delta Y_t$ follows an autoregressive or a moving average process, dynamic cost minimization gives a form of an ECM.

Many previous empirical models on the money demand were based on a partial adjustment model. As shown in Section 2.1.1, a real adjustment model and a nominal adjustment models are of the forms, respectively:
In past twenty years, these two adjustment models have been estimated and tested through standard econometric methods. Because these two models are non-nested and not directly statistically comparable, empirical research has been focused on the consistency of these models with economic theory and the stability of the estimated parameters. Although there have been many empirical investigations since then, no concrete conclusions have emerged. For example, according to Thornton's estimation (Thornton, 1985) over the data of 1951:2 - 1984:2, all these models show some problems either in parameter stability or theory compatibility.

Recently economists have realized these kinds of estimation and testing methods may have some problems. If many macroeconomic variables, including a real balance or a GNP, are not stationary and have unit root as shown in Nelson and Plosser (1982), standard regression theories cannot be applied to estimate the money demand function and to make inference about estimated parameters. In recent years, many new econometric theories have been developed, especially about the unit root and cointegration, and they make it possible to approach the above
problems in a different way. In this section, this new approach is employed to compare those two money demand function in a model specification or a hypothesis test.

A PAM equation (2.2.24) can be rewritten as below:

\[(2.2.38) \quad Y_t - Y_{t-1} = \Delta Y_t = b_0 + b_1(Y_t^* - Y_{t-1}^*) + b_1(Y^*_{t-1} - Y_{t-1})\]

With an equilibrium equation \(Y_t^* = \sum_{i=0}^{n} a_i X_{it}\), the PAM can be expressed as below:

\[(2.2.39) \quad \Delta Y_t = b_0 + b_1 \sum_{i=1}^{n} a_i \Delta X_{it} + b_1(\sum_{i=1}^{n} a_i X_{it-1} - Y_{t-1}) + \epsilon_t\]

where \(b_1 = b_2\). This equation is a restricted form of an error-correction model as equation 2.2.36, such that both \(b_1\) and \(b_2\) are the same but significantly different from 0. Therefore, testing that the PAM is a proper dynamic adjustment model is the same as testing two null hypotheses \(H_{01}: b_1 = 0, b_2 = 0\) and \(H_{02}: b_1 = b_2\) in equation (2.2.39).

If \(q\) lags are allowed, it has a form of:

\[(2.2.40) \quad \Delta Y_t = b_0 + \sum_{j=0}^{q} b_j \sum_{i=1}^{n} a_i \Delta X_{it-j} + \sum_{j=1}^{q} b_j(\sum_{i=1}^{n} a_i X_{it-j} - Y_{t-j}) + \epsilon_t\]
then the null hypotheses become $H_0^1: \sum_{j=1}^{q} b_{1j} = \sum_{j=1}^{q} b_{2j} = 0$ and $H_0^2: \sum_{j=1}^{q} b_{1j} = 0, j=1,2,\ldots,q$. Provided that both $Y_t$ and $X_t$ have a unit root and $Y_t$ and $X_t$ are cointegrated, $\Delta X_t$ and $\Delta Y_t$ are stationary and also $(\sum_{i=0}^{n} a_i X_{it} - Y_t)$ is stationary where $a = (a_1, a_2, \ldots, a_n)$ is a cointegrating vector. Therefore, all variables in equation (2.2.40) are stationary and standard regression theory can be applied.

In order to test those hypotheses, a non-linear least squares method suggested by Stock (1987) or Phillips and Loretan (1991) and a two step ordinary least squares method suggested by Engle and Granger (Engle and Granger 1987) are employed.

At the first step, a univariate error-correction model is estimated by applying non-linear least squares method to find long-run equilibrium coefficients or a cointegrating vector. Following Phillips and Loretan (1991), a general type of univariate error-correction model as below is used:

$$\Delta Y_t = b_0 + \sum_{i=0}^{q} b_{1i} (Y_{t-1-i} - Y_{t-1}) + \sum_{i=q}^{n} \sum_{j=0}^{2i} b_{2ij} \Delta X_{jt-i}$$

where $Y_t^* = \sum_{i=0}^{n} a_i X_{it}$, $q$ is a number of leads and lags of ECM and $n$ is the number of independent variables. Since this ECM is different from
standard ECMs in equation (2.2.26) and equation (2.2.27), it is called 'generalized ECMs' (Phillips and Loretan 1991). In this generalized ECM, q leads and lags of explanatory variables are included. As shown in Phillips and Loretan (1992), this estimator is consistent and asymptotically efficient.

At the second step, those estimated cointegrating vectors are regarded as given parameters and error-correction models such as equations (2.2.26) and (2.2.27) are estimated by an OLS method and a parsimonious form is searched for.

Actually this method combines both a non-linear least squares method and a two-step OLS method and it has several advantages. First, as Phillips and Loretan (1991) showed, this non-linear least squares estimator of a cointegrating vector in the general ECM is both consistent and asymptotically efficient and has less bias in small samples than a simple OLS estimator. Second, standard tests such as a t-test or a F-test can be applied to test the estimates of a cointegrating vector. Third, it is very easy to estimate and search for a parsimonious form of ECM or to test a hypothesis and a restriction on ECM because only a linear model is used at the second step estimation. A standard statistical package such as SAS or RATS is enough for a model estimation or a hypothesis test.

To find a very parsimonious form of an ECM, Hendry's 'general to specific' methodology is employed because there are too many parameters
to estimate in a general long lag ECM. That is, first a very general ECM with long lags is estimated and next, a more parsimonious form of ECM is searched. Throughout this process, several statistics are examined to find the 'best' equation, which is consistent with an economic theory, has desired statistical properties, and represents the dynamic structure of the data but as a parsimonious form.

As indicated in Hendry (1980) and in Hendry and Ericsson (1991), the following four conditions may be important criteria by which to judge the model specification:

1. Model's encompassment;
2. Error term's being white noise;
3. Parameter constancy;
4. Weak exogeneity of current conditioning variables.

If the suggested empirical model fails to satisfy any of above conditions, it should be rejected because those are the necessary conditions for a good model specification.

To evaluate model's encompassment, goodness of fit statistics, such as R-square or standard error, are examined. To check the error term's distribution, several statistics such as DW statistics, X² test statistics for serial correlation etc. are examined. To test parameter constancy, Chow tests are employed. As in cointegrating vector estimation in the above sector, a recursive least squares estimation method is also applied to examine the parameter constancy of the error-correction model in the 1970s.

In the money demand estimation, two types of adjustment variables, real balance and a nominal balance are considered and estimation
equations of partial adjustment models of real and nominal adjustment
types are of the forms, respectively:

(i). Real Adjustment Type:

\[
\Delta m_t = b_0 + \sum_{j=0}^{q} b_{1j} (a_1 \Delta t_{-j} + a_2 \Delta Y_{t-j}) + \sum_{j=0}^{q} b_{2j} (a_1 Y_{t-j-1} + a_2 Y_{t-j-1} - m_{t-j-1})
\]

(ii). Nominal Adjustment Type:

\[
\Delta m_t = b_0 + \sum_{j=0}^{q} b_{1j} (a_1 \Delta r_{t-j} + a_2 \Delta Y_{t-j} + a_3 \Delta P_{t-j}) + \sum_{j=0}^{q} b_{2j} (a_1 r_{t-j-1} + a_2 Y_{t-j-1} + a_3 P_{t-j-1} - m_{t-j-1})
\]

where \(r_t\) and \(Y_t\) are interest rate and real GNP respectively and null
hypotheses are \(H_0: \sum_{j=1}^{q} b_{1j} = \sum_{j=1}^{q} b_{2j}, H_0: \sum_{j=1}^{q} b_{1j} = 0, \sum_{j=1}^{q} b_{2j} = 0.\)

Generalized Error-correction models of real or nominal adjustment
types are also written as follow respectively:

(iii). Real Adjustment Type:
At the first step, non-linear least squares methods are used to estimate equation (2.2.44) and (2.2.45) and to estimate cointegrating vector and at the second step, tests are performed on equations (2.2.44) and (2.2.45). Finally, Hendry's method are employed to find a parsimonious form of money demand function as an ECM.

(b) Vector Error-correction Model

In addition to this univariate ECM, a vector error-correction model (VECM) is also employed to examine the monetary dynamic of this cointegrated economic variables. The VECM is defined as follows (Engle and Granger 1987, Hylleberg and Mizon 1989): A n-vector time series $X_t$ has an vector error-correction representation if it can be expressed as:

\begin{align*}
\Delta m_t &= b_0 + \sum_{j=1}^{q} b_j \left( a_1 Y_{t-j-1} + a_2 Y_{t-j-1} - m_{t-j-1} \right) \\
&\quad + \sum_{i=q+1}^{1} c_i \Delta r_{jt-i} + c_{i+1} \Delta Y_{jt-i} \\
\end{align*}

\begin{align*}
\Delta m_t &= b_0 + \sum_{j=0}^{q} b_j \left( a_1 Y_{t-j-1} + a_2 Y_{t-j-1} + a_3 P_{t-j-1} - m_{t-j-1} \right) \\
&\quad + \sum_{i=q+1}^{1} c_i \Delta r_{jt-i} + c_{i+1} \Delta Y_{jt-i} + c_{i+2} \Delta P_{jt-i} \\
\end{align*}
where $B$ is a lag operator, $A(B)$ is a $K$ (nxn) matrix ($A_{1}^{ij}(B)$, $A_{2}^{ij}(B)$, 
..., $A_{K}^{ij}(B)$), for $i = 1, 2, ..., n$, $j = 1, 2, ..., n$, and $K$ is the num-
ber of lags, $u_{t}$ is a stationary multivariate disturbance term, with $A(0)$
$= I$, all elements of $A(1)$ are finite, $Z_{t} = \sum_{i=1}^{n} a_{i} \cdot X_{it}$ and at least one
of $a_{i} \neq 0$, $i = 1, 2, ..., n$.

The main advantage of this VECM is to capture the time series
properties of variables in the system of cointegrated variables,
through the complex lag-structure allowed, while at the same time in-
corporating an economic theory of an equilibrium type.\(^{21}\) In addition,
with a cointegrated economic variables, the existence of VECM is
 guaranteed by Granger's representation theorem: (Engle and Granger
1987, Hylleberg and Mizon 1989): If a $n$ variable vector $X_{t}$ is coin-
tegrated with order $d$, $b$, denoted $X_{t} \sim CI(d, b)$, with $d = 1$ and $b = 1$,
and with cointegration rank $r$, then there exists an error-correction
representation with $Z_{t} = a \cdot X_{t}$ and with an ($r \times 1$) vector of stationary ran-
dom variables:

(2.2.47) \[ \mathbf{A}(B) \cdot (1 - B) \cdot \mathbf{X}_t = \sum_{i=1}^{p-1} \mathbf{A}_i \cdot \mathbf{B}^i \cdot (1 - B) \cdot \mathbf{X}_t = -\gamma \cdot \mathbf{Z}_{t-1} + \mathbf{d}(B) \cdot \mathbf{e}_t. \]

where \( \mathbf{A}(0) = \mathbf{I} \).

With this VECM (2.2.47), several things can be examined. For example, with a four variable VECM system of real GNP, nominal balance, price level, and interest rate, each equation has an error-correction term in the right hand side. Therefore, it can be shown which variable actually adjusts to the monetary disequilibrium and tested which adjustment model actually holds. Moreover, some methodologies of the vector autoregressive model such as causality tests can also be applied to the VECM.

A causality test means only a test of zero restrictions on the VECM and similarly, neutrality means that the sum of coefficients of lagged variables are zero in the equation (Stock and Watson 1989). Therefore, a causality test from the \( j \)-th variable to the \( i \)-th variable is a test for \( H_0: \sum_{k=1}^{k_0} \mathbf{A}_{ij} = 0 \) for all \( k \) in equation (2.2.47) and a neutrality test from the \( j \)-th variable to the \( i \)-th variable is a test for \( H_0: \sum_{k=1}^{k_0} \mathbf{A}_{ij} = 0 \) in equation (2.2.47). 22

22. This definition is different from that of Toda and Phillips (1991). Toda and Phillips consider not only the causality defined in this section but also the significance of the error-correction term. As a result, their test method is not a two step method and test statistics does not follow a F-distribution or \( \chi^2 \) distribution.
To estimate this VECM, a two step method as in the univariate model is employed as suggested by Engle and Granger (1987): in the first step, the cointegrating equation and $\hat{\beta}_{t-1}$ are estimated and in the next step, the VECM (2.2.47) is estimated by an OLS method. It is shown that two step estimators of a single equation of the error-correction system have the same limiting distribution as the MLE using true value of the cointegrating vector $\alpha$ and $\hat{\beta}_{t-1}$.

To specify and find a more parsimonious form of the VECM, one of the multivariate time series specification method is used. When $n$ variable VAR is given by:

$$(2.2.48) \quad A(B)X_t = (I - \sum_{i=1}^{P} \hat{A}_i^* B^i)X_t = \epsilon_t$$

where $\text{det}(A(B)) = 0$ has $d < n$ unit roots, it can be also rewritten as an error-correction model as equation (2.2.47) by a simple manipulation. Therefore, $q$ lag VAR can be rewritten as $q-1$ lag VECM.

In this lag selection procedure, the rank of autoregressive coefficient matrices in VAR is estimated and the number of VAR lag is selected. For example, when ranks of some autoregressive coefficients, $\hat{A}_i^*$ in VAR of equation (2.2.48) become 0, they can be eliminated in estimation and the VAR become more parsimonious. To test the rank of
matrices \( A_i^* \), Tiao and Tsay's \( C(k,s) \) statistic (1985) is utilized as below:

\[
C(k,s) = -(T-k-1) \cdot \sum_{i=1}^{s} \log(1- \hat{\lambda}_i(k))
\]

where \( X_{kt} = (X_{t}', X_{t-1}', \ldots, X_{t-k}')' \), and \( \hat{\lambda}_i(k) \) is the \( i \)-th smallest eigenvalue of \( \hat{V}(k) = (\Sigma X_{kt} \cdot X_{kt})^{-1} \cdot (\Sigma X_{kt} \cdot X_{kt-1}) \cdot (\Sigma X_{kt-1} \cdot X_{kt-1})^{-1} \cdot (\Sigma X_{kt-1} \cdot X_{kt}) \). If \( C(k,s) \) statistics is less then the critical value, then it is not rejected that rank of \( \Gamma_k \) is less than \( (n-s) \). As shown in Tiao and Tsay (1985), \( C(k,s) \) is asymptotically a chi-squared random variable with \( s^2 \) degree of freedom if rank of \( \Gamma_k < n-s \).

2.3 Structural Break Test

As indicated in the Introduction of this dissertation, one of the interesting questions in the money demand literature is the 'missing money' in the 1970s. Therefore, to examine the possible structural break of a money demand function in the 1970s, structural break tests are performed. In this section several break tests are reviewed.

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23. Eigenvalue \( A \) is the same as a squared partial canonical correlation of \( x_t \) and \( x_{t-k} \) given \( x_{t-1}, x_{t-2}, \ldots, x_{t-k-1} \).
Many tests for a structural break have been proposed in the literature. In this dissertation, following Andrews and Ploberger (1991), Andrews et al. (1992) and Bai et al. (1992), three tests statistics, considering the maximum of the process of Wald test statistics, $F_t$, are employed:

- Sup Wald Test Statistic: $F_s = \sup F_t$,
- Mean Wald Test Statistic: $F_m = \text{Mean } F_t$,
- Exp-mean Wald Test Statistic: $F_e = \text{Mean } \exp(F_t)$,

where $F_t$ is a Wald test statistic for a zero-one dummy variable at time $t$ and $t/T \in (\delta, 1 - \delta)$. As shown in Andrews and Ploberger (1991), mean Wald test and Exp mean test statistics are considered as the most powerful tests.

As a parameterized model, an autoregressive model with a zero-one dummy intercept variable is selected, so that the estimated equation is of the form:

$$\Delta X_t = \sum_{j=1}^{P} a_j \Delta X_{t-j} + \lambda_i D_i + \epsilon_t,$$

where $D_i$ is 0 if $t < i$ and 1 if $t \geq i$ and $i \in (0.15XT, 0.85XT)$. A Wald test ($F$-test) statistic on the dummy intercept for $H_0: \lambda_i = 0$ for each $i$
is estimated. It is known that $F_t$ and $g(F_t)$ have limiting distributions of $F^*$ and $g(F^*)$ respectively, where $g(\cdot)$ denote the sup $F$, mean $F$ and Exp-mean $F$ functionals respectively given in above and $F^*(\delta) = \{\delta(1-\delta)\}^{-1/2} \tilde{W} \tilde{W}$ and $\tilde{W}(\cdot)$ is one dimensional two sided Brownian motion on $(-\infty, \infty)$ (Andrews 1991 and Bai et al. 1992). Since both $F^*$ and $g(F^*)$ are non-standard distributions and their small sample distributions are not known, the significance levels are tabulated by simulations in Bai et al. (1991).

In this dissertation, structural break tests are performed on each individual variable in a money demand equation and on the residuals of a cointegration equation and their statistics are compared with the significant level. The former is to detect the break of each individual series and the latter is to find the break in a cointegration equation.

A confidence interval of the timing of a possible structural break is also constructed. As shown in Bai (1991) and Bai et al. (1992), the distribution of the timing $i$ of a possible structural break is determined as follows: $(\lambda \Sigma_{\xi}^{-1} \lambda)(i - i_0) \Rightarrow V^*$, where $\Sigma_{\xi}$ is a variance covariance matrix of error term in equation (2.2.50), $V^* = \arg\max_v (-\infty, \infty)(W(v) - 1/2|v|)$ and $W(\cdot)$ is one dimensional two sided Brownian motion on $(-\infty, \infty)$. In addition, when $\hat{\lambda}$ and $\hat{\Sigma}_{\xi}$ are the Gaussian maximum likelihood estimators of $\lambda$ and $\Sigma_{\xi}$ evaluated at $\hat{i}$, $(\hat{\lambda} \hat{\Sigma}_{\xi}^{-1} \lambda)(\hat{i} - i_0) \Rightarrow V^*$. 
The limiting distribution of \( V \) is 
\[
1.5 e^{-|x|/0.5} \Phi(-1.5\sqrt{|x|}) - 0.5 \Phi(0.5)/|x|,
\]
where \( \Phi(\cdot) \) is the cumulative normal distribution function (Picard 1985) and the 10% and 5% two-sided critical values are 4.67 and 7.63, respectively (Bai et al. 1992). Based on this distribution, a 100(1-\( \alpha \))% confidence interval of the timing of a structural break is constructed as follows:

\[
(2.2.51) \quad i \pm c_\alpha / (\hat{\alpha} \Sigma^{-1} \hat{\alpha}),
\]

where \( c_\alpha \) is the \( \alpha \) percentile of the distribution of \( v^* \).
Chapter III. Empirical Results

This chapter presents the empirical study of the money demand function of the United States. As indicated in the Introduction, three major empirical research results are presented. In Section 3.1, a long-run money demand and a short-run monetary dynamic are examined with the data from 1953 to 1991 by a cointegration theory and an error-correction model. Section 3.2 examines the relationship of a partial adjustment model and a more general error-correction model and presents several tests results on them. In Section 3.3, the money demand in the 1970s and the 'missing money' are revisited and sequences of cointegration tests and sequences of cointegrating vector are estimated to see the changes of cointegration relationship of money demand in the 1970s.

The data used in this empirical research are seasonally adjusted quarterly data from 1953 to 1991:2. M1 is currency plus checkable deposits and is measured as seasonally adjusted monthly averages of February, April, August and November of each year. A scale variable,  

\( Y_t \), is measured by a real GNP and a price level, \( P_t \), by the implicit GNP price deflator. As opportunity costs of holding money, the 10 year treasury bond rate (TB), the time deposit rate (TD) and the inflation rate (\( \Pi \)) are adopted. Real balances, \( m_1 \), are defined as M1 deflated by the price level and all variables are in logarithms except the inflation rate.\(^{25}\)

3.1 The Money Demand in 1953 - 1991

As a standard form of the money demand function as economic theories suggest (Baumol 1952 and Tobin 1956), money demand is assumed to be a stable function of a scale variable and an opportunity cost of holding money. Therefore, a cointegration relationship of real balances, real GNP and treasury bond rate is considered and it is estimated with the data of 1953 - 1991. Since all variables in this model (real balances, real GNP and the interest rate) are in log form, and the tested equation is in linear form, the long-run money demand

\(^{25}\) All data were obtained from Citibase. The Citibase M1 series (FM1) was used for 1959:1 - 1991:3 and the earlier M1 data were formed by splicing the M1 series reported in Banking and Monetary Statistics, 1941-1970 (Board of Governors of the Federal Reserve System) to the Citibase data 1959 average. The price deflator was obtained as the ratio of nominal GNP and real GNP (GNP and GNP82 in Citibase). Interest rate is 10 year treasury bond rate (GYGN3 in Citibase).
equation is implicitly of the log linear form: \( m = A Y^\alpha T^\beta \), where \( A \), \( \alpha \), and \( \beta \) are constant.

As the first step of this empirical examination, unit root tests are performed on each individual series and test results are discussed in Section 3.1.1. Provided that those variables are I(1) series, several cointegration tests are performed on those possible cointegrated variables and their relationship is addressed in Section 3.1.2. Finally, a vector error-correction model is estimated based on the cointegrated relationship of money demand, and a final short-run money demand function is specified as a single equation reduced form in Section 3.1.3.

3.1.1 Unit Root Tests

The unit root tests on the above economic variables are performed by the method suggested in Section 2.1.1 and the test statistics, ADF\((a)\) and ADF\((t)\) test statistics and Phillips and Perron's Z\((a)\) and Z\((t)\) statistics are reported in Table 1 and Table 2 respectively. As shown in Table 1, ADF test statistics of real balances, real GNP, and the treasury bill rate are greater than their critical values at the 5% significance level whether trend is included or not in the test equation so that the null hypothesis of unit root is not rejected. In
addition, Phillips and Perron tests in Table 2 do not reject the null hypothesis at the 5% significance level. However, tests for the inflation rate are less clear. Its ADF statistics in 0 or 1 AR lag model are greater than the 5% critical value but with more lags they are not. In other words, ADF test with fewer lags rejects the unit root of the inflation rate but that with longer lags does not reject. With this ambiguous result of different lag structure, Akaike Information Criterion and Sawa’s Bayesian Information Criterion are estimated and the number of lags is selected as 1. Therefore, it is considered as stationary. According to the Phillips and Perron test, test statistics are quite stable and the unit root of the inflation rate is uniformly rejected.

In conclusion, the unit root test statistics in Table 1 and Table 2 indicate that real balances, the treasury bill rate and real GNP are integrated series but the inflation rate is a little ambiguous. However, throughout this dissertation, the inflation rate will be regarded not as I(1) but as an I(0) series.

3.1.2 Cointegration Tests

In the cointegration tests on money demand equation, real balances, real GNP and the treasury bond rate are included. The inflation rate is not included yet because it is not clear whether the inflation rate is I(0) or I(1). If it is a stationary series, then it
would be meaningless to add it in cointegration tests and even if it is I(1), it would be reasonable not to include it because usually the nominal interest rate and the inflation rate tend to move together and they may be cointegrated. However, the inflation rate will be included when estimating a money demand function as an error-correction model type.

First, residual based tests of Engle and Granger's ADF tests and Phillips Ouliaris's $Z(a)$ and $Z(T)$ tests are performed and their test statistics are reported in Table 3. Since the standard statistics for a lag selection procedure, such as Akaike Information Criterion (AIC) and Sawa's Bayesian Information Criterion (BIC), indicated that the ADF statistic with no lag is the best, only that statistic is considered. The estimated ADF test statistic is -3.494. Since the critical values at the 5% and 10% significance levels provided by Phillips and Ouliaris (1991) are -3.768 and -3.449 respectively, no cointegration among real balances, real GNP and a treasury bond rate is rejected at the 10% significance level but not at the 5% significance level.

Phillips and Ouliaris's $Z(a)$ statistic and $Z(a)$ statistic are -13.237 and -2.688, respectively. These two statistics are greater than their critical values of the 5% significant level, -26.094 and -3.768, respectively, and they do not reject the null hypotheses of no cointegration.

Second, Stock and Watson's common trend test statistics are estimated. First, $C(k, a)$ statistics are employed to specify the lag
of VAR model because these tests are based on the VAR model and two lag VAR model is selected to filter the data. Finally, eigenvalue and their $q_f$ statistic are estimated and is reported in Table 4. The estimated $q_f(3, 2)$ statistic is -17.48, which is greater than -26.0, a critical value of the 5% significance level. Therefore, three common trend or no cointegration among them is not rejected at the 5% significance level.

Third, Johansen tests based on likelihood ratio test principle are performed. First, The $C(k,s)$ statistics for the ranks of lag coefficient matrix in the VAR of real balances, the interest rate and real GNP are estimated and they are reported in Table 5. As shown in Table 5, test statistic of the 7th lag autoregressive coefficient matrix is 20.792 and it is greater than than 16.29, the critical value at the 5% significance level. The null hypotheses that the rank of 5th lag coefficient matrix is zero is not rejected at the 5% significant level and consequently, 7th lag should be included in this VAR. However, test statistics of the 8th and 9th lag coefficient matrices are 15.72 and 9.208, respectively, and they are not significant at the 5% level. The null hypotheses that the rank of 8th lag and 9th lag coefficient matrix is zero is not rejected at the 5% significant level. Therefore, from these test results, VAR model with 7 lags or VECM with 6 lags is selected for further analyses. However, to see the
robustness of cointegration test statistics, other lag structures are also considered.

Next, with this VAR model, partial squared canonical correlations and Johansen and Juselius's trace test statistics and maximum eigenvalue test statistics are estimated and they are reported in Table 6 along with their critical values.

Trace test statistics and maximum eigenvalue test statistics for the 4 lag VAR model are 50.846 and 32.227. Since their critical values at the 5% significance levels are 40.20 and 26.41, respectively, the null hypothesis of no cointegration among these three variables is rejected under the 5% significant level. However, the test statistics for a null hypothesis of more than one cointegrating vector are less than the 5% significant level. These statistics are also estimated with different lags and are reported in Table 6. They confirm that these statistics are relatively stable with different lags and that the model has only one cointegrating vector.

In conclusion, Engle and Granger's ADF tests and Johansen's trace test and maximal eigenvalue tests based on maximum likelihood estimation do reject the null hypothesis of no cointegration, but Stock and Watson's common trend test and Phillips and Ouliaris's $Z(t)$ and $Z(a)$ tests do not.

One of the reasons for this different test result is that all these tests are based on their asymptotic distribution and they have different statistical powers in small samples. Since Monte Carlo study
suggests that tests based on maximum likelihood estimation outperform Stock and Watson's common trend test and Phillips and Ouliaris's $Z(a)$ and $Z(t)$ tests, it may be concluded that real money balances, real GNP and the short-run interest rate are cointegrated.

Estimation of the cointegrating vectors during 1953 - 1991:2 are performed by several methods described in Section 2.2.2 and their estimates are reported in Table 7. As for the long-run income elasticity, an OLS estimator by the Engle and Granger method is 0.586 and dynamic OLS and GLS estimators are between 0.286 and 0.676. Estimates of elasticity of interest rate are also in a wide range. A static OLS estimation gives 0.312 and dynamic OLS and dynamic GLS estimators are between 0.075 and 0.386. Despite that those estimators are consistent, they look different in this small sample model.

Even though there are several available estimates of the cointegrating vector, only one estimate is selected and will be used for a further analysis. For this purpose, residuals of each cointegration equation are derived and ADF tests are performed on these residuals to check that residuals based on the estimated cointegrating vector show stationarity. If the estimate of the cointegrating vector is close to the true cointegrating vector, then residuals of this cointegrating vector should be stationary.

The ADF test statistics on these estimated residuals are reported in Table 8. Since the 5% and 10% critical value of the ADF test statistics on residuals of OLS cointegration equation are 3.768 and
3.449 respectively, several of the estimated ADF test statistics are not significant, indicating that residuals have unit roots and estimated cointegrating vectors are not close to the true value. The ADF statistic of dynamic OLS with 3 lags and leads is 3.975 and it is also greater than the 5% significance level, meaning that this estimate is close to the true cointegrating vector. In the rest of this chapter, this dynamic OLS estimator with 3 lags and leads will be used as a cointegrating vector. According to it, the long-run elasticities of real money balance with respect to real GNP and treasury bond rate are 0.676 and -0.386 respectively. Therefore, the long-run money demand function is specified as follows:

\[(3.1.1) \quad m_t = 1.631 + 0.676 Y_t - 0.386 TB_t \]

\[ (0.027) \quad (0.019) \]

Figure 1 shows residuals of the cointegration equation estimated by the dynamic OLS method with 3 lags and leads. Even though it looks well-fitted in the long-run, it shows some problems such as consistent over-prediction in the late 70's and under-fitting in the late 80's.
To examine it, a short-run model is estimated by error-correction model.

3.1.3 Error-correction Model and The Short-run Money Demand Function

To analyze the short-run dynamics of money demand function, a vector error-correction model (VECM) of 4 macroeconomic variables of real GNP, money balance, price level and interest rate is estimated by a two-step method. For this purpose, as indicated in Section 3.1.2, a dynamic OLS estimator with 3 lags and leads are selected and estimation results of this VECM are reported in Tables 9, 10 and 11. As indicated in Johansen’s cointegration test, 4 lag VECM is selected. There are several monetary adjustment models in the literature as explained in Section 2.1. They are classified as (1) a real adjustment model, (2) a nominal adjustment model, and (3) a price adjustment model, according to which variable actually adjusts to the monetary disequilibrium. One of the advantages of this VECM is that they can be compared with each other in this VECM.

First, as shown in Table 9, the sign of the error-correction term in the nominal balance equation is negative, indicating that the model is stable. Second, the error-correction term in the nominal balances is significant but that of the price equation is not, meaning that only
nominal balances adjust to the monetary disequilibrium. In other words, empirically, a nominal adjustment model is favored by this VECM. Third, to see more detail of this VECM, a Granger causality test and neutrality test are performed and test results are reported in Tables 10 and 11. As defined in section 2.2.3, a causality test means only a test of zero restrictions on the VECM and similarly, neutrality means that the sum of coefficients of lagged variables are zero in the equation (Stock and Watson 1989). According to these test statistics, several interesting things are noticed.

One of the most controversial debates in the last two decades is a debate of money-income causality. According to causality test in this VECM, it is not rejected that change in nominal balances do not Granger-cause change of real GNP. That is to say, nominal money balances do not help predict the movement of real GNP. However, an interesting relationship between these two variables shows up from this test result. According to the test statistics in Table 10, both hypotheses that change of nominal balance does not Granger-cause the change of interest rate and that the change of the interest rate does not Granger-cause the change of GNP, are rejected. In other words, changes in nominal balances may cause changes in the interest rate and those of interest rate may cause changes in GNP. This mechanism is the same as the standard Keynesian transmission theory as found in standard macroeconomics textbook.
In the neutrality tests, it is not rejected that changes in nominal balance do not have any effect on output or on interest rate or that changes in interest rate do not have any effect on output. From these two tests, it may be concluded that the causality form nominal balance to the interest rate and that from the interest rate to real GNP hold only in the short-run but not in the long-run.

These test results seem to contradict Stock and Watson's (1989). According to their test results, money clearly causes industrial production. However, to interpret the statistical result in this section, one should be a little cautious because there are two channels through which economic variables can be related in this VECM. Even though causality test statistics or neutrality test statistics of these VECMs show that nominal money balances do not have any impact on real output, it should be remembered that the error-correction term is significant and that money, real GNP, interest rate and price level are cointegrated. In other words, they share a common trend in the long-run and they move together; as a result, the current change in nominal balances is partly the result of their movement into alignment with the trend value of real GNP or the interest rate. Therefore, the above result can be indirect evidence that money has an impact on GNP not only in the short-run but also in the long-run.

Since the demand for money function is expressed as a single equation in the literature, a univariate ECM is set up from the above
VECM and Hendry's 'general to specific' methodology is adopted to find a more parsimonious form. In addition, the inflation rate, which was not included in cointegration analysis, is added to estimate the short-run money demand function because it is considered as one of the opportunity cost of holding money. At the first step, the unrestricted general ECM is estimated and reported in Table 12 and next Hendry's 'general to specific' principle explained in Section 2.2.3 is applied.

Finally, a below equation is selected:

\[
\begin{align*}
\Delta m_t &= 0.058 - 0.032 \text{ECM}_{t-1} + 0.163 \Delta m_{t-1} + 0.219 \Delta m_{t-2} \\
&+ 0.116 \Delta m_{t-6} - 0.019 \Delta TB_t - 0.048 \Delta TB_{t-1} - 0.038 \Delta TB_{t-2} \\
&- 0.038 \Delta TB_t - 0.328 \pi_t \\
\end{align*}
\]

\[
\begin{align*}
\Delta m_t &= (0.022) (0.011) (0.070) (0.068) \\
\text{ECM}_{t-1} &= (0.055) (0.009) (0.009) \\
\Delta m_{t-1} &= (0.009) (0.094) \\
\Delta m_{t-2} &= (0.009) (0.009) \\
\Delta m_{t-6} &= (0.009) (0.009) \\
\Delta TB_t &= (0.009) (0.009) \\
\Delta TB_{t-1} &= (0.009) (0.009) \\
\Delta TB_{t-2} &= (0.009) (0.009) \\
\Delta TB_t &= (0.009) (0.009) \\
\pi_t &= (0.009) (0.009) \\
\end{align*}
\]

\[DW = 1.836, \quad R^2 = 0.622,\]

\[\xi_1,1 = F(1, 139) = 3.42, \quad \xi_1,4 = F(4, 136) = 2.16,\]

\[\xi_2 = F(18, 118) = 2.87,\]

\[ARCH 2 F(2, 133) = 2.24, \quad ARCH 4 F(4, 129) = 1.95\]

\[\xi_3 = F(1, 139) = 3.03\]

\[\xi_4 = X^2(2) = 1.64\]

In equation (3.2.1), the dynamic OLS cointegrating vector of 3 lags and leads is used. All the coefficients in this equation are statistically significant at the 5% level and their signs are compatible with economic theory. (i.e. those of error-correction terms, the interest rate and the inflation rates are negative and those of real GNP are positive.)
\( f_{1,i} \) is a Lagrange multiplier test statistic for residual autocorrelation of order \( i \) in F-test form, \( F(i, T-i) \) and \( f_2 \) is a Lagrange multiplier test statistic for heteroscedasticity in F-test form. \( f_3 \) is a RESET (regression specification test) statistic for omitted variables, F-test form and \( f_4 \) shows the test statistics for the normality of residuals as a \( x^2 \)-test form. As all the above test statistics show, there seems little problem with serial correlation or ARCH. In addition, residuals of this equation seems to follow a normal distribution and inference based on the t-distribution or F-distribution works well. Figure 2 shows the actual and fitted value of \( \Delta m_t \) from equation (3.1.1). It fits relatively well and does not show any disturbance through the whole period.

With these final models and general unrestricted models, structural stabilities are tested through the period when most money demand models show structural breakdowns such as 'missing money' period in 74:1 - 76:2, velocity decline period in 81:4 - 83:2 and M1 explosion period in 85:1 - 86:4. The Chow test statistics are reported in Table 13. At the 5% significance level, none of these test statistics are bigger than the critical values and none of them reject the constancy of the parameters.

3.2 Partial Adjustment Model and Error-Correction Model.
This section presents an empirical study of dynamic adjustment model of the money demand function in the United States from 1953 to 1991. In a possible dynamic adjustment model, a real adjustment model, a nominal adjustment model and an error-correction model are considered.

All variables are in logarithms except the interest rate and the inflation rate. The long-run money demand function or a possible cointegration relationship is estimated as a linear equation and it is specified as both a real balance equation and a nominal balance equation. In addition, they are estimated with and without the inflation rate because it can be considered as another opportunity cost of holding money in addition to interest rate.

3.2.1 Real Adjustment Model and Nominal Adjustment Model

As the first step of this empirical investigation, the real and nominal adjustment models based on PAM are compared and for this purpose, equations (2.2.38) and (2.2.39) are estimated as restricted forms of an ECM by a nonlinear least squares method. Table 14 shows the empirical results of this estimation including estimates of coefficients and standard errors. According to it, whether inflation rate is included or not in a long-run money demand function, the long-run elasticity of a nominal money balance with respect to the price level
is close to 1. This is called the homogeneity property of long-run money demand function and it is considered as one of the necessary conditions of well-specified money demand function (Goldfeld and Sichel 1987).

The long-run elasticity of money demand with respect to real GNP ranges from 0.521 to 0.657 and interest semi-elasticity ranges from 0.026 to 0.055. Figure 3 and 4 show the actual and fitted value of real and nominal balances from these partial adjustment models estimations. In Figure 3, explanatory variables are GNP and interest rate and in Figure 4, they are GNP, interest rate and price level. As Goldfeld indicated, a real adjustment model seriously under-predicts real money balances after the 1973 and shows the 'missing money' problem. A nominal adjustment model does not seem to have the 'missing money' problem in late 70s but does not fit very well after 1980. More about the stability of the estimated parameters are discussed in parameter constancy tests.

Table 15 shows the nonlinear least squares estimates of cointegrating vectors from unrestricted error-correction models with some past and future lags of explanatory variables. These estimators are not only consistent but also efficient as shown in Phillips and Loretan (1991). The elasticity of money demand with respect to the price level is close to 1 and also the null hypothesis that it is one is not rejected by t-tests. It seems that estimates of cointegrating vector
in Table 15 are close to those of PAMs in Table 14. The long-run elasticity of real money demand with respect to GNP is between 0.506 and 0.800 and a semi-elasticity with respect to the treasury bill rate is between 0.20 to 0.54. The former is relatively stable whether the inflation rate is included or not but the latter is not. If the inflation rate is included, it is around 0.025 but without it, it goes up to 0.057.

With these estimates of cointegrating vectors, \( \hat{a} \), in Table 15, restrictions of PAMs, \( H_0^1: \Sigma b_{1j} = 0, \Sigma b_{2j} = 0 \), and \( H_0^2: \Sigma b_{1j} = \Sigma b_{2j} \) in equations (2.2.39) and (2.2.40) are tested and their test statistics are reported in Table 15. According to it, the first null hypothesis \( H_0^1: \Sigma b_{1j} = 0, \Sigma b_{2j} = 0 \), i.e. sum of adjustment coefficients are zero, is clearly rejected at the 5% significance level whether inflation is included in long-run equation or not and whether the number of lags is different. The second null hypothesis such that \( H_0^2: \Sigma b_{1j} = \Sigma b_{2j} \) is certainly rejected at the 5% significance level except for 3 cases.

Next, parameter constancy tests of PAM are performed and test statistics are reported in Table 16. The considered period is 1953:1 to 1991:2 with a split point of 1974:1 or 1979:1. This division is a
little arbitrary but considering that the 'missing money' problem hap-
pened during 1974:1 - 1976:2 and financial de-regulation began at that
time, it has some aspects which make it desirable.

Among these 12 estimated equations, none of them passes the con-
stancy tests and it is clearly rejected that all the parameters are
constant through the considered period. The nominal adjustment model
passed these tests when the considered period is 1953:1 to 1976:4 or
1953:1 to 1978:1 with a split point of 1974:1. This is the same as
Goldfeld's argument (1976) that a nominal adjustment model can reduce
the forecast errors of a real adjustment model during the 'missing
money' period of 1974:1 to 1976 and that the former fits better than
the latter does.

In conclusion, it can be said that both a real adjustment model
and a nominal adjustment model based on a PAM do not satisfy the
desirable properties that a money demand function should have and con-
sequently, a less restricted and more general ECM is recommended.

3.2.2 Error-correction Model

At the first step of ECM estimation, estimated cointegrating vec-
tors in Table 15 are carefully examined and six of them are selected
based on the significance level of estimated coefficients. They are
equations (1), (2), (3), (4), (7) and (9). Next, with these cointegrating vector estimates, a general unrestricted ECM which consists of up to 4 lags of the differences of each cointegrating variable are estimated by an ordinary least squares method. Finally some parsimonious forms of ECMs are chosen by Hendry's principle and they are called Model I and Model II.

These models are four variable cointegrating equations including real balances, GNP, interest rate and an inflation rate. To find a error-correction term, estimated cointegrating vectors of equations (2) and (4) in Table 15 are adopted. In the Model I, the estimates of the long-run elasticity on real money demand with respect to GNP is 0.58 and the semi-elasticities with respect to the treasury bill rate and an inflation rate are 0.027 and 0.036. In the Model II, they are 0.074, 0.021 and 0.042, respectively. As Goldfeld and Sichel find in 1987, the inflation rate is significant in the cointegration equation of real money balances.

The estimates of parsimonious ECMs are reported in Table 17. In addition, ECM estimates of two sub-periods 1953:1 to 1973:4 and 1974:1 to 1991:2 are reported. As seen in Table 17, these two equations look well-specified. Both cointegrating vectors and estimated coefficients in ECMs have the anticipated sign and also are significant at the 5% level. Even in the sub-period models, all coefficients have anticipated signs except the lagged value of real balances. Figure 5 and
6 are the actual and fitted values of real balances from 1953:1 to 1991:2. It seems that both models have neither the 'missing money' problem nor any significant under-prediction or over-prediction during the sample period. In addition, as several statistics show, they do not have any significant serial correlation problem.

In order to test the stability of these specifications through time, parameter constancy tests (Chow Tests) are performed with these two equations and test statistics are reported in Table 18. The first column in Table 18 shows the Chow test statistics for the 'missing money' period 1974:1 - 1976:4 and the third and fourth columns are those for 1953:1 - 1991:2 with a split point of 1974:1 and 1979:1. As this table shows, all these statistics are not significant at the 5% level and parameter constancy for these periods is not rejected. In conclusion, it can be said that these two ECMs have desirable properties which money demand function should have.

3.3 Money Demand in the 1970s.

The 1970s are an important period in the money demand literature. First, a conventional money demand function is for the first time noticed to be unstable and second, the prediction based on it shows significant and consistent errors for several years. In this section,
the money demand of this period will be extensively examined and es-
timation results are presented by three parts: (1) cointegration tests,
(2) error-correction model and (3) structural break tests.

In this section, a four variable economic model is considered
using real balances (m1), real GNP, the treasury bond rate (TB) and
time deposit interest rate (TD). As seen, this model is different from
the three variable model in Section 3.1. There are some reasons for
including time deposit rate. First, until 1980s, the time deposit in-
terest rate was controlled by the Federal Reserve Board according to
Regulation Q, so that it did not change frequently and it is a form of
a step function. Therefore, TD rate looks as if it is not cointegrated
with treasury bond rate and it is also confirmed by several cointegra-
tion tests. Second, before the 1980s, the saving account was
considered as the most important substitute for a checking account and
it is reasonable to include this to analyze its relationship with money
balance. In addition, it would be reasonable to use the same variables
as Goldfeld or other previous researchers did because one of the objec-
tives of this dissertation is to compare this new money demand function
with the previously estimated models in the 1970s.

3.3.1 ‘Missing Money’ in the 1970s.
With these data, a conventional Goldfeld-type real adjustment model like equation (2.2.3) is estimated and its results are reported in Table 19. It is estimated with the sample period of 1953:1 to 1973:4 and with the sample period of 1953:1 to 1979:4 by an OLS estimation method and a GLS estimation method allowing a first order serial correlation. Since it is known that the 'missing money' problem occurred in the mid 1970s, these two estimation results will be enough to show how the coefficients on a conventional money demand function changed in the 1970s.

Figure 7 shows the dynamic simulation of real money balance since the 1950s. As seen clearly in this figure, the backward simulated value before 1973 is very close to the actual value and any consistent and significant errors do not appear. However, after 1974, a consistent and significant over-forecast, a so-called 'missing money' problem is shown.

Table 19 shows several important things about the conventional money demand function in the 1970s. First, the coefficient on the lagged dependent variable increases, indicating long adjustment lags. The OLS estimate and the GLS estimate increase from 0.889 to 0.954 and from 0.855 to 0.920, respectively. Second, the impact elasticity for real income significantly decreases. The OLS estimate decreases from 0.076 to 0.044 and the GLS estimate also decreases from 0.091 to 0.058. Third, long-run elasticities with respect to income and interest rates
rise. The OLS and GLS estimates of long-run elasticities with respect to real GNP rise from 0.691 to 0.956 and 0.628 and 0.806 respectively. Those with respect to treasury bond rate also rise from 0.263 to 0.674 and 0.200 to 0.458 respectively. Therefore, it can be said from these results that a conventional money demand relationship changed in the 1970s.

3.3.2 Cointegration Tests and the Long-run Coefficients

In order to examine how the money demand relationship changed in 1970s, sequences of cointegration tests are performed on it and their test statistics are plotted in Figure 8. The graph (a) in Figure 8 shows the sequence of ADF test statistics and the graph (b) and (c) in Figure 8 show the sequences of Johansen's trace and maximal eigenvalue test statistics, respectively, for a null hypothesis of no cointegration with the data from 1953:1 to each period. In ADF test, no lag and 1 autoregressive lag terms are selected according to AIC or BIC statistics and in Johansen's tests, 4 lag VAR model is selected. As clearly seen in the graph (a) of Figure 8, the ADF test statistics are stable until 1975 but sharply rise soon after 1975. As seen in the graph (b) and (c), both trace test statistics and maximal eigenvalue test statistics for the null hypothesis of no cointegration are higher than the 5% significant levels until the mid-1970s but soon after, they become less
than the 5% significant levels. Therefore, it can be said from these sequences of cointegrating test statistics, ADF and Johansen's test statistics, that the cointegration relationship of money demand, which holds well until the early 1970, deteriorates after the mid-1970s. In addition to these tests, the null hypothesis that there may be more than one cointegrating vector is also tested by both trace and maximal eigenvalue tests. Even though test statistics are not reported here, it has been determined that there does not exist more than one cointegration relationship among these variables.

Since the above three test statistics indicate that the cointegration relation of money demand holds before 1974 but does not afterward, a cointegrating vector using the data from 1953:1 to 1973:4 is estimated by a dynamic OLS method and a dynamic GLS method and estimated residuals from those cointegrating vectors are examined.

First, as reported in Table 20, all the signs of these estimates of cointegrating vectors are as expected and all the coefficients are significant at the 5% level. Several estimates of long-run elasticity of real money balances with respect to real GNP are around 0.5 and support the theory of economies of scale in money holding. The estimates of the long-run elasticities with respect to the treasury

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26. See Baumol (1952) and Tobin (1956)
bond rate and the saving interest rate are also similar to each es-
imate and are around 0.1 and 0.2, respectively. These estimates are a
little different from those of a real adjustment model in Table 19. In
particular, the cointegrating coefficient of the treasury bond rate is
less than that of the saving interest rate. It seems natural that the
former is less than the latter because a saving account is regarded as
a more close substitute to a narrow money (ml) than a treasury bond.27

Based on these estimated cointegrating vectors, the residuals of
cointegration equation from 1953 to 1979 are derived and they are
plotted. Since there are ten possible estimates of cointegrating vec-
tors, there can be ten different residuals of cointegration equation
and it would be tedious to plot all the residuals. Therefore, unit
root tests are performed on the estimated cointegration residuals as
done in Section 3.2.2 and test statistics are reported in Table 21.
Two estimates of cointegrating vectors, equation 4 and equation 8 (a
dynamic OLS estimate and a dynamic GLS estimate with 2 lags and leads)
are selected and residuals of those cointegration equations are plotted
in Figure 9.

However, all graphs look similar and it would make little dif-
ference if other estimates of cointegrating vectors were used. As seen

27. In Goldfeld (1974) or Goldfeld and Sichel (1991), the elasticity
of saving interest rate is greater than that of treasury bill rate or
that of commercial paper rate.
in both graph (a) and (b), patterns of the estimated residuals of cointegration equations are quite similar. They had been stable until 1974 and sharply dropped after 1975 and became stable again after 1977. Therefore, it may be inferred that a structural break in this cointegration relationship possibly occurred around 1974 or 1975.

With this information, structural break tests introduced in Section 2.3 are performed. First, structural break tests are performed on each series and three test statistics, (a) sup Wald test statistic, (b) mean Wald test statistic and (c) exp-mean Wald test statistic, are estimated and reported in Table 22. For these tests, 4 lag autoregressive model, equation (2.2.46) with $p = 4$, is selected.

As shown in Table 22, the structural break test statistics for all four variables, real balance, real GNP, and the two interest rates, are less than 5% significant level and the null hypothesis of no structural break of each individual series is not rejected through the sample period of 1953 to 1979. However, when the same tests are performed on the estimated residuals of several cointegration equations, the opposite result obtains. As reported in Table 22, all sup-Wald test statistics are greater than the 1% significance level and mean Wald test statistics and exp-mean Wald test statistics are also significant at the 5% level. Therefore, it can be said from these test results that a structural change of a cointegration relationship among
real money balances, a real GNP and interest rates occurred within the sample period.

Figure 10 show the sequence of Wald test statistics on the residuals of estimated cointegration equations 4 and 8. According to it, it seems that a break does happen in the mid-1970s. To find the timing of this structural change of the cointegration equation, a confidence interval is constructed. Since the supremum of the Wald test statistic happens at 1974:3, this may be a possible break point. With this information, confidence intervals are constructed by way of equation (2.2.46) and their estimates are reported in Table 22. Unanimously, the 95% confidence interval is from 1974:1 to 1975:1. This estimate of a break time is very close to Hafer and Hein's previous model, which find the break time at 1974:2 (Hafer and Hein 1982). Considering that the U.S. economy received a big supply shock and experienced a high rate of inflation and unemployment at this period, it is not surprising that the money demand function shifted during this period.

To find a money demand function with a structural change, cointegration tests from 1953:1 to 1979:4 are performed on the money demand equation with a dummy intercept at each different period. If a structural change happens at time t, then the dummy intercept should be significant and at the same time the cointegration relationship of money demand with the dummy intercept should hold.
The ADF test statistics and Johansen's trace and maximal eigenvalue test statistics with the dummy intercept at each different period are estimated and the resulting sequences of test statistics are examined. In Figure 11, sequences of ADF test statistics and Johansen's trace test statistics and maximal eigenvalue test statistics with the data from 1973 to 1979 are plotted when the dummy intercept is added in each different period. When a dummy intercept variable is added around 1975, the ADF statistics fall sharply and the trace and maximal eigenvalue test statistics rise rapidly and the null hypothesis of no cointegration is rejected at the 5% significance level in both tests. In other words, with a simple zero-one dummy intercept at the 1974:3, the cointegration relationship among real money balances, GNP and two interest rates from 1953 to 1979:4 still holds and it is also stable.

To see the change of the value of the each of the cointegrating coefficients, sequences of dynamic OLS estimates and dynamic GLS estimates of cointegrating vectors with a zero-one dummy variable at 1974:3 are estimated with the data from 1953 to each period until 1980 by a recursive least squares method. Since it is too tedious to show the changes of all different estimates of cointegrating vector, only

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28. According to the simulation study, critical values of ADF test statistics with a dummy variable is close to those of equation where the dummy variable is considered as a usual independent variable (Granger and Lee 1991).
two estimates, dynamic OLS estimates and dynamic GLS estimates with 2 lags and leads are reported.

Figures 12 and 13 show sequences of the dynamic OLS estimates and dynamic GLS estimates of elasticities of real money balance with respect to real GNP and interest rates in the 1970s, respectively. The elasticity of real money balance with respect to real GNP is around 0.5 and those with respect to treasury bond rate and saving interest rate are around -0.15 and -0.25 throughout the 1970s. The coefficients on the dummy intercept are around 0.07 and this shift in intercept only accounts for 1% of the money demand. Therefore, it can be said that the shift in the long-run money demand in 1970s is relatively small and most of over-forecasting errors in 1970s are due to the mis-specification of short-run dynamic adjustment model.

As seen in these Figures, contrary to popular opinion, the cointegrating coefficient of each variable does not show any big change when a dummy intercept is include. This also supports the hypothesis that the long-run elasticities of money demand were robust throughout the 1970s.

From the data from 1953 to 1979:4, the long-run money demand function with a dummy shift (d_t) in 1974:3 is specified as follows.

\[ (3.3.1) \quad m_t = 3.60 + 0.553 y_t - 0.164 T_B - 0.150 T_D - 0.076 d_t \]
\[ (0.049) \quad (0.027) \quad (0.038) \quad (0.006) \]
According to the above equation, the amount of structural shift is 0.076 and is significantly different from 0.15 of the Hafer and Hein's previously estimated value (Hafer and Hein 1982). In other words, with the correctly specified model, the shift of money demand function in the mid-1970s are significantly lower than previously estimated value.

3.3.3 Error-correction Model and the Dynamic Coefficients

With these estimates of cointegrating vectors, error-correction models are estimated to examine the dynamic property of the money demand function and to find the impact coefficient of real GNP and interest rates. To compare estimated coefficients of an ECM with those of a partial adjustment model in equation (2.1.3), a simple ECM with 1 lag as in equation (2.2.37) is selected. To see the changes of an estimated coefficient of each explanatory variable through 1970, a sequence of ECM models with the data from 1953 to each period is estimated by a recursive least squares method and sequences of estimated coefficients are plotted. As above, dynamic OLS estimates and dynamic GLS estimates with 2 lags and leads, respectively, are used as estimated cointegrating vectors.

Figures 14 and 15 show the sequence of estimated coefficients of ECM. Throughout the decade, the impact elasticity of real money balance with respect to real GNP is around 0.3 and those with respect
to treasury bond rate and saving interest rate are -0.02 and -0.03, respectively. The coefficient on the error-correction term is around -0.12, which means that 12% of last period’s error is adjusted through a current period.

As seen in Figures 14 and 15, contrary to conventional beliefs indicated in Section 3.3.1 or in Judd and Scadding (1982), the impact coefficients on real GNP and interest rates do not decrease after 1974. In addition, the coefficients on the error-correction term do not significantly change, indicating that the coefficients on the lagged dependent variable does not increase. In conclusion, all the coefficients are quite stable through the decade and do not show any big change.
Chapter IV Conclusion

It is widely believed that the U.S. money demand function of M1 has been very unstable and consequently, it is very hard to have reliable predictions. Moreover, one of the widespread opinions is that the fundamental relationship of money demand and the key macroeconomic variables changed in the 1970s. Throughout this dissertation, these old questions are reexamined by cointegration theory and an error-correction model. Fortunately, it looks as though this attempt is successful and there is a stable money demand function, contrary to the prevailing view. In conclusion, a few things can be claimed through these empirical examinations.

First, since the macroeconomic variables included in a money demand equation are integrated series, a cointegration approach and an error-correction model should be used in model estimating and hypotheses testing in money demand analysis.

Second, a long-run relationship of real balance with a scale variable and interest rate such as GNP and a treasury bond rate respectively holds well from 1953 to 1991 and it can be expressed as a cointegration equation.
Third, new tests show that a partial adjustment model as a restricted form of an ECM does not fit well as a short-run model of monetary dynamics. As several statistics show, it does not have the desired properties of money demand function either in real adjustment type or nominal adjustment type. In addition, as an ECM shows, the dynamic structure of short-run money demand is more complicated than a simple partial adjustment model as in Goldfeld’s work (Goldfeld 1973, 176). Following Hendry’s methodology, a well defined short-run money demand function as an error-correction model is specified.

Fourth, a money demand equation before 1974 can be expressed as a cointegration equation between a scale variable such as GNP and interest rates such as a treasury bond rate and time deposit interest rate respectively, and its long-run relationship is well defined. However, it seems that a structural break occurred around 1974:3 and its confidence interval is estimated as (1974:1 - 1975:1).

Fifth, even though a cointegration relationship of money demand does not hold after 1975 until 1980, it seems that the change of money demand function is a simple shift of the intercept term and the shift amount is less than previously estimated value. With a zero-one dummy intercept variable in 1974:3, the cointegration relationship still holds until 1980 and the cointegrating vectors are also stable. Therefore, it can be said that the fundamental relationship of money demand and the key macroeconomic variables did not change in the 1970s.
Furthermore, estimated coefficients of an error-correction model based on the cointegrating equation of money demand do not show any shift or change through 1970. Contrary to prevailing beliefs, they were stable in the 1970s.

Last, this empirical investigation only confirms the 'missing money' problem and it does not try to find the possible reason for the 'missing money' problem. Considering that a shift occurred in the intercept, a possible explanation is that there may be some omitted variables in the money demand equation. To address this, further research may be needed.
Table 1. Unit Root Tests 1:
Augmented Dickey-Fuller Tests (1953-1991:2)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Statistics</th>
<th>Trend</th>
<th>ADF(a) Statistics (Number of Lags)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>M1</td>
<td>ADF(a)</td>
<td>no</td>
<td>-0.39</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>no</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>yes</td>
<td>-1.24</td>
</tr>
<tr>
<td>GNP</td>
<td>ADF(a)</td>
<td>no</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>no</td>
<td>-0.29</td>
</tr>
<tr>
<td></td>
<td>ADF(a)</td>
<td>yes</td>
<td>-11.24</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>yes</td>
<td>-2.33</td>
</tr>
<tr>
<td>Treasury</td>
<td>ADF(a)</td>
<td>no</td>
<td>-8.80</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>no</td>
<td>-2.13</td>
</tr>
<tr>
<td>Bond</td>
<td>ADF(a)</td>
<td>yes</td>
<td>-21.25</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>yes</td>
<td>-3.17</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>no</td>
<td>-3.18</td>
</tr>
<tr>
<td>Inflation</td>
<td>ADF(a)</td>
<td>yes</td>
<td>-22.83</td>
</tr>
<tr>
<td></td>
<td>ADF(tₐ)</td>
<td>yes</td>
<td>-3.33</td>
</tr>
</tbody>
</table>

1. The critical value at the 5% significant level for ADF(a) and ADF(tₐ) without linear time trend are -13.7 and -2.89 respectively and those with linear time trend are -20.7 and -3.45 respectively when sample size is 100. (Fuller 1976)
Table 2. Unit Root Tests 2: Phillips-Perron Tests (1953-1991:2)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Statistics</th>
<th>Trend</th>
<th>PP Statistics (Number of Lags)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>M1</td>
<td></td>
<td>no</td>
<td>-1.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no</td>
<td>-1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yes</td>
<td>-1.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yes</td>
<td>-0.65</td>
</tr>
<tr>
<td>GNP</td>
<td></td>
<td>no</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no</td>
<td>-0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yes</td>
<td>-6.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yes</td>
<td>-1.62</td>
</tr>
<tr>
<td>Treasury</td>
<td></td>
<td>no</td>
<td>-7.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no</td>
<td>-1.94</td>
</tr>
<tr>
<td>Bond</td>
<td></td>
<td>yes</td>
<td>-15.78</td>
</tr>
<tr>
<td>Rate</td>
<td></td>
<td>yes</td>
<td>-2.80</td>
</tr>
<tr>
<td>Inflation</td>
<td></td>
<td>no</td>
<td>-33.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no</td>
<td>-5.87</td>
</tr>
<tr>
<td>Rate</td>
<td></td>
<td>yes</td>
<td>-39.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>yes</td>
<td>-4.98</td>
</tr>
</tbody>
</table>

1. The critical values at the 5% significant level for Z(a) and Z(t_a) are the same as those of ADF tests (Fuller 1976)
Table 3. Residual Based Cointegration Tests
(With a Simple OLS Equation)

<table>
<thead>
<tr>
<th>ADF Statistics (Number of Lags)</th>
<th>Phillips and Ouliaris Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>


The critical values at the 10%, 5% and 1% significant level for ADF statistics are 3.449, 3.767, and 4.308, respectively. Those of \( Z(a) \) and \( Z(t) \) at 10%, 5% and 1% are -22.195, -26.094, and -34.169 and 3.449, 3.768, and 4.308, respectively (Engle and Yoo 1987, and Phillips and Ouliaris 1991).

Table 4. Stock and Watson's Cointegration Test Statistics

<table>
<thead>
<tr>
<th>( \lambda_3 )</th>
<th>m1, Y, TB</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>( q_f(3, 2) )</td>
<td>0.87</td>
<td>-17.48 (p = 2)</td>
</tr>
</tbody>
</table>

1. The critical values are from Stock and Watson (1989).
2. \( p = \) the number of lag in vector autoregressive correction.
<table>
<thead>
<tr>
<th>Number of Lags (k)</th>
<th>Eigenvalues</th>
<th>The number of Observations</th>
<th>C(k, s)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>s = 3</td>
<td>s = 2</td>
<td>s = 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.999 0.981 0.636</td>
<td>153</td>
<td>1819.783 750.067</td>
<td>152.707</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.261 0.119 0.004</td>
<td>152</td>
<td>64.897 19.652</td>
<td>0.622</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.287 0.010 0.001</td>
<td>151</td>
<td>52.056 1.731</td>
<td>0.164</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.108 0.015 0.010</td>
<td>150</td>
<td>19.083 2.234</td>
<td>0.036</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.062 0.020 0.007</td>
<td>149</td>
<td>13.386 3.925</td>
<td>0.974</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.054 0.046 0.003</td>
<td>148</td>
<td>15.485 7.424</td>
<td>0.492</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.129 0.038 0.000</td>
<td>147</td>
<td>25.792 5.702</td>
<td>0.035</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.091 0.010 0.000</td>
<td>146</td>
<td>15.724 1.714</td>
<td>0.110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.045 0.016 0.001</td>
<td>145</td>
<td>9.208 2.513</td>
<td>0.089</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Three variable (n = 3) VAR model is considered.
2. The rank of VAR coefficient matrix is n - s.
3. C(k, s) follows $X^2$ distribution with $s^2$ degree of freedom. The critical values of $X^2(9)$, $X^2(4)$, and $X^2(1)$ at 1% significant level are 16.191, 9.488, and 3.841, respectively.
Table 6. Trace Test and Maximum Eigenvalue Test Statistics

<table>
<thead>
<tr>
<th>Number of Lags</th>
<th>Number of Observations</th>
<th>Trace Tests r = 3</th>
<th>r = 2</th>
<th>r = 1</th>
<th>Maximum Eigenvalue Tests r = 3</th>
<th>r = 2</th>
<th>r = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>152</td>
<td>95.753</td>
<td>26.500</td>
<td>3.575</td>
<td>69.252</td>
<td>22.925</td>
<td>3.575</td>
</tr>
<tr>
<td>2</td>
<td>151</td>
<td>52.981</td>
<td>17.867</td>
<td>4.031</td>
<td>35.114</td>
<td>13.836</td>
<td>4.031</td>
</tr>
<tr>
<td>4</td>
<td>149</td>
<td>44.757</td>
<td>15.959</td>
<td>6.161</td>
<td>28.798</td>
<td>9.797</td>
<td>6.161</td>
</tr>
<tr>
<td>6</td>
<td>147</td>
<td>48.531</td>
<td>19.225</td>
<td>5.335</td>
<td>29.306</td>
<td>13.890</td>
<td>5.335</td>
</tr>
<tr>
<td>7</td>
<td>146</td>
<td>51.556</td>
<td>17.801</td>
<td>5.618</td>
<td>33.754</td>
<td>12.186</td>
<td>5.608</td>
</tr>
</tbody>
</table>

1. When \( n \) is the number of variables in VAR, the number of cointegrating vector is \( n - r \). i.e. \( r = 3 \) means no cointegration and \( r = 1 \) or \( r = 2 \) mean that there exist at most 1 cointegrating vector or 2 cointegrating vectors.

2. 5% critical values of trace tests are 35.068, 20.168 and 9.904, when \( r \) is 3, 2, and 1 respectively. Those of Maximum eigenvalue tests are 21.89, 15.75, and 9.094 respectively (Johansen and Juselius 1990).
Table 7. Estimated Cointegrating Vectors (1953 - 1991:2)

<table>
<thead>
<tr>
<th>Estimation Method</th>
<th>Real GNP</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static Estimators</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.586</td>
<td>-0.312</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>GLS</td>
<td>0.286</td>
<td>-0.075</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.018)</td>
</tr>
<tr>
<td><strong>Dynamic OLS Estimators</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(q = 0)</td>
<td>0.613</td>
<td>-0.331</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>(q = 1)</td>
<td>0.638</td>
<td>-0.352</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>(q = 2)</td>
<td>0.653</td>
<td>-0.368</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>(q = 3)</td>
<td>0.676</td>
<td>-0.386</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.019)</td>
</tr>
<tr>
<td><strong>Dynamic GLS Estimators</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(q = 0)</td>
<td>0.356</td>
<td>-0.126</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>(q = 1)</td>
<td>0.437</td>
<td>-0.188</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>(q = 2)</td>
<td>0.478</td>
<td>-0.227</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>(q = 3)</td>
<td>0.521</td>
<td>-0.263</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.029)</td>
</tr>
</tbody>
</table>
1. ( ) are standard errors.
2. The estimation equation for a dynamic OLS estimator and a dynamic GLS estimator has the form of:

\[ x_{2t} = \theta x_{1t} + d(L) \Delta x_{1t} + c_{22}(L) \varepsilon_{2t} \]

where \( x_t = (x_{1t}, x_{2t}) \) is a vector of \( n \) integrated time series, \( \varepsilon_t \) is i.i.d. with mean 0 and variance matrix \( \Sigma \) and \( d(L) = \sum_{i=-q}^{q} d_i L^j \)

where \( L \) is a lag operator and \( k \) is the number of leads and lags.
Table 8. ADF Test Statistics on the Residuals of Estimated Cointegration Equation

<table>
<thead>
<tr>
<th>Cointegration Equation</th>
<th>q</th>
<th>Number of Lags of AR terms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Static OLS</td>
<td>0</td>
<td>3.478</td>
</tr>
<tr>
<td>Dynamic OLS</td>
<td>0</td>
<td>3.650</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.805</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.900</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.975</td>
</tr>
<tr>
<td>Static GLS</td>
<td>0</td>
<td>0.794</td>
</tr>
<tr>
<td>Dynamic GLS</td>
<td>0</td>
<td>1.287</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1.985</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.444</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.905</td>
</tr>
</tbody>
</table>

1. Estimated Cointegrating vector in Table 7 are used to derive residuals.
2. ADF tests in Section 2.2.2 are employed.
3. q is the number of leads and lags in dynamic OLS and dynamic GLS. In the Johansen's estimator, q is the number of VAR lags.
4. The critical values at the 10%, 5%, and 1% significant level for ADF statistics of a simple OLS cointegrating vector in three variable linear model are 3.449, 3.767 and 4.308, respectively.
Table 9. Estimate of Vector Error-correction Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\Delta M_t$</th>
<th>$\Delta P_t$</th>
<th>$\Delta R_t$</th>
<th>$\Delta Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.120 (0.032)</td>
<td>0.017 (0.017)</td>
<td>6.049 (2.299)</td>
<td>0.107 (0.037)</td>
</tr>
<tr>
<td>error correction</td>
<td>-0.069 (0.019)</td>
<td>-0.010 (0.010)</td>
<td>-3.715 (1.357)</td>
<td>-0.059 (0.022)</td>
</tr>
<tr>
<td>$\Delta M_{t-1}$</td>
<td>0.023 (0.091)</td>
<td>0.028 (0.048)</td>
<td>-11.531 (6.614)</td>
<td>-0.077 (0.106)</td>
</tr>
<tr>
<td>$\Delta M_{t-2}$</td>
<td>0.345 (0.087)</td>
<td>-0.003 (0.046)</td>
<td>11.293 (6.365)</td>
<td>0.190 (0.102)</td>
</tr>
<tr>
<td>$\Delta M_{t-3}$</td>
<td>0.155 (0.093)</td>
<td>0.067 (0.049)</td>
<td>5.442 (6.739)</td>
<td>-0.096 (0.108)</td>
</tr>
<tr>
<td>$\Delta M_{t-4}$</td>
<td>0.029 (0.094)</td>
<td>0.036 (0.050)</td>
<td>-22.510 (6.845)</td>
<td>-0.029 (0.110)</td>
</tr>
<tr>
<td>$\Delta M_{t-5}$</td>
<td>0.171 (0.084)</td>
<td>-0.060 (0.045)</td>
<td>-3.764 (6.122)</td>
<td>0.128 (0.099)</td>
</tr>
<tr>
<td>$\Delta M_{t-6}$</td>
<td>0.120 (0.076)</td>
<td>0.008 (0.040)</td>
<td>25.791 (5.537)</td>
<td>0.116 (0.089)</td>
</tr>
<tr>
<td>$\Delta P_{t-1}$</td>
<td>0.072 (0.179)</td>
<td>0.388 (0.094)</td>
<td>21.696 (12.962)</td>
<td>-0.076 (0.208)</td>
</tr>
<tr>
<td>$\Delta P_{t-2}$</td>
<td>-0.148 (0.185)</td>
<td>0.154 (0.098)</td>
<td>1.066 (13.442)</td>
<td>-0.103 (0.216)</td>
</tr>
<tr>
<td>$\Delta P_{t-3}$</td>
<td>0.038 (0.188)</td>
<td>0.109 (0.100)</td>
<td>-1.407 (13.673)</td>
<td>-0.255 (0.220)</td>
</tr>
<tr>
<td>$\Delta P_{t-4}$</td>
<td>0.009 (0.181)</td>
<td>0.137 (0.096)</td>
<td>8.723 (13.168)</td>
<td>0.051 (0.212)</td>
</tr>
<tr>
<td>$\Delta P_{t-5}$</td>
<td>0.129 (0.171)</td>
<td>0.035 (0.090)</td>
<td>-20.384 (12.382)</td>
<td>-0.308 (0.199)</td>
</tr>
<tr>
<td>$\Delta P_{t-6}$</td>
<td>-0.116 (0.161)</td>
<td>0.005 (0.085)</td>
<td>-5.984 (11.703)</td>
<td>0.122 (0.188)</td>
</tr>
<tr>
<td>$\Delta R_{t-1}$</td>
<td>-0.005 (0.001)</td>
<td>0.001 (0.001)</td>
<td>0.080 (0.098)</td>
<td>0.003 (0.001)</td>
</tr>
<tr>
<td>$\Delta R_{t-2}$</td>
<td>0.006 (0.001)</td>
<td>0.000 (0.000)</td>
<td>-0.052 (0.105)</td>
<td>-0.003 (0.002)</td>
</tr>
<tr>
<td>$\Delta R_{t-3}$</td>
<td>0.002 (0.002)</td>
<td>0.000 (0.001)</td>
<td>0.074 (0.112)</td>
<td>0.002 (0.002)</td>
</tr>
<tr>
<td>$\Delta R_{t-4}$</td>
<td>0.001 (0.001)</td>
<td>0.001 (0.001)</td>
<td>0.264 (0.114)</td>
<td>0.001 (0.002)</td>
</tr>
<tr>
<td>$\Delta R_{t-5}$</td>
<td>0.002 (0.002)</td>
<td>0.001 (0.001)</td>
<td>-0.268 (0.117)</td>
<td>-0.003 (0.002)</td>
</tr>
<tr>
<td>$\Delta R_{t-6}$</td>
<td>-0.043 (0.082)</td>
<td>0.010 (0.043)</td>
<td>6.949 (5.943)</td>
<td>0.187 (0.100)</td>
</tr>
<tr>
<td>$\Delta Y_{t-1}$</td>
<td>0.055 (0.082)</td>
<td>-0.016 (0.043)</td>
<td>-1.078 (5.948)</td>
<td>0.060 (0.100)</td>
</tr>
<tr>
<td>$\Delta Y_{t-2}$</td>
<td>-0.144 (0.082)</td>
<td>0.052 (0.043)</td>
<td>-6.148 (5.931)</td>
<td>-0.118 (0.095)</td>
</tr>
<tr>
<td>$\Delta Y_{t-3}$</td>
<td>-0.141 (0.082)</td>
<td>-0.015 (0.043)</td>
<td>-1.448 (5.943)</td>
<td>-0.073 (0.096)</td>
</tr>
<tr>
<td>$\Delta Y_{t-4}$</td>
<td>0.002 (0.082)</td>
<td>-0.039 (0.043)</td>
<td>-4.291 (5.968)</td>
<td>-0.138 (0.096)</td>
</tr>
<tr>
<td>$\Delta Y_{t-5}$</td>
<td>-0.153 (0.077)</td>
<td>0.057 (0.041)</td>
<td>-1.989 (5.611)</td>
<td>-0.014 (0.090)</td>
</tr>
</tbody>
</table>

1. ( ) are standard errors.
2. The dynamic OLS cointegrating vector with 3 lags and leads are used to estimate this VECM.
Table 10. Granger Causality Test

<table>
<thead>
<tr>
<th>Equation</th>
<th>$\Delta M_{t-i}$</th>
<th>$\Delta P_{t-i}$</th>
<th>$\Delta R_{t-i}$</th>
<th>$\Delta Y_{t-i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta M_t$</td>
<td>7.811 (0.00)</td>
<td>0.297 (0.94)</td>
<td>7.137 (0.00)</td>
<td>1.985 (0.07)</td>
</tr>
<tr>
<td>$\Delta P_t$</td>
<td>0.808 (0.57)</td>
<td>10.789 (0.00)</td>
<td>0.929 (0.48)</td>
<td>0.625 (0.71)</td>
</tr>
<tr>
<td>$\Delta R_t$</td>
<td>4.967 (0.00)</td>
<td>1.052 (0.40)</td>
<td>2.459 (0.03)</td>
<td>0.657 (0.68)</td>
</tr>
<tr>
<td>$\Delta Y_t$</td>
<td>1.519 (0.18)</td>
<td>1.419 (0.21)</td>
<td>2.440 (0.03)</td>
<td>1.898 (0.09)</td>
</tr>
</tbody>
</table>

1. ( ) are marginal significance level for the F-test on the hypothesis.
2. The dynamic OLS cointegrating vector with 3 lags and leads are used to estimate the VECM.
3. Tests are performed on 6 lag VECM.
Table 11. Neutrality Test

<table>
<thead>
<tr>
<th>Equation</th>
<th>Variables</th>
<th>$\Delta h_{t-i}$</th>
<th>$\Delta p_{t-i}$</th>
<th>$\Delta r_{t-i}$</th>
<th>$\Delta y_{t-i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta h_t$</td>
<td>38.952 (0.00)</td>
<td>0.195 (0.66)</td>
<td>1.082 (0.30)</td>
<td>5.086 (0.03)</td>
<td></td>
</tr>
<tr>
<td>$\Delta p_t$</td>
<td>1.106 (0.17)</td>
<td>57.085 (0.00)</td>
<td>1.368 (0.24)</td>
<td>0.236 (0.62)</td>
<td></td>
</tr>
<tr>
<td>$\Delta r_t$</td>
<td>0.232 (0.31)</td>
<td>0.061 (0.81)</td>
<td>0.002 (0.99)</td>
<td>0.344 (0.56)</td>
<td></td>
</tr>
<tr>
<td>$\Delta y_t$</td>
<td>2.174 (0.37)</td>
<td>5.722 (0.02)</td>
<td>0.005 (0.94)</td>
<td>0.193 (0.66)</td>
<td></td>
</tr>
</tbody>
</table>

1. ( ) are marginal significance level for the F-test on the hypothesis.
2. The dynamic OLS cointegrating vector with 3 lags and leads are used to estimate the VECM.
3. Tests are performed on 6 lag VECM.
Table 12. General Unrestricted ECM

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Number of Lags</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td></td>
<td>0.098</td>
<td>0.057</td>
<td>0.000</td>
<td>0.300</td>
<td>0.156</td>
<td>-0.047</td>
<td>0.158</td>
</tr>
<tr>
<td>ECM</td>
<td></td>
<td></td>
<td>(0.037)</td>
<td>(0.022)</td>
<td>(0.089)</td>
<td>(0.087)</td>
<td>(0.090)</td>
<td>(0.094)</td>
</tr>
<tr>
<td>Δm</td>
<td></td>
<td></td>
<td>0.178</td>
<td>-0.059</td>
<td>0.061</td>
<td>-0.176</td>
<td>-0.127</td>
<td>0.064</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td></td>
<td></td>
<td>(0.082)</td>
<td>(0.087)</td>
<td>(0.088)</td>
<td>(0.088)</td>
<td>(0.090)</td>
</tr>
<tr>
<td>AR</td>
<td></td>
<td></td>
<td>-0.009</td>
<td>-0.048</td>
<td>-0.043</td>
<td>0.014</td>
<td>-0.000</td>
<td>0.006</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td></td>
<td></td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>ΔY</td>
<td></td>
<td></td>
<td>-0.481</td>
<td>-0.072</td>
<td>-0.079</td>
<td>0.115</td>
<td>-0.171</td>
<td>0.342</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td></td>
<td></td>
<td>(0.183)</td>
<td>(0.201)</td>
<td>(0.200)</td>
<td>(0.201)</td>
<td>(0.197)</td>
</tr>
</tbody>
</table>

1. ( ) are standard errors.
2. The dynamic OLS cointegrating vector with 3 lags and leads are used to derive the error-correction term.

Table 13. Parameter Constancy Tests for Parsimonious ECMs

<table>
<thead>
<tr>
<th>Periods</th>
<th>Dynamic OLS Cointegrating Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldfeld's 'Missing Money' 1974:1-</td>
<td>1.802 (0.079)</td>
</tr>
<tr>
<td>Velocity 1982:1-</td>
<td>1.251 (0.288)</td>
</tr>
<tr>
<td>Decline 1983:2</td>
<td></td>
</tr>
<tr>
<td>Money 1985:1-</td>
<td>1.805 (0.084)</td>
</tr>
<tr>
<td>Explosion 1986:4</td>
<td></td>
</tr>
</tbody>
</table>

1. ( ) are p-values.
Table 14. Non-linear Least Squares Estimates of Real and Nominal Adjustment Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Long-run Coefficients</th>
<th>Dynamic Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q</td>
<td>y</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Adjustment Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.657</td>
<td>-0.055</td>
</tr>
<tr>
<td></td>
<td>(0.099)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>2</td>
<td>0.574</td>
<td>-0.027</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>3</td>
<td>0.640</td>
<td>-0.050</td>
</tr>
<tr>
<td></td>
<td>(0.135)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>4</td>
<td>0.563</td>
<td>-0.026</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>5</td>
<td>0.530</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(0.150)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>6</td>
<td>0.500</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Nominal Adjustment Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.521</td>
<td>-0.031</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>8</td>
<td>0.542</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td>(0.107)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>9</td>
<td>0.525</td>
<td>-0.031</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>10</td>
<td>0.529</td>
<td>-0.030</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>11</td>
<td>0.531</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(0.175)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>12</td>
<td>0.524</td>
<td>-0.030</td>
</tr>
<tr>
<td></td>
<td>(0.170)</td>
<td>(0.008)</td>
</tr>
</tbody>
</table>
1. ( ) are standard errors.
2. q is the number of lags in below models
3. Estimated models are equations (2.2.42) and (2.2.43).

(i). real adjustment type:

$$\Delta m_t = b_0 + \sum_{j=0}^{q-1} b_j (a_1 \Delta r_{t-j} + a_2 \Delta Y_{t-j}) + \sum_{j=0}^{q-1} b_j (a_1 r_{t-j-1} + a_2 Y_{t-j-1} - m_{t-j-1})$$

(ii). nominal adjustment type:

$$\Delta W_t = b_0 + \sum_{j=0}^{q-1} b_j (a_1 \Delta r_{t-j} + a_2 \Delta Y_{t-j} + a_3 \Delta P_{t-j}) + \sum_{j=0}^{q-1} b_j (a_1 r_{t-j-1} + a_2 Y_{t-j-1} + a_3 P_{t-j-1} - W_{t-j-1})$$

where $r_t$ and $Y_t$ are interest rate and real GNP respectively.
<table>
<thead>
<tr>
<th>Model and leads in the ECM</th>
<th># of lag</th>
<th>Cointegrating Vector</th>
<th>Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$y$</td>
<td>$r$</td>
</tr>
<tr>
<td>1 * 0</td>
<td>0</td>
<td>0.677</td>
<td>-0.054</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.099)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>2 * 0</td>
<td>0</td>
<td>0.580</td>
<td>-0.027</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.063)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>3 * 1</td>
<td>1</td>
<td>0.800</td>
<td>-0.057</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.266)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>4 * 1</td>
<td>1</td>
<td>0.574</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.074)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>5 2</td>
<td>2</td>
<td>0.888</td>
<td>-0.043</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.666)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>6 2</td>
<td>2</td>
<td>0.562</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.100)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>7 * 0</td>
<td>0</td>
<td>0.506</td>
<td>-0.030</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.108)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>8 0</td>
<td>0</td>
<td>0.521</td>
<td>-0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.107)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>9 * 1</td>
<td>1</td>
<td>0.593</td>
<td>-0.024</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.162)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>10 1</td>
<td>0</td>
<td>0.584</td>
<td>-0.020</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.151)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>11 2</td>
<td>2</td>
<td>0.601</td>
<td>-0.014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.228)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>12 2</td>
<td>2</td>
<td>0.575</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.161)</td>
<td>(0.010)</td>
</tr>
</tbody>
</table>
1. ( ) in cointegrating vectors are standard errors and ( ) are p-
values of significance level for F-test.
2. * means that all variables in the cointegrating equation are
significant at the 5% significance level.
3. Estimation equations for cointegrating vectors are generalized ECMs
such that:

\[
\Delta m_t = b_0 + \sum_{j=0}^{q} b_j (a_1 r_{t-j-1} + a_2 y_{t-j-1} - m_{t-j-1})
\]

\[
+ \sum_{i=-q}^{q} (c_{1i} \Delta r_{jt-i} + c_{2i} \Delta y_{jt-i}).
\]

(iv). Nominal Adjustment Type:

\[
\Delta M_t = b_0 + \sum_{j=0}^{q} b_j (a_1 r_{t-j-1} + a_2 y_{t-j-1} + a_3 p_{t-j-1} - M_{t-j-1})
\]

\[
+ \sum_{i=-q}^{q} (c_{1i} \Delta r_{jt-i} + c_{2i} \Delta y_{jt-i} + c_{3i} \Delta p_{jt-i}).
\]

4. Estimated models in the tests are of the forms:

\[
\Delta m_t = b_0 + \sum_{i=0}^{q} b_{1i} (m_{t-i} - m_{t-i-1}) + \sum_{i=0}^{q} b_{2i} (m_{t-i} - \hat{m}_{t-i-1})
\]

\[
\Delta M_t = b_0 + \sum_{i=0}^{q} b_{1i} (M_{t-i} - M_{t-i-1}) + \sum_{i=0}^{q} b_{2i} (M_{t-i} - \hat{M}_{t-i-1})
\]

where \( \hat{m}_t \) is an estimated equilibrium real balance, \( \hat{M}_t \) is an
estimated equilibrium nominal balance and \( \hat{a} \) is an estimate of a
cointegrating vector from Phillips and Loretan's generalized ECM.

\( H_{01} \) is a F-test for \( \sum_{i=1}^{q} b_{1i} = 0, \) \( \sum_{i=1}^{q} b_{2i} = 0 \) and \( H_{02} \) is a F-test for
\( \sum_{i=1}^{q} b_{1i} = \sum_{i=1}^{q} b_{2i}. \)
Table 16. Parameter Constancy Tests (Chow Tests) for PAMs

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>Model</td>
<td>Model</td>
<td>Model</td>
<td>Model</td>
</tr>
<tr>
<td>1</td>
<td>4.91 (0.00)</td>
<td>3.18 (0.00)</td>
<td>1.21 (0.30)</td>
<td>12.96 (0.00)</td>
</tr>
<tr>
<td>2</td>
<td>1.55 (0.14)</td>
<td>1.18 (0.29)</td>
<td>2.73 (0.07)</td>
<td>5.23 (0.00)</td>
</tr>
<tr>
<td>3</td>
<td>3.62 (0.00)</td>
<td>2.36 (0.00)</td>
<td>0.81 (0.52)</td>
<td>5.73 (0.00)</td>
</tr>
<tr>
<td>4</td>
<td>1.54 (0.14)</td>
<td>1.25 (0.23)</td>
<td>2.93 (0.02)</td>
<td>4.87 (0.00)</td>
</tr>
<tr>
<td>5</td>
<td>3.86 (0.00)</td>
<td>2.39 (0.00)</td>
<td>2.91 (0.01)</td>
<td>4.44 (0.00)</td>
</tr>
<tr>
<td>6</td>
<td>3.45 (0.00)</td>
<td>2.34 (0.00)</td>
<td>4.14 (0.00)</td>
<td>7.31 (0.00)</td>
</tr>
<tr>
<td>7</td>
<td>1.12 (0.36)</td>
<td>0.92 (0.56)</td>
<td>2.11 (0.12)</td>
<td>12.68 (0.00)</td>
</tr>
<tr>
<td>8</td>
<td>1.02 (0.43)</td>
<td>0.87 (0.62)</td>
<td>1.91 (0.15)</td>
<td>11.34 (0.00)</td>
</tr>
<tr>
<td>9</td>
<td>2.07 (0.04)</td>
<td>1.65 (0.06)</td>
<td>3.53 (0.01)</td>
<td>8.15 (0.00)</td>
</tr>
<tr>
<td>10</td>
<td>1.99 (0.05)</td>
<td>1.61 (0.07)</td>
<td>3.47 (0.01)</td>
<td>8.02 (0.00)</td>
</tr>
<tr>
<td>11</td>
<td>2.09 (0.04)</td>
<td>1.68 (0.05)</td>
<td>2.62 (0.02)</td>
<td>5.37 (0.00)</td>
</tr>
<tr>
<td>12</td>
<td>1.93 (0.05)</td>
<td>1.58 (0.08)</td>
<td>2.43 (0.03)</td>
<td>5.62 (0.00)</td>
</tr>
</tbody>
</table>

1. ( ) are p-values of significance level for F-test.
Table 17. Estimates of Parsimonious ECMs

<table>
<thead>
<tr>
<th></th>
<th>Equation I</th>
<th></th>
<th>Equation II</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>73:4</td>
<td>91:2</td>
<td>91:2</td>
<td>73:4</td>
<td>91:2</td>
<td>91:2</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0762</td>
<td>0.0756</td>
<td>0.0813</td>
<td>0.0701</td>
<td>0.0743</td>
</tr>
<tr>
<td></td>
<td>(0.0176)</td>
<td>(0.0334)</td>
<td>(0.0174)</td>
<td>(0.0164)</td>
<td>(0.0303)</td>
</tr>
<tr>
<td>EC_t-1</td>
<td>0.0380</td>
<td>0.0401</td>
<td>0.0345</td>
<td>0.0367</td>
<td>0.0379</td>
</tr>
<tr>
<td></td>
<td>(0.0086)</td>
<td>(0.0168)</td>
<td>(0.0087)</td>
<td>(0.0080)</td>
<td>(0.0149)</td>
</tr>
<tr>
<td>ΔY_t</td>
<td>0.2002</td>
<td>0.1835</td>
<td>0.1686</td>
<td>0.2058</td>
<td>0.2006</td>
</tr>
<tr>
<td></td>
<td>(0.0700)</td>
<td>(0.1552)</td>
<td>(0.0760)</td>
<td>(0.0697)</td>
<td>(0.1503)</td>
</tr>
<tr>
<td>Δr_t-1</td>
<td>-0.0016</td>
<td>-0.0037</td>
<td>-0.0032</td>
<td>-0.0017</td>
<td>-0.0038</td>
</tr>
<tr>
<td></td>
<td>(0.0011)</td>
<td>(0.0012)</td>
<td>(0.0008)</td>
<td>(0.0011)</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>Δr_t-2</td>
<td>-0.0025</td>
<td>-0.0030</td>
<td>-0.0028</td>
<td>-0.0027</td>
<td>-0.0031</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0011)</td>
<td>(0.0008)</td>
<td>(0.0012)</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>Δr_t-4</td>
<td>-0.0012</td>
<td>-0.0017</td>
<td>-0.0015</td>
<td>-0.0014</td>
<td>-0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.0011)</td>
<td>(0.0008)</td>
<td>(0.0013)</td>
<td>(0.0011)</td>
</tr>
<tr>
<td>ΔΠ_t-2</td>
<td>-0.0015</td>
<td>-0.0012</td>
<td>-0.0014</td>
<td>-0.0013</td>
<td>-0.0015</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0008)</td>
<td>(0.0004)</td>
<td>(0.0004)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>Δm_t-2</td>
<td>-0.0165</td>
<td>0.3026</td>
<td>0.2376</td>
<td>-0.0041</td>
<td>0.2934</td>
</tr>
<tr>
<td></td>
<td>(0.0924)</td>
<td>(0.0983)</td>
<td>(0.0639)</td>
<td>(0.0925)</td>
<td>(0.0980)</td>
</tr>
<tr>
<td>R²</td>
<td>0.563</td>
<td>0.568</td>
<td>1.933</td>
<td>1.950</td>
<td>1.860</td>
</tr>
<tr>
<td>DW</td>
<td>2.648</td>
<td></td>
<td>4.467</td>
<td></td>
<td>4.190</td>
</tr>
<tr>
<td>X₁(2)</td>
<td>4.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X₁(4)</td>
<td>4.190</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. ( ) are standard errors.

2. X₁²(p) is a $X^2$ test statistic for p-th order serial correlation.
### Table 18. Parameter Constancy Tests (Chow Tests) for Parsimonious ECMs

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Split Point</th>
<th>Test Statistics</th>
<th>Equation I</th>
<th>Equation II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
<td></td>
</tr>
<tr>
<td>1953:1 - 1978:4</td>
<td>1974:1</td>
<td>1.31</td>
<td>1.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.20)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>1953:1 - 1991:4</td>
<td>1974:1</td>
<td>0.79</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.60)</td>
<td>(0.57)</td>
<td></td>
</tr>
<tr>
<td>1953:1 - 1991:4</td>
<td>1979:1</td>
<td>1.82</td>
<td>1.78</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td>(0.10)</td>
<td></td>
</tr>
</tbody>
</table>

1. ( ) are p-values of significance level for F-test.
### Table 19. Real Adjustment Model Estimates

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>$m_t$</td>
<td>$m_t$</td>
</tr>
<tr>
<td>Estimation Method</td>
<td>OLS</td>
<td>GLS</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.978</td>
<td>0.962</td>
</tr>
<tr>
<td>Independent Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.178</td>
<td>0.282</td>
</tr>
<tr>
<td></td>
<td>(0.137)</td>
<td>(0.177)</td>
</tr>
<tr>
<td>Real GNP ($Y_t$)</td>
<td>0.076</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Treasury Bond Rate ($TB_t$)</td>
<td>-0.029</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Saving Interest Rate ($TD_t$)</td>
<td>-0.014</td>
<td>-0.024</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$m_{t-1}$</td>
<td>0.889</td>
<td>0.855</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>Long-run Elasticities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GNP ($Y_t$)</td>
<td>0.691</td>
<td>0.628</td>
</tr>
<tr>
<td>Treasury Bond Rate ($TB_t$)</td>
<td>-0.263</td>
<td>-0.200</td>
</tr>
<tr>
<td>Saving Interest Rate ($TD_t$)</td>
<td>-0.127</td>
<td>-0.166</td>
</tr>
<tr>
<td>Adjustment Coefficient ($\alpha$)</td>
<td>0.110</td>
<td>0.145</td>
</tr>
</tbody>
</table>
1. ( ) are standard errors
2. The estimation equation is a real adjustment model such as

\[(2.1.5) \quad m_t = \beta_0 + \beta_1 y_t + \beta_2 r_t + \beta_3 m_{t-1} + e_t\]

where $\beta_0 = \alpha a_0$, $\beta_1 = \alpha a_1$, $\beta_2 = \alpha a_2$ and $\beta_3 = (1 - \alpha)$ and $a_1$ and $a_2$ are long-run elasticities with respect to real GNP and interest rate respectively and $\alpha$ is a dynamic adjustment coefficient.
3. In the GLS estimation, error terms are assumed to be AR(1).

<table>
<thead>
<tr>
<th>Number of Lags</th>
<th>Equation Y</th>
<th>TB</th>
<th>TD</th>
<th>Equation Y</th>
<th>TB</th>
<th>TD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simple OLS Estimates</td>
<td>Simple GLS Estimates</td>
<td></td>
<td>Dynamic OLS Estimates</td>
<td>Dynamic GLS Estimates</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.469</td>
<td>-0.080</td>
<td>-0.205</td>
<td>6</td>
<td>0.252</td>
<td>-0.033</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.017)</td>
<td>(0.026)</td>
<td></td>
<td>(0.035)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>q = 0</td>
<td>2</td>
<td>0.504</td>
<td>-0.098</td>
<td>-0.212</td>
<td>7</td>
<td>0.317</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.017)</td>
<td>(0.027)</td>
<td></td>
<td>(0.042)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>q = 1</td>
<td>3</td>
<td>0.534</td>
<td>-0.104</td>
<td>-0.234</td>
<td>8</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.020)</td>
<td>(0.030)</td>
<td></td>
<td>(0.051)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>q = 2</td>
<td>4</td>
<td>0.564</td>
<td>-0.104</td>
<td>-0.264</td>
<td>9</td>
<td>0.481</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.022)</td>
<td>(0.035)</td>
<td></td>
<td>(0.057)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>q = 3</td>
<td>5</td>
<td>0.615</td>
<td>-0.113</td>
<td>-0.302</td>
<td>10</td>
<td>0.540</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.023)</td>
<td>(0.039)</td>
<td></td>
<td>(0.061)</td>
<td>(0.033)</td>
</tr>
</tbody>
</table>

1. ( ) are standard errors.
2. The estimation equation for a dynamic OLS estimator and a dynamic GLS estimator has the form of:

\[(2.2.23) \quad x_{2t} = \Theta x_{1t} + d(L) \Delta x_{1t} + c_{22}(L) e_{2t}\]

where \( x_t = (x_{1t}, x_{2t}) \) is a vector of \( n \) integrated time series, \( e_t \) is i.i.d. with mean 0 and variance matrix \( \Sigma \) and \( d(L) = \sum_{i=-k}^{k} d_i L^i \)

where \( L \) is a lag operator and \( k \) is the number of leads and lags.
3. In the dynamic GLS estimation, error terms are assumed to be AR(1) when \( q = 0 \) and 1 and AR(2) when \( q = 2 \).
Table 21. ADF Test Statistics on the Residuals of Estimated Cointegration Equation (1953 - 1973)

<table>
<thead>
<tr>
<th>Cointegration Equation</th>
<th>Number of Lags In Dynamic Equation</th>
<th>Number of Lags of AR terms 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static OLS</td>
<td>0</td>
<td>3.860</td>
<td>3.456</td>
<td>3.225</td>
<td>2.871</td>
<td>2.815</td>
</tr>
<tr>
<td>Dynamic OLS</td>
<td>0</td>
<td>3.980</td>
<td>3.543</td>
<td>3.330</td>
<td>2.935</td>
<td>2.857</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4.209</td>
<td>3.703</td>
<td>3.491</td>
<td>3.069</td>
<td>2.994</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.680</td>
<td>4.025</td>
<td>3.841</td>
<td>3.430</td>
<td>3.454</td>
</tr>
<tr>
<td>Static GLS</td>
<td>0</td>
<td>2.168</td>
<td>2.460</td>
<td>2.311</td>
<td>2.128</td>
<td>2.148</td>
</tr>
<tr>
<td>Dynamic GLS</td>
<td>0</td>
<td>2.394</td>
<td>2.524</td>
<td>2.382</td>
<td>2.163</td>
<td>2.147</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.962</td>
<td>2.814</td>
<td>2.632</td>
<td>2.278</td>
<td>2.213</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.601</td>
<td>3.258</td>
<td>3.061</td>
<td>2.643</td>
<td>2.547</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.271</td>
<td>3.737</td>
<td>3.527</td>
<td>3.092</td>
<td>3.023</td>
</tr>
</tbody>
</table>

1. Estimated Cointegrating vector in Table 20 are used to derive residuals.
2. ADF tests in Section 2.2.2 are employed.
### Table 22. Structural Break Test Statistics

<table>
<thead>
<tr>
<th>Variables</th>
<th>Sup Wald</th>
<th>Mean Wald</th>
<th>Exp-mean Wald Test</th>
<th>Break Point</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Money Balances</td>
<td>4.81</td>
<td>0.82</td>
<td>1.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real GNP</td>
<td>1.88</td>
<td>0.45</td>
<td>0.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treasury Bond Rate</td>
<td>2.36</td>
<td>0.45</td>
<td>0.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saving Interest Rate</td>
<td>7.72</td>
<td>0.42</td>
<td>3.23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Residuals of Cointegration Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sup Wald</th>
<th>Mean Wald</th>
<th>Exp-mean Wald Test</th>
<th>Break Point</th>
<th>Confidence Interval</th>
</tr>
</thead>
</table>

1. The critical values of the 5% significance level of the sup Wald test, mean Wald test and the exp-mean Wald test are 8.76, 2.99 and 2.08 respectively. Those of 10% significance level are 7.18, 2.17 and 1.52 respectively and those of 1% significance level are 12.33, 4.71 and 3.50 respectively.

2. To derive the residuals of cointegration equations, estimates of cointegration vector in Table 20 are used. In equation 1, the simple OLS estimates of Cointegration vector in Table 20 is used and in equations 2, 3, 4, and 5, dynamic OLS estimates of cointegrating vectors are used. By same way, the simple GLS and dynamic GLS estimates are used in equation 6, 7, 8, 9 and 10.
Figure 1. Residuals of Cointegration Equation

1. Cointegrating vector is estimated by a dynamic OLS method with 3 lags and leads.
Figure 2. Actual and Fitted Values from Equation (3.2.1)

1. Real adjustment model estimated by OLS method in Table 19 is used to simulate the real balance.
Figure 3. Actual and Fitted Values of Real Money Balance by Real Adjustment Model
Figure 4. Actual and Fitted Values of Real Money Balance by Nominal Adjustment Model
Figure 5. Actual and Fitted Values of Real Money Balance by Error-Correction Model I.
Figure 6. Actual and Fitted Values of Real Money Balance by Error-Correction Model II.
Figure 7. Dynamic Simulation of Real Adjustment Model in the 1970s.
Figure 8. Sequence of Cointegration Test Statistics
(a) Residuals of Cointegration Equation 4

(b) Residuals of Cointegration Equation 8

Figure 9. Residuals of Cointegration Equations
Figure 10. Wald Test Statistics for Structural Break of Residuals of Cointegration Equations
Figure 11. Sequence of Cointegration Test Statistics
With a Dummy Intercept
Figure 12. Sequence of Cointegration Coefficients: 
(Equation 4)
Figure 13. Sequence of Cointegration Coefficients:
(Equation 8)
Figure 14. Sequence of Coefficients of ECM:
(Equation 4)
Figure 15. Sequence of Coefficients of ECM:
(Equation 8)
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