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A novel hybrid full-wave analysis method for planar transmission lines embedded in multilayered dielectrics: The WH/GSMT

Chou, Ling-Miao, Ph.D.

The Ohio State University, 1992
A Novel Hybrid Full-Wave Analysis Method for Planar Transmission Lines Embedded in Multilayered Dielectrics – the WH/GSMT

A Dissertation
Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

by

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1992

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Dedication

to my dear parents
Acknowledgement

I would like to thank my advisor, Professor Pathak, for his support from the start through to the completion of this work. The careful review of this work by Professor Munk and Professor Newman is also appreciated. In addition, I want to express my deepest gratitude to my supervisor, Dr. Rojas. Without his patient listening and relentless critique, this dissertation would not be possible.

Also, my gossip-mate, Dr. to-be Mimi’s amusing romance brought lots of fun during these hard working days. Of course, I cannot forget to thank my boyfriend, Chien-Hao, for reminding me to work all the time.

Finally, I would like to thank all the people at the ElectroScience Laboratory. It has been a very pleasant, fruitful and important six years in my life.
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CHAPTER I
Introduction

Circuits at microwave/millimeter wave frequencies are now typically made up of microstrip lines or coplanar waveguides instead of the traditional rectangular waveguides because of their lower cost, lighter weight and ease of manufacturing, especially when a large number of elements are involved as in the microwave integrated circuit or large antenna phased array applications. Recently, the trend of building monolithic microwave/millimeter wave integrated circuits (MMIC) where all the passive and active devices are fabricated together in a single crystal makes the characterizations and modelings of these planar circuits even more important.

In this dissertation, a novel approach which is based on a hybrid procedure referred to as the WH/GSMT is employed for solving planar transmission line problems. It is a rigorous, full-wave analysis method. The WH in the WH/GSMT stands for the Wiener-Hopf [26] based solution of an appropriate canonical problem (as depicted in Figure 1) and GSMT refers to the well known generalized scattering matrix technique [26, 2]. In the present work, the transmission lines are assumed to be perfect electric conductors (PEC) of infinitesimal thickness which are embedded in multi-layered isotropic dielectric slabs. The effect of the inhomogeneity and anisotropy of the substrate material as well as the effect of finite thickness and finite conductivity of the metal lines has not been taken into account in the present analysis. The propagation constant, characteristic impedance, field distribution, possible lateral leakage of the dominant mode as well as of some higher order modes for some
common planar transmission lines, including single microstrip line (SMS), coupled microstrip lines (CMS), single conductor-backed slotline (SSL), finline (FIL) and single coplanar waveguide (CPW) will be investigated. In high-density microwave integrated circuits, the lateral leakage which may cause the undesirable cross talk between neighboring lines can not be neglected. Despite the research related to planar transmission lines, especially microstrip lines, started long time ago and have led to numerous publications in the past two decades [3], the mechanism of radiation into space wave or leakage into surface wave has not attracted much attention until recently [4, 5, 6]. Very few numerical results dealing rigorously with the leakage have been presented [6, 7, 8]. Moreover, the research devoted to the analysis of coplanar waveguides has been considerably less than that devoted to microstrip lines. However, the coplanar waveguide is gaining increasing attention due to easy insertion of active and passive devices into such waveguides where both the signal and ground lines lie in the same plane; this is especially true as the operating frequency is pushed higher and higher to the millimeter wave range where the idea of MMIC becomes popular. The conductor-backed coplanar waveguide (CBCPW) will be studied in great detail in this work with emphasis on the effects of double layered substrates and lateral ground planes of finite-extent. This important CPW geometry is particularly suited to the present WH/GSMT method of analysis.

Since most of the planar transmission lines used do not allow closed form analytical expressions, the use of numerical techniques is inevitable. Trade-offs have to be made when considering the CPU time, memory space, versatility and other factors [2]. For ideal transmission line problems as considered here, the spectral domain approach (SDA) appears to be the most widely used full-wave analysis method [12, 13, 14, 19]. Although there are other more versatile numerical techniques that can be employed, such as finite element or finite difference methods [17, 18], usually
they require a much larger amount of CPU time and memory space and thus will
not be advantageous for dealing with these kinds of ideal transmission line prob­
lems. The integral equation technique, which includes the popular SDA, and the
mode matching method will be briefly reviewed here and compared with the cur­
rently used method, namely, the WH/GSMT. For a more complete literature review
on the planar transmission lines or the components created from them, one can
refer to the collection of papers in [3] which covers many important publications
during the past two decades or the classic book by Gupta et al. [9] which discusses
in detail the analysis, modeling and design considerations for microstrip as well as
other planar transmission lines. Also, the book entitled "Numerical Techniques for
Microwave and Millimeter-wave Passive Structures" (edited by T. Itoh) [2] contains
a wealth of useful information in this regard.

When applying the integral equation technique for solving planar transmission
line problems, an integral equation is formulated where the electric charge or cur­
rent distribution on the conducting strips (or magnetic currents in the slots for
slotlines) are assumed to be unknowns. The kernel of the integral equation is the
Green's function for the line source excited PEC grounded multi-layered dielectric
slab which takes into account all the boundary conditions except those on the con­
ducting strips. After imposing the boundary condition on the conducting strips, the
integral equation is then solved by the method of moments (MoM), which in turns
leads to a system of simultaneous linear equations that can be solved numerically.

Most of the planar transmission lines cannot support a truly tangential electric
and magnetic (TEM) mode due to the presence of the dielectric substrate. However,
the fields of the dominant mode for many of them are very close to TEM at low
frequencies, hence the analysis based on a quasi-TEM approximation can work well
in this frequency range. Under the quasi-TEM approximation, the transmission line
problem reduces to an electrostatic one where the governing equation is the Possion equation while the capacitance and charge distribution are the unknowns [20]. The printed transmission line in this case can be modelled by distributed capacitance, inductance and resistance where all the parameters are frequency independent. Also, the conformal mapping technique is often used to solve the quasi-TEM transmission line problem.

As the operating frequency is increased, the phenomena of frequency dispersion and leakage (or radiation) become much more pronounced, and a full-wave analysis such as the integral equation/MoM which takes into account all the frequency dependent effects is required to accurately model the circuits. When using MoM to solve the integral equation, one can either choose subdomain basis functions (like pulse or piecewise sinusoidal functions) [15, 16], or the entire domain basis functions which encompasses the whole conducting strip. In the SDA, Galerkin’s MoM is used where both the basis and testing functions are entire domain functions having closed form expressions in the Fourier transform (or spectral) domain. The testing procedure results in a system of linear equations where only one fold infinite integrations in the spectral domain are required. The condition that the determinant of the matrix for these simultaneous linear equations be equal to zero yields the propagation constants of the transmission line modes. The numerical efficiency of the SDA is mainly attributed to a good selection of the basis functions for the current distribution, which not only incorporate the edge condition, but are sufficiently close to the transmission line modes by themselves such that a small number of basis functions is enough to ensure the accuracy of the result. The variational property guaranteed by the Galerkin’s MoM also enhances the accuracy of the propagation constant.

In contrast to the SDA where the conducting strip is viewed as a surface current sheet, it is part of the parallel plate waveguide in the spatial mode matching method,
where the electric and magnetic fields are expanded in terms of the parallel plate waveguide modes in each region with their amplitudes as unknowns. A relatively big system of linear equations resulting from the field continuity condition between the boundary of waveguides is then solved to yield the propagation constant of the transmission lines. Although the mode matching method is limited to closed region problems, it can rigorously take into account the finite thickness of the conducting strips.

The method used here, referred to as the WH/GSMT, consists of two key steps: first, the edge of a PEC half plane embedded in multilayered dielectric slabs as depicted in Figure 1, is characterized by a scattering matrix which results from solving the canonical scattering problem with the Wiener-Hopf (WH) method; and then the complete solution is built up by taking into account the wave interactions between the various edges in a self-consistent fashion as contained in the generalized scattering matrix technique (GSMT), where the transverse resonance condition determines the allowable modal propagation constants of the transmission line structure. The present research can be viewed as an important extension of Oliner and K. S. Lee's work [23] where a simplified model is used to study a wide microstrip line. A top PEC cover plane is included in the geometry in order to simplify the factorization process in the Wiener-Hopf procedure which otherwise involves a complicated integral due to the branch cut in the integrand of the corresponding open region problem. However, as long as the circuit is used for guided waves rather than radiation, the effect of the top cover is negligible if it is placed far enough.

The WH/GSMT can be viewed as a more efficient version of the spatial mode matching method since in the WH/GSMT the scattering by a single edge is analyzed and the solution is expressed in closed forms while in the spatial mode matching
method the modal amplitudes, which constitute the unknowns, are solved numerically on the computer and the scattering by a single edge is therefore solved numerically [2, 31]. On the other hand, the interaction between edges is calculated numerically with the GSMT for both methods. Both the WH/GSMT and the spatial mode matching method provide a clearer physical picture for the guided wave propagation than does the integral equation technique. In contrast to the SDA, where narrower lines favour the need for fewer basis functions, the WH/GSMT is suited for transmission lines of wider lateral dimensions (as would happen at higher frequencies) since fewer evanescent modes need to be included. When the ratio between the width of the lines and the height of the waveguide is neither too small nor too large, the CPU time and memory space for the WH/GSMT are comparable to those for the SDA. However, the WH/GSMT requires a longer pre-analysis and also, the versatility of this method is relatively limited. Yet in some particular situations, e.g., the conductor backed coplanar waveguide (CBCPW) with lateral ground planes of finite-extent, the SDA would not be as effective as the WH/GSMT.

The format of this dissertation is as follows. In Chapter II, the canonical problem of the scattering of a parallel plate waveguide mode when it is obliquely incident on the edge of a PEC half plane which is embedded in multilayered dielectric slabs that fill the parallel plate waveguide, as depicted in Figure 1, is solved via the WH technique. The solution is then written in a compact scattering matrix form from which all the relevant scattering matrix elements for the interior half plane edge discontinuity can be identified. The scattering matrix can then be easily incorporated in the formulation of the GSMT. The transverse resonance relations which are developed from the GSMT formulation for obtaining the modal propagation constants of for the planar transmission line structures treated here are listed in Chapter III. In addition, a discussion on the advantages/disadvantages of the WH/GSMT along
with some numerical considerations that one has to be aware of when using the WH/GSMT is also given in that chapter. Chapter IV consists of two parts: in the first part, the results based on the WH/GSMT are obtained for many different transmission lines and then compared with independent published results based on other methods to illustrate the accuracy and the versatility of the WH/GSMT. In the second part of Chapter IV, the study is focused on the dispersion and lateral leakage of the CBCPW with a one or two layered substrate, and lateral ground planes with finite or infinite extent. The effect of the top PEC cover plane which is included to simplify the factorization process in the Wiener-Hopf procedure is studied in Chapter V. A simple rule of thumb is given for the height of the top cover which is required in the closed region case to simulate a corresponding open region problem. Chapter VI contains some conclusions and possible future work. Note that the $e^{j\omega t}$ time convention is used and suppressed throughout the dissertation.
Figure 1: The geometry of the canonical problem: A PEC half plane embedded in a multilayered dielectric region bounded by PEC planes forming a parallel plate waveguide.
CHAPTER II
Scattering of an Obliquely Incident Parallel Plate Mode by a PEC Half Plane Embedded in Stratified Media Within the PEC Parallel Plate Region

In this chapter, the canonical problem of the scattering of a parallel plate mode (PPM) which is obliquely incident on the edge of a perfect electric conducting (PEC) half plane embedded in a multilayered dielectric within the PEC parallel plate region is solved via the Wiener-Hopf technique. The geometry of this canonical problem along with its side and top views are depicted in Figures 1, 2 and 3. Referring to Figure 2, if we denote the region $y < 0$, $0 < z < A$ as region 1 terminating in port 1; $y > 0$, $0 < z < D$ as region 2 terminating in port 2; and, $y > 0$, $D < z < A$ as region 3 terminating in port 3, respectively, then the interior half plane discontinuity can be viewed as a three-port circuit element where each individual region, terminating in its respective port is a parallel plate waveguide filled with dielectric layers. Since there are two independent PPM sets associated with each port; namely, TEz and TMz modes, the edge discontinuity of the half plane can be fully characterized after solving six scattering problems where each problem has a different kind of excitation which is either TEz or TMz type PPM incident from one of the three regions. Since the Wiener-Hopf formulation for analyzing all the six problems are quite similar, only one of them, in particular, the TMz mode incidence from region 2, will be addressed in detail in the second section of this chapter, while the key steps for the other five cases will be given in appendix B. The results found from the Wiener-Hopf analysis are then written into a compact generalized scattering
matrix form in the last section of this chapter so that it can be easily incorporated in the formulation of the generalized scattering matrix technique (GSMT) for studying planar transmission line problems in chapter III.

2.1 The PPM Mode in a Parallel Plate Waveguide Region Filled With Multilayered Dielectric

D. C. Chang and E. F. Kuester [24] employed the Wiener-Hopf technique to analyze the scattering of an obliquely incident TEM wave by the edge of a semi-infinite PEC half plane on a grounded dielectric slab of infinite extent. The analysis of the corresponding but more general multilayered geometry of Figure 1 being considered here is similar to that in [24] except that it is more involved due to the additional complexity resulting from the presence of the multilayers. A top PEC cover is added here in order to simplify the factorization process in the Wiener-Hopf procedure.
Before we begin to solve the scattering problem of an obliquely incident PPM by the PEC half plane, some notation and functions are introduced first such that the fields of a PPM in a PEC parallel plate waveguide filled with multilayered dielectric as considered here can be described easily. It is well known that for a source-free multilayered region with arbitrary homogeneous material such as ferrite, anisotropic dielectric or chiral media, we have to solve a system of four simultaneous equations at the interface of any two layers which comes from the condition that the total tangential electric field $E_t$, and total tangential magnetic field $H_t$, be continuous across the interface [10, 11] (note that bold face letters are used to denote vector quantities and the subscript t denotes tangential components). However, for the homogeneous isotropic dielectric material with scalar permittivity $\epsilon$ and scalar permeability $\mu$ as considered here, the system of four simultaneous equations can be decoupled into two systems of two simultaneous equations if the fields are expressed in terms of the
electric and magnetic vector potentials normal to the interface (z-directed potentials here) [2, 25, 27].

Take one of the TMz PPMs in region 2 as an example. If it is assumed to have the travelling wave phase dependence $e^{-jk_0ax}$ and $e^{-jk_0\lambda_my}$ in the x and y directions, respectively, then the electric and magnetic fields of that mode at any point $(x,y,z)$ can be expressed as follows:

\[
E_z(x,y,z) = \frac{\psi_e(\alpha, \lambda_m, z)}{\varepsilon_r(z)} e^{-jk_0ax} e^{-jk_0\lambda_my} \tag{2.1}
\]

\[
E_t(x,y,z) = \frac{1}{k_0} \frac{(\alpha x + \lambda_m y)}{(\alpha_x^2 + \lambda_m^2)} \frac{\partial \psi_e(\alpha, \lambda_m, z)}{\partial z} e^{-jk_0ax} e^{-jk_0\lambda_my} \tag{2.2}
\]

\[
H_t(x,y,z) = \frac{1}{\eta_0} \frac{(\lambda_m x - \alpha y)}{(\alpha^2 + \lambda_m^2)} \psi_e(\alpha, \lambda_m, z) e^{-jk_0ax} e^{-jk_0\lambda_my} \tag{2.3}
\]

where $\varepsilon_r(z)$ is a piece-wise constant function which assumes the value $\varepsilon_{r_i}$ if $z$ is in the i-th layer, $k_0$ is the free space wave number and $\eta_0$ is the intrinsic impedance of free space. In this case, the normal electric field $E_z$ is proportional to the z-directed magnetic vector potential. As one can see, the condition that the tangential electric and tangential magnetic fields be continuous for any $z$ is equivalent to $\psi_e(\alpha, \lambda_m, z)$ and \(\frac{1}{\varepsilon_r(z)} \frac{\partial}{\partial z} \psi_e(\alpha, \lambda_m, z)\) being continuous for any $z$. If we let $\psi_e(\alpha, \lambda_m, z)$ and \(\frac{1}{\varepsilon_r(z)} \frac{\partial}{\partial z} \psi_e(\alpha, \lambda_m, z)\) play the same roles as voltage and current in microwave circuit theory, and if we consider the multilayers as sections of transmission lines of different characteristic impedances in series, then $\psi_e(\alpha, \lambda_m, z)$ and \(\frac{1}{\varepsilon_r(z)} \frac{\partial}{\partial z} \psi_e(\alpha, \lambda_m, z)\) can be written down as follows where the propagation matrix $\overline{P}_e(\alpha, \lambda_m, z)$ is analogous to the ABCD matrix. If the point lies in the i-th layer of region 2, $0 < z < D$, $y > 0$, as depicted in Figure 2, then

\[
\begin{bmatrix}
\psi_e(\alpha, \lambda, z) \\
\frac{1}{\varepsilon_{r_i}} \frac{\partial \psi_e(\alpha, \lambda, z)}{\partial z}
\end{bmatrix} = \overline{P}_e(\alpha, \lambda, z - z_0)\overline{P}_e(\alpha, \lambda, d_{i-1}) \cdots \overline{P}_e(\alpha, \lambda, d_{-M}) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{2.4}
\]

\[
z_0 = \sum_{j=-M}^{i-1} d_j , \quad \tag{2.5}
\]

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where

$$\overline{P}_e(\alpha, \lambda, d_j) = \begin{bmatrix} \cos(kz_j d_j) & \frac{\varepsilon_{r_j}}{kz_j} \sin(kz_j d_j) \\ -\frac{kz_j}{\varepsilon_{r_j}} \sin(kz_j d_j) & \cos(kz_j d_j) \end{bmatrix}, \quad (2.6)$$

and $\varepsilon_{r_j}$ and $\mu_{r_j}$ are the relative permittivity and relative permeability, respectively, of the corresponding dielectric layer which has thickness $d_j$. The column vector $[1, 0]^T$ (T is the transpose operator) in Equation (2.4) comes from the boundary condition that $E_t = 0$ on the PEC at $z=0$. Note that $\frac{\partial}{\partial z} \psi_e(\alpha, \lambda, z)$ is calculated from the multiplication of the propagation matrices as in Equation (2.4) instead of taking a partial derivative of $\psi_e(\alpha, \lambda, z)$ with respect to $z$. There are many different ways to express the fields in a multilayered dielectric region. The reason we choose to write the propagation matrix $\overline{P}_e(\alpha, \lambda, d_j)$ in this way is such that $\overline{P}_e$ itself is an entire and even function of $\alpha$, $\lambda$ and $kz_j$ explicitly. That is, it does not have any singularities like poles and branch cuts. The importance of these properties will become more pronounced in the next section where the Wiener-Hopf analysis requires a rigorous knowledge of the analytical behavior of the functions involved.

By employing the matrix $\overline{P}_e$ to express the fields, the continuity of the tangential electric and tangential magnetic fields in the dielectric layers is automatically implied. The other boundary condition that $E_t = 0$ on the PEC at $z=D$ puts a limitation on the values that $\alpha^2 + \lambda_m^2$ can assume; namely, $\alpha$ and $\lambda_m$ must satisfy the following equation

$$[0, 1] \overline{P}_e(\alpha, \lambda_m, d_{-1}) \overline{P}_e(\alpha, \lambda_m, d_{-2}) \cdots \overline{P}_e(\alpha, \lambda_m, d_{-M}) \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0. \quad (2.8)$$
Similarly, the electric and magnetic fields for a TEz PPM in a parallel plate waveguide filled with multilayered dielectric like in region 2 can be expressed as follows if it is assumed to have the dependence $e^{-j k_0 \alpha x}$ and $e^{-j k_0 \lambda n y}$ in the $x$ and $y$ directions:

$$H_z(x, y, z) = \frac{\psi_h(\alpha, \lambda_n, z)}{\mu_r(z)} e^{-j k_0 \alpha x} e^{-j k_0 \lambda n y}$$  \hspace{1cm} (2.9)

$$H_t(x, y, z) = -\frac{1}{k_0} \frac{\partial}{\partial z} \psi_h(\alpha, \lambda_n, z) e^{-j k_0 \alpha x} e^{-j k_0 \lambda n y}$$  \hspace{1cm} (2.10)

$$E_t(x, y, z) = -\eta_0 \frac{\lambda_n \hat{x} - \alpha \hat{y}}{\alpha^2 + \lambda_n^2} \psi_h(\alpha, \lambda_n, z) e^{-j k_0 \alpha x} e^{-j k_0 \lambda n y}$$  \hspace{1cm} (2.11)

where

$$\begin{bmatrix}
\psi_h(\alpha, \lambda, z) \\
\frac{1}{\mu_r} \frac{\partial \psi_h(\alpha, \lambda, z)}{\partial z}
\end{bmatrix} = \bar{P}_h(\alpha, \lambda, z - z_0) \bar{P}_h(\alpha, \lambda, d_{i-1}) \cdots \bar{P}_h(\alpha, \lambda, d_{-M}) \begin{bmatrix}
0 \\
1
\end{bmatrix}$$  \hspace{1cm} (2.12)

if the point is in the $i$-th layer and

$$\bar{P}_h(\alpha, \lambda, d_j) = \begin{bmatrix}
\cos(k_{zj} d_j) & \frac{\mu_r}{k_{zj}} \sin(k_{zj} d_j) \\
-\frac{k_{zj}}{\mu_r} \sin(k_{zj} d_j) & \cos(k_{zj} d_j)
\end{bmatrix}$$  \hspace{1cm} (2.13)

are defined in Equations (2.5) and (2.7), respectively. Likewise, the column vector $[0, 1]^T$ in Equation (2.12) comes from the boundary condition that $E_t = 0$ on the PEC at $z=0$. Again, $\alpha$ and $\lambda_n$ must satisfy the following equation

$$[1, 0] \bar{P}_h(\alpha, \lambda_n, d_{-1}) \bar{P}_h(\alpha, \lambda_n, d_{-2}) \cdots \bar{P}_h(\alpha, \lambda_n, d_{-M}) \begin{bmatrix}
0 \\
1
\end{bmatrix} = 0$$  \hspace{1cm} (2.14)

With all the functions defined above, the fields of any PPM incident from regions 1, 2 and 3 can be expressed easily. Here they are listed in order to avoid any confusion. As mentioned in the beginning of this chapter, the Wiener-Hopf analysis is fairly involved due to the presence of the multilayers. However, with the compact and
systematic notation defined above, it is possible to easily identify the corresponding counterparts between single layered and multilayered cases and thus extend the single layered solution to the multilayered one in a straight-forward manner.

It is assumed that there are M and N layers of dielectric below and above the PEC half plane, respectively. Figures 1 and 2 show the manner in which the layers are numbered. For the sake of convenience, the subscripts a, d and b are used interchangeably with 1, 2 and 3, respectively, to denote the regions, since the subscripts 1, 2, 3,... are also sometimes used to denote a particular layer of the multilayer slab or the mode sequence of the PPM in certain regions. Let A, D and B be the heights of regions 1, 2 and 3; namely,

\[ D = \prod_{j=-M}^{-1} d_j \]  
(2.15)

\[ B = \prod_{j=1}^N d_j \]  
(2.16)

\[ A = D + B \]  
(2.17)

Define the modal functions \( e_{in} \) and \( h_{in} \), \( i=a, d \) or \( b \) for the n-th \( TM_z \) and \( TE_z \) polarized PPM mode in region 1, 2 or 3, respectively, as follows:

\[ e_{an} = \frac{\psi_{de}(\alpha, \lambda_{ae_n}, z)}{\epsilon_r(z)} \]  
(2.18)

\[ e_{dn} = \frac{\psi_{de}(\alpha, \lambda_{de_n}, z)}{\epsilon_r(z)} \]  
(2.19)

\[ e_{bn} = \frac{\psi_{be}(\alpha, \lambda_{be_n}, z)}{\epsilon_r(z)} \]  
(2.20)

\[ h_{an} = \frac{\psi_{dh}(\alpha, \lambda_{ah_n}, z)}{\mu_r(z)} \]  
(2.21)

\[ h_{dn} = \frac{\psi_{dh}(\alpha, \lambda_{dh_n}, z)}{\mu_r(z)} \]  
(2.22)

\[ h_{bn} = \frac{\psi_{bh}(\alpha, \lambda_{bh_n}, z)}{\mu_r(z)} \]  
(2.23)
where

\[
\begin{bmatrix}
\psi_{de}(\alpha, \lambda, z) \\
\frac{1}{\varepsilon_{r_i}} \frac{\partial \psi_{de}(\alpha, \lambda, z)}{\partial z}
\end{bmatrix} = \begin{bmatrix} P_e(\alpha, \lambda, z - z_d) P_e(\alpha, \lambda, d_{i-1}) \cdots P_e(\alpha, \lambda, d_{-M}) \\
0
\end{bmatrix},
\]

\[ \psi_{dh}(\alpha, \lambda, z) = \begin{bmatrix} P_h(\alpha, \lambda, z - z_d) P_h(\alpha, \lambda, d_{i-1}) \cdots P_h(\alpha, \lambda, d_{-M}) \\
0
\end{bmatrix}, \tag{2.24} \]

\[ z_d = \sum_{j=-M, j \neq 0}^{i-1} d_j \tag{2.26} \]

and

\[
\begin{bmatrix}
\psi_{be}(\alpha, \lambda, z) \\
\frac{1}{\varepsilon_{r_i}} \frac{\partial \psi_{be}(\alpha, \lambda, z)}{\partial z}
\end{bmatrix} = \begin{bmatrix} P_e(\alpha, \lambda, z - z_b) P_e(\alpha, \lambda, -d_{i+1}) \cdots P_e(\alpha, \lambda, -d_N) \\
0
\end{bmatrix},
\]

\[ \psi_{bh}(\alpha, \lambda, z) = \begin{bmatrix} P_h(\alpha, \lambda, z - z_b) P_h(\alpha, \lambda, -d_{i+1}) \cdots P_h(\alpha, \lambda, -d_N) \\
0
\end{bmatrix}, \tag{2.27} \]

\[ z_b = A - \sum_{j=i+1, j \neq 0}^{N} d_j \tag{2.29} \]

if the point \((x,y,z)\) is in the \(i\)-th layer.

If the amplitudes of the \(n\)-th incoming and outgoing PPM modes in region \(i\) are denoted by \(A_{-\alpha}^{i} n_p\) and \(A_{+\alpha}^{i} n_p\), \(p=e, h\), respectively, then the \(z\)-components of the total electric and magnetic fields in different regions can be expressed as follows. Other components of the fields can be obtained easily from Equations (2.1)-(2.3) and (2.9)-(2.11). The \(E_z\) and \(H_z\) in region 1 are

\[
E_z(x, y, z) = e^{-jk_0 \alpha x} \sum_n \left[ A_{+\alpha n} e^{jk_0 \alpha n y} + A_{-\alpha n} e^{-jk_0 \alpha n y} \right] e_n, \tag{2.30}
\]

\[
H_z(x, y, z) = e^{-jk_0 \alpha x} \sum_n \left[ A_{+\alpha n} e^{jk_0 \alpha n y} + A_{-\alpha n} e^{-jk_0 \alpha n y} \right] h_n; \tag{2.31}
\]

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while

\[ E_z(x, y, z) = e^{-jk_0\alpha x} \sum_n \left[ A_{de_n}^+ e^{-jk_0\lambda_{den} y} + A_{de_n}^- e^{jk_0\lambda_{den} y} \right] \varepsilon_{dn} , \quad (2.32) \]

\[ H_z(x, y, z) = e^{-jk_0\alpha x} \sum_n \left[ A_{dh_n}^+ e^{-jk_0\lambda_{dhn} y} + A_{dh_n}^- e^{jk_0\lambda_{dhn} y} \right] h_{dn} \quad (2.33) \]

in region 2 and

\[ E_z(x, y, z) = e^{-jk_0\alpha x} \sum_n \left[ A_{be_n}^+ e^{-jk_0\lambda_{ben} y} + A_{be_n}^- e^{jk_0\lambda_{ben} y} \right] \varepsilon_{bn} , \quad (2.34) \]

\[ H_z(x, y, z) = e^{-jk_0\alpha x} \sum_n \left[ A_{bh_n}^+ e^{-jk_0\lambda_{bh} y} + A_{bh_n}^- e^{jk_0\lambda_{bh} y} \right] h_{bn} \quad (2.35) \]

in region 3.

For a fixed \( \alpha \), the normalized propagation constant in the y direction (\( \lambda_{ip_n} \)) is the n-th zero of \( D_{ip}(\lambda) = 0 \) where the subscripts i=d, b or a, denotes the region; p=e, h, denotes TMz or TEz mode, respectively. The functions \( D_{ip}(\lambda) \) are given below:

\[ D_{de}(\lambda) = [0,1] \bar{P}_e(\alpha, \lambda, d_{-1}) \bar{P}_e(\alpha, \lambda, d_{-2}) \cdots \bar{P}_e(\alpha, \lambda, d_{-M}) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2.36) \]

\[ D_{be}(\lambda) = [0,1] \bar{P}_e(\alpha, \lambda, -d_1) \bar{P}_e(\alpha, \lambda, -d_2) \cdots \bar{P}_e(\alpha, \lambda, -d_N) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2.37) \]

\[ D_{ae}(\lambda) = [0,1] \bar{P}_e(\alpha, \lambda, d_N) \cdots \bar{P}_e(\alpha, \lambda, d_2) \bar{P}_e(\alpha, \lambda, d_1) \]

\[ \bar{P}_e(\alpha, \lambda, d_{-1}) \bar{P}_e(\alpha, \lambda, d_{-2}) \cdots \bar{P}_e(\alpha, \lambda, d_{-M}) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2.38) \]

\[ D_{dh}(\lambda) = [1,0] \bar{P}_h(\alpha, \lambda, d_{-1}) \bar{P}_h(\alpha, \lambda, d_{-2}) \cdots \bar{P}_h(\alpha, \lambda, d_{-M}) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2.39) \]

\[ D_{bh}(\lambda) = [1,0] \bar{P}_h(\alpha, \lambda, -d_1) \bar{P}_h(\alpha, \lambda, -d_2) \cdots \bar{P}_h(\alpha, \lambda, -d_N) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2.40) \]
As mentioned before, since \( \alpha \) and \( \lambda \) always appear together as a term \( \alpha^2 + \lambda^2 \) in the propagation matrices \( \overline{P}_e \) and \( \overline{P}_h \), it is possible to numerically search for the zeros of \( D_{ip}(\lambda) \) by treating \( \alpha^2 + \lambda^2 \) as a single variable and then subtract \( \alpha^2 \) to get the corresponding \( \lambda^2 \). Therefore, the root searching process only has to be done once as long as the substrate geometry and the operating frequency are the same. For the lossless case where all the permittivities and permeabilities are real numbers, the values of \( \alpha^2 + \lambda^2 \) are always real with an upper bound equal to the maximum of the product \( \varepsilon_r \mu_r \) in that region. Thus, for a given \( \alpha \), the sequence of \( \lambda_{ip_n} \) as \( n=1,2,3,... \) can be numerically found easily. Also, because all the \( D_{ip}(\lambda) \) are even functions of \( \lambda \), the zeros \( \lambda_{ip_n} \) are symmetrically paired with respect to \( \lambda = 0 \) for a fixed \( \alpha \). We denote the zeros in the lower half \( \lambda \) plane by \( \lambda_{ip_n} \) and the ones in the upper half \( \lambda \) plane by \( -\lambda_{ip_n} \). For a lossless material and a real \( \alpha \), specifically, \( \lambda_{ip_n} \) denotes the zeros which are either real positive or negative imaginary.

### 2.2 Scattering of a TMz PPM Obliquely Incident From Region 2 by the PEC Half Plane

Still referring to the same geometry as depicted in Figures 1, 2 and 3, consider the incident fields which correspond to the m-th TMz polarized PPM obliquely incident onto the edge of the PEC half plane from region 2; these incident electric and magnetic fields are given by

\[
E_{z}^{i}(x, y, z) = \frac{\psi_{de}(-\lambda_{dem}, z)}{\varepsilon_r(z)} e^{jk_0 \lambda_{dem} y} e^{-jk_0 \alpha x}
\]

\[
E_{t}^{i}(x, y, z) = -\frac{j}{k_0} \left( \frac{\alpha \hat{x} - \lambda_{dem} \hat{y}}{\alpha^2 + \lambda_{dem}^2} \right) \frac{1}{\varepsilon_r(z)} \frac{\partial \psi_{de}(-\lambda_{dem}, z)}{\partial z} e^{jk_0 \lambda_{dem} y} e^{-jk_0 \alpha x}
\]
\[ H_i(x, y, z) = -\frac{1}{\eta_0} \frac{(\lambda_{dem} \hat{x} + \alpha \hat{y})}{(\alpha^2 + \lambda^2)} \psi_{de}(-\lambda_{dem}, z) e^{jk_0 \lambda_{dem} y} e^{-jk_0 \alpha x} \]  

Equation (2.44)

for \(-\infty < y < \infty\), \(0 \leq z < D\). This field can be referred to as the unperturbed field because it is the field that would exist if the PEC half plane is extended to \(y \to -\infty\). All the scattered fields must also have the same dependence \(e^{-jk_0 \alpha x}\) in the \(x\) direction since the geometry has a uniform cross section along the edge direction, and the phase has to be matched for every \(x\). The normalized propagation constant along the edge direction, \(\alpha\), is assumed to be a complex number with positive real part and non-positive imaginary part to ensure that the field propagates in the \(x\) direction and is finite when \(x \to \infty\). Assuming a fixed \(\alpha\), the \(e^{-jk_0 \alpha x}\) factor will be suppressed in most of the development for the sake of convenience. Also, in the following analysis, the argument \(\alpha\) will be omitted for some functions defined in the previous section, such as \(\overline{P}_{e,h}(\alpha, \lambda, z)\), \(\psi_{de, dh}(\alpha, \lambda, z)\) and \(\psi_{be, bh}(\alpha, \lambda, z)\) since \(\alpha\) is assumed to be fixed. In this section, \(\lambda_m\) and \(\lambda_{dem}\) will be used interchangeably to simplify the notation.

To begin with the analysis, it is important to introduce the Fourier transform of \(f(y)\) as \(\tilde{f}(\lambda)\), which is defined as usual by

\[ \tilde{f}(\lambda) = \frac{k_0}{2\pi} \int_{-\infty}^{\infty} f(y) e^{jk_0 \lambda y} dy \]  

Equation (2.45)

where the “\(-\)” will be used to denote Fourier transformed functions. Also, the one sided Fourier transforms \(\tilde{f}_{\pm}(\lambda)\) are defined as

\[ \tilde{f}_{+}(\lambda) = \frac{k_0}{2\pi} \int_{0}^{\infty} f(y) e^{jk_0 \lambda y} dy \]  

Equation (2.46)

\[ \tilde{f}_{-}(\lambda) = \frac{k_0}{2\pi} \int_{-\infty}^{0} f(y) e^{jk_0 \lambda y} dy \]  

Equation (2.47)

The corresponding inverse transform of (2.45) is given by

\[ f(y) = \int_{-\infty}^{\infty} \tilde{f}(\lambda) e^{-jk_0 \lambda y} d\lambda \]  

Equation (2.48)
Of course,
\[ \tilde{f}(\lambda) = \tilde{f}_+(\lambda) + \tilde{f}_-(\lambda) \] (2.49)

It is known from Fourier transform theory that if \( f(y) \sim e^{k_0r^-y} \) as \( y \to \infty \) and \( f(y) \sim e^{k_0r^+y} \) as \( y \to -\infty \), then \( \tilde{f}_+(\lambda) \) and \( \tilde{f}_-(\lambda) \) are regular in the upper and lower half \( \lambda \) plane defined by \( \tau > \tau_- \) and \( \tau < \tau_+ \), respectively, where \( \tau = \text{Im}(\lambda) \).

The integration path for the inverse Fourier transform of \( \tilde{f}(\lambda) \) should lie in a strip \( \tau_- < \tau < \tau_+ \) where both \( \tilde{f}_+(\lambda) \) and \( \tilde{f}_-(\lambda) \) are regular. Usually, in the Wiener-Hopf analysis, a small loss in the material is introduced first and then the lossless case is treated as the limiting case of vanishing loss afterwards. Here, the lossless case is considered directly for simplicity. However, this should by no means be a source of confusion. For the problem being considered here, if the dielectric materials are lossless and \( \alpha \) is real, then the integration path is the real \( \lambda \) axis as shown in Figure 4. Now, we can begin to use the Wiener-Hopf technique to solve the canonical scattering problem. The formulation used here is based on Jones' method.

The total field can be written as the superposition of the fields, namely,
\[ E = E^i + E^s \] (2.50)
\[ H = H^i + H^s \] (2.51)

where the \( E^i, H^i \) are the incident (unperturbed) fields and \( E^s \) and \( H^s \) are the scattered electric and magnetic fields, respectively. Note that \( E^s \) and \( H^s \) are the fields that take into account the absence of a PEC conductor at \(-\infty < y < \infty, z = D\).

Assume the spectral (Fourier transform domain) representation of the scattered field for \( z < D \) to be
\[ \tilde{E}_z^s(\lambda, z) = E_d(\lambda) \frac{\psi_{de}(\lambda, z)}{\psi_{de}(\lambda, D)} \] (2.52)
\[ \tilde{H}_z^s(\lambda, z) = H_d(\lambda) \frac{\psi_{dh}(\lambda, z)}{\psi_{dh}(\lambda, D)} \] (2.53)
Figure 4: The integration path in the $\lambda$ plane.
where the superscript \( s \) denotes the 'scattered' field, and the functions \( \psi_{de}, \psi_{dh}, \psi_{be}, \psi_{bh} \) along with their partial derivatives with respect to \( z \) are defined in Equations (2.24)-(2.28). As in the previous section, \( \epsilon_r(z) \) and \( \mu_r(z) \) are piecewise constant functions which assume the values \( \epsilon_{ri} \) and \( \mu_{ri} \), respectively, in the \( i \)-th layer. \( E_d(\lambda), H_d(\lambda), E_b(\lambda) \) and \( H_b(\lambda) \) are the unknown functions to be solved. The scattered fields in (2.52)-(2.59) already satisfy the boundary conditions on the PECs at \( z=0 \) and \( z=A \), and also across the interface of any two dielectric layers for regions \( z < D \) and \( z > D \). Therefore, the remaining boundary condition that needs to be enforced is at \( z = D \) where the PEC half plane resides; namely,

\[ E_t(y, z = D) = 0 \quad ; \quad 0 < y < \infty \]  

(2.60)
\[
E_t(y, z = D_-) = E_t(y, z = D_+) \quad ; \quad -\infty < y < 0 \quad (2.61)
\]
\[
H_t(y, z = D_-) = H_t(y, z = D_+) \quad ; \quad -\infty < y < 0 \quad (2.62)
\]

along with the edge condition near \( y=0, z=D \) and the radiation condition for \( y \to \infty \) and \( y \to -\infty \). Thus, with all the functions defined in the previous section, the scattering problem of a PPM by a PEC half plane embedded in a multilayered dielectric region is similar to the corresponding single layered one. Notice that in the spectral domain, for fixed \( \lambda \) and \( z \), the \( \hat{z} \)-directed electric and magnetic vector potentials (which are proportional to the \( z \) components of the magnetic and electric fields, respectively) always give rise to tangential fields in two mutually orthogonal directions; namely, the tangential fields lie either in the
\[
\hat{u} = \frac{(\lambda \hat{x} - \alpha \hat{y})}{\sqrt{\alpha^2 + \lambda^2}} \quad (2.63)
\]
direction or in the
\[
\hat{v} = \frac{(\alpha \hat{x} + \lambda \hat{y})}{\sqrt{\alpha^2 + \lambda^2}} \quad , \quad (2.64)
\]
direction. This property makes the decoupling of the boundary condition at \( z = D \) possible.

Boundary conditions (2.60) and (2.61) together imply that
\[
\tilde{E}_t^i(\lambda, z = D_-) = \tilde{E}_t^i(\lambda, z = D_+) \quad , \quad (2.65)
\]
since \( \tilde{E}_t^i(y, z = D_-) = \tilde{E}_t^i(y, z = D_+) = 0 \) for \(-\infty < y < \infty \). From Equations (2.54) and (2.58), one obtains two relations
\[
\frac{\varepsilon_{r,-1} E_d(\lambda) D_{de}(\lambda)}{\psi_{de}(\lambda, D)} = \frac{\varepsilon_{r,1} E_b(\lambda) D_{be}(\lambda)}{\psi_{be}(\lambda, D)} \quad (2.66)
\]
and
\[
\mu_{r,-1} H_d(\lambda) = \mu_{r,1} H_b(\lambda) \quad . \quad (2.67)
\]
The boundary condition (2.62) can be written as

\[ \mathbf{H}_t(y, z = D_+) - \mathbf{H}_t(y, z = D_-) = -\hat{z} \times J_s(y), \quad -\infty < y < \infty \quad (2.68) \]

where \( J_s(y) \) is the total induced surface current on the PEC half plane which can be written as the sum of two parts:

\[ J_s(y) = \begin{cases} J_s^i(y) + J_s^a(y), & y > 0 \\ 0, & y < 0 \end{cases} \quad (2.69) \]

The first part \( J_s^i(y) \) is the surface current induced by the incident field and defined as

\[ J_s^i(y) = \begin{cases} -\hat{z} \times H_t(y, z = D_-), & y > 0 \\ 0, & y < 0 \end{cases} \quad (2.70) \]

The other part \( J_s^a(y) \) is the unknown surface current induced by the scattered field. Substituting (2.69) and (2.70) into Equation (2.68) and taking the Fourier transform yields

\[ \mathbf{H}_t^T(\lambda, z = D_+) - \mathbf{H}_t^T(\lambda, z = D_-) = \mathbf{H}_t^T(\lambda, z = D_-) - \hat{z} \times \tilde{J}_{s+}^a(\lambda) \quad (2.71) \]

where the subscripts + and – denote the one sided Fourier transformed functions. From Equations (2.44) and (2.47), one has

\[ \tilde{J}_{s-}^i(\lambda, z = D_-) = \frac{1}{2\pi \eta_0 \left( \alpha^2 + \frac{\lambda_m^2}{\lambda^2} \right)} \psi_{de}(-\lambda_m, D) \quad (2.72) \]

Equation (2.71) actually consists of two conditions since it is in a vector form. Applying (2.71) in the \( \hat{u} \) direction gives

\[ \frac{\varepsilon_{r1} E_b(\lambda) D_{be}(\lambda)}{k_0 \psi_{be}(\lambda, D) Q_e(\lambda)} = \frac{-j}{2\pi} \left( \frac{1}{\lambda + \lambda_m} - \frac{\lambda_m}{\alpha^2 + \lambda_m^2} \right) \psi_{de}(-\lambda_m, D) \]

\[ + \eta_0 (\alpha \hat{x} + \lambda \hat{y}) \cdot \tilde{J}_{s+}^a(\lambda) \quad (2.73) \]

where

\[ Q_e(\lambda) = \frac{D_{be}(\lambda) D_{de}(\lambda)}{k_0 D_{ae}(\lambda)} \quad (2.74) \]

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Similarly, applying (2.71) in the \( \hat{v} \) direction gives

\[
\frac{-3 \mu_r H_b(\lambda)}{Q_h(\lambda)} = \frac{1}{2 \pi \eta_0} \frac{\alpha}{\lambda_m^2 + \alpha^2} \psi_{de}(-\lambda_m, D) + (-\lambda \hat{x} + \alpha \hat{y}) \cdot \tilde{J}_{s+}^s(\lambda)
\]  

(2.75)

where

\[
Q_h(\lambda) = k_0 D_{bh}(\lambda) D_{dh}(\lambda)
\]

(2.76)

In deriving Equations (2.73)-(2.76), two relations have been employed, namely,

\[
D_{ae}(\lambda) = D_{de}(\lambda) \psi_{be}(\lambda, z = D) - D_{be}(\lambda) \psi_{de}(\lambda, z = D)
\]  

(2.77)

and

\[
D_{ah}(\lambda) = \frac{1}{\mu_r} \frac{\partial}{\partial z} \psi_{bh}(\lambda, z = D) D_{dh}(\lambda) - \frac{1}{\mu_{r-1}} \frac{\partial}{\partial z} \psi_{dh}(\lambda, z = D) D_{bh}(\lambda)
\]  

(2.78)

\( Q_e(\lambda) \) and \( Q_h(\lambda) \), which are closely related to the Green’s function kernels for the multilayered dielectric slab, are functions to be factorized. Since all the \( D_{ip}(\lambda) \) (i=a, b, d; p=e, h) functions are regular and an even valued functions of \( \lambda \) with simple zeros, the WH factorization is quite easy. The details of the factorization procedure are given in Appendix A. Equations (2.75) and (2.73) are the two Wiener-Hopf equations that need to be solved. The boundary condition (2.60) implies that \( \bar{E}_b^s \) is regular in the lower half \( \lambda \) plane, or equivalently, \( E_b(\lambda) D_{be}(\lambda)/\psi_{be}(\lambda, D) \) and \( H_b(\lambda) \) are regular in the lower half plane. From the edge condition [26], we have

\[
E_b(\lambda) \sim \lambda^{-p} \text{ as } |\lambda| \to \infty,
\]

(2.79)

\[
H_b(\lambda) \sim \lambda^{-p} \text{ as } |\lambda| \to \infty,
\]

(2.80)

\[
\tilde{J}_{s+}^s \cdot \hat{u} \sim \lambda^{-p} \text{ as } |\lambda| \to \infty
\]

(2.81)

and

\[
\tilde{J}_{s+}^s \cdot \hat{v} \sim \lambda^{-1-p} \text{ as } |\lambda| \to \infty
\]

(2.82)
where $0 < p < 1$. Also, it can be checked from the definitions that

$$Q_e(\lambda) \sim \lambda \text{ as } |\lambda| \to \infty,$$

(2.83)

$$Q_h(\lambda) \sim \lambda^{-1} \text{ as } |\lambda| \to \infty,$$

(2.84)

and

$$\frac{D_{be}(\lambda)}{\psi_{be}(\lambda, D)} \sim \lambda \text{ as } |\lambda| \to \infty.$$  

(2.85)

Thus, both (2.73) and (2.75) can be written into a standard form of the Wiener-Hopf equations where one side of the equation is regular in the lower half $\lambda$ plane while the other side is regular in the upper half $\lambda$ plane. Note that there must be an overlapping region between these two half planes, which shrinks to the real axis of the $\lambda$ plane if $\alpha$ is real and if all the dielectrics are lossless. By invoking Liouville’s theorem[26] and upon examining the behavior of the various terms in the Wiener-Hopf equations, it is concluded that both equations which result from (2.73) and (2.75), respectively, must be equal to a constant. These two Wiener-Hopf equations are given as follows. In particular, the one obtained from equation (2.73) is

$$\frac{e_r E_b D_{be}(\lambda)}{k_0 \psi_{be}(\lambda, D)Q_{e-}(\lambda)} + \frac{1}{2\pi (\lambda + \lambda_m)} \psi_{de}(-\lambda_m, D)Q_{e+}(-\lambda_m)$$

$$= -\frac{1}{2\pi (\lambda + \lambda_m)} (Q_{e+}(\lambda) - Q_{e+}(-\lambda_m)) + \frac{\lambda_m}{2\pi (\alpha^2 + \lambda_m^2)} \psi_{de}(-\lambda_m, D)Q_{e+}(\lambda)$$

$$+ \eta_0 (\alpha \hat{x} + \lambda \hat{y}) \cdot \vec{J}_{n+}^e Q_{e+}(\lambda)$$

$$= \frac{1}{2\pi \psi_{de}(-\lambda_m, D)Q_{e+}(-\lambda_m)C_1}$$

(2.86)

and the other one from equation (2.75) is

$$-\frac{\mu r E_b H_b}{Q_{h-}(\lambda)} = \frac{1}{2\pi \eta_0 (\alpha^2 + \lambda_m^2)^2} \psi_{de}(-\lambda_m, D)Q_{h+}(\lambda) + (-\lambda \hat{x} + \alpha \hat{y}) \cdot \vec{J}_{n+}^h Q_{h+}(\lambda)$$

$$= -\frac{1}{2\pi \eta_0 \psi_{de}(-\lambda_m, D)Q_{e+}(-\lambda_m)C_2}$$

(2.87)
where $C_1$ and $C_2$ are constants to be determined. Rearranging equations (2.86) and (2.87) gives

$$\frac{\varepsilon_r E_b(\lambda) D_{be}(\lambda)}{k_0 \psi_{be}(\lambda, D)} = \frac{j}{2\pi} \psi_{de}(-\lambda_m, D) Q_{e+}(-\lambda_m) \left( C_1 - \frac{1}{\lambda + \lambda_m} \right) Q_{e-}(-\lambda_m)$$

and

$$\mu_r H_b(\lambda) = -\frac{j}{2\pi \eta_0} \psi_{de}(-\lambda_m, D) Q_{e+}(-\lambda_m) C_2 Q_{h-}(\lambda).$$

The remaining task is to find the two unknown constants $C_1$ and $C_2$. Substituting (2.88) and (2.89) into (2.58), and using the condition that $\tilde{E}_t^s(\lambda, D)$ is regular in the lower half $\lambda$ plane, which means it cannot have a pole at $\lambda = -j\alpha$, yields

$$C_1 Q_{e+}(j\alpha) + C_2 Q_{h+}(j\alpha) = \frac{Q_{e+}(j\alpha)}{-j\alpha + \lambda_m}. \tag{2.90}$$

Another condition comes from expressing $\tilde{J}_{s+}^s(\lambda)$ as a sum of its components in the $\hat{u}$ and $\hat{v}$ directions which can be obtained from Equations (2.86) and (2.87), and then requiring that $\tilde{J}_{s+}^s(\lambda)$ be regular in the upper half plane, which implies that $\tilde{J}_{s+}^s(\lambda)$ cannot have a pole at $\lambda = j\alpha$. This condition yields

$$\frac{C_1}{Q_{e+}(j\alpha)} + \frac{C_2}{Q_{h+}(j\alpha)} = \frac{1}{(j\alpha + \lambda_m) Q_{e+}(j\alpha)}. \tag{2.91}$$

From the above two equations, $C_1$ and $C_2$ can be solved:

$$C_1 = \frac{\lambda_m}{(\lambda_m^2 + \alpha^2)} + \frac{j\alpha}{(\lambda_m^2 + \alpha^2)} \Omega_1 \tag{2.92}$$

and

$$C_2 = -\frac{j\alpha}{(\lambda_m^2 + \alpha^2)} \Omega_2 \tag{2.93}$$

where

$$\Omega_1 = \frac{Q_{e+}^2(j\alpha) + Q_{h+}^2(j\alpha)}{Q_{e+}^2(j\alpha) - Q_{h+}^2(j\alpha)} \tag{2.94}$$

and

$$\Omega_2 = \frac{2Q_{e+}(j\alpha)Q_{h+}(j\alpha)}{Q_{e+}^2(j\alpha) - Q_{h+}^2(j\alpha)}. \tag{2.95}$$
Substituting the result of \( C_1 \) and \( C_2 \) back to Equations (2.88) and (2.89), the expressions for the functions \( E_b(\lambda) \) and \( H_b(\lambda) \) are obtained. \( E_d(\lambda) \) and \( H_d(\lambda) \) can be then be deduced from equations (2.66) and (2.67), respectively. Consequently, all the terms for the scattered fields in the spectral domain as defined in Equations (2.52)-(2.59) are known. The last step is to take their inverse Fourier transforms to obtain the scattered field back in the spatial domain. This inversion integral can be evaluated using the Cauchy Residue Theorem after closing the original integration path (along the real \( \lambda \) axis) by a half circle of an infinitely large radius in the lower half \( \lambda \) plane when \( y > 0 \), and likewise in the upper half \( \lambda \) plane when \( y < 0 \).

2.3 The Generalized Scattering Matrix for the PEC Half-plane

To fully characterize the half-plane edge, one has to solve five other canonical scattering problems, which are similar to the one solved in the previous section except that a differently polarized PPM is now incident from the different regions. The key steps in the Wiener-Hopf analysis for those five problems are given in Appendix B.

With the solutions obtained to the six scattering problems, one can then describe the PEC half plane edge discontinuity as a six port circuit element and express the properties of the discontinuity in terms of the generalized scattering matrix which characterizes the six port circuit model. The generalized scattering matrix is defined in the same way as the usual scattering matrix used in microwave circuits except that the generalized scattering matrix includes the evanescent modes in addition to the propagating modes [2, 26]. The fields of the evanescent modes will decay exponentially away from the PEC half plane edge discontinuity located at \( y=0 \) as \(|y| \) increases because of the factor \( e^{\pm jk_0 \lambda_{ipn} y} \), where \( \lambda_{ipn} \) is an imaginary number for the evanescent modes. Therefore, most of the evanescent modes are not important.
except a few leading terms. Same as in the usual scattering matrix, the elements in the generalized scattering matrix will be different as the definition of the modal functions in each region varies or the reference plane for each port shifts. If the reference planes for all the six ports are fixed at \( y=0 \) and the definition for modal functions and amplitudes of modes as given in Equations (2.18)-(2.35) are followed, then one has

\[
\begin{bmatrix} [A_{ae}^+, [A_{ah}^+, [A_{de}^+, [A_{dh}^+, [A_{be}^+, [A_{bh}^+] \end{bmatrix}^T
\]

\[= [S] \begin{bmatrix} [A_{ae}^-, [A_{ah}^-, [A_{de}^-, [A_{dh}^-, [A_{be}^-, [A_{bh}^-] \end{bmatrix}^T \quad (2.96)
\]

where \([A_{ip}^\pm], (i=a, d, b; p=e, h)\) is a vector containing the corresponding modal amplitudes,

\[ [A_{ip}^\pm] = [A_{ip_1}^\pm, A_{ip_2}^\pm, \ldots, A_{ip_n}^\pm, \ldots] \quad (2.97) \]

and the scattering matrix \([S]\) consists of 6x6 submatrices \([S_{ip,jq}]\), namely,

\[
[S] = \begin{bmatrix}
[S_{1e,1e}] & [S_{1e,1h}] & [S_{1e,2e}] & [S_{1e,2h}] & [S_{1e,3e}] & [S_{1e,3h}] \\
[S_{1h,1e}] & [S_{1h,1h}] & [S_{1h,2e}] & [S_{1h,2h}] & [S_{1h,3e}] & [S_{1h,3h}] \\
[S_{2e,1e}] & [S_{2e,1h}] & [S_{2e,2e}] & [S_{2e,2h}] & [S_{2e,3e}] & [S_{2e,3h}] \\
[S_{2h,1e}] & [S_{2h,1h}] & [S_{2h,2e}] & [S_{2h,2h}] & [S_{2h,3e}] & [S_{2h,3h}] \\
[S_{3e,1e}] & [S_{3e,1h}] & [S_{3e,2e}] & [S_{3e,2h}] & [S_{3e,3e}] & [S_{3e,3h}] \\
[S_{3h,1e}] & [S_{3h,1h}] & [S_{3h,2e}] & [S_{3h,2h}] & [S_{3h,3e}] & [S_{3h,3h}]
\end{bmatrix} \quad (2.98)
where each submatrix
\[
[S_{ip,jq}] = \begin{bmatrix}
S_{ip,jq}(1,1) & S_{ip,jq}(1,2) & \cdots & S_{ip,jq}(1,n) & \cdots \\
S_{ip,jq}(2,1) & S_{ip,jq}(2,2) & \cdots & S_{ip,jq}(2,n) & \cdots \\
\vdots & \vdots & \ddots & \vdots & \ddots \\
S_{ip,jq}(m,1) & S_{ip,jq}(m,2) & \cdots & \vdots & \ddots \\
\end{bmatrix} \tag{2.99}
\]

The dimension of each individual submatrix \([S_{ip,jq}]\) depends on how many evanescent PPMs are retained. Thus, if the ‘n’-th, ‘q’-polarized PPM with amplitude \(A_{j_{qn}}^-\) is incident from region ‘j’, then the scattered field amplitude \(A_{ip_m}^+\) coupled to the ‘m’-th, ‘p’-polarized PPM in region ‘i’ is
\[
A_{ip_m}^+ = S_{ip,jq}(m,n)A_{j_{qn}}^-. \tag{2.100}
\]

After inspecting the results of the six scattering problems, it is found that there exists a certain kind of symmetric relationship which allows all the elements of different scattering submatrices to be expressed in a single compact form instead of listing them case by case; namely,
\[
S_{ip,jq}(m,n) = \Psi_{ip}(m)\mathcal{F}(m,n; ip, jq)\Phi_{jq}(n) \tag{2.101}
\]

where \(i=1, 2, 3; p=e, h; m,n=1, 2, 3,...\) The functions \(\Psi_{ip}(m), \mathcal{F}(m,n; ip, jq)\) and \(\Phi_{jq}(n)\) are given below.

\[
\Psi_{1e}(m) = -D_{be}(-\lambda_{ae_m}) \left[ \frac{d}{d\lambda} D_{ae}(\lambda = -\lambda_{ae_m}) \right]^{-1} \tag{2.102}
\]

\[
\Psi_{1h}(m) = -k_0D_{bh}(-\lambda_{ah_m}) \left[ \frac{d}{d\lambda} D_{ah}(\lambda = -\lambda_{ah_m}) \right]^{-1} \tag{2.103}
\]

\[
\Psi_{2e}(m) = k_0 \left[ \frac{d}{d\lambda} D_{de}(\lambda = \lambda_{de_m}) \right]^{-1} \tag{2.104}
\]

\[
\Psi_{2h}(m) = \left[ \frac{d}{d\lambda} D_{dh}(\lambda = \lambda_{dh_m}) \right]^{-1} \tag{2.105}
\]
\[ \Psi_{3e}(m) = k_0 \left[ \frac{d}{d\lambda} D_{be}(\lambda = \lambda_{bm}) \right]^{-1} \quad (2.106) \]
\[ \Psi_{3h}(m) = \left[ \frac{d}{d\lambda} D_{bh}(\lambda = \lambda_{bh_m}) \right]^{-1} \quad (2.107) \]

where the functions \( D_{ip} \) \((i=a, d, b; p=e, h)\) are given in Equations (2.36)-(2.41) and the derivative \( d/d\lambda \) comes from evaluating the residue at the corresponding poles.

The function \( F(m, n; ip, jq) \) consists of two parts,

\[ F(m, n; ip, jq) = F_1(m, n; ip, jq) F_2(m, n; ip, jq) \quad , \quad (2.108) \]

where the first part \( F_1(m, n; i, j; p, q) \) is the product of two \( Q_+ \) functions,

\[ F_1(m, n; i, j; p, q) = \begin{cases} 
Q_{p+}(-\lambda_{ipm}) & \text{if } i = 2, 3 \\
Q_{q+}(-\lambda_{jqm}) & \text{if } j = 2, 3 \\
\frac{1}{Q_{p+}(-\lambda_{ipm})} & \text{if } i = 1 \\
\frac{1}{Q_{q+}(-\lambda_{jqm})} & \text{if } j = 1
\end{cases} \quad (2.109) \]

where \( Q_{e+} \) and \( Q_{h+} \) are given in Appendix A, and the second part \( F_2(m, n; ip, jq) \) is

\[ F_2(m, n; ip, jq) = \begin{cases} 
\frac{[\lambda_R \lambda_I - \alpha^2 + j\alpha(\lambda_R + \lambda_I)\Omega_1]}{(\lambda_R + \lambda_I)\alpha^2 + \lambda_I^2} & \text{if } p = e, q = e \\
\frac{1}{\eta_0(\alpha^2 + \lambda_I^2)} & \text{if } p = h, q = e \\
-\eta_0\frac{\alpha\Omega_2}{(\alpha^2 + \lambda_I^2)} & \text{if } p = e, q = h \\
\frac{[\lambda_R \lambda_I - \alpha^2 - j\alpha(\lambda_R + \lambda_I)\Omega_1]}{(\lambda_R + \lambda_I)\alpha^2 + \lambda_I^2} & \text{if } p = h, q = h
\end{cases} \quad (2.110) \]

\[ \lambda_R = \begin{cases} 
\lambda_{ipm} & \text{if } i = 2, 3 \\
-\lambda_{ipm} & \text{if } i = 1
\end{cases} \quad ; \quad \lambda_I = \begin{cases} 
\lambda_{jqm} & \text{if } j = 2, 3 \\
-\lambda_{jqm} & \text{if } j = 1
\end{cases} \quad (2.111) \]
where $\Omega_1$ and $\Omega_2$ are defined in Equations (2.94) and (2.95), respectively. The last term of $S_{p,j}(m,n), \Phi_{j}(n)$, is

\[
\begin{align*}
\Phi_{1e}(n) &= \frac{1}{k_0 \varepsilon_{r-1}} \frac{\partial}{\partial z} \psi_{de}(\lambda_{en}, z = D) \\
\Phi_{1h}(n) &= \psi_{dh}(\lambda_{hn}, z = D) \\
\Phi_{2e}(n) &= \psi_{de}(-\lambda_{en}, z = D) \\
\Phi_{2h}(n) &= \frac{1}{k_0 \mu_{r-1}} \frac{\partial}{\partial z} \psi_{dh}(-\lambda_{hn}, z = D) \\
\Phi_{3e}(n) &= -\psi_{be}(-\lambda_{en}, z = D) \\
\Phi_{3h}(n) &= -\frac{1}{k_0 \mu_{r-1}} \frac{\partial}{\partial z} \psi_{bh}(-\lambda_{hn}, z = D)
\end{align*}
\]

where the functions $\psi_{jq}(j = a, b, d; q = e, h)$ are defined in Equations (2.24)-(2.28).

For convenience of incorporation in the GSMT formulation, the half plane edge discontinuity will be viewed as a three port circuit element in Chapter III instead of a six port one. Namely, the $TM_z$ and $TE_z$ polarized PPMs in each region will be grouped together and the region 1, 2 and 3 depicted in Figure 2 will be viewed as terminated by port 1, 2 and 3, respectively. The amplitudes of outgoing and incoming PPMs in each region are renamed as $b_i$ and $a_j$, respectively, as follows.

\[
\begin{align*}
[a_1] &= \begin{bmatrix} A_{ae}^- & A_{ah}^- \end{bmatrix}^T \\
[a_2] &= \begin{bmatrix} A_{de}^- & A_{dh}^- \end{bmatrix}^T \\
[a_3] &= \begin{bmatrix} A_{be}^- & A_{bh}^- \end{bmatrix}^T
\end{align*}
\]

and

\[
\begin{align*}
[b_1] &= \begin{bmatrix} A_{ae}^+ & A_{ah}^+ \end{bmatrix}^T \\
[b_2] &= \begin{bmatrix} A_{de}^+ & A_{dh}^+ \end{bmatrix}^T \\
[b_3] &= \begin{bmatrix} A_{be}^+ & A_{bh}^+ \end{bmatrix}^T
\end{align*}
\]
Thus, Equations (2.96) and (2.98) can be expressed as

\[
\begin{bmatrix}
  [b_1] \\
  [b_2] \\
  [b_3]
\end{bmatrix}
= \begin{bmatrix}
  [S_{11}] & [S_{12}] & [S_{13}] \\
  [S_{21}] & [S_{22}] & [S_{23}] \\
  [S_{31}] & [S_{32}] & [S_{33}]
\end{bmatrix}
\begin{bmatrix}
  [a_1] \\
  [a_2] \\
  [a_3]
\end{bmatrix},
\]

(2.120)

where

\[
[S_{ij}] = \begin{bmatrix}
  [S_{ie,je}] & [S_{ie,jh}] \\
  [S_{ih,je}] & [S_{ih,jh}]
\end{bmatrix}.
\]

(2.121)

The above equation (2.120) will be just simply written as

\[
b_i = \sum_{j=1}^{3} S_{ij} a_j, \ i = 1, 2, 3.
\]

(2.122)

in Chapter III for the sake of brevity.
CHAPTER III
Application of the WH/GSMT Analysis to Planar Transmission Lines

In this chapter, the WH/GSMT based analysis is applied to find the modal propagation constants and other related information, like characteristic impedances, field strengths, etc. associated with planar transmission lines.

The planar transmission line problem to be solved could be very general, where an arbitrary number of PEC strips of infinitesimal thickness are embedded in a multilayered dielectric region, as depicted in Figure 5. The top and bottom PEC planes have to be included since they are contained in the solution to the canonical PEC half plane problem considered in Chapter II. In practical situations, there almost always exists a bottom conducting plane which works as a ground plane, mechanical support, electric shield and a heat sink, etc.. The presence of the top PEC plane will, besides acting as a shield, have only very little effect on the performance of the whole circuit as long as the transmission line does not give rise to space wave radiation and the top PEC plane is put far enough from the dielectric region containing the transmission lines. This point will be discussed in detail in Chapter V.

Since one can by no means be exhaustive in studying all the cases which can be handled by the procedure developed here, the transmission lines commonly used in microwave integrated circuits will be emphasized, namely, microstrip lines, slotlines, coplanar waveguides and finlines. As in the usual rectangular waveguide or coaxial transmission lines, there exists a set of modes together with their modal propagation
constants for the above mentioned planar transmission lines. The latter propagation constants may be real, imaginary or complex numbers for each circuit configuration. Here, one has to be careful not to confuse the transmission line modes with the PPMs where the former ones satisfy all the boundary conditions pertinent to the whole circuit while the latter ones basically serve as basis functions to expand the fields in individual parallel plate regions. If the dielectric medium is assumed to be lossless, then a real or imaganary modal propagation constant corresponds to the propagating or evanescent mode, respectively, for boxed planar transmission lines. Although there are some reports about the complex modes in boxed planar transmission lines, the physical meaning of these modes is till not very clear [29, 30].

For planar transmission lines in an open region or just partly open, as in the cases considered here where top and bottom covers are included but laterally unbounded, a complex propagation constant means that the corresponding mode is propagating and attenuating at the same time because of the energy carried away either by space wave radiation or by surface wave (or PPM) leakage in the transverse directions [33]. Although one can, in principle, find all the possible modal propagation constants pertinent to a particular planar transmission line structure with the WH/GSMT provided that process for searching the roots (or propagation constants) is done carefully, we are only interested here in the modal propagation constants which are either real or complex but with small imaginary part since they are really the important ones from a practical point of view.

One feature of the planar transmission line embedded in multi-layered dielectrics is that it does not allow a TEM propagating mode as for the homogeneous medium case. Nonetheless, the fundamental mode for many of the planar transmission lines is close to a TEM mode, usually called quasi-TEM, when frequency is relatively low.
Therefore, the modal voltage, current and characteristic impedance for a quasi-TEM mode can be defined in a way similar to the TEM mode.

In the first section of this chapter, the results based on the scattering matrix derived in Chapter II for the PEC half plane edge discontinuity are extended to some basic discontinuities which will then be used as building blocks for developing the transverse resonance relation for other geometries. A complete description of how the WH/GSMT works is given in the second section while the transverse relations for some common planar transmission line structures are listed in the third section. Some numerical considerations when applying the WH/GSMT are given in the last section.
3.1 Scattering Matrices for Some Basic Discontinuities

In this section, the scattering matrices are given for some basic discontinuities which are illustrated in Figures 6-8. Note that in Chapter II, the fields are described in terms of a common (or global) coordinate system which is fixed while here the scattering matrices are defined with respect to some local reference planes. However, even though the origin is not specified, the directions of coordinate axes are kept the same as in Figure 2. In particular, \( \hat{x} \) still denotes the longitudinal direction of the transmission line and it is the outward-pointing direction in the following figures while \( \hat{y} \) and \( \hat{z} \) denote the right-handed lateral and upward normal directions, respectively. A variation \( e^{-jk_0x} \) is assumed where \( \alpha \) is a given (fixed) number in this section. All the figures in this section (Figures 6-8) and in section 3.3 (Figures 10-16) are side views where the dotted vertical lines indicate the reference planes for defining the scattering matrices and the amplitudes of the PPMs. For simplicity, the presence of the multi-layered substrate is not shown in Figures 6-8 and Figures 10-16. However, the effects of the multilayers are indeed contained for all the scattering matrices developed here because they are in turn built up from the canonical scattering matrices for the PEC half plane of Figure 2 which were obtained in Chapter II.

3.1.1 The Scattering Matrices \( S^R \) and \( S^L \) for a PEC half-plane

Referring to Figures 6(a) and (b), the \( S^R \) and \( S^L \) are the scattering matrices for the PEC half-plane edge discontinuity when the half-plane is located in the right or left hand side, respectively, within the parallel plate region. With the notation introduced at the end of Chapter II, one can express

\[
b_i = \sum_{j=1}^{3} S_{ij}^R a_j , \quad i = 1, 2, 3 \quad (3.1)
\]
for Figure 6 (a), where $S_{ij}^R$ is nothing but the matrix $S_{ij}$ in Equation (2.122), and

$$b_i = \sum_{j=1}^{3} S_{ij}^L a_j , \quad i = 1, 2, 3$$

(3.2)

for Figure 6 (b). The matrix $S_{ij}^L$ is identical to $S_{ij}^R$ except for the sign difference in the cross terms; namely,

$$S_{ie,je}^L = S_{ie,je}^R , \quad S_{ih,jh}^L = S_{ih,jh}^R , \quad S_{ie,jh}^L = -S_{ie,jh}^R , \quad S_{ih,je}^L = -S_{ih,je}^R .$$

(3.3)

### 3.1.2 The matrices $S_{RW}^R$ and $S_{LW}^R$ for a PEC half-plane terminated by a PEC or PMC wall

The $S_{RW}^R$ and $S_{LW}^R$ are the scattering matrices when the PEC half plane inside the parallel plate is terminated by a PEC or PMC wall in the right or left, respectively, as depicted in Figure 7(a) and (b). In Figure 7(b),

$$b_1 = S_{LW}^R(s,w)a_1 ,$$

(3.4)

where

$$S_{LW}^R(s,w) = T_a(s)S_{LW}^R(0,w)T_a(s) ,$$

(3.5)
The function $T_a(s)$ is a diagonal modal propagation delay matrix with the diagonal terms equal to the phase change of each PPM due to the shift of the reference plane by a distance $s$,

$$T_a(s) = \begin{bmatrix}
ed^{-jk_0\lambda_{ae_1}s} & 0 & 0 & \cdots & \cdots & \cdots \\
0 & ed^{-jk_0\lambda_{ae_2}s} & 0 & 0 & \cdots & \cdots \\
\cdots & 0 & \cdots & 0 & 0 & \cdots \\
\cdots & 0 & 0 & ed^{-jk_0\lambda_{ah_1}s} & 0 & 0 \\
\cdots & \cdots & 0 & 0 & ed^{-jk_0\lambda_{ah_2}s} & 0 \\
\cdots & \cdots & \cdots & 0 & 0 & \cdots 
\end{bmatrix} \quad (3.6)$$

In the following, $T$ will be used to denote this kind of translation matrix while the subscript $a$, $b$ or $d$ will denote the region $i$ to which $\lambda_{ip_n}$ corresponds.

$$S_{LW}(0, w) = S_{11}^L + \left[S_{12}^L R_2, S_{13}^L R_3\right] \begin{bmatrix}
I - S_{22}^L R_2, & -S_{23}^L R_3 \\
-S_{32}^L R_2, & I - S_{33}^L R_3
\end{bmatrix}^{-1} \begin{bmatrix}
S_{21}^L \\
S_{31}^L
\end{bmatrix} \quad (3.7)$$

where $I$ is the identity matrix while $R_2$ and $R_3$ are the reflection matrices from the walls. Both $R_2$ and $R_3$ are diagonal matrices with diagonal terms of the form

$$ed^{-jk_02\lambda_{ip_n}w} \cdot \begin{cases}
-1 \\
+1
\end{cases} ; \quad p = e, h, \quad n = 1, 2, 3, ... \quad (3.8)$$

$i = d, b$ for $R_2$ and $R_3$, respectively; $-1(1)$ is picked when the corresponding PPM is TM$_z$ polarized and the terminating wall is a PEC (PMC) or when that PPM is TE$_z$ polarized and the terminating wall is a PMC (PEC). As one can see, $S_{LW}(0, w)$ consists of two parts: the first part $S_{11}^L$ is the direct reflection from the leading edge of the PEC half plane while the second part takes into account the wave interactions between the edge and the side wall.
Figure 7: A half plane terminated by a PEC or PMC wall

If all $S_{ij}^L$ in (3.7) are replaced by $S_{ij}^R$, then $S_{RW}(0, w)$ will be obtained. Furthermore, $S_{RW}(s, w)$ is given by (3.5) upon replacing the $S_{LW}(0, w)$ therein by $S_{RW}(0, w)$.

3.1.3 The scattering matrix $S^{AA}$ for an iris

The iris is shown in Figure 8. If one defines [2]

$$
\begin{bmatrix}
  b_1 \\
  b_4
\end{bmatrix} = S^{AA}(s_1, w, s_2)
\begin{bmatrix}
  a_1 \\
  a_4
\end{bmatrix}
$$

then

$$
S^{AA}(s_1, w, s_2) = \begin{bmatrix}
  T_a(s_1) & 0 \\
  0 & T_a(s_2)
\end{bmatrix} S^{AA}(0, w, 0) \begin{bmatrix}
  T_a(s_1) & 0 \\
  0 & T_a(s_2)
\end{bmatrix}
$$

(3.10)

where

$$
S^{AA}(0, w, 0) = \begin{bmatrix}
  S_{11}^{AA}(0, w, 0), & S_{14}^{AA}(0, w, 0) \\
  S_{41}^{AA}(0, w, 0), & S_{44}^{AA}(0, w, 0)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
  \tilde{S}_{12} & \tilde{S}_{13} \\
  S_{12}^L & S_{13}^L
\end{bmatrix} \begin{bmatrix}
  I - \tilde{S}_{22}, & -\tilde{S}_{23} \\
  -\tilde{S}_{32}, & I - \tilde{S}_{33}
\end{bmatrix}^{-1} \begin{bmatrix}
  S_{21}^{R'}, \tilde{S}_{21} \\
  S_{31}^{R'}, \tilde{S}_{31}
\end{bmatrix} + \begin{bmatrix}
  S_{11}^{R'}, \tilde{S}_{11} \\
  0, & S_{11}^L
\end{bmatrix},
$$

(3.11)
Figure 8: The iris

\[
\tilde{S}_{ij} = \sum_{k=2}^{3} S_{ik}^{R'} S_{kj}^{L'},
\]

(3.12)

\(S^{R'}\) and \(S^{L'}\) are just \(S^{R}\) and \(S^{L}\) after being shifted by \(w/2\); namely,

\[
S^{R'} = T S^{R} T, \quad S^{L'} = T S^{L} T
\]

(3.13)

and

\[
T = \begin{bmatrix}
I & 0 & 0 \\
0 & T_d(w/2) & 0 \\
0 & 0 & T_b(w/2)
\end{bmatrix}
\]

(3.14)

Although it is not obvious in (3.11), the matrices \(S_{11}^{AA}(0, w, 0)\) and \(S_{44}^{AA}(0, w, 0)\) are identical except for the sign difference in the cross terms, and this is likewise true of \(S_{14}^{AA}(0, w, 0)\) and \(S_{41}^{AA}(0, w, 0)\). \(S^{AA}(0, w, 0)\) can also be written in a more symmetric form, but the matrix in (3.11) takes less computer time to calculate.

3.2 The WH/GSMT Scheme

There are two key steps in the WH/GSMT. First, one finds the scattering matrix of a single edge by solving a canonical problem with the Wiener-Hopf technique, as is done in Chapter II. In this part, the normalized propagation constant \(\alpha\) is treated as
a fixed parameter temporarily. Then, in the second step, one develops the transverse resonance relation based on the generalized scattering matrix technique where the cross section of the transmission line is viewed as a network consisting of many pieces of different transmission lines. The transverse resonance condition yields a system of linear homogeneous equations where the coefficients of these equations are functions of \( \alpha \) and the amplitudes of the PPMs in different regions are unknowns in these equations. To have a non-trivial solution to the simultaneous equations, the determinant of the coefficient matrix must vanish. Thus, the modal propagation constant of the transmission line, \( \alpha \), is obtained by searching for the value of \( \alpha \) which makes the determinant equal to zero. From a physical point of view, this amounts to finding suitable modal ray launching angles of the PPMs such that the PPM waves bouncing back and forth between edges can build a constructive interference (or resonance). The corresponding eigenvector yields the amplitudes of the PPMs in different regions for this transmission line mode.

The flowchart of the WH/GSMT scheme is given in Figure 9 where \( \alpha \) is the unknown modal propagation constant to be found. First, one has to search for the zeros of \( D_{ip} \) (i=a, b, d; p=e, h) defined in Equations (2.36)-(2.41) by treating the term \( \alpha^2 + \lambda^2 \) as a single variable. Then pick an initial guess for \( \alpha \) and calculate \( Q_e + (-\lambda_{ipm}) \), \( Q_h + (-\lambda_{ipm}) \), i=a, b, d; p=e, h, m=1,2,3,... and the basic scattering matrices \( S^R \) and \( S^L \). The next step is to build the transverse resonance relation and calculate \( Det[\bar{\Omega}(\alpha)] \) where \( \bar{\Omega}(\alpha) \cdot \bar{a} = \bar{0} \) symbolically denotes the transverse resonance relation and \( \bar{a} \) denotes the vector consisting of unknown amplitudes of the PPMs. If \( Det[\bar{\Omega}(\alpha)] = 0 \), then that value of \( \alpha \) is the desired modal propagation constant of the transmission line, otherwise one has to keep trying other values of \( \alpha \) till the proper value of \( \alpha \) is found. Other related information like characteristic impedance

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Figure 9: The flowchart for finding the propagation constant $\alpha$. 

- **search roots of** $D_{ip}(a^2 + \lambda^2)$
- **pick initial value** $a$
- **calculate** $S^R(a), S^L(a)$
- **calculate** $Q_{e+}, Q_{h+}$
- **transverse resonance relation** $\bar{T}(a) \cdot \bar{a} = 0$
  
  \{ SMS, CPW, CPW_FG, ..., FIL \}

- **Det[\bar{T}(a)] = 0?**
  - **yes** 
    - $Z_0$, field
and the field distribution for that transmission line mode can be calculated after the propagation constant $\alpha$ is obtained.

In the WH/GSMT analysis, the only thing that needs to be changed for different transmission line structures is the transverse resonance relation if the substrate structure is the same. The transverse resonance relation can be developed in a brute force manner for a general transmission line as depicted in Figure 5. One just assumes the amplitudes of the PPMs in each region as unknowns, then the scattering matrices corresponding to all the discontinuities will prescribe relations among these PPM amplitudes and yield a system of linear homogeneous equations. However, the dimension of the resulting matrix is relatively large if no simplification is done at all. This may cause numerical inefficiency and instability. For some common planar microwave/millimeter wave transmission lines such as a single microstrip line (SMS), a single slotline (SSL), or a single conductor-backed coplanar waveguide (SCPW), etc., the transverse resonance relation $\bar{T}(\alpha) \cdot \bar{a} = 0$ can be greatly simplified by eliminating some unknowns from the system of linear equations and thus reducing the size of the matrix $\bar{T}(\alpha)$. These transverse resonance relations are listed in the next section.

3.3 The Transverse Resonance Relations for Some Common Planar Transmission Lines

3.3.1 Analysis of a single microstrip line (SMS)

The configuration of a SMS is depicted in Figure 10(a). For Figure 10(a), the transverse resonance condition is achieved when

$$
\begin{bmatrix}
I - \tilde{S}_{22}, & -\tilde{S}_{23} \\
-\tilde{S}_{32}, & I - \tilde{S}_{33}
\end{bmatrix}
\begin{bmatrix}
a_2 \\
a_3
\end{bmatrix}
= 0
$$

(3.15)

where $\tilde{S}_{ij}$ is defined in Equation (3.12).
Figure 10: The single microstrip line (SMS)

Due to the symmetry of the structure, the even or odd modes can be found separately by including a PMC or PEC wall bisection, respectively, which is placed in the middle of the original SMS, as shown in Figure 10(b). In this case, the resonance condition is achieved when

\[
\begin{bmatrix}
1 - S_{22}^R R_2, & -S_{23}^R R_3 \\
-S_{32}^R R_2, & 1 - S_{33}^R R_3
\end{bmatrix}
\begin{bmatrix}
a_2 \\
a_3
\end{bmatrix} = 0
\]

(3.16)

where \( R_2 \) and \( R_3 \) are similar to what is defined Equation (3.8) except the phase delay is now for a distance \( w/2 \) rather than \( w \).

3.3.2 Analysis of a boxed single microstrip line (SMS_WW)

The SMS_WW geometry is shown in Figure 11. For the bisected version of this geometry which is depicted in Figure 11 (b), the transverse resonance relation can be written in two different forms; the first one is given by

\[
\left\{
\left[
\begin{bmatrix}
S_{22}^R & S_{23}^R \\
S_{32}^R & S_{33}^R
\end{bmatrix}
\right] + \left[
\begin{bmatrix}
S_{21}^R \\
S_{31}^R
\end{bmatrix}
\right] R_1 \left(1 - S_{11}^R R_1\right)^{-1} \left[
\begin{bmatrix}
S_{12}^R & S_{13}^R
\end{bmatrix}
\right] + \left[
\begin{bmatrix}
R_2 \\
0
\end{bmatrix}
\right]
\right) = 0
\]

(3.17)
where $R_2$ and $R_3$ are similar to what is defined Equation (3.8) except that $w$ is replaced by $w/2$. $R_1$ is a diagonal matrix which takes into account the reflection from the PEC wall on the left hand side and the phase shift for the round trip distance $2s$. The second form of the transverse resonance relation is

$$[I - S^{RW}(0,w/2)R_1]b_1 = 0,$$  \hspace{1cm} (3.18)\

where $S^{RW}(0,w/2)$ was defined previously.

The second form given in Equation (3.18) looks more compact than the first one given in (3.17). However, a more compact formula does not necessarily require a smaller amount of computer execution time, although it does provide a more systematic and structured form which is easier to code. The essential difference in these two transverse resonance forms lies in the matrix that needs to be inverted. One has to make sure that the matrix to be inverted is far from singular, otherwise it may cause numerical instability. Although the inverse operation is theoretically valid as long as the matrix is not exactly singular, it may nevertheless cause some numerical problems when taking the inverse of a nearly singular matrix. From a physical point of view, it is important to know how many possible feedback loops the whole circuit contains, and how the resonance condition is achieved for each
case. Take the SMS_WW as an example, the side walls modify the SMS modes corresponding to the circuit without side walls as well as producing extra parasitic waveguide modes. For these modified microstrip line modes, most of the fields concentrate under the strip. If the second relation in Equation (3.18) is used, the term

\[
\begin{bmatrix}
I - S_{22}^R R_2 & -S_{23}^R R_3 \\
-S_{32}^R R_2 & I - S_{33}^R R_3
\end{bmatrix}
\]

contained in \( S_{RW}(0, w/2) \) might be close to being singular. Thus, if the mode of interest is a microstrip line mode, the first resonance equation in (3.17) is preferred. However, the second resonance form can work well for parasitic waveguide modes associated with the outside shielding box.

3.3.3 Analysis of coupled microstrip lines (CMS)

For closely coupled microstrip lines as shown in Figure 12(a), the transverse resonance relation may be written as follows.

\[
[I - S_{44}^{AA}(0, w_1, s/2)S_{11}^{AA}(s/2, w_2, 0)] b_4 = 0 .
\]

(3.19)

If the two coupled lines are identical, that is, \( w_1 = w_2 = w \), then the problem reduces to a bisected version as shown in Figure 12(b). In this case, if the coupling is strong enough, usually this condition is satisfied, then the resonance condition is achieved when

\[
[I - S_{44}^{AA}(0, w, 0)R_4] b_4 = 0 ,
\]

(3.20)

where \( R_4 \) is a diagonal matrix which takes into account the reflection from the PEC or PMC wall to the right and the phase change for the round trip distance \( 2(s/2) \).

However, if an extremely weakly coupled case is considered where most of the fields concentrate under each individual strip instead of the slot region between the
Figure 12: The coupled microstrip lines (CMS)

two lines, then a transverse resonance relation

\[
\begin{bmatrix}
S_{22}^{R'}, S_{23}^{R'} \\
S_{32}^{R'}, S_{33}^{R'}
\end{bmatrix}
\begin{bmatrix}
S_{22}' \\
S_{23}' \\
S_{32}' \\
S_{33}'
\end{bmatrix}
= \begin{bmatrix}
S_{11}' \\
S_{12}'
\end{bmatrix}
\begin{bmatrix}
R_4 \\
I - S_{11}' R_4
\end{bmatrix}^{-1}
\begin{bmatrix}
S_{12}' \\
S_{13}'
\end{bmatrix}
\]

\[
-\begin{bmatrix}
a_2 \\
a_3
\end{bmatrix}
= 0
\]  

(3.21)

is preferred where \(S^{R'}\) and \(S^{L'}\) are the same as defined in Equations (3.13) and (3.14), respectively.

### 3.3.4 Analysis of a single slotline (SSL)

The transverse resonance relation for the SSL in Figure 13 can be written as

\[
[I - Ta(s/2)S_{11}^{R}Ta(s)S_{11}^{L}Ta(s/2)] a_1 = 0.
\]  

(3.22)

The even or odd mode for SSL can be obtained by putting a PMC or PEC wall bisection at the middle of the slot, respectively.
3.3.5 Analysis of a single coplanar waveguide (SCPW)

For an asymmetric SCPW with different slot width $s_1$ and $s_2$ as depicted in Figure 14(a), the transverse resonance condition is achieved when

\[
\begin{bmatrix}
I - S_{11}^{AA}(s_1/2, w, s_2/2)T_a(s_1/2)S_{11}^L T_a(s_1/2) \\
-S_{44}^{AA}(s_1/2, w, s_2/2)T_a(s_1/2)S_{11}^R T_a(s_1/2) \\
-S_{14}^{AA}(s_1/2, w, s_2/2)T_a(s_2/2)S_{11}^R T_a(s_2/2) \\
I - S_{44}^{AA}(s_1/2, w, s_2/2)T_a(s_2/2)S_{11}^L T_a(s_2/2)
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_4
\end{bmatrix} = 0. \quad (3.23)
\]

For a SCPW with equal slot width $s$, only half of the geometry, as shown in Figure 14(b), needs to be considered. The resonance relation for this bisection case is

\[
\begin{bmatrix}
I - S_{11}^{RW}(s/2, w/2)T_a(s/2)S_{11}^L T_a(s/2)
\end{bmatrix}
\begin{bmatrix}
b_1
\end{bmatrix} = 0. \quad (3.24)
\]

Placing a PMC wall bisection at the middle yields the normal CPW mode while a PEC wall yields the so called slotline mode.

Both of the above two resonance relations work well as long as $s$ and $w$ are not too large. This condition is satisfied in most practical situations.
3.3.6 Analysis of a single coplanar waveguide with finite lateral ground planes (SCPW_FG)

The configuration is shown in Figure 15. Besides the two modes corresponding to CPW and slotline modes which exist when there are infinitely large lateral ground planes, there is one additional mode, called coplanar microstrip (CPM) mode. Assume that $s_1 = s_2 = s$ and $w_1 = w_2$ such that only the bisected configuration needs to be considered. Two kinds of resonance relations can be obtained for this case; namely,

$$[I - S^{RW}(s/2, w/2)S_{44}^{AA}(0, w_1, s/2)]b_1 = 0$$  \hspace{1cm} (3.25)

and

$$\left\{ \begin{array}{l}
S_{22}' , S_{23}' \\
S_{32}' , S_{33}' \\
S_{32}' , S_{33}' \\
\end{array} \right\} \left( \begin{array}{l}
S_{22}' , S_{23}' \\
S_{32}' , S_{33}' \\
\end{array} \right) \left[ \begin{array}{l}
S_{21}' \\
S_{31}' \\
\end{array} \right] S^{RW}(s, w/2) \left( I - S_{11}' S^{RW}(s, w/2) \right)^{-1}$$

\[ \cdot \left[ S_{12}' , S_{13}' \right] - I \right\} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = 0 , \hspace{1cm} (3.26)\]

where $S^{R'}$ and $S^{L'}$ are the same as defined in Equations (3.13) and (3.14), respectively, except that $w/2$ is replaced by $w_1/2$. Theoretically, the first resonance relation given in (3.25) is suited for the CPW mode while the second one in (3.26) is preferred.
for the CPM mode. However, since \( s \) and \( w \) are small in practical situations, either one can work well for the CPW or CPM mode.

### 3.3.7 Analysis of a finline (FIL)

There are many different kinds of finlines. Figure 16(a) shows a configuration which could represent an antipodal finline without overlapping, or a unilateral finline. The transverse resonance condition for this case is obtained when

\[
[I - S_{LW}(s/2, w_1)S_{RW}(s/2, w_2)] b_1 = 0 .
\] (3.27)

Notice that two sets of parameters \( \lambda_{ipn} \), \( Q_{e+} \), \( Q_{h-} \), and half-plane scattering matrices \( S^R, S^L \) have to be calculated if the two fins are not located in the same horizontal plane.

Figure 16(b) is an overlapped antipodal finline. Denote the upper fin as fin U and the lower one as fin V. Note that the overlapped region corresponds to region 2 for fin U while it corresponds to region 3 for fin V. For the upper fin U, the equivalent reflection matrix \( R_{UE} \) is defined as follows.

\[
b_2 = R_{UE} a_2
\] (3.28)
Figure 16: The non-overlapping finline (FIL_NOVLP) and overlapping finline (FIL_OVLP)

where

\[
R_{UF} = S_{22}^{U} + \begin{bmatrix} S_{21}^{U} R_{1}^{U}, S_{23}^{U} R_{3}^{U} \end{bmatrix} \begin{bmatrix} I - S_{11}^{U} R_{1}^{U}, -S_{13}^{U} R_{3}^{U} \\ -S_{31}^{U} R_{1}^{U}, I - S_{33}^{U} R_{3}^{U} \end{bmatrix}^{-1} \begin{bmatrix} S_{12}^{U} \\ S_{32}^{U} \end{bmatrix}. \quad (3.29)
\]

The subscript \( U \) (\( V \)) is used to denote the scattering matrices pertinent to fin \( U \) (\( V \)).

\( R_{1}^{U} \) and \( R_{3}^{U} \) are diagonal reflection matrices for port 1 and 3 of fin \( U \) with the round trip phase shift \( 2(w_{2} - s) \) and \( 2w_{1} \), respectively. For the lower fin \( V \), the equivalent reflection matrix \( R_{VF} \) is defined as

\[
b_{6} = R_{VF} a_{6} \quad (3.30)
\]

where

\[
R_{VF} = S_{33}^{V} + \begin{bmatrix} S_{31}^{V} R_{1}^{V}, S_{32}^{V} R_{2}^{V} \end{bmatrix} \begin{bmatrix} I - S_{11}^{V} R_{1}^{V}, -S_{12}^{V} R_{2}^{V} \\ -S_{21}^{V} R_{1}^{V}, I - S_{22}^{V} R_{2}^{V} \end{bmatrix}^{-1} \begin{bmatrix} S_{13}^{V} \\ S_{23}^{V} \end{bmatrix}. \quad (3.31)
\]

and \( R_{1}^{V} \) and \( R_{2}^{V} \) are diagonal reflection matrices for port 1 and 2 of fin \( V \) with round trip phase shift \( 2(w_{1} - s) \) and \( 2w_{2} \), respectively.

When considering the relation between \( b_{2} \) and \( a_{6} \), or similarly, between \( b_{6} \) and \( a_{2} \), an additional transformation matrix has to be included besides the phase change.
for the overlapping distance $s$ because the overlapped region is viewed as region 2 of fin $U$ but region 3 of fin $V$; namely,

$$a_6 = T_b^V(s)X_{bd}b_2,$$  
(3.32)

$$a_2 = T_d^U(s)X_{db}b_6,$$  
(3.33)

where the transformation matrices $X_{bd}$ and $X_{db}$ are diagonal matrices,

$$X_{bd} =$$

$$\begin{bmatrix}
\psi_{de}(\lambda_{de_1}, h) & 0 & 0 & \cdots & \cdots & \cdots \\
0 & \psi_{de}(\lambda_{de_2}, h) & 0 & 0 & \cdots & \cdots \\
0 & 0 & \ddots & 0 & 0 & \cdots \\
\cdots & 0 & 0 & \psi_{dh}(\lambda_{dh_1}, h) & 0 & 0 \\
\cdots & \cdots & 0 & 0 & \psi_{dh}(\lambda_{dh_2}, h) & 0 \\
\cdots & \cdots & \cdots & 0 & 0 & \ddots \\
\end{bmatrix}$$

(3.34)

and

$$X_{db} = X_{bd}^{-1}$$  
(3.35)

where $h$ is the height of the overlapped part, as shown in Figure 16(b), $T_d^U(s) = T_b^V(s)$ are diagonal matrices corresponding to the phase change for the overlapped width $s$, and $\psi_{de}, \psi_{dh}$ are functions $\psi_{de}, \psi_{dh}$ as defined in Equations (2.24) and (2.25) but with respect to fin $U$.

The transverse resonance condition of the overlapped finline is achieved when

$$[I - T_d^U(s)X_{db}R_{VE}T_b^V(s)X_{bd}R_{UE}]a_2 = 0.$$  
(3.36)

### 3.4 Numerical Considerations

The following considerations are important in the numerical evaluation of various functions and parameters when applying the WH/GSMT to solve planar transmission line problems.
1. As mentioned in Chapter II, when searching for the propagation constants of the PPMs, we search for the values of $\alpha^2 + \lambda^2$ which make $D_{ip}(\alpha) = 0$ rather than the $\lambda$ which makes $D_{ip}(\alpha) = 0$ while keeping $\alpha$ fixed because $\alpha^2 + \lambda^2$ always appears together as a term in the function $D_{ip}(\alpha)$, $(i=a, b, d; p=e, h)$. Doing so not only saves the computer execution time since we do not repeat the root searching process for different $\alpha$, but also brings some important numerical advantage. The reason is as follows. Although $D_{ip}(\alpha)$ never becomes singular no matter what values $\alpha$ and $\lambda$ are assumed, its value oscillates rapidly because of the trigonometric functions $\sin$ and $\cos$ involved. This makes the Newton-Raphson root searching process difficult to apply since precise initial guesses for the roots have to be given in order not to miss any root. However, for lossless materials, one can see from the definition of $D_{ip}(\alpha)$ that $D_{ip}(\alpha)$ are always real valued if $\alpha^2 + \lambda^2$ is real and, more important, the values $\alpha^2 + \lambda^2$ which can make $D_{ip}(\alpha) = 0$ are always real [31]. This useful property makes the root searching of $D_{ip}(\alpha)$ for lossless materials as simple as that for real valued functions with real arguments. In the codes, the bisection method [28] is applied to search the roots which turns out to be much more stable than the Newton-Raphson method.

If the dielectrics are lossy, the zeros as well as $D_{ip}(\alpha)$ become complex valued. In this case, the zeros for the lossless case can be used as an initial guess and then the Newton-Raphson method is applied to accurately locate the roots.

2. The $Q_{e+}(\lambda)$ and $Q_{h+}(\lambda)$ given in Appendix A involve some infinite products $P_{ip}(\lambda)$ $(i=a, d, b; p=e, h)$ which have to be truncated somewhere numerically. To determine whether the number of terms used is enough or not, the relative error functions $E_e$ and $E_h$ are defined for $Q_{e+}(\lambda)$ and $Q_{h+}(\lambda)$, respectively as
follows.

\[
E_e(\lambda; N_{ae}, N_{be}, N_{de}) = \frac{|Q_{e+}(\lambda; N_{ae}, N_{be}, N_{de}) Q_{e+}(-\lambda; N_{ae}, N_{be}, N_{de}) - Q_e(\lambda)|}{|Q_e(\lambda)|}
\]

(3.37)

and

\[
E_h(\lambda; N_{ah}, N_{bh}, N_{dh}) = \frac{|Q_{h+}(\lambda; N_{ah}, N_{bh}, N_{dh}) Q_{h+}(-\lambda; N_{ah}, N_{bh}, N_{dh}) - Q_h(\lambda)|}{|Q_h(\lambda)|}
\]

(3.38)

where \(N_{ip} (i=a, d, b; p=e, h)\) is the number of terms retained for calculating the product \(P_{ip}\). The \(Q_e(\lambda)\) and \(Q_h(\lambda)\) are given in Equations (2.74) and (2.76), respectively; they are independent of \(N_{ip}\). Note that the argument \(\lambda\) can not be too close to any of \(\pm \lambda_{ipn}\), otherwise some of the \(Q\) functions will blow up.

After performing some numerical tests, the following things are observed. Usually, for a fixed \(\alpha\), the error \(E_e\) and \(E_h\) will increase for the same number of \(N_{ip}\) as the magnitude of \(\lambda\) increases. Also, as the electrical heights of the parallel plate waveguides increase, more \(N_{ip}\) terms are required in general to achieve the same accuracy for \(E_e\) and \(E_h\). Although a relative convergence phenomenon is not particularly observed [26], the rule which is required for convergence if the mode matching method is applied, namely, that \(N_{bp}/B \sim N_{dp}/D \sim N_{ap}/A; p=e, h\), is still recommended. Usually, the relative errors \(E_e\) and \(E_h\) first drop quite fast as the number of terms \(N_{ip}\) increases, then the errors reach stable values. After a certain point, keeping more and more terms just will not help to reduce the errors. Sometimes, the errors \(E_e\) and \(E_h\) may even increase dramatically if a very large number of terms are used, because all the numbers dealt with are too large to be calculated accurately by the computer.
In conclusion, the number of terms \( N_{ip} \), which plays a role similar to the end points for the infinite integral in the spectral domain approach, are not very critical as long as they large enough. For most of the cases considered here, 40 to 80 terms of \( N_{ip} \) are enough [48]. However, before calculating anything else, one should always check the relative error for the largest \(|\lambda|\) to be used and increase \( N_{ip} \) till \( E_e \) and \( E_h \) reach relatively stable values.

3. The scattering matrix for the canonical PEC half plane problem contains a term \( \alpha^2 + \lambda_j^2 \) in the denominator (refer to Equation (2.110)). Therefore, if a zero of \( \alpha^2 + \lambda^2 \) in \( D_{ip} \) happens to be very close to 0, then one has to be careful. Actually, the fields of a PPM with \( \alpha^2 + \lambda^2 = 0 \) are rather peculiar; namely, there is neither an \( E_z \) nor an \( H_z \) component for this case. Thus the present PPM can not be simply classified as a TM\( z \) or TE\( z \) polarized one. Furthermore, the tangential electric field \( E_t \) associated with this PPM is always parallel to the tangential magnetic field \( H_t \) for every point in the space, which means the Poynting vector is always equal to zero, and neither real power nor active power is involved. The canonical scattering problem when the incident wave is such a peculiar PPM is still under investigation.

In practice, we can either adjust the parameters to move the term \( \alpha^2 + \lambda_j^2 \) away from the origin or just interpolate the neighboring results and avoid that troublesome point for which \( \alpha^2 + \lambda^2 = 0 \) to totally bypass the problem. It should be quite safe as long as all the zeros \( \alpha^2 + \lambda_j^2 \) of the PPMs have magnitudes greater than 0.1.

One thing worth mentioning is that although some of the elements of the scattering matrix for the canonical half plane problem will blow up when \( \alpha^2 + \lambda_j^2 \) goes to 0, the functions \( Q_{e+}(\lambda) \) and \( Q_{h+}(\lambda) \) are not affected and remain well
behaved. Therefore, if the fields of the trouble-causing mode are so small that they do not need to be included in the GSMT formulation, then the computer codes can still work without any need for additional corrective measures.

4. It is desirable to find a good initial guess of the transmission line modal propagation constant $\alpha$. The results based on the quasi TEM assumption or other approximate models can serve as a rough estimate for this purpose. For the lossless dielectrics, a sweep of $\alpha$ from a maximum of $\sqrt{\varepsilon_r \mu_r}$ to a minimum of $\sqrt{\varepsilon_r \mu_r}$ (i=-M, -M+1,..., -1, 1, 2,..., N) can give us all the bounded propagating modes, including the fundamental and some possible higher order ones. For a leaky mode where $\alpha$ is complex, we can increase the frequency to make the leakage vanish, if possible, to get a rough estimate of the real part of $\alpha$ and then trace back to find the complex $\alpha$. A contour plot of $\text{Det}[^{\alpha}(\alpha)]$ for some complex $\alpha$ region can also give us some idea of the initial guess of $\alpha$.

The Newton-Rapson method is used to locally search for $\alpha$ after picking an initial value of $\alpha$. In general, both the determinant of the matrix for the transverse resonance relation $\text{Det}[^{\alpha}(\alpha)]$ and $\alpha$ are complex. Although it is known theoretically that the value of $\alpha$ which makes $\text{Det}[^{\alpha}(\alpha)]$ equal to zero is the modal propagation constant of that transmission line, it turns out in practice that the $\text{Det}[^{\alpha}(\alpha)]$ rarely goes to 0 exactly; this means that a threshold has to be set. If the threshold set is too large, it may yield an $\alpha$ that is not very accurate; whereas a threshold that is too small may result in a situation where a propagation constant is never found. If everything works fine, the root searching algorithm should converge within 20 iterations and often much faster. Somehow, bounded modes usually converge faster than leaky modes. Also, $\text{Det}[^{\alpha}(\alpha)]$ behaves much worse if $\alpha$ is very close to one of the zeros.
\(\alpha^2 + \lambda^2_{ipn}\) of the \(D_{ip}(\lambda)\), or equivalently, one of the \(\lambda_{ipn}\) is very close to zero. It means that there is one PPM at the boundary of propagating and evanescent regions in the lateral directions.

5. In the WH/GSMT, the PPMs are used as basis functions to expand the fields in each parallel plate region. To determine how many evanescent PPMs are required for inclusion in the generalized scattering matrix formulation, one can estimate from the factor \(e^{-jk_{0}\lambda_{ipn}W}\) where \(W\) is the lateral dimension between two discontinuities and the asymptotic behavior of \(\lambda_{ipn}\) is given in (A.7). If the fields associated with that PPM has decayed enough before reaching another edge, then it can be neglected. The larger the lateral dimension is, the fewer evanescent PPMs are required. The criterion \(N \geq H/W\) can usually be applied where \(N\) is the number of evanescent PPMs required and \(H\) is the height of the corresponding parallel plate waveguide. In contrast to the spectral domain approach where narrower lines or slots are preferred since less basis functions are required, the WH/GSMT is more suitable for a transmission line with larger lateral dimensions. The heights of the parallel plate waveguides also play a crucial role when determining the number of the evanescent PPMs. How far the top cover should be placed such that the whole circuit can simulate an open region problem will be studied in chapter V.

6. In order to find the characteristic impedance of the transmission line, one has to calculate the transmission line modal current, voltage and power flow through the cross section. Usually the calculation of the transmission line modal power is quite time consuming because the fields at the whole cross section have to be known and summed together. For the subroutine developed
here to calculate the power flow, the integration along the lateral (y) direction is written into closed from expressions and only the integration along the normal (z) direction is done numerically. This saves lots of computer execution time. The transmission line modal voltage and current are not uniquely defined for non-TEM fields. Here, following the conventional definitions, the modal transmission line current only includes the longitudinal current while the path of integration when calculating the transmission line modal voltage will be specified case by as in Chapter IV.
CHAPTER IV
Numerical Results and Discussions

Some numerical results based on the WH/GSMT developed in Chapter III are presented in this chapter for various planar microwave/millimeter wave transmission lines. First, the accuracy of the WH/GSMT method is assessed by comparing the results based on this method with those available in the literature. Since there are many published results based on other independent approaches which can serve as references for checking the validity of the scheme developed here, the cases we pick not only can verify the accuracy of the WH/GSMT and show the versatility of the method, but also simultaneously illustrate some interesting physical phenomena. The numerical results for these examples, including the single microstrip line (SMS), single slotline (SSL), coupled microstrip lines (CMS) and antipodal finline, will be given in the first section of this chapter. After initially assessing the validity of the WH/GSMT in the first section, we will concentrate on the results for the conductor-backed coplanar waveguide (CBCPW) in the second section. The dispersion and lateral power leakage of a CBCPW on one or two layered substrate and the effect of the lateral ground planes when they are of finite extent will be emphasized.

In this chapter, the quantity $\alpha$, which in general is a complex number, still denotes the normalized propagation constant of the transmission line, while $Re[\alpha]$ and $Im[\alpha]$ denote the normalized phase and attenuation constant, respectively. Note that for the sake of convenience, the sign of $Im[\alpha]$ is neglected in the results shown here.
4.1 Results Compared With References

4.1.1 Example I: SMS

A wide SMS which was previously studied by Ermert [32, 33] and Oliner et al. [5, 23] is considered in the first example. $Re[\alpha]$ and $Im[\alpha]$ for this case are plotted as functions of frequency in Figures 17 and 18 where the curves for $N=0, 1$ and 2 correspond to the dominant, first and second higher order mode, respectively. The dashed line in Figure 17 is the normalized phase constant of the lowest order PPM between the top and bottom PEC planes, which will become the $TM_0$ surface wave mode when the top cover is removed. The two dotted lines correspond to values of $\sqrt{\varepsilon_{r1}}$ and $\sqrt{\varepsilon_{r2}}$, respectively. The dashed line and the $\sqrt{\varepsilon_{r2}}$ dotted line are the thresholds of surface wave leakage and space wave radiation, respectively when the top cover is removed. As shown in the figures, the dominant mode is bounded for all frequencies since most of the fields for this mode are concentrated under the strip and thus have a higher effective dielectric constant. As the frequency increases, $Re[\alpha]$ of this mode will approach the limit $\sqrt{\varepsilon_{r1}}$. The first and second higher order modes will have power leakage in the lateral directions when their phase velocities are faster than that of the PPM; the cut-off frequency for these higher order modes is around 14.5 GHz and 29.0 GHz, respectively.

The curves for $Re[\alpha]$ shown in Figure 17 agree with [23] where a simplified model is used, while $Im[\alpha]$ given in Figure 18 tends to be larger than the one given in [23]. Note that it is usually more difficult to achieve the same accuracy for $Im[\alpha]$ as $Re[\alpha]$.

4.1.2 Example II: stripline with two dielectrics

As a second example, a stripline in two different dielectrics which was studied previously by Das and Pozar [7] is considered. The effective dielectric constant
Figure 17: The normalized phase constant $Re[\alpha]$ of a SMS versus frequency.

**Figure 17**: The normalized phase constant $Re[\alpha]$ of a SMS versus frequency.

- $\varepsilon_r_1 = 9.8$
- $\varepsilon_r_2 = 1.0$
- $w = 3.0 \text{ mm}$
- $t = 0.635 \text{ mm}$
- $h = 2.54 \text{ mm}$
Figure 18: The normalized attenuation constant $\text{Im}[\alpha]$ of a SMS versus frequency.
$\epsilon_{eff} = Re[\alpha^2]$ and $Im[\alpha]$ are plotted as functions of the dielectric constant $\epsilon_r^2$ in Figure 19.

The results for $\epsilon_{eff}$ obtained here agree with those in [7]. When the two dielectric layers of the stripline are of equal thickness, the stripline mode will remain purely bounded no matter $\epsilon_r^1$ or $\epsilon_r^2$ is larger. The glitch in the solid line of $Im[\alpha]$ when $\epsilon_r^2$ is close to $\epsilon_r^1$ is due to a numerical instability. This difficulty arises because the expression in the WH/GSMT solution are obtained for the general case of a multilayered geometry and the general solution will become unstable for the limiting special case of a single homogeneous region. If the two dielectrics are of different thickness, then the stripline mode may become leaky when the dielectric constant of the thinner layer is smaller than that of the thicker layer. This interesting phenomenon is explained in [7]. The dashed curves in Figure 19 also clearly illustrate this point.

4.1.3 Example III: SSL

$Re[\alpha]$ and $Im[\alpha]$ of a single conductor-backed slotline as functions of the normalized slot width are given in Figures 20 and 21 along with the corresponding results published in [7] and [6]. The solid line is obtained by the present WH/GSMT while the dotted and the dashed lines are calculated with the spectral domain approach [7] and the spatial domain mode matching method [6], respectively. Both the dotted and dashed lines are recovered here by reading the curves published in [7] point by point and then connecting the points together. The results from three different sources agree well generally for both the real and imaginary parts of $\alpha$. The deviation when $d/\lambda_0$ is small or large is probably due to the numerical disadvantage inherited in the method used; namely, the spectral domain approach will require a large number of basis functions when the slot width $d$ is large while for the spatial
Figure 19: The effective dielectric constant, $\varepsilon_{eff}$, and the normalized attenuation constant, $\text{Im}[\alpha]$, as functions of the dielectric constant $\varepsilon_r$ for a stripline in two dielectrics.
domain mode matching method and the WH/GSMT, many evanescent PPMs have to be included when d is very small.

4.1.4 Example IV: CMS

In the previous three examples, the accuracy of both the real and imaginary parts of α obtained by the WH/GSMT was checked against results found in the literature. For the CMS considered here, which was studied before in [35], the ability for the WH/GSMT to model planar transmission lines embedded in an arbitrary number of dielectric layers is examined. In Figure 22, the even and odd mode dispersion characteristics for three different types of CMS are given where the solid, dashed and dotted lines are for conventional CMS, CMS with dielectric overlay and inverted CMS, respectively. Although the top PEC cover is put infinitely far in [35], the result in Figure 22 shows good agreement with [35] with $H_3 = 4H_1$ in our case. For the overlayed CMS, the $\varepsilon_{r1}$ is chosen to be the same as $\varepsilon_{r2}$, and it is noted for this case that the normalized phase constants of the even and odd mode are both quite close to $\sqrt{\varepsilon_{r1}}$ ($= \sqrt{\varepsilon_{r2}}$) for all the frequencies because most of the fields are confined in those two dielectric layers. This feature makes the overlayed CMS well suited for the design of a directional coupler, as suggested in [35]. The dispersion curve for the inverted CMS has the largest variation among the three types of CMS for the range of frequencies considered.

The characteristic impedance may be defined in different ways. Let the power, current and voltage associated with the transmission line be defined as follows:

$$P = \frac{1}{2} \int_{\text{cross section}} \mathbf{E}_t \times \mathbf{H}^* \cdot ds,$$  \hspace{1cm} (4.1)

$$I = \int_{\text{strip}} J_\sigma d\ell,$$ \hspace{1cm} (4.2)
Figure 20: The normalized phase constant, $\text{Re}[\alpha]$, vs. the normalized slot width $d/\lambda_0$. 

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$\varepsilon_r = 2.25$
$t/\lambda_0 = 0.267$
$h/\lambda_0 = 3.33$
Figure 21: The normalized attenuation constant, $\text{Im}[\alpha]$, vs. the normalized slot width $d/\lambda_0$. 

- WH/GSMT
- Das & Pozar
- Shigesawa, Tsuji & Oliner
\[ V = - \int_{C_1} \mathbf{E}_t \cdot d\ell \]  

where \( \mathbf{H}_t^* \) is the complex conjugate of \( \mathbf{H}_t \), \( J_s \) is the surface current density along the longitudinal direction and \( C_1 \) is a vertical path from the bottom PEC plane to the center of the strip. The results for the characteristic impedance based on three different definitions; namely, \( Z_0(1) = P/|I|^2 \), \( Z_0(2) = |V/I| \) and \( Z_0(3) = |V|^2/P \) are given in Figures 23, 24 and 25, respectively. Although the general shape of the curves are similar, the values are different when obtained from the three expressions for \( Z_0 \) since the field is not purely TEM. The agreement for the normalized phase constant values based on the WH/GSMT with that given in [35] is better than that for the characteristic impedance. It is easier to obtain an accurate phase constant than an accurate characteristic impedance; this is analogous to the situation for numerically computing eigenvalues and eigenfunctions where it turns out that the eigenvalues are usually less sensitive to numerical inaccuracies than the eigenfunctions.

4.1.5 Example V: antipodal finline

The antipodal finline depicted in Figure 26 is chosen as an example mainly to demonstrate the versatility of the WH/GSMT. The same parameters as in [36] are used in order to make a comparison. The normalized wavelength \( \lambda_g/\lambda_0 (= 1/\text{Re}[\alpha]) \) and the characteristic impedance \( Z_0 \) versus the overlapped or separation length \( s \) of two fins of the antipodal finline are given in Figure 27. The antipodal finline without overlapping, as depicted in Figure 26(b), is similar to a unilateral finline, and it also resembles a partially filled waveguide as the two fins recede to the side walls. When the fins are overlapped, as depicted in Figure 26(a), it behaves like a parallel plate waveguide with most of the fields concentrated in the overlapped region. Since the characteristic impedance of the unilateral finline is usually higher
Figure 22: Even and odd mode dispersion characteristics for three different types of coupled microstrip lines.

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$H_1 = H_2 = W$  $H_3 = 4 H_1$

$S = 0.4W$  $\varepsilon_{r_3} = 1.0$
Figure 23: Characteristic impedance defined by $P/I^2$ versus frequency for the three types of CMS.
Figure 24: Characteristic impedance defined by $V/I$ versus frequency for the three types of CMS.
Figure 25: Characteristic impedance defined by $V^2/P$ versus frequency for the three types of CMS.
than that of other planar transmission lines, the use of overlapped (or staggered) antipodal finline suggests a solution for lower impedance levels [36].

For the overlapping case, the results for both $\lambda_g/\lambda_0$ and $Z_0$ agree well with [36]. For the non-overlapping case, the agreement is reasonable for both $\lambda_g/\lambda_0$ and $Z_0$ when the operating frequency is 40 GHz. But for the other frequency, namely for 27 GHz, which is close to the $TE_{10}$ cut-off frequency (26.35 GHz) for the rectangular waveguide (WG-22) enclosure, the result is quite different from that in [36] for both $\lambda_g/\lambda_0$ and $Z_0$. The impedance curves shown in Figure 27(b) is obtained from the expression $V^2/P$ where the power P is defined in Equation (4.1) except that the factor 1/2 is dropped. The integration path chosen for computing the voltage V is a vertical path connecting the centers of the overlapped sections of the fins for the overlapping case while it is a direct line from the edge of one fin to the second fin for the non-overlapping case. Actually, the non-overlapping case is not favorable for the WH/GSMT when the aspect ratio S/B(=0.5 S/A) is too small since many evanescent PPMs have to be included. This is why there is a gap between the overlapping and non-overlapping curves when S/A is small. However, when S/A is around 0.3 to 0.6, the results based on the WH/GSMT should be quite reliable whereas the spectral domain approach, used in [36], may require a large number of basis functions. But without further data from measurements or from other numerical methods, it is difficult to state which result is more accurate.

4.2 A Study of the CBCPW

The coplanar waveguide, because of its advantage resulting from the ease of parallel and series insertion of both active and passive components, has been suggested by many microwave engineers [40, 43, 42] as an alternative to the microstrip line in
Figure 26: (a) Overlapped and (b) non-overlapped antipodal finline geometry and parameters used.

\[ \text{WG} = 22 \quad (A = 2.845\text{mm}, \quad B = 5.69\text{mm}) \]

\[ T = 0.127\text{mm}, \quad \varepsilon_r = 2.22 \]
Figure 27: (a) Normalized wavelength $\lambda_g/\lambda_0$ and (b) characteristic impedance $Z_0$ of the antipodal finline
microwave integrated circuits. This is especially true at millimeter wave frequencies, where via holes that are used to ground the active devices in microstrip circuits can introduce significant parasitic inductance and degrade circuit performance.

There are many variations of coplanar waveguides where the ideal form consists of a center conducting strip with two semi-infinite side conductors on the surface of an infinitely thick dielectric substrate [42]. The type considered here is the conductor-backed coplanar waveguide (CBCPW) because the Wiener-Hopf analysis discussed in Chapter II is particularly amenable to treating this geometry. In practice, the additional bottom ground plane is used to separate the circuit from the environment and improve mechanical strength as well as the heat dissipation capability. However, the bottom ground plane may also cause power leakage of the CPW mode into the PPMs in the lateral directions, and thus result in cross talk or undesired coupling to neighboring lines [6].

Although there have been many discussions on CBCPW, most of them are results based on the quasi-TEM assumption [43, 44] and others present only qualitative studies on their general properties [41, 42]. There are very few numerical results based on the rigorous, full-wave analysis of the CBCPW; this is probably due to the numerical complexity involved in rigorously analyzing the CBCPW [8]. Since the dimensions of the CPW circuit are still quite small in terms of the wavelength in most practical applications, the quasi-TEM methods usually provide accurate enough data for circuit modelling and designing. However, there are some physical phenomena exclusively associated with the wave or frequency dependent characteristic, such as the dispersion of the propagation constant or the power leakage due to surface wave or space radiation, which can not be accounted for with just a quasi-TEM analysis. Moreover, as the operating frequency is pushed higher and higher, it is important to know what the limits of the quasi-TEM methods are.
In this section, the results from the WH/GSMT will be presented for the following four kinds of CBCPW with single or double layered substrate, and with infinite or finite lateral ground planes. They will be referred to as CBCPW(1), CBCPW(2), CBCPW_FG(1) and CBCPW_FG(2), respectively, where the number inside the parenthesis denotes the number of substrate layers and FG means finite lateral ground planes. The power leakage into the PPM and the effect of finite lateral conducting planes will be taken into account in the analysis in a rigorous manner. Actually, a geometry with large lateral dimensions like the finite lateral ground planes is favorable for the WH/GSMT. On the other hand, the spectral domain approach may require a large number of basis functions for the same case. We hope that, with the numerical results shown here, the basic characteristics of the CBCPW will become even clearer. Also, these results can serve as a check on the validity of some quasi-TEM results as well as on other approximate analytical expressions found in the literature.

In the first subsection, the magnitude of the PPM power leakage of a CBCPW(1) as a function of the width of the center conductor, the substrate thickness and frequency will be examined. The problem of how to eliminate the undesired lateral power leakage by using a double layered substrate will be investigated in the second subsection. In the last two subsections, the effect of the lateral ground planes of finite-extent will be examined. The coupled slotline mode where the electric fields in the slots point in the same direction is not considered here since they can usually be suppressed with air bridges. The modes associated with the top cover plate will be neglected, too, since the top cover is placed sufficiently far from the dielectric layers.
4.2.1 CBCPW(1)

For a CBCPW(1) as depicted in Figure 28, there is always power leakage of the CPW mode into the PPM in the lateral directions because the phase velocity of the PPM between the side and bottom conducting planes is slower than that of the CPW mode. Hence, the propagation constant of the CPW mode will be a complex number. The solid lines in Figures 28 and 29 show the $Re[\alpha]$ and $Im[\alpha]$ of CBCPW(1) obtained by the WH/GSMT as functions of the center conductor width $W$ and substrate thickness $t_{-1}$. The aspect ratio $W/(W+2G)$ is fixed at 0.5.

There is an approximate expression for $Im[\alpha]$ which is derived by using the reciprocity theorem [42]; namely,

$$Im[\alpha] = \frac{1}{k_0 2t_{-1} \eta_d} \frac{Z_0 (W + 2G)^2}{(\lambda_d)^2} \sqrt{1 - \frac{\epsilon_{eff}}{\epsilon_{r_{-1}}}}$$

(4.4)

where $Z_0$ is the CPW impedance, $\eta_d = 377/\sqrt{\epsilon_{r_{-1}}}$ is the wave impedance of the substrate dielectric material and $\lambda_d = \lambda_0/\sqrt{\epsilon_{r_{-1}}}$ is the wavelength inside the substrate dielectric. Since the expression is so simple, it would be very useful if it can yield accurate results. All the quantities in Equation (4.4) which are necessary to calculate $Im[\alpha]$ can be easily found except for $Z_0$ and $\epsilon_{eff}$. The value of $Z_0$ is especially difficult to find because the fields extend laterally out to infinity, and the value of the guided power $P$ at any cross section as defined in Equation (4.1) will become singular for a mode with lateral power leakage. Therefore, the CPW characteristic impedance, $Z_0$, which could have been defined as $V^2/P$ where $V$ is the voltage across the slot becomes ill-defined in this case. In order to at least roughly check the validity of Equation (4.4), the quasi-TEM $\epsilon_{eff}$ and $Z_0$ from Equations (7)-(10) in [43] are used in (4.4) and the results are plotted in in Figures 28 and 29 for $Im[\alpha]$ with dashed lines together with the results based on the WH/GSMT. Also, the quasi-TEM $\epsilon_{eff}$ from [43] are plotted in $Re[\alpha]$ of Figures 28 and 29 with dashed
lines to compare against the WH/GSMT results. As one can see from the \( Re[\alpha] \) part of the figures, the results of the quasi-TEM method differ by less than 0.5% from by the WH/GSMT result when \( W \) is small or \( t_{-1} \) is large. This is expected since the quasi-TEM method is designed for a CPW rather than a MS. As \( W \) gets too large or \( t_{-1} \) gets too small, the CBCPW will approach the characteristics of the MS. As for the \( Im[\alpha] \) part, the values from the approximate expression (4.4) with the quasi-TEM \( Z_0 \) and \( \epsilon_{eff} \) are about twice as large as the WH/GSMT result, but the shapes of the curves agree with each other when \( W \) is small or \( t_{-1} \) is large.

This transition phenomenon for a CBCPW from CPW to MS is shown clearly in Figures 30 and 31 where the dashed lines in Figure 30 are the quasi-TEM results from Equations (9a) and (9b) in [43]. When the size of \( W+2G \) is relatively small, \( Re[\alpha] \) has little dispersion, and \( Im[\alpha] \) increases linearly as the frequency increases as predicted by Equation (4.4) and also shown in the results of [8]. But when \( W+2G \) gets larger and larger, the \( Re[\alpha] \) becomes as dispersive as for a MS and the power leakage rate \( Im[\alpha] \) has a totally different shape.

In conclusion, although the exact values of \( Im[\alpha] \) cannot be verified because \( Z_0 \) is not well defined, the functional dependance of \( Im[\alpha] \) for a CBCPW(1) on \( W+2G \), the substrate thickness \( t_{-1} \) and the frequency as prescribed in Equation (4.4) appear to be quite reliable as long as the CBCPW works like a CPW rather than a MS. Similarly, the quasi-TEM method is suited for the CPW with small \( W+2G \), large \( t_{-1} \) and lower frequencies.

4.2.2 CBCPW(2)

From the discussion above, one notices that the dominant CPW mode of a CBCPW(1) will always have the lateral power leakage although increasing the substrate thickness or decreasing the ground to ground spacing \( W+2G \) can reduce the
Figure 28: The normalized propagation constant $\alpha$ of a CBCPW(1) as a function of the center conductor width $W$. 
Figure 29: The normalized propagation constant $\alpha$ of a CBCPW(1) as a function of the substrate thickness $t_{-1}$. 

Eq. (4.4)
Figure 30: The real part of the normalized propagation constant $\alpha$ of a CBCPW(1) as functions of frequencies for various center conductor width $W$. 
Figure 31: The imaginary part of the normalized propagation constant $\alpha$ of a CBCPW(1) as functions of frequencies for various center conductor width $W$. 

- [1] $W = 0.1\text{mm}$
- [2] $W = 0.2\text{mm}$
- [3] $W = 0.3\text{mm}$
- [4] $W = 0.5\text{mm}$
- [5] $W = 2.0\text{mm}$
leakage rate. However, in practical situations, the substrate may not be made too thick. Also, the center conductor can not be too narrow because the ohmic loss will then become large [40]. The idea of adding one layer of low permittivity material such as quartz between the original semiconductor substrate and the bottom ground plane to lower the phase constant of the PPM and eliminate the lateral power leakage has been suggested by Jackson [40]. Here, some numerical data and a simple design rule based on this idea are presented.

In practice, one has limited choices for the substrate materials. Therefore, the design goal is to find optimal substrate thicknesses of the CBCPW(2) for given dielectric constants to eliminate the PPM leakage while still preserving the feature of small frequency dispersion. First, the thicknesses that can make $Im[\alpha]$ go to zero are examined. Then, a minimum thickness for the top layer of the substrate which will make the $\epsilon_{eff}$ of the CPW insensitive to the type of material and the thickness of the bottom layer is investigated. Finally, the result over a wide band of frequencies will be shown.

In general, the phase constant for a CPW mode as well as the PPM are complicated functions of many parameters. But for low frequencies, its value can be approximated by a simple expression which is independent of frequency. Refer to CBCPW(2) depicted in Figure 32. The effective dielectric constant of the CPW mode is approximately equal to the average of the dielectric constants of the two layers in which the CPW is sandwiched if $W+2G$ is small and both $t_{-1}$ and $t_1$ are large; namely,

$$
\epsilon_{eff} = Re[\alpha]^2 \approx \frac{(\epsilon_{r_{-1}} + \epsilon_{r_1})}{2}.
$$

(4.5)
Meanwhile, the normalized phase constant of PPM mode, denoted by \( k_s/k_0 \), can be approximated by the following expression

\[
(k_s/k_0)^2 \approx \frac{\epsilon_{r-1}\epsilon_{r-2}}{\epsilon_{r-1}\rho + \epsilon_{r-2}(1 - \rho)} , \quad \rho = \frac{t_{-2}}{(t_{-1} + t_{-2})}
\]

for low frequencies \( t_{-1} + t_{-2} \leq 0.1\lambda_d \) is a conservative estimate for the working range). Equating (4.5) and (4.6) yields the critical thickness ratio \( \rho_c \) which makes \( k_s/k_0 = Re[\alpha] \) for fixed values of \( \epsilon_{r_1}, \epsilon_{r-1}, \) and \( \epsilon_{r-2} \).

The \( Re[\alpha] \) and \( Im[\alpha] \) of the CBCPW(2) based on the WH/GSMT are plotted in Figures 32 as functions of the thickness ratio between the two substrate layers for different total substrate thickness. The normalized phase constant \( k_s/k_0 \) for the PPM between the side and bottom conductors with different total substrate thickness are plotted with dotted lines. The two dotted lines in Figures 32 nearly overlap with each other, and both are very close to the values calculated with Equation (4.6). The curves for \( Re[\alpha] \) of the CPW mode are relatively flat if \( t_{-1} \) is large and the values are close to the constant calculated with Equation (4.5) in the flat region. When \( k_s/k_0 > Re[\alpha] \), the \( Im[\alpha] \) shows a relatively large value as expected. There will be two intersection points, in general, between the \( Re[\alpha] \) of the CPW mode and the \( k_s/k_0 \) curves if \( \epsilon_{r_1} < \epsilon_{r-2} < (\epsilon_{r-1} + \epsilon_{r_1})/2 \). The smaller the \( \epsilon_{r-2} \) is, the smaller the critical thickness ratio \( \rho_c \) is and thus the region without leakage is larger. The left intersection point is predicted by equating Equations (4.5) and (4.6); whereas, the right intersection point when \( t_{-2}/(t_{-1} + t_{-2}) \) is close to 1 implies that \( t_{-1} \) can not be too small, otherwise \( Re[\alpha] \) may drop so rapidly as to be smaller than \( k_s/k_0 \) and the leakage will turn on again. Figure 33 shows a similar result where all the parameters are the same as in Figure 32 except that \( \epsilon_{r-2} \) has a smaller value. The region without lateral leakage for the case in Figure 33 is larger than that in Figure 32, as expected.
Figure 32: The normalized phase constant $Re[\alpha]$ and the normalized attenuation constant $Im[\alpha]$ of CBCPW(2) versus the substrate thickness ratio.
Figure 33: The normalized phase constant $\text{Re}[\alpha]$ and the normalized attenuation constant $\text{Im}[\alpha]$ of CBCPW(2) versus the substrate thickness ratio.
After choosing a suitable thickness ratio which can eliminate the PPM leakage, the next step is to find a proper $t_{-1}$. We want to keep $t_{-1}$ small because $t_{-2}$ can also be small in this way. However, $Re[\alpha]$ tends to be more sensitive to circuit dimensions as well as to bottom layer material and its thickness, if $t_{-1}$ is too small. This point is illustrated by Figures 34 and 35, and it can also be observed from the quasi-TEM expressions [43, 45].

Figure 34 shows the $Re[\alpha]$ of the CBCPW(2) for various $W$ and $G$. $Im[\alpha]$ is equal to zero for all the cases considered here. When $t_{-1}$ is smaller, more fields penetrate down to the bottom layer and it results in a smaller effective dielectric constant. As $t_{-1}$ increases, all the three curves reach almost the same stable values. The minimum thickness for $Re[\alpha]$ to reach a stable value prescribed by [43] is $1.5(W+2G)$. This criterion agrees with the full-wave WH/GSMT result given in Figure 34. Figure 35 demonstrates the effect of the dielectric thickness again where $Re[\alpha]$ is plotted as a function of the thickness $t_{-2}$ for various $t_{-1}$ and $\varepsilon_{r_{-2}}$. Similarly, $Im[\alpha]$ is equal to zero for all the cases shown. When $t_{-1}$ gets larger and larger, the solid and dashed lines which correspond to different $\varepsilon_{r_{-2}}$ get closer and closer. It implies that the effective dielectric constant of the CPW mode will be insensitive to the material and thickness of the bottom layer provided that the top layer thickness $t_{-1}$ is large enough. Also, when $t_{-2}$ gets larger, all the curves will reach respective stable values. This suggests that a conventional CPW without the bottom ground plane can be simulated by a CBCPW with a double layered substrate. In the last example for this subsection, the dispersion of $Re[\alpha]$ as frequency changes is investigated. In Figure 36, the WH/GSMT result is plotted with solid lines while the quasi-TEM result (refer to [45] and Appendix C) is given by the dashed lines. None of the cases have PPM leakage for the frequency range considered. When the condition
Figure 34: The normalized phase constant $Re[\alpha]$ of CBCPW(2) versus the dielectric thickness $t_{-1}$ for various $W$ and $G$. 
Figure 35: The normalized phase constant \(Re[\alpha]\) as of CBCPW(2) a function of the thickness \(t_2\) for various \(t_1\) and \(\epsilon_{r_2}\).
\[ t_{-1} > 1.5(W + 2G) \] is satisfied (curve[1]) then \( Re[\alpha] \) has very little dispersion over a wide band while it becomes much more dispersive as the circuit dimensions increase.

In conclusion, by adding an additional layer of substrate, choosing a suitable thickness ratio and making \( t_{-1} \geq 1.5(W + 2G) \), one can eliminate the lateral power leakage pertinent to CBCPW(1) while still preserving the low dispersion characteristic over a wide frequency range. For higher frequencies, the method presented here can serve as the initial guess for the design.

4.2.3 CBCPW_FG(1)

When used in the microwave integrated circuit, the CPW usually can not have infinitely large lateral ground planes. The CBCPW_FG structure as depicted in Figure 37 can be viewed as three coupled microstrip lines. Hence, there exist three normal modes with zero cutoff frequency; namely, the additional CPM (coplanar microstrip line) mode besides the slotline and the CPW mode corresponding to the CBCPW with infinite lateral ground planes. Among the three normal modes, the slotline mode is even (in terms of the transverse electric field pattern) and it can be suppressed by connecting the lateral ground planes with airbridges, hence it will not be considered here. A simple descriptive picture of the field pattern of the other two odd modes, i.e., the CPW and CPM mode, is given in Figure 37. If the lateral ground planes are finite but large, the field pattern of these two odd modes will be quite different. Although there are some studies on the CBCPW_FG [41, 42, 44], only very few numerical results based on the full-wave analysis method have been presented for this case. Besides the overmoding problem caused by the additional CPM mode, the higher order CPM modes[41] will emerge if the width of the side conductors \( W_{\ell} \) is too large [40]. However, if \( W_{\ell} \) is too small, the CPW mode may differ too much from the case with infinite lateral ground planes and there will be
Figure 36: The dispersion curves of CBCPW(2) versus frequencies for various W.
finite potentials on the lateral ground planes [44] which may cause some problems when the active devices are inserted. The question of how to find a suitable $W_f$ is the main concern of this subsection. Before answering this question, we have to first decide which parameter or quantity will be used to measure the similarity or difference between modes.

One simple yet interesting quantity to examine is the voltage on the three conductors with respect to the bottom ground plane for the CPW and CPM mode. This approach was used in [44] where the inductance and capacitance of the CBCPW.FG(1) are calculated under the quasi-TEM assumption. One then ends up with an eigenvalue problem where the eigenvalues yield the effective dielectric constant for each normal mode while the eigenvectors yield the voltages on the conductors. The three normal modes are expressed as follows [44]:

$$
V_1 = \begin{pmatrix} -1 \\ 0 \\ +1 \end{pmatrix}, \quad V_2 = \begin{pmatrix} a \\ 1 \\ a \end{pmatrix}, \quad V_3 = \begin{pmatrix} -b \\ 1 \\ -b \end{pmatrix}
$$

(4.7)

where $V_1$, $V_2$ and $V_3$ correspond to the voltages of the slotline, CPM and CPW mode, respectively; here, '$a$' is a positive number which is a little smaller than 1, and '$b$' is a positive number close to zero. If a signal is sent on the center conductor while
the two side conductors are kept at ground potentials, one will need a combination of two normal modes; namely,

\[
V_0 = \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix} = \frac{b}{(a + b)} V_2 + \frac{a}{(a + b)} V_3.
\] (4.8)

Therefore, the smaller \( b \) is, the closer the CPW mode is to the ideal \( V_0 \) mode. However, rigorously speaking, neither the CPW nor the CPM mode is purely TEM. Sometimes, the axial components of electric or magnetic fields are not small. Although for a non-TEM case like this, the values of the voltages are not uniquely defined and they depend on the path of the integration, it is nevertheless true as one can see from the figures shown later that the voltages \( a \) and \( b \) are still quite meaningful numbers.

Another quantity we want to investigate is the mode coupling coefficient which is defined by the normalized field overlap integral as follows:

\[
C_{12} = \frac{\int E_1 \times H_2^* \cdot ds}{[\int E_1 \times H_1^* \cdot ds]^{1/2} [\int E_2 \times H_2^* \cdot ds]^{1/2}}
\] (4.9)

where subscripts 1 and 2 correspond to the two interacting modes and the integral is evaluated over the cross section of the waveguide. Although the mode coupling coefficient expression defined this way is rigorous, the value obtained from it will be a relative quantity. That is, unlike the voltage vector defined in Equation (4.7) where a voltage equal to 0.5 is five times as large as a voltage 0.1, a coupling coefficient equal to 0.5 or 0.1 can tell us no more than the fact that the coupling is stronger for one case as compared to the other. Therefore, although the coupling coefficient can be used to measure the similarity or difference between two modes, it is not obvious where one should put the threshold to decide the range for suitable lateral conductor widths \( W_t \).
Since a description of both the voltage and coupling coefficients mentioned above have certain advantages and disadvantages, we therefore decide to use both. The agreement between the results assure the validity of both methods. Figure 38 shows the dispersion curves of CPW and CPM modes as functions of the side conductor width \( W_l \), where CPM1, CPM2 denote the lowest, and first higher order CPM mode, respectively. There is no PPM power leakage in this case. When \( W_l \) is small, there are only CPM1 and CPW modes. As \( W_l \) increases, CPM2 will emerge and there results a strong coupling between CPM2 and CPW, which are shown in curves (2) and (3) for values of \( W_l \) around 2.0mm. Usually, by examining the field distribution, it is quite clear whether a mode should be called CPW or CPM except in the strongly coupled region where both modes are hybrids of CPW and CPM modes. As \( W_l \) gets larger and larger, more and more higher order CPM modes will begin to manifest and couple with the CPW mode when they are phase matched.

The plots of voltage ratio and coupling coefficient as functions of the side conductor width \( W_l \) are given in Figure 39(a) and (b), respectively. \( V_{lb} \) and \( V_{cb} \) are voltages on the lateral and center conductor, respectively, both of which are defined with respect to the bottom ground plane. (The integration paths used here are vertical lines from the bottom ground plane to the center of each conductor.) The value of \( |V_{lb}/V_{cb}| \) for CPM1 and CPW curves, is actually the voltage a and b in Equation (4.7), respectively. The voltage \( a \) for the CPM1 mode is always close to 1 while the voltage \( b \) of the CPW mode drops first as \( W_l \) increases and then keeps a small value for a while as expected. The initial drop implies that, as \( W_l \) gets larger, the CPW mode with finite-extent lateral ground planes will be more similar to the CPW mode with infinite ones. The voltage ratio for the CPW mode begins to increase when the CPW mode begins to transition to the CPM2 mode. In Figure 39(b), the coupling coefficient curve \( C_{ij} \) is obtained by substituting Equation (4.9) with the
mode (i) and (j) labelled in the curves of Figure 38. Comparing Figure 38 and Figure 39(b), one finds that the peak in curve $C_{23}$ clearly indicates the strong coupling between CPW and CPM$_2$ modes when $W_i$ is about 2.0 mm. Curve $C_{12}$ drops first as curve (1) and (2) in Figure 38 gets farther apart, reaches a flat region, and then increases again when curve (2) gets close to the coupling region.

Comparing (a) and (b) in Figure 39, it is found that the CPW curve in (a) and the $C_{12}$ curve in (b) have very similar shapes. Note that actually the voltage 'b' measures the change of the CPW mode itself when the width of the lateral ground planes changes while the coupling coefficient $C_{12}$ is the relation between the CPW and CPM$_1$ modes. However, both methods yield similar results as the widths of the lateral ground planes are concerned. It was concluded that one must have $W_i \geq 1.5(W + 2G)$ in order for line-to-line coupling to be less than 40 dB for a CPW without the bottom ground plane [43]. From Figure 39, it is found that $W_i = 2.0(W + 2G)$ can make the voltage $b=0.15$ and the coupling coefficient $C_{12} = 0.05$, while $b$ drops to 0.10 and $C_{12}$ decreases to about 0.02 when $W_i = 3.0(W + 2G)$. After that point, both $b$ and $C_{12}$ do not change significantly. Therefore, $W_i$ should be at least equal to $2(W+2G)$ for this case, and the upper bound for $W_i = 1.2$ mm, which is determined by the cutoff of the first higher order CPM mode.

4.2.4 CBCPW\_FG(2)

As mentioned in the previous section and in [41], there are two problems associated with the CBCPW\_FG; namely, first, the overmoding problem because of the existence of the CPM$_1$ mode which has a zero frequency cutoff and second, the higher order CPM mode may be closely phase matched with the CPW mode at certain frequencies for the wide band application. In this section, one dielectric layer
Figure 38: The dispersion curves of \( \text{Re}\left[\alpha\right] \) of CBCPW\_FG(1) as functions of the side conductor width \( W_{t} \).

- \( \epsilon_{r1} = 1.0 \)
- \( \epsilon_{r-1} = 13.0 \)
- \( W = 0.1\text{mm} \)
- \( t_{1} = 0.6\text{mm} \)
- \( t_{-1} = 0.2\text{mm} \)
- \( G = 0.05\text{mm} \)
- \( \text{freq} = 30.0\text{GHz} \)
Figure 39: (a) The voltage ratio and (b) coupling coefficient of CBCPW FG(1) as functions of the side conductor width $W_f$. 
with lower dielectric constant is added between the original substrate and the bottom conducting plane such that all the CPM modes will have smaller phase constant than the CPW mode and no phase match can exist, at least for lower frequencies. Therefore, the second problem will be solved to some extent.

The first layer thickness $t_{-1}$ for the CBCPW_FG(2) is chosen to be larger than 1.5(W+2G) such that the second layer will not affect the CPW mode much. The circuit can then be viewed as consisting of two wide microstrip lines coupled through the center conductor and two slots. It is known that the phase constant of a SMS mode will always be smaller than the phase constant of the PPM between the strip and the ground plane. The phase constant of the CPM mode will be a little larger than that of the SMS mode for each side conductor due to the mutual coupling. However, the phase constant of the PPM calculated with Equation (4.6), can still provide a good approximation for that of the CPM mode.

The phase constants $Re[\alpha]$ of the CPW and CPM modes for the CBCPW_FG(2) as functions of the side conductor width $W_{\ell}$ are given in Figure 40. The result for the CBCPW_FG(1) is also plotted for a comparison. Again, there is no lateral PPM leakage for all the cases considered here. The curves for $Re[\alpha]$ of the CPW mode of CBCPW_FG(1) and CBCPW_FG(2) almost overlap one another while the phase constant for the CPM (more specifically, the CPM$_1$) mode becomes smaller as the dielectric constant $\varepsilon_{r-2}$ of the bottom layer decreases. Therefore, the phase match phenomenon between the CPW and CPM$_1$ modes will not occur for the double layered substrate case no matter how large $W_{\ell}$ is. Since the phase constant of the PPM between the side conductor and the bottom ground plane is smaller than that of the CPW mode, the field will decay laterally in the side conductor region in this case. In other words, the outer edge of the side conductor will be seen less and thus the CPW with finite-extent lateral ground planes will behave more like a CPW with
infinitely wide lateral ground planes. However, this effect is not very prominent because the operating frequency is so low. It is interesting to examine the voltage ratio and the coupling coefficient of the CBCPW\_FG(2) in a similar manner to that done in the previous section. The results for this are given in Figure 41. The coupling coefficients of the double layered cases decrease a little faster than the single layered one while the voltage ratios of CPW mode are almost the same. For the case with double layered substrate, a value of 2(W+2G) seems to be a reasonable choice for the minimum W\_L. The maximum allowable W\_L does not need to be chosen critically as in the case of a single layered substrate, since it is not subject to the mode coupling problem any more.

In conclusion, the dispersion curves over a wide frequency band of the four cases considered in the section, namely, CBCPW(1), CBCPW(2), CBCPW\_FG(1) and CBCPW\_FG(2) are summarized together in Figure 42. The parameters for CBCPW(2), CBCPW\_FG(1) and CBCPW\_FG(2) are chosen in such a way that there is no lateral PPM leakage and only very small effect for the finite-extent ground planes while the low dispersion characteristic is kept and the phase constants Re[\alpha] are close to that of the CBCPW(1) case. As one can see from the results shown, based on the design rules discussed in this section, the dispersion is very small from 4 to 34 GHz for all the four cases and the relative difference among these curves is within 1.5% for the frequency range considered.
Figure 40: The phase constant $\text{Re}[\alpha]$ for CBCPW.FG(1) and CBCPW.FG(2) as functions of the width $W_f$. 
Figure 41: (a) The voltage ratio and (b) coupling coefficient of CBCPW.FG(1) and CBCPW.FG(2) as functions of the side conductor width $W_f$. 

$\epsilon_{\perp 2} = 2.25$

$\epsilon_{\perp 2} = 1.0$

$t_{\perp 2} = 0$

$t_{\perp 2} = 0.1 \text{mm}$
Figure 42: The normalized phase constant \( Re[\alpha] \) versus frequencies for different kinds of CBCPW.
CHAPTER V
The Effect of the PEC Top Cover Height

In this chapter, the question of how far away a PEC top cover should be placed in order to have a negligible effect on the otherwise open region transmission lines will be addressed. The effective dielectric constant $\varepsilon_{eff}$ and characteristic impedance $Z_0$ are used to measure the effect of the top cover. The smallest top cover height which makes the differences of both $\varepsilon_{eff}$ and $Z_0$ between a closed region problem and a corresponding open region one smaller than a certain prescribed percentage error is called the critical height. Generally speaking, it is almost impossible to find a rigorous closed form expression for the critical height. On the other hand, even deducing certain kind of rules from numerical results is not an easy task since many parameters are involved. However, starting with a rough analysis where the field, which is originally expressed as a summation of waveguide modes, is written into a series of rays\[46\], we come up with an assumption that a factor, which is a function of all the parameters, may be predominant in predicting the critical height. To test this assumption, some numerical experiments are performed and a rule of thumb is given after examining the available results.

Consider the same canonical PEC half plane scattering problem as solved in Chapter II but with only a single layered substrate, as depicted in Figure 43. The $z$ component of the scattered electric field in the region $d < z < d + b$, $y > 0$ when the field is a TMz polarized PPM incident from the region $0 < z < d$, $y > 0$ is
given by

\[ E_z^s(x, y, z) = e^{-jk_0 \alpha x} \int_{\lambda = -\infty}^{\infty} \frac{f(\lambda) \cos(\xi k_0(d + b - z))}{\xi \sin(\xi k_0 b)} e^{-jk_0 \lambda y} \, d\lambda \]  

(5.1)

where \( f(\lambda) \) is a slow varying function of \( \lambda \) and

\[ \xi = \sqrt{1 - (\alpha^2 + \lambda^2)} \]  

(5.2)

with \( k_0, \alpha, \lambda \) being defined earlier in Chapter II. Although the integrand in Equation (5.1) is an even function of \( \xi \), and thus either choice of the branch of the square root function in Equation (5.2) will be acceptable, we let \( \xi \) in (5.2) denote the root with \( \text{Re}[\xi] \geq 0 \) and \( \text{Im}[\xi] \leq 0 \). \( \alpha \) is assumed to be real positive and less than 1 temporarily in following analysis. In chapter II, the scattered field \( E_z^s(x, y, z) \) is calculated by contour deformation and summation of residues at the zeros of the resonant denominator \( \xi \sin(\xi k_0 b) \). To change the mode representation to a ray representation, one can expand the resonant denominator into a power series and rewrite part of the integrand as follows [46]:

\[
\frac{\cos(\xi k_0(d + b - z))}{\sin(\xi k_0 b)} = \frac{j \left( e^{-j\xi k_0(z-d)} + e^{-j\xi k_0[2b-(z-d)]} \right)}{1 - e^{-2j\xi k_0 b}} \left( 1 + e^{-2j\xi k_0 b} + e^{-4j\xi k_0 b} + \cdots \right)
\]  

(5.3)

if \( k_0 \) is assumed to have a vanishingly small negative imaginary part. Then, using the stationary phase method to asymptotically evaluate the integral, the scattered field \( E_z^s(x, y, z) \) can be approximated by a sum of rays emanating from the edge of the PEC half plane:

\[
E_z^s(x, y, z) \sim \frac{e^{j\pi/4}}{(1 - \alpha^2)^{1/4} \sqrt{k_0}} \left\{ \sum_{n=0}^{\infty} \frac{f(\lambda) = \frac{y}{\rho_{1n}} \sqrt{1 - \alpha^2}}{\rho_{1n}} \frac{e^{-jk_0\sqrt{1 - \alpha^2} \rho_{1n}}}{\sqrt{\rho_{1n}}} + \sum_{n=1}^{\infty} \frac{f(\lambda) = \frac{y}{\rho_{2n}} \sqrt{1 - \alpha^2}}{\rho_{2n}} \frac{e^{-jk_0\sqrt{1 - \alpha^2} \rho_{2n}}}{\sqrt{\rho_{2n}}} \right\} e^{-jk_0 \alpha x}
\]
\[
\begin{align*}
\approx \frac{e^{j\pi/4} f(\lambda = 0)}{(1 - \alpha^2)^{1/4}} \sqrt{\frac{2\pi}{k_0}} \left\{ \sum_{n=0}^{\infty} \frac{e^{-j k_0 \sqrt{1-\alpha^2} \rho_{1n}}}{\sqrt{\rho_{1n}}} + \sum_{n=1}^{\infty} \frac{e^{-j k_0 \sqrt{1-\alpha^2} \rho_{2n}}}{\sqrt{\rho_{2n}}} \right\} e^{-j k_0 \alpha x} \\
\text{(5.4)}
\end{align*}
\]

where
\[
\rho_{1n} = \sqrt{[ (z - d) + 2nb]^2 + y^2}, \quad \rho_{2n} = \sqrt{[2nb - (z - d)]^2 + y^2} \\
\text{(5.5)}
\]
are the distances from the edge to the observation point \((x, y, z)\) for multiply bouncing rays, as is shown in Figure 43. That is, the total effect may be simulated by an equivalent line source located at the edge of the half plane. If the geometry is a microstrip line instead of a semi-infinite PEC half plane, the main contribution to the fields can be thought of as being generated by a line source located at each of the two strip edges. For other more complicated planar transmission lines, one expects to obtain similar results.

By analytical continuation, the result given in Equation (5.4) is assumed to be valid if \(\alpha\) is bigger than 1. However, the exponential factor \(e^{-j k_0 \sqrt{1-\alpha^2} \rho_{1n}}\) and \(e^{-j k_0 \sqrt{1-\alpha^2} \rho_{2n}}\) become \(e^{-k_0 \sqrt{\alpha^2 - 1} \rho_{1n}}\) and \(e^{-k_0 \sqrt{\alpha^2 - 1} \rho_{2n}}\). Therefore, instead of multiply bouncing rays, the scattered field \(E_z(x, y, z)\) in this case is the sum of many exponentially decaying terms where each term contains a factor \(e^{-k_0 \sqrt{\alpha^2 - 1} \rho} / \sqrt{\rho}\) in which \(\rho\) is the distance from the source or its image points to the observation point. If the direct contribution from the source to the observation point is much larger than all the other terms which are from the image points to the observation point such that it is accurate enough to retain only the \(n=0\) term in Equation (5.4), then the top cover can be considered as totally removed. Therefore, it seems that the most important factor in deciding how far away the top cover should be placed such that the problem can simulate an open region one is

\[
\gamma = k_0 \text{Re} \left[ \sqrt{\alpha^2 - 1} \right] 2b \\
\text{(5.6)}
\]
In the following, we perform numerical tests for different cases and try to see if there does exist certain kind of relationship between the relative percentage error and the factor $\gamma$.

First, the simplest case, namely, a single microstrip line (SMS) on a single layer substrate as depicted in Figure 44(a), is tested. There are four parameters that can be varied: strip width $w$, substrate permittivity $\varepsilon_r$, substrate thickness $d$ and frequency. In Tables 1, 2 and 3, the results for different $w$, $\varepsilon_r$, and $d$ are given for a fixed frequency. All the numerical results given in this chapter are obtained from the package PCAAMT[47] by Das and Pozar, which is based on a full-wave spectral domain moment method analysis. This package is used because it can calculate the closed region problem as well as the open one. If the results obtained with the WH/GSMT are compared with what is obtained with the PCAAMT for closed region problems, it is found that the maximum relative difference is about 0.2% for $\varepsilon_{eff}$. The difference between the results is larger for the characteristic impedance $Z_0$.

In the following tabulated results, the $\varepsilon_{eff}$ for a corresponding open region problem is used to calculate $\gamma$ in Equation (5.6), but it will only make slight differences if the $\varepsilon_{eff}$ for the closed region problem is used. As one can see from the tabulated results in Tables 1, 2 and 3, the relative error, $e\%$, for $\varepsilon_{eff}$, is around 1% when $\gamma$ is about $\pi/2$ and decreases to 0.1% to 0.2% or smaller when $\gamma$ increases to about $\pi$. When $\gamma$ increases to an even larger number, such as $3\pi$, the $\varepsilon_{eff}$ for the closed problem and the open problem are almost identical. The relative error of $Z_0$ for the same $\gamma$ is larger than that of $\varepsilon_{eff}$. In most of the cases, the relative error $e\%$ for $Z_0$ is less than 0.5% when $\gamma$ is around $\pi$, and it reduces to less than 0.05% when $\gamma$ increases to around $3\pi$. The $\gamma$ required to achieve the same level of relative percentage error $e\%$ are quite similar for all the three tables. This supports the assumption that the factor $\gamma$ seems to bear a certain relationship with the critical height.
Figure 43: The path and the geometry of the PEC half plane.

Figure 44: (a) The geometry of SMS for Tables 1-4. (b) The geometry of CMS for Tables 5 and 6.
Table 1: The effective dielectric constant $\varepsilon_{eff}$ and the characteristic impedance $Z_0$ of a SMS. $\varepsilon_r=10.0$, $d=0.05\text{cm}$, $freq=30.0\text{GHz}$

<table>
<thead>
<tr>
<th>w (cm)</th>
<th>$b=\infty$</th>
<th>b=0.05cm</th>
<th>b=0.1cm</th>
<th>b=0.3cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{eff}$</td>
<td>$\varepsilon_{eff}$</td>
<td>$%$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>0.01</td>
<td>6.764</td>
<td>6.695</td>
<td>1.021.51</td>
<td>6.7580.093.02</td>
</tr>
<tr>
<td>0.05</td>
<td>7.715</td>
<td>7.600</td>
<td>1.491.63</td>
<td>7.7030.153.26</td>
</tr>
<tr>
<td>0.10</td>
<td>8.389</td>
<td>8.278</td>
<td>1.321.71</td>
<td>8.3770.143.42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>w (cm)</th>
<th>$b=\infty$</th>
<th>b=0.05cm</th>
<th>b=0.1cm</th>
<th>b=0.3cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Z_0(\Omega)$</td>
<td>$Z_0(\Omega)$</td>
<td>$%$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>0.01</td>
<td>95.67</td>
<td>95.11</td>
<td>0.591.51</td>
<td>95.650.023.02</td>
</tr>
<tr>
<td>0.05</td>
<td>52.56</td>
<td>51.57</td>
<td>1.881.63</td>
<td>52.420.273.26</td>
</tr>
<tr>
<td>0.10</td>
<td>35.84</td>
<td>35.12</td>
<td>2.011.71</td>
<td>35.720.333.42</td>
</tr>
</tbody>
</table>
Table 2: The effective dielectric constant $\varepsilon_{eff}$ and the characteristic impedance $Z_0$ of a SMS. $\varepsilon_r=10.0$, $d=0.1\text{cm}$, $\text{freq}=30.0\text{GHz}$

<table>
<thead>
<tr>
<th>w (cm)</th>
<th>$\varepsilon_{eff}$</th>
<th>$\gamma$</th>
<th>$%$</th>
<th>$\varepsilon_{eff}$</th>
<th>$\gamma$</th>
<th>$%$</th>
<th>$\varepsilon_{eff}$</th>
<th>$\gamma$</th>
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<td>0.94</td>
<td>8.041</td>
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<tr>
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<td>1.73</td>
<td>0.91</td>
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<td>9.113</td>
<td>10.74</td>
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<table>
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<tr>
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<th>$Z_0$(Ω)</th>
<th>$Z_0$(Ω)</th>
<th>$%$</th>
<th>$\gamma$</th>
<th>$Z_0$(Ω)</th>
<th>$%$</th>
<th>$\gamma$</th>
<th>$Z_0$(Ω)</th>
<th>$%$</th>
<th>$\gamma$</th>
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</tr>
<tr>
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<td>93.91</td>
<td>2.02</td>
<td>1.67</td>
<td>92.21</td>
<td>0.17</td>
<td>3.34</td>
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<td>41.58</td>
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<td>3.58</td>
<td>41.60</td>
<td>0.02</td>
<td>10.74</td>
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Table 3: The effective dielectric constant $\varepsilon_{eff}$ and the characteristic impedance $Z_0$ of a SMS. $\varepsilon_r=2.2$, $d=0.05\text{cm}$, freq=$30.0\text{GHz}$

<table>
<thead>
<tr>
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<th>b=∞</th>
<th>b=0.1cm</th>
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<th>b=0.3cm</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{eff}$</td>
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<td>$\gamma$</td>
</tr>
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<td>1.731</td>
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<td>1.822</td>
<td>1.804</td>
<td>0.99</td>
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<tr>
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<td>1.898</td>
<td>1.875</td>
<td>1.21</td>
<td>1.19</td>
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<table>
<thead>
<tr>
<th>w (cm)</th>
<th>b=∞</th>
<th>b=0.1cm</th>
<th>b=0.2cm</th>
<th>b=0.3cm</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$Z_0(\Omega)$</td>
<td>$Z_0(\Omega)$</td>
<td>e%</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>0.01</td>
<td>173.96</td>
<td>171.58</td>
<td>1.37</td>
<td>1.07</td>
</tr>
<tr>
<td>0.05</td>
<td>97.40</td>
<td>96.37</td>
<td>1.01</td>
<td>1.14</td>
</tr>
<tr>
<td>0.10</td>
<td>67.54</td>
<td>66.28</td>
<td>1.87</td>
<td>1.19</td>
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Next, the frequency sweep test is done for SMS and coupled microstrip lines (CMS) operating at either even or odd modes. The geometry is depicted in Figure 44(a) and (b) and the results are given in Tables 4, 5 and 6, respectively. For higher frequencies, similar to Tables 1 to 3, ε% of $\varepsilon_{\text{eff}}$ is always less than 0.3% and ε% of $Z_0$ is no more than 0.8% most of the time when $\gamma$ is equal to $\pi$. While $\gamma$ increases to $2\pi$, there is almost no observable error at all. As the frequency goes lower, the $\gamma$ required is much smaller. One has to use other expression to determine the critical top cover height since the absolute height is more important than the height in terms of effective wavelength at the low frequency region.

From the six Tables given here, we conclude that: if 0.5% error of $\varepsilon_{\text{eff}}$ and 1.0% error of $Z_0$ is acceptable, then $\gamma = \pi$ or equivalently, b is about 1/4 the effective wavelength, is enough. If one wants even smaller error percentage, $\gamma = 2\pi$ or b is about 1/2 the effective wavelength will be more than enough. However, if the top cover height b obtained this way is more than 5 to 10 times the substrate thickness or widths of lines, one may consider other quasi-TEM formulas since the b predicted here is probably too large. As mentioned before, the propagation constant $\alpha$ in Equation (5.6) does not have to be very accurate since the b predicted here is just a rough estimate. Also, as one can see from the simple derivation of Equation (5.6), this rule of thumb can be applied to a multi-layered geometry as long as the top layer has smaller permittivity than the effective dielectric constant of the transmission line. For the case with the lateral leakage like conductor-backed coplanar waveguide discussed in Chapter IV, the rule of thumb can still be used. However, if $\text{Re}[\alpha] < 1$ and there is space wave like radiation, this rule of thumb cannot be applied.
Table 4: The effective dielectric constant $\epsilon_{eff}$ and the characteristic impedance $Z_0$ of a SMS. $\epsilon_r=10.0$, $d=0.05\text{cm}$, $w=0.05\text{cm}$

<table>
<thead>
<tr>
<th>freq (GHz)</th>
<th>$\epsilon_{eff}$</th>
<th>$Z_0(\Omega)$</th>
<th>$\epsilon_{eff}$</th>
<th>$Z_0(\Omega)$</th>
<th>$\epsilon_{eff}$</th>
<th>$Z_0(\Omega)$</th>
<th>$\epsilon_{eff}$</th>
<th>$Z_0(\Omega)$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$b=\infty$</td>
<td>$b=\lambda_0/4/\sqrt{\epsilon_{eff} - 1}$ ($\gamma = \pi$)</td>
<td>$b=\lambda_0/2/\sqrt{\epsilon_{eff} - 1}$ ($\gamma = 2\pi$)</td>
<td>$b=3\lambda_0/4/\sqrt{\epsilon_{eff} - 1}$ ($\gamma = 3\pi$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b(\text{cm})$</td>
<td>$\epsilon_{eff}$</td>
<td>$e%$</td>
<td>$b(\text{cm})$</td>
<td>$\epsilon_{eff}$</td>
<td>$e%$</td>
<td>$b(\text{cm})$</td>
<td>$\epsilon_{eff}$</td>
</tr>
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<td>1.042</td>
<td>6.751</td>
<td>0.01</td>
<td>2.084</td>
<td>6.752</td>
<td>0.00</td>
<td>3.127</td>
</tr>
<tr>
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<td>6.802</td>
<td>0.623</td>
<td>6.801</td>
<td>0.01</td>
<td>1.245</td>
<td>6.802</td>
<td>0.00</td>
<td>1.868</td>
</tr>
<tr>
<td>10.0</td>
<td>6.965</td>
<td>0.307</td>
<td>6.961</td>
<td>0.06</td>
<td>0.614</td>
<td>6.965</td>
<td>0.00</td>
<td>0.921</td>
</tr>
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<td>0.149</td>
<td>7.335</td>
<td>0.14</td>
<td>0.298</td>
<td>7.344</td>
<td>0.01</td>
<td>0.447</td>
</tr>
<tr>
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<td>7.701</td>
<td>0.18</td>
<td>0.193</td>
<td>7.714</td>
<td>0.01</td>
<td>0.289</td>
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<tr>
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<td>0.071</td>
<td>8.030</td>
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<td>0.141</td>
<td>8.044</td>
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<td>0.17</td>
<td>0.111</td>
<td>8.325</td>
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<td>0.166</td>
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<tr>
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<td>0.045</td>
<td>8.548</td>
<td>0.16</td>
<td>0.091</td>
<td>8.561</td>
<td>0.01</td>
<td>0.136</td>
</tr>
</tbody>
</table>

| freq (GHz) | $b=\infty$      | $b=\lambda_0/4/\sqrt{\epsilon_{eff} - 1}$ ($\gamma = \pi$) | $b=\lambda_0/2/\sqrt{\epsilon_{eff} - 1}$ ($\gamma = 2\pi$) | $b=3\lambda_0/4/\sqrt{\epsilon_{eff} - 1}$ ($\gamma = 3\pi$) |
|-----------|------------------|---------------|------------------|---------------|------------------|---------------|------------------|---------------|
|           | $b(\text{cm})$  | $Z_0(\Omega)$ | $e\%$           | $b(\text{cm})$  | $Z_0(\Omega)$ | $e\%$           | $b(\text{cm})$  | $Z_0(\Omega)$ | $e\%$           |
| 3.0       | 48.53            | 1.042         | 48.52            | 0.02           | 2.084            | 48.53            | 0.00             | 3.127            | 48.53            | 0.00             |
| 5.0       | 48.52            | 0.623         | 48.51            | 0.02           | 1.245            | 48.52            | 0.00             | 1.868            | 48.52            | 0.00             |
| 10.0      | 48.70            | 0.307         | 48.64            | 0.12           | 0.614            | 48.70            | 0.00             | 0.921            | 48.70            | 0.00             |
| 20.0      | 50.05            | 0.149         | 49.91            | 0.28           | 0.298            | 50.04            | 0.02             | 0.447            | 50.05            | 0.00             |
| 30.0      | 52.56            | 0.096         | 52.39            | 0.32           | 0.193            | 52.56            | 0.00             | 0.289            | 52.56            | 0.00             |
| 40.0      | 56.00            | 0.071         | 55.86            | 0.25           | 0.141            | 56.00            | 0.00             | 0.212            | 56.00            | 0.00             |
| 50.0      | 60.12            | 0.055         | 60.04            | 0.13           | 0.111            | 60.12            | 0.00             | 0.166            | 60.12            | 0.00             |
| 60.0      | 64.72            | 0.045         | 64.75            | 0.05           | 0.091            | 64.72            | 0.00             | 0.136            | 64.72            | 0.00             |
Table 5: The effective dielectric constant $\varepsilon_{\text{eff}}$ and characteristic impedance $Z_0$ of even mode of coupled microstrip lines $\varepsilon_r=10.0$, $d=0.05\text{cm}$, $w=0.05\text{cm}$, $s=0.02\text{cm}$

<table>
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<td>7.365</td>
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<td>0.591</td>
<td>7.441</td>
<td>1.182</td>
<td>1.772</td>
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<tr>
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<td>7.669</td>
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<td>0.871</td>
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<td>$\gamma = 3\pi$</td>
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Table 6: The effective dielectric constant $\varepsilon_{\text{eff}}$ and characteristic impedance $Z_0$ of odd mode of coupled microstrip lines. $\varepsilon_r=10.0$, $d=0.05\text{cm}$, $w=0.05\text{cm}$, $s=0.02\text{cm}$

<table>
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<td>$\varepsilon_{\text{eff}}$</td>
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</thead>
<tbody>
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<td></td>
<td>$Z_0(\Omega)$</td>
<td>$b(\text{cm})$</td>
<td>$Z_0(\Omega)$</td>
<td>$e%$</td>
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<td>60.0</td>
<td>110.49</td>
<td>0.049</td>
<td>110.15</td>
<td>0.31</td>
</tr>
</tbody>
</table>
CHAPTER VI

Conclusions

In this dissertation, a novel approach which is based on a hybrid procedure referred to as the WH/GSMT is employed for solving planar transmission line problems embedded in multilayer dielectrics. It is a rigorous, full-wave analysis method which consists of two key steps. First, the canonical half plane problem which models the edges of the planar transmission lines embedded in a parallel plate region is analyzed with the Wiener-Hopf technique, and then the interaction between the edges is taken into account with the generalized scattering matrix technique. Note that the lateral power leakage is taken into account rigorously.

The analysis of the canonical scattering problem is given in Chapter II, where the otherwise relatively involved analysis is done in a simple and systematic manner with the use of some compact notation. The solution is then expressed as a scattering matrix which fully characterizes the edge of the half plane embedded in a multilayer dielectric sandwiched between parallel plates. In Chapter III, a flow chart describing the WH/GSMT scheme is given along with some transverse resonance relations for common planar transmission lines, which include SMS, CMS, SSL, CPW,... etc.. Also, a discussion on the advantages/disadvantages of the WH/GSMT analysis is presented along with some numerical considerations which one must be aware of when implementing this procedure a computer code. Chapter IV contains two parts: in the first part, the numerical results based on the WH/GSMT for different planar
transmission lines are shown and compared with the results based on other independent approaches available in the literature to assess the accuracy of the WH/GSMT. In the second part of Chapter IV, a detailed study on the dispersion and lateral leakage of conductor backed coplanar waveguides (CBCPW) on the one or two layered substrate, with finite or infinite lateral ground planes is performed with the WH/GSMT. Some simple design rules are given such that the lateral leakage for the CBCPW can be suppressed and the effect of lateral ground planes of finite extent can be minimized while still preserving the low dispersion characteristic over a wide band of frequencies. The effect of the top cover which is included to simplify the factorization process in the Wiener-Hopf procedure is discussed in Chapter V. After a rough but intuitively valid analysis and some numerical tests, a rule of thumb is deduced for the critical top cover height necessary to simulate an open region problem with a closed region one.

Presently, only the lateral power leakage of the transmission lines is rigorously included in the WH/GSMT. The dielectric loss can be incorporated with moderate modifications in the computer codes where all the functions remain in the same forms. However, to deal rigorously with the conductor loss due to finite conductivity is not that straightforward. One possible approach is to replace the PEC half plane with a resistive card and then solve the half plane scattering problem with the Wiener-Hopf technique. This is apparently quite challenging work because the Wiener-Hopf equations are expected to become coupled in this case. The finite thickness of the conductors can also be treated in a similar manner where the finite thickness can be modelled by the generalized boundary conditions [51, 52].

At present, the subroutine for the transverse resonance relation is programmed case by case in the computer code. To be more versatile, one can write a general code to deal with the more complicated transmission line configurations. Also, the
code is not very user-friendly in the present stage. However, since the code contains structured formats, it can be improved without tremendous effort. The trouble causing situation mentioned in page 56 when $\alpha^2 + \lambda_j^2$ of a PPM is close to zero still needs more investigation. Also, the effect of the top cover height when there is space wave like radiation (i.e., $Re[\alpha] < 1$) remains to be studied.

Compared with the widely used spectral domain approach (SDA), the WH/GSMT furnishes a different physical insight, is suited for the planar transmission line configuration with relatively wide lateral dimensions, requires a similar amount of CPU time and memory storage space but requires a lengthier pre-analysis. The versatility of the WH/GSMT is usually relatively limited because the complexity of the transverse resonance relations will increase rapidly as the transmission line configurations become more complicated. However, in some particular situations where the areas covered with and without the PEC strips for the same dielectric interface are comparable, e.g., the conductor backed coplanar waveguide, it is much more effective than the SDA.
APPENDIX A

Factorization of $Q_e(\lambda)$ and $Q_h(\lambda)$

As defined in (2.74) and (2.76),

$$Q_e(\lambda) = \frac{D_{be}(\lambda)D_{de}(\lambda)}{k_0D_{ae}(\lambda)} ,$$

and

$$Q_h(\lambda) = \frac{k_0D_{bh}(\lambda)D_{dh}(\lambda)}{D_{ah}(\lambda)} .$$

There is more than one way to factorize $Q_e$ and $Q_h$ such that

$$Q_e(\lambda) = Q_{e+}(\lambda)Q_{e-}(\lambda)$$

and

$$Q_h(\lambda) = Q_{h+}(\lambda)Q_{h-}(\lambda) ,$$

where $Q_{e+}(\lambda)$ and $Q_{h+}(\lambda)$ are free of poles, branch points and zeros in the upper half $\lambda$ plane while $Q_{e-}(\lambda)$ and $Q_{h-}(\lambda)$ are free of poles, branch points and zeros in the lower half $\lambda$ plane.

We also require

$$Q_{e+}(-\lambda) = Q_{e-}(\lambda)$$

and

$$Q_{h+}(-\lambda) = Q_{h-}(\lambda) .$$

It is known that when $m$ is large,

$$\lambda_{de_m} \sim -j \frac{(m-1)\pi}{k_0 D} , \quad \lambda_{be_m} \sim -j \frac{(m-1)\pi}{k_0 B} , \quad \lambda_{ae_m} \sim -j \frac{(m-1)\pi}{k_0 A} ,$$

$$\lambda_{dh_m} \sim -j \frac{m\pi}{k_0 D} , \quad \lambda_{bh_m} \sim -j \frac{m\pi}{k_0 B} , \quad \lambda_{ah_m} \sim -j \frac{m\pi}{k_0 A} .$$
Based on the above asymptotic behavior of these roots, a factorization form which converges fast concerning the infinite product terms [48, 49] is given as follows:

\[
Q_{e+}(\lambda) = [Q_e(\lambda = 0)]^{1/2} e^{\chi(\lambda)} \frac{(1 - \frac{\lambda}{\lambda_{de1}})(1 - \frac{\lambda}{\lambda_{be1}}) \mathcal{P}_{de}(\lambda) \mathcal{P}_{be}(\lambda)}{(1 - \frac{\lambda}{\lambda_{ae1}}) \mathcal{P}_{ae}(\lambda)} g(\lambda) \quad (A.8)
\]

where

\[
\chi(\lambda) = \frac{j\lambda k_0}{\pi} [D \ln(A/D) + B \ln(A/B)] \quad (A.9)
\]

\[
g(\lambda) = \frac{\Gamma(1 - j\lambda k_0 A/\pi)}{\Gamma(1 - j\lambda k_0 B/\pi) \Gamma(1 - j\lambda k_0 D/\pi)} , \quad (A.10)
\]

\(\Gamma\) is the Gamma function and

\[
\mathcal{P}_{de}(\lambda) = \frac{\prod_{n=1}^{\infty} (1 - \lambda/\lambda_{de_{n+1}})}{\prod_{n=1}^{\infty} (1 - j\lambda k_0 D/n\pi)}
\]

\[
\mathcal{P}_{be}(\lambda) = \frac{\prod_{n=1}^{\infty} (1 - \lambda/\lambda_{be_{n+1}})}{\prod_{n=1}^{\infty} (1 - j\lambda k_0 B/n\pi)}
\]

\[
\mathcal{P}_{ae}(\lambda) = \frac{\prod_{n=1}^{\infty} (1 - \lambda/\lambda_{ae_{n+1}})}{\prod_{n=1}^{\infty} (1 - j\lambda k_0 A/n\pi)} \quad (A.11)
\]

Similarly,

\[
Q_{h+}(\lambda) = [Q_h(\lambda = 0)]^{1/2} e^{\chi(\lambda)} \frac{\mathcal{P}_{dh}(\lambda) \mathcal{P}_{bh}(\lambda) \mathcal{P}_{ah}(\lambda)}{\mathcal{P}_{ah}(\lambda)} g(\lambda) \quad (A.12)
\]

where

\[
\mathcal{P}_{dh}(\lambda) = \frac{\prod_{n=1}^{\infty} (1 - \lambda/\lambda_{dh_{n}})}{\prod_{n=1}^{\infty} (1 - j\lambda k_0 D/n\pi)}
\]

\[
\mathcal{P}_{bh}(\lambda) = \frac{\prod_{n=1}^{\infty} (1 - \lambda/\lambda_{bh_{n}})}{\prod_{n=1}^{\infty} (1 - j\lambda k_0 B/n\pi)}
\]

\[
\mathcal{P}_{ah}(\lambda) = \frac{\prod_{n=1}^{\infty} (1 - \lambda/\lambda_{ah_{n}})}{\prod_{n=1}^{\infty} (1 - j\lambda k_0 A/n\pi)} \quad . \quad (A.13)
\]
APPENDIX B  

Key Steps of the Wiener-Hopf Formulation

In Chapter II section 1, the scattering problem for a TMz PPM incident wave from region 2 is addressed. Among the other five kinds of incidence, TEz from region 2, TMz and TEz from region 3 are very similar to the previous case while TMz and TEz from region 1 are slightly different. The key steps are given below.

B.1 TEz PPM Obliquely Incident From Region 2

For an incident wave

\[ H_U(y, z) = \psi_{dh}(-\lambda dh_m, z) e^{jk_0 \lambda dh_m y}, -\infty < y < \infty, 0 \leq z < D, \]  

(B.1)

\( \tilde{H}_i^- (\lambda, z = D_-) \) in Equation (2.71) becomes

\[ \tilde{H}_i^-(\lambda, z = D_-) = -\frac{1}{2\pi k_0} \frac{(\alpha \hat{x} - \lambda dh_m \hat{y})}{(\alpha^2 + \lambda^2_{dh_m})} \frac{1}{(\lambda + \lambda_{dh_m}) \mu_{r-1}} \frac{\partial \psi_{dh}(-\lambda dh_m, D)}{\partial z}. \]  

(B.2)

Thus, the two Wiener-Hopf Equations (2.86) and (2.87) become

\[ \frac{\epsilon_{r_1} E_b(\lambda) D_{be}(\lambda)}{k_0 \psi_{be}(\lambda, D) Q_{e-}(\lambda)} = -\frac{1}{2\pi k_0} \frac{\alpha}{(\alpha^2 + \lambda^2_{dh_m}) \mu_{r-1}} \frac{1}{(\lambda + \lambda_{dh_m}) \mu_{r-1}} \frac{\partial \psi_{dh}(-\lambda dh_m, D)}{\partial z} Q_{e+}(\lambda) + (\alpha \hat{x} + \lambda \hat{y}) \cdot \tilde{f}_{s+} Q_{e+}(\lambda) \]  

(B.3)

and

\[ -j \mu_{r_1} H_b(\lambda) \frac{1}{Q_{h-}(\lambda)} + \frac{1}{2\pi k_0} \frac{1}{(\lambda + \lambda_{dh_m}) \mu_{r-1}} \frac{1}{(\lambda + \lambda_{dh_m}) \mu_{r-1}} \frac{\partial \psi_{dh}(-\lambda dh_m, D)}{\partial z} Q_{h+}(-\lambda dh_m) \]  

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\[
\frac{1}{2\pi k_0 \mu_{r-1}} \frac{\partial \psi_{dh}(-\lambda_{dhm}, D)}{\partial z} \frac{1}{(\lambda + \lambda_{dhm})} (Q_{h+}(\lambda) - Q_{h+}(-\lambda_{dhm})) \\
+ \frac{1}{2\pi k_0 \mu_{r-1}} \frac{\partial \psi_{dh}(-\lambda_{dhm}, D)}{\partial z} \frac{\lambda_{dhm}}{(\alpha^2 + \lambda_{dhm}^2)} Q_{h+}(\lambda) + (-\lambda \hat{x} + \alpha \hat{y}) \cdot \tilde{J}_{s+}^s(\lambda) Q_{h+}(\lambda) \\
= \frac{1}{2\pi k_0 \mu_{r-1}} \frac{\partial \psi_{dh}(-\lambda_{dhm}, D)}{\partial z} Q_{h+}(-\lambda_{dhm}) C_4 
\]  

where \( C_3 \) and \( C_4 \) are constants.

**B.2 TMz PPM Obliquely Incident From Region 3**

For an incident wave

\[
E^i_z(y, z) = \frac{\psi_{be}(-\lambda_{be_m}, z)}{e_{r}(z)} e^{jk_0\lambda_{be_m}y}, \quad -\infty < y < \infty, \quad D < z \leq A
\]

all the formulation is the same as in the case of the TMz polarized PPM obliquely incident from region 2 except that the \( \tilde{H}^t_z(\lambda, z = D_-) \) in Equation (2.71) is replaced by \( -\tilde{H}^t_z(\lambda, z = D_+) \) where

\[
\tilde{H}^t_z(\lambda, z = D_+) = -\frac{1}{2\pi \eta_0} \frac{(\lambda_{be_m} \hat{x} + \alpha \hat{y})}{(\alpha^2 + \lambda_{be_m}^2)} \frac{1}{(\lambda + \lambda_{be_m})} \psi_{be}(-\lambda_{be_m}, D).
\]

**B.3 TEz PPM Obliquely Incident From Region 3**

For an incident wave

\[
H^i_z(y, z) = \frac{\psi_{bh}(-\lambda_{bh_m}, z)}{\mu_r(z)} e^{jk_0\lambda_{bh_m}y}, \quad -\infty < y < \infty, \quad D < z \leq A
\]

if \( \tilde{H}^t_z(\lambda, z = D_-) \) in the case of TEz polarized PPM obliquely incident from region 2 is replaced by \( -\tilde{H}^t_z(\lambda, z = D_+) \) where

\[
\tilde{H}^t_z(\lambda, z = D_+) = \frac{1}{2\pi k_0} \frac{(\alpha \hat{x} - \lambda_{bh_m} \hat{y})}{(\alpha^2 + \lambda_{bh_m}^2)} \frac{1}{(\lambda + \lambda_{bh_m})} \frac{1}{\mu_{r1}} \frac{\partial \psi_{bh}(-\lambda_{bh_m}, D)}{\partial z}.
\]
B.4 TMz PPM Obliquely Incident From Region 1

For an incident wave
\[ E^i_z(y, z) = \frac{\psi_{de}(\lambda_{aem}, z)}{\epsilon_r(z)} e^{-jk_0\lambda_{aem}y}, \quad -\infty < y < \infty, \quad 0 \leq z \leq A, \quad (B.9) \]
the boundary condition (2.60) that the total tangential field \( E \) vanishes on the PEC half plane implies
\[ \tilde{E}^i_t(\lambda, z = D) = -\hat{E}^i_t(\lambda, z = D) \quad (B.10) \]
where
\[ \tilde{E}^i_t(\lambda, z = D) = \frac{1}{2\pi k_0} \frac{\alpha x + \lambda_{aem} \dot{y}}{\alpha^2 + \lambda_{aem}^2} \frac{1}{\lambda - \lambda_{aem}} \frac{1}{\epsilon_{r-1}} \frac{\partial \psi_{de}(\lambda_{aem}, D)}{\partial z}. \quad (B.11) \]

In this case, the boundary condition (2.62) yields
\[ \tilde{H}^i_t(\lambda, z = D+) - \tilde{H}^s_t(\lambda, z = D-) = -\dot{z} \times \tilde{J}^s_{s+}(\lambda). \quad (B.12) \]

Since
\[ \tilde{E}^i_t(\lambda, z = D) = \tilde{E}^i_t(\lambda, z = D) + \tilde{E}^i_t(\lambda, z = D) = -\tilde{E}^i_t(\lambda, z = D) + \tilde{E}^i_t(\lambda, z = D), \quad (B.13) \]
\( \tilde{E}^i_t(\lambda, z = D) \) is regular in the lower half \( \lambda \) plane except a simple pole at \( \lambda = \lambda_{aem} \) coming from the part \( \tilde{E}^i_t(\lambda, z = D) \). Considering two mutually orthogonal directions \( \hat{u} \) and \( \hat{v} \) separately, it is found that
\[ \tilde{E}^i_t(\lambda, z = D) \cdot (\alpha \hat{x} + \lambda \hat{y}) + \frac{1}{2\pi k_0} \frac{1}{\lambda - \lambda_{aem}} \frac{1}{\epsilon_{r-1}} \frac{\partial \psi_{de}(\lambda_{aem}, D)}{\partial z} \]
and
\[ \tilde{E}^i_t(\lambda, z = D) \cdot (-\lambda \hat{x} + \alpha \hat{y}) \]

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are both regular in the lower half \( \lambda \) plane. Therefore, the Wiener-Hopf equations (2.86) and (2.87) become

\[
\frac{\varepsilon_{r_1} E_b(\lambda) D_{be}(\lambda)}{k_0 \psi_{be}(\lambda, D) Q_{e-}(\lambda)} + \frac{j}{2\pi k_0 \varepsilon_{r-1}} \frac{1}{\partial z} \psi_{de}(\lambda_{ae_m}, D) \frac{1}{(\lambda - \lambda_{ae_m}) Q_{e+}(-\lambda_{ae_m})} = \\
\eta_0 (\alpha \hat{x} + \lambda \hat{y}) \cdot \tilde{J}_{s+}^e Q_{e+}(\lambda) + \frac{j}{2\pi k_0 \varepsilon_{r-1}} \frac{1}{\partial z} \psi_{de}(\lambda_{ae_m}, D) \frac{1}{(\lambda - \lambda_{ae_m}) Q_{e+}(-\lambda_{ae_m})} \\
= \frac{j}{2\pi k_0 \varepsilon_{r-1}} \frac{1}{\partial z} \psi_{de}(\lambda_{ae_m}, D) \frac{1}{Q_{e+}(-\lambda_{ae_m})} C_5
\]

and

\[
-3 \mu_{r_1} H_b(\lambda) = (\lambda \hat{x} + \alpha \hat{y}) \cdot \tilde{J}_{h-}^e Q_{h-}(\lambda) \\
= \frac{j}{2\pi k_0 \eta_0 \varepsilon_{r-1}} \frac{1}{\partial z} \psi_{de}(\lambda_{ae_m}, D) \frac{1}{Q_{e+}(-\lambda_{ae_m})} C_6 , \quad (B.15)
\]

where \( C_5 \) and \( C_6 \) are constants.

**B.5 TEz PPM Obliquely Incident From Region 1**

For an incident wave

\[
H_z^i(y, z) = \frac{\psi dh(\lambda a_{hm} z)}{\mu_r(z)} e^{-j k_0 \lambda a_{hm} y} , \quad -\infty < y < \infty , \quad 0 \leq z \leq A , \quad (B.16)
\]

\[
\tilde{E}_t^i(\lambda, z = D) = \frac{j}{2\pi} \eta_0 (\lambda a_{hm} \hat{x} + \alpha \hat{y}) \frac{1}{(\alpha^2 + \lambda_{ahm}^2)} \psi dh(\lambda a_{hm} D) . \quad (B.17)
\]

In this case,

\[
\tilde{E}_t^e(\lambda, z = D) = (\alpha \hat{x} + \lambda \hat{y})
\]

and

\[
\tilde{E}_t^e(\lambda, z = D) \cdot (\lambda \hat{x} - \alpha \hat{y}) - \frac{j}{2\pi} \eta_0 \frac{1}{(\lambda - \lambda_{ahm})} \psi dh(\lambda a_{hm} D)
\]

are both regular in the lower half \( \lambda \) plane. The Wiener-Hopf equations become

\[
\frac{\varepsilon_{r_1} E_b(\lambda) D_{be}(\lambda)}{k_0 \psi_{be}(\lambda, D) Q_{e-}(\lambda)} = \eta_0 (\alpha \hat{x} + \lambda \hat{y}) \cdot \tilde{J}_{s+}^e Q_{e+}(\lambda) \\
= \frac{-j}{2\pi} \eta_0 \psi dh(\lambda a_{hm} D) \frac{1}{Q_{h+}(-\lambda_{ahm})} C_7 \quad (B.18)
\]

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\[
\begin{align*}
\frac{-j\mu_1 H_b(\lambda)}{Q_{h-}(\lambda)} + \frac{1}{2\pi} \psi_{dh}(\lambda_{ah_m}, D) & \frac{1}{(\lambda - \lambda_{ah_m}) Q_{h+}(\lambda_{ah_m})} \frac{1}{Q_{h-}(-\lambda_{ah_m})} \\
= (-\lambda \hat{x} + \alpha \hat{y}) \cdot \tilde{J}^n_{s+} Q_{h+}(\lambda) + \frac{1}{2\pi} \psi_{dh}(\lambda_{ah_m}, D) & \frac{1}{(\lambda - \lambda_{ah_m}) Q_{h+}(-\lambda_{ah_m})} \\
= \frac{1}{2\pi} \psi_{dh}(\lambda_{ah_m}, D) & \frac{1}{Q_{h+}(-\lambda_{ah_m})} C_8 , \quad (B.19)
\end{align*}
\]

where \( C_7 \) and \( C_8 \) are constants.
APPENDIX C
Quasi-TEM $\varepsilon_{\text{eff}}$ for CBCPW(2)

Using the same algorithm as prescribed in [45], the quasi-TEM $\varepsilon_{\text{eff}}$ for CBCPW(2) in Figure 36 can be calculated as follows:

$$\varepsilon_{\text{eff}} = q_1\varepsilon_1 + q_{-1}\varepsilon_{-1}q_{-2}\varepsilon_{-2} \quad (C.1)$$

where $q_i$ is the filling factor,

$$q_1 = \frac{C_{1}^a}{C_t^a} \quad (C.2)$$
$$q_{-1} = \frac{C_{-1}^a}{C_t^a} \quad (C.3)$$
$$q_{-2} = \frac{(C_{-2}^a - C_{-1}^a)}{C_t^a} \quad (C.4)$$

$C_t^a$ is the total capacitance per unit length when replacing all the dielectric materials by air,

$$C_t^a = C_T^a + C_{-1}^a + C_{-2}^a \quad (C.5)$$

and

$$C_t^a = 2\pi\epsilon_0 \frac{K(k_i)}{K(k_i)} , \quad i = 1, -1, -2 \quad (C.6)$$

where

$$k_1 = \frac{\tanh(\frac{\pi w}{4t_1})}{\tanh\left[\frac{\pi(w + 2G)}{4t_1}\right]} \quad (C.7)$$
$$k_{-1} = \frac{\sinh(\frac{\pi w}{4t_{-1}})}{\sinh\left[\frac{\pi(w + 2G)}{4t_{-1}}\right]} \quad (C.8)$$
$$k_{-2} = \frac{\tanh(\frac{\pi w}{4(t_{-1} + t_{-2})})}{\tanh\left[\frac{\pi(w + 2G)}{4(t_{-1} + t_{-2})}\right]} \quad (C.9)$$
with $K(k)$ and $K(k')$ as the complete elliptic integral of the first kind and its complement, and $k'_t = \sqrt{1 - k_t^2}$. 
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