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Part I. A taxonomic paradigm from Boethius' "De divisione" applied to the eight modes of music. Part II. "Arioso and Toccata" for euphonium solo, wind ensemble, harp, and percussion. [Original composition]

Deason, William David, D.M.A.

The Ohio State University, 1992
PART I: A TAXONOMIC PARADIGM
FROM BOETHIUS' DE DIVISIONE APPLIED TO
THE EIGHT MODES OF MUSIC

PART II: ARIOSO AND TOCCATA FOR EUPHONIUM SOLO,
WIND ENSEMBLE, HARP, AND PERCUSSION

A Dissertation
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Musical Arts in the
Graduate School of The Ohio State University
by

William David Deason, B. M., M. M.

** ** ** **

The Ohio State University

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INTRODUCTION

Carolingian music theory treatises are practical handbooks rather than theoretical treatises.¹ Though Boethius' *De musica* was the most frequently quoted theoretical source in tenth and eleventh century treatises, there is little in his work which would assist Medieval singers in the learning of chant repertory. A perusal of the cyclic organization of the *De institutione harmonische* of Hucbald,² the Dasia notation of the *Musica enchiriadis*,³ and the mnemonic aids given to help memorize chant in the *Epistola de ignoto cantu* of Guido,⁴ shows that the purpose of Carolingian theorists was primarily utilitarian. Their focus was on practical ways of uniformly and efficiently performing and teaching chant. A rationale based on pure theory could scarcely provide the means to accomplish this goal. What theoretical structures Carolingian music writers did create can be described as first-order theory in that theoretical relationships were used directly as an aid to establishing uniform performance. A predilection for pure theory is less demonstrable among Carolingian music writers than among later writers.

Beginning in the eleventh century a more purely theoretical way of looking at music appeared in treatises written by German theorists. This new theoretical way saw music theory as a science that required empirical observation and demonstration. For example,

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¹Crocker (1972) says that the Frankish musician was singer, teacher, and theorist in that order. See Richard Crocker, "Hermann's Major Sixth," *Journal of the American Musicological Society* 25 (1972), p. 29.
³*Musica et scholia enchiriadis una cum aliquibus tractatulis adiunctis*, Bayerische Akademie der Wissenschaften, Veröffentlichungen der Musik historischen Kommission 3 (Munchen, 1981).
⁴The solmisation syllables *ut*, *re*, *mi*, *fa*, *sol*, and *la* from the hymn *Ut queant laxis* were provided by Guido as an aid in the singing of chant. See the *Epistola der ignoto cantu* in *Gerbert Scriptores*, II, pp. 43-50. For an English translation see Guido of Arezzo, *Epistola de ignoto cantu*, translated by Oliver Strunk in *Source Readings in Music History*. (New York: W. W. Norton and Co., 1950), pp. 121-5.
around the year 1050 Hermannus stated that, in order to be a musician, one must know the "science of rationally composing songs."  

Starting with Berno of Reichenau in the early eleventh century, theorists were preoccupied with organizational methods, the use of a new vocabulary of descriptive terms, and the inner structural relationships of the Medieval gamut. Above all, this concern with the identical modal correspondences which may be found by dividing the gamut into identical hexachords testifies to a more theoretical approach.

This predilection for theorizing has been noted in modern scholarship. In the case of Hermannus, for example, Oesch (1961) criticized his theoretical preoccupation as theorizing for the sake of theorizing. Fellerer (1956) noted Wilhelm of Hirsau’s persistent use of demonstration for proving theoretical concepts.

Early Medieval German writers on music were theorists as well as teachers, and they did create a theory of music. They explored the structure of the gamut and modal relationships in such a systematic and logical manner that it is easy to see that a new conceptualization was emerging in early Medieval German music theory.

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5 Musica Hermanni Contracti, translated by Leonard Ellinwood (Rochester: Eastman School of Music, 1936, p. 47: "Oportet autem nos scrire, quod omnis musicæ rationis ad hoc spectat intentio, ut cantilenæ rationabiliter componendæ...")


Purpose of this study

This study will demonstrate that a component of Aristotelian logic, the science of division, influenced the ways German theorists of the early Medieval period rationalized the division of plainchant into eight modes or tones. Discussions of the modes in German theory frequently focused on how the Medieval gamut could be divided so that a unified modal system resulted. Division as a science was elevated to a position of extreme utility. By the eleventh century, German music theorists had gained knowledge about the tools of Aristotelian division, and that they used these tools in their description of the authentic/plagal pairings which were known later as the “maneriae.”

During the early Medieval period, the Aristotelian science of division was primarily known through Boethius’ translations and monographs. One of his monographs, the De divisione, so concisely outlines Aristotle’s division categories, that it can serve as the basis for the present study. A summary of Boethius’ division types will be presented, and a taxonomic paradigm will be constructed.

Division was not unique to the Greeks. It is reasonable to assume that the process of division would have to be a necessary part in any civilized society. However, Plato and Aristotle regarded the process of division as a science. In doing so, they elevated division to a point where its nature could be examined and demonstrated like any other empirical science.

Thus, treatises by five early Medieval German theorists will be examined for terminological usage and conceptual evidence that they were influenced by Aristotelian logic in the division of the modes into authentic and plagal octave species, and the division of these octaves into species of fourths and fifths. The five treatises are the Prologus in

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tonarium of Berno of Reichenau (ca. 1021-48),^9 the Musica of Hermannus Contractus (ca.1048-54),^10 the Musica of Wilhelm of Hirsau (ca. 1069),^11 the De musica of Aribio (1068-78),^12 and the Musica of Engelbert of Admont (before 1325).^13 Although other German theorists of this period also used species theory, these five writers were specifically chosen for inclusion in this document because they employed a new concept. They rationalized the modes by dividing the gamut according to hexachords^14 and by further subdividing hexachords into tetrachords. I will argue that this new approach to the division of the gamut, which Crocker described as a second order theoretical concept,^15 is an illustration of Boethius' division of accidents into other accidents. Although the Medieval diatonic gamut can be conceptually divided into hexachords or tetrachords, neither the hexachord nor the tetrachord is essential to the existence of the gamut.

However, Boethius' De divisione is more than merely Aristotelian. Boethius used anthropomorphism to illustrate Aristotle's view of the opposition of relationship. In this way Boethius grafted a Platonic concept onto Aristotelian division. Anthrpomorphism, which can be traced from the Pythagoreans to Plato, provided the common illustration that the five German theorists used when they discussed the relationship between the authentic and plagal modes.

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^9 Berno of Reichenau, Prologus in Tonarium , Scriptores ecclesiastici de musica sacra potissimum , edited by Martin Gerbert (St. Blasien, 1784), [GS], II, pp. 69-93
^10 Musica Hermanni Contracti , op. cit .
The *Organon* of Aristotle

Aristotelian logic is comprised of several distinct parts, each of which received a measure of attention at various times during the Medieval period. His logical writings, known cumulatively as the *Organon* (or instrument), include the *Categories*, *De interpretatione*, *Prior analytics*, *Posterior analytics*, the *Topics*, and the *Sophistici elenchi*. Only the *Categories*, the *De interpretatione*, and the *Topics* are important to the present study.

Prior to the twelfth century only Aristotle's *Categories* and *De interpretatione* were known to Medieval scholars. The *Categories* discusses the divisions of reality as expressed through words that are not combined into sentences. The *De interpretatione* discusses the linguistic components from which statements are constructed and judgments made. In both treatises Aristotle relies on division to make his tenets clear.

Though division came under the rubric of logic for Aristotle, as an aspect of intellectual thought it was not neatly compartmentalized within any standard academic discipline. From Plato to Boethius and well into the early Medieval period, the science of division was virtually synonymous with dialectic. Plato clearly made this association in the *Sophist*, and Boethius rearticulated this association in his *Commentary on Cicero's*...
Since both logic and dialectic were parts of the Medieval Trivium, the science of division had a guaranteed position of importance.

Aristotelian Logic in the later Medieval period

The sphere of intellectual activity centering in France and especially Paris during the eleventh and twelfth centuries produced great thinkers ranging from Peter Abelard (1079-1142) to Thomas Aquinas (1225-1274), all of whom are indebted to Aristotle.

During the twelfth and thirteenth centuries, when the *Organon* was disseminated in Europe, exciting musical developments were taking place. Polyphony reached a high degree of development, the precedents of modern tonality were well underway, and musical notation reached an amazing degree of complexity.

Because Aristotle had become the philosophical authority for the later Medieval period, music theorists, like scholars in other disciplines, strove to verify their arguments through the adoption of Aristotelian terminology. Among those who were influenced in one way or another by Aristotle, were Johannes de Garlandia, the Anonymous of

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21 *Boethius In topica Ciceronis Commentarium*: "Plato calls dialectic a faculty which is able to separate into many parts that which is one thing just as it often separates a *genus* through its own different parts down to the final *species* and often collects things which exist as many into one thing through the reason of their *genera*." *Boethius In topica Ciceronis Commentarium, Patrologiae cursus completus, series latina*, accurante J. P. Migne (Paris, 1891), vol. 64, Lib, 1, 1045B 1-8: "Plato etiam dialecticam vocat facultatem quae id quod unum est possit in plura partiri, velut solet *genus* per proprias *differentias* usque ad ultimas *species* separari, atque ea quae multa sunt, in unum *generam* ratione colligere." Also see Eleonore Stump, *Dialectic and It's Place in the Development of Medieval Logic* (Ithaca: Cornell University Press, 1989), p. 2.

22 Abelard's primary work to be influenced by Aristotelian logic was his *Dialectica*, edited by J. M. de Rijk (Wijsgerigeteksten en studies, I). Of all the pre-Organon logicians, Abelard is conceded to have been the most influenced by Aristotle. See Martin Tweedale, "Abelard and the Culmination of the Old Logic," *The Cambridge History of Later Medieval Philosophy*, op. cit., p. 143.


Anonymous IV, Jacques de Liege, and Marchetto de Padua. Recent studies by Yudkin (1990), Huglo (1990), and Wingell (1990) have focused on the influence of Aristotelian logic and philosophy on music theorists during the later Medieval period.

Aristotelian Logic in the Early Medieval Period

The premise of this study is that the Aristotelian science of division influenced early Medieval music theorists in the same way that his other logical works influenced later theorists. Although less is known about the availability of Aristotelian texts on logic than about other texts, it is possible to show that even in the early Medieval period some scholars had a rudimentary familiarity with Aristotle's views.

Following models instituted by Alcuin and his pupil Rhabanus Maurus (776-856), cultural centers were founded during the late eighth and early ninth centuries at the abbeys of Saint-Gall, Fulda, Reichenau, and others. The few texts on logic available in these new cultural centers included Boethius' translations of the first two works of the Organon and treatises attributed to Augustine and Alcuin. Though logic was not a central part of the

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writings of Augustine and Alcuin, their authority as clerics may have been responsible for
the popularity of their treatises during the eighth, ninth, and tenth centuries.33

On the other hand, a catalogue of holdings at the monastic library of Reichenau as
early as the year 822 records copies of the Isagoge of Porphyry, the De interpretatione of
Aristotle,34 and a version of Boethius' translation of Aristotle's Categories. These logical
treatises will be discussed during the course of this document. At the beginning of his
Musica, Hermannus specifically refers to the Isagoge of Porphyry and the Categories of
Aristotle,35 two important logical texts. It is clear that these texts of the Organon were
available to the earliest German music theorists.

The Problem for early Medieval theorists

Despite Aristotle's influence on theorists during the later Medieval period, the eight
mode system, which had been formulated and adopted centuries earlier, was still the
classificational norm for polyphonic music.36 The system was the same one which
Carolingian theorists used in the classification of the chant repertory in their desire to
impose an order and unity on the large repertory of monophonic chant. Their goal was to
transmit a comprehensive picture of the total chant repertory through representative

80-1. The most studied work on logic from Alcuin onward was the Categoriae decem, which had been
attributed during the Middle Ages to Augustine. The Categoriae decem is a Latin epitome on Aristotle's
Categories. Alcuin's Dialectica shows knowledge, through Boethius' commentaries, of the Isagoge of
Porphyry and the De interpretatione of Aristotle. See Marenbon, p. 47.
34 Marenbon, op cit., p. 53.
35 Trans. Ellinwood, Musica Hermanni Contracti, op. cit., p. 19: "For, to make a long story short, just as
Grammar is reduced to eight varieties, the Isagoge to five, and the Categories to ten. . ." Musica
Hermanni Contracti, p. 19: "Ut enim ex multis dicamus, sicut grammatica ad VIII isagoge ad V
prae dicamenta ad X varietates rediguntur. . ." 
36 In this document the eight modes are regarded as a single system in which they are arranged in four pairs
of two, called the authentic and plagal modes, respectively. In each pair the first mode, or the authentic
mode, is regarded as superior to the second mode, or the plagal. Yet the two modes in each pair mingle
together as one unit.
examples, for it would have been an impossible task to unify performance practices without some kind of logical classification model. The problem for early Medieval theorists, then, was to find a taxonomic paradigm that would enable them to classify the chant repertory according to the eight modes.

Organization of this document

The goal of this document is to demonstrate the influence of the Aristotelian science of division on the organization of the eight modes of music by early German theorists.

Chapter I presents the science of division as it is defined by Aristotle. The discussion differentiates between qualitative and quantitative division, and gives examples of each. Because Aristotelian division features a specialized terminology called the predicables, Aristotle's definitions follow. Also included are Boethius' definitions of the predicables whenever they differ or amplify Aristotle's definitions. An understanding of the predicables is necessary because Boethius used them in the *De divisione*. Similarly I will use the predicables in the taxonomic paradigm when it is applied to the eight church modes.

Because quaternary division was especially common in Aristotelian division, examples of quaternary division in Aristotle's writings are included. Chapter I concludes with a comparison of quaternary division examples by the Pythagoreans and by the Medieval philosopher John Scottus Eriugena. Although the two divisions are separated by about fourteen hundred years, they show such a remarkable similarity that I use them here to show the pervasiveness of quaternary division from the Greek to the early Medieval periods.

Chapter II discusses the background of the *De divisione*, and includes comments on the preservation and transmission of this treatise.
Boethius discusses six categories of division in the *De divisione*. Five of these categories were used by German theorists in their modal discussions. Chapter III summarizes these five division categories.

Chapter IV presents a taxonomic paradigm derived from the *De divisione*, which adds musical nomenclature to the logical terminology.

Chapter V discusses the eight modes of music, focusing on the two primary ways early Medieval theorists rationalized the modes: 1) as a nucleus of tones and semitones around a specific pitch called a "final", and 2) as octave *species* constructed of *species* of fourths and fifths. *Species* theory was the manner in which early German theorists rationalized the modes. The different ways in which Byzantine and early Western music theorists divided the eight modes using quaternary division is also discussed.

Chapter VI applies the taxonomic paradigm of the *De divisione* to the eight modes of music. The purpose of this chapter is to show how the modal system can be rationalized through Boethius' division types.

Chapter VII discusses the terminological and conceptual usage of Aristotelian division in the eight mode systems of Berno, Hermannus, Wilhelm, Aribo, and Englebert. Quotations from their treatises are included as evidence of the influence of Aristotelian division. This chapter shows the degree to which these five theorists applied the Boethian concept of anthropomorphic opposition in their divisions of the eight modes by means of the division of *genus* into *species* through *differentiae*. 
CHAPTER I

The Aristotelian Science of Division

Division begins by amassing a collection of things to be classified. The collection is variously called the subject, the genus, or the whole. Once collection is completed, division begins and progresses to the point where further division is impossible.\(^1\)

Aristotle’s recognition of the value of division, as well as his observation about the limits of its usefulness, can be found in several logical and biological works.\(^2\) The most important extant treatise on division after Aristotle is Boethius’ *De divisione*. Since the *De divisione* is based on Aristotelian division, the discussion in this chapter will focus on Aristotle’s views about division.

Aristotle’s discussions on division feature a specialized set of terms called predicables, and these terms were adopted by Boethius in his logical monographs, including the *De divisione*. Because the Aristolian science of division cannot be comprehended without an understanding of the predicables, definitions of them are presented before discussing division.

This chapter is concerned with two types of division which are called here qualitative and quantitative division. Though Aristotle did not specifically name quantitative division, he said in the *Categories* that genus and species signify substance qualitatively differentiated,\(^3\) and his discussions suggest that division is either qualitative or quantitative, depending on the goal. For instance, if one wishes to reach a *definition* of


the subject which is divided, then qualitative division is necessary. If the goal of division is to reach a plurality of parts with no hierarchy, then quantitative division is required.

The number of parts into which either the subject or the genus is to be divided can vary with the individual doing the dividing. Examples of binary, ternary, and quaternary division can be found in Aristotle's writings. However, if frequency of usage is any indication of usefulness, then quaternary division was the most useful division type for Aristotle. To illustrate this point examples of quaternary division by Aristotle are included in later discussion.

Although Boethius' De divisione is a summary of Aristotelian division, Boethius also incorporated an important Platonic device in his treatise: the use of an anthropomorphic illustration to show a division type. Plato derived anthropomorphism from the Pythagoreans, and Boethius got it from Plato. Medieval scholars acquired Pythagorean anthropomorphism from both Plato and Boethius.

The chapter will conclude with an example of quaternary division by the Pythagoreans which shows a striking one-to-one correspondence with a division application by one of the most important early Medieval philosophers, John Scottus Eriugena (d. 877). Both the Pythagoreans and John Scottus accompany their quaternary divisions with anthropomorphic illustrations. After the Pythagorean example is given, a short discussion follows which suggests that the quaternary divisions of the Pythagoreans and John are in reality examples of dual division. The example is given as propeadeutic to the discussion of the dual yet quaternary division of the eight church modes given in Chapter VI.

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4 Chadwick, Boethius, op. cit., p. 201.
The Predicables

Medieval writers on division inherited from Aristotle and Porphyry a set of six terms which are used in division. These terms, called predicables, were developed by Aristotle for use in structuring propositions. The definitions of the predicables given here are based on those of Aristotle and Boethius.

Kretzmann and Stump (1990) define a predicable as a universal attribute which can be asserted of more than one thing. They list six predicables: 1) definition, 2) genus, 3) species, 4) difference, 5) accident, and 6) property. With the exception of accident, all of the predicables were used in the discussions of mode by the German theorists that I will examine in chapter VII.

*Genus* and *species* refer to things or attributes of things. *Genus* is a comprehensive class which is comprised of *species*. For Aristotle a *genus* is "what is predicated in the category of essence of a number of things exhibiting differences in kind." The *genus* is always of wider denotation than the subclass *species*. In the *De divisione*, Boethius defined *genus* as that which is predicated of more than one thing differing in *species* in respect of what it (the *genus*) is. In other words, for Aristotle and Boethius, *genus* and *species* share some common attribute or attributes.

*Species* is a sub-class of *genus*. Though a *species* usually contains several members which are individualized by some agent or *difference*, Aristotle said that the *species* can also be synonymous with individuals. In the *De divisione* Boethius equated

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8Ibid., 121b, 3-5.
9Boethius, *De divisione*, Patrologiae latina [PL], accurante J.-P. Migne, 880A13-15: "Genus est quod praedicatur de pluribus species differentibus in eo quod est..."
species with genus, since "genus is always the proper whole of the species, but is more general in nature."11

A difference is the means by which we present one thing as standing apart from another.12 Differences are used to divide the genus into species. A difference is part of the essence of a subject that distinguishes one species from other species in the same genus. Boethius went a step further and recognized two types of difference: constitutive and divisive. The "constitutive" difference allows a new species to stand in the place of the genus. In this instance, the new species then becomes a genus which can be divided into still other species. The new species is called a subaltern genus. The "divisive" difference simply divides the genus.

Accident is an inessential property that may be attributed to a subject. Accident is defined by Aristotle as something which may or may not belong to some "self-same thing", such as in "sitting," or in possessing the quality of being "white."13 For Boethius, if an attribute is not predicated on the subject's essence, it is termed an accident.14

Property is an accident that has a special relationship to the subject: it must be capable of being predicated convertibly of the subject.15 As an example of this, Aristotle says a property of man is that he (man) is capable of learning grammar. If a subject is capable of learning grammar, then he is a man.16 It can be seen that property is an attribute which is unique to a specific subject.

11 Ibid., PL 878D13-879A1: "...genus semper speciei propriae totum est, ut universalius in natura."
12 Ibid., PL 880A15-B1: "Differencia, quia alius ab alio distare proponimus."
14 Boethius, De topicis differentiis, Lib. II, PL 1186C9-12: "Solumvero inesse ad accident inesse necesse est, nam cum neque ut genus, neque ut diffinitio, neque ut proprium inest, sed inest tamen, ut accident inesse necesse est."
16 Ibid.
A definition is comprised of a genus plus one or more differentia. In the De divisione, Boethius said that division is necessary for a full definition of species, and that a single definition is made of divisions joined together.\(^\text{17}\)

In summary, the predicables can be divided into two classes. On the one hand, genus and species refer to things or attributes of things. On the other, difference, accident, and property are the means by which we assign things or attributes of things to their proper genus or species. This process creates a definition. The predicables, then, provide a general rationale for the construction of theoretical descriptions\(^\text{18}\) (or definitions), and this rationale creates the necessity for class-inclusion (as in genus or species), and class exclusion (as in difference, accident, or property).

Qualitative and quantitative division

Aristotle’s discussions of division differentiate two categories of division: 1) qualitative division, and 2) quantitative division.

In qualitative division, a hierarchical order of differentiae is successively added\(^\text{19}\) to the genus being divided.\(^\text{20}\) The goal of qualitative division is to reach a definition of the divided subject which reveals the nature of kinds of things of which the subject is an example.\(^\text{21}\) When dividing qualitatively, one establishes a series of greater or lesser degrees of differentiae, so that the addition of a difference to the genus “intensifies that

\(^{\text{17}}\)Boethius, *De divisione*, op. cit., PL 660C13414: “Nam divisionibus junctis una compositur definitio.”


\(^{\text{19}}\)To use the term "added" in a discussion of division would seem to be a contradiction. Yet Aristotle makes it clear that the addition of differentiae to a genus is the means for isolating the uniqueness of a particular subject which is to be defined. In the *Topics* he said that "for the frame of a definition should first place the object in its genus, and then append its differences." *The Works of Aristotle*, edited by S. D. Ross (London: Oxford University Press, 1928). *Topica*, translated by W. A. Pickard-Cambridge, 139a, 28-30.


\(^{\text{21}}\)Desclauriers, *Plato and Aristotle on Division and Definition*, op. cit., p. 208.
character which it has as a given." Aristotle said that the genus should fall under the same division as the species, meaning that both the genus and species are to be divided quantitatively or qualitatively. For Aristotle, the purpose of division is to discover "kinds" of entities which contain the same differentia. In reaching the definition, it is possible to include as many differentiae as one wishes as long as they form a consistent line of division.

Another purpose of qualitative division is to show logical relations between classes. An example of this is can be seen in the Medieval Porphyrian Tree. The Porphyrian Tree is an example of qualitative division of the genus Substance. Substance is divided into the species animate and inanimate bodies. Under animate body comes rational and irrational animal. Under rational animal is man, and under man are Socrates, Plato, and particular men.

When deciding whether to divide qualitatively or not, one should study the class of predicates to see if they admit an increase of degree. If they do, then qualitative division is the most useful division type. Diagram one shows how a definition is reached through the successive conjunction of differentiae to a genus creating species. The diagram shows how a species is sought through the addition of a differentia to the highest genus. If a definition of the species is not reached, another differentia must be added to the existing differentiae. A definition is reached when enough differentiae have been added to the highest genus to reach the definition of a species.

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24 *Ibid.*, 143a34-b1
Diagram one. The creation of a *definition* through the qualitative addition of *differentiae* to a *genus* as an example on qualitative division.

Quantitative division yields parts of things, not kinds of things, and these parts are related, although not hierarchically. The many parts of a divided whole may be listed or regarded in any order. This process is the reverse of qualitative division in that qualitative division proceeds from many parts to one part (*the definition*), while quantitative division proceeds from a singular entity (the whole) to many parts. If the class of predicates of a whole represent quantity rather than quality, that is, if the predicates do not admit an increase of degree, then one must use quantitative division. Diagram two illustrates quantitative division of a whole into parts.

Diagram two. The quantitative division of a whole into parts.

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29Ibid., 152a, 37-152b,5.
Both qualitative and quantitative divisions were used by the Medieval German theorists in their discussions of the modes.

The Stages of division

Division involves three stages. The first stage consists of collecting the things to be classified. Stage two establishes the differences among the things through the addition of *differentiae*. For Aristotle, these *differentiae* must be based on naturally occurring conditions. Stage three consists of classifying the results of the division according to an external taxonomic paradigm.

After division is completed, representative examples of each of the categories are selected for demonstration. This process is called induction, and the greater the number of divisional categories, the more accurate will be the inference concerning a particular category.

Both qualitative and quantitative division were basic to that part of Aristotelian logic known to early Medieval German music theorists. As I will show in Chapter IV, both division types were prominent in the *De divisione* of Boethius.

Examples of division in Aristotle's writings

Quaternary division was the division type most frequently applied by Aristotle. A few representative examples of this division type will demonstrate its importance.

As shown in Dumitriu's *History of Logic*, Aristotle's entire corpus of writings can be divided into four areas: 1) logic, 2) rational philosophy, 3) metaphysics, and 4) practical sciences.

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In his *Topics*, Aristotle divided propositions into four types. The elements from which propositions or problems are formed are 1) *property*, 2) *definition*, 3) *genus*, and 4) *accident*. According to the nature of their predicates, any proposition belongs to one of these four *species* of propositions.\(^{33}\)

In his *Physics*, Aristotle outlined four universal conditions which are common to all natural phenomena. The four conditions are 1) *movement*, 2) *place*, 3) *void*, and 4) *time*.\(^{34}\)

In his earlier work, the *Categories*, Aristotle spoke of the four types of “quality,” which include 1) states or conditions, 2) things in virtue of a natural capacity or incapacity, 3) affective qualities and affections, and 4) the shape and figure or external of a thing.\(^{35}\) Also in the *Categories* he also spoke of the four types of “things that are.” These include 1) things that are said of a subject and in a subject, 2) things that are in a subject but not of a subaltern subject, 3) things that are both of a subject and in a subject, and 4) things that are neither in a subject nor said of a subject.\(^{36}\)

In book II of the *Posterior analytics*, Aristotle said that there are four questions which cover the whole sphere of knowledge. These are 1) is the connection of an attribute with a thing a fact, 2) is there a reason for the connection, 3) does a thing exist, and 4) what is the nature of a thing.\(^{37}\)

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\(^{36}\) Ibid., p. 4.
Aristotle's famous four primary "causes" for the understanding of particular things are listed in the *Metaphysics*. The four "causes" are: 1) formal, 2) material, 3) efficient, and 4) final.\(^{38}\)

In summary, many examples of quaternary division can be found in Aristotle's thinking. It is reasonable to infer that, if Aristotle's definition of division was adopted by Medieval scholars, then his predilection for quaternary division may also have been equally influential.

The Influence of Greek Quaternary Division on John Scottus Eriugena

Quaternary division can be seen in Medieval philosophical treatises, particularly in those by writers trained in logic.\(^{39}\) Moreover, it is evident that the manner in which some Medieval writers divided a subject was so similar to Greek procedures, that a direct influence is likely. One remarkable example of the commonality between Medieval and Greek quaternary division will illustrate this point.

A striking similarity can be seen between the Pythagorean quaternary division of numbers and the quaternary division of the Universe by the important early Medieval philosopher John Scottus Eriugena (d. 877). John, who was associated with Charles the Bald, grandson of Charlemagne, conjoined logic with his theological system in a major philosophical work entitled the *Periphyseon* (c. 870).\(^{40}\) This text, although primarily a Neo-Platonic work,\(^{41}\) shows that John possessed a substantial knowledge of Aristotelian


\(^{41}\) Some of the Neoplatonic elements in the *Periphyseon* include: 1) the trichotomy of the Universe into God, the (Platonic) Forms, and Sensible Matter, 2) a regard for the movement of universal nature as consisting of three aspects: permanence, procession, and return, 3) an echo of Plato's World Soul (*PL* 122, 476C-7A), and 4) the Soul (human soul) is responsible for creating all that lies below the the intelligible world. See Marenbon, *Early Medieval Philosophy*, pp. 60-70.
logic. The entire content of the *Periphyseon* is structured around a quaternary division which is similar to the quaternary division of numbers by the Pythagoreans. Shown below is a comparison of the quaternary division of numbers by the Pythagoreans with an "anthropomorphic" division of the Universe given in the *Periphyseon* of John:

<table>
<thead>
<tr>
<th>Pythagoreans</th>
<th><em>Periphyseon</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Monad: That which begets without being begotten</td>
<td>1. That which creates and is not created.</td>
</tr>
<tr>
<td>Tetrad: That which is begotten and does not beget</td>
<td>2. That which is created and creates.</td>
</tr>
<tr>
<td>Ogdoad: A three dimensional construct of the sensible world.</td>
<td>3. That which is created and does not create.</td>
</tr>
<tr>
<td>Hebdomad: That which is neither begotten nor begets.</td>
<td>4. That which neither creates nor is created.</td>
</tr>
</tbody>
</table>

Table one. A comparison of the quaternary division of numbers by the Pythagoreans and the division of the Universe in the *Periphyseon*.

Hadot has suggested that this Pythagorean division was known to John through a letter which included a dispute between Candidus and Marius Victorinus over the issue of the Hebdomad, in which God, who is Immutable, is neither created nor creates. John seems to have made some notations in this letter.

If one looks closely at the quaternary division of the Pythagoreans and of John in the *Periphyseon*, it seems that two of the divisions are unnecessary. These are the Pythagorean Ogdoad and the Hebdomad, and John's divisions two and four. All that exists can be justified within the Pythagorean Monad and the Tetrad, and divisions one and three of the *Periphyseon*. For the Pythagoreans, the Tetrad (the Universe) exists because

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42 This view of the Pythagoreans was transmitted through Philo, whose writings indicated that this Pythagorean quaternary division was known to Plotinus and Philolaus, and was hence known in Neoplatonist circles. See *The Cambridge History of Later Greek and Early Medieval Philosophy*, ed. Armstrong, p. 522.


of the existence of the Monad (God). The Monad is the only division that can create (beget). An identical relation can be seen between divisions one and three of the *Periphriseon*. In effect, all existence, whether it is God or the Universe, can be assigned to these two divisions.

In summary, the quaternary division examples of the Pythagoreans and John Scottus can be seen as actually constituting a dual division type. The Monad of the Pythagoreans and John’s division number one are “perfect” and “immutable” because they are capable of creating without themselves being created. The Tetrad and John’s number three are “imperfect” and “mutable” because they are created without creating anything themselves.

This point is stressed here because it reflects the Medieval conception of the relationship between the authentic and plagal modes which are discussed in chapters V and VI of this document. In this conception, the authentic modes are seen as “immutable” and “perfect”, while the plagal modes are seen as “imperfect” and “mutable.”

**Conclusion**

Through his definitions and examples, Aristotle demonstrated his valuation of division. Not only his prescriptions of division for others, but his own division of the sciences shows the practical ways he uses division.

The predicables may seem ambiguous to us today, yet Aristotle’s careful definitions of them, as well as the amount of attention he spent explaining division, may help explain why the Medievals regarded division so highly.
CHAPTER II

The History of Boethius' De divisione

Boethius' De divisione is a technical treatise on the application of classification techniques. Like technical books written for the two other disciplines comprising the early Medieval trivium, grammar and rhetoric, the De divisione was written for a specific practical purpose. Its purpose is to show how to divide and classify a subject through observation of similarities and differences among the subject's constituent parts, and thereby reach a definition of the subject.

Boethius' De divisione, a treatise written between 505 and 509, is relatively short, if it is compared with some of his better known texts, the De institutione musica, the De arithmetica, and the Consolation of Philosophy, as well as some lesser known texts, the Commentaries on Aristotle's Categories and the De interpretatione. It would be easy to underestimate the importance of the De divisione. No English translation was available until 1990, although an Italian translation and commentary appeared in 1969. Scholars interested in the text must still consult the Latin text found in the Migne edition of the Patrologia latina. However, the De divisione, as well as similar early texts on classification, had a long and pervasive influence on Medieval logic and taxonomy. Both this treatise and the De topicis differentiis, another monograph of Boethius with which the

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3See the Trattato sulla Divisione, translated by Lorenzo Pozzi (Padua: Liviana, 1969).
De divisione was frequently paired, were part of the "logica vetus," or "old logic," the primary course of study for scholars from the fifth to the twelfth centuries. (The De topicis differentiis is a detailed study of dialectical and rhetorical topics for the purposes of argumentation.)

Boethius' sources for the De divisione

At the beginning of the De divisione, Boethius attested to the importance of his subject by briefly discussing some of his precedents. He began by referring to a lost treatise on division by Andronicus of Rhodes, a Peripatic philosopher of the first century B.C. Boethius also referred to Porphyry's commentary Plato's Sophist, also lost. From these statements, Chadwick has suggested that a major source for the De divisione was probably this Porphyrian commentary, since it was evidently designed to synthesize the Platonic dialectic of the Sophist with that of Andronicus on division. Magee (1991) agrees that the Porphyrian commentary was probably a source, even though the text of the De divisione mentions no specific Platonic dialogues, but questions the work of Andronicus, due to slim evidence of its existence.

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7 Boethius's De topicis differentiis, trans. Stump, p. 221.
8 Boethius, De divisione, PL 875D1-4: "Quam magnos studiosis afferat fructus scientia dividendi, quamque apud peripateticam disciplinam, semper haec fuerit in honore notilia, docet at Andronici, diligentissimi senis, de divisione liber editus." Andronicus was also the first editor of the Aristotelian Organon. See Ebbesen, "Ancient Scholastic Logic," The Cambridge History of Later Medieval Philosophy, p. 105.
9 Ibid., PL 876D1-2.
10 Chadwick, Boethius, op. cit., p. 164.
12 Ibid.
The divisional paradigm used by Boethius can, however, be found in other writers who preceded him, including Albinus, Sextus Empiricus, and Clement of Alexandria. Reiss (1982) suggested that in the De divisione Boethius intended to “correct” Aristotle’s view on the value of division. In the preface to an Italian translation of the De divisione Pozzi (1969) stated that the Boethian monograph is an original synthesis of the Stoic, Platonic, and Aristotelian conceptions of division.

The fact that the De divisione and the De topicis differentiis were considered original monographs by Boethius may have contributed to their great prestige during the Medieval period. This is especially the case during the early Medieval period, before the large number of manuscript copies of the De musica and the De arithmetica were circulated. It was on these latter works that much of Boethius’ later reputation was to rest. Through Boethius’ logical monographs the Medievals received their basic philosophical concepts, particularly the terminology used in Medieval Logic.

When the entire Aristotelian Organon became available in the early twelfth century in new translations, only the De divisione and the De topicis differentiis were retained in the curriculum at the University of Paris, but modern scholars agree that Boethius was one

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13 Chadwick, Boethius, op. cit. In Book I, Chapter XII of the De institutione musica, Boethius also mentioned Albinus. In this chapter, Boethius referred to the method of division of voices. He described it as a twofold division, which included: 1) Continuous voice which is interrupted by intervals, and 2) Continuous voice as when speaking or reciting a prose oration. Boethius said that Albinus added a third type which included an intermediate voice, such as in the recitation of heroic poems as in a singing manner. See the De institutione musica, translated by Calvin Bower.


of the main philosophical sources for the early Scholastics, and that he must be considered as second only to Augustine as an authority among Christian philosophers.\(^\text{18}\)

**Preservation and transmission of the *De divisione***

The preservation and transmission of the *De divisione* is difficult to trace during the centuries immediately after Boethius' death. A senator and friend of Boethius, Martius Novatus Renatus, may have been responsible for its earliest preservation.\(^\text{19}\) Renatus' name appears in a series of scribal subscriptions in several early manuscripts of the dialectical treatises. A manuscript of the tenth or eleventh century from Fleury\(^\text{20}\) contains comments of Theodorus the librarian; this fact indicates that the *De divisione* had been included. Theodorus claimed to have "corrected" the codex of Renatus.\(^\text{21}\) Renatus, who was evidently an important figure during Boethius' life, commissioned Theodorus to copy the dialectical writings at Constantinople.\(^\text{22}\)

According to Magee, Boethius' logical monographs were circulated as a corpus probably until the Carolingian period, when individual treatises began to be copied separately.\(^\text{23}\)

In the *Bibliotheca bibliothecarum* II of 1739, Montfaucon said that he had seen a codex which had been drawn by the scribe Theodorus from an autograph belonging to Flavian, a pupil of Priscian.\(^\text{24}\) This codex contained a copy of the *De divisione*. According to

\(^{18}\) *Boethius on Division*, *The Cambridge Translations*, eds. Kretzmann and Stump, p. 11.

\(^{19}\) Chadwick, *Boethius*, p. 255.

\(^{20}\) Orleans 267; Paris Bibl. Nat. nouv. acq. 1611 (libri 31). fo. 51r.

\(^{21}\) Chadwick, *Boethius*, p. 256.

\(^{22}\) Reiss, *Boethius*, op. cit., p. 135.

\(^{23}\) Magee, *Boethius' De divisione, text and sources*, op. cit.

Chadwick, the manuscript research of Lorenzo Minio-Paluello has produced two recensions of Boethius’ translation of Aristotle’s *Topics*. A fragment of the second translation was inserted into the middle of the *De divisione*, and was preserved in twelve manuscripts. Of these, the earliest one dates from the tenth or eleventh century. Manuscript copies of the *De divisione* may be found in the Arundel Manuscript Collection at the British Library, as well as in the Harleian Collection, also at the same library. Abelard’s short glosses on the *De divisione* are preserved in the Bibliothèque Nationale in Paris, and the *De divisione* was one of the direct sources for Abelard’s *Dialectica*. Yet, since the *De topicis differentiis* existed in about 170 extant manuscripts from the tenth to the eleventh centuries, it seems reasonable to infer that its companion, the *De divisione*, must also have been viewed as an important work. Confirmation of this view came from recent research by Magee, who showed that the *De divisione* was copied at least 195 times, and was widely circulated throughout Europe and Britain.

**Conclusion**

Boethius’ *De divisione* is a synthesis of division types given by the Greeks. Its structure suggests that it was a handbook designed for practical application. Since the treatise was known and copied many times during the Medieval period, it must have been considered a useful text in the division and classification of things.

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26Ibid., p. 201.
27*Bibliothecae Arundeliana*ae, No. 383, fol. 54, and No. 392, fol. 151.
28*Bibliothecae Harleiana*ae, No. 3272, f. 5.
29*Biblitheque Nationale*, MS. Lat. 13.368, fol. 146r-156r.
32Magee, "Boethius’ *De divisione*, text and sources,” *op. cit.*
However, information concerning the location of manuscripts and the availability of the treatise to scholars of the early Medieval period is still lacking. More research needs to be done in this area.
CHAPTER III

Summary of the De divisione

Boethius describes six types of division in the De divisione. Only five of these were used by the German theorists discussed in Chapter VII of this study, but since this chapter summarizes the De divisione, all six division types will be included. The taxonomic paradigm which is presented in Chapter IV is based on the five division types that were actually used by the theorists.

The De divisione is not Boethius' only treatise which features the division types presented here. For example, in Boethius' Commentaries, three to five division types can be found. However, it is only in his second Commentary on Porphyry's Isagoge that all six types are found, though they are listed in a different order than in the De divisione.¹

General categories of division

In the De divisione, Boethius presents two general categories of division, each of which is subdivided into three types. Each of the six types has specific criteria that identify the type of division to be employed. Of Boethius' six types of division, four use the predicables, and though the other two do not, they show an obvious indebtedness to Aristotelian terminology.

A. Division \textit{secundum se.}²

1. The first general type of division \textit{secundum se} is the division of a genus into two or more species³ by the appending of a difference.⁴ A difference is part of the essence of a subject that distinguishes a species from all other species in the same genus. The more differentiae that are added to the subject, the more specific the definition of the species becomes. This is the basis of qualitative division in which species are hierarchically related.

2. The second type of general division \textit{secundum se} is that of a whole divided into its proper parts:⁵ "the number of times we separate each out of the whole."⁶ One may divide the whole into as many parts as needed. Though this division type appears obvious and simplistic, it is distinct from the division of a genus into species, since the division of a whole into parts is an example of quantitative division. Thus the parts are not hierarchically related. Boethius stresses that the parts are already present in the whole, and that the whole continues to exist after the parts are determined. (Boethius' wording for this division type should be considered carefully, since the division of a whole into parts can be seen as contingent on the "paradox of Increase," a principle articulated by Henry⁷ after Abelard's description of the principle in the \textit{Dialectica}. The paradox states that it is impossible for anything to increase by the addition of parts, because when further parts are adjoined to a

²Boethius, \textit{De divisione}, \textit{PL} 878D5-6: "Omnis enim vocis et generis et totius divisio, \textit{secundum se} divisione nuncupatur."
³\textit{PL} 877B8-9: "Est enim divisio \textit{generis} in \textit{species}."
⁴\textit{PL} 879D1: "Nam sicut \textit{species} ex \textit{genere} constat et \textit{differentia}..
⁵\textit{PL} 877B9-10: "Est nuncus divisio cum totum in proprias dividit unum et resolvimus..."
⁶\textit{PL} 877C12-13: "Totum in partes dividitur quoties in ea ex quibus est compositum unum quodque resolvimus..." Normore points out that, in Medieval philosophy, the application of the principle that numerically distinct things are wholly distinct - that is, they contain no overlap of parts - was a pervasive concept in metaphysics. He cites Abelard, who, in the \textit{Theologia Christiana}, Bk. III, pars. 148-53, stated that having no overlap of parts is what characterises numerical distinctness. See Calvin G. Normore, \textit{The Tradition of Medieval Nominalism}, \textit{Studies in Medieval Philosophy}, p. 214.
⁷D. P. Henry, \textit{Medieval Logic and Metaphysics} (London: Hutchinson University Library, 1972), p. 120.
thing, neither that to which the parts are adjoined, nor the adjoined parts themselves, increase in the sense that they have more parts than they had before. I make this point here in order to show that for Boethius the number of parts contained in a whole is a fixed and inevitable quantity, which can be divided in one and only one way.)

3. The third type of division secundum se is that of an expression (or utterance) that signifies many things when it is examined and the plurality of its significations is disclosed. This division type is subdivided into two groups, nouns and expressions which consist of nouns and verbs. Boethius calls the proper signification of a noun or name a partition of equivocation, while the distribution of an expression is a discrimination of ambiguity. The division of a whole into parts and the division of an expression are the two types which do not feature the predicables.

B. Division secundum accidentis.

1. The first type in this category is the division of a subject into its accidents. Boethius exemplified this division type by saying that of (the subject) Man, some are black, some white, and some of an intermediate color.

8 Petrus Abaelardus, Dialectica, ed. De Rijk, p. 421, 25-37: "Neque enim cum aliquid alicui apponitur, allud quod appositum est, crevit neque illud cui appositum est, cum plures partes quam prius non habet."
9 De divisione, PL 877B10-11: "Est alia cum vox multa significans in signicationes propria recipit sectionem."
10 I am following the example of de Rijk, who translates 'nomen' as 'noun'. See de Rijk, "On the Chronology of Boethius' Works on Logic I," p. 46.
11 De divisione, PL 878A1-2: "Aut enim unum nomen multa significat, aut oratio jam nomibus verbisque composita. . . ." PL 877B12-13: "...est alia divisio quae secundum accidentes fieri dicitur. There are two types of accident: separable and inseparable. In the Aristoteles latinus, these two kinds of accident are given, but they are distinguished as 'common' and 'particular'.
13 PL 878A14-15: "...ut cum dicimus omnium hominum ali sunt nigrī, alii candidi, alii medii coloris."
2. The second division secundum accidens is the opposite of the first type. This is
the division of an accident into subjects. Boethius exemplified this type by saying that,
among desirable things, some are located in the soul while others are in the body, since
what is sought is an accident of the soul and the body, which are subjects of a goodness
located in them.

3. The final division of this type is that of an accident into other accidents. Boethius said this type is illustrated by the division of objects into solid and liquid
objects. He also said that this type of division becomes transformed into one of the other
two types of division by accident, if both accidents are possessed by the same subject.

The Oppositions

Near the end of the treatise, Boethius says that the division by accident involves
contrariety. Thus, the separation of a species from a genus occurs according to one of
four Oppositions:

1. Oppositions which are antithetical to each other, such as good and evil.
2. Oppositions which are due to a possession or privation, as when we apply the
difference of possession in the division of a genus.
3. Oppositions which occur through affirmation or negation. Concerning this type Boethius said it is often necessary to devise a species by negation, if a single word does not exist to give (the species) a simple (affirmative) name.

4. Oppositions which occur through relationship, as in father, son, master, slave.

Although the Oppositions in the *De divisione* are the same as Aristotle's in the *Categories*, Boethius listed them in a different order. However, Boethius did use Aristotle's order in his *Commentary on Aristotle's Categories*. Thus, it is possible that maintaining the Aristotelian order had some importance for Boethius in the *Commentary*. Of far more importance than the order of listing of the Oppositions is the fact that Boethius chose to illustrate two of the Oppositions with different examples than Aristotle's. Of these two, the example that Boethius used to illustrate the Opposition of relationship - father, son, master, slave - bears no resemblance to Aristotle's illustration of relationship. Aristotle exemplified the Opposition of relationship by comparing of a "double and a half", and "knowledge with the knowable." The fact that Boethius used an anthropomorphic example to illustrate the Opposition of relationship is significant for music theory because Medieval theorists viewed the authentic and plagal modes as existing in an anthropomorphic relationship. For example the authentic modes were often called "teacher" or "master," while the Plagal modes were called "pupil" or "servant."

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26 *PL 881D12-882A1*: "Tertia oppositio est quae est secundum affirmationem et negationem. . ."
27 *PL 882B6-9*: "Necesse est autem saepe species negatione componere, cum ea quam simplici nomine species volumus assignare nullo vocabulo nuncupatur. . ."
28 *PL 882A2-3*: "Quarta secundum relationem, ut pater, filius, dominus, servus."
29 In Chapter 10 of the *Categories*, Aristotle listed the four Oppositions as follows:
   1. relationship
   2. contrariety
   3. privation and possession
   4. affirmation and negation
   See Aristotle's *Categories and De Interpretatione*, trans. Ackrill, p. 31. In Chapter 11 of the *Categories*, Aristotle said that all contraries must either be in the same genus, or in contrary genera, or be themselves genera.
Boethius stressed the importance of applying the Oppositions to the division of genus into species. He said that the division by applying Oppositions as differentiae to genera are numerous, and, in most cases, we equate Oppositions with differentiae. The concern with the division of genus into two species through the difference of Opposition has important consequences for the four-fold division of the eight church modes into authentic and plagal species. This is because the authentic and plagal modes, which are comprised of species of fourths and fifths, are distinct due to the inverted positioning of the fourths and fifths. As I will discuss in Chapter VI, this inverted positioning is an example of Boethius' first opposition of contraeity.

The Division of a Whole into Parts

Concerning the division of a whole into parts, Boethius says that what we call a whole has many meanings, which include qualities of being continuous, non-continuous, universal, or consisting of some powers. Later he said that a whole is classified as a substance and a form. In the process of division, which may be seen as an opposite process from the construction of a definition (discussed below), Boethius said that a genus can be considered to be a whole, while the species can be regarded as a part. Differentiae may also be seen as wholes, while a species may be regarded as a part.
Finally Boethius asserted that, in some instances (such as the division of syllables into letters), things are perceived as "parts of the parts."  

No matter which of the six types of division one uses, whether: 1) the division of a genus into species, 2) the division of a whole into parts, 3) the division of an expression (or utterance) into many meanings, 4) a subject into accidents, 5) accidents into subjects, or 6) accidents into other accidents, the resulting subdivisions must exhibit some property which properly confirms the appropriateness of the division. Hence the species of a genus must possess the property of being in one of the four modes of Opposition. The genus must be seen as having been divided into certain "offsprings of itself", meaning that the division must exhibit the essential nature of the genus, while it must also exhibit a difference. Moreover, the division of a whole is not to be considered as arbitrary, but must be divided into a specific number of parts, since this division is done by quantity. Finally, the parts which constitute the whole should precede their completion, sometimes naturally, and also temporally.

Construction of a definition

After indicating that the demonstration of a definition is discussed in Aristotle's Posterior Analytics, Boethius gives a short but important synopsis of the rules for creating a definition. Boethius restricted his construction of a definition to intermediate

\[41 \text{PL 888B3-4: } "...alio tamen modo acceptae non partes totius, sed partes partium sunt." \]
\[42 \text{PL 881A4-5: } "Illa vero quae (differentiae) per se sunt sola ad divisionem generis apta sunt. Haec enim informant et perficiunt uniuscujusque substantiam. ..." \]
\[43 \text{PL 879B3-5: } "Generis quoque sectio a totius distributione sejungitur, quod totius divisio secundum quantitatem fit." \]
\[44 \text{PL 879B14-C1: } "Partes quae totum jungunt compositi sui perfectionem alias natura tantum, alias ratione quoque temporis antecedunt." \]
\[45 \text{PL 885D7-11: } "Et illud quidem, an illa possit diffinitio demonstrari, et quemadmodum per demonstrationem valeat inveniri, et quaecunque de ea subtilius in Postremis Analyticis ab Aristotele tractata sunt praetermittam." In Book II, Ch. 10 of the Posterior Analytics, Aristotle discussed the types of definition. He defined definition as: a) an indemonstrable statement of essential nature, or b) a syllogism of essential nature differing from demonstration in grammatical form, or c) the conclusion of a demonstration giving essential nature. \]
things, since these can be predicated of other genera, species, or individuals. It is significant that Boethius said no definition can be found for either the “higher” genera, or the “lower” things, such as individuals, since these lack specifying differentiae. His purpose in this section of the De divisione was to demonstrate that the definition of a species consists in the conjoining of a genus with a sufficient number of differentiae to ‘equal’ the species. As an example, he applied his method to the definition of a “Noun.” This process is to be done through the division of the differentiae of a genus.

As if it were a summary statement, Boethius clarified the relationship among the genus, the species, and the definition. He said that a genus is a whole in division, but [the species] may be a part in a definition. Boethius went on to say that, in the construction of a definition, the genus and the differentiae are parts, while the species which results from this process can now be considered a whole. Boethius was suggesting that the distinctions among genera, species, whole, and parts are not in fact rigid, but that the terms may sometimes be used interchangeably, depending upon whether division of a whole is into quantitative parts or the construction of a definition is being sought.

46De divisione, PL 886A5-6: "Mediae igitur quae et habeat generae, et de aliis.
47PL 886A6-8: "...vel generibus, vel de speciebus, vel de individuis praedicantur sub differentiam cadere possunt.
49PL 886A2-4: "Porro autem inferiores ut sunt individua, ipsa quoque specificis differentiis caret.
50PL 886B8-11: "...et postremo toties differentias differentiis distribuimus usque dum omnes juntae generi spectem aequali diffinitione describant. Boethius' method of constructing a definition is similar to Aristotle's discussion on the division of a genus, found in Bk. II, Ch. 13 of the Posterior Analytics. Especially pertinent are the three objects that Aristotle said were to be kept in view during the establishment of a definition by division. These are 1) one should admit only elements in the definable form, 2) these elements must be arranged in the right order, and 3) no elements must be excluded.
51PL 886B11-12: "Si est nobis diffinire nomen..."
52PL 887B13-14: "...quod genus in divisione totum est in diffinitione pars..."
53PL 887C16-D2: "In diffinitione vero et genus et differentiae partes sunt. Diffinita vero species totum."
Criteria for division

A. In the divisional paradigm suggested by Boethius, the following conditions hold for division secundum se. In some cases the conditions must hold for the division. In other cases the conditions may hold for themselves.

The genus:
1. Must be capable of division into two or more species.
2. Must be divisible into certain “offsprings of itself”, whereby the resulting species retain the nature of the genus.
3. Must be more universal than the species.
4. Must exist prior to the species.
5. Must be divisible by differentiae which are not separable (thus not using differentiae which are separable accidents, e.g. sleeping, standing, etc.).
6. Must be divisible according to definite criteria.
7. Must be capable of qualitative division.
8. May be most commonly divided by the first Opposition: contraeity.

The species:
1. Must possess differentiae which will distinguish it from other members of the same genus.
2. Must be the same in substance as the genus.
3. Must be capable of being identified by the name and definition of the genus.
4. May be based on negation only if a simple (affirmative) name does not exist.
5. May be substituted by a difference if no suitable name exists for the species.
6. May resemble a part in a definition.
The whole:
1. Must be divisible into parts which are continuous, non-continuous, universal, or possessing of certain powers.
2. Must be capable of quantitative division.
3. May exhibit the property that the parts themselves are capable of being divided.
4. May resemble the genus before division.

B. The following conditions must hold for division secundum accidens:

The subject:
1. May contain accidents which can be divided by each other if they are possessed by the same subject.

The accident:
1. Is usually capable of being contrasted with an opposite accident.

Conclusion

This chapter presented the six division types which Boethius gave in the De divisione. It is important to keep in mind the distinction between qualitative and quantitative division, since it is in qualitative division where the differentiae establish the definition of the subject. Although the differentiae are usually one of Boethius' four Oppositions, the decision as to what constitutes a single differentia is a matter of discretion. Two people may not agree on what constitutes a specific differentia or on the total number of differentiae needed for a definition.
CHAPTER IV

A Taxonomic Paradigm Derived from Boethius' *De divisione*

In this chapter two diagrams are presented. The first diagram shows the relationship among five of Boethius' six types of division. The second diagram is a taxonomic paradigm which will be applied to the eight church modes. In the taxonomic paradigm the predicables are combined with modal terminology used in German music treatises.

THE FIVE DIVISION TYPES USED IN MEDIEVAL GERMAN THEORY

Diagram three shows the relationships among five of Boethius' division types given in the *De divisione*.

Diagram three. Relationships among five division types of the *De divisione*. 39
Division Types in Diagram Three

I. Genus into species...

The division of genus into two or more species through the appending of differentiae to the genus is shown at the left of the diagram. Since the genus exists prior to species, genus is placed above species in the diagram. The differentiae used in the division of genus into species are qualitative, which means that, as differentiae are added to the genus, each differentia brings us closer to identifying the true nature of the genus. In qualitative division, then, successive differentiae are added to the genus until a satisfactory definition of a particular species is reached. Differentiae are added as needed to reach the definition of the species. This procedure is illustrated by the arrow pointing to the left at the top of the diagram.

The designation of a difference as either constitutive or divisive is shown on the left side of the diagram. If a difference is divisive, it distinguishes the various species of a genus from each other. If a difference is constitutive, it becomes or constitutes a species. In some cases a difference can be both constitutive and divisive, which is shown by two small reciprocal arrows.

When a genus is divided into species, these species may be regarded as subaltern genera, if these species are capable of division into their own species. A subaltern genus is a genus which is lower than the highest genus, but higher than the lowest species. The division of a genus into subaltern genera is shown on the lower left side of the diagram.

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2Division continues until no further division is possible.
Since subaltern genera are included under the highest genus, subaltern genera are placed under the genus, as shown at the lower left side of the diagram.

A species exhibits one or more properties, which are attributes unique to that specific species. The division of species by properties is shown on the bottom left side of the diagram. A property may also be an accident, if the property can be removed from a subject without destroying the subject. The far left side of the diagram shows a property which is also an accident.

II. Whole into Parts

The bottom right side of the diagram shows the division of a whole into parts. Since the whole is destroyed if any part is destroyed, the existence of the whole is posterior to the parts. Thus the whole is listed below the parts on the diagram, and this division type should be read from the bottom up. As a quantitative division type, it represents the division of a whole into integral parts which are not hierarchically related to each other. The division of the whole into parts, then, does not reveal the nature or essence of the whole. Since quantitative division is the opposite of qualitative division, it is illustrated by the arrow moving toward the right at the bottom of the diagram.

In certain cases, however, a part may be equivalent to a species. This equivalence is possible when the part itself is capable of being further divided into parts, each of which bears some commonality with the part which was divided. The equivalence of a part with a species is shown in the middle of the diagram.

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3See Chapter III, notes 42 and 43.
III. Subject into *accidents*

The division of a subject into *accidents* is shown on the right side of the diagram. The *accidents* are attributes of the subject which may be removed from the subject without destroying it. The arrow pointing to the right signifies that the division of subject into *accidents* may continue until all the *accidents* have been recognized.

IV. *Accident* into subjects

The right portion of the diagram also shows the division of an *accident* into subjects. In this division type, the *accident* is used to distinguish among subjects which feature that particular *accident*.

V. *Accident* into other *accidents*

The far right side of the diagram shows the division of an *accident* into other *accidents*. In some cases an *accident* and a *difference* may be regarded as interchangeable, just as an *accident* and a *property* may also be interchangeable. The interchangeability of an *accident* with a *difference* is illustrated in diagram three by small contrary arrows in the upper right side of the diagram.

A TAXONOMIC PARADIGM

Diagram four is similar to diagram three, except that terminology used in discussions of modes in music treatises is added to the logical nomenclature. Diagram four is the actual taxonomic paradigm which will be applied to the eight church modes in chapter V.
Diagram four. The taxonomic paradigm of the *De divisione*.
Discussion of diagram four

The top of the diagram shows the genus Protus divided into the species authentic and plagal modes. The genus Protus, which exists prior to the species, is divided qualitatively through the difference of the contrary positioning of the first species of fourth and first species of fifth. This difference can be seen as both a divisive and a constitutive difference, since it both divides the genus Protus, and constitutes the authentic and plagal species. (Obviously this same process can also be applied to the Deuterus, Tritus, and Tetrardus modes by the differentiae of other species of fourths and fifth.)

The bottom of the diagram shows the Protus, Deuterus, Tritus, and Tetrardus modes as parts of a quantitative whole. In the later Medieval period, the term “maneriae” was used to designate the whole. The whole in this diagram is shown to exist posterior to the parts. Just as the designation as Protus, Deuterus, Tritus, and Tetrardus are “parts,” the subdivision of them into constituent authentic and plagal octave species are “parts of the parts.”

The right side of the diagram shows the division of a Subject into accidents through the application of the synemmenon tetrachord. The appearance of the second species of fourth in the Protus authentic mode, which is shown in the middle of the diagram, illustrates the division of Subject Protus into accidents, since two forms of the Protus

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4 The tone.semitone construction of the three species of fourth are: first : TST; second: STT, third: TTS. There is a fourth species of fourth which is a duplicate of the first species (TST), but is located in a different position on the gamut. According to Hermannus, this species of fourth has a different constitution and power. See Musica Hermannus Contractus, trans. Ellinwood, p. 27: "Quarta (species ), D, G, in positione prima, in constitutione et postestate quarta..." The four species of fifth are: first: TSTT; second: STTT; third: TTTS; and fourth: TTST.

5See Chapter 2, note 31, for an explanation of the logical origin of the term 'maneriae'.

6The synemmenon tetrachord, which was derived form the Greek Lesser Perfect System and adapted to the Medieval gamut by Hucbald, features a tone.semitone.tone construction on the pitch g. This results in a non diatonic pitch, or b flat.
authentic mode result. Both of these forms feature the first species of fifth as the bottom fifth, but have two distinct species of fourth at the upper level: the first fourth and the second fourth. Yet both modal forms could be designated Protus authentic.

The far left side of the diagram shows how the division of accident into subjects may be applied to the octave species on a. The usual designation for the octave species on a is as Protus plagal mode. This mode is comprised of a first species fourth (TSS) placed below a first species fifth. However, if the b flat is included, the first species fourth becomes a second species fourth (STT). This results in a second (transposed) Deuterus authentic mode, which means that the Deuterus authentic mode has a second modal seat on a in addition to its normal seat on e. Thus the second fourth divides the octave species on a into two modes: the Protus plagal and a (transposed) Deuterus authentic. This was a process known as transformation.7 (The division of accident into Subjects is also applied to the octave species on g, shown on the far right side of the diagram.) Therefore the second species of fourth, which results from the synemmenon tetrachord, is also illustrative of an accident (the b flat) which is equivalent to a difference (since it creates two modes).

The top portion of the diagram shows the division of accident into other accidents. I am applying this division type to the division of the hexachord into tetrachords.8 The uppermost line on the diagram shows the construction of a hexachord, which is comprised of two tones, a semitone, and two tones. The tetrachord features a tone, a semitone, and a tone. Thus there is a nuclear tetrachord found in the hexachord. In this sense, it is possible to "divide" the hexachord by the tetrachord.

7Guido was among the first writers on music to use transformatio as a descriptive term, which refers to the 'transposition' from one of the usual modal seats found in the finales tetrachord. In chapter 9 of the Micrologus, Guido said: "Uptote si quis vellet antiphonam cuius principium essts in .D., .E., vel in .F., quae sunt alterius voces, incipere, mox auditu perciperet quanta diversitatis transformatio fieret."

8The creation of a hexachord by the addition of a whole tone at either end of a tetrachord was a unique theoretical construction of the five German theorists included in this study.
This division is shown on the second line at the top of the diagram with the letters T and S. Against the Medieval gamut, the tetrachord and hexachord can both regarded as 
*accidents*, since each can be removed from the gamut without destroying it.

**Conclusion**

Boethius' *De divisione* neatly summarizes the Aristotelian science of division. In spite of its brevity it's method is sufficiently comprehensive to apply to many disciplines. It is perhaps for this reason that the treatise was so valued during the Medieval period.

The remainder of this document will show how Boethius’ division types can be applied to the eight church modes.
CHAPTER V

The Eight Church Modes

This chapter will discuss the eight church modes from two perspectives: 1) mode as a nucleus of tones and semitones surrounding the final pitches of chants, and 2) mode as scalar species. Though there were other ways of identifying the mode of a chant, such as the initial tone or tones, medial cadences, range, and modal repercussion (repetition of motivic patterns within a chant), mode as nucleus and mode as species were the two primary means of modal determination in Medieval theory.

There were also two primary Carolingian means of modal determination. The first was to view the concluding pitch of a particular chant ended as the one which determined the mode.¹ In the other Carolingian conception which I call here mode as nucleus, the final tone is combined with the adjacent tones and semitones which lay on either side of it.

Although the Carolingian *Alia musica*² conjoined the eight modes with the Boethian seven octave species, the theory of mode as species developed only with Medieval German theorists. When the eight modes were borrowed by Western scholars from the Byzantines, the Carolingian view of mode as nucleus was the prevailing view. Yet, the later Medievals adopted the German conception of mode as species.³ Indeed, by the early sixteenth century, the conception of mode as octave species had become

¹ *Dialogues de musica*, *Source readings in Music History*, selected and annotated by Oliver Strunk (New York: W. W. Norton and Co., 1950), i, p.
The chapter concludes with a discussion on the use of anthropomorphic illustrations to show the relationship between the authentic and plagal modes. Though anthropomorphic descriptions are not part of Aristotelian logic, Boethius added this Platonic element in the *De divisione*. Anthropomorphism was used frequently by Medieval theorists as a descriptive device, and I suggest that its use possibly originated with the Pythagoreans, and was transmitted to Boethius through the *Timaeus* of Plato.

The "Octoechoi"

The eight church modes comprise part of a system used in the division and classification of monophonic chant during the Medieval period. The earliest Western writers to use the eight modes in the organization of chant were Carolingian. Borrowed by the Carolingians from the Byzantines, the eight mode system appeared in the West during the eighth and ninth centuries. As used by the Byzantines, the modes are known as the "octoechoi." The Byzantine limitation of the modes to eight may have had non-musical origins. Werner has stated that the limitation of the modes to eight may have been due to a calendric influence.\(^5\)

Carolingian scholars were faced with the task of classifying hitherto orally transmitted chant, since it was Charlemagne's wish to unify performance practice.\(^6\)

Because notation was only beginning to evolve, the problem of establishing uniformity in

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performance was difficult. The hope was that the “octoechoi” used as a model would prove useful in the division and classification of chant. Since the system of eight modes lasted for over 600 years, it can be seen that the Carolingians were successful in their quest for uniformity.

Mode as a nucleus of tones and semitones around a final

The earliest known Western treatise to divide chant according to the eight modes was the *Musica disciplina*. In this treatise, no discussion of an external construction of mode is given, since no such construction could be done without musical notation. Recognition of the modes had to be done through memorization of selected representative chants.

Other early Medieval treatises which discuss the division of chant according to the eight modes are those of Hucbald, the *Musica and Scholia enchiriadis* treatises, the *Alia musica*, the *Dialogus de musica* of Pseudo-Odo, and the *Commemoratio brevis*. Except for the *Alia musica*, the primary means of modal designation in these treatises referred to the nucleus of tones and semitones found around four primary “finals” or ending pitches. The four “finals” were identical to the tones of the “finales” tetrachord of the Medieval gamut, and the modal “finals” as a group constitute a tone/semitone/tone

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7 *Aureliani Reomensis Music disciplina*, ed Gushee, op. cit. In his Introduction to the *Musica disciplina*, Gushee adds that he is no longer inclined to grant priority to Aurelian as the first Carolingian theorist. That the *Musica disciplina* is chronologically the first treatise we possess has little bearing on whether it was the actually the first treatise written during this period. See Gushee, pp. 12-3.


10 Heard, *"Alia musica,*, op. cit.

11 *Dialogues de musica*, op. cit., 1, p. 114.

pattern which is found on the pitches \(D, E, F, \) and \(G\). The four nuclei around these four "finals" were designated by the Greek nomenclature Protus, Deuterus, Tritus, and Tetrardus, and the classification of a particular chant was made according to which of these nuclei the chant corresponded. As Crocker observed, modal classification focused upon the identification of the pitches immediately preceding the final in a chant, since this portion would be fresh in the ear when the chant concluded.\(^{13}\)

In the later Medieval period the nuclei of tones and semitones around the final came to be designated as the "maneriae," a term derived from Medieval logic.\(^ {14}\) In the ninth century *Alia musica*, the Greek terms Hypodorian, Hypophrygian, Hypolydian, Hypomixolydian, Dorian, Phrygian, Lydian, Mixolydian, and Hypermixolydian were used in modal designation.\(^ {15}\) In the earliest tonaries, the modes were identified by numbering them from I to VIII.\(^ {16}\) Modes I and II are the Protus authentic and plagal modes, III and IV the Deuterus modes, V and VI the Tritus modes, with VII and VIII the Tetrardus modes.

Chants of the same modal designation may have ranges that center them above or below the primary nucleus of tones and semitones. Thus one chant might have a tessitura lower than the final, while another chant might have a tessitura higher than the final. For this reason, it was necessary to divide each of the four primary modes into two modes.

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\(^{13}\)Richard Crocker, "Hermanns's Major Sixth," *op cit.*, p. 30.


\(^{15}\)Heard, *Alia musica*, *op. cit.*, p. 79.

\(^{16}\)Michel Huglo, *Les Tonaires*, p. 58: "Mais ce qui est plus important pour l'histoire des tonaires est de remarquer que l'*Alia musica* est, a notre connaissance le premier auteur medieval qui ait tente de lancer un pont entre ce qu'on appelle les 'modes grecs' antiques et le huit tons ecclesiastiques cette 'relation' etable par 'auteur principal entre les dorien, hypodorien, phrygien, etc. et les tons I, II, II, etc. se retrouvera consignee plus tard dans nombre de tonaires et de traites."
yielding the authentic and plagal modes. The Protus, Deuterus, Tritus, and Tetrardus designations, then, each refer to an authentic and plagal pair.

Although Byzantine and Western conceptions of mode featured similar nomenclatures, the manner of dividing the eight modes was different. The Byzantines ordered the eight modes into two sets of four, while Western writers divided the eight modes into four pairs of two modes.\textsuperscript{17} Diagram five compares the Byzantine and Western manner of ordering the modes with both the modal names and numbers.

<table>
<thead>
<tr>
<th>Byzantine</th>
<th>Western</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protus authentic</td>
<td>Tetrardus plagal</td>
</tr>
<tr>
<td>Deuterus authentic</td>
<td>Tritus plagal</td>
</tr>
<tr>
<td>Tritus authentic</td>
<td>Protus plagal</td>
</tr>
<tr>
<td>Tetrardus plagal</td>
<td>Deuterus plagal</td>
</tr>
<tr>
<td>Tritus plagal</td>
<td>Protus plagal</td>
</tr>
</tbody>
</table>

Diagram five. A Comparison of Byzantine and Western modal organizations.

There is always a second location in the diatonic octave where the nucleus of tones and semitones is identical with the nuclei surrounding the finals. This second location is found at the interval of a fifth above or a fourth below the authentic finals or seats, and was later called the affinal position. Theorists recognized that if a chant remained in the higher portion of its modal range it could conclude on the affinal pitch, or the secondary modal seat. The identical nuclear similarity of tones and semitones around the final and affinal positions was a critical factor in the pairing of the authentic and plagal modes.

It can be seen that, from the standpoint of intervallic similarity, the authentic/plagal modal pairs are related. In describing the relationship between the authentics and plagals, theorists frequently used anthropomorphic nomenclature, such as "teacher and pupil,"\textsuperscript{18} "author" or "master,"\textsuperscript{19} or, in the case of the authentic mode, the one "endowed with authority."\textsuperscript{20} The term "inferior" was often applied to the plagal modes, which meant that a plagal mode was lower than its corresponding authentic mode.

Diagram six shows the four authentic modal seats (finals), the plagal secondary seats (afffinals), and the pattern of tones and semitones which surround each. The primary seats are encased within the pitch set $C$ to $a$, while the secondary seats are encased within the set $G$ to $e$. Each pitch set is comprised of two tones, a semitone, and two tones.\textsuperscript{21}

\textsuperscript{18}Musica disciplina, ed. Gushee, \textit{op. cit.}, p. 79: "...quos quidam latus, quidam autem discipulpus nuncupant. Quod est evidentius apparent, si volueris segregare a magistro discipulum... ."

\textsuperscript{19}Dialogues de musica, \textit{GS}, vol. I, p. 258: "M. Quarta D. in quam terminatur modus, qui dicitur authentus protus, id est, auctor vel princeps primus,... ."

\textsuperscript{20}Bemonis, \textit{Prologus in tonarium}, \textit{GS}, vol. II, p. 68: "Authentos enim Graeci magistros dicunt, quasi auctoratos, id est, autoritate praeditos, quorum praecellit auctoritas." See also \textit{Musica Hermanni Contracti}, trans. Ellinwood, p. 32: "But this is done according to their position on the monochord, not according to their dignity, for each one of them is authentic, i.e. author or master, in itself." \textit{Musica Hermanni Contracti}, p. 32: "Sed hoc secundum monochordi positionem fit, non secundum dignitatem, nam unusquisque eorum per se authenticus est, hoc est auctor sive magister."

\textsuperscript{21}I am using primary seats to represent the modal finals, and secondary seats for the cofinals.
Mode as species

In the early Medieval period, there developed another way of viewing the eight modes. This was to regard the modes as octave species which are in turn comprised of species of fourths and fifths. Prominent among those who regarded mode in this way were early German theorists, including Berno, Hermannus, Wilhelm, Aribo, and Englebert.

In species theory, the patterns of whole tones and semitones which surround modal seats are viewed within an identical interval of fourth or fifth. The perfect fourth or fifth becomes a boundary interval. Although the boundary interval is the same for all nuclei contained within it, the differing pattern of whole tones and semitones contained within the

Diagram six. The four authentic modal seats and the four secondary seats with surrounding whole tones and semitones.
boundary interval means that several scalar varieties of fourths and fifths result. These varieties are called *species*. Gushee points out that *species* theory was the hottest theoretical topic during the eleventh century.²²

Diagram seven shows the three *species* of fourth and the four *species* of fifth.

![Diagram of species of fourth and fifth](image)

**Diagram seven.** The *species* of fourths and fifths.

In *species* theory, *species* of fourths and fifths are combined to form octave *species*. Seven *species* of octave may be created by this combination. Only some, however, were considered acceptable. In the Medieval modal view these were octave *species* that were created from similarly numbered *species* of fourths and fifths. The Protus authentic mode (Dorian), for example, is comprised of a first *species* fifth, or D-a, placed below a first *species* fourth, or A-D. The Protus plagal mode (Hypodorian) has the same *species* of fourth and fifth, but their positions are inverted. The Deuterus modes (Phrygian and Hypophrygian) feature the second *species* of fourth, or B-E, and second

species of fifth, or E-b. The Tritus modes (Lydian and Hypolydian) feature the third species of fourth, or C-F, and third species of fifth, or F-c.

The Tetrardus modes (Mixolydian and Hypomixolydian) have two problematic characteristics. The first characteristic of the Tetrardus modes is that they are constructed with a fourth species of fifth, T T S T T as the bottom element, but with a first species of fourth, T S T, as the upper element. The latter is placed on the pitch G instead of D, as it is in the Protus modes. Hence the Tetrardus modes are constructed of unlike species of fourth and fifth. The second more interesting characteristic is that the Tetrardus plagal mode, which extends from D to d, is exactly the same octave species as the Protus plagal mode. Since Medieval theorists sought a single identity for each octave species, a dual designation of the D-d octave posed a problem for the modal nucleus theory, but could be accommodated by the species theory.

Diagram eight shows the eight mode system divided into four pairs of two modes. Each pair shows the primary modal designation, such as Dorian, Phrygian, etc. on the top of each staff. Under each staff appear the related plagal modes, each of which is given the prefix “Hypo.” The brackets show the octave species for each mode.
The anthropomorphic view of the authentic/plagal pairs

We now turn to a discussion of the Medieval view of the relationship between the authentic and plagal modes. The fact that anthropomorphic nomenclature was used in describing the modal pairs has been discussed earlier in this chapter. This anthropomorphic view of modes had a philosophical origin.

This origin is in the Pythagorean concept of duality. The Pythagoreans had established a series of ten ‘oppositions’ as a means of understanding reality. Foremost among these oppositions was the concept of odd versus even. Odd represented

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23See pp. 47-8.
conceptually the Creator, and would hence signify such attributes as "unity," "indivisibility," and "immutability." Even represented conceptually the Creation, and represented "divisibility," "alterity," or a "determined" nature. The Pythagoreans also applied dualism to numbers directly. Odd numbers represented "immutability" and "indivisibility," while even numbers represented "divisibility" and "alterity."

The Pythagorean conception of "unity" versus "alterity" as applied to odd and even numbers can be seen in Plato’s *Timaeus*. This cosmological treatise was the most influential work of Plato during the early Medieval period, and it exhibits Pythagorean influence. In the *Timaeus*, Plato said that all Creation displayed alterity as distinct from the Creator, who alone displayed Immutability. Yet for Plato the Creator and the Created existed in an environment of ‘friendship’ or ‘kinship.’ The Pythagoreans and Plato described the relationship between odd and even numbers in an anthropomorphic manner.

The philosophical opposition of Unity (or Immutability) and Duality (or Mutability) was known to Boethius, who used it in his *De arithmetica*. This treatise likely transmitted the anthropomorphic conception of duality to early Medieval thinking.

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29 Ibid., p. 91.
30 Ibid., p. 105: "And God set soul in the midst thereof and spread her through all its body...but for its excellence it was able to be company."
Conclusion

Even when the advent of polyphonic music created difficulty in the use of the eight modes as a taxonomic system, the eight mode paradigm remained the normative one among theorists and composers until Glareanus (1547) suggested its replacement by the twelve mode system.\(^2\)

Powers (1981) has noted that, even with the obvious superiority for polyphonic music of Glareanus' new system, Palestrina continued to use the eight mode system when ordering a group of works according to a modal system.\(^3\)

Medieval theorists divided the modes into authentic and plagal pairs for practical considerations. Yet the description of related authentic and plagal modes as having anthroporphic relationships may also have been regarded from a theological perspective. This perspective was derived from the Pythagoreans and transmitted through Plato. It was picked up by Boethius and conjoined with Aristotelian division in the *De divisione*.


\(^3\)Harold Powers, "Tonal Types and Modal Categories in Renaissance Polyphony," *Journal of the American Musicological Society* 34 (Fall, 1981), p. 467. Powers says that this conservative stance, which took place along with an increased interest in modality during the sixteenth century, shows that the 'evolutionist' view of a historical succession from modality to tonality fails.
CHAPTER VI

The taxonomic paradigm of the *De divisione* applied to the eight church modes

This chapter will show how Boethius' division of *genus* into *species*, a whole into parts, and subject into *accident.s* can be applied to the eight church modes. It will also explain how these division tools can rationalize irregular modes of the system. Examples of irregular modes include transposed modes resulting from the process of *transformatio*, and octave *species* that are created from dissimilar *species* of fourths and fifths.¹ (Chapter VII will offer evidence that these division tools were employed by five Medieval German theorists in their discussions on the eight church modes.)

*Genus* into *species*

In the *De divisione*, Boethius said that: "*genus* is divided not in meanings, but in offsprings of itself."² His implication is that whatever results from the division of the *genus* retains the “essence” or “nature” of the original *genus*. The *genus* “creates” the *species* over which the *germs* rules. Example one illustrates the qualitative³ division of the *genera* Protus, Deuterus, Tritus, and Tetrardus (the maneriae) into authentic and plagal *species*. The left side of the example shows the number of each mode. The odd numbered modes are listed above the staff in the example, while the even numbered modes are positioned under the staff. The right side of the example shows the authentic modes as

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¹Guido was among the first writers on music to use *transformatio* as a descriptive term, referring to a move from one of the original ‘seats’ of the tetrachord of the finals. In chapter 9 of the *Micrologus*, Guido said: "Utpote si quis vellet antiphonam cuius principium esset in D., E., vel in F., quae sunt alterius modi voces, incipere, mox auditu perciperet quantâ diversitatis *transformatio* fieret."

²Boethius, *De divisione*, PL 878D10-11: "*Genus* vero non in significationes sed in quasdam a se quodammodo procreationes disjungitur."

³Ibid., PL 879B-7: "*Generis* vero distributio qualitate perficitur."
"immutable" or "perfect," while the plagal modes are "mutable" or "imperfect." In an anthropomorphic description, this concept is an example of Boethius' division through the opposition of contraeity.

Example one. The division of genus into species applied to the Protus, Deuterus, Tritus, and Tetrardus modes.

At another place in the treatise Boethius specified that the division of a genus into species, "is in the conjunction of a difference." He then further clarified the nature of this division type when he said that the division of a genus into species must be made either through a divisive or constitutive difference. That is, the divisive difference divides the

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4The conception of the opposition of "perfection" and "imperfection" was a Pythagorean doctrine which was transmitted to the Medieval West through the Timaeus of Plato.
5See chapter III, page 28 of this document. Boethius actually illustrated the opposition of contraeity as "good" and "evil".
6Ibid., PL 879D3: "... species vero a genere, differentiae conjunctione."
7See Chapter IV, note 3. It is in the De differentitis topicis where Boethius expressly stated that differentiae will be either constitutive or divisive. See Boethius, De differentitis topicis. Lib. I, PL 1178 B9-10: "Aut enim constitutiva erit differentiae, aut divisiva: sed si constitutiva fuerit, quasi genus obinet locum..." 1178 B13-14: "At si divisiva fuerit, velut species consideratur..."
genus into species, while the constitutive difference becomes a species. In some instances the same difference can be both divisive and constitutive. In other words, the difference which actually causes the division of the genus can itself constitute a species. Example two shows the difference which is used to divide the Protus into authentic authentic and plagal species. This difference is the reversed position of the first species of fourth and fifth.

Example two. The division of genus into species through a difference applied to the Protus maneria.

Both the species of fifths and fourths can now be viewed as subaltern genera, which are genera that are lower in qualitative status than the highest genus, but are higher than the lowest species. Example three takes the fifth as an illustration and shows the four species of fifths. Each fifth belongs to a different authentic/plagal modal pair. The differentiae which divide each species of fifth from the others are specific arrangements of whole tones and semitones. Since the fifths were one of the differentiae which divided the

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8In the De divisione, Boethius said that, "in general, all differentiae that are of such a sort that there can be no species without them, but in virtue of them [are the ones that] must be used in the division of the genus or in the definition of the species." De divisione, PL 881C8-12: "...et universaliter dicendum est, quae cunque differentiae hujusmodi sunt, ut non modo praeter has species esse non possit, sed per eas solas sit, haec vel in divisione generis, vel in speciei definitione sumendae sunt."

9See Chapter V, note one.
modal pairs from each other, the fifth itself could seen as a divisive difference. Since each species of fifth has its own constitutive difference, specific patterns can be seen as species of a new subaltern genus: the Fifth (as a general class).

Example three. The division of the (subaltern) genus fifth into species through the differentiae of whole tones and semitones.

The division of a genus into species is accomplished by one of Boethius' oppositions. Therefore it can be assumed that the oppositions are equivalent to differentiae. Of the four oppositions, Boethius suggested that the first opposition, contraeity, is the most prevalent. In the words of Boethius, "much of the division of genera occurs in connection with contraries, for we bring almost all differentiae down to contraries." As I indicated earlier in this document, Boethius' source of the four

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10See Chapter four for a discussion of the four oppositions.
11Boethius, *De divisione*, PL 883C2-5: "In contrariis autem generis multo divisio est. Fere enim cuncias differentias in contraria deducimus."
12See Chapter 4, section II.
oppositions was Aristotle’s *Categories*.\textsuperscript{13} However, Boethius ordered the four oppositions differently, and more significantly, he used an analogy different than Aristotle’s to illustrate the fourth opposition. Boethius created an anthropomorphic illustration for this type of opposition, by suggesting that “the fourth opposition occurs through relationship, as father, son, master, slave...”\textsuperscript{14}

Example four, given below, is similar to example two. However, in this example Boethius’ concept of opposition is applied to the division of the Protus modes into authentic and plagal *species*. The example shows that, although the *species* of fifth and fourth which comprise the two modes are identical, they are in inverted order. Because of this, they show contraeity, which is Boethius’ first opposition. The authentic mode has the fourth placed above the fifth; the plagal has the fifth placed above the fourth. Hence this mutual but reversed positioning of the first fourth and fifth shows the divisive *difference* between the two modes. At the same time it can be seen that the fourth and fifth are related, since the fourth contains the same pattern of tones and semitones as the fifth (the fifth contains the fourth within itself). Because the fourth and fifth are related in this way, this division also shows Boethius’ fourth type of Opposition: relationship.\textsuperscript{15}

\textsuperscript{13}Aristotle’s *Categories and De interpretatione*, trans. Ackrill, pp. 31-2: “Things are said to be opposed to one another in four ways: as relatives or as contraries or as privation and possession or as affirmation and negation.”

\textsuperscript{14}Boethius, *De divisione*, PL 882A2-3: “Quarta secundum relationem, ut pater, filius, dominus, servus.” In the *De topicis differentiis*, Boethius gave an even clearer definition of opposition through relationship. He said that “properties of opposites which are related to each other are themselves also related to each other.” See Boethius’s *De topicis differentiis*, trans. Stump, p. 56. Boethius, *De differentiis topicis*, Lib. II, PL 1191D5-6: “… oppositorum ad se relatorum prorsa, et ipsa ad se referuntur.” Aristotle illustrated opposition by relation as in “the double and the half.” See Aristotle’s *Categories and De interpretations*, trans. Ackrill, p. 31.

\textsuperscript{15}See note 17 of Chapter 1 for a discussion of the anthropomorphic nature of relationship in the *Periphyseon* of John Scottus Eriugena.
If a *difference* can be separated from the subject without changing the nature of the subject, this *difference* can be regarded as a separable *accident*. In the eight mode system, the use of the b flat as an *accident* could be applied to the Protus authentic and the Tritus authentic and plagal modes only. If the flat were used in the Protus plagal and Tetrardus authentic modes, then the flat would have to be regarded as a *difference*, because the Medievals after Guido viewed the flat in these modes as changing the nature of the subject. Example five compares the Protus authentic with the Protus plagal modes, both of which include the b flat. In the Protus authentic, the flat constitutes an *accident*, since it does not change the designation of the mode as Protus authentic. In the Protus plagal, the flat constitutes a *difference*, since it changes the designation from Protus plagal to Deuterus authentic. (This was called *transformatio*, and will be discussed in the section on division *secundum accidens* later in this chapter.)

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16 This was an attribute of *accident* with which Aristotle, Porphyry, and Boethius agreed. In the Medieval gamut of music, only the b flat or b natural exhibit this feature, except for the gamut of the *Musica enchiriadis*, where other chromatic inflections are used.
Example five. A comparison of the Protus authentic and plagal modes with b flat as an *accident* and as a *difference*.

Whole into parts

In the *De divisione*, Boethius thoroughly discusses the division of a whole into its constituent parts. He specified four principal ways of division. These include the division of a whole into:

1) continuous parts (such as in a body into its parts)\(^\text{17}\)
2) non-continuous parts (such as in the "whole" crowd, the "whole" population, the "whole" army)\(^\text{18}\)
3) the whole which is universal (as in "man" or "horse")\(^\text{19}\)
4) the division of a whole that consists of powers (such as in the division of abstract entities, like the soul, there is the capacity for "understanding," another for "sensing," and another for "living."\(^\text{20}\))

\(^\text{17}\)Boethius, *De divisione*, PL 887D6-7: "Totum namque est quod continuum est, ut corpus..."

\(^\text{18}\)Ibid., PL 887D8-10: "Dicimus quoque totum quod continentum non est, ut totum gregem, vel totum populum, vel totum exercitum."

\(^\text{19}\)Ibid., PL 887D10-11: "Dicimus quoque totum quod universale est, ut hominem vel equum."

\(^\text{20}\)Ibid., PL 888A1-3: "Dicitur quoque totum quod ex quibusdam virtutibus constat, ut animae alia est potentia sapiendi, alia sentiendi alia vegetandi."
It should be kept in mind that, for Boethius, dividing a whole into parts is a quantitative division. The implication is that this type of division, unlike dividing a genus into species, is not hierarchical. For this reason, Boethius may have felt obliged to include a type of division in which all the parts can be regarded as integral (or collective) parts, rather than as divisive (or distributive) parts, that is, a division which would be more closely aligned with the division of genus into species.

Example six illustrates the division of a whole into non-continuous or discrete parts applied to the eight modes. The parts of the whole are the four pairs of authentic and plagal modes. Since Boethius also said that multiple divisions exist, the parts themselves may be divided into ‘parts of the parts’. Hence in this type of division, the individual modes can be regarded as these ‘parts of the parts’.

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Example six. The division of the whole into parts applied to the eight modes.

The Medievals had known Latin grammatical theory from Priscian and Donatus, among others. Since early logic and grammar had a common origin, both of these

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21 The division of a whole is into integral parts. In the division of genus into species, the species are hierarchically related.
22 Boethius, De divisione, PL 885B-8: "Fit autem generis ejusdem divisio multipliciter..."
23 The 'parts of speech' for Priscian were name (comprised of a noun plus an adjective), verb, participle, pronoun, preposition, adverb, interjection, and conjunction. See D. P. Henry, Commentary on De grammatico: The Historical-Logical Dimensions of a dialogue of St. Anselm's (Reidel: Synthese
disciplines dealt with "parts of speech." The difference was that logic, as inherited from Aristotle, focused on two of the eight "parts of speech," the name/verb as a signifier of the subject, and the predicate concepts of the proposition under consideration. In the De divisione, Boethius followed the example of Aristotle by also granting a higher status to names and verbs as "parts of speech." What is significant for the present discussion is that, during the Medieval period, there was an attempt to bridge the gap between logic and grammar. Hence the analogy of using "parts of speech" to illustrate the division of a whole into parts with regard to modal division shows not only the influence of grammar, but of logic as well.

**Division secundum accidentes**

Division secundum accidentes operates on three levels: 1) a subject into accidents, 2) accidents into subjects, and 3) accidents into other accidents. In division secundum accidentes, the accidents are present in the subject, but they may be removed from the subject without destroying the subject. In this document I regard the b-flat, through its availability in the synemmenon tetrachord, as an accident, since the Protus authentic and the Tritus

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Historical Library 8), pp. 262-3. Also see D. P. Henry, "Predicables and Categories," The Cambridge History of Later Medieval Philosophy, p. 133. In the De syllogismo categorico, Boethius contrasts the philosopher's view that names and verbs are primary, with the other six parts being merely supplementary, with that of the grammarians, who simply see eight parts of speech. Boethius, De syllogismo categorico, PL 798C7-14: "Haec enim ex nomine et verbo componi videtur: sed prius utrum nomen et verbum solae partes orationis sint consideremus, an etiam aliae sex, ut grammaticorum opinio fert, an aliquae ex his in verbi et nominis jura vertantur; quod nisi prius constitutum sit, tota propositionum ac deinceps ea ipsa quae ex propositionibus componitur syllogismorum ratio titubabit." The principal grammatical texts studied were the Institutiones grammaticae of Priscian, and the Ars maior and Ars minor of Donatus. During the Carolingian period these were supplemented by texts for school use taken from Bede, Alcuin, and others. The subjects of the Trivium were studied more widely than the Quadrivium during the Carolingian period. Of the Trivium, grammar was the most studied. See Laistner, Thought and Letters in Western Europe, p. 170.


authentic and plagal modes would have the same designation if the b flat were not present. In other words, the b flat sometimes is extrinsic to the subject “diatonic gamut.”

Near the end of the *De divisione*, Boethius says that anything belonging to the division by *accident* involves contraeity. As an illustration of the first division level, or a subject into *accidents*, Boethius uses the example of the subject ‘men’ being divided into black, white, and a intermediate color, because “… these are *accidents* for men, not *species* of men, and Man is their subject, not their *genus*.”

The implication of this level of division is that the “nature” or “essence” of the subject remains intact during the division process. The *accident* or *accidents* may be removed from the subject without the destruction of the subject. Moreover, since the division by *accident* involves contraeity, the subject cannot possess an *accident* and its opposite simultaneously.

**Subject into *accidents***

Example seven shows the division of a subject into *accidents* as applied to the Protus authentic mode. The subject “Protus authentic” is divided into two versions: 1) with b flat, and 2) with b natural. Since the flat and the natural are both *accidents*, and are therefore extrinsic attributes of the subject, the choice to use one or the other would be made with the knowledge that the designation of the subject as “Protus authentic” remains unchanged. This is another way of saying that the *accident* which is present in the subject, may be removed without destroying the subject.

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26Boethius, *De divisione*, PL 890 D10-891A1: "Nunc de his divisionibus dicemus quae per *accidens* sunt. Harum autem commune praeceptum est, quidquid ipsorum dividitur in opposita disgregari. . . ."

27See Chapter 4, note 14 of this document.

28Ibid., PL 878A15-878B2: "Haec enim *accidentia* sunt hominibus, non homininum *species*, et homo his subjectum, non horum *genus* est."
Example seven. The division of the subject into *accidents* applied to the Protus authentic mode.

The eight mode system provides another illustration of the division of a subject into *accidents*. Since the eight mode paradigm uses seven discrete diatonic pitches, it is necessary to feature two designations for one of the octave *species*. The octave *species* which received this dual designation during the Medieval period was the octave *species* on D. In some treatises of the Medieval period, this octave *species* was designated as both Protus authentic and Tetrardus plagal. The distinction between the designations was based on the two different 'mean' pitches of the D octave *species*. The Protus authentic mode has A as it's "mean" pitch. The Tetrardus plagal mode, on the other hand, has the pitch G, the final, as its "mean" pitch. Both of these modes have, of course, the same combination of first *species* fifth and fourth as their constituent parts, but the lesser *species* are in reversed order. Hence in terms of *species* the Tetrardus plagal mode would appear to be exactly the same as the Protus plagal mode. The difference lies, however, in the "mean" pitch. Example eight shows the Protus authentic and Tetrardus plagal modes with their respective "mean" pitches. The dual designation of these two modes represents the division of Subject into *accidents*. 
Example eight. The division of the octave species on D into the accidents Protus authentic and Tetrardus plagal

The division of a subject into accidents is, of course, similar to the division of a genus into species through a difference, when that difference can be viewed as an accident.

Just as the Protus authentic may be divided into two accidents, so may the subject "Tritus authentic" be divided into two accidents by the synemmenon tetrachord. However, there is an important distinction. If the pitch class b in the Tritus authentic is flatted, then the species of fourth in that mode is changed, and the tritone above the final is eliminated. This change does not affect the division type, but it may explain the ubiquity of the flat in this mode during the Medieval period. It may also explain why, in the sixteenth century, Glareanus considered the use of the flat in the Tritus authentic mode as creating a different mode (i.e. transposed Ionian). For Glareanus, the accident would in this instance be regarded as a difference, since the division type would be different. It would constitute the division of genus into species. Example nine, which is a comparison between the Protus authentic and the Tritus authentic modes with the flat present, shows how the flat affects

\[\text{Example eight. The division of the octave species on D into the accidents Protus authentic and Tetrardus plagal.}\]
the Tritus mode's *species* of fourth, and converts it from three whole tones into two whole tones and a semitone.

Example nine. The subjects Protus authentic and Tritus authentic compared.

**Accidents into subjects (transformatio)**

If the b flat is used in the octave *species* on a, or the normal Protus plagal mode, then a second Deuterus authentic mode results. This constitutes a process which was known as *transformatio*, and the process results in two seats for the Deuterus authentic mode. This process is therefore an illustration of how the *accident* b flat can be divided into two subjects: the original Deuterus mode on e, and the transposed one on a. Example ten shows a comparison among the Protus plagal, the Deuterus authentic, and the new (transposed) Deuterus authentic modes.
Example ten. The division of accident into subjects applied to the Deuterus authentic mode.

The difficulty for music theorists was, of course, that the Deuterus authentic and Protus authentic modes already had identities in the eight mode system. Hence the result of transformatio led to a dual view of the Deuterus authentic, as well as the Protus authentic modes, thereby forcing an expansion of the eight mode system. This dual view is similar to the dual view that the Medievals had inherited from the Greeks concerning the relation between odd and even numbers. The application of the attitude toward the authentic versus plagal modes, with the former as representing “immutability” or perfection, and the latter as representing “mutability” or imperfection, was then transferred to modal transposition. The new Deuterus authentic on a was “less perfect” than the same mode on its original seat, which, of course, was on e. This may have been among the reasons why Guido considered transformatio as an unacceptable practice.30 His view remained valid until the late Medieval period.

Accident into other accidents (Hexachords into Tetrachords)

According to Aristotelian logic, a property and an accident may, under certain conditions, be equivalent predicables, and used interchangeably. Such a view is taken in this document with respect to the division of the gamut into tetrachords and hexachords.

The Medieval gamut featured two important attributes which theorists closely associated with the division of the gamut into eight modes: the gamut could be divided into its component tetrachords or hexachords. From Hucbald forward, theorists discussed the gamut and its division into the eight modes with reference to one or the other of these attributes.

The first of these attributes to be discussed was the arrangement of the gamut into tetrachords. The gamut was typically shown to contain four tetrachords, of which the “primary” tetrachord was the familiar “tetrachord of the finals.” The finals of the eight modes constituted this tetrachord, which features a tone/semitone/tone intervallic structure. This particular intervallic structure was found on two locations within the gamut, at the “graves” and “excellentes” tetrachords. In the Music enchiriadis the gamut was organized so that all four tetrachords reflected the same structure as the “graves” tetrachord. Example eleven shows a comparison between the division of the gamut according to

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31 Trans. Pickard-Cambridge, Topica, 102b 21-3: "It is clear on the face of it that there is nothing to prevent an accident from becoming a temporary or a relative property."


33 Ibid., pp. 89. For a discussion concerning how the vitia (upward or downward semitonal inflections of the middle pitch of each tetrachord) of the Enchiriadis gamut could be used to rationalize "accidentals" without having to employ transformatio, see Charles M. Atkinson, "From 'Vitium' to 'Tonus acquistus. On the Evolution of the Notational Matrix of Medieval Chant,'" Cantus Planus, Budapest, 1990, pp. 181-97.
tetrachords in the *Musica enchiriadis* and that found in the *De harmonica institutione* of Hucbald.

Example eleven. The division of the gamut by tetrachord according the the *Musica enchiriadis* and the *De harmonica institutione* of Hucbald.\(^{34}\)

The second division of the gamut was through the hexachord. In the later theoretical literature, the hexachord actually came to be designated by the predicable property.\(^{35}\) The outline of a hexachordal view was first given by Guido, who located it on two positions in the gamut by placing a whole tone on either side of the “graves” and “superius” tetrachords.\(^{36}\) The result was to create identical hexachords from C to a, and from G to e. These became later known as the “natural” and “hard” hexachords, respectively. Both Hermannus Contractus in his *Musica*,\(^{37}\) and Wilhelm of Hirsau, in his

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\(^{35}\)Pesce reminds us that the actual word 'hexachord' was not used until the sixteenth century. See Dolores Pesce, *The Affinities and Medieval Transposition* (Bloomington: Indiana University Press, 1987), p. 174.


\(^{37}\)For a critical text and English translation, Leonard Ellinwood, *Musica Hermanni Contracti* (Rochester: Eastman School of Music, 1936.)
treatise of the same title, described the “seats of the finals” within the framework of identical hexachords. Wilhelm recognized the potential of a third hexachord within the interval set F to d through the synemmenon tetrachord, which became known as the “soft” hexachord. Hence the eight modes, which had their primary “seats” based in the “natural” hexachord, were described with respect to their position within identical hexachords.

Movement within a specific mode was determined according to one’s position within a particular hexachord by the use of hexachordal syllables. To move from one hexachord to another was called mutation. Example twelve shows the division of the gamut according to identical hexachords.

Example twelve. The division of the gamut according to hexachords.

We now come to the reason why Boethius’ division of accident into other accidents is applicable in this instance. If the tetrachord and hexachord are both regarded as accidents, then it is possible to show that the hexachord, as an accident, can be “divided” into tetrachords. In the same manner, the tetrachord can be “divided” into the hexachord.

[Diagram of hexachords]

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This division type is possible because the tetrachord is encased within the hexachord. This is precisely the way in which Hermannus Contractus derived his hexachord. Example thirteen shows how the tetrachord is encased within the hexachord.

Example thirteen. The division of *accident* into other *accidents* applied to the tetrachord and hexachord.

**Conclusion**

This chapter has attempted to show how five of the six division types outlined in Boethius’ *De divisione* could illustrate the division of the eight church modes. Two aspects of this division process should be noted, since they will be significant in the next chapter, in which selected music treatises are examined for evidence of logic. The first aspect is terminological, in which the logical predicables are the semantic tools used to describe the divisional components. The second aspect is conceptual, in that Boethius provided at least one model which could be used to rationalize the way in which the division was

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39Trans. Ellinwood, *Musica Hermanni Contracti*, p. 57: "Take any tetrachord you wish... add a tone at both ends; you then have the limits of the tonal patterns which form the basis of the modes." *Musica Hermanni Contracti*, p. 57: "Accipe tetrachordum quocumque voleris... addito utrinque eto, habes terminus modorum qui fiunt sedes troporum."
undertaken. This conceptual model is in Boethius' use of an anthropomorphic illustration to describe his fourth opposition, that is, the opposition of relationship. The next chapter will show that this illustration was repeated by five German theorists in their efforts to describe the relationship between the authentic and plagal modes.
CHAPTER VII

Examples of Division in Five German Music Treatises

The eight church modes were established in the Medieval West during the Carolingian period. Yet the suggestion that Carolingian music writers were influenced by Aristotelian logic is not directly supported by theoretical writings. However, when we turn to the early Medieval German writers on music, the influence of logic is apparent. This chapter presents terminological and conceptual evidence that the Aristotelian science of division was used by these writers in their discussions on mode. In support of this tenet, quotations from five German treatises are presented. These treatises are the Prologus in tonarium of Bemo of Reichenau, the Musica of Hermannus Contractus, the Musica of Wilhelm of Hirsau, the De musica of Aibo, and the Musica of Engelbert of Admont.¹ Each treatise is examined for evidence of an awareness of the five division types given in the taxonomic paradigm presented in Chapter VI.

Of the five theorists, only two mention directly Aristotelian logic or logical texts which were Peripatetic. These two theorists are Hermannus and Wilhelm. As I indicated in the Introduction to this document, a listing of the monastic library at Reichenau from the year 822 reveals that several important books on logic were available.² Both Hermannus and Bemo were in an excellent position to have come into contact with Aristotelian logic. In the case of Hermannus, confirmation of this contact comes directly from the author. At the beginning of his Musica, he specifically refers to the Isagoge of Porphyry, and the Categories of Aristotle.³

¹The dates of these treatises are given on page 3 of this study.
²See page 4.
³Trans. Ellinwood, Musica Hermanni Contracti, op. cit., p. 19: "For, to make a long story short, just as Grammar is reduced to eight varieties, the Isagoge to five, and the Categories to ten...". Musica
Although Wilhelm did not mention any logical texts by name, he suggested that an association exists between Peripatetic logic and tetrachordal formation in music. He said that "ancient and modern authorities of musical art wrote many commendable things, many useful and extremely necessary, (since) they invented the first paths of this same art, they also found the clearest source itself, that is, the principal tetrachord, which is a kind of Peripatetic logic..."\(^4\)

**Genus into Species through Differentiae**

Examples one and two of Chapter VI illustrate the division of the *genera* Protus, Deuterus, Tritus, and Tetrardus into authentic and plagal *species*. Since the authentic and plagal *species* have been divided under the highest *genus* (the Protus, Deuterus "maneria," etc.), these *species* can now be called subaltern *genera*. The authentic and plagal *species* (or subaltern *genera*) can be further divided into lesser *species*. Example three of Chapter VI showed the division of the subaltern *genus* "fifth" into four *species* of fifths according to the *differentiae* of varying tone and semitone combinations.

The term *species* was generally not used in Carolingian music treatises. Yet all of the German theorists discussed in this chapter used the predicabale *species* in their discussions of fourths, fifths, and octaves. The consistent use of *species* among the Germans testifies to the value which they placed on this logical term.\(^5\)

\(^4\) *Hermanni Contracti*, p. 19: "Ut enim ex multis pauc a dicamus, sicut grammatica ad VIII isagoge ad V praedicamenta ad X varietetates rediguntur..."

\(^4\) *Willehelmi Hirsauensis Musica*, ed Harbison, *op. cit.*, p. 26: "W. Anti qui et moderni musicae artis auteores multa scripsero probabilia, multa utilia et valde necessaria, quin immo primas eiusdem artis invenere venas, ipsum quoque limpidissimum fontem, id est principale tetrachordum quodam Peripateticae rationis ex parte detexere fossorio..."

\(^5\) In section five his *Prologus*, Bemo discusses the *species* of fourths, fifths, and octaves. *Berno Augensis Prologus in tonarium*, GS, II, *op. cit.*, p. 67: "Habeat enim diatessaron *species* tres, diapente quatuor, diapason a VII." In his *Musica*, Hermannus devotes chapter five to the *species* of fourth, chapter six to
However, it is not the mere use of *species* that shows the influence of Aristotelian logic. It is in the conceptual view of the German theorists through which specific *species* of fourths, fifths, and octaves are regarded as members of a larger class of *species* of fourth, *species* of fifth, and *species* of octave (which we may call subaltern *genera*), that the most striking influence of Aristotelian logic is shown.

By the time of *De musica* of Engelbert, one can see the terminology of *genus* into *species* applied to the modes. In Engelbert’s words: “according to the quality of their *genus*, (the modes) are called authentic and plagal, that is, the principal and secondary, or authoritative and subjugal.”

*Genus into Species through the First Opposition of Contraeity*

The division of *genus* into *species* through Boethius’ first opposition of contraeity applied to the modes is a conceptual view which was shared by Berno and Hermannus. Their emphasis on symmetrical properties of the eight mode system shows that both theorists valued symmetry in their theory of the modes. Moreover, the symmetrical properties of the modes has little to do with the chant repertory. Instead the concern of Berno and Hermannus for symmetry shows that they were interested in the internal
properties of their theoretical system. This concern for internal structural symmetry of the modes can be seen, then, as a second order of logic and is a unique characteristic of Medieval German theory.

For Hermannus the Protus and Tetrardus authentic modes are symmetrical in that the Protus authentic mode is characterized by the descent of a tone and the ascent of a first species fifth, while the Tetrardus authentic is the opposite in that it ascends by a tone, but descends by a fourth species fifth. The Protus and Tetrardus authentic modes are described within the range of a sixth, or C to A. Similarly, the Deuterus mode descends by a ditone and ascends a fourth, while the Tritus mode ascends by a ditone and descends by a fourth. The opposition of these two modes exists within the same sixth, or C to A as do the Protus and Tetrardus authentic modes. It can be seen that this symmetry has its own internal logic and elegance, and has little to do with the chant repertory. Powers (1980) has also noted the elegance of this symmetrical attribute in the modal theory of Hermannus.

Berno was even more specific in his view of the modes as being opposite, for he simply said that “Protus and Tetrardus are opposites...” and that “the Deuterus and Tritus are entirely opposites.”

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7See note 14 of the Introduction to this document.
8Trans. Ellinwood, *Musica Hermanni Contracti*, op. cit., p. 57: "Primus modus vocum est qui tono deponitur et prima specie diapente intenditur..."
9Ibid., p. 58: "Qarrtum modum vocum tono intensum et quarta specie diapente remissum tetrardo aptamus."
11*Berno Augiensis Prologus in Tonarium*, op cit., p. 78: "Protus...et Tetrardus contrarii sunt..."
12Ibid., p. 78.; "Deuterus et Tritus omnino sunt contrarii..."
Genus into Species through the Fourth Opposition of Relationship

The other opposition used by the German theorists was Boethius' opposition of relationship. As I indicated in Chapter I, Boethius used an anthropomorphobic illustration to show the opposition of relationship, and, in doing so, he conjoined a Platonic concept with Aristotelian division. Anthropomorphobic illustrations were frequently used by Medieval German theorists to clarify the relationship between the authentic and plagal modes. The use of anthropomorphobic illustration is to show that, though the authentic and plagal modes are separate, they are related enough to mingle with each other in an atmosphere of kinship or friendship.

Berno employed Boethius' opposition of relationship of authentic and plagal modes when he said, "so...we use Latin names, and call the higher (authentic) ones teachers, and the lower (plagal) ones pupils."

Aribo applies a unique anthropomorphobic illustration in his description of the authentic and plagal modes as male and female respectively, and asserts that these modes mingle by sharing a marriage chamber. Thus in Aribo's words, "the authentics are concordant and discordant with the plagals, as if the four modest brides with their grooms from the same marriage beds came forth and linked two circles of dances, so that those very wedding beds were the centers for their wedding choruses."

Wilhelm employed a distinctly anthropomorphobic illustration in his description of the relationship among the authentic and plagal modes. His description became increasingly graphic as he discussed each modal pair of the "maneriae." He said, concerning the

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13 See pp. 18-9 of this Document.
14 In Chapter V I suggested that the source of this view was Platos' Timaeus.
15 Prologus in Tonarium, op. cit., p. 69: "Sed...latinis utamur nominibus, et excelsiores magistros, inferiores vero discipulos nominemus."
16 Aribonis De musica, ed. Smits van Waesberghe, p. 17-18: "Concordant discordantque autenti cum plagis, quomodo si procederent de quatuor thalamis totidem nuptae modestae cum suis sponsis, copularentque duos chorearum circulos, ut ipsi thalami matronali choro essent centra."
relationship between the Protus authentic and plagal: "... if these two are joined;"\textsuperscript{17} Deuterus: "... if these two are united;"\textsuperscript{18} Tritus: "... if these two are mated;"\textsuperscript{19} and Tetrardus: "if these two are intertwined."\textsuperscript{20} Wilhelm stated that he used anthropomorphic illustrations rather than the musical ones, because he thought that such a process would aid in comprehension of modal relationships. Thus he said, "I have visually and thus more comprehensively recast the important elements of these figures, making them physical more than musical."\textsuperscript{21}

**Whole into Parts**

In the treatises selected for this study, the terminology of a "whole into parts" as a division type which is quantitatively based is less used than the qualitative division of a genus into species. Yet the terminology of the division of a "whole into parts" can be seen in three of the five treatises. The evidence is strong enough to show that the conception of a whole which is divided into quantitative parts was a concept that was clearly understood during the early Medieval period.

Hermannus and Engelbert applied the division of a whole into parts in their description of the "maneriae," while Aribo used the description of the whole into parts in his discussion of the species of fourths and fifths.

In section eight of his *Musica* Hermannus defines mode as "an inflection of many pitches within any one octave, as determined by fixed intervals and fitted into one whole."\textsuperscript{22} From his wording, Hermannus shows that his conception of a whole exists

\textsuperscript{17}Willehelmi Hirsauagensis Musica, ed. Harbinson, *op. cit.*, p. 33: "Qui si iungantur... ."
\textsuperscript{18}Ibid., "Qui si coadunentur... ."
\textsuperscript{19}Ibid., "Qui si copulentur... ."
\textsuperscript{20}Ibid., "Qui si conectantur... ."
\textsuperscript{21}Ibid., p. 34: "... manifestus adhuc tam corporis quam chordis visui figurae huius perspicacia subiciatur."
\textsuperscript{22}Trans. Ellinwood, *Musica Hermanni Contracti*, *op. cit.*, p. 37: "Tropus est inter unum quoque diapason multarum vocum ratis effecta intervallis apta in unum corpus modulatio."
only after consideration of the parts. This was an important aspect of Boethius’ division of a whole into parts.

Aribo combined species with the division of a whole into parts and added an ontological hue to this relationship. In chapter eighty seven of his De musica, he said, “since every complete thing receives its true name from its parts, we ought to consider the same for the fourth and fifth. We know that there are no more than four species of diatessaron and diapente, but since the species of a diapason is made up of the species of its diapente and diatessaron, it cannot be more than the species of its parts.” The key phrase in Aribo’s statement is “since every complete thing receives its true name from its parts.” Aribo is suggesting that since a whole cannot exist without all of its parts, the number of octave species (as a whole) cannot exist without their constituent species of fourths and fifths (as parts). This conception is similar to Boethius’ division of a whole into its proper parts, “the number of times we separate each out of the whole.” Although Boethius’ statement is ambiguous, the implication of both Boethius and Aribo is that, though one may divide a whole into as many parts as one wishes, the whole as an entity cannot exist without all of its parts.

Engelbert’s wording in book four of his Musica shows that he, too, understood Boethius’ view that the whole succeeds the parts. In chapter ten of book four, where he discusses why there are eight modes, he says that, “... in music, lesser consonances create and compound greater ones, just as the parts create their whole... and the diatessaron and the diapente, and again the diapente and the diatessaron exist in accordance with their

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23 Smits van Waesberghe, Aribonis De musica, op. cit., p. 35: “Cum enim omne totum de suis partibus nominetur veraciter, idem in tota diapason, et in suis partibus, diatessaron scilicet ac diapente, considerare debemus. Species diatessaron et diapente non plures quam quatuor scimus esse. Sed cum species diapason constituantur ex speciebus diapente et diatessaron, non possunt fieri plures, quam partium species.”

24 See page 27.
species, and make up their species of diapason..."\textsuperscript{25} It can be seen that Engelbert, like Aribö, combined the division of a genus into species with the division of a whole into parts. By doing so, it is easy for us to see that both Aribö and Engelbert regarded these two division types as being inherently different.

Subject into accidents

If a subject features an attribute which can be removed without the destruction of the subject, then that attribute is an accident. An accident is not essential to the subject in the sense that the subject is dependent on the accident for its definition, as would be the case if the attribute were a difference. Although the term accident was not used by the theorists selected for this study, the evidence is clear that they understood the nature of an attribute which is not essential to the definition of a subject.

The following two cases are offered as an example of the division of a subject into accidents: 1) the division of the subject “octave species on D” into the accidents Protus authentic and Tetrardus plagal, and 2) the division of the subject “octave species on D into the accidents Protus authentic with and without the B flat. If either of the two modal designations as Protus authentic or Tetrardus plagal were removed, or if the B flat were removed, the subject “octave species on D” remains.

In the first case, the octave species on D, which is comprised of a first species of fifth plus a first species of fourth, was designated in Medieval theory as both a Protus authentic mode and as a Tetrardus plagal mode. The subject “octave species on D” is divided into two accidents, the Protus authentic and Tetrardus plagal modes.

\textsuperscript{25}Engelberti abbatis Admontensis de musica, op. cit., p. 345: “Quod sicut in musica minores consonantiae saciunt, et componunt maiores tanquam partes suum totum, videlicet toni at semitonia, ditonos, et diatessaron, et diapenti, et iterum diatessaron, et diapenti, et iterum diatessaron et diapenti secundum duas species constituant et componunt species diapason..."
The second case of this division type is in the regard of two octave *species* on D the second of which includes the *synemmenon* tetrachord. The *species* of fifth would be the same for both octave *species*, but the *species* of fourth for the octave containing B natural is a first *species* of fourth, whereas the *synemmenon* tetrachord (giving the B flat) provides a second *species* of fourth for the second octave *species*. Yet the designation as Protus authentic applies to both octave *species*.

Concerning the first case of this division type, Hermannus discussed in section twelve of his *Musica* the two modes found on D, the Protus auththentic (the Dorian), and the Tetrardus plagal, (the Hypomixolydian). These two modes have exactly the same range.

For Hermannus, the distinction between the two modes could be seen in their respective “mean” pitches.26 These two mean pitches form the point of disjunction for the integral *species* of fifths and fourths for these two modes.

For the Protus authentic mode, the mean pitch is A, or the cofinal of that mode. For the Tetrardus plagal mode, the “mean” pitch is G, or the final of the Tetrardus modes. The distinction between the two modes would be made according to the frequency of one of these mean pitches in colons, commas, and conclusions of chants. Frequent use would emphasize the *species* of fourth, or (D to G), or *species* of fifth, or )D to a). Hermannus felt that repetition of these two “mean” pitches was sufficient for modal identification, for “these two middle pitches therefore produce, within one octave, the two different modes... in this manner.”27

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27 Ibid.: “Hae igitur mediatates inter unum diapason duos diversos de quibus agitur modos hoc ratione coniciunt.”
Engelbert explained that the distinction between these two modes was in their constituent *species* of fifth. Thus he said that Protus authentic mode is comprised of a first *species* of fifth and a first *species* of fourth, while the Tetrardus plagal mode is comprised of a first *species* of fourth plus a fourth *species* of fifth.28

Concerning the second case of the division of subject into *accidents*, we turn to Engelbert. In chapter sixteen of book four of the *De musica*, he referred to the two diapasons that can be formed on D, the second of which is divided through the synemmenon tetrachord. These two diapasons "can pertain to two different types of songs."29 The synemmenon tetrachord creates two octave *species* on D, and "allows two forms of ascent and descent as possible in that diapason, that is through b fa or through # mi, as one can see."30

**Accidents into subjects**

The following is offered as an illustration of Boethius' division of an *accident* into subjects. In this example the four *species* of fifth are regarded as *accidents*, since it is possible to change any *species* of fifth into another without altering the designation "*species* of fifth." The *accident* "*species* of fifth" (first, second, third, and fourth) can be found at three positions in the gamut: 1) on the pitches D, E, F, G; 2) on the pitches A, B, C, and D; and 3) on the pitches G, A, B flat, and C using the synemmenon tetrachord.

Diagram seven in Chapter V showed the four *species* of fifth.31 In the diatonic gamut they are found on the pitches D, E, F, and G. Each *species* of fifth also exists at a

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28 *Engelerti abbitis Admontensis de musica*, op. cit., p. 348: "Item diapason, quod est a D. in d. deservuit cantui authenti proti, i.e. toni primi, in quantum componitur ex prima *specie* diapente, et prima *specie* diatesseron, et cantui plagalis tetrardi, i.e. toni octavi, in quantum componitur ex prima *specie* diatesseron, et quarta *specie* diapente."

29 Ibid., p. 347: "...in ipsa potest pertinere ad duplicum cantuum."

30 Ibid.: "...et admittit duplex ascensus et descensus possibilis in illo diapason, videlicet per b fa, vel per mi, ut patet."

31 See page 48.
perfect fifth above each of these pitches, or on the pitches A, B, C, and D. This is because the pattern of tones and semitones on these groups of pitches is identical. However, if the pitch B flat is substituted for B natural, a new position on the gamut arises on which a third series of fourths and fifths may be constructed. The B flat is, of course, available through the synemmenon tetrachord.

Hermannus recognized the third positioning of the four species of fifths through the synemmenon tetrachord. Concerning this, he said, "... a mere form of species (of fifth) may be produced if the Greek gamma is added and the synemmenon is inserted, so that there will be a kind of first species, C, A, b, C, D; a second, A, b, C, D, E; a third, b, C, D, E, F, and a fourth, C, D, E, F, G."  

**Accidents into other Accidents**

Example thirteen of chapter VI showed how the tetrachord of the finals, which features a tone/semitone/tone construction, is encased within a hexachord, since a whole tone is added at both ends of the tetrachord. It is possible to say that the tetrachord can be "divided" into the hexachord. The term "divided" is used in this instance to mean that, since the hexachord contains the tetrachord, the tetrachord can be divided into hexachord with two tones remaining. This is similar to the division of 5 by 4 with 1 as the remainder. If both the hexachord and the tetrachord are regarded as accidents of the gamut, meaning that they can be removed from the gamut without destroying it, I suggest that this can be seen as an example of Boethius' division of accidents into other accidents. I submit that the application of the division of accident into other accidents is valid here, because the theorists included in this study discussed the tetrachord first, then

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32 Trans. Ellinwood., Musica Hermanni Contracti, op. cit., p. 30: "Ergo, quod constitutio repudiât, forma tantum si gamma Grecum apponatur synemmenon quoque interponatur fieri poterit, ut sit quasi prima C, A, b, C, D, secunda A, b, C, D, E, tercia b, C, D, E, F, quarta C, D, E, F, G."
the hexachord. This division type is valid also because the theorists associated the
hexachord with modal recognition.

The division of *accident* into other *accidents* as applied to the encasement of the
tetrachord within the hexachord is a divisional perspective which all five German theorists
shared. This perspective appears to have been unique to them,\(^{33}\) and is the primary reason
why these five writers were chosen for this study.

The best example of this division type can be seen in Hermannus' *Musica*. In the
section on the *modi vocum*, he associated the “tetrachord of the finals” with its
“encasement” within a hexachord by adding a whole tone at both ends of this tetrachord.
Hermannus also recognized that this process applies to the identical hexachord found in the
affinal position, or at the upper fifth The process of adding whole tones to both ends of
these two tetrachords resulted in a hexachord on c, later called the “natural” hexachord, and
a hexachord on g, later called the “hard” hexachord. For Hermannus, this process was the
basis of modal construction: “take any tetrachord you wish...add a tone at both ends. You
then have the limits of the tonal patterns which form the basis of the modes.\(^{34}\) It is evident
that Hermannus associated the hexachord with modal recognition, for he later said, “thus it
is clear that this recognition of the modes can be perfectly accomplished in any tetrachord
by adding a tone at both ends."\(^ {35}\)

In diagram two of chapter IV, I showed how an *accident* can be equivalent to a
*property* if the *property* can be taken from the subject without destroying the subject. This,
of course, is the definition of an *accident*.

\(^{35}\)Ibid., p. 60: Ergo hanc agnitionem troporum in quodlibet tetrachordo adiecto utrinque tone perfecte fieri manifestum est."
In his *De musica*, Aribo applied the predicable *property* in his description of the interval of the sixth in which Protus, Deuterus, Tritus, and Tetrardus modal seats are found.\(^{36}\) “The *property* of the Protus (authentic and plagal) exists so that they are in agreement in their height through the diapente and in their disposition across the tone. The *property* of the Deuterus (authentic and plagal) exists with a complete diatessaron through its extending and a ditone when it returns. The *property* of the Tritus (authentic and plagal) exists in such a way that they are concordant during the extension of a ditone and through a diatessaron during its return. The *property* of the Tetrardus (authentic and plagal) exists in such a way that they are countered by the Proti, so much as the height of the Proti is equal to the depth of them.”\(^{37}\) In this last statement, where he contrasts the height of the Protus mode with the depth of the tetrardus, Aribo suggests that the Protus authentic mode is opposite to the Tetrardus authentic. The ascending Protus authentic mode ascends by two tones, a semitone, and two tones in its initial *species* of fifth. The Tetrardus authentic mode descends by two tones, a semitone, and two tones in its initial *species* of fifth.

In his *Musica*, Wilhelm also recognized the identity of modal seats as identical tetrachords ‘encased’ within a hexachordal formation, and he expanded this divisional conception to include a whole tone on either side of the synemmenon tetrachord. However, Wilhelm took this division type a step further when he identified the synemmenon tetrachord as another *species* of fourth which, if placed within a framework that included a whole tone at either end, would yield an identical hexachord to the two indicated by Hermannus. In Wilhelm's words, “To whichever of those tetrachords (graves, finales, superiores, and excellentes) you add a tone from either part, the first limit is from

\(^{36}\)A *property* is an attribute which is capable of predicated convertibly of the subject.

\(^{37}\)Aribonis *De musica*, ed. Smits van Waesberghe, op. cit., p. 32: "*Proprietas* est protorum, ut concordent in elevatione per diapente in dispositione per tonum. *Proprietas* est dueterorum cum integra diatesseron per intensionem, ditono per remissionem. *Proprietas* est tritorum, ut concordent intensione ditono, diatesseron remissione. *Proprietas* est tetrachordum, ut protis opponentur, quatemus horum elevatio istorum sit remissio."
gamma to E., for the finales, the second limit is from C to G, from the superiores, the third limit is from G to e, for the excellentes, the fourth limit is from c to (Z), the fifth is able to be found through the synemmenon from F to d. As Pesce has pointed out, this was the first recognition of what was later termed the "soft" hexachord.

From quotations of five German theorists this chapter has shown that the science of division as inherited from Aristotle and transmitted by Boethius was a new approach to rationalizing the eight modes. Of the six predicables, *genus*, *species*, *property*, *difference*, and *definition* were used by the theorists. Although *accident* was not used, it is evident that the conception of *accident* as a *property* which is not essential to the subject was understood. Finally, the quotations in this chapter show that Boethius’ five division types were, to varying degrees, applied by these five German writers.

38 Willehelmi Hirsaugensis Musica, ed. Harbinson, *op. cit.*, p. 59-60: "Secundum melodiae modum vel institutionem troporum quatuor tetrachorda in monochordo firmantur, id est Gravium A.B.C.D., Finalium D.E.F.G., Superium a.b.c.d., Excellentium d.e.f.g. Cuicumque horum tetrachordorum tonum ex utraque parte addideris, oritur tibi aliqua meta troporum utpote a Gravibus prima meta est a (gamma) in E., a Finalibus secunda a C., in a., a Superiortibus tertia a G. in e., ab Excellentibus quarta a c. in G., quinta per synemmenon nasci poterit as F. in d."

CONCLUSION

Thanks to recent research by Magee, it has now been established that Boethius' *De divisione* was a widely copied treatise during the Medieval period. In the absence of testimony that any music writer of the early Medieval period had a copy of the *De divisione* and had actually read it, this study can at best speculate that Aristotelian logic was known to these writers through the Boethian monograph. More research needs to be done on the availability of this treatise to music theorists. *Species* theory and division reached German theorists of the early Medieval period from some source. I suggest that the *De divisione* was the likely source.

In summary, the method of classifying of the eight church modes adopted by early Medieval German music theorists, the terminology used in this classification, and the manner of rationalizing problematic chants, all show such an obvious similarity to the methods proposed by Boethius, that it seems plausible to assume an influence, direct or indirect from the *De divisione*.

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1 See chapter II, note 33.
BIBLIOGRAPHY


________. *Dialogi in Porphyrium*. *Patrologiae latina*, Vol. 64, 9A-47B.

________. *In topica Ciceronis commentariorum*. *Patrologiae latina*, Vol. 64, 1040D-1174B.


Engleberti Admontensis *De musica*. *Gerbert Scriptores* 2, pp. 287-369.


PART II: ARIOSO AND TOCCATA FOR EUPHONIUM SOLO,
     WIND ENSEMBLE, HARP, AND PERCUSSION
Note: All notes are to the major second above.