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Digital photogrammetric approach to ice-flow determination in Antarctica

Tseng, Yi-Hsing, Ph.D.
The Ohio State University, 1992
DIGITAL PHOTOGRAMMETRIC APPROACH TO ICE-FLOW DETERMINATION IN ANTARCTICA

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

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* * * * *

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To My Parents
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Chapter I
Introduction

1.1 Background and Motivation

The motion of the Antarctic ice sheet is directly connected to changes of the global climate and sea level. The mass balance of the ice sheet and its future behavior are two urgent issues being addressed by Antarctic glaciologists. Monitoring and understanding motions of large ice streams in Antarctica requires vast, continuous, and accurate observations of ice flow. Until now, most of the active ice streams in Antarctica have not been observed carefully and understood because existing measuring technologies are inefficient or inaccurate in determining ice flow for a large Antarctic ice stream. Thus, an efficient, accurate, and economic measuring method is anxiously expected.

Velocities and strain rates are two of many types of important data needed for the study of geometrical changes of an ice sheet. Velocities of ice flow are the most wanted data. Measured velocities show the main feature of ice stream flow, and the mass-balance of an ice stream is calculated mainly by using velocities. Further, in order to understand the present day behavior and to predict the effect of any changes of an ice sheet, closely spaced and extensive velocities and strain rates in heavily crevassed areas are required. These data provide us an opportunity to understand ice flow in detail and to analyze stresses and physical controls on ice flow.
Crevassed areas are often the most interesting sites. Large and irregular strain rates commonly appear in such areas and show the evidences of forces acting on a glacier. However, it is inefficient and too dangerous to apply field surveys in crevassed areas. Photogrammetric and remote sensing techniques are therefore considered to be the most feasible ways to obtain suitable measurements efficiently and safely.

Repeat photogrammetry has been applied successfully by Whillans et al. [submitted] in the study of ice stream B in West Antarctica. Aerial photographs from two consecutive years of 1985 and 1986 were obtained and measured. The photo blocks were controlled by using approximately 10 TRANSIT tracking stations. Displacements were obtained by visually tracking natural features on individual crevasses from one year to the next using an analytical plotter - a photogrammetric stereo measuring instrument. Satisfactory measurements have been obtained by using this human approach. The results have been analyzed and have proven to be a significant achievement in understanding the dynamics of Antarctic ice streams.

Although satisfactory results can be obtained by using the human approach of repeat photogrammetry, a major difficulty is the need for a large amount of indoor work. An experienced operator needs about 20 hours on average to measure a pair of stereo models, on which about 50 displacements are determined. In addition, humans tend to mistrack corresponding features due to deformed, similar, or blurred features.

Thanks to the advance of computer technology, digital photogrammetry emerged in 1980s. Automatic correlation techniques have been developed and widely applied for stereo matching to generate digital elevation models (DEM) or ortho-photos efficiently [Gruen and Baltsavias, 1986; Hahn and Föstner, 1988]. These successful applications
of digital photogrammetry induce the motivation of the digital approach to matching multi-temporal images to determine ice flow automatically. If digital photogrammetry is applied, efficiency and low cost are the considerable benefits to be anticipated.

Remotely sensed images and digitized aerial photographs are considered to be the most feasible image resources for this application. Remote sensing imagery directly provides images in digital form, so digital matching is a natural application. Bindschadler and Scambos [1991] have successfully derived velocities by matching Landsat Thematic Mapper (TM) images. A remaining issue is whether the limited resolution of remotely sensed images (28.5 m for TM and 10 m for SPOT) is adequate for the need to obtain accurate data for the study of ice kinematics. Matching digitized aerial photographs has the potential in obtaining accurate data efficiently and automatically. Hahn and Föstner [1988] have demonstrated a matching accuracy as good as an operator can achieve. Perhaps this digital approach is the best solution to the difficulty of the human approach in applying repeat photogrammetry.

Two objectives are discussed in this study. The first one is to develop an automatic image matching process which can efficiently and accurately determine velocities and strain rates using multi-temporal digital images. To achieve this objective, the primary stage is to develop a matching method which is adaptive to ice deformation and directly generates velocities and strain rates. Seeking methods and criteria to control and assess data quality is the next stage. The second objective is to test and demonstrate the proposed theory and methods for remote sensing images and digitized aerial photographs. These experiments may reveal important clues for a glaciologist to determine the future direction on an Antarctic project.
1.2 General Approach and Anticipated Problems

In order to match images automatically and obtain accurate data, some accessory processes are required. Before matching, images need to be pre-processed in order to meet required matching conditions. After matching, quality control is necessary to reject bad data resulting from mismatches. If a matching process which does not determine velocities and strain rates directly is applied, simple algebra is needed to derive wanted data. Figure 1.1 shows the proposed general approach.

Figure 1.1: Proposed general approach.
Images obtained at different epochs may be quite different in radiometry and geometry. However, similar radiometry and geometry are required for most matching techniques. In addition, in order to determine displacements and deformations from matched image patches, transforming images to a common coordinate system is also required. Pre-processing is therefore needed to adjust radiometric and geometric differences between images.

Sun-angle difference and projection error are two problems often encountered in matching images. These two problems may cause systematic errors or even create mismatches. Pre-processing should also correct these two problems. However, sensor position and orientation as well as terrain relief are required for correcting the problems, and they are often unknown or poorly calculated. Thus, such problems often can only be roughly corrected or remain incorrect. In order to avoid mismatches caused by these problems, images should have similar sun directions, and large-scale projection errors should be corrected.

At the start of the matching process, finding conjugate image areas is the first approximation problem. After pre-processing, images are in a common coordinate system. If ice surface is static, conjugate points can be obtained by looking for the same coordinates. However, ice motion (the maximum speed is about 400 m/yr) may result in conjugate areas several tens of pixels apart. The matching procedure should find correct matches even if initial approximations are poor. The method of cross-correlation [Gambino and Crombie, 1974] has proven to be very effective in finding corresponding areas. The only requirement for this method is that the size of a search window in the second image should be large enough to cover the conjugate area of a matching patch in the first image. This method also needs a large amount of
computation time. In order to decrease the need of the computation time, the size of search window needs to be reduced by using roughly known prior information of ice flow.

Deformation and rotation of ice features due to ice motion may disturb a matching process and lead to a poor match if the algorithm cannot adapt to the deformation and rotation. A cross-correlation method can not adapt to geometrical differences, so it is limited in its accuracy achievements. A least-squares matching is highly adaptive [Ackermann, 1984] to geometrical difference and has the reputation of obtaining matching results with high accuracy. Its small pull-in range (1 or 2 pixel) [Gruen and Baltsavias, 1986], which is the capability of converging to correct match, is also well known. Thus a two-step matching, a cross-correlation followed by a least-squares matching, is proposed to solve this problem.

In order to determine velocities and strain rates directly, traditional least-squares matching needs to be modified. Shaping parameters (the parameters of an affine transformation are commonly used) should be replaced by the parameters of ice motion (two components of velocity, three strain rates, and a rotation rate). In addition to the advantage of direct estimation, this modification allows us to determine the theoretical accuracy of final results more easily and to match more than two multi-temporal images in one step.

A pair of image patches may be mismatched due to many factors. For example, image patches are too noisy, blurred by ground fog, covered by clouds, or do not contain enough textures to match. Consequently, a procedure of quality control is needed to reject data resulting from mismatches. Because biased data greatly disturb
the interpretation and analysis, a strict quality control is necessary. Because incorrect data are not distinguishable, the strategy of the quality control is to reject all suspicious data. Some bad data may still remain after quality control. Thus, a consistency check is needed to identify inconsistent data and to estimate the relative data accuracy.

The theoretical accuracy of matching results can be assessed according to the theory of least-squares estimation. An optimal patch size for matching can also be determined by analyzing changes of the a posteriori estimation of the variance of unit weight versus changes of patch size.

1.3 Organization

This report is composed of six chapters including this introductory chapter. Chapters II, III, and IV discuss the theory and methods of pre-processing, matching, quality control and assessment respectively. Chapter V describes experiments and compares results with manual data. The last chapter concludes this study and suggests future research.

In chapter II, the principles and methods of radiometric adjustment and geometrical correction used to pre-process images for the matching process are discussed. Based on the properties of remotely sensed images and digitized aerial photographs, different strategies and methods of pre-processing are developed.

Chapter III deals with the matching theory. In order to show how velocities and strain rates can be determined directly by the matching process, the fundamental theory and derivations of ice motion are discussed first. The technique of cross-correlation
used to obtain reliable but less accurate matches is described next. The central theme of this chapter is the theory of the modified least-squares matching which matches images with high accuracy and determines velocities, strain rates, and rotation rate directly. The function of the matching process is then extended to match more than two multi-temporal images in one step. Finally, a method to derive strain rates using data (displacements or the parameters of an affine transformation) resulting from a traditional matching method.

Chapter IV covers the methods of quality control and assessment. For quality control, an SNR test, a similarity check, a uniqueness check, and a consistency check are devised. The relative and theoretical accuracy are used to assess data quality. A method for determining the optimal patch size for matching is also developed according to the principle of theoretical accuracy.

Two pairs of Antarctic images mostly covering the same area are tested, and the results are compared with manually measured data. The first experiment matches a pair of SPOT images, and a pair of digitized aerial photographs are matched in the second experiment. The matching results of the both experiments are shown by using isoline maps and are compared with the same sort of maps obtained from manual data which are results of repeat photogrammetry by measuring the same aerial photographs digitized.

The last chapter summarizes the study, achievements, and shortages. In the end, it concludes this research with suggestions for practical works and future research.
Chapter II
Pre-Processing

2.1 Introduction

To achieve the automation of image matching and to reduce matching errors are the primary motivations of image pre-processing. The automatic matching process would not be feasible, if images are different in orientation and scale. Thus, the first mission of pre-processing is to adjust the orientation and scale differences between images. In addition, bad matching conditions may lead images to be mis-matched. For examples images might be noisy, different in gray-tone and contrast, or inherent with projection errors. Improving the matching conditions is therefore necessary. According to their usage, the pre-processing techniques can be divided into two categories - radiometric adjustment and geometrical correction. Their theories and effects are discussed in sections 2.2 and 2.3 respectively.

The needs of pre-processing are subject to requirements of a matching process and characteristics of the images. A primary requirement for image matching is that images should be similar in brightness, orientation, and scale. To improve the quality of matching results, further refinements to images may be needed. In this study, images obtained at different epochs are to be matched. These images may be quite different in brightness and contrast due to the use of different sensors or the change of illumination systems. In addition, the images are often different in orientation and
scale due to varying sensor positions and altitudes. Thus, the techniques of pre-processing are principally developed to adjust the radiometric and geometrical differences between images. The techniques are also extended to obtain better quality of the matching results, if these are feasible.

There are two different approaches in developing the pre-processing techniques. Normalizing images according to the physical conditions of image formation is the first approach. This approach is rigorous, but requires the knowledge about the sensor orientation and terrain relief. Modifying the image shadows to be seen as if images were taken under a fictitious sun direction is an example. Rectifying images to be free from projection errors is another example. The second approach is to modify the differences of the images relatively, without considering the physical condition of image formation. For example, one can simply adjust images to be similar in brightness and orientation, so that the minimum requirements for a matching process are provided. Although this approach is simple, some systematic errors may occur in the matching results.

Projection errors caused by sensor-orientation and shadow differences resulting from sun-angle changes are considered to be two major influencing factors in obtaining accurate matching results. It is desired to follow the first approach to correct these differences. However, some unknown physical conditions confine the implementation of the first approach. The following paragraphs discuss the approaches for the cases of the images to be tested.

For SPOT images, neither correction is feasible due to the poorly known sensor orientation and terrain relief. In fact, the corrections are not critical in matching SPOT images. Projection errors of SPOT images are small due to high altitude of the sensor
(832 km) and the small field of view [Richards, 1986]. And shadow differences do not cause significant problems in either SPOT image. This has been proven by visually checking the images and by matching some conjugate image patches of the images.

For digitized aerial photographs, both corrections are possible, because the sensor position and orientation are estimated by aerial triangulation, and terrain relief can be determined by matching the stereo image pairs. However, the determination of terrain relief is difficult. First, generating a digital elevation model (DEM) by matching stereo images is a time-consuming and computation-intensive task. In addition, the DEM obtained from stereo images may not be reliable in some subtle or vague image areas. Thus, we use a rough DEM formed by using the estimated coordinates of tie points resulting from the computations of aerial triangulation to rectify large-scale projection errors. Because the DEM is too rough, correction for shadow differences is not possible. This might be the reason why the matching results of digitized aerial photographs are not as good as that of SPOT images.

### 2.2 Radiometric Adjustment

Radiometric equalization and noise reduction are two techniques used to adjust the gray-tone and contrast of images. The technique of radiometric equalization tunes up images to be similar in their gray-tones and contrast. The noise-reduction process increases the correlation coefficient between images by smoothing gray-levels. These two techniques and their effects are discussed in the following two sub-sections.
2.2.1 Radiometric Equalization

Tuning up images to have similar gray-tones and contrast is the primary goal of radiometric equalization. This can be achieved by using two different approaches. A relative adjustment that modifies the brightness values of one image to be similar to that of the other image is the first approach. The second approach is to normalize each image to a preconceived distribution of gray values. This approach adjusts each image individually without the need of knowing the brightness values of the other image. In addition, it permits modification of images to be optimal in vision, with mid-gray tones and maximum contrast. For these reasons, the second approach is adopted.

A histogram of the distribution of brightness values is used to assess or adjust the tonal or radiometric quality of an image. Figure 2.1 shows two example images A and B and their histograms. Image B is an enhanced version of image A by stretching the histogram. This example illustrates that the visual impact of an image can be dramatically improved by simply stretching its histogram.

A process of contrast modification is the mapping of brightness values, in which the brightness value of a particular histogram bar is re-specified more favorably. The mapping of brightness values associated with contrast modification can be described as \( y = f(x) \), where \( x \) is the original brightness value of a particular bar in the histogram and \( y \) is the corresponding new brightness value [Richards, 1986]. A simple contrast modification can be accomplished by using a closed mathematical form of mapping function, for example a linear \( y = ax + b \), a logarithmic \( y = b \cdot \log(ax) + c \), or an exponential \( y = b \cdot e^{ax} + c \) contrast enhancement et cetera. Figure 2.1 is an example of a linear contrast modification.
Figure 2.1: Two example images and their histograms. Image (B) is an enhanced version of image (A).

Instead of using a closed mathematical form of the mapping function, the second approach of radiometric equalization matches an image histogram to a preconceived diagram, so that it is called a histogram equalization. The mapping function is altered case by case subject to the shape of the image histogram and the reference diagram. In this study, a Gaussian function is used as the reference diagram. With gray levels ranging from 0 to 255, the Gaussian function has its mean, $\mu$, at the gray level of 127.
with a standard deviation of $\sigma = 42$, so that $\pm 3\sigma$ is at 0 and 255. Thus, it is expressed as:

$$f(x) = \frac{1}{\sqrt{2\pi \sigma}} e^{\frac{(x-\mu)^2}{2\sigma^2}}, \quad 0 < x < 255 \quad \text{(2.1)}$$

Figure 2.2 shows the Gaussian function. This equalization process yields an image with a saturated contrast and in which most detail is contained in the mid-gray range.

The histogram equalization is implemented by adjusting the cumulative histogram of an image to match the cumulative Gaussian function. A look-up table is built to record the newly assigned gray-level of each original brightness. New gray-levels are determined by comparing the cumulative histogram with the cumulative Gaussian function. The image is then modified according to the look-up table. Figure 2.3 depicts the procedure of a histogram equalization, in which a new gray-level $y$ replaces the original gray-level $x$. And figure 2.4 shows the equalized version of image A in figure 2.1 and its new histogram. The histogram of the equalized image is a little skewed, because the original histogram is not symmetrical.
Although the process of histogram equalization enhances the contrast of an image, its inherent noises have also increased. This can be seen by investigating a profile of image A in figure 2.1 and that of the equalized version. Figure 2.5 shows the central profile (at row number 100) of the original image, and figure 2.6 shows that of the equalized image. It can be seen that the equalization process amplifies all the gray-level changes including the noise. Changes in the right portion of the image profile of a flat ice surface reveal the amount of noise increased.
Figure 2.5: Central profile of the original image.

Figure 2.6: Central profile of equalized image.

The process of histogram equalization will not affect the matching process of cross-correlation to be discussed in chapter III, because the correlation is normalized with variances of images. This can be seen by checking the correlation functions (to be defined in section 3.3) between two image patches (a template and a search window to be defined in section 3.1) before and after the process. The peak profiles which
contain the maximum correlation coefficients of the correlation functions are compared in figure 2.7. There are no significant differences between the profiles.

![Graph showing peak profiles of the correlation functions before and after equalization.](image)

Figure 2.7: Peak profiles of the correlation functions before and after histogram equalization.

On the other hand, histogram equalization does improve the matching condition for least-squares matching to be mentioned in chapter III. Least-squares matching is very sensitive to radiometric differences between images, because it searches for the maximum in un-normalized correlation functions - the covariance function. This can be proved by investigating differences of the covariance functions before and after the process. Figure 2.8 compares peak profiles of the covariance functions. Significant changes, about a factor of 200 times, between the profiles can be recognized. This may explain that least-squares matching fails to match original images, but matches equalized images very well.
2.2.2 Noise-Reduction process

Reducing the noise of images improves the matching conditions [Ehlers, 1980]. A noise-reduction process is particularly useful in suppressing amplified noises resulting from the process of radiometric enhancement. Thus, the technique of noise reduction is applied to images after radiometric equalization.

The desired information and unwanted noise are not readily distinguishable. But the analysis of the image power spectrum done by Helava [1976] shows that the low-frequency spectrum contains the most important information for image correlation. It is therefore appropriate to assume that noises are mainly high-frequency random signals. Based on this assumption, a low-pass filter is used to reduce these noises.

A simple low-pass filter can be implemented by convolving an image with an equal-coefficient window. Figure 2.9 shows an example of a 3 by 3 filter in which the total...
weight of 1 is equally distributed among all cells. More elaborate filters can be found in many signal processing books, for example Young's [1986] and Hamming's [1989]. Most commercially available image-processing software provides various options for filters, thus discussions on the theory of filters are not covered here.

```
1/9 1/9 1/9
1/9 1/9 1/9
1/9 1/9 1/9
```

Figure 2.9: A simple low-pass filter.

A low-pass filter reduces noises and increases the correlation coefficient between matched image patches. By applying the 3 by 3 filter shown in figure 2.9 to the equalized image A, its profile in figure 2.6 will be smoothed as in figure 2.10. Figure 2.11 shows peak profiles of the correlation functions of the equalized image patches before and after the filtering process. The peak is obviously sharpened after applying the low-pass filter. This concludes that a low-pass filter enhances similarity between images, so that more reliable matches can be obtained. It also reveals that noises dominate the short-wave-length (< 30 meters in ground scale) information of the images.
2.3 Geometric Correction

Adjusting orientation and scale differences between images and correcting projection errors resulting from image formation are the primary objectives of
geometric correction. The orientation and scale correction process validates the automation of the matching process in finding conjugate image patches. The projection correction process improves the accuracy of matching results.

As mentioned in the introductory section, two approaches are involved in implementing geometric corrections. For SPOT images, only the two-dimensional orientation and scale differences between images can be adjusted, because the sensor orientation and position are not known accurately. This can be achieved by transforming the coordinate system of one image to that of the other using a similarity transformation. This approach warrants the matching process to find the conjugate image patches, but it cannot correct for projection errors to improve the matching accuracy. For digitized aerial photographs, we follow the first approach to adjust orientation differences and projection errors. Images are transformed to the ground coordinate system and projection errors are rectified according to the sensor orientation and terrain relief.

The resampling process and corrections to SPOT images and digital aerial photographs are discussed in the following sub-sections. The resampling process is discussed first because corrections using forward or backward transformation and rectification are subject to the strategy of resampling.

2.3.1 Resampling

The objective of resampling is to construct a grid structure in the transformed or rectified image. Geometrical corrections destroy the original grid structure of an image. The resampling procedure is required to obtain a grid in the new image.
Resampling and geometrical corrections of images can be done directly (forward) or indirectly (backward) [Hössler, 1980]. The direct method transforms or rectifies original pixel positions (in grid structure) to a designated coordinate system in which the transformed or rectified positions are no longer in grid structure. Then a new grid is constructed in the designated coordinate system, and gray-levels of new grid positions are interpolated from those of the transformed or rectified positions. Figure 2.12 shows the procedures of the direct method. The indirect method performs resampling and geometrical corrections in reverse. A grid is firstly constructed in the designated coordinate system, then the grid positions are backward transformed or rectified to the original image coordinate system. Gray-levels of each transformed positions are interpolated from those of the image pixels and assigned back to the corresponding grid positions in the designated coordinate system. Figure 2.13 shows the procedures of the indirect method.

Figure 2.12: Direct geometrical correction and resampling method.
Figure 2.13: Indirect geometrical correction and resampling method.

The indirect method is superior to the direct method in implementation. The indirect method resamples an image in its original coordinate system in which known gray-levels are in the original grid structure. Gray-levels of non-grid positions are interpolated from known values of the grid positions. Thus, a simple bi-linear interpolation method can be used. In addition, gray-levels of the four nearby grid positions are needed for interpolating each non-grid position, so that only a small amount of computer central memory is necessary. On the other hand, the direct method resamples an image in the designated coordinate system in which known gray-levels are in the transformed non-grid positions. Gray-levels of the reconstructed grid positions are interpolated from those of the non-grid positions. Because the known gray-levels are in irregular structure, the interpolation process is complicated. Also, more space is needed to store the coordinates of non-grid positions in the central memory for interpolation. The indirect method is thus adopted for this study for the proceeding reasons.
Using the indirect method, geometrical corrections need to be performed indirectly as well. Thus, geometrical corrections described in the following sub-sections for both SPOT images and digitized aerial photographs are all backward.

2.3.2 Relative Transformation for SPOT Images

A similarity transformation is used to adjust the orientation and scale differences between SPOT images in order to fulfill the matching requirements. Projection errors are not corrected due to the poorly known sensor position and orientation. These errors are actually so small (due to high altitude of the sensor and small field of view) that they will not disturb the matching process.

The relative orientation, scale, and translation between SPOT images are unknown. It means that parameters of the similarity transformation for the geometrical correction are not known. These parameters need to be determined in order to proceed the geometrical correction. This can be done by using the coordinates of two or more manually measured conjugate points.

The conjugate points should be selected in a static area of the ice surface. However, a static area often lacks distinct features to identify conjugate points, so that the portion of a shear zone next to an interstream ridge, which is moving very slowly compared to other crevassed areas, is considered to be the best area in selecting conjugate points. The estimated transformation parameters may contain errors, if the conjugate points are not static. This leads to imperfect geometrical correction. Because velocities, strain rates, and rotation rate are sensitive to geometrical errors, systematic errors may occur due to imperfect geometrical correction.
Three methods can be used to measure the conjugate points. First, the points are measured on a computer screen by using a pointing device. Because image features of an ice surface may be subtle or lack sharp boundaries, conjugate points may not be coregistered accurately. Second, a cross-correlation method can be used to achieve one-pixel accuracy. A template and its conjugate search window (to be defined in chapter III) can be specified on a computer screen, then the matched pixels are determined by using cross-correlation. By using this method, it is required that images not be too different in scale and orientation in order to correlate; otherwise, one of the images should be resampled according to the approximately known scale factor and orientation. The third method is to apply a two-step matching: a cross-correlation followed by a least-squares matching. This method offers an accuracy at the sub-pixel level. The same requirements as the second method are needed. This method is more elaborate but may not be necessary because movements of the selected conjugate points may cause errors larger than one pixel.

Having measured the coordinates of the conjugate points, transformation parameters can be calculated from these coordinates. According to the resampling principle, backward transformation parameters are to be calculated. For instance, if image B needs to be resampled in the coordinate system of image A, the transformation parameters from image A to B are to be calculated. Let \((x_{ai}, y_{ai})\) and \((x_{bi}, y_{bi})\) be coordinates of the \(i\)th conjugate point in images A and B respectively, and \((a, b, \Delta x, \Delta y)\) be the backward transformation parameters. Then, a similarity transformation can be formulated as:

\[
x_{bi} = a \cdot x_{ai} + b \cdot y_{ai} + \Delta x
\]
\[
y_{bi} = a \cdot y_{ai} - b \cdot x_{ai} + \Delta y
\]  
(2.2)
If there are only two measured conjugate points, the parameters are computed as follows:

\[
\begin{bmatrix}
    a \\
    b \\
\end{bmatrix}
= \begin{bmatrix}
    x_{A1} & y_{A1} & 1 & 0 \\
    y_{A1} & -x_{A1} & 0 & 1 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
    x_{B1} \\
    y_{B1} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \Delta x \\
    \Delta y \\
\end{bmatrix}
= \begin{bmatrix}
    x_{A2} & y_{A2} & 1 & 0 \\
    y_{A2} & -x_{A2} & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
    x_{B2} \\
    y_{B2} \\
\end{bmatrix}
\]

If more than two conjugate points are measured, a least-squares adjustment can be used to determine the optimal transformation parameters.

The parameters \( \Delta x \) and \( \Delta y \) are the components of translation. The scale factor, \( S \), and rotation angle, \( \theta \), can be calculated from parameters \( a \) and \( b \) as follows:

\[
S = \sqrt{a^2 + b^2}
\]

\[
\theta = \tan^{-1}\left(\frac{b}{a}\right)
\]

The quantities \( \Delta x, \Delta y, S \) and \( \theta \) present the geometrical differences between images A and B.

Knowing the transformation parameters, a grid structure can be constructed in image A, and each grid position is transformed to the coordinate system of image B by using:

\[
\begin{bmatrix}
    x_B \\
    y_B \\
\end{bmatrix}
= \begin{bmatrix}
    a & b \\
    -b & a \\
\end{bmatrix}
\begin{bmatrix}
    x_A \\
    y_A \\
\end{bmatrix}
+ \begin{bmatrix}
    \Delta x \\
    \Delta y \\
\end{bmatrix}
\]

After the transformation, if point \((x_B, y_B)\) is located inside the boundaries of image B, the gray-level of \((x_B, y_B)\) can be bi-linearly interpolated from its nearby pixels and assigned back to \((x_A, y_A)\). Otherwise, a zero gray-level is assigned.
2.3.3 Geometrical Corrections for Digitized Aerial Photographs

Rigorous geometrical corrections are feasible for digitized aerial photographs. Two projective relationships are involved in correcting the geometry. First, digitizing the aerial photographs by using a CCD camera involves a 2-D to 2-D projection, i.e., from the photo plane to the sensor array matrix. This projective relationship can be retrieved by using the measured image coordinates and known photo coordinates of the fiducial marks. The second projective relationship is a 3-D (ground) to 2-D (photo) projection. This relationship is computed from aerial triangulation. Knowing these two projective relationships, digitized images can be transformed to the ground coordinate system and rectified to remove most of the projection errors.

The primary requirements for matching images (similar scale and orientation) are ultimately fulfilled by transforming all images to the ground coordinate system. The completeness of projection error correction is subject to the amount of detail and accuracy of the terrain relief data. As mentioned in the introductory section, a rough DEM is formed by using coordinates of the tie points computed from aerial triangulation. Thus, only large-scale projection errors can be corrected.

In the following four sub-sections, characteristics of the involved coordinate systems are discussed first, such that transformations between them can be made more transparent. Then, the projective transformation from the photo to the image coordinate system is discussed. Third, the collinearity transformation from the ground to the photo coordinate system is presented. The last sub-section describes the overall procedures of rectification and resampling.
Coordinate systems

Three coordinate systems are involved in correcting the geometry of digitized aerial photographs. They are the digital-image, the photo, and the ground coordinate systems. In different fields, different definitions for each coordinate system may exist. Here, the discussion is focused on the one we used in this study.

Figure 2.14 shows the digital-image coordinate system and an $m$ by $n$ image. This system is defined as a left-handed system, because an image is often scanned from left to right in columns and from top to bottom in rows. A right-handed system can be obtained by changing the sign of the row numbers.

![Figure 2.14: Digital-image coordinate system.](image)

The photo coordinate system can be defined as a 2-D or 3-D system. A 2-D system is used in retrieving the photo geometry, because the projection is plane to plane. The definition of a 3-D relationship between the photo and ground coordinate systems is needed to retrieve the geometry of ground coordinates. Thus, it is necessary to define a 3-D photo coordinate system.

Figure 2.15 depicts the 2-D photo coordinate system. This system is conventionally defined by the fiducial marks. The principal point is taken as the origin. The $x$-axis is chosen to coincide with the line connecting the origin and the fiducial most nearly
aligned with the direction of flight. And the y-axis is chosen to be perpendicular to the x-axis to form a right-handed system.

![Figure 2.15: 2-D photo coordinate system.](image)

The 3-D photo coordinate system is illustrated in figure 2.16. The origin of the 2-D system is shifted to become the perspective center, and the z-axis is chosen to be perpendicular to the photo plane to form a right-handed system. Because the distance between the perspective center and the principal point is the focal length \( f \), the z coordinate of a photo point is always \(-f\).

![Figure 2.16: 3-D photo coordinate system.](image)
A locally defined 3-D Cartesian system has been used as the ground coordinate system for the manually measured data by Whillans et al. [submitted] using repeat photogrammetry. In order to obtain consistent results, the same ground coordinate system is used here. Figure 2.17 illustrates the relationship between the local coordinate system and the global coordinate systems - the geographical and geocentric coordinate systems. The origin of the local coordinate system is chosen at the point with the geographic coordinates as follows:

- **latitude**: $S 83^\circ 28'00"$
- **longitude**: $W 138^\circ10'00"$
- **height**: 340 meters,

which is near by the Up-B Camp. The up-direction at the origin is assigned as the Z-axis, the direction pointing to the South Pole is the Y-axis, and the X-axis is chosen to be perpendicular to the Z and Y-axes to form a right-handed system. This local coordinate system can be easily transformed from a global coordinate system.

![Diagram of coordinate systems](image)

Figure 2.17: Relationships between the local and global coordinate systems.
Projective Transformation from Photo to Image Coordinate System

Figure 2.18 shows the projective relationship between the digital-image and photo coordinate systems. The projective transformation from the photo to the digital-image coordinate system can be expressed as:

\[
\begin{bmatrix}
  c \\
  r
\end{bmatrix} = S \begin{bmatrix}
  a_1 & a_2 & a_3 \\
  a_4 & a_5 & a_6 \\
  1 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\] \hspace{1cm} (2.6)

in which \( c \) and \( r \) are the column and row numbers of an image pixel respectively, \( x \) and \( y \) are photo coordinates, and \( S \) is the scale factor. Eight parameters \( a_1, a_2, \ldots, a_8 \) are involved in this transformation [Shih, 1989]. This transformation can be simplified as:

\[
c = \frac{a_1 \cdot x + a_2 \cdot y + a_3}{a_7 \cdot x + a_8 \cdot y + 1} \]

\[
r = \frac{a_4 \cdot x + a_5 \cdot y + a_6}{a_7 \cdot x + a_8 \cdot y + 1} \]

This backward transformation is intentionally designed according to the principle of resampling.
Figure 2.18: Relationship between the image and photo coordinate systems.

The transformation parameters can be calculated by using the measured image coordinates and the known photo coordinates of the fiducial marks [Chen, 1992]. If eight fiducials are available in each photograph, there are 16 measurements, and 8 unknowns are to be solved. A least-squares adjustment can be used to solved this redundant system.

**Collinearity Transformation from Ground to Photo Coordinate System**

Figure 2.19 illustrates the perspective relationship between the photo and ground coordinate systems. According to the collinearity function commonly used in photogrammetry, the transformation from the ground to the photo coordinate system is expressed as:
\[ x = f \frac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \]

\[ y = f \frac{r_{21}(X - X_0) + r_{22}(Y - Y_0) + r_{23}(Z - Z_0)}{r_{31}(X - X_0) + r_{32}(Y - Y_0) + r_{33}(Z - Z_0)} \]  

(2.8)

where

\((x,y)\) : photo coordinates

\(f\) : focal length

\((X,Y,Z)\) : ground coordinates

\((X_0,Y_0,Z_0)\) : the perspective center

\(r_{11}, r_{12}, \ldots, r_{33}\) : elements of the rotation matrix

The focal length is obtained from camera calibration. The coordinates of the perspective center and the elements of the rotation matrix are computed from an aerial triangulation. By given \(X, Y,\) and \(Z,\) the transformed photo coordinates can be calculated.

Figure 2.19: Relationship between the photo and ground coordinate systems.
Rectification and Resampling Procedures

According to section 2.3.1, the indirect method of rectification and resampling is applied. A 2-D grid structure is first constructed in the ground coordinate system. For each grid position, its elevation is interpolated from the DEM. These 3-D coordinates are then transformed to the image coordinate system through the photo coordinate system. The resampling process is performed in the image coordinate system, and the resampled gray-levels are assigned back to the corresponding grid points. Figure 2.20 summarizes the overall procedure.

Figure 2.20: Procedure of geometrical correction for digitized aerial photographs.
Chapter III
Matching Process and Estimation of Ice Motion

3.1 Introduction

This chapter deals with the theory of the matching process and shows how ice motion is determined directly or indirectly by matching multi-temporal images of Antarctica ice stream. It contains six sections including this introductory section. Two velocity components, three strain rates, and a rotation rate are the parameters used to describe ice motion. They are termed the parameters of ice motion in this study, and are defined and derived in the second section. A two-step matching process is proposed to obtain optimal results. Less precise but more reliable results are first obtained by applying the matching technique of cross-correlation. These results offer good approximations to the next matching step - least-squares matching. The third section covers the theory and procedure of the cross-correlation method, and the forth section shows how traditional least-squares matching was modified to determine the parameters of ice motion directly. The function of the matching process is then extended to match images from three or more epochs. This efficient and accurate multi-image matching procedure is described in the fifth section. The last section notes how the parameters of ice motion can be determined indirectly by using existing image-matching software.
At this stage in the procedure, it is assumed that the images are referenced to the same coordinate system and are similar in brightness, as described in the preceding chapter. As far as feasible, projection errors should also have been corrected. Consequently, geometrical differences between matched image patches represent the deformation of the ice surface between epochs. The matching can be done in either an image coordinate system (as done here with SPOT imagery) or in an earth-based system (as done here with aerial photography).

Some general remarks about matching procedures are appropriate, before proceeding to the following sections. For the case of matching two images, the image from the first epoch is divided into square or rectangular patches of equal size. These patches are called templates. Associated with each template, a larger image patch is selected in the second image. This is called search window here. A search window should cover the conjugate image area of its associated template. The matching process compares one conjugate pair of a template and search window at a time. By applying the cross-correlation method, displacements can be determined in one-pixel accuracy. These displacements can be used to derive the parameters of ice motion using the equations described in section 3.6, or provide the needed approximations for least-squares matching (LSM). By using the modified least-squares matching to be described in section 3.4, the parameters of ice motion can be determined directly. If a traditional least-squares matching is applied, the parameters of ice motion can be derived from the estimated parameters of deformation, which are also called shaping parameters. The derivations are described in section 3.6. Figure 3.1 illustrates the overall procedure. A similar procedure can be followed for the case of matching more than two images. It is described in section 3.5.
Figure 3.1: Image matching process.
The sizes of template and search window are crucial in obtaining reliable matches efficiently. Theoretically, the larger template size yields the higher accuracy. However, on the one hand, the mathematical functions used to model the deformation of a finite area of ice surface may fail to model that of a larger area due to the assumption of linear property mentioned in section 3.2. On the other hand, computation time needed for matching increases exponentially with an increase in the size. Therefore an optimal template size should be determined before matching. The size of a search window is determined such that it cover the possible conjugate area of its associated template. If the center of a search window is chosen to have the same coordinates as that of the center of its conjugate template, the size of search window should be twice the anticipated maximum ice displacement plus the template dimension. Figure 3.2 shows the relationship between the dimensions of a template and search window, where the dimension of the template is $a$ by $b$, and $D_{\text{max}}$ is the anticipated maximum displacement. The size of a search window can be reduced if the velocity of the covered ice surface can be predicted prior to the matching process. This can dramatically reduce the computation time to about 20% of the original need.

\[ D_{\text{max}} \quad a \quad D_{\text{max}} \]

\[ D_{\text{max}} \]

\[ b \]

\[ D_{\text{max}} \]

\[ \text{template} \]

\[ \text{search window} \]

Figure 3.2: Relationship between the dimensions of template and search window.
Because the parameters of ice motion are determined by a template and its matched patch, it is preferable to derive the parameters which are referenced to the geometrical center of the template, which is called the centroid of the template. In fact, strain rates and rotation rate are independent of a reference point, whereas the components of a velocity are changed subject to the use of a reference point. When a map is needed to show determined velocities, it makes more sense to show the velocities which are referenced to the centroids of their associated templates.

In order to cope with geometrical differences between the images caused by ice deformation, an adaptive matching is needed to achieve an optimal match [Markarovic, 1980 and Förstner, 1984]. Among existing methods of area-based matching using gray-levels, the technique of least-squares matching has been proven the most efficient method in this aspect [Gruen, 1985]. The least-squares method also offers an important advantage that the mathematical function used to model geometrical differences can be modified to fit the physical conditions. This provides us an opportunity to model geometrical differences using the parameters of ice motion. Thus, the technique of least-squares matching is adopted in order to obtain optimal matching results.

It should be noted that least-squares matching cannot work alone in this application due to its small pull-in range of 2-3 pixels [Schewe and Förstner, 1986], which is the distance from which a matching process can converge to the correct match. A very good first approximation is required. Thus, the image should first be matched by a cross-correlation method in order to provide least-squares matching good approximations. The approach of cross-correlation exhaustively searches for the maximum correlation coefficient of all possible matches. It generally offers a reliable match, unless the images are homogenous or contain very different features.
Therefore, a two-step matching, a cross-correlation followed by a least-squares matching, is suggested.

### 3.2 Derivation of Ice-Motion Parameters

The parameters of ice motion are two components of velocity, three strain rates, and a rotation rate, whereas image matching measures displacements of matched image points or parameters of deformation of matched image patches. Thus, the mathematical relationship between ice-motion parameters and measured displacements or deformation parameters needs to be derived. The following two sub-sections discuss the derivations of ice motion from displacements and deformation respectively.

#### 3.2.1 Deriving Ice Motion from Displacements

According to the fundamental concept of rock mechanics [Jaeger and Cook, 1976], the components of strain and rotation are defined in terms of gradients of displacement as:

\[
\varepsilon_{xx} = \frac{\partial D_x}{\partial x} \\
\varepsilon_{yy} = \frac{\partial D_y}{\partial y} \\
\varepsilon_{xy} = \frac{1}{2} \left[ \frac{\partial D_y}{\partial x} + \frac{\partial D_x}{\partial y} \right] \\
\omega = \frac{1}{2} \left[ \frac{\partial D_y}{\partial x} - \frac{\partial D_x}{\partial y} \right]
\] (3.1)
where
\[ \varepsilon_{xx}, \varepsilon_{yy}, \text{ and } \varepsilon_{xy} : \text{components of strain} \]
\[ \omega : \text{component of rotation} \]
\[ D_x, D_y : x \text{ and } y \text{ components of displacement} \]

The components of strain and rotation in equation (3.1) are, by definition, infinitesimal. They are defined based on the assumption that the components of strain and rotation are determined from the displacements of two points which are so close that their second derivatives can be neglected.

The definitions of the components of strain and rotation can be extended to define strain rates and rotation rate in terms of gradients of velocity. Velocities are derived from displacements and the time interval between the epochs of image formation as:

\[ U_x = \frac{D_x}{T} \]
\[ U_y = \frac{D_y}{T} \]

where
\[ U_x, U_y : x \text{ and } y \text{ components of velocity} \]
\[ T : \text{time interval} \]

Replacing gradients of displacement by gradients of velocity in equation (3.1), strain rates and rotation rate are determined as:

\[ \dot{\varepsilon}_{xx} = \frac{\partial U_x}{\partial x} \]
\[ \dot{\varepsilon}_{yy} = \frac{\partial U_y}{\partial y} \]
\[ \dot{\omega} = \frac{\partial U}{\partial y} \]
\[ \epsilon_{\perp} = \frac{1}{2} \left[ \frac{\partial u_x}{\partial x} + \frac{\partial u_x}{\partial y} \right] \]

\[ \omega = \frac{1}{2} \left[ \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right] \]

where

\[ \epsilon_{\perp}, \epsilon_{\|}, \text{and } \epsilon_{\perp} \]: components of strain rate

\[ \omega \]: component of rotation rate, positive anti-clockwise

Equations (3.2) and (3.3) determine the parameters of ice motion by using estimated displacements. If a matching process yields displacements, such as the cross-correlation method applied by Bindschadler and Scambos [1991] and the normalized cross-correlation method to be mentioned in the next section, these equations can be used to derive the parameters of ice motion from the displacements. This indirect scheme is addressed in section 3.6.

### 3.2.2 Deriving Ice Motion from Deformation

Ice motion can also be determined by using measured parameters of deformation in a finite area. Considering a small area of ice surface, the deformation in this area can then be expressed by using a Taylor expansion of the first degree. This is based on a hypothesis that the imaged area of a template is so small that the assumption made for the definitions of equation (3.1) is still valid. In other words, the deformation in this small area is assumed to be linear. Thus, the deformation can be expressed as displacement functions of coordinates in the small area:
In equation (3.4), the deformation is represented by two linear displacement functions of \(x\) and \(y\) coordinates, in which gradients of displacement and the displacement of the origin are parameters of deformation.

Similar to the derivation of equation (3.3), deformation rate can be obtained by dividing equation (3.4) by time interval \(T\) as:

\[
U_x(x, y) = U_{ox} + x \cdot \frac{\partial U_x}{\partial x} + y \cdot \frac{\partial U_x}{\partial y}
\]

\[
U_y(x, y) = U_{oy} + x \cdot \frac{\partial U_y}{\partial x} + y \cdot \frac{\partial U_y}{\partial y}
\]  

(3.5)

where

\(U_{ox}, U_{oy}\) : velocity of the origin of the coordinate system

In equation (3.5), the deformation rate is represented by two linear velocity functions of \(x\) and \(y\) coordinates, in which gradients of velocity and the velocity of the origin are parameters of the deformation rate.

Parameters of the deformation rate in equation (3.5) are referenced to the origin of the coordinate system. However, it is preferable to derive the parameters which are referenced to the geometrical center or the centroid of the template. Gradients of
velocity are actually independent of the reference point, so that they are invariant. The velocity referenced to the centroid can be obtained by substituting equation (3.5) by the centroid coordinates \((\bar{x}, \bar{y})\) of a template as:

\[
U_z(\bar{x}, \bar{y}) = U_{0z} + \bar{x} \cdot \frac{\partial U_z}{\partial x} + \bar{y} \cdot \frac{\partial U_z}{\partial y} \tag{3.6}
\]

\[
U_y(\bar{x}, \bar{y}) = U_{0y} + \bar{x} \cdot \frac{\partial U_y}{\partial x} + \bar{y} \cdot \frac{\partial U_y}{\partial y} \tag{3.7}
\]

Let \(\bar{U}_x = U_x(\bar{x}, \bar{y})\) and \(\bar{U}_y = U_y(\bar{x}, \bar{y})\), then equation (3.5) can be revised as:

\[
U_x(x, y) = \bar{U}_x + (x - \bar{x}) \frac{\partial U_x}{\partial x} + (y - \bar{y}) \frac{\partial U_x}{\partial y} \tag{3.8}
\]

\[
U_y(x, y) = \bar{U}_y + (x - \bar{x}) \frac{\partial U_y}{\partial x} + (y - \bar{y}) \frac{\partial U_y}{\partial y}
\]

According to equation (3.3), strain rates and rotation rate are defined by gradients of velocity. Thus, equation (3.7) can be substituted by equation (3.3) to obtain:

\[
U_x(x, y) = \bar{U}_x + (x - \bar{x}) \cdot \dot{\epsilon}_u + (y - \bar{y}) \cdot \dot{\epsilon}_v - (y - \bar{y}) \cdot \dot{\omega} \tag{3.8}
\]

\[
U_y(x, y) = \bar{U}_y + (x - \bar{x}) \cdot \dot{\epsilon}_w + (x - \bar{x}) \cdot \omega + (y - \bar{y}) \cdot \dot{\omega}
\]

Equation (3.8) shows that deformation rate is modeled by the parameters of ice motion. This equation can be used to modify a least-squares matching to be a matching process that can directly estimate the parameters of ice motion. This will be discussed in detail in section 3.4.

The parameters of ice motion can also be indirectly determined by using a traditional least-squares matching which estimates the deformation between matched image patches with the parameters of an affine transformation. The parameters of ice
motion can be derived from estimated parameters of transformations. This indirect estimation scheme is discussed in section 3.6.

3.3 Cross-Correlation

The cross-correlation method correlates a template all over the conjugate search window. A coefficient of correlation is calculated from each possible match. Coefficients of all possible matches form a correlation function. The best match is achieved by searching for the maximum of the correlation function. One-pixel accuracy of matching can be obtained by simply locating the match that shows the maximum correlation coefficient. Sub-pixel accuracy can be achieved by interpolating the highest peak in the correlation function.

Applying the cross-correlation method to multi-temporal images offers displacements of matched points. The parameters of ice motion then can be derived from estimated displacements. This method has been applied successfully by Bindschadler and Scambos [1991] to match Landsat Thematic Mapper images of ice stream E in Antarctica. Although this method is feasible, the matching results may contain systematic errors due to the deformation of the ice surface. In order to avoid the systematic errors, an adaptive matching, such as a least-squares matching, is required, whereas a good approximation is needed to solve the pull-in-range problem. Thus, in this study, the cross-correlation method is applied to obtain the first approximation for the least-squares matching.

Many algorithms have been proposed to calculate the coefficient of correlation [Gambino and Crombie, 1974, Ehlers and Wrobel, 1980, and Göpfert, 1980], such as normalized correlation coefficient, residual, covariance, and Fourier transform.
correlation. The residual algorithm has been applied by Bindschadler and Scambos [1991]. This method and the covariance algorithm are sensitive to the brightness differences between the image patches, and the implementation of Fourier transformation is complicated. Here, the most commonly used algorithm, normalized correlation coefficient, is applied. It is invariant to brightness differences between the image patches and is very adaptive to varying contrasts [Fürstner, 1984]. The only shortcoming of this algorithm is that it is computationally intensive.

The coefficient of cross-correlation is a measure of the similarity of two image patches. According to the algorithm of normalized correlation, the coefficient is calculated by using the variances and covariance of the image patches. Let $\sigma_{LL}, \sigma_{RR}$ be the variances of a left and right image patches, $g_{Li}, g_{Ri}$ be gray-levels of the $i$th point of a left and right image patches, and $\sigma_{LR}$ be the covariance between the image patches. Then a coefficient of cross-correlation, $C$, is calculated as follows:

$$C = \frac{\sigma_{LR}}{\sqrt{\sigma_{LL} \cdot \sigma_{RR}}}$$  \hspace{1cm} (3.9)

where

$$\sigma_{LR} = \frac{1}{(n-1)} \sum_{i=1}^{n} [(g_{Li} - \bar{g}_L) \cdot (g_{Ri} - \bar{g}_R)]$$

$$\sigma_{LL} = \frac{1}{(n-1)} \sum_{i=1}^{n} (g_{Li} - \bar{g}_L)^2$$

$$\sigma_{RR} = \frac{1}{(n-1)} \sum_{i=1}^{n} (g_{Ri} - \bar{g}_R)^2$$

$n$ : number of pixels in the image patch

$\bar{g}$ : average gray-level of the image patch

The maximum correlation coefficient is 1, and -1 is the minimum. Changes in the brightness of images, equivalent to a shift of the image histogram, will not affect the
correlation coefficient after the normalization. But changing the contrast of images does slightly change the correlation.

One-pixel accuracy is obtained by simply locating the maximum correlation coefficient. Sub-pixel accuracy is not needed at this stage. The matching accuracy will be refined by least-squares matching to be discussed in the next section.

3.4 Direct Estimation of Ice Motion Using Least-Squares Matching

In this section we will describe how traditional least-squares matching is modified to estimate the parameters of ice motion directly. Based on the concept of traditional least-squares matching, mathematical functions of the geometrical transformation are modified to be modeled by the parameters of ice motion.

The advantages of directly estimating the parameters of ice motion can be recognized in three aspects. First, matching results can be used directly for the study of ice mechanics. Second, the accuracy of the estimated parameters can be directly determined by using the normal equations of the modified least-squares matching. Third, more than two images can be matched all at once.

The theory of the least-squares approach to matching gray-levels of two image patches is presented by Förstner [1982]. Its performance and potential applications in photogrammetry is investigated by Ackermann [1984]. Its potential for obtaining high accuracy results of matching is reported to be about 0.05 pixel size. This has attracted many photogrammetrists to follow or elaborate on the algorithm for many applications, such as point transfer and DEM generation [Ackermann and Schneider, 1986, Gruen
and Baltsavias, 1986, and Hahn and Förstner, 1988}. Nowadays this algorithm is employed widely in the field of digital photogrammetry.

Conventionally, functions of an affine transformation are used in least-squares matching. The parameters of these geometrical transformations are called *shaping parameters* which do not clearly show geophysical meanings. The concerted approach is applied by using the mathematical functions of equation (3.8) for geometric changes. Thus, geophysically significant parameters of ice motion are estimated directly.

The observation equations of a least-squares matching can be formulated based on the gray-level differences between images. Minimizing the sum of squares of the gray-level differences between two conjugate image patches is fundamental to least-squares matching. Let an image patch be a template, and its conjugate image patch be resampled from the associated search window based on given approximations of the shaping parameters. Then each pair of conjugate pixels of the image patches form an observation equation. It is also possible to apply a radiometric transformation to a least-squares matching. If a linear transformation of radiometry is applied, the observation equation is formulated [Ackermann, 1984] as:

\[ g_t(x_t, y_t) + n(x_t, y_t) = h_0 + h_1 \cdot g_s(x_s, y_s) \]  

(3.10)

where

- \( g_t(x_t, y_t) \): gray-level value of a template pixel at \((x_t, y_t)\)
- \( g_s(x_s, y_s) \): gray-level value of a search-window pixel at \((x_s, y_s)\)
- \( n(x_t, y_t) \): gray-level residual
- \( h_0, h_1 \): a shift and a scale factor of the gray-level values
The radiometric transformation may not be needed however, if radiometric differences between images are not significant. In this study, we normalize the images prior to the matching process, so that these images have similar radiometry. In addition, Rosenholm [1987] showed that reliability and accuracy of the matching results are not significantly improved by applying a radiometric transformation in a least-squares matching. Consequently, the observation equation shown in equation (3.10) can be simplified as:

\[ g_r(x_i, y_i) + n(x_i, y_i) = g_r(x_s, y_s) \] (3.11)

The observation equation is nonlinear, because \( g_r \) is determined subject to changes in \((x_i, y_i)\) coordinates, and \((x_s, y_s)\) are functions of the unknown shaping parameters. A recursive approach to the solution is therefore needed.

In order to complete the observation equation, geometrical transformations should be formulated. Because image coordinates are in a common coordinate system after the pre-processing, the coordinate differences between matched image points represent the ice deformation. Thus, transformed coordinates are obtained by adding deformation to the original coordinates. Let \( T \) be the time interval that the images were obtained, then geometrical transformations can be formulated as:

\[
x_s = x_i + T \cdot U_x(x_i, y_i)
\]

\[
y_s = y_i + T \cdot U_y(x_i, y_i)
\] (3.12)

In order to model geometrical transformations by using the parameters of ice motion, \( U_x \) and \( U_y \) are eliminated by combining equations (3.12) and (3.8):

\[
x_s = x_i + T \cdot \vec{U}_x + T \cdot (x_i - \bar{x}_i) \cdot \dot{\epsilon}_x + T \cdot (y_i - \bar{y}_i) \cdot \dot{\epsilon}_y - T \cdot (y_i - \bar{y}_i) \cdot \omega
\]

\[
y_s = y_i + T \cdot \vec{U}_y + T \cdot (x_i - \bar{x}_i) \cdot \dot{\epsilon}_y + T \cdot (x_i - \bar{x}_i) \cdot \omega + T \cdot (y_i - \bar{y}_i) \cdot \dot{\epsilon}_x
\] (3.13)
By combining equations (3.11) and (3.13), a complete observation equation is formed. The ice-motion parameters, $\overline{U}_x, \overline{U}_y, \dot{e}_x, \dot{e}_y, \ddot{e}_x, \ddot{e}_y$, and $\dot{\omega}$, are unknowns. Each set of estimated parameters of ice motion has the reference to the centroid of the template - $(\overline{x}, \overline{y})$.

A Newton-Raphson scheme can be applied to solve nonlinear equations iteratively. First, observation equations are linearized by using a Taylor expansion. According to the assumption made in section 3.3, terms of second derivatives of the Taylor expansion can be neglected. Let $\dot{\overline{U}}_x, \dot{\overline{U}}_y, \dot{\dot{e}}_x, \dot{\dot{e}}_y$ and $\dot{\omega}$ be the initial values of the parameters, then the conjugate image patch of the template can be resampled from the search window according to the transformed pixel coordinates $(x_s, y_s)$, where

\[
\begin{align*}
\dot{x}_s &= x_i + T \cdot \dot{\overline{U}}_x + T \cdot (x_i - \overline{x}) \cdot \dot{e}_x + T \cdot (y_i - \overline{y}) \cdot \dot{e}_y - T \cdot (y_i - \overline{y}) \cdot \dot{\omega} \\
\dot{y}_s &= y_i + T \cdot \dot{\overline{U}}_y + T \cdot (x_i - \overline{x}) \cdot \dot{e}_y + T \cdot (x_i - \overline{x}) \cdot \dot{\omega} + T \cdot (y_i - \overline{y}) \cdot \dot{e}_y
\end{align*}
\]  

(3.14)

The observation equation is linearized as

\[
g_i(x_i, y_i) - g_s(x_s, y_s) + n(x_i, y_i) =
\]

\[
\frac{\partial g_i}{\partial U_x} d\overline{U}_x + \frac{\partial g_i}{\partial U_y} d\overline{U}_y + \frac{\partial g_i}{\partial \dot{e}_x} d\dot{e}_x + \frac{\partial g_i}{\partial \dot{e}_y} d\dot{e}_y + \frac{\partial g_i}{\partial \dot{\omega}} d\dot{\omega} + \frac{\partial g_i}{\partial \ddot{e}_x} d\ddot{e}_x + \frac{\partial g_i}{\partial \ddot{e}_y} d\ddot{e}_y + \frac{\partial g_i}{\partial \ddot{\omega}} d\ddot{\omega}
\]  

(3.15)

where

\[
\frac{\partial g_i}{\partial U_x} = \frac{\partial g_i}{\partial x_i} \frac{\partial x_i}{\partial \overline{U}_x} = T \cdot \frac{\partial g_i}{\partial \overline{U}_x} \equiv T \cdot g_i'
\]

\[
\frac{\partial g_i}{\partial U_y} = \frac{\partial g_i}{\partial y_i} \frac{\partial y_i}{\partial \overline{U}_y} = T \cdot \frac{\partial g_i}{\partial \overline{U}_y} \equiv T \cdot g_i'
\]
The symbols of \( g'_x \) and \( g'_y \) are gray-level gradients of the template. According to the derivations shown above, the original gradients were formed by using the resampled image patch from the search window. The replacement of the original gradients by the template gradients is based on the assumption that the resampled image patch is similar to the template. It has been proven that this procedure does affect the matching results [Li, 1988], where two advantages can be recognized. This assumption saves on computation, because the normal matrix does not need to be updated for each iteration, and furthermore, an a priori analysis is possible concerning the reliability and accuracy of the matching results using template alone.

Making the substitutions in equation (3.15) the linearized observation equation is:

\[
[g_i(x_i, y_i) - g_r(x_r, y_r)] + n(x_i, y_i) =
T \cdot g'_x \cdot d\bar{U}_x + T \cdot g'_y \cdot d\bar{U}_y + T(x_i - x_r)g'_x \cdot d\bar{e}_x + T(y_i - y_r)g'_y \cdot d\bar{e}_y +
T[(y_i - \bar{y}_i)g'_x + (x_i - \bar{x}_i)g'_y] \cdot d\bar{e}_x + T[(x_i - \bar{x}_i)g'_y + (y_i - \bar{y}_i)g'_x] \cdot d\bar{e}_y
\]

Each pair of conjugate pixels forms an observation equation. If a template contains 20 \( \times 20 \) pixels, 400 observation equations can be formed. Combining all observation equations, a matrix form is obtained as:

\[
L - e = AX
\]

where
\( L \) : observation vector formed by all \( g_i(x_i, y_i) - g_i(x_t, y_t) \)
\( e \) : error vector formed by all \( n(x_i, y_i) \)
\( A \) : design matrix formed by all the coefficients in equation (3.11)
\( X \) : unknown vector - \([dU_x, dU_y, d\hat{e}_x, d\hat{e}_y, d\hat{e}_\gamma, d\omega]^T\)

The number of unknowns is 6, so that the dimension of \( A \) will be \( n \) by 6, if there are \( n \) pixels in a template. The redundancy, \( r \), is \( n-6 \).

The system of equation (3.17) is a homogenous Gauss-Markov model [Koch, 1988], if the error vector, \( e \), fulfills the following assumptions:

\[
\begin{align*}
E(e) &= 0 \\
D(e) &= \sigma_o^2 I
\end{align*}
\]  
(3.18)

where

\( E \) : expectation
\( D \) : dispersion
\( \sigma_o^2 \) : variance of unit weight
\( I \) : \( n \times n \) identity matrix

This model implies that the observation equations are independent, i.e., \( D(L) = \sigma_o^2 I \), and leads the least-squares estimation to be unbiased.

According to the theory of least-squares estimation, the unknown vector and the variance of unit weight can be estimated by using the following equations:

\[
\hat{X} = (A^T A)^{-1} A^T L = N^{-1} A^T L
\]
\[
\hat{\sigma}_o^2 = \frac{1}{r} \nu^T \nu
\]  
(3.19)

where
\( \hat{X} \): estimated unknown vector

\( V \): residual vector = \( AX - L \)

\( \sigma^2_0 \): a posteriori estimation of the variance of unit weight

\( N \): normal matrix

Estimated unknowns can be used to update the approximations for the next iteration. Another iteration is performed based on the new approximations, until the norm of the estimated unknown vector is smaller than a pre-defined criterion, or the iteration number is larger than a limit.

The least-squares solution will not exist, if \( N^{-1} \) does not exist. This means that the structure of the image data is not feasible in determining some of the unknowns. This is an extreme case actually. In practice, very few images will lead the \( N^{-1} \) to be nonexistent. However, an ill-condition \( N \) matrix is often encountered, which means some rows of the \( N \) matrix commonly are correlated in certain degree. Under these circumstances, the solution may converge to a mismatch or even diverge. Therefore, the structure of the \( N \) matrix is associated with the reliability of the solution. It is important to analyze the \( N \) matrix in order to provide a check to the matching reliability. This aspect will be discussed extensively in the next chapter.

The \( \sigma^2_0 N^{-1} \) matrix is the covariance matrix of the estimated unknown vector \( \hat{X} \). Thus, the variances and covariances of the estimated parameters can be obtained from the elements of \( \sigma^2_0 N^{-1} \) matrix. The quantity \( \hat{d}_0 \) describes the differences in gray-levels between of the matched image patches, and is often called a root mean square (RMS) error. It should be about equal to the noise. If the value is much larger than expected, the matching may have failed, or the mathematical functions do not fit the physical model. This will also be discussed in detailed in the next chapter.
3.5 Matching Images from Three or More Epochs

So far, the discussions concern the case of matching a pair of images. In many cases, however, there may be more than two images obtained at different epochs in hand. Under this circumstance, all the images may have to be matched to determine the ice motion optimally. Although it is possible to solve this problem by matching the images pair by pair and averaging the results, matching all the images in one step is the most efficient and accurate method.

Three advantages of matching multi-images in one step can be recognized. First, the accuracy of matching results can be rigorously estimated. Although it is also possible to estimate the accuracy of averaged results by using the error propagation rule for pair by pair matching, the unknown correlation (due to the fact that an image may be used to match two or more images) between two sets of matching results may lead to a biased estimation. Second, the reliability of matching results can be improved. According to the concept of statistics, the reliability of an estimation can be improved if more independent observations are used. So, one can expect that the pull-in-range and the convergence speed of the least-squares matching can be improved by matching multiple-images. Third, matching all images at once is more efficient in computation than pair by pair matching. Matching multiple-images using the least-squares method will yield a normal matrix of size six by six, because the same number of unknowns are needed. Thus, the computation needed for matching multi-images is only slightly more than that for matching a pair of images.

Because the method of cross-correlation cannot be used to match more than two images at once, images are matched pair by pair during the matching process of cross-
correlation. Under this condition, the best procedure is to use the image from the first epoch as the base image to match the other images one by one. After the first pair of images was matched, one can take advantage of the dramatically reduced search window (size-wise) for matching the other images, because the velocity pattern estimated by the first pair of images provides accurate prediction of the conjugate area for other images. This decreases a large amount of the computation time needed.

More than two images can be matched in one step by using the approach of the modified least-squares matching, after the first approximations are provided by the matched image pair using cross-correlation. The observation equations described in section 3.4 equations (3.11) and (3.13) can also be used for this case, except that a template will match with more than one search window. Let a template be a sub-image of the base image, then the search windows, \( s_1, s_2, \ldots, s_n, \ldots, s_n \), are obtained from the other images with respect to the time sequence that the images were taken, where \( n \) is the number of images to be matched. Consequently, a pixel of the template and its conjugate pixel in each search window, \( s_i \), form an observation equation as follows:

\[
g_i(x_i, y_i) + n_i(x_i, y_i) = g_n(x_n, y_n)
\] (3.20)

where

\[
x_n = x_i + T_i \cdot \bar{U}_i + T_i(x_i - \bar{x}_i) \cdot \hat{e}_{xx} + T_i(y_i - \bar{y}_i) \cdot \hat{e}_{xy} - T_i(\dot{y}_i - \ddot{y}_i) \cdot \omega
\]

\[
y_n = y_i + T_i \cdot \bar{U}_i + T_i(x_i - \bar{x}_i) \cdot \hat{e}_{yy} + T_i(\dot{x}_i - \ddot{x}_i) \cdot \omega + T_i(\dot{y}_i - \ddot{y}_i) \cdot \hat{e}_{yy}
\]

The linearized observation equation will be

\[
[g_i(x_i, y_i) - g_n(x_n, y_n)] + n_i(x_i, y_i) =
\]

\[
T_i \cdot g' \cdot d\bar{U}_i + T_i \cdot g' \cdot d\bar{U}_i + T_i(x_i - \bar{x}_i)g' \cdot d\hat{e}_{xx} + T_i(y_i - \bar{y}_i)g' \cdot d\hat{e}_{xy} + T_i[(y_i - \bar{y}_i)g' + (x_i - \bar{x}_i)g'] \cdot d\hat{e}_{yy} + T_i[(x_i - \bar{x}_i)g' - (y_i - \bar{y}_i)g'] \cdot d\omega
\] (3.21)
It can be seen that the number of unknowns is still six, in spite of an increase in the number of observations. The same procedure of least-squares approach described in the preceding section can be followed to estimate the parameters and the variance of unit weight.

### 3.6 Indirect Estimation of Ice Motion

In some cases, well developed commercial software for matching general images may be available. However, these programs usually determine displacements or shaping parameters instead of the parameters of ice motion. Thus, some simple derivations are needed.

The techniques of cross-correlation and least-squares matching have been commonly implemented in various computers. By applying cross-correlation software, coordinate differences between matched image points are obtained. In this case, these coordinate differences represent displacements of points on the ice surface. The derivation of ice motion from estimated displacements is needed. If software of least-squares matching is used, shaping parameters of an affine transformation are usually obtained. These parameters represent the deformation between matched image patches. Deriving ice motion from estimated deformation is therefore required. The following two subsections describe the derivations of ice motion from displacements and deformation respectively.
3.6.1 Indirect Estimation of Ice Motion from Displacements

Displacements referenced to grid points are obtained, if images are matched patch by patch using the method of cross-correlation. Let \( i \) be the column number, and \( j \) be the row number of the grid. The velocity at grid point \((x_i, y_j)\) can be derived from the displacement as:

\[
U_x(x_i, y_j) = \frac{D_x(x_i, y_j)}{T} \tag{3.22}
\]

\[
U_y(x_i, y_j) = \frac{D_y(x_i, y_j)}{T}
\]

Knowing the velocities of all grid points, gradients of velocity can be calculated by using the velocities nearby. For example, if we know the velocities of grid points shown in figure 3.3, the gradients of velocity at \((x_i, y_j)\) can be computed by using

\[
\frac{\partial U_x}{\partial x} = \frac{U_x(x_{i+1}, y_j) - U_x(x_{i-1}, y_j)}{x_{i+1} - x_{i-1}}
\]

\[
\frac{\partial U_y}{\partial y} = \frac{U_y(x_i, y_{j+1}) - U_y(x_i, y_{j-1})}{y_{j+1} - y_{j-1}}
\]

\[
\frac{\partial U_x}{\partial y} = \frac{U_x(x_i, y_{j+1}) - U_x(x_i, y_{j-1})}{y_{j+1} - y_{j-1}} \tag{3.23}
\]

\[
\frac{\partial U_y}{\partial x} = \frac{U_y(x_{i+1}, y_j) - U_y(x_{i-1}, y_j)}{x_{i+1} - x_{i-1}}
\]

Then strain rates and rotation rate referred to \((x_i, y_j)\) can be calculated by using equation (3.3).
By applying this indirect estimation method, three disadvantages should be noted. First, calculations of strain rates and rotation rate from the velocities nearby may violate the assumption of the definition of infinitesimal strain made in the derivation of equations (3.2) and (3.3). Since the velocity of each point is determined by matching a template with its conjugate image patch, an area of size three times larger than the template is involved in the calculation of strain rates and rotation rate. If changes of the velocities in this area are not linear, the estimation may contain some biases. Second, the cross-correlation method is not adaptive to geometrical differences between images resulting from deformation of ice surface, so that estimated displacements may contain systematic errors. Third, the accuracy of the estimated strain rates and rotation rate is difficult to be determined, because the correlation of the estimated velocity components is unknown.

Figure 3.3: Referenced grid points of velocities.
3.6.2 Indirect Estimation of Ice Motion from Deformation

A set of shaping parameters is yielded from a match of a pair of image patches, if a least-squares matching is applied. Commonly, they are parameters of an affine transformation which model the geometrical differences between a template and its conjugate image patch resampled from the search window. Thus, the geometrical transformations from a template to the conjugate image patch can be expressed as:

\[
x_s = a_0 + a_1 \cdot x_i + a_2 \cdot y_i
\]
\[
y_s = b_0 + b_1 \cdot x_i + b_2 \cdot y_i
\]

where \(a_0, a_1, a_2, b_0, b_1, b_2\) are the estimated shaping parameters.

The deformation function can be obtained by subtracting the first function in equation (3.24) by \(x_i\) and subtracting the second function by \(y_i\). Let

\[
D_x(x_i, y_i) = x_s - x_i
\]
\[
D_y(x_i, y_i) = y_s - y_i
\]

then equation (3.24) becomes:

\[
D_x(x_i, y_i) = a_0 + (a_1 - 1) \cdot x_i + a_2 \cdot y_i
\]
\[
D_y(x_i, y_i) = b_0 + b_1 \cdot x_i + (b_2 - 1) \cdot y_i
\]

Dividing equation (3.25) by time interval - \(T\), it becomes:

\[
U_x(x_i, y_i) = \frac{a_0}{T} + \frac{(a_1 - 1)}{T} \cdot x_i + \frac{a_2}{T} \cdot y_i
\]
\[
U_y(x_i, y_i) = \frac{b_0}{T} + \frac{b_1}{T} \cdot x_i + \frac{(b_2 - 1)}{T} \cdot y_i
\]

Comparing equations (3.5) and (3.26), the following relationships are obtained:

\[
U_{0x} = \frac{a_0}{T}
\]
\[
\frac{\partial U_x}{\partial x} = \frac{(a_1 - 1)}{T}
\]
\[
\frac{\partial U_y}{\partial y} = \frac{a_2}{T}
\]
\[
U_{xy} = \frac{b_0}{T}
\]
\[
\frac{\partial U_x}{\partial x} = \frac{b_1}{T}
\]
\[
\frac{\partial U_y}{\partial y} = \frac{(b_2 - 1)}{T}
\]

Substituting equation (3.3) by (3.27), strain rates and rotation rate are obtained:

\[
\dot{\varepsilon}_{xx} = \frac{(a_1 - 1)}{T}
\]
\[
\dot{\varepsilon}_{yy} = \frac{(b_2 - 1)}{T}
\]
\[
\dot{\varepsilon}_{xy} = \frac{(a_2 + b_1)}{2T}
\]
\[
\dot{\omega} = \frac{(b_1 - a_2)}{2T}
\]

The velocity of the centroid of the template can be derived by substituting equation (3.6) by (3.27) as:

\[
U_x(\bar{x}, \bar{y}) = \frac{a_0}{T} + \frac{(a_1 - 1)}{T} \cdot \bar{x} + \frac{a_2}{T} \cdot \bar{y}
\]
\[
U_y(\bar{x}, \bar{y}) = \frac{b_0}{T} + \frac{b_1}{T} \cdot \bar{x} + \frac{(b_2 - 1)}{T} \cdot \bar{y}
\]

Matching images with existing software offers two advantages. First, it is more economic than designing new software. Second, well developed commercial software
makes production more efficient. However, two disadvantages should be noted. First, the accuracy of derived parameters of ice motion cannot be determined, if the applied software does not provide the inverted normal matrix. Second, matching multi-images at once may not be possible, if this function is not designed in the software.
4.1 Introduction

A pair of image patches may be mismatched due to many factors. For example, the image patches may be too noisy or not contain significant or similar features to match, or there may be more than one possible match between them. As mismatches greatly disturb the interpretation and analysis of entire data, a quality control to matching results is necessary. The next section describes how mismatches can be removed. In addition, in order to provide a confident study of ice mechanics, the precision of matching results should be assessed, after mismatches are removed. Section 4.3 discusses how the theoretical accuracy of matching results is assessed.

4.2 Quality Control

Unbiased study of ice mechanics relies on the correctness of data. Misestimated data will largely disturb the interpretation and analysis of entire data. Thus, strict quality control is necessary. The strategy of quality control is to reject all suspicious data according to some criteria. Because incorrect data are not exactly distinguishable, some good data may be rejected.
It is well known that image matching is an ill-posed problem [Tikhonov and Arsenin, 1977 and Poggio and Koch, 1985], which means that solutions may not be existent, unique, or stable. Thus, constraints are required to find the best solution. The cross-correlation method finds the maximum correlation coefficient as the constraint. The constraint of the least-squares matching is to find the least-square sum of gray-level differences between images. In some cases, these constraints are too weak to obtain correct matches. For matching stereo images, some additional constraints, such as epipolar geometry [Helava and Chapelle, 1972] or collinearity conditions [Gruen and Baltavias, 1988], can be used to confine solutions to an a priori solution space. However, in the case of multi-temporal images, there are no additional constraints available. Thus, criteria need to be set up using the characteristics of the images to detect and remove suspicious matching results.

Image patches which provide poor matching conditions may easily be mismatched. In fact, some pairs of image patches are inherently not possible to be matched correctly. For example, image patches may cover a flat ice surface, may be blurred by ground fog or cloud, or contain some ice features which are completely deformed between epochs. In addition, mismatches may occur if their solutions are not unique because of similar ice features. All of these factors are noted as poor matching conditions. Therefore, the matching results obtained under a poor matching condition should not be used.

Three parameters can be used to measure the matching conditions of a pair of image patches. They are the signal-to-noise ratio, the similarity between image patches, and the uniqueness of the match. These parameters can be quantified according to characteristics of the image patches. Using the quantified parameters, some criteria can be set up to reject suspicious matches.
In addition to checking each single match, the consistency between matches is another useful check. This check is applied based on the physical property of ice motion that the motion varies gradually with respect to location changes. For instance, if a velocity obtained from a match of a pair of image patches is largely different in direction and scale from the velocities obtained from nearby patches, it may be a bad estimation resulting from a mismatch. Visually investigating a vector map of estimated velocities is the most simple way to check data consistency. A statistic approach to checking residuals between determined values and interpolated values from nearby points is also possible.

The first three of the four following sub-sections discuss the three measuring parameters of matching conditions respectively. The methods used to quantify the factors and set the criteria will be described as well. The last sub-section covers how data consistency is checked.

4.2.1 Signal-to-Noise Ratio (SNR)

The SNR of a signal is a standard engineering parameter used to measure the ratio of the power of a signal to that of its noise. It is often defined as the ratio of the variance of a signal to that of its noise [Whalen, 1971] as:

\[
SNR = \frac{\sigma_s}{\sigma_n}
\]  

(4.1)

The SNR of an image is defined in the same fashion, where signal is represented by the brightness values of an image. In general, the SNR of an image is a quantitative measure of useful information contained in the image.
Image content is the most important factor in matching images. A pair of image patches can be successfully matched if only they contain enough useful information. In other words, the SNR of image patches indicates whether they can be matched reliably. For instance, low SNR image areas, such as a flat ice surface or foggy area, are obviously not possible to match. Thus, in order to maintain the reliability of matching results, low SNR image patches should not be used.

A rigorous estimation of the SNR of an image patch may not be possible, because noises are not distinguishable from brightness values. Whereas, a rough estimation is rather easy and useful in this case. According to its definition, the SNR of an image patch is determined by the ratio of its brightness variance to the variance of noise. The variance of brightness can be calculated from gray-levels by ignoring their noises. If noises are randomly distributed, a calculated value should be close to the true value. The variance of the noises can be calculated by using the gray-levels of a homogenous image area which covers an area of flat ice surface. This scheme actually estimates the ratio of the image content of a patch to that of a homogenous image patch. Although it is not a rigorous estimation of SNR, it offers an appropriate quantitative judgment to discriminate between matchable and unmatchable image patches.

A threshold of SNR is needed to reject unmatchable image patches. Ehlers [1982] and Föstner [1984] stated that stereo images with SNR higher than 2 offer reliable matches. By the experience of matching ice images, a higher threshold, SNR > 3, is needed to ensure the correctness of matching results. A test image in figure 4.1 is used to demonstrate how the threshold works. It can be seen that the portion of the right lower corner of the image is an area of flat ice surface. The image size is 512 pixels by 512 pixels. In order to test the image patch by patch, it is divided into patches of 32 pixels by 32 pixels. Patches whose SNRs are lower than 3 will be blocked in figure
4.2. Comparing figures 4.1 and 4.2, it can be seen that homogenous image patches are blocked.

Figure 4.1: An example image for SNR test.
Figure 4.2: Remaining patches after SNR test.
In order to show how an SNR test can improve the quality of matching results, a test is demonstrated for a matching example here. Figure 4.3 shows a vector map of all the displacements obtained by matching the image in figure 4.1 with its conjugate image. The blank area is non-overlap area between the images. More detailed procedures of matching these two images can be seen in section 5.2. Investigating figure 4.1 and 4.3, it can be seen that vectors in the crevassed area show a nice consistency, whereas vectors in the inter-overlap zone and flat ice surface are basically chaotic. These inconsistent vectors may result from mismatches. Mismatches occur in the inter-overlap zone due to the fact that some image patches from the second image in this area are partially out of the image boundaries. Such mismatches will be checked in the next section. Here, an SNR test is used to remove all the suspicious matches in the flat-ice area. Figure 4.4 shows a new vector map after an SNR test with the criterion of SNR > 3. In this map all the vectors in the flat-ice area are eliminated. It should be noted that although some of the vectors in the flat-ice area seem to be consistent with the consistent vectors in the crevassed area, they should not be considered to be correct matches in order to maintain the reliability of matching results.
Figure 4.3: A vector map of displacements resulting from all the matches of two example images.
Figure 4.4: A vector map of displacements after SNR check.
4.2.2 Image Similarity

The substance of image matching is to find the highest similarity between a template and all possibly matched patches resampled from its conjugate search window. The match which yields the highest similarity is determined as the best match. Matching results are then obtained by measuring the geometrical differences between the image patches of the best match.

The matching results are of interest only if they are generated from a pair of matched image patches which have a similarity higher than an anticipated level. However, in some cases, the similarity between matched patches may be lower than the anticipated level. This occurs when one or both of the image patches are covered partially or entirely by cloud or ground fog, or are partially out of image boundaries or on a flat-ice area, or some of the imaged features are completely deformed between epochs due to snow falls or newly created crevasses. The matching results obtained from such matches are not reliable. Thus, a similarity check is needed to reject such sort of suspicious matching results.

Setting up a similarity criterion to check extreme cases, that similarity of matched patches is very high or very low, is easy. Unfortunately, in practice, most matches in critical areas, such as inter-overlap, inter-clouded, inter-foggy, or inter-crevassed areas, have similarities in the middle range. Some of these matches are obviously correct or incorrect, but many of them are inbetween. Therefore, a proper criterion is of importance, if one wants to eliminate all incorrect matches but to keep correct matches as many as possible.

One needs to know how to judge the similarity between a pair of matched image patches, before a criterion can be set up. Through the matching process, two measures
of similarity are available. They are the maximum coefficient of correlation obtained from the best match of cross-correlation and the RMS error resulting from the least-squares matching mentioned in section 3.4. Each of these two measures has a drawback in evaluating image similarity. The former one is often under-estimated, because the cross-correlation method is not adaptive to ice deformation. Thus, in some areas where ice deformation is large a correct match may be misjudged. The latter one is sensitive to brightness differences between the image patches, so that only relative judgments are available. In order to overcome the drawbacks, the normalized correlation coefficient calculated by using the template and the resampled patch according to the geometrical differences determined by the least squares matching is used to judge the similarity between matched patches. This measurement is invariant to brightness differences due to the normalization, and is adaptive to ice deformation. It is defined as the similarity measurement, SM, of a pair of matched image patches.

In general, the similarity measurement, SM, of a pair of obviously matchable patches is larger than 0.7, and that of unmatchable patches is smaller than 0.3. Patches with similarity measurements ranged in between 0.3 and 0.7 are usually distributed in critical areas mentioned above. Figure 4.5 shows the isoline map of the similarity measurements obtained by matching the image in figure 4.1 and its conjugate image, on which a vector map of the determined displacements is overlaid. It can be seen that vectors distributed in the region of SM > 0.7 (crevassed area) are consistent. Some of the vectors distributed in the areas of 0.3 < SM < 0.7 (critical areas) are consistent with those vectors in crevassed area, but some are not. An extremely strict quality control can be proceeded by setting up a criterion that SM > 0.7. Although this criterion maintain high quality of matching results, many good data are sacrificed. In
order to save as many qualified data as possible, more elaborate scheme of quality control is needed.

Figure 4.5: An isoline map of similarity measurement (isoline interval = 0.2) overlaid by a vector map of displacement.
Image patches in critical areas cannot be matched well due to the fact that they may be partially out of image boundaries or partially cover unmatchable areas. It makes sense to assume that correct matches can be obtained if more than half of the area of matched patches is matchable area. Because the average similarity measurement in crevassed area is about 0.8, the criterion that $SM > 0.4$ can be used to accept the data resulting from patches whose matchable areas are more than half of their imaged areas. This criterion saves most qualified data in critical areas, but a few mismatches may be accepted.

In order to eliminate accepted mismatches due to the loose criterion of similarity check, an accessory criterion is needed. This criterion is established based on the principle that the similarity measurement of a correct match should be larger than the correlation coefficient resulting from cross-correlation due to the geometrical adaptivity of least-squares matching. In other words, if the similarity between matched image patches after the least-squares matching is lower than that achieved by cross-correlation, this match tends to be a mismatch. Let $C$ be the correlation coefficient resulting from cross-correlation, then the accessory criterion can be set as $SM > C$.

By applying the criteria, $SM > 0.4$ and $SM > C$, most inconsistent vectors in figure 4.5 are removed. Figure 4.6 shows the vector map after applying the similarity check. Only one displacement seems to be inconsistent with the other vectors. This can be removed by checking the uniqueness of match to be mentioned in the next subsection.
Figure 4.6: A vector map of displacements after an SNR and a similarity check.
4.2.3 Uniqueness and Stability of Match

Matches determined under the constraints of cross-correlation and least-squares matching may not be unique or stable. This can occur if some similar ice features are closely distributed in the imaged areas or the image contents do not provide enough information for solving for the unknown shaping parameters. Such non-uniqueness or unstable matches often occur in low SNR image areas, so that few of them remain after the SNR check. However, even if there is only one mismatch accepted, it may largely affect further studies. In order to ensure the matching quality, it is worth to check the uniqueness and stability of each match.

The uniqueness of a match is tested by checking if there exists any secondary possible match in the solution space. This can be achieved by checking the peak difference of the highest and second highest peaks of a correlation function generated from cross-correlation. A criterion of the peak difference larger than 0.2 is set to reject non-unique matches. This is based on the assumption that image noises affect the calculation of correlation coefficient by ±0.1, so that the maximum effect on the peak difference is 0.2.

The stability test is to check the linear system sensitivity of observation equations, \( AX=L \), formed during least-squares matching. This can be accomplished by checking the condition number \( k(A) \) of the observation equations. The condition number is a quantity which measures the sensitivity of the solution \( X \) to errors in matrix \( A \). If the elements in \( A \) are accurate to \( e \), then the solution of \( X \) can only accurate to \( k(A) \cdot e \) [Dongarra et al., 1979 and Golub and Van Loan, 1989]. Accordingly, if the condition number goes to infinite, the linear system becomes singular, which means the system is not solvable because \( A \) is numerically rank deficient. A singular system is rarely seen.
in image matching, but an unstable system may occur when $k(A)$ is large. If the elements in $A$ are accurate to $10^{-6}$ and we want to calculate the solution $X$ accurate to $10^{-3}$, the condition number should be smaller than $10^3$. Thus, $k(A) < 10^3$ is set as the criterion to check the stability of a match.

The remaining data after the SNR and similarity tests in figure 4.6 are tested by using these two criteria. Some non-unique matches are rejected by the criterion of peak difference including the inconsistent one shown in figure 4.6. None are rejected due to the check of stability. Figure 4.7 shows a clean vector map of displacements after the tests of SNR, similarity, and uniqueness.
Figure 4.7: A clean vector map of displacements after the test of SNR, similarity, and uniqueness.
4.2.4 Consistency Check

After the tests of SNR, similarity, and uniqueness, the remaining data should vary gradually with respect to location changes. This consistency property is based on the phenomena of ice motion. According to this property, a parameter of ice motion determined by a match should be similar to that interpolated from the nearby data. A test on this property is called consistency check. Data that fail to pass the consistency check possibly result from mismatches.

Visually investigating a vector map of displacements or isoline maps of the parameters of ice motion is the simplest way to check data consistency. In a vector map, if a vector is largely different in direction or scale from nearby velocities, it may originate from a mismatch. In an isoline map, an inconsistent value causes a sharp mountain or deep valley. One can easily detect obviously bad data by using visual checks, but it is difficult to judge slightly inconsistent data visually. Thus, a more elaborate check is needed.

A statistic approach to checking data consistency provides a more rigorous check. The basic idea of this approach is to check the residuals between original data and interpolated data from nearby points. The RMS error of the residuals between original and interpolated data can be calculated as an indication of data consistency or relative accuracy. A general statistic rule that residuals larger than 3 times the RMS error are considered as outliers can be used to detect inconsistent data.

A difficulty may be encountered in implementing the statistic approach, because some interpolated data may be biased. Theoretically, the interpolation can be done easily by averaging the data of the 8 surrounding grid points, because data resulting from the matching process are in a grid structure. However, some data may not have all the 8
surrounding grid points, because some data be rejected by the previous steps of quality control or the data point is on the border of a data region. Because of ice deformation, interpolated data may be biased, if some of the surrounding grid data are not available. Thus, data points on the border of a data region or sparsely distributed points tend to be misjudged in this approach.

Using the same example shown in the previous sub-sections, the consistency of the remaining results after the tests of SNR, similarity, and uniqueness are checked. Table 4.1 shows the RMS error, maximum residual, and number of inconsistent points of each parameter of ice motion. Converting the RMS errors of velocity components to the ground scale, a relative accuracy of about 1 meter/yr is obtained which is three times better than that of human approach shown by Whillans and Jackson [1992].

Table 4.1: Results of a consistency check

<table>
<thead>
<tr>
<th>Parameter</th>
<th>RMS</th>
<th>Max. Residual</th>
<th>No. of Inconsistent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>pixel/yr</td>
<td>pixel/yr</td>
<td>Points</td>
</tr>
<tr>
<td>$U_x$</td>
<td>0.097</td>
<td>0.402</td>
<td>2</td>
</tr>
<tr>
<td>$U_y$</td>
<td>0.109</td>
<td>0.406</td>
<td>3</td>
</tr>
<tr>
<td>$\dot{e}_{xx}$</td>
<td>0.011</td>
<td>0.050</td>
<td>2</td>
</tr>
<tr>
<td>$\dot{e}_{yy}$</td>
<td>0.016</td>
<td>0.060</td>
<td>4</td>
</tr>
<tr>
<td>$\dot{e}_{xy}$</td>
<td>0.009</td>
<td>0.043</td>
<td>3</td>
</tr>
<tr>
<td>$\dot{\phi}$</td>
<td>0.008</td>
<td>0.033</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 4.8 shows the detected inconsistent vectors. Each of them is indicated using a cross sign. It can be seen that they are mainly distributed on the border of the data region where interpolated data are often biased. In addition, most of the inconsistent data have residuals just a little more than three times RMS error, so that they actually can be accepted.
4.3 Quality Assessment

The understanding of data accuracy is crucial to obtain confident results from the study of ice mechanics. The most rigorous method of quality assessment is to check data with a set of more accurate and independently measured check data. However, such data are not available in most cases. Alternatively, a theoretical accuracy or relative accuracy can be determined to assess data quality. The theoretical data accuracy can be determined according to the theory of least-squares matching. The relative accuracy can be obtained from the check of data consistency mentioned in section 4.2.4.

According to the theory of least-squares matching mentioned in section 3.4, the variance-covariance matrix of the estimated parameters of ice motion is $\mathbf{C}$ $\mathbf{N}^{-1}$. Thus the diagonal elements in the matrix are the variances of the parameters which represent the internal accuracy (or theoretical accuracy) of the estimated parameters. The off-diagonal terms in the matrix are covariances between the parameters. They reveal the correlations between the parameters.

The internal accuracy can be improved theoretically by increasing the number of observations. In image matching, the number of observations can be increased by increasing the template size. Table 4.2 and figure 4.9 show the assessed accuracy of the parameters of ice motion in a crevassed area versus the increase of the template size. It can be seen that the accuracy can always be improved when the template size is increased. However, the improvements do not make sense when the template size is too large to meet the assumption made in chapter III that the deformation inside a template should be linear. Thus, in order to obtain appropriate results, an optimal
template size (the maximum template size can be used under the assumption) should be
determined before matching.

Table 4.2: Theoretical accuracy achieved by using different template sizes.

<table>
<thead>
<tr>
<th>Template Size</th>
<th>$U_x$ (pixel/yr)</th>
<th>$U_y$ (pixel/yr)</th>
<th>$\dot{\epsilon}_{xx}$ (yr$^{-1}$)</th>
<th>$\dot{\epsilon}_{yy}$ (yr$^{-1}$)</th>
<th>$\dot{\epsilon}_{xy}$ (yr$^{-1}$)</th>
<th>$\omega$ (yr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15x15</td>
<td>0.110</td>
<td>0.068</td>
<td>0.025</td>
<td>0.017</td>
<td>0.020</td>
<td>0.011</td>
</tr>
<tr>
<td>20x20</td>
<td>0.068</td>
<td>0.046</td>
<td>0.015</td>
<td>0.009</td>
<td>0.009</td>
<td>0.007</td>
</tr>
<tr>
<td>25x25</td>
<td>0.050</td>
<td>0.035</td>
<td>0.007</td>
<td>0.006</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>30x30</td>
<td>0.041</td>
<td>0.032</td>
<td>0.005</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>35x35</td>
<td>0.033</td>
<td>0.029</td>
<td>0.004</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>40x40</td>
<td>0.029</td>
<td>0.026</td>
<td>0.003</td>
<td>0.005</td>
<td>0.004</td>
<td>0.003</td>
</tr>
<tr>
<td>45x45</td>
<td>0.026</td>
<td>0.023</td>
<td>0.002</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>50x50</td>
<td>0.023</td>
<td>0.021</td>
<td>0.002</td>
<td>0.004</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Figure 4.9: Relationship between theoretical accuracy and template size.

An optimal template size can be determined by monitoring the $a$ posteriori variance
of unit weight, $\sigma^2_0$, versus that of the template size. The quantity of $\sigma^2_0$ is equal to the
mean-square gray-level differences between matched image patches. When the mathematical model fits the physical model, i.e., the assumption of linear deformation is valid, an increase of the template size will decrease $\sigma_0^2$, which means the internal accuracy is improved. However, when the template is so large that it violates the assumption, $\sigma_0^2$ will be increased by increasing the template size. Table 4.3 and figure 4.10 show the quantities of $\sigma_0^2$ determined by using different template sizes for a crevassed area. It can be seen that an increase of the template size decreases $\sigma_0^2$ when the template size is smaller than 25x25. On the other hand, it increases $\sigma_0^2$ when the template size is larger than 25x25. Two important messages are revealed from this experiment. First, the optimal template size is about 25x25. Second, the linear property of ice deformation is only valid in the range of 250 meters, because an image pixel covers about 10x10 square-meters in the ground scale. Therefore, one should not use a template that covers an ice surface larger than 250 meters by 250 meters.

Table 4.3: Quantities of $\sigma_0^2$ (graylevel$^2$) determined by using different template sizes.

<table>
<thead>
<tr>
<th>Template Size</th>
<th>$\sigma_0^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15x15</td>
<td>20.01</td>
</tr>
<tr>
<td>20x20</td>
<td>19.03</td>
</tr>
<tr>
<td>25x25</td>
<td>18.06</td>
</tr>
<tr>
<td>30x30</td>
<td>18.76</td>
</tr>
<tr>
<td>35x35</td>
<td>19.26</td>
</tr>
<tr>
<td>40x40</td>
<td>20.00</td>
</tr>
<tr>
<td>45x45</td>
<td>20.46</td>
</tr>
<tr>
<td>50x50</td>
<td>20.45</td>
</tr>
</tbody>
</table>
If the parameters of ice motion are not determined directly from the matching process but derived from other estimated quantities, error propagation rules can be used to derive the theoretical accuracy of the derived parameters. By using this approach, one should watch the correlation between the originally determined data in order to obtain an unbiased estimation of accuracy. One should also keep the assumption made for the derivation of the parameters of ice motion in mind, otherwise biased data or misassessed accuracies may be obtained.
Chapter V
Experiments and Results

5.1 Introduction

A pair of SPOT images and a pair of digitized aerial photographs were tested. In order to compare results, the images were selected such that they mostly cover the same ice surface. In this area, manually measured velocities obtained from repeat photogrammetry using an analytical plotter are available. These manual data provide a check for the matching results of a digital approach. The matching results of SPOT images cannot be checked rigorously due to poorly known sensor position and orientation, but a visual check by comparing the isoline maps of determined parameters with that derived from manual data is possible. The matching results of digitized aerial photographs can be compared visually and numerically with the manual data.

The experiments were implemented in an IBM compatible personal computer with a 80386 20 MHz CPU. Although it takes about 10 hours to match a pair of 512 pixel by 512 pixel images, it is still faster than the human approach mentioned in the introductory chapter. The automatic matching process offers a warrant to build up an efficient and economic production line for ice-flow determination.

The following two sections describe the experiments and analyze the results. Section 5.2 discusses the experiments and matching results of SPOT images, and section 5.3 discusses that of digitized aerial photographs.
5.2 Matching SPOT Images

This section describes the procedures of matching SPOT images obtained. It is composed of four sub-sections. First, the specifications of the images are described. Then the procedures of pre-processing and the matching process are shown sequentially. The final results are discussed in the last sub-section.

5.2.1 Specifications of the SPOT Images

The images were acquired nearly two years apart. The first image was scanned on the date of 5th February, 1988, and the second image was obtained on 15th December, 1989. They cover a portion of dragon on ice stream B in West Antarctica.

A test area which contains shore margins, crevassed areas, less crevassed areas, and flat ice surfaces of the ice stream was selected. This area is also covered by the aerial photographs to be tested. A pair of 512 by 512 pixel sub-images which cover the test area were cut out from the original images. Figures 5.1 and 5.2 show the first and second sub-images and their histograms respectively.

Visually investigating the images reveals the need of pre-processing techniques. First, it can be seen that the contrasts of the images are quite low, and their gray-tones are different. Their histograms clearly show their inherent properties of radiometry. Second, they are obviously different in orientation. The shore lines of the ice stream on the images can only be visually matched by rotating one of the images about 15 degrees. In order to achieve automation and obtain reliable matching results, the processing of radiometric adjustment and geometrical correction is needed before matching the images.
Figure S.1: A 512 by 512 SPOT image scanned on 5th February, 1988 and its histogram.
Figure 5.2: A 512 by 512 SPOT image scanned on 15th December, 1989 and its histogram.
5.2.2 Pre-processing

Radiometric Adjustment

The image radiometry was firstly adjusted by using the technique of histogram equalization mentioned in chapter II. Figures 5.3 and 5.4 show the enhanced versions and their new histograms of the first and second images respectively. Textures in the images became very clear, and they are shown mainly in the mid-gray range. This can be proved by investigating the new histograms. That the shapes of the new histograms are similar to that of the preconceived Gaussian function is the evidence.

The process of radiometric adjustment not only provides a better matching condition, but also offers the possibility for us to select some pairs of conjugate image patches in determining the orientation differences between the images. This procedure will be described in detail in next sub-section.

After histogram equalization, a low-pass filter was used to reduce the image noise. A 3 by 3 window with equal coefficients (figure 2.9) was used to convolve the images. This process does not make obvious changes in vision on the images, but it does improve the matching condition as mentioned in chapter II.
Figure 5.3: Enhanced version of the first image and its histogram.
Figure 5.4: Enhanced version of the second image and its histogram.
Geometrical Correction

According to chapter II, the images need to be transformed to a common coordinate system, and their projection errors should be corrected if possible. However, in this case, the SPOT images can only be adjusted relatively for their orientation differences by using a similarity transformation because their sensor position and orientation are poorly known. By doing this, the relative transformation parameters between the images should be determined first. Then, one image can be transformed to the coordinate system of the other image according to the transformation parameters.

The transformation parameters can be calculated using two or more than two measured conjugate points which are static or moving very slowly. The coordinates of conjugate points should be measured accurately in order to reduce systematic errors of matching results. Manually measuring conjugate points on a computer screen by using a pointing device is the simplest method, but two disadvantages are recognized. First, a high resolution monitor and proper image processing software are required. Second, it is difficult to achieve good accuracy by measuring images manually on a computer screen. The other approach is to obtain accurate conjugate points by matching selected patches of conjugate image areas. Because recognizing conjugate areas is much easier than pointing accurate conjugate points on a computer screen, a low resolution monitor can be used. A matching method, such as cross-correlation or least-squares matching, can be used to match selected conjugate patches. One-pixel or better accuracy can be achieved easily by using this approach. Therefore, this method was used to measure conjugate points for the test images.

In order to determine the ice deformation based on the coordinate system of the first image, the second image needs to be transformed to the coordinate system of the first image. Here, it is assumed that crevasses next to the interstream shore ridge are static
compared to the other crevassed area. On the computer screen, four templates were selected from the first image in the shore margin area, and four conjugate search windows were identified the second image. In order to obtain accurate parameters, two conjugate pairs more than the need for solving the parameters were selected. A two-step matching process, a cross-correlation followed by a least-squares matching, was performed for each pair of image patches. If sub-pixel accuracy is not required, it is not necessary to apply a least-squares matching. From the matching results, the image coordinates of four conjugate points were obtained. Table 5.1 lists the coordinates in pixel unit.

Table 5.1: The coordinates of measured conjugate points between the images.

<table>
<thead>
<tr>
<th>Pt. No.</th>
<th>x1 (pixel)</th>
<th>y1 (pixel)</th>
<th>x2 (pixel)</th>
<th>y2 (pixel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>501.5</td>
<td>179.0</td>
<td>365.5</td>
<td>54.7</td>
</tr>
<tr>
<td>2</td>
<td>421.5</td>
<td>278.0</td>
<td>311.5</td>
<td>168.6</td>
</tr>
<tr>
<td>3</td>
<td>325.5</td>
<td>393.5</td>
<td>247.4</td>
<td>301.2</td>
</tr>
<tr>
<td>4</td>
<td>233.5</td>
<td>484.0</td>
<td>178.5</td>
<td>419.0</td>
</tr>
</tbody>
</table>

The transformation parameters were calculated by using the measured coordinates. They are listed as follows:

- **X translation** -167.471 pixels
- **Y translation** 1.582 pixels
- **Rotation** 14.148 degrees
- **Scale factor** 1.005

These parameters can be used to transform the coordinate system of the first image to that of the second image. These backward transformation parameters were computed, because they are needed for the procedure of indirect resampling mentioned in chapter II.

An indirect resampling procedure was performed on the second image according to the transformation parameters. The coordinates of each pixel in the first image were
transformed to the coordinate system of the second image. If a transformed position is inside the boundaries of the second image, a gray-level of the position is bilinearly interpolated by using the gray-levels of nearby pixels. A zero gray-level is assigned otherwise. Figure 5.5 shows the resampled image of the second image. The black area is non-overlapping area. By overlapping the image on the first image, one can see that the imaged shore margin are nicely matched.

Figure 5.5: Resampled image from the second image.
5.2.3 Matching Process and Quality Control

According to matching process mentioned in chapter III, the images were firstly matched by using the cross-correlation method. The matching was then refined by using the least-squares matching. The parameters of ice motion were estimated directly using the least-squares matching. Some criteria were specified to control the quality of the matching results.

First at all, the size of the template and the search window have to be determined. A grid on the first image was constructed. The interval between the grid lines was 20 pixels. Centered on each grid point, we opened a template. The optimal template size, 25 by 25 pixels, was used according to section 4.3. According to the predicted maximum displacement (about 500 meters) and the pixel size in the ground scale (about 10 by 10 square meters), the search-window size should be 125 by 125 pixels. Working with such big search windows, the matching process may take more than 50 hours for a 80386 personal computer. A look-up table which records roughly estimated displacements of the grid points was used to reduce the computation time. This approach reduced the size of the search window down to 45 by 45 pixels, and decreased the need of computation time down to about 3 hours.

The images were matched first by using the cross-correlation method. Based on the data resulting from the cross-correlation, the least-squares matching was performed to determine the parameters of ice motion. The results (displacements) of cross-correlation were used as the first approximations for least-squares matching. The approximations of velocity components were obtained by using the estimated displacements divided by the time interval. The other parameters are usually small so that their initial values were set to be zeros.
As described in chapter IV, some data may result from mismatches. Thus, a quality control process is needed. According to the analysis in chapter IV, the criteria for the quality control were set as follows:

- \( \text{SNR} > 3 \)
- \( \text{SM} > 0.4 \)
- \( \text{Peak Diff.} > 0.2 \)
- \( \text{SM - C} > 0 \)
- \( k(A) > 10^3 \)

Nearly 40% of the data were rejected. After the quality control, a vector map of the remaining displacements was plotted which has been shown in figure 4.7 in chapter IV. Those vectors seem to be consistent visually. Table 4.1 and figure 4.8 have showed the results of a consistency check in chapter IV. Although there are some inconsistent data found (figure 4.8) through the check procedure, they can be accepted because they are mainly distributed on the border of data region where data are tend to be misjudged in checking consistency.

### 5.2.4 Matching results

Parameters of ice motion were determined for each successfully matched pair of image patches. Table 5.2 lists the average theoretical accuracy and relative accuracy (RMS errors of the consistency check). As anticipated, the theoretical accuracy is more optimistic than the relative accuracy due to two reasons. First, the theoretical accuracy tends to be over-estimated due to the assumption of linear property of ice deformation mention in chapter III. Second, the relative accuracy tends to be underestimated because larger RMS errors of consistency check than it should be may be obtained in areas where the ice motion is not uniform. Table 5.2 shows reasonable discrepancies between the estimated theoretical and relative accuracy.
Table 5.2: Theoretical and relative accuracy of the matching results of SPOT images.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Theoretical accuracy</th>
<th>Relative accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_x$</td>
<td>0.033 pixel/yr</td>
<td>0.097 pixel/yr</td>
</tr>
<tr>
<td>$U_y$</td>
<td>0.032 pixel/yr</td>
<td>0.109 pixel/yr</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{xx}$</td>
<td>0.005 yr$^{-1}$</td>
<td>0.011 yr$^{-1}$</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{yy}$</td>
<td>0.005 yr$^{-1}$</td>
<td>0.016 yr$^{-1}$</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{xy}$</td>
<td>0.003 yr$^{-1}$</td>
<td>0.009 yr$^{-1}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.003 yr$^{-1}$</td>
<td>0.008 yr$^{-1}$</td>
</tr>
</tbody>
</table>

Each determined parameter of ice motion is presented by using an isoline map. Figures 5.6 to 5.11 show the isoline maps of $U_x, U_y, \dot{\varepsilon}_{xx}, \dot{\varepsilon}_{yy},$ and $\phi$ respectively, in which small dots represent measurement points. Large shearing rates should appear in areas where velocity component changes rapidly. According to the map of $x$ component of velocity in figures 5.6, large strain rate $\dot{\varepsilon}_{xx}$ should appear along the isoline of -5 pixel/yr. A nice correspondence can be seen in figure 5.8. A similar agreement can be found in figures 5.7 and 5.9. According to the velocity maps, small quantities of $\dot{\varepsilon}_{xy}$ are also expected. Figure 5.10 shows this agreement.

In order to compare the results with manually measured data, the data should be transformed to the ground coordinate system. The transformation parameters can be determined roughly by measuring two or more than two conjugate image points between the first SPOT image (figure 5.3) and the resampled image from digitized aerial photograph (figure 5.20) which is referenced to the ground coordinate system. From the coordinates of 4 pairs of measured conjugate points, the parameters of a similarity transformation are calculated as follows:
Figures 5.12 to 5.16 show the isoline maps of the transformed velocities and strain rates in the ground coordinate system. A nice consistency can be found by visually comparing these maps with that obtained from manually measured data shown in figure A1 to A5 in the appendix. Some common properties can be found from the comparison: for example, the X component of velocity increases steadily toward the center of ice stream, the Y component of velocity changes in a small range from -10 to 10 m/yr, the quantity of \( \dot{\varepsilon}_x \) contains the most strain rate (about 70%) of the ice stream surface (its maximum is about 0.05 yr\(^{-1}\)), and the longitudinal and lateral extension rates, \( \dot{\varepsilon}_l \) and \( \dot{\varepsilon}_l \), are nearly zero.

A numerical check has also been performed by calculating residuals between the data obtained from the matching process and interpolated data from manual data. Table 5.3 shows the RMS error and average residual for each parameter. Large average residuals of X and Y components of velocity reveal systematic errors due to imperfect transformation. The RMS errors of strain rates are surprisingly consistent to the measures of relative accuracy listed in table 5.2.

Table 5.3: Numerical comparison between matching results and manual data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>RMS error</th>
<th>Average error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_x )</td>
<td>9.785 m/yr</td>
<td>-8.030 m/yr</td>
</tr>
<tr>
<td>( U_y )</td>
<td>7.891 m/yr</td>
<td>7.413 m/yr</td>
</tr>
<tr>
<td>( \dot{\varepsilon}_{xx} )</td>
<td>0.012 yr(^{-1})</td>
<td>-0.004 yr(^{-1})</td>
</tr>
<tr>
<td>( \dot{\varepsilon}_{yy} )</td>
<td>0.017 yr(^{-1})</td>
<td>0.001 yr(^{-1})</td>
</tr>
<tr>
<td>( \dot{\varepsilon}_{xy} )</td>
<td>0.011 yr(^{-1})</td>
<td>0.001 yr(^{-1})</td>
</tr>
</tbody>
</table>
Figure 5.6: x component of velocity. Isoline interval: 1 pixel/yr.
Figure 5.7: $y$ component of velocity. Isoline interval: 1 pixel/yr.
Figure 5.8: Strain rate $\dot{\varepsilon}_x$. Isoline interval: 0.01 yr$^{-1}$. 
Figure 5.9: Strain rate $\dot{\varepsilon}_{yy}$. Isoline interval: 0.01 yr$^{-1}$. 
Figure 5.10: Strain rate $\dot{\varepsilon}_{xy}$. Isoline interval: 0.01 yr$^{-1}$. 
Figure 5.11: Rotation rate $\omega$. Isoline interval: 0.01 yr$^{-1}$. 
Figure 5.12: Longitudinal velocity. Isoline interval: 20 m/yr.
Figure 5.13: Transverse velocity. Isoline interval: 2 m/yr.
Figure 5.14: Rate of longitudinal extension. Isoline interval: 0.01 yr$^{-1}$. 
Figure 5.15: Rate of lateral extension. Isoline interval: 0.01 yr⁻¹.
Figure 5.16: Strain rate $\dot{e}_{xy}$. Isoline interval: 0.01 yr$^{-1}$. 
5.3 Matching Digitized Aerial Photographs

This section describes the procedures of matching two digitized aerial photographs, and compares the results with manually measured data. The following sub-sections describe the specifications of photographs and digitization, pre-processing, and matching process sequentially. The last subsection compares the results with manually measured data.

5.3.1 Specifications of Photographs and Digitization

The photographs were taken about one year apart. The first photograph is one of the photographs of the photo block obtained on the date of 22nd January, 1985. The second photograph belongs to the photo block taken on 25th January, 1986. Both photo blocks were taken by using a Wild model RC-8 camera with a standard wide-angle lens (focal length of 152 mm) and a photo size of 230x230 square mm. The flying height was about 8,000 meters, so that the resulting photographs are at a scale of about 1:50,000.

An aerial-triangulation process has been completed for each photo block, so that the perspective center and orientation of the selected photographs are known. In addition, terrain relief of the imaged area can be roughly determined by using the coordinates of tie points resulting from aerial triangulation. These data provide us the possibility to transform photos to the ground coordinate system and correct projection errors.

The photographs were scanned by using an Eikonix EC850 camera which scans objects by using a linear 4096 element charge-coupled device (CCD) array. Its scanning capability is 8 bits per pixel, which means the radiometric resolution is 256
gray-scale. The maximum spatial resolution is 4096x4096 pixels. Scanning with this maximum resolution generates a 16 MB file. In order to reduce the needs of storage space and processing time, a resolution of 2048x2048 pixels is used. Figures 5.17 and 5.18 show the pair of the scanned images.

Figure 5.17: Scanned image from the first photograph.
Each photograph has 8 fiducial marks whose photo coordinates can be obtained from the calibration reports of the camera. These fiducial marks provides us an opportunity to correct projective distortions resulting from the scanning process (see section 2.3.3).
5.3.2 Pre-processing

Because the scanned images are only partially overlapping and are enclosed by imaged camera frame, it is improper to perform radiometric adjustment for the entire image. Overlapping images which cover a target area should be cut out from the original images before applying radiometric adjustments. Because the original images are different in scale and orientation, resampling procedure is required to obtain target images. This procedure can be combined into the procedure of the geometrical correction mentioned in chapter II. A target area is first selected based on the ground coordinate system, and a grid structure is constructed according to a specified resolution. Then each grid point is transformed backward and to the original images for resampling. After resampling, the process of radiometric adjustment is performed on the resampled images.

A test area which is covered by the test images was selected. This area is also mostly covered by the test SPOT images, and manually measured data are also available in this area. The size of this area is 5 by 5 square kilometers and is in the range of -25,000 m < X < -20,000 m and 5,000 m < Y < 10,000 m in the ground coordinate system. A similar resolution to SPOT images, 10 by 10 square meters, was used to resample the target images.

Equations (2.7) and (2.8) can be used to transform and rectify ground coordinates of each grid point to each image coordinate system for resampling. The parameters in equation (2.8) can be obtained according to known sensor position and orientation and terrain relief. Table 5.4 lists the perspective centers and orientation of the selected images, and figure 5.19 shows the terrain relief determined from the coordinates of tie points.
Table 5.4: Sensor position and orientation of the selected images.

<table>
<thead>
<tr>
<th></th>
<th>$X_0$</th>
<th>$Y_0$</th>
<th>$Z_0$</th>
<th>$\omega$</th>
<th>$\varphi$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image 1</td>
<td>-2264.30 m</td>
<td>8062.99 m</td>
<td>7215.15 m</td>
<td>-0° 09'0°.774</td>
<td>-0° 42'9°.937</td>
<td>-83° 51'6°.543</td>
</tr>
<tr>
<td>Image 2</td>
<td>-24577.41 m</td>
<td>8528.49 m</td>
<td>7226.35 m</td>
<td>-0° 06'13°.504</td>
<td>-0° 06'37°.977</td>
<td>97° 37'17°.127</td>
</tr>
</tbody>
</table>

Figure 5.19: Terrain relief used for geometrical correction.
The parameters in equation (2.7) can be calculated from measured image coordinates and known photo coordinates of the fiducial marks. Tables 5.5 and 5.6 list the measured image coordinates and known photo coordinates of fiducial marks for both of the test images respectively as well as the calculated transformation parameters.

Table 5.5: Fiducial coordinates and transformation parameters of the first image.

<table>
<thead>
<tr>
<th>Image coordinates (pixel)</th>
<th>Photo coordinates (mm)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>101.90</td>
<td>129.68</td>
<td>-105.996</td>
</tr>
<tr>
<td>984.50</td>
<td>99.50</td>
<td>0.009</td>
</tr>
<tr>
<td>1864.89</td>
<td>137.54</td>
<td>106.013</td>
</tr>
<tr>
<td>1901.51</td>
<td>1017.40</td>
<td>109.998</td>
</tr>
<tr>
<td>1872.48</td>
<td>1898.23</td>
<td>106.001</td>
</tr>
<tr>
<td>990.50</td>
<td>1929.50</td>
<td>-0.004</td>
</tr>
<tr>
<td>107.50</td>
<td>1894.50</td>
<td>-105.991</td>
</tr>
<tr>
<td>71.35</td>
<td>1011.59</td>
<td>-109.993</td>
</tr>
</tbody>
</table>

Table 5.6: Fiducial coordinates and transformation parameters of the second image.

<table>
<thead>
<tr>
<th>Image coordinates (pixel)</th>
<th>Photo coordinates (mm)</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>169.15</td>
<td>150.24</td>
<td>105.991</td>
</tr>
<tr>
<td>1055.57</td>
<td>117.71</td>
<td>-0.012</td>
</tr>
<tr>
<td>1938.49</td>
<td>152.54</td>
<td>-106.002</td>
</tr>
<tr>
<td>1981.97</td>
<td>1036.06</td>
<td>-110.001</td>
</tr>
<tr>
<td>1955.17</td>
<td>1921.81</td>
<td>-106.003</td>
</tr>
<tr>
<td>1068.52</td>
<td>1958.51</td>
<td>0.004</td>
</tr>
<tr>
<td>177.66</td>
<td>1924.51</td>
<td>106.008</td>
</tr>
<tr>
<td>139.48</td>
<td>1036.31</td>
<td>109.991</td>
</tr>
</tbody>
</table>
The target images were transformed and resampled according to the determined parameters. The resampled images were then adjusted by using the technique of histogram equalization. Figures 5.20 and 5.21 show the obtained first and second images respectively.

Figure 5.20: Resampled first image.
Figure 5.21: Resampled second image.
5.3.3 Matching Process and Quality Control

The same matching process as described in section 5.2.3 was followed to match the digitized aerial photographs. Quality control is performed by using the same criteria too. More than 60% of the data were rejected by the procedure of quality control. The results of consistency check show a poor consistency. Table 5.7 shows the RMS errors of consistency check for the determined parameters of ice motion. They are much worse than in the case of matching SPOT images (table 4.1). After excluding inconsistent data, about 25% of the original data have remained.

Table 5.7: Consistency check of the matching results of digitized photographs.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>RMS errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U_x)</td>
<td>2.51 m/yr</td>
</tr>
<tr>
<td>(U_y)</td>
<td>2.02 m/yr</td>
</tr>
<tr>
<td>(\varepsilon_{xx})</td>
<td>0.025 yr(^{-1})</td>
</tr>
<tr>
<td>(\varepsilon_{yy})</td>
<td>0.021 yr(^{-1})</td>
</tr>
<tr>
<td>(\varepsilon_{xy})</td>
<td>0.016 yr(^{-1})</td>
</tr>
<tr>
<td>(\omega)</td>
<td>0.015 yr(^{-1})</td>
</tr>
</tbody>
</table>

5.3.4 Matching Results

Because the matching results are referred to the ground coordinate system, a rigorous comparison with manually measured data is possible. Figures 5.22 to 5.27 show the isoline maps of the determined parameters of ice motion. These maps can be visually compared with the maps obtained from manual data in figure A1 to A6 in
appendix. Table 5.8 shows numerical comparisons of the matching results and manual data.

Table 5.8: Numerical comparisons for the matching results and manual data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>RMS error</th>
<th>Average error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_x$</td>
<td>8.754 m/yr</td>
<td>6.491 m/yr</td>
</tr>
<tr>
<td>$U_y$</td>
<td>4.006 m/yr</td>
<td>-0.693 m/yr</td>
</tr>
<tr>
<td>$\dot{\epsilon}_{xx}$</td>
<td>0.026 yr$^{-1}$</td>
<td>-0.010 yr$^{-1}$</td>
</tr>
<tr>
<td>$\dot{\epsilon}_{yy}$</td>
<td>0.022 yr$^{-1}$</td>
<td>0.001 yr$^{-1}$</td>
</tr>
<tr>
<td>$\dot{\epsilon}_{xy}$</td>
<td>0.023 yr$^{-1}$</td>
<td>0.015 yr$^{-1}$</td>
</tr>
<tr>
<td>$\dot{\phi}$</td>
<td>0.027 yr$^{-1}$</td>
<td>-0.022 yr$^{-1}$</td>
</tr>
</tbody>
</table>

By investigating the isoline maps, one can find that the matching results are not consistent with the manual data. They are also very noisy compared to the manual data and the results of SPOT images. Two possible factors may degrade the matching results. First, the second image was not focused well during digitization. It can be seen that the image quality of the second image (figure 5.21) is much poorer than the first image (figure 5.20). Second, the sun directions of the images are different in about 90 degrees according to the shadow differences between the images. A large average error in X component of velocities shown in table 5.8 reveals a systematic error. This systematic error may result from imperfect geometrical correction.

Although the matching results are not as good as we expected, the matching process was proceeded successfully. If shadow differences can be adjusted and image can be digitized properly, reliable and precise matching results should be expected.
Figure 5.22: Longitudinal velocity. Isoline interval: 20 m/yr.
Figure 5.23: Transverse velocity.  Isoline interval: 2 m/yr.
Figure 5.24: Rate of longitudinal extension. Isoline interval: 0.01 yr⁻¹.
Figure 5.25: Rate of lateral extension. Isoline interval: 0.01 yr\(^{-1}\).
Figure 5.26: Strain rate $\dot{\varepsilon}_y$. Isoline interval: 0.01 yr$^{-1}$. 
Figure 5.27: Rotation rate $\dot{\omega}$. Isoline interval: 0.01 yr$^{-1}$.
6.1 Summary and Conclusions

An automatic, accurate, and economic method was developed to determine velocity, strain rates, and rotation rate of a large ice stream by matching multi-temporal digital images. A two-step matching process (a cross-correlation followed by a least-squares matching) was devised to achieve automation of image matching. Modifying traditional least-squares matching such that multi-temporal images of ice streams can be matched accurately and the parameters of ice motion can be determined directly is one of the major achievements of this study. Quality control and assessment are also designed to ensure data quality.

The developed matching process and quality control have been applied to SPOT images and digitized aerial photographs successfully. The data resulting from matching SPOT images show a nice consistency with manually measured data, and their relative accuracy (about 1 meter/yr on velocity and 0.01 yr⁻¹ on strain rates and rotation rate) is very satisfactory. The matching results from digitized aerial photographs are not as good as that of SPOT images possibly due to improper digitizing procedure or sun-angle differences.

The following conclusions are made with respect to the procedures of pre-processing, matching process, and quality control and assessment:
Pre-processing

- Radiometric adjustment is necessary in order to obtain reliable matches.
- Sun-angle difference may disturb the matching process, whereas its correction is very difficult. Thus, images should be taken under similar sun directions.
- Geometrical correction is needed to achieve automation of image matching and to determine ice motion accurately. An ultimate procedure of geometrical correction is to transform images to a common coordinate system. Automatic searching for conjugate image patches becomes possible and simple, if images are in a common coordinate system. Ice motion and deformation are determined by measuring geometrical differences between matched image patches in a common coordinate system. Any imperfect coordinate transformation will cause systematic errors.

Matching Process

- In order to model ice deformation, an adaptive matching process is required. Traditional least-squares matching (LSM) is modified to obtain best match by estimating and adapting ice deformation using the parameters of ice motion. Because of the inherent small pull-in range of LSM, a cross-correlation method is needed to provide a good first approximation for LSM.
- Three or more than three multi-temporal images can be matched in one step by using the modified LSM.
- A traditional LSM can also be applied to determine the parameters of ice motion indirectly.
Quality Control and Assessment

- Quality control is needed to reject mismatches. Three quantitative checks - the SNR, similarity, and uniqueness check, are devised to control data quality. A consistency check is used to detect inconsistent data and to estimate relative accuracy.
- The theoretical accuracy of matching results can be determined according to the theory of least-squares matching.
- The optimal template size which offers the best theoretical accuracy and does not violate the assumption made for deriving the parameters of ice motion is about 250 by 250 square meters in the ground scale.

6.2 Recommendations

For a practical application, a well developed software is needed. Three programs can be designed with respect to the procedures of pre-processing, matching process, and quality control. The matching process and quality control basically are automatic processes. The programs designed in this study for these processes can be easily used for practical applications. However, for some procedures of pre-processing, some sophisticated programs are needed to handle interactive processes, such as measuring conjugate points. Programs for such processes designed for experiments are not user friendly. Therefore, for practical applications, commercial software for image processing may be needed. A high resolution monitor may also be required, if the size of applied images is large.

Images with 10 meter resolution have been tested in this study. In practice, lower resolution images, such as TM images (28.5 meter), may be applied. In this case, the
optimal template size becomes quite small (about 10 by 10 pixels for TM images). Under this circumstance, the accuracy of matching results could be largely degraded. To solve this problem, one can use a large template size to obtain better accuracy. However, the theoretical accuracy will become too optimistic to judge data quality. In this case, the relative accuracy should be used to assess data quality instead of using theoretical accuracy.

Sun-angle difference is a major problem which has not been solved in this study. This problem may degrade matching results or even cause mismatches, so that it is worth to extend this research in solving this problem. Horn [1986] showed a theory in reconstructing object surface from the shading of a single image. Obtaining object surface using this method, a new sun direction then can be introduced to generate a fictitious image which is simulated as taken under the specified sun direction. A detailed object surface reconstructed for sun-angle correction is in fact also very useful for glaciological study. Thus, this could be a more important motivation to investigate the sun-angle problem.
Appendix

Isoline Maps of the Parameters of Ice Motion Generated from Manually Measured Data

Figure A1: X component of velocity. Isoline interval: 20 m/yr.
Figure A2: Y component of velocity. Isoline interval: 2 m/yr.
Figure A3: Strain rate $\dot{\varepsilon}_{xx}$. Isoline interval: 0.01 yr$^{-1}$. 
Figure A4: Strain rate $\dot{\varepsilon}_{\text{yr}}$. Isoline interval: 0.01 yr$^{-1}$. 
Figure A5: Strain rate $\dot{\varepsilon}_y$. Isoline interval: 0.01 yr$^{-1}$. 
Figure A6: Rotation rate $\omega$. Isoline interval: 0.01 yr$^{-1}$. 
References


