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The effectiveness of a novel direct instructional approach on math word problem solving skills of elementary students with learning disabilities

Lee, Jeanette Wooster, Ph.D.
The Ohio State University, 1992

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THE EFFECTIVENESS OF A NOVEL DIRECT INSTRUCTIONAL APPROACH
ON MATH WORD PROBLEM SOLVING SKILLS
OF ELEMENTARY STUDENTS WITH LEARNING DISABILITIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * *

The Ohio State University
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Advisor
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To all the boys and girls considered difficult-to-teach, and to all their instructors who know they can learn, and who are teaching them
ACKNOWLEDGEMENTS

Thank you, Dr. Gwendolyn Cartledge, faculty advisor, Dr. Larry Miller, Dr. Ralph Gardner III, and Dr. Emmalou Norland for your careful and faithful consideration throughout this project. You had many responsibilities and many advisees, yet you treated each of us as if there was only one of us. From your classes, I broadened my knowledge base in content. From your model, I learned the greatness of serving. To my friend, Ms. Sharon Powell, who cared enough to inquire along the way, I thank you for your interest and concern. From you, I have recognized the importance and the impact of a faithful friend. To my parents, Mrs. Hazel Wooster and Mr. Jerome Wooster, I thank you for being my first and ongoing teachers. From you, I learned the meaning of the term "advocate." Thank you for providing the foundation that continues to stimulate and nurture spiritual, emotional, and intellectual growth. To my husband, Mr. Gus Lee, Jr., I am endeared for your spontaneous and unswerving support and love. From you I have learned how much freedom and growth can result from genuine togetherness. Thank you for giving unselfishly and for making this endeavor worth all the sacrifices.
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PUBLICATION


FIELDS OF STUDY

Major Field: Education

Studies in Specific Learning Disabilities, Behavior Disorders, Mental Retardation, and Psychology
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CHAPTER I
INTRODUCTION

Every day, problem situations that require the use of mathematical skills are encountered. Adults compare and buy, balance their checkbooks, estimate time requirements, pay bills, measure, and so forth. To a large extent, a person's level of independence is determined by his or her ability to apply and perform computations accurately. The objective of the school mathematics curriculum is to teach youngsters such math competencies.

While children know basic arithmetic skills, and can compute (Bley & Thornton, 1989; Kane, Byrne, and Hater, 1974), they have much difficulty incorporating those skills in real life problem situations. From elementary school through high school, teachers lament that students do not make connections between previously learned principles and existing problems (Kane et al., 1974; Darch, Carnine, and Gersten, 1983; Strickland & Denitto, 1989).

Both current and previous research studies have indicated that the difficulty not only has a national scope, but has been pervasive over time, and for students of all ages and ability levels (Darch, Carnine, and Gersten, 

Akey (in Calwell, 1992) stated that "computers and calculators can compute much faster and probably more accurately than most people." Because of the widespread availability and use of calculators today, many suggest that selecting the correct process and recognizing a reasonable answer are now more important than computing (e.g. Bell, Fischbein & Greer, 1984; Bley & Thornton, 1989; Cawley, 1984; Grouws & Good, 1987; Peterson, 1973). The technology of calculators is useless if students don't know when to use addition, subtraction, multiplication or division. Without a basic understanding of the concepts of each operation, students will not know how to get an estimated answer, and will be lost if they push a wrong button, or if a machine goes awry.

The difficulties of children in the application of math skills have prompted a host of studies which offer several different solutions to the problem (Bley & Thornton, 1989;
Case & Harris, 1988; Cawley, 1984; Darch et al., 1983; Fuson & Willis, 1988; Grouws & Good, 1987; Kane et al., 1974; Kresse, 1985; Moore & Carnine, 1989; Shultz & Leonard, 1989; Sibbert, Carnine & Stein, 1990; Synder, 1988; Sowder, 1988; Strickland & Denitto, 1989; Thomas, 1988; Thornton, 1989; Tsai & Derry, 1987; and VanDevender & Harris, 1987). These solutions represent widely divergent philosophies of instruction for teaching students in mainstream educational programs. In spite of the host of proposals, appeals from the national level for more focus on word problems have rarely prompted the desired results (Marzola, 1987). Teachers have recognized the importance, but have tended to spend little instructional time on the specific objective (Herrmann, 1989; Case & Harris, 1988). Many teachers have self-reported that they lack confidence in implementing instruction that explains cognitive reasoning processes associated with successful word problem solving strategies (Herrmann, 1989; Grouws & Good, 1987). Direct instruction is therefore avoided. For those who do teach problem solving in the mathematics curriculum, presentations have often been poorly made, causing many students to develop a dislike for the task (Grouws & Good, 1987; Vaidya, Nasuti, and Rockhill, 1981).

Additional sources have been identified as accounting for poor student performance with word problems. In regular education, the sources named include:
Textbooks

Not enough conceptual work has been included in many basal texts, although writers have provided more exercises recently than in years past. Although the quantity of word problem exercises has increased, the quality has been questioned. For instance, Sowder's 1988 review of mathematics texts concluded that textbook translations usually started with a picture and ended with symbols when the procedure should be reversed. Missing were exercises that called for students to make a drawing for given facts. He felt that this activity was crucial in helping students to translate terms into numerical problems.

Another problem in textbook design has been in the distribution of examples. Many books have placed word problems at the end of a unit that incorporated a specific computational skill. When students realize that all the problems can be solved by the same method, they stop reading and analyzing each example (Carpenter, Corbitt, Kepner, Lindquist, and Reys, 1981). They simply lift the data provided in the text and add, subtract, multiply or divide, depending on the most recently taught computation skill.

Reading Material

It follows that children who have difficulty identifying and comprehending basic words would concurrently have difficulty reading and solving word problems. However, the language of math is unique, and even good readers often have trouble reading math word problems (Kane, 1974).
Thomas (1988) confirmed this notion by saying that "(w)ord
problems make special linguistic demands on the reader ... 
(that) involve both technical terminology and ordinary 
language" (p. 245).

The Students

In a study by VanDevender & Harris (1987), 83% of 
teachers surveyed believed that students made many 
careless mistakes. Moore & Carnine (1989) found that 
students did not check their answers for reasonableness. 
Direct observations in research by Marzola (1987) revealed 
that students were very haphazard in their approach. Many 
simply scanned a problem for the numbers and selected an 
operation arbitrarily. Kresse (1985) concluded that 
children used no process for searching and reasoning. 
However, Sowder (1988) identified a catalogue of 
strategies used by students. Over 70 interviews of average 
and above average students from 29 different classrooms 
revealed that a student might:

a. Find the numbers and perform the most recently 
taught operation, or perform the operation s/he 
felt most competent doing.

b. Simply make a guess at the operation to be used.

c. Look at the size of the numbers and determine a 
likely operation.

d. Try all the operations and choose the most 
reasonable answer.

e. Look for key words or phrases.
f. Determine if the answer would be smaller or larger than the given numbers, then try subtraction or division for reductions, and addition or multiplication for enlargements, or

g. Choose the operation whose meaning fit the strategy. (This strategy was rarely used by the students interviewed in Sowder's study.)

Students with Learning Disabilities

While much research has been conducted with students in the regular curriculum regarding problem solving strategies, little has been done in comparison with children identified as having specific learning disabilities. A battery of assessment instruments have identified these children as exceptional learners. They have average or above average intelligence, but exhibit a puzzling array of learning problems. Disorders are exhibited in one or more of the psychological processes involved in understanding or using spoken or written language. Problems may be manifested in disorders of listening, thinking, talking, reading, writing, spelling, or arithmetic. Children with learning disabilities comprise the majority of individuals labeled as exceptional. The identification distinguishes them as needing some specialized teaching strategies in order to be successful in school.

A host of factors may contribute to the unique needs of students with learning disabilities in relation to difficulties in solving word problems. Mann & Suiter (1974)
noted that these children may have difficulty in making symbol-value associations, in estimating, in comprehending or remembering the purpose of the process signs, in understanding words—including vocabulary dealing with distance and measurement, space and time, and words with dual meanings. Cawley (1984) also suggested that for children with learning disabilities, language or reading comprehension deficiencies may negatively impact the acquisition of mathematics concepts and skills.

Bley and Thornton (1989) outlined specific learning disabilities and their effect on mathematics performance under categories such as a) perceptual problems (figure-ground, discrimination, reversals, spatial orientation); b) memory problems (short-term, long-term, sequential); c) integrative deficits (closure, expressive language, receptive language, abstract reasoning); and d) behavioral difficulties (distractibility, perseveration, impulsivity, and reading problems). Unlike a number of other educators, Bley and Thornton (1989) classified reading problems under behavioral difficulties because "when they interfere with mathematics performance, they can affect behavior" (p. 21).

Because of the unique characteristics and learning styles of children with specific disabilities, a number of investigators have examined alternative approaches to math instruction. Engelmann and Carnine (1992) have field tested in a variety of classrooms a different set of
instructional strategies for teaching math concepts, including word problems. They suggest that the different strategies are particularly effective with students having learning disabilities or who are at-risk in math. For word problem solving the Englemann-Carnine way, students learn to represent a problem graphically before they attempt to calculate. Specific instructions are outlined for teachers to follow to ensure that students master the skill sequence which leads to a real understanding of the connections between the operations of mathematics and problem situations. This recently defined novel approach is packaged in a new basal mathematics program called Connecting Math Concepts.

Statement of the Problem

The process of solving dilemmas presented in math story problems has proven difficult for students in the regular program. For those with learning disabilities, the challenge has been even greater. Numerous authors have proposed methods for helping students, but no strategy has received widespread recognition as effective, generalizable, or practical for long-term implementation.

A unique program that holds much promise for developing mathematical reasoning has been developed by Siegfried Engelmann and Douglas Carnine (1992). The program describes a strategy that follows the theoretical framework of direct instruction. This program outlines a carefully sequenced
series of demonstrations, instructions and questions which
guide students into selecting an operation appropriate to a
problem situation. A visual image is used as the basis for
the task. The image—a arrow—is concise and is
generalizable to all types of addition and subtraction word
problems. The process simplifies the role of abstract
reasoning in that students need only to determine the
greatest total number value, and then the smaller number
values identified in a story problem. In *Connecting Math
Concepts*, the strategy used to teach word problem solving
is unlike all others. Students are not simply directed to
decide on an operation—they learn *how* to make such
decisions. All students master the technique during
interactive presentations with the teacher, and following
many opportunities to practice the technique.

Another advantage of the new teaching model is its ease
of implementation for classroom instructors. Scripts that
tell teachers what to say and what to do are provided for
each lesson. Presentations are lively and designed with
audible signals for choral responding. The pace and
format facilitate attending even among distractible
youngsters. In many instances, the teacher is able to
work with an entire classroom of students and reach a
great number of students in an efficient manner. Lessons
are designed to promote active student listeners who
respond to the frequently asked questions which assess
comprehension during the instructional phase. The concise
plan addresses many of the issues to be considered when teaching students with special needs—classroom management, unique learning styles, and characteristics of learners that interfere with the typical learning process.

Hypotheses

This study will investigate the differences between a treatment group and a comparison group related to the criterion variable of interest—skill in solving one-step addition and subtraction word problems. The hypotheses guiding the investigation are that

1) instruction using a generalizable graphic representation during directed instructional sessions will be positively related to problem solving abilities; and

2) skills learned from the novel teaching strategy for solving single-step addition and subtraction word problems will be maintained over time.

Objectives of the Study

1) Determine whether the novel approach for teaching word problem solving will make a difference between the treatment group of learning disabled students and the comparison group of learning disabled students in math performance.

2) Investigate the maintenance of word problem skills over time.

3) Analyze the type of errors that are made by students in both the treatment and comparison groups, to help
determine whether types are altered as a result of the novel teaching approach.

Definition of Terms

A. Math word problems

1. Constitutive definition

Math word problems are short stories with contrived dilemmas, presented in written form, for which a student must perform certain calculations to arrive at a correct answer. Bley & Thornton (1989) asserted that word problems represented the principal reason for studying math: for the process of applying previously acquired knowledge to new and unfamiliar situations.

According to the guide edited by Barkey (1988), the arithmetic skills necessary for the successful completion of math word problems include number recognition, concepts of counting, calculations, the use of measuring tools to determine dimension, quality, or capacity, and the use of those skills to facilitate independence in functional situations.

2. Operational definition

Math word problem solving ability is defined by the number of correct answers on a 15-item criterion referenced test designed by the researcher to assess skill mastery in choosing the addition or subtraction operation appropriately, and in making correct computations with problems having addends, subtrahends, and minuends no larger than four-digit numerals.
B. Traditional teaching approach

The traditional teaching approach is a reference to the method and time of presentation for math word problems using typical basal math programs. In this study, the traditional approach was based on the Addison-Wesley mathematics program adopted for the public schools of Kanawha County, West Virginia. Students in the comparison group encountered math word problems with each different unit of math instruction. Directions were provided orally by the teacher, and a sample problem was demonstrated before the class. Students were directed to draw pictures if they needed to, and were reminded to refer to a chart posted in the classroom that was labeled the "5-Point Checklist" for problem solving. The chart read:

1. Question
2. Data
3. Plan
4. Answer
5. Check

The list on the chart was an abbreviation for a five step remembering strategy which the textbook authors highlighted for word problem solving mastery.

Point 1 stood for "understand the question." Pictures and problems with short sentences were used in this program to help students in the early grades (1, 2, 3) to focus on the question. In the later grades (4, 5, 6) students were given data or an equation from which they were to
formulate a question. Point 2 stood for "find the needed data." Students were directed to locate from tables, pictures, menus, lists, or story descriptions, the specific numbers which were needed for problem solutions. Point 3 reminded the students to "plan what to do." Exercises were given which directed children to make a table, draw a picture, or guess and check. Point 4 meant "find the answer." In the Addison-Wesley program, a computational skill was developed and practiced first. Then students solved problems using the developed skill. Point 5 stood for "check back." Having completed a computation, students were encouraged to reread and assess if the answer made sense. Estimation skills were taught to assist students in making such assessments concerning their answers. In addition to the poster, the textbooks consistently printed the 5-Point Checklist in chart form at the top of each workbook exercise involving word problems. Students in the comparison group had received instruction explaining the 5-Point Checklist on several occasions prior to this study.

A typical word problem lesson began with a whole class introduction by the teacher. A sample problem was worked out orally. Having reminded students to refer to the 5-Point Checklist, the teacher then directed students to read, solve, and label exercises independently. One lesson usually consisted of 10 problems.

D. The novel teaching approach

The instructional plan for this project was a
modification of the instructional strategy outlined by Engelmann and Carnine (1992) in the recently published basal math series called *Connecting Math Concepts*. The foundation of the program was the notion that numbers come in families. All math facts were taught using number families. Each number family was represented visually around an arrow which pointed to the right. Two numbers "sat" above the arrow, and the sum of those numbers was written at the end of the arrow. Four number facts were generated from one diagram of three numbers. For instance, from the pictorial representation below, students learned that the four facts $4 + 3 = 7$, $3 + 4 = 7$, $7 - 3 = 4$, and $7 - 4 = 3$ all belonged to the same family.

\[
\begin{array}{c}
3 \\
\downarrow
\end{array} \quad \begin{array}{c}
4 \\
\rightarrow
\end{array} \quad 7
\]

Having this background, students were taught to solve math word problems, also by using an arrow (symbolic of a number line). In lessons carefully sequenced with structured worksheets, students identified the number values in a problem and placed them in their appropriate places around an arrow. They were taught that a problem in math occurs when one number that belongs in the family was missing.

Scripts provided for the teacher led the students to determine the sum total in a situation. If the number of the sum total was provided in the story, the student wrote it at the end of the arrow. If that greatest number was not given, the students wrote the lesser number elements given
above the arrow.

Only two rules needed to be learned: a) If a lesser number was missing, subtract the other two numbers provided, and b) If the greatest number at the end of the arrow was missing, add the other numbers to find it.

The novel teaching method controlled student attention with signals given during class instruction for choral responding. It used structured worksheets, required the same simple figure for all situations, and had a standardized text for instruction. Engelmann and Carnine (1992) had analyzed addition and subtraction word problems and found that they could be categorized as either comparison, action, or classification types. The same arrow drawing strategy was generalizable to all these types of addition and subtraction problems. For students, the strategy simplified the role of abstract reasoning by having them think only of a greatest total number value, and lesser number values identified in a story problem. The simple graphic representation provided a concrete visual aid for the organization of elements. Unlike any other program, this model showed students how to decide on an operation. For teachers, the model provided specific directions for what to say and do—procedures for teaching which were lacking in attempts by other authors to help students make decisions when word problems were encountered.

The researcher made some modifications of the original program, e.g.,
1. Personalized the basic concept by expanding on the notion of numbers coming in families
   a. Created cartoon numbers to represent family members
   b. Likened the arrow around which the numbers were placed to the home of the family
   c. Called the numeral of greatest value "Mr. Big"
   d. Created the scenario of a missing person. When a math problem occurred, it was because "one family member was missing."

2. Composed structured worksheets to help students instantly recognize
   a. If a small number was missing
   b. If a large number was missing
   c. If you added to solve the mystery
   d. If you subtracted to solve the mystery
   e. Where numbers were placed around the arrow

3. Made and used transparencies to accompany whole group presentations

4. Used terms to generate more interest in math problems:

   This                      Instead of
   mystery                   problem
   Mr. Big                   big number
   help!                     what's missing
   who                       number

5. Eliminated from worksheets review exercises for basic computation
6. Regarding worksheet layout:
   a. Enlarged numerals
   b. Put additional space between practice problems
   c. Added a name and date line to the top of each sheet for a structured heading guide
   d. Added stimuli to some sheets (e.g. 4.3, 6.3) to help students organize their work on the page
      (added lines or boxes to separate problems, dotted the outline of arrows, made thin lines to help students keep numbers in columns)

7. Added script

8. Modified some of the established script, i.e., explained that the term "big" as in "big number" meant the "number having the largest value."

9. Made color-coded families of cartoon numbers

10. Made a three-dimensional arrow house with manipulative numbers which fastened to a display board with velcro strips

11. For each lesson, built in a review of previous lessons for reinforcement and clarification of concepts

12. Used auditory, visual and tactile measures to instill the rule "When a small number is missing, subtract"
   a. Auditory: Said rule, had students echo the chant several times
   b. Visual: Used S-S symbol, matching small (number) with subtract
c. Tactile: "Wrote" S and S on the backs of students while saying the subtraction rule

13. Added people figures to transparencies 5.1, 5.12, 5.13 to portray number family members as people

14. Used rhyming sound association as a memory aid, e.g., "more" and "for": When a problem tells about getting more, you put the values in forward along the arrow.
(See Day 7.)

Limitations of the Study

For the current research, only those problems which required a single operation (one-step) were used. Also, to help minimize the interference of reading disabilities which could account for math errors, and the interference of difficult computations, situations from second and third level math texts were used. The new teaching method was primarily structured to help students set up numbers correctly and to chose an appropriate operation. These two skills were considered the most crucial first steps toward reaching solutions to word problems. The principal point of interest for this investigation, therefore, was whether students chose correct operations, rather than whether errors were made because of faulty computations.
CHAPTER II
REVIEW OF LITERATURE

The introduction to the research indicated that word problems have been troublesome for students because of their own careless mistakes, or because of poorly designed basal textbooks, the specialized readability of printed story situations, and weak teacher-directed presentations. This chapter presents previous research studies which have identified additional variables that might account for the difficulty students experience in solving word problems.

Gender Differences

Research has indicated that one's gender makes a difference in relation to performance in mathematics. In studies that spanned the past 20 years, it has been found that females were more likely to be successful with computations, while males have outperformed females in word problem solving. Johnson (1984) reported that the widespread agreement of male superiority in the skill was largely based on the works of students of Donald Taylor, a professor at Stanford University and later at Yale University. Sweeney's (1953) study concluded that general reasoning and the ability to restructure problems were
responsible for the male advantage. However, because subjects in his reasoning experiment did no problem solving, the only conclusion that can be reliably drawn from his research was that there was a definite male advantage.

Attitudes. In addition to scoring higher, males had a more positive attitude toward word problems than female subjects (Carey, 1955, 1958). Fennama and Sherman (1977) labeled the attitude of females as fearful. They believed that fear toward the math experience accounted for the low percentage of women in math related careers. The poor attitudes could be reversed, however. When small group discussions, aimed at improving attitudes toward the math skill, were held, female scores improved in Carey's 1955 and 1958 investigations. Fennama & Sherman (1977, 1978) and Sherman (1982) noted that females did not always possess a negative attitude toward mathematics. At the elementary level, they found girls were as likely as boys to rate mathematics as a favorite subject. Differing attitudes or perceptions of math as a masculine domain emerged around the 10th grade level.

In a more recent study, Genshaft and Naglieri (1988) addressed the subject of remediating math anxiety in girls. They selected 45 junior high eighth graders, and 45 tenth grade high school girls from a large urban school district for a six-week, 12 lesson treatment project. The specific techniques for improving attitudes and reducing anxiety were described in A Mindset for Math. In addition to outlining
activities for students, the guide provided practical suggestions for administrators who might organize similar programs.

Feminine Context. Milton (1958, 1959) simply reworded word problems into ones with a feminine context, and found that female college students then scored as well as males. The results of his two experiments have been widely cited, although in his third experiment using high school students, no such effects of rewording were found. In Milton's third study, males outperformed females on problems written from both a masculine and a feminine orientation. Additional studies refuted the significance of composing scenarios from a feminine perspective.

Hoffmann & Maier (1966), Role (1970), Heyn, and Berry & Pollack (1978) all have reported no effect of rewording on the performance of females with word problems. Berger and Gold (1979) composed gender-neutral problems, but still found a male advantage for those over 25 years in age.

Prior Experience. The search for variables to account for sex differences has been ongoing. Fennema (1977) stated that girls took fewer math courses and were less apt because of poor preparation. Have social changes since that time impacted differences? Johnson's (1984) series of nine experiments investigated sex differences among college students, to make such a determination. The sexes continued to differ, he found. Males had an advantage of 35% over females--the same level as reported in 1950.
Prior experience with similar word problems had no effect on performance, although it was believed that spatial ability contributed to sex differences in math word problem solving (Benbow & Stanley, 1983; Barnett, Lane & Dratt, 1979; Hyde, Geiringer & Yen, 1975; Sherman, 1967).

Other Factors. Marshall (1984) studied the scores of nearly 300,000 sixth grade students in California. She controlled or eliminated several factors previously found to be related to gender differences in math. The first factor was instructional experience. Instructional experience referred to the number and type of courses in mathematics training a student had received. At the elementary level, boys and girls have an identical history of instructional experience. At the secondary level, however, boys generally enroll in a greater number of mathematics courses than do girls.

The second factor which Marshall found related to gender differences in math was puberty. In her study, Marshall (1984) involved no children who had reached adulthood. Differences due to hormonal changes could not be applied.

A third factor was attitudes. By involving all sixth graders, Marshall eliminated the interference of negative feelings about math. At the sixth grade level, girls were as likely as boys to rate mathematics as a favorite subject. Most studies noted that differing attitudes or perceptions of math as a masculine domain emerged around the 10th grade.
level (See, for example, Fennama & Sherman, 1977, 1978; and Sherman, 1982).

A fourth factor Marshall eliminated was socioeconomic status (SES). No SES group of either sex had a particular advantage or disadvantage with word problems. The average probabilities of solving word problems for all children ascended as SES classification increased (Marshall, 1984). Having controlled or eliminated these factors, Marshall's investigations still confirmed earlier studies that gender was a statistically significant factor in solving math word problems.

Nevertheless, all these claims of various non-intellectual variables that negatively affected scores for females—attitudes, wording of problems, general reasoning ability, and field dependence—were disputed by the results of Berry's (1958, 1959) experiments.

Structure of the Curriculum

The spiral curriculum in the United States has been blamed as a cause of poor student performance on math word problems. Described as a low-intensity curriculum, the spiral curriculum is based on the introduction of various topics beginning in grade one, and repeating instruction on those topics (in greater depth) annually. According to Engelmann, Carnine, and Steely (1991), though the intent has been to add depth, the result has been "the rapid, superficial coverage of a large number of topics each
Yearly reintroductions have largely been review sessions for many students who have failed to master prerequisite skills. Suggested pacing guidelines from basal manuals have been too fast, introducing too many new concepts simultaneously (Steely, Carnine, Engelmann, 1990). As an illustration, Porter (1989) reported that 70% or more of the topics covered in math textbooks received less than 30 minutes of instructional time. The rationale has been described as "teaching for exposure." Important content has been reviewed each year, but the review has been too delayed, and too infrequent. Carnine (in press) found that in one program, new material was reviewed an average of only once every 20 days.

Specific Learning Disabilities

Cawley (1984) has given attention to the unique learning strategies of children classified as learning disabled. Enright (1989) similarly has identified special student characteristics that have impeded the acquisition of math reasoning skills. While the reader is reminded that the learning disabled category represents a complex heterogeneous group with varying patterns and inconsistencies, certain manifestations are highlighted in relation to poor performance with word problem solving. Impulsive behavior, a tendency for hyperactivity, and distractibility result in frustration. Difficulty in reading, understanding, and remembering terminology have
been negative factors. Additional characteristics include poor short- and long-term memory, difficulty transferring thoughts to written form, and inattention to detail. Silbert, Carnine and Stein (1990) referred to such youngsters as "instructionally naive." For these, complex tasks must be simplified by teaching smaller component skills explicitly, in sequential order.

**Recommended Strategies**

Scholars have proposed various techniques for teaching word problem solving. The range of techniques has been widespread and represents a diverse and often contradictory host of ideas.

**Pictures.** A number of authors suggested the need for pictorial representations as an aid for students. Bley and Thornton (1989) noted that many textbooks presented sequences that directed students to simply picture mentally what was happening in a story. Various forms of mental imagery have been said to facilitate learning, making abstract information more concrete and imaginable (Willows and Houghton, 1987). For students with learning disabilities, however, Bley and Thornton stated that the graphic presentation should first be made for them. During the initial stages of learning, students could choose the appropriate picture out of a multiple choice arrangement. Once this aid was faded, Bley and Thornton proposed that
students could later draw their own pictures. Others also stressed that students should be encouraged to draw their own illustrations (Kane, Byrne, and Hater, 1974; Thornton, 1989). Regardless of the age of the student, Bley and Thornton believed that the use of pictures drawn by students was appropriate.

Thomas (1988) and Larson (1980) added that diagrams, tables, and graphs were other helpful visual representations which students should be encouraged to make. Because students with visual-memory or visual discrimination deficits might have trouble retrieving the correct sign for an operation, Bley and Thornton (1989) suggested that a printed strip of the operation signs be placed before students as a visual reminder of the alternative operations.

Concrete Objects. The manipulation of concrete materials adds the tactile and kinesthetic channels to the visual-only mode of pictures. For all students, manipulatives might enhance motivation and perhaps meaning (Sowder, 1988; VanDevender and Harris, 1987). Having worked with students in regular programs, Sowder (1988) noted:

When concrete materials of some sort were used during the teaching experiment, the students noticeably picked up. Hands-on work rather than sole reliance on paper and pencil and symbolic procedures has been advocated for years. ... (A) greater centrality of natural settings of all sorts in math lessons would appear to
diminish the gap between students' ability to compute and their ability to use the operations in the settings which story problems represent" (p. 9).

Cawley (1984) advocated the use of manipulatives because they helped special students focus on the notion of activity. He asserted that the use of manipulatives, and the substituting of numbers in a story with indefinite quantifiers (some, a, bunch, a group, a set, etc.) would lead students to attend to the information source for choosing an operation.

**Role Play.** Allowing students to physically act out a situation was an alternative strategy proposed (Kane, Byrne and Hater, 1974). Genshaft and Naglieri (1988) suggested similar math parlor games for aiding in the comprehension of math vocabulary. Dumping essential mathematical terms in a bag and having students choose one to act out could serve the dual purpose of assessing understanding and reinforcing concepts, they suggested.

Besides the students themselves, LeBlanc (1982) encouraged teachers to show the different methods of solving a problem. Students could act out solutions using paper clips and other small objects as comprehension aids.

**Reading Decoding and Comprehension.** Much discussion has centered on the issue of language skills for successful word problem solving. Cawley (1984) stated that a most basic need for solving math word problems was reading—the ability to interpret language statements. Bley
and Thornton (1989) explained that students must understand what questions were being asked and what information was missing. Such basic requirements of sorting essential information from non-essential information requires receptive and expressive language skills—difficulties for many students with learning disabilities. Prerequisites to word problem solving include word recognition (the association of a word to its appropriate meaning), general comprehension (Grouws and Good, 1987), vocabulary usage, the translation of written problems into mathematical sense (Bley and Thornton, 1989), reading decoding, and receptive and expressive language skills (Cawley, 1984).

The readability of word problems has been especially troublesome because of the high percentage of words that appear only once, according to research by Panchyshyn & Enright (1981). They studied five different basal math series in grades 1 through 8 and found that the majority of words found in sentences resulting in computation do not appear in the Dolch list. In addition, over 70% of the words appeared only once in a math series. Since comprehension is dependent, in part, on a child's sight vocabulary, they suggested that it is reasonable to assume that vocabulary is often an obstacle in word problem solving.

Sullivan (1981) isolated 50 vocabulary words that comprised 51% of words selected from kindergarten through sixth grade math texts. She conducted a study to determine
if vocabulary instruction would make a difference in the performance of children's word problem solving. Thirty-eight 4th, 5th and 6th graders received two hours of math instruction each day, over a three week summer math enrichment program. The control group was drilled in basic facts, while the experimental group got instruction in specific math vocabulary. All students made gains, but no significant differences were found in the scores of the two groups. Kane, Byrne, and Hater (1974) stated:

There are important similarities between ordinary written English and the language of mathematics. Thus, many of the skills taught in school reading programs can be applied to the task of reading in the language of math. On the other hand, a number of differences exist which suggest additional reading skills needed and modifications of reading skills useful in ordinary English. (p. 10)

For example, Kresse (1985) stated that students should be shown that the reading of story problems is opposite the reading of most paragraphs that begin with a topic, give details, and often end with the main idea. He promoted teaching students how to search the end (question) of a story problem first in order to know the purpose of the exercise. Thomas (1988) suggested that instructors should help students break down the original problem into a series of short simple sentences.
Another example of the differences in reading was addressed by Bley and Thornton (1989). They said that when a student looked at a mathematical sign s/he must think one thing but say another. For instance, a child seeing (+), must think "add," and then say "plus." The integration of concepts might come readily with students in the regular math program, but may need to be directly taught to some students with learning disabilities. They suggested the use of a chart to help youngsters translate symbols.

Reading Rate. Kane, Byrne and Hater (1974) pointed out that reading math word problems was unlike reading a novel or newspaper. Because it requires detailed and careful attention, students should be taught to read at relatively slow rates. First they should skim for general information, reading the selection quickly to determine what the story is about. Next, however, students should reread the story slowly to see how ideas are related.

Color Cues. Once the story has been read, color cues may help. For the learning disabled problem solver, Bley and Thornton (1989) suggested the use of one color underlining the subject of the situation, and another color underlining the action. They advocated that students X-out information that is not needed.

Think Out Loud. Thomas (1988) advocated large group instruction with students reading after the teacher. He suggested directly taught lessons where the teacher reads the story aloud, then thinks aloud the problem solving
strategy. When students get to hear the teacher think out loud, they learn by modeling the method of categorizing a problem and testing the choice of a schema, he believed. Kress et al. (1985) called this type of modeling strategy guided practice in inference awareness. He stated that when the teacher orally reasons through numerous word problems, students learn metacognitive strategies; they learn how to visualize and label—an important first step in the process of problem solving.

Herrmann (1989) conducted research using the think out loud strategy as an aid in solving math word problems. Following nine hours of intensive training with lectures, video tape recorded lessons, demonstration lessons, and simulated teaching activities, four elementary level teachers taught 32 students during a five week supervised clinical practicum. Each teacher had eight students, three of whom were target students. The teachers used explicit instructional statements to demonstrate the method of solving different types of story problems. During pre and post problem solving interviews, each target student was asked to think out loud while solving a set of word problems. Students were more effective following teacher-directed presentations. The explicit explanations were:

1) Introduction statements. The teacher gave information on when a particular strategy should be used and why it was important to use it.

2) Modeling strategies. Instead of having students
memorize rules, the teachers verbalized how students were to visualize the problem.

3) Interactive statements. Teachers elaborated on explanations, and clarified concepts by re-explaining and asking students questions to check for understanding.

VanDevender and Harris (1987) also noted an advantage of having students think aloud. When every decision was verbalized, faulty steps would come to the surface. Teachers then would have specific diagnostic information on which remediation techniques could be formulated.

Key Words. The attention to detail within the written structure of the problem, namely, key words, has been an issue of much debate among scholars. Grouws and Good (1987) said that special attention to terminology should be a routine part of math instruction. Teachers should write any new word on the board, pronounce it, have the students repeat the pronunciation, and then discuss the word's meaning. Many others advocated such a focus on key words. Bley and Thornton (1989) noted trouble words such as "each" (meaning one) and "many" (8 or 43 but never 0 or 1). They suggested that key words and their meanings be placed just before each problem and highlighted within the problem. In different situations, the same terms might call for different operations, they noted. "Return" means remove or take away in one situation such as:
"I have 8 pencils.
I return 6 of them.
Now I have ___ pencils."
The same word calls for addition in a different situation:
"I have 8 pencils.
My friend returns 2 pencils to me.
Now I have ___ pencils."
The posting of a chart that matches troublesome terms with their meaning was recommended.

Thomas (1988) also identified troublesome terms that have different meanings in different contexts. For words such as "product" which may mean a good or the sum of multiplication, Thomas urged teachers to caution students and to solicit from students their informal definitions and interpretations of important phrases.

Kresse (1985) defined key words differently. Often, the key word might be a noun found in the problem question. In a situation that ends with "How many buses are needed?", for instance, he identified "bus" as the key word. He also identified plurals and articles (a, an, the) as major keys to comprehension which must be pointed out and taught.

Other scholars have disagreed with the whole notion of identifying key words or phrases (Cawley, 1984; Sowder, 1988; Tsai & Derry, 1987; Webb, 1986). Webb (1986) believed that if relying on key words, students could easily be tricked and would often be led to choose an incorrect operation. He further asserted that key words
were ineffective for solving complex problems.

Sowder (1988) considered it "unfortunate" that some "well-meaning teachers" used the key word strategy—unaware of its "defects." Cawley's (1984) remarks were even more pointed:

Tasks that direct the child to seek key words (i.e. left means to subtract) or to use computational rules (e.g. 3 or more different numerals suggest addition) as aids to the selection of an operation are not word-problem activities. They are, in reality, instances of negative learning for the child. (p.83)

Listing Steps. Case and Harris (1988) conducted research with four students with learning disabilities who were enrolled in self-contained classes because of math difficulties. Tasks for the students consisted of sets of 14 one-step word problems. A folder for each student marked progress, listed strategy steps, vocabulary, and self-instruction statements. Each student received training directly from the teacher two or three days a week in 35 minute sessions. A number of instructional strategies were implemented in this research: The teachers encouraged students to use covert speech; the teacher modeled problem solving processes and gave students specific feedback; students had opportunities for controlled practice; and students kept a record of progress with daily graphs. The most effective strategy, however, according to the researchers, was the use of a chart which served as a prompt
for the students. The chart listed five steps to be followed when solving a story problem: 1) Read the problem aloud, 2) Look for important words and circle them, 3) Draw pictures to tell what is happening, 4) Write down the math sentence, and 5) Write down the answer.

Others offered similar lists (See, for example, LeBlanc, 1982). Bley and Thornton (1989) broke down the sequence of steps with reminders to: 1) Read the problem, 2) Picture what is happening, 3) Think: What is the question? 4) Tell what you do to answer it (add, subtract, multiply or divide), 5) Compute the answer, and 6) Check. Is the answer reasonable?

"RIDGES" is the mnemonic code composed by Synder (1988) for upper elementary to high school students:

Read the problem.

I know statement (List all information given.)

Draw a picture (which need not be realistic).

Goal statement. (What do you want to know?)

Equation development

Solve the problem.

Marzola (1987) transferred similar steps onto prompt cards with success. He worked with 30 fifth and sixth grade children living in New York city and classified as learning disabled. The systematic word problem solving plan, coupled with calculator use, resulted in posttest scores for students in the experimental group going from a mean of 66% or below to 90% and above. The one exception in the group
was a student later reclassified as mildly mentally impaired.

**Direct Instruction for Special Populations**

Much of the research related to effective math strategies with special populations has come under the classification of direct instructional models. "Direct Instruction" identifies a structured, teacher-directed environment with carefully sequenced explanations and questions that are designed to enable all students to acquire and retain new learning. Developed by Carnine and Engelmann (1982), direct instruction presentation techniques maximize the amount of time students spend on task. Teacher demonstrations are presented before groups at a rapid pace, and are punctuated by numerous signals for students to answer questions in unison. Choral responding is a method that promotes student attentiveness and participation. Students are immediately reinforced for correct answers, while immediate corrections are made if answers are in error. In addition to group responding, individual students are called on from time to time following a question. Scripts for lessons are written to insure that teachers accurately implement the program. They are outlined so that information is presented in the most succinct yet explicit manner possible.

Research studies have testified to the wide-scale orientation of direct instruction techniques with learners
of various backgrounds. After 11 periods of direct instruction, 73 skill-deficient fourth graders in Oregon in six regular classrooms translated word problems into equation form better than students who were taught by more traditional methods (Darch, Carnine & Gersten, 1984). In another recent study, a brain injured 8-year-old boy who had been comatose for approximately 3 weeks, was tutored by the direct instruction approach. After 13 sessions, Thomas' accuracy in solving addition and subtraction story problems improved from 11.4% correct at baseline to 91.25% correct after instruction. Completion of basic math facts increased from six facts a minute at baseline to 11.5 facts a minute after treatment (Glang, Singer, Cooley & Tish, 1991).

Economically disadvantaged primary grade students have also benefitted in reading and math with direct instruction programs (Bereiter & Kurland, 1978; Brophy & Evertson, 1976), as well as preschoolers (Carnine, 1981). Kelly, Gersten & Carnine (1990) found direct instruction to be superior to traditional approaches when teaching fractions to high school students with learning disabilities and to high school students in remedial programs. In three other studies by Kameenui, Carnine, Darch & Stein (1986), the effectiveness of following the direct instruction model for teaching fractions, subtraction, and division was assessed. Students in the control groups, taught by the direct instruction model, outperformed comparison groups which followed a traditional approach.
In a study by Darch, Carnine and Gersten (1984), elementary teachers used direct instruction or active teaching techniques with 73 skill deficient fourth graders. Students in this study were taught word story problems involving multiplication and division. The direct instruction approach, with semi-scripted lessons, was contrasted to a traditional approach based on four state-adopted basal series. Students in experimental groups and in comparison groups received instruction with identical problems. Sessions were structured with 15 minutes of teacher-led instruction and 15 minutes of independent seat work.

In the classrooms following a traditional approach, the teacher used group discussion to introduce different problems. Guided questions were used to help the students locate information and to find solutions to the problems. Situations within the interests of the students were included. The teacher modeled alternative methods of choosing a correct operation if students were not successful.

In the experimental groups, prerequisite skills were taught in a sequential manner and problem solving strategies were taught explicitly using direct instruction techniques. Students first learned to discriminate multiplication problems from addition problems. Word problems were then introduced where students had to decide whether multiplication or addition was used. The concept of
division was then taught, and the final step required students to discriminate among word problems that required all four operations. The direct instructional program was marked by high rates of question-answer exchanges between the teachers and students. Answers were given orally in unison by the students, and precise feedback followed.

On a posttest containing 26 story problems, students taught problem solving with the direct instruction approach performed at a significantly higher level than students taught with traditional approaches. This result remained consistent even when compared to traditional groups which were provided up to eight additional remedial lessons. Ten days after the posttest was given a delayed posttest was administered to measure whether skills were retained. In this case, only the direct instruction students who received extra review lessons performed significantly better than comparison groups.

In another review, Sommers (1991) evaluated the gains of at-risk students in basic skills classes who scored below the 50th percentile on the Gates-MacGinitie Reading Test and the Stanford Mathematics standardized tests of achievement. Students were sixth-, seventh-, and eighth-graders in a middle school, and participated for a school year in math (and other) programs which followed principles of direct instruction. Using the Corrective Mathematics Multiplication and Division programs, they were taught how to do three- and four-step multiplication and division
problems. To develop an understanding of the concept of fractions, students began with the Basic Fractions Mathematics Module. In this program they learned how to add, subtract, multiply and divide fractions, including mixed numbers and equivalent fractions. The more advanced Fractions-Decimals-Percents Mathematics Module presented division and reduction of fractions, and the basic operations with decimals.

Grade equivalent gains in math were assessed with the KeyMath Diagnostic Inventory. Seventh graders using Corrective Math gained an average of 1.2 grade levels following four to eight months of instruction. Three groups of sixth graders gained .77, 1.06 and .66 years in math skills. Eighth graders using Corrective Math Basic Fractions-Decimals-Percents gained .57 years in math achievement. Another group of eighth graders using Corrective Math Ratios and Equations gained a mean average of .8 years. Finally, for two groups of sixth graders and one group of seventh graders using Corrective Mathematics Multiplication, Division, Basic Fractions, and the Fractions-Decimals-Percents Modules, gain scores in grade equivalents were 1.1, .9, and .71.

The largest body of data which has supported direct instruction is called the Follow-Through study. Described as the largest educational experiment ever conducted, the study involved over 15,000 disadvantaged children from 60 different communities. Each community chose an
instructional model for its children to follow from grades K - 3. Programs were based on either individualized instruction, discovery learning, Piaget's levels of cognitive development, open education, the language-experience model, direct instruction, or child development theory. Gains in cognitive skills were assessed using the Metropolitan Achievement Test. Scores revealed significant differences in groups. Programs which emphasized individualization and psychodynamics yielded low gains in student achievement. Students whose communities sponsored direct instruction and applied behavior analysis had an advantage of 1/2 a standard deviation above the next highest sponsor in total math, and 3/4 a standard deviation above the next highest sponsor in language skills. Direct instruction produced better gains than eight other approaches in both math computation and problem solving. On the average, the achievement level of low income primary level students was raised to the level of national norms. Direct instruction was the only program which also benefitted middle income students.

Children who completed the full program were tested again at grades 5 and 6 to determine the long-term effects of the program. Significant effects were noted in math problem solving and to a lesser degree, in math concepts. The 10 year federally funded program revealed an increase in self esteem as well. The Coopersmith Self-Esteem Inventory and the Intellectual Achievement Responsibility Scale were
used for assessment in the affective domain.

The success of direct instruction with children in disadvantaged circumstances has led the authors to make bold claims. Englemann (1980) stated:

Because of its broad orientation, direct instruction applies to nearly all instructional problems, from the teaching of very unfamiliar behaviors to a handicapped youngster to turning on older students who are not easy to motivate. It applies to the teaching of college level skills and to the teaching of subjects not taught well through traditional approaches--reading, spelling, arithmetic, and the sciences. (p. 3)

In spite of its success in improving student performance, however, direct instruction is not without its critics. Heshusius (1991) denounced the approach as being too rigid and for ignoring the personal needs of individual students. He stated that the student, by following a carefully sequenced lesson which calls for specific answers, is simply "responding to the logic of the presentation."

Heshusius stated that real learning does not occur in such an organized fashion:

Multiple plausible responses exist to many questions and problems. To superimpose known outcomes is in many cases a consequence of a synthetic or artificial construction of knowledge.
... Authentic learning does not occur in a stable, steadily progressing manner: rather, its visible outcomes are variable. (p. 325)

In attempting to explain why direct instruction is not more widely used, Jones & Krouse (1985) also cited the low emphasis given to the importance of individual differences. Additionally, they noted complaints by some teachers that the structured sequencing is inflexible, not allowing teachers to be creative, and also time-consuming in regard to preparations necessary before the instructional phase. Jones and Krouse (1985) further noted the time cost of training and supervising teachers before the program could be accurately implemented.

That presentations are usually made before the whole class rather than to small groups is consistent with the findings of Effective Schools Research (Reynolds, 1988). The direct instruction model maximizes academic learning time (ALT) for all students, with teacher-directed lessons requiring high rates of opportunities for students to practice skills. While choral responses are given, the sequencing of concept presentations gives extensive coverage of content, and insures that students understand at every step. "For this reason, direct instruction is often recommended as a procedure of benefit to handicapped students and other students at risk of academic failure" (Pendarvis & Howley, 1988, p. 20).
The structured script and training required for teachers is also considered essential. Darch, Carnine & Gersten (1983) found that without such preparation, teachers were not as effective in promoting students' mastery of math skills. Besides the advantage to students by insuring clear communication of material in a most efficient, interesting and sound manner, Engelmann (1980) described the advantages of such structure to the teacher:

The program provides the tasks and the script, which means that the teacher does not have to worry about designing these and about how these interface with one another (both during a given lesson and over long periods of time.) The teacher can concentrate on executing the tasks—presenting and communicating skills to the children. After all, why should the teacher be required to be a curriculum designer? What training has been provided to assure even modest results in this enterprise? And when, precisely, is the teacher supposed to find time to analyze and develop a full-blown curriculum? A far more manageable endeavor would be to reduce the number of things the teacher is required to do and to concentrate on that which is central to the teaching role—presenting. (p. 80)

The implication from research studies is that children with learning disabilities can be taught to be proficient and more analytical in their approaches to word problems,
and can learn to differentiate between several different problem types. Marzola (1987) stated that:

Educators can begin to explore the development of a variety of comprehensive problem solving programs for children with learning disabilities and examine the task variables that contribute to problem solving success within this group. (p. 12)
CHAPTER III

METHOD

In this chapter, the procedures for conducting the investigation will be presented. The characteristics of the students involved, how they were selected for the study, and the setting and background of the researcher will be described. The research design will be listed and explained, and the measurement instrument development will be given. In addition, this chapter will present the sequence and method by which the data were analyzed, and the instructional sequence for the treatment and comparison groups.

Subjects

All students who participated in this study were enrolled in Kanawha County, West Virginia, public schools, in fourth-, fifth-, or sixth-grade level special classrooms, having been classified as specific learning disabled under the regulations of West Virginia Policy 2419. Criteria for identification of specific learning disabilities in the state specifies that the general intellectual functioning of a student must be average or above average; that there is a severe discrepancy between the child's intellectual ability
and his/her achievement (in basic reading decoding skills, reading comprehension, written expression, mathematics calculation or mathematics reasoning, listening comprehension or oral expression); and that deficits in memory, perception and conceptualization are exhibited. The educational performance of such a child is therefore adversely affected. The label does not include children who have significant discrepancies academically as a result of mental impairments, visual, hearing or motor handicaps, emotional disturbances, cultural differences or economic disadvantages (Policy 2419, 1991).

The number of students for this study totalled 49. Two classrooms participated in the pilot testing of the measurement instruments, two other classrooms comprised the treatment group, and two more classrooms served as the comparison group. No two classrooms were from the same school; six different schools were involved. One classroom in the treatment group was a resource room. In this model, the special students had a homeroom with regular education peers. At least 40% of these special student's instructional time was delivered in the regular education program. All other classrooms in the study were self-contained. In the self-contained placements, students were taught in separate classrooms by special education teachers for more than 60% of the instructional day. Whether assigned to a self-contained classroom or to a resource room, every student in this study had a severe discrepancy
between intellectual ability and achievement in mathematics calculation or mathematics reasoning. Most had severe discrepancies in other areas as well. Having a disability in math skills, however, was a common denominator for all subjects.

The researcher received permission from the superintendent of Kanawha County schools to conduct the study (Appendix A), cooperating school principals, teachers, and parents. Permission letters and consent forms for each group of students are found in Appendices B, C, and D.

Pilot-Test Subjects. Of the students (16) in the pilot test group, 19% (3) were fourth graders, 50% (8) were fifth graders, and 31% (5) were sixth graders. No females were enrolled in either of the classrooms used for the pilot test. The age range in years and months was 11-1 to 13-6. The mean age was 12-3. Eighty-one percent of the youngsters came from low socioeconomic homes, based on their eligibility in the federally funded free lunch program. Reading grade equivalent scores, all measured by the Woodcock-Johnson Psychoeducational Battery, ranged from 1.2 to 6.0. However, the mode was 1.2, and the mean reading grade equivalent was 2.3. I.Q. scores were measured according to the Weschler Intelligence Scale for Children - Revised (WISC-R). Based on the most recent assessment, intelligence quotients of the pilot test group ranged from 84 to 103. The mean I.Q. was 94.
Treatment Group Subjects. Of the students (18) in the treatment group, 6% (1) were enrolled in the fourth grade, 44% (8) were fifth graders, while 50% (9) were sixth graders. Males predominated, comprising 95% of the sample. The age range in years and months for these students was from 11-3 to 14-4. The mean age was 12-5. Seventy-two percent of the students in the treatment group were from low socioeconomic families. Reading grade equivalent scores, from the Woodcock-Johnson Psychoeducational Battery, ranged from 1.3 to 6.3. The mode reading grade equivalent score was 1.6, and the mean score was 2.6. The range of I.Q. scores for youngsters receiving the treatment was 80 to 106. The mean I.Q. score was 93.

Comparison Group Subjects. The comparison group consisted of 15 students with 13% (2) in the fourth grade, 53% (8) in the fifth grade, and 33% (5) in the sixth grade. Males outnumbered females at 94% of the sample. Ages of children in the comparison group ranged in years and months from the youngest at 11-2 to the oldest at 13-7. The mean age was 12-4. As was typical of students in the pilot test and treatment groups, the majority (87%) of students represented low socioeconomic level families. Reading grade equivalent scores ranged from 1.1 to 4.9, with a mean of 2.5. On the WISC-R, the measured range of I.Q. scores was from 81 to 120. The mean I.Q. score for the comparison group was 93. Table 1 provides a composite of student characteristics.
Table 1

Characteristics of Students

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Grade</th>
<th>Mean</th>
<th>Low</th>
<th>Mean</th>
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<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>Male</td>
</tr>
<tr>
<td>Pilot</td>
<td>16</td>
<td>19%</td>
<td>50%</td>
<td>31%</td>
<td>100%</td>
</tr>
<tr>
<td>Treatment</td>
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<td>6%</td>
<td>44%</td>
<td>50%</td>
<td>95%</td>
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<tr>
<td>Comparison</td>
<td>15</td>
<td>13%</td>
<td>53%</td>
<td>33%</td>
<td>87%</td>
</tr>
</tbody>
</table>

The administration of a pretest was a control measure for subject selection. Control variables considered included gender, socioeconomic status, reading level, grade enrolled, age, and intelligence quotient.

Assignment of treatment to groups was done randomly. Random assignment of subjects to groups was not done randomly because of the nature of the students with learning disabilities (specifically, the need for routine and structure). Random assignment of subjects to groups would involve shifting students from other classrooms and possibly from other schools to generate desired numbers in the treatment group. Novelty and disruption would then have been major threats to internal validity. The results obtained, in such a case, might not have been generalizable to situations where the treatment was institutionalized.
Controlling for mortality, the study occurred toward the end of the school term. In the researcher's experience, family relocation is rare during the final semester. Parents and guardians often delay moving plans so their children can finish the year at the home school. Attendance was monitored during each day of the treatment. Two students in the comparison group were lost because of erratic and very poor attendance during the treatment period. Both were present for the pretest but absent on the day of the posttest. It was believed that their scores would not have reflected achievement from instruction in their classrooms. Therefore, no makeup posttest was attempted for either student. Pretest scores were likewise not averaged with other pretest scores of students in the comparison group. It was thought that inclusion of the scores of these students would have a negative impact on the comparison group mean, and would not have represented the average student, with a more perfect attendance record.

History was controlled by administering the pretests, treatment, and posttests on the same dates for the experimental and control groups.

Setting

By keeping classrooms intact, the researcher was able to implement the treatment during the regularly scheduled math period of each classroom, teaching the
same students who had studied together and identified as a group since the beginning of the school term. Location threat was thus controlled.

The facilities used at each school included the classroom in which students received special math instruction, with an overhead projector. With the exception of one classroom, all programs were in rooms of typical class size dimensions. Pilot school A classroom was 24 feet long by 31 feet wide; pilot school B classroom was 30 feet by 25 feet; treatment school A classroom was 35 feet by 25 feet; treatment school B classroom was 16 by 19.5 feet; comparison school classroom A was 36 by 21 feet; and comparison school classroom B was 32 by 22 feet. In addition to student desks, classrooms typically included one or two teacher desks, long tables, a computer center, television, bookshelves, file cabinets, and utility carts with equipment such as overhead projectors and record players.

**Experimenter**

The researcher had the active role of administering the tests and implementing the treatment. Special training in direct instructional strategies for the program outlined in *Connecting Math Concepts* was received during the summer of 1991. The teacher trainer/workshop coordinator was Bernadette Kelly, one of the authors for an advanced level of the program upon which this research was based.
The researcher has permanent certification from the state of West Virginia Board of Education for teaching children (levels K - 12) who have specific learning disabilities, behavior disorders, or mental retardation. Teaching experience has covered a span of 17 years, and has consisted of instruction in a self-contained unit with children having specific learning disabilities, in a multicategorical (for the behavior disordered, mentally impaired, and learning disabled) resource room, and home-bound instruction with students in the public school system. Current employment is as an assistant professor in the Department of Education at West Virginia State College.

In an effort to avoid novelty effects, the researcher was introduced to the treatment groups as an assistant math teacher, and acted in that capacity on two occasions prior to any solo contact with the entire classroom. The two days fell within the span of ten consecutive school days.

Procedures

Research Design

In order to investigate the effectiveness of the novel teaching strategy for this research study (objective one of this study), the nonequivalent control group design was used. This design is recommended in real world settings when it is not possible or convenient to randomly
assign individual subjects to the levels of the active independent variable. The design involves an experimental (treatment) group and a comparison group. The experimental group received the treatment (X1)—learning how to solve one-step addition and subtraction math word problems following a new and unique teaching approach—while the comparison group was taught how to solve similar problems under the traditional teaching approach and schedule. Therefore, the independent variable was method of instruction with two levels: 1) experimental (X1), and 2) traditional (X2). The dependent variable was word problem solving ability. Two subskills (selection of the appropriate operation, and finding the correct answer) were identified to measure word problem solving ability. An additional math skill—computation—was measured on the assessment instruments.

The sequence of research activities was:
1) administered pretest to both groups,
2) conducted intervention with the experimental group,
3) administered posttest to both the experimental and comparison groups.

\[ 0 \ X1 \ 0 \]

\[ 0 \ X2 \ 0 \]

Mean scores on the post tests were then analyzed to determine if there was a statistically significant difference between the two sets of scores. A delayed
posttest was given to the treatment group 10 days after instruction to assess the strength of the treatment over time.

The groups of subjects were termed non-equivalent because students remained in intact classrooms instead of being assigned to randomly equated ones.

**Instrument Development**

"No Problem!" is the name given to the test developed to assess how well children solved math word problems. Three forms of the instrument were designed: Form A as a pretest; Form B for the first posttest; and Form C for the delayed posttest as a check for maintenance and generalization of skills. Each form had the same number of items, format, instructions, discrimination ability, range, and level of difficulty, but the actual problems were not the same. Although a number of standardized instruments which assess math skills are on the market, such as the KeyMath (American Guidance Service), Brigance Inventory of Basic Skills (Curriculum Associates), the Stanford Diagnostic Mathematics Test (Psychological Corporation), and the Diagnostic Test of Arithmetic Strategies (Pro-ed), none were designed to assess problem solving abilities alone.

Story problems for the current instrument were generated from situations presented in four basal math programs: Holt Mathematics (Levels 2 and 3); Scott Foresman Mathematics (Level 3); SRA Mathematics
These levels were chosen for the fourth-, fifth-, and sixth-level students because children with disabilities in mathematics are often achieving two levels below actual grade placement. In addition, for many, reading skills are also weak. Since the purpose of this study was to investigate how children with learning disabilities apply math skills, the selection further reflected an effort to control for errors made as a result of reading decoding or reading comprehension weaknesses. According to the Fog Index (in Norland, 1991), the readability of Form A is level 2.6; of Form B is 2.5; of Form C is 2.8. The differences are considered negligible.

All problems were first copied verbatim onto separate slips of paper with a notation of the source and operation required. All the subtraction problems were placed in one bag, and all the addition problems were placed in another bag. For each test, nine subtraction problems and six addition problems were drawn randomly. Out of each set of 15 problems, slips were selected in the blind, so that although each test form had the same number of addition and subtraction problems, the sequential order of those problems varied.

Modifications were made on some of the sample stories to facilitate text reading. Proper names deemed difficult to read (e.g. Diego, Juanita, Blanca, Antonio, Kiku) and "dated" or uncommon names were omitted. These
were replaced by names considered currently fashionable according to Price (1985) and Nurnberg (1984). Some of the story character names were replaced with male names, simply because boys typically dominate the field of learning disabilities. Interest in reading is better generated at this age with same-sex personalities in texts. In other cases, phrases were replaced with complete sentences, or phrases were added to generate more information for passage content. When in the original text, the numbers were aligned, care was taken to reposition the numbers to become part of the text. The purpose in this change was to construct a contrast from the typical rows of computations presented on daily math skill sheets.

In addition, numerals were often replaced with the written representations to discourage students from looking immediately at figures while ignoring text. Age-appropriateness of activities was another consideration. Scenarios that had girls playing with dolls or children handling puppets were not included, for instance. Bley and Thornton (1989) posited that attention and comprehension were enhanced when instructors used interesting stories within youngsters' experience. See Appendix E to compare original stories to the modifications that were made.

Several authors (Sowder, 1988; Carpenter, Corbitt, Kepner, Lindquist, Reys, 1981) have noted that children often simply added all numbers when presented with word problems, since this was the operation in which they felt
most competent. Other children read the lead problem carefully, then used the same operation for the remaining problems, which were read carelessly. (This procedure is understandable, since a review of the basal programs revealed that in most instances, the same operation was required of all problems on a page.) Supposing that children do follow these adaptations, the writer determined that having an equal number of addition and subtraction problems would give all children a 50% chance of scoring correctly. The rationale for having each test consist of an odd number of problems (15), and the favoring of subtraction (9 problems) over addition (6 problems), is thus provided.

Following a developmental sequence that begins with the most basic form of word problems, only situations that involved a single operation (one-step) for solving were included on the No Problem! instrument. The following problems were eliminated from the pool of situations from the basal texts because they did not fit the researcher's judgement of simplicity:

1. Ninety-three tickets for the puppet show were made. The boys sold 39. The girls sold 49. How many tickets were left to sell? (Scott Foresman Level 3)

2. Dean had 13 clear marbles. He traded 6 of them for 4 blue marbles. What does he have now? (Houghton Mifflin Level 3)

4. Kelly had 7 dimes and 8 pennies. He spent 45c. What coins does he have left? (Houghton Mifflin Level 3)

The total number of stories for each form was limited in accordance with the literature (e.g. Bley & Thornton, 1989) and experience regarding the attention span of children with learning disabilities. Only five stories were printed on a page in consideration of needs of the group in the areas of:

a) motivation and interest
b) visual tracking, and
c) figure ground discrimination.

Directions were typed in boldface letters at the top of each page as a reminder for those with short term memory difficulties. Simple line drawings related to some of the story problems were placed on the pages of each form to stimulate interest, and to avoid the appearance of a test. The title given the instrument was written positively, without the term "test," for the benefit of those with test-anxiety.

In addition to the written directions on each page, an example of a completed problem was presented to show children that two responses were required for each problem. They were directed to circle a sign, and also to write out
the problem with its answer. Rather than leaving a blank for students to print the operation sign, the symbols were printed to aid students who have difficulty retrieving information (Bley and Thornton, 1989). See the testing instruments in Appendix F. For further clarification, the investigator presented explanations and demonstrations, and provided opportunities for the students to ask questions about the directions prior to the testing. Part of the instructional period was used to clarify directions for taking the test. Students were not given help in reading or solving problems. The purpose was to control for the internal validity threat termed "practice effect of testing." A script was written and followed to ensure standardized procedures in administration. See Appendix G. Direct instructional procedures have proven more effective with this population than written instructions/examples alone.

Validity and Reliability

A panel of experts to establish content validity consisted of five individuals—a certified reading specialist, a certified math specialist, two certified teachers of the learning disabled, and one certified psychologist. Members examined the format of the tests, content of items, readability level, and gender orientation of the three instruments.
Modifications made to the instrument format, following suggestions by the panel, included 1) adding a bold line to separate the directions from the test questions, and 2) enlarging the boxes in which students were to write problems and answers. Changes in descriptions of story situations included some tense alterations, word substitutions, or reordering of sentence structure in 8 out of the 45 printed scenerios. Readability level was confirmed suitable for intermediate level students with significant reading difficulties.

An assessment of gender orientation was made by rating each of the 45 story problems. An item was rated to be of male, female, or neutral orientation, based on the key persons in the story, or based on the main activity of the situation. On test form A, 27% of the stories were rated "female," 46% were rated "male," and 27% were rated "neutral." On test form B, 7% of the situations were rated "female," 40% were rated "male," while 53% were rated "neutral." On test form C, 7% of the situations were rated "female," 27% were rated "male," and 66% were considered "neutral."

Pilot Testing

Pilot tests were administered on all three forms as a reliability check, prior to the actual study. A desired reliability level was set at .6 a priori. The tests were administered to 16 students with learning disabilities
as defined and identified by the Department of Exceptional Students in Kanawha County, West Virginia. These students were not a part of the experimental or comparison groups for the actual research project, but were similar in characteristics: grade level (4th, 5th, and 6th graders); exceptionality (specific learning disabled); skill needs (deficient in math achievement); placement for service delivery (taught in resource rooms or in self-contained classrooms with a special educator presenting math instruction), ages; and location (public schools in Kanawha County, West Virginia).

The parallel forms technique was used to correlate the results of the three equivalent forms of the test administered to the same individuals. Form A was administered on a Wednesday. Form B was administered on the Friday of the same week, while Form C was administered on Monday of the next week.

The mean, standard deviation, and range of raw scores for students who participated in the pilot testing of the measurement instruments are reported in Table 2. Scores were classified by test form and type of skill measured. Comparisons across test scores revealed that for each word problem skill score, the means generally fell within a span of a few points. The same was true for the standard deviations and range of scores.

The scores supported the findings of Ballow and Cunningham (1982) that students could compute correctly at a
higher level than they could set up problems. The total mean score for computation with the present group was 12.69, compared to a total mean average of 11.59 for having chosen the correct operation, and a total mean score of 10.09 for having written the correct answer to a story problem. Once a problem was set up, students were likely to find its solution. However, the numerical selection or numerical organization often did not correspond to the situation written in the accompanying story text.

Table 2

<table>
<thead>
<tr>
<th>Test/Score Name</th>
<th>Mean</th>
<th>SD</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form A/Operation Selection</td>
<td>11.38</td>
<td>2.28</td>
<td>8-15</td>
</tr>
<tr>
<td>Form B/Operation Selection</td>
<td>11.40</td>
<td>3.29</td>
<td>6-15</td>
</tr>
<tr>
<td>Form C/Operation Selection</td>
<td>12.00</td>
<td>2.97</td>
<td>5-15</td>
</tr>
<tr>
<td>Form A/Answer</td>
<td>9.50</td>
<td>3.35</td>
<td>4-15</td>
</tr>
<tr>
<td>Form B/Answer</td>
<td>10.07</td>
<td>3.86</td>
<td>3-15</td>
</tr>
<tr>
<td>Form C/Answer</td>
<td>10.69</td>
<td>3.40</td>
<td>3-15</td>
</tr>
<tr>
<td>Form A/Computation</td>
<td>12.75</td>
<td>2.11</td>
<td>9-15</td>
</tr>
<tr>
<td>Form B/Computation</td>
<td>12.27</td>
<td>2.05</td>
<td>8-15</td>
</tr>
<tr>
<td>Form C/Computation</td>
<td>13.06</td>
<td>1.79</td>
<td>9-15</td>
</tr>
</tbody>
</table>
Separate coefficients of equivalence were determined for each of the summated scores. Correlations were examined against the .6 alpha set a priori to assess whether the three forms of the test, *No Problem!*, were indeed parallel forms.

Based on standardized variables, each score distinction within each of the three test forms, with the exception of the operation selection score on form B, fell at or above the .6 coefficient criterion of acceptability. Refer to Tables 3 and 4.

Table 3

*Internal Consistency by Form and Domain*

<table>
<thead>
<tr>
<th>Test Form/Score</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>A/Operation Selection</td>
<td>0.69</td>
</tr>
<tr>
<td>B/Operation Selection</td>
<td>0.72</td>
</tr>
<tr>
<td>C/Operation Selection</td>
<td>0.59</td>
</tr>
<tr>
<td>A/Answer</td>
<td>0.92</td>
</tr>
<tr>
<td>B/Answer</td>
<td>0.83</td>
</tr>
<tr>
<td>C/Answer</td>
<td>0.91</td>
</tr>
<tr>
<td>A/Computation</td>
<td>0.76</td>
</tr>
<tr>
<td>B/Computation</td>
<td>0.89</td>
</tr>
<tr>
<td>C/Computation</td>
<td>0.83</td>
</tr>
</tbody>
</table>
Table 4

Correlation Matrices

Operation Selection Scores

<table>
<thead>
<tr>
<th></th>
<th>Form A</th>
<th>Form B</th>
<th>Form C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form A</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Form B</td>
<td>0.42</td>
<td>1.00</td>
<td>-</td>
</tr>
<tr>
<td>Form C</td>
<td>0.56</td>
<td>0.53</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Answer Scores

<table>
<thead>
<tr>
<th></th>
<th>Form A</th>
<th>Form B</th>
<th>Form C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form A</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Form B</td>
<td>0.83</td>
<td>1.00</td>
<td>-</td>
</tr>
<tr>
<td>Form C</td>
<td>0.71</td>
<td>0.85</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Computation Scores

<table>
<thead>
<tr>
<th></th>
<th>Form A</th>
<th>Form B</th>
<th>Form C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form A</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Form B</td>
<td>0.70</td>
<td>1.00</td>
<td>-</td>
</tr>
<tr>
<td>Form C</td>
<td>0.81</td>
<td>0.62</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Conditions of Testing

The researcher administered all tests to students in the study. The regularly assigned special classroom teachers were present during the sessions, and had the role of writing notations on script copies during the pretest, to indicate interruptions or deviations from the prepared outline. Intervention periods and posttest sessions were all recorded on Sony HF cassette tapes, using a Bell and Howell Educator Series cassette recorder model 3085.

Although the term "test" was not used in the presence of the students, a testing environment was established. A "Do Not Disturb" sign was posted outside the closed classroom door. After directions are explained by the researcher, the researcher and the classroom teacher monitored the students by observation, rather than by proximity while moving around the rows of seats. One adult was seated at the front of the room, while the other adult was seated in the rear.

Rules for the students included "Work quietly," "Raise your hand," and "Stay in your seat." These rules were printed on individual cards, discussed and posted in the front of the room before directions were given. Math puzzle worksheets were distributed prior to the test administration. Students were directed to work on their math puzzles if they finished "No Problem!" before their peers. When all students had completed the test, or at the end of the period, the researcher and classroom teacher
collected papers. These measures were taken to minimize the movement and noise in the classroom—distractions for children with specific learning disabilities which might have constituted situation-induced (unique intersession history) sources of test error.

Students had 30 minutes to complete the 15 problems on the test. A timer was used to insure adherence to this condition of testing for all groups. Directions for taking the test required approximately 10 minutes. The entire session therefore occurred within the 45 minute period designated for math instruction. The sessions were recorded. In an effort to avoid the stereotype of making math a super-serious subject (Genshaft and Naglieri, 1988) the stated imposition of a maximum time for completing the activity was eliminated, and students were simply told that papers would be collected at the end of the math period. The conditions for the delayed posttest were identical to the conditions for the first posttest.

Based on observations made during the pilot testing, the instrument was not modified thereafter. Students did not have difficulty confining problems within the area of the boxes. All were able to complete each form within the time limitations. None indicated confusion with the directions by seeking assistance after independent work time began. None appeared to be frustrated with the task, and scores earned by students on all three forms of the test were similar, as one would anticipate for parallel forms of
Data Analysis

Three sets of information were generated from each instrument. There was a summated score related to the operation chosen (15 points possible). A second summated score (15 points possible) reflected the number of correct answers to the problems presented in the story situation. The third summated score (15 points possible) gave students credit for performing computations correctly, even though problems they set up may not have been appropriate to the scenario printed on the test. The identification of a computation score enabled the researcher to compare results with the findings of Ballow and Cunningham (1982) asserting that 60% of students can compute correctly at a higher level than they can set up problems.

To analyze differences between the treatment group and comparison groups on word problem solving ability, two separate t-tests were run on pretest points; one t-test on mean scores for operation selection, and another t-test on means for the answer score. The t-test compares means from two different groups to determine if they are the same or if they are different. Alpha was set at .05 a priori.

To analyze the significance of the differences of posttest scores from delayed posttest scores within the treatment group, the ANOVA procedure was employed. This procedure tested the second hypothesis—namely, that skills
learned from the novel teaching strategy would be maintained over time. The operation selection scores and the answer scores for posttests and delayed posttests were tested.

**Scoring procedures.** Considering the possibility of instrumentation threat, the researcher and classroom teachers scored each of the instruments as an interobserver reliability measure. (The percentage of agreement between raters was 95%) A key with the answers and appropriate operations was provided by the researcher for "O" (operation selection) and "A" (answer) scores. Computation ("C") scores were checked individually. If a student wrote a problem and computed it accurately, s/he earned a "C" point, even if the problem did not reflect the situation described. Calculators served as aids to the scorers as needed for the computation raw totals.

**Error analysis.** An analysis was made of the computation errors, with a qualitative assessment of possible reasons for student responses. The information delineating student responses is outlined on Tables 15 and 16 in Chapter 4. Were the digits of a two-digit number reversed? Did the child carry the wrong number? Were individual numerals reversed, indicative of visual discrimination problems? Did children have trouble differentiating the problem number from numbers in the problem? Was the right problem written in the wrong box, indicating problems with figure ground orientation? Were numbers properly aligned? Were numerals printed within the
confines of the box? For some children with specific learning disabilities, difficulty keeping figures within a designated space may be symptomatic of spatial disabilities or fine motor difficulties. Were computations of basic facts inaccurate, indicating memory or processing deficits? Did the student use the correct operation for the first problem, and perseverate, using that same operation for the remaining story situations? The qualitative assessment of student errors is reviewed in Chapters 4 and 5.

Instructional Sequence

Assessment. After the researcher had interacted in the treatment group classrooms on at least two occasions as an aide, the teacher explained to the students that there would be a shifting of roles: the researcher would now be the math teacher, while the regular teacher would act as her assistant during upcoming math periods. The researcher then provided instructions to the whole group regarding the completion of No Problem! Form A, the researcher-designed test that would generate information on math problem solving ability, based on a summated computation score and a summated problem interpretation score for each child. The complete script for administering the pretest is found in Appendix C. Students completed Form A on that same day, during their math class period. Following directions, no assistance was given to the students. The classroom teacher took notes on a separate copy of the script, indicating any
deviations from the intended interactions between researcher and students.

For students in the comparison groups, the researcher in like manner, administered the pretest during the same week, while the classroom teachers assisted. The same script was followed for all classrooms. The role of the teachers during the pretest included noting beginning and ending times, following the script, and writing down any deviations from the planned activity (e.g. intercom interruptions, special needs of the children, visitors, emergency drills, etc.).

Math instruction. Implementation of the novel teaching strategy commenced on the day immediately following the pretest, and continued each consecutive school day for a total of 9 instructional days/sessions. Instruction and practice exercises took place solely during the regularly scheduled math period for 45 minutes.

For children in the treatment groups, no other mathematics lessons were presented on the days of the project. A summary of the intervention yields the following basic learnings. The novel instruction used as its foundation, the concept that numbers in each computation fact belonged to one number family. To solve addition and subtraction math word problems, an arrow was drawn, like a number line. Numbers in a family were positioned around that arrow. The smaller numbers in the family were always placed above the arrow, while the number
of greatest value was written at the end of the arrow (which pointed to the right), e.g., the numbers 9, 6, and 3 all belong to a number family. They would be written around an arrow like the following model:

\[
\begin{array}{c}
6 \\
\hline
3 \\
\rightarrow 9
\end{array}
\]

The number of greatest value in addition or subtraction families equalled the sum of the smaller values.

Students learned that a math problem occurred because one of the numbers in the family was missing. In order to find the missing number and solve the problem, only two simple rules were followed:

a. If a small number was missing, you subtracted to find the missing number.

b. If the "big" number was missing, you added the small numbers.

The process of abstract reasoning was simplified. The method was unique, for it showed students how to decide on an operation using one figure design. The method also was unique in giving explicit directions for the presenter to follow. The reader is referred to Appendix H for the complete script which describes the specific direct instructional method of presentation, including materials needed, references to student activity sheets, verbal exchanges, and movements of the presenter. Appendix I presents the student activity sheets.

The plan controlled for student attention with auditory
signals for choral responding. There were structured worksheets for students, the standardized script for the presenter, and an arrow figure drawn as a visual cue that was generalizable to all categories of addition and subtraction word problems. Each instructional session lasted 45 minutes. There were 9 sessions.

For those in the comparison group, math instruction was presented by the classroom teacher for the same duration (45 minutes) each day of treatment. The researcher obtained copies of the lesson plans written by teachers of students in the comparison group, and consulted with those teachers for a complete description of the a) number of occasions when math word problems were presented or assigned during the treatment period, b) the methods used for presenting the math word problems, and c) a copy of the specific word problems completed by the students during the project treatment period.

The traditional math instruction provided the comparison students for this study was based on the Addison-Wesley Mathematics program adopted for students in regular education by the county of Kanawha in West Virginia. In both comparison groups, the special education teachers also used this basal series for their students with specific learning disabilities. Different (lower) level texts of this series were followed in the special classrooms, however. In comparison classroom A, Level 3 of the program was used with the intermediate grade students,
while in comparison classroom B, the students were divided into two groups— one in Level 4, and the other in Level 5 of the Addison-Wesley series.

Each classroom had a chart posted with five steps listed in abbreviated form, to help students with math word problems. From earlier sessions with the teacher, they had been taught that Step 1, "Question," was a reminder to "understand the question;" Step 2, "Data," was a reminder to "find the needed data;" Step 3, "Plan," was a reminder to "plan what to do;" Step 4, "Answer," was a reminder to perform computations to find the answer;" and Step 5, "Check," was a reminder to reread, or check back over their work. In addition to the poster, the Addison-Wesley textbook consistently printed the "5-Point Checklist" in chart form at the top of each workbook exercise that involved word problems.

During the nine-day treatment period, a total of three periods were assigned to word problem solving in the comparison group classrooms. In comparison group A, the first instructional period with word problems as the skill focus came from the Level III text on page 238. Printed at the top of the page was "There are 5 boxes of apples. 4 apples are in each box. There are 20 apples in all." A picture which corresponded to this information was on the right of the printed information. The teacher, following directions in the manual, asked students to describe the boxes of apples, directing them to provide a multiplication
sentence for the picture \((5 \times 4 = 20)\). Referring to the printed page in the text, the teacher reminded students of the relationship between multiplication and division, and wrote \(20 \div 4 = 5\) and \(20 \div 5 = 4\) on the chalkboard. It was explained that the first division fact was used to find how many boxes there were, while the second division fact was used to tell how many apples there were in each box. Following this lesson development, students worked semi-independently to solve five problems (called exercises) on the page. The teacher monitored during independent work time, helping some with words in the stories. Exercises 1 - 4 were all single-step problems involving short division with no remainder. Exercises 1 and 2 both described four sets of three \((12 \div 4; 12 \div 3)\). Exercise 5 required two-steps: addition, then short division with no remainder. When most of the students had completed the first four problems, the teacher addressed the whole group again, giving guidance for solving the challenging last problem. Circles were drawn on the chalkboard to represent apples from the problem situation.

The second instructional period with word problem solving as the skill focus for comparison group A, came from the text on pages 300 and 301. The exercise consisted of two "warm-up" problems with the teacher, then nine additional problems which students worked independently. The recently reviewed computation skill--long division with a remainder--was the primary operation to be used for all
but one of the situations. Problems 1 - 7 involved the single step operation for solutions. Problems 8 and 9 were two-step exercises. For problem 8, students needed to multiply the data first, then divide. For problem 9, students needed to add pairs of numbers from a column of six to find the pair which totalled 30 points. The answer to this last problem consisted of the names of two children who earned the points which totalled 30.

In comparison classroom B, students completed a page of problem solving exercises during the treatment period. From level IV of the Addison-Wesley Math Program, page 207 was completed totally with teacher direction for each of the nine problems. For each problem, students had to refer to data on a chart at the top of the page which was titled, "Aluminum Can Drive." The 5-Point Checklist at the top of the page was reviewed first. The teacher then read through the chart of data with the students, explaining its title and organization. A different student was called on to read aloud one question at a time. Others offered answers as the teacher presented guided questions to lead students to each solution. Time was provided for each student to write the solution and label the data on his/her paper. Students were encouraged to "stay with the group" and not try to work ahead. Situations included one-step and two-step problems. Operations required for solutions varied slightly:
Problem 1 - Column addition (of four numbers)
Problem 2 - Column addition and division (getting the average)
Problem 3 - Column addition (of four numbers)
Problem 4 - Getting the average of four numbers
Problem 5 - Column addition for six sets of four numbers; then identify the largest sum
Problem 6 - Column addition (of six numbers)
Problem 7 - Getting the average of six numbers
Problem 8 - Getting the average of four numbers
Problem 9 - Multiplication and addition

Assessment. On the school day following the treatment, *No Problem! Form B* was given as a posttest to the students in the experimental and comparison groups. Conditions of testing were identical to those described for the pretest. Refer to Appendix G for the script followed for the posttest administration. Posttest sessions were audiotaped.

A delayed posttest to assess the maintenance and generalization of skills was given to students in the experimental group 10 days following the final day of special instruction. This test was *No Problem! Form C* found in Appendix F. Again, conditions for testing were identical to pretest conditions. The script for teacher directions was identical to the script for the posttest.
CHAPTER IV

RESULTS

This chapter presents the data generated from the pre-, post-, and delayed posttests administered to students in the treatment and comparison groups. Mean scores were analyzed between groups and within groups to determine whether significant differences existed. Statistical tests of inference used in group comparison research are described. Graphic displays, used widely in single-subject research, are also presented, allowing the reader to look directly at the data and to make a visual analysis of change scores. In addition to examinations based on the number of errors made, an analysis of the types of errors that students made is presented.

Treatment and Comparison Group Scores

In order to show initial group equivalence in word problem solving ability, a pretest analysis was done. Pretest (Form A) scores were examined to assess similarities between the treatment and comparison groups prior to the intervention period of the research. Mean test scores, as presented in Table 5, demonstrated little variation in a) operation selection, b) answer
scores, or c) computation scores. (See Table 5 for pretest mean scores.)

Table 5

Pretest Mean Scores

<table>
<thead>
<tr>
<th>Score</th>
<th>Treatment Group</th>
<th>Comparison Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean / SD</td>
<td>Mean / SD</td>
</tr>
<tr>
<td>Operation Selection</td>
<td>9.39 / 2.83</td>
<td>9.93 / 2.99</td>
</tr>
<tr>
<td>Answer</td>
<td>7.17 / 4.49</td>
<td>7.40 / 3.79</td>
</tr>
<tr>
<td>Computation</td>
<td>10.33 / 4.17</td>
<td>10.00 / 3.07</td>
</tr>
</tbody>
</table>

Note. The computation score was not used to measure word problem solving ability.

Were the mean scores of students in the treatment group statistically different (at alpha level .05) from the mean scores of students in the comparison group before the intervention began? The null hypothesis was that there was no difference between the two groups at the onset. The alternative hypothesis was that there was a significant difference between the groups involved. To test the hypotheses, the t-test procedure was used, because this study had one independent variable, with two levels called
groups, and one dependent variable. The dependent variable—word problem solving ability—was measured by two skills (operation selection and answer score). Therefore, two separate t-tests were run for the two different scores. The homogeneity of variance assumptions for the use of t-test were met.

Students also earned computation scores for answering problems they wrote, even when the figures were not appropriate ones for the accompanying word problem. For this reason, computation scores were not used to assess word problem solving ability in this study. The computation score was identified and measured on the test solely to compare results with earlier findings (Ballow and Cunningham, 1982) concerning the difference between children's computation abilities and their problem set-up ability. (See Table 6 for t-tests on pretest scores.)

Table 6 shows that the null hypothesis was accepted for the groups at pretest. The probability of the observed sample value was greater than the maximum allowable risk at the .05 level of significance. There was no statistically significant difference in math word problem solving ability (based on operation selection scores and on answer scores) of students in the treatment group and students in the comparison group before the treatment period.
Although pretest scores were similar for the students in the two groups, differences were observed in word problem solving skills following intervention. For operation selection (the primary objective emphasized with the novel teaching method), youngsters in the treatment group made a mean score gain of 2.61 points (28%) on the posttest (from 9.39 to 12.0). In contrast, the mean operation selection score for students in the comparison group dropped slightly, from 9.93 to 9.36, a difference of .6 mean score points (-6%) from a total possible of 15.

Table 6

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
<th>SE</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>18</td>
<td>9.39</td>
<td>2.83</td>
<td>0.66</td>
<td></td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.60</td>
</tr>
<tr>
<td>Comparison</td>
<td>15</td>
<td>9.93</td>
<td>2.99</td>
<td>0.77</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Answer |    |      |     |     |     |       |
| Treatment | 18 | 7.17 | 4.49| 1.06|     | 0.16  |
|          |    |      |     |     |     | 0.87  |
| Comparison | 15 | 7.40 | 3.79| 0.98|     |       |

\( \alpha < .05 \)

Note. The assumptions for the use of a t-test were met.
Although computation skills were not directly taught with the novel instructional model presented to students in the treatment group, their answer scores and computation scores also rose. The answer score mean increased from 7.16 to 10.22 (43%) at posttest, and the computation score increased from 10.33 to 13.22 (28%) at posttest.

For students in the comparison group, the correct answer was also written more often, from a pretest mean of 7.40 to a posttest mean of 7.50 (up 1%). The mean computation score remained unchanged (11.0) from the pretest to posttest examinations for students in the comparison group.

Following treatment, were the mean scores of students in the experimental group statistically different from the mean scores of students in the comparison group? Were differences greater than would be expected by chance alone? The null hypothesis was that there was no significant difference (at alpha level .05) between the means of the groups following treatment. The alternative hypothesis was that there was a statistically significant difference between the group means following treatment. As before, the t-test procedure was used to test the hypotheses about differences between the group means. Two separate t-tests were run; one on "operation selection" mean scores, and the other on the "answer" score. These two scores were used to measure the dependent variable—word problem solving ability. The computation score was not used to assess word problem solving in this study. (See Table 7 for t-tests.)
Table 7

Posttest Scores: t-tests

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
<th>SE</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Operation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>18</td>
<td>12.00</td>
<td>2.85</td>
<td>0.67</td>
<td>*-2.32</td>
<td>0.03</td>
</tr>
<tr>
<td>Comparison</td>
<td>14</td>
<td>9.36</td>
<td>3.61</td>
<td>0.96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **Answer** |    |      |     |     |       |       |
| Treatment  | 18 | 10.22| 3.37| 0.79| *-1.99| 0.06  |
| Comparison | 14 | 7.50 | 4.38| 1.17|       |       |

* $\alpha < .05$

**Note.** The assumptions for the use of a t-test were met.
Table 7 shows that the null hypothesis was rejected for the groups at posttest. The probability of the observed sample value was smaller than the maximum allowable risk at the .05 level of significance. There was a statistically significant difference in math word problem solving ability of students in the treatment group and students in the comparison group after the intervention period.

Ten days following the posttest, students in the treatment group took a delayed posttest to determine if skills were retained. All mean scores on the delayed posttest declined from posttest levels, although scores remained slightly higher than pretest levels.

Comparing pretest to delayed posttest mean scores, levels of competence increased 6% for operation (O) selection (9.39 - 10.0 mean score); increased 11% in finding the correct (A) answer (7.16 - 7.92 mean score); and increased 7% in computation (C) skills (10.33 - 11.07 mean score). (See Table 8 and Figures 1, 2, and 3.)
Table 8

Comparison of Change Scores

<table>
<thead>
<tr>
<th>Score</th>
<th>Pretest Means</th>
<th>Posttest Means</th>
<th>% Change</th>
<th>Delayed Posttest Means</th>
<th>% Change From Pretest Means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment Group</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>9.39</td>
<td>12.00</td>
<td>+28%</td>
<td>10.00</td>
<td>+ 6%</td>
</tr>
<tr>
<td>A</td>
<td>7.16</td>
<td>10.22</td>
<td>+43%</td>
<td>7.92</td>
<td>+11%</td>
</tr>
<tr>
<td>C</td>
<td>10.33</td>
<td>13.22</td>
<td>+28%</td>
<td>11.07</td>
<td>+ 7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comparison Group</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>9.93</td>
<td>9.35</td>
<td>- 6%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>7.40</td>
<td>7.50</td>
<td>+10%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>11.00</td>
<td>11.00</td>
<td>UNC</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. O = Operation selection score

A = Answer score

C = Computation score
Figure 1. Operation selection score comparison.
Figure 2. Answer score comparison.
Figure 3: Computation score comparison.
There was a noticeable decline in mean scores from posttest to delayed posttest. In order to examine score differences statistically, Duncan's multiple range test procedure for ANOVA was performed. The operation selection scores and the answer scores within the treatment group were analyzed. While the decline in scores at the delayed posttest was noticeable, ANOVA procedures indicated that the mean score differences were not significant. For students in the treatment group, the means for operation selection and answer scores were not statistically different from posttest to delayed posttest. Nevertheless, scores of students dropped when there was a time lapse following treatment. See Tables 9 and 10.

Table 9

Anova: Within-treatment group answer scores

Posttest vs. delayed posttest

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4.32</td>
<td>1</td>
<td>4.32</td>
<td>0.24</td>
</tr>
<tr>
<td>Error</td>
<td>529.42</td>
<td>29</td>
<td>18.26</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>533.74</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 10

*Within-treatment group operation scores*

**Posttest vs. Delayed Posttest**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>30.19</td>
<td>1</td>
<td>30.19</td>
<td>2.92</td>
</tr>
<tr>
<td>Error</td>
<td>300.00</td>
<td>29</td>
<td>10.34</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>330.19</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In summary, the statistical measures revealed significant differences across groups in operation selection and answer score, comparing posttest score means. Students taught by the direct instruction method outperformed students taught by the traditional method in selecting the appropriate operation for solving one-step addition and subtraction word problems, and in calculating to obtain the correct answer. The difference in mean scores was statistically significant. With 95% confidence, the results indicate that the treatment accounted for the difference.

Skills gained by students in the treatment group were not maintained over time. Mean scores at delayed
posttest administration were lower than mean scores at posttest. Despite the decline, delayed posttest scores remained slightly higher than pretest scores.

Error Analysis

Knowledge of such raw data enables an instructor to compare scores of one student or group with another, and can yield information on the mastery level of subject matter in general terms. However, scores alone are of little help for specifying particular skills which must be honed. In order to design individualized instructional programs for special students, teachers need to know exactly what errors students made, in addition to knowing that errors were made. As Enright (1989) stated, "The incorrect answers that students provide to ... assessment items yield important information that points to the procedures for correcting skill needs in mathematics" (xiii).

To accomplish an in-depth assessment of student performance, an error analysis was completed from data gathered in this study. Error analysis has been defined as "the detailed inspection of a student's incorrect responses on a work sample to ascertain if a pattern of errors exists that is contributing to the wrong answers" (Howell & Morehead, 1986; Siegler, 1983). Conventional wisdom holds that errors are not just the opposite of correct responses; that most errors do not occur randomly
but in recognizable and correctable form (Howell & Kaplan, 1980; Howell & Morehead, 1986). Having identified patterns of mistakes, an instructor can then establish a remedial mathematics program within the classroom.

The identification and classification of errors made by students in this study was conducted to point out weak skill areas to classroom teachers, and to provide the reader with specific information related to sources of student errors while finding answers to word problems. Additionally, by examining errors by group and by test, conjectures might be made about the direction of errors and the possible impact of the treatment on computing skills.

An "0" score on each test counted the frequency of errors made because students chose the wrong operation. Other errors were made in computation or in the "set-up" of the numbers. The researcher examined those other mistakes made on the measurement instruments of students in the treatment and comparison groups, and categorized them. In addition to having three different raw scores noted on each test, each missed problem was labeled according to obvious error sources. The codes used by the researcher on the test instruments are identified, along with the descriptions that match:

Students did not borrow (DB) when necessary, but subtracted digits in a column without regard to the
position of the largest number. Note an example:

\[
\begin{align*}
9,461 \\
- 8,642 \\
1,221
\end{align*}
\]

In some cases where numbers were spelled, rather than written in Arabic form, faulty reading (R) translations resulted in incorrect numerals printed in the answer boxes. Students often guessed a number based on the initial consonant sound of the word. Some examples follow:

<table>
<thead>
<tr>
<th>PRINTED IN TEXT</th>
<th>STUDENTS WROTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>five</td>
<td>4</td>
</tr>
<tr>
<td>twelve</td>
<td>20</td>
</tr>
<tr>
<td>eleven</td>
<td>8</td>
</tr>
<tr>
<td>eight</td>
<td>18</td>
</tr>
<tr>
<td>thirteen</td>
<td>30, 3</td>
</tr>
<tr>
<td>forty-seven</td>
<td>56, 46, 7, 57</td>
</tr>
<tr>
<td>fourteen</td>
<td>15</td>
</tr>
<tr>
<td>firefighters</td>
<td>55</td>
</tr>
</tbody>
</table>

When spellings were not a factor, with all variables printed in Arabic form, students sometimes transcribed (T) the numbers incorrectly from the text to the box. Reversals were included in this category, as when a youngsters wrote 51 for 15.

Other errors resulted when students did not align
numbers (A) properly when setting up a problem,

\[ 87 + 146 \]

or when they placed the largest number (L) on the bottom and attempted to subtract:

\[ 47 - 204 \]

\[ 243 \]

Within one problem, some mixed (M) operations, adding numerals in one column while subtracting numerals in the adjacent column. Some circled one sign (S), but performed a different operation.

\[(M) \ 18 + 17 \]

\[21\]

\[(S) \ 9, 461 - 8, 642 \]

\[18, 103\]

Careless handwriting (H) made it difficult for some to read their intended notations. Others wrote the correct numbers for the situation, but didn't recognize and write answers to the basic addition and subtraction facts (F).

Two types of cases were observed when students did not carry when appropriate. Some placed two numerals in one column of the sum, for example:
Others who did not carry omitted the number to be carried altogether:

\[
\begin{align*}
54 + 28 &= 72 \\
\end{align*}
\]

The numerical code "0" indicated that students were stumped with zeros in the minuend when borrowing was necessary. A common attempt when borrowing from the adjacent column that contained a zero looked like:

\[
\begin{align*}
10 & \quad 10 \\
3 & \quad 0 \quad 0 \quad 7 \\
\underline{2642} \\
1465 \\
\end{align*}
\]

"P" indicated a partial listing of the data in a problem, while "B" designated a blank answer box. In most instances of blank boxes, it appeared that youngsters did not attempt to solve a problem. For a few, time simply ran out. Other blank boxes had notations that had been erased by the students.

The final category of errors was identified with the letter "O" for other. Included in this category were instances when students borrowed from a number, and replaced that "host" with a larger number, such as:
Others borrowed or carried when there was no need, subtracted for a three digit column addition problem, or wrote the problem while they did not solve it. Errors that had no obvious rationale were also included in this "0" category. As an assessment of the scope of the different sources of error, and the test occasion for each mistake, the number of occasions for each mistake are listed in Tables 11 and 12. Note that the percentage of operation selection errors was based on the total possible operation points, while the percentages for the other errors were based on the total points possible for answer scores and computation scores for pretests and posttests. (See Tables 11 and 12 for data on student errors.)
<table>
<thead>
<tr>
<th>Type</th>
<th>Treatment Group</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>%</td>
</tr>
<tr>
<td>* Operation selection</td>
<td>101</td>
<td>37%</td>
</tr>
<tr>
<td>DB: Did not borrow</td>
<td>12</td>
<td>14%</td>
</tr>
<tr>
<td>B: Blank box</td>
<td>26</td>
<td>30%</td>
</tr>
<tr>
<td>F: Facts not known</td>
<td>11</td>
<td>13%</td>
</tr>
<tr>
<td>O: Other various errors</td>
<td>16</td>
<td>19%</td>
</tr>
<tr>
<td>S: Chose one sign, performed another</td>
<td>4  5%</td>
<td>8 11%</td>
</tr>
<tr>
<td>L: Wrote largest number on bottom to subtract</td>
<td>4  5%</td>
<td>3  4%</td>
</tr>
<tr>
<td>R: Reading difficulties</td>
<td>4</td>
<td>5%</td>
</tr>
<tr>
<td>DC: Did not carry</td>
<td>4</td>
<td>5%</td>
</tr>
<tr>
<td>A: Wrong alignment</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>M: Mixed operations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>within one problem</td>
<td>2</td>
<td>2%</td>
</tr>
<tr>
<td>Ø: Zeros in minuend</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>T: Transcription</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>H: Handwriting</td>
<td>1</td>
<td>1%</td>
</tr>
<tr>
<td>P: Partial data listing</td>
<td>1</td>
<td>1%</td>
</tr>
</tbody>
</table>

Note. Operation errors based on total operation points
### Table 12

**Analysis of Errors on Posttest**

<table>
<thead>
<tr>
<th>Type</th>
<th>Treatment Group</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>%</td>
</tr>
<tr>
<td>* Operation selection</td>
<td>50</td>
<td>22%</td>
</tr>
<tr>
<td>DB: Did not borrow</td>
<td>4</td>
<td>10%</td>
</tr>
<tr>
<td>B: Blank box</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>F: Facts not known</td>
<td>9</td>
<td>22%</td>
</tr>
<tr>
<td>O: Other various errors</td>
<td>4</td>
<td>10%</td>
</tr>
<tr>
<td>S: Chose one sign,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>performed another</td>
<td>3</td>
<td>8%</td>
</tr>
<tr>
<td>L: Wrote largest number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>on bottom to subtract</td>
<td>2</td>
<td>5%</td>
</tr>
<tr>
<td>R: Reading difficulties</td>
<td>6</td>
<td>15%</td>
</tr>
<tr>
<td>DC: Did not carry</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>A: Wrong alignment</td>
<td>2</td>
<td>5%</td>
</tr>
<tr>
<td>M: Mixed operations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>within one problem</td>
<td>2</td>
<td>5%</td>
</tr>
<tr>
<td>Ø: Zeros in minuend</td>
<td>6</td>
<td>15%</td>
</tr>
<tr>
<td>T: Transcription</td>
<td>2</td>
<td>5%</td>
</tr>
<tr>
<td>H: Handwriting</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>P: Partial data listing</td>
<td>0</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Note.** Operation errors based on total operation points.
At pretest, the skill most problematic for youngsters in the treatment group and in the comparison group was in selecting the operation to use. In the treatment group, 37% of the answers were wrong because students added when they should have subtracted, or subtracted when they should have added. In the comparison group, 40% of the answers were wrong for the same reason. The second most prevalent type of error was counted because students in the treatment group did not complete a problem, leaving boxes blank. In contrast to the treatment group, for the comparison group, the second most prevalent source of errors occurred because students did not know basic addition and subtraction facts.

In order of frequency, the remaining top sources of errors made by treatment group students fell in the following categories: "Other various errors" (19%), "Did not borrow" (14%), and "Facts not known" (13%). In addition to incorrect operation selections and facts not known, comparison group students made most errors because they did not borrow when it was necessary (19%), made other various errors (18%), chose one sign but performed another (11%), and left blank boxes (11%). Refer to Table 11 for other sources of errors that resulted in incorrect answers to addition and subtraction word problems at pretest.

For the specific skill directly taught in the
treatment—operation selection—students in the treatment group improved by 15%. Following the traditional method, students in the comparison group improved in operation selection by 5%. Although the novel teaching method did not provide instruction or practice time in how to compute, i.e., the mechanics of addition and subtraction, the number of errors in computation dramatically decreased among students in the treatment group. At the posttest, the number of computation errors dropped 53% overall for the treatment group, compared to the overall drop of 14% less computation errors in the comparison group. Compare pretest scores in Tables 11 to posttest scores in Table 12.

The delayed posttest that was given to students in the treatment group revealed that student errors at that time covered a smaller range of categories than errors made on pretest or posttest instruments. Nine categories of errors made by treatment students were observed, as opposed to 12 categories of errors by the treatment students at pretest and 11 categories of errors at posttest. In addition to operation selection, the categories of errors included "blank boxes" (25% of errors), "other various errors" (25%), "did not borrow" (15%), "facts not known" (15%), "chose one sign, performed another" (6%), "wrote largest number on bottom to subtract" (6%), "reading difficulties" (6%), and "handwriting" (2%).
It is considered noteworthy that fifty-four percent of the blank boxes during the delayed posttest were indicated for one student who was still working on his test at the close of the period. Students from other groups who had left blank boxes at pretest and posttest had considered themselves finished, based on the fact that they completed other problems that appeared later in the instrument, and had begun working on the math puzzles as directed for early finishers. In contrast, the treatment group student stayed on task with the test for the entire period, but completed only the first 8 out of the 15 problems.

Out of 195 raw score points possible in operation selection, the treatment group made 65 (33%) errors. The percentage of errors in selecting the correct operation was a slight improvement over the percentage of errors in operation selection (37%) at pretest.
CHAPTER V

DISCUSSION

This final chapter briefly reviews the purposes of the investigation and analyzes results. The limitations of the study are outlined, followed by a discussion of implications of the findings—their meaning and significance. Suggestions for further research and follow-up studies which might be conducted in the future to advance knowledge in the field of teaching mathematics to students with specific learning disabilities are presented.

Objective 1

There were three objectives guiding this investigation. The first objective was to determine whether the novel approach for teaching word problem solving would make a difference between the treatment group of students with learning disabilities and the comparison group of students with learning disabilities. Would instruction using a generalizable graphic representation during directed instructional session be positively related to problem solving abilities?

The results of the investigation supported this hypothesis. Following nine consecutive days of a novel direct instruction method, students with math learning
disabilities in the treatment group made significantly better scores than did similar students of a comparison group in word problem solving. Not only did the treatment group students outperform comparison youngsters in choosing the correct operation, but they also made superior scores in deriving correct answers in their calculation of the problem. These findings were consistent with other studies involving the use of direct instruction teaching to skill-deficient youngsters. For instance, Darch, Carnine & Gersten (1984) studied the effectiveness of a direct instruction method that taught 73 skill-deficient fourth graders how to translate word story problems into mathematical equations. Posttest results indicated a significant positive effect for the direct instruction model, as opposed to a traditional approach. In another model, Moore and Carnine (1989) found success with what they termed ATCD, for active teaching with empirically validated curriculum design. The ATCD curriculum was based on the principles of direct instruction by Engleman and Carnine (1982). In the Moore and Carnine (1989) study, high school students in general education classrooms were taught for 20 fifty minute sessions. The ATCD group received explicit instruction in problem solving. They were taught explicit rules and strategies, were presented with a range of examples, and were presented with examples and non-examples for discrimination practice. A comparison group followed active teaching with basals (ATB). The main difference
between the group treatments was the presentation of explicit strategies in the ATCD group. The ATB group was taught several procedures for solving different word problems, but these procedures were not broken down into small steps. Other researchers who used direct instruction procedures successfully for teaching math skills to low-achieving students include Engelmann (1980); Carnine (1981); Gersten & Carnine (1984); Darch & Stein (1986); and Sommers, 1991).

Specific features of the novel approach in the present study may be explored individually as probable factors for the differences in posttest scores between groups. In addition to the components of direct instruction, this model presented a unique method of interpreting a problem, was presented in nine consecutive school days, was limited to addition and subtraction word problems, and employed a detailed script for the teacher.

**Use of the Arrow.** The consistent use of the arrow, with its accompanying rules, might have been the feature that enabled students to decode a word problem to properly add or subtract. Students only had to memorize and internalize two simple rules. They learned how to position the numbers contained in word problems around an arrow so that they would be able to determine which sign to use. The positioning of the numbers actually was an exercise in determining the relative size of the numbers in a problem situation. The complex skill of critical
thinking was broken down into simple steps. Students were exposed to a feature of consistency. An arrow was always drawn first after the reading of a story. No circles, lines, or other marks were drawn at times. No objects were manipulated at other times. No steps were outlined in list form and followed. No search ensued for key words or phrases. An arrow was drawn first, no matter how a story problem read. The issue of sameness may have aided students who typically have short term memory weaknesses. The issue has been reiterated by Bley and Thornton (1989) who have urged teachers to capitalize on patterns or other associations that reinforce retention for children with learning disabilities.

Schedule of Consecutive Days. The concentration of attention in a nine-day unit schedule must also be considered. Following the treatment, students in the experimental group spent more time reading and solving word problems on the posttest than did students in the comparison group, even though both groups were given identical test directions, and an identical number of minutes in which to work. Perhaps the previous focus on word problem solving, to the exclusion of all other math skills, resulted in students being more attentive, conscientious, or confident about their ability. Much can be speculated about the efficacy of the continuous schedule of lessons in the treatment program versus the intermittent schedule for presenting word problems that the traditional
program outlined. Children with learning disabilities are often characterized by deficiencies in retaining previously acquired skills. For this reason, it has been important for teachers to provide frequent and distributed practice following skill acquisition. A consistent schedule for reviewing and reteaching material has helped to ensure that important skills were not lost over time. The emphasis on word problem solving in the treatment group, with nine consecutive days of instruction, may be considered a factor in the superior performance of those students in selecting the correct sign with which to compute.

**Combination of Addition and Subtraction Problems.** Because daily exercises included both addition and subtraction word problems, students in the treatment group were forced to make discriminations with each example. Contrary to most basal texts, entire pages of exercises were not restricted to problems that required the same operation for their solution. This structuring may have induced students to read and think more carefully than students in the comparison group.

**Detailed Script.** Another factor which may have influenced differences in performance with the two groups was the use of a script versus the use of a manual for instructors. The script was carefully sequenced to maximize comprehension of subject matter. It detailed every verbal and nonverbal interaction with the students. In contrast to the detailed script which was followed by
the researcher, comparison group classroom special educators followed a manual which only presented guidelines for the instructional sequence. The interpretation and execution of those guidelines might have varied widely, since directions for teachers following the traditional approach were not as explicit as those for the researcher.

**Direct Instruction Components.** The use of a sequenced script is one of the components of the direct instruction model. In addition to the structured script, the rapid pacing of lessons, the many opportunities for students to respond, the feedback and correction given frequently and specifically, and the maximum use of instructional time reflect the principles of the direct instruction method. For special populations, a structurally oriented approach has been needed to facilitate learning. This model reflects the principles of behaviorism which have been successfully applied to improve academic achievement of such youngsters. Skills are broken down into manageable steps that can be reached in a reasonable amount of time.

**Objective 2**

At the onset of the study, the researcher sought to investigate whether skills learned from the novel teaching strategy for solving single-step addition and subtraction word problems would be maintained over time.
The results of the delayed posttest for students in the treatment group showed that they did not maintain and apply skills learned from the treatment model at the same level of accuracy as they demonstrated at posttest, once the special instruction had ceased for a span of 10 days. The mean operation selection score and mean answer score of treatment group students declined at the delayed posttest administration. Delayed posttest scores did remain above pretest levels for operation selection and answer scores.

The lack of skill maintenance for students taught by direct instruction is not consistent with prior research studies. See, for instance, studies by other researchers in comparable investigations (e.g. Darch, Carnine & Gersten, 1984 and Moore & Carnine, 1989). A number of factors in this investigation may have contributed to score decline after the time lapse following treatment.

Timing of the Delayed Posttest. Scheduling the delayed posttest on the last day of the school year might have resulted in numerous errors that were situation-induced. The administration of the delayed posttest might have been under the worst possible conditions. A day earlier, or a week earlier might have found very different scores.

Latency Period. Also, the lack of a period of distributed practice following the treatment may have been a factor. None of the studies that have used similar
procedures indicated what, if anything, might have been done with students in the interim to maintain skills. Under typical circumstances, classroom teachers frequently review new material with students after its initial presentation. Teachers often remind students informally, or prompt them with guided questions about new subject matter. Throughout the school day in various activities, students may have opportunities to interact with newly learned skills. Review exercises may include such diverse activities as having each student give a "password" (answer to a skill check question) to get in line for lunch, assigning tasks for cooperative groups, or simply asking related questions on a frequent basis.

Objective 3

The third objective of this study was to analyze the type of errors that were made by students in both the treatment and comparison groups, to help determine whether types of errors were altered as a result of the novel teaching approach. The error analysis revealed that students in the treatment group not only improved in their ability to choose the correct operation to solve a word problem, but they also improved in computation skills.

Following treatment, the number of computation errors, and also the range of error categories was reduced for students who participated in the treatment. The improvement in computation skills was an unanticipated gain,
since the novel approach included no instruction, correction, or practice time with basic computation procedures. It may be speculated that the students in the treatment group gained confidence in word problem solving ability, and approached the testing situation without the impulsive behavior that often is characteristic of students with specific learning disabilities.

Limitations

Additional or alternative arrangements might have yielded stronger results. Issues include the length of the treatment period, the time lapse before the delayed posttest, the timing of the delayed posttest, the scheduling of word problems in the traditional approach, the lack of basic skill review in the treatment approach, and the administration of a delayed posttest to one group.

Length of Treatment Period. The treatment period was limited to 9 days of instruction. Considering the scope of skills taught, and considering the novelty of the approach, the time allocated for this intervention may have been inadequate. Additional periods in the classroom would have enabled the researcher to provide individualized assistance to students who required extra help, and would have enabled all students to rehearse skills and interact with the material and instructor to maximize retention of learnings.
Time Lapse Before Delayed Posttest. The delayed posttest was administered to students in the treatment group 10 days after the posttest. This time lapse duplicated the delay in previous studies with special populations (e.g., Darch, Carnine & Gersten, 1984 and Moore & Carnine, 1989). For students in mainstream educational programs, a 10 day delay may not have been sufficient to test the maintenance of skills learned. Other investigators may question whether the 10-day delay was sufficient to test skill maintenance with students having learning disabilities. That is, if tested as much as two months after instruction, would test behaviors have eroded to baseline levels? Perhaps a real issue is the need to put in place strategies that help ensure the maintenance of skills acquired of learners with learning disabilities.

One suggestion is to encourage teachers to begin daily math classes with one or two word problems, using situations designed around the student names and activities of particular interest to them. One problem might be demonstrated by the teacher to the whole group. The second problem might be the challenge to individual or team workers.

To rehearse crucial rules that enable students to decide which operation to use, chanting in unison in a rhythmic manner each period might make the facts more retrievable. In addition, self-correcting flash cards
should be available for independent study. Public posting
the names of classroom members who are mastering basic
addition and subtraction facts has been an effective
strategy for motivating reluctant learners to pursue
additional skills (VanHouten, 1980).

Teachers may also need to restructure worksheets so
that different required operations are represented on a
single page of word problems. Students will be required to
be more discriminating readers by such an arrangement. The
current organization of basal mathematics textbooks does
not challenge students to be discriminating readers of word
problems.

Timing of the Delayed Posttest. An additional
consideration was that students in the treatment group took
the delayed posttest on the last day of the school year.
Numerous distractions—typical of the last day of
school—might have induced many of the errors made on the
final instrument. One student from the resource room was
dismissed from the exercise because he simply could not
concentrate on a test while his regular class peers were
watching a special video. His resource room teacher's
unexpected and unannounced absence on the same day was
another consideration. Throughout the building, activities
included special games, parties, and visitors. On a
noncognitive level was the excitement and anticipation of
summer vacation.
Scheduling of Word Problems in Traditional Group.
The two approaches widely differed in terms of the number of days that word problems were taught. The comparison group received only 3 days of instruction on word problems during the treatment period, as opposed to 9 days of continuous instruction on word problems for the treatment group. For the comparison group, the 3 days were scattered intermittently during the 9-day period.

Lack of Basic Skills Review. The novel instructional approach did not allow the researcher to control for number fact deficits, or for reading and language deficits. Readiness skills were not preassessed or reviewed in preparation of the more advanced skills involved in word problem solving.

Administration of Delayed Posttest to One Group.
The delayed posttest might have been administered to students in the comparison group in order to determine comparable competence in word problem solving following a time lag in instruction. The present study used the delayed posttest to investigate the treatment method. This limited the analysis of scores from the delayed posttest to within-group comparisons. Had the students in the comparison group also been administered the delayed posttest, between-group comparisons on skill maintenance may have been made as well.
Implications for Classroom Teachers

The results of this study support other investigations which imply that children with learning disabilities can be taught to be skilled and more analytical in their approaches to word problems. From the study, observations have resulted in recommendations for special educators, textbook authors, and program planners.

Worksheet Designs. When designing worksheets for students with specific learning disabilities, it is recommended that generous space be allowed between and around problems. Day three worksheets for the treatment group were troublesome for students because computations could often not be confined to the area provided. The same page layout that regular populations had used in field tests successfully, was an issue with these special children.

Familiarity with Script. Thorough review of the script prior to presentation was a prerequisite. Because of the familiarity with the script, the researcher was able to make frequent visual exchanges with students. However, reading before the students felt awkward. Constant eye contact was the learned and preferred modus operandi. Nevertheless, in spite of the concerns about reading a full script before students, the regularly assigned classroom teachers in all participating schools commented that students did not seem to notice or mind.
Necessity of a Detailed Script. The script was a helpful prompt and necessary guide to ensure a standard text for duplication of efforts for research purposes. However, when working with more than one classroom of students, the researcher found that different groups necessitated modifications in pacing. The verbatim script, at times, impeded the sequence of verbal interchanges that might have resulted in more clarity and comprehension of content. Rather than a detailed script which addressed each teacher and student response, an abbreviated text which outlined key objectives would have been more practical and effective for a teacher who was skilled in the techniques of direct instruction, and who was knowledgeable of the objectives and techniques in the Connecting Math Concepts approach to word problem solving.

Choral Responding. The concept of choral responding was well received by the students and their teachers. Students were eager to tell members who were absent on the presentation day "how the clicker worked," i.e., how students were to answer in unison after the sound. Several asked to see the clicker and requested to click it once. Their teachers noted that the students were attentive and enjoyed speaking with the group after the clicker. Some said that they planned to use the technique with their classes in the future.
The researcher was similarly convinced of the effectiveness of an audible signal to precede choral responding with classes of students. The direct instructional model, which included choral responding, enabled students to interact frequently with the content. With high rates of questioning throughout the presentation, students stayed focused. Also, judging from the facial expressions, students had fun in the process. The inclusion of frequent questions during the presentation stage of a lesson, with the requirement of choral responses by students, is highly recommended.

Areas for Remediation. In addition to choosing the wrong operation to solve a word problem, students made errors in a number of skill categories necessary for successful word problem solving. The quantity of categories lends credence to the complexity of steps and skills required for word problem solving. Implications for remedial instruction in mathematics may be derived by examining the list of errors presented in Table 15. For instance, many students did not have instant recognition of basic addition and subtraction facts. They were observed counting on their fingers or counting on areas of the printed numbers. It is recommended that teachers help students master this basic objective. Activities might include the provision of daily timed drills, the posting of progress charts, the enlistment of parents and/or tutors to help in home enrichment sessions, and
instructional games for review.

An additional concern was based on the observation of students writing the greater number below the minuend. Such improper problem writing indicated a lack of understanding in the basic concept of subtraction. It is recommended that structured worksheets be designed to help students focus on correct and incorrect models. It is recommended that a criterion level of mastery be the immediate visual recognition (at 100% accuracy) of subtraction problems set up properly— with the minuend larger or equal in value to the subtrahend. The mechanics of borrowing or regrouping should be introduced only after this concept is firm.

Classroom Management. A behavior management system was a prerequisite to instruction with classrooms of students having specific learning disabilities. In one treatment classroom, a clearly defined system of rewards and consequences was in place prior to the treatment. The researcher followed the model, with the addition of individual rule cards, and as able to instruct, giving little attention to redirecting behavior. In the second treatment classroom, the researcher found it necessary to establish a simple program of rules and consequences. With the program in place, the harder-to-motivate became more involved. Instructional sessions flowed with students spending more time on task than before the behavior management system.
The superior performance of students in the treatment group warrants the attention of teachers and researchers who are interested in strategies that help children learn. The most critical step toward the solution of a dilemma must be deciding what to do with data. In the skill of choosing an operation, treatment group children significantly outperformed children taught by a traditional method. One who would choose to compile figures in a problem that calls for decrease lacks a most fundamental understanding of a situation. For such a one, computation or calculator skills could be likened to tools in the hands of an unskilled layman. Mathematical skills are connected to real life experiences. We must empower all students to see and make the connections.

Although students in the treatment group scored higher in operation selection than those in the comparison group, it was noted that not all drew the arrow and placed numbers on test forms as the instructional model had been presented. On the first posttest, 56% of the students made drawings for some or all of the word problems, while 25% of the students made drawings on the delayed posttest forms. During the instructional phase, arrows were drawn for each of the problems as an important first step. It is doubtful whether the illustration with proper number placements could have been internalized by the students considering the time span of the treatment. Perhaps the attention on the skill resulted in students working more carefully.
Students in the treatment group did spend more minutes on examinations following the intervention stage than they had during pretest (13 minutes versus 20 - 25 minutes), and beyond the average number of minutes (14 - 16 minutes), spent by students in the comparison group on the posttest. One student in the treatment group was still reading and working at the bell on the delayed posttest, having completed only 8 problems. The same student had finished early during previous tests.

Summary of Procedures and Findings

The purpose of this study was to investigate the effectiveness of a novel teaching method for helping students solve one-step math word problems. The subjects were elementary level students with specific learning disabilities in mathematics. Two intact classrooms for students with learning disabilities comprised a treatment group which was taught by the researcher. One of these classrooms was a self-contained unit for children with learning disabilities, while the other classroom was a resource room of children with learning disabilities. Two additional intact self-contained classrooms for students with learning disabilities comprised the comparison group. The regularly assigned special education teachers instructed their students in the comparison group classrooms. The period of treatment covered a span of three weeks. Instruction was given on 9
consecutive school days.

To determine initial group equivalence, a pretest was administered prior to the treatment period. After the treatment, all students were administered a posttest on word problems that required either addition or subtraction, and which had one-step solutions. The tests generated two scores related to word problem solving ability—an operation selection score, and the answer score. Following the example of two previous studies (Darch, Carnine & Gersten, 1984 and Moore & Carnine, 1989), a delayed posttest was given 10 days after the posttest with the experimental group to see how skills were maintained.

The novel teaching method followed the direct instruction format. The uniqueness of the strategy was primarily attributed to the use of an arrow around which numbers in a problem were to be positioned. The strategy was based on a technique developed by Engelmann and Carnine (1992), and has been outlined in a new basal math series called *Connecting Math Concepts*. Through a series of carefully sequenced lessons, students in an experimental group learned how to position numbers in word problems around an arrow, and subsequently how to determine if a problem situation required addition or subtraction. The use of the arrow was applied to all types of addition and subtraction word problems: those labeled action problems, problems of comparison, or classification problems. The
strategy did not require students to make any other drawings, nor to manipulate objects. Similarly, the strategy did not present or isolate key words or phrases within story problems. The objective of the novel teaching method was to enable children to interpret word problems so that they would appropriately add or subtract. Drills on basic math facts were not included. Related homework assignments were not given. The lessons were whole group presentations which took place in the regular math classroom of the special students, during the regular math period.

During the treatment period, a comparison group of learning disabled students were taught math in a traditional method according to the Addison-Wesley Math Program. In contrast to the treatment group, students in the comparison group encountered three days of instruction which were devoted to math story problems. This exposure was in synchrony with the sequence of lessons regularly assigned from the text, and followed the objectives of the regular (traditional) math program. For word problems, comparison group students were aided with a 5-Point Checklist that highlighted the essential steps for completing any story problem. At the beginning of each lesson, the teacher modeled the method for working a sample word problem. Students then solved a page of exercises independently. Help was given with unknown words. During the other six days of treatment, students in the
comparison groups practiced additional math skills, namely, finding the area of an object, getting the volume, having zeros in the quotient, dividing money, finding averages, reviewing subtraction, completing long divisions (with and without remainders), and identifying polygons.

There were some features that the two instructional models had in common. Both groups received 45 minutes of math instruction each day. The use of a manual guided teachers in the comparison groups, and the use of a script guided the researcher in the treatment groups.

Pretest scores indicated that the children in the treatment group were similar to the children in the comparison group in word problem solving ability prior to intervention. Mean pretest scores between groups were not statistically different.

Following the treatment, students in the experimental group outperformed those in the comparison group on both measures of word problem solving ability. They chose the correct operation more often, and found the correct answer to the problems more frequently than did students in the comparison group. Both scores were found to be statistically different for the two groups.

Scores for students in the treatment group dropped on the delayed posttest. The delayed posttest scores, however, were higher than the treatment students' pretest scores. The lower scores at delayed posttest were not statistically different from posttest scores,
however. The results indicate that for this investigation, hypothesis 1 was supported—instruction using a generalizable graphic representation during directed instructional sessions was positively related to problem solving ability. Hypothesis 2 was not supported. Skills learned from the novel teaching strategy were not maintained over time.

**Future Studies**

This study was limited to one-step word problems that involved only subtraction or addition skills. Engelmann and Carnine (1992) have modified the arrow for more complex problems that require multiplication or division for solutions. Future studies might investigate the effectiveness of the program with other special populations, with other age levels, when more than a single step is required, or when students must choose between the four basic operations.

This alternative model appears to have the potential of helping even the most difficult-to-teach. In view of the gains made by students in the treatment group in choosing the correct operation, future studies might investigate results when students have worked during a longer intervention period, with more opportunities for practice and feedback. Few educators have disagreed with the notion that special learners require many successful repetitions before skills are mastered. Enright (1989)
suggested that periodic review needed to be done with special learners as often as every few days. With that consideration, future studies might investigate the effectiveness of different schedules of instruction: Following the same instructional model, would students retain more when presented information in a continuous sequence of lessons (e.g., an intensive 10, 20, or 30-day unit) versus when lessons come with intermittent frequency? For word problems, in most basal programs, including the novel approach in its original form, an intermittent end-of-unit schedule had been outlined. Would student performance have differed significantly if a continuous schedule of lessons in word problems had been presented to students in the comparison group, following the traditional program?

Conclusions

A special strategy has been developed to help students with the difficult task of solving math word problems. In a relatively short time frame, students with learning disabilities who were taught to use this strategy made significant improvements in their ability to decide on an operation—a most critical step in the word problem solving process. Treatment group students outperformed students with learning disabilities who were instructed according to a traditional method. The results of this investigation must be noteworthy to teachers who have
self-reported insecurity in teaching students how to solve word problems. (This instructional procedure included a complete script with directions and student activity sheets that could be implemented with little prior training.) The results must be worth noting because students with significant cognitive difficulties became more successful in performing a complex and crucial task. Can one argue with Enright's (1989) summation that "(...) problem solving is the central focus of the mathematics curriculum (and) all other mathematics skills should be learned to facilitate the solving of problems" (p. xiii)?

Cawley (1984) reminded us that from both the teaching and research perspectives, there are numerous alternatives: "Whatever we do ... (we must keep in mind that) if one approach does not work, try another" (p. 26). Many of our efforts to help children with math word problems have been ineffective. This direct instruction model is another way. It is a solution strategy that has a variety of applications. It appears to be an option worth trying.
References


APPENDIX A

PERMISSION TO CONDUCT RESEARCH
April 20, 1992

Ms. Jeanette W. Lee
1518 Jackson Street
Charleston, W.V. 25311

Dear Ms. Lee:

I am pleased to tell you that your research request, "Connecting Math Concepts: Using An Arithmetic Word Problem Solving Model With Students Having Learning Disabilities", was approved by the Kanawha County School Board at their meeting on April 16, 1992. Approval of your request was limited to Edison, Elkview, Glenwood, Rand and Taft Elementary schools. Since Kanawha County has engaged Johns Hopkins University to conduct a three year, building-wide program, "Success for All" at Chandler Elementary, this school was excluded from participation in your study. You may contact Ms. Sandra Barkey, the Director of the Department of Exceptional Students (348-6640) to suggest a replacement school.

Although you are familiar with many of the teachers, students, and parents who will participate in your study, please observe the following requirements in conducting your research:

1. Introduce yourself to the principals of schools selected for your study prior to contacting the teaching staff. Give a copy of this letter to the school principals and identify yourself as a Professor of Teacher Education from West Virginia State University and a Doctoral Candidate from Ohio State University.

2. Provide the principals and teachers with a brief, simple, and easy to read description of your study and why it could be of value for the teaching staff to participate in your research.

3. A completed copy of your research study must be sent to the Office of Research and Evaluation, Kanawha County Schools, after your thesis is approved.

We look forward to receiving the results of your study and feel that your work in this area will be very helpful to Kanawha County Schools. If you have any questions, please contact Dr. Kay Warner, Committee Chair (348-6115).

Sincerely,

Carolyn Meadows
Superintendent

cc: Sandra Barkey, Dept. of Exceptional Students
Nancy Villers, Edison Elementary
John Eagle, Elkview Elementary
Karen Simon, Glenwood Elementary
Michael Pack, Rand Elementary
Sandra Morris, Taft Elementary
APPENDIX B

LETTER OF INTENT AND CONSENT FORM: PILOT GROUP
Dear Parents:

I am an assistant professor at West Virginia State College, and also am a graduate student working on my doctoral degree in special education at The Ohio State University. For sixteen years I have been in Kanawha County, teaching children in special programs. My dissertation study centers on helping children solve math word problems.

There are already many tests for math, but few, if any, are on a child's ability to solve word problems. I have written a math test limited to word or story problems. Now I wish to see how useful this test is with children. Questions that need to be answered include:
1. How much time is needed for students to finish the test?
2. Are boxes large enough for students to write in problems?
3. Can students understand the directions that are given?
4. Are there too few or too many problems on a page?
5. What comments do students have about the test?

There are three versions of the test. Each contains 15 problems written in story form. I want to give these tests to the children in your child's class. A test will be given to the whole group at once, during the regular math period for your child's class. The scores that each child earns on the tests will not affect his/her math grade in any way. They will be used only for research purposes.

In addition to giving tests in math word problems, I desire to see the school records in order to note your child's grade level, age, gender, and standard scores on ability tests. A summary of this information will be written in a report, but no names will be used. Numbers will be reported by schools only. At your request, I will be happy to give you a copy of the report.

If you are willing for your child to participate in this study, please sign the permission form and return it in the enclosed stamped envelope. If you have any questions or concerns, please feel free to call me at 766-3314 or 344-0263. Thank you!

Sincerely,

Jeanette V. Lee
PERMISSION SLIP

I give permission for my child.

(Print child's name.)

to be tested on word problem solving ability.

In addition, I give Mrs. Lee permission to look at school records to get my child's grade level, age, gender, and previous standard scores on tests of ability.

I understand that Mrs. Lee will not use my child's name in any records to report results of her study.

At my request, I understand that I will be given a copy of the results of the study.

(Parent signature) (Date)
APPENDIX C

LETTER OF INTENT AND CONSENT FORM: TREATMENT GROUP
Dear Parents:

I am an assistant professor at West Virginia State College, and also am a graduate student working on my doctoral degree in special education at The Ohio State University. As you probably know, many children have difficulty with math—especially with solving word problems. However, based on some recent research, there is a promising new method for teaching children how to better solve math word problems.

With your permission, I want to teach this new method to your child's class. Your child's school day will not be disturbed. This instruction will be provided during the regular math period. Also, these lessons will occur for only three days a week, for three weeks.

Your child's teacher, the school principal, and the Kanawha County Board of Education have approved this request. However, it is also necessary to receive your permission. In addition to teaching your child this word problem method, I would desire to see the school records in order to note your child's grade level, age, gender, and standard scores on tests of ability. A summary of this information will be given in a report. Be assured that your child will not be identified. Numbers will be reported by schools only. At your request, I will be happy to share with you the report of the study results.

If you are willing to permit your child to receive this math instruction, then please sign the permission form and return it in the enclosed stamped envelope. If you have any questions or concerns about this study, please feel free to call me at 766-3314 or 344-0263. Thank you!

Sincerely,

Jeanette V. Lee
PERMISSION SLIP

I give permission for Jeanette W. Lee to teach my child, 

______________________________
(Print child’s name.)

a different way to solve addition and subtraction math word problems. This instruction will occur as part of my child’s regular math group instruction.

In order to determine if the new method is effective, I grant Mrs. Lee permission to test my child before and after the special teaching occurs in the classroom.

In addition, I give Mrs. Lee permission to look at the school records to get my child’s grade level, age, gender, and standard scores on previous tests of ability.

I understand that Mrs. Lee will not use my child’s name in any records to report results of her study.

At my request, I understand that I will be given a copy of the results of the study.

______________________________  ______________________________
(Parent signature)  (Date)
APPENDIX D

LETTER AND CONSENT FORM: COMPARISON GROUP
April 24, 1992

Dear Parents:

I am an assistant professor at West Virginia State College, and also an graduate student working on my doctoral degree in special education at The Ohio State University. There is a promising new method for teaching children how to solve math word problems. I am interested in finding out whether that new program is more effective than the one your child is using now.

I want to see how well your child's class solves addition and subtraction word problems. The children will be given tests, and their scores will be compared to children who are being instructed by the new method.

Every step will be taken to keep from disrupting your child's regular school routine. The tests will be given by your child's regular math teacher during the regular math period. Only three math periods would be involved.

The scores your child earns on the tests will not affect his/her math grade in any way. They will only be used for the research study. Your child's teacher, the school principal, and the Kanawha County Board of Education have approved this request. It is also necessary to receive your approval. In addition to testing your child in math word problems, I would desire to see the school records in order to note your child's grade level, age, gender, and standard scores on tests of ability. A summary of this information will be written in a report, but no names will be used. Numbers will be reported by schools only. At your request, I will be happy to give you a copy of the report.

If you are willing to allow your child's math teacher to give your child these math tests, then please sign the permission form and return it in the enclosed stamped envelope. If you have any questions or concerns, please feel free to call me at 766-3314 or 344-0263. Thank you!

Sincerely,

Jeanette V. Lee
PLEASE NOTE:

Page(s) missing in number only; text follows.
Filmed as received.
APPENDIX E

MODIFICATIONS OF ORIGINAL WORD PROBLEMS
Ellen promised herself Houghton Mifflin that she would read every afternoon. She read 118 pages on Monday, 204 pages on Tuesday, and 193 pages on Wednesday. How many pages did Ellen read altogether?

Todd promised himself that he would read every afternoon. He read 118 pages on Monday, 204 pages on Tuesday, and 193 pages on Wednesday. How many pages did Todd read altogether?

The Greys had 9 cans of orange juice and 5 cans of tomato juice. They had how many cans of juice?

The Greys had nine cans of orange juice and five cans of tomato juice. They had how many cans of juice?

There were 593 tropical fish and 276 goldfish. How many fish were there in all?

At the pet store, there were 593 tropical fish and 276 goldfish. How many fish were there in all?

The girls made 37 puppets. The boys made 18 puppets. The girls made how many more puppets than the boys?

Brett's scout troop made 37 model cars. Seth's scout troop made 18 models. Brett's troop made how many more model cars than Seth's troop?

After lunch 7 boys and 6 girls played ball together. How many children played ball?

After lunch seven boys and six girls played ball together. How many children played ball?

Wade found 13 red rocks. He gave away 5. How many rocks did he have then?

Matt found thirteen red rocks. He gave away six of them. How many rocks did he have then?
Jerry has 7 records and Jean has 8 records. How many do they have in all?

8,642 people were at the game. 1,375 left before the end. How many people stayed?

Jerry collected 75 shells. He gave 43 to his brother. How many shells did Jerry have left?

There are 11 cats and 7 fewer dogs are there? How many fewer dogs are there?

57 letters done 28 letters yet to do How many letters in all?

Sandra found 138 shells last year and 115 shells this year. How many more shells did she find last year?

14 children at the park. 9 people leave. How many children are left?

Adam has 17 records and Ronnie has 8 records. How many do they have in all?

At the basketball tournament, 8,642 people were counted. Before the end of the game, how many people stayed?

The animal shelter now has 11 cats and 7 dogs. How many fewer dogs are there?

Mom asked us to put stamps on her letters. 57 letters are done. We have 28 letters yet to do. How many letters will we stamp in all?

Mary found 138 shells last year and 115 shells this year. How many more shells did she find last year?

14 children were at the park. 9 children leave. How many children are left?
The Greys had 9 boxes of raisens. They ate 5 of them. How many boxes did the Greys have left?

The Smiths had packed 9 boxes of raisens for their camping trip. They ate 5 of them. How many boxes did the Smiths have left?

Jack had 9 marbles. He bought 6 more. How many did he have in all?

Jack had 9 matchbox cars. He bought 6 more. How many does he have in all?

A flock of swans had 3007 males and 1642 females. How many more were males?

A flock of swans had 3007 males and 1642 females. How many more swans were male?

Kay walked 3 miles before lunch and 4 miles after lunch. She walked how many miles in all?

Kay walked three miles before lunch and four miles after lunch. She walked How many miles in all?

28 animals. 14 are fed. How many remain?

Mr. Walker is in charge of feeding 28 animals. He has fed 14 of them. How many remain to be fed?

Kuri painted 17 pictures. He gave 8 of them to friends. How many pictures does he have left?

Andrew painted 17 pictures. He gave 12 of them to friends. How many pictures does he have left?

Karla had 13 acorns. Nancy had 3 acorns. How many more acorns did Karla have?

Carl had 13 Nintendo cartridges but Jason had 3. How many more cartridges did Carl have?

There are 15 books in 1 pile and 8 books in another

There are 15 books in one pile and 8 books in another
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are 9 history books and 3 music books. How many books are there</td>
<td>12</td>
</tr>
<tr>
<td>all?</td>
<td></td>
</tr>
<tr>
<td>There were 10 children. There 6 baseball caps. How many more caps were</td>
<td>4</td>
</tr>
<tr>
<td>needed?</td>
<td></td>
</tr>
<tr>
<td>There were 20 cupcakes. 21 are eaten. How many remain?</td>
<td>11</td>
</tr>
<tr>
<td>On the Ferris wheel there are 87 people. 146 people are waiting in line</td>
<td>230</td>
</tr>
<tr>
<td>How many people are there in all?</td>
<td></td>
</tr>
<tr>
<td>There were 5 birds eating crackers on the picnic table. Then 3 more</td>
<td>8</td>
</tr>
<tr>
<td>birds came along. How many birds were there in all?</td>
<td></td>
</tr>
<tr>
<td>There were 8 oranges and 5 were eaten. How many were left?</td>
<td>3</td>
</tr>
<tr>
<td>Ed caught 5 fish and Pat caught 8 fish. How many fish did they catch?</td>
<td>13</td>
</tr>
<tr>
<td>Question</td>
<td>Answer</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>14 firefighters.</strong> 6 went to a fire. How many were left?</td>
<td>They catch in all? 14 firefighters were at the station but only 6 went to a fire. How many were left at the station?</td>
</tr>
<tr>
<td>The book has 92 pages. Chita read 39 pages. How many left?</td>
<td>The book has 92 pages. Nikki read 39 pages already. How many are left to be read?</td>
</tr>
<tr>
<td>The coach had 18 baseballs. He bought 12 more baseballs. How many baseballs in all?</td>
<td>The coach had 18 baseballs. He bought 12 more baseballs. How many baseballs does he have in all?</td>
</tr>
<tr>
<td>It is 85 miles to Bluefield. You have driven 31 miles. How many miles remain?</td>
<td>It is 85 miles to Bluefield. You have driven 31 miles. How many miles remain?</td>
</tr>
<tr>
<td>At the city park we counted 17 park benches and 8 fountains. How many more park benches?</td>
<td>At the city park we counted 17 park benches and 8 fountains. How many more park benches?</td>
</tr>
</tbody>
</table>
APPENDIX F

TEST INSTRUMENTS
Read each story.
Decide if you should add or subtract.
Circle the right sign.
Write your problem and answer in the box.
Follow the example given here.

1. Matt found thirteen red rocks. He gave away six of them. How many rocks did he have left?

2. After lunch, seven boys and six girls played kickball. How many children children played ball?

3. At the beach, Jerry collected 75 shells. He gave 43 shells to his brother. How many shells did Jerry have left?

4. Mom asked us to put stamps on her letters. We have done 57 letters. We have 28 more letters to do. How many letters will we stamp in all?

5. The Greys have nine cans of orange juice and five cans of tomato juice. How many cans of juice do they have?
Read each story.
Decide if you should add or subtract.
Circle the right sign.
Write your problem and answer in the box.
Follow the example given here.

6. Todd promised himself that he would read every afternoon. He read 118 pages on Monday, 204 pages on Tuesday, and 193 pages on Wednesday. How many pages did Todd read altogether?

7. Hattie has a spoon collection. Out of 50 spoons, 10 are made of silver. How many spoons are not silver?

8. Brett's scout troop made 37 model cars. Seth's scout troop made 18 models. Brett's troop made how many more model cars than Seth's troop?

9. At the pet store, there were 593 tropical fish and 276 goldfish. How many fish were there in all?

10. Mrs. Grey had twelve candles. She used eight of them. How many candles were left?
Read each story.
Decide if you should add or subtract.
Circle the right sign.
Write your problem and answer in the box.
Follow the example given here.

11. At the basketball tournament 8,642 people were counted. Before the end of the game 1,375 people left. How many people stayed?

12. The animal shelter now has eleven cats and seven dogs. How many fewer dogs are there?

13. Mary found 136 shells last year and 115 shells this year. How many more shells did she find last year?

14. Danny picked seven red flowers and six yellow flowers. How many more red flowers did he pick?

15. Adam has 17 records and Ronnie has 18 records. How many do they have in all?
1. At the picnic, the boys used nine plates and the girls used seven plates. How many plates did they use in all?

2. Kay walked three miles before lunch and four miles after lunch. She walked how many miles in all?

3. A flock of swans had 3,007 males and 1,642 females. How many more swans were male?

4. There were 15 books in one pile and eight books in another pile. How many books were there in all?

5. Carl had 13 Nintendo cartridges but Jason had 8. How many more cartridges did Carl have?
Read each story.
Decide if you should add or subtract.
Circle the right sign.
Write your problem and answer in the box.
Follow the example given here.

6. There are nine pizzas with mushrooms and eight pizzas with pepperoni. + or -
How many pizzas are there in all?

7. Mr. Walker is in charge of feeding 28 animals. He has fed 14 of them. + or -
How many remain to be fed?

8. Andrew painted 17 pictures. He gave 12 of them to friends. + or -
How many pictures does he have left?

9. Peter collected 62 rocks. He painted funny faces on 39 of them. + or -
He wants to paint them all and sell them. How many more rocks does he have to paint?

10. Jack had nine matchbox cars. He bought six more. + or -
How many cars does he have in all?
Read each story.
Decide if you should add or subtract.
Circle the right sign.
Write your problem and answer in the box.
Follow the example given here. 

11. Team A scored 5 points. Team B scored 3 points. How many more points did Team A score?

12. Alex collects stamps. He has 54 from the United States and 26 from other countries. How many stamps does he have in all?

13. The Smiths had packed nine boxes of raisins for their camping trip. They ate five of them. How many boxes did the Smiths have left?

14. Fourteen children were at the park. Nine children leave. How many children are left?

15. Forty-seven tickets were sold in the morning and 204 were sold in the afternoon. How many more tickets were sold in the afternoon?
Read each story. Decide if you should add or subtract. Circle the right sign. Write your problem and answer in the box. Follow the example given here. 

1. At the city park we counted 17 benches and 8 fountains. How many more benches are there than fountains? + or -

2. It is 85 miles to Bluefield. You have driven 31 miles. How many miles remain? + or -

3. The coach had 18 baseballs. He bought 12 more baseballs. How many baseballs does he have in all? + or -

4. There were 9,461 tickets. 8,642 were sold. How many were not sold? + or -

5. The book has 92 pages. Nikki has read 39 pages already. How many pages are left to be read? + or -
Read each story.
Decide if you should add or subtract.
Circle the right sign.
Write your problem and answer in the box.
Follow the example given here.

6. Fourteen firefighters were at the station but only six went to a fire. How many were left at the station?

7. Ed caught five fish and Pat caught eight fish. How many fish did they catch in all?

8. Wade saw 14 robins and 6 crows. He saw how many fewer crows than robins?

9. There were 10 children ready for practice. There were 6 baseball caps. How many more caps were needed?

10. Don saw eight white rabbits and seven brown rabbits in Coonskin Park. How many rabbits did he see in all?
Read each story.
Decide if you should add or subtract.
Circle the right sign.
Write your problem and answer in the box.
Follow the example given here. ————>

11. Xom baked 32 cupcakes. 21 were eaten. How many cupcakes remain?

12. On the Ferris wheel there are 87 people. 146 people are waiting in line. How many people are there in all?

13. In one tank there are 195 fish and in another tank there were 64 fish. There were how many fish in all?

14. There were eight oranges in the fruit bowl and five were eaten. How many were left?

15. There were 12 birds eating crackers on the picnic table. Then 3 more birds came along. How many birds were there in all?
APPENDIX G

SCRIPT FOR ADMINISTERING TESTS
Preparation: Remind the principal of the testing time so that no messages are relayed over the intercom, and no emergency drills are planned. Post the "Do Not Disturb" sign on the outside of the classroom door. Close the door.

Materials to have ready:
- bell or timer
- rule cards ("Sit," "Listen," "Work Quietly," "Raise Your Hand")
- "Do Not Disturb" sign
- damp paper towel or sponge
- clock or watch
- clicker
- masking tape
- Vis-a-Vis marker
- overhead projector
- transparency of sample questions
- test forms
- math puzzles

Form A

Classroom Teacher: Mrs. Lee and I will be switching roles today during our math period. She will be your teacher, and I will be the assistant teacher for math time. (Put up the rule cards 'Sit' and 'Listen.') Remember to look and listen carefully so you will
learn. Stay in your seats while Mrs. Lee is talking.

(Take a seat near the rear of the room and note on the
script which the researcher is to follow—any
deviations from the text.)

Researcher: Today for math we are going to do something
that is different. We are interested in how you would
solve some problems. As I explain how to do the
assignment today, I will be asking you questions. You
will answer as a group—all together—after you hear
the signal. (Show and sound the clicker.) Do you talk
before or after the clicker? (Click one time.)

Students: After

Researcher: That's right. When I ask you a question,
everybody will answer at the same time after the
clicker. You will be given three sheets. On each page
there will be five short stories with problems to
solve. First, you are to read a story to yourself.
After you read a story, decide if you should add or
subtract. What should you do first? (Click)

Students: Read a story.

Researcher: Good remembering. You will read a story to
yourself first. Next you should decide if you should
add or subtract to solve the problem. (Show
transparency which has the response column duplicated.)
If you think that you should add to find an answer to
the problem, you would circle the plus sign, like this.
(Circle the appropriate sign.) If you think that you
should subtract to solve the problem, then you would circle the minus, or take-away sign. (Wipe off the circle around the plus sign and draw a ring around the minus sign). You will circle only one sign--(Point to the transparency as you say...)--the plus sign or the minus or take-away sign. Do you circle one or two signs, class? (Click)

Students: One

Researcher: Good. You circle only one sign. The last thing to do is to write your problem and answer in the box. Here is a problem I am writing as an example. (Write 56 - 21 = 35 in column form in the box). Notice that I put all the numbers and the operation sign inside the box. As a quick review, you read, (Point to the sample story on the transparency.) circle, and write. Read, circle and write. Now it's your turn to say those three steps. (Click)

Students: Read, circle, write.

Researcher: Good. Are there any questions? (Answer any queries.) Let's do some sample problems together. (Pass out sample worksheet to students. Without referring to any specific story situations, explain how students should read each problem first. Circle an operation sign on the sample worksheet [presented on the transparency] and have students model the movement on their sheets. Write an addition or subtraction problem on the transparency, and direct students to
Researcher: Let's pass out your main papers now. When you get your paper, put your name on the top line, and then put your pencil on the floor beside your desk. We want to start working at the same time. (Pass out papers and make sure that the directions are followed.) If you forget the directions, they are printed at the top of each page in bold letters. Look at the examples at the top of each page.

Students: (Turn to the second and third page of the test.)

Researcher: When you finish problem five on the first page, you can go right on to the next page. When you finish problem 10, turn on over and do the last page. You will have all our math time to do these sheets. It is important that you work carefully. You may go back and check your work during this period. If you finish early, you will stay in your seats and have these math puzzles to work on. (Pass out puzzle pages, placing them on the floor face down beside each student's desk.)

If you finish before the math period is over, will you get out of your seat for any reason? (Click)

Students: No

Researcher: Thank you. We will all stay seated and work quietly so everyone can concentrate. I will tell you when the math period is over. Are there any questions now? (Answer any questions except those which relate
directly to the words or operations needed for the problems on the test.)
You may pick up your pencils and begin. (Put up "Work Quietly" rule card.)
(Teacher and researcher should note the time. Student work time will be 30 minutes. The teacher will set the timer for 30 minutes, or signal the researcher at the end of 30 minutes to ring a bell.
The researcher and teacher are to remain seated as they monitor students visually. Both may collect papers when the timer rings off.)

The bell is the signal that tells us that math time is over. Please put down your pencils. If you did not get to finish your puzzles, you may work on them later today.
Thank you for following directions. I like how you stayed in your seats during math. You should be proud of yourselves.
Researcher: (Put up the rule cards "Sit" and "Listen.")

I like how you are staying in your seats, and looking this way. These are the rules (gesture toward the rule cards) that we will follow for this period, like before. You may remember that when I ask you questions, you are to answer as a group at the same time after you hear the signal.

(Show and sound clicker.)

Do you talk before or after the clicker? (Click one time.)

Students: After

Researcher: (Give feedback.) Everybody will answer at the same time after the clicker sound. Sometimes I may call on one of you by name to answer by yourself after the clicker. Be ready!

Just as before, you will be given three sheets. Today's sheets have different problems on them.

First, you are to read a story to yourself. After you read a story, decide if you should add or subtract.

What should you do first? (Click)

Students: Read a story.

Researcher: (Give praise.) You will read a story to yourself first. (Show transparency with sample questions.)

Next you decide if you should add or subtract to solve
the problem. If you think you should add to find the answer, you would circle the plus sign, like this.
(Circle the appropriate sign.)
If you think that you should subtract to solve the problem, then you would circle the minus, or take-away sign.
(Wipe off the circle around the plus sign and draw a ring around the minus sign.)
Do you circle one or two signs, (Call one student by name.)? (Click)

Student: One
Researcher: (Give feedback.) You will circle only one sign for each problem--the plus sign or the minus or take-away sign. The last thing to do is to write your problem and answer in the box. Here is a problem I am writing as an example.
(Write 56 - 21 = 35 in column form in the box.)
Notice that I put all the numbers and the operation sign inside the box. As a quick review, you read, (Point to the sample story on the transparency.) circle, and write. Read, circle and write. Now it's your turn to say those three steps. (Click)

Students: Read, circle, write.
Researcher: (Give feedback.) Sometimes when you read, you may come across a word that is hard to read. If that happens, read the other words in the sentence that you do know. You may be able to figure out the strange
word. If you aren't sure, make a good guess and try to solve the problem anyway. You must work on your own. Will you ask your neighbor to help you with a hard word? (Click)

Students: No

Researcher: Thank you. For this period, you are to work all by yourself. Make every effort to put something in every box. That shows that you have tried your best. Should any boxes be empty? (Click)

Students: No

Researcher: (Give feedback.) Let's pass out your papers now. When you get your paper, put your name on the top line, and then put your pencil on the floor beside your desk. We want to start working at the same time. (Pass out papers and make sure that the directions are followed.)

If you forget the directions, they are printed at the top of each page in bold letters. Look at the examples at the top of each page.

Students: (Turn to the second and third pages of the test.)

Researcher: When you finish problem five on the first page, you can go right on to the next page. When you finish problem 10, turn on over and do the last page. You will have all your math period to do these sheets. Take your time as before to do your best. You may go back and check your work during the period.
(Put up the "Raise Your Hand" rule card.)

Are there any questions? (Answer any queries.)

Researcher: If you finish early, you will stay in your seats and have these math puzzles to work on. (Pass out puzzle pages, placing them on the floor face down beside each student's desk.)

If you finish before the math period is over, will you get out of your seat for any reason? (Click)

Students: No

Researcher: (Give feedback.) We will all stay seated and work quietly so everyone can concentrate. I will tell you when the math period is over today. Are there any questions now? (Answer any questions except those which relate directly to the words or operations needed for the problems on the test. Post "Work Quietly" rule card.)

You may pick up your pencils and begin.

Students: (Begin test.)

Teacher: (Note the time, or set the timer for 30 minutes. The researcher and teacher are to remain seated as they monitor students visually.)

Researcher: The bell is the signal that tells us that math time is over. Please put down your pencils.

Students: (Put pencils down.)

Researcher: (Give feedback while collecting papers.)

Thank you for following directions. If you did not get to finish your puzzles, you may have time later
today to work on them.
APPENDIX H

SCRIPT FOR DIRECT INSTRUCTION
Intervention Day 1: Getting Started

Materials: Cassette tape (60 minutes)
Clicker
Overhead projector
Rule cards (Sit, Listen, Raise Your Hand)
Student worksheets (1.2)
Tape player
Three different colored Vis-a-Vis markers
Transparencies (1.1, 1.2, 1.3)

(Post rule cards. Give feedback to students who are attentive. Turn on tape player to record classroom interactions.)

Starting today, we are going to cover some things in math that are very different from what you have studied before. You will have to work hard, but you can do it. You will learn a lot. For you to learn well, you must follow my instructions. I will be giving you lots of instructions. Try to follow them exactly. You will have to remember the things we cover, because you will use everything I show you. Many times, I will ask you questions. Sometimes, I will call on one person by name to answer. Since you will not know who I will call, you each need to carefully listen and think. Often, I will expect everyone to answer at the same time. This clicker (Show clicker.) will be used to signal that it is your time to talk as a group. When you hear this
Introduction to Number Families

a. You will learn something each day that will make it easier to understand math. You will soon learn how to read a story and figure out an answer to a problem.

(Show transparency 1.1 with a number line drawn.)

Here is a number line. This figure is very important. It will help us in math. Watch what I do with it.

If we have 4 units on the line, like this (Draw in units.), and we add 2 more, (Add in a different color.) how many do we have in all? (Signal.) 6

Right. I'm going to write 6 here at the end of the arrow. (Write 6.)

b. (Show a different number line.)

Here I have 4 units on the line (Draw as in a.)

Now I have 3 more. (Draw.)

How many do I have in all? (Signal.) 7

Good. I'm going to write 7 at the end of the arrow. (Write 7.)

c. Let's try it another way. (Show another number line.)

I have 8 in all. (Write 8 at the end of the line.) I have 5 units here. (Write 5 at the top of the arrow.) How many more units do I need? (Signal.)
Three is correct. I will write three on top of the number line with the 5. (Write.)

Now imagine the number line.

Here are some problems.

\[ \begin{align*}
  6 & \rightarrow 8 \\
  3 & \rightarrow 5 \\
  4 & \rightarrow 6 \\
  4 & \rightarrow 7 \\
  6 & \rightarrow 7
\end{align*} \]

As you may have noticed, the small numbers always go on top of the arrow. The big number is always at the end of the arrow.

d. We call this (Circle the last one.) a number family.

All these number families have 1 big number and 2 small numbers.

In a number family, where is the big number always? (Signal.)

At the end of the arrow.

Yes. The big number is always at the end of the arrow.

Where are the smaller numbers? (Signal.)

On top of the arrow.

Good. Smaller numbers always go on top of the arrow.

e. Number families will help us in addition and subtraction. You will see. Every number family has at least three members. Whenever you have a problem in math, it is because one number in the family is
missing. If the big number is missing in a problem, (Cover the big number with finger.) we add the two small numbers to find that big number. If the big number is missing, what do we do with the two smaller numbers to find that big number? (Signal.)

We add them.

That's right. [See Level C, Lesson 3, Exercise 2: Number Families (page 10).]

(Pass out student sheets, page 5 for Lesson 3 - 1.2.

f. Put your name on your sheet. (Observe students.)

Find part 2. You are going to write the addition problem for every family that has a missing big number. Look at each family. If the big number is missing, copy the addition problem that ends with the box. You will not answer the problem. Just copy it. If a small number is missing, don't write anything. Raise your hand when you're finished.

(Observe students and give feedback.)

g. Check your work. (Show transparency 1.2. Point to the appropriate problem as students answer.)

What's the letter beside the first family you worked?

(Signal.) C

Good.

Read the problem. Get ready. (Signal.)

8 plus 9 equals box.

What's the letter beside the next family you wrote?

(Signal.) E
(Give feedback.) Read the problem. Get ready.

(Signal.) 12 plus 16 equals box.

That's right.

What's the letter beside the next family you wrote?

(Signal.) F

F is correct. Read the problem. Get ready. (Signal.)

2 plus 3 equals box.

Good.

What's the letter beside the next family you wrote?

(Signal.) I

Right. Read the problem. Get ready. (Signal.)

1 plus 7 equals box. Good job.

(Project transparency 1.3.)

Now look at Part 4 of your sheet. One number in each family is missing. Is the missing number a big number or a small number? (Signal.) A big number.

That's right. What can we do to the small numbers to find the missing big number? (Signal.) Add.

Yes, we add the two smaller numbers to find the big number. Take time now and try to find the missing big number in each family. Put the number in the box at the end of the arrow. (Signal. Observe students as they work and provide assistance as needed. When most have finished, check together!)

Let's see how well you did.

2 plus 7 equals box. What number goes in the box, class?
(Signal.) 9. Right.

You could also say the numbers in another order: 7 plus 2 equals 9. For number family b we would read: 1 plus 7 equals box. What big number goes in the box, (Call on one student by name.)?

(Signal.) 8. Good thinking.

(Follow the same script to check problems c through m. Call on individual names to answer problems d, g, i, and k. Ask for choral responses for the other problems.)

h. Listen: Number families also show you how to write subtraction problems. When you subtract, you start with a big number and end up with a smaller number. So if a family has a missing small number, you always subtract to find the missing number.

Once more: If a family has a missing small number, you subtract.

i. Look at your sheet again. Put your finger on Part 2. (Observe all students.)

What is the letter beside the first family with a missing small number?

(Signal.) A. Right.

What’s the letter beside the next family with a missing small number?

(Signal.) B. Right again.

What’s the letter beside the next family with a missing small number?

(Signal.) D. You’re getting good at this.
What's the letter beside the next family with a missing small number?
(Signal.) G. (Give feedback.)

What's the letter beside the next family with a missing small number?
(Signal.) H. (Give feedback.)

j. (Write on the board or transparency:)

[] 4
--------> 5

Here's family A. To say the subtraction problem for this family, you just start with the big number and go backwards along the arrow. Remember, that's how you subtract. You start with the big number and go backwards along the arrow.

Everybody, what's the big number in this family?
(Signal.) 5. Right.

My turn to say the subtraction problem: 5 minus 4 equals box.

Your turn: Say the subtraction problem for family A.
Get ready. (Signal.) 5 minus 4 equals box. Good job.

k. Touch family B.

You're going to say the subtraction problem for family B. Remember, you start with the big number and go backwards along the arrow.

l. Everybody, say the subtraction problem for family B.
Get ready.
(Signal.) 17 minus 9 equals box. (Give feedback.)
(Repeat step m until firm.)

m. Touch family D.

Everybody, say the subtraction problem. Get ready.
(Signal.) 28 minus 14 equals box. That's right.

n. Touch family G.

Everybody, say the subtraction problem. Get ready.
(Signal.) 6 minus 4 equals box. Good.

o. Now touch family H.

Everybody, say the subtraction problem. Get ready.
(Signal.) 18 minus 3 equals box. That's right.

I'll show you how to write the family on the board.

Watch. (Write on the transparency!)

a. 5 - 4 = []

There it is: 5 minus 4 equals box.

Your turn. Write the subtraction problem for family A.

Then write the subtraction problem for every other family with a missing small number. You will only write the problems—not the answers. Raise your hand when you're finished.

(Observe students and give feedback.)

p. Write on the overhead projector:

b. 17 - 9 = []

d. 28 - 14 = []

g. 6 - 4 = []

h. 18 - 3 = []

Check your work. Here's the problem you should have written for each family.
Read what you should have for family B. (Signal.)
17 minus 9 equals box. (Give feedback.)

Read what you should have for family D. (Signal.)
28 minus 14 equals box. Right.

Read what you should have for family G. (Signal.)
6 minus 4 equals box. (Give feedback.)

Read what you should have for family H. (Signal.)
18 minus 3 equals box.

Raise your hand if you got all the problems right.
If you made any mistakes, fix up those you made. (Allow time.)

You learned some important things today. Numbers come in families. You can show the family around a number line, and make addition and subtraction problems with one family.

If two small numbers are known in a family, do you add or subtract to find the big number?
(Signal.) Add. That's right.

If a small number is missing in a family, do you add or subtract the other numbers to find that missing number?
(Signal.) Subtract. Good job. You're smarter already.
Intervention Day 2: Solving Column Problems in Number Families

Materials: Cassette tape (60 minutes)
Clicker

Number people drawn on transparency squares:
- Sample envelope - 2, 4, 6, cardboard arrow
- Envelope a - 5, 3, 8
- Envelope b - 2, 5, 3
- Envelope c - 10, 6, 4
- Envelope d - 3, 7, 4
- Envelope e - 5, 2, 7

Overhead projector

Rule cards (Sit, Listen, Raise Your Hand)
Student worksheets (2.1, 2.2)
Tape player
Transparencies (2.3, 2.4)
Vis-a-Vis markers

a. (Post rule cards. Give feedback to students who are attentive. Turn on tape player to record classroom interactions.)

Like people, numbers come in families. There are usually three members in a number family. (Project the three orange colored numbers with faces from the sample envelope.) The home of a number family is represented by an arrow.
(Show arrow, then place on the overhead screen.)
The arrow points to the right.

How many members are usually in a number family?
(Signal.)

**Three**
(Give feedback.)

Now what does the house of the number family look like?
(Signal.)

**An arrow**
(Give feedback.)

b. Every member has a special place in the home.
The number of greatest value—the largest of the three
digits—is always at the end of the arrow. Let's call
that largest number "Mr. Big."
In this family (Point to the three numbers on the
screen [2, 6, 4]), which is Mr. Big? (Signal.)

**Six**
(Give feedback.)

Watch me place him in his special spot.
(Move the number 6 to the end of the arrow.)

c. The two smaller numbers sit above the arrow.
(Move the two smaller numbers to rest on the segment
of the arrow.)

d. Now all the family members are in their special places.
Let's see how you can help some number family members
to get home.
(Distribute Worksheet 2.1.)
Put your name and the date on your paper when you get it. (Observe students and give feedback.)

e. (Remove the numbers 3, 8, 5 from envelope A, and place on the screen.)

Which number is Mr. Big in this family? (Signal.)

Eight

(Give feedback.)

Where is his special place in the house? (Signal.)

At the end of the arrow

(Give feedback.)

Put him there on your sheets by house a. (Observe students.)

The two small numbers have a special place to sit.

Place them where they belong. (Observe students.)

If you put them above the arrow, like this, (Demonstrate on the overhead.) you remembered well.

The order of the small numbers doesn't matter. If you have the small numbers above the arrow, you are right.

f. Here are the number family members for house b.

(Show the numbers and follow the script above for houses b, c, d, and e.)

g. (Give feedback. Show transparency 2.3.)

You have done so well that you are now ready to be detectives. There is a missing number person in each of these houses. You tell me if the missing person is a small number or the largest number—Mr. Big—in the family. As I point, say "small" or "Mr. Big".
(Point as the signal for students to answer problem situations on the overhead. Give feedback at each line.)

h. Now you are hot on the track to locating the missing member. Here's the important information that will take you right to the number person: When a small number is missing, you subtract the two other numbers, one from another. (Repeat this statement, writing an "S" as you say "small" and writing another "S" as you say "subtract.")

When the small number is missing, you ... (Signal.)

Subtract

(Give feedback.)

When Mr. Big is missing, you add the two small numbers together.

(Call on individual students to circle the plus or minus sign beside each problem. Have students verbalize, e.g., "The small number is missing, so I subtract." Give feedback.)

i. (Distribute student sheets Level C, Lesson 5, Part 2-2.2. Direct students to head their papers.)

You have learned that numbers come in families, and that we write the family members (the numbers), around an arrow. We know that a math situation is a problem when one of the numbers is missing. You have learned to write addition problems and subtraction problems to figure out
the missing number from a number family.
(Refer to Lesson 5, pages 19 & 20.)

Listen: What do you do to find the big number when it is missing? (Signal.) Add. Good remembering.

What do you do to find a small number when it is missing? (Signal.) Subtract. That's right.

You can write a column problem if the number family has 2-digit numerals.

j. Touch family A.

What's missing in that family? (Signal.)

A small number. That's right.

Everybody, say the problem for family A. Get ready.

(Signal.) 36 minus 21 equals box. Very good!

(Write on the board or overhead:)

\[
36 - 21 = \ 
\]

Here's the problem written in a column:

36 minus 21 equals box. When the problem is in a column, I don't have to write the box.

(Erase the box.)

Remember to keep your digits lined up.

k. Your turn: Copy the problem in the box for A. Raise your hand when you are finished. We are not trying to solve the problem now. We just want to write it down correctly.
(Proceed after all have finished.)

1. Touch family B.

   What's missing in that family? (Signal.)

   The big number. Correct.

   Everybody, say the problem for family B. Get ready.

   (Signal.) 32 plus 16 equals box. (Give feedback.)

m. Your turn: Write the column problem for B. You don’t have to write the box. Raise your hand when you are finished writing the problem. We are not trying to solve the problem now. We just want to write it down correctly.

   (Observe students and give feedback.)

   (Write on the board or overhead:)

   \[
   \begin{array}{c}
   32 \\
   \hline
   + 16
   \end{array}
   \]

n. Check your work. Here's the column problem for family B. 32 plus 16. Raise your hand if you got that right.

   (Observe students and give feedback.)

o. Your turn: Write the column addition problem or column subtraction problem for the rest of the families in part 2. Remember, if the big number is missing, you add. If a small number is missing, you subtract. Mr. Big will be the number on top—the one you write first—when you subtract.

Write the plus sign or the minus sign so we will know which operation you choose. Raise your hand when you are finished. (Observe students and give feedback.)
p. Check your work.

Family A. What's missing in the family--the big number or a small number? (Signal.)
A small number. (Give feedback.)
Read the problem you wrote. (Signal.)
36 minus 21. Good job. You subtract when a small number is missing.

Family B. What's missing in the family--the big number or a small number? (Signal.)
The big number. (Give feedback.)
Read the problem you wrote. (Signal.)
32 plus 16. That's fine. When the big number is missing, you add the two small numbers.

Family C. What's missing in the family--the big number or a small number? (Signal.)
A small number. Right.
Read the problem you wrote. (Signal.)
71 minus 51. You are learning very well.

Family D. What's missing in the family--the big number or a small number? (Signal.)
The big number. Good for you.
Read the problem you wrote. (Signal.)
12 plus 63. That's the right problem.

Family E. What's missing in the family--the big number or a small number, (Call on one student by name.)?
(Signal.)
A small number. You're on the ball.
Read the problem you wrote, everybody. (Signal.)
85 minus 64. That's good. Small—subtract (Write S-S on the board or overhead. Point to each S as you say "small"—"subtract."
(Point to the first s.) If the missing number is small,
(Point to the second s.) you ... (Signal.)
Subtract. (Give feedback.)

Everybody, make sure you've written the correct problems, then write the answers to the problems in part 2. Raise your hand when you're finished.
(Observe students and give feedback.)

Check your work. Read each problem and the answer.
Problem A, (Call one student by name.) (Signal.)
36 minus 21 equals 15. Very good.
Problem B. (Signal.) 32 plus 16 equals 48.
(Give feedback.)
Problem C, (Call one student by name.) (Signal.)
71 minus 51 equals 20. Good thinking.
Problem D. (Signal.) 12 plus 63 equals 75.
That's right, class.
Problem E. (Signal.) 85 minus 64 equals 21.
(Give feedback.)

Raise your hand if you got all the answers right.
(Look for hands.) Good for you. (If some of the hands are not raised, add:) If you missed a few of the problems, you may fix up any mistakes.
Intervention Day 3: Number Families/Puzzles

Materials: Cassette tape (60 minutes)
Clicker
Damp paper towel
Overhead projector
Rule cards (Sit, Listen, Raise Your Hand)
Student worksheets (3.2, 3.3)
Tape player
Transparencies (3.1, 3.2, 3.3)
Vis-a-Vis markers

a. (Post rule cards. Give feedback to students who are attentive. Turn on tape player to record classroom interactions.)

We have learned that numbers come in families. Tell me if you remember how many numbers are in a complete number family. Get ready. (Signal.) 3. That's right. We learned that there are three numbers in a complete number family. In a math problem, one of those numbers is missing from the family. We wrote the numbers in the family around a number line. To show the missing number, we drew a box. Today, we will use a letter just like the box to show or stand for the missing number.

(Pass out Lesson 23 sheet, Part 1, from textbook Level C - 3.2.)

Write your name on your paper.
(Observe students and give feedback.)

Everybody, find part 1 on your worksheet. These
problems tell about two numbers and a letter that goes in a number family. Remember, the letter works just like a box. If the missing number in a problem is the big number, you add to find it. If the missing number is a small number, you subtract to find it.

Your turn: If the missing number is the big number, how can you find it? (Signal.) Add (Give feedback.)

If the missing number is a small number, how do you find it? (Call on one student.)? (Signal.) Subtract. (Give feedback.)

b. Touch problem A.
I'll read it. The small numbers are B and 21. The big number is 95.

Your turn: Write an arrow and put the numbers and the letter in family A. Raise your hand when you are finished. (Observe students and give feedback.)

c. (Write on overhead:)

\[ \begin{array}{c}
  & B & 21 \\
\hline
\text{–} & & 95
\end{array} \]

Here's number family A. Is the missing number the big number or a small number? (Signal.)

A small number. (Give feedback.)

So do we add or subtract to find the missing number? (Signal.) Subtract. That's right.

d. Do it. Write the subtraction problem. When you figure out the answer, cross out B in the number family and write the answer above it. Raise your hand when you're
Everybody, read the subtraction problem and the answer.

(Signal.) 95 minus 21 equals 74. (Give feedback.)

So what did you write for B in the number family?

(Signal.) 74.

(Write to show:)

\[
\begin{array}{c}
74 \\
\hline
21 \rightarrow 95
\end{array}
\]

Here's what your number family should look like.

Raise your hand if you got everything right for problem A. (Observe hands and give feedback.)

f. (Guide students to write letters around the arrow for problem b.)

g. (Write on the board or overhead:)

b. 341 \quad 199 \quad \rightarrow \quad T

Here's the number family.

Read the addition problem and the answer. (Signal.)

341 plus 199 equals 540. (Give feedback.)

So what did you write for T in the number family?

(Signal.) 540. (Give feedback.)

(Write to show:)

b. 341 \quad 199 \quad \rightarrow \quad 540
h. (Guide students through problems c and d.

Write on transparency 3.2:

c. 19 \[\xrightarrow{72}\] 31

d. 381 \[\xrightarrow{402}\] 783

(Give students assistance and feedback.)

Comparison Word Problems

a. (Refer to Level C, Lesson 25, Exercise 7, Teacher's Manual pages 106, 107.)

. (Write on the board or overhead:)

You're going to work word problems that are just like number problems, but they have only letters.

Listen to this sentence: A is more than B. That sentence tells about two numbers—A and B in a number family. You don't know whether A is 20 or 17 or 56. But you do know that A is more than B. So if A is 20, B is less than 20.

Listen again. A is more than B. Which is more, A or B? (Signal) A. (Give feedback.)

Here's a rule about A and B. One of these numbers is the big number in the number family. Which is the big number? (Signal) A. That's right.

So I write A for the big number and B for the small
number.

(Write to show:)

\[ B \rightarrow A \]

That family shows us that the number for A is more than the number for B.

(Erase letters:)

b. New problem. Listen: C is less than D. One of these numbers is the big number.

Listen again and think about which letter is the big number and which is the small number. C is less than D.

Is C a small number or the big number? (Signal.)

A small number.

Yes, C is less. So it must be the small number. D is the big number.

(Write to show:)

\[ C \rightarrow D \]

That family shows us that the number for C is less than the number for D.

(Erase letters:)

------>

c. One more. R is smaller than T. Listen again and think about the number family. R is smaller than T.

Is R a small number or the big number? (Signal.)

A small number.
Yes, R is smaller. It's a small number, so T must be the big number.

(Write:) 

\[ R \quad \longrightarrow \quad T \]

Here's what you write for R is smaller than T.

(Pass out student worksheet 3.3 and direct students to head papers. Project matching transparency 3.3.)

**Workbook Practice**

a. Your turn. Turn over your sheet to lesson 25 and find part 1. (Observe students.) Sentence A: J is less than M.

Write the letters in the number family. Write the small number close to the big number. Raise your hand when you are finished.

b. Check your work.

(Write on the board or overhead:)

\[ J \quad \longrightarrow \quad M \]

Everybody, what's the big number for sentence A? (Signal.) M. Right.

And J is a small number.

c. Sentence B: R is more than P.

Write the letters in the number family.

d. Check your work.

(Write on the board:)

\[ P \quad \longrightarrow \quad R \]
Everybody, what's the big number for sentence B?
(Signal.) R. Good.

What's the small number? (Signal.) P. Good.

e. Work the rest of the problems in part 1. Raise your hand when you're finished.
(Observe students and give feedback.)

f. (Write on the board:)

\[
\begin{align*}
    & J \\
    \rightarrow & P \\
    \rightarrow & J \\
\end{align*}
\]

g. Check your work.

Here's what you should have for problems C and D. Raise your hand if you got both of them right.
(Observe and acknowledge students.)

Day 4: Comparisons

Materials: Cassette tape (60 minutes)

Clicker

Overhead projector

Rule cards (Sit, Listen, Raise Your Hand)

Student worksheets (4.2, 4.3)

Tape player

Transparencies (4.1, 4.2, 4.3)

Vis-a-Vis markers

(Post rule cards. Give feedback to students who are attentive. Turn on tape player to record classroom
interactions.)

(Show transparency 4.1 with Post-It notes covering letters.)

We have learned that a problem in math occurs when one number in a number family is missing. As you see on the screen, a box was first put in the special place that belonged to the missing number. The box in this number family (Point to the first number family on the transparency.) is on top of the arrow house. Is the missing number a small number, or is it Mr. Big? (Signal.)

A small number (Give feedback.)

(Review in like manner for the second and third number families presented on the transparency.)

Yesterday, the missing number was represented by something other than a box. Think for a minute, then raise your hand if you remember what was used instead of a box in number families that had missing persons. (Provide wait time. Prompt students if necessary.)

Letters

(Uncover the Post-It notes which cover the letters in the three number families.)

T stands for a missing number in this first family. Is T a small number or a large number? (Click.)

Small number (Give feedback.)

How do you know that T is a small number? (Call on an individual student by name.)
You see the letter H here where a number belongs.  
(Point to the letter on transparency 4.1.)
What kind of number must be H? A big number or a smaller one? (Signal.)

A big number (Give feedback. Review in like manner what A in the third word family represents.)

(Draw a line through each letter as students figure out the number which the letter represents. Write the number above the letter stand-in.)

a. For math today, you are going to put letters and numbers in number families. This will help us get ready for word problems.

(Distribute workbook lesson 28 - Sheet 4.2. Direct students to neatly print names and date on papers. Project matching transparency 4.2.)

Each problem on your sheet has a circled number in it.

b. Touch problem A.

I'll read the whole sentence, including the circled number. Listen: J is 5 less than K.

Your turn: Say the whole sentence. (Signal.)

J is 5 less than K. Very good.

Now I will read the sentence without the circled number.

(Cover the circled number with finger.)

Listen: J is less than K.

Your turn: Say that. (Signal.) J is less than K.

(Give feedback.)
The sentence you just said tells us how to put J and K in the number family. Listen again: J is less than K. Look at the first part of that sentence up here on the screen. "J is less..."

Is J the big number or a small number? (Signal.)

A small number. (Give feedback.)

(Write to show;)

\[ J \quad \Rightarrow \quad K \]

That shows that J is less than K. Listen to the whole sentence again: J is 5 less than K.

We know where to put the 5. It has to be a small number, because that's the only place it can go.

(Write to show;)

\[ 5 \quad J \quad \Rightarrow \quad K \]

So here's the number family for J is 5 less than K.

d. Your turn: Complete the number family for problem A.

e. Touch problem B.

J is 18 more than K.

Your turn to say the sentence without the 18.

(Signal.) J is more than K. That's right.

f. Make the number family for that much of the problem. J is more than K. Raise your hand when you are finished.

(Observe students and give feedback.)

(Write on the transparency;)
Here's what you should have so far. Listen to sentence B again: J is 18 more than K.

g. Put 18 in the only place that it can go in the number family. It's the first small number.

(Write on the transparency:)

\[
\begin{align*}
18 & \quad K \\
\longrightarrow & \quad J
\end{align*}
\]

Here's the family for the sentence. J is 18 more than K.

h. Touch problem C.

P is less than T.

Your turn: Say the sentence without the 9.

(Signal.) P is less than T. (Give feedback.)

Put those letters in the number family. Then put the 9 in the only place it can go. Raise your hand when you are finished.

(Observe students and give feedback.)

(Write on the overhead:)

\[
\begin{align*}
9 & \quad P \\
\longrightarrow & \quad T
\end{align*}
\]

Check your work. Here's what you should have. P is less than T. P is a small number. T is the big number. The first small number is 9. Raise your hand if you got it right.

i. Touch problem D.

H is 12 larger than F.
Say the problem without the 12. (Signal.)

*H is larger than F.*

Write those letters in the number family. Then put the 12 in the number family. Raise your hand when you're finished.

(Write on the board:)

\[
\begin{array}{c}
12 \\
\hline
F \\
\hline
\rightarrow
\end{array}
\]

Here's what you should have. \(H\) is the big number. \(F\) is a small number and 12 is the first small number. Raise your hand if you got it right.

(Observe students and give feedback.)

j. Touch problem E. \(H\) is 17 less than \(Y\).

Say the problem without the 17. (Signal.)

*\(H\) is less than \(Y\).* Good.

Write those letters in the number family. Then put the 17 in the number family. Raise your hand when you are finished.

(Write on the transparency:)

\[
\begin{array}{c}
17 \\
\hline
H \\
\hline
\rightarrow
\end{array}
\]

Here's what you should have. \(Y\) is the big number. \(H\) is a small number and 17 is the first small number. Raise your hand if you got it right.

(Observe and acknowledge hands.)

k. Raise your hand if you got all the number families right. (Observe and acknowledge hands. Collect papers.)
Comparisons: Solving for a letter

a. (Refer to Lesson 30, Exercise 1, Teacher's Manual pages 125, 126.)
Pass out student worksheets 4.3 (lesson 30, part 1, textbook lesson 30, student page 44.)

These are some more word problems for practice. Each problem tells about two numbers and a letter. One of the numbers is circled. You work the problems just like the others you've worked.

b. Problem A: F is 12 more than 56.
Say the problem without the 12. (Signal.)
F is more than 56. Good.

c. Make the number family for problem A.
Remember, F is more than 56. Raise your hand when you're finished.
(Observe students and give feedback.)
(Write on the board or overhead:)

\[
\begin{array}{c}
12 \\
56 \\
\hline
F
\end{array}
\]

Check your work. Here's what you should have.
d. You can solve this problem to figure out what number F is. Is F a small number or the big number? (Signal.)
Big number. That's right.

What do you do to figure out a missing big number? (Signal.) Add. Good. You add when the missing number is the big number in a problem.

Write the addition problem to find out what number F is.
Notice the thin vertical or up and down lines on your papers. Those lines can help you keep your numbers lined up for each column. Write your problems using those lines. When you solve the mystery, cross out F in the number family and write the missing number above it. (Give individual assistance as needed.)

e. Everybody, read the addition problem and the answer.
   (Signal.) 12 plus 56 equals 68.
   (Write to show:)

[\[
\begin{array}{c}
12 \\
\downarrow \\
56 \\
\hline
68
\end{array}
\]\]

Yes, if F is 12 more than 56, F is 68.

Raise your hand if you got it right.

(Observe and acknowledge hands.)

f. Problem B: 96 is 17 less than F.

Say the problem without the 17. (Signal.)

96 is less than F. (Give feedback.)

Make the number family.

(Observe students and give feedback.)

(Write on board or overhead:)

[\[
\begin{array}{c}
17 \\
\downarrow \\
96 \\
\hline
F
\end{array}
\]\]

Check your work. Here's the number family for problem B.

g. Now write the number problem and figure out what number F is. Raise your hand when you're finished. (Observe students and give feedback.)
Check your work. Read the problem and the answer.
(Signal.) 17 plus 96 equals 113. (Give feedback.)
(Write to show:)
\[
\begin{array}{c}
17 \\
\hline
96 \\
\hline
113
\end{array}
\]

If 96 is 17 less than F, F is 113. Raise your hand if you got it right.

h. Problem C: 96 is 17 more than F. Your turn: Make the number family. Then write the addition or subtraction problem and figure out what number F is.
(Observe students and give feedback.)
(Write on board:)
\[
\begin{array}{c}
17 \\
\hline
F \\
\hline
96
\end{array}
\]

Check your work. Here's the number family for C.

Everybody, read the subtraction problem and the answer.
(Signal.) 96 minus 17 equals 79. (Give feedback.)
(Write to show:)
\[
\begin{array}{c}
17 \\
\hline
F \\
\hline
79
\end{array}
\]

If 96 is 17 more than F, F is 79. Raise your hand if you got it right.

i. Problem D: F is 27 less than 53. Your turn: Make the number family. Then write the addition or subtraction problem and figure out what number for F.
(Observe students and give feedback.)
(Write on the board:)

Check your work. Here's the number family for D.

Everybody, read the problem and the answer. (Signal.)

59 minus 27 equals 32. (Give feedback.)

(Write to show:)

\[
\begin{array}{c}
27 \\
\hline
F \\
\hline
32 \\
\hline
59
\end{array}
\]

If F is 27 less than 59, F is 32.

Raise your hand if you got it right.

(Observe students and give feedback. As you collect papers, "write" two 8 shapes with your finger on the backs of students as you say "Small number missing—subtract". Have students repeat the rule.)

Intervention Day 5: Comparison Word Problems

Materials: Cassette tape (60 minutes)

Clicker

Overhead projector

Rule cards (Sit, Listen, Raise Your Hand)

Student worksheets (5.1, 5.2)

Tape player

Transparencies (4.1, 5.1, 5.12, 5.13, 5.2, 5.3)

(Post rule cards. Give feedback to students who are attentive. Turn on tape player to record classroom interactions.)

(Project transparency 4.1.)

You have learned that math problems may contain numbers
and letters. The problem or mystery is solved only when we end up with all numbers. Look at the family on top of the screen. What kind of number is missing--small or big? (Signal.)

Small (Give feedback.)

Let's solve that mystery. It's a small number that's missing, so we subtract. Small number missing, subtract. You repeat that rule. (Signal.)

Small number missing--subtract (Give feedback.)

Remember that you move backward in the house, starting at Mr. Big. Say the subtraction problem with me. (Signal.)

Six minus two equals T. (Give feedback.)

It is easy when the numbers are small like these. What if there were larger numbers in the same kind of problem. (Write on the projector:)

\[
\begin{array}{c}
32 \\
\hline
T \\
\end{array}
\rightarrow
\begin{array}{c}
46 \\
\end{array}
\]

(Show how larger numbers must be put in columns to be computed. When the answer is given, cross out the T and write 14 above it.)

As far as the mystery goes, the case is closed when all three numbers are in the house.

a. (Refer to Level C, Lesson 41, Exercise 2, page 165.)

Today, I'm going to say sentences that have the names Wendy and Tom.

(Show figures on transparencies 5.12 and 5.13.)

You will tell me which person has the
bigger number. That's the person who would be the big
number in a number family.

Remember, the person who has more or does more has the
bigger number.

b. Wendy is shorter than Tom. Who has the bigger number?
(Signal.) *Tom.* (Give feedback.)

(Show figures of Wendy and Tom on transparency 5.1 at
appropriate places around the arrow for this and the
following examples.)

Wendy runs farther than Tom. Who has the bigger number?
(Signal.) *Wendy.* (Give feedback.)

Wendy is higher than Tom. Who has the bigger number?
(Signal.) *Wendy.* (Give feedback.)

Tom eats more than Wendy. Who has the bigger number?
(Signal.) *Tom.* (Give feedback.)

Wendy is older than Tom. Who has the bigger number?
(Signal.) *Wendy.* (Give feedback.)

Wendy eats less than Tom. Who has the bigger number?
(Signal.) *Tom.* (Give feedback.)

(Repeat step b until firm.)

c. You did a good job listening to sentences and thinking
of numbers. Now we will look at some sentences and
change them into number families. (Distribute Lesson
41, textbook sheet page 81, part 2 - [5.2]. Have
students write their names on the top.)

We are going to write letters in number families. What
have we always written around the number line?
Sometimes in math, problems that involve numbers are hidden in stories. You may read words, but the words stand for numbers. Today you will learn how to change those words into numbers. Look at part 2 on your sheet.

I will read what it says in the box. Follow along:

When you make number families from word problems, you can use letters to stand for the names. You can write the first letter for each name in the problem.

Here's a sentence: Wendy has more seeds than Tom.

Here's the number family: Wendy is the W. She's the big number because she has more. Tom is the T. He's a small number in this problem.

There are five sentences, a through e, at the bottom of your paper. Beside each sentence, write its number family. Remember to use the first letter of each name. Remember to write the letters around the number line or arrow. Raise your hand when you are finished. (Observe students, give assistance and feedback.)

Let's check your work. Look at the screen. (Show transparency 5.2. For sentence a, the letter for Ann is ... (Signal.) A (Draw a line under the A in Ann.) and the letter for Debby is ... (Signal) D (Draw a line under the D in Debbie. Give feedback.) (Draw an arrow under the sentence.)

Ann is older than Debbie. Which letter is the big
number? (Call on one student, then signal.) A (Give feedback.) So where would the A be placed around the arrow, class? (Signal.) At the end of the arrow. (Give feedback.)

D must be a small number. Where do the small numbers go in relation to the arrow? (Signal.) On top. (Give feedback.) (Point to the example.)

If your number family for "a" looks like this, raise your hand. (Observe students and give feedback.)

f. (Show next sentence on the chalkboard.)
Doug runs less than Jerry. What letter will stand for Doug? (Signal.) D. (Give feedback. Draw a line under the D.)

What letter stands for Jerry? (Signal.) J. (Give feedback. Draw a line under the J.)

Who runs more? (Signal.) Jerry. (Give feedback.) So the J will go at the end of the arrow. (Draw an arrow and write J at the end.)

Where will we put the D? (Signal.) On top of the arrow. (Give feedback.)

g. Repeat the sequence for sentences c, d, and e, as described in steps e or f.

Comparing Height
(Refer to Lesson 44, Exercise 3, Level C, teacher's manual, pages 176, 177. Distribute sheet 5.3 and direct students to head.)
a. (Student worksheet: Textbook page 30, Lesson 44, Level C)
These are word problems that tell about people who are
taller and people who are shorter. Remember this: the
taller person is the big number.

The first sentence of each problem tells you how to make
the family with two letters and a number.

The second sentence tells you which letter to replace
with a number. The question in the problem tells you
which letter to figure out.

b. I'll read problem A: Tim was 7 inches taller than Bill.
Bill was 54 inches tall. How tall was Tim?

We are going to write the number family for the
first sentence of problem A. The first thing to do
with any word problem is to draw the home.

(Draw on transparency 5.3 and direct students to copy.)
Let's read that first sentence again and leave out the
number. The words alone will give us the ideas we need.

(Cover "7 inches" from sentence a. Signal.)
Tim was taller than Bill. (Give feedback.)
(Direct students to write the letters around the arrow.)

(Write on the board:)

\[
\begin{array}{c}
7 \quad B \\
-----------> T
\end{array}
\]

Here's the number family.

c. Let's read the rest of problem A.

Bill was 54 inches. How many inches tall was Tim?

(Guide, so students can replace one of the letters
with a number.).

(Write on the board:)

\[
\begin{array}{c}
7 \\
\rightarrow \\
54
\end{array}
\]

Here's the completed number family for problem A.

d. I'll read the rest of the problems in part 3:

Problem B: Fran was 13 inches shorter than Ginger. Ginger was 54 inches tall. How many inches tall was Fran?

Problem C: Debby was 21 inches taller than Billy. Billy was 37 inches tall. How many inches tall was Debby?

Problem D: Donnie was 17 inches shorter than Greg. Greg was 73 inches tall. How many inches tall was Donnie?

e. For the rest of the problems in part 3, write the number family with two letters. Then replace one of the letters with a number. Raise your hand when you are finished.

(Observe students and give feedback.)

f. (Write on board or overhead:)

\[
\begin{array}{c}
b. \ 13 \ F \\
\rightarrow \\
\theta
\end{array}
\]

\[
\begin{array}{c}
c. \ 21 \ B \\
\rightarrow \\
D
\end{array}
\]
Check your work. Here are the number families for the problems.

g. Now figure out the answers for the problems. Write the addition problem or subtraction problem and the answer. Raise your hand when you are finished.

(Observe students and give feedback.)

h. Check your work.

Problem A. Tim was 7 inches taller than Bill. Bill was 54 inches tall. How many inches tall was Tim? Everybody, what's the answer? (Signal.) 61 inches.

(Give feedback.)

Problem B. Fran was 13 inches shorter than Ginger. Ginger was 54 inches tall. How many inches tall was Fran? What's the answer? (Signal.) 41 inches.

Problem C. Debby was 21 inches taller than Billy. Billy was 37 inches tall. How many inches tall was Debby? What's the answer? (Signal.) 58 inches.

(Give feedback.)

Problem D. Donnie was 78 inches shorter than Greg. Greg was 78 inches tall. How many inches tall was Donnie? What's the answer? (Signal.) 61 inches.

(Give feedback.)

You are doing some very hard word problems. Great day!

*Intervention Day 6: Action Word Problems*
Materials: Cassette tape (60 minutes)
Clicker
Overhead projector
Rule cards (Sit, Listen, Raise Your Hand)
Student worksheets (6.1, 6.2)
Tape player
Transparencies (6.1, 6.2, 6.3)
Vis-a-Vis markers

(Refer to Level C, Lesson 57, Exercise 1, pages 224 - 226 teacher's manual.)

. (Post rules cards. Give feedback to students who are attentive. Turn on tape player to record classroom interactions.)

a. (Distribute sheets: 6.1, 6.2 - Textbook lesson 57, pages 133, 134.)
Today you are going to learn about a new kind of problem. The missing number is not at the end of these problems. I'll read what it says in the box. Follow along: Here's a problem. The problem has a box in it. I'll read the box as some. (Read from transparency 6.1.) "You have some. You find 31 more. You end up with 76.

The problem tells you that you are getting more. So you put the values forward along the number-family arrow." You can see the number family. "You have some, and you find 31 more." A box will be written first, and then
31 is the next number. "You end up with 76." 76 is the big number. "That's what you do if the problem tells about getting more."

If the problem tells about getting less, you go backward (when writing) along the arrow."

"Here's a problem about getting less: You have some. You lose 76. You end up with 31."

"You put the values backward along the number-family arrow." You can see the number family. You have some. You lose 76. The box is the big number for some. You lose 76. That's the next number backward along the arrow. You end up with 31. That's the number at the beginning of the arrow.

Remember, each problem has three parts. If the problem tells about getting more, you put the values forward along the arrow. If the problem tells about getting less, you put the values backward along the arrow.

If the problems tells about getting less, do you go forward or backward along the arrow, class? (Signal.)

Backward (Give feedback.)

d. (Show transparency 6.2."

Turn to the next page. Find the sample problems on your sheet. (Observe students.)

Each problem has three parts. You will tell me what to write for each part. Then you will tell me whether you go forward along the arrow or backward along the arrow.

d. Touch sample problem 1. You have 81. You lose some.
You end up with 59.

. Everybody, what should I write for the first part? (Signal.) 81. (Give feedback.)

. What do you write for the next part? (Signal.) Box. (Give feedback.)

. And what do you write for the last part? (Signal.) 59. (Give feedback.)

. You write parts of the problem in the same order in which they come in the story.

f. Now you have to figure out whether you put the values in forward along the arrow or backward. Remember if you get more, you put them in forward along the arrow. If you get less, you put them in backward along the arrow.

. Listen to the first part of the problem: You have 81. You lose some. Are you getting more or getting less? (Signal.) Less. (Give feedback.)

. So do you move forward along the arrow or backward along the arrow? (Signal.) Backward. (Give feedback.) You go backward along the arrow, starting with the first value in the problem.

g. (Write on the board or overhead:)

\[
81 \quad \rightarrow \quad 59
\]

. Tell me what to write first. (Signal.) 81. (Give feedback.)

. (Write 81:)

\[
81 \quad \rightarrow \quad 59
\]

. Tell me what to write next. (Signal.) Box.
Tell me what to write last. (Signal.) 59.

(Give feedback.)

(Write 59!)

59 [ ]

------------- > 81

Here's the number family.

h. Touch sample problem 2: You have 81. You find some. You end up with 129.

What do you write for the first part? (Signal.) 81.

(Give feedback.)

What do you write for the next part? (Signal.) Box.

(Give feedback.)

What do you write for the last part? (Signal.) 129.

(Give feedback.)

i. Now you have to figure out whether you put the values in forward along the arrow or backward along the arrow. Remember, if you get more, you put them forward along the arrow. If you get less, you put them backward along the arrow.

Listen to the first part of the problem: You have 81. You find some. Are you getting more or less? (Signal.) More. (Give feedback.)

So do you go forward along the arrow or backward along
the arrow? (Signal.)

Forward. (Give feedback.)

Tell me what to write first. (Signal.) 81.

(Give feedback.)

(Write on the board or overhead:)

\[
\begin{array}{c}
81 \\
\end{array}
\]

Tell me what to write next. (Signal.) Box.

(Give feedback.)

(Write:)

\[
\begin{array}{c}
81 \\
\end{array}
\]

Tell me what to write last. (Signal.) 129.

(Write 129:)

\[
\begin{array}{c}
81 \\
\end{array}
\] \rightarrow 129

There's the number family. Remember that you write the values for the parts in the same order they are in the problem. All you have to do is figure out whether you go backward along the arrow or forward along the arrow.

Practice

a. (Refer to Level C textbook, Lesson 57, page 134, part 2.)

Your turn. Find problem A.

b. Problem A: You have 538. You lose some. You end up with 390.

What do you write for the first part? (Signal.)
538. (Give feedback.)

What do you write for the next part? (Signal.) Box.

(Give feedback.)

What do you write for the last part? (Signal.) 390.

c. Listen: Make a number family arrow. Then write the value for the first part where it belongs. Just write the value for the first part of that problem. Raise your hand when you are finished.

(Observe students and give feedback.)

(Write on the board:)

Here's what you should have for a.

You're getting less, so the value for the first part is the big number. The other values will go backward along the arrow. Put the value just for the first part.

Remember, if you get more, you go ... (Signal.)

Forward. (Give feedback.)

If you get less, you go ... (Signal.) Backward.

(Give feedback.)

Write all the number values around a number line. Raise your hand when you are finished with the problem.

(Observe students and give feedback individually.)

Read problem B. Decide if you are getting more or getting less. Then put 285—the first part—where it belongs on the arrow.

(Write on the board or overhead:)
Here's what you should have. You're getting more. So the value for the first part is the first small number. You're going forward along the arrow.

e. Your turn: For the rest of the problems in part 1, make a number family arrow, and write the first value. Raise your hand when you are finished.

(Observe students, giving assistance as needed.)

(Write on the board or overhead:)

\[
\begin{align*}
\text{c. } & \quad \longrightarrow \quad \square \\
\text{d. } & \quad \longrightarrow \quad 56 \\
\text{e. } & \quad \square \\
& \quad \longrightarrow
\end{align*}
\]

In problem C, you are getting less, so the values go backward along the arrow starting with the box. That's the big number.

In problem D you are getting less, so the values go backward along the arrow, starting with 56.

In problem E you are getting more, so the values go forward along the arrow, starting with the box.

f. Now you are going to put the other values in each number family.

g. Touch problem A.

What's the value for the first part? (Signal.) 538.

(Give feedback.)
What's the value for the next part? (Signal.) Box. (Give feedback.)

What's the value for the last part? (Signal.) 390. (Give feedback.)

Those values go backward along the arrow. Complete the number family so it has all three values. Raise your hand when you're finished. (Observe students and give feedback.)

(Write to show:)

\[
a. \quad \begin{array}{c}
390 \\
\hline
538
\end{array}
\]

Here's what you should have for family A.

h. Touch problem B. The value for the first part is already in the family. Complete family B so it has all three values. Raise your hand when you are finished. (Observe students and give feedback.)

(Write to show:)

\[
\begin{array}{c}
285 \\
\hline
299
\end{array}
\]

Here's what you should have for family B.

i. Your turn: Complete the rest of the families so they have two numbers and a box. Raise your hand when you are finished. (Observe students and give feedback.)

(Write on the board or overhead:)

\[
c. \quad \begin{array}{c}
177 \\
56
\end{array}
\]

\[
d. \quad \begin{array}{c}
19 \\
\hline
56
\end{array}
\]
Here's what you should have for families C, D, and E.

Raise your hand if you got everything right.

Now figure out the missing number in each family. Write the missing addition or subtraction problem and the answer. Copy the missing number in your number family.

Raise your hand when you are finished.

(Observe students and give feedback.)

Check your work. Read each problem and the answer.

Problem A. (Signal.) 538 minus 390 equals 148.

(Give feedback.)

Problem B. (Signal.) 299 minus 285 equals 14.

(Give feedback.)

Problem C. (Signal.) 177 plus 56 equals 233.

(Give feedback.)

Problem D. (Signal.) 56 minus 19 equals 37.

(Give feedback.)

Problem E. (Signal.) 738 minus 231 equals 507.

(Give feedback.)

Day 7: Classification Word Problems

Materials: Cardboard squares (3)

Cassette tape (60 minutes)

Clicker

Overhead projector
Rule cards (Sit, Listen, Raise Your Hand)

Student worksheets (7.2, 7.3)

Tape player

Transparencies (5.1, 7.1, 7.2, 7.3, 7.4)

Vis-a-Vis markers

(Post rule cards. Give feedback to students who are attentive. Turn on tape player to record classroom interactions.)

(Review A)

To solve math word problems, we know now how to draw an arrow first. We know that each problem has at least three parts, or family members.

If the problem tells about getting more or about getting less, there is a certain order and direction to be followed when writing the family members down.

If the problem tells about getting more, you put the values in forward along the arrow home. More...Forward

Your turn to say it. If the problem tells about getting more, you write the values in which direction—forward or backward? (Signal.) Forward (Give feedback.)

(Show transparency 7.1. Help students decide on the direction for writing numbers in the sample problems.)

(Review B)

In math problems that involve getting more or getting less, we said that it is important to put the number family members down in a certain order.

(Show transparency 5.1.)
Sometimes the person who belongs here (Place one cardboard square on top of the arrow on transparency 5.1 on the left-hand side.) comes home first. If he comes home first, guess who always will come home next.

Do you think it's Mr. Big, or the other small number? (Signal.)

Small number (Give prompts and feedback as appropriate.)

If Mr. Big gets home first, (Place a cardboard square in the spot for the largest number.) who comes in next: the small number here (Place a square above the arrow closest to Mr. Big.), or the small number there? (Remove the square above the arrow and place a square on the top left portion of the arrow.)

Is this one home next, class? (Signal.)

No (Give feedback. Demonstrate with three persons holding hands to show progressive order, e.g. "The numbers have to be close enough to hold hands.")

If a story problem tells about getting more, or getting less, the numbers come home in a certain order.

Let me tell you a secret about home that will help you. (Explain that the arrow home represents a number line.)

Have students count as you write numbers 1 - 5 on transparency 5.1 above the arrow.

Ask the students to name a bigger number. Write the large number at the end of the arrow.
Explain that when you get more of something, you are talking about small numbers trying to go forward in the direction of the big numbers.

Explain that when you get less of something, you are talking about a big number that is losing something, or moving in the direction of the small numbers.

Distribute student sheet 7.3.
Assist as students place numbers and boxes around the number lines drawn.

a. (Refer to Level C, Lesson 93, Teacher's Manual pages 122, 123. Distribute worksheet 7.2 [Lesson 93 worksheet, page 246, Part 1.] Direct students to write in the appropriate heading.)

You have learned how to work several different kinds of word problems. (Give name of classroom teacher.) and I are very proud of you. You should be proud of yourselves for what you have learned. Today you are ready for a new kind of word problem. Find part 1 on your sheets.

(Show word problem on transparency 7.2.)

I'll read what it says in the box. You follow along:

Here's a problem. There were 60 cars in all. 14 were red cars. The rest were blue cars. How many blue cars were there? (Draw an arrow below the word problem.)
You write **R** for red cars (Write **R** above the arrow.), **B** for blue cars (Write **B** above the arrow, to the right of **R**.), and **All** for all cars. (Write "**All**" at the end of the arrow.)

The word **cars** goes under the number family. (Write.)

Then you put in the numbers you know: 14 were red cars. (Strike out the **R** and write 14 above it.) There were 60 cars in all. (Strike out the word "**All**" and write 60 above it.)

Now you have a number family with two numbers. You can figure out the missing number.

Is **B** a small number or the big number in this family? (Signal.) **Small number.** (Give feedback.)

When a small number is missing in a family, do you add or subtract? (Signal.) **Subtract.** (Give feedback.)

Your turn: Write the number family inside the box. Make sure that it has letters, numbers, and the word **cars** below. Figure out the answer. Raise your hand when you are finished.

(Observe students and give feedback.)

Everybody, how many blue cars were there? (Signal.) 46. (Give feedback.)

Follow along as I read problem A. Tim had red marbles and green marbles. How many marbles did he have in all?

Listen: Are we talking about cars in this problem? (Signal.) **No.** (Give feedback.)

What word tells about all the things in this problem?
Your turn: Make the arrow for the number family. Write marbles below the number family. Write R for red marbles, G for green marbles and All for all marbles. Raise your hand when you have a number family with the words and two letters.

(Observe students and give feedback.)

(Write on the board or overhead:)

\[
\begin{array}{c}
\text{a. } \hspace{1cm} \text{(Marbles)} \\
\text{R} \hspace{1cm} \text{G} \hspace{1cm} \text{All}
\end{array}
\]

Here's what you should have. The small numbers are R and G, because you have to add the red marbles and the green marbles to get all the marbles.

Now read the problem and put in the numbers. Figure out the number of marbles in all. Raise your hand when you are finished. (Observe students and give feedback.)

(Write to show:)

\[
\begin{array}{c}
\text{a. } \hspace{1cm} \text{(Marbles)} \\
\text{14} \hspace{1cm} \text{16} \hspace{1cm} \text{All}
\end{array}
\]

Here's what you should have for number family A.

Everybody, how many marbles are in all? (Signal.) 30.

(Give feedback.)

d. Problem B: Dora had big jars and little jars. She had 35 jars in all. 14 were big jars. How many were little jars?
What word tells about all the things in this problem? (Signal.) Jars. (Give feedback.)

Make the arrow for the number family. Then write jars below the number family. Use B for big jars, L for little jars and All for all jars. Raise your hand when you have a family with the words and two letters. (Observe students and give feedback.)

(Write on the board or overhead:)

\[
\begin{align*}
\text{b. B} & \quad \text{L} \\
\phantom{\text{b. B}} & \rightarrow \text{All} \\
\text{jars}
\end{align*}
\]

Here's what you should have.

Now read the problem, put in the numbers and figure out the number of little jars. Raise your hand when you are finished. (Observe students and give feedback.)

(Write to show:)

\[
\begin{align*}
\text{b.} & \quad 14 \\
\phantom{\text{b.}} & \rightarrow \text{35} \\
\text{jars}
\end{align*}
\]

Here's what you should have for number family B.

How many little jars were there? (Signal.) 21. (Give feedback.)

**Day 8: Action and Comparison Problems**

Materials: Cassette tape (60 minutes)

Clicker

Overhead projector
(Post rule cards. Give feedback to students who are attentive. Turn on tape player to record classroom interactions.)

a. (Refer to lesson 67, exercise 7: Problem Solving)
You have learned how to solve math problems by using number families around an arrow. Today we will review two different types of problems.
(Pass out sheets 8.1 and 8.2 which are based on textbook level C, Lesson 67, part 4, page 164, and Lesson 69, part 4, page 170. Direct students to head papers.)

Some of these problems tell what happened first and what happened next. Other problems tell who had more or who had less.

b. Problem A tells that something weighed less. The thing that weighed less is the small number.

Problem B tells that one object is longer than another object. The object that is longer is the big number.

c. Your turn: Read each problem. Write the number family around the arrow for each problem. Raise your hand when you have a number family for each problem.
(Observe students and give feedback individually.)

(Write on the board:)
Check your work.
Here's what you should have for each problem. Raise your hand if you got all of them right.
(Observe and acknowledge hands.)

d. Now write the addition problem or the subtraction problem for each number family and figure out the answer. (Observe students and give feedback.)
e. Problem A: A cow weighed 196 pounds less than a horse. The cow weighed 741 pounds. How many pounds did the horse weigh? Say the number problem and the answer.
(Signal.) 196 plus 741 equals 937. (Give feedback.)

In this problem, we wanted to know about pounds. That word is written right after the numbers in the problems: 196 pounds, and 741 pounds. We say that the unit of interest in this problem is pounds.

What's the unit name? (Signal.) Pounds.
How many pounds did the horse weigh? (Signal.)
537 pounds. (Give feedback.)

Problem B: The post was 37 inches longer than the board. The post was 219 inches long. How long was the board? Say the number problem and the answer.
(Signal.) 219 minus 37 equals 182. (Give feedback.)
The numbers refer to what unit name? (Signal.) Inches. (Give feedback.)

When you answer a word problem, you always want to refer to the number with its unit. In other words, you would not only say 182 as the answer to problem b, but you would say that the answer is 182 inches. Now let me ask you the question in that number problem. You give me the complete answer, which is the number with its unit name—182 inches.

How long was the board? (Signal.) 182 inches.
(Give feedback.)

Problem C: A train started with some tons of concrete. The train unloaded 16 tons of concrete. The train ended up with 986 tons of concrete. How many tons of concrete did the train start with? Say the number problem and the answer. (Signal.) 986 plus 16 equals 1,002.
(Give feedback.)

What's the unit name now? (Signal.) Tons.
(Give feedback.)

So how many tons did the train start with? (Signal.) One thousand two tons. (Give feedback.)

Problem D: A toy store had 109 dolls. Then the store
bought some more dolls. The store ended up with 525 dolls. How many dolls did the store buy? Say the number problem and the answer. (Signal.)

525 minus 109 equals 416. (Give feedback.)

What's the unit name? (Signal.) Dolls. (Give feedback.)

How many dolls did the store buy? (Signal.) 416 dolls.

f. Raise your hand if you got most of those problems right.

(Observe and acknowledge hands.)

Independent Practice: Action and Comparison Word Problems

a. Turn to your next sheet. It is labeled "Part 4" on the top. You are going to make number families for these word problems.

Problem A tells about something that is closer. The thing that is closer is a small number.

Problem B tells who worked longer. The person who worked longer is the big number.

Remember, when you make number families for problems that tell who had more or less, you need two letters in the family.

b. Your turn: Make families for all problems in part 4. Raise your hand when you are finished. We will check them together.

(Observe students and give feedback individually.)

c. (Write on the board or overhead:)

\[ \begin{array}{c}
\text{b} \quad \text{c} \\
\hline
\end{array} \]

\[ a. \quad 36 \quad \rightarrow \quad \frac{48}{\text{c}} \]
Check your work.
Here's what you should have for each family.

d. Now write the number problem and the answer for each family. Raise your hand when you're finished.
   (Observe students and give feedback.)

e. Check your work.
   Here's what you should have for each family.

d. Now write the number problem and the answer for each family. Raise your hand when you're finished.
   (Observe students and give feedback.)

e. Check your work.

Problem A: The lake is 36 miles closer than the city. The lake is 43 miles away. How far away is the city?
Say the number problem and the answer. (Signal.)
36 plus 43 equals 84. (Give feedback.)
What's the unit name? (Signal.) Miles. (Give feedback.)
How far away is the city? (Signal.) 84 miles.

Problem B: Jan worked 47 hours longer than Ted worked. Jan worked 56 hours. How many hours did Ted work?
Say the number problem and the answer. (Signal.)

56 minus 47 equals 9. (Give feedback.)

What's the unit name? (Signal.) Hours

(Give feedback.)

How many hours did Ted work? (Signal.) 9 hours.

(Give feedback.)

Problem C: A company had some trucks. The company bought 160 more trucks. The company ended up with 255 trucks. How many trucks did the company start with?

Say the number problem and the answer. (Signal.)

255 minus 160 equals 95. (Give feedback.)

What's the unit name? (Signal.) Trucks.

(Give feedback.)

How many trucks did the company start out with?

(Signal.) 95 trucks. (Give feedback.)

Problem D: The mountain is 41 miles closer than the park. The park is 120 miles away. How far away is the mountain? Say the number problem and the answer.

(Signal.) 120 minus 41 equals 79. (Give feedback.)

What's the unit name? (Signal.) Miles.

(Give feedback.)

How far away is the mountain? (Signal.) 79 miles.

Raise your hand if you got most of those right.

(Observe hands. Give feedback.)

Day 9: Mixed review: Independent practice

Materials: Cassette tape (60 minutes)
Clicker

Overhead projector

Poster with attachable pieces:
  Arrow, Numbers

Rule cards (Sit, Raise Your Hand, Work Quietly)

Student worksheets (9.1)

Tape player

a. Three weeks ago, I explained to you that you would be learning how to work hard problems. I told you that you could do it, and you have. You have learned how to read different types of word problems. You know that the numbers in problems come in families, and you know how to find a missing number in a family.

We have learned to write the numbers in a family around a special symbol. What is that special symbol? (Signal.) An arrow. (Give feedback.)

(Show an arrow on a visual aid.)

From now on, whenever you read a word problem in math, you can draw an arrow to help you. You must remember that there is a place for each number in a family.

Where does the big number in a family go? (Signal.) At the end of the arrow. (Give feedback.)

(Use poster to show a large number at the end of the arrow.)

Where do the small numbers in a number family go? (Signal.) On top of the arrow. (Give feedback.)

(Use a poster to place two smaller numbers on top of
b. We know that each math problem has a missing number. In order to find that missing number, we may have to add or subtract the numbers given. If we know the big number in a family, (Remove one small number from the model.) do we add or subtract? (Signal.) Add. (Give feedback.)

If one of the small numbers is missing in a number family, ... (Replace the big number and remove one small number.) do we add or subtract the remaining numbers to find that missing number? (Signal.) Subtract. (Give feedback.)

c. For some problems, it is important to write the numbers in a certain order. If a problem tells about getting more, you put the values forward along the arrow.
If the problem tells about getting less, you put the number values backward along the arrow.

Your turn: Let's pretend that you have some baseball cards, but then you lose some. Are you getting more, or are you getting less? (Signal.) Less. (Give feedback.)
Would you write the numbers forward or backward in that problem? (Signal.) Backward. (Give feedback.)

d. Today for math you are going to practice solving problems on your own. Read each story. Draw an arrow
for each story and decide where the numbers go. Then
 decide whether to add or subtract. Write answers to
 your problems. While you are working, I will be walking
 around and watching you and helping if you need it. If
 you are not sure how to solve a problem, raise your hand
 for me, or you may talk it over quietly with your study
 buddy. Stay in your seats.

(Distribute papers and monitor while students are
 working. Give individual assistance as needed, and
 provide feedback.)
Part 2

a. □ 4 → 5
b. □ 9 → 17
c. 8 → 9 □
d. □ 14 → 28
e. 12 → 16 □
f. □ 2 → 3

g. □ 4 → 6
h. □ 3 → 18
i. □ 1 → 7

Part 4

a. 2 → 7  b. 1 → 7  c. 2 → 10  d. 1 → 1

e. 1 → 3  f. 1 → 10  g. 2 → 4  h. 1 → 6

i. 2 → 8  j. 1 → 4  k. 2 → 9  l. 1 → 5

m. 2 → 5
Part 2

a. 21 \rightarrow 36

b. 32 \rightarrow 16

c. \rightarrow 71

d. 12 \rightarrow 63

e. \rightarrow 85
Part I

Make the number family. Work the addition problem or the subtraction problem.

a. The small numbers are 21 and 21. The big number is 95.
b. The second small number is 199. The big number is T. The first small number is 341.
c. The big number is 91. The small numbers are 19 and M.
d. The first small number is 381. The second small number is H. The big number is 783.
Lesson 25

Part 1

a. J is less than M.

b. R is more than P.

c. P is more than J.

d. W is less than J.

Part 2

<table>
<thead>
<tr>
<th>a. 6 9</th>
<th>e. 6 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. 6 8</td>
<td>f. 6 9</td>
</tr>
<tr>
<td>c. 6 7</td>
<td>g. 6 6</td>
</tr>
<tr>
<td>d. 6 6</td>
<td>h. 6 8</td>
</tr>
</tbody>
</table>
Lesson 28

Part 1

a. J is 5 less than K.

b. J is 18 more than K.

c. P is 9 less than T.

d. H is 12 larger than F.

e. H is 17 less than Y.
Make a number family for each problem. Then write the addition or subtraction problem and figure out what number $F$ is.

a. $F$ is 12 more than 56.

b. 96 is 17 less than $F$.

c. 96 is 17 more than $F$.

d. $F$ is 27 less than 59.
When you make number families from word problems, you can use letters to stand for the names. You can write the first letter for each name in the problem.

Here's a sentence: Wendy has more seeds than Tom.
Here's the number family: \[ T \rightarrow W \]
Wendy is the W. She's the big number because she has more.
Tom is the T. He's a small number in this problem.

Write the number family for each sentence.

a. Ann is older than Debby.

b. Doug runs less than Jerry.

c. Jan has more paper than Carol.

d. Tony is shorter than Jack.

e. Fran eats less than Don.
Part 3

For each problem, make a number family with two letters. Replace one of the letters with a number. Then write the addition problem or subtraction problem and the answer.

a. Tim was 7 inches taller than Bill. Bill was 54 inches. How many inches tall was Tim?

b. Fran was 13 inches shorter than Ginger. Ginger was 54 inches tall. How many inches tall was Fran?

c. Debby was 21 inches taller than Billy. Billy was 37 inches tall. How many inches tall was Debby?

d. Reggie was 17 inches shorter than Greg. Greg was 78 inches tall. How many inches tall was Reggie?
Here's a problem: You have □.

You find 31 more.

You end up with 76.

The problem tells you that you're getting more. So you put the values in **forward** along the number-family arrow.

You have □. You find 31 more. You end up with 76.

That's what you do if the problem tells about getting more.

If the problem tells about getting less, you go **backward** along the arrow.

Here's a problem about getting less: You have □.

You lose 76.

You end up with 31.

You put the values in backward along the number-family arrow.

You end up with 31. You lose 76. You have □.
Remember, each problem has three parts. If the problem tells about getting more, you put the values in forward along the arrow.

- If the problem tells about getting less, you put the values in backward along the arrow.

**Sample problem 1:**
- You have 81.
- You lose □.
- You end up with 59.

**Sample problem 2:**
- You have 81.
- You find □.
- You end up with 129.

**a.** You have 538.
- You lose □.
- You end up with 390.

**b.** You have 285.
- You find □ more.
- You end up with 299.

**c.** You have □.
- You lose 56.
- You end up with 177.

**d.** You have 56.
- You lose □.
- You end up with 19.

**e.** You have □.
- You find 231 more.
- You end up with 738.
Here's a problem: There were 60 cars in all. 14 were red cars. The rest were blue cars. How many blue cars were there?

You write R for red cars, B for blue cars and All for all cars. The word cars goes under the number family.

\[
\begin{array}{c}
R & B & \text{All} \\
cars & & \\
\end{array}
\]

Then you put in the numbers you know: 14 were red cars. There were 60 cars in all.

\[
\begin{array}{c}
14 & 60 \\
R & B & \text{All} \\
cars & & \\
\end{array}
\]

Now you have a number family with two numbers. You can figure out the missing number.

a. Tim had red marbles and green marbles. He had 14 red marbles. He had 16 green marbles. How many marbles did he have in all?

b. Dora had big jars and little jars. She had 35 jars in all. 14 were big jars. How many were little jars?
Write the number family for each problem. Then write the addition or subtraction problem for each number family and figure out the answer.

a. A cow weighed 196 pounds less than a horse. The cow weighed 741 pounds. How many pounds did the horse weigh?

b. The post was 37 inches longer than the board. The post was 219 inches long. How long was the board?

c. A train started with some tons of concrete. The train unloaded 16 tons of concrete. The train ended up with 986 tons of concrete. How many tons of concrete did the train start with?

d. A toy store had 109 dolls. Then the store bought some more dolls. The store ended up with 525 dolls. How many dolls did the store buy?
Part 4  
Make the number family for each problem and figure out the answer.

a. The lake is 36 miles closer than the city. The lake is 43 miles away. How far away is the city?

b. Jan worked 47 hours longer than Ted worked. Jan worked 56 hours. How many hours did Ted work?

c. A company had some trucks. Then the company bought 160 more trucks. The company ended up with 255 trucks. How many trucks did the company start with?

d. The mountain is 41 miles closer than the park. The park is 120 miles away. How far away is the mountain?
Name: ___________________________ Date: ________________ 9.1

a. Read each word problem.

b. Draw an arrow for each problem and put the numbers in the number family where they belong around the arrow.

c. Decide whether to add or subtract to find a missing number.

d. Solve the problem.

1. Rita had 56 feet of string. Al had 19 feet of string. How much less string did Al have than Rita?

2. Sid weighed 150 pounds. Donna weighed 22 pounds less than Sid weighed. How much did Donna weigh?

3. A car was 24 feet long. A truck was 59 feet long. How much shorter was the car than the truck?
4. Jose had 95 cans. Rita had 14 more cans than Jose had. How many cans did Rita have?

5. There were 137 bugs in a barn. 78 were flies. The rest were not flies. How many of the bugs were not flies?

6. A bowl contains red marbles and blue marbles. There are 37 blue marbles. If there are 124 marbles altogether, how many red marbles are in the bowl?

7. All the fish that live in a pond are black or gold. There are 65 black fish and 72 gold fish in the pond. How many fish live in the pond?

8. A tractor had 176 quarts of gasoline. Then some more gasoline was put in the tractor. The tractor ended up with 330 quarts. How many quarts were added?
9. A sports car could go 145 miles per hour. A race car could go 189 miles an hour. How many miles an hour faster was the race car than the sports car?

10. A pig weighed 136 pounds more than a fox. The fox weighed 59 pounds. How much did the pig weigh?

11. A elm tree was 15 feet shorter than a pine tree. The pine tree was 84 feet tall. How tall was the elm tree?

12. A bus was 14 feet wide. A house was 55 feet wide. How much wider was the house than the bus?

13. A dog started out with some fleas. Then the dog got rid of 48 fleas. The dog ended up with 99 fleas. How many fleas did the dog start out with?
14. Marcus earned 23 dollars selling papers. He collected 12 dollars from the houses of some of his customers. How many dollars in sales were mailed into the office?

15. Ashley and Todd went on a nature walk. Ashley found 217 rocks and Todd found 52 rocks. How many rocks did they find all together?