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Study of weldment fracture behavior using the J-integral

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The Ohio State University, 1992
STUDY OF WELDMENT FRACTURE BEHAVIOR
USING THE J-INTEGRAL

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

The Ohio State University
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To my parents
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ABSTRACT

Residual stresses are formed in a welded plate due to a nonlinear temperature history caused by a thermal load. High tensile residual stress around a weld area is a major factor for cracking problems in a welded plate. Therefore, analysis of welding residual stress and fracture behavior is important for the evaluation of static or fracture strength of a welded structure.

A fracture assessment procedure and modeling techniques were developed for welded plates. Finite element method was used to predict a residual stress distribution and the J-integral values for combined residual stress and external loads. The thermal model includes welding heat input and deposition of weld beads. A lumped model was developed for multi-pass welding of thick plates.

A crack driving force estimation procedure was developed for a welded plate subjected to combined residual stress and external loads. Finite element formulation was conducted to include the effect of initial plastic strain caused by welding on the J-integral. Residual stress in the welded plate was determined without a crack. Generalized plane strain assumption was used for a residual stress analysis to consider the strain of a cross section in the welding direction. A crack was introduced in the
next step, which resulted in the redistribution of the residual stress field. An external load was superimposed on the redistributed residual stress field to evaluate the J-integral for combined loads.
CHAPTER I
INTRODUCTION

Fusion welding produces a nonlinear thermal loading due to a localized heating which results in a nonuniform temperature distribution in weldments. After welding, residual stresses are created in the structure due to plastic strains caused by localized heating of the structure. As the plate thickness increases the number of weld passes required to complete a full penetration weld also increases. Therefore, the plate is subjected to multiple complex thermal cycles and inelastic strain patterns, creating a more severe and accumulated residual stress distribution through the thickness of the plate. Two major effects of welding residual stresses are distortion and fracture of welded structures.

Residual stress increases crack driving force and largely affects the strength and resistance to brittle fracture of the structure. Tensile residual stress around the weld area is a major factor contributing to cracking and fracture problems in heavy structures with welded thick plates. When there is no data for residual stress distribution, yield stress is often used as a residual stress magnitude for as-welded structures in fracture assessment guidelines. The yield stress value is conservative and
sometimes results in unnecessary repair of discontinuities. It is necessary to accurately assess the magnitude and distribution of residual stresses throughout the thickness of the plate.

There is little data available pertaining to the magnitude and distribution of the residual stresses occurring in multi-pass welding and even less on the distribution of residual stresses through-the-thickness of the plate. Sectioning techniques are used for measuring the through-thickness residual stress distribution [1, 2]. The residual stresses released during sectioning of the test plate into small pieces are calculated by measuring the released elastic strains. The disadvantage of these techniques for measuring residual stresses through-the-thickness is that they are destructive and time consuming. Therefore, numerical analysis methods are necessary for the prediction of residual stresses in thick plate welding to save time and cost.

The thermal and mechanical response of a weldment is a three-dimensional problem that requires a considerable amount of computing time. The computational time required to simulate a multi-pass weld increases in proportion to the number of passes. Therefore it is necessary to develop a simulation model to reduce the computational time without jeopardizing model accuracy. To reduce the computational time, a two-dimensional analysis is usually conducted with appropriate simplifying assumptions depending on the problem. For thin plates, heat flow in the through-thickness direction is neglected to reduce the problem to a two-dimensional plane stress analysis. Thick plate or pipe girth welding can be
reduced to a two-dimensional problem by assuming symmetry along the cross section [3, 4, 5].

Linear elastic fracture mechanics can be used for weldment fracture analysis. Stress intensity factors for a residual stress field and external loading can be linearly superimposed for combined loading. However, the use of linear elastic fracture mechanics is limited to small scale yielding only. Another approach to fracture assessment of weldments is the concept of crack tip opening displacement. CTOD design curves can be used for determination of acceptable sizes of defects in welded plates, which sometimes result in a very conservative analysis.

With the development of finite element techniques, the J-integral can be very effectively used for fracture analysis of structures. The J-integral was developed by Rice [6] as a fracture parameter in elastic-plastic fracture mechanics. Many investigators suggested other energy integral expressions to expand the J-integral to include incremental plasticity or the effect of thermal loading. Wilson and Yu [7] developed a modified expression of the J-integral for a combined thermal and external loading in an elastic body. Blackburn et al. [8] and Aoki et al. [9, 10] have proposed new path-independent integrals, $J^*$ and $\hat{J}$, respectively, for elastic-plastic problems. Shih [11] developed a domain integral expression of the J-integral for a three dimensional crack front under thermal stresses.

Another approach to elastic-plastic fracture mechanics was developed by the General Electric Company. They developed an engineering procedure for the calculation of the J-integral for strain
hardening materials [12], which was further developed for a combined secondary stress and external loading [13]. Another approximation procedure was suggested by Chell [14] to incorporate secondary stresses in the J-integral evaluation.

Residual stresses can be easily incorporated in linear elastic fracture mechanics by the superposition method. However, there is no guideline available to accurately evaluate the crack driving force of a weldment subjected to combined residual stress and external loading in elastic-plastic fracture mechanics, where large plastic deformation is produced around the crack tip. The goal of this research is to develop a guideline procedure for the calculation of welding residual stresses and to determine the J-integral values for combined residual stress and external loading of a welded plate.

The modeling and analysis of residual stress and the J-integral were carried out in three steps:

1) Thermal analysis: A ramp heat input function was used to control the peak temperature and required time to reach a peak temperature around the weld area. The best ramp time was selected from the comparison of numerical results with the experimental results. Heat flow in the welding direction was neglected to reduce the problem to a two-dimensional analysis. Heat flow was limited to a cross section perpendicular to the welding direction.

2) Residual stress analysis: The generalized plane strain assumption was implemented in this study to consider the strain of the cross section in the
welding direction. For thick plate welding, symmetry along the cross section is usually assumed to reduce the problem to a two-dimensional plane strain analysis. Adequate results with the plane strain assumption have been reported by many investigators [3, 4]. According to our preliminary study, the plane strain analysis showed an unrealistically large tensile stress zone which increased as the plate thickness and weld passes were increased.

The idea of grouping weld passes into several layers was suggested by some investigators [1, 15, 16, 17]. In this research, A lumped pass model was developed to reduce the total number of weld passes simulated, thus reducing computational cost. One layer or two layers of weld bead were considered as one lumped pass in the lumped pass model. The results were compared with a non-lumped model and experimental results.

The ABAQUS commercial finite element analysis package was used for the residual stress analysis because of its availability and its ability to solve non-linear problems [18]. It was successfully used for previous welding problems [19]. Temperature history at the surface was obtained by thermocouples; the residual stress distribution was determined using the blind hole drilling method. Experimental temperature fields and residual stresses for 1/2 inch and 1 inch thick plates were compared with the finite element results.

3) The J-integral analysis: A crack was introduced to a specimen subjected to a residual stress field. The introduction of a crack caused redistribution of residual stress around the crack tip. An external loading was
superimposed on the residual stress field to calculate the J-integral for combined loading. A finite element formulation was conducted for a path independent domain integral with combined residual stress and external loading. The J-integral analysis was carried out for a welded thin plate and multi-pass welding of thick plate subjected to combined loading.
CHAPTER II
LITERATURE REVIEW

2.1. Welding Heat Flow Analysis

A fusion welding process produces thermal heat input to a welded plate which causes residual stress and distortion in the plate as a result of complex thermal cycles. The heat transferred to the workpiece through a molten weld pool spreads to the plate through conduction. The analysis of welding heat flow includes heat generation by the welding arc, heat loss by convection and radiation, and heat conduction with boundary conditions and initial conditions.

With an assumption of a quasi-stationary state, a moving coordinate system can be used for thermal analysis of a welding problem with the location of the moving arc as an origin (See Figure 2.1). The early work toward an analytical solution of welding heat flow was conducted by Rosenthal [20, 21]. He derived an analytical solution for a quasi-stationary state with assumptions of constant material properties and a point heat source. Rosenthal's solutions are still widely used for welding heat flow analysis. Analytical studies of welding heat flow were
Figure 2.1 Schematic diagram of welding thermal model
expanded by many investigators for quasi-stationary state or non-stationary state. Wells [22] studied the average size of the fusion zone in single pass butt welds through theoretical analysis and experimental verification using Rosenthal's solution. In the work of Adams [23], engineering relationships were derived for centerline cooling rate and peak temperature distribution in fusion welding with mathematical approximations. Jhaveri [24] developed charts for the effect of plate thickness, thermal properties, and welding parameters on cooling rate and peak temperature distribution of welded plates. In 1965, Christensen [25] conducted theoretical and experimental analysis for fused metal cross section, peak temperature, and cooling rate. He established average arc efficiencies for various welding processes. Eager [26] also conducted theoretical predictions for welding heat flow by using the Gaussian distribution for a moving heat source on a semi-infinite plate. In 1980, the finite source theory was suggested by Tsai [27] as an analytical solution of welding heat flow which incorporated a welding heat source with a skewed Gaussian distribution and finite plate thickness.

Analytical solutions for transient analysis of a welded plate were developed by Naka [28] and Masubuchi [29]. Tsai [30] also developed an analytical solution for transient heat flow in a welded plate to incorporate Gaussian heat distribution using the principle of superposition. A closed form solution was presented for the transient
analysis of thermal behavior in a welded plate using pulsed current gas tungsten arc welding.

A numerical approach to heat flow analysis during welding using finite difference method was investigated by many researchers. Pavelic [31] used a finite difference method to predict temperature distribution in a welded plate using gas tungsten arc welding under two dimensional conditions. The shape of the molten pool was used as a boundary condition. Temperature profiles were compared with experimental data from thermocouples. A specific computer program was developed by Stoeckinger [32] for a finite difference method to predict temperature distribution. A three dimensional program was used for heat transfer analysis of gas tungsten arc welding of aluminum and titanium. Experiments were performed using thermocouples to verify the analytical solution of temperature profiles. The procedure was also used to determine welding variables and their effects on heat transfer across the base metal and weld tooling interface [33].

With the advancement of computer technology, there has been improvement in the computer aided analysis of welding heat flow using the finite element method. In 1973, Hibbitt [34] developed a numerical model for the finite element analysis of a weldment with temperature dependent material properties. An uncoupled thermal and mechanical formulation was derived which treats the thermal and mechanical parts of the procedure separately. Example analyses were carried out for one and two dimensional models. Friedman [35] also conducted finite
element analysis for thermal and mechanical behavior of a welded plate. Two dimensional analysis was performed for a longitudinal butt weld with temperature dependent material properties. The procedure can be applied to plane or axisymmetric analysis of weldments under quasi-stationary conditions. Paley [36] developed a computer program for the heat flow analysis of welds with temperature dependent material properties using actual weld shapes. The program was verified by experimental analysis of a bead-on-plate weld using thermocouples.

Recently, many commercial finite element packages are available for thermal analysis of weldments. Tekriwal [37] used a commercial finite element package, ABAQUS, to develop a heat transfer model for gas tungsten arc welding. Heat input from the welding arc was modeled using symmetric Gaussian distribution and the user subroutine DEFLUX. Parametric studies were conducted for fusion zone and heat affected zone size, which were compared with experimental data. The finite element model was modified later to predict thermal history of welded plate with the gas metal arc welding process [38]. The metal transfer in gas metal arc welding was modeled by adding elements at each time step corresponding to filler metal addition.

The procedure for thermal analysis of a thick plate with a multi-pass welding process was developed by Tsai et. al. [19, 39]. Heat input from welding arc was modeled using a ramp function. Ramp time was determined by comparison between numerical results and experimental data using thermocouples. Bead elements were initially removed and
activated as the actual weld bead was accumulated. Separate programs were developed to monitor temperature profiles and the change of molten pool shapes. Analysis of cooling rate and heat affected zone size was also included in the program.
2.2. Welding Residual Stress Analysis

Residual stresses are formed in a structure by fabrication processes, such as rolling, casting, machining, or welding. In any welding process, non-uniform temperature changes produce inelastic strains. Those inelastic strains cause residual stresses and distortion after welding. Therefore, analysis of welding residual stress includes heat conduction, thermal stress during welding, and determination of final residual stress and distortion.

Many techniques have been developed for measurement of welding residual stresses. Early work on experimental analysis for residual stress was done by numerous investigators. In 1939, Harter [40] performed experiments for residual stress measurement. A relaxation technique was used by Jonassen [41, 42] with strain gages for experimental measurement of residual stresses in 1 inch thick plate with a double V or single V groove. Wilson [43] also conducted an experimental analysis of a butt welded plate using a sectioning method with strain gages. In 1963, Burdekin [44] measured residual stress distribution on cylinders with circumferential butt welds in the as-welded condition and after heat treatment using the Gunnert gauge method. Another experimental measurement was conducted for welded I beam connections by Dawes [45]. Both elastic and elastic-plastic strains were measured by the change of gage length of 20 mm.
One of the most widely used techniques for residual stress measurement is the stress relaxation method by sectioning or hole drilling. Alpsten [46] experimentally investigated the distribution of initial and final welding residual stresses in heavy column shapes built up by welding of plates using the sectioning method. The residual stress distribution, through thickness, was measured by the change of strains before and after sectioning and slicing of the specimen using an extensometer. Both initial stresses caused by manufacturing and welding residual stresses were measured through the thickness of the plate. The effects of fabrication procedure, geometry of the plates, and weld procedures on the residual stress distribution were also studied. In 1972, Bjorhovde [47] reported an experimental investigation of the distribution of residual stresses in heavy steel plates. The plate thickness varied from 1 1/2 inch to 6 inch. Weld beads were placed along the center or along the edges, to simulate component plates of welded, built-up shapes. A sectioning method was employed for the residual stress measurement using a mechanical extensometer. The effect of plate geometry, manufacturing method, and welding on the residual stress distribution was also studied. Another method for measuring three dimensional residual stresses was suggested by Ueda [2] based on the theory of inherent strain using sectioning methods.

Experiments were also performed for the effect of residual stresses on brittle fracture or fatigue strength. Robellotto [48] reported some experimental study of residual stress measurement and experiments for
fatigue crack growth in a residual stress field. Ikeda [49] studied the brittle fracture initiation characteristics of welded high strength steel. The specimens were welded with various heat input and evaluated by a deep notch test. The effect of welding residual stress on the brittle fracture initiation temperature was investigated.

Over the past 20 years, many investigators have developed analytical and numerical models to predict welding temperature and residual stresses. Early studies of theoretical solutions for welding residual stresses were limited to simple models because of difficulty in the mathematical formulation of welding problems. These studies generally used closed form analytical solutions for the heat source, such as point or line heat source and ignored distribution of heat flux over a weld bead. Another attempt was made by Masubuchi [50] for an analytical method to evaluate welding residual stress and distortion, which could be applied in more practical problems. The concept was based on an analogy between the theory of residual stresses and the theory of vortex motion in fluid dynamics. Theory of residual stress was developed from the treatment of residual stress as a boundary problem of elasticity similar to the wing theory.

With the development of powerful computers and numerical techniques, it became possible to analyze more complicated welding problems and various weld processes. One of the first numerical thermal-mechanical models for welding was developed by Hibbitt [34]. He analyzed gas metal arc welding with a two dimensional thermal
model and a plane strain assumption using uncoupled finite element method. Basic assumptions for uncoupling of the thermal and mechanical analysis are:

1. The dimensional changes during welding are negligible.
2. Mechanical work done during welding is insignificant compared to the thermal energy changes.

Hibbitt showed both assumptions were reasonable for the modeling of welded steel plate [34].

During the past 20 years, many researchers have studied residual stress analysis of welding problems. Argyris [3] studied thermo-mechanical response of bead on steel plate using finite element method. The two dimensional heat flow analysis was conducted with the transient diffusion package SMART II. The mechanical response of a welded plate was calculated by thermoelastic-viscoplastic analysis with a plane strain assumption. Volume changes and latent heat effects during recrystallization were included in the analysis. In 1989, Free [4] analyzed residual stress in multi-pass weldments with the finite element package PAFEC. A typical cross section was modeled assuming axisymmetric or plane strain condition. Room temperature material properties were used for both the thermal and mechanical analysis. Each weld pass was modeled individually rather than grouped together in layers. Experimental verification was performed using high temperature strain gages, hole drilling method, and extensometer. Through thickness residual stresses were measured by the sectioning
method. A coupled, thermal-mechanical analysis was performed by Mahin [51] for gas tungsten arc welding of a circular 304L stainless steel specimen. Finite element package PASTA2D was employed in the numerical analysis. A two dimensional analysis was conducted with an axisymmetric assumption. In his work, neutron diffraction techniques were used to experimentally measure the residual elastic strains through the thickness up to 0.27".

As the plate thickness of a structure increases, the number of weld passes required to complete a welded joint also increases, which requires extended computing time for the analysis of thick plates. The idea of saving computer time by grouping the actual weld passes in a large lumped pass was attempted by many researchers. Ueda [1] analyzed residual stress in a multi-pass welded pressure vessel. The sectioning method was used with wire strain gages to read the change of elastic strains before and after sectioning. The finite difference method was employed for heat conduction analysis, and the finite element method was used for residual stress analysis. Two layers of actual weld passes were regarded as one lumped pass without loss of accuracy.

In the work of Rybicki [15], a lumped model was used for a residual stress analysis of multi-pass, girth-butt welded pipes. Two dimensional analysis was conducted with an axisymmetric assumption. Temperature fields were calculated using point heat source theory for an infinite solid. Finite element method was used for stress analysis with temperature dependent mechanical properties. A mechanical model
with a lumped pass of each weld layer was suggested in this research as follows: The highest temperature for selected points was determined by the temperature distribution from each weld pass. These maximum temperatures were used for the thermal loading of mechanical analysis. Further attempts grouped layers of weld passes into a large lumped pass, but the result of residual stresses did not agree with the experimental data. In 1990, Leung [16, 17] developed a modeling technique for thermal and mechanical analysis of a welded plate using a commercial finite element package ABAQUS. AISI 316L stainless steel specimens were used for the analysis with submerged arc welding. Uncoupled analysis was employed and a Gaussian heat flux distribution was used for the weld pool. A two dimensional cross section perpendicular to the welding direction was used for both thermal and residual stress analysis. Several techniques for lumping of weld passes for stress analysis were attempted.
2.3. Weldment Fracture Analysis

Tensile residual stresses created around a weld area cause significant reduction in resistance to fracture. In a welded structure, fractures may occur from flaws smaller than the critical size due to tensile stresses around the weld area. Many theoretical and experimental approaches were performed for fracture and fatigue problems in a welded structure.

2.3.1 Linear Elastic Fracture Mechanics

A simple and widely used method for fracture assessment of a weldment is linear elastic fracture mechanics. Early studies for the effect of residual stress on the brittle fracture were conducted by many investigators, including Kihara [52], Masubuchi [53], and Kammer [54]. Masubuchi [53] analyzed the effects of residual stress on brittle fracture of weldments using the strain energy released during the crack opening stages in a residual stress field. The released strain energy was calculated from the residual stress before cracking and dislocation or crack opening. A schematic diagram for the effect of residual stress on unstable fracture is shown in Figure 2.2 [54]. The rate of strain energy release without residual stress is shown as line OA. When a crack is located in a residual stress field, the rate of strain energy release increases rapidly due to the effect of tensile residual stress as shown
Figure 2.2 Schematic diagram for effects of residual stress on unstable fracture [54]
curve OR. As the crack length increases, the effect of residual stress on the rate of energy release decreases due to compressive residual stress away from the weld area. For a given critical value of energy release rate, \( G_c \), it can be shown from the figure that failure of a structure occurs from a flaw smaller than the critical flaw size.

2.3.2. Crack Tip Opening Displacement (CTOD)

Another approach to fracture assessment of weldments is the concept of crack tip opening displacement. In 1963, Wells [55] proposed crack opening displacement as a characterizing parameter for fracture behavior in the vicinity of a crack. He employed the concept of crack opening displacement as the parameter for crack extension, which can be applied beyond general yielding. In his work, he recognized that under plane stress conditions and below general yielding, crack opening displacement was proportional to the crack driving force and inversely proportional to the yield strength of the material. Beyond general yielding, crack opening displacement was proportional to the plastic strain. Dugdale [56] advanced this CTOD concept by providing a closed form solution for plane stress conditions. He derived an equation for the length of the plastic zone using a yield strip model. British Standard PD6493 [57] provides guidance on acceptable levels for defects in fusion welded joints using CTOD design curves. The document suggests a
method for assessment of defects for brittle fracture and failure by fatigue based on fitness-for-purpose concepts. It considers various types of weld defect and their effect on fracture and fatigue.

2.3.3. The J-integral

The J-integral was developed by Rice [6] as a fracture parameter in elastic-plastic fracture mechanics. A path independent integral was derived for a material represented by deformation plasticity. The J-integral was interpreted as the rate of change in potential energy for nonlinear materials. A method to evaluate the J-integral was suggested by Bucci [58] which was applicable for test specimens. Begley [59] showed that the critical value of the J-integral could be used as a fracture criterion.

The fracture assessment procedure using the J-integral can be simplified by applying the J-design curve. Turner [60, 61] developed a J-design curve based on analytical studies, which was similar to the concept of the CTOD design curve. The data used for the design curve was based on finite element calculations using incremental plasticity.

For a finite element technique of a J-integral evaluation, the virtual crack extension method is generally used [62, 63, 64]. Parks [62, 63] suggested the use of the virtual crack extension method for a finite element formulation based on the energy release rate during crack
extension. The method requires only one elastic-plastic finite element solution. Variations in the potential energy can be calculated by changing nodal point positions of the elements forming selected contours for the calculation of the J-integral.

An engineering approach to the J-integral analysis for elastic-plastic material was developed by the General Electric Company. They developed an engineering procedure for the calculation of the J-integral in strain hardening materials [12], which was further developed for a combined secondary stress and external loading [13]. Another approximation procedure was developed by Chell [14, 65] to incorporate secondary stresses in the J-integral evaluation. Chell suggested a J estimation procedure for combined mechanical, thermal, and residual stresses. An approximation solution for the J-integral was proposed, based on material dependent functions and linear elastic value of the J-integral.

Other investigators developed energy integral expressions to expand the J-integral and include incremental plasticity, thermally loaded structures, surface traction, or dynamic fracture. In 1979, Wilson [7] derived an expression for the calculation of stress intensity factor in a thermally stressed body. The J-integral was used to derive the stress intensity factor using a relationship between the J-integral and the stress intensity factor for linear elastic fracture mechanics. Because the J-integral loses its path independency in a thermal stress field, a new formulation was developed to calculate the J-integral value at the crack
tip, which was used for the calculation of stress intensity factor. For a non-linear elastic material, Blackburn [8, 66] proposed a path independent integral, $J^*$. Rice's $J$-integral was modified to include creep or unloading of a elastic-plastic material. Atluri [67] performed another work to extend the $J$-integral to include finite and infinitesimal elasticity, rate-dependent incremental plasticity, and elasto-viscoplasticity. The conservation law was generalized to finite deformations when body force, inertia, and arbitrary crack face tractions were present.

In 1986, Shih [11] developed a domain integral expression for the $J$-integral in a thermally stressed body, based on an energy integral in dynamic fracture [68]. A general three dimensional expression was derived based on a line integral expression for the energy release rate for a deformation theory solid. Two dimensional and axisymmetric specialization of the $J$-integral were also presented. Test calculations were performed for thermo-elastic and thermo-elastic-plastic problems. For an elastic-plastic material, Kishimoto and Aoki [9, 10, 69] developed a path independent integral $\hat{J}$ as an energy flux rate during crack extension. This integral was presented as an extension of the $J$-integral by Rice to include the effect of plastic deformation, body force, thermal strains and inertia of material. A path independent line integral was derived based on the energy balance during crack extension. They concluded that the $\hat{J}$ integral was an energy release rate of any material due to crack extension and had physical significance as a crack driving force. When the elastic-plastic behavior of a material is represented by
deformation plasticity theory, the $\mathbf{J}$ integral is the same as the $J$ value at the crack tip.
CHAPTER III
RESIDUAL STRESS ANALYSIS OF WELDED PLATES

Finite element models were developed for thermal and residual stress analysis. They are used to evaluate temperature and thermal stress history of a welded plate. The heat transfer and stress analysis of weldments are nonlinear, time dependent, and require large amounts of computer time. Though computer technologies are greatly advanced, reducing the computational time required, it remains a major user task to reduce the computational time for finite element analysis of welding problems. Therefore, a main concern in model development is to develop a finite element model for multi-pass welding of thick plates which reduces computational time without sacrificing analysis accuracy. Computer time for the analysis of a multi-pass welding is directly proportional to the number of weld passes, thus the lumped model which groups several weld passes into a large lumped pass is used for both thermal and residual stress analysis. The accuracy of the model may be checked by temperature measurements using thermocouples and hole drilling residual stress measurements at the surface of the plate.
An uncoupled thermo-mechanical analysis was performed. The thermal analysis was performed and the temperature history was stored, for later use as a thermal loading input in the subsequent stress analysis. Assuming symmetry along the cross section, the welding problem was reduced to a two dimensional analysis. Generalized plane strain elements were used in the mechanical analysis for the strain of a cross section in the welding direction. The finite element mesh and time steps were identical for both heat transfer and stress analysis.

The material used in the model development of residual stress analysis was ASTM A36 mild steel. Temperature dependent thermal and mechanical properties are shown in Figure 3.1 [70]. Mechanical properties are assumed constant above 1400°F.

The size and number of the finite element mesh greatly affects the accuracy and computational cost of the analysis. Smaller meshes are required for weld areas to obtain accurate stress distribution, since rapid change of temperature distribution causes large tensile stresses and most cracking problems. From previous experience [5], the element size at the weld area should be smaller than or equal to the depth of the weld bead. The smallest mesh sizes used were 0.05 inch for a 1/2 inch thick plate, 0.0625 inch for a 1 inch thick plate, and 0.125 inch for a 2 inch thick plate. The mesh size was increased outside the weld area because the temperature field tends to change linearly. The largest mesh sizes used were 0.25 inch for a 1/2 inch thick plate, 0.5 inch for a 1 inch thick plate, and 1 inch for a 2 inch thick plate. Since a butt welded plate is symmetric
about the weld line, only one half of the plate was modeled. Two-
dimensional finite element meshes used for both heat transfer and stress
analysis are shown in Figure 3.2 - 3.6. The area with oblique lines
represents layers of weld bead, and was determined by the actual weld
bead shape. Eight-noded rectangular elements were used for both
analyses. The total number of elements for each specimen was 151, 192,
198, and 172 for 1/2 inch thick plate, 1 inch thick plate with a double V
groove, 1 inch thick plate with a single V groove, and 2 inch thick plate
with a double V groove, respectively. The development of the finite
element model required three parts: thermal model, mechanical model,
and lumped pass model.
Figure 3.1 Temperature dependent material properties of A36 steel
Figure 3.2  Finite element mesh for the 1/2 inch thick plate

Figure 3.3  Finite element mesh for the 1 inch thick plate with a double V groove
Figure 3.4 Finite element mesh for the 1 inch thick plate with a single V groove

Figure 3.5 Finite element mesh for the 2 inch thick plate (the first lumped model)

Figure 3.6 Finite element mesh for the 2 inch thick plate (the second lumped model)
3.1. Thermal Model

Since a three dimensional problem was reduced to a two dimensional analysis by assuming symmetry along the cross section, heat flow was limited to the cross section perpendicular to the welding direction. Fusion zone elements were divided into groups as shown in Figure 3.2 - 3.6 to numerically represent layers of weld beads. These fusion zone elements were initially removed and activated for each corresponding weld pass to simulate the deposition of weld beads. A heat flux for each bead was applied to the top surface of these newly activated bead elements.

Time increments used in the analysis were dependent on the magnitude of the temperature gradient. Time increments were small enough to describe the thermal history of the model accurately. The maximum allowable temperature change between time increments was limited to 200°F. The temperature data for each time increment was saved for thermal loading to use in the mechanical analysis. Elapsed time between weld passes was assumed to be 600 seconds, comparing favorably with the experimental data. An arc efficiency of 85 percent was used for the net heat input to the plate for the GMAW process [25]. Heat losses or gains from phase transformation were neglected. A free convection boundary condition was assumed for both top and bottom surfaces of the plate. A heat convection coefficient of 1.0 E-5 BTU/in² F was used for all surfaces.
A heat input to the plate was calculated by the following equation:

\[ Q = \frac{60\eta EI}{1055Vb} \]  

(3.1)

where

- \( Q \) = heat input, BTU/in\(^2\)
- \( \eta \) = arc efficiency
- \( E \) = arc voltage, volts
- \( I \) = arc current, amps
- \( V \) = welding speed, inches/min
- \( b \) = width of bead element deposited, inches
- 60 = conversion factor, minute to seconds
- 1055 = conversion factor, Joules to BTU

(E, I and V are measured values from experiments)

A ramp heat input function was developed to apply heat flux to the model gradually with variable ramp times. The ramp heat input model was used to avoid numerical convergence problems caused by an instantaneous increase in temperature near the fusion zone. It also includes the effect of a moving arc in the two-dimensional plane. From Eqn. (3.1), the magnitude of \( \frac{Q}{60} \) was used as the amplitude of heat flux with the magnitude of \( V \) as heat input time. The general amplitude-time curve for the ramp input model is shown in Figure 3.7. It shows the variation of heat flux with time. Heat flux was uniformly distributed over the length of each weld layer (b). It was increased linearly up to \( q_{\text{max}} \) during ramp time \( t_1 \) and decreased from \( q_{\text{max}} \) to zero during time \( t_3 \).
same time period was used for both $t_1$ and $t_3$. Actual welding time for the arc to travel across the unit thickness of the finite element model was $t_1 + t_2$. Ramp times ($t_1$ and $t_3$) from 10% to 100% of the total actual heat input time ($t_1 + t_2$) were considered in the study of ramp time effect for a 1/2 inch thick plate. The total area under these various ramp heat curves is constant to insure the same total heat input to the model.

Thermal analysis was conducted for a 1/2 inch thick plate to select the best ramp time for a heat input model. Experimental results of temperature profiles from thermocouples were compared with numerical data. Figure 3.8 shows the temperature profiles at 1/4 inch from the weld centerline for the first pass of the 1/2 inch plate with the ramp heat input time of 20, 50, and 100 percent of actual heat input time. The general trend shows larger ramp times to slightly decrease peak temperatures and cooling rates. The temperature profile for the large ramp time shifted to the right as the ramp time increased, indicating more time was required to reach peak temperature. The maximum difference in times to reach peak temperature was 2 seconds. The maximum temperature difference during the heating cycle was 180°F. The effect of ramp time on temperature profile during the cooling cycle was small compared to the heating cycle.

Numerical temperature profiles were compared with experimental data at 1/2 inch and 1 inch from the centerline, at the top surface of the plate as shown in Figure 3.9 and Figure 3.10. The general trend of the ramp time effect was the same as the temperature profiles at 1/4 inch from the centerline. The effect of ramp time was less dominating as the
distance from the centerline increased. As shown in the figures, a ramp time of 20% of the actual weld time ($t_1+t_2$) provides the best correlation with the experimental data. Consequently, this value was used for the remainder of the analysis. Ramp times affect the high temperature zone around the weld bead only. Therefore, residual stresses are not sensitive to a ramp time due to low mechanical properties at high temperature, such as Young's modulus or yield stress.
Figure 3.7 Ramp heat input model
Figure 3.8 Effect of ramp times for the first pass of 1/2 inch thick plate (1/4 inch from the centerline on the top surface)
Figure 3.9 Effect of ramp times for the first pass of 1/2 inch thick plate (1/2 inch from the centerline on the top surface)
Figure 3.10 Effect of ramp times for the first pass of 1/2 inch thick plate (1.0 inch from the centerline on the top surface)
3.2. Mechanical Model

Residual stresses are the final state of internal stress caused by permanent plastic strains accumulated during multiple heating and cooling cycles of a welding process. Therefore, a complete history of the temperature distribution throughout the plate is required for the calculation of residual stresses. A generalized plane strain theory was used for the stress analysis to consider the strain of the cross section in the direction of welding [71]. The generalized plane strain theory assumes that the model lies between two initially parallel planes in the thickness direction (welding direction of the model). These planes may move as rigid bodies with respect to each other, which cause axial strain. This strain varies linearly throughout the cross section with respect to position in the planes. ABAQUS ten node generalized plane strain elements were used in the stress analysis [18]. The same two extra nodes were used for all elements for the axial strain and rotation of the cross section.

For the modeling of a multi-pass welding process, fusion zone elements were incrementally activated following the pass sequence to model the deposition of weld beads. The temperature history obtained from the thermal analysis was used as a thermal loading into the structural model to calculate thermal strains and stresses for each time increment. These thermal strains and stresses were accumulated to produce the final state of residual stress. Free boundary conditions were assumed for all the free surfaces except at the center line of the cross
section, where symmetry conditions existed. Volume changes due to phase transformations were neglected. Initial stresses and strains were assumed as zero, since the test specimens were stress relieved before welding.
3.3. Lumped Model

After mechanical constrains are built up by initial weld passes, following passes affect residual stress distribution in local area only. Based on this phenomena, the idea of grouping one or two layers of weld passes into one lumped pass was used to reduce the computing cost. Lumped pass models were used for both thermal and stress analysis of 1 inch and 2 inch thick plates. Lumped pass models can be very efficiently used to reduce computational cost with reasonable results of residual stress field, especially for thick plates, which require many weld passes for a complete joint [1, 15].

Two models were used for the analysis of the 1 inch thick plate. The first model (not lumped) assumed the heat flux of each individual pass was distributed over the top surface of one layer of weld bead. The result of this temperature analysis for each pass was used as thermal loading in the stress analysis. The analysis was repeated for each pass to complete one layer. The second model (lumped pass model) used lumped passes. Each layer of weld bead was considered as one lumped pass in this model. Heat fluxes for each pass in that layer were added and distributed over the top surface of the layer. A total of eleven weld passes were lumped into six passes in the 1 inch thick plate with a double V groove; seventeen passes were lumped into seven passes in the 1 inch thick plate with a single V groove as shown in Figure 3.3 and 3.4.
Three models were used for the analysis of the 2 inch thick plate with a double V groove. The first model (not lumped) analyzed each weld pass individually. The second model considered each layer of weld bead as one lumped pass. Thirty two weld passes were lumped into fourteen passes as shown in Figure 3.5. The third model assumed two layers of weld bead as one lumped pass. Thirty two weld passes were reduced to eight lumped passes as shown in Figure 3.6. The results of residual stress distribution for three models were compared with experimental data in section 3.6.
3.4. Experimental Verification

Experiments were conducted for temperature and residual stress measurements to verify numerical results. Test specimens were prepared by butt welding ASTM A36 steel plates. A single bevel preparation was used for the 1/2 inch thick plate, and both single and double V grooves were used for the 1 inch thick plate. The sizes of the specimens were 15" x 8" for the 1/2 inch thick plate and 12" x 18" for the 1 inch thick plate. A gas metal arc welding system and an automatic traveling unit were combined to make an automatic welding system. To accurately measure total power, a volt-meter and ampere-meter were attached to the welding system between earth shunt and torch cable. An AWS E70S-3 type electrode was used as recommended by the American Welding Society. The shielding gas was composed of 98% Ar - 2% O₂ gas. A spray transfer mode was obtained during welding. Pass sequences and welding parameters for each specimen were shown in Table 3.1.

The temperature histories and residual stresses were measured at several points near the weld on the surface of the test specimens. Temperature profiles were obtained through the use of thermocouples, and residual stress distribution with strain gages attached to a data acquisition system developed at the Department of Welding Engineering, Ohio State University [19]. The results of these experiments were used for the verification of temperature and residual stress distributions calculated by the finite element analysis.
A data acquisition system was used to monitor temperature changes during welding and strain relaxation during blind hole drilling measurement. Four major components of this system were: an IBM AT computer, an HP3497A data acquisition/control unit, an IBM graphic printer, and an HP6214B power supply as shown in Figure 3.11. The overall maximum speed of the system was 10 channels per second. Generally, this speed was sufficient to monitor temperature and strain changes during welding.

Surface temperature changes were measured for 1/2 inch and 1 inch thick specimens using ANSI type K thermocouples. Thermocouples were attached after locally polishing the surface of the specimens. All thermocouples were mounted on the surface of the specimen near the weld centerline, where large variations of temperature profiles were expected. Quantitative data at these locations was collected via the thermocouples and data acquisition system and compared with finite element results.

Residual stresses were measured for 1 inch thick specimens with double V and single V groove. The plates were annealed before welding to minimize the influence of initial stresses due to prior manufacturing processes. The blind hole drilling method was used to experimentally determine the surface residual stresses in the weldment [72, 73]. The diameter of a drilled hole was 1/16 inch. The strain gages were designed and manufactured for the blind hole drilling method by Measurement Group, Inc. (gage pattern: EA-06-062RE-120). A high speed air turbine drill was used to reduce drilling induced stresses. The strain relaxation
data used to convert strains to residual stresses was obtained when the hole depth reached 1.2 times of the hole diameter, as recommended by the strain gage manufacturer. To obtain accurate blind hole drilling results, that reflect only the residual stress from welding, the drilling must be performed slowly with minimal pressure, otherwise additional thermal and mechanical strains will be superimposed during the drilling process. To achieve this goal, the drilling process usually took approximately 6 minutes to complete. Residual stress measurements were obtained at locations around the middle transverse cross section of the specimens.
Table 3.1 Pass sequence and welding parameters of each pass

<table>
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<tr>
<th>Pass Sequence</th>
<th>Pass No.</th>
<th>Welding Parameters</th>
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<th>Voltage</th>
<th>Speed</th>
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Figure 3.11 Data acquisition system for temperature and strain measurement
3.5. Thermal Analysis of Thick Plates with Multi-Pass Welding

A thermal analysis was conducted for 1/2 inch, 1 inch, and 2 inch thick plates. Eqn. (3.1) in section 3.1 was used to determine the magnitude of heat flux and heat input time. Ramp heat input times, length of weld layer (b), and magnitudes of heat flux (q\text{max}) for 1/2 inch and 1 inch thick plates are shown in Table 3.2. Heat inputs for the top and bottom groove of the 2 inch thick plate were assumed similar to the heat input of the 1 inch thick plate with a single V groove. A ramp time consisting of 20 percent of the total heat input time was used, which was determined from a study of the effect of ramp time on the temperature profile. Eight-node rectangular elements were used for the two dimensional thermal analysis. The temperature dependent material properties of ASTM A36 mild steel were used as shown in Figure 3.1. An uncoupled thermo-mechanical analysis was performed. The thermal analysis was initially performed and the temperature history was stored and used as the thermal loading for the stress analysis. Each welding pass modeled in the thermal analysis was divided into small time increments. The maximum temperature change between any two time increments was less than 200°F.

Figure 3.12 shows the numerically calculated temperature versus time plots for the first pass of the 1/2 inch thick plate at 1/4 inch from the centerline on the top surface. The maximum temperature was 690°F. Experimental results from the thermocouples were plotted for comparison. The results of numerical analysis showed higher peak temperature and
lower temperature during the cooling cycle. The difference between numerical and experimental results for the maximum temperature was 50°F with 7 percent deviation. The temperature profile for the same weld pass at 1/2 inch from the centerline on the top surface is plotted in Figure 3.13. The differences between the peak temperatures calculated by the thermal analysis and experimental results were less than 10°F (2 percent error). As shown in the figures, the temperature profiles from the numerical analysis have good correlation with the experimental data near the weld area. This is important since high temperature in this area usually results in large plastic strains and thermal stresses.

Figure 3.14 shows the temperature profiles for the first pass of the 1 inch thick plate with a double V groove at 1/2 inch from the centerline on the top surface. The experimental data has a higher peak temperature and lower cooling rate than the numerical result. The difference of peak temperatures between two results was 10 percent. The temperature history at 3/4 inch from the centerline on top surface is shown in Figure 3.15. The maximum deviation of the peak temperature between experimental and numerical results is less than 3 percent. As shown in the figures, the temperature distribution at the area far from the weldline agree better with the experimental results. The temperature data for each increment was saved and used as a thermal loading to the stress analysis.
Table 3.2 Ramp heat input times and maximum heat flux

A) 1/2 inch plate

<table>
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<th>t2 (sec)</th>
<th>t3 (sec)</th>
<th>b (inch)</th>
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B) 1 inch plate with double V groove

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<th>t3 (sec)</th>
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C) 1 inch plate with single V groove

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Figure 3.12 Temperature profiles for the first pass of the 1/2 inch thick plate (1/4 inch from the centerline on the top surface)
Figure 3.13 Temperature profiles for the first pass of the 1/2 inch thick plate (1/2 inch from the centerline on the top surface)
Figure 3.14 Temperature profiles for the first pass of the 1 inch thick plate (1/2 inch from the centerline on the top surface)
Figure 3.15 Temperature profiles for the first pass of the 1 inch thick plate (3/4 inch from the centerline on the top surface)
3.6. Residual Stress Analysis of Thick Plates with Multi-Pass Welding

Residual stress analysis was conducted for 1 inch and 2 inch thick plates using temperature histories obtained from thermal analysis. The same finite element meshes and time steps were used for both thermal and stress analysis. ABAQUS ten node generalized plane strain elements were used for the stress analysis [18]. The blind drilling hole method was used for residual stress measurements around the weld area, where large tensile stress and transition to compressive stress were expected. The shape of the weld bead and welding parameters are shown in Table 3.1. Temperature dependent material properties are shown in Figure 3.1.

Figure 3.16 shows the longitudinal (welding direction) stresses at the top surface of the 1 inch thick plate with a double V groove. The first model (not lumped) analyzed every welding pass sequentially. High tensile stresses exist around the weld area with a maximum value of 43 ksi and a tensile stress zone of about 0.8 inch as shown in the figure. The stress gradually changed to a compressive stress at a distance farther from the centerline. The maximum compressive stress was about -10 ksi. A total of eleven weld passes were lumped into six passes in the lumped pass model. The result showed tensile yield stresses around the weld centerline, but the tensile zone was increased by about 1/3 inch in the lumped pass model. The lumped pass model showed higher longitudinal compressive stresses outside the weld area than the non-lumped model. The results of both
analyses were compared with experimental data. Generally, experimental data was between the two analytical predictions.

Transverse (perpendicular to welding direction) stresses at the top surface of the 1 inch thick plate with a double V groove are plotted in Figure 3.17. The first model (non-lumped) showed higher tensile transverse stress around the weld centerline than the lumped pass model. The tensile stress zone was slightly increased in the lumped model. The tensile stresses were gradually decreased to zero at 1.5 inch from the centerline. Experimental results showed a larger tensile transverse stress zone than the results of the numerical analysis.

Figure 3.18 and 3.19 shows longitudinal and transverse stresses at the top surface of the 1 inch thick plate with a single V groove. Seventeen passes were reduced to seven lumped passes in the lumped pass model. The lumped pass model showed slightly lower tensile longitudinal stresses around the weld centerline, but the longitudinal tensile stress zone was increased by 1/2 inch in the lumped pass model. Transverse stresses were also reduced in the weld area and the tensile zone was increased in the lumped pass model. The general trend of the longitudinal stress distribution was similar to the results of the 1 inch thick plate with a double V groove, indicating that the joint geometry on a flat plate is not a major factor influencing the longitudinal stress distribution on the surface.

There was good agreement between the theoretical analysis and the experimental results for both transverse and longitudinal stresses in the 1 inch thick plates. This provided confidence in the application of the
generalized plane strain theory to the calculation of residual stresses in multi-pass weldments. Also, the stress distributions obtained by the lumped pass model compared favorably with the results of the non-lumped model and experiments. Therefore, further reduction of weld passes were tested in the 2 inch thick plate with a double V groove.

The longitudinal stresses at the top surface of the 2 inch thick plate with a double V groove are shown in Figure 3.20. Thirty two actual weld passes were reduced to fourteen lumped passes in the second model, and eight lumped passes in the third model. The tensile stress zone was increased by 1/2 inch in the second model, and 5/8 inch in the third model compared to the first (non-lumped) model due to the increase of plastic zone caused by large heat input in the lumped models. Maximum tensile stresses were reduced by 5 ksi in the third model. General trends were similar to the 1 inch thick plates, but tensile zones increased when compared to the 1 inch thick plates. The transverse stresses at the top surface of the 2 inch thick plate with a double V groove are shown in Figure 3.21. Maximum tensile stresses were reduced by 7 ksi in both lumped pass models compared to the first model. Tensile stress zone increased for the lumped models, which was similar to the longitudinal stress distributions.

Figure 3.22 and 3.23 show the longitudinal and transverse stress variations at the weld centerline through the thickness of the 2 inch thick plate with a double V groove. General trends were similar for all three models, but stresses were decreased at both surfaces in lumped pass
models. The tensile transverse stresses at both surface areas were balanced by the compressive stresses in the middle of the plate. Both transverse and longitudinal stress distributions showed similar shape, but transverse stresses in the middle of the plate were shifted to compressive stresses. Maximum tensile stress existed near the surfaces of the plate and was gradually decreased in the middle of the plate.

Figure 3.24 - 3.27 illustrate stress contours for the cross sections near the weld centerline for the 1 inch thick plates with a single and double V groove. The comparison of the figures yields useful information that the trends of the stress distribution through the thickness are largely affected by the shape of the weld groove. The plate welded with a double V groove showed high tensile longitudinal stress at the bottom of the plate and high tensile transverse stress at the top of the plate, but the plate welded with a single V groove showed high tensile longitudinal stress at the top of the plate and high tensile transverse stress at the bottom of the plate. The stress distribution through the thickness is very useful for fracture assessment of weldments. The magnitude and type (tensile or compressive) of residual stress around a crack tip should be considered in the fracture analysis of a welded structure.

Thermal and mechanical models were developed for fillet welding of a plate. Specimen geometry and finite element mesh are shown in Figure 3.28. Generalized plane strain elements were used for mechanical analysis. A single pass weld was laid on each side of the plate as shown in
the figure. Welding parameters for the gas metal arc welding used in the analysis were as follows:

- Current: 190 Amp.
- Voltage: 25 Volts
- Welding speed: 8.0 ipm
- Weld sequence: Right toe - Left toe

The same modeling techniques and procedure for the thermal and stress analysis of butt welding of thick plates were used. Gap elements were used to consider the gap between the horizontal and vertical plates. Figure 3.29 and 3.30 show longitudinal (Sz) and transverse (Sx) stress distributions at the centerline and fillet toe cross sections. Through thickness stress (Sy) was almost negligible compared to other stress components. High tensile transverse stress (Sx) was shown at the top surface of both fillet toe cross sections, which could be a major factor for initiation and propagation of a longitudinal fillet toe crack.
Figure 3.16 Longitudinal stress at the top surface of the 1 inch thick plate with a double V groove
Figure 3.17 Transverse stress at the top surface of the 1 inch thick plate with a double V groove
Figure 3.18 Longitudinal stress at the top surface of the 1 inch thick plate with a single V groove
Figure 3.19 Transverse stress at the top surface of the 1 inch thick plate with a single V groove
Figure 3.20 Longitudinal stress at the top surface of the 2 inch thick plate with a double V groove
Figure 3.21  Transverse stress at the top surface of the 2 inch thick plate with a double V groove
Figure 3.22 Longitudinal through-thickness stress for the 2 inch thick plate with a double V groove at weld centerline
Figure 3.23 Transverse through-thickness stress for the 2 inch thick plate with a double V groove at weld centerline
Figure 3.24 Transverse stress contour for the 1 inch thick plate with a double V groove

Figure 3.25 Longitudinal stress contour for the 1 inch thick plate with a double V groove
Figure 3.26 Transverse stress contour for the 1 inch thick plate with a single V groove

Figure 3.27 Longitudinal stress contour for the 1 inch thick plate with a single V groove
Weld Sequence:
Right (A) - Left (B)

Figure 3.28 Finite element mesh for the fillet weld specimen
Figure 3.29 Through-thickness distribution of transverse stress (Sx) for fillet weld
Figure 3.30 Through-thickness distribution of longitudinal stress (Sz) for fillet weld
CHAPTER IV
THE J-INTEGRAL ANALYSIS FOR COMBINED
PRIMARY AND SECONDARY STRESSES

In this chapter, the J-integral was evaluated for a material subjected to combined primary and secondary stresses. The virtual crack extension method [62, 63, 64] is generally used in finite element calculations, and the decrease in potential energy is calculated with respect to the virtual crack advancement. The J-integral loses its path-independency when there is an inelastic strain or a crack surface traction. Shih [11] developed an alternative J-integral expression for a thermally stressed body. A commercial finite element package ABAQUS was used in this research, incorporating Shih's procedure for the calculation of J-integral values for combined external and thermal loads [71]. Kishimoto and Aoki [9, 10] developed a path independent integral $\hat{J}$ as an energy flux rate for an elastic-plastic material during crack extension. This integral was suggested as an extension of the J-integral by Rice and includes the effect of plastic deformation, body force, thermal strains and inertia of the material.
4.1. Theoretical Background

4.1.1 The J-integral

The J-integral was proposed by Rice [6] as a strain energy release rate in nonlinear elastic materials. Consider the two dimensional nonlinear elastic body shown in Figure 4.1. The potential energy of the body is given as:

$$\Pi(a) = \int_A W dA - \int_{\Gamma} T_i u_i ds$$

(4.1)

where, \( W \) : Strain energy density,

$$W = W(\varepsilon) = \int_0^\varepsilon \sigma_{ij} \varepsilon_{ij}$$

(4.2)

\( \Gamma \) : a contour for the line integral

\( A \) : the area enclosed by \( \Gamma \) as shown in Figure 4.1.

Differentiation of Eqn. (4.1) by crack length, \( a \), results in

$$\frac{d\Pi}{da} = \int_A \frac{dW}{da} dA - \int_{\Gamma} T_i \frac{du_i}{da} ds$$

(4.3)
From the relation \( \frac{d}{da} = \frac{\partial}{\partial a} - \frac{\partial}{\partial x_i} \) and the divergence theorem, the energy release rate from Eqn. (4.3) becomes:

\[
J = -\frac{d\Pi}{da} = \int_r (W n_i - T_i \frac{\partial u_i}{\partial x_1}) ds
\]

(4.4)

The J-integral has a path-independent value in a nonlinear, elastic body. For a linear elastic material, the J-integral is related to a stress intensity factor:

\[
J = G = K_i^2 / E'
\]

(4.5)

where, \( G \) : energy release rate

\( E' = E \) for plane stress

\( E' = E / (1-v^2) \) for plane strain

The J-integral is interpreted as a measure of the HRR (Hutchinson, Rice and Rosengren) field intensity, where the stress-strain relation is characterized by a power law [74].
4.1.2 The Virtual Crack Extension Method

An alternative method to calculate the $J$-integral, instead of the line integral method, is the virtual crack extension method [62, 63, 64]. By interpreting the $J$-integral as the potential energy difference between two identically loaded bodies with different crack sizes, the $J$-integral can be calculated from:

$$J = -\frac{1}{B} \frac{\partial U}{\partial a}$$  \hspace{1cm} (4.6)$$

where,
- $U$ : strain energy
- $B$ : specimen thickness
- $a$ : crack length

as shown in Figure 4.2. From this relationship, the $J$-integral can be calculated, using load-displacement curves from two finite element solutions, with slightly different crack lengths.

A new finite element method for elastic-plastic evaluation of the $J$-integral was presented by Parks [63], which requires only a single finite element solution. The energy comparison of two specimens with slightly changed crack lengths was accomplished by moving the nodal positions along a contour and processing the data from the initial configuration points.
From a two dimensional elastic-plastic finite element analysis with deformation plasticity, the potential energy is given by:

\[ \Pi = \sum_{\text{elements}} \int WdV - [T(X)]'U = \sum_{\text{elements}} W(X, U) - [T(X)]'U \quad (4.7) \]

where,  
- \( W \): strain energy density  
- \( X \): nodal point coordinates  
- \( U \): displacement vector  
- \( T \): nodal force vector

The decrease in the potential energy, with respect to the crack length, is:

\[ -\delta\Pi = J\delta\ell = - \sum_{\text{elements}} (\partial W_i / \partial X)' \delta X \quad (4.8) \]

The summation should be carried out for all elements between contour \( \Gamma_0 \) and \( \Gamma_1 \), as shown in Figure 4.3. The virtual crack extension method can be applied to incremental plasticity, with \( W \) dependent on the loading history.
4.1.3 The J-integral in a thermally stressed body

The J-integral can be interpreted as the energy release rate for a nonlinear elastic material. It can also be used for the material characterized by incremental theory of plasticity with the change of the strain energy density to the path dependent stress work density. However, the J-integral loses its path-independency when there is an inelastic strain or a crack surface traction. Alternative expressions for the J-integral were suggested by many researchers to obtain path independent J values for combined primary and secondary stresses [7, 8, 9, 11].

Shih [11] developed an alternative J-integral expression for a thermally stressed body. The energy release per unit of crack advancement for the quasi-static condition is:

\[
J = \lim_{r \to 0^+} \int_\Gamma (W\delta_{ii} - \sigma_{ij}u_{ji})n_i \, ds
\]

Eqn. (4.9) becomes:

\[
J = \int_C (\sigma_{ij}u_{ji} - W\delta_{ii})m_i q_i \, ds - \int_{C^+ + C^-} \sigma_{ij}u_{ji} m_i q_i \, ds
\]

where \( C = C_i + C^* + C^- - \Gamma \). \( q_i \) is a smooth function that is unity on \( \Gamma \) and vanishes on \( C_i \), and \( m_i \) is the outward normal, as shown in Figure 4.4. Applying the divergence theorem and
\[ \sigma_{ij,i} + f_i = 0, \quad W_j = q_{ij} \varepsilon_{ij} - \alpha \sigma_{kk} \theta_i \]  

Eqn (4.10) becomes:

\[ J = \int_{A} \left( (\sigma_{ij} u_{j,i} - W \delta_{ii}) q_{i,i} + (\alpha \sigma_{ii} \theta_{i,i} - f_i u_{i,i}) \right) dA - \int_{C_{end}} t_i u_{i,i} q_i ds \]  

\[ J = \int_{A} [(\sigma_{ij} u_{j,i} - W \delta_{ii}) q_{i,i} + (\alpha \sigma_{ii} \theta_{i,i} - f_i u_{i,i})] dA - \int_{C_{end}} t_i u_{i,i} q_i ds \]  

A is the area enclosed by C, \( f_i \) is a body force, and \( t_i \) is a crack face traction. Eqn. (4.12) is a path-independent expression for evaluation of the J-integral.

4.1.4. The \( \tilde{J} \)-integral

Aoki developed a path independent line integral, \( \tilde{J} \), to consider the effect of plastic deformations and thermal strains [9, 10]. From the energy balance of a solid [9]:

\[ \int_{S} T_i \dot{u}_i dS + \int_{V} F_i \dot{u}_i dV = \int_{V} \rho \ddot{u}_i dV + \int_{V} \sigma_{ij} \dot{\varepsilon}_{ij} dV \]  

\[ \int_{S} T_i \dot{u}_i dS + \int_{V} F_i \dot{u}_i dV = \int_{V} \rho \ddot{u}_i dV + \int_{V} \sigma_{ij} \dot{\varepsilon}_{ij} dV \]  

A fracture process region is defined as \( A_{end} \), where fracture occurs and continuum mechanics do not work. \( \Gamma_{end} \) is the contour surrounding \( A_{end} \) as shown in Figure 4.5. Consider the energy balance of the
material during its virtual crack extension with the change in the propagating crack length, \( l \), as the time-like variable. From Eqn. (4.13),

\[
\int_{r_*}^{r_1} T_i \frac{du_i}{dl} dl + \int_A F_i \frac{du_i}{dl} dA = \int_A \sigma_{ij} \frac{d \epsilon_{ij}}{dl} dA + \int_A \frac{\rho u_i}{dA} dl + J
\]

(4.14)

where, \( J \) represents the rate of energy change in the fracture process region. Using Gauss' theorem and Eqn. (4.13), Eqn. (4.14) reduces to:

\[
\dot{J} = \int_A \left( (\sigma_{ij} \frac{du_i}{dl})_j + (F_i - \rho \ddot{u}_i) \frac{du_i}{dl} - \sigma_{ij} \frac{d \epsilon_{ij}}{dl} \right) dA + \int_{r_{\infty}}^{r_1} T_i \frac{du_i}{dl} dl
\]

(4.15)

With a moving coordinate system, and the origin at the crack tip:

\[
\frac{du_i}{dl} = \frac{\partial u_i}{\partial t} + \frac{\partial u_i}{\partial x_i} \frac{\partial x_i}{\partial t} = \frac{\partial u_i}{\partial t} - \frac{\partial u_i}{\partial x_i}
\]

(4.16)

Because the process region is assumed to move with the same speed as the crack tip, \( \partial u_i / \partial x_i = 0 \) on \( \Gamma_{\text{end}} \). From Eqn. (4.15)

\[
\dot{J} = \int_{r_{\infty}}^{r_1} T_i \frac{du_i}{dl} dl = -\int_{r_{\infty}}^{r_1} T_i \frac{\partial u_i}{\partial x_i} dl
\]

(4.17)
For arbitrary contour $\Gamma$, Eqn. (4.17) becomes

$$
\dot{J} = -\int_{r_{\text{in}}} T_{ij} \frac{\partial u_{i}}{\partial x_{j}} d\Gamma = \int_{r+} T_{ij} \frac{\partial u_{i}}{\partial x_{j}} d\Gamma - \int_{r-} T_{ij} \frac{\partial u_{i}}{\partial x_{j}} d\Gamma
$$

$$
= \iint_{A} \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_{i}} dA - \int_{r+} T_{i} \frac{\partial u_{i}}{\partial x_{j}} d\Gamma
$$

(4.18)

We can change the strain $\varepsilon_{ij}$ into the sum of elastic, thermal and plastic strains:

$$
\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^t + \varepsilon_{ij}^p
$$

(4.19)

With no surface traction, Eqn. (4.18) becomes:

$$
\dot{J} = \int_{\Gamma} W^{*} dx_{2} - \int_{r} T_{i} \frac{\partial u_{i}}{\partial x_{j}} d\Gamma + \iint_{A} \sigma_{ij} \frac{\partial (\varepsilon_{ij}^t + \varepsilon_{ij}^p)}{\partial x_{i}} dA
$$

(4.20)

$W^{*}$ is the elastic strain energy. Assuming that $\Gamma$ is very small and the material behavior is non-linear elastic, Eqn. (4.20) becomes [10]

$$
\dot{J} = \lim_{\rho \to 0} \left[ \int_{\Gamma} W dx_{2} - \int_{r} T_{i} \frac{\partial u_{i}}{\partial x_{j}} d\Gamma \right] = \lim_{\rho \to 0} J
$$

(4.21)
J is the Rice's J-integral and

\[ W = \int_{0}^{r_0} \sigma_{ij} \, d\varepsilon_{ij} \]  

(4.22)
Figure 4.1 A two dimensional cracked body
Figure 4.2 Schematic diagram for interpretation of the J-integral
Figure 4.3 Schematic diagram for virtual crack extension method [63]
Figure 4.4 A two dimensional thermally stressed cracked body
Figure 4.5 Configuration of crack tip and fracture process region [9]
4.2. The EPRI Engineering Solution

An engineering solution for elastic-plastic fracture analysis was developed by the General Electric Company [12]. It was generalized later to include the effect of secondary stresses [13].

For a Ramberg-Osgood material characterized by:

\[
\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha\left(\frac{\sigma}{\sigma_y}\right)^n \tag{4.23}
\]

The J-integral can be expressed as a sum of linear elastic and fully plastic solutions.

\[
J = J^e(a_e) + J^p(a,n) \tag{4.24}
\]

where, \(J^e\) : elastic contribution to J

\(J^p\) : fully plastic solution of J

a : crack length

\(\alpha, n\) : strain hardening constant

\(a_e\) : effective crack length, \(a_e = a + \Phi r_y\)

\[
r_y = \frac{1}{\beta \pi} \left[\frac{n-1}{n+1}\right] \left(\frac{K_i}{\sigma_y}\right)^2
\]

\[
\Phi = \frac{1}{1 + (P/P_0)^\beta}
\]

and

\(\beta = 2\) for plane stress, \(\beta = 6\) for plane strain
P : generalized load per unit thickness

$P_0$ : limit load per unit thickness

The elastic contribution can be easily calculated from a LEFM solution of $K_I$:

$$J^e = \frac{K_I^2}{E'}$$  \hfill (4.25)

For a fully plastic, cracked body, the $J$-integral can be expressed as

$$J_p = \alpha \varepsilon, \sigma, c, g(a/b)h_1(a/b,n)(P/P_0)^{n+1}$$  \hfill (4.26)

where,
- $a$ : crack length
- $c$ : uncracked ligament
- $b$ : plate width
- $g$ : dimensionless function
- $h$ : function of $a/b$, and $n$

The fully plastic solution is calculated using tables developed for several geometries; compact specimen, center cracked plate, single edge cracked plate, double edge cracked plate, and flawed cylinders.

Since secondary stresses do not influence the fully plastic solution, it is proposed that only the elastic contribution needs to be modified to include the effect of secondary stresses. $K_I$ is evaluated by
\[ K_I = K_{1M} + K_{1S} \]  
\hspace{1cm} (4.27)

where

- \( K_{1M} \): stress intensity factor due to mechanical load
- \( K_{1S} \): stress intensity factor due to secondary stress
Another J-integral estimation procedure for combined primary and secondary stresses was developed by Chell [14]. Chell used an approximation equation:

\[ J = J_e F(\sigma_{\text{ref}}) \]  

(4.28)

where, \( J_e \): the linear elastic value of \( J \):

\[ J_e = \frac{(K^{ps})^2}{E'} \]

\( K^{ps} \): stress intensity factor for combined stress

\( \sigma_{\text{ref}} \): reference stress related to a reference strain, \( \varepsilon_{\text{ref}} \), through the uniaxial true stress - true strain relationship, \( \sigma_{\text{ref}} = \sigma / m(a,t) \)

\( F(\sigma_{\text{ref}}) \): material dependent function

The function \( F(\sigma_{\text{ref}}) \) is given by:

\[ F(\sigma_{\text{ref}}) = \frac{E \varepsilon_{\text{ref}}}{\sigma_{\text{ref}}} + \frac{1}{2} \frac{(\sigma_{\text{ref}})}{\sigma_y} / \left( \frac{E \varepsilon_{\text{ref}}}{\sigma_{\text{ref}}} \right) \]  

(4.29)

Using the parameter \( L_r = \sigma_{\text{ref}} / \sigma_y \), the function \( F \) becomes

\[ F(L_r) = \frac{E \varepsilon_{\text{ref}}(L_r \sigma_y)}{\sigma_y L_r} + \frac{1}{2} \frac{L_r^3 \sigma_y}{E \varepsilon_{\text{ref}}(L_r \sigma_y)} \]  

(4.30)
\( L_r \) is determined from

\[
L_r = \begin{cases} 
L_r^P + L_r^S - L_r^P L_r^S / L_r^* & L_r^P < L_r^* \\
L_r^P & L_r^P > L_r^* 
\end{cases}
\]

(4.31)

where, \( L_r^* = \min(\sigma_f/\sigma_y, 1.25) \)
\( \sigma_f \): flow stress

\( L_r^S \) satisfies the equation \( J_{FOP}^S = J_e^S F(L_r^S) \). The FOP refers to a quantity evaluated using an Irwin type, first order plastic correction with an effective crack length.

When the sum of peak values for primary and secondary loads exceeds the yield stress, \( J_e \) should be evaluated using the elastic-plastic stress distribution in the uncracked structure. A simple method to determine elastic-plastic stress distribution was suggested.
4.4. The J-integral Analysis for Combined Thermal and Mechanical Loads

The J-integral analysis for a thermally stressed body can be conducted using ABAQUS, including Shih's procedure for the thermal loading case [71]. A finite element model was developed for the fracture analysis of a thermally stressed body. Thermal and mechanical modeling techniques, described in Chapter 3, were used for the J-integral analysis of combined thermal stress and external loads. Eight node rectangular elements were used for two dimensional analysis with focused mesh around the crack tip. Quarter point singular elements were used to model singularities around the crack tip.

4.4.1. Example calculation of the EPRI section 7.4

A sample calculation of section 7.4 of the Electric Power Research Institute (EPRI) report [13] was tested for verification of the FEM model with combined thermal and external loads. The specimen contained a single edge crack of length 0.5 inch. The geometry of the specimen and crack are shown in Figure 4.6. Material properties, bilinear stress-strain curve, and thermal gradient applied to the model are also shown. The maximum temperature deviation from the linear temperature field, which does not contribute to thermal stress, was 80°F at the center of the plate. Figure 4.7 shows a finite element mesh for the specimen. Half the
plate was used, due to symmetry along the crack plane. One hundred fifty two (152) eight-node plane stress elements were used for the analysis.

A thermal load was applied to the model as the initial stress field using *Temperature command in ABAQUS, and a uniform tensile load was added to a thermal stress field. Results of the J-integral calculation are shown in Figure 4.8 for combined thermal and external loads. The J value for pure thermal load was 2.0 psi-in, which was equivalent to an external load of 5 ksi without the thermal stress field. The J-integral for combined thermal stress and external load of 60 ksi was 1703 psi-in. The J value for an external load was plotted in the figure to reveal the effect of thermal stress on the J value. The effect of the thermal load on the J-integral gradually decreased as the applied load increased. The difference between two J values was 3.8 percent at an external load of 60 ksi.
Figure 4.6 Specimen geometry and material properties of the EPRI example 7.4

\[ E = 30 \times 10^3 \text{ ksi} \]
\[ \sigma_y = 60 \text{ ksi} \]
\[ \beta = 7.3 \times 10^{-6} /F \]
\[ v = 0.3 \]

Thermal Loading:
\[ T(x) = 200 + 351x - 88x^2 \]

\[ E = 30 \times 10^3 \text{ ksi} \]
\[ E_T = 4.25 \times 10^3 \text{ ksi} \]
Figure 4.7 Finite element mesh for the EPRI example 7.4
Figure 4.8 The J-integral value for the EPRI example 7.4
4.4.2. Example calculation of the EPRI section 7.6

Test calculations for section 7.6 of the EPRI report [13] were performed to verify the finite element model for combined loads. It was used to compare finite element results and EPRI solutions. The same geometry and material properties of the previous example were used, with different thermal loads and stress-strain relationships as shown in Figure 4.9. The same finite element mesh for the EPRI section 7.4 example, was used, as shown in Figure 4.6. Thermal stress without a crack was calculated along the cross section, at the center of the plate, and plotted as thermal load 1 in Figure 4.10. The crack tip was located in the compressive stress zone as shown in the figure. The maximum tensile, thermal stress without a crack was 14.4 ksi. Results of the J-integral calculated by FEM are shown in Figure 4.11. The J value was increased up to 2272 psi-in at an external load of 60 ksi. EPRI solutions were also plotted for comparison. EPRI solutions showed a lower value than the numerical results. The difference between the two solutions was 4.2 percent with an external load of 60 ksi. The J-integral values obtained by finite element analysis and EPRI solution show good agreement as shown in Figure 4.11.

Further analysis revealed the effect of larger thermal loads on the accuracy of the EPRI solution. The thermal load was increased until the maximum stress reached a yield stress magnitude, shown as thermal load 2 in Figure 4.10. The maximum tensile stress was 56.6 ksi at both
ends of specimen. The $J$-integral value of thermal load case 2 is shown in Figure 4.12. The $J$ value of the thermal stress field (without external load) was increased from 2.6 psi-in to 58.5 psi-in compared to the thermal load case 1 due to the increase of thermal stresses. From the figure, the results of finite element analysis and EPRI solutions show good agreement for combined large thermal stress and external loads up to the yield stress.
Thermal Loading:

\[ T(x) = 125 + 400x - 100x^2 \]

Stress-strain curve:

\[ \frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + 0.5 \left( \frac{\sigma}{\sigma_y} \right)^5 \]

E = 30 X 10^3 ksi

\( \sigma_y = 60 \) ksi

\( \beta = 7.3 \times 10^{-6} /F \)

\( v = 0.3 \)

Figure 4.9 Specimen geometry and material properties of the EPRI example 7.6
Figure 4.10 Distribution of thermal loads for the EPRI example 7.6
Figure 4.11 The J-integral values of combined thermal and external loads for the EPRI example 7.6 (thermal load 1)
Figure 4.12 The $J$-integral values of combined thermal and external loads for the EPRI example 7.6 (thermal load 2)
4.5. The J-integral Analysis for Combined Residual Stress and Mechanical Loads

The procedure for J-integral analysis with combined thermal and external loads cannot be directly applied to welded plates with a residual stress field. Initial stresses and plastic strains caused by welding create path-dependency of the J-integral value at different contours. To evaluate the J-integral value at the crack tip for combined residual stress and external loads, very small elements are needed around the crack tip. The J value at the nearest contour to the crack tip can be used as an approximate solution for the J-integral with combined residual stress and external loads.

The J-integral estimation of a welded plate, subjected to the combined loads of a residual stress field and an external force, was performed in three steps as follows:

1) Thermal analysis: Temperature history of the plate was calculated from heat input to the model.
2) Residual stress analysis: Residual stress was evaluated using the temperature history obtained from the previous step.
3) The J-integral analysis: A crack was introduced in the residual stress field and an external load was applied to calculate a J-integral value for the combined loads.

An overall procedure for this analysis is shown in Figure 4.13. A commercial finite element package ABAQUS [18] was used for the
thermal and mechanical analysis. The J-integral analysis for a two dimensional plane stress problem was conducted for the verification of this procedure. The results were compared with available engineering solutions. Three dimensional analysis was also conducted for a single edge notched specimen.
ANALYSIS PROCEDURE

FEM Modeling

Thermal Analysis

Temperature Field → Thermocouple

Stress Analysis

Residual Stress → Blind Hole Drilling Method

J-integral Analysis

J-integral Value for Combined Loading → EPRI Engineering Solution

Figure 4.13 Analysis procedure of the J-integral for combined residual stress and external loads
4.5.1 Welded Thin Plate with a Center Crack

A J-integral analysis was conducted for a center cracked plate of A515 grade 70 steel subjected to combined residual stress and external loads. Temperature dependent thermal and mechanical properties were used shown in Figure 4.14. The Ramberg-Osgood stress-strain relationship was used for the analysis, which was selected to closely represent the stress-strain curve of A515 steel (see Figure 4.15). Figure 4.16 shows the geometry of the specimen with a center crack. A line heat source was used to generate the welding thermal load at the centerline of the plate. Welding current, voltage, and speed were assumed as 200A, 25V, and 12 ipm, respectively. The arc efficiency was 0.85 for the GMAW process. Due to symmetry, one quarter of the plate was modeled. The finite element mesh contained ninety eight elements as shown in Figure 4.17. The plane stress assumption was used for residual stress and the J-integral analysis.

Temperature histories and residual stress distributions were obtained from thermal loads along the centerline of the plate. The longitudinal (welding direction) residual stress distribution at the crack plane is shown in Figure 4.18. The maximum tensile residual stress was 48 ksi at the centerline and the size of tensile stress zone was 0.7 inch. The crack tip was located in the tensile residual stress zone as shown in the figure. A crack was introduced in the next step which
resulted in the redistribution of the residual stresses. An external load was applied to the model and J-integral values were calculated.

Figure 4.19 shows comparisons of J-integral values computed using FEM, EPRI solution and Chell's approximation solution. Because the J-integral loses its path-independency in the residual stress field, the J value at the nearest contour to the crack tip was used as a finite element solution. The engineering solution from the EPRI procedure was also calculated and plotted for comparison. The two results show good agreement.

Chell's approximation solution [14] was calculated and compared with FEM and EPRI solutions. For the calculation of the J-integral, Eqn. (4.30) was used with the following assumptions:

\[
\frac{\varepsilon}{\varepsilon_y} = \frac{\varepsilon}{\varepsilon_y} + 4\left(\frac{\varepsilon}{\varepsilon_y}\right)^5
\] (Ramberg-Osgood relation) \hspace{1cm} (4.32)

\[
F(Lr^*) = \alpha(\sigma_{ref}/\sigma_y)^{n-1}
\] \hspace{1cm} (4.33)

where, \(\alpha = 4.0\) and \(n=5\)

\(m(a,t) = 2[1-(a/t)]/ 3\)

\(L_r^* = 1.25\)

\(\sigma_y = 41.1\) ksi
Because the sum of stresses from external loads and the peak value of residual stress exceed the yield stress of the material, redistributed stresses for an uncracked plate were calculated using the finite element method. Those redistributed stresses were used to calculate the displacement field with an elastic stress-strain relationship. Subsequently, stress intensity factors for combined primary and secondary loads \( (K^{ps}) \) were calculated by the displacement method. Elastic solutions of the J-integral were calculated from those stress intensity factors. Additionally, J-integral values calculated from linearly superimposed primary and secondary stress intensity factors \( (K^p + K^s) \) were also plotted, which overestimated the J-integral based on \( K^{ps} \) \( (K^p + K^s > K^{ps}) \). Chell's solution showed lower J-integral values at large external loads above 30 ksi when compared to the FEM or EPRI solutions.
Figure 4.14 Temperature dependent material properties of A515 steel
Figure 4.15  Engineering stress-strain curve and the Ramberg-Osgood relationship of A515 steel

\[ \frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + 4 \left( \frac{\sigma}{\sigma_y} \right)^5 \]
Figure 4.16 Specimen geometry of a welded thin plate with a center crack
Figure 4.17  Finite element mesh for a quarter of a welded plate with a center crack.
Figure 4.18 Longitudinal (welding direction) residual stress distribution of a welded thin plate without a crack
Figure 4.19 Comparison of the J-integral vs. external loads by finite element method, EPRI solution, and Chell's solution
4.5.2. Three Dimensional Analysis of a Bead on Plate Weld

A J-integral analysis was conducted for a three dimensional model of a bead on plate weld with a single edge notch. The geometry and dimensions of the specimen are shown in Figure 4.20. The thickness of the plate is 0.25 inch with a 0.09 inch deep single edge crack throughout the width of the plate. Due to symmetry, half of the plate was used for a finite element mesh as shown in the figure. Seven hundred thirty five (735) twenty-node three dimensional elements were used for both thermal and mechanical analysis. Quarter point singular elements were used, with a focused mesh around the crack tip. Material properties for A515 grade 70 were used in the analysis (see Figure 4.14). The engineering stress-strain curve of A515 was used for the analysis [75], shown in Figure 4.15.

The gas tungsten arc process was used for deposition of the bead on the plate. Welding parameters are as follows:

- Current : 200 Amp.
- Voltage : 10 Volts
- Welding speed : 5.24 ipm

An arc efficiency of 0.35 was selected for the GTAW process. A heat flux was applied to the centerline elements of the top surface at the same time. A surface heat input to the unit length of the plate was calculated using Eqn. (3.1) as follows:
\[ Q = \frac{60\eta EI}{1055bV} \quad \text{(4.34)} \]

where

- \( Q \) = heat input, BTU/in\(^2\)
- \( \eta \) = arc efficiency (0.35)
- \( E \) = arc voltage, volts (10V)
- \( I \) = arc current, amps (200 A)
- \( b \) = total width of bead elements being deposited (0.075 inch)
- \( V \) = welding speed, inches/min (5.24 ipm)
- 60 = conversion factor, minutes to second
- 1055 = conversion factor, Joule to BTU

The calculated total heat input to the unit volume of the plate was \( Q = 50.65 \) BTU/in\(^3\) for half of the specimen. The time required to pass the bead width (0.15 inch) was considered the heat input time. Therefore, applied heat flux to the model was \( Q = 29.45 \) BTU/in\(^3\)/sec \( \times \) 1.72 sec.

Uncoupled thermal-mechanical analysis was used. Thermal analysis was performed first, and then the residual stress was analyzed using the temperature history from the thermal analysis. Free boundary conditions were assumed for the analysis, except at the centerline of the plate, where a symmetry condition was applied. Figure 4.21 shows the through thickness distribution of longitudinal (welding direction) stress at different distances from the weldline. The maximum tensile stress was 80.3 ksi at the centerline and gradually decreased along the
transverse direction. The size of the tensile stress zone was 0.4 inch, as shown in the figure.

The J-integral value was evaluated as a function of the crack tip location. The change of J-integral values along the Z-direction for external loads of 20, 30, and 35 ksi (without residual stress) were shown in Figure 4.22. The J-integral showed almost the same value along the length, but sharply decreased at the end of the plate due to the effect of the free surface.

A J-integral analysis was also performed for combined residual stress and external loads. A crack was introduced to the residual stress field by changing the boundary conditions, and then external loads were applied to calculate the J-integral for the combined loads. The J value at the nearest contour to the crack tip was assumed as a finite element solution for the J-integral. The results of these J values are plotted in Figure 4.23 as a function of the crack tip location along the Z-direction (perpendicular to the weldline) for various external loads up to 30 ksi. The J-integral had a maximum value at the centerline of the plate and then gradually decreased along the Z-direction, due to the decrease of tensile residual stresses. Without an external load (residual stress only), the J-integral showed a maximum value of 63.4 psi-in at the centerline, which was equivalent to an external load of 26 ksi without residual stress.

EPRI solutions, with a plane strain assumption, were calculated and compared with the numerical results in Figure 4.24. The Ramberg-
Osgood relation used for the EPRI solution was selected to closely represent the engineering stress-strain curve of A515 steel (see Figure 4.15). EPRI solutions showed a lower value than the finite element solution for the external load above 15 ksi. The difference between the two solutions gradually increased as the external load increased up to 28% for an external load of 35 ksi. The plane stress EPRI solution was plotted for reference purposes and showed higher J-integral values compared with the finite element or plane strain EPRI solution for the external loads above 20 ksi.
Figure 4.20 Three dimensional finite element mesh for a bead on plate
Figure 4.21 Through thickness distribution of transverse (perpendicular to welding direction) residual stresses for a bead on plate at different distance from the weldline
Figure 4.22  The change of the J-integral value along crack location of a bead on plate for different loads (external loads only) (S : external load)
Figure 4.23 The change of the J-integral value along crack location of a bead on plate for different loads (combined residual stress and external loads) (S: external load)
Figure 4.24 Comparison of the J-integral for combined residual stress and external loads between finite element analysis and EPRI solution
5.1. The J-integral for Combined Residual stress and External Loads

Welding processes create nonlinear thermal cycles which produce thermal strains and stresses. After welding, plastic strains are accumulated around the weld area which result in residual stress distribution. These initial plastic strains cause path-dependency of the J-integral. In Section 4.5, small elements were used around a crack to calculate the J value at the crack tip, which caused increased number of elements and possible errors. Therefore, an alternative expression for the J-integral is necessary to consider initial plastic strains produced by welding.

From Section 4.1.4 a J value for combined primary and secondary stresses at the crack tip can be expressed as:

\[
\lim_{\rho \to 0} J = \int_{\Gamma} -T_{i} \frac{\partial u_{i}}{\partial x_{i}} d\Gamma
\]

(5.1)
where $\Gamma_1$ is a contour surrounding the fracture process region. With a smooth function $q_i$, which has unit value at $\Gamma_1$ and zero at $\Gamma_2$, Eqn. (5.1) becomes:

$$\lim_{p \to 0} J = \int_{\Gamma + \Gamma - \Gamma} \sigma_{ij} n_j \frac{\partial u_i}{\partial x_1} q d\Gamma$$

when there is no crack face traction.

Crack tip configuration as shown in Figure 5.1, applied with the divergence theorem, Eqn. (5.2) becomes:

$$\lim_{p \to 0} J = \int_A (\sigma_{ij} \frac{\partial u_i}{\partial x_1} q)_j dA$$

$$= \int_A \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1} q dA + \int_A \sigma_{ij} \frac{\partial u_i}{\partial x_1} q_j dA$$

(5.3)

When the fracture process region is diminishing, the area $A$ includes the crack tip region. The total strain $\varepsilon_{ij}$ is the sum of elastic strain, $\varepsilon^{e,ij}$, plastic strain from mechanical loads, $\varepsilon^{p,ij}$, and initial plastic strain after welding, $\varepsilon^{i,p,ij}$.

$$\varepsilon_{ij} = \varepsilon^{e,ij} + \varepsilon^{p,ij} + \varepsilon^{i,p,ij}$$

(5.4)
From Eqn. (5.3) and (5.4), we get a path-independent expression for $J$:

$$ J = \int_A \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1} q dA + \int_A \sigma_{ij} \frac{\partial u}{\partial x_1} q, dA + \int_A \sigma_{ij} \frac{\partial (\varepsilon_{ij} + \varepsilon^{ip}_{ij})}{\partial x_1} q dA $$

(5.5)

Using the relation

$$ \frac{\partial W_e}{\partial \varepsilon_{ij}} = \sigma_{ij} $$

(5.6)

where $W_e$ is an elastic strain energy density. Eqn. (5.5) becomes:

$$ J = \int_A \frac{\partial W}{\partial x_1} q dA + \int_A \sigma_{ij} \frac{\partial u}{\partial x_1} q, dA + \int_A \sigma_{ij} \frac{\partial (\varepsilon_{ij} + \varepsilon^{ip}_{ij})}{\partial x_1} q dA $$

(5.7)

Isoparametric elements can be used for finite element calculation of the $J$-integral with combined residual stress and external loads. Eqn. (5.7) can be easily changed to a finite element formulation using isoparametric elements.

For isoparametric elements, the coordinates and displacements within an element are defined by [76]

$$ x_i = \sum N_i X_i, \quad u_i = \sum N_i U_i $$

(5.8)

where, $X_i$ : nodal values of coordinates
\( U_i \): nodal values of displacements

\( N_i \): shape function

For a field quantity \( \Phi \), upon interpolation:

\[
\Phi = \sum N_i \Phi_i \quad \text{or} \quad \Phi = [N][\Phi_e]
\]

(5.9)

where \( \Phi_i \) or \( \{\Phi_e\} \) is a nodal value of \( \Phi \). Derivatives of \( \Phi \) are given as

\[
\begin{bmatrix} \Phi_x \\ \Phi_y \end{bmatrix} = [B][\Phi_e]
\]

(5.10)

\[
[B] = \begin{bmatrix} N_{i,x} & N_{2,x} & N_{3,x} & N_{4,x} \\ N_{i,y} & N_{2,y} & N_{3,y} & N_{4,y} \end{bmatrix}
\]

(5.11)

for bilinear isoparametric elements. The function \( q \) can be changed following the same formulation:

\[
q = \sum N_i Q_i \quad \text{and} \quad \begin{bmatrix} q_x \\ q_y \end{bmatrix} = [B][Q_e]
\]

(5.12)

where, \( Q_i \) or \( \{Q_e\} \) is a nodal value of \( q \). Using Eqns. (5.10), (5.11), and (5.12), Eqn (5.7) becomes:
\[ J = \sum_{\text{Elements GaussPoint}} \sum_{\text{Elements GaussPoint}} \left[ \frac{\partial W}{\partial x_i} q + \sigma_{ij} \frac{\partial u_i}{\partial x_i} q + \sigma_{ij} \frac{\partial (\varepsilon_{ij} + \varepsilon_{ij}^{ep})}{\partial x_i} q \right] D w \]

(5.13)

where,

\( D \): a determinant of the Jacobian matrix

\( w \): weight factor
Figure 5.1 A two dimensional cracked body for the \( J \)-integral evaluation
5.2 The J-integral Analysis of a Welded Thin Plate with a Center Crack

A J-integral analysis was performed for a welded, A515 thin steel plate with a center crack. Thermal and mechanical properties of the A515 steel are shown in Figure 4.14. The engineering stress strain curve for A515 steel which was used in this analysis is shown in Figure 4.15. The geometry of the specimen and its crack size were identical to the specimen used in Section 4.5.1 (see Figure 4.16). Welding current, voltage, and travel speed were 200A, 25V, and 12 ipm. A line heat source was applied to the centerline of the plate. An arc efficiency of 0.85 was used for GMAW. Due to symmetry along both the x and y axis, one quarter of the plate was modeled. Figure 5.2 shows the finite element mesh of the specimen. Four hundred thirty two (432) four-node plane stress elements were used for both the residual stress and J-integral analysis. No singular element was used around the crack tip as shown in the figure.

The first contour contained two elements adjacent to the crack tip. The second contour included elements of the first contour and one more layer of elements. Subsequent contours were defined by the same procedure. Eight contours were used to calculate the J-integral. The value of $Q$ for Eqn. (5.12) was defined as unity for every node, except the boundary nodes forming each contour for the J-integral calculation. $Q$ was defined as zero at the boundary nodes, therefore, $Q$ decreased from unity to zero at the boundary elements.
A stress analysis for the combined residual stress and external loads of a cracked plate was conducted using ABAQUS. The results of stress, elastic strain energy, displacement and plastic strains were used for a J-integral analysis using the finite element formulation shown in Section 5.1. A FORTRAN program for an isoparametric formulation is shown in Appendix A. Gauss points for each element were selected to match those used for the ABAQUS solution [18], so that the solution of stresses at each Gauss point could be easily transferred from ABAQUS results into the J-integral calculation.

Figure 5.3 shows the redistribution of longitudinal (welding direction) residual stresses at the centerline of the plate (crack plane) due to crack initiation and growth. The maximum tensile stress of 45.9 ksi, without a crack, increased slightly around the crack tip, at the crack initiation stage. This tensile stress gradually decreased as the crack propagated, from the effects of compressive residual stresses outside the weld area.

For verification of the finite element formulation for the J-integral, a test analysis of the J-integral was conducted with an external load of 10 ksi (without a residual stress field) for a linear elastic material behavior. The results of the J-integral values are plotted in Figure 5.4. The J values show path independent values of 5.1 psi-in, except at the nearest contour to the crack tip, where stress singularity caused a large error. This error for the first contour could be reduced, if proper singular elements were used around the crack tip. Calculated J values are compared with linear
elastic fracture mechanics solutions using stress intensity factor as shown in Figure 5.4, which showed good correlation. The J values were also calculated for a refined mesh to verify the effect of mesh size. Half of the element size was used around the crack tip for the refined mesh. Two solutions showed very close results.

The J-integral value of a residual stress field (without external loads) is shown in Figure 5.5. This result shows a path independent value of 26.9 psi-in, which is equivalent to an external load of 22 ksi, without the residual stress field. Deviation of J values for different contours was less than 2.6 percent except the first contour. The ABAQUS solutions for second and third contours were plotted for comparison, and revealed path dependency.

A finite element analysis was performed in several steps for combined residual stresses and external loads. A crack was introduced in the residual stress field, resulting in a redistribution of the residual stresses. An external load was applied to the model and the J-integral values were calculated. The J values for combined residual stresses and external loads are plotted in Figure 5.6. The external loads increased gradually from zero to 30 ksi. The J-integral values for the external loads of 0, 10, 20, 30 ksi were 26.9, 43.6, 66.9, and 108.5, psi-in, respectively. The deviation of the J values for different contours increased gradually as the external load increased as shown in the figure. It is noted from the figure, that the area affected by stress and strain error at the crack tip became greater for large external loads. From this error, the J value at the
first and second contour for an external load of 30 ksi showed significant deviation from other contours.

The effect of initial plastic strain caused by welding on J-integral values at different contours is shown in Figure 5.7, for the residual stress field without external loads. Because initial plastic strains were determined before the crack introduction, crack tip singularity of initial plastic strain does not exist. Therefore, the contribution from the initial plastic strain, shown as J2 in the figure, converged to zero at the crack tip where the area for the integral was almost negligible. The effect of initial plastic strain gradually increased at outer contours up to 68 percent of the total J value at the 8th contour. The J1 term shown in the figure represents the difference between the total J value and the contribution from initial plastic strain, J2, which decreased gradually for outer contours.

Figure 5.8 shows the effect of initial plastic strain on the J value for combined residual stress and an external load of 30 ksi. The general trend of the initial plastic strain effect was similar to the previous figure, but the initial plastic strain effect decreased when compared to the case of residual stress only. The contribution of initial plastic strain, which caused path-dependency of the J-integral, shows a maximum value of 24 percent at the last contour.

Results of the J-integral in the residual stress field (without external load) were compared with Masubuchi's analytical methods for brittle fracture of weldments [53]. Elastic strain energy released due to crack
formation was calculated from the residual stress field without a crack and vertical displacement (location) along the crack surface caused by cracking. Two finite element meshes with slightly different crack lengths were used for calculation of the elastic strain energy release rate. When the increase of crack length for the second finite element mesh was less than $10^{-3} \times S$, where $S$ is the side length of the finite element, round off error for the displacement field became too large. Therefore, an increase of the crack length was taken as $(5 \times 10^{-3}) \times S$. The result of the elastic strain energy release rate was 26.9 psi-in (see Figure 5.9). The difference between these two solutions was 10 percent. The error is attributed to the assumption of elastic material behavior in Masubuchi’s method.
Figure 5.2 Finite element mesh for a welded thin plate with a center crack
Figure 5.3 Redistribution of longitudinal (welding direction) residual stresses due to a crack propagation.
Figure 5.4 The J-integral values for a welded thin plate for an external load of 10 ksi (Linear elastic)
Figure 5.5 The J-integral values for a welded thin plate for residual stress field (without external load)
Figure 5.6 The J-integral values for a welded thin plate for combined residual stress and external loads (S : external load)
Figure 5.7 Components of the J-integral values for a welded thin plate for residual stress field (without external load)
Figure 5.8 Components of the J-integral values for a welded thin plate for combined residual stress and external load of 30 ksi
Figure 5.9 Comparison of the J-integral value for a welded thin plate with Masubuchi's method
5.3 The J-integral analysis of a Welded Thick Plate with a Single Edge Crack

The J-integral was evaluated for a 1 inch thick A515 steel plate with a double V-groove for combined residual stress and external loads. The material properties and engineering stress strain curve used in this problem were identical to the previous problem in Section 5.2. The plate geometry and crack, after welding, are shown in Figure 5.10. A single edge crack of 1/8 inch depth is located at the centerline of the top surface. Welding parameters used in this analysis, were the same values from the experiment with the 1 inch thick A36 steel plate with a double V groove, discussed in Section 3.5. Welding parameters, geometry of groove, and bead shape are in Table 3.1.

Half of the plate was used for the model, due to symmetry along the centerline. Figure 5.11 shows the finite element mesh of the model. One hundred and eighty five (185) four-node rectangular elements were used including twenty eight (28) focused mesh around the crack tip. Seven (7) contours were defined using focused mesh around the crack tip for the J-integral calculation. The same procedure used in the previous section was adapted for the definition of Q value.

Generalized plane strain elements [18] were used for both the residual stress and the J-integral analysis. Eleven weld passes were analyzed individually for the calculation of the temperature field. As discussed in Section 3.1, twenty percent of ramp time was used for a heat input to the model. Residual stress distribution was calculated using
thermal loads from the temperature analysis. The general procedure for thermal and residual stress analysis was described in Chapter 3. A crack was introduced in the next step to calculate the redistribution of the residual stress field and the J-integral.

Figure 5.12 shows the through thickness distribution of transverse residual stresses (perpendicular to the crack) at the plate centerline. Maximum tensile stress of 43.3 ksi occurred at the top surface, with a tensile zone depth of 0.2 inch. This stress gradually decreased to a compressive stress near the middle of the plate. The crack tip was located in the tensile stress zone as shown in the figure. Redistributed residual stress distribution by the crack is plotted in the figure. The maximum tensile stress increased to 83.4 ksi at the crack tip, and the tensile zone was reduced to 0.1 inch.

ABAQUS solutions for the J-integral values for an external load (without residual stress field) calculated using generalized plane strain and plane strain elements are compared in Figure 5.13. As shown in the figure, solutions from two types of elements were almost identical up to a 30 ksi of external load. The difference between these two solutions increased gradually above 30 ksi load. The deviation of these solutions was less than 10 percent, to 35 ksi load, but increased rapidly above 35 ksi. The large deviation of the solutions is attributed to the large plastic deformation produced around the crack tip for the generalized plane strain model.
Figure 5.14 shows the J-integral values with an external load of 20 ksi, without a residual stress field. The results show path independent values except for the contours near the crack tip, where crack tip singularity caused some errors for the J-value. The J-integral at the last contour was 9.2 psi-in. The maximum deviation of the J values is less than 5 percent, except the first contour which contains crack tip elements, where the difference was 7.6 percent. ABAQUS solutions are also plotted for comparison.

The J-integral for a residual stress field without external load is plotted in Figure 5.15. The J value for the last contour was 16.0 psi-in, which was nearly equivalent to the external load (without residual stress field) of 26 ksi, calculated from the J-integral analysis of external loads only. ABAQUS solutions are also plotted in the figure, which show path dependent values for different contours. Therefore, the J value close to the crack tip could be used as an approximate solution for the residual stress field. These two solutions show similar results around the crack tip, as expected.

Both results of the J-integral for residual stress and combined residual stress and external loads are shown in Figure 5.16. The J values at the last contour were 16.0, 24.1, 38.1, 59.7, and 116.9 psi-in for external loads of 0, 10, 20, 30, and 41.1 ksi, respectively. Generally, the deviation of J values at each different contour increases for large external loads.

Figure 5.17 shows the effect of initial plastic strain imposed by welding. The effect of initial plastic strain (J2 in the figure) was almost
zero around the crack tip, as described in the previous section. The magnitude of the contribution from the initial plastic strain increased to 47 percent of the total J value at the last contour.
Figure 5.10 Specimen geometry of a 1 inch thick plate with a single edge crack

Figure 5.11 Finite element mesh of a 1 inch thick plate with a single edge crack
Figure 5.12 Transverse (perpendicular to welding direction) Residual stress distribution of a 1 inch thick plate and redistributed stress by a single edge crack.
Figure 5.13 Comparison of the J-integral for generalized plane strain and plane strain elements for a 1 inch thick plate.
Figure 5.14 The J-integral values for a 1 inch thick plate for an external load of 20 ksi
Figure 5.15 The J-integral values for a 1 inch thick plate for residual stress field (without external load)
Figure 5.16 The $J$-integral values for a 1 inch thick plate for combined residual stress and external loads ($S$: external load)
Figure 5.17 Components of the $J$-integral values for 1 inch thick plate for residual stress field (without external load)
CHAPTER VI
CONCLUSIONS

For evaluation of weldment fracture, it is necessary to understand thermal and mechanical behavior of a structure during welding. The J-integral was used in this study as an elastic-plastic fracture parameter for a welded plate. A general procedure and modeling techniques were developed for weldment fracture analysis using the J-integral. Finite element models were developed for the two dimensional or three dimensional thermal and mechanical analysis of bead on plate, fillet weld, and multi-pass welded thick plates.

The thermal model includes welding heat input and deposition of weld beads. A ramp heat input function was used in the two dimensional thermal model to simulate the effect of a moving arc. Twenty percent of the actual heat input time required to pass a unit length was used as a ramp time, which was determined by the comparison of experimental results and finite element solutions. Temperature profiles for 1/2 inch and 1 inch thick plates were verified by experimental results using thermocouples.
A generalized plane strain assumption was used for the two dimensional residual stress analysis to allow strain to occur in the welding direction. The residual stress distributions of 1 inch thick plates with a double or single V groove were compared with experimental results from blind drilling hole method, which showed good agreement.

The effect of joint geometry on residual stress was studied using 1 inch thick plates with a single V groove or double V groove. Longitudinal and transverse residual stresses were the primary concern of the analysis because the shear stress was negligibly small. The stress distribution through the plate thickness was obtained from the finite element analysis. It was noted that the maximum stress existed near the surface. The longitudinal stress showed high tensile stress through the thickness but the transverse stress changed from tensile stress at the surface to compression near the middle of the plate.

A lumped model was developed for thick plates to reduce computing time. One or two layers of weld beads were grouped into one large lumped pass. The results showed reasonable correlation with experimental data and numerical results obtained by a non-lumped model.

A finite element model was developed for the J-integral analysis of a welded plate subjected to combined residual stress and external loads. The J-integral analysis of a welded plate was carried out in three steps:

1) Thermal analysis: Temperature history was calculated using the heat input from the arc to the plate.
2) Residual stress analysis: Residual stress field was determined from thermal loads obtained in the previous step.

3) The J-integral analysis: The J-integral values were evaluated for combined residual stress field obtained from the previous step and external loads.

The J-integral of combined residual stress and external loads was calculated for a welded thin plate with a center crack. The J value at the nearest contour to the crack tip was assumed as a finite element solution for the J-integral. The EPRI solution and Chell's solution were compared with finite element results. The EPRI solution agreed well with finite element solution, but Chell's approximation solution showed lower J values than other solutions for external loads above 30 ksi.

Three dimensional analysis was carried out for the J-integral analysis of a bead on plate. The J values were determined as a function of the location at the crack front. The EPRI plane strain solution showed lower value than the finite element solution for external loads above 15 ksi.

Because the J-integral loses its path-independency in a residual stress field, finite element formulation was performed for the expression of the J-integral to include the effect of initial plastic strains caused by welding. Test calculations were performed for a welded thin plate and multi-pass welding of a 1 inch thick plate with a double V groove. The results showed path-independent J values for combined residual stress and external loads except for the first contour, where the stress singularity at the crack tip created errors. The contribution of initial plastic strain to the
J value for combined loads gradually decreased as external loads increased.
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APPENDIX

C ==================================================
C CALCULATION OF THE J-INTEGRAL FOR COMBINED
C RESIDUAL STRESS AND EXTERNAL LOADS
C ==================================================

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION XL(4),YL(4),DN(2,4),VJAC(2,2),SH(4)
DIMENSION JNODE(10,1000),JEL(10,1000)
DIMENSION XX(1000), YY(1000)
DIMENSION VJACDN(2,4),WE(1000),Q(1000,8)
C
DIMENSION N(1000,4),NB(8,4),
* S11(1000,4),S22(1000,4),S12(1000,4),S33(1000,4),
* EIP11(1000),EIP22(1000),EIP12(1000),EIP33(1000),
* U1(1000),U2(1000),
* XJ1(1000,8),XJ2(1000,8),XJ3(1000,8),XJ4(1000,8),
* XJ(1000,8),TJ1(8),TJ2(8),TJ3(8),TJ4(8),TJ(8),
* GP(4,2)
C==============================================
OPEN (UNIT=5,FILE='JEL.DAT',STATUS='OLD')
OPEN (UNIT=6,FILE='JNODE.DAT',STATUS='OLD')
OPEN (UNIT=7,FILE='JSE.DAT',STATUS='OLD')
OPEN (UNIT=8,FILE='JS.DAT',STATUS='OLD')
OPEN (UNIT=9,FILE='JEIP.DAT',STATUS='OLD')

167
OPEN (UNIT=10,FILE='JEP.DAT',STATUS='OLD')
OPEN (UNIT=11,FILE='JU.DAT',STATUS='OLD')
OPEN (UNIT=12,FILE='JOUT.DAT',STATUS='NEW')

C===============================================
C    ** READ INPUT DATA **
C===============================================

NEL=228
NODE=664
JCON=7
Jtip=200

C DO 100 I=1,NEL
     READ(5,*) J,N(J,1),N(J,2),N(J,3),N(J,4)
100 CONTINUE
C
DO 130 I=1,NODE
     READ(6,*) J,XX(J),YY(J)
130 CONTINUE
C===============================================
C DO 210 I=1,NEL*4
     READ(7,*) J,K,S11(J,K),S22(J,K),S33(J,K),S12(J,K)
210 CONTINUE
C
DO 220 I=1,NODE
     READ(8,*) J,WE(J)
220 CONTINUE
C
DO 230 I=1,146
     READ(9,*) J,EIP11(J),EIP22(J),EIP33(J),EIP12(J)
230 CONTINUE
C
DO 240 I=1,NODE
EP11(J)=EP11(J)-EIP11(J)
EP22(J)=EP22(J)-EIP22(J)
EP33(J)=EP33(J)-EIP33(J)
EP12(J)=EP12(J)-EIP12(J)
240 CONTINUE
C
DO 250 I=1,NODE
READ(11,*) J,U1(J),U2(J)
250 CONTINUE
C
C==========================================================================
C ** DEFINE Q FOR EACH NODE, EACH CONTOUR **
C==========================================================================
DO 410 I=1,1000
DO 410 J=1,JCON
Q(I,J)=1.0
410 CONTINUE
C
DO 420 J=1,5
Q(1+(J+1)*100,1)=0.0
Q(2+(J+1)*100,2)=0.0
Q(4+(J+1)*100,3)=0.0
Q(8+(J+1)*100,4)=0.0
Q(16+(J+1)*100,5)=0.0
Q(32+(J+1)*100,6)=0.0
Q(64+(J+1)*100,7)=0.0
420 CONTINUE
C==========================================================================
C ** CALCULATION OF J1, J2, J3, J4 **
C==========================================================================
\[ \begin{align*}
GP(1,1) &= -1.0/3.0^{0.5} \\
GP(2,1) &= 1.0/3.0^{0.5} \\
GP(3,1) &= -1.0/3.0^{0.5} \\
GP(4,1) &= 1.0/3.0^{0.5} \\
\end{align*} \]

\[ \begin{align*}
C \quad GP(1,2) &= -1.0/3.0^{0.5} \\
GP(2,2) &= -1.0/3.0^{0.5} \\
GP(3,2) &= 1.0/3.0^{0.5} \\
GP(4,2) &= 1.0/3.0^{0.5} \\
\end{align*} \]

\[ \text{DO 470 K=1,JCON} \]
\[ \text{DO 470 L=1,1000} \]
\[ XJ1(L, K) = 0.0 \]
\[ XJ2(L, K) = 0.0 \]
\[ XJ3(L, K) = 0.0 \]
\[ XJ4(L, K) = 0.0 \]
\[ XJ(L, K) = 0.0 \]
\[ 470 \text{ CONTINUE} \]

\[ \text{C} \]
\[ \text{C=} \]
\[ \text{= FOR EACH CONTOUR} \]
\[ \text{DO 500 IC=1,JCON} \]
\[ \text{C=} \]
\[ \text{= FOR EACH ELEMENT} \]
\[ \text{DO 530 IE=1,NEL} \]

\[ \text{C} \]
\[ \text{IF(N(IE,1).EQ.0) GO TO 530} \]

\[ \text{C} \]
\[ N1=N(IE,1) \]
\[ N2=N(IE,2) \]
\[ N3=N(IE,3) \]
\[ N4=N(IE,4) \]

\[ \text{C} \]
\[ XL(1)=XX(N1) \]
\[
\begin{align*}
XL(2) &= XX(N2) \\
XL(3) &= XX(N3) \\
XL(4) &= XX(N4) \\
C \\
YL(1) &= YY(N1) \\
YL(2) &= YY(N2) \\
YL(3) &= YY(N3) \\
YL(4) &= YY(N4) \\
C \\
C\text{=}\text{==================== FOR EACH GAUSS POINT} \\
\text{DO 550 IG}=1,4 \\
C \\
\text{DO 560 K}=1,2 \\
\text{DO 560 L}=1,4 \\
VJACDN(K,L)=0.0 \\
560 \text{ CONTINUE} \\
C \\
\text{PXI}=GP(IG,1) \\
\text{PET}=GP(IG,2) \\
C \\
\text{CALL SHAPE}(\text{PXI, PET, XL, YL, SH, DN, VJAC, DETJAC}) \\
C \\
\text{DO 570 J}=1,2 \\
\text{DO 570 L}=1,4 \\
\text{DO 570 K}=1,2 \\
VJACDN(J,L)=VJACDN(J,L)+VJAC(J,K)\times DN(K,L) \\
570 \text{ CONTINUE} \\
C \\
C\text{=}\text{================================:========================J1} \\
QG=SH(1)\times Q(N1,IC)+SH(2)\times Q(N2,IC)+SH(3)\times Q(N3,IC)+ \\
* SH(4)\times Q(N4,IC) \\
C
\[ W_{EX} = VJACDN(1,1) \times WE(N1) + VJACDN(1,2) \times WE(N2) + VJACDN(1,3) \times WE(N3) + VJACDN(1,4) \times WE(N4) \]
\[ C \]
\[ X_{J11} = W_{EX} \times QG \]
\[ X_{J11} = X_{J11} \times DETJAC \]
\[ C \]
\[ U_{1X} = VJACDN(1,1) \times U1(N1) + VJACDN(1,2) \times U1(N2) + VJACDN(1,3) \times U1(N3) + VJACDN(1,4) \times U1(N4) \]
\[ C \]
\[ U_{2X} = VJACDN(1,1) \times U2(N1) + VJACDN(1,2) \times U2(N2) + VJACDN(1,3) \times U2(N3) + VJACDN(1,4) \times U2(N4) \]
\[ C \]
\[ Q_X = VJACDN(1,1) \times Q(N1,IC) + VJACDN(1,2) \times Q(N2,IC) + VJACDN(1,3) \times Q(N3,IC) + VJACDN(1,4) \times Q(N4,IC) \]
\[ C \]
\[ Q_Y = VJACDN(2,1) \times Q(N1,IC) + VJACDN(2,2) \times Q(N2,IC) + VJACDN(2,3) \times Q(N3,IC) + VJACDN(2,4) \times Q(N4,IC) \]
\[ C \]
\[ X_{J22} = S11(IE,IG) \times U_{1X} \times Q_X + S22(IE,IG) \times U_{2X} \times Q_Y \]
\[ * \]
\[ S12(IE,IG) \times U_{1X} \times Q_Y + S12(IE,IG) \times U_{2X} \times Q_X \]
\[ X_{J22} = X_{J22} \times DETJAC \]
\[ C \]
\[ QG = SH(1) \times Q(N1,IC) + SH(2) \times Q(N2,IC) + SH(3) \times Q(N3,IC) + SH(4) \times Q(N4,IC) \]
\[ C \]
\[ EP{11X} = VJACDN(1,1) \times EP11(N1) + VJACDN(1,2) \times EP11(N2) + VJACDN(1,3) \times EP11(N3) + VJACDN(1,4) \times EP11(N4) \]
\[ C \]
\[ EP{22X} = VJACDN(1,1) \times EP22(N1) + VJACDN(1,2) \times EP22(N2) + VJACDN(1,3) \times EP22(N3) + VJACDN(1,4) \times EP22(N4) \]
\[ C \]
\[ EP{12X} = VJACDN(1,1) \times EP12(N1) + VJACDN(1,2) \times EP12(N2) + VJACDN(1,3) \times EP12(N3) + VJACDN(1,4) \times EP12(N4) \]
*          VJACDN(1,3)*EIP12(N3)+VJACDN(1,4)*EIP12(N4)
C
XJ33=S11(IE,IG)*EP11X*QG+S22(IE,IG)*EP22X*QG+
*     S12(IE,IG)*EIP12X*QG*2.0
XJ33=XJ33*DETJAC
C===============================================
QG=SH(1)*Q(N1,IC)+SH(2)*Q(N2,IC)+SH(3)*Q(N3,IC)+
*     SH(4)*Q(N4,IC)
C
EIP11X=VJACDN(1,1)*EIP11(N1)+VJACDN(1,2)*EIP11(N2)+
*     VJACDN(1,3)*EIP11(N3)+VJACDN(1,4)*EIP11(N4)
C
EIP22X=VJACDN(1,1)*EIP22(N1)+VJACDN(1,2)*EIP22(N2)+
*     VJACDN(1,3)*EIP22(N3)+VJACDN(1,4)*EIP22(N4)
C
EIP12X=VJACDN(1,1)*EIP12(N1)+VJACDN(1,2)*EIP12(N2)+
*     VJACDN(1,3)*EIP12(N3)+VJACDN(1,4)*EIP12(N4)
C
XJ44=S11(IE,IG)*EIP11X*QG+S22(IE,IG)*EIP22X*QG+
*     S12(IE,IG)*EIP12X*QG*2.0
XJ44=XJ44*DETJAC
C=================================================================
XJ1(IE,IC)=XJ1(IE,IC)+XJ11
XJ2(IE,IC)=XJ2(IE,IC)+XJ22
XJ3(IE,IC)=XJ3(IE,IC)+XJ33
XJ4(IE,IC)=XJ4(IE,IC)+XJ44
550 CONTINUE
C
XJ(IE,IC)=XJ1(IE,IC)+XJ2(IE,IC)+XJ3(IE,IC)+XJ4(IE,IC)
530 CONTINUE
500 CONTINUE
C
C===============================================
C ** TOTAL J FOR EACH CONTOUR (1 - JCON) **
C===============================================
C
DO 610 K=1,JCON
  TJ1(K)=0.0
  TJ2(K)=0.0
  TJ3(K)=0.0
  TJ4(K)=0.0
  TJ(K)=0.0
610 CONTINUE
C
C===============================================
C
CALL CRACK(JNODE,JEL,JCON,JTIP,N)
C
DO 600 IC=1,JCON
  DO 630 J=1,NEL
    IE=JEL(IC,J)
    IF(IE.EQ.0) GO TO 640
    TJ1(IC)=TJ1(IC)+XJ1(IE,IC)
    TJ2(IC)=TJ2(IC)+XJ2(IE,IC)
    TJ3(IC)=TJ3(IC)+XJ3(IE,IC)
    TJ4(IC)=TJ4(IC)+XJ4(IE,IC)
    TJ(IC)=TJ(IC)+XJ(IE,IC)
  630 CONTINUE
  640 CONTINUE
640 CONTINUE

C
  TJ1(IC)=TJ1(IC)*2.0
  TJ2(IC)=TJ2(IC)*2.0
  TJ3(IC)=TJ3(IC)*2.0
  TJ4(IC)=TJ4(IC)*2.0
TJ(IC)=TJ(IC)*2.0

C
600 CONTINUE
C=======================================
C ** OUTPUT ***
C======================================================== CRACK
   DO 710 I=1,JCON
      WRITE(12,51) I
   51 FORMAT(//' CONTOUR =' ,I3/)
   DO 715 J=1,NEL
      IF(JEL(I,J).EQ.0) GO TO 715
      WRITE(12,52) J,JEL(I,J)
   52 FORMAT(2I6)
715 CONTINUE
710 CONTINUE
C
   DO 720 I=1,JCON
      WRITE(12,51) I
   DO 725 J=1,NODE
      IF(JNODE(I,J).EQ.0) GO TO 725
      WRITE(12,52) J,JNODE(I,J)
725 CONTINUE
720 CONTINUE
C======================================================== Q
   DO 730 I=1,JCON
      WRITE(12,51) I
   DO 735 J=1,NODE
      IF( Q(J,I).EQ.1.0 ) GO TO 735
      WRITE(12,61) J,Q(J,I)
   61 FORMAT(I5,F10.1)
735 CONTINUE
730 CONTINUE
C================================================================================== OUTPUT
WRITE(12,30)
30 FORMAT(/1X,'CONTOUR',3X,'TJ1',7X,'TJ2',
* 7X,'TJ3',7X,'TJ4',7X,'J'/)
DO 700 I=1,JCON
   WRITE(12,40) I,TJ1(I),TJ2(I),TJ3(I),TJ4(I),TJ(I)
40 FORMAT(I6,5F10.4)
700 CONTINUE
C==================================================================================
STOP
END
C
C==================================================================================
C
C==================================================================================

SUBROUTINE SHAPE (PXI,PET,XL,YL,SH,DN,VJAC,DETJAC)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION XL(4),YL(4),DN(2,4),VJAC(2,2),SH(4)
DIMENSION XI(4),ET(4)
C
DATA XI/-1.0,1.0,1.0,-1.0/
DATA ET/-1.0,-1.0,1.0,1.0/
C
** SHAPE FUNCTION & DERIVATIVE **
C
DO 810 L=1,4
   DUM1=(1.0+XI(L)*PXI)/4.0
   DUM2=(1.0+ET(L)*PET)/4.0
   SH(L)=4.0*DUM1*DUM2
   DN(1,L)=XI(L)*DUM2
   DN(2,L)=ET(L)*DUM1
810 CONTINUE
C==============================================
C ** JACOBIAN AND DETERMINANT **
C==============================================

VJAC(1,1)=0.0
VJAC(1,2)=0.0
VJAC(2,1)=0.0
VJAC(2,2)=0.0

C
DO 830 L=1,4
   VJAC(1,1)=VJAC(1,1)+DN(1,L)*XL(L)
   VJAC(1,2)=VJAC(1,2)+DN(1,L)*YL(L)
   VJAC(2,1)=VJAC(2,1)+DN(2,L)*XL(L)
   VJAC(2,2)=VJAC(2,2)+DN(2,L)*YL(L)
830 CONTINUE
C
DETJAC=VJAC(1,1)*VJAC(2,2)-VJAC(2,1)*VJAC(1,2)
C==============================================
C ** REPLACE JACOBIAN BY ITS INVERSE **
C==============================================
C
   DUM1=VJAC(1,1)/DETJAC
   VJAC(1,1)=VJAC(2,2)/DETJAC
   VJAC(1,2)=-VJAC(1,2)/DETJAC
   VJAC(2,1)=-VJAC(2,1)/DETJAC
   VJAC(2,2)=DUM1

C
840 CONTINUE
C
RETURN
END
C
C==============================================
SUBROUTINE CRACK (JNODE,JEL,JCON,JTIP,N)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION JNODE(10,1000),JEL(10,1000),N(1000,4)

C
C USE JNODE(IC,IN), JEL (IC,IE)
C
DO 150 K=1,10
DO 150 L=1,1000
JEL(K,L)=0
JNODE(K,L)=0
150 CONTINUE

C
DO 170 K=1,7
DO 170 L=1,K
JEL(K,1+(L-1)*4)=200+K-L+1
JEL(K,2+(L-1)*4)=207+K-L+1
JEL(K,3+(L-1)*4)=214+K-L+1
JEL(K,4+(L-1)*4)=221+K-L+1
170 CONTINUE

C
RETURN
END