INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
Analysis of damage to food particles during pumping

Rahardjo, Budi, Ph.D.
The Ohio State University, 1992
ANALYSIS OF DAMAGE TO FOOD
PARTICLES DURING PUMPING

DISSERTATION
Presented in Partial Fulfillment of the Requirement for
the Degree of Doctor of Philosophy in the Graduate
School of the Ohio State University

By
Budi Rahardjo

The Ohio State University
1992

Dissertation Committee:
Dr. Sudhir K. Sastry
Dr. Winston D. Bash
Dr. Robert J. Gustafson
Dr. Robert G. Holmes

Approved by:
Adviser
Department of Agricultural Engineering
Dedicated to
my parents and gurus,
for their inspiration,
and encouragement.
ACKNOWLEDGEMENTS

I wish to express my sincere appreciation to Dr. Sudhir K. Sastry for his patience, valuable guidance, constructive criticisms, and inspiration throughout my entire course of graduate study. I acknowledge gratefully Dr. Winston D. Bash, Dr. Robert J. Gustafson, and Dr. Robert G. Holmes, for their useful suggestions and comments. I am thankful to Brian F. Heskitt, Carl Cooper, and Dusty Baumann for their technical assistance. I well appreciate the Indonesian Ministry of Education for the financial support. I am grateful the MUCIA-JMOSO’s staffs for their encouragement.

I express my special thanks to my parents and parents-in-law for their support and encouragement. Finally, I heartily appreciate my wife, Nunuk, and children, Bram, Dhian, and Astri, for their love, patience, and moral support.
VITA

October 14, 1944 ............ Born - Yogyakarta, Indonesia

1967 ........................ B. Sc., Gadjah Mada University, Yogyakarta, Indonesia.

1968 - 1971 ................. Assistant lecturer, Gadjah Mada University, Yogyakarta, Indonesia.


1976 - 1977 ................. Lecturer, Gadjah Mada University, Yogyakarta Indonesia.

1980 - 1986 ................. Lecturer, Gadjah Mada University, Yogyakarta Indonesia.

FIELDS OF STUDY

Major Field: Agricultural Engineering

Studies in Chemical Engineering, Dr. Robert S. Brodkey, Dr. Harry C. Hershey, Dr. Kent S. Knaebel.

Studies in Food Process Engineering, Dr. Sudhir K. Sastry, Dr. John L. Blaisdell.

Studies in Food Science and Technology, Dr. Winston D. Bash, Dr. Santi R. Bhowmik, Dr. Poul M. Hansen, Dr. Kurt L. Wiese.
# TABLE OF CONTENT

| DEDICATION | ................................................................. | ii |
| ACKNOWLEDGEMENT | ............................................................... | iii |
| VITA | ................................................................. | iv |
| LIST OF TABLES | ............................................................... | vii |
| LIST OF FIGURES | ............................................................... | x |

**CHAPTER**

| I  INTRODUCTION | ................................................................. | 1 |
| References | ................................................................. | 5 |

| II  KINETICS OF SOFTENING OF POTATO TISSUE DURING COOKING | ................................................................. | 6 |
| Abstract | ................................................................. | 6 |
| Introduction | ................................................................. | 6 |
| Materials and Methods | ................................................................. | 8 |
| Results and Discussions | ................................................................. | 16 |
| Conclusions | ................................................................. | 27 |
| Nomenclature | ................................................................. | 28 |
| References | ................................................................. | 30 |

| III  DAMAGE TO FOOD PARTICLES DURING PUMPING | ................................................................. | 33 |
| Abstract | ................................................................. | 33 |
| Introduction | ................................................................. | 33 |
| Materials and Methods | ................................................................. | 35 |
| Results and Discussions | ................................................................. | 45 |
| Conclusions | ................................................................. | 52 |
| Nomenclature | ................................................................. | 53 |
| References | ................................................................. | 55 |

| IV  PARTICLE-WALLS COLLISIONS IN SOLID-LIQUID MIXTURES DURING PUMPING | ................................................................. | 57 |
| Abstract | ................................................................. | 57 |
| Introduction | ................................................................. | 58 |
| Materials and Methods | ................................................................. | 60 |
| Results and Discussions | ................................................................. | 68 |
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>TABLE</th>
<th>DESCRIPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Ranges of assigned cooking temperature and cooking time.</td>
<td>12</td>
</tr>
<tr>
<td>2.</td>
<td>Average peak temperature and heating time on potato samples during cooking.</td>
<td>19</td>
</tr>
<tr>
<td>3.</td>
<td>Estimation of the cooking rate constant ( k_T ) of the lower temperature range of the fast phase.</td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>Estimation of the activation energy ( E_0 ) and the frequency factor ( A ) of the lower temperature range of the fast phase.</td>
<td>21</td>
</tr>
<tr>
<td>5.</td>
<td>Estimation of the cooking rate constant ( k_T ) of the higher temperature range of the fast phase.</td>
<td>21</td>
</tr>
<tr>
<td>6.</td>
<td>Estimation of the activation energy ( E_0 ) and the frequency factor ( A ) of the higher temperature range of the fast phase.</td>
<td>22</td>
</tr>
<tr>
<td>7.</td>
<td>Cooking rate constants of the slow rate phase.</td>
<td>24</td>
</tr>
<tr>
<td>8.</td>
<td>Transition points from the fast rate phase to the slow rate phase.</td>
<td>24</td>
</tr>
<tr>
<td>9.</td>
<td>Pertinent variables for particle damage during pumping.</td>
<td>36</td>
</tr>
<tr>
<td>10.</td>
<td>Rheological properties of fluids used in pumping experiments.</td>
<td>40</td>
</tr>
<tr>
<td>11.</td>
<td>Calculated strength of potato particles cooked at 90 and 100°C for given cooking time.</td>
<td>41</td>
</tr>
<tr>
<td>12.</td>
<td>Dimensions of the rotor cavity.</td>
<td>44</td>
</tr>
<tr>
<td>13.</td>
<td>Number of particles and replicates.</td>
<td>45</td>
</tr>
<tr>
<td>14.</td>
<td>Pertinent variables for particle collision with wall during pumping.</td>
<td>61</td>
</tr>
</tbody>
</table>
15. Calculated average failure strength $\sigma_m$ of potato cubes with size of 10, 15 and 20 cm cooked at 90° and 100°C for cooking time 3, 6, 9, and 12 minutes. ........................................ 90

16. Measured failure strength $\sigma_m$ and Young's modulus of elasticity $E$ to failure of potato cooked at 90°C for cooking time of 0-20 minutes. ...................... 91

17. Summary of stress distribution on an uncooked potato cube under a compression pressure of 0.1 MPa and the whole bottom is supported by a flat surface. ........................................ 93

18. Summary of stress distribution on an uncooked potato cube under a cutting pressure of 0.1 MPa and the bottom is partially supported by a flat surface. ...................... 95

19. Estimated total compressive forces to failure on cooked potato cubes with size of 10, 15, and 20 mm. ............................................ 98

20. Estimated total cutting forces to failure on cooked potato cubes with size of 10, 15, and 20 mm. .... 99

21. Estimated torques and powers to break uncooked and cooked potato cubes with size of 10, 15 and 20 mm at pumping speeds of 90, 130, and 180 rpm under cutting forces with case of loading area of 5 elements. ............................... 100

22. Physical and thermal properties of aluminum and potato. ........................................ 114

23. Stress failure of potato cylinder cooked at temperature of 70°C. ............................ 114

24. Stress failure of potato cylinder cooked at temperature of 80°C. ............................ 115

25. Stress failure of potato cylinder cooked at temperature of 90°C. ............................ 115

26. Stress failure of potato cylinder cooked at temperature of 100°C. ............................ 116

27. Stress failure of potato cylinder cooked at temperature of 105°C. ............................ 116

28. Stress failure of potato cylinder cooked at temperature of 110°C. ............................ 117
29. Stress failure of potato cylinder cooked at temperature of 115°C. ......................... 117
30. Stress failure of potato cylinder cooked at temperature of 120°C. ......................... 118
31. Stress failure of potato cylinder cooked at temperature of 125°C. ......................... 118
32. Stress failure of potato cylinder cooked at temperature of 130°C. ......................... 119
33. Data of damage to particles during pumping. ...... 121
34. Data of particle collisions with the wall during pumping. ................................. 125
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURES</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Schematic diagram of the mini retort for cooking the potato samples.</td>
<td>11</td>
</tr>
<tr>
<td>2. Typical failure stress ratio $\sigma_m/\sigma_0$.  (a). Cooking temperature 70°-90°C.  (b). Cooking temperature 100°-115°C.  (c). Cooking temperature 120°-128°C.</td>
<td>17</td>
</tr>
<tr>
<td>4. Cooking rate constant of the fast rate phase.</td>
<td>20</td>
</tr>
<tr>
<td>5. Cooking rate constant of the slow rate phase.</td>
<td>23</td>
</tr>
<tr>
<td>6. Duration $t_d$ of the fast rate phase, determined by the intersection of the fast rate phase and slow rate curves.</td>
<td>25</td>
</tr>
<tr>
<td>7. Transition point $\sigma_d/\sigma_0$ from the fast rate phase to the slow rate phase.</td>
<td>26</td>
</tr>
<tr>
<td>8. Setup of the pumping experiment.</td>
<td>42</td>
</tr>
<tr>
<td>9. Detailed pump rotor.</td>
<td>43</td>
</tr>
<tr>
<td>10. Relationship between broken particle fraction with particle size, density ratio $D_r$, Reynolds number $Re$, Froude number $Fr$, Euler number $Eu$, and particle volumetric fraction $V_f$.</td>
<td>46</td>
</tr>
<tr>
<td>11. Relationship between broken particle fraction with particle size, Reynolds number $Re$, Froude number $Fr$, Euler number $Eu$, and particle volumetric fraction $V_f$ (less than 0.01).</td>
<td>49</td>
</tr>
</tbody>
</table>
13. Views of the pump and rotor assembly used for particle motion observation. ................. 65

14. Relation of the collision velocity ratio with the dimensionless groups. ....................... 69

15. Relation of the collision velocity ratio with Reynolds number Re, Froude number Fr and particle volumetric fraction Vf. ......................... 70

16. Relation of the number of particle collisions with the dimensionless groups. ................. 72

17. Relation of the number of particle collisions with particle size ratio, Reynolds number Re, Froude number Fr, and particle volumetric fraction Vf. 73

18. Model of cube under compression load. The cube is meshed into 10x10 elements. The loads are applied at the top surface and the whole bottom surface is supported by a flat surface. ...................... 88

19. Model of cube under cutting load. The cube is meshed into 10x10 elements. The loads are applied at the top surface and the bottom surface is partially supported by a flat surface. ............... 89

20. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area $A_c$ of 10 mm$^2$ or at two nodal points and the bottom is supported by a flat surface. .... 92

21. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area $A_c$ of 50 mm$^2$ or at 6 nodal points and the bottom is supported by a flat surface. .... 92

22. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area $A_c$ of 10 mm$^2$ or at two nodal points and the bottom is supported on ten nodal points. 94

23. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area $A_c$ of 50 mm$^2$ or at 6 nodal points and the bottom is supported on 6 nodal points. .... 94

24. Stress distribution on a cube of uncooked potato under a compression pressure of 0.2 MPa at a compression area $A_c$ of 50 mm$^2$ or at 6 nodal points and the bottom is supported by a flat surface. .... 96
25. Stress distribution on a cube of uncooked potato under a compression pressure of 0.025 MPa at a compression area $A_c$ of 200 mm$^2$ or at 6 nodal points and the bottom is supported by a flat surface. .... 97

26. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 20 mm$^2$ or at 3 nodal points and the bottom is supported by a flat surface. .......... 128

27. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 30 mm$^2$ or at 4 nodal points and the bottom is supported by a flat surface. .......... 128

28. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 40 mm$^2$ or at 5 nodal points and the bottom is supported by a flat surface. .......... 129

29. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 60 mm$^2$ or at 7 nodal points and the bottom is supported by a flat surface. .......... 129

30. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 70 mm$^2$ or at 8 nodal points and the bottom is supported by a flat surface. .......... 130

31. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 80 mm$^2$ or at 9 nodal points and the bottom is supported by a flat surface. .......... 130

32. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 90 mm$^2$ or at 10 nodal points and the bottom is supported by a flat surface. .......... 131

33. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 20 mm$^2$ or at 3 nodal points and the bottom is supported on 9 nodal points. .......... 131

34. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 30 mm$^2$ or at 4 nodal points and the bottom is supported on 8 nodal points. .......... 132
35. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 40 mm$^2$ or at 5 nodal points and the bottom is supported on 7 nodal points. ....... 132

36. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area 60 mm$^2$ or at 7 nodal points and the bottom is supported on 5 nodal points. .............. 133

37. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 70 mm$^2$ or at 8 nodal points and the bottom is supported on 4 nodal points. ...... 133

38. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 80 mm$^2$ or at 9 nodal points and the bottom is supported on 3 nodal points. ...... 134

39. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 90 mm$^2$ or at 10 nodal points and the bottom is supported on 2 nodal points. ...... 134

40. Stress distribution on a cube of uncooked potato under a compression pressure of 0.5 MPa at a compression area of 50 mm$^2$ or at 6 nodal points and the bottom is supported by a flat surface. .. 135

41. Stress distribution on a cube of uncooked potato under a compression pressure of 0.044 MPa at a compression area of 112.5 mm$^2$ or at 6 nodal points and the bottom is supported by a flat surface. .. 135

xiii
CHAPTER I
INTRODUCTION

The primary objective of food processing is to extend the shelf lives of the food products while retaining nutritional values. Demands for high quality of food products challenge food scientists to continually improve methods of food processing. Today conventional thermal processing of food involves filling the products into containers and then heating the containers at a recommended temperature for certain time to achieve commercial sterility of the products. Results of experimental studies indicated that thermal processing destroys microorganisms, but reduces the nutritional value of the products. It was revealed that the thermal degradation rate of the microorganisms at high temperature is higher than that of the food nutrient. Based on these thermal resistance differences, it is suggested that food should be processed at high temperature for short time (HTST) or at ultra high temperature (UHT).

Aseptic processing has been applied successfully for liquid foods. In this processing method, food products and containers are sterilized separately and the foods are packaged in sterile environment. The sterilization of food
products can be accomplished by the application of a high temperature short time process in continuous sterilizers (UHT) (Chandarana et al., 1986). Aseptic processing reduces significantly the cost of food packaging and offers food products that are desirable to consumers. The success of aseptic processing for liquid foods has motivated the investigation to apply this aseptic technology for liquid food containing large particles (≥2 cm). However, processing of products containing particles at high temperatures is the main restraining factor. The constraint in the development of thermal processing for particulates is the lack of knowledge of the flow behavior and convective heat transfer during sterilization. Many intensive studies on these fields are being conducted (Nelson et al., 1987).

Current system designs of aseptic processing involve the use of a positive displacement pump to move product through heating scraped-surface heat exchanger, a hold tube, cooling scraped-surface heat exchanger, and a back pressure valve or a positive displacement timing pump positioned after the cooler to maintain the pressure within the system. The back pressure must be kept at a level needed to keep the products boiling temperature higher than the processing temperature (Chandarana et al., 1986). Furthermore, the sterilization time for heterogeneous products is related to the rate at which the products is moving through the system, the physical and rheological properties of the carrier medium and particle,
and the design components. Thus pumps are a critical consideration in the design of a continuous aseptic processing system. The metering pump preferably is a positive displacement type since this pump type is less sensitive to pressure drop than are centrifugal pumps. Selection of the type of positive displacement pump is dependent on the characteristics of the product such as viscosity and presence of particulate, pressure drops in the system, and cost (Nelson et al., 1987).

Subjecting food particles to pumping may reduce their quality via deformation and breakage. Furthermore, most foods are treated with heat prior to pumping (Nelson et al., 1987). This treatment reduces the particle strength and consequently increases the number of the deformed particles. The damage phenomena of the food particles during pumping qualitatively is already known. However, since this knowledge cannot be used to relate the damage occurrences with the existing processing conditions, it is necessary to understand quantitatively the mechanical damage to food particles. This information is useful in estimating the damage occurrence in food processing especially damage due to mechanical equipment. Since consumers acceptance of products will determine the success of the processing methods, mechanical damage to food particles needs to be quantified.

Early studies in pumping of liquid-particle mixtures were primarily concerned with the pump performances (Brebner, 1964;
Walker and Goulas, 1984; Ou et al. 1987). Very few studies have been conducted to analyze the degradation of the particle during pumping (Minemura et al. 1986; Nassar, 1987). Kinetics of thermal softening of food have been investigated intensively, but the relationships to mechanical damage have not been addressed (Huang and Bourne, 1983; Rao and Lund, 1987). Mechanical damage of food products has been studied intensively, but cooked product integrity has not been addressed (Mohsenin, 1977). Phenomena of fluid particle transport has become of major interest for studies during the past several years (Sastry and Zuritz, 1987). Useful literature on solid liquid flow, capsule flow, motion of particles in pipe flow and mathematical modeling are accessible.

Accordingly, the objectives of this dissertation were:

1. To investigate the kinetics of softening of vegetable tissue during cooking.
2. To investigate experimentally the damage to particles during pumping and its relationship with the pumping variables.
3. To study the particle-wall collisions in solid-liquid mixtures during pumping and their relationships with the pumping variables.
4. To determine analytically the maximum expected stresses for cooked food particles under compressive loads associated with pumping.
References


CHAPTER II

KINETICS OF SOFTENING OF POTATO TISSUE DURING COOKING

Abstract

Kinetics of softening of potato tissue under heat treatment was investigated. Failure stress was found to be expressed by two first order kinetic equations; fast rate phase and slow rate phase. The reaction rate constants of the fast and slow rate phases were found to be described by two Arrhenius equations; one each at low and high temperature ranges. The logarithm of the duration of the fast rate phase was linearly related to the inverse of the cooking temperature.

Introduction

Softening of food tissue during thermal processing is a constraint in the development of continuous processes for food particles. These particles are often broken or deformed during processing (Nelson, 1988).
In general, time and temperature dependencies of the softening of food tissue during cooking are adequately described using reaction kinetic laws and Arrhenius equations. Loh and Breene (1981, 1982) indicated that the first order kinetic model was applicable for most vegetables. Harada et al. (1985a, 1985b) mentioned that for process time exceeding the optimal cooking time, textural softening was best described by a second order kinetic equation. Paulus and Saguy (1980) found that the effect of heat treatment on the quality of carrot followed first order kinetics. The cooking rate constants found from the kinetic model in general complied with the Arrhenius equation. However, Suzuki et al. (1976) found that the cooking rate constant of rice can be expressed using two Arrhenius equations sequentially in which the slopes changed at a cooking temperature of about 110°C. The size of the samples for kinetic studies were adequately small (Harada et al., 1985a, Paulus and Saguy, 1980). However significant lags existed between the sample and the fluid temperatures. In order to eliminate the lag time effects, Loh and Breene (1981) eliminated data points in the initial heating stages on the determination of kinetic parameters for potatoes. Harada et al. (1985) developed an approximation equation to calculate the average temperature of the samples. They defined the cooking time as the time elapsing from the moment when the product reached an average temperature of 80°C to the end of the process time. Suzuki et al. (1976) in their
kinetic studies on cooking of rice corrected the cooking time by converting the come up time of the vessel cooker center temperature to the experimental temperature into the cooking time by an integration method. Konanayakam and Sastry (1986) developed a procedure to determine the kinetic parameters using sample average temperatures and iterative correction for thermal lags. The average temperatures of the samples were determined from the center temperature data and the relation between the average and center temperature were found from Heisler charts.

It is desirable to investigate a softening model of vegetable tissue during cooking. The model should be able to estimate the strength of the food particle during processing for a given cooking time and temperature. Accordingly, the objectives of this study were to determine kinetic parameters for thermal softening of potatoes particles.

**Materials and Methods**

**Theory.** The kinetic equation for thermal softening can be expressed as (Lund, 1983; Rao and Lund, 1986):

\[
\frac{d\sigma}{dt} = -k_T \sigma^n
\]  

(1)

For the first order reaction or \( n = 1 \) equation (1) becomes:
\[
\ln \left( \frac{\sigma_f}{\sigma_i} \right) = -k_r t
\]

(2)

The reaction rate constant \( k_r \) is dependent on the cooking temperature which can be expressed using the Arrhenius equation as:

\[
\ln \left( \frac{k_r}{A} \right) = -\frac{E_a}{R} \frac{1}{T}
\]

(3)

For a first order reaction, equations (2) and (3) can be combined as:

\[
\ln \left( \frac{\sigma_f}{\sigma_i} \right) = -A t e^{-\frac{E_a}{R} \frac{1}{T}}
\]

(4)

The change of food properties during heating or cooling in which the average sample temperature is time dependent, are evaluated using the relation:

\[
\ln \left( \frac{\sigma_i}{\sigma_0} \right) = \int_0^{t_h} -A e^{-\frac{E_a}{R} \frac{1}{T_a(t)}} dt
\]

(5)

Similarly, the food properties before cooling are evaluated using the relation:

\[
\ln \left( \frac{\sigma_m}{\sigma_f} \right) = \int_0^{t_c} -A e^{-\frac{E_a}{R} \frac{1}{T_a(t)}} dt
\]

(6)

The correction can be performed by approximating the heating and cooling curves as a succession of \( N \) constant.
temperature treatments of short time $\Delta t$ (Konanakayam and Sastry, 1986). For heating, equation (5) can be expressed as:

$$\ln\left[\frac{\sigma_f}{\sigma_0}\right] = \sum_{n=1}^{N_h} -A e^{-\frac{E_a}{RT_a(t_n)}} \Delta t$$  \hspace{1cm} (7)

Similarly equation (6) becomes:

$$\ln\left[\frac{\sigma_f}{\sigma_m}\right] = \sum_{n=1}^{N_c} -A e^{-\frac{E_a}{RT_a(t_n)}} \Delta t$$  \hspace{1cm} (8)

Sample Material. Potatoes were chosen as the sample material in these kinetic studies because of their homogenous properties (Matthews and Hall, 1968). Samples of Idaho potatoes (variety unknown) were purchased from a grocery store. Samples were stored at a temperature of about 7°C.

Sample Preparation. Potatoes were sliced to a thickness of 10 mm. Cylindrical samples were prepared using a cork bore (diameter 10 mm) mounted on a small stand of an electrical drill.

Cooking Procedure. Samples were cooked either using water in a bath or steam in a mini retort. The water bath (Brookfield Ex200, Brookfield Engineering Laboratories Inc., Stoughton, MS) was used to heat the samples at a temperature range of 70 - 100°C. The mini retort (Fig. 1) was used to heat the samples to temperature above 100°C. The temperature was
Figure 1. Schematic diagram of the mini retort for cooking potato samples.

measured using thermocouple (probe diam. 1/16 in., wire gauge 24, Omega Engineering Inc., Stamford, CT). The temperature history of the heating was recorded using a data logger (21X Micrologger, Campbell Scientific Inc., Logan, UT). Ranges of time and temperature combinations are presented in Table 1.

Compression Test Procedure. All tests were conducted using Instron Universal Testing Instrument (metric model). The cylindrical specimen was placed in between two compression plates lubricated with a thin film of oil to reduce the
Table 1. Ranges of assigned cooking temperature and cooking time.

<table>
<thead>
<tr>
<th>Cooking Temperature, °C.</th>
<th>Assigned cooking time, min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0, 15, 30, 45, 60, 75</td>
</tr>
<tr>
<td>80</td>
<td>0, 10, 20, 30, 40, 50</td>
</tr>
<tr>
<td>90</td>
<td>0, 5, 10, 15, 20, 25</td>
</tr>
<tr>
<td>100</td>
<td>0, 3, 6, 9, 12, 15</td>
</tr>
<tr>
<td>105</td>
<td>0, 2, 6, 10, 14, 18</td>
</tr>
<tr>
<td>110</td>
<td>0, 2, 4, 6, 8, 10</td>
</tr>
<tr>
<td>115</td>
<td>0, 2, 4, 6, 8, 10</td>
</tr>
<tr>
<td>120</td>
<td>0, 2, 4, 6, 8, 10</td>
</tr>
<tr>
<td>125</td>
<td>0, 2, 3, 4, 5, 6</td>
</tr>
<tr>
<td>130</td>
<td>0, 2, 3, 4, 5, 6</td>
</tr>
</tbody>
</table>

Effects of the shear stresses due to contact between the load plates and the flat ends of the sample (Finney, 1963; Finney et al., 1964; Finney and Hall, 1967). The strain rate was slow (1 cm/min) as suggested by Diehl and Hamann (1979). Compression was stopped at a strain level of 0.6 mm/mm. Each test was conducted using five samples.

Failure was defined as a point where the force reached its peak and was then followed by decreasing compression force (Mohsenin, 1977, 1986). A computer program was developed to scan the curve and to locate the first peak point. The failure stress was defined as the stress at the peak point. It was calculated from the peak force divided by the apparent (initial) cross sectional area of the samples.
Lag Correction for Heat Transfer. The Biot number of the potato samples was expected to be greater than 0.1; thus during heating and cooling significant lags existed between the sample and fluid temperatures. It was observed that significant softening of tissue occurred during heating. To investigate the time lag correction, the center temperatures of the samples were measured during cooking. Copper constantan thermocouples (probe diam. 1/16 in., wire gauge 24, Omega Engineering Inc. Stamford, CT) were imbedded into the center of the potato cylinder and temperatures were recorded every second using a data logger during heating and cooling.

The lag time of the sample to reach the fluid temperature during heating or to reach the water temperature during cooling was evaluated by utilizing Heisler charts for a finite cylinder and using the Biot number of the samples (Konanayakam and Sastry, 1986). The temperature of the center $T_c$ of a finite cylinder is given as:

$$\frac{T_c - T_s}{T_i - T_s} = \left(\frac{T_c - T_s}{T_i - T_s}\right)_{tc} \times \left[\frac{T_c - T_s}{T_i - T_s}\right]_{is}$$

Similarly the sample average temperature $T_a$ is given as:

$$\frac{T_a - T_s}{T_i - T_s} = \left(\frac{T_a - T_s}{T_i - T_s}\right)_{tc} \times \left[\frac{T_a - T_s}{T_i - T_s}\right]_{is}$$

Quantities of the center and average temperature ratios of the finite cylinder were evaluated using Heisler charts (Henderson and Perry, 1976). By comparing both values, the
average sample temperatures can be calculated from the measured center sample temperatures. A program was developed to calculate the average sample temperature. The Heisler charts were digitized and attached to the program. The temperature ratio for given Biot and Fourier numbers was calculated using interpolation between two known successive points.

The period of the constant temperature estimated using Heisler charts is the constant temperature of the particle between time at \( \sigma_i \) and time at \( \sigma_f \) as shown by equation 4. Thus using \( \sigma_i \) and \( \sigma_f \), the activation energy \( E_a \) and the frequency factor \( A \) can be determined. The real cooking time is the period between the time at \( \sigma_i \) and the time at \( \sigma_f \). The values of \( \sigma_i \) and \( \sigma_f \) are found from the correction equations (7) and (8). A program written in BASIC (Turbo Basic, Borland International, Inc., CA) was used to evaluate the initial and final failure stresses. The activation energy \( E_a \) and the frequency factor \( A \) were determined using regression analyses (Minitab Statistical Software, Minitab, Inc., College Park PA).

**Determination of Heat Transfer Coefficient.** Measurements of the convective heat transfer coefficient \( h_s \) was conducted using aluminum cylinders with a diameter and thickness of 10 mm. The samples were heated with steam or with hot water, and then cooled with water. The heat transfer coefficients were
calculated from the temperature-time curves based on the following equation (Heldman and Singh, 1980):

\[ \frac{dT_p(t)}{dt} = \frac{A_s h_s}{m c_p} [T_s - T_p(t)] \quad (11) \]

For constant fluid temperature \( T_s \), the integration of equation (11) gave:

\[ \ln \left( \frac{T_s - T_p(t)}{T_s - T_p(0)} \right) = -\frac{A_s h_s}{m c_p} t \quad (12) \]

For time dependent fluid temperature \( T_s(t) \), equation (11) was modified into:

\[ \frac{T_{p,n+1} - T_{p,n}}{t_{n+1} - t_n} = \frac{A_s h_s}{m c_p} \left[ T_{s,n+1} + T_{s,n} - \frac{T_{p,n+1} - T_{p,n}}{2} \right] \quad (13) \]

The time and temperature data were recorded using a data logger (21X Micrologger, Campbell Scientific Inc., Logan, UT) at an interval time of 0.2 seconds. A computer program was developed to scan the curves and then to calculate the heat transfer coefficients. The measurements of the convective heat transfer coefficients were conducted at a temperature range of 70-100°C for water and 100-130°C for steam. The convective heat transfer coefficient \( h_s \) of heating in water and in steam were found to be 1016 w/m²°C (std. dev. 99.2 w/m²°C) and 1194 w/m²°C (std. dev. 121.4 w/m²°C) respectively. The heat transfer coefficient \( h_s \) of cooling in water was found to be 928 w/m²°C (std. dev. 36.2 w/m²°C).
Results and Discussion

The measured failure stress $\sigma_m$ of cooked potato cylinders are presented in Appendix A. The average stress failure $\sigma_0$ of uncooked samples was found to be 1.063 MPa. The average of the ratio of the failure stress $\sigma_m/\sigma_0$ of the samples to that of the uncooked samples are shown in Figure 2 (a through c). From the graph it can be observed that the tissue softening did not occur at a cooking temperature of 70°C. For cooking at temperature higher than 70°C, the tissue softening occurred at an initial fast reaction rate constant which was characterized by a negative steep slope (fast cooking rate constant). Then the curve was followed by a shallow negative slope straight line (slow cooking rate constant). This result is similar to other observations found in the literature. Huang and Bourne (1983) confirmed that the rate of softening of canned vegetables during the retort process composed of two first order processes occurring at different rates. Konanayakam and Sastry (1986) observed that the shrinkage of mushroom tissue during blanching occurred in three phases of first order kinetic equations.

Typical temperature history of the samples are shown in Figures 3(a) and (b). The time for the sample average temperature to reach within 5°C of the fluid temperature are shown in Table 2. The time for the sample average temperature to cool below 5°C of the fluid temperature was very small and
Figure 2. Typical failure stress ratio $\sigma_m/\sigma_0$. (a). Cooking temperature 70°-90°C. (b). Cooking temperature 100°-115°C. (c). Cooking temperature 120°-128°C.
Figure 3. Typical average sample temperature. (a). Cooking temperature 100°C. (b). Cooking temperature 110°C.

e negligible compared to that of heating. Thus the actual cooking time is equal to the assigned cooking time minus the heating time. Since sample temperature histories for heating and cooling were nearly identical for all treatments at given process temperature, the same correction was applied to all these treatments.
Table 2. Average peak temperature and heating time on the potato samples during cooking.

<table>
<thead>
<tr>
<th>Average sample peak temperature, °C.</th>
<th>Average sample heating time, s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>29</td>
</tr>
<tr>
<td>80</td>
<td>38</td>
</tr>
<tr>
<td>90</td>
<td>44</td>
</tr>
<tr>
<td>100</td>
<td>49</td>
</tr>
<tr>
<td>103</td>
<td>52</td>
</tr>
<tr>
<td>110</td>
<td>60</td>
</tr>
<tr>
<td>115</td>
<td>65</td>
</tr>
<tr>
<td>120</td>
<td>72</td>
</tr>
<tr>
<td>124</td>
<td>75</td>
</tr>
<tr>
<td>128</td>
<td>78</td>
</tr>
</tbody>
</table>

The results of the first estimation of the activation energy $E_a$ and frequency factor $A$ of the fast rate phase are shown graphically in Figure 4. These results suggest that the cooking rate constants of the fast phases cannot be represented by a single Arrhenius equation. Suzuki et al. (1976) in their study of kinetics of rice cooking suggested the use of two Arrhenius equations, one for cooking temperatures below 110°C and one for temperatures above 110°C. It is likely that two equations are applicable to the cooking rate constants of the potato samples. Therefore the correction procedure was first applied for the lower cooking temperatures, 70 - 110°C, until the activation energy calculated in two successive iterations differed less than 2%. Using the results of the lower range part, the correction of
Figure 4. Cooking rate constant of the fast rate phase.

The activation energy and the frequency factor of the lower temperature range changed slightly after the correction procedure was done (about 0.7% from the first estimate). In contrast, the corrected activation energy of the upper temperature range was significantly different (about 27%) from the uncorrected value (58.2 kJ/mol from 74.1 kJ/mol; Tables 5 and 6). The transition point from the lower part with the high activation energy to the upper part with the lower activation energy occurs at a temperature of about 103°C.
Table 3. Estimation of the cooking rate constant $k_T$ of the lower temperature range of the fast phase.

<table>
<thead>
<tr>
<th>Cooking Temp., °C</th>
<th>First Estimate $k_T$, 1/s</th>
<th>Third Estimate $k_T$, 1/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.000079</td>
<td>0.000078</td>
</tr>
<tr>
<td>80</td>
<td>0.000812</td>
<td>0.000807</td>
</tr>
<tr>
<td>90</td>
<td>0.00323</td>
<td>0.00318</td>
</tr>
<tr>
<td>100</td>
<td>0.00891</td>
<td>0.00858</td>
</tr>
<tr>
<td>103</td>
<td>0.00911</td>
<td>0.00870</td>
</tr>
</tbody>
</table>

Table 4. Estimation of the activation energy $E_a$ and the frequency factor $A$ of the lower temperature range of the fast phase.

<table>
<thead>
<tr>
<th></th>
<th>First Estimate</th>
<th>Third Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_a$, kJ/mol</td>
<td>152.5</td>
<td>151.4</td>
</tr>
<tr>
<td>$A$, 1/s</td>
<td>$2.12 \times 10^{19}$</td>
<td>$1.42 \times 10^{19}$</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.951</td>
<td>0.948</td>
</tr>
</tbody>
</table>

The slow cooking rate phase kinetics were determined using the last three or four cooking times. The fourth or the third data point was used as the origin point to determine the cooking rate constant of the slow phase. As with the fast rate phase, the cooking rate constants of the slow phase can be divided into two Arrhenius equations. The correction does not produce a significant change in the activation energy $E_a$ and the frequency factor $A$ of the slow phase. The cooking
Table 5. Estimation of cooking rate constant $k_T$ of the higher temperature range of the fast phase.

<table>
<thead>
<tr>
<th>Cooking Temp., °C</th>
<th>First Estimate $k_T$, 1/s</th>
<th>Fourth Estimate $k_T$, 1/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>103</td>
<td>0.0091</td>
<td>0.0087</td>
</tr>
<tr>
<td>110</td>
<td>0.0160</td>
<td>0.0142</td>
</tr>
<tr>
<td>115</td>
<td>0.0188</td>
<td>0.0150</td>
</tr>
<tr>
<td>120</td>
<td>0.0262</td>
<td>0.0191</td>
</tr>
<tr>
<td>124</td>
<td>0.0346</td>
<td>0.0253</td>
</tr>
<tr>
<td>128</td>
<td>0.0406</td>
<td>0.0289</td>
</tr>
</tbody>
</table>

Table 6. Estimation of the activation energy $E_a$ and frequency factor $A$ of the higher temperature range of the fast phase.

<table>
<thead>
<tr>
<th></th>
<th>First Estimate</th>
<th>Fourth Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_a$, kJ/mol</td>
<td>74.1</td>
<td>58.2</td>
</tr>
<tr>
<td>$A$, 1/s</td>
<td>$1.97 \times 10^8$</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>$r^2$</td>
<td>0.988</td>
<td>0.969</td>
</tr>
</tbody>
</table>

rate constants of the slow rate phase are shown in Figure 5.

Unlike the fast rate phase, the change from the higher activation energy to the lower activation energy of the slow rate phase occurs at a temperature of about 115°C. The activation energy and the frequency factor of the lower temperature range are about 101.8 kJ/mol and $1.96 \times 10^{11}$ 1/s respectively (Table 7). The cooking rate constants at
temperatures higher than 115°C do not change as the temperature increases (p=0.469). It is likely that the cooking rate constant becomes constant after a certain cooking level is reached.

The transition point from fast rate phase to slow rate phase is expressed by duration $t_d$ of fast rate phase and transition stress ratio $\sigma_d/\sigma_i$ (Table 8). The duration $t_d$ of the fast rate phase was found from the intersection of two lines that are expressible using the inverse temperature. Therefore the duration $t_d$ of the fast rate phase should be proportional to the inverse of the cooking temperature. Figure 6 shows that the logarithm of the duration $t_d$ is proportional with the inverse cooking temperature.

**Figure 5.** Cooking rate constant of the slow rate phase.
Table 7. Cooking rate constants of the slow rate phase.

<table>
<thead>
<tr>
<th>Cooking Temp., °C</th>
<th>Position of origin</th>
<th>Cooking Rate constant.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>σ_r/σ_i</td>
<td>t_r, s</td>
</tr>
<tr>
<td>70</td>
<td>0.890</td>
<td>2671</td>
</tr>
<tr>
<td>80</td>
<td>0.271</td>
<td>1762</td>
</tr>
<tr>
<td>90</td>
<td>0.119</td>
<td>756</td>
</tr>
<tr>
<td>100</td>
<td>0.048</td>
<td>491</td>
</tr>
<tr>
<td>103</td>
<td>0.030</td>
<td>548</td>
</tr>
<tr>
<td>110</td>
<td>0.030</td>
<td>300</td>
</tr>
<tr>
<td>115</td>
<td>0.048</td>
<td>295</td>
</tr>
<tr>
<td>120</td>
<td>0.089</td>
<td>168</td>
</tr>
<tr>
<td>124</td>
<td>0.093</td>
<td>105</td>
</tr>
<tr>
<td>128</td>
<td>0.087</td>
<td>82</td>
</tr>
</tbody>
</table>

The calculated transition stress ratios σ_d/σ_i are shown in Table 8. The transition stress ratio σ_d/σ_i can be modified to σ_d/σ_0. Figure 7 shows the plot of the modified transition stress σ_d/σ_0 versus cooking temperature. The plot indicates that at temperatures higher than 95°C the modified transition stress ratio σ_d/σ_0 is constant at about 0.04.

The softening of potato during cooking has been related to the physical properties of potatoes. Early studies (Linehan and Hughes, 1969a, 1969b and 1969c) revealed that tubers with high starch content were significantly firmer than those of low starch content. However, Loh et al. (1982) revealed that starch had minor role in depressing fracturability, it only had a substantial influence in other textural changes occurring during cooking. They found that
Table 8. Transition points from the fast rate phase to the slow rate phase.

<table>
<thead>
<tr>
<th>Cooking Temp., °C</th>
<th>Init. Failure Stress σ_i, MPa</th>
<th>σ_d / σ_i</th>
<th>Transition point σ_d / σ_0</th>
<th>t_d, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>1.062</td>
<td>0.890</td>
<td>0.890</td>
<td>2671</td>
</tr>
<tr>
<td>80</td>
<td>1.059</td>
<td>0.278</td>
<td>0.276</td>
<td>1580</td>
</tr>
<tr>
<td>90</td>
<td>1.044</td>
<td>0.063</td>
<td>0.062</td>
<td>863</td>
</tr>
<tr>
<td>100</td>
<td>0.988</td>
<td>0.055</td>
<td>0.052</td>
<td>335</td>
</tr>
<tr>
<td>103</td>
<td>0.978</td>
<td>0.040</td>
<td>0.037</td>
<td>361</td>
</tr>
<tr>
<td>110</td>
<td>0.799</td>
<td>0.035</td>
<td>0.026</td>
<td>234</td>
</tr>
<tr>
<td>115</td>
<td>0.596</td>
<td>0.070</td>
<td>0.039</td>
<td>173</td>
</tr>
<tr>
<td>120</td>
<td>0.495</td>
<td>0.102</td>
<td>0.048</td>
<td>128</td>
</tr>
<tr>
<td>124</td>
<td>0.451</td>
<td>0.089</td>
<td>0.038</td>
<td>121</td>
</tr>
<tr>
<td>128</td>
<td>0.384</td>
<td>0.089</td>
<td>0.032</td>
<td>84</td>
</tr>
</tbody>
</table>

σ_0 = 1.063 MPa

The fracturability of plant tissue was dependent on the combined mechanical strength of the cell and the adhesive forces which cement them. Warren and Woodman (1973) and Warren et al. (1975) revealed that the reduction of tissue strength which occurred during the cooking was due to water uptake by the polysaccharides of the cell wall. This increased the thickness of the cell wall and reduced the viscosity of the cell wall matrix. Both effects reduced stress required to separate the cell wall. These observations concurred with the explanation provided by Loh et al. (1982). Further literature in the relation of tissue softening and the micro structural changes is limited. It may be that the wall thickening and the wall matrix viscosity reduction described
Figure 6. Duration $t_d$ of the fast rate phase, determined by the intersection of the fast rate phase and slow rate phase curves.

by Warren et al. (1975) occur with different reaction rate constants. The combined effects may cause the observed two rate phases.

Conclusions

The softening of the potato tissue can be modeled as a combination of two first order kinetic equations consisting of fast rate and slow rate phases. For each of the phases, the cooking constant rates can be expressed using two Arrhenius equations. The transition points of the activation energy for
the fast rate phase and the slow rate phase occur at 105°C and 115°C respectively. The duration of the fast rate phase can be expressed in terms of the exponential of the inverse of the cooking temperature. The stress ratio of the transition point decreases with increasing temperature, and is constant above 95°C.

The results of the analyses of the softening potato tissue indicate that, except for cooking at short time and at narrow temperature range, tissue softening cannot be specified using single first order kinetic and single Arrhenius equation.
Nomenclature

Symbol

A : Frequency factor, 1/s.
A_s : Surface area, m².
c_p : Heat capacity, kJ/kg.
E_a : Activation energy, kJ/mol.
h_s : Coefficient of convective heat transfer, w/m²°C.
k : Thermal conductivity, w/m°C.
k_t : Cooking rate constant, 1/s.
m : Mass, kg.
N : Number of segment of lag correction.
R : Gas constant, kJ/°K mol.
T : Absolute temperature, °K.
T_a : Average temperature of the sample, °C.
T_c : Center temperature of the sample, °C.
T_i : Initial temperature of the sample, °C.
T_s : Fluid or surrounding temperature, °C.
t : Cooking time, s.

Greek Letter

σ : Failure stress, Mpa

Subscripts related to N, t, and σ symbols

c : Cooling.
d : Duration.
h : Heating.
i : Initial.
f : Final.
r : Origin or reference.
0 : Uncooked.
References


CHAPTER III

DAMAGE TO FOOD PARTICLES DURING PUMPING

Abstract

Damage to food particles during pumping was studied using potatoes shaped into cubes, cylinders and spheres. The variables studied included particle size ratio, density ratio, particle volumetric fraction, Reynolds number, Froude number, and Euler number. The results indicated that the particle size ratio, the Reynolds number, the Froude number, and the Euler number significantly affected the broken particle fraction. The effect of the volumetric fraction of particle was found to be significant when it was less than 0.01. The density ratio and the flow direction of liquid did not have significant effect on the broken particle fraction.

Introduction

Aseptic processing is characterized by a continuous flow of product and a continuous flow of packages. The continuous flow of product depends on pumps. These pumps are a critical
considerations in the design of an aseptic processing system. As stated by FDA regulations, a metering pump should be located upstream from the holding tube and should be operated to maintain the required rate of product flow. A positive displacement type metering pump is preferred since this pump type is less sensitive to pressure drop than are centrifugal pumps. Selection of the type of positive displacement pump is dependent on the characteristics of the product such as viscosity and presence of particulate, pressure drops in the system, and cost (Nelson et al., 1987).

Subjecting food particles to pumping may reduce their quality via deformation and breakage. However, the damage to the particles during pumping has not yet been intensively studied. Most of the early studies on the pumping of fluid-particle mixtures were mainly related to the pump characteristics. Brebner (1964) investigated the behavior of a wood-chip suspension pumped through an aluminum pipe and evaluated the modes of conveyance and friction losses. Walker and Goulas (1984) observed the performance characteristics of centrifugal pumps when handling non Newtonian homogeneous slurries. Minemura et al. (1985) analyzed the characteristics of centrifugal pumps handling air-water mixtures. Ou et al. (1987) studied the pumping characteristics of chopped sorghum slurries. None of them reported the particle degradation during pumping.
Few studies on pumping were related to the particle degradation during pumping. Minemura et al. (1986) studied the behavior of solid particles in a radial flow pump impeller. The equations of motion of solid particles in a pump impeller at low specific speed were solved numerically. They obtained the particle trajectories and forces acting on the particles for different flow capacities. Nassar et al. (1987) analyzed the particle degradation in a semi continuous flow system containing solid particles. The particle was assumed to break into two fragments of any size. They concluded that all variances and covariances of the particle sizes approach zero in the limits.

It is desirable to study the damage to food particles during pumping. This knowledge may be used to prevent excessive reduction of the visual quality of the food during processing. The objectives of these studies were to determine the pertinent variables of pumping liquids containing food particles and to find their relationships with the number of broken particles.

**Materials and Methods**

**Theory.** The relationship between damage to particles during pumping and the pertinent pumping variables are determined using dimensionless analysis. The identified pertinent
Table 9. Pertinent variables for particle damage during pumping.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Exponent</th>
<th>Name</th>
<th>SI Unit</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_o$</td>
<td>$\alpha$</td>
<td>Rotor outside diameter</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>$D_i$</td>
<td>$\beta$</td>
<td>Rotor inside diameter</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>$h$</td>
<td>$\gamma$</td>
<td>Rotor thickness</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>$N_r$</td>
<td>$\delta$</td>
<td>Number of cycles in sinusoidal rotor compartment</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$V_c$</td>
<td>$\varepsilon$</td>
<td>Pump cavity</td>
<td>$m^3$</td>
<td>$L^3$</td>
</tr>
<tr>
<td>$N_s$</td>
<td>$\zeta$</td>
<td>Rotor speed</td>
<td>s$^{-1}$</td>
<td>$T^{-1}$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\eta$</td>
<td>Fluid viscosity</td>
<td>kg $m^{-1}s^{-1}$</td>
<td>ML$^{-1}$T$^{-1}$</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>$\theta$</td>
<td>Fluid density</td>
<td>kg $m^{-3}$</td>
<td>ML$^3$</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>$\iota$</td>
<td>Particle density</td>
<td>kg $m^{-3}$</td>
<td>ML$^3$</td>
</tr>
<tr>
<td>$a$</td>
<td>$\kappa$</td>
<td>Particle size</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>$Z_r$</td>
<td>$\lambda$</td>
<td>Aspect ratio</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$V_p$</td>
<td>$\nu$</td>
<td>Total volume of particles</td>
<td>$m^3$</td>
<td>$L^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\xi$</td>
<td>Particle strength</td>
<td>kg $m^{-1}s^{-2}$</td>
<td>ML$^{-1}$T$^{-2}$</td>
</tr>
<tr>
<td>$g$</td>
<td>$\omega$</td>
<td>Gravity</td>
<td>m $s^{-2}$</td>
<td>LT$^{-2}$</td>
</tr>
<tr>
<td>$N_b$</td>
<td>$\pi$</td>
<td>Broken particle fraction</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Pumping variables that can affect the damage to the particles are tabulated in Table 9 (Ellis, 1964; Cooper and Wolf, 1967; Lawler and Lu, 1971; Roco and Shook, 1983; Soo, 1984). The dimensional analysis of the damage to particles during pumping is determined using the Rayleigh method (Brodkey and Hershey, 1988).
If all the variables raised to the appropriate power are multiplied together and grouped, these groups must be equal a constant that will have no dimension.

\[ D_0^\alpha D_1^\beta h^\gamma N_r^\delta V_c^\epsilon N_s^\zeta \mu^\eta \rho_r^\rho \rho_p^\rho a^\kappa Z_r^\lambda V_p^\nu \sigma^\xi g^\sigma N_b^\tau = \text{constant} \]  \hspace{1cm} (14)

Substituting the dimensions of the pertinent variables into equation (14), a dimension relationship is obtained as:

\[ L^\alpha L^\beta L^\gamma L^3 T^\zeta M^\eta L^\mu L^\tau M^\rho L^3 M^\lambda L^\epsilon T^\nu L^\sigma T^\phi = M^\mu L^\nu T^\rho \]  \hspace{1cm} (15)

The Pi theorem states that for \( n \) unknown variables with \( m \) known dimensions, there will be \((n-m)\) number of dimensionless groups. Three dimensions appear in equation (15) with 15 unknown exponents. Thus three equations are available to solve 3 unknown exponents. Accordingly twelve dimensionless groups will be formed from equation (15). Applying the three equations to solve \( \alpha, \zeta \) and \( \theta \), a relationship of the dimensionless groups is found as:

\[ \left( \frac{D_i}{D_o} \right)^\beta \left( \frac{h}{D_o} \right)^\gamma N_r^\delta \left( \frac{V_c}{D_o} \right)^\epsilon \left( \frac{\mu}{D_o} \right)^\eta \left( \frac{\rho_r}{D_o} \right)^\rho \left( \frac{\rho_p}{D_o} \right)^\rho \left( \frac{a}{D_o} \right)^\kappa \left( \frac{Z_r}{D_o} \right)^\lambda \left( \frac{V_p}{D_o} \right)^\nu \left( \frac{\sigma}{D_o} \right)^\xi \left( \frac{g}{D_o} \right) \right)^\sigma N_b^\tau = \text{constant} \]  \hspace{1cm} (16)

Rearranging the dimensionless groups and taking the \( \pi \)th root of both sides of equation (16), the broken particle fraction \( N_b \) is related to the other dimensionless groups by an expression as:
The first four dimensionless groups on the right side of equation (17) are the pump design parameters, while the last seven groups are the operational pumping parameters. Only the effects of the operational pumping parameters were investigated at this study. These were namely particle size ratio \( \frac{a}{D_o} \), Reynolds number \( \text{Re} \), particle fluid density ratio \( D_r \), Froude number \( \text{Fr} \), Euler number \( \text{Eu} \), particle volumetric fraction \( V_f \), and aspect ratio. For non-Newtonian liquid, the Reynolds number \( \text{Re} \) was calculated using the generalized Reynolds number (Brodkey and Hershey, 1988) given as:

\[
\text{Re} = \frac{\rho_p N_s^2 D_o^2}{2^{n-3} m (\frac{3n+1}{n})^n} 
\]

For a geometrically regular particle, the particle size is defined as the diameter of a sphere having the same volume as the particle (Govier and Aziz, 1972). The equivalent diameter of a particle can be expressed as:

\[
a_{eq} = \sqrt[3]{\frac{6V}{\pi}} 
\]

Using the equivalent diameter \( a_{eq} \), the particle size ratio therefore becomes \( a_{eq}/D_o \). For cubes and cylinders (aspect
ratio = 1) of size a, the equivalent diameters are 1.24a and 1.14a respectively.

\( Z_r \) was set equal to 1. Consequently, the number of the operational dimensionless groups was reduced to six. The constant and the powers of the relationship of the dimensionless groups were estimated by varying one variable of each dimensionless groups, i.e.: fluid density \( \rho_f \), particle size \( a_{eq} \), fluid viscosity \( \mu \), pump speed \( N_p \), particle strength \( \sigma \), and total volume of the particles \( V_p \).

**Sample Materials.** The material of the samples used for the experiments were potatoes bought from a grocery. Food grade salt was used to adjust the specific gravity of the liquid. Carboxymethyl Cellulose (CMC, Aldrich Chemical Co. Inc., Milwaukee, WI) was used to alter the fluid viscosity. CMC was chosen because small changes in CMC concentration in water could cause significant changes in the fluid viscosity with negligible changes in fluid density. The rheological properties of CMC and salt solutions used for the experiments are shown in Table 10.

**Sample preparations.** Potato flesh was cut into cubes, cylinders and spheres. A slicer was used to cut potatoes into slices and cubes. A set of cork borers with diameter up to 35 mm mounted on an electrical drill were used to prepare cylindrical samples. A potato shaper consisting of two half
Table 10. Rheological properties of fluids used in pumping experiments.

<table>
<thead>
<tr>
<th>Solution % weight</th>
<th>Consistency Coef., m, mPa</th>
<th>Flow Behavior Index n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.5</td>
<td>0.978</td>
</tr>
<tr>
<td>CMC 0.05%</td>
<td>2.1</td>
<td>0.927</td>
</tr>
<tr>
<td>CMC 0.1%</td>
<td>9.7</td>
<td>0.839</td>
</tr>
<tr>
<td>CMC 0.2%</td>
<td>40.1</td>
<td>0.745</td>
</tr>
<tr>
<td>CMC 0.4%</td>
<td>144.7</td>
<td>0.727</td>
</tr>
<tr>
<td>CMC 0.6%</td>
<td>643.9</td>
<td>0.645</td>
</tr>
<tr>
<td>CMC 0.8%</td>
<td>995.3</td>
<td>0.513</td>
</tr>
<tr>
<td>Salt 10% and CMC 0.05%</td>
<td>2.7</td>
<td>0.966</td>
</tr>
<tr>
<td>Salt 25%</td>
<td>2.5</td>
<td>0.998</td>
</tr>
</tbody>
</table>

Spherical cups was used to form the potatoes into spheres.

**Sample Treatment.** Potato samples were cooked in water to modify the strength of the particle. The strengths were estimated using the kinetic model of tissue softening (Chapter II). The average temperatures of the cubes, finite cylinders and spheres were calculated using Heisler charts for infinite slab, infinite cylinder and sphere (Henderson and Perry, 1976). The calculated strength of the cooked particles are shown in Table 11.

**Pumping Experiments.** The setup of the pumping experiment is shown in Figure 8. It consisted of a continuous positive displacement progressive cavity pump (Sine Pump, The Kontro
Table 11. Calculated strength of potato particles cooked at 90° and 100° for given cooking time.

<table>
<thead>
<tr>
<th>Particle size, mm</th>
<th>Cooking Temp, °C and time, min.</th>
<th>Avg. particle strength $\sigma$, MPa</th>
<th>Cube</th>
<th>Cylinder</th>
<th>Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>90°C, 6 min</td>
<td>0.53</td>
<td>0.52</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>100°C, 6 min</td>
<td>0.12</td>
<td>0.11</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>100°C, 3 min</td>
<td>0.82</td>
<td>0.78</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>100°C, 6 min</td>
<td>0.24</td>
<td>0.21</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>100°C, 9 min</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>100°C, 12 min</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

Co., Inc., Orange, MA) and a piping system. The displacement of the fluid occurred because of the axial movement of the rotor shape relative to the pump house. The rotor resembled a sine curve in its tangential direction (Figure 9). The rotor dimensions are listed in Table 12. Its maximum speed was about 198 rpm. The pump was powered by a three phase AC, 208 V electrical motor which provided 2 HP with a maximum possible speed of 1730 rpm.

The piping system consisted of a suction line, a discharge line, a tank and a flow meter. The tank was open and contained a screen to collect samples flowing out from the discharge pipe. At the suction port was a screen to prevent broken samples from flowing into the rotameter. The volume of the entire piping system was about 80 liters.
Experimental Procedures. The shapes of the potato particles used for the experiments were cubes, spheres and cylinders (aspect ratio = 1). The experiments were conducted by placing the potato samples into a sample feeder allowing them to flow through the pump, collecting them downstream, and evaluating damage.

Studies on the effect of the number of particles in flow (particle volumetric fraction) were conducted using potato particles with size of 1.0, 1.5 and 2.0 cm of each shape. The potatoes were cooked at 90°C for 6 minutes for particle size of 1.0 cm and at 100°C for 6 minutes for particle sizes of 1.5
Figure 9. Detailed pump rotor (dimension in inches).

and 2.0 cm. The maximum number of particles was limited by the space of the sample feeder. The number of particles of each size used in this experiment are shown in Table 13.

The density experiments were conducted using three levels of liquid density; 1.0, 1.07 and 1.15 kg/l. The pump speeds used were 90, 130 and 175 rpm. The flow directions were upward and downward (clockwise and counterclockwise pump rotor rotation, Figures 8 and 9). The samples used were cylindrical potatoes of size 2.0 cm cooked at 100°C for 6 minutes. Each run was conducted using 5 samples with two replicates.

The effect of particle strength was observed using uncooked potato and cooked potato samples. Cylindrical
Table 12. Dimensions of the rotor cavity.

<table>
<thead>
<tr>
<th>Description</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside diameter, $D_0$</td>
<td>6.5 inches (165.1 mm)</td>
</tr>
<tr>
<td>Inside diameter, $D_1$</td>
<td>3.56 inches (90.4 mm)</td>
</tr>
<tr>
<td>Rotor thickness, $h$</td>
<td>1.69 inches (42.9 mm)</td>
</tr>
<tr>
<td>Height of rotor cavity</td>
<td>1.47 inches (37.3 mm)</td>
</tr>
<tr>
<td>Width of rotor cavity</td>
<td>1.32 inches (33.5 mm)</td>
</tr>
<tr>
<td>Number of cycles in rotor compartment, $N_r$</td>
<td>2</td>
</tr>
<tr>
<td>Cross sectional area of the rotor cavity, $A_r$</td>
<td>$1.23 \times 10^{-4}$ m$^2$</td>
</tr>
<tr>
<td>Volume of rotor cavity</td>
<td>0.005 m$^3$</td>
</tr>
</tbody>
</table>

Potatoes with size of 2.0 cm were cooked at 100°C for 3, 6, 9, and 12 minutes. The experiments were run using three pump speed levels at 90, 130 and 185 rpm. Each run was using 5 samples with 4 replicates.

Observations of the effect of fluid viscosity were conducted using samples of potato cylinder cooked at 100°C for 6 minutes. The viscosity experiments were performed using the CMC solution at concentrations of 0.0, 0.1, 0.2, 0.4, 0.6 and 0.8% (Table 10). The experiments were conducted using three pump speed levels at 90, 130 and 170 rpm. Each run involved 5 samples with 4 replicates. Totally the experiments were conducted using 92 treatments.

Damage to each particle was defined by the existence of a visible broken part of the sample, and was determined by the fraction of particles that were broken. The relationship
Table 13. Number of particles and replicates.

<table>
<thead>
<tr>
<th>Size, mm</th>
<th>Number of particles</th>
<th>Replicates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>2</td>
</tr>
</tbody>
</table>

between the broken particle fractions and the dimensionless groups were calculated using linear regression and factorial analysis (Minitab Statistical Software, Minitab, Inc., State College, PA).

Results and Discussion

The observed broken particle fraction $N_b$ at several levels of dimensionless groups are presented in Appendix B. The results of the multiple regression analysis of the broken particle fraction $N_b$ with all dimensionless groups indicate
that among the pertinent dimensionless groups, the particle density ratio $D_r$ and the volumetric fraction $V_f$ of the particles do not have significant effects to the broken particle fraction (Figure 10). The relationship of the broken particle fraction $N_b$ with all dimensionless groups is found as:

$$N_b = 1000 \left( \frac{a_{eq}}{D_o} \right)^{2.38} D_r^{0.71} Re^{-0.24} Fr^{-0.39} Eu^{-0.19} V_f^{0.17}$$

(20)

$$r^2 = 0.843$$

**Figure 10.** Relationship between the broken particle fraction with particle size ratio, density ratio, Reynolds number $Re$, Froude number $Fr$, Euler number $Eu$ and particle volumetric fraction.
Excluding the particle density ratio from the equation (17), the results of the multiple linear regression of the broken particle fraction $N_b$ with the other dimensionless groups show that the volumetric particle fraction $V_f$ does not affect $N_b$ significantly (Figure 11). The relationship of the broken particle fraction $N_b$ with the dimensionless groups, neglecting the density ratio, is:

$$N_b = 1023 \left( \frac{a_{eq}}{D_0} \right)^{2.36} Re^{-0.24} Fr^{0.39} Eu^{-0.19} V_f^{0.17}$$  

$$r^2 = 0.845$$

The relations of the broken particle fraction $N_b$ with and without the density ratio are not changed significantly. The constant and the exponents of the dimensionless groups are slightly changed (equations 20 and 21).

From the experiment it was observed that the effect of the particle volumetric fraction to the broken particle fraction tended to be constant at a certain level. Using only the data of the volumetric particle fraction $V_f$ which are less than 0.01, the effect of the volumetric fraction of the particles to the damage to particles is found to be significant (Figure 12). The relationship of the broken particle fraction $N_b$ with the particle size ratio, the Reynolds number, the Froude number, the Euler number and the particle volumetric fraction (less than 0.01) is found as:
Figure 11. Relationship between the broken particle fraction and particle size ratio, Reynolds number Re, Froude number Fr, Euler number Eu, and particle volumetric fraction Vf.

\[ N_b = 1023 \left( \frac{a_{eq}}{D_o} \right)^{2.36} Re^{-0.24} Fr^{0.39} Eu^{-0.19} V_f^{0.17} \]

\[ r^2 = 0.845 \]

\[ Nb = 1174 \left( \frac{a_{eq}}{D_o} \right)^{2.29} Re^{-0.23} Fr^{0.36} Eu^{-0.21} V_f^{0.23} \]

for \( V_f \leq 0.01 \)  \hspace{1cm} (22)

\[ r^2 = 0.857 \]

The results of the multiple linear regression with all data indicate that the particle volumetric fraction is not significant. However, it is significant when the analysis is conducted using only the particle volumetric fraction that are less than 0.01. Comparison of equation (22) with equation (21) shows that the exponent of the particle volumetric fraction which is less than 0.01 is greater than that when
using all data (0.23 compared to 0.17). It seems that the effect of the particle volumetric fraction tends to be constant at a particle volumetric fraction greater than 0.01. This limitation might be due to the pipe size used in pumping. The pipe size determines the maximum volumetric rate of the particles entering the pump and it might cause the effect of the particle volumetric fraction to be constant.

The total volume $V_p$ of the particles can be replaced with the number $N_p$ of particles and the volume of a single particle. Accordingly the volumetric fraction of the particles can be expressed as:

$$N_b = 1174 \left( \frac{a_{eq}}{D_o} \right)^{2.29} Re^{-0.23} Fr^{0.36} Eu^{-0.21} V_f^{0.23}$$

$R^2 = 0.857$

**Figure 12.** Relationship of the broken particle fraction with size ratio, Reynolds number Re, Froude number Fr, Euler number Eu and particle volumetric fraction (less than 0.01).
\[ V_f = \frac{\pi N_p a_{eq}^3}{6D_o^3} \]  \hspace{1cm} (23)

Regrouping the pertinent variables of each dimensionless group as shown by equations (17) and (23), the broken particle fraction \( N_b \) can be related with the pertinent variables. The power of the particle density \( \rho_p \) was found to be small (0.02). Neglecting the effect of the particle density \( \rho_p \), the broken particle fraction \( N_b \) can be expressed as:

\[ N_b = 1015 \frac{N_p^{0.23} a_{eq}^{2.97} N_s^{0.92} \mu^{0.23}}{D_o^{2.65} g^{0.36} a^{0.21}} \]  \hspace{1cm} (24)

for \( \left( \frac{\pi N_p a_{eq}^3}{6D_o^3} \right) \leq 0.01 \)

Equation (24) indicates that the broken particle fraction \( N_b \) increases as the fluid viscosity \( \mu \) and the pump speed \( N_s \) increase. It is likely that the broken particle fraction can be related to the drag force which is expressed as (Brodkey and Hershey, 1988):

\[ F_d = 6\pi r_p U_e \mu \]  \hspace{1cm} (25)

The drag force \( F_d \) is equal to the force that holds the particle when it is cut. An increase of the holding force will increase the chance of the particle to be cut. Referring to the drag force equation, an increase in the fluid viscosity \( \mu \) and the pump speed \( N_s \) will increase the magnitude of the
drag force or the holding force. Consequently it will increase the broken particle fraction.

Equation (24) shows that the broken particle fraction $N_b$ increases as the equivalent diameter $a_{eq}$ increases. However, the size of the particle that can be delivered by the pump is limited by the size of the rotor cavity. From the observation when the equivalent diameter $a_{eq}$ of the particle is greater than the rotor cavity (33.5 mm), the particle is always broken. Thus, it can be stated that the broken particle fraction $N_b$ equals 1 when $a_{eq}$ is greater than 33.5 mm.

The effects of the density ratio $D_r$ to the broken particle fraction $N_b$ is found to be insignificant. Moreover, neither the fluid density nor the particle density appears in equation (24). Results of the factorial analysis of the effects of the density ratio, the flow direction, and the pumping speed to the broken particle fraction showed that the flow direction and the density ratio are less significant than that of the pumping speed. The experiments were conducted with a narrow range of fluid density and with an unfixed particle density. These conditions might not show the effect of the density ratio. Further studies using a wide range of fluid density and a fixed particle density are suggested.

Although the pump design parameters were already identified in equation (17), some design parameters such as sharp corners and abrupt turns were not included in the dimensional analysis. Avoiding sharp corners and chute
shaping the flow passage might potentially reduce the broken particle fraction. Further studies on the dynamics of particle being compressed by a rotor and the flow dynamics of the fluid particle mixtures are needed.

**Conclusion**

Particle size ratio, Reynolds number, Froude number, and Euler number significantly affected the broken particle fraction. The effect of the volumetric fraction of the particle was significant when it was less than 0.01 and less significant when it was higher than 0.01. The effect of density ratio and flow direction to the broken particle fraction were found to be not significant. Further investigation on the effect of the pump design parameters are necessary in order to reduce the damage.
Nomenclature

Symbol

\( A_r \) = Rotor cross sectional area at flow direction, \( \text{m}^2 \).

\( a \) = Particle size, length or diameter, \( \text{m} \).

\( a_{eq} \) = Equivalent diameter, \( \text{m} \).

\( C \) = Constant.

\( D_i \) = Inside rotor diameter, \( \text{m} \).

\( D_o \) = Outside rotor diameter, \( \text{m} \).

\( D_r \) = Density ratio, \( \rho_p/\rho_f \).

\( E_u \) = \( \sigma/\rho_p N_s^2 D_o^2 \), Euler number.

\( F_d \) = Drag force, \( \text{N} \).

\( F_r \) = \( N_s^2 D_o^2 /g \), Froude number.

\( g \) = Gravitation, \( \text{m/s}^2 \).

\( h \) = Rotor thickness, \( \text{m} \).

\( m \) = Coefficient of consistency, \( \text{kg/m s}^2 \).

\( N_b \) = Broken particle fraction.

\( = \) Number of broken particle/number of sample.

\( N_p \) = Number of particles.

\( N_r \) = Number of cycles in sinusoidal rotor compartment.

\( N_s \) = Rotor speed or pumping speed, \( \text{rad/s} \).

\( n \) = Rheological property, flow behavior index.

\( Re \) = \( \rho_p N_s D_o^2 /\mu \), Reynolds number.

\( r_p \) = Particle radius, \( \text{m} \).

\( V \) = Volume, \( \text{m}^3 \)

\( V_c \) = Pump cavity, \( \text{m}^3 \).
\( V_f = \frac{V_p}{D_0^3} \), volumetric fraction of particle, \( m^3/m^3 \).

\( V_p \) = Total volume of particles, \( m^3 \).

\( Z_r \) = Aspect ratio.

**Greek Characters**

\( \alpha = \text{Exponent} \).

\( \beta = \text{Exponent} \).

\( \gamma = \text{Exponent} \).

\( \delta = \text{Exponent} \).

\( \epsilon = \text{Exponent} \).

\( \zeta = \text{Exponent} \).

\( \eta = \text{Exponent} \).

\( \theta = \text{Exponent} \).

\( \iota = \text{Exponent} \).

\( \kappa = \text{Exponent} \).

\( \lambda = \text{Exponent} \).

\( \nu = \text{Exponent} \).

\( \xi = \text{Exponent} \).

\( \omicron = \text{Exponent} \).

\( \pi = \text{Exponent} \).

\( \phi = \text{Constant} \).

\( \rho_f = \text{Fluid density, kg/m}^3 \).

\( \rho_p = \text{Particle density, kg/m}^3 \).

\( \sigma = \text{Particle strength, kg/m s}^2 \).

\( \mu = \text{Fluid viscosity, kg/m s} \).
References


CHAPTER IV
PARTICLE-WALL COLLISIONS IN SOLID-LIQUID MIXTURES
DURING PUMPING

Abstract

Motion of potato cubes, cylinders and spheres during pumping were observed. The collision velocity and the number of collisions per particle with the pump wall were determined by videotaping the particle motion. Both collision parameters were related to the dimensionless groups of the pertinent pumping variables namely particle fluid density ratio, particle size ratio, particle volumetric fraction, Reynolds number, and Froude number. The results indicated that the Reynolds and the Froude numbers, and the particle volumetric fraction significantly determined both collision parameters. The effects of the particle size ratio were found to be significant to the number of particle collisions and insignificant to the collision velocity. The density ratio and flow direction did not have significant effects on the motion of the particle.
Introduction

Aseptic processing of liquid food has shown a significant improvement for retaining the nutritional values of the food while reducing the processing cost. These successes have motivated studies to apply this method for liquid food containing particles. However, aseptic processing requires a metering pump installed upstream from the holding tube and operated to maintain the required rate of product flow. This pump may cause the food particles to be deformed or broken.

Few studies on pumping have dealt with the particle motion during pumping. Minemura et al. (1986) investigated the motion of solid particles in a radial flow pump impeller. They assumed that the number of particles entrained in water and the particle diameters were small, and therefore the flow condition was substantially unaltered. The equations of motion of solid particles in a pump impeller at low specific speed were solved numerically. Other studies dealing with the pump were mostly related to the pump performance (Brebner, 1964; Walker and Goulas, 1984).

The motion of a solid in liquid flows has been investigated intensively. Rubinow and Keller (1961) and Saffman (1965) have developed analytical estimations of the forces acting on a sphere moving in a viscous fluid. Later, using those analytical estimations, Lawler and Lu (1971) developed a model to predict radial migration of particle in
a pipe flow. However, the model was applicable only for a small particle in which the particle was assumed to not to alter the field flow of the fluid. Ellis (1964a, 1964b, 1964c) used dimensional analysis to determine the relation of the capsule velocity with pertinent variables. He observed the capsule velocity ratio and its relationship with dimensionless groups. However, he did not determine the numerical relationships of the capsule velocity with the pertinent groups. Sastry and Zuritz (1987) reviewed literatures of particle behavior in tube flow. They indicated that particles in tube flow experience a number of forces causing migration to and from tube walls.

The lateral migration of the particles might cause collision of particles with walls or particle to particle collision. An excessive collision could deform or break food particles. Particle collisions with a hard surface such as pump wall may cause more damage to the particles than with a soft surface. Hence it is desirable to investigate the collision velocity and the number of collisions of food particles with walls during pumping. The purposes of these studies was to determine the pertinent variables that have significant effects on the collision velocity and number of collisions per particle and then to investigate their relationships with the pertinent variables.
Materials and Methods

Theory. The relationship of the collision velocity and the number of collisions per particle with the pertinent variables is investigated using dimensional analysis. The identified pertinent variables for the number and velocity of collisions per particle are shown in Table 14 (Rubinow and Keller, 1961; Ellis, 1964a; Saffman, 1965; Cooper and Wolf, 1967; Lawler and Lu, 1971; Soo, 1984).

The pertinent variables are grouped into dimensionless groups using the Raleigh method (Brodkey and Hershey, 1988). If all the variables raised to the appropriate power are multiplied together and grouped, these groups must equal a dimensionless constant.

\[ D_o \delta D_r \hbar N_r \rho_c n_c \mu \theta \rho_p \alpha \beta \gamma \zeta V_p U_c \xi g^* = \text{constant} \quad (26) \]

Substituting the dimensions of the pertinent variables into equation (26), a dimensional relationship is obtained as:

\[ L^\alpha L^\beta L^\gamma L^\delta T^\epsilon L^\zeta M^\theta L^\mu T^\nu = M^0 L^0 T^0 \quad (27) \]

Three dimensions appear in equation (27) involving 14 unknown exponents. Accordingly, there will be three equations available to solve three exponents, and eleven dimensionless groups will be formed. Solving the exponents \( \alpha, \zeta \) and \( \theta \) using the available three equations, a relationship of the
Table 14. Pertinent variables for particle collision with the wall during pumping.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Exponent</th>
<th>Name</th>
<th>SI Unit</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>D₀</td>
<td>α</td>
<td>Rotor outside diameter</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>D₁</td>
<td>β</td>
<td>Rotor inside diameter</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>h</td>
<td>γ</td>
<td>Rotor thickness</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>Nᵣ</td>
<td>δ</td>
<td>Number of cycles in sinusoidal rotor compartment</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Vₖ</td>
<td>ε</td>
<td>Rotor cavity</td>
<td>m³</td>
<td>L³</td>
</tr>
<tr>
<td>Nₛ</td>
<td>ζ</td>
<td>Rotor speed</td>
<td>s⁻¹</td>
<td>T⁻¹</td>
</tr>
<tr>
<td>µ</td>
<td>η</td>
<td>Fluid viscosity</td>
<td>kg m⁻¹s⁻¹</td>
<td>ML⁻¹T⁻¹</td>
</tr>
<tr>
<td>ρₗ</td>
<td>θ</td>
<td>Fluid density</td>
<td>kg m⁻³</td>
<td>ML⁻³</td>
</tr>
<tr>
<td>ρₚ</td>
<td>υ</td>
<td>Particle density</td>
<td>kg m⁻³</td>
<td>ML⁻³</td>
</tr>
<tr>
<td>a</td>
<td>κ</td>
<td>Particle size</td>
<td>m</td>
<td>L</td>
</tr>
<tr>
<td>Zᵣ</td>
<td>λ</td>
<td>Aspect ratio</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Vₚ</td>
<td>ν</td>
<td>Total volume of particles in rotor</td>
<td>m³</td>
<td>L³</td>
</tr>
<tr>
<td>Vₖ</td>
<td>ξ</td>
<td>Collision velocity</td>
<td>m s⁻¹</td>
<td>LT⁻¹</td>
</tr>
<tr>
<td>Nₖ</td>
<td>η</td>
<td>Number of collisions per particle</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>g</td>
<td>π</td>
<td>Gravity</td>
<td>m s⁻²</td>
<td>LT⁻²</td>
</tr>
<tr>
<td>ω</td>
<td>τ</td>
<td>Angle of collision observation</td>
<td>rad</td>
<td>-</td>
</tr>
</tbody>
</table>

Dimensionless groups is found to be:

\[
\left( \frac{D₁}{D₀} \right) ^ β \left( \frac{h}{D₀} \right) ^ γ Nᵣ δ \left( \frac{Vₖ}{V_c} \right) ^ ε \left( \frac{µ}{D₀} \right) ^ η \left( \frac{ρₚ}{ρₗ} \right) ^ υ \left( \frac{a}{D₀} \right) ^ κ Zᵣ ^ λ \left( \frac{Vₚ}{D₀} \right) ^ ν \left( \frac{Uₖ}{D₀} \right) ^ ξ \left( \frac{g}{D₀} \right) ^ τ = \text{constant}
\] (28)

Rearranging the dimensionless groups and taking the ξth root of both sides of equation (28), the collision velocity
ratio $U_c / N_s D_o$ is related to the other dimensionless groups by an expression:

$$
\frac{U_c}{N_s D_o} = \phi \left\{ \left( \frac{D_i}{D_o} \right)^{a'} \left( \frac{h}{D_o} \right)^{b'} \left( \frac{V_c}{D_o} \right)^{c'} \left( \frac{a}{D_o} \right)^{d'} \left( \frac{N_s D_o^2 \rho_P}{\mu} \right)^{e'} \left( \frac{\rho_P}{\rho_f} \right)^{f'} \right\}
$$

$$
\left( \frac{N_s^2 D_o}{g} \right)^{g'} \left( \frac{V_P}{D_o^3} \right)^{h'} Z_r^{i'}
$$

(29)

The collision velocity $U_c$ is defined as the component of the particle velocity vector $U_p$ normal to the pump wall when the particle collides the wall.

In addition to the pertinent variables of the collision velocity $U_c$, the dimensional analysis of the number of collisions per particle $N_c$ includes the angle $\omega$ of the particle collision observation. Using the same procedure, the number of collisions $N_c$ per particle with the pump wall is related to the other dimensionless groups as:

$$
N_c = \psi \left\{ \left( \frac{D_i}{D_o} \right)^{a''} \left( \frac{h}{D_o} \right)^{b''} \left( \frac{V_c}{D_o} \right)^{c''} \left( \frac{a}{D_o} \right)^{d''} \left( \frac{N_s D_o^2 \rho_P}{\mu} \right)^{e''} \left( \frac{\rho_P}{\rho_f} \right)^{f''} \right\}
$$

$$
\left( \frac{N_s^2 D_o}{g} \right)^{g''} \left( \frac{V_P}{D_o^3} \right)^{h''} Z_r^{i''}
$$

(30)

The first four dimensionless groups on the right side of equation (29) and the first five groups of equation (30) are the pump design parameters, while the last six groups are the operational pumping parameters. Only the effects of the operational pumping parameters will be investigated in this study. They were namely particle size ratio $a/D_o$, Reynolds
number Re, density ratio Dr, Froude number Fr, particle volumetric fraction Vf, and aspect ratio Zr.

For non-Newtonian liquid, the Reynolds number Re is calculated using the generalized Reynolds number (Brodkey and Hershey, 1988) given as:

\[ Re = \frac{\rho_p N_s^{2-n} D_0^2}{2^{n-3} m \left( \frac{3n+1}{n} \right)^n} \]  

(31)

The particle size of a geometrically regular particle is defined as the equivalent diameter of a sphere having the same volume as the particle (Govier and Aziz, 1972). The equivalent diameter of a particle can be represented as:

\[ a_{eq} = \sqrt[3]{\frac{6V}{\pi}} \]  

(32)

Therefore the particle size ratio is given as \( a_{eq}/D_c \). The equivalent diameters \( a_{eq} \) for cubes and cylinders (aspect ratio = 1) of size a respectively are 1.24a and 1.14a.

The aspect ratio Zr was set equal to 1. Hence, the operational dimensionless groups was reduced to five. The constant and the exponents of the equations (29) and (30) were evaluated by altering the fluid density \( \rho_f \), the particle size \( a_{eq} \), the fluid viscosity \( \mu \), the pump speed \( N_s \), and the total volume of the particles \( V_p \).
Sample Materials. Food particles used for the observation were cubes, cylinders and spheres made from potatoes bought from a grocery. The fluids used for pumping were water, salt solutions, and Carboxymethyl Cellulose (CMC, Aldrich Chemical Co. Inc., Milwaukee, WI) solutions. Food grade salt and CMC were used to vary the water density and viscosity respectively. The rheological properties of the liquids are listed in Table 10.

Sample Preparations. Potatoes were shaped into cubes using a slicer. Potato cylinders were made using a set of cork borers with diameter up to 35 mm mounted on a small stand of an electrical drill. Potato spheres were formed using a tool consisting of two half spherical cups.

Pumping Experiments. The pump used for the experiment was a continuous positive displacement pump (Sine Pump Industry, The Kontro Co., Orange, MA). The pump house and the liners were transparent plexiglass replicas of the original steel parts (Figure 13). The radial projection of the rotor resembled a sine curve in its axial direction. Its dimensions are listed in Table 12. The possible maximum pumping speed was 198 rpm. The motions of the food particles inside the pump house were videotaped using video camera recorder (SVHS, Panasonic) which could produce 60 picture frames. The pump was powered by a three phase AC, 208 V electrical motor which provided 2 HP
Figure 13. Views of the pump and the rotor assembly. The flow directions were upward and downward for clockwise and counterclockwise rotor rotation respectively.

with a maximum possible speed of 1730 rpm.

Experimental Procedures. The experiments were conducted by placing the potato particles into the liquid flows and then videotaping the particle motion inside the pump house. The velocity and the number of particle collisions with the pump wall were determined from the videotaped particle motions.

The effect of the particle size was observed from the motion of 1 to 2 cm cubes, cylinders and spheres at pump speeds of 90, 130 and 165 rpm. The effect of the particle
volumetric fraction were obtained by placing 50 1 cm
particles, 20 1.5 cm particles or 10 2 cm particles in flow,
and then observing the number of particles in the rotor
compartment at any given time and the collisions of each
particle. The effects of the liquid viscosities were obtained
using CMC solution at concentration of 0.0, 0.1, 0.2, 0.4, 0.6
and 0.8% (Table 10) with pump speeds of 90, 130 and 170 rpm.
The effects of the liquid density were determined using fluid
specific gravities of 1.0, 1.07 and 1.15 at pump speeds of 90,
130 and 175 rpm. The flow directions were upward and downward
(Figure 13).

The particle velocity vector $\mathbf{U}_p$ was determined from the
change rate of the particle position. Using polar coordinate,
the particle position vector $\mathbf{r}_p$ was expressed as:

$$\mathbf{r}_p = r \mathbf{e}_r$$

(33)

The particle velocity $\mathbf{U}_p$ was defined as the derivative of
the position vector as:

$$\mathbf{U}_p = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt} \mathbf{e}_r + r \frac{d\mathbf{e}_r}{dt}$$

(34)

From mechanics (Meriam and Kraige, 1986), the derivative
of the unit vector $\mathbf{e}_r$ with respect to time was given as:

$$\frac{d\mathbf{e}_r}{dt} = \frac{d\theta}{dt} \mathbf{e}_\theta$$

(35)
Inserting the derivative of the radial unit vector into Equation (34), the particle velocity $v_p$ therefore can be expressed as:

$$v_p = \frac{dr}{dt} e_r + r \frac{d\theta}{dt} e_\theta$$  \hspace{1cm} (36)$$

Equation (36) was modified to estimate the velocity $v_p$ using the observed discrete particle positions $r_p$. The velocity $v_p$ of the particle at a certain time was represented by an equation as:

$$v_{p,n} = \frac{r_n - r_{n-1}}{\Delta t} e_r + \left( \frac{r_n + r_{n-1}}{2} \right) \frac{\theta_n - \theta_{n-1}}{\Delta t} e_\theta$$

$$= U_{r,n} e_r + U_{\theta,n} e_\theta$$  \hspace{1cm} (37)$$

Since the pump rotor and the pump house were cylinders, using the rotor axis as the origin of the polar coordinate, the normal component of the particle velocity or the collision velocity $U_c$ was the radial component $U_r$ of the particle velocity vector. Correspondingly, the tangential component of the particle velocity was the angular component $U_\theta$ of the particle velocity vector.

The collision velocities of the particle were calculated from three or four points of the particle positions before they collided with the pump wall. The collision frequencies of the particle were determined from the number of the particle collision with the wall in between the liners ($130 \leq \theta \leq 320$, Figure 13). Ten collision velocities and ten
collision frequencies of each treatment were collected from the observation. A total of 82 treatments were collected from this study. The relations of the velocity ratio and the number of collisions per particle with the dimensionless groups were calculated using linear regression and factorial analysis (Minitab Statistical Software, Minitab, Inc., College Park, PA).

**Results and Discussion**

The velocity ratio and the number of collisions per particles at several levels of dimensionless groups are shown in Appendix B. The relationship between the collision velocity ratio \( \frac{U_c}{N_s D_0} \) and the operational dimensionless groups is shown in Figure 14. The results of the regression analysis indicate that the effects of the particle size ratio \( \frac{a_{eq}}{D_0} \) and the density ratio \( D_r \) are not significant. The relation of the collision velocity ratio \( \frac{U_c}{N_s D_0} \) and the operational dimensionless groups is found as:

\[
\frac{U_c}{N_s D_0} = 0.015 \left( \frac{a_{eq}}{D_0} \right)^{0.06} D_r^{0.44} R_e^{0.08} F_r^{-0.28} V_f^{-0.11}
\]

\[r^2 = 0.782\]  

The relation of the collision velocity ratio \( \frac{U_c}{N_s D_0} \) with the Reynolds number \( R_e \), the Froude number \( F_r \) and the particle volumetric fraction \( V_f \) is shown in Figure 15. The numerical
relationship between the collision velocity ratio $U_c/N_sD_o$ with the pertinent dimensionless groups can be expressed as:

$$\frac{U_c}{N_sD_o} = 0.016 \text{Re}^{0.07} \text{Fr}^{-0.27} V_f^{-0.11}$$

(39)

$$r^2 = 0.778$$

Comparison of equation (38) with equation (39) indicates that excluding the density ratio $D_r$ and the particle size ratio $a_{eq}/D_o$ does not change the constant and the exponents of the Reynolds number $\text{Re}$, the Froude number $\text{Fr}$ and the particle volumetric fraction $V_f$ significantly.

The particle volumetric fraction $V_f$ can be expressed using the number of particles $N_p$ in the rotor, the particle
Figure 15. Relation of the collision velocity ratio with Reynolds number \( \text{Re} \), Froude number \( \text{Fr} \) and particle volumetric ratio \( V_f \).

Replacing the dimensionless groups in equation (39) with the pertinent variables shown in Table 13 and equation (40), the collision velocity \( U_c \) of particle can be expressed as:

\[
U_c = 0.017 \frac{N_p a_{eq}^3}{D_0^3} \rho_p^{0.08} D_0^{1.2} g^{0.27} N_b^{0.53} \mu^{0.08} N_p^{0.11} a_{eq}^{0.33}
\]

Equation (41) indicates that as the particle size decreases, the collision velocity \( U_c \) will increase. Hypothetically the collision velocity \( U_c \) of a particle of the
same size of a rotor space will be zero. If a clearance is available, the centrifugal force will accelerate the particle until it reaches an equilibrium with the resistance forces. The results also indicate that the collision velocity $U_c$ decreases as the fluid viscosity increases. The decrease of the collision velocity $U_c$ for more viscous fluid might be due to the increase of the drag force required to move the particle toward the wall. Inversely the collision velocity increases as the pump speed increases. It might be due to an increase of the centrifugal force available to accelerate the particle toward the wall. An increase of the number $N_p$ in the rotor compartment reduces the particle collision velocity $U_c$.

The combination of the effects of the density ratio, flow direction and flow rate were analyzed using factorial analysis. The results indicate that the effects of the density ratio and flow direction on the collision velocity $U_c$ are much less significant than that of the flow rate. It seems that the effects of the density ratio and the flow direction are negligible.

The relation of the number of collisions per particle with all dimensionless groups is shown in Figure 16. The results of the linear regression of the number of collisions per particle $N_c$ with all operational parameters indicate that the density ratio $D_r$ does not affect it significantly. The relation can be expressed as:
The relation of the number of collisions per particle $N_c$ with the dimensionless groups excluding the density ratio is shown in Figure 17. The relation of the particle number of collisions per particle $N_c$ with the pertinent dimensionless groups can be expressed as:

$$N_c = 0.35 \left( \frac{a_{eq}}{D_o} \right)^{1.52} D_r^{-0.33} Re^{0.14} Fr^{-0.09} V_f^{-0.46}$$

$$r^2 = 0.831$$

(42)
Figure 17. Relation of the number of particle collisions with particle size ratio, Reynolds number Re, Froude number Fr and particle volumetric fraction $V_f$.

The results of the multiple linear regression indicate that excluding the density ratio does not change the constant and the exponents of the pertinent dimensionless groups significantly.

Replacing the dimensionless groups with the pertinent variables shown in Table 13 and equation (40), the number of collisions per particle can be expressed as:

$$N_c = 0.35 \left( \frac{a_{eq}}{D_o} \right)^{1.54} Re^{0.14} Fr^{-0.09} V_f^{-0.46}$$

$$r^2 = 0.831$$

Equation (44) shows that as the particle size increases the number of collisions per particle increases.
Theoretically a particle of the same size as the rotor space is always in contact with the pump wall thus the number of collisions per particle of the particle is not quantifiable. The results also indicate that the number of collisions per particle $N_c$ increases as the fluid viscosity decreases. The decrease of the number of collisions per particle $N_c$ for higher viscosity agrees with the decrease of the collision velocity $U_c$. In contrast, as the pump speed increases the number of collisions per particle decreases while the collision velocity increases. An increase of the number $N_p$ of particles in the rotor compartment reduces the number of collisions per particle with the pump wall.

The combination of the effects of the density ratio, flow direction and pump speed were analyzed using factorial analysis. The results indicate that the effects of the density ratio and the flow direction on the number of collisions per particle are much less significant than that of the pump speed. It seems that effect of density ratio and the flow direction are negligible.

The effects of the fluid-particle density ratio on the collision velocity $U_c$ and the number of collisions per particle $N_c$ are not found to be significant. Studies on the flow of a fluid containing particles in a tube demonstrated that the density ratio affected the particle velocity and the particle trajectory (Ellis, 1964b, Lawler and Lu, 1971). The experiment to study the effects of the density was conducted
using potato which varied slightly and using a small range of liquid density. These variations might cause the effects of the density to be unobservable. Further studies using particles with a fixed density and a wide range of fluid density are suggested.

The video camera used for the observation has a recording speed of 60 frames per second. It was not possible to determine the collision acceleration of the particle during contact with the wall. Further studies using a higher speed camera is suggested. The collision velocities were determined from the videotaped motion and then were determined manually by measuring the particle position. Since digital image processing technology has developed significantly in recent years, it's use in conjunction with a high speed camera could be helpful in determining the particle positions. A computer program then can be developed to determine the collision velocity and the number of collisions per particle appropriately.

Conclusion

The Reynolds and Froude numbers, and the particle volumetric fraction have significant effects on both collision parameters. The collision velocity ratio and the number of collisions per particle increase as the Reynolds number
increases, and as the Froude number or the volumetric fraction decreases. The particle size ratio only has significant effect on the number of collisions per particle. The number of collisions per particle increases as the particle size increases. The density ratio and the flow direction do not affect the collision velocity or the number of collisions per particle.
Nomenclature

Symbol

\( A_r \) = Rotor cross sectional area at flow direction, \( \text{m}^2 \).

\( a \) = Particle size, length for cubes or diameter for cylinder or sphere, \( \text{m} \).

\( a_{eq} \) = Equivalent diameter, \( \text{m} \).

\( C \) = Constant.

\( D_0 \) = Rotor outside diameter, \( \text{m} \).

\( D_i \) = Rotor inside diameter, \( \text{m} \).

\( D_r \) = Density ratio, \( \rho_p/\rho_f \).

\( \mathbf{e}_r \) = Unit vector in radial direction.

\( \mathbf{e}_\theta \) = Unit vector in angular direction.

\( Fr \) = \( N_s^2D_0/g \), Froude number.

\( g \) = Gravitation, \( \text{m/s}^2 \).

\( h \) = Rotor thickness, \( \text{m} \).

\( m \) = Coefficient of consistency, \( \text{kg/m s}^2 \).

\( N_c \) = Number of collisions per particle.

\( N_p \) = Number of particles.

\( N_r \) = Number of cycles in sinusoidal rotor compartment.

\( N_s \) = Rotor speed or pumping speed, \( \text{rad/s} \).

\( n \) = Rheological property, flow behavior index.

\( Re \) = \( \rho_pN_sD_0^2/\mu \), Reynolds number.

\( r_p \) = Particle radius, \( \text{m} \).

\( \mathbf{r}_p \) = Vector of particle position.

\( U_c \) = Collision velocity of particle, \( \text{m/s} \).
$U_r = \text{Radial component of particle velocity, m/s.}$

$U_\theta = \text{Tangential component of particle velocity, m/s.}$

$U_p = \text{Vector of particle velocity, m/s.}$

$V = \text{Volume, m}^3$

$V_c = \text{Pump cavity, m}^3.$

$V_f = \frac{V_p}{D_o^3}$, volumetric fraction of particle, m$^3$/m$^3$.

$V_p = \text{Total volume of particles, m}^3.$

$Z_r = \text{Aspect ratio.}$

**Greek Characters.**

$\alpha = \text{Exponent.}$

$\beta = \text{Exponent.}$

$\gamma = \text{Exponent.}$

$\delta = \text{Exponent.}$

$\epsilon = \text{Exponent.}$

$\zeta = \text{Exponent.}$

$\eta = \text{Exponent.}$

$\theta = \text{Exponent.}$

$\iota = \text{Exponent.}$

$\kappa = \text{Exponent.}$

$\lambda = \text{Exponent.}$

$\nu = \text{Exponent.}$

$\xi = \text{Exponent.}$

$\omicron = \text{Exponent.}$

$\pi = \text{Exponent.}$

$\tau = \text{Exponent.}$
\[ \phi = \text{Constant.} \]
\[ \psi = \text{Constant.} \]
\[ \rho_f = \text{Fluid density, \( \text{kg/m}^3 \).} \]
\[ \rho_p = \text{Particle density, \( \text{kg/m}^3 \).} \]
\[ \sigma = \text{Particle strength, \( \text{kg/m s}^2 \).} \]
\[ \mu = \text{Fluid viscosity, \( \text{kg/m s} \).} \]
\[ \omega = \text{Angle of collision observation, rad.} \]
References


CHAPTER V
STRESS ANALYSIS OF FOOD PARTICLE
UNDER COMPRESSION LOADS

Abstract

Stress distribution on potato cubes under compressive force was investigated using the finite element method assuming the potato was Hookean material. Two load cases were used; load on a cube with the whole bottom supported and load on a partially supported cube. The failure forces were calculated using the linear relationships between the maximum stress, the particle size, and the particle strength.

When the whole cube bottom was supported and the cube was under a uniform pressure, varying the loading area slightly changed the maximum stress and its location. It can be considered that the maximum stress was directly proportional with the magnitude of the total compressive forces. Decrease in the support area increased the maximum stress and therefore decreased the total force required to break the cubes. It was evident that once the particle was caught by rotating blades, breakage was not avoided since the available power exceeded the particle strength.
Introduction

One of the constraints in the development of continuous processing for liquid food products containing particles is that of damage. The deformed particle is most likely due to the compressive or shearing loads upon being caught by moving parts such as blades and rotor. Furthermore, heat treatment which is applied to the particle during processing reduces the particle strength and consequently increases the number of broken particles.

Failure of food particles occurs when the critical state of stress is exceeded. Failure for uniaxial, biaxial, or triaxial states of stress or strain are basically based on maximum permissible values of normal stresses, shear stresses, or distortion energies (Shigley and Mitchell, 1983). Miles and Rehkugler (1973) reported that shear stress was the most significant failure parameter for apples. Fridley et al. (1968) found that bruises on peaches and pears occurred in the area of maximum shear stress. Mohsenin (1986) stated that the shear stress failure theory was probably suitable for many biological materials. Misra (1978) used the maximum shear stress to be the criteria for failure of soybean during drying. Shigley and Mitchell (1983) described that the maximum shear stress theory is used to predict yielding, and hence it applies only to ductile materials.
The entire stress distribution on a particle can be calculated using numerical methods such as finite element analysis. Using this method, the conditions under which the critical stress is exceeded can be determined. Finite element methods have been applied to model and investigate the physical and mechanical problems of agricultural materials. Misra (1978) applied the finite element method to estimate the shrinkage stress of soybean during drying. He assumed that the soybean was a linear elastic material (Hookean material). Yano et al. (1987) used the finite element method to predict the instantaneous longitudinal elasticity of a porous gel. They applied the stress relaxation theory for the finite element formulation. Haghighi and Segerlind (1988a, 1988b, 1988c) applied the finite element approach to simultaneously predict temperature, moisture, and stress profiles on an isotropic sphere during drying. In addition to the equilibrium and strain displacement relationships, they employed the viscoelastic constitutive equation for their finite element formulation which was then to be used to simulate the stress in soybeans.

It is desirable to analyze the stress distribution within a food particle under compressive loads and to determine the maximum stresses which exceed the strength of the food particle. The objectives of this study were to determine the maximum stress and failure forces for a cooked food particle under compressive loads.
Theory. Strain stress relationship of most food materials are generally not linear (Mohsenin, 1986). However, Finney (1963) and Finney and Hall (1967) indicated that the strain-stress curves of uncooked potato during loading were approximately linear. Such relationship for cooked potato is not reported yet. In addition, food material is considered to be a viscoelastic material which is characterized by its time dependency. Time dependence of viscoelastic materials are recognized as stress relaxation under constant strain or creep deformation under constant force (Mohsenin, 1986). For convenience in analysis, linear elastic behavior was assumed in this study.

The linear elasticity problem consists of variables such as displacement $u$, strain $\varepsilon$, stress $\sigma$, and force $F$. Their relationships determine the governing equations for the finite element formulation that can be expressed as follows (Becker et al., 1981).

**Equilibrium equation:**

$$\sigma_{ij,j} + F_i = 0$$

**Hooke's law:**

$$\sigma_{ij} = E_{kl} ij \varepsilon_{kl}$$
Strain-displacement relationship:

\[ \epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (47) \]

The boundary conditions:

\[
\begin{align*}
\sigma_{ij}n_j &= t_i \quad \text{on } S_1 \\
u_i &= \delta_i \quad \text{on } S_2 \\
\sigma_{ij}n_j &= S_i \quad \text{on } S_{1i} \\
u_i n_j &= d_{ij} \quad \text{on } S_{2i} 
\end{align*}
\quad (48) \]

Using the variational calculus, the functional for the governing equations is defined as:

\[
V = \int_V \left[ u_i (-\sigma_{ji,j} - 2F_i) + \epsilon_{ij} (E_{kl} \epsilon_{kl} - \sigma_{ij}) + \sigma_{ij} (u_{i,j} - \epsilon_{ij}) \right] dV - \int_{S_1} u_i (\sigma_{ij}n_j - 2t_i) dS - \int_{S_2} \sigma_{ij}n_j (u_i - 2\delta_i) dS - \int_{S_{1i}} u_i (\sigma_{ij}n_j - 2S_i) dS - \int_{S_{2i}} \sigma_{ij} (u_i n_j - 2d_i) dS \quad (49) \]

The functional possesses the property that any function which makes it a minimum also satisfies the governing equations and the boundary conditions. The finite element method is based on the minimization of this functional over a set of elements. The solution of solid mechanics problem can be approached by a numerical procedure based on a principle which states that the displacement at the equilibrium position occurs such that the potential energy of a stable system is a minimum value (Segerlind, 1984).
This study was not intended to develop a finite element program, therefore the two-dimensional boundary value problem on linear elastic system was solved using a software package of Finite Element System (Algor Interactive Systems, Inc., Pittsburgh, PA).

**Models of Stress Analysis.** Stress analysis was conducted for potato cubes with two load cases; compressive pressure with the whole cube bottom supported (case #1, Figure 18) and cutting pressure in which the cube bottom is supported partially (case #2, Figure 19). Compressive loads on the cubical food particle due to pump rotor or due to collision were simplified by assuming that the loads were distributed uniformly in the direction of the particle thickness. Consequently, the problem could be reduced to a two-dimensional problem. The reduction of a three-dimensional problem to a two-dimensional problem could be done in two ways; plane stress or plane strain. Since the load was uniform in the thickness direction, plane stress was used to solve this problem.

For the case of compressive load (case #1), the nodal points of the bottom surface were constrained only in the z direction (Figure 18). The nodal points at the compression area were not constrained. For the case of cutting load (case #2), the nodal point at the edge of the compression area was constrained in the x and y directions. At the support edge
Figure 18. Model of cube under compression load. The cube is meshed into 10x10 elements. The loads are applied at the top surface and the whole bottom surface is supported by a flat surface.

the nodal point was constrained in the x, y, and z directions. The other nodal points were constrained only in z direction (Figure 19).

The cubes were meshed into 10x10 square elements. The loading area was varied by increasing the number of the elements under the load. In the case of the compressive load, the loading area was varied from 1 to 10 elements on the top surface. In the case of the cutting load, the maximum number of elements under load was varied from 1 to 9 elements.

A computer program was used to analyze the stress distribution on cubes (Algor Interactive Systems, Inc.,
Figure 19. Model of cube under cutting load. The cube is meshed into 10x10 elements. The loads are applied at the top surface. The bottom surface is partially supported by a flat surface.

Pittsburgh, PA). The element formulation chosen for this problem was the Constant Linear Strain Triangle (CLST). The integration method used was the second order. The method to calculate the stress at nodal points was the von Mises stress yield criterion. Using the principle stresses, it can be expressed as shown on the following equation (Desai and Abel, 1972; Shigley and Mitchell, 1983).

\[ \sigma = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \] (50)
Table 15. Calculated average failure strength $\sigma_m$ of potato cubes with size of 10, 15 and 20 mm cooked at 90° and 100° for cooking time of 3, 6, 9 and 12 minutes.

<table>
<thead>
<tr>
<th>Cooking time, minutes.</th>
<th>Particle strength $\sigma_m$, MPa</th>
<th>Cube size and cooking temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10mm(90°C) 15mm(100°C) 20mm(100°C)</td>
</tr>
<tr>
<td>Uncooked</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td>3</td>
<td>0.80</td>
<td>0.57</td>
</tr>
<tr>
<td>6</td>
<td>0.52</td>
<td>0.12</td>
</tr>
<tr>
<td>9</td>
<td>0.34</td>
<td>0.02</td>
</tr>
<tr>
<td>12</td>
<td>0.22</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Shigley and Mitchell (1983) explained that it is the best theory to use for ductile materials. The failure was determined by comparing the von Mises maximum stresses with the strength of the uniaxial test of the material.

Sample Material. The material used for the model was potato tissue. The strength of cooked potato was estimated using the kinetic model of tissue softening (Chapter II). The Young's modulus of elasticity of cooked potatoes were obtained from the observation. These properties of the sample cooked at certain cooking temperature and cooking time are presented in Tables 15 and 16. The Poisson's ratio for uncooked potato has been reported as 0.4 (Mohsenin, 1986). Finney and Hall (1967) using hydrostatic pressure found the Poisson's ratio for whole potatoes was 0.492. The authors observed that the Poisson's ratio of potato was not significantly affected by cooking.
Table 16. Measured failure strength $\sigma_n$ and Young's modulus of elasticity $E$ to failure of potato cooked at 90°C for cooking time of 0 - 20 minutes.

<table>
<thead>
<tr>
<th>Cooking time, minutes</th>
<th>Failure strength, Mpa.</th>
<th>Elasticity to failure, Mpa.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncooked</td>
<td>1.10</td>
<td>2.84</td>
</tr>
<tr>
<td>5</td>
<td>0.44</td>
<td>1.48</td>
</tr>
<tr>
<td>10</td>
<td>0.19</td>
<td>0.99</td>
</tr>
<tr>
<td>15</td>
<td>0.12</td>
<td>0.75</td>
</tr>
<tr>
<td>20</td>
<td>0.12</td>
<td>0.71</td>
</tr>
</tbody>
</table>

The average Poisson's ratio was found to be 0.341 (stand. dev. = 0.102) and was used for this analysis.

Results and Discussion

The stress distribution on a cube with size of 10 mm supported by a flat surface and subjected to a pressure of 0.1 MPa (case #1) with compression area $A_c$ at 1 element and 5 elements are shown in Figures 20 and 21 respectively. The summary of the maximum stress with the compression area $A_c$ varying from 1 to 10 elements is presented in Table 17. It shows that for given a constant uniform pressure the maximum stress changes generally by a small value as the compression area increases. It seems that under a constant pressure the maximum stress does not change considerably as the compression
Figure 20. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 10 mm² or at 2 nodal points and the bottom is supported by a flat surface.

Figure 21. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 50 mm² or at 6 nodal points and the bottom is supported by a flat surface.

area increases. Referring to Figures 20 and 21, and Figures 26 through 32 on Appendix D, it is shown that the maximum stress moves from the left top corner to the left bottom
Table 17. Summary of stress distribution on an uncooked potato cube under a compression pressure of 0.1 MPa and the whole bottom is supported by a flat surface.

<table>
<thead>
<tr>
<th>No. of elements</th>
<th>Area, $A_c$, mm$^2$</th>
<th>Total Force, $F_c$, N</th>
<th>Max Stress, $\sigma_m$, MPa</th>
<th>Stress Conc. factor, $C_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.13</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>2</td>
<td>0.12</td>
<td>1.15</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>3</td>
<td>0.11</td>
<td>1.14</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>4</td>
<td>0.12</td>
<td>1.17</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>5</td>
<td>0.13</td>
<td>1.25</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>6</td>
<td>0.13</td>
<td>1.32</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>7</td>
<td>0.13</td>
<td>1.33</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>8</td>
<td>0.13</td>
<td>1.29</td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>9</td>
<td>0.12</td>
<td>1.17</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>10</td>
<td>0.10</td>
<td>1.00</td>
</tr>
</tbody>
</table>

corner as the loading area increases.

The stress distribution on a cube subjected to cutting pressure of 0.1 MPa (case #1) with the compression area $A_c$ at 1 element and 5 elements are presented in Figures 22 and 23 respectively. The summary of the maximum stress with compression area $A_c$ varying from 1 element to 9 elements is listed in Table 18. It indicates that the maximum stress on cubes under uniform pressure increases with the increase of the compression area (Figures 33 through 39 on Appendix D). In other words, the maximum stress increases as the support area decreases. As the support area decrease, the force is concentrated in a smaller area.

Since the stress strain relationship was assumed to be a linear elastic system, the maximum stress increases
Figure 22. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 10 mm$^2$ or at 2 nodal points and the bottom is supported on 10 nodal points.

Figure 23. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 Mpa at a compression area of 50 mm$^2$ or at 6 nodal points and the bottom is supported on 6 nodal points.

proportionally to the increase of the compressive pressure (Figures 21 and 24, and Figure 40 on Appendix D). Similarly for a linear system, the stress at any point is proportional
Table 18. Summary of stress distribution on uncooked potato cube under cutting force of 0.1 MPa and the bottom is partially supported by a flat surface.

<table>
<thead>
<tr>
<th>No. of elements</th>
<th>Area, ( A_c ) mm²</th>
<th>Total Force, ( F_c ) N</th>
<th>Max Stress, ( \sigma_m ) MPa</th>
<th>Stress Conc. factor, ( C_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>1</td>
<td>0.13</td>
<td>1.27</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>2</td>
<td>0.14</td>
<td>1.35</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>3</td>
<td>0.19</td>
<td>1.89</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>4</td>
<td>0.23</td>
<td>2.33</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>5</td>
<td>0.33</td>
<td>3.28</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>6</td>
<td>0.43</td>
<td>4.29</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>7</td>
<td>0.54</td>
<td>5.42</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>8</td>
<td>0.67</td>
<td>6.72</td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>9</td>
<td>0.91</td>
<td>9.06</td>
</tr>
</tbody>
</table>

with the inverse of the square of the cube size (Figures 21 and 25, and Figure 41 on Appendix D).

Referring to Figures 20 through 25, and Appendix D, for identical shape particles with different size (homologous particles), the stress at any nodal point is related to the total compressive force and particle size as shown by equation (51).

\[
\sigma_i = c_i \frac{F_c}{a^2}
\]

The values of stress concentration factor \( C_m \) for both cases are included in Tables 17 and 18. Using the value of \( C_m \), for a known particle failure strength \( \sigma_f \), the total failure force \( F_f \) that could break the potato cube was calculated using
Figure 24. Stress distribution on a cube of uncooked potato under a compression pressure of 0.2 MPa at a compression area of 50 mm$^2$ or at 6 nodal points and the bottom is supported by a flat surface.

\[
F_f = \frac{\sigma_f a^2}{C_m}
\]  

(52)

For potato cubes with size of 1 cm cooked at temperature of 90°C, and cubes with size of 1.5 and 2 cm cooked at temperature of 100°C for cooking time 3, 6, 9, and 12 minutes (Table 15), the estimated failure forces using equation (52) for both cases are shown in Tables 17 and 18.

Table 17 shows that for a cube with its whole bottom supported, failure always occurs when the total force exceeds the failure force regardless of its location. In contrast, when the bottom is supported partially, the failure force decreases as the support area decreases. It indicates that collision of a particle with a corner or an edge increases the
Figure 25. Stress distribution on a cube of uncooked potato under a compression pressure 0.025 MPa at a compression area of 200 mm² or at 6 nodal points and the bottom is supported by a flat surface.

For the case of cutting load, the torque required by the rotor to cut the cube was estimated using the relationship as shown in equation (53).

\[ T_r = \int_0^r ap_c dr \approx RxF_c \]

Since the cutting pressure was uniform, the torque was calculated from the magnitude of the total cutting force and the radius of rotor center line. Using the rotor radius \( R \) (63.9 mm) and the total failure force \( F_r \) with loading area at 5 elements, the estimated torques and powers \((w)\) required to break the cubes for cutting load case are shown in Table 19.
Table 19. Estimated total compressive forces to failure on cooked potato cubes with size of 10, 15 and 20 mm.

<table>
<thead>
<tr>
<th>Cube size (mm)</th>
<th>Cooking</th>
<th>Total failure force $F_f$, N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Temp. °C</td>
<td>Time min.</td>
</tr>
<tr>
<td>10 Uncooked</td>
<td>90</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>41.60</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>27.20</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>17.60</td>
</tr>
<tr>
<td>15 Uncooked</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>102.60</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>21.60</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3.60</td>
</tr>
<tr>
<td>20 Uncooked</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>262.40</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>76.80</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>16.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.20</td>
</tr>
</tbody>
</table>

The power provided by the motor used for the experiment was about 1500 w (2 HP). If about 40% of the motor power was used for pumping, the hydraulic power was about 600 w. Therefore the breaking power varied from 0% to 10% of the hydraulic power. It seems that as the food particle was caught by the pump rotor, the breaking of the particle was not avoidable. The rotor provided sufficient torque to cut the particle. Therefore the only possible solution to avoid the damaging the particle was to minimize the chances of the particle being caught by the rotor.
Table 20. Estimated total cutting forces to failure on cooked potato cubes with size of 10, 15 and 20 mm.

<table>
<thead>
<tr>
<th>Cube size</th>
<th>Cooking</th>
<th>Total failure force ( F_f ), N</th>
<th>Number of compressed elements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Temp.</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>°C</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>83.46</td>
<td>56.08</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>62.99</td>
<td>42.33</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>40.94</td>
<td>27.51</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>26.77</td>
<td>17.99</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>17.32</td>
<td>11.64</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>187.80</td>
<td>126.19</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>100.98</td>
<td>67.86</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>21.26</td>
<td>14.29</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3.54</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>1.77</td>
<td>1.19</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
<td>333.86</td>
<td>224.34</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>258.27</td>
<td>173.54</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>75.59</td>
<td>50.79</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>15.75</td>
<td>10.58</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3.15</td>
<td>2.12</td>
</tr>
</tbody>
</table>

The increase of torque due to the cutting of a particle can be sufficiently detected by a power sensor. Two possible approaches are detailed below. A torque sensor can be installed on a pump rotor. As soon the sensor detects an increase in torque magnitude, the motor could be stopped momentarily to give time for the particle to escape from the trap. To reduce the inertia of the pump shaft and rotor it is suggested to operate the pump at a low speed. It agrees with the relation of the pump speed to the number of broken particles which suggests that operating the pump at a low
Table 21. Estimated torque and powers to break uncooked and cooked potato cubes with size of 10, 15 and 20 mm at pumping speed of 90, 130, and 180 rpm under cutting forces with case of loading area at 5 elements.

<table>
<thead>
<tr>
<th>Cube size (mm)</th>
<th>Cooking Temp. (°C)</th>
<th>Time (min)</th>
<th>Failure torque (Nm)</th>
<th>Power at failure (w) at pumping speed (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Uncooked</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.759</td>
<td>7.16</td>
<td>10.34</td>
<td>14.31</td>
</tr>
<tr>
<td>6</td>
<td>0.573</td>
<td>5.40</td>
<td>7.80</td>
<td>10.80</td>
</tr>
<tr>
<td>9</td>
<td>0.372</td>
<td>3.51</td>
<td>5.07</td>
<td>7.02</td>
</tr>
<tr>
<td>12</td>
<td>0.244</td>
<td>2.30</td>
<td>3.32</td>
<td>4.59</td>
</tr>
<tr>
<td>15 Uncooked</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1.708</td>
<td>16.10</td>
<td>23.26</td>
<td>32.20</td>
</tr>
<tr>
<td>6</td>
<td>0.919</td>
<td>8.66</td>
<td>12.51</td>
<td>17.32</td>
</tr>
<tr>
<td>9</td>
<td>0.193</td>
<td>1.82</td>
<td>2.63</td>
<td>3.65</td>
</tr>
<tr>
<td>12</td>
<td>0.032</td>
<td>0.30</td>
<td>0.44</td>
<td>0.61</td>
</tr>
<tr>
<td>20 Uncooked</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>3.037</td>
<td>28.63</td>
<td>41.35</td>
<td>57.25</td>
</tr>
<tr>
<td>6</td>
<td>2.350</td>
<td>22.14</td>
<td>31.99</td>
<td>44.29</td>
</tr>
<tr>
<td>9</td>
<td>0.688</td>
<td>6.48</td>
<td>9.36</td>
<td>12.96</td>
</tr>
<tr>
<td>12</td>
<td>0.143</td>
<td>1.35</td>
<td>1.95</td>
<td>2.70</td>
</tr>
</tbody>
</table>

Pumping speed reduces damage (Chapter III).

Another possible way to reduce excessive pressure to the food particle is to use a rotor with a soft surface. Using rubber or soft polymer materials to cover the rotor surface might reduce the pressure to the food particles.

The finite element analysis used to analyze the stress distribution ignored the effect of large deformation which likely occurred in most food particles. The analysis also assumed that the potato sample was a Hookean material.
However, the elasticity of cooked material is not linear. It is also dependent on the cooking temperature and cooking time. Further investigation of the stress distribution on cooked food materials is suggested.

The stress analysis approach would be helpful in determining whether or not a given collision of known force results in failure, since force data are unavailable, this must await further study.

**Conclusion**

The maximum stress on a cube with the whole bottom supported is determined by the total magnitude of the pressure. The location of the force is less significant than its magnitude. For a partially supported cube, the maximum stress increases as the support area decreases. Once the particle is caught by the pump rotor, the breakage of the particle is not avoided. To avoid the damaging the food particle, the chances of the particle of being caught must be minimized.
Nomenclature

Symbol

$A_c$ = Compression area, m$^2$.

$a$ = Particle size, length for cubes, m.

$c_i$ = Constant at nodal point $i$.

$D$ = Rotor size or diameter, m.

$E$ = Young's modulus elasticity, MPa.

$E_{klij}$ = Matrix of the material properties.

$F_c$ = Compressive force, N.

$F_f$ = Force to failure, N.

$N$ = Rotor speed or pumping speed, rpm.

$P_f$ = Power to break the food particle, w.

$P_h$ = Hydraulic power, w.

$p$ = Pressure, MPa.

$R$ = Rotor radius, m.

$T$ = Torque, Nm.

$u$ = Displacement, m.

Greek Characters

$\delta$ = Compressive strain, mm/mm.

$\varepsilon$ = Strain, mm/mm.

$\sigma$ = Particle strength, MPa.

$\sigma_m$ = Maximum stress, MPa.

$\mu$ = Poisson's ratio.
References


CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. The softening of potato tissue can be modeled by two first order kinetics equations consisting of fast and slow rate phases. The cooking rate constants can be expressed using two Arrhenius equations.

2. The duration of the fast rate phase is expressible in terms of the inverse of the cooking temperature.

3. The damage to food particles is significantly influenced by particle size ratio, Reynolds number, Froude number, and Euler number.

4. The damage to the particles is not significantly affected by the particle volumetric fraction when it is higher than 0.01.

5. The collision velocity of the particles to the walls is determined by the Reynolds number, Froude number and particle volumetric fraction.

6. The number of particle collision to walls is influenced by the particle size ratio, Reynolds number, Froude number and particle volumetric fraction.
7. Particle-fluid density ratio and flow direction are not significantly related to the damage to particles, particle collision velocity, and number of particle collision.

8. Breakage of cooked food particles is not avoidable when it is caught by rotating blades, since the available power can exceed the particle strength.

Recommendations

1. Studies on the relationships of micro structural and physical changes with the tissue softening during cooking.

2. Acquires data of kinetics of softening of various food for further studies on the damage to food particles during pumping.

3. Studies on the flow dynamics of fluid-particle mixtures during pumping in order to reduce the chance of the particles being caught by rotor.

4. Further studies on damage to particles with irregular shapes.

5. Studies on effects of pump design parameters to damage to particles and particle collisions.
6. Further studies on collisions of particle to walls and particle to particle to determine the particle collision forces

7. Investigation of failure of food particles under collision forces.

8. Development of finite element analyses for non linear elastic materials and with large deformation to determine food particle failure.
LIST OF REFERENCES


Appendix A
Data Relative to Chapter II
Table 22. Physical and thermal properties of aluminum and potato.

<table>
<thead>
<tr>
<th></th>
<th>Aluminum</th>
<th>Potato</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, g/cm³</td>
<td>2.707</td>
<td>1.05-1.07</td>
</tr>
<tr>
<td>Moisture content, %</td>
<td>-</td>
<td>81.5</td>
</tr>
<tr>
<td>Specific heat, kJ/kg°C</td>
<td>0.896</td>
<td>3.517</td>
</tr>
<tr>
<td>Thermal conductivity, w/m°C</td>
<td>237</td>
<td>0.554</td>
</tr>
</tbody>
</table>


Table 23. Stress failure of potato cylinder cooked at temperature of 70°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.147</td>
<td>0.813</td>
<td>0.934</td>
<td>1.024</td>
<td>0.975</td>
<td>1.053</td>
</tr>
<tr>
<td></td>
<td>1.214</td>
<td>0.870</td>
<td>1.025</td>
<td>0.922</td>
<td>0.973</td>
<td>0.805</td>
</tr>
<tr>
<td></td>
<td>0.983</td>
<td>1.015</td>
<td>0.913</td>
<td>0.917</td>
<td>0.798</td>
<td>1.127</td>
</tr>
<tr>
<td></td>
<td>1.066</td>
<td>0.896</td>
<td>1.035</td>
<td>1.044</td>
<td>0.888</td>
<td>1.125</td>
</tr>
<tr>
<td></td>
<td>1.350</td>
<td>1.029</td>
<td>0.900</td>
<td>0.819</td>
<td>0.967</td>
<td>1.019</td>
</tr>
<tr>
<td>Mean</td>
<td>1.152</td>
<td>0.925</td>
<td>0.961</td>
<td>0.945</td>
<td>0.920</td>
<td>1.026</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.126</td>
<td>0.084</td>
<td>0.057</td>
<td>0.082</td>
<td>0.069</td>
<td>0.118</td>
</tr>
</tbody>
</table>
Table 24. Stress failure of potato cylinder cooked at temperature of 80°C.

<table>
<thead>
<tr>
<th>Cooking time, minutes</th>
<th>Stress failure, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.240   0.957  0.332  0.216  0.209  0.330</td>
</tr>
<tr>
<td>10</td>
<td>1.178   0.737  0.417  0.362  0.279  0.256</td>
</tr>
<tr>
<td>20</td>
<td>1.377   0.604  0.349  0.394  0.274  0.214</td>
</tr>
<tr>
<td>30</td>
<td>1.104   0.792  0.413  0.200  0.297  0.125</td>
</tr>
<tr>
<td>40</td>
<td>1.014   0.733  0.460  0.264  0.287  0.266</td>
</tr>
<tr>
<td>50</td>
<td>0.123   0.114  0.047  0.078  0.031  0.067</td>
</tr>
</tbody>
</table>

Mean: 1.183  0.764  0.394  0.287  0.269  0.238
Std. Dev.: 0.123  0.114  0.047  0.078  0.031  0.067

Table 25. Stress failure of potato cylinder cooked at temperature of 90°C.

<table>
<thead>
<tr>
<th>Cooking time, minutes</th>
<th>Stress failure, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.244   0.339  0.238  0.132  0.085  0.048</td>
</tr>
<tr>
<td>5</td>
<td>1.079   0.487  0.111  0.101  0.150  0.075</td>
</tr>
<tr>
<td>10</td>
<td>1.098   0.456  0.216  0.103  0.149  0.069</td>
</tr>
<tr>
<td>15</td>
<td>1.276   0.579  0.188  0.136  0.063  0.092</td>
</tr>
<tr>
<td>20</td>
<td>0.824   0.352  0.186  0.148  0.160  0.106</td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

Mean: 1.104  0.443  0.188  0.124  0.121  0.078
Std. Dev.: 0.160  0.089  0.043  0.019  0.040  0.020
Table 26. Stress failure of potato cylinder cooked at temperature of 100°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.164</td>
<td>0.311</td>
<td>0.073</td>
<td>0.048</td>
<td>0.039</td>
<td>0.038</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.100</td>
<td>0.035</td>
<td>0.025</td>
<td>0.019</td>
<td>0.010</td>
<td>0.009</td>
</tr>
</tbody>
</table>

1.250 0.370 0.042 0.082 0.029 0.026
1.075 0.295 0.051 0.049 0.055 0.051
1.310 0.329 0.075 0.046 0.049 0.043
1.130 0.275 0.083 0.029 0.034 0.028
1.053 0.286 0.112 0.034 0.030 0.041

Table 27. Stress failure of potato cylinder cooked at temperature of 105°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>14</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.984</td>
<td>0.255</td>
<td>0.078</td>
<td>0.030</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.036</td>
<td>0.025</td>
<td>0.011</td>
<td>0.005</td>
<td>0.004</td>
<td>0.001</td>
</tr>
</tbody>
</table>

1.049 0.245 0.067 0.030 0.022 0.014
0.978 0.222 0.094 0.021 0.012 0.017
0.973 0.239 0.087 0.030 0.018 0.014
0.985 0.278 0.068 0.032 0.017 0.016
0.937 0.290 0.072 0.035 0.012 0.017

Mean                                        | 0.984 | 0.255 | 0.078 | 0.030 | 0.016 | 0.016 |
| Std.Dev.                                    | 0.036 | 0.025 | 0.011 | 0.005 | 0.004 | 0.001 |
### Table 28. Stress failure of potato cylinder cooked at temperature of 110°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.958</td>
<td>0.319</td>
<td>0.067</td>
<td>0.024</td>
<td>0.023</td>
<td>0.013</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.044</td>
<td>0.030</td>
<td>0.015</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
</tr>
</tbody>
</table>

### Table 29. Stress failure of potato cylinder cooked at temperature of 115°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.003</td>
<td>0.248</td>
<td>0.044</td>
<td>0.029</td>
<td>0.016</td>
<td>0.013</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.136</td>
<td>0.025</td>
<td>0.009</td>
<td>0.006</td>
<td>0.001</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Table 30. Stress failure of potato cylinder cooked at temperature of 120°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.125</td>
<td>0.196</td>
<td>0.045</td>
<td>0.023</td>
<td>0.012</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>0.973</td>
<td>0.219</td>
<td>0.034</td>
<td>0.022</td>
<td>0.012</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>0.893</td>
<td>0.204</td>
<td>0.045</td>
<td>0.023</td>
<td>0.019</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>0.936</td>
<td>0.201</td>
<td>0.063</td>
<td>0.027</td>
<td>0.011</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>1.052</td>
<td>0.196</td>
<td>0.035</td>
<td>0.020</td>
<td>0.018</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.996</td>
<td>0.203</td>
<td>0.044</td>
<td>0.023</td>
<td>0.014</td>
<td>0.018</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.083</td>
<td>0.008</td>
<td>0.010</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 31. Stress failure of potato cylinder cooked at temperature of 125°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.073</td>
<td>0.080</td>
<td>0.049</td>
<td>0.018</td>
<td>0.031</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>1.127</td>
<td>0.067</td>
<td>0.041</td>
<td>0.029</td>
<td>0.036</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>1.017</td>
<td>0.083</td>
<td>0.043</td>
<td>0.033</td>
<td>0.043</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>1.159</td>
<td>0.081</td>
<td>0.036</td>
<td>0.034</td>
<td>0.032</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>1.017</td>
<td>0.089</td>
<td>0.043</td>
<td>0.043</td>
<td>0.029</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.079</td>
<td>0.080</td>
<td>0.042</td>
<td>0.031</td>
<td>0.034</td>
<td>0.023</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.057</td>
<td>0.007</td>
<td>0.004</td>
<td>0.008</td>
<td>0.005</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Table 32. Stress failure of potato cylinder cooked at temperature of 130°C.

<table>
<thead>
<tr>
<th>Stress failure, MPa at Cooking time, minutes</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.969</td>
<td>0.067</td>
<td>0.041</td>
<td>0.026</td>
<td>0.022</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>1.081</td>
<td>0.040</td>
<td>0.028</td>
<td>0.024</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.905</td>
<td>0.042</td>
<td>0.027</td>
<td>0.031</td>
<td>0.030</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>0.990</td>
<td>0.046</td>
<td>0.030</td>
<td>0.034</td>
<td>0.029</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>1.166</td>
<td>0.045</td>
<td>0.028</td>
<td>0.028</td>
<td>0.028</td>
<td>0.024</td>
</tr>
<tr>
<td>Mean</td>
<td>1.022</td>
<td>0.048</td>
<td>0.031</td>
<td>0.029</td>
<td>0.027</td>
<td>0.024</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.091</td>
<td>0.010</td>
<td>0.005</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Appendix B
Data Relative to Chapter III
Table 33. Data of damage to particles during pumping.

<table>
<thead>
<tr>
<th>( N_b )</th>
<th>( a_{eq}/D_o )</th>
<th>( D_r )</th>
<th>Re</th>
<th>Fr</th>
<th>Eu</th>
<th>( V_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.15</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.02</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.15</td>
<td>0.0009</td>
</tr>
<tr>
<td>0.02</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.15</td>
<td>0.0018</td>
</tr>
<tr>
<td>0.03</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.15</td>
<td>0.0034</td>
</tr>
<tr>
<td>0.00</td>
<td>0.11</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>20.76</td>
<td>0.0006</td>
</tr>
<tr>
<td>0.05</td>
<td>0.11</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>20.76</td>
<td>0.0012</td>
</tr>
<tr>
<td>0.04</td>
<td>0.11</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>20.76</td>
<td>0.0029</td>
</tr>
<tr>
<td>0.04</td>
<td>0.11</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>20.76</td>
<td>0.0059</td>
</tr>
<tr>
<td>0.06</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>20.76</td>
<td>0.0118</td>
</tr>
<tr>
<td>0.00</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>39.64</td>
<td>0.0014</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>39.64</td>
<td>0.0028</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>39.64</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>413000</td>
<td>1.60</td>
<td>386.07</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>200.66</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.15</td>
<td>0.14</td>
<td>1.07</td>
<td>830000</td>
<td>6.25</td>
<td>98.63</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>413000</td>
<td>1.60</td>
<td>283.29</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>147.24</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.15</td>
<td>0.14</td>
<td>1.07</td>
<td>830000</td>
<td>6.25</td>
<td>72.37</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>413000</td>
<td>1.60</td>
<td>76.27</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>39.64</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.25</td>
<td>0.14</td>
<td>1.07</td>
<td>830000</td>
<td>6.25</td>
<td>19.48</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>413000</td>
<td>1.60</td>
<td>14.53</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>7.55</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.40</td>
<td>0.14</td>
<td>1.07</td>
<td>830000</td>
<td>6.25</td>
<td>3.71</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>413000</td>
<td>1.60</td>
<td>3.63</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.30</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>1.89</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.70</td>
<td>0.14</td>
<td>1.07</td>
<td>830000</td>
<td>6.25</td>
<td>0.93</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>743000</td>
<td>1.56</td>
<td>77.94</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>108000</td>
<td>3.17</td>
<td>38.44</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.30</td>
<td>0.14</td>
<td>1.07</td>
<td>149000</td>
<td>5.78</td>
<td>21.06</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.15</td>
<td>0.14</td>
<td>1.07</td>
<td>358000</td>
<td>1.27</td>
<td>95.76</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>568000</td>
<td>2.66</td>
<td>45.82</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.30</td>
<td>0.14</td>
<td>1.07</td>
<td>873000</td>
<td>5.27</td>
<td>23.10</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>96000</td>
<td>1.07</td>
<td>144.21</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.30</td>
<td>0.14</td>
<td>1.07</td>
<td>151000</td>
<td>2.19</td>
<td>55.52</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.30</td>
<td>0.14</td>
<td>1.07</td>
<td>241000</td>
<td>4.55</td>
<td>26.76</td>
<td>0.0070</td>
</tr>
</tbody>
</table>
Table 33. (Continued) Data of damage to particles during pumping.

<table>
<thead>
<tr>
<th>N_b</th>
<th>a_{eq}/D_o</th>
<th>D_r</th>
<th>Re</th>
<th>Fr</th>
<th>Eu</th>
<th>V_f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.14</td>
<td>1.07</td>
<td>3700</td>
<td>1.46</td>
<td>83.28</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.25</td>
<td>0.14</td>
<td>1.07</td>
<td>4600</td>
<td>2.00</td>
<td>60.99</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.35</td>
<td>0.14</td>
<td>1.07</td>
<td>7800</td>
<td>4.32</td>
<td>28.18</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.30</td>
<td>0.14</td>
<td>1.07</td>
<td>3000</td>
<td>0.85</td>
<td>142.66</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.35</td>
<td>0.14</td>
<td>1.07</td>
<td>5400</td>
<td>1.92</td>
<td>63.40</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.55</td>
<td>0.14</td>
<td>1.07</td>
<td>9700</td>
<td>4.21</td>
<td>28.94</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>353000</td>
<td>1.50</td>
<td>81.44</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>529000</td>
<td>3.07</td>
<td>39.64</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.30</td>
<td>0.14</td>
<td>1.07</td>
<td>724000</td>
<td>5.72</td>
<td>21.30</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.00</td>
<td>209000</td>
<td>1.21</td>
<td>100.54</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.00</td>
<td>314000</td>
<td>2.66</td>
<td>45.81</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.00</td>
<td>447000</td>
<td>5.27</td>
<td>23.10</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.00</td>
<td>0.14</td>
<td>0.93</td>
<td>211000</td>
<td>1.37</td>
<td>89.19</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>0.93</td>
<td>304000</td>
<td>2.84</td>
<td>42.90</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>0.93</td>
<td>420000</td>
<td>5.40</td>
<td>22.65</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.07</td>
<td>353000</td>
<td>1.50</td>
<td>81.44</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>529000</td>
<td>3.07</td>
<td>39.64</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.07</td>
<td>724000</td>
<td>5.72</td>
<td>21.30</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.00</td>
<td>209000</td>
<td>1.21</td>
<td>100.54</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>1.00</td>
<td>314000</td>
<td>2.66</td>
<td>45.81</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>1.00</td>
<td>447000</td>
<td>5.27</td>
<td>23.10</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.00</td>
<td>0.14</td>
<td>0.93</td>
<td>211000</td>
<td>1.37</td>
<td>89.19</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>0.93</td>
<td>304000</td>
<td>2.84</td>
<td>42.90</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>0.93</td>
<td>420000</td>
<td>5.40</td>
<td>22.65</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.00</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>100.05</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.00</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>100.05</td>
<td>0.0004</td>
</tr>
<tr>
<td>0.04</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>100.05</td>
<td>0.0011</td>
</tr>
<tr>
<td>0.06</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>100.05</td>
<td>0.0022</td>
</tr>
<tr>
<td>0.04</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>100.05</td>
<td>0.0056</td>
</tr>
<tr>
<td>0.04</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>100.05</td>
<td>0.0111</td>
</tr>
<tr>
<td>0.05</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>22.65</td>
<td>0.0008</td>
</tr>
<tr>
<td>0.05</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>22.65</td>
<td>0.0014</td>
</tr>
<tr>
<td>0.08</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>22.65</td>
<td>0.0038</td>
</tr>
<tr>
<td>0.08</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>22.65</td>
<td>0.0074</td>
</tr>
<tr>
<td>0.06</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>22.65</td>
<td>0.0149</td>
</tr>
<tr>
<td>0.08</td>
<td>0.15</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>45.30</td>
<td>0.0018</td>
</tr>
<tr>
<td>0.08</td>
<td>0.15</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>45.30</td>
<td>0.0036</td>
</tr>
<tr>
<td>0.15</td>
<td>0.15</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>45.30</td>
<td>0.0089</td>
</tr>
<tr>
<td>0.25</td>
<td>0.15</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>45.30</td>
<td>0.0178</td>
</tr>
</tbody>
</table>
Table 33. (Continued) Data of damage to particles during pumping.

<table>
<thead>
<tr>
<th>$N_b$</th>
<th>$a_{eq}/D_o$</th>
<th>$D_r$</th>
<th>Re</th>
<th>Fr</th>
<th>Eu</th>
<th>$V_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.16</td>
<td>0.0001</td>
</tr>
<tr>
<td>0.00</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.16</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.00</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.16</td>
<td>0.0006</td>
</tr>
<tr>
<td>0.00</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.16</td>
<td>0.0012</td>
</tr>
<tr>
<td>0.01</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.16</td>
<td>0.0029</td>
</tr>
<tr>
<td>0.01</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>98.16</td>
<td>0.0058</td>
</tr>
<tr>
<td>0.00</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>16.99</td>
<td>0.0004</td>
</tr>
<tr>
<td>0.00</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>16.99</td>
<td>0.0008</td>
</tr>
<tr>
<td>0.00</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>16.99</td>
<td>0.0020</td>
</tr>
<tr>
<td>0.00</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>16.99</td>
<td>0.0039</td>
</tr>
<tr>
<td>0.03</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>16.99</td>
<td>0.0079</td>
</tr>
<tr>
<td>0.00</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>32.09</td>
<td>0.0009</td>
</tr>
<tr>
<td>0.00</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>32.09</td>
<td>0.0019</td>
</tr>
<tr>
<td>0.10</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>32.09</td>
<td>0.0047</td>
</tr>
<tr>
<td>0.00</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
<td>3.07</td>
<td>32.09</td>
<td>0.0093</td>
</tr>
</tbody>
</table>
Appendix C
Data Relative to Chapter IV
Table 34. Data of particle collision with the wall during pumping.

<table>
<thead>
<tr>
<th>$U_c/N_sD_0$ Mean</th>
<th>$N_c$ Mean</th>
<th>$a_{eq}/D_o$ Mean</th>
<th>$D_r$</th>
<th>Re</th>
<th>Fr</th>
<th>$V_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.081 0.020 2.2</td>
<td>0.7 0.06</td>
<td>1.07 413000 1.597</td>
<td>0.0001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.076 0.015 2.0</td>
<td>0.9 0.09</td>
<td>1.07 413000 1.597</td>
<td>0.0004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.076 0.010 1.8</td>
<td>0.8 0.12</td>
<td>1.07 830000 6.251</td>
<td>0.0001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.063 0.012 1.9</td>
<td>0.7 0.06</td>
<td>1.07 830000 6.251</td>
<td>0.0009</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.061 0.003 1.9</td>
<td>0.5 0.09</td>
<td>1.07 830000 6.251</td>
<td>0.0004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.056 0.011 1.9</td>
<td>0.7 0.12</td>
<td>1.07 830000 6.251</td>
<td>0.0009</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.072 0.010 1.9</td>
<td>0.6 0.12</td>
<td>1.07 413000 1.597</td>
<td>0.0007</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.075 0.016 1.8</td>
<td>0.6 0.11</td>
<td>1.07 413000 1.597</td>
<td>0.0006</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.057 0.009 1.9</td>
<td>0.5 0.11</td>
<td>1.07 830000 6.251</td>
<td>0.0007</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.077 0.015 2.2</td>
<td>0.8 0.14</td>
<td>1.07 413000 1.597</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.062 0.018 1.8</td>
<td>0.6 0.14</td>
<td>1.07 577000 3.072</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.057 0.009 1.9</td>
<td>0.7 0.14</td>
<td>1.07 830000 6.251</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.075 0.015 1.7</td>
<td>0.5 0.14</td>
<td>1.07 74300 1.563</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.059 0.007 1.7</td>
<td>0.5 0.14</td>
<td>1.07 108000 3.168</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.047 0.011 1.5</td>
<td>0.6 0.14</td>
<td>1.07 149000 5.784</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.073 0.028 1.6</td>
<td>0.5 0.14</td>
<td>1.07 35800 1.272</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.059 0.019 1.5</td>
<td>0.7 0.14</td>
<td>1.07 56800 2.659</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.044 0.015 1.4</td>
<td>0.6 0.14</td>
<td>1.07 87300 5.273</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.061 0.012 1.3</td>
<td>0.7 0.14</td>
<td>1.07 9600 1.066</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.052 0.012 1.3</td>
<td>0.9 0.14</td>
<td>1.07 15100 2.193</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.043 0.012 1.3</td>
<td>0.9 0.14</td>
<td>1.07 24100 4.551</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.049 0.016 1.1</td>
<td>0.6 0.14</td>
<td>1.07 3700 1.460</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.050 0.009 0.9</td>
<td>0.7 0.14</td>
<td>1.07 4600 1.997</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.044 0.010 1.1</td>
<td>0.6 0.14</td>
<td>1.07 7790 4.321</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.053 0.013 0.9</td>
<td>0.8 0.14</td>
<td>1.07 2950 0.854</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.049 0.009 0.9</td>
<td>0.8 0.14</td>
<td>1.07 5400 1.921</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.044 0.006 1.0</td>
<td>0.7 0.14</td>
<td>1.07 9670 4.209</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.077 0.022 2.0</td>
<td>1.0 0.14</td>
<td>1.07 353000 1.495</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.063 0.022 1.8</td>
<td>0.8 0.14</td>
<td>1.07 529000 3.072</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.051 0.006 2.0</td>
<td>0.7 0.14</td>
<td>1.07 724000 5.719</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.077 0.018 1.8</td>
<td>0.8 0.14</td>
<td>1.00 209000 1.211</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.059 0.012 2.0</td>
<td>0.6 0.14</td>
<td>1.00 314000 2.659</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.050 0.018 1.9</td>
<td>0.8 0.14</td>
<td>1.00 447000 5.273</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.065 0.054 1.8</td>
<td>0.7 0.14</td>
<td>0.93 211000 1.365</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.058 0.004 1.8</td>
<td>0.9 0.14</td>
<td>0.93 304000 2.839</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.050 0.009 1.8</td>
<td>0.8 0.14</td>
<td>0.93 420000 5.398</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 34. (Continued) Data of particle collision with the wall during pumping.

<table>
<thead>
<tr>
<th>$U_c/N_sD_o$ Mean</th>
<th>N$_c$ Mean</th>
<th>$a_{eq}/D_o$ Mean</th>
<th>Dr</th>
<th>Re</th>
<th>Fr</th>
<th>$V_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SD</td>
<td>SD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.076</td>
<td>0.011</td>
<td>1.9</td>
<td>0.7</td>
<td>0.14</td>
<td>1.00</td>
<td>209000</td>
</tr>
<tr>
<td>0.060</td>
<td>0.007</td>
<td>1.8</td>
<td>0.8</td>
<td>0.14</td>
<td>1.00</td>
<td>314000</td>
</tr>
<tr>
<td>0.050</td>
<td>0.005</td>
<td>1.7</td>
<td>0.9</td>
<td>0.14</td>
<td>0.93</td>
<td>447000</td>
</tr>
<tr>
<td>0.077</td>
<td>0.012</td>
<td>1.8</td>
<td>0.9</td>
<td>0.14</td>
<td>0.93</td>
<td>211000</td>
</tr>
<tr>
<td>0.057</td>
<td>0.004</td>
<td>1.9</td>
<td>0.5</td>
<td>0.14</td>
<td>0.93</td>
<td>304000</td>
</tr>
<tr>
<td>0.050</td>
<td>0.008</td>
<td>1.8</td>
<td>0.6</td>
<td>0.14</td>
<td>0.93</td>
<td>420000</td>
</tr>
<tr>
<td>0.075</td>
<td>0.014</td>
<td>1.5</td>
<td>0.6</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.071</td>
<td>0.014</td>
<td>1.3</td>
<td>0.7</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.069</td>
<td>0.020</td>
<td>0.9</td>
<td>0.7</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.058</td>
<td>0.023</td>
<td>1.0</td>
<td>0.7</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.057</td>
<td>0.015</td>
<td>0.8</td>
<td>0.7</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.058</td>
<td>0.016</td>
<td>0.7</td>
<td>0.6</td>
<td>0.08</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.079</td>
<td>0.020</td>
<td>1.8</td>
<td>0.8</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.070</td>
<td>0.021</td>
<td>1.2</td>
<td>0.4</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.050</td>
<td>0.010</td>
<td>1.0</td>
<td>0.6</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.057</td>
<td>0.022</td>
<td>2.0</td>
<td>0.9</td>
<td>0.15</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.058</td>
<td>0.017</td>
<td>1.5</td>
<td>0.6</td>
<td>0.15</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.075</td>
<td>0.018</td>
<td>1.4</td>
<td>0.8</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.069</td>
<td>0.012</td>
<td>1.1</td>
<td>0.7</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.065</td>
<td>0.018</td>
<td>1.3</td>
<td>0.6</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.059</td>
<td>0.016</td>
<td>1.3</td>
<td>0.9</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.057</td>
<td>0.012</td>
<td>0.8</td>
<td>0.4</td>
<td>0.07</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.079</td>
<td>0.016</td>
<td>1.8</td>
<td>0.8</td>
<td>0.11</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.068</td>
<td>0.019</td>
<td>1.2</td>
<td>0.6</td>
<td>0.11</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.057</td>
<td>0.012</td>
<td>1.4</td>
<td>0.9</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.054</td>
<td>0.012</td>
<td>1.3</td>
<td>0.5</td>
<td>0.14</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.089</td>
<td>0.015</td>
<td>1.9</td>
<td>0.9</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.084</td>
<td>0.025</td>
<td>1.4</td>
<td>0.8</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.083</td>
<td>0.017</td>
<td>0.9</td>
<td>0.8</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.078</td>
<td>0.018</td>
<td>1.0</td>
<td>0.9</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.078</td>
<td>0.017</td>
<td>0.9</td>
<td>0.8</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.066</td>
<td>0.023</td>
<td>0.8</td>
<td>0.7</td>
<td>0.06</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.082</td>
<td>0.018</td>
<td>1.9</td>
<td>0.7</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.063</td>
<td>0.011</td>
<td>1.0</td>
<td>0.6</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.074</td>
<td>0.032</td>
<td>0.9</td>
<td>0.6</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.065</td>
<td>0.008</td>
<td>0.8</td>
<td>0.7</td>
<td>0.09</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.080</td>
<td>0.019</td>
<td>2.1</td>
<td>0.8</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.063</td>
<td>0.012</td>
<td>1.3</td>
<td>0.5</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
</tr>
<tr>
<td>0.059</td>
<td>0.014</td>
<td>1.3</td>
<td>0.6</td>
<td>0.12</td>
<td>1.07</td>
<td>577000</td>
</tr>
</tbody>
</table>
Appendix D
Data Relative to Chapter V

127
Figure 26. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 20 mm² or at 3 nodal points and the bottom is supported by a flat surface.

Figure 27. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 30 mm² or at 4 nodal points and the bottom is supported by a flat surface.
Figure 28. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 40 mm² or at 5 nodal points and the bottom is supported by a flat surface.

Figure 29. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 60 mm² or at 7 nodal points and the bottom is supported by a flat surface.
Figure 30. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 70 mm² or at 8 nodal points and the bottom is supported by a flat surface.

Figure 31. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 80 mm² or at 9 nodal points and the bottom is supported by a flat surface.
Figure 32. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 90 mm$^2$ or at 10 nodal points and the bottom is supported by a flat surface.

Figure 33. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 20 mm$^2$ or at 3 nodal points and the bottom is supported on 9 nodal points.
Figure 34. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 30 mm\(^2\) or at 4 nodal points and the bottom is supported on 8 nodal points.

Figure 35. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 40 mm\(^2\) or at 5 nodal points and the bottom is supported on 7 nodal points.
Figure 36. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 60 mm² or at 7 nodal points and the bottom is supported on 5 nodal points.

Figure 37. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 70 mm² or at 8 nodal points and the bottom is supported on 4 nodal points.
Figure 38. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 80 mm² or at 9 nodal points and the bottom is supported on 3 nodal points.

Figure 39. Stress distribution on a cube of uncooked potato under a compression pressure of 0.1 MPa at a compression area of 90 mm² or at 10 nodal points and the bottom is supported on 2 nodal points.
Figure 40. Stress distribution on a cube of uncooked potato under a compression pressure of 0.5 MPa at a compression area of 50 mm² or at 6 nodal points and the bottom is supported by a flat surface.

Figure 41. Stress distribution on a cube of uncooked potato under a compression pressure of 0.044 MPa at a compression area of 112.5 mm²/mm or at 6 nodal points and the bottom is supported by a flat surface.