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Theoretical and experimental analysis of residual stress formation after implant resistance welding of polycarbonate

Park, Joon Boo, Ph.D.

The Ohio State University, 1992
THEORETICAL AND EXPERIMENTAL ANALYSIS OF RESIDUAL STRESS FORMATION AFTER IMPLANT RESISTANCE WELDING OF POLYCARBONATE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

By

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To My Wife Heykyung and

Daughter Eunhye
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Finally and ultimately, all the glories to my Lord in the heaven !. Thank you Jesus, Amen !
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ABSTRACT

Formation of residual stress during thermoplastic welding causes detrimental effects to the joint quality under both dynamic and static loading conditions. Residual stress can reduce the solvent resistance of polymers as well as the tensile strength and fatigue life of the joint. Therefore, it is vital to predict and measure the level of residual stresses. Here, the formation of thermal and residual stresses during implant resistance welding of polycarbonate was studied.

Thermocouples and an infrared temperature sensor were used to measure the temperature history and temperature distribution in the parts during welding. Heat flow analysis during implant resistance welding was done using Finite Element Method (FEM) and Finite Difference Method (FDM) which are connected with FEM and a simplified modeling analysis so-called "multi-bar analogy" respectively for stress analysis. FEM and FDM predictions of heat flow analysis were in good agreement with experimental measurements.

The formation of thermal and residual stress was predicted using 2-D finite element analysis and multi-bar analogy in conjunction with non-isothermal linear viscoelasticity for a thermorheologically simple material. The residual stresses in the parts were measured using both photoelasticity and moire interferometry. Sectioning method utilizing
moire interferometry was used to measure residual stress. FEM prediction of residual stress was in good agreement with photoelasticity measurement and moire interferometry measurement. Residual stress formation in the weld was predicted by multi-bar analogy modeling analysis and multi-bar analogy prediction was in good agreement with FEM prediction.

Heat treatment to reduce residual stress after welding was performed. Residual stress distribution after heat treatment was predicted using FEM. The FEM prediction was in good agreement with photoelasticity measurement and moire interferometry measurement.

This methodology for prediction, measurement and reduction of residual stress can be incorporated into the design, analysis, and welding procedures for plastic and composite joints. This will result in stronger and more reliable welds.
CHAPTER I
INTRODUCTION

1.1 Joining of Plastics

The usage of engineering thermoplastics in both structural and nonstructural applications is rapidly increasing. Most plastics offer much more flexibility to manufacturing industries than metals. The flexibly deformable state at elevated temperature enable them to be shaped into very complex geometries to meet the service requirements of parts. The forming procedures of plastics are inexpensive, quick and easy, and suitable for mass production with good quality. Moreover plastics offer many advantages, such as high stiffness and strength per unit weight, high toughness per unit weight, corrosion resistance, excellent insulation, variation of colors and more. Therefore, plastics are selected for many good engineering application. They are used more and more in many industries such as aerospace, automobile, electric, electronic, building, pipe industries, and major and minor domestic appliances. As the demand for these plastics increases, so do the requirements for joining. There are various methods of joining plastic parts together. Development of effective joining methods to meet the service requirements is very important. Joining methods for plastics can be classified into three categories: mechanical joining, adhesive bonding, and welding.
Most structural mechanical joints require an expensive multi-step procedure (drilling, inserting, and tightening). This process introduces holes into parts, which weakens them. Holes and inserted bolts act as stress concentrators which can induce failure. Also inserting metallic fasteners induce weight gain in the structures which can reduce the joint efficiency. However mechanical joining offers the following advantages: it does not require surface preparation, it is not adversely affected by thermal cycles and environmental conditions, it is easy to inspect and repair, and it produces good joint strength [1]. Adhesive bonding is a joining process where adhesive material is placed between two parts (adherends) to produce a joint. Both thermoplastics and thermosets can be adhesively joined. Adhesive bonds have more uniform load distribution along the joint. However, in cases of shear loading of adhesive joints, high stress concentration can occur at the edges. Surface preparation is very crucial for a good adhesive joint. In some cases, finding an appropriate adhesive for specific materials is very difficult. Inspection is required after adhesive joining. Another major drawback of adhesive bonding is that adhesives are very susceptible to adverse environmental conditions, like high temperature and moisture.

Thermoplastics and thermoplastic composites can also be joined by welding. Welding is advantageous because it results in more uniform loading than mechanical fastening and it does not introduce any new materials as in the case of adhesive bonding [2]. Welding of thermoplastics is a single step process which can be divided into five subprocesses: surface preparation, heating, pressing, intermolecular diffusion, and cooling. Since a thermal cycle is involved in welding,
careful attention should be paid to maintain the optimal welding parameters for good quality of welds. In comparison with other joining procedures, welding has several advantages: welding introduces no new materials, it is fast, economical, and with proper design of weld joints local stress concentration can be minimized. However in welding, thermally induced residual stresses can have a detrimental effect on joint quality and strength.

Welding techniques available for plastics can be classified into two groups: those in which heat is generated by mechanical movement between the parts to be welded (e.g. ultrasonic, friction and vibration welding) and those which involve an external heating sources (e.g. hot gas, heating tools (infrared, laser, hot plate etc), microwave, resistance, induction etc.) [3]. Joining is a critical step in manufacturing of plastics and composites. Therefore, careful consideration of the joining methods and joint design will result in the production of cost effective and efficient joints.

1.2 Prediction and Measurement of Residual Stress in Welds

The analysis of thermal and residual stresses during plastics welding has been of increasing concern in many applications. This is because the formation of residual stress during plastic welding can adversely affect the quality of the weld strength and corrosion resistance of the joints [4 - 7]. Residual stress is very important when evaluating the strength of weldments, especially when they include defects. Since polymers are viscoelastic materials, prediction of thermal and residual stresses must include time in the temperature history, making the analysis
more complex. In this work, the experimental measurements and theoretical predictions of thermal and residual stresses during implant resistance welding were performed.

Implant resistance welding is a joining technique which is widely used in the pipe industry. During implant resistance welding, a conductive resistance wire mesh is placed at the welding interface. The mesh is electrically heated while the polymer is melted with the parts being pressed together. At the end of welding, the mesh remains imbedded at the joint interface. An implant resistance welder was set up for this research. Figure 1.1 shows a schematic diagram of implant resistance welding.

Moire interferometry and photoelasticity method were used to measure residual stress after welding. Moire interferometry is a whole-field measurement method of deformed displacements with very good accuracy. It can be used for the measurement of stress and strain for complex geometries of very small parts. Another technique used for measuring residual stress is photoelasticity. It relies on the principle that many amorphous transparent materials which are optically isotropic become optically anisotropic when they are stressed. Therefore, under stress they exhibit optical characteristics similar to crystals. This effect normally persists while the loads are maintained but vanishes almost instantaneously after the loads are removed [8]. Therefore by using the stress optic law along with the value of the stress optic coefficient, whole field measurement of stress distribution in the specimen is possible.

In this research, the total strain in polycarbonate (PC) after welding was measured using moire interferometry and compared with
Figure 1.1 Schematic of Implant Resistance Welding Process
nonisothermal viscoelasticity analysis of Finite Element Method (FEM). The residual stress distribution after welding was also measured using sectioning method along with moire interferometry. Residual stress prediction of FEM and a simplified modeling analysis (multi-bar analogy) for implant resistance welding were compared with photoelasticity measurement and moire interferometry measurements. Heat flow analysis during welding was done with FEM and Finite Difference Method (FDM) and compared with measurement. For complete analysis, heat flow prediction was fed to nonisothermal viscoelastic stress analysis during welding.

Heat treatment was performed to relieve some of the residual stress in the weld. After heat treatment, residual stress formation was measured using photoelasticity and moire interferometry with the sectioning method. FEM analysis for the heat treatment procedure was performed and FEM predictions were compared with experimental measurements.

1.3 Objectives of Research

The safety and durability of a load-bearing parts depends on the relationship between the stresses developed in the parts and the maximum allowable stress in the material. The stresses in the part include the combination of the externally applied stresses and residual stresses. Modern methods of theoretical and experimental stress analysis allows the applied stress to be determined very precisely even for complex configurations. But the residual stresses are often ignored simply because they are unknown and difficult to evaluate. Clearly, the
availability of reliable methods of measuring and predicting residual stresses would allow these stresses to be incorporated into design and structural analysis procedures and accordingly provide for safer and more dependable structures and greater efficiencies in material utilization. Another important reason for predicting and measuring residual stresses is to help in the evaluation and to guide in the development of new manufacturing processes. It has been frequently found that relatively minor changes in heat treatment schedules, machining rates, welding procedures etc. can make the difference between parts with severe residual stress and relatively residual-stress-free parts [9,10]. Therefore prediction and measurement of thermal and residual stress formation during welding is very important. Nevertheless, little research was done to predict and measure thermal and residual stress formation during thermoplastic welding and to investigate the effect of thermal and residual stresses on the weld. In this work, the experimental measurement and theoretical prediction of thermal and residual stress during implant resistance welding was studied. The experimental and theoretical approached used for the this research can be used for other welding procedures. The moire interferometry method can be used to measure the residual stress in other manufacturing methods with very good accuracy. An annealing heat treatment was performed to reduce the level of residual stress in the weld. This annealing technique can be incorporated into welding schedules to reduce the level of residual stress in plastic welding procedures.
CHAPTER II
LITERATURE REVIEW

2.1 Theoretical Approaches

Thermally-induced residual stresses are internal self-equilibrium stresses which remain in a part after final cooling to room temperature. Development of residual stress in viscoelastic polymers is strongly dependent on the temperature history of the parts. Viscoelastic material properties of polymers such as PMMA, polyurethan, polystyrene etc. were determined by many scientists [11 - 16]. In their work, the master relaxation curves at the reference temperatures were experimentally determined including the shifting effect based on time-temperature superposition principle. Due to the lack of experimental data for the stress relaxation of polycarbonate, it was approximated. The stress relaxation curve of PMMA was modified to describe the behavior of polycarbonate after observing that the shapes of relaxation curves for viscoelastic materials are similar. Schapery suggested an efficient numerical method called "collocation method" to characterize the viscoelastic behavior with mechanical model using a series of springs and dampers [17]. This collocation method was used to characterize the viscoelastic behavior of polycarbonate using the generalized Maxwell
Residual stress formation in injection molding was predicted using Leonov theory in which evaluation of the static viscoelastic properties are determined by dynamic measurement [18,19]. In many cases, dynamic measurement of viscoelastic properties is done first and it is converted into static properties using this Leonov theory. Dirichlet series creep function was used to analyze the aging effects of concrete [20]. This analysis was achieved by the rate-type formulation of the creep law which is based on Kelvin chain model.

Prediction of residual stress in quenching of a glass disk, which behaves like a typical viscoelastic material at elevated temperatures, was performed by Woo and coworkers [21 - 24]. In their work, concepts of hardening time to reduce the numerical computation time was introduced. They suggested the numerical time and space discretization scheme in viscoelasticity analysis; this results in better accuracy in severe working conditions such as in the quenching process. They compared the analytical predictions with experimental measurements of residual stresses [25,26]. Viscoelastic analysis of moisture-induced damage in graphite/epoxy composites was done, indicating that drying and moisture cycling are more detrimental than moisture absorption. Theoretical prediction of moisture induced damage was compared with experimental measurement [27]. Theoretical approach used in this graphite/epoxy composites can be used for thermal stress analysis problem in composites because the effect of moisture is exactly the same as the thermal effect in viscoelastic material.
A method to obtain solution for viscoelastic problem using quasi-elastic scheme was suggested by Schapery [28,29]. This quasi-elastic equilibrium concept was used for FEM formulation by many scientists [30-32]. Thus, FEM became a very successful method to predict thermal and residual stress in viscoelastic materials. Here too, the FEM for nonisothermal viscoelastic stress analysis for the prediction of thermal and residual stress after welding was based on the quasi-elastic equilibrium formulation.

2.2 Experimental Approach

Methods for the measurement of residual stress can be divided into two classes: destructive methods (layer removal method, drilling hole method etc.) and nondestructive methods (photoelasticity, X-ray, ultrasonic, moire interferometry). Residual stress is a self-equilibrium stress. Therefore, removing a portion of a residually stressed body results in the deformation of the body to form a new self equilibrium. Measuring the resulting deformation provides valuable information for the measurement of residual stress. The disadvantages of destructive methods are that the measured data is sometimes very difficult to interpret analytically and determining actual values of residual stress are very painstaking and tedious. Another limitation is that destructive methods do not provide the spatial distribution of residual stress. To have complete information about residual stress, several sectioning or hole drillings should be performed. The most serious disadvantage of the destructive methods is that they are destructive to the parts being
analyzed and thus the samples which are tested can not be put back into service.

Nondestructive methods for measuring residual stresses can be used for every piece in a production lot not only during manufacturing but also in service conditions. In comparison with destructive techniques, some of these methods are relatively rapid and inexpensive. The major drawback with some nondestructive methods is that the measured data are indirect, which means the measurement of physical quantities influenced by residual stress. Thus interpretation into actual values of residual stress should be followed, which is not an easy job [33 - 38].

**Hole Drilling Method**

This is a destructive method which is widely used for the measurement of residual stress. It involves drilling a hole in the surface of the specimen and measurement of strain variation due to the hole. Measurement of the resulting strain can be interpreted as residual stress redistribution around the hole from elasticity theory. The strains can be measured with strain gage rosette, photoelasticity and moire interferometry. This technique has a number of limitations: measuring the residual stress at a large number of point on a surface can be time consuming and when the surface is not flat, this method cannot be used. The drilling operation itself can create heat generation and plastic deformation which leads to the formation of residual stress. Sometimes plastic deformation effects limit the reliability of the hole drilling method [33,34].
Layer Removal Method

This method is applied to sheet materials and involves removing successive uniform layers from the surface of a sample and measuring the resulting curvature. To use this method, the stress should not vary in the plane of the specimen but only through the thickness and the removal of surface layers should not disturb the stress in the remaining material. Therefore, this method is not suitable for detecting critical local stress concentrations [35].

X-ray Diffraction

This is an advanced and widely used method for the measurement of residual stress nondestructively [36]. It is based on measurements of lattice strains or of changes in the spacings between crystallographic lattice planes which are caused by stress. Portable equipment is commercially available, and it permits these measurement to be made very quickly. The elastic constants of crystals vary with orientation and they differ from the bulk elastic constants which are not easily measured on polycrystalline samples. Therefore, measured values for crystals are not always available [36].

Ultrasonics

Ultrasonic methods for evaluating residual stresses are based on the changes in the velocity of ultrasonic waves due to stress [37]. High-order elastic constants are generally required in order to relate the ultrasonic velocity measurement to stresses. These constants are not
generally available and must be experimentally determined for the particular samples [38,39]. The usefulness of the ultrasonic approached is very limited in highly attenuating materials.

Photoelasticity

Photoelasticity method for experimental stress analysis is a well-established method. Photoelasticity can be used for the measurement of residual stress when the object of interest itself is transparent. If the object is not transparent, a reflective coating needs to be applied to the surface of the object for reflective photoelasticity method. At elevated temperatures the reflective coating can separate from the surface of the object and this method cannot be used. Even in the case of transparent materials, if the material properties vary with time and temperature, photoelasticity measurement of stress at elevated temperatures is very difficult. In the case of viscoelastic polymeric materials, measurement at a range of temperatures must include relations between birefringence coefficient of stress and strain and time and temperature [40-43]. Despite such deficiencies and since the polycarbonate is transparent and photoelasticity is a well-established method, it was used for the estimation of residual stress at room temperature in this project.

Moire Interferometry

Moire interferometry is used to measure in-plane displacements, U and V, on a flat surface. It extends the sensitivity of moire methods into the subwavelength range, making it suitable for the analysis of localized
deformations with very good accuracy [44]. Moire interferometry has very high displacement sensitivity to the defined directions, the direction perpendicular to the lines of reference grating [44]. It can be used for the measurement of stresses and strains for complex geometries and very small parts. For example, the stress concentration and distortion problem in the different layers of composite structures can be approached with moire interferometry [45,46]. Moire interferometry can also be used for the crack propagation in dynamic fracture analysis [47]. Usage of moire interferometry can also be extended to micromechanics [48]. Moire interferometry is a direct measurement method of transient thermal and residual strains and stresses. Moire interferometry nicely fills the gap in capabilities of other experimental techniques, because it can be used in zones of very high strain gradients and it can be used to determine shear strains as readily as normal strains [44]. This moire interferometry set-up was established at The Ohio State University for measurement of residual strains and residual stresses in viscoelastic plastics for this research. For the measurement of residual stress after welding, the sectioning method was used. This sectioning method along with moire interferometry was used for measurement of residual strains through the thickness direction in composite cylinders after curing. [49]

In the sectioning method, sectioning in the middle of a specimen creates the free surface along the sectioning line which results in complete release of residual stress along the sectioning line in the direction perpendicular to the sectioning line. Moire interferometry can measure the change in the deformed displacements due to release of the
residual stress. The sectioning method utilizing moire interferometry is a new technique for the measurement of residual stress with very high accuracy. If the zone of interest for the measurement of residual stress is flat and large enough to replicate the moire grating (as small as few square millimeters), this technique can be used for most materials including metals.

2.3 Effect of Residual Stress on Welds

The adverse effect of residual stresses in a structure cannot be observed until external loading or environmental conditions are applied to a structure. Either or both the external loading and the environmental conditions may cause noticeable damage to the structure. Due to the existence of residual stresses, fracture can be produced under external loads which by themselves would not have done so. Residual stress reduces the static and dynamic failure stress of a structure [4, 50 - 56]. Environments which by themselves are not particularly damaging to a material cause detrimental effects to the material in the presence of residual stresses, i.e. stress corrosion [4,55]. In some cases, residual stresses can be beneficial. The presence of properly oriented compressive residual stresses are beneficial to the strength of structures under tensile external loads [4].

In order to demonstrate the effect of residual stresses on the impact behavior of polycarbonate, notched Izod impact tests were conducted at room temperature [50,51]. The effects of residual stresses on the tensile-yield strength, fracture toughness, and impact strength of
polycarbonate were studied [50 - 53]. Cold rolling of amorphous polymer such as polycarbonate was carried out on conventional rolling mill. Since the rolling process was done at room temperature and the dimensional change is permanent, roll reduction introduce plastic deformation in the part after rolling process. Roll reduction percent is defined by:

\[
\frac{\text{initial thickness} - \text{final thickness}}{\text{initial thickness}} \times 100
\]

Therefore, more percent roll reduction includes more plastic deformation and resultantly, larger roll reduction results in larger residual stress in the part. It was reported that impact strength observed in polycarbonate is influenced by both molecular orientation and residual stress [51]. To determine the impact strength after rolling, the izod impact test was performed by Broutman and coworkers [50,51]. As shown in Figure 2.1, the impact strength decreases considerably as percent roll reduction. At other cases of roll directions (0 degrees and 45 degrees), the impact strength also decreases as percent of the roll reduction increases. Figure 2.2 shows the impact strength for 0.186 inch polycarbonate sheets as a function of roll reduction [51]. A maximum in impact strength occurs at 2 percent roll reduction. At higher percent roll reduction, the impact strength decreases. As the percent roll reduction is increased, the induced residual stress is increased due to increased plastic deformation and entrapped elastic strains. As shown in Figure 2.2, residual stress has a considerable effect on the impact strength considerably. Effects of residual stresses on the crazing of polymers
was studied by Legrand [52]. The effect of heat treatment of the molded polycarbonate specimen, which can reduce the residual stress, is summarized in Table 1. Legrand [54] found that suppression of craze initiation in advance of the crack in polycarbonate specimen was observed due to the existence of residual stress. As shown in Table 1, crazing was observed in all annealed specimens. Thus it was concluded that annealing can reduce the residual stress in the molded parts. In conjunction with this, it was observed that the increased strength achieved after heat treatment of molded polycarbonate samples in the temperature range 80 °C - 130 °C is due to ordering of the amorphous regions and reducing residual stress. Figure 2.3 shows the effect of heat treatment on the strength of molded polycarbonate parts. It shows that higher heat treatment temperatures (below the glass transition temperature) increase the yield strength remarkably. Hence Table 1 and Figure 2.3 say that the induced-residual stress in the parts reduces the yield strength.

Effects of weld residual stress on distortion and stress corrosion were studied by Liede [55]. Distortion may occur slowly and therefore it may not be apparent until the assembly of parts is initiated. Heat treatment, resonance vibration, and overstressing were suggested as residual stress relieving methods for metals [55]. Structures with welding residual stress are more susceptible to chemicals, sea water and moisture than ones without residual stress. This susceptibility to corrosion eventually leads to stress corrosion cracking in the structure [55]. Quantitative study about the effect of residual stress in welds on
the fatigue strength was performed by Gurney [54]. Figure 2.4 shows the quantitative effect of residual stress on fatigue life of weld. While this data is for grade 50B steel, the effect of residual stress on fatigue life of plastics such as polycarbonates is expected to be similar. He concluded that fatigue design stresses for as welded structure should consider the presence of residual stress in the weld. It was found that residual stress can significantly reduce the buckling strength of columns. If the column also has initial distortion, its buckling strength will be reduced further [4].
Figure 2.1. Variation of Notched Izod Impact Strength With Percent Roll Reduction for Polycarbonate. Original Sheet Thickness = 0.128 inches [51].
Figure 2.2. Variation of Notched Izod Impact Strength With Percent Roll Reduction for Polycarbonate. Original Sheet Thickness = 0.186 inches [51]
Table 1. Effect of Residual Stress on The Crazing and Fracture [52].

<table>
<thead>
<tr>
<th>Sample</th>
<th>Temperature</th>
<th>Fracture</th>
<th>Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notched unannealed</td>
<td>25 °C</td>
<td>ductile</td>
<td>uncrazed</td>
</tr>
<tr>
<td>Notched unannealed</td>
<td>Liquid N₂</td>
<td>brittle</td>
<td>crazed</td>
</tr>
<tr>
<td></td>
<td>-180 °C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Notched annealed @ 125 °C</td>
<td>25 °C</td>
<td>brittle</td>
<td>crazed</td>
</tr>
<tr>
<td>Notched annealed @ 125 °C</td>
<td>Liquid N₂</td>
<td>brittle</td>
<td>crazed</td>
</tr>
<tr>
<td></td>
<td>-180 °C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cold drawn notch</td>
<td>25 °C</td>
<td>brittle</td>
<td>uncrazed</td>
</tr>
<tr>
<td>Cold drawn notch</td>
<td>Liquid N₂</td>
<td>brittle</td>
<td>uncrazed</td>
</tr>
<tr>
<td></td>
<td>-180 °C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.3 Flexural Yield Strength vs. Time of Heating at Various Temperatures (°C) [53]
Figure 2.4. Results for Transverse Non-Load-Carrying Fillet Welded Specimens Subjected to Tensile Loading (R = 0)

○ - as-welded; △ - stress relieved

[54]
3.1 Introduction

Since the thermal history during implant resistance welding plays a major role on the formation of thermal and residual stresses, precise prediction of heat flow is mandatory for stress and strain analysis. Two theoretical modeling approaches for heat flow analysis of implant resistance welding were performed by virtue of the Finite Difference Method (FDM) and the Finite Element Method (FEM). For FDM heat flow analysis during implant resistance welding, explicit 1-D formulation based on the energy balance method and central difference method of higher order differentiation was developed along with the assumption that the temperature distribution is uniform in the direction parallel to the weld line [57,58]. Convection heat loss was not considered in the analysis. For FEM analysis 2-D 8-noded serendipity heat transfer elements were used. Transient and steady heat transfer analysis were done in both FEM and FDM analysis. The theoretical and experimental techniques used for the implant resistance welding procedure can be applied to other welding methods with minor modifications in modeling.
The major temperature variation during implant resistance welding occurs in the direction perpendicular to the weld line, because a constant heat flux is produced from the resistance wire mesh which remains imbedded at the welding interface. In this kind of temperature distribution, the major thermally-induced residual stress is in the direction parallel to the weld line [4,59 - 61]. During welding, the hotter region tends to elongate, while the colder regions constrain it resulting in the formation of thermal and residual stresses. When the major temperature variation is in the direction perpendicular to the weld line, shear stresses are small and the dominating thermally induced stress is in the direction parallel to the weld line. Hence, the multi-bar analogy modeling (see Figure 3.1) analysis can predict the thermally induced stress formation in the direction parallel to the weld line. The multi-bar analogy modeling analysis was used for thermal and residual stress prediction in quenching of glass disks where the major temperature variation is in the direction perpendicular to the center line. Closed form solutions of this problem [21-24] were in good agreement with experimental measurement [25,26]. At elevated temperature, glass exhibits a typical viscoelastic behavior. Multi-bar analogy prediction of residual stress in the glass disk quenching problem was compared with the closed form solution and experimental data. FDM heat flow analysis was connected with the multi-bar analogy analysis for combined thermal and residual stress analysis.

Polycarbonate (PC) was selected as a specimen material because PC is one of the few materials which allow both photoelasticity and moire
interferometry measurement. It is generally understood that mechanical models comprised of springs and dashpots can be used to describe mechanical behavior of linear viscoelastic materials. It is not necessary to use a general array of springs and dashpots but only the simple arrangement of Voigt elements in series (Kelvin model) or Maxwell elements in parallel [62,63]. Viscoelastic mechanical behavior of PC was done using a generalized Maxwell model with thirteen elements (See Figure 3.2). Viscoelastic stress relaxation curve characterized by a generalized Maxwell model with thirteen elements was described with a Dirichlet series [64]. To determine the elastic constants of the generalized Maxwell model, the collocation method suggested by Schapery [17,65,66] was used. The William-Landel-Ferry (WLF) equation was used for time-temperature superposition of a thermorheologically simple material (TSM) in nonisothermal viscoelasticity [67,68].

Nonisothermal viscoelastic FEM modeling analysis for the prediction of thermal and residual stress during implant resistance welding was performed using 8-noded serendipity plane stress element with biquadratic interpolation function to describe the temperature and stress field more accurately [69,70]. Appropriate initial and boundary conditions were implemented to model the implant resistance welding procedure (See Figure 3.3).
Figure 3.1 Schematics of Multi-Bar Analogy Modeling Analysis.
Figure 3.2. Viscoelastic Generalized Maxwell Model With Thirteen Elements.
Pressure (It is removed during cooling.)

Figure 3.3 FEM Mesh and Boundary Conditions for Viscoelastic Thermal and Residual Stress Analysis
3.2 Heat Flow Analysis

3.2.1 FDM Heat Flow Analysis

Figure 3.4 shows the FDM modeling scheme for heat flow analysis of implant resistance welding. During implant resistance welding, the edge of the PC substrate is exposed to a uniform heat flux, it was assumed that the temperature is uniform through the thickness and the width of the part. This was verified by infrared temperature measurements. The major temperature variation was in the direction perpendicular to the weld line. Therefore, any points with the same distance from the weld line have the same temperature history during welding. Thus, the implant resistance welding procedure was modeled as a 1-D heat flow problem with constant heat flux at one end and no convective losses at the other edges (see Figure 3.4). Explicit 1-D FDM formulation based on energy balance and central difference method of differentiation was performed. The size of the time increment was checked for stability at every time step. The substrate was divided into N nodes. The energy balance equations for the 1-st node, intermediate nodes, and the N-th node differ due to boundary conditions.

The finite difference form of the energy balance equation at node 1 is:

\[ q \Delta t = A_c \left( \lambda \frac{T_{1}^{p+1} - T_{1}^{p}}{\Delta x} \right) - \frac{A_C}{2} \rho C \frac{\Delta x}{\Delta t} \]

where \( q \) is heat flow rate,

\( A_C \) is cross sectional area,

\( \lambda \) is thermal conductivity,
Descritize the specimen into 1-D FDM elements

Figure 3.4 Schematics of FDM Modeling for Heat Flow Analysis of Implant Resistance Welding.
\( \rho \) is density
\( C \) is heat capacity,
\( T_1^p \) is the temperature of node 1 at time step \( p \),
\( T_2^p \) is the temperature of node 2 at time step \( p \),
\( T_1^{p+1} \) is temperature at time step \( p+1 \).

To predict \( \dot{q} \), the electric current \( (I) \) and voltage \( (V) \) applied to the Nichrome wire mesh during welding were measured and then \( \dot{q} \) was calculated as:

\[
\dot{q} = I \cdot V
\]

Control volume for the 1-st node energy balance, was set up half of intermediate nodes (see Figure 3.5). After some simplification, the finite difference equation for the temperature of the 1-st node is:

\[
T_1^{p+1} = \dot{q} \frac{2 \Delta x}{k} M + (1 - 2M) T_1^p + 2M T_2^p
\]  

(2)

where \( M = \frac{\alpha \Delta t}{\lambda \Delta x^2} \)

where \( \alpha = \frac{\lambda}{\rho c} \), \( \alpha \) is thermal diffusivity.

For stability \( 1 - 2M > 0 \)  

(3)
Figure 3.5 Control Volume And Energy Balance For 1-st Node.

\[ q_c = \lambda A_c \frac{T_1^p - T_2^p}{\Delta x}, \quad \dot{q} = \frac{l \cdot V}{A} \], where A is cross sectional area of specimen.
The energy balance equation for intermediate nodes is the 1-D conduction equation (see Figure 3.6 for control volume and energy balance.):

\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{4}
\]

The finite difference form of Equation 4 is:

\[
T_{k+1}^p = M (T_{k+1}^p + T_{k-1}^p) + (1 - 2M)T_k^p \tag{5}
\]

For stability of solution

\[1 - 2M > 0\] \tag{6}

As was the case of 1-st node, control volume for energy balance of last node \( n \) was chosen as half of intermediate nodes (see Figure 3.7). Since no convective heat loss was considered in heat flow analysis, the energy balance finite difference equation for the last node (node \( n \)) is as follows:

\[
\lambda A_c \frac{T_{n-1}^p - T_n^p}{\Delta x} = \frac{\Delta x}{2} A_c \rho C \frac{T_{n-1}^p - T_n^p}{\Delta t} \tag{7}
\]

After some simplifications, Equation 7 becomes:

\[
T_n^{p+1} = 2M T_{n-1}^p + (1 - 2M)T_n^p \tag{8}
\]
Figure 3.6 Control Volume And Energy Balance For Intermediate Nodes.

\[ q_c @ x = \lambda A_c \frac{T_{k-1}^p - T_k^p}{\Delta x} \]

\[ q_c @ x + \Delta x = \lambda A_c \frac{T_k^p - T_{k+1}^p}{\Delta x} \]
Figure 3.7 Control Volume And Energy Balance For last Node \( n \).

\[
q_c = \lambda A_c \frac{T_{n-1}^p - T_n^p}{\Delta x}
\]
For stability of the solution

\[ 1 - 2 \, M > 0 \quad (9) \]

Therefore, time stepping using Equations 2, 5, and 8 provides the temperature history of all of the nodes. Appendix A includes the FDM program for heat flow analysis.

3.2.2 FEM Heat Flow Analysis

For FEM analysis, 2-D 8-noded heat transfer element with biquadratic interpolation function was used. Boundary conditions used for FEM heat transfer analysis are shown in Figure 3.8. The implant resistance welding procedure was modelled as a constant heat flux at the bottom edge of the substrate during heating. Therefore, face 1 of the elements at the bottom edge include a constant heat flux during heating. Changes in thermal properties during heating and cooling were not considered in the analysis. Newton-Raphson and Gauss-Jordan method were used to solve the equations.

Both transient and steady state heat transfer analysis were done. In both transient and steady-state heat transfer analysis, an automatic time increment scheme was used where the maximum temperature change is controlled. In the automatic time increment scheme, the time increment is controlled by the specified error limit and the
2-D 8-noded plane stress elements

Figure 3.8  FEM Mesh and Boundary Conditions for Heat Flow Analysis.
specified maximum temperature change. In the automatic time increment scheme, the minimum usable time step is controlled by

\[
\Delta t > \frac{\rho C \Delta l^2}{6\lambda}
\]  

(10)

where \( \Delta t \) is the automatic time increment,

\( \Delta l \) is an element dimension.
3.3 Viscoelasticity

3.3.1 Constitutive Equation

The stress and strain constitutive equation of a viscoelastic Thermorheologically Simple Material (TSM) is given as follows [62-64]:

\[
\sigma_{ij} = \int_0^t G_{ijkl} (\xi - \xi') \frac{\partial}{\partial \tau} [\varepsilon_{kl}^{t} - \delta_{ij} \varepsilon_{kl}^{0}] d\tau
\]

(11)

where \(\sigma_{ij}\) are stresses,

\(G_{ijkl}\) are viscoelastic stress relaxation functions,

\(t\) and \(\tau\) are ultimate time and intermediate time respectively,

\(\delta_{ij}\) is Kronecker delta function,

\(\varepsilon_{kl}^{t}\) are total strains,

\(\varepsilon_{kl}^{0}\) are thermal strains, and

\(\xi\) and \(\xi'\) are reduced times defined by:

\[
\xi = \int_0^t \frac{dt}{\chi}
\]

(12)

\[
\xi' = \int_0^\tau \frac{dt}{\chi}
\]

where \(\chi\) is a shifting function of linear viscoelasticity defined by the WLF (Williams-Landel-Ferry) equation for temperatures above the glass transition temperature:
\[
\text{Log}_{10} (\chi(T)) = -\frac{C_1 \left(T - T_0\right)}{C_2 + (T - T_0)}
\] (13)

where

\[
C_1 = 17.44,
\]
\[
C_2 = 51.6,
\]
\[
T_0 \text{ is the reference temperature.}
\]

An increase in temperature accelerates molecular and segmental motion, bringing the system more rapidly to equilibrium and accelerating all types of viscoelastic relaxation processes. A convenient way of formulating this effect of temperature on the mechanical behavior of a viscoelastic polymer is in term of a shift function \(\chi(T)\). \(\chi(T)\) is a ratio of the time constant of a particular response at a specific temperature to the response time constant at the reference temperature. To describe this time-temperature superposition principle, the WLF equation was used with the reference temperature being the glass transition temperature. Since PC was assumed isotropic, Equation 11 reduces to the following:

\[
s_{ij} = \int_0^t G_1 (\xi - \xi') \frac{\partial}{\partial \tau} (e_{ij}) d\tau
\] (14)

\[
\sigma_{kk} = \int_0^t G_2 (\xi - \xi') \frac{\partial}{\partial \tau} (\epsilon_{kk}^1 - \epsilon_{kk}^0) d\tau
\] (15)

where \(S_{ij}\) is deviatoric stress defined by
\( S_{ij} = \sigma_{ij} - \frac{\sigma_{kk} \delta_{ij}}{3} \) \hspace{1cm} (16)

\( \sigma_{kk} \) is defined by:

\[ \sigma_{kk} \delta_{ij}/3 = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3 \] \hspace{1cm} (17)

\( e_{ij} \) is deviatoric strain defined by:

\[ e_{ij} = \varepsilon_{ij} - \varepsilon_{kk} \delta_{ij}/3 \] \hspace{1cm} (18)

\( \varepsilon_{kk} \) is defined by:

\[ \varepsilon_{kk} \delta_{ij}/3 = (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})/3 \] \hspace{1cm} (19)

By assuming the Poisson's ratio \( (\nu) \) is constant

\[ G(t) = \frac{E(t)}{2(1 + \nu)} \] \hspace{1cm} (20)

\[ G_1(t) = 2G(t) \] \hspace{1cm} (21)

\[ G_2(t) = 3 \lambda(t) + 2G(t) \] \hspace{1cm} (22)

where \( E(t) \) is the stress relaxation modulus,

and \( \lambda(t) \) is Lame's constant which is defined by:

\[ \lambda(t) = \frac{\nu E(t)}{2(1 + \nu)(1 - 2\nu)} \] \hspace{1cm} (23)
3.3.2 Characterization of Mechanical Behavior of PC

Viscoelastic constitutive equations of stress and strain were defined using a generalized Maxwell model and including the time-temperature superposition principle as described by the WLF equation. For the characterization of the mechanical behavior of PC, the collocation method and Gauss-Siedal elimination method were used to determine the elastic constants of the thirteen elements in the generalized Maxwell model [17,71]. The basic idea in the collocation method is to select a set of collocation times \( t_j \) and to force the model to agree with data at each collocation time i.e.

\[
E(t_j) - E_\infty = \sum_{i=1}^{N} E_i \exp\left(-\frac{t_j}{\tau_i}\right)
\]

(24)

where \( \tau_i \) is relaxation time \( \tau_i = \eta_i/E_i \), see Figure 3.2), \( E(t) \) is the stress relaxation curve at the reference temperature, and \( E_\infty \) is \( E(t) \) at \( t = \infty \). The relaxation times \( \tau_i \) can be selected arbitrarily; they are related to the collocation times in such a way that one of the experimental terms in the series is active at each collocation time. For each collocation time, \( t_j \), \( \tau_j \) is selected so that \( E_j \exp\left(-\frac{t_j}{\tau_j}\right) \) is primarily responsible for the response in the neighborhood of \( t_j \). In order to guarantee that the initial "elastic-like" response is modeled, the first collocation time needs to be selected at 0. When \( t_j \) is zero, Equation 24 becomes
\[ E(0) - E_\infty = \sum_{i=1}^{N} E_i \]  

(25)

The remaining collocation times are then selected arbitrarily on the \( \text{Log} \ (t) \) domain covering the range of variation of the experimental data. Due to exponential terms, the spacing of the collocation times should be such that \( \text{Log} \ (E) \) is roughly linear between pairs of collocation times. Since each exponential term is active over roughly one decade, this spacing should never be larger than one decade. A one decade spacing is usually sufficient. After \( N \) collocation times are selected and those are substituted in Equation 24, there exists an \( N \) by \( N \) linear system with \( N \) unknowns \( (E_1, E_2, \ldots, E_N) \). Since \( \tau_i = \frac{t_i}{E_i} \), \( E_i \)'s for arbitrarily selected \( \tau_i \)'s only need to be determined to define the unknowns in the generalized Maxwell model. Gauss-Siedal method was used to solve these equations.

To solve a system of \( N \) linear equation, the rows are rearranged so that the diagonal elements have as large a magnitude as possible relative to the magnitude of other coefficients in the same row. The rearranged system is defined as \( [A] \{x\} = \{b\} \). Beginning with an initial approximation to the solution vector, \( x_i^1 \), each component of \( x_i^{n+1} \) is found for \( i = 1, 2, \ldots, N \) by iteration of
\[ x_i^{n+1} = \frac{b_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{n+1} - \sum_{j=i+1}^{N} \frac{a_{ij}}{a_{ii}} x_j^n \]  

(26)

A sufficient condition for convergence is that

\[ |a_{ii}| \sum_{j=1}^{N} |a_{ij}| > |a_{ij}| \quad i = 1, 2, \ldots, N \]  

(27)

When the sufficient condition is met, \( x^n \) will converge to the solution no matter what initial vector is used.

After successful completion of the collocation procedure, all constants (\( E_\infty, E_i, \) and \( \tau_i \)) can be determined. Then, the stress relaxation modulus (\( E(t_j) \)) of PC can be described as Prony series as follows [61-63];

\[ E(t_j) = E_\infty + \sum_{i=1}^{N} E_i \exp\left(\frac{-t_j}{\tau_i}\right) \]  

(28)

Appendix B includes the programs for the collocation method.
3.4 Multi-Bar Analogy Analysis

During implant resistance welding, the major temperature variation is only in the direction perpendicular to the weld line. The temperature varies very little in the direction parallel to the weld line is was verified by infrared temperature sensor measurements [60,61]. Pressure is applied to both ends of the PC substrate during heating and cooling (see Figure 1.1) but this external pressure has little effect on the stress development in the direction parallel with the weld line. Additionally, the most detrimental residual stress is the one developed in the direction parallel with the weld line [60,61]. Therefore, without loosing too much generality, stress developed in the direction parallel with the weld line can be predicted using geometrical compatibility, and force and moment equilibrium equations in that direction. During heating and cooling in implant resistance welding, the hotter regions of the PC substrate are constrained by colder regions. These concepts yield the multi-bar analogy modeling analysis where the movable rigid boundary all over the bars stands for the equivalent internal constraints during welding. A schematic of the multi-bar analogy is shown in Figure 3.1. In the multi-bar analogy, the middle bar represents the hottest zone or the region at the weld line. Side bars (next to the middle bar) represent the zones adjacent to the weld line. Each bar will experience the same temperature history as the material in the zone represented by the bar. During the implant resistance welding process, thermal loading is a major cause for the development of stress. Therefore, in multi-bar analogy analysis,
each bar is assumed to experience the same thermal history as the equivalent material zone. Increasing the number of bars will certainly increase the accuracy of prediction in multi-bar analogy modeling analysis. When the temperature variation is sharp, the number of bars should be increased to a sufficient number to more accurately describe the temperature field.

In our cases, the number of bars selected for the prediction of thermal and residual stress during implant resistance welding was eleven. From moment equilibrium, force equilibrium, and geometric compatibility, the following equation can be obtained:

\[
A_m \int_0^1 E(\xi - \xi') \frac{d}{d\tau} [\varepsilon^t_m - \varepsilon^0_m] d\tau + \sum_{i=1}^5 \left\{ 2A_{si} \int_0^1 E(\xi - \xi') \frac{d}{d\tau} [\varepsilon^t_{si} - \varepsilon^0_{si}] d\tau \right\} = 0 \tag{29}
\]

where \(A_m\) is the cross sectional area of the middle bar
\(A_{si}\) is the cross sectional area of the side bars
\(\varepsilon^t_s(t)\), and \(\varepsilon^0_s(t)\) are total strain and thermal strain of side bars respectively
\(\varepsilon^t_m(t)\), and \(\varepsilon^0_m(t)\) are total strain and thermal strain of the middle bar respectively.

And,

\[
\varepsilon^0(t) = \gamma \theta = \gamma (T - T_0) \tag{30}
\]

where \(\gamma\) is coefficient of thermal expansion,
\(T\) is the temperature at time \(t\),
\(T_0\) is the initial temperature.
Equation 29 can be rewritten as follows:

\[ A_m \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} \varepsilon_m \, d\tau + \sum_{i=1}^{5} 2A_{s_i} \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} \varepsilon_{s_i} \, d\tau \]

\[ = A_m \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} \varepsilon_m \, d\tau + \sum_{i=1}^{5} 2A_{s_i} \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} \varepsilon_{s_i} \, d\tau \]

(31)

Looking at the left and right sides of Equation 31, one can develop a generalized procedure to integrate each term in the equation. Examining the general integral term and using integration by parts give:

\[ \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} \varepsilon(\tau) \, d\tau = E_n \varepsilon(t) - \int_0^t \frac{\partial}{\partial \tau} (E(\xi - \xi')) \varepsilon(\tau) \, d\tau \]

(32)

where \( E_n \) is \( E(\xi - \xi') \) at \( \tau = t \).

For finite difference integration of Equation 32, time should be discretized. Dividing the time interval into \( (n-1) \) intervals from \( t_1 = 0 \) to \( t_n = t \), with \( t_i \) as an intermediate value and writing \( \varepsilon_i = \varepsilon(t_i) \), Equation 32 becomes:

\[ E_n \varepsilon_i + 0.5 \sum_{i=1}^{n-1} (\varepsilon_i + \varepsilon_{i+1}) (E_i - E_{i+1}) = \]

\[ \{ E_n + 0.5 (E_{n-1} - E_n) \} \varepsilon_n + 0.5 \varepsilon_{n-1} (E_{n-1} - E_n) + \]

\[ 0.5 \sum_{i=1}^{n-2} (\varepsilon_i + \varepsilon_{i+1}) (E_i - E_{i+1}) \]

(33)
where \( E_i = E \left( \zeta(t_i) - \zeta(t_n) \right) \)

In our case, the cross-sectional area of all side bars is the same, and the cross-sectional area of the middle bar is half that of the side bars. Hence, \( A_{si} = A_s = \text{constant} \) and \( A_m = 0.5 A_s \), where \( i = 1, 2, \ldots, 5 \).

By implementing this integration scheme for each term in Equation 31, the total strain in the multi-bar analogy model, \( \varepsilon^t_n \) ( = \( \varepsilon^t_{sn} = \varepsilon^t_{mn} \)) is found to be,

\[
\varepsilon^t_n = \frac{R(t_n) - LP_m - \sum_{i=1}^{5} P_{si}}{L q_m + \sum_{i=1}^{5} q_{si}} \tag{34}
\]

where

\[
R(t_n) = \sum_{s=1}^{5} (E_{sn} \gamma_s \theta_{sn} + \sum_{i=1}^{n-1} \gamma_s (\theta_{si} + \theta_{si+1})(E_{si} - E_{si+1})) \tag{35}
\]

where

\[
\theta_i = \theta(t_i)
\]

\[
q = 0.5 \left( E_{n-1} + E_n \right)
\]

\[
P = 0.5 \varepsilon^t_{n-1} \left( E_{n-1} - E_n \right) + 0.5 \sum_{i=1}^{n-2} (\varepsilon^t_i + \varepsilon^t_{i+1}) (E_i - E_{i+1})
\]

\[
L = A_m / 2A_s
\]

\[
E_i = E \left( \xi(t_n) - \xi(t_i) \right)
\]
All subscripts "m" and "s" stand for middle bar and side bar respectively. For example, \( P_{si} \) means \( P \) of \( i \)-th side bar.

As shown in Equation 34, the total strain, \( \varepsilon^t_n \), at any time can be determined from all parameters in the previous time period. Once the strain is found, the stress is calculated using the 1-D viscoelasticity constitutive equation:

\[
\sigma(t) = \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} (\varepsilon^t - \varepsilon^\theta) \, d\tau
\]

\[
= \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} \varepsilon^t \, d\tau - \int_0^t E(\xi - \xi') \frac{\partial}{\partial \tau} \varepsilon^\theta \, d\tau
\]

where \( \varepsilon^\theta(t) = \gamma \theta(t) = \gamma \theta \)

Equation 36 is solved using the same partial integration procedure along with the numerical integration scheme used in Equation 33. Then the thermal stress at any time is given by:

\[
\sigma(t) = P_n \varepsilon^t_n + P + E_n \gamma \theta_n + 0.5 \sum_{i=1}^{n-2} (\lambda \theta_i + \lambda \theta_{i+1}) (E_i - E_{i+1})
\]

(37)

In Equation 37, all terms are determined at the previous time steps and \( \varepsilon^t_n \) is determined from Equation 34.

When the body is cooled down to the final equilibrium temperature the residual stress field, which is generated by the cooling process, is formed. In terms of viscoelasticity parameter \( \xi \) (reduced time defined by
Equations 12 and 13), residual stress is formed as long as $\xi$ does not change because viscoelastic stress relaxation is not permitted any longer. As shown in Figure 3.9, $\xi$ of the middle bar (and similarly for the different side bars) approaches a constant value. Since $\xi$ is constant, there are no more viscoelastic effects (see constitutive Equation 11) and a permanent residual stress is developed. Explicitly $\xi(t)$ is constant when $t \geq t_h$, where $t_h$ is defined as the "hardening time". $\xi(t)$ is defined by integration of WLF equation, which is strongly dependent on the temperature history of the point of interest. Thus the hardening time also depends on temperature history of the point of interest. When using the WLF equation to describe the temperature shifting effects on stress relaxation, $t$ becomes $t_h$ immediately after the temperature drops below the glass transition temperature.

Considering the hardening time ($t_h$), Equation 36 can be integrated with time in two parts as follows:

$$\int_0^{t_h} \text{Equation 36} \, d\tau + \int_{t_h}^t \text{Equation 36} \, d\tau$$

and then Equation 36 can be written as

$$\sigma(t) = \sigma_0^{t_h} + \sigma_{t_h}^t$$
Figure 3.9 Variation of the Reduced Time (\( \xi \)) at the Middle Bar During the glass Disk Quenching.
But $\sigma_{t_h}^t$ is elastic stress because the reduced time, $\xi(t)$, is constant and there is no viscoelastic effect after the "hardening time", $t_h$. Since residual stress is formed due to elastic stress, then $\sigma_{t_h}^t = - \sigma_e(t_h)$.

Consequently, the residual stress in viscoelastic stress analysis in multi-bar analogy is:

$$\sigma_{\text{residual}}(x, \infty) = \sigma(t_h) - \sigma_e(t_h)$$ \hspace{1cm} (38)

where $\sigma_e$ is the elastic stress

$$\sigma_e(t_h) = E_0 (\varepsilon_{t_e}(t_h) - \gamma \theta_n)$$ \hspace{1cm} (39)

where $E_0$ is stress relaxation modulus at room temperature, 
$\varepsilon_{t_e}$ is total elastic strain at hardening time.

Again from geometry compatibility, force equilibrium, and moment equilibrium, the elastic strain can be determined by

$$\varepsilon_{t_e} = \frac{\sum_{s=1}^{5} (E_{s0} \gamma \theta_s(t)) + LE_{m0} \gamma \theta_m(t)}{\sum_{s=1}^{5} E_{s0} + LE_{m0}}$$ \hspace{1cm} (40)

where $E_{m0} = E_{s0} = E_0$ is stress relaxation modulus at room temperature.
By substituting Equation 40 into Equation 39, $\sigma_e$ can be determined. Then, from Equation 38, the residual stress can be predicted. Using the hardening time will reduce the computation time considerably because integration of the viscoelasticity constitutive equation for time greater than the hardening time is not necessary.

This multi-bar analogy model was used in the glass disk quenching problem [72,73], where the major temperature variation is in the direction perpendicular to disk center; it was found to be in good agreement with the closed form solution of Woo et al [21-24] (see Figure 3.10) in the case of cooling from 588 °C down to 138 °C. In the case of cooling from 613 °C down to 138 °C, there is some deviation between multi-bar analogy prediction and the closed form solution. This is because the temperature gradients are greater and shifting effects are greater requiring increase in the number of bars. Modeling of the disk quenching problem using multi-bar analogy is shown in Figure 3.8. The temperature profiles for implant resistance welding and for disk quenching are similar. Therefore, multi-bar analogy modeling analysis could also be successfully used for implant resistance welding. Appendix C includes the multi bar analogy analysis program.
Figure 3.10 Comparison Between Multi-Bar Analogy and Closed-Form Solution of Residual Stress in Disk Quenching Problem (Solution 1: Max. Temp.: 613 °C, Min. Temp.: 138 °C, Sol. 2: Max. Temp.: 588 °C, Min. Temp.: 138 °C). Woo & Lee's Data Were Taken From Reference [72].
Figure 3.11 Modeling of The Glass Disk Quenching Problem Using Multi-Bar Analogy.
3.5 FEM Analysis for the Prediction of Stress

For the approximate spatial reduction by variational principle in FEM, the viscoelasticity constitutive equation (see section 3.3.1) is written as a set of simultaneous integral equations. Hence, in order to solve the viscoelasticity problem, a numerical method should be used to discretize the viscoelastic constitutive equations in time. The stress relaxation function \( G_1(t) \) and \( G_2(t) \) (see Equations 20, 21, 22, and 28) can be defined by Prony series as follows:

\[
G_1(t) = G_1^\infty + \sum_{i=1}^{m} G_{1i} \text{Exp} \left( -\frac{t}{\tau_i} \right) \quad (41)
\]

\[
G_2(t) = G_2^\infty + \sum_{i=1}^{m} G_{2i} \text{Exp} \left( -\frac{t}{\tau_i} \right) \quad (42)
\]

From Equation 14

\[
S_{ij} = \int_0^t (G_1^\infty + \sum_{i=1}^{m} G_{1i} \text{Exp} \left( \frac{(\tau - t)}{\tau_i} \right) \frac{\partial e_{ij}}{\partial \tau}) d\tau = G_{10} \left( e_{ij} - \sum_{i=1}^{m} \phi_{i1} e_i \right) \quad (43)
\]

\[
G_{10} = G_1^\infty + \sum_{i=1}^{m} G_{1i}
\]

where

\[
\phi_{i1} = \frac{G_{1i}}{G_{10}} \quad (44)
\]

\[
e_i = \int_0^t \left( 1 - \text{Exp} \left( \frac{(\tau - t)}{\tau_i} \right) \right) \frac{\partial e_{ij}}{\partial \tau} d\tau
\]
To perform this integration, one can assume that during time interval $\Delta t$, $e_{ij}$ varies linearly and thus $\partial e_{ij} / \partial \tau = \Delta e_{ij} / \Delta \tau$. If total elapsed time $t$ is divided into $n$ intervals, $t_1 = 0, \ldots, t_{n+1} = t$.

$$e_i^{n+1} = \int_0^{t_n} (1 - \text{Exp} \left( \frac{\tau - t_{n+1}}{\tau_i} \right)) \frac{\partial e_{ij}}{\partial \tau} d\tau + \int_{t_n}^{t_{n+1}} (1 - \text{Exp} \left( \frac{\tau - t_{n+1}}{\tau_i} \right)) \frac{\partial e_{ij}}{\partial \tau} d\tau$$

(45)

Considering

$$1 - \text{Exp} \left( \frac{\tau - t_{n+1}}{\tau_i} \right) = 1 - \text{Exp} \left( -\frac{\Delta \tau}{\tau_i} \right) + (1 - \text{Exp} \left( \frac{\tau - t_{n+1}}{\tau_i} \right)) \text{Exp} \left( -\frac{\Delta \tau}{\tau_i} \right)$$

(46)

Using Equation 46 and the approximation of $\partial e_{ij} / \partial \tau = \Delta e_{ij} / \Delta \tau$ gives,

$$e_i^{n+1} = \left(1 - \text{Exp} \left( -\frac{\Delta \tau}{\tau_i} \right) \right) \int_0^{t_n} \frac{\partial e_{ij}}{\partial \tau} d\tau + \text{Exp} \left( -\frac{\Delta \tau}{\tau_i} \right) \int_{t_n}^{t_{n+1}} (1 - \text{Exp} \left( \frac{\tau - t_n}{\tau_i} \right)) \frac{\partial e_{ij}}{\partial \tau} d\tau$$

$$+ \frac{\Delta e_{ij}}{\Delta t} \int_{t_n}^{t_{n+1}} (1 - \text{Exp} \left( -\frac{\Delta \tau}{\tau_i} \right)) d\tau$$

(47)

The first and last integrals are readily evaluated. The second integral is from Equation 43, which represents viscous strain in the $i$-th term only in Prony series of stress relaxation function. Hence,
\[ e_i^{n+1} - e_i^n = \Delta e_i \]

\[ = (1 - \text{Exp}(-\Delta t/\tau_i)) e_i^n + (\text{Exp}(-\Delta t/\tau_i) - 1) e_i^n + (\Delta t - \tau_i (1 - \text{Exp}(-\Delta t/\tau_i))) \frac{\Delta e_{ij}}{\Delta t} \]

\[ = \frac{\tau_i}{\Delta t} (\frac{\Delta t}{\tau_i} + \text{Exp}(-\Delta t/\tau_i) - 1) \Delta e_i + (1 - \text{Exp}(-\Delta t/\tau_i))(e_i^{n+1} - e_i^n) \]

(48)

From Equation 43

\[ \Delta S_{ij} = 2G_{10}(\Delta e_{ij} - \sum_{i=1}^{m} \phi_{ij}(e_i^{n+1} - e_i^n)) \]

(49)

With the same procedure starting from Equation 15,

\[ \Delta \sigma_{kk} = 2G_{20}(\Delta e_{kk} - \sum_{i=1}^{m} \beta_i (e_{kki}^{n+1} - e_{kki}^n)) \]

(50)

\[ G_{20} = G_{2\omega} + \sum_{i=1}^{m} G_{2i} \]

(51)

where

\[ \beta_i = \frac{G_{2i}}{G_{2\omega}} \]

\[ e_{kki} = \int_0^t (1 - \text{Exp}((\tau - t)/\tau_i)) \frac{\partial e_{kk}}{\partial \tau} d\tau \]
From Equations 49 and 50, the tangential moduli for $\Delta S_{ij}$ and $\Delta \sigma_{kk}$ can be obtained by differentiating $\Delta S_{ij}$ and $\Delta \sigma_{kk}$ with respect to $\Delta \varepsilon_{ij}$ and $\Delta \varepsilon_{kk}$ respectively:

$$G_1^T = G_{10} [1 - \sum_{i=1}^{m} \phi_i \left( \frac{\Delta \tau}{\tau_i} + \exp(-\frac{\Delta \tau}{\tau_i}) - 1 \right)]$$

$$G_2^T = G_{20} [1 - \sum_{i=1}^{m} \beta_i \left( \frac{\Delta \tau}{\tau_i} + \exp(-\frac{\Delta \tau}{\tau_i}) - 1 \right)]$$

These tangential moduli can be used to find $S_{ij}$ and $\sigma_{kk}$ with the Newton-Rapson method in FEM.

For FEM analysis, 8-noded serendipity plain strain element with biquadratic interpolation function and Gaussian reduced integration method were used. Since coupled analysis of temperature and displacement was performed, the same mesh was used for heat transfer and stress analysis. Due to symmetry, a quarter of the geometry with appropriate boundary conditions was used for the analysis. The boundary conditions and mesh for FEM stress analysis are shown on Figure 3.3. During analysis the external pressure was removed at the same time that the pressure was removed during actual welding.
3.6 Photoelasticity

Maxwell found that the change in the refractive indices in the medium are linearly proportional to the loads and follows the relationships;

\[ n_1 - n_2 = f_\sigma (\sigma_2 - \sigma_1) \]
\[ n_3 - n_2 = f_\sigma (\sigma_2 - \sigma_3) \]  \hspace{1cm} (53)
\[ n_1 - n_3 = f_\sigma (\sigma_3 - \sigma_1) \]

where \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are principal stresses at a point and \( n_1, n_2, \)
\( n_3 \) are refractive indices in the principal stress direction, \( f_\sigma \) is the stress optic coefficient.

In most photoelastic analyses, stress distribution is quite complex. Knowing the precise value of the stress optic coefficient \( f_\sigma \) is mandatory to determine the stress distribution accurately. Although the value of \( f_\sigma \)
is reasonably accurate, the stress optic coefficient \( f_\sigma \) varies with the supplier, batch of resin, and temperature. Therefore, it is always necessary to calibrate the value of \( f_\sigma \) before the photoelastic measurement is performed.

Consider the case of a plane-stress model inserted into the circular polariscope (see figure 3.12). The plane-polarized light beam emerging from the polarizer can be described as;

\[ E_{py} = k \cos \omega t \]  \hspace{1cm} (54)
Figure 3.12 Schematic of Photoelasticity Measurement Set-up (Circular Polariscop) [8]
where \( E_{py} \) is the polarized light in the direction of polarizer axis,

- \( k \) is the magnitude of light intensity,

and \( \omega \) is the circular frequency for light in the visible spectrum (approximately 1015 rad/s).

When the light enters into the first quarter-wave plate, it is resolved into two light components, \( E_f \) and \( E_s \), whose axis of vibration are parallel to the fast and slow axis, respectively. Since the axis of the quarter-wave plate is oriented 45 degree from polarizer axis,

\[
E_f = -\frac{\sqrt{2}}{2} k \cos \omega t \\
E_s = -\frac{\sqrt{2}}{2} k \cos \omega t
\]  

(55)

As the components pass through the quarter-wave plate, a relative phase shift between fast and slow axis \( \Delta = \pi/2 \) will be developed and the light components emerging from quarter-wave plate can be described as;

\[
E'_f = \frac{\sqrt{2}}{2} k \cos \omega t \\
E'_s = \frac{\sqrt{2}}{2} k \cos (\omega t - \frac{\pi}{2}) = \frac{\sqrt{2}}{2} k \sin \omega t
\]  

(56)

where \( E'_f \) is the light component in fast axis emerging from the first quarter-wave plate, and \( E'_s \) is the light component in slow axis emerging from the first quarter-wave plate.

After leaving the quarter-wave plate, the components enters the model in the way shown in Figure 3.13. Since the stressed specimen is behaving
Figure 3.13 Resolution of The Light Components as They Enter The Stressed Model [8]
as a temporary quarter-wave plate, the components $E'_f$ and $E'_s$ are resolved in $E_1$ and $E_2$. Thus by substituting Equation (56) into the description for $E_1$ and $E_2$:

$$E_1 = \frac{\sqrt{2}}{2} k [ \cos \omega t \cos (\pi/4 - \alpha) + \sin \omega t \cos (\pi/4 - \alpha) ]$$
$$= \frac{\sqrt{2}}{2} k \cos (\omega t + \alpha - \pi/4) \quad (57.a)$$

$$E_1 = \frac{\sqrt{2}}{2} k [ \sin \omega t \cos (\pi/4 - \alpha) - \cos \omega t \cos (\pi/4 - \alpha) ]$$
$$= \frac{\sqrt{2}}{2} k \sin (\omega t + \alpha - \pi/4) \quad (57.b)$$

The two components $E_1$ and $E_2$ propagate through the model with different velocities. The additional relative retardation $\Delta$ after passing the stressed model is

$$\Delta = \frac{2\pi c}{\lambda} (\sigma_1 - \sigma_2) \quad (58)$$

where $h$ is the specimen thickness,

$c = c_2 - c_1$ is the relative stress-optic coefficient, and

$\lambda$ is wave length.

Thus the wave emerging from the stresses model can be expressed as

$$E'_1 = \frac{\sqrt{2}}{2} k \cos (\omega t + \alpha - \pi/4) \quad (59.a)$$

$$E'_2 = \frac{\sqrt{2}}{2} k \cos (\omega t + \alpha - \pi/4 - \Delta) \quad (59.b)$$
The light emerging from the model propagates to the second quarter-wave plate in the fashion shown in Figure 3.14. Utilizing equation (59.a) and (59.b), the fast and slow axis components of light can be expressed as follows:

\[ E_f = \frac{\sqrt{2}}{2} k \left[ \cos(\omega t + \alpha - \pi/4) \sin(\pi/4 - \alpha) + \sin(\omega t + \alpha - \pi/4 - \Delta) \cos(\pi/4 - \alpha) \right] \] (60.a)

\[ E_s = \frac{\sqrt{2}}{2} k \left[ \cos(\omega t + \alpha - \pi/4) \cos(\pi/4 - \alpha) - \sin(\omega t + \alpha - \pi/4 - \Delta) \sin(\pi/4 - \alpha) \right] \] (60.b)

As the light passes through the second quarter-wave plate, a relative phase shift \( \Delta = \pi/2 \) is developed between fast axis and slow axis of quarter wave plate. Thus the light components emerging from the second quarter-wave plate can be described as follows:

\[ E_f = \frac{\sqrt{2}}{2} k \left[ \cos(\omega t + \alpha - \pi/4) \sin(\pi/4 - \alpha) \right. \\
+ \left. \sin(\omega t + \alpha - \pi/4 - \Delta) \cos(\pi/4 - \alpha) \right] \] (61.a)

\[ E_s = \frac{\sqrt{2}}{2} k \left[ \cos(\omega t + \alpha - \pi/4) \cos(\pi/4 - \alpha) \right. \\
+ \left. \cos(\omega t + \alpha - \pi/4 - \Delta) \sin(\pi/4 - \alpha) \right] \] (61.b)

Finally, the light enters the analyzer in the manner as shown in Figure 3.15. Thus \( E_{ax} \) (light polarized in the direction of the analyzer after passing analyzer) can be describes as;
\[ E_{ax} = \frac{k}{2} \left[ \sin(\omega t + \alpha - \pi/4)\cos(\pi/4 - \alpha) + \cos(\omega t + \alpha - \pi/4 - \Delta)\sin(\pi/4 - \alpha) \right] \]
\[-\left[ \cos(\omega t + \alpha - \pi/4)\sin(\pi/4 - \alpha) - \sin(\omega t + \alpha - \pi/4 - \Delta)\cos(\pi/4 - \alpha) \right]\]
\[= k \sin \Delta/2 \sin (\omega t + 2\alpha - \Delta/2) \quad (62) \]

Since the intensity of light is proportional to the square of the amplitude of the light wave, the intensity \( I \) of the light emerging from the analyzer of a circular polariscope is given by

\[ I = K \sin^2 (\Delta/2) \quad (63) \]

Equation 63 indicates that the intensity of light emerging from analyzer of circular polariscope is a function dependent only on the difference between principal stresses. From Equation 63, \( I = 0 \) only when \( \Delta/2 = n\pi \), where \( n = 0,1,2, \ldots \). Hence from Equations 57 and 63,

\[ n = n_2 - n_1 = \frac{\Delta}{2\pi} = \frac{hc}{\lambda} (\sigma_1 - \sigma_2) \quad (64) \]

where \( n = 0,1,2,3, \ldots \)

Equation 64 can be redefined as follows:

\[ \sigma_1 - \sigma_2 = \left( \frac{\lambda}{c} \right) n = \frac{f_o}{h} (n_2 - n_1) \quad (65) \]

where stress optic coefficient is defined by \( f_o = \frac{\lambda}{c} \)
Figure 3.14 Resolution of The Light Components as They Enter The Second Quarter-Wave Plate [8]
Figure 3.15 Components of The Light Vectors Which Are Transmitted Through The Analyzer (Dark Field) [8].
3.7 Moire-interferometry

Moire-interferometry can measure in-plane displacements, U and V, on flat surfaces. It can be used to determine shear strains as easily as normal strains. The sensitivity of moire-interferometry can be extended to subwavelength range, which means moire-interferometry can be used in both macromechanics and micromechanics. Moire-interferometry method can be used in zone of very high strain gradients such as stress and strain distribution analysis in composite laminates. This method can fill the gap in other experimental methods so nicely that it can solve problems which cannot be solved by other experimental methods. Moire-interferometry apparatus has been set up at the Ohio State University to be used in this work.

The moire-interferometry method can be used extensively for the experimental approach of stress and strain determination in viscoelastic materials under various loading conditions. Moire-interferometry can be used for measurements in both macromechanics and micromechanics domain. Moire-interferometry is an optical method utilizing interference of expanded and ultimately parallelized laser beam field. It provides whole-field contour maps of in-plane displacements with subwave length sensitivity. The abundance of displacement permits reliable determination of normal and shear strains. A schematic of a typical optical arrangement set-up is shown in Figure 3.16. The specimen surface is covered with a diffraction grating of frequency f/2 lines/mm. A parallel beam of laser light arrives to the specimen at angle -α, while a portion of
the same beam is redirected by a plane mirror to arrive at the symmetrical angle $+\alpha$. Since they are mutually coherent, the two incident beams generate an interference pattern comprised of alternating bright bands (constructive interference) and dark bands (destructive). The bands are very closely spaced and they function as a reference grating. It is not a real or physical grating, but instead, it is a virtual reference grating. Its frequency is $f$ lines per mm, which is related to wavelength $\lambda$ and angle $\alpha$ by

$$\sin \alpha = \frac{\lambda f}{2} \quad (66)$$

For a He-Ne laser, $\lambda = 633$ nm, $\alpha = 49.5$, $f = 2400$ lines/mm = 60,960 lines/inch and the corresponding initial or no-load frequency of the specimen grating is 1200 lines/mm.

The specimen grating and reference grating interact to form a moire pattern. It is photographed by the camera, which is focused on the plane of the specimen. The moire fringe pattern is essentially a null field (uniform intensity across the field, devoid of fringes, see Figure 3.17) before loads are applied to the specimen. After loading, the moire fringe pattern seen in the camera is a contour map in which the fringe order at any point is proportional to in-plane displacement at the point [74]. Figure 3.18 shows a typical example of moire fringe pattern. From moire fringe pattern, the displacement $U$ and $V$ of every point can be calculated by;
\[ U = \frac{1}{f} N_x \quad (67) \]
\[ V = \frac{1}{f} N_y \quad (68) \]

where \( N_x \) and \( N_y \) are fringe orders at the corresponding points, 
\( f \) is the frequency of the reference grating which is 
2400 lines/mm.

From the equations of strain and displacement, the normal and shear strains can be determined by;

\[ \varepsilon_x = \frac{\partial U}{\partial x} = \frac{1}{f} \left[ \frac{\partial N_x}{\partial x} \right] \quad (69) \]
\[ \varepsilon_y = \frac{\partial V}{\partial y} = \frac{1}{f} \left[ \frac{\partial N_y}{\partial y} \right] \quad (70) \]
\[ \gamma_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} = \frac{1}{f} \left[ \frac{\partial N_x}{\partial y} + \frac{\partial N_y}{\partial x} \right] \quad (71) \]

where \( \varepsilon_x \), \( \varepsilon_y \) and \( \gamma_{xy} \) are normal strains and shear strain respectively.
Figure 3.16 4-Beam Moire Interferometry That Utilizes a Beam-Splitter to Produce The $N_x$ and $N_y$ Patterns

- LB - laser beam
- BE - beam expander
- M1-M4 - plane mirrors
- PM - parabolic mirror
- BS - beam splitter
- S - specimen and specimen grating
- LF - loading fixture
- CL - camera lens
- P - linear polarizer
- CB - camera back
Figure 3.17 Picture of Typical Moire Null Field.
Figure 3.18 Typical Example of Moire Fringe Pattern (Courtesy of Dr. Han).
3.8 Sectioning Method

In the sectioning method for the measurement of the residual stress, free surface is created in the middle of the specimen. The free surface created causes total stress release in the direction perpendicular to the free surface because the free surface cannot support the stress in the direction perpendicular to the free surface. Figure 3.19 shows a schematic diagram of the sectioning method. Thus by measuring the strain changes at the zone adjacent to the free surface after sectioning, the residual stress distribution in the direction perpendicular to sectioning line can be measured. Since moire interferometry allows the whole-field measurement of strains, moire interferometry was used to measure strain distribution along the sectioning line.

Once entrapped elastic strains due to residual stress are measured, residual stress can be determined by stress and strain constitutive equation. Since the welded specimen is flat and thin, the plane stress constitutive equation was used for the determination of residual stress as follows:

\[
\sigma_{xx} = \frac{E}{1-\nu^2}(\varepsilon_{xx} + \nu\varepsilon_{yy}) \quad (72)
\]

\[
\sigma_{yy} = \frac{E}{1-\nu^2}(\varepsilon_{yy} + \nu\varepsilon_{xx}) \quad (73)
\]

\[
\tau_{xy} = \frac{E_0}{2(1+\nu)}\gamma_{xy} \quad (74)
\]
Sectioning line

Sectioning using low speed diamond cutter

Residual stressed specimen

Free surface is generated and residual stress adjacent the free surface is released.

Figure 3.19 Schematic Diagram of Sectioning Method for The Measurement of Residual Stress.
where $E$ is Young's modulus of polycarbonate at the room
temperature (2.38 GPa),
and $v$ is Possion's ratio of polycarbonate (0.35).

Moire interferometry utilizing sectioning method can be used for
measurement of the residual stress distribution at small size of part with
very good accuracy. This technique allows the measurement of the
residual stress distribution in the direction perpendicular to the sectioning
line along the sectioning line.

In case of drilling hole method, there should be enough space to
put a strain rosette whose diameter is about 0.2 inches then get only one
measurement at that point. Therefore, the drilling hole method is not a
feasible method for small parts. Also numerical analysis needs to be
done to create the calibration curve shown in Figure 3.20. Figure 3.20
illustrates that this calibration curve can be used only for the specific
cases where this calibration applies. To make a calibration curve, trial
and error type of work in both numerical and experimental analysis should
be done. The reliability and accuracy of drilling hole method depend on
many parameters such as drilling speed, diameter of hole, depth of hole,
and measurement distance from center of hole. Therefore, these
parameters should be controlled very carefully to use the calibration
curve, which is very difficult in reality. The appropriate drilling speed for
material depends on the stiffness of the material. For example, hard
materials like steel require very high drilling speeds (about 200,000 rpm)
while for soft materials like plastics, very low drilling speed (about 60 rmp or less) are desirable. Hence, the proper drilling speed is often determined by trial and error. Unlike the drilling hole method, moire interferometry does not depend on so many parameters. In most cases, Buehlar low speed cutting machine with water or oil cooling works very well. Sensitivity of moire interferometry measurement, which was used for this work, has limitation in strain measurement. When the strain value is less than about $8 \times 10^{-5}$ ($=1/2400 \times 1/5$), moire interferometry does not discern the strain value. In case of strong materials like steel, small strain values can result in large stress and accordingly, larger amount of error may be introduced in moire interferometry measurement. However, in case of soft materials like plastics, the error is reduced. This problem can be circumvented by using grating with higher grating frequency. Using grating with 20,000 line per mm in combination with high resolution optical microscope, strains with very small values were measured [48].
Figure 3.20 Calibration Curve in Drilling Hole Method to Determine The Stress From Strain Rosette Measurement. The Coefficients Should Be Determined By Numerical Analysis [75].
4.1 Implant Resistance Welding

An implant resistance welder was designed and built for this research. The implant resistance welder consists of a mini air press, air pressure controller, power supplier, and fixture to hold the specimen. (see Figure 4.1) The resistance wire mesh was placed at the weld interface and it stays at the weld interface after welding. Figure 4.2 shows the implant- resistance-welded specimen. The resistance wire mesh was connected to an electric power supplier to provide the electric current to the wire. The time for current input to the resistance wire mesh was controlled by timer and switch. An ampere meter was connected in series with the mesh and the current input was measured. A voltage meter was connected in parallel with the mesh and the voltage drop across wire mesh was measured. These two parameters were used to estimate the heat generation rate of the resistance wire mesh during welding. This implant resistance welder was mounted on a mounting-base-kit table to allow the horizontal and vertical rotations of welder. These rotations are necessary for the fine alignment and carrier of rotation techniques in moire interferometry. Before welding was performed, the polycarbonate substrate was dried in the oven for 24
hours at 100 °C to remove any moisture in parts. When this drying process was not performed, a lot of air bubble appeared after welding and heat treatment.
Figure 4.1 Implant Resistance Welder Designed and Built
Figure 4.2 Implant Resistance Welded Specimen.
4.2 Temperature Measurement

Temperature distribution across the weld was measured using infrared temperature sensor (Inframetric model 525). Inframetric 525 consists of an infrared scanner, electronic control unit, a power supply and interconnecting cables. Figure 4.3 shows schematic of system. Inframetric 525 is a small lightweight field instrument which produces a TV compatible video output signal of the thermal patterns in the scene that it is viewing. As the specimen is scanned, the naturally emitted infrared radiation is detected and converted by a liquid nitrogen cooled mercury, cadmium, telluride (Hg Cd Te) detector to an electric signal which is processed into a TV image of the temperature pattern in the scene. Line scanning mode was set up to see the temperature profile of any horizontal line in the specimen. The system could not provide absolute values of temperature without knowing the emissivity of PC. It was used to get a qualitative measure of thermal concours and the temperature distribution.

Absolute values of temperature were measured with K-type thermocouples and Hewlett Packard data acquisition system (HP 3497A). Small holes were drilled on the surface of the specimen and thermocouples were embedded in them (see Figure 4.4). The thermocouples were connected to an HP 3497A and data from the thermocouples was collected and transformed from analog signals to digital signals. Finally an IBM AT computer was used to convert the data to actual temperature values. Figure 4.5 shows the picture of the data acquisition system and Figure 4.6 shows a schematic of it.
Figure 4.3 Schematic of Infrared Temperature Measurement Set-Up
Figure 4.4 Polycarbonate Substrate With Embedded Thermocouples.
Figure 4.5 Data Acquisition System for Temperature Measurement Using Thermocouples
Figure 4.6 Schematic of Temperature Measurement Set-Up
By performing temperature measurement with both infrared temperature sensor and K-type thermocouples, temperature profiles in the direction parallel to weld line in specimen and actual temperature value were measure respectively.

4.3 Photoelasticity

Circular polariscope with dark field fringe pattern set-up, was used for the measurement of residual stress, because dark field fringe pattern set-up gave us better resolution of the fringe map than the light field fringe set-up. The 060 series polariscope, which consists of light source, polarizer, analyzer, and quarter wave plates, was used. Figure 4.6 shows the circular polariscope which was used in this research. A schematic of photoelastic measurement system is shown on Figure 3.12. The first element following the light source is the polarizer, which converts the ordinary light into plane-polarized light. The second element is a quarter-wave plate set at angle $\beta = \pi /4$ to the axis polarization. The quarter wave plate converts the plane-polarized light into circular-polarized light. The third component is the second quarter-wave plate which is set up so that the its slow axis is parallel with the fast axis of the first quarter-wave plate. The purpose of this is to convert circular-polarized light to plane-polarized light, which is vibrating in the vertical axis. The last element is the analyzer with its polarization axis in the horizontal direction. Its major function is to convert light into the component whose intensity is solely dependent on the principal stress difference.
Figure 4.7  Circular Polariscope for Photoelasticity Measurement
To photograph the photoelastic fringe map, a 35mm camera was mounted on a tripod and a light sensor was used to select the exposure time.

For the calibration of the stress optic coefficient, $f_\alpha$, the tensile specimen was made based on ASTM standard D638-86 [76] (see Figure 4.8). External tensile load was applied using a fixture, a mechanical spring, and a loading frame. While the load is applied in increments, the photoelastic fringes and loads were recorded and plotted. Since the load was applied within the elastic limit of PC, the slope of the load versus fringe order curve should be the stress optic coefficient of PC. The stress measurement of photoelasticity was determined by Equation 65. The calibration technique requires a specimen where the stress distribution is easy to predict and the configuration of which is easy to machine and simple to load. Among several methods of calibration, the tensile test method was selected, because machining of a tensile specimen is easy and the theoretical stress distribution can be accurately determined. From these experimental data, the material fringe value $f_\alpha$ can be determined as follows;

By nature of photoelasticity as was discussed, relation between stress and photoelastic fringes is

$$\sigma_1 - \sigma_2 = \frac{N f_\alpha}{h} \tag{75}$$

During a tensile test, the stress condition is unidirectional. Therefore, $\sigma_2$ in Equation 75 is zero.
Then

\[ \sigma_1 = \frac{P}{Wh} \quad \text{and} \quad \sigma_2 = 0 \quad (76) \]

where \( P \) is the external load, \( W \) is the width of the specimen, and \( h \) is the thickness of the specimen.

From Equations 75 and 76, the stress optic coefficient is determined by

\[ f_\alpha = \frac{P}{WN} \quad (77) \]

In photoelasticity measurement, the fringe sign can be determined with inspection rather than with certainty which works well for every case. There exist several methods for the determination of photoelasticity fringe sign such as nail test, using of photoelastic fringe compensator, and using of a white light light source (see reference 77 pages 121 and 122 for further details). Among these methods, a white light was used as light source for the determination of sign, and an increase or decrease in the fringe order was determined by observing the color sequence in the fringe pattern. Appendix D includes the color sequence table. This technique can be used successfully for the determination of photoelastic fringe sign even when the part geometry is complex and small.
Figure 4.8 Polycarbonate Tensile Specimen Used For Photoelasticity Calibration of Stress Optic Coefficient.
4.4 Moire-interferometry Measurement

4.4.1 Reproduction of Submaster Grating Molds

Figure 4.9 shows the moire interferometry system which was set up at the Ohio State University. There are two submaster molds for moire-grating replication on a specimen. Those gratings are replicated on glass, because the glass can keep grating consistant over a wide range of temperatures. Silicon rubber (General Electric RTV 615) submaster gratings can be reproduced from Ultraglow (Epoxy) gratings. Silicon rubber does not stick to glass very well, therefore a silicon primer (General Electric SS4120) is placed on the glass substrate and dried for 12 hours at room temperature. An effective method called "drag method" is used for priming; after an optical tissue is laid on the glass, the tissue is wetted with a few drops of silicon primer (see Figure 4.10). Then the tissue is manually dragged to wet the entire surface of the glass. To make the silicon rubber mold, resin and hardener are mixed. When the silicon rubber resin and hardener are mixed, many air bubbles are generated and remain entrapped in the mixture. To eliminate these air bubbles before mixing, the resin and hardener are placed in a vacuum dessicator (Thomas Scientific 3751-H10) under vacuum which is applied by high-power vacuum pumping machine (Thomas Scientific 1057-Q10 direct drive air pump). After all entrapped air bubbles disappeared, the silicon rubber resin and hardener were mixed throughly and vacuum-pumped again to eliminate air bubbles generated during the mixing process. After successful elimination of air bubbles in the mixture, a
small pool of silicon rubber is poured on the Ultraglow grating and the silicon-primed glass substrate is lowered slowly so as not to entrap air bubbles (see Figure 4.11). Then the silicone rubber is cured for 24 hours. For ease of separation of the two molds after curing, device shown in Figure 4.12 was designed and built. As shown in Figure 4.12, while substrates A is holding mold B, raising up mold C with bolts D applies shear loading at the interface between mold B and mold C. The shear loading at interface between mold B and C separates two molds. Ultraglow grating mold also can be reproduced from silicon rubber grating mold with a similar procedure to that of reproduction of silicon rubber grating. Ultraglow epoxy grating can be used disposably for grating replication on the specimen.

4.4.2 Specimen Grating Replication

Crossed-linek gratings, having 1200 lines/mm perpendicular to both the x and y axes, are used. Before replication of the grating on specimen, surface cleaning is very important. Cleaning with solvents is normally sufficient. Methy alchol is usually used for cleaning the PC surface and both methyl alchol and aceton are used for the surface preparation of the glass pieces. Specimen grating are produced by a replication procedure. In our case, Epoxy adhesive (Micro Measurement Group PC10C) works well for grating replication on polycarbonate. Basically the specimen grating replication procedure consists of casting a thin film of adhesive material between the specimen and Ultraglow master grating.
The submaster mold grating has high frequency grooves that makes up the diffraction grating. When mold and specimen are separated, the specimen is left with a thin film of grating which is firmly stuck to the surface of the specimen by the adhesive. The procedure for specimen grating replication was as follows;

The Ultraglow grating was coated with a thin layer of pure Aluminum (Fisher Scientific A557-500 Aluminum wire) by using vacuum deposition. This first layer of Aluminum should be thin. Then a parting agent (Kodak Photo-Flow 200) is applied to the thin Aluminum layer; this is done by dipping the submaster mold with the aluminum layer in a solution of Photo-Flow (diluted Photo-Flow and water by 200:1 in volume ratio) and drying it in the vertical position in a dust-free environment for 24 hours. Then this grating is coated with the second Aluminum layer using vacuum deposition. This Aluminum coated grating is now ready for specimen grating replication. Before replication to the specimen, the grating is aligned with respect to the specimen using the device shown in Figure 4.13. Mold B needs to be aligned first to insure that the two reflected laser beam have the same height with respect to the bottom plate C. Then the bar A is aligned with respect to the bottom plate C and stuck to mold A. Then the bar rests along the edge of the specimen for grating-to-specimen alignment. Then moire grating on the specimen is replicated with respect to this bar which is the reference position.
Figure 4.9 Moire Interferometry Set-Up
Figure 4.10 Applying Silicon Primer Using The Drag Method
Figure 4.11 Lowering Method to Minimize Entrapping Air Bubbles in the Adhesive.
Figure 4.12 Device For Separating Gratings.
Figure 4.13 Specimen Grating Alignment. (a) Use of The Alignment Bar. (b) Alignment Fixture (c) Alignment Bar Cemented to Mold
4.4.3 Alignment Procedure

Since the tolerance of moire-interferometry is several wavelengths of relative motion, relative small motion of virtual grating pitch (1/f) costs the loss of moire fringe pattern. Vibration control of moire-interferometry set-up is very important for good contrast of moire fringe pattern. Moire interferometry was set up on the Honeycomb vibration free optic table (Newport Research Series Plus). And this iptic table was mounted on the vibration isolation legs.

Alignment of moire-interferometry is very crucial and many fine techniques are involved in alignment. A summary of allignment techniques, which were carried out at our laboratory, is available from Reference 43.

Prior to critical allignment of the optical system, the components are set up with an angle of approximately \( \alpha = 49.5^\circ \). This angle is the angle required for \( \lambda = 633 \) nm (Helum-Neon laser) and \( f = 2400 \) lines/mm. A permanent white card with a small hole (\( \approx \) 0.1 in diameter) is positioned in plane I and a temporary white card is located in plane II (see Figure 4.14). Observing plane I while adjusting the angle of mirror A, two bright dots will appear on plane I. The mirrors are adjusted until the two dots superimpose to form one dot that is located near the aperture (hole in white card). This means that mirror A is perpendicular to the plane of the specimen grating. Next the two bright dots on plane II appear as illustrated in Figure 4.15. Axes x' and y' are parallel to the x and y axes defined in Figure 4.14. The specimen is rotated in its plane, together with the loading fixture, until the dots lie on the x' and y"
axis. This adjustment moves the dots, vertically in opposite directions, and makes the lines of the specimen grating parallel to those of the virtual reference grating. Next, the angle is adjusted until the two dots merge and become one. Small adjustments can be made by translating the parabolic mirror perpendicular to its axis, or by rotating it about a central axis parallel to the y axis. If a collimating lens is used, a can be adjusted by translating the lens perpendicular to its optical axis. The card from plane II is removed and the lateral position of the camera lens and the camera back are adjusted for concentricity with the beam reflected from the specimen. The position is selected for the desired magnification and critical focus of the specimen plane on the film.

The initial or no-load fringe pattern will be a true null field only if the specimen grating is perfect and the virtual grating is in perfect alignment with the specimen grating. In general, a small number of fringes is observed across the field. These will be negligible compared to load-induced fringes in most cases. Otherwise, the fringe order of the initial field should be subtracted from that of the final loaded field.
Figure 4.14  Moire Optical System For Laser Beam Collimation Using Mirrors
Figure 4.15  Two Bright Dots on Plane II Are Brought Together By Interferometer Adjustment.
4.5 Sectioning Method

Moire grating was replicated on the surface of the specimen after welding and a Buehler low speed cutting machine was used for sectioning. Leco VC-50 cutting oil was used as coolant during sectioning. Susceptibility of polycarbonate to the cutting oil was tested. Polycarbonate was soaked in the cutting oil for 72 hours and the weights before and after soaking were compared. A fixture to hold the weld specimen to the Buehler low speed cutting machine was designed and used. To eliminate the machining induced residual stress during cutting, the cutting speed was maintained below level 4 of the cutting machine. Stress-free specimen was tested to observe whether this cutting procedure generate any machining-induced stress. After sectioning the specimen, cleaning the grating surface of specimen is very important to obtain a good fringe pattern. Poor illumination due to improper cleaning results in very poor contrast of fringe pattern. After sectioning, the specimen was soaked in a bath of clean coolant again for 10 minutes, which removes most of the polycarbonate dusts from the surface. Then the specimen is positioned vertically and any remaining dust is blown off by an air gun (Dust Off made by Falcon). Then scotch tape (3M Magic tape 810) was used to clean off remaining dusts and coolant oil from the surface. This work needs to be done very carefully to avoid peeling the moire grating from the specimen surface. Scotch tape was applied to the specimen section by section and removed very carefully and slowly (See Figure 4.16). The moire interferometry optical system was aligned for the undeformed grating and a set of null field fringe patterns
were recorded for reference. Then the sectioned specimen was positioned in the aligned moire interferometry optic system and the deformed displacement field was measured.
Figure 4.15 Removal of Coolant Oil Stains With Scotch Tape.
4.6 Heat Treatment for Residual Stress Relieving

The welded polycarbonate plates were placed in oven with temperature control and a thermocouple was used to measure the temperature during heat treatment.

Polycarbonate weld was annealed above (170 °C) and below (100 °C) the glass transition temperature ($T_g = 150 \, \text{oC}$). When annealing was performed above the glass transition temperature, careful attention was paid to supporting the specimen because above the glass transition temperature, plastics get very flexible and soft. For the implant-resistance-welded polycarbonate specimen, glass substrates and mold release (Frekote 34 made by Dexter corporation) were used (see Figure 4.17) to support the specimen. The mold release was sprayed on the glass substrate and was dried for the easy separation of the specimen after annealing.
Figure 4.17 Fixture Used to Support The Polycarbonate During Heat Treatment At Temperatures Above the Glass Transition Temperature.
CHAPTER V
RESULTS AND DISCUSSION

The heat flow in PC during implant resistance welding is a dominating factor for formation of thermal and residual stress. Therefore, measurement and prediction of temperature is very important for sound prediction of thermal and residual stresses. Heat flow analysis was performed with both FDM and FEM and those predictions are compared with experimental data.

Thermal and residual stress predictions incorporated with heat flow analysis were performed using both multi-bar analogy and FEM. Multi-bar analogy prediction of residual stress was compared with FEM prediction. FEM predictions of residual stress were compared with photoelasticity measurements and the sectioning method measurement utilizing moire interferometry. FEM prediction of residual strains were compared with moire-interferometry measurement.

5.1 Heat Flow Prediction During Implant Resistance Welding

The temperature profile across a PC weld during implant resistance welding was measured using Inframetric model 525 infrared temperature sensor. Infrared measurement revealed that temperature variation in the direction parallel to the weld line is negligible except near the edge of the
sample (see Figure 5.1). At the edge of the specimen the temperature drops due to convective losses to the air. Therefore, the temperature variation in the direction parallel to the weld line could be assumed to be uniform (zones of variations at the edges are very small). This uniformity in the direction parallel to the weld line provides the justification for 1-D modeling analysis using FDM. For FEM and FDM, the measured voltage and current were used to calculate the heat generation rate in the mesh. Without a calibration curve for emissivity of PC in the range of temperatures experienced during welding, the infrared temperature sensor cannot provide an absolute value for temperature. To measure the actual values of temperatures, K-type thermocouple wires were embedded in the sample and data was collected through a Hewlett Packard 3497A data acquisition system. Figure 5.2 shows a comparison between experimental measurements and FEM predictions. The heating time was 180 seconds with a current of 12.6 ampere and voltage of 0.57 volt which resulted in a heat flux of 22300 Watt/m². The points nearest to the weld line was heated fastest and they show good agreement with FEM predictions. For the points at 10 mm and 15 mm from the weld line, the deviation between FEM and the measurement after the maximum temperature is greater. Both show the same behavior the FEM predictions are higher than experimental measurements, possibly due to variation in the thermal properties with temperature, neglecting convective losses on the surface in FEM heat flow analysis, and contact resistance between the mesh and PC and between the thermocouples and
Figure 5.1 Infrared Measurement Of Temperature Distribution in the Direction Parallel With the Weld Line.
Figure 5.2. Comparison Between FEM Prediction Of Heat Flow And Temperature Measurement (Marked Line Is Experimental Data And Solid Line Is FEM Prediction)
the plastic. Temperature histories at the same points were also predicted by 1-D FDM analysis. Figure 5.3 shows good agreement between predictions of 1-D FDM and 2-D FEM. This good agreement provides assurance in the assumption of uniform temperature in the direction parallel to the weld line. FDM and FEM heat flow analysis were connected to multi-bar analogy analysis and FEM stress analysis respectively.

5.2 Mechanical Characterization of PC

Characterization of viscoelastic mechanical behavior of PC was done using a generalized Maxwell model with thirteen elements. To determine the constants in the model, the collocation method and Gauss-Siedal elimination method were used. Collocation method is a numerical method which works very effectively for curve fitting of experimental measurement of stress relaxation modulus and creep compliance. If experimental data of viscoelastic stress relaxation or creep compliance is available, the collocation method can be used for the characterization of viscoelastic mechanical behavior using either the generalized Maxwell model or the generalized Kelvin model. The results of the collocation method provides the full expansion of Prony series to describe the stress relaxation modulus. To obtain better accuracy with respect to experimental data, larger number of collocation points need to be put in the rapid transition regions of stress relaxation or creep compliance. Increasing the number of collocation points increases the agreement of collocated curve with experimental data. Figure 5.4 shows a good
Figure 5.3 Temperature Prediction Comparison Between FDM And FEM.
agreement between experimental data [12,15] and collocated curve. Thirteen points were selected as collocated points. To have good agreement between collocated data and experimental data, an appropriate set of relaxation times and collocation points are selected. If the relaxation times and the collocation time are not selected properly, the collocated curve will deviate considerably from the experimental data between the collocation points. This appropriate set of collocation points can be only determined by trial and error.

The experimental data to describe the viscoelastic mechanical behavior of PC was not available from the literature and was difficult to determine experimentally. RDA-700 machine was used in an attempt to determine the viscoelastic mechanical behavior of PC such as stress relaxation modulus as a function of time and temperature, but it was not successful. When the oven temperature was above the glass transition temperature, the PC specimen deformed too easily for the RDA-700 machine to capture any data for stress relaxation. When the oven temperature was below the glass transition temperature, the RDA-700 machine was not sensitive enough to detect the stress relaxation behavior of PC. Assuming the shape of the stress relaxation curve for most viscoelastic polymers is similar, experimental data for the stress relaxation modulus curve of PMMA [12,15,78,79] was used and the upper plateau of stress relaxation curve was modified to agree with that of PC (the upper plateau of stress relaxation curve is the value of tensile
Figure 5.4. Comparison Between Collocated Curve And Woo’s Experimental Data.
modulus at room temperature.). Since PC is an amorphous polymer, stress relaxation effects below the glass transition temperature were neglected.

5.3 Residual Stress Measurement and Prediction In The Weld

5.3.1 Photoelasticity Measurement of Residual Stress

As discussed previously, the stress optic coefficient of PC was determined by a calibration procedure. Photoelastic fringe variation and external loading were recorded. As discussed in section 4.3, the slope of the straight line (see Figure 5.5) is the stress optic coefficient of PC, \( f_0 = 37.4 \text{ lb/in} \). This value of the stress optic coefficient was measured at room temperature (20 °C) using a tensile test based on ASTM standard D638-86. This value of stress optic coefficient is only valid for room temperature and it is assumed that stress relaxation in PC at temperature near room temperature is negligible. This value of stress optic coefficient was used to measure the residual stress after welding of PC. For the measurement of residual stress, the PC weld was placed in the circular polariscope with dark fringe format. As discussed earlier, the photoelastic fringes in circular polariscope give the principal stress difference. Thus the principal residual stress difference \( \Delta \sigma_{\text{res}} ( = \sigma_{\text{res} 1} - \sigma_{\text{res} 2} ) \) in the middle of specimen could be measured using photoelasticity. For comparison, therefore, \( \Delta \sigma_{\text{res}} \) in the middle of PC was predicted with FEM. As shown in Figure 5.6, the highest and most concentrated level of residual stress developed in the regions adjacent
Figure 5.5 Calibration Curve For The Determination Of Stress Optic Coefficient Of Polycarbonate.
Figure 5.6 Photoelastic Fringe Pattern Of Residual Stress In a Polycarbonate Weld (Welding Time Is 6 Minutes).
to the weld line and lower level of residual stress developed in the regions away from the weld line. As shown in Figures 5.7 and 5.8, photoelasticity measurement and FEM prediction are in very good agreement with each other. Figures 5.7 and 5.8 also show that despite the assumed stress relaxation modulus of PC, the FEM prediction of residual stress formation in PC were in good agreement with the photoelastic measurement. Therefore, it was observed that photoelasticity can be used to measure residual stress in the thermoplastic welding, assuming the specimen is transparent. Since FEM prediction of residual stress was in good agreement with photoelasticity measurement, FEM modeling of implant resistance welding was successful. Hence this approach could be used for prediction of residual stress in other thermoplastic welding processes with minor modification in FEM modeling.

5.3.2 Moire interferometry measurement of residual stress using sectioning method

FEM simulation of sectioning was performed to observe the feasibility of this method. Direction and location of cutting to produce a free surface, which leads to the release of residual stress adjacent to the cutting line, are shown in Figure 5.9. Boundary and initial conditions for FEM simulation of the sectioning method are shown in Figure 5.10. For the FEM simulation of sectioning method, implant resistance welding was simulated first and accordingly an FEM model of polycarbonate with
Figure 5.7 Comparison Between Photoelasticity Measurement And FEM Prediction Of Residual Stress Induced In Polycarbonate Welds (Heating Time Is 6 Minutes.).
Figure 5.8  Comparison Between Photoelasticity Measurement And FEM Prediction Of Residual Stress Induce In Polycarbonate Weld. (Heating Time Is 7 Minutes.)
Figure 5.9 Direction And Location Of Cutting For FEM Simulation Of Sectioning Method.
Y-displacement is fixed to zero at this node.

These elements are removed for sectioning.

Figure 5.10 Boundary And Initial Conditions For FEM Simulation Of The Sectioning Method.
residual stress was generated. Like in the experiment, FEM modeling was done by removing the experiments adjacent to the weld. The y-displacement at the top left corner (see Figure 5.10) was fixed to avoid rigid body motion. By doing so, the sectioning II (see Figure 5.9) process was simulated to observe how the residual stresses are redistributed after sectioning. As shown in Figure 5.11, residual stress, which is perpendicular to and adjacent to the sectioning line, is released completely after sectioning. Thus by measuring the strain change adjacent to the sectioning line utilizing moire interferometry, the residual stress along the sectioning line can be determined. Figure 5.12 shows that the residual stress parallel to the sectioning line changes only slightly despite sectioning. Hence from FEM simulation of sectioning method, it was observed that sectioning needs to be done at the specific location perpendicular to the direction of the measured residual stress.

To measure the residual stress using sectioning, the cutting should not generate any machining-induced stress. For soft materials like plastics, low speed cutting is recommended [78] to eliminate machining-induced stress. A stress-free specimen, as determined by photoelasticity, was cut using Buehler low-speed diamond cutter and it developed no moire and photoelastic fringe patterns as a result of cutting. Thus, it was determined that a Buehler low-speed diamond cutter is suitable for the sectioning method.

Cleaning the surface of the specimen, where the moire grating was replicated before cutting and which was covered with the coolant oil
Sectioning position

- Original residual stress
- Releaved residual stress

This value is measured by moire interferometry.

Figure 5.11 $\sigma_y$ Distribution After Sectioning (FEM Prediction).
This value is measured by moire interferometry.

Figure 5.12 $\sigma_x$ Distribution After Sectioning (FEM Prediction).
during cutting, was very difficult. If cleaning was not done properly, the moire fringe pattern from sectioned specimen could not be observed due to poor reflection of the laser light from the surface. Several approaches were used for cleaning the specimen. 1,1,1 Trichloroethane made by Fisher Scientific is a good degreaser to clean the surface. 1,1,1 Trichloroethane was applied on the specimen several times until the specimen surface was clean enough to have a good moire grating. However, 1,1,1 Trichloroethane was so strong that it softened the polycarbonate specimen which resulted in release of residual stress. Therefore instead of using 1,1,1 Trichloroethane as a degreaser, the cleaning method discussed in section 4.5 was used and it worked well.

Figures 5.13 and 5.14 show the moire fringe pattern of polycarbonate welds (heating time is 7 minutes) after sectioning I (See Figure 5.9). Both figures show that higher level of residual stress was released near the weld line where highest levels of tensile residual stress developed. Figure 5.13 shows that due to higher residual stress release in x direction ($\sigma_x$), larger release of entrapped elastic strains in x direction ($\varepsilon_x$) were observed at regions near to the weld line. This is because major variation of $\sigma_x$ is mainly in the direction of y direction (see Figure 5.15.a). In Figure 5.14, despite sectioning I, release of higher entrapped residual strains ($\varepsilon_y$) in y direction was also observed. This is because unlike $\sigma_x$, $\sigma_y$ varies in both x and y directions (see Figure 5.15.b). Figures 5.15, and 5.16 show moire fringe pattern of polycarbonate weld (heating time is 7 minutes) after sectioning II (See Figure 5.9). Both figures show that higher level of residual stress was released near the
weld line where higher level of tensile residual stress developed. In the case of section II, \( \varepsilon_x \) was not fully released because major variation of \( \sigma_x \) is in the direction of y direction (see Figure 5.15.a). However, \( \varepsilon_y \) was released very well because \( \sigma_y \) cannot be supported along the sectioning line after sectioning II. From these moire fringe patterns, the residual stresses along the sectioning lines I (\( \sigma_x \)) and the sectioning line II (\( \sigma_y \)) were determined using Equations 67 to 71. Figures 5.18 and 5.19 show comparisons between moire interferometry measurements using sectioning method and FEM predictions of residual stress for a welding time was 7 minutes. Figures 5.20 and 5.21 show comparison between moire interferometry measurements using the section method and FEM predictions when welding time was 6 minutes. These four figures show that this moire interferometry using sectioning methods worked very well for the measurement of residual stress in the weld. Figures 5.18 and 5.20 shows that high tensile residual stress developed in the region adjacent to the weld line and compressive residual stress developed in the region away from the weld line. Figure 5.20 shows that region of tensile residual stress in x-direction appeared at \( Y/L = 0.1 \) but Figure 5.18 shows that the region of tensile residual stress did not appear down to \( Y/L = 0.05 \). Thus these two figure shows that as heat input to the weld increases, the higher tensile residual stress in the direction parallel to the weld line is generated and the larger is the tensile region of residual stress. Figures 5.19 and 5.21 show that the higher level of residual
Figure 5.13. Moire Fringe Pattern ($N_x$) After Sectioning I.
(Heating Time Is 7 Min.)
Figure 5.14. Moire Fringe Pattern ($N_y$) After Sectioning I. (Heating Time Is 7 Min..)
Figure 5.15 Typical Distribution of Residual Stress in Butt Weld [4].
Figure 5.16. Moire Fringe Pattern ($N_x$) After Sectioning II. (Heating Time Is 7 Minutes.)
Figure 5.17. Moire Fringe Pattern ($N_y$) After Sectioning II.
(Heating Time Is 7 Minutes.)
Figure 5.18 Comparison Between FEM Prediction And Moire Interferometry Measurement Of Residual Stress ($\sigma_x$).
(Heating Time Is 7 Minutes.)
Figure 5.19 Comparison Between FEM Prediction And Moire Interferometry Measurement Of Residual Stress ($\sigma_y$).
(Heating Time Is 7 Minutes.)
Figure 5.20 Comparison Between FEM Prediction And Moire Interferometry Measurement Of Residual Stress ($\sigma_x$).
(Heating Time Is 6 Minutes.)
Figure 5.21 Comparison Between FEM Prediction And Moire Interferometry Measurement Of Residual Stress ($\sigma_y$).
(Heating Time is 6 Minutes.)
stress in the direction perpendicular to the weld line is also generated as
the more heat input is applied to the weld. Figures 5.18, 19, 20, and 21
show that errors between moire interferometry measurement and FEM
prediction are about 40% or more. Reasons for these errors could
result from several simplifying assumptions in FEM modeling analysis
such as approximate approach in determining stress relaxation modulus of
polycarbonate and shifting function in describing time-temperature
shifting effect, and not considering squeeze flow effect during welding
and viscoplastic deformation during welding. However, it was identified
that moire interferometry utilizing sectioning method can be a very good
experimental approach to measure residual stress after welding. In the
case of implant resistance welding, measurement of residual stress at the
region close to the weld line could not be achieved because of weld flash
and wire mesh. To measure the residual stress at the region adjacent to
the weld line, flash needs to be grinded off to replicate moire grating.
But the remained wire mesh at the weld interface makes the grinding
process very difficult. In other welding processes, where the weld flash
can be grinded off easily, measurement of residual stress close to the
weld line can be achieved, as far as the specimen geometries have flat
areas to replicate moire grating. The procedures used for the residual
stress measurement of implant resistance welding process can be used
for other welding processes in the same manner. When the specimen
geometry is cylindrical, special devices and techniques suggested by
Boeman [79] can be used for the measurement of residual stress.
Moire interferometry can be used to metals and composites as well as plastics for the measurement of process-induced stress as well as residual stress. As far as there exists flat area on the specimen to replicate the moire grating, the sectioning method utilizing moire interferometry can be also used for the part with complex geometry for the measurement of residual stress and process-induced stress in various manufacturing processes such as sheet mold compounding (SMC), injection molding, metal forming etc..

5.3.3 Moire Interferometry Measurement of the Residual Strains After Welding

Moire interferometry was used to measure the total residual strains after welding. This work provides the first step in measuring residual strains in thermoplastic welding using moire interferometry. Moire interferometry can be used to measure the residual strains in opaque materials and it can be used even for complex geometries with small size parts. Thermally induced residual deformed displacements in x and y direction (U and V) were measured using moire interferometry and they were converted to residual strains ($\varepsilon_x$ and $\varepsilon_y$) using displacement and strain relationships (Equations 67 to 71). Figures 5.21, and 22 illustrate the moire fringe patterns of the residual deformed displacement fields, after welding when heating time is 7 minutes. In Figure 5.21, larger deformation in the direction parallel to the weld line developed at the region near the weld line, which resulted from larger viscoelastic and plastic deformation in that area. In Figure 5.22, larger deformation in the
direction perpendicular to the weld line also developed at the region near the weld line. The reasons for larger deformation at the region near the weld line in both directions are that during cooling process, viscoelastic deformation was frozen and larger plastic deformation developed due to the higher temperature gradient at the region near the weld line. As shown in Figures 5.23 and 5.24, moire interferometry measurement of residual strains $\varepsilon_x$ and $\varepsilon_y$, are in surprisingly good agreement with FEM prediction of $\varepsilon_x$ and $\varepsilon_y$ considering all of the assumptions which are incorporated into the FEM analysis. As shown in both figures, more severe deformation in x and y directions occurred at the regions adjacent to the weld line. In linear viscoelasticity analysis using FEM for residual strain prediction, flash formation due to the squeeze flow and viscoplastic deformation, and approximated approaches in determining stress relaxation modulus and shifting function were not considered. These may lead to the considerable deviation between moire interferometry measurement and FEM prediction. Nevertheless, moire interferometry is a good experimental method to determine displacements and strains. In case that sectioning method is not applicable for the measurement of process-induced stresses (for example, strain or stress measurement in very small part where sectioning cannot be used), the procedure used for residual strain measurement can be a very good experimental technique to measure strains. Moire interferometry can be a nice experimental method to identify the numerical modeling analysis predictions.
Figure 5.22  Moire Fringe Pattern Developed Due To Induced Residual Strains ($N_X$.)
Figure 5.23 Moire Fringe Pattern Developed Due To Induced Residual Strains ($N_y$.)
Figure 5.24 Comparison Between Moire Interferometry Measurement And FEM Prediction Of Residual Strains ($\varepsilon_x$) (Heating Time is 7 Minutes.)
Figure 5.25 Comparison Between Moire Interferometry Measurement and FEM Prediction of Residual Strains ($\varepsilon_y$)
(Heating Time is 7 Minutes.)
5.4 FEM and Multi-Bar Analogy Predictions for Typical Welds

FEM predictions of residual stress were in very good agreement with photoelastic measurement and with moire interferometry measurement and FEM prediction of residual stress ($\sigma_x$ and $\sigma_y$). Residual strains ($\varepsilon_x$ and $\varepsilon_y$), were in good agreement with moire-interferometry measurements. FEM modeling analysis worked very well for the prediction of residual stress formation during implant resistance welding. Figures 5.26 and 5.27 show residual stresses in $x$ and $y$ direction respectively. Since plastics are nonconductive materials, temperature gradient is very high at the region near the weld line and is low at the region away from the weld line. Hence, Figure 5.26 shows that highly concentrated residual stress formation in $x$ direction ($\sigma_x$) developed at the region adjacent to the weld line. FEM analysis predicted that the maximum level of residual stress is 14.2 Mpa which is 23 % of yield strength of polycarbonate (62.1 MPa), when heating time is 7 minutes. In this case, heating (7 minutes) and cooling (30 minutes) processes during welding were slow but still the level of residual stress is about a quarter of the yield strength of polycarbonate, which will reduce the impact and static strength of the weld. Unlike $\sigma_x$, the residual stress in the direction perpendicular to the weld line ($\sigma_y$) is relatively wide spread except at the outer edge near the weld line. Figures 5.26 and 5.27 show that major variation of $\sigma_x$ is in $y$ direction but that $\sigma_y$ is varying both in $x$ and $y$ directions. Both figures also shows that maximum level of $\sigma_y$ is lower than that of $\sigma_x$. 
As shown in Figure 5.28, residual stress developed in the direction parallel to the weld line ($\sigma_x$) is about three to ten times larger than the stress in the direction perpendicular to the weld line ($\sigma_y$). Therefore, residual stress developed in the direction parallel to the weld line is the most detrimental residual stress. As the heat input for welding increases, the residual stress in the direction perpendicular to weld line increases due to the constraint imposed in the direction perpendicular to the weld line due to the applied pressure (see Figure 1.1). Figure 5.29 shows that the maximum $\sigma_x$ is developed at the center portion of the weld. This is because a greater constraint exists at the center portion of the weld rather than at the edges.

Figures 5.30 and 5.31 show $\sigma_x$ and $\sigma_y$ formation in the middle of the weld where the maximum $\sigma_x$ is developed. Both figures show typical variation of thermal and residual stress development. At the end of heating, maximum thermal stress, which was compressive, developed at
Figure 5.26  FEM Prediction Of Residual Stress Distribution In Polycarbonate Weld (σ_x) (Heating Time is 7 Minutes.).
Figure 5.27  FEM Prediction Of Residual Stress Distribution In Polycarbonate Weld (\( \sigma_y \)) (Heating Time 7 Minutes.).
Figure 5.28 Ratio of $\sigma_x / \sigma_y$ Along the Weld Line.
Figure 5.29  $\sigma_x$ Variation In The Direction Parallel To The Weld Line
Figure 5.30  FEM Prediction Of Stress History Deveopled In The Weld (σx) (Heating time is 7 minutes.).
Figure 5.31  FEM Prediction Of Stress History Developed In The Weld ($\sigma_Y$) (Heating time was 7 minutes.).
the region near to the weld line but during cooling, level of thermal stress decreased and finally the residual stress was formed. $\sigma_x$ and $\sigma_y$ are compressive at regions adjacent to the weld line and tensile at the regions away from the weld line. After cooling, $\sigma_x$ and $\sigma_y$ are tensile near the weld line and compressive away from it. During heating, as the higher level of thermal stress develops, the higher level of residual stress will develop in the weld.

Figure 5.32 shows comparison between multi-bar analogy prediction of $\sigma_x$ and FEM prediction. Multi-bar analogy predictions are in good agreement with FEM prediction, which means multi-bar analogy modeling analysis works well for the prediction of thermal and residual stress during implant resistance welding. Therefore, multi-bar analogy analysis is a simplified but good approach to predict the dominating thermal and residual stresses during implant resistance welding. Multi-Bar analogy modeling analysis can be a good approach to predict residual stress for other welding procedures such as induction welding, infrared welding, ultrasonic welding etc. where the heat flux applied to the weld is similar to that in implant resistance welding.
Figure 5.32 Comparison Of Multi-Bar Analogy And FEM Prediction Of Residual Stress After Welding.
5.5 Residual Stress Reduction Method by Heat Treatment

To reduce the induced residual stress after welding, annealing at elevated temperature was performed. Since polycarbonate is susceptible to moisture, the specimen was dried in an oven 130 °C for 24 hours before welding. The polycarbonate weld was annealed below glass transition temperature (130 °C) for 36 hours and it was exposed to the photoelastic polariscope to observe the any changes in photoelastic fringe pattern due to residual stress change after annealing. However there was no noticeable change in photoelastic fringe pattern after annealing. For the specific conditions used for welding used for this work, annealing the weld below the glass transition temperature may take longer time than 36 hours for reduction of the residual stress in the weld. Therefore, it was found that annealing below glass transition temperature is not an effective approach to achieve reduction of the residual stress in the weld.

A polycarbonate weld was annealed above the glass transition temperature at 170 °C for 1.5 hours and photoelastic measurement showed remarkable change in the residual stress distribution (see Figure 5.33). When annealing is performed above the glass transition temperature, careful attention needs to be paid to support the specimen to avoid distortions due to softening of the polymers. The major reason for formation of the residual stress in the weld is that the relatively rapid cooling during welding procedure freezes polymeric chains into unfavored position and the bulky nature of the chains leads to difficulties in segmental alignment and results in poor packing and unoccupied volume.
in the glass state [80,81]. At the temperature above glass transition temperature, polymer becomes soft and rubbery. On application of heat and maintaining whole weld at the same temperature above glass transition temperature, movement of polymeric chains become possible and they move in more favored position. Hence, polymeric chains are easy to move to favored position and resultanty, induced-residual stress can be reduced. These annealing procedure was simulated using FEM and the predictions of redistributed residual stress after annealing were compared with photoelastic measurement and moire interferometry measurement using sectioning method. For FEM simulation of heat treatment, first the model was used to simulate welding resulting in the formation of residual stress. Then, in the model, the temperature was uniformly raised to the annealing temperature for the annealing time. Then weld was slowly cooled down to room temperature. Heat generation rate (235000 Watt/m³) for annealing during heating process of annealing in the oven was determined by comparing experimental temperature measurement and FEM heat flow analysis. Temperature variation during heating was measured using thermocouple. The elapsed time when surface temperature of the specimen reached the set-up temperature (170 °C) was also measured. Using presumed value of heat generation rate, FEM simulation for heating process was performed by trial and error until two parameters of FEM predictions (temperature variation during heating and the elapsed time for the set-up temperature) agree with those of thermocouple measurement.
Figure 5.34 shows that photoelastic measurement of residual stress is in good agreement with FEM predictions. Figure 5.35 shows that moire interferometry measurement is in good agreement with FEM prediction. Photoelasticity measurement is applicable only to transparent materials with the stress optic coefficient which is small enough to allow photoelastic fringes. Unlike photoelasticity, moire interferometry can be used most of materials as far as moire grating can be replicable on the surface of specimen.

Figure 5.36 shows that the residual stress was reduced considerably after annealing at the temperature above the glass transition temperature. Figure 5.36 shows that under the specific annealing condition (described in the Figure 5.36), the maximum tensile residual stress in the direction parallel to the weld line which is the most critical residual stress, is reduced to one third of the original residual stress. From experimental measurements and FEM analysis, it was found that annealing the plastic welds to reduce residual stress should be done at the temperature above the glass transition temperature and that this annealing technique works very well for the reduction of the residual stress. The annealing technique used here can be applied to other welding processes as well as other manufacturing products in order to reduce the residual stress.
Figure 5.33 Photoelastic Measurement of Residual Stress Distribution After Annealing at the Temperature Above the Glass Transition Temperature (170 °C). Heating time is 7 minutes. Annealing Time is 1.5 Hour.
Figure 5.34 Comparison Between Photoelastic Measurement and FEM Prediction Of Residual Stress Distribution Before and After Annealing. Heating Time is 7 Minutes.
Figure 5.35 Moire interferometry measurement and FEM prediction ($\sigma_x$). Heating Time is 7 Minutes.
Annealing Time is 1.5 Hours @ 170 °C.
Figure 5.36  FEM Prediction of the Residual Stress ($\sigma_x$) Distribution Before And After Annealing. Heating time is 7 minutes. Annealing Temperature and Time is 170 °C and 1.5 Hour.
CHAPTER VI
CONCLUSION AND RECOMMENDATION

Formation of the residual stress in welding process is unavoidable. Residual stress affects the strength of the weld under both static and dynamic loadings. Therefore, prediction and measurement of residual stress in the weld is very important. In this work, theoretical and experimental approach to predict and measure the residual stress after implant resistance welding of polycarbonate were performed. Theoretical and experimental methods used in this work can be applied other welding procedures. Heat treatment to reduce the residual stress in the weld was also performed.

Since FDM and FEM predictions of temperature field were in good agreement with experimental measurements, formulation and modeling of FDM and FEM for heat flow analysis during implant resistance welding were done properly. 1-D lumped FDM heat flow prediction was in good agreement with 2-D FEM heat flow prediction.

FEM predictions of residual stress were in good agreement with photoelasticity measurement and with moire-interferometry measurement. Residual strain after implant resistance welding was also measured using moire interferometry. FEM predictions of residual strains were in good agreement with moire interferometry measurement.
Moiré-interferometry can be used extensively for measurement of thermal and residual stress during plastic welding and thermally induced stress in other manufacturing processes. Since FEM predictions are in good agreement with experimental work, numerical procedures and formulations involved in FEM analysis seem to be done properly. Both photoelasticity and moiré interferometry methods are good experimental approaches for the measurement of residual stress after welding.

FEM prediction revealed that residual stress developed in the direction parallel (\( \sigma_x \)) is most dominating detrimental stress. Maximum residual stress developed at the center portion of weld. Therefore, \( \sigma_x \) at the center portion of weld is the most critical residual stress in the implant resistance weld. With minor changes in modeling, FEM analysis used for implant resistance welding can be used for other welding processes for prediction of thermal and residual stress in the weld.

Multi-bar analogy prediction of residual stress in the direction parallel to the weld line, which is most detrimental, was in good agreement with FEM prediction. Multi-bar analogy is a simplified and good methodology for the prediction of the most detrimental thermal and residual stress during implant resistance welding and other problem where temperature profile is similar with the implant resistance welding procedure.

Annealing the specimen above the glass transition temperature is an effective heat treatment method to reduce the residual stress in the weld. The heat treatment method used in this work can be used in reality at the factory site.
In FEM modeling analysis for the welding procedure, effect of squeeze flow needs to be included for the better accuracy of residual stress and distortion analysis. During the welding procedure, viscoplastic deformation and nonlinearity of viscoelasticity analysis may also be considered for the better prediction of residual stress and distortion.

The techniques developed here for measurement of the thermal and residual stress during plastic welding were very successful. Now, the level of residual stresses for all welding techniques (not just implant resistance welding) can be measured, and predicted. Its effects on the joint quality can be investigated. Techniques to minimize the residual stress was developed. This project provides the methodology necessary to incorporate the effects of residual stress in the design process. This should results in better design and manufacturing of welded joints.

To increase the sensitivity of moire interferometry measurement, moire grating with higher frequency may be used.

Since theoretical and experimental approaches to measure and predict residual stress were developed, quantitative and qualitative approaches to investigate the effect of residual stress on weld strength in plastic welding can be performed. The optimal heating and cooling procedure to minimize the residual stress can be also tried to find.
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78. Personal Discussion and Recommendation With Mr. Paul D. Millard Working at Micro Measurement Group (919-365-3800)


Appendix A

F.D.M. Program For Heat Flow Analysis During Implant Resistance Welding
THIS PROGRAM IS FOR HEAT TRANSFER ANALYSIS IN THERMOPLASTIC WELDING PROCEDURE WITH F.D.M..

DIMENSION TEMP(3000,20)
OPEN (UNIT = 8, FILE = 'TEMP.DAT', STATUS = 'NEW')

DEFINING FUNDAMENTAL PARAMETERS

ALPH = 7.75E-5
DEN = 1200.
C = 1260.
AK = 0.4683 I0.195 I0.4683
DO I = 1,20
TEMP(I,1) = 20.
ENDDO
DELT = 2.
DELX = 0.005 I5.08E-3

PAR1 = AK*DELT/(DEN*C*(DELX**2.))
QT = 8000. !10364. !8592. !10364.
H = 4.4
TEM = 20.
AS = (2. + 0.25)*2*2.54E-2*DELT
AC = 2.*0.25*(2.54E-2)**2
PAR2 = AS/AC
P = AS/DELX
DO I=2,900
DO J=1,20

DETERMINING TEMP. AT NODE 1

IF(I.GT.120) THEN
QT=0.
ENDIF
IF(J.EQ.1) THEN
PARA1 = 1. - 2.*PAR1 - PAR2*(H/AK)*DELX*PAR1
IF(PARA1.LE.0.) GO TO 1000
TEMP(I,J) = QT*(2.*DELX/AK)*PAR1 + PARA1*TEMP(1,J-1)
& + 2.*PAR1*TEMP(2,J-1) + PAR2*(H/AK)*DELX*PAR1*TEM
WRITE(8,*), TEMP(I,J)
GO TO 3000
ENDIF

DETERMINING TEMP. AT END NODE

IF(J.EQ.20) THEN
PARA3 = 1. - 2.*PAR1 - H*(AS/2. + AC)/AC*DELX/AK*2.*PAR1
IF(PARA3.LE.0) GO TO 1000
TEMP(I,J) = 2.*PAR1*TEMP(19,J-1) + PARA3*TEMP(20,J-1) &
+ H*(0.5*PAR2 + 1.)*(DELX/AK)*2.0*PAR1*TEM
C WRITE(8,*),TEMP(I,J)
GO TO 3000
ENDIF
C
C DETERMINING TEMP. AT INTERMINATE NODES
C
PARA2 = 1. - 2.*PAR1 - H*P*PAR1*(DELX**2)/(AK*AC)
IF(PARA2.LE.0) GO TO 1000
TEMP(I,J) = PAR1*(TEMP(I + 1,J-1) + TEMP(I-1,J-1) ) + &
PARA2*TEMP(I,J-1) + H*P/(AK*AC)*PAR1*(DELX**2)* TEM
C WRITE(8,*),TEMP(I,J)
3000 CONTINUE
IF(J.EQ.20) THEN
TIME = 2. * I
C WRITE(8,*),TIME
ENDIF
ENDDO
ENDDO
GO TO 2000
1000 CONTINUE
WRITE(*,100)
100 FORMAT(/, '*** WARNING ; SOLN IS UNSTABLE !! ***')
2000 CONTINUE
STOP
END
Appendix B

Flow Chart And Programs Of Collocation Method

For The Characterization Of Polycarbonate
Program EQ.FOR
(Generates set of relaxation times and \( E_i \)'s for coll. Method.)

COE.DAT

Program GS.FOR
(Solve set of equations for coll. method.)

PC.DAT
(Experimental Data)

SOLDAT

Program PME.FOR
(Generate relaxation modulus master curve.)

RXE.DAT

Figure B.1 Flow Chart for Collocation Method
C THIS PROGRAM IS FOR THE DETERMINATION OF RELAXATION
C TIME (t_i) AND E_j
C FOR COLLOCATION METHOD.
C
C Program EQ.FOR
C
DIMENSION A(30,30), T(30), TL(30), PA(30), PAR(30)
OPEN(UNIT = 7, FILE = 'PMMA40.DAT', STATUS = 'OLD')
OPEN(UNIT = 9, FILE = 'PMMA40.TIM', STATUS = 'OLD')
OPEN(UNIT = 8, FILE = 'COE.DAT', STATUS = 'NEW')
PA0 = 2.63
DO I = 1, 13
READ(7, *) PA(I)
ENDDO
DO I = 1, 13
PAR(I) = PA(I) - PA0
ENDDO
DO I = 1, 13
TI(I) = 10. ** (I - 1)
ENDDO
DO I = 1, 13
T(I) = 10. ** (I - 3)
IF (I.EQ.1) THEN
T(I) = 10. ** (-4.)
ENDIF
IF (I.EQ.2) THEN
T(I) = 10. ** (-2.)
ENDIF
IF (I.EQ.13) THEN
T(I) = 10. ** 11.
ENDIF
DO I = 1, 13
READ(9, *) T(I)
T(I) = 10.0 ** T(I)
WRITE(*, *) T(I)
ENDDO
DO I = 1, 13
DO J = 1, 13
A(I, J) = EXP(-T(I)/TI(J))
WRITE(8, *) A(I, J)
ENDDO
WRITE(8, *) PAR(I)
A(I, 13 + I) = 1.
WRITE(8, *) A(I, J = 1, 13), PAR(I)
ENDDO
CLOSE (7)
CLOSE (8)
CLOSE (9)
STOP
END
PROGRAM GS.FOR

THIS PROGRAM PROVIDES THE SOLN. OF
MATRIX OF EQUATIONS USING GAUSS-SIEDEL METHOD

INTEGER FLAG
DIMENSION A(20,20), X(20)

OPEN(UNIT = 7, FILE = 'COE.DAT', STATUS = 'OLD')
OPEN(UNIT = 8, FILE = 'SOL.DAT', STATUS = 'NEW')

READ PARAMETERS

1 CONTINUE
READ(7,*) N, ITMAX, EPS
NPI = N + 1
DO I = 1, N
READ(7,*)(A(I,J), J = 1, NPI)
WRITE(7,*)(A(I,J), J = 1, NPI)
ENDDO

READ(7,*) (X(I), I = 1, N)
WRITE(7,*)(X(I), I = 1, N)

NORMALIZE THE DIAGONAL ELEMENTS

DO I = 1, N
ASTAR = A(I,I)
DO J = 1, NPI
A(I,J) = A(I,J)/ASTAR
ENDDO
ENDDO

BEGIN GAUSS-SIEDEL ITERATION

DO 9 ITER = 1, ITMAX
FLAG = 1
DO 7 I = 1, N
XSTAR = X(I)
X(I) = A(I,NPI)
7 CONTINUE

FIND NEW SOL VALUE, X(I)

DO 5 J = 1, N
IF(I.EQ.J) GO TO 5
X(I) = X(I) - A(I,J)*X(J)
5 CONTINUE
C TEST X(I) FOR CONVERGENCE
C
IF(ABS(XSTAR- X(I)).LE.EPS) GO TO 7
FLAG = 0
7 CONTINUE
IF (FLAG.NE.1) GO TO 9
C GO TO 1
9 CONTINUE
C GO TO 1
DO I = 1, N
WRITE(8, *) X(I)
ENDDO
STOP
END
PROGRAM PME.FOR

DIMENSION T(1000), TI(1000), E(1000), G(50)
OPEN (UNIT = 7, FILE = 'SOL.DAT', STATUS = 'OLD')
OPEN (UNIT = 8, FILE = 'RXE.DAT', STATUS = 'NEW')

GO = 2380.
DO I = 1, 13
   READ (7, *) GO
   TI(I) = 10. * 10. ** (I - 1)
ENDDO

DO I = 1, 15
   DO I = 1, 1000
      DELT = 0.015
      TIME = DELT * I
      T(I) = 10. ** TIME
   IF (I.EQ.1) THEN
      T(I) = 0.00001
   ENDIF
   IF (I.EQ.2) THEN
      T(I) = 0.0001
   ENDIF
   IF (I.EQ.3) THEN
      T(I) = 0.001
   ENDIF
   IF (I.EQ.4) THEN
      T(I) = 0.01
   ENDIF
   T(I) = 10. ** (I - 3)
   SUM = 0.
   DO J = 1, 13
      SUM = SUM + G(J) * EXP(-1. * T(I) / TI(J))
   ENDDO
   E(I) = 2.630 + SUM
   T(I) = LOG10(T(I))
   WRITE (8, *) T(I), E(I)
ENDDO
STOP
END
Appendix C

Program For Multi-Bar Analogy
PROGRAM TPC.FOR

THIS PROGRAM IS FOR THE ESTIMATION OF RESIDUAL STRESS 
IN THERMOPLASTIC (P/C) WELD BASED ON THE FIVE-BAR ANALOGY.

FOR THE DETERMINATION OF TEMP. FIELD

DIMENSION X(20),AL(6),TEMP(900,20),ETA(900,11), 
& THTA(900,6),LINE(11),CHAR(11),ETEMP(900,6) 
& ,XX(900,6),Y(900,1),NPTS(11), 
& ETA1(900,11),DETA1(900,11),RLXM(900,6),PM(900,6) 
& , PPM(900,6),SIGM(900,6),RTM(900,6),RT(900),EPS(900) 
& ,ST(900,1),RXSIGM(20),RSIGM(20) 
CHARACTER*7 LINE 
CHARACTER*1 CHAR 

OPEN(UNIT = 8,FILE = 'PCC.DAT',STATUS = 'NEW')

DTHCK = 0.032 
DTIME = 4.0

**********************************************************************

TEMPERATURE FIELD GENERATION WITH F.D.M.

**********************************************************************

THIS PROGRAM IS FOR HEAT TRANSFER ANALYSIS IN THERMOPLASTIC 
WELDING PROCEDURE WITH F.D.M..

DIMENSION TEMP(3000,20)
OPEN (UNIT = 8,FILE = 'TEMP.DAT',STATUS = 'NEW')

DEFINING FUNDAMENTAL PARAMETERS

ALPH = 6.85E-5 
DEN = 1200. 
C = 1260. 
AK = 0.195 !0.4683 
DO I = 1,20 
TEMP(1,I) = 20. 
ENDDO 
DELT = 4.0 
DELX = 0.005 !5.08E-3
PAR1 = AK*DELT/(DEN*C*(DELX**2))
QT = 15750.*0.47 135100. 110364. 18592. 110364.
H = 8.392
TEM = 20.
AS = (2. + 0.25)*2*2.54E-2*DELX
AC = 2.*0.25*(2.54E-2)**2
PAR2 = AS/AC
P = AS/DELX
DO I = 2,900
DO J = 1,11
C DETERMINING TEMP. AT NODE 1
C
IF(I.GT.96) THEN
QT = 0.
ENDIF
IF(J.EQ.1) THEN
PARA1 = 1. - 2.*PAR1 - PAR2*(H/AK)*DELX*PAR1
IF(PARA1.LE.0.) GO TO 10000
TEMP(I,J) = QT*(2.*DELX/AK)*PAR1 + PARA1*TEMP(I-1,1)
& + 2.*PAR1*TEMP(I-1,2) + PAR2*(H/AK)*DELX*PAR1*TEM
C WRITE(8,*)I,TEMP(I,J)
GO TO 30000
ENDIF
C DETERMINING TEMP. AT END NODE
C
IF(J.EQ.11) THEN
PARA3 = 1. - 2.*PAR1 - H*(AS/2. + AC)/AC*DELX/AK*2.*PAR1
IF(PARA3.LE.0) GO TO 10000
TEMP(I,J) = 2.*PAR1*TEMP(I-1,9) + PARA3*TEMP(I-1,10)
& + H*(0.5*PAR2 + 1.)*(DELX/AK)*2.0*PAR1*TEM
C WRITE(8,*)I,TEMP(I,J)
GO TO 30000
ENDIF
C DETERMINING TEMP. AT INTERMINATE NODES
C
PARA2 = 1. - 2.*PAR1 - H*P*PAR1*(DELX**2)/(AK*AC)
IF(PARA2.LE.0) GO TO 10000
TEMP(I,J) = PARA1*(TEMP(I-1,J + 1) + TEMP(I-1,J-1)) +
& PARA2*TEMP(I-1,J) + H*P/(AK*AC)*PAR1*(DELX**2)*
& TEM
IF(I.EQ.900) THEN
WRITE(8,*)I,TEMP(I,J)
ENDIF
30000 CONTINUE
C IF(J.EQ.20) THEN
C TIME = 2. * I
C WRITE(8,*)TIME
C ENDIF
ENDDO
ENDDO
GO TO 20000
10000 CONTINUE
WRITE(*,100)
100 FORMAT(/,'*** WARNING ; SOLN IS UNSTABLE !! ****')
20000 CONTINUE
C
C******************************************************************************
C
DO J = 1,11
ETA(1,J)=0.
ETA1(1,J)=0.
DETA1(1,J)=0.
ENDDO
C
C DETERMINATION OF EQUIVALENT TEMP.
C
DO J = 1,11
ETA(1,J)=0.
XX(1,J) = 0.
ENDDO
C
DO I = 1,900
ETEMP(I,1) = (TEMP(I,1) + TEMPO,2))/2.
ETEMP(I,2) = (TEMP(I,2) + 2*TEMP(I,3) + TEMP(I,4))/4
ETEMP(I,3) = (TEMP(I,4) + 2*TEMP(I,5) + TEMP(I,6))/4
ETEMP(I,4) = (TEMP(I,6) + 2*TEMP(I,7) + TEMP(I,8))/4
ETEMP(I,5) = (TEMP(I,8) + 2*TEMP(I,9) + TEMP(I,10))/4
ETEMP(I,6) = (TEMP(I,10) + TEMP(I,11))/2
WRITE(*,*)ETEMP(I,6)
ENDDO
C
C DETERMINATION OF REDUCED TIME OF EQUIV. TEMP.
C
DO J = 1,6
THTA(1,J)=0.
XX(1,J) = 0.
ENDDO
DO I = 2,900
C TIME = DTIME*(I-1)
THTA(I,1) = 0.5*DTIME*(SHIFT(ETEMP(I,1)) + SHIFT(ETEMP(I-1,1)))
& + THTA(I-1,1)
THTA(I,2) = 0.5*DTIME*(SHIFT(ETEMP(I-1,1)) + SHIFT(ETEMP(I,2)))
& + THTA(I-1,2)
THTA(I,3) = 0.5*DTIME*(SHIFT(ETEMP(I,3)) + SHIFT(ETEMP(I-1,3)))
& + THTA(I-1,3)
THTA(I,4) = 0.5*DTIME*(SHIFT(ETEMP(I,4)) + SHIFT(ETEMP(I-1,4)))
& + THTA(I-1,4)
THTA(I,5) = 0.5*DTIME*(SHIFT(ETEMP(I,5)) + SHIFT(ETEMP(I-1,5)))
&
THTA(I,6) = 0.5*DTIME*(SHIFT(ETEMP(I,6)) + SHIFT(ETEMP(I-1,6))
& + THTA(I-1,6)
C
XX(I,1) = LOG10(THTA(I,1))
C
XX(I,2) = LOG10(THTA(I,2))
C
XX(I,3) = LOG10(THTA(I,3))
C
XX(I,4) = LOG10(THTA(I,4))
C
XX(I,5) = LOG10(THTA(I,5))
C
XX(I,6) = LOG10(THTA(I,6))
ENDDO

DETERMINATION OF STRESS AND STRAIN WHEN I = 2

GDD = THTA(900,1) - THTA(1,1)
GD = THTA(900,1) - THTA(2,1)
CALL RLX(GDD,RM1)
CALL RLX(GD,RM2)
PM2 = (RM1 + RM2)/2.
RT2 = RM2*ALPH*(ETEMP(2,1) + ETEMP(2,2) + ETEMP(2,3) +
& ETEMP(2,4) + ETEMP(2,5) + ETEMP(2,6)) +
& (RM1 - RM2)*0.5*ALPH*(ETEMP(2,1) + ETEMP(1,D) +
ETEMP(2,2) + ETEMP(1,2) + ETEMP(2,3) + ETEMP(1,3) +
ETEMP(2,4) + ETEMP(1,4) + ETEMP(2,5) + ETEMP(1,5) +
ETEMP(2,6) + ETEMP(1,6)))
EPS(2) = RT2/(6.0*PM2)
C
WRITE(*,*)EPS(2),PM2,ALPH
EPS(1) = 0.
DO J = 1,3
IF(J.EQ.1) THEN
WRITE(7,*) 'SOLUTION (A)'
ELSE IF(J.EQ.2) THEN
WRITE(7,*) 'SOLUTION (B)'
ELSE IF(J.EQ.3) THEN
WRITE(7,*) 'SOLUTION (C)'
ENDIF
ENDDO

DO N=3,900
N = 900
DO I=3,900
XSA1 = THTA(900,1) - THTA(I,1)
CALL RLX(XSA1,RLXM(I,1))
XSA1 = THTA(900,2) - THTA(I,2)
CALL RLX(XSA1,RLXM(I,2))
XSA1 = THTA(900,3) - THTA(I,3)
CALL RLX(XSA1,RLXM(I,3))
XSA1 = THTA(900,4) - THTA(I,4)
CALL RLX(XSA1,RLXM(I,4))
XSA1 = THTA(900,5) - THTA(I,5)
CALL RLX(XSA1,RLXM(I,5))
XSA1 = THTA(900,6) - THTA(I,6)
CALL RLX(XSA1,RLXM(I,6))

C WRITE(*,*) RLXM(I,1)
ENDDO
DO I = 3,900
C MIDDLE-BAR
PM(I,1) = (RLXM(I,1) + RLXM(I-1,1))/2.
SUM = 0.0
DO J = 1, I-2
SUM = SUM + (EPS(J) + EPS(J + 1))*(RLXM(J,1) - RLXM(J + 1,1))/2.
ENDDO
PPM(I,1) = EPS(I-1)*(RLXM(I-1,1) - RLXM(I,1))/2 + SUM
C SIDE-BAR 1
PM(I,2) = (RLXM(I,2) + RLXM(I-1,2))/2.
HAP = 0.
DO K = 1, I-2
HAP = HAP + (EPS(K) + EPS(K + 1))*(RLXM(K,2) - RLXM(K + 1,2))/2.
ENDDO
PPM(I,2) = EPS(I-1)*(RLXM(I-1,2) - RLXM(I,2))/2 + HAP

C SIDE-BAR 2
PM(I,3) = (RLXM(I,3) + RLXM(I-1,3))/2.
HAP = 0.
DO K = 1, I-2
HAP = HAP + (EPS(K) + EPS(K + 1))*(RLXM(K,3) - RLXM(K + 1,3))/2.
ENDDO
PPM(I,3) = EPS(I-1)*(RLXM(I-1,3) - RLXM(I,3))/2 + HAP
C SIDE-BAR 3
PM(I,4) = (RLXM(I,4) + RLXM(I-1,4))/2.
HAP = 0.
DO K = 1, I-2
HAP = HAP + (EPS(K) + EPS(K + 1))*(RLXM(K,4) - RLXM(K + 1,4))/2.
ENDDO
PPM(I,4) = EPS(I-1)*(RLXM(I-1,4) - RLXM(I,4))/2 + HAP
C SIDE-BAR 4
PM(I,5) = (RLXM(I,5) + RLXM(I-1,5))/2.
HAP = 0.
DO K = 1, I-2
HAP = HAP + (EPS(K) + EPS(K + 1))*(RLXM(K,5) - RLXM(K + 1,5))/2.
ENDDO
PPM(I,5) = EPS(I-1)*(RLXM(I-1,5) - RLXM(I,5))/2 + HAP
C SIDE-BAR 5
PM(I,6) = (RLXM(I,6) + RLXM(I-1,6))/2.
HAP = 0.
DO K = 1, I-2
HAP = HAP + (EPS(K) + EPS(K + 1))*(RLXM(K,6) - RLXM(K + 1,6))/2.
C WRITE(*,*) PM(I,1),PM(I,2),PM(I,3)
ENDDO
PPM(I,6) = EPS(I-1)*(RLXM(I-1,6) - RLXM(I,6))/2 + HAP

DETERMINATION OF THERMAL STRESS EFFECT

MIDDLE BAR

SUM = 0.
DO L = 1, I-1
SUM = SUM + ALPH*(ETEMP(L,1) + ETEMP(L+1,1))*(RLXM(L,1) - RLXM(L+1,1))/2.
ENDDO
RTM(I,1) = RLXM(I,1)*ALPH*ETEMP(I,1) + SUM

SIDE-BAR 1
HAP = 0.
DO M = 1, I-1
HAP = HAP + ALPH*(ETEMP(M,2) + ETEMP(M+1,2))*(RLXM(M,2) - RLXM(M+1,2))/2.
ENDDO
RTM(I,2) = RLXM(I,2)*ALPH*ETEMP(I,2) + HAP

SIDE-BAR 2
HAP = 0.
DO M = 1, I-1
HAP = HAP + ALPH*(ETEMP(M,3) + ETEMP(M+1,3))*(RLXM(M,3) - RLXM(M+1,3))/2.
ENDDO
RTM(I,3) = RLXM(I,3)*ALPH*ETEMP(I,3) + HAP

SIDE-BAR 3
HAP = 0.
DO M = 1, I-1
HAP = HAP + ALPH*(ETEMP(M,4) + ETEMP(M+1,4))*(RLXM(M,4) - RLXM(M+1,4))/2.
ENDDO
RTM(I,4) = RLXM(I,4)*ALPH*ETEMP(I,4) + HAP

SIDE-BAR 4
HAP = 0.
DO M = 1, I-1
HAP = HAP + ALPH*(ETEMP(M,5) + ETEMP(M+1,5))*(RLXM(M,5) - RLXM(M+1,5))/2.
ENDDO
RTM(I,5) = RLXM(I,5)*ALPH*ETEMP(I,5) + HAP

SIDE-BAR 5
HAP = 0.
DO M = 1, I-1
HAP = HAP + ALPH*(ETEMP(M,6) + ETEMP(M+1,6))*(RLXM(M,6) - RLXM(M+1,6))/2.
RTM(I,6) = RLXM(I,6)*ALPH*ETEMP(I,6) + HAP

RT(I) = RTM(I,1) + RTM(I,2) + RTM(I,3) + RTM(I,4)
& + RTM(I,5) + RTM(I,6)

WRITE(7,*))RT(I)

EPS(I) = (RT(I) - PPM(I,1) - PPM(I,2) - PPM(I,3)
& - PPM(I,4) - PPM(I,5) - PPM(I,6))/
& (PM(I,1) + PM(I,2) + PM(I,3) + PM(I,4)
& + PM(I,5) + PM(I,6))

ST(I,1) = EPS(I)

MIDDLE BAR
SIGM(I,1) = PM(I,1)*EPS(I) + PPM(I,1) -RTM(I,1)
XX(I,1) = SIGM(I,1)

SIDE-BAR
SIGM(I,2) = PM(I,2)*EPS(I) + PPM(I,2) -RTM(I,2)
SIGM(I,3) = PM(I,3)*EPS(I) + PPM(I,3) -RTM(I,3)
SIGM(I,4) = PM(I,4)*EPS(I) + PPM(I,4) -RTM(I,4)
SIGM(I,5) = PM(I,5)*EPS(I) + PPM(I,5) -RTM(I,5)
SIGM(I,6) = PM(I,6)*EPS(I) + PPM(I,6) -RTM(I,6)
XX(I,2) = SIGM(I,2)
XX(I,3) = SIGM(I,3)
XX(I,4) = SIGM(I,4)
XX(I,5) = SIGM(I,5)
XX(I,6) = SIGM(I,6)

DETERMINATION OF RESIDUAL STRESS

DETERMINATION OF ELASTIC STRAIN & STRESS

PA = 0.0
CALL RLX(PA,RM)
ELEPS = RM * ALPH * (ETEMP(655,1) + ETEMP(655,2)
& + ETEMP(655,3) + ETEMP(655,4) + ETEMP(655,5)
& + ETEMP(655,6) ) / (6.0*RM)

DO I = 1,6
RXSIGM(I) = RM* (ELEPS - ALPH*ETEMP(655,I))
RSIGM(I) = SIGM(655,I) - RXSIGM(I)
WRITE(8,*)) i,RSIGM(I)
ENDDO
CLOSE(UNIT = 8)
STOP
SUBROUTINE RLX(X,Y)

DIMENSION T(20)
REAL II
DO I=1,13
II = I-1
T(I) = 10**II
ENDDO

Y = 4.5 -11.57*EXP(-X/T(1)) + 88.02*EXP(-X/T(2)) + 76.98*EXP(-X/T(3)) + 241.95*EXP(-X/T(4)) + 228.12*EXP(-X/T(5)) + 220.89*EXP(-X/T(6)) + 256.36*EXP(-X/T(7)) + 290.24*EXP(-X/T(8)) + 457.66*EXP(-X/T(9)) + 325.64*EXP(-X/T(10)) + 147.13*EXP(-X/T(11)) + 38.90*EXP(-X/T(12)) + 15.18*EXP(-X/T(13))
Y = Y*100.
RETURN
END

FUNCTION SHIFT(VAL)
IF(VAL.LT.150) THEN
SHIFT = 1.
GO TO 1000
ENDIF

PARA = 17.44*(VAL - 150.)/(51.6 + (VAL - 150.))
SHIFT = 10.**PARA
1000 CONTINUE
RETURN
END
Appendix D

List Of Color Sequence In Photoelasticity Measurement
Table D.1 Sequence of Colors produced in a Dark-Field Polariscope With White Light [8]

<table>
<thead>
<tr>
<th>Color</th>
<th>Retardation, nm</th>
<th>Fringe order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Gray</td>
<td>160</td>
<td>0.28</td>
</tr>
<tr>
<td>White</td>
<td>260</td>
<td>0.45</td>
</tr>
<tr>
<td>Yellow</td>
<td>350</td>
<td>0.60</td>
</tr>
<tr>
<td>Orange</td>
<td>460</td>
<td>0.79</td>
</tr>
<tr>
<td>Red</td>
<td>520</td>
<td>0.90</td>
</tr>
<tr>
<td>Tint of passage no. 1†</td>
<td>577</td>
<td>1.00</td>
</tr>
<tr>
<td>Blue</td>
<td>620</td>
<td>1.06</td>
</tr>
<tr>
<td>Blue-green</td>
<td>700</td>
<td>1.20</td>
</tr>
<tr>
<td>Green-yellow</td>
<td>800</td>
<td>1.38</td>
</tr>
<tr>
<td>Orange</td>
<td>940</td>
<td>1.62</td>
</tr>
<tr>
<td>Red</td>
<td>1,050</td>
<td>1.81</td>
</tr>
<tr>
<td>Tint of passage no. 2†</td>
<td>1,150</td>
<td>2.00</td>
</tr>
<tr>
<td>Green</td>
<td>1,350</td>
<td>2.33</td>
</tr>
<tr>
<td>Green-yellow</td>
<td>1,450</td>
<td>2.50</td>
</tr>
<tr>
<td>Pink</td>
<td>1,550</td>
<td>2.67</td>
</tr>
<tr>
<td>Tint of passage no. 3†</td>
<td>1,730</td>
<td>3.00</td>
</tr>
<tr>
<td>Green</td>
<td>1,800</td>
<td>3.10</td>
</tr>
<tr>
<td>Pink</td>
<td>2,100</td>
<td>3.60</td>
</tr>
<tr>
<td>Tint of passage no. 4†</td>
<td>2,300</td>
<td>4.00</td>
</tr>
<tr>
<td>Green</td>
<td>2,400</td>
<td>4.13</td>
</tr>
</tbody>
</table>

† The tint of passage is a sharp dividing zone between red and blue in the first-order fringe, red and green in the second-order fringe, and pink and green in the third-, fourth-, and fifth-order fringes. Beyond five fringes, white-light analysis is not practical since the colors become very pale and difficult to distinguish.