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Periodic moment method solutions for scattering from arrays of lossy dielectric bodies

Yang, Chang-Fa, Ph.D.
The Ohio State University, 1992
PERIODIC MOMENT METHOD SOLUTIONS
FOR SCATTERING FROM
ARRAYS OF LOSSY DIELECTRIC BODIES

A Dissertation
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of the Ohio State University

by

Chang-Fa Yang

* * * * *

The Ohio State University
1992
DEDICATION

To Jesus Christ my Lord.
Also, to my parents, my wife Guey-Ling, and my daughter Jennifer.
ACKNOWLEDGEMENTS

I would like to thank my advisor, Professor W. D. Burnside, and also Professor R. C. Rudduck for their guidance in this research and for their kindness. I would also like to express my special appreciation to Professor B. A. Munk for his kindness and many helpful discussions, to Professor E. H. Newman for giving me his MM codes, to Dr. H. Pues and Mr. C. M. Robinson of Emerson & Cuming, Inc. (a GRACE company) for furnishing the absorber materials, and the Ohio Supercomputer Center for providing the computer resources. I wish to thank my wife and daughter, and also my parents and family for their love and supports.
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CHAPTER I
Introduction

Wedge and pyramidal microwave absorber have been extensively used for many years to line indoor measurement facilities. To design and arrange absorber properly, several major research efforts have been undertaken to investigate the scattering properties of absorber. Heuristic UTD solutions were applied by Dewitt and Burnside [1][2] to analyze the edge and tip scattering from homogeneous lossy dielectric wedges and pyramids. The calculated and measured results from their research were then employed to determine the absorber placement for compact range applications [3]. Later, Joseph [4] developed a similar solution for the scattered fields from pyramidal absorber tips, and then used this solution to study the OSU/ESL compact range chamber. Also, several multilayer wedges were designed, fabricated, and measured by Munk and Burnside [5].

To enhance measurements at low signal levels and/or reduce the size of the compact range chamber, absorber with good scattering performance is required. Such absorber can be realized by introducing appropriate multiple layers or specially contoured surfaces (curved or serrated) to obtain smoother impedance transitions between air and absorber. The complex geometry of the absorber led to the use of Moment Method (MM); however, it could only be used to analyze a finite absorber section. Since the truncated ends of a finite absorber panel may produce a scattering so strong that the reflection performance of a typical wall of absorber cannot be determined from MM solutions, another approach had to be found. Because an
infinite wall of identical dielectric bodies better represents an absorber wall, Periodic Moment Method (PMM) solutions are developed as part of this study.

In the past, the scattering from a doubly-periodic array of rectangular metal plates was investigated by Ott, Kouyoumjian, and Peters [6]. More general formulations were developed by Chen [7] for scattering from a doubly-periodic array of arbitrary metal plates along any two coordinates. Cwik and Mittra [8] had attacked the problem of an infinite array of free-standing arbitrarily shaped perfectly conductive or resistive patches. A spectral formulation was derived by Munk, Burrell, and Kornbau [9] [10] to analyze arrays of piecewise linear wires in a stratified medium. A PMM program based on the above theory was developed by Henderson [11] to solve doubly-periodic arrays of arbitrary shaped wires or slots in a lossy stratified dielectric medium. The problem of scattering from a periodic array of dielectric rectangular cylinders was studied by Bertoni, Cheo, and Tamir [12]. The fields in the dielectric cylinders were expand by the modes in infinite parallel layers with the same dielectric constants and thicknesses as those of the rectangles. Recently, PMM was applied to a singly-periodic array of lossy strip structures and structured slabs with double periodicity by Jorgenson and Mittra [13],[14]. A similar spectral domain formulation as that derived in [9] was used to calculate a free space periodic Green's function.

In this dissertation, PMM solutions for scattering from singly and doubly periodic arrays of inhomogeneous lossy dielectric bodies are developed. The spectral domain formulation of the fields from periodic arrays of sources [9] [10] and the MM volume formulation [15] [16] are applied to obtain the expressions for scattering from the periodic arrays of lossy dielectric bodies. PMM is an ideal tool for analyzing and designing microwave wedge and pyramidal absorber since accurate solutions for the reflection and transmission properties of an infinite absorber wall can be obtained.
for a wide variety of geometries and material properties. The required height, shape, and loading of the absorber are first evaluated using the reflection coefficient; then, the loss in one pass through the absorber wall is examined from the transmission coefficient.

A side view of the OSU/ESL anechoic chamber is shown in Figure 1. As discussed in [3], pyramidal absorber is used in the backwall sector since it has better specular reflection performance for near normal incidence. On the other hand, wedge absorber excites a low (ideally zero) backscatter for an incident wave propagating obliquely along the wedge axis, and therefore the wedges are used in the target sector. However, the specularly reflected waves shown in the figure may cause a serious problem in the measurements. Thus, one of the purposes of this study is to design absorber that can provide both the advantages of wedge and pyramidal absorber.
In Chapter II, the PMM solution for a TM polarized plane wave incident upon a singly-periodic array of two-dimensional dielectric bodies is discussed. Then, Chapter III derives the expressions for the scattering from a doubly-periodic array of three-dimensional dielectric objects illuminated by a plane wave with an arbitrary incident angle and polarization. To have an accurate modelling of the absorber wall, the dielectric constants of the absorber materials should be determined too. In Chapter IV, an approach for extracting the material parameters is described, and some measured results are shown. Then, Chapter V presents numerous wedge and pyramidal absorber designs. Calculated results from PMM are compared with measured responses to verify the PMM solutions. As will be shown, very good agreement between calculations and measurements has been obtained. A summary of this dissertation is given in Chapter VI. Lastly, a two-dimensional periodic Green’s function is derived in Appendix A, and closed form integrations for the two and three dimensional cases are shown in Appendix B and C, respectively.
CHAPTER II

A Periodic Moment Method Solution for TM Scattering from a Singly-Periodic Array of Lossy Dielectric Bodies

2.1 Introduction

In this chapter, a PMM solution for TM scattering from a singly-periodic array of lossy dielectric bodies is developed for studying wedge absorber. First, the method of moments is reviewed. Then, the volume equivalence theorem and a singly-periodic Green's function are described. Next, periodic moment method is introduced, and a hybrid approach for designing the absorber is discussed. Lastly, a method for evaluating a single infinite sum and some plots showing the convergence of the sum are presented.

2.2 Basic Moment Method Formulation

The Moment Method is a numerical technique for solving a linear operator equation by transforming it into a system of simultaneous algebraic equations. Many linear problems of engineering and scientific interest can be formulated into a linear operator equation given by

\[ \mathcal{L}y(\bar{x}) = P(\bar{x}), \bar{x} \in \mathcal{R} \tag{2.1} \]

where \( P(\bar{x}) \) is the known excitation function, \( \mathcal{L} \) is a linear (differential or integral) operator, \( y(\bar{x}) \) is the unknown response, and \( \mathcal{R} \) is the domain of the problem. To approximate the unknown response, a finite set of basis functions is chosen to expand
\[ y(\bar{x}) \text{; i.e.,} \]
\[ y(\bar{x}) = \sum_{n=1}^{N} I_n f_n(\bar{x}) \]  
(2.2)

where \( f_n(\bar{x}), \ n = 1, 2, \ldots, N \) are the known basis functions, and \( I_n, \ n = 1, 2, \ldots, N \) are unknown coefficients. Substituting the above equation into the linear operator equation and noting the approximation, one obtains
\[ \sum_{n=1}^{N} I_n F_n(\bar{x}) - p(\bar{x}) = \epsilon(\bar{x}; I_1, I_2, \ldots, I_N) \]
(2.3)

where \( F_n(\bar{x}) = Lf_n(\bar{x}) \), the result of the linear operator operating on the chosen basis function \( f_n(\bar{x}) \), and \( \epsilon(\bar{x}; I_1, I_2, \ldots, I_N) \) are error functions due to the finite nature of the summation.

To transform Equation (2.3) into \( N \) simultaneous algebraic equations, a set of \( N \) test functions \( \Psi_m(\bar{x}) \) are chosen to enforce \( N \) weighted averages of the linear operator equation by setting
\[ \int_{\mathcal{R}} \Psi_m(\bar{x}) \epsilon(\bar{x}; I_1, I_2, \ldots, I_N) d\bar{x} = 0, \ m = 1, 2, \ldots, N. \]
(2.4)

Then, Equation (2.3) becomes
\[ \sum_{n=1}^{N} I_n \int_{\mathcal{R}} \Psi_m(\bar{x}) F_n(\bar{x}) d\bar{x} = \int_{\mathcal{R}} \Psi_m(\bar{x}) p(\bar{x}) d\bar{x}, \ m = 1, 2, \ldots, N \]  
(2.5)

and in matrix form
\[ [Z][I] = [V]. \]  
(2.6)

Note that
\[ [I] = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \text{ an unknown coefficient vector} \]  
(2.7)
\[ [V] = \begin{bmatrix}
\int_R \psi_1(x)p(x)dx \\
\int_R \psi_2(x)p(x)dx \\
\vdots \\
\int_R \psi_N(x)p(x)dx
\end{bmatrix}, \text{ a known excitation vector} \quad (2.8) \]

and

\[ [Z] = \begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1N} \\
Z_{21} & Z_{22} & \cdots & Z_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{N1} & Z_{N2} & \cdots & Z_{NN}
\end{bmatrix}, \text{ a square impedance matrix} \quad (2.9) \]

in which

\[ Z_{mn} = \int_R \psi_m(x)f_n(x)dx. \quad (2.10) \]

The unknown coefficient vector, \([I]\), is then equal to

\[ [I] = [Z]^{-1}[V] \quad (2.11) \]

which can be used to determine the response function.

### 2.3 Scattered Electric Fields from a Singly-Periodic Array of Lossy Dielectric Bodies

In Figure 2, a time harmonic \((e^{j\omega t})\) TM polarized plane wave is incident upon a singly-periodic array of identical lossy dielectric wedges in free space. The scattering performance of the wedge absorber can be determined from the reflected and transmitted plane waves such as those shown in the previous figure. The equations for calculating the scattered fields from the singly-periodic array of lossy dielectric bodies are derived in this section. First, the volume equivalence theorem is reviewed. Then, a periodic Green's function in plane wave expansion is described.
Figure 2: A plane wave incident on an infinite array of identical wedges which excites a specular reflected plane wave, a specular transmitted plane wave, and some possible grating lobes.
2.3.1 The Volume Equivalence Theorem

From the volume equivalence theorem [17], materials in free space can be substituted with free space and equivalent volume currents (scattered currents). In Figure 3(a), electric and magnetic impressed sources radiate in the presence of inhomogeneous bodies. From Maxwell's equations, the E and H fields for the original problem are solutions to the following coupled vector equations:

\[
\begin{aligned}
\nabla \times \mathbf{H} &= j\omega \varepsilon \mathbf{E} + \mathbf{J}^i, \text{ and} \\
\nabla \times \mathbf{E} &= -j\omega \mu \mathbf{H} - \mathbf{M}^i
\end{aligned}
\]

which can be written as

\[
\begin{aligned}
\nabla \times \mathbf{H} &= j\omega \varepsilon_o \mathbf{E} + j\omega (\varepsilon - \varepsilon_o) \mathbf{E} + \mathbf{J}^i \\
&\equiv j\omega \varepsilon_o \mathbf{E} + \mathbf{J}^s + \mathbf{J}^i, \text{ and} \\
\nabla \times \mathbf{E} &= -j\omega \mu_o \mathbf{H} - j\omega (\mu - \mu_o) \mathbf{H} - \mathbf{M}^i \\
&\equiv -j\omega \mu_o \mathbf{H} - \mathbf{M}^s - \mathbf{M}^i
\end{aligned}
\]
where \( \varepsilon = \varepsilon_1 \) and \( \mu = \mu_1 \) in \( V \), the materials, and \( \varepsilon = \varepsilon_0 \) and \( \mu = \mu_0 \) in free space. These two coupled vector equations describe the equivalent problem shown in Figure 3(b). Therefore, the equivalent volume currents replacing the inhomogeneous bodies are given by

\[
J^s = j\omega(\varepsilon_1 - \varepsilon_0)E
\]

and

\[
M^s = j\omega(\mu_1 - \mu_0)H
\]

where \( E \) and \( H \) are the total electric and magnetic fields inside the materials. In the equivalent problem, the impressed sources radiate in free space and produce incident fields, \( E^i \) and \( H^i \); the scattered currents, \( J^s \) and \( M^s \), also radiate in free space and excite scattered fields, \( E^s \) and \( H^s \). Since the total fields in the presence of the materials are the superposition of the incident and scattered fields, one has

\[
E = E^i + E^s
\]

and

\[
H = H^i + H^s.
\]

Substituting Equation (2.14) into Equation (2.16) and Equation (2.15) into Equation (2.17), one obtains

\[
- E^s + \frac{J^s}{j\omega(\varepsilon_1 - \varepsilon_0)} = E^i
\]

and

\[
- H^s + \frac{M^s}{j\omega(\mu_1 - \mu_0)} = H^i
\]

where \( J^s \) and \( M^s \) are unknown because \( E \) and \( H \) are the total fields to be determined. If the materials are dielectrics; i.e., \( \mu_1 = \mu_0 \), then only Equation (2.18) is needed and \( E^s \) is in general calculated from an integration over the unknown, \( J^s \). Therefore, Equation (2.18) is an integral equation to be employed as the linear operator equation for solving the scattering from inhomogeneous lossy dielectric bodies.
2.3.2 Singly-Periodic Green's Function in Plane Wave Expansion

For the E-field polarized along the \( \hat{z} \) direction (TM case), the incident plane wave is given by

\[
E'(R) = \hat{z}e^{-jk_o\hat{z} \cdot R} = \hat{z}e^{-jk_o(s_x x + s_y y)} \tag{2.20}
\]

where \( \hat{z} \) is the unit directional vector of the incident plane wave. For this case, Equation (2.14) reduces to

\[
J_z^s = j\omega(\epsilon - \epsilon_o)E_z \tag{2.21}
\]

where \( \epsilon \) is used instead of \( \epsilon_j \) for simplicity. Due to the periodic structure and the incident plane wave, the scattered current at any point of the wedge absorber segments differs from that in a selected reference wedge by only a known phase shift. For the case shown in Figure 2, the scattered current in the \( n \)th wedge is given by

\[
J_z^s(R' + \hat{z}D_x n) = J_z^s(R')e^{-jk_o D_x ns_z} \tag{2.22}
\]

where \( n \) is an integer numbering the wedges, \( D_x \) is the interelement distance, and \( R' \) is a position vector located in the cross section of the reference wedge. Thus, the scattering of a plane wave from a periodic structure is solved once the equivalent current over a reference wedge is determined; therefore, only a finite number of unknowns is needed for the solution.

The E-field from a singly-periodic array of electric filaments (a periodic Green's function) in spectral domain formulation (plane wave expansion) is derived in Appendix A. By applying superposition, the scattered E-field from the singly-periodic array of the equivalent currents replacing the wedges, \( J_z^s \), is given by

\[
E_z^s(r) = \frac{Z_o}{2D_x} \sum_{i=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-jk_o(R-R')^2} \frac{J_z^s(R')}{r_y} \, dx' \, dy' \tag{2.23}
\]

where

\[
R' = (x', y'), \text{ the source points on the cross section of the reference wedge}
\]
\( \mathbf{R} = (x, y) \), the field points

\[
\tau_{\pm} = \hat{s}(s_x + \frac{i\lambda}{D_x}) \pm \hat{y} r_y, \ y \geq y'
\]

(2.24)

\[
r_y = \begin{cases} 
\sqrt{1 - (s_x + \frac{i\lambda}{D_x})^2}, & (s_x + \frac{i\lambda}{D_x})^2 \leq 1 \\
-j\sqrt{(s_x + \frac{i\lambda}{D_x})^2 - 1}, & \text{otherwise}
\end{cases}
\]

(2.25)

and

\[
Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}, \text{ the free space characteristic impedance.}
\]

In the spectral domain formulation, the scattered E-field is calculated from an infinite sum of the plane waves and double integrals over the cross section of the reference wedge. Note that the plane wave propagates if \( r_y \) is real but becomes evanescent when \( r_y \) is imaginary. The scattered far field (\( y \rightarrow \pm \infty \)) solution for the periodic array illuminated by a plane wave is composed of a finite number of propagating plane waves which are simply the terms with real \( r_y \) in the summation. However, to calculate the scattered fields near or inside the array, the infinite summation needs to be evaluated. In practice, all the propagating waves and a finite number of evanescent waves with smaller \(|r_y|\) are included to obtain an approximate result. Because of the exponential decay with respect to \(|y - y'|\) for \( r_y \) imaginary, only a few evanescent waves are needed to have an accurate solution. Since the number of the propagating modes increases and \(|r_y|\) varies slower with respect to the summation index for larger \( D_x/\lambda \), the summation converges faster for smaller interelement distance. On the other hand, the summation of Hankel functions in the spatial domain formulation converges faster for larger \( D_x/\lambda \). However, for problems such that the cross sections of the bodies increase with the interelement distance, only small \( D_x/\lambda \) is practical in applying the Method of Moments. Besides, the double integrals in Equation (2.23) can be carried out in closed form as shown in Appendix B, and in the spatial domain formulation, numerical integration is usually
required. Also, the scattered far fields of the periodic layer illuminated by a plane wave are composed of a finite number of propagating plane waves which are the direct result of the plane wave expansion solutions. Therefore, the spectral domain formulation is the better choice for Periodic Moment Method. The convergence of the single infinite sum will be further discussed in Section 2.6.

2.4 Periodic Moment Method Solution

A PMM solution for TM scattering from the singly-periodic array of lossy dielectric bodies is developed in this section.

From Equations (2.18), (2.20), (2.21) and (2.23), the integral equation for the periodic moment method solution is given by

\[
\frac{Z_o}{2D_x} \sum_{i=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-jk_0(R-R')_y} J_z^s(R') \, dx' \, dy' + \frac{J_z^s(R)}{j\omega(\epsilon' - \epsilon_o)} = e^{-jk_0z}R \tag{2.26}
\]

where the unknown is the equivalent current density, \( J_z^s \), and the incident plane wave is set unity at the origin. To solve this equation with the Moment Method, first \( J_z^s \) is expanded into a set of basis functions, \( J_{zn}^s, n = 1, 2, \ldots, N \); i.e.,

\[
J_z^s = \sum_{n=1}^{N} I_n J_{zn}^s. \tag{2.27}
\]

Then, Equation (2.26) is integrated with test functions, \( w_m, m = 1, 2, \ldots, N \), and transformed into \( N \) simultaneous linear algebraic equations such that

\[
\sum_{n=1}^{N} I_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \left( -E_{zn}^s(x, y) + \frac{J_{zn}^s(x, y)}{j\omega(\epsilon(x, y) - \epsilon_o)} \right) w_m(x, y) \right] \, dx \, dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w_m(x, y) e^{-jk_0(s(z+y) - s'y)} \, dx \, dy, \quad m = 1, 2, \ldots, N \tag{2.28}
\]

where

\[
E_{zn}^s(x, y) = -\frac{Z_o}{2D_x} \sum_{i=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-jk_0(R-R')_y} J_{zn}^s(R') \, dx' \, dy'. \tag{2.29}
\]
and \( I_n \) are \( N \) unknown coefficients to be solved from these \( N \) linear independent equations. Note that \( E_n^s \) is the scattered E-field from the singly-periodic sources with \( J_n^s \) as the reference element.

Due to the complicated field distribution inside a dielectric body, pulse functions for \( J_n^s \) and delta functions for \( w_m \) are usually employed in the moment method volume formulation. The cross section of the reference wedge is divided into \( N \) quadrilateral cells which correspond to \( N \) basis functions. In Figure 4, an example of the cells is shown. Triangular cells are generated by a quadrilateral with two corners at the same position. Also, two sides of the cells have been made parallel to the \( x \) axis to improve the program efficiency in calculating the summation. However, the formula in Appendix B apply to a general quadrilateral. The basis functions are defined by

\[
J_n^s = \begin{cases} 
\frac{1}{A_n}, & \text{in Cell } n, \text{ and} \\
0, & \text{otherwise}
\end{cases} \tag{2.30}
\]

where \( A_n \) is the area of Cell \( n \). Since the integration of the pulse functions over the reference wedge is equal to 1, \( E_n^s \) depends less on the cell size. The test functions are chosen such that

\[
w_m = \delta(x - x_m)\delta(y - y_m), \ m = 1, 2, \ldots, N \tag{2.31}
\]

where \((x_m, y_m)\) is the centroid of Cell \( m \). Therefore, Equation (2.28) is satisfied only at the centers of the cells, and only \( N \) linearly independent pulse basis functions are included to expand the scattered current. Thus, a pulse basis and point-matching formulation is used, and the \( N \) simultaneous linear algebraic equations become

\[
\sum_{n=1}^{N} I_n[-E_n^s(x_m, y_m) + J_n^s(x_m, y_m) \frac{J_n^s(x_m, y_m)}{j\omega(\epsilon(x_m, y_m) - \epsilon_0)}] = e^{-jk_0(s_x x_m + s_y y_m)}, \ m = 1, 2, \ldots, N. \tag{2.32}
\]
A Four-Layer Wedge Absorber
Max Segment Size = $0.125 \lambda_d$
Frequency = 2 GHz

Figure 4: An example of PMM cells dividing the cross section of a multi-layer wedge absorber design with four different LS layers attached to an 8" commercial wedge.
In matrix form, the above expression can be written as

\[ [Z + \Delta Z][I] = [V] \] (2.33)

where

\[
[I] = \begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
\]

\[
[V] = \begin{bmatrix}
E_z^i(x_1, y_1) \\
E_z^i(x_2, y_2) \\
\vdots \\
E_z^i(x_N, y_N)
\end{bmatrix}
\]

\[
[Z] = \begin{bmatrix}
-E_{z1}^s(x_1, y_1) & -E_{z2}^s(x_1, y_1) & \cdots & -E_{zN}^s(x_1, y_1) \\
-E_{z1}^s(x_2, y_2) & -E_{z2}^s(x_2, y_2) & \cdots & -E_{zN}^s(x_2, y_2) \\
\vdots & \vdots & \ddots & \vdots \\
-E_{z1}^s(x_N, y_N) & -E_{z2}^s(x_N, y_N) & \cdots & -E_{zN}^s(x_N, y_N)
\end{bmatrix}
\] (2.36)

and

\[
[\Delta Z] = \begin{bmatrix}
\frac{1}{j\omega(\epsilon(x_1, y_1) - \epsilon_0)A_1} & 0 & \cdots \\
0 & \frac{1}{j\omega(\epsilon(x_2, y_2) - \epsilon_0)A_2} & \cdots \\
\cdots & \cdots & \ddots & \cdots \\
0 & \cdots & \cdots & \frac{1}{j\omega(\epsilon(x_N, y_N) - \epsilon_0)A_N}
\end{bmatrix}
\] (2.37)

The unknown current vector, \([I]\), is then given by

\[ [I] = [Z + \Delta Z]^{-1}[V] \] (2.38)

which can be used to determine the scattered field.
The scattered field is composed of an infinite number of plane waves. In the far zone, only the propagating (specular and grating lobes) waves are significant. Therefore, the reflection coefficient for the infinite absorber wall is calculated from the specular and grating lobe reflected plane waves, which determine the scattering performance of the absorber. The required height and loss of the absorber can be estimated from the specular and grating lobe transmitted plane waves; i.e., the transmission coefficients of the periodic layer. Each coefficient is calculated from a single term of the summation in Equation (2.23) with real \( r_y \) and is simply a closed-form integration over the cross section of the reference wedge.

### 2.5 A Hybrid Approach for Designing Absorber

In this section, an approach for calculating the scattering from a periodic layer mounted on a flat layer as shown in Figure 5(a) is described. The flat layer can be multi-layer, metal backed, or a ferrite; while, the periodic layer is composed of lossy dielectric materials and can be either singly or doubly periodic. This is called a hybrid approach because results from two different methods are used to solve the total problem. PMM is employed for calculating the reflection and transmission coefficients on both the front and back faces of the periodic layer alone in free space as illustrated in Figure 5(b). For the flat layer shown in Figure 5(c), a computer code written by Richmond [18] is applied to evaluate the reflection and transmission coefficients of the layer. Note that \( R_i \) and \( T_i \), for \( i = 1, 2, 3 \) as shown in Figure 5 correspond to the reflection and transmission coefficients of the front and back faces of the periodic layer, and the front face of the flat layer, respectively. In all cases, the phase reference is located at \( O \). By superposing all the specular plane waves bouncing between the back face of the periodic layer and the front face of the flat
Figure 5: A hybrid approach for calculating the reflection and transmission coefficients of an absorber wall.

Layer, the specular reflection and transmission coefficients of the periodic layer backed with the flat layer for the case without any grating lobes are given by

\[
R = R_1 + T_1 R_3 T_2 e^{-j2k_o d \cos \theta} [1 + \sum_{n=1}^{\infty} (R_2 R_3 e^{-j2k_o d \cos \theta})^n], \quad \text{or}
\]

\[
R = R_1 + \frac{T_1 R_3 T_2 e^{-j2k_o d \cos \theta}}{1 - R_2 R_3 e^{-j2k_o d \cos \theta}}
\]

and

\[
T = \frac{E_1^1(\theta)e^{-j \xi o d \cos \theta} [1 + \sum_{n=1}^{\infty} (R_2 R_3 e^{-j2k_o d \cos \theta})^n] T_3}{E_1^1(\theta)e^{-j \xi o d \cos \theta}}, \quad \text{or}
\]

\[
T = \frac{T_1 T_3}{1 - R_2 R_3 e^{-j2k_o d \cos \theta}}
\]
where $d$ is the gap between the layers, $\theta$ is the angle of incidence, $k_o$ is the free space propagation constant, and the phase reference is at $O$ of Figure 5(a). Note that the phase shift along the vertical ($\hat{y}$) direction for a complete bounce is $e^{-j2k_o d\cos\theta}$, and the phases of the plane waves at one point inside the gap along the horizontal direction are all the same.

To design absorber, one may first employ PMM to determine the scattering from both the front and back faces of the periodic layer. Then, the multi-layer program can be applied repeatedly together with Equations (2.39) and (2.40) to search for a good flat layer (base) to match the periodic layer. Since the CPU time for running the multi-layer program is much less than that for PMM, many cases can be tested for one periodic layer.

For a proper absorber design, it is usually necessary to attenuate the incident wave to such a level that the scattering from any object behind the absorber is negligible compared to that from the absorber itself. To estimate the required height and loss of the absorber, one may try several different base thicknesses with and without a metal ground plane on the back face of the absorber by applying the hybrid approach. One should choose the base thickness so that increasing the thickness further will not significantly change the reflection coefficient of the absorber wall. However, if the base of the absorber is too thick, a more lossy material should be used to reduce the height of the absorber. Since the attenuation of the incident wave is stronger at higher frequencies, only the performance at lower frequencies needs to be examined to determine the base thickness or material loss. Note that grating lobes are not normally observed at lower frequencies, and therefore Equations (2.39) and (2.40) can be used for the hybrid approach.

Another method for estimating the height and loss of the absorber is to calculate the maximum backface reflection (the maximum return from the objects behind the
absorber wall). By mounting the absorber on a metal ground plane, according to Equation (2.39) the maximum backface reflection of the absorber wall is equal to

$$\frac{|T_1 T_2|}{1 - |R_2|}.$$  \hspace{1cm} (2.41)

The reflection coefficient from the back face of the absorber, $R_2$, can be calculated from PMM. However, $R_2$ is approximately equal to the reflection coefficient from the absorber bottom only, because the absorber wall provides a good impedance match to free space on the top side. Also, from the reciprocity relationship [19], one finds that

$$\int E_2 \cdot J_1 \, dv = \int E_1 \cdot J_2 \, dv \hspace{1cm} (2.42)$$

or that a plane wave incident upon the front face of the absorber in free space has the same transmission coefficient as a plane wave coming into the back face of the absorber along the reverse direction. Therefore, to determine the height and loss of the absorber, one can apply PMM to calculate the reflection and transmission coefficients from only the front side of the absorber at the low frequency end. The reflection coefficient on the absorber backface can be approximated by the reflection coefficient from a flat interface of air and a medium which corresponds to the absorber base. The maximum backface reflection calculated from Equation (2.41) should be several dB (say 10 dB) lower than the reflection coefficient on the absorber frontface. Therefore, to design absorber one may employ PMM to determine the reflection and transmission coefficients for an infinite wall of absorber in free space. Then, Equations (2.39) and (2.40) or Equation (2.41) can be used to estimate the height and loss of the absorber so that the absorber has the designed performance.
2.6 Convergence of the Single Infinite Sum

In this section, attention is given to an approach for evaluating the following single infinite sum in the impedance element:

\[ Z_{mn} = -E_{z n}^s(R_m) = \frac{Z_0}{2D_x A_n} \sum_{i=-\infty}^{\infty} \int \int_{cell_n} e^{-j\omega(R_m-R') \cdot \hat{r}_{\pm}} \frac{dz' dy'}{r_y} \]  

(2.43)

where

\[ R' = (x', y'), \]  the source points over the nth cell in the reference wedge

\[ R_m = (x_m, y_m), \]  the center of the the mth cell

\[ \hat{r}_{\pm} = \hat{x}(s_x + \frac{i\lambda}{D_x}) \pm \hat{y} y, \]  for \( y > y' \)  

(2.44)

and

\[ r_y = \begin{cases} \sqrt{1 - (s_x + \frac{i\lambda}{D_x})^2}, & (s_x + \frac{i\lambda}{D_x})^2 \leq 1 \\ -j\sqrt{(s_x + \frac{i\lambda}{D_x})^2 - 1}, & \text{otherwise.} \end{cases} \]  

(2.45)

Because of the exponential decay for evanescent plane waves, the single infinite sum converges to a finite value and the magnitude of each summation term depends on \( r_y \), the y component of the unit directional vectors of the plane waves. For \( |\frac{i\lambda}{D_x}| > |s_x| \)  and \( r_y \) imaginary, the magnitude of the summation terms decreases with increasing \( |i| \). Therefore, to calculate the single infinite sum, all propagating plane waves and at least two lowest order evanescent plane waves are included first. Then, with \( |i| \) increased step by step, two terms having the same \( |i| \) are added to the current sum. Next, a convergence check is performed by comparing the new sum with the previous sum. When the percentage difference between the adjacent sums is less than some small value (say 0.1%), the sum is terminated and a convergent result is assumed. To determine that a convergent result is obtained, one may try two maximum percentage differences (say 1% and 0.1%), and compare the calculated reflection and transmission coefficients.
In Figures 6-17, several plots of convergence curves are shown for the wedge geometry given in Figure 4. Note that T and R in the figures correspond to the Transmitting (nth) and Receiving (mth) cells, respectively. The imaginary part of the sum normalized to that of the last sum is plotted with respect to the positive index of the sum. All plots show that the single infinite sum converges rather fast, and the approach for evaluating the single infinite sum reaches the convergent value with only a few terms for a maximum percentage difference of 0.1%. Also, the infinite sums for the rectangular cells converge faster than those for the triangular cells, and usually more terms should be included for the self impedance elements. Note that all curves converge smoothly except the one shown in Figure 8, where the curve oscillates around a convergent value. This occurs when the distance between the centers of the transmitting cell and receiving cell is equal to $\frac{D_0}{2}$, and the phase of $e^{-jk_0(R_m-R'_m)f_\pm}$ is close to either 0 or $\pi$. Nevertheless, a fast convergent curve is still observed for this case.
Figure 6: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the self impedance of the rectangular cell at the left corner of the wedge.
Figure 7: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the adjacent rectangular cells on the bottom.
Figure 8: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the rectangular cells on the bottom.
Normal Incidence

Figure 9: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the rectangular cells on the bottom.
Figure 10: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the self impedance of the triangular cell.
Figure 11: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the cells shown in the inlet.
Figure 12: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the self impedance of the triangular cell.
Figure 13: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the cells shown in the inlet.
Figure 14: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the cells shown in the inlet.
Figure 15: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the self impedance of the cell marked S on the left side of the wedge.
Figure 16: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the cells shown in the inlet.
Figure 17: A convergent check for the wedge geometry shown in Figure 4. This curve corresponds to the mutual impedance between the cells shown in the inlet.
CHAPTER III
A Periodic Moment Method Solution for Scattering from a Doubly-Periodic Array of Lossy Dielectric Bodies

3.1 Introduction

In this chapter, a periodic moment method solution for a three-dimensional (3-D) lossy dielectric array with double periodicity is developed. First, an expression for the fields from a doubly-periodic array of electrical volume currents is derived with two different approaches. The expression is complete and can be applied to the regions with or without the sources. Next, the moment method formulation for the periodically distributed dielectric bodies is discussed, and a 3-D PMM formulation for an arbitrary incident plane wave is presented. Finally, an approach for evaluating double infinite sums in the PMM formulation is reported.

3.2 Fields from a Doubly-Periodic Array of Electric Currents

In Figure 18, a time harmonic \( e^{j\omega t} \) plane wave with an arbitrary incident angle and polarization illuminates an infinite wall of pyramidal absorber. From the moment method volume formulation, the dielectric materials are replaced by a set of equivalent electrical volume current densities (scattered currents), which are given by

\[
J^s = j\omega (\epsilon - \epsilon_0)E
\]

where \( E \) is the total E field, \( \epsilon \) is the dielectric constant of the material, and \( \epsilon_0 \) is the free space dielectric constant. Due to the periodicity and plane wave incidence,
Figure 18: An arbitrarily polarized plane wave is incident upon an infinite wall of pyramidal absorber with interelement spacings of $D_x$ and $D_z$. 
these equivalent currents must satisfy the following:

\[ J^s(R' + \hat{z}D_z m + \hat{z}D_z n) = J^s(R') e^{-jk_0(D_z ms + D_z ns)} \tag{3.2} \]

where \( m \) and \( n \) are integers numbering the periodic elements with \( m = n = 0 \) being the reference element, \( D_z \) and \( D_z \) are the interelement distances along \( x \) and \( z \) directions, \( \hat{s} \equiv \hat{x}s_x + \hat{y}s_y + \hat{z}s_z \) is the directional vector of the incident plane wave, and \( R' \) is inside the reference element of the dielectric array. From [9], the vector potential created by a doubly-periodic array of the scattered currents, with phase matching relative to the incident plane wave, is given by

\[
A^s(R) = \frac{1}{2jk_0D_zD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} J^s(R') e^{-jk_0(R-R') \hat{r}_\pm} \frac{u(y'-y)}{r_y} dv' \tag{3.3}
\]

or

\[
A^s(R) = \frac{1}{2jk_0D_zD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} u(y'-y') J^s(R') e^{-jk_0(R-R') \hat{r}_+} \frac{u(y'-y)}{r_y} dv' + u(y'-y') J^s(R') e^{-jk_0(R-R') \hat{r}_-} \frac{u(y'-y)}{r_y} dv' \tag{3.4}
\]

where

\[
u(\xi) = \begin{cases} 1 , & \xi > 0 \\ 0 , & \xi < 0 \end{cases} \tag{3.5}\]

\[
\hat{r}_\pm = \hat{z}(s_x + \frac{k\lambda}{D_z}) \pm \hat{y}r_y + \hat{z}(s_z + \frac{l\lambda}{D_z}) \text{, for } y \geq y' \tag{3.6}\]

\[
r_y = \begin{cases} \sqrt{1 - (s_x + \frac{k\lambda}{D_z})^2 - (s_z + \frac{l\lambda}{D_z})^2} , & (s_x + \frac{k\lambda}{D_z})^2 + (s_z + \frac{l\lambda}{D_z})^2 - 1 \\ -j\sqrt{(s_x + \frac{k\lambda}{D_z})^2 + (s_z + \frac{l\lambda}{D_z})^2 - 1} , & \text{otherwise} \end{cases} \tag{3.7}\]

\( D_z \) and \( D_z \) are the interelement spacings as shown in Figure 18, and

\[
k_0 = \omega/\sqrt{\mu_0\varepsilon_0}, \text{ the free space propagation constant.} \]

The scattered H-field is calculated from
\[-\frac{1}{2j\varepsilon D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} \nabla \times [u(y' - y)J^s(R') \frac{e^{-jko(R-R')}}{r_y}] + \nabla \times [u(y' - y)J^s(R') \frac{e^{-jko(R-R')}}{r_y}] \, dv'.\] (3.8)

Applying the vector identity:
\[
\nabla \times (wA) = w \nabla \times A - A \times \nabla w
\] (3.9)

and substituting
\[
\nabla u(y) = \hat{y}\delta(y)
\] (3.10)

into Equation (3.8), one finds that
\[-\frac{1}{2j\varepsilon D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} u(y' - y) \nabla \times \left[ J^s(R') \frac{e^{-jko(R-R')}}{r_y} \right]
+ u(y' - y) \nabla \times \left[ J^s(R') \frac{e^{-jko(R-R')}}{r_y} \right]
- J^s(R') \frac{e^{-jko(R-R')}}{r_y} \times \hat{y}\delta(y - y')
+ J^s(R') \frac{e^{-jko(R-R')}}{r_y} \times \hat{y}\delta(y' - y) \, dv'.\] (3.11)

Because
\[-\nabla \times \left[ J^s(R') \frac{e^{-jko(R-R')}}{r_y} \right] = -jko\hat{r}_\pm \times \left[ J^s(R') \frac{e^{-jko(R-R')}}{r_y} \right]\] (3.12)

and
\[-\iint_{\text{ref}} J^s(R') \frac{e^{-jko(R-R')}}{r_y} \times \hat{y}\delta(y - y') + J^s(R') \frac{e^{-jko(R-R')}}{r_y} \times \hat{y}\delta(y' - y) \, dv'
= \iint_{\text{ref}} J^s(R') \frac{e^{-jko(R-R')}}{r_y} \times \hat{y} \bigg|_{y'=y} \, dx' \, dz'
= 0\] (3.13)
Equation (3.11) reduces such that

\[
\mathbf{H}^s(\mathbf{R}) = \frac{-1}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} u(y - y') \hat{\mathbf{r}}_+ \times \mathbf{J}^s(\mathbf{R}') e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_+} \frac{1}{r_y} dy' + u(y - y') \hat{\mathbf{r}}_- \times \mathbf{J}^s(\mathbf{R}') e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_-} \frac{1}{r_y} dy'.
\] (3.14)

Then, the scattered E-field can be determined from one of Maxwell's equations and is given by

\[
\mathbf{E}^s(\mathbf{R}) = \frac{1}{j\omega \varepsilon_0} [\nabla \times \mathbf{H}^s(\mathbf{R}) - \mathbf{J}^s(\mathbf{R})].
\] (3.15)

Following similar steps to derive the scattered H-field, one obtains that

\[
\nabla \times \mathbf{H}^s(\mathbf{R}) = \frac{1}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} u(y - y') jk_0 \hat{\mathbf{r}}_+ \times \mathbf{J}^s(\mathbf{R}') e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_+} \frac{1}{r_y} dy' + u(y - y') jk_0 \hat{\mathbf{r}}_- \times \mathbf{J}^s(\mathbf{R}') e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_-} \frac{1}{r_y} dy' + [\hat{\mathbf{r}}_+ \times \mathbf{J}^s(\mathbf{R}')] \times \hat{y} \delta(y - y') e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_+} \frac{1}{r_y} dy' - [\hat{\mathbf{r}}_- \times \mathbf{J}^s(\mathbf{R}')] \times \hat{y} \delta(y - y') e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_-} \frac{1}{r_y} dy'.
\] (3.16)

or

\[
\nabla \times \mathbf{H}^s(\mathbf{R}) = \frac{jk_0}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} \hat{\mathbf{r}}_\pm \times \mathbf{J}^s(\mathbf{R}') e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_\pm} \frac{1}{r_y} dy' + \frac{1}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iint_{\text{ref}} [\hat{\mathbf{r}}_+ \times \mathbf{J}^s(\mathbf{R}')] \times \hat{y} e^{-jk_0(\mathbf{R} - \mathbf{R}') \cdot \hat{\mathbf{r}}_+} \frac{1}{r_y} dy' dz' dx'.
\] (3.17)

Employing the vector identity:

\[
(A \times B) \times C = (C \cdot A)B - (C \cdot B)A
\] (3.18)
in Equation (3.17), one finds that

\[ \nabla \times \mathbf{H}^s(\mathbf{R}) = \frac{jk_0}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iiint_{\text{ref}} \hat{r}_z \times \hat{r}_z \times J^s(\mathbf{R}') e^{-jk_0(\mathbf{R}-\mathbf{R}')} \rho_{yz} \, dv' \]

\[ + \frac{1}{D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iiint_{\text{ref}} (\overline{I} - \hat{y}\hat{y}) \cdot J^s(\mathbf{R}') \rho_{xy} \bigg|_{y'=y} e^{-jk_0((z-z')r_x + (z-z')r_z)} \, dx' \, dz' \]  

(3.19)

or

\[ \nabla \times \mathbf{H}^s(\mathbf{R}) = \frac{jk_0}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \iiint_{\text{ref}} \hat{r}_z \times \hat{r}_z \times J^s(\mathbf{R}') e^{-jk_0(\mathbf{R}-\mathbf{R}')} \rho_{yz} \, dv' \]

\[ + \frac{1}{D_x D_z} \int_{\text{ref}} (\overline{I} - \hat{y}\hat{y}) \cdot J^s(\mathbf{R}') \rho_{xy} \bigg|_{y'=y} \sum_{k=-\infty}^{\infty} e^{-jk_0((z-z')s_z + \frac{kl}{D_z})} \sum_{l=-\infty}^{\infty} e^{-jk_0((z-z')s_z + \frac{kl}{D_z})} \, dx' \, dz' \] 

(3.20)

where \( \overline{I} \) is a unit dyad which is defined by

\[ \overline{I} = \hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}. \]

(3.21)

From Equation (A.4) and the Fourier transform pair, \( \mathcal{F}[\delta(t)] = 1 \), one obtains that

\[ \sum_{k=-\infty}^{\infty} e^{-jk_0((z-z')s_z + \frac{kl}{D_z})} = e^{-jk_0((z-z')s_z} \sum_{k=-\infty}^{\infty} e^{j\frac{kl}{D_z}(x'-x)} \]

(3.22)

or

\[ \sum_{k=-\infty}^{\infty} e^{-jk_0((z-z')s_z + \frac{kl}{D_z})} = e^{-jk_0((z-z')s_z} D_z \sum_{m=-\infty}^{\infty} \delta(x'-x - mD_z) \]

(3.23)

and similarly

\[ \sum_{i=-\infty}^{\infty} e^{-jk_0((z-z')s_z + \frac{li}{D_z})} = e^{-jk_0((z-z')s_z} D_z \sum_{n=-\infty}^{\infty} \delta(z'-z - nD_z). \]

(3.24)

Substituting Equations (3.23) and (3.24) into Equation (3.20), one finds that
\[ \nabla \times H^s(R) = \frac{jk_o}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \int \int \int \hat{r}_{\pm} \times [\hat{r}_{\pm} \times J^s(R')] e^{-jk_o(R-R') \cdot \hat{r}_{\pm}} \, dv' \]
\[ + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \int \int (\hat{l} - \hat{y} \cdot \hat{y}) \cdot J^s(R') \bigg|_{y'=y} e^{-jk_o[(x-x')s_x + (z-z')s_z]} \delta(x' - x - mD_x)\delta(z' - z - nD_z) \, dx'dz' \] (3.25)

or

\[ \nabla \times H^s(R) = \frac{jk_o}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \int \int \int \hat{r}_{\pm} \times [\hat{r}_{\pm} \times J^s(R')] e^{-jk_o(R-R') \cdot \hat{r}_{\pm}} \, dv' \]
\[ + (\hat{l} - \hat{y} \cdot \hat{y}) \cdot J^s(R + \hat{z}m'D_x + \hat{z}n'D_z)e^{jk_o(m'D_x s_x + n'D_z s_z)} \] (3.26)

where \((m', n')\) corresponds to the only non-zero term in the double integrals because of the \(\delta\) functions. Applying Equation (3.2) to Equation (3.26), one finds that

\[ \nabla \times H^s(R) = \frac{jk_o}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \int \int \int \hat{r}_{\pm} \times [\hat{r}_{\pm} \times J^s(R')] e^{-jk_o(R-R') \cdot \hat{r}_{\pm}} \, dv' \]
\[ + J^s(R) - \hat{y} \cdot J^s(R). \] (3.27)

Substituting Equation (3.27) into Equation (3.15) yields

\[ E^s(R) = \frac{Z_o}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \int \int \int \hat{r}_{\pm} \times [\hat{r}_{\pm} \times J^s(R')] e^{-jk_o(R-R') \cdot \hat{r}_{\pm}} \, dv' \]
\[ - \frac{1}{j\omega e_o} \hat{y} \cdot J^s(R) \] (3.28)

or

\[ E^s(R) = E^s'(R) - \frac{1}{j\omega e_o} \hat{y} \cdot J^s(R) \] (3.29)

where \(E^s'(R)\) is the electric field valid outside the source region and is given by

\[ E^s'(R) = \frac{Z_o}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \int \int \int \hat{r}_{\pm} \times [\hat{r}_{\pm} \times J^s(R')] e^{-jk_o(R-R') \cdot \hat{r}_{\pm}} \, dv'. \] (3.30)
Note that $Z_o$ is the free space characteristic impedance, and Equation (3.29) agrees with Equation (82) of [20].

Equation (3.29) can also be derived from the following method. In Figure 19, an infinitesimally thin free-space layer perpendicular to $\hat{y}$ is inserted at the field point $R$. Because only an infinitesimally thin layer of equivalent volume current is removed, the E-field outside the layer will remain the same as that without the free-space layer. Also, from the boundary conditions between two dielectric media, one has

\[
E_x(R) = E_x(R - \hat{y}\delta) \quad \text{(3.31)}
\]
\[
E_y(R) = \frac{\varepsilon}{\varepsilon_0} E_y(R - \hat{y}\delta), \text{ and} \quad \text{(3.32)}
\]
\[
E_z(R) = E_z(R - \hat{y}\delta). \quad \text{(3.33)}
\]

Substituting Equation (3.1) into Equation (3.32), one finds that

\[
E_y(R) = E_y(R - \hat{y}\delta) + \frac{1}{j\omega\varepsilon_0} \hat{y} \cdot J^s(R - \hat{y}\delta). \quad \text{(3.34)}
\]

Since $R$ is inside the free-space layer, Equation (3.30) can be used to determine $E(R)$; i.e.,

\[
E(R) = E^s(R) + E^i(R). \quad \text{(3.35)}
\]

Applying Equations (3.31), (3.33), and (3.34) to the above equation, one obtains

\[
E(R - \hat{y}\delta) + \frac{1}{j\omega\varepsilon_0} \hat{y} \cdot J^s(R - \hat{y}\delta) = E^s(R) + E^i(R). \quad \text{(3.36)}
\]

which reduces to Equation (3.29) as $\delta \rightarrow 0$ (the free-space layer vanishes).

### 3.3 Periodic Moment Method Solution

From Equation (2.18), the integral equation of the MM volume formulation used to solve for the scattering from the doubly-periodic array of 3-D lossy dielectric
Figure 19: An infinitesimally thin free-space layer replaces the same infinitesimally thin dielectric layer at $R$.

bodies is given by

$$- \mathbf{E}^s(R) + \frac{\mathbf{J}^s(R)}{j\omega(\epsilon(R) - \epsilon_0)} = \mathbf{E}^i(R)$$ \hspace{1cm} (3.37)

where $\mathbf{E}^s(R)$ is the scattered E-field determined from Equation (3.29), $\mathbf{J}^s(R)$ is the unknown equivalent volume current density, and $\mathbf{E}^i(R)$ is the known incident plane wave defined by $\mathbf{E}^i(O)e^{-jko(s_x x + s_y y + s_z z)}$. For 3-D lossy dielectric bodies, three sets of basis functions are needed to expand the scattered current which is given by

$$\mathbf{J}^s = \hat{x} J^s_x + \hat{y} J^s_y + \hat{z} J^s_z.$$ \hspace{1cm} (3.38)

Dividing the reference element of the array into $N$ cells and employing pulse basis functions, one may approximate the scattered current with the following expressions:

$$J^s_x = \sum_{n=1}^{N} I_{zn} J^s_{zn}$$ \hspace{1cm} (3.39)


\[ J_y^s = \sum_{n=1}^{N} I_{yn} J_{yn}^s, \text{ and} \]

\[ J_z^s = \sum_{n=1}^{N} I_{zn} J_{zn}^s \]  

(3.40) 

(3.41) 

where

\[ J_{zn}^s = J_{yn}^s = J_{zn}^s = \begin{cases} 
\frac{1}{V_n} & \text{in Cell } n, \\
0 & \text{otherwise.} 
\end{cases} \]  

(3.42) 

Note that \( V_n \) is the volume of Cell \( n \), and \( I_{zn}, I_{yn} \) and \( I_{zn} \) are the unknown coefficients to be solved. Substituting Equations (3.39)-(3.42) into Equation (3.30) yields

\[ E^s' = \sum_{n=1}^{N} I_{zn} E_{zn}^s + I_{yn} E_{yn}^s + I_{zn} E_{zn}^s \]  

(3.43) 

where

\[ E_{zn'}(R) = \frac{Z_o}{2D_x D_z V_n} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \frac{1}{r_y} \int \int_{ref} \hat{r}_\pm \times (\hat{r}_\pm \times \hat{z}) e^{-jk_o (R-R')} dv' \]  

(3.44) 

Similar equations can be obtained for \( E_{yn'}(R) \) and \( E_{zn'}(R) \) by replacing \( \hat{x} \) in Equation (3.44) with \( \hat{y} \) and \( \hat{z} \). Note that \( E_{zn'}^s, E_{yn'}^s, \) and \( E_{zn'}^s \) are excited from the doubly-periodic arrays of the electrical currents with \( \hat{x} J_{zn}^s \), \( \hat{y} J_{yn}^s \), and \( \hat{z} J_{zn}^s \) being the reference elements of the arrays, respectively. Applying Equations (3.29) and (3.43) to the integral equation given in Equation (3.37) yields

\[ \sum_{n=1}^{N} I_{zn} [\frac{\hat{x} J_{zn}^s}{j\omega(\epsilon - \epsilon_o)} - E_{zn}^s] + I_{yn} [\frac{\hat{y} J_{yn}^s}{j\omega(\epsilon - \epsilon_o)} - E_{yn}^s] + I_{zn} [\frac{\hat{z} J_{zn}^s}{j\omega(\epsilon - \epsilon_o)} - E_{zn}^s] = E^t. \]  

(3.45) 

By enforcing the above equations at the centers of the cells, the integral equation is then transformed into \( 3N \) simultaneous linear algebraic equations given by

\[ \sum_{n=1}^{N} \frac{I_{zn} [\hat{x} J_{zn}^s(R_m) - E_{zn}^s(R_m)] + I_{yn} [\frac{\hat{y} J_{yn}^s(R_m)}{j\omega(\epsilon(R_m) - \epsilon_o)} - E_{yn}^s(R_m)]}{j\omega(\epsilon(R_m) - \epsilon_o)} + I_{zn} [\frac{\hat{z} J_{zn}^s(R_m)}{j\omega(\epsilon(R_m) - \epsilon_o)} - E_{zn}^s(R_m)] = E^t(R_m), \ m = 1, 2, \ldots, N \]  

(3.46)
where $\mathbf{R}_m$ is at the centroid of Cell $m$. Thus, a pulse basis and point-matching formulation is used here.

In matrix form, Equation (3.46) can be written as

$$[Z + \Delta Z][I] = [V]$$  \hspace{1cm} (3.47)

where

$$[I] = \begin{bmatrix}
I_{x1} \\
I_{y1} \\
I_{z1} \\
\vdots \\
I_{xN} \\
I_{yN} \\
I_{zN}
\end{bmatrix}$$  \hspace{1cm} (3.48)

$$[V] = \begin{bmatrix}
E^i_x(\mathbf{R}_1) \\
E^i_y(\mathbf{R}_1) \\
E^i_z(\mathbf{R}_1) \\
\vdots \\
E^i_x(\mathbf{R}_N) \\
E^i_y(\mathbf{R}_N) \\
E^i_z(\mathbf{R}_N)
\end{bmatrix}$$  \hspace{1cm} (3.49)

$$[Z] = \begin{bmatrix}
\ddots & \vdots & \ddots & \vdots \\
\cdots & -E^i_{xn}(\mathbf{R}_m) \cdot \hat{x} - E^i_{yn}(\mathbf{R}_m) \cdot \hat{y} - E^i_{zn}(\mathbf{R}_m) \cdot \hat{z} & \cdots & \cdots \\
\cdots & -E^i_{xn}(\mathbf{R}_m) \cdot \hat{y} - E^i_{yn}(\mathbf{R}_m) \cdot \hat{y} - E^i_{zn}(\mathbf{R}_m) \cdot \hat{y} & \cdots & \cdots \\
\cdots & -E^i_{xn}(\mathbf{R}_m) \cdot \hat{z} - E^i_{yn}(\mathbf{R}_m) \cdot \hat{y} - E^i_{zn}(\mathbf{R}_m) \cdot \hat{z} & \cdots & \cdots \\
\vdots & \ddots & \vdots & \ddots
\end{bmatrix}$$  \hspace{1cm} (3.50)

and
The unknown current vector is given by

\[
[I] = [Z + \Delta Z]^{-1}[V]
\]  

which can be used to determine the scattered field.

In the Method of Moments, the most time-consuming procedure is usually in calculating the impedance matrix, \([Z]\). As shown in Equations (3.44) and (3.50), double infinite sums of the plane waves and triple integrals over each PMM cell of the reference dielectric body are needed to evaluate each matrix element in \([Z]\). Fortunately, the triple integrals can be carried out in closed form, as shown in Appendix C. The closed-form integration is obtained by selecting truncated pyramid cells which have two rectangular faces aligned with the \(\hat{z}\) and \(\hat{\xi}\) axes. In Figure 20, examples of the cells are plotted. Note that the sizes and positions of the rectangular faces are arbitrary.

### 3.4 Evaluation of the Double Infinite Sums

An approach for determining the double infinite sums in the elements of the impedance matrix is discussed in this section. From Equations (3.44) and (3.50), one finds that the elements are given by

\[
[Z_{mn}] = -\frac{Z_0}{2D_x D_z V_n} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \int_{cell_n} \int_{cell_{n'}} \int_{cell_n'} [P_{\pm}] e^{-jk_0(\bar{R}_m - \bar{R}')} d\nu' d\nu' d\nu' \quad (3.53)
\]
Figure 20: PMM cells used in the doubly-periodic moment method so that the triple integrals of the impedance element can be evaluated in closed form.

where

\[
[P_{\pm}] = \begin{bmatrix}
  r_x^2 - 1 & \pm r_x r_y & r_x r_z \\
  \pm r_x r_y & r_y^2 - 1 & \pm r_y r_z \\
  r_x r_z & \pm r_y r_z & r_z^2 - 1
\end{bmatrix}
\tag{3.54}
\]

\[
\hat{r}_{\pm} = \hat{x}(s_x + \frac{k\lambda}{D_z}) \pm \hat{y}r_y + \hat{z}(s_z + \frac{l\lambda}{D_z}), \text{ for } y_m \geq y'
\tag{3.55}
\]

\[
r_y = \begin{cases}
  \sqrt{1 - (s_x + \frac{k\lambda}{D_z})^2 + (s_z + \frac{l\lambda}{D_z})^2}, & (s_x + \frac{k\lambda}{D_z})^2 + (s_z + \frac{l\lambda}{D_z})^2 \leq 1 \\
  -j\sqrt{(s_x + \frac{k\lambda}{D_z})^2 + (s_z + \frac{l\lambda}{D_z})^2 - 1}, & \text{otherwise}
\end{cases}
\tag{3.56}
\]

\[
\overline{R}_m = (x_m, y_m, z_m), \text{ the center of Cell } m \text{ (a receiving cell), and}
\]

\[
\overline{R}' = (x', y', z'), \text{ the source point in Cell } n \text{ (a transmitting cell).}
\]

In the double sums, the magnitude of each summation term reduces exponentially with respect to \(|r_y|\) when \(r_y\) becomes imaginary; i.e., \((s_x + \frac{k\lambda}{D_z})^2 + (s_z + \frac{l\lambda}{D_z})^2 > 1\).

Thus, the double infinite sums converge and only a finite number of summation terms with smaller \(|r_y|\) need to be included to obtain an accurate solution. Also,
in order to have a proper convergence check in selecting these summation terms to be included, it is essential that terms with larger magnitude should be added first. Since the imaginary part of $r_y$ determines the magnitude of the summation terms, the double sums can be evaluated radially in $k - l$ space according to $r_y$. First, terms with

$$\frac{B_{k-2} + B_{k-1}}{2} \leq |r_y| < \frac{B_{k-1} + B_k}{2}$$  \hspace{1cm} (3.57)

are grouped together, where

$$|r_y| = \sqrt{|(s_x + \frac{k\lambda}{D_x})^2 + (s_z + \frac{l\lambda}{D_z})^2 - 1|}$$  \hspace{1cm} (3.58)

$$B_k = \sqrt{(s_x + \frac{k\lambda}{D_x})^2 + s_z^2 - 1}, k \geq k_i$$  \hspace{1cm} (3.59)

$$B_{k_i-1} = -B_{k_i}$$  \hspace{1cm} (3.60)

$$k = k_i + p, p = 1, 2, \cdots, \text{and}$$  \hspace{1cm} (3.61)

$$k_i = [((\sqrt{2 - s_z^2} + |s_x|)\frac{D_x}{\lambda} + 1].$$  \hspace{1cm} (3.62)

Note that $k_i$ is chosen such that all propagating modes and some evanescent waves are included first as an initial value for the double sums and $|r_y|$ increases monotonously with respect to $p$, a radial parameter. An example for grouping the terms is shown in Figure 21. Note that $B_k$ corresponds to $(k, l=0)$ terms, and boundary lines for $|r_y|$ lie midway between two adjacent $B_k$'s. Therefore, the radial and grouping parameters become the summation indexes for the double infinite sums. Finally, the infinite sums have to be truncated for practical reasons, and a method for determining the convergent result of the sums is required. In Figure 22-28, the imaginary part of $-E_{zn}^t(R_m) \cdot \hat{z}$ normalized to a convergent value of the sums is plotted with respect to $p$. An infinite wall of 10' high pyramidal absorber at 150 MHz is studied, and the geometry of the pyramid is shown in the insert of the figures. Note that T and R in the figures correspond to the Transmitting (nth)
and Receiving (mth) cells, respectively. From these plots, one finds that for the cases with the receiving cells above or below the transmitting ones, an overdamping convergence is expected; for the cells located at the same y coordinate, an underdamping behavior is observed. The overdamped cases converge considerably faster than the underdamped one. For the former, the infinite sums may be terminated when the percentage difference between the current and previous results is less than a small value. This approach is similar to that used for the singly periodic arrays. On the other hand, the underdamped cases require a more sophisticated method to extract the convergent solution accurately and efficiently. For a function oscillating around a center value, the convergent result may be estimated from an average over several periods. Since the self-impedance element not only has the smallest distance between source and field points but also has the source points surrounding the field point (center), its convergence curve should be the smoothest. Thus, one may choose
a period of the oscillation for the self impedance and use this period as the range for taking the average. In the examples shown, the average over the points between the 3rd and the 5th peaks (derivative = 0) is performed. For the other parallel cells with the same transmitting cell as that of the self impedance, the range for the self term is used to take the average and determine the convergent result. From Figures 24, 27, and 28, one observes that more summation terms are needed for a less symmetric cell. Figure 26 shows a rapid oscillation in the convergence of the double sums. This occurs because the distance between the centers of the cells along the x direction is equal to $D_e$. Therefore, the phase of $e^{-jk_0(R_m-R')f_{\pm}}$ is close to either 0 or $\pi$ so that the curve becomes zigzagged.

The plane wave spectrum formulation, the closed form integration and the approach for determining the double infinite sums are important in writing an efficient and practical PMM program. To further improve the PMM program, two other properties of the impedance element given in Equation (3.53) have been applied. First, although there are nine elements in $[Z_{mn}]$, they are calculated simultaneously so that the double sums and triple integrals are carried out only once for determining $[Z_{mn}]$. Note that $[Z_{mn}]$ is also a symmetric matrix. Secondly, the triple integrals need to be performed only three times at most per $(k, l)$ for each transmitting cell. This is because the integrals can be written as

$$e^{-jk_0(R_m-R')f_{\pm}} \iiint_{cell \ n} [P_{\pm}] e^{jk_0(R'R'y')y'} \, dv'$$

(3.63)

for all the field points above or below the nth cell. Also, if the y coordinate at the center of the receiving cell is the same as that at the center of the transmitting cell, the triple integrals become

$$e^{-jk_0(x_m r_z + z_m r_z)} \iiint_{cell \ n} [P_{\pm}] e^{jk_0(z'r_z \pm (y'-y_n)r_y + z'r_z)} \, dv'.$$

(3.64)
Thus, the centers of parallel cells should be located at the same y coordinate to reduce the calculation effort. From Equations (3.63) and (3.64), it is clear that only two different integrals need to be calculated for a transmitting cell at the top or bottom of the whole reference body, whereas, three are needed for the other cases.
Figure 22: Convergence of the double sums versus a radial parameter \( p \) for the transmitting and receiving cells shown in the insert. An infinite wall of 10' pyramidal absorber is studied.
Figure 23: Convergence of the double sums versus a radial parameter \((p)\) for the transmitting and receiving cells shown in the insert.
Figure 24: Convergence of the double sums versus a radial parameter \( p \) for the cell (self impedance) shown in the insert.
Figure 25: Convergence of the double sums versus a radial parameter (p) for the transmitting and receiving cells shown in the insert. The curve for the self impedance given in the previous figure is repeated for comparison.
Figure 26: Convergence of the double sums versus a radial parameter \( p \) for the transmitting and receiving cells shown in the insert. Note that the spacing between the cells is equal to half of the interelement distance.
Figure 27: Convergence of the double sums versus a radial parameter (p) for the cell (self impedance) shown in the insert.
Frequency = 150 MHz
Max Segment Size = 0.125 Wld
Dielectric Constant = 3 - j4

The (1, 1) Element of the Zmn Matrix

Figure 28: Convergence of the double sums versus a radial parameter (p) for the cell (self impedance) shown in the insert.
CHAPTER IV
Measurements of Material Constitutive Parameters

4.1 Introduction

The constitutive relationships for linear isotropic materials in the frequency domain are given by

\[ D = \varepsilon(\omega)E, \quad \text{and} \]
\[ B = \mu(\omega)H. \]  \hspace{1cm} (4.1)

This chapter describes an approach for extracting the permittivity (\(\varepsilon\)) and permeability (\(\mu\)) of samples in free space. In Section 4.2, a theory for measuring the material parameters is discussed. The relation between the induced voltage of the antenna and the scattered E and H fields over the surfaces of the samples in free space is derived. Then, Section 4.3 describes two experimental setups for determining the dielectric constants of non-magnetic materials (\(\mu = \mu_0\)). To extract the dielectric constants, the Physical Optics (PO) approach is applied to the theory discussed in Section 4.2. Finally, the measured dielectric constants of some carbon-loaded foam materials are reported in Section 4.4.
4.2 Theory

From Equation (7.12) [21] or (3.35) [19], the Lorentz’ reciprocal relation is given by

\[ \int_S \left( \mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1 \right) \cdot \hat{n} \, da = \int_V \left( -\mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{E}_2 \cdot \mathbf{J}_1 + \mathbf{H}_1 \cdot \mathbf{M}_2 - \mathbf{H}_2 \cdot \mathbf{M}_1 \right) \, dv \]  

(4.3)

where \((\mathbf{E}_1, \mathbf{H}_1)\) and \((\mathbf{E}_2, \mathbf{H}_2)\) are fields from \((\mathbf{J}_1, \mathbf{M}_1)\) and \((\mathbf{J}_2, \mathbf{M}_2)\), respectively, all materials in \(V\) are linear matter, \(S\) encloses \(V\), and \(\hat{n}\) is a unit vector directed along the outward normal to \(S\). In Figure 29, Surfaces \(S_a\) and \(S_c\) enclose the impressed sources, \(\mathbf{J}^i\) and \(\mathbf{M}^i\), and Surface \(S_b\) encloses the scattered sources, \(\mathbf{J}^s\) and \(\mathbf{M}^s\), which are equivalent sources replacing the scatters outside the antenna system. Because there is no source and no nonlinear medium between \(S_a + S_c\) and \(S_b\), from
Figure 30: Incident and scattered modal voltages on the cross section of the cylindrical waveguide, $S_c$.

Equation (4.3), one has

$$
\int \int_{S_a+S_b+S_c} (E^i \times H^s - E^s \times H^i) \cdot \hat{n} \, da = 0 \tag{4.4}
$$

where $(E^i, H^i)$ and $(E^s, H^s)$ are fields excited from $(J^i, M^i)$ and $(J^s, M^s)$, respectively. Choosing $S_a$ along the metal surface of the antenna system or in any null field region yields

$$
\int \int_{S_b} (E^i \times H^s - E^s \times H^i) \cdot \hat{n} \, da = - \int \int_{S_c} (E^i \times H^s - E^s \times H^i) \cdot \hat{n} \, da \tag{4.5}
$$

where $S_c$ is a cross section of the cylindrical waveguide connected to a circulator as shown in Figure 29 and 30. Note that $S_a$ and $S_c$ just have to enclose the impressed sources, not the whole antenna system, and both the impressed and scattered sources radiate in the presence of the antenna system in free space. If only the dominant mode is allowed to propagate in the waveguide, then from Chapter 8 of [19], Equation (4.5) reduces to

$$
\int \int_{S_b} (E^i \times H^s - E^s \times H^i) \cdot \hat{n} \, da = -V^i_o I^s_o + V^s_o I^i_o \tag{4.6}
$$
where $V_o$ and $I_o$ are modal voltage and modal current of the dominant $E$ and $H$ fields in the cylindrical waveguide. The modal voltage $V_o$ and modal current $I_o$ are of the general forms

\[ V_o = V_o^+ e^{-\gamma o z} + V_o^- e^{\gamma o z} \]  
\[ I_o = I_o^+ e^{-\gamma o z} + I_o^- e^{\gamma o z} \]

or

\[ I_o = \frac{1}{Z_c} (V_o^+ e^{-\gamma o z} - V_o^- e^{\gamma o z}) \]

where $Z_c$ and $\gamma_o$ are the characteristic impedance and propagation constant of the dominant mode, the shape of the cylindrical waveguide is independent of $z$, and superscript $+$ and $-$ represent the dominant wave components traveling along $+z$ and $-z$ directions, respectively.

Substituting the above equations into Equation (4.6), one obtains

\[ \int \int (E^i \times H^s - E^s \times H^i) \cdot \hat{n} \, da \, \approx \, \frac{2}{Z_c} (V_o^{i+} V_o^{s-} - V_o^{i-} V_o^{s+}). \]  

If a well matched circulator is used, then

\[ V_o^{s+} \approx 0 \]

and Equation (4.10) becomes

\[ \int \int (E^i \times H^s - E^s \times H^i) \cdot \hat{n} \, da \, \approx \, \frac{2}{Z_c} V_o^{i+} V_o^{s-}. \]

Thus, the received voltage, $V_o^{s-}$, is proportional to a surface integral over the closed surfaces of the measured objects.

To determine both $\varepsilon$ and $\mu$ of a sample, two independent equations in the complex domain, and thus two different measurements for the sample are required.
A reference is also needed to determine the proportional factor, \( \frac{2}{Z_c} V_{o}^{i+} \). In Figure 31(a), a metal plate is used as the reference which from Equation (4.12) provides

\[
\iint_{S_1} (E^i \times H^s - E^s \times H^i) \cdot n_1 \, d\alpha = \frac{2}{Z_c} V_{o}^{i+} V_{o}^{s-}(\text{plate}) \tag{4.13}
\]

where a perfectly matched circulator is employed. Then, in Figure 31(b) and 31(c), the sample with and without a metal backing are measured to obtain two independent equations given by

\[
\iint_{S_2} (E^i \times H^s - E^s \times H^i) \cdot n_2 \, d\alpha = \frac{2}{Z_c} V_{o}^{i+} V_{o}^{s-}(\text{sample on plate}) \tag{4.14}
\]

and

\[
\iint_{S_3} (E^i \times H^s - E^s \times H^i) \cdot n_3 \, d\alpha = \frac{2}{Z_c} V_{o}^{i+} V_{o}^{s-}(\text{sample alone}). \tag{4.15}
\]

Taking the metal plate as the reference, one obtains, by dividing the above two equations with Equation (4.13), the following two linearly independent equations:

\[
\frac{\iint_{S_2} (E^i \times H^s - E^s \times H^i) \cdot n_2 \, d\alpha}{\iint_{S_1} (E^i \times H^s - E^s \times H^i) \cdot n_1 \, d\alpha} = \frac{V_{o}^{s-}(\text{sample on plate})}{V_{o}^{s-}(\text{plate})} \tag{4.16}
\]

and

\[
\frac{\iint_{S_3} (E^i \times H^s - E^s \times H^i) \cdot n_3 \, d\alpha}{\iint_{S_1} (E^i \times H^s - E^s \times H^i) \cdot n_1 \, d\alpha} = \frac{V_{o}^{s-}(\text{sample alone})}{V_{o}^{s-}(\text{plate})} \tag{4.17}
\]

where \( V_{o}^{i+} \) is assumed to be independent of \( V_{o}^{s-} \). Then, these two complex equations can be used to extract the \( \epsilon \) and \( \mu \) for the sample under test.
(a) A metal plate reference

(b) A sample on a metal plate

(c) The sample alone in free space

Figure 31: Three different measurements are needed to extract $\epsilon$ and $\mu$ associated with a sample under test.
4.3 Two Experimental Setups for Determining the Dielectric Constants of Non-magnetic Materials

In this section, two experimental setups for extracting the material properties of dielectric flat sheets are discussed. One setup uses spherical wave incidence, and the other employs plane wave incidence.

4.3.1 Spherical Wave Incidence

An experimental setup for measuring the material constitutive parameters is shown in Figure 32 where a $2' \times 2'$ absorber sheet backed with a metal plate is put on a foam table. An AEL horn is mounted 64" above the table and points normally to the center of the absorber sheet. Below the table, 18" pyramidal absorber is used to eliminate the scattering from the floor. The frequency responses of the metal plate and the absorber sheet with the metal backing are measured to obtain the ratio $\frac{V_o^s(a) \text{ (absorber on plate)}}{V_o^s(a) \text{ (plate)}}$ which is the right side of Equation (4.16). Also, background signals are subtracted from each response, and a smoothing process is performed after taking the ratio to remove the scattering outside the target region. Because the metal plate shields the section of the foam table behind it, its background should be measured with the foam table removed.

To determine the dielectric constants, the measured AEL horn patterns in free space are used as the incident fields. Also, the PO approach is employed to calculate the scattered E and H fields over the front surfaces of the absorber sheets. The surface integral over the back of the sheets with the metal backing is zero, and the returns from the four edge surfaces are neglected. Then, the ratio on the left side of Equation (4.16) is obtained. No multiple bouncing rays between the absorber sheet and the antenna system need to be included since this contribution is removed during the measurement. Equation (4.5) still holds because the higher order scattering can
Figure 32: An experimental setup for measuring the dielectric constants of the absorber sheets.
be considered as fields excited from another scattered source, and the superposition is applicable in a linear space.

For non-magnetic materials \((\mu = \mu_o)\), the dielectric constants may be determined from either Equation (4.16) or (4.17). The metal-backed setup is chosen because the metal plate shields the center part of the foam table. Therefore, in calculating the scattered fields, it is not necessary to include the reflection from the 5" thick foam table.

In the PO approach, the scattered \(E\) and \(H\) fields are obtained by multiplying the incident fields by the effective reflection coefficients at the incident points, such that

\[
E' = \hat{n}_\perp R_\perp E^i_\perp + \frac{1}{j\omega\epsilon_o} \nabla \times (\hat{n}_\perp R_\parallel H^i_\perp) \tag{4.18}
\]

or

\[
E' = \hat{n}_\perp R_\perp E^i_\perp + \frac{-j\kappa_o(\hat{s}_o - 2\hat{s}_o \cdot \hat{n}\hat{n})}{j\omega\epsilon_o} \times (\hat{n}_\perp R_\parallel H^i_\perp) \tag{4.19}
\]

and

\[
H^s = \frac{1}{Z_o}(\hat{s}_o - 2\hat{s}_o \cdot \hat{n}\hat{n}) \times E^s. \tag{4.20}
\]

Note that the locally plane wave approximation is used for the spherical wave incidence from the AEL horn, and as shown in Figure 33, \(\hat{s}_o\) is the directional vector of the incident ray, \(\hat{n}\) is the normal unit vector of the front face, \(\hat{s}_o - 2\hat{s}_o \cdot \hat{n}\hat{n}\) is the directional vector of the reflected ray, \(\hat{n}_\perp\) is a unit vector perpendicular to the plane of incidence (i.e., \(\hat{n}_\perp = \frac{\hat{s}_o \times \hat{n}}{|\hat{s}_o \times \hat{n}|}\)), \(E^i_\perp = \hat{n}_\perp \cdot E^i\), \(H^i_\perp = \hat{n}_\perp \cdot H^i\), \(R_\perp\) and \(R_\parallel\) are \(E\) perpendicular and \(E\) parallel effective reflection coefficients at the front face of the absorber sheets, \(\kappa_o = \omega\sqrt{\mu_o\epsilon_o}\), and \(Z_o = \sqrt{\mu_o/\epsilon_o}\).

The effective reflection coefficients are defined to be the ratio between the total reflected wave and the incident wave on an interface. For the case shown in Figure 34,
Figure 33: Rays excited from an AEL horn are incident upon a flat sheet.

one has [9]

\[
R \perp = \frac{R_{11} + R_{12} e^{-j2k_1 d_1 s_1 y}}{1 - R_{12} R_{10} e^{-j2k_1 d_1 s_1 y}} \quad (4.21)
\]

and

\[
R \parallel = \frac{R_{11} + R_{12} e^{-j2k_1 d_1 s_1 y}}{1 - R_{12} R_{10} e^{-j2k_1 d_1 s_1 y}} \quad (4.22)
\]

where

\[
R_{ij} \perp = \frac{Z_j s_{iy} - Z_i s_{jy}}{Z_j s_{iy} + Z_i s_{jy}} \quad (4.23)
\]

\[
R_{ij} \parallel = \frac{Z_i s_{iy} - Z_j s_{jy}}{Z_i s_{iy} + Z_j s_{jy}} \quad (4.24)
\]

\[
s_{iy} = \sqrt{1 - s_{iz}^2 - s_{ix}^2} = \sqrt{1 - \frac{\varepsilon_0}{\varepsilon_i} \left( s_{0z}^2 + s_{0x}^2 \right)} \quad (4.25)
\]
\[ \begin{align*}
\varepsilon_0 \mu_0 & \quad \varepsilon_1 \mu_1 & \quad \varepsilon_2 \mu_2 \\
\hat{s}_0 & \quad \hat{s}_1 & \quad \hat{s}_2 \\
- d_1 - & \quad \gamma
\end{align*} \]

Figure 34: Rays reflected from and transmitted thought a single layer.

\[ s_0, s_1, s_2 \] are the directional vectors of the rays shown in Figure 34 with

\[ s_i = \hat{x}s_{ix} + \hat{y}s_{iy} + \hat{z}s_{iz} \]  \hspace{1cm} (4.26)

\[ k_1 = \omega \sqrt{\mu_1 \varepsilon_1} \]  \hspace{1cm} (4.27)

d_1 \text{ is the thickness of the absorber sheet, and}

\[ Z_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}. \]  \hspace{1cm} (4.28)

In extracting the complex dielectric constants of the non-magnetic materials,

\[ \frac{\int (E' \times H' - E' \times H') \cdot \hat{n}_2 \, da}{\int (E' \times H' - E' \times H') \cdot \hat{n}_1 \, da} \]

first a table of the surface integral ratios, \( \frac{\hat{s}_2}{\hat{s}_1} \), is generated with respect to the real (\( \varepsilon_r \)) and imaginary (\( \varepsilon_i \)) parts of the complex dielectric constants, and the frequency. An approach derived by Lin [22] is applied to perform the numerical integration. Only 5 \times 5 points over a quarter of the \( 2' \times 2' \) sheet area are
required to obtain a convergent result for frequencies ranged from 2 to 18 GHz. Due to the symmetry of the incident fields and the absorber sheets, the integral is carried out over a quarter of the sheet area. By using this approach, it took about 2 CPU hours on VAX 8550 to generate a table for $\varepsilon_r$ varying from 1 to 3 at a step of 0.02, $\varepsilon_i$ from 0 to 3 at a step of 0.02, and frequency from 2 to 18 GHz at a step of 0.5 GHz. This table is then used to determine the dielectric constants for any non-magnetic material whose dielectric constant is within the $\varepsilon$ domain of the table and is measured under the same experimental setup. To extract the complex dielectric constants, the measured response ratio, $\frac{V_0^T-(\text{absorber on plate})}{V_0^T-(\text{plate})}$, is compared with the calculated surface integral ratio stored in the table (using both magnitude and phase) to search for any dielectric constant having the difference, $|\frac{\iint_{S_2} (E'^x \times H'^y - E'^y \times H'^x) \cdot \hat{n}_2 \, da}{\iint_{S_1} (E'^x \times H'^y - E'^y \times H'^x) \cdot \hat{n}_1 \, da}|$ less than some threshold (max error) at each frequency. Then, an average $\varepsilon$ weighted by the inverse of the difference is calculated. However, multiple solutions having similar responses but considerably different $\varepsilon$'s may occur at some frequencies. By comparing the dielectric constants with those at adjacent frequencies and knowing that the material parameters should vary smoothly with the frequency, one can solve the ambiguity.

The advantages of this approach for extracting the material parameters are:

1. a non-destructive method is used.
2. overall results weighted by the incident wave pattern are calculated.
3. alignment of the sample is not critical for spherical wave incidence.

However, the PO approach is better for a large, flat, and thin sheet under normal incidence. In our measurements, $2' \times 2' \times 0.5''$ flat sheets were used for determining the dielectric constants at frequencies between 2 and 18 GHz and the horn antenna was pointed normally to the absorber sheets.
4.3.2 Plane Wave Incidence

Another experimental setup is shown in Figure 35 where a 2' × 2' absorber sheet is glued to a 2' × 2' metal plate mounted vertically on a pedestal in the OSU/ESL compact range. The mounting device allows computer control of both azimuth and elevation rotations, and the compact range provides a better measurement system than the previous setup. Since a plane wave is incident upon the flat sheets, good alignment is required. This was accomplished by finding a center peak in both azimuth and elevation scans over the reference metal plate. A similar procedure to that used in the other setup can obviously be used to extract the dielectric constants. However, because the fields are uniform over the absorber surfaces for PO approach, no integration is needed in calculating Equation (4.16). Thus, the CPU time required
for evaluating this equation is more than 25 times less than that with the spherical wave incidence. Therefore, a search algorithm can be directly used to find the dielectric constants.

4.4 Dielectric Constants of Carbon-loaded Foam Materials

This section discusses the dielectric constants of some foam materials loaded with carbon particles. These materials are usually used to make microwave absorber for anechoic chamber applications. In order to determine the absorber performance accurately with PMM, a good estimation of the material parameters is essential. Both the spherical and plane wave setups have been applied in these measurements, and the absorber materials studied here are the E&C LS flat sheets, Rantec 8.5" black wedges, and E&C 8" blue wedges.

4.4.1 E&C LS Flat Sheets

With the experimental setup shown in Figure 32, the dielectric constants of E&C LS materials were determined. Four different samples of $2' \times 2' \times \frac{1}{2}''$ LS flat sheets with loading varying from LS-12 (special LS) to LS-30 were measured. Some of the results are plotted in Figures 36-40. The measurements of the first sample were repeated, and the dielectric constants of LS-12 to LS-20 determined from these measurements are shown in Figure 41. These two measurements of the first sample are very similar except at the low frequency end and 11 GHz where the data indicate characteristics associated with the measurement system. Therefore, a better calibration of the system is required. To check the measurement accuracy, two multi-layer reflection tests were performed and the results are shown in Figures 42 and 43. In this case, the calculated reflection coefficients of the multi-layers using the
measured dielectric constants are compared with the measured reflection coefficients given by \( |\frac{\varepsilon_r^{(\text{multi-layer})}}{\varepsilon_r^{(\text{plate})}}| \) for normal incidence. The results indicate that the measured dielectric constants are still accurate enough to predict the reflection level.

Sample # 3 is also measured in the OSU/ESL compact range with the setup shown in Figure 35. The dielectric constants of the LS materials measured with both polarizations are shown in Figures 44-47 for the two different thicknesses used in the calculations. Note that the solutions obtained from both polarizations are very close to each other. This indicates that the LS flat sheets are very uniform in thickness and loading, and the alignment of the metal plate was accurate. Note that the curves with the thicknesses of the samples chosen to be 0.51" have smoother variations than those with 0.50" thick layers. More detailed discussion about selecting the thickness will be addressed later. Comparing these results with those plotted in Figure 38 for the spherical wave setup, one finds that the agreement is good except at the low frequency end and around 13 GHz. The curves in Figure 44 and 45 vary more smoothly than those in Figure 38 because the compact range provides better measured results.
Figure 36: Complex dielectric constants of E&C LS materials measured with spherical wave incidence. Sample # 1
Figure 37: Complex dielectric constants of E&C LS materials measured with spherical wave incidence. Sample # 2
Figure 38: Complex dielectric constants of E&C LS materials measured with spherical wave incidence. Sample # 3
Figure 39: Complex dielectric constants of E&C LS materials measured with spherical wave incidence. Sample # 4
Figure 40: Complex dielectric constants of E&C LS-12 (special) materials measured with spherical wave incidence.
Figure 41: Complex dielectric constants of E&C LS materials measured with spherical wave incidence. Sample # 1 (Repeated)
Figure 42: A reflection test on measured dielectric constants using a multilayer panel - Example 1. The results given in Figure 41 are used in the calculations.
Figure 43: A reflection test on measured dielectric constants using a multilayer panel - Example 2. The results given in Figure 41 are used in the calculations.
Figure 44: Complex dielectric constants of E&C LS materials measured with a vertically polarized plane wave for Sample # 3. The layers are all assumed to be 0.50" thick in the calculations.
Figure 45: Complex dielectric constants of E&C LS materials measured with a horizontally polarized plane wave for Sample # 3. The layers are assumed to be 0.50" thick in the calculations.
Figure 46: Complex dielectric constants of E&C LS materials measured with a vertically polarized plane wave for Sample # 3. The layers are assumed to be 0.51" thick in the calculations.
Figure 47: Complex dielectric constants of E&C LS materials measured with a horizontally polarized plane wave for Sample # 3. The layers are assumed to be 0.51" thick in the calculations.
4.4.2 Commercial Wedge Absorber Materials

The dielectric constants of two commercial wedge materials, which are identified as the black and blue wedge samples, were measured with the plane-wave setup shown in Figure 35. Three pieces of $2' \times 2' \times 1''$ flat absorber sheets were cut from the $2' \times 2' \times 2''$ bases of a black wedge and two blue wedge panels. After taking measurements of these three $1''$ thick sheets, they were split into $2' \times 2' \times 0.5''$ flat pieces in order to study the effects of thickness on extracting the dielectric constant and to examine the uniformity of the absorber materials. All these $0.5''$ thick absorber samples (2 sets) were measured in the compact range. Then, a subset of these samples was selected to be further cut into $2' \times 2' \times 0.25''$ layers which were also measured. Both vertical and horizontal polarizations were employed to study the consistency of the measured results using different field polarizations. The uniformity of the thickness, the homogeneity of the materials, and the alignment of the metal plate on the mounting device can all cause variations in the results.

As shown in Figure 48, the dielectric constants of the $1''$-thick sheet are plotted with respect to frequency for the sheet thickness varied from $1.00''$ to $1.03''$. The results vary significantly even with a 1% increment of thickness, especially at frequencies where the thickness approaches a multiple of a half wavelength (6 GHz) in free space. Although an accurate measurement of the absorber thickness is possible, the absorber sheets cut from the wedge panels do not have uniform thickness in most cases. Thus, by trying several thicknesses and selecting the one with the smoothest behavior, one may obtain a solution that models the material properties more closely and minimizes the error due to thickness variations. The curve of $1.025''$ shows the smoothest response among these curves.

In Figure 49, dielectric constants measured with horizontally and vertically polarized plane waves are compared with the results measured with the absorber
sheet rotated by 90° from M1 to M2 shown in the inset of the figure. Note that the curves match rather well when the E-field is along the same direction relative to the absorber sample, but show some discrepancy for the other case. This indicates that the alignment of the metal plate was fine but the absorber sheet was not uniform in the thickness and/or material. The absorber sheets were aligned with M2 for the rest of the data shown in this chapter. The measured dielectric constants for two 0.5'' and two 0.25'' black wedge flat sheets with vertically polarized plane wave incidence are shown in Figures 50-53. The 1'' sample was split into # a and # b samples, and # a was further cut into # aa and # ab samples. The curves with the smoothest behavior for the 1'', 0.5'', and 0.25'' cases are plotted in Figure 54 and 55 for vertical and horizontal polarizations, respectively. The 0.25'' thick flat sheets are too thin electrically at the lower frequencies to have enough materials to accurately determine their dielectric constants. Except for this problem, the results for different thicknesses are essentially consistent. In addition, the black wedge material appears to be quite uniform since the dielectric constants of the two 0.5'' thick samples match with each other very well.

The measured dielectric constants for two blue wedge samples are shown in Figures 56-65 for various thicknesses. The curves with the smoothest variations are compared in Figures 66-69. Examining the dielectric constants among these cases, especially between two 0.5'' sheets, one finds that the dielectric constant of the blue wedge material varies significantly across the depth of the base. Some difference is also observed between vertical and horizontal polarizations, which may be due to the variations in the thickness as well as material properties.

From these results, one observes that the curves determined from the 1'' thick samples are not smooth, especially near the frequencies where the thickness approaches a multiple of a half wavelength in free space. As for the 0.25'' thick sheets,
they are not only too thin to be measured properly at the lower frequencies, but also do not have the frequency region sensitive to the variation of the thickness to be used in determining the average thickness for the best result. However, the $2' \times 2' \times 0.5''$ samples appear to be the best choice for extracting the dielectric constant for frequencies between 2 GHz and 18 GHz. With this in mind, the best data for the black wedge and two blue wedge samples are given in Figures 70 and 71 for both polarizations. Note that the two blue wedge materials are considerably different which implies that the material properties for the blue wedge absorber panels can vary significantly. In addition, the blue and black wedge materials are quite different, especially in the real part.
Figure 48: Complex dielectric constants of a 2' × 2' × 1" black wedge material measured with a horizontally polarized plane wave. Several thicknesses have been tested to search for a smooth response.
Figure 49: Complex dielectric constants of a 2' × 2' × 1" black wedge material measured with horizontally and vertically polarized plane waves. The results with the absorber sample rotated by 90° from M1 to M2 are also shown, and the thickness is chosen to be 1.025".
Figure 50: Complex dielectric constants of a $2' \times 2' \times 0.5''$ black wedge material (Sample # a) measured with a vertically polarized plane wave.
Figure 51: Complex dielectric constants of a 2' × 2' × 0.5" black wedge material (Sample # b) measured with a vertically polarized plane wave.
Figure 52: Complex dielectric constants of a $2' \times 2' \times 0.25''$ black wedge material (Sample # aa) measured with a vertically polarized plane wave.
Figure 53: Complex dielectric constants of a 2' × 2' × 0.25" black wedge material (Sample # ab) measured with a vertically polarized plane wave.
Figure 54: Complex dielectric constants of the black wedge material measured with a vertically polarized plane wave. The best results for all the five samples are plotted for comparison.
Figure 55: Complex dielectric constants of the black wedge material measured with a horizontally polarized plane wave. The best results for all the five samples are plotted for comparison.
Figure 56: Complex dielectric constants of a $2' \times 2' \times 1''$ blue wedge material (Sample #1) measured with a vertically polarized plane wave.
Figure 57: Complex dielectric constants of a $2' \times 2' \times 0.5''$ blue wedge material (Sample # 1a) measured with a vertically polarized plane wave.
Figure 58: Complex dielectric constants of a $2' \times 2' \times 0.5''$ blue wedge material (Sample # 1b) measured with a vertically polarized plane wave.
Figure 59: Complex dielectric constants of a 2' × 2' × 0.25" blue wedge material (Sample # 1aa) measured with a vertically polarized plane wave.
Figure 60: Complex dielectric constants of a 2' × 2' × 0.25" blue wedge material (Sample # 1ab) measured with a vertically polarized plane wave.
Figure 61: Complex dielectric constants of a $2' \times 2' \times 1''$ blue wedge material (Sample #2) measured with a horizontally polarized plane wave.
Figure 62: Complex dielectric constants of a $2' \times 2' \times 0.5''$ blue wedge material (Sample # 2a) measured with a vertically polarized plane wave.
Figure 63: Complex dielectric constants of a 2' × 2' × 0.5" blue wedge material (Sample # 2b) measured with a vertically polarized plane wave.
Figure 64: Complex dielectric constants of a $2' \times 2' \times 0.25''$ blue wedge material (Sample # 2ba) measured with a vertically polarized plane wave.
Figure 65: Complex dielectric constants of a 2' × 2' × 0.25" blue wedge material (Sample # 2bb) measured with a vertically polarized plane wave.
Figure 66: Complex dielectric constants of the first blue wedge material measured with a vertically polarized plane wave. The best results for all the five #1 samples are plotted for comparison.
Figure 67: Complex dielectric constants of the first blue wedge material measured with a horizontally polarized plane wave. The best results for all the five #1 samples are plotted for comparison.
Figure 68: Complex dielectric constants of the second blue wedge material measured with a vertically polarized plane wave. The best results for all the five \#2 samples are plotted for comparison.
Figure 69: Complex dielectric constants of the second blue wedge material measured with a horizontally polarized plane wave. The best results for all the five #2 samples are plotted for comparison.
Figure 70: Comparison of the black and blue wedge materials measured with a vertically polarized plane wave. Only the 0.5" thick samples are included here since they give the smoothest result.
Figure 71: Comparison of the black and blue wedge materials measured with a horizontally polarized plane wave.
5.1 Introduction

In this chapter, results of the PMM calculations and Radar Cross Section (RCS) measurements for several microwave wedge and pyramidal absorber designs are presented. These absorber materials include commercial wedge absorber, wedges with different dielectric losses, wedge widths and heights, wedges with curved surfaces, multi-layer wedge absorber, serrated wedge absorber (patented), commercial pyramids, straight pyramids, and curved pyramidal absorber. Both TM (H-field traverse to the wedge axis) and TE (E-field traverse to the wedge axis) cases are considered for the wedge absorber examples. The 2-D PMM solution described in Chapter II can only be used to calculate the TM case. For the TE case, the 3-D PMM solution discussed in Chapter III is applied with the reference element of the array being a thin section of the wedge (say $10^{-6}$ inches long). The reflection and transmission coefficients of an infinite absorber wall are determined from the PMM solutions in order to study the performance of anechoic chamber absorber. The Cray Y-MP supercomputer at The Ohio Supercomputer Center and the VAX 8550 at The ElectroScience Laboratory have provided the required computer resources. Also, the absorber RCS measurements were performed in OSU/ESL compact range to compare with calculated backscatter RCS levels. In the measurements, either a smoothing process or a
time-gating window was used to remove the scattering from the truncated ends and back faces.

To obtain the calculated backscatter RCS of an absorber panel, the magnitude of the specular reflection coefficient from the PMM solution for an infinite wall of absorber is multiplied by the calculated backscatter RCS of a metal plate having the same physical base dimension as the panel (24" × 24" for most cases). Note that for a 24" × 24" panel at 2 GHz and normal incidence, the reflection levels are about 19 dB lower than the RCS levels in dB relative to a square meter. In addition, calculated and measured time domain responses obtained from the inverse Fourier transform of the frequency domain responses are also shown for the TM case.

5.2 Commercial Wedge Absorber

This section shows the backscatter performance of one 18" and two 8" commercial wedge panels at normal incidence. Measured results are compared with calculated ones to validate the use of the PMM solutions in designing wedge absorber.

5.2.1 Black Wedge Absorber

Figure 72 shows the TM result for a plane wave normally incident upon a 24" × 24" commercial wedge panel which is identified as black wedge absorber. The dimensions of the wedge are described in the inset of the figure. One should especially note the very good agreement obtained between the RCS measurements and PMM calculations. The magnitude and phase of the calculated specular reflection coefficients for the TM case are plotted in Figure 73 with the phase reference chosen
at the edges of the wedges. In this case, the smooth variations in both the magnitude and phase versus frequency indicate that the leading edge diffraction term dominates for the black wedge panel for normal incidence. This is also observed in the time domain responses obtained by taking the inverse Fourier transform of the frequency domain responses. The time domain responses without windowing are plotted in Figure 74. In the measurements, the frequency range is from 2 to 18 GHz with a step of 0.01 GHz, and in the calculations, it is from 2 to 8 GHz with a step of 0.1 GHz. Because the phase reference is on the plane containing the edges of the wedges and the spike occurs at the origin, the dominant scattering center is clearly identified as the edge. Therefore, PMM can also be used to identify the scattering centers associated with an absorber panel. In Figure 75, the measured and calculated frequency responses for the TE case are compared. Again, very good agreement is obtained. Note that the levels of the TM responses are higher than that for the TE case. Thus, one should emphasize the TM case during the design phase.

In the calculations, the dielectric constant of the black wedge material shown in Figure 70 have been used. The good agreement between the RCS measurements and PMM calculations suggests that the dielectric constant was accurately determined. Figure 76 shows the CPU time on the Cray Y-MP (8 CPU's) supercomputer and the number of cells used in running the 2-D PMM code for the TM case. Note that smaller cells were employed for the lower frequencies which implies more accuracy for these cases. Only 1 minute of CPU time is needed for filling a 500 × 500 complex matrix, and the CPU time to solve the matrix is negligible compared to the total CPU time because a highly vectorized matrix solver has been used.
Figure 72: Comparison between RCS measurements and PMM calculations for an 8" commercial black wedge. A TM polarized plane wave is incident normally on a 24" x 24" black wedge panel.
Figure 73: The specular reflection coefficient versus frequency for an infinite wall of the 8" commercial black wedge absorber illuminated by a TM polarized plane wave.
Figure 74: The calculated and measured impulse responses for the TM case of the 8" commercial black wedge.
Figure 75: Comparison between RCS measurements and PMM calculations for the 8" commercial black wedge. A TE polarized plane wave is incident normally on a 24" x 24" black wedge panel.
Figure 76: The CPU time on the Cray Y-MP supercomputer and the number of cells in running the PMM program for the black wedge absorber.
5.2.2 Blue Wedge Absorber

Another 24" × 24" commercial wedge panel is shown in Figure 77 where both the TM and TE polarized plane waves illuminate the absorber at normal incidence. This wedge panel is identified as the blue wedge throughout this report. The TM response appears to have a larger return than the TE response which is normally the case for wedge absorber. Good agreement between the RCS measurements and PMM calculations is again achieved. However, larger differences occur at both the lower and higher frequencies. This may be due to contributions from the truncated ends and/or the material property variations across the depth of the measured wedges. The dielectric constants of the blue wedge material which have been used in the PMM calculations are plotted in Figure 70. The magnitude and phase of the calculated specular reflection coefficient (SRC) for the TM case are shown in Figure 78 with the phase reference set at the edges. Note that there is one deep null point between every two peaks. This indicates that there are two scattering centers of about the same magnitude interacting with each other. The TM measured and calculated time domain responses for the blue wedge panel without windowing are plotted in Figure 79 which clearly shows two scattering centers with similar magnitude. These centers are separated by about 1 ns in return time which implies two scattering centers separated by about 6" in free space. Since the phase reference is chosen at the edges of the wedges and the depth of the blue wedge panel is 6", it can be assumed that the dominant scattering comes from the edges and valleys. The measured response has a longer delay for the valley scattering because the depth of the measured blue wedge panel was actually deeper than 6".
Figure 77: RCS measurements and PMM calculations for an 8" commercial blue wedge.
Figure 78: The specular reflection coefficient versus frequency for an infinite wall of the 8" commercial blue wedge absorber illuminated by a TM polarized plane wave.
Figure 79: The calculated and measured impulse responses for the TM case of the 8\textquotedbl{} commercial blue wedge.
5.2.3 18" Commercial Wedge Absorber

The backscatter performance of an 18" commercial wedge panel is shown in Figure 80. This wedge was made from a foam material similar to the LS-18 flat sheet; thus, the corresponding dielectric properties of the LS-18 absorber given in Figure 46 were used in the calculations. Without measuring the dielectric constant of this wedge sample, one should not expect excellent agreement between measurements and calculations. In spite of the large wedge size, the performance of the 18" commercial wedge is only about 10 dB better than that of the 8" commercial wedges.
Figure 80: The RCS measurements and PMM calculations for an 18" commercial wedge for both the TM and TE polarizations.
5.3 Homogeneous Wedge Absorber

In this section, the performance of some uniformly doped wedges are analyzed with PMM. Variations in doping levels, wedge widths and wedge heights are discussed for the TM case only. Also, the measured and calculated results for a 16" wedge absorber are compared for both the TM and TE cases.

5.3.1 Wedge Absorber with Different Dopings

Comparison of wedges with different dielectric losses is given in Figure 81 where the specular reflection coefficients calculated from PMM are plotted versus frequency for the TM case and normal incidence. Based on these results, one should expect about a 7dB improvement by using the LS-16 material instead of LS-20. Note that the dielectric constants plotted in Figure 41 have been employed in these calculations.

For a proper absorber design, the reflection from the absorber backface should be small compared to that from the leading edges. To satisfy this criterion, the transmission coefficient can be examined to determine if the material has enough loss. For this purpose, the specular transmission coefficients for both LS-16 and LS-20 wedges are presented in Figure 82. From Equation (2.41), the maximum backface reflection is \(-47\) dB \((2 \times -25 + 3\) dB) for the LS16 wedges at 2 GHz which is about 15 dB lower than the reflection from the absorber itself. Therefore, the doping of the 8" commercial blue wedge absorber can be reduced to that of the LS-16 material for use above 2 GHz.
Figure 81: Comparison of the reflection coefficients for wedges with two different dielectric constants.
Figure 82: Comparison of the transmission coefficients for the wedges with two different dielectric constants.
5.3.2 Wedges with Different Widths

In Figure 83, the specular reflection coefficients of wedges with different widths are compared for the TM case and normal incidence. In all cases, the wedge total height is 10", the base is 2" thick, and the doping for the wedges is assumed to be that of the LS-16 material given in Figure 41. The calculated results indicate that for wedge widths between 1.8" and 6.7", the reflection levels of the constant height wedges are about the same. Although a narrower wedge has a sharper edge and therefore a smaller RCS, the density of the edges increases to cause more scattering. Thus, as long as the wedge angle is not too large, the wedge width is not an important factor for the performance of the absorber shown here. The transmission coefficients for the same cases are shown in Figure 84. Those wedges have sufficient absorption of the incident waves to avoid any significant scattering from the absorber backfaces.

5.3.3 Wedges with Different Heights

In Figure 85, wedges having the same width but different heights are compared. The wedge width is 4", the base is 2" thick, and the LS-16 material (see Figure 41) is used for all cases. By increasing the wedge height from 8" to 25", the reflection coefficient can be reduced by about 10 dB. As the wedge becomes higher, a better impedance match between the absorber and air is obtained.
Figure 83: Comparison of wedges with different wedge widths. The specular reflection coefficient for the TM case and normal incidence is plotted versus frequency. The wedge height is 10" and the base thickness is 2" for all cases.
Figure 84: Comparison of wedges with different wedge widths. The specular transmission coefficient for the TM case and normal incidence is plotted versus frequency.
Figure 85: Comparison of wedges with different wedge heights for the TM case and normal incidence.
5.3.4 16" Wedge Absorber

In Figures 86 and 87, the measured results for two samples of a 16" wedge are compared with the calculated results for both the TM and TE cases at normal incidence. Because of the variations in the material properties as well as the cutting errors, the RCS measurements for these two samples differ considerably, especially at higher frequencies. Nevertheless, the PMM calculations using the LS-16 material given in Figure 46 predict the performance of the first sample reasonably well. From the calculated results, one can see that the 16" wedge is slightly better than the 18" commercial wedge shown in Figure 80.
Figure 86: The RCS measurements and PMM calculations for a 16" straight wedge with the E-field polarized along the wedge axis. Two samples of the 16" wedge were measured.
Figure 87: The RCS measurements and PMM calculations for the 16" wedge with the H-field polarized along the wedge axis. Two samples of the 16" wedge were measured.
5.4 Wedges with Curved Surfaces

Some curved pyramid designs had been fabricated and measured by DeWitt [1]. He concluded that the overall scattering levels of the curved pyramids he designed did not improve significantly due to the scattering from the absorber material inhomogeneities, and some flaws in cutting the curved surfaces. In this section, results of PMM calculations and RCS measurements are given for both a 10" and 13" curved wedge design. Curved pyramidal absorber will be discussed later in another section. Since a theoretical model is used to analyze the curved wedges, these wedges can have uniform doping and accurate shaping. Note that the curved surfaces are modelled with piecewise linear segments.

5.4.1 10" Curved Wedge Absorber

Results for a curved wedge absorber panel that was designed using the PMM solution are shown in Figure 88. In order to preserve the material properties of the curved wedges, they were cut from the 8" commercial blue wedges examined earlier. A complementary geometry was chosen so that a single cut through a foam block can make two similar wedge panels to maximize factory production. The curved wedge consists of a 3" base width, a 2" base thickness, an 8" wedge height, and a 16" radius of curvature on the wedge surfaces. Several radii of curvature, wedge widths, and wedge heights have been examined. The best design is shown in the figure. By comparing Figures 77 and 88, one should observe about a 10 dB improvement in performance of the curved wedge relative to the commercial one. Due to cutting errors and material property variations across the measured samples, one should not expect perfect agreement between the calculated and measured results. Nevertheless, the PMM calculations still follow the same trends as the RCS measurements.
The magnitude and phase of the calculated specular reflection coefficient for the TM case are shown in Figure 89 with the phase reference set at the edges. Also, the measured and calculated time domain responses for the curved wedge are plotted in Figure 90. The calculated result shows two major scattering centers but the measured response indicates only one. The missing second strong spike in the measured result may be due to cutting errors and misalignment of the valleys. Since the calculated centers are separated by about 1 ns in return time and the phase reference is chosen at the edges of the wedges, the dominant scattering comes from the edges and about 2" above the valleys in the PMM calculations. Thus, the pointed edge and narrow valley reduce the scattering from the edge and valley, but the curved surface appears to cause some additional return, although it is very small.
Figure 88: RCS measurements and PMM calculations for a 10'' curved wedge.
Figure 89: The specular reflection coefficient versus frequency for the curved wedge with the E-field polarized along the wedge axis.
Figure 90: The calculated and measured impulse responses for the 10" curved wedge with the E-field polarized along the wedge axis.
5.4.2 13" Curved Wedge Absorber

The frequency and time domain responses for a 13" curved wedge are shown in Figures 91-93. The dielectric constant of the wedge is assumed to be that of the LS-16 material given in Figure 46. The curved wedge has a complementary shape with a 4" base width, a 2" base thickness, a 11.3" wedge height, and a 20" radius of curvature on the wedge surfaces. Note that several radii of curvature have been examined, and the best one is shown here. Two samples of the curved wedge were fabricated and measured; however, both samples had cutting errors. Therefore, the agreement between measurements and calculations is not very good, especially at the higher frequencies. The measured impulse responses indicate a much stronger scattering from the edges than that estimated with PMM. One should realize that it becomes more and more difficult to achieve these low scattering levels because the material property variations and cutting errors become more important. In Figure 94, calculated results for two different segment sizes in modelling the curved surfaces are compared. Note that 1" is equal to 0.6 \( \lambda_d \) at 6 GHz (\( \lambda_d = \lambda_0 / \sqrt{|\varepsilon_r|} \), the wavelength in the material), and the calculated results with the smoother surfaces show lower reflection levels at the higher frequencies. This indicates that smooth cutting is necessary to fabricate a good curved wedge design.
Figure 91: The RCS measurements and PMM calculations for a 13\textdegree
curved wedge with the E-field polarized along the wedge axis. Two samples were measured.
Figure 92: The RCS measurements and PMM calculations for the 13'' curved wedge with the H-field polarized along the wedge axis. Two samples were measured.
Figure 93: The calculated and measured impulse responses for the 13° curved wedge with the E-field polarized along the wedge axis.
Figure 94: The PMM calculations with two different segment sizes in modelling the curved surfaces for the 13'' curved wedge.
5.5 Multi-Layer Wedge Absorber

In this section, several multi-layer designs are presented. The concept of using multi-layer wedges was previously investigated on an experimental basis by Munk and Burnside [5]. A better impedance match between air and absorber can be achieved by attaching layers of materials with proper dielectric losses to the wedge surfaces. With PMM, the multi-layer absorber can be analyzed numerically on the computer and many cases may be tested to find an optimized solution.

5.5.1 Four-Layer Wedge Absorber

A multi-layer wedge made by attaching four different LS layers to the commercial blue wedge is shown in Figure 95. The four layers are LS-12, LS-14, LS-16, and LS-18 from free space to the blue wedge. These materials have increasing dielectric losses from LS12 to the blue wedge; thus, these layers provide an impedance match between the air and the blue wedge. In addition, the fields inside the wedge can be absorbed sufficiently in the blue wedge for frequencies above 2 GHz. According to this design, a 23" long single four-layer wedge section has been fabricated. The measured dielectric constants of these layers are plotted in Figure 41. Because the dielectric constants of the LS-16 and LS-18 samples are very close to each other, this multi-layer wedge is virtually composed of 4 different materials, including the blue wedge. Measured and calculated results, together with those for a 23" long single commercial blue wedge section, are shown in Figure 96 for the TM case and normal incidence. In the measurements, a smoothing process was performed to remove the scattering from the truncated ends and bottom of the wedge section. In the calculations, the dielectric constant of the LS-20 sample given in Figure 41 was used for the blue wedge, and the reflection coefficients were multiplied by the calculated RCS of
a 4" × 23" metal plate to compare with the measured RCS. As long as the edge scattering dominates (including the internal edges), the calculated and measured results should be close. In fact, as shown in Figure 96, the agreement between calculations and measurements is good. With four layers of the LS materials on the blue wedge, one is able to improve the performance of the blue wedge by about 10-15 dB for frequencies between 2 GHz and 12 GHz.

A 24" × 24" four-layer wedge panel was also fabricated and measured, as shown in Figure 97 for both the TM and TE polarizations. In this case, the measured responses are considerably higher than the calculated ones. This may be due to differences in material properties as opposed to those modelled in the PMM program. Note that the dielectric constant of the LS12 material given in Figure 44 and those in Figure 46 were used in the calculations for the four layers and the internal wedge.
Without determining the dielectric constants, one should not expect close agreement between measured and calculated results. In Figure 98, both the measured and calculated time domain responses are shown, which indicate four major scattering centers due to the four absorber layers. Although the multi-layer wedge has more scattering centers than the homogeneous wedge, these returns are not only smaller but also separated in space to cancel each other at most frequencies. Therefore, improvement over the ordinary wedge absorber can be achieved with the multi-layer approach.
Figure 96: Comparisons between the measurements and calculations for the commercial blue wedge and the four-layer design for a 23" long single wedge section.
Figure 97: The RCS measurements and PMM calculations for a 24" × 24" four-layer wedge panel.
Figure 98: The calculated and measured impulse responses for the four-layer wedge with the E-field polarized along the wedge axis.
5.5.2 Eight-Layer Wedge Absorber

For the four-layer design, a peak occurs around 3 GHz due to the constructive phasing of the scattering from each of the layers. To remove this peak and design a wedge with lower reflection levels, more layers with smaller thicknesses are required. Thus, a design with eight $\frac{1}{8}$" layers on LS16 wedges as shown in Figure 99 was tested. Note that the dielectric constants of the layers progressively increase from the outer layers to the inner ones using the data given in Figure 41. The calculated results for the TM case and normal incidence are plotted in Figure 100 together with those for the first four-layer design and the commercial wedge. This new design is better than the four-layer design by about 10 dB for frequencies above 2.5 GHz, but has a high return at the low frequency end. It was found that the thicknesses of the layers were too thin for good phase cancellation at the low frequencies. From plots of the PMM modal current density for this 8-layer design, it was determined that the scattering from the inner layers dominated, especially at the low frequency end. Thus, another design with the thickness of the last layer increased to $\frac{3}{8}$" as plotted in Figure 101 was examined. The specular reflection coefficient for this design is given in Figure 102 which shows the desired improvement at the low frequency end and has similar performance to the previous 8-layer design at other frequencies. Note that the reflection coefficient of the second 8-LS-layer design is below $-50$ dB for frequencies at and above 2 GHz. As shown in Figure 103, the performance at the low frequencies can be further improved by using four-$\frac{1}{8}$", three-$\frac{1}{4}$", and one-$\frac{3}{8}$" layers.
Eight $\frac{1}{8}''$-thick Layers

Dielectric Constants of the Layers are $\frac{1}{4}$ LS12, $\frac{1}{2}$ LS12, $\frac{3}{4}$ LS12, LS12, $\frac{1}{3}$ between LS12 and LS14, $\frac{2}{3}$ between LS12 and LS14, LS14, and Half between LS14 and LS16 from Air to the LS16 Wedges.

Figure 99: An eight-layer design which has eight $\frac{1}{8}''$ thick LS layers on a wedge with the dielectric constant of the LS16 material.
Figure 100: Comparison of the 8-layer design (all \( \frac{1}{8}'' \)) with the four-layer design and commercial wedge.
Dielectric Constants of the Layers are $1/4$ LS12, $1/2$ LS12, $3/4$ LS12, LS12, $1/3$ between LS12 and LS14, $2/3$ between LS12 and LS14, LS14, and Half between LS14 and LS16 from Air to the LS16 Wedges.

Figure 101: An eight-layer design which has seven $\frac{1}{8}$" and one $\frac{3}{8}$" thick LS layers on a wedge with the dielectric constant of the LS16 material.
Figure 102: Comparison between two 8-layer designs.
Figure 103: Comparison between two 8-layer designs.
5.6 Serrated Wedge Absorber

To fabricate the multi-layer wedges, it is necessary to have careful quality control in loading the proper doping level for each layer, and numerous layers are needed to achieve extremely low reflectivity levels. In addition, the multi-layer design requires adhesive between layers, which may cause some scattering. Thus, an alternative approach for designing the impedance matching layer on the wedge surfaces by using serration was studied.

An example of the serrated wedge design is shown in Figure 104 where ten fins are fabricated on each wedge. Also, a 0.6" vertical shift is designed to make the fins on one side of the wedge to lie midway between the fins on the other side so that the locations of the fins on each wedge are distributed evenly to have better phase cancellation along the vertical direction. Note that serration provides an impedance transition from air to the interior wedges, and serrated wedge absorber is a double impedance matching design using wedges. One matching scheme is the interior wedges, and the other is the tiny wedges or fins that make up the serrated surfaces. Note that a patent right has been filed for serrated electromagnetic absorber by E&C and OSU.

In Figure 105, the PMM calculations for the serrated wedge are plotted for the TM case and normal incidence. The material properties of the serrated wedge are chosen to be that of the LS-16 sample shown in Figure 41. Comparing this design with the four-layer wedge, one should note about a 10 dB improvement at most frequencies. Because the minimum distance between fins along the direction of the incident plane wave is 0.6", the returns from the fins become in phase at a multiple of 9.8 GHz. However, the first peak at 9.8 GHz is still 50 dB lower than that from an equivalent metal ground plane. Also, the peak occurs because the periodic array
of identical wedges has perfect panel alignment and uniform material properties. In an anechoic chamber, there will be some random factors associated with the panel position and material loading which will actually help the performance of the absorber at high frequencies due to phase cancellation. Note that at 9.8 GHz a quarter of wavelength is only 0.3". For symmetrical serration, the first peak will occur at 4.9 GHz. A comparison of the serrated designs with and without the 0.6" shift is given in Figure 106. Note that the 0.6" shift does improve the performance of the wedges considerably at frequencies around 5 GHz.

Figure 107 presents another serrated wedge design which has eight fins on every wedge and a 0.75" shift. Two samples of 20" x 24" panels have been fabricated according to this design. The calculated and measured frequency domain responses for the TM and TE cases at normal incidence are plotted in Figures 108 and 109, respectively. Note that the material properties of this serrated wedge is assumed to be that of the LS-16 sample given in Figure 46. The time domain responses for both the calculations and measurements are compared in Figure 110. In the calculated response, eight dominated scattering centers are clearly identified and they correspond to the eight fins equally spaced along the vertical direction. On the other hand, the measured response shows more scattering centers with unequal spacings because of cutting errors, and some returns are also seen in the 2" base region due to the inhomogeneity of the carbon-loaded foam material. Therefore, without an accurate controlled cutting routine and homogeneous foam material, one should not expect perfect agreement between measurements and calculations. Comparison between asymmetry and symmetry designs for this serrated wedge absorber is given in Figures 111 and 112. Again one can clearly see the importance of using the shift. Note that for the asymmetry design, the first peak occurs at a frequency near 8 GHz and the reflection level of the peak is still below −50 dB.
The reflection performance of the first serrated wedge design with two different doping levels is shown in Figure 113. With the material properties of the serrated wedge varied from that of LS15 to midway between LS14 and LS16, only a small difference is seen in the specular reflection coefficients. Therefore, the loading of the serrated wedge absorber with a uniform dielectric constant is not a critical factor in terms of its reflection performance. However, a thicker base was used for the less lossy material in order to avoid the scattering from the absorber backface.

In Figure 114, the serrated wedges with different numbers of fins are compared. The material properties of these serrated wedges are chosen to be that of the LS-16 sample shown in Figure 41. Because the height of the interior wedge is 6", the peaks due to the constructive phasing of the fin scattering centers occur at frequencies near a multiple of $N$ GHz, where $N$ is the number of fins on each wedge with the fins on the left side of the wedge shifted by half of the vertical distance between two adjacent fins on one side. Thus, the positions of the peaks can be controlled by the number of fins. Note that the reflection levels at low frequencies do not change significantly for the cases shown.

Comparison of the serrated wedge absorber for various lengths of the fins is presented in Figure 115. These wedges have ten fins attached to each wedge, and the dielectric constant of the LS-16 sample plotted in Figure 41 is employed. As is expected, the longer the fins are, the better the reflection performance is, especially at low frequencies. Thus, the serrated wedge design has the versatility in its capability of arranging the number, lengths, and positions of the fins depending upon the desired performance for the absorber.

In fabricating the fins, the edges of the fins may be dull because of cutting errors. To study this problem, the serrated wedges with flat edges were examined. In Figure 116, the cases with $\frac{1}{16}$" and $\frac{1}{32}$" wide flat fins are plotted together with
the sharp serrated wedges. As indicated in Figure 117, the $\frac{1}{32}''$ case shows only a small increase in the reflection coefficient; while, the performance of the $\frac{1}{16}''$ case is degraded by about 5 dB at low frequencies and 10 dB at high frequencies for the TM case and normal incidence. Since the flat edges become larger in wavelength at a higher frequency, the dull-fin problem is more serious at high frequencies.
Serrated Wedge Absorber
(A Patent Filed by E&C and OSU)

Figure 104: A serrated wedge design with ten fins on each wedge. The length of the fins is 4" from the higher valley to the edge and there is a 0.6" vertical shift on the left fins.
Figure 105: Comparison of the reflection performance for the serrated wedge design, the four-layer design, and the commercial blue wedge.
Figure 106: Comparison between the asymmetry and symmetry designs for the TM case and normal incidence.
Serrated Wedge Absorber

(A Patent Filed by E&C and OSU)

Figure 107: A serrated wedge design with eight fins on each wedge. The length of the fins is 4" from the higher valley to the edge and there is a 0.75" vertical shift on the left fins.
Figure 108: The measured and calculated RCS's of the serrated wedge panel with eight 4" long fins on each wedge for the TM case and normal incidence.
Figure 109: The measured and calculated RCS's of the serrated wedge panel with eight 4" long fins on each wedge for the TE case and normal incidence.
Figure 110: The measured and calculated time domain responses of the serrated wedge panel with eight 4" long fins on each wedge for the TM case and normal incidence.
Figure 111: Comparison between the asymmetry and symmetry designs for the TM case and normal incidence.
Figure 112: Comparison between the asymmetry and symmetry designs for the TE case and normal incidence.
Figure 113: Comparison of the serrated wedges with different dielectric constants.
Figure 114: Comparison of the serrated wedges with different numbers of fins.
Figure 115: Comparison of the serrated wedges with different lengths of the fins.
Figure 116: Geometry of the serrated wedges with pointed and flat fins.
Figure 117: Comparison between the serrated wedges with pointed and flat fins.
5.7 Commercial Pyramidal Absorber

In Figure 118, the backscatter results for an 8" commercial pyramid is presented for both vertical (VP) and horizontal (HP) polarizations. Due to symmetry, one would expect the same result for both polarizations; however, the differences observed can be easily attributed to inhomogeneities and potential misalignment. Nevertheless, good agreement between measurements and calculations was obtained at the lower frequency end. On the other hand, a considerable difference is seen around 3.5 GHz, which may be due to the discrepancy of the dielectric properties between the calculated and measured results. Note that the dielectric constants of the 8" commercial blue wedge were employed in the PMM calculations for the pyramid.

The backscatter RCS of an 18" commercial pyramid is plotted in Figure 119. The dielectric constant of the pyramid is assumed to be that of an LS-16 sample shown in Figure 46. Due to the large pyramid size and the limited but valuable supercomputer time and memory, the results were only calculated from 2 to 3 GHz. As shown in the figure, the calculations and measurements agree very well for this case. Also, the performance of the 18" pyramid is about 15 dB better than that of the 8" pyramid for the lower frequencies.

In Figure 120, the measured responses for the commercial 8" and 18" pyramids are compared with the calculated response for the 13" serrated wedge plotted in Figure 107. Since there were quite a few cutting errors in fabricating the serrated wedge panels, the theoretical result is used for this comparison to show the performance that can be achieved. From this comparison, one can see that the scattering performance of the 13" serrated wedge absorber is similar to that of the 18" pyramidal absorber for frequencies between 2.5 GHz and 6 GHz, and is more than 10 dB.
better than the 8" pyramid for frequencies between 2 GHz and 6 GHz. As mentioned earlier, the peak around 8 GHz has a reflection level below -50 dB and will be further reduced by the misalignment of panels in a real chamber. Besides, the location of the peak can be pushed to even higher frequencies by using more fins on the wedge surfaces. For the low frequency performance, the reflection level is around -50 dB for this serrated wedge, which is excellent for a wedge structure. Turning our attention now to the 8" pyramid, one should notice the superior scattering performance of the 13" serrated wedge. In fact, it is expected that the 13" serrated wedge will perform as well as a 13" pyramid at normal incidence. That being the case, one of our major design goals has been achieved.
Figure 118: The RCS measurements and PMM calculations for an 8" commercial pyramid.
Figure 119: The RCS measurements and PMM calculations for an 18" commercial pyramid.
Figure 120: Comparison between the serrated wedge and commercial pyramidal absorber.
5.8 Pyramidal Absorber for Low Frequency Applications

In this section, several pyramidal absorber designs for low frequency applications are presented. Scale models are considered so that one can select the proper absorber geometry depending on the desired operational frequency range. Note that the dielectric constants employed in this simulation should be the values used for the full scale absorber since only the geometry and frequency are scaled.

5.8.1 72” Pyramidal Absorber

The PMM calculations for a \( \frac{1}{30} \) scale model of a 72” pyramidal absorber which is commercially available is shown in Figure 121. The specifications of the absorber from Rantec are also given in the figure. As one compares these two results, he should note that the calculated results show a much lower reflection level than the specifications at the higher frequencies. In the calculations, the dielectric constant of this material has been assumed to be eight times that of the LS20 material shown in Figure 44 at the scale frequencies; i.e., \( 8 \times (\varepsilon_r - 1) + 1 \) at 2 GHz and above. The dielectric constants of the material for the pyramid are shown in Figure 122. As indicated in Figure 123, the material properties are chosen so that the 72” absorber can have sufficient absorption for frequencies above 100 MHz. Note that the commercial material may have a higher dielectric loss than that used in the program simulation. This may be one of the reasons that the absorber is specified to have a larger reflection level than that predicted from the PMM calculations. The CPU time on the Cray Y-MP supercomputer needed to run the 3-D PMM code for this absorber is plotted in Figure 124, together with the number of cells used in modelling the pyramid.
In Figure 125, 72" pyramids with a 12" width are compared for various base thicknesses. Note that the thickness of the base determines the location of the valley with respect to the tip of the pyramid. This causes the peak to shift toward the lower frequencies as the distance between the tip and valley increases; i.e., the thickness decreases. Overall, the reflection levels for the thickness varied from 15" to 30" are about the same. The case with a 10" thick base has a high return at the low frequency end, which may be due to the scattering from the absorber backface. The specular transmission coefficients for these pyramids are shown in Figure 126. The results indicate that the loss in one pass through the pyramid wall decreases as the base thickness is reduced, and the case with a 10" thick base does show a significant level of penetration. The CPU time needed to calculate the pyramid with a 10" thick base is shown in Figure 127. With a narrower width, not only the double infinite sum converges faster, but also the number of the cells is less. Figures 128-131 plot the PMM results for different dielectric constants. With a less lossy material, the reflection coefficient tends to increase at the lower frequencies but decrease at the higher frequencies, and the transmission coefficient becomes larger at all frequencies, as expected.
An Infinite Wall of Pyramid Absorber
1/30 Scale Model

Figure 121: The specular reflection coefficients of a commercial 72" pyramidal absorber from PMM calculations and company specifications. A $\frac{1}{30}$ scale model is employed in the calculations.
Figure 122: Dielectric constant of a material, which is child times more lossy than...

- Imaginary Part of Dielectric Constant
- Real Part of Dielectric Constant

Frequency (GHz)
Figure 123: The calculated specular transmission coefficient of the commercial 72" pyramidal absorber.
Normal Incidence
Material: 8 X LS20
An Infinite Wall of Pyramid Absorber

Figure 124: The CPU time on the Cray Y-MP supercomputer and the number of cells in running the PMM program for the commercial 72'' pyramidal absorber.
Figure 125: The calculated specular reflection coefficients of a 72" pyramidal absorber with a 12" wide base for various base thicknesses. A $\frac{1}{30}$ scale model is employed in the calculations.
Figure 126: The calculated specular transmission coefficients of the 72" pyramidal absorber with a 12" wide base for various base thicknesses. A $\frac{1}{30}$ scale model is employed in the calculations.
Figure 127: The CPU time on the Cray Y-MP supercomputer and the number of cells in running the PMM program for the 72" pyramidal absorber with a 12" wide and 10" thick base.
Figure 128: Comparison of the 72" pyramids with different dielectric constants for the specular reflection coefficient. The thickness of the base is 30".
Figure 129: Comparison of the 72" pyramids with different dielectric constants for the specular transmission coefficient. The thickness of the base is 30".
Figure 130: Comparison of the 72" pyramids with different dielectric constants for the specular reflection coefficient. The thickness of the base is 10".
Figure 131: Comparison of the 72" pyramids with different dielectric constants for the specular transmission coefficient. The thickness of the base is 10".
5.8.2 75" Curved Pyramidal Absorber

A 75" curved pyramid design is shown in Figure 132. The top section of the pyramid has a 780" concave radius of curvature on each surface to make the tip more pointed, and the bottom section has a 180" convex radius of curvature to make the valley more gradual. Thus, a better impedance match between air and absorber can be achieved. Note that many cases with a 60" pyramidal section and a 12" base width have been tested, and the best one is presented here. For the pyramidal absorber, better results can be obtained with the junction of the top and bottom curved sections located about a quarter of the pyramidal section above the valley. Unfortunately, this is not a complementary shape which means that it will be a little more costly to produce. In Figure 133, the curved pyramids with a 15" and 30" thick bases in free space, and also a 15" thick base on a metal ground plane are compared. The hybrid approach discussed in Section 2.5 was applied to calculate these three cases. Note that there is approximately a 5 dB difference between the cases with and without metal backing. Figure 134 shows the comparison between the curved and straight pyramids of the same height, width and material. The curved pyramid provides a considerable improvement over the straight one for frequencies near 120 MHz where the curved shape produces a null response. This was accomplished by optimizing the radii of curvature and the location of the junction to give a minimum reflection around 120 MHz which is where the straight pyramid starts to deteriorate. Thus, the curved surfaces provide additional design parameters which can be used to control the location of the first null point at lower frequencies; however, the changes are rather subtle and require the use of a numerical solution to solve for the optimized shape.

In Figure 135, the transmission coefficients for the curved and straight pyramids are compared. Since the volumes and shapes of these two pyramids are quite similar,
the attenuation losses from these infinite walls of pyramids are essentially the same. Comparisons between the hybrid and PMM (using PMM only) approaches are shown in Figures 136 and 137 for the curved pyramid, and in Figures 138 and 139 for the straight pyramid. Although there are some differences between these two approaches due to numerical errors, the results of these approaches follow the same trends, especially for the transmission coefficient.
Figure 132: A 75" curved pyramid with a 15" thick and 12" wide base.
Figure 133: Comparison of the curved pyramidal absorber for two different base thicknesses in free space or with metal backing.
Figure 134: Specular reflection coefficient comparison between the straight and curved pyramidal absorber.
Figure 135: Specular transmission coefficient comparison between the straight and curved pyramidal absorber.
Figure 136: Comparison between the Hybrid and PMM results for the specular reflection coefficient of the 75" curved pyramidal absorber.
Figure 137: Comparison between the Hybrid and PMM results for the specular transmission coefficient of the 75\" curved pyramidal absorber.
Figure 138: Comparison between the Hybrid and PMM results for the specular reflection coefficient of the 75" straight pyramidal absorber.
Figure 139: Comparison between the Hybrid and PMM results for the specular transmission coefficient of the 75" straight pyramidal absorber.

Max Segment Size
0.125 λ (3-4.5 GHz)
0.15 λ (4.8-6 GHz)
5.8.3 120" Curved Pyramidal Absorber

Another curved pyramid design, which is a 120" pyramid with a 24" × 24" width and 30" base thickness, is shown in Figure 140. Note that the dielectric constant of the absorber material has been chosen to be four times that of the LS20 material at the frequencies for a \( \frac{1}{3} \) scale model. The reflection performance of this curved pyramid was determined using the hybrid approach so that various base designs could be efficiently evaluated. Some results of this study are shown in Figure 141 such as a 30" and 60" thick bases in free space as well as a 30" thick base on a metal ground plane. These results indicate that a 30" base is thick enough to avoid a significant backface reflection. In order to illustrate the performance improvement achievable using this analysis tool, the 10' curved pyramid and a 10' straight pyramid are compared in Figure 142. One can see that the reflection level of the curved pyramid is about 15 dB lower than that of the straight one at frequencies around 100 MHz, and approximately 5 dB better at the higher frequencies. Note that the curved shapes not only reduce the scattered fields from both tip and valley, but also cause the scattered fields to cancel each other at 100 MHz which is where the straight pyramid starts to deteriorate. The transmission coefficients for the curved and straight pyramids are shown in Figure 143. As is expected, there is only a very slight difference between the transmission coefficients of these two pyramids.

To design pyramidal absorber for use at frequencies as low as 300 MHz, a \( \frac{1}{3} \) scale model of the pyramid shown in Figure 140 has been employed. According to this model, a total of eight 40" curved pyramids with an 8" × 8" width and 10" base thickness have been cut from two 72" commercial pyramids. These curved pyramids have been measured in a parallel-plate waveguide system at OSU/ESL. The measured results are given in Figure 144 together with the PMM calculations using the hybrid approach. Note that the high return around 300 MHz in the
measurements is due to range clutter errors. In Figure 145, the performance of the 40" curved pyramid is compared with those of the 36" and 72" commercial pyramidal absorber specified by the manufacturer. Clearly, the 40" curved pyramid is better than the 72" commercial pyramid even though the curved one is much smaller.
Figure 140: A 120" curved pyramid with a 30" thick and 24" x 24" wide base.
Figure 141: Comparison of the curved pyramidal absorber for two different base thicknesses in free space or with metal backing.
Figure 142: Specular reflection coefficient comparison between the straight and curved pyramidal absorber.
Figure 143: Specular transmission coefficient comparison between the straight and curved pyramidal absorber.
Figure 144: The reflection measurements and PMM calculations for the 40" curved pyramidal absorber.
Figure 145: Comparison between the PMM calculations of the 40" curved pyramid and the absorber specifications for two commercial pyramid designs.
5.9 Bistatic and Backscatter Patterns

Figures 146-149 show the specular reflection coefficient with respect to the angle of incidence for the 10" curved wedge, 8" pyramid, and 13" serrated wedge at 2 GHz. Both the perpendicular and parallel polarizations are shown for $\phi = 0$ and $\phi = 90^\circ$ planes of incidence, where the former is transverse to the wedge axes, and the latter is parallel. Note that the 13" serrated wedge illuminated by a perpendicularly polarized plane wave with the plane of incidence along the wedge axes appears to have the best performance for wide angles of incidence.

The backscatter patterns versus angle of incidence for the 8" commercial wedge, 10" curved wedge, and 13" serrated wedge at 6 GHz are shown in Figures 150 and 151 for TM and TE polarizations in the $\phi = 0$ plane of incidence. These backscattered results occur when the specular reflected waves or the grating lobes propagate along the reverse direction to the incident plane wave. Thus, the angles of the backscattered plane waves are given by

$$\theta = \sin^{-1} \left( \frac{i\lambda_0}{2D} \right)$$

where $D$ is the interelement spacing, $\lambda_0$ is the free space wavelength, and $i = 0, 1, 2, \cdots$. In Figure 152, the backscatter patterns for the 8" and 18" commercial pyramids are presented. In this case, the frequencies have been chosen to include backscattered results at the angle normal to the pyramidal side surfaces (approximately 77°). As can be easily seen from these results, both pyramids backscatter very strong signals for this angle of incidence. On the other hand, there is no backscattered plane wave for an incident wave propagating obliquely along the axis of a uniform wedge absorber wall. As a result, wedge absorber is a much better choice for such situations as shown by DeWitt and Burnside [2] earlier. Note that
the 13" serrated wedge also provides similar backscatter performance at normal incidence as compared with the 18" commercial pyramid. Therefore, the serrated wedge is an ideal absorber material in that it has the advantages of both the commercial wedge and pyramid.
Figure 146: The calculated bistatic patterns for the 10° curved wedge and 8° commercial pyramid for the E-field perpendicular to the plane of incidence at 2 GHz.
Figure 147: The calculated bistatic patterns for the 10° curved wedge and 8° commercial pyramid for the E-field parallel to the plane of incidence at 2 GHz.
Figure 148: The calculated bistatic patterns for the serrated wedge absorber with eight 4\" fins for the E-field perpendicular to the plane of incidence at 2 GHz.
Figure 149: The calculated bistatic patterns for the serrated wedge absorber with eight 4" fins for the E-field parallel to the plane of incidence at 2 GHz.
Figure 150: The calculated backscatter patterns for the 8" commercial wedge, the 10" curved wedge, and the 13" serrated wedge with eight 4" fins for TM case at 6 GHz.
Figure 151: The calculated backscatter patterns for the 8" commercial wedge, the 10" curved wedge, and the 13" serrated wedge with eight 4" fins for TE case at 6 GHz.
Figure 152: The calculated backscatter patterns for the 8" and 18" commercial pyramids. The frequencies are chosen such that the backscattered plane waves occur at the angle normal to one of the pyramidal surfaces.
CHAPTER VI
Summary and Conclusions

The periodic moment method solutions for scattering from infinite walls of lossy
dielectric bodies with single or double periodicities has been developed in this dis­
sertation. The spectral domain formulation [9] [10] and the moment method volume
polarization current approach [15] [16] have been applied to derive the expressions
used to determine the scattered fields. With these expressions, PMM programs have
been developed for analyzing and designing wedge and pyramidal absorber. To im­
prove the PMM efficiency, closed form integrals have been derived, and an accurate
algorithm has been employed to evaluate the infinite sums in the PMM formulations.
In the three-dimensional PMM formulation, a complete expression for fields inside
or outside the source regions has been derived from both a rigorous method similar
to that given in [20] and an approach of inserting a free-space layer through the
field point and parallel to the dielectric array. Also, a hybrid approach using PMM
and a solution for planar layers has been described. This method can be used to
determine the scattering from a periodic array of lossy dielectric bodies backed by
multiple layers of dielectric or magnetic materials and terminated with or without a
metal ground plane.

To determine the dielectric constants of the microwave absorber materials, a free
space method has been developed for extracting the material constitutive param­
eters. From the Lorentz' reciprocal relationship [21], the antenna received voltage
is proportional to an integral over the closed surfaces of the measured objects. A
PO approximation has been used to calculate the scattered fields on the front face of the metal-backed absorber sheets illuminated by an incident wave. Both the spherical-wave and plane-wave setups have been employed to determine the material properties. For frequencies between 2 and 18 GHz, the measured results for 24" × 24" × 0.5" absorber sheets gave reasonably smooth variations. These results were also verified by the very good agreement achieved between PMM calculations and RCS measurements for various absorber examples.

PMM is an ideal tool for studying microwave absorber for anechoic chamber applications since accurate solutions for the reflection and transmission properties of an infinite absorber wall can be obtained for a wide variety of geometries and material properties. The required height, shape and loading of absorber are first evaluated using the reflection coefficient; then, the loss in one pass through the absorber wall is examined based on the transmission coefficient. Both RCS measurements and PMM calculations have been presented for ordinary wedges, new curved wedges, new four-layer wedges, patented serrated wedges, commercial pyramids and a new 40" curved pyramid. By considering the variables associated with material measurements, very good agreement between measurements and calculations has been obtained. Also, PMM solutions have been given for some low-frequency pyramids, several eight-layer wedges, and wedges with various dopings, widths and heights. For diagnostic purposes, the calculated impulse responses have been compared to measured impulse responses for the TM case and normal incidence. The time domain responses are obtained from the inverse Fourier transform of the frequency domain responses. These results show that the PMM solutions can be employed to identify the most significant scattering centers associated with an absorber wall. Finally, bistatic and backscatter patterns for both the wedge and pyramidal absorber walls have been reported, which illustrate the versatility of this technique.
The objective of this research has been to develop an analysis tool that could be used to design better absorber than presently available. This goal has been accomplished as verified by the new curved wedges and pyramids, multilayer wedge, and patented serrated wedge absorber designs which perform better than commercially available materials. The blue wedge material was used to fabricate the 10° curved wedge, so that the material properties were as similar as possible. This was done to illustrate the effect of shaping alone as proposed by the PMM solution. Both the measurements and calculations indicate that the 10° curved wedge provides at least 10 dB improvement over the commercial blue wedge. The 13° curved wedge absorber has an even lower reflection level than the 10° curved wedge because of the larger size and the smaller dielectric constant. However, smooth cutting is essential to achieve such a low radar cross section. For the first four-layer design, LS absorber layers were added to the commercial blue wedge in this case to illustrate the effect of material variations. The reflection level was reduced by 10-15 dB with four layers attached to the blue wedge. As shown by the analysis, the reflection level could have been lowered by another 10 dB if an eight-layer design is employed. However, the multi-layer wedge absorber requires good control of the dielectric constants of each layer to maintain the desired performance. Also, the glue between adjacent layers can become a significant problem if it is not carefully controlled. It was also shown that by carefully cutting the wedge surfaces into serration, one can design a 13° serrated wedge which has similar performance as that of the 13° eight-layer design for frequencies above 2 GHz. The number, lengths, and positions of the fins making the serrated surfaces provide the design parameters that one can adjust to achieve the desired reflection level. Due to the complicated shape of serration with pointed fins, a computer controlled cutting machine and rigid absorber materials may be needed to produce the serrated wedge properly. Note that the serrated wedge absorber
design has both the advantages of the wedge and pyramid. It has low backscatter reflection performance for broadside as well as near grazing incidence if illuminated in the plane of the fins. Because of its superior performance, a patent right has been filed for the serrated electromagnetic absorber design.

Low frequency absorber materials are of special interest because they normally are very large which impacts the overall chamber size and ultimately its cost. At the lower frequencies, the PMM numerical solution becomes more efficient which makes even more appropriate for design purposes. A commercial 6' pyramidal absorber has been analyzed with the PMM tool. Good agreement between the PMM calculations and absorber specifications has been obtained at frequencies around 120 MHz, but the calculated results indicate a lower reflection level than the specifications at the higher frequencies. The actual material may be more lossy than that modelled in the calculations and the material inhomogeneities can be a determining factor at the higher frequencies because of the low reflection level of the pyramid itself. Several other 6' pyramids have been compared for different base thicknesses and material properties. In addition, 6' and 10' curved pyramids have also been designed with the PMM program. The radii of curvature for the curved pyramids are chosen to produce a reflection coefficient null at the lowest operational frequency which is where the straight pyramids of the same height start to degrade. For the 10' curved pyramid design, 15 dB improvement at the low frequencies has been achieved. A 1/30 scale model was used for those studies because the material properties were not available below 2 GHz. However, the scale model can provide many absorber designs for applications at different frequency ranges with only a single PMM simulated data set. Therefore, by employing a 1/3 scale model over the data calculated for the 10' curved pyramid, a 40'' curved pyramidal absorber has been designed for use at frequencies above 300 MHz. Good agreement between calculations and measurements has been
obtained for the 40" curved pyramid. At frequencies above 300 MHz, the 40" curved pyramid performs even better than the commercial 72" pyramid.

As for the future, these PMM solutions could be extended to treat dielectric bodies in the presence of dipole arrays or slots on a metal ground plane. On the other hand, one could modify these solutions to treat magnetic and/or anisotropic materials. A major challenge in attacking these topics will be to improve the program efficiency, especially in terms of the large memory size needed to run these PMM codes.
APPENDIX A

The E-Field from Singly-Periodic Array of Electric Filaments

In this appendix, the E-field excited from a singly-periodic array of electric filaments in free space is derived. As shown in Figure 153, the array is located along the x axis with the filaments flowing in the z direction. The interelement distance is \( D_x \) and the reference element of the array is positioned at the origin. From Equation (5-83) in [19], the vector potential from all the filaments is given by

\[
A(R) = \frac{\hat{z}}{4j} \sum_{n=-\infty}^{\infty} I_n H_0^{(2)}(k_o R_n)
\]  (A.1)

where \( R_n = \sqrt{(x - nD_x)^2 + y^2} \) is the distance between the the field point, \( R = (x, y) \), and the nth filament, \((n D_x, 0)\), in the x-y plane. According to Equation (2.22), the filaments have a periodic variation which is given by

\[
I_n = I_o e^{-jk_o n D_x z}
\]  (A.2)

where \( I_o \) is the reference element. Then, Equation (A.1) becomes

\[
A(R) = \frac{\hat{z}I_o}{4j} \sum_{n=-\infty}^{\infty} e^{-jk_o n D_x z} H_0^{(2)}(k_o R_n). 
\]  (A.3)

To transform the above spatial domain formulation into a spectral domain formulation and obtain a faster converging summation for the vector potential, an approach discussed in Appendix A of [9] is applied here.

From the Poisson sum formula [23], one has

\[
\sum_{n=-\infty}^{\infty} e^{j n \omega_o t} F(n \omega_o) = T \sum_{i=-\infty}^{\infty} f(t - iT)
\]  (A.4)
where $T = \frac{2\pi}{\omega_0}$ and $F(\omega)$ is the Fourier transform of $f(t)$; i.e., $F(\omega) = \mathcal{F}[f(t)]$. Also, a transform pair from (A18) of [9] for the Hankel function states

\[ H_0^2(k_0\sqrt{(\omega_2 - \omega)^2 + |y|^2}) = \mathcal{F}[e^{j\omega_2 t}e^{-j|y|\sqrt{k_0^2 - t^2}}] \]

where $\omega$ and $t$ are the variables for the transform pair. Applying Equation (A.5) to Equation (A.4), and then substituting $\omega_0$ with $D_x$ in Equation (A.4) and also $\omega$ with $nD_x$ and $\omega_2$ with $x$ in Equation (A.5), one obtains

\[ \sum_{n=-\infty}^{\infty} e^{jnD_xt} H_0^2(k_0\sqrt{(x - nD_x)^2 + y^2}) = \frac{2\pi}{D_x} \sum_{n=-\infty}^{\infty} e^{jz(t-i\frac{2\pi}{D_x})} \frac{e^{-j|y|\sqrt{k_0^2 - (t-i\frac{2\pi}{D_x})^2}}}{\pi \sqrt{k_0^2 - (t-i\frac{2\pi}{D_x})^2}} \]

(A.6)
Then, employing this equation in Equation (A.3) and replacing \( t \) with \( -k_0 s_x \), one finds that

\[
A(R) = \frac{jI_o}{j2k_o D_x} \sum_{i=-\infty}^{\infty} e^{-j k_o x(s_x + i2\pi\frac{2\pi}{k_o D_x})} e^{-j k_o y \sqrt{1 - (s_x + i\frac{2\pi}{k_o D_x})^2}} \sqrt{1 - (s_x + i\frac{2\pi}{k_o D_x})^2} (A.7)
\]

or

\[
A(R) = \frac{jI_o}{j2k_o D_x} \sum_{i=-\infty}^{\infty} e^{-j k_o R \cdot \hat{r}_\pm} \frac{e^{-j k_o y \hat{y}}}{r_y} (A.8)
\]

where

\[
k_o = \frac{2\pi}{\lambda} \text{ the free space propagation constant}
\]

\[R = (x, y), \text{ the field points}
\]

\[
\hat{r}_\pm = \hat{x}(s_x + \frac{i\lambda}{D_x}) \pm \hat{y} r_y, \text{ for } y \geq 0, \text{ and}
\]

\[
r_y = \begin{cases} 
\sqrt{1 - (s_x + \frac{i\lambda}{D_x})^2}, & (s_x + \frac{i\lambda}{D_x})^2 \leq 1 \\
-j \sqrt{(s_x + \frac{i\lambda}{D_x})^2 - 1}, & \text{otherwise.}
\end{cases} (A.10)
\]

The E-field due to the electric filaments is then given by

\[
E(R) = \frac{1}{j\omega \epsilon_0} \nabla \times [\nabla \times A(R)] (A.11)
\]

or

\[
E(R) = \frac{-I_o}{2k_o D_x \omega \epsilon_0} \sum_{i=-\infty}^{\infty} \nabla \times [\nabla \times (\hat{z} e^{-j k_o R \cdot \hat{r}_\pm} / r_y)]. (A.12)
\]

Applying

\[
\nabla \times (\hat{z} e^{-j k_o R \cdot \hat{r}_\pm} / r_y) = -j k_o \hat{r}_\pm \times (\hat{z} e^{-j k_o R \cdot \hat{r}_\pm} / r_y) (A.13)
\]

to Equation (A.12), one obtains

\[
E(R) = -\frac{Z_0 I_o}{2 D_x} \sum_{i=-\infty}^{\infty} \hat{z} e^{-j k_o R \cdot \hat{r}_\pm} / r_y (A.14)
\]

where \( Z_0 \) is the free space characteristic impedance, \( I_o \) is the reference filament at the origin, and \( \hat{r}_\pm \times (\hat{r}_\pm \times \hat{z}) = -\hat{z} \) is used.
For an array with a reference filament located at $R'$, the E-field at $R$ excited from this array is equal to the E-field at $R - R'$ produced by the array having the reference filament at the origin. Thus, the E-field from an infinite array of electric filaments with the reference element located at $R'$ and the phase following Equation (A.2) is given by

$$E(R) = -\frac{Z_0 I_0}{2D_x} \sum_{i=-\infty}^{\infty} z e^{-jk_0(R-R') \cdot \hat{r}_\pm} \frac{e^{-jk_0 r_y}}{r_y}. \quad (A.15)$$
APPENDIX B

Closed-Form Integration for the 2-D PMM Impedance Matrix

In this appendix, a closed-form integral is derived for the elements of the 2-D PMM impedance matrix derived in Chapter II. From Equation (2.29) and (2.36), the matrix elements are given by

\[
Z_{mn} = -E_z^n(R_m) = \frac{Z_o}{2D_xA_n} \sum_{i=-\infty}^{\infty} \iint_{cell\,n} e^{-jk_o(R_m-R')\cdot\hat{r}_\pm} \frac{dx'dy'}{r_y} \tag{B.1}
\]

or

\[
Z_{mn} = \frac{Z_o}{2D_xA_n} \sum_{i=-\infty}^{\infty} e^{-jk_0xy} \int_{cell\,n} \int e^{jk_o(x'y'-xy')} dx'dy' \tag{B.2}
\]

where

\[
R' = (x',y'), \text{ a source point in the } n\text{th cell of the reference wedge}
\]

\[
R_m = (x_m,y_m), \text{ the center of the } m\text{th cell}
\]

\[
\hat{r}_\pm = \hat{x}(s_x + \frac{i\lambda}{D_x}) \pm \hat{y}r_y, y > y'
\]

\[
r_y = \begin{cases} 
\sqrt{1 - (s_x + \frac{i\lambda}{D_x})^2}, & (s_x + \frac{i\lambda}{D_x})^2 \leq 1 \\
-j\sqrt{(s_x + \frac{i\lambda}{D_x})^2 - 1}, & \text{otherwise}
\end{cases} \tag{B.3}
\]

\[
A_n \text{ is the cross-section area of the } n\text{th cell, and}
\]

\[
Z_o = \sqrt{\frac{\mu_0}{\varepsilon_0}}, \text{ the free space characteristic impedance.}
\]

To solve the 2-D integration in a closed form, the geometry of the PMM cells is chosen to be a quadrilateral as that shown in Figure 154. Note that the formula derived below can be applied to a general quadrilateral with the four corners of the
cells labeled counterclockwise. First, the cell is divided into four triangles and one rectangle as indicated in Figure 154. The integral is then calculated from

\[ \int \int_{\text{cell}} e^{jk_0 (x' r_z - |y - y'| r_y)} \, dx' dy' = \sum_{l=1}^{5} I_l \]  

(B.5)

where \( I_l \) is the integral over Region \( l \) of the cell. For the first region, the integral is given by

\[ I_1 = \int_{y_1}^{y_4} \int_{x_1 + \frac{x_4 - x_1}{y_4 - y_1} (y' - y_1)}^{x_1} e^{jk_0 (x' r_z - |y - y'| r_y)} \, dx' dy' \]  

(B.6)

or

\[ I_1 = \frac{e^{jk_0 r_x z_1}}{j k_0 r_x} \int_{y_1}^{y_4} e^{-jk_0 r_y |y - y'|} [1 - e^{jk_0 r_x \frac{y_4 - y_1}{y_4 - y_1} (y' - y_1)}] \, dy'. \]  

(B.7)

Because of \(|y - y'|\), three different cases have to be considered. They are (1) \( y \geq y_1 \) and \( y_4 \), (2) \( y \leq y_1 \) and \( y_4 \), and (3) \( y \) between \( y_1 \) and \( y_4 \). Thus, the closed-form integral for \( I_1 \) is given by
\[
I_1 = \frac{e^{j k_0 r z_1 + r_y (y_1 - y)}}{j k_0 r z_1 (y_4 - y_1)} \{EC[j k_0 r_y(y_4 - y_1)] - EC[j k_0 r_y(y_4 - y_1)] \}
\]

\[
= \frac{e^{j k_0 r z_1 - r_y (y_1 - y)}}{j k_0 r z_1 (y_4 - y_1)} \{EC[-j k_0 r_y(y_4 - y_1)] - EC[-j k_0 r_y(y_4 - y_1)] \}
\]

\[
\text{for } y \geq y_1 \text{ and } y_4 \quad \text{(B.8)}
\]

\[
= \frac{e^{j k_0 r z_1}}{j k_0 r z_1} \{(y - y_1)[EC(-j k_0 r_y|y - y_1|) - e^{-j k_0 r_y|y - y_1|} EC(j k_0 r_z x_4 - x_1 (y - y_1) + j k_0 r_y|y - y_1|) + (y_4 - y)[EC(-j k_0 r_y|y - y_4|) - e^{j k_0 r z_1 (y_4 - y_1) x_4 - x_1 (y_4 - y) - j k_0 r_y|y - y_4|)]},
\]

\[
\text{for } y \text{ between } y_1 \text{ and } y_4 \quad \text{(B.10)}
\]

\[
\equiv I_1(x_1, y_1, x_4, y_4)
\quad \text{(B.11)}
\]

where

\[
EC(x) = \frac{e^x - 1}{x}
\quad \text{(B.12)}
\]

and

\[
\lim_{x \to 0} EC(x) = 1.
\quad \text{(B.13)}
\]

Note that these expressions apply to both \(y_1 > y_4\) and \(y_1 < y_4\). Also, to avoid numerical overflow or underflow, the difference between two \(y\) coordinates (\(y, y_1\) and \(y_4\)) is kept in the exponent, since \(r_y\) can be imaginary.

For the other three triangles, the integrals are given by

\[
I_2 = \int_{y_1}^{y_2} \int_{x_1}^{x_2} \frac{e^{j k_0 r z_1 + r_y (y_1 - y)}}{r y_1 (y_2 - y_1)} e^{j k_0 r_z x = -|y - y'| r y} \, dz' dy'
\quad \text{(B.14)}
\]

\[
I_3 = \int_{y_2}^{y_3} \int_{x_2}^{x_3} \frac{e^{j k_0 r z_1 + r_y (y_1 - y)}}{r y_2 (y_3 - y_2)} e^{j k_0 r_z x = -|y - y'| r y} \, dz' dy'
\quad \text{(B.15)}
\]

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and

\[ I_4 = \int_{y_4}^{y_3} \int_{x_3 + \frac{x_4 - x_3}{y_4 - y_3}}^{x_3} e^{jk_0(z' r_x - |y-y'|r_y)} \, dx' \, dy'. \]  

(B.16)

By comparing with Equation (B.6) and referring to Equation (B.11), one has

\[ I_2 = -I_1(x_1, y_1, x_2, y_2) \]  

(B.17)

\[ I_3 = +I_1(x_3, y_3, x_2, y_2) \]  

(B.18)

and

\[ I_4 = -I_1(x_3, y_3, x_4, y_4). \]  

(B.19)

For the fifth region, the integration is over the rectangle and is given by

\[ I_5 = \int_{y_2}^{y_4} \int_{x_1}^{x_3} e^{jk_0(z' r_x - |y-y'|r_y)} \, dx' \, dy' \]  

(B.20)

or

\[ I_5 = \int_{x_1}^{x_3} e^{jk_0 z' r_x} \, dz' \int_{y_2}^{y_4} e^{-jk_0 |y-y'|r_y} \, dy' \]  

(B.21)

where

\[ \int_{x_1}^{x_3} e^{jk_0 z' r_x} \, dx' = (x_3 - x_1)e^{jk_0 r_x z_1} EC[jk_0 r_x (x_3 - x_1)] \]  

(B.22)

and

\[ \int_{y_2}^{y_4} e^{-jk_0 |y-y'|r_y} \, dy' = \]

\[ \begin{cases} e^{-jk_0 r_y (y_4 - y_2)}EC(\pm jk_0 r_y (y_4 - y_2)), & y \geq \text{both } y_2 \text{ and } y_4 \\ (y - y_2)EC(-jk_0 |y_2 - y|r_y) + (y_4 - y)EC(-jk_0 |y_4 - y|r_y), & \text{otherwise.} \end{cases} \]  

(B.23)

A numerical error will occur in the expressions for \( I_1 \) given in Equations (B.8)-(B.10), when \( r_x \) is approaching or equal to 0. To solve this problem, \( \lim_{r_x \to 0} I_1 \) is calculated from the following relationship,

\[ \lim_{z \to 0} \frac{p(z)}{q(z)} = \lim_{z \to 0} \frac{\partial p(z)}{\partial z} \frac{\partial q(z)}{\partial z}. \]  

(B.24)
The results are

\[
\lim_{\tau_{x} \to 0} I_1 = \begin{cases} 
(y_4 - y_1)(x_1 - x_4)e^{-\xi_1 EC'(\pm \xi_2)}, & \text{for } y \geq \text{ both } y_1 \text{ and } y_4 \\
\frac{\tau_{x} - \tau_{y} + 2}{\tau_{y} - \tau_{y} + 1} \{(y - y_1)^2 e^{-\xi_1 EC'(\xi_1)} + \\
(y_4 - y)[(y - y_1)EC(\xi_3) + (y_4 - y)EC'(\xi_3)]\}, & \text{otherwise}
\end{cases}
\]  

\text{(B.25)}

where

\[
\xi_1 = jk_0 \tau_y |y - y_1| \\
\xi_2 = jk_0 \tau_y (y_4 - y_1) \\
\xi_3 = -jk_0 \tau_y |y - y_4| \\
EC'(x) = \frac{\partial EC(x)}{\partial x} = \frac{(x - 1)e^x + 1}{x^2}
\]

\text{(B.26) to (B.29)}

and

\[
\lim_{x \to 0} EC'(x) = 0.5.
\]  

\text{(B.30)}
APPENDIX C

Closed-Form Integration for the 3-D PMM Impedance Matrix

In this appendix, a closed-form integral is derived for the elements of the 3-D PMM impedance matrix. From Equations (3.44) and (3.50), one finds that the elements of the impedance matrix are given by

\[ [Z_{mn}] = -\frac{Z_0}{2D_x D_z V_n} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \frac{1}{r_y} \int \int [P_{\pm}] e^{-jk_o(R_m - R') \cdot \hat{r}_{\pm}} \, dv' \] (C.1)

where

\[ [P_{\pm}] = \begin{bmatrix}
\hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{x}) \cdot \hat{x} & \hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{y}) \cdot \hat{x} & \hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{z}) \cdot \hat{x} \\
\hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{x}) \cdot \hat{y} & \hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{y}) \cdot \hat{y} & \hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{z}) \cdot \hat{y} \\
\hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{x}) \cdot \hat{z} & \hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{y}) \cdot \hat{z} & \hat{r}_{\pm} \times (\hat{r}_{\pm} \times \hat{z}) \cdot \hat{z}
\end{bmatrix} \] (C.2)

a 3 x 3 polarization matrix

\[ \hat{r}_{\pm} = \hat{x}(s_z + \frac{k\lambda}{D_x}) \pm \hat{y} r_y + \hat{z}(s_z + \frac{l\lambda}{D_z}), \text{ for } y_m < y' \] (C.3)

\[ r_y = \begin{cases} 
\sqrt{1 - (s_z + \frac{k\lambda}{D_x})^2 - (s_z + \frac{l\lambda}{D_z})^2}, & (s_z + \frac{k\lambda}{D_x})^2 + (s_z + \frac{l\lambda}{D_z})^2 \leq 1 \\
-j\sqrt{(s_z + \frac{k\lambda}{D_x})^2 + (s_z + \frac{l\lambda}{D_z})^2 - 1}, & \text{otherwise}
\end{cases} \] (C.4)

\[ R_m = (x_m, y_m, z_m), \text{ the center of Cell m, the receiving cell} \]

\[ R' = (x', y', z_m), \text{ a source point in Cell n, the transmitting cell} \]

\[ k_o = \omega \sqrt{\mu_o \varepsilon_o}, \text{ the free space propagation constant} \]

\[ D_x \text{ and } D_z \text{ are the interelement spacings as shown in Figure 18} \]

\[ V_n \text{ is the volume of Cell n, and} \]

\[ Z_0 = \sqrt{\frac{\mu_o}{\varepsilon_o}}, \text{ the free space characteristic impedance}. \]
Because of Equation (C.3), \([P_\pm]\) depends on \(y'\) and therefore is kept inside the integral. By applying the vector identity,

\[
A \times (B \times C) = (A \cdot C)B - (A \cdot B)C
\]  

(C.5)

the polarization matrix can be reduced to

\[
[P_\pm] = \begin{bmatrix}
  r_x^2 - 1 & \pm r_y r_z & r_x r_z \\
  \pm r_x r_y & r_y^2 - 1 & \pm r_y r_z \\
  r_x r_z & \pm r_y r_z & r_z^2 - 1
\end{bmatrix}
\]  

(C.6)

where only four of the matrix elements change sign with \(y_m - y'\) and the matrix is symmetric.

In order to obtain a closed-form solution for \([Z_m]\), the PMM cells are chosen to be a truncated pyramid with two rectangular faces of arbitrary sizes parallel to the x-z plane and aligned with \(\hat{x}\) and \(\hat{z}\) as shown in Figures 20 and 155. Thus, the triple integrals in Equation (C.1) become

\[
\int \int \int_{\text{cell } n} [P_\pm] e^{-j k_0 (R_m - R')} \, dv' =
\]

\[
e^{-j k_0 (r_x x_m + r_z z_m)} \int_{x_{10}}^{x_{12}} \int_{y_{10}}^{y'} \int_{z_{10}}^{z_{11}} [P_\pm] e^{j k_0 (r_x x' - r_y y')} \, dz' \, dy' \, dx' \]  

(C.7)

where as shown in Figure 155, \((x_{i}, y_{i}, z_{i}), i = 1, 8,\) are the corners of Cell \(n\). Also, Points 9 to 12 correspond to the corners of a rectangle perpendicular to \(y\) and inside Cell \(n\), and are defined by

\[
x'_{10} = x_2 + \frac{x_6 - x_2}{y_5 - y_1} (y' - y_1') \equiv a_1 + m_1 y'
\]

with \(a_1 \equiv x_2 - m_1 y_1'\) and \(m_1 \equiv \frac{x_6 - x_2}{y_5 - y_1}\)  

(C.8)

\[
x'_{12} = x'_4 + \frac{x_8 - x'_4}{y_5 - y_1} (y' - y_1') \equiv a_2 + m_2 y'
\]

with \(a_2 \equiv x'_4 - m_2 y_1'\) and \(m_2 \equiv \frac{x_8 - x'_4}{y_5 - y_1}\)  

(C.9)
Figure 155: The PMM cell for the doubly-periodic moment method to have a closed form integration in calculating the impedance matrix.

\[
z_{11} = z_3 + \frac{z_7 - z_3}{y_5 - y_1} (y' - y_1') \equiv a_3 + m_3 y'
\]

with \( a_3 \equiv z_3 - m_3 y_1 \) and \( m_3 \equiv \frac{z_7 - z_3}{y_5 - y_1} \) \hspace{1cm} (C.10)

and

\[
z_9' = z_1' + \frac{z_5' - z_1'}{y_5' - y_1'} (y' - y_1') \equiv a_4 + m_4 y'
\]

with \( a_4 \equiv z_1' - m_4 y_1' \) and \( m_4 \equiv \frac{z_5' - z_1'}{y_5' - y_1'} \). \hspace{1cm} (C.11)

Because the integrations in \( x \) and \( z \) are separable, one has
\[
\int_{x_{10}^{l}}^{x_{12}^{l}} \int_{y_{1}^{l}}^{y_{5}^{l}} \int_{z_{9}^{l}}^{z_{11}^{l}} e^{ijk_{0}(rz_{5}z_{r}z_{r}^{' r}z_{r}^{' r}z_{r}^{' r}y_{r}^{' r}y_{m}^{' r})} \, dz' \, dy' \, dx' \\
= \int_{y_{1}^{l}}^{y_{5}^{l}} [P_{\pm}] e^{-jko_{y}y_{r}y_{m}^{' r}} \int_{x_{10}^{l}}^{x_{12}^{l}} e^{ijk_{0}z_{r}z_{r}'} dx' \int_{z_{9}^{l}}^{z_{11}^{l}} e^{ijk_{0}z_{r}z_{r}'} dz' \, dy' \\
= -\frac{1}{k_{0}^{2}r_{z}r_{z}} \int_{y_{1}^{l}}^{y_{5}^{l}} [P_{\pm}] e^{-jko_{y}y_{r}y_{m}^{' r}} \left[ e^{ijk_{0}(rz_{5}z_{r}z_{r}^{' r}z_{r}^{' r}z_{r}^{' r}y_{r}^{' r}y_{m}^{' r})} + e^{ijk_{0}(rz_{5}z_{r}z_{r}^{' r}z_{r}^{' r}z_{r}^{' r}y_{r}^{' r}y_{m}^{' r})} \right] \, dy' \\
= -\frac{1}{k_{0}^{2}r_{z}r_{z}} \left[ I(x_{12}^{l}, z_{11}^{l}) + I(x_{10}^{l}, z_{9}^{l}) - I(x_{10}^{l}, z_{11}^{l}) - I(x_{12}^{l}, z_{9}^{l}) \right] (C.13) \\
= \frac{1}{k_{0}^{2}r_{z}r_{z}} \left[ I(x_{12}^{l}, z_{11}^{l}) + I(x_{10}^{l}, z_{9}^{l}) - I(x_{10}^{l}, z_{11}^{l}) - I(x_{12}^{l}, z_{9}^{l}) \right] (C.14)
\]

where

\[
I(x_{12}^{l}, z_{11}^{l}) \equiv \int_{y_{1}^{l}}^{y_{5}^{l}} [P_{\pm}] e^{-jko_{y}y_{r}y_{m}^{' r}} e^{ijk_{0}(rz_{5}z_{r}z_{r}^{' r}z_{r}^{' r}z_{r}^{' r}y_{r}^{' r}y_{m}^{' r})} \, dy'. 
\]

For \( y_{m} \geq y_{5}^{l} \) or \( y_{m} \leq y_{1}^{l} \), and \( y_{5}^{l} > y_{1}^{l} \), the first integral in Equation (C.14) is given by

\[
I(x_{12}^{l}, z_{11}^{l}) = [P_{\pm}] e^{ijk_{0}(rz_{2}z_{2}^{r}z_{3}^{r}z_{3}^{r}z_{3}^{r}y_{3}^{r}y_{m}^{r})} \int_{y_{1}^{l}}^{y_{5}^{l}} e^{jko_{y}(y_{r}^{r}y_{m}^{r})} e^{jko_{z}(rz_{2}^{r}z_{2}^{r}z_{3}^{r}z_{3}^{r}z_{3}^{r}y_{3}^{r}y_{m}^{r})} \, dy'. 
\]

For \( y_{5}^{l} > y_{m} > y_{1}^{l} \), one has

\[
I(x_{12}^{l}, z_{11}^{l}) = e^{ijk_{0}(rz_{2}z_{2}^{r}z_{3}^{r}z_{3}^{r}z_{3}^{r}y_{3}^{r}y_{m}^{r})} \left\{ \int_{y_{1}^{l}}^{y_{5}^{l}} e^{jko_{y}(y_{r}^{r}y_{m}^{r})} e^{jko_{z}(rz_{2}^{r}z_{2}^{r}z_{3}^{r}z_{3}^{r}z_{3}^{r}y_{3}^{r}y_{m}^{r})} \, dy' \right\} \\
+ [P_{-}] \int_{y_{m}^{l}}^{y_{5}^{l}} e^{-jko_{y}(y_{r}^{r}y_{m}^{r})} e^{jko_{z}(rz_{2}^{r}z_{2}^{r}z_{3}^{r}z_{3}^{r}z_{3}^{r}y_{3}^{r}y_{m}^{r})} \, dy' 
\]

or

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\[ I(x'_{12}, z'_{11}) = [P_+]e^{jk_0(r_x x'_4 + r_y (y'_1 - y_m) + r_z z'_3)(y_m - y'_1)} \times \]
\[ EC[jk_0(r_x m_2 + r_y + r_z m_3)(y_m - y'_1)] \]
\[ + [P_-]e^{jk_0(r_x x'_5 + r_y (y_m - y'_5) + r_z z'_4)(y'_5 - y_m)} \times \]
\[ EC[jk_0(r_x m_2 - r_y + r_z m_3)(y_m - y'_5)]. \quad (C.20) \]

Note that similar equations can be obtained for \( I(x'_{10}, z'_9), I(x'_{10}, z'_{11}) \) and \( I(x'_{12}, z'_9) \) by comparing Equations (C.8)-(C.11). Finally, if \( r_x \) or \( r_z \) are approaching or equal to 0, Equation (B.24) can be applied to determine the proper expression for Equation (C.14).
BIBLIOGRAPHY


