INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
Numerical and experimental studies of liquid injection molding with preplaced fiber mats

Young, Wen-Bin, Ph.D.
The Ohio State University, 1991

Copyright ©1991 by Young, Wen-Bin. All rights reserved.
NUMERICAL AND EXPERIMENTAL STUDIES OF LIQUID INJECTION MOLDING WITH PREPLACED FIBER MATS

DISSERTATION

Presented In Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

Wen-Bin Young, B.S., M.S.

* * * * *

The Ohio State University
1991

Dissertation Committee:
Ming J. Liou
S. Nakamura
L. James Lee

Approved by

Ming J. Liou
Adviser
Department of Mechanical Engineering
ACKNOWLEDGMENTS

I would like to express my sincere appreciation to my adviser, Professor Ming J. Liou, and co-adviser, Professor L. James Lee, for their continuous encouragement and thoughtful guidance through this research. Also, I wish to express my appreciation to other members of my dissertation committee, Dr. S. Nakamura and Dr. C. Mobley, for offering their time and suggestions on the dissertation. Appreciation is extended to the Engineering Research Center and Super Computer Center of The Ohio State University.

I would like to thank my parents, Jender and Wheamay Young, and My wife, I-Chyun Liu, for their understanding and emotional support during this research.
VITA

May 1, 1961 ............................................................. Born, Taiwan, R.O.C.

1983 ......................................................................... B.S., National Taiwan University, Taiwan, R.O.C.

1985 ......................................................................... Teaching Assistant, National Taiwan University, Taiwan, R.O.C.

1987-1988 .............................................................. Research Assistant, The Ohio State University, Columbus, Ohio.

1988 ......................................................................... M.S., The Ohio State University, Columbus, Ohio.

1988-present ............................................................. Research Assistant, The Ohio State University, Columbus, Ohio.

PUBLICATIONS


FIELDS OF STUDY

Major Field: Mechanical Engineering
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ................................................................. ii
VITA .................................................................................................. iii

## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Literature Review</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Scope of Study</td>
<td>6</td>
</tr>
<tr>
<td>II. 2-D MOLD FILLING SIMULATION USING CONTROL VOLUME FINITE DIFFERENCE METHOD</td>
<td></td>
</tr>
<tr>
<td>2.1 Formulations</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Numerical Formulations</td>
<td>11</td>
</tr>
<tr>
<td>2.3 Experimental</td>
<td>15</td>
</tr>
<tr>
<td>2.4 Mold Filling With Various Fiber Stacks in the Mold Cavity</td>
<td>24</td>
</tr>
<tr>
<td>2.5 Conclusions</td>
<td>42</td>
</tr>
<tr>
<td>III. MOLD FILLING SIMULATION FOR IRREGULAR MOLD GEOMETRY</td>
<td></td>
</tr>
<tr>
<td>3.1 Formulations</td>
<td>44</td>
</tr>
<tr>
<td>3.2 Boundary Conditions and Computational Schemes</td>
<td>55</td>
</tr>
<tr>
<td>3.3 Experimental</td>
<td>58</td>
</tr>
<tr>
<td>3.4 Two Dimensional Isothermal Mold Filling</td>
<td>59</td>
</tr>
<tr>
<td>IV. THREE DIMENSIONAL MOLD FILLING SIMULATION</td>
<td></td>
</tr>
<tr>
<td>4.1 Formulations</td>
<td>74</td>
</tr>
<tr>
<td>4.2 Boundary Conditions and Computational Schemes</td>
<td>82</td>
</tr>
<tr>
<td>4.3 Three Dimensional Isothermal Mold Filling</td>
<td>83</td>
</tr>
</tbody>
</table>
4.3 Three Dimensional Isothermal Mold Filling

V. MESH REFINEMENT OF CONTROL VOLUME FINITE ELEMENT METHOD IN SOLVING MOVING BOUNDARY PROBLEMS

5.1 Introduction
5.2 Moving Front Mesh Refinement
5.3 Numerical Implementation
  5.3.1 Mesh Refinement
  5.3.2 Refinement of Source Nodes
  5.3.3 Determination of Filled Base Control Volumes
  5.3.4 Flow Chart of Moving Front Mesh Refinement Method
5.4 Non-isothermal Mold Filling Simulation in RTM
5.5 Non-isothermal Mold Filling Simulation in RIM
5.6 Conclusions

VI. CONCLUSIONS AND RECOMMENDATIONS

6.1 Numerical Simulation
6.2 Permeability of Fiber Mats

APPENDICES

A. COORDINATE TRANSFORMATION IN THE 2-D FORMULATION

B. PERMEABILITY MEASUREMENT OF FIBER MATS
  B.1 Scope of Study
  B.2 Permeability Measurements in the Planar Directions
  B.3 Permeability Measurements in the Transverse Direction

LIST OF REFERENCES
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Conditions for each experiments</td>
<td>23</td>
</tr>
<tr>
<td>2. Process conditions and thermal, kinetic, and rheological parameters used in RTM simulation (from Lin et al.[6])</td>
<td>114</td>
</tr>
<tr>
<td>3. Comparison of CPU time, temperature and conversion predicted for different mesh configurations (resin temperature = 20 °C, wall temperature = 75 °C)</td>
<td>120</td>
</tr>
<tr>
<td>4. Process conditions and thermal, kinetic, and rheological parameters used in RIM simulation (from Castro and Mocosko[8])</td>
<td>125</td>
</tr>
<tr>
<td>5. Comparison of CPU time, pressure, temperature and conversion for different mesh configurations (resin temperature = 60 °C, wall condition = adiabatic)</td>
<td>132</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A schematic diagram of a control volume</td>
</tr>
<tr>
<td>2. Schematic of a mold filling apparatus</td>
</tr>
<tr>
<td>3. Schematic of test setup used to obtain pictures of the flow front at the top and bottom surfaces of the mold at the same time with a single camera</td>
</tr>
<tr>
<td>4. Permeabilities as a function of porosity and superficial velocity for random fiber mat</td>
</tr>
<tr>
<td>5. Permeabilities as a function of porosity and superficial velocity for bidirectional fiber mat</td>
</tr>
<tr>
<td>6. Fiber mat arrangement in three cases. (1) 6 random fiber mats (2) 4 random fiber mats with two more in two regions (3) 4 random and 6 bidirectional fiber mats side-by-side in a mold of 40 cm x 13.5 cm</td>
</tr>
<tr>
<td>7. Experimental and computer simulation of flow front position as a function of time for flow in uniform random reinforcement mat in a center gated rectangular (13.5 cm X 40 cm) mold cavity (porosity = 82%, cavity thickness = .23 in, flow rate = 22.4 ml/sec)</td>
</tr>
<tr>
<td>8. Experimental and 3D computer simulation of flow pressure as a function of time for flow in uniform random reinforcement mat in a center gated rectangular mold cavity (porosity = 82%, cavity thickness = .23 in, flow rate = 22.4 ml/sec)</td>
</tr>
<tr>
<td>9. Experimental and computer simulation of flow front position as a function of time for flow in uniform random reinforcement mat with patches of extra random reinforcement mat in a center gated rectangular (13.5 cm X 40 cm) mold cavity (normal porosity = 85%, patch porosity = 80%, cavity thickness = .20 in, flow rate = 22.4 ml/sec)</td>
</tr>
<tr>
<td>10. Experimental and 2D computer simulation of flow pressure as a function of time for flow in uniform random reinforcement mat with patches of extra random reinforcement mat in a center</td>
</tr>
</tbody>
</table>
11. Experimental and computer simulation of flow front position as a function of time for flow in random [4R] and bidirectional [6B] reinforcement mat side by side in a center gated rectangular (13.5 cm X 40 cm) mold cavity (R porosity = 85%, B porosity = 67%, cavity thickness = .185 in, flow rate = 22.4 ml/sec) .......................................................................................... 31

12. Experimental and 2D and 3D computer simulations of flow pressure as a function of time for flow in random [4R] and bidirectional [6B] reinforcement mat side by side in a center gated rectangular mold cavity (R porosity = 85%, B porosity = 67%, cavity thickness = .185 in, flow rate = 22.4 ml/sec) .................................................................................. 32

13. Experimental and computer simulation of flow front position as a function of time for [6R-8B-6R] stacking sequence in a center gated 13.5 cm X 40 cm X 1.5 cm rectangular mold cavity (flow rate = 22.4 ml/sec) ..................................................... 34

14. Experimental flow front positions and the side view of computer simulation of flow front positions for [6R-8B-6R] stacking sequence in a center gated rectangular mold (flow rate=22.4 ml/sec) ............................................................................ 35

15. Experimental and computer simulation of flow front position as a function of time for [4B-12R-4B] stacking sequence in a center gated 13.5 cm X 40 cm X 1.5 cm rectangular mold cavity (flow rate = 22.4 ml/sec) .................................................................................. 37

16. Experimental flow front positions and the side view of computer simulation of flow front positions for [4B-12R-4B] stacking sequence in a center gated rectangular mold (flow rate=22.4 ml/sec) ............................................................................. 38

17. Experimental and 3D computer simulation of flow pressure as a function of time for the [6R-8B-6R] and [4B-12R-4B] stacking sequence in a center gated 13.5 cm x 40 cm x 1.5 cm rectangular mold cavity (flow rate = 22.4 ml/sec) ..................................................... 39

18. Experimental inlet pressure for filling a disk-shaped metal mold with a 4B-8R-4B or 4R-8B-4R fiber stack (mold thickness = 1.27 cm, porosity: R=82.6%, B=57.6%, flow rate = 41.9 ml/sec)......................................................... 41

19. The local coordinate in a curved thin shell model.................................. 48

20. Control volumes in a triangular mesh .................................................. 51
21. A typical two dimensional triangular element ........................................53
22. Simulated flow front, pressure distribution, and velocity field with 45° bidirectional fiber mat (flow rate = 2.75e-5 m^3/s, porosity = 0.7, k_y/k_x = 0.8, mold dimension 40 x 13.5 x 0.3 cm) ........................................................................................................61
23. Experimental and numerical comparison of flow front positions with 45° bidirectional fiber mat ..........................................................63
24. Simulated flow front, pressure distribution, and velocity field with random fiber mat (injecting pressure = 1.86E6 N/m^2, porosity = 0.7, mold dimension 40 x 13.5 x 0.3 cm) ..................................................64
25. Simulated flow front, pressure distribution, and velocity field with random fiber mat in a circular mold (flow rate = 2.771e-4 m^3/s, porosity = 0.7, mold thickness = 1.323 cm, radius = 0.2 m) ..........................................................................................................................66
26. A finite element model of an automotive inner hood ................................67
27. Simulated pressure distribution and flow front profiles with random fiber mats and a center gate mold filling ...............................68
28. Simulated pressure distribution and flow front profiles with random fiber mats and an end gate mold filling ........................................69
29. Comparison of simulated weldline positions in a mold with (a) center gate and (b) end gate ..............................................................71
30. Comparison of flow front profiles and weldline formation for different thickness and permeability in the insert areas ......................72
31. A typical wedge type element and corresponding normalized element ................................................................................................75
32. An "U" shape three dimensional finite element model ......................84
33. Mold filling simulation of flow front and pressure distribution with an anisotropic fiber mat aligned 45° along the mold (flow rate = 2.24E-5 m^3/s, porosity = 0.7, k_x : k_y : k_z = 10 : 5 : 1) ......................85
34. Comparison of numerical and actual flow fronts ..................................89
35. Base and refined elements and control volumes ..................................91
36. Three stages of element status .........................................................93
37. Initial mesh refinement for mold filling .............................................95
38. Inlet nodes 1 and 3 are filled at this point ................................................. 96
39. Inlet nodes are all filled at this point ......................................................... 98
40. Further change of element configuration ................................................. 99
41. Numbering of refined elements and nodes ............................................. 101
42. A loop configuration of source nodes ..................................................... 103
43. The configuration of refined control volumes in base control volumes .......................................................................................................................... 106
44. A schematic block diagram of the program flow chart ................................ 108
45. The discretized meshes of a simple mold ............................................... 115
46. The predicted flow front positions by using different mesh configurations in RTM simulation ................................................................. 116
47. The predicted temperature distribution by using different mesh configurations for a RTM simulation; the inlet temperature is 20 °C and the contour spacing is 10 °C; 1 = 30 °C ............................................. 117
48. The predicted conversion distribution by using different mesh configurations for a RTM simulation; the contour spacing is 0.004; 1 = 0.004 .................................................................................... 119
49. The discretized meshes of a simple rectangular mold with a line type end gate .................................................................................................................. 126
50. The predicted temperature distribution by using different mesh configurations for a RIM simulation; the inlet temperature is 60 °C, wall is adiabatic and the contour spacing is 10 °C; 1 = 70°C ....................................................................................................... 127
51. The predicted conversion distribution by using different mesh configurations for a RIM simulation; the contour spacing is 0.1; 1 = 0.1 .................................................................................... 128
52. The predicted temperatures at x=0.15 m and near the wall by using different meshes for a RIM simulation ......................................................... 130
53. The predicted conversions at x=0.15 m and near the wall by using different meshes for a RIM simulation ......................................................... 131
54. Schematic diagrams of the flow field in the kx device and the modification .......................................................................................................................... 137
55. Coordinate transformation and fiber orientation in a simple 2-D model ................................................................. 141
56. Drawing of a rectangular aluminum mold used for permeability measurement in the planar directions ....................... 146
57. Dry and wet fiber filling pressures of random fiber mats (CertainTeed U750) in the planar directions with porosity = 0.73 ........................................................................................................... 149
58. Dry fiber filling pressure of random fiber mats (CertainTeed U750) with respect to superficial velocity at different porosities ........................................................................................................ 150
59. Wet fiber filling pressure of random fiber mats (CertainTeed U750) with respect to superficial velocity at different porosities ........................................................................................................ 151
60. Permeability of CertainTeed U750 random fiber mat in the planar directions ................................................................. 152
61. A typical inlet pressure curve in a transverse mold filling using k_x device ........................................................................ 154
62. Transverse impregnation pressures of bidirectional fiber mats (Cofab A1118B) with different fiber stacking thickness but the same porosity, 0.6 .................................................................................. 155
63. Transverse impregnation pressure per unit length in the flow direction (Cofab A1118B, porosity = 0.6) .................................................. 156
CHAPTER I

INTRODUCTION

1.1 Introduction

Structural reaction injection molding, SRIM, is a process that effectively combines the advantages of reaction injection molding, RIM, and continuous fiber reinforcement. The SRIM process uses conventional RIM equipment and molds, and maintains production rates nearly as high as the RIM process. In SRIM, a preformed continuous or woven fiber mat is placed within the mold prior to filling. The reacting resin stream is then injected into the closed mold, allowing the resulting resin matrix and mat system to be demolded as a composite after curing. The mold filling only takes a few seconds, therefore, the injected resin must have low viscosity in order to facilitate the mold filling process. The viscosity of the resin should remain at a low value until the completion of the mold filling stage, and then increase quickly to minimize the mold curing time.

Resin transfer molding (RTM) is a process very similar to structural RIM. During resin transfer molding, resin is injected into mold cavities that have previously been filled with fibrous preforms. One of the major differences between the two processes is that RTM uses a lower filling rate and pressure than SRIM because the resins used in RTM are typically of much higher viscosity than those used in SRIM.
In a SRIM or RTM process, the effects of fiber mat type, injection velocity, and injection pressure on the in-mold flow are essential for the final molded part quality. To understand the mold filling process is very important for process control in SRIM and RTM. In mold filling, knowledge of the mechanism of the air entrapment, the flow front profile, the pressure distribution, and the wetting of the fibers are essential for the mold design, clamping force estimation, and product quality control. Currently, most of the commercial products made by SRIM are developed based on experience and repeated experiments, which are time consuming and inefficient. In order to optimize the manufacturing process and reduce the cost, studies on the issues related to the mold filling process are necessary.

1.2 Literature Review

The mold filling process of SRIM or RTM can be analyzed by either a microscopic or a macroscopic method. In the microscopic approach, the flow through the fibrous mat is studied based on the boundary conditions which account for all the fibers in the flow field. Little work has been done in this area because of computational limitations and the incomplete description of fibers in the fibrous mat, especially for the random fibrous mat. One approach to this problem is to simplify the fibrous mat as a set of parallel tubes or capillaries [1, 2, 3]. By solving the Navier-Stokes equation and the corresponding boundary conditions, the flow front and pressure distributions in the mold can be obtained. Chan and Hwang [2] developed a 1-D model to simulate the mold-filling process in a rectangular mold. Non-Newtonian flow with the power law response was taken into account in their studies to characterize the resin viscosity.
Instead of using Darcy's law, a set of parallel capillaries with constant diameters was employed to model the fiber mat. Isothermal, laminar, and fully developed flow in the capillaries was considered. It was concluded that as the inlet pressure or porosity was lower than a critical value, a short shot could be expected. They also showed the strong dependence of the inlet pressure on the shear rate dependent viscosity. However, this model can be misleading because the fibrous mats are often more complex than parallel tubes.

In macroscopic studies, the mold filling process can be regarded as a process of flow through porous media. There is much literature concerning the problems of flow through porous media with applications in the petroleum, agriculture, filtration, heating and refrigeration industries. However, the research in these fields is not directly applicable to the mold filling processes in SRIM and RTM.

The basic concerns in mold filling of SRIM/RTM are the resin flow through the fiber mats in a mold cavity and heat transfer between the resin and fibers. Following that, the heat of reaction during cure must be considered. Darcy's law for flow through porous media is the most commonly used equation for describing flow through the fibrous reinforcements. Darcy's law assumes plug flow and therefore does not account for any drag effects from the walls of the mold. Eckler and Rust [4] have reported that under most production conditions the flow resistance from the walls is much smaller than that of the porous media and can therefore be ignored. The exception to this is when the mold cavity is very thin and the porosity of the reinforcement is high.
Several types of mold filling have been studied. Gonzalez, et al.[5, 6] studied resin transfer molding in a disk-shaped mold. They decoupled the reaction from the mold filling and used both analytical and numerical methods to solve a 1-D mold filling and curing problem. They neglected the chemical reaction and heat transfer during the filling stage, while in the curing stage only heat conduction in the mold thickness direction was considered. It was assumed that no viscous dissipation or molecular diffusion occurred and, at each point within the mold, the fiber and resin temperatures were the same.

Reboredo and Rojas [7] considered the heat transfer and heat generation in their 1-D simulation of the SRIM process in a rectangular mold. Heat convection was considered in the filling stage while heat conduction was included in the curing stage. They used a modified Darcy's law with a second-order term in velocity to describe the momentum balance. The conversion, temperature, and pressure profiles were predicted. However, only 1-D flow was considered and no experimental work was reported to verify their results.

Most of the studies in SRIM/RTM were based on the assumption that the fiber mats were isotropic, which may not be true depending on the type of fiber mat. Coulter and Guceri [8] studied the effects of anisotropic reinforcements on resin flow in the RTM process. With the use of boundary-fitted curvilinear coordinate systems, a 2-D finite difference method was used to analyze the in-plane flow in molds with irregular shapes. However, when different types of fiber mats are stacked together, the porosity and permeability in the mold thickness direction may vary. Studies relating to the effects of fiber stacks were not reported in their work.
Permeability variations in the fiber stack may occur for several reasons. For instance, if a part must be stronger near an attachment point and the thickness cannot be increased, extra reinforcement can be added to increase the strength at that point. If sufficient strength cannot be obtained using one type of fiber mat, then several mat types can be incorporated in the mat stack. Coulter and Guceri [8] also studied the molding flow with permeability variations in the planar direction.

In macroscopic studies of SRIM/RTM processes mentioned earlier, the basic flow model was Darcy's law. By regarding the flow through fiber mats as a flow through porous media, the permeability of the fiber mat is an important parameter for accurately determining the flow resistance and molding pressure. Adams et. al. [9, 10] used a simple model to study the radial flow through fiber mats. Based on the analytical solution for the radial flow field, they could determine the fiber mat permeabilities in both principal directions. They also investigated the runner effect of the fiber stacks with different types of fiber mats and the anisotropic behavior of stitched fiber mats. Gauvin et. al. [11, 12] measured the permeability of fiber mat at various porosity. They again confirmed the basic assumption of Darcy’s law, pressure drop through fiber mat is proportional to the superficial velocity. A simple sum-up rule was used by them to estimate the permeability of fiber stack with different types of fiber mats. Molnar et. al. [13] performed a series of experiments to study the fiber wetting process with various fiber types and flow rates. They also found the runner effect of fiber stack with different types of fiber mats. Trevino et. al. [14] had a systemic study of principal permeabilities using random, bidirectional and unidirectional fiber mats. Experiments were conducted to investigate flow front variations with various types of
fiber mats either stacked together or put side by side. The permeability in the transverse
direction was found to depend not only on porosity but also on thickness. The fiber
elastic response under uniform pressure was also measured in their experimental work.

1.3 Scope of Study

In this study, both 2-D and 3-D numerical simulation models for the mold
filling process are developed. The flow front progression and transient filling pressure
are predicted and compared with the corresponding experimental results. Specifically,
the permeability variations both in the plane and through the thickness of the part are
examined. Local permeability variations in the plane of the part were formed by either
changing the type of reinforcement or by adding more layers of the reinforcement in
that area. Permeability variations through the thickness were accomplished by using
multiple types of reinforcement in the stacking sequence.

Chapter II uses a simple model to study the mold filling flow in a rectangular
mold. The model was applied to study the effect on the mold filling mechanism with
permeability variations in both planar and transverse directions. Chapter III develops a
numerical model for simulating the mold filling flow with complicated mold geometry.
The model also deals with the mold filling process with preplaced anisotropic fiber
mats. Chapter IV extends the model to three dimensional type of flow, which can
simulate a complete 3-D type of Darcy's flow in complicated 3-D flow domain.
Chapter V presents a numerical method which improves the computing efficiency in
solving moving boundary types of problems.
2.1 Formulations

In order to simulate the filling process of RTM or SRIM several assumptions must be made to simplify the problem. In this study, the preplaced fiber mats in the mold cavity are assumed rigid and no deformation occurs during mold filling. Capillary and inertia effects are neglected due to the low Reynolds number of the resin flow. Also, surface tension is considered negligible compared to the dominant viscous force.

The mold cavity is assumed to be much larger than the pore size of the fiber mat, therefore, Darcy's law for flow through porous media can be used to replace the momentum equation. It can be written in the form:

\[ \mathbf{v} = -\frac{\mathbf{k}}{\mu} \cdot \nabla P \]  

(2.1)

where \( \mu \) is the viscosity and \( \bar{\mathbf{k}} \) is the permeability tensor. In a three dimensional flow field under Cartesian coordinates, the velocity vector \( \mathbf{v} \) consists of three components,
u, v and w, in the x, y and z direction, respectively. Equation 2.1 can then be written as:

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} = -\frac{1}{\mu} \begin{bmatrix}
  k_{xx} & k_{xy} & k_{xz} \\
  k_{yx} & k_{yy} & k_{yz} \\
  k_{zx} & k_{zy} & k_{zz}
\end{bmatrix} \begin{bmatrix}
  \frac{\partial P}{\partial x} \\
  \frac{\partial P}{\partial y} \\
  \frac{\partial P}{\partial z}
\end{bmatrix}
\]

\[(2.2)\]

where \(k_{ij}\) (\(i, j = x, y, z\)) are the components of the permeability tensor. For an incompressible fluid, the continuity equation can be reduced to the form:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

\[(2.3)\]

If the coordinate axes are so chosen that their directions match the principal directions of the fiber mats, the cross terms in the permeability tensor become zero. Therefore, non-zero terms of the permeability tensor only exist in the diagonal. Substituting equation 2.2 into equation 2.3 and aligning the axes to the principal directions of the fiber mat yield:

\[
\frac{\partial}{\partial x} \left( \frac{k_{xx}}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{k_{yy}}{\mu} \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{k_{zz}}{\mu} \frac{\partial P}{\partial z} \right) = 0
\]

\[(2.4)\]

The pressure field during mold filling can be determined by solving equation 2.4. If the permeability and viscosity are assumed constant throughout the flow field, equation 2.4 is reduced to a Laplace equation, which can be solved with various numerical techniques. During mold filling the position of the flow front varies with
time and equation 2.4 turns out to be a quasi-steady-state problem with moving boundaries.

In this chapter a control volume finite difference method was used [15]. This method facilitates the physical interpretation of mass conservation within the control volume. The pressure is defined at each nodal point as shown in Figure 1. The control volume is formed by a quadrilateral with each of its sides located midway between node (i,j,k) and the adjacent nodes. The velocity components u, v, and w are defined as in Figure 1.

Equation 2.4 can be integrated over a control volume leading to:

\[ \iiint_V \nabla \cdot \left( \frac{k_{xx}}{\mu} \frac{\partial p}{\partial x} \mathbf{i} + \frac{k_{yy}}{\mu} \frac{\partial p}{\partial y} \mathbf{j} + \frac{k_{zz}}{\mu} \frac{\partial p}{\partial z} \mathbf{k} \right) \, dV = 0 \]  \hspace{1cm} (2.5)

By using the Divergence theorem, Equation 2.5 can be rewritten as:

\[ \iint_S \left( \frac{k_{xx}}{\mu} \frac{\partial p}{\partial x} \mathbf{i} + \frac{k_{yy}}{\mu} \frac{\partial p}{\partial y} \mathbf{j} + \frac{k_{zz}}{\mu} \frac{\partial p}{\partial z} \mathbf{k} \right) \cdot \mathbf{n} \, dS = 0 \]  \hspace{1cm} (2.6)

where S is the boundary surface of the volume V, and \( \mathbf{n} \) is the unit vector perpendicular to the boundary surface. If a control volume shown in Figure 1 is considered, equation 2.6 combined with equation 2.2 can be written as:

\[ \iint_S (u \mathbf{i} + v \mathbf{j} + w \mathbf{k}) \cdot \mathbf{n} \, dS = 0 \]  \hspace{1cm} (2.7)
Figure 1  A schematic diagram of a control volume
and

\[ q_{i-1,j,k} + q_{i,j,k} + q_{i+1,j,k} + q_{i,j-1,k} + q_{i,j,k+1} + q_{i,j,k-1} + q_{i,j,k+1} = 0 \]  (2.8)

where \( q \) is the flow rate across the boundary surface of the control volume. Equation 2.8 is the working equation used for mold filling simulation in the control volume finite element method.

2.2 Numerical Formulations

In the mold cavity, the viscosity and permeability may vary due to non-Newtonian effects and different types of fiber mat. The velocity defined on the border of a control volume is not continuous between two adjacent control volumes with different permeability. An averaging scheme was used to define an average permeability which will ensure the velocity to be continuous in the computation.

If a velocity component, \( u_{i-1/2,j,k} \), is considered in Figure 1, the following equations can be established by using equation 2.1

\[
 u_{i-1/2,j,k} = -\frac{k_{xx;i-1,j,k}}{\mu_{i-1,j,k}} \frac{(P_{i-1/2,j,k} - P_{i-1,j,k})}{(\Delta x_{i-1/2} / 2)} \]  (2.9)

\[
 u_{i-1/2,j,k} = -\frac{k_{xx;i,j,k}}{\mu_{i,j,k}} \frac{(P_{i,j,k} - P_{i-1,j,k})}{(\Delta x_{i-1/2} / 2)} \]  (2.10)
By combining equations 2.9 and 2.10 and eliminating $P_{i-1/2,j,k}$, the velocity can be expressed as

$$u_{i-1/2,j,k} = -\xi_{i-1/2,j,k} \frac{(P_{i,j,k} - P_{i-1,j,k})}{(\Delta x_{i-1/2})}$$

(2.11)

where

$$\xi_{i-1/2,j,k} = \frac{2}{\xi_{i-1,j,k} + \xi_{i,j}}$$

(2.12)

Therefore, the flow rate at the left boundary of the control volume of node $(i,j,k)$ can be defined as

$$q_{i-1/2,j,k} = -\xi_{i-1/2,j,k} \frac{(P_{i,j,k} - P_{i-1,j,k})}{\Delta x_{i-1/2}} \Delta f_j \Delta f_k$$

(2.13)

$$\Delta f_j = \frac{\Delta y_{j-1/2} + \Delta y_{j+1/2}}{2}$$

$$\Delta f_k = \frac{\Delta z_{k-1/2} + \Delta z_{k+1/2}}{2}$$

(2.14)

where $\Delta y_{j+1/2}$ is the distance between nodes $(i,j,k)$ and $(i,j+1,k)$ in the y-direction, $\Delta z_{k+1/2}$ is the distance between nodes $(i,j,k)$ and $(i,j,k+1)$ in the z-direction and $\Delta x_{i-1/2}$ is the distance between nodes $(i,j,k)$ and $(i-1,j,k)$. Based on equation 2.13, equation 2.8 can be written as
\[
\begin{align*}
\zeta_{i-\frac{1}{2},j,k} & \cdot \frac{(P_{i-1,j,k} - P_{i,j,k})}{\Delta x_{i-\frac{1}{2}}} \Delta f_j \Delta f_k \\
+ \zeta_{i+\frac{1}{2},j,k} & \cdot \frac{(P_{i+1,j,k} - P_{i,j,k})}{\Delta x_{i+\frac{1}{2}}} \Delta f_j \Delta f_k \\
+ \zeta_{i,j-\frac{1}{2},k} & \cdot \frac{(P_{i,j-1,k} - P_{i,j,k})}{\Delta y_{j-\frac{1}{2}}} \Delta f_j \Delta f_k \\
+ \zeta_{i,j+\frac{1}{2},k} & \cdot \frac{(P_{i,j+1,k} - P_{i,j,k})}{\Delta y_{j+\frac{1}{2}}} \Delta f_j \Delta f_k \\
+ \zeta_{i,j,k-\frac{1}{2}} & \cdot \frac{(P_{i,j,k-1} - P_{i,j,k})}{\Delta z_{k-\frac{1}{2}}} \Delta f_j \Delta f_k \\
+ \zeta_{i,j,k+\frac{1}{2}} & \cdot \frac{(P_{i,j,k+1} - P_{i,j,k})}{\Delta z_{k+\frac{1}{2}}} \Delta f_j \Delta f_k \\
\end{align*}
\]

\[\text{(2.15)}\]

and

\[
\begin{align*}
(k_w + k_x + k_y + k_n + k_z) (P_{i,j,k} = & k_w P_{i-1,j,k} + k_x P_{i+1,j,k} \\
+ & k_y P_{i,j-1,k} + k_n P_{i,j+1,k} + k_z P_{i,j,k-1} + k_x P_{i,j,k+1} \\
\end{align*}
\]

\[\text{(2.16)}\]

where

\[
\begin{align*}
k_w & = \frac{\zeta_{i-\frac{1}{2},j,k} \Delta f_j \Delta f_k}{\Delta x_{i-\frac{1}{2}}} \\
k_x & = \frac{\zeta_{i+\frac{1}{2},j,k} \Delta f_j \Delta f_k}{\Delta x_{i+\frac{1}{2}}} \\
k_y & = \frac{\zeta_{i,j-\frac{1}{2},k} \Delta f_j \Delta f_k}{\Delta y_{j-\frac{1}{2}}} \\
k_n & = \frac{\zeta_{i,j+\frac{1}{2},k} \Delta f_j \Delta f_k}{\Delta y_{j+\frac{1}{2}}} \\
k_z & = \frac{\zeta_{i,j,k-\frac{1}{2}} \Delta f_j \Delta f_k}{\Delta z_{k-\frac{1}{2}}} \\
k_x & = \frac{\zeta_{i,j,k+\frac{1}{2}} \Delta f_j \Delta f_k}{\Delta z_{k+\frac{1}{2}}}
\end{align*}
\]
Equation 2.16 can be written for all nodes in the flow field, which results in a set of linear algebraic equations. Boundaries of the flow domain in mold filling include the mold walls, inlets, and flow front. At the mold walls there is no flow in the direction normal to the wall. A constant flow rate is assigned to the control volumes of the inlet nodes. In other words, the inlet nodes are treated as source nodes which generate constant flow flux. At the flow front, a parameter $f$ is used to represent the status of each control volume in the flow domain [15]. If the control volume is empty, $f$ is equal to zero. If the control volume is filled with liquid, $f$ is equal to 1. If the control volume is partially filled, $f$ is equal to the volume fraction of the fluid occupying the element. The control volumes with $f$ values between 1 and 0 are considered the flow front nodes. The nodal pressures in the partially filled flow front control volumes are set to be zero. With these described boundary conditions, the set of linear algebraic equations developed previously can be solved to determine the pressure field during mold filling. The velocity field can then be computed with Darcy's law.

Although the above formulation is steady state, the mold filling process is not. However, since the time derivative term can be neglected in the low Reynolds number flow, the mold filling can be regarded as a quasi-steady state process by assuming a steady state condition at each time step. In other words, the problem is solved by considering the transient solution to be a sequence of steady state solutions separated by small time increments. The new flow front can be estimated according to the velocity
vector in the flow front and the time step. If \( \Delta t \) is the time step, \( q \) is the liquid flux into each control volume, and \( V \) is the total volume of a control volume, \( f \) can be defined as

\[
f^{n+1} = f^n + \Delta t \cdot q^n / V
\]

(2.18)

where \( n \) represents each quasi-steady state and \( q \) is determined by the velocity field. In equation 2.18, \( q \) is equal to zero for all the control volumes located at the liquid domain because of the continuity condition applied in this domain and has a non zero value for the flow front control volumes since constant pressure boundary condition is applied at the flow front.

The selection of the size of a time increment for each quasi-steady state is based on the consideration that each time increment allows only one control volume to be completely filled. In some cases, more than one control volume may be filled simultaneously. This restriction of the time increment ensures the stability of the quasi-steady state approximation. This procedure of solving a steady state problem and flow front propagation is repeated until the mold is filled completely.

2.3 Experimental

The experimental work was conducted with the mold filling apparatus developed in our laboratory [14] and diagramed in Figure 2. A constant speed pump was set up by mounting a 8.26 cm diameter hydraulic cylinder in the test section of an Instron Universal Testing Instrument (Model 1137). The cylinder was filled by a hand operated rotary feed pump. After the cylinder was filled and the valve to the transfer
Figure 2  Schematic of a mold filling apparatus
pump closed, a valve to the mold was opened and the Instron's crosshead was set to
descend at a constant speed. The descending crosshead pushed the cylinder ram back
into the body of the cylinder, thereby forcing the nonreactive fluid (DOP oil) into the
mold. The fluid flow rates were varied by changing the speed of the crosshead.

The 20.32 cm by 80.01 cm platens of the mold were constructed out of 1.27 cm
thick clear acrylic sheets so progress of the flow front could be observed during filling.
The mold cavity thickness was varied by inserting different rectangular spacers. The
mold was sealed by gluing 0.79 mm rubber gasket to the spacer so that when the mold
was closed, the rubber gasket would seal the seams between the mold halves and the
spacer. The assembly was clamped together with steel angle irons and C-clamps. Steel
angle iron or flat plate was used between the C-clamps and the platens to spread the
force applied by the C-clamps and to prevent damage to the acrylic platens from the
concentrated force of the clamps. When visual inspection of the flow front progress
was not required from one or both sides of the mold, the platen on that side was
reinforced with 0.635 cm thick steel plates to reduce bending of the platen from the mat
compression force. When platen bending occurs, an average thickness of the cavity
was measured after the mold was assembled. The mold filling pressure was measured
with a pressure gauge (Omega, Model PX302, 0-150 psi) mounted near the inlet of the
filling line. A setup was used to take pictures of the top and bottom surfaces of the
mold at the same time with only one camera. Figure 3 shows the schematic diagram of
this setup.

Two types of fiber mats, a continuous random fiber mat (OCF - M8610) and a
stitched bidirectional fiber mat (Cofab - A1118B), were used as reinforcements in this
Figure 3  Schematic of test setup used to obtain pictures of the flow front at the top and bottom surfaces of the mold at the same time with a single camera.
The permeabilities in the flow directions ($k_x$ and $k_y$) and the transverse direction ($k_z$) for different types of glass fiber mats as functions of flow rate and mat porosity have been reported [14]. Several functions were proposed by fitting the curves of permeability versus the fiber mat porosity and the superficial velocity. The results are shown in equations 2.19 to 2.23.

Random fiber mat:

$$k_x = k_y = 241 \nu' + 40 \ e^{5.45 \phi}$$  \hspace{1cm} (2.19)

$$k_z = \frac{62 \phi^3}{(1 - \phi)^2}$$  \hspace{1cm} (2.20)

Bidirectional fiber mat:

$$k_x = 47 \ e^{5.92 \phi} + (2018 \phi - 592) \nu'$$  \hspace{1cm} (2.21)

$$k_y = \frac{4}{3} k_x$$  \hspace{1cm} (2.22)

$$k_z = \frac{272 \phi^3}{(1 - \phi)^2}$$  \hspace{1cm} (2.23)

where $\phi$ is the porosity, $\nu'$ (cm/sec) is the superficial velocity, and the unit for the permeability is the darcy. Equations 2.19 and 2.20 are applicable in the porosity range from 0.54 to 0.95, and equations 2.21 to 2.23 are applicable in the porosity range from
0.45 to 0.67. The applicable superficial velocity range is from 1 cm/sec to 11 cm/sec. Figures 4 and 5 show the comparison of the experimental results (stars) and fitted curves (solid lines). Equations 2.19 to 2.23 were used for calculating the permeability of the fiber mats in the computer simulations.

Two sets of experiments were run in this study. The first set included three different arrangements of fiber mats in the planar direction as shown in Figure 6. In case 1, the whole cavity was preplaced with six layers of random fiber mats. In case 2, four layers of random fiber mats were preplaced in the cavity and two more layers of random fiber mats were added to two small rectangular areas as shown in Figure 6. Therefore, porosity and permeability variations were created in the plane. In case 3, 4 layers of random and 6 layers of bidirectional fiber mats were placed side-by-side in the mold cavity and the porosity as well as permeability was different in these two regions. In cases 1 and 3, two runs were conducted with the same fiber mat arrangement. For one run, a 2.5 cm hole was cut through the mats under the gate so that there would be no flow resistance in the thickness direction at the entrance. The flow pattern resembles a 2-D (i.e. x and y direction) mold filling. The second run was done without the hole through the mats under the gate, and the flow pattern resembles a 3-D mold filling. The detailed information for each case is listed in Table 1.

The second set included experiments using the same layers of fiber mats (i.e. 12 layers of random fiber mats, R, and 8 layers of bidirectional fiber mats, B) but different stacking order to investigate the importance of stacking sequence. Two stacking sequences were employed (6R-8B-6R and 4B-12R-4B) and photographs were taken during mold filling from the top and bottom of the mold to determine the flow front
Figure 4 Permeabilities as a function of porosity and superficial velocity for random fiber mat.
Figure 5  Permeabilities as a function of porosity and superficial velocity for bidirectional fiber mat.
Figure 6  Fiber mat arrangement in three cases. (1) 6 random fiber mats (2) 4 random fiber mats with two more in two regions (3) 4 random and 6 bidirectional fiber mats side-by-side in a mold of 40 cm x 13.5 cm.

Table 1  Conditions for each experiment.

<table>
<thead>
<tr>
<th>Fiber mats combination</th>
<th>Porosity</th>
<th>Mold thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 6 R</td>
<td>$\phi_R = 0.82$</td>
<td>0.58 cm with hole and without hole</td>
</tr>
<tr>
<td>Case 2 4R with 6R patches</td>
<td>$\phi_{4R} = 0.85$</td>
<td>0.50 cm with hole</td>
</tr>
<tr>
<td>Case 3 4R and 6B side-by-side</td>
<td>$\phi_{4R} = 0.85$</td>
<td>0.47 cm with hole and without hole</td>
</tr>
<tr>
<td>SET 2 6R-8B-6R fiber stacks</td>
<td>$\phi_R = 0.80$</td>
<td>1.50 cm without hole</td>
</tr>
<tr>
<td>4B-12R-4B fiber stacks</td>
<td>$\phi_B = 0.555$</td>
<td>1.50 cm without hole</td>
</tr>
</tbody>
</table>
position at each mold face. The mold cavity thickness was 1.5 cm for the second set of experiments.

As mentioned before, the mold platens tended to deform when the mold was clamped together. Therefore, the thickness of the mold cavity in each experiment was determined by averaging the measured thickness at the center and the side of the mold after the mold was clamped. For the second set of experiments, when bidirectional and random fiber mats were clamped together, the individual thickness of each layer was calculated based on the method presented in [14]. The porosity of the fiber mat in each run was determined using the mold cavity volume and the fiber weight and density.

2.4 Mold Filling With Various Fiber Stacks in the Mold Cavity

The effects of permeability variations in the fibrous reinforcement on the mold filling of RTM/SRIM are studied here. Specifically, the permeability variations both in the plane and through the thickness of the fiber mat are examined. Local permeability variations in the plane of the part were formed by either changing the type of reinforcement or adding more layers of the same reinforcement in some areas, thereby changing the porosity and therefore the permeability of those areas. Permeability variations through the thickness are formed by using multiple types of reinforcement in the stacking sequence. The variations in type and/or amount of reinforcement would create permeability variations in the flow field of the mold cavity. Permeability variations in a fiber stack may occur for several reasons. If a part needs to be stronger near an attachment point, without changing the thickness of that part, extra reinforcement can be added to increase the fiber content in that area. If strength is not
enough in a stacking sequence using random fiber mats, some of the random fiber mats can be replaced by directional fiber mats.

Figure 7 is a plot of the predicted position of the flow front versus time and the actual position of the flow front from experiment at three different times for case 1 (without hole). There is good agreement between the simulation and the experiment. The shape of the experimental flow front at 8 seconds showed some distortion, which was probably due to flow channelling between the reinforcements and the edge of the mold. This often occurs even if the reinforcements are completely against the wall as they were in this case. Figure 8 is a plot of the transient pressure prediction from the 3-D computer simulation and pressure measurements for this case. The predicted pressure is matched reasonably well with the measurements. The flow contacted the side wall of the mold after several seconds and it can be seen that there is a transition in the relationship between pressure and time. This is because the filling is a radial flow before the fluid contacts the side walls and it becomes a unidirectional flow after contacting the side walls. The difference in measured pressure between the experiment with a hole under the gate and that without a hole was small, indicating that the flow resistance through the random fiber mat in the thickness direction is relatively small and does not add significant pressure during mold filling.

In the second case, extra reinforcement mats were added in specific areas of the mold to study the effects of local permeability variations. In industrial practice, these variations could be either intentional or due to variations in the porosities of the mats. The permeability variations were created by adding two more layers of random fiber mats. The two patches were an equal distance away from the flow entrance. The
Figure 7 Experimental and computer simulation of flow front position as a function of time for flow in uniform random reinforcement mat in a center gated rectangular (13.5 cm X 40 cm) mold cavity (porosity = 82%, cavity thickness = .23 in, flow rate = 22.4 ml/sec).
Inlet Pressure
6R, Porosity=82%, Center Gate

![Pressure vs Time Graph](image)

Figure 8 Experimental and 3D computer simulation of flow pressure as a function of time for flow in uniform random reinforcement mat in a center gated rectangular mold cavity (porosity = 82%, cavity thickness = .23 in, flow rate = 22.4 ml/sec).
position of the patches along with the predicted and actual flow front positions, can be seen in Figure 9. A hole was cut through the mats under the gate in this case. It can be seen that by increasing the fiber content by 50% in the area of the patches the permeability in that area was not altered enough to create a dry (resin free) area in the part. In fact, the porosity decrease only slightly slowed the advancement of the flow front. The pressure traces of the 2-D computer simulation and experimental measurements are presented in Figure 10. Again, the simulation matched reasonably well with experimental results, considering that the porosity of the random fiber mats may vary as much as 15% [14].

Experiments were also run by using random and bidirectional fiber mats side by side in the mold cavity. Since the random fiber mat in this case has a higher permeability, the flow front expands more slowly in the bidirectional fiber mats than in the random fiber mats as shown in Figure 11. The pressure traces for both experiments and the computer simulations (2-D and 3-D) are plotted in Figure 12. The simulation matched the experimental results. The deviation of the predicted inlet pressure from the measured was due to the error of the used permeability, which came from the porosity variation of the random fiber mats [14] or the error inherited from the permeability curve fitting.

Large parts formed by RTM/SRIM are commonly center gated to reduce the resin flow lengths and to properly center the forces in the clamping press. By center gating instead of edge gating the mold, the resin must pass through the plane of the top layers of reinforcement in the mat stack before flowing away from the gate in the in-
Figure 9 Experimental and computer simulation of flow front position as a function of time for flow in uniform random reinforcement mat with patches of extra random reinforcement mat in a center gated rectangular (13.5 cm X 40 cm) mold cavity (normal porosity = 85%, patch porosity = 80%, cavity thickness = .20 in, flow rate = 22.4 ml/sec).
Figure 10  Experimental and 2D computer simulation of flow pressure as a function of time for flow in uniform random reinforcement mat with patches of extra random reinforcement mat in a center gated rectangular mold cavity (normal porosity = 85%, patch porosity = 80%, cavity thickness = .20 in, flow rate = 22.4 ml/sec).
Figure 11  Experimental and computer simulation of flow front position as a function of time for flow in random [4R] and bidirectional [6B] reinforcement mat side by side in a center gated rectangular (13.5 cm X 40 cm) mold cavity (R porosity = 85%, B porosity = 67%, cavity thickness = .185 in, flow rate = 22.4 ml/sec).
Figure 12 Experimental and 2D and 3D computer simulations of flow pressure as a function of time for flow in random [4R] and bidirectional [6B] reinforcement mat side by side in a center gated rectangular mold cavity (R porosity = 85%, B porosity = 67%, cavity thickness = .185 in, flow rate = 22.4 ml/sec).
plane directions of the reinforcement. When studying mold filling near the gate, flow in both the radial and thickness directions must be considered.

Figure 13 shows experimental results of the flow front positions at the top and bottom mold surfaces along with the flow front predictions by the 3-D computer simulations for the 6R-8B-6R mat stack. Both experiment and simulation show the lead-lag in flow front positions between the top and bottom surfaces in the regions near the gate. The simulations agree well with the experiments for the top surface but not as well for the bottom surface in the area near the gate. Since the thickness is much smaller than the planar dimensions (x,y), a small inaccuracy in transverse flow (z direction) prediction may result in poor flow front predictions on the bottom surface. The lead-lag in flow front positions between the top and bottom surfaces of the mold is clearly shown in Figure 14, which shows the experimental flow front positions and the side view of the 3-D simulation.

The x direction permeability of the random fiber mat is higher than that of the bidirectional fiber mat and, therefore, the fluid would prefer to flow through the random fiber mat in the in-plane direction and then fill the bidirectional fiber mat across the mold thickness. The fluid, however, must first reach the random fiber mat near the mold gate. In the 6R-8B-6R stacking sequence the fluid enters directly into the upper group of random fiber mats and can flow easily in the in-plane directions without having to cross the middle group of low permeability bidirectional fiber mats. In other words, the bidirectional fiber mats tend to form a flow barrier in the thickness direction that prevents the fluid from reaching the lower group of random fiber mats. This subsequently causes the difference in flow front positions at the top and bottom mold
Flow Front Contour
6R-8B-6R, Porosity: R=80% ; B=55.5%

Figure 13 Experimental and computer simulation of flow front position as a function of time for [6R-8B-6R] stacking sequence in a center gated 13.5 cm X 40 cm X 1.5 cm rectangular mold cavity (flow rate = 22.4 ml/sec).
Flow Front Contour
6R-8B-6R, Porosity: R=80.0%, B=55.5%

Figure 14 Experimental flow front positions and the side view of computer simulation of flow front positions for [6R-8B-6R] stacking sequence in a center gated rectangular mold (flow rate=22.4 ml/sec)
surfaces observed in Figures 13 and 14. As the distance from the mold entrance is increased, the ratio of this cross flow resistance to the total flow resistance decreases and the differences in flow front position between the top and bottom surfaces also decreases.

In the 4B-12R-4B stack, the flow situation (Figures 15 and 16) is different. The flow must cross the first group of bidirectional fiber mats to reach the higher permeability random fiber mats in the center. As soon as the fluid reaches the middle random fiber mats, it can easily cross these mats and reach the bottom fiber layers. This, in conjunction with the high initial filling pressure, results in a relatively small difference in flow front position at the top and bottom mold surfaces as shown in Figure 15. Also, Figure 16 provides a clear comparison of the flow front positions between the top and bottom surfaces of the mold. In this case, the predictions match the experiments very well. Comparing Figures 13 and 15, the stacking sequence can be seen to play a major role in how the flow front advances near the gate region. Two distinctly different flow regions can be seen. At and very near the center gate, there is a transition zone. The length of this transition zone depends on the fiber stacking sequence for the same combination of fiber mats. Farther away from the gate the resin has already penetrated through the thickness of the mat stack and will not flow perpendicular to the reinforcement layers unless induced to do so by permeability variations. Flow away from the gate in a center gated mold can often be modeled as a planar flow.

Figure 17 is a plot of the pressure traces for the two experiments and their corresponding 3-D computer simulations. In both cases, the simulated pressure trace
Flow Front Contour

4B-12R-4B, Porosity: R=80%; B=55.5%

Figure 15: Experimental and computer simulation of flow front position as a function of time for [4B-12R-4B] stacking sequence in a center gated 13.5 cm X 40 cm X 1.5 cm rectangular mold cavity (flow rate = 22.4 ml/sec).
Flow Front Contour

4B-12R-4B, Porosity: B=55.5%, R=80.0%

Figure 16 Experimental flow front positions and the side view of computer simulation of flow front positions for [4B-12R-4B] stacking sequence in a center gated rectangular mold (flow rate=22.4 ml/sec)
Figure 17 Experimental and 3D computer simulation of flow pressure as a function of time for the [6R-8B-6R] and [4B-12R-4B] stacking sequence in a center gated 13.5 cm x 40 cm x 1.5 cm rectangular mold cavity (flow rate = 22.4 ml/sec).
fell below the experimental data. This may result from the inaccuracy of $k_z$
measurement or the non-uniform mold thickness due to mold bending. Since the mold
thickness along the mold edges is less than the average thickness used in the
simulation, the experimental inlet pressure may be higher than that predicted by the
simulation. In order to further study the effect of the mat stack on the inlet pressure,
experiments were conducted using a disk-shaped metal mold (1.27 cm in thickness)
and two different mat stacks out of the same fiber mats (8 random fiber mats and 8
bidirectional fiber mats, $\phi_R = 82.6\%$, $\phi_B = 57.6\%$). Two runs were performed for the
same mat stack, one with a hole cut through the mat stack under the inlet gate and the
other without a hole. The measured inlet pressures were shown in Figure 18. When a
hole was cut through the fiber stack, the inlet pressure was about the same for
experiments with different fiber stacking sequences. In the experiments with a 4R-8B-
4R fiber stack, the inlet pressure did not change as a hole was cut through the mat stack
because, in both cases, the major pressure drop was due to the fluid through the
random fiber mats in the planar direction. In the cases with a 4R-8B-4R fiber stack,
whether a hole was cut through the fiber mats or not, the flow through thickness
direction of the bidirectional fiber mats was not important. However, in the case with a
4B-8R-4B fiber stack, the flow through the thickness of the bidirectional fiber mats
played an important role. And, the transverse permeability is not equal to the
summation of the transverse permeability of each layer. Thus, the simulation pressure,
which used the summation of the transverse permeability of each layer, was less than
measured inlet pressure in the experiment with a 4B-8R-4B fiber stack and without a
hole cut through it.
Figure 18 Experimental inlet pressure for filling a disk-shaped metal mold with a 4B-8R-4B or 4R-8B-4R fiber stack (mold thickness = 1.27 cm, porosity: R=82.6%, B=57.6%, flow rate = 41.9 ml/sec).
It is also clear that the transverse flow permeability is sensitive to the mat stacking sequence. The simulated pressure trace as well as the measured result for the 4B-12R-4B experiment was higher than that of 6R-8B-6R due to the differences in flow paths near the gate. The overshoot and subsequent decline of the pressure recorded for the 4B-12R-4B experiment are not fully understood. It is possible that under a high filling pressure the transverse permeability of the fiber mat changes with time due to fiber deformation because the fiber mats under the inlet gate were loose compact due to the mold bending. And, the overshoot was not sensed in the experiments using an uniform thickness metal mold (Figure 18). Further studies are being conducted to understand this phenomenon.

2.5 Conclusions

In this study, the numerical simulation yields a good prediction of the flow front positions during mold filling. The two dimensional computer model was found to predict the filling pressure well when the random fiber mats, which have high permeability in the thickness direction, were used or when a hole was cut through the mat stack under the flow entrance. The inlet pressure, however, is not well matched between simulation and experimental results for cases where the flow through the thickness direction of the bidirectional fiber mats plays a major role. It appears that the thickness direction permeability of the bidirectional fiber mat depends not only on the porosity but also other effects, such as the interface between two fiber mats, the total stack thickness, and mat compressibility. Since the numerical simulation used here is based on quadrilateral meshes, the number of control volumes near the inlet of a center-gated mold may be too small to accurately model the flow near the entrance, which may
also introduce some error in the prediction of the inlet pressure. A simulation using triangular meshes is currently under development in order to increase the accuracy of prediction.

In a center-gated mold the flow enters from the top or bottom of the fiber stack. Therefore, in areas near the flow entrance, the flow must pass through the thickness of the mat stack. The permeability in the thickness direction has a large effect on the total flow resistance of the part. In areas not near the gate of a center-gated mold the stacking sequence does not affect the filling pressure or the progress of the flow front.
CHAPTER III
MOLD FILLING SIMULATION
FOR IRREGULAR MOLD GEOMETRY

3.1 Formulations

In the previous chapter, a control volume finite difference method was used to simulate the molding flow with preplaced fiber mats. However, this method is limited to a regular mesh, i.e. regular mold shape. In reality, the mold geometry can be quite complicated and the previous method is no longer adequate in the actual applications. Therefore, a control volume finite element method is introduced to incorporate this necessity since this method is suitable for any complicated flow domain.

The flow through preplaced fiber mat is modelled as flow through porous medium with appropriate permeability, which is defined by experiments. The same governing equations can be used to compute the mold filling processes with irregular mold geometry. Equation 2.3 can be manipulated and integrated over a control volume and leads to:

\[ \iiint_V \nabla \cdot (u\mathbf{i} + v\mathbf{j} + w\mathbf{k}) \, dV = 0 \] (3.1)

By using the Divergence theorem, equation 3.1 can be written as:
\[
\iint_S (u\vec{i} + v\vec{j} + w\vec{k}) \cdot \vec{n} \, dS = 0 \quad (3.2)
\]

or
\[
\iint_S \begin{bmatrix} n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \, dS = 0 \quad (3.3)
\]

where \( n_x, n_y, \) and \( n_z \) are the components of the vector normal to the surface of the control volume. Introducing equation 2.2 into equation 3.3 yields:

\[
\iint_S \frac{1}{\mu} \begin{bmatrix} n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial P}{\partial y} \\ \frac{\partial P}{\partial z} \end{bmatrix} \, dS = 0 \quad (3.4)
\]

Equation 3.4 is the working equation for solving the problems of flow through anisotropic porous media and is basically a mass balance equation. In this study, equation 3.4 is used to solve the transient flow field during mold filling by using control volume finite element method. Although the equation is steady state, the mold filling process is not. The mold filling can be regarded as a quasi-steady state process by assuming a steady state condition at each time step. In other words, the problem is solved by considering the transient solution to be a sequence of steady state solutions separated by many small time increments.
The permeability tensor is used to represent the anisotropic material characteristics of the fiber mats. The permeabilities \([k_{ij}]\) in equation 3.4 are the components of a permeability tensor defined in a three-dimensional Cartesian coordinate system. Since the measured permeabilities of a fiber mat are always in its principal directions, a transformation is necessary in order to transform the permeability tensor from the principal directions of the fiber mat to the defined coordinate directions.

By assuming \(x, y, \) and \(z\) are the local coordinate axes, and \(x_1, x_2, \) and \(x_3\) are the coordinates along the three principal directions of the fiber mat, the transformation can be written as:

\[
\begin{bmatrix}
k_{xx} & k_{xy} & k_{xz} \\
k_{yx} & k_{yy} & k_{yz} \\
k_{zx} & k_{zy} & k_{zz}
\end{bmatrix} =
\begin{bmatrix}
l_{11} & l_{12} & l_{13} \\
l_{21} & l_{22} & l_{23} \\
l_{31} & l_{32} & l_{33}
\end{bmatrix}
\begin{bmatrix}
k_{11} & 0 & 0 \\
0 & k_{22} & 0 \\
0 & 0 & k_{33}
\end{bmatrix}
\begin{bmatrix}
l_{11} & l_{12} & l_{13} \\
l_{21} & l_{22} & l_{23} \\
l_{31} & l_{32} & l_{33}
\end{bmatrix}
\]

(3.5)

where \(l_{ij}\) are the directional cosines of local coordinates \(x, y, \) and \(z\) with respect to the principal axes \(x_1, x_2, \) and \(x_3\) are the principal permeabilities of the fiber mat in three directions. A fiber orientation angle \(\alpha\) was used to calculate the directional cosines, \(l_{ij}\), which will be discussed in Appendix A. For each type of fiber mat, the principal permeabilities in the three directions can be measured [14]. By using equation 3.5, the permeability tensor in any coordinate system can be derived.

In many RTM or SRIM applications, the mold geometry is often considered to be two-dimensional, which means that the dimension of the thickness is much smaller than the dimensions in the planar directions. Therefore, mold filling in a thin cavity can be modeled as a two dimensional flow. In other words, the local coordinates are
chosen in such a way that the z axis is always along the thickness direction. Thus, the formulation is based on a local coordinate system and the geometry data are mapped to this local coordinate system along the part surface as shown in Figure 19. Since the pressure gradient in the thickness direction is negligibly small as compared to those in other directions, the pressure is assumed constant in the z direction. By assuming pressure be a function of x and y only, equation 2.2 can be written in a two dimensional form:

\[
\begin{bmatrix}
    u(x,y,z) \\
v(x,y,z)
\end{bmatrix} = -\frac{1}{\mu(x,y,z)} \begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix} \begin{bmatrix}
\frac{\partial P(x,y)}{\partial x} \\
\frac{\partial P(x,y)}{\partial y}
\end{bmatrix}
\] (3.6)

In order to eliminate the independent variable z in the above equation, the velocity is averaged through the z direction [16]:

i.e.

\[
\begin{bmatrix}
\bar{u}(x,y) \\
\bar{v}(x,y)
\end{bmatrix} = \frac{1}{h_z} \int_{-\frac{h_z}{2}}^{\frac{h_z}{2}} \begin{bmatrix}u(x,y,z) \\
v(x,y,z)
\end{bmatrix} dz
\] (3.7)

where \(h_z\) is the part thickness. Introducing equation 3.6 into equation 3.7 leads to:

\[
\begin{bmatrix}
\bar{u}(x,y) \\
\bar{v}(x,y)
\end{bmatrix} = -\frac{1}{h_z} \int_{-\frac{h_z}{2}}^{\frac{h_z}{2}} \frac{1}{\mu(x,y,z)} \begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix} \begin{bmatrix}
\frac{\partial P(x,y)}{\partial x} \\
\frac{\partial P(x,y)}{\partial y}
\end{bmatrix} dz
\] (3.8)
Figure 19 The local coordinate in a curved thin shell model
where

\[
\begin{bmatrix}
  s_{xx} & s_{xy} \\
  s_{yx} & s_{yy}
\end{bmatrix} = \frac{1}{h_z} \int_0^1 \frac{1}{\mu(x,y,z)} \begin{bmatrix}
  k_{xx} & k_{xy} \\
  k_{yx} & k_{yy}
\end{bmatrix} \, dz
\]

(3.9)

where \( s_{ij} \)'s are the flow coefficients with average values of viscosity and permeability in the thickness direction. The part thickness, \( h_z \), in equation 3.9 may vary from one element to another within a finite element model. By substituting equation 3.8 into equation 3.2, equation 3.3 can be re-written in a two dimensional form as:

\[
\int_C h_z \begin{bmatrix}
  n_x & n_y
\end{bmatrix} \begin{bmatrix}
  s_{xx} & s_{xy} \\
  s_{yx} & s_{yy}
\end{bmatrix} \begin{bmatrix}
  \frac{\partial P}{\partial x} \\
  \frac{\partial P}{\partial y}
\end{bmatrix} \, dL = 0
\]

(3.10)

The integrand in the above equation is independent of \( z \). Therefore, the surface integral \( dS \) in equation 3.2 can be expressed by \( h_z dL \) in equation 3.10 where \( h_z \) is also a function of \( x \) and \( y \) since the thickness of a part may vary from one location to another. Equation 3.10, which includes independent variables \( x \) and \( y \), is the formulation for two dimensional mold filling.

The entire flow field is divided into a number of three-node triangular elements. The control volume is formed based on the nodes of adjacent elements. Each element is
divided into three sub-areas by the lines connecting the centroid of the element to the midpoint of each side. A control volume, as shown in Figure 20, is composed of several sub-areas, which have a common node at the center of the control volume.

By using linear shape functions [17] for a three-node triangular element, an approximate solution of equation 3.10 can be given by

\[
P = \sum_{i=1}^{3} P_i N_i
\]

where

\[
P_i = \text{node pressure}
\]

\[
N_i = \frac{1}{2A} (\alpha_i + \beta_i x + \gamma_i y)
\]

(3.12)

\[
\alpha_i = x_j y_k - x_k y_j \quad i \neq j \neq k, \text{ and } i, j, \text{ and } k
\]

\[
\beta_i = y_j - y_k
\]

\[
\gamma_i = x_k - x_j
\]

permute in a natural order

\[A : \text{area of an element}\]

Differentiating equation 3.11 with respect to \(x\) and \(y\) results in:

\[
\begin{bmatrix}
\frac{\partial P}{\partial x} \\
\frac{\partial P}{\partial y}
\end{bmatrix}
= \frac{1}{2A} \begin{bmatrix}
\beta_1 & \beta_2 & \beta_3 \\
\gamma_1 & \gamma_2 & \gamma_3
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix}
\]

(3.13)

Substituting the above equation into equation 3.10 yields:
Figure 20  Control volumes in a triangular mesh
In equation 3.14, the line integral C is along the boundary of a control volume. As shown in Figure 21, the control volume boundary within each element consists of two straight line segments. Therefore, if a control volume consists of m elements, equation 3.14 can be expressed as:

\[ \sum_{i=1}^{m} \left\{ \frac{h_x}{2A} \left[ l_{ac} \left[ n_x \ n_y \right]_{ac} + l_{bc} \left[ n_x \ n_y \right]_{bc} \right] \left[ \begin{array}{cccc} s_{xx} & s_{xy} & \beta_1 & \beta_2 & \beta_3 \\ s_{yx} & s_{yy} & \gamma_1 & \gamma_2 & \gamma_3 \end{array} \right] \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} \right\} dL = 0 \] (3.15)

where \( l_{ac} \) and \( l_{bc} \) are the length between points a and c, and b and c, shown in Figure 21, respectively. In the above equations, the integration along the control volume boundary is divided into m parts, and each part integrates along \( l_e \) in an element. Since the only quantity which depends on \( dL \) is the normal vector, the integration can be written as the normal vector times the boundary length (i.e. \( l_{ac} \) and \( l_{bc} \)). After expansion and manipulation, equation 3.15 can be rewritten as:

\[ \sum_{i=1}^{m} \left\{ \left( F_{1x} \beta_1 + F_{1y} \gamma_1 + F_{2x} \beta_1 + F_{2y} \gamma_1 \right) P_1 + \left( F_{1x} \beta_2 + F_{1y} \gamma_2 + F_{2x} \beta_2 + F_{2y} \gamma_2 \right) P_2 + \left( F_{1x} \beta_3 + F_{1y} \gamma_3 + F_{2x} \beta_3 + F_{2y} \gamma_3 \right) P_3 \right\} = 0 \] (3.16)
Figure 21  A typical two dimensional triangular element
\[
F_{lx} = \frac{1}{2A} h_x (n_x s_{xx} + n_y s_{yx})_{bc}
\]
\[
F_{ly} = \frac{1}{2A} h_x (n_x s_{xy} + n_y s_{yy})_{bc}
\]
\[
F_{2x} = \frac{1}{2A} h_x (n_x s_{xx} + n_y s_{yy})_{ac}
\]
\[
F_{2y} = \frac{1}{2A} h_x (n_x s_{xy} + n_y s_{yy})_{ac}
\]

Equation 3.16 can be written for all control volumes in the flow field, which results in a set of linear algebraic equations. Together with the appropriate boundary conditions, the pressure in the flow field can then be solved. The detailed technique for determining the moving flow front during mold filling will be discussed later. In this two dimensional model, thin parts with non-uniform thickness can also be handled by introducing different thickness, \(h_z\), in each element.

As mentioned earlier, the two dimensional formulation is based on the local coordinate which is chosen in such a way that the \(z\) direction is always along the thickness direction. If the geometry is planar, all the local coordinates are the same as the global coordinates. However, for a three dimensional thin object, the local coordinates of each element may be different from each other. A detailed discussion of the local coordinate for each element with respect to the global coordinate as well as determination of fiber orientation is given in Appendix A.
3.2 Boundary Conditions and Computational Schemes

Boundaries of the flow domain in mold filling include the mold walls, inlets, and flow front. At the mold walls there is no flow in the direction normal to the wall, which means that the first derivative of pressure normal to the wall is zero. For the case of specified injecting flow rate, a specified flow rate is assigned to the control volumes enclosing the inlet nodes. In other words, the inlet nodes are treated as source nodes which generate specified flow flux. For the case of specified injecting pressure, a specified pressure is assigned to the inlet nodes. At the flow front, a parameter $f$, which has been used in FAN [15], is used to represent the status of each control volume in the flow domain. If the control volume is empty, $f$ is equal to zero. If the control volume is filled with liquid, $f$ is equal to 1. If the control volume is partially filled, $f$ is equal to the volume fraction of the fluid occupying the control volume. The control volumes with $f$ values between 1 and 0 are considered the flow front elements. The node pressures in the partially filled flow front control volumes are set to zero.

With the above described boundary conditions, the set of linear algebraic equations developed in both the two and three dimensional formulations can be solved to determine the pressure field during mold filling. The velocity field can then be computed with Darcy's law (i.e. equation 2.2).

As mentioned earlier, the mold filling process is regarded as a quasi-steady state process by assuming a steady state condition at each time step. The selection of a time increment for each quasi-steady state is based on the constraint that each time increment allows only one control volume to be completely filled. In some cases, more than one control volume may be filled simultaneously. This restriction of the time increment
ensures the stability of the quasi-steady state approximation. The new flow front in each time step can be estimated according to the velocity vector in the flow front and the time increment after the pressure field is determined.

At the beginning of mold filling, the mold filling simulation program (MFS) assumes that the control volumes enclosing the inlet nodes are filled with fluid. Then, with the boundary conditions described earlier, the pressure calculation is performed to determine the pressures of the nodes which are currently filled with fluid. For the case of specified flow rate, the pressure at the inlet nodes are assumed the same, and is solved with the specified flow rate as the source term. For the case of specified pressure, the pressure at the inlet nodes is the value being specified.

After the pressure field is determined, the velocity field can be evaluated according to Darcy's law. If the permeability is a function of velocity, it is corrected based on the calculated velocity field. For a selected time increment, the volume of resin flow into each flow front control volume \((0 < f < 1)\) is calculated based on the velocity field. The calculated volume of resin inflow is added to the original volume of resin in the flow front control volume. If the total resin volume in a control volume is equal to the volume of the control volume, that control volume is considered "full" \((f = 1)\). If the total resin volume in a control volume is less than the volume of the control volume, \(f\) is calculated as the volume fraction of the resin in the control volume. The time increment is selected in such a way that a control volume will be filled in each time step. Sometimes, several control volumes can be filled simultaneously. After the value \(f\) is updated, another pressure computation is performed for all the control volumes which are considered full. The procedure is repeated until the whole mold is filled.
During the mold filling, the time for a control volume to be half filled can be determined. This time is assumed to be the exact time when the flow front reaches the node within the control volume. In this way, the time for the flow front to reach each node within the mold is known. And, the flow front profile can be determined by interpolating and connecting the positions with the same filling time.

The control volume method, instead of the regular finite element method, was used in the formulation because it is more robust and more user friendly (e.g. no need of remeshing). These are essential requirements for users who need to deal with various parts with complicated geometry in composite manufacturing. The node numbering and shape function techniques in the formulation were based on the finite element method because it is more convenient to use when dealing with complicated geometries. The formulation of pressure field resulted in the same discretization equations as those derived from Galerkin or variational techniques [18]. Therefore, the numerical scheme used in this study can be considered as a control volume finite element method.

Although the triangular element is less accurate than the quadrilateral element, it was still chosen in the formulation based on the following reasons: (1) Complicated geometries can be more easily divided into triangular meshes. (2) Triangular mesh is free from numerical integration in computing the element matrices, which may save a great deal of computation time. (3) Mass balance near the flow front is easily maintained using triangular mesh, especially for cases where the flow is anisotropic. (4) In composite processing, the variations in resin chemistry, fiber mat properties and
molding conditions are often large enough that using quadrilateral elements in simulation may not make much improvement in prediction accuracy. And, (5) better accuracy can also be obtained by using more triangular meshes in the flow domain.

3.3 Experimental

The experimental work was conducted with a mold filling apparatus built in our laboratory. A constant speed pump was set up by mounting an 8.26 cm diameter hydraulic cylinder in the test section of an Instron Universal Testing Instrument (Model 1137). The cylinder was filled by a hand operated rotary feed pump. After the cylinder was filled and the valve to the transfer pump closed, a valve to the mold was opened and the Instron's crosshead was set to descend at a constant speed. The descending crosshead pushed the cylinder ram back into the body of the cylinder, thereby forcing the nonreactive fluid (DOP oil) into the mold. The fluid flow rates were varied by changing the speed of the crosshead. Detailed experimental set-up has been described in a previous work [14].

The 20.32 cm by 80.01 cm platens of the mold were constructed out of 1.27 cm thick clear acrylic sheets so progress of the flow front could be observed during filling. The mold cavity thickness was varied by inserting different rectangular spacers. The mold was sealed by glueing 0.79 mm rubber gasket to the spacer so that when the mold was closed, the rubber gasket would seal the seams between the mold halves and the spacer. The assembly was clamped together with steel angle irons and C-clamps. Steel angle iron or flat plate was used between the C-clamps and the platens to spread the force applied by the C-clamps and to prevent damage to the acrylic platens from the
concentrated force of the clamps. When visual inspection of the flow front progress was not required from one or both sides of the mold, the platen on that side was reinforced with 0.635 cm thick steel platens to reduce bending of the platen from the mat compression force. When platen bending did occur, an average thickness of the cavity was measured after the mold was assembled. The mold filling pressure was measured with a pressure gauge (Omega, Model PX302, 0-150 psi) mounted near the inlet of the filling line.

A stitched bidirectional fiber mat (Cofab-A1118B), were used as reinforcements in this study. The bidirectional fiber mat is anisotropic in all three directions. The nonreactive fluid for the mold filling experiments was DOP oil (diphenyl-octyl-phthalate) with viscosity of 80 cp. The Instron's crosshead was set up at a constant descending speed so that a constant flow rate was maintained at the mold inlet. Experiments were run under room temperature isothermal conditions. Since the mold filling rate and liquid viscosity are low the viscous heating effect was considered negligible.

3.4 Two Dimensional Isothermal Mold Filling

The proposed simulation model is able to simulate the mold flow patterns for mats under different fiber orientations and different resin injecting methods, such as a specified injecting pressure or flow rate. The rectangular mold described earlier was used to generate flow patterns for anisotropic fiber mats with fiber orientation differing from the flow direction. The results were compared with the simulation results.
Figure 22 shows the flow front, pressure distribution, and the velocity field resulting from the 2-D mold flow simulation by using the rectangular mold preplaced with bidirectional fiber mats. The bidirectional fiber mats were placed in 45° with respect to the mold side wall. The flow rate was set at 2.75e-5 m²/s. The porosity of the bidirectional fiber mat used in the simulation was 70%. The simulated flow front profiles shown in Figure 22 are the flow front positions at different time steps. Since the gate is located at the center of the mold, the fluid comes in at the center and flows out radially. At each time step, a closed curve, including the mold boundary, is drawn to represent the flow front position. The assigned number near each flow front curve indicates a specified time step. Six time steps are drawn in the graph and each one has a corresponding time written near the right hand side of the graph.

As one would expect, the flow pattern orients to the same direction as the fiber mat. The elliptic shape of flow fronts is due to the difference of permeability in the two principal directions. At 70% porosity, the measured permeability in one direction is larger than that in the other direction \((k_y/k_x = 0.8)\) so that the resin flows faster in one direction than the other. During the mold filling process, pressure distribution keeps changing with time due to the moving of the flow front. The pressure distribution shown in Figure 22 corresponds to the time near the completion of the mold filling. Six levels of pressure isobars are drawn in the figure. Near the center inlet gate, the pressure is the highest and decreases gradually to zero at the flow front.

The pressure isobars also orient to the same direction as the fiber mat due to a slight rotation in the velocity field and flow path. The velocity field is represented by arrows pointing to the direction of the velocity vector with the length corresponding to
Figure 22  Simulated flow front, pressure distribution, and velocity field with 45° bidirectional fiber mat (flow rate = 2.75e-5 m³/s, porosity= 0.7, k_y/k_x = 0.8, mold dimension 40 x 13.5 x 0.3 cm)
the magnitude of the velocity. Higher velocity is shown near the center gate location in
the figure.

In order to verify these simulation results, experiments using the same
processing conditions were conducted. The flow front patterns were recorded by
taking pictures at pre-specified times. Figure 23 shows the comparison of experimental
and simulation flow fronts where the solid lines are numerical predictions and the dash
lines are the experimental results. In general, the predicted flow front follows the same
trend with the experimental results.

As mentioned earlier, MFS can handle various injection conditions. A constant
flow rate mold filling was demonstrated in the previous case. A mold filling simulation
was also run by using the rectangular mold with random fiber mats and constant
injecting pressure. Figure 24 shows the simulated mold filling results including the
flow fronts, pressure distribution, and velocity field. The pressure at the center gate is
kept constant throughout the entire filling process. The flow resistance is small at the
beginning and becomes higher and higher as more and more fiber mats are impregnated
by the fluid. Thus, the filling of the mold is faster at the beginning and gradually slows
down as the resistance increases, showing an uneven distribution of flow fronts. Six
levels of flow fronts are drawn in the figure with the corresponding times shown near
the right hand side.

The pressure distribution shown in the figure is near the completion of the mold
filling. The pressure is the highest near the center gate position and drops gradually to
zero at the flow front. The four corners of the mold are the outlets, and consequently,
Flow front of mold filling with bidirectional fiber mat oriented 45 degree

Figure 23 Experimental and numerical comparison of flow front positions with 45° bidirectional fiber mat
Figure 24  Simulated flow front, pressure distribution, and velocity field with random fiber mat (injecting pressure = 1.86E6 N/m², porosity = 0.7, mold dimension 40 x 13.5 x 0.3 cm)
have the lowest pressure and relatively higher velocity as shown in the pressure
distribution and velocity field plots.

In addition to the rectangular mold, a circular mold made of aluminum was also
built in our laboratory. Figure 25 shows the simulated mold filling results using the
circular mold. A center gate and constant flow rate were used in this simulation. Since
random fiber mats were used here, the corresponding flow front, velocity field and
pressure distribution were symmetric.

One of the major features of 2-D MFS is that it can simulate the mold filling of a
thin part with complicated geometry. In order to show this ability, an automotive inner
hood mold was used for the mold filling simulation. The inner hood is a reinforcement
for the outer hood and is bounded to the outer hood in the actual production. Figure 26
shows a finite element model for this automotive inner hood, where two dimensional
triangular elements were used to constitute the skin of the part. The mold shown in
Figure 26 is an angle view. Since the inner hood is used as reinforcement, some places
are cut out and form large holes. This tends to complicate the flow pattern during mold
filling.

A gate located at the center of the mold was used in the first mold filling
simulation. Figure 27 shows the resulting flow front and pressure distribution near the
completion of mold filling. Again, the highest pressure region is near the inlet gate,
which is at the center of the mold. Figure 28 shows the simulation results for the same
mold and processing conditions except that the gate was moved to the front end. The
resulting maximum mold filling pressure is higher when using an end gate compared to
Figure 25. Simulated flow front, pressure distribution, and velocity field with random fiber mat in a circular mold (flow rate = 2.771e-4 m³/s, porosity = 0.7, mold thickness = 1.323 cm, radius = 0.2 m)
Figure 26  A finite element model of an automotive inner hood
Figure 27  Simulated pressure distribution and flow front profiles with random fiber mats and a center gate mold filling
Figure 28  Simulated pressure distribution and flow front profiles with random fiber mats and an end gate mold filling
a center gate. The reason is that, for the end gate, the flow path inside the mold is much longer than for the center gate. Thus, when an end gate is used, a higher pressure is needed to inject the resin through a longer flow path. Consequently, the highest pressure region is distributed near the edge of the mold, where the gate is located. High pressure near the mold edge implies a need of higher clamping force or a potential leakage problem during the mold filling process. Therefore, a center gate would be the better design for thin parts with large surface area.

From the simulated flow front profiles, one can also estimate the formation of weldlines, which usually indicate the locations of needed vent lines in order to avoid trapping air inside the molded part. Figure 29 shows the predicted weldline locations for the hood mold by using both center gate and end gate. The weldlines are estimated by connecting the junctions as two flow front lines meet together. By the choice of gate location, the weldline locations can be shifted as shown in Figure 29.

Although the weldline position can be predicted by the computer simulation, it is impractical to construct a mold with such a complicated vent distribution. One way is to design the mold with thinner thickness at the hole locations instead of actually putting inserts there. After the molding process, these locations can be punched out. In this way, the resin flow can be manipulated to either eliminate the weldline formation or guide the weldlines to the thinner locations which will be punched out eventually.

Figure 30 shows different part designs, where $k_{\text{thin}}$ means the permeability for the thinner locations and $k_{\text{thick}}$ is the permeability for the main part. The case, when $k_{\text{thin}}/k_{\text{thick}}$ equals to zero, means putting inserts in the mold. The results show that for
Figure 29 Comparison of simulated weldline positions in a mold with (a) center gate and (b) end gate
Figure 30  Comparison of flow front profiles and weldline formation for different thickness and permeability in the insert areas
the case \( k_{\text{thin}}/k_{\text{thick}} = 0.01 \) and the thickness ratio of thin/thick = 1/6, the flow passes around the thinner areas and traps the air into these areas. However, when the permeability of the thinner area is increased to \( k_{\text{thin}}/k_{\text{thick}} = 0.1 \), the weldlines are formed outside the thinner areas. If the condition of the thinner area is further changed to thin/thick = 4/6 and \( k_{\text{thin}}/k_{\text{thick}} = 0.6 \), the flow front becomes smooth and no weldlines are formed.

Figure 30 shows two ways to avoid the weldline formation by applying either very high or very low permeability (i.e. \( k_{\text{thin}}/k_{\text{thick}} = 0.6 \) or 0.01) in the thin areas. The drawbacks of these methods are that, for very low permeability, more fiber mat is needed to decrease the permeability in the thinner areas, which causes more waste of material. While, for higher permeability, thicker sections need to be cut off from the part.
CHAPTER IV

THREE DIMENSIONAL MOLD FILLING SIMULATION

4.1 Formulations

Although the two dimensional model considers the viscosity, permeability, and velocity variations in the thickness direction, only the average values of those are used to calculate the pressure field. The resulting pressure field is two dimensional and is constant throughout the whole thickness direction. The error of using average values in the simulation may become very large when the parts are no longer 'thin'. In order to solve this type of geometry, a true 3-D model is developed in this section.

Six-node wedge-type elements shown in Figure 31 with linear shape functions are used in this model. The corresponding approximation solution of equation 3.4 is given by [18]

\[ P = \sum_{i=1}^{6} P_i N_i \]  \hspace{1cm} (4.1)

where
Figure 31  A typical wedge type element and corresponding normalized element
\[\begin{align*}
N_1 &= (1 - \xi - \eta)(1 - \zeta) \\
N_2 &= \xi (1 - \zeta) \\
N_3 &= \eta (1 - \zeta) \\
N_4 &= (1 - \xi - \eta) \zeta \\
N_5 &= \xi \zeta \\
N_6 &= \eta \zeta
\end{align*}\]  
(4.2)

and \(\xi, \eta,\) and \(\zeta\) are the axes of the transformed normal coordinates. The transformation maps an irregularly shaped element into a regular shaped element as shown in Figure 31. Thus, the coordinate of any point within an element can be expressed by the node coordinates and shape functions:

\[\begin{align*}
x &= \sum_{i=1}^{6} x_i N_i \\
y &= \sum_{i=1}^{6} y_i N_i \\
z &= \sum_{i=1}^{6} z_i N_i
\end{align*}\]  
(4.3)

A Jacobian matrix can then be defined for the normalized coordinate shown in Figure 31.

\[
[J] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
\]  
(4.4)
With equations 4.2 and 4.3, the Jacobian matrix can be expressed as:

\[
[J] = \begin{bmatrix}
\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} & \frac{\partial N_5}{\partial x} & \frac{\partial N_6}{\partial x} \\
\frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} & \frac{\partial N_5}{\partial \xi} & \frac{\partial N_6}{\partial \xi} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} & \frac{\partial N_5}{\partial \eta} & \frac{\partial N_6}{\partial \eta} \\
\frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \frac{\partial N_3}{\partial \zeta} & \frac{\partial N_4}{\partial \zeta} & \frac{\partial N_5}{\partial \zeta} & \frac{\partial N_6}{\partial \zeta} \\
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\
y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\
z_1 & z_2 & z_3 & z_4 & z_5 & z_6
\end{bmatrix}
\]

(4.5)

\[
= [N] [X]
\]

The derivatives of pressure with respect to the global coordinate can be derived by using the chain rule and the shape functions.

\[
\begin{bmatrix}
\frac{\partial P}{\partial x} \\
\frac{\partial P}{\partial y} \\
\frac{\partial P}{\partial z}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} & \frac{\partial N_5}{\partial x} & \frac{\partial N_6}{\partial x} \\
\frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y} & \frac{\partial N_5}{\partial y} & \frac{\partial N_6}{\partial y} \\
\frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \frac{\partial N_3}{\partial z} & \frac{\partial N_4}{\partial z} & \frac{\partial N_5}{\partial z} & \frac{\partial N_6}{\partial z}
\end{bmatrix} \begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_4 \\
P_5 \\
P_6
\end{bmatrix}
\]

(4.6)
Substituting equation 4.6 into equation 3.4 yields:

$$\iint \frac{1}{\mu} [n_x \ n_y \ n_z] [K] [J^{-1}] [N] [P] \ dS = 0 \quad (4.7)$$

where

$$[K] = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}$$

If a control volume consists of \( m \) sub-areas defined earlier, the surface integral of equation 4.7 can be evaluated by summing up the \( m \) sub-area surface integrals of the control volume. The boundary surface of a sub-area in a control volume is also shown in Figure 31, where the boundary surface is divided into three regions, A, B and C. The surface integral is evaluated in each region.

Based on Figure 31, a vector space of the normalized coordinate with respect to the global coordinate can be defined as follows:

$$\bar{e}_t = \frac{\partial x}{\partial \xi} \hat{i} + \frac{\partial y}{\partial \xi} \hat{j} + \frac{\partial z}{\partial \xi} \hat{k}$$

$$\bar{e}_n = \frac{\partial x}{\partial \eta} \hat{i} + \frac{\partial y}{\partial \eta} \hat{j} + \frac{\partial z}{\partial \eta} \hat{k} \quad (4.8)$$

$$\bar{e}_s = \frac{\partial x}{\partial \zeta} \hat{i} + \frac{\partial y}{\partial \zeta} \hat{j} + \frac{\partial z}{\partial \zeta} \hat{k}$$
**Regions A and B:**

In order to evaluate the surface integral in region A, let \( \hat{n}_A \) be the normal vector pointing out of surface A as shown in Figure 31, and \( \vec{e}_r \) be the vector on surface A along line de. Then, by defining a line coordinate \( r \) along line de, the relationships between \( r \) and the normalized coordinate are

\[
\xi = \frac{5 - r}{12}, \quad \eta = \frac{1 + r}{6}
\]

\[
\frac{\partial \xi}{\partial r} = -\frac{1}{12}, \quad \frac{\partial \eta}{\partial r} = \frac{1}{6}
\]

(4.9)

where \( r \) is a parameter along the line de. Therefore, \( r = -1 \) is corresponding to the point \( d, (1/2, 0, 0) \), and \( r = 1 \) is corresponding to the other point \( e, (1/3, 1/3, 0) \) as shown in Figure 31. By the definition of equation 4.9, a vector \( \vec{e}_r \) with respect to the global coordinate can be defined along the line de as:

\[
\vec{e}_r = \frac{\partial x}{\partial r} \hat{i} + \frac{\partial y}{\partial r} \hat{j} + \frac{\partial z}{\partial r} \hat{k}
\]

\[
= (\frac{\partial x}{\partial \xi} \frac{\partial \xi}{\partial r} + \frac{\partial x}{\partial \eta} \frac{\partial \eta}{\partial r}) \hat{i} + (\frac{\partial y}{\partial \xi} \frac{\partial \xi}{\partial r} + \frac{\partial y}{\partial \eta} \frac{\partial \eta}{\partial r}) \hat{j} + (\frac{\partial z}{\partial \xi} \frac{\partial \xi}{\partial r} + \frac{\partial z}{\partial \eta} \frac{\partial \eta}{\partial r}) \hat{k}
\]

(4.10)

A small surface area, d\( r \) by d\( \zeta \), times the normal vector in region A can thus be defined as [19]:

\[
\begin{bmatrix}
    n_x \\
    n_y \\
    n_z
\end{bmatrix}
\times
(\vec{e}_r \times \vec{e}_\zeta)
\]

\[
dS = (\vec{e}_r \times \vec{e}_\zeta) \ dr \ d\zeta
\]
With the same procedure, a B matrix in region B can be derived:

\[
[B] = \frac{1}{12} \left[ \begin{array}{ccc}
-2 \frac{\partial y}{\partial \xi} + \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} \\
\frac{\partial z}{\partial \xi} & -2 \frac{\partial x}{\partial \xi} + \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \eta} \\
\frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & -2 \frac{\partial y}{\partial \xi} + \frac{\partial y}{\partial \eta}
\end{array} \right]
\] (4.12)

Region C:

Following the same procedure, a small surface area, \( dS \) by \( d\eta \), times the normal vector in region C can be defined as:

\[
\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \cdot \left( \frac{\partial (\hat{e}_x \times \hat{e}_\eta)}{\partial \xi} \right) d\xi \ d\eta = \begin{bmatrix} \frac{\partial y}{\partial \xi} \frac{\partial z}{\partial \eta} - \frac{\partial z}{\partial \xi} \frac{\partial y}{\partial \eta} \\
\frac{\partial z}{\partial \xi} \frac{\partial x}{\partial \eta} - \frac{\partial x}{\partial \xi} \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta}
\end{bmatrix} d\xi \ d\eta \] (4.13)

\[= [C] \ d\xi \ d\eta \]
Equation 4.7 can then be expressed in a matrix form using equations 4.11 to 4.13.

\[
\sum_{i=1}^{m} \left\{ \int_{s_i} \frac{1}{\mu} \left( [A^T] \, d\zeta + [B^T] \, d\zeta + [C^T] \, d\xi \, d\eta \right) [K] [J^{-1}] [N] [P] \right\}_i = 0 \quad (4.14)
\]

or

\[
\sum_{i=1}^{m} \left\{ \left( \int_{i-2}^{i} [A^*] \, d\zeta + \int_{0}^{i} [B^*] \, d\zeta + \int_{0}^{i} [C^*] \, d\eta \right) [P] \right\}_i = 0 \quad (4.15)
\]

where

\[
[A^*] = \frac{1}{\mu} [A^T][k][J^{-1}][N]
\]

\[
[B^*] = \frac{1}{\mu} [B^T][k][J^{-1}][N]
\]

\[
[C^*] = \frac{1}{\mu} [C^T][k][J^{-1}][N]
\]

The integration with respect to \( S_e \) in equation 4.14 is over the control volume surface in an element (i.e. regions A, B and C). As shown in Figure 31, the limits of integration for each region in equation 4.14 can be expanded according to the normalized coordinate, which results in equation 4.15. Equations 4.15 and 4.16 are written for one control volume. The formulation can be applied to all the control volumes within the flow field, which yields a set of linear algebraic equations. Gauss integral method is used to evaluate the integral in the equation.
4.2 Boundary Conditions and Computational Schemes

The boundary conditions are basically the same as those used in 2-D mold filling simulation. Also, the same parameter $f$ is used to track the status of each control volume (empty $f = 0$, flow front $0 < f < 1$, and filled $f = 1$). The computational scheme follows the same procedures described in 2-D simulation. First, one can determine the pressure field according to the flow equations derived earlier. Then, the mass flow into each flow front control volume can be calculated and used to choose the time increment and update the volume fraction value, $f$, for each control volume.

The time for a control volume to be half filled is also determined during the computation. This time is assumed to be the exact time when the flow front reaches the node within the control volume. Therefore, the isobar based on those node half filled time data is equivalent to the flow front contour during the mold filling.

The proposed 3-D model is used for parts with complicated three dimensional geometry. Since the 2-D model neglects the details in the thickness direction, the computation time is ten times shorter than that of the 3-D model. However, using the 3-D model will have better accuracy when the change in the thickness direction is significant. In actual applications, the users may make a choice according to their specific requirements.
4.3 Three Dimensional Isothermal Mold Filling

In RTM and SRIM, the thickness direction permeability may play an important role in the mold filling process [20], especially, for a thick part where the thickness of the part is no longer small compared to the planar dimensions. The thin part assumption is no longer valid and the two dimensional mold filling program is not suitable for the simulation. In these cases, the three dimensional mold filling simulation is necessary.

A three dimensional finite element model for a rectangular "U" shape mold is shown in Figure 32 and is used for 3-D mold filling simulation. The finite element model is composed of six node wedge type elements as discussed in the previous chapter. The model shown in Figure 32 is an angle view so that the three dimensional perspective can be easily observed. A coordinate system is also shown in the lower right corner of the figure. A center gate located at the top surface of the mold was used in the simulation. The permeability of the fiber mat was assumed to be $k_x : k_y : k_z = 10 : 5 : 1$. The fiber mat was aligned 45° along the mold side line.

The predicted flow fronts and pressure distribution are shown in Figure 33. The plots are again shown in angle views so that the difference near the top and bottom mold surfaces can be seen. The flow front distribution shows a large lead-lag between the top and bottom surfaces due to the low permeability of the fiber mat in the thickness direction. When the fluid comes in from the top surface inlet gate, it tends to spread out instead of penetrating the fiber mats in the thickness direction because the flow
Figure 32  An "U" shape three dimensional finite element model
Figure 33   Mold filling simulation of flow front and pressure distribution with an anisotropic fiber mat aligned 45° along the mold (flow rate = 2.24E-5 m²/s, porosity = 0.7, kₓ : kᵧ : kz = 10 : 5 : 1)
resistance is lower in the planar direction. Thus, when the fluid reaches the short side of the mold, the bottom surface of the mold has not been wetted yet.

Also, the two-arm part of the mold is filled earlier than the other end of the mold. Consequently, as shown in the pressure distribution plot, the pressure is built up in the two-arm part of the mold and decreases gradually to the other end. The shape of the flow fronts and the pressure distribution are oriented in the same direction as the fiber mat due to the anisotropic effect of the fiber mat.
CHAPTER V

MESH REFINEMENT OF CONTROL VOLUME FINITE ELEMENT METHOD IN SOLVING MOVING BOUNDARY PROBLEMS

5.1 Introduction

The control volume finite element method has been used in solving moving boundary problems [21~25]. Since this method constructs a finite element mesh in the entire flow domain, no re-mesh is needed during computation. This makes the numerical scheme more robust and more user friendly than the conventional finite element method. Another advantage of the control volume finite element method is its ability to handle complex flow domains, which makes the method very attractive in dealing with many manufacturing processes, such as injection molding and compression molding [21~23].

In a moving boundary problem such as injecting a liquid into an empty cavity, the flow front changes as a function of time. A given control volume in the cavity may be empty, partially filled or fully filled with liquid at a given time depending on the flow pattern. Using the control volume finite element method a volume fraction, f, is adapted to specify the status of a control volume in the flow domain (i.e., f = 1 for fully filled, f = 0 for empty, and 0 < f < 1 for partially filled). Only the control volumes which are fully filled with liquid are considered in computation at each time.
step. The volume fractions of the control volumes located at the flow front (i.e. partially filled) are then updated at each time step based on the calculated flow field.

The boundary conditions are applied at the nodes inside each flow front control volume. In most cases, the nodes are located near the center of the partially filled control volumes. The actual flow front, however, may not exactly lie on these nodes. The locations of computational and the actual flow fronts are shown in Figure 34. Consequently, applying the flow front boundary conditions at these nodes (computational flow front) may introduce errors in the calculation. In the computation of the pressure field, since the pressure is at its minimum near the flow front boundary, the error may be negligible. However, in the computation of heat and mass transfer, these errors may be too large to result in an acceptable temperature field and mass distribution, especially near the flow front region.

In structural reaction injection molding (SRIM) and resin transfer molding (RTM) processes, temperature and conversion usually have the maximum values near the flow front during resin injection because of heat exchange between the resin and the fiber there. In thermoplastic injection molding (TIM) and reaction injection molding (RIM) processes, a well-known fountain flow effect occurs at the flow front, which again has a strong influence on the subsequent temperature and mass distribution. For the problems of melting and solidification, the moving interface between the liquid and the solid phases is the major area for heat exchange. When performing the numerical analysis of these types of problems, the moving front area requires better accuracy than other areas. In conventional finite element methods, a finer mesh size can be assigned
Figure 34 Comparison of numerical and actual flow fronts.
at this region at each time step (i.e. re-mesh). In the control volume finite element method, a fixed finite element mesh can not fulfill this requirement.

In this study, a moving front mesh refinement method is proposed to increase the numerical accuracy near the flow front region without refining the mesh of the entire flow domain. Its application to RTM/SRIM mold filling and RIM mold filling is demonstrated.

5.2 Moving Front Mesh Refinement

The major feature of the control volume finite element method is that no computational effort is needed for re-mesh during the analysis. In order to preserve this advantage, the mesh refinement method proposed here follows the following approach. First, the mesh is created in two levels, one is the base mesh and the other is the refined mesh based on the base mesh. Therefore, during the numerical analysis, no re-mesh is necessary except the switch between base and refined meshes. The base mesh is used for fully filled and empty control volumes, while the refined mesh is used for partially filled control volumes (i.e. flow front region). As the moving front boundary moves into a new position, the base mesh in this position will be refined into a smaller mesh size. After the moving front moves away, the mesh returns to its original size, and the mesh at the new flow front region will be refined.

Figure 35 shows a triangular element and three possible refinements, the first with eighteen refined elements (b), the second with twelve refined elements (c), and the last with six refined elements (d). The mesh refinement is set up according to the
Figure 35 Base and refined elements and control volumes.
element and control volume border lines. The control volume border lines for each node (e.g. point 1) are defined by connecting the center of this element (i.e. point 16) to the middle points (i.e. points 5 and 11) of the element border lines (i.e. lines 1-2 and 1-3). The reason for choosing this particular method of mesh refinement is that it links the base and refined meshes without destroying the original node locations. In Figure 35 (b), three equally spaced refined nodes are chosen on each element border line. The eighteen refined elements are set up by connecting the refined nodes in the manner shown in the figure. In addition to the nine refined nodes located on the three element border lines, four internal refined nodes (i.e. points 13-16) are generated this way. Figure 35 (c) shows the twelve refined elements which follow a similar pattern as the eighteen refined elements except that only one refined node is defined in each of the element border line. Figure 35 (d) defines six refined elements by connecting the base nodes (points 1-3) and the mid-points of the element border lines (points 5, 8, 11) to the element center point (point 16). A triangular base element which is automatically refined into eighteen small elements is chosen as an example in this study.

As the moving front moves into a control volume, the elements in that control volume will be switched to the refined elements as shown in Figure 35. When the flow front moves through these refined elements, there are three possible stages as shown in Figure 36. Stage I is when one base node (i.e. node 1) in the element is filled according to its base control volume. At this stage, the refined eighteen elements are relaxed to the form of fourteen elements. Stages II and III correspond to two (i.e. nodes 1 and 3) and all three nodes being filled according to their base control volumes, respectively. The refined elements will be further relaxed to either nine refined
Figure 36  Three stages of element status.
elements or one element as shown in Figure 36. Stage III is essentially the original base mesh.

In the down stream region, no computation is needed since there is no fluid in this region. Therefore, the refinement of an empty base element can be done at any time before the flow front moves in this element and can be directly refined to eighteen elements without going through different stages. One way to conduct the refinement is to refine the entire domain before the calculation, as shown in this study. As for the problems of solidification and welding, the above mentioned three stages need to be used for both upstream and downstream regions since both sides of the interface are involved in the calculation.

The following example shows the change of element status when the flow front moves through a region shown in Figure 37. Fluid is injected into a rectangular cavity from nodes 1, 2 and 3 (source nodes). After the element status has been changed to the refined mesh, six extra nodes (a - f) are generated between the three source nodes. Since the original three source nodes are chosen to represent a line source from one side of the cavity, these six extra nodes (a - f) are also considered as source nodes at this stage of calculation in order to preserve a line source boundary condition.

The calculation starts based on the refined meshes shown in Figure 37 (the detailed calculation procedure is given in the next section). After a few time steps, the moving front moves to a new position shown in Figure 38. At this time, nodes 1 and 3 are completely filled according to their base control volumes. The mesh configuration is changed to a new status, where elements 142 and 263 are in stage I. When the
Figure 37  Initial mesh refinement for mold filling.
Figure 38 Inlet nodes 1 and 3 are filled at this point.
moving front moves further to a location shown in Figure 39, node 2 is also filled. At this time, elements 142 and 263 are in stage II and elements 245 and 256 are in stage I. When the moving front is at a location shown in Figure 40, nodes 1 to 6 are filled and elements 142, 245, 256 and 263 are in stage III, while elements 475 and 596 are in stage II, and elements 578 and 589 are in stage I.

As mentioned earlier, the mesh is set up at two levels, one is the base mesh and the other is the refined mesh. During the calculation, the node location (either base node or refined node) is not changed. Therefore, the nodal quantity (e.g. pressure, temperature, conversion, etc.) stays the same despite the changes of mesh configuration. There is no need for nodal interpolation as in the case of conventional finite element methods where mesh deformation and re-mesh often make nodal interpolation a necessary step during numerical calculation.

5.3 Numerical Implementation

5.3.1 Mesh Refinement

The triangular base elements (base mesh) are generated by using I-DEAS (supported by Structural Dynamics Research Corporation). After the base mesh is set up, the refined mesh is generated based on the element and control volume border lines of the base mesh as described in Section 5.2. During mesh refinement, the element numbering is changed while the base node numbering is kept the same. For an element "n", the refined eighteen small elements are numbered according to the following rule:
Figure 39  Inlet nodes are all filled at this point.
Figure 40  Further change of element configuration.
100

\[ e_{ni} = (n - 1) \times 18 + i \quad i = 1 \sim 18 \quad (5.1) \]

where \( e_{ni} \) is the individual element number for each of the eighteen refined elements in the base element \( n \).

The generation of refined nodes is more complicated than that of refined elements. First, the entire base mesh is scanned over to determine the border lines which constitute the base mesh. Then, three refined nodes are assigned to each border line at equal distance. These refined nodes are numbered according to the following rule:

\[ n_{ji} = n_o + (j - 1) \times 3 + i \quad i = 1 \sim 3, \quad j = 1 \sim l_o \quad (5.2) \]

where \( n_{ji} \) is the refined node number on the border line "\( j \)", \( n_o \) is the total number of base nodes, and \( l_o \) is the total number of border lines within the base mesh. After the refined nodes on the element border lines have been assigned, four more refined nodes are placed inside each of the base elements; one is located at the center of the base element, and the others are placed at middle points of the three lines connecting the base element center and the three base nodes (see Figure 35). The numbering of refined nodes inside an element "\( j \)" is based on:

\[ n_{ji} = n_o + 3 \times l_o + (j - 1) \times 4 + i \quad i = 1 \sim 4, \quad j = 1 \sim e_o \quad (5.3) \]

where \( e_o \) is the total number of base elements in the original mesh. An example of refined elements and nodes with their numbering is shown in Figure 41.
5.3.2 Refinement of Source Nodes

The base nodes located in the gate region are considered as source nodes which provide fluid flux to fill the entire flow domain. During mesh refinement, three refined nodes are defined on each base element border line. If both base nodes of a border line are source nodes, the three refined nodes on this border line are also considered as source nodes. The total inward flux is set to be the summation of the flow flux in each of these source nodes. While the pressure of each source node is assumed to be the same, the inflow flux for each source node can be determined by using mass and momentum balance.

One problem associated with the above mentioned source nodes determination method is that, sometimes, the resulting set of refined nodes may not cover all the source nodes if the gate configuration forms a loop and there is a mesh inside the loop like the one shown in Figure 42. Here, the three base source nodes happen to constitute an element. The program will define the refined source nodes located on the border lines connecting the base source nodes. However, the four refined nodes inside the element are also source nodes if the entire element is considered to be a gate. Without considering these source nodes, the flow will proceed in both inward and outward directions initially as shown in Figure 42. In most situations, the inward flow will fill the element after the initial several time steps if the enclosed area is not large. After that, the flow will move outward only as shown in Figure 42, which is the correct mold filling pattern.
Initial stage

Inside control volumes are filled

Figure 42 A loop configuration of source nodes.
5.3.3 Determination of Filled Base Control Volumes

The example shown in Figure 41 has four base nodes (control volumes) and two base elements. Each base control volume is composed of several refined control volumes. However, the boundary of the refined control volume is not necessarily the same as that of the base control volume. One way to determine if a base control volume is filled is to check if all the refined control volumes which have all or part of their area lying inside the base control volume are filled or not. If all these refined control volumes are filled, the base control volume is considered filled.

In order to determine the status of a base control volume, the number of refined control volumes which lie inside the base control volume is determined using the following equations.

\[
\begin{align*}
    c_{ri} &= 4 \times e_{ce} + 1 \\
    c_{rb} &= 4 \times e_{ce} + 3
\end{align*}
\]

for interior node \hspace{1cm} (5.4)

for boundary node \hspace{1cm} (5.5)

where \( e_{ce} \) is the number of base elements which are adjacent to the base node of the base control volume, and \( c_{ri} \) is the number of refined control volumes inside this base control volume. Since the equations used for interior and boundary nodes are different, the boundary nodes in the flow domain must be determined. The following algorithm is used to search for the boundary nodes inside a complex flow domain.

1. Pick an element adjacent to the base node and assign the other two nodes on the element as \( n1 \) and \( n2 \).
2. Search for the rest of the adjacent elements to find an element which also connects to n2.

3. If no element is found in step 2, this base node is a boundary node.

4. If an element is found in step 2, assign the third node on this element to be the new n2. If new n2 is the same as n1, the base node is an interior node. Otherwise, repeat step 2.

During mesh refinement, each refined node is assigned to one or more parent nodes. If a refined node (control volume) is located completely inside a base control volume, its parent node is defined as the base node of the control volume. If a refined node covers more than one base control volume, all the base nodes of the shared control volume are defined as the parent nodes of the refined node. At each time step of calculation, when a refined control volume is filled, a count is added to each of its parent nodes. When the count of a base control volume is equal to \( c_{rc} \), it is considered filled as defined earlier.

The drawback of this method is that the calculated filling time is longer than the actual filling time because the total area of the involved refined control volumes is larger than that of the corresponding base control volume as shown in Figure 43. The element switching time will be delayed, which usually increases the computational effort, but the accuracy will not be affected.
Figure 43  The configuration of refined control volumes in base control volumes
5.3.4 Flow Chart of Moving Front Mesh Refinement Method

To initiate calculation, the finite element mesh is generated and read in by the program as shown in Figure 44. The processing conditions are also read in by the program. After the data interface, the refined mesh and the refined inlet nodes are defined as described earlier. Then, the main calculation routine is called. After each calculation, the pressure and velocity fields are determined. The derived flow flux at the flow front is used to move the front to a new position. The status of each base control volume is checked. If no base control volume is filled, calculation for the next time step is activated. If some base control volumes are filled, an element switch routine is called to reduce the element numbers according to the proper stages (see Figure 36). If the flow domain is not completely filled, calculation for the next time step is activated.

5.4 Non-isothermal Mold Filling Simulation in RTM

Resin transfer molding (RTM) is a process of injecting reactive resin into a closed mold with preplaced fiber mats or preforms to produce a continuous fiber reinforced polymer composite. During resin injection (known as the filling stage), heat transfer between the fiber and the resin and chemical reaction of the resin may occur. A numerical scheme based on the control volume finite element method has been developed in our laboratory [26]. It was found that when the heat transfer coefficient between the fiber mat and the resin fluid is large, the resin flow is slow, or the solid grain is very fine, local thermal equilibrium can be assumed. If the x and y coordinates are defined along the planar directions of a 3-D thin cavity and the z coordinate is
Figure 44 A schematic block diagram of the program flow chart.
defined along the gapwise direction, the governing equations of momentum, energy and species balance in the 3-D thin cavity are as follows:

momentum:
\[
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y}
\end{bmatrix}
= \begin{bmatrix}
s_{xx} & s_{xy} \\
0 & s_{yy}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial y}
\end{bmatrix}
\tag{5.6}
\]

continuity:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\tag{5.7}
\]

and
\[
\begin{bmatrix}
s_{xx} & s_{xy} \\
0 & s_{yy}
\end{bmatrix}
= \frac{1}{h_2^2}
\begin{bmatrix}
h_2 & 1 \\
\mu(x,y) & k_{xy}
\end{bmatrix}
\tag{5.8}
\]

energy:
\[
\frac{\partial}{\partial t}(\rho c_p + \rho \phi c_r) + \rho \frac{\partial T}{\partial x} + \phi \frac{\partial T}{\partial y} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \phi \Delta \dot{m}
\tag{5.9}
\]

species:
\[
\frac{\partial \alpha}{\partial t} + (\rho \frac{\partial \alpha}{\partial x} + \phi \frac{\partial \alpha}{\partial y}) = \dot{m}
\tag{5.10}
\]

where $\bar{u}$ and $\bar{v}$ are the components of gapwise average velocity in the planar directions, $T$ is temperature, $\alpha$ is resin conversion, $h_z$ is the thickness of the 3-D thin cavity, $\mu$ is viscosity, $\dot{m}$ is the mass generation rate, $\Delta H$ is the heat of reaction, $[k_{ij}]$ is the permeability matrix of the fiber mat defined in the local element coordinate system, and

\[
c_p = c_{pr} \rho_r + c_{pf} \rho_f
\]

\[
\rho = (\rho_r \rho_f) / (\rho_r \rho_f + \rho_r \rho_f)
\]

\[
k = (k_r k_f) / (k_r \rho_r + k_f \rho_f)
\tag{5.11}
\]
where \( \rho \) is density, \( c_p \) is specific heat coefficient, \( k \) is thermal conductivity, and \( \phi \) is the fiber mat porosity. The thermal parameters used in equations 5.9 and 5.11 are represented by using subscript "f" for the fiber mat and "r" for the resin fluid.

Equation 5.6 follows Darcy's law for flow through porous media without considering the inertia effect. The continuity equation 5.7 is a consequence of the incompressible fluid assumption, where the velocity components are the gapwise average values in the planar directions. Equation 5.9 is the energy equation. The first term on the LHS is the rate of internal energy change, while the second term on the LHS is the advective term which is the energy convected by the fluid. The first term on the RHS is the energy transferred by heat conduction. The second term on the RHS is the heat generation by means of chemical reaction or other heat sources. Equation 5.10 describes the mass balance of chemical species. The assumption made here is that the resin density does not change, despite chemical reaction, so equations 5.6 to 5.8 hold. For our interest, the mass generation will be assumed only via the polymerization process. The polymerization process can be either step growth polymerization (usually in saturated monomers), chain growth polymerization (typically in unsaturated monomers), or both.

In the mold filling stage of RTM, the viscous dissipation is usually insignificant compared with other contributions such as heat transfer between the resin and the fiber
mat or the reaction exotherm. The mass diffusion is also negligible during the polymerization process because the chemical reaction rate is often much higher than the mass diffusion rate. For a thin mold cavity, the flow is simplified to a two-dimensional problem, but heat transfer is still in the three-dimensional form because heat convection in the planar direction and heat conduction in the gapwise direction are both significant.

In this study, the mass generation term is assumed

$$\dot{m} = (k_1 + k_2 \alpha^m)(1 - \alpha)^{m_2}$$

(5.12)

The rate constants $k_1$ and $k_2$ are expressed in the Arrhenius form:

$$k_1 = A_1 e^{-E_1/RT_{abs}}$$

(5.13)

$$k_2 = A_2 e^{-E_2/RT_{abs}}$$

(5.14)

where $A_1$ and $A_2$ are constants, and $E_1$ and $E_2$ are activation energies associated with the rate constants $k_1$ and $k_2$ respectively. $R$ is the gas constant, and $T_{abs}$ is the absolute temperature. The above mentioned kinetic model can fit polymerizations which have a bell-shaped isothermal reaction rate profile [27]. The resin viscosity is assumed to be a function of temperature and conversion only. Initially, the viscosity decreases with the increase of resin temperature. When the reaction starts, the resin viscosity will increase rapidly due to the formation of polymer network. The resin viscosity change for a thermosetting polymer can be expressed by a widely used model [28]:
where \( \alpha_g \) is gel conversion, \( E_\mu \) is the activation energy, and \( A_\mu, a \) and \( b \) are constants.

\[ \mu = A_\mu e^{E_\mu / RT_a \left( \frac{\alpha_e - \alpha_g}{\alpha_e} \right)^a \exp(b)} \]  

(5.15)

The momentum and energy equations are coupled through the viscosity since the viscosity depends on the resin temperature and conversion. The energy equation and balance of species are also coupled through the rate of reaction. Equations 5.6 to 5.10 are solved following the same method used by Lin et. al. [26]. The pressure and velocity fields are obtained using the same procedures described by Young et. al. [23]. Temperature and conversion are solved by considering the convection term implicitly, and the conduction and reaction terms explicitly. The heat conduction in the gapwise direction is discretized by using the collocation method [26].

For each time step, the time interval \( \Delta t \) and the new flow region are determined from the previous velocity field. Then, an initial guess of the viscosity distribution is defined in the flow equation to calculate the pressure and velocity fields. The equivalent Courant number is then calculated and used to determine the sub-time interval, which is used as a time sub-increment to calculate the temperature and conversion [26]. The viscosity distribution is updated based on the new temperature and conversion fields. This updated viscosity distribution is used to calculate the new pressure and velocity profiles. All the procedures are iterated until the convergence of viscosity distribution is achieved, then the new flow front and time interval are determined for the next time step. These procedures are repeated until the mold is completely filled.
The moving front mesh refinement scheme is applied to the calculation and the results are compared to the predictions without mesh refinement. Table 2 lists the process conditions and kinetic and rheological constants used in this study.

A mold shape shown in Figure 45 is chosen as the flow domain. A line type end-gate is located at the upper left corner of the mold. The mold was discretized into three different mesh configurations with 34, 260 and 396 nodes respectively. As shown in Figure 45, the mesh with 396 nodes has generally the same size of element as that with 260 nodes except the region near the inlet gate where the former has a much finer mesh. Therefore, the simulated results from the mesh with 396 nodes would be more accurate than those from the other two mesh configurations. Non-isothermal mold filling simulation was performed for each mesh configuration. The mesh with 34 nodes was also used to run the mold filling simulation with the moving front mesh refinement scheme.

The predicted flow front positions from several mesh configurations are shown in Figure 46. The difference among them are not very large because the flow front position depends on pressure distribution and the pressure is at its minimum at the flow front. Nevertheless, the predicted flow front positions from the mesh refinement method \((n = 34r)\) are closer to those from using finer mesh \((n = 260)\). A clear indication is around the middle corner of the mold as marked in Figure 46.

Figure 47 shows the isotherms at the mid-plane of the mold predicted by using different mesh configurations. The resin is injected into the gate at 20 °C with the mold
Table 2  Process conditions and thermal, kinetic, and rheological parameters used in RTM simulation (from Lin et al.[6])

<table>
<thead>
<tr>
<th>Process conditions</th>
<th>Thermal and Kinetic Parameters</th>
<th>Rheological Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_w = 75 , ^\circ C$</td>
<td>$c_{pr} = 1680 , (J/kg , K)$</td>
<td>$A_{\mu} = 2.78e-4 , (Pa , sec)$</td>
</tr>
<tr>
<td>$T_o = 20 , ^\circ C$</td>
<td>$c_{pf} = 670 , (J/kg , K)$</td>
<td>$E_{\mu} = 18000 , (J/mole)$</td>
</tr>
<tr>
<td>$\phi = 0.8$</td>
<td>$k_r = 0.168 , (W/m , K)$</td>
<td>$\alpha_g = 0.1$</td>
</tr>
<tr>
<td>$Q = 22.4 , \text{cm}^3/\text{s}$</td>
<td>$k_f = 0.0335 , (W/m , K)$</td>
<td>$a = 1.5$</td>
</tr>
<tr>
<td>$h = 0.003 , \text{m}$</td>
<td>$A_1 = 3.79e5 , (1/sec)$</td>
<td>$b = 1.0$</td>
</tr>
<tr>
<td></td>
<td>$A_2 = 6.79e5 , (1/sec)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_1 = 13000 , (J/mole)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$E_2 = 12000 , (J/mole)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\rho_r = 1100 , (kg/m**3)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\rho_f = 2560 , (kg/m**3)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Delta H = 1.8148e8 , (J/m**3)$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 45 The discretized meshes of a simple mold.
Figure 46  The predicted flow front positions by using different mesh configurations in RTM simulation
Figure 47 The predicted temperature distribution by using different mesh configurations for a RTM simulation; the inlet temperature is 20 °C and the contour spacing is 10 °C; 1 = 30 °C.
and initial fiber temperature at 75 °C. The contour spacing is 10 °C in Figure 47. The temperature distributions predicted by using meshes with 396 and 260 nodes are about the same. The predicted temperatures by using a coarse mesh with 34 nodes are, in general, lower as compared to the results of using finer mesh sizes. Moving front mesh refinement improves the prediction of temperature distribution significantly. It should be mentioned that the contour lines of n = 34r are not as smooth as those using a finer mesh because the former ones are plotted based on the base mesh instead of the refined mesh.

The prediction of conversion distributions is shown in Figure 48, where the contour spacing is 0.004. In Figure 48, the conversion values of n= 34 are severely underestimated as compared to the results of the other cases. By comparing the conversion distributions of n= 260, n = 34r and n =396, it can be seen that the predictions of n = 34r have better accuracy near the mold outlet region, but are less accurate in the upstream region when compared with n = 260. This is because a fine mesh is only used in the flow front region.

Table 3 lists the comparison of resin temperature and conversion at the outlet position, CPU time and number of nodes used. The predicted values from n = 396 nodes are considered as the best results. In fact, this has been checked by using an even finer mesh to run the same simulation. The results are about the same as those obtained using 396 nodes. Based on this, the predicted values from using n = 260 and n = 34r are about the same order of accuracy. However, the CPU time of using mesh refinement is much less than that of using finer mesh (n = 260).
Figure 48 The predicted conversion distribution by using different mesh configurations for a RTM simulation; the contour spacing is 0.004; $1 = 0.004$. 

$n = 396$

$n = 260$

$n = 34$

$n = 34$
Table 3  Comparison of CPU time, temperature and conversion predicted for different mesh configurations (resin temperature = 20 °C, wall temperature = 75 °C)

<table>
<thead>
<tr>
<th>number of nodes</th>
<th>regular mesh</th>
<th>mesh refined at flow front</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 34</td>
<td>n = 260</td>
<td>n = 396</td>
</tr>
<tr>
<td>Cray CPU (sec)</td>
<td>6.1</td>
<td>156.0</td>
</tr>
<tr>
<td>Resin Temperature (°C) at outlet</td>
<td>72.66</td>
<td>75.35</td>
</tr>
<tr>
<td>Resin Conversion at outlet</td>
<td>.019</td>
<td>.024</td>
</tr>
</tbody>
</table>
5.5 Non-isothermal Mold Filling Simulation in RIM

Reaction injection molding (RIM) is a process in which reactive resin is injected into a closed mold and cured. Usually, an isocyanate and a polyol are injected at high speed into a mixing head for complete mixing. The mixing head is mounted onto the gate of the mold so that the resin can be injected into the mold through the gate right after mixing. In general, the mold temperature is set slightly higher than the resin temperature. Since the filling process is very fast, the mold wall can be considered as adiabatic without too much error.

Several researchers have analyzed the mold filling and curing process of RIM [28~30]. Most of them were limited to simple mold geometry. For complex mold geometry, Garcia et. al. [31] used the finite difference method with a domain mapping approach to solve the flow field and temperature distribution. Recently, Wang et al. [32] used the control volume finite element method to solve the same problem. In this study, the moving front mesh refinement method is incorporated into the control volume finite element method for RIM simulations and the results are compared with several mesh configurations.

The Hele-Shaw model describes the flow of viscous fluid through two parallel plates with small gap. The original model was developed to analyze the lubrication problem. The same model was applied to the thermoplastic injection molding process [16, 21, 33, 34] and the reaction injection molding process [31,32]. The Hele-Shaw model assumes a slow flow motion in a narrow channel gap. In such a flow configuration, the inertia force becomes negligible as compared to the dominate viscous
force. If the flow in the gapwise direction, molecular diffusion, viscous dissipation, and heat conduction in the planar direction are neglected, the thermal properties are assumed constant, and the resin is assumed to be an incompressible Newtonian fluid, the governing equations can be written as [35]:

\[
\frac{\partial}{\partial x} \left( s \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( s \frac{\partial P}{\partial y} \right) = 0
\]  
(5.16)

\[
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial z^2} + \Delta H \dot{m}
\]  
(5.17)

\[
\frac{\partial \alpha}{\partial t} + u \frac{\partial \alpha}{\partial x} + v \frac{\partial \alpha}{\partial y} = \dot{m}
\]  
(5.18)

\[
s = \int_0^h \frac{(z - h)}{2} \frac{\partial z}{\partial x} \text{dz}
\]  
(5.19)

\[
u = \int_{\frac{h}{2}}^{h} \frac{z^* \partial P}{\mu} \text{dz}^*
\]  
(5.20)

\[
u = \int_{\frac{h}{2}}^{h} \frac{z^* \partial P}{\mu} \text{dz}^*
\]  
(5.21)

\[
\dot{m} = A e^{\frac{E}{RT} \alpha} (1 - \alpha)^2
\]  
(5.22)

\[
\mu = A_p e^{\frac{E_p}{RT} \alpha \left( \frac{\alpha_{eq} - \alpha}{\alpha_{eq}} \right)^{\gamma + \delta}}
\]  
(5.23)

A second order kinetics with Arrhenius temperature dependence is used in equation 5.22. For polyurethane chemical systems, a second order equation worked well as shown by Castro and Macosko [36].
Equation 5.16 is of elliptic nature which requires specification of appropriate boundary conditions along the entire domain boundaries. At the mold walls, the normal derivative of pressure must vanish. At the moving front, a constant ambient pressure is assumed. The initial temperature and conversion are specified at the inlet gate. Since the diffusion and conduction in the planar direction are not considered, no boundary conditions need to be specified at the mold walls. The upper and lower mold wall temperatures are specified as isothermal or adiabatic conditions.

Near the moving front, there exists a complex flow field. The material near the center line flows faster than the progression of the moving front. Therefore, when the material near the center line is approaching the moving front, it will slow down and spread out to the mold wall, which is known as the fountain flow effect. For RIM process modeling, the fountain flow effect strongly determines the thermal and reaction history of each material element. Neglecting this effect might result in a wrong conclusion as reported by Castro and Macosko [28]. The fountain flow effect was modeled here by setting the temperature and conversion profiles at the flow front uniform and equal to the temperature and conversion values at the center line of the node immediately upstream from the front. This was suggested by Lord and Williams [37] and was proved a very accurate method by Garcia et al. [31].

The momentum, energy, and species balance equations are coupled through the viscosity and rate of reaction. Equations 5.16 to 5.18 were solved following the same procedures used in the previous section for non-isothermal RTM simulation. However, an upwind scheme was used to deal with the convection term in the energy
and species balance equations. Temperature and conversion were solved by considering the convection term implicitly, and the conduction and reaction terms explicitly. Iterations among pressure, temperature, and conversion were performed until the viscosity converged at each time step (see previous section). The moving front mesh refinement method was also incorporated to the control volume finite element method to solve the problem. The results with and without mesh refinement are compared based on the mesh size, accuracy and required CPU time. Table 4 lists the process conditions and kinetic and rheological parameters used in the flow simulations.

A rectangular mold (41.6 x 10 x 0.64 cm) with a line type end gate was used for a series of flow simulations. The mold was discretized into three kinds of mesh configurations, one coarse mesh with 90 nodes, one middle mesh with 378 nodes and one fine mesh with 527 nodes. Figure 49 shows these three mesh configurations and the moving front mesh refinement for the coarse mesh. Non-isothermal mold filling of the RIM process was simulated by using all mesh configurations.

Since flow is unidirectional, the fluid will progress uniformly and result in a uniform distribution in the width direction (y) for all quantities. Figures 50 and 51 show the temperature and the conversion contours of half of the mold in the gapwise and flow directions (x-z) for all cases. The contour spacing is 10 °C for temperature and 0.1 for conversion. Because only four data points were used for contour plotting in the z-direction, the resulting contour lines were not very smooth. In Figures 50 and 51, it can be seen that a hot spot results near the walls due to the fountain flow effect. The contour shapes for all cases do not vary much. But, the resulting temperatures and conversions from the coarse mesh configuration are lower than the values predicted by
Table 4  Process conditions and thermal, kinetic, and rheological parameters used in RIM simulation (from Castro and Macosko [8])

<table>
<thead>
<tr>
<th>Process conditions</th>
<th>Thermal and Kinetic Parameters</th>
<th>Rheological Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0 = 60 , ^{\circ}\text{C}$</td>
<td>$c_p = 1840 , (\text{J/kg K})$</td>
<td>$\mu = 10.3\times10^{-8} , (\text{Pa sec})$</td>
</tr>
<tr>
<td>$Q = 1.9424\times10^{-4} , (\text{m}^3/\text{sec})$</td>
<td>$k = 0.17 , (\text{W/m K})$</td>
<td>$E_\mu = 41300 , (\text{J/mole})$</td>
</tr>
<tr>
<td></td>
<td>$E_1 = 53200 , (\text{J/mole})$</td>
<td>$a = 1.5$</td>
</tr>
<tr>
<td></td>
<td>$A = 2.545\times10^7 , (1/\text{sec})$</td>
<td>$b = 1.0$</td>
</tr>
<tr>
<td></td>
<td>$\Delta H = 2.32\times10^8 , (\text{J/m}^3)$</td>
<td>$\alpha_g = 0.65$</td>
</tr>
<tr>
<td></td>
<td>$\rho = 1000 , (\text{kg/m}^3)$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 49  The discretized meshes of a simple rectangular mold with a line type end gate
Figure 50 The predicted temperature distribution by using different mesh configurations for a RIM simulation; the inlet temperature is 60 °C, wall is adiabatic and the contour spacing is 10 °C; 1 = 70 °C
Figure 51  The predicted conversion distribution by using different mesh configurations for a RIM simulation; the contour spacing is 0.1; 1 = 0.1
using the fine mesh configuration. By incorporating the mesh refinement method, the results improved significantly and were close to the results from the fine mesh configuration.

Since the mesh refinement assumes that the moving front is the most important area and refinement is focused on this area only, the predicted results have good accuracy downstream and near the wall. However, in the RIM process, the material near the center line is moving faster than the moving front. The approximation error of using coarse mesh in the upstream region is more severe near the center line due to the high convective nature in this area.

Figures 52 and 53 show the comparisons of temperature and conversion at the hot spot, at the location $x = 0.15$ m from the inlet gate and near the wall, where the conversion has a maximum value and the improvement for using mesh refinement is most significant. Table 5 lists the comparison of maximum temperature and conversion, outlet temperature and conversion, average conversion, inlet pressure, and required CPU time for all cases. The prediction accuracy of the $n = 90r$ case is better than that of the $n = 90$ case, but is not as good as those of the $n = 378$ and $n = 527$ cases. The method of using 18 refined elements at the flow front region fails to provide as much improvement in RIM simulation as in RTM simulation. This is probably due to the high convective effect near the mold center in the RIM process, which increases the influence of the changes in the upstream area on the entire resin flow.

A simulation using 6 refined elements in the flow front region is also performed. This reduces the mesh density in the flow front region and allows the use
Figure 52 The predicted temperatures at $x=0.15$ m and near the wall by using different meshes for a RIM simulation.
Figure 53 The predicted conversions at $x=0.15\, \text{m}$ and near the wall by using different meshes for a RIM simulation.
Table 5  Comparison of CPU time, pressure, temperature and conversion for different mesh configurations (resin temperature = 60 °C, wall condition = adiabatic)

<table>
<thead>
<tr>
<th></th>
<th>regular mesh</th>
<th>mesh refined at flow front (18)</th>
<th>mesh refined at flow front (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Nodes</td>
<td>n = 90</td>
<td>n = 378</td>
<td>n = 90 r</td>
</tr>
<tr>
<td>Cray CPU (sec)</td>
<td>8</td>
<td>263</td>
<td>195.0</td>
</tr>
<tr>
<td>Pressure (kpa)</td>
<td>5.45</td>
<td>6.3</td>
<td>6.08</td>
</tr>
<tr>
<td>Outlet Temp. (°C)</td>
<td>94.71</td>
<td>99.46</td>
<td>98.0</td>
</tr>
<tr>
<td>Max. Temp. (°C)</td>
<td>115.4</td>
<td>124.9</td>
<td>123.3</td>
</tr>
<tr>
<td>Outlet Conversion</td>
<td>.273</td>
<td>.309</td>
<td>.300</td>
</tr>
<tr>
<td>Max. Conversion</td>
<td>.465</td>
<td>.546</td>
<td>.531</td>
</tr>
<tr>
<td>Average Conversion</td>
<td>.241</td>
<td>.279</td>
<td>.268</td>
</tr>
</tbody>
</table>
of finer elements in the upstream region without exceeding computational time. By using \( n = 184r \) for the same rectangular mold, the simulated results in Table 5 show the same accuracy as that of \( n = 378 \), with less CPU time.

5.6 Conclusions

Many manufacturing processes involve a transient type of moving front. The control volume finite element method has been shown to be very efficient in dealing with this type of problem, especially for complex flow domains. However, the control volume finite element method usually inherents some approximation error near the flow front region. The proposed moving front mesh refinement method has provided a way to reduce this error.

The simulation of RTM and RIM processes using the mesh refinement method shows an improved accuracy, especially near the flow front region. In the RIM process, high convective flow near the center line makes the upstream area more important. Therefore, fine mesh size is also required in the upstream region. This suggests that, for the case where the flow front region is not dominant, the degree of mesh refinement (i.e. ratio of the mesh size in the flow front region to that in the upstream region) should not be too high (e.g. 6 refined elements instead of 18 refined elements).
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Numerical Simulation

The control volume finite element method (CV/FEM) was used to simulate the isothermal mold filling in RTM and SRIM. Darcy's law was used to describe the resin flow. Both 2-D and 3-D simulation models were developed in this study. The anisotropic behavior of the fiber mats was also considered. The major advantage of using CV/FEM was its ability to deal with complicated mold geometry. A flow front mesh refinement method was also proposed to improve the simulation accuracy near the flow front region in moving boundary problems. The nonisothermal mold filling processes of both RTM and RIM were analyzed to demonstrate the efficiency of this refinement method.

The numerical simulations reported in this study, both isothermal and nonisothermal, did not consider the effect of gravity force during the mold filling process. For high or medium pressure mold filling processes, the gravity force could be neglected. However, for the low pressure RTM process, such as vacuum driving mold filling used in aerospace industry, the gravity force may play an important role during the filling stage. The gravity force may need to be included in the numerical simulation model. As for the nonisothermal mold filling model, the mold surface
boundary conditions were currently assumed either isothermal or adiabatic. In actual molding processes, the mold surface conditions depends on the mold materials used. A heat transfer model for the mold itself may be necessary in order to determine the thermal state of the mold during both the filling and curing stages.

The 3-D numerical simulation of the mold filling process reported in this study was limited to the isothermal conditions. In some applications, a non-isothermal mold filling simulation may be necessary. By adding the energy and species balance equations to the model, a 3-D non-isothermal mold filling simulation with chemical reaction can be developed based on the isothermal model described here.

Further study in the simulations of RTM and SRIM processes may include the curing stage and the estimation of shrinkage and thermal stress in the composites. A CAD system may also be developed by using the simulation tools to determine the optimal process conditions for actual composite manufacturing.

6.2 Permeability of Fiber Mats

One of the important characteristics of the fiber mat is the permeability. The mold filling simulation performed in this study was based on the permeabilities reported by Trevino et. al. [14]. Some of the comparisons between the numerical and experimental results reported in Chapter II show large discrepancy. This was believed to be caused by the inadequate permeabilities measured and used in the simulations. The method used to measure the transverse permeability did not yield satisfactory results due to the leakage problem and the difficulty of generating a true unidirectional
flow in the transverse direction (see Figure 54a). One way to modify the $k_z$ device is to put a mylar sheet between two fiber mats. Each mylar sheet is cut a hole in the center. In this way, the mylar sheet can prevent any transverse flow across the fiber mat in the area other than the center hole. A unidirectional flow can thus be achieved. Figure 54b shows the schematic diagram of this modification.

One other way to measure the transverse permeability of the fiber mat is to use the back calculation curve fitting method. Since the permeability in the planar directions can usually be measured accurately, the transverse permeability can be determined by a curve fitting method. The inlet pressure can be measured by a pressure transducer during the mold filling of a circular mold with a center gate. If the permeabilities in the planar directions can be determined by the experimental methods, the transverse permeability can be obtained by fitting the inlet pressure curve using the 3-D mold filling simulation program and a trial-and-error method. This method to determine the transverse permeability of the fiber mat can completely eliminate the problems of leakage and non-uniform flow encountered in the current measuring method.

In the RTM and SRIM processes, the fiber mats often need to be preformed to the part shape before molding. During the preforming process, the fiber mat may sustain different degrees of local stretching and deformation. The permeability and porosity of the preformed fiber mat may vary from location to location. A study on the fiber mat preforming process will help to determine the actual permeability and porosity of the preformed fiber mats.
Figure 54  Schematic diagrams of the flow field in the $k_z$ device and the modification
In some preliminary studies (see Appendix B), the wetting process was found to be a combination of both micro and macro flows. During liquid injection, the measured inlet pressure did not reach the maximum value after the mold has been completely filled. Instead, the inlet pressure continued to increase as more liquid was pumped through the fiber mats. This effect was believed to be the consequence of fiber wetting during the filling process. The pressure variations may be caused by the change of porosity and micro pore distribution. However, the exact mechanism is still unknown and needs to be further studied.
APPENDIX A

COORDINATE TRANSFORMATION IN THE 2-D FORMULATION
A simple 2-D finite element model based on four triangular elements (i.e. ABD, BCD, DEF and CED) and the corresponding control volumes are shown in Figure 55, where X, Y and Z are the global coordinate system and x, y and z are the local coordinate system. Lines showing inside the element are the fiber orientation. In general, the global coordinate system is chosen when the mesh is generated. Each node within a finite element mesh is described by the three dimensional global coordinate system. Since, in the 2-D formulation, the z-direction is not considered, the global node coordinate must be transformed to the local two dimensional coordinate system before any computation. The reason that the local coordinate system is two dimensional is that the z-direction of the local coordinate system is always aligned with the surface normal to the corresponding element so that all nodes in all elements have the same z coordinate. As shown in Figure 55, the local node coordinates of the element ABD have the same z coordinate (i.e. zero), therefore, only x and y coordinates need to be specified.

Since the quantity inside the summation of equation 3.15 is a scalar, the local coordinate system for each element can be selected independently. One way to do so is to directly rotate the Z axis of the global coordinate system to the surface normal to the element. Figure 55 shows this rotation. If a node has global coordinates X, Y and Z, and the corresponding local coordinates x, y and z, the following is the equation which transforms the global coordinates to the local coordinates:
Figure S5: Coordinate transformation and fiber orientation in a simple 2-D model.
where $n_x$, $n_y$ and $n_z$ are the components of the element surface normal vector based on the global coordinates.

The reason for defining the local coordinate system as described in Figure 55 is to facilitate the representation of fiber orientation in the formulation. When a fiber mat is stamped into a preform, the local deformation of the fiber mat can be thought as a rotation of the surface normal to the fiber mat at every location. If the fiber has an angle $\alpha$ against the X axis of the global coordinate system before preforming, it is assumed that the same angle $\alpha$ maintains with the x axis of the local coordinate system after preforming. In this way, a single parameter $\alpha$ is sufficient to describe the fiber orientation, and the directional cosines in equation 3.5 can be expressed as:

$$
\begin{bmatrix}
1_{11} & 1_{12} & 1_{13} \\
1_{21} & 1_{22} & 1_{23} \\
1_{31} & 1_{32} & 1_{33}
\end{bmatrix}
=\begin{bmatrix}
\cos\alpha & -\sin\alpha & 0 \\
\sin\alpha & \cos\alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

(A.2)

This assumption, however, is invalid if the mat is also re-oriented in the x-y plane during preforming (e.g. preforming to semi-spherical or conical shapes). In such cases, the actual fiber angle in each element may need to be defined individually in
order to accurately describe the fiber orientation. In our program, the parameter \( \alpha \) based on the local coordinates can be varied from element to element so that it can handle cases where the fiber mat has different orientation in different regions after preforming.
APPENDIX B

PERMEABILITY MEASUREMENT OF FIBER MATS
B.1 Scope of Study

The development of a sound mold filling model for RTM/SRIM requires an accurate description of the fiber mat characteristics. The fiber mats are usually anisotropic because of the variations of fiber orientation and stacking sequence. In general, the fiber mat is treated as a porous medium, and the permeability of the fiber mat is determined experimentally based on Darcy's law. A preliminary experimental study has been performed to investigate the fiber wetting phenomenon and how it affects mat permeability.

The experimental work was conducted using a mold filling apparatus built in our laboratory. A constant speed pump was set up by mounting an 8.26 cm diameter hydraulic cylinder in the test section of an Instron Universal Testing Instrument (Model 1137). The cylinder was filled by a transfer gear pump. After the cylinder was filled and the valve to the transfer pump closed, a valve to the mold was opened and the Instron's crosshead was set to descend at a constant speed. The descending crosshead pushed the cylinder ram back into the body of the cylinder, thereby forcing a nonreactive fluid (DOP oil) into the mold. The fluid flow rates were varied by changing the speed of the crosshead. The mold filling pressure was measured with a pressure gauge (Omega, Model PX302, 0-150 psi) mounted near the inlet of the filling line.

A rectangular mold made of two 1.5 inches thick aluminum plates was used to measure the permeability of fiber mats in the planar directions. Figure 56 shows the drawing of the mold. Two windows located at the upper mold were used to visualize
Figure 56 Drawing of a rectangular aluminum mold used for permeability measurement in the planar directions.
the flow front approaching and moving away from the fiber mats. The one inch diameter inlet and outlet holes were located at the lower mold. The mold cavity near the inlet was designed to be a diverging channel so that the injected liquid can quickly develop to an uniform flow and not trap any air bubbles near the inlet region. A transverse permeability measurement device [14] developed in our laboratory was used to study the permeability and wetting in the thickness direction.

B.2 Permeability Measurements in the Planar Directions

The random fiber mat used in this permeability study was CertainTeed U750. The fiber mat was cut into a rectangular shape with dimensions 30 by 14 cm. Different numbers of fiber mats were stacked together to produce fibrous reinforcement with various porosities. The thickness of the mold cavity was measured after the assembly of each run and it had the value of 0.53±0.02 cm. The reason for the thickness variation was due to the entrapment of some fiber bundles near the edge of the mat in the O-ring slot. However, no leakage problem was observed during the experiments. By using 2, 4 and 6 layers of fiber mats stacked together, approximated porosities of 0.91, 0.82 and 0.73 can be produced after the mold was assembled.

The Instron machine and hydraulic cylinder set-up described earlier was used as the liquid injection device. Four different speeds were used to fill the mold and the inlet pressure was measured by a pressure transducer. For each run, both dry and wet fiber filling pressures were measured. The dry fiber filling pressure was defined as the inlet pressure measured as the liquid flow front passing through the fiber mats. After the fiber mats were filled, the liquid was pumped through the fiber mat again with the same
speed and the maximum pressure measured was defined as the wet fiber filling pressure. Figure 57 shows the comparison of dry and wet fiber filling pressures with porosity = 0.73. In general, the wet fiber filling pressure is higher than the dry fiber filling pressure. In the mold filling process, the inlet pressure did not reach the maximum value as the liquid passing through the fiber mat. Instead, the inlet pressure would slowly increase as more liquid was pumped through the fiber mats. This effect was believed to be the consequence of fiber wetting during the filling process. The pressure variations may be caused by the change of porosity and micro pore distribution. However, the exact mechanism is still largely unknown.

Figures 58 and 59 show the dry and wet fiber filling pressures with respect to various superficial velocities at different porosities. The pressure drop is proportional to the superficial velocity under different porosities, which confirms the assumption of Darcy's law. The dry fiber filling pressures were used to determine the permeability of U750 random fiber mat. Figure 60 shows the permeability with respect to various porosities. The circles are the experimental data and the solid line is the numerical fitting using the Kozeny-Carman equation. The fitting equation is

$$k_x = k_y = 1.50775 \times 10^{-10} \frac{\phi^3}{(1-\phi)^2} \text{ (m}^2) \tag{B.1}$$

B.3 Permeability Measurements in the Transverse Direction

The transverse permeabilities of random and bidirectional fiber mats were measured by Trevino et al.[14] who verified the linear relationship between the
(\Delta x = .3 \text{ m}, \text{ viscosity} = 0.08 \text{ Pa s, porosity} = .7325)

- o wet fiber filling pressure
- + dry fiber filling pressure

Figure 57: Dry and wet fiber filling pressures of random fiber mats ( CertainTeed U750) in the planar directions with porosity = 0.73
Dry fiber filling pressure
(CertainTeed fiber mat U750)

$\Delta P$ (N/m$^2$)

Superficial velocity (cm/sec)

Figure 58  Dry fiber filling pressure of random fiber mats (CertainTeed U750) with respect to superficial velocity at different porosities
Wet fiber filling pressure  
(CertainTeed fiber mat U750)

\[ \Delta P = \frac{1}{2} \frac{k}{\rho} u^2 \]  
\( (\Delta x = 0.3 \text{ m}, \text{viscosity} = 0.08 \text{ Pa s}) \)

- porosity = 0.9067
- porosity = 0.8234
- porosity = 0.7325

Superficial velocity (cm/sec)

Figure 59  Wet fiber filling pressure of random fiber mats (CertainTeed U750) with respect to superficial velocity at different porosities
Figure 60  Permeability of CertainTeed U750 random fiber mat in the planar directions
superficial velocity and the impregnation pressure. However, the difference of dry and wet fiber filling pressures was not mentioned in their study. Also, the effect of fiber stack thickness on the impregnation pressure was not reported. Studies regarding to these phenomena are reported here.

The fiber mat used in the transverse impregnation was the Cofab A1118B bidirectional stitched fiber mat. A continuous gear pump was used as the injection device. A transverse permeability measurement device [14] was used to hold the fiber stack together. As in the planar directions, the inlet pressure did not reach the maximum value after the fiber mat was completely wetted. Instead, the inlet pressure continued to increase as more liquid was pumped through the fiber mats. In all the experiments conducted, it took less than 3 seconds for the liquid to wet the entire fiber mats. However, the inlet pressure kept increasing. In general, 5 to 10% pressure variations could be observed as the liquid was pumped through the fiber mats for 5 minutes (see Figure 61). In the study of transverse permeability, a gear pump was used and the pressure measured after 2 minutes of injection was chosen as the wet fiber filling pressure.

Three different thicknesses, 0.4625, 0.9525, and 1.415 cm, were used for fiber mat stacks. The porosity in all cases was kept at a constant value of 0.6. Figure 62 shows the wet fiber filling pressure with respect to different injection flow rates for various stacking thicknesses. A linear relationship is observed for all three fiber mat stacking thicknesses. Figure 63 shows the pressure per unit length in the flow direction with respect to different flow rates. The three lines in Figure 62 fall into a
Figure 61 A typical inlet pressure curve in a transverse mold filling using $k_z$ device

(porosity = 0.6; flow rate = 24.58 ml/sec)

wet fiber filling pressure
Pressure drop during impregnation of fiber mat with porosity = 0.6

![Graph showing pressure drop vs flow rate for different fiber stacking thicknesses.]

Figure 62: Transverse impregnation pressures of bidirectional fiber mats (Cofab A1118B) with different fiber stacking thickness but the same porosity, 0.6
$\Delta p/\Delta z$ during impregnation of fiber mat with porosity = 0.6

Figure 63  Transverse impregnation pressure per unit length in the flow direction (Cofab A1118B, porosity = 0.6)
single line in Figure 63, which indicates that the permeability is actually independent of the fiber stacking thickness.
LIST OF REFERENCES