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Information in the cash market and stock index futures market

Chan, Kalok, Ph.D.
The Ohio State University, 1990
INFORMATION IN THE CASH MARKET AND STOCK INDEX FUTURES MARKET

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of the Ohio State University

by

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* * * * *

The Ohio State University

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CHAPTER I
OVERVIEW

The relation between the stock index futures market and underlying cash market has received much attention over the last few years. Ever since its introduction in 1982, volume in index futures has increased dramatically. By 1987, the average daily dollar volume in the S&P 500 futures contracts alone exceeded the dollar volume of cash S&P 500 trade by a factor of about two, while the dollar value of the daily net change in the total open interest is about 8% of the S&P 500 stock dollar volume.

Recently, there has been much controversy among academics and regulators about the effect of index futures trading on the cash market. On the one hand, since cash index futures provide a low cost vehicle for trading, this will attract more informed traders so that futures trading can provide information for the cash market. On the other hand, if the futures market is dominated by uninformed speculators, futures trading can destabilize the cash market.

The unprecedented stock market crash of 1987 further sparked attacks against index futures trading. Some regulators blamed the market activities in the futures market as a catalyst for the decline. They claimed that on "Black Monday" of the crash, the transactions
associated with portfolio insurance started in the futures market and then by means of index arbitrages transmitted the pressure to individual stocks in the cash market. The worry about the destabilizing effect of futures trading has led the regulators to propose some restrictions in the futures market. The Chicago MERC advocated a 12 point daily limit on S&P 500 contract price movements; the SEC proposed trading halts in both futures and cash markets; and the Brady Commission recommended uniform margin requirements in two markets.

The purpose of this dissertation is to study the informational role of the futures market and discuss how it is affected by the market structures. Several related issues are examined in the dissertation. First, is the futures market redundant or is it providing information to the cash market? Second, how will the institutional characteristics affect the price discovery process in the two markets? Third, why is the behavior of futures prices different from that of cash index prices? Fourth, what kind of information advantage does the futures market have?

The dissertation consists of two major chapters. Chapter II discusses why the behavior of the futures prices and cash index prices might be different. We argue that the difference in market structures is important. This is because while individual stocks are traded in the cash market, futures contracts (or equivalently, baskets of stocks) are traded in the futures market. It is demonstrated that if
Market makers are not informed but just determine the prices based on observed order flows, their rational pricing decisions result in different behavior for futures and cash index prices. Further, profit maximizing informed traders have different incentives to collect market-wide and firm-specific information in the two markets.

Chapter III examines the lead-lag relation between prices of the two markets to see which market reacts faster to the information. It also provides evidence about how the lead-lag relation might be affected by institutional factors. It is found that futures prices might lead, or lag, cash index prices. Further, the feedback from the futures market into the cash market is larger when there are more stocks moving together. Therefore, it suggests that the futures market has a price discovery role, especially in market-wide information.
A. INTRODUCTION

The relation between futures prices and the underlying cash index prices has been studied extensively. Since futures and cash index prices reflect the aggregate values of the underlying stocks, they are expected to exhibit similar behavior. However, previous studies find that the two exhibit quite different behavior. Stoll and Whaley [1987] report frequent discrepancies between futures and cash index prices. MacKinley and Ramaswamy [1988] find that the autocorrelation of price changes is zero for the futures but positive for the cash index. Amihud and Mendelson [1989] document that the variance of futures price changes is larger than that of cash index price changes. Other studies examine time series relationships: Ng [1987], Kawaller, Koch and Koch [1987], Stoll and Whaley [1988] and Harris [1989] find that S&P 500 futures price changes lead cash index price changes.

Very few papers distinguish the difference between futures and cash index prices from the viewpoint of different structures for
trading the securities in the two markets. One difference in the market structure is that while individual stocks are traded in the stock market, futures contracts (or equivalently, baskets of stocks) are traded in the futures market. Two papers examine the difference between the two markets from this perspective. Kumar and Seppi [1989] explain the existence of arbitrage opportunities with heterogenous information in the two markets. Specifically, floor traders in the futures market have information about the value of the aggregate index while specialists have information about the relative values of their individual stocks. Short-lived arbitrage opportunities arise as a result of lags in the transmission of price information between markets. Subrahmanyam [1989] explains the popularity of trading in baskets of securities by demonstrating that this allows liquidity traders to realize their trades more efficiently because their losses to informed traders are lower in baskets than in the individual securities.$^3$

The aim of this chapter is to underscore the importance of the different structures in affecting the behavior of futures and cash index prices. Because of the different settings, while market makers in the cash market trade individual stocks, those in the futures market trade futures contracts (baskets of stocks). When market makers are not informed but rational, they will take into account the amount of uncertain information in pricing the securities. Therefore, the market makers in the futures market takes into account the variance of underlying values of futures contracts (i.e. variances of
individual stock payoffs plus covariances among stock payoffs) in determining futures prices, but those in the cash market just takes into account the variances of individual stock payoffs in determining stock prices. Therefore, even though the market makers are rational, futures prices may be different from the aggregate of stock prices (cash index prices).  

This chapter develops a model to derive the equilibrium prices in the cash and futures markets. We extend the rational expectations framework in Kyle [1985] and Admati and Pfleiderer [1988] to multiple security markets and the futures market. In each market, there are informed traders who acquire information about the securities. Based on their information, they submit orders along with noise traders to market makers who set equilibrium prices conditional upon the net order flow. In choosing the quantities that they would trade, informed traders make strategic decisions and take into account the effect of their quantities traded on the prices. A rational equilibrium can then be determined such that the conjectures of the traders and market makers are self-fulfilling.

The model shows that cash index prices behave quite differently from futures prices even though the two markets have the same number of informed traders. It is demonstrated that futures prices are more volatile, so that they are less efficient in reflecting information than cash index prices. This stems from the different pricing behavior in the two markets. In the futures market, market makers take into account the correlated information (covariances among stock
payoffs) in adjusting futures prices in response to the order flows. But in the stock market, market makers do not take into account the correlated information in adjusting individual stock prices to the order flows, where the liquidity trading comprising the order flows in each stock is also uncorrelated. When the model is extended to multiperiod analysis, it implies that: 1) the variance of futures price changes is larger than the variance of cash index price changes; 2) even though price changes of individual stock and futures are serially uncorrelated, cash index price changes are positively autocorrelated.

This chapter further considers the profits earned by informed traders who acquire market wide and firm specific information and trade in the cash and futures market. It is shown that even in the absence of transaction costs, a market wide informed trader earns a larger profit by trading the futures contracts than by trading a portfolio of stocks, and a firm specific informed trader earns a larger profit by trading individual stocks than by trading the futures contracts. Therefore, if information acquisition is endogenous, there is a larger incentive to collect market wide information but a lower incentive to collect firm specific information in the futures market than in the cash market. This has interesting implications for the lead-lag behavior between price movements in two markets. 5

The chapter is organized as follows. Section B derives the equilibria in the cash and futures market. We compare the efficiency of cash index and futures prices in reflecting information and examine
the profits earned by market wide and firm specific informed traders. Section C considers the entry of informed traders, and discusses the relation between the number of informed traders in the cash and futures market. Section D extends the model to a multi-period world, where we examine the time series properties and the temporal relationship between cash index and futures prices. This section is followed by conclusions in Section E.
B. THE MODEL

1. Two-Period Framework

Consider a simple two period model where the traded assets consist of \( m \) stocks and a futures contract on an index of one share of each stock. In the first period, the stocks are traded at \( P_i \), \( i = 1, 2, \ldots, m \), and the futures contract is priced at \( F \). In the second period, each share of the stock \( i \) pays off \( V_i \), and therefore the index value is \( I = \sum_{i=1}^{m} V_i \).

It is assumed that there are some random variables that affect payoffs of the stocks, such that \( V_i = \tilde{V} + \tilde{W} + \tilde{S}_i \), \( i = 1, 2, \ldots, m \), with

1) \( \tilde{V} \) is a constant and the same for \( m \) stocks and known to all traders;

2) \( \tilde{W} \) is a market wide information component that affects all stocks together, normally distributed with mean zero and variance \( \sigma_w^2 \);

3) \( \tilde{S}_i \) is a firm specific information component that affects stock \( i \), normally distributed with mean zero and variance \( \sigma_{S_i}^2 \), and \( \tilde{S}_i \) is independent of \( \tilde{S}_j \), \( i \neq j \);

4) \( \tilde{W} \) is uncorrelated with \( \tilde{S}_i \).

In the market, there are informed traders who are risk neutral and observe some signals that are informative about \( V_i \). Some are market wide informed traders (indexed by \( \tilde{W} \)) who observe \( \tilde{W} \) perfectly; the others are firm specific informed traders (indexed by \( \tilde{S}_i \)) who observe \( \tilde{S}_i \) perfectly. For convenience of analysis, we assume that no trader is informed of more than one random variable. Therefore, no informed
trader has perfect information about $\bar{V}_i$, but each receives a perfect signal about one random variable. This model differs from those of previous papers which have informed traders with diverse information. In our model, the signals of different types of traders are uncorrelated, and each signal reveals just part of the information. Therefore, the total uncertainty about $\bar{V}_i$ is $\sigma^2_w + \sigma^2_s$, and each informed trader resolves the uncertainty by possessing the signal about the actual realization of one random variable. The size of information that the trader has about $\bar{V}_i$ can be measured by the amount of uncertainty that he resolves. Thus, the size of the information trader $W$ has is $\sigma^2_w$; the size of information that trader $S_i$ has is $\sigma^2_s$.

In the futures contracts, the total uncertainty about $\bar{1}$ is $m^2\sigma^2_w + m^2\sigma^2_s$. The size of information that trader $W$ has is $m^2\sigma^2_w$ and the size of information that trader $S_i$ has is $\sigma^2_s$.

In this section, we assume that there is one market wide informed trader $W$ in the cash and futures market respectively, and one firm specific informed trader $S_i$ in stock trading post $i$ and the futures trading pit. Based on their information, informed traders submit the orders by choosing the number of shares or futures contracts in order to maximize the expected profits. Along with the informed traders, there is one liquidity (noise) trader who has a non-informationally based trading of exogenous random order $\bar{z}_i$ in stock trading post $i$ and $\bar{z}_f$ in the futures trading pit. $\bar{z}_1, ..., \bar{z}_m$ and $\bar{z}_f$ are normally and independently distributed with means zero and variance $\sigma^2_{z_i}$, and uncorrelated with $\bar{W}$ and $\bar{S}_i$. 
In addition to the traders, there is a risk neutral market maker who sets the equilibrium prices and clears the orders in each stock trading post and the futures pit. The market makers have no information about \( \bar{v}_i \) and \( \bar{I} \), and do not observe individual quantities traded by each trader separately. Further, each market is separated such that market makers can only observe the net order flows inside their own markets. Based on the net order flows, they set the prices to clear the orders and satisfy the zero expected profit condition.

2. Pricing Strategy

Following the models in Kyle (1985) and Admati and Pfleiderer (1988), we derive the Nash equilibria in the cash and futures market. In each trading post/pit, market wide and firm specific informed traders make strategic decisions and submit market orders to the market maker, taking the strategies of all other traders as given, and taking into account the effect of their chosen quantities on the prices.

The role of market makers is to determine the equilibrium prices. We assume that the market makers' pricing response is a linear function of net order flows, and as will be shown, this is consistent with the zero expected profit condition. Further, the equilibrium thus derived is rational as the conjectures of the traders and market makers about each other behavior are self-fulfilling. The equilibrium conditions in the stock and futures market are stated below.
Stock Market

In stock trading post i, let $\tilde{w}_i$ and $\tilde{s}_i$ be the quantities traded by market wide informed trader $\tilde{W}$ and firm specific informed trader $\tilde{S}_i$, respectively. Along with the orders ($\tilde{z}_i$) from the liquidity trader, net order flow ($\tilde{q}_i$) to the market maker is

$$\tilde{q}_i = \tilde{w}_i + \tilde{s}_i + \tilde{z}_i \quad (1)$$

and the linear pricing strategy of the market makers implies that

$$\tilde{p}_i = \tilde{V} + \lambda \tilde{q}_i$$
$$\tilde{V} + \lambda (\tilde{w}_i + \tilde{s}_i + \tilde{z}_i) \quad (2)$$

To derive the equilibrium condition, we first make some conjectures about the quantities traded by each informed trader. Later, we show that the conjectures are self-fulfilling in equilibrium, and therefore rational. Suppose we conjecture that the quantities traded by informed traders are linear functions of their information, so that

$$\tilde{w}_i = \alpha_w \tilde{W} \quad (3)$$
$$\tilde{s}_i = \alpha_s \tilde{S}_i \quad (4)$$

we have the following equilibrium results.

Proposition 1

In stock trading post i, if the market maker follows a linear pricing strategy, then in equilibrium, the orders submitted by market wide informed trader $\tilde{W}$ and firm specific informed trader $\tilde{S}_i$ are
It is noted that the linearity of the strategies is a derived result (because of the profit maximization of the quadratic objective function) in the sense that they are still optimal even when nonlinear strategies are allowed. Further, the equilibrium is unique when we constrain the strategies to be linear. This is because the second order condition of profit maximization rules out a solution with $\lambda$ being negative. We do not, however, investigate whether equilibria with non-linear strategy functions exist; such an investigation is beyond the scope of this paper.

The above equilibrium condition is an extension of the result in Kyle (1985). If there is only one source of uncertainty, the result will be reduced to Kyle's. Similar to Kyle's intuition, $1/\lambda$ measures the "depth" of the market, i.e., the order flow necessary to induce prices to rise or fall by one dollar. It is noted that $1/\lambda$ decreases with the size of information ($\sigma_w^2$ and $\sigma_s^2$), and increases with the amount of noise trading ($\sigma_z^2$). The intuition is that the larger the size of information, the more likely that orders are information related, and therefore the larger the price movements are in response to order flows and the lower the market depth. On the other hand, the larger the amount of noise trading, the more likely that the orders
are noise related, and therefore the lower the price movements are in response to the order flows and the higher the market depth.

**Futures Market**

In the futures trading pit, we derive the equilibrium condition similarly. To distinguish the notations from those in the stock market, we add a superscript \( f \) to them. Let \( \tilde{w}^f \) and \( \tilde{s}^f_1 \) be the quantities traded by market wide informed trader \( W^f \) and firm specific informed trader \( S^f_1 \), \( i=1,2,...,m \). Therefore, net order flows are

\[
\tilde{q}^f = \tilde{w}^f + \sum_{i=1}^{m} \tilde{s}^f_i + \tilde{z}^f
\]

and from the linear pricing strategy, the market maker sets the futures price to be

\[
F = m\tilde{w} + \lambda^f q^f \\
= m\tilde{w} + \lambda^f (\tilde{w}^f + \sum_{i=1}^{m} \tilde{s}^f_i + \tilde{z}^f) \tag{9}
\]

Again, we conjecture that the quantities traded by informed traders are linear functions of their information,

\[
\tilde{w}^f = \alpha^f_w m\tilde{w} \\
\tilde{s}^f_i = \alpha^f_s S^f_i
\]

we have the following equilibrium results.

**Proposition 2**

In the futures trading pit, if the market maker follows a linear pricing strategy, then in equilibrium, the orders submitted by market
wide informed trader $\bar{w}^f$ and firm specific informed trader $s_i^f$, $i=1,2,...,m$, are

$$\bar{w}^f = \frac{1}{2} \lambda^f m \bar{w}^f$$  \hspace{1cm} (11)$$

$$s_i^f = \frac{1}{2} \lambda^f s_i$$  \hspace{1cm} (12)$$

$$\lambda^f = \left\{ \frac{1}{4\sigma_w^2} \left[ m^2 \sigma_w^2 + m \sigma_s^2 \right] \right\}^{1/2} \hspace{1cm} (13)$$

Proof. See Appendix.

The equilibrium in the futures trading pit is very similar to that in stock trading post $i$. However, there are some noticeable differences. First, in stock trading post $i$, market wide informed trader $W$ submits an order $\bar{w}$ which is a function of $\bar{w}$. But in the futures trading pit, market wide informed trader $\bar{w}^f$ submits an order $\bar{w}^f$ which is a function of $m \bar{w}$. Second, the parameters for the market depths are different, as $1/A^f$ is a function of the uncertainty about $\bar{V}_i$, while $1/\lambda^f$ is a function of the total uncertainty about $\bar{I}$.

3. Stock Price Efficiency

This subsection examines the efficiency of individual stock prices to reflect each piece of information. This is to see how well the information is impounded into the prices. For ease of exposition, we measure the efficiency of prices by the posterior precision of the random variable conditional on prices. The efficiency of stock price $P_i$ to reveal $\bar{w}$ (denoted by $\Gamma(\bar{w}|P_i)$) is measured by the posterior precision of $\bar{w}$ conditional on $P_i$, and the efficiency of stock price $P_i$
to reveal $\tilde{S}_i$ (denoted by $\Gamma(\tilde{S}_i | \tilde{P}_i)$) is measured by the posterior precision of $\tilde{S}_i$ conditional on $\tilde{P}_i$. We also introduce $\Gamma_w$ and $\Gamma_s$ to represent prior precisions of $\tilde{W}$ and $\tilde{S}_i$, where $\Gamma_w = \text{Var}^{-1}(\tilde{W})$ and $\Gamma_s = \text{Var}^{-1}(S_i)$.

By definition, $\Gamma(\tilde{W} | \tilde{P}_i) = \text{Var}^{-1}(\tilde{W} | \tilde{P}_i)$ and $\Gamma(\tilde{S}_i | \tilde{P}_i) = \text{Var}^{-1}(\tilde{S}_i | \tilde{P}_i)$. By evaluating $\text{Var}(\tilde{W} | \tilde{P}_i)$ and $\text{Var}(\tilde{S}_i | \tilde{P}_i)$ and inverting the expression,

$$
\Gamma(\tilde{W} | \tilde{P}_i) = (1 + d_w)\Gamma_w 
$$

$$
\Gamma(\tilde{S}_i | \tilde{P}_i) = (1 + d_s)\Gamma_s 
$$

where $d_w = \sigma^2_w/(\sigma^2_w + \sigma^2_s)$, $d_w = 1 - d_s$. 

(14) and (15) show that the posterior precision of prices is not affected by the amount of noise trading $\sigma^2_z$. This is similar to the results in Kyle [1985] and Admati and Pfleiderer [1988]. When the informed traders are risk neutral, an increase in noise trading brings forth more informed trading to retain the informativeness. $d_w$ and $d_s$ help us to analyze the difference in the posterior precision of prices when the payoff is affected by two factors instead of one factor. If $\tilde{W}$ is the only piece of information, $\sigma^2_s$ is zero and $d_w$ is one. Thus market wide informed trader $\tilde{W}$ contributes $\Gamma_w$ to the posterior precision. Similarly, if $\tilde{S}_i$ is the only piece of information, $\sigma^2_s$ is zero and $d_s$ is one, and firm specific informed trader $S_i$ contributes $\Gamma_s$ to the posterior precision. Therefore, when there is only one piece of information, each informed trader can contribute his information to the posterior precision completely. But when there are two pieces of information, $d_w$ and $d_s$ are less than one, and each informed trader contributes his information to the posterior precision
incompletely. It can be immediately seen that how much an informed trader can contribute to the posterior precision depends on the relative sizes of the two pieces of information. For example, the higher the ratio \( \frac{\sigma_w^2}{\sigma_s^2} \), i.e., the larger the relative size of market wide information, the more information a market wide informed trader contributes to the posterior precision. On the other hand, the lower the ratio \( \frac{\sigma_w^2}{\sigma_s^2} \), i.e., the larger the relative size of firm specific information, the more information a firm specific informed trader contributes to the posterior precision.

4. Comparison of Price Efficiency

This subsection compares the efficiency of individual stock prices, cash index, and futures prices in reflecting market wide and stock specific information. For ease of comparison, efficiency is measured by the variance of the random variable conditional on the prices. The larger the conditional variance, the less efficient are the prices reflecting the information.

The efficiency of the prices to reflect market wide information are as follows,

\[
\text{Var}(\tilde{W} | \tilde{P}_1) = \sigma_w^4 - \frac{\sigma_w^4}{2(\sigma_w^2 + \sigma_s^2)} \tag{16}
\]

\[
\text{Var}(\tilde{W} | \tilde{I}) = \sigma_w^4 - \frac{m^2 \sigma_w^4}{(m^2 + m)\sigma_w^2 + 2(m^2 \sigma_s^2)} \tag{17}
\]

\[
\text{Var}(\tilde{W} | \tilde{F}) = \sigma_w^4 - \frac{m^2 \sigma_w^4}{2(m^2 \sigma_w^2 + m \sigma_s^2)} \tag{18}
\]
It can immediately be seen that $\text{Var}(\bar{W} \mid F) > \text{Var}(\bar{W} \mid \bar{P}) > \text{Var}(\bar{W} \mid \bar{I})$.

It should not be surprising to see that individual stock prices are the least efficient in reflecting market wide information. Since market wide information is systematic and stock specific information is not, the relative size of market wide information underlying futures prices and cash index prices is larger than those underlying individual stock prices. Therefore, information is reflected more precisely by cash index and futures prices. The reason that cash index prices are more efficient than futures prices is as follows. In the futures market, market makers take into account the correlated information in adjusting futures prices in response to the order flows. But in the stock market, market makers do not take into account the correlated information in adjusting individual stock prices to the order flows, where the liquidity trading comprising the order flows in each stock is also uncorrelated. As a consequence, the impact of liquidity trading on the futures market is larger. Futures prices are more volatile in response to the liquidity trading and become less efficient in reflecting information than cash index prices.

The efficiency of the prices to reflect firm specific information are,

$$\text{Var}(\bar{S}_i \mid \bar{F}_i) = \sigma_s^2 - \frac{\sigma_s^4}{2(\sigma_w^2 + \sigma_s^2)}$$  \hspace{1cm} (19)$$

$$\text{Var}(\bar{S}_i \mid \bar{I}) = \sigma_s^2 - \frac{\sigma_s^4}{(m^2 + m) \sigma_w^2 + 2(m \sigma_s^2)}$$  \hspace{1cm} (20)$$
\[ \text{Var}(\tilde{S}_i|\mathbb{F}) = \sigma_s^2 - \frac{\sigma_s^4}{2(m^2 \sigma_w^2 + m \sigma_s^2)} \]  \hspace{1cm} (21)

Evaluating the above equations, we have the following inequality:
\[ \text{Var}(\tilde{S}_i|\mathbb{F}) > \text{Var}(\tilde{S}_i|\mathbb{F}_r) > \text{Var}(\tilde{S}_i|\mathbb{F}_r). \]
Individual stock prices are the most efficient in reflecting firm specific information. This is because the relative size of firm specific information underlying individual stock prices is the largest. Futures prices are less efficient than cash index prices in reflecting stock specific information because they are more volatile.

To conclude this subsection, we find that individual stock prices are the most efficient in reflecting firm specific information but the least efficient in reflecting market wide information. More interestingly, cash index prices are more efficient than futures prices in reflecting any piece of information, because futures prices are more volatile in response to the liquidity trading.

5. Expected Profits of Informed Traders

This subsection analyzes the expected profits of informed traders in the cash and futures market. For trader \( W \) and \( S_i \) who trade in the cash market, their expected profits are denoted \( \pi_w \) and \( \pi_{s,i} \). For trader \( \tilde{W}_f \) and \( \tilde{S}_{i,f} \) who trade in the futures market, the expected profits are denoted \( \tilde{\pi}_w \) and \( \tilde{\pi}_{s,i} \). The expected profits of trader \( W \) and \( S_i \) are given by,

\[ \pi_w = E[ \sum_{i=1}^{n} \tilde{W}_i (\tilde{V}_i - \tilde{P}_i) | \tilde{W} ] \] \hspace{1cm} (22)

\[ \pi_{s,i} = E[ \frac{S_i}{\tilde{S}_i} (\tilde{V}_i - \tilde{P}_i) | \tilde{S}_i ] \] \hspace{1cm} (23)
Substituting the price function in (2) and order functions in (5) and (6) into the above profit functions and taking the expectation, we have

\[ \pi_w = \frac{m \sigma_w \sigma_z}{2(\sigma_w^2 + \sigma_z^2)^{1/2}} \]  \hspace{1cm} (24)

\[ \pi_{s,i} = \frac{\sigma_s \sigma_z}{2(\sigma_w^2 + \sigma_s^2)^{1/2}} \]  \hspace{1cm} (25)

For those traders in the futures market, the expected profits are

\[ \pi_w^f = E[\tilde{\bar{W}}^m(X_{i=1}^m \tilde{V}_i - \bar{P}) | \tilde{V}] \]  \hspace{1cm} (26)

\[ \pi_{s,i}^f = E[\tilde{\bar{S}}^m(X_{i=1}^m \tilde{V}_i - \bar{P}) | \tilde{S}_i] \]  \hspace{1cm} (27)

Substituting the price function in (9) and the order functions in (11) and (12) into (26) and (27) and taking the expectation, we have

\[ \pi_w^f = \frac{2 \sigma_w \sigma_z}{2(m \sigma_w^2 + m \sigma_s^2)^{1/2}} \]  \hspace{1cm} (28)

\[ \pi_{s,i}^f = \frac{\sigma_s \sigma_z}{2(m \sigma_w^2 + m \sigma_s^2)^{1/2}} \]  \hspace{1cm} (29)

It can be observed that a market wide informed trader earns a larger profit by trading futures contracts than by trading a portfolio of stocks, and a firm specific informed trader earns a larger profit by trading individual stocks than by trading futures contracts. The different expected profits is related to the
difference in relative sizes of market wide and firm specific information underlying futures and individual stock prices. In a rational expectation framework, the size of information has a feedback on the pricing process, and affects the profits of informed traders. A market wide informed trader finds that he is better off in exploiting his information in the futures market because the diversification effect of firm specific information has a smaller feedback on the pricing process. On the other hand, a firm specific trader finds that he is worse off in trading in the futures market because the systematic effect of market wide information, together with the additional effect of other firm specific information, induce a larger feedback on the pricing process.
C. MULTIPLE INFORMED TRADERS

1. Pricing Strategy

In the previous section, the number of informed traders is taken as fixed. This assumption is relaxed in this section. We first examine how the number of informed traders affects the pricing strategy and the equilibrium results, and then see how the equilibrium number of informed traders is obtained. We first examine the equilibrium conditions in the stock market and futures market.

Stock Market

**Proposition 3**

In the stock trading post $i$, where there are $N_w$ market wide and $N_s$ firm specific informed traders, and $\tilde{w}_i$ and $\tilde{s}_i$ shares traded by each market wide and firm specific informed trader respectively, the equilibrium conditions are

$$\tilde{w}_i = \alpha_w \tilde{w}$$

$$\alpha_w = \frac{1}{\lambda (N_w + 1)}$$

$$\tilde{s}_i = \alpha_s \tilde{s}_i$$

$$\alpha_s = \frac{1}{\lambda (N_s + 1)}$$

$$\lambda = \left\{ \frac{1}{2} \left[ \eta_w \sigma_w^2 + \eta_s \sigma_s^2 \right] \right\}^{1/2}$$

$$\eta_w = \frac{N_w}{(N_w + 1)^2}$$

$$\eta_s = \frac{N_s}{(N_s + 1)^2}$$

**Proof.** See Appendix.
Futures Market

Proposition 4

In the futures trading pit, where there are \( N_w^e \) market wide informed traders and \( N_s^e \) firm specific informed traders (in each stock), and \( w^e \) and \( s^e \) contracts traded by each market wide and firm specific informed trader, the equilibrium conditions are,

\[
\begin{align*}
\widetilde{f}^e_w &= \alpha_w^e m_w^e \\
\gamma^e_s &= \alpha_s^e g_j^e \\
\gamma^e &= \left\{ \frac{1}{\sigma_z^2} \left[ \eta_w^e \sigma_w^2 + \eta_s^e \sigma_s^2 \right] \right\}^{1/2} \\
\eta_w^e &= \frac{N_w^e}{(N_w^e + 1)^2} \\
\eta_s^e &= \frac{N_s^e}{(N_s^e + 1)^2}
\end{align*}
\] (33) (34) (35)

Proof. See Appendix.

Proposition 3 and 4 are general cases of proposition 1 and 2.

For example, for proposition 3, in addition to the relations discussed under proposition 1, the market depth, \( 1/\lambda \), is also inversely related to \( \eta_w \) and \( \eta_s \). Since \( \eta_w \) and \( \eta_s \) decrease with \( N_w \) and \( N_s \) respectively, the market depth thus increases with the number of informed traders. The reason for this increase is that although the market makers face more informed traders, these informed traders are of the same type and observe the same information. Competition among informed traders allows the market makers to infer the information contained in the order flows more easily, thus increasing the market depth.
\( \alpha_w \) and \( \alpha_s \) represent the sensitivities of the quantities traded by the informed traders to their information. Since \( \alpha_w \) and \( \alpha_s \) are affected by \( 1/\lambda \), which is in turn affected by \( \eta_w \) and \( \eta_s \), the sensitivities of the quantities traded are thus related not only to the number of its own type of informed traders, but also to the number of the other type of informed traders. Specifically, more market wide (firm specific) informed traders increase the market depth and thus increase the sensitivities of firm specific (market wide) informed traders. On the other hand, although more informed traders of the same type increases the market depth and thus increases the sensitivities indirectly, (30) and (31) show that more informed traders competing for the same information also decreases the sensitivities directly. A closer examination reveals that the latter effect dominates, so that more market wide (firm specific) informed traders decreases the sensitivities of market wide (firm specific) informed traders to the information.

2. Price Efficiency

The efficiency of the prices to reveal information is again defined by the posterior precision of a random variable. Making use of the conditional variance, we have the following results about the posterior precision.
Proposition 5

Let $\Gamma(\overline{W} | \overline{P}_i)$ and $\Gamma(\overline{S}_i | \overline{P}_i)$ denote the posterior precision of stock prices to reflect market wide and firm specific information. In equilibrium,

\[
\Gamma(\overline{W} | \overline{P}_i) = (1 + \frac{1}{1+\delta_w} N_w) \Gamma_w, \quad \delta_w = (\kappa_w/\eta_w)(\sigma_w^2/\sigma_w^2)
\]

\[
\Gamma(\overline{S}_i | \overline{P}_i) = (1 + \frac{1}{1+\delta_s} N_s) \Gamma_s, \quad \delta_s = (\kappa_s/\eta_s)(\sigma_s^2/\sigma_s^2)
\]  

where $\kappa_s = N_s/(N_s + 1)$, $\kappa_w = N_w/(N_w + 1)$,

\[
\eta_s = N_s/(N_s + 1)^2, \quad \eta_w = N_w/(N_w + 1)^2
\]

Proof. See Appendix.

Equation (36) and (37) are general cases of equation (14) and (15). $\delta_w$ and $\delta_s$ are discount factors for the contribution of informed traders to the posterior precision. The lower the discount factors, the more an informed trader can contribute to the posterior precision. When there is only one source of uncertainty, the discount factors are reduced to zero. If $\overline{W}$ is the only source of uncertainty, $\sigma_s^2$ becomes zero, and $\Gamma(\overline{W} | \overline{P}_i) = (1 + N_w) \Gamma_w$. Thus each market wide informed trader contributes $\Gamma_w$ to the posterior precision. Similarly, if $\overline{S}_i$ is the only source of uncertainty, $\sigma_s^2$ becomes zero, and $\Gamma(\overline{S}_i | \overline{P}_i) = (1 + N_s) \Gamma_s$, so that each firm specific informed trader contributes $\Gamma_s$ to the posterior precision.

Note that the discount factors depend not only on $(\sigma_w^2/\sigma_s^2)$, but also on $N_w$ and $N_s$. First, $\delta_w$ increases with $N_w$ and $\delta_s$ increases with $N_s$, suggesting that when there are more informed traders competing for the same information, each of them will trade less aggressively,
leading to a larger discount factor for that information. However, since the number of informed traders is much higher, the posterior precision of prices to reflect one piece of information still increases with the number of traders possessing that information.

Second, \( \delta_w \) decreases with \( N_s \) and \( \delta_g \) decreases with \( N_w' \). Therefore, the posterior precision of prices to reflect market wide (firm specific) information decreases with the number of firm specific (market wide) informed traders.

The efficiency of cash index and futures prices is not discussed, as they have the same implications. Therefore, if there are more market wide informed traders but fewer stock specific informed traders in the futures market, as we will show later, this implies another relation between the efficiency of cash index and futures prices. Relative to cash index prices, there is an increase in the efficiency of futures prices to reflect market wide information but a decrease in the efficiency to reflect firm specific information.

3. Endogenous Information Acquisition

This subsection considers how market competition determines the number of informed traders. We assume that the information is acquired at some costs and traders choose to be informed only if it is profitable. Therefore, the number of informed traders becomes endogenous as traders will acquire the information when the expected profits exceed the costs.

In the cash market, expected profits of traders \( W \) and \( S_1 \) are,
We examine the effects of the number of informed traders on the expected profits. Not surprisingly, expected profits decrease when there are more informed traders who share the same information. On the other hand, since the number of informed traders affects the market depth, which in turn affects expected profits of the traders, it is found that the number of the first type of informed traders also affects expected profits of the second type of informed traders. Specifically, when there are more market wide (firm specific) informed traders, expected profits of stock specific (market wide) informed traders are also higher.

In the futures market, expected profits of informed traders \( W^f \) and \( S^f_i \) are,

\[
\pi^f_w = \frac{2 \sigma_w^2 \sigma_z^2}{2(\eta_w \sigma_w^2 + \eta_s \sigma_s^2)^{1/2}(N_w + 1)^2}
\]

\[
\pi^f_{s,i} = \frac{\sigma_s^2 \sigma_z^2}{2(\eta_w \sigma_w^2 + \eta_s \sigma_s^2)^{1/2}(N_s + 1)^2}
\]

where \( \eta_s = N_s/(N_s + 1)^2 \), \( \eta_w = N_w/(N_w + 1)^2 \)

Suppose the cost of acquiring information about \( W \) is \( C_w \), and the cost of acquiring information about \( S_i \) is \( C_s \). In equilibrium, the
number of informed traders should be such that each trader can only earn enough profit to cover the cost of acquiring the information. There is thus a relation between the number of informed traders in the two markets.

**Proposition 6**

Let $N_w$ and $N_s$ represent the number of market wide informed traders and firm specific informed traders (in each stock) in the cash market, and $N^f_w$ and $N^f_s$ represent the number of informed traders in the futures market. If the cost of acquiring either market wide or firm specific information is the same in both markets, then in equilibrium such that each informed trader can only cover the information acquisition cost, $N_w < N^f_w$ and $N_s > N^f_s$.

**Proof.** See Appendix.

This result should not be surprising. When the number of informed traders is the same, we show that a market wide informed trader earns a larger profit in the futures market than in the cash market, and vice versa for a firm specific informed trader. Therefore, if information acquisition is endogenous, this will attract more market wide informed traders into the futures market and more firm specific informed traders into the cash market.

This result will remain unchanged even when the expected profits of stock specific informed traders in the futures market are too low and cannot cover the information acquisition cost. In that case, no one will acquire firm specific information so that the market maker is
aware that there is no order submitted by firm specific informed traders in the futures market. Therefore, in equation (40), $\sigma_s^2$ becomes zero and $\pi^f_w$ is larger than before. This will attract more market wide informed traders to the futures market and make the result stronger.

The conclusion may change if we change our assumption about the cost of information acquisition. Suppose there is an economies of scale in information acquisition, such that when there are more traders acquiring one type of information, it will lower the cost of acquiring either types of information. In that case, a higher number of market wide informed traders will lower both $C_w$ and $C_s$ in the futures market, and a higher number of firm specific informed traders also lower $C_w$ and $C_s$ in the cash market. Therefore, the relation between $N_w$ and $N^f_w$, $N_s$ and $N^f_s$, may become ambiguous and depend on the exact cost function.
D. MULTIPERIOD MODEL

1. Price Behavior

This section extends the model to more than two periods. The stocks and futures contracts are now traded over a span of $T$ periods. It is assumed that the payoffs of the stocks in period $T$ are exogenously given by

$$\bar{V}_i = \bar{v} + \sum_{t=1}^{T}(\bar{w}_t + \bar{s}_it)$$

(42)

Informed traders $W$ (or $W^f$) and $S_i$ (or $S_i^f$) observe $\bar{w}_{t+1}$ and $\bar{s}_{i\tau+1}$, respectively, at period $t$. At period $t+1$, these become public information, so the information the informed traders have is only useful for one period. At each period, there is the noise trading $\bar{z}_{it}$ in stock trading post $i$ and $\bar{z}_{it}^f$ in the futures pit. $\bar{w}_t$, $\bar{s}_i$, $\bar{z}_it$ (or $\bar{z}_{it}^f$), $i=1,2,...,m$, are normally and independently distributed, and serially uncorrelated with means zero and variances $\sigma_w^2$, $\sigma_s^2$, $\sigma_z^2$.

We can derive the equilibria as in the two period model. In equilibrium, the price functions are represented as follows:

\[
\begin{align*}
\bar{P}_{i,t} &= \bar{v} + \sum_{\tau=1}^{T}(\bar{w}_\tau + \bar{s}_{i\tau}) + \kappa_w \bar{w}_{t+1} + \kappa_s \bar{s}_{i\tau+1} + \lambda \bar{z}_{it}, \\
& \quad i=1,2,...,m \\
\bar{Y}_t &= m\bar{v} + \sum_{i=1}^{m}\sum_{\tau=1}^{T}(\bar{w}_\tau + \bar{s}_{i\tau}) + \kappa_w m\bar{w}_{t+1} + \kappa_s \sum_{i=1}^{m} \bar{s}_{i\tau+1} + \\
& \quad \lambda \sum_{i=1}^{m} \bar{z}_{i\tau+1},
\end{align*}
\]

(43)
The price functions are very similar to those in the two period model, except that now the public information includes the private information that has been released prior to that period. Without proof, it should be noted that all the results we discuss in the two period framework will apply here. Also, under this model, no arbitrage activities are profitable. Before the trading at period $t$, for a pure arbitrager who does not observe $\tilde{W}_{t+1}$ and $\tilde{S}_{it+1}$, the expected value of the basis $(\tilde{I}_t - \tilde{F}_t)$ is zero. The arbitrager cannot submit orders at period $t$ and take offsetting positions in the two markets to exploit the basis difference. The basis difference developed at period $t$ will not persist at period $t+1$, as $\tilde{W}_{t+1}$ and $\tilde{S}_{it+1}$ become public information.

2. Time Series Properties

From the price functions generated in the previous subsection, we can derive a number of results about the time series properties regarding cash index and futures prices. The price changes in the individual stocks, cash index, and futures are first generated, with

$$\Delta \tilde{P}_{it} = \tilde{P}_{it} - \tilde{P}_{it-1}, \Delta \tilde{I}_t = \tilde{I}_t - \tilde{I}_{t-1}, \Delta \tilde{F}_t = \tilde{F}_t - \tilde{F}_{t-1},$$

$$\Delta \tilde{P}_{it} = (1-\kappa_w)\tilde{W}_t + (1-\kappa_s)\tilde{S}_{it} + \kappa_w \tilde{W}_{t+1} + \kappa_s \tilde{S}_{it+1} +$$

$$\lambda (\tilde{Z}_{it} - \tilde{Z}_{it+1}) \quad i=1,2\ldots m \quad (46)$$
From these, variances of price changes can be computed,
\[
\text{Var}(\Delta I_t) = (1-\kappa_s)\sum_{i=1}^m \bar{S}_i + \kappa_s \sum_{i=1}^m \bar{S}_i + 1 + \kappa \sum_{i=1}^m \bar{S}_i \quad (49)
\]
\[
\text{Var}(\Delta F_t) = (1-\kappa_f)\sum_{i=1}^m \bar{S}_i + \kappa_f \sum_{i=1}^m \bar{S}_i + 1 + \kappa \sum_{i=1}^m \bar{S}_i \quad (50)
\]

The variances of the price changes of individual stocks and futures are therefore equal to the variances of the information, and independent of the noise trading. This is because the increase in variance of price changes due to noise trading exactly offsets the decrease in variance of price changes due to partial adjustment to the information. On the other hand, \text{Var}(\Delta I_t) is not equal to \text{Var}(\Delta F_t).

We now compute the autocovariances of the price changes,
\[
\text{Cov}(\Delta P_{t-1}, \Delta P_t) = 0 \quad (52) \\
\text{Cov}(\Delta F_{t-1}, \Delta F_t) = 0 \quad (53) \\
\text{Cov}(\Delta I_{t-1}, \Delta I_t) = m(m-1)\eta_w \sigma_w^2 \quad (54)
\]

It is noted that price changes of individual stock prices and futures prices are serially uncorrelated. This is because price changes due to partial adjustment to information are positively autocorrelated, while price changes due to noise trading are negatively autocorrelated. In our model, the two effects exactly offset each other. As for cash index, because of the uncorrelated noise trading, the negative serial correlation due to noise trading is smaller than the positive serial correlation due to partial adjustment to information. Therefore, cash index price changes are positively autocorrelated.\(^{11}\)

To conclude, the results here show that (1) the variance of futures price changes is larger than the variance of cash index price changes; (2) futures price changes are less autocorrelated than cash index price changes; and (3) cash index price changes are positively autocorrelated even if the individual stock price changes are uncorrelated. This is consistent with the evidence in previous studies (See Mackinlay and Ramaswamy [1988], Lo and Mackinlay [1988] and Amihud and Mendelson [1989]). In those papers, various reasons are proposed to explain the evidence, such as nonsynchronous trading of the stocks, specialist smoothing and large transaction costs in the cash market. This subsection thus adds one more explanation to the
list. We show that even without any kind of friction, the two markets would have different behavior in a rational expectation framework.

3. Lead-Lag Relationship

The lead-lag relationship is examined by the cross correlation between the two time series. The correlations between futures price changes and lead and lag of cash index price changes are equivalently compared by their cross covariance terms,

\[
\text{Cov}(\Delta F_t, \Delta I_{t+1}) = m^2 \kappa_w (1-\kappa_w) \sigma_w^2 + m \kappa_s (1-\kappa_s) \sigma_s^2
\]  \tag{55}

\[
\text{Cov}(\Delta F_t, \Delta I_{t-1}) = m^2 \kappa_w (1-\kappa_w) \sigma_w^2 + m \kappa_s (1-\kappa_s) \sigma_s^2
\]  \tag{56}

The covariance terms thus involve two terms: the first term is due to covariance of market wide information and the second term is due to covariance of firm specific information. From proposition 6, since \( N^f_w > N_w \) and \( N^f_s < N_s \), we have

\[
m^2 \kappa_w (1-\kappa_w) \sigma_w^2 > m^2 \kappa_w (1-\kappa_w) \sigma_w^2
\]  \tag{57}

\[
m \kappa_s (1-\kappa_s) \sigma_s^2 < m \kappa_s (1-\kappa_s) \sigma_s^2
\]  \tag{58}

Without knowing \( \sigma_w^2 \) and \( \sigma_s^2 \), it is difficult to tell the relation between the two covariance terms. If market wide information dominates, futures price changes lead cash index price changes more than they lag. If firm specific information dominates, we observe the reverse. Generally, since market wide information is systematic, when the number of stocks is large, market wide information dominates and futures price changes lead cash index price changes more.
E. CONCLUSION

This chapter examines the difference between cash index prices and futures prices, which both reflect values of underlying stocks. One distinction lies in the fact that individual stocks are traded in the cash market, while futures contracts (portfolios of stocks) are traded in the futures market. Therefore, while stock values are priced first and then aggregated in the cash market, they are aggregated first and then priced in the futures market. This distinction will be important when there is correlated information among securities. This is because market makers in the futures market take the correlated information into account when pricing the futures, but those in the cash market would not take them into account when pricing the stocks because their portfolios are just individual stocks.

This chapter shows that futures prices will more volatile than cash index prices, so that futures prices are less efficient than cash index prices in reflecting information. When the model is extended to more than two periods, there are two implications. First, the variance of futures price changes is larger than that of cash index price changes. Second, even though price changes of individual stocks and futures are serially uncorrelated, cash index price changes are positively autocorrelated. We also demonstrate that a market wide informed trader earns a higher profit by trading futures contracts than by trading a portfolio of stocks, and that a firm specific informed trader earns a larger profit by trading the individual stock than by trading futures contracts. Thus, there will be a larger incentive to collect market wide information but a lower incentive to
collect firm specific information in the futures market than in the cash market. Therefore, futures prices may react faster to market wide information than to cash index prices.

Some previous papers, for example, Mackinley and Ramaswamy [1988] and Amihud and Mendelson [1989] propose some frictions in the market to account for the differences between futures and cash index prices. It is suggested that nonsynchronous trading among the individual stocks will cause the cash index price changes to be positively autocorrelated even when the futures price changes are not autocorrelated. Further, the smoothing of prices by the specialists will reduce the volatility of cash index price changes relative to that of futures price changes. In this chapter, we demonstrate that even in the absence of these frictions, cash index and futures prices may still exhibit different behaviors under a rational expectation framework. Further studies should identify empirically how much the different behavior between the two markets can be explained by the frictions. This can provide an indirect test of the hypotheses of this chapter.

It should be reminded that the difference between index futures prices and cash index prices is a result of the rational behavior of market makers under asymmetric information in the two markets. For any other basket securities or index funds, their price behaviors are also expected to be different from cash index prices. Examination of their price behaviors can help to verify the hypotheses.
ENDNOTES

1. We assume that the interest rate and dividend yield are constant and thus we do not have to consider their behaviors in affecting the futures prices.

2. The discrepancies still occur after they have adjusted for the interest cost and dividend yield.

3. Some other papers consider the difference of the other trading practices in the cash and futures market. Fishman and Longstaff [1989] show that the dual trading practice in the futures market makes futures prices incorporate information more rapidly than stock prices. George, Kaul and Nimalendran [1990] show that while asymmetric information and divergent beliefs are important determinants of trading volume in a Walrasian market (e.g. futures market), the expected liquidity trading is the major component of trading volume in a specialist market (e.g. NYSE).

4. The deviation of futures prices from cash index prices is short-lived. Given sufficient time and low transaction cost, the arbitrage activities will bring the two into alignment.

5. Subrahmanyam [1989] argues that because of the transaction costs, the futures and cash market will not have equal access to market wide and firm specific information. This paper thus reinforces the argument. In the model, we endogenize the decision to acquire market wide or firm specific information because of the different expected profits earned by the informed traders in the two markets. We show that even without transaction costs or other market frictions, there are still different incentives to acquire the information in the two markets.

6. In the previous literature, e.g. Grossman [1976], traders are assumed to have diverse information, such that each trader observes the realization of the random variable plus a noise term independent of the other. For example, the jth trader observes \( \tilde{v}_{ij} = \tilde{v}_i + \tilde{e}_{ij} \), where \( \tilde{e}_{ij} \)'s are independently and identically distributed. Therefore, although the signals are diverse, they are still driven by a common factor \( \tilde{v}_i \), so that the diverse information of each trader is not independent of each other.

7. This assumption is not critical for the derivation of our results. However, if market makers can observe the net order flows in other trading posts/pits, they will set the prices based also on the net order flows in other markets. Nevertheless, this would not change the implications.
8. However, if we aggregate the order at \( m \) stock trading posts, the total order that market wide informed trader \( \hat{W} \) submits is a function of \( m\hat{W} \).

9. Since \( \hat{W} \) and \( \hat{P}_i \) are jointly normal, \( \text{Var}(\hat{W}|\hat{P}_i) = \text{Var}(\hat{W}) - \frac{\text{Cov}(\hat{W},\hat{P}_i)^2}{\text{Var}(\hat{P}_i)} \). \( \text{Var}(\hat{S}_i|\hat{P}_i) \) is calculated similarly.

10. It should be remembered that futures prices are more volatile because there is information correlated among securities. If there is no correlated information (market wide information), behavior of market makers in the cash and futures market will be the same, and futures and cash index prices are equally efficient in reflecting information.

11. It can be shown that \( \text{Cov}(\Delta\hat{I}_{t-1}, \Delta\hat{I}_t) = 0.5[\text{Var}(\Delta\hat{F}_t) - \text{Var}(\Delta\hat{F}_t)] \). Therefore, \( \rho(\Delta\hat{I}_{t-1}, \Delta\hat{I}_t) = 0.5([\text{Var}(\Delta\hat{F}_t)/\text{Var}(\Delta\hat{I}_t)] - 1) \).
CHAPTER III

A FURTHER ANALYSIS OF THE LEAD-LAG RELATIONSHIP BETWEEN THE CASH MARKET AND STOCK INDEX FUTURES MARKET

A. INTRODUCTION

The lead-lag relation between price movements in the stock index futures market and underlying cash market has always been of interest to academics, investors, and regulators. The lead-lag relation is of interest because it illustrates how fast one market reflects new information relative to the other, and how well the two markets are linked. The lead-lag relation has recently attracted much new attention when volatility in these markets increases, as there is concern that uninformed speculative trade in the futures market is fueling volatile price swings in both markets.

Several studies examine the temporal relationship between price changes of the cash index and futures. Ng [1987] investigates the interday price behavior of the S&P 500 index futures and finds that it can predict spot price levels. Finnerty and Park [1987], Kawaller, Koch and Koch [1987], and Stoll and Whaley [1988] consider the lead-lag relation between intraday cash index and futures prices and find that futures price changes lead cash index price changes.¹ Harris

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[1989a] adjusts for nonsynchronous trading in the S&P 500 stocks and observes that the cash market is led by the futures market in the ten-day period surrounding the stock market crash in October 1987. As there is extensive evidence that futures lead cash index and not vice versa, it is widely accepted that information is disseminated in the futures market faster than in the cash market.

The problem associated with the study of the S&P 500 index is that the trading of some component stocks is not heavy enough to allow the prices to update information on a minute to minute basis. When the lead-lag relation is analyzed based on intraday price changes, the staleness of component stock prices can cause futures prices to appear to lead cash index values. Thus, the evidence that futures lead cash index may be simply due to infrequent trading of component stocks, not to delay in the adjustment of cash index prices. Although Harris [1989] adjusts for the infrequent trading, his result just pertains to the ten days surrounding the market crash period.

In this chapter, we analyze the intraday relation between cash and futures price changes of the Major Market Index (MMI). There are two interesting features about the MMI. First, on average, those stocks underlying the MMI are more actively traded than S&P 500 stocks. If the prices of these MMI stocks can update information fast enough, it is possible that cash index would not lag behind futures. Even though there might still be lead-lag relation due to nonsynchronous trading of futures and component stocks, we do not expect futures prices to lead cash index predominately. Second, while S&P 500 index contains 500 stocks, there are only twenty stocks underlying MMI. Index arbitrage is therefore relatively cheap, easy,
and fast for MMI due to the small number of component stocks. The ease of index arbitrage can eliminate any discrepancies between the prices in two markets instantaneously. In this case, we will not observe the lead-lag behavior if we analyze the data over a longer time interval basis. In brief, the study of MMI cash and futures prices allows us to examine the relation between the two markets when those frictions (infrequent trading, transaction costs) that favor lead-lag behavior are much reduced.

This chapter also investigates whether the lead-lag patterns are affected by institutional factors. It is claimed that the futures market reacts to new information faster than the cash market, due to the absence of short sales constraints and lower transaction costs in trading futures contracts, but these have not yet been examined. This chapter will provide empirical evidence as to how these institutional factors influence lead-lag patterns. Specifically, we stratify the observations and examine whether the lead-lag patterns change in accordance with: i) bad news versus good news; ii) the intensity of trading activity in the two markets; and iii) the extent of market wide movement. The results will shed some light on the determinant(s) of the lead-lag relation.

The results of this chapter show that while past futures price changes are correlated with cash index price changes, past cash index price changes are also correlated with futures price changes. The lead-lag pattern can hardly be explained completely by nonsynchronous trading. First, the feedback from the futures market to the cash
market seems to be larger than the reverse. This is not related to the fact that futures are traded more frequently than some less active component stocks underlying the index, as the relation also holds between futures prices and the most heavily traded stocks. Further, for those stocks which trade at almost every five-minute interval, they can still be led by the futures for more than two intervals (ten minutes). More interestingly, the evidence indicates that when there are more stocks moving together (market wide information), futures prices lead cash index more, while the feedback from the cash market into the futures market remains the same. This suggests that the futures market is able to update market wide information faster than the cash market.

Chapter III is organized as follows. Section B discusses the relationship between cash index and futures prices and the influences of some institutional factors on the lead-lag relation. Section C describes the data and preliminary statistics. Section D presents empirical results. This is followed by conclusions in Section E.
B. THE LEAD-LAG RELATIONSHIP BETWEEN THE TWO MARKETS

Both cash index and futures prices reflect aggregate values of the underlying stocks. Since the futures prices have to be adjusted by the cost of carrying forward, cash index and futures prices would not be the same. But if interest rates and dividend yields are nonstochastic, contemporaneous price changes in the two markets should be perfectly correlated, and there would be no lead-lag relation between them. It is only when one market reacts faster to information than the other market that the lead-lag relation will be observed. Nevertheless, if there is not a significant lag in the transmission of information between the two markets, the lead-lag relation should not persist for too long. In the extreme case, if one market reacts instantaneously to the price movement in the other market, the lead-lag relation will not be observed.

The existence of a lead-lag relation does not necessarily imply that the price movement in one market provides information for the other market. The discrepancies between the prices in the two markets would bring in index arbitrage, which induces corresponding price movement in the other market. Suppose futures prices react faster to information and deviate too much from the cash index level, then there will be arbitrage opportunities, so that arbitrageurs come into the two markets, buying the cheaper one and selling the more expensive. Therefore, even though the traders in the cash market do not react to price movement in the futures market, index arbitrage causes cash index prices to follow the past price movement in the futures market.
There are several institutional factors which can influence how fast the two markets will reflect information, and thus affect the lead-lag relation. The first one is the short sale constraints in the cash market. Examples of short sale constraints include legal or contractual prohibitions of shorting by certain institutional investors and corporate insiders, the inability to borrow stock to short, and the 'no short sale on a down-tick' rule, which prohibits short sales at prices below the last price. Diamond and Verrecchia [1987] examine the effects of short-sale constraints on the speed of adjustment of security prices, showing that prohibiting traders from shorting reduces the adjustment speed of prices to private information, especially with respect to private bad news. On the other hand, there is no short sale constraint in the futures market, so futures prices are symmetrical in reflecting private good news and bad news. Therefore, the lead-lag relation would not be the same under the bearish and bullish market; futures prices lead cash index more under bad news.

The lead-lag relation can also be affected by the intensity of trading activity in the two markets. Admati and Pfleiderer [1988] model a market with four types of traders: informed investors, random liquidity traders, discretionary liquidity traders, and a specialist. They show that in general, trades of both discretionary liquidity traders and informed traders will cluster together, with each group preferring to trade when the market is thick. This clustering of trades causes more information to be released when the level of trading activity is higher. Therefore, the lead-lag relation may vary
with the relative intensity of trading activity in the two markets.\footnote{5} Stephan and Whaley [1990] study the intraday relation in the stock and stock option market. They find that it is not only price changes in the stock market that lead price changes in the option market, but that the trading activity (proxied by the number of transactions and trading volume) in the two markets also bears the same kind of lead-lag relation. This provides evidence that price discovery and trading activity are related.

Lead-lag patterns may also depend on whether the information is market wide or not. Subrahmanyam [1990a] suggests that some informed traders are precluded from the cash and futures markets due to fixed costs of trading or budget constraints. For instance, market wide informed traders do not trade in individual securities because such trading would require a larger capital outlay than trading in the futures contracts. On the other hand, security-specific informed traders do not trade on the basket (futures contracts) because the security-specific information is trivial in determining the futures prices. Further, Chapter II shows that the profits of informed traders depend on the amount of information they have relative to other informed traders. Because of the diversification effect of firm specific information, a market wide informed trader earns a higher profit by trading in futures contracts than by trading in individual securities. Therefore, if information acquisition is endogenous, there is a higher incentive to collect market wide information in the futures market than in the cash market. Overall, the above discussion suggests that the futures market should be better in
reflecting market wide information than the cash market. When there are more stocks moving together due to market wide information, futures prices should lead cash index more.
C. DATA AND PRELIMINARY STATISTICS

1. Data

The Major Market Index (MMI) is a price-weighted index based on 20 widely held and highly capitalized blue-chip stocks traded on the New York Stock Exchange. MMI futures contracts are traded at the Chicago Board of Trade (CBOT). Our study period is from August 1984 to July 1986. On every trading day, only the nearby futures contracts are examined because it is more actively traded.

Two sets of data are used to investigate the lead-lag relationship between the cash index and futures prices. The first, the MMI futures price data, obtained from the CBOT, consists of transaction price data recorded whenever the transaction price differs from the previous one. The second, the Fitch data, obtained from Francis Emory Fitch Inc., consists of a time-ordered record of every transaction of twenty component stocks of the MMI. For each transaction, the date, time, price and number of shares traded are available. It is from these Fitch transaction data that we compute the MMI levels.

In addition to this data, we also analyze the lead-lag relation using the cash index which is reported by the CBOT. Potentially, the reported cash index may lag behind the transaction prices. We therefore want to see whether the reporting bias affects the lead-lag pattern significantly.

To examine the intraday lead-lag relation between the cash index and futures prices, five-minute price changes of the cash index and
futures are first generated. Every day, trading hours are partitioned into five-minute intervals. In each interval, the last price observation for the futures and component stocks is identified. If no price is observed within that five-minute span, the last price in the previous five minutes is recorded for this interval. The cash index is calculated by aggregating prices of twenty stocks and adjusting aggregate prices by a divisor.\textsuperscript{9}

Some observations are excluded from the sample. First, in the sample period, component stocks are traded in the NYSE between 9.00 am and 3.00 pm (CST), while MMI futures contracts are traded in the CBOT between 8.45 am and 3.15 pm (CST). Since no price observation is available for the component stocks when the stock market is closed, we confine our study to the trading hours in the NYSE. Second, since some stocks have delayed openings, we wait until all the stocks have started trading. On average, it takes about 25 minutes to have all stocks begin trading. Therefore, roughly speaking, observations in the first half an hour are excluded from the sample. Third, we also exclude overnight returns as they span a time interval different from the other intraday five-minute returns.

2. Preliminary Statistics

Table 1 reports the trading frequency and non-trading probability for futures contracts and component stocks. Trading frequency is the average number of trades in the five-minute interval. Non-trading probability is the proportion of the intervals in which there is no trade. Since the futures price dataset records only the trade whenever the price is different from the previous one, there is
a downward bias in the trading frequency and an upward bias in the non-trading probability. Before July 1985, there is an average of 5 trades (that record price changes) and 6% probability that there will be no trade (that records price changes) in the five-minute interval. After that, there is a decrease in futures trading volume, with less than two trades every five minutes and 32% non-trading probability. Thus, infrequent trading of the futures may distort the lead-lag relationship in the second half of the period. For the component stocks, the most actively traded one is IBM, which has 7.83 trades and 0.5% nontrading probability in one five-minute interval. This is followed by AT&T, Exxon, and Sears. Although the component stocks are blue chip companies, not all of them are traded every five minutes. Half of the component stocks have more than 10% probability that there will be no trade in the five-minute interval. Therefore, it is possible that nonsynchronous trading of component stocks and futures can account for the lead-lag relation.

Table 2 presents the autocorrelation structures of five-minute price changes of the MMI cash index and futures. The first order autocorrelation of the MMI futures is small (0.037). In contrast, the first and second order autocorrelation of cash index price changes are quite high (0.295 and 0.115). This can be explained by the nonsynchronous trading of the underlying stocks. It also suggests that past price changes of the cash index can be used to explain a systematic portion of current price changes that might be due to the lag in price adjustment of some less active stocks. Later, we will see whether the cash index still lags behind futures when we filter a
systematic portion of cash index price changes that can be explained by the past price changes of cash index.

Table 3 shows the cross correlation between price changes of cash index and futures. This gives us a preliminary look of the lead-lag relation between the two markets and helps us to determine the number of leads and lags we use in the regressions analysis in Table 4. In the first period, the contemporaneous correlation is 0.506, suggesting that the two time series are not perfectly correlated. The cross correlations between price changes of cash index and leads and lags of futures are quite high. The correlation coefficients at the first three lags are 0.4108, 0.2022, and 0.0761, respectively. The correlation coefficient is also quite high at the first lead (0.1583), but becomes much smaller after that. In the second subperiod, when the futures are less frequently traded, futures prices become stale so that futures price changes are more correlated with past cash index price changes than before.
D. EMPIRICAL RESULTS

1. Lead-lag relationship

The lead-lag behavior between the cash and futures market is examined by the estimation of the following regression,

$$\Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t}$$

where $\Delta S_t$ and $\Delta F_t$ are five minute cash index and futures price changes at time $t$. The coefficients with negative subscripts ($b_{-3}, b_{-2}, b_{-1}$) are lag coefficients, and those with positive subscripts ($b_1, b_2, b_3$) are lead coefficients. If the lag coefficients are significant, cash index prices lag futures. If the lead coefficients are significant, cash index prices lead futures. The choice of three leads and lags is based on Table 3 which shows that cross coefficients of longer leads and lags are small. Since three leads and lags are required for each observation, we lose six more observations every day. As cash index price changes are serially autocorrelated, we estimate the regression with a two pass (Yule-Walker) procedure to correct for the first order autocorrelation in the residuals.

Regression results are presented in Table 4(a). Model 1 estimates regression by using the raw cash index price changes. In the first subperiod, the contemporaneous coefficient $b_0$ is 0.380 and is the largest among all coefficients, suggesting that the two markets are reacting to much of the information simultaneously. The estimates of lag coefficients ($\hat{b}_{-3}, \hat{b}_{-2}, \hat{b}_{-1}$) are respectively 0.070, 0.143, 0.294, and are significant at the 0.1% level. This suggests that futures lead cash index prices. On the other hand, the significance of the
first lead coefficient $\hat{b}_1$ (the estimate is 0.106) suggests that cash index also lead futures. Evidence indicates that in the second period, when the futures contracts are less frequently traded, futures prices become stale so that lag coefficients are smaller and lead coefficients are larger. Since the lag coefficients can be biased upward due to nonsynchronous trading of component stocks and futures, we estimate the lead-lag relation with a two stage procedure in Model 2. In the first stage, we fit an AR(1) model for cash index price changes and extract the residuals. In the second stage, we estimate regression using the extracted residuals. The two stage procedure filters out some systematic portion of cash index price changes that may be due to nonsynchronous trading of component stocks before we estimate the lead-lag relation.\(^{12}\) This allows us to infer the lead-lag behavior when the bias due to nonsynchronous trading of stocks and futures is reduced. Not surprisingly, the three lag coefficients $(\hat{b}_{-3}, \hat{b}_{-2}, \hat{b}_{-1})$ are now reduced to 0.022, 0.052, and 0.176. Nevertheless, the economic significance of $\hat{b}_{-1}$ indicates that futures prices still lead cash index. Overall, the result contrasts with those of previous studies which find that futures prices lead cash index values predominately. Instead, it is found that while past futures price changes are correlated with cash index price changes, past cash index price changes are also correlated with futures price changes.

Table 4(b) reports the estimates of regression models using the MMI reported by the CBOT. It can be seen that the lead-lag relation using reported MMI is different from the previous one, whether we use
the raw cash price changes or residual price changes. In both regressions, the lag coefficients are larger and the lead coefficients are smaller when we use the reported index than the coefficients when we use the transaction prices data. Therefore, the reporting bias seems to be nontrivial, and the use of the reported index may affect the lead-lag relation.

Since there is a bias associated with the reported index, we use only the transaction price data in our following tests. Also, because futures prices seem to become stale in the second subperiod, our tests are based on the data in the first subperiod.

We also examine the lead-lag relation between price changes of the futures and individual stocks. This is to verify whether it is the infrequent trading of some component stocks that causes futures prices to lead cash index. If true, we should observe futures prices leading only those infrequently traded stocks, not those heavily traded stocks. Table 5 shows the regression results where the dependent variables now are price changes of individual stocks. Since the price changes of individual stocks and futures are of different scales, they are first standardized by their variances. It is noted that the $R^2$ of each regression varies widely across stocks, ranging from 0.37 for IBM to 0.02 for AT&T. This indicates that the association between each stock and the futures market differs from stock to stock. It does not appear that futures prices lead only the prices of less actively traded stocks. For the component stocks like IBM, AT&T, and Exxon, which are actively traded, the lag coefficients are also economically significant. Further, the lag coefficients are much larger than the
lead coefficients. For example, for IBM, the estimate of $b_{-1}$ is almost two times as large as that of $b_1$ (0.216 vs. 0.127). Since the trading activity of IBM doesn't seem to be lower than that of MMI futures (in terms of the number of trades), it is therefore beyond what nonsynchronous trading can explain. More interestingly, those heavily traded stocks (like AT&T and Exxon) which trade almost every five minutes, are also led by the futures up to two intervals (ten minutes). Overall, it seems that the futures market is fast in updating information, but the cash market is slow in reacting to information disseminated in the price changes in the futures market.

2. Relation under Bad News and Good News

To examine whether short sale constraints affect the lead-lag relation, the observations are sorted by the magnitude of cash index price changes. We partition the trading hours into 30-minute intervals (i.e. each interval contains 6 observations) and stratify observations into 5 quintiles based on the cash index price changes within 30 minutes, and estimate the regression for 5 quintiles. We choose the 30-minute interval because it is short enough so that there are not many bits of information affecting the lead-lag relation differently, and long enough to allow the information effect to have an impact on the lead-lag relation of some observations in the interval.13

Table 6 reports the regression results for five quintiles. Quintile 1 is the bad news group, containing observations of the smallest price changes. Quintile 5 is the good news group, containing
observations of the largest price changes. Quite interestingly, there is a U-shaped pattern for $R^2$ to move across the quintiles, so that the association between two markets is the largest when returns are abnormally negative (quintile 1) or positive (quintile 5). However, there is no evidence to suggest that short sale constraints will affect lead-lag relation. First, it does not seem that cash index prices lag futures prices only under bad news. Instead, in quintile 5, the economic significance of the first two lag coefficients indicates that futures prices also lead cash index prices also under good news. Second, cash index prices do not lag futures prices more under bad news. In fact, the lag coefficients $\hat{b}_{-1}$ and $\hat{b}_{-2}$ decrease from 0.328 and 0.131 in quintile 5 (good news) to 0.276 and 0.113 in quintile 1 (bad news).

3. Relation under different intensity of trading activity

Since we do not have the intraday futures trading volume, we measure the intensity of trading activity by the number of trades in the cash and futures market, respectively. It should be remembered, however, that since the trades in the futures market are recorded only when prices are different from previous ones, the recorded number of trades is not the true number of trades. But, if buy and sell orders are random over time, the recorded number of trades can reflect the level of trading activity.

Based on the number of trades in the two markets, we form subgroups that allow the level of trading activity in one market to vary, while keeping the level in the other market constant.
Observations are first sorted into three groups based on the number of trades per stock in the corresponding 30-minute interval. Within each group, observations are further sorted into three subgroups based on the recorded number of trades in the futures market. Regressions are then estimated for each subgroup, and reported in Table 7. The average trading frequency is calculated for each subgroup, and indicates that the levels of trading activity in two markets are correlated. This is especially true for the last three subgroups; the sorting procedure is not quite successful in controlling the level of trading activity in the cash market when we allow the number of trades in the futures market to increase.

One interesting pattern we observe is that when we control the level of cash market trading activity, a rise in the level of futures trading activity increases the $R^2$ for the regression, or the association between the two markets. This indicates more common information is generated when the level of trading activity is higher. However, whether this will result in the futures market reacting to information faster relative to the cash market seems to be uncertain. For the first 3 subgroups ([1,1], [1,2], [1,3]), $b_{-1}$ increases and $b_1$ decreases when the level of futures trading activity is higher. However, this can be just because futures prices are less stale than before. In the last 3 subgroups ([3,1], [3,2], [3,3]), when the trading activity in the futures markets is generally high, a further increase in the trading activity does not induce significant changes in $\hat{b}_{-1}$ and $\hat{b}_1$. 
4. Relation under market wide movement

It is hypothesized that futures prices lead cash index more when there is market wide information that affects component stocks moving together in prices. This is because traders find that it is cheaper to trade in futures contracts than to trade in a portfolio of stocks. Therefore, the degree to which market wide information will affect the lead-lag relation can be measured by the relative advantage of trading in the futures market as opposed to the cash market.

Suppose a trader has private information about each component stock, such that he knows that the price change of stock i is $\Delta S_i$. He can exploit his information by taking position in one unit of a futures contract, or one share of each stock in the cash market. The gross profits he earns (before transaction costs) are therefore $|\sum_{i=1}^{20} \Delta S_i|$ if he exploits the information in the futures market, and $\sum_{i=1}^{20} |\Delta S_i|$ if he exploits the information in the cash market. Assume also that the transaction cost is fixed and is the same for trading in a futures contract or a component stock. Since transaction costs are fixed, the relative advantage of trading in futures contracts can be measured by the gross profit earned in the futures market relative to that earned in the cash market. Consider an extreme case where there is market wide information that affects all stock prices moving together in the same direction, so that $|\sum_{i=1}^{20} \Delta S_i|$ equals $\sum_{i=1}^{20} |\Delta S_i|$. Under this situation, gross profits earned in the two markets are the same, but the net profit earned in the futures market is higher because the total transaction cost is lower there. Consequently, the futures contracts are much more attractive. However, when there are some stocks moving
in opposite direction so that $|\Sigma_{i=1}^{20} \Delta S_i| < \Sigma_{i=1}^{20} |\Delta S_i|$, the relative advantage of trading in futures contracts becomes smaller.

Therefore, the relative advantage of futures contracts can be measured by the co-movement ratio $|\Sigma_{i=1}^{20} \Delta S_i| / \Sigma_{i=1}^{20} |\Delta S_i|$. This co-movement ratio measures the extent of market wide movement. The more the price changes of individual stocks moving in the same direction, the higher the co-movement ratio. In the extreme, if the prices of component stocks all move in the same direction, the co-movement ratio will be one.

We stratify observations into 5 quintiles based on the co-movement ratio within the 30-minute interval. The higher the quintile, the more likely that there is market wide information affecting the stocks moving together. Table 8(a) contains the regression results for each quintile. It is first noted that the $R^2$ increases across higher quintile, suggesting that the correlation between the two markets is higher under market wide movement. More interestingly, when we move across higher quintiles, while the lead coefficients remain about the same, the lag and contemporaneous coefficients increase dramatically. For example, the estimates of $b_{-2}$, $b_{-1}$, $b_0$ are 0.096, 0.228, and 0.305, respectively, in quintile 1, and increase to 0.149, 0.353, and 0.437 in quintile 5. Certainly, when there is a higher association between the two markets, the existence of nonsynchronous trading of futures and component stocks would seem to imply that the lag and contemporaneous coefficients increase. However, if it is really due to the nonsynchronous trading, we also expect the lead coefficients to increase. The fact that the lead coefficients do not increase
proportionally, however, rules out this explanation. In fact, it seems that there is some information (market wide information) to which futures price react faster than cash index prices. When there are more stocks moving together, futures prices lead cash index prices more.

One problem associated with the co-movement ratio is that it is a price-weighted measure. If there are a few stocks which have much bigger prices than the others, the magnitudes of price changes of these stocks will have a decisive impact on the co-movement ratio. In this regard, we sort the observations based on a ratio, \(|N_u - N_d|/(N_u + N_d + N_z)\), where \(N_u\), \(N_d\) and \(N_z\) are the number of stocks moving upward, downward and with no change in the 30 minute interval. The ratio measures the net proportion of stocks moving together. It will just depend on the direction, not the magnitudes of price changes of the component stocks. Again, the higher the quintile, the more likely that there is market wide information affecting the stocks moving together. Table 8(b) contains the results for each quintile. In general, the pattern is similar to that in Table 8(a). The lead coefficients remain about the same in each quintile, while the lag and contemporaneous coefficients increases across higher quintiles.

The stratification of observations based on the co-movement ratio can pose a problem if the cash market trading volume is correlated with the extent of market wide information. This is very likely to occur when there is market wide information that affects all stocks moving together, which will trigger trades for each stock and affect trading volume in the whole market. To distinguish the effect of market wide information from the effect of trading volume, we therefore control the
trading volume in the cash market. We sort observations into subgroups first based on the trading volume and then on the co-movement ratio, and estimate regressions for the subgroups. The results are presented in Table 9. In the last three groups ([3,1], [3,2], [3,3]), the sorting procedure is not quite successful in controlling cash market trading volume when we allow the co-movement ratio to increase. But in general, it is found that when we move across subgroups of higher co-movement ratio, while the increase in \( \hat{b}_1 \) is trivial, the increase in \( \hat{b}_{-1} \) is much larger. For example, in the first three groups where the trading volume is controlled, the estimates of \( \hat{b}_{-1} \) in the subgroup [1,1], [1,2], and [1,3] are 0.146, 0.191, and 0.237, respectively. This confirms that the change in lead-lag patterns is due to the effect of market wide information.
E. CONCLUSION

This chapter investigates the lead-lag relation between the intraday cash and futures prices of the Major Market Index (MMI) during the period August 1984 through July 1986. We employ the transaction prices data of the futures and the component stocks, and examine whether the past price movement in one market is correlated with the price movement in the other market. The chapter also sorts the observations to see if the lead-lag pattern would change in accordance with: i) bad news versus good news; ii) the intensity of trading activity in the two markets; and iii) the extent of market wide movement.

The results of this chapter show that while the past price movement in the futures market can predict price movement in the cash market, past price movement in the cash market can also predict price movement in the futures market. However, the lead-lag relation is not symmetrical. While past futures price movement predict cash index price movement up to fifteen minutes, past cash index price movement only predict futures price movement up to five minutes. This thus has implications about previous studies that examine the S&P 500 cash index and futures. These studies all find that futures prices lead cash index prices, not vice versa. This is probably because they study the S&P 500 index. Since some stocks underlying the S&P 500 index are not actively traded, infrequent trading may cause stock prices to be unable to update information immediately. In this chapter, when we study the MMI, which comprises actively traded
stocks, it is not only found that the lead-lag relation persists for a shorter interval, but also that cash index prices predict futures prices.

It is possible that nonsynchronous trading of futures and component stocks can explain why cash index prices lead futures prices. However, nonsynchronous trading can hardly explain completely the overall lead-lag pattern. As indicated by the magnitude of the lead and lag coefficients, the feedback from the futures market into the cash market seems to be larger than the reverse. Further, such a lead-lag pattern also holds between the futures and the most heavily traded stocks. The more puzzling phenomenon, however, is that those heavily traded stocks which trade in almost every five-minute interval, can be led by the futures for more than two intervals (ten minutes). The evidence thus suggests that futures prices are fast in updating information, while individual stocks require some time in reacting to information disseminated by the futures prices.

Short sale constraints seem not to affect the lead-lag relationship. Even under good news, the cash index still lags futures. Further, there is no additional feedback from the futures market into the cash market under bad news. The variation in intensity of trading activity in two markets also does not affect the lead-lag patterns consistently. Instead, it is found that the lead-lag pattern vary with the extent of market wide movement. When there are more stocks moving together (market wide information), futures prices lead cash index prices more, but the feedback from the cash market into the futures market remains about the same. The effect
still persists even after controlling the trading volume in the cash market.

The findings in this chapter provide some support to the hypothesis that the cash and futures markets do not have symmetrical access to information. This is consistent with Subramanyam [1990] and the results in Chapter II that argue because of transaction costs and differential expected profits, the futures market would be the main discovery of market wide information while the cash market is the main discovery of firm specific information. This, together with the existence of some lags in information transmission from the futures market to the cash market, are responsible for causing futures prices to lead cash index prices more when there are more stocks moving together (market wide information).

In brief, this chapter identifies two forces that cause futures prices to lead cash index prices predominately. First, the futures market is fast in processing information, while the cash market is slow in reacting to new information contained in price changes in the futures market. Second, the futures market seems to be better in reflecting market wide information, a major component that determines the futures prices or underlying cash index values. Future research should investigate further the informational roles of the two markets and the transmission lags between them.
ENDNOTES


2. Although Finnerty and Park [1987] and Stoll and Whaley [1988] study MMI futures contracts, they just examine whether MMI futures lead cash index, and not whether cash index lead futures. Therefore, they exclude the possibility of feedback between the two markets.

3. For details about the index arbitrage in the MMI, see Chung [1990].

4. There are two other institutional differences that might explain why the futures lead cash index, but which are not discussed in the chapter. One is that the stabilization activities by specialists may smooth price adjustments in the stock market relative to the futures market. Another is that the dual trading practice in the futures market makes futures prices incorporate information more rapidly than in the stock market. (See Fishman and Longstaff [1989])

5. Subrahmanyam [1990] shows that if liquidity traders are risk averse, they may prefer to trade in a dispersed fashion during the day, as opposed to their behavior in the concentrated trading equilibrium of Admati and Pfleiderer [1988]. George, Kaul and Nimalendran [1990] show that in a specialist market (e.g. NYSE), the expected liquidity trading is the major component of trading volume, while the informed trading is only of second order importance. These results would imply that the trading volume in the cash market is not related to release of information.

6. MMI futures contract was introduced in July 23, 1984. The original contract was denominated as 100 times the index value and had minimum price increments of 0.10 index points. On July 7, 1985, the CBOT introduced a second futures contract, the Maxi MMI, which is denominated as 250 times the index value and has minimum price increments of 0.05 index points. The two contracts were traded simultaneously, and the smaller contract diminished in trading volume. On September 19, 1986, the smaller contract was discontinued. Since only the smaller contract is available in the dataset, we focus on the more heavily traded period, i.e., the period before July 1985.

7. The futures prices are reported through an advanced computer system called the Market Price Reporting and Information System (MPRIS). At each trading pit, the Exchange-employed market reporters enter the prices into the MPRIS through a computer terminal. Operating "online in real-time," the MPRIS accepts transactions at the rate of four or five per second in a typical market situation, and sends it back onto the trading floor's electronic wallboards.

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8. Stoll and Whaley [1988] explain how the cash index is recorded: "AMEX computes and disseminates the MMI level at fifteen second intervals. The time stamps that appear on the CBOT data base are the times at which the prices are received and recorded by the CBOT."

9. MMI is a price weighted index and can be expressed as

$$ I_t = \sum_{i=1}^{20} p_{i,t} / d, $$

where $p_{i,t}$ is the price of ith stock at time t and d is the adjustment divisor. The divisor will be adjusted for any stock splits or stock dividends.

10. The use of the term "lead and lag" does not necessarily mean that price movement in one market causes the other market. It is more appropriately interpreted as one market reacting faster to information and the other market lags behind and catches up.

11. We correct only for the first-order autocorrelation in the residuals because Table 2 shows that the partial autocorrelations of cash index returns are much smaller after the first lag.

12. The filtering procedure may, however, purge away some information that can be explained by past price changes in the futures market, so that the lag coefficients can be biased downward.

13. We also use one hour interval, but the results remain the same.
CHAPTER IV
CONCLUSION AND FUTURE RESEARCH

This dissertation studies the informational roles of the cash market and stock index futures market, and examines how they are affected by market structures.

Chapter II extends the rational expectation framework in Kyle [1985] and Admati and Pfleiderer [1988] to include both markets. Because of the different setting, while market makers in the cash market trade individual stocks, those in the futures market trade futures contracts (baskets of stocks). Therefore, stock values are priced first and then aggregated in the cash market, but they are aggregated first and then priced in the futures market. This distinction will be important when there is information correlated information among securities. This is because market makers in the futures market take the correlated information into account when pricing the futures, but those in the cash market do not take them into account when pricing the stocks because their portfolios are just individual stocks.

It is found that futures prices are more volatile than cash index prices. This will imply that: 1) futures prices are less efficient in reflecting information than cash index prices; 2) the
variance of futures price changes is larger than that of cash index price changes; 3) even though price changes of individual stocks and futures are serially uncorrelated, cash index price changes are positively autocorrelated. This research also demonstrates that a market wide informed trader earns a higher profit by trading futures contracts than by trading a portfolio of stocks, and that a firm specific informed trader earns a larger profit by trading the individual stock than by trading futures contracts. Thus, there will be a larger incentive to collect market wide information but a lower incentive to collect firm specific information in the futures market than in the cash market.

Chapter III examines the lead-lag relationship between the cash and futures prices of the Major Market Index (MMI) by employing the transaction price data of the futures and component stocks. The evidence indicates that the past price movement in the cash market and futures market can predict current price movement in the other market. However, the lead-lag relation is not symmetrical. While past futures price changes predict cash index price changes up to fifteen minutes, past cash index price changes only predict futures price changes up to five minutes. Nonsynchronous trading can hardly explain completely the overall lead-lag pattern. First, the feedback from the futures market into the cash market seems to be larger than the reverse, and that relation also holds between the futures and the most heavily traded stocks. Second, those heavily traded stocks which trade in almost every five-minute interval, can be led by the futures for more than two intervals (ten minutes). The evidence thus suggests that futures prices are fast in updating information, while individual
stocks require some time in reacting to information disseminated by the futures prices.

There is also evidence that the lead-lag pattern varies with the extent of market wide movement. When there are more stocks moving together (market wide information), futures prices lead cash index prices more, but the feedback from the cash market into the futures market remains about the same. The findings in this chapter provide some support to the hypothesis that the cash and futures markets do not have symmetrical access to the information.

Certainly, even though the two markets do not have symmetrical access to the information, if one market reacts instantaneously to the price movement in the other market, we will not observe this lead-lag relation. The evidence that futures prices lead cash index more under market wide information suggests also that there are some lags in information transmission from the futures market to the cash market. Future research should investigate further the informational roles of the two markets and the transmission lags between them.
Proof of Proposition 1

The net order flow \( q^i \) to the market maker is assumed to be equal to \( q^i = \tilde{w}_1 + \tilde{s}_1 + \tilde{z}_1 \). We conjecture that the quantities traded by informed traders are linear functions of their information, so that \( \tilde{w}_1 = \alpha_w \tilde{w} \) and \( \tilde{s}_1 = \alpha_s \tilde{s} \). We also assume that the market maker follows a linear pricing strategy, such that \( \tilde{P}_1 = \tilde{V} + \lambda \tilde{q}_1 \).

Consider the market wide informed trader \( w \). He chooses \( \tilde{w}_1 \) to maximize the expected profit conditional on his signal about \( \tilde{w} \), which is

\[
E \left[ \tilde{w}_1 (\tilde{V}_1 - \tilde{P}_1) \left| \tilde{w} \right]\right] \quad (A.1)
\]

Substituting the conjectured orders, the net order flows function, and the pricing function into (A.1), trader \( w \) is to maximize

\[
E \left[ \tilde{w}_1 (\tilde{V} - \lambda (\tilde{w} + \tilde{s}_1 + \tilde{z}_1)) \left| \tilde{w} \right]\right] \quad (A.2)
\]

Similarly, the stock specific informed trader \( s \) chooses \( \tilde{s}_1 \) to maximize the expected profit conditional on his signal about \( \tilde{s} \), which is

\[
E \left[ \tilde{s}_1 (\tilde{V}_1 - \tilde{P}_1) \left| \tilde{s}_1 \right]\right] \quad (A.3)
\]

Through substitution, (A.3) can be reduced to

\[
E \left[ \tilde{s}_1 (\tilde{V}_1 - \lambda (\tilde{w} + \tilde{s}_1 + \tilde{z}_1)) \left| \tilde{s}_1 \right]\right] \quad (A.4)
\]
Maximizing (A.2) and (A.4) respectively gives \( \tilde{w}_1 = \frac{1}{2} \lambda \bar{w}, \tilde{s}_i = \frac{1}{2} \lambda \bar{s}_i \).

We now determine the value of \( \lambda \). Since the market maker earns zero expected profit, \( \bar{p}_i \) is set equal to \( E(\bar{V}_i | \bar{q}_i) \). Now \( E(\bar{V}_i | \bar{q}_i) = E(\bar{V}_i) + \frac{\text{Cov}(\bar{V}_i, \bar{q}_i)}{\text{Var}(\bar{q}_i)} [\bar{q}_i - E(\bar{q}_i)] = \bar{V} + \frac{\text{Cov}(\bar{V}_i, \bar{q}_i)}{\text{Var}(\bar{q}_i)} \bar{q}_i \). Therefore, \( \bar{p}_i = E(\bar{V}_i | \bar{q}_i) \) requires \( \lambda = \frac{\text{Cov}(\bar{V}_i, \bar{q}_i)}{\text{Var}(\bar{q}_i)} \).

Evaluating the latter would give the value for \( \lambda \) as in Proposition 1.

Q.E.D.

Proof of Proposition 2

We skip the proof as the derivation is very similar to that in Proposition 1.

Proof of Proposition 3

If \( N_w \) and \( N_s \) represent the number of market wide and stock specific informed traders, then the net order flow \( \bar{q}_i \) to the market maker is equal to \( \bar{q}_i = N_w \bar{w}_i + N_s \bar{s}_i + \bar{z}_i \). We conjecture that the quantities traded by informed traders are linear functions of their information, so that \( \bar{w}_i = \alpha_w \bar{w} \) and \( \bar{s}_i = \alpha_s \bar{s}_i \). We also retain the assumption that the market maker follows a linear pricing strategy, such that \( \bar{p}_i = \bar{V} + \lambda \bar{q}_i \).
Consider the kth market wide informed trader ($\tilde{W}_k$). He chooses $\tilde{w}_k$ in order to maximize the expected profit, conditional upon his signal about $\tilde{W}$, which is

$$E \left[ \tilde{w}_k (\tilde{V}_k - \tilde{P}_k) | \tilde{W} \right] \quad (A.5)$$

Since the kth trader knows the other $(N_w - 1)$ market wide informed traders also have perfect information about $\tilde{W}$, he expects them to submit the market orders based on the same information that he has. By substituting the conjectured orders, the net order flows function, and the pricing function into (A.5), $\tilde{W}_k$ is to maximize

$$E \left[ \tilde{w}_k \left( \tilde{W} - \lambda (\tilde{w}_k + (N_w - 1)\alpha \tilde{W}_k + \tilde{s}_k + \tilde{z}_k) \right) \right] \quad (A.6)$$

Similarly, the hth stock specific informed trader ($\tilde{S}_h$), chooses $\tilde{s}_h$ to maximize the expected profits conditional upon his signal about $\tilde{S}_h$, which is

$$E \left[ \tilde{s}_h (\tilde{V}_h - \tilde{P}_h) | \tilde{S}_h \right] \quad (A.7)$$

Through substitution, (A.3) can be reduced to

$$E \left[ \tilde{s}_h \left( \tilde{S}_h - \lambda (\tilde{w} + \tilde{s}_h + (N_s - 1)\alpha \tilde{S}_h + \tilde{z}_h) \right) \right] \quad (A.8)$$

Maximizing (A.2) and (A.4), respectively, gives the values for $\tilde{w}_k$ and $\tilde{s}_h$. To determine the value of $\lambda$, a rational pricing strategy again requires that $\lambda = \text{Cov}(\tilde{V}_i, \tilde{q}_i)/\text{Var}(\tilde{q}_i)$. Evaluating the expression would give the value of $\lambda$ as in Proposition 3.

Q.E.D.

Proof of Proposition 4

We skip the proof as the derivation is very similar to that in Proposition 3.
Proof of Proposition 5

Because of the linear pricing strategy, the price function is

$$\bar{P}_i = \bar{V} + \lambda \bar{q}_i$$

$$= \bar{V} + \lambda (N_w\bar{w}_i + N_s\bar{s}_i + \bar{z}_i) \quad (A.9)$$

Substituting the order functions for $\bar{w}$ and $\bar{s}_i$ into (A.9),

$$\bar{P}_i = \bar{V} + \lambda (N_w\alpha_w\bar{w} + N_s\alpha_s\bar{s}_i + \bar{z}_i) \quad (A.10)$$

Now $\Gamma(\bar{w}|\bar{P}_i) = \text{Var}^{-1}(\bar{w}|\bar{P}_i,\bar{s}_i)$. From the conditional variance, we can evaluate $\text{Var}(\bar{w}|\bar{P}_i,\bar{s}_i)$, and its inverse, $\Gamma(\bar{w}|\bar{P}_i)$, so that

$$\Gamma(\bar{w}|\bar{P}_i) = \frac{1}{\sigma_w^2} + \frac{N_w^2\alpha_w^2}{\sigma_z^2} \quad (A.11)$$

Expressing $\alpha_w^2$ in terms on $\lambda$, and substituting the values for $\lambda$ into (A.11), we can then eliminate the terms for $\sigma_z^2$. Rearranging and representing (A.11) in terms of the prior precision, we obtain the results in Proposition 5.

Q.E.D.

Proof of Proposition 6

In equilibrium, the expected profits of informed traders are equal to the cost of information acquisition. Since the cost of information acquisition is the same in the cash and futures markets, we require the expected profits of informed traders of the same category in the two markets to be the same.

Equating $\pi_w$ with $\pi_w^f$ and $\pi_{s,i}$ with $\pi_{s,i}^f$, we have

$$(N_w + 1)^2 \left[ N_w\sigma_w^2 + N_s\frac{(N_w+1)^2}{(N_s+1)^2} \sigma_s^2 \right]$$
\[ (N_{g+1})^2 \left[ N_{g} \sigma^2 + \frac{N_{f+1}}{(N_{f+1})^2} \sigma_{w}^2 \right] = (N_{f+1})^2 \left[ N_{f+1} \sigma_{w}^2 + \frac{N_{f+1}}{(N_{f+1})^2} \sigma_{w}^2 \right] \quad (A.12) \]

Further, we divide (38) by (39) and (40) by (41), and equate the two ratios,

\[ \frac{(N_{w}+1)^2}{(N_{s}+1)^2} = \frac{m^2}{m(N_{s}+1)^2} \quad (A.14) \]

We want to show \( N_{w} < N_{f+1} \), \( N_{s} > N_{s} \). We prove the inequalities by contradiction.

Case 1 : \( N_{w} \geq N_{f+1} \), \( N_{s} > N_{s} \). Now \( (N_{w}+1)^2 \geq (N_{f+1}+1)^2 \), and because of (A.14), L.H.S. > R.H.S. in (A.12). Therefore, (A.12) is contradicted.

Case 2 : \( N_{w} < N_{f+1} \), \( N_{s} \leq N_{s} \). Now \( (N_{s}+1)^2 \leq (N_{s}+1)^2 \), and (A.14) implies \( (N_{w}+1)^2/(N_{s}+1)^2 > (N_{f+1}+1)^2/[m^2(N_{f+1}+1)^2] \), so L.H.S. < R.H.S. in (A.13). Therefore, (A.13) is contradicted.

Case 3 : \( N_{w} \geq N_{f+1} \), \( N_{s} \leq N_{s} \). Now (A.14) is contradicted.

Q.E.D.
TABLE 1

Trading Frequency and Non-trading Probability of the MMI Futures and Component Stocks in the Five Minute Interval
Sample period is from August 1984 to July 1986

<table>
<thead>
<tr>
<th>Futures/Stocks</th>
<th>N</th>
<th>Trading Frequency&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Non-Trading Probability&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Futures (84/08-85/06)</td>
<td>14316</td>
<td>5.01</td>
<td>6.4%</td>
</tr>
<tr>
<td>(85/07-86/07)</td>
<td>17684</td>
<td>1.84</td>
<td>31.6%</td>
</tr>
<tr>
<td>American Express</td>
<td>32000</td>
<td>3.07</td>
<td>10.7%</td>
</tr>
<tr>
<td>AT &amp; T</td>
<td>32000</td>
<td>7.13</td>
<td>1.0%</td>
</tr>
<tr>
<td>Chevron</td>
<td>32000</td>
<td>2.16</td>
<td>15.4%</td>
</tr>
<tr>
<td>Coco Cola</td>
<td>32000</td>
<td>1.85</td>
<td>29.3%</td>
</tr>
<tr>
<td>Dow Chemical</td>
<td>32000</td>
<td>3.23</td>
<td>7.4%</td>
</tr>
<tr>
<td>Du Pont</td>
<td>32000</td>
<td>2.16</td>
<td>16.5%</td>
</tr>
<tr>
<td>Eastman Kodak</td>
<td>32000</td>
<td>2.89</td>
<td>12.3%</td>
</tr>
<tr>
<td>Exxon</td>
<td>32000</td>
<td>4.30</td>
<td>3.2%</td>
</tr>
<tr>
<td>General Electric</td>
<td>32000</td>
<td>3.39</td>
<td>7.6%</td>
</tr>
<tr>
<td>General Motor</td>
<td>32000</td>
<td>3.52</td>
<td>9.8%</td>
</tr>
<tr>
<td>IBM</td>
<td>32000</td>
<td>7.93</td>
<td>0.5%</td>
</tr>
<tr>
<td>International Paper</td>
<td>32000</td>
<td>1.32</td>
<td>36.8%</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
<td>32000</td>
<td>2.91</td>
<td>11.1%</td>
</tr>
<tr>
<td>Merck &amp; Co.</td>
<td>32000</td>
<td>1.57</td>
<td>32.4%</td>
</tr>
<tr>
<td>Minnesota Mining &amp; Mfg.</td>
<td>32000</td>
<td>1.71</td>
<td>25.8%</td>
</tr>
<tr>
<td>Mobil Oil</td>
<td>32000</td>
<td>3.83</td>
<td>5.1%</td>
</tr>
<tr>
<td>Philip Morris</td>
<td>32000</td>
<td>2.33</td>
<td>18.1%</td>
</tr>
<tr>
<td>Proctor &amp; Gamble</td>
<td>32000</td>
<td>1.62</td>
<td>26.5%</td>
</tr>
<tr>
<td>Sears</td>
<td>32000</td>
<td>3.67</td>
<td>6.3%</td>
</tr>
<tr>
<td>U.S. Steel</td>
<td>32000</td>
<td>1.75</td>
<td>25.3%</td>
</tr>
<tr>
<td>Stocks' Average</td>
<td>32000</td>
<td>3.12</td>
<td>15.1%</td>
</tr>
</tbody>
</table>

<sup>a</sup> Trading frequency is the average number of trades in the five-minute interval.
<sup>b</sup> Non-trading probability is the proportion of the intervals in which there is no trade.
<table>
<thead>
<tr>
<th>Lag</th>
<th>MMI Cash Index</th>
<th>MMI Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACF</td>
<td>PACF</td>
</tr>
<tr>
<td>1</td>
<td>0.2953</td>
<td>0.2953</td>
</tr>
<tr>
<td>2</td>
<td>0.1151</td>
<td>0.0306</td>
</tr>
<tr>
<td>3</td>
<td>-0.0064</td>
<td>-0.0531</td>
</tr>
<tr>
<td>4</td>
<td>-0.0466</td>
<td>-0.0378</td>
</tr>
<tr>
<td>5</td>
<td>-0.0360</td>
<td>-0.0070</td>
</tr>
<tr>
<td>6</td>
<td>-0.0270</td>
<td>-0.0095</td>
</tr>
<tr>
<td>7</td>
<td>-0.0207</td>
<td>-0.0112</td>
</tr>
<tr>
<td>8</td>
<td>-0.0012</td>
<td>0.0074</td>
</tr>
<tr>
<td>9</td>
<td>0.0082</td>
<td>0.0083</td>
</tr>
<tr>
<td>10</td>
<td>0.0326</td>
<td>0.0227</td>
</tr>
<tr>
<td>11</td>
<td>0.0248</td>
<td>0.0058</td>
</tr>
<tr>
<td>12</td>
<td>0.0132</td>
<td>0.0006</td>
</tr>
</tbody>
</table>
TABLE 3

Cross correlation coefficients of five-minute price changes of the MMI cash ($\Delta S_t$) and MMI futures ($\Delta F_t$). Sample period is from August 1984 to June 1986.

<table>
<thead>
<tr>
<th>Lag k</th>
<th>$\rho_k(\Delta S_t, \Delta F_{t-k})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>84/08-85/06</td>
</tr>
<tr>
<td>-6</td>
<td>-0.0082</td>
</tr>
<tr>
<td>-5</td>
<td>-0.0120</td>
</tr>
<tr>
<td>-4</td>
<td>-0.0346</td>
</tr>
<tr>
<td>-3</td>
<td>-0.0389</td>
</tr>
<tr>
<td>-2</td>
<td>-0.0228</td>
</tr>
<tr>
<td>-1</td>
<td>0.1583</td>
</tr>
<tr>
<td>0</td>
<td>0.5215</td>
</tr>
<tr>
<td>1</td>
<td>0.4108</td>
</tr>
<tr>
<td>2</td>
<td>0.2022</td>
</tr>
<tr>
<td>3</td>
<td>0.0761</td>
</tr>
<tr>
<td>4</td>
<td>-0.0058</td>
</tr>
<tr>
<td>5</td>
<td>-0.0293</td>
</tr>
<tr>
<td>6</td>
<td>-0.0197</td>
</tr>
</tbody>
</table>
Regression\textsuperscript{a} of five-minute MMI cash price changes on leads and lags of five-minute futures price changes. The standard errors are shown in parentheses. Sample period is from August 1984 to July 1986.\textsuperscript{b}

$$\Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t}$$

<table>
<thead>
<tr>
<th>Period</th>
<th>$\hat{b}_{-3}$</th>
<th>$\hat{b}_{-2}$</th>
<th>$\hat{b}_{-1}$</th>
<th>$\hat{b}_0$</th>
<th>$\hat{b}_1$</th>
<th>$\hat{b}_2$</th>
<th>$\hat{b}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>84/08</td>
<td>0.070</td>
<td>0.143</td>
<td>0.294</td>
<td>0.380</td>
<td>0.106</td>
<td>-0.005</td>
<td>-0.008</td>
</tr>
<tr>
<td>-85/06</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N=14320</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R\textsuperscript{2}=0.51</td>
</tr>
<tr>
<td>85/07</td>
<td>0.004</td>
<td>0.034</td>
<td>0.209</td>
<td>0.517</td>
<td>0.234</td>
<td>-0.014</td>
<td>-0.003</td>
</tr>
<tr>
<td>-86/07</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N=18005</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>R\textsuperscript{2}=0.55</td>
</tr>
</tbody>
</table>

**Based on residual price changes**\textsuperscript{b}

<table>
<thead>
<tr>
<th>Period</th>
<th>$\hat{b}_{-3}$</th>
<th>$\hat{b}_{-2}$</th>
<th>$\hat{b}_{-1}$</th>
<th>$\hat{b}_0$</th>
<th>$\hat{b}_1$</th>
<th>$\hat{b}_2$</th>
<th>$\hat{b}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>84/08</td>
<td>0.022</td>
<td>0.052</td>
<td>0.176</td>
<td>0.347</td>
<td>0.107</td>
<td>-0.002</td>
<td>-0.008</td>
</tr>
<tr>
<td>-85/06</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td></td>
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<td>R\textsuperscript{2}=0.51</td>
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\textsuperscript{a} Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

\textsuperscript{b} The dependent variable is the residual cash index price changes estimated from the AR(1) model for the raw data.
Regression$^a$ of five-minute reported MMI cash price changes on leads and lags of five-minute futures price changes. The standard errors are shown in parentheses. Sample period is from August 1984 to July 1986.

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t} \]

Based on raw price changes

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<tr>
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Based on residual price changes$^b$

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<td>-85/06</td>
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</table>

a. Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

b. The dependent variable is the residual cash index price changes estimated from the AR(1) model for the raw data.
Regression of five-minute standardized price changes of component stocks on leads and lags of five-minute standardized futures price changes, arranged in ascending order of trading frequency. The standard errors are shown in parentheses. Sample period is from August 1984 to June 1985.

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t} \]

<table>
<thead>
<tr>
<th>Rank</th>
<th>Company</th>
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<th>( \hat{b}_{-1} )</th>
<th>( \hat{b}_0 )</th>
<th>( \hat{b}_1 )</th>
<th>( \hat{b}_2 )</th>
<th>( R^2 )</th>
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<td>0.450</td>
<td>0.127</td>
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<tr>
<td>2</td>
<td>AT &amp; T</td>
<td>0.044</td>
<td>0.047</td>
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<tr>
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<td>Exxon</td>
<td>0.059</td>
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<td>0.003</td>
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<tr>
<td>4</td>
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<td>0.053</td>
<td>0.098</td>
<td>0.016</td>
<td>-0.002</td>
<td>0.03</td>
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<tr>
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<td>Dow Chemical</td>
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<tr>
<td>8</td>
<td>General Motor</td>
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<td>0.248</td>
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<tr>
<td>10</td>
<td>Johnson &amp; Johnson</td>
<td>0.050</td>
<td>0.099</td>
<td>0.125</td>
<td>0.008</td>
<td>0.012</td>
<td>0.05</td>
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</tbody>
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(cont'd)
Regression\(^a\) of five-minute standardized price changes of component stocks on leads and lags of five-minute standardized futures price changes, in ascending order of trading frequency. The standard errors are shown in parentheses. Sample period is from August 1984 to June 1985.\(^b\)

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t} \]

<table>
<thead>
<tr>
<th>Rank</th>
<th>Company</th>
<th>(\hat{b}_{-2})</th>
<th>(\hat{b}_{-1})</th>
<th>(\hat{b}_0)</th>
<th>(\hat{b}_1)</th>
<th>(\hat{b}_2)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
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<td>Eastman Kodak</td>
<td>0.077 (0.008)</td>
<td>0.200 (0.008)</td>
<td>0.248 (0.008)</td>
<td>0.091 (0.008)</td>
<td>-0.001 (0.008)</td>
<td>0.16</td>
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<tr>
<td>12</td>
<td>Chevron</td>
<td>0.048 (0.008)</td>
<td>0.078 (0.008)</td>
<td>0.106 (0.008)</td>
<td>0.024 (0.008)</td>
<td>-0.013 (0.008)</td>
<td>0.04</td>
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<tr>
<td>13</td>
<td>Du Pont</td>
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<td>0.107 (0.008)</td>
<td>0.148 (0.008)</td>
<td>0.060 (0.008)</td>
<td>-0.015 (0.008)</td>
<td>0.07</td>
</tr>
<tr>
<td>14</td>
<td>Philip Morris</td>
<td>0.098 (0.008)</td>
<td>0.146 (0.008)</td>
<td>0.155 (0.008)</td>
<td>0.029 (0.008)</td>
<td>-0.000 (0.008)</td>
<td>0.08</td>
</tr>
<tr>
<td>15</td>
<td>U.S. Steel</td>
<td>0.049 (0.008)</td>
<td>0.077 (0.008)</td>
<td>0.100 (0.008)</td>
<td>0.046 (0.008)</td>
<td>-0.008 (0.008)</td>
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</tr>
<tr>
<td>16</td>
<td>Minnesota Min. &amp; Mfg</td>
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<td>0.180 (0.008)</td>
<td>0.205 (0.008)</td>
<td>0.069 (0.008)</td>
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<td>Proctor &amp; Gamble</td>
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<td>0.110 (0.008)</td>
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<td>-0.004 (0.008)</td>
<td>0.07</td>
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<td>18</td>
<td>Coco Cola</td>
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<td>0.168 (0.008)</td>
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<td>0.09</td>
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<tr>
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<td>0.150 (0.008)</td>
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<td>0.053 (0.008)</td>
<td>0.009 (0.008)</td>
<td>0.08</td>
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</tbody>
</table>

\(^a\) Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

\(^b\) The number of observations is 14322.

\(^c\) The company is arranged by their rankings of trading frequency.
Regression of five-minute MMI cash price changes on leads and lags of five-minute futures price changes, sorted by cash index price changes within the 30-minute interval. The standard errors are shown in parentheses. Sample period is from August 1984 to June 1985.

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t} \]

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<th>( \hat{b}_1 )</th>
<th>( \hat{b}_2 )</th>
<th>R²</th>
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</tbody>
</table>

a. Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

b. The number of observations ranges from 2928 to 3072 in 5 quintiles.

c. Average price change is the mean of cash index price changes within 30 minutes.
TABLE 7

Regression\(^a\) of five-minute MMI cash price changes on leads and lags of five-minute futures price changes, sorted by the cash and futures market trading frequency within the 30-minute interval. The standard errors are shown in parentheses. Sample period is from August 1984 to June 1985.\(^b\)

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t} \]

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<th>Group</th>
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<th>Futures Market Frequency</th>
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<th>(\hat{b}_2)</th>
<th>(R^2)</th>
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<tbody>
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<td>[1,1]</td>
<td>1.99</td>
<td>1.6</td>
<td>0.125</td>
<td>0.155</td>
<td>0.270</td>
<td>0.162</td>
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<td></td>
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<td>(0.018)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.015)</td>
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<td>[1,2]</td>
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<td>3.5</td>
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<td>0.271</td>
<td>0.096</td>
<td>-0.011</td>
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<td>0.264</td>
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<td>-0.011</td>
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<td>0.378</td>
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<td>(0.014)</td>
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<tr>
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<td>-0.003</td>
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<td>(0.015)</td>
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<td>(\cdot )</td>
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\(a\). Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

\(b\). The number of observations ranges from 798 to 2874 for each group.
c. Cash market trading frequency is the average number of trades per component stock in five minutes.
d. Futures market trading frequency is the recorded number of trades for the futures in five minutes.
Regression\textsuperscript{a} of five-minute MMI cash price changes on leads and lags of five-minute futures price changes, sorted by the co-movement ratio within the 30-minute interval.\textsuperscript{b} The standard errors are shown in parentheses. Sample period is from August 1984 to June 1985.\textsuperscript{c}

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta P_{t+k} + \epsilon_{s,t} \]

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Ratio</th>
<th>( \hat{b}_2 )</th>
<th>( \hat{b}_{-1} )</th>
<th>( \hat{b}_0 )</th>
<th>( \hat{b}_1 )</th>
<th>( \hat{b}_2 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
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<td>0.342</td>
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<td>0.42</td>
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<td>(0.010)</td>
<td>(0.010)</td>
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</table>

\textsuperscript{a} Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

\textsuperscript{b} Co-movement ratio is measured by \(|\sum_{i=1}^{20} \Delta S_i| / \sum_{i=1}^{20} |\Delta S_i|\), where \( \Delta S_i \) is the price change of component stock \( i \) within the 30-minute interval.

\textsuperscript{c} The number of observations ranges from 2982 to 3042 for the quintiles.
Regression\(^a\) of five-minute MMI cash price changes on leads and lags of five-minute futures price changes, sorted by the net proportion of stocks moving together within the 30-minute interval.\(^b\) The standard errors are shown in parentheses. Sample period is from August 1984 to June 1985.\(^c\)

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t} \]

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Average Net</th>
<th>(\hat{b}_2)</th>
<th>(\hat{b}_{-1})</th>
<th>(\hat{b}_0)</th>
<th>(\hat{b}_1)</th>
<th>(\hat{b}_2)</th>
<th>(R^2)</th>
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<td>1</td>
<td>0.15</td>
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<td>(0.011)</td>
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<tr>
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<td>0.271</td>
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<td>0.093</td>
<td>-0.023</td>
<td>0.42</td>
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<td>(0.010)</td>
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<td>(0.011)</td>
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</tr>
<tr>
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<td>0.73</td>
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<td>(0.010)</td>
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</tr>
</tbody>
</table>

\(^a\) Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

\(^b\) The net proportion of stocks moving together is measured by \(|N_u - N_d|/|N_u + N_d + N_z|\) where \(N_u\), \(N_d\) and \(N_z\) are the number of stocks moving upward, downward and with no change in the 30-minute interval.

\(^c\) The number of observations ranges from 2982 to 3042 for the quintiles.
TABLE 9

Regression\(^a\) of five-minute MMI cash price changes on leads and lags of five-minute futures price changes, sorted by the cash market trading volume and co-movement ratio within the 30-minute interval.\(^b\) The standard errors are shown in parentheses. Sample period is from August 1984 to June 1985.\(^c\)

\[ \Delta S_t = a + \sum_{k=-3}^{3} b_k \Delta F_{t+k} + \epsilon_{s,t} \]

<table>
<thead>
<tr>
<th>Group</th>
<th>Average Trading Volume</th>
<th>Average Co-movement</th>
<th>Average Volume</th>
<th>Average Ratio</th>
<th>(\hat{b}_{-3})</th>
<th>(\hat{b}_{-2})</th>
<th>(\hat{b}_{1})</th>
<th>(\hat{b}_{2})</th>
<th>R(^2)</th>
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<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.014)</td>
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<tr>
<td>[1,2]</td>
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<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>[1,3]</td>
<td>326.7</td>
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<td>0.131</td>
<td>0.237</td>
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<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.013)</td>
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<tr>
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<td>(0.014)</td>
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<td>(0.014)</td>
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</tr>
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</table>

\(^a\) Regressions are estimated with a two-pass (Yule-Walker) procedure which corrects for first order autocorrelation in the residuals.

\(^b\) Trading volume is the total number of shares traded for the 20 stocks.
Co-movement ratio is measured by $|\sum_{i=1}^{20} \Delta S_i|/|\sum_{i=1}^{20} |\Delta S_i|$, where $\Delta S_i$ is the price change of component stock i within the 30-minute interval.

c. The number of observations ranges from 1374 to 1860 for each group.
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