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Computer modeling and analysis of anisotropic material flow in compression molding of short-fiber-reinforced composite material

Kim, Junil, Ph.D.
The Ohio State University, 1990
COMPUTER MODELING AND ANALYSIS OF
ANISOTROPIC MATERIAL FLOW IN COMPRESSION MOLDING OF
SHORT-FIBER-REINFORCED COMPOSITE MATERIAL

A Dissertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

by

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1990

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To My Parents and my late sister Miran
ACKNOWLEDGEMENTS

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• Studies in Computer Aided Manufacturing in Forming of Polymers and Composites

• Studies in Systems Theory and Modeling
Publications


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CHAPTER I. INTRODUCTION

The objective of the study was threefold. First, this research was concerned with an experimental study of material flow in compression molding, using short-fiber-reinforced polymeric composites to manufacture parts with a plate-rib type geometry. Second, a finite element model was to be developed together with a measurement procedure to adequately reflect the flow properties of the material. Third, the validity of the proposed model was to be evaluated by comparing its predictions with experimental results.

1.1. Problem Definition

A polymeric composite material is defined as a combination of materials which are different in composition on a macroscopic scale. The two distinctive constituents that compose a composite material are usually reinforcing fiber and binder (or matrix). The reinforcing fibers carry the applied load and the binder maintains the integrity of the composite body. The use of reinforcing fibers enhances the mechanical properties of the articles manufactured with composites without significantly increasing the weight. The recent surge in the use of composite materials is due to the growing industrial demand for lighter, energy efficient, and mechanically superior materials.
Reinforced plastic composites such as thermoset and thermoplastic composites are gaining popularity since they meet such industrial demands quite well at relatively low cost.

Depending on the desired mechanical properties and the tolerance requirements, reinforced polymeric composites (thermoplastic or thermoset) are fabricated through such processes as injection molding, resin transfer molding, autoclave, casting, thermoforming, and compression molding. The present study is concerned with polymeric composites reinforced with short, randomly distributed glass fibers and their flow during compression molding process. Polymeric composites reinforced with short randomly distributed fibers are suitable for mass production for they are easy to handle. This study was concerned with polymeric composites with short, randomly distributed glass fibers and their flow during molding. Major attention was paid to the material referred to as Sheet Molding Compound (SMC). However, the results of the research can be applied to other composites that are reinforced with short glass fibers.

I.1.1. SMC and the Compression Molding Process

SMC refers to a variety of polyester based pre-pregnated thermosetting molding materials reinforced with random short glass strands. In general, it exhibits a combination of stiff and strong mechanical properties. In addition, SMC is relatively easy to handle and its material cost is low. As such, SMC has recently become a good candidate for replacing sheet metal in automotive
industries in an effort to produce lighter, stronger and noncorrosive automotive parts.

Compression molding is commonly employed in the processing of SMC to manufacture certain desired parts. Usually, the compression molding of SMC can be delineated by four steps: 1) material preparation, 2) mold filling, 3) curing, and 4) ejection of parts and cooling.

The first step, material preparation consists of mixing, paste feed, glass chopping, impregnation, and wind-up as illustrated in Figure 1.1. Mixing involves the blending of resin with a filler (calcium carbonate) and additives (initiator, inhibitor, thickening agent, etc.) to make paste. A typical SMC recipe is given in Table 1.1. The filler is added in particle forms to make up the volume, which reduces the use of expensive polyester. The initiator triggers the chemical reaction (between polyester and styrene in the compound shown in Table 1.1) of the thermosetting polymer. The inhibitor is used to control the timing of the initiation of the chemical reaction. The role of the thickening agent is to increase the viscosity of the compound so that it is easy to handle.

Paste feed refers to an operation of spreading a layer of paste with an accurate thickness on polyethylene or nylon films. Glass chopping is the cutting of a continuous strand fiberglass to a desired length. The chopped fiber glass is then dropped on the paste coated polyethylene or nylon film. Impregnation involves bringing the two paste coated films, one having the chopped fibers, together. Wind-up refers to the spooling of a compound sheet onto a
Figure 1.1. Process of manufacturing SMC
Table 1.1. Typical SMC recipe [1]

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<th>Ingredients</th>
<th>Parts by weight</th>
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<tr>
<td>Unsaturated polyester</td>
<td>33.5</td>
</tr>
<tr>
<td>Styrene</td>
<td>48.5</td>
</tr>
<tr>
<td>Low profile additive</td>
<td>18.0</td>
</tr>
<tr>
<td>Thickening agent</td>
<td>5.0</td>
</tr>
<tr>
<td>Lubricant (zinc stearate)</td>
<td>5.0</td>
</tr>
<tr>
<td>CaCO$_3$</td>
<td>150.0</td>
</tr>
<tr>
<td>1&quot; glass fiber</td>
<td>84.0</td>
</tr>
<tr>
<td>Initiator</td>
<td>2.0</td>
</tr>
<tr>
<td>Inhibiter</td>
<td>0.1</td>
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turret winder after impregnation. The wound compound sheet then thickens until it becomes viscous enough to be molded. Once the thickening has been achieved, a charge is cut to a desired size from the sheet. Due to the impregnation of the glass fibers, the flow of the charge so-prepared exhibits directionality as illustrated in Figure 1.2. This is an important property of fiber-reinforced polymeric composite materials, which affects processability during molding, and is the topic of major interest in this study. A detailed discussion on how this material characteristic is modeled is given in Chapter IV.

Mold filling starts when a charge is placed on a hot mold at a nominal temperature of about 150 °C. The mold is closed rapidly until it nearly touches the charge surface, and is then closed slowly to the desired height at the nominal speeds between 254 and 508 mm./min. The flow of charge is influenced by the orientation and distribution of the glass fiber in the charge. During this period, heat conduction from the mold surface to the charge may result in non-isothermal flow which can additionally influence the orientation and distribution of glass fibers across the work piece. Due to the inclusion of the inhibitor which prevents the curing from occurring by consuming the initiator in the compound, the amount of curing during the molding stage is negligible. This makes it possible for the mold filling to be isolated from the curing stage. The effect of glass fibers in SMC is accounted for by considering anisotropic flow properties along the thickness and plane directions of SMC during molding.
Differences in

Flow property in plane direction
Flow property in thickness direction

Figure 1.2. Fiber orientation in SMC
Once the mold is filled, a constant force is maintained in the mold, and the curing takes place. The unsaturated polyester resin is cured by free radical crosslinking copolymerization. Due to its fast and highly exothermic nature, there is a great deal of interaction between heat transfer and reaction kinetics. When reaction starts, a large amount of heat is released during a very short time period due to the fast cure reaction. The poor heat conductive characteristics of SMC makes it difficult for the heat to dissipate from the center of the charge towards the mold.

The temperature in the interior of the part often exceeds the mold temperature, and causes thermal degradation in thick parts. For obtaining better physical properties, a high level of conversion from monomer to polymer is required. From the viewpoint of mass production, the cycle time of compression molding should be short since it affects the manufacturing cost of the product. Thus an optimization in determining the desirable cycle time without sacrificing the quality of final product is desirable. Once the curing is complete, the part is ejected from the mold and is allowed to cool.

Due to the complexity resulting from the interaction between flow mechanism, heat transfer and curing reaction, a comprehensive analysis of the SMC molding process is extremely difficult. As indicated, the flow and the curing stages can be modeled independently. In the flow stage, by assuming that no cure reaction takes place, flow and heat transfer are two governing phenomena; while in the curing stage, which starts after the mold is filled, curing reaction and heat transfer are the two important phenomena.
I.1.2. The Problem

The proposed research attempts to model the flow in the compression molding of a fiber-reinforced composite material without considering curing and heat transfer. Although the actual composite material is a mixture of short glass fibers and resin, the material is modeled as a homogeneous continuum. The mixture also has been known to exhibit mechanical response similar to that of visco-plastic material, i.e., the amount of force required to achieve deformation depends on the rate of deformation. The applications of the visco-plastic assumption have enjoyed success in the modeling of flat-plate type geometry by assuming isotropic material properties. The molding of flat-plate type geometry has been successfully treated as flow between two parallel plates with small gap distance. However, this isotropic visco-plastic assumption did not turn out to be a valid one in modeling compression molding of rib-plate type geometry. In this study, the composite material is assumed to have an anisotropic visco-plastic flow property.

The anisotropic material flow is believed to be associated with the formation of defects in the finished parts, called 'sink mark', as illustrated in more detail in the following literature review. The sink mark is one of the major problems to be solved in manufacturing parts made from SMC. Although a sink mark is essentially an aesthetic problem, especially, when observed on the hood, roof, or trunk lid of a car, it is a critical one. Currently, auto body panels are manufactured in two-piece construction (flat outer bonded with structural
inner) in order to avoid the formation of sink marks. This two-piece construction method adds additional time, equipment, and labor to the entire production cycle. As such, the ultimate goal is to manufacture a large flat SMC part with reinforcing substructures in one-piece construction without a sink mark. This goal can be accomplished by understanding and successful modeling of the flow mechanism through experimental and numerical studies.

1.2. Scope and Objectives of the Research

The material flow in SMC during compression molding of plate-rib type geometry was considered via experimental and numerical studies under isothermal conditions. As will be described in the literature review, the previous studies discovered that the nonuniformity of the flow resulting from the interaction between the reinforcing fibers and the resin during molding creates uneven distribution of fibers in the area around the rib entrance, a resin rich area and a fiber rich area as shown in Figure 1.3. Consequently, these two areas exhibit different thermophysical properties, and this difference results in differential shrinkage which becomes a sink mark through polymerization and cooling. Therefore, the ability to model the complex flow pattern during molding of the short-fiber-reinforced composite material is critical in furthering the investigation.

As the fibers are considered to be the source of anisotropic material flow, the amount and the length of the reinforcing fibers in relation to material anisotropy was investigated. The influences of the interfacial friction between
Figure 1.3. Potential causes of sink mark

Undesirable Fiber Flow

Shrinkage due to
Polymerization (Volume Change)
Thermal Shrinkage
SMC charge and the mold surfaces and the mold closing speed on the flow were also examined. A measurement procedure which transforms this empirical information to a numerical structure was required for a mathematical model. Finally, a computer model, which incorporates the results of measurements, was developed, and the validation was performed via comparison with the molding experiments.

There were five steps to be taken to achieve the objectives of the current study. First, a series of experimental studies were conducted for the molding of plate-rib type geometry to obtain information on material flow behavior on the particular molding process of interest. Second, an anisotropic material flow model which accounts for the material flow of the short fiber-reinforced composites was developed. Third, a series of material tests which determine the parameters of the computer model were performed. Fourth, the fundamentals of measurement theory was contemplated in an attempt to validate the conduct of measurements performed in the current study. Finally, the predictions from the computer model were compared with molding experiments, and a discussion is presented pertaining to how to improve the compression molding of plate-rib type geometry.

With the above mentioned objectives in mind, Chapter II presents a literature review in three parts: 1) flow of SMC with flat geometry, 2) flow of SMC with plate-rib type geometry, and 3) modeling of anisotropic material flow. Chapter III presents results of the experimental molding of plate-rib type geometry, which renders an insight as to what aspects of the molding are to be
modelled. Chapter IV is a description of the constructed anisotropic finite element model. Chapter V presents the background and the procedure of the material characterization employed in the study. Chapter VI examines the foundation of the measurements conducted in the present study. Chapter VII summarizes and evaluates the current approach and is followed by the suggestions for future work in Chapter VIII.
CHAPTER II. LITERATURE REVIEW

This chapter reviews previous studies on experimental and numerical modeling of SMC compression molding in three parts, they are: 1) flow of SMC with flat geometry, 2) flow of SMC with plate-rib type geometry, and 3) modeling of anisotropic material flow.

II.1. Flow of SMC in Molding a Flat-Plate Type Geometry

The kinematics of flow across the charge thickness and the flow boundary condition at the mold/charge interface in flat-plate compression has drawn a great deal of discussion over the past ten years. Experimentally, several earlier studies [1,2] reported significant preferential flow near the mold surface when SMC was deformed under slow compression rates. This preferential flow is a result of the heat transfer through the mold surface and has a great effect on the flow pattern during mold filling. More recent work [3,4,7], however, showed that preferential flow occurs only in thick charges molded at very slow closing speeds. Under production conditions (i.e., closing speeds of 4-8 mm/sec.), the transverse velocity profile resembles a plug flow while the
material slips at the mold surface. In a visualization study using carbon black as tracer, Kau [5] suggested a partial-slip boundary condition at the mold surface based on the observed length of the carbon black streaks and the location of their end points.

The earliest models for the flow of SMC [6-9] were based on isothermal lubrication theory and were motivated by the successful application of the Generalized Hele-Shaw (GHS) models to thermoplastic injection molding [10]. In these papers, SMC was modeled as an incompressible isotropic, Newtonian or power law fluid. Normal stresses and inertia terms were neglected and the no-slip boundary condition was applied at the mold surface. These assumptions lead to a parabolic velocity profile through the cavity thickness. These models yielded good agreement between calculated flow front progression and experimental results for thin charges. For thicker charges, experimental results showed a substantially different flow front progression than the calculated profile which was supposed to be independent of cavity thickness. In addition to this discrepancy, these models may not be applicable for pressure prediction because of the isothermal assumption and the lack of consideration of the friction force at the mold/charge interface. Recently, Lee and Tucker [11] included normal stresses and heat transfer in their simulation. The effect of heat transfer on preferential flow was analyzed. However, the effect of friction at the mold surface was still not considered. Several researchers also imposed a transverse viscosity gradient to simulate the formation of preferential flow [11,12].
Another approach of modeling was proposed by Barone and Caulk [4,13]. They treated SMC as a two-dimensional sheet which stretches uniformly through the cavity thickness and slips on the mold surface. The momentum balance equation can be expressed by three terms in a two-dimensional frame: a hydrodynamic pressure resultant through the cavity thickness, a planar stress resultant embodying the material response to extensional deformation and the friction force between the SMC and the mold. This approach is more consistent with the observed kinematics in that representing SMC as two-dimensional isotropic sheets instead of a three-dimensional isotropic fluid is more reasonable. The plug-like flow front profile agrees better with experimental observation than a profile which results from the no-slip boundary condition. The transient heat conduction in SMC during mold filling can also be easily incorporated into this type of model since the assumption of plug flow can reduce the energy balance equation into a pseudo-conduction problem with a warped time [1,14]. Some applications based on this approach have been reported in the last several years [15-22]. There are, however, several limitations associated with this approach. First, a slip boundary condition (no friction) at mold surface may not be accurate as mentioned in a previous report [5]. Secondly, although the macroscopic view of the flow front profile resembles a plug-like flow, the small amount of preferential flow near the mold surface may play an important role in determining the surface related properties or defects. Finally, Fan et al. [23] showed in their recent study that the velocity profile, instead of being an assumption in the model, arises as a consequence of the interaction between heat transfer, flow and temperature dependent flow properties.
Compared to the relatively extensive study in mold filling experiments and modeling, the rheological characterization of molding compounds and the friction measurement at mold surface have not yet received much attention. Even for mold filling experiments and modeling, most of the comparison is limited to the flow front progression only. Comparisons of temperature and pressure distributions, and the detailed transverse velocity profile have only been reported recently [23].

Because of the presence of long fiber strands, the flow properties of SMC cannot be measured by conventional rheometers. Several researchers [1,15,21] employed squeeze flow rheometer to measure the flow resistance of SMC under compression molding. By creating a specific mold surface condition, e.g., by applying sand paper to generate no-slip boundary condition (shear deformation) or employing a layer of lubricant (elongational deformation), the two different flow resistances at different deformation modes were measured. The response was modeled following the Newtonian law or the power law, which essentially assumed the material is a three-dimensional isotropic continuum. By adapting the approach proposed by Barone and Caulk [4,13], Castro et al. [19,20,21] related the measured or employed compression force as a function of two terms, a flow resistance of SMC and a friction force at the mold surface. Compression force vs. closing speed from a molding experiment yielded the extensional viscosity and friction coefficient.
II.2. Flow of SMC in Molding a Rib-Plate Type Geometry

The flow of SMC in molding plate-rib type geometry has been mostly investigated because of a particular problem called sink mark. A sink mark is a small depression created on the surface opposite to the ribs or bosses. It has been experimentally shown [24-28] that the complicated flow pattern around the rib entrance has much to do with this type of defect.

Rabenold [24], Jutte [25], Boyd [26], and Ampthor [27] conducted experimental investigations in compression mold SMC to form a rib-plate type geometry. Their studies were focused on isolating the causes of sink marks and discovering the optimal mold geometry to eliminate sink marks. Although no one succeeded in eliminating this defect, the following observations were made about the relation between sink size and process variables:

- compression pressure, cure time, and cooling time during molding do not influence sink mark formation,
- lower mold temperature (nominal temperature is 150 °C) reduces sink mark depth,
- in some instances, charges made of multiple layers of SMC reduced sink mark depth,
- in some instances, short fiber SMC (nominal fiber length is 25.4 mm) reduced sink marks,
- combination of long (58 mm) and short (6.35 mm) fibers was found to reduce sink marks significantly,
• rib draft angle larger than 1° increased sink mark depth,
• increasing rib corner radius increased sink mark depth,
• rib width between 3.2 and 10 mm was recommended for smaller sink marks.

The above authors also investigated the causes of a sink mark, and gave the following conclusions based on their experimental observations. The potential factors that cause a sink mark are:

1) polymerization shrinkage,
2) thermal shrinkage due to cooling,
3) shrinkage due to entrapped air.

They observed that the effect of these three factors were greatly amplified by the disruptive flow around the rib entrance. That is, the nonuniformity of the flow resulting from the interaction between the reinforcing fibers and the resin during molding creates uneven distribution of fibers in the area around the rib entrance, a resin rich area and a fiber rich area as shown in Figure 1.3. Consequently, these two areas exhibit different thermophysical properties, and this difference results in differential shrinkage which becomes a sink mark through polymerization and cooling. As a result, the resin rich area at the joint of plate and rib tends to have high coefficient of thermal shrinkage.

Smith et al. [28] also conducted an experimental investigation on the effect of the rib geometry on sink depth. They also pointed out that a square (no corner radius) rib corner results in a smaller sink depth than a round rib corner. The authors also attempted to model the flow as Newtonian viscous flow using
the finite difference method assuming the thermal and polymerization shrinkage as the causes of sink mark.

Keown [29] described the design and the performance guidelines for various fiberglass reinforced plastic materials. Keown recommended a 1° rib draft angle, 3.18 mm rib width, and 3.18 mm rib depth for reducing sink in SMC compression molding. McCluskey et al. [30] suggested means of obscuring sink mark by putting styling lines along the traces of the sink mark. Ross [31] of Ford Motor Company explored a method of manufacturing functionally acceptable hood panel which would provide weight reduction of 30~50% without significant compromise in appearance. He investigated a two-piece sandwich hood design instead of one-piece design. Therefore, adhesives had to be applied to join flat SMC parts and fiber reinforced plastic (F.R.P) frame. He concluded that at the current time FRP/SMC hood design was not recommendable as there was not enough weight saving and surface quality was poor. Ross recommended the use of flat reinforced plastic surface and metal frame instead. Burch et al. [32] studied one-piece SMC hood for Ford Motor's Econoline vans. The paper compared one-piece ribbed design and two-piece stamped hood construction. His one-piece hood design showed acceptable result with draft angle of 0.5°, sharp corner radii, thickness to depth ratio of almost 1. He also found that the fiber length of 50.8 to 63.5 mm, and reduced pressure would yield optimum results. This one-piece ribbed design was attempted because the SMC part that was examined was the roof of the vehicle, instead of fenders or doors where surface finish is critical. Ehnert [33] of Volkswagen tried a door panel design with combination fibers of different
length and different resins. The long fibers were designed to remain near the surface so that no resin rich area was created near the surface. The combination of long and short fibers and high and low viscosity resin showed a successful result in molding passenger car doors with SMC.

In summary, the investigation of the previous studies showed that the following rib geometry minimizes sink marks:

- square rib corner,
- rib width between 0.32 and 1 cm,
- small rib draft angle (0.5 to 1 °).

These factors were considered in the experimental molding study presented in Chapter III. However, none of the previous studies investigate the structural design criteria for ribbed structures: where to place a rib, what size of ribs for what kind of stiffness requirement. This is an important subject for future study.

Smith et al. [28] and Naka et al. [34] conducted mathematical modeling of sink mark formation in compression molding and injection molding with a plate-rib type geometry, respectively. The hypothesis established by Smith et al. [28] about the cause of the sink mark was that the resin rich area created by the nonuniform flow during filling contributed to the area's different material properties such as thermal and polymerization shrinkage coefficients, and tensile modulus. Therefore, if this resin rich area could be eliminated, then the sink mark would not be formed. Smith et al., postulated the sink mark formation mechanism as a flow induced phenomenon. They claimed that the triangular resin rich area under the surface of the flat upper plate could be reduced by
controlling the flow, and the flow could be controlled by dimensional factors. Their specific interest was rib entrance geometry such as shape and radius of the rib entrance corner and rib width. They modeled the SMC as a Newtonian fluid with high viscosity. As they postulated, the sink depth was related to the rib entrance geometry. That is, the sharper the corner radius, the smaller was the sink depth. Thus, they concluded that the change of mold geometry could significantly reduce the sink depth, although it did not completely eliminate the sink mark. Naka et al. [34] modelled the sink mark problem in injection molding of a thermoplastic material, ABS. They claimed that sink mark in injection molding was created by a nonuniform shrinkage history of the resin in the mold during the cooling stage. Based on this assumption, Naka et al. predicted sink mark using state equation which utilized a P-V-T relationship. The conclusions were:

1) a sink mechanism can be characterized by total strain evaluation involving P-V-T relationship,

2) in the rib, the highest temperature location is the intersection of the rib and plate,

3) packing pressure and rib width are the important factors.

The two analytical studies introduced above are relatively simple isotropic mathematical models constructed to tackle the sink mark problem. The results of these two studies could not quantify the experimental observations. This means that the mathematical model that can stretch and compliment the experimental work in FRP molding should be more comprehensive than an isotropic continuum model.
II.3. Modeling of Anisotropic Material Flow

The flow model that accounts for material anisotropy has been rather rare in composite forming research. The following section reviews studies on anisotropy in SMC compression molding.

Lee et al. [1] showed experimentally that the shear and elongational viscosities are not the same in compression molding of SMC. According to them, elongational viscosity is several orders higher than shear viscosity. Lee and Tucker [3] also pointed out the need to develop anisotropic viscosity model which better describes the reality in SMC compression molding. All these studies considered SMC flow during molding only in flat geometries where the effect of material anisotropy is not very important.

Three interesting approaches [35-37] which attempted to capture the anisotropic and non-homogeneous properties of composite materials are described as follows. Hojo et al. [35] conducted a study to account for the distribution of short reinforcing fibers during compression molding. They assumed that in composite materials that contains short fibers, separation between matrix material and fiber particles would take place at certain flow velocities. Thus, they developed a model which can calculate distribution of fiber content as a function of distance from the origin assuming short fibers as spheres or cylinders. They simplified compression molding as one dimensional flow in a long slot. Measured fiber weight at certain distances from the origin
were compared with calculated fiber content. In spite of good agreements between experiments and theoretical calculation, it is considered that the simplification of three dimensional compression molding to one dimensional slot flow limits the validity of their model. Hojo et al. [36] expanded this work by putting the fiber/matrix separation and orientation together. Viscosity was expressed in terms of fiber orientation to account for non-homogeneity and anisotropy together. Rectangular specimen with random and unidirectional fibers were compressed experimentally, and compared with the model predictions. The proposed model was applied only to flat geometry. In the later study, the authors claimed that they used finite element method to obtain solutions for the velocity field without mentioning any details. The amount of fiber orientation and its effect on viscosity was considered in terms of plane anisotropy, but the anisotropy in thickness direction was not handled with the proposed fiber orientation model.

Malinowski et al. [37] proposed a method to treat non-homogeneous material deformation using finite element method. The model material was considered to have a finite number of rigid-plastic bodies with strain hardening. In other words, different material flow properties (flow stress) were assigned to different element of the finite element model. The result of the simulations of a cylinder compression suggested that the materials with hard core deform more uniformly than the isotropic material. The authors applied their model only for simple compression of a cylindrical specimen. Whether this idea can be extended to the compression of rib-plate type geometry is the subject of a further study.
Hirai et al. [38] proposed an anisotropic finite element model as a design aid for the problems of surface cavity and rib brittleness. Flow resistance of the material in normal and parallel directions are differentiated to reflect the effect of reinforcement. Experimentally, BMC with 0.635 cm (0.25 ") fiber and plasticine were compared by molding in a mold with rib-plate type geometry. They have noted that, the material tended to flow more in the horizontal direction when it showed anisotropy, which seemed to reduce the surface cavity. The authors recommended smaller rib corner radius and higher anisotropy to avoid surface cavity, but they did not mention how anisotropic material flow resistance could be represented and measured. Hirai et al. [39] furthered their work to investigate the effect of material anisotropy on viscosity in flow propagation into the rib and the velocity pattern in rib-plate type geometry. Their work was purely numerical, and how the flow property can be measured was not mentioned.

The review of the literature on anisotropic material flow in SMC forming suggests the following:

1) sink mark formation is affected by material anisotropy,
2) anisotropic numerical models have been proposed and partially validated via comparisons with experiments,
3) there have been no suggestions regarding how anisotropic flow resistance can be measured,
4) there has not been a comprehensive experimental validation comparing various parameters during molding of rib-plate type geometry.
Thus, the study presented was intended to develop an anisotropic flow model along with measurement procedure, numerical model based on finite element method, and experimental validation.
CHAPTER III. EXPERIMENTAL STUDY OF COMPRESSION MOLDING

The objective of this chapter was twofold. First, the flow mechanism of the short-fiber-reinforced composite materials in compression molding was investigated using SMC with different fiber weight contents and lengths. Second, important parameters are derived from the understanding of the experimental observations so that they can be utilized in the subsequent finite element modeling.

III.1. Compression Molding of Plate-Rib Type Geometry

Six model materials were compression molded under various conditions to investigate the material flow of short-fiber-reinforced composites during compression molding. An experimental mold with a transparent front wall was devised for flow observation. Molding experiments were conducted using an Instron 1322 hydraulic testing machine at 20 °C.
III.1.1. Model Materials

For a systematic investigation, six model materials with different fiber lengths and weight contents were used. Table 3.1 summarizes the compositions of five model materials. Silly putty was used to represent an isotropic material which does not contain reinforcing glass fibers. SMC's with different weight contents and fiber lengths were used to investigate the effect of glass fibers versus anisotropic flow property. SMC model materials used in these experiments were provided by GenCorp.

The initial charge had a dimension of 50.8 (w) x 12.7 (H) x 101.6 (L) mm. SMC charges were made by stacking multiple layers. Silly putty charges were molded using a rectangular mold on a small manual press. SMC charges were made by stacking individual 50.8 (w) x 101.6 (L) mm pre-cut sheets to make up 12.7 mm thickness. SMC charges were not pre-compacted since in actual production, they are used in stacked form without compaction. On the average, it took 8 to 9 plies of SMC to make 12.7 mm with model materials 1 and 2 (6.35 and 12.7 mm fibers with 10% by weight fiber content), 4 to 5 plies for material 3, and it only took 2 to 3 plies with model materials 4 and 5 (12.7 and 25.4 mm fibers with 20 and 30% by weight fiber content, respectively). For SMC's with only 10% by weight fiber content, the variation in the thickness was up to 40%.

The air voids are unavoidable due to the way that SMC is prepared. There are three stages during which air can be mixed into SMC. First, calcium
Table 3.1. Summary of Model Materials

<table>
<thead>
<tr>
<th>Model material number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber length (mm)</td>
<td>6.35</td>
<td>12.7</td>
<td>25.4</td>
<td>12.7</td>
<td>25.4</td>
<td>silly putty</td>
</tr>
<tr>
<td>Fiber weight content (%)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>
carbonate filler in the particulate form is mixed with other chemical ingredients using an agitator. Due to the motion of the agitator, air is mixed with calcium carbonate particle and other ingredients. Thus, the amount of air entrapment depends on the mixing speed. Secondly, while the short fibers are being sprinkled and sandwiched between SMC layers, a certain amount of air is entrapped. The reader can refer to Figure 1.1 which illustrates the SMC manufacturing process. Thirdly, when the charge is made up of multiple layers of SMC, a small amount of air is entrapped between layers since individual SMC ply has a rough irregular surface.

The entrapment of air and mixing of different ingredients, including short glass fibers, makes SMC an anisotropic, compressible, and inhomogeneous material. However, most studies have treated such materials as isotropic, incompressible, and homogeneous continua. This chapter probes experimentally what kind of assumptions are valid in the modeling of the process.

III.1.2. Equipment

The following describes the design of the mold, the testing machine, and the molding conditions.
A modular experimental mold was machined from Aluminium 6061 T20. Aluminium 6061 was chosen for its machinability since the hardness of the mold is not a critical factor. Figure 3.1 is a photograph of the mold before it is assembled for molding. Figure 3.2 shows the projected top view of the lower mold. The dashed area indicates the area occupied by the initial charge. The mold was 101.6 mm (4 inches) long and 127 mm (5 inches) wide. It was observed in a previous study [23] that the pressure could go up to 5.5 N/mm² (800 p.s.i) in the compression molding of SMC with 30 % fiber weight content. Since initially only a 8800 Kg load cell was available, this is why the area of the mold was determined not to exceed 13,000 mm².

A Kistler type 6157A pressure transducer was flush mounted in the center of the top mold in order to measure the pressure on top of the rib entrance. The characteristics of the pressure transducer are summarized in Table 3.2. The pressure transducer is applicable to either 2000 bar (1 bar= 10⁶ Nm²) or 200 bar range, and 200 bar range was used for the experiment. A Kistler type 5080 amplifier was used for this purpose. A chart recorder was connected to the amplifier to record the change of the pressure during molding. The measurement of the pressure on top of the rib entrance region was
Table 3.2  Technical data of pressure transducer (Kistler, type 6157A)

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum measuring range:</td>
<td>0 to 2000 bar</td>
</tr>
<tr>
<td>Over load:</td>
<td>2500 bar</td>
</tr>
<tr>
<td>Sensitivity nominal (pC/bar)</td>
<td>-9.0</td>
</tr>
<tr>
<td>Maximum tool temperature:</td>
<td>200 °C</td>
</tr>
<tr>
<td>Linearity nominal:</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Figure 3.1. Experimental mold before assembly
Figure 3.2. Top and front views of the bottom mold

- Material flow is blocked in this direction to satisfy the plane strain condition
- Initial charge
- Material flow
- 127 mm
- 101.6 mm
- 9.5 mm
- 6.35 mm
conducted because in the previous work [23] it was discovered to be an important indicator regarding the filling of a rib. The previous observations are presented together with the results of the current study in the following sections.

An Instron 1322 hydraulic testing machine with 22,000 Kg (50,000 lbs) load cell was employed to apply required force for deformation. The specification of the testing machine is given in Table 3.3. In order to achieve constant mold closing speed, the control was set in stroke mode using a linear ramp function.

Figure 3.3 is a schematic of the experimental set up. As indicated in Figure 3.3, load and the pressure at the top of the compressed charge opposite to the rib location were simultaneously measured. In summary, the followings were observed during molding:

• Load vs. displacement,
• Pressure above the rib vs. displacement,
• Rib filling pattern,
• Deformity of fibers around the rib entrance.

A clear front wall machined from PMMA (Polymethylmethacrylate) allowed observation of the deformation pattern of the deforming charge as indicated in Figure 3.4. Figure 3.5 shows the entire experimental set up.
<table>
<thead>
<tr>
<th>Force Rating</th>
<th>100 kN (22 kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stroke</td>
<td>127 mm (5 inches)</td>
</tr>
<tr>
<td>Maximum Velocity</td>
<td>360 mm/minute</td>
</tr>
<tr>
<td>Minimum Velocity</td>
<td>0.000016 mm/minute</td>
</tr>
</tbody>
</table>

Cross Head
(Constant Velocity)

Load Cell

Pressure Transducer

Upper Mold

Transparent Window

Composite Charge

Ribbed Lower Mold

Measurements of

• Load vs. displacement
• Pressure above the rib vs. displacement
• Rib filling pattern
• Deformity of fibers around the rib entrance

Figure 3.3. Schematic of experimental set up
Figure 3.4. Assembled mold with SMC charge
Figure 3.5 Photograph of experimental set up
III.1.3. Molding Results

The experimental results of the compression molding of short-fiber-reinforced composites for the rib-plate type geometry are summarized in this chapter. The contents of this section include: the void content estimation, deformed charge shape, measurement of resin rich triangular region, and center pressure versus charge height.

The compression molding process for rib-plate type geometry was idealized as a plane strain process. This idealization is further explicated in Chapter IV. Various molding conditions were applied with six model materials: SMC with 25.4 mm long fibers and 30 % weight content and silly putty were used to represent commercial compound and its paste, respectively. SMC's of 6.35, 12.7, and 25.4 mm long fibers with 10 and 20 % fiber weight contents were also used to investigate the effect of fiber length and content on material flow. The applied experimental conditions are summarized in Table 3.4. The actual deformation of SMC during a compression molding is probably similar to the deformation under lubricated mold surface since the process is non-isothermal (hot mold / cold charge) and the molten resin at the interface functions as a lubricant. The lubricated condition of the actual non-isothermal compression molding was simulated here by applying a layer of automotive grease at the charge/mold interface. For comparison, unlubricated interfacial conditions were also applied in this study.
Table 3.4. Summary of Experimental Conditions

<table>
<thead>
<tr>
<th>Unlubricated</th>
<th>Lubricated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant mold closing speed of $V=12.7$ mm/min.</td>
<td>Constant mold closing speed of $V=12.7$ mm/min.</td>
</tr>
<tr>
<td>30% reduction of charge height</td>
<td>30% reduction of charge height</td>
</tr>
<tr>
<td>60% reduction of charge height</td>
<td>60% reduction of charge height</td>
</tr>
<tr>
<td>Constant mold closing speed of $V=50.8$ mm/min.</td>
<td>Constant mold closing speed of $V=50.8$ mm/min.</td>
</tr>
<tr>
<td>30% reduction of charge height</td>
<td>30% reduction of charge height</td>
</tr>
<tr>
<td>60% reduction of charge height</td>
<td>60% reduction of charge height</td>
</tr>
</tbody>
</table>
In the preliminary molding experiment, it was observed that the rib could be filled at 60% reduction in charge height. Load, pressure, and deformed charge shape were recorded at 30 and 60% reduction in charge height.

### III.1.3.1. Estimation of the Void Content

As discussed previously, short-fiber-reinforced composites such as SMC contain a certain amount of air. This void content may affect the processability of the charge during molding. Thus, this section attempts to estimate the void contents using five different model materials: 6.35 mm-10% by weight fiber content, 12.7 mm-10% by weight fiber content, 25.4 mm-10% by weight fiber content, 12.7 mm-20% by weight fiber content, and silly putty. Silly putty represents the material without inhomogeneity such as fibers. Since the fibers are added to SMC by weight, the volume occupied by different fibers depends on the cut lengths. Table 3.5 shows the results of the molding experiments. The table was generated by comparing the initial and final cross-sectional areas of the plane of plane strain. Each condition was repeated three times and the calculated areas were averaged.

As shown in Table 3.5, the 6.35 mm-10% by weight fiber content and 25.4 mm-10% by weight fiber content SMC exhibited about 4% loss in volume after 60% reduction in height. Thus, 12.7 mm turns out to be the void prone fiber length. When the length of the fiber is as short as 6.35 mm, the wetting of fibers by resin seems easier. Compared to SMC with 12.7 mm fibers, there are
Table 3.5. Summary of estimated volume loss after 60% deformation

Initial Charge

Deformed Charge

<table>
<thead>
<tr>
<th>Materials</th>
<th>6.35mm10% SMC</th>
<th>25.4mm10% SMC</th>
<th>12.7mm10% SMC</th>
<th>12.7mm20% SMC</th>
<th>Silly putty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss of Volume (%)</td>
<td>4.4</td>
<td>4.3</td>
<td>12.1</td>
<td>21.3</td>
<td>1</td>
</tr>
</tbody>
</table>
fewer fiber strands in SMC with 25.4 mm fibers since fibers are added by the weight. This is why SMC with 12.7 mm-10% fibers showed higher void content. The 6.35 mm-10% and 25.4 mm-10% SMC's are the best candidates for the model material, in using the mathematical model assuming incompressibility.

III.1.3.2. Comparison of the Deformed Shape

The effect of fibers on material flow was investigated using the five model materials. Due to the volume loss discussed in the previous section, the simple comparison of the planar flow and the rib flow at 30 and 60% height reduction was not possible. Since the center pressure and the load were measured simultaneously, the pressure information was readily available for the actual initial charge height. For each material, the height of the charge from which pressure reading started was considered the actual initial charge height. Therefore, different model materials had different initial heights.

Consequently, a parameter "D" was devised. D represents the ratio between the depth of the material flown into the rib (T) the half width of the deformed charge (W). Thus D is intended to be an index of whether the material is more likely to flow toward the planar or downward normal to the planar direction (rib direction for short) at the given state of deformation. For example, a material with a larger D value tends to flow better in the rib direction compared with the material with a smaller D value at that particular state of
Table 3.6. Summary of deformed shapes in terms of rib vs. planar flow

![Diagram showing initial and deformed charge with dimensions](50.8 mm, 12.7 mm)

**Symbol Designation**

\[ D = \frac{T}{W} \]

<table>
<thead>
<tr>
<th></th>
<th>Unlubricated</th>
<th>Lubricated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30%</td>
<td>60%</td>
</tr>
<tr>
<td><strong>v=12.7 (mm/min)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.35 mm10%</td>
<td>.118</td>
<td>.195</td>
</tr>
<tr>
<td>12.7 mm10%</td>
<td>.076</td>
<td>.188</td>
</tr>
<tr>
<td>25.4 mm10%</td>
<td>.09</td>
<td>.193</td>
</tr>
<tr>
<td>12.7 mm20%</td>
<td>.057</td>
<td>.219</td>
</tr>
<tr>
<td>Silly putty</td>
<td>.083</td>
<td>.118</td>
</tr>
<tr>
<td><strong>v=50.8 (mm/min)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.35 mm10%</td>
<td>.118</td>
<td>.178</td>
</tr>
<tr>
<td>12.7 mm10%</td>
<td>.137</td>
<td>.188</td>
</tr>
<tr>
<td>25.4 mm10%</td>
<td>.178</td>
<td>.193</td>
</tr>
<tr>
<td>12.7 mm20%</td>
<td>.036</td>
<td>.225</td>
</tr>
<tr>
<td>Silly putty</td>
<td>.091</td>
<td>.116</td>
</tr>
</tbody>
</table>
deformation. By comparing the values of D in Table 3.6, the following observations were gained:

• materials flowed better in the rib direction under an unlubricated condition compared to the lubricated condition,

• in both lubricated and unlubricated moldings, the materials with fibers flowed better into the rib compared to silly putty at 60 % height reduction for both mold closing speeds. This is thought to be the anisotropic effect of fiber,

• in lubricated moldings, the filling of the rib (rib depth was 9.525 mm) was never complete in any conditions,

• at lower closing speed, the rib flow of materials with fibers longer than 6.35 mm was hindered initially since fibers tended to form a bridge over the rib entrance (i.e., comparing 30 and 60 % height reduction for column under unlubricated conditions),

• from the comparison of 12.7 mm 20 % SMC and silly putty, SMC shows a differential filling (i.e., once the blockage of the fibers is overcome, SMC flows faster into the rib than silly putty).

III.1.3.3. Comparison of the Fiber Deformity

As mentioned previously, the major motivation for the study of the molding of composite materials with rib-plate type geometry was the sink mark which blemishes the surface finish of the molded part. Using the current experimental results, a set of new parameters were proposed which were useful for the analysis of the sink mark formation.
10 % by weight fiber content SMC's with 6.35, 12.7, and 25.4 mm fiber lengths were employed in the experiments. Moldings were performed at two different velocities (12.7 and 50.8 mm/min.), lubrication conditions, and deformation percentages. Finished specimens were ejected from the mold, and dried until they became rigid. The cross-section was cut and photographed. Photographs are listed in Appendix.

Three dimensions were measured from the photograph of the charge cross-section, h1, h2, and T. h1 represents the thickness of the region where fibers are undeformed. h2 is the total current thickness of the deformed charge, and T indicates the amount of rib flow as indicated in Table 3.7. A parameter R1 was devised to indicate the sink resistance. The value of R1 is 1.0 initially, and becomes zero when all fibers are bent during the rib flow. Thus, the larger R1 values indicate that there is a thick layer of undeformed fibers which would camouflage the triangular resin rich area. As previously reported [23], if fibers can bridge over the rib area, the sink mark can be masked.

The combination of the parameters R1 and T can be used to represent the possibility for sink mark formation. That is, if T reaches the rib depth and R1 is still relatively large, the part is less likely to have a sink mark although it may have triangular resin area under neath the flat surface. Table 3.7 exhibits that the unlubricated molding condition yields higher values of T and R1 (in most cases). The parameter R1 alone can be misleading since its values are high in lubricated conditions due to the small amount of rib flow.
Table 3.7. Summary of fiber deformity using the parameters R1 and T

![Diagram of fiber deformation](image)

Parameters: $R_1 = \frac{h_1}{h_2}$ (0 ≤ $R_1$ ≤ 1) and $T$

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Reduction % Velocity (mm/min)</th>
<th>Fiber Length (mm)</th>
<th>Fiber Content (wt. %)</th>
<th>Interfacial Condition</th>
<th>$R_1 (= \frac{h_1}{h_2})$</th>
<th>$T$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30%</td>
<td>6.35</td>
<td>10</td>
<td>L</td>
<td>0.33</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>UL</td>
<td>0.54</td>
<td>2.74</td>
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<tr>
<td></td>
<td></td>
<td>12.7</td>
<td>10</td>
<td>L</td>
<td>0.83</td>
<td>0.56</td>
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<td>0.78</td>
<td>1.60</td>
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<tr>
<td></td>
<td></td>
<td>25.4</td>
<td>10</td>
<td>L</td>
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<td>1.25</td>
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<td></td>
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<td>UL</td>
<td>0.69</td>
<td>1.52</td>
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<tr>
<td></td>
<td>60%</td>
<td>6.35</td>
<td>10</td>
<td>L</td>
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<td>7.32</td>
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<tr>
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<td>0.52</td>
<td>9.53</td>
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<td></td>
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<td>12.7</td>
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<td>L</td>
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<td>6.66</td>
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<td>0.45</td>
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<td></td>
<td>UL</td>
<td>0.62</td>
<td>8.51</td>
</tr>
</tbody>
</table>
The optimal molding condition suggested by Table 3.7 is that the lower surface of the charge should be unlubricated so that the rib can be quickly filled and the upper surface is lubricated so that the fibers near the surface will not deform. Thus, combined with the material flow pattern mentioned in the previous section, a process with high values of D, R1, and T gives the best material for the sink mark free rib-plate type geometry.

III.1.3.4. Comparison of Pressure Level

The following presents experimental results on the measured pressure and the charge height. The measured pressure at the top of the surface of the deforming charge opposite to the location of the rib yields interesting information on the fiber deformation. Previous work [23] gave these discoveries on the measured pressure under various molding conditions, and they are introduced in Figures 3.6, 3.7, and 3.8.

Figure 3.6 compares the experimentally measured pressure levels during compression molding of rib-plate type geometry using silly putty and SMC (25.4 mm 30 %). As mentioned, silly putty represented the material without the reinforcing fibers. Pressures were measured at the top charge surface opposite to the rib location (designated as p1) and the location 50.8 mm away from it (designated as p2). The pressure level obtained from silly putty compression showed monotonic increase while that of SMC showed three regions. The pressure pattern obtained from SMC initially it increased as if it is a compression between two flat plate molds. This was because the fiber
Figure 3.6. Comparison of pressure vs. charge height between 25.4 mm-30 % by weight fiber content SMC and silly putty. P1: pressure measured at the center, P2: pressure measured 50.8 mm away from the center. Closing speed = 12.7 mm/min.
around the rib entrance could sustain the applied pressure. Until sufficient pressure built up, the pressure increased monotonically, and then decreased gradually. This gradual decrease of pressure was an indication that the fibers around the rib entrance were gradually bent into the rib. When the rib was completely filled, the pressure rose sharply again as indicated in Figure 3.6. The second pressure reading, p2 namely, exhibited a smooth curve for silly putty molding, while that of SMC molding showed some fluctuation. The reason for the fluctuation (p2) in SMC molding was understood as the repeated contact of either fiber rich compound or the resin rich compound with the tip of the pressure transducer during molding.

Figure 3.7 shows the effect of mold closing speed on the pressure level. Figure 3.8 illustrates the effect of charge thickness by showing the pressure curves of charges made of different number of SMC plies. For thin charges the third stage of the pressure curve pattern is non-existent since the rib filling was not complete. Similar results were duplicated in the current study, and they are shown in Figure 3.9.

Figures 3.10 through 3.13 displays the pressure versus charge height curves for all the molding conditions using four SMC model materials. In molding under unlubricated interfacial conditions, SMC model materials completed the filling except 12.7 mm-20 % SMC. Consequently, the curves obtained from unlubricated molding display the three stages demonstrated by Figure 3.6. Figures 3.10 and 3.11 show the pressure level when no lubrication
Figure 3.7. Pressure vs. charge height curves at various mold closing speeds. Initial circular charge diameter=108 mm, at 20 °C, 4 plies of SMC with 25.4 mm-30 % by weight fiber content.
Figure 3.8. Pressure vs. charge height curves with different number of charge plies. Initial circular charge diameter=108 mm, at 20 °C, SMCs with 25.4 mm-30 % by weight fiber content. Closing speed=2.54 mm/min.
Figure 3.9. Comparison of center pressure between SMC charge with 6.35 mm fiber length and 10% weight content, and silly putty. Mold closing speed=12.7 mm/min., No lubrication, 60% reduction in charge height.
Figure 3.10. Center pressure versus charge height during molding of four SMC charges. Mold closing speed=50.8 mm/min., No lubrication, 60% reduction.
Molding Unlubricated v=12.7mm/min  
Center Pressure vs. Height

Figure 3.11. Center pressure versus charge height during molding of four SMC charges. Mold closing speed=12.7 mm/min., No lubrication, 60% reduction.
Figure 3.12. Center pressure versus charge height during molding of four SMC charges. Mold closing speed=50.8 mm/min., Lubricated with grease, 60% reduction in height.
Figure 3.13. Center pressure versus charge height during molding of four SMC charges. Mold closing speed=12.7 mm/min., Lubricated with grease, 60% reduction in height.
was applied on the mold surface at mold closing speeds of 50.8 and 12.7 mm/min. respectively. On the other hand, in the molding under lubricated interfacial conditions, the pressure level was similar to the one from compression between flat mold since enough fibers are bridging over the rib to sustain the pressure. Figures 3.12 and 3.13 show almost monotonically increasing pressure curves, and the ribs were never completely filled. Thus, the pressure curve pattern shown in Figures 3.10 and 3.11 can be used as an indicator for the completion of the rib filling.

III.2. Summary and Conclusion

In this chapter, various molding experimental results were described. As discussed, short-fiber-reinforced composites such as SMC are non-homogeneous, compressible, and anisotropic materials. The compressibility and anisotropic flow properties were examined. Two parameters, $D$ and $R_1$, were devised to be used as indicators for the anisotropic flow characteristics and the sink resistance for compression molding of rib-plate type geometry.

It has been shown from the molding experiments using different SMC model materials that SMC with 6.35 mm-10% could be compared with an incompressible anisotropic flow model since it exhibits a relatively small amount of volume loss and little or no bridge effect.
CHAPTER IV. THE PROPOSED MODEL

This chapter presents a mathematical description of the model which accounts for the anisotropic material flow of short-fiber-reinforced composites during compression molding. The material was observed to exhibit normal anisotropy, and was assumed to obey Hill's anisotropic yield criterion with its associated flow rule. Finite element method was applied to solve the governing partial differential equations of the proposed model. The two-dimensional finite element model was applied to the plane strain compression of a plate with a stiffening rib to substantiate the effect of the anisotropy induced by reinforcing fibers in the charge on the material flow during deformation.

IV.1. Introduction

In what follows, a review of literature pertaining to the current modeling work is presented. Although SMC is a composite material which contains unsaturated polyester and chopped glassfiber as major constituents, most modeling studies conducted [11,13,15,21] treated SMC as a homogeneous isotropic continuum. The reported success of the models was limited to simple compression between two flat plate molds,
isothermal or non-isothermal. For thin SMC charges deforming between flat plates, the Hele-Shaw model or lubrication assumption captured the reality reasonably. However, Barone et al. [13] pointed out the nonhomogeneous anisotropic nature of SMC, and the difficulty of modeling the flow of such materials. That is, due to the fiber presence, the flow modeling becomes very difficult when the velocity normal to the direction of the plane parallel to the alignment of fibers is not negligible.

Smith et al. [28] was one of the first to study material flow during compression molding of SMC parts with a stiffening rib. The authors considered the composite material as an isotropic Newtonian fluid in their numerical model, and compared the numerical simulation results with experiments to investigate effects of rib geometry on the material flow around the rib entrance. Due to the treatment of a nonhomogeneous anisotropic continuum as a homogeneous isotropic Newtonian fluid, the authors were only able to give a qualitative description of experimental results. Hirai et al. [38] attempted to model anisotropic material flow of short fiber reinforced composites by differentiating the viscosity of a power law fluid between the plane direction and the normal direction. The authors demonstrated that material flow in the plate-rib type geometry was influenced by the anisotropy induced by the fibers. However, their model was validated only with hypothesized anisotropy factors. Thus, the authors gave only a qualitative account of the anisotropic mold flow. In their recent study, Hirai et al. [39] furthered their numerical modeling of anisotropic material flow of molding parts with a stiffening rib. They delineated anisotropy by initial anisotropic stiffness and anisotropic viscosity during flow.
The authors' study, however, was limited to numerical modeling without providing an anisotropy measurement procedure or experimental validation.

The deficiency of isotropic continuum model was well manifested in the study of the SMC compression molding presented in Chapter III. The molding results demonstrated that there are multiple phenomena to be modelled in compression molding of SMC. First, the flow of SMC was affected by the short fibers, and was different from that of the material without short fibers. The flow of SMC into the rib was hindered by fibers which acted as a bridge over the rib. The fiber bridge gradually collapsed as the molding load was continuously applied. Finally, the void contents were not negligible for materials with more than 10 % by weight fiber contents. These pieces of evidence suggested that modeling the deformation of short-fiber-reinforced materials as isotropic continuum would not be adequate.

In order to appropriately model the flow of fiber reinforced composite material in forming parts with stiffening rib, these three problems should be accommodated properly. As Hirai et al. [39] noted, the fiber bridge formation was not observed when the length of the reinforcing fiber was 6.35 mm. Same result was also observed in the molding experiment described in Chapter III. For SMC with 6.35 mm-10 % by weight fiber content, the estimated void content was less than 5 %. This study, therefore, intends to establish a continuum model which can account for the anisotropy resulting from the presence of the fibers. The treatment of fiber bridge formation and void content are not considered in the current study, and are left as future work.
IV.2. Governing Equations

It was assumed that the dimensions of SMC charge under consideration were large enough to make the continuum assumption still a valid one. As the fibers were distributed randomly in the direction of plane, the flow property of the material was considered to be isotropic in the plane direction, but exhibits anisotropy in the direction normal to the plane. Figure 4.1 shows the coordinate system of choice. X-axis represents the rolling direction of SMC, Z-axis is the transverse direction, and Y-axis is the direction normal to the plane spanned by X and Z-axes. The elastic response of SMC was neglected. In addition to these assumptions, the deformation process was assumed quasi-static, and body force was neglected. Then the following set of equations suffices to describe the mechanics of the deforming body.

- Equilibrium equations,

\[ \sigma_{ij,j} = 0. \]  \hspace{1cm} (1)

- Compatibility equation and incompressibility:

\[ \dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \text{ and } \dot{\epsilon}_{v} = \dot{\epsilon}_x + \dot{\epsilon}_y + \dot{\epsilon}_z = 0. \]  \hspace{1cm} (2)
Figure 4.1. Coordinate system and direction of anisotropy
• Yield criterion and constitutive equation

\[
2f(\sigma_{ij}) = f(\sigma_y - \sigma_z)^2 + g(\sigma_z - \sigma_x)^2 + h(\sigma_x - \sigma_y)^2 + 2l\tau_{yz}^2 + 2m\tau_{zx}^2 + 2n\tau_{xy}^2 \\
= 2 \bar{\sigma}^2
\]

\[
\dot{\varepsilon}_{ij} = \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}} \lambda \text{ constitutive equation}
\]

where \( \lambda \) is a nonnegative constant.

• Boundary conditions \((S_U + S_I = S)\):

The velocity vector \( \mathbf{u} \) is prescribed \((\mathbf{u}_i = \mathbf{u}^*_i)\) on part of the boundary \((S_U, \delta u_i = 0 \text{ on } S_U - \text{essential boundary condition})\).

The traction vector \( \mathbf{F} \) is prescribed \((f_i = f^*_i)\) on the rest of the boundary \((S_I, \sigma_{ij}n_j = 0 \text{ on } S_I - \text{natural boundary condition}, n_j \text{ is a unit outward normal})\).

IV.3. Yield Criterion and the Flow Rule

Finite element formulation for anisotropic material flow is derived in the following. A similar derivation for sheet metal forming can be found in Kobayashi et al.’s [40]. The yield criterion suggested by Hill in 1948 [41] is adopted in the current approach. The yield function \( f(\sigma_{ij}) \) has a dimensionless form of

\[
2f(\sigma_{ij}) = f(\sigma_y - \sigma_z)^2 + g(\sigma_z - \sigma_x)^2 + h(\sigma_x - \sigma_y)^2 + 2l\tau_{yz}^2 + 2m\tau_{zx}^2 + 2n\tau_{xy}^2 \\
= 2 \bar{\sigma}^2
\]
where $\sigma_{ij}$ is the stress tensor, $f$, $g$, $h$, $l$, $m$, and $n$ are anisotropy parameters characterizing the current state of anisotropy, and $\bar{\sigma}$ is the effective stress.

The quadratic yield criterion reduces to von Mises yield criterion when anisotropy $l=m=n=3f=3g=3h$. The state of anisotropy is represented by the three mutually orthogonal planes of symmetry at every point. Anisotropy parameters $f, g, h, l, m,$ and $n$ can be proportional, depending on the selection of $\bar{\sigma}$ as a material property.

In order to determine the anisotropy parameters, load is usually applied along three principal directions of anisotropy. If $Y_x$, $Y_y$, and $Y_z$ are simple compressive (or tensile) yield stresses in the three principal directions, the following expressions can be drawn from equation (4):

$$Y_x^2 = \frac{2\bar{\sigma}^2}{g + h}$$
$$Y_y^2 = \frac{2\bar{\sigma}^2}{f + h}$$
$$Y_z^2 = \frac{2\bar{\sigma}^2}{f + g}.$$

Taking inverse proportion and summing together, we have

$$\frac{f \pm g \pm h}{\bar{\sigma}^2} = \left( \frac{1}{Y_x^2} + \frac{1}{Y_y^2} + \frac{1}{Y_z^2} \right). \quad (5)$$

As mentioned previously, the selection of $\bar{\sigma}$ decides the proportional ratios of anisotropy parameters. The effective stress, $\bar{\sigma}$, should be determined from experiments, such as simple compression. For example, if the simple
compression (or tensile) yield stress $Y_x$ is defined as $\sigma$ in equation (5), then $f+g+h = (1 + \left(\frac{Y_x}{Y_y}\right)^2 + \left(\frac{Y_x}{Y_z}\right)^2)$. Other definitions are also possible, and the value of $(f+g+h)$ changes accordingly.

The hypothesis is also made that $f(\sigma_{ij})$ is the plastic potential for rigid viscoplastic material. Applying the associated flow rule the strain rate tensor is expressed as:

$$
\begin{align*}
\dot{e}_x &= \lambda \{ h (\sigma_x - \sigma_y) + g (\sigma_x - \sigma_z) \}, \\
\dot{e}_y &= \lambda \{ f (\sigma_y - \sigma_z) + h (\sigma_y - \sigma_x) \}, \\
\dot{e}_z &= \lambda \{ g (\sigma_z - \sigma_x) + f (\sigma_z - \sigma_y) \}, \\
\dot{e}_{yz} &= \lambda \{ f T_{yz} \}, \\
\dot{e}_{xz} &= \lambda \{ k T_{zx} (\dot{e}_{x} - \dot{e}_{y} - \dot{e}_{z}) \}, \\
\dot{e}_{xy} &= \lambda \{ n T_{xy} \}
\end{align*}
$$

where $\lambda$ is a proportionality constant and $\dot{e}_{ij}$ is the strain tensor.

The effective strain rate, $\dot{\epsilon}$, is defined such that the rate of and the plastic work per unit volume is $\dot{\sigma} = \sigma_{ij} \dot{e}_{ij} = \bar{\sigma} \dot{\epsilon}$. The definitions of the effective stress, $\bar{\sigma}$, and effective strain rate, $\dot{\epsilon}$, are written as:

$$
\bar{\sigma} = \sqrt{\frac{f(\sigma_y - \sigma_z)^2 + g(\sigma_z - \sigma_x)^2 + h(\sigma_x - \sigma_y)^2 + 2l_1 \tau_{yz}^2 + 2m \tau_{zx}^2 + 2n \tau_{xy}^2}{2}},
$$

$\dot{\epsilon} = \sqrt{2 \left( f \dot{\epsilon}_{yz} - \dot{\epsilon}_{xz} \right)^2 + g \left( \dot{\epsilon}_{xz} - \dot{\epsilon}_{xy} \right)^2 + h \left( \dot{\epsilon}_{xy} - \dot{\epsilon}_{yz} \right)^2 + \ldots + \frac{2 \dot{\epsilon}_{xy}^2}{2n}}$

It can also be derived that $\lambda = \frac{\dot{\epsilon}}{2\bar{\sigma}}$*. It should be noted that elastic deformation is neglected in all the derivations.

* Derivation of $\lambda$:

For isotropically hardening material, the plastic work-rate per unit volume is calculated by:

$$
\dot{\sigma} = \sqrt{\frac{f(\sigma_y - \sigma_z)^2 + g(\sigma_z - \sigma_x)^2 + h(\sigma_x - \sigma_y)^2 + 2l_1 \tau_{yz}^2 + 2m \tau_{zx}^2 + 2n \tau_{xy}^2}{2}},
$$

$\dot{\epsilon} = \sqrt{2 \left( f \dot{\epsilon}_{yz} - \dot{\epsilon}_{xz} \right)^2 + g \left( \dot{\epsilon}_{xz} - \dot{\epsilon}_{xy} \right)^2 + h \left( \dot{\epsilon}_{xy} - \dot{\epsilon}_{yz} \right)^2 + \ldots + \frac{2 \dot{\epsilon}_{xy}^2}{2n}}$

It can also be derived that $\lambda = \frac{\dot{\epsilon}}{2\bar{\sigma}}$. It should be noted that elastic deformation is neglected in all the derivations.
IV.4. Finite Element Formulation

The following finite element formulation is composed of three parts. First, the functional is derived from the weak form of the equilibrium equation. Second, the entire domain is approximated by finite elements. Finally, anisotropic flow rule for plane strain deformation is incorporated into the discretization form.

IV.4.1. Derivation of the Functional

The process of forming flat parts with a rib can be realized as plane strain deformation. In plane strain deformation, flow motion is independent of the third action. For example, given a coordinate system in Figure 4.1, Z is the independent axis and flow is everywhere parallel to a given plane (xy plane).

By multiplying arbitrary perturbation of the velocity vector $\delta u_i$ to equation (1) and integrating over the domain enclosed by $S$, the following weak form is obtained:

$$
\int_V \sigma_{ij,j} \delta u_i \, dV + \int_{S_f} (f^* - f) \delta u_i \, dS = 0 .
$$

(7)

$\dot{\omega} = \sigma_{ij} \dot{e}^{ij} \cdot \ddot{e} \cdot \dot{e}$. Using the flow rule

$\dot{\omega} = \sigma_{ij} \dot{e}^{ij} = \sigma_{ij} \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}} \lambda$.

Since $f(\sigma_{ij})$ is a homogeneous function of degree 2 (i.e., $\sigma_{ij} \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}} = 2f(\sigma_{ij})$),

$\dot{\omega} = \sigma_{ij} \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}} \lambda = 2 f(\sigma_{ij}) \lambda = 2 \sigma^2 \lambda = \dot{\sigma}^2$. Thus, $\lambda = \frac{\dot{\sigma}}{2\dot{\sigma}}$. 
Using integration by parts, the above equation becomes

\[
\mathbf{\nabla} \cdot \int_{V} \left\{ (\sigma_{ij} \delta_{ij})_{,j} - \sigma_{ij} \delta_{ij,ij} \right\} dV + \int_{S_f} f_i \delta u_i dS - \int_{S_f} f_j \delta u_j dS = 0. \tag{8}
\]

Utilizing the divergence theorem, the following virtual work statement is obtained:

\[
\int_{V} n_j \sigma_{ij} \delta u_i dV - \int_{V} \sigma_{ij} \delta u_i,j dV + \int_{S_f} f_i \delta u_i dS = 0 \tag{9-a}
\]

\[
\int_{V} \sigma_{ij} \delta u_{ij,i} dV - \int_{S_f} f_i \delta u_i dS = 0 \tag{9-b}
\]

\[
\int_{V} \sigma_{ij} \delta \varepsilon_{ij} dV - \int_{S_f} f_i \delta u_i dS = 0. \tag{9-c}
\]

It should be noted that the tensor notation is replaced by vectors and matrices from hereafter. The convention for the notation is as follows:

Tensor quantities are expressed with tensor indices (e.g., \( \sigma_{ij} \) and \( \varepsilon_{ij} \)),

Vectors are highlighted (e.g., \( \sigma \): stress tensor, \( \varepsilon \): strain rate tensor),

Matrices are bold faced and underlined (e.g., \( \mathbf{D} \) and \( \mathbf{B} \)).

In the vector and matrix form, equation (9-c) can be written as:

\[
\int_{V} \sigma^T \delta \varepsilon \, dV - \int_{S_f} \mathbf{F}^T \delta \mathbf{u} \, dS = 0 \tag{10}
\]
IV.4.2. Discretization

Equation (10) is discretized using the Galerkin method, where the shape function is a testing function itself. Nodal values are used for finite element approximation. The convention for the notation is as follows:

\[
\mathbf{v} = \mathbf{N} \hat{\psi} \quad (\mathbf{N} \text{ is a shape function matrix}), \\
\hat{\mathbf{e}} = \mathbf{B} \hat{\psi} \quad (\mathbf{B} \text{ is a differential operator matrix}),
\]

where \( \hat{\psi} \) represents discretized nodal velocity vector.

More detailed explanation is available by Kobayashi et al. [40].

The discretized form of equation (10) is:

\[
\int_{\Omega} \sigma^T \delta (\mathbf{B} \hat{\psi}) \, dV - \int_{S_f} \mathbf{F}^T \delta (\mathbf{N} \hat{\psi}) \, dS = 0 \quad (11).
\]

Making use of the arbitrariness of \( \delta \hat{\psi} \), equation (11) becomes

\[
\int_{\Omega} \mathbf{B}^T \sigma \, dV - \int_{S_f} \mathbf{N}^T \mathbf{F} \, dS = 0 \quad (12)
\]

IV.4.3. Anisotropic Plane Strain Derivation

The strain rate tensor \( \dot{\mathbf{e}} \) is derived below using flow rule:
In the plane strain case ($\dot{\varepsilon}_z = \dot{\varepsilon}_{yz} = \dot{\varepsilon}_{zx} = 0$), the above equation reduces to

\[
\dot{\varepsilon} = \frac{\dot{\varepsilon}}{2\sigma} \begin{pmatrix}
(g+h) & -h & -g & 0 & 0 & 0 \\
-h & (f+h) & -f & 0 & 0 & 0 \\
-g & -f & (f+g) & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & m & 0 \\
0 & 0 & 0 & 0 & 0 & n
\end{pmatrix} \begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_yz \\
\tau_zx \\
\tau_{xy}
\end{pmatrix}
\]

(13)

Namely, $\dot{\varepsilon} = D^{-1}(\dot{\varepsilon},\tilde{\sigma})\sigma$. Note that $D^{-1}(\dot{\varepsilon},\tilde{\sigma})$ is a singular matrix. Therefore, the condition of incompressibility is introduced to circumvent this difficulty. The incompressibility constraint can be achieved by applying the penalty method. The relationship between pressure (P) and volumetric strain rate ($\dot{\varepsilon}_v$) can be written as:

\[
\dot{\varepsilon}_v = \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z = -\frac{P}{K} = -\frac{\sigma_x + \sigma_y + \sigma_z}{3K} \left( \begin{array}{c}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{array} \right)
\]

(14)

(15)

where $K$ is a penalty constant.

It is assumed that the volumetric strain rate $\dot{\varepsilon}_v$ is so small that $\dot{\varepsilon}$ can be approximated as

$\dot{\varepsilon} = \{\dot{\varepsilon}_x+\dot{\varepsilon}_v, \dot{\varepsilon}_y+\dot{\varepsilon}_v, \dot{\varepsilon}_z+\dot{\varepsilon}_v, \dot{\varepsilon}_{xy}\}^T$. 

\[ \dot{\varepsilon} = \{ \dot{\varepsilon}_x + \dot{\varepsilon}_v, \dot{\varepsilon}_y + \dot{\varepsilon}_v, \dot{\varepsilon}_z + \dot{\varepsilon}_v, \dot{\varepsilon}_{xy} \}^T. \]

By imposing plane strain condition, \( \dot{\varepsilon}_z = 0 \), \( \sigma_z \) can be written as

\[ \sigma_z = \frac{(g - \frac{2\delta}{\varepsilon} K)\sigma_x + (f - \frac{2\delta}{\varepsilon} K)\sigma_y}{f+g+\frac{2\delta}{\varepsilon} K}. \]  

(16)

Making use of equations (14), (15), and (16), \( \dot{\varepsilon} \) is written as

\[ \dot{\varepsilon} = \begin{pmatrix} \dot{\varepsilon}_x + \dot{\varepsilon}_v \\ \dot{\varepsilon}_y + \dot{\varepsilon}_v \\ \dot{\varepsilon}_z + \dot{\varepsilon}_v \\ \dot{\varepsilon}_{xy} \end{pmatrix}. \]

\[ \begin{pmatrix} g+h+\frac{2\delta}{\varepsilon} K & \frac{(g - \frac{2\delta}{\varepsilon} K)^2}{1+g+\frac{2\delta}{\varepsilon} K} & \frac{(f - \frac{2\delta}{\varepsilon} K)(g - \frac{2\delta}{\varepsilon} K)}{f+g+\frac{2\delta}{\varepsilon} K} \\
\frac{(f - \frac{2\delta}{\varepsilon} K)(g - \frac{2\delta}{\varepsilon} K)}{1+g+\frac{2\delta}{\varepsilon} K} & \frac{(f - \frac{2\delta}{\varepsilon} K)^2}{1+g+\frac{2\delta}{\varepsilon} K} & \frac{(f - \frac{2\delta}{\varepsilon} K)}{f+g+\frac{2\delta}{\varepsilon} K} \\
\frac{(f - \frac{2\delta}{\varepsilon} K)}{1+g+\frac{2\delta}{\varepsilon} K} & \frac{(f - \frac{2\delta}{\varepsilon} K)^2}{1+g+\frac{2\delta}{\varepsilon} K} & 0 \\
0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{pmatrix}. \]

(17)

The inverted form of the equation (17) is equation \( \sigma = D\dot{\varepsilon} \):
The effective stress, \( \tilde{\sigma} \), and the effective strain rate, \( \tilde{\varepsilon} \), for a plane strain case are

\[
\tilde{\sigma}^2 = \frac{f(\sigma_y - \sigma_z)^2 + g(\sigma_z - \sigma_x)^2 + h(\sigma_x - \sigma_y)^2 + 2\tau_{xy}^2}{2} \quad (19)
\]

\[
\tilde{\varepsilon}^2 = \left( \frac{2}{(f+g+h)^2} \{ g^2(f+g)\varepsilon_y^2 + f^2(g+h)\varepsilon_x^2 - 2 fh \varepsilon_x \varepsilon_y \} + \frac{\varepsilon_{xy}^2}{2n} \right) \quad (20)
\]

From equation (12), the following is obtained by replacing \( \sigma \) with \( D \varepsilon \):

\[
\Phi = \int_V \mathbf{B}^T D (\varepsilon, \tilde{\sigma}) \varepsilon \, dV - \int_{S_f} \mathbf{N}^T \mathbf{F} dS = 0 \quad (21)
\]

\[
= \int_V \mathbf{B}^T D (\varepsilon, \tilde{\sigma}) \mathbf{B} \varepsilon \, dV - \int_{S_f} \mathbf{N}^T \mathbf{F} dS = 0 \quad (22)
\]
IV. 4.4. Integration of Volumetric Term

The matrix $D$ can be decomposed into $D_1$ and $D_2$.

$$
\Phi = \int_V \mathbf{B}^T D_1(\dot{\varepsilon}, \bar{\sigma}) \mathbf{B} \, v \, dV + \int_V \mathbf{B}^T D_2(\dot{\varepsilon}, \bar{\sigma}) \mathbf{B} \, v \, dV - \int_S \mathbf{N}^T F \, dS
$$

(23)

where $D_1$ and $D_2$ are defined as

$$
D_1(\dot{\varepsilon}, \bar{\sigma}) = \frac{\bar{\sigma}}{\dot{\varepsilon}} \begin{pmatrix}
\frac{2(4f+g+h)}{9(fg+gh+hf)} & -\frac{2(2f+2g-h)}{9(fg+gh+hf)} & 0 & 0 \\
-\frac{2(2f+2g-h)}{9(fg+gh+hf)} & \frac{2(1+4g+h)}{9(fg+gh+hf)} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2}{n}
\end{pmatrix}
$$

$$
D_2(\dot{\varepsilon}, \bar{\sigma}) = \frac{\bar{\sigma}}{\dot{\varepsilon}} \begin{pmatrix}
\frac{K\dot{\varepsilon}}{9\bar{\sigma}} & \frac{K\dot{\varepsilon}}{9\bar{\sigma}} & 0 & 0 \\
\frac{K\dot{\varepsilon}}{9\bar{\sigma}} & \frac{K\dot{\varepsilon}}{9\bar{\sigma}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
$$

The regular integration of $\Phi$ results in inaccuracy as we pose incompressibility [42,43]. The incompressibility constraint, $\dot{\varepsilon}_V = -\frac{P}{K}$, imposed on each element demands a stringent condition in the numerical calculation. It was found that the regular integration results in a "locking" problem. To resolve this difficulty, reduced integration is performed [44]. That is, the part of the matrix that enforces incompressibility in the matrix $D$, namely $D_2$, is computed with reduced integration. The rest of the matrix, namely, $D_1$, which tends to be significantly smaller is computed with regular integration. The validity of this approach is demonstrated by Hughes and Malkus [44,45].
Regular Gauss quadrature integration is performed for the first term in equation (23) and one point Gauss quadrature integration is performed for the second term in equation (23).

Matrix $\mathbf{D}_1$ is represented using the parameter $r$. For plane strain and planar isotropy case, $r$ is defined as $r = \frac{g}{n} = \frac{g}{f}$, and $\xi = f+g+h = 2 + \left( \frac{Y_x}{Y_y} \right)^2 = \frac{2f+4r}{f+1}$. Chapter V covers the derivation and the physical meaning of the ratio $r$.

\[
\mathbf{D}_1 = \frac{2}{9} \frac{\tilde{\sigma} (2+r)}{\tilde{\xi} \tilde{\dot{e}} (1+2r)} \begin{pmatrix}
5+r & - (1+2r) & 0 & 0 \\
- (1+2r) & 2+4r & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

(24)

In addition to $\tilde{\sigma}$ and $\tilde{\dot{e}}$, the other quantities needed to be measured are just $r$ and $\frac{n}{f}$ in order to determine the above matrix $\mathbf{D}_1$. The parameter $\frac{n}{f}$ was assumed 3. Chapter V discusses the details of the measurements. The expressions for the effective stress and the effective strain rate are also modified accordingly as:

\[
\tilde{\sigma}^2 = \frac{\xi}{2} \frac{1}{2+ r} \left( (\sigma_y - \sigma_z)^2 + r (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 2 \frac{n}{f} \tau_{xy}^2 \right)
\]

(25)

\[
\tilde{\dot{e}}^2 = \frac{2}{\xi} \frac{2+r}{(1+2r)^2} \left( (1+r) \dot{\varepsilon}_x^2 + 2 r^2 \dot{\varepsilon}_y^2 - 2 r \dot{\varepsilon}_x \dot{\varepsilon}_y + \frac{(1+2r)^2}{2 \frac{n}{f}} \dot{\varepsilon}_{xy}^2 \right)
\]

(26)
IV.4.5. Interpretation of D Matrix

The constitutive equation for a viscous materials can be expressed as

\[ \sigma'_{ij} = 2 \eta \dot{\varepsilon}_{ij} \]  \hspace{1cm} (27)

where \( \eta \) is a viscosity of the fluid.

Comparing equations (27) and (18), the matrix \( D \) in equation (18) can be construed as viscosity. From this perspective, the elements in matrix \( D \) can be replaced by coefficients of fluid viscosity.
CHAPTER V. CHARACTERIZATION OF THE MATERIAL

The objective of the work described in this chapter is twofold. First, the flow resistance of short-fiber-reinforced materials was measured via a compression test. Second, the anisotropic parameters introduced in the proposed mathematical model were determined via tension test. These two techniques are frequently used in metal forming [46] to characterize the flow behavior of metals during various manufacturing processes. In the current study, the possibility of adapting these techniques in the modeling of compression molded short-fiber-reinforced composites was explored.

V.1. Model Material

SMC with 6.35 mm long fibers and 10 % fiber weight content was selected as the model material for the following characterizations. As discussed in Chapter III, this particular composition of SMC exhibits about 4 % volume loss after 60 % deformation, and the fibers are short enough not to block the rib flow when molded with a 6.35 mm wide rib. These are necessary conditions since the proposed model deals with incompressible material only, and does not consider the fiber blocking effect.
Other materials that showed strong influence of factors other than anisotropy were not selected. For example, SMC with 12.7 mm long fibers with 10 and 20% fiber weight contents were not chosen because they both displayed over 10% volume loss and fibers hindering the rib flow.

V.2. Measurement of Flow Resistance

V.2.1. Background

The model material was considered a rate-dependent viscoplastic continuum. Thus, the material's response was characterized in terms of a stress-strain rate relation. Conventionally, a squeeze flow rheometer has been employed in determining the flow resistance of SMC [2,46,47]. However, in the current study, an anisotropic yield function suggested by Hill and its associated flow rule are employed because the observed mold filling could not be explained by the conventional isotropic approach. To this end, the compression test which is used in metal forming is applied to determine the flow resistance of short-fiber-reinforced composite materials for processing.

Figure 5.1 is a schematic of the compression test. The compression test was applied to determine flow stress data (true stress and true strain rate relationships) at various temperatures and strain rates. In order to prevent the influence of friction, appropriate lubrication was applied. Therefore, the mold surface was covered initially with mylar sheet, and automotive grease was
Load
Mold Closing Speed

Gap Height

Charge/Mold
Lubrication
(Grease)

Radius of the
Cylindrical Charge

\[
\dot{\varepsilon} = \frac{d \varepsilon}{dt} = \frac{dh}{h dt} = \frac{v}{h} \quad \sigma = \frac{Load}{Area}
\]

\[
\sigma = C (\dot{\varepsilon})^m
\]

Figure 5.1. Schematic of compression test
applied to simulate a no friction condition. The relationships for stress, strain, and strain rate are as follows:

stress: \[ \sigma = \frac{\text{Load}}{\text{Area}} \],

strain: \[ \varepsilon = \ln \frac{h_0}{h} \],

strain rate: \[ \dot{\varepsilon} = \frac{d \varepsilon}{dt} = \frac{dh}{hdt} = \frac{v}{h} \].

Here, \( h_0 \) and \( h \) represent the initial and instantaneous heights of the cylindrical charge, respectively. \( V \) and \( t \) represent mold closing velocity and time respectively.

The strain rate dependency of the material was assumed as follows:

\[ \sigma = C (\dot{\varepsilon})^m \]

\[ \ln(\sigma) = \ln C + m \ln (\dot{\varepsilon}) \].

Therefore the relationship is specified by parameters \( C \) and \( m \) which were estimated by taking the natural logarithm of the above equation.
V.2.2. Test Procedure and Results

The compression test was conducted on an Instron 1322 hydraulic testing machine. The load cell and the pressure transducer used in the test have been described in Tables 3.1 and 3.2. A set of flat molds made of Aluminium 6061 T20 were used. In order for the parameters C and m to be estimated, the strain rate during compression should be maintained to be constant. However, the control mechanism for the constant strain rate was not available. Thus, four different constant mold closing velocities were applied as an alternative. The obtained load versus displacement curve was utilized to calculate flow stress at a specific strain and strain rate.

A cylindrical charge of 38.1 mm diameter and 12.5 mm height was prepared using the model material: SMC with 6.35 mm long fibers with 10 % by weight fiber content. SMC sheets were stamped out using a circular cutter and a manual hydraulic press (88 Kg capacity). The individual SMC plies had a thickness of 1.5 ± 0.2 mm. Consequently 8 to 9 plies of circular SMC were stacked to make up 12.7 mm height. This stacked cylindrical charge was used without compaction.

Initially, the flow stress was obtained by incremental loading of 10 % deformation at a time up to 60 % deformation. A preliminary compression test was conducted using an SMC with 12.7 mm long fiber and 10 % by weight fiber content. The specimen with the initial height and diameter of 12.6 and 38.1 mm was compressed to the final height of 6.1 mm with the increment of 10 %
deformation at a time. The charge/mold interface was relubricated before each incremental loading. Figure 5.2 displays the results of this measurement. The data graphed are: center pressure measured from the flush mounted pressure transducer at the center of the top mold surface, flow stress obtained by dividing measured load with measured area (corrected stress), and flow stress obtained by measured instantaneous charge height and the volume constancy assumption (uncorrected stress). It should be noted that the SMC samples used were "compressible". As a result, the measured strain included the ram displacement necessary to compact the specimen, i.e. to squeeze the air out of the compressed specimens.

The corrected stress exhibited better agreement with the measured center pressure. However, the degree of agreement was very sensitive to the accuracy of the diameter measurement. It was realized that the measurement of accurate charge diameter was extremely difficult due to the charge's irregular circumference as it deforms. This discrepancy at the end of compression was not understood at first. As the charge was compacted to become denser, better agreement between pressure and stress was expected. It was later discovered that the specimen near the edge either buckled or cracked. Thus, due to the difficulty in accurate radius measurement and the observed buckling, the incremental loading was not attempted.

Instead of load measurement for estimating true flow stress, the center pressure was utilized as the surrogate for flow stress. Again, a preliminary lubrication test was conducted. Figure 5.3 is a comparison of pressure levels
Figure 5.2. Three way comparison of stress levels in incremental loading. 12.7 mm-10% by weight fiber content SMC. V=12.7 mm/min., Lubricated.
Figure 5.3. Comparison of the pressure levels obtained from pressure transducer between continuous loading and incremental loading.
between continuous and incremental compression using the same type of specimen in the above discussion. The charge height was reduced 20% at a time and the pressure was compared with that of a continuous run. As expected, the pressure level for the continuous run was slightly higher toward the end of the compression. However, the incremental compression frequently gave inconsistent readings due to the slight difference in the charge placement. Therefore, it was decided that the pressure reading of the continuous compression to 60% of charge height reduction was used as a surrogate for the measurement of flow stress. However, continuous compression may not be acceptable for high viscosity SMC (i.e., high fiber content SMC) due to the large difference of viscosity between SMC and lubricant.

Figures 5.4 and 5.5 show the flow stress versus strain and flow stress versus strain rate curves at four mold closing velocities obtained from the compression tests. The values of C and m at various strains were obtained from the logarithmic flow stress versus logarithmic strain rate using S.A.S (Statistical Analysis System). Figure 5.6 is the logarithmic graph used in parameter estimation. Table 5.1 summarizes calculated values of C and m at various strains.

The flow stress versus strain curve shown in Figure 5.4 is different from that of metals because it does not show an apparent yield stress. Considering the differences in rate dependency for various ram velocities, shown in Figure 5.4, it would have been useful to conduct two or more tests at ram velocities between 101.6 mm/min. and 50.8 mm/min., and include a "compaction" stage,
Figure 5.4. Flow stress versus strain curves obtained by compression test
Surface condition: lubricated (grease on mylar surface)
Closing velocities: 12.7, 25.4, 50.8, and 101.6 mm/min.
Charge diameter: 38.1 mm, Charge height: 12.7 mm
Figure 5.5. Flow stress versus strain rate curves obtained by compression test. Surface condition: lubricated (grease on mylar surface). Closing velocities: 12.7, 25.4, 50.8, and 101.6 mm/min. Charge diameter: 38.1 mm, Charge height: 12.7 mm.
Figure 5.6. Logarithmic flow stress versus strain rate curves obtained by compression test.
Surface condition: lubricated (grease on mylar surface)
Closing velocities: 12.7, 25.4, 50.8, and 101.6 mm/min.
Charge diameter: 38.1 mm, Charge height: 12.7 mm
Table 5.1. Values of C and m at various strains

<table>
<thead>
<tr>
<th>Strain</th>
<th>C</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>.18</td>
<td>.52</td>
</tr>
<tr>
<td>.1</td>
<td>.46</td>
<td>.59</td>
</tr>
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<td>.16</td>
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</tr>
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</tr>
<tr>
<td>.34</td>
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<tr>
<td>.98</td>
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<td>1.0</td>
</tr>
</tbody>
</table>
where with increasing strain or ram displacement the load or flow stress does not increase. The material's dependency on strain and strain rate is well illustrated by Figure 5.6 and Table 5.1. As the SMC charge was being compressed, the rate dependency index \( m \) has values increased from 0.5 to 1, which indicates that it becomes a Newtonian viscous material at large strains. The results summarized in Table 5.1 are utilized in the finite element model to provide the strain and strain rate dependent material flow behavior.

V.3. Measurement of Anisotropy

As discussed, Hill's yield function and its associated flow rule were applied to represent the anisotropic flow property of the short-fiber-reinforced material. Hill's yield function has been used in sheet metal forming studies to represent the anisotropy of the sheet metals. Compared to sheet metals, the SMC model material exhibits a far greater degree of dimensional irregularity, which caused difficulty in measuring the dimensional changes. This was reflected in the data presented this section.

V.3.1. Background

The anisotropy measurement of sheet material in plastic deformation is contingent upon the applied yield function. The following shows how a simple tension test can be used to measure the material anisotropy derived from the yield function.
V.3.2. Determination of \( r \)

As mentioned previously, the yield function suggested by Hill is a plastic potential representing the state of anisotropy of the material.

\[
2f(\sigma_{ij}) = f(\sigma_y - \sigma_z)^2 + g(\sigma_z - \sigma_x)^2 + h(\sigma_x - \sigma_y)^2 + 2\tau_{yz}^2 + 2m\tau_{xz}^2 + 2n\tau_{xy}^2
\]

\[
= 2\bar{\sigma}^2
\]

Applying its associated flow rule we obtain:

\[
de_{x} = d\lambda \{ h (\sigma_x - \sigma_y) + g (\sigma_x - \sigma_z) \}
\]

\[
de_{y} = d\lambda \{ f (\sigma_y - \sigma_z) + h (\sigma_y - \sigma_x) \}
\]

\[
de_{z} = d\lambda \{ g (\sigma_z - \sigma_x) + f (\sigma_z - \sigma_y) \}
\]

where \( d\lambda \) is a non-negative proportionality constant.

Figure 5.7 shows the coordinate system used in the current discussion. If a simple tension is applied in the z-direction (\( \sigma_x = \sigma_y = 0 \)) to a strip lying in the \((x,z)\) plane and cut parallel to the z-axis, the stress-strain relation reduces to the following form:

\[
de_{x} = d\lambda \{ -g \sigma_z \}
\]

\[
de_{y} = d\lambda \{ -f \sigma_z \}
\]

\[
de_{z} = d\lambda \{ (g + f) \sigma_z \} 
\]
Figure 5.7. Assumed coordinate system for a sheet of short-fiber-reinforced material
The ratio of thickness strain ($d\varepsilon_y$) to width strain ($d\varepsilon_x$) is

$$r_0 = \frac{d\varepsilon_x}{d\varepsilon_y} = \frac{\sigma_y}{\sigma_x}$$

If a simple tension is applied in the x-direction ($\sigma_y = \sigma_z = 0$) to a strip lying in the $(x,z)$ plane and cut parallel to the z-axis, the stress-strain relation reduces to the following form:

$$d\varepsilon_x = d\lambda \{ (h + g) \sigma_x \}$$
$$d\varepsilon_y = d\lambda \{ -h \sigma_x \}$$
$$d\varepsilon_z = d\lambda \{ -g \sigma_x \}$$

The ratio of thickness strain ($d\varepsilon_y$) to width strain ($d\varepsilon_z$) is

$$r_{90} = \frac{d\varepsilon_z}{d\varepsilon_y} = \frac{g}{h}$$

where the subscript 90 refers to the 90-degree direction.

Thus, $\frac{r_0}{r_{90}} = \frac{f}{g} = \frac{h}{f}$. Since the short-fiber-reinforced material is assumed to show no anisotropy in the plane, this ratio $\frac{r_0}{r_{90}} = \frac{h}{f} = 1$. That is, $r_0 = r_{90} = r$. 

V.3.3. Test Procedure and Results

Based on the derivation introduced in the previous section, uniaxial tension tests were conducted to measure the anisotropy factor via thickness and width strain measurements.

V.3.3.1 Tension Test

An Instron electric tensile tester 1362 with 8800 Kg load cell was employed in the determination of r. Figure 5.8 schematically shows the experimental set up. ASTM B557 (Standard Tension Test for Wrought Steel), D3039 (Standard Tension Test for Plastic), D638 (Standard Test for Matrix Material of Composites), and D882 (Standard Test Methods for Thin Sheets or Films) were referenced, although there is no available reference directly related to the materials discussed here. Figure 5.9 illustratively explains the measurement of r discussed in the previous section.

The anisotropy factor r is defined as the ratio between the width strain and the thickness strain. Prior to the test, SMC specimens were pulled to define a range where deformation is rather uniform. Anisotropy factor r was measured within a 2 % strain range because the specimen tended to break at larger strains. Thus, the r measurement conducted represents the material anisotropy only at a small strain range.
Schematic of Tensile Test

Figure 5.8. Schematic of tension test equipment set up
Figure 5.9. Schematic of r measurement

\[ r = \frac{\text{de}_z}{\text{de}_y} = \frac{\ln (W_0/W)}{\ln (t_0/t)} = \frac{a}{f} = \frac{a}{h} \]
Due to the expected non-uniformity of the specimen, accurate measurements of the thickness and the width change were quite difficult. Three methods were tried to measure dimensional changes of the tensile specimen. First, direct measurements of width and thickness were performed using digital micrometer (Mitotoyo, 6 Inch.). The r values shown in Table 5.2 were obtained by this method. The r values obtained by this method yielded values that are consistently smaller than 1.0. However, the value of r could change severalfold for the specimens from the same batch. Second, the deformed tensile specimens were photographed at different longitudinal strains, and the negatives were magnified using microscopes. This attempt did not enhance the precision of the measurement at all. Moreover, the irregularity of the specimens was more exaggerated by optical magnification. Finally, specimens were photographed using slide films at various strains for the magnification. The developed slides were projected in order to obtain magnification of 25 times. This also amplified the inconsistency of the tensile specimen. Therefore, the average r values obtained from the direct micrometer measurement were selected. The value of r used in the finite element simulation was 0.2, which is an average of the r values at two different strain rates.

Width and thickness strains were measured at three locations of the specimen during tests. Each value of r represent an average from these three measurements. Specimens were cut in rolling, transverse to rolling, and 45 degree to the rolling direction to assess the assumption of plane isotropy. Table 5.3 summarizes estimated r values under different conditions.
Table 5.2. Measured values of r for 6.35 mm fibers with 10 % by weight fiber content

<table>
<thead>
<tr>
<th></th>
<th>Strain rate (0.003)</th>
<th>Strain rate (0.012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-direction</td>
<td>0.39, 0.14, 0.10</td>
<td>0.24, 0.16, 0.13</td>
</tr>
<tr>
<td>90°-direction</td>
<td>0.26, 0.10, 0.49</td>
<td>0.13, 0.29, 0.15</td>
</tr>
<tr>
<td>45°-direction</td>
<td>0.30, 0.44, 0.36</td>
<td>0.13, 0.23, 0.21</td>
</tr>
<tr>
<td>average</td>
<td>0.29</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Results indicate that the $r$ values of the tested specimen were always less than 1.0. However, as indicated in Table 5.3, the data scattered over a wide range, and they were sample and location dependent. Further investigation is required to be more conclusive about the $r$ value of SMC charge. For future investigation, improvement in thickness measurement is desired.
CHAPTER VI. MEASUREMENT THEORY AND THE PROPOSED MODEL

This chapter attempts to lay a foundation pertaining to the particular conduct of measurement described in the study. The discussion builds around the notion of a quantity and a scale. These two concepts render a useful understanding toward the synthesis and the analysis of the measurement.

VI.1. Fundamentals of Measurement

A definition of measurement and the rudimentary terminology as they are used in the current study are presented.

VI.1.1. Function and Procedures of Measurement

The function of measurement is to assign symbols which are to represent certain attribute of a matter according to the specified rules. The matters of interest are referred to as objects, and measurement relates objects to symbols of certain structure. Numbers are usually used, among other symbols, because one can take advantage of their well established and clearly understood structure. As such, measurement enables a more precise description of the relational structure and the use of mathematics [49].
The use of mathematics is particularly important in scientific inquiry since it is a very effective tool for reasoning.

The three procedures that are employed in making measurements are ordering, unit counting, and inequality solving [50]. The simplest of the three is ordering, also referred to as ordinal measurement. This procedure is carried out by comparing the competing attributes of the objects. However, if the difference of the attribute between the entities are too small or obscure, it is difficult to establish orders. The second procedure of measurement is unit counting. A standard unit is constructed in order for the counting to be performed. The standard unit is constructed such that the attributes of objects are compared as how many copies of the standard unit would make them equivalent. This construction of a standard unit inevitably involves the description of the procedure which can lead to the same count over and over. The third kind of measurement procedure is inequality solving. This method is applicable to the situation where the number of objects are limited and standard unit construction is impractical.¹

¹ The current study is mostly concerned with the second procedure of measurement where the construction of a scale is required. The conduct of measurement, especially by use of a scale, conveniently transforms empirical relational structure to numerical structure such that the use of mathematics is possible.

* Krantz, pp1-6
**Kaplan, pp 184-186
VI.1.2. Lexicon of Terms in Measurement

An attempt is made to present rudimentary definitions of few terms that appear often in the current discussion. These terms are compiled from the previous work conducted by Kaplan [49], Ellis [51], and Miller [52].

Objects

Objects are differentiated from things. In the context of the current discussion, objects are things upon which the purpose or intension of the investigation is bestowed [52]

Structure

"A structure is an equivalence class of isomorphs." [49]
"Formally, a structure is one or more sets together with a relation defined on the sets." [52]

Representation

"Representation refers to the means by which two structures are compared. More specifically, it refers to the condition which must be met if one structure is to serve as a representation of a second." "A representation of a structure is a second structure together with a map (or set of maps) which defines how the elements of the two are related." [52]
Empirical relational structure
"This is a term often used in the theory of measurement and the theory of modelling. It refers to a set of objects or phenomena which are known to exist from empirical evidence." [52]

Measurement
Assignment of measures to magnitudes according to certain rules [49].
"Measurement is a special case of representation in which the represented structure is an empirical relational structure and the representing structure is (generally) a numerical one. Numbers are used to represent objects and the properties of numbers are used to represent relations. Generally, operational correspondences are used to represent an object numerically so that the relevant properties are preserved." [52]

Measure
The number assigned to a particular object in measurement. [49]

Attribute
The measurable property of a matter.

Scale
"rule of assignment," or "designation of the logical structure of the procedure of assignment" [49].
Quantity

The property of objects to which assignments are made. Quantity is the kind of property which permits the notion of degree. In other words, quantity is a kind of property of a matter which has magnitude [49,51].

Interpretation

"An interpretation is the association of the symbols and predicates of a formal theory with objects and relations of a structure in such a way that the statements of a theory are true in the structure." [52]

Calibration

Calibration is a process of comparing an object with a standard. Generally, secondary or tertiary standards are used in measurement. It is the purpose of calibration to check if the secondary or tertiary standards are equivalent to the original standard by comparing the standard used in measurement and the original one [49].

VI.2. Quantity and Scale

This section introduces the concepts of a quantity and a scale that base the discussion throughout. The recognition of the existence of a quantity and the understanding of a scale deserve quintessential importance in the present discussion.
VI.2.1. The Concept of Quantity

Both quality and quantity are kinds of properties that admit the notion of degree. However, the understanding and representation of a quality are often very difficult. Frequently, quality is a composite image of two or more quantities. On the other hand, quantity refers to the property which can be distinctively identified from other attributes of the matter. Nonetheless, just how to verify the existence of a quantity is not a simple task. The following tries to explain how the existence of a quantity is construed in the present study.

Ellis [51] argued that the existence of a quantity can be demonstrated by establishing linear ordering relationship among objects without describing rules for obtaining numerical assignments. In other words, the existence of linear ordering relation is an evidence for the existence of quantity. He took this position to conveniently escape the besiegement of the metaphysical burden of the empiricism. However, his argument is self-contradictory. On one hand, it makes sense to say that if one can establish a linear ordering relationship with respect to one particular attribute among objects, one may claim that the existence of the quantity is proved. On the other hand, how can one order things in any way without any rules which prescribe the comparison. Articulated or not, one applies a rule to come to a judgement of a comparison.

Ordering (or ordinal measurement to be more clear) can preserve the empirical relational structure better than unit counting because the empirical observation frequently is the composite image of two or more quantities. In this
regard, ordinal measurement is often a qualitative measurement compared to *unit counting*. When one cannot provide explicit rules regarding how the numbers are assigned, one applies an implicit rule one does not verbalize. When it is difficult to specify what constitutes the particular empirical information, one is apt to resort to ordinal measurement. Should such specification become necessary, one can resort to the measurement of an associated property. For example, in measurements which involve human beings, one tends to select an associated physical quantity to stay away from the theoretical difficulty.* However, when one constructs a numerical structure using unit counting of the corresponding empirical relational structure even when the identification of a quantity is not possible, the information presented by the empirical observation may be lost.

In the current study, establishing an order relation is considered as another kind of measurement, and is called ordinal measurement. As such, the problem of "how the existence of the quantity could be vindicated ?" is yet to be resolved. In order to present the standpoint for such argument and to successfully defend it, three philosophical positions are briefly contemplated: realism, operationalism, and constructive empiricism.

The critique on naive realism that the assertion of the pre-existing property of a matter in isolation from the surroundings fails to recognize the basically relational nature of the measurement is probably only partially valid [50]. More fundamentally, measurement deals with empirical facts and seeks a

* Krantz, pp32
contingent account for the observed phenomenon. This realist's assertion of the pre-existing properties does not help the conduct of measurement since a pragmatic account providing is of immediate concern.

On the other hand, operationalism can be very convenient when the circumstance is such that the identification of a quantity is extremely difficult. Moreover, operationalism fundamentally does not give an ontological status to a quantity. It recognizes only the measuring operations and their results [51]. Any discussion of the property of the matter comes after performing measuring operations. Operationalism recognizes the relational nature of the measurement properly, but it may fail to establish a consistent theory. It is important to realize that the reason for this difficulty in quantity identification is frequently related to the fact that the contingent empirical fact is a composite image of two or more quantities. Thus, in spite of its convenience, the operationalism poses a great deal of danger.

Finally, the view of the constructive empiricism is adopted in the current study for the following reasons. As measurement deals with empirical facts, providing the pragmatic account is of primary importance. As such, one should strive to see in what range the particular construction of the numerical structure provides an adequate explanation of the phenomenon under investigation [53].

* The validity of the number assigned to a quantity is referred to as the analyticity in this discussion. One must also see to it that the building blocks of the measurement is such that the development of the consistent theory

* van Fraassen, pp 42-53.
development is possible. These goals can be achieved by paying constant attention to the measurement's explanatory capability.

In summary, an ontological status is granted to a quantity on the metaphysical level. The matter of great concern is to investigate why things can be arranged by the way suggested by the particular conduct of measurement. This involves the examination on the ranges where the numerical structure resulted from a measurement is valid.

VI.2.2. The Concept of a Scale

A scale refers to the particular rule by which numerical assignments are arrived. In other words, a scale predicates the structure of a particular measurement so that one can interpret the numbers assigned by the measurement operation [49]. The construction of a scale means the procedure of establishing the rule to define a standard unit rather than the unit itself. Therefore, the validity and the adequacy of a measurement is greatly affected by the way a scale is constructed.

VI.2.3. Scale and Calibration

Calibration and scale are sometimes confused in the practical conduct of measurement. This brief description is intended to distinguish these two different concepts. The term calibration refers to the conduct of comparison according to the same rule used in the scale construction. The pseudo
standard unit to be used in actual measurement is compared with the presumed standard. Rarely is the case, when one makes the comparison with the genuine standard. What is usually available for comparison is the secondary or tertiary standard which is presumably equivalent* to the genuine standard. As quoted by Kaplan [49] "the purpose of calibration is to equate the number assignments, which are made on a given occasion, to the number assignment that would be made of the same things on any similar occasion with the same instructions."

VI.2.4. Application of Arithmetic and a Scale

This section portrays the relation between arithmetic and the scale. Arithmetic operations are performed on the numbers assigned by the rule of measurement. The understanding of the role of scale in arithmetic application is useful in conducting measurement toward its synthetic end. It guides one to construct a measurement which is logically valid.

Arithmetic application can be delineated as pure and applied. Pure arithmetic is a precise, perfect, formal and conceptual system whereby the conditions for the possibility of applying such arithmetic are ensured [51]. Applied arithmetic is a system whereby possibility of application is not logically guaranteed. For example, suppose that a+b=c is an arithmetic proposition. If a+b=c is a pure arithmetic proposition, it is known to be true or false a priori

* Kaplan, pp 178-180, and 186-187
** Ellis, Chapter I
with absolute certainty. However, the proposition of applied arithmetic $a+b=c$ is not known true or false a priori. If, for example, applied arithmetic proposition $a+b=c$ represents a concatenation of lengths $a\text{ cm} + b \text{ cm} = c \text{ cm}$, the truth or falsity of this arithmetic proposition is an empirical question. Consequently, the applied arithmetic proposition is an empirical proposition. In other words, applied arithmetic is used in correlating the results of measurements under certain conditions [50]. Applied arithmetic, thus, is a contingent fact, and it is subject to an interpretation for the truth to be judged. This process of interpretation involves examination of scales used in measurement.

There is an important question about the analyticity in applied arithmetic. For instance, the density of water measured by using 5 lbs. and 5 tons of water would be different. The numerical law that describes density is expressed as $\rho$ (density) = $m$ (mass) / $v$ (volume). The volume measured with 5 tons of water is different from that measured with 5 lbs. of water because water exhibits small amount of compressibility. For the numerical value of the density of water to be valid, the range of the mass that is applied should be known. As such, one has to specify the range where the analyticity of the particular measurement holds. This is different from the measurement of length and mass. This distinction, however, is not the fundamental characteristics of certain quantities because there are alternative ways in arriving at the numeral assignment of the same quantity. The propositions resulting from the measurement of temperature and density are different from those of length and mass measurements. The procedures used in measuring length and mass are called fundamental
measurement. The procedures used in density and temperature measurement are called derived and associative measurement respectively.

**VI.3. Classification of Measurement Based on Scale**

Classification of measurements is presented according to the scales. The reckoning of the analyticity that results from the particular conduct of measurement is an important issue since it is a good indicative of the limits of the measurement.

**VI.3.1. Fundamental Measurement**

Fundamental measurement, by definition, does not require measurements of any other quantity*. The analyticity is guaranteed regardless of the range. Fundamental measurement is also referred to as extensive or direct measurement in the sense that the existence of other quantities is not required. Examples of fundamental measurement are length and mass among others. There are two conditions that are necessary for the fundamental measurements** [51] to be possible. The first condition is the existence of the independent criterion for ordering. The second condition is the existence of the standard unit (Archimedean postulate). That is, if an order among objects can be established by a particular quantity shared by them without having to rely on other quantities, and a well bound standard unit can be constructed,

---

* Ellis, pp53-57  
** Ellis, pp 74-75
fundamental measurement is possible. Trivial as these two conditions may
look, they qualify well as necessary conditions. Construction of a fundamental
measurement is the most difficult of the three types introduced here.

VI.3.2. Derived Measurement

Derived measurement is considered here as a preliminary to the
discussion of associative measurement. Derived measurement arrives at the
numerical assignment by way of measurement of other quantities that are
measured fundamental or otherwise. The quantity measured by derived
measurement is related to other quantities by a numerical law, empirical or
logical [49,51]. As such, derived measurement is an indirect measurement.
The examples for the numerical laws are Ohm's law of electrical resistance and
Newton's law of viscosity. There are numerous examples of derived
measurements, such as, measurements of density, viscosity, and electrical
resistance.

Derived measurements can extend the conceptual span beyond the
reach originally conceived by fundamental measurement by the use of the law
[49]. However, this expansion of meaning hinges upon the discovery of such
laws. The discovery of the law frequently goes side by side with the
construction of derived measurement. As discussed previously with the
measurement of water density, the analyticity of the measures obtained by
derived measurement is not guaranteed. Depending on the amount of water
used in measurement, same numerical law can yield different number as the density of water.

VI.3.3. Associative Measurement

Associative measurement is often considered as part of derived measurement. However, there is a subtle yet significant difference between derived and associative measurements. A simplistic view is that the existence of numerical law is of paramount importance for the former, and the selection of an associated quantity is the matter of great concern for the latter. Numerical laws are yet required in associative measurement to relate quantities that appear in the associated property of the choice. In comparison, the numerical laws in the derived measurement relate the quantity of interest to other quantities that are measurable otherwise.

For example, it is usually conceived that the temperature scale represents even divisions between the freezing point and boiling point of water. The arithmetic operation can only be validly performed on the differences of the assigned numbers, not the numbers themselves*. That is, the temperature of 30 °F is not three times as high as the temperature of 10 °F. It indicates that 30 °F is at three times the distance from 0 °F than that of 10 °F. On the other hand, one material can be three times as denser than the other. Often, the properties represented by associative measurement are more likely to be intensive properties.

* Kaplan, pp 195
If the numerical law that relates quantities in the associative property satisfies the *operational criteria*, associative measurement is possible. That is, if the procedure of measurement can comply with the physical phenomena that are observable, the measurement passes the *operational criterion*. However, the view taken in the present study is such that the analyticity of the measures generated by the particular associative measurement should be examined carefully. This investigation is helpful in probing the limits.

**VI.4. Yield Function and the Associative Measurement**

The yield function contingent upon the model described in Chapter IV and V are explicated in this section in the context of associative measurement.

**VI.4.1. Yield Function in Plasticity**

The following is concerned with the identity and the role of yield function in the present study. To this end, a discussion of plasticity where the use of yield function was first started is presented.

Plasticity is concerned with the motion of a body during the course of an irreversible deformation upon certain loading conditions. This irreversible deformation takes place when the material is stressed beyond its reversible limit (elastic limit). However, there are aspects of the plastic deformation that make the mathematical approach rather difficult. That is, the plastic deformation is
not only affected by the initial and the final stage of the deformation, but also is influenced by the history as to the final stage is achieved. Therefore the criterion which rigorously describes the onset of the plastic deformation is extremely difficult to establish. Also, there is no reasonably simple numerical law which relates stress and strain as opposed to Hookean law in elasticity. The treatment of material property such as anisotropy is extremely difficult for these reasons [54]. The practical objective of the study of plasticity is to better understand the deforming process so that required load to achieve desired geometry, manufacturability, and defect formation can be predicted. The models require experimental validation to be put to use for any practical purposes. However, the three problems previously mentioned should be overcome. They are: describing the onset of the plastic deformation, devising a law to relate stress and strain, and reflecting the material property such as anisotropy.

Yield criterion is a numerical law which describes the onset of plastic deformation. Since it is construed that the phenomenon of plastic yielding is caused by the imposed stresses, the yield function is composed of components of stress [54]. A flow rule is a numerical law based on empirical observation, which depicts the relation between the stress and the increment of strain. By combining yield criterion and the flow rule, the treatment of the first two problems come in handy [55]. Since the empirical knowledge showed that

* Dieter, pp77-81. The debate about the influences of deviatoric stress an static stress are omitted to avoid unnecessarily length discussion.

** \[ \frac{\partial f(c_{ij})}{\partial \sigma_{ij}} d\lambda, \] where \( f(c_{ij}) \) is a yield criterion and \( d\lambda \) is a nonnegative constant.
the ratio between the increment of strain and the stress is a non-negative constant, a mathematical postulate is made such that the increment of the components of the plastic strain is obtained by taking partial derivatives of the scalar function. The yield criterion of choice is a scalar function, and is called plastic potential. This way of treating plastic deformation has been proven empirically adequate in metal forming which mainly deals with materials that show no direction dependent properties.

When the material does not exhibit direction dependent properties (stress strain relation in this discussion), the second invariant of the stress tensor is adopted as yield criterion. The reason for this choice is that for isotropic material the function used as the criterion should be independent of the choice of the axes. Thus, the term invariant represents the invariance with respect to the rotation. The assumed yield criterion presupposes that the plastic yielding takes place if the combined stress states reach a certain constant value. The constant can be now estimated by the simple mechanical testing such as uniaxial tension in which only one component of the stresses is present. If an isotropic material is subject to a uniaxial tension, the stress strain relation should not be affected by the direction of loading and the lateral contraction is the same in every direction perpendicular to the tension loading direction. This phenomenon works as an operational criterion. This leads to the conclusion that the yield criterion should be a function of invariants of the

\[ J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 6\tau_{yz}^2 + 6\tau_{zx}^2 + 6\tau_{xy}^2 \]

\[ (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 6\tau_{yz}^2 + 6\tau_{zx}^2 + 6\tau_{xy}^2 = \text{Constant} \]
stress tensor. For isotropic material, the criterion which uses the second invariant of the stress tensor \( J_2 \) is used and is suggested by von Mises. The use of \( J_2 \) as yield criterion results in reasonable agreement with empirical knowledge. The quadratic function \( J_2 \) meets the formal requirement of being independent of the axes system. It also satisfies the operational criterion for isotropic material (i.e., equal contraction in two directions perpendicular to the simple tensile loading situation).

The anisotropic yield function adopted in the current study is the one suggested by Hill in 1949. This is a convenient generalization of the second invariant of the stress tensor \( J_2 \) which is used for isotropic material such as metals. The parameters of non-unity are multiplied to each term to account for the directional variation. This choice of yield criterion satisfies the operational condition. That is, the difference in the amount of contraction in two directions perpendicular to the loading direction can be represented by Hill’s yield criterion as presented in Chapter V. However, no other form of restriction is existent regarding the form and the conditions adequate for yield criterion. Consequently, the anisotropic yield function by Hill is valid only in loose sense for the proposed model. Hill’s anisotropic yield criterion is not designed to reflect the nature of the composite material which is the source of the anisotropy. It merely seems to satisfy the operational criterion. The following discusses a postulate on why the operational criterion is met.

\[
F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = \text{Constant}
\]

* Dieter, pp77

**** \( F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = \text{Constant} \)
In sheet metals which exhibits rather obvious anisotropy, the source of anisotropy is the differential hardening in different directions. In the type of short-fiber-reinforced composites, the anisotropy results from the alignment of the fibers and the presence of resin mixed in the compound. As illustrated in Figure 6.1, the displacement in the direction perpendicular to the fiber alignment represents the reduction of the amount of resin between fiber strands. In the direction of fiber alignment, the displacement is difficult to take place due to the fibers' resistance against motion. This simple hypothesis about the motion of the composite material under given loading condition agrees with the observed phenomenon.

A measure which represents the degree of anisotropy is given as the ratio between the increments of strains in thickness and width directions. Depending on the amount of resin between the fiber strands, the anisotropy varies. As such, the measure of anisotropy should vary according to the stage of deformation. However, the measure of anisotropy is treated as constant for mathematical simplicity. This little hypothetical story enables the adoption of Hill's yield criterion in the current study. The view taken in this study is that, without a plausible explanation behind the observed physical phenomenon, the mere satisfaction of the operational criterion is an acceptable ground for the choice of measurement.
Figure 6.1. Idealization of the motion of the composite body under compression
VI.4.2. Anisotropic Yield Function and the Associative Measurement

The yield criterion provides a means of conveniently avoiding the theoretical difficulties in plasticity. Together with the use of the flow rule, the yield criterion is involved in describing the onset of the plastic yielding, stress-strain relation, and anisotropy. However, only a weak explanation is provided as to why the operational criterion is satisfied. Like temperature, the anisotropy is not an extensive property of the material. In the measurement of temperature, the notion of thermal equilibrium is borrowed and the change of heat state is equated with the change of temperature. The concept of the onset of the plastic yielding in plasticity is what the notion of heat state to temperature measurement. Because the anisotropy is not to be directly dealt with, plastic yielding replaces its place as an associated property.

The modeling approach attempted in this study utilizes measurement devised and developed in metal forming. As described, the mathematical description of the plastic deformation is a difficult task even for metals. The current study strives to apply it to mathematically model the anisotropic behavior of composite material. During the course of this attempt, how one should synthesize the measurement which would arrange things according to the particular concept of anisotropy is reviewed in the light of measurement theory.
CHAPTER VII. RESULTS, DISCUSSIONS, AND MODEL VALIDITY

This chapter uses the mathematical model and the experimental results discussed in the previous chapters to assess the validity of the anisotropic flow modeling. A finite element code derived from ALPID (Analysis of Large Plastic Incremental Deformation) was developed in order to implement the finite element formulation discussed in Chapter IV. Validity assessment was attempted by comparing the results between the finite element model and the experimental results introduced in Chapter III.

VII.1. Background

As explained, the model material selected was an SMC with 6.35 mm long fibers with 10 % by weight fiber content since it exhibited small amount of volume loss and no fiber blocking. The flow resistance and the anisotropy factor measured in Chapter V were used in the finite element simulation. The same conditions applied in the molding experiments were applied in the simulation. The comparisons were made between the predicted and measured load, and the predicted and measured deformed shape.
VII.2. Comparison between the Proposed Model and the Experimental Results

The following compares the results between the simulation based on the constructed finite element model and the experimental molding results that were presented in Chapter III. Deformed shape, load level during compression, and final strain distribution were compared to assess the validity of the proposed model.

VII.2.1. Simulation Conditions

Then the following set of equations were used to describe the mechanism of the deforming body. The conditions applied in the finite element simulation were pertinent to these equations and their parameters.

• Equilibrium equations,
  \[ \sigma_{ij,j} = 0. \]

• Compatibility equation and incompressibility:
  \[ \dot{\varepsilon}_{ij} = \frac{1}{2} ( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} ), \text{ and } \dot{\varepsilon}_v = \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z = 0. \]

• Yield criterion and constitutive equation
  \[ 2f(\sigma_{ij}) = f(\sigma_y - \sigma_z)^2 + g(\sigma_z - \sigma_x)^2 + h(\sigma_x - \sigma_y)^2 + 2\tau_{yz}^2 + 2m\tau_{zx}^2 + 2n\tau_{xy}^2 \]
  \[ = 2 \bar{\sigma}^2 \]
\[ \dot{\epsilon}_{ij} = \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}} \lambda \text{ constitutive equation} \]

where \( \lambda \) is a nonnegative constant.

• Boundary conditions \((S_u + S_f = S)\):

The velocity vector \( u \) is prescribed \((u_i = u_i^*)\) on part of the boundary \((S_u, \delta u_i = 0 \text{ on } S_u - \text{ essential boundary condition})\).

The traction vector \( F \) is prescribed \((f_i = f_i^*)\) on the rest of the boundary \((S_f, \sigma_{ij}n_j = 0 \text{ on } S_f - \text{ natural boundary condition, } n_j \text{ is a unit outward normal})\).

Interfacial friction was handled with constant shear friction model, \( \tau = m \kappa \) [40]. Here, \( \tau \) is the frictional stress, \( m \) is the friction factor, and \( \kappa \) is the local yield stress. Friction was given as traction at the charge/mold interface. Boundary conditions are illustrated in Figure 7.1.

The eight experimental molding conditions shown in Table 3.4 were applied in the simulation using the proposed anisotropic finite element formulation. The flow resistance given in Table 5.1 was incorporated in the proposed model to provide the flow stress-strain-strain rate relationship of the model material. The average of the anisotropy factor, \( r = 0.2 \), was used as described in Chapter V. The conditions applied to the finite element simulation are summarized in Table 7.1. As indicated in the table, \( s(1) \) through \( s(8) \) represent moldings with the conditions specified.
Figure 7.1 Plane strain finite element mesh system of the initial charge

Number of nodes = 270

Number of elements = 255
Table 7.1.  Summary of simulation conditions

<table>
<thead>
<tr>
<th>Unlubricated</th>
<th>Lubricated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant mold closing speed of $V=12.7$ mm/min.</td>
<td>Constant mold closing speed of $V=12.7$ mm/min.</td>
</tr>
<tr>
<td>30% reduction of charge height $(s_1)$</td>
<td>30% reduction of charge height $(s_5)$</td>
</tr>
<tr>
<td>60% reduction of charge height $(s_2)$</td>
<td>60% reduction of charge height $(s_6)$</td>
</tr>
<tr>
<td>Constant mold Closing speed of $V=50.8$ mm/min.</td>
<td>Constant mold Closing speed of $V=50.8$ mm/min</td>
</tr>
<tr>
<td>30% reduction of charge height $(s_3)$</td>
<td>30% reduction of charge height $(s_7)$</td>
</tr>
<tr>
<td>60% reduction of charge height $(s_4)$</td>
<td>60% reduction of charge height $(s_8)$</td>
</tr>
</tbody>
</table>

anisotropy factor $r = 0.2$
friction factor $m$ for the constant shear model
$m=0.0$ for the lubricated condition
$m=0.8$ for the unlubricated case

$s(1)$ through $s(8)$ designate each molding case
The layout of the plane strain finite element mesh system is given in Figure 7.1. Due to the symmetry, only the right half of the charge cross-section was simulated. Eight Patran generated 255 element mesh systems were necessary to simulate s(1) through s(8). Although the actual rib corner radius was zero, the rib corner radius in simulation was assumed 1 mm in order to avoid too many remeshing. Remeshing was still necessary due to severe element distortion around the rib corner for s(2), s(4), s(6), and s(8) at the later stage of the simulation. Constant shear model was used for the friction model. The friction factor for lubricated cases was assumed as $m=0.0$ since there was only a small difference between measured center pressure and the calculated stress ($\text{Load/} \text{Area}$). The friction factor for the unlubricated cases was assumed as $m=0.8$ for it was used in the previous studies [48,56]. However, the exact charge/mold interface condition and an adequate model to represent it should be further explored in the future.

The finite element code based on the formulation presented in Chapter IV was run on CRAY/YMP-2. The pre and post processing of the calculated results were done using PATRAN and IDEAS on micro VAX and VAX 8550, respectively.

**VII.2.2. Comparison in Terms of D**

This section evaluates the constructed model by comparing the deformed shapes between predicted and measured in terms of the parameter $D$ introduced in Chapter III.
The parameter $D$ represents the ratio between the half width of the deformed charge ($W$) and the depth of the material flown into the rib. $D$ is used as an index of the flow propensity of the short-fiber-reinforced composite at the particular stage of deformation. The value of $D$ is contingent upon the particular stage of deformation (e.g. effective strain), thus $D$ is not a constant for a given material.

Figures 7.2 is the deformed geometry of the model material obtained from the finite element simulation at 30 % reduction in charge height under unlubricated surface condition. The friction factor of $m=0.8$ was applied to represent the unlubricated surface condition. The bulging flow front in Figure 7.2 shows the characteristics of the deformation under high friction. However, bulging flow front was not as clear in the actual experimental molding since multi-layered charge did not have clear silhouette. Figure 7.3 is a result of lubricated compression at 60 % reduction in charge height. The friction factor of $m=0.0$ was applied to represent the lubricated surface condition. The flat flow front shown in Figure 7.3 exhibits the characteristics of the well lubricated deformation. The friction factors remained constant throughout the simulation whereas in reality the friction increased for the lubrication layer became thinner. The realistic representation of the charge/mold interfacial friction condition is a very important problem yet to be solved. However, the phenomenon of friction was not the immediate concern of the study presented here. Values of the parameter $D$ obtained from the finite element simulation were compared with experimentally measured ones.
Figure 7.2. Deformed geometry for 6.35 mm long fiber-10 % by weight fiber content SMC. V=12.7 mm/min., unlubricated (m=0.8), 30 % deformation.
Figure 7.3. Deformed geometry for 6.35 mm long fiber-10 % by weight fiber content SMC. \( V=12.7 \text{ mm/min.} \), lubricated \((m=0.0)\), 60 % deformation.
The results of the finite element simulation are summarized in Table 7.2 in comparison with the molding results of Chapter III. The values of D show good agreement except for the lubricated cases at 60 % deformation (by reduction in charge height). In the case of lubricated molding up to 60 % deformation, the frictional effect was construed to be significant. When lubrication effect was estimated using a cylindrical charge with 38.1 mm diameter, the difference between measured center pressure and the average stress obtained from load measurement was small. In the case of plane strain molding, the initial charge area (50.8 X 101.6 mm) where the load was applied was larger compared to the cylinder compression. The ratio of the areas between the cylinder compression and the plane strain compression was $1/4.52$ (1140 mm$^2$ divided by 5161 mm$^2$). As the plane strain compression progressed the increased area incurred the stronger influence of the charge/mold interface friction. Therefore, experimental results at 60 % deformation represented a poor surface lubrication, while model simulation assumed a perfect lubrication.

The anisotropic finite element simulation was also compared with isotropic finite element formulation where the anisotropy factor $r$ was 1.0. A lubricated compression and an unlubricated compression at 12.7 mm/min. closing velocity (namely, cases s(1), s(2), s(5), and s(6)) were simulated and compared with the experimental results. In all cases, the anisotropic formulation gave a better prediction to the experimental observations.
Table 7.2. Comparison of values $D$ for the 6.35 mm long -10 % by weight fiber content SMC model material.

<table>
<thead>
<tr>
<th>Symbol Designation</th>
<th>$D = \frac{T}{W}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial charge</td>
<td></td>
</tr>
<tr>
<td>deformed charge</td>
<td></td>
</tr>
</tbody>
</table>

Values of $D$ obtained for 6.35 mm 10% SMC

<table>
<thead>
<tr>
<th>$v$ (mm/min)</th>
<th>Experiment</th>
<th>Simulation Anisotropic</th>
<th>Simulation isotropic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v=12.7$</td>
<td>0.118</td>
<td>0.117</td>
<td>0.109</td>
</tr>
<tr>
<td>($v=50.8$</td>
<td>0.118</td>
<td>0.101</td>
<td>0.109</td>
</tr>
</tbody>
</table>

Unlubricated | Lubricated

<table>
<thead>
<tr>
<th></th>
<th>30%</th>
<th>60%</th>
<th>30%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>0.118</td>
<td>0.195</td>
<td>0.055</td>
<td>0.100</td>
</tr>
<tr>
<td>60%</td>
<td>0.185</td>
<td>0.178</td>
<td>0.069</td>
<td>0.157</td>
</tr>
<tr>
<td>30%</td>
<td>0.109</td>
<td>0.185</td>
<td>0.029</td>
<td>0.054</td>
</tr>
<tr>
<td>60%</td>
<td>0.183</td>
<td>0.178</td>
<td>0.051</td>
<td>0.060</td>
</tr>
</tbody>
</table>
The predictions of the load levels during compression molding from the finite element simulation were also compared with the experimental observations. This comparison of load levels supplements the quantitative assessment of the validity of the present anisotropic modeling approach.

Figure 7.4 is a three way comparison of load level versus stroke between the experimental data and the simulation results. This is the case designated as s(1) for which the D value comparisons were made. The predicted load levels by both isotropic and anisotropic simulations were higher than the experimental data. The use of the friction factor $m=0.8$ was probably an overestimation of the actual situation. The value of $m=0.8$ was estimated originally from the ring compression test of silly putty [48]. However, the ring compression test was not applicable to SMC since accurate measurement of the ring diameter was not possible due to the irregularity of the SMC specimen.

Figure 7.5 compares the load level of the lubricated compression at the 60% reduction in charge height. As expected, the experimentally obtained load curve rises higher than the curves obtained from the simulation.

Figure 7.6 (a) compares two experimental load readings of the lubricated and unlubricated interfacial conditions at the mold closing velocity of 12.7 mm/min. up to 60% reduction in charge height.
Figure 7.4. Comparison of load versus stroke diagram for 6.35 mm long fiber-10% fiber weight content SMC. V=12.7 mm/min., unlubricated (m=0.8), 30% deformation.
Figure 7.5. Comparison of load versus stroke diagram for 6.35 mm long -10 %by weight fiber content SMC. $V=12.7$ mm/min., lubricated ($m=0.0$), 60 % deformation.
Figure 7.6. (a) Comparison of experimental load readings between lubricated and unlubricated conditions for $v=12.7$ mm/min. at 60% deformation.
(b) Comparison of load predictions by simulation between lubricated ($m=0.0$) and unlubricated ($m=0.4$ and $m=0.8$) conditions for $v=12.7$ mm/min. at 60% deformation,
In simulation, the friction factors $m=0$, $m=0.4$, and $m=0.8$ were used to represent lubricated and two different unlubricated conditions, respectively.

The comparison of the predictions is given in Figure 7.6 (b). The load prediction for the lubricated case with friction factor $m=0.0$ showed a good agreement. For the unlubricated case, the load was dependent on friction factor used in the simulation. That is, when the friction factor $m=0.4$ was applied in the simulation, a closer agreement with the experimental result was achieved compared to the case where $m=0.8$ was used.

Figure 7.7 (a) compares two experimental load readings at the mold closing velocity of 12.7 and 50.8 mm/min. up to 60% reduction in charge height. As shown in the figure, the load reading at the higher mold closing speed was higher. Equivalent conditions were imposed in the simulation. The comparison of the predictions is given in Figure 7.7 (b). The load prediction for the closing speed 50.8 mm/min. was overpredicted. The load prediction at closing speed 12.7 mm/min. showed a reasonable degree of agreement.
Comparison of experimental load readings between v=12.7 and 50.8 mm/min., lub.

Comparison of load predictions between v=12.7 and 50.8 mm/min., lub.

Figure 7.7. (a) Comparison of experimental load readings at v=12.7 and 50.8 mm/min. at 60% deformation under lubricated condition
(b) Comparison of load prediction by simulation at v=12.7 and 50.8 mm/min. at 60% deformation under lubricated condition
VII.3. Conclusions

The assumption of anisotropic material flow, results of the material characterizations, and the constructed finite element model were all incorporated in this chapter in an attempt to assess the validity of the present approach. A reasonable degree of agreement was obtained by comparing deformed shape. The evaluation of the model's validity will become a basis for the further modeling that can render an accurate prediction of the problems like sink mark formation. For example, the information regarding strain distribution can be utilized to establish the criterion for sink mark formation. The sink resistance parameter, R1, introduced in Chapter III indicated that the material is less sink-prone at the higher interface friction. Figure 7.8 compares the strain distributions for the compression at the mold closing velocity of 12.7 mm/min.

As suggested by Table 3.7, the measured value of the parameter R1 is higher for the unlubricated compression. That is, it is more likely that there would be undeformed fibers that would camouflage the sink-prone resin rich triangular area. The comparison of Figures 7.8. (a) and (b) shows that the surface region opposite to the rib location experiences a higher strain in the lubricated compression when compared at the same reduction in height. It is also shown that the rib is not completely filled in the lubricated molding. In the comparison of Figures 7.8 (b) and (c), the strain of the surface area opposite to the rib location for the anisotropic simulation is estimated higher than that of the isotropic simulation. This point has to be further investigated for the accurate prediction of the sink mark and the suggestions for remedy using the simulation.
Figure 7.8. Comparison of strain distributions between the unlubricated and lubricated compression simulations at $v=12.7$ mm/min. until 60% deformation.
CHAPTER VIII. CONCLUSIONS AND SUGGESTIONS

This chapter summarizes the results of the experimental and the numerical studies. Suggestions for the future studies are also presented following the recommendations for tooling and process design.

VIII.1. Conclusions

VIII.1.1 Recommendation for Tooling

As indicated by the parameters D and R1 in Chapter III, the increase in charge/mold interfacial friction results in faster rib filling. The current tooling design is such that rib slots formation are machined on the bottom mold. It takes about 10 seconds for the upper mold to contact the charge surface once the charge is placed on the bottom mold. Since the compression molding process of short-fiber-reinforced polymeric composites such as SMC is conducted under non-isothermal condition, i.e., hot mold and cold charge, significant amount of heat transfer takes place between the bottom mold and the charge. As a result of heat transfer, more resin paste near the bottom surface melts, and acts as lubricant. The effect of differential heat transfer has been mentioned in the previous study [48].

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Figure 8.1 schematically shows the recommended tooling design that would increase interface friction on the side where the rib is formed.

**VIII.1.2. Recommendation for Process Design**

From the experimental molding results summarized in Tables 3.6 and 3.7, it can be concluded that the rib filling should be completed before the charge becomes too thin, i.e. rib filling before side filling. As illustrated in Figure 8.2, if the rib flow takes place after the plate portion of the closed mold is completed, the fibers near the surface opposite to the location of the rib are more likely to deform because of the thin charge thickness. This deformation of the fibers will result in the formation of a resin rich area near the part surface, which is undesirable. With the help of the finite element simulation capability, a proper molding condition may be designed to generate the recommended flow pattern shown in Figure 8.2.

The tooling shown in Figure 8.3 can also be tried. The purpose of the tooling illustrated in Figure 8.3 is that if the deformation of the fibers is inevitable, pressure or vibration can be applied to straighten the deformed fibers at the end of the mold filling.
Figure 8.1. Suggested tooling design for faster rib filling
Figure 8.2. Suggested process design to prevent fiber deformation
Figure 8.3. Potential tooling design for sink mark free part
VIII.2. Suggestions for Future Study

The following summarizes the suggestions for future studies in three categories: part design, material characterization, and process modeling.

VIII.2.1. Part Design Criteria

Although a great deal of effort has been spent on the investigation of the material flow of the short-fiber-reinforced composites for parts with stiffening ribs, the problems regarding why and how such parts should be designed have not been exploited. The questions that ought to be studied are:

- how to calculate the required stiffness of the ribbed part,
- how to calculate the number, placement, shape, and the size of the ribs that provide desired stiffness.

VIII.2.2. Material Characterization

As discussed in Chapter V, the anisotropy and flow stress measurements should be improved. The following should be studied to better represent the nonhomogeneous and compressible nature of the short-fiber-reinforced composite materials:

- investigation on the more adequate form of the constitutive model,
- use of compression test, after initial compaction, to obtain the flow stress data,
• introduction of optical measurement device for improved measurement of the anisotropy factor,
• estimation of the charge/mold interfacial friction,
• better instrumentation for accurate load and pressure readings, and constant strain rate control.

VIII.2.3. Process Modeling

The anisotropic model presented in this study considers only the material flow under isothermal conditions. Therefore, the following should be included for complete modeling of the molding process with short-fiber-reinforced composite materials:

• numerical modeling of the deformation by assigning different flow stress to different layers with isotropic formulation as shown in Figure 8.4, a previous investigation has shown that the problem associated with tracing the layers after remeshing was difficult to solve,
• physical modeling using layers of plasticines with different flow stresses for material flow study,
• physical and numerical modeling to actually generate sink marks.

Based on the results of the above investigations, more sophisticated models should include the following for complete representation of the actual molding process:

• tracking of the change in fiber orientation during molding,
Modeling with layers of different flow stresses

Figure 8.4. Schematic of the modeling using different flow stress for layers
• incorporation of heat transfer and chemical reaction for non-isothermal and reactive molding,

• inclusion of the volume change using the pressure-volume-temperature-conversion relation,

• incorporation of the compressible material behavior.
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APPENDIX: PHOTOGRAPHS OF THE MOLDED CHARGES

Figure 9.1. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 30% deformation, unlubricated surface condition, v=12.7 mm/min.
Figure 9.2. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 60% deformation, unlubricated surface condition, v=12.7 mm/min.
Figure 9.3. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 30% deformation, unlubricated surface condition, v=50.8 mm/min.
Figure 9.4. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 60% deformation, unlubricated surface condition, \( v = 50.8 \text{ mm/min} \).
Figure 9.5. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 30% deformation, lubricated surface condition, v=12.7 mm/min.
Figure 9.6. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 60% deformation, lubricated surface condition, v=12.7 mm/min.
Figure 9.7. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 30% deformation, lubricated surface condition, v=50.8 mm/min.
Figure 9.8. Cross-section of 6.35 mm-10 % by weight fiber content SMC at 60% deformation, lubricated surface condition, v=50.8 mm/min.