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Superfluid turbulence in two-fluid flow of helium II

Courts, Samuel Scott, Ph.D.
The Ohio State University, 1988
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U·M·I
SUPERFLUID TURBULENCE
IN TWO-FLUID FLOW OF HELIUM II

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Samuel Scott Courts, B.Sc., M.S.

*****

The Ohio State University
1988

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SUPERFLUID TURBULENCE
IN TWO-FLUID FLOW OF HELIUM II

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The Ohio State University, 1988

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The completion of measurements of the dissipation for independently varied normal-fluid and superfluid velocities is presented. These measurements show the propagation of the first and second critical velocities into the \((V_n, V_s)\) plane above the thermal counterflow line and provide a data base with which to compare theoretical predictions.

The steady state dissipation \(\Delta T\) was measured across the ends of a flow tube \((d=134\mu m)\) during laminar and turbulent flow. These flow states were produced using a combination of thermal counterflow procedures and using a very fine fiberglass bundle to produce a pure superflow via film flow of the superfluid component.

These data show that the first critical velocity is actually a closed boundary separating laminar flow at small normal-fluid and superfluid velocities from turbulent flow at higher combinations. It was also found that the second critical velocity propagates into the \((V_n, V_s)\) plane above thermal counterflow. The transition becomes increasingly abrupt as \(V_n\) is increased. These two boundaries define three regions of turbulence in the \((V_n, V_s)\) plane. The turbulence in regions for high \(V_n\) appears to be
homogeneous fitting a simple mutual friction model. The TI/TII Boundary separates this region from one of low-level dissipation. These flow states are very complicated and the dissipation here can oscillate between two values. The first critical velocity separates laminar flow states from a turbulent state obeying a modified mutual friction model suggested by Baehr and Tough. This type of turbulence exists both above and below the thermal counterflow line.
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"Combined Heat and Mass Transport in Superfluid Helium-II"

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63. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4\text{K}$ for $V_n=21.5\text{ cm/s}$ for the roughened flow tube. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.120$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

64. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.4\text{K}$ for $V_{sm}=0\text{ cm/s}$ for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$.

65. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.4\text{K}$ for $V_{sm}=2.25\text{ cm/s}$ for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. 
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67. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=0.5$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

68. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=1$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

69. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=1.5$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

70. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=2$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

71. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=2.75$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

72. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=3.25$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
73. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=3.75$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

74. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=4.8$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

75. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=5.7$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

76. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=6.4$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

77. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=7.5$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

78. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=7.7$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

79. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=9$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
80. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n =11.5$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L =0.162$. $V_{am} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_a =0$ and $V=0$ are marked with vertical dashed lines.

81. Excess dissipation as a function of superfluid velocity $V_{am}$ at $T=1.6K$ for $V_n =15$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L =0.162$. $V_{am} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_a =0$ and $V=0$ are marked with vertical dashed lines.

82. Excess dissipation as a function of superfluid velocity $V_{am}$ at $T=1.6K$ for $V_n =18$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L =0.162$. $V_{am} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_a =0$ and $V=0$ are marked with vertical dashed lines.

83. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{am} =0$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.168$.

84. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{am} =0.8$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.168$.

85. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{am} =2$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.168$.

86. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{am} =3$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.168$.

87. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{am} =4$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.168$.

88. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n =0$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.160$. $V_{am} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_a =0$ and $V=0$ are marked with vertical dashed lines.

89. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n =1$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.160$. $V_{sm} =0$
corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V = 0$ are marked with vertical dashed lines.

90. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n = 2 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V = 0$ are marked with vertical dashed lines.

91. Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n = 3.5 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V = 0$ are marked with vertical dashed lines.

92. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.8K$ for $V_{sm} = 0 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$.

93. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.8K$ for $V_{sm} = 3 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$.

94. Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.8K$ for $V_{sm} = 4 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$. 

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CHAPTER I

INTRODUCTION

As is often the case in science, the discovery of the superfluid (He II) phase of liquid helium in 1938 was quite accidental.\textsuperscript{1,2} The discovery of this quantum fluid led to plethora of experimental and theoretical worked aimed at understanding the phenomenon. Five decades have now passed and thanks to this work our knowledge of superfluid helium has increased greatly. We now know that many of the flow properties of He II are due to the existence of quantized vortex lines in the fluid. The turbulent flow states of He II are modeled as a random tangle mass of these vortex lines in the fluid. However, this model does not accurately predict the dissipation produced by this turbulence nor does it account for all of the changes in the form of the dissipation ("critical velocities") encountered as the flow of He II through flow tubes is altered. This chapter presents an overview of the experimental work and theoretical models of superfluid helium.

1.1 Early Experiments and Theories of Superfluid Helium

Thirty years after the liquification of helium,\textsuperscript{3} experiments designed to measure the viscosity of the fluid yielded puzzling, contradictory results. Measurements of the viscous flow resistance of He II forced to flow through a narrow channel yielded a viscosity of virtually zero.\textsuperscript{1,2,4} At the same time measurements of the viscous drag on a body moving through the fluid yielded a viscosity almost identical of that of \( ^4 \text{He} \) gas.\textsuperscript{5,6}
Tisza resolved these experimental inconsistencies by proposing that He II was composed of two inseparable interpenetrating fluids. This became known as the two fluid model. In this model the superfluid component carries no entropy and is nonviscous allowing dissipationless flow. All entropy is carried by the normal-fluid component which behaves like an ordinary viscous fluid. The total density of the fluid is the sum of the densities of the superfluid and normal-fluid components:

$$\rho = \rho_n + \rho_s.$$  \hfill 1.1

The dynamical equations governing the flows of these fluid components were derived by Landau in three dimensions using thermodynamic arguments:

$$\rho_n \frac{\partial \vec{v}_n}{\partial t} + \rho_n (\vec{v}_n \cdot \nabla) \vec{v}_n = \eta \nabla^2 \vec{v}_n - \frac{\rho_n}{\rho} \nabla P - \rho_s S \nabla T$$ \hfill 1.2

and

$$\rho_s \frac{\partial \vec{v}_s}{\partial t} + \rho_s (\vec{v}_s \cdot \nabla) \vec{v}_s = -\frac{\rho_s}{\rho} \nabla P - \rho_s S \nabla T$$ \hfill 1.3

where $\vec{v}_n = \vec{v}_n(\vec{r}, t)$ and $\vec{v}_s = \vec{v}_s(\vec{r}, t)$ represent the normal-fluid and superfluid velocities at position $\vec{r}$ and time $t$ respectively. In these equations $\eta$ is the normal-fluid viscosity, $P$ is the pressure at position $\vec{r}$, $s$ is the specific entropy of the normal-fluid, and $T$ is the temperature of the fluid at position $\vec{r}$.

Experiments showed that eqns. 2 and 3 adequately described the flow of He II for small normal-fluid and superfluid velocities where the measured dissipation was that expected from the flow of the viscous normal-fluid component. The model failed for higher velocities, however, where the measured dissipation was much higher than could be accounted for by the model. Gorter and Mellink explained this unexpected
dissipation phenomenologically by assuming the existence of an interaction between
the two fluid components in turbulent flow\(^9\). They called this interaction a mutual
friction force \(F_{sn}\). Introducing this interaction into the two equations yields

\[
\rho_n \frac{\partial \vec{v}_n}{\partial t} + \rho_n (\vec{v}_n \cdot \vec{\nabla}) \vec{v}_n = \eta \vec{\nabla}^2 \vec{v}_n - \frac{\rho_n}{\rho} \nabla P - \rho_s S \nabla T + \vec{F}_{sn} \tag{1.4}
\]

and

\[
\rho_s \frac{\partial \vec{v}_s}{\partial t} + \rho_s (\vec{v}_s \cdot \vec{\nabla}) \vec{v}_s = -\frac{\rho_s}{\rho} \nabla P - \rho_s S \nabla T - \vec{F}_{sn}. \tag{1.5}
\]

From experiments measuring the temperature difference across the ends of a flow
tube Gorter and Mellink found

\[
F_{sn} = \rho_s \rho_n A(T) V^3 \tag{1.6}
\]

where \(A(T)\) is the Gorter-Mellink (or mutual friction) coefficient and \(V\) is the average
relative velocity between the normal-fluid and superfluid components.

In 1955, Feynman suggested that this dissipative force could be due to the exis-
tence of a disordered tangle of vortex lines in the fluid\(^{10}\). On a microscopic level, the
turbulence in superfluid helium is the result of a random distribution of vortex lines.
The mutual friction force is then viewed as the scattering of normal-fluid excitations
by vortex lines.\(^{11}\) Vinen suggested from first principles that for a steady state vortex
line density the mutual friction \(F_{sn}\) was given by

\[
F_{sn} = \frac{B\kappa\rho_s \rho_n}{3\rho} V L_o \tag{1.7}
\]

where \(\kappa = h/m = 9.92 \times 10^{-4}\) cm\(^2\)/sec is the quantum of circulation and \(B\) is an
experimentally determined, temperature dependent coefficient. The quantity \(L_o\) is
the steady state length of vortex line per unit volume. This connection between the mutual friction force and the vortex line density allowed comparisons between experimental and theoretical results to be made.

Vinen describes the length of length of vortex line density per unit length, L, in his phenomenological model.\textsuperscript{1,2} He assumes a disordered, homogeneous, isotropic vortex tangle which drifts with the superfluid. The time rate of change of L depends on two competing processes, a line production process and a line annihilation process, yielding

\[
\frac{dL}{dt} = g(T)V L^{\frac{3}{2}} - a(T)L^2
\]

where \(g(T)\) and \(a(T)\) are experimentally determined parameters which characterize the dynamic processes and geometrical effects. Setting \(dL/dt=0\) he arrives at the steady-state line equation\textsuperscript{13}

\[
L_0 = \gamma^2 V^2
\]

where \(\gamma = g(T)/a(T)\). Experimental evidence existed for a critical velocity below which \(F_n\) (therefore L) was zero. To account for this Vinen modified his steady-state equation to include a wall effect: \textsuperscript{14}

\[
L_{\text{eff}} = \gamma(T)V - \left(\frac{\alpha}{d}\right)
\]

where \(d\) is the dimension of the flow channel and \(\alpha\) is a constant on the order of 1. Not only did this model give reasonable agreement with experiment for the turbulence, it had the added feature of predicting a critical velocity when \(L=0\) or \(V=(\frac{\alpha}{\gamma d})\).

In 1978 Schwarz attempted to model the behavior of superfluid turbulence on a microscopic level utilizing computer simulations.\textsuperscript{15} He begins with a homogeneous
distribution of vortex lines constrained to obey classical vortex dynamics. Using various approximations he arrives at an equation with no adjustable parameters. He finds a homogeneous vortex line density independent of initial conditions with a value which is very close to that found in turbulent pure superfluid flow. However, this model did not explain the critical velocity found in this type of flow.

The success of these models gave much evidence as to the existence of vortex lines in turbulent He II flow. Irrefutable evidence was given by the development of an imaging system which produced the photographs of quantized vortices in figure 1. Figure 1a-1 show stable vortex arrays (white circles) in a rotating bucket of He II for angular velocities varying from 0.3 to 0.9 rad/sec.16,17 The photographs were taken from the rotating frame of reference and the bucket was 2mm in diameter.

1.2 Flow Measurements

In practice most measurements of the temperature difference $\Delta T$ and pressure difference $\Delta P$ are spatially averaged, steady state measurements. Therefore equations 1.4 and 1.5 can be simplified by taking averages over time and space yielding

$$0 = \eta < \nabla^2 v_n > - \frac{\rho n}{\rho} < \nabla P > - \rho_s S < \nabla T > + F_{sn}$$

1.11

and

$$0 = - \frac{\rho_s}{\rho} < \nabla P > - \rho_s S < \nabla T > - F_{sn}.$$  

1.12

where the brackets $<>$ indicate a space-time average of the axial component of the enclosed vector quantity. These two equations can be solved for the thermodynamic quantities which gives the difference in these quantities between the two ends of the flow:
Figure 1

Quantized Vortex Lines in He II \textsuperscript{16,17}
\[ \langle \bar{V}P \rangle = \eta \langle \bar{V}^2 \bar{v}_n \rangle; \quad \text{(1.13)} \]

\[ \langle \bar{V}T \rangle = \frac{\eta}{\rho S} \langle \bar{V}^2 \bar{v}_n \rangle + \frac{F_{sn}}{\rho_s S}; \quad \text{(1.14)} \]

and

\[ \langle \bar{V}\mu \rangle = -S \langle \bar{V}T \rangle + \frac{\langle \bar{V}P \rangle}{\rho} = \frac{F_{sn}}{\rho_s}. \quad \text{(1.15)} \]

The equation for \( \langle \bar{V}P \rangle \) is the Poiseuille equation for \( \langle V_n \rangle \) which can be solved for a circular tube of radius \( a \) to give

\[ \langle \bar{v}_n \rangle = 2V_n \left(1 - \frac{r^2}{a^2}\right) \hat{z}, \quad \text{(1.16)} \]

where \( \hat{z} \) is the axial direction and \( r \) is the radial distance from the tube axis. \( V_n \) is the average of \( \bar{v}_n \) across the flow channel. The temperature difference is defined as

\[ \Delta T = |\langle \bar{V}T \rangle| \quad \text{(1.17)} \]

which, along with the above equation for \( \bar{v}_n \), allows the total temperature difference to be written as

\[ \Delta T = \Delta T_\eta + \Delta T' \quad \text{(1.18)} \]

where

\[ \Delta T_\eta = \frac{32\eta_n \ell}{d^2 \rho ST V_n} \quad \text{(1.19)} \]
and

\[ \Delta T' \equiv \frac{\ell F_{sn}}{\rho_s S}. \]  \hspace{1cm} (1.20)

\( \Delta T_n \) is the temperature difference due to normal fluid flow. It is small and proportional to both the average normal-fluid velocity \( V_n \) and the normal-fluid viscosity \( \eta \). The quantity \( \Delta T' \) is the friction term due to the interaction between the normal fluid and the vortex lines in the superfluid.

Many experiments have been performed to study superfluid turbulence. Many of these involve simply the measure of the temperature difference, pressure difference, or chemical potential difference across a flow tube for different flow states. The most common of these experiments are those done using an apparatus which produces either pure superflow \(^{18-26,48}\) or thermal counterflow\(^{18,19,27-36,49-51}\) and abundant data exist for both types of flow.

Pure superflow (where the normal-fluid is immobilized) is the simplest flow state to study. Figure 2 shows a typical superflow apparatus. The superleak is a volume of closely packed material which is impermeable to normal fluid flow but allows the superfluid component to pass through unimpeded. The superleak also acts as a chemical potential short so that the temperature and pressure on both sides of the superleak assume values so that the chemical potentials on both sides are equal.

The data obtained from such experiments show laminar (dissipationless) flow for low values of \( V_s \). There is a discontinuous onset of dissipation at a critical value of \( V_s \) (sometimes labelled \( V_{c4} \)) indicating the transition from laminar flow to superfluid turbulence. Only one transition is seen in this type of flow and this turbulent state has been labelled TIV. For pure superflow equation (1.18) reduces to

\[ \Delta T = \frac{\ell F_{sn}}{\rho_s S}. \]  \hspace{1cm} (1.21)
Figure 2
Typical Pure Superflow Apparatus
and the vortex line density is predicted with excellent agreement by

\[ L_{0_{TV}} = (\gamma_4 V)^2 \]  

1.22

where \( \gamma_4 \) is a temperature dependent quantity. The line density depends only on \( V(=V_a) \) and is independent of tube size indicating the turbulence for pure superflow is homogeneous. This agreed with the results of computer simulations performed by Schwarz although his results did not show a critical velocity at that time.\(^{15}\)

Figure 3 shows a typical thermal counterflow apparatus and typical dissipation as a function of the normal-fluid velocity \( V_n \).\(^{32}\) A closed cell containing a heater and thermometer is connected to a helium reservoir by a flow tube of length \( \ell \) and area \( A \). The temperature of the reservoir is kept constant. Energizing the heater with power \( \dot{Q} \) produces a flow of helium away from the cell and toward the reservoir. Since no entropy is carried by the superfluid component, the heat is carried away only by the normal-fluid component. The average normal-fluid velocity is given by the Two-Fluid Model as

\[ V_n = \frac{\dot{Q}}{\rho S T A} \]  

1.23

where \( A \) is the cross-sectional area of the flow tube and \( T \) is the temperature of the helium reservoir. The condition of conservation of mass current requires a superfluid velocity which satisfies

\[ V_{s_{GF}} = -\frac{\rho_n}{\rho_s} V_n \]  

1.24

where \( V_{s_{GF}} \) is the superfluid flow due only to thermal counterflow.

The data for thermal counterflow show two transitions along the trajectory. The first transition (traditionally labelled \( V_{c1} \)) separates laminar flow states from a
Figure 3

Typical Thermal Counterflow Apparatus and Data
turbulent region labelled TI. It is possible to maintain laminar flow in the TI region if you begin in laminar flow. The transition is completely reproducible only if you begin in the TI state and move into the laminar flow state. In this TI turbulent region the vortex line density has the form

\[ L_{\alpha_I} = \left( \gamma_1 V - \frac{\alpha_1}{d} \right)^2 \]  

1.25

where \( \gamma_1 \) is a function of temperature and \( \alpha_1 \) is a temperature independent, slightly diameter-dependent quantity of order 1. At higher \( V_n \) this form of turbulence changes to a second form of turbulence TII at a critical velocity, traditionally labelled \( V_{c2} \). In this region (for \( V \) greater than or about \( 2V_{c2} \) the line density is given by

\[ L_{\alpha_{II}} = \left( \gamma_2 V - \frac{\alpha_2}{d} \right)^2 \]  

1.26

where \( \gamma_2 \) is a function of temperature and \( \alpha_2 \) is a temperature independent, slightly diameter dependent quantity of order 10. At all temperatures \( \alpha_2 < \alpha_1 \). The dissipation for this TII turbulent region is in excellent agreement with the computer simulations of Schwarz. Awschalom, Millikan, and Schwarz have also shown this TII turbulent state to be homogeneous.

The results of these measurements are depicted in figure 4. A critical velocity (filled squares) occurs along \( V_n = 0 \) for both directions of pure superfluid flow indicating a transition from laminar flow to superfluid turbulence (TIV). There are two critical velocities (filled squares) occurring along the thermal counterflow trajectory. The first separates laminar flow from a turbulent state TI. The second separates this TI turbulent state from a second turbulent state TII which appears to be nearly homogeneous.
Critical Velocities in the \((V_n, V_s)\) Plane.
1.3 Flow Measurements for Independently varied $V_n$ and $V_s$

With a different description for the dissipation in each region of the $(V_n, V_s)$ plane it was impossible to really understand the processes involved with the turbulent flow of He II. It was hoped that more understanding could be obtained with measurements of the dissipation for more general flows of $V_n$ and $V_s$.

Many experiments for independently varied $V_n$ and $V_s$ have been performed to measure the temperature difference, chemical potential difference, pressure difference, and second sound attenuation. Whenever possible, data from these experiments are compared to data from the present experiment. These comparisons will be found in later chapters.

Workers at the University of Leiden have produced an amazing amount of data for independently varied normal and superfluid velocities. Kramers was the first of these workers measuring properties of He II using second sound techniques. Data of this sort is somewhat open to interpretation and not readily comparable to later direct measurements of the temperature difference, pressure difference, and chemical potential difference across the flow tube.

Van der Heijden followed Kramers and made quasistatic measurements of the temperature difference and chemical potential difference across the ends of a capillary of $\ell=14.6\text{ cm}$, $d=294\text{ \mu m}$. He was the first to observe oscillation regions in the $(V_n, V_s)$ plane, i.e. combinations of $V_n$ and $V_s$ for which stable flow states do not exist. Van der Heijden also finds that $F_{sn}=0$ when $V=0$ and that an additional friction force $F_s$ should exist which is associated only with the flow of the superfluid component of He II. His data also show the existence of a line just above and parallel to $V=0$ for which $\Delta T=0$. Unfortunately, Van der Heijden was unable to compare his results to those of Kramers.

De Haas followed Van der Heijden performing essentially the same experiments in a slightly different apparatus. De Haas made quasistatic measurements of the
temperature difference and chemical potential difference across the ends of a d=216 μm flow tube. (Data were also obtained for stainless steel and copper tubes but each metal tube appears to exhibit its own individual, characteristic turbulent states; therefore these data have not been included.) He obtains data for flow states both above and below the thermal counterflow line with trajectories both vertical (fixed $V_n$) and parallel to the thermal counterflow line. The range of his data extends outward to about the value of $V_n$ where the turbulence changes from the TI to the TII turbulent state in thermal counterflow. De Haas sees $F_{sn}$ vary smoothly over the $(V_n, V_s)$ plane. He also sees a transition from turbulent to laminar flow but the sensitivity of his apparatus was unable to yield an accurate value of the position of this transition in the $(V_n, V_s)$ plane.

Slegtenhorst, de Haas' successor, performed the same types of experiments at much higher values of $V_n$. He made quasistatic measurements of the temperature difference and chemical potential difference in both a stainless steel capillary ($d=206 \, \mu m, \ell=8.5 cm$) and a glass capillary ($d=216 \, \mu m, \ell=8.5 cm$). Above the thermal counterflow line he again saw the oscillation regions seen previously. He also saw flow states where the $\Delta T$ and $\Delta \mu$ changed drastically for small changes in either the normal or superfluid velocity. He called the locus of these points the Steep Branch I and the Steep Branch II. Both branches appeared in most, but not every, case. It was unclear as to the relationship between these branches and the transition from the TI to the TII turbulent state found in thermal counterflow. Below the thermal counterflow line Slegtenhorst found the $\Delta T$ to vary smoothly as a function of $V$.

At The Ohio State University Opatowsky performed pure superflow and thermal counterflow experiments in a variety of glass flow tubes of different geometry. Among them was a circular glass tube with $d=134 \, \mu m$ and $\ell=9.9 \, cm$. He obtained the first unambiguous results showing the functional form of $L_o$ agrees with the Vinen-Schwarz homogeneous relation.
Other measurements taken in the various geometry tubes demonstrated that superfluid turbulence in pure superflow was independent of geometry proving turbulent superflow to be the homogeneous turbulent flow Vinen and Schwarz describe in their theories. In thermal counterflow, however, Opatowsky found the dissipation to be highly dependent upon the tube geometry indicating nonhomogeneous turbulence. For thermal counterflow in circular glass tubes he found a laminar state, a turbulent state TI, and a turbulent state TII. From his results Opatowsky was able to conclude that reliable pure superflow data could be obtained only from flow tubes exhibiting those three distinct states found in thermal counterflow.

Baehr, Opatowsky's successor, performed experiments in the same circular glass flow tube. By utilizing a fountain pump/thermal counterflow apparatus she measured the dissipation for flow states below the thermal counterflow line. In pure superflow, Baehr found a transition from laminar flow to superfluid turbulence (sometimes labelled $V_c$) which was independent of temperature. She also obtained an extensive set of measurements for dissipation below the thermal counterflow line at 1.4K and 1.6K yielding three major results. First, the transition from laminar flow to superfluid turbulence in pure superflow and the transition from laminar flow to the TI turbulent state are part of the same transition boundary. The results of her measurements showed that this critical velocity was temperature independent and could be described reasonably well by

$$V_c(V_n) = V_o + mV_n$$

where $V_o = 1.5 \text{ cm/sec}$ and $m = 0.785$. Second, her data showed that the transition separating the turbulent states TI and TII propagates below the thermal counterflow.
line at 1.6K. She was unable to see this at propagation 1.4K, however, but this is probably due to the limits of the sensitivity her apparatus. Third, up until this time a successful, quantitative description of the dissipation encountered in combined normal and superfluid flow had not be found. Based on her data, Baehr and Tough suggested a modified equation linking the vortex line density to the mutual friction force with excellent results for flow states lying below the thermal counterflow line in the \((V_n, V_s)\) plane.

Marees continued the investigation of He II flow states at Leiden.\(^{44-47}\) He used the same capillary tubes as Slegtenhorst but placed several thermometers along the length of the tube to measure the quasistatic temperature gradient. He found that for flow states above the thermal counterflow line with \(V_n \approx V_{nc2}\) there existed a nonlinear temperature profile along the length of the tubes. He also found that the lower steep branch Slegtenhorst had found corresponded well with \(V_s \approx 0\) (i.e. pure normal-fluid flow). In other measurements with glass tubes of lengths 0.085m, 8.88m, 0.145m, and 10.64m and diameters 216 , 217, 133, and 133 \(\mu m\) respectively Marees shows the existence of the transition from laminar flow to turbulent flow for small \(V_n\) above the thermal counterflow line in the \((V_n, V_s)\) plane. He locates the transition both above and below the thermal counterflow line and also finds a critical normal fluid velocity \(V_{nc}^ll\) that defines the right-hand boundary of the laminar region. Like Baehr, Marees finds that the boundary below the thermal counterflow line to be linear. However, his data show it to be temperature-dependent obeying

\[
V_{sc}(V_n) = a(T)V_n + V_{sc}^{(o)}
\]

where

\[
a(T) = \frac{3(\rho_s \rho_n)^{1/2}}{\rho}.
\]
These data show the origin of the \((V_n, V_s)\) plane to be enclosed by a laminar region which is separated from turbulent flow states by these boundaries.

1.4 Recent Theories of Superfluid Turbulence

Schwarz has modified his original theory of superfluid turbulence to allow for reconnection processes. First he allows vortex lines that are sufficiently close to one another to reconnect which allows the vortex tangle to be self-sustaining. Next he includes boundary effects whereby vortex lines sufficiently close may reconnect to the channel wall. By assumption the turbulence depends only on the relative velocity \(V\). This results in a critical velocity below which the vortex tangle is no longer self-sustaining and \(L_o\) drops discontinuously to zero. This is exactly what is seen in experimental measurements of the first critical velocity. For pure superflow, where one would expect this calculation to work best, this critical velocity is a factor of 2.5 too large. Increasing the distance from which the vortex lines reconnect to the wall can increase the critical velocity but the smallest possible value is simply too large. Even worse, the temperature dependence of this critical velocity is too strong at lower temperatures. Since the critical velocity depends only on \(V\), the addition of a normal-fluid velocity simply moves the superfluid critical velocity into the \((V_n, V_s)\) plane along a trajectory defined by \(V=\text{constant}\) as shown in figure 5. It is interesting to note that this critical velocity does agree well with the first critical velocity in thermal counterflow where it has both the correct magnitude and the correct temperature dependence. This result is most likely coincidental. Schwarz then includes additional boundary effects whereby vortex lines may pin or depin at imperfections on the boundary and assumes explicit dependence on both \(V_n\) and \(V_s\). The addition of these assumptions yields a critical velocity which is an order of magnitude too small in superflow, but does yield a nonsymmetric closed boundary.
Figure 5
The critical velocities calculated in the Schwarz theory. The critical velocities with line reconnections and $V$ dependence are shown as $V=\text{constant}$ lines. The critical velocities given by depinning processes and explicit $V_n$ and $V_s$ dependence are shown as the closed boundary about the origin.
about the origin as shown in figure 5. This critical velocity is physically different than those measured in thermal counterflow or pure superflow as it represents the velocity needed to depin vortex lines from the imperfections on the boundary and sweep them out the channel.

1.5 Motivation of Research

This introduction has discussed a number of fascinating features existing in the \((V_n, V_a)\) plane. The research presented in this paper attempts to provide understanding of some of the questions previously unanswered. First, how does the onset of superfluid turbulence from laminar flow vary in the \((V_n, V_a)\) plane? Second, does the "critical velocity" separating the two turbulent states TI and TII occur only in thermal counterflow and below? If not, how does this transition propagate through the \((V_n, V_a)\) plane above thermal counterflow? Third, how does the surface roughness of the flow channel affect the transition from laminar flow to superfluid turbulence and from the TI to the TII turbulent state? Fourth, it has been shown by Baehr and Tough that the measured dissipation for flow states below the thermal counterflow line can be fit using a modified formula linking the vortex line density to the dissipation. Is it possible that this modification provides a unified description of the dissipation in all regions of the \((V_n, V_a)\) plane?
CHAPTER II

APPARATUS

Figure 6 shows a schematic diagram of the apparatus used in these experiments. This probe is similar to the probe used by both L. B. Opatowsky \textsuperscript{52} and M. L. Baehr \textsuperscript{53} and is described in detail in their dissertations. Only a brief overview of the apparatus will be presented here with a detailed description of the modifications made for this experiment.

This experiment requires a well characterized flow tube, a method for producing and measuring independent superfluid and normal-fluid velocities, and a sensitive measurement of the dissipation. In figure 6 a flow tube connects a large helium reservoir regulated at temperature $T_1$ to a small lower chamber containing a heater $H_2$. This chamber in turn is connected to a small upper chamber via a superleak. The superleak offers no resistance to the flow of superfluid but is impermeable to the flow of normal-fluid. A bundle of fiberglass fibers wrapped on a heater $H_1$ is used to siphon superfluid from the main bath through the superleak, through the flow tube, and into the reservoir. This device will be referred to as a film-flow transfer apparatus, or FFTA.

2.2 Flow Tube

The flow tube used in this experiment is the same one used previously. It is glass strengthened by a fiberglass sheath and coated with Stycast 1266 epoxy resin.\textsuperscript{54} The
Figure 6

Schematic Diagram of the Experimental Apparatus
tube is \( l = 9.9 \text{cm} \) long and the diameter was determined to be \( d = 1.34 \times 10^{-2} \text{cm} \).

After an extensive set of measurements was obtained for this flow tube the apparatus was warmed to room temperature, the inner surface of the tube was "roughened", and a second set of measurements was obtained. The roughening was produced by plating the inner tube surface with 1 \( \mu \text{m} \) diameter spheres polystyrene spheres. These spheres were suspended in methyl alcohol and drawn into the flow tube via capillary action. The polystyrene spheres remained fixed to the wall of the flow tube after the alcohol evaporated. Examination of the surface of sample flow tubes with a scanning electron microscope showed that the spheres tend to clump together in islands of about 25 spheres with a typical distance between islands of about 25\( \mu \text{m} \). Figure 7 shows a photograph of the polystyrene spheres on the surface of a sample flow tube taken by a camera attached to a scanning electron microscope. The magnification is 1450\text{x}. The grouping of the spheres and distribution of these groups along the length of the flow tube was consistent. Further investigation with the sample flow tubes showed that the spheres do not detach from the flow tube wall after being cooled to liquid nitrogen temperature.

2.3 Superleak

The superleak was made by packing Linde B aluminum oxide powder into a thin-walled CuNi tube of length 4 cm and inner diameter 5.5 mil. The aluminum oxide powder has a grain size of 0.05 \( \mu \text{m} \) and the packing fraction for this superleak was about 0.28 to 0.45.

2.4 Cell Thermometer

The temperature \( T_2 \) in the lower chamber was measured with a CG500 carbon glass thermometer. At 1.4K this device had a resistance of roughly 240 K-ohms and
Figure 7

Polystyrene Spheres on the Surface of a Sample Flow Tube
a sensitivity

\[ \frac{dR}{dt} \frac{1}{T} = 5.48 K^{-1}. \]

The sensitivity was extremely stable and the resistance drift was on the order of 1 ohm/hour. This resistance was monitored with an SHE 120 picowatt resistance bridge with an excitation voltage set for maximum sensitivity with minimum self-heating.

2.5 Temperature Regulation System

The reservoir temperature was regulated electronically using standard techniques. This regulation system is identical to the one utilized by Baehr\(^5\) and a complete description may be found in Appendix D of her dissertation. Fluctuations in this regulated temperature were less than about 5 \(\mu K\) when averaged over three seconds.

2.6 Film-Flow Transfer Apparatus

Pure superfluid flow through the flow tube was produced by the FFTA. The FFTA consists of a heater, \(H_1\), and a fiberglass bundle. Five feet of Evanohm wire\(^5\) (200 ohm/ft) were wrapped around a 5cm long, 6mm o.d., 4mm i.d. copper tube. The wire was then coated with GE varnish to secure it to the tube. After drying, the fiberglass bundle was wrapped over the Evanohm wire 15 times. The fiberglass bundle was wrapped with a mylar which was secured in place by thread. The entire heater assembly was attached to the stainless steel vacuum can pumping line by means of a Stycast 1266 fitting which also provided thermal isolation. The resistance of the heater is 1034 ohms. This heater was energized by a Hewlett Packard 6177C DC Current Source and the current through the heater was monitored by an HP 3478A Multimeter. Typical values of current through the heater ranged from about 1-25 mA.
Superfluid helium is siphoned over the fiberglass bundle from the main bath, over the heater $H_1$, and finally into the upper chamber. The heater $H_1$ acts as a control valve by increasing the local temperature of the helium film. The sensitivity of film-flow to changes in temperature results in very fine control of the superfluid transfer rate. The superfluid velocity through the flow tube due to the FFTA alone is labelled $V_{sm}$, i.e. this velocity results from a net mass transfer of superfluid into the reservoir can.

An order of magnitude estimate of $V_{sm}$ can be obtained using the film-flow rate $R$ over clean glass. At 1.5K,\(^{57}\)

$$R(cm^3/sec) = 0.75 \times 10^{-4}(cm^3/cm/sec) \times P$$ \hspace{1cm} 2.1

where $P$ is the minimum perimeter over which the film flows. In the FFTA the minimum perimeter is the total perimeter of the fiberglass fibers in the bundle. The bundle\(^{37}\) consists of 7200 fibers of approximately $10^{-3} cm$ diameter (compared to a film thickness of about $10^{-6} cm$). The minimum perimeter is then roughly $P=22 cm$ which yields a transfer rate $R = 1.7 \times 10^{-3} cm^3/sec$. If the flow tube area is $A$, then $V_{sm}$ is given by

$$V_{sm} = \frac{R}{A} = 13cm/sec.$$ \hspace{1cm} 1.2

The highest value of $V_{sm}$ seen in this experiment was about 40 cm/sec. This is consistent with a film transfer rate of about $3R$ over the fiberglass, a value more typical of a rough metal surface.

2.7 Capacitor

The actual superfluid velocity due to the FFTA is determined from the rate of change of the level in the reservoir as superfluid was transferred from the bath. A
cylindrical level-sensing capacitor was monitored with a General Radio 1615 capacitance bridge connected to an Ithaco 393 lock-in amplifier. The capacitance is directly proportional to the height of the liquid helium in the reservoir can. A detailed discussion of this capacitor is found in Baehr's dissertation. A small computer was programmed to calculate $V_{sm}$ after reading the output from the lock-in for up to five minutes. The lock-in output was also connected to a Hewlett Packard 7044A X-Y Chart Recorder to preserve time records of the reservoir level. Estimates of sources of error in the calculation of $V_{sm}$ indicate a maximum systematic error of 5% and a maximum random error of 0.06 cm/sec.
CHAPTER III

DATA ACQUISITION

The experimental procedure consisted of measuring the temperature difference across the flow tube \( \Delta T = \Delta T_2 - \Delta T_1 \) as a function of the two velocities \( V_n \) and \( V_s \). Figure 8 shows a simplified diagram of the experimental apparatus with the FFTA represented as a controlled helium current source. Figure 9 shows the region of the \((V_n, V_s)\) plane accessible to the apparatus.

The total superfluid velocity through the flow tube \( V_s \) is the sum of the counterflow induced component \( V_{s\, CF} \) and the mass flow component \( V_{sm} \):

\[
V_s = V_{sm} + V_{s\, CF}
\]

In pure counterflow, the superfluid current source is turned off so that \( V_{sm} = 0 \) and \( V_s = V_{s\, CF} \) (Eqn. 3.1). Increasing the power \( P \) in the heater \( H_2 \) increases \( V_n \) (Eqn. 1.23) and the trajectory given by equation (1.23) is followed into the \((V_n, V_s)\) plane. This trajectory is labelled as “Thermal Counterflow Line” in figure 9. By adjusting the heater \( H_1 \), the superfluid current source can be controlled to give values of \( V_{sm} \) ranging from 0 to about 25 cm/sec. Appropriate combinations of counterflow and superflow from the FFTA can then provide access to the region of the \((V_n, V_s)\) plane shown in figure 9.

In day to day operation one of two methods would be followed in obtaining data. In the first method the value of the normal-fluid velocity \( V_n \) would be set at some
Figure 8

A Simplified Schematic of the Experimental Apparatus
Thermal Counterflow

\[ \rho_n V_n = -\rho_s V_s \]

Figure 9

Region of the \((V_n, V_s)\) Plane Accessible to this Apparatus
fixed value using heater $H_2$. A large superfluid velocity would then be added by means of the FFTA. The superfluid velocity $V_{sm}$ would then be reduced in steps. At each step the temperature $T_2$ was monitored until a steady state was reached and $\Delta T$ was recorded. Beginning with a large $V_{sm}$ insured that superfluid turbulence was generated in the flow tube. Steady-state measurements of this type thus follow the vertical trajectories depicted in figure 9. In a second method the superfluid velocity $V_{sm}$ due to the FFTA was held constant. The normal fluid velocity $V_n$ and superfluid velocity induced by counterflow $V_{scf}$ were varied by controlling the power dissipated in $H_2$. The temperature difference was then measured in steps yielding trajectories as shown in figure 9 parallel to the thermal counterflow line. This procedure was more difficult to follow and so only a limited amount of data was taken in this manner.
CHAPTER IV

MEASUREMENTS OF THE FIRST CRITICAL VELOCITY

FOR INDEPENDENTLY VARIED \( V_n \) AND \( V_s \)

University of Leiden measurements of the first critical velocity showed as many as three different transitions defining the laminar region about the origin. Is this the case or is there one continuous boundary separating laminar flow from only one type of turbulence which varies slightly as the normal-fluid velocity is increased from zero? This chapter attempts to answer this question.

4.1 Data

Measurements of the temperature difference \( \Delta T \) as a function of \( V_n \) and \( V_s \) were taken at reservoir temperature of 1.4K, 1.6K, and 1.8K. At 1.4K I followed vertical trajectories of \( V_n = 0, 1, 2, 3, 4, 5, \) and 6 cm/sec. Data taken in trajectories parallel to the thermal counterflow line were taken at \( V_{sm} = 0, 2.25, 3, \) and 4.5 cm/sec. At 1.6K we followed vertical trajectories of \( V_n = 0, 0.5, 1, 1.5, 2, 2.75, 3, 3.25, 3.75, \) and 4.8 cm/sec. Trajectories parallel to the counterflow line were taken at \( V_{sm} = 0, 0.8, 2, 3, \) and 4 cm/sec. At 1.8K we followed vertical trajectories of \( V_n = 0, 0.8, 2, 3, \) and 4 cm/sec. Trajectories parallel to the counterflow line were taken at \( V_{sm} = 2, 3, \) and 4 cm/sec.

A second set of data was taken at \( T=1.4K \) with the tube walls "roughened" to study the effect of surface roughness on the transition to superfluid turbulence.
These data were taken in vertical trajectories of \( V_n = 0, 2, 3, \) and 4 cm/sec and with trajectories parallel to the counterflow line at \( V_{sm} = 0 \) and 3 cm/sec.

The total temperature difference is a sum of a viscous term \( \Delta T_\eta \) (Eqn. 1.19) and a mutual friction term \( \Delta T' \) (Eqn. 1.20):

\[
\Delta T = \Delta T_\eta + \Delta T'
\]

Equation (1.19) is used to calculate the small viscous term. The excess temperature difference \( \Delta T' \) is then obtained from the \( \Delta T \) data and equation (4.1).

4.2 Experimental Results

Figure 10 shows a typical run taken using the first method described in Chapter 3. These data were taken at 1.6K in a vertical trajectory with \( V_n = 2.75 \) cm/sec. (This trajectory is depicted as the dashed line in figure 13.) Figure 10 is a plot of the excess temperature difference \( \Delta T' \) as a function of \( V_t \). As \( V_t \) is reduced from about 6 cm/sec, \( \Delta T' \) decreases sharply becoming zero below \( V_t = 3.9 \) cm/sec. Since the form of the dissipation is not known here, we simply take the transition from superfluid turbulence to laminar flow to occur at \( V_t = (4.14 \pm 0.25) \) cm/sec. In the laminar region where \( \Delta T' = 0 \) there is no excess dissipation due to turbulence. The point \( (V_n = 2.75 \text{ cm/sec}, V_t = 4.15 \text{ cm/sec}) \) then denotes part of the boundary in the \((V_n, V_t)\) plane separating a laminar region from superfluid turbulence.

Figure 11 shows a typical run taken using the second method. These data were taken at 1.8K in a trajectory parallel to the thermal counterflow line with \( V_{sm} = 4.0 \) cm/sec. This trajectory is shown as a dashed line in Figure 14. In the region between \( V_n = 1.0 \text{ cm/sec} \) and \( V_n = 4.0 \text{ cm/sec} \) there is no excess dissipation. There are two transitions to superfluid turbulence along this trajectory which occur at \( V_n = (0.75 \)
The excess dissipation $\Delta T'$ as a function of $V_s$ for fixed $V_n = 2.75 \text{ cm/s}$ at $T=1.6\text{K}$. The transition to superfluid turbulence occurs at $V_s = (4.15 \pm .25) \text{ cm/s}$. The trajectory for these data in the $(V_n,V_s)$ plane is shown by the dashed line in figure 13.
The excess dissipation $\Delta T'$ as a function of $V_n$ for fixed $V_{sm} = 4.0$ cm/s at $T = 1.8$ K. The transitions to superfluid turbulence occur at $V_n = (0.75 \pm 0.25)$ and $V_n = (4.24 \pm 0.25)$ cm/s. The trajectory for these data in the $(V_n, V_s)$ plane is shown by the dashed line in figure 14.
\( V_n = (4.25 \pm 0.25) \text{ cm/sec} \) and \( V_n = (4.25 \pm 0.25) \text{ cm/sec} \) respectively. Therefore the points 

\( (V_n = 0.75 \text{ cm/sec}, V_{sm} = 4.0 \text{ cm/sec}) \) (or \( V_s = 3.65 \text{ cm/sec} \)) and \( (V_n = 4.24 \text{ cm/sec}, V_{sm} = 4 \text{ cm/sec}) \) (or \( V_s = 1.93 \text{ cm/sec} \)) also lie on the boundary separating laminar flow from superfluid turbulence.

Figures 12-14 show the collection of these boundary points at the three temperatures studied. Points shown below the thermal counterflow line were obtained previously \(^{50,51}\) while points above the line are the present extension of this earlier experiment. The solid circles in figure 12 are the data obtained from the roughened flow tube. No discernable difference was seen between the results for the roughened and smooth flow tube. The collection of boundary points in figures 12-14 may be taken to define a critical line in the \((V_n,V_s)\) plane separating the region of superfluid turbulence from laminar, vortex free flow. Since the data from quadrants I and IV can be mapped into the physically equivalent quadrants II and III, the results show that the critical line is a closed curve about the origin.

The transition to superfluid turbulence in the \((V_n,V_s)\) plane has also been observed by Marees et al. \(^{48,47}\) at the University of Leiden. They used glass flow tube of diameter \( d = 1.3 \times 10^{-2} \text{ cm} \) and produced independent normal and superfluid velocities \( V_n \) and \( V_s \) using a combination of thermal counterflow and mass flow. Their experiment differs from ours, however, in the means of producing the superfluid mass flow \( V_{sm} \). Instead of the FFTA (figures 6 and 8) which produces a constant value of \( V_{sm} \) they essentially fill the large tube rising from the upper chamber and allow the helium to continuously drain through the superleak and flow tube into the reservoir. Their upper chamber is also held at the reservoir temperature \( T_1 \) so that the level difference \( \Delta z \) between the large tube and the reservoir is equal to the chemical potential drop \( \Delta \mu \) across the flow tube. Data for \( \Delta z \) and \( \Delta T' \) as a function of time are shown in figure 15 for a short tube (\( \ell = 14.5 \text{ cm} \)) and a very long tube (\( \ell = 10.64 \text{ m} \)). The superfluid mass flow velocity \( V_{sm} \) at any time is obtained from the time
Figure 12

The boundary of superfluid turbulence in the \((V_n, V_s)\) plane at 1.4K. The data points mark the transition between superfluid turbulence and laminar flow and fall on a critical line surrounding the origin. Points below the thermal counterflow line are from our previous results (reference 50). Solid symbols are results obtained with the artificially roughened flow tube.
The boundary of superfluid turbulence in the \((V_n, V_s)\) plane at 1.6K. The data points mark the transition between superfluid turbulence and laminar flow and fall on a critical line surrounding the origin. Points below the thermal counterflow line are from our previous results (reference 50).
Figure 14

The boundary of superfluid turbulence in the \((V_n,V_s)\) plane at 1.8K. The data points mark the transition between superfluid turbulence and laminar flow and fall on a critical line surrounding the origin.
derivative of Δz. The transition from superfluid turbulence to laminar flow is defined to occur when the chemical potential Δμ (proportional to Δz) first goes to zero. In the case of the short tubes (figure 15a) the excess temperature difference ΔT′ also goes to zero at the same time and velocity. The thermal resistance of the long tube is so large, however, that this is not the case as is shown in figure 15b.

We compare our data for the boundary between superfluid turbulence and laminar flow with those of Marees and van Beelen in figures 16-18. We have rendered all velocities dimensionless using the definitions

\[ V_n^* = \frac{V_n d}{4\pi \kappa} \]

and

\[ V_s^* = \frac{V_s d}{4\pi \kappa} \]

where \( \kappa \) is the quantum of circulation. (Some data from Leiden were also obtained from glass tubes of 2.17 x 10^{-2} cm diameter. Because there is a substantial discrepancy between results obtained with long and short tubes of this diameter, we have not included these data in our discussion.) In these figures the present data are shown as open circles and the Leiden data as square (open squares for the short tube and filled squares for the long tube). The vertical dot-dashed line in these figures represents the right hand side of the boundary between superfluid turbulence and laminar flow as found in the Leiden experiments.

The best overall agreement between the two sets of data occurs at 1.6K (figure 17). Here one can picture a smooth critical line in the \((V_n^*, V_s^*)\) plane separating laminar flow from superfluid turbulence. As noted earlier, the data are duplicated in the second and third quadrants so the critical line forms a closed boundary about
In the experiments of Marees et.al. (references 46 and 47) the chemical potential difference $\Delta \mu$ (or the level difference $\Delta z$) and the excess temperature difference $\Delta T'$ decay in time as shown in these representative data for a short tube (a) and a very long tube (b).
Figure 16

The boundary of superfluid turbulence in the \((V^*_n, V^*_s)\) plane at 1.4K. The circles are our data (also shown in figure 12). The squares are the data of Marees et.al. for the short tube (open symbols, reference 47) and the long tube (closed symbols, reference 46). The vertical dot-dashed line is the estimate in reference 47 for the right hand limit of the boundary.
Figure 17

The boundary of superfluid turbulence in the \((V_n^*, V_s^*)\) plane at 1.6K. The circles are our data (also shown in figure 13). The squares are the data of Marees et.al. (reference 47) for the short tube. The vertical dot-dashed line is the estimate in reference 47 for the right hand limit of the boundary.
The boundary of superfluid turbulence in the $(V_n^*, V_s^*)$ plane at 1.8K. The circles are our data (also shown in figure 14). The squares are the data of Marees et.al. for the short tube (open symbols, reference 47) and the long tube (closed symbols, reference 46). The vertical dot-dashed line is the estimate in reference 47 for the right hand limit of the boundary.
the origin. Clearly the concept of a single critical velocity for the transition to superfluid turbulence is oversimplified. Critical velocities observed in pure superflow or in thermal counterflow actually represent the intersection of the experimental trajectory with the critical line in the \((V_n, V_s)\) plane. Theories of the critical velocity will have to consider the stability of the superfluid turbulent state as a function of \(V_n\) and \(V_s\) together.

The agreement between the present data and the Leiden results is worse at 1.4K (figure 16) and really terrible at 1.8K (figure 18). The discrepancies could be due to the fact that the Leiden experiments are never truly in a steady state. Dynamic effects near the critical line could mask the actual transition. Marees and van Beelen consider such an effect to be a plausible explanation for the discrepancies in their large tube data. On the other hand their thermal counterflow data point is obtained in a true steady state measurement and differs from our result by a factor of two at 1.8K. Flow tube length and surface roughness also do not seem to play a role. Their data are at slightly different temperature than ours but the correction for this difference should be minor. (The data of references 46 and 47 were actually taken at 1.41K, 1.61K, and 1.81K.) Yamaguchi et al.\(^{59}\) have recently shown that the shape of the tube entrance can have a major influence on the critical velocity. Apparently the stability of very low levels of superfluid turbulence is influenced by effects that are currently beyond experimental control. On the positive side, there is clear evidence for a critical line in the \((V_n, V_s)\) plane, however poorly defined, separating superfluid turbulence from laminar flow.

4.3 Theory

A vast number of theories have been proposed for the critical velocities in superfluid helium. Most of these give a critical value of a single velocity \((V_n, V_s,\) or
the relative velocity $V_n - V_s$) and are therefore inconsistent with the present data. One promising result has been recently obtained by Schwarz. He considers the problem of remanent vortex lines pinned across the flow tube and subject to an imposed combined normal and superfluid flow. The pinning sites (imperfections on the flow tube surface) are taken to be on the order of 1/100 the tube size. The calculations lead to values of $V_{s,pin}$ and $V_{n,pin}$ at which these remanent vortex lines depin and are swept away. The collection of these depinning points forms a closed boundary about the origin in the $(V_n, V_s)$ plane. Although this calculation bears a superficial resemblance to our data, and is important in that it includes $V_n$ and $V_s$ independently, it cannot be taken as the explanation for our observations. In the first place we find no effect on the critical velocities when imperfections of the size used by Schwarz are introduced into the flow tube. Secondly, the calculated depinning velocities are an order of magnitude smaller than the observed critical velocities. Finally, and most important, the critical line in the $(V_n, V_s)$ plane that we have described marks the boundary between superfluid turbulence and vortex-free laminar flow. There is no clear physical connection between this boundary and the condition for vortex depinning. Indeed if the calculations are correct, then the remanent lines in the flow tube would all be depinned and swept away at velocities well below those defining the boundary for superfluid turbulence.

Figure 19 shows a comparison of the critical velocities calculated by Schwarz with those obtained from this experimental apparatus. The closed depinning boundary is physically different from our Superfluid Turbulence Boundary and is an order of magnitude too small. This boundary is the small boundary at the origin in figure 19. On the other hand, the original critical velocity he calculates with line reconnections and wall effects is physically equivalent to what we measure. This critical velocity should agree in pure superflow but it does not. The temperature dependence is too extreme. Paradoxically, it does agree with the first critical velocity measured in
thermal counterflow where it appears to have the correct temperature dependence. This agreement is probably coincidental, however. The boundary formed by the propagation of this calculated critical velocity into the \((V_n, V_s)\) plane is shown as the dot-dashed line in figure 19. Note that it is a factor of about 2.5 times too large to agree in pure superflow.
Figure 19

Critical velocities obtained from the Schwarz theory.
CHAPTER V

DISSIPATION IN THE \((v_{n}, v_{s})\) PLANE AND

MEASUREMENTS OF THE SECOND CRITICAL VELOCITY

FOR INDEPENDENTLY VARIED \(v_{n}\) AND \(v_{s}\)

5.1 DATA

The measurements of the temperature difference \(ΔT\) as a function of \(v_{n}\) and \(v_{s}\) taken at reservoir temperatures of 1.4K, 1.6K, and 1.8K were extended to include flow states with much higher normal-fluid velocities \(v_{n}\). Additional data sets were taken at 1.4K in vertical trajectories at \(v_{n} = 7, 8, 9.45, 10, 11, 12, 14, 16,\) and 20 cm/sec. At 1.6K additional data were taken in vertical trajectories of \(v_{n} = 5.7, 6.4, 7.5, 7.7, 9, 11.5, 15,\) and 18 cm/sec. At 1.8K additional vertical trajectories of \(v_{n} = 0, 1, 2,\) and 3.5 cm/sec were followed. As discussed in Chapter 4 a second set of data was taken to study the effects of surface roughness on superfluid turbulence. These data were taken with high normal-fluid velocities at \(T=1.4K\) in vertical trajectories of \(v_{n} = 10, 14, 16,\) and 21.5 cm/sec.

5.2 EXPERIMENTAL RESULTS

Figure 20 shows the essential features of the \((v_{n}, v_{s})\) plane at 1.4K. The thermal counterflow line is labelled and two "critical velocities" along this trajectory
have been depicted as closed squares in the figure. The first has been shown to be the intersection of the thermal counterflow line with the Superfluid Turbulence Boundary which encloses the origin (Chapter 4). There is no superfluid turbulence within this boundary. The second closed square marks the transition from a region TI described by the equation suggested by Baehr and Tough to a highly developed region of turbulence TII. This TII state has been shown to be nearly homogeneous.\cite{38}

We will begin the discussion of our results with four representative sets of data at small $V_n$ which follow the trajectories indicated by the vertical dashed lines in figure 20 at $V_n = 0, 2, 7, \text{ and } 10 \text{ cm/sec.}$

5.2.1 Dissipation at small $V_n$: Modified Mutual Friction

As stated earlier, the measured total temperature difference $\Delta T$ is the sum of two contributions, a small viscous term $\Delta T_n$ from normal fluid flow and an excess temperature difference $\Delta T''$ which is a mutual friction term due to the interaction between the normal fluid and the vortex lines in the superfluid (equation 1.18). The viscous term is proportional to both the normal fluid velocity $V_n$ (or the power $\dot{Q}$ by equation 1.23) and the normal fluid viscosity $\eta_n$ and is calculated using equation (1.19). The excess temperature difference $\Delta T''$ due to superfluid turbulence is then obtained by subtracting the small viscous temperature difference from the measured total temperature difference using Eqns. (1.23) and (1.19). This excess temperature difference is related to the mutual friction force $F_{sn}$ by equation (1.20). For homogeneous superfluid turbulence $F_{sn}$ is related to the vortex line density by Vinen's original equation (Eqn. 1.7) with a slight modification: \cite{12}

$$F_{sn} = \frac{2}{3} \left( \frac{\rho_n \rho_s}{2\rho} \right) \kappa BV L_o,$$  \hspace{1cm} 5.1

where $V$ is the magnitude of the relative velocity between the superfluid and normal-fluid given by

$$V = V_s - V_n,$$  \hspace{1cm} 5.2
Figure 20

Essential features of the \((V_n, V_s)\) plane at 1.4K. There is no superfluid turbulence in the region enclosed by the Superfluid Turbulence Boundary. The two "critical velocities" found along the thermal counterflow trajectory are indicated as filled squares. The vertical dashed lines show the trajectories of several data runs to be used as examples in this paper.
and \( L_o \) is the homogeneous vortex line density. Schwarz has shown that

\[
L_o = c_L^2 \left( \frac{V^2}{\beta^2} \right)
\]

where

\[
\beta = \frac{\kappa}{4\pi} \log \left( \frac{0.267 \times 10^9}{L_o^3} \right).
\]

Here \( c_L \) is a universal function of temperature and pressure which has been determined by Schwarz in his computer simulations of a homogeneous vortex tangle. We call the model for the dissipation given by equations (1.18)-(1.20) and (5.1)-(5.6) the "Simple Mutual Friction (SMF)" model and compare it directly with our data in figure 19. These figures show data for \( \Delta T' \) as a function of \( V_s \) for constant \( V_n \), that is, along the trajectories shown by the vertical dashed lines in figure 20. The solid line is the excess temperature difference predicted by the SMF model using equations (1.18)-(1.20) and (5.1)-(5.6).

Figure 20a shows \( \Delta T' \) as a function of \( V_s \) for pure superflow. The SMF model is in excellent agreement with the experimental results as has been shown previously. The scale of the figure is such that the transition to superfluid turbulence at \( V_s = 1.5 \) cm/sec is not evident. Figure 21b shows the data with \( V_n = 2 \) cm/sec. The fit is still good although the data are starting to pull away from the predicted values at higher \( V_s \). The data in figure 21c are for \( V_n = 7 \) cm/sec and begin on the thermal counterflow line in the turbulent state TI. The discrepancy between experimental results and the SMF model is much more evident here. The agreement is even worse at \( V_n = 10 \) cm/sec as shown in figure 21d. There is a clear discrepancy with the SMF model that increases systematically with \( V_n \).

The discrepancy of the data with the SMF model could mean that the "actual homogeneous line density" is much less than the homogeneous value \( L_o \) given by equation (5.3). In this case we could use equations (1.2) and (5.1) with the data to
Figure 21

Excess dissipation as a function of total superfluid velocity at $T=1.4K$. The solid line is the predicted dissipation calculated using the SMF model. The velocity combinations corresponding to $V=0$ and Thermal Counterflow are marked with vertical dashed lines.
determine the "equivalent homogeneous line density" which must depend on both $V_n$ and $V_s$, not just the relative velocity $V$. Another interpretation of the discrepancy, however, is to treat equation (5.1) as incomplete. In this model the line density retains the homogeneous value $L_o$ as given by equation (5.3), but equation (5.1) for $F_m$ is modified by introducing a coupling constant $\alpha$ representing the coupling between the vortex lines and the normal fluid flow:

$$F_m = \frac{2}{3} \left( \frac{\rho_n \rho_s}{2 \rho} \right) \kappa BV L_o(V) \alpha(V, V_n).$$

5.8

Baehr and Tough fit their data assuming that the coupling parameter is given by

$$\alpha = \left( 1 - \frac{V_n}{V} \right).$$

5.9

where $a$ is a temperature-dependent quantity on the order of 1 determined experimentally. We call the model for the dissipation using equations (5.5) and (5.6) in place of equation (5.1) the "Modified Mutual Friction (MMF)" Model. This model has been shown to be in excellent agreement with measured dissipation for velocity combinations below the thermal counterflow line (see figure 20). We find this modification also yields excellent agreement for velocity combinations that are above the $V=0$ line.

Figure 22 shows the same data as figure 21 with the solid line now calculated using the MMF model with $a=0.80 \pm 0.05$. Obviously there is no difference between the MMF and SMF models for pure superflow as $V_n = 0$ and $\alpha = 1$ as shown in figure 20a. At $V_n = 2$ cm/sec, figure 22b, the MMF model still is not significantly better than the SMF model but by $V_n = 7$ cm/sec and $V_n = 10$ cm/sec (figures 22c and 22d) the MMF model yields much better agreement.

This agreement appears to be qualitatively correct over the entire range of normal fluid velocities from thermal counterflow at negative $V$, through $V=0$ to large
Figure 22

Excess dissipation as a function of total superfluid velocity at $T=1.4K$. The solid line is the predicted dissipation calculated using the MMF model. The velocity combinations corresponding to $V=0$ and Thermal Counterflow are marked with vertical dashed lines.
positive $V$ (see also figure 20). Baehr and Tough also find excellent agreement with this MMF model for negative $V_s$, from thermal counterflow to large negative $V_s$. As will be discussed below, however, the MMF model does not give a good description of the low-level dissipation from approximately $V_s = 0$ to $V = 0$.

The data shown in figure 22 represent only a small fraction of the present experimental results. In figures 23-25 we show all of our data for $V > 0$ in a form which displays the overall qualitative agreement with the MMF model. The excess dissipation (the measured temperature difference $\Delta T$ minus the small viscous term) is used with equations (1.20) and (5.3)-(5.6) to determine the vortex line density $L_0$. If the MMF model is appropriate, the line density determined from runs at every value of $V_n$ should agree. Further, the line density should be the homogeneous line density given by the Schwarz theory. Our results are given in figures 23-25 for the three temperatures studied: 1.4K; 1.6K; and 1.8K. The dimensionless quantity $L_0^{\frac{1}{2}} d$ is given as a function of the relative velocity $V$. The solid lines are calculated from equations (5.3) and (5.4) for the homogeneous vortex line density using the values of $c_L$ given in the figures. The values of $c_L$ are in good agreement with the values determined by Schwarz in numerical simulations and with values determined from other experiments in homogeneous superfluid turbulence in pure superflow. The values of $a$ (Eqn. 5.6) used in the MMF model are also given in these figures. At 1.4K and 1.6K the values are identical to those found by Baehr and Tough. No other data are available at 1.8K for comparison, but it seems that $a \to 1$ at higher temperatures. The results shown in figures 23-25 indicate that the Schwarz theory of superfluid turbulence can be phenomenologically extended to include a wide variety of flow states in the $(V_n, V_s)$ plane.

5.2.2 Low-Level Dissipation Region
Figure 23

The dimensionless quantity $L_0^{1/2}d$ as a function of $V$ calculated using the MMF model at $T=1.4K$. The solid line is calculated from the Schwarz theory with the appropriate value of $c_L$ listed.
The dimensionless quantity $L_0^{\frac{3}{2}} d$ as a function of $V$ calculated using the MMF model at $T=1.6$K. The solid line is calculated from the Schwarz theory with the appropriate value of $c_L$ listed.

Figure 24
The dimensionless quantity $L_0^{1/2}d$ as a function of $V$ calculated using the MMF model at $T=1.8K$. The solid line is calculated from the Schwarz theory with the appropriate value of $c_L$ listed.
Figure 26

Excess dissipation as a function of total superfluid velocity at $T=1.4\,\text{K}$. The solid line is calculated from the Schwarz theory with the appropriate value of $c_L$ listed.
The results shown in figure 22 suggest that the MMF model yields goods results for all combinations of the velocities \( V_n \) and \( V_s \) shown but this is not the case. The apparent fit in figure 22 is deceiving because a large scale has been used in order to show data for higher values of \( V_s \). There is poor agreement in the region outside the superfluid turbulence boundary, approximately between the \( V_s =0 \) line and the \( V=0 \) line (see figure 20). Figure 26 shows an enlargement of this region for \( V_n = 7 \) and \( 10 \) cm/sec. In figure 26a the MMF model shown as the solid line deviates substantially from the data from about \( V_s =2 \) cm/sec to about \( V_s =9 \) cm/sec. The agreement in figure 26b for \( V_n =10 \) cm/sec is poor until \( V_s \) is about \( 12 \) cm/sec. Not only is the agreement poor in this region, but even the sign of the dissipation is wrong! We refer to this region of the plane as the "Low-Level Dissipation (LLD) Region".

Extensive data obtained in this LLD region by workers at the University of Leiden show that the flow states here are extremely complicated.\(^{34,41-44}\) The superfluid turbulence is inhomogeneous and very small but nonzero. Since \( V \) is small in this region any effects depending explicitly on \( V \) will be small. However, both \( V_n \) and \( V_s \) can be large in this region so effects depending upon either or both of these velocities will dominate here. To further emphasize this point we show in figure 27 our data for \( \Delta T' \) along the \( V=0 \) line. In order to compress the range of \( \Delta T' \) these data are presented as the cube root of \( \Delta T' \) as a function of \( V_n \) (or \( V_s \) ). Once outside the laminar region surrounding the origin (figure 20) the dissipation along this line increases smoothly indicating no abrupt transition from one type of dissipation to another. Using several temperature probes along the length of the flow tube the Leiden workers are able to see the inhomogeneity of turbulence along the length of the tube. They observe turbulence form at one end of the tube, grow, and finally break away to be swept out the other end of the tube. They have also observed oscillations between two turbulent states. This LLD region persists to higher values of \( V_n \) as shown in figure 28 for \( V_n =16 \) cm/sec and \( V_n =21.5 \) cm/sec. The LLD region for \( V_n \)
Figure 27
Excess dissipation as a function of total superfluid velocity for the trajectory \( V=0 \).
=16 cm/sec occurs for \( V_s \geq 4 \) cm/sec. At \( V_n = 21.5 \) cm/sec the LLD region occurs for \( V_s \geq 9 \) cm/sec.

5.2.3 Extension of the TI/TII Transition

Figure 28 also reveals a feature which is not present at smaller values of \( V_n \) (figure 22). There is a sharp transition from the state of low-level turbulence to another state of much higher dissipation. In figure 28a for \( V_n = 16 \) cm/sec this transition is located roughly at \( V_s \approx 3 \) cm/sec. In figure 28b at \( V_n = 21.5 \) cm/sec this transition occurs at \( V_s \approx 8 \) cm/sec. Above these values of \( V_s \) the dissipation is small while below the dissipation becomes much larger. The significance of this transition can be understood by plotting the \( V_n \) and \( V_s \) coordinates of the transition in the \( (V_n, V_s) \) plane. Figure 29a shows our data at 1.4K, including results for the "roughened" flow tube. The locus of these points forms a boundary that intersects the thermal counterflow line exactly at the TI/TII transition (see figure 20). Apparently the transition we have observed in the more general flow states (figure 28) is simply an extension of the TI/TII transition into the \( (V_n, V_s) \) plane. Figure 28b shows similar data at 1.6K including points below the thermal counterflow line obtained in previous experiments with the same flow tube. The transition at 1.6K cannot be resolved for \( V_n \) less than or about 6 cm/sec but becomes increasingly more abrupt as \( V_n \) is increased.

This transition has also been seen in the Leiden experiments,\textsuperscript{34,41,42,44} but two types of behavior are found as shown in figure 30a. The first behavior which they label the "Steep Branch II" is equivalent to what our data show (figure 30b). At some value of \( V_s \), \( \Delta T' \) drops sharply from a very high value (high dissipation) to a very small value (low-level dissipation). They also see a second type of behavior they label the "Steep Branch I" which we do not see. Here \( \Delta T' \) drops rapidly at a smaller value
Figure 28
Excess dissipation as a function of total superfluid velocity at $T=1.4K$. The solid line is the predicted dissipation calculated using the MMF model.
The location of the TI/TII transition in the $(V_n, V_s)$ plane. Open squares are for data in which the flow tube had been "roughened".

Figure 29
Figure 30

(a) The transition to the LLD state observed in the Leiden experiments. Two types of behavior are observed labelled the "Steep Branch I" and the "Steep Branch II".
(b) The transition observed in the present experiments.
of $V_\ast$, becomes unstable, and finally stabilizes again at a higher value of $V_\ast$. Our TI/TII Boundary coincides with their Steep Branch-II. Comparisons of these results are shown in figure 31 with references and flow tube characteristics given in tables 1 and 2. The normal and superfluid velocities have been reduced to dimensionless form using equations (4.2) and (4.3). The agreement between the present results and those from Leiden is quite good considering the variety of tube materials, diameters, lengths, and surface roughness. In contrast with the Leiden experiments, however, we have never observed the "Steep Branch I" in our apparatus. This behavior appears to be associated with axially inhomogeneous turbulence in the flow tube, but there are no obvious experimental conditions that can be identified with this effect.

5.2.4 The Simple Mutual Friction Region

The transition revealed in figure 31 (identified above as the extension of the TI/TII transition) is between the state of low-level dissipation (LLD) and a state of much higher dissipation. The high dissipation state along the thermal counterflow line is the state TII which has been shown to be homogeneous superfluid turbulence. This suggests that the entire region in the $(V_n, V_\ast)$ plane below the TI/TII boundary (figure 31) could also be homogeneous superfluid turbulence with a dissipation given by simple mutual friction. To test this idea we compare the dissipation in this region with the simple mutual friction model (Eqns. 1.18-1.20 and 5.1-5.4). We have replotted the data given in figure 28 for $V_n = 16$ cm/sec and 21.5 cm/sec in figure 32. The solid line is calculated from the equations (1.18)-(1.20) and (5.1)-(5.4) and is in reasonable agreement with the high-level dissipation data. To explore this agreement further we have analyzed all of our data in this high dissipation region at 1.4K and 1.6K. The temperature difference data are reduced to give the vortex line density $L_\circ$ using equations (1.18)-(1.20) and (5.1) and the results are given in figures 33 and 34.
Figure 31

A comparison of the TI/TII boundary in the $(V^*_n, V^*_s)$ plane with previous results.
Table 1

References, tube size, material, and length for the collected data shown in figure 31a for $T=1.4K$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Ref.</th>
<th>Tube Size ($\mu$ m)</th>
<th>Material</th>
<th>$l$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>Present Work</td>
<td>134</td>
<td>Glass</td>
<td>9.9</td>
</tr>
<tr>
<td>□</td>
<td>Present Work</td>
<td>134</td>
<td>Glass (Roughened)</td>
<td>9.9</td>
</tr>
<tr>
<td>●</td>
<td>42</td>
<td>216</td>
<td>Glass</td>
<td>8.5</td>
</tr>
<tr>
<td>▽</td>
<td>42</td>
<td>206</td>
<td>Metal</td>
<td>8.5</td>
</tr>
<tr>
<td>◊</td>
<td>44</td>
<td>216</td>
<td>Glass</td>
<td>8.5</td>
</tr>
<tr>
<td>△</td>
<td>44</td>
<td>206</td>
<td>Metal</td>
<td>8.5</td>
</tr>
<tr>
<td>Symbol</td>
<td>Ref.</td>
<td>Tube Size ($\mu$m)</td>
<td>Material</td>
<td>$l$ (cm)</td>
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<tr>
<td>o</td>
<td>Present Work</td>
<td>134</td>
<td>Glass</td>
<td>9.9</td>
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<td>□</td>
<td>44</td>
<td>216</td>
<td>Glass</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Table 2

References, tube size, material, and length for the collected data shown in figure 31b for $T=1.6K$. 
Figure 32
Excess dissipation as a function of total superfluid velocity at high normal-fluid velocities at $T=1.4$K. The solid line is the predicted dissipation calculated using the MMF model with the Schwarz theory with $c_L=0.120$. 

(a) $V_n=16$ cm/s

(b) $V_n=21.5$ cm/s
as the dimensionless quantity $L_0^3 d$ as a function of the relative velocity $V$. The solid lines are calculated with the Schwarz theory using equations (5.3) and (5.4) with the values of $c_L$ given in the figures. These values of $c_L$ are typical of thermal counterflow results, which in general are slightly smaller than found in superflow. The TI/TII transition is obvious in these data, and the rather limited data for the high dissipation state are in reasonable agreement with the Schwarz theory. It seems likely then that the entire region below the TI/TII boundary (figure 31) can be described by simple mutual friction.

5.3 Theory

Computer simulations by Schwarz of a random vortex tangle have produced important results. Our results show that the dissipation over a wide range of the $(V_n, V_s)$ plane can be associated with the homogeneous vortex line density of the Schwarz simulations (figures 23-25, 33, and 34). In the MMF regions we have used a phenomenological coupling of the vortex lines to the velocity. In his most recent paper $^{61}$ Schwarz finds the mutual friction force is related to the vortex line density by

$$F_m = \left( \frac{\rho_n \rho_s}{2\rho} \right) \kappa B (I_\parallel - c_L I_\perp) LV$$

5.7

where

$$\bar{L} = c_L^2 \left( \frac{V}{\beta} \right)^2.$$  

5.8

The quantities $I_\parallel$ and $I_\perp$ represent a measure of anisotropy of the vortex tangle. If the tangle is isotropic then $I_\parallel = 2/3$ and $I_\perp = 0$ thus reducing equation (5.7) to equation (5.1), the result for simple mutual friction.

Donnelly and coworkers have determined the anisotropy of the TII state in thermal counterflow by measuring attenuation coefficients of second-sound and ion currents. $^{63}$ The results are in good agreement with Schwarz's computer simulations.
Figure 33

The dimensionless quantity $L_0^{1/2}d$ as a function of $V$ at $T=1.4K$ calculated using the MMF model. The solid line is calculated from the Schwarz theory with the appropriate value of $c_L$ listed.
The dimensionless quantity $L_0^{1/2}d$ as a function of $V$ at $T=1.6K$ calculated using the MMF model. The solid line is calculated from the Schwarz theory with the appropriate value of $c_L$ listed.

**Figure 34**
In his simulations Schwarz treats the vortex tangle as homogeneous but having an anisotropy imposed by the fluid flow. The relative velocity $V_r - V_n$ is taken to be uniform across the flow tube. It is certainly possible that in the more general flows considered in our experiments there will be regions in the $(V_n, V_r)$ plane where the relative velocity is not uniform. It is tempting to speculate that in these regions the anisotropy could depend on $V_n$, and that our coupling constant $\alpha$ (eqn. 5.6) is a measure of this anisotropy and is equivalent to the quantity $(\tilde{I}_|| - c_L \tilde{I}_\perp)$ in equation (5.7).

5.4 Conclusion

Our data have evidenced three types of dissipation that occur in the flow of Helium II. Figure 35 shows a summary of these regions. Homogeneous turbulence occurs along the trajectory $V_n = 0$ cm/s (pure superflow) once outside the Superfluid Turbulence Boundary. A normal-fluid velocity added to this superfluid flow modifies the homogeneous turbulence giving rise to the "Modified Mutual Friction" regions in both the first and fourth quadrants as shown in figure 35. The second type of dissipation is the "Low-Level Dissipation" which is found in the region where the relative velocity $V$ (i.e. $F_n$) is small. Effects which depend explicitly on either $V_n$ or $V_r$ will be seen only in this region. The third type of turbulence is one of "Simple Mutual Friction". This type of dissipation occurs at higher values of $V_n$. Dissipation in this region appears to be homogeneous although the value of $c_L$ in Schwarz's model that fits the data is smaller than the homogeneous value obtained from pure superflow experiments. The LLD region and the SMF region are separated by the TI/TII boundary which is the extension of the TI/TII transition found in thermal counterflow into the $(V_n, V_r)$ plane.
Figure 35

The $(V_n, V_s)$ plane at $T=1.4$ showing the types of dissipation in each region.
CHAPTER VI

CONCLUSIONS

The goals of this research were (1) measure the first critical velocity above the thermal counterflow line; (2) measure the second critical velocity above the thermal counterflow line; (3) complete a measurement of dissipation for independently varied superfluid and normal-fluid velocities above the thermal counterflow line; (4) determine the effects of surface roughness on superfluid turbulence; and (5) determine whether the model developed by Baehr and Tough agree with these measurements of dissipation and thus constitute a unified description for dissipation in turbulent He II for all flow states. This chapter reviews the findings.

6.1 Measurements of the first critical velocity

The measurements of the first critical velocity from laminar flow to turbulent flow show its propagation above the thermal counterflow line. Keeping in mind that quadrants I and IV are physically equivalent to quadrants II and III, it appears that the first critical velocity forms a closed boundary about the origin of the \((V_n, V_s)\) plane. This boundary has been named the Superfluid Turbulence Boundary. The critical velocities found in thermal counterflow and pure superflow merely represent the intersection of those trajectories with the Superfluid Turbulence Boundary. Comparision with other experiments show qualitative agreement although the actual shape of the boundary is still in question. This boundary has been described by
linear equations below the thermal counterflow line with success by both Baehr and Maress, but this is probably coincidental and not physically meaningful. Schwarz's findings of critical velocity combinations, $V_{n,pin}$ and $V_{s,pin}$, where vortex lines depin from the surface do not seem to have any physical connection with this boundary.

6.2 The TI/TII transition

Previous experiments have shown that the TI/TII transition exists below the thermal counterflow line. The present research shows that this transition exists also above the thermal counterflow line and becomes increasingly abrupt for higher $V_n$. This transition is equivalent to the Steep Branch II seen by workers at Leiden and agreement of the location of this transition is quite good. They also see another steep branch (Steep Branch I) which appears to occur where $V_s \approx 0$, but we have not seen this transition in any of our experiments. This Steep Branch I does not seem to depend on tube material, tube size, tube length, tube surface roughness, or type of driving mechanism for the superfluid flow. Apparently this transition depends on some factor presently unaccounted for.

6.3 Regions of Turbulence

As shown in figure 33, our data indicate the existence of three types of turbulence. For small $V_n$ there is no turbulence. This laminar flow region is bounded by the Superfluid Turbulence Boundary. The homogeneous turbulence found in pure superflow experiments becomes increasing modified as a normal–fluid velocity is imposed, resulting in the Modified Mutual Friction Regions. For large $V_n$ (past the TI/TII transition boundary) the turbulence is found to be nearly homogeneous. The TI/TII transition boundary separates highly developed turbulence from a Low-Level Dissipation Region. In this region $V$ is small so only effects explicitly due to $V_n$.
and/or \( V_s \) alone will be seen. This region is separated from the Modified Mutual Friction Regions by the the \( V=0 \) line and the \( V_*=0 \) line respectively.

### 6.4 Effects of Surface Roughness on Superfluid Turbulence

Throughout Chapters 4 and 5 data for both “unroughened” and “roughened” flow tube surfaces have been presented. Comparisons of these data seem to indicate there is no effect on the onset of superfluid turbulence or the measured dissipation due to surface roughness when the roughness is \( \approx 1/100 \) of the tube diameter. In his computer simulations Schwarz finds critical velocity combinations \( V_{n,\text{pin}} \) and \( V_{s,\text{pin}} \) for which vortex lines depin and are swept out of the flow tube. In his computer simulations, these effects are dependent upon the size of pinning sites (or surface roughness) disagreeing with results of this research.

### 6.5 The Modified Mutual Friction Model

This research shows that the model for dissipation suggested by Baehr and Tough which successfully fits the dissipation for flow states below the thermal counterflow line does work above the thermal counterflow line in a restricted region. This region lies above the line defined by \( V=0 \). Unfortunately it does not constitute a unified description for the dissipation in all regions of the \( (V_n,V_s) \) plane. Two regions of turbulence (the Low-Level Dissipation Region and the Simple Mutual Friction Region) exist between \( V=0 \) and the thermal counterflow line whose dissipation does not follow this model.

**Future Studies of Superfluid Turbulence**

The complexity of the flow states of Helium II for independently varied superfluid and normal fluid velocities is undeniable. Much progress has been made in
understanding these flows and some success has been achieved in simulating them on a computer but there still must be a very fundamental concept which is missing. This is evident because present models for dissipation include the relative velocity alone and predict zero dissipation along $V=0$. The dissipation encountered along the trajectory $V=0$, however, shows this must be incorrect. Any model which yields results completely consistent with present data must include a dissipation term which depends explicitly on $V_n$ and/or $V_s$. Continued investigation of these flow states will provide this important missing concept. In the meantime, it is apparent that the dissipation encountered in the flow of Helium II is a two-fluid phenomena and must be modeled as such.
APPENDIX A

This appendix contains plots of all data runs obtained in the experiments reported in this dissertation. Two types of plots are presented here. The first type is one where the excess dissipation $\Delta T'$ is plotted as a function of the superfluid velocity due only to the helium wick $V_{sm}$. These plots are the data taken in the vertical trajectories shown in figure 9. The second type of plot is one where the excess dissipation $\Delta T'$ is plotted as a function of the normal-fluid velocity $V_n$. These plots are the data taken in trajectories parallel to the thermal counterflow line as shown in figure 9.

In each case, the solid line in the plot shows the predicted dissipation calculated using the appropriate model. For data lying mostly above $V=0$, the calculation uses the Modified Mutual Friction Model. For data in the region beyond the TI/TII Boundary the predicted dissipation is obtained from the Simple Mutual Friction Model. The caption for each figure indicates which model is plotted as well as the value of $c_L$ used in the Schwarz calculation. All plots for data obtained in trajectories parallel to the thermal counterflow line are shown with the Modified Mutual Friction Model.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n = 0$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.136$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=1$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 38

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=2$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 30
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n =3$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_1 =0.136$. $V_{sm} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s =0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=4$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=5$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n = 6$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.136$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_* = 0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=7$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 44
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n =8$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.136$. $V_{sm} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s =0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=9.45$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_*=0$ and $V=0$ are marked with vertical dashed lines.
Figure 40

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n = 10$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.136$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V=0$ are marked with vertical dashed lines.
Figure 47

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=11$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.120$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 48
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n = 12$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L = 0.120$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V = 0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n =14$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L =0.120$. $V_{sm} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s =0$ and $V=0$ are marked with vertical dashed lines.
Figure 50

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=16$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.120$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=20$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.120$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 52
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.4K$ for $V_{sm}=0$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Figure 53
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.4K$ for $V_{sm}=2.25$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Figure 54

Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.4K$ for $V_{sm}=3$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Figure 55

Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.4K$ for $V_{sm}=4.5 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4\,\text{K}$ for $V_n=0\,\text{cm/s}$ for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 57

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=2$ cm/s for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 58

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n =3$ cm/s for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_L =0.136$. $V_{sm} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s =0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n = 4 \text{ cm/s}$ for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.136$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=10$ cm/s for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.136$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n =14$ cm/s for the roughened flow tube. The solid line is the predicted dissipation calculated using the SMF model with $c_L =0.120$. $V_{sm} =0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s =0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4$K for $V_n=16$ cm/s for the roughened flow tube. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.120$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.4K$ for $V_n=21.5$ cm/s for the roughened flow tube. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.120$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 64
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.4\text{K}$ for $V_{sm}=0 \text{ cm/s}$ for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with $c_r=0.136$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Excess dissipation as a function of normal-fluid velocity \( V_n \) at \( T=1.4\text{K} \) for \( V_{sm}=2.25 \text{ cm/s} \) for the roughened flow tube. The solid line is the predicted dissipation calculated using the MMF model with \( c_L=0.136 \). The velocity combination corresponding to \( V=0 \) is marked with a vertical dashed line.

Figure 65
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=0$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=0.5 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6\text{K}$ for $V_n=1\text{ cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

Figure 68
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=1.5$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=2$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 71

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6$K for $V_n=2.75$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 72

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=3.25$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=3.75$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=4.8$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=5.7$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.168$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6\,\text{K}$ for $V_n=6.4\,\text{cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=7.5$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6$ K for $V_n=7.7$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Figure 79

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=9$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_*=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6\text{K}$ for $V_n=11.5\text{ cm/s}$. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.162$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=15 \text{ cm/s}$. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.162$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.6K$ for $V_n=18$ cm/s. The solid line is the predicted dissipation calculated using the SMF model with $c_L=0.162$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{sm}=0$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{sm}=0.8$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.

Figure 84
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6\,\text{K}$ for $V_{sm}=2\,\text{cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Figure 86
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{sm}=3$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Figure 87
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.6K$ for $V_{sm}=4$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.168$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Figure 88

Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n = 0$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V = 0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n = 1$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V = 0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n = 2$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$. $V_{sm} = 0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s = 0$ and $V=0$ are marked with vertical dashed lines.
Excess dissipation as a function of superfluid velocity $V_{sm}$ at $T=1.8K$ for $V_n=3.5$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $e_L=0.160$. $V_{sm}=0$ corresponds to the Thermal Counterflow Line while the velocity combinations corresponding to $V_s=0$ and $V=0$ are marked with vertical dashed lines.

Figure 91
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.8\,\text{K}$ for $V_{sm}=0\,\text{cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.160$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Figure 93

Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.8K$ for $V_{sm} = 3$ cm/s. The solid line is the predicted dissipation calculated using the MMF model with $c_L = 0.160$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
Excess dissipation as a function of normal-fluid velocity $V_n$ at $T=1.8\,\text{K}$ for $V_{\text{sm}}=4\,\text{cm/s}$. The solid line is the predicted dissipation calculated using the MMF model with $c_L=0.160$. The velocity combination corresponding to $V=0$ is marked with a vertical dashed line.
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