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An investigation of dichroic surfaces of staggered tuned arrays in an infinite medium and a stratified dielectric medium

Tokarz, Frederick A., Ph.D.
The Ohio State University, 1989
AN INVESTIGATION OF DICHROIC SURFACES OF
STAGGERED TUNED ARRAYS IN AN INFINITE MEDIUM
AND A STRATIFIED DIELECTRIC MEDIUM

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

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* * * * *

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CHAPTER I
INTRODUCTION

A. BACKGROUND AND MOTIVATION

In the recent past, there has been a sustained effort to investigate scattering properties of periodic surfaces. The term "periodic surface" is generally associated with two dimensional infinite periodic arrays of metallic dipoles or slots (holes in a ground plane), even though practical periodic surface elements rarely consist of linear dipoles or slots. The periodic surfaces are also referred to in the literature as dichroic surfaces or Frequency Selective Surfaces (FSS). Dichroic surfaces are commonly used in the design of Radio Frequency (RF) antenna systems to selectively "filter" specific frequencies of interest. An example of the use of a dichroic surface is shown in Figure 1. This particular antenna is designed to reflect energy in three separate frequency bands (bands 1, 2, 3). To accomplish this, subreflector 1 consists of a specially designed dichroic surface embedded in dielectric. It is designed to reflect energy in frequency bands (2) and (3) but be transparent in band (1).

Several individual arrays can be combined with dielectric layers to produce an integrated multiplanar FSS. According to Munk
Figure 1. Multiple frequency dual reflector antenna demonstrating a dichroic surface for subreflector 1 which is designed to reflect energy in bands 2 and 3 and is transparent in band 1.
and Stosic [1] a multiplanar FSS can produce a flatter resonance and shorter cut-off outside resonance than a monoplanar array. Previous investigators such as Munk and English [2] have concentrated on biplanar array designs where the two individual arrays are identical. For the FSS example shown in Figure 1, a dual frequency reflection resonance is required. If both of the arrays are identical then one or more of the reflection bands will not have the proper resonance frequency. To overcome this problem, one requires a biplanar array where each individual array is resonant at a different design frequency. The individual array resonant frequencies would correspond to one of the reflection bands of the overall dual frequency reflecting FSS. This non-symmetrical biplanar array would have a resonance occurring in each of the design reflection bands (2 and 3). This type of a biplanar array is referred to as a 'staggered tuned' biplanar array. Different resonances for each array can occur when the array geometries are varied. For instance, the resonance will change if the length or width of the elements in each array are different or if the dielectric environment is different for each array.

Staggered tuned biplanar arrays may have significant advantages over identical biplanar arrays. First, it is possible to design a dual-stopband filter where each of the arrays is resonant at a different frequency. Second, the non-identical arrays may be designed to have a larger combined bandwidth than the identical arrays without seriously affecting cutoff outside the resonance band. Third, since the individual resonance frequencies may vary with
angle of incidence, this behavior may be exploited to give the designer more options in designing a FSS. The negative aspect is that the design problem is more complex with non-identical arrays. Furthermore, each resonance band has different requirements for achieving stable resonances with angle of incidence, properly referred to as "scan compensation".

Munk [3] has investigated the resonant properties of staggered tuned arrays when placed in an infinite medium. The arrays were tuned to resonate at different frequencies by placing different loads on the elements in each array. In this investigation, the arrays are tuned to resonate at different frequencies by changing the element geometry of the array. In addition to the stopbands, design methods for creating a passband in a staggered tuned array of dipoles are investigated.

Munk and Kornbau [4] have investigated the conditions necessary for a dichroic surface to be scan compensated. They and others have applied the conditions to symmetrical biplanar arrays (resonant frequencies of the two arrays are the same). This dissertation will investigate the conditions necessary for dichroic surfaces of staggered tuned arrays to be scan compensated (resonant frequencies of the arrays are not the same). It will also investigate the degree of scan compensation achieved in the passband of a staggered tuned biplanar array of dipoles.

In summary this dissertation will investigate the impedance, reflection, and transmission properties of dichroic surfaces of
staggered tuned arrays in a stratified dielectric medium as well as develop practical design tools for applying FSS analysis techniques to the biplanar array design problem.

B. ANALYSIS TECHNIQUE

The basic theory of periodic surfaces has been developed by Munk, Burrell, and Kornbau [5] and has been updated by Munk [6]. The arrays can consist of thin wire elements or flat elements and also allows for dielectric layers. English [7] analyzed FSS designs consisting of infinite arrays of dipole antennas with arbitrary orientation. Kornbau [8] developed designs which reduced the interelement spacing by tilting the dipole arrays. Both of the methods result in FSS designs which were scan compensated. In [7,8], the specific theory utilized assumes that the elements in a given array are identical and have the same orientation. The general theory of dichroic surfaces, however, is not restricted to the element geometries and element orientations which vary in the different arrays. The fundamental Plane Wave Expansion (PWE) technique can be used to analyze very general cases. For convenience the theory is summarized in the following paragraphs. The PWE is very general since it is a modified moment method solution.

In order to apply the PWE, it is assumed that all elements in a given array are identical and have the same orientation (the element geometries and element orientations in different arrays are not
necessarily related). Consider the infinite biplanar periodic array of dipoles embedded in a finite series of dielectric layers as illustrated in Figure 2. The number of dielectric layers is denoted by the variable MM. The mth slab has thickness \(d_m\) and constitutive parameters \((\varepsilon_m, \mu_m)\). The y coordinate of the interface between the mth and m+1 dielectric slab is denoted as \(b_m\).

From Figure 3, the individual array elements of the nth array are orientated in the \(\hat{p}^n\) direction and the interelement spacings are \(D_x\) and \(D_z\). The array is contained in the x-z plane which results in \(p_y = 0\). For all cases investigated in this dissertation the array will be restricted to the x-z plane. The total length of the elements in the nth array is \(2l^n\) and the width \(w^n\). The distance from the \(y = 0\) plane to the nth array is \(y^n\) (see Figure 2). With the geometry defined, assume the array is illuminated by a plane wave propagating in the direction \(\hat{s}_\pm\), where

\[
\hat{s}_\pm = s_x\hat{x} \pm s_y\hat{y} \pm s_z\hat{z}
\]  

(1.1)

and

\[
s_y = -\cos(\alpha)\sin(\eta)
\]  

(1.2)

\[
s_z = -\sin(\alpha)\sin(\eta)
\]  

(1.3)

with
Figure 2. Periodic array of dipoles embedded in a stratified dielectric medium.
Figure 3. A planar array (the ith array) of linear dipoles orientated in the $\hat{p}_i$ direction.
The angles alpha ($\alpha$) and eta ($\eta$) specify the plane and angle of incidence as illustrated in Figure 4. The plane of incidence is determined by $\alpha$ and contains $\hat{s}$ and the -$y$ axis. The angle of incidence is determined by $\eta$ and is measured between the -$y$ axis and $\hat{s}$. The incident field is broken down into parallel ($\parallel E^i$) and orthogonal ($\perp E^i$) components with respect to the plane of incidence. The scattered fields are similarly decomposed in the direction $\parallel E^s$ and $\perp E^s$. This decomposition greatly simplifies the analysis of a stratified
The incident electric field, $\vec{E}_i$, will induce currents on every array element. Since every element in the array is identical with a periodic interelement spacing, Floquet's theorem can be used to model the behavior of the induced currents. Floquet's theorem requires that (1) the shape or amplitude of the current on each element be identical and (2) the phase of the current (from element to element) vary linearly across each array. In other words the currents in each element differ only by a phase constant and the phase constant between adjacent elements must be identical for each array. In general this requires the interelement spacing to be constant for each array. Also the theorem requires that the phase of the current for each element match the incident field. If the amplitude and phase are known for one element, from boundary conditions all information about the array is known. Assume we know the proper current modes to put on this one element (called the reference element), then we can relate the current on the reference element to any other element $(q,m)$. Thus

$$I_{q,m}(l) = I_{0,0}(l)e^{-j\beta(qD_xs_x + mD_zs_z)}$$

(1.5)

where $D_x$ is the interelement spacing in the x-direction and $D_z$ is the interelement spacing in the z-direction and $q$ and $m$ are arbitrary
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integers. Therefore if we know the parameters of the incident wave, the geometry of the array, and the current shape on one element, we can find the currents on all the array elements.

The currents on each element will reradiate scattered fields, $\tilde{E}^s$. Since the total tangential electric field must vanish on the perfectly conducting array elements (except at the element terminals), then

$$\tilde{E}_{\text{tan}}^{\text{total}}(\vec{R}) = \tilde{E}_{\text{tan}}^i(\vec{R}) + \tilde{E}_{\text{tan}}^s(\vec{R}) \quad (1.6)$$

where $\vec{R}$ is an arbitrary point on the surface of the reference element of the nth array and $\tilde{E}_{\text{tan}}^i$ is the incident field at $\vec{R}$. $\tilde{E}_{\text{tan}}^s$ is the tangential component from the scattered field from all the arrays. From Kent [9], $\tilde{E}_{\text{tan}}^s$ may be written as,

$$\tilde{E}_{\text{tan}}^s(\vec{R}) = \sum_{n'=1}^{NN} I_{n'}(\vec{R}n')\tilde{e}_{\text{tan}}^{n'}(\vec{R}) \quad (1.7)$$

where $I_{n'}(\vec{R}n')$ is the terminal current on array $n'$ and $\tilde{e}_{\text{tan}}^{n'}$ is a plane wave expansion function for array $n'$. Therefore (1.6) can be expressed as:

$$\tilde{E}_{\text{tan}}^{\text{total}}(\vec{R}) = \tilde{E}_{\text{tan}}^i(\vec{R}) + \sum_{n'=1}^{NN} I_{n'}(\vec{R}n')\tilde{e}_{\text{tan}}^{n'}(\vec{R}) = 0 \quad (1.8)$$
Next we require that $E^{\text{total}}_{\text{tan}}(\tilde{R})$ vanish as a weighted average along a filament on the wire surface. (1.8) becomes,

$$
\frac{1}{w(\tilde{R}^n)} \int_{E^{\text{tan}}_{\text{lin}}(\tilde{R})} \frac{1}{w(\tilde{R}^n)} \int_{E^{\text{lin}}_{\text{lin}}(\tilde{R})}^{E^{\text{lin}}_{\text{lin}}(\tilde{R})} \sum_{n'=1}^{N} \frac{1}{w(\tilde{R}^n)} \int_{E^{\text{lin}}_{\text{lin}}(\tilde{R})}^{E^{\text{lin}}_{\text{lin}}(\tilde{R})} E^{\text{lin}}_{\text{lin}}(\tilde{R}) \, dl \, l_{n'}(\tilde{R}^n')
$$

(1.9)

where $w(\tilde{R})$ is the weight or testing function and $w(\tilde{R}^n)$ is the function evaluated at the terminals. Kent [9] identifies the first integral as the forcing function in the nth array, $V_{0}^{n}$, denoted

$$
V_{0}^{n} = \frac{1}{w(\tilde{R}^n)} \int_{E^{\text{lin}}_{\text{lin}}(\tilde{R})}^{E^{\text{lin}}_{\text{lin}}(\tilde{R})} E^{\text{lin}}_{\text{lin}}(\tilde{R}) \, dl
$$

(1.10)

Kent [9] defines the second integral the self impedance of the array reference element $Z^{nn'}$ as

$$
Z^{nn'} = \frac{1}{w(\tilde{R}^n)} \int_{E^{\text{lin}}_{\text{lin}}(\tilde{R})}^{E^{\text{lin}}_{\text{lin}}(\tilde{R})} E^{\text{lin}}_{\text{lin}}(\tilde{R}) \, dl
$$

(1.11)

According to Schelkunoff [10], $w(\tilde{R})$ should approximate the current distribution under transmitting conditions. This will result in
the induced voltage expression of (1.10) being exact. Thus (1.9) can be written as

\[ V_n(\hat{R}_n) = [Z_{nn'}][I_{n'}(\hat{R}_n')] \]  

(1.12)

The solution takes the form as a typical moment method problem, in that \( V_n(\hat{R}_n) \) and \( Z_{nn'} \) can be calculated. \( I_{n'}(\hat{R}_n') \), the unknown current, can then be found by a simple matrix inversion, thus the PWE is a moment method type of solution [9].

In summary, to find the terminal current \( I_{n'}(\hat{R}_n') \), one proceeds as follows: 1). calculate the induced voltage in all the arrays, 2). calculate the impedance matrix \( [Z_{nn'}] \) and 3). solve equation (1.12) for the terminal currents. Once the terminal currents are known the scattered field can be found.

This investigation is broken into three parts. The first part, Chapter II, gives a summary of the basic theory of periodic surfaces used in this dissertation. It summarizes the development of expressions for the induced voltage, the mutual impedance, and the self-impedance. The total forward and backscattered fields are then developed for a biplanar array embedded in a stratified dielectric medium. It also shows that an array of dipoles can be modeled as a shunt impedance (surface impedance) on a transmission line and it relates the surface impedance of the array to the input impedance of the array so that the analysis can be simplified.
The second part of this investigation, Chapter III, deals with the analysis of staggered tuned biplanar arrays of dipoles in free space. In Chapter III, we investigate the effect on the reflection bandwidth by having each of the individual arrays resonant at a slightly different frequency. Design methods for creating a two band biplanar array are presented in the second part. The problems with designing a transmission band for a two band biplanar array are presented along with solutions to the problems. It is shown that a two band biplanar array with a passband can be designed for one scan angle. As the scan angle is varied the resonant frequency and reflection and transmission bandwidth change. Calculations are presented illustrating the analysis presented in Chapter III. The investigation presented here applies only to dipoles.

The third part of this dissertation, Chapter IV, is an analysis of staggered tuned biplanar arrays of dipoles embedded in dielectric. While Chapter III is mainly concerned with the interaction between the dipole arrays, Chapter IV is concerned with the interaction of the dipole arrays and the dielectric layers. The investigation here also only applies to dipoles. Design guidelines on achieving some degree of scan compensation by the use of dielectric slabs is applied to the staggered tuned biplanar array. The problems of obtaining a reasonable degree of scan compensation for staggered tuned biplanar arrays is presented in the third part. The results should be useful in the design of such structures. Calculations are also presented in this part.
Chapter V summarizes the results and provides conclusions and recommendations. The main report is followed by appendices which explore in detail several specific aspects of the problem.
CHAPTER II
PERIODIC SURFACE THEORY

In order to analyze the properties of a staggered tuned biplanar array, three important quantities must be calculated. The quantities are: 1) the induced voltage in all the arrays, 2) the impedance matrix, and 3) the terminal currents. Once the terminal currents are known the scattered field can then be calculated.

A. INDUCED VOLTAGE

The induced voltage is the voltage which appears at the open circuited terminals of the array reference element which may be located in a stratified medium. Periodicity simplifies the calculation for the induced voltage in that the magnitude of the induced voltage will be the same for every element of the array and the phase of the induced voltage will vary linearly across the array elements (Floquet's theorem). By calculating the induced voltage on the reference element, the voltage will be known for every element.

To calculate the induced voltage at the reference element, the field at any point within any individual dielectric slab must be determined. Figure 5 illustrates this situation. From Figure 5 the
Figure 5. Incident electric field, $E_{inc}(R)$, at point $R$ in slab $m$. 
layered slabs are illuminated by an externally incident plane wave traveling in the \( \hat{s}_{0+} \) direction, where

\[
\hat{s}_{0+} = s_{0x} \hat{x} + s_{0y} \hat{y} + s_{0z} \hat{z}
\]  

(2.1)

where \( s_{0x}, s_{0y}, s_{0z} \) are defined in Appendix A. Looking at Figure 5, some of the incident energy propagates to the \( m \)th slab where one plane wave propagates in the \( -y \) direction (\( \hat{s}_{m-} \)) and one propagates in the \( +y \) direction (\( \hat{s}_{m+} \)). The expression for \( \hat{s}_{m\pm} \) is given by

\[
\hat{s}_{m\pm} = s_{mx} \hat{x} \pm s_{my} \hat{y} + s_{mz} \hat{z}
\]  

(2.2)

where \( s_{mx}, s_{my}, s_{mz} \) are found from \( \hat{s}_0 \) and matching boundary conditions at the slab interfaces as explained in Appendix A.

The total electric field in slab \( m \) due to the externally impressed plane wave at point \( \tilde{R} \) has been derived by Munk, Kornbau, Burrell [11] and is given by

\[
\tilde{E}(\tilde{R}) = \left\{ [\hat{E}(0,0,0) \cdot \hat{n}_{0+}]_m \hat{\tau}_{0,m}^+ [\hat{\gamma}_{0,-}^+ e^{-j\beta_m \hat{x}} (\hat{R} \cdot \hat{y} b_{m-1}) \hat{s}_{m+}^+ + \right.
\]

\[
\left. \hat{n}_{m-} \cdot \hat{\gamma}_{m+}^+ e^{-j\beta_m d_m s_{my} e^{-j\beta_m (\hat{R} \cdot \hat{y} b_{m-1}) \hat{s}_{m-}^+} + \right. \]

\[
\left. [\hat{E}(0,0,0) \cdot \hat{n}_{0+}]_{s_{my}} [\hat{\gamma}_{0,+}^+ e^{-j\beta_m d_m s_{my} e^{-j\beta_m (\hat{R} \cdot \hat{y} b_{m-1}) \hat{s}_{m+}^+} + \right. \]

\[
\left. \hat{n}_{m-} \cdot \hat{\gamma}_{m+}^+ e^{-j\beta_m d_m s_{my} e^{-j\beta_m (\hat{R} \cdot \hat{y} b_{m-1}) \hat{s}_{m-}^+} \} \Phi_{1,m-1} \right. \]

(2.3)
where we define

\[ \tau_{0,m}^+ = \prod_{i=0}^{m-1} \tau^i \]  

and \( \tau^i \) is the effective transmission coefficient for the orthogonal or parallel electric field from slab \( i \) to slab \( i+1 \) traveling in \( +\hat{y} \) direction. Also \( \Gamma_{i,e}^+ \) and \( \Gamma_{i,e}^- \) are the effective Fresnel reflection coefficients at the interface between slab \( i \) and slab \( i+1 \) for a right going wave. The effective reflection and transmission coefficients are presented in Appendix B and were derived by Munk, Kornbau, and Burrell [12]. \( \Phi_{1,m-1} \) represents the total phase delay through the \( m-1 \) slabs and is given by

\[ \Phi_{1,m-1} = \prod_{i=1}^{m-1} e^{-\beta_i d_{i+1}} \]  

The unit vectors \( \hat{n}_{m+}, \hat{n}_{m+}, \hat{n}_{m-}, \hat{n}_{m-} \) are associated with the orthogonal and parallel components of the electric field. They are also discussed in Appendix A. Figure 6 shows the \( \hat{n}_{m\pm} \) vectors.

With the incident field known at point \( \hat{R} \) in slab \( m \), the induced voltage can be calculated. According to Schelkunoff [13], the induced voltage at the terminals of a wire element, exposed to an electric field \( \hat{E} \), is given by,
Figure 6. Normalized electric field vectors and Poynting vectors in the mth slab.
\[ V_n = \frac{1}{I_{n,t}(\hat{R}^n)} \int_{\text{ref}} \mathbf{E} \cdot \hat{p} \cdot I_{n,t}(p) \, dp \]  

(2.6)

where \( \mathbf{E} \) is given by (2.3), \( \hat{p}^n \) is the orientation of the element, \( I_{n,t}(p) \) is the current distribution along the element under transmitting conditions, and \( I_{n,t}(\hat{R}^n) \) is the terminal current. Note that \( \hat{R}^n \) locates the antenna terminals of the reference element and is a fixed vector while \( \hat{R} \) is a variable. \( \hat{R} \) may be written as,

\[ \hat{R} = \hat{R}^n + \hat{p}^n \quad -\ln < p < \ln \]  

(2.7)

where \( p \) is the variable of integration in (2.6). Figure 7 shows the relationship between the reference element and \( \hat{R}, \hat{R}^n, \) and \( \hat{p}^n \). Note also that \( \mathbf{E} \cdot \hat{p}^n \) is the tangential component of the incident electric field along the elements which agrees with Kent's [14] moment method solution.

Substituting equation (2.7) into (2.5) and substituting the result into (2.6) yields the induced voltage according to Munk [15] and English [16],
Figure 7. Reference element of the nth array embedded in the mth slab.
where \( \mathbf{R}^n \) can be written as

\[
\mathbf{R}^n = \mathbf{R}_x^n + \mathbf{R}_y^n + \mathbf{R}_z^n
\]  

(2.9)

and

\[
\mathbf{P}^{n,t}_{m\pm} = \mathbf{P}^n \cdot \mathbf{n}_{m\pm} \mathbf{P}^{n,t}_{m\pm}
\]

(2.10)

\[
P^{n,t}_{m\pm} = \frac{1}{\ln(t(\mathbf{p} = 0))} \int_{-\ln}^{\ln} \ln(t(\mathbf{p}) \mathbf{e}^{-j\beta m \mathbf{P}^n \mathbf{n}_{m\pm}} d\mathbf{p}
\]

(2.11)

where \( \mathbf{P}^{n,t}_{m\pm} \) is referred to as the generalized pattern factor and \( P^{n,t}_{m\pm} \) is the radiation pattern.

In equations (2.8) and (2.11) the undefined quantity is the antenna current under transmitting conditions. For elements which are normally \( \lambda/2 \) in length, the transmit current can be approximated by Schellkunoff [17] as
\[ I_{m,n}(p) = \sin(\beta_d(l_e^n - |p|)) \]  

(2.12)

where \( \beta_d \) is the effective propagation constant along the array, \( l_e^n \) is the effective half-length of the element and depends on \( l^n \) and the wire radius of the element. For dipole elements, \( l_e^n \) is always greater than \( l^n \).

By substituting (2.12) into (2.11) a very good approximation for the radiation pattern can be found. From English [18] this radiation pattern is

\[ P_{m+n} = \frac{1}{\sin(\beta_d l_e^n)} \]

\[ \left\{ \frac{1}{\beta_d^n + \beta_m (\hat{\mathbf{p}} \cdot \hat{\mathbf{s}}_{m\pm})} \left[ \cos(\beta_d^n (l_e^n - l^n) - \beta_m l^n (\hat{\mathbf{p}} \cdot \hat{\mathbf{s}}_{m\pm}) - \cos(\beta_d^n l_e^n) \right] + \right. \]

\[ \left. \frac{1}{\beta_d^n - \beta_m (\hat{\mathbf{p}} \cdot \hat{\mathbf{s}}_{m\pm})} \left[ \cos(\beta_d^n (l_e^n - l^n) + \beta_m l^n (\hat{\mathbf{p}} \cdot \hat{\mathbf{s}}_{m\pm}) - \cos(\beta_d^n l_e^n) \right] \right\} . \]

(2.13)

It should be noted that if the array is contained in the plane of the array (i.e. \( \hat{\mathbf{y}} \cdot \hat{\mathbf{p}}^n = 0 \) then,
\[ \frac{1}{\parallel}_{m^+} P_{m^+,t}^{n,t} = \frac{1}{\parallel}_{m^-} P_{m^-,t}^{n,t} \quad \text{for} \quad \mathbf{\hat{y}} \cdot \mathbf{\hat{p}}^n = 0. \quad (2.14) \]

Equation (2.8) will also be simplified if (2.14) holds. Again for all cases considered in this dissertation, the individual array elements will be restricted to the x-z plane (p_y=0).

**B. Calculating the Mutual Impedance Z_{n,n'}**

The mutual impedance, Z_{n,n'}, has been defined by English [19] as "the open circuit voltage induced at the terminals of the reference element of array n due to the field radiated by currents on the entire n'th array." Munk [20] has shown that if the currents on n'th array are Floquet type currents, array n' radiates a discrete spectrum of plane waves. Since the arrays considered in this investigation are periodic and infinite in extent, the currents on the array are Floquet type currents.

Figure 8 depicts an infinite array of infinitesimal elements of length dp' orientated in the \( \mathbf{\hat{p}}^n \) direction, embedded in slab \( m' \). The reference element is located at \( \mathbf{R}' \) and the field point at \( \tilde{R} \). From Munk et al [21], the scattered field when the field point \( \tilde{R} \) is to the right of the array is given by,

\[ dE(\tilde{R}) = I(\mathbf{R'}) dp' \sum_{k,n} \sum e^{-j\beta_{m'}(\tilde{R} - \mathbf{R'}) \cdot \mathbf{r}_{m'k}} \frac{Z_{m'}}{2 D_x D_z} e_+ \quad (2.15) \]
Figure 8. Infinite array of infinitesimal dipole elements.
When the field point is to the left of the array it is given by,

\[ dE(\mathbf{R}) = I(\mathbf{R'})d\mathbf{p'} \sum_{k,n} \frac{Z_{m'} e^{-j\beta_{m'}(\mathbf{R} - \mathbf{R'}) \cdot \hat{n}_{m'}}}{2D_x D_z r_{m'y}} e. \quad (2.16) \]

where \( \hat{n}_{m'} \) are defined in Appendix A. Also,

\[ Z_{m'} = \sqrt{\frac{\mu_{m'}}{\varepsilon_{m'}}}. \quad (2.17) \]

As shown in Appendix A, we decompose \( \hat{e}_{m'} \) into two orthogonal components, one parallel to the plane of incidence, \( \parallel n_{m'} \), the other perpendicular to the plane of incidence, \( \perp n_{m'} \). This decomposition will simplify the analysis. From (A.12),

\[ \hat{e}_{m'} = -\parallel n_{m'} (\perp n_{m'} \cdot \hat{p}) + \parallel n_{m'} (\parallel n_{m'} \cdot \hat{p}) . \quad (2.18) \]

Substituting (2.18) into (2.15) or (2.16) results in,

\[ dE_{\pm}(\mathbf{R}) = -I(\mathbf{R'})d\mathbf{p'} \sum_{k,n} \frac{Z_{m'} e^{-j\beta_{m'}(\mathbf{R} - \mathbf{R'}) \cdot \hat{n}_{m'}}}{2D_x D_z r_{m'y}} \]

\[ \left[ \parallel n_{m'} (\perp n_{m'} \cdot \hat{p}) + \parallel n_{m'} (\parallel n_{m'} \cdot \hat{p}) \right] . \quad (2.19) \]
where $\pm$ is chosen based on the following convention

$$
+ \text{ if } \hat{y}(\tilde{R} - \tilde{R}^n) > 0 \\
- \text{ if } \hat{y}(\tilde{R} - \tilde{R}^n) < 0 .
$$  

(2.20)

If the infinitesimal elements of Figure 8 are replaced by elements with length $2n'$ (2.19) becomes

$$
E_x(\tilde{R}) = \frac{Z_m'}{2D_xD_z} \sum_k \sum_n \epsilon^{-j\beta_m'(\tilde{R} - \tilde{R}') \cdot \hat{r}_{m'}^+} \\
[\hat{n}_{m'}^\pm (\hat{n}_{m'}^\pm \cdot \hat{p}^n) - \|\hat{n}_{m'}^\pm \|\hat{n}_{m'}^\pm \cdot \hat{p}^n)] \\
\int_{-l^n}^{l^n} I(p') e^{-j\beta_m'p' \cdot (\hat{n}_{m'}^\pm \cdot \hat{r}_{m'}^\pm)} dp' 
$$  

(2.21)

where $\tilde{R}^n'$ is defined as

$$
\tilde{R}' = \tilde{R}^n' + \hat{p}^n'p' \quad -l^n' < p' < l^n' .
$$  

(2.22)

Figure 9 depicts the $n'$th array embedded in slab $m'$ of a stratified medium. The observation point is to the right of the array, in the $m$th slab. Munk [22] has shown that the total field in slab $m$ is composed of four path contributors (Path a is direct, paths b and c are single bounces, and path d is a double bounce). The total field is,
Figure 9. Four bounce paths from an infinite array in a stratified medium.
\[ \mathbf{E}_T(\mathbf{R}) = \mathbf{E}_a(\mathbf{R}) + \mathbf{E}_b(\mathbf{R}) + \mathbf{E}_c(\mathbf{R}) + \mathbf{E}_d(\mathbf{R}) \]  

(2.23)

where \( \mathbf{E}_a(\mathbf{R}) \) is the scattered field that initially is in the \( \hat{r}_{m'} \) direction and arrives at the field point in the \( \hat{r}_m \) direction. \( \mathbf{E}_b(\mathbf{R}) \) is the total scattered field that initially is in \( \hat{r}_{m'} \) and arrives at the field point in the \( \hat{r}_m \) direction. \( \mathbf{E}_c(\mathbf{R}) \) is the scattered field that initially is in the \( \hat{r}_{m'} \) direction and arrives at the field point in the \( \hat{r}_m \) direction. \( \mathbf{E}_d(\mathbf{R}) \) is the scattered field that initially is in the \( \hat{r}_{m'} \) direction and arrives at the field point in the \( \hat{r}_m \) direction.

The field at point \( \mathbf{R} \) was derived by English [23] and is,

\[
\mathbf{E}(\mathbf{R}) = \frac{1}{2\pi D_x D_z} \sum_{m,n} e^{-j\beta_m} \left[ \begin{array}{c}
\sum_{m',n'} e^{-j\beta_{m'}} \frac{Z_{m'} \sum_{k,n} e^{-j\beta_{m'}(\hat{y}b_{m'} - \hat{R}_{13}) \cdot \hat{r}_{m'}}}{r_{m'y}} \\
\left( 1 + \frac{1}{r_{m'y}} \right) e^{-j2\beta_{m'}(y_{m'-1} - b_{m'-1}) r_{m'y}}
\end{array} \right]
\]

(2.24)
where

\[ \Phi_{m'+1,m-1} = e^{j\beta m' + 1 d m' + 1 r(m'+1)y} e^{j\beta m'+2 d m' + 2 r(m'+2)y} \cdots e^{-j\beta m-1 d m-1 r(m-1)y}. \]  

Equation (2.3) represents the total electric field in slab m due to an incident plane wave. Equation (2.24) represents the field at \( \tilde{R} \) due to plane wave spectrum radiated from the n'th array. If we now introduce a test element as shown in Figure 10, the induced voltage in that test element can be calculated using (2.6). If we make this test element the reference element of array n, the mutual impedance between array n and array n' can be calculated as [24],

\[ Z_{n,n'} = \frac{V_{n,n'}(\tilde{R}^n)}{I_{n'}(\tilde{R}^{n'})}. \]  

(2.26)

If \( \tilde{R} \) is restricted in (2.24) to the domain of the test element, then

\[ \hat{R} = \tilde{R}^n + pp^n \quad -ln < p < ln. \]  

(2.27)
Figure 10. Test element in slab m.
Substituting (2.27) into (2.24), the induced voltage can be found by using (2.6). That result is substituted into (2.26) which from [25] results in,

\[ Z_{n,n'} = \frac{Z_{m'}}{2D_x D_z} \sum_{k,n} e^{-j \beta m' (\hat{y}_{b m'} - \hat{R}_{n'}) \hat{r}_{m' y}} \left[ 1 \pm \tau_{m',m} P_m^{n'} P_m^{n, t} (1 + \pm \Gamma_{m',e} e^{-j 2 \beta m (b_m - y_n) r_{m y}}) (1 + \pm \Gamma_{m',e} e^{-j 2 \beta m (y_n - b_m' - 1) r_{m y}}) \right] \]

\[ 1 - \pm \Gamma_{m',e} \Gamma_{m',e} e^{-j 2 \beta m' d_m r_{m' y}} \]

\[ \text{If} \quad n > n' \text{, the result is} \]

\[ Z_{n,n'} = \frac{Z_{m'}}{2D_x D_z} \sum_{k,n} e^{-j \beta m' (\hat{y}_{b m'} - \hat{R}_{n'}) \hat{r}_{m' y}} \left[ 1 \pm \tau_{m',m} P_m^{n'} P_m^{n, t} (1 + \pm \Gamma_{m',e} e^{-j 2 \beta m (b_m - y_n) r_{m y}}) (1 + \pm \Gamma_{m',e} e^{-j 2 \beta m (y_n - b_m' - 1) r_{m y}}) \right] \]

\[ 1 - \pm \Gamma_{m',e} \Gamma_{m',e} e^{-j 2 \beta m' d_m r_{m' y}} \]

\[ \text{The above expression is valid for} \quad y_n > y_n'. \text{ For} \quad y_n < y_n' \text{ the result is} \]

\[ Z_{n,n'} = \frac{Z_{m'}}{2D_x D_z} \sum_{k,n} e^{-j \beta m' (\hat{y}_{b m'} - \hat{R}_{n'}) \hat{r}_{m' y}} \left[ 1 \pm \tau_{m',m} P_m^{n'} P_m^{n, t} (1 + \pm \Gamma_{m',e} e^{-j 2 \beta m (y_n - b_m - 1) r_{m y}}) (1 + \pm \Gamma_{m',e} e^{-j 2 \beta m' (b_m' - y_n) r_{m y}}) \right] \]

\[ 1 - \pm \Gamma_{m',e} \Gamma_{m',e} e^{-j 2 \beta m' d_m r_{m' y}} \]

\[ \frac{r_{m' y}}{r_{m y}} \Phi_{m',1,m+1} e^{-j \beta m' (\hat{R}_{n'} - \hat{y}_{b_m}) \hat{r}_{m'}.} \quad (2.28) \]

\[ \frac{r_{m' y}}{r_{m y}} \Phi_{m',1,m+1} e^{-j \beta m' (\hat{R}_{n'} - \hat{y}_{b_m}) \hat{r}_{m'}.} \quad (2.29) \]
C. THE ARRAY SELF-IMPEDANCE CALCULATION

To determine the self-impedance of array n with itself, the expression for the mutual impedance is slightly modified in the following manner. The test element is identical to the reference element of the n'th array and placed parallel to the reference element one wire radius away in the same dielectric slab. Kornbau [26], English [27], and others have shown that under these conditions, the mutual impedance accurately approximates the array self-impedance if the wire radius (a) is $\ll \lambda$. Figure 11 shows the placement of the element $R^n$, and the reference element, $R^n'$. Also shown are the five wave paths which contribute to the scattered field. Path 1 will be called the direct path. Path 2 is a single bounce path going to the right. Path 3 is a single bounce mode going to the left. Paths 4 and 5 are double bounce paths with path 4 going to the left and path 5 going to the right. Since there are five contributing paths to the scattered field there are five contributions to the total self impedance. The self-impedance is the sum of the five components

$$Z^{n,n'} = Z_1^{n,n'} + Z_2^{n,n'} + Z_3^{n,n'} + Z_4^{n,n'} + Z_5^{n,n'}.$$  \hspace{1cm} (2.30)

The self-impedance terms have been determined by English [28] and will be summarized here
Figure 11. Five bounce paths of the self-impedance term.
\[ Z_{2}^{n,n'} = \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{-j\beta_{m'}(\tilde{R}_{n}^{n'} - \tilde{R}_{n'})} \frac{r_{m',y}}{r_{m',e}} \]

\[
\frac{\perp P_{m'}^{n} \perp P_{m'}^{n,t} \perp \Gamma_{m',e}}{1 - \perp \Gamma_{m',e}^{+} \perp \Gamma_{m',e}^{-} e^{-j2\beta_{m'}d_{m'}r_{m',y}}} e^{-j2\beta_{m'}(y_{n'} - b_{m'} - 1)r_{m',y}} \quad (2.31)
\]

\[ Z_{3}^{n,n'} = \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{-j\beta_{m'}(\tilde{R}_{n}^{n'} - \tilde{R}_{n'})} \frac{r_{m',y}}{r_{m',e}} \]

\[
\frac{\perp P_{m'}^{n} \perp P_{m'}^{n,t} \perp \Gamma_{m',e}}{1 - \perp \Gamma_{m',e}^{+} \perp \Gamma_{m',e}^{-} e^{-j2\beta_{m'}d_{m'}r_{m',y}}} e^{-j2\beta_{m'}(y_{n'} - b_{m'} - 1)r_{m',y}} \quad (2.32)
\]
\[ Z_{4}^{n,n'} = \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{-j\beta_{m'} (\tilde{R}^{n} - \tilde{R}^{n'}) \cdot \hat{r}_{m'} +} \frac{1}{r_{m'} y} \]

\[
\frac{\perp P_{m'}^{n'} \perp P_{m'}^{n',t} \perp \Gamma_{m'}^{+}, \perp \Gamma_{m'}^{-}, e + \parallel P_{m'}^{n'} \parallel P_{m'}^{n',t} \parallel \Gamma_{m'}^{+}, \parallel \Gamma_{m'}^{-}, e}{1 - \perp \Gamma_{m'}^{+}, e \perp \Gamma_{m'}^{-}, e e^{-j2\beta_{m'} d_{m'} r_{m'} y}} + \frac{1}{1 - \parallel \Gamma_{m'}^{+}, \parallel \Gamma_{m'}^{-}, e e^{-j2\beta_{m'} d_{m'} r_{m'} y}} \]

\[ e^{-j2\beta_{m'} (y^{n'} - b_{m'} - 1) r_{m'} y} e^{-j2\beta_{m'} (b_{m'} - y^{n}) r_{m'} y} \quad (2.33) \]

\[ Z_{5}^{n,n'} = \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{-j\beta_{m'} (\tilde{R}^{n} - \tilde{R}^{n'}) \cdot \hat{r}_{m'} -} \frac{1}{r_{m'} y} \]

\[
\frac{\perp P_{m'}^{n'} \perp P_{m'}^{n',t} \perp \Gamma_{m'}^{+}, \perp \Gamma_{m'}^{-}, e + \parallel P_{m'}^{n'} \parallel P_{m'}^{n',t} \parallel \Gamma_{m'}^{+}, \parallel \Gamma_{m'}^{-}, e}{1 - \perp \Gamma_{m'}^{+}, e \perp \Gamma_{m'}^{-}, e e^{-j2\beta_{m'} d_{m'} r_{m'} y}} + \frac{1}{1 - \parallel \Gamma_{m'}^{+}, \parallel \Gamma_{m'}^{-}, e e^{-j2\beta_{m'} d_{m'} r_{m'} y}} \]

\[ e^{-j2\beta_{m'} (y^{n'} - b_{m'} - 1) r_{m'} y} e^{-j2\beta_{m'} (b_{m'} - y^{n}) r_{m'} y} \quad (2.34) \]

\[ Z_{1}^{n,n'} = \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{-j\beta_{m'} (\tilde{R}^{n} - \tilde{R}^{n'}) \cdot \hat{r}_{m'} +} \frac{1}{r_{m'} y} \]

\[
\frac{\perp P_{m'}^{n'} \perp P_{m'}^{n',t} + \parallel P_{m'}^{n'} \parallel P_{m'}^{n',t}}{1 - \perp \Gamma_{m'}^{+}, \parallel \Gamma_{m'}^{-}, e e^{-j2\beta_{m'} d_{m'} r_{m'} y}} \quad (2.35) \]

The above result are only for \( \tilde{R}^{n} > \tilde{R}^{n'} \).
Notice in (2.34) that the direct path \( Z_{1}^{n,n'} \) does not include any higher order bounces as do paths 2 through 5. Kent [29] and English [30] have previously pointed this out and concluded that the contribution from \( Z_{1}^{n,n'} \) will have the strongest evanscent field coupling. The consequences of this are discussed in Appendix C.

The cases investigated in this paper have planar array elements (\( \hat{\mathbf{y}} \cdot \hat{\mathbf{p}}^{n} = 0 \)), so path 5 consists only of higher order reflections of path 1. Summing the self-impedance contributions of \( Z_{1}^{n,n'} \) and \( Z_{5}^{n,n'} \) results in:

\[
Z_{1}^{n,n'} + Z_{5}^{n,n'} = \frac{Z_{m'}^{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{j \beta_{m'}(\hat{R}_{n}^{n'} - \hat{R}_{n}^{n'})^{+} r_{m'} y_{m'}}
\]

\[
= \left[ \frac{\perp P_{m'}^{n'} \perp P_{m'}^{n,t}}{1 - \perp \Gamma_{m'}^{+} e^{j 2 \beta_{m'} m_{m'} r_{m'} y_{m'}}} \frac{\parallel P_{m'}^{n'} \parallel P_{m'}^{n,t}}{1 - \parallel \Gamma_{m'}^{+} e^{j 2 \beta_{m'} m_{m'} r_{m'} y_{m'}}} \right]^{+}
\]

(2.36)

where

\[
\hat{R}_{n}^{n'} - \hat{R}_{n}^{n'} = ay
\]

(2.37)

and \( a \) is the effective radius of the element. The effective radius is discussed in Appendix G.

Since \( \hat{\mathbf{y}} \cdot \hat{\mathbf{p}}^{n} = 0 \),

\[
P_{m'}^{n'} = P_{m'}^{n'} \hspace{1cm} P_{m'}^{n,t} = P_{m'}^{n,t}
\]

(2.38)
Summing up (2.31), (2.32), (2.33), and (2.36) and making use of (2.37) and (2.38) results in,

$$Z_{n,n'}^{n} = \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} \sum_{m} e^{-j\beta m' a r m'y} \left[ P_{m'}^{n'} \left\langle P_{m}^{n}, \right| T_{m'}^{+} \left| P_{m'}^{n}, \right\rangle + \left\langle P_{m'}^{n}, \right| P_{m'}^{n}, \right\rangle T_{m'}^{+} \right]$$

(2.39)

where

$$\text{I}_\parallel T_{m'}^{+} = \left[ \frac{(1 + \Gamma_{m', m'}^{+} e^{-j2\beta m'(y_n^{n'} + b_m^{m'} - 1)y m'y})(1 + \Gamma_{m', m'}^{+} e^{-j2\beta m'(b_m^{m'} - y_n^{n'})y m'y})}{1 - \Gamma_{m', m'}^{+} e^{j2\beta m' y m'y}} \right]$$

(2.40)

Equation (2.39) is equivalent to the four path description of Munk [31].

D. CALCULATING THE ARRAY SCATTERED FIELDS

In this section expressions for the scattered near fields from a biplanar array will be developed. The location of the field point \( \hat{R} \) will be \( \hat{R} = \hat{y} b_{MM} \) for the forward scattered field and \( \hat{R} = (0,0,0) \) for the backscattered field. Both of the field points are in free space outside of the stratified media. Therefore, each of these scattered fields will consist of only two wave modes. Figure 12 illustrates the
Figure 12. Path contributions to the field at the point $R - b_{MM} \hat{y}$.
two path contributions to the electric field at $\hat{R} = \hat{y}b_{MM}$. The scattered field due to the single bounce path ($E_{b}^{(1')}^{(1)}$) from array one is given by English [32] as,

$$
\xi_{b}^{(1'),(1)}(y b_{MM}) = \mathbf{I}^{(1')}^{(0)} \sum_{m,n} \frac{e^{-j\beta m(y b_{MM} - \hat{R}^{(1')}^{(1)}, \hat{r}_{m,y})}}{r_{m,y}}
$$

$$
\xi_{d}^{(1')}^{(1)}(y b_{MM}) = \mathbf{I}^{(1')}^{(0)} \sum_{m,n} \frac{e^{-j\beta m(y b_{MM} - \hat{R}^{(1')}^{(1)}, \hat{r}_{m,y})}}{r_{m,y}}
$$

$$
\Phi_{m'+1,MM} e^{-j2\beta m'(y n' - b_{m'} - 1)r_{m,y}}
$$

where

$$
\Phi_{m'+1,MM} = \prod_{i=m'+1}^{MM} e^{-j\beta d_{i} r_{i,y}}
$$

The scattered field due to the direct path ($E_{a}^{(1')}$) from array one is given by [32],
\[
\tilde{E}_{a}^{(1)}(y^{b}_{\text{MM}}) = -\hat{I}^{(1)}(0) \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{-j\beta_{m'}(\hat{y}^{b}_{m'} - \hat{R}^{(1)})} \frac{r_{m'y}}{r_{m'y}}
\]

\[
\left[ \frac{\sum_{\mathbb{N}_{0} \perp \mathbb{R}_{m',0} P_{m'}^{(1)}}}{1 - \mathbb{I}_{m',e} \mathbb{I}_{m',e} e^{j2\beta_{m'}d_{m'}r_{m'y}}} \right] + \frac{\sum_{\mathbb{R}_{m',0} P_{m'}^{(1)}}}{1 - \mathbb{I}_{m',e} \mathbb{I}_{m',e} e^{j2\beta_{m'}d_{m'}r_{m'y}}}
\]

\[
\Phi_{m'+1,\text{MM}}
\]  

(2.43)

The sum of (2.41) and (2.43) results in

\[
\tilde{E}_{a}^{(1)}(y^{b}_{\text{MM}}) = -\hat{I}^{(1)}(0) \frac{Z_{m'}}{2D_{x}D_{z}} \sum_{k,n} e^{-j\beta_{m'}(\hat{y}^{b}_{m'} - \hat{R}^{(1)})} \frac{r_{m'y}}{r_{m'y}}
\]

\[
\left[ \frac{\sum_{\mathbb{N}_{0} \perp \mathbb{R}_{m',0} P_{m'}^{(1)}}}{1 - \mathbb{I}_{m',e} \mathbb{I}_{m',e} e^{j2\beta_{m'}(y^{(1)} - b_{m'-1})r_{m'y}}} \right] + \frac{\sum_{\mathbb{R}_{m',0} P_{m'}^{(1)}}}{1 - \mathbb{I}_{m',e} \mathbb{I}_{m',e} e^{j2\beta_{m'}(y^{(1)} - b_{m'-1})r_{m'y}}}
\]

\[
\Phi_{m'+1,\text{MM}}
\]  

(2.44)

where

\[
\mathbb{I}_{m'} = \frac{1}{1 - \mathbb{I}_{m',e} \mathbb{I}_{m',e} e^{j2\beta_{m'}d_{m'}r_{m'y}}}
\]  

(2.45)
Likewise the scattered near field from array 2 at $\mathbf{R} = \mathbf{y}_{\text{b}_{\text{MM}}}$ is given by:

$$
\mathbf{E}^{(2')}(\mathbf{y}_{\text{b}_{\text{MM}}}) = -I^{(2')}(0) \frac{Z_{m'}}{2D_1D_2} \sum_k \sum_n e^{-j\beta_{m'}(\mathbf{y}_{b_{m'}} - \mathbf{R}^{(2')})_m} \cdot \mathbf{r}_{m',y}
$$

$$
= \left[ \frac{\hat{\mathbf{\Omega}}_{m',0}\mathbf{P}_{m'}^{(2')}(1 + \mathbf{r}_{m',0} e^{-j2\beta_{m'}(y^{(2')}-b_{m'-1} r_{m',y})})}{r_{m',y}} \right] + \left[ \frac{\hat{\mathbf{\Omega}}_{m',0}\mathbf{P}_{m'}^{(2')}(1 + \mathbf{r}_{m',0} e^{-j2\beta_{m'}(y^{(2')}-b_{m'-1} r_{m',y})})}{r_{m',y}} \right]
$$

$$
\Phi_{m'+1,\text{MM}}
$$

(2.46)

Figure 13 illustrates the two path contributions to the electric field at $\mathbf{R} = (0,0,0)$. The scattered near field from array one due to the direct path ($\mathbf{E}_{c}^{(1')}$) is,

$$
\mathbf{E}_{c}^{(1')}(0,0,0) = -I^{(1')}(0) \frac{Z_{m'}}{2D_1D_2} \sum_k \sum_n e^{-j\beta_{m'}(\mathbf{y}_{b_{m'-1}} - \mathbf{R}^{(1')})_m} \cdot \mathbf{r}_{m',y}
$$

$$
= \left[ \frac{\hat{\mathbf{\Omega}}_{m',0}\mathbf{P}_{m'}^{(1')}(1 + \mathbf{r}_{m',0} e^{-j2\beta_{m'}(y^{(1')}-b_{m'-1} r_{m',y})})}{r_{m',y}} \right] + \left[ \frac{\hat{\mathbf{\Omega}}_{m',0}\mathbf{P}_{m'}^{(1')}(1 + \mathbf{r}_{m',0} e^{-j2\beta_{m'}(y^{(1')}-b_{m'-1} r_{m',y})})}{r_{m',y}} \right]
$$

$$
\Phi_{m'-1,1}
$$

(2.47)
Figure 13. Path contributions to field at the point $\mathbf{R}=(0,0,0)$. 
Likewise the scattered near field from array one due to a single bounce path ($\tilde{E}_d^{(1')}$) is given by

$$\tilde{E}_d^{(1')} (0,0,0) = -i^{(1')} (0) \frac{Z_{m'}}{2D_x D_z} \sum_{k,n} e^{j \beta_{m'} (y_{b_{m'}-1} - R^{(1')}) \cdot r_{m'}}$$

$$\left[ \sum_{\perp m'} P_{m',0}^{(1')} \tau_{m',0}^{m'} \Gamma^{m'} e^{-j2\beta_{m'} (b_{m'}-y^{(1')}) r_{m'}} + \sum_{\parallel m'} P_{m,0}^{(1')} \tau_{m,0}^{m'} \Gamma^{m'} e^{-j2\beta_{m'} (b_{m'}-y^{(1')}) r_{m'}} \right]$$

$$\Phi_{m'-1,1} \quad (2.48)$$

The sum of $\tilde{E}_c^{(1')} + \tilde{E}_d^{(1')}$ is the sum of the scattered near fields from array one at $\tilde{R} = (0,0,0)$ and is given by

$$\tilde{E}^{(1')} (0,0,0) = -i^{(1')} (0) \frac{Z_{m'}}{2D_x D_z} \sum_{k,n} e^{j \beta_{m'} (y_{b_{m'}-1} - R^{(1')}) \cdot r_{m'}}$$

$$\left[ \sum_{\perp m'} P_{m'} \tau_{m'} \Gamma^{m'} (1 + \perp m') e^{-j2\beta_{m'} (b_{m'}-y^{(1')}) r_{m'}} + \sum_{\parallel m'} P_{m} \tau_{m} \Gamma^{m'} (1 + \parallel m') e^{-j2\beta_{m'} (b_{m'}-y^{(1')}) r_{m'}} \right]$$

$$\Phi_{m'-1,1} \quad (2.49)$$
A similar expression could easily be derived for \( \mathbf{E}^{(2')} (0,0,0) \).

The total scattered field at \( \mathbf{R} = (0,0,0) \) is due to the superposition of the reradiated fields from both arrays plus the direct reflected field \( \mathbf{E}^{DR}(0,0,0) \). The direct reflected field is the field reflected from the stratified media with all arrays absent. The total scattered field at \( \mathbf{R} = (0,0,0) \) is,

\[
\mathbf{E}(0,0,0) = \mathbf{E}^{DR}(0,0,0) + \mathbf{E}^{(1')} (0,0,0) + \mathbf{E}^{(2')} (0,0,0) \quad (2.50)
\]

where

\[
\mathbf{E}^{DR}(0,0,0) = \hat{n}_0 \cdot (\mathbf{E}^{1}(0,0,0) \cdot \hat{n}_0) \Gamma^{+}_{0,e} + \hat{n}_0 \cdot (\mathbf{E}^{(1)}(0,0,0) \cdot \hat{n}_0) \|\Gamma^{+}_{0,e} \quad (2.51)
\]

The total transmitted field at \( \mathbf{R} = \hat{y}_{b_{MM}} \) is due to the superposition of the reradiated fields from both arrays plus the direct transmitted field \( \mathbf{E}^{DT}(\hat{y}_{b_{MM}}) \). The direct transmitted field is the field transmitted through the stratified media with all arrays absent. The total scattered field in the forward direction at \( \mathbf{R} = \hat{y}_{b_{MM}} \) is

\[
\mathbf{E}(\hat{y}_{b_{MM}}) = \mathbf{E}^{DT}(\hat{y}_{b_{MM}}) + \mathbf{E}^{(1)}(\hat{y}_{b_{MM}}) + \mathbf{E}^{(2)}(\hat{y}_{b_{MM}}) \quad (2.52)
\]

where
The only unknowns in (2.52) and (2.50) are the terminal currents of the reference elements. Assuming we have only one current mode on each of the elements, a 2x2 impedance matrix would result for a biplanar array. The terminal currents of the reference elements may now be calculated using the following relations

\[
\begin{bmatrix}
V^{(1)} \\
V^{(2)}
\end{bmatrix} =
\begin{bmatrix}
Z^{1,1} & Z^{1,2} \\
Z^{2,1} & Z^{2,2}
\end{bmatrix}
\begin{bmatrix}
I^{(1)}(0) \\
I^{(2)}(0)
\end{bmatrix}
\]

(2.54)

\[
\begin{bmatrix}
I^{(1)}(0) \\
I^{(2)}(0)
\end{bmatrix} =
\begin{bmatrix}
Z^{1,1} & Z^{1,2} \\
Z^{2,1} & Z^{2,2}
\end{bmatrix}^{-1}
\begin{bmatrix}
V^{(1)} \\
V^{(2)}
\end{bmatrix}
\]

(2.55)

\[
\begin{bmatrix}
Z^{1,1} & Z^{1,2} \\
Z^{2,1} & Z^{2,2}
\end{bmatrix}^{-1} = \frac{1}{\text{DET}}
\begin{bmatrix}
Z^{2,2} & -Z^{1,2} \\
-Z^{2,1} & Z^{1,1}
\end{bmatrix}
\]

(2.56)

where

\[
\text{DET} = Z^{1,1}Z^{2,2} - Z^{1,2}Z^{2,1}
\]

(2.57)

By calculating (2.50) and (2.52) the reflection and transmission properties of the entire periodic surface can be analyzed. Note that (2.50) and (2.52) include propagating modes as well as evanescent modes and is therefore an equation for the total near field. If only
the far fields are of interest, only the propagating modes need to be summed (the evanescent modes decay to zero in the far field). If there are no grating lobes (e.g. interelement spacing < \lambda/2), only the \( k=n=0 \) term need to be calculated in the far field.

Even though equations (2.50) and (2.52) are good for all polarizations, frequencies, and angles of incidence, the expressions are messy and not easy to work with. However, considerable simplification is obtained if only one array is embedded in an infinite medium and only the principal planes are considered. Then (2.50) becomes

\[
E(0,0,0) = E^{(1)}(0,0,0)
\]

(2.58)

where \( E^{(1)}(0,0,0) \) is given by (2.49) and becomes for this case

\[
E^{(1)}(0,0,0) = -I^{(1)}(0) \frac{Z_l}{2D_xD_z} \sum_{k,n} \sum_{r_1 y} e^{-j \beta_1 (\hat{R}^{(1)} \cdot \hat{r}_1)} \left[ \hat{\nabla}_1 P^{(1)}_{1} + N \hat{\nabla}_1 P^{(1)}_{1} \right]
\]

(2.59)

where from (2.56) \( I^{(1)}(0) \) becomes

\[
I^{(1)}(0) = \frac{V^{(1)}}{Z^{l,1}}
\]

(2.60)

and from (2.8) \( V^{(1)} \) is
\[ V^{(1)} = [(\hat{E}_{i}(0,0,0)\cdot n_{0})P_{1}^{1,t} + (\hat{E}_{i}(0,0,0)\cdot |n_{0}|)P_{1}^{1,t}]e^{-\beta_{i}R_{1,1}'s_{1}} \] (2.61)

using (2.39) \( Z_{1,1}' \) is,

\[ Z_{1,1}' = \frac{Z_{1}}{2D_{x}D_{z}} \sum_{k,n} \frac{e^{-j\beta_{1}ar_{1,y}}}{r_{1,y}} \left[ \perp P_{1}^{(1)}P_{1}^{1,t} + \| P_{1}^{(1)}P_{1}^{1,t} \right] . \] (2.62)

This may also be broken up into the real and imaginary parts as follows

\[ Z_{1,1}' = R_{1,1} + jX_{1,1} \] (2.63)

where \( R_{1,1} \) is,

\[ R_{1,1} = \frac{Z_{1}}{2D_{x}D_{z}} \frac{1}{r_{1,y}} \left[ \perp P_{1}^{(1)}P_{1}^{1,t} + \| P_{1}^{(1)}P_{1}^{1,t} \right] \] (2.64)

where the separation between the test element and the reference element is zero for the \( k=n=0 \) term. \( jX_{1,1} \) is

\[ jX_{1,1} = \frac{Z_{1}}{2D_{x}D_{z}} \sum_{(k,n)\neq(0,0)} \frac{e^{-j\beta_{1}ar_{1,y}}}{r_{1,y}} \left[ \perp P_{1}^{(1)}P_{1}^{1,t} + \| P_{1}^{(1)}P_{1}^{1,t} \right] . \] (2.65)
The $R^{1,1}$ term corresponds to the propagating mode ($k=n=0$) for the case of no grating lobes. $jX^{1,1}$ corresponds to the evanescent waves $k$ and $n$ such that $r_{1y}$ is imaginary and will only produce imaginary terms.

Next consider the special case where the dipoles are $\hat{z}$ directed (i.e. $\hat{p}^0=\hat{z}$) and the angle of incidence is restricted to the x-y plane. Then (2.59) becomes for the reflected field $\tilde{R}=(0,0,0)

$$
\Gamma_{\text{array}} = \frac{E^{(1)}(0,0,0)}{E^i} = -\frac{Z_1}{2D_xD_z} \frac{1}{r_{1y}^2} Z^{1,1} P^{(1)}_{1\perp} | P^{1,1}_{1\perp} |_{k=n=0} \tag{2.66}
$$

where $| P^{(1)}_{1\perp} | = 0$ for $| \hat{p}_{1\perp} \hat{E}_i = 0$, and $\Gamma_{\text{array}}$ is the array reflection coefficient.

Substituting (2.64) into (2.65) results in

$$
\Gamma_{\text{array}} = \frac{jE^{(1)}(0,0,0)}{E^i} = \frac{R^{1,1}}{Z^{1,1}} = -\frac{1}{1+jX^{1,1}} \tag{2.67}
$$

For incidence in the yz plane, a similar result is obtained

$$
\Gamma_{\text{array}} = \frac{\|E^{1}(0,0,0)}{E^i} = \frac{R^{1,1}}{Z^{1,1}} = -\frac{1}{1+jX^{1,1}} \tag{2.68}
$$

It is obvious from (2.67) and (2.68) that if $X^{1,1}$ is zero the reflection coefficient is unity.
E. SIMPLIFYING THE ANALYSIS USING A TRANSMISSION LINE ANALOGY

It is helpful in the analysis of periodic surfaces to use an equivalent transmission line model of a dipole array. The model shown in Figure 14 is a matched transmission line with characteristic impedance $Z_0$ and a shunt impedance, $Z_{sh1}$. The shunt impedance is also a surface impedance which is completely imaginary so $Z_{sh1}$ is real. Transmission line theory does not model the evanescent mode component of the mutual coupling between different periodic arrays. To use the transmission line model correctly, the evanescent fields must be tightly bound to the periodic surface so that evanescent mode coupling between different arrays can be neglected. To achieve this, the elements within a given array should be closely spaced. Also, transmission line theory only models the one propagating mode which means there are no grating lobes. No grating lobes can be achieved by making sure the interelement spacing is less than $\lambda/2$.

From Figure 14, the combined impedance, $Z_1$, of the infinite transmission line and $Z_{sh1}$ is given by

$$Z_1 = \frac{j Z_0 Z_{sh1}}{Z_0 + j Z_{sh1}}$$

(2.69)
Monoplanar array in an infinite medium

Transmission line equivalent.

Figure 14. Transmission line model of a monoplanar array.
where the prescripts $\perp$ and $\parallel$ refer to the polarization of the angle of incidence.

The reflection coefficient $\Gamma_1$ is given by

$$\parallel \Gamma_1 = \frac{\parallel Z_1 - \perp Z_0}{\parallel Z_1 + \perp Z_0} \quad .$$  \hspace{1cm} (2.70)

Substituting (2.69) into (2.70) results in

$$\parallel \Gamma_1 = \frac{1}{2 \parallel j \frac{1}{\parallel Z_{sh1}} + \parallel \frac{1}{\parallel Z_0}} \quad .$$  \hspace{1cm} (2.71)

Comparing this to (2.68), the shunt impedance is related to the real and imaginary parts of the input impedance by

$$\parallel \frac{1}{\parallel Z_{sh1}} = \frac{X^{11}}{2R^{11}} \quad .$$  \hspace{1cm} (2.72)

From the theory developed in sections A thru D, we can find the input impedance of a monoplanar array. The input impedance can then be used to find the surface impedance ($Z_{sh}$). The surface impedance can be used to quickly come up with an initial design for a biplanar array, via transmission line techniques. This can be done for all polarizations and scan angles [33].
From the above discussions, it can be seen that surface impedances are very useful in the design of periodic surfaces. By looking at the surface impedance at one frequency, the shape of the resonance can be estimated, along with the resonance frequency for each incident direction. Otherwise a series of computations for the complete range of frequencies would be required. The computations may not give the desired response of the surface. Without this computational method, obtaining the desired response could involve a great deal of expensive and time consuming experimentation.

F. SUMMARY

This chapter summarized the development for an expression for the induced voltage which appears at the terminals of an arbitrary array \( n \) located in slab \( m \) due to an externally impressed plane wave. Next an expression for the mutual impedance was summarized for planar arrays located in different dielectric layers. The self-impedance was approximated by using the mutual impedance concept and placing the reference element one wire radius away from the test element. The total forward and backscattered fields were then developed for a biplanar array embedded in a stratified dielectric medium. Finally it was shown that an array of dipoles can be modeled as a shunt impedance on a transmission line. From this analogy, we can determine the surface impedance if we know the input impedance of the arbitrary array \( n \).
A. INTRODUCTION

In this chapter, the methods of the previous chapter are used to investigate the reflection and transmission properties of staggered tuned biplanar arrays of straight dipoles. Staggered tuned biplanar arrays have a different resonance region for each of the individual arrays. The array is resonant at a frequency where the reflection coefficient is unity. Staggered tuned biplanar arrays are not identical. The arrays are nonidentical in that one or more of the following are different: a) the element dimensions, b) interelement spacing, c) the dielectric environment or loading. The three cases are shown in Figure 15. Different element dimensions with different interelement spacing will be investigated in this chapter.

The motivation for investigating staggered tuned biplanar arrays is to determine if the bandwidth of the resonant region can be increased or if a dual stopband can be constructed from non-identical arrays. Conceptually this is straightforward. If array 1 resonates at $f_1$ and array 2 resonates at $f_2$, and $f_2$ is larger than $f_1$, then the combination of $f_1$ and $f_2$ should provide a dual resonance.
Figure 15. Staggered tuned biplanar arrays.
region. If \( f_2 \) is only slightly larger than \( f_1 \), the combination may provide a larger resonance region than if both arrays resonated at \( f_1 \).

Before beginning the analysis, a discussion regarding grating lobes is presented. The grating lobe mechanism is depicted in Figure 16. The interelement spacing is \( D \). The grating lobe condition for an array in free space is

\[
D \cos \theta_i + D \cos \theta_g = n\lambda \quad \text{n=1,2,3..... (3.1)}
\]

where \( \lambda \) is the free space wavelength, \( \theta_i \) is the angle between the array plane and incident vector, and \( \theta_g \) is the angle between the array plane and the grating lobe. Grating lobes are usually undesirable in that energy is propagating in a direction which is not specular. The onset of grating lobes occurs when \( \theta_g=0 \) and equation (3.1) becomes,

\[
D(1 + \cos \theta_i) = \lambda \quad . \quad \text{ (3.2)}
\]

With grating lobes present the bistatic reflection coefficient cannot reach unity and true resonance cannot be met at the FSS design frequency. It is obvious that the way to eliminate grating lobes from (3.2) is to decrease the interelement spacing. For an array in free space the interelement spacing must be less than the element length since the element length is approximately \( \lambda/2 \) at resonance. In order to satisfy the need for decreased interelement spacing, the linear
Grating lobe condition: $D \cos \theta_i + D \cos \theta_g = n \lambda$

Figure 16. Grating lobe illustration.
elements are rotated. In this way the interelement spacing can be much less than the element length. Although the theory presented in Chapter II accounts for all grating lobe effects, the analysis presented here will try to eliminate cases were the frequencies are above the onset of grating lobes. This puts a major restriction on the interelement spacing in staggered tuned biplanar arrays.

B. STAGGERED TUNED BIPLANAR ARRAYS WITH SLIGHTLY DIFFERENT RESONANT FREQUENCIES

In this section, biplanar arrays where each element in array 1 has a different length than the elements in array 2 will be investigated. The configuration of the biplanar array is shown in Figure 17. Notice that although the element lengths are different in the two arrays, the interelement spacings are identical. The interelement spacings need to be the same for each array since the mutual impedance calculated by (2.27) and (2.28) assumed that the reference element in array n had the same interelement spacing as the reference element in array n'. This limitation in design freedom has its drawbacks but can be somewhat overcome as will be shown in Section D.

To simplify the analysis several assumptions are made. The incident direction is in the principal planes of the linear dipole elements. The interelement spacing is chosen so that there are no grating lobes present (only one propagating mode). Finally the biplanar array is in an infinite medium.
Figure 17. Staggered tuned biplanar array with different element dimensions for each array.
As shown in Chapter II, Section D, the reflection from this type of structure comes from the two dipole arrays only. From (2.48) the reflection coefficient may be written as

$$\Gamma_{0,e}^{a+} = \frac{\tilde{E}(0,0,0)}{\tilde{E}_i} = \frac{\tilde{E}(1)(0,0,0) + \tilde{E}(2)(0,0,0)}{\tilde{E}_i} \quad . \quad (3.3)$$

For two arrays in an infinite medium, $\tilde{E}(1)$ and $\tilde{E}(2)$ can be written from (2.47) as

$$\tilde{E}(1) = \frac{Z_0 K(1)}{2D_x D_z} \sum_{k,n} \frac{1}{2\pi} \left[ \frac{\hat{\lambda}_{01} p_{0}^{(1)} }{\parallel \hat{n}_0 \parallel} + \frac{\hat{\lambda}_{01} p_{0}^{(1)} }{\parallel \hat{n}_0 \parallel} \right] \quad (3.4)$$

$$\tilde{E}(2) = \frac{Z_0 K(2)}{2D_x D_z} \sum_{k,n} \frac{1}{2\pi} \left[ \frac{\hat{\lambda}_{01} p_{0}^{(2)} }{\parallel \hat{n}_0 \parallel} + \frac{\hat{\lambda}_{01} p_{0}^{(2)} }{\parallel \hat{n}_0 \parallel} \right] e^{-j\phi_0} \quad (3.5)$$

where

$$\phi_0 = \beta_1 d_1 r_{0y} \quad r_{1y} = r_{0y} \quad . \quad (3.6)$$

The expression above is a doubly infinite sum of plane waves. The $k=n=0$ term is the specularly reflected field in which we are interested. Terms other then $k=n=0$ express the near field and are of no interest here, since we are concerned only with the reflected far
field. Looking only at the reflected field and confining incident direction to the principal planes (3.4) and (3.5) become

\[
\tilde{E}^{(1)} = \frac{-Z_0 I^{(1)} p^{(1)}}{2D_x D_{z0y}}
\]

(3.7)

\[
\tilde{E}^{(2)} = \frac{-Z_0 I^{(2)} p^{(2)}}{2D_x D_{z0y}} e^{-i\phi_1}
\]

(3.8)

where from (2.55) \( I^{(1)} \) and \( I^{(2)} \) are given by

\[
I^{(1)} = \frac{Z^{22} V^{(1)} - Z^{12} V^{(2)}}{\det}
\]

(3.9)

\[
I^{(2)} = \frac{Z^{11} V^{(2)} - Z^{21} V^{(1)}}{\det}
\]

(3.10)

where

\[
\det = Z^{22} Z^{11} - Z^{12} Z^{21}
\]

(3.11)

Combining (3.9) and (3.10) into (3.4) and (3.5) results in (3.3) becoming
\[ \Gamma_{0,e} = \frac{-Z_0}{(DE)^2 D_x D_z r_{0y}} \sum \sum \frac{e^{-j \beta_0 r_{0y}}}{r_{0y}} \left[ P_{01}^{(1)} \perp P_0^{(1)} + \parallel P_0^{(1)} \parallel P_0^{(1)} \right] + j X^{11} \quad (3.12) \]

The impedance term \( Z_1, Z_2, Z_2, \) and \( Z_1 \) can be broken into two parts, the propagating mode and the evanescent modes. The propagating modes are the terms for \( k=n=0 \) and are the real space terms. The evanescent modes account for the remainder of the sum. Due to the condition for no grating lobes, the additional terms are imaginary. Recall from (2.39) that \( Z_1 \) can be written as

\[ Z_1 = \frac{Z_0}{2D_x D_z} \sum \sum \frac{e^{-j \beta_0 r_{0y}}}{r_{0y}} \left[ \perp P_0^{1,t} P_0^{(1)} + \parallel P_0^{1,t} \parallel P_0^{(1)} \right] = Z_1(0) + j X^{11} \quad (3.13) \]

where

\[ Z_1(0) = \frac{Z_0}{2D_x D_z} \left[ \perp P_0^{1,t} P_0^{(1)} + \parallel P_0^{1,t} \parallel P_0^{(1)} \right] \quad (3.14) \]

\[ j X^{11} = \frac{Z_0}{2D_x D_z} \sum \sum \frac{e^{-j \beta_0 r_{0y}}}{r_{0y}} \left[ \perp P_0^{1,t} P_0^{(1)} + \parallel P_0^{1,t} \parallel P_0^{(1)} \right]. \quad (3.15) \]

In a similar manner \( Z_2^2 \) can be represented as

\[ Z_2^2 = Z_2^2(0) + j X^{22} \quad (3.16) \]
Note that $Z_{11}(0)$, $Z_{22}(0)$, $X_{11}$, and $X_{22}$ are real quantities. From (2.28) we can write $Z^{21}$ as

\[ Z^{21} = \frac{Z_0}{2D_x D_z} \sum_{k,n} \frac{e^{-j2\phi_1}}{r_{0,y}} \left[ \frac{P_0^{2,t} \cdot P_0^{(1)}}{\perp P_0} + \frac{P_0^{2,t} \cdot P_0^{(1)}}{\parallel P_0} \right] = Z^{21}(0) + jX^{21} \quad (3.17) \]

where

\[ Z^{21}(0) = \frac{Z_0}{2D_x D_z} \frac{e^{-j2\phi_1}}{r_{0,y}} \left[ \frac{P_0^{2,t} \cdot P_0^{(1)}}{\perp P_0} + \frac{P_0^{2,t} \cdot P_0^{(1)}}{\parallel P_0} \right] \quad (3.18) \]

\[ jX^{21} = \frac{Z_0}{2D_x D_z} \sum_{(k,n) \neq (0,0)} \frac{e^{-j2\phi_1}}{r_{0,y}} \left[ \frac{P_0^{2,t} \cdot P_0^{(1)}}{\perp P_0} + \frac{P_0^{2,t} \cdot P_0^{(1)}}{\parallel P_0} \right] \quad (3.19) \]

$X^{21}$ will go to zero if $\phi_1$ gets large. Since $\phi_1 = \beta_1 d_1 r_{1,y}$, if we make the spacing $d_1$ large than $X^{21}$ will be small.

In a similar manner $Z^{12}$ can be represented as

\[ Z^{12} = Z^{12}(0) + jX^{12} \quad (3.20) \]

Note that $X^{12}$ and $X^{21}$ are real quantities but $Z^{21}(0)$ and $Z^{12}(0)$ are in general complex. Using (3.13), (3.16), (3.17), and (3.20) in (3.12) results in
\[ \Gamma_{0,e}^{a+} = \frac{-Z_0}{\text{DE}2D_x D_y 0_y} E_1 [P_0^{(1)} (Z^{22}(0)V^{(1)} + jX^{22}(0)V^{(1)} - Z^{12}(0)V^{(2)} + jX^{12}V^{(2)}) + P_0^{(2)} (Z^{11}(0)V^{(2)} + jX^{11}V^{(2)} - Z^{21}(0)V^{(1)} + jX^{21}V^{(1)}) e^{-j\phi_1}] \] (3.21)

Due to the fact that \( P_0^{(1)} \neq P_0^{(2)} \) and \( V^{(1)} \neq V^{(2)} \) for a staggered tuned biplanar array, equation (3.21) is quite complex and does not result in a simpler expression or much insight into the problem. To simplify the problem and gain useful insight into this configuration, transmission line theory will be used. For transmission line theory to apply \( X^{21} = 0 \) and \( X^{12} = 0 \). From (3-19), \( X^{21} \) goes to zero when \( \phi_1 \) gets large. According to Bingham [34], if \( d_1 \geq \lambda \), then in most cases transmission line theory can be used.

The reflected field at the boundary of array 1 from Appendix D equation (D.6) for principal plane incidence is

\[ \Gamma_{0,e}^{a+} = \frac{Z_0 (1 - e^{-j\beta_0 d_1 r_1 y}) + 2jZ_{sh1} + 2jZ_{sh2} e^{-j2\beta_0 d_1 r_1 y}}{(Z_0 + 2jZ_{sh1})(1 + \frac{2jZ_{sh2}}{Z_0}) - Z_0 e^{-j2\beta_0 d_1 r_1 y}} \] (3.22)

where the prescripts \( \perp \) and \( \parallel \) have been dropped for notational economy. For polarization \( \perp \) to the plane of incidence \( \perp Z_0 = Z_0/r_{0y} \) and for polarization \( \parallel \) to the plane of incidence \( \parallel Z_0 = Z_0r_{0y} \).

Notice that \( Z_{sh1} \) and \( Z_{sh2} \) are real quantities so that the surface impedance \( jZ_{sh1} \) or \( jZ_{sh2} \) is imaginary only. The magnitude squared of the total reflection coefficient is found by multiplying by the complex conjugate of \( \Gamma_{0,e}^{a+} \).
Substituting (3.22) into (3.23) and simplifying results in

\[
|\Gamma_{0,e}^{a+}|^2 = \frac{2Z_0^2(1-\cos^2\phi_1) + 4(Z_{sh1}^2 + Z_{sh2}^2) + 8Z_{sh1}Z_{sh2}\cos^2\phi_1 + 4Z_0\sin^2\phi_1(Z_{sh1} + Z_{sh2})}{Z_{sh1}^2 Z_{sh2}^2 + \frac{Z_0^2}{16}}
\]  

(3.24)

where \( \phi_1 = \beta_0 d_1 r_1 y \)

For perfect reflection

\[
|\Gamma_{0,e}^{a+}|^2 = 1
\]

(3.25)

and (3.24) can be written under the condition of (3.25) as
The only term that is different on both sides of (3.26) is

\[
\frac{Z_{sh1}Z_{sh2}}{Z_0/16}
\]

Therefore in order to obtain perfect reflection \( Z_{sh1} \) or \( Z_{sh2} = 0 \), which is what we expect from Chapter Two Section E. In order to have a large power reflection band (3.27) must be small over a large frequency band compared to the sum of the other terms of the denominator of (3.25).

To increase the reflection band the interelement spacing \((D_x, D_z)\) can be decreased. This would result in \( Z_{sh} \) being small over a larger frequency band. This is illustrated in Figure 18. The plots show the surface impedance as a function of frequency. Resonance occurs around 7.2 GHz and the array is in free space. The interelement spacings are 2.5 cm for one case and 1.5 cm for the other case. The dipoles are tilted 10 degrees from the z axis.
Figure 18. Surface impedance of individual arrays where the interelement spacing is varied.
If each of the arrays is resonant at a slightly different frequency, will this increase the reflection coefficient over a larger band? The answer is not a simple yes as might be expected. Referring to Figure 19, the surface impedance of two configurations of biplanar arrays is plotted. In configuration 1, both arrays are resonant at the same frequency \( f_0 \). In configuration 2, one of the arrays is resonant at \( f_1 \) which is less than \( f_0 \) and the other is resonant at \( f_2 \) which is greater than \( f_0 \). In region 1 (frequencies less than \( f_1 \)), all the surfaces impedances are less than zero and

\[
|Z_{sh2}| > |Z_{sh0}| > |Z_{sh1}| \quad \text{for } f < f_1 \quad . \quad (3.28)
\]

Using (3.26), \( |\Gamma_{0,e}^{a+}| \) for the two configurations is

\[
|\Gamma_{0,e}^{a+}|^2 = \frac{2Z_0^2(1-\cos 2\phi_1)+4(Z_{sh1}^2+Z_{sh2}^2)+8Z_{sh1}Z_{sh2}\cos 2\phi_1+4Z_0\sin 2\phi_1(Z_{sh1}+Z_{sh2})}{2Z_0^2(1-\cos 2\phi_1)+4(Z_{sh1}^2+Z_{sh2}^2)+8Z_{sh1}Z_{sh2}\cos 2\phi_1+4Z_0\sin 2\phi_1(Z_{sh1}+Z_{sh2})+\frac{Z_0^2}{Z_0/16}}
\]

Configuration 2 \quad (3.29)
Figure 19. Surface impedance for two biplanar array configurations.  
Configuration 1: Both arrays resonant at \( f_0 \). Configuration 2: Array 1 resonant at \( f_1 \) and array 2 resonant at \( f_2 \).
\[ |\Gamma_{0,e}^{a+}|^2 = \]
\[
\frac{2Z_0^2(1-\cos2\phi_1)+8Z_{sh0}^4(1+\cos2\phi_1)+8Z_0\sin2\phi_1Z_{sh0}}{2Z_0^2(1-\cos2\phi_1)+8Z_{sh0}^4(1+\cos2\phi_1)+8Z_0\sin2\phi_1Z_{sh0}+\frac{Z_{sh0}^4}{Z_0/16}}
\]

Configuration 1 \hspace{1cm} (3.30)

The common term in the numerator and the denominator of (3.29) is

\[ A_2 = 2Z_0^2(1-\cos2\phi_1)+4(Z_{sh1}^2+Z_{sh2}^2)+8Z_{sh1}Z_{sh2}\cos2\phi_1+4Z_0\sin2\phi_1(Z_{sh1}+Z_{sh2}) \]

\hspace{1cm} (3.31)

Likewise the common term in the numerator and denominator of (3.30) is

\[ A_1 = 2Z_0^2(1-\cos2\phi_1)+8Z_{sh0}^4(1+\cos2\phi_1)+8Z_0\sin2\phi_1Z_{sh0} \]

\hspace{1cm} (3.32)

The first term of \( A_1 \) (3.32) and \( A_2 \) (3.31) is the same. If we assume in region 1 that

\[ Z_{sh2} - Z_{sh0} = Z_{sh0} - Z_{sh1} \]

\hspace{1cm} (3.33)
then the last terms of $A_2$ and $A_1$ are also the same. This is not a bad assumption in that the interelement spacing and the surrounding dielectric environment are the same for both configurations. The interelement spacing is the primary function in determining the slope of the surface impedance versus frequency. Also the resonant frequencies $f_0$, $f_1$, and $f_2$ are only slightly different and we expect the slopes of the surface impedance versus frequency to be similar. This leaves the following terms in $A_2$ and $A_1$,

$$B_2 = 4(Z_{sh1}^2 + Z_{sh2}^2) + 8Z_{sh1}Z_{sh2}\cos2\phi_1$$

(3.34)

$$B_1 = 8Z_{sh0}^2(1 + \cos2\phi_1)$$

(3.35)

If (3.33) applies then

$$B_1 = B_2 \quad \text{for } |\cos2\phi_1| = 1$$

(3.36)

For all other values of $\cos2\phi_1$ (assuming $Z_{sh1} \neq Z_{sh2}$)

$$B_2 > B_1 \quad \text{for } |\cos2\phi_1| \neq 1$$

(3.37)

Therefore

$$A_2 \geq A_1 \quad \text{for } Z_{sh2}-Z_{sh0}=Z_{sh0}-Z_{sh1}$$

(3.38)
Equations (3.29) and (3.30) can be written in terms of $A_1$ and $A_2$,

$$|\Gamma_{0,e}^{a+}|^2 = \frac{1}{16Z_{sh1}^2 Z_{sh2}^2} \frac{1}{A_2Z_0^2}$$  \hspace{1cm} \text{Configuration 2} \hspace{1cm} (3.39)$$

$$|\Gamma_{0,e}^{a+}|^2 = \frac{1}{16Z_{sh0}^4} \frac{1}{A_1Z_0^2}$$  \hspace{1cm} \text{Configuration 1} \hspace{1cm} (3.40)$$

Since

$$\frac{Z_{sh1}^2 Z_{sh2}^2}{A_2} < \frac{Z_{sh0}^4}{A_1} \hspace{1cm} (3.41)$$

the following inequality results

$$|\Gamma_{0,e}^{a+}|_{\text{conf2}}^2 > |\Gamma_{0,e}^{a+}|_{\text{conf1}}^2 \hspace{1cm} (3.42)$$
Thus the reflection band in region 1 for the staggered tuned biplanar array will be larger than for the single resonant biplanar array if the interelement spacings are the same for all the arrays. A similar analysis for region 3 can also show that the reflection will be larger for the dual resonant biplanar array.

In region 2 it is not obvious whether configuration 1 or configuration 2 has a larger reflection coefficient. It depends on the frequency of interest, the incident angle, the frequency separation between the two resonant frequencies, and the distance separating the two arrays. When at the resonant frequency of configuration 1, \( f_0 \), transmission line theory tells us that the power reflection coefficient is

\[
|\Gamma_{0,e}^{a+,\text{conf1}}|^2 = 1 \quad \text{at } f_0
\]

while

\[
|\Gamma_{0,e}^{a+,\text{conf2}}|^2 < 1 \quad \text{at } f_0
\]  \hspace{1cm} (3.43)

Operating at a frequency other than \( f_0 \) in region 2 depends on other considerations. What we expect in region 2 for configuration 2 is two distinct resonance regions and a minimum in the reflection coefficient between \( f_1 \) and \( f_2 \).
A computer program written by Henderson, called Periodic Moment Method (PMM) [35] is used to generate the results presented here. The computer program calculates the reflection and transmission coefficients in a general stratified medium. PMM can model any type of element shape with one or multiple current modes on the element. Basically this computer program calculates the induced voltage, the impedance matrix, and the terminal currents from (2.55). Finally, the reradiated fields are calculated from (2-49), (2-50), and (2-51). From the calculations, we can determine the specular reflection and transmission coefficients assuming no grating lobes.

If the calculations have been performed correctly and the dielectric and arrays are lossless, energy should be conserved if the array is illuminated by an incident plane wave. That is,

\[ |\Gamma_{0,e}^{a+}|^2 + |T_{0,e}^{a+}|^2 = 1 \]  

(3.45)

where \(\Gamma_{0,e}^{a+}\) and \(T_{0,e}^{a+}\) are the specular reflection and transmission coefficients respectively. Equation (3.45) is a statement of energy conservation and acts as a check of the calculated results. To produce accurate conservation of energy, the impedance double summation of (2.29) and (2.39) must be calculated precisely. Although the program PMM can calculate impedances to any accuracy, it requires large amounts of computer time to do so. Since
we are only interested in general performance characteristics, the accuracy can be relaxed somewhat.

There are four different configurations of elements used in the biplanar arrays. This is shown in Figure 20. Table 1 lists the conditions investigated in this section. In each biplanar array the interelement spacings are $D_x=D_z=1.5$ cm. The elements are rotated 10 degrees in a square grid. The spacing between the arrays is $.8$ cm ($2\lambda_0$, where $\lambda_0$ is the wavelength at resonance).

Table 1. Summary of cases investigated in Chapter III Section B.

| Figure | $|T|\perp \lambda_0 \parallel \eta$ (deg) | $D_x=D_z$ (cm) | $L_1$ (cm) | $L_2$ (cm) |
|--------|--------------------------------|----------------|------------|------------|
| 21     | $|T|$ $\perp \lambda_0 \parallel$ | 0.0            | 1.5        | varied     | varied     |
| 22     | $|T|$ $\perp$                        | 45.0           | 1.5        | varied     | varied     |
| 23     | $|T|$ $\parallel$                    | 60.0           | 1.5        | varied     | varied     |
| 24     | $|T|$ $\parallel$                    | 30.0           | 1.5        | varied     | varied     |
| 25     | $|T|$ $\parallel$                    | 75.0           | 1.5        | varied     | varied     |
| 26     | $|T|$ $\perp \lambda_0 \parallel$   | 0.0            | 1.5        | 1.97       | 2.08       |

The quantity $|T_0^{a+}\eta|^2$ can be used to indicate more clearly what the reflection band actually looks like. For this reason, the transmission loss versus frequency was plotted rather than the reflection loss versus frequency. Five different incident angles are investigated. Figure 21 is for normal incidence and includes plots for four different element lengths. From Figure 21, we can see, as shown in the analysis previously discussed, that the reflection coefficient is higher for frequencies below $f_1$ and above $f_2$. Remember $f_1$ is the resonant frequency of array 1 and $f_2$ is the resonant frequency of
Figure 20. Different biplanar array configurations.
Figure 21. Transmission coefficient for biplanar arrays where the individual array element lengths are varied. Arrays resonant around 7.2 GHz.
array 2. In most of the region between $f_1$ and $f_2$, the staggered tuned biplanar arrays have a higher reflection coefficient. For the case where the element lengths in array 1 are 1.97 cm. and the element lengths in array 2 are 2.08 cm., there is only one resonant region in the figure which we do not expect from transmission line theory. For the other two staggered tuned array cases, the separation between the two resonant frequencies is large enough so that two distinct frequency bands are apparent. The power reflection coefficient is still greater than .99 for all cases, that is

$$|\Gamma_{0,e}^a|^2 > .99 \quad \text{for } f_2 > f > f_1 . \quad (3.46)$$

Figures 22 and 23 are for the E-field perpendicular to the plane of incidence and at incident angles of 45° and 60° respectively. In Figure 22, the configuration with $L_1=1.875$ cm and $L_2=2.206$ cm has a larger reflection coefficient then the symmetrical biplanar array for most of the region between $f_1$ and $f_2$ which was not the case in Figure 21. This is due to the resonance curve of each array becoming broader as the incident angle increases for polarizations perpendicular to the plane of incidence [36]. In Figure 23 where the incident angle is 60°, there is no apparent increase in the reflection coefficient between $f_1$ and $f_2$. For the staggered tuned biplanar array where $L_1=1.875$ cm and $L_2=2.206$ cm, the reflection coefficient is larger at $f_0$ than when $L_1=L_2$. Notice that as the resonance band
Figure 22. Transmission coefficient for biplanar arrays where the individual array element lengths are varied. Arrays resonant around 7.2 GHz.
Figure 23. Transmission coefficient for biplanar arrays where the individual array element lengths are varied. Arrays resonant around 7.2 GHz.
becomes broader, there is less of an increase in the bandwidth between having both arrays at the same resonance frequency and having them at slightly different frequencies. We conclude that the more the scan angle increases for perpendicular polarization, the more likely our staggered tuned biplanar array will show only one resonant region.

Figures 24 and 25 are cases where the E-field is parallel to the plane of incidence. From Munk and Kornbau [36], we know that the bandwidth should decrease for increasing incident angle. In Figure 24 the incident angle is 30° and we start to see two distinct frequency bands when the resonant frequencies of both arrays are not the same. For the biplanar array where the resonant frequencies are 6.5 GHz and 7.9 GHz, the power reflection coefficient is less than .99 between 6.9 GHz and 7.4 GHz. When the incident angle is 75° (Figure 25), the reflection bandwidth is significantly reduced. Also notice that the resonant frequencies are also changing with incident angle. This broadening and narrowing of the resonance curve is typical of designs of biplanar arrays in an infinite medium. This is not necessarily true for biplanar arrays in a stratified dielectric medium. We can conclude that the more the scan angle is increased for parallel polarization the more likely our staggered tuned biplanar array will have two resonant regions.

The distance between both arrays also has an effect on the overall resonance curve. For the case where array 1 is resonant at 6.8 GHz and array 2 is resonant at 7.3 GHz, the spacing between the
Figure 24. Transmission coefficient for biplanar arrays where the individual array element lengths are varied. Arrays resonant around 7.2 GHz.
Figure 25. Transmission coefficient for biplanar arrays where the individual array element lengths are varied. Arrays resonant around 7.2 GHz.
two arrays was varied from 0 cm to 2 cm. This is illustrated in Figure 26. For the case where the spacing is .8 cm (.2λ₀ where λ₀ is the resonant wavelength) the resonant curve is as expected. When the spacing is 1.6 cm (.4λ₀), there is a null produced around 8 GHz which is due to the interaction of both arrays. When the spacing is 2.0 cm (λ₀/2) there is a null around 7.3 GHz which is between both resonant frequencies. Going back to (3.24) for the case when φ₁=π (r₀y=1, d₁=λ/2), (3.24) becomes

\[
|I^{a+}_{0,e}|^2 = \frac{Z_{sh1}^2 + Z_{sh2}^2 + 2Z_{sh1}Z_{sh2}}{Z_{sh1}^2 - Z_{sh2}^2 + 2Z_{sh1}Z_{sh2} + \frac{Z_{sh1}^2 Z_{sh2}^2}{Z_0^2/4}}
\]

and

\[
|I^{a+}_{0,e}|^2 = 0 \quad \text{when } Z_{sh1} = -Z_{sh2} \text{ and } φ₁ = π
\]

This null is referred to as a modal interaction null. Since Z_{sh1} has the opposite sign of Z_{sh2}, this null will always occur between the resonant frequencies f₁ and f₂. Munk [37] also found that a null will occur between the two resonant frequencies if both arrays are in the same plane. It is obvious that careful spacing is necessary to avoid any modal interactions.
Figure 26. Transmission coefficient for staggered tuned biplanar arrays where the distance between the arrays are varied. Array 1 resonant around 6.8 GHz. Array 2 resonant at 7.3 GHz.
C. TRANSMISSION

If all we desired was a reflection band for one frequency region and the rest of the frequencies were a don't care situation, then a simple ground plane could be used instead of a frequency selective surface. The point is there usually is a transmission band associated with a biplanar array of dipoles and it is necessary to investigate the transmission characteristics of a biplanar array also. In this section we will investigate the transmission characteristics of a biplanar array.

Since we are dealing with a lossless medium

\[ |\Gamma_{0,e}|^2 + |T_{0,e}|^2 = 1 \]  \hspace{1cm} (3.49)

If \( |\Gamma_{0,e}|^2 = 0 \) then \( |T_{0,e}|^2 = 1 \) \hspace{1cm} (3.50)

Setting (3.24) equal to zero results in a unity transmission coefficient or

\[ 2Z_0^2(1-\cos^2\phi_1)+4(Z_{sh1}^2+Z_{sh2}^2)+8Z_{sh1}Z_{sh2}\cos2\phi_1+4Z_0\sin2\phi_1(Z_{sh1}+Z_{sh2}) = 0 \]  \hspace{1cm} (3.51)

As shown in Appendix E, a transmission coefficient of unity can occur if the following happen,
\[ Z_{sh1} = -Z_{sh2} \quad (3.52) \]

or

\[ Z_{sh1} = Z_{sh2} = -Z_0/2\tan\phi_1 \quad \text{when } \cos\phi_1 \neq 0, \sin\phi_1 \neq 0 \quad (3.53) \]

As mentioned in Appendix E, (3.52) can only occur between the resonant frequencies of array 1 and array 2 (refer to Figure 27). We usually do not desire a transmission band in this region. Equation (3.53) can occur in region 1 or 3 in Figure 27. If both arrays have the same interelement spacing then the slopes of the surface impedance versus frequency will be similar and \( Z_{sh1} \neq Z_{sh2} \) as illustrated in Figure 27. This does not mean that we cannot have a transmission band for those conditions, but only that the transmission coefficient cannot be one.

Using (3.49) and (3.24) the transmission coefficient can be written as

\[ |T_{0,e}|^2 = \]

\[ \frac{Z_{sh1}^2 + Z_{sh2}^2}{Z_0^2/16} \]

\[ = \frac{2Z_0^2(1-\cos2\phi_1) + 4(Z_{sh1}^2 + Z_{sh2}^2) + 8Z_{sh1}Z_{sh2}\cos2\phi_1 + 4Z_0\sin2\phi_1(Z_{sh1} + Z_{sh2}) + Z_{sh1}^2Z_{sh2}^2}{Z_0^2/16} \]

\[ (3.54) \]
Figure 27. Surface impedance for two arrays (array 1 resonant at $f_1$ and array 2 resonant at $f_2$).
Let

\[ A_2 = 2Z_0^2(1 - \cos \phi_1) + 4(Z_{sh1}^2 + Z_{sh2}^2) + 8Z_{sh1}Z_{sh2}\cos \phi_1 + 4Z_0 \sin \phi_1 (Z_{sh1} + Z_{sh2}) \]  

\[ (3.55) \]

Substituting (3.55) into (3.54) results in

\[ |T_{0,e}^a|^2 = \frac{Z_{sh1}^2 Z_{sh2}^2}{Z_0^2/16} \frac{Z_{sh1}^2 Z_{sh2}^2}{A_2 + \frac{Z_{sh1}^2 Z_{sh2}^2}{Z_0^2/16}} \]  

\[ (3.56) \]

In order for $|T_{0,e}^a|^2$ to be large, the term $A_2$ must be small compared to $Z_{sh1}^2 Z_{sh2}^2/(Z_0^2/16)$. If $|Z_{sh1}|$ and $|Z_{sh2}|$ are much greater than $Z_0$, then the transmission coefficient will be large since $A_2$ will be small compared to $Z_{sh1}^2 Z_{sh2}^2/(Z_0^2/16)$. The magnitude of $Z_{sh}$ increases as the interelement spacing increases. However, if the interelement spacing is made too large, grating lobes may occur at the higher frequencies. Also the bandwidth of the resonance region may become too small if the interelement spacing is large. The requirements for the reflection band (small interelement spacing) are working against the requirements for the transmission band.
(large interelement spacing). It may not be possible to make $|Z_{sh}| \gg Z_0$ in the transmission band. To resolve this problem the conditions of (3.53) need to be approximately met. Set

$$\Delta = Z_{sh2} - Z_{sh1} . \quad (3.57)$$

Substituting (3.57) for $Z_{sh2}$ results in (3.55) becoming

$$A_2 = \frac{2Z_0^2(1-\cos \phi_1)+4(2Z_{sh1}^2+2\Delta Z_{sh1}+\Delta^2)+8\cos \phi_1(Z_{sh1}^2+\Delta Z_{sh1})+4Z_0\sin \phi_1(2Z_{sh1}+\Delta)}{2} . \quad (3.58)$$

Rearranging terms (3.58) becomes

$$A_2 = \frac{2Z_0^2(1-\cos \phi_1)+8Z_{sh1}(1+\cos \phi_1)\Delta^2+8\Delta Z_{sh1}(1+\cos \phi_1)+4\Delta Z_{sh1} \sin 2\phi_1+4\Delta Z_0 \sin 2\phi_1}{2} . \quad (3.59)$$

Let

$$Z_0 = -2Z_{sh1} \cot \phi_1 \quad \text{and} \quad \cos \phi_1 \neq 0, \sin \phi_1 \neq 0 . \quad (3.60)$$

The above relation is from (3.53) which is one of the conditions for a unity transmission coefficient. Substituting (3.60) into (3.59) results in
A_2 = 8Z_{sh1}^2 \cot \phi_1 (1 - \cos^2 \phi_1) + 8Z_{sh1}^2 (1 + \cos^2 \phi_1) + 8\Delta Z_{sh1} (1 + \cos^2 \phi_1) + \nonumber \\
4\Delta^2 - 16Z_{sh1}^2 \cot \phi_1 \sin 2\phi_1 - 8\Delta Z_{sh1} \cot \phi_1 \sin 2\phi_1 \quad \text{for } \cos \phi_1 \neq 0, \sin \phi_1 \neq 0 
\nonumber 
(3.61)

The following trigometric identities will be used in (3.61)

\begin{align*}
1 - \cos 2\phi &= 2\sin^2 \phi \\
1 + \cos 2\phi &= 2\cos^2 \phi \\
\sin 2\phi &= 2\sin \phi \cos \phi
\end{align*}

resulting in

\begin{align*}
A_2 &= 4\Delta^2 \quad \text{for } Z_0 = -2Z_{sh1} \cot \phi_1 \text{ and } \cos \phi_1 \neq 0, \sin \phi_1 \neq 0 
\nonumber 
(3.65)
\end{align*}

Equation (3.56) becomes

\[ |T_{0,e}^a|^2 = \frac{16Z_{sh1}^2 (Z_{sh1}^2 + 2\Delta Z_{sh1} + \Delta^2)}{4Z_{sh1}^2 \cot^2 \phi_1} \]

\[ + \frac{4\Delta^2}{16Z_{sh1}^2 (Z_{sh1}^2 + 2\Delta Z_{sh1} + \Delta^2)} \]

\[ + \frac{4\Delta^2}{4Z_{sh1}^2 \cot^2 \phi_1} \]

for \( Z_0 = -2Z_{sh1} \cot \phi_1, \cos \phi_1 \neq 0, \sin \phi_1 \neq 0 \) \quad (3.66)
Simplifying this results in

\[
|T_{0,e}^a|^2 = \frac{Z_{sh1}^2 + 2\Delta Z_{sh1} + \Delta^2}{\Delta^2 \cotan^2 \phi_1 + Z_{sh1}^2 + 2\Delta Z_{sh1} + \Delta^2}
\]

for \( Z_0 = -2Z_{sh1} \cotan \phi_1, \cos \phi_1 \neq 0, \sin \phi_1 \neq 0 \) \hspace{1cm} (3.67)

Even if \( Z_{sh1} \) is small compared to \( Z_0 \), one can obtain a large transmission coefficient if

\( Z_0 = -2Z_{sh1} \cotan \phi_1 \quad \text{and} \quad \cos \phi_1 \neq 0, \sin \phi_1 \neq 0 \) \hspace{1cm} (3.68)

and

\( \Delta \ll Z_{sh1} \) \hspace{1cm} (3.69)

Next we consider an application of the principles discussed in this section. We will try to construct a two band biplanar array that has a reflection band centered at 8.7 GHz and 5.8 GHz as well as a transmission band centered at 2.9 GHz. One of the arrays is tuned to be reflective in the 8.7 GHz band and the other array is tuned to be reflective in the 5.8 GHz band. Figure 28 shows the transmission loss of biplanar arrays with interelement spacings of 1.5, 2.0, 2.5 cm. The bandwidth is larger for the smaller interelement spacings. Figure 29 shows the reflection loss of the biplanar arrays. The largest
Figure 28. Transmission coefficient for staggered tuned biplanar arrays where interelement spacing is varied. Arrays resonant at 5.8 and 8.7 GHz.
Figure 29. Reflection coefficient for staggered tuned biplanar arrays where interelement spacing is varied. Arrays resonant at 5.8 and 8.7 GHz.
interelement spacing has the largest reflection loss in the
transmission band. There is no null in the transmission band in
Figure 29, just a gradual rolloff. What is occurring is that the surface
impedance of each array has a significantly different value for
frequencies in the transmission band. Figure 30 is a plot of the
surface impedance versus frequency for the three biplanar arrays
considered in Figures 28 and 29. There are two lines that are solid,
two that are dotted, and two that are dashed. The pair of lines
correspond to one of the three biplanar arrays where each line is
resonant at 5.8 GHz or 8.7 GHz. The larger the interelement spacing,
the larger the slope of the surface impedance. The interelement
spacing of 2.5 cm has the largest surface impedance variation. At the
frequencies of 5.8 and 8.7 GHz there is an intersection of three of the
surface impedances. This corresponds to the resonant frequency of
the arrays and has the value of $Z_{sh}=0$. The three lines are arrays
with the same resonant frequency but different interelement spacings. At the transmission band, there is a large difference in the
shunt impedance for the two arrays,

$$Z_{sh2} - Z_{sh1} = \text{large} \quad (3.70)$$

This violates condition (3.69). It is the reason that there is no null in
the transmission band. How we can get a null in the transmission
band is the subject of the next section.
Figure 30. Surface impedance of individual arrays where the interelement spacing is varied.
D. STAGGERED TUNED BIPLANAR ARRAY OF DIPOLES WITH DIFFERENT INTERELEMENT SPACINGS IN EACH ARRAY PLANE

In order to create a null in the transmission band for a biplanar array that is staggered tuned, each of the surface impedances of the biplanar array should be of the same magnitude in the transmission band. For the case where the transmission band is below the resonant frequencies of both arrays, two methods of adjusting the surface impedance include either introducing different interelement spacings in each array plane or changing the width of the element. The staggered tuned biplanar array with different interelement spacing will be investigated in this section. The wide dipole biplanar array will be investigated in the next section.

A straightforward method of creating a null in the transmission band for a biplanar array is to have both arrays identical. In this way $Z_{sh1}$ and $Z_{sh2}$ are equal for all frequencies and their difference will be zero in the transmission band. By carefully choosing the spacing between the two arrays a null in the desired transmission band can be created. For staggered tuned dipoles where the interelement spacings and element widths are the same, $Z_{sh1}$ and $Z_{sh2}$ are not the same since they are resonant at different frequencies. The trick is to get the condition of (3.69) enforced in the transmission band, that is

$$Z_{sh2} - Z_{sh1} = \Delta < Z_{sh1} \quad (3.71)$$
If there is a large difference between the resonant frequencies of each array and the interelement spacing and width are the same for both arrays, condition (3.71) cannot be met. To meet the conditions of (3.71), we will investigate changing the interelement spacing of the arrays. This is illustrated in Figure 31. Since the array with the higher resonant frequency ($f_2$) is not packed as tightly as the array with the lower resonant frequency ($f_1$), by reducing the interelement spacing of the array resonant at $f_2$ the slope of $Z_{sh2}$ will be reduced and condition (3.71) may be met. An intersection between $Z_{sh1}$ and $Z_{sh2}$ is now possible. However, the mutual impedance calculation described in (2.28) and (2.29) assumes that both arrays have the same interelement spacing between them. This is a result of the Floquet currents assumed on each of the array elements not changing in phase by the same amount from adjacent element to adjacent element for each of the individual arrays. So by having different interelement spacings in each of the individual arrays, the Floquet currents may be different in array 1 compared to array 2.

To resolve this problem and get the reduced interelement spacing in array 2 and not violate Floquet's theorem, an identical array may be placed in the same plane as the original array. This is shown in Figure 32. Each of the arrays that are in the same plane have the same interelement spacing but one of the arrays is offset from the other array. In Figure 32, the offset is $D_x/2$. A system of
Condition 1: Interelement spacing of both arrays are the same.

Condition 2: Interelement spacings are not the same for both arrays.

Figure 31. Surface impedance of two biplanar arrays in free space resonant at $f_1$ and $f_2$. 
Figure 32. Single and two array configurations in the same plane.
more than two dipole arrays in the same plane can also be considered. However as the number of arrays in the same plane increases, the size of the impedance matrix increases. Thus the total computation time increases by $N^2$, where $N$ is the number of arrays in the same plane. Going from one array to three arrays in the same plane, our impedance matrix now contains 9 terms and our computation time increases by a factor of nine. For this reason the number of arrays in the same plane should be held to a minimum.

A system of two dipole arrays in the same plane has been shown in Figure 32. Consider that each of the reference elements has 3 current modes so we have a total of six current modes and thus a 6x6 impedance matrix.

$$ [V] = [Z][I] \quad (3.73) $$

which expands to

$$
\begin{pmatrix}
V(1) \\
V(2) \\
V(3) \\
V(4) \\
V(5) \\
V(6)
\end{pmatrix} =
\begin{pmatrix}
Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} \\
Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} \\
Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} \\
Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} \\
Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} \\
Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66}
\end{pmatrix}
\begin{pmatrix}
I(1) \\
I(2) \\
I(3) \\
I(4) \\
I(5) \\
I(6)
\end{pmatrix}. \quad (3.74)
$$

Whether this configuration is considered as two arrays with three modes or one array with six modes is a matter of definition. Both representations are mathematically equivalent. For physical reasons
we will consider this as two arrays with three modes each. Computational-wise, it is more efficient to represent it as one array with six modes.

One has to be careful in picking the number of current modes to be used on the simple dipole element when there are multiple arrays in the same plane. Kornbau [38] found that one current mode is sufficient as long as the element length is less than \(0.7\lambda\). His results were for a single array in a plane. When we have more than one array in the same plane it was found that in some cases one mode is not sufficient and that three modes were required.

Figure 33 is a plot of the surface impedance versus frequency for three arrays. The first configuration is represented by a solid line with the array resonant at 8.7 GHz and an interelement spacing of 2.5 cm \((Z_{sh2b})\). The dashed line is an array also resonant at 8.7 GHz but there are two arrays in the same y plane \((Z_{sh2a})\). It has a interelement spacing of \(D_x=1.25\) cm., \(D_z=2.5\) cm. The third configuration is a single array resonant at 5.8 GHz with an interelement spacing of 2.5 cm \((Z_{sh1})\). The two array configuration which is resonant at 8.7 GHz has a surface impedance in the transmission band which is closer to the surface impedance of the array resonant at 5.8 GHz. The condition in (3.71) can now be met since

\[
Z_{sh2a} - Z_{sh1} = \Delta_a < Z_{sh1} \quad . \tag{3.74}
\]
Surface impedance in free space
interelement spacing, $D_z = 2.5$ cm, $D_x = 2.5$ or 1.25 cm

---

Figure 33. Surface impedance of three individual arrays:

- **Solid**: (8.7 GHz) $D_x = D_z = 2.5$ cm.
- **Dotted**: (5.8 GHz) $D_x = D_z = 2.5$ cm.
- **Dashed (8.7 GHz)** $D_x = 1.25$ cm, $D_z = 2.5$ cm.
For the single array resonant at 8.7 GHz there is a large difference between $Z_{sh2b}$ and $Z_{sh1}$ so that

$$Z_{sh2b} - Z_{sh1} = \Delta_b > Z_{sh1} \quad (3.75)$$

With proper spacing between the array resonant at 8.7 GHz and the array resonant at 5.8 GHz, a null in the reflection coefficient can occur in the desired transmission band.

Table 2 provides a summary of the cases investigated in this section. Figure 34 compares the transmission coefficient of two staggered tuned biplanar arrays. For both of the biplanar arrays, one of the arrays is resonant at 5.8 GHz and the other is resonant at 8.7 GHz. The array resonant at 8.7 GHz has either an interelement spacing of $D_x$ and $D_z$ for one of the biplanar arrays or an interelement spacing of $D_x/2$ and $D_z$ for the other biplanar array. The biplanar array with the closer packed elements ($D_x/2$ and $D_z$) has a larger reflection bandwidth at 8.7 GHz than the biplanar array were the interelement spacing is $D_x$ and $D_z$. The reflection bandwidth at 5.8 GHz is similar for both of the biplanar arrays. Figure 35 is a plot of the reflection coefficient for both biplanar array configurations. As expected a null in the reflection coefficient occurs in the transmission band (centered at 2.9 GHz) for the closer packed biplanar array configuration. Its transmission characteristics are superior to the biplanar array where the interelement spacing is the same for both arrays.
Figure 34. Transmission coefficient of two staggered tuned biplanar array configurations. Arrays resonant at 5.8 and 8.7 GHz.
Figure 35. Reflection coefficient of two staggered tuned biplanar array configurations. Arrays resonant at 5.8 and 8.7 GHz.
Table 2. Summary of cases investigated in Chapter III Section D.

| Figure | $|$ or $|$ | Polar $\perp$ or $\parallel$ | $\eta$ (deg) | $Dx_1=\Dz_1$ (cm) | $Dx_2=\Dz_2$ (cm) |
|--------|------------|----------------|-------------|------------------|------------------|
| 34     | $|$         | $\perp$ or $\parallel$ | 0.0         | 2.5              | 1.25 or 2.5      |
| 35     | $|$         | $\perp$ or $\parallel$ | 0.0         | 2.5              | 1.25 or 2.5      |
| 37     | $|$         | $\perp$ or $\parallel$ | 0.0         | 2.0              | 1.00 or 2.0      |
| 38     | $|$         | $\perp$ or $\parallel$ | 0.0         | 2.0              | 1.00 or 2.0      |
| 39     | $|$         | $\perp$ or $\parallel$ | 0.0, 45.0   | 2.0              | 1.00 or 2.0      |
| 40     | $|$         | $\perp$ or $\parallel$ | 0.0, 45.0   | 2.0              | 1.00 or 2.0      |

The surface impedance for the three configurations of arrays presented in Figure 33 is shown in Figure 36 with the difference being the interelement spacing of 2.0 cm. (or 1.0 cm.) instead of 2.5 cm. (or 1.25 cm.). Once again $Z_{sh_a}$ is close to $Z_{sh_1}$ at the transmission band frequencies but the overall magnitude of $Z_{sh_1}$ is less. Figure 37 is a plot of the power reflection coefficient for the two biplanar arrays. Although the null in the reflection band is not as wide for the biplanar arrays it is still greater than 0.01 throughout the band. This corresponds to a transmission coefficient between 2.6 GHz and 3.2 GHz of

$$|T_{0,e}^{a+}|^2 > .99$$

Figure 38 is a plot of the power transmission coefficient for the two biplanar array configurations. The biplanar array with a closer interelement spacing for the array resonant at 8.7 GHz has a superior
SURFACE IMPEDANCE IN FREE SPACE
INTERELEMENT SPACING, DZ=2.0 CM, DX=1.0 OR 2.0 CM

RESONANT FREQUENCY
SOLID: 8.7 GHz, DX=DZ=2.0
DASHED: 8.7 GHz, DZ=2.0, DX=1.0
DOT-DASH: 5.8 GHz, DX=DZ=2.0

FREQUENCY (GHz)

IMAG (ωεω/2π)

1 2 3 4 5 6 7 8 9 10

Figure 36. Surface impedance of three individual arrays.
Figure 37. Reflection coefficient of two staggered tuned biplanar array configurations. Arrays resonant at 5.8 and 8.7 GHz.
Figure 38. Transmission coefficient of two staggered tuned biplanar array configurations. Arrays resonant at 5.8 and 8.7 GHz.
reflection band performance to the other biplanar array. Its reflection coefficient in the band from 5.6 to 6.0 GHz is

\[ |\Gamma^{a+}_{0,e}|^2 > 0.975 \]  

(3.77)

while its reflection coefficient in the 8.4 GHz to 9.0 GHz band is

\[ |\Gamma^{a+}_{0,e}|^2 > 0.99 \]  

(3.78)

The design just discussed has shown reasonable performance in the frequency bands of interest, but it is only for one scan angle. Plots of the reflection and transmission power coefficients for scan angles of 0° and 45° (with \( \hat{E} \) perpendicular to the plane of incidence and \( \hat{E} \) parallel to the plane of incidence) are shown in Figures 39 and 40. There is a noticeable change in the bandwidth as the scan angle is varied. With the \( \hat{E} \) field perpendicular to the plane of incidence, the larger the scan angle the larger the bandwidth. With \( \hat{E} \) parallel to the plane of incidence, the larger the scan angle the smaller the bandwidth. As mentioned previously in this chapter this is not unexpected [39]. We also notice that the resonance frequency is changing as the scan angle is changed in Figure 39. Variations in bandwidth with scan angle and changes in the resonance frequency are undesirable characteristics for most applications. Figure 40 shows that the transmission bandwidth and center frequency are varying with scan angle which is also undesirable. It will be shown
Figure 39. Transmission coefficient of a staggered tuned biplanar array. Arrays resonant at 5.8 and 8.7 GHz.
Figure 40. Reflection coefficient of a staggered tuned biplanar array. Arrays resonant at 5.8 and 8.7 GHz.
in Chapter IV that the use of dielectric slabs adjacent to a periodic structure of dipole elements can reduce the bandwidth variations inherent in dichroic surfaces.

E. WIDE DIPOLE ELEMENTS IN A BIPLANAR ARRAY

As mentioned in the previous section another method for creating a null in the transmission band for a biplanar array that is staggered tuned is by using wide elements for one of the individual arrays. A typical wide dipole array is shown in Figure 41. The width of the array is about $\lambda/10$ at the resonant frequency of the array. Thiele [40] states that as the width of a dipole increases its resonance region bandwidth increases and its resonant frequency decreases. If the resonance region bandwidth increases, than the magnitude of the surface impedance of that array will also be smaller as shown in Figure 42. This is what is desired to create a null in the transmission band. Essentially by increasing the width of the dipole, the array is becoming more closely packed. A discussion of the current modes and the equivalent radius for a wide thin dipole is discussed in Appendix G. Our analysis here will be limited to the principal planes.

In Figure 42, the width of the normal dipoles is .03 cm. Three array configurations are plotted in this figure. The solid line represents an array resonant at 8.7 GHz with a normal width ($Z_{sh2b}$). The dashed line is an array resonant at 8.7 GHz with a wide width ($Z_{sh2a}$). The dot-dash line is an array resonant at 5.8 GHz with a
Figure 41. Dipole array with wide elements.
Figure 42. Surface impedance of three individual arrays.
normal width \((Z_{\text{sh}})\). The wide array configuration has a surface impedance which is close to the surface impedance of the array resonant at 5.8 GHz. By widening the dipole, the surface impedance changes less with frequency and a null in the reflection coefficient is now possible.

Table 3 is a summary of the cases investigated in this section. Figure 43 is a plot of the reflection coefficient of two biplanar arrays. The biplanar arrays were designed to have a dual reflection band (one array resonant at 5.8 GHz the other at 8.7 GHz) and a passband (centered at 2.9 GHz). The designs are not optimum but are presented to illustrate performance characteristics. For both of the biplanar arrays, the interelement spacing is 2.0 cm. The solid line represents a biplanar array with wide dipole elements in the array that is resonant at 8.7 GHz. The dashed line represents a biplanar array with a normal width dipole elements for both arrays. There is a small null in the reflection coefficient in the transmission band (2.9 GHz). The performance in the passband of the biplanar array with wide elements for array 2 is superior to the normal width biplanar array.
Figure 43. Reflection coefficient of two staggered tuned biplanar array configurations. Arrays resonant at 5.8 and 8.7 GHz.
Table 3. Summary of cases investigated in Chapter III Section E.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Polar</th>
<th>Eta, η (deg)</th>
<th>Dx=Dz (cm)</th>
<th>Width (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>Lor</td>
<td>0.0</td>
<td>2.0</td>
<td>0.03</td>
</tr>
<tr>
<td>44</td>
<td>Tor</td>
<td>0.0</td>
<td>2.0</td>
<td>0.03</td>
</tr>
<tr>
<td>45</td>
<td>Land</td>
<td>0.0,45.</td>
<td>2.0</td>
<td>0.03</td>
</tr>
<tr>
<td>46</td>
<td>Land</td>
<td>0.0,45.</td>
<td>2.0</td>
<td>0.03</td>
</tr>
<tr>
<td>49</td>
<td>Lor</td>
<td>0.0</td>
<td>2.0</td>
<td>0.03</td>
</tr>
<tr>
<td>50</td>
<td>Tor</td>
<td>0.0</td>
<td>2.0</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Figure 44 is a plot of the transmission coefficient of the two biplanar arrays discussed above. The biplanar array with a wide dipole array has a larger reflection region at 8.7 GHz than the biplanar array with a normal width for the individual arrays. Both of the biplanar arrays have similar reflection performance at the resonant region of 5.8 GHz.

As with the two arrays in one plane, the wide dipole can have reasonable performance in the frequency bands of interest for one scan angle. For other scan angles the performances varies as shown in Figure 45. This is not unexpected as previously mentioned in this chapter. The resonant frequency and bandwidth are changing as a function of scan angle. When the \( \vec{E} \) field is parallel to the plane of incidence and at an incident angle of 45° (\( α=80^\circ \) and \( η=45^\circ \)) the bandwidth is reduced. When the \( \vec{E} \) field is perpendicular to the plane of incidence and at an incident angle of 45° (\( α=-10^\circ \) and \( η=45^\circ \)) the bandwidth increases. Of more concern is the change in resonant frequency as the scan angle changes. Figure 46 illustrates that the
Figure 44. Reflection coefficient of two staggered tuned biplanar array configurations. Arrays resonant at 5.8 and 8.7 GHz.
Array 1 (5.8 GHz)  Array 2 (8.7 GHz)

Figure 45. Transmission coefficient of a staggered tuned biplanar array. Arrays resonant at 5.8 and 8.7 GHz.
performance in the transmission band (2.9 GHz) is also reduced as the scan angle is varied. The variation in bandwidth and resonant frequency due to changing scan angle will be investigated in Chapter IV.

As a final case investigated, let us suppose that a staggered tuned biplanar array has reduced interelement spacing for the individual array resonant at 8.7 GHz and has wide dipole elements. The width is adjusted so that the surface impedance of this array is equal to the surface impedance of the array resonant at 5.8 GHz at approximately 3 GHz. This biplanar array is shown in Figure 47. The surface impedance of each of the arrays is plotted in Figure 48. Notice that the surface impedance of array 1 intersects the surface impedance of array 2 in the transmission band (2.9 GHz). In other words \( Z_{sh1} = Z_{sh2} \) in the transmission band which satisfies the condition in (3.69). The reflection and transmission coefficients for this biplanar array configuration are plotted in Figure 50 along with three other configurations of staggered tuned biplanar arrays which are shown in Figure 49. The other arrays consist of one array resonant at 5.8 GHz which is the same for all the configurations and an array resonant at 8.7 GHz which either has reduced interelement spacing, wide element dipoles, or normal dipoles. The performance in the transmission band is superior for the wide element reduced interelement spacing array configuration which is due to the fact that \( Z_{sh1} = Z_{sh2} \) in the transmission band. By adjusting the width of the dipoles in array 2 so that \( Z_{sh1} = Z_{sh2} \), we can maximize our
Arrays resonant at 5.8 and 8.7 GHz.

Figure 46. Reflection coefficient of a staggered tuned biplanar array.

**Array 1 (5.8 GHz)**

**Array 2 (8.7 GHz)**
Figure 47. Staggered tuned biplanar array in free space.
SURFACE IMPEDANCE IN FREE SPACE
INTERELEMENT SPACING: DZ=2.0 CM, DX=1.0 OR 2.0 CM

Figure 48. Surface impedance of two individual arrays.
Array 1. (5.8 GHz)
Common Array: \(D_X = D_Z = 2.0 \text{ cm.}
Normal Width

Array 2. (8.7 GHz)
Solid: \(D_X = D_Z/2 = 1.0 \text{ cm.}
Wide width

Dashed: \(D_X = D_Z/2 = 1.0 \text{ cm.}
Normal width

Dotted: \(D_X = D_Z = 2.0 \text{ cm.}
Wide width

Dot-Dash: \(D_X = D_Z = 2.0 \text{ cm.}
Normal width

Figure 49. Geometry of the individual arrays used in Figure 50.
Figure 50. Transmission and reflection coefficients of four staggered tuned biplanar array configurations. Arrays resonant at 5.8 and 8.7 GHz. Geometry of the individual arrays is in Figure 49.
performance in the transmission band. The biplanar array with reduced interelement spacing and wide dipoles in array 2 has the largest reflection bandwidth at the 8.7 GHz resonant region. The dipoles for that configuration are packed closer than the other array configurations.

E. SUMMARY

In this chapter, the performance of staggered tuned biplanar arrays in an infinite medium were investigated. By having each of the arrays resonant at a slightly different frequency the reflection bandwidth was increased. A staggered tuned biplanar array with slightly different resonant frequencies may have only one resonant region instead of the expected two. For polarizations perpendicular to the plane of incidence, increases in scan angle make it more likely that only one resonant region will occur. For polarizations parallel to the plane of incidence, increases in scan angle make it more likely that there will be two resonant regions.

A dual band biplanar array can be designed by tuning each of the arrays to be resonant at different frequencies if the frequencies are sufficiently separated. A passband can be created if the surface impedances of each individual array are similar in the transmission band. If the widths and interelement spacings of each array of a staggered tuned biplanar array are the same, the surface impedances of each array will be different and a passband may not be realizable.
To make the surface impedances the same at a specific frequency, design procedures were developed which changed the width, interelement spacing, or a combination of the two on one of the individual arrays.

It was demonstrated that a staggered tuned biplanar array with a passband can be designed for one scan angle. As the scan angle is varied, the resonant frequency and reflection and transmission bandwidth change.
CHAPTER IV
INVESTIGATION OF STAGGERED TUNED BIPLANAR ARRAYS EMBEDDED IN DIELECTRIC SLABS

A. INTRODUCTION

In this chapter, the impedance and transmission properties of a periodic structure consisting of two different dipole arrays sandwiched between one or three dielectric slabs is analyzed. The introduction of dielectric slabs will demonstrate a reduction in the variations in frequency response with changing incident angles. The intent of this chapter is not to repeat previously published analyses [41-46] investigating scan compensation of dichroic surfaces, but to apply existing concepts for investigating a staggered tuned biplanar array in a stratified medium.

In order to produce the results described in this chapter, the computer program PMM was used with slight modifications. In this dissertation, it is not the purpose to exhaustively characterize the behavior of staggered tuned biplanar arrays in dielectric slabs but to look at three particular cases. The cases will effectively demonstrate three important points for a staggered tuned biplanar array:

1). How to reduce resonant frequency variation due to different scan angles by using dielectric slabs.
2). How to control the bandwidth variations due to different scan angles with dielectric compensation.

3). How to reduce the amount of variation of the transmission bandwidth and transmission center frequency due to scan angle.

The three cases investigated in this chapter are: 1). staggered tuned biplanar arrays with slightly different resonant frequencies. 2). Staggered tuned biplanar arrays with a wide difference in resonant frequencies in a single dielectric slab. 3). Staggered tuned biplanar arrays with a wide difference in resonant frequencies in three dielectric slabs.

B. STAGGERED TUNED BIPLANAR ARRAYS WITH SLIGHTLY DIFFERENT RESONANT FREQUENCIES IN A SINGLE DIELECTRIC SLAB.

In this section, we will look at biplanar arrays where a straight dipole is used for the element in the array. The elements in array 1 may have a slightly different length than the elements in array 2. The configuration of this biplanar array is shown in Figure 51. The interelement spacing of each array are the same even though the lengths may be different. The incident direction is in the principal plane of the linear dipole element. There are no grating lobes present. There are several quantities of interest in Figure 51; including the thickness of the dielectric slabs (d₁, d₂, d₃) and their respective dielectric constants (ε₁, ε₂, ε₃). In this section ε₁=ε₂=ε₃ to reduce the number of variables.
Figure 51. Staggered tuned biplanar array embedded in dielectric.
The input impedance for a monopanel array may be written as

\[ Z_{in} = R + jX \quad . \quad (4.1) \]

For a fixed incident angle and polarization, \( R \) is relatively constant while \( X \) increases with frequency. For a fixed frequency and interelement spacing less than \( \lambda/2 \), the variation in \( X \) with angle of incidence is small. The variation in \( R \) with incident angle for a fixed frequency depends on the polarization. For polarizations perpendicular to the plane of incidence, \( R \) increases with scan angle. For polarizations parallel to the plane of incidence, \( R \) decreases with scan angle. From Chapter II we know the reflection coefficient depends on the ratio \( X/R \). The smaller \( R \) is the more dependent the reflection coefficient is on \( X \). By adding the appropriate dielectric layers to both sides of the array, the variation of \( R \) with incident angle can be decreased. This is well known among designers of periodic surfaces. Munk and Kornbau [47] found that for a dielectric constant of less than 2 with around \( \lambda/4 \) spacing on both sides of each array, optimum stabilization of the bandwidth of the band stop filter can be achieved.

Table 4 summarizes the cases to be investigated in this section. The spacing between the arrays shall be slightly more than \( \lambda/4 \). The spacing of the two outer layers will be around \( \lambda/4 \) thick. Since the array is embedded in a dielectric layer the resonant wavelength and
element dimensions must be modified by taking the square root of the appropriate dielectric constant.

\[ \lambda_\varepsilon = \frac{\lambda_0}{\sqrt{\varepsilon_1}} \quad (4.2) \]

\[ L_{1\varepsilon} = \frac{L_1}{\sqrt{\varepsilon_1}} \quad (4.3) \]

\[ L_{2\varepsilon} = \frac{L_2}{\sqrt{\varepsilon_1}} \quad (4.4) \]

\[ D_{x\varepsilon} = \frac{D_x}{\sqrt{\varepsilon_1}} \quad (4.5) \]

\[ D_{z\varepsilon} = \frac{D_z}{\sqrt{\varepsilon_1}} \quad (4.6) \]

Equations (4.2) to (4.6) produce the corresponding dimensions for the cases to follow.

Table 4. Summary of cases investigated in Chapter IV Section B.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Case</th>
<th>Element 1 ε</th>
<th>Element 2 ε</th>
<th>ε 1</th>
<th>d1/λε</th>
<th>d2/λε</th>
<th>d3/λε</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>B.1</td>
<td>1.65</td>
<td>1.65</td>
<td>1.5</td>
<td>.25</td>
<td>.27</td>
<td>.25</td>
</tr>
<tr>
<td>53</td>
<td>B.2</td>
<td>1.61</td>
<td>1.70</td>
<td>1.5</td>
<td>.25</td>
<td>.27</td>
<td>.25</td>
</tr>
<tr>
<td>54</td>
<td>B.3</td>
<td>1.57</td>
<td>1.75</td>
<td>1.5</td>
<td>.25</td>
<td>.27</td>
<td>.25</td>
</tr>
<tr>
<td>55, 56</td>
<td>B.1,2,3</td>
<td>varied</td>
<td>varied</td>
<td>1.5</td>
<td>.25</td>
<td>.27</td>
<td>.25</td>
</tr>
</tbody>
</table>
Before we discuss the results, let us discuss what we expect to happen when the dipole lengths of the individual arrays are different for each array. From Appendix H, (H.22), we expect a unity reflection coefficient when

\[ Z_{sh1} = 0 \text{ or } Z_{sh2} = 0 \]  

(4.7)

For a staggered tuned biplanar array, we expect to see two distinct resonant regions with a minimum between the resonant frequencies based on transmission line theory. However, recall that in Chapter III we found that no saddle extended between the resonant frequencies if the element lengths of each individual array were slightly different.

We are finally ready to discuss the calculated results. Let us begin with the most basic case where the element lengths of both individual arrays are identical (case B.1) and are embedded in a dielectric slab. The transmission loss for this configuration is shown in Figure 52 for perpendicular and parallel polarization for various scan angles (1, 30, 60, 70. deg). The bandwidth is still changing as the scan angle is increased, but the amount of variation has been reduced. Further optimization to reduce the bandwidth variation is possible. The resonant frequency is also changing with scan angle but the amount of variation is less when compared to the free space case (Figures 21 to 25). Also for parallel polarization there are two
Figure 52. Transmission coefficient for a biplanar array in a dielectric slab ($L_1=L_2=1.65$ cm., case B.1). Arrays resonant around 7.2 GHz.
distinct resonant regions at the higher scan angles even though both arrays are identical. Kornbau [48] discusses this phenomenon and refers to it as the overdamped case.

Next we make the elements of each individual array different by changing the length of each element (Case B.2, \( L_1 = 1.61, L_2 = 1.70 \)). Figure 53 shows the transmission loss for both polarizations. The reflection bandwidth variation with incident angle has also been reduced compared to the free space case by placing the biplanar array in a dielectric slab. As with the free space case in Chapter III, we do not see two distinct resonant regions as predicted by transmission line theory for perpendicular polarization. For parallel polarization at the scan angles of 30, 60, 70 degrees, we are starting to see two distinct resonant regions.

When the differences in element lengths between the individual arrays is increased even more (Case B.3, \( L_1 = 1.57, L_2 = 1.75 \)), two distinct resonant regions are seen for all polarizations as shown in Figure 54. The bandwidth and resonant frequency variations with scan angle has been reduced when the biplanar array is placed in a dielectric slab. Dielectric stabilization works equally well whether the biplanar array has identical elements or slightly different elements.

Next we compare the three cases investigated above to each other for purposes of determining if the reflection band has increased for a staggered tuned biplanar array. Figure 55 shows the transmission loss for normal incidence for the three cases. Notice
Figure 53. Transmission coefficient for a biplanar array in a dielectric slab (L₁=1.70 cm. and L₂=1.61 cm., case B.2). Arrays resonant around 7.2 GHz.
Figure 54. Transmission coefficient for a biplanar array in a dielectric slab ($L_1=1.75$ cm. and $L_2=1.57$ cm., case B.3). Arrays resonant around 7.2 GHz.
Figure 55. Comparison of transmission coefficients for cases B.1, B.2, B.3. Arrays resonant around 7.2 GHz.
that the reflection bandwidth increases with larger differences between the elements of each array. In other words, case B.3 ($L_1=1.57, L_2=1.75$) has a greater bandwidth than case B.2 ($L_1=1.61, L_2=1.70$) which has a greater bandwidth than case B.1 ($L_1=1.65, L_2=1.65$). Two distinct resonant bands are starting to show up for case B.3 in that the reflection coefficient decreases in the frequency band between the two resonant frequencies of case B.3.

Figure 56 plots the transmission loss for the three cases when the scan angle is 60 degrees ($\eta=60$) for perpendicular and parallel polarization. For perpendicular polarization, the reflection bandwidth is larger for when there is a larger difference between the elements of each individual array. For parallel polarization, two distinct reflection bands are evident for all cases. Still, the transmission loss in the frequency region between the resonant frequencies of the two arrays is large and would be adequate for most applications. In the frequency region outside of the resonant frequencies of each individual array, the reflection bandwidth is larger for staggered tuned arrays as was the case in Chapter III.

In summary, a staggered tuned biplanar array where the individual resonant frequencies of each array are slightly different, produces a larger reflection bandwidth in comparison to a case where the individual resonant frequencies of each array are the same. Using a dielectric slab reduces the bandwidth variation caused by changing the scan angle. The dielectric stabilization appears to
Figure 56. Comparison of transmission coefficients for cases B.1, B.2,
B.3. Arrays resonant around 7.2 GHz. Scan angle is 60
degrees.
work equally as well for staggered tuned biplanar arrays as compared to symmetrical biplanar arrays.

C. STAGGERED TUNED BIPLANAR ARRAYS WITH A WIDE SEPARATION IN RESONANT FREQUENCIES IN A SINGLE DIELECTRIC SLAB.

In this section, a periodic surface consisting of two "staggered tuned" dipole arrays embedded in a dielectric slab is investigated. Figure 57 shows the geometry of the structure. The dielectric layer has a relative permittivity of $\varepsilon=2$ or 3. The thickness of the dielectric layer is $d_0=d_1+d_2+d_3$ where $d_1$ is the distance from the left boundary of the dielectric slab to the first array, $d_2$ is the distance between the arrays, and $d_3$ is the distance from the second array to the right boundary of the dielectric slab. The dipoles in each array are tilted 10 degrees from the z axis. Four-legged loaded elements are used in the dipole arrays. A four-legged loaded element is shown in Figure 58. The individual arrays are resonant at 8.7 GHz (array 1) and 5.8 GHz (array 2). The biplanar array is designed to have a passband at 2.9 GHz.

The reflection from this type of structure comes from three sources. The first two are the dipole arrays in the presence of the dielectric, and the third is that from the dielectric layer itself. From equation (2.50) the total scattered field can be represented as

$$\mathbf{E}(0,0,0) = \mathbf{E}^{DR}(0,0,0) + \mathbf{E}^{(1)}(0,0,0) + \mathbf{E}^{(2)}(0,0,0) \quad . \quad (4.8)$$
Figure 57. Staggered tuned biplanar array embedded in a single dielectric slab. Array 1 resonant at 8.7 GHz. Array 2 resonant at 5.8 GHz.
Figure 58. Physical dimension for a four-legged loaded element.
\( \mathbf{E}^{\text{DR}}(0,0,0) \) is the field reflected from the dielectric medium with all arrays absent. The dielectric helps compensate for the scan impedance variation. To design a scan compensated staggered tuned biplanar array, we might take the approach as we did in Section B and make the spacing \( (d_1, d_2, d_3) \) to be \( \equiv \lambda/4 \) where \( \lambda \) is the wavelength at the resonant frequency. In this case we have two resonant frequencies. Let \( \lambda_1 \) be the resonant wavelength of array 1 and \( \lambda_2 \) be the resonant wavelength of array 2. For arrays embedded in dielectric the wavelengths are given by

\[
\lambda_1 = 2.44 \text{ cm} \ (\varepsilon=2), \ 1.99 \text{ cm} \ (\varepsilon=3) \tag{4.9}
\]

\[
\lambda_2 = 3.66 \text{ cm} \ (\varepsilon=2), \ 2.99 \text{ cm} \ (\varepsilon=3) \ . \tag{4.10}
\]

\( \lambda_1 \) and \( \lambda_2 \) are related by

\[
\frac{\lambda_1}{4} = \frac{\lambda_2}{6} \tag{4.11}
\]

\[
\frac{\lambda_2}{4} = \frac{3}{8} \lambda_1 \tag{4.12}
\]

The spacing between the arrays cannot be optimum for both resonant frequencies (around \( \lambda/4 \)). Further complicating the problem is the fact that there is a passband at 2.9 GHz. Let \( \lambda_t \) be the wavelength at the passband. The passband wavelength is
\[ \lambda_t = 7.31 \text{ cm (}\varepsilon=2), \ 5.97 \text{ cm (}\varepsilon=3) \]  

The following relations between \( \lambda_t \) and \( \lambda_1 \) or \( \lambda_2 \) apply

\[ \frac{\lambda_1}{4} = \frac{\lambda_t}{12} \]  
\[ \frac{\lambda_2}{4} = \frac{\lambda_t}{8} \]  

The total transmitted field is also due to the superposition of the reradiated fields from both arrays plus the direct transmitted field, \( \hat{E}^{DT}(\hat{y}_{b_{MM}}) \). \( \hat{E}^{DT}(\hat{y}_{b_{MM}}) \) is the field transmitted through the dielectric media with all arrays absent. From (2.52) the total scattered field in the forward direction at \( \hat{R} = \hat{y}_{b_{MM}} \) is

\[ \hat{E}(\hat{y}_{b_{MM}}) = \hat{E}^{DT}(\hat{y}_{b_{MM}}) + \hat{E}^{(1')}(\hat{y}_{b_{MM}}) + \hat{E}^{(2')}(\hat{y}_{b_{MM}}) \]  

From Munk and Kornbau [47], we know that \( r_{1y} \) changes with scan angle. The higher the dielectric the less the change in \( r_{1y} \) as incident angle is varied. However, as the dielectric constant increases the bandwidth of the passband decreases due to the reflection coefficient \( \Gamma_1^+ \) increasing as the dielectric increases. In other words the center frequency of our passband will vary less with a higher dielectric constant but the bandwidth will decrease.

From (H.24) a unity transmission coefficient can occur when
where $|\Gamma^a_{1,e}|$ is the effective reflection coefficient at array 1 and includes the reflection contribution from the array. If $\cos(2\phi_1+\theta_1) \neq 1$, then from inspection, (4.17) cannot be satisfied since $|\Gamma^a_{1,e}|$ would be complex. Equation (4.17) is satisfied when

$$|\Gamma^a_{1,e}| = \frac{Z_0-Z_1}{Z_0+Z_1} \text{ and } \cos(2\phi_1+\theta_1)=1 \text{ for } |T^a_{1,e}|^2 = 1 \quad (4.18)$$

For the transmission coefficient to be large $Z_1$ and $Z_0$ should be about the same value and $|\Gamma^a_{1,e}|$ should be small. From our analysis in Chapter III we conclude that $Z_{sh1}$ and $Z_{sh2}$ should be as large as possible and their magnitude should be approximately equal.

The geometry for the three cases investigated in this section is shown in Figure 59. The two band biplanar array is designed to be reflective at 5.8 and 8.7 GHz with a passband at 2.9 GHz. One of the dipole arrays is resonant at 5.8 GHz the other at 8.7 GHz. Array 1 has half the interelement spacing of array 2. The surface impedances of each array are equal in the passband region. Table 5 summarizes the three cases to be investigated in this section. Only the parallel polarization cases are investigated in this section due to limitations in the current version of the PMM code for rotated four legged loaded elements.
Figure 59. Dual band biplanar array in a single dielectric slab. Array 1 resonant at 8.7 GHz. Array 2 resonant at 5.8 GHz.
Table 5. Summary of cases investigated in Chapter IV Section C.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Case</th>
<th>$\varepsilon_1$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>C.1</td>
<td>2</td>
<td>$\lambda_t/4$</td>
<td>$\lambda_2/4$</td>
<td>$\lambda_t/4$</td>
<td>$\parallel$</td>
</tr>
<tr>
<td>61</td>
<td>C.2</td>
<td>2</td>
<td>$\lambda_1/4$</td>
<td>$\lambda_t/4$</td>
<td>$\lambda_2/4$</td>
<td>$\parallel$</td>
</tr>
<tr>
<td>62</td>
<td>C.3</td>
<td>3</td>
<td>$\lambda_1/4$</td>
<td>$\lambda_t/4$</td>
<td>$\lambda_2/4$</td>
<td>$\parallel$</td>
</tr>
</tbody>
</table>

Let us begin our discussion of the calculated results by looking at case C.1 when the two band biplanar array is embedded in dielectric with a relative permittivity of $\varepsilon=2$. Three scan angles are investigated ($\eta=1, 30, 45$ degrees). The geometry of the arrays in the x-z plane are shown in Figure 60 as well as a plot of the surface impedance. From Figure 61, we can see that the stopbands (5.8 and 8.7 GHz) show a small change in resonant frequency as the scan angle is varied, but the amount of variation has been reduced from the free space case of Chapter III. The bandwidth is fairly constant as the scan angle changes. We can conclude that the biplanar array is fairly scan compensated in the stopbands. However, in the passband there is a large variation in bandwidth and center frequency. At a scan angle of $\eta=1$ and 30 degrees the performance is acceptable, but at $\eta=45$ degrees the results are marginal at best. The critical factor here is the spacing between the arrays and the dielectric slab as will be demonstrated from looking at the calculated results of case C.2.

Case C.2 has identical parameters as in case C.1 except for the distances between the arrays and the dielectric slab. For case C.1, $d_1=d_3=\lambda_t/4$ and $d_2=\lambda_2/4$ (refer to Figure 57) where $\lambda_t$ is the
Figure 60. Geometry and surface impedance of the arrays used in case C.1. The arrays are in the x-z plane.
Figure 61. Reflection and transmission coefficients for a dual band staggered tuned biplanar array embedded in a single dielectric slab (Case C.1).
wavelength of the passband and $\lambda_2$ is the wavelength of array 2. In case C.2, $d_1=\lambda_1/4$, $d_2=\lambda_1/4$, and $d_3=\lambda_2/4$ where $\lambda_1$ is the resonant wavelength of array 1. The geometry of the arrays in the x-z plane is illustrated in Figure 62. From Figure 63, the stopbands at 5.8 and 8.7 GHz show similar performance to the staggered tuned biplanar array in case 1. That is the biplanar array is fairly scan compensated in the stopbands. What is different from case C.1 is that the passband (2.9 GHz) shows a reduced variation for bandwidth and in the location of the center frequency as the scan angle is changed. This variation in passband parameters is small enough for most applications of interest. The bandwidth of the passband increases slightly as the scan angle increases for parallel polarization. This is the opposite of what happens at the stopbands of the biplanar array where the bandwidth decreases as the scan angle is increased. Overall case C.2 is an acceptable design for most applications. We can conclude from comparison with case C.1 that the distance parameters ($d_1$, $d_2$, $d_3$) should be picked carefully in the design process.

Case C.3 is identical to case C.2 except the relative permittivity of the dielectric slab has been changed to $\varepsilon=3$. Figure 64 shows the geometry and surface impedance of the arrays for case C.3. Figure 65 shows a plot of the reflection and transmission coefficients for case C.3. The change in resonant frequency as the scan angle is varied has been reduced at the stopband at 8.7 GHz in comparison to case C.2. The performance at 5.8 GHz is similar to case C.2. At the passband, the variation in the center frequency has also been
Figure 62. Geometry and surface impedance of the arrays used in case C.2. The arrays are in the x-z plane.
Figure 63. Reflection and transmission coefficients for a dual band staggered tuned biplanar array embedded in a single dielectric slab (Case C.2).
Figure 64. Geometry and surface impedance of the arrays used in case C.3. The arrays are in the x-z plane.
Figure 65. Reflection and transmission coefficients for a dual band staggered tuned biplanar array embedded in a single dielectric slab (Case C.3).
reduced slightly in comparison to case C.2. However, the bandwidth of the passband is smaller for case C.3 in comparison to case C.2 even though the bandwidth variations are less for case C.3. Case C.3 is an acceptable design with slight changes in performance parameters due to scan angle changes.

In summary all cases showed a certain degree of scan compensation due to stabilization from the dielectric slab. The resonant frequency variation was slight in comparison to the free space cases and the bandwidth variation was reduced by use of the dielectric slab. Case C.1 shows the need to carefully pick the thickness parameters \((d_1, d_2, d_3)\) when designing a biplanar array of dipoles with a passband. Cases C.2 and C.3 resulted in acceptable performance including the passband when the thickness parameters were carefully chosen. When the dielectric slab had a permittivity of \(\varepsilon=3\), the biplanar array exhibited little variation in array performance parameters especially in the passband compared to the other cases.
D. STAGGERED TUNED BIPLANAR ARRAYS WITH WIDE SEPARATION IN RESONANT FREQUENCIES IN A STRATIFIED DIELECTRIC MEDIUM.

In this section, a periodic surface consisting of two staggered tuned dipole arrays embedded in three dielectric slabs is investigated. Figure 66 shows the geometry of the structure. The geometry is similar to that in Section C except there are three dielectric layers. Layers one and three are identical (ε=3.3 or 1.5), while layer two is different (ε=2.2). The arrays are located in the crack between layers one and two (array 1) and layers two and three (array 2). Four legged loaded elements are once again used in the dipole arrays. The individual arrays are resonant at 8.7 GHz (array 1) and 5.8 GHz (array 2). The biplanar array has a passband at 2.9 GHz.

Three dielectric layers give the designer more parameters to work with and potentially a better design can be achieved. Table 6 summarizes the four cases to be investigated in this section. The plane containing array 1 consists of either one or two arrays. Figure 67 shows the geometry for the four cases. Only cases where the incident angles are parallel to the plane of incidence are investigated.
Figure 66. Staggered tuned biplanar array embedded in a stratified dielectric medium. Array 1 resonant at 8.7 GHz. Array 2 resonant at 5.8 GHz.
Figure 67. Dual band biplanar array in three dielectric slabs. Array 1 resonant at 8.7 GHz. Array 2 resonant at 5.8 GHz.
Table 6. Summary of cases investigated in Chapter IV Section D.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Case</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$Z_{sh}$'s array 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>D.1</td>
<td>3.3</td>
<td>2.2</td>
<td>3.3</td>
<td>$3\lambda_1/8$</td>
<td>$3\lambda_t/16$</td>
<td>$\lambda_2/4$</td>
<td>≠    one</td>
</tr>
<tr>
<td>66</td>
<td>D.2</td>
<td>3.3</td>
<td>2.2</td>
<td>3.3</td>
<td>$3\lambda_1/8$</td>
<td>$3\lambda_t/16$</td>
<td>$\lambda_2/4$</td>
<td>=    two</td>
</tr>
<tr>
<td>67</td>
<td>D.3</td>
<td>1.5</td>
<td>2.2</td>
<td>1.5</td>
<td>$\lambda_1/4$</td>
<td>$\lambda_t/4$</td>
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</tr>
<tr>
<td>68</td>
<td>D.4</td>
<td>1.5</td>
<td>2.2</td>
<td>1.5</td>
<td>$\lambda_t/4$</td>
<td>$\lambda_t/4$</td>
<td>$\lambda_1/4$</td>
<td>=    two</td>
</tr>
</tbody>
</table>

The basic design in case D.1 was provided by Kornbau [49]. The design was optimized by use of a Monte Carlo method. The only difference between case D.1 and case D.2 is the surface impedance of array 1 (resonant at 8.7 GHz). For case D.1, array 1's surface impedance does not equal array 2's in the passband (2.9 GHz). By reducing the interelement spacing in array 1 and by adjusting the width of the elements, the surface impedance of array 1 and array 2 are equal at the passband frequency for case D.2. This is the same procedure that was used in Chapter III in providing a large transmission band. This is illustrated in Figures 68 and 69 for cases D.1 and D.2 respectively. Figure 70 presents the calculated results for case D.1. It is the most scan compensated of all the designs investigated in this dissertation. There is very little variation in the biplanar array's performance parameters for the stopbands at 5.8 and 8.7 GHz. The passband shows a little variation in the center frequency as the scan angle is varied. Case D.2 almost has the same scan compensated performance (Figure 71). There are larger variations in resonant frequency as the scan angle is varied as
Figure 68. Geometry and surface impedance of the arrays used in case D.1. The arrays are in the x-z plane.
Figure 69. Geometry and surface impedance of the arrays used in case D.2. The arrays are in the x-z plane.
Figure 70. Reflection and transmission coefficients for a dual band staggered tuned biplanar array embedded in three dielectric slabs (Case D.1).
Figure 71. Reflection and transmission coefficients for a dual band staggered tuned biplanar array embedded in three dielectric slabs (Case D.2).
compared to case D.1. However, case D.2 has a larger bandwidth than case D.1 for the stopbands at 5.8 and 8.7 GHz. This larger bandwidth overcomes any shortcomings in scan compensation for case D.2. The passband (2.9 GHz) also has a slightly larger bandwidth than case D.1. The variation in center frequency with scan angle in the passband for case D.1 reduces the overall performance in the passband. As was the case in Chapter III, we can increase the bandwidth of the passband and the bandwidth of the stopband by making the surface impedance of both arrays equal. This was accomplished by making array 1 (resonant at 8.7 GHz) consist of two arrays in the same plane and by adjusting the width of the elements in array 1. Both biplanar arrays have an acceptable performance for most applications.

Cases D.3 and D.4 have changed the outer dielectric layers from a relative permittivity of 3.3 to 1.5. Case D.3 has the same type of distance parameters \((d_1, d_2, d_3)\) that was investigated in Section C \((d_1=\lambda_1/4, d_2=\lambda_1/4, d_3=\lambda_2/4)\). The interelement spacing of the arrays and a plot of the surface impedance are illustrated in Figure 72 for case D.3. This design is fairly scan compensated in the stopbands (5.8 and 8.7 GHz), and provides a fairly large bandwidth in the passband (2.9 GHz). The reflection and transmission coefficients are shown in Figure 73. The center frequency of the passband varies more with scan angle than case D.1 but the bandwidth of the passband is larger. This is same result we observed in Section C. The higher the relative permittivity of the dielectric slab the less
Figure 72. Geometry and surface impedance of the arrays used in case D.3. The arrays are in the x-z plane.
Figure 73. Reflection and transmission coefficients for a dual band staggered tuned biplanar array embedded in three dielectric slabs (Case D.3).
variation there is in the center frequency of the passband. However, the bandwidth is smaller.

Case D.4 was designed to get a large bandwidth in the passband. It is analogous to the $\lambda/4$ transformer in that the outer dielectric slabs are $\lambda_{t}/4$, and the relative permittivity of slabs one and three is

$$
\varepsilon_1 = \varepsilon_3 = \sqrt{\varepsilon_0 \varepsilon_2}
$$

(4.19)

The geometry and surface impedance of the arrays used in case D.4 are shown in Figure 74. The reflection and transmission coefficients are plotted in Figure 75. The passband has the largest bandwidth of all cases investigated in this dissertation. However, this design shows larger variations in the center frequency of the passband as the scan angle is changed. The resonant frequency of the stopbands also show larger changes as the scan angle is changed.

In summary, a staggered tuned biplanar array placed between three dielectric slabs allows the designer more options in meeting his performance requirements. It also complicates the design. Several designs were investigated in this section. Each design emphasized different performance parameters. A designer may pursue a configuration such as case D.1 if scan compensation in the passband and stopbands are of primary interest. A larger bandwidth in the passband could be obtained by using a design similar to case D.2 while sacrificing very little in scan compensation. A design such as
Figure 74. Geometry and surface impedance of the arrays used in case D.4. The arrays are in the x-z plane.
Figure 75. Reflection and transmission coefficients for a dual band staggered tuned biplanar array embedded in three dielectric slabs (Case D.4).
case D.3 may be of interest if an even larger passband was desired but the center frequency of the passband would show more change as the scan angle is varied. Case D.4 would provide a large bandwidth in the passband but would sacrifice the scan compensation of the biplanar array.

E. SUMMARY

As mentioned before, previous investigators have analyzed the scan compensation of biplanar arrays in dielectric. The biplanar arrays they analyzed had identical resonant frequencies for each of the individual arrays. In this investigation we applied the scan compensation concepts to staggered tuned arrays. It was not the purpose to exhaustively characterize the staggered tuned biplanar array's performance when embedded in a stratified dielectric medium. Instead, a few cases were examined to get an understanding of the performance of the staggered tuned biplanar array when embedded in dielectric. Even so, we do have enough information to make the following general conclusions.

1. Placing a staggered tuned biplanar array in dielectric with the proper choice of dielectric constant ($\varepsilon$) and thickness ($d$) improved the scan compensation of the array.

2. For a staggered tuned biplanar array in dielectric where the individual array resonant frequencies are only slightly different, only one resonant region may result.
3. For a staggered tuned biplanar array with slightly different resonant frequencies in only one dielectric layer, the scan compensated performance is similar to a biplanar array where each of the individual arrays are identical.

4. A two band biplanar array with a passband embedded in three dielectric layers gives the designer more options in achieving the desired performance.

Although a fair amount of design work still needs to be done, it is clear that staggered tuned biplanar arrays can achieve a certain degree of scan compensation.
CHAPTER V

CONCLUSIONS

This dissertation analyzed the impedance, reflection, and transmission properties of staggered tuned arrays of dipoles in an infinite medium and embedded in a stratified dielectric medium. This document demonstrated that one can use the plane wave expansion technique for determining the reflection and transmission coefficients of staggered tuned arrays. Munk [3] first introduced the methodology for analyzing staggered tuned arrays. Munk's work on staggered tuned arrays was only in an infinite medium and did not look at the conditions necessary for scan compensation. Munk and Kornbau [4] and several others have investigated the conditions necessary for a symmetrical biplanar array to be scan compensated. This work has combined the above efforts and investigated the conditions necessary for a staggered tuned biplanar array to be scan compensated. To be specific, we wish to know to what extent each resonance band can achieve stable resonances with angle of incidence. In addition, we investigated the conditions necessary for designing a passband with a staggered tuned biplanar array.

In Chapter III, we demonstrated that the reflection bandwidth of the biplanar array can be increased by having each of the individual arrays resonant at a slightly different frequencies. This
was shown by use of a transmission line analogy as well as calculated computer results using the plane wave expansion method. However, transmission line analogy tells us that there should be two resonant regions if each of the individual arrays in the biplanar array is resonant at a different frequency. This was not always the case and reveals that there are limitations in the transmission line analogy. Also in Chapter III, it was shown by analysis and calculated results that a large transmission band could be obtained for a staggered tuned biplanar array of dipoles if the surface impedance of the individual arrays are nearly equal at the passband frequency. As the scan angle is changed for a biplanar array in an infinite medium, the resonant frequencies, the frequency of the passband, and the bandwidths change. By placing the staggered tuned biplanar array in dielectric with the proper choice of $\varepsilon$ and $d$, the scan compensation of the array was improved. This was demonstrated in Chapter IV. For a staggered tuned biplanar array in a dielectric slab where the individual array resonant frequencies are only slightly different, only one resonant region may result. This was the same as the results obtained when the staggered tuned biplanar array was in an infinite medium (Chapter III).

An additional observation, somewhat detached from the main focus of this dissertation, is that a significant amount of CPU time can be saved when the direct path of the self-impedance term is recalled from a previous calculation. The CPU time savings decreases when
the arrays are close to the slab boundaries. It is recommended that all periodic surface design codes incorporate this time saving feature.

A final observation concerns what additional research needs to be investigated. There are several items which were beyond the scope of this initial investigation. They include the following:

1. A rigorous examination of the conditions that result in only one resonant region for a staggered tuned biplanar array. This would include an experimental investigation of the impedance properties as well as the reflection and transmission coefficients.

2. An examination of what the conditions are from periodic surface theory that result in a passband for a staggered tuned biplanar array.

3. Design of a wide reflection band using staggered tuned biplanar arrays with a passband. A design that had a reflection band from 6-16 GHz with a passband at 2.1 GHz would be extremely beneficial.
APPENDIX A
DIRECTION OF PROPAGATION VECTORS AND ELECTRIC FIELD VECTORS

The electric field radiated from a periodic structure can be expressed as an infinite sum of discrete plane waves. Munk [50], English [51], and many others have investigated the propagation directions of these plane waves. Each plane wave propagates in a different direction denoted by the summation indices \( k \) and \( n \). The plane waves will radiate in discrete directions given by (Figure 76)

\[
\hat{r}_{m\pm} = \frac{\hbar}{\lambda} \pm \frac{\lambda}{\lambda} \hat{x} m_x + \hat{y} m_y + \hat{z} m_z \quad (A.1)
\]

where

\[
r_{mx} = s_{mx} + \frac{k\lambda}{D_x} \quad (A.2)
\]

and

\[
r_{mz} = s_{mz} + \frac{n\lambda}{D_z} \quad (A.3)
\]

and
Figure 76. Plane wave directions.
$r_{my} = \sqrt{1-(r_{mx})^2-(r_{mz})^2}$ \quad (A.4)

\(\hat{s}_m\) is the direction of propagation of the incident externally impressed plane wave and is given by,

$$\hat{s}_{m\pm} = \hat{x}s_{mx} \pm \hat{y}s_{my} + \hat{z}s_{mz} \quad (A.5)$$

when

$$1-(r_{mx})^2-(r_{mz})^2 < 0 \quad (A.6)$$

The (-j) branch of the square root is chosen which produces an evanescent wave rather than a growing wave. When (A.4) is real, that is (A.6) is greater than zero, we get a propagating wave. If (A.6) is real for values of \(k\) and \(n\) other then \((k,n)=(0,0)\), this corresponds to a grating lobe. This is usually undesirable and can be controlled by making the interelement spacing \(D_x\) and \(D_z\) small enough in order to prevent the onset of grating lobes.

The subscript \(m\) in (A.1) through (A.6) refers to the dielectric medium where both \(\hat{s}_{m\pm}\) and \(\hat{r}_{m\pm}\) are evaluated. From (A.2) and (A.3) we see that for \(k=n=0\)

$$\hat{s}_{m\pm} = \hat{r}_{m\pm} \mid_{k=n=0} \quad (A.7)$$
This corresponds to the dominating propagating mode.

In order to fully illustrate the direction of propagation, refer to Figure 76. In the figure, $s_{0+}$, $s_{0-}$, and $s_{m+}$ are the incident, reflected, and transmitted Poynting vector directions. The other directions are grating lobes for $k \neq 0$ and $n \neq 0$.

The components of $\hat{r}$ in different mediums (m and m' in this case) are related by

$$r_{m'x} = \frac{\beta_m}{\beta_{m'}} r_{mx}$$

(A.8)

and

$$r_{m'z} = \frac{\beta_m}{\beta_{m'}} r_{mz}$$

(A.9)

The electric field can be decomposed into components parallel and perpendicular to the plane defined by $\hat{r}_{m+}$ and the -y axis. In this way Fresnel reflection and transmission coefficients can be used to analyze the behavior of periodic surfaces in a dielectric medium [52]. From Munk, Burrell, and Kornbau [53], the electric field direction $\hat{e}_{m\pm}$ is defined as

$$\hat{e}_{m\pm} = [\hat{p} \times \hat{r}_{m\pm}] \times \hat{r}_{m\pm}$$

(A.10)
for a linear element orientated in the \( \hat{p} \) direction. This can be written as

\[
\varepsilon_{m\pm} = (\hat{r}_{m\pm} \cdot \hat{p}) \hat{r}_{m\pm} - \hat{p} \quad . \tag{A.11}
\]

Decomposing (A.11) into parallel and perpendicular components to the plane containing \( \hat{r}_{m\pm} \) and \( -y \) axis results in

\[
\varepsilon_{m\pm} = \perp \hat{n}_{m\pm} \cdot \hat{p} - \parallel \hat{n}_{m\pm} \cdot \hat{p} \quad (A.12)
\]

where \( \perp \hat{n}_{m\pm} \) is normal to the propagating mode direction \( \hat{r}_{m\pm} \).

Therefore,

\[
\parallel \hat{n}_{m\pm} \cdot \hat{r}_{m\pm} = 0 \quad (A.13)
\]

where

\[
\perp \hat{n}_{m\pm} = \frac{-\hat{y} \times \hat{r}_{m\pm}}{\hat{y} \times \hat{r}_{m\pm}} = \frac{-x_{r_{mz}} + z_{r_{mx}}}{\sqrt{(r_{mx})^2 + (r_{mz})^2}} \quad (A.14)
\]

and

\[
\parallel \hat{n}_{m\pm} = \pm \perp \hat{n}_{m\pm} \times \hat{r}_{m\pm} = \frac{-x_{r_{mx}r_{my}} \pm \Lambda((r_{mx})^2 + (r_{mz})^2)}{\sqrt{(r_{mx})^2 + (r_{mz})^2}} - z_{r_{mz}r_{my}} \quad . \tag{A.15}
\]
Figure 77. Normalized electric field vectors and Poynting vectors in the $m$th and $m+1$ slabs.
Figure 77 shows the relative orientations of $\hat{n}_\perp$, $\hat{n}_\parallel$, and $\hat{r}_\pm$ for a particular (k,n)th plane wave.
APPENDIX B

EFFECTIVE REFLECTION AND TRANSMISSION COEFFICIENTS

Munk [54] has derived the effective reflection and effective transmission coefficients. Figure 78 shows a stratified medium illuminated by a plane wave. Each layer will have a right going and a left going propagating plane wave. Defining the ratio of the reflected field, $\frac{\tilde{E}^-}{\tilde{E}^+}$, to the incident electric field, $\frac{\tilde{E}^+}{\tilde{E}^+}$, as the effective reflection coefficient, $\frac{\Gamma^+}{\Gamma^-}$ for a right going wave is,

$$\frac{\tilde{E}^-}{\tilde{E}^+} = \frac{\frac{1}{\Gamma^+} + \frac{1}{\Gamma^+} e^{-j\beta m+1d_{m+1}r(m+1)y}}{1 - \frac{1}{\Gamma^+} \frac{1}{\Gamma^+} e^{-j\beta m+1d_{m+1}r(m+1)y}}$$

where

$$\frac{\Gamma^+}{\Gamma^-} = \frac{Z_{m+1}\Gamma_{my} - Z_{m}r_{m+1}y}{Z_{m+1}\Gamma_{my} + Z_{m}r_{m+1}y}$$

(B.1)

and

$$\frac{\Gamma^+}{\Gamma^-} = \frac{Z_{m+1}\Gamma_{m+1}y - Z_{m}\Gamma_{m+1}y}{Z_{m+1}\Gamma_{m+1}y + Z_{m}\Gamma_{m+1}y}$$

(B.2)
Figure 78. Multiple reflections in a stratified dielectric medium.
where $\chi_0$ and $\gamma_0$ are the regular Fresnel reflection coefficients occurring between two semi-infinite mediums $m$ and $m+1$. The complex impedance is related to the complex permittivity and permeability by

$$Z_m = \sqrt{\frac{\mu_m}{\varepsilon_m}}.$$  \hfill (B.4)

Also note that for the regular Fresnel reflection coefficients only

$$\chi_{m+1} = -\chi_m.$$  \hfill (B.5)

In order to use (B-1), $\chi_{m+1,e}$ must be known prior to calculating $\chi_{m,e}$. Therefore the effective reflection coefficient must be calculated in an iterative manner, beginning at the last slab (MM)

$$\chi_{MM,e} = \chi_{MM}$$  \hfill (B.6)

and working right to left using (B.1).

Unlike the regular Fresnel reflection coefficients, the effective reflection coefficients for left going waves are not in general related to the effective reflection coefficients for right going waves in each
dielectric slab. For a left going wave the effective reflection coefficient is

\[ \Gamma_{m+1,e}^- = \frac{\frac{\tilde{E}^r_m}{\tilde{E}^i_m} + \frac{\Gamma^-_{m+1}}{\Gamma^-_{m,e}} e^{-j2\beta_m d_m r_m y}}{1 - \frac{\Gamma^-_{m,e}}{\Gamma^+_{m}}} \]  \hspace{1cm} (B.7)\]

where

\[ \Gamma_{m+1}=\Gamma_m \]  \hspace{1cm} (B.8)\]

Equation (B.7) is also an iterative solution, so calculating the first slab by equating

\[ \Gamma_{1,e}^- = \Gamma_{1}^- \]  \hspace{1cm} (B.9)\]

then proceed with the remaining slabs working left to right.

The effective transmission coefficient from slab m to slab m+1 is the ratio of transmitted wave in slab m+1 to incident wave in slab m.
\[ \tau_m^+ = \frac{\frac{i}{\|} E_{\text{in}}^{l+} - e^{j2\beta_{\text{in}} d_m + iy}}{1 - \frac{i}{\|} \Gamma_m^+ e^{j2\beta_{\text{in}} d_m + iy}} \cdot \Gamma_m. \] (B.10)

For left-going waves the transmission coefficient for a transmitted wave in slab m due to incident wave in slab m+1 is

\[ \tau_{m+1}^- = \frac{\frac{i}{\|} E_{\text{out}}^{l-} - e^{j2\beta_{\text{out}} d_{m+1} + iy}}{1 - \frac{i}{\|} \Gamma_{m+1}^- e^{j2\beta_{\text{out}} d_{m+1} + iy}} \cdot \Gamma_{m+1}. \] (B.11)
APPENDIX C
DIRECT PATH CALCULATION OF SELF-IMPEDANCE

In Chapter II, five paths contributed to the self-impedance as shown in Figure 79. This could be reduced to four path contributions if the elements in the array were only contained in the x-z plane. That is,

\[ \mathbf{p}^n \cdot \mathbf{y} = 0. \]  \hspace{1cm} (C.1)

Since this research deals only with planar elements (\( \mathbf{p}^n \cdot \mathbf{y} = 0 \)), why identify 5 path contributions when four will do? The answer lies in the self-impedance term \( Z_{1,n'}^{n,n} \). From (2.35)

\[
Z_{1,n'}^{n,n} = \frac{Z_{m'}^{m}}{2D_x D_z k_{n,n'}} \sum \frac{e^{j \beta m' r_m' y}}{r_m' y} [\perp P_{m',+}^{n,n} + \parallel P_{m',+}^{n,n}] (C.2)
\]

where \( \hat{R}_n - \hat{R}_n' = a \hat{y} \), and \( a \) is the wire radius of the element.

As Kent [55] points out, \( Z_{1,n'}^{n,n} \) impedance contribution due to path 1 has the strongest evanescent wave coupling and will take the most terms to converge. Note also that \( Z_{1,n'}^{n,n} \) does not contain any reflection or transmission terms (\( \Gamma_{m,e}^+, \Gamma_{m}^+, \tau_{m',e}^+ \)), so \( Z_{1,n'}^{n,n} \) is independent of the surrounding dielectric layers. Thus \( Z_{1,n'}^{n,n} \) can be
Figure 79. Five bounce paths of the self-impedance term.
calculated and filed permanently for future use. If the dielectric surrounding the array ($\varepsilon_{m'}$) and the interelement spacings ($D_x$ and $D_z$) are not changed from one design to another, $Z^{n,n'}_1$ does not need to be recalculated. This implies that as the thickness and dielectric constants of neighboring dielectric layers are changed, only the other four path contributions need to be calculated ($Z^{n,n'}_2$, $Z^{n,n'}_3$, $Z^{n,n'}_4$, $Z^{n,n'}_5$).

Kent [55] recommended modifying existing codes to incorporate this time saving feature. This feature was incorporated for the periodic surface code FORPOLE for use in this Appendix.

The question arises, how much CPU time is saved when the direct mode is called from a previous calculation rather then recalculated? The approach used in answering that question was to first calculate the direct path self-impedance term as well as the four bounce paths. Upon completion of determining the impedance terms, the scattered field was determined and the CPU time was noted. Recalculating the four bounce paths and recalling the stored data of the direct path, the impedance matrix was determined again. The scattered field was found once again and the CPU time was noted. In this way, the CPU time savings was recorded as a savings in determining the reflected and transmitted fields, the end product of all periodic surface design codes.

Two cases of interest were investigated regarding the time savings of calling up a previously calculated value for the direct path self-impedance term. The first case consists of a monplanar array embedded in dielectric. This is illustrated in Figure 80. Table 7 lists
Figure 80a. Monoplanar array used in determining CPU time savings for the five bounce paths.

Figure 80b. Biplanar array used in determining CPU time savings for the five bounce paths (arrays in end slabs).

Figure 80c. Biplanar array used in determining CPU time savings for the five bounce paths (arrays in middle slab).
the CPU time for determining the reflected and transmitted field when the direct path of the self-impedance term is calculated or when it is recalled from a previous calculation. The percent difference in CPU time is also noted in Table 7 for various distances to the slab boundary. Figure 81 also plots the percentage of CPU time saved versus distance to the slab interface. The reflected and transmitted fields were calculated from 39.5 GHz to 40.5 GHz in .1 GHz steps for the monoplane array.

Table 7. Comparison of CPU time when calculating the reflected and transmitted fields for a monoplane array

<table>
<thead>
<tr>
<th>Distance to Slab Interface (Wire Radius)</th>
<th>CPU Time $Z_{dir}$ Calculated (sec)</th>
<th>CPU Time $Z_{dir}$ Recalled (sec)</th>
<th>% Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>70.74</td>
<td>54.50</td>
<td>22.96</td>
</tr>
<tr>
<td>2.</td>
<td>65.20</td>
<td>39.69</td>
<td>39.13</td>
</tr>
<tr>
<td>2.61</td>
<td>65.01</td>
<td>37.04</td>
<td>43.02</td>
</tr>
<tr>
<td>5.22</td>
<td>67.96</td>
<td>29.77</td>
<td>56.19</td>
</tr>
<tr>
<td>10.44</td>
<td>69.28</td>
<td>25.00</td>
<td>63.91</td>
</tr>
<tr>
<td>20.88</td>
<td>80.77</td>
<td>22.83</td>
<td>71.71</td>
</tr>
<tr>
<td>41.76</td>
<td>79.73</td>
<td>22.31</td>
<td>72.02</td>
</tr>
</tbody>
</table>

As can be seen from Figure 81 and Table 7 there is a substantial time savings when recalling a previously calculated direct path self-impedance term. When the array was $\lambda/4$ away from the slab boundary there is over a 70% CPU time savings. When the array was as close as two wire radius away there is almost a 40% savings in CPU time. It is only when the array is a wire radius away from the slab boundary that the CPU time savings is less than 25%.
Figure 81. CPU time saved when recalling Zdir in comparison to calculating Zdir for a monoplanar array in dielectric slab. Distance to slab boundary is varied.
When the array is one wire radius away from the slab boundary, the reference element is in the crack between the dielectric slab and free space. That is the reference element is centered on the y-axis.

\[ \hat{R}_n - \hat{R}_{n'} = ay \quad . \]  

(C.3)

The reference element does not need to be located on the y-axis, but is usually put there for convenience.

Figure 82 illustrates the situation when the reference element is placed one wire radius away from the slab boundary. Notice that path 3 has a path length equal to that of the direct path (path 1). Recalling (2-32)

\[ Z_{3}^{n,n'} = \frac{Z_{m'}^{m} \sum_{n} e^{-j\beta_{m'}(\hat{R}_n - \hat{R}_{n'})} \hat{r}_{m'}}{D_{x}' D_{z}'} \]

\[ = \frac{1}{D_{x}' D_{z}'} \sum_{n} \frac{e^{-j\beta_{m'}(\hat{R}_n - \hat{R}_{n'})} \hat{r}_{m'}}{r_{m'} y} \]

Using (C.3) and

\[ b_{m'} - y_{n'} = a \quad . \]  

(C.4)
Figure 82. "Path" contributions for self-impedance calculation, when array is one wire radius from slab boundary.
equation (2.32) becomes

\[
Z_{3}^{n,n'} = \frac{Z_{m'}}{2D_{x}D_{z}k_{n}} \sum_{m} e^{-j\beta_{m'} r_{m'}} y
\]

Also from (2.35) for this case

\[
Z_{1}^{n,n'} = \frac{Z_{m'}}{2D_{x}D_{z}k_{n}} \sum_{m} e^{-j\beta_{m'} r_{m'}} y \left[ P_{m'}^{n,r} + P_{m'}^{n,t} \right] . \tag{C.6}
\]

\(Z_{1}\) and \(Z_{3}\) have the same exponential term, \(e^{-j\beta_{m'} r_{m'} y}\), as mentioned before and will decay at a similar rate. From this analysis it is obvious that the closer a neighboring dielectric is to the array of interest the less CPU time savings will be realized by recalling a previously calculated \(Z_{1}\) term. The evanescent wave coupling is stronger as the distance to neighboring dielectric layers is made smaller.

The second case investigated consisted of a biplanar array embedded in a stratified dielectric medium. This is illustrated in Figures 80b and 80c. The percent difference in CPU time is listed in
Table 8, and shown in Figure 83. The two arrays are either embedded in the outer dielectric slabs or both of the arrays are in the inner dielectric slab. The reflected and transmitted fields were calculated from 10 to 20 GHz in 1 GHz steps.

Table 8. Comparison of CPU time in calculating the reflected and transmitted fields for a biplanar array.

<table>
<thead>
<tr>
<th>Distance to Slab Interface (Wire Radius)</th>
<th>CPU Time $Z_{dir}$ Calculated (sec)</th>
<th>CPU Time $Z_{dir}$ Recalled (sec)</th>
<th>% Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. (left)</td>
<td>258.52</td>
<td>244.25</td>
<td>5.52</td>
</tr>
<tr>
<td>1. (inner)</td>
<td>264.54</td>
<td>248.57</td>
<td>6.04</td>
</tr>
<tr>
<td>1. (end)</td>
<td>251.11</td>
<td>193.13</td>
<td>23.09</td>
</tr>
<tr>
<td>1. (end)</td>
<td>275.02</td>
<td>192.09</td>
<td>23.09</td>
</tr>
<tr>
<td>2. (inner)</td>
<td>237.42</td>
<td>118.05</td>
<td>50.28</td>
</tr>
<tr>
<td>2. (end)</td>
<td>282.60</td>
<td>120.52</td>
<td>57.35</td>
</tr>
<tr>
<td>4. (inner)</td>
<td>237.43</td>
<td>83.16</td>
<td>64.97</td>
</tr>
<tr>
<td>4. (end)</td>
<td>274.88</td>
<td>84.00</td>
<td>69.44</td>
</tr>
<tr>
<td>8. (inner)</td>
<td>252.27</td>
<td>66.98</td>
<td>73.45</td>
</tr>
<tr>
<td>8. (end)</td>
<td>270.14</td>
<td>68.98</td>
<td>74.47</td>
</tr>
<tr>
<td>13. (inner)</td>
<td>263.36</td>
<td>64.89</td>
<td>75.36</td>
</tr>
<tr>
<td>13. (end)</td>
<td>257.35</td>
<td>66.98</td>
<td>73.45</td>
</tr>
<tr>
<td>20. (inner)</td>
<td>282.05</td>
<td>79.53</td>
<td>71.80</td>
</tr>
<tr>
<td>20. (end)</td>
<td>257.29</td>
<td>58.24</td>
<td>77.36</td>
</tr>
</tbody>
</table>

For a biplanar array, the mutual impedance terms must be calculated. With the addition of the mutual impedance terms, the time savings realized in recalling $Z_1$ will be almost as large for a biplanar array as for a monoplanar array. As long as the spacing between the two arrays is large ($\lambda/8$), the mutual impedance terms will converge rapidly and the CPU time savings will be similar to a
Figure 83. Cpu time saved when recalling Zdir in comparison to calculating Zdir for two biplanar arrays each in a dielectric slab. Distance to slab boundary is varied.
monoplane case. When the arrays are in the outer layers there should be more time savings than when in the inner layer. This difference is due to the mutual coupling of the two arrays. When both arrays are in the inner layer, they are closer together and hence there is stronger wave coupling between them which takes longer to converge.

Consider the case when both arrays are in the inner slab .127 cm from the slab boundaries as shown in Figure 84. The CPU time increases by 15 seconds for the $Z_1$ recalled case compared to the case when the arrays are .0849 cm from the slab boundaries. The two arrays are closer to each other than they are to the slab boundaries. This results in a larger CPU time than when the arrays were closer to the boundary.

A final case investigated is when both arrays are located in the crack between the slab boundaries. This is shown in Figure 85. Notice that the path length for $Z_2$ for both arrays is the same as the path length for $Z_1$. When both arrays where located in the inner slab one wire radius away from the slab boundary, the path length for $Z_3$ of the second array had the same path length as $Z_1$. This is shown in Figure 86. The configuration with two path lengths equivalent to $Z_1$ will take longer to converge than the case with one path length equivalent to $Z_1$. Table 9 compares the two configurations along with a third configuration which is equivalent to the case when both arrays are in the crack. This case is shown in Figure 87. Both arrays are one wire radius to the left of the slab boundaries. The $Z_3$ path for
Figure 84. Two Arrays in the Inner Slab Closer to Each Other than to the Slab Boundaries.
Figure 85. Self Impedance Bounce Paths When Both Arrays Are in the Cracks Between Dielectric Slabs
Figure 86. Self Impedance Paths When Both Arrays Are in the Inner Slab One Wire Radius (a) From The Slab Boundaries $b_1$ and $b_2$. 
Figure 87. Self Impedance Paths When Both Arrays are One Wire Radius to the Left of the Slab Boundaries.
both arrays has the same path length has the $Z_1$ path. When both arrays are in the inner slab there is a larger saving in CPU time. For the other two configurations, the CPU time that was saved was minimal.

Table 9. Comparison of CPU time in calculating the reflected and transmitted fields for a biplanar array.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>CPU Time $Z_{dir}$ Calculated (sec)</th>
<th>CPU Time $Z_{dir}$ Recalled (sec)</th>
<th>% Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both arrays in crack (Figure 85)</td>
<td>258.52</td>
<td>244.25</td>
<td>5.52</td>
</tr>
<tr>
<td>Arrays left of slabs (Figure 87)</td>
<td>264.54</td>
<td>248.57</td>
<td>6.04</td>
</tr>
<tr>
<td>Arrays in inner slab (Figure 86)</td>
<td>251.11</td>
<td>193.13</td>
<td>23.09</td>
</tr>
</tbody>
</table>

In summary, a significant amount of CPU time can be saved when the direct path of the self-impedance term is recalled from a previous calculation. The CPU time savings decreases when the arrays are close to the slab boundaries. It is recommended that all periodic surface design codes which utilize the plane wave expansion technique incorporate this time saving feature.
APPENDIX D

REFLECTION COEFFICIENT OF A BIPLANAR ARRAY USING TRANSMISSION LINE ANALOGY

In this appendix the equivalence between the reflection coefficient for a biplanar array of dipoles and two reactances across a transmission line model is investigated. The transmission line model is shown in Figure 88.

From (2.71), the reflection coefficient is given by,

\[ \Gamma_1 = -\frac{1}{1 + \frac{2jZ_{sh1}}{Z_0}} \]  

(D.1)

where the subscripts \( \perp \) and \( \parallel \) (the incident electric field is either \( \perp \) or \( \parallel \) to the plane of incidence) have been dropped for economy of notation.

The impedance \( Z_1 \) just right of \( Z_{sh2} \) yields the transformed impedance, \( Z_1(y_1) \)

\[ Z_1(y_1) = Z_0 \frac{1 + \Gamma_1 e^{-j2\beta_0 y_1}}{1 - \Gamma_1 e^{-j2\beta_0 y_1}} \]  

(D.2)

The total impedance \( Z_2 \) observed at \( Z_{sh2} \) looking right is,
Biplanar array in an infinite medium

Transmission line equivalent of biplanar array in an infinite medium.

Figure 88. Transmission line model of a biplanar array.
\[ \frac{1}{Z_2} = \frac{1}{jZ_{sh2}} + \frac{1}{Z_1(y_1)^\ast} = \frac{1}{jZ_{sh2}} + \frac{1 - \Gamma_1 e^{-j2\beta_0y_1}}{Z_0(1 + \Gamma_1 e^{-j2\beta_0y_1})}. \quad (D.3) \]

The corresponding reflection coefficient \( \Gamma_2 \) is given by,

\[ \Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{1 - \frac{Z_0}{Z_2}}{1 + \frac{Z_0}{Z_2}}. \quad (D.4) \]

Substituting (D.3) into (D.4) results in

\[ \Gamma_2 = \frac{\frac{Z_0}{jZ_{sh2}} - \frac{1 - \Gamma_1 e^{-j2\beta_0y_1}}{1 + \Gamma_1 e^{-j2\beta_0y_1}}}{1 + \frac{Z_0}{jZ_{sh2}} + \frac{1 - \Gamma_1 e^{-j2\beta_0y_1}}{1 + \Gamma_1 e^{-j2\beta_0y_1}}}. \quad (D.5) \]

Substituting for \( \Gamma_1 \) and simplifying results in

\[ \Gamma_2 = \frac{(Z_0 + 2jZ_{sh1}) + 2jZ_{sh2} e^{-j2\beta_0y_1} - Z_0 e^{-j2\beta_0y_1}}{(Z_0 + 2jZ_{sh1})(1 + \frac{2jZ_{sh2}}{Z_0}) - Z_0 e^{-j2\beta_0y_1}}. \quad (D.6) \]
APPENDIX E

CONDITIONS FOR A UNITY TRANSMISSION COEFFICIENT FOR A BIPLANAR ARRAY

For a lossless medium, the transmission coefficient is related to the reflection coefficient by

\[ |T_{0,e}^a|^2 = 1 - |\Gamma_{0,e}^a|^2 \]  \hspace{1cm} \text{(E.1)}

From (3.24), (E.1) can be written as

\[ |T_{0,e}^a|^2 = 1 - \frac{2Z_0^2(1-\cos^2\phi_1)+4(Z_{sh1}^2+Z_{sh2}^2)+8Z_{sh1}Z_{sh2}\cos^2\phi_1+4Z_0\sin^2\phi_1(Z_{sh1}+Z_{sh2})}{Z_{sh1}^2Z_{sh2}^2 + \frac{Z_0^2}{16}} \]  \hspace{1cm} \text{(E.2)}

To have \(|T_{0,e}^a|^2 = 1\) implies that
\[ 2Z_0^2(1-\cos^2\phi_1)+4(Z_{sh1}^2+Z_{sh2}^2)+8Z_{sh1}Z_{sh2}\cos2\phi_1+4Z_0\sin2\phi_1(Z_{sh1}+Z_{sh2}) = 0 \]  \hspace{1cm} (E.3)

Rearranging (E.3) in powers of \(Z_{sh1}\)

\[ 4Z_{sh1}^2+Z_{sh1}(8Z_{sh2}\cos2\phi_1+4Z_0\sin2\phi_1)+2Z_0^2(1-\cos^2\phi_1)+4Z_0Z_{sh2}\sin2\phi_1+4Z_{sh2}^2 = 0 \]  \hspace{1cm} (E.4)

Let the coefficient of \(Z_{sh1}^2\) be, \(a\), the coefficient of \(Z_{sh1}\) be \(b\), and the coefficient of \(Z_{sh1}^0\) be \(c\). Since \(Z_{sh1}\) is a real quantity then

\[ b^2 - 4ac \geq 0 \]  \hspace{1cm} (E.5)

Substituting (E.4) into (E.5) results in

\[ (4Z_{sh2}\cos2\phi_1+2Z_0\sin2\phi_1)^2 - 8[Z_0^2(1-\cos^2\phi_1)+2Z_0Z_{sh2}\sin2\phi_1+2Z_{sh2}^2] \geq 0 \]  \hspace{1cm} (E.6)

Rearranging (E.6) results in

\[ 16Z_{sh2}^2(\cos^2\phi_1-1)+16Z_0Z_{sh2}\sin2\phi_1(\cos2\phi_1-1)+4Z_0^2(\sin^22\phi_1+2\cos2\phi_1-2) \geq 0 \]  \hspace{1cm} (E.7)
The trigometric identities

\[
\begin{align*}
\cos^2\phi_1 - 1 &= -\sin^2\phi_1 \\
\cos 2\phi_1 - 1 &= -2\sin^2\phi_1 \\
\sin^2\phi_1 &= 4\sin^2\phi_1 \cos^2\phi_1 \\
\cos^2\phi_1 - 1 &= -\sin^2\phi_1
\end{align*}
\] (E.8) (E.9) (E.10) (E.11)

will be substituted into (E.7) resulting in

\[-16Z_{sh2}\sin^2\phi_1 - 32Z_0 Z_{sh2}\sin^2\phi_1 \sin^2\phi_1 + 16Z_0^2 \sin^2\phi_1 (\cos^2\phi_1 - 1) \geq 0 \] (E.12)

Using (E.11) and (E.10) again results in

\[-64Z_{sh2}^2 \sin^2\phi_1 \cos^2\phi_1 - 64Z_0 Z_{sh2} \cos^2\phi_1 \sin^2\phi_1 - 16Z_0^2 \sin^4\phi_1 \geq 0 \] (E.13)

This may also be written as

\[-(8Z_{sh2} \sin\phi_1 \cos\phi_1 + 4Z_0 \sin^2\phi_1)^2 \geq 0 \] (E.14)

It is obvious by inspection of (E.14) that this quantity can never be greater than zero and the only solution is zero. This occurs when

\[Z_{sh2} = \frac{-Z_0 \tan\phi_1}{2} \quad \cos\phi_1 \neq 0\] (E.15)
or when

$$\sin \phi_1 = 0 \quad . \quad (E.16)$$

When the transmission coefficient is unity the quadratic equation for $Z_{sh1}$ is $b^2-4ac=0$. Solving for $Z_{sh1}$ when the transmission coefficient is unity results in

$$Z_{sh1} = \frac{-8Z_{sh2} \cos \phi_1 + 4Z_0 \sin 2\phi_1}{8} \cos \phi_1 \neq 0 \quad . \quad (E.17)$$

Solving (E.15) for $Z_0$ and substituting into (E.17) results in

$$Z_{sh1} = -Z_{sh2} (\cos 2\phi_1 - \cot \phi_1 \sin 2\phi_1) \cos \phi_1 \neq 0 \quad . \quad (E.18)$$

Simplifying (E.18) results in

$$Z_{sh1} = Z_{sh2} \quad \text{for } |T_{0,c}^+|^2 = 1 \text{ and } \cos \phi_1 \neq 0, \sin \phi_1 \neq 0 \quad . \quad (E.19)$$

If $\sin \phi_1 = 0$ then,

$$Z_{sh1} = -Z_{sh2} \quad \sin \phi_1 = 0 \quad . \quad (E.20)$$

is also a solution.
Referring to Figure 89, a transmission coefficient of 1 occurs in region 2 when the conditions of (E.20) happen. The case where $\phi_1=0$ (both arrays are in the same plane) was discussed in Chapter III and is called a modal interaction null. We are not interested in having a null in this region.

In regions 1 and 3, a null can occur if the conditions of (E.19) are met. If the slopes of the surface impedance versus frequency are similar for both arrays and the arrays have different resonant frequencies then a transmission coefficient of unity is not possible in regions 1 and 3 (Figure 89). If both arrays are the same, it is possible.

In addition if $\cos\phi_1=0$, $|T_0^a|^2$ cannot be unity. So if one were to illuminate the biplanar array at normal incidence, and the arrays were spaced $d_1=\lambda/4$ apart then

$$\phi_1 = \beta_1 d_1 = \pi/2$$

(E.21)

and $|T_0^a|^2=1$ is not possible.
b. Sign and magnitude of surface impedance for three regions.

Figure 89. Surface impedance for two arrays (array 1 resonant at $f_1$ and array 2 resonant at $f_2$).
APPENDIX F
COMMENTS ON SLOT ARRAYS

As mentioned in Chapter I, the methodology for the staggered tuned dipoles is not applicable for slot arrays. In this appendix, an explanation will be provided explaining why slots cannot be staggered tuned in most cases.

By the equivalence principle, slots can be modeled as a ground plane sandwiched between two oppositely directed magnetic dipoles. With the proper use of dual quantities (magnetic fields for electric fields, voltages for currents, and admittance for impedance) the scattering from slot arrays can be determined using the same formulas and steps used for dipoles in Chapter II. For multiple slot arrays the problem simplifies further because the ground plane effectively eliminates all interactions except those between adjacent arrays. In other words admittances such as $Y_{1,3}$ or $Y_{1,4}$ will be zero. Only self and adjacent mutual admittances such as $Y_{1,2}$ will be non-zero.

Consider the system of two slot arrays shown in Figure 90. The matrix equation for this system is

$$[I] = [Y][V] \quad \text{(F.1)}$$
THE PHYSICAL SLOT ARRAYS...

Array #1 Array #2

SLOTS

INCIDENT H-FIELD $H_i$

ARE MODELLED AS...

Array #1 Array #2

BACKSCATTER

$\vec{H}_{S1}^-$

DIRECT REFLECTED

$\vec{H}_{DIR1}$

$V_a^{(1)}$ $V_{a}^{(2)}$

$V_b^{(1)}$ $V_{b}^{(2)}$

GROUND PLANES

MAGNETIC CURRENTS, $M_S$

Figure 90. Modeling of a biplanar slot array.
which expands to

\[
\begin{pmatrix}
I^{(1)}_1 \\
0
\end{pmatrix} = 
\begin{pmatrix}
Y^{11} & Y^{12} \\
Y^{21} & Y^{22}
\end{pmatrix}
\begin{pmatrix}
V^{(1)}_1 \\
V^{(2)}
\end{pmatrix}
\]  \quad \quad \text{(F.2)}

We note that in this case we have an admittance matrix, an incident field inducing a current, and the solution contains voltages. The incident field only induces currents into array 1, the other array never sees the incident field because of the ground plane of array 1. The magnetic currents on opposite sides of the arrays have equal magnitude but opposite directions. That is \( V^{(1)}_a = -V^{(1)}_b \) and \( V^{(2)}_a = -V^{(2)}_b \). The transmitted field is due only to the outside voltage \( V^{(2)}_b \) since the incident field is blocked by the first ground plane and fields radiated by inside voltages \( V^{(1)}_a \) and \( V^{(2)}_a \) are blocked by the last ground plane. Fields reflected from array 1 include both the field radiated by \( V^{(1)}_b \) (backscatter) and the field reflected by the ground plane (direct reflected).

Next let's consider that array 1 is resonant at \( f_1 \) and array 2 is resonant at \( f_2 \) with \( f_2 > f_1 \) and \( f_2 \neq n\lambda \) where \( n \) is an integer. This is illustrated in Figure 91. If the frequency of the incident \( \tilde{H} \) field is \( f_1 \) then at the first array

\[ |\tilde{H}_{11}| = |\tilde{H}^{+}_{s1}| \quad \text{and} \quad |\tilde{H}_{\text{dir}} + \tilde{H}^{+}_{s2}| = 0 \]  \quad \text{(F.3)}
Figure 91. Biplanar slot array, with one array resonant at $f_1$ and the other array at $f_2$ (Incident $\vec{H}$ field at $f_1$).
When $\vec{H}_{s1}^+$ is incident on the second array the field is reflected at the array since $f_1 \neq f_2$ and

$$|\vec{H}_{s2}^-|/|\vec{H}_i| \equiv |\vec{H}_{s2}^+|/|\vec{H}_i| << 1 \quad (F.4)$$

and most of the energy is reflected off of array 2 and passes through array 1. Very little of the energy passes through array 2 and hence this array configuration is primarily reflective. Likewise, if the frequency of the incident $\vec{H}$ field is $f_2$ then at the first array

$$|\vec{H}_{s1}^-|/|\vec{H}_i| = |\vec{H}_{s2}^+|/|\vec{H}_i| << 1 \quad (F.5)$$

and most of the incident energy is reflected with very little passing through the biplanar array. So instead of having a two band biplanar slot array, we have a no pass biplanar slot array.

The question arises if $f_1$ and $f_2$ are only slightly different, will this result in an increased transmission bandwidth? The answer is no, since the transmission coefficient of just one array will always be greater or equal to the transmission through a biplanar array configuration.
APPENDIX G

EFFECTIVE RADIUS OF FLAT CONDUCTING ELEMENTS

In this paper, the dipoles are flat elements. Although the general theory can be applied to flat elements, computational time can be saved by using an equivalent wire radius. Hallen [56] used a formulation relating flat elements to cylindrical elements. A graph of this relationship from [57] is shown in Figure 92. It is valid for widths less than $\lambda/8$.

It is well known that the circuit $Q$ of an antenna increases with a smaller dipole radius [58]. The bandwidth of the dipole will increase as the radius of the dipole is increased. Equivalently, increasing the width of a flat element will enlarge the bandwidth since the equivalent radius will also increase. This is an important point in the design of staggered tuned biplanar arrays and is discussed in Chapter III.

The theory developed in Chapter II applies to linear elements, i.e., elements with current distribution of the type $\hat{p}I(l)$. As the width of the dipole increases a more accurate representation may be a two dimensional surface current density of the form,

$$\tilde{J}_s(\tilde{R}) = \hat{p}J_s(\tilde{R})$$  \hspace{1cm} (G.1)
Figure 92. Effective radius "a" for a thin, flat conductor of width w and thickness t.
where \( \mathbf{R} \) denotes an arbitrary source point on the reference element and \( \mathbf{p} \) indicates the direction of current flow. This representation has been developed by Munk [59]. If one is interested in all possible planes of interest then the development by Munk should be followed. However, if only the behavior in the principal planes is of interest, than one is really only interested in the current flow along the length of the dipole and a one dimensional representation should be adequate. To achieve this one dimensional representation, the wide dipole was broken up into normal width dipoles placed side by side to each other as shown in Figure 93. It was found for dipole widths less than \( \lambda/10 \), that there is no significant difference between one dipole and three dipoles side by side. The cases are plotted in Figure 94 and is representative of the resonance region.
Wide dipoles

Three dipoles side by side equivalent to 1 wide dipole

Figure 93. Wide dipole element broken up into normal width dipoles placed side by side.
Figure 94. Transmission coefficient of a monoplanar array. Segments and modes are varied.
APPENDIX H

CONDITIONS FOR A UNITY REFLECTION AND TRANSMISSION COEFFICIENT FOR A STAGGERED TUNED BIPLANAR ARRAY IN A STRATIFIED MEDIUM USING TRANSMISSION LINE THEORY.

In this appendix, an analysis of a staggered tuned biplanar array of dipoles in a stratified medium using a transmission line model is performed. The transmission line model is shown in Figure 95. Our procedure is to find the input impedance, $Z_{in,i}$, at each slab interface and then find the effective reflection coefficient, $\Gamma_{i,e}^a$ at the interface. The input impedance at boundary $b_3$ is,

$$Z_{in,3} = Z_0$$

(H.1)

which is the characteristic impedance of the medium to the right of boundary $b_3$. The effective reflection coefficient at boundary $b_3$ is

$$\Gamma_{3,e}^+ = \Gamma_3^+ = \frac{Z_{in,3} - Z_1}{Z_{in,3} + Z_1} = \frac{Z_0 - Z_1}{Z_0 + Z_1}$$

(H.2)

where $Z_1$ is the characteristic impedance of the third slab.

The input impedance at boundary $b_2$ (Figure 95) can be expressed as
Biplanar array in a stratified medium

Transmission line equivalent of a biplanar array in a stratified medium.

Figure 95. Transmission line model of a staggered tuned biplanar array in a stratified medium.
where \( jZ_{sh2} \) is the surface impedance of the second array and \( \phi_3 \) is

\[
\phi_3 = \beta_3 d_3 r_3 y
\]  

(Simplify (H.3)) results in

\[
Z_{in,2} = \frac{jZ_{sh2}Z_1}{\left(1-e^{-j\phi_3}\right)
\left(jZ_{sh2}Z_0 + Z_1^2\right) + \left(1+e^{-j\phi_3}\right)
\left(jZ_{sh2}Z_1 + Z_1 Z_0\right)}.
\]  

The effective reflection coefficient at boundary \( b_2 \) (Figure 95) is

\[
\Gamma_{2,e} = \frac{Z_{in,2} - Z_2}{Z_{in,2} + Z_2}
\]  

where the superscript \( a \) in \( \Gamma_{2,e} \) indicates that the effective reflection coefficient includes the reflection from array 2. Substituting (H.5) into (H.6) results in
when $Z_{sh2} = 0$ (array 2 is at resonance) then

$$\Gamma_{2,e}^{a^+} = -1 \quad \text{for } Z_{sh2} = 0 \quad \text{(H.8)}$$

Next we find $\Gamma_{1,e}^{a^+}$ (effective reflection coefficient at boundary $b_1$) in terms of $\Gamma_{2,e}^{a^+}$. First we find $Z_{in,1}$ (input impedance at $b_1$) in terms of $\Gamma_{2,e}^{a^+}$.

$$Z_{in,1} = \frac{1+\Gamma_{2,e}^{a^+}e^{-j2\phi_2}}{1-\Gamma_{2,e}^{a^+}e^{-j2\phi_2}} \frac{jZ_{sh1}Z_2}{1+\Gamma_{2,e}^{a^+}e^{-j2\phi_2}} \frac{1+\Gamma_{2,e}^{a^+}e^{-j2\phi_2}}{1-\Gamma_{2,e}^{a^+}e^{-j2\phi_2}} \frac{jZ_{sh1}+Z_2}{1-\Gamma_{2,e}^{a^+}e^{-j2\phi_2}} \quad \text{(H.9)}$$

where $jZ_{sh1}$ is the surface impedance of array 1 and

$$\phi_2 = \beta_2 d_2 r_2 y \quad \text{(H.10)}$$
Rearranging (H.9) we have

\[ Z_{in,1} = \frac{jZ_{sh2}Z_2(1+\Gamma_{2,e}^{a+}e^{-j2\phi_2})}{jZ_{sh1}(1-\Gamma_{2,e}^{a+}e^{-j2\phi_2})+Z_1(1+\Gamma_{2,e}^{a+}e^{-j2\phi_2})} \quad \text{(H.11)} \]

Next we find \( \Gamma_{1,e}^{a+} \)

\[ \Gamma_{1,e}^{a+} = \frac{Z_{in,1}-Z_1}{Z_{in,1}+Z_1} \quad \text{(H.12)} \]

Substituting (H.11) into (H.12)

\[ \Gamma_{1,e}^{a+} = \frac{(jZ_{sh1}Z_2-Z_1^2)(1+\Gamma_{2,e}^{a+}e^{-j2\phi_2})-jZ_{sh1}Z_1(1-\Gamma_{2,e}^{a+}e^{-j2\phi_2})}{(jZ_{sh1}Z_2-Z_1^2)(1+\Gamma_{2,e}^{a+}e^{-j2\phi_2})-jZ_{sh1}Z_1(1-\Gamma_{2,e}^{a+}e^{-j2\phi_2})} \quad \text{(H.13)} \]

When \( Z_{sh1}=0 \) (array 1 at resonance) then

\[ \Gamma_{1,e}^{a+} = -1 \quad \text{for } Z_{sh1}=0 \quad \text{(H.14)} \]

It can also be shown that if \( \Gamma_{2,e}^{a+} = -1 \), then

\[ |\Gamma_{1,e}^{a+}| = 1 \quad \text{for } \Gamma_{2,e}^{a+} = -1 \quad \text{(H.15)} \]
Next we find $\Gamma_{0,e}^{a+}$ (effective reflection coefficient at $b_0$) in terms of $\Gamma_{1,e}^{a+}$. First we find $Z_{\text{in},0}$ (input impedance at $b_0$) in terms of $\Gamma_{1,e}^{a+}$

\[
Z_{\text{in},0} = \frac{Z_1(1+\Gamma_{1,e}^{+}e^{-j2\phi_1})}{1-\Gamma_{1,e}^{+}e^{-j2\phi_1}}
\]

(H.16)

where $\phi_1$ is

\[
\phi_1 = \beta_1 d_{1y}
\]

(H.17)

$\Gamma_{0,e}^{a+}$ is given by

\[
\Gamma_{0,e}^{a+} = \frac{Z_{\text{in},0}-Z_0}{Z_{\text{in},0}+Z_0}
\]

(H.18)

Substituting (H.16) into (H.18) results in

\[
\Gamma_{0,e}^{a+} = \frac{Z_1(1+\Gamma_{1,e}^{a+}e^{j2\phi_1})-Z_0(1-\Gamma_{1,e}^{a+}e^{-j2\phi_1})}{Z_1(1+\Gamma_{1,e}^{a+}e^{j2\phi_1})+Z_0(1-\Gamma_{1,e}^{a+}e^{-j2\phi_1})}
\]

(H.19)

Multiplying (H.19) by the complex conjugate
\[ (Z_1 - Z_0)^2 + |\Gamma_{1,c}^{a+}|^2 (Z_1 + Z_0)^2 + (Z_1^2 - Z_0^2)[(\Gamma_{1,c}^{a+})^* e^{j2\phi_1} + \Gamma_{1,c}^{a+} e^{-j2\phi_1}] \]

\[ |\Gamma_{0,c}^{a+}|^2 = \frac{(Z_1 + Z_0)^2 + |\Gamma_{1,e}^{a+}|^2 (Z_1 - Z_0)^2 + (Z_1^2 - Z_0^2)[(\Gamma_{1,e}^{a+})^* e^{j2\phi_1} + \Gamma_{1,e}^{a+} e^{-j2\phi_1}]}{(Z_1 + Z_0)^2 + |\Gamma_{1,c}^{a+}|^2 (Z_1 + Z_0)^2 + (Z_1^2 - Z_0^2)[(\Gamma_{1,c}^{a+})^* e^{j2\phi_1} + \Gamma_{1,c}^{a+} e^{-j2\phi_1}]} . \] (H.20)

To have a unity reflection coefficient implies that

\[ (Z_1 - Z_0)^2 + |\Gamma_{1,c}^{a+}|^2 (Z_1 + Z_0)^2 = (Z_1 + Z_0)^2 + |\Gamma_{1,e}^{a+}|^2 (Z_1 - Z_0)^2 \]

for \( |\Gamma_{0,c}^{a+}|^2 = 1 \). (H.21)

This can only occur if \( |\Gamma_{1,c}^{a+}|^2 = 1 \). This condition is met from (H.8), (H.14), and (H.15) when \( Z_{sh1} \) or \( Z_{sh2} = 0 \). Therefore,

\[ |\Gamma_{0,c}^{a+}|^2 = 1 \quad \text{when} \quad Z_{sh1} = 0 \quad \text{or} \quad Z_{sh2} = 0 . \] (H.22)

At the resonant frequency of either of the arrays, transmission line theory tells us that we have a unity reflection coefficient. This is the same conditions that apply for a staggered tuned biplanar array in free space. If the arrays are staggered tuned and resonant at different frequencies than our biplanar array should have two resonances according to transmission line theory.

The transmission coefficient is related to the reflection coefficient in a lossless medium by
By setting $|\Gamma_{0,e}^{a+}|^2 = 0$ in (H.20), we can determine the conditions that will result in a unity transmission coefficient. This results in

$$
(Z_1-Z_0)^2 + |\Gamma_{1,e}^{a+}|^2 (Z_1+Z_0)^2 + (Z_1-Z_0)^2 [ (\Gamma_{1,e}^{a+})^* e^{i2\phi_1} + \Gamma_{1,e}^{a+} e^{-j2\phi_1} ] = 0
$$

for $|T_{0,e}^{a+}|^2 = 1$. (H.24)

Let,

$$
\Gamma_{1,e}^{a+} = |\Gamma_{1,e}^{a+}| e^{-j\theta_1}
$$

(H.25)

Substituting (H.25) into (H.24)

$$
(Z_1-Z_0)^2 + |\Gamma_{1,e}^{a+}|^2 (Z_1+Z_0)^2 + 2(Z_1-Z_0)^2 |\Gamma_{1,e}^{a+}| \cos(2\phi_1 + \theta_1) = 0
$$

for $|T_{0,e}^{a+}|^2 = 1$. (H.26)

Solving (H.26) for $|\Gamma_{1,e}^{a+}|$,

$$
|\Gamma_{1,e}^{a+}| = \frac{-2(Z_1-Z_0)^2 \cos(2\phi_1 + \theta_1) \pm \sqrt{4(Z_1-Z_0)^2 \cos^2(2\phi_1 + \theta_1) - 4(Z_1+Z_0)^2 (Z_1-Z_0)^2}}{2(Z_1+Z_0)^2}
$$

(H.27)
The quantity in the square root in (H.27) must be greater or equal to zero. From inspection, the radical in (H.27) cannot be greater than zero. Equation (H.27) is equal to zero when \( \cos^2(2\phi_1 + \theta_1) = 1 \) or \( \cos(2\phi_1 + \theta_1) = \pm 1 \). Under the above conditions (H.27) becomes

\[
|\Gamma_{1,e}^{a+}| = \frac{Z_0 - Z_1}{Z_0 + Z_1} \quad \text{for} \quad \cos(2\phi_1 + \theta_1) = \pm 1. \tag{H.28}
\]

For \( Z_0 > Z_1 \)

\[
|\Gamma_{1,e}^{a+}| = \frac{Z_0 - Z_1}{Z_0 + Z_1} \quad \text{for} \quad \cos(2\phi_1 + \theta_1) = 1. \tag{H.29}
\]

\[
|T_{0,e}^{a+}|^2 = 1 \quad \text{if} \quad |\Gamma_{1,e}^{a+}| = \frac{Z_0 - Z_1}{Z_0 + Z_1} = \Gamma_1^- \quad \text{and} \quad \cos(2\phi_1 + \theta_1) = 1. \tag{H.30}
\]

This is different than for a staggered tuned biplanar array in an infinite medium, where for a unity transmission coefficient, \( |\Gamma_{1,e}^{a+}|^2 = 0 \). For a staggered tuned biplanar array in a stratified medium, a unity transmission coefficient is obtained no matter what the value of the permittivity in the dielectric slab is, if (H.30) is fulfilled.
LIST OF REFERENCES


