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A hybrid computer-aided engineering system for process sequence design in axisymmetric sheet metal forming

Sitaraman, Suresh Kumar, Ph.D.
The Ohio State University, 1989
A HYBRID COMPUTER-AIDED ENGINEERING SYSTEM FOR PROCESS SEQUENCE DESIGN IN AXISYMMETRIC SHEET METAL FORMING

A Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

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To my parents
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Fields of Study

Publications


# TABLE OF CONTENTS

DEDICATION ii  
ACKNOWLEDGMENTS iii  
VITA iv  
LIST OF TABLES x  
LIST OF FIGURES xi  

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II LITERATURE REVIEW AND RESEARCH OBJECTIVES</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Summary</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Expert Systems in Manufacturing</td>
<td>4</td>
</tr>
<tr>
<td>2.3 Knowledge-Based Approach to Cold Forging and Sheet Metal Forming</td>
<td>6</td>
</tr>
<tr>
<td>2.3.1 FORMing EXpert (FORMEX)</td>
<td>6</td>
</tr>
<tr>
<td>2.3.2 Automatic Generation of Multitechnology Process Outlines (AGMPO) System</td>
<td>8</td>
</tr>
<tr>
<td>2.3.3 Review of Other Literature on Sheet Metal Process Sequence Design</td>
<td>10</td>
</tr>
<tr>
<td>2.4 Process Modeling in Sheet Metal Forming</td>
<td>10</td>
</tr>
<tr>
<td>2.5 Objectives and Scope of the Current Research</td>
<td>13</td>
</tr>
<tr>
<td>III TECHNICAL BACKGROUND ON SHEET FORMING PROCESSES</td>
<td>20</td>
</tr>
<tr>
<td>3.1 Summary</td>
<td>20</td>
</tr>
<tr>
<td>3.2 Classification of Sheet Metal Operations</td>
<td>20</td>
</tr>
<tr>
<td>3.2.1 Operations For Producing Blanks</td>
<td>20</td>
</tr>
<tr>
<td>3.2.2 Operations For Cutting Holes</td>
<td>21</td>
</tr>
<tr>
<td>3.2.3 Operations For Progressive Forming</td>
<td>21</td>
</tr>
<tr>
<td>3.2.4 Operations For Size Control</td>
<td>24</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Classification of Cutting Operations</td>
<td>22</td>
</tr>
<tr>
<td>3.2 Classification of Sheet Metal Forming Operations</td>
<td>23</td>
</tr>
<tr>
<td>4.1 Direct Element Library</td>
<td>49</td>
</tr>
<tr>
<td>4.2 Input Object Geometry Files for Test Cases</td>
<td>58</td>
</tr>
<tr>
<td>5.1 Comparison between Artificial Intelligence and Traditional Languages</td>
<td>40</td>
</tr>
<tr>
<td>6.2 Summary of Tool Dimensions, Material Properties, and Process Conditions for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel</td>
<td>113</td>
</tr>
<tr>
<td>6.3 Summary of Tool Dimensions, Material Properties, and Process Conditions for Ellipsoidal Punch Stretching of 321 Stainless Steel</td>
<td>121</td>
</tr>
<tr>
<td>7.1 Summary of Tool Dimensions, Material Properties, and Process Conditions for Hemispherical Punch Drawing of Copper</td>
<td>137</td>
</tr>
<tr>
<td>7.2 Summary of Tool Dimensions, Material Properties, and Process Conditions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel</td>
<td>143</td>
</tr>
<tr>
<td>7.3 Summary of Tool Dimensions, Material Properties, and Process Conditions for Flat-Bottom Cylindrical Punch Drawing of Brass</td>
<td>144</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Integrated Design and Manufacturing</td>
<td>14</td>
</tr>
<tr>
<td>2.2</td>
<td>CAD/CAM/CAE System</td>
<td>16</td>
</tr>
<tr>
<td>2.3</td>
<td>The Blank and Successive Drawing Operations for Producing Steel Shell</td>
<td>17</td>
</tr>
<tr>
<td>2.4</td>
<td>Second, Third, Fourth, and Eighth Stations in the Drawing of a Differential Gear Case</td>
<td>17</td>
</tr>
<tr>
<td>2.5</td>
<td>CAE System Architecture</td>
<td>19</td>
</tr>
<tr>
<td>3.1</td>
<td>Shearing Operations</td>
<td>24</td>
</tr>
<tr>
<td>3.2</td>
<td>Cutoff Operations</td>
<td>24</td>
</tr>
<tr>
<td>3.3</td>
<td>Parting Operations</td>
<td>25</td>
</tr>
<tr>
<td>3.4</td>
<td>Blanking Operations</td>
<td>25</td>
</tr>
<tr>
<td>3.5</td>
<td>Perforating Operations</td>
<td>27</td>
</tr>
<tr>
<td>3.6</td>
<td>Notching, Seminotching and Lancing Operations</td>
<td>27</td>
</tr>
<tr>
<td>3.7</td>
<td>Trimming Operations</td>
<td>28</td>
</tr>
<tr>
<td>3.8</td>
<td>Brake Bending Operations</td>
<td>30</td>
</tr>
<tr>
<td>3.9</td>
<td>Stretch Forming Operations</td>
<td>30</td>
</tr>
<tr>
<td>3.10</td>
<td>Deep Drawing</td>
<td>31</td>
</tr>
<tr>
<td>3.11</td>
<td>Rubber-Diaphragm Hydroforming</td>
<td>31</td>
</tr>
<tr>
<td>3.12</td>
<td>Dimpling</td>
<td>33</td>
</tr>
<tr>
<td>3.13</td>
<td>Embossing</td>
<td>33</td>
</tr>
<tr>
<td>3.14</td>
<td>Drop Hammer Forming</td>
<td>34</td>
</tr>
<tr>
<td>3.15</td>
<td>Shape Classification of Sheet Metal Parts</td>
<td>35</td>
</tr>
<tr>
<td>4.1</td>
<td>Rule-Based Architecture</td>
<td>44</td>
</tr>
<tr>
<td>4.2</td>
<td>Object Geometry Input - Direct Geometry</td>
<td>50</td>
</tr>
<tr>
<td>4.3</td>
<td>Object Geometry Input - Direct and Reverse Geometry</td>
<td>51</td>
</tr>
<tr>
<td>4.4</td>
<td>Object Geometry Input - Direct, Reverse, and Direct Geometry</td>
<td>52</td>
</tr>
</tbody>
</table>
6.12 Theoretical Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Various Friction (Punch Height = 25.0 mm)

6.13 Tool Geometry for Flat-Bottom Cylindrical Punch Stretching

6.14 Radial Strain Distributions for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Oil Lubrication (Punch Height ~ 22 mm)

6.15 Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Oil Lubrication (Punch Height ~ 22 mm)

6.16 Strain Distributions for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Various Friction Conditions (Punch Height ~ 22 mm)

6.17 Punch Force Variation for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Oil Lubrication

6.18 Ellipsoidal Punches

6.19 Mid Surface Radial Strain Distributions for 2:1 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 21.8 mm, 37.3 mm, and 47.8 mm)

6.20 Mid Surface Circumferential Strain Distributions for 2:1 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 21.8 mm, 37.3 mm, and 47.8 mm)

6.21 Mid Surface Radial Strain Distributions for 1:2 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 27.9 mm, 36.1 mm, and 50.3 mm)

6.22 Mid Surface Circumferential Strain Distributions for 1:2 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 27.9 mm, 36.1 mm, and 50.3 mm)

6.23 Deformed Workpiece Geometries for Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 21.8 mm and 37.3 mm)

7.1 Deformation Zones in Deep Drawing

7.2 Drawbead Modeling in Deep Drawing Analysis

7.3 Mid Surface Circumferential Strain Distributions for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication (Punch Heights = 17.2 mm, 21.2 mm, 25.4 mm, 29.1 mm, and 37.8 mm)
7.4 Mid Surface Thickness Strain Distributions for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication 
(Punch Heights = 17.2 mm, 21.2 mm, 25.4 mm, 29.1 mm, and 37.8 mm) 139

7.5 Punch Force Variation for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication 141

7.6 Deformed Workpiece Geometry for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication 
(Punch Heights = 17.2 mm, 21.2 mm, 25.4 mm, 29.1 mm, and 37.8 mm) 142

7.7 Radial Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication 
(Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) 145

7.8 Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication 
(Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) 146

7.9 Radial Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Various Friction Conditions (Punch Height = 40.1 mm) 148

7.10 Punch Force Variation for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication 149

7.11 Deformed Workpiece Geometry for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication 
(Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) 151

7.12 Radial Strain Distributions for Flat-Bottom Cylindrical Punch Drawing Brass under Oil Lubrication (Punch Height = 30.0 mm) 153

7.13 Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Brass under Oil Lubrication (Punch Height = 30.0 mm) 154

7.14 Punch Force Variation for Flat-Bottom Cylindrical Punch Drawing Brass under Oil Lubrication 155

7.15 Deformed Workpiece Geometry for Flat-Bottom Cylindrical Punch Drawing of Brass under Oil Lubrication (Punch Height = 30.0 mm) 156

7.16 Radial Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication 
(Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) (Drawbead Simulation) 158
7.17 Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication
(Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) (Drawbead Simulation) 159
C.1 Flat-Bottom Cylindrical Punch 188
CHAPTER I

INTRODUCTION

Die design in sheet forming, after many years of practice, today still remains more an art than a science. This is due to the complexities involved in the die designing procedure where several mutually interacting factors must be considered. Therefore, it is commonly believed by most in industry that die designing is essentially a trial and error process, and that only from experience and observation can one become a die design expert. So far, such a belief has fueled industries to invest substantial amount of time, energy, and money in training beginners, but not much is done in developing a scientific approach to die design. Therefore, the state of the art is such that much of the die design is done empirically by human experts with little or no computer aids. Thus, the conventional noncomputerized techniques are time consuming and costly. Also, when a die design expert retires or otherwise leaves the company, valuable experience and knowledge worth many man-years is lost forever. An integrated Computer-Aided Design and Computer-Aided Manufacturing (CAD/CAM) system for die design would result in quantifiable cost and time savings by improved standardization and could provide a permanent and confidential source of expertise. Furthermore, such a system could serve as a valuable consultant for experts and as a dependable training aid for beginners.

In some of the companies from industrialized countries initial attempts to computerize die designing methodology and integrate it with computer-aided manufacturing have
already proven to be largely successful. For example, Fujitsu Ltd., Tokyo using their Integrated Computer Aided Design/Sheet MetaL manufacturing system (ICAD/SML) has shortened the manufacturing process and set up time by about 75%, and consequently, a three-fold increase in their overall productivity has been estimated [Sfiligoj, 1983]. Hitachi Ltd., in cooperation with designers from ten industries that manufacture sheet products, has developed a CAD system for progressive dies which has resulted in an amazing 80% reduction in die design time [Murakami, 1980]. Pressed Steel Fisher [Kovacs, 1978], widely uses CAD/CAM techniques for single and double action draw dies, blanking and piercing dies, trimming and flanging dies, and also progressive dies, and they report time savings of 40% in design and manufacture. Toyota Motor Corporation [Ishigaki, 1984; Higashi, 1985; Takahashi, 1985 and 1988; Okamoto, 1988] reports that their "integrated" CAD/CAM/CAE system has already resulted in 50% reduction in man-hours in die design by employing powerful geometric functions to design die faces and to evaluate press forming feasibility, 30% reduction in man-hours in the manufacture of die faces by eliminating measuring work for model fabrication and the preparation of NC processing data, and 30% reduction in man-hours in die tryout and modifications by understanding better press forming severity and by upgrading press die accuracy.

It is admitted by many experts that improvements in the stamping industries in the U. S. A. cannot be attributed to any enhancement in die design skills, but rather to the progress made in material uniformity, to the discoveries in new tool materials, tool surface treatments, and to reduced tooling costs due to electro-discharge machining and numerically controlled contour milling machines. At present, about one-third or more of the material coming into a given stamping plant goes out as scrap, and the energy spent in running the forming presses is much larger than that goes into the useful plastic deformation of sheet
[Duncan, 1985]. Therefore, there is enormous room for improvements in sheet metal forming techniques.

This document is organized into eight chapters. Chapter 2 provides a review of the literature on expert systems in manufacturing with emphasis on expert systems for sheet metal forming operations and also a review of literature on process modeling of sheet metal forming. The scope and objectives of the current research is also provided in Chapter 2. Chapter 3 summarizes commonly used techniques in sheet metal operations, effects of material properties on process feasibility, and provides a wide classification of sheet metal parts. Chapter 4 provides a detailed description of the knowledge-based system that has been as part of the research project and presents example process sequences from industrial practice and compares them with the process sequences outputted by the knowledge-based system. Chapter 5 provides the mathematical background for sheet forming analysis. Chapter 6 presents the simulation of stretch forming with a detailed look at the mathematical procedure and compares theoretical predictions with experimental data. Chapter 7 presents the simulation of deep drawing with an exhaustive description of the mathematical procedure and compares theoretical predictions with experimental data. Chapter 8 provides concluding remarks, identifies the research contributions of this work, and presents an outline for future research work.
CHAPTER II

LITERATURE REVIEW AND RESEARCH OBJECTIVES

2.1 Summary

This chapter presents a review of literature from two broad areas: knowledge-based systems in manufacturing and mathematical modeling of sheet metal forming processes. The literature review on knowledge-based systems in manufacturing places emphasis on systems for designing process sequence in sheet metal forming. The literature review on mathematical modeling of sheet metal forming processes focuses primarily on finite-difference based technique to simulate sheet forming processes. The last section of this chapter outlines the scope and objectives of the current research and explains how the research fits into the overall research goals of the Engineering Research Center for Net Shape Manufacturing at The Ohio State University.

2.2 Expert Systems in Manufacturing

Research in artificial intelligence aims at, among other things, developing 'intelligent' computer systems that can mimic the reasoning of human experts in solving decision making problems. Such computer systems, called expert systems\(^1\) or knowledge-based systems, use a knowledge-base and an inference engine to solve problems that were normally thought to require human expertise for solving. Computer programs

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\(^1\)Terms knowledge-based system and expert system will be used interchangeably throughout this document.
implementing such systems, unlike other common algorithmic programs that merely assist human beings in tedious and repetitive calculations, supplement designers in decision making processes. Today's expert systems can perform better than or at least as well as highly skilled experts in medical diagnosis and therapy, mass-spectrogram interpretation, predicting crop disease, locating mineral and oil deposits, computer configuration, and other diagnostic and design applications. In manufacturing, where process planning is primarily a decision making process, researchers have developed several experimental expert systems. A brief survey of some of the popular expert systems in manufacturing is provided in the following discussion.

Expert systems for manufacturing generally adopt either 'Variant' or 'Generative' approach for generating process plans. The Computer-Aided Process Planning (CAPP) family of systems are based on the 'variant' approach which consists of:

1. Classifying the part into a family of objects employing certain part coding/grouping technique (Group Technology),
2. Retrieving a predefined plan associated with this family, and
3. Modifying the plan to meet specific requirements of the part.

Although this approach has been found successful in several industries, it has some inherent drawbacks. Among them, a) it is not always possible to categorize every part using the part coding system and b) solutions can only be a subset of old solutions, thus it does not facilitate developing new approaches. In the sheet metal area, the Boeing Process Planning System (BUCCS) and Brigham Young's decision tree system [Allen, 1980] are some of the CAPP systems that are based (not completely) on Group Technology.

The 'Generative' approach, on the other hand, has been pursued by many systems realizing the drawbacks associated with the variant approach. In the generative approach process planning is represented in logic, and the knowledge and the control are separated
so that knowledge can be easily updated without having to modify the control modules. For example, XPS-1, GECAPP (General Electric), and GENPLAN (Lockheed-Georgia) [Eshel, 1986] are based on the generative approach. Also in the ICAM project, some semi-generative process planning systems for sheet metal parts have been developed.

GARI [Descotte, 1984], an expert system for generating machining plans for mechanical parts, employs constraint-satisfaction strategy. The planner in GARI generates a loosely constrained (considering only a few of the given constraints) initial machining plan for the given object geometry. This initial plan is later iteratively constrained to produce a 'refined' final plan. When there is a conflict among constraints, the planner finds a compromise among such "antagonist preferences" based on the weightage attached to different constraints.

2.3 Knowledge-Based Approach to Cold Forging and Sheet Metal Forming

Following sections describe in detail two forming process sequence design advisors, namely, FORMEX (for cold forging) and AGMPO (for deep drawing and machining). These two systems were chosen for review because of their relevance to the current research. FORMEX was developed as part of a Ph. D. dissertation at The Ohio State University, and AGMPO is one of the first expert systems developed for designing the process sequence for axisymmetric\(^1\) deep drawing and machining. A brief discussion on other research projects in progress is also presented.

2.3.1 FORMing EXpert (FORMEX)

FORMEX [Sevenler, 1986a and 1986b] is a knowledge-based system implemented in PROLOG for establishing the forming sequence for cold forming axisymmetric parts. The

\(^1\) Throughout this document axisymmetric will mean rotationally symmetric.
FORMEX approach does not depend on a detailed classification of object geometries to determine the forming sequence. Instead a broad object geometry classification is used in the initial stages, and a knowledge about forming sequence design represented in specific production rules is used in later stages to differentiate very similar object geometries from each other and to develop a suitable process planning sequence.

FORMEX consists of seven top level, sequential subtasks as given below:

Subtask 1: Check if the given part can be cold formed using the given material and machine type.

Subtask 2: Broadly classify the part geometry into solid parts with decreasing diameter on one end, solid parts with decreasing diameters on both ends, and hollow parts with cylindrical cavity and different diameters on the outside.

Subtask 3: Convert the input geometry, given in (X,Y) coordinate pairs into a 2-D and 3-D internal object geometry representation.

Subtask 4: Select the billet diameter based on the largest of the diameters of the zones having a length to diameter ratio greater than or equal to 1, and also keeping the maximum number of extrusions required to extrude the smallest diameter of the part from this chosen diameter less than or equal to 3. Compute the billet cutoff length by volume constancy.

Subtask 5: Establish forward extrusion sequence without violating process and geometry variables based on material mechanical properties.

Subtask 6: Develop the upsetting sequence for producing diameters that are larger than the initial billet diameter. The upsetting can be done in one blow without buckling of the workpiece, if the length to diameter ratio of
the undeformed zone is less than 2.5. If not, an intermediate 'taper' upsetting should be done before achieving required final diameter.

Subtask 7: After establishing extrusion and upsetting sequences separately and independently, combine them suitably on the same station so as to reduce the total number of stations.

Each of these subtasks themselves consist of similar lower level subtasks.

The broad classification approach used in FORMEX facilitates easy modifications to include hollow parts which the system currently cannot handle. The system is also being interfaced with a commercial CAD system for reading the input geometry drawn on the CAD system and also to display intermediate object geometries on the same system.

2.3.2 Automatic Generation of Multitechnology Process Outlines (AGMPO) System

This is an experimental system for developing a multitechnology process outline for the manufacture of axisymmetric, monotonic cups of nominal uniform wall thickness parts that are deep drawn and machined [Eshel, 1985 and 1986]. The input to the system is a CAD representation of the cross-section of the final object geometry and the output from the system is the process outline to manufacture the part. Main components of the system are:

(1) the Automatic Circumscription by a Deep-drawable Preform (ACDP) subsystem that produces the formable preform from which the final part can be machined. The automatic design of the formable preform is formulated as a computational geometry problem where the circumscription is similar to "demachining" and
the Automatic Generation of Forming Process Outline (AGFPO) subsystem that generates the process outline to deep-draw the preform, given by ACDP, from a circular blank.

AGFPO is implemented in PROLOG, and runs on a UNIX operating system. It employs Generate and Test and Rectify (G&TR) strategy to establish the forming sequence. Using the Design or Generate category of rules, the initial sequence of deep drawing operations is planned. Then the Test category of rules check the feasibility of this process outline employing certain test variables such as the reduction ratio, height to diameter ratio of the finished cup, die profile and punch fillet radius to thickness ratio, etc. If one or more of the operations of the process outline cannot be completed without any defect/failure, that is, if one of the test variables is violated, then the Rectification category of rules rectify the process outline modifying the factors affecting the violated test variables. A Computational category of rules is also introduced for efficiency purposes. According to the designers, "generate and test is the natural way a process planning expert draws from his experience." G&TR works on the assumption that the first-guess solution of the expert may not be correct, nevertheless it is essentially good; exact solutions can then be derived from the first solution with minor modifications.

The system, in the present form, can handle only monotonic geometries. In other words, it cannot handle geometries requiring reverse operations. The system relies only on experience-based die-design guidelines for its process sequence design. The worthiness of experience-based guidelines is as good as the expertise of the die designer(s) from whom they are obtained, and it is almost impossible to obtain experience-based guidelines encompassing all material, process, and geometry variables.
2.3.3 Review of Other Literature on Sheet Metal Process Sequence Design

Tisza's [Tisza, 1986a, 1986b, 1986c, and 1987] work describes a CAD/CAM system, in progress, for developing deep-drawing process sequence and designing tools for the manufacture of sheet metal components having axisymmetric and rectangular cross-sections. The system would be modular. It would consist primarily of a Geometric Description Module for reading in the object geometry, a Blank Module for determining the optimum shape, size, and nesting of blanks, a Technological Design Module for designing process sequence based on empirical rules and technological parameters, a Tool Design Module for designing the tools and selecting a tool of standard size, and an NC/CNC Post Processor Module for preparing control programs for NC/CNC manufacturing of tool elements. Karima and Richardson [Karima, 1987] have proposed a knowledge-based framework to systems in metal forming. The system design is based on manipulating the knowledge to determine what action to take to achieve the desired goals, keeping in perspective the various design constraints.

2.4 Process Modeling in Sheet Metal Forming

Techniques which are available for modeling sheet metal processes include Finite-Element Method, Finite-Difference Method, Shell-Decomposition Method, Geometric Modeling, Method of Characteristics, Tractrix-Profile Approach, etc. [Duncan, 1981 and 1985]. The complex geometry of the sheet metal stampings combined with the difficulty associated with representing tool-sheet interface friction conditions and plastic properties of materials makes almost all techniques gross simplification of the actual process. However, the improved cost, time, and energy savings in die design and less chances of failures during die tryouts due to the introduction of these techniques have justified the use of process modeling in sheet metal industries. The following sections present a brief review
of analytical tools available for modeling sheet metal forming processes. The emphasis is primarily on the finite-difference based techniques with which the present research is concerned.

The history of process modeling of sheet metal processes is more than fifty years old, and can be attributed to the pioneering work done by Sachs, Swift, and Hill [for example, Sachs, 1934; Swift, 1939; Hill, 1950] in the analysis of stretch forming and deep drawing of axisymmetric cups employing hemispherical shaped punches. Their analysis focused primarily on non-hardening materials without much consideration to the effects of anisotropy, bending, friction over the die profile, and blankholding force.

Process modeling of sheet metal forming has gained improved importance over the past three decades in pace with the advances made in computational facilities. Chung and Swift [Chung, 1951] have done extensive semiempirical analysis of deep drawing of cylindrical cups taking into consideration the effects of bending, thickness changes, strain-rate hardening, blankholding force, and die profile friction but not of anisotropy of the material. Their analysis assumed that the blankholder force was applied only at the rim of the flange. Furthermore, no analysis was done for the cup wall. Woo [Woo, 1964a and 1968] employed finite-difference method to study deep drawing of cylindrical cups taking into consideration the stretch forming of the material over the punch head. Woo's analysis, based on incremental theory, was academically fundamental for its time and was free from empirical justifications. The predicted stresses and strains compared successfully with experimental results. However, anisotropy effects were not included in the analysis. The work by Vemuri et al. [Vemuri, 1986], based on Woo's approach, predicted stresses and strains in deep drawing of a cylindrical cup taking into consideration the effect of normal anisotropy also. Reissner et al. [Reissner, 1987] have analyzed the drawing and redrawing processes during the manufacture of two-part preserve cans employing finite-difference
method similar to Woo's approach. Their work was based on a rigid-plastic material model with normal anisotropy and isotropic workhardening.

Significant work has been done in the analysis of stretch forming. Woo [Woo, 1965] has analyzed stretch forming of an isotropic von Mises material over a hemispherical punch under plane stress conditions using finite-difference technique. Chakrabarty's [Chakrabarty, 1970] analysis on an isotropic, rigid plastic material predicts punch load and hoop strains. The punch was assumed to be well lubricated, and therefore, frictional effects were neglected. Kaftanoglu and Alexander [Kaftanoglu, 1970] have developed a comprehensive analysis for axisymmetric punch stretching taking into account the effects of anisotropy, friction, non-linear strain hardening, and pre-strain and have reported "good" agreement between theoretical predictions and experimental results with ferrous and non-ferrous materials employing hemispherical as well as ellipsoidal punches. Under varying frictional conditions they observed through their analysis that the location of the plastic instability point moves away from the punch with increasing coefficient of friction, as found earlier by experiments. They have reported that the instability criterion based on 'Strain Propagation' was the best in successfully predicting the onset instability rather than 'Maximum Load' or 'Maximum Pressure' criteria. Kim and Kim's [Kim, 1984] semi-empirical analysis, based on Woo's method, has used Hill's new yield function to predict stresses and strains in an axisymmetric stretch forming analysis over a hemispherical punch. Their analysis uses friction coefficient as a variable to match the theoretical and experimental thickness strains at the pole. The experimental thickness strains measured elsewhere in the cup were found to be less than their analysis-predicted values which they attribute to the possible draw-in during experiments and to errors in measurements. Gerdeen's analysis [Gerdeen, 1984 and 1986] of forming axisymmetric sheet metal parts is based on a shell theory approach. Gerdeen divides the final formed geometry into shells
and maps points from the formed geometry onto the starting blank geometry by assuming a linear thickness strain distribution along the radius. From the initial and final geometry mapping, it is then possible to calculate the total strains, and thus the feasibility of forming can be determined from the forming limit diagram of the material. From the calculated strain distributions, stresses and forming forces can also be determined. This analysis has produced results of reasonable accuracy in predicting forming strains and loads for Fukui cups and wheel disks.

The research by Duncan et al. [Duncan, 1981 and 1985] and the work at Toyota Motor Corporation [Ishigaki, 1984; Higashi, 1985; Takahashi, 1985 and 1988; Okamoto, 1988] provide a detailed description of geometry-based techniques. An extensive survey of literature on process modeling of sheet metal forming with emphasis on finite-element method has been done by Lee et al. [Lee, 1988].

2.5 Objectives and Scope of the Current Research

The research strategy of the Engineering Research Center for Net Shape Manufacturing (ERC/NSM) is guided by the belief that part design and manufacturing should be truly integrated [Altan, 1988]. As outlined in Figure 2.1, given the functional parameters such as loading patterns, strength, environment, assembly constraints, etc., the product designer designs the product (shape, size, surface finish and tolerances, and material). The decisions made at this stage also determine the overall manufacturing, maintenance, and support costs associated with the specific product. The designer then explores other alternative designs and manufacturing costs associated with them before selecting the optimal design that not only will satisfy functional requirements but also will be cost and time efficient from manufacturing, maintenance, and support standpoints (concurrent engineering or simultaneous engineering).
Figure 2.1 Integrated Design and Manufacturing
For a given product design alternative, the methodology for manufacturing can be automated as shown in Figure 2.2 [Altan, 1985]. The Sheet Metal Thrust Area¹ at the ERC/NSM is currently investigating several aspects of sheet metal forming operations and working on developing a stand-alone system as outlined in Figure 2.2. The present research aims to implement stages 4 through 7 of the system, that is, to develop a knowledge-based system for process planning for manufacturing axisymmetric sheet metal parts and interface it with analytical modules.

In other words, the current research aims at developing a hybrid Computer-Aided Engineering (CAE) system for designing process sequence for the manufacture of axisymmetric sheet metal components. The input to the CAE system is the final sheet metal object geometry that needs to be manufactured, and the output from the system is the process sequence with intermediate object geometries. Some typical process sequence designs are shown in Figures 2.3 and 2.4, which the system will aim to come up with. It was decided to focus primarily on axisymmetric parts because of their wide scope of applicability and reasonable amount of complexity, and this would be the first step in developing a general-purpose system to handle a broad range of three-dimensional and other complex geometries.

Two main components of the hybrid CAE system are a knowledge-based expert system module (symbolic module) and a process-modeling analysis module (numeric module). The expert system module will first generate an initial-guess process sequence based on experience-based die design guidelines, and this process sequence will then be tested for defects and failures by mathematically modeling the sheet metal forming process

¹ Brief outline of research goals of all thrust areas of the ERC/NSM can be found in the Center’s annual report [Altan, 1988]. Detailed descriptions of individual projects can be obtained from respective thrust area coordinators.
Figure 2.2 CAD/CAM/CAE System
Figure 2.3  The Blank and Successive Drawing Operations for Producing Steel Shell [Jones, 1941]

Figure 2.4  Second, Third, Fourth, and Eighth Stations in the Drawing of a Differential Gear Case [Jones, 1941]
using the analysis module. The analysis module formulates mechanics of metal forming and predicts stresses and strains in the deformed geometry and the punch load versus the displacement. Figure 2.5 shows the architecture of the hybrid CAE system.

The present research initially proposes to keep the expert system module and the analysis module separate and stand-alone. The user would first run the expert system module to design an initial process sequence which is designed using experience-based die design guidelines and empirical formulas. The user would then run the analysis module having the geometry outputted by the expert system as the input to the analysis module and determine the stress and strain distribution. If the predicted strain distribution is within agreeable limits (perhaps as suggested by certain failure-prediction criterion for the material), the process sequence needs no further modifications. If not, the user will add 'dynamic' directive rule(s) to the knowledge base of the expert system to be able to obtain a different process sequence and repeat the procedure until a satisfactory sequence is obtained. The directive rule(s) can be deleted from the knowledge base before starting on a new or a different problem.

Later, in a follow-up project [Kinzel, 1989] as indicated by the dashed line in Figure 2.5, the expert system and analysis modules will be integrated to produce a computer-aided engineering system in which the expert system will communicate with the analysis module to test and rectify the generated process sequence.
Figure 2.5 CAE System Architecture
CHAPTER III

TECHNICAL BACKGROUND ON SHEET FORMING PROCESSES

3.1 Summary

This chapter presents an overview and examples of several sheet metal operations with examples. It discusses briefly the shape classification of different sheet metal object geometries. The last section of this chapter lists some of the important mechanical properties of materials and gives a brief definition of each one of them. This chapter is intended for readers having little or no familiarity with sheet metal operations; other readers may directly proceed to Chapter 4 without any loss of continuity.

3.2 Classification of Sheet Metal Operations

Sheet metal operations can be broadly classified into: 1) Cutting Operations and 2) Forming Operations as shown in Tables 3.1 and 3.2 [Eary, 1974; Altan, 1983]. The following sections describe briefly some of the operations in each category.

3.2.1 Operations For Producing Blanks
- Shearing

The cutting action is along a straight line. Shearing is used to cut wide coils into large blanks and into narrow strips and to square the edges of large sheets to produce accurate blanks (Figure 3.1).
- Cutoff

The cutting action may be along straight, angular, jogged or a curved line. Blanks must nest perfectly together before a cutoff operation is possible (Figure 3.2).

- Parting

When blanks do not nest perfectly they are cut from sheet-metal strip by a parting operation. Two lines of cutting are generally used (Figure 3.3).

- Blanking

The cutting operation is along a complete or enclosed contour. This operation results in excessive scrap material (Figure 3.4).

### 3.2.2 Operations For Cutting Holes

- Punching

A punching operation involves cutting holes in the blank and/or in the parts surrounding the blank. (Piercing is producing holes by a tearing action. The piercing punch is normally bullet shaped. No slug is produced during piercing.)

- Slotting

Slotting is commonly used for cutting elongated and rectangular holes.

- Perforating

When many holes are punched (in a specific pattern), then the operation is known as perforating (Figure 3.5).

### 3.2.3 Operations for Progressive Forming

- Notching and Seminotching

Notching is removing a piece of scrap metal from the edge of the strip (usually in a progressive die). Seminotching is making a notchlike cutouts in the central
Table 3.1 Classification of Cutting Operations [Eary, 1974]

<table>
<thead>
<tr>
<th>Operations for Producing Blanks</th>
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<tbody>
<tr>
<td>Shearing</td>
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<td>Cutoff</td>
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<tr>
<td>Parting</td>
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<td>Blanking</td>
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<tr>
<th>Operations for Cutting Holes</th>
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<td>Punching</td>
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<tr>
<td>Slotting</td>
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<tr>
<td>Perforating</td>
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<tr>
<th>Operations for Progressive Working</th>
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<tbody>
<tr>
<td>Notching</td>
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<td>Seminotching</td>
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<td>Lancing</td>
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<td>Parting</td>
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<td>Cutoff</td>
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<th>Operations for Size Control</th>
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<tr>
<td>Trimming</td>
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<td>Slitting</td>
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<td>Shaving</td>
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Table 3.2 Classification of Sheet Metal Forming Operations [Altan, 1983]

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<th>Bending and Straight Flanging</th>
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<td>Brake Bending</td>
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<td>Roll Bending</td>
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<th>Surface Contouring of Sheet</th>
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<tr>
<td>Contour stretch forming</td>
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<td>(Stretch forming)</td>
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<td>Androforming</td>
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<td>Age forming</td>
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<td>Creep forming</td>
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<td>Die-quench forming</td>
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<td>Bulging</td>
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<td>Vacuum forming</td>
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<th>Linear Contouring</th>
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<td>Linear stretch forming</td>
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<td>(Stretch forming)</td>
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<td>Linear roll forming</td>
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<td>(Roll forming)</td>
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<tr>
<th>Deep recessing and flanging</th>
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<tr>
<td>Spinning (and roller flanging)</td>
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<tr>
<td>Deep drawing</td>
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<tr>
<td>Rubber pad forming</td>
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<td>Marform process</td>
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<td>Rubber diaphragm hydroforming</td>
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<table>
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<th>Shallow recessing</th>
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<tr>
<td>Dimpling</td>
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<tr>
<td>Drop hammer forging</td>
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<tr>
<td>Electromagnetic forming</td>
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<tr>
<td>Explosive forming</td>
</tr>
<tr>
<td>Joggling</td>
</tr>
</tbody>
</table>
Figure 3.1 Shearing Operations [Eary, 1974]

Figure 3.2 Cutoff Operations [Eary, 1974]
Figure 3.3 Parting Operations [Eary, 1974]

Figure 3.4 Blanking Operations [Eary, 1974]
portion of the progressive strip to permit part attachment along the outer edges of the skeleton (Figure 3.6).

- Lancing

When a cut is made partway through the metal, and no scrap is freed from the strip the operation is called lancing. (Frequently used for bending to form tabs.)

3.2.4 Operations for Size Control

- Trimming

Cutting off the excess or damaged metal after a drawing operation is called trimming (Figure 3.7).

- Shaving

This involves cutting off metal in a chip fashion similar to a broaching operation to obtain accurate dimensions and also to remove the rough fractured edge of the sheet metal.

3.2.5 Bending and Straight Flanging

- Brake Bending

It is widely used for forming flat sheets into linear sections, such as angles, channels and hats. In air bending, the workpiece is supported only at the outer edges so that the length of the ram stroke determines the angle of bend of the part. In die bending, the sheet is forced into a female die cavity of the required part angle (Figure 3.8).
Figure 3.5  Perforating Operations [Eary, 1974]

Figure 3.6  Notching, Seminotching and Lancing Operations [Eary, 1974]
Figure 3.7 Trimming Operations [Eary, 1974]
3.2.6 Surface Contouring of Sheet

- Stretch Forming

Stretch forming is a recessing process in which the edge of the workpiece is clamped rigidly. The surface area of the workpiece is increased through a reduction in sheet thickness. The load is applied through a rigid punch (Figure 3.9). This process is used in particular for forming sheet metal components with a surface area of more than 540 sq. ft. which cannot be formed by conventional forming processes because of the necessary machine size.

3.2.7 Deep Recessing and Flanging

- Deep Drawing

In deep drawing, a sheet blank (hot or cold), usually subjected to a peripheral hold-down pressure, is forced by a punch into and through a die to form a deep recessed part having a wall thickness substantially the same as that of the blank. This process is used to produce cylindrical or prismatic cups with or without a flange on the open end. Cups or tubes can be sunk or redrawn to increase their length and to reduce their lateral dimensions (Figure 3.10).

- Rubber-Diaphragm Hydroforming

In this process the blank is held between a diaphragm, which closes the ram pressure chamber, and a blank holder. A male punch works against the diaphragm, and the metal is shaped by balancing the pressure of the ram chamber against the pressure of the press base chamber on which the punch is mounted (Figure 3.11).
Figure 3.8   Brake Bending Operations: (A) air bending, (B) air rounding, (C) die bending, (D) die rounding [Altan, 1983]

Figure 3.9   Stretch Forming Operations [Lange, 1985]
Figure 3.10  Deep Drawing: (A) first draw, (B) redraw, (C) reverse draw [Altan, 1983]

Figure 3.11  Rubber-Diaphragm Hydroforming [Altan, 1983]
3.2.8 Shallow Recessing

- Dimpling

Dimpling is a process for producing small conical flanges around holes in sheet metal parts that are to be assembled with flush or flat-headed rivets. Dimpling is most commonly applied to sheets that are too thin for countersinking (Figure 3.12).

- Embossing

Embossing is a recessing process in which a rigid punch pushes the workpiece into a rigid die as shown in Figure 3.13. The depth of the recess is very small compared to workpiece dimensions. Embossing is most frequently used for producing beads in a part which would increase the stiffness of the part.

- Drop Hammer Forming

This process produces shapes by the progressive deformation of sheet metal in matched dies under repetitive blows of a gravity drop or power drop hammer (Figure 3.14).

3.3 Shape Classification of Parts

In automobile and aircraft industries, a large number of asymmetrical, difficult-to-produce sheet-metal components are required. A shape classification of these parts according to Sachs [Lange, 1985] is given in Figure 3.15. Production of these components require a combination of bending, deep drawing and stretch forming operations.

3.4 Mechanical Properties of Materials

Commercially produced sheet metal strips used in stamping plants must meet certain specific requirements to ensure that the finished part possesses desired mechanical
Figure 3.12 Dimpling [Altan, 1983]

Figure 3.13 Embossing [Altan, 1983]
Figure 3.14 Drop Hammer Forming [Altan, 1983]
Figure 3.15 Shape Classification of Sheet Metal Parts [Lange, 1985]
properties, dimensional tolerances, chemical composition, and is also free from flaws. Parts which must confirm to definite strength requirements in service demand a high level of uniformity in chemical composition, mechanical properties, and especially in dimensional accuracies. The following sections describe some of the common material properties, and how they influence the sheet metal forming processes [Taylor, 1983].

3.4.1 Modulus of Elasticity

The initial slope of the stress-strain curve in the elastic region before any plastic deformation has occurred is the modulus of elasticity of the material. This property affects the springback and shape distortion at low strains.

3.4.2 Yield Strength

The stress at which the material starts yielding, that is, start to deform plastically, is known as the yield strength of the material. This determines the load necessary to initiate deformation in a forming operation.

3.4.3 Tensile Strength

The maximum stress observed in the tensile test is known as the tensile strength or the ultimate strength. This determines the maximum load that can be applied in a forming operation.
3.4.4 Uniform Elongation

The engineering strain at the point of ultimate strength is known as the uniform elongation. Prior to this point, the material elongates uniformly, and after this point, necking begins and ultimately leads to fracture.

3.4.5 Percentage Elongation

The elongation per unit gage length of the tensile specimen at the point of fracture is known as the percentage elongation. This has been used extensively as an approximate indication of sheet metal formability. However, no single property can be taken as a reliable indicator of formability under all conditions.

3.4.6 Percentage Reduction in Area

This is the percentage reduction in the cross-sectional test area of a tensile specimen at fracture. It is not uncommon for tensile specimens to break very close to or outside the gage length. In such cases, the percentage reduction in area is a more reliable indicator of material formability than percentage elongation.

3.4.7 Strain-Hardening Exponent

This, also known as work-hardening exponent (n), is determined by the slope of true stress and true strain (effective stress and effective strain in the case of multiaxial loading) curve in a log-log plot. A commonly used power-law equation is given as

\[ \sigma = K(\varepsilon + \varepsilon_0)^n \]  

(3-1)
where $\bar{\sigma}$ is the equivalent stress, $\bar{\varepsilon}$ is the equivalent strain, $\varepsilon_0$ is the initial strain or pre-strain, and $K$ and $n$ are the strain-hardening coefficient and exponent, respectively. In materials with a high strain-hardening exponent, the flow stress increases rapidly with strain; also such materials possess a good formability in stretching operations.

3.4.8 Strain-rate Sensitivity

The strain-rate sensitivity ($m$) is a measure of flow stress as a function of strain rate:

$$\bar{\sigma} = K \left( \bar{\varepsilon} + \varepsilon_0 \right)^n \bar{\varepsilon}^m$$

(3-2)

A positive strain-rate sensitivity indicates that the flow stress increases as the rate of deformation increases. This has two consequences. Higher stresses are required to form parts at higher rates. Also, at regions where the strain rates are higher, the material resists further deformation due to the increase in flow stress level, and thus results in a uniform strain distribution. Higher $n$ and $m$ values lead to good formability in stretching operations, but have little effect on drawability. In a drawing operation, metal in the flange must be drawn in without causing fracture in the wall. In this instance, high $n$ and $m$ values strengthen the wall, which is beneficial, but they also strengthen the flange and make it harder to draw in.

3.4.9 Anisotropy Factor

The normal or thickness anisotropy factor ($R$) is defined as the ratio of the true width strain to the true thickness strain in the uniform elongation of a tensile test and can be expressed as
where R is the anisotropy factor and $\varepsilon_2$ and $\varepsilon_3$ are true width and thickness strains, respectively.

Anisotropy factor is a measure of the ability of a material to resist thinning. In drawing, the material in the flange is stretched in one direction (radially) and compressed in the perpendicular direction (circumferentially). A high anisotropy value indicates a material with good drawing properties.

$$R = \frac{\varepsilon_2}{\varepsilon_3}$$

(3-3)
CHAPTER IV

KNOWLEDGE-BASED APPROACH TO PROCESS SEQUENCE DESIGN

4.1 Summary

This chapter presents a detailed description of the knowledge-based system. The chapter is organized into six sections. Section 4.2 discusses the various techniques employed as part of this research to acquire knowledge for constructing the knowledge base. Section 4.3 provides a brief description of a generic rule-based system or a production system and explains why the knowledge-based system was constructed as a rule-based system. Section 4.4 presents a comparison between conventional programming languages and artificial intelligence languages and concludes that the numeric module of the hybrid system be implemented in FORTRAN and the symbolic module (knowledge-based module) of the hybrid system be implemented in MPROLOG (Modular PROgramming in LOGic). Section 4.5 provides a detailed description of the knowledge-based system with sections on the object geometry input scheme, the knowledge base, and the inference engine. Section 4.6 presents example process sequences from industrial practice and compares them with the process sequences outputted by the knowledge-based system.

4.2 Knowledge Acquisition from Domain Experts

The power of a 'knowledge-based' system, as the name implies, often lies on the strength of its knowledge. The acquisition of knowledge and the representation of
knowledge are two of the most important tasks in the construction of expert systems. Knowledge acquisition is essentially collecting knowledge, experience, rules of thumb or empirical rules, and other important relevant information from domain experts or other reliable sources. Domain experts refer to people who have considerable expertise in the chosen problem domain. For the present expert system for process sequence in sheet metal forming, well-experienced and reputed die designers can be thought of as domain experts. Available literature on building expert systems [Goodall, 1985; Waterman, 1986] has outlined various techniques that can be adopted for acquiring knowledge. The following sections briefly describe the various sources that the research team employed to obtain knowledge for constructing the knowledge-based system.

- **Knowledge through literature reviews:** Review of literature on die design for sheet metal processes provided detailed, academically fundamental information on die design techniques. Although, the information obtained from the literature is not always the same as what is currently being practiced in industry, literature survey is an easy and inexpensive mode of knowledge acquisition. Information found in the open literature was reviewed and the available knowledge was formalized to be able to be added to the knowledge base.

- **Knowledge from industrial brochures:** This information is a compromise between the academically fundamental knowledge obtained through literature reviews and the practical, experience-based knowledge obtainable from industrial experts.

- **Knowledge through industrial visits:** This knowledge, not always quantifiable, was useful in getting a better 'feel' for the problem domain and in understanding
the common terminology used on the shop floor, and thus facilitated constructing a practical expert system that would cater to industrial needs and also be equipped with a friendly user interface. An expert system with such a front-end is expected to have a relatively easy learning curve and therefore, is likely to be accepted more readily by the industrial users.

- **Knowledge from industrial experts**: The process of knowledge acquisition from industrial experts would involve presenting a few typical problems to the expert(s) and letting the expert(s) talk through the solution. Some typical axisymmetric sheet metal parts would be chosen for this purpose. During the verbal analysis, the expert(s) would be questioned/interrupted to explain why a particular decision was reached. This would enable him/her to identify what parameters influenced what decisions. This analysis would be recorded, transcribed, and then checked with the expert(s) before formalizing the rules. Besides this, the expert(s) may be observed while working and may be asked to comment on solutions done by others to obtain a more comprehensive knowledge.

The current knowledge base is constructed based on the knowledge obtained from the first three aforementioned sources. The literature that was reviewed for knowledge acquisition includes [Jones, 1941], [Sachs, 1951], [Schuler, 1966], [Schubert, 1967], [Eary, 1974], [Newby, 1976], [Keyes, 1980 and 1982], and [Eshel, 1986]. Knowledge from industrial experts is being acquired through interviews and practical training [Ahmetoglu, 1989] and will be used to expand the current knowledge base in the follow-up project.
4.3 Description of Rule-Based Systems/ Production Systems

It was found through literature review and talks during industrial visits that domain experts can best express their problem solving strategies in terms of condition-action rules, or if... then... rules. Rule-based systems probably constitute the best means available today for codifying the problem solving know-how of human expert. Rule-based systems are also known as production systems.

A sample production system consists of three parts (Figure 4.1): (a) a rule base or permanent knowledge base consisting of production rules (condition-action pairs) (b) an interpreter or inference engine that controls the system's activity, and (c) a buffer-like data structure known as context or short-term memory buffer or dynamic data buffer [Barr, 1981].

The syntax of a production rule is

\[
\begin{align*}
\text{If} & \quad \text{<condition>} \\
\text{then} & \quad \text{<action>}
\end{align*}
\]

The condition portion of a production rule, sometimes called LHS (Left-Hand Side), contains one or more conditions, while the action portion, sometimes called RHS (Right-Hand Side), contains one or more actions. During the execution of the production system, a production rule whose condition portion is satisfied can 'fire', that is, the action portion will be executed.

The interpreter has the task of deciding which production rule to fire next and essentially controls the execution of the production system. The condition portion of a production rule must be present in the context or short-term memory buffer before that production rule can be fired. Once the production rule is fired, the actions of the
Figure 4.1 Rule-Based System Architecture
production rule may change the contents of the short-term memory buffer, and this would result in the conditions of other rules being satisfied.

A detailed discussion on production systems and their merits and demerits is beyond the scope of the current study. Interested readers can refer to exhaustive discussions given by Winston [Winston, 1977], Davis and King [Davis, 1977], Nilsson [Nilsson, 1980], Barr and Feigenbaum [Barr, 1981], and Buchanan and Shortliffe [Buchanan, 1984].

4.4 Selection of Knowledge-Based System Building Tools/Languages

Having decided that the knowledge-based system be implemented as a rule-based architecture, the selection of an appropriate language/tool for building the system is the next issue.

Problem-oriented or conventional programming languages such as FORTRAN, Pascal, C, etc. as well as symbol-manipulation or artificial intelligence programming languages such as Lisp, PROLOG, etc. can be used for building expert systems. The high level nature of artificial intelligence languages and their inherent suitability for knowledge-based programming tasks decrease the overall development time, initial prototype development time, and debugging and testing time. Ease of knowledge base expansion, ease of program restructuring, and the ease with which the programs can be understood even by people who do not have much exposure to programming, make artificial intelligence programming languages attractive for designing expert systems. On the other hand, the relatively low-level languages such as FORTRAN, Pascal, and C have convenient features for performing numeric computations applicable to scientific, mathematical, and statistical problem areas. They, however, are not geared toward manipulating lists of descriptors; rather, they are geared to manipulating numbers. Artificial intelligence languages provide a means of actually describing nonnumeric aspects
of the design process. Furthermore, they provide facilities for separating knowledge base from the control flow (to some extent). This makes expansion or modification of the knowledge base a relatively easy task.

Based on the above factors, it was decided that the conventional expert system components, the knowledge base and the inference engine, be constructed in a symbol-manipulation language such as PROLOG and the analysis modules be implemented in a number-manipulation language such as FORTRAN (See Figure 2.5). This would result in a faster computation without sacrificing the convenient facilities offered by the artificial intelligence language. This would also enable replacement of the current analysis module with more versatile FEM modules in the future, if necessary.

Above paragraphs did not discuss the possibility of employing commercially available expert system tools for constructing the knowledge-based system. The selection of PROLOG to implement the knowledge-based system is based also on logistical grounds. Availability of in-house expertise in PROLOG, proven success of PROLOG to implement knowledge-based systems for designing process sequence [Eshel, 1985 and 1986; Sevenler, 1986a], and availability of a well-tested MPROLOG (Modular PROLOG; a dialect of PROLOG) compiler were some of the factors that contributed to the selection of PROLOG for implementing the knowledge-based system.

4.5 Description of Knowledge-Based System

4.5.1 Object Modeling and Interfacing

The final object geometry for which a process sequence needs to be designed is the input to the expert system. The following sections describe briefly the various methods available for inputting the geometric information to the knowledge-based system.
1) Initial Graphics Exchange Specification (IGES)-Based Input: The object geometry can be generated on any commercially available CAD system that can translate the object geometry data into an IGES format which can be read in by the input module of the expert system. The intermediate object geometries outputted by the expert system will be in IGES format which can be 'retranslated' to be able to be displayed on the same CAD system. Nearly all commercially available CAD systems provide facilities for transcribing to and from IGES data files. This procedure is useful particularly for complex object geometries where other simpler techniques cannot be employed.

2) Primitives-Based Input: In this technique complicated cross-sections can be decomposed into simpler surface or volume elements known as primitives. The user can choose items from a menu containing such primitives, define the attributes for the primitives, enlarge, rotate, and/or translate to produce the desired final object geometry. This technique is quite similar to Solid/Surface Modeling.

3) Two-Dimensional Graphic Primitives-Based Input: Two-dimensional elements such as lines, arcs, etc. can also be used to define the input object geometry [Tisza, 1987]. This is particularly useful in representing thin axisymmetric sheet metal geometries where meridional section profiles are sufficient to define the geometry.

4) Alphanumeric Input: This involves the description of the geometry in alphanumeric terms. This scheme provides much more a priori information to the knowledge-based system than the two-dimensional graphic primitives-based input scheme. It identifies the orientation of line segments (horizontal, vertical, tapered, or taper-reduced), the nature of arc elements (convex, concave, convex-reduced, or concave-reduced), and the features they represent (flange, bottom, or wall). Such a detailed information results in quantifiable savings in computational time, since the knowledge-based system does not have to do the task of identifying the geometric primitives and recognizing the features the geometric
primitives represent. This scheme has been successfully used by Eshel and Barash [Eshel, 1985].

The present research employs the alphanumeric scheme for inputting the geometry. For axisymmetric thin sheet metal geometries, it is sufficient to input the meridional profile. The input object geometry can be thought of as a concatenation of elements, and an element is defined as a geometric primitive with a fillet radius at its end. An element is represented as a record or a structure\(^1\) (implemented as a list in MPROLOG) as

\[(\text{Element_name, Element_type, T, ID, OD, H, R, FR})\]

where \text{Element_name} is the feature (wall, bottom, flange, etc.) that the element represents, \text{Element_type} is the class of elements (h1, v1, c1, etc.) to which the element belongs (Table 5.1), \text{T} is the thickness of the element, \text{ID} is the inner diameter of the element, \text{OD} is the outer diameter of the element, \text{H} is the Height of the element, \text{R} is the radius of the element, \text{FR} is the fillet radius of the element. The input object geometry is a list of such elements. Table 4.1 provides a library of elements. Figures 4.2, 4.3, and 4.4 show three input object geometries and their representations. Geometric elements shown in Figure 4.2 are called "direct" or "monotonic" since the diameter and cumulative height of elements have a nondecreasing variation when traversed from the pole (bottom center of the object geometry) to the orifice. However elements in Figure 4.3 present a "reverse" and "direct" variation, while elements in Figure 4.4 show a direct, reverse, and direct variation. It may also be seen that reverse elements have nonpositive heights and that an element is represented as tapered or taper-reduced, convex or convex-reduced, or concave or concave-reduced depending on whether it is direct or reverse and vice versa.

---

\(^1\) We refrain from referring to the element representation as \text{Frame} as in AI knowledge-representation terminology, since there is no concept of default values and/or inheritance here.
Table 4.1 "Direct" Element Library

<table>
<thead>
<tr>
<th>Element Geometry</th>
<th>Conventional Name</th>
<th>Element Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>horizontal**</td>
<td>h1</td>
</tr>
<tr>
<td></td>
<td>vertical**</td>
<td>v1</td>
</tr>
<tr>
<td></td>
<td>tapered**</td>
<td>a1</td>
</tr>
<tr>
<td></td>
<td>tapered-reduced*</td>
<td>a2</td>
</tr>
<tr>
<td></td>
<td>convex**</td>
<td>r1</td>
</tr>
<tr>
<td></td>
<td>convex-reduced*</td>
<td>r2</td>
</tr>
<tr>
<td></td>
<td>concave**</td>
<td>c1</td>
</tr>
<tr>
<td></td>
<td>concave-reduced*</td>
<td>c2</td>
</tr>
</tbody>
</table>

* Elements that require 'nosing' (complementary) operations.

** Elements that are formed by main operations such as drawing, redrawing, stretching, etc. and are the focus of the current study.

1 Figures 4.3 and 4.4 illustrate how "reverse" elements are represented.
Object Geometry Input File

[bottom, h1, 0.0625e0, 0, 0.9e0, 0, 0, 0],
[wall3, v1, 0.0625e0, 1, 1, 1.85e0, 0, 0.05e0],
[wall2, c1, 0.0625e0, 1, 2.3e0, 0.85e0, 1.25e0, 0],
[wall1, v1, 0.0625e0, 2.375e0, 2.375e0, 2.65e0, 0, 0.2e0]
}

highductile(steel)
[british, E, Nu, Sigy, Sigu, K, N, M]

Figure 4.2 Object Geometry Input - Direct Geometry
Object Geometry Input File

[bottom, h1, 0.0625e0, 0, 0.875e0, 0, 0, 0],
[wall4, r1, 0.0625e0, 1.100e0, 1.625e0, -0.65e0, 1.250e0, 0.250e0],
[wall3, h1, 0.0625e0, 1.825e0, 2.350e0, 0, 0, 0.100e0],
[wall2, v1, 0.0625e0, 2.550e0, 2.550e0, 1.875e0, 0, 0.100e0],
[wall1, a1, 0.050e0, 2.625e0, 3.675e0, 1.000e0, 0, 0, 0.375e0],
[flange, h1, 0.0625e0, 4.000e0, 4.500e0, 0, 0, 0.250e0]

highductile(brass)
[british, E, Nu, Sigy, Sigu, K, N, M]

Figure 4.3 Object Geometry Input - Direct and Reverse Geometry
Object Geometry Input File

```
[bottom,h1,8,0,40,0,0],
[wall1,v1,8,48,48,9,0,4],
[wall2,a1,8,52,75,9,0,4],
[wall3,h1,8,82,139,0,0,16],
[wall4,r1,8,155,270,-10,60.7e0,8],
[wall5,h1,8,270,360,0,0,0],
[wall6,v1,8,396,396,11,0,10]
]
highductile(steel)
[metric,E,Nu,Sigy,Sigu,K,N,M]
```

Figure 4.4  Object Geometry Input - Direct, Reverse, and Direct Geometry (Converter Cover)
4.5.2 Control Strategy

The inference engine employs a two-pass, depth-first control strategy to design the process sequence for the given object geometry and the material. The first pass employs a geometry-based approach to design an initial process sequence, and the second pass employs a formability-based approach to refine this initial process sequence in communication with the material knowledge base. Following sections describe in detail the two techniques with an outline of the rules employed.

4.5.2.1 Geometry-Based First Pass

Geometry-based first pass determines the intermediate object geometries in the reverse or backward direction starting from the final object geometry based only on the geometric information of the final object geometry. In other words, it establishes the previous object geometry from which the current object geometry can be possibly formed in one station or in one hit taking into consideration only the geometry information of the current geometry without considering the material formability. This backward-stepping procedure is performed recursively until the current geometry is a flat circular blank. Appendix D provides a list of production rules that are associated with the first pass.

The process sequence obtained through the geometry-based first pass may not be practical and feasible since it did not take into consideration the formability of the material.

4.5.2.2 Formability-Based Second Pass

Input to the formability-based second pass is the process sequence obtained from the first pass, and the output from the second pass is a "rectified" process sequence that is a superset of the first-pass process sequence. The second pass involves following steps:
1) Arrange the process sequence obtained from the first pass as a list of object geometries with the blank as the head member and the final object geometry as the end member.

2) Consider the first two members of this list, the blank and the first deformed geometry.

3) For the two members under consideration, test for forming-limit violation in communication with the material module. If forming limits such as maximum draw ratio, maximum height to diameter ratio, maximum taper height, etc. are exceeded, then suggest rectification. If rectification is suggested, then it can be accomplished either by introducing intermediate geometries between the pair of adjacent geometries under consideration or by annealing. If intermediate successive redrawings are to be included as part of rectification, then use the maximum reduction ratio possible for each station and form to desired diameter in the last station. Insert the intermediate object geometries in the process sequence list between the two members under consideration.

4) Consider the next two geometries in the process sequence list.

5) Repeat steps 3 and 4 until the end of process sequence list is reached.

6) The new process sequence list with the inserted intermediate object geometries and/or suggestions for annealing is the rectified process sequence.

Appendix E outlines rules employed in the second pass.

4.5.3 Material Database Module

Material database module contains material formability information in the form of PROLOG clauses. Appendix F provides a description of clauses from the material module.
As seen from Appendix F, materials are broadly classified into "high ductile", "medium ductile" and "low ductile." Materials such as aluminum-killed steel, stainless steel, copper, brass, etc. may typically be classified as high ductile; medium to high carbon steels are typically medium ductile; refractory materials are typically low ductile. The input object geometry file to the knowledge-based system, as shown in Figures 4.2 through 4.4, should include such a broad classification for the material.

The material database module, at the present stage, has forming parameter values primarily for high ductile materials. This category comprises nearly 90% of the materials commonly used in sheet metal forming. The database contains only limited information for medium and low ductile materials, since they were not available in the open literature. However, the material database can be expanded to include forming limit information for medium and low ductile materials without any modifications to any other module.

The database is constructed based on the hypothesis that the materials belonging to the same category (high ductile, medium ductile, or low ductile) exhibit almost similar forming limit characteristics; in other words, their limiting values are almost equal. This may be a questionable assumption for some users. The database, however, can be easily modified to reflect individual characteristics of materials, rather than grouping them into three broad categories. Figure 4.5 shows how the material database can be modified.

4.6 System Testing

The knowledge-based system was tested against several test cases found in die design handbooks and industrial brochures. Sections 4.6.1 through 4.6.5 compare process sequences outputted by the knowledge-based system with corresponding ones from industrial practice, and section 4.6.6 presents a cumulative discussion. Table 4.2 presents the input object geometry files for the test cases.
module material.
%
% This module gets the limiting values for various parameters
% such as draw ratio, height to diameter ratio, etc.
%
all_global.

/*$eject*/

body.

get_limit(draw_ratio,Worklist_local,Blankdia,

      highductile(_),
      cup,Stagenumber,Limit_value) :-

get_t_over_blankd(Worklist_local,Blankdia,T_over_blankd),
T_over_blankd >= 0.015e0, !,
(Stagenumber=1,Limit_value=2.0e0;
Stagenumber=2,Limit_value=1.33e0;
Stagenumber=3,Limit_value=1.28e0;
Stagenumber=4,Limit_value=1.25e0;
Stagenumber>=5,Limit_value=1.22e0).

.
.
.

endmod. /* material */

(a) Material module before modification; bold characters shall be modified.
All high ductile materials have same values for draw ratio.

Figure 4.5 Material Database Modification
get_limit(draw_ratio, Worklist_local, Blankdia, 
    highductile(rimming_steel), 
    cup, Stagenumber, Limit_value) :-
    get_t_over_blankd(Worklist_local, Blankdia, T_over_blankd),
    T_over_blankd >= 0.015e0, !,
    (Stagenumber=1, Limit_value=2.15e0; 
    Stagenumber=2, Limit_value=1.36e0; 
    Stagenumber=3, Limit_value=1.33e0; 
    Stagenumber=4, Limit_value=1.28e0; 
    Stagenumber>=5, Limit_value=1.25e0).

(b) Material module after modification; reflects individual characteristics of materials.

get_limit(draw_ratio, Worklist_local, Blankdia, 
    rimming_steel, 
    cup, Stagenumber, Limit_value) :-
    get_t_over_blankd(Worklist_local, Blankdia, T_over_blankd),
    T_over_blankd >= 0.015e0, !,
    (Stagenumber=1, Limit_value=2.15e0; 
    Stagenumber=2, Limit_value=1.36e0; 
    Stagenumber=3, Limit_value=1.33e0; 
    Stagenumber=4, Limit_value=1.28e0; 
    Stagenumber>=5, Limit_value=1.25e0).

(c) Material module after modification; reflects individual characteristics of materials.

---

1 In modification (b) rimming_steel is classified as a high ductile material; however the draw ratio values are different from (a). If values for other formability parameters (height to diameter ratio, taper height, etc.) are not available, generic values for high ductile materials will be used by the program. For modification (c) the user has to make sure that all formability parameters are available for the material. Modification (b) is recommended over (c).
### Table 4.2  Input Object Geometry Files for Test Cases

| Bottom, H1, 0.0625e0, 0.3.6e0, 0, 0, 0, 0, 0, 0, 0, 0.2e0 |
| Wall1, V1, 0.0625e0, 4.4, 8.425e0, 0, 0.2e0 |
| High Ductile (Steel) |
| British, E, Nu, Sigy, Sigu, K, N, M |

*Object Geometry Input File for Figure 4.7*

| Bottom, H1, 0.065e0, 0.1.125e0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.2e0 |
| Wall3, V1, 0.065e0, 1.3125e0, 0.3125e0, 2.025e0, 0, 0.1e0 |
| Wall2, A1, 0.065e0, 1.4125e0, 2.025e0, 0.25e0, 0, 0.20e0 |
| Wall1, V1, 0.065e0, 2.125e0, 2.125e0, 0.75e0, 0, 0.20e0 |
| Flange, H1, 0.065e0, 2.425e0, 2.90e0, 0, 0, 0.15e0 |
| High Ductile (Steel) |
| British, E, Nu, Sigy, Sigu, K, N, M |

*Object Geometry Input File for Figure 4.9*

| Wall4, V1, 0.15e0, 1.15e0, 1.15e0, 1.48e0, 0, 0.575e0 |
| Wall3, C1, 0.15e0, 1.15e0, 4.33e0, 2.52e0, 3.36e0, 0 |
| Wall2, V1, 0.15e0, 4.7125e0, 4.7125e0, 0, 0.375e0 |
| Wall1, H1, 0.15e0, 4.8325e0, 7.02e0, 0, 0, 0.06e0 |
| High Ductile (Steel) |
| British, E, Nu, Sigy, Sigu, K, N, M |

*Object Geometry Input File for Figure 4.11*

---

1. All real values must be represented in exponential form in MPROLOG to distinguish them from lists.

2. E is the modulus of elasticity, Nu is the Poisson's ratio, Sigy is the yield strength, Sigu is the ultimate strength, K is the strain-hardening coefficient, N is the strain-hardening exponent, and M is the strain-rate sensitivity.
Table 4.2 (continued)

Object Geometry Input File for Figure 4.13


Object Geometry Input File for Figure 4.14

Object Geometry Input File for Figure 4.15
4.6.1 Process Sequence for a Deep-Drawn Cylindrical Shell

Figure 4.6 shows the process sequence from industrial practice for a 4 in. deep-drawn vertical cylindrical shell, and Figure 4.7 presents the process sequence outputted by the knowledge-based system. It can be seen that the number of stations predicted by the knowledge-based system is the same as the number found in the industrial practice. The blank of diameter 13.12 in. and thickness 0.0625 in. suggested by the knowledge-based system is 3% more in diameter than the blank used in industrial practice.

4.6.2 Process Sequence for a Flanged Shell Having Two Diameters

Figure 4.8 shows the process sequence from industrial practice for a flanged shell having two diameters with a sheet thickness of 0.065 in., and Figure 4.9 presents the predicted sequence. The blank of diameter 5.55 in. and thickness 0.065 in. suggested by the knowledge-based system agrees within 2% with the industrial practice of 5.44 in. diameter and 0.065 in. thickness. However, industrial practice is to employ six stations, while the knowledge-based system predicts that the same task can be accomplished in five stations. According to the knowledge-based system intermediate geometry 2 (steel shell with a diameter of 3.125 in.) in the industrial practice can perhaps be removed. Rule 47 from the material database module (Appendix F) states that the maximum possible draw ratio for the first station for a high ductile workpiece with a thickness to blank diameter ratio greater than 0.01 is 1.89. The present workpiece has a thickness to blank diameter ratio of about 0.012, and therefore, it is likely to be feasible to draw a 2.87 in. shell from a 5.55 in. blank without having through an intermediate geometry of about 3.125 in. diameter. Excepting this difference, the process sequence outputted by the knowledge-based system agrees qualitatively with the industrial practice process sequence.
Figure 4.6  Process Sequence for a Cylindrical Shell - Industrial Practice [Jones, 1941]
Figure 4.7 Process Sequence for a Cylindrical Shell - KBS
Figure 4.8 Process Sequence for a Flanged Shell with Two Diameters - Industrial Practice [Jones, 1941]
Figure 4.8 (continued)
Figure 4.9 Process Sequence for a Flanged Shell with Two Diameters - KBS
4.6.3 Process Sequence for Tapering Concave Shell

Figure 4.10 shows the process sequence from industrial practice for a tapering concave shell, and Figure 4.11 gives the process sequence outputted by the knowledge-based system. The blank of diameter 9.32 in. suggested by the system agrees well with the blank of diameter 9.25 in. employed in industrial practice. The knowledge-based system suggests eight stations to manufacture the concave shell, and the industrial practice is also to employ the same number of stations.

4.6.4 Process Sequence for a Deep Conical Shell

Figure 4.12 presents the process sequence for a deep conical shell from industrial practice, and Figure 4.13 presents the process sequence outputted by the knowledge-based system. The blank of diameter 11.48 in. and thickness 0.064 in. suggested by the system is about the same as the blank of diameter 11.50 in. and thickness 0.064 in. from the industrial practice. The number of stations is six in both cases. However, the process sequences differ in certain geometric aspects. The vertical wall section in intermediate geometries 2, 3, and 4 from industrial practice is connected to the horizontal bottom surface through tapered wall elements; the knowledge-based system suggests that these intermediate geometries can be formed with an optimal fillet radius (about 5 to 10 times the sheet thickness) between the vertical wall element and the horizontal bottom element. Both designs are practical and feasible; however, the former design has a higher factor of safety.
Figure 4.10  Process Sequence for a Tapering Concave Shell - Industrial Practice [Jones, 1941]
Figure 4.11  Process Sequence for a Tapering Concave Shell - KBS
Figure 4.12  Process Sequence for a Deep Conical Shell - Industrial Practice [Eary, 1974]
Figure 4.13 Process Sequence for a Deep Conical Shell - KBS
4.6.5 Process Sequences for Reverse Geometries

Few preliminary process sequence designs were done for reverse geometries. Figure 4.14 presents a process sequence for a double-walled reverse geometry shown in Figure 4.3.

Figure 4.15 presents a process sequence for a shallow converter cover. The converter cover geometry was obtained from Borg-Warner. The input object geometry is shown in Figure 4.4.

4.6.6 Discussion on Comparisons

The blank diameter suggested by the knowledge-based system is about the same as that used in industrial practice. However, in some cases, the blank diameter suggested by the knowledge-based system is about 1 to 2% more than the diameter used in industrial practice. This means that our system tends to be conservative as far as the trimming region is concerned. Rule 3, which estimates the trimming area in the knowledge base, can be modified, if required.

The number of stations suggested by the knowledge-based system usually agrees with the number obtained from industrial practice. In some cases, the knowledge-based system suggests one or two less number of stations than that used in industrial practice indicating that certain stations can be eliminated and still the final object geometry can be formed without any defects or failures. This, according to the knowledge-based system, is possible by taking advantage of maximum draw or redraw ratio in each station. For certain other cases involving a deep concave or tapered section, the knowledge-based system suggests one station more than the industrial practice.

The knowledge base was constructed with rules obtained from several different sources, and the process sequences outputted by the system were tested against process
Figure 4.14 Process Sequence for a Reverse Geometry - KBS
Figure 4.15  Process Sequence for a Converter Cover - KBS
sequences designed by several other die designers. Therefore, as seen from the examples, it is likely that there are bound to be certain disagreements between the two designs. However, the differences are minor, and therefore, it was concluded that the knowledge base needs no further modification. The user of the system, however, can modify the rule base if deemed necessary for his/her choice of case(s).
CHAPTER V

MATHEMATICAL MODELING OF
SHEET METAL FORMING PROCESSES

5.1 Summary

This chapter provides a mathematical background for sheet forming analysis. Section 5.2 describes Hill's two yield criteria for anisotropic materials and discusses how they can be used. Section 5.3 presents the equilibrium equation for a general, axisymmetric profile. The plasticity equations, the equilibrium equations, and the volume constancy equations (discussed later) form the basis for analysis of sheet metal forming in the current research. Section 5.4 presents an overview of the analysis procedure.

5.2 Classical Theory of Plasticity

Hill's [Hill, 1948 and 1950] theory (commonly known as Hill's 'old' theory) of anisotropic plasticity, based on modifications to von Mises quadratic yield function and its associated flow rule, is probably the most widely used theory today [Wagoner, 1980]. Assuming isotropy in the plane of the sheet, the theory requires only one anisotropy parameter, the average anisotropy factor R. Isotropic hardening is assumed indicating that all plastic parameters are independent of the current stress and strain states. Furthermore, the plastic properties are considered uniform and homogeneous in the material, and the effect of hydrostatic stress is not included. However, this 'old' theory has been able to...
predict the plastic behavior of most of the materials with reasonable success except for materials such as rimmed steel, zinc, copper and 70/30 brass sheet which have an average anisotropy factor less than 1. A newer version of Hill's theory [Hill, 1979] involves two anisotropy parameters, \( R \) and \( M \), and has been increasingly used today.

Based on Hill's 'old' theory (Appendix A) it can be shown that

\[
\begin{align*}
\sigma_r - \sigma_\theta &= \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\bar{\varepsilon}}[2d\varepsilon_\theta + d\varepsilon_t] \\
\sigma_t - \sigma_\theta &= \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\bar{\varepsilon}}[d\varepsilon_\theta + (1 + R)d\varepsilon_t] \\
d\bar{\varepsilon}^2 &= \left(\frac{1 + R}{1 + 2R}\right)(2d\varepsilon_\theta^2 + (1 + R)d\varepsilon_t^2 + 2d\varepsilon_\theta d\varepsilon_t) 
\end{align*}
\]

where \( \sigma_r \), \( \sigma_\theta \), and \( \sigma_t \) are principal radial, circumferential, and thickness Cauchy stresses, respectively, \( d\varepsilon_r \), \( d\varepsilon_\theta \), and \( d\varepsilon_t \) are principal radial, circumferential, and thickness strain increments, respectively, \( d\bar{\varepsilon} \) is the equivalent strain increment and \( R \) is the average anisotropy factor. Based on Hill’s 'new' theory it can be shown that (Appendix B)

\[
d\bar{\varepsilon} = \left[\frac{2(1 + R)}{2}\right]^{\frac{1}{M - 1}} \left\{ \frac{2d\varepsilon_\theta + d\varepsilon_t}{(1 + 2R)^{\frac{1}{M - 1}}} \right\}^{\frac{M - 1}{M}} + \left(\frac{1 + R}{1 + 2R}\right)^{\frac{1}{M - 1}} [d\varepsilon_t]^{\frac{1}{M - 1}}
\]

\[
\sigma_r - \sigma_\theta = \pm \left(\frac{1 + R}{1 + 2R}\right)^{\frac{1}{M - 1}} \frac{\bar{\sigma}}{d\bar{\varepsilon}^{\frac{1}{M - 1}}} [d\varepsilon_t + 2d\varepsilon_\theta]^{\frac{1}{M - 1}}
\]

where \( + \) is used when \( d\varepsilon_r \) is greater than or equal to \( d\varepsilon_\theta \) and

\( - \) is used when \( d\varepsilon_r \) is less than \( d\varepsilon_\theta \)
\[\sigma_r - \sigma_t = \frac{1}{2}(1 + R) \frac{1}{R+1} \pm \left[ \left( \frac{2 \sigma_\theta + \sigma_t}{2R + 1} \right)^{\frac{1}{2}} + \left| \sigma_t \right|^{\frac{1}{2}} \right] \] (5-3)

where + (first) is used when \(d_\sigma \) is greater than or equal to \(d_\theta \) and
- (first) is used when \(d_\sigma \) is less than \(d_\theta \)
+ (second) is used when \(d_\theta \) is less than or equal to zero and
- (second) is used when \(d_\theta \) is greater than zero

and where \(R\) and \(M\) are Hill's 'new' anisotropy parameters. When \(M = 2\), equations (5-2) and (5-3) become equation (5-1).

5.3 Equilibrium Equations Based on Tool Profiles

The equilibrium equation for a general, axisymmetric punch profile shown in Figure 5.1 can be shown (Appendix C) to be

\[r \frac{d}{d\alpha} (\sigma_r \ t) + \frac{d}{d\alpha} (\sigma_r \ t) = \frac{\sigma_\theta \ t}{r} (\cos \alpha + \mu \ sin \alpha) + \sigma_r \mu \ t \] (5-4)

where \(\mu\) is the friction coefficient
\(t\) is the thickness of the material
\(p_1 = p + \frac{t}{2}\) = meridional radius of curvature
\(r = r + \frac{t}{2} \ sin \alpha\) = current radius

and also the punch pressure \(p\) can be evaluated using the expression

\[p = \frac{1}{p_1} [\sigma_1 r + \sigma_2 p_1 \ sin \alpha] \] (5-5)

As shown in Appendix C, the equilibrium equation for elements over the punch profile of a flat-bottomed cylindrical punch can be written in a finite-difference form as
Figure 5.1 Deformation over a Curved Profile
\[
(\sigma, t)_{i+1} = (\sigma, t)_i + \int_t^{t+1} \left[ \sigma \left[ \cos \phi + \mu \sin \phi - \sigma, t \cos \phi \right] d\phi + \int_t^{t+1} \sigma, t d\phi \right]
\]

where \( \phi = \pi - \alpha \)

\[
r' = r_c + \rho_1 \sin \phi = r_c + (\rho_1 + \frac{1}{2}) \sin \phi
\]

and similarly for a hemispherical punch where \( r_c = 0 \) it can be shown that (Appendix C)

\[
(\sigma, t)_{i+1} = (\sigma, t)_i + \int_t^{t+1} \left[ \sigma \left[ \cot \phi + \mu \right] - \sigma, t \cot \phi \right] d\phi + \int_t^{t+1} \sigma, t d\phi
\]

\[5.4 \text{ Analysis Overview} \]

The analysis module which formulates the mechanics of metal forming and simulates sheet metal forming processes such as stretch forming and deep drawing is called SHEET_FORM. SHEET_FORM uses incremental deformation theory and assumes that the sheet material is rigid plastic. The sheet material is further assumed to be isotropic in the plane of the sheet and anisotropic in the normal or the thickness direction. Hill's anisotropic yield function is used to define the yield surface, and Swift's power-law equation is used to describe the hardening curve of the material. The tool-workpiece interface friction is modeled using modified Coulomb friction theory. SHEET_FORM
determines the stress and strain distributions in the workpiece at every instant by modeling the forming process as a quasi-static process. SHEET_FORM is implemented in FORTRAN, runs on a VAX/VMS operating system, and uses PLTPAK (The Ohio State University) for graphics.

5.4.1 Rigid-Plasticity

SHEET_FORM assumes the material to be rigid plastic neglecting elastic strain component. This assumption is valid in most of the metal forming operations where the elastic strain component is significantly small and thus negligible compared to the plastic strain component. This approach has been successfully employed by, for example, Kobayashi and Kim [Kobayashi, 1978] and Wang [Wang, 1982]. On the other hand, elastoplastic approach is CPU time expensive, and has been pursued by researchers to alleviate unloading problem with the rigid plastic material, to determine residual stresses, and to estimate springback. For a detailed review of all available approaches, the reader is referred to the exhaustive survey done by Lee et al. [Lee, 1988].

5.4.2 Friction

The tools (the punch and the dies) are assumed to be rigid when compared to the workpiece undergoing deformation. The interface friction between the workpiece and the tools is modeled using modified Coulomb friction theory. Although this model is somewhat crude, it has been found to be appropriate in most of the sheet forming cases [Saran, 1989a; Germain, 1987].

According to Coulomb friction theory, for a slipping node, the tangential friction force ($F_t$) can be estimated from the normal force ($F_n$) as
In the modified Coulomb friction theory, the friction coefficient ($\mu$) is expressed as a function of the relative movement of the workpiece node with respect to the tool, and the tangential friction force can be estimated from the normal force as (Figure 5.2)

$$F_{t} = \mu f(\Delta u_{t}) F_{n}$$  \hspace{1cm} (5-9)

The term $f(\Delta u_{t})$ has a value between 0 and 1, and is determined using

$$f(\Delta u_{t}) = 1 \text{ if } \|\Delta u_{t}\| \geq \delta$$

$$f(\Delta u_{t}) = \frac{\|\Delta u_{t}\|}{\delta} \text{ if } \|\Delta u_{t}\| < \delta$$  \hspace{1cm} (5-10)

where $\Delta u_{t}$ is the relative tangential movement of a workpiece node with respect to the tool between two adjacent time steps, and $\delta$ is set equal to $1 \times 10^{-3} \times \Delta h$, where $\Delta h$ is the incremental punch depth.

5.4.3 Contact Boundary

The contact boundary, the node at which the workpiece leaves the punch or die surface, can be determined by looking at the sign of the normal force $F_{n}$. Typically the normal force is non-negative or acting out of the tool surface. When the normal force is negative or the direction of the normal force is toward the tool surface, it would mean that the tool surface pulls the workpiece node towards itself in the normal direction, a
Figure 5.2 Modified Coulomb Friction Model [Wagoner, 1988]
physically unrealistic condition [Wagoner, 1988]. Therefore, the workpiece nodes for which \( F_n < 0 \) are released from the tool surface and treated as free or unsupported nodes.

5.4.4 Unloading Phenomenon in Rigid-Plastic Analysis

When an element in the workpiece undergoes no further or negligible deformation or when \( d\varepsilon \leq \delta \) where \( \delta \) is set to \( 1.25 \times 10^{-4} \), roughly an order of magnitude smaller than the elastic strain of most metals, certain indeterminacy problems arise in rigid-plastic formulation of (rate-insensitive) materials [Germain, 1987] due to the unloading phenomenon. The effective stress for every node on the workpiece stays on the hardening curve (Figure 5.3) until the node starts unloading (for example at strain \( \varepsilon_A \)). The effective stress for this node should lie somewhere between 0 and \( \sigma_A \) to satisfy the equilibrium condition in the whole body. If the material is rate-sensitive, then the unloading phenomenon is automatically taken care of by the rate-sensitivity of the material since effective strain increment rate also decreases.

The effect of material unloading phenomenon can be explained, for example, using a sheet material undergoing tensile deformation. Elements which experience strain localization or necking will have less load-bearing capacity due to decreasing cross-sectional area, and this leads to unloading phenomenon in adjacent regions where further incremental strains are smaller, and therefore, stresses need to be smaller to maintain equilibrium due to less thinning of the material.

The unloading phenomenon is handled using a linear hardening law as shown in Figure 5.3. When the effective strain for a node is less than or equal to \( \delta \), then the linear hardening law is employed, and when the effective strain is greater than \( \delta \), then the usual nonlinear power hardening law is employed. For a more detailed discussion on this approach, the reader is referred to the work by Germain et al. [Germain, 1987].
Figure 5.3 Unloading Model [Germain, 1987]
5.4.5 Bending Correction

When the sheet is drawn or stretched over a curved surface, there is additional tensile (compressive) straining on the convex (concave) surface. Current analysis estimates stresses and strains along the midsurface of the sheet, and therefore, is not capable of predicting local perturbation in the strain distribution at the location of curvatures. According to bending theory [Hosford, 1983], the increment in true strain on the outer and inner\(^1\) surfaces of the sheet over the midplane of the sheet can be determined using

\[
\Delta e = \ln \left(1 \pm \frac{t}{2p}\right)
\]

(5-11)

where \(p\) is the local radius of curvature, \(t\) is the sheet thickness, \(+\) is for the outer surface of the sheet and \(-\) is for inner surface of the sheet. This means that when \(p\) is positive, the outer surface is tensile and the outer surface strains are greater than the midsurface strains; the inner surface is compressive and the inner surface strains are less than the midsurface strains. In the current analysis, radii of curvature (both meridional and circumferential) are available at all nodes as part of the analysis and no separate calculation needs to be done for estimating the radii. It must be emphasized that the bending effect is included only in the computation of the strain distribution and is not included in the stress computations, and therefore, is limited in its scope. A more elaborate, integrated bending correction approach has been successfully employed by Stoughton [Stoughton, 1985] and Wenner [Wenner, 1983].

\(^1\) Inner surface refers to the punch-contact surface. Outer surface refers to the die-contact surface. Refer also to Figure 6.1. Note that the punch corner radius is positive and the die corner radius is negative.
6.1 Summary

This chapter provides a detailed description of the stretch forming simulation procedure. Sections 6.2, 6.3, and 6.4 provide the plasticity equations, the equilibrium equations, and the volume constancy equations, respectively. Section 6.5 describes the solution algorithm. Section 6.6 outlines the punch calculation procedure. Section 6.7 gives a brief account of bending correction technique. Some of the equations found in the following sections have already been discussed in the previous chapter. However, they are repeated here for the sake of continuity. Section 6.8 presents several numerical examples and a discussion.

6.2 Plasticity Equations

Based on Hill's (1950) 'old' theory it can be shown that (Appendix A)

\[
\sigma_e - \sigma_r = \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\bar{e}}[2d\varepsilon_e + d\varepsilon_i]
\]

\[
\sigma_i - \sigma_r = \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\bar{e}}[d\varepsilon_e + (1 + R)d\varepsilon_i]
\]

\[
d\bar{e}^2 = \left(\frac{1 + R}{1 + 2R}\right)\left(2d\varepsilon_e^2 + (1 + R)d\varepsilon_i^2 + 2d\varepsilon_e d\varepsilon_i\right)
\]
where \( \sigma_r, \sigma_\theta, \) and \( \sigma_t \) are principal radial, circumferential, and thickness Cauchy stresses, respectively, \( \overline{\sigma} \) is the equivalent stress, \( \Delta \varepsilon_r, \Delta \varepsilon_\theta, \) and \( \Delta \varepsilon_t \) are principal radial, circumferential, and thickness strain increments, respectively, and \( d \overline{\varepsilon} \) is the equivalent strain increment. It is implicitly assumed in this equation that the elastic part of the strain increment is negligible, and therefore, the total strain increment is equal to the plastic strain increment. The thickness and the circumferential strains at a node are defined as

\[
\varepsilon_t = \ln \left( \frac{t}{t_0} \right) \\
\varepsilon_\theta = \ln \left( \frac{r'}{r} \right)
\]

where \( t \) is the current sheet thickness at the node, \( t_0 \) is the initial thickness of the blank, \( r' \) is the current radius of the node, and \( r \) is the initial radius of the node.

The volume constancy equation is satisfied as

\[
\varepsilon_t + \varepsilon_\theta + \varepsilon_r = 0
\]

The material is assumed to obey the isotropic strain-hardening equation [Swift, 1952]

\[
\overline{\sigma} = K(\overline{\varepsilon} + \varepsilon_0)^n
\]

where \( \overline{\varepsilon} \) is the total equivalent strain, \( \varepsilon_0 \) is the pre-strain, and \( K \) and \( n \) are the strain-hardening coefficient and exponent, respectively. \( \overline{\varepsilon} \) can be computed by summing the equivalent strain increments as
\[ \varepsilon = \int \varepsilon = \Sigma \varepsilon_j \quad (6-5) \]

6.3 Equilibrium Equations

Deformation zones. For an axisymmetric punch stretching operation, the sheet can be divided into the following three deformation zones (Figure 6.1)

Zone 1: Punch-contact zone
Zone 2: Unsupported zone
Zone 3: Die-contact zone

Before discussing the equilibrium equations for these zones, a description of punch-profile representation in the input module to SHEET_FORM using two-dimensional graphic 'primitives' is necessary.

6.3.1 Punch-Head Representation

An axisymmetric punch head is a shell of revolution of two-dimensional punch profile about an axis. A generic two-dimensional punch profile can be created by a combination of line, arc and curve segments. A line segment can be defined by its end points, an arc segment can be defined by its end points and the radius of curvature, and a curve segment can be defined by a set of arbitrary points. As seen from Figure 6.2, a hemispherical punch head is composed of only one arc segment, a cylindrical punch with a flat bottom is defined by a line and an arc segment, and a generic punch profile is represented as a curve segment or a combination of arc and line segments. Although a curve segment provides a 'universal' representation capability, the arc and the line representations are included to facilitate easy input of punch profile. (An arc segment is a special case of a curve segment...
Figure 6.1 Deformation Zones in Stretch Forming
Figure 6.2 Punch-Head Representation in SHEET_FORM
with a constant meridional radius of curvature, while a line segment is a special case of an arc segment having zero curvature.)

6.3.2 Zone 1: Punch-Contact Zone

When the workpiece is in contact with a punch profile having an arbitrary contour or an arc with a constant meridional radius of curvature, the equilibrium equation (Appendix C) can be written as

$$\frac{d}{d\phi}(\sigma, t) = \frac{\rho_\phi}{r_\phi}[(\sigma_\phi t(\cos \phi + \mu \sin \phi) - \sigma, t \cos \phi)] + \sigma, \mu t$$

(6-6)

where

$$\phi = \pi - \alpha$$

$$\mu$$ is the Coulomb friction coefficient

$$\rho_\phi = \rho_\phi + \frac{1}{2} = \text{meridional radius of curvature}$$

and

$$r = r + \frac{1}{2} \sin \phi = \text{current radius}$$

and the punch pressure $$p$$, normal to the punch surface, can be evaluated as

$$p = \frac{l}{\rho_\phi r}[\sigma_\phi t' + \sigma_\phi \rho_\phi' \sin \alpha]$$

(6-7)

For a curve segment, a cubic spline is fitted through the given set of points and the meridional radius of curvature in equation (6-6) is computed using

$$\rho_\phi = \frac{(1 + y'^2)^{1.5}}{y''}$$

(6-8)

and the angle $$\phi$$ can be determined by
\[ \phi = \tan^{-1}(y') \]  

(6-9)

where \( y' \) and \( y'' \) are the first and second derivatives of \( y \) with respect to \( x \). A curve segment will have negative \( \rho_\phi \) in concave regions and positive \( \rho_\phi \) in convex regions. The current work aims to study forming employing convex punch heads where \( \rho_\phi \) is positive everywhere.

When the workpiece is in contact with a flat line segment of the punch profile, as in a flat-bottom cylindrical punch, the equation of equilibrium in the radial direction can be written as

\[ \frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} \]  

(6-10)

and the equation of equilibrium in the normal direction is nonexistent, since the pressure normal to the punch surface is zero for a flat membrane element with zero local curvature.

6.3.3 Zone 2: Unsupported Zone

In the unsupported zone the normal pressure is zero, and therefore, the equilibrium equation in the direction normal to the sheet surface can be derived from equation (6-7) as

\[ \frac{\sigma_r}{\rho_\phi} + \frac{\sigma_\theta}{\rho_\phi} = 0 \]  

(6-11)

where \( \rho_\theta \), circumferential radius of curvature, and \( \rho_\phi \) are defined as
\[
\rho_\theta = \frac{r}{\sin \phi} \\
\rho_\phi = \frac{dr}{d\phi} \left( \frac{1}{\cos \phi} \right)
\]  

(6-12)

For a stretch forming operation, since both \(\sigma_t\) and \(\sigma_\theta\) are tensile (positive), \(\rho_\theta\) and \(\rho_\phi\) need to have opposite signs to be able to satisfy equation (6-11). When substituted with the current values of \(r\) and \(\phi\), \(\rho_\theta\) turns out to be positive and \(\rho_\phi\) turns out to be negative.

The equation of equilibrium in the vertical direction can be derived from equation (C-2) (Appendix C) as

\[
\sigma_t r \ t \ \sin \phi = \text{constant} \quad (6-13)
\]

6.3.4 Zone 3: Die-Contact Zone

It is assumed that the die profile is defined by an arc with a constant radius of curvature \(\rho_\phi\), and therefore, the equation of equilibrium for this zone simplifies to

\[
\frac{d}{d\phi}(\sigma_t) = -\left[ \sigma_t (\cos \phi + \mu \sin \phi) - \sigma_t \cos \phi \right] \frac{\rho_\phi}{r_c - \rho_\phi \sin \phi + \mu \sigma_t} \quad (6-14)
\]

where \(r_c\) is the distance between the central axis and the center of die profile radius.

6.4 Volume Constancy Equations

For a sufficiently small element, the volume constancy relation of an element can be written as

\[
(R_{i-1}^2 - R_i^2) t_0 = \frac{(r_{i-1}^2 - r_i^2)(t_{i-1} + t_i)}{2} \frac{1}{\cos \left( \frac{\phi_{i-1} + \phi_i}{2} \right)} \quad (6-15)
\]
where subscripts 'i' and 'i-1' refer to the inner and outer boundaries of the element. When the element is in the die-contact region, the volume constancy equation can be written as

\[
(r_{i-1}^2 - r_i^2) t_0 = \left( \rho_\phi + \frac{t_{i-1} + t_i}{4} \right)^2 (t_{i-1} + t_i) \left[ \frac{r_e}{t_{i-1} + t_i} \left( \phi_i - \phi_{i-1} \right) + \cos \phi_i - \cos \phi_{i-1} \right]
\]

(6-16)

6.5 Solution Algorithm

The blank is divided into N elements of equal width \( \Delta \bar{r} \), and the elements are numbered 1 to N from the blank periphery to the pole. For N elements, there are N+1 boundaries or nodes, and the nodes are numbered likewise from 1 to N+1 from the periphery to the pole. The stretch forming process is modeled stage by stage as a quasistatic process, and for a given stage j of the punch displacement, all strains and stresses throughout the workpiece at the end of previous stage j-1 are known. At the start of computation (j=0), all strains and stresses are initialized to zero. The finite-difference algorithm consists of the following steps for each stage.

6.5.1 Punch-Contact Zone

1) Assume \( \Delta \varepsilon_r \) at the pole (\( \bar{r} = 0; i = N+1 \)).

2) Calculate \( \Delta \varepsilon_r = \Delta \varepsilon_{\theta} = -0.5 \Delta \varepsilon_i \) at the pole.

3) Compute \( \Delta \bar{\varepsilon} \) using equation (6-1)

4) Compute \( \varepsilon_r, \varepsilon_\theta, \varepsilon_i \) and \( \bar{\varepsilon} \) as the algebraic sum of the respective incremental strains.

5) Determine \( \bar{\sigma} \) using equation (6-4).
6) Calculate \( \sigma_i \) and \( \sigma_0 \) from equation (6-1) substituting \( \sigma_t = 0 \)

7) For the next node (i-1) assume the workpiece thickness \( t_{i-1} \) to be equal to \( t_i \)

8) Compute \( \phi_{i-1} \) and \( r'_{i-1} \) using volume constancy equation (6-16) and the punch profile, respectively.

9) Compute \( (\varepsilon_t)_{i-1} \) and \( (\varepsilon_r)_{i-1} \) and thus \( (\varepsilon)_{i-1} \) using equations (6-2) and (6-3).

10) Determine \( (d\varepsilon_t)_{i-1} \), \( (d\varepsilon_r)_{i-1} \), and \( (d\varepsilon)_{i-1} \) by subtracting respective strains at the end of previous stage (j-1) from current strains.

11) Compute \( (d\varepsilon)_{i-1} \), \( (\varepsilon)_{i-1} \), \( (\varepsilon)_{i-1} \), \( (\sigma_t)_{i-1} \), and \( (\sigma_0)_{i-1} \) as outlined in steps 3 through 6.

12) Determine \( (\sigma_t)_{i-1} \) as the product of \( (\sigma_t)_{i-1} \) and \( t_{i-1} \). Compute also \( (\sigma_t)'_{i-1} \) using equilibrium equation (6-10) or (6-6), written in a finite-difference form (Appendix C), depending on whether the element is in contact with a line segment or an arc/curve segment of the punch profile. Check if \( (\sigma_t)_{i-1} \) and \( (\sigma_t)'_{i-1} \) agree within ±0.5%. If not, determine a new value for thickness \( t_{i-1} \) using

\[
\Delta\varepsilon' = \frac{1}{1 + R} \left[ \frac{(1 + 2R)}{(1 + R)} \frac{\Delta\varepsilon (\sigma_t)'}{t_{i-1}} + d\varepsilon_{\theta_{i-1}} \right]
\]

where

\[
t_{i-1} = t_0 e^{\Delta\varepsilon'}
\]

and repeat steps 8 through 12. If the two values agree within ±0.5%, store the current stress and strain values for node (i-1). Compute punch normal pressure \( p \) using equation (6-7). If \( p \) is not negative, decrement \( i \) by 1 and repeat steps 7 through 12 for the next node. If \( p \) is negative, start the computations for the next zone.
6.5.2 Unsupported Zone

13) For the next node (i-1) assume the workpiece thickness $t_{i-1}$ to be equal to $t_i$.

14) Determine an initial guess value for $(\varepsilon_{\theta})_{i-1}$ using

$$
(\varepsilon_{\phi})_{i-1} = (\varepsilon_{\phi})_i + \int_{r_i}^{r_{i-1}} \frac{1}{r} \left[ \varepsilon^r - \varepsilon^\theta \cos \phi - 1 \right] r d\Gamma
$$

(6-18)

15) Calculate $(de_r)_{i-1}$ and $(de_\theta)_{i-1}$.

16) Determine $r'_{i-1}$ using equation (6-2).

17) Compute $(d\bar{\varepsilon})_{i-1}$, $(\bar{\varepsilon})_{i-1}$, $(\bar{\sigma})_{i-1}$, $(\sigma_r)_{i-1}$ and $(\sigma_\theta)_{i-1}$ as outlined in steps 3 through 6.

18) Calculate $\sin \phi_{i-1}$ and hence $\phi_{i-1}$ using equation (6-13).

19) Calculate a new value for $r'_{i-1}$ using volume constancy equation (6-15). If this new value does not agree with the $r'_{i-1}$ computed in step 16 within $\pm 0.05\%$, calculate a new value for $(de_\theta)_{i-1}$, repeat steps 17 through 19 until the agreement is reached.

20) Calculate $\rho_\theta$ and $\rho_\phi$ using equation (6-12).

21) Compute $(\sigma_r)_{i-1}'$ using equilibrium equation (6-11) substituting $(\sigma_\theta)_{i-1}$ obtained in step 17. Check if this value agrees with $(\sigma_r)_{i-1}$ calculated in step 17 within $\pm 0.5\%$. If not, compute a new value for the thickness as outlined in step 12. Using this $(de_r)_{i-1}$ and $(de_\theta)_{i-1}$ obtained in step 19, repeat steps 17 to 21 until the agreement is reached. As before, store the current stresses and strains for node $i-1$. Check if the material touches the die surface. If not, decrement $i$ by 1 and repeat steps (13) through (21). If the material touches the die surface, start the computations for the next zone.
6.5.3 Die-Contact Zone

The procedure for this zone is similar to the procedure for the punch-supported zone, except for the appropriate equilibrium and volume constancy equations.

22) Do computations until the held periphery of the blank is reached. Check if the absolute value of the circumferential strain for the node on the held boundary, \(|(\varepsilon_\theta)_1|\), is less than 0.5%. If not, move the assumed punch-contact boundary by one or two nodes until \(|(\varepsilon_\theta)_1|\) is less than 0.5%.

23) Compute the total punch displacement (cup height). Check if the cup height is more than the desired cup height. If so, write the results to an output file and exit the program. If not, repeat steps 1 through 23 until desired cup height is reached.

6.6 Punch Force Calculation

At any stage during deformation, the punch force required can be determined from the stresses and the sheet geometry in Zone 2 because this zone serves as a link for transmitting the punch force to the held boundary. For each step, the punch force can be calculated by multiplying equation (6-13) by \(2\pi\). Punch force can also be calculated by integrating the vertical component of punch pressure (equation 6-7) over the punch surface area.

6.7 Bending Correction

From the midsurface strains, the outer and inner surface radial and circumferential strains can be calculated using (see also Section 5.4.5)

\[
(\varepsilon_r)_{\text{outer, inner}} = (\varepsilon_r)_{\text{mid}} + \ln \left(1 \pm \frac{t}{2\rho_\theta}\right) \\
(\varepsilon_\theta)_{\text{outer, inner}} = (\varepsilon_\theta)_{\text{mid}} + \ln \left(1 \pm \frac{t}{2\rho_r}\right)
\]  

(6-19)
and the thickness strain can be determined using volume constancy equation (6-3).

6.8 Numerical Examples

Predicted results from SHEET_FORM were compared with several experimental data available from in-house experiments as well as from the open literature. Hemispherical punch stretching, flat-bottom cylindrical punch stretching, elliptical punch stretching, parabolic punch stretching, and general convex profile punch stretching were simulated. The following sections present some selected examples.

6.8.1 Hemispherical Punch Stretching

The hemispherical punch stretching experiments were carried out using 114 x 180 mm blanks held firmly around the periphery [Knibloe, 1988 and 1989b]. The blanks were made of three kinds of Al-killed low-carbon steel, namely uncoated steel, 1.5 galvanized steel (1.5 galvanized means coating on one side is thicker than the other side, "A-body" steel used for autobody hoods) and Zn-Fe electrogalvanized steel ("C-body" steel used for deck lids). The constitutive equations were determined employing data from uniaxial tension and plane-strain tests. The hemispherical punch stretching experiments were carried out under three lubrication conditions using the same punch and die configuration. The punch stretching machine is similar to the one used in an earlier study [Hecker, 1975] and was designed and built at the General Motors Research Laboratories and later donated to The Ohio State University Department of Materials Science and Engineering. For each combination of material and lubrication, a series of tests were performed using four to six samples deformed to a variety of punch heights up to and including the point of fracture. Table 6.1 gives a summary of tool dimensions, material properties, and process conditions.

---

1 terms used in the Chrysler Corporation
Table 6.1  Summary of Tool Dimensions, Material Properties, and Process Conditions for Hemispherical Punch Stretching of Uncoated Steel [Knibloe, 1988]

<table>
<thead>
<tr>
<th>Tool Dimensions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Punch Diameter</td>
<td>101.6 mm (4.00 in.)</td>
</tr>
<tr>
<td>Punch Nose Radius</td>
<td>50.8 mm (2.00 in.)</td>
</tr>
<tr>
<td>Die Opening Diameter</td>
<td>105.7 mm (4.16 in.)</td>
</tr>
<tr>
<td>Die Rounding Radius</td>
<td>6.35 mm (0.25 in.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material Properties and Blank Dimensions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R (Anisotropy Factor)</td>
<td>1.92</td>
</tr>
<tr>
<td>K (Strain-Hardening Coeff.)(^1)</td>
<td>520.7 MPa (75.5 Ksi)</td>
</tr>
<tr>
<td>n (Strain-Hardening Exp.)</td>
<td>0.21</td>
</tr>
<tr>
<td>(\varepsilon_0) (Pre-strain)</td>
<td>0.0</td>
</tr>
<tr>
<td>Blank Diameter</td>
<td>118.4 mm (4.66 in.)</td>
</tr>
<tr>
<td>Blank Thickness</td>
<td>1.04 mm (0.041 in.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process Conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction Coefficient</td>
<td>0.33 (Dry)</td>
</tr>
<tr>
<td>(for best match between prediction and experiments)</td>
<td>0.09 (Polyethylene plus Oil)</td>
</tr>
<tr>
<td></td>
<td>0.06 (Teflon)</td>
</tr>
<tr>
<td>Cup Heights</td>
<td>21.5, 25.0, and 28.3 mm</td>
</tr>
<tr>
<td></td>
<td>(0.85, 0.98, and 1.11 in.)</td>
</tr>
</tbody>
</table>

\(^1\) \(\bar{\sigma} = K(\bar{\varepsilon} + \varepsilon_0)^n\)
Figure 6.3 shows the tool geometry and dimensions. The circumferential and radial strains after stretching were measured using the circle-grid technique. The experiments were conducted at the Department of Materials Science and Engineering, The Ohio State University, as part of the Engineering Research Center for Net Shape Manufacturing Sheet Metal Research program.

Figures 6.4, 6.5, and 6.6 show theoretical and experimental outer surface true radial and circumferential strain distributions against original radial distance from the punch pole. These results are for hemispherical punch stretching of uncoated steel for cup heights 21.5 mm, 25.0 mm, and 28.3 mm, respectively, under dry or unlubricated condition. A friction coefficient of 0.33 was chosen for all heights to obtain the best possible match between theoretical predictions and experimental data. Comparison between the theoretical predictions and experimental data appear to be good. Figure 6.7 shows the deformed workpiece geometry for the three punch heights.

Figures 6.8 and 6.9 present outer, mid, and inner surface theoretical strains due to bending correction and experimental data against original radial distance from the punch pole. These results are for hemispherical punch stretching of uncoated steel for cup heights of 25.0 mm and 28.3 mm, respectively, under dry lubrication condition. For the present hemispherical punch stretching the $P_\phi / t$ ratio is more than 40 in most of the locations except near the die corner radius where it is about 6, and therefore, bending effects are almost negligible everywhere in the workpiece except near the die corner. The radial strain distribution for the outer, mid, and inner surfaces is roughly about the same in most part of the workpiece except near the die corner radius where the three radial strain curves separate. It may be seen also that the three curves intersect immediately after the punch-contact boundary region. The inner surface radial strains are greater than the mid and outer surface radial strains in most of the unsupported and die-contact regions and less than
Figure 6.3 Tool Geometry for Hemispherical Punch Stretching
Figure 6.4 Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Dry Friction (Punch Height = 21.5 mm) (Experimental Data from [Knibloe, 1988])
Figure 6.5 Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Dry Friction (Punch Height = 25.0 mm) (Experimental Data from [Knibloe, 1988])
Figure 6.6 Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Dry Friction (Punch Height = 28.3 mm) (Experimental Data from [Knibloe, 1988])
Figure 6.7 Deformed Workpiece Geometry for Punch Heights = 21.5, 25.0, and 28.3 mm
Figure 6.8  Outer, Mid, and Inner Surface Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Dry Friction (Punch Height = 25.0 mm) (Experimental Data from [Knibloe, 1988])
Figure 6.9 Outer, Mid, and Inner Surface Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Dry Friction (Punch Height = 28.3 mm) (Experimental Data from [Knibloe, 1988])
the mid and outer surface radial strains in the punch-contact region. This is because the meridional line of the workpiece is convex in the punch-contact region and concave in the unsupported and die-contact regions.

Figures 6.10 and 6.11 show theoretical and experimental radial and circumferential strain distributions plotted against original radial distance from the pole for hemispherical punch stretching of uncoated steel for a cup height of about 25.0 mm under polyethylene plus oil and teflon lubrication conditions, respectively. Again, the friction coefficients were chosen to obtain the best possible match between theoretical predictions and experimental data. It can be seen that teflon gives the best lubrication condition followed by polyethylene plus oil and dry conditions, respectively. Figure 6.12 shows theoretical outer surface radial and circumferential strain distributions for uncoated steel for eight different friction coefficients varying from 0.0 to 0.40 for a cup height of 25.0 mm. It can be seen that as the friction coefficient increases or lubrication decreases, the strain-localization point moves from the punch pole toward the crown or punch contact boundary region. In other words, under well-lubricated conditions the material over the punch stretches easily, and it is likely to neck and fracture near the pole; under low or unlubricated conditions, the material over the punch gets locked, and necking and fracture occurs near the crown or punch contact boundary region. It can be observed also that the area under the radial strain curve (when plotted against the original radial distance) is approximately the same for all friction conditions for the same punch height. This is to be expected since this area is essentially proportional\(^1\) to the length of the sheet from the held boundary to the punch pole.

\[ \epsilon_r = \log \left( \frac{dl}{dl} \right) = \log \left( \frac{dl}{d\theta} \right); l = \int e^\gamma d\theta \] where \( dl \) is the length of an element and \( l \) is the length of deformed geometry.
Figure 6.10 Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Polyethylene plus Oil Lubrication (Punch Height = 25.0 mm) (Experimental Data from [Knibloe, 1988])
Figure 6.11 Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Teflon Lubrication (Punch Height = 24.6 mm) (Experimental Data from [Knibloe, 1988])
Figure 6.12 Theoretical Strain Distributions for Hemispherical Punch Stretching of Uncoated Steel under Various Friction (Punch Height = 25.0 mm)
6.8.2 Flat-Bottom Cylindrical Punch Stretching

Theoretical predictions for flat-bottom cylindrical punch stretching were compared with experimental data obtained from Saran et al. [Saran, 1989b]. Table 6.2 gives a summary of tool dimensions, material properties, and process conditions. Figure 6.13 shows the tool geometry. According to Saran et al. the blanks were firmly clamped by serrated dies to avoid draw-in during stretching.

Figures 6.14 and 6.15 show theoretical and experimental radial and circumferential strain distributions, respectively, against original radial distance from the pole for deep drawing quality steel for a cup height of roughly about 22 mm when drawing with oil lubrication. A friction coefficient of 0.20 was chosen to obtain the best possible match between theoretical predictions and experimental data. Theoretical predictions have been shown for inner, mid, and outer sheet surfaces, since it was not known whether the experimental data were measured on the outer or the inner surface or average values were computed from inner and outer surface strains to roughly correspond to midsurface strains. The theoretically predicted radial strain distribution seems to agree reasonably well with the experimental data; however, the circumferential strain does not agree that well. Figure 6.16 presents a sensitivity comparison for radial strain, varying the friction coefficient between 0.0 and 0.25. As before, with the increasing friction the necking is more pronounced and occurs near the punch nose radius. Figure 6.17 presents punch force variation against punch displacement, and it can be seen that the predicted values agree well with the experimental data.

6.8.3 Stretching using Punches with General Profiles

In addition to hemispherical punch stretching and flat-bottom cylindrical punch stretching, SHEET_FORM was also used to simulate stretching using punches with

<table>
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<tr>
<th>Tool Dimensions</th>
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<tr>
<td>Punch Diameter</td>
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<td>(for best match between prediction and experiments)</td>
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<tr>
<td>Cup Height</td>
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</table>
Figure 6.13 Tool Geometry for Flat-Bottom Cylindrical Punch Stretching
Figure 6.14 Radial Strain Distributions for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Oil Lubrication (Punch Height ~ 22 mm) (Experimental Data from [Saran, 1989b])
Figure 6.15  Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Oil Lubrication (Punch Height ~ 22 mm) (Experimental Data from [Saran, 1989b])
Figure 6.16  Strain Distributions for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Various Friction Conditions (Punch Height ~22 mm) (Experimental Data from [Saran, 1989b])
Figure 6.17  Punch Force Variation for Flat-Bottom Cylindrical Punch Stretching of Deep-Drawing Quality Steel under Oil Lubrication (Experimental Data from [Saran, 1989b])
general meridional profiles. For example, theoretical predictions for stretching with axisymmetric ellipsoidal punches were compared with experimental data obtained from Biondich et al. [Biondich, 1986]. Figure 6.18 shows ellipsoidal punches with the axis ratio of 1:2, 1:1, 2:1 and 4:1. Axis ratio is defined as the ratio between the length of the axis in the horizontal direction (perpendicular to the direction of punch travel) and the length of the axis in the vertical direction (parallel to the direction of punch travel). The punch with an axis length ratio of 1:1 is a hemispherical punch having a radius 50.8 mm (2 in.). Table 6.3 gives a summary of tool dimensions, material properties, and process conditions. According Biondich et al., the lubrication in the tool-workpiece interface was excellent.

Figures 6.19 and 6.20 show theoretical and experimental radial and circumferential strain distributions, respectively, against original radial distance from the pole for 321 stainless steel for punch depths 21.8 mm, 37.3 mm, and 47.8 mm under teflon and grease lubrication. These results are for 2:1 ellipsoidal punch for friction coefficients 0.01 and 0.04. The agreement between experimental data and theoretical predictions is very good for punch depths 21.8 mm and 37.3 mm. However, theoretical simulation could not be carried out for 47.8 mm, since the internal deformation forces could not reach equilibrium with the external forces, thus suggesting possible necking and fracture. For a 2:1 punch which is blunter than a hemispherical punch, the workpiece is likely to get locked over the punch more than a hemispherical punch. Although this helps preventing excessive sliding of material near the punch pole due to excellent lubrication and although this results in a uniform radial strain distribution, the maximum cup height obtainable using a 2:1 punch is typically less than that obtainable using a hemispherical punch. In experiments, it is likely that the sheet boundary probably drew in as the punch advanced, and therefore, the experiments could be carried out for a greater cup depth than the theoretical simulation.
Figure 6.18  Ellipsoidal Punches [Biondich, 1986]

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<td>n (Strain-Hardening Exp.)</td>
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<tr>
<td>ε₀ (Pre-strain)</td>
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<td>Cup Heights (2:1 Punch)</td>
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<tr>
<td>Cup Heights (1:2 Punch)</td>
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<td></td>
<td>(1.10, 1.42, and 1.98 in.)</td>
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</table>
Figure 6.19 Mid Surface Radial Strain Distributions for 2:1 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 21.8, 37.3, and 47.8 mm) (Experimental Data from [Biondich, 1986])
Figure 6.20  Mid Surface Circumferential Strain Distributions for 2:1 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 21.8, 37.3, and 47.8 mm) (Experimental Data from [Biondich, 1986])
Figures 6.21 and 6.22 show theoretical and experimental radial and circumferential strain distributions, respectively, against original radial distance from the pole for stainless steel for punch depths 27.9 mm, 36.1 mm, and 50.3 mm under teflon and grease lubrication. These results are for the 1:2 ellipsoidal punch for friction coefficients 0.01, 0.04, and 0.09. The agreement between experimental data and theoretical predictions is poor. However, the trend predicted by the simulation is as would be expected. When friction is lower (0.01), excessive stretching occurs near the sharp-curvature punch pole, and when friction is relatively higher (0.09), the maximum moves away from the punch pole. Figure 6.23 shows the deformed geometry for stretching using the two ellipsoidal punches for punch depths 21.8 and 37.3 mm. Although some results were presented here for ellipsoidal punches, more comparative studies between experimental data and theoretical predictions are needed to evaluate the validity of SHEET_FORM for general punch profiles.
Figure 6.21 Mid Surface Radial Strain Distributions for 1:2 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 27.9, 36.1, and 50.3 mm) (Experimental Data from [Biondich, 1986])
Figure 6.22 Mid Surface Circumferential Strain Distributions for 1:2 Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 27.9, 36.1, and 50.3 mm) (Experimental Data from [Biondich, 1986])
Figure 6.23 Deformed Workpiece Geometries for Ellipsoidal Punch Stretching of 321 Stainless Steel under Teflon and Grease Lubrication (Punch Heights = 21.8 and 37.3 mm)
CHAPTER VII

SIMULATION OF DEEP DRAWING

7.1 Summary

This chapter provides a detailed description of the deep drawing simulation procedure. The deep drawing analysis was developed using certain subroutines obtained from the DEEPDW program [Vemuri, 1984 and 1986]. Section 7.2 provides the plasticity equations, the equilibrium equations, and the volume constancy equations. Section 7.3 describes the solution algorithm. Section 7.4 describes drawbead modeling. Section 7.5 discusses the punch force calculation and the bending correction. Section 7.6 presents numerical examples and discussion. Section 7.7 provides a cumulative discussion on the CPU-time efficiency of the analysis program.

7.2 Plasticity, Equilibrium, and Volume Constancy Equations

Any cup drawing process can be divided into two major stages: embossing stage and drawing stage. In the embossing stage, the punch is merely impressed on the flat blank, and therefore, the punch load required is small. In the drawing stage, the flange is compressed, the material is bent over the die radius and then unbent and straightened to form the wall of the cup. During this stage, the workpiece can be divided into five zones, as seen in Figure 7.1,

Zone 1: Blankholder-contact zone

Zone 2: Radial drawing zone not in contact with blank holder
Figure 7.1 Deformation Zones in Deep Drawing
Zone 3: Die-contact zone
Zone 4: Unsupported wall zone
Zone 5: Punch-contact zone

It may be seen that Zone 3, Zone 4, and Zone 5, respectively, correspond to Zone 3, Zone 2, and Zone 1 in the stretchforming analysis discussed in the previous chapter. Discussion in this chapter will focus on issues that were not discussed earlier.

7.2.1 Zone 1: Blankholder-Contact Zone

The force equilibrium equation in radial direction is

$$\frac{d\sigma_r}{dr} = \frac{\sigma_0 - \sigma_r}{r'} + \frac{2\mu \sigma_r}{t}$$ (7-1)

where, as defined before, $r'$ and $t$ are the current radius and thickness, respectively, $\sigma_r$, $\sigma_\theta$, and $\sigma_z$ are the principal radial, circumferential, and thickness stresses, respectively, and $\mu$ is the friction coefficient between the sheet and blankholder and die surfaces. The volume constancy equation can be written as

$$(R_i^2 - R_{i+1}^2) t_0 = \frac{1}{2} (r_i^2 - r_{i+1}^2) (t_i + t_{i+1})$$ (7-2)

where subscripts 'i' and 'i+1' refer to the outer and inner boundaries of an element, respectively. The above equation is the same as equation (6-15) with $\phi = 0$ for a horizontal element. Subscripts 'i' and 'i+1' in equation (7-2) correspond to 'i-1' and 'i' in equation (6-15). The difference in the notation is intentional, and the reason will be explained in the following sections.
7.2.2 Zone 2: Radial Drawing Zone Not in Contact with Blankholder

The force equilibrium equation in the radial direction is

\[
\frac{d\sigma_r}{dr'} = \frac{\sigma_0 - \sigma_r}{r'}
\]  

(7-3)

The above equation is the same as equation (7-1) except for the friction term due to the blankholder.

7.3 Solution Algorithm

As explained in the previous chapter, the blank is divided into \( N \) elements of equal width \( \Delta \tilde{R} \), and the elements are numbered 1 to \( N \) from the blank periphery to the pole. For \( N \) elements, there are \( N+1 \) boundaries or nodes, and the nodes are numbered likewise from 1 to \( N+1 \) from the periphery to the pole. The deep drawing process is modeled stage by stage as a quasistatic process, and for a given stage \( j \) of the punch displacement, all strains and stresses throughout the workpiece at the end of previous stage \( j-1 \) are known. At the start of the computation \( (j=0) \), all strains and stresses are initialized to zero. The finite-difference algorithm consists of the following steps for each stage.

7.3.1 Zone 1: Blankholder-Contact Zone

1) Assume that the punch has moved through a finite vertical displacement \( \Delta h \).
2) Compute the outer radius \( r'_i \) (\( i=1 \)) or draw-in of the first element using total volume constancy of the blank [Chung, 1951]
3) Assume an initial thickness value \( t \).
4) Compute \( (e_{\theta})_i \) and \( (e_r)_i \) and thus \( (e_{r'})_i \) using equations (6-2) and (6-3).
5) Determine \((\epsilon_r)_{i-1}, (\epsilon_\theta)_{i-1}\), and \((\epsilon_z)_{i-1}\) by subtracting the respective strains at the end of previous stage \((j-1)\) from current strains.

6) Compute \((d\epsilon_r)_i\), \((d\epsilon_\theta)_i\), \((d\epsilon_z)_i\) using equations (6-1), (6-5), and (6-4).

7) Calculate \((\sigma_r)_i\) and \((\sigma_\theta)_i\) from equation (6-1) substituting \((\sigma_r)_i = 0\) for the first node since the circumferential boundary of the blank is traction free.

8) For the next node \((i+1)\), assume the workpiece thickness \(t_{i+1}\) to be equal to \(t_i\).

9) Compute \(r'_{i+1}\) using volume constancy equation (7-2).

10) Compute \((\sigma_r)_{i+1}\) for the next node using force equilibrium equation (7-1).

11) Repeat steps 4 through 10. In step 7 use \(\sigma_r\) obtained in step 10 for nodes greater than 1.

12) Terminate the analysis for Zone 1 if one of the following conditions is reached:
   
   (a) the thickness stress becomes positive or equal to zero
   
   (b) the current radius \(r'\) becomes less than the blankholder inner radius or the die lip radius.

13) The solution is considered satisfactory if the thickness stress integrated over the surface under the blank holder approximates the applied blankholding force within certain limits. Otherwise the entire procedure (steps 1 through 13) is repeated starting with a new guess value for the thickness.

### 7.3.2 Zone 2: Radial Drawing Zone Not in Contact with Blankholder

This is a relatively thin zone between the blankholder inner boundary and the die lip.

14) For the first element, the outer radius \(r'_i\) and thickness \(t_i\) are known from Zone1.
15) As before, for this element, the inner boundary thickness $t_{i+1}$ is assumed to be equal to $t_i$ as a first approximation. Now the inner radius of this element $r'_{i+1}$ can be calculated using the volume constancy equation (7-2).

16) Compute $(\varepsilon_{r})_{i+1}$ and $(\varepsilon_{\theta})_{i+1}$ and thus $(\varepsilon_{\varphi})_{i+1}$ using equations (6-2) and (6-3).

17) Determine $(d\varepsilon_{r})_{i+1}$, $(d\varepsilon_{\theta})_{i+1}$, and $(d\varepsilon_{\varphi})_{i+1}$ by subtracting respective strains at the end of previous stage $(j-1)$ from current strains.

18) Compute $(d\varepsilon_{r})_{i+1}$, $(d\varepsilon_{\theta})_{i+1}$, $(d\varepsilon_{\varphi})_{i+1}$, $(\sigma_{r})_{i+1}$, $(\sigma_{\theta})_{i+1}$, and $(\sigma_{\varphi})_{i+1}$ assuming $(\sigma_{r})_{i+1} = 0$, as before, using equations (6-1), (6-5), and (6-4).

19) Determine $(\sigma_{r})_{i+1}$ as the product of $(\sigma_{r})_{i+1}$ and $t_{i+1}$. Compute also $(\sigma_{r})'_{i+1}$ using equilibrium equation (7-3). Check if $(\sigma_{r})_{i+1}$ and $(\sigma_{r})'_{i+1}$ agree within ±0.5%. If not, determine a new value for thickness $t_{i+1}$ using

$$\Delta \varepsilon_i = - \frac{1}{1 + R} \frac{\Delta \varepsilon}{\sigma} \left( \frac{1 + 2R}{1 + R} \right) \frac{\sigma}{t_{i+1}} + \frac{d\varepsilon_{\varphi}}{t_{i+1}}$$

$$t_{i+1} = t_i e^{\Delta \varepsilon_i}$$

and repeat steps 15 through 19. If the two values agree within ±0.5%, store the current stress and strain values for node $(i+1)$.

20) The analysis for Zone 2 is terminated if the current radius $r_i$ becomes less than the die lip radius $r_c$.

### 7.3.3 Zone 3: Die-Contact Zone

The solution procedure for this zone is similar to the solution procedure described for Zone 3 in the previous chapter. The procedure employed in deepdrawing traverses from the outer boundary of an element to the inner boundary while in stretchforming it is from the inner boundary to the outer boundary. The analysis for Zone 3 is terminated if the
angle $\phi_{e+1}$ of the next element becomes greater than the angle $\Phi'$ under which the blank touches the die radius. The angle $\Phi'$ itself is calculated for each stage, based on the analysis by Chung and Swift [Chung, 1951]. The end node of Zone 3 will be the matching boundary node and will be called $i_{\text{match}}$. Also $\epsilon_0$, $\epsilon$, and $\sigma_r$ for node $i_{\text{match}}$ are stored as the matching boundary values.

### 7.3.4 Zone 5: Punch-Contact Zone and Zone 4: Unsupported Zone

Analysis starts from the punch pole assuming a thickness strain increment at the pole, and steps 1 through 21 from the stretchforming solution algorithm in Chapter 6 are executed from node N+1 through $i_{\text{match}}$, and values for $\epsilon_0$, $\epsilon$, and $\sigma_r$ for node $i_{\text{match}}$ are compared with values determined in the previous section. If all three values do not agree within about 0.5%, the thickness strain at the punch pole is varied and the analysis procedure for Zone 5 and Zone 4 are repeated until convergence is obtained. The punch height is then computed and $\Delta h$ is added to the punch height for the next step until the desired cup height is reached.

### 7.4 Drawbead Modeling

In addition to the blankholder restraint, SHEET_FORM is also capable of modeling the drawbead restraint. Drawbead is modeled as a normal traction force (see Figure 7.2) at the sheet boundary (drawbead force = $2\pi rt\sigma_r$). The analysis procedure involves only four zones since there is no Zone 1. The solution procedure is identical to the one discussed for blankholder restraint except for the boundary node (i=1) where $\sigma_r$ computed from drawbead force is used as the boundary stress.
Figure 7.2  Drawbead Modeling in Deep Drawing Analysis
7.5 Punch Force Calculation and Bending Correction

The procedure is identical to that of the stretchforming analysis discussed in Chapter 6.

7.6 Numerical Examples and Discussion

Predicted deep drawing results from SHEET_FORM were compared with several experimental data available from open literature. The following sections present hemispherical punch deep drawing and flat-bottom cylindrical punch deep drawing.

7.6.1 Hemispherical Punch Deep Drawing

Theoretical predictions for deep drawing were compared with Woo's [Woo, 1968] experimental data for hemispherical punch drawing of copper with graphite-in-tallow lubrication. Table 7.1 gives a summary of tool dimensions, material properties, and process conditions. Figures 7.3 and 7.4 show mid-surface theoretical and experimental circumferential and thickness strain distributions against current radial distance from the pole. These results are for five cup heights 17.2 mm, 21.2 mm, 25.4 mm, 29.1 mm, and 37.8 mm corresponding to \( \frac{r' f}{r f_{1}} \) (draw-in ratio; ratio of current workpiece outer boundary radius to blank radius) = 0.969, 0.951, 0.925, 0.903, and 0.827, respectively. A punch friction coefficient of 0.04 and a die friction coefficient of 0.12 were chosen and was found to provide good agreement for circumferential strains at all cup heights. The agreement between theoretical and experimental thickness strain distributions, however, is weak especially in the punch contact region. This is because the current axisymmetric analysis assumes the material to be isotropic in the plane of the sheet, and therefore the radial and circumferential strains at the punch pole are equal and are equal to half the negative value of the thickness strain. However, in the experiments radial and circumferential strains are not equal at the pole for an anisotropic sheet, and the ratio between the strains roughly equals
Table 7.1  Summary of Tool Dimensions, Material Properties, and Process Conditions for Hemispherical Punch Drawing of Copper [Woo, 1968]

**Tool Dimensions**
- Punch Diameter = 50.8 mm (2.0 in.)
- Punch Nose Radius = 25.4 mm (1.0 in.)
- Die Opening Diameter = 53.9 mm (2.12 in.)
- Die Rounding Radius = 12.7 mm (0.50 in.)

**Material Properties and Blank Dimensions**
- R (Anisotropy Factor) = 1.00
- K (Strain-Hardening Coeff.) = 336.5 MPa (48.8 Ksl)
- n (Strain-Hardening Exp.) = 0.375
- $\varepsilon_0$ (Pre-strain) = 0.04
- Blank Diameter = 112.8 mm (4.44 in.)
- Blank Thickness = 0.89 mm (0.035 in.)

**Process Conditions**
- Friction Coefficient = 0.04 (Punch); 0.12 (Die)
  (for best match between prediction and experiments)
- Blankholder Force = 4.44 kN (1.0 Klb)
- Cup Heights = 17.2, 21.2, 25.4, 29.1, 37.8 mm
  (0.68, 0.83, 1.00, 1.15, 1.49 in.)
Figure 7.3  Mid Surface Circumferential Strain Distributions for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication (Punch Heights = 17.2 mm, 21.2 mm, 25.4 mm, 29.1 mm, and 37.8 mm; Draw-in Ratios = 0.969, 0.951, 0.925, 0.903, 0.827) (Experimental Data from [Woo, 1968])
Figure 7.4  Mid Surface Thickness Strain Distributions for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication (Punch Heights = 17.2 mm, 21.2 mm, 25.4 mm, 29.1 mm, and 37.8 mm; Draw-in Ratios = 0.969, 0.951, 0.925, 0.903, 0.827) (Experimental Data from [Woo, 1968])
the ratio of anisotropic factors in the two directions. The current analysis procedure cannot incorporate planar anisotropy (it handles normal anisotropy, however, using average anisotropy factor.) for it would violate the axisymmetric assumption. Figure 7.5 compares theoretical punch force prediction with experimental data. Figure 7.6 presents the deformed workpiece geometry at the five punch heights; workpiece geometries are staggered by 3 mm to illustrate the draw-in of the sheet boundary. A kink appears in the deformed workpiece geometry at a current radial distance of 27 mm for a cup height of 35 mm corresponding to the discontinuity seen in the circumferential strain distribution at the matching boundary (Figure 7.3; radius = 27 mm, strain = -0.28, draw-in ratio = 0.827).

7.6.2 Flat-Bottom Cylindrical Punch Deep Drawing

Theoretical predictions for flat-bottom cylindrical punch drawing were compared with experimental data obtained from Saran et al. [Mattiasson, 1987 and Saran, 1989a]. Two example cases were analyzed: one with low-strength steel deep drawn to produce a cup with a tapering wall (large clearance between the punch and the die throat; see Table 7.2 for tool dimensions, material properties, and process conditions) and the other with brass deep drawn to produce a cup with vertical wall (narrow clearance between the punch and the die throat; see Table 7.3 for tool dimensions, material properties, and process conditions) having sharp die and punch corner radii. According to Saran et al. the latter tool setup would present a severe test for computations and is of interest from industry standpoint.

Figures 7.7 and 7.8 show theoretical and experimental radial and circumferential strain distributions against original radial distance from the pole for low-strength steel for cup heights of 19.7 mm, 40.1 mm, and 64.0 mm with mineral oil lubrication and boundary adhesive. A friction coefficient of 0.15 was chosen to obtain best possible match between
Figure 7.5  Punch-Force Variation for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication (Experimental Data from [Woo, 1968])
Figure 7.6  Deformed Workpiece Geometry for Hemispherical Punch Drawing of Copper under Graphite-in-Tallow Lubrication (Punch Heights = 17.2 mm, 21.2 mm, 25.4 mm, 29.1 mm, and 37.8 mm) (Plots have been staggered by 3 mm to illustrate draw-in of the workpiece boundary.)

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<td>Cup Heights</td>
<td>19.7, 40.1, and 64.0 mm (0.78, 1.58, and 2.52 in.)</td>
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<tr>
<td>Punch Diameter</td>
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<tr>
<td>Punch Nose Radius</td>
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<tr>
<td>Die Opening Diameter</td>
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</tr>
<tr>
<td>Die Rounding Radius</td>
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<td>R (Anisotropy Factor)</td>
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<tr>
<td>K (Strain-Hardening Coeff.)</td>
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</tr>
<tr>
<td>n (Strain-Hardening Exp.)</td>
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<tr>
<td>ε₀ (Pre-strain)</td>
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<tr>
<td>Blank Diameter</td>
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<tr>
<td>Blank Thickness</td>
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<td>Friction Coefficient</td>
<td>0.06</td>
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<td>(for best match between prediction and experiments)</td>
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<tr>
<td>Blankholder Force</td>
<td>100.0 kN (22.5 Klb)</td>
</tr>
<tr>
<td>Cup Height</td>
<td>30.0 mm (1.18 in.)</td>
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Figure 7.7  Radial Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication (Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
Figure 7.8  Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication (Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
theoretical predictions and experimental data. The theoretical predictions are shown for inner, mid, and outer sheet surfaces, since it was not known from literature whether the experimental data were measured on the outer or the inner surface or average values were computed from inner and outer surface strains to roughly correspond to midsurface strains. As seen from Figures 7.7 and 7.8, the theoretical prediction is able to capture the radial strain maxima and circumferential strain minima fairly well, and the agreement between theoretical prediction and experimental data is good. Figure 7.9 presents a sensitivity comparison for a cup height of 40.1 mm varying the punch and the die friction coefficients from 0.0 to 0.35. The friction coefficient under the blankholder was not changed and therefore, the tangential restraining force due to blankholder was kept constant. As seen from Figure 7.9, there is no significant difference in the midsurface radial strain distribution except when $\mu = 0.35$. In drawing operation the friction over the punch and the friction over the die are counteractive. Higher friction over the punch locks the sheet and tries to draw the flange in, while lower friction enhances stretching of the material over the punch and results in less draw-in of the flange. On the other hand, higher friction over the die corner radius restricts the drawing of the flange and tries to stretch the material in the unsupported region and over the punch head; lower friction over the die corner radius allows easy drawing of the flange, and does not necessarily stretch the material in the unsupported region and over the punch head. Therefore, up to about 40 mm, the radial strain distribution exhibits almost similar variation for friction values 0.0 through 0.25. However for higher heights (50 mm and above) and higher friction (0.35 and above), the material gets locked over the punch head and also over the die and results in radial strains of the order of 100-150% suggesting excessive necking and fracture near the punch corner radius. Figure 7.10 presents the punch force variation against displacement for a friction
Figure 7.9 Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Various Friction Conditions (Punch Height = 40.1 mm) (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
Figure 7.10  Punch Force Variation for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
value of 0.15. Although the comparison is very good between theoretical and experimental punch forces up to about a punch displacement of 52 mm, after 52 mm the experimental punch force remains almost constant and suddenly starts decreasing at about 64 mm, while the theoretical prediction is still rising after 52 mm, however, very gradually up to about 55 mm and starts decreasing suddenly at about 67 mm. According to the experiments, the sheet was completely drawn out of the blankholder at the punch depth of 64 mm, and the tangential frictional restraining force is almost zero at 64 mm. Furthermore, the sheet boundary is almost at the die throat, and there is no further work done for flange compression. Therefore the experimental punch force starts decreasing at 64 mm and eventually be zero. However, in the simulation, the sheet boundary is still under the blankholder. This is also evidenced by the location of a minimum point shown in the theoretical circumferential strain distribution for the cup height of 64 mm. Circumferential strain minimum occurs at the die corner radius for cup drawing. In the experimental strain distribution, the minimum is at the sheet boundary (100 mm) for 64 mm cup height indicating that the sheet boundary has moved to the die corner and therefore, it is no longer under the blankholder. In the theoretical strain distribution, the minimum is at 93 mm, and the sheet boundary is still under the blankholder for a punch height of 64 mm. Therefore, radial frictional restraining force combined with continued flange compression prevents the punch force from decreasing. However, at about 67 mm the sheet boundary is drawn out of the blankholder, and with no further flange compression, the predicted punch force starts falling rapidly. Figure 7.11 presents the deformed workpiece geometry for the three cup heights, as before, staggered by 3 mm to illustrate the draw-in of the sheet boundary. The unsupported cup wall region is angular or tapered due to the relatively
Figure 7.11  Deformed Workpiece Geometry for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication (Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) (Plots have been staggered by 3 mm to illustrate draw-in of the workpiece boundary.)
large clearance between the die and the punch. As before, kinks appear at the intersection of die-contact and unsupported zones at higher punch depths due to the discontinuities in the mid-surface circumferential strain distributions at the matching boundary (Figure 7.8).

Figures 7.12 and 7.13 show theoretical and experimental radial and circumferential strain distributions against original radial distance from the pole for brass for a cup height of 30.0 mm. Theoretical predictions have been shown for inner, mid, and outer sheet surfaces. As seen from Figures 7.12 and 7.13, theoretical prediction is able to capture the maxima and minima fairly well and the agreement between mid-surface theoretical prediction and experimental data is good. Figure 7.14 presents punch force variation against punch displacement, and the comparison is very good between theoretical and experimental punch forces. Figure 7.15 presents the deformed workpiece geometry, and it can be seen that the wall is almost vertical due to the narrow clearance between the die throat and the punch.

7.6.3 Drawbead Simulation

Since SHEET_FORM is capable of simulating drawbead restraint, the above cases were rerun replacing the blankholder force by equivalent drawbead tangential restraining force at the boundary using the following equation:

$$DBF = 2\mu BHF$$  \hspace{1cm} (7-4)

where DBF is the drawbead force, BHF is the blankholder force, and \(\mu\) is the friction coefficient between the blankholder and the sheet. Figures 7.16 and 7.17 show theoretical and experimental radial and circumferential strain distributions against original radial distance from the pole for low-strength steel for cup heights of 19.7 mm, 40.1 mm, and
Figure 7.12  Radial Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Brass under Oil Lubrication (Punch Height = 30.0 mm) (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
Figure 7.13  Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Brass under Oil Lubrication (Punch Height = 30.0 mm) (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
Figure 7.14  Punch Force Variation for Flat-Bottom Cylindrical Punch Drawing of Brass under Oil Lubrication (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
Figure 7.15 Deformed Workpiece Geometry for Flat-Bottom Cylindrical Punch Drawing of Brass under Oil Lubrication (Punch Height = 30.0 mm)
64.0 mm using mineral oil lubrication with boundary adhesive. The blankholder-restraint comparisons were shown earlier in Figures 7.7 and 7.8, and the material, tool, and process information was provided in Table 7.2. As seen from Figures 7.16 and 7.17, the drawbead simulation presents equally good comparison between the experimental data and theoretical predictions.

7.7 CPU-Time Efficiency

The analysis module appears to be extremely CPU-time efficient. It took 36 CPU seconds on VAX-8550 to simulate the hemispherical punch stretching to a height of 28.3 mm (Figure 6.6). It took 61 CPU seconds to simulate the flat-bottom cylindrical punch stretching to a height of 22 mm (Figures 6.14 and 6.15). It took 91 and 102 CPU seconds to simulate the 2:1 ellipsoidal punch stretching to a height of 37.3 mm with friction coefficients of 0.01 and 0.04, respectively (Figures 6.19 and 6.20). The simulation of 1:2 ellipsoidal punch stretching to a height of 50.3 mm took 226, 231, and 189 seconds, respectively, for friction coefficients of 0.01, 0.04, and 0.09 (Figures 6.21 and 6.22). The difference in CPU time is about 20% for the different friction conditions and can be attributed to the number of iterations needed to achieve convergence in each time step. Usually, it was found that under the same material properties, blank dimensions, and process conditions, the hemispherical punch stretching took the least amount of time followed by the flat-bottom cylindrical punch stretching and the general-profile punch stretching.

It took 78 seconds to simulate hemispherical punch drawing to a height of 37.8 mm (Figures 7.3 and 7.4) and 152 seconds to simulate cylindrical punch drawing with a blankholder to a height of 64.0 mm (Figures 7.7 and 7.8).
Figure 7.16  Radial Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication (Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) (Drawbead Simulation) (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
Figure 7.17  Circumferential Strain Distributions for Flat-Bottom Cylindrical Punch Drawing of Low-Strength Steel under Mineral Oil Lubrication (Punch Heights = 19.7 mm, 40.1 mm, and 64.0 mm) (Drawbead Simulation) (Experimental Data from [Mattiasson, 1987 and Saran, 1989a])
8.1 Knowledge-Based Approach to Process Sequence Design

Industrial visits and the literature survey show that several metal forming companies have already gained considerable savings in time, efforts, and money, and also an improved quality of products by employing some of the Computer-Aided Design/Computer-Aided Manufacture/Computer-Aided Engineering (CAD/CAM/CAE) techniques. These include maintaining a common product data base, modeling and understanding better plastic flow of metals, employing numerically controlled contour milling machines, using sophisticated computer-interfaced contour measurement devices, etc. However, process planning and die designing still remains to be a trial-and-error process, and therefore no fully integrated system exists today in industry.

Talks with experts in industry reveal that automatic generation of process outline is a difficult but important requirement in developing a practical computer-integrated manufacturing system. They see that such a goal is attainable only gradually through a determined and huge effort. As a first step, the present research has developed an automatic process sequence system for a specific domain of problems, namely axisymmetric geometries.
The system's performance is comparable to that practiced in industry today. The process sequences outputted by the system compare very well with those from industrial practice as described in Chapter 4.

The present knowledge-based system is likely to result in quantifiable savings in time, effort and cost. For example, it would take about 20 man-hours for a die designer to design a typical process sequence shown in Chapter 4 using conventional means. On the other hand, the same process sequence can probably be designed using the developed knowledge-based system in about half an hour, and nearly 90% of this time is for preparing the input object geometry file.

8.2 Analysis of Sheet Forming Operations

An analysis module SHEET_FORM to simulate axisymmetric sheet metal forming operations such as stretch forming and deep drawing has been developed. Strains and punch force values predicted by SHEET_FORM agree with experimental data within engineering accuracy. SHEET_FORM can be used as a reliable stand-alone tool for evaluating press forming severity in axisymmetric sheet metal forming. It has been shown that it is possible to model tool-workpiece interface friction conditions, to model anisotropic behavior of metals, to account for bending effects, etc. using finite-difference approach. It has also been shown that the present program can successfully model sheet forming using nonconventional convex punch profiles.

8.3 Outline of Future Research Work

• Integration of the KBS and SHEET_FORM: Currently the knowledge-based system and the analysis module stand separate. Therefore, the user has to run
the knowledge-based system, obtain a process sequence, and then run the
analysis module to finetune the process sequence. The future work will look at
integrating the two modules so that the knowledge-based system will opaquely
communicate with the analysis module to rectify the initial-guess process
sequence [Kinzel, 1989].

- **Simulation of Axisymmetric Direct and Reverse Redrawing:** The current
analysis module is not complete because it can simulate only those forming
processes where the starting geometry is a flat sheet. In other words, it can
simulate only the first station of the process sequence. As part of the future
work, the analysis program will be extended to model axisymmetric direct
redrawing and reverse redrawing operations [Ahmetoglu, 1989]. Reissner and
Ehrismann's [Reissner, 1987] work, discussed earlier, has employed finite-
difference technique to model such direct and reverse redrawing operations for
cylindrical geometries. The finite-difference approach, however, is limited in
scope and cannot possibly analyze forming from one complex geometry to
another complex geometry. For such tasks, the finite-element approach may be
employed.

- **IGES-Based Input and Output:** Currently, alphanumeric scheme is employed to
input object geometry to the knowledge-based system. Although it is relatively
easy to input object geometries through this scheme, this scheme will require a
'human in the chain' in an integrated design and manufacturing system.
Therefore, the input-output module of the knowledge-based system needs to be
modified so that it can accept input object geometry information in a neutral file
format (for example, IGES - Initial Graphics Exchange Specification) and output intermediate object geometries in the process sequence in the same format [Kinzel, 1989].

- **Process Sequence Design for Nonaxisymmetric Geometries:** The current knowledge base can be extended for designing process sequence for nonaxisymmetric sheet metal geometries [Kinzel, 1990].

- **Plane-Strain Analysis:** The current analysis procedure can also be extended to model plane-strain stretching and drawing [Ahmetoglu, 1989]. This analysis module will then be linked with the expanded knowledge-based system discussed in the previous section [Kinzel, 1990]. The scope of finite-difference procedure is limited to two-dimensional analysis such as axisymmetric and plane-strain analysis. For complex three-dimensional geometries which cannot be analyzed by the plane-strain model, the finite-element procedure will be employed.

### 8.4 Summary of Research Contributions

The current research has following contributions from a practical as well as an academic standpoint:

1) The hybrid computer-aided engineering system for process planning in die design for the manufacture of axisymmetric sheet metal parts is probably one of very first systems in the domain. The presence of analytical modules, when integrated, will make the expert system more powerful and flexible than conventional semi-generative or generative rule-based expert systems in other
areas of manufacturing which utilize only empirical formulas and other rules of thumb for reasoning.

2) Techniques for acquiring and representing knowledge in a complex domain like the sheet metal die design are still in their infancy. The current research has made a significant contribution in knowledge acquisition and representation.

3) Expert systems, developed so far, acquire case-specific knowledge from the users through a question-answer interactive session, which is relatively easy to be 'understood' by the expert system. The current knowledge-based system, however, has to deal with features-and-attributes based input scheme to understand the input object geometry.

4) It is still widely believed by most in industry that die design is essentially a trial and error process and probably will remain so in the future due to the complexities involved in the die designing procedure. The current computer-aided engineering system, if found acceptable in the industry, would pave way in eliminating such a long-held belief and would certainly stimulate further research and development into the next few decades.

5) Mathematical modeling of sheet metal forming processes is a challenging task because of the difficulty associated with representing tool-sheet interface friction conditions, contact boundaries, and plastic properties of materials. Although it has taken advantage of several earlier work in mathematical modeling of sheet forming processes, the current analysis procedure has successfully utilized the finite-difference procedure to forming using general
convex punch profiles, has employed modified coulomb friction model, has modeled unloading phenomenon for rigid-plastic material, has accounted for bending effect, has employed recent developments in the theory of plasticity, etc. Results predicted by the current analysis approach compare very well with experimental data, and the CPU time is about one order of magnitude less than that needed by finite-element method. Ease of data input is another added advantage. The present approach satisfies equilibrium, plasticity and compatibility equations at every stage of the process and is free from empirical justifications. The techniques employed by the current research and its findings, being academically fundamental and at par with recent progresses, are therefore likely to become an important addition to the expanding realm of literature on process modeling of sheet metal forming.


ERC/NSM-87-9, Engineering Research Center for Net Shape Manufacturing, The Ohio State University, Columbus, Ohio 43210, July 1987.

Germain, 1989

Goodall, 1985

Haller, 1986

Hecker, 1975

Higashi, 1985

Hill, 1948

Hill, 1950

Hill, 1979

Hosford, 1983

Ishigaki, 1984

Jones, 1941

Kaftanoglu, 1970


Lee, 1988

Mattiasson, 1987

Miedema, 1982

Murakami, 1980

Nilsson, 1980

Okamoto, 1988

Reissner, 1987

Sachs, 1934

Sachs, 1951

Saran, 1989a

Saran, 1989b

Saran, 1989c

Schuler, 1966  

Sevenler, 1986a  

Sevenler, 1986b  

Sfiligoj, 1983  

Slater, 1977  

Stoughton, 1985  

Swift, 1939  

Swift, 1952  

Takahashi, 1985  

Taylor, 1983  

Tisza, 1986a  


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APPENDIX A

PLASTICITY EQUATIONS BASED ON HILL'S OLD THEORY
The following sections provide a detailed outline for obtaining the plasticity equations for anisotropic materials using Hill's 'old' yield function [Wang, 1982; Germain, 1989]

The yield function \( f \) is given as [Hill, 1950]

\[
f = F(\sigma_2 - \sigma_3)^2 + G(\sigma_1 - \sigma_3)^2 + H(\sigma_1 - \sigma_2)^2 \tag{A-1}
\]

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses and \( F, G, \) and \( H \) are constants.

Assuming normal anisotropy in the \( x_3 \) direction and planar isotropy in the \( x_1-x_2 \) plane, \( \sigma_1 \) and \( \sigma_2 \) can be interchanged in equation (A-1) and can be written as

\[
f = F(\sigma_1 - \sigma_3)^2 + G(\sigma_2 - \sigma_3)^2 + H(\sigma_2 - \sigma_1)^2 \tag{A-2}
\]

Comparing equation (A-2) with (A-1) and substituting \( \sigma_1 = \sigma, \sigma_2 = 0, \) and \( \sigma_3 = 0 \) for a uniaxial loading, it can be seen that

\[
F = G = \alpha \tag{A-3}
\]

and

\[
f = \alpha(\sigma_1 - \sigma_3)^2 + \alpha(\sigma_2 - \sigma_3)^2 + H(\sigma_1 - \sigma_2)^2 = (\alpha + H)\bar{\sigma}^2 \tag{A-4}
\]

where \( \alpha \) is an arbitrary constant and \( \bar{\sigma} \) is the equivalent stress.

The flow rule is written as

\[
de^p_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \tag{A-5}
\]
where $\Delta e_{ij}^p$ is the strain tensor. Under large plastic deformation, the elastic strain component can be neglected and therefore $\Delta e_{ij} = \Delta e_{ij}^p$. Using equation (A-5), the constitutive equations between principal stresses and strains can be derived as

$$\frac{\Delta e_1}{\alpha(\sigma_1 - \sigma_3) + H(\sigma_1 - \sigma_2)} = \frac{\Delta e_2}{\alpha(\sigma_2 - \sigma_3) + H(\sigma_2 - \sigma_1)} = \frac{\Delta e_3}{\alpha(2\sigma_3 - \sigma_1 - \sigma_2)} = 2\alpha d\lambda$$ (A-6)

Since normal anisotropy factor $R$ is defined to be $\Delta e_2^p / \Delta e_3^p$ under uniaxial normal loading in $x_1$ direction, from equation (A-6) it can be shown that

$$\frac{\Delta e_2}{\Delta e_3} = \frac{H}{\alpha} = R$$ (A-7)

Equation (A-6) can be rewritten using the above relation as

$$\frac{\Delta e_1}{(1 + R)\sigma_1 - R\sigma_2 - \sigma_3} = \frac{\Delta e_2}{(1 + R)\sigma_2 - R\sigma_1 - \sigma_3} = \frac{\Delta e_3}{(2\sigma_3 - \sigma_1 - \sigma_2)} = 2\alpha d\lambda$$ (A-8)

The equation for equivalent plastic work can be written as

$$\bar{\sigma}d\varepsilon = \sigma_1\Delta e_1 = \sigma_1\Delta e_1 + \sigma_2\Delta e_2 + \sigma_3\Delta e_3$$ (A-9)
Using volume constancy relation $d\varepsilon^+ + d\varepsilon^2 + d\varepsilon^3 = 0$ and using equation (A-8) an expression for $d\lambda$ can be derived as

$$d\lambda = \frac{d\varepsilon}{2\alpha(1 + R)\bar{\sigma}}$$

(A-10)

and $\bar{\sigma}$ can be shown to be

$$\bar{\sigma} = \left\{ \frac{1}{1 + R} \left[ (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 + R(\sigma_1 - \sigma_2)^2 \right] \right\}^{0.5}$$

(A-11)

The inverse normality equations between principal strain increments and principal stresses can be derived from the above equations as

$$d\varepsilon_1 = \frac{d\varepsilon}{(1 + R)\bar{\sigma}} [(1 + R)\sigma_1 - R\sigma_2 - \sigma_3]$$

$$d\varepsilon_2 = \frac{d\varepsilon}{(1 + R)\bar{\sigma}} [(1 + R)\sigma_2 - R\sigma_1 - \sigma_3]$$

(A-12)

$$d\varepsilon_3 = \frac{d\varepsilon}{(1 + R)\bar{\sigma}} [2\sigma_3 - \sigma_1 - \sigma_2]$$

From equation (A-12), it is can be shown that

$$\sigma_2 - \sigma_1 = \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\varepsilon} [d\varepsilon_2 - d\varepsilon_1]$$

$$\sigma_3 - \sigma_2 = \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\varepsilon} [Rd\varepsilon_3 - d\varepsilon_2]$$

(A-13)

$$\sigma_3 - \sigma_1 = \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\varepsilon} [Rd\varepsilon_3 - d\varepsilon_1]$$
Using equations (A-9) and (A-13) and eliminating all stress terms, a relation between the equivalent strain and principal strains can be established as

\[
d\varepsilon^2 = \left( \frac{1 + R}{1 + 2R} \right) \left( d\varepsilon_1^2 + d\varepsilon_2^2 + R d\varepsilon_3^2 \right)
\]  

(A-14)

In polar coordinates, \( d\varepsilon_i = d\varepsilon_r, \) \( d\varepsilon_2 = d\varepsilon_\theta, \) \( d\varepsilon_3 = d\varepsilon_i, \) and \( \sigma_i = \sigma_r, \) \( \sigma_2 = \sigma_\theta, \) \( \sigma_3 = \sigma_i. \) Hence

\[
\sigma_\theta - \sigma_r = \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\varepsilon} [2d\varepsilon_\theta + d\varepsilon_i]
\]

\[
\sigma_r - \sigma_\theta = \frac{(1 + R)\bar{\sigma}}{(1 + 2R)d\varepsilon} [d\varepsilon_\theta + (1 + R)d\varepsilon_i]
\]

(A-15)

\[
d\varepsilon^2 = \left( \frac{1 + R}{1 + 2R} \right) \left( 2d\varepsilon_\theta^2 + (1 + R)d\varepsilon_i^2 + 2d\varepsilon_\theta d\varepsilon_i \right)
\]
APPENDIX B

PLASTICITY EQUATIONS BASED ON HILL'S NEW THEORY
The following sections provide a detailed outline for obtaining the plasticity equations for anisotropic materials using Hill's 'new' yield function.

The yield function $f$ according to Hill's 'New' Theory [Hill, 1979] is

$$f = F|\sigma_2 - \sigma_3|^M + G|\sigma_3 - \sigma_1|^M + H|\sigma_1 - \sigma_2|^M + A|2\sigma_1 - \sigma_2 - \sigma_3|^M + B|2\sigma_2 - \sigma_3 - \sigma_1|^M + C|2\sigma_3 - \sigma_1 - \sigma_2|^M$$  \hspace{2cm} (B-1)

where $F$, $G$, $H$, $A$, $B$, and $C$ are constants, $\sigma_1$, $\sigma_2$, and $\sigma_3$ are principal stresses and $M$ is a material mechanical property constant. According to Hill, a simplified form of this equation can be written as

$$f = H|\sigma_1 - \sigma_2|^M + C|2\sigma_3 - \sigma_1 - \sigma_2|^M$$  \hspace{2cm} (B-2)

when $A = 0 = B$ and $F = 0 = G$. Assuming $\sigma_1 > \sigma_2 > \sigma_3$ and substituting $\sigma_1 = \sigma$, $\sigma_2 = 0$, and $\sigma_3 = 0$ under uniaxial loading, the above equation can be written as

$$f = H(\sigma_1 - \sigma_2)^M + C(\sigma_1 + \sigma_2 - 2\sigma_3)^M = (H + C) \sigma^M$$  \hspace{2cm} (B-3)

From the flow rule

$$\text{de}_{ij} = d\lambda \left( \frac{df}{d\sigma_{ij}} \right)$$  \hspace{2cm} (B-4)

the constitutive equations between principal stresses and strains can be established as
\[
\frac{de_1}{H(\sigma_1 - \sigma_2)^{M-1} + C(\sigma_1 + \sigma_2 - 2\sigma_3)^{M-1}} = \frac{de_2}{-H(\sigma_1 - \sigma_2)^{M-1} + C(\sigma_1 + \sigma_2 - 2\sigma_3)^{M-1}} = \frac{de_3}{-2C(\sigma_1 + \sigma_2 - 2\sigma_3)^{M-1}} = M d\lambda
\]

(B-5)

As explained in Appendix A, it can be shown that

\[
\frac{H}{C} = (2R + 1)
\]

(B-6)

Using equation (B-6), equation (B-5) can be rewritten as

\[
\frac{de_1}{(2R + 1)(\sigma_1 - \sigma_2)^{M-1} + (\sigma_1 + \sigma_2 - 2\sigma_3)^{M-1}} = \frac{de_2}{-(2R + 1)(\sigma_1 - \sigma_2)^{M-1} + (\sigma_1 + \sigma_2 - 2\sigma_3)^{M-1}} = \frac{de_3}{-2(\sigma_1 + \sigma_2 - 2\sigma_3)^{M-1}} = C M d\lambda
\]

(B-7)

Using the above equation, the equivalent plastic work equation (A-9) and the principle of volume constancy, an expression for \(d\lambda\) and \(\bar{\sigma}\) can be derived as

\[
d\lambda = \frac{d\bar{\sigma}}{2(1 + R) C M \bar{\sigma}^{M-1}}
\]

(B-8)

and

\[
\bar{\sigma} = \left\{ \frac{1}{2(1 + R)} \left[ (1 + 2R)(\sigma_1 - \sigma_2)^{M} + (\sigma_1 + \sigma_2 - 2\sigma_3)^{M} \right] \right\}^{\frac{1}{M}}
\]

(B-9)
and the equivalent strain increment can be expressed as a function of the principal strain increments as

\[
d\bar{e} = \frac{[2(1+R)]^\frac{1}{2}}{2}(\frac{(de_1 - de_2)}{(1+2R)^{\frac{1}{2-p}}} + (de_1 + de_2))^{\frac{1}{2-p}}
\]  

(B-10)

The stress-strain relations can then be expressed as

\[
\sigma_1 - \sigma_2 = \left(\frac{1+R}{1+2R}\right)^\frac{1}{2-p}\bar{\sigma}\left(-de_3 - 2de_2\right)^\frac{1}{2-p}
\]  

(B-11)

and

\[
\sigma_1 - \sigma_3 = \frac{1}{2}(1+R)^{\frac{1}{2-p}}\bar{\sigma}\left[\left(-2de_2 - de_3\right)^\frac{1}{2-p} + (-de_3)^\frac{1}{2-p}\right]
\]  

(B-12)

In polar coordinates \(de_1 = de_r\), \(de_2 = de_\theta\), and \(de_3 = de_t\), equations (B-10) to (B-12) can be rewritten as

\[
d\bar{e} = \frac{[2(1+R)]^\frac{1}{2}}{2}\left\{\frac{2de_\theta + de_t^\frac{1}{2-p}}{(1+2R)^{\frac{1}{2-p}}} + [de_t]^\frac{1}{2-p}\right\}
\]  

(B-13)

\[
\sigma_r - \sigma_\theta = \left(\frac{1+R}{1+2R}\right)^\frac{1}{2-p}\bar{\sigma}\left(-de_t - 2de_\theta\right)^\frac{1}{2-p}
\]
Equations (B-13) and (B-14) are applicable only when \( \sigma_r > \sigma_\theta > \sigma_t \). A more generalized form can be derived using the above approach as

\[
\sigma_r - \sigma_t = \frac{1}{2}(1 + R) \frac{\bar{\sigma}}{d\varepsilon^{\nu - 1}} \left[ \left( \frac{-2d\varepsilon_\theta - d\varepsilon_t}{2R + 1} \right)^{\frac{1}{\nu-1}} + (d\varepsilon_t)^{\frac{1}{\nu-1}} \right] \tag{B-14}
\]

\[
d\varepsilon = \frac{[2(1 + R)]^{\frac{1}{\nu}}}{2} \left\{ \frac{|2d\varepsilon_\theta + d\varepsilon_t|}{(1 + 2R)^{\frac{1}{\nu - 1}}} \right\} \tag{B-15}
\]

\[
\sigma_r - \sigma_\theta = \pm \left( \frac{1 + R}{1 + 2R} \right) \frac{\bar{\sigma}}{d\varepsilon^{\nu - 1}} |d\varepsilon_t + 2d\varepsilon_\theta|^{\frac{1}{\nu - 1}}
\]

where + is used when \( d\varepsilon_r \) is greater than or equal to \( d\varepsilon_\theta \) and

- is used when \( d\varepsilon_r \) is less than \( d\varepsilon_\theta \)

\[
\sigma_r - \sigma_t = \frac{1}{2}(1 + R) \frac{\bar{\sigma}}{d\varepsilon^{\nu - 1}} \left[ \pm \left( \frac{2d\varepsilon_\theta + d\varepsilon_t}{2R + 1} \right)^{\frac{1}{\nu - 1}} \pm |d\varepsilon_t|^{\frac{1}{\nu - 1}} \right] \tag{B-16}
\]

where + (first) is used when \( d\varepsilon_r \) is greater than or equal to \( d\varepsilon_\theta \) and

- (first) is used when \( d\varepsilon_r \) is less than \( d\varepsilon_\theta \)

+ (second) is used when \( d\varepsilon_t \) is less than or equal to zero and

- (second) is used when \( d\varepsilon_t \) is greater than zero.
APPENDIX C

EQUILIBRIUM EQUATIONS BASED ON A GENERAL PUNCH PROFILE
The following sections present a detailed outline for deriving the equilibrium equations for the workpiece undergoing deformation over a curved profile.

Referring to Figure 5.1 for an axisymmetric geometry, the force balance equation along the radial direction [Kaftanoglu, 1970] is

\[
-\left(\sigma_1 - \frac{\delta\sigma_1}{2}\right)(r - \frac{\delta r}{2})\delta\theta \left(t - \frac{\delta t}{2}\right)\cos\left(\alpha - \frac{\delta\alpha}{2}\right) + \\
\left(\sigma_1 + \frac{\delta\sigma_1}{2}\right)(r + \frac{\delta r}{2})\delta\theta \left(t + \frac{\delta t}{2}\right)\cos\left(\alpha + \frac{\delta\alpha}{2}\right) + \\
p_r \delta\theta \rho_1 \delta\alpha \sin\alpha - \mu p_r \delta\theta \rho_1 \delta\alpha \cos\alpha - 2\sigma_2 \rho'_1 \delta\alpha t \sin\frac{\delta\theta}{2} = 0
\]

where \(\mu\) is the friction coefficient, 
\[\rho'_1 = \rho_1 + \frac{t}{2},\]

and 
\[r' = r + \frac{t}{2} \sin \alpha\]

Similarly in the vertical direction the force equilibrium equation can be written as

\[
\left(\sigma_1 - \frac{\delta\sigma_1}{2}\right)(r - \frac{\delta r}{2})\delta\theta \left(t - \frac{\delta t}{2}\right)\sin\left(\alpha - \frac{\delta\alpha}{2}\right) - \\
\left(\sigma_1 + \frac{\delta\sigma_1}{2}\right)(r + \frac{\delta r}{2})\delta\theta \left(t + \frac{\delta t}{2}\right)\sin\left(\alpha + \frac{\delta\alpha}{2}\right) + \\
p_r \delta\theta \rho_1 \delta\alpha \cos\alpha + \mu p_r \delta\theta \rho_1 \delta\alpha \sin\alpha = 0
\]

For a small element \(\delta\alpha\) and \(\delta\theta\) are small and therefore, in the limiting case, equations (C-1) and (C-2) can be rewritten, respectively, as

\[
\frac{1}{r} \frac{d}{dr}(\sigma_1 t r') + \left[\rho'_1 p_r \left(\tan \alpha - \mu\right) - \sigma_1 t \tan \alpha - \sigma_2 \frac{\rho'_1 t}{r \cos \theta}\right] \frac{d\theta}{dr} = 0
\]
\[
\frac{1}{r} \frac{d}{d\alpha} (\sigma_1 r) + \left[ \sigma_1 r \cot \alpha - \rho_1 \frac{r}{r} (\cot \alpha + \mu) \right] = 0
\] (C-4)

From the above two equations an expression for \( p \) can be obtained as

\[
p = \frac{1}{\rho_1} \left[ \sigma_1 r + \sigma_2 p_1 \sin \alpha \right]
\] (C-5)

and eliminating \( p \) from equation (C-3) using equation (C-5), a semi-general-purpose equilibrium equation in terms of principal stresses and geometric parameters for elements over the punch or die rounding radius can be derived to be

\[
\frac{1}{r} (\sigma_1 t) \frac{dr}{d\alpha} + \frac{d}{d\alpha} (\sigma_1 t) = \frac{\sigma_2 p_1 t}{r} (\cos \alpha + \mu \sin \alpha) + \sigma_1 \mu t
\] (C-6)

In polar coordinates \( \sigma_1 = \sigma_r \) and \( \sigma_2 = \sigma_\theta \) and therefore, equation (C-6) can be rewritten as

\[
\frac{1}{r} (\sigma_r t) \frac{dr}{d\alpha} + \frac{d}{d\alpha} (\sigma_r t) = \frac{\sigma_\theta p_1 t}{r} (\cos \alpha + \mu \sin \alpha) + \sigma_r \mu t
\] (C-7)

Equation (C-7) can be modified for different punch geometries. For an axisymmetric geometry

\[
\frac{dr}{d\alpha} = \rho_1' \cos \alpha
\] (C-8)

and therefore the equilibrium equation for elements over the punch profile of a flat-bottomed cylindrical punch (Figure C.1) can be written in a finite-difference form as
\[ (\sigma_r, t)_{i+1} = (\sigma_r, t)_i + \int_{\phi_1}^{\phi_2} \rho_1^2 \left[ \sigma_e t (\cos \phi + \mu \sin \phi) - \sigma_r t \cos \phi \right] d\phi + \int_{\phi_1}^{\phi_2} \sigma_r \mu t \ d\phi \]

where \( \phi = \Pi - \alpha \)

\[ r' = r_c + \rho_1^2 \sin \phi = r_c + \left( \rho_1 + \frac{1}{2} \right) \sin \phi \]

and similarly for a hemispherical punch where \( r_c = 0 \), it can be shown that

\[ (\sigma_r, t)_{i+1} = (\sigma_r, t)_i + \int_{\phi_1}^{\phi_2} \left[ \sigma_e t (\cot \phi + \mu) - \sigma_r t \cot \phi \right] d\phi + \int_{\phi_1}^{\phi_2} \sigma_r \mu t \ d\phi \]
Figure C.1  Flat-Bottomed Cylindrical Punch
APPENDIX D

PRODUCTION RULES ASSOCIATED WITH FIRST PASS
[Rule 1] If the input final object geometry is a concatenation of elements of varying thicknesses, then select the maximum thickness to be the blank thickness.

[Rule 2] If the input final object geometry is axisymmetric, then the blank is circular.

[Rule 3] If the blank size is computed using equivalent surface area, then the actual blank area is 1.128 times the equivalent blank area.

[Rule 4] If elements of the input object geometry have monotonically nondecreasing diameters and heights from the bottom center¹ of the cup, then the elements constitute a set of direct elements.

[Rule 5] If elements of the input object geometry have monotonically nondecreasing diameters and monotonically nonincreasing heights from the bottom center of the cup, then the elements constitute a set of reverse elements.

[Rule 6] If an element belongs to the h1 class, then the element can be either reverse or direct.

¹ Bottom center refers to the pole of the cup. Top refers to the orifice or the mouth of the cup. An element X is said to be above an element Y, if X is closer to the top of the cup. An element X is said to be followed by an element Y, if X is the immediate element above Y.
[Rule 7] If a set of elements is direct,
then they can be formed by a combination of [cupping, direct
redrawing, tapering, stretchforming, sizing].

[Rule 8] If a set of elements is reverse,
then they can be formed by a combination of [reverse redrawing,
stretchforming, sizing].

[Rule 9] If the input object geometry is a concatenation of sets of direct and
reverse elements,
then the process sequence is a concatenation of process sequences for
each set.

[Rule 10] If a set of elements is reverse,
then the process sequence for the reverse set can be obtained by
changing the sign of individual element heights (from negative to
positive).

[Rule 11] If a process sequence is obtained for a set of reverse elements
then change the sign of individual element heights for the intermediate
object geometries (from positive to negative).

[Rule 12] If set of direct elements is followed by a set of reverse elements, and
a process sequence is obtained for a set of reverse elements
then append the process sequence for the set of reverse elements to the set of direct elements.

[Rule 13] If a set of direct elements is followed by a set of reverse elements or vice versa, then the starting deformation zone "blank" is converted into equivalent cylindrical cup and appended to the other set.

[Rule 14] The total material volume of a set of deformed elements, before and after deformation, is constant (or the total meridional surface area of a set of deformed elements, for all practical purposes, is constant).

[Rule 15] If the current deformed elements are known, then the previous deformed elements would be either a blank or a cylindrical cup which is a concatenation of h1 and v1 elements from the bottom of the cup.

[Rule 16] If previous deformed elements need to designed, then compute the dimensions of the previous deformed elements based on equivalent surface area of the current deformed elements.

[Rule 17] If the previous deformed cylindrical cup needs to designed, and if the diameter of the v1 element is less than twice the optimal radius,
then the bottom of the cup is hemispherical having a radius equal to half of the diameter of the vl element.

[Rule 18] If the current object geometry has three or less number of elements, then the previous object geometry is a blank.

[Rule 19] If the set of elements in the current object geometry starting from the bottom of the cup is [E1,E2,E3,E4] where
E1 is an element of type h1 or r1 or a1,
E2 is an element of type vl or a1 or r1,
E3 is an element of type h1 or a1 or r1 or c1, and
E4 is an element of type vl,
then the set of elements constitute a set of current deformed elements.

[Rule 20] If the set of elements in the current object geometry starting from the bottom of the cup is [E1,E2,E3] where
E1 is an element of type h1 or r1 or a1,
E2 is an element of type vl or a1 or r1, and
E3 is an element of type h1 or a1 or r1 or c1,
then the set of elements constitute a set of current deformed elements.

[Rule 21] If the set of elements in the current object geometry starting from the bottom of the cup is [E1,E2,E3] where
E1 is an element of type r1 or a1,
E2 is an element of type h1, and
E3 is an element of type \( v_1 \),

*then* the set of elements constitute a set of current deformed elements.

**[Rule 22]** *If* the set of elements in the current object geometry starting from the bottom of the cup is \([E_1, E_2, E_3]\) where

\[ E_1 \text{ is an element of type } h_1 \text{ or } r_1 \text{ or } a_1, \]
\[ E_2 \text{ is an element of type } c_1, \]
\[ E_3 \text{ is an element of type } v_1, \]

*then* the set of elements constitute a set of current deformed elements.

**[Rule 23]** *If* the set of elements in the current object geometry starting from the bottom of the cup is \([E_1, E_2, E_3]\) where

\[ E_1 \text{ is an element of type } v_1 \text{ or } a_1, \]
\[ E_2 \text{ is an element of type } h_1 \text{ or } a_1 \text{ or } r_1 \text{ or } c_1, \]
\[ E_3 \text{ is an element of type } v_1 \text{ or } a_1, \]

*then* the set of elements constitute a set of current deformed elements.

**[Rule 24]** *If* the set of elements in the current object geometry starting from the bottom of the cup is \([E_1, E_2]\) where

\[ E_1 \text{ is an element of type } h_1 \text{ or } r_1 \text{ or } a_1, \]
\[ E_2 \text{ is an element of type } v_1 \text{ or } a_1 \text{ or } r_1, \]

*then* the set of elements constitute a set of current deformed elements.

**[Rule 25]** *If* the set of elements in the current object geometry starting from the bottom of the cup is \([E_1, E_2]\) where
E1 is an element of type r1 or a1, and
E2 is an element of type h1,
\textit{then} the set of elements constitute a set of current deformed elements.

[Rule 26] \textit{If} the set of elements in the current object geometry starting from the bottom of the cup is [E1,E2] where
E1 is an element of type h1 or r1 or a1, and
E2 is an element of type c1,
\textit{then} the set of elements constitute a set of current deformed elements.

[Rule 27] \textit{If} the set of elements in the current object geometry starting from the bottom of the cup is [E1,E2] where
E1 is an element of type v1 or a1, and
E2 is an element of type h1 or a1 or r1 or c1,
\textit{then} the set of elements constitute a set of current deformed elements.

[Rule 28] \textit{If} the final object geometry has a flat flange, and the outer diameter of the flange is less than 1.45 cup diameter,
\textit{then} form the flange as an angular element and size it in the last station.

[Rule 29] \textit{If} the final object geometry has a flat flange, and the outer diameter of the flange is more than or equal to 1.45 cup diameter,
\textit{then} size the outside diameter of the flange in the first draw operation.
[Rule 30] If an element of an intermediate geometry is not part of the final object geometry, then design the fillet radius of this element to be optimal.

[Rule 31] If a fillet radius (die or punch round radius) is to be optimal, then make the fillet radius to be at least five times the element thickness.

[Rule 32] If a fillet radius (die or punch round radius) cannot be designed large enough to be optimal because of the geometric constraints of the adjacent elements, then make the fillet radius as large as possible.

[Rule 34] If the angle between two adjacent elements which are either both direct or a direct with a reverse above is less than 180 degrees (or convex), then the fillet radius between the two elements is formed using the punch nose radius.

[Rule 35] If the angle between two adjacent elements which are either both direct or a direct with a reverse above is more than 180 degrees (or concave), then the fillet radius between the two elements is formed using the die corner radius.
Rule 36] If the adjacent elements are either both reverse or a reverse with a direct above, then invert Rule 34 and Rule 35.

[Rule 37] The list of object geometries having the blank and the final object geometry as the end members after the first pass is the first_process_sequence.
APPENDIX E

PRODUCTION RULES ASSOCIATED WITH SECOND PASS
[Rule 38] If the first_process_sequence is to be tested against material formability, then every pair of adjacent geometries from the process sequence needs to be tested against material formability.

[Rule 39] If a pair of adjacent geometries is to be tested against material formability, then every deformed element in the current object geometry needs to be tested against material formability, and an element being tested is called as test_element.

[Rule 40] A test_element is required to satisfy all the appropriate formability parameters.

[Rule 41] The set of material formability parameters includes [Draw_ratio, Redraw_ratio, Height_to_diameter_ratio, Height_taper].

[Rule 42] If the current station is 1, then the test_element will be tested for Draw_ratio, Height_to_diameter_ratio, and Height_taper.

[Rule 43] If the current station is greater than 1, then the test_element will be tested for Redraw_ratio and Height_taper.
[Rule 44] If Draw\_ratio exceeds Limiting\_draw\_ratio
Or if Height\_to\_diameter\_ratio exceeds Limiting\_height\_to\_diameter\_ratio,
then a tear is predicted near the punch corner radius.

[Rule 45] If Redraw\_ratio exceeds Limiting\_redraw\_ratio,
then a tear is predicted near the punch corner radius.

[Rule 46] If Height\_taper exceeds Limiting\_height\_taper
then a puckering may occur near the die corner radius.

[Rule 47] If the test\_element exceeds the limiting value of any of the appropriate material formability parameters,
then it needs to be rectified.

[Rule 48] If the test\_element belongs to the h1 class,
then no testing needs to be done.

[Rule 49] If rectification is suggested,
then it can be accomplished either by introducing intermediate geometries between the pair of adjacent geometries under consideration or by annealing.

[Rule 50] If intermediate successive redrawing is included as part of the rectification,
then use the maximum reduction ratio possible for each station and form to desired diameter in the last station.

[Rule 51] If intermediate successive redrawing is to be included as part of the rectification, then increment the station numbers correspondingly.

[Rule 52] If the test_element is a v1 class element (vertical cylindrical element), and if it is the top element in the set of current deformed elements, and if rectification is suggested, then introduce cylindrical-shell-like intermediate geometry(s).

[Rule 53] If the test_element is a v1 class element, and if it is not the top element in the set of current deformed elements, and if rectification is suggested, and if the intermediate shell diameter is greater than the outer diameter of the element above then introduce cylindrical-shell-like intermediate geometry(s).

[Rule 54] If the test_element is a v1 class element, and if it is not the top element in the set of current deformed elements, and if rectification is suggested, and if the intermediate shell diameter is less than the outer diameter of the element above by less than twice the optimal fillet radius,
then introduce intermediate cylindrical-shell-like geometry(s) which is connected to the element above by almost zero-length 45 degree tapered element.

[Rule 55] If an intermediate object geometry has a c1 class element followed by a v1 class element, then there need not be any fillet radius between the two elements.

[Rule 56] If the test_element is an a1, r1, or c1 class element (tapered, convex, or concave section), and if rectification is suggested, then form it from an intermediate stepped (inverted stairway) geometry.

[Rule 57] If a stepped geometry needs to be formed as an intermediate stage for forming a deep tapered, convex, or concave section, then the total height of the stepped geometry should never exceed the height of the tapered, convex, or concave section and shall be about 80% of the height of the tapered, convex, or concave section.

[Rule 58] If a stepped geometry needs to be formed, then form it through successive redraw operations with equal step heights.
[Rule 59] If a stepped geometry needs to be formed, 

and if the top outer diameter of the stepped geometry cannot be formed in one station,

then form it using rules applicable to v1 class element.

[Rule 60] If a stepped geometry needs to be formed, 

then the adjacent steps have a diameter ratio of 1.4.

[Rule 61] If a stepped geometry needs to be formed, 

and if the difference between the adjacent diameters is more than twice the optimum die radius 

then the adjacent vertical cylindrical sections will be connected through horizontal elements (h1 class).

[Rule 62] If a stepped geometry needs to be formed, 

and if the difference between the adjacent diameters is less than twice the optimum die radius 

down then the adjacent steps will be connected through almost-zero-length 45 degree tapered element (a1 class).

[Rule 63] Draw_ratio is $\frac{d_{blank}}{d_{cup}}$.

[Rule 64] Redraw_ratio is $\frac{d_{cup(\text{current\_station})}}{d_{cup(\text{next\_station})}}$.

[Rule 65] Height_ratio is $\frac{h_{cup}}{d_{cup}}$. 
[Rule 66] The list of object geometries having the blank and the final object geometry as the end members after the second pass is the rectified_process_sequence.
APPENDIX F

PRODUCTION RULES ASSOCIATED WITH MATERIAL MODULE
[Rule 67] If a material is high ductile, and if the thickness to blank dia. ratio is greater than or equal to 0.020, and

if stage number is 1,
then the limiting draw_ratio is 2.08;
else if the stage number is 2,
then the limiting redraw_ratio is 1.36;
else if the stage number is 3,
then the limiting redraw_ratio is 1.32;
else if the stage number is 4,
then the limiting redraw_ratio is 1.28;
else if the stage number is greater than 4
then the limiting redraw_ratio 1.25.

[Rule 68] If a material is high ductile, and if the thickness to blank dia. ratio is greater than or equal to 0.015, and

if stage number is 1,
then the limiting draw_ratio is 2.00;
else if the stage number is 2,
then the limiting redraw_ratio is 1.33;
else if the stage number is 3,
then the limiting redraw_ratio is 1.28;
else if the stage number is 4,
then the limiting redraw_ratio is 1.25;
else if the stage number is greater than 4
then the limiting redraw_ratio 1.22.

[Rule 69] If a material is high ductile,
and if the thickness to blank dia. ratio is greater than or equal to 0.010,
and
if stage number is 1,
then the limiting draw_ratio is 1.89;
else if the stage number is 2,
then the limiting redraw_ratio is 1.32;
else if the stage number is 3,
then the limiting redraw_ratio is 1.27;
else if the stage number is 4,
then the limiting redraw_ratio is 1.23;
else if the stage number is greater than 4
then the limiting redraw_ratio 1.19.

[Rule 70] If a material is high ductile,
and if the thickness to blank dia. ratio is greater than or equal to 0.006,
and
if stage number is 1,
then the limiting draw_ratio is 1.82;
else if the stage number is 2,
then the limiting redraw_ratio is 1.28;
else if the stage number is 3,
then the limiting redraw_ratio is 1.25;
else if the stage number is 4,
then the limiting redraw_ratio is 1.22;
else if the stage number is greater than 4
then the limiting redraw_ratio 1.18.

[Rule 71] If a material is high ductile,
and if the thickness to blank dia. ratio is greater than or equal to 0.003,
and
if stage number is 1,
then the limiting draw_ratio is 1.72;
else if the stage number is 2,
then the limiting redraw_ratio is 1.27;
else if the stage number is 3,
then the limiting redraw_ratio is 1.23;
else if the stage number is 4,
then the limiting redraw_ratio is 1.20;
else if the stage number is greater than 4
then the limiting redraw_ratio 1.16.

[Rule 72] If a material is high ductile,
and if the thickness to blank dia. ratio is greater than or equal to 0.0015,
and
if stage number is 1,
then the limiting draw_ratio is 1.59;
else if the stage number is 2,
then the limiting redraw_ratio is 1.25;
else if the stage number is 3,
then the limiting redraw_ratio is 1.22;
else if the stage number is 4,
then the limiting redraw_ratio is 1.18;
else if the stage number is greater than 4
then the limiting redraw_ratio 1.15.

[Rule 73] If a material is high ductile,
and if the thickness to blank dia. ratio is less than 0.0015,
and if stage number is 1,
then the limiting draw_ratio is 1.58;
else if the stage number is 2,
then the limiting redraw_ratio is 1.22;
else if the stage number is 3,
then the limiting redraw_ratio is 1.19;
else if the stage number is 4,
then the limiting redraw_ratio is 1.16;
else if the stage number is greater than 4
then the limiting redraw_ratio 1.14.

[Rule 74] If a material is high ductile,
and if the stage number is 1,
then the limiting height_ratio is 0.90.

[Rule 75] If a material is low ductile,  
and if the stage number is 1,  
then the limiting height_ratio is 0.19.

[Rule 76] If the test_element is taper, convex or concave element,  
then the maximum height it can be drawn to in one station can be found as:

\[
H_{\text{max}} = 0.54 \times \text{Punch\_diameter} - 0.16 \times \text{Die\_diameter} + 0.58 \times \text{Punch\_nose\_radius} + 46.8 \times \text{Thickness} - 0.36
\]

(all dimensions are in inch)

\[
H_{\text{max}} = 0.54 \times \text{Punch\_diameter} - 0.16 \times \text{Die\_diameter} + 0.58 \times \text{Punch\_nose\_radius} + 46.8 \times \text{Thickness} - 9.144
\]

(all dimensions are in mm)